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Some insights into a sequential resource allocation mechanism for en route air traffic management

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Abstract

This paper presents a game theoretic model of a sequential capacity allocation process in a congestible transportation system. In this particular application, we investigate the governing principles at work in how airlines will time their requests for en route resources under capacity shortfalls and uncertain conditions, when flights are not able to take their preferred route at their preferred departure time slot due to the shortfalls. We examine a sequential "First Submitted First Assigned" (FSFA) capacity allocation process within an en route air traffic flow management (ATFM) program such as the Collaborative Trajectory Options Program (CTOP), which is a Federal Aviation Administration initiative that aims to manage en route capacity constraints brought on by inclement weather and capacity/demand imbalances. In the FSFA process, flights are assigned the best available routes and slots available at the time flight operators submit their preference requests during the planning period, in a sequential manner. Because flight operators compete with one another for resources, in such an allocation process they would be expected to make their requests as early as possible. However, because weather and traffic conditions – and therefore, the values of resources – can change significantly, flight operators may prefer to request resources later in the process rather than earlier. We use a game theoretic setup to understand how flight operators might tradeoff these conflicts and choose an optimal time to submit their preferences for their flights, as submission times are competitive responses by flight operators looking to maximize their benefits. We first develop a loss function that captures the expected utility of submitting preferences under uncertainty about operating conditions. Then, a conceptual model of the FSFA process is constructed using a simultaneous incomplete information game, where flight operators compete for the "prizes" of having submitted their inputs before others. A numerical study is performed in which it is demonstrated that preference submission times are heavily influenced by the general uncertainty surrounding weather and operational conditions of the ATFM program, and each flight operator's internal ability to handle this uncertainty. A key finding is that, in many of the scenarios presented, an optimal strategy for a flight operator is to submit their preferences at the very beginning of the planning period. If air traffic managers could expect to receive more submissions at the beginning of the planning period, they could more easily coordinate the ATFM program with other ATFM programs taking place or scheduled to take place, and they would have more opportunity to call another FSFA allocation route before the ATFM program begins, should conditions change enough to warrant this. Outputs of the model may provide some general insights to flight operators in planning submission strategies within competitive allocation processes such as FSFA. Also, this work may have a broader application to other sequential resource allocation strategies within congestible and controlled transportation systems.

Keywords

air transportation, en route air traffic flow management (ATFM), Collaborative Trajectory Options Program (CTOP), sequential resource allocation, airline competition, applied game theory.

List of acronyms

AFP	Airspace Flow Program
ATFM	Air Traffic Flow Management
CDM	Collaborative Decision Making
CTOP	Collaborative Trajectory Options Program
FAA	Federal Aviation Administration
FCA	Flow Constrained Area
GDP	Ground Delay Program
NAS	National Airspace System
RBS	Ration-by-Schedule
TOS	Trajectory Options Set

List of notation

$V_{n,s}$	Utility of slot s to a flight n, where $s \in [1, S]$ and includes all available slots in the en route ATFM program
V_n^*	Utility of the highest utility slot to flight n
$U_{n,s}(t)$	Estimated utility of slot s to n at time t
$\gamma_{n,r(s)}(t)$	Stochastic term representing n's imprecise knowledge about the route conditions of a particular slot at t, distributed type 1 extreme value (Gumbel)
$p_{n,s}(t)$	Probability of n choosing slot s at time t
$\omega(t)$	Scale parameter of $\gamma_{n,r(s)}(t)$
k	Parameter capturing the unpredictability of weather and operating conditions of an en route ATFM program
$L_n(t)$	n's loss in (true) utility resulting from its decisions at t
$l_n(t)$	$= L_n(t)/L_n^{max}$, where L_n^{max} is the maximum loss possible for n (due to incomplete information)
$E[\pi_n]$	Expected payoff
$R_{x\|t_n}$	True utility that n gains by submitting t t and being xth in the submission order, relative to the utility of submitting last, $R(N)$
$C(t_n)$	Cost (due to uncertainty) n incurs in making a preference submission at time t
q_n	Time n submits during the planning period as a proportion of the total ATFM program planning period (T), such that $q_n = (T - t_n)/T$
h_n	n's uncertainty level, which determines the rate at which n's uncertainty decreases during the planning period; $h_n \sim U(h_{min}, h_{max})$

1. Introduction

This paper presents a game theoretic model of a sequential capacity allocation process in a congestible transportation system. We investigate the governing principles at work in how airlines will time their requests for resources under capacity shortfalls and uncertain conditions, when an Air Traffic Flow Management (ATFM) program is in place. This particular application involves the allocation of constrained en route resources (in the form of departure time "slots" on specified routes) to flights, when flights are not able to take their preferred route at their preferred departure time slot. In the "First Submitted First Assigned" (FSFA) process, operators of impacted flights submit their en route resource preference requests to air traffic managers during the planning period, which are then used to allocate the best available routes and slots available at the time they make their request, in a sequential manner. Because flight operators compete with one another for resources, in such a sequential allocation mechanism they would be expected to make their requests as early as possible. However, weather and operating conditions can change significantly, which will impact both the true and perceived values of resources to flight operators over time. Weather will change both the set of routes available to a flight, as well as the relative value of each route. Operating conditions that can change include fuel loading requirements, which in turn depend on planned routes as well as passenger counts, the latter which will shift as airlines work to minimize ATFM impacts to customers by reassigning and rescheduling passengers to flights. In addition, crew shift schedules may also be impacted as crews time out with flight delays. As a result of these possible changes, flight operators may prefer to request resources later in the process rather than earlier. Therefore we ask the question: how might uncertainty influence a flight operator's decision about when to make their resource requests in this competitive environment?

The Federal Aviation Administration (FAA) operates ATFM programs to reduce the scale and cost of disruptions to flight operators during times of adverse weather and heavy traffic demands. ATFM programs developed to handle problems in the en route airspace have been quite successful in mitigating the costs of disruptions, although their success has been limited due to inflexibilities in incorporating flight operators' specific needs and adapting to changing weather and traffic conditions. As a result, the FAA has recently implemented a new ATFM program called the Collaborative Trajectory Options Program (CTOP)(FAA, 2014). CTOP is similar to previous en route ATFM programs in that it aims to safely and efficiently meter aircraft flow through and around capacity constrained airspace regions. However, CTOP differs in that it considers flight operators' submitted en route resource preferences (delayed departure times and reroutes) through an electronic negotiation process when assigning these resources.

Within ATFM programs like CTOP, there are many potential designs for the processes by which flight operators can express resource preferences and the rules by which air traffic managers assign available resources. In the current CTOP, flights are assigned resources using an algorithm that accounts for system constraints, with slots to fly through the constrained area assigned using Ration-by-Schedule (RBS)(FAA, 2012). However, the rules of allocation in that algorithm are somewhat unclear. Also, Advisory Circular No. 90-115 states, "While a TOS (Trajectory Options Set) may be submitted at any time, there are many advantages to submitting a TOS well in advance of a planned flight."(FAA, 2014). This statement implies that there are operational advantages for airlines to plan well in advance, rather than actual rewards offered to airlines for submitting early. If such incentives were to be offered, how would airlines time their submissions in order to maximize their benefits? The FSFA allocation scheme introduced above is a channel through which this question can be explored(Kim &

Hansen, 2013). However, in Kim & Hansen (2013) it was assumed that flight operators submit complete resource preference inputs in an arbitrary and random order in the FSFA, without considering flight operators' competitive responses to the FSFA allocation rule.

In this paper we investigate the impacts competition might have on program outcomes, by presenting a game theoretic treatment of airline preference submission behavior within the FSFA allocation process. We first present a loss function that captures the expected benefits of submitting preferences under uncertainty regarding operating conditions. We then develop a conceptual model of the FSFA resource preference submission process. The model is an N-player simultaneous game, where flight operators compete for resources through their input submission times and not, for instance, the inputs themselves. The process can be modeled as a simultaneous game because flight operators do not know the outcome of all allocations until the end of the ATFM program planning period. A flight operator's submission decision is the result of a tradeoff between the flight operator's desire to win a resource of higher utility versus minimizing the level of uncertainty (due to changing weather and operational conditions) about the utilities of the resources themselves. The former encourages flight operators to submit their preference inputs earlier while the latter would encourage later submissions. Numerical examples illustrate some potential outcomes this competition may induce. A key outcome is that flight operators will be motivated to submit their inputs earlier during the planning period if operational and weather uncertainty levels are lower and competition for resources is high. In fact, in many of the examples explored, many operators are expected to make preference submissions at the very beginning of the planning period. This outcome may be beneficial to air traffic managers in that first-stage strategic planning decisions can be made earlier rather than later, providing opportunities for later ATFM program revisions and better coordination with other ATFM programs. Flight operators would typically submit early in situations when they had less uncertainty; however, it is clear that if early submissions were made under high uncertainty, then resource allocations could be highly suboptimal.

This paper is organized as follows: Section 2 describes ATFM programs, including the Collaborative Trajectory Options Program (CTOP), and provides a review of the existing literature on aviation resource allocation programs. Section 3 introduces the analysis framework and resulting model properties. Section 4 presents numerical results from example model applications, and Section 5 provides conclusions and a discussion of the research.

2. Background

During times of en route capacity shortfalls, flights are rerouted, delayed on the ground (at origin airports), and delayed in the air (Miles-in-Trail) as needed. Prior to 2006, flights were instructed to wait on the ground and/or reroute around constraints as instructed by FAA air traffic managers. Multiple Ground Delay Programs (GDPs) were called to handle en route constraints; this could result in inefficient and inequitable ground delay allocations, in that ground delays could be assigned to flights destined for a GDP airport whether they were scheduled to fly through the constrained en route airspace or not. Airlines could, subject to FAA approval, reroute their own flights; selected reroutes from a standard set of playbook routes were assigned(Wilmouth & Taber, 2005) if airlines did not reroute flights on their own. In 2006, the Airspace Flow Program (AFP) was launched. In an AFP, flights scheduled to fly through capacity constrained airspace are assigned a delayed departure time on the original filed route, using the Ration-by-Schedule (RBS) algorithm. The flight operator can either accept the delayed departure time, reject it to reroute around the constrained airspace, or cancel the flight altogether. As slots to fly through the constrained airspace are vacated through flight cancelations and reroutes, the schedule is compressed such that remaining flights are moved into earlier slots as available. Currently, miles-in-trail restrictions, GDPs, and standardized reroutes continue to be used along with AFPs to handle en route constraints.

The Collaborative Trajectory Options Program (CTOP) is an en route air traffic flow management (ATFM) initiative that became operational in mid-2014(FAA, 2014). The purpose of CTOP is to more efficiently and equitably utilize en route capacity during times of capacity shortfalls caused by inclement weather and flight capacity-demand imbalances. It offers flight operators combinations of reroute and delayed departure time options, and allows operators to communicate their preferences regarding the offered options. CTOP is similar to earlier ATFM programs in that it aims to safely meter aircraft flow through capacity constrained airspace regions designated Flow Constrained Areas (FCAs). However, unlike other programs, it allows flights to be allocated reroutes and delayed departure times simultaneously, and most notably, incorporates operators' preferences regarding the available resources in the resource allocation decisions.

The CTOP applies RBS to delay and/or reroute flights at the entry points of an FCA(FAA, 2012). RBS is used to allocate constrained airport capacity to flights during a GDP by assigning necessary delays in the order flights are scheduled to arrive at the airport for which the GDP was issued. RBS is a well-accepted allocation scheme that has been used in GDPs since the mid-1990s, and has received extensive attention in both research(Vossen & Ball, 2005) and practice. Under CTOP, when flights receive their delayed "departure" times through RBS, flight operators are also notified of the reroute options available. Each reroute option has an allowed departure time associated with it. Flight operators are asked to submit a Trajectory Options Set (TOS) for each flight, which consists of a set of options "weighted" with a relative cost. The options in a TOS can therefore be ranked in any given situation, and the most desirable available option identified and assigned(FAA, 2014). The FAA will then use the TOS' in an algorithm that includes use of RBS to allocate constrained slots within the FCA(FAA, 2012). The operators may submit edits and changes to their TOS during the planning period, as available options and en route conditions change.

Through its use of RBS and incorporation of flight operator preferences, CTOP actualizes the Collaborative Decision Making (CDM) concept(FAA, 2012). CDM is a joint government and industry initiative that aims to improve ATFM in the National Airspace System by encouraging information sharing between stakeholders. This information exchange has been shown to greatly benefit ATFM; however, information exchange within allocation schemes where airlines must compete for scarce resources can also encourage gaming/strategic behaviors(Ball, Futer, Hoffman, & Sherry, 2002; Hoffman, Burke, Lewis, Futer, & Ball, 2005). As a result, ATFM programs should be designed to discourage this behavior when possible, or at least account for it, whenever it is detrimental to the system.

Resource allocation schemes for en route air traffic management, which propose a more structured approach to information exchange and coordination by stakeholders in the spirit of CDM, have been considered since the mid-1990s. Goodhart looked at the incorporation of user preferences to the en route resource allocation process(Goodhart, 2000). As an alternative to the application of GDPs (and therefore, RBS) for en route resource allocation(Jakobovits, Krozel, & Penny, 2005), Hoffman et al. (2007) and Ball et al. (2010) introduced the Ration-by-Distance (RBD) allocation method. They demonstrated that RBD could be more efficient than RBS under early GDP cancellation, but less equitable. As a result of this finding, they introduced an equity-based

RBD algorithm, which is a constrained version of RBD that imposes an upper bound value on a pre-defined equity metric. Equity and fairness are important considerations in ATFM program design, as ATFM programs that promote equitable treatment of airlines are less likely to encourage gaming behavior by a highly competitive industry (Hoffman, Burke, Lewis, Futer, & Ball, 2005). Failing to consider equity, as well as potential competitive behavior, in ATFM program design may be detrimental to an otherwise well-designed program. Hoffman et al. (2005) proposed a method tailored to addressing the en route resource allocation problem through the simultaneous rationing of multiple resources. They discuss the greater efficiency of this strategy compared to GDPs. Their strategy was also designed to incorporate changing user preferences and real-time operational decisions by the FAA. Pourtaklo and Ball (2009) propose an algorithm to equitably allocate airspace slots specifically within the AFP context using flight operator preference information and randomization.

Market-based methods for rationing constrained airport capacity have been studied extensively (Ball, et al., 2007; Swaroop, Zou, Ball, & Hansen, 2012), and attention has also been given to en route resource allocation from a competitive/market-based perspective. Waslander et al. (2008a) propose a market mechanism-based approach in allocating en route resources to competing airlines, and incorporate this rationing mechanism into an ATFM model. They show that it is in the airlines' best interest to participate in the airspace resource allocation market; they can do no worse by participating than if they do not participate and allow the central decision maker to assign them resources without taking their preferences into account. The resources in this study consisted of access to airspace sectors during particular time intervals. Another study (Waslander, Roy, Johari, & Tomlin, 2008b) considers a scheme in which airlines submit maximum lump-sum bids for resources in a market, which in turn influences resource prices. They show that a Nash equilibrium and a bound on the worst efficiency loss exist for utility maximizing players that anticipate how their bids will affect resource prices. Another study looks at the implementation of a market for constrained en route or airport resources, whereby competing airlines can pay for delay reductions and receive payments for delay increases, by trading slots initially allocated by a first planned first served policy(Castelli, Pesenti, & Ranieri, 2011). The authors show that the resulting slot allocation allows for all flights to be better off economically, and that it maximizes efficiency from a social welfare perspective.

There is little published literature on airline resource request processing under changing NAS conditions. Ball et al. (2005) compare the results of a batch-oriented periodic process to a fast-response asynchronous process within the slot credit substitution mechanism. The authors note that a fast-response process may have applications beyond airports slots, with a particularly promising application in flexible flight planning. Kim (2011) proposed and compared several different methods of en route resource allocation possible in ATFM programs (RBS being one of several) that are in line with the CDM philosophy. One allocation method investigated was a sequential method called "First Submitted First Assigned" (FSFA), in which an impacted flight is assigned the best available resource at the time the flight's operator submits the required preference inputs. When the ATFM program is announced, traffic managers provide all operators of flights scheduled to enter the FCA with up-to-date information about the constrained airspace (location, start time, duration, etc.) and the reroute/delay options available. Flight operators are then requested to submit their resource preferences to traffic managers at some point before a specified deadline; at the time of a flight's preference submission, traffic managers allocate to that flight the best available resources according to their submission information. As a result, a flight is rewarded for earlier submissions through the increased probability of obtaining a desired resource before their competitors do. However, NAS operating conditions, and an airline's actions in response to these conditions, may change rapidly prior to a flight's scheduled and/or actual departure time. A flight operator becomes more certain of its resource valuations closer to departure time. As a result, the utility of a flight's resource assignment will depend on both the resources available at the time of their preference input submission, as well as the certainty regarding the preference inputs submitted to obtain a resource. The time that the operator of a flight submits their complete route preference values should be a calculated balance of these two opposing considerations.

Furthermore, because a flight is assigned the best resources available at the time of their preference submission in FSFA, flight operators have no incentive to submit untruthful information. They should desire to submit the most truthful information possible to obtain the highest utility option available to them; the competition for resources plays out in the times that flight operators submit, rather than the submission information itself. Kim & Hansen (2013) assessed the relative costs of the FSFA scheme under the assumption that the preference submissions order is random and independent of flight operator characteristics. However, because the value of a flight's resource allocation may be highly dependent on its place in the preference submission order, the assumption that submission order is completely random and independent of the factors that define the cost (flight characteristics, operational conditions of the ATFM program, etc.) is likely invalid. For instance, we would expect earlier scheduled flights, flights with higher unit airborne costs, and flights that strongly prefer particular routes to submit their preference inputs earlier. Although the allocation framework currently used in the CTOP differs from FSFA, FSFA can provide some insights into how airlines might time their TOS submissions if the allocation algorithm were to provide clear rewards for earlier submission. Given that FSFA controls the number of times that operators can submit preferences (whereas the current CTOP allocation procedure does not), it can be applied to entire CTOPs or even to subsets of flights and airspace within the larger CTOP framework, where and when there is heavy competition for the same resources.

In this paper we explore how flight operators will time their requests for scarce resources in FSFA under uncertain conditions using a game theoretic setup, as submission times are competitive responses by flight operators looking to maximize their expected benefits through this submission. We explore how uncertainty about changing NAS conditions might influence an operator's decision to submit its flight's route preference information later when the operator has better information. We first define the expected utility of submitting preferences under uncertainty about operating conditions, and then develop a competition model of submission times during an ATFM program planning period.

3. Competition in a sequential allocation program

This section discusses the methodology developed to assess competitive response strategies within the FSFA resource allocation process. As described in the previous section, flight operators must choose when to submit their en route resource preferences within the planning period, for use in the allocation process. The resulting submission times will be their best responses in balancing two opposing objectives: the desire to submit earlier than their competitors in order to obtain a more desirable, lower cost resource; their desire to submit later in the submissions process, particularly when uncertainty regarding evolving NAS weather and operational conditions is high. We first introduce a model describing how resources costs change under evolving NAS conditions, and then introduce the competition model framework and resulting preference submission strategy. Please note that from this point forward, when we state "a flight prefers X", "a flight's X", etc., we are actually stating "flight operator prefers X", "flight operator's X", etc., and have removed "operator" for brevity.

3.1 Impacts of submission time choice in evolving conditions

Say there is a capacity shortfall in a portion of en route airspace, due to inclement weather and/or excessive demands. Flights that are scheduled to enter the airspace cannot do so at their originally scheduled times, and must be delayed and/or rerouted. An ATFM program is initiated for that airspace, for the period over which demand-capacity imbalances are anticipated. The program will employ the FSFA process to allocate the constrained resources to the N impacted flights. Say that the operator of flight $n \in [1, N]$ is the first to submit its resource preferences during the planning period. Flight n, therefore, has available to it all departure slots on all routes whose departure time from Fix A is not earlier than n's original scheduled departure time. Say that $V_{n,s}$ represents the utility of slot s to a flight n, where $s \in [1, S]$ and includes all available slots. We assume that $V_{n,s}$ consists of both airborne and ground delay costs(Kim & Hansen, 2013). If the operator of flight n had perfect information (during the program planning period) about how weather and operating conditions in the NAS will evolve, it would also know the true utilities of each available departure slot for n, and be able to identify the available slot of highest utility (or, alternately, lowest cost):

$$V_n^* = max(V_{n,1}, V_{n,2}, \ldots, V_{n,s}, \ldots, V_{n,S}) \quad (1)$$

where for any slot s that is prior to n's original departure time (and therefore is unavailable to n), $V_{n,s} = -\infty$. However, n is unlikely to have perfect information throughout the planning period, as NAS conditions can change rapidly in ways that cannot be predicted by flight operators or traffic managers. These conditions include weather and traffic, as well as traffic management actions taken by the FAA. Airline internal operational situations can also change rapidly in response to NAS conditions or for other reasons, due to changing passenger loads, fuel requirements, crew scheduling, and others. Therefore, at some time t during the planning period, the operator of flight n will estimate the utility of slot s to be $U_{n,s}(t)$, rather than its true value, $V_{n,s}$[1]. Thus we write:

$$U_{n,s}(t) = V_{n,s} + \gamma_{n,r(s)}(t) \quad (2)$$

The stochastic term $\gamma_{n,r(s)}(t)$ represents flight n's imprecise knowledge about the route conditions of a particular slot (on that route)[2] at t, and we assume it is distributed type 1 extreme value (Gumbel). We emphasize that n only knows $U_{n,s}(t)\ \forall s$ at time t, not $V_{n,s}$ or $\gamma_{n,r(s)}(t)$, $\forall s$. However, because the true slot utilities are $V_{n,s}$, n can actually expect to obtain a true utility at t as follows:

$$E[U_n(t)] = \sum_{s \in S} p_{n,s}(t) \cdot V_{n,s} \quad (3)$$

where $E[U_n(t)]$ is the expected true utility n can expect to obtain from the entire set of ATFM program resources available, $p_{n,s}(t)$ is the probability of n choosing slot s based on its understanding at t that the utility of s is $U_{n,s}(t)$. Given that $\gamma_{n,r(s)}(t)$ is Gumbel distributed, this probability has the standard logit expression:

$$p_{n,s}(t) = exp\big(V_{n,s}/\omega(t)\big) / \sum_{s \in S} exp\big(V_{n,s}/\omega(t)\big) \quad (4)$$

[1] We exploit the properties of random utility to model uncertainty about slot values, but do so in the "opposite" way that random utility is used to understand choice behavior. Typically, V is the "explained" utility of a choice, rather than what it is here, which is the true utility of a choice. Similarly, U is typically the "true" utility, rather than the perceived utility of a choice at a time t.

[2] We make the assumption that γ depends only on changing information (over t) about the route of slot s, and not departure times. In other words, we assume that the information used to consider an early slot or later slot on that same route, at some time t in the planning period, is the same.

where $\omega(t)$ is the scale parameter of $\gamma_{n,r(s)}(t)$, and indicates the variance of $\gamma_{n,r(s)}(t)$ (Ben-Akiva & Lerman, 1994).

Uncertainty about the true utility loss in assigning a given slot to a given flight is likely to be greatest at the beginning of the program planning period ($t = 0$) and decreases as it progresses to the end of the planning period, T. We capture this idea by assuming that the variance of $\gamma_{n,r(s)}(t)$, and therefore its scale parameter $\omega(t)$, decreases linearly with respect to t:

$$\omega(t) = k \cdot (T - t), \forall t \leq T \tag{5}$$

where k is a parameter capturing the overall unpredictability of weather and operating conditions of the particular ATFM program in question. Equation (5) assumes that flight n's information about conditions is perfect by T, so that $\omega(T)$ =0. Therefore, as the submission time approaches T, (4) shows that the probability of n correctly identifying the resource of true highest utility to itself – V_n^* – approaches 1. When $t < T$, flight n believes slot s is valued at $U_{n,s}(t)$, and with this information can only expect to gain $E[U_n(t)]$ with its choice. Flight n's loss in (true) utility resulting from its decisions at t can be expressed as:

$$L_n(t) = V_n^* - E[U_n(t)] \tag{6}$$

Recall that $E[U_n(t)] \to V_n^*$, and therefore $L_n(t) \to 0$, as $t \to T$. Note that (6), which we call the "loss function", assumes that all slots are available to all flights at any time.

If $\omega(t)$ is very large, $\gamma_{n,r(s)}(t)$ is highly variable, and in turn $U_{n,s}(t)$ becomes a very poor reflection of the true utility $V_{n,s}$. Therefore, according to (4), slot choice probabilities become nearly equal among all available slots, and $E[U_n(t)]$ approaches $\overline{V_n}$ (the average deterministic utility of all slots, to n). Following this, the maximum value that $L_n(t)$ can take is $L_n^{max} = V_n^* - \overline{V}_n$. We can now represent the utility loss function as a proportion of the maximum loss possible:

$$l_n(t) = L_n(t)/L_n^{max}\ ,\ l_n(t) \in (0,1] \tag{7}$$

The shape of $L_n(t)$, and therefore $l_n(t)$, is highly dependent on the values of $\omega(t)$ and $V_{n,s}$, as well as the set of available slots S. It may be convex within the planning period depending on how $\omega(t)$ is specified, or it could have an inflection point after which the function becomes concave. If only one resource is available to n, $l_n(t) = 0\ \forall t$. However, if there are several resources of differing utilities, $l(t)$ is strictly decreasing and differentiable(Kim, 2011).

This section has introduced a functional form for a flight's loss in utility (or, increase in costs) caused by uncertainty regarding NAS conditions during an ATFM (in particular, CTOP) program planning period. The loss function is used in the following competition model.

3.2 Payoff function and equilibrium submission strategy

We set up the FSFA preference submissions and resource allocation process as a competition in which the operator of each flight must decide when to submit their en route preference information during the program planning period. Let us assume that each flight (referred to as players from this point forward) can be distinguished from another by its operator's level of uncertainty during a particular planning period. For instance, some players may have schedules that are less easily disrupted than other players, and/or more robust operational recovery plans. Players will also have different capabilities with regards to how they handle information about external uncertainties through their internal operations processes, and some will have larger and more experienced operations groups. These features can increase or decrease a player's "uncertainty" compared with other players. We assume that players' uncertainty levels – which determine their submission strategies – are continuously and identically distributed over the player population, and players' own uncertainty levels are known only to themselves. Players are not informed about their competitors' actions and allocation outcomes, or resource availability status, during the planning period. The FSFA competition process can then be modeled as a simultaneous incomplete information game, where each player is uncertain about their competitors' uncertainty levels and resulting strategies (Gibbons, 1992). We assume that all players are rational, in that they will employ strategies to maximize their own payoff (or benefit) based on what they know about themselves and their competitors. Each player believes that their competitors are also rational, and believe that other players believe that they believe they are rational, and so on. As a result of the above assumptions, we have a symmetric game where the expected payoff function and equilibrium submission time strategy are identical for all players.

To preserve tractability, further assumptions are required. Firstly, we ignore the potential effects of correlation among flights operated by a single entity, such as an airline, and consider each flight to be a single non-cooperative player in the competition. Secondly, all players will submit their preference inputs sometime during the ATFM program planning period. Thirdly, each player is informed about the resource they are allocated immediately after making their submission, and are not permitted to swap or modify it during the allocation process. This rule is enforced to prevent players from submitting inputs at the very start of the planning period simply for the purpose of reserving a slot they consider at that time to be desirable, with the idea that they can submit again later without cost. Situations like this may in fact arise when there are very few desirable resources and very similar flights (with respect to say, aircraft size, origin and destination, departure/arrival times, etc.) competing for them. Fourthly, we assume that players are not informed as to when other players submit, what they submit, or slot availability (and therefore, what allocations other players receive) at any time during the planning period. The result is that players have no information about their competitors' actions. All players in the FSFA process are assumed to submit truthful route preferences as their strategies play out through the time at which they submit their preferences, rather than the preference information itself (it is in their best interest to submit truthful inputs). Finally, all players assign identical costs to each slot under conditions of perfect information about NAS conditions. They differ from one another only in how their uncertainty levels change over the planning period. One again might imagine a similar situation might arise where very similar flight operators desire very similar resources. However, this is a restrictive assumption that should be relaxed in future work. All the assumptions listed ensure that the FSFA allocation situation is a competitive one. If for instance, it were true that players did not have the same cost functions (and players learned of these over, say, many iterations of the same ATFM program in the same problematic airspace), they might not desire the same resources, thereby dampening competition. However, this is an entirely different situation not in the scope of this analysis. The construct (and assumptions) of our game limits the analysis and results presented in this paper to competitive situations only.

There exist several methods in the game theory and auctions literature for modeling the FSFA competition process as described above (Krishna, 2002). For this analysis we use a sporting contest analogy, where players' effort levels, or "bid" strategies, are dependent on prize values, their personal ability levels, and their probabilities of winning those prizes(Moldovanu & Sela, 2001). We assume an N-player game setup with $N-1$ prizes, where prize values are ordered from largest to smallest. The last player to submit does not win anything.

This is analogous to the last player to submit winning the prize of lowest utility, because we define prizes only by their relative values to one another. There is no information lost when we employ these relative utility values; we are only concerned with players' actions when faced with one choice against another in the set. Therefore we can set the prize for the last submitter N as zero[3]. Player n would like to maximum its expected payoff, which can be expressed as:

$$E[\pi_n] = \sum_{x\in[1,N-1]} R_{x|t_n} \cdot P(n = x|t_n) - C(t_n),\ n = 1, \dots, N \tag{8}$$

where $C(t_n)$ is the cost (due to uncertainty) player n incurs in making a preference submission at time t; $R_{x|t_n}$ is the true utility that players gain by being xth in the submission order given they submitted at t, relative to the utility of being last, $R(N)$, if they have perfect information about NAS conditions; $P(n = x|t_n)$ is the probability of n being xth in the submission order given they submitted at t. For example, if the expected utility of being first is $R(1)$, then $R_1 = R(1) - R(N)$. Equation (8) also assumes that the expected utility of being in a given place in the submission order is identical for all players in a particular ATFM program, and is common knowledge. This follows from a previously stated assumption that all players have identical flight cost functions when they have perfect information. Also, it is shown in Kim (2011) that over many program instances, the xth player to submit will have a greater expected utility (or lower expected cost) than that of the $(x + 1)$th submitter. Therefore, $R_1 \geq R_2 \geq \cdots \geq R_{N-1} \geq 0$. Furthermore, the set of R_n values will depend on the supply and demand characteristics of the flow constrained areas (FCAs) and CTOP.

If player n submitted early in the planning period, n's true expected utility by being xth to submit will certainly be lower than R_x. Equation (8) therefore states that the amount by which n's expected payoff is degraded by uncertainty at the time of preference submission is additive, and equals $C(t_n)$. $C(t_n)$ is a linear function of the loss function $L(q_n)$ introduced in (6). If q_n is the time n submits during the planning period as a proportion of the total planning period (T), such that $q_n = (T - t_n)/T$, $q_n \in [0,1]$, then

$$C(q_n) = L(q_n) \cdot h_n \tag{9}$$

A larger value for q indicates an earlier submission time, in turn representing a costlier submission due to higher uncertainty. As $L(q_n)$ is rewritten as a function of q_n instead of t_n, it is increasing; $L(0) = 0$, and $L'(q_n) \geq 0$. The loss function is identical for all players, in that they all observe the same changing information about weather, demand, and ATFM actions captured by parameter k (equation 5). h_n represents player n's uncertainty level, which determines the rate at which n's informational uncertainty decreases during the planning period as $t_n \to T$. The rationale is that players have different capabilities for processing and incorporating this changing information into their strategic planning decisions, and this is captured by their uncertainty level h. A player with a lower h can better handle changing conditions, and therefore incurs smaller losses as a result of the uncertainty represented by $L(\cdot)$. Conversely, a player with a high h suffers high losses when subject to $L(\cdot)$. We will assume that h is continuously and uniformly distributed between h_{min} and h_{max}, and all players know this. If $h_n = h_{min}$, player n's internal operations can handle NAS uncertainties best among all players, and n's cost of submitting at some early time will be smaller than any other player. As a result, n is likely to submit well before the end of the planning period, T. If $h_n = h_{max}$, the opposite is true, and we assume that n would prefer to

[3] This is analogous to setting $N - 1$ alternative specific constants in a discrete choice model.

submit their inputs as close to T as possible. Since $L(0) = 0$ it follows that $C(0) = 0$, implying no loss from imperfect information if a player's submission is made at T.

Figure 1 displays the submission cost function C for two k values and three arbitrary h values. Recall that in (5), k captures the overall unpredictability of weather and operating conditions in a particular ATFM program, by setting the value of the scale parameter $\omega(t)$ of the stochastic term representing information uncertainty in the loss function. Again, h represents a player's uncertainty level. We have shown C to be a function of t (rather than q, such that time increases towards the right), and normalized to L^{max}, previously defined as the maximum utility loss that any flight n can sustain in the competition. The examples shown represent a simple three-player game, where there are also three slot choices in the set S, with utilities $V_1 = 10$, $V_2 = 4$ (and $V_3 = 0$; the relationships between these values are analogous to those between R_1, R_2, and R_3). The two values of k are proportions of V_1: $k = V_1$ or $2V_1$. We will use these values for some of the numerical examples illustrating the equilibrium strategy in Section 4.

Place Figure 1 here.

The submission cost function $C(t)$ is shown to be monotonic and differentiable, that is also strictly decreasing over the strategy space of our game, $t \in (0, T]$ (or, $q \in (0,1]$). Since this is a symmetric game, the functional form of the submission strategy is identical for all players and can be represented as a function of a player n's uncertainty level.

The probability that Player 1 submits before Player 2 is:

$$P(t_1 < t_2) = P(q_1 > q_2) = P\big(g(h_1) > g(h_2)\big) = 1 - F\big(g^{-1}(q_1)\big) \tag{10}$$

For (10) to hold, we must assume a-priori that $g(h_n)$ is monotonic and differentiable, and verify afterwards that our assumption is correct. We know that this assumption holds as Moldovanu and Sela (2001) show that their bid function is strictly increasing and differentiable, and that it maximizes expected payoff.

It was previously stated that players do not know their competitors' submission strategies because they do not know their competitors' uncertainty levels, and it follows that a player's probability of winning against one competitor is independent of the probability of winning against another. Therefore, the probability of being first to submit (i.e., beating the other $N-1$ players) is $\Big(1 - F\big(g^{-1}(q_n)\big)\Big)^{N-1}$, the probability of being second to submit (i.e., beating $N-2$ players but losing to one) is $\frac{(N-1)!}{1!((N-1)-1)!}\Big(1 - F\big(g^{-1}(q_n)\big)\Big)^{N-2} \cdot F\big(g^{-1}(q_n)\big)$, and the probability of being xth to submit is $\frac{(N-1)!}{(x-1)!((N-1)-(x-1))!}\Big(1 - F\big(g^{-1}(q_n)\big)\Big)^{N-x} \cdot F\big(g^{-1}(q_n)\big)^{x-1}$. If we also normalize all utilities such that the choice of highest deterministic utility is one, then we rewrite the payoff function of (8), using (9) and (10), as follows:

$$E[\pi_n(q_n)]$$
$$= \sum_{x\in[1,N-1]}\left[\frac{(N-1)!}{(x-1)!((N-1)-(x-1))!} \cdot \frac{r_x}{L^{max}} \cdot \Big(1 - F\big(g^{-1}(q_n)\big)\Big)^{N-x} \cdot F\big(g^{-1}(q_n)\big)^{x-1}\right] - h_n l(q_n) \tag{11}$$

where r_x is the utility of having submitted xth, normalized to the utility of the most valuable slot. Our aim is to solve (11), in order to obtain the submission time strategy $q_n = g(h_n)$ that maximizes player n's expected payoff with respect to the conditions of the ATFM program, the information n has about its competitors, and n's own information about itself. $g(h_n)$ is also the equilibrium submission strategy, meaning that n cannot achieve a higher payoff by deviating from this strategy. The payoff function in (11) is concave with respect to the submission strategy in $q \in (0,1]$ (Moldovanu & Sela, 2001). Depending on the conditions of the ATFM program (defined by v_s, q_n and k), equilibrium submission strategies may lie on the boundaries of the strategy space.

We solve (11) assuming the three-player game assumption made previously for *Figure 1*. It was determined to be a reasonable assumption because it does not detract from the insights that can be obtained from the competition setup, but (11) can be solved with relative ease. Therefore, by using equations (3) through (7) for $l(q_n)$, rearranging terms, and assuming that players' uncertainty levels are uniformly distributed where $h_n \sim U(h_{min}, h_{max})$, (11) is solved for a three-player game:

$$\begin{aligned} &1 - \left(\sum_{s\in S} exp\left(\frac{v_s}{q_n k}\right)\right)^{-1} \cdot \sum_{s\in S} v_s\, exp\left(\frac{v_s}{q_n k}\right) \\ &= 2\left[\left(r_1 h_{max} - r_2(h_{min} + h_{max})\right)(ln\, h_{max} - ln\, h_n) + (r_1 - 2r_2)(h_n - h_{max})\right]/(h_{max} - h_{min})^2 \end{aligned} \tag{12}$$

where v_s are the normalized "true" slot utilities, such that the most valuable slot has utility $v_1 = 1$. See the Appendix for the derivation of (12) from (11). The above expression cannot be expressed for q_n (the equilibrium submission strategy) in closed form, but it is possible to find solutions numerically. A third-order Taylor series yielded poor approximations of the function at the boundaries of the planning period; as a result it was not used. The reason that it cannot be solved in closed form is because the loss function $L(q_n)$ is non-linear. However, if $L(q_n)$ took on a simpler (i.e., linear or quadratic) form, we would have a closed form expression for the submission time strategy (see Appendix).

Depending on the values of parameters $h_{min}, h_{max}, v_{s\in S}, r_1$, and r_2, there may not exist a solution to (12) for values of $h \in [h_{min}, h_l], h_l \leq h_{max}$. This is because the left side of (11) can only take a maximum value of 1. However, this threshold h_l is not critical; there is another (higher) value h_0 where $h_l \leq h_0$, which can be defined as a submission decision threshold for players. If $h_n < h_0$, n is positioned to gain more from submitting as early as possible, because n's disutility due to uncertainty is relatively smaller than its potential gain from being among the first to submit. As a result, when $h_n < h_0$, n will always submit as soon as possible, i.e., at the start of the planning period. Player n with $h_n > h_0$ will want to submit at $t > 0$, depending on the parameters of the submission strategy function h_{min}, h_{max}, r_1, and r_2. We find both h_l and h_0 numerically as they also cannot be expressed in closed form.

Once we find q_n, the time that n submits during the planning period (of total duration T) can be obtained:

$$t_n = max(T(1 - q_n), 0)\,,\ \ t_n \in [0, T]\ \forall n \tag{13}$$

4. Numerical study

Figure 2 displays submission strategies for seven values of k shown at the bottom of the figure, for a scenario where $v_s = [1,0.5,0]$, $r_1 = 0.8$, $r_2 = 0.8v_2$, $h_{min} = 0.5$, $h_{max} = 1.5$, and $T = 2$ hours. These values were chosen arbitrarily, but are meant to be representative of conditions in an ATFM program such as the CTOP. Recall that k is a parameter that captures the overall uncertainty regarding weather and operating conditions of the particular program in question, and h represents an individual player's uncertainty level. The x-axis represents values of h from h_{min} to h_{max}, and the y-axis represents the ATFM program planning period. It is observed that the submission time strategies increase with h and t (decreasing when expressed in q) and are differentiable. Although we are using three-player game examples, we will discuss the submission strategy functions shown in this section by referring to the flight population. In addition, it is worth mentioning again that the results shown are the outcomes of a highly competitive FSFA process, based on the assumptions laid out at the beginning of Section 3.2.

Place Figure 2 here.

The figure shows that submission times are very sensitive to k, which represents the overall uncertainty regarding weather and operating conditions. The general uncertainty level in an ATFM program such as CTOP can vary from one program to the next depending on the characteristics of the adverse weather causing the program and how traffic is managed in response to it, both of which can be represented in k. Given how much strategies can differ with respect to k, the results in *Figure 2* suggest that players' (or, flights') preference submission behavior may vary significantly from one CTOP to the next. When $k \geq 0.5$, traffic managers would observe a fairly slow arrival of submissions after a clump of arrivals at $t = 0$. When $h \approx h_{max}$, submission strategies become very sensitive such that very small increases in h result in large increases in t.

Flights with uncertainty levels $h \leq h_0$ would expect to maximize their payoff by submitting at some time before or at the very start of the planning period ($t = 0$); however, because it is not possible to make preference submissions before the planning period begins, all players with $h \leq h_0$ submit at $t = 0$. As k decreases, h_0 increases, so more flights will be inclined to submit at the very beginning. This is of course assuming that h are uniformly distributed over the flight population. Again, a smaller k would result in many flights submitting as soon as the planning period begins, which is expected. Traffic managers would then anticipate receiving more submissions at the beginning of the planning period, the majority of the CTOP planning could be completed early, and the CTOP could be more readily coordinated with other air traffic flow management programs planned or in progress. For the examples shown in *Figure 2*, a significant proportion of flights would submit their preference inputs at the beginning of the CTOP planning period.

We now further explore the sensitivity of strategies to a player's individual uncertainty level h and the overall program uncertainty level k, as well as v_2 values. Recall that we normalized the utility values to be between 0 and 1 such that v_1 is always 1. *Figure 3* shows the resulting submission strategies for the same examples shown in *Figure 2*, except that we have swapped the values represented by the x-axis and the lines. As a result, the x-axis now represents values of k from 0.25 to 3, while each curve represents a value of h as identified in the legend at the bottom.

Place Figure 3 here.

When both k and h (overall uncertainty level and individual player's uncertainty level, respectively) are high, the slopes of the lines grow smaller, indicating that the equilibrium submission strategy changes little with respect to changes in k. Conversely, at lower values of both k and h, the equilibrium strategy changes much faster in response to changes in k (beyond the threshold where a player would submit at the very beginning of the planning period). This is more readily observed in the following figure (*Figure 4*), where the contours represent the submission strategy (by time, t, in hours).

Place Figure 4 here.

The x-axis was truncated at $h = 0.8$ instead of at $h_{min} = 0.5$, to reduce the black space shown. The figure shows that under situations of higher uncertainty (i.e. high k and h, overall uncertainty level and individual player's uncertainty level, respectively), flights will become increasingly motivated to submit near the end of the planning period. In addition, when k and h levels are high, strategies are more robust to changes in both compared to when k and h are lower. More robust submission strategies at high k and h levels are illustrated by the wider (and lighter colored) contours. Contours become narrower as k and/or h decrease, indicating that small increases in uncertainty will have a larger impact on the flights' payoff functions, thereby having a greater effect on the submission strategy. The black areas represent situations where resource preference submissions ought to be made at the very beginning of the planning period. It covers a fairly large portion of the figure, indicating that airlines should submit at $t = 0$ for many combinations of k and h.

Figure 5 displays submission strategies for v_2 – the normalized "true" utility of the second most valuable slot – ranging from 0 to 1 and overall program condition uncertainty level k ranging from 0.25 to 3, when a player's individual uncertainty level is $h = 1$. The left plot of *Figure 5* shows submission strategy t with respect to k, where each curve is generated using a single value of v_2. The frontier curve is for $v_2 = 0$, with curves of v_2 values increasing in the direction of the arrow. The right plot of *Figure 5* displays the identical example but with submission time plotted against v_2. Each curve represents a value of k, with curves of k values increasing in the direction of the arrow. The frontier curve represents $k = 3$.

Place Figure 5 here.

Figure 5 suggests that flights will submit earlier in the CTOP planning period as the normalized utility of the second most valuable slot (v_2) increases. This observation is intuitive in that when v_2 is higher, the expected relative utility of being anything other than last (third) to submit is also larger. Therefore, flights are pressured by greater competition and therefore have more motivation to submit earlier. For instance, if we were to draw a vertical line at $k = 1$ in the left plot, when $v_2 = 0.5$ it is an optimal strategy to submit inputs at $t = 0.2$ hours. However, when $v_2 = 0.2$ it is best to submit at $t = 1$ hour. This is a significant difference; according to the shape of the curves these differences generally decrease as k increases, but can still be significant at larger values of k and when v_2 is large. More easily observable from the right graph is that for all values of k at player uncertainty level $h = 1$, if v_2 is larger than approximately 0.65, the players' submission strategy will be to submit at the beginning of the planning period. At smaller values of v_2 the submission strategy varies significantly with respect to the range of k values shown. However, when k is large, the submission strategy will not change significantly with a unit increase in k. This last observation is similar to that of *Figure 4*.

Figure 6 shows submission strategies when the overall uncertainty level is $k = 1$, the players' individual uncertainty levels are uniformly distributed between $[h_{min}, h_{max}] = [0.5, 1.5]$, and again v_2 (the normalized "true" utility of the second most valuable slot) ranges from zero to one.

Place Figure 6 here.

The main observations from *Figure 6* confirm the observations from the previous figures. When a player's uncertainty level (h) is high, and the second most valuable slot (v_2) is closer in value to that of the most valuable slot ($v_1 = 1$), the optimal strategy is to submit later than when v_2 is closer in value to the third and least valued slot. As observed in *Figure 5*, when v_2 is high, more flights will be incentivized to submit earlier, and at the very beginning of the planning period. We observe at the top of the figure that for a given value of h, the submission strategy t is not monotonic with respect to v_2. This behavior contradicts the idea that players are more likely to submit earlier when the prize values are higher. However, the behavior is only observed at very high h values; throughout these examples it has been shown that the submission strategy is highly sensitive at high h, and we should investigate this further to determine the cause of the non-monotonicity. The function appears to behave as expected for all other combinations of h and v_2.

Figure 7 displays h_0 – the submission decision threshold – as a function of v_2 (normalized "true" utility of second most valuable slot) and k (overall conditions uncertainty level). Recall that if $h_n \leq h_0$, it is best for flight n to submit preferences at the beginning of the planning period ($t = 0$). However, if $h_n > h_0$, then the optimal strategy will be to submit at some time $t \in (0, T]$.

Place Figure 7 here.

Figure 7 provides information about FSFA submissions process outcomes for different scenarios as represented by v_2 and k. For any combination of v_2 and k, the corresponding h_0 value can be found from the figure above. For instance if $k = 1.5$ and $v_2 = 0.4$, then $h_0 \approx 0.85$. A graphic like *Figure 7* can be used to quickly reduce a flight's strategy set in an ATFM program. For instance, if $h_n \leq h_0$ (meaning that n's uncertainty level is less than the submission decision threshold uncertainty level), flight n knows immediately to submit at the beginning of the planning period without further analysis efforts. If n's uncertainty level is greater than the threshold, or $h_n > h_0$, n knows they should submit at a later time to minimize their costs, and further analysis is required to determine exactly when. Air traffic managers can benefit from such a graphic as it gives some indication of the predictability of the FSFA submissions process in a given program instance. If h_0 is very high within the range (h_{min}, h_{max}), traffic managers can expect to receive more submissions at the beginning of the planning period, as it indicates that the utility of being first or second is greater than the utility loss incurred by submitting early for a large proportion of airlines. As a result, the ATFM program planning process can be completed early, and this may help traffic managers in coordinating the program with other programs taking place or scheduled to take place. It can also provide traffic managers more time to call another FSFA allocation round before the program begins, should conditions change enough to warrant it. Although early planning efforts may be irrelevant and highly suboptimal under greater uncertainty about NAS conditions, we know that submission strategies implicitly account for this as airlines are less likely to submit early when facing greater uncertainty.

5. Concluding remarks

In the sequential "First Submitted First Assigned" (FSFA) resource allocation process, participating flights are requested to submit their en route preferences during the ATFM planning period. The earlier a flight submits, the more likely it is to receive a desired resource. However, the flight is also likely to face greater uncertainty about weather and operational conditions in the NAS; in turn, the flight will have more uncertainty identifying which resource is indeed of highest value to it, and therefore, which resource to request. To understand how flights tradeoff these conflicts and choose optimal preference submission times, we presented a game theoretic treatment of submission behavior in the FSFA process. A numerical study demonstrated that preference submission times are heavily influenced by the weather and operational conditions of the ATFM program, and each flight operator's ability to handle uncertainty (which varies throughout the flight population). A notable finding is that in many scenarios investigated, a large proportion of flights would find that submitting their preferences at the very beginning of the ATFM program planning period is an optimal strategy. Indeed, if air traffic managers could expect to receive more submissions at the beginning of the planning period, some benefits may arise. Firstly, the ATFM program could be more easily coordinated with other ATFM programs taking place or scheduled to take place. Secondly, there may be time to call another resource allocation round before the program start, should conditions change enough to warrant it. On the other hand, early submission also means that flight operators are more likely to select resources that in retrospect are suboptimal, because the utility loss that is incurred by being "late" to submit in a high competitive environment exceeds the utility loss of obtaining a resource of uncertain value. In less competitive scenarios – that may arise in future work when some key assumptions are relaxed – we may find that airlines are somewhat less inclined to submit as early as they do here.

Outputs of the model developed in this paper may provide some general, high-level insights to airlines in planning submission strategies within competitive allocation processes such as FSFA. Tools – Figure 7 being a simple example of one – may be developed to reduce the number of potential strategies within different ATFM programs, or even quickly determine that a submission made at the very beginning or very end of an ATFM planning period is best. As discussed previously there are numerous factors that can determine flight costs (and therefore flight scheduling decisions) in times of capacity shortfalls, which are not explicitly accounted for here. For instance, in this model we have represented both general ATFM program situation uncertainty as well as airline uncertainty levels. However, each form of uncertainty can be further broken down by cause and type, and may result in different input submission timing decisions. As a result, although this analysis can provide some insights about submission timing in an FSFA process, much more analysis as well as experience would be necessary to make such decisions in a real-life scenario.

The model developed in this paper may have applications to other sequential resource allocation strategies within congestible and controlled transportation systems operating under uncertain and/or changing conditions. One possible example may be a northern shipping channel operating under heavy demand/capacity imbalances and variable environmental conditions due to climate change.

This analysis assumed that all flights would participate in the submission system, but this may not necessarily be true. If a flight is scheduled towards the end of an ATFM program, under highly unstable and rapidly changing conditions its operator may feel that a wait-and-see approach is more desirable than a premature allocation. Also, although an operator may submit information and receive an allocation for their flight, they might ultimately cancel the flight and fail to inform air traffic managers. Schedule compression as done in GDPs, or a credit system (that extends beyond a single CTOP) may discourage this behavior. These assumptions should be relaxed or addressed further in future work.

There are several key ways by which this work can be extended and improved upon. Firstly, the model in (11) can be solved for flight populations greater than three. Secondly, an alternate auction/contest analogy may be sought, where "prize" values and their devaluations due to uncertainty at the time of preference submission) are not additive. In addition, we should revisit and relax some of the restrictive assumptions made for the analysis. We cannot say exactly how relaxing the assumptions would impact the results presented; however, we could surmise that relaxing certain assumptions may dampen competition, resulting in less situations where flights are motivated to submit at the very beginning of the planning period. An alternate formulation that incorporates flight heterogeneity not only regarding how flight operators handle uncertainty, but also in terms of their "true" resource valuations (which were assume to be identical for all flights in this paper), would be useful. If a large proportion of flights do not necessarily desire the same resources in an ATFM program, and they are all aware of this fact, then the competition would dampen and flights may submit later than the model introduced would indicate. The FSFA process may be unnecessary in situations such as this, and it would be helpful to identify the threshold at which the process is inefficient. Also, we may revisit the assumption that players' uncertainty levels are uniformly distributed over the population. Given that the airline industry can be segmented by legacy carriers, low cost carriers, regional airlines, etc., investigating the applicability of other distributions is warranted. Finally, the model would benefit from relaxing the assumption that each flight belongs to an individual airline; players may be characterized as flight operators with a set of flights (possibly designated as flows) for which en route resources in the ATFM program are sought.

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Appendix: Equilibrium airline preference submission strategy

Here we show how we obtained the (optimal) player strategy shown in equation 12 from the payoff function of equation 11.

Recall that $P(q_1 > q_2)$ is the probability that Player 1 submits earlier than Player 2. Also recall $P(q_1 > q_2) = P(t_1 < t_2)$. If we assume a-priori that $g(h_n)$ is monotonic and differentiable, then we can say that:

$$\begin{aligned} P(q_1 > q_2) &= P\big(g(h_1) > g(h_2)\big) \\ &= P\big(g^{-1}(q_1) < g^{-1}(q_2)\big) \\ &= 1 - P\big(g^{-1}(q_2) < g^{-1}(q_1)\big) \\ &= 1 - F\big(g^{-1}(q_1)\big) \end{aligned}$$

where $q_n = g(h_n)$ is the submission time strategy for player n.

We assume that the probabilities of winning or losing against other players are independent. Therefore, the probability of being first to submit (i.e., beating the other $N-1$ players) is $\Big(1 - F\big(g^{-1}(q_n)\big)\Big)^{N-1}$, and the probability of being xth to submit is $\frac{(N-1)!}{(x-1)!\left((N-1)-(x-1)\right)!}\Big(1 - F\big(g^{-1}(q_n)\big)\Big)^{N-x} \cdot F\big(g^{-1}(q_n)\big)^{x-1}$. The payoff function for n becomes:

$$E[\pi_n(q_n)] = \sum_{x \in [1, N-1]} \left[\frac{(N-1)!}{(x-1)!\big((N-1)-(x-1)\big)!} \cdot \frac{r_x}{L^{max}} \cdot \Big(1 - F\big(g^{-1}(q_n)\big)\Big)^{N-x} \cdot F\big(g^{-1}(q_n)\big)^{x-1} \right] - h_n l(q_n)$$

If $N = 3$, the above becomes:

$$E[\pi_n(q_n)] = \frac{r_1}{L^{max}} \cdot \Big(1 - F\big(g^{-1}(q_n)\big)\Big)^2 + \frac{2r_2}{L^{max}} \cdot \Big(1 - F\big(g^{-1}(q_n)\big)\Big) \cdot F\big(g^{-1}(q_n)\big) - h_n l(q_n)$$

Set $g^{-1}(q_n) = h_n = y_n$.

$$E[\pi_n(q_n)] = \frac{r_1}{L^{max}} \cdot \left(1 - F(y_n)\right)^2 + \frac{2r_2}{L^{max}} \cdot \left(1 - F(y_n)\right) \cdot F(y_n) - h_n l(q_n)$$

And find the partial derivative of the expected value of π_n with respect to q_n:

$$\frac{\partial E[\pi_n]}{\partial q_n} = -\frac{2r_1}{L^{max}} \cdot \left(1 - F(y_n)\right) \cdot F'(y_n) \cdot y_n' - \frac{2r_2}{L^{max}} \cdot F'(y_n) \cdot y_n' \cdot F(y_n)$$

$$+ \frac{2r_2}{L^{max}} \cdot \left(1 - F(y_n)\right) \cdot F'(y_n) \cdot y_n' - y_n \cdot l'(q_n) = 0$$

$$\Leftrightarrow l'(q_n) = -\frac{2r_1}{y_n L^{max}} \cdot \left(1 - F(y_n)\right) \cdot F'(y_n) \cdot dy_n - \frac{2r_2}{y_n L^{max}} \cdot (2F(y) - 1) \cdot F'(y_n) \cdot dy_n$$

Now we determine boundary conditions. If $h = h_{max}$ (drop the subscript n), where h_{max} is the highest uncertainty level possible, we conjecture that n will submit as late in the CTOP planning period as possible, at T (or $q \approx 0$). Otherwise, when $h < h_{max}$, $q > 0$. Therefore,

$$\int_q^0 l'(x)dx = -\frac{2r_1}{L^{max}} \int_h^{h_{max}} x^{-1} \cdot \left(1 - F(x)\right) \cdot F'(x)dx - \frac{2r_2}{L^{max}} \int_h^{h_{max}} x^{-1} \cdot (2F(x) - 1) \cdot F'(x)dx$$

We know that the operators' h take values that are uniformly distributed between h_{min} and h_{max}. If $a = h_{min}$ and $b = h_{max}$, $F(x) = \frac{x-a}{b-a}$ and $F'(y) = \frac{1}{b-a}$.

$$l(q) = \frac{2r_1}{L^{max}} \int_{h_n}^{h_{max}} \frac{1}{(b-a)x} \left(1 - \frac{x-a}{b-a}\right) dx + \frac{2r_2}{L^{max}} \int_{h_n}^{h_{max}} \frac{1}{(b-a)x} \left(\frac{2(x-a)}{b-a} - 1\right) dx$$

$$= \frac{2}{L^{max}(b-a)^2} \left(r_1(b \ln x - x) + r_2(2x - (a+b)\ln x)\right)\Big|_h^{h_{max}}$$

Evaluate and replace a and b to obtain:

$$l(q) = \frac{2}{L^{max}(h_{max} - h_{min})^2} \left[\left(r_1 h_{max} - r_2 \cdot (h_{min} + h_{max})\right) \cdot (\ln h_{max} - \ln h) + (r_1 - 2r_2) \cdot (h - h_{max})\right]$$

We know, according to equations (3) through (7), that:

$$l(q) = \left[1 - \left(\sum_{s \in S} exp\left(\frac{v_s}{\omega(q)}\right)\right)^{-1} \cdot \sum_{s \in S} v_s \cdot exp\left(\frac{v_s}{\omega(q)}\right)\right] / L^{max}$$

Therefore:

$$1 - \left(\sum_{s \in S} exp\left(\frac{v_s}{qk}\right)\right)^{-1} \cdot \sum_{s \in S} v_s \cdot exp\left(\frac{v_s}{qk}\right)$$

$$= \frac{2\left[\left(r_1 h_{max} - r_2 \cdot (h_{min} + h_{max})\right)(ln\, h_{max} - ln\, h) + (r_1 - 2r_2)(h - h_{max})\right]}{(h_{max} - h_{min})^2}$$

where of course $\omega(q) = qk$.

Recall that we assumed the submission strategy to be monotonic and differentiable a-priori. Numerical examples in Kim (2011) show that submission strategies increase through the planning period. All players that desire to submit before the planning period submit at $t = 0$. Moldovanu and Sela (2001) also prove that the bid function is strictly increasing and differentiable, and that it maximizes expected payoff.

If $r_2 = 0.5r_1$ then (12) becomes:

$$1-\left(\sum_{s\in S} exp\left(\frac{v_s}{qk}\right)\right)^{-1}\cdot\sum_{s\in S} v_s\cdot exp\left(\frac{v_s}{qk}\right)=\frac{r_1(h_{max}-h_{min})(ln\, h_{max}-ln\, h)}{(h_{max}-h_{min})^2}$$

And if $L(q)$ were linear with a form such as $L(q)=qk$, we would have a closed form solution:

$$t=\max\left(T\left(1-\frac{2\left[\left(r_1 h_{max}-r_2\cdot(h_{min}+h_{max})\right)(\ln h_{max}-\ln h)+(r_1-2r_2)(h-h_{max})\right]}{(h_{max}-h_{min})^2\cdot k}\right),0\right)$$

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List of Figures

4. Figure

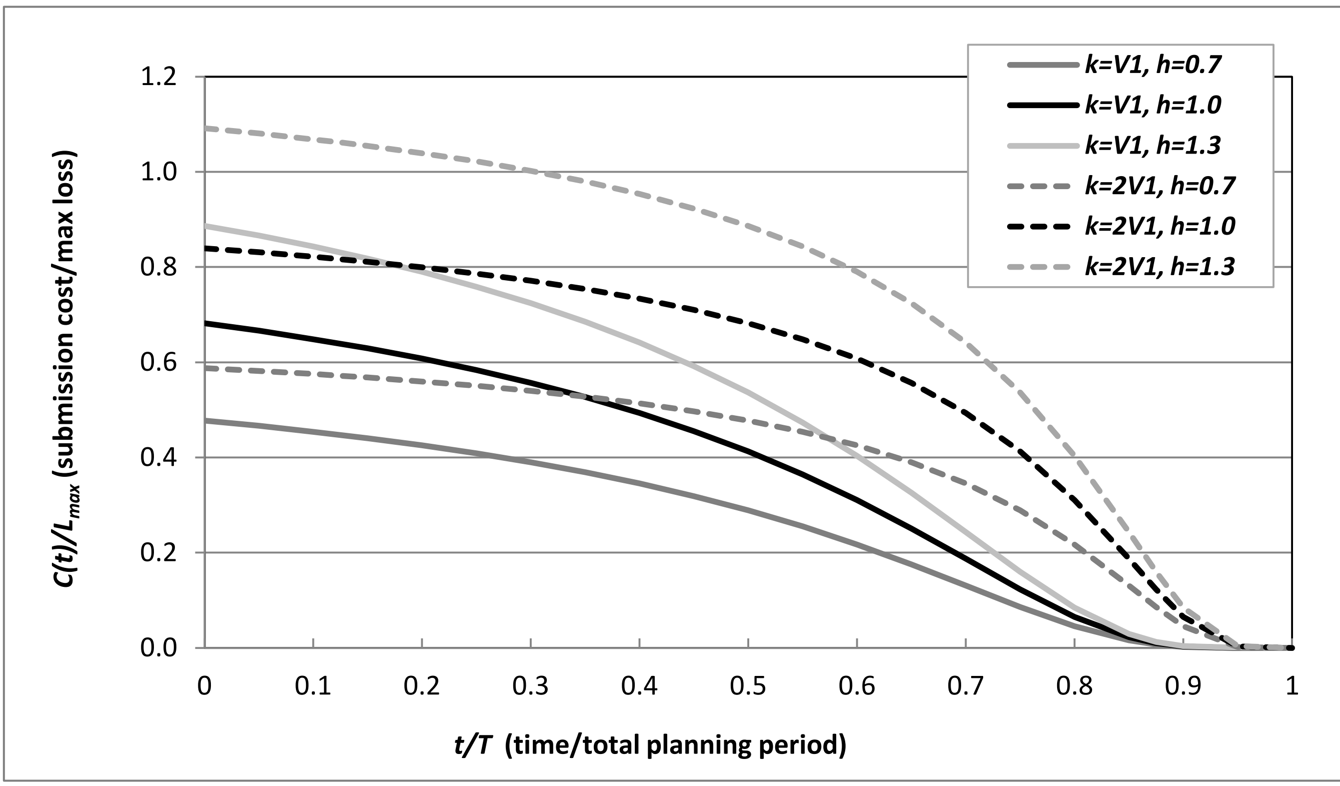

Figure 1. Submission cost function $C(t)$ (normalized to the maximum loss L_{max}), over the planning period T

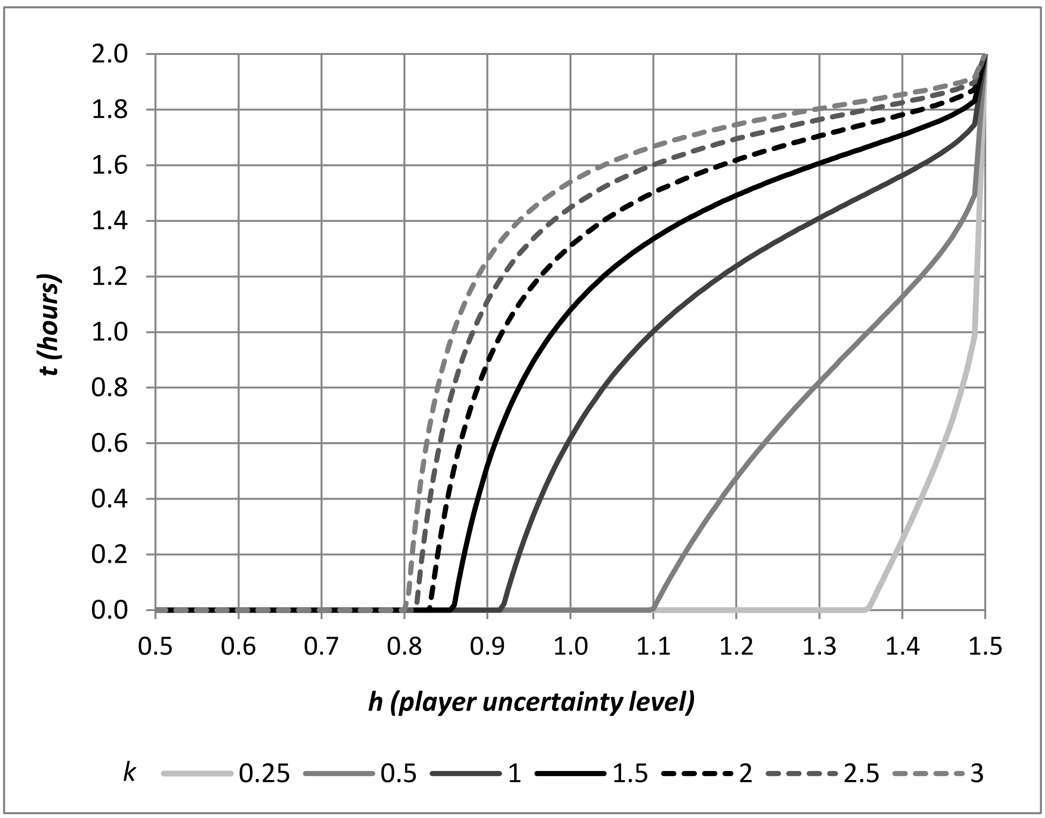

Figure 2. Equilibrium submission strategies by k (conditions uncertainty parameter)

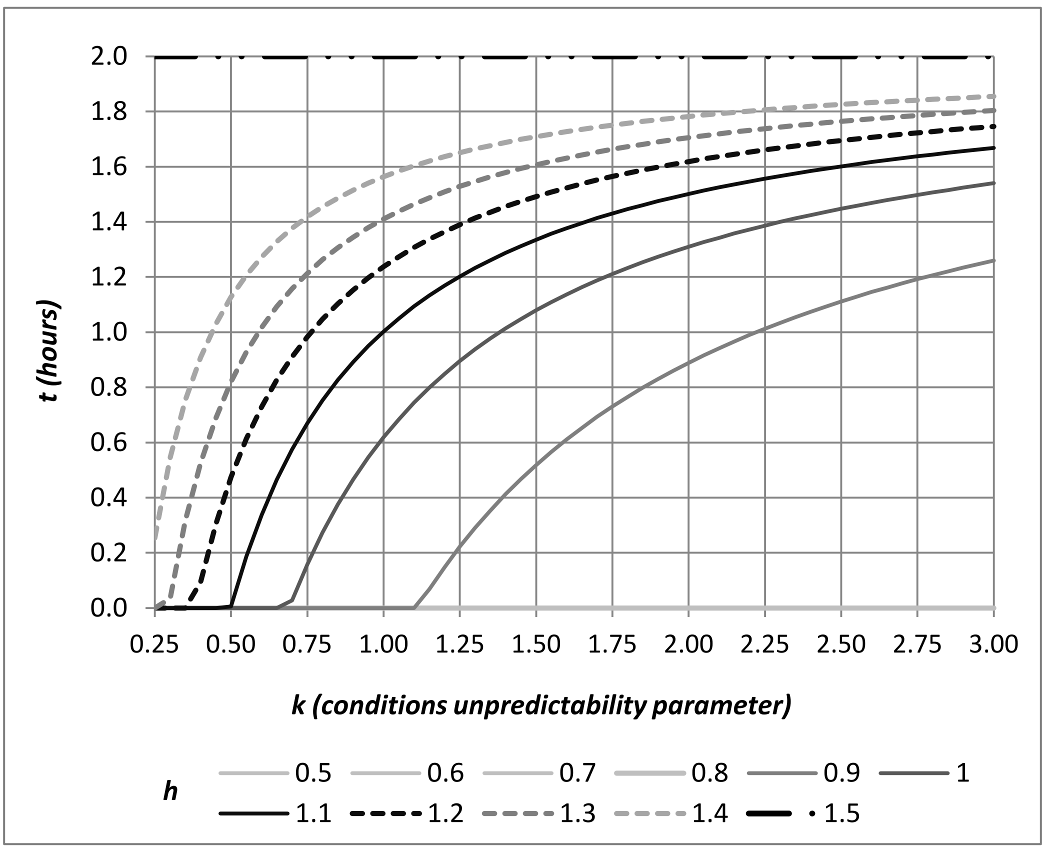

Figure 3. Equilibrium submission strategies by h (player uncertainty level)

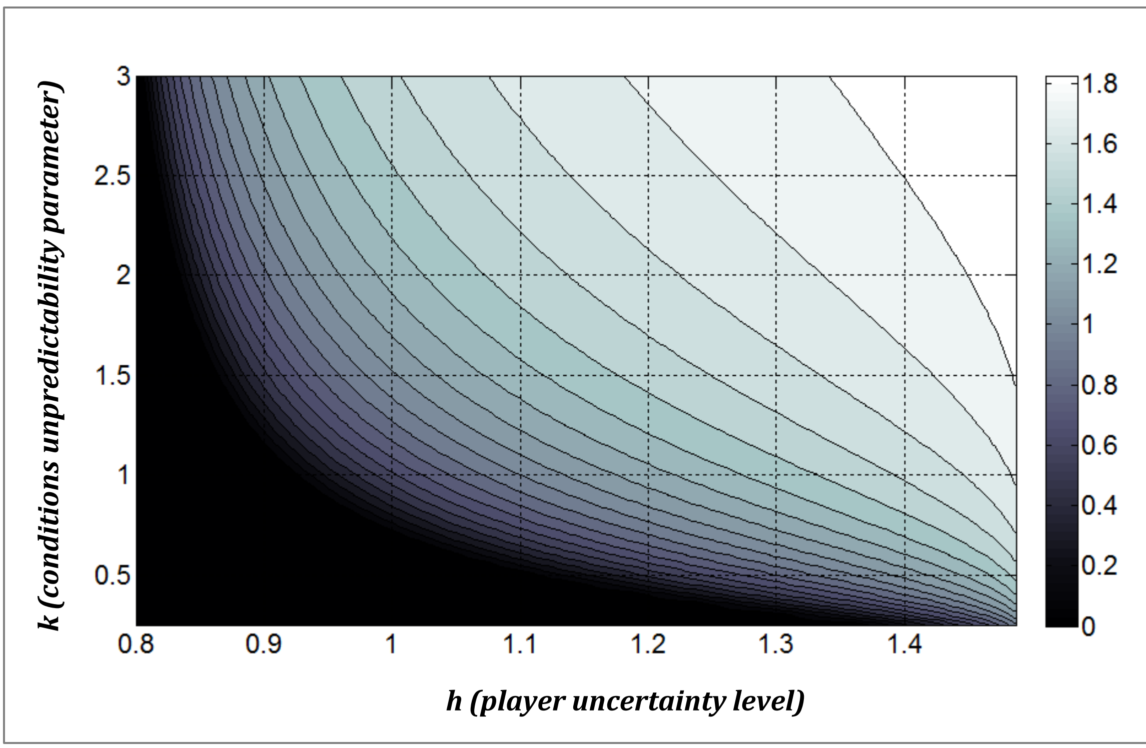

Figure 4. Equilibrium submission strategy (t hours, scale shown on right)

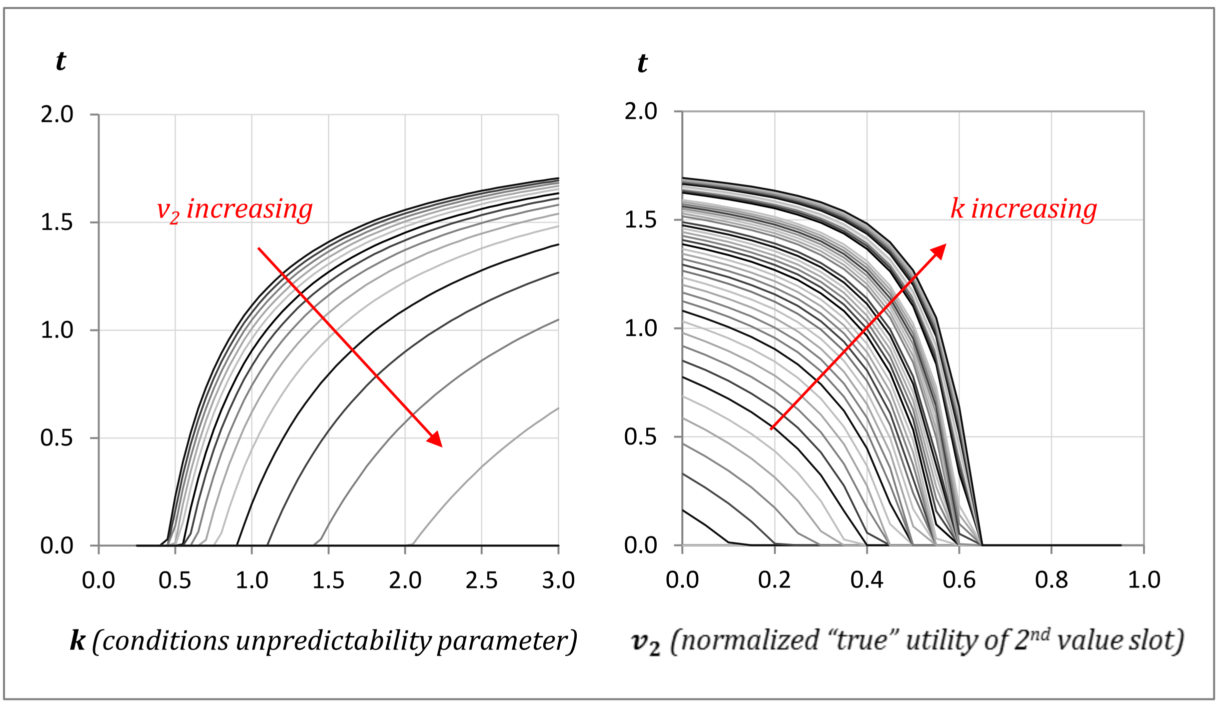

Figure 5. Equilibrium submission strategies (t, hours), player uncertainty level $h = 1.0$

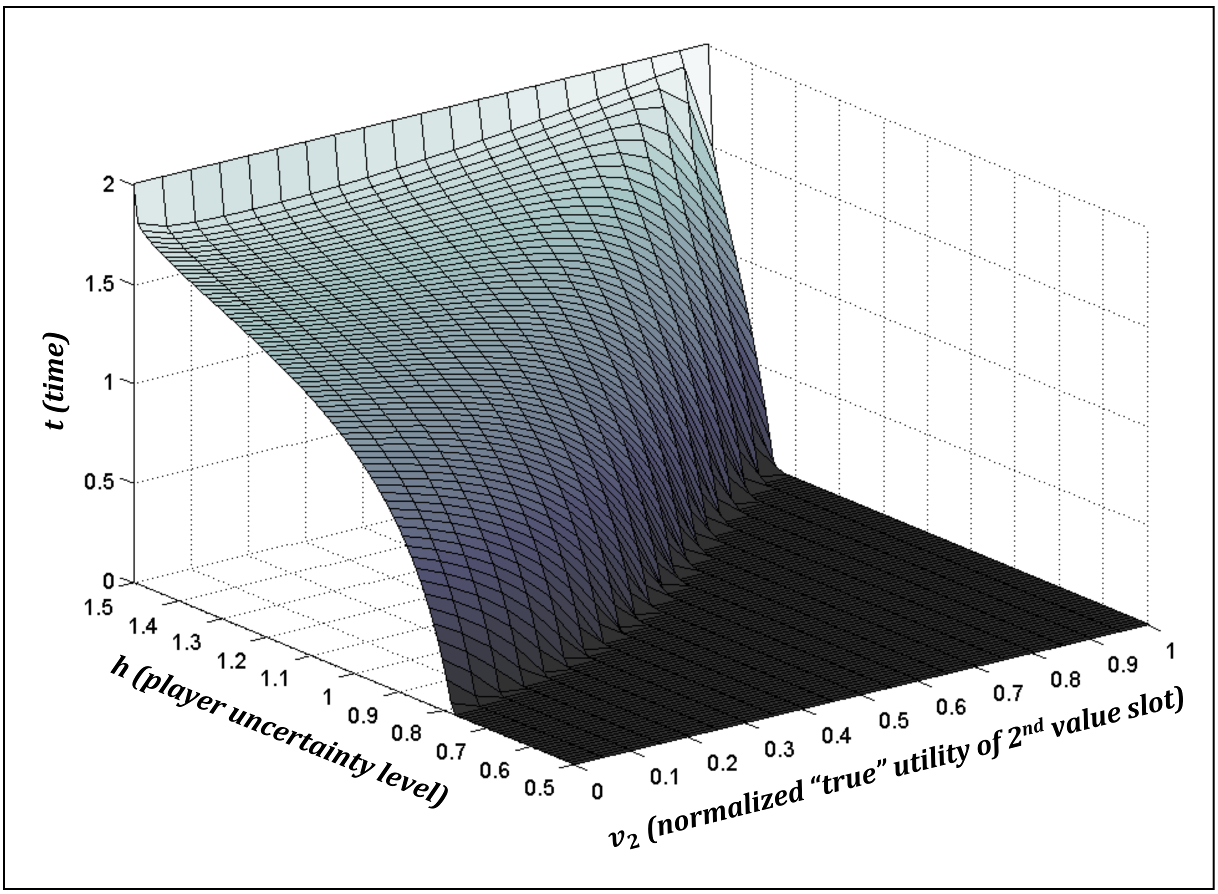

Figure 6. Equilibrium submission strategy (t hours), with conditions uncertainty parameter $k = 1.0$

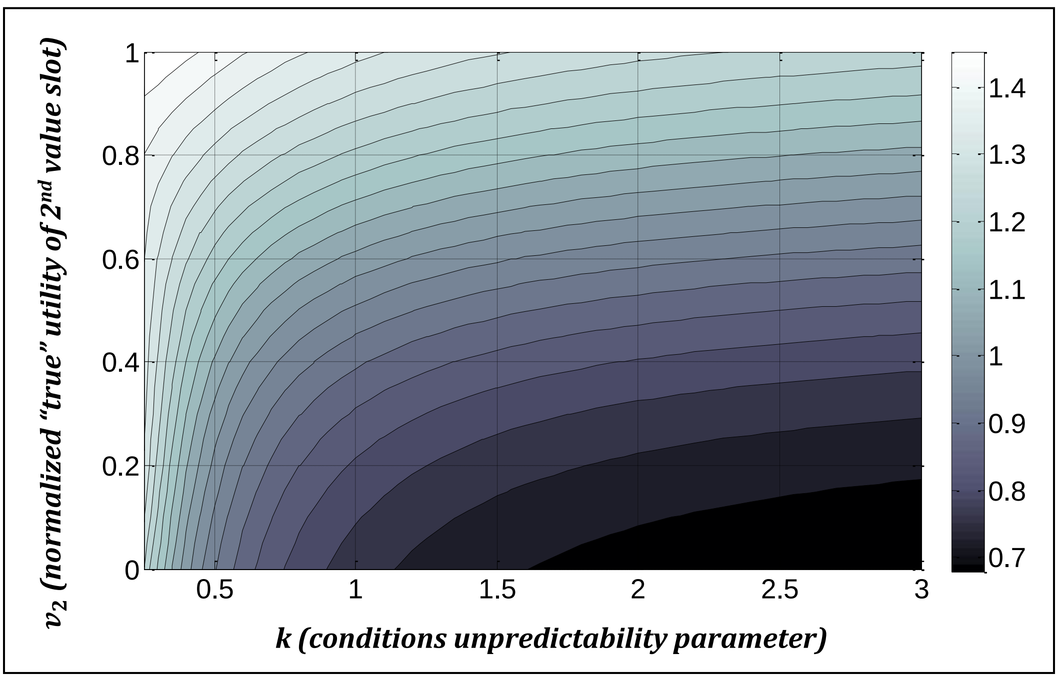

Figure 7. Submission strategy based on submission decision threshold (h_0, scale shown on right)