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Acceleration of Transient Stability Simulation for Large-Scale Power Systems on Parallel and Distributed Hardware

by

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To

my Grandpa

Haaj Jalil Jalili-Marandi

and

my Parents

for their Everlasting Supports

Abstract

Transient stability analysis is necessary for the planning, operation, and control of power systems. However, its mathematical modeling and time-domain solution is computationally onerous and has attracted the attention of power systems experts and simulation specialists for decades. The ultimate promised goal has been always to perform this simulation as fast as real-time for realistic-sized systems.

In this thesis, methods to speedup transient stability simulation for large-scale power systems are investigated. The research reported in this thesis can be divided into two parts. First, real-time simulation on a general-purpose simulator composed of CPU-based computational nodes is considered. A novel approach called Instantaneous Relaxation (IR) is proposed for the real-time transient stability simulation on such a simulator. The motivation of proposing this technique comes from the inherent parallelism that exists in the transient stability problem that allows to have a coarse grain decomposition of resulting system equations. Comparison of the real-time results with the off-line results shows both the accuracy and efficiency of the proposed method. It is demonstrated that a power system with 80 synchronous generators and 312 buses can be successfully modeled in detail and run in real-time for the transient stability study by using 8 nodes of the PC-cluster based simulator.

In the second part of this thesis, Graphics Processing Units (GPUs) are used for the first time for the transient stability simulation of power systems. Data-parallel programming techniques are used on the single-instruction multiple-date (SIMD) architecture of the GPU to implement the transient stability simulations. Several test cases of varying sizes are used to investigate the GPU-based simulation. The largest system that was implemented on a single GPU consists of 1280 buses and 320 generators all modeled in detail. The simulation results reveal the obvious advantage of using GPUs instead of CPUs for large-scale problems.

In the continuation of part two of this thesis the application of multiple GPUs running in parallel is investigated. Two different parallel processing based techniques are imple-

mented: the IR method, and the incomplete LU factorization based approach. Practical information is provided on how to use multi-threaded programming to manage multiple GPUs running simultaneously for the implementation of the transient stability simulation. The implementation of the IR method on multiple GPUs is the intersection of data-parallelism and program-level parallelism, which makes possible the simulation of very large-scale systems with 7020 buses and 1800 synchronous generators.

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List of Acronyms

CPU Central Processing Unit
CUDA Compute Unified Device Architecture
GPU Graphics Processing Unit
ILU Incomplete LU
IR Instantaneous Relaxation
MIMD Multiple-Instruction-Multiple-Data
RTW Real-Time Workshop
SFU Special Function Unit
SIMD Single-Instruction-Multiple-Data
SM Streaming Multiprocessor
SP Stream Processor
TPC Thread Processing Cluster
WR Waveform Relaxation
XHP eXtra High Performance

1 Introduction

Electric power systems are large and complex. The complexity of power systems arises from the interactions of several devices that are involved in the system such as generators, transmission and distribution networks, and electrical loads. The generators are interconnected via the transmission lines and cover vast geographical areas. Continuous growth in electricity demand and consequent expansion of the power systems are creating newer and larger problems. Therefore, power engineers are always exploring methods for quick and efficient solutions for these problems.

Maintaining system stability is very important in order to have a secure and continuous operation of the power system. Loss of supply following system instability would result in massive economic losses to both the power producers as well as customers. Dynamic stability analysis preforms simulations of the impact of potential electric grid fault conditions after a grid disturbance (contingency) in a transient time frame, which is normally up to about 10 seconds after a disturbance. When a grid is subjected to a disturbance, active and reactive powers of generators oscillate for a few seconds following the disturbance. These oscillations must be damped out to regain a stable operating condition. Contingency conditions studied include "normal" transmission line outages and/or power plant outages caused by acts of nature or equipment (e.g., due to lightning), "wear and tear" (e.g., equipment age failures) and outage conditions caused by human error or potential equipment failures. Any specific contingency simulation analysis can take several minutes of computer time, even when simulating only a few seconds of grid response after a "what if"

disturbance. Thus, analyzing hundreds or more of such "what if" contingencies can take hours of calculations [1].

A major issue facing the electric utility industry today is to perform the aforementioned calculations for a large-scale power system in a much shorter time interval so that the calculations can be performed on-line as changes occur based on real-time data, rather than performing the calculations off-line during days, weeks or even months ahead of time. This large amount of computer time occurs because the transient phenomena have to be calculated over a 5 to 10 seconds time interval for a large interconnected power system based on detailed dynamic mathematical models of grid components. These analyses are currently conducted off-line since the simulation must be run for each condition of a large set of all contingency conditions that might occur. Using a software tool developed by EPRI [2], the computer time to perform hundreds of contingency simulations was reduced to about 20 to 30 minutes. However, improved numerical methods and computer systems are still needed today, to reduce this computational time to less than 5 to 10 minutes. This will meet the requirements for the promised real-time dynamic stability analysis, which could then become a powerful tool for system operation. By using such a tool in the energy management centers the operators would quickly evaluate a large number of potential harmful contingencies and determine which ones could cause unacceptable system instabilities. Thus, operators would have the opportunity to figure out appropriate control actions that could prevent the grid from having a regional or multi-regional cascading blackout.

1.1 General Terms and Definitions

In this section[1] the important terms used in this thesis are defined to clearly identify the scope of work done in this research.

1.1.1 Transient

The IEEE Standard Dictionary defines a transient phenomena as [3]:

Pertaining to or designating a phenomenon or a quantity that varies between two consecutive steady states during a time interval that is short compared to the time scale of interest. A transient can be a unidirectional impulse of either polarity or a damped oscillatory wave with the first peak

[1]Material from this section has been published: V. Jalili-Marandi, V. Dinavahi, K. Strunz, J. A. Martinez, and A. Ramirez, "Interfacing techniques for transient stability and electromagnetic transient programs," *IEEE Trans. on Power Delivery*, vol. 24, no. 4, pp. 2385-2395, Oct. 2009.

occurring in either polarity.

Overvoltages due to lightning and capacitor energization are examples of events that cause impulsive and oscillatory transients, respectively. Some of the most common types of transient phenomena in power systems include energization of transmission lines, switching off of reactors and unloaded transformers, linear resonance at fundamental or at a harmonic frequency, series capacitor switching and sub-synchronous resonance, and load rejection [4].

1.1.2 Stability

From a system point of view there exist several types of stability definitions such as: Lyapunov stability, input-output stability, stability of linear systems, and partial stability [5]. Kimbark has classically defined stability related to power systems in [6], however, this definition was restricted to synchronous machines, and their being "in step". The IEEE/CIGRE Joint Task Force on Stability Terms and Definitions [5] adopted the following definition:

Power system stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded so that practically the entire system remains intact.

Instability in power systems can be caused by either small or large disturbances. During a small disturbance the set of equations which describe the perturbed power system can be linearized; however, during the large disturbance these equations cannot be linearized for the purpose of analysis [7]. Typical examples of small disturbances are a small change in the scheduled generation of one machine, or a small load (say 1/100 of system capacity or less) disconnected or added to the network [8,9]. Severe perturbations such as short-circuit faults and loss of generation events are representative of large disturbances. Additionally, phenomena which cause instability problems in power systems have been sub-classified based on their duration. Two types of time frame are recognizable: short term and long term. The period of interest to stability assessment of a network perturbed by a short term instability event is in the order of few seconds (3 to 20 seconds); however, this time span extends to several or many minutes for the long term one [5].

1.1.3 Transient Stability

Power system stability phenomena can be categorized into three major classes: rotor angle stability, voltage stability, and frequency stability. If an interconnected network has been

subjected to a perturbation; the ability of this power system to keep its machines in synchronism, and to maintain voltages of all buses as well as the frequency of the whole network around the steady-state values is the basis for the above mentioned classification [10]. Each form of stability phenomena may be caused by a small or large disturbance.

Although in the literature the term *transient stability* has been used to refer to the large-disturbance rotor angle stability phenomenon [7,8], some authors have used this term as a general purpose stability study of the given network with a particular disturbance sequence [11]. The IEEE/CIGRE task force report has categorized both the small and large disturbance rotor angle stability phenomena as short-term events. Furthermore, it recommends the term transient stability for large-disturbance rotor angle stability phenomenon, with a time frame of interest in the order of 3 to 5s following the disturbance. This time span may increase up to 10-20s in the case of very large networks with dominant inter-area swings [5].

The complete power system model for transient stability analysis can be mathematically described by a set of first-order differential equations and a set of algebraic equations. The differential equations model dynamics of the rotating machines while the algebraic equations represent the transmission system, loads, and the connecting network [12]. Chapter 2 provides details of the basic approach and numerical methods required for the solution of the transient stability problem.

A complete description of the power network dynamic behavior requires a very large number of equations. For instance, consider a realistic inter-connected power system which includes over 3000 buses and about 400 power stations which are feeding 800 loads. Assuming that the transmission system and loads are modeled by algebraic equations, and the generation stations are modeled by a set of 20 first-order differential equations each. The transient stability analysis of the described network needs solving of 8000 differential equations and about 3500 algebraic equations [9,13]. To make this solution as time-efficient as possible usually a time-step in the range of a few milliseconds is chosen for the simulation. In transient stability studies it is assumed that voltage and current waveforms more or less remain at power frequency (60 or 50 Hz). Thus, for modeling the electrical parts of the power system steady-state voltage and current phasors can be used. Moreover, transient stability study is a positive-sequence single-phase type of analysis [4,14].

1.1.4 Real-Time Simulation

The term "real-time" has been traditionally used by the computer industry to refer to interactive systems where the computer response is sufficiently fast enough to satisfy human users. A more rigorous definition is applied to digital control schemes where the computer response must occur at specific time intervals. In the case of power system simulation, this implies that the computer must solve the model equations within the model time step [15]. In general, real-time digital simulation may be defined as a faithful reproduction of output waveforms, by combining systems of hardware and software, that would be identical to the waveforms or effects produced by the real power system being modeled. Depending on the time taken by the computer to complete the computation of state outputs for each time-step two situations can arise. As shown in Figure 1.1(a), if the execution time, T_e, for the simulation of any time-step is smaller or equal to the time-step used, the simulation is said to be real-time simulation. On the other hand, as shown in Figure 1.1(b), if T_e for any time-step is greater than its time-step, the simulation is said to be non-real-time or off-line and in that case, execution time overruns take place. If such a situation is observed, the simulation time-step should be increased or the system model should be modified to fit the execution time within the time-step.

Analog scaled-down simulator also known as Transient Network Analyzers (TNAs), were the predecessors of fully digital real-time simulators. However, realization of large-scale power systems with a high level of complexity, non-linearity, and sophisticated dynamic elements using TNAs is practically impossible [16]. Real-time digital simulation is a state-of-the-art technique for simulation of power systems and their components. During the last ten to fifteen years, significant efforts have been made to develop real-time digital simulators of power system networks. Developments of high speed computers and other devices accelerated the research in this area. The approach in digital simulation provides accuracy in component modeling and flexibility in component interconnection for representation of a power system. The system is modeled with the help of a software using graphical interface on a workstation or a personal computer (PC) and then simulated on a powerful parallel processor based PC cluster.

Real-time simulation can be classified into two categories [17]: (1) Fully digital real-time simulation and (2) Hardware-In-the-Loop (HIL) real-time simulation. A fully digital real-time simulation requires the entire system (including control, protection and other accessories) to be modeled inside the simulator and the simulation to be completed within

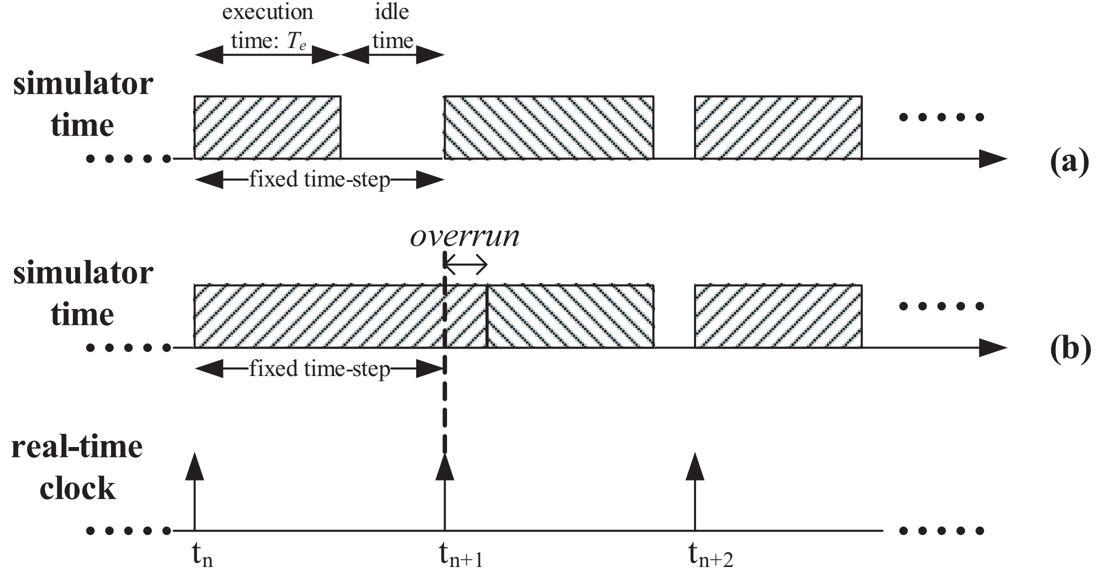


Figure 1.1: (a) Real-Time and (b) Non-Real-Time or Off-Line simulation.

the specified time-step. In this type of simulation, no I/Os or external interfacing is necessary (except those required for monitoring the simulation results). On the other hand, Hardware-In-the-Loop (HIL) simulation refers to a simulation, where parts of the fully digital real-time simulation have been replaced with actual physical components. In this case, the simulation proceeds with the device-under-test connected through input and output interfaces such as filters, Digital-to-Analog (D/A) and Analog-to-Digital (A/D) converters, signal conditioners etc. The simulation can also be modified with the user defined control inputs, for example closing or opening of switches to connect or disconnect the components in the simulated power system.

Fully digital simulation is often used for the understanding of behavior of a system under certain circumstances resulting from external or internal dynamic influences, however, HIL simulation is used to minimize the risk of investment through the use of a prototype once the underlying theory is established with the help of fully digital real-time simulation. Fully digital simulation is the type of real-time simulation that is used in this thesis.

1.2 Literature Review

Transient instability has long been recognized as the dominant problem in power system operation. It has been extensively studied since the 1920s and a lot of knowledge and experience is available in the literature [5]. Transient stability study is important for planning, design, operation, control, and post-disturbances analysis in power systems [10].

From the system theory viewpoint, power systems transient stability is a strongly nonlinear problem. To assess it accurately, first it should be mathematically described by a set of differential-algebraic equations (DAEs) as follows:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{V}, t) \tag{1.1}$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{V}, t) \tag{1.2}$$

$$\mathbf{x}(t_0) = \mathbf{x_0} \tag{1.3}$$

where $\mathbf{x}$ is the vector of state variables, $\mathbf{x_0}$ is the initial values of state variables, and $\mathbf{V}$ is the vector of bus voltages. Equation (1.1) describes the dynamic behavior of the system, while equation (1.2) describes the network constraints on (1.1). Solution of these equations in time-domain requires employing numerical integration methods. Historically, time-domain methods have been used even before the advent of numerical computers where calculations of simplified or reduced versions of the system dynamic equations were carried out manually to extract the machines' rotor angle evolution with time, known as swing curves [18]. Another approach of evaluating transient stability is a graphical method, popularized in 1930s, as known as "equal-area criterion". This method deals with a one-machine system connected to an infinite bus or any two-machine system, whether it actually has only two machines or more than two machines reduced to two-machine equivalent. The method studied stability by using the concept of energy. The equal-area method is still used to provide insight into the physical concept of the transient stability phenomena, and for evaluating the various system parameters [19, 20]. Further information about other types of approaches that have been developed based on the energy concept is available in [21].

Exploring the transient stability literature reveals that the efforts on the acceleration of the simulation have been twofold. One is on the algorithms and numerical method, and the other is on the hardware architecture. From the algorithm viewpoint there has been extensive research done on solving equations (1.1) and (1.2) with accuracy and efficiency. The overall solution can be classified into two groups [12]: partitioned and simultaneous solution approaches. In the partitioned solution the differential equation set (1.1) is solved separately for the state variables and the equation set (1.2) is solved for the algebraic variables, and these solutions are then alternated. In the simultaneous approaches, however, an implicit integration method converts the (1.1) into a set of algebraic equations, and then this set is lumped into the (1.2) resulting in a larger set of algebraic equations including all

the variables. In each of these approaches one can use various type of integration methods. The explicit Runge-Kutta [22], predictor-corrector method [23], implicit multi-step integration [24] are some example methods that have been exploited for the partitioned approaches. For the simultaneous approaches the multi-step integration methods and the trapezoidal rule have been used widely [25, 26].

The complexity of power systems simulation has increased with the system size. It was found that a single computer cannot handle the simulation of ever expanding power systems since it is time critical. Therefore, the need for parallel processing in the transient stability simulation of realistic systems was recognized to reduce computational complexity. The diakoptics method introduced by Kron [27] tears the problem into several subtasks that can be run concurrently on parallel computers. In parallel-in-space [28] and waveform relaxation [29] methods the system is decomposed into smaller subsystems and the computation is allocated to parallel computers. There are also parallel-in-time methods [30,31] which concurrently solve multiple time steps on parallel processors. While *parallel processing* usually refers to simulation techniques in which closely coupled processors are simultaneously working on the transient stability computation, the concept of *distributed processing* employs a number of loosely coupled and geographically distributed computers to simulate large-scale power systems [32].

From the hardware point of view several types of MIMD (Multiple-Instruction Multiple-Data) and SIMD (Single-Instruction Multiple-Data) parallel architectures [33] have been employed to accelerate transient stability simulation. Supercomputers [34], multiprocessor networks [35, 36], array-processors [37, 38], and PC-cluster based real-time simulators [39] all have been examined and reported for this application. Although these hardware-based approaches helped to speed-up the simulations, they were stymied by significant drawbacks. The cost of the supercomputers, communication issues and difficulties in the control of multiprocessor-based systems, difficulties in the programming and required algorithms for fitting in array-processors, and limitation on maximum system size of the simulated networks in real-time simulators are some of the noteworthy bottlenecks which limited their widespread application.

The new capability of the modern GPU, as a massively parallel processor, for general purpose high performance computation is the beginning of a new era in computing science. Although at first glance the SIMD architecture of the GPUs and that of array-processors might look similar, there are significant differences between these two technologies. Moreover, issues related to processing elements' communication overhead, program-

ming complexity, and cost effectiveness have been solved for the GPU [40]. The advantages of modern GPUs, the demand for fast simulation, and the structure of the transient stability computation makes the GPU very suitable for this application.

1.3 Motivation for This Work

In power system operations, the operators strive to operate the systems with a high degree of *reliability*. Reliability refers to the ability of the system to supply adequate electricity service on a nearly continuous basis, with few interruptions over an extended time period [5]. The key to have a reliable system operation is to maintain satisfactory *security* at all times. Unlike reliability that is measured as performance over a period of time, security refers to the degree of risk in a power system's ability to survive contingencies without interruption to customer service at any instant of time. The security of a power system can be assessed by simulating potential disturbances and determining if the disturbances will cause any adverse impacts that could result in unsafe condition in the network. However, in many cases a "static security assessment" or "static contingency analysis" can not achieve the necessary security under changing grid and generation conditions. For this purpose the "dynamic security assessment" (DSA) tools are employed, which take a snapshot of the system condition, perform comprehensive security assessment, and provide operators with warning of abnormal situations as well as remedial recommendations. The main objective of DSA tools is to determine if the system can tolerate a set of major contingencies, which falls under purview of transient stability analysis [41].

The need of real-time assessment of dynamic stability analysis was highlighted by the recent black out in USA (August 14, 2003) and Italy (September 28, 2003). The August 14 black-out in USA and Canada affected 50 million people. It took a day to restore power to New York City, and almost two days to restore power to Detroit. The Italian black-out, the worst blackout in Europe that affected 57 million people, started with 6545 MW import to Italy. In less than 3 minutes cascading phenomena isolated the Italian system from Europe, loss of generation in Italy and insufficient load shedding resulted in system black-out. The phenomena occurred in less than 3 minutes, but it has been proceeded by about 15 minutes within which the problem evolved from a normal situation to an alert and then to an emergency state with a restoration time of 19 hours. A list of evolving factors was collected from which it was identified that improved power system monitoring and preventive actions are the most important items [42]. Presently, in most utilities, the dynamic security

analyses are conducted by off-line studies. However, there is an increasing demand for fast and real-time simulations which can be incorporated within the energy management system to determine the critical system limits based on the current conditions of the system. This is a particular application where speed of simulation is vital.

The need to accelerate transient stability simulations for realistic size power systems is the main driving force for this research. The speed of transient stability simulations can be improved by three approaches:

- Developing new algorithmic methods
- Exploiting parallel and distributed processors
- Utilizing faster processors

While the transient stability simulation tools have been improved over the last two decades, the improvement has been mainly made in the modeling complexity and user friendliness rather than in the structure of the algorithm. It was predicted in 1993 that the impact of new mathematical methods or algorithms in power system analysis will be at best evolutionary and not revolutionary [43]. It was a true prediction at least for the transient stability simulation. Although adaptations such as dishonest Newton-Raphson, innovations such as sparsity handling and optimal ordering, and efficient coding brought computer analysis of large-scale power systems into practical use, the basic time domain algorithm for transient stability simulation remained the most reliable method for the commercial and industry software developers [44]. Therefore, after many years of experience in the transient stability simulation methods, as described in the literature review, no one expects that a novel approach on a single processor could significantly alleviate the simulation time, unless it somehow exploits a specific hardware architecture. Against the gradual improvements of the algorithmic methods, the hardware improvements have been revolutionary. These improvements includes processor architecture design, such as evolving multi-core CPUs and GPUs, the processor's speed, and advancements in the peripheral technology such as storage devices and communication equipment.

This thesis aggregates all three aforementioned approaches to accelerate of transient stability simulation of large-scale power systems. A novel algorithmic method is proposed and implemented on two different types of processors. One is a general purpose CPU-processor based state-of-the-art real-time digital simulator, and the other is the massively parallel Graphics Processing Unit (GPU). The CPU-based processor has a sequential architecture, while GPU has a data-parallel design.

1.4 Thesis Objectives

The previous sections gave a glimpse of the transient stability problem and the wide efforts made in this area. It can be concluded that in the transient stability analysis fast and reliable simulation is never enough and more is always required by the industry. The objective of this thesis is to investigate the use of parallel processing based approaches to accelerate transient stability simulation of large-scale systems. To achieve this purpose first we will focus on the real-time simulation on a PC-cluster real-time simulator by introducing and implementing a novel method. This can lead us to configure a real-time simulator that is specifically designed for transient stability analysis, such as existing ones for the electromagnetic studies, but one that is much more cost effective. In the second part of this thesis the use of single and multiple GPUs for the large-scale transient stability simulations is investigated. It is predicted that GPUs will be at the core of the near future massive computational engines. Therefore, power system software developers should be aware of the GPU applications in the power system computations and exploit it.

1.5 Thesis Outline

The thesis consists of six chapters. Each chapter discuses a particular topic of relevance to the thesis and the contributions made are described.

- **Chapter 1: Introduction** - The general terms used in this thesis are described in this chapter to highlight the scope of the research. The background work done in this area since several decades ago is summarized by considering both software and hardware developing aspects. The applications of transient stability analysis in the planning and operation of power systems are discussed which justified the need for faster transient stability simulations. The desire to accelerate the transient stability simulation for large-scale power systems is the main motivation of this thesis.

- **Chapter 2: Parallel Transient Stability Simulation Methods** - The purpose of this chapter is to provide a basis for the thesis. It begins with the transient stability problem formulation and the standard solution method to model this phenomena in power systems. However, the focus in this chapter is to review the application of parallel processing based technology used up to date of preparing this dissertation. This application includes both the algorithmic aspects as well as hardware advancements.

- **Chapter 3: The Instantaneous Relaxation (IR) Method** - The Instantaneous Relaxation (IR) method is introduced and implemented in this chapter. The objective is to revisit the application of real-time digital simulators to the transient stability problem. By exploiting the parallelism inherent in the transient stability problem, a parallel solution algorithm can be devised to maximize the computational efficiency of the real-time simulator. This would reduce the cost of the required hardware for a given system size or increase the size of the simulated system for a fixed cost and hardware configuration. To demonstrate the performance of the IR method, three case studies have been implemented on a PC-Cluster based real-time simulator and the results are validated by the PSS/E software. Several comparisons verified the accuracy and efficiency of the IR method.

- **Chapter 4: Single GPU Implementation: Data-Parallel Techniques** - In this chapter we discuss GPU-based transient stability simulation for large-scale power systems. The mathematical complexity along with the large data crunching need in the transient stability simulation, and the substantial opportunity to exploit parallelism are the motivations to use GPU in this area. However, since the GPU's architecture is markedly different from that of a conventional CPU, it requires a completely different algorithmic approach for implementation. This chapter investigates the potential of using a GPU to accelerate this simulation by exploiting its SIMD architecture. Two SIMD-based programming models to implement the standard method of the transient stability simulation were proposed and implemented on a single GPU. The simulation codes are written entirely in C++ integrated with GPU-specific functions.

- **Chapter 5: Multi-GPU Implementation of Large-Scale Transient Stability Simulation** - The main goal in this chapter is to demonstrate the practical aspects of utilizing multiple GPUs for large-scale transient stability simulation. Two parallel processing based techniques are implemented on a Tesla S1070 unit. The techniques used here are from tearing and relaxation categories, explained in Chapters 2 and 3. The experimental results revealed that program level decomposition, as it happens in the IR method, is more efficient than task level decomposition.

- **Chapter 6: Summary and Conclusions** - The contribution of this research are summarized in this chapter. Some plans for the future work are suggested here.

Part I

Real-Time Transient Stability Simulation on CPU-based Hardware

2

Parallel Transient Stability Simulation Methods

2.1 Introduction

In Chapter 1, the applications of transient stability analysis in the planning and operation of power systems are discussed which justified the needs for faster transient stability simulations. As mentioned, for several decades it was known that the single-processor based methods are not effective for the simulation of large-scale power systems. Thus, to achieve substantial improvement in the speed of transient stability simulation parallel processing approaches have been chosen as the most promising methods. In this chapter we will discuss the issues related to the parallel simulation of the transient stability on two fronts (1) parallel processor's hardware architecture, and (2) parallel processing based transient stability algorithms.

The chapter starts with the transient stability problem formulation and numerical methods for solution in the time-domain. Then, it will discuss the general classifications existing for hardware architecture of the parallel processors, and continue with introducing state-of-the-art hardware utilized in this thesis. A review of the parallel-processing-based algorithms for the solution of differential-algebraic equations, (DAEs) and their specific application for the transient stability problem will be described in the remainder of this chapter.

2.2 Standard Method for Transient Stability Modeling

The AC transmission network responds rapidly to any change in load or network topology. The time constant associated with the network variables are extremely small and can be assumed to be negligible in transient stability analysis without significant loss of accuracy [45]. In transient stability the concern is electromechanical oscillation, that is the variation in power output of machines as their rotors oscillate. The time constants associated with the rotors are of the order 1 to 10 seconds. Therefore, the differential equations that are relevant in this analysis are dominated by those having time constants of this order.

A widely used method for detailed modeling of the synchronous generator for the transient stability simulation is the Park's equations with an individual dq reference frame fixed on the generator's field winding [8]. The network side, including transmission lines and loads, is modeled using algebraic equations in a common DQ reference frame. Representation of AVR and PSS increases the number of differential equations and hence the complexity of the model. However, the validity of the dynamic response in a network with a lot of interconnections and in a time frame of a few seconds highly depends on the accuracy of the generator model and other components which can have effects on the dynamics of the system. Realistic interconnected power systems are generally supervised and maintained regionally by the control centers located in different geographical places. Therefore, fully detailed models for the transient stability studies are imperative for both online and offline simulations [46]. In this work the detailed model of synchronous generator including AVR and PSS is used. Each generating unit is modeled using a 9th order Park's model with an individual dq reference frame fixed on the generator's field winding [8]. The network, including transmission lines and loads, is modeled using algebraic equations in a common DQ reference frame. The complete system representation used in this thesis is summarized here:

1. Equations of motion (swing equations or rotor mechanical equations):

$$\dot{\delta}(t) = \omega_R.\Delta\omega(t) \tag{2.1}$$

$$\dot{\Delta\omega}(t) = \frac{1}{2H}[T_e(t) + T_m - D.\Delta\omega(t)].$$

2. Rotor electrical circuit equations: This model includes two windings on the d axis (one excitation field and one damper) and two damper windings on the q axis.

$$\dot{\psi}_{fd}(t) = \omega_R.[e_{fd}(t) - R_{fd}i_{fd}(t)] \tag{2.2}$$

$$\dot{\psi}_{1d}(t) = -\omega_R.R_{1d}i_{1d}(t)$$

$$\dot{\psi}_{1q}(t) = -\omega_R.R_{1q}i_{1q}(t)$$

$$\dot{\psi}_{2q}(t) = -\omega_R.R_{2q}i_{2q}(t).$$

3. Excitation system: Figure 2.1 shows ST1A type excitation system [47]. This system includes an AVR and PSS.

$$\dot{v}_1(t) = \frac{1}{T_R}[v_t(t) - v_1(t)] \tag{2.3}$$

$$\dot{v}_2(t) = K_{stab}.\dot{\Delta\omega}(t) - \frac{1}{T_w}v_2(t)$$

$$\dot{v}_3(t) = \frac{1}{T_2}[T_1\dot{v}_2(t) + v_2(t) - v_3(t)].$$

4. Stator voltage equations:

$$e_d(t) = -R_a i_d(t) + L''_q i_q(t) - E''_d(t) \tag{2.4}$$

$$e_q(t) = -R_a i_d(t) - L''_d i_d(t) - E''_q(t)$$

where

$$E''_d \equiv L_{aq}[\frac{\psi_{q1}}{L_{q1}} + \frac{\psi_{q2}}{L_{q2}}] \tag{2.5}$$

$$E''_q \equiv L_{ad}[\frac{\psi_{fd}}{L_{fd}} + \frac{\psi_{d1}}{L_{d1}}].$$

5. Electrical torque:

$$T_e = -(\psi_{ad}i_q - \psi_{aq}i_d) \tag{2.6}$$

where

$$\psi_{ad} = L''_{ad}[-i_d + \frac{\psi_{fd}}{L_{fd}} + \frac{\psi_{d1}}{L_{d1}}] \tag{2.7}$$

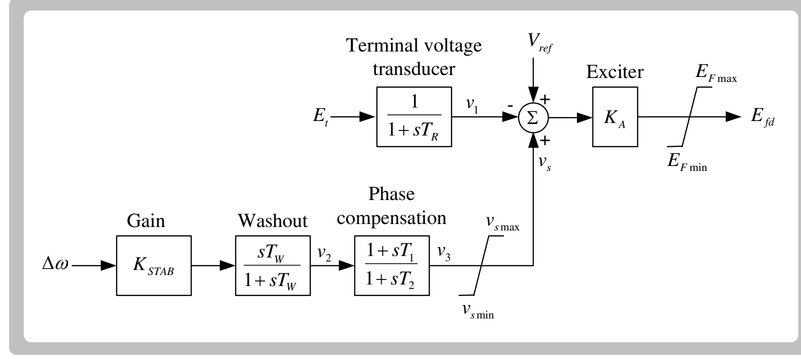


Figure 2.1: Excitation system with AVR and PSS [47].

$$\psi_{aq} = L''_{aq}[-i_q + \frac{\psi_{q1}}{L_{q1}} + \frac{\psi_{q2}}{L_{q2}}].$$

where ω_R, H, D, R_{fd}, R_{1d}, R_{1q}, R_{2q}, R_a, L_{fd}, L_{d1}, L_{q1}, L_{q2}, L''_d, L''_q, L_{ad}, L_{aq}, L''_{ad}, L''_{aq}, T_R, T_w, T_1, T_2, and K_{stab} are constant system parameters whose definition can be found in [9].

6. Network equations [8]: Stator equations are solved together with the network equations. All nodes except the generator nodes are eliminated and the admittance matrix for the reduced network is obtained. The procedure for network reduction is shown below. The nodal equation for the network can be written as:

$$I = YV \tag{2.8}$$

where $I_{(n+r)\times 1} = [I_{n\times 1} \vdots 0_{r\times 1}]^t$. n denotes the number of generator nodes and r denotes the number of remaining nodes. The matrices Y and V are partitioned as:

$$\begin{bmatrix} I_n \\ 0_r \end{bmatrix} = \begin{bmatrix} Y_{nn} \; Y_{nr} \\ Y_{rn} \; Y_{rr} \end{bmatrix} \begin{bmatrix} V_n \\ V_r \end{bmatrix}. \tag{2.9}$$

Expanding (2.9) to find I_n based on V_n we obtain:

$$I_n = (Y_{nn} - Y_{nr}Y_{rr}^{-1}Y_{rn})V_n \tag{2.10}$$

Thus,

$$I_n = Y_R.E_n \tag{2.11}$$

where the matrix $Y_R = Y_{nn} - Y_{nr}Y_{rr}^{-1}Y_{rn}$ is the desired reduced matrix. It has the dimensions $n \times n$ where n is the number of generators. This matrix must be computed for steady-state, during the transient state, and after the clearing of the transient phenomena.

As the components in the network common DQ frame are of interest the above complex matrix equation can be written as two separate real matrix equations:

$$I_{Dn} = G_R.E_{Dn} - B_R.E_{Qn} \tag{2.12}$$

$$I_{Qn} = G_R.E_{Qn} + B_R.E_{Dn}. \tag{2.13}$$

To relate the components of voltages and currents expressed in the d, q reference of each individual machine to the common reference frame (DQ), the following reference frame transformation is used:

$$i_{dq} = I_{DQ} \cdot \exp(-j\delta) \tag{2.14}$$

$$e_{dq} = E_{DQ} \cdot \exp(-j\delta) \tag{2.15}$$

where δ is the rotor angle of the synchronous machine.

The kth iteration current components in the common reference frame can be expressed as [48]:

$$I_D(k) = \frac{1}{A_5}\left(S_1 + S_2 + A_7\right) \tag{2.16}$$

$$I_Q(k) = \frac{1}{A_6}\left(S_3 + S_4 + A_8\right) \tag{2.17}$$

where the S and A parameters are defined as below:

$$S_1 = \sum_{\substack{j=1 \\ j\neq k}}^{n} I_D(j).A_1,\ S_2 = \sum_{j=1}^{n} I_Q(j).A_2 \tag{2.18}$$

$$S_3 = \sum_{j=1}^{n} I_D(j).A_3,\ S_4 = \sum_{\substack{j=1 \\ j\neq k}}^{n} I_Q(j).A_4$$

and

$$A_1 = G_R(k,j) \cdot u_1(j) - B_R(k,j) \cdot u_3(j) \tag{2.19}$$
$$A_2 = G_R(k,j) \cdot u_2(j) - B_R(k,j) \cdot u_4(j)$$
$$A_3 = G_R(k,j) \cdot u_3(j) - B_R(k,j) \cdot u_1(j)$$
$$A_4 = G_R(k,j) \cdot u_4(j) + B_R(k,j) \cdot u_2(j)$$
$$A_5 = 1 + B_R(k,j) \cdot u_3(j) - G_R(k,j) \cdot u_1(j)$$
$$A_6 = 1 - G_R(k,j) \cdot u_4(j) - B_R(k,j) \cdot u_2(j)$$
$$A_7 = \sum_{j=1}^{n} G_R(k,j) \cdot u_5(j) - B_R(k,j) \cdot u_6(j)$$
$$A_8 = \sum_{j=1}^{n} G_R(k,j) \cdot u_6(j) + B_R(k,j) \cdot u_5(j)$$

where

$$u_1 = \frac{\omega}{\omega_0}\left(L''_{ad} - L''_{aq}\right)\cos(\delta) \cdot \sin(\delta) - R_a \tag{2.20}$$
$$u_2 = \tfrac{\omega}{\omega_0}\left(L''_{ad}\sin^2(\delta) + L''_{aq}\cos^2(\delta) + L_l\right)$$
$$u_3 = -\tfrac{\omega}{\omega_0}\left(L''_{ad}\cos^2(\delta) + L''_{aq}\sin^2(\delta) + L_l\right)$$
$$u_4 = -\tfrac{\omega}{\omega_0}\left(L''_{ad} - L''_{aq}\right)\cos(\delta) \cdot \sin(\delta) - R_a$$
$$u_5 = -\cos(\delta) \cdot E''_d - \sin(\delta).E''_q$$
$$u_6 = \cos(\delta) \cdot E''_q - \sin(\delta).E''_d.$$

Having the above parameters and thus I_D and I_Q, the components of the bus voltages can be expressed as:

$$E_D(k) = I_D(k).u_1(k) + I_Q(k).u_2(k) + u_5(k) \tag{2.21}$$

$$E_Q(k) = I_D(k).u_3(k) + I_Q(k).u_4(k) + u_6(k). \tag{2.22}$$

The general form of DAEs which describe the dynamics of a multi-machine power system is given as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{V}, t) \tag{2.23}$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{V}, t) \tag{2.24}$$

$$\mathbf{x}(t_0) = \mathbf{x_0} \tag{2.25}$$

where according to the aforementioned formulations the vector of state variables ($\mathbf{x}$) in (4.1) and (4.2) for the synchronous generator is given as:

$$\mathbf{x} = [\delta \ \Delta\omega \ \psi_{fd} \ \psi_{1d} \ \psi_{1q} \ \psi_{2q} \ v_1 \ v_2 \ v_3]^t \tag{2.26}$$

and $\mathbf{x_0}$ is the initial values of state variables, and $\mathbf{V}$ is the vector of bus voltages. (4.1) describes the dynamic behavior of the system, while (4.2) describes the network constraints on (4.1). The standard approach to solve these nonlinear and coupled DAEs involves three steps [12]:

- **Step 1**. The continuous-time differential equations are first discretized and converted to discrete-time algebraic equations. Using the implicit Trapezoidal integration method, discretizing (4.1) results in a new set of non-linear algebraic equations:

$$0 = \frac{h}{2}\left[\mathbf{f}(\mathbf{x}, \mathbf{V}, t) + \mathbf{f}(\mathbf{x}, \mathbf{V}, t-h)\right] - (\mathbf{x}(t) - \mathbf{x}(t-h)) \tag{2.27}$$

  where h is the integration time-step.

- **Step 2**. The existing non-linear algebraic equations are linearized by the Newton-Raphson method (for the jth iteration) as:

$$J(\mathbf{z}_{j-1}) \cdot \Delta\mathbf{z} = -\mathbf{F}(\mathbf{z}_{j-1}) \tag{2.28}$$

  where J is the Jacobian matrix, $\mathbf{z} = [\mathbf{x}, \mathbf{V}]$, $\Delta\mathbf{z} = \mathbf{z}_j - \mathbf{z}_{j-1}$, and $\mathbf{F}$ is the vector of nonlinear function evaluations.

- **Step 3**: The resulting linear algebraic equations are solved to obtain the system state. (4.5) is solved using the LU factorization followed by the forward-backward substitution method.

2.3 Parallel Processor Architecture

A sequential computer with one CPU (central processing unit) includes only one control instruction unit. Apart from its limitation to single instruction execution at any time, there were two main obstacles with this technology: slow memory access and fundamental limitations such as overheating with compact circuits. These issues limited the achievable speed of serial computers even with the growth of the hardware technology. Therefore, the parallel processing techniques were seriously taken into account as the main alternative approach. As reported in the IEEE committee report [49]:

"Parallel processing is a form of information processing in which two or more processors together with some form of inter-processor communication system, co-operate on the solution of a problem".

In parallel processing the single CPU is replaced by multiple CPUs (even if they are individually slower than the presumed single CPU) whose overall parallel performance accelerates the simulation.

Chronologically, there are two famous taxonomies for classification of the parallel processing architecture hardware. The first one was made by Flynn [33] in which computing machines are characterized by the number of simultaneously active instruction and data streams. The two practically used groups are Single-Instruction Multiple-Data (SIMD) and Multiple-Instruction Multiple-Data (MIMD) architectures. In a SIMD-based technology the parallelism is exploited by performing the same operation concurrently on many pieces of data, while in the MIMD architecture different operations may be performed simultaneously on many pieces of data. The SIMD model works best on a certain set of problems such as image processing, and MIMD is suitable for general purpose computation. Vector processors and array processors are examples of the SIMD-based architecture, multi-processor and PC clusters have an MIMD architecture.

The other taxonomy was made by Gurd [50] in which rather than concentrating on the number of active instruction streams, the focus is on the relationship between processing elements and memory modules. Based on this taxonomy there are two classes of parallel processing architectures: distributed memory, and shared memory. In the former, there is no memory in the system other than the local memory on each processing element, and the processors communicate with each other by sending and receiving messages in a network with topologies such as mesh, ring, or hypercube. An example of these processors is Intel iPSC/2 hypercube machine which also has been used in the transient stability simulation of power systems. It consists of a host computer as the cube-manager, and 32 processors (nodes). Each node is directly connected to only $d-1$ nodes, where d is the cube dimension. The host processor loads the execution program into all nodes and sends all the necessary data to each processor, where the solution is performed in parallel. The results are sent back to the host. Successful simulation on these machines requires the decomposition of the problem into loosely coupled tasks and distribute them among the processors. The communication between nodes is by sending messages.

In the shared memory processors, however, there is a central memory accessible from any of the processing units, regardless of existing local memory on each processing units. The common memory is used to make communications between processors in shared memory architecture. The Alliant FX/8 is an example of these machines that contains 8 Computational Elements and 64MB of shared memory.

This dissertation involves two state-of-the-art parallel hardware architecture: PC-Cluster based real-time simulator, and Graphics Processing Unit (GPU). The former is a CPU-based simulator whose details on architecture and configuration will be explained in this section. The architecture of GPU, however, is substantially different from that of the CPU-based processors. Thus, GPU will be introduced in this chapter, and it will be explained in detail in Chapter 4.

2.3.1 PC-Cluster Based Real-Time Simulator

The real-time simulator existing in the RTX-LAB at the University of Alberta is manufactured by OPAL-RT Technologies Inc. using commercial-off-the-shelf components. It mainly comprises of two groups of computers known as *target* nodes, and *hosts*. Target nodes are the computational cores which carry out the simulation, and each of them is powered by a dual 3.0GHz Intel Xeon^{TM} processor. Each host is a high-performance computer which has a 3.0GHz Intel Pentium IV CPU to offer fast loading and compilation of the developed models, and providing the interface between the user and the simulator. The high-speed communication links connect targets, as well as hosts and targets. External hardware can also be connected to the simulator via the FPGA-based (Field-Programmable Gate Array) analog/digital inputs/outputs.

The hardware architecture of the real-time simulator is shown in Figure 2.2. The two processors, i.e. CPUs, in one target communicate with each other through shared memory. The targets is also capable of eXtreme High Performance (XHP) mode execution, in which one CPU is dedicated entirely to the computation while the other CPU is running real-time operating system tasks and schedulers. Several state-of-the-art computer networking technologies have been utilized to achieve the best communication throughput:

- Shared memory for inter-processor communication in one target. It has the lowest latency.
- InfiniBand architecture for inter-target communication. It has low latency (from several to several-ten microsecond) depending on communication data size.
- SignalWire which only links adjacent two targets. It has only several-microsecond level of latency.
- Giga-speed Ethernet which mainly connects between targets and hosts, or among hosts.

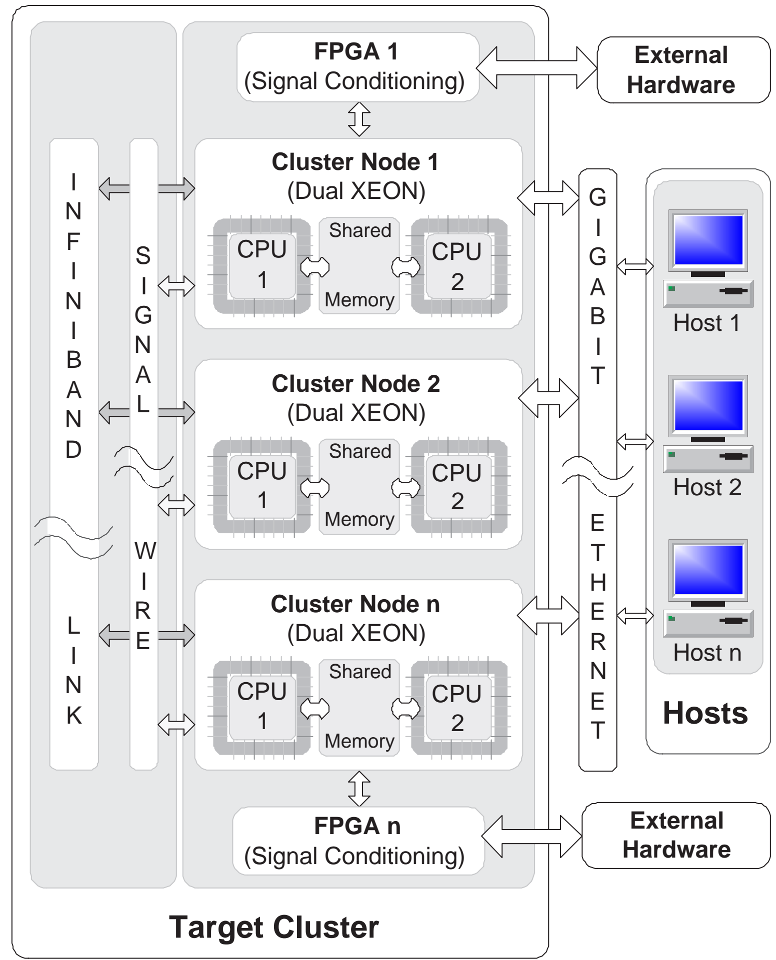


Figure 2.2: Hardware architecture of the RTX-LAB real-time simulator [51].

This high-performance PC-cluster based real-time simulator, which has a shared-memory MIMD architecture, enables any general purpose parallel processing based simulation and specifically the digital real-time simulation. Other companies such as RTDS Technologies Inc. and Hypersim have also manufactured similar real-time using distributed sequential processors. The philosophy of the real-time simulation, its necessity, and requirement will appear in the next chapter.

2.3.2 Graphics Processing Unit

Recently, Graphics Processing Units (GPUs), which were originally developed for rendering detailed real-time visual effects in the video gaming industry, have become programmable to the point where they are a viable general purpose programming platform. General purpose programming on the GPU (also called GPGPU) is currently getting a lot of attention in the various scientific communities due to the low cost and huge compu-

tational horsepower of the recent GPUs. The use of GPGPU techniques as an alternative to the parallel CPU-based cluster of computers in simulations that need highly intensive computations has become a real possibility.

Figure 2.3 illustrates schematic of how the GPU and CPU hardware are connected. As shown in this figure, GPU is mounted to the PC motherboard similar to the other add-in peripheral cards. The fundamental idea of the GPU is exploiting the parallel processing. The GPU executes independently from the CPU but it is controlled by CPU. Application program running on CPU uses the driver software to communicate with the GPU. The many-core architecture of GPU, that will be discussed in detail in Chapter 4, is especially suited for problems that can be expressed as fine-grained data-parallel computations. Except the field of image rendering, which GPU was originally designed for, several other fields from the signal processing and physics simulation to computational finance and biology have also exploited GPUs to accelerate their simulations.

The modern GPU consists of multiprocessors which map the data elements to the parallel processing threads. The multiprocessor creates, manages, and executes concurrent threads in hardware with zero scheduling overhead. The fast barrier synchronization with lightweight thread creation and zero-overhead thread scheduling supports fine-grained parallelism. To manage hundreds of threads the multiprocessors map each thread to one scaler processor core, and each scaler thread executes independently with its own instruction. There is a global device memory that all the multiprocessors can have access to. Also, each multiprocessor has its own on-chip memory that is accessible individually. Overall, the GPU can be categorized as an SIMD and shared memory processor. However, there are significant differences between SIMD structure of the GPU and that of the array processors which will be explained in Chapter 4.

2.4 Parallel Solution of Large-Scale DAEs Systems

A common approach for time-domain simulation of a physical system, described by a set of non-linear DAE consists of three steps: an integration method (e.g. trapezoidal rule) for discretizing the differential equations, an iterative method (e.g. Newton-Raphson) for solving the non-linear algebraic equations, and a linear equation solver such as Gaussian Elimination and Back Substitution. This traditional approach is referred to as the *standard* or *direct* simulation approach [52]. Both the storage and CPU time required by the standard approach grow rapidly with the size of the system, measured in terms of its components

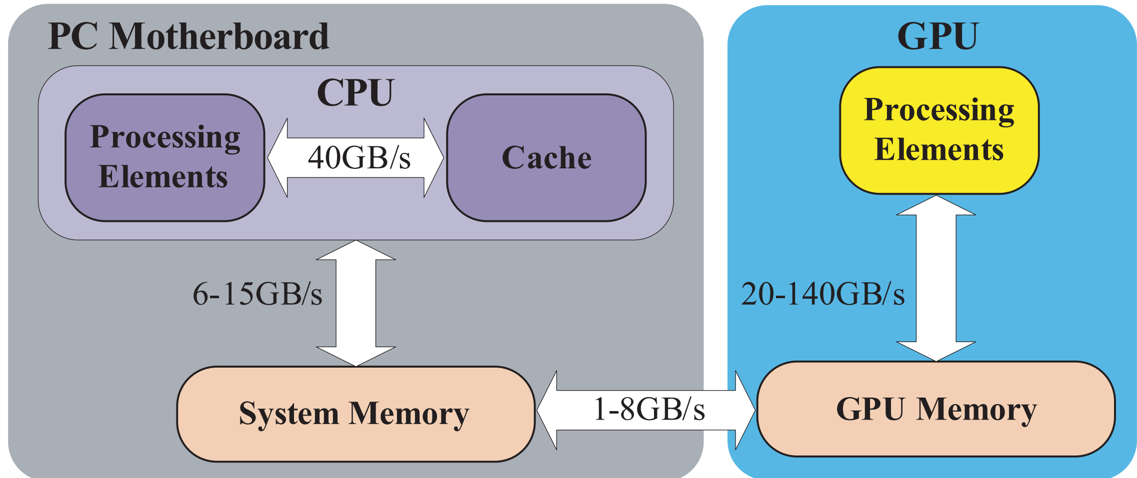


Figure 2.3: Connection of the GPU to a PC motherboard using the PCIe bus.

(i.e. generators in the case of a power system), increases. The demand for simulating ever larger systems brought the use of parallel architectures to the forefront of researchers minds. Clearly, exploitation of such parallel hardware was not possible unless appropriate software was developed that fits the architecture. Moreover, in a large system of DAEs different variables change at different rates. In the standard approach the integration method is forced to discretize all the differential equation with the same time-step which must be small enough to capture the fastest dynamics in the system. As such simulating realistic-size large-scale systems using the standard approach became very time consuming. To address these problems, a family of techniques known as *domain decomposition* was developed.

Domain decomposition refers to any technique that divides a system of equations into several subsets that can be solved individually using conventional numerical methods. To solve a set of non-linear differential-algebraic equations domain decomposition can be applied at any of the three levels of equations, i.e. differential equations, non-linear algebraic equations, and linear algebraic equations. In these techniques the system of equations at each level is viewed as a composition of several subsystems at the same level that have interaction together. The subsystem is a subset of system variables. When the system is decomposed into subsystems, the solution of each subsystem is carried out by using the conventional numerical techniques existed for each level of equations. The advantage of decomposition techniques is that they are suitable for parallel hardware architectures since several subsystems can be solved simultaneously.

To describe the structure of a system the notion of the *dependency matrix* (D) is used.

For a system with n equations and n unknown variables, D is an $n \times n$ matrix whose elements are 1 or 0. If the ith equation involves the jth variable, then $D(i, j)$ is 1, otherwise $D(i, j) = 0$.

Two different approaches were proposed in the literature to perform domain decomposition: *tearing* and *relaxation*.

2.4.1 Tearing

Tearing (introduced as the *diakoptics* method by G. Kron [27]) is the approach that takes advantage of the block structure of the system of equations. For a system of equations in which the dependency matrix is sparse, i.e. D has a small percentage of 1's, tearing can be used to achieve decomposition while maintaining the numerical properties of the method used to solve the system. The Bordered Blocked Diagonal (BBD) form is one specific structure suitable for this approach. Tearing decomposition at the level of linear algebraic equations can be implemented as the *Block LU Factorization* method, and at the level of non-linear algebraic equations as the *Multilevel Newton-Raphson* method [53].

It should be noted that the computational efficiency of this approach over the standard approach depends critically on the structure of the system, and it does not increase when system dependency matrix is dense. However, the numerical properties of the tearing approach are the same as those of the standard numerical methods applied to the system without using decomposition. For example, in nonlinear algebraic equations the Multilevel Newton-Raphson method still has the same local quadratic rate of convergence the same as that of the conventional full Newton-Raphson method.

2.4.2 Relaxation

Relaxation [54] is an approach which is not restricted to a particular system structure. In this approach the system is partitioned into a number of subsystems based on either the system equations or component connectivity. Solving these subsystems is always easier than solving the original system. Therefore, the complexity will be reduced regardless of the system sparsity. Within each subsystem the variables to be solved for are called *internal variables* and the other variables involving in that subsystem are referred as *external variables*, which are internal variables of other subsystems. To solve a subsystem for its internal variables the values of its external variables are first guessed and then updated through an iterative procedure.

Two well known iterative schemes used for relaxation decomposition are the Gauss-

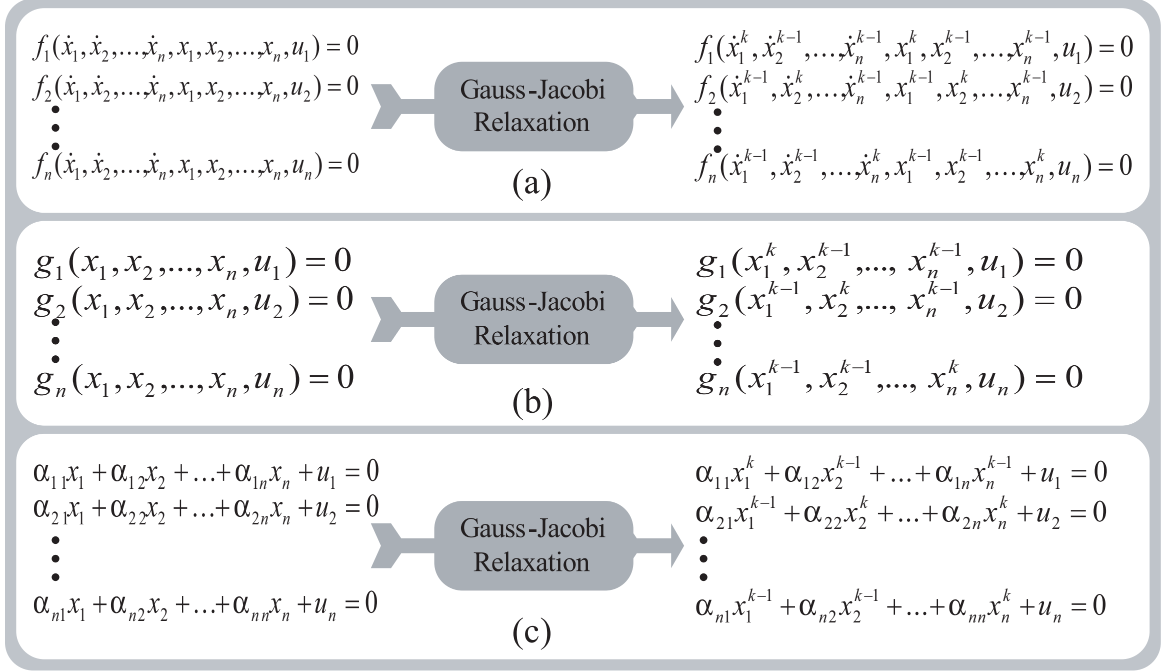


Figure 2.4: Applying Gauss-Jacobi relaxation at different level of equations: (a) differential equations, (b) non-linear algebraic equations, (c) linear algebraic equations.

Seidel and Gauss-Jacobi methods [55]. Relaxation can be used at each level of the solution. Figure 2.4 gives an example of using Gauss-Jacobi relaxation at different levels of equations. The application of this approach for nonlinear algebraic equations can be found in [28, 30]. Relaxation can be used at the level of differential equations as well, but it is not straightforward. In this case, the system is broken into subsystems in a way that the components inside of each subsystem (internal variables) are strongly interdependent while the dependency between components in two different subsystems (internal and external variables) is weak enough to ignore their interconnection. In other words, the subsystems can be *relaxed*. Therefore, each part of the system is still a system of differential equations but with a smaller size that can be solved in the time-domain using the standard approach. The relaxation approach applied to the differential equations' level has been known as the Waveform Relaxation (WR) method, which is discussed later in this chapter.

2.5 Power System Specific Approaches

The previous section provided a review of the parallel-processing-based computational approaches for a general case of DAEs describing the behavior of a dynamical system. This section introduces the ideas and approaches that have been specifically proposed for

the transient stability computation. Although, there is no specific classification for these methods they appear here chronologically.

2.5.1 Diakoptics

In the 1950s G. Kron developed a solution method for large networks called "diakoptics" [27]. The basic idea of diakoptics is to solve a large system by tearing it apart into smaller subsystems. These subnetworks are then analyzed independently as if they were completely decoupled, and then to combine and modify the solutions of the torn parts to yield the solution of the original problem. The solution of the entire network can be obtained by injecting back the link currents into the corresponding nodes.The result of the procedure is identical to one that would have been obtained if the system had been solved as one.

The advantages of diakoptics were at least twofold. Firstly, larger systems can be solved efficiently by the use of diakoptics on a given computer by processing the torn parts through the computer serially. Secondly, diakoptics employs a multiplicity of computers which essentially operate in parallel, and thus provide more speed of execution than by the use of a single computer. The computers can be physically next to each other, thus forming a cluster of computers, or they can be miles apart. Each computer in the latter application can work on the solution of a given part [56].

2.5.2 Parallel-in-Space Methods

The parallel-in-space algorithms are step-by-step methods based on partitioning the original system into subsystems and distributing them among the parallel processors. These subsystems should be loosely coupled or independent parts. In the literature of transient stability simulation "parallel-in-space" usually addresses the task-level parallelism in which serial algorithms are converted into various smaller and independent tasks that may be solved in parallel. In the transient stability calculation of a large-scale power system the obvious part that parallelism can be exploited in is the solution of linear algebraic equations.

The most significant early work in this area is described in [57] where the Trapezoidal Rule was used to discretize the differential equations, and then the parallelism was applied to solve the algebraic equations. The algorithm presented in [58] that uses the Runge-Kutta method is a typical parallel-in-space approach, which distributes solutions of the nonlinear equations of each time step into multiprocessors.

Suppose the set of differential-algebraic equations that describe the dynamics of the power system are given as following:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{V}) \\ \mathbf{I} &= \mathbf{Y}.\mathbf{V} \end{aligned} \tag{2.29}$$

where vectors $\mathbf{x}$ and $\mathbf{V}$ are the state variables and bus voltages of the system. Applying the implicit trapezoidal integration method to the differential equations and rearranging them result in a set of algebraic equations:

$$\begin{aligned} \mathbf{F} &= \mathbf{x}^k - \mathbf{x}^{k-1} - \tfrac{h}{2}\left[\mathbf{f}^k + \mathbf{f}^{k-1}\right] = \mathbf{0} \\ \mathbf{G} &= \mathbf{I} - \mathbf{Y}.\mathbf{V} = \mathbf{0} \end{aligned} \tag{2.30}$$

Applying the Newton-Raphson method to these equations, we obtain a set of linear algebraic equations:

$$\begin{bmatrix} \mathbf{F} \\ \mathbf{G} \end{bmatrix} = -\begin{bmatrix} \mathbf{J}_1 & \mathbf{J}_2 \\ \mathbf{J}_3 & \mathbf{J}_4 \end{bmatrix} \begin{bmatrix} \Delta\mathbf{x} \\ \Delta\mathbf{V} \end{bmatrix} \tag{2.31}$$

where $\mathbf{J}_1$, $\mathbf{J}_2$, $\mathbf{J}_3$, and $\mathbf{J}_4$ are the Jacobian coefficient sub-matrices and are defined as following:

$$\begin{aligned} \mathbf{J}_1 &= \tfrac{\partial \mathbf{F}}{\partial \mathbf{x}} & \mathbf{J}_2 &= \tfrac{\partial \mathbf{F}}{\partial \mathbf{V}} \\ \mathbf{J}_3 &= \tfrac{\partial \mathbf{G}}{\partial \mathbf{x}} & \mathbf{J}_4 &= \tfrac{\partial \mathbf{G}}{\partial \mathbf{V}} \end{aligned} \tag{2.32}$$

Applying the Gaussian elimination to equations (2.31) we get:

$$\begin{bmatrix} \mathbf{F} \\ \hat{\mathbf{G}} \end{bmatrix} = -\begin{bmatrix} \mathbf{J}_1 & \mathbf{J}_2 \\ \mathbf{0} & \hat{\mathbf{J}}_4 \end{bmatrix} \begin{bmatrix} \Delta\mathbf{x} \\ \Delta\mathbf{V} \end{bmatrix} \tag{2.33}$$

where

$$\begin{aligned} \hat{\mathbf{G}} &= \mathbf{G} - \mathbf{J}_3\mathbf{J}_1^{-1}\mathbf{F} \\ \hat{\mathbf{J}}_{\mathbf{4}} &= \mathbf{J}_4 - \mathbf{J}_3\mathbf{J}_1^{-1}\mathbf{J}_2 \end{aligned} \tag{2.34}$$

Therefore, equation (2.33) can be decoupled and solved with the Gauss-Jacobi iterative

scheme as:

$$\Delta\mathbf{V}^{(k)} = -\hat{\mathbf{J}}_4^{-1}.\hat{G}^{(k-1)}$$
$$\Delta\mathbf{x}^{(k)} = -\mathbf{J}_1^{-1}(\mathbf{F}^{(k-1)} + \mathbf{J}_2\Delta\mathbf{V}^{(k-1)}) \tag{2.35}$$

or with the Gauss-Seidel iterative scheme:

$$\Delta\mathbf{V}^{(k)} = -\hat{\mathbf{J}}_4^{-1}.\hat{\mathbf{G}}^{(k-1)}$$
$$\Delta\mathbf{x}^{(k)} = -\mathbf{J}_1^{-1}(\mathbf{F}^{(k-1)} + \mathbf{J}_2\Delta\mathbf{V}^{(k)}) \tag{2.36}$$

where k is the iteration index. Equation (2.35) and (2.36) can be solved to update $\mathbf{V}$ and $\mathbf{x}$ at each time-step. Therefore, the work associated with each time-step can be distributed among the parallel processors and run simultaneously.

Note that in (2.31) $\mathbf{J}_4$ actually is the admittance matrix of the interconnected network, and $\mathbf{J}_1$ is diagonally blocked, i.e.:

$$\mathbf{J}_1 = diag\left[\mathbf{J}_{1i}\right], \qquad i = 1, ..., n_{gen}$$

where n_{gen} is the number of generators, and obviously:

$$\mathbf{J}_1^{-1} = diag\left[\mathbf{J}_{1i}^{-1}\right], \qquad i = 1, ..., n_{gen}$$

Therefore, the computation of $\mathbf{J}_{1i}^{-1}$s can be assigned to parallel CPUs in any order. In the transient stability simulation different machines may have different models, for example in a machine using the classical model the corresponding $\mathbf{J}_{1i}$ is a 2×2 block while for a machine using a detailed model including exciter and PSS, the corresponding $\mathbf{J}_{1i}$ may reach 9×9 or even higher depending on the complexity of the element models. Thus, in the parallel-in-space simulation, it is important to care about balancing the CPU loads to achieve better parallel gain.

For improved computational efficiency some variations of the Newton-Raphson's method such as Very Dishonest Newton (VDHN) or Decoupled Newton method have been suggested to be used. In VDHN method the Jacobian matrices is held constant unless the convergence slows down. In [59] authors proposed to keep $\hat{\mathbf{J}}_4$ fixed unless the number of iterations exceeds a threshold value, convergence slows down, or the system undergoes topology changes, while other Jacobian sub-matrices, i.e. $\mathbf{J}_1$, $\mathbf{J}_2$, and $\mathbf{J}_3$ are updated at each iteration.

In the Decoupled Newton method, in equation (2.31), the sub-matrices $\mathbf{J}_2$ and $\mathbf{J}_3$ are ignored, and equations are directly decomposed as:

$$\begin{aligned}\Delta\mathbf{V}^{(k)} &= -\mathbf{J}_4^{-1}.\mathbf{G}^{(k-1)} \\ \Delta\mathbf{x}^{(k)} &= -\mathbf{J}_1^{-1}.\mathbf{F}^{(k-1)}\end{aligned} \tag{2.37}$$

To avoid the time consuming matrix inversion operation some parallel iterative methods have been proposed. The Successive-Over-Relaxation Newton method uses an approximated Jacobian matrix containing only diagonal elements:

$$\frac{\partial f_i(z)}{\partial z_j} = \begin{cases} \frac{\partial f_i(z)}{\partial z_j} & i = j \\ 0 & i \neq j \end{cases} \tag{2.38}$$

where z presents both the state and algebraic variables. The iterative equation to obtain individual z at each time-step can be stated as:

$$z_i^{(k)} = z_i^{(k-1)} - w_i \frac{f_i(z_i^{(k-1)})}{\partial f_i(z_i^{(k-1)})} \tag{2.39}$$

where w_i is the relaxation factor for the z_i. Since it is not desirable to change the algorithm for every case, in [59] authors proposed to use $w_s = 0.9$ for static and $w_d = 1.9$ for the dynamic variables instead of using different values for each variable.

2.5.3 Parallel-in-Time Methods

Despite the sequential character of the initial value problem which derives from the discretization of differential equations, parallel-in-time approaches have been proposed for parallel processor implementation. The idea of exploiting the parallelism-in-time in power system applications was first proposed in [31] to concurrently find the solution for multiple time-steps. In this method simulation time is divided into a series of blocks that each of them contains a number of steps that lead to the solution of the system. In other words, this technique concurrently solves many time-steps. Suppose there is a set of differential equation in the compact form of (2.40):

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{f}(t) \tag{2.40}$$

In which $\mathbf{f}$ is an explicit function of time. Applying the trapezoidal rule to (2.40) results in a set of algebraic equation:

$$\mathbf{x}^k = \mathbf{x}^{k-1} + \frac{h}{2}\left[\mathbf{A}(\mathbf{x}^k + \mathbf{x}^{k-1}) + \mathbf{f}^k + \mathbf{f}^{k-1}\right] \tag{2.41}$$

Rearranging equation (2.41) as:

$$(\mathbf{I} - \frac{h}{2}\mathbf{A})\mathbf{x}^k = (\mathbf{I} + \frac{h}{2}\mathbf{A})\mathbf{x}^{k-1} + \frac{h}{2}(\mathbf{f}^k + \mathbf{f}^{k-1}) \tag{2.42}$$

For each time-step the whole right-hand-side of the equation (2.42) can be explicitly evaluated to determine the value of $\mathbf{x}^k$ by solving a set of linear algebraic system. However, in the parallel-in-time method the vector $\mathbf{x}$ is determined for $\mathbf{T}$ time-steps simultaneously, where $\mathbf{T}$ is the number of time-steps for which the output results are required. Rearranging equation (2.42) for a set of $\mathbf{T}$ equations can be written as:

$$\begin{aligned}
&(\mathbf{I} - \tfrac{h}{2}\mathbf{A})\mathbf{x}_1 - (\mathbf{I} + \tfrac{h}{2}\mathbf{A})\mathbf{x}_0 = \tfrac{h}{2}(\mathbf{f}_1 + \mathbf{f}_0) \\
&(\mathbf{I} - \tfrac{h}{2}\mathbf{A})\mathbf{x}_2 - (\mathbf{I} + \tfrac{h}{2}\mathbf{A})\mathbf{x}_1 = \tfrac{h}{2}(\mathbf{f}_2 + \mathbf{f}_1) \\
&\quad\cdot \\
&\quad\cdot \\
&\quad\cdot \\
&(\mathbf{I} - \tfrac{h}{2}\mathbf{A})\mathbf{x}_T - (\mathbf{I} + \tfrac{h}{2}\mathbf{A})\mathbf{x}_{T-1} = \tfrac{h}{2}(\mathbf{f}_T + \mathbf{f}_{T-1})
\end{aligned} \tag{2.43}$$

where the subscribe denotes the individual time-step. A way of parallelizing this class of algorithms is to apply Gauss-Jacobi relaxation in order to exploit the parallel-in-time formulation. Therefore T time-steps can be solved simultaneously. A comprehensive research in this area has been done by M. La Scala et al. [28, 30, 60, 61]

2.5.4 Waveform Relaxation

The Waveform Relaxation (WR) method, was the first attempt to exploit both space and time parallelism in the transient stability problem. The WR method is an iterative approach for solving the system of DAE over a finite time span. In this method the original DAE, which usually has a large scale, is partitioned into smaller weakly coupled subsystems that can be solved independently. Each subsystem uses the previous iterate waveforms of other subsystems as guesses for its new iteration. After each iteration, waveforms are exchanged between subsystems, and this process is repeated until convergence is gained. This method is based on the Gauss-Seidel or Gauss-Jacobi iterative approaches explained earlier in this chapter.

The WR method was first introduced in [62] for VLSI circuit simulation. The first application of the WR algorithm in the power system area was suggested in [63] and with further development it was used for transient stability study analysis in [64] in 1989. In [65] the simulation time between the sequential WR method and direct method has been compared for some study cases. For example, a simulation interval of $2s$ in a network with 20 synchronous generators (all represented by the *classical* model) and 118 buses took $11829.5s$ and $1403.4s$ using the direct and sequential WR method, respectively. The promise of adopting parallel computers to implement the WR method was mentioned in [65], but the first time that the WR method has been used on a parallel machine was in 1997. In [66] several comparisons have been shown to clarify the efficiency of this method for parallel processing. For instance, a network with 195 synchronous generators and 970 buses has been modeled on several CPUs existing in a parallel machine. The minimum achieved execution time (not including communication time) for a simulation interval of $1.02s$ was $36.61s$ (for each CPU) in which 12 CPUs have been run in parallel; the time for the direct method was $689.75s$ (using one CPU). Although it was a big speedup, but it is still too far from real-time simulation. The useful outcome resulted from both sequential [65] and parallel [66] implementations of the WR method is that this algorithm is more efficient for larger systems.

The general form of DAE in the transient stability study of power systems described with equations (2.29). The time-domain standard method to solve this set of DAE was previously described through steps 1, 2, and 3 at the beginning section of this chapter. However, in the WR method first the system of nonlinear DAEs is decomposed into decoupled subsystems, and each subsystem is solved separately for the entire simulation time interval using waveforms from the previous iteration of the other subsystems. To achieve the convergence several iterations may be required, where each of the subsystems exchange waveforms and are then solved with updated data collected from other subsystems. This process is repeated until all waveforms converge with the necessary accuracy. To describe these explanations mathematically, suppose that equation sets (4.1) and (4.2) can be partitioned into r weakly coupled subsystems as equations (2.44):

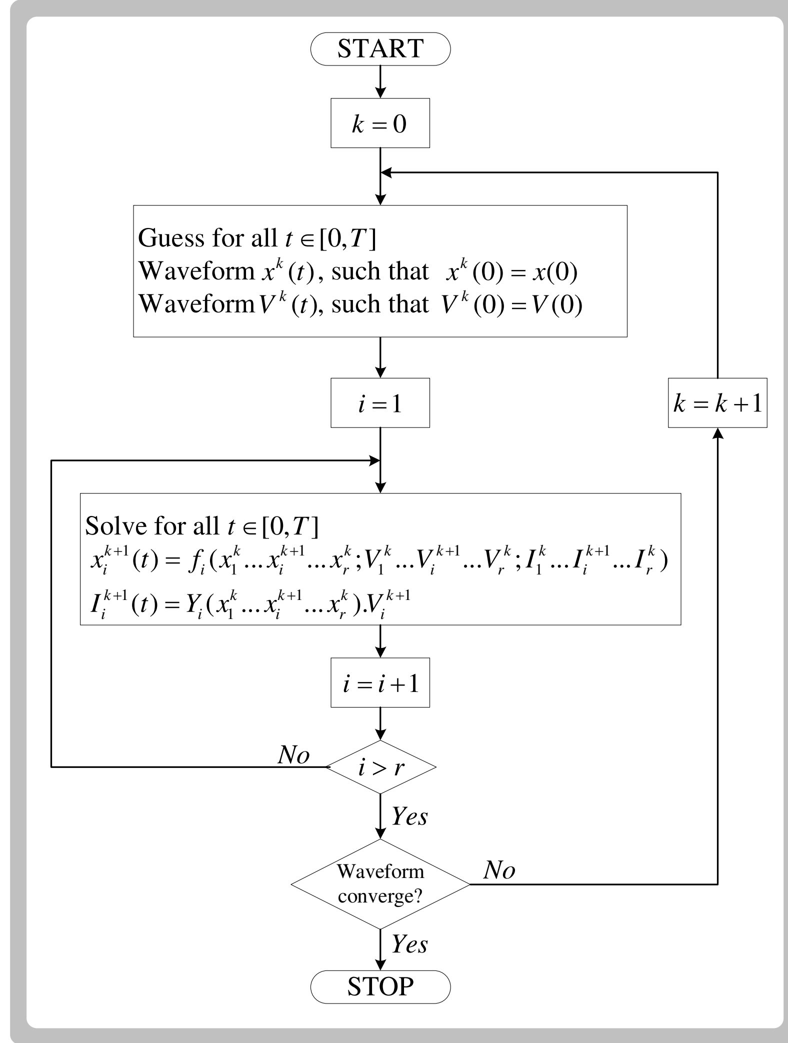


Figure 2.5: The Gauss-Jacobi WR algorithm; k: the number of iteration, i: the number of subsystem.

$$
\begin{aligned}
\dot{\mathbf{x}}_{\mathbf{1}} &= f(\mathbf{x_1} ..., \mathbf{x_r};\ \mathbf{V_1} ..., \mathbf{V_r};\ \mathbf{I_1} ..., \mathbf{I_r}) \\
\mathbf{I_1} &= Y(\mathbf{x_1} ..., \mathbf{x_r}).\mathbf{V_1} \\
&. \\
&. \\
&. \\
\dot{\mathbf{x}}_{\mathbf{r}} &= f(\mathbf{x_1} ..., \mathbf{x_r};\ \mathbf{V_1} ..., \mathbf{V_r};\ \mathbf{I_1} ..., \mathbf{I_r}) \\
\mathbf{I_r} &= Y(\mathbf{x_1} ..., \mathbf{x_r}).\mathbf{V_r}
\end{aligned}
\tag{2.44}
$$

The WR method can be based on either Gauss-Jacobi or Gauss-Seidel algorithms. The flowchart of Gauss-Jacobi WR algorithm for a time interval of $[0, T]$ is depicted in Figure 2.5. As can be seen, in each iteration each subsystem is being solved independently of other

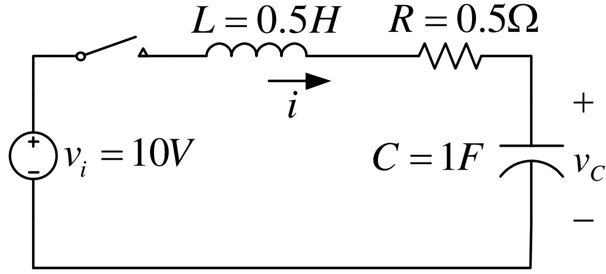


Figure 2.6: The RLC circuit.

subsystems. Thus, this method can be implemented on parallel CPUs, so that each CPU solves one of the subsystems. In the Gauss-Seidel based WR method, the ith subsystem uses the current iterate waveform from subsystems $(1, ..., i-1)$ and the previous iterate waveforms from subsystems $(i+1, ..., r)$ as inputs. This algorithm is therefore sequential in nature. In both algorithms, the subsystems are discretized and solved independently. The method exploits time parallelism over the simulation period since subsystems are solved concurrently. The space parallelism also is inherited due to the system decomposition shown in equations (2.44).

To show the procedure of the WR method and its related issues a simple example will be demonstrated here. Consider the *RLC* circuit shown in Figure 2.6, in which the switch is closed at $t = 0$, and $v_c(0) = 0$. By choosing the voltage of capacitor (v_c) and the current of inductor (i) as the state variables, the mathematical description of this circuit would be as follows [67]:

$$\dot{i}(t) = 20 - 2v_c(t) - i(t) \tag{2.45}$$

$$\dot{v}_c(t) = i(t) \tag{2.46}$$

The voltage waveform resulting from the solution of this set of ordinary differential equations (ODEs) achieved from the direct method has been plotted in Figure 2.7 by the solid line. To apply the WR method to this system first it must be broken into subsystems. In this example there are two differential equations; thus, the system is divided into two subsystems. Subsystem I includes the equation (2.45) and Subsystem II includes the equation (2.46). Applying a Gauss-Jacobi iterative scheme, in the kth iteration the Subsystem I is being solved by considering $v_c^{(k-1)}$ and Subsystem II is being solved by taking $i^{(k-1)}$. After computations are done over the given simulation time interval in both sub-

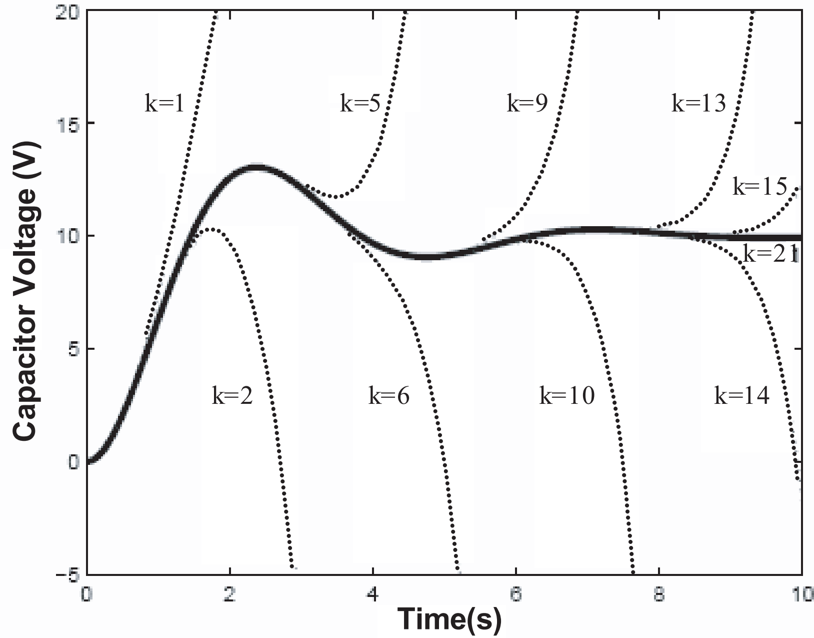


Figure 2.7: Response of the RLC circuit for the capacitor voltage, k: the number of WR iterations.

systems, waveforms would be exchanged; then, the two subsystems are ready to be solved for the next iteration with the new waveform data. This procedure will be continued until the resulting waveforms converge within required accuracy. In Figure 2.7 the response of several iterations has been superimposed on the direct method response with dash lines. As k increases, at the end of each iteration the resulting waveform converges toward the direct method response more than the previous iteration.

From this example one important property of the WR method can be explained. It can be observed in Figure 2.7 that as the number of iterations increases, the time interval in which the resulting waveform is close to the exact one becomes larger. In other words, the method works well for a certain interval, but it is inaccurate outside of this span. So, instead of applying the method in each iteration over the whole simulation time, it is more effective to divide the simulation time into small intervals (with the length of win) and solve equations piece by piece within each interval, as shown in Figure 2.8. This technique, known as *windowing*, decreases the number of iterations required within each interval for achieving the certain accuracy [52]. The complete flowchart of WR method including the windowing technique is depicted in Figure 2.9. Windowing also reduces the required memory space, because the iterative waveforms need to be stored only for small time intervals instead of the whole simulation time [67]. In the large-scale networks if the

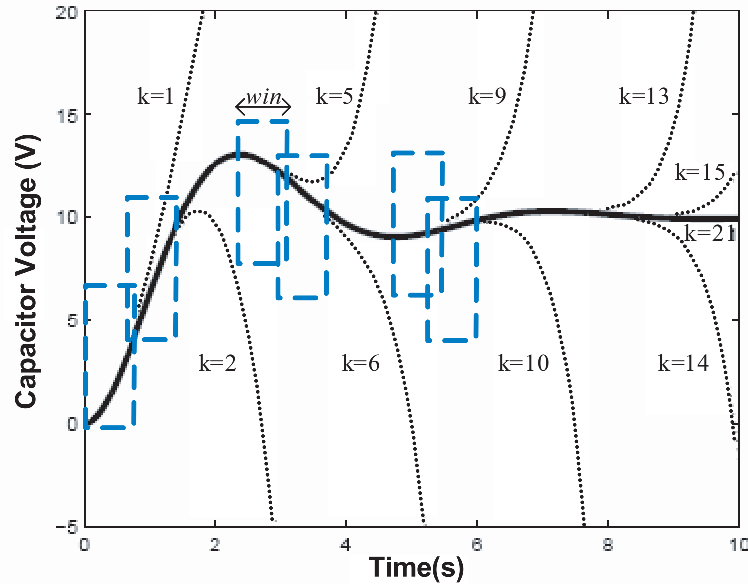


Figure 2.8: Application of windowing technique in the WR method, k: the number of WR iterations, win: window length.

intervals are too small, the advantages of the WR method are lost due to the increase in communication time among the partitions [29].

The same as other parallel computation algorithms, the first requirement in utilizing the WR method is to partition the problem into smaller tasks that can be distributed among the several processors. Partitioning can be classified into fine-grain and coarse-grain. In the former the problem is divided into many small tasks, and consequently, the communication between the processors is broad. In the later the problem is divided into a few but large tasks or subsystems which impose less communication on the processors. Thus, choosing the partitioning approach, is mainly limited by the parallel processor hardware. If the available hardware cannot provide a fast capability among its processors, the coarse-grain method is more preferable. A full section discussing this issue appears in the Chapter 3.

2.6 Power System Partitioning

Various approaches based on criteria such as computation loads, network topology, and dynamic behavior of the system can be used to partition power system for transient stability simulation. For example the system can be decomposed to split the computation burden among parallel processors based on the total number of equations, or by consid-

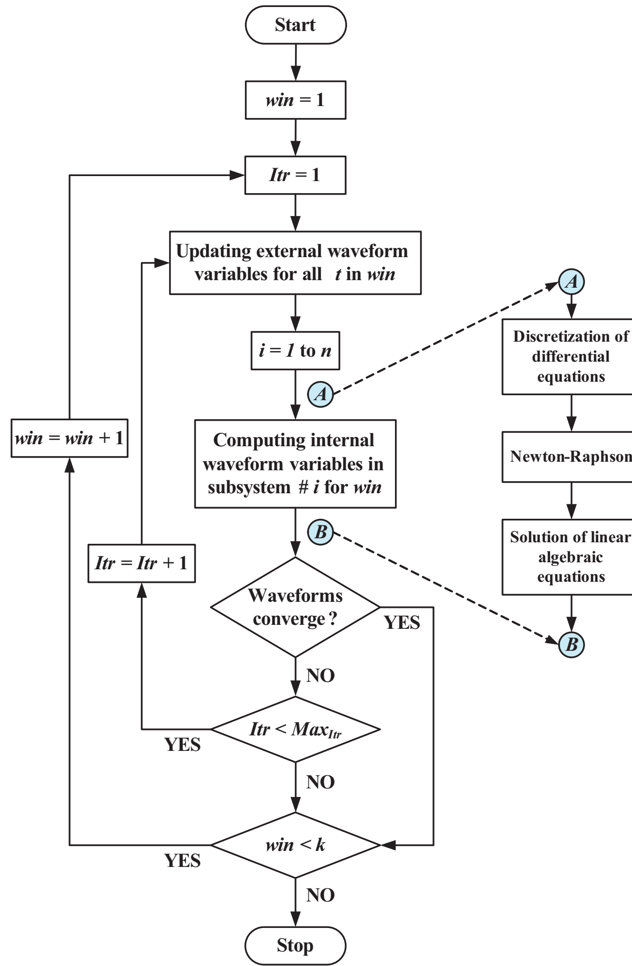


Figure 2.9: Flowchart of the WR method for the duration of $[0, T]$ which is divided into k window intervals. n: the total number of subsystems; Itr: counter of iterations in each window; Max_{Itr}: the maximum allowable number of iterations in each window; win: counter for windows.

ering the complexity of the generator models and the connectivity of the buses. There are methods based on graph theory for network partitioning for use in the block iterative solution of linear systems. In [68] this technique has been used for load flow study in power systems.

In the approaches that are based on the system dynamics the coherency characteristics of the generators are used. To find the coherent generators two methods are available. One is based on time-domain simulation, and the other method is to use eigenvalue analysis. The best way of determining generator coherency is through observing the swing curves generated by numerical integration of the system dynamic equations. In a power network a pair of generators are called coherent if the difference between their rotor angles (δ_i and

δ_j) remains constant over time:

$$\delta_i(t) - \delta_j(t) = \Delta\delta_{ij} \pm \varepsilon \qquad\qquad 0 < t < T \tag{2.47}$$

where $\Delta\delta_{ij}$ is a constant value, and ε is a small positive number. In case that $\varepsilon = 0$ generators i and j are perfectly coherent. However, in the time-domain approach the computation involved for a large-scale system is intensive because it requires solution of the system dynamic equations. In [70] the time-domain simulation method to find out coherent groups of generators has been described based on two assumptions that coherent generators are independent from the size of disturbance and also the detailed of modeling. These assumptions have been validated based on observations of many simulations. Therefore, a linearized classical model of the synchronous machines can be used to find out rotor angle trajectories based on criteria in (2.47). Slow coherency partitioning method, reported in [71], has been applied in power systems. In this method instead of time-domain simulation, the concept of tow-time-scale model is used to decompose the system. Two time scales related to difference between the inter-area and local oscillations happening in a power system following a fault. This approach requires the calculation of modes and eigenvalues of the given system. In [72] a direct method of coherency determination through the use of Taylor series expansion of the faulted and post faulted systems has been proposed. There are also methods that decompose the system based on the concept of the electrical distance between busbars, and is independent of the systems operating condition [73]. In this thesis the slow coherency method has been used for partitioning large-scale systems.

2.7 Types of Parallelism Used in This Thesis

We have used three types of parallelism's:

- **Algorithm-level**: This is a *top-level* or *coarse-grain* parallelism which happens before any numerical method starts solving the system equations. It is also known as *program-level* parallelism [29]. Here the objective is not to address task definition and scheduling, but the parallelism inherent in the overall algorithm. The WR method is an example of this type of parallelism.

- **Task-level**: In this type of parallelism the traditional serial algorithm is converted into various smaller and independent tasks which may be solved in parallel. For

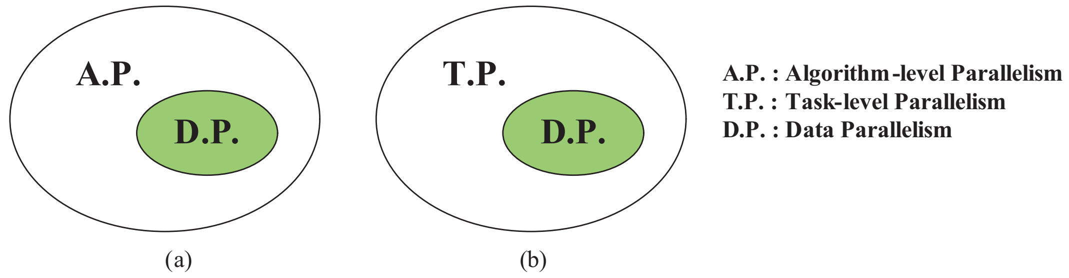


Figure 2.10: Integrating various types of parallelism: (a) algorithm-level method on a data-parallel hardware, (b) task-level method on a data-parallel hardware.

example, to solve a linear set of equations in the form of $Ax = b$, the task-level parallelism entails decomposing the matrix A into various other matrices that can be solved in parallel. Using the sparsity related solution methods, or converting matrix A into a block-bordered diagonal matrix are examples of the task-level parallelism approach. The parallel-in-space methods explained earlier also falls into this category.

- **Data-parallelism**: This is the most *fine-grained* type of parallelism that can be used on the SIMD-based architectures such as vector processors or GPUs. A given problem must have the capability to be expressed in the data-parallel format in order to take advantages of the SIMD hardware.

Both the algorithm-level and task-level approaches can take advantage of data-parallelism techniques as shown in Figure 2.10. In Chapter 5 it will be shown how the proposed algorithm-level parallel method, i.e. Instantaneous Relaxation, and a task-level parallel method, known as Incomplete LU factorization, can be implemented on data-parallel architecture of GPUs.

2.8 Summary

In this chapter the subject of parallel processing based computation applicable for transient stability simulation of large-scale power systems has been discussed. To this end we started with the general classification of parallel processors hardware architectures to describe two state-of-the-art available processor techniques (PC-Cluster based real-time simulator and GPUs) in the RTX-LAB. The second portion of this chapter was allocated to the parallel solution methods for the differential-algebraic equations, and approaches specific to the transient stability simulation of power systems have been discussed. In Chapter

3 the general idea of the WR method suitable for real-time simulation is discussed. However, the original WR method contains drawbacks which offer serious obstacles for implementation in real-time space. The proposed approach in this thesis is the Instantaneous Relaxation method that will be comprehensively described in Chapter 3. As the Instantaneous Relaxation and WR methods have a common mathematical background, the WR method was explained with more details in this chapter to prepare the reader for Chapter 3.

3

The Instantaneous Relaxation (IR) Method

3.1 Introduction

The objective of this chapter [1] is to revisit the application of real-time digital simulators to the transient stability problem. As mentioned in the previous chapter, transient stability simulation of realistic-size power systems involves computationally onerous time-domain solution of thousands of nonlinear differential algebraic equations (DAE's). Furthermore, from the point of view of dynamic security assessment which is required for safe system operation and control, several transient stability cases need to be run in a short period of time for analyzing multiple contingencies and to initiate preventive control actions.

Currently available commercial real-time simulators such as RTDS [74], RT-LAB from OPAL-RT Technologies Inc. [51], and Hypersim [75] address these needs to a large extent by using multiple racks or clusters of multi-processor architectures. The question that arises, however, is whether this approach is the most efficient and cost-effective, given the prevalent practice of using a real-time simulator, originally designed and built for electromagnetic transient studies, for transient stability simulations. This is done, of course, using simpler models and larger time-steps. For example, nominal-pi models are used instead of frequency-dependent models for transmission lines, and the simulator time-step is chosen to be in the range of milliseconds instead of microseconds. Nevertheless, there is underlying sequentiality in the electromagnetic transient simulation algorithm [76] that precludes

[1] Material from this chapter has been published: V. Jalili-Marandi, V. Dinavahi, "Instantaneous relaxation-based real-time transient stability simulation," *IEEE Trans. on Power Systems*, vol. 24, no. 3, August 2009, pp. 1327-1336.

an efficient utilization of the hardware capabilities for transient stability simulation. By exploiting the parallelism inherent in the transient stability problem, a parallel solution algorithm can be devised to maximize the computational efficiency of the real-time simulator. This would reduce the cost of the required hardware for a given system size or increase the size of the simulated system for a fixed cost and hardware configuration.

In this chapter we propose a fully parallel *Instantaneous Relaxation* (IR) method for real-time transient stability simulation. The idea of using relaxation-based solution of DAE's is certainly not new and has been explored before. The Waveform Relaxation (WR) method, described in the previous chapter, was first introduced in [62] for VLSI circuit simulation. Then in [63] this method was applied to the power systems area and used comprehensively for off-line transient stability simulation [65]. The classical model of the synchronous machine was used in these simulations. Although this algorithm was implemented sequentially, it was predicted that it will accelerate the simulation by exploiting parallel processors [29]. Later in [66] the WR method was implemented on parallel computers.

As will be explained later, although the WR method is a parallel method successfully implemented for off-line simulations, there are inefficiencies that surface when it is implemented in real-time. Therefore, the IR method which overcomes these limitations is proposed for real-time implementation.

The chapter begins with a discussion about the obstacles of the WR method for real-time implementation. The algorithm of the proposed IR method will be explained, and the approach for partitioning a power system for performing IR method will be discussed. Real-time simulation results and their comparative analysis with off-line simulations will be shown at the end.

3.2 Limitations of WR method for Real-Time Transient Stability Simulation

The WR method was explained in the Chapter 2. The most important advantages of this method as a parallel-processing application where parallelism happens at the program level and not at the task level, are:

- it is an inherently parallel method.
- multi-rate integration methods can be used.
- each subsystem can be solved independently

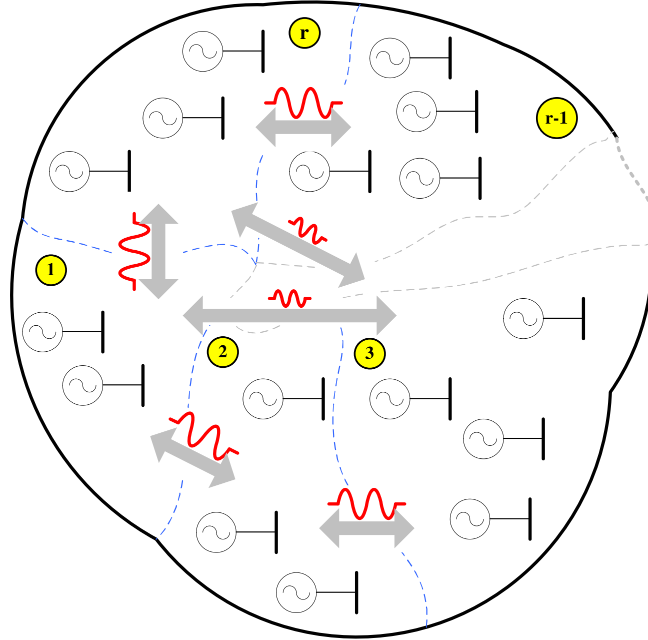


Figure 3.1: Partitioning a large system into subsystems.

- subsystems become smaller than the original large system, it takes less time to solve.
- different levels of accuracy for modeling in each subsystem can be used.

Figure 3.1 schematically shows a large-scale power system partitioned into r smaller subsystems to conduct a parallel simulation by using the concept of WR method. The outstanding difference between the WR and other classical decomposition methods for solving linear and non-linear algebraic equations is that in this method during each iteration each subsystem is analyzed for the entire time interval $[0, T]$. In other words, the elements in this technique are waveforms of the variables rather than their instantaneous values. In each iteration of the WR method each subsystem is solved by using the three basic steps of the transient stability *standard* approach for all $t \in [0, T]$.

It was discussed in [67] and [29] that the WR method works well for a certain interval, but it is inaccurate outside of this span. So, instead of applying the method in each iteration over the whole simulation time, i.e. $[0, T]$, it is more effective to divide the simulation time into k small intervals or windows, i.e. $[0, T_1]$, $[T_1, T_2]$,..., $[T_k, T]$, and solve equations piece by piece within each interval. This technique, known as *windowing*, decreases the number of iterations required within each interval for achieving required accuracy [52]. Furthermore, windowing reduces the required memory space, because the iterative waveforms need to be stored only for small time intervals instead of the whole simulation time.

The waveform-based property of the WR method is one issue that needs to be changed for real-time simulation. There are two reasons. First, in real-time simulation and specifically in hardware-in-the-loop simulation the instantaneous value of each variable at each time-step is required and not the complete waveforms. Second, if waveforms are going to be used as numerical elements, all waveforms of the variables such as bus voltages or generator angles must be computed for the entire simulation interval, say $20s$, in the first time-step of the simulation, say $1ms$. Clearly, this is not practical for a large-scale system with thousands of variables. To overcome this restriction the windowing technique can be used. So, the whole simulation time is divided into small intervals, and each interval is computed in one time-step. The time intervals must be small enough so that the computation tasks can be performed during one time-step. Although windowing can help maintain the waveform property; however, working with waveforms in real-time is not as efficient as in off-line simulation.

Suppose the simulation interval $[0, T]s$ is divided into k windows of length $m \times h$ milliseconds, h being the time-step. When the simulation starts, all waveforms for the interval of the first window must be computed during the first time-step. Then, there are two options. In the first option (Figure 3.2) the real-time simulator is idle during the remaining length of the first window, i.e. for $(m - 1) \times h$ milliseconds, when it just sends out instant values of variables at each time-step. After this period simulator resumes computation for the interval of the second window, and again becomes idle. This process is repeated until the end of simulation time. In the second option, depicted in Figure 3.3, the simulator continues the computation for each window in the subsequent time-steps while it also sends out the instantaneous values of variables at each time-step. Therefore, the computation finishes in k consecutive time-steps, and after that the simulator becomes idle when it sends out instant values at each time-step. It can be concluded that in both options the simulator performs the entire computation in k time-steps and then remains idle for $(m-1) \times k$ time-steps. In other words, the computation load has not been distributed among the time-steps equally. Thus, real-time implementation of the native WR method can be inefficient from resource utilization point of view.

3.3 Instantaneous Relaxation

To overcome the limitations of the WR for real-time implementation we propose the *point-wise* or *Instantaneous Relaxation* (IR) technique. It is simply the WR method with a window

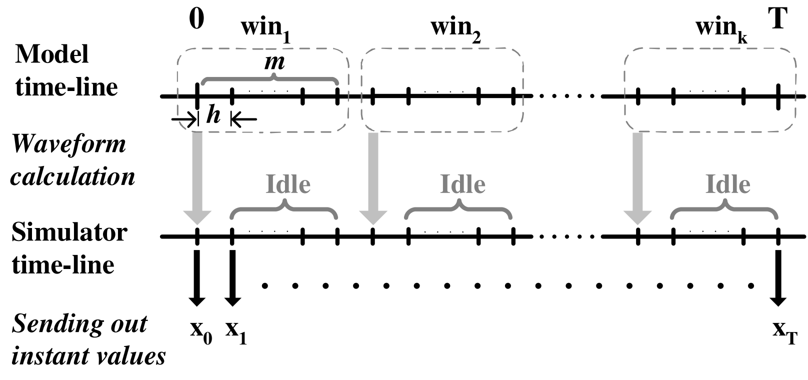


Figure 3.2: Real-time implementation of the WR method: Option 1.

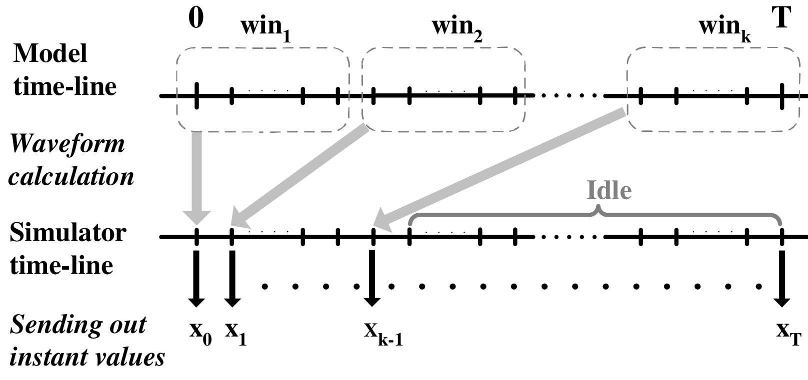


Figure 3.3: Real-time implementation of the WR method: Option 2.

length of one time-step, i.e. $m = 1$. It has been verified in the off-line implementation of WR method that the smaller the length of window, the faster the convergence. On the other hand, if the window is made too small the overall communication time among subsystems increases which causes a loss of the advantages of the WR method. However, the communication latency between computation nodes in currently available real-time simulators is in the order of a few microseconds. This latency is small compared to the time-step required for transient stability and can therefore be neglected. Based on the previous experience with the WR method and the arguments made in the pervious section, it can be concluded that the IR method not only inherits the advantages of the WR method but is also efficient from the real-time simulation point of view.

To apply relaxation methods at the level of differential equations the preliminary step is clustering variables into groups which can be solved independently. This will be specifically discussed for the transient stability application in the next section. After partitioning the system into n subsystems, the set of DAE's equations (i.e. (3.1) and (3.2)) are prepared

to describe the dynamics of each subsystem:

$$\dot{\mathbf{x}}^{int} = \mathbf{f}(\mathbf{x}^{int}, \mathbf{x}^{ext}, \mathbf{V}^{int}, \mathbf{V}^{ext}, t), \tag{3.1}$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}^{int}, \mathbf{x}^{ext}, \mathbf{V}^{int}, \mathbf{V}^{ext}, t), \tag{3.2}$$

where $\mathbf{x}^{int}$ and $\mathbf{V}^{int}$ are state and algebraic variables that define the dynamic behavior of Subsystem i, and $\mathbf{x}^{ext}$ and $\mathbf{V}^{ext}$ are the state and algebraic variables that defining all subsystems excluding Subsystem i. Therefore, $\mathbf{x} = \mathbf{x}^{int} \bigcup \mathbf{x}^{ext}$ and $\mathbf{V} = \mathbf{V}^{int} \bigcup \mathbf{V}^{ext}$ are the set of all state and algebraic variables that describe the original-size system. Using the Gauss-Jacobi iteration scheme the pseudo code for the **GJ-IR** method is as follows:

```
guess initial values for x^int and V^int at each subsystem
t=0;
repeat{
    t=t+1;
    for each subsystem solve
        ẋ_t^int = f(x_t^int, x_{t-1}^ext, V_t^int, V_{t-1}^ext)
        0 = g(x_t^int, x_{t-1}^ext, V_t^int, V_{t-1}^ext)
}until (t>T)
```

and using the Gauss-Seidel iteration scheme the **GS-IR** method is as follows:

```
guess initial values for x^int and V^int at each subsystem
t=0;
repeat{
    t=t+1;
    for each subsystem solve
        ẋ_t^int = f(x_t^int, x_t^ext, V_t^int, V_t^ext)
        0 = g(x_t^int, x_t^ext, V_t^int, V_t^ext)
}until (t>T)
```

From the above mentioned algorithms, the obvious difference between GJ-IR and GS-IR is that in the GS-IR at each time-step the Subsystem i has to wait until Subsystems 1 to $i-1$ get solved, so that Subsystem i can use the last values of the $\mathbf{x}^{ext}$ and $\mathbf{V}^{ext}$ to solve $\mathbf{x}^{int}$ and $\mathbf{V}^{int}$. However, in the GJ-IR the solution of Subsystem i for the current time-step

is fully independent from the solution of other subsystems. Therefore, the GS-IR has a sequential nature while the GJ-IR is fully parallel that makes it suitable for our purpose. From now on, in this thesis the IR method refers to the GJ-IR.

To solve each subsystem we first start with discretizing (3.1) that results in a new set of non-linear algebraic equations. In this work we used the trapezoidal rule as the implicit integration method to discretize the differential equations as follows:

$$0 = \mathbf{x}^i - \frac{h}{2}\left[\mathbf{f}^i(\mathbf{x}^i, \mathbf{V}^i, t) + f^i(\mathbf{x}^i, \mathbf{V}^i, t-h)\right], \tag{3.3}$$

where $i = 1, 2, \ldots, n$ indicates the subsystem, and h is the integration time-step. (3.2) and (4.4) can be linearized by the Newton-Raphson method (for the jth iteration) as:

$$J(\mathbf{z}^i_{j-1}) \cdot \Delta\mathbf{z}^i = -\mathbf{F}^i(\mathbf{z}^i_{j-1}), \tag{3.4}$$

where J is the Jacobian matrix, $\mathbf{z}^i = [\mathbf{x}^i, \mathbf{V}^i]$, $\Delta\mathbf{z}^i = \mathbf{z}^i_j - \mathbf{z}^i_{j-1}$, and $\mathbf{F}^i$ is the vector of nonlinear function evaluations. (4.5) is a set of linear algebraic equations that can be solved with Gaussian Elimination and back substitution method. Benchmarking revealed that a majority of execution time in a transient stability simulation is spent for the nonlinear solution. By using the IR method, however, and by distributing the subsystems over several parallel processors, a large-scale system is divided into individual subsystems whose matrix sizes are smaller resulting in faster computations.

To clarify the differences between the WR and IR methods, the flowcharts of IR algorithm is shown in Figure 3.4 which can be compared with Figure 2.9 in Chapter 2. Practically in the WR method it does not seem efficient to perform several iterations of the Newton-Raphson. Let the exact solution of a waveform be $x(.)$, and the result of the kth iteration of the WR method be $x^k(.)$. Depending on the length of window some iterations will be required for $x^k(.)$ to converge to $x(.)$; however, the starting iterations for $x^k(.)$ are poor approximations of $x(.)$. Thus, it is superfluous to perform Newton-Raphson iterations for computing a close approximation to $x^k(.)$ which itself is a poor approximation of $x(.)$. The convergence rate of the IR method is higher than that of WR, because its window length is minimum. Therefore, performing several iterations of Newton-Raphson, as shown in Figure 3.4, increases the accuracy of IR.

Following the convergence of iterative solutions in all subsystems, the state and algebraic variables calculated from the last time-step are updated. The state variable description of generators used in this work was defined in Chapter 1. These state variables and voltages of generator buses must be exchanged between all interconnected subsystems.

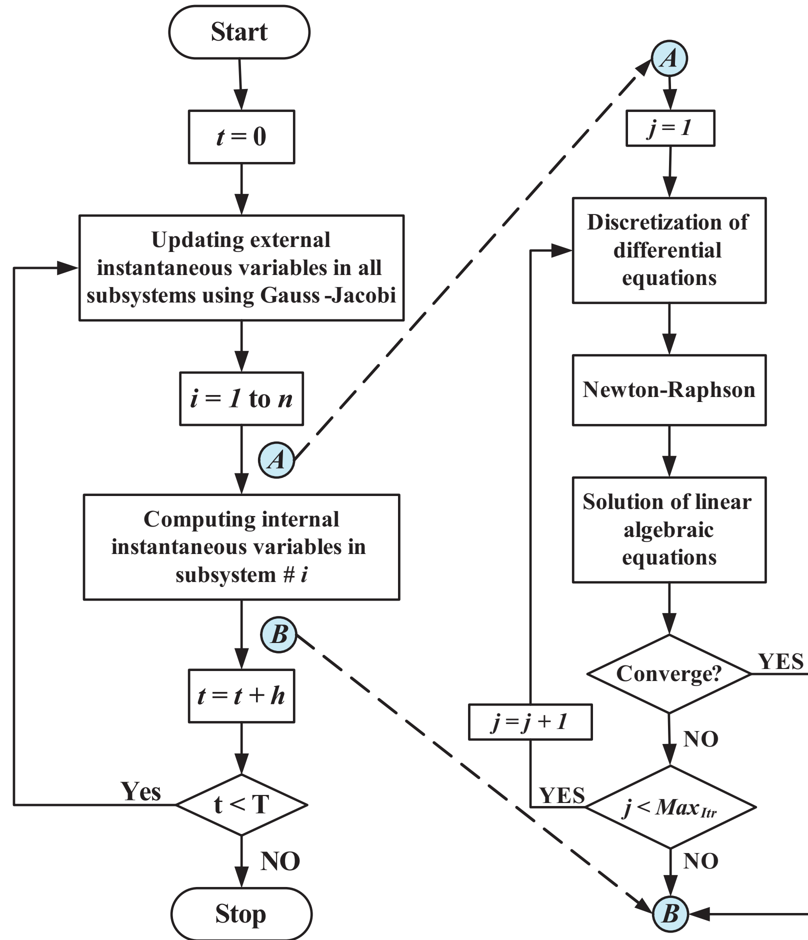


Figure 3.4: Flowchart of the proposed IR method for the duration of $[0, T]$ with a time-step of h. j: counter for iterations of the Newton-Raphson; Max_{Itr}: the maximum allowable number of iterations for the Newton-Raphson in each time-step.

3.4 Coherency Based System Partitioning for the IR Method

One way to partition a power system for parallel processing is to distribute equal numbers of generators and buses among the processors. However, this is not an efficient method because the network buses have different connectivity and the generator models vary in both size and complexity. This gives rise to the load balancing problem in a parallel multi-processor simulator architecture. Another option is to split the computation burden among processors based on the total number of equations; however, this approach will increase both the programming and communication complexity. A more efficient method is to partition the system by considering the complexity of the generator models and the connectivity of the buses. In this case, different number of generators and buses are assigned to parallel processors, and the computation burden is roughly even; however, the draw-

back of this method is that it cannot be used for a general-purpose program at least in the off-line sense [77].

The primary requirement for successfully using the relaxation methods at the level of differential equations is to divide the system into subsystems in which tightly coupled variables are grouped together. In [52] it was shown that the WR method will converge for any chosen partitioning scheme; however, the rate of convergence is highly dependent on the method of partitioning [29]. In spite of all that, it is important to ask this question: is transient stability simulation of a large-scale network by IR method restricted by this prerequisite? In other words, whether the partitioning scheme's dependence on system modeling or the characteristics of the disturbance such as its severity or location, will influence the convergence of the IR or WR methods.

Determination of tightly coupled variables or simply partitioning the system can find a physical meaning from the power system point of view. Following a large disturbance in the system, some generators lose their synchronism with the network. Thus, the system is naturally partitioned into several areas in which generators are in step together while there are oscillations among the different areas. Generators in each of these areas are said to be *coherent*. The coherency characteristic of the power system reflects the level of dependency between generators. Coherent generators can be grouped in the same subsystem which can be solved independently from other subsystems with the WR or IR methods. The partitioning achieved using the coherency property has two characteristics which make it appropriate for our study. The coherent groups of generators are independent of: (1) the size of disturbance and (2) the level of detail used in the generators. Therefore, the linearized model of the system and the simple classical model of generators can be used to determine coherency. Furthermore, slow coherency based grouping is insensitive to the location of disturbance in the power system [78]. These features of slow coherency lead us to use this partitioning method in this thesis.

3.5 Implementation of IR Method

The IR method was implemented using a customized MATLAB S-function. S-function is a computer language description of a SIMULINK block that can be written in either C, C++, Fortran, Ada, or MATLAB, and it is compiled as a `mex` file to dynamically link into MATLAB. The most common use of S-function is to create a custom SIMULINK block. This block may include a new general purpose application, a hardware device driver, or

describing a system as a set of mathematical equation. In our case, the purpose was to implement the IR method. Thus, we use the S-function to mathematically model the power system equipments and solve it using the proposed IR method.

The customized S-function block is solved at every time-step assigned by SIMULINK but the time-step can be changed from outside the S-function in the main model. The change in time-step can be passed to the S-function structure by defining it as a parameter of the S-function. The 'discrete solver' option is used for the simulation, even though selection of different solver in SIMULINK does not influence the S-function as the solver is chosen inside the S-function structure. The models developed in SIMULINK could be solved with either variable or fixed time-step. Due to the model validation process, only the fixed-step solver was utilized. In this research we used the C programming language to prepare the SIMULINK S-function which will be explained in this section.

3.5.1 Building of C-based S-Function

The general structure of the S-function written in C is shown in Figure 3.5. The SIMULINK S-Function block is invoked at each simulation time-step, and any changes in the S-function inputs can be passed to it even while the simulation is running. As shown in Figure 3.5 flowchart, the essential functions in C-based S-function to be used are the follows.

Function `static void mdlInitializeSizes(SimStruct *S)`

This is the first invoked function in the S-function, and it is used to specify the basic characteristics of the block, such as number of inputs, outputs, the port width of each input and output, and the number of parameters of the S-function block. It is worthwhile to mention that while the simulation is running, the s-function parameters cannot be changed but its inputs can be changed. In this case, three input and nine output ports are defined. The input corresponds to external machines ID, and variables that are required to be exchanged among subsystems.

Function `static void mdlInitializeSampleTimes(SimStruct *S)`

Sample time of the S-function block is initialized here, which is retrieved from the first S-function parameter by C function `mxGetScalar(ssGetSFcnParam(S, 0))`.

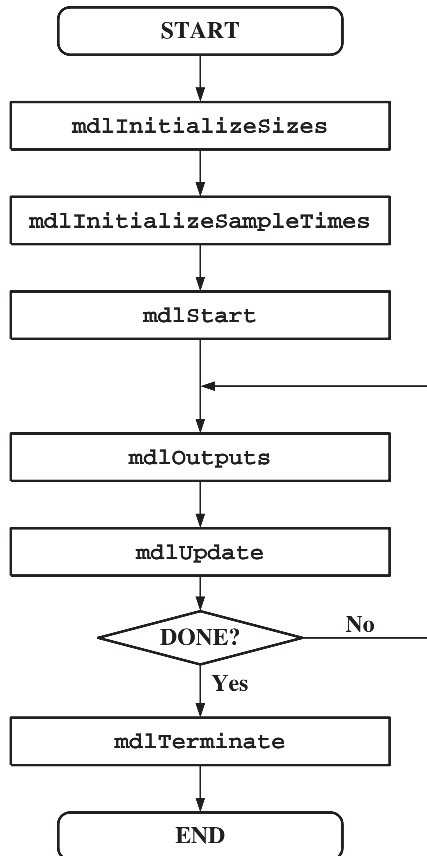


Figure 3.5: SIMULINK S-function flowchart

Function `static void mdlStart(SimStruct *S)`

All initializations including the network's steady state parameters, machines data, and network's admittance matrix are placed in this function. The data files in the PSS/E *.raw file format are read and interpreted, and corresponding models are initialized. Moreover, in order to reduce computational time in the simulation loop, all admittance matrices corresponding to the switching states are uploaded here for future access in simulation loop.

Function `static void mdlOutputs(SimStruct *S, int_T tid)`

This is the function invoked in every simulation loop. Outputs requested to be either monitored, saved, or exchanged between subsystems are sent to S-function block output. The outputs are calculated in `mdlUpdate` function discussed below.

Function `static void mdlUpdate(SimStruct *S, int_T tid)`

This is the main function implementing all the transient stability numerical calculations. Invoked in each time step, the function first reads from the S-function block input the external variables that need to be updated for the current time-step. Then, in case that the topology of the network has changed, the appropriate admittance matrix will be loaded. Hereafter, the right hand side of flowchart shown in Figure 3.4 is run. Finally, this function calculates requested outputs and saves them to be used by the function `mdlOutputs` for S-function block output.

Function `static void mdlTerminate(SimStruct *S)`

In this function, memory blocks allocated for storing transient stability models are freed to ensure no memory leakage in the C program.

3.5.2 Off-Line Implementation of the IR Method

To implement the IR method, an S-function block that mathematically models power system transient stability computations is developed. This is a general block which can be specified by setting parameters to model a portion of large-scale power system. To model the complete system, several of these blocks can be simply placed and connected in SIMULINK environment. Each block represents a subsystem identified by the partitioning method such as slow coherency method or geographic partitioning. The required parameters of the current version of the developed S-function block are: the numbers of machines included in the system and in the individual subsystem, the total number of areas, the ID numbers of internal and external machines. For instance, suppose a power system is decomposed into three areas each is represented by an S-function block. Figure 3.6 shows the top lay-out of a three-area system called `Area1`, `Area2`, and `Area3`. Inside the subsystem `Area1` is depicted in Figure 3.7, where the S-function block is identified by `area1`. This S-function block involves three input ports, `U`, `I`, and `IDs`, and nine output ports. The output ports are of two types: the *measuring* outputs, which are directly sent to the `Monitoring` block in Figure 3.6, to be saved or seen on scopes, as illustrated in Figure 3.8, and the *data* outputs, which are sent to the external subsystems, i.e. `Area2` and `Area3`, to update them by the last computations of `Area1`.

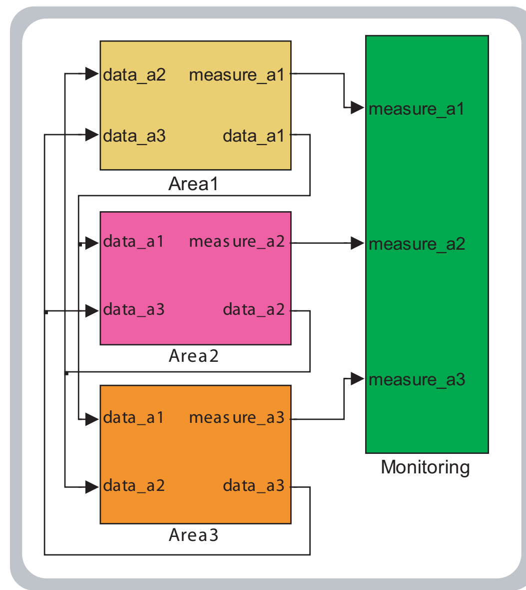


Figure 3.6: Top lay-out of a three-area decomposed system using developed S-function blocks in SIMULINK for **off-line** simulation.

3.5.3 Real-time Implementation of the IR Method

The real-time simulation software package provided by OPAL-RT came in the form of three MATLAB/SIMULINK based toolboxes:

- ARTEMIS: This toolbox provides additional discretization methods, improved solving algorithms, and a pre-computation mechanism to boost the overall performance of the SimPowerSystem (SPS) blockset that was built into MATLAB/SIMULINK.
- RT-EVENTS: This toolbox targets specifically the power electronics modeling, operation, compensation, and optimization.
- RT-LAB: RT-LAB contains the modules that are required for the pre-compilation of the source code, RT hardware and software interface, and simulation result acquisition.

The OPAL-RT software package in the RTX-LAB at the University of Alberta requires two operating systems: Windows XP on the host computers, and RT Linux on the target nodes. Power system models could be developed off-line in the SIMULINK graphical environment on the host. In case those models are not available in SIMULINK, user defined S-function block can also be incorporated into the models (as explained in the previous

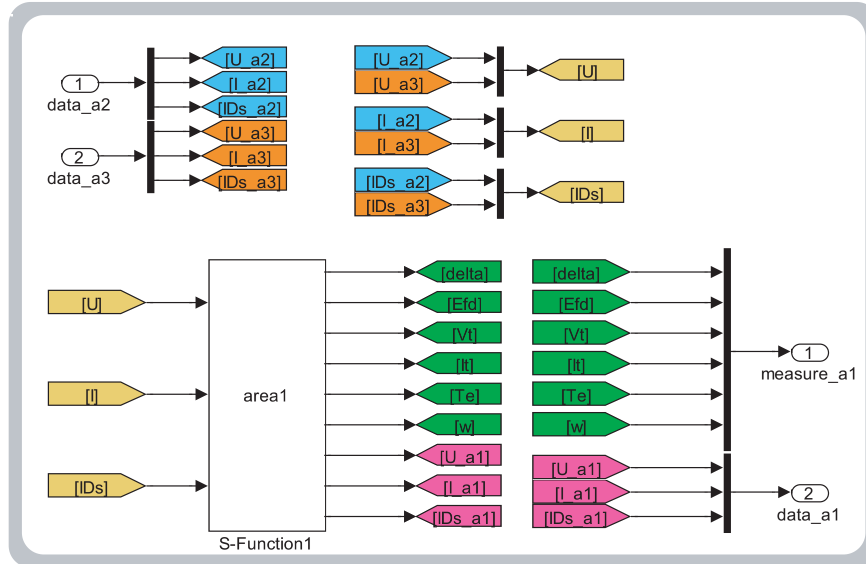


Figure 3.7: Placing S-function in subsystem `Area1` for **off-line** simulation.

section). When the model optimizations are completed and verified with off-line simulations, the RT-LAB utility is used to transfer the pre-compiled source code into the RT Linux operating system, which resides on the nodes of the simulator. The execution of the loaded model could be directly controlled through the Windows based utility without dealing with the Linux OS.

The target runs on a Linux based real-time operating system which offers optimized performance through a single programming environment and direct control of all system operations by using single kernel design. It offers eXtra High Performance (XHP) mode operation through CPU shielding where one CPU is dedicated for the simulation while the other CPU is load with RT Linux operation system to manage data feeding to the shared memory and to interact with the peripheral hardware. The target is responsible for real-time execution of the model, data transfer between nodes and the host, and data communication with external hardware through I/Os. The target is also required to compile source code generated by MATLAB/SIMULINK Real-Time Workshop (RTW) to executables. The hosts are installed with the RT-LAB software provided by Opal-RT Technologies Inc. to coordinate all hardware engaged for the simulation. The hosts are mainly used to create, edit and verify models in SIMULINK, compile SIMULINK blocks into C code by RTW, control and configure real-time simulations in targets, manipulate model parameters in real time as well as acquire real-time simulation results.

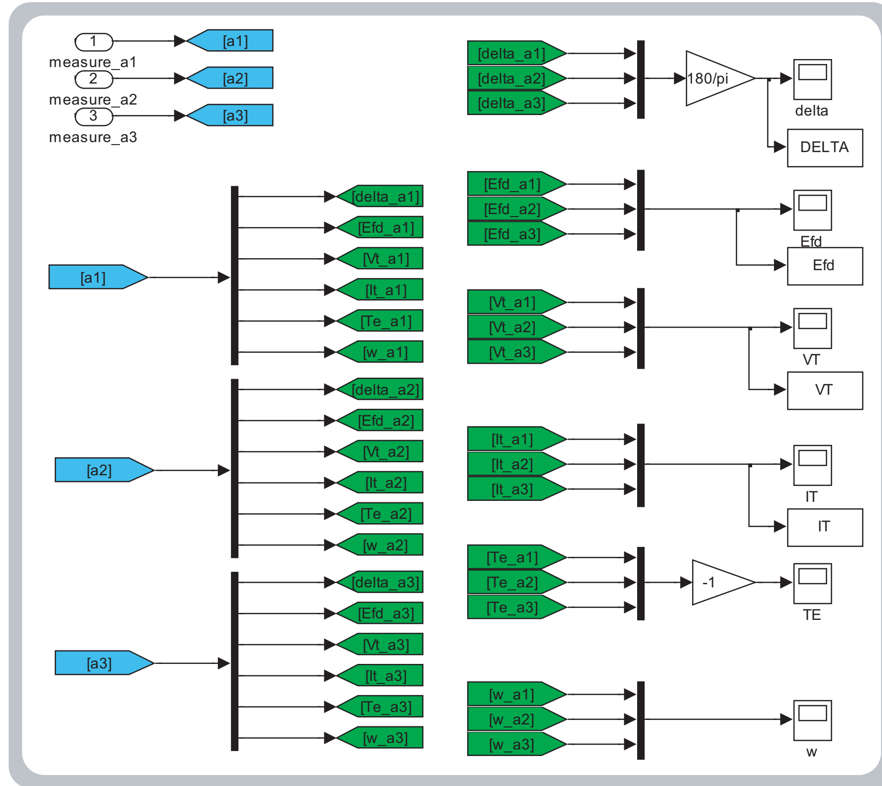


Figure 3.8: Monitoring and saving outputs of the decomposed system.

A view of the real-time simulator target nodes and the host workstation existing in the RTX-LAB at the University of Alberta is illustrated in Figure 3.9. Each cluster node consists of a dual Intel $Xeon^{TM}$ shared-memory PC running at $3.0GHz$ on a real-time Linux operating system. The inter-node communication is through InfiniBand with a $10Gb/s$ data transferring rate. As shown in this picture, a model, which in our case is a power system, can be partitioned and distributed among the cluster nodes. In the case of using multiple nodes, one node is the Master and other nodes operates as Slaves. The name of the master subsystem, in the top lay-out of the SIMULINK model, must be prefixed with 'SM', and the name of the slave subsystems must be prefixed with 'SS'. Moreover, the monitoring block, where the required outputs are being saved and scopes are placed, must be prefixed by 'SC', as shown in Figure 3.10.

Before compiling the SIMULINK model into C-code, the **OpComm** block from RT-LAB/OPAL-RT must be added into the subsystems. OpComm is a communication block that must be used in subsystems receiving signals from other subsystems, all input ports must go through this communication block being connected, as shown in Figure 3.11.

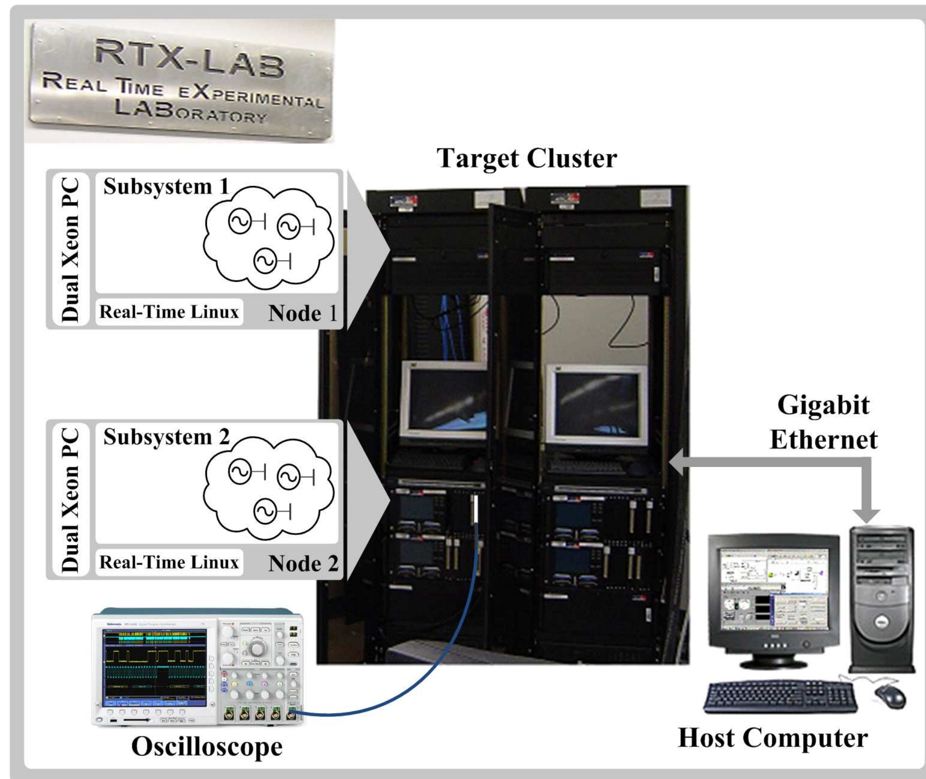


Figure 3.9: Configuration of the PC-cluster based real-time simulator in the RTX-LAB at the University of Alberta.

3.6 Experimental Results

In this section we will demonstrate results to verify the efficiency of the IR method for real-time simulation. To do so we have chosen three case studies. One is the Kundur's 4 machine and 11 bus system found in [9]. The other case study is the IEEE 39 bus New England test system [80]. There is also a large-scale power system made case study to fully occupy the computing capacity of the existing real-time simulator. The real-time results for these case studies have been validated using the PSS/E software program.

To incorporate IR method S-function blocks into RTX-LAB simulator, all the required `*.c` and `*.h` files, as well as the data files must be transferred to targets in RT-LAB for real-time simulation. Together with existing C code generated by RTW, all the `*.c` codes are compiled by GCC/G++ in targets to generate real-time executables. The complete C source code of S-function program is demonstrated in Appendix A. To compile the source code into the MEX-function (executable for MATLAB with extension `*.dll` in Microsoft Windows platform), the following command is used for all the S-function codes in MATLAB:

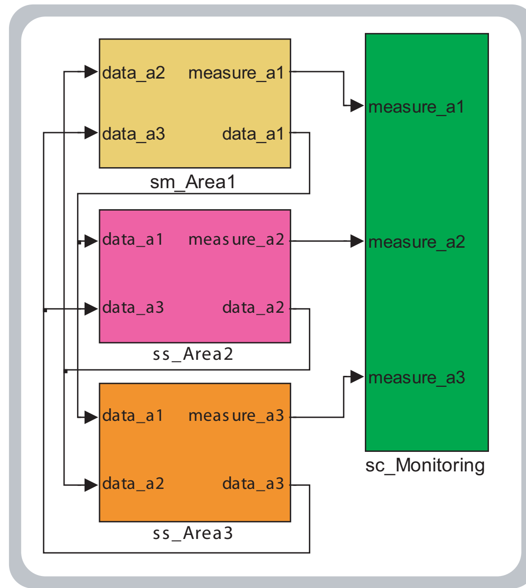


Figure 3.10: Top lay-out of a three-area decomposed system using developed S-function blocks in SIMULINK for **real-time** simulation.

```
>> mex -g area1.c
```

or

```
>> mex area1.c
```

The `-g` option is used to include debugging information in the MEX-function. More information on debugging SIMULINK S-function is available in [79].

3.6.1 Case Study 1

Figure 3.12 illustrates the test system used as the first case study. Each synchronous generator is equipped by an exciter and PSS. A set of 6 differential equations model mechanical rotation, field winding, and 3 damper windings of each synchronous generator as given in Chapter 2. The complete system can be described by 36 non-linear differential and 8 algebraic equations. Since generators $\{1, 2\}$ and $\{3, 4\}$ are coherent, the system can be partitioned into two subsystems. This coherency relation can also be observed later in the simulation results. These two subsystems are distributed across two cluster nodes as seen in Figure 3.13. The simulation time-step is chosen to be $1ms$. Using the IR method, once the steady-state has been reached, a three-phase fault at Bus 8 is imposed at $t = 5s$ and is cleared in $80ms$. The real-time simulation results are recorded on an external oscilloscope

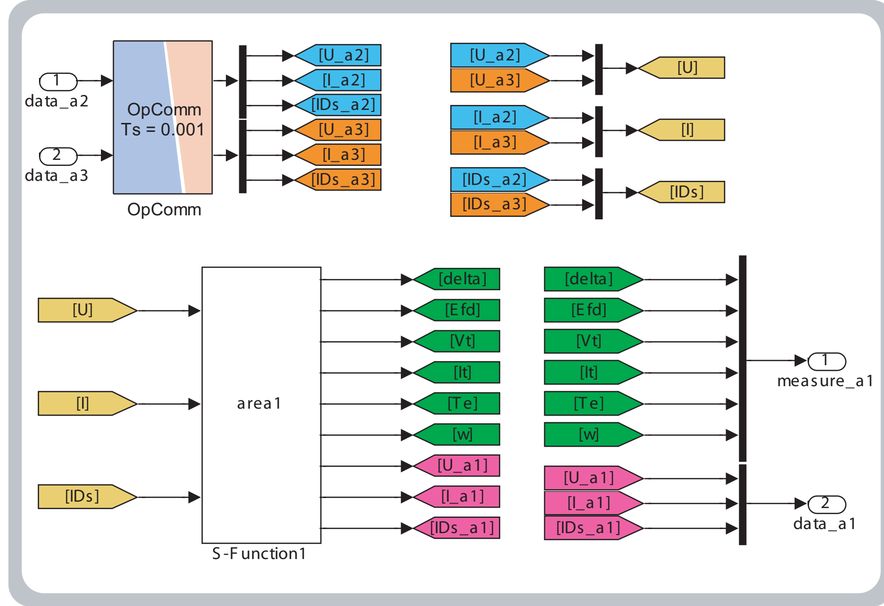


Figure 3.11: Placing S-function in subsystem `Area1` for **real-time** implementation.

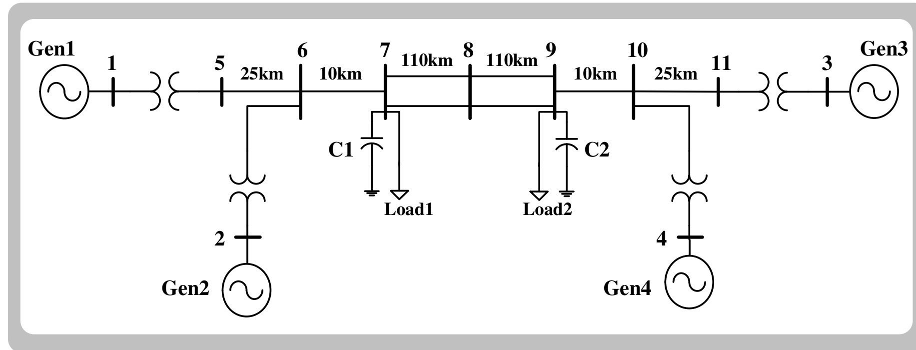


Figure 3.12: One-line diagram for Case Study 1.

and saved. The relative machine angles are shown in Figure 3.14 in which $Gen4$'s angle is selected as the reference. The real-time results are superimposed on the results found from PSS/E. As can be seen the IR method is completely stable during the steady-state of the system, i.e. $t < 5sec$. During the transient state and also after the fault is cleared, the real-time results closely follow the results from PSS/E. The maximum discrepancy between real-time simulation and PSS/E was found to be 0.93%, based on (3.5):

$$\varepsilon_\delta = \frac{max|\delta_{PSS/E} - \delta_{IR}|}{\delta_{PSS/E}}. \tag{3.5}$$

where $\delta_{PSS/E}$ and δ_{IR} were defined as the relative machine angles from PSS/E and IR method respectively. It can be further observed from Figure 3.14 that following the fault,

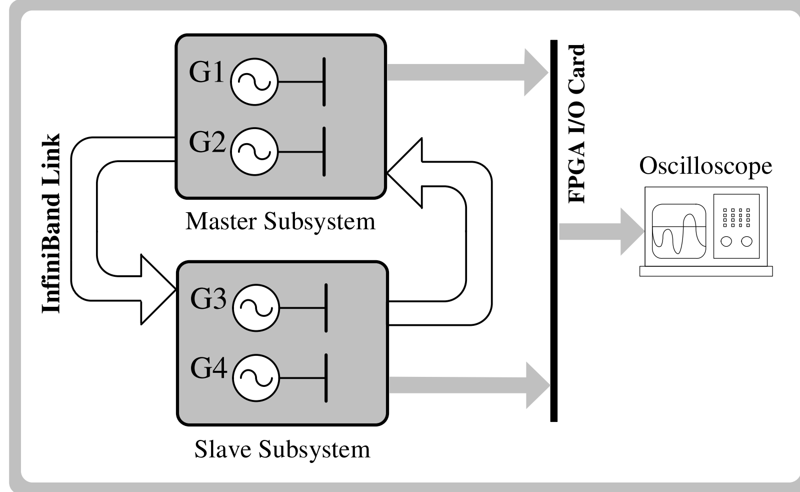


Figure 3.13: Distribution of subsystems of Case Study 1 among cluster nodes of the real-time simulator.

the oscillations of $Gen4$ are closer to the oscillations of $Gen3$ rather than to those of $Gen1$ or $Gen2$. If the reference generator is switched to $Gen1$ or $Gen2$ instead of $Gen4$, it was found that the oscillations of $Gen1$ are closer to the oscillations of $Gen2$ rather than to those of $Gen3$ or $Gen4$. This observation practically demonstrates the coherency relation existing in this system.

To investigate the effect of the fault location and the partitioning scheme on the performance of the IR method, the following scenario was simulated. Suppose that the fault happens at Bus 5. Two different patterns of partitioning have been applied for this case. The first is based on the coherency property of the system, i.e. $\{1, 2\}$ and $\{3, 4\}$. The second pattern is based on the fact that since $G1$ is the closest generator to the fault location it will accelerate faster than other generators in the system; therefore, the system is divided into two subsystems: $\{1\}$ and $\{2, 3, 4\}$. These two patterns have been simulated in real-time using the IR method, and then the results were compared with those of PSS/E. Results of both patterns are close to those from PSS/E, but the maximum error in the second pattern is larger than the maximum error when coherency-based partitioning was used. Several other combinations of fault location and partitioning patterns have also been examined in this system, and it was concluded that slow coherency partitioning based IR method are the closest to the PSS/E's results.

Table 3.1 shows the timing performance of Master and Slave nodes of the real-time simulator running under the XHP execution mode during one time-step ($1ms$). This table shows that the tasks of computation and communication are done in less than $60\mu s$. These

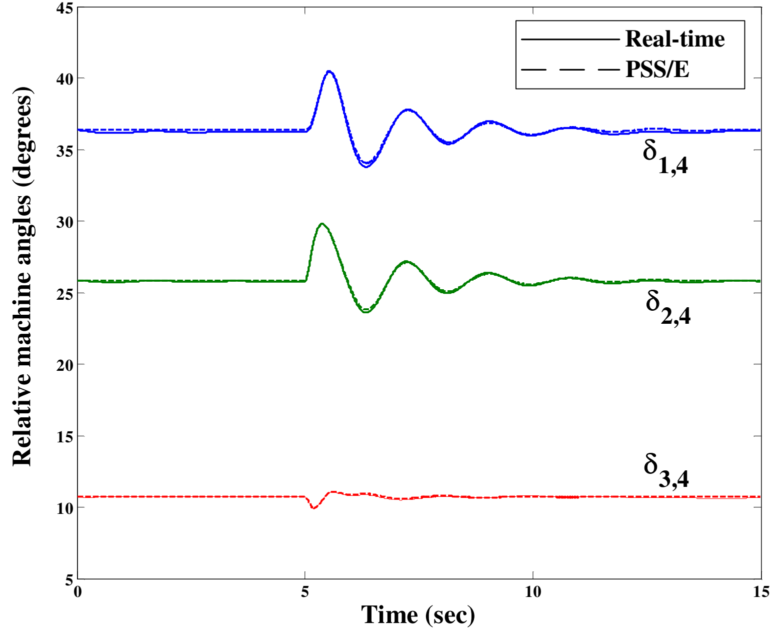


Figure 3.14: Comparison of relative machine angles collected from real-time simulator and PSS/E simulation for Case Study 1: $\delta_{i,4} = \delta_i - \delta_4$; $i = 1, 2, 3$.

Table 3.1: Performance log for real-time simulation of Case Study 1

	Duration (μs)	
Task	Master	Slave
Computation	47.91	47.20
Communication	11.55	8.12
Idle Time	939.45	940.07
Other	1.09	4.61
Total Step Size	1000	1000

execution times were sampled across many time-steps, and it was found that the idle times of the processors were uniform throughout those time-steps.

3.6.2 Case Study 2

The one-line diagram of IEEE's New England test system is shown in Figure 3.15. As in the previous case study, all generator models are detailed and equipped with AVR and PSS. The system data in PSS/E format is given in Appendix E. Using the partitioning pattern mentioned in [80], the system has been divided into 3 subsystems: $\{1, 8, 9\}$, $\{2, 3, 4, 5, 6, 7\}$, and $\{10\}$. These 3 subsystems were distributed on three cluster nodes of the real-time simulator: one Master and two Slaves, as illustrated in Figure 3.16. A question which may arise here is about the uneven loading of CPUs. Although it is possible to add $\{10\}$ to sub-

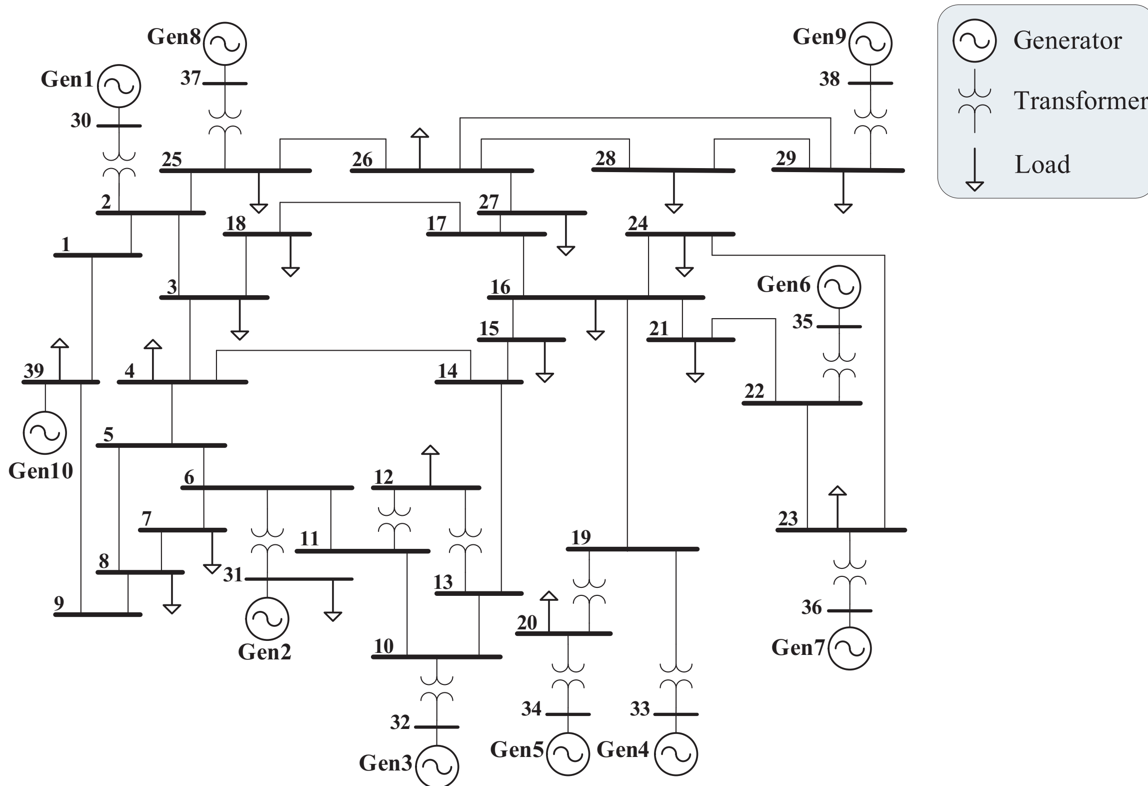


Figure 3.15: One-line diagram for Case Study 2.

system 1 and to use only 2 computation nodes, our intent in this study was to demonstrate the implementation of IR in three parallel cluster nodes. Several fault locations have been tested and the results were compared with those of PSS/E; in all cases results from the IR method match very well. In this section a sample of these results are presented. A three-phase fault happens at Bus 21, at $t = 1s$ and it is cleared after $100ms$. $Gen10$ is the reference generator and the relative machine angles are shown in Figure 3.17 and Figure 3.18. The maximum deviation of IR real-time simulation result from the PSS/E result based on (3.5) is 1.51%.

Table 3.2 shows the timing performance of Master and two Slave nodes in the real-time simulation during one time-step ($1ms$). Again the sampled idle times were found to be uniform across several time-steps. In the PC-cluster architecture the Master node is responsible for communicating with the host computer and also for organizing the communication among the Slaves. This explains why the Master's communication time in Tables 3.1 and 3.2 is larger than Slaves' communication time. Moreover, it can be seen that the computation time in both case studies is not very high, since the computation load is distributed equally among all time-steps. From the idle time duration in both Tables it is

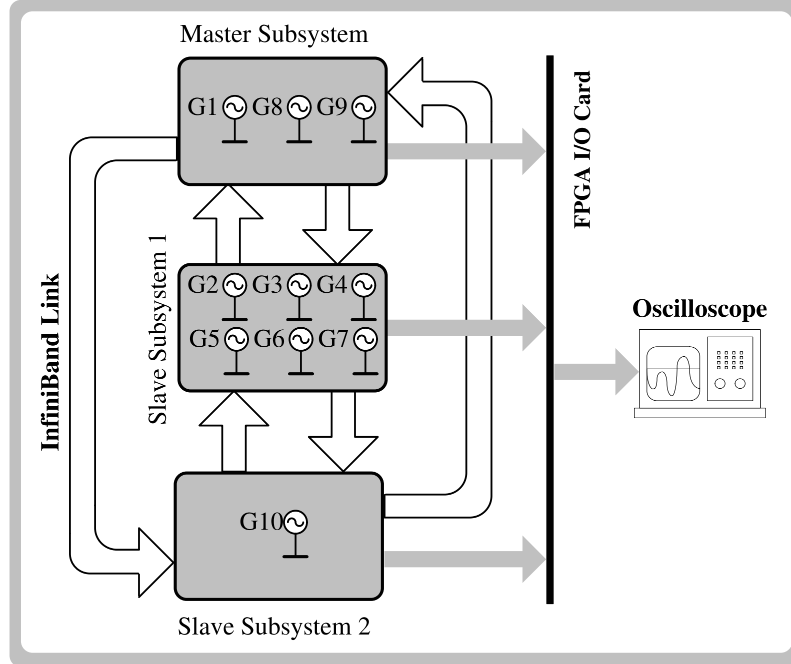


Figure 3.16: Distribution of subsystems of the Case Study 2 among cluster nodes of the real-time simulator.

Table 3.2: Performance log for real-time simulation of Case Study 2

	Duration (μs)		
Task	Master	Slave1	Slave2
Computation	212.77	348.13	17.27
Communication	13.44	7.30	4.51
Idle Time	770.93	631.73	974.12
Other	2.86	12.84	4.1
Total Step Size	1000	1000	1000

concluded that larger subsystems can be implemented on each node, and that faster than real-time simulation is also possible.

The accuracy of the IR method is analyzed by varying the time-step and calculating the error in (4.24). The results are presented in Table 3.3. As expected when the time-step increases the computation error increases as well. Nevertheless, it can be predicted that with larger time-steps larger systems can be simulated on this hardware using the IR method. It can however be seen that the maximum error depends not only on the time-step but also on the size of the system. For instance, the time-step of $5ms$ results in the maximum error of 1.32% and 3.29% in the 4 and 10 generator systems, respectively.

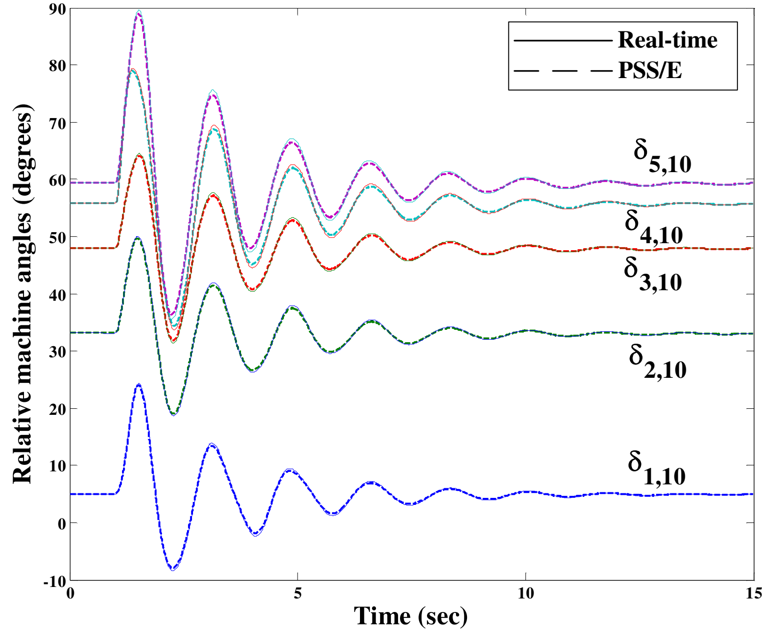


Figure 3.17: Comparison of relative machine angles collected from real-time simulator and PSS/E simulation for Case Study 2: $\delta_{i,10} = \delta_i - \delta_{10}$; $i = 1 \dots 5$.

Table 3.3: Relation between time-step and accuracy of the IR method

	Maximum error $\varepsilon_\delta\%$	
Time-Step (ms)	Case study 1	Case study 2
1	0.93	1.51
2	1.04	1.70
5	1.32	3.29
10	1.88	4.2

3.6.3 Case Study 3: Large-Scale System

In the previous two case studies it was concluded that larger subsystems can be implemented on each node of the real-time simulator. By performing several tests with various sizes of power systems it was realized that each target node of the existing simulator can be filled with 10 generators all modeled in detailed while still running in real-time with a time-step of 4ms. As we have 8 target nodes available, the largest system that can be modeled in real-time is an 80 generators system. This system was made by expanding the IEEE test system. For this purpose, the IEEE 39 bus system was duplicated 8 times in the PSS/E software environment, and then interconnected by transmission lines to build a large-scale network. The result is a 312 bus and 80 generator power system. The steady-state and dynamic stability of this system has been examined and verified in PSS/E. This

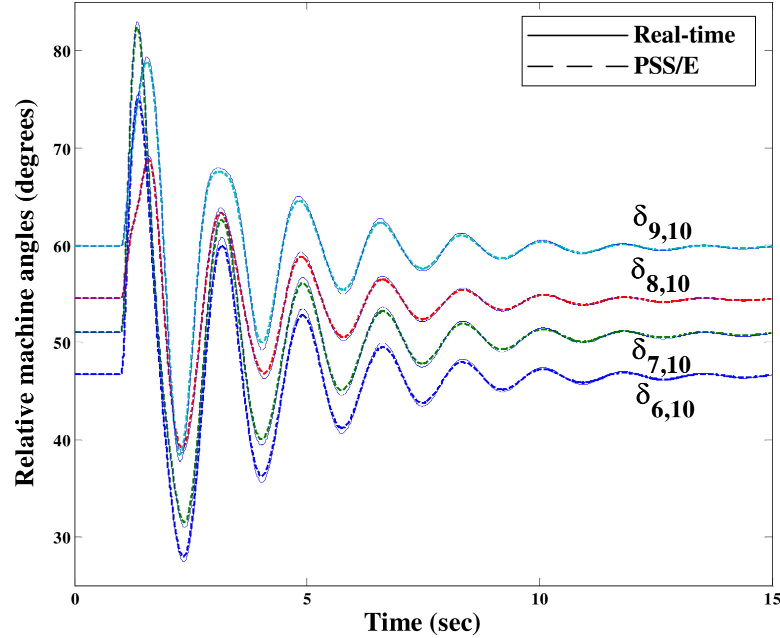


Figure 3.18: Comparison of relative machine angles collected from real-time simulator and PSS/E simulation for Case Study 2: $\delta_{i,10} = \delta_i - \delta_{10}$; $i = 6 \ldots 9$.

system was partitioned and distributed in all 8 target nodes of the simulator running in parallel and in XHP mode. Figure 3.19 illustrates the top lay-out of this system in the SIMULINK environment prepared to be run on the real-time simulator. As explained in the previous section one target node must be the master while others are slave nodes (i.e. 7 nodes in this case). The performance log of this simulation is shown in the Appendix B for randomly selected five successive time steps. These results are directly reported by `MonitoringViewer` feature of the RT-LAB software that shows in detailed the timing of each operation in each target node during the simulation period. For each target node, this report includes an item called `Number of Overruns` which counts the number of time steps that for any reason the simulation could not satisfy the real-time requirements and lasted more than the fixed time-step. In Appendix B the report includes the timing of just two nodes: master (`sm_area1`), and one of the slaves (`ss_area2`). The report declares that the number of overruns for all nodes is 0, which means the simulation was performed successfully in real-time.

3.7 Summary

This chapter presented a parallel processing method known as instantaneous relaxation (IR) for the real-time transient stability simulation of large-scale power systems. Although

it is possible to utilize real-time simulators based on the electromagnetic transient simulation approach to perform transient stability analysis, the size and cost of the simulator is usually prohibitive especially for simulating large-scale systems. The motivation behind this work is to test the real-time feasibility of a fully parallel method that could alleviate these limitations. The waveform relaxation (WR) method was investigated in this chapter for implementation in real-time. However, it was found that WR method has some restrictions for real-time simulation due to the following reasons:

- The WR method provides a set of values in the form of a complete waveform. However, real-time simulation especially hardware-in-the-loop simulation requires instantaneous values of variables.
- Implementation of the WR causes uneven computation loads among the time-steps. This results in execution time overrun in some time-steps and excessive idling time in the others. An overrun, which describes a situation when the simulator requires a larger time-step than the specified fixed time-step to finish its task, is not acceptable in hard real-time systems.

These problems are overcome by the proposed IR method. It inherits all the advantages of the WR method but is also efficient for real-time implementation. The two main differences between the IR and WR methods are:

- In the IR method the instantaneous values of the variables are being used and not their waveforms.
- To achieve the required accuracy several iterations of the Newton-Raphson within each time-step are performed.

To demonstrate the performance of the IR method, three case studies have been implemented on a PC-Cluster based real-time simulator and the results are validated by the PSS/E software. Several comparisons verified the accuracy and efficiency of the IR method. In addition, the performance of the slow coherency method as the partitioning tool was analyzed, and it was concluded that for different fault locations in the system results derived from this method had lower amounts of error.

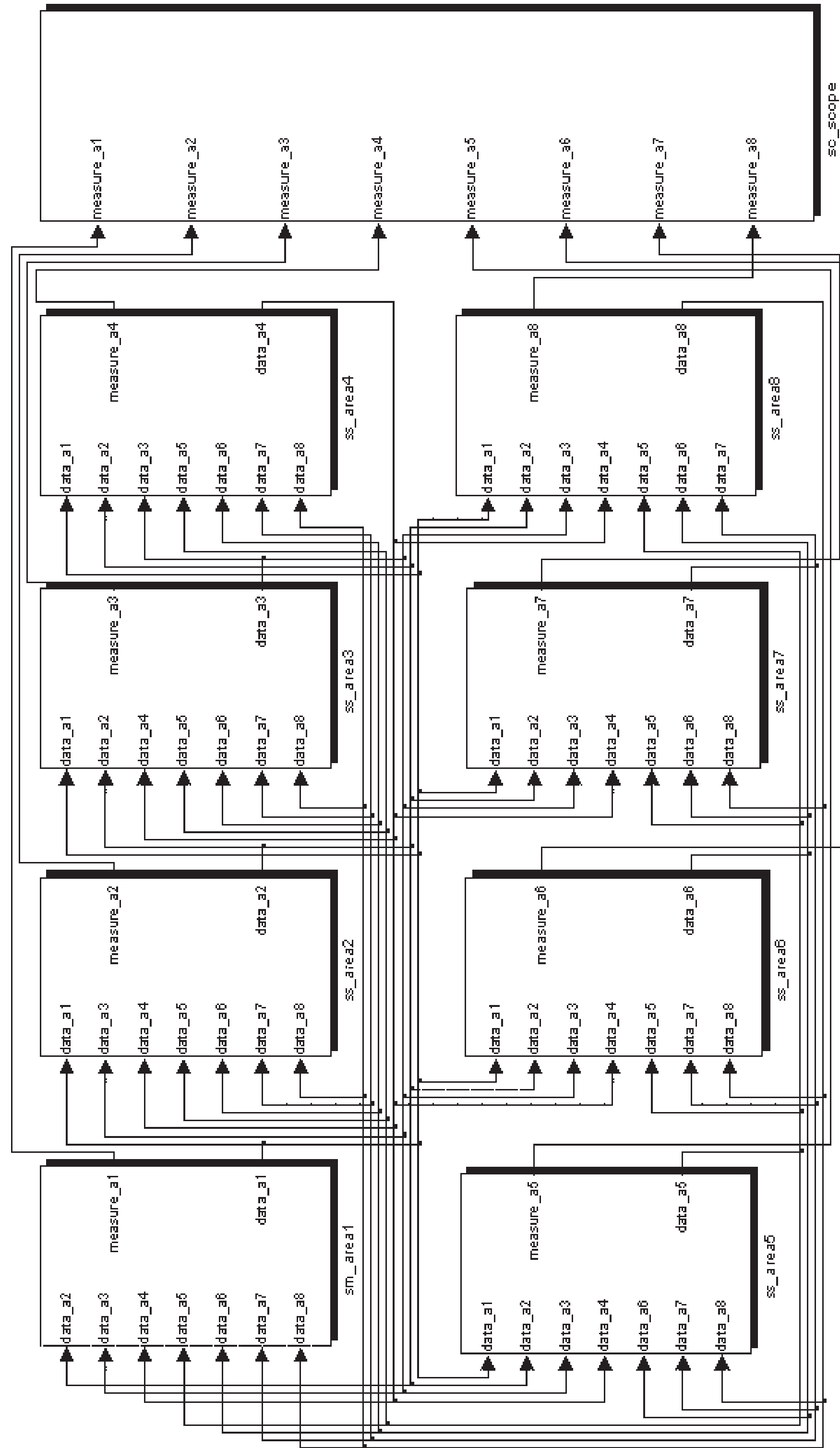


Figure 3.19: Implementation of a large-scale power system for parallel real-time simulation using the IR method.

Part II

Large-Scale Transient Stability Simulation on GPU-based Hardware

4

Single GPU Implementation: Data-Parallel Techniques

4.1 Introduction

Part I of this thesis discussed the real-time simulation of transient stability. The Instantaneous Relaxation method was proposed and successfully implemented on the real-time simulator. It was mentioned that since decades ago several approaches have been developed to perform transient stability simulation faster for large-scale systems on parallel computing hardware [49]. We categorized these methods into two major groups: tearing and relaxation [81]. The commonality of these methods is *task-parallelism* which executes the component subsystems on parallel and distributed hardware composed of clusters of CPUs. Nevertheless, there are two limitations in these methods that contribute to the speed bottleneck: (1) data-sequential computation on the CPUs of the cluster, and (2) limited data bandwidth, and latency of the inter-processor communication channels of the cluster.

Recently, the Graphics Processing Unit (GPU), as introduced in Chapter 2, has revealed the potential to revolutionize state-of-the-art research in *data-parallel* computing. A data-parallel application consists of large streams of data elements in the form of matrices and vectors that have identical computation codes (kernels) applied to them. The data-parallel characteristic of the GPU gives it a single-instruction-multiple-data (SIMD) architecture.

In Part II of this thesis, i.e. this chapter[1] and the next one, we will discuss GPU-based

[1]Material from this chapter has been published: V. Jalili-Marandi, V. Dinavahi, "SIMD-based large-scale transient stability simulation on the graphics processing unit," *IEEE Trans. on Power Systems*, pp. 1-10, 2010.

transient stability simulation for large-scale power systems. The motivation for this work is twofold: the mathematical complexity along with the large data-crunching need in this simulation, and the substantial opportunity to exploit parallelism. Both these characteristics are uniquely suited to the GPU. However, since the GPU's architecture is markedly different from that of a conventional CPU, it requires a completely different algorithmic approach for implementation. The GPU thrives on applications that have a large computational requirement, and where data-parallelism can be exploited. Therefore any computation that is desired to be implemented on the GPU must be in the SIMD format, otherwise the GPU cannot deliver its computational benefits. Here we propose SIMD-based programming models to exploit the GPU's resources for transient stability simulation.

4.2 GPU Overview

As GPU and its applications are new topics in the Power and Energy Society (PES), before going further into GPU's application for transient stability simulation, it is important to first become familiar with GPU's architecture and programming paradigm.

4.2.1 GPU Evolution

The earliest ancestors of the dedicated graphics processors were originally designed to accelerate the visualization tasks in the research labs and flight simulators. Later they found their way to commercial workstations, personal computers, and entertainment consols [82]. By the end of 2000s, the PC add-in graphics cards were developed as fixed-function accelerators for graphical processing operations (such as geometry processing, rasterization, fragment processing, and frame buffer processing). Around this time, the term of "graphics processing unit" (GPU) arose to refer to this hardware used for graphics acceleration [83].

In the early 2000's the GPU was a fixed-function accelerator originally developed to meet the needs for fast graphics in the video game and animation industries [82]. The demand to render more realistic and stylized images in these applications increased with time. The existing obstacle in the fixed-function GPU was the lack of generality to express complicated graphical operations such as shading and lighting that are imperative for producing high quality visualizations. The answer to this problem was to replace the fixed-function operations with user-specified functions. Developers, therefore, focused on improving both the application programming interface (API) as well as the GPU hardware.

The result of this evolution is a powerful programmable processor with enormous arithmetic capability which could be exploited not only for graphics applications but also for general purpose computing (GPGPU). Taking advantage of the GPUs' massively parallel architecture, the GPGPU applications quickly mushroomed to include intensive computations such as those in molecular biology, image and video processing, n-body simulations, large-scale database management, and financial services [83].

The especial architecture of the GPU, explained in this section, made it a successful accelerator/processor in particular applications in which a large amount of computations is required to be performed on a data-parallel structure of input elements, the same as graphics applications. A data-parallel application consists of large streams of data elements in the form of matrices and vectors that identical computation codes (kernels) are applied to them. Moreover, the data communication required to compute the output streams is small. Through the GPUs evolution, as their main duty was for different application demands than the CPU, the architecture of the GPU has progressed in a different direction than that of the CPU. A life-like rendering of images requires billions of pixels per second and each pixel requires hundreds or more operations. Therefore, the fundamental design of the GPU is in such a way to deliver an enormous amount of computations for its best efficiency in a massive data-parallel application. Later it will be observed that the size of computation is a key-point to achieve maximum efficiency in a GPGPU application.

4.2.2 GPU Hardware Architecture

To efficiently use and program a GPU it is instructive to learn the GPU's internal architecture. In this work we have used NVIDIA's GeForce GTX 200 series. The following section briefly explains the architecture and hardware specifications that we require for programming purposes.

Figure 4.1 illustrates the architecture of the GPU [84] plugged into the motherboard of a 2.5GHz quad-core AMD Phenom CPU supported by 4GB of RAM. The GPU accesses the main memory of the system, i.e. RAM, via the PCIe 2.0 bus (Peripheral Component Interconnect Express). This version of GPU has 16 links in its PCIe interface that each link has a bandwidth capability f 0.5GB/s in each direction simultaneously. Thus, the PCIe 2.0 bus shown in Figure 4.1 supports up to 8GB/s transfer rate. This rate between the CPU and RAM is typically in the 12GB/s range. The GPU runs its own specified instructions independently but it is controlled by the CPU. The computing element in the GPU is called a *thread*. When a GPU instruction is invoked, blocks of threads (with the maximum size of

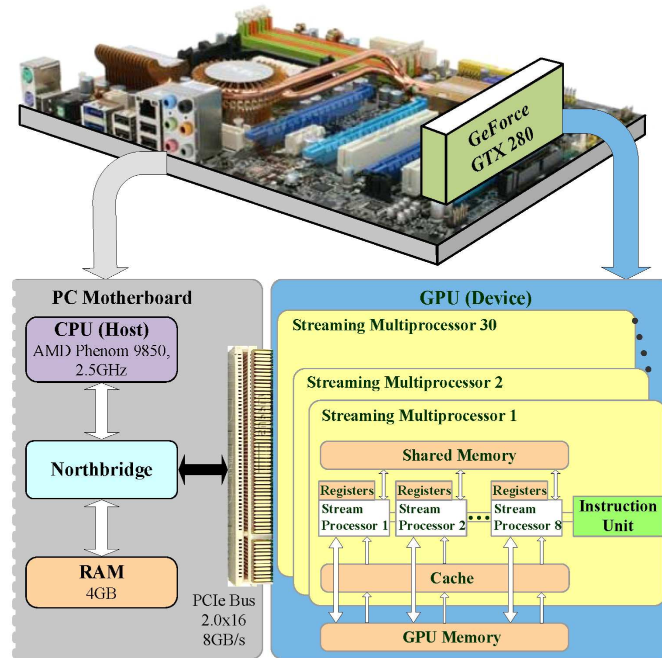


Figure 4.1: Hardware architecture of GPU mounted on the PC motherboard.

512 threads per block) are defined to assign one thread to each data element. All threads in one block run the same instruction on one streaming multiprocessor (SM).

Each SM includes 8 stream processor (SP) cores, an instruction unit, and on-chip memory that comes in three types: registers, shared memory, and cache. Threads in each block have access to the shared memory in the SM, as well as to a global memory in the GPU. Unlike a CPU, the architecture of the GPU is developed in such a way that more transistors are devoted to data processing rather than data caching and flow control. When a SM is assigned to execute one or more thread blocks, the instruction unit splits and creates groups of parallel threads called *warps*. The threads in one warp are managed and processed concurrently on the eight stream processors.

The target GPU specifications in this study are given in Tables 4.1. Threads are assigned by the thread scheduler which talks directly to each SM through a dedicated instruction unit which in turn assigns the tasks to the eight stream processors. Depending on the GPU model, 2 or 3 SM's can be clustered to build a thread processing cluster (TPC). Moreover two special function units (SFUs) have been included in each SM to execute transcendental calculations (e.g. sin, cosine), attribute interpolation and for executing floating point instructions. Figure 4.2 illustrates these units schematically.

Table 4.1: GeForce GTX 280 GPU specifications

Number of SM's	30
Number of SP's	240
Device memory	1GB
Clock rate	1.3GHz
Warp size	32 threads
Active warps/SM	32
Bandwidth:	
Host-to-Device	1.35GB/s
Device-to-Host	1.62GB/s
Device-to-Device	114.00GB/s

4.2.3 GPU Programming

After becoming familiar with the GPU's architecture, in this section the GPU programming models are reviewed to understand the GPU's computational resources. Depending on the application, the GPU programming paradigms can be divided into two main categories: graphics related programming, and GPGPU programming. In both these categories the GPU follows the SIMD model.

Graphical functionality

From the graphics point of view the GPU has two types of programmable processors: *vertex* and *fragment* processors [85]. Vertex processors process streams of vertices (made up of positions, colors, and other attributes) which are the elements that build a polygonal geometric model. In computer graphics 3D objects are typically represented with triangular meshes. The vertex processors apply a vertex program (also called a vertex shader) to transform each vertex based on its position relative to the camera, and then each set of three vertices is used to compute a triangle from which streams of fragments are generated. A fragment contains all information, such as color and depth, needed to generate a shaded pixel in the final image. The fragment processors apply a fragment program (also called a pixel shader) to each fragment in the stream to compute the final color of each pixel. This functionality of GPUs is out of the scope of this research and we do not refer to that anymore.

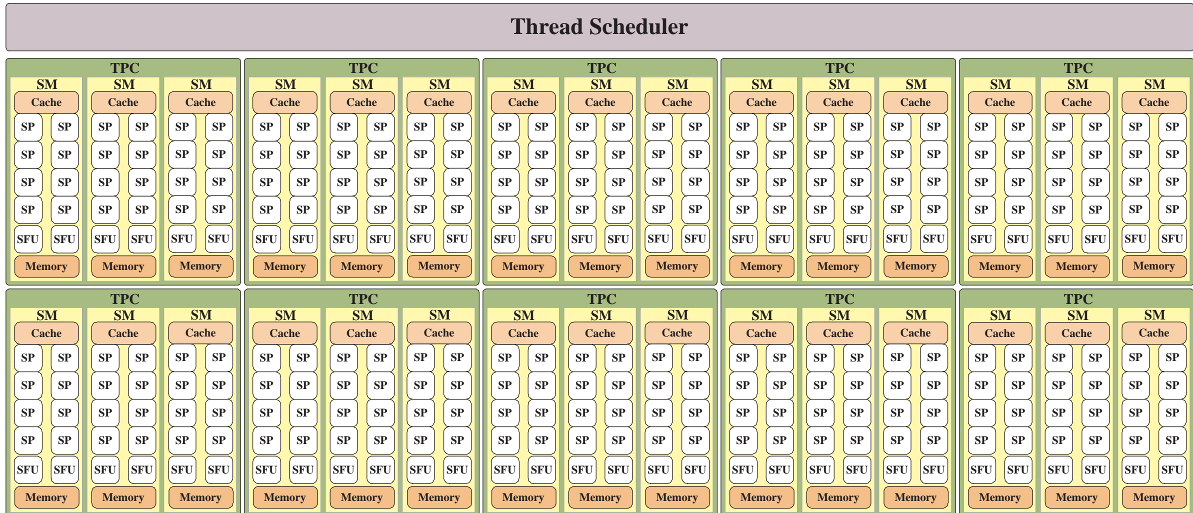


Figure 4.2: The GTX 280 hardware architecture: it consists of 10 TPCs, 3 SMs per TPC, and a total of 240 SPs; TPC: thread processing cluster, SM: streaming multiprocessor, SP: stream processor.

General purpose functionality

Early GPGPU programming directly used the existing graphics API's to express computations in terms of an image. In other words, the computational elements (i.e. vectors and matrices) were mapped onto the graphical elements (i.e. vertices or pixels) which are the components of shading programs. Shading languages used for this purpose include Cg, HLSL, and OpenGL. Although this technique has been successfully used in many research areas, including power system load flow computations [86], it was largely unapproachable by the common programmer. Firstly, because there is a mismatch between traditional programming and the graphics API's, and secondly, because it is difficult to debug or modify these programs.

High level programming languages

Along with the evolution of the programmable GPU's hardware architecture, the developers designed higher level languages that were specifically for general computation purposes. Dedicated GPU programming tools that bypassed the graphics functions of the GPU were created for general purpose computing, starting with Brook and Sh, and ultimately leading to commercial tools such as AMD's HAL and NVIDIA's CUDA.

CUDA (Compute Unified Device Architecture) provides a C-like syntax to execute and manage computations on the GPU as a data-parallel computing device. A CUDA program consists of multiple phases that are executed on either the CPU (*host*) or the GPU (*device*).

The phases of the program that exhibit little or no data-parallelism are run in the host-code after compiling with the host's standard C compiler, whereas the phases that exhibit fine-grained parallelism are implemented in the device-code in the form of *kernels*, the synonym for GPU functions. Host-code uses a CUDA-specific function-call syntax to invoke the kernel code. Calling a kernel distributes the tasks among the available multiprocessors to be simultaneously run on a large number of parallel threads. The programmer organizes these threads into a grid of thread blocks. Thread creation, scheduling, and resource management are performed in hardware. Each thread of a CUDA program is mapped to a physical thread resident in the GPU, and each running thread block is physically resident on a SM.

Moreover, a library of the basic linear algebra subprograms (BLAS) is provided that allows the integration with C++ code. By using this library, called CUBLAS, portions of a sequential C++ program can be executed in SIMD-form on the GPU, as shown in Figure 4.3, while other parts of the code are executed sequentially on the CPU [87]. Wherever a kernel is invoked a grid consisting of several blocks with equal numbers of threads/block is created. Each block within a grid, and each thread within a block are identified by individual indices that make them accessible via the built-in variables in CUDA. Threads determine the task they must do and the data they will access by inspecting their own thread and block IDs. Therefore in an application with highly intensive computations, the onerous computation tasks can be offloaded to the GPU, and performed faster in parallel, whereas mundane tasks such as the flow control of the program, required initial calculations, or the updating and saving of the variables can be done by the CPU. This co-processing configures a hybrid GPU-CPU simulator.

4.3 Data-Parallel Computing

Here the fundamental difference in the computing model of a GPU and a CPU is discussed, which leads us to a new methodology to implement the transient stability simulation on the data-parallel architecture of GPU. From the programmer's perspective a GPU is a parallel machine, whereas the CPU is a serial machine. A CPU executes one instruction after the another, and each instruction does one thing, for instance, adding the contents of two memory locations. A GPU, on the other hand, can add many pairs of numbers at the same time.

Let us look at a simple example. Suppose we want to evaluate $z = x + y$, where x, y,

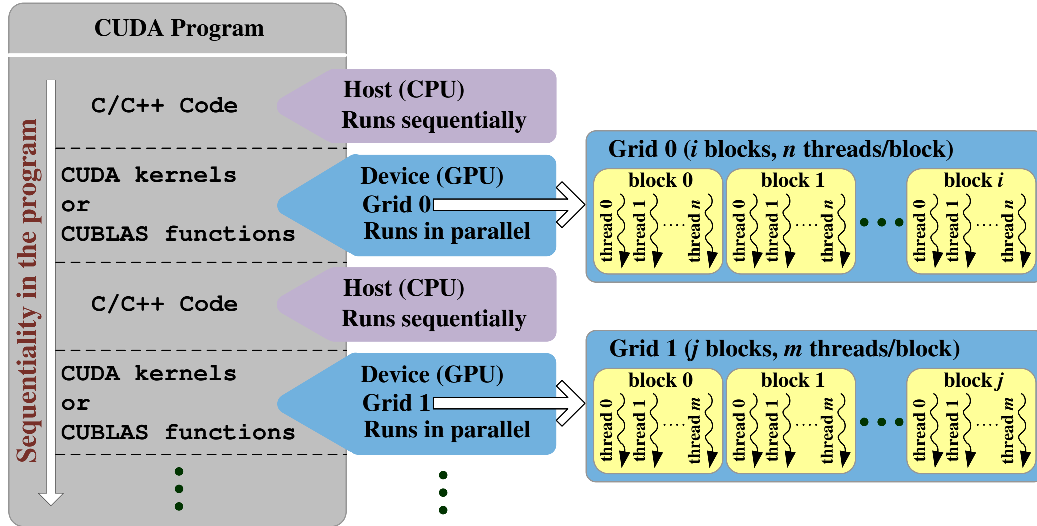


Figure 4.3: Cooperation of the host and device to execute a CUDA program, and the hierarchy of computing structure in a GPU.

and z are $n \times 1$ real vectors. On the CPU, a single for loop is typically used over all array elements as follows:

$for(i = 0; i < n; i++)$

$z[i] \leftarrow x[i] + y[i]$

In this model two levels of computation are active: outside the loop, the loop counter i is increasing and compared with the length of vectors n, whereas inside the loop, the arrays are accessed at a fixed position determined by the loop counter and the actual computation is performed (addition on each data element). The calculations performed on each data element in the vectors are independent of each other, i.e., for a given output position, distinct input memory locations are accessed and there are no data dependencies between elements in the result vector.

If we had a vector processor capable of performing operations on whole vectors of length n or even n CPUs, we would not need the for loop at all. This is the core idea of SIMD programming. The computation on the GPU is performed by separating the outer loop from the inner calculations. The inner loop calculations are extracted into a computational kernel as follows:

$if(index < n)$

$z[index] \leftarrow x[index] + y[index]$

where $index$ is the ID of the threads assigned to elements of the vectors. Note that the kernel is no longer a vector expression but a scalar template of the underlying math that

forms a single output value from a set of input values. For a single output element, there are no data dependencies with other output elements, and all dependencies to input elements can be described relatively. Whenever a kernel is called, the driver logic hardware schedules each data item into the different multiprocessors (SM)–this is not programmable. Although internally the computation is split up among the available SMs, one cannot control the order in which they are working. One can therefore assume that all work is done in parallel without any data interdependence.

Here is a simple example to show how a CUDA-based SIMD kernel looks like. Suppose we want to model a limiter to control the maximum and minimum amounts of a signal as shown in Figure 4.4. In the transient stability simulation of power systems these limiters are widely used in the model of excitation and PSS devices for synchronous generators, and depending on specifications of each machine the up and down limits are different. Suppose the output signals of the transfer function that we want to pass them through the limiter are saved in a vector, V_2. We also need two vectors whose elements are the maximum and minimum limits of the signals at each individual machine, i.e. v_{max} and v_{min}. The implementation of this model by using the *function* concepts in C++ running sequentially on the CPU is as follows:

```
void
limiterFunc(float* V_in, float* V_max, float* V_min,
            const int numElems)
{
       for (int i=0; i < numElems; i++)
       {
           if (V_in[i] >= V_max[i])
               V_in[i] = V_max[i];
           else if (V_in[i] < V_min[i])
               V_in[i] = V_min[i];
       }
}
```

where in this code `V_in` is the vector of input signals, i.e. V_2 in Figure 4.4, `V_max` and `V_min` are the maximum and minimum limits of each limiter, and `numElems` is the length of the input signal. The porting of this function in CUDA language running in SIMD format on the GPU is a kernel as below:

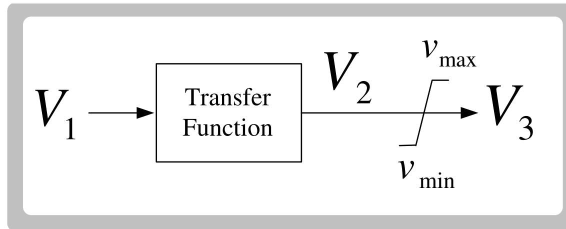


Figure 4.4: An schematic model for a limiter.

```
__global__ void
limiterKernel(float* DEVICE1, float* DEVICE2, float* DEVICE3,
           const int numElems)
{
       int idx = threadIdx.x + blockDim.x * blockIdx.x;
       if (idx < numElems)
       {
           if (DEVICE1[idx] >= DEVICE2[idx])
               DEVICE1[idx] = DEVICE2[idx];
           else if (DEVICE1[idx] < DEVICE3[idx])
               DEVICE1[idx] = DEVICE3[idx];
       }
}
```

In this kernel the `global` qualifier in the first line declares a function as being a kernel. Such a function is callable from the CPU and is executed on GPU. The up and down limits for each limiter are saved and transferred to the GPU in two vectors: `DEVICE2` and `DEVICE3`. The values of input signal, which also has been transferred to GPU, i.e. `DEVICE1`, is compared with these limits to meet the required conditions. We can see in this kernel there is not any for loop, and instead of that a new parameter, called `idx`, is defined to control the execution of the kernel. To invoke this kernel from a CPU-based code we need to add a syntax as below:

```
limiterKernel<<<dimGrid, dimBlock>>>(DEVICE1, DEVICE2, DEVICE3, N);
```

where the first parameter of the $<<< ... >>>$ specifies the dimension of the grid, and the second parameter defines the size of the thread block. The index of each block within the grid and its dimension are identified and accessible within the kernel through the built-in

variables, i.e. `blockIdx` and `bloclDim` respectively. Each thread inside the thread block is also identified by a built-in index called threadIdx. Therefore, each thread in the grid can be addressed by an index calculated based on these built-in variables. This is what the parameter `idx` does in the `limiterKernel` code. Comparing `idx` and the length of the vectors specifies the number of required active threads and controls running the code.

4.4 SIMD-Based Standard Transient Stability Simulation on the GPU

4.4.1 Standard Transient Stability Simulation

The general form of DAEs which describe the dynamics of a multi-machine power system discussed in Chapter 2 are repeating here for easy referencing:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{V}, t) \quad (4.1)$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{V}, t) \quad (4.2)$$

$$\mathbf{x}(t_0) = \mathbf{x_0} \quad (4.3)$$

where $\mathbf{x}$ is the vector of state variables, $\mathbf{x_0}$ is the initial values of state variables, and $\mathbf{V}$ is the vector of bus voltages. The standard approach to solve these nonlinear and coupled DAEs involves three steps [12]:

- **Step 1**. The continuous-time differential equations are first discretized and converted to discrete-time algebraic equations. Using an implicit numerical integration method, discretizing (4.1) results in a new set of non-linear algebraic equations:

  $$0 = \frac{h}{2}\left[\mathbf{f}(\mathbf{x}, \mathbf{V}, t) + \mathbf{f}(\mathbf{x}, \mathbf{V}, t-h)\right] - (\mathbf{x}(t) - \mathbf{x}(t-h)) \quad (4.4)$$

  where h is the integration time-step.

- **Step 2**. The existing non-linear algebraic equations are linearized by the Newton-Raphson method (for the jth iteration) as:

  $$J(\mathbf{z}_{j-1}) \cdot \Delta\mathbf{z} = -\mathbf{F}(\mathbf{z}_{j-1}) \quad (4.5)$$

  where J is the Jacobian matrix, $\mathbf{z} = [\mathbf{x}, \mathbf{V}]$, $\Delta\mathbf{z} = \mathbf{z}_j - \mathbf{z}_{j-1}$, and $\mathbf{F}$ is the vector of nonlinear function evaluations.

- **Step 3**: The resulting linear algebraic equations are solved to obtain the system state. (4.5) is solved using the LU factorization followed by the forward-backward substitution method.

Chapter 2 describes how to solve the network equations.

4.4.2 SIMD Formulation for Transient Stability Solution

The CPU and the GPU have radically different architectures, so they require different programming approaches. The generator and the network model shown in Chapter 2 are suitable for sequential computation on the CPU. To be able to perform these computations on the GPU, the equations must be expressed in the SIMD format, i.e. instead of using single-element values vectors or matrices of them must be used. This is straightforward for the generator equations and is accomplished by replacing all variables and parameters with the vectors whose elements relate to each generator. For instance, the rotor angle equation for the ith generator, as given in (2.1), is discretized by (4.4) as follows:

$$0 = \omega_R^i \frac{h}{2}\left[\Delta\omega^i(t) + \Delta\omega^i(t-h)\right] - (\delta^i(t) - \delta^i(t-h)). \tag{4.6}$$

Allocating vectors whose length are the number of existing generators, this equation can be expressed for SIMD computing as below:

$$0 = \omega_R^v \frac{h}{2}\left[\Delta\omega^v(t) + \Delta\omega^v(t-h)\right] - (\delta^v(t) - \delta^v(t-h)) \tag{4.7}$$

where the superscript v indicates the vector-format variable, and

$$\delta^v = \left[\delta^1,\ \delta^2, ..., \delta^i ..., \delta^n\right]^t \tag{4.8}$$

and

$$\Delta\omega^v = \left[\Delta\omega^1,\ \Delta\omega^2, ..., \Delta\omega^i ..., \Delta\omega^n\right]^t. \tag{4.9}$$

Similarly other differential equations described in Chapter 2, i.e. (2.1) to (2.3), can be implemented in the SIMD format.

For the network side, expressing the computations in SIMD format is more complicated and requires a revision of the equations. For the n generators in the system, the objective in the network side equations is to compute two vectors whose elements are the I_D and I_Q of each generator given by (2.16) and (2.17). In the sequential computing model, as shown by (2.18)-(2.20), I_D and I_Q are constructed based on the parameters S_1 to S_4, and A_1 to A_8.

Therefore, these parameters must first be computed in the SIMD format to get the I_D and I_Q vectors. We start with the allocation of six vectors on the GPU as follows:

$$U_k^v = \left[u_k^1,\, u_k^2, \ldots, u_k^i, \ldots, u_k^n\right]^t; \;\; k = 1..6 \tag{4.10}$$

where the superscript v indicates the vector-format variable. u_k^i for the ith generator is computed based on (2.20). Having these vectors on the GPU, a kernel was prepared to lay these vectors on the main diagonal of a square matrix. Thus, six matrices are built on the GPU whose diagonal elements are the U_k^v and whose off-diagonal elements are zero, as:

$$U_k^m = diag(U_k^v); \;\; k = 1..6 \tag{4.11}$$

where superscript m indicates the matrix-format variable. From the reduced admittance matrix shown in (2.10) two sets of vectors and matrices are then extracted. The vectors are the real and imaginary parts of the diagonal elements of the $Y_R = G_R + jB_R$:

$$G^v = [G_{11},\, G_{22}, \ldots, G_{ii}, \ldots, G_{nn}]^t \tag{4.12}$$

$$B^v = [B_{11},\, B_{22}, \ldots, B_{ii}, \ldots, B_{nn}]^t. \tag{4.13}$$

The two matrices contain the off-diagonal elements of G_R and B_R:

$$G^m_{off-diag} = G_R - diag(G^v) \tag{4.14}$$

$$B^m_{off-diag} = B_R - diag(B^v). \tag{4.15}$$

Note that vectors G^v and B^v, and matrices $G^m_{off-diag}$ and $B^m_{off-diag}$ can be built and saved off-line for any number of contingencies and transferred to the GPU during the initialization step. Once all of the above vectors and matrices are allocated on the GPU, the SIMD computation for S_1 to S_4 can be expressed as follows:

$$S_1^v = (U_1^m \cdot G^m_{off-diag} - U_3^m \cdot B^m_{off-diag}) \cdot I_D^v \tag{4.16}$$

$$S_2^v = (U_2^m \cdot G_R - U_4^m \cdot B_R) \cdot I_Q^v \tag{4.17}$$

$$S_3^v = (U_3^m \cdot G_R - U_1^m \cdot B_R) \cdot I_D^v \tag{4.18}$$

$$S_4^v = (U_4^m \cdot G_{off-diag}^m + U_2^m \cdot B_{off-diag}^m) \cdot I_Q^v. \tag{4.19}$$

The SIMD computations of A_5 to A_8 is given as:

$$A_5^v = I^v + B^v * U_3^v - G^v * U_1^v \tag{4.20}$$

$$A_6^v = I^v - G^v * U_4^v - B^v * U_2^v \tag{4.21}$$

$$A_7^v = G_R \cdot U_5^v - B_R \cdot U_6^v \tag{4.22}$$

$$A_8^v = G_R \cdot U_6^v + B_R \cdot U_5^v \tag{4.23}$$

where $I^v = [1, 1, ..., 1]_{n \times 1}^t$, and the star operation '$*$' refers to a kernel that multiplies two equal-length vectors element by element. Based on (4.16) to (4.23) the SIMD format of equations I_D and I_Q, and then E_D and E_Q is achieved.

4.5 GPU-Based Programming Models

The GPU thrives on applications that have a large computational requirement, and where data-parallelism can be exploited. A data-parallel application consists of large streams of data elements in the form of matrices and vectors that have identical computation codes (kernels) applied to them. The data-parallel characteristic of the GPU gives it a single-instruction-multiple-data (SIMD) architecture. Therefore any computation that is desired to be implemented on the GPU must be in the SIMD format, otherwise the GPU cannot deliver its computational benefits. Here we propose two SIMD-based programming models to exploit the GPUs resources for transient stability simulation. In the first model the GPU is used as a co-processor for the CPU to offload certain computational tasks (hybrid GPU-CPU simulation), whereas in the second model the GPU works as a stand-alone processor and the CPU only controls the flow of the simulation (GPU-only simulation).

4.5.1 Hybrid GPU-CPU Simulation

The standard method of transient stability explained in the previous section was implemented in a hybrid GPU-CPU configuration. Benchmarking revealed that a majority of

execution time in the simulation was spent for **Step 2** and **Step 3**, i.e. the nonlinear iterative solution using Newton-Raphson, and the linear algebraic equation solution. Therefore these two steps of the simulation were off-loaded to the GPU to be processed in parallel, while the remaining tasks such as discretizing, updating, and computation of intermediate variables were executed sequentially on the CPU. The entire simulation code was developed in C++ integrated with CUDA.

In this programming model discretization of the differential equations, and building of the Jacobian matrix are done on the CPU, whereas the network algebraic equations and the Jacobian matrix solutions are performed on the GPU. At each time-step, the Jacobian matrix is transferred to the GPU. It should be noted that CUDA stores matrices on the GPU in a column-major format. Therefore after the Jacobian matrix is constructed on CPU, it needs to be transposed before or after being transferred to the GPU. The other option is to construct the Jacobian matrix in the column-major format from the beginning on the CPU obviating the need for an extra transpose operation and thus saving computation time. Here the latter option has been adopted in this programming model.

With the Jacobian matrix in the proper format on the GPU, the SIMD-format of the LU factorization method was used on the GPU to handle large matrix decomposition.This factorization scheme was implemented using a blocked algorithm, employing bulk matrices, specifically suited for data-parallelism [88]. After transforming the Jacobian matrix into its upper and lower triangular matrices, the BLAS2 function $cublasStrsv()$ of the CUBLAS library is used to solve the equation $L \cdot U \cdot \Delta\mathbf{x} = -\mathbf{F}(\mathbf{x}_{j-1})$ for $\Delta\mathbf{x}$. The results are transferred back to the CPU to update the state variables for the next iteration's calculations. Then the computations are continued on the GPU for the network side solution as described by (4.10) to (4.23). Once the iteration process converges, the time-step is advanced. These steps are illustrated in the flowchart given in Figure 4.5.

4.5.2 GPU-Only Simulation

In the second programming model the transient stability simulation was carried out as a GPU-only computation. In this model the CPU initializes the GPU with the system data, and then all the 3 steps of the simulation are done on the GPU, while the CPU monitors and controls the flow of the simulation. In this programming model the Jacobian matrix is completely constructed on the GPU. Constructing the Jacobian matrix for a multi-machine power system is well suited for exploiting data-parallel programming. For example, in the row-major-saved Jacobian matrix, the first column is the derivative of all non-linear

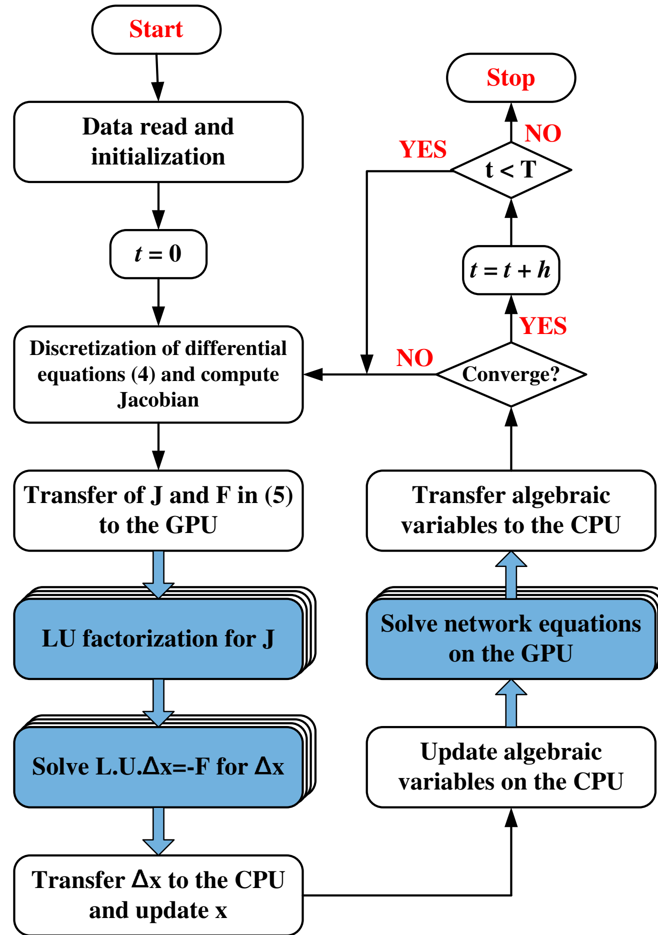


Figure 4.5: Flowchart for the hybrid GPU-CPU transient stability simulation. The colored boxes refer to the GPU operations, and white boxes refer to the CPU operations.

functions with respect to one of the variables. Although these functions have different formulations, they re-occur for all generators in the system. Therefore the derivatives of all identical functions (that have the same formulation but different data-input) can be evaluated in a data-parallel model. After the sub-matrices of the Jacobian are calculated they are combined together to form the full Jacobian matrix on the GPU, and the simulation continues the same as hybrid GPU-CPU model.At the end of simulation, the value of the required variables is transferred to the CPU memory to be saved or plotted. The flowchart in Figure 4.6 illustrates this process. The C++ source code for this model has been shown in Appendix C.

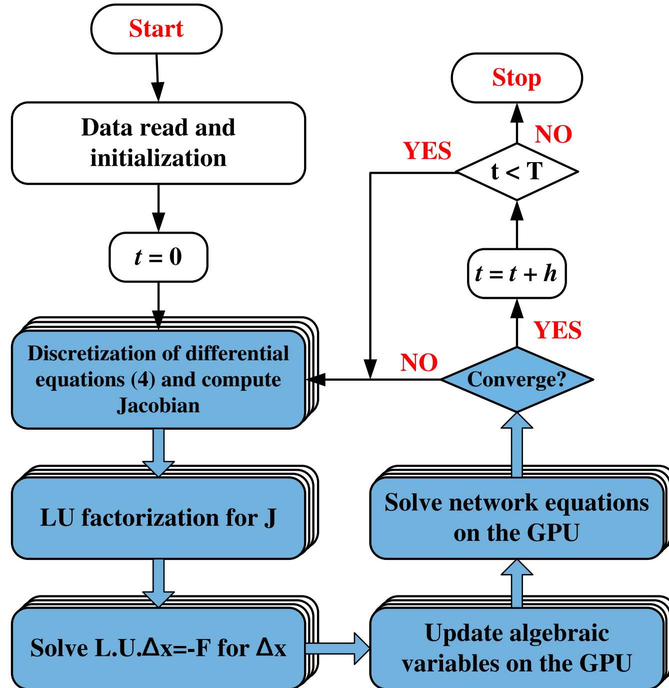


Figure 4.6: Flowchart for the GPU-only transient stability simulation. The colored boxes refer to the GPU operations, and white boxes refer to the CPU operations.

4.6 Experimental Results

In this section we demonstrate results to verify the accuracy and efficiency of the SIMD-based programming models described in the previous section for the transient stability simulation of large-scale power systems on the GPU. As mentioned before, the target GPU in this work is connected to the motherboard of a quad-core CPU. Although the CPU has 4 cores, to precisely control the execution of the CPU and to have a clear comparison, thread programming was used to force the CPU to run the C++ code on only one of the 4 cores. Otherwise, the operating system decides each core's execution at each instant which leads to a vague evaluation. We selected Windows XP-64 as the host's operating system. There were three reasons for choosing this platform: first, we needed a 64-bit operating system, in order to utilize 8GB of RAM. Second, we expected fewer driver issues on Windows compared to Linux. Third, within the Windows product line, Windows Vista was not supported by the NVIDIA GPU Computing platform, leaving Windows XP as the only choice. For development, we used Microsoft Visual Studio 2005.

4.6.1 Simulation Accuracy Evaluation

The accuracy of the programming models was validated using PTI's PSS/E software program. The case study used in this section is the IEEE 39 bus New England test system whose one-line diagram was shown in Figure 3.15 in Chapter 3. The system data in the PSS/E format is also given in Appendix E. The complete system can be described by 90 non-linear differential equations and 20 algebraic equations. Several fault locations have been tested and the results were compared with those of PSS/E. In all cases results from the proposed programming models match the PSS/E results very well. A sampling of these results obtained from the hybrid GPU-CPU simulation is presented here. A three-phase fault happens at Bus 21, at t=1s and it is cleared after 100ms. $Gen10$ is the reference generator and the relative machine angles are shown in Figure 4.7 and Figure 4.8. For comparison PSS/E results are superimposed in these two figures. As can be seen the transient stability code is completely stable during the steady-state of the system, i.e. t<1s. During the transient state and also after the fault is cleared, the program results closely follow the results from PSS/E. The maximum discrepancy between generator angles from GPU-CPU co-processing and the PSS/E simulation was found to be 1.46%, based on (4.24):

$$\varepsilon_{\delta} = \frac{max|\delta_{PSS/E} - \delta_{GPU-CPU}|}{\delta_{PSS/E}} \tag{4.24}$$

where $\delta_{PSS/E}$ and $\delta_{GPU-CPU}$ are the relative machine angles from PSS/E and GPU-CPU co-processing simulation, respectively.

4.6.2 Computational Efficiency Evaluation

To investigate the efficiency of the proposed SIMD-based programming models for the transient stability simulation, we show comparative results in this section. Several test systems of increasing sizes have been used for this evaluation whose specifications are listed in Table 4.2. The $Scale\ 1$ system is the IEEE's New England test system, illustrated in Figure 3.15 and verified in the previous section. The $Scale\ 1$ system was duplicated several times to create systems of larger scales. Thus, we obtained test systems of 78, 156, 312, 624, and 1248 buses. In these systems a flat start was used, i.e. voltage and angle of all buses set to $1.0\angle 0°$p.u., and they were modeled in the PSS/E software to find the load flow results. These results were then fed into the prepared simulation codes.

Three separate simulation codes were prepared: the first code is purely in C++ to be run sequentially on the CPU (CPU-only), the second is C++ integrated with CUDA to be

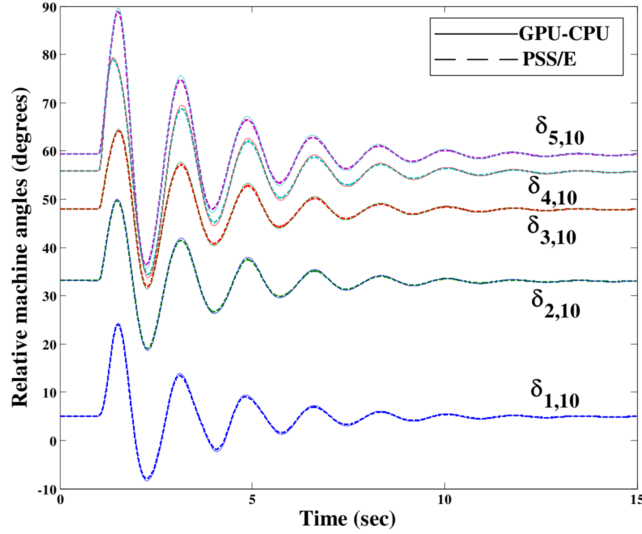


Figure 4.7: Comparison of relative machine angles collected from hybrid simulator and PSS/E simulation for IEEE 39 bus test system: $\delta_{i,10} = \delta_i - \delta_{10}$; $i = 1 \ldots 5$, for a three-phase fault at Bus 21.

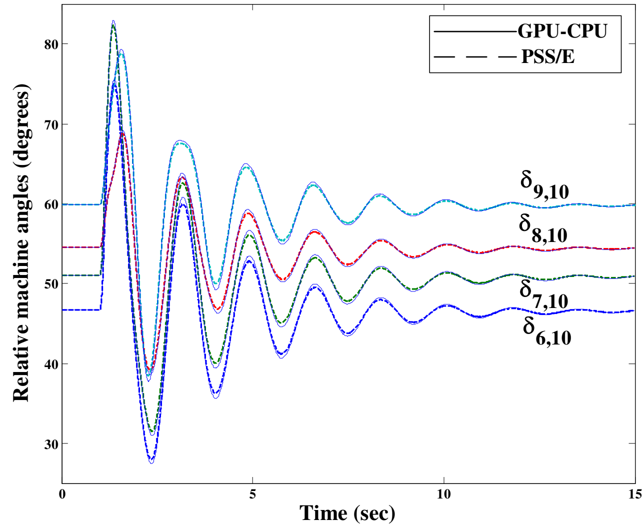


Figure 4.8: Comparison of relative machine angles collected from hybrid simulator and PSS/E simulation for IEEE 39 bus test system: $\delta_{i,10} = \delta_i - \delta_{10}$; $i = 6 \ldots 9$, for a three-phase fault at Bus 21.

completely run on the GPU (GPU-only), and the third is also the integration of C++ and CUDA, however, it uses GPU as the coprocessor (GPU-CPU). The difference between the second and third programming models was explained in Section III. The execution time of these three codes was compared for the test systems. In Table 4.2 the columns indicated by CPU-only, GPU-only, and GPU-CPU list the computation time of each programming model to simulate a duration of 1580ms with a time-step of 10ms for all systems. The CPU

Table 4.2: System scale versus computation time for various configurations for a simulation duration of 1580ms

System scale	Gens.	Buses	State, Alg. variables	CPU-only	GPU-only	GPU-CPU	PSS/E Program
1	10	39	90, 20	0.9s	5.5s	2.8s	0.35s
2	20	78	180, 40	6.4s	7.7s	5.2s	0.40s
4	40	156	360, 80	49.8s	12.3s	10.5s	0.43s
8	80	312	720, 160	7.2min	21.5s	21.1s	0.46s
16	160	624	1440, 320	1hr	41.0s	44.8s	0.55s
32	320	1248	2880, 640	10hr	1min15.2s	1min44.4s	0.83s

execution time of PTI's PSS/E software program is also included in Table 4.2 for reference. Figure 4.9 plots the computation times with respect to the system size.

The application of GPU (in both GPU-only and GPU-CPU models) is truly advantageous for parallel computing on a large set of input data. For small size of data, the communication overhead and memory latency in the GPU are not insignificant compared to the computation time. As such, we did not expect better performance for $Scale$ 1 and $Scale$ 2 systems. When the size of system increases, however, the latency is dwarfed by the computation time, and involving the GPU into the simulations results in a significant acceleration. For example, for $Scale$ 32, the GPU-CPU takes 1 min 44.4 s for simulation, whereas the CPU-only needs 10 hrs. Table 4.3 lists the speed-up factors, defined by (4.25) and (4.26), for the two GPU-based simulations:

$$\beta_{GPU-only} = \frac{CPU_only\ processing\ time}{GPU_only\ processing\ time} \tag{4.25}$$

$$\beta_{GPU-CPU} = \frac{CPU_only\ processing\ time}{GPU_CPU\ processing\ time}. \tag{4.26}$$

The speed-up factors for the two simulations are plotted in Figure 4.10. As can be seen for the $Scale$ 32 system GPU-CPU co-processing is more than 340 times faster than CPU-only processing.

4.7 Discussion

The tabulated results and graphs reveal that for small systems the hybrid GPU-CPU programming model is faster than the GPU-only model, whereas for large-scale systems GPU-only model is faster. This result is consistent with the performance of a single GPU in

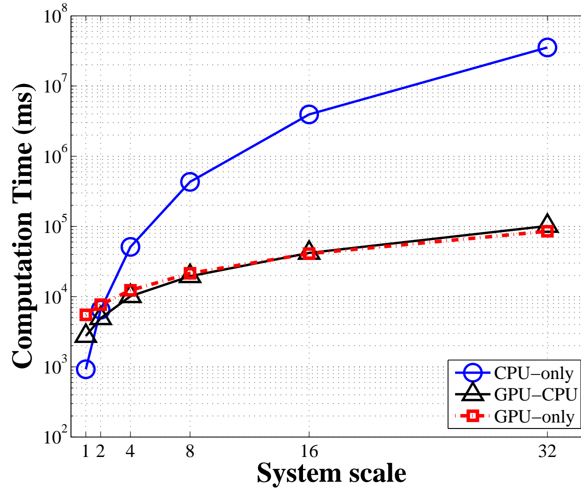


Figure 4.9: Computation time variation with respect to system scale.

other reported applications. However, there are applications where a GPU may need to communicate with other entities or use services which may not be accessible to a GPU, for example in a multiple GPU application. In such cases and with the currently available GPU technologies, the hybrid GPU-CPU programming model is not only useful but also unavoidable. Similar to all programmable processors, efficient GPU programming needs a good understanding of its hardware architecture. Managing the number of required active threads for each kernel call, and the number of device-host interactions are essential to make a timesaving program.

Another useful observation found from the achieved results is the scalability of the proposed hybrid simulator. A system whose performance improves after adding a specific hardware component, proportionally to the capacity added, is said to be a scalable system. In a single GPU expanding the size of data-under-process asks for the co-operation of more SPs which translates to adding more computing resources. In our experiments the size of test systems and the hybrid simulators' elapsed computation time change approximately at the same rate. In the CPU-only simulation cases, however, the computation time increases at a rate that is approximately the cube of the system size increment rate.

The last column in Table 4.2 indicates PSS/E's computation time for the test systems. It is important to put the lower execution times of PSS/E into proper perspective so as not to misjudge the performance of the GPU. PSS/E is a mature software program developed over several decades which incorporates specialized techniques to optimize computational efficiency for large-scale system simulation. In terms of the numerical methods, the key

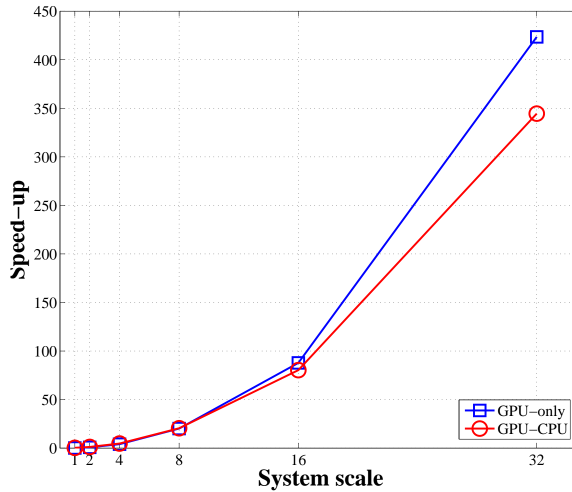


Figure 4.10: Speed-up of GPU-based processing.

Table 4.3: Speed-up comparison

System scale	$\beta_{GPU-only}$	$\beta_{GPU-CPU}$
1	$\times$0.16	$\times$0.3
2	$\times$0.83	$\times$1.2
4	$\times$4.05	$\times$4.7
8	$\times$20.1	$\times$20.5
16	$\times$87.8	$\times$80.3
32	$\times$423.5	$\times$344.8

factors that contribute to PSS/E's speed are as follows:

1. PSS/E uses the Modified Euler integration algorithm which is an explicit integration technique (Chapter 26 in [89]). This is a simple and fast method, however, it can suffer from numerical instability. For a given system configuration, depending on the time-step value the simulation may either converge or diverge. If it did not converge, the time-step must be decreased and program must be rerun [90]. In the proposed GPU-based simulation, the Trapezoidal Rule, an implicit integration algorithm, was used because it is more accurate and avoids numerical instability. However this method requires an iterative solution of the resulting nonlinear algebraic equations at each time-step.

2. The outputs of an explicit integrator are not instantaneous functions of their inputs, and a straightforward non-iterative procedure would be enough to find the derivatives of states, as it happens in PSS/E. However the proposed GPU-based simulator

uses the Newton-Raphson method which consumes a large portion of the simulation time, albeit it gives accurate convergence. At each time-step the Jacobian matrix is calculated, LU factorized, and then solved using forward-backward substitutions.

3. PSS/E takes advantage of the sparsity of the system matrices. This is an important factor in its fast and successful management of computer storage. However it should be noted that presenting PSS/E with a power system network model, whose original admittance matrix has a number of nonzero elements that exceed the program's allocated capacity, results in an error condition that prevents the use of some, but not all, of the available iteration algorithms (Chapter 5 in [89]). The purpose in this chapter was to show how the same implementation of a transient stability algorithm on a GPU is advantageous with respect to a CPU implementation. Therefore sparsity was not used in any of the proposed programming models.

4.8 Summary

Transient stability simulation of large-scale power systems is computationally very demanding. This chapter investigated the potential of using a GPU to accelerate this simulation by exploiting its SIMD architecture. Two SIMD-based programming models to implement the standard method of the transient stability simulation were proposed and implemented on a single GPU. The simulation codes are quite flexible and extensible; they are written entirely in C++ integrated with GPU-specific functions. The accuracy of the proposed methods were validated by the PSS/E software. The efficiency was evaluated for several large test cases. Based on the results obtained, it can be concluded that:

- Using a GPU for transient stability simulations is highly advantageous when the system size is large. As such, for simulating realistic-size power systems the application of GPU looks promising.
- For small-scale systems the hybrid GPU-CPU simulation was faster than the GPU-only simulation, while for large-scale systems the GPU-only model was faster.

5

Multi-GPU Implementation of Large-Scale Transient Stability Simulation

5.1 Introduction

In Chapter 4 the GPU hardware architecture, based on a NVIDIA GPU, and the SIMD programming models were explained in detail. The transient stability simulation was implemented on a single GPU, and the results revealed the excellent capabilities of including GPU as a powerful processor in the power system simulations. However, similar to a CPU, a GPU also has some computing capacity limitations, that restrict the maximum size of the problem which can be implemented on it. Simulations of realistic power system often involve large problem size with onerous calculations. Therefore, this chapter explores the use of multiple GPUs working in parallel and parallel processing based techniques for solving large-scale systems.

The chapter[1] begins with a review of the multi-GPU simulation application, and continues by describing the details of the computing system architecture that has been utilized in this research. Managing multiple GPUs working in parallel to perform a simulation requires specific programming skills which will be highlighted in this chapter. Two parallel processing based techniques (tearing and relaxation) are implemented on multiple GPUs, and a comparison between the two methods is presented. The chapter is concluded by experimental results and discussions.

[1]Material from this chapter has been submitted: V. Jalili-Marandi, V. Dinavahi, "Multiple GPU implementation for large-scale transient stability simulation," *IEEE Trans. on Power Systems*, pp. 1-9, 2010.

5.2 Multi-GPU Overview

5.2.1 Applications

The obvious reason for utilizing multiple GPUs for general purpose computing is to achieve a higher simulation speed, as is the goal in this research. However, compared to the host main memory, GPU has limited device memory. For applications which require a lot of storage, device memory limitations can be a major bottleneck for GPU. For this reason, sometimes running algorithms on multiple GPUs is not only required for faster applications but also necessary to overcome memory bottlenecks.

There are two possible hardware configurations to use multiple GPUs for the computing purpose. The first is to mount multiple individual GPUs internally to the motherboard of the host[2] normally using the PCIe buses [92]. The use of multiple GPUs as CUDA devices working on an application is only guaranteed to work if these GPUs are of the same type [40]. It is important to note that in addition to the GPUs such as GTX 280, described in Chapter 4, that are applicable to both the graphical tasks as well as general purpose computing, there are GPU-architecture-based cards which are specifically designed for the high performance computation with single and double precision floating point (e.g. Tesla C1060). The Tesla series, introduced by NVIDIA in 2007, is the first GPU generation that are fully dedicated to general purpose computing.

The second hardware configuration is to use an external computing unit equipped with multiple GPUs. In this case all the GPUs are compute-specific. For instance the NVIDIA Tesla S1070 (server version) integrates four Tesla T10 GPUs. This multi-GPU configuration is used in this research.

5.2.2 Computing System Architecture

The computing system used in this research is one unit of Tesla S1070 manufactured by NVIDIA [91]. The "Tesla" series of cards are compute-only devices without video output connectors. The Tesla S1070 is equipped with four independent T10 processors (GPU), and the programmer decides how many GPUs must be employed in a simulation and the tasks each GPU must perform. Each of the T10 GPUs is integrated with 4.0GB of memory so that the total memory of the S1070 unit is 16GB. Delivering a theoretical peak performance of 4 teraflops has made Tesla S1070 an energy efficient teraflop processor. Table 5.1 summarizes

[2] A comprehensive and practical work is the FASTRA done at the University of Antwerp, Belgium. http://fastra.ua.ac.be

Table 5.1: Tesla T10 Processor specifications

Number of SM's	30
Number of SP's	240
Device memory	4GB
Clock rate	1.3GHz
Warp size	32 threads
Active warps/SM	32

the important specifications of a T10 processor. For more hardware architecture information of Tesla S1070 the manufacturer data sheets are attached in Appendix D. Each of the T10 GPUs includes 240 stream processors; thus the Tesla S1070 has a total of 960 stream processors which can execute thousands of concurrent threads. Therefore, this computing system that mixes the multi-core CPUs any multiple GPUs provides a heterogenous computing environment with the optimized performance.

Figure 5.1 illustrates the Tesla S1070 from front and top views. The inside architecture of Tesla S1070 is shown in Figure 5.2. As depicted in this figure, each pair of the GPUs are connected to one input/ouput NVIDIA Switch. On the host side, i.e. on the PC motherboard, a Host Interface Card (shown in Figure 5.3) must be plugged into the PCIe bus. Then by using the NVIDIA PCIe cable connected between the NVIDIA Switch and Host Interface Card the link between one pair of GPUs of Tesla S1070 and PC host is established. To connect the other pair of GPUs, the host computer must have an extra PCIe slot mounted with another Host Interface Card. Otherwise, to have concurrent access to 4 GPUs two hosts must be used so that each pair of GPUs connects to one host. Figure 5.4 shows the possible configurations to connect a unit of Tesla S1070 to host computers. The configuration used in this thesis is the same as in Figure 5.4(a) The maximum transfer bandwidth between the host system and the Tesla processors is 12.8 GB/s.

With all of the highly advanced technology used in Tesla S1070, however, the direct intercommunication capability of the 4 GPUs (such as using shared memory) is a missed but promised feature. This means that Tesla S1070 cannot behave as a unified processor. Therefore, to circulate data among the 4 GPUs of the Tesla S1070, data on one GPU is first transferred to the host's main memory (i.e. RAM), and then it is uploaded to the other GPUs. By using the CPU threads, explained in the next section, this deficiency can be recovered, but it should be noted that manipulating intensive computation requires distributing it among existing GPUs.

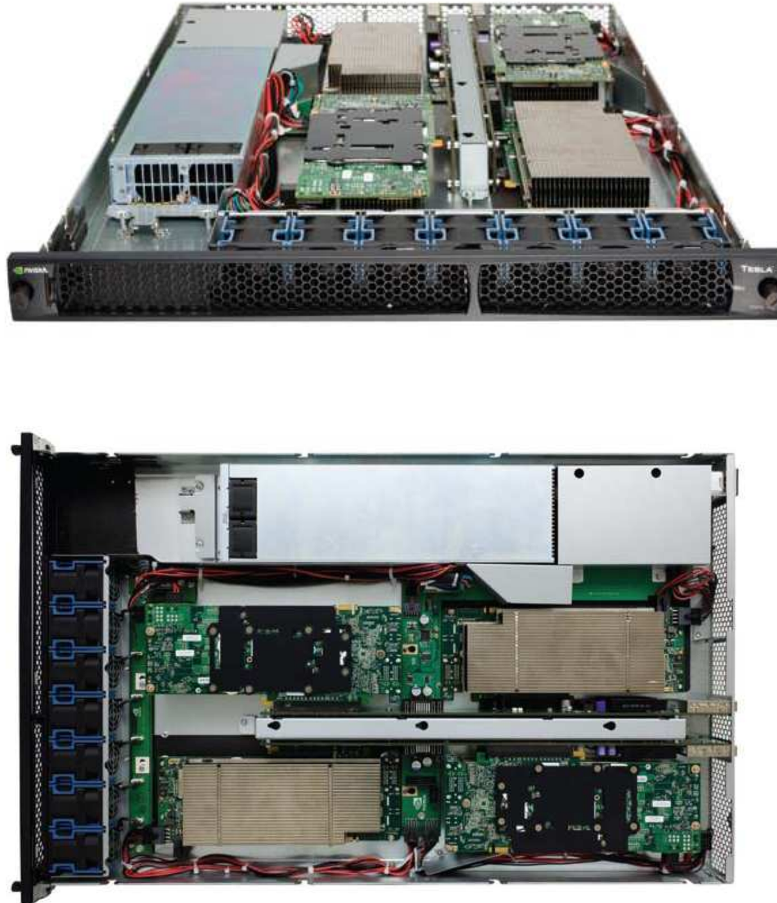


Figure 5.1: Front and top views of Tesla S1070 [91].

5.2.3 Multi-GPU programming

The Tesla S1070 is a CUDA-enabled device. Therefore, all the CUDA programming functionalities explained for the GTX 280 in Chapter 4 are also valid for the Tesla S1070. It was previously mentioned that the GPU runs its own specified kernel independently but is controlled by the CPU. A single GPU can execute only one kernel at any given time; whereas a multi-GPU server such as the Tesla S1070 can run multiple kernels (4) simultaneously. In this chapter we refer to these paradigms *serial kernel* and *parallel kernel* executions, respectively Figure 5.5. Thus to have four GPUs working in parallel, in a Tesla S1070 or any multi-GPU architecture, we require the same number of CPU cores to manage and control the GPUs simultaneously. This minimizes the overhead that occurs in data copying and kernel invocation.

Microsoft Visual C++ provides support for creating multi-thread applications with

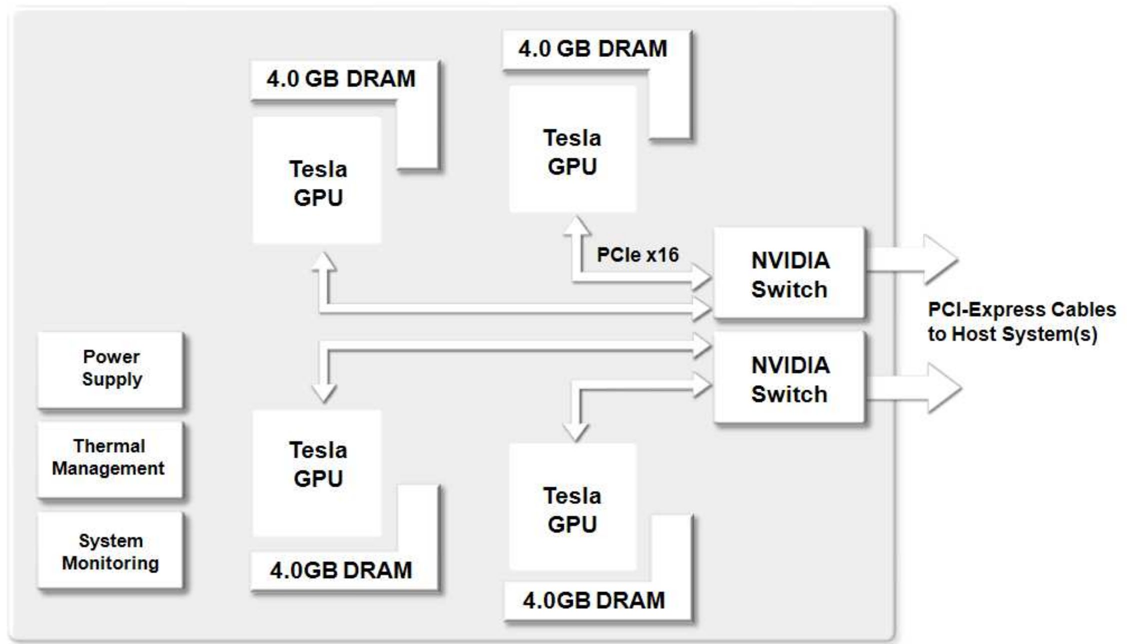


Figure 5.2: Inside architecture of the Tesla S1070 computing system [91].

Figure 5.3: Host Interface Card installing in the host side [91].

Windows XP. There are two ways to program with multiple threads: use the Microsoft Foundation Class (MFC) library or the C run-time library and the Win32 API [93]. There also exist some packages and libraries, such as *Boost C++ Libraries* [94], that can be installed and utilized for the CPU multi thread programming. In this work, the C-run-time library was used to have a full control on the synchronization of GPU data-transfer. Writing and debugging a multi threaded program is inherently a complicated and tricky undertaking, because the programmer must ensure that objects are not accessed by more than one CPU thread at a time. Synchronizing the resource access between CPU threads is a common problem when writing CPU multi threaded applications [93]. Having two or more CPU threads simultaneously access the same data can lead to undesirable and unpredictable results. For example, one CPU thread could be updating the contents of a structure while

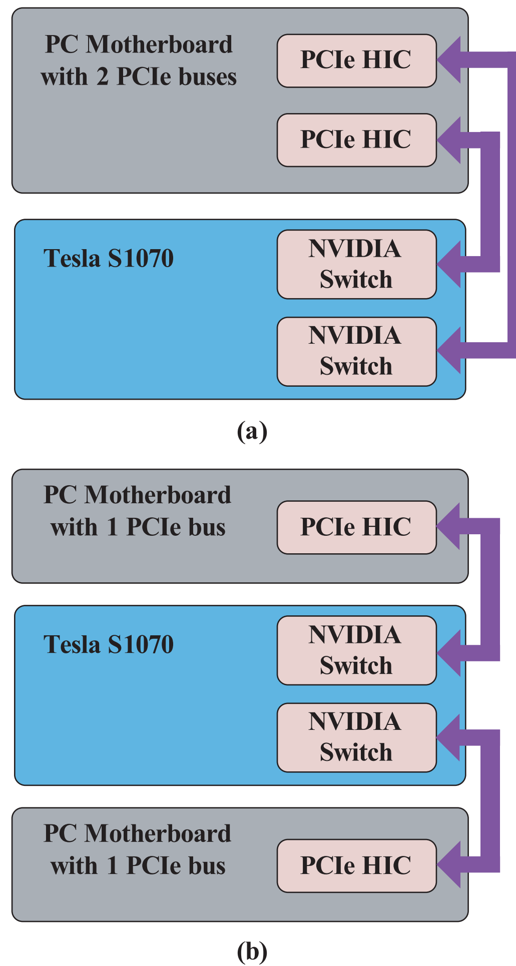


Figure 5.4: Possible configurations of connecting Tesla S1070 to host CPU system: (a) using one host, (b) using two hosts.

another CPU thread is reading the contents of the same structure. It is unknown what data the reading thread will receive: the old data, the newly written data, or possibly a mixture of both. To control synchronization among the CPU threads, *events* are used. Events allow CPU threads to be synchronized by forcing them to pause until a specific event be set or reset.

Figure 5.6 illustrates how the application of multi thread programming is managed to operate up to 4 GPUs connected to a quad-core CPU. The main thread on the CPU first creates four child threads which are responsible for handling one GPU each. Creating the CPU threads is a time-consuming operation; thus once they are created they should be used for the whole of the simulation time duration. The main thread also creates two event arrays called *Start* and *End*. Each element of these arrays is used as a notification signal between the main thread and one of the child threads to determine the initiation or

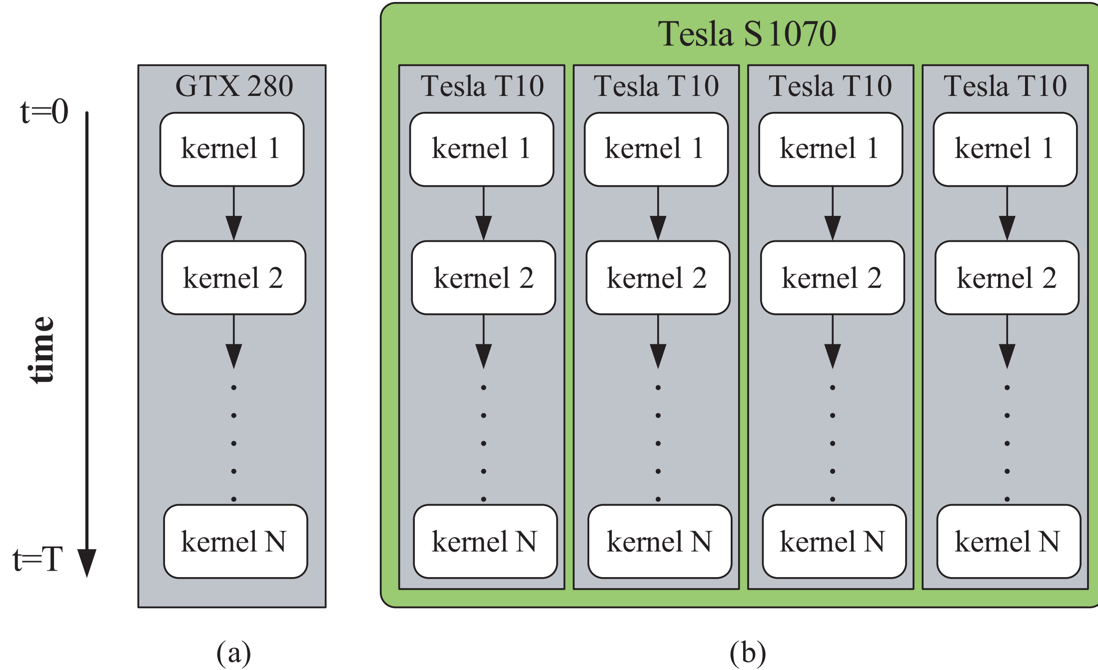


Figure 5.5: (a) Serial kernel execution on a single GPU, (b) Parallel kernel execution on a multi-GPU system.

termination of a specific action. Child threads begin execution by setting one individual device (GPU) and initializing it, and then wait until the Start event is set. As soon as the main thread sets the Start event the computations in the individual GPU belonging to each child thread are activated. The main thread goes into a waiting state until all the threads are done by the GPU computations and the End event is set. Thereafter, four child threads will exchange and update the required data with the help of the CPU, and the simulation continues. At each child thread when the GPU computation is done, in addition to setting the End event, it is also required to reset the Start event to prevent running of the child threads until the data update is done by the main thread and the Start command is released. Therefore, by appropriate use of the Start and End events the synchronization between child threads and consequently the multiple GPUs is orchestrated.

5.3 Implementation of Parallel Transient Stability Methods on Tesla S1070

In Chapter 2 two categories of the parallel processing based techniques, i.e. tearing and relaxation methods, were explained. In this section one method belonging to each category is selected to be implemented on the parallel architecture of Tesla S1070. From the cate-

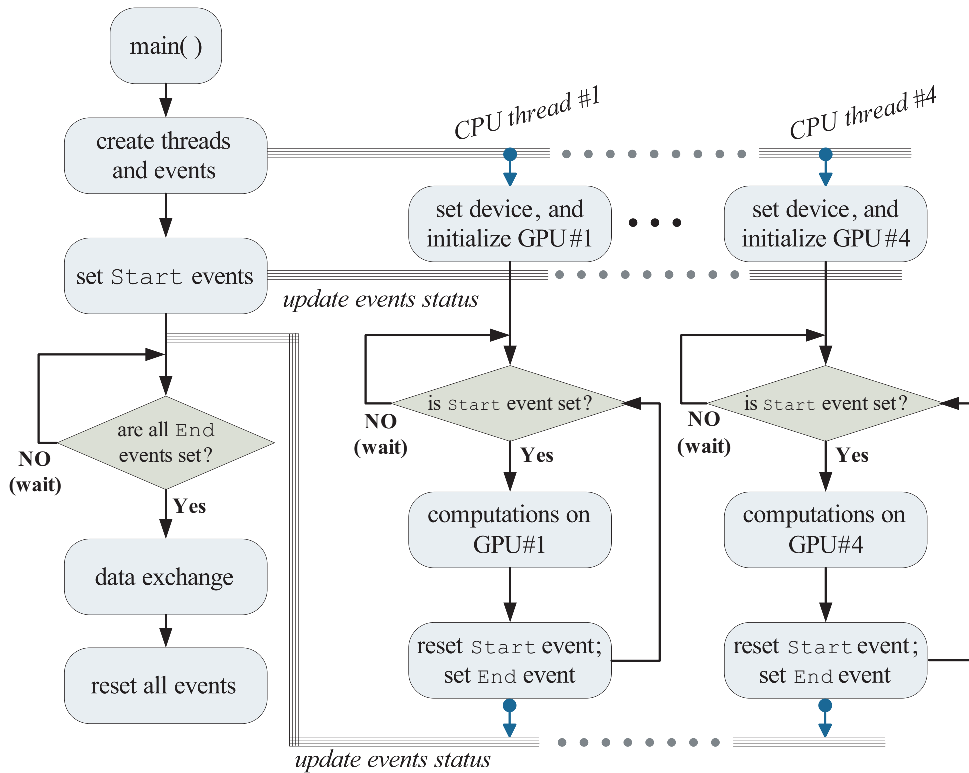


Figure 5.6: The programming application for general purpose multi-GPU computation using a quad-core CPU.

gory of tearing methods, the Block Incomplete LU decomposition method, and from the relaxation group the Instantaneous Relaxation method, described in Chapter 3, are being examined.

5.3.1 Tearing Methods on Multi GPU

In the GPU-only programming model described in Chapter 4, three major parts can be discerned: (1) integration of differential equations and Jacobian matrix computation, (2) LU decomposition of the Jacobian matrix, and (3) solution of the linear algebraic equations, updating and saving the required variables, and GPU-CPU communication time. To get an estimate of the timing load of each part, the results of the elapsed time for these specific parts as a fraction of the total simulation time, and for various system sizes used in Chapter 4 are listed in Table 5.2. These results reveal that LU decomposition (i.e. PART2 in Table 5.2) is the most time-consuming part of the simulation. Thus, the part that would benefit most from using multiple GPUs to reduce the simulation time is the LU factorization. In this section we will use parallel processing to significantly improve this timing.

Tearing methods take advantage of the block structure of the system of equations. In a

Table 5.2: System scale versus fraction of simulation time elapsed for each major part. PART1: integration and Jacobian matrix computations, PART2: LU decomposition, and PART3: linear algebraic solution, updating and saving variables, and communication time.

System scale	PART1	PART2	PART3
1	23%	55%	22%
2	14%	70%	16%
4	8%	81%	12%
8	4%	88%	8%
16	2%	91%	7%
32	1%	92%	7%

system of equations where the dependency matrix is sparse, tearing can be used to achieve decomposition while maintaining the numerical properties of the method used to solve the system [62]. The Jacobian matrix resulting from the linearization step of the transient stability simulation is a case that fits this condition well. The Jacobian matrix of a large-scale power system for transient stability study has a sparse diagonally-blocked structure.

The Incomplete LU factorization (ILU) is widely recognized as effective method for a preconditioned iterative sparse linear system solution. A preconditioner is any form of implicit or explicit modification of an original linear system that makes it easier to solve by an iterative method. For example, scaling all rows of a linear system to make the diagonal elements equal to one is an explicit form of preconditioning. The ILU method entails a decomposition of the form $A = LU - R$, where L and U have the same nonzero structure as the lower and upper parts of A, and R is the residual or error of the factorization [95]. This incomplete factorization, known as ILU(0), is easy and inexpensive to compute, but it often leads to a rough approximation and imposes more iterations to the applied iterative scheme. Generally, more accurate ILU factorizations require fewer iterations to converge. Thus, several alternatives of this approach have been developed by allowing more fill-ins in the L and U matrices. One alternative that is useful for sparse matrices is to perform the LU-decomposition only at locations where A originally has non-zeros. A variant of this approach is the Block ILU in which the decomposition is conducted only along the main diagonal [95]. Figure 5.7 shows the original sparse matrix divided into diagonal sub-matrices which locally are decomposed using ILU. Because these blocks do not communicate with each others and can be individually decomposed, this scheme fits well in a parallel processing based architecture.

Considering the multi-GPU programming concepts explained in the previous section

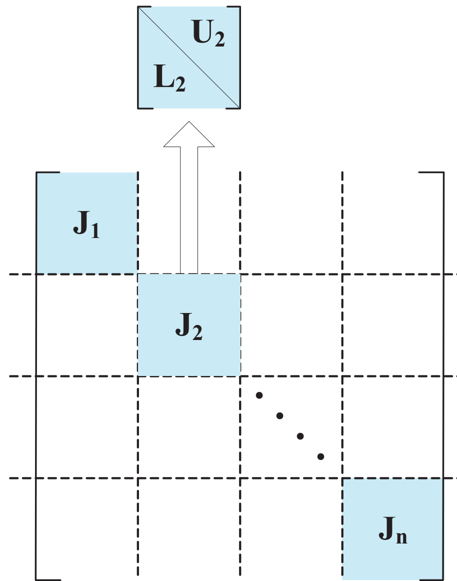


Figure 5.7: Sparse diagonally blocked matrix is a suitable case for the ILU method.

(such as creating and managing CPU threads and events), Figure 5.8 illustrates the distribution of computations among the GPUs of a Tesla S1070 unit. As shown in the pseudo code, the discretization of the DAEs and computation of the Jacobian matrix for the whole system are done on the main thread on the CPU. The Jacobian matrix is then decomposed into n sub-matrices which are then transferred to each of the GPUs. GPUs concurrently decompose the sub-matrices into their L and U factors. Due to the parallel operation of GPUs, their computation loads must be equal to have an efficient simulation. Thus, the sub-matrices must have equal or approximately equal size. Moreover, the size of sub-matrices is an important factor: if they are too small the overall efficiency of using GPUs will be lost. After all GPUs are done with the factorization steps, the L-U factors are transferred to the CPU main memory, and then uploaded to one of the GPUs to build the original-size LU factor and perform the remaining calculations.

5.3.2 Relaxation Methods on Multiple GPUs

In Chapter 3 the Instantaneous Relaxation (IR) method was proposed and implemented on the PC-cluster based simulator made up of multiple CPUs. The important advantage of this method in the transient stability study of large-scale power systems is its ability to make a coarse-grain decomposition which allows breaking down the large DAE systems into smaller pieces and distributing the individual computations to specified processors. These processors need to communicate only once at each time-step.

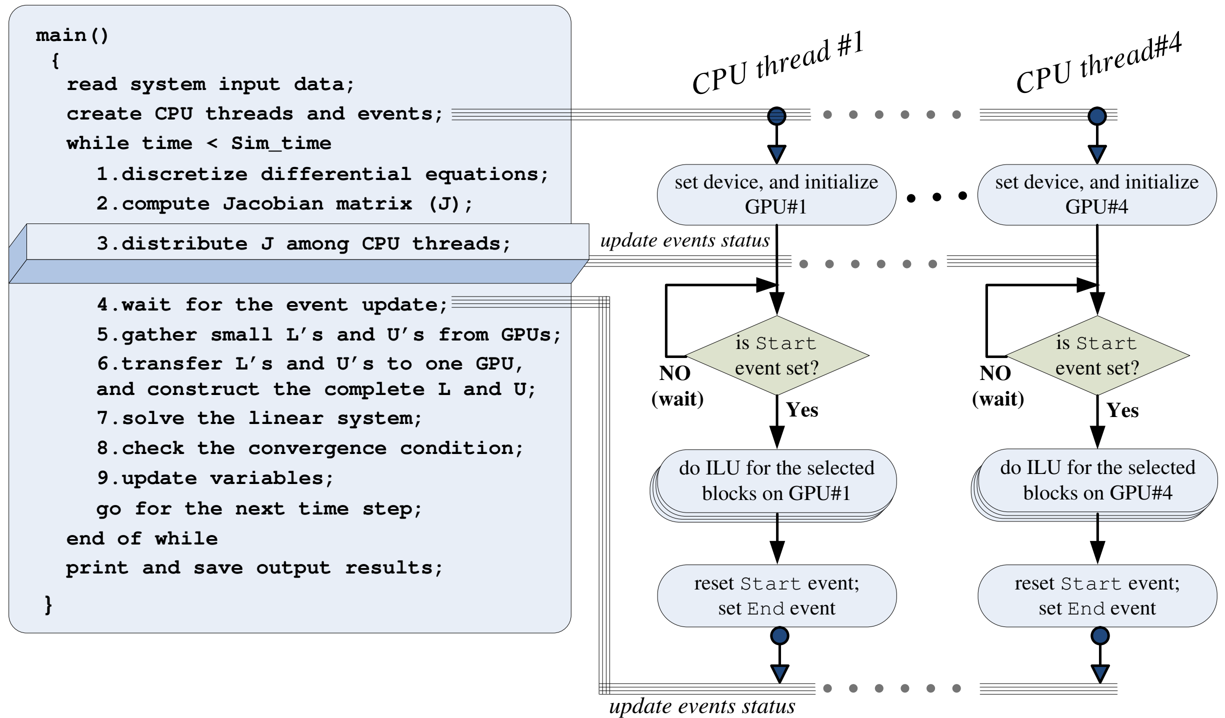


Figure 5.8: ILU-based tearing method implementation on multiple GPUs.

In this section the IR method will be implemented on the multi-GPU architecture of Tesla S1070. This provides a high degree of parallelization for the transient stability study. First, the solution algorithm is inherently parallel, and second, this algorithm is implemented on up to 4 GPUs each with a massively data-parallel architecture. In other words, implementing of the IR method on a multiple GPUs is actually the intersection of algorithm-level-parallelism and data-parallelism.

Figure 5.9 illustrates how the Instantaneous Relaxation method is implemented on the 4 GPUs of a Tesla S1070 system. Comparing this figure with Figure 5.8 it can be seen that in the IR method the system decomposition and consequently the parallelization happen at the top level of computations, while in the tearing based approach parallelization is in the inner loop of the procedure. Thus, in case the convergence condition in the ILU-based tearing method is not fulfilled, the loop must be repeated which multiplies the communication time between the Tesla S1070 and CPU memory. Whereas, from the GPU programming perspective efficient programs are ones which require the minimum amount of GPU-CPU communication. To practically confirm this statement, Table 5.3 lists the experimental results showing the maximum bandwidth rate of the Tesla T10 processors inside the Tesla S1070 system. In this Table "Device" refers to a single Tesla T10 processor, and

Table 5.3: Tesla T10 processor bandwidth test results

Host-to-Device	1.35GB/s
Device-to-Host	1.62GB/s
Device-to-Device	73.1GB/s

"Device-to-Device" means the operations inside one GPU, i.e. not the interconnection between 2 GPUs. As it can be seen, the Host-Device bandwidth for both directions is lower than that of the device to device rate. Therefore, especially in the application of multiple GPUs where data transfer is unavoidable, the GPU-CPU communication must be reduced to minimize simulation time. This is what the IR method provides perfectly. In the next section, a comprehensive efficiency comparison between the tearing and relaxation types parallelization will be shown and discussed.

In addition to the high parallelization offered by the IR method, its implementation on both single GPU (serial kernel) as well as multiple GPUs (parallel kernel) covers the maximum size limitation imposed by CUDA/CUBLAS application. In the current version of CUBLAS API, BLAS2 functions are specified for the single or double precision matrix-vector operations. For example, $cublasStrsv()$ solves a system of equations of the form:

$op(A) * x = b$,

where $op(A) = A$ or $op(A) = A^T$,

b and x are n-element single-precision vectors, and A is an $n \times n$, upper or lower triangular matrix consisting of single-precision elements. This function plays an important role in the solution of a system of linear algebraic equations that have already been LU factorized. $cublasStrsv()$ takes n as an input argument that determines the number of rows and columns of the matrix A. In the current implementation of CUBLAS, n must not exceed 4070 for the single-precision entries. For the double-precision function n is limited to 2040 [87]. This is an unexplained boundary, imposed by the GPU manufacturer, that does not allow the programmer to go beyond systems that may include matrices with the dimension larger than these limits. For instance, the size of Jacobian matrix in the transient stability stability simulation of a power network within m machines each of which is modeled by 9 state variables is $9m \times 9m$. Thus, the largest system that can be modeled in a single GPU with the traditional approach cannot include more than 450 machines. In the IR method, however, the system is decomposed into subsystems which are solved individually. Consequently, regardless of its serial or parallel implementation of the IR method, each subsystem can use the maximum compute capacity of the available hard-

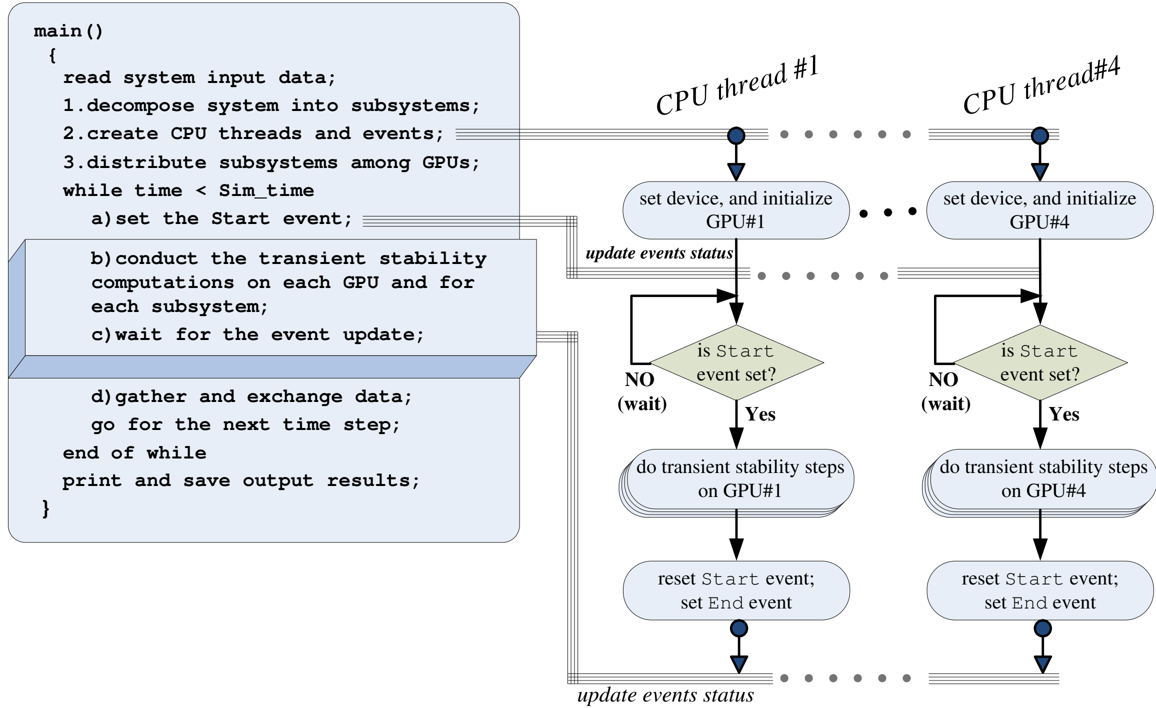


Figure 5.9: IR method implementation on multiple GPUs.

ware. This is the direct result of decomposing the system of DAE from the differential equations level and before performing the discretization step (i.e. top level). In the ILU-based tearing method described in the previous section, although the Jacobian matrix is torn into smaller sub-matrices, eventually they must be rebuild the original-size LU matrix to be used in $cublasStrsv()$ function.

5.4 Experimental Results

5.4.1 Work-station and Test Systems

In this section we demonstrate results to verify and compare the efficiency of the parallel processing based techniques described in the previous section. As mentioned before, the target computing system is a unit of Tesla S1070 that is connected to the motherboard of a quad-core CPU supported by 8GB of memory (RAM). Depending on the size of computations being done on the multi-GPU system, using the same amount of CPU memory (RAM) as the GPUs have is recommended to achieve better performance. For example, in the case of running 4 GPUs of the Tesla S1070, the CPU should be supported by 16GB of RAM; however, in our case the host motherboard can support a maximum of 8GB of RAM. Thus, the simulation time results that are shown in this section can be improved simply by

Table 5.4: Test System Scales

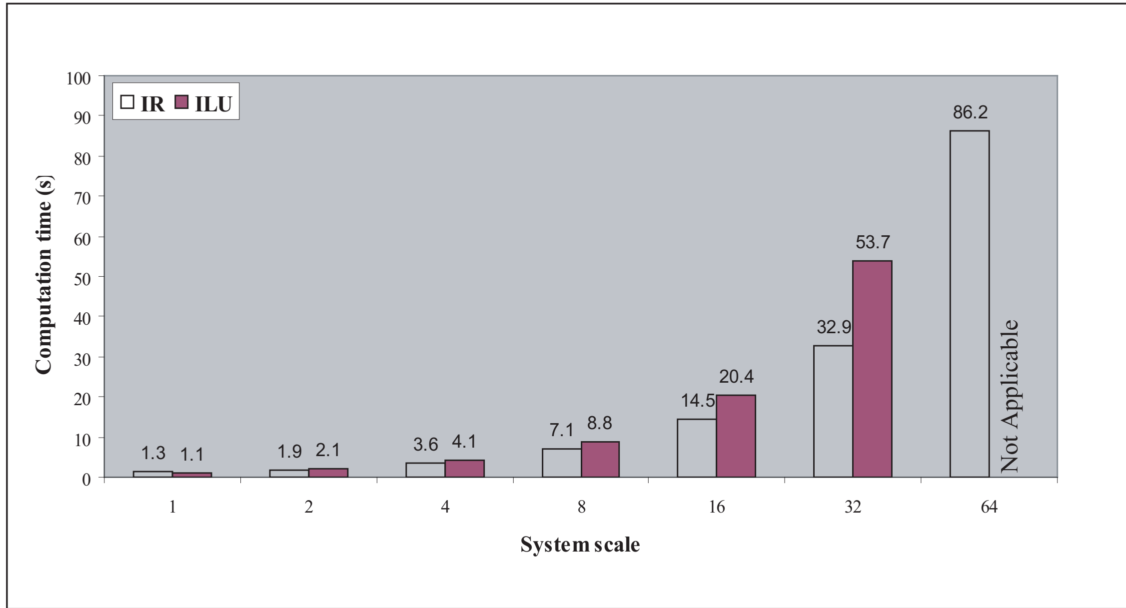
System scale	Generators	Buses
1	10	39
2	20	78
4	40	156
8	80	312
16	160	624
32	320	1248
64	640	2496
128	1280	4992
180	1800	7020

upgrading the host-side memory.

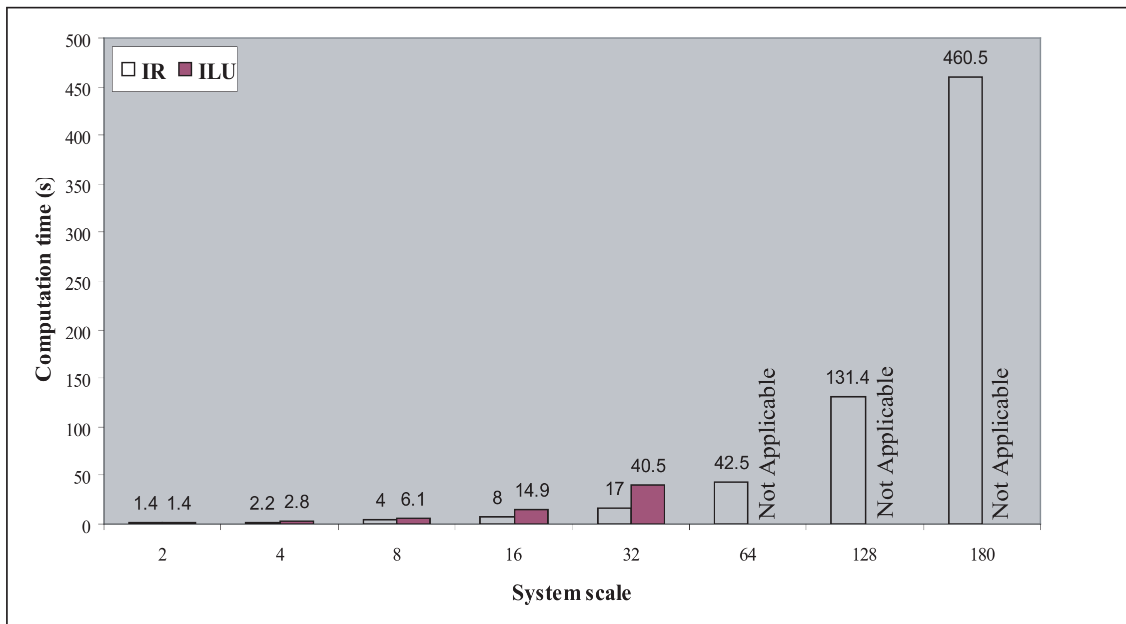
The accuracy of the simulation models has been investigated and discussed for the single GPU case in Chapter 4. In this chapter also, we use the IEEE's New England test system and develop its multiplicands by using PTI's PSS/E software. The steady-state and dynamic stability of these systems have been examined and verified. Although these are fictitious systems, the important advantage of using them as test cases is that they let us to explore the performance of the GPU-based simulation statistically and gradually. In addition to the systems used in Chapter 4, in this chapter systems with $Scales$ 64, 128, and 180 have been developed. The single-line diagrams for these systems are given in the Appendix E, and Table 5.4 summarizes the specifications of these cases. The $Scale$ 180 is constructed based on the discussion provided in the previous section that the maximum computing capacity of each GPU for the prepared model is 450 generators. This system is built so that all GPUs of the Tesla S1070 become fully populated ($4 \times 450 = 1800$ generators). The admittance matrix and load flow results of PSS/E simulating these systems were saved in a text format file to be fed into the prepared programs for the tearing and IR methods.

5.4.2 Transparency

Both parallel processing based methods are implemented to work with 1 to 4 GPUs, depending on the required computing capacity. This issue in the parallel processing software development is referred as *transparency*, which means the ease with which software written for a set number of processors can be reformulated for another number of processors [49]. Although the SIMD hardware architecture of GPU and the way kernels are invoked and run by many threads offer a high degree of transparency, the developed software is ef-



(a)



(b)

Figure 5.10: Multi-GPU simulation: (a) 2 GPUs, (b) 4 GPUs.

fectively controllable by changing one variable at the compile time to work with various numbers of parallel GPUs. In this section the 2 and 4 GPUs implementation will be shown.

Graphs shown in Figure 5.10 compare the computation time of the IR and ILU-based tearing methods. In Figure 5.10.a the results for utilizing 2 GPUs illustrates that as the system enlarges the IR method accelerates more than ILU method, so that for the largest

applicable system, i.e. $Scale$ 32, the IR is 1.6 times faster than ILU method. Moreover, $Scale$ 64 and higher are not implementable with the ILU method, due to GPU hardware restriction, while the IR overcomes the limitation and can simulate larger system. These systems that are not implementable are shown as “Not Applicable” in graphs. The largest system that can be implemented by exploiting 2 parallel GPUs is $Scale$ 90, which includes 900 fully detailed generator models, not shown in this figure.

Figure 5.10.b shows the computation time for the case of using 4 parallel GPUs. In this case the $Scale$ 1 is ignored, because the communication and computation times are too similar to reveal any computing advantage of including multiple GPUs. From the achieved results it is clear that IR implementation is faster than ILU-based simulation, so that for the largest applicable system for both methods, i.e. $Scale$ 32, the IR is 2.4 times faster than ILU method. Furthermore, very large-scale systems as $Scales$ 64, 128, and 180 have been simulated by the IR method. $Scale$ 180 is the system that entirely occupies Tesla S1070 to solve 19800 DAEs.

5.4.3 Scalability

The other important observation is the scalability characteristic of the IR method. The Scaling Factor (SF) is defined as:

$$SF = \frac{computation\, time\, of\, Single\, GPU}{computation\, time\, of\, Multiple\, GPUs} \tag{5.1}$$

The SF reveals how much the parallel multi-GPU simulation is efficient compared with the single-GPU simulation. Ideally, and for an available hardware, we expect that in case of using n parallel processors simultaneously running to solve a problem which takes T seconds on a single processor, the simulation time would break down to $\frac{T}{n}$ seconds. However, this is not true in practice due to several software development issues such as task scheduling, processors’ communication, and the parallel processing algorithm. Thus, SF is always less than n, and the closer it is to n, the higher is the efficiency of multi processing based technique. This factor is computed for the IR as well as ILU-based methods, for 2 and 4 GPUs, and for the system scales that are applicable to both methods. The results are depicted in Figs. 5.11 and 5.12. For the IR method in both the 2 and 4 GPUs, as the test system expands, the SF factor grows closer to the number of parallel GPUs in use. This means that the IR method is scalable, and if somehow the communication time is reduced, for example by upgrading the hardware or advancement in GPU cluster technology, the SF will

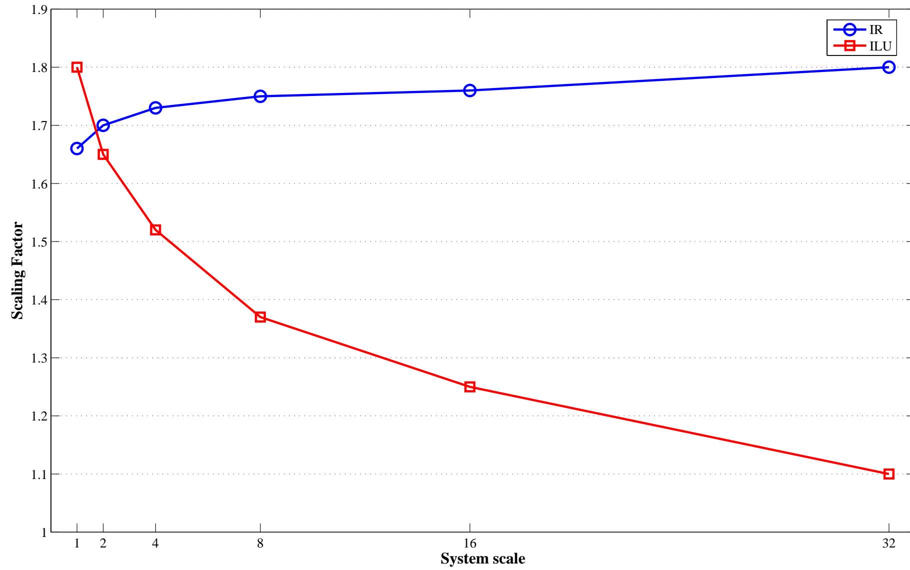


Figure 5.11: Scaling factor for the IR and ILU methods using 2 GPUs.

increase further. On the other hand, in the ILU-based method as the system scale increases the SF decreases. One reason that can explain this low SF in the ILU-based method is that as the system enlarges the size of data that needs to be transferred among the GPUs increases which leads in-turn to a rise of the communication time, and in case of requiring iterations the situation becomes worse. As explained before, the IR method is a algorithm-level parallel method where decomposition happens at the top level so that the GPUs are required to communicate only once at each time-step. The advantage of algorithm-level decomposition over the task-level decomposition is obvious from this experiment.

5.4.4 LU Factorization Timing

As explained in the previous section and shown in the Table 5.2, a significant portion of the simulation time is elapsed for the LU factorization of the Jacobian matrix. In the ILU method the Jacobian matrix is torn into several sub-matrices to reduce the LU factorization time by exploiting parallel GPUs. Table 5.5 lists the elapsed time for the LU factorization part in case of using multiple GPUs and compares it with the single GPU application. This timing for the multi-GPU applications includes four major tasks: (1) decomposition of the Jacobian matrix into sub-matrices, (2) communication time between GPUs and main memory to transfer these sub-matrices, (3) LU factorization for each sub-matrix on each GPU, and (4) reformation of the original size Jacobian matrix. The extraneous tasks, i.e. tasks 1, 2, and 4, cause the LU factorization time in the 2 and 4 GPU implementations to be

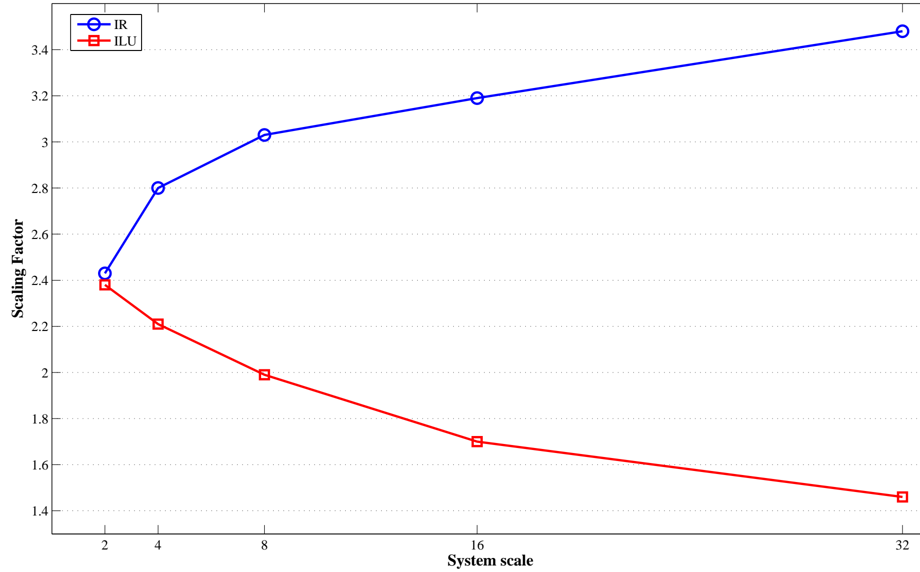


Figure 5.12: Scaling factor for the IR and ILU methods using 4 GPUs.

Table 5.5: System size versus LU decomposition elapsed time (in seconds) for single, 2, and 4 GPUs applications.

System scale	1 GPU	2 GPUs	4 GPUs
1	1.1	0.7	Not Applied
2	2.4	1.6	0.9
4	5.1	3.3	2.2
8	10.8	7.8	5.2
16	23.3	17.5	12.3
32	54.6	44.5	33.1

more than half or quarter of that in the single GPU application.

5.5 Summary

The main goal in this chapter was to demonstrate the practical aspects of utilizing multiple GPUs. A parallel processing technique can be implemented on any parallel processor based hardware, for example using a cluster consisting of hundreds of PC's. However, employing a large cluster has some drawbacks as well: it is quite expensive, is not always available, takes a lot of space and requires considerable maintenance. By including GPU units in the simulation and appropriate programming of GPUs, a regular PC can have the performance of a super computer with much less price and energy usage. Moreover, these GPUs are affordable for personal, research, and industry applications.

In this chapter, two parallel processing based techniques implemented on a Tesla S1070 unit. The techniques used here are from tearing and relaxation categories, explained in Chapters 2 and 3. The experimental results revealed that algorithm-level decomposition, as it happens in the Instantaneous Relaxation method, is more efficient than task level decomposition.

Although the multi-GPU server with its current technology looks very capable, its API still is in its infancy. Performance of the multi-GPU algorithm can be further increased by direct GPU to GPU communication. We await NVIDIA's CUDA support for direct inter-GPU communication instead of communication through the CPU.

6 Summary and Conclusions

Transient stability analysis is a major requirement for the planning and safe operation of power systems. It is the core of any DSA tool in the energy management system. However, time domain transient stability simulation of large-scale power systems is computationally very demanding. The transient stability problem is rich in parallelism which makes it very suitable for applying parallel processing based techniques.

The goal in this thesis is to accelerate the transient stability simulation. To do so, the investigation in this thesis branched into two directions. In part one the focus is the real-time implementation of transient stability simulation. As the real-time simulator has a parallel processor architecture, it is unavoidable to have a parallel processing based technique to use the simulator's capacity. The IR method was proposed and successfully implemented for this purpose, and is shown to give both accuracy and efficiency for PC-cluster based architecture. The main outcome of this approach is that it uses the program-level parallelism that inherently exists in the nature of the transient stability problem. In the second part the objective is still the acceleration of the transient stability simulation for larger systems but by using the general purpose computing capacity of the GPUs. The use of GPU requires the rethinking and re-engineering of the problem solution in a SIMD-based programming model. The results obtained in this part revealed the advantage of GPU-based simulation over the CPU-based one for large-scale systems. By using single GPU it became possible to model large-scale systems that take several hours of CPU time in a few minutes. Integrating the IR method with the multiple GPUs was the other achievement of this thesis

that not only increased the scale of largest implementable system but also overcame the limitation existed in the current GPUs.

6.1 Contributions of This Thesis

The main contributions of this thesis are summarized as follows:

- This thesis has a significant practical value. Although transient stability has been formulated and it has a rich literature resources, in this thesis the effort was to aggregate the abstract of all the required equations, for synchronous machine and network modeling, and step-by-step numerical methods to solve these equations. Moreover, in both real-time implementation as well as single/multiple GPU programming the achieved experiences and difficulties that may exist were clearly explained. This type of knowledge cannot be usually found in power systems literature.
- The IR method proposed in Chapter 3 showed an efficient performance for both real-time simulation as well as multiple GPU implementation. Integrating this method with the slow coherency based partitioning and its implementation on a general purpose state-of-the-art real-time simulator resulted in an inexpensive and efficient real-time simulator. In the GPU applications, the IR method was also very helpful to overcome the technology restriction existing in the present GPUs. By the use of this method it was possible to conduct the transient stability simulation of large-scale power systems which are modeled in detail.
- The software developed for both real-time simulation as well as GPU application, is designed in such a way to be easy to expand to include more models of power system elements in its library. It is also easy in both cases to change the number of processor nodes or GPUs which are running in parallel.
- This is the first time that GPU is used for dynamic computations in power systems. This research will introduce GPUs to the power systems experts and inspire them to use GPUs for other burdensome computations such as load flow and electromagnetic transient studies.

6.2 Directions for Future Work

This research can be continued in several ways for the transient stability simulation of power systems. First, the software developed for both real-time simulation as well as GPU

codes have the capacity to be further improved. For instance, including more models for synchronous machines, AVR, PSS, and loads, adding more elements such as protection devices, as well as the ability for performing various types of contingency studies all are cases that lead to provide a complete set of transient stability tool. Having this tool and using the IR algorithm as the core of parallel processing method the general purpose real-time simulator can be utilized as a real-time transient stability simulator.

It is anticipated that the GPU can play an important role in realizing the ultimate goal of online or real-time dynamic security assessment in the energy control center. The transient stability simulation algorithm in this research work employed the implicit Trapezoidal integration method along with the iterative Newton-Raphson procedure. However, as reported in Chapter 1, there exist many other possibilities for solving the nonlinear DAEs applicable to the transient stability problem, such as:

- explicit or implicit integration methods
- iterative and non-iterative solvers
- simultaneous or partitioned solution approaches
- application of sparse methods to solve linear algebraic equations [96].

In addition, it is also possible to implement other parallel processing based techniques to investigate higher speed-ups. It is predicted that if a method accelerates the CPU-based simulation, it would also accelerate the GPU-based model, if that approach was efficiently implemented on the GPU.

Existing real-time simulators used in the power system area are clusters made up of CPU or DSP based hardware. Each rack of these simulators costs several hundreds of dollars and are not affordable for all industry and academic customers. Moreover, the maximum size of the system that can be simulated by a rack of PC-cluster is very limited. The price of the GPU-cluster is much lower than the PC-cluster simulator while its computing capacity is much higher. Therefore, a worthwhile research goal is to move toward using GPUs for the real-time simulations. Certainly, GPU has the capacity for this purpose, but the important points are how to manage or force a GPU code to run in real-time and second, how to perform the hardware-in-the-loop simulations. The former is mainly the matter of software development while the later is the hardware issue.

Recently, NVIDIA represented next-generation of CUDA architecture GPUs named *Fermi*. The complexity of the Fermi architecture is managed by a multi-level program-

ming model that allows software developers to focus on algorithm design rather than the details of how to map the algorithm to the hardware, thus improving productivity. The most important difference of this new generation of GPUs in comparison with existing ones is that Fermi supports simultaneous execution of multiple kernels from the same application, each kernel being distributed to one or more SMs on the device. This capability avoids the situation where a kernel is only able to use part of the device and the rest goes unused [97]. Exploiting this technology would let us implement a program or task level parallel transient stability simulation algorithm (such as Instantaneous Relaxation or ILU methods) on a single Fermi GPU in the future.

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area1.c S-function Complete Source Code

This is the main S-function program that implements area1.c S-function initialization, simulation loop, and output.

area.c

```
1 /*
2 To make a new area:
3  * "save as" this file with another name, e.g area2.c
4  * Set the S_FUNCTION_NAME
5  * Set N, N_a1, N_o_A
6  * Update the Gens array with the id-numbers of machines in this area
7  * Generate the mex file
8
9  * This sfunction is for partitioning the system just in 2 areas
10 * The admittance matrix must be saved in the folder as Y_matrix.mat,
11   and simin.m must be run to generate required files
12 */
13 #include <stdio.h>
14 #include <stddef.h>
15 #include <stdlib.h>
16 #include <math.h>
17 #include "nrutil.h"
18 #define NR_END 1
19 #define FREE_ARG char*
20
21 #define S_FUNCTION_NAME  area1
22 #define S_FUNCTION_LEVEL 2
23
24 #include <string.h>
25 #include "simstruc.h"
26 /*************************************************************************/
27 #define U_a2(element) (*uPtrs1[element])  /* Pointer to Input Port0 */
28 #define I_a2(element) (*uPtrs2[element])  /* Pointer to Input Port3 */
29 #define ID_num_a2(element) (*uPtrs3[element])  /* Pointer to Input Port3 */
30
31 #define N 10 //The total number of all generators
32 #define N_a1 3  //The number of generators in this area
33 #define N_o_A 2 //The total number of all outside areas
34
35 #define pi 3.14159265
36 #define epsilon 0.05
37 #define h 0.01
38 #define wR 2*pi*50
39
40 #define duration1 1/h
41 #define duration2 10
42 #define duration duration1+duration2
43
44 #include <stdio.h>
45
46 static int Gens[] = {0,1,8,9};//The Id number of generators in this area,
47                               //the first one is always 0
48 static int Gens_a2[] = {2,3,4,5,6,7,10};//The Id number of external generators
49
50 struct machine{
51     //last iteration values
52     float delta, slip, sfd, sd1, sq1, sq2, V1, V2, V3;
53     float id, iq, vd, vq, vt;
54     float ifd, efd, Efd, Vref, id1, iq1, iq2;
55     float s_ad, s_aq;
56     float Eds, Eqs;
57     float Te, Tm, Pt, Qt;
58     float w;
59     float u1, u2, u3, u4, u5, u6;
60     float H, D;
61     //fundamental parameters which are calculated based on the Standard params
62     float Lad, Laq, Lfd, Ld1, Lq1, Lq2, Rfd, Rd1, Rq1, Rq2, Lad_sec, Laq_sec;
63
64     //previous time step values
65     float pre_delta, pre_slip, pre_sfd, pre_sd1, pre_sq1, pre_sq2;
66     float pre_V1, pre_V2, pre_V3;
67     float pre_Te, pre_vt;;
68     float pre_ifd, pre_efd, pre_id1, pre_iq1, pre_iq2;
69
70     float KA, TR, Efd_max, Efd_min;
71     float Kstab, TW, T1, T2, vs_max, vs_min;
72
73     int EXC, PSS;
74 };
75
76 struct BUS{
77     float ID, IQ, It;
78     float VD, VQ;
79 };
80
81 struct G{
82   float pref[N+1][N+1];
83     float fault[N+1][N+1];
84   float postf[N+1][N+1];
85 }G;
86
87 struct B{
88   float pref[N+1][N+1];
89     float fault[N+1][N+1];
90   float postf[N+1][N+1];
91 }B;
92
93 static struct machine   Gen[N_a1+1];
94 static struct BUS   bus[N_a1+1];
95
96 static float r_Te[9][N_a1+1], r_ifd[9][N_a1+1], r_efd[9][N_a1+1], r_id1[9][N_a1+1];
97 static float r_iq1[9][N_a1+1], r_iq2[9][N_a1+1], r_Vt[9][N_a1+1];
98
99 static float U1[N+1], U2[N+1], U3[N+1], U4[N+1], U5[N+1], U6[N+1], ID[N+1], IQ[N+1];
100
101 static int time, PS, EX, no_EXPS;
102
103 static int N_out;
104
105 #define NPARAMS   0
106 /*====================*
107  * S-function methods *
108  *====================*/
109
110 #define MDL_CHECK_PARAMETERS
111 #if defined(MDL_CHECK_PARAMETERS) //&& defined(MATLAB_MEX_FILE)
112   /* Function: mdlCheckParameters =============================================
113    * Abstract:
114    *    Validate our parameters to verify they are okay.
115    */
116   static void mdlCheckParameters(SimStruct *S)
117   {
118 }
119 #endif /* MDL_CHECK_PARAMETERS */
120
121 /*================*
122  * Build checking *
123  *================*/
124
125 /* Function: mdlInitializeSizes ===============================================
126  * Abstract:
127  *   Setup sizes of the various vectors.
128  */
129 static void mdlInitializeSizes(SimStruct *S)
130 {
131
132     N_out=N-N_a1;
133
134     ssSetNumSFcnParams(S, NPARAMS);  /* Number of expected parameters */
135     if (ssGetNumSFcnParams(S) == ssGetSFcnParamsCount(S)) {
136         mdlCheckParameters(S);
137         if (ssGetErrorStatus(S) != NULL) {
138             return;
139         }
140     } else {
141         return; /* Parameter mismatch will be reported by Simulink */
142     }
143
144     if (!ssSetNumInputPorts(S, 3)) return;
145     ssSetInputPortWidth(S, 0, 6*N_out);//U_a2
146     ssSetInputPortWidth(S, 1, 2*N_out);//ID_a2
147     ssSetInputPortWidth(S, 2, N_out);//ID-numbers of 2'nd area
148
149     ssSetInputPortDirectFeedThrough(S, 0, 0);
```

```
150     ssSetInputPortDirectFeedThrough(S, 1, 0);
151     ssSetInputPortDirectFeedThrough(S, 2, 0);
152
153     ssSetNumContStates(S, 0);
154     ssSetNumDiscStates(S, 0);
155
156     if (!ssSetNumOutputPorts(S,9)) return;
157     ssSetOutputPortWidth(S, 0, N_a1);//delta
158     ssSetOutputPortWidth(S, 1, N_a1);//Efd
159     ssSetOutputPortWidth(S, 2, N_a1);//Vt
160     ssSetOutputPortWidth(S, 3, N_a1);//It
161     ssSetOutputPortWidth(S, 4, N_a1);//Te
162     ssSetOutputPortWidth(S, 5, N_a1);//w
163     ssSetOutputPortWidth(S, 6, 6*N_a1);//u1_a1, u2_a1, u3_a1, u4_a1, u5_a1, u6_a1
164     ssSetOutputPortWidth(S, 7, 2*N_a1);//ID_a1, IQ_a1
165     ssSetOutputPortWidth(S, 8, N_a1);//ID-number of generators include in this area
166
167     ssSetNumSampleTimes(S, 1);
168     ssSetNumRWork(S, 0);
169     ssSetNumIWork(S, 0);
170     ssSetNumPWork(S, 0);
171     ssSetNumModes(S, 0);
172     ssSetNumNonsampledZCs(S, 0);
173
174     /* Take care when specifying exception free code - see sfuntmpl_doc.c */
175     ssSetOptions(S,
176                  SS_OPTION_WORKS_WITH_CODE_REUSE |
177                  SS_OPTION_EXCEPTION_FREE_CODE |
178                  SS_OPTION_USE_TLC_WITH_ACCELERATOR);
179 }
180
181 /* Function: mdlInitializeSampleTimes =========================================
182  * Abstract:
183  *    Specifiy that we inherit our sample time from the driving block.
184  */
185 static void mdlInitializeSampleTimes(SimStruct *S)
186 {
187         ssSetSampleTime(S, 0, h);
188         ssSetOffsetTime(S, 0, 0.0);
189         ssSetModelReferenceSampleTimeDefaultInheritance(S);
190 }
191
192 #define MDL_START  /* Change to #undef to remove function */
193 #if defined(MDL_START)
194   /* Function: mdlStart =======================================================
195    * Abstract:
196    *    This function is called once at start of model execution. If you
197    *    have states that should be initialized once, this is the place
198    *    to do it.
199    */
200 static void mdlStart(SimStruct *S)
201 {
202     char tab;
203     int i,j,k;
204     float temp;
205     int ID_num;
206     float H, D;
207     float xd, xq, xdp, xqp, xds, xqs, xl, Ra, Tdo_p, Tdo_s, Tqo_p, Tqo_s;
208     float KA, TR, Efd_max, Efd_min;
209     float Kstab, TW, T1, T2, vs_max, vs_min;
210     int EXC, PSS;
211
212     float delta,slip,sfd,sd1,sq1,sq2,v1,v2,v3;
213     float iq,id,vq,vd,Iq,Id;
214     float ifd,efd,Ef,id1,iq1,iq2;
215     float sad,saq,Vref,Eds,Eqs;
216     float Te,Tm,Pt,Qt;
217     float u1,u2,u3,u4,u5,u6;
218
219     FILE *fp;
220
221     time = 0;
222     PS=0; EX=0; no_EXPS=0;
223     /*****************************************************************
224     *reading files and filling matrices*******************************
225     *****************************************************************/
226     //1- Generators
227     j=0;
228   fp=fopen("Gen_params.txt","r");
229   for(i=1;i<N+1;i++){
230         fscanf(fp,"%d%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f\
231       %c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%d%c%d%c%",
232             &ID_num,&tab,
233             &H,&tab,&D,&tab,
234             &xd,&tab,&xq,&tab,&xdp,&tab,&xqp,&tab,&xds,&tab,&xl,&tab,&Ra,&tab,
235             &Tdo_p,&tab,&Tdo_s,&tab,&Tqo_p,&tab,&Tqo_s,&tab,
236             &KA,&tab,&TR,&tab,&Efd_max,&tab,&Efd_min,&tab,
237             &Kstab,&tab,&TW,&tab,&T1,&tab,&T2,&tab,&vs_max,&tab,&vs_min,&tab,&EXC,&tab,&PSS,&tab);
238
239         for(k=1;k<N_a1+1;k++)
240       //Checking if the machin with this ID number includes in this area
241             if (ID_num==Gens[k]){
242                 j=j+1;
243
244                 Gen[j].H = H;
245                 Gen[j].D = D;
246
247                 xqs=xds;
248                 Gen[j].Lad = xd - xl;
249                 Gen[j].Laq = xq - xl;
250
251                 Gen[j].Lfd = Gen[j].Lad*(xdp-xl)/(Gen[j].Lad-xdp+xl);
252                 Gen[j].Lq1 = Gen[j].Laq*(xqp-xl)/(Gen[j].Laq-xqp+xl);
253                 Gen[j].Ld1 = Gen[j].Lad*Gen[j].Lfd*(xds-xl)/
254           (Gen[j].Lad*Gen[j].Lfd-(xds-xl)*(Gen[j].Lad+Gen[j].Lfd));
255                 Gen[j].Lq2 = Gen[j].Laq*Gen[j].Lq1*(xqs-xl)/
256           (Gen[j].Laq*Gen[j].Lq1-(xqs-xl)*(Gen[j].Laq+Gen[j].Lq1));
257
258                 Gen[j].Rfd = (Gen[j].Lad+Gen[j].Lfd)/(Tdo_p*wR);
259                 Gen[j].Rd1 = (Gen[j].Ld1+(Gen[j].Lad*Gen[j].Lfd/(Gen[j].Lad+Gen[j].Lfd)))/(Tdo_s*wR);
260                 Gen[j].Rq1 = (Gen[j].Laq+Gen[j].Lq1)/(Tqo_p*wR);
261                 Gen[j].Rq2 = (Gen[j].Lq2+(Gen[j].Laq*Gen[j].Lq1/(Gen[j].Laq+Gen[j].Lq1)))/(Tqo_s*wR);
262
263                 Gen[j].Lad_sec = 1/((1/Gen[j].Lad)+(1/Gen[j].Lfd)+(1/Gen[j].Ld1));
264                 Gen[j].Laq_sec = 1/((1/Gen[j].Laq)+(1/Gen[j].Lq1)+(1/Gen[j].Lq2));
265
266                 Gen[j].KA=KA; Gen[j].TR=TR;
267                 Gen[j].Efd_max=Efd_max; Gen[j].Efd_min=Efd_min;
268
269                 Gen[j].Kstab=Kstab; Gen[j].TW=TW;
270                 Gen[j].T1=T1; Gen[j].T2=T2;
271
272                 Gen[j].EXC = EXC;
273                 Gen[j].PSS = PSS;
274
275                 if (Gen[j].PSS==1){
276                     Gen[j].vs_max=vs_max;
277                     Gen[j].vs_min=vs_min;
278                 }else if (Gen[j].EXC==1 && Gen[j].PSS==0){
279                     Gen[j].vs_max=0;
280                     Gen[j].vs_min=0;
281                 }
282             }
283     }
284     fclose(fp);
285
286     j=0;
287     fp=fopen("ss_calc.txt","r");
288   for(i=1;i<N+1;i++){
289         fscanf(fp,"%d%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f\
290           %c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%\
291           f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%f%c%",
292             &ID_num,&tab,
293             &delta,&tab,&slip,&tab,&sfd,&tab,&sd1,&tab,&sq1,&tab,&sq2,&tab,&v1,&tab,&v2,&tab,&v3,&tab,
294             &iq,&tab,&id,&tab,&vq,&tab,&vd,&tab,&Iq,&tab,&Id,&tab,
295             &ifd,&tab,&efd,&tab,&Ef,&tab,&id1,&tab,&iq1,&tab,&iq2,&tab,
296             &sad,&tab,&saq,&tab,&Vref,&tab,&Eds,&tab,&Eqs,&tab,
297             &Te,&tab,&Tm,&tab,&Pt,&tab,&Qt,&tab,
298             &u1,&tab,&u2,&tab,&u3,&tab,&u4,&tab,&u5,&tab,&u6,&tab);
299
300         U1[i] = u1;
301         U2[i] = u2;
```

```
302        U3[i] = u3;
303        U4[i] = u4;
304        U5[i] = u5;
305        U6[i] = u6;
306
307        IQ[i] = Iq;
308        ID[i] = Id;
309
310        for(k=1;k<N_a1+1;k++)
311      //Checking if the machin with this ID number includes in this area
312            if (ID_num==Gens[k]){
313                j=j+1;
314                Gen[j].delta = delta;
315                Gen[j].slip = slip;
316                Gen[j].sfd = sfd;
317                Gen[j].sd1 = sd1;
318                Gen[j].sq1 = sq1;
319                Gen[j].sq2 = sq2;
320                Gen[j].V1 = v1;
321                Gen[j].V2 = v2;
322                Gen[j].V3 = v3;
323
324                Gen[j].iq = iq;// rotor ref.
325                Gen[j].id = id;
326                Gen[j].vq = vq;
327                Gen[j].vd = vd;
328                bus[j].IQ = Iq;//network ref.
329                bus[j].ID = Id;
330                bus[j].It = sqrt(Iq*Iq+Id*Id);
331                Gen[j].vt = sqrt(vq*vq+vd*vd);
332
333                Gen[j].ifd = ifd;
334                Gen[j].efd = efd;
335                Gen[j].Efd = Ef;
336                Gen[j].id1 = id1;
337                Gen[j].iq1 = iq1;
338                Gen[j].iq2 = iq2;
339
340                Gen[j].s_ad = sad;
341                Gen[j].s_aq = saq;
342                Gen[j].Vref = Vref;
343                Gen[j].Eds = Eds;
344                Gen[j].Eqs = Eqs;
345
346                Gen[j].Te = Te;
347                Gen[j].Tm = Tm;
348                Gen[j].Pt = Pt;
349                Gen[j].Qt = Qt;
350
351                Gen[j].u1 = u1;
352                Gen[j].u2 = u2;
353                Gen[j].u3 = u3;
354                Gen[j].u4 = u4;
355                Gen[j].u5 = u5;
356                Gen[j].u6 = u6;
357
358                //previous time step values
359            Gen[j].pre_delta = Gen[j].delta;    Gen[j].pre_slip = Gen[j].slip;
360                Gen[j].pre_sfd = Gen[j].sfd;    Gen[j].pre_sd1 = Gen[j].sd1;
361                Gen[j].pre_sq1 = Gen[j].sq1;    Gen[j].pre_sq2 = Gen[j].sq2;
362                if (Gen[j].PSS==1){
363                    Gen[j].pre_V1 = Gen[j].V1;
364                    Gen[j].pre_V2 = Gen[j].V2;
365                    Gen[j].pre_V3 = Gen[j].V3;
366                }else if (Gen[j].EXC==1 && Gen[j].PSS==0){
367                    Gen[j].pre_V1 = Gen[j].V1;
368                    Gen[j].pre_V2 = 0;
369                    Gen[j].pre_V3 = 0;
370                }
371                Gen[j].pre_Te = Gen[j].Te;
372                Gen[j].pre_ifd = Gen[j].ifd;       Gen[j].pre_efd = Gen[j].efd;
373                Gen[j].pre_id1 = Gen[j].id1;       Gen[j].pre_iq1 = Gen[j].iq1;
374        Gen[j].pre_iq2 = Gen[j].iq2;
375                Gen[j].pre_vt = Gen[j].vt;
376            }
377    }
378      fclose(fp);
379
380      //3- G matrices
381      fp=fopen("G_pref.txt","r");
382    for(i=1;i<N+1;i++){
383      for(j=1;j<N+1;j++){
384        fscanf(fp,"%f%c",&temp,&tab);
385        G.pref[i][j]=temp;
386      }
387    }
388    fclose(fp);
389
390      fp=fopen("G_fault.txt","r");
391    for(i=1;i<N+1;i++){
392      for(j=1;j<N+1;j++){
393        fscanf(fp,"%f%c",&temp,&tab);
394        G.fault[i][j]=temp;
395      }
396    }
397    fclose(fp);
398
399      fp=fopen("G_postf.txt","r");
400    for(i=1;i<N+1;i++){
401      for(j=1;j<N+1;j++){
402        fscanf(fp,"%f%c",&temp,&tab);
403        G.postf[i][j]=temp;
404      }
405    }
406    fclose(fp);
407
408    //4- B matrices
409      fp=fopen("B_pref.txt","r");
410    for(i=1;i<N+1;i++){
411      for(j=1;j<N+1;j++){
412        fscanf(fp,"%f%c",&temp,&tab);
413        B.pref[i][j]=temp;
414      }
415    }
416    fclose(fp);
417
418    fp=fopen("B_fault.txt","r");
419    for(i=1;i<N+1;i++){
420      for(j=1;j<N+1;j++){
421        fscanf(fp,"%f%c",&temp,&tab);
422        B.fault[i][j]=temp;
423      }
424    }
425    fclose(fp);
426
427      fp=fopen("B_postf.txt","r");
428    for(i=1;i<N+1;i++){
429      for(j=1;j<N+1;j++){
430        fscanf(fp,"%f%c",&temp,&tab);
431        B.postf[i][j]=temp;
432      }
433    }
434    fclose(fp);
435  }
436  #endif /*  MDL_START */
437
438  #define MDL_INITIALIZE_CONDITIONS
439  /* Function: mdlInitializeConditions ========================================
440   * Abstract:
441   *    Initialize both discrete states to one.
442   */
443  static void mdlInitializeConditions(SimStruct *S)
444  {
445  }
446
447  /* Function: mdlOutputs =======================================================
448   * Abstract:
449   */
450  static void mdlOutputs(SimStruct *S, int_T tid)
451  {
452      real_T            *y0    = ssGetOutputPortRealSignal(S,0);
453      real_T            *y1    = ssGetOutputPortRealSignal(S,1);
```

```
454     real_T            *y2    = ssGetOutputPortRealSignal(S,2);
455     real_T            *y3    = ssGetOutputPortRealSignal(S,3);
456     real_T            *y4    = ssGetOutputPortRealSignal(S,4);
457     real_T            *y5    = ssGetOutputPortRealSignal(S,5);
458     real_T            *y6    = ssGetOutputPortRealSignal(S,6);
459     real_T            *y7    = ssGetOutputPortRealSignal(S,7);
460     real_T            *y8    = ssGetOutputPortRealSignal(S,8);
461
462     int k,j;
463
464     for(k=1;k<N_a1+1;k++){
465         j=k-1;
466         y0[j]=Gen[k].delta;
467         y1[j]=Gen[k].Efd;
468         y2[j]=Gen[k].vt;
469         y3[j]=bus[k].It;
470         y4[j]=Gen[k].Te;
471         y5[j]=(1+Gen[k].slip)*wR;
472
473         y6[j*6]=Gen[k].u1;
474         y6[j*6+1]=Gen[k].u2;
475         y6[j*6+2]=Gen[k].u3;
476         y6[j*6+3]=Gen[k].u4;
477         y6[j*6+4]=Gen[k].u5;
478         y6[j*6+5]=Gen[k].u6;
479
480         y7[j*2]=bus[k].ID;
481         y7[j*2+1]=bus[k].IQ;
482
483         y8[j]=Gens[k];//sending out the ID-number of machines include in this area
484     }
485 }
486
487 #define MDL_UPDATE
488 /* Function: mdlUpdate =====================================================
489  * Abstract:
490  */
491 static void mdlUpdate(SimStruct *S, int_T tid)
492 {
493     InputRealPtrsType uPtrs1 = ssGetInputPortRealSignalPtrs(S,0);
494   //u1_a2, u2_a2, u3_a2, u4_a2, u5_a2, u6_a2
495     InputRealPtrsType uPtrs2 = ssGetInputPortRealSignalPtrs(S,1);
496   //ID_a2
497     InputRealPtrsType uPtrs3 = ssGetInputPortRealSignalPtrs(S,2);
498   //ID-number of other areas
499
500     int i,j,k,k2,p,k_a,k_g;
501     int  del,sl,sf,d1,q1,q2,v1,v2,v3;
502
503     float **G_NN, **B_NN;
504     float r_Eds[9][N_a1+1], r_Eqs[9][N_a1+1], r_u1[9][N_a1+1], r_u2[9][N_a1+1],
505       r_u3[9][N_a1+1], r_u4[9][N_a1+1], r_u5[9][N_a1+1], r_u6[9][N_a1+1];
506     float r_A1[9][N_a1+1], r_A2[9][N_a1+1], r_A3[9][N_a1+1], r_A4[9][N_a1+1],
507       r_A5[9][N_a1+1], r_A6[9][N_a1+1], r_A7[9][N_a1+1], r_A8[9][N_a1+1];
508     float r_ID[9][N_a1+1], r_IQ[9][N_a1+1], r_id[9][N_a1+1], r_iq[9][N_a1+1],
509       r_sad[9][N_a1+1], r_saq[9][N_a1+1];
510     float r_VtD[9][N_a1+1], r_VtQ[9][N_a1+1];
511
512     float A5[N_a1+1], A6[N_a1+1], A7[N_a1+1], A8[N_a1+1], A9[N_a1+1], A10[N_a1+1];
513     float sigma1[N_a1+1], sigma2[N_a1+1], sigma3[N_a1+1], sigma4[N_a1+1];
514
515     float max_err,err3,err1,err2;
516     float ID_temp, IQ_temp, v_s;
517
518   float *deltaX;
519   float *F,**J,**invJ;
520     int size;
521     int in_area;
522     int end_data, N_A, Area, *addr, *temp_addr, IDs_out[N];
523     FILE *fp;
524
525     UNUSED_ARG(tid); /* not used in single tasking mode */
526
527     for(k=0;k<N_out;k++){
528         IDs_out[k]=ID_num_a2(k);
529     }
530
531   if(!ssIsFirstInitCond(S)){
532
533         p=0;
534         for(k=1;k<N+1;k++){
535             in_area=0;
536             for(j=1;j<N_a1+1;j++){
537                 if(k==Gens[j]){
538                     in_area=1;
539                     k_g=j;
540                 }
541             }
542             if (in_area){
543                 U1[k]=Gen[k_g].u1;
544                 U2[k]=Gen[k_g].u2;
545                 U3[k]=Gen[k_g].u3;
546                 U4[k]=Gen[k_g].u4;
547                 U5[k]=Gen[k_g].u5;
548                 U6[k]=Gen[k_g].u6;
549
550                 ID[k]=bus[k_g].ID;
551                 IQ[k]=bus[k_g].IQ;
552             }
553         }
554
555         for(j=1;j<N_out+1;j++){
556             k=IDs_out[j-1];
557
558             U1[k]=U_a2(p*6);//u1
559             U2[k]=U_a2(p*6+1);//u2
560             U3[k]=U_a2(p*6+2);//u3
561             U4[k]=U_a2(p*6+3);//u4
562             U5[k]=U_a2(p*6+4);//u5
563             U6[k]=U_a2(p*6+5);//u6
564
565             ID[k]=I_a2(p*2);//ID
566             IQ[k]=I_a2(p*2+1);//IQ
567
568             p++;
569         }
570     }
571
572     time++;
573
574     del=0; sl=1; sf=2; d1=3; q1=4; q2=5; v1=6; v2=7; v3=8;
575
576   //Newton's stuffs =>  J.deltaX=-F ,
577   G_NN=matrix(1,N,1,N);
578   B_NN=matrix(1,N,1,N);
579
580     size = 9*N_a1-3*no_EXPS-2*EX;
581     F=vector(1,size);
582   J=matrix(1,size,1,size);
583   invJ=matrix(1,size,1,size);
584   deltaX=vector(1,size);
585
586     for(i=1;i<=size;i++){
587         F[i]=0;
588     deltaX[i]=1.0;
589         for(j=1;j<=size;j++){
590             J[i][j]=0;
591             invJ[i][j]=0;
592         }
593     }
594
595     if(time<duration1){
596         for(i=1;i<N+1;i++){
597             for(j=1;j<N+1;j++){
598                 G_NN[i][j]=G.pref[i][j];
599                 B_NN[i][j]=B.pref[i][j];
600             }
601         }
602     }else if ((time >= duration1) && (time < duration1+duration2)){
603         for(i=1;i<N+1;i++){
604             for(j=1;j<N+1;j++){
605                 G_NN[i][j]=G.fault[i][j];
```

```
606                 B_NN[i][j]=B.fault[i][j];
607             }
608         }
609     }else if (time >= duration1+duration2){
610         for(i=1;i<N+1;i++){
611             for(j=1;j<N+1;j++){
612                 G_NN[i][j]=G.postf[i][j];
613                 B_NN[i][j]=B.postf[i][j];
614             }
615         }
616     }
617
618   for(k=1;k<N_a1+1;k++){
619     //previous time step values
620     Gen[k].pre_delta = Gen[k].delta;    Gen[k].pre_slip = Gen[k].slip;
621     Gen[k].pre_sfd = Gen[k].sfd;    Gen[k].pre_sd1 = Gen[k].sd1;
622     Gen[k].pre_sq1 = Gen[k].sq1;    Gen[k].pre_sq2 = Gen[k].sq2;
623     Gen[k].pre_V1 = Gen[k].V1;          Gen[k].pre_V2 = Gen[k].V2;
624     Gen[k].pre_V3 = Gen[k].V3;
625
626         Gen[k].pre_Te = Gen[k].Te;
627
628         Gen[k].pre_ifd = Gen[k].ifd;       Gen[k].pre_efd = Gen[k].efd;
629         Gen[k].pre_id1 = Gen[k].id1;       Gen[k].pre_iq1 = Gen[k].iq1;
630       Gen[k].pre_iq2 = Gen[k].iq2;       Gen[k].pre_vt = Gen[k].vt;
631     }
632
633     max_err=1;
634   while(max_err>epsilon){
635         j=1;
636     for(k=1;k<N_a1+1;k++){
637         F[j]=-((Gen[k].delta-0.5*h*wR*Gen[k].slip) -
638             Gen[k].pre_delta - 0.5*h*wR*Gen[k].pre_slip);
639         j++;
640         F[j]=-((2*Gen[k].H+Gen[k].D*h)*Gen[k].slip +
641             (Gen[k].D*h-2*Gen[k].H)*Gen[k].pre_slip -
642             0.5*h*(Gen[k].Te+Gen[k].pre_Te) - h*Gen[k].Tm);
643         j++;
644         F[j]=-((Gen[k].sfd-Gen[k].pre_sfd) -
645             0.5*h*wR*(Gen[k].efd+Gen[k].pre_efd) +
646             0.5*h*wR*Gen[k].Rfd*(Gen[k].ifd+Gen[k].pre_ifd));
647         j++;
648         F[j]=-((Gen[k].sd1-Gen[k].pre_sd1) +
649             0.5*h*wR*Gen[k].Rd1*(Gen[k].id1+Gen[k].pre_id1));
650         j++;
651         F[j]=-((Gen[k].sq1-Gen[k].pre_sq1) +
652             0.5*h*wR*Gen[k].Rq1*(Gen[k].iq1+Gen[k].pre_iq1));
653         j++;
654         F[j]=-((Gen[k].sq2-Gen[k].pre_sq2) +
655             0.5*h*wR*Gen[k].Rq2*(Gen[k].iq2+Gen[k].pre_iq2));
656         j++;
657         if (Gen[k].EXC==1 && Gen[k].PSS==1){
658           F[j]=-((Gen[k].TR+0.5*h)*Gen[k].V1 -
659               0.5*h*(Gen[k].vt+Gen[k].pre_vt) +
660               (0.5*h-Gen[k].TR)*Gen[k].pre_V1);
661           j++;
662           F[j]=-((Gen[k].TW+0.5*h)*Gen[k].V2 -
663               Gen[k].Kstab*Gen[k].TW*(Gen[k].slip-Gen[k].pre_slip) +
664               (0.5*h-Gen[k].TW)*Gen[k].pre_V2);
665           j++;
666           F[j]=-((Gen[k].T2+0.5*h)*Gen[k].V3 -
667               (Gen[k].T1+0.5*h)*Gen[k].V2 + (0.5*h-Gen[k].T2)*Gen[k].pre_V3 +
668               (Gen[k].T1-0.5*h)*Gen[k].pre_V2);
669           j++;
670         }
671     }
672
673       /**********************************
674       *Jacobian Matrix*******************
675       **********************************/
676
677        if(ssIsFirstInitCond(S)){
678
679            for(k=1;k<N_a1+1;k++){
680                 J[9*(k-1)+1][9*(k-1)+1] = 1;
681                 J[9*(k-1)+1][9*(k-1)+2] = -0.5*h*wR;
682
683                 J[9*(k-1)+2][9*(k-1)+1] = 0;
684                 J[9*(k-1)+2][9*(k-1)+2] = 2*Gen[k].H+Gen[k].D*h;
685                 J[9*(k-1)+2][9*(k-1)+3] = 0;
686                 J[9*(k-1)+2][9*(k-1)+4] = 0;
687                 J[9*(k-1)+2][9*(k-1)+5] = 0;
688                 J[9*(k-1)+2][9*(k-1)+6] = 0;
689
690                 J[9*(k-1)+3][9*(k-1)+1] = 0;
691                 J[9*(k-1)+3][9*(k-1)+3] = 1+0.5*h*wR*Gen[k].Rfd*(1/Gen[k].Lfd);
692                 J[9*(k-1)+3][9*(k-1)+4] = 0;
693                 J[9*(k-1)+3][9*(k-1)+5] = 0;
694                 J[9*(k-1)+3][9*(k-1)+6] = 0;
695                 if (Gen[k].EXC==1 && Gen[k].PSS==1){
696                     J[9*(k-1)+3][9*(k-1)+7] = -0.5*h*wR*(-Gen[k].KA*Gen[k].Rfd/Gen[k].Lad);
697                     J[9*(k-1)+3][9*(k-1)+9] = -0.5*h*wR*(Gen[k].KA*Gen[k].Rfd/Gen[k].Lad);
698                 }
699
700                 J[9*(k-1)+4][9*(k-1)+1] = 0;
701                 J[9*(k-1)+4][9*(k-1)+3] = 0;
702                 J[9*(k-1)+4][9*(k-1)+4] = 1+0.5*h*wR*Gen[k].Rd1*(1/Gen[k].Ld1);
703                 J[9*(k-1)+4][9*(k-1)+5] = 0;
704                 J[9*(k-1)+4][9*(k-1)+6] = 0;
705
706                 J[9*(k-1)+5][9*(k-1)+1] = 0;
707                 J[9*(k-1)+5][9*(k-1)+3] = 0;
708                 J[9*(k-1)+5][9*(k-1)+4] = 0;
709                 J[9*(k-1)+5][9*(k-1)+5] = 1+0.5*h*wR*Gen[k].Rq1*(1/Gen[k].Lq1);
710                 J[9*(k-1)+5][9*(k-1)+6] = 0;
711
712                 J[9*(k-1)+6][9*(k-1)+1] = 0;
713                 J[9*(k-1)+6][9*(k-1)+3] = 0;
714                 J[9*(k-1)+6][9*(k-1)+4] = 0;
715                 J[9*(k-1)+6][9*(k-1)+5] = 0;
716                 J[9*(k-1)+6][9*(k-1)+6] = 1+0.5*h*wR*Gen[k].Rq2*(1/Gen[k].Lq2);
717
718                 if (Gen[k].EXC==1 && Gen[k].PSS==1){
719                     J[9*(k-1)+7][9*(k-1)+1] = 0;
720                     J[9*(k-1)+7][9*(k-1)+3] = 0;
721                     J[9*(k-1)+7][9*(k-1)+4] = 0;
722                     J[9*(k-1)+7][9*(k-1)+5] = 0;
723                     J[9*(k-1)+7][9*(k-1)+6] = 0;
724                     J[9*(k-1)+7][9*(k-1)+7] = Gen[k].TR+0.5*h;
725
726                     J[9*(k-1)+8][9*(k-1)+2] = -Gen[k].Kstab*Gen[k].TW;
727                     J[9*(k-1)+8][9*(k-1)+8] = Gen[k].TW+0.5*h;
728
729                     J[9*(k-1)+9][9*(k-1)+8] = -(Gen[k].T1+0.5*h);
730                     J[9*(k-1)+9][9*(k-1)+9] = (Gen[k].T2+0.5*h);
731                 }
732             }
733         }
734         //*******************************
735         else{
736             for(k=1;k<N_a1+1;k++){
737                 J[9*(k-1)+1][9*(k-1)+1] = 1;
738                 J[9*(k-1)+1][9*(k-1)+2] = -0.5*h*wR;
739
740                 J[9*(k-1)+2][9*(k-1)+1] = -0.5*h*r_Te[del][k];
741                 J[9*(k-1)+2][9*(k-1)+2] = 2*Gen[k].H+Gen[k].D*h;
742                 J[9*(k-1)+2][9*(k-1)+3] = -0.5*h*r_Te[sf][k];
743                 J[9*(k-1)+2][9*(k-1)+4] = -0.5*h*r_Te[d1][k];
744                 J[9*(k-1)+2][9*(k-1)+5] = -0.5*h*r_Te[q1][k];
745                 J[9*(k-1)+2][9*(k-1)+6] = -0.5*h*r_Te[q2][k];
746
747                 J[9*(k-1)+3][9*(k-1)+1] = 0.5*h*wR*Gen[k].Rfd*r_ifd[del][k];
748                 J[9*(k-1)+3][9*(k-1)+3] = 1+0.5*h*wR*Gen[k].Rfd*r_ifd[sf][k];
749                 J[9*(k-1)+3][9*(k-1)+4] = 0.5*h*wR*Gen[k].Rfd*r_ifd[d1][k];
750                 J[9*(k-1)+3][9*(k-1)+5] = 0.5*h*wR*Gen[k].Rfd*r_ifd[q1][k];
751                 J[9*(k-1)+3][9*(k-1)+6] = 0.5*h*wR*Gen[k].Rfd*r_ifd[q2][k];
752                 if (Gen[k].EXC==1 && Gen[k].PSS==1){
753                     J[9*(k-1)+3][9*(k-1)+7] = -0.5*h*wR*r_efd[v1][k];
754                     J[9*(k-1)+3][9*(k-1)+9] = -0.5*h*wR*r_efd[v3][k];
755                 }
756
757                 J[9*(k-1)+4][9*(k-1)+1] = 0.5*h*wR*Gen[k].Rd1*r_id1[del][k];
```

```
758                 J[9*(k-1)+4][9*(k-1)+3] = 0.5*h*wR*Gen[k].Rd1*r_id1[sf][k];
759                 J[9*(k-1)+4][9*(k-1)+4] = 1+0.5*h*wR*Gen[k].Rd1*r_id1[d1][k];
760                 J[9*(k-1)+4][9*(k-1)+5] = 0.5*h*wR*Gen[k].Rd1*r_id1[q1][k];
761                 J[9*(k-1)+4][9*(k-1)+6] = 0.5*h*wR*Gen[k].Rd1*r_id1[q2][k];
762
763                 J[9*(k-1)+5][9*(k-1)+1] = 0.5*h*wR*Gen[k].Rq1*r_iq1[del][k];
764                 J[9*(k-1)+5][9*(k-1)+3] = 0.5*h*wR*Gen[k].Rq1*r_iq1[sf][k];
765                 J[9*(k-1)+5][9*(k-1)+4] = 0.5*h*wR*Gen[k].Rq1*r_iq1[d1][k];
766                 J[9*(k-1)+5][9*(k-1)+5] = 1+0.5*h*wR*Gen[k].Rq1*r_iq1[q1][k];
767                 J[9*(k-1)+5][9*(k-1)+6] = 0.5*h*wR*Gen[k].Rq1*r_iq1[q2][k];
768
769                 J[9*(k-1)+6][9*(k-1)+1] = 0.5*h*wR*Gen[k].Rq2*r_iq2[del][k];
770                 J[9*(k-1)+6][9*(k-1)+3] = 0.5*h*wR*Gen[k].Rq2*r_iq2[sf][k];
771                 J[9*(k-1)+6][9*(k-1)+4] = 0.5*h*wR*Gen[k].Rq2*r_iq2[d1][k];
772                 J[9*(k-1)+6][9*(k-1)+5] = 0.5*h*wR*Gen[k].Rq2*r_iq2[q1][k];
773                 J[9*(k-1)+6][9*(k-1)+6] = 1+0.5*h*wR*Gen[k].Rq2*r_iq2[q2][k];
774
775                 if (Gen[k].EXC==1 && Gen[k].PSS==1){
776                     J[9*(k-1)+7][9*(k-1)+1] = -0.5*h*r_Vt[del][k];
777                     J[9*(k-1)+7][9*(k-1)+3] = -0.5*h*r_Vt[sf][k];
778                     J[9*(k-1)+7][9*(k-1)+4] = -0.5*h*r_Vt[d1][k];
779                     J[9*(k-1)+7][9*(k-1)+5] = -0.5*h*r_Vt[q1][k];
780                     J[9*(k-1)+7][9*(k-1)+6] = -0.5*h*r_Vt[q2][k];
781                     J[9*(k-1)+7][9*(k-1)+7] = Gen[k].TR+0.5*h;
782
783                     J[9*(k-1)+8][9*(k-1)+2] = -Gen[k].Kstab*Gen[k].TW;
784                     J[9*(k-1)+8][9*(k-1)+8] = Gen[k].TW+0.5*h;
785
786                     J[9*(k-1)+9][9*(k-1)+8] = -(Gen[k].T1+0.5*h);
787                     J[9*(k-1)+9][9*(k-1)+9] = (Gen[k].T2+0.5*h);
788                 }
789             }
790             /*end of Jacobian*******************/
791         }//else of first...
792
793         invJ = inverse_of_matrix(J,size);
794         deltaX = mtrx_vect_multiply(invJ,size,size,F,size);
795         j=1;
796         for(k=1;k<N_a1+1;k++){
797           Gen[k].delta = Gen[k].delta + deltaX[j];
798           j++;
799           Gen[k].slip = Gen[k].slip + deltaX[j];
800           j++;
801           Gen[k].sfd = Gen[k].sfd + deltaX[j];
802           j++;
803           Gen[k].sd1 = Gen[k].sd1 + deltaX[j];
804           j++;
805           Gen[k].sq1 = Gen[k].sq1 + deltaX[j];
806           j++;
807           Gen[k].sq2 = Gen[k].sq2 + deltaX[j];
808           j++;
809
810           if (Gen[k].EXC==1 && Gen[k].PSS==1){
811             Gen[k].V1 = Gen[k].V1 + deltaX[j];
812             j++;
813             Gen[k].V2 = Gen[k].V2 + deltaX[j];
814             j++;
815             Gen[k].V3 = Gen[k].V3 + deltaX[j];
816             j++;
817           }
818
819         Gen[k].Eds = Gen[k].Laq_sec*
820           ((Gen[k].sq1/Gen[k].Lq1)+(Gen[k].sq2/Gen[k].Lq2));
821         Gen[k].Eqs = Gen[k].Lad_sec*
822           ((Gen[k].sfd/Gen[k].Lfd)+(Gen[k].sd1/Gen[k].Ld1));
823         Gen[k].u5 = -cos(Gen[k].delta)*Gen[k].Eds -
824           sin(Gen[k].delta)*Gen[k].Eqs;
825         Gen[k].u6 = cos(Gen[k].delta)*Gen[k].Eqs -
826           sin(Gen[k].delta)*Gen[k].Eds;
827       }
828
829       for(k=1;k<N+1;k++){
830         in_area=0;
831         for(j=1;j<N_a1+1;j++){
832           if(k==Gens[j]){
833             in_area=1;
834             k_g=j;
835           }
836         }
837         if (in_area){
838           U1[k]=Gen[k_g].u1;
839           U2[k]=Gen[k_g].u2;
840           U3[k]=Gen[k_g].u3;
841           U4[k]=Gen[k_g].u4;
842           U5[k]=Gen[k_g].u5;
843           U6[k]=Gen[k_g].u6;
844         }
845       }
846
847         for(k=1;k<N_a1+1;k++){
848             k_a = Gens[k];
849             A7[k]=0; A8[k]=0; A9[k]=0; A10[k]=0;
850             A5[k]=1 + B_NN[k_a][k_a]*Gen[k].u3 - G_NN[k_a][k_a]*Gen[k].u1;
851             A6[k]=1 - G_NN[k_a][k_a]*Gen[k].u4 - B_NN[k_a][k_a]*Gen[k].u2;
852
853             for(k2=1;k2<N+1;k2++){
854                 A7[k]=A7[k] + G_NN[k_a][k2]*U5[k2] - B_NN[k_a][k2]*U6[k2];
855                 A8[k]=A8[k] + G_NN[k_a][k2]*U6[k2] + B_NN[k_a][k2]*U5[k2];
856             }
857
858             err3=1;
859             while (err3 > 0.001){
860                 sigma1[k]=0; sigma2[k]=0; sigma3[k]=0; sigma4[k]=0;
861                 ID_temp = bus[k].ID;
862                 IQ_temp = bus[k].IQ;
863                 for (k2=1;k2<N+1;k2++){
864
865                     if(k2!=k_a){
866                         sigma1[k] = sigma1[k] + ID[k2]*(G_NN[k_a][k2]*U1[k2]-B_NN[k_a][k2]*U3[k2]);
867                         sigma4[k] = sigma4[k] + IQ[k2]*(G_NN[k_a][k2]*U4[k2]+B_NN[k_a][k2]*U2[k2]);
868                     }
869                     sigma2[k] = sigma2[k] + IQ[k2]*(G_NN[k_a][k2]*U2[k2]-B_NN[k_a][k2]*U4[k2]);
870                     sigma3[k] = sigma3[k] + ID[k2]*(G_NN[k_a][k2]*U3[k2]-B_NN[k_a][k2]*U1[k2]);
871                 }
872                 bus[k].ID = (sigma1[k] + sigma2[k] + A7[k] + A9[k])/A5[k];
873                 bus[k].IQ = (sigma3[k] + sigma4[k] + A8[k] + A10[k])/A6[k];
874
875                 ID[Gens[k]]=bus[k].ID;
876                 IQ[Gens[k]]=bus[k].IQ;
877
878                 err1=fabs(bus[k].ID-ID_temp);
879                 err2=fabs(bus[k].IQ-IQ_temp);
880                 if(err1>=err2)
881                     err3=err1;
882                 else
883                     err3=err2;
884             }
885             //in rorot ref.
886             Gen[k].iq = bus[k].IQ*cos(Gen[k].delta) - bus[k].ID*sin(Gen[k].delta);
887             Gen[k].id = bus[k].ID*cos(Gen[k].delta) + bus[k].IQ*sin(Gen[k].delta);
888             // in commone reference i.e. Network side
889             bus[k].It=sqrt(bus[k].ID*bus[k].ID+bus[k].IQ*bus[k].IQ);
890             bus[k].VD = bus[k].ID*Gen[k].u1 + bus[k].IQ*Gen[k].u2 + Gen[k].u5;
891             bus[k].VQ = bus[k].ID*Gen[k].u3 + bus[k].IQ*Gen[k].u4 + Gen[k].u6;
892
893             Gen[k].vt = sqrt(bus[k].VQ*bus[k].VQ + bus[k].VD*bus[k].VD);
894         }
895
896         for(k=1;k<N_a1+1;k++){
897             Gen[k].s_ad = Gen[k].Lad_sec*(-Gen[k].id+(Gen[k].sfd/Gen[k].Lfd)+(Gen[k].sd1/Gen[k].Ld1));
898             Gen[k].s_aq = Gen[k].Laq_sec*(-Gen[k].iq+(Gen[k].sq1/Gen[k].Lq1)+(Gen[k].sq2/Gen[k].Lq2));
899
900             Gen[k].ifd = (Gen[k].sfd-Gen[k].s_ad)/Gen[k].Lfd;
901             Gen[k].id1 = (Gen[k].sd1-Gen[k].s_ad)/Gen[k].Ld1;
902             Gen[k].iq1 = (Gen[k].sq1-Gen[k].s_aq)/Gen[k].Lq1;
903             Gen[k].iq2 = (Gen[k].sq2-Gen[k].s_aq)/Gen[k].Lq2;
904
905             if (Gen[k].EXC==1 && Gen[k].PSS==1){
906                 v_s = Gen[k].V3;
907                 if (v_s >= Gen[k].vs_max)      v_s = Gen[k].vs_max;
908                 else if (v_s < Gen[k].vs_min)  v_s = Gen[k].vs_min;
909
```

```
910             Gen[k].Efd = Gen[k].KA*(Gen[k].Vref-Gen[k].V1+v_s);
911             if (Gen[k].Efd >= Gen[k].Efd_max)      Gen[k].Efd = Gen[k].Efd_max;
912             else if (Gen[k].Efd < Gen[k].Efd_min)  Gen[k].Efd = Gen[k].Efd_min;
913             Gen[k].efd = Gen[k].Rfd*Gen[k].Efd/Gen[k].Lad;
914         }else{
915             Gen[k].Efd=0;
916         }
917         Gen[k].Te = -(Gen[k].s_ad*Gen[k].iq - Gen[k].s_aq*Gen[k].id);
918     }
919
920 /*********************************
921 *Rond Calculation*****************
922 *********************************/
923
924     for(k=1;k<N_a1+1;k++){
925         k_a=Gens[k];
926         r_Eds[q1][k]=Gen[k].Laq_sec/Gen[k].Lq1;
927         r_Eds[q2][k]=Gen[k].Laq_sec/Gen[k].Lq2;
928         r_Eqs[sf][k]=Gen[k].Lad_sec/Gen[k].Lfd;
929         r_Eqs[d1][k]=Gen[k].Lad_sec/Gen[k].Ld1;
930
931         r_u1[del][k] = 0;
932         r_u2[del][k] = 0;
933         r_u3[del][k] = 0;
934         r_u4[del][k] = 0;
935
936         r_u5[del][k] = sin(Gen[k].delta)*Gen[k].Eds-cos(Gen[k].delta)*Gen[k].Eqs;
937         r_u5[sf][k] = -sin(Gen[k].delta)*r_Eqs[sf][k];
938         r_u5[d1][k] = -sin(Gen[k].delta)*r_Eqs[d1][k];
939         r_u5[q1][k] = -cos(Gen[k].delta)*r_Eds[q1][k];
940         r_u5[q2][k] = -cos(Gen[k].delta)*r_Eds[q2][k];
941
942         r_u6[del][k] = -sin(Gen[k].delta)*Gen[k].Eqs-cos(Gen[k].delta)*Gen[k].Eds;
943         r_u6[sf][k] = cos(Gen[k].delta)*r_Eqs[sf][k];
944         r_u6[d1][k] = cos(Gen[k].delta)*r_Eqs[d1][k];
945         r_u6[q1][k] = -sin(Gen[k].delta)*r_Eds[q1][k];
946         r_u6[q2][k] = -sin(Gen[k].delta)*r_Eds[q2][k];
947
948         r_A1[del][k] = 0;
949         r_A2[del][k] = 0;
950         r_A3[del][k] = 0;
951         r_A4[del][k] = 0;
952         r_A5[del][k] = 0;
953         r_A6[del][k] = 0;
954
955         r_A7[del][k] = G_NN[k_a][k_a]*r_u5[del][k] - B_NN[k_a][k_a]*r_u6[del][k];
956         r_A7[sf][k] = G_NN[k_a][k_a]*r_u5[sf][k] - B_NN[k_a][k_a]*r_u6[sf][k];
957         r_A7[d1][k] = G_NN[k_a][k_a]*r_u5[d1][k] - B_NN[k_a][k_a]*r_u6[d1][k];
958         r_A7[q1][k] = G_NN[k_a][k_a]*r_u5[q1][k] - B_NN[k_a][k_a]*r_u6[q1][k];
959         r_A7[q2][k] = G_NN[k_a][k_a]*r_u5[q2][k] - B_NN[k_a][k_a]*r_u6[q2][k];
960
961         r_A8[del][k] = G_NN[k_a][k_a]*r_u6[del][k] + B_NN[k_a][k_a]*r_u5[del][k];
962         r_A8[sf][k] = G_NN[k_a][k_a]*r_u6[sf][k] + B_NN[k_a][k_a]*r_u5[sf][k];
963         r_A8[d1][k] = G_NN[k_a][k_a]*r_u6[d1][k] + B_NN[k_a][k_a]*r_u5[d1][k];
964         r_A8[q1][k] = G_NN[k_a][k_a]*r_u6[q1][k] + B_NN[k_a][k_a]*r_u5[q1][k];
965         r_A8[q2][k] = G_NN[k_a][k_a]*r_u6[q2][k] + B_NN[k_a][k_a]*r_u5[q2][k];
966
967         r_ID[del][k] = (((bus[k].IQ*r_A2[del][k])+r_A7[del][k])*A5[k]-
968        (sigma1[k]+sigma2[k]+A7[k]+A9[k])*r_A5[del][k])/(pow(A5[k],2));
969         r_ID[sf][k] = r_A7[sf][k]/A5[k];
970         r_ID[d1][k] = r_A7[d1][k]/A5[k];
971         r_ID[q1][k] = r_A7[q1][k]/A5[k];
972         r_ID[q2][k] = r_A7[q2][k]/A5[k];
973
974         r_IQ[del][k] = (((bus[k].ID*r_A3[del][k])+r_A8[del][k])*A6[k]-
975        (sigma3[k]+sigma4[k]+A8[k]+A10[k])*r_A6[del][k])/(pow(A6[k],2));
976         r_IQ[sf][k] = r_A8[sf][k]/A6[k];
977         r_IQ[d1][k] = r_A8[d1][k]/A6[k];
978         r_IQ[q1][k] = r_A8[q1][k]/A6[k];
979         r_IQ[q2][k] = r_A8[q2][k]/A6[k];
980
981         r_id[del][k] = r_ID[del][k]*cos(Gen[k].delta)-bus[k].ID*sin(Gen[k].delta)+
982                        r_IQ[del][k]*sin(Gen[k].delta)+bus[k].IQ*cos(Gen[k].delta);
983         r_id[sf][k] = r_ID[sf][k]*cos(Gen[k].delta)+r_IQ[sf][k]*sin(Gen[k].delta);
984         r_id[d1][k] = r_ID[d1][k]*cos(Gen[k].delta)+r_IQ[d1][k]*sin(Gen[k].delta);
985         r_id[q1][k] = r_ID[q1][k]*cos(Gen[k].delta)+r_IQ[q1][k]*sin(Gen[k].delta);
986         r_id[q2][k] = r_ID[q2][k]*cos(Gen[k].delta)+r_IQ[q2][k]*sin(Gen[k].delta);
987
988         r_iq[del][k] = r_IQ[del][k]*cos(Gen[k].delta)-bus[k].IQ*sin(Gen[k].delta)-
989                       r_ID[del][k]*sin(Gen[k].delta)-bus[k].ID*cos(Gen[k].delta);
990         r_iq[sf][k] = r_IQ[sf][k]*cos(Gen[k].delta)-r_ID[sf][k]*sin(Gen[k].delta);
991         r_iq[d1][k] = r_IQ[d1][k]*cos(Gen[k].delta)-r_ID[d1][k]*sin(Gen[k].delta);
992         r_iq[q1][k] = r_IQ[q1][k]*cos(Gen[k].delta)-r_ID[q1][k]*sin(Gen[k].delta);
993         r_iq[q2][k] = r_IQ[q2][k]*cos(Gen[k].delta)-r_ID[q2][k]*sin(Gen[k].delta);
994
995         r_sad[del][k] = -Gen[k].Lad_sec*r_id[del][k];
996         r_sad[sf][k] = Gen[k].Lad_sec*(-r_id[sf][k]+(1/Gen[k].Lfd));
997         r_sad[d1][k] = Gen[k].Lad_sec*(-r_id[d1][k]+(1/Gen[k].Ld1));
998         r_sad[q1][k] = Gen[k].Lad_sec*(-r_id[q1][k]);
999         r_sad[q2][k] = Gen[k].Lad_sec*(-r_id[q2][k]);
1000
1001        r_saq[del][k] = -Gen[k].Laq_sec*r_iq[del][k];
1002        r_saq[sf][k] = Gen[k].Laq_sec*(-r_iq[sf][k]);
1003        r_saq[d1][k] = Gen[k].Laq_sec*(-r_iq[d1][k]);
1004        r_saq[q1][k] = Gen[k].Laq_sec*(-r_iq[q1][k]+(1/Gen[k].Lq1));
1005        r_saq[q2][k] = Gen[k].Laq_sec*(-r_iq[q2][k]+(1/Gen[k].Lq2));
1006
1007        r_Te[del][k] = -(r_sad[del][k]*Gen[k].iq + Gen[k].s_ad*r_iq[del][k] -
1008                       r_saq[del][k]*Gen[k].id - Gen[k].s_aq*r_id[del][k]);
1009        r_Te[sf][k] = -(r_sad[sf][k]*Gen[k].iq + Gen[k].s_ad*r_iq[sf][k] -
1010                      r_saq[sf][k]*Gen[k].id - Gen[k].s_aq*r_id[sf][k]);
1011        r_Te[d1][k] = -(r_sad[d1][k]*Gen[k].iq + Gen[k].s_ad*r_iq[d1][k] -
1012                      r_saq[d1][k]*Gen[k].id - Gen[k].s_aq*r_id[d1][k]);
1013        r_Te[q1][k] = -(r_sad[q1][k]*Gen[k].iq + Gen[k].s_ad*r_iq[q1][k] -
1014                      r_saq[q1][k]*Gen[k].id - Gen[k].s_aq*r_id[q1][k]);
1015        r_Te[q2][k] = -(r_sad[q2][k]*Gen[k].iq + Gen[k].s_ad*r_iq[q2][k] -
1016                      r_saq[q2][k]*Gen[k].id - Gen[k].s_aq*r_id[q2][k]);
1017
1018        r_ifd[del][k] = -r_sad[del][k]/Gen[k].Lfd;
1019        r_ifd[sf][k] = (1-r_sad[sf][k])/Gen[k].Lfd;
1020        r_ifd[d1][k] = -r_sad[d1][k]/Gen[k].Lfd;
1021        r_ifd[q1][k] = -r_sad[q1][k]/Gen[k].Lfd;
1022        r_ifd[q2][k] = -r_sad[q2][k]/Gen[k].Lfd;
1023
1024        r_efd[del][k] = Gen[k].Rfd*r_ifd[del][k];
1025        r_efd[sf][k] = Gen[k].Rfd*r_ifd[sf][k];
1026        r_efd[d1][k] = Gen[k].Rfd*r_ifd[d1][k];
1027        r_efd[q1][k] = Gen[k].Rfd*r_ifd[q1][k];
1028        r_efd[q2][k] = Gen[k].Rfd*r_ifd[q2][k];
1029
1030        r_id1[del][k] = -r_sad[del][k]/Gen[k].Ld1;
1031        r_id1[sf][k] = -r_sad[sf][k]/Gen[k].Ld1;
1032        r_id1[d1][k] = (1-r_sad[d1][k])/Gen[k].Ld1;
1033        r_id1[q1][k] = -r_sad[q1][k]/Gen[k].Ld1;
1034        r_id1[q2][k] = -r_sad[q2][k]/Gen[k].Ld1;
1035
1036        r_iq1[del][k] = -r_saq[del][k]/Gen[k].Lq1;
1037        r_iq1[sf][k] = -r_saq[sf][k]/Gen[k].Lq1;
1038        r_iq1[d1][k] = -r_saq[d1][k]/Gen[k].Lq1;
1039        r_iq1[q1][k] = (1-r_saq[q1][k])/Gen[k].Lq1;
1040        r_iq1[q2][k] = -r_saq[q2][k]/Gen[k].Lq1;
1041
1042        r_iq2[del][k] = -r_saq[del][k]/Gen[k].Lq2;
1043        r_iq2[sf][k] = -r_saq[sf][k]/Gen[k].Lq2;
1044        r_iq2[d1][k] = -r_saq[d1][k]/Gen[k].Lq2;
1045        r_iq2[q1][k] = -r_saq[q1][k]/Gen[k].Lq2;
1046        r_iq2[q2][k] = (1-r_saq[q2][k])/Gen[k].Lq2;
1047
1048        //Excitation
1049        if (Gen[k].EXC==1 && Gen[k].PSS==1){
1050            r_VtD[del][k]=r_ID[del][k]*Gen[k].u1+bus[k].ID*r_u1[del][k] +
1051                          r_IQ[del][k]*Gen[k].u2+bus[k].IQ*r_u2[del][k] +
1052                          r_u5[del][k];
1053            r_VtD[sf][k]=r_ID[sf][k]*Gen[k].u1+r_IQ[sf][k]*Gen[k].u2+r_u5[sf][k];
1054            r_VtD[d1][k]=r_ID[d1][k]*Gen[k].u1+r_IQ[d1][k]*Gen[k].u2+r_u5[d1][k];
1055            r_VtD[q1][k]=r_ID[q1][k]*Gen[k].u1+r_IQ[q1][k]*Gen[k].u2+r_u5[q1][k];
1056            r_VtD[q2][k]=r_ID[q2][k]*Gen[k].u1+r_IQ[q2][k]*Gen[k].u2+r_u5[q2][k];
1057
1058            r_VtQ[del][k]=r_ID[del][k]*Gen[k].u3+bus[k].ID*r_u3[del][k] +
1059                          r_IQ[del][k]*Gen[k].u4+bus[k].IQ*r_u4[del][k] +
1060                          r_u6[del][k];
1061            r_VtQ[sf][k]=r_ID[sf][k]*Gen[k].u3+r_IQ[sf][k]*Gen[k].u4+r_u6[sf][k];
```

```
1062                r_VtQ[d1][k]=r_ID[d1][k]*Gen[k].u3+r_IQ[d1][k]*Gen[k].u4+r_u6[d1][k];
1063                r_VtQ[q1][k]=r_ID[q1][k]*Gen[k].u3+r_IQ[q1][k]*Gen[k].u4+r_u6[q1][k];
1064                r_VtQ[q2][k]=r_ID[q2][k]*Gen[k].u3+r_IQ[q2][k]*Gen[k].u4+r_u6[q2][k];
1065
1066                r_Vt[del][k]=(bus[k].VQ*r_VtQ[del][k]+bus[k].VD*r_VtD[del][k])/
1067                    Gen[k].vt;
1068                r_Vt[sf][k]=(bus[k].VQ*r_VtQ[sf][k]+bus[k].VD*r_VtD[sf][k])/Gen[k].vt;
1069                r_Vt[d1][k]=(bus[k].VQ*r_VtQ[d1][k]+bus[k].VD*r_VtD[d1][k])/Gen[k].vt;
1070                r_Vt[q1][k]=(bus[k].VQ*r_VtQ[q1][k]+bus[k].VD*r_VtD[q1][k])/Gen[k].vt;
1071                r_Vt[q2][k]=(bus[k].VQ*r_VtQ[q2][k]+bus[k].VD*r_VtD[q2][k])/Gen[k].vt;
1072
1073                r_efd[v1][k]=-Gen[k].KA*Gen[k].Rfd/Gen[k].Lad;
1074                r_efd[v3][k]=Gen[k].KA*Gen[k].Rfd/Gen[k].Lad;
1075            }
1076        }
1077        //end of rond calculation//////////////////////////
1078        for(i=1;i<=size;i++)
1079            deltaX[i]=fabs(deltaX[i]);
1080
1081        max_err = my_max(deltaX, 1, size);
1082    }//while
1083
1084  free_matrix(G_NN,1,N,1,N);
1085  free_matrix(B_NN,1,N,1,N);
1086
1087  free_matrix(J,1,size,1,size);
1088  free_matrix(invJ,1,size,1,size);
1089  free_vector(F,1,size);
1090  free_vector(deltaX,1,size);
1091 }
1092
1093 /* Function: mdlTerminate =====================================================
1094  * Abstract:
1095  *    No termination needed, but we are required to have this routine.
1096  */
1097 static void mdlTerminate(SimStruct *S)
1098 {
1099     UNUSED_ARG(S); /* unused input argument */
1100 }
1101
1102
1103 #ifdef  MATLAB_MEX_FILE    /* Is this file being compiled as a MEX-file? */
1104 #include "simulink.c"      /* MEX-file interface mechanism */
1105 #else
1106 #include "cg_sfun.h"       /* Code generation registration function */
1107 #endif
```

B
Performance Log for Real-time Simulation of a Large-Scale System

The performance log in this Appendix shows the RTX-LAB simulator nodes timing schedule for simulation of a large-scale power system described in Chapter 3. For this power system, all the 8 nodes of the simulator have been used; however, to save space, here just 2 nodes are being demonstrated: the master node (sm_area1), and one of the slave nodes (ss_area2). Exploring this report is useful to understand how the parallel architecture of the real-time simulator works. The sequence of events, the duration of each event, and the probable obstacles that might be in the simulation can be extracted from the performance log. However, the `Number of Overruns` is the most important item in this report. If this item is 0 for all nodes the simulation was indeed running in real-time.

File version: 1
Model name: rt_312b_8a.mdl
File created on: Friday-April 23-2010 at 12:12:16
Model sample time: 0.004s
Time factor: 1

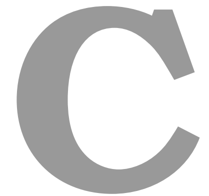
Model step			1772	1773	1774	1775	1776
sm_area1	3000.000 Mhz						
	Status Update						
		Duration	0.712667	0.705	0.785	0.712333	0.712333
		Start	27.21485	27.21885	27.22285	27.22685	27.23085
		Stop	27.21485	27.21885	27.22285	27.22685	27.23085
	Data Acquisition						
		Duration	3.07	3.355	3.157667	3.082333	3.092667
		Start	27.21692	27.22104	27.22493	27.22892	27.23292
		Stop	27.21692	27.22105	27.22493	27.22892	27.23292
	Multi-Receive						
		Duration	1899.548	1901.052	1770.955	1890.463	1899.613
		Start	27.21295	27.21695	27.22108	27.22496	27.22895
		Stop	27.21485	27.21885	27.22285	27.22685	27.23085
	Handle target requests						
		Duration	0.085	0.077333	0.082333	0.075	0.082333
		Start	27.21692	27.22105	27.22493	27.22892	27.23292
		Stop	27.21692	27.22105	27.22493	27.22892	27.23292
	Handle host requests						
		Duration	0.04	0.055	0.04	0.042667	0.04
		Start	27.21692	27.22105	27.22493	27.22892	27.23292
		Stop	27.21692	27.22105	27.22493	27.22892	27.23292
	Pre-execution computation time						
		Duration	0.035	0.035	0.032667	0.032333	0.035
		Start	27.21292	27.21692	27.22105	27.22493	27.22892
		Stop	27.21292	27.21692	27.22105	27.22493	27.22892
	Major computation time						
		Duration	2068.946	2196.785	2078.678	2069.99	2067.637
		Start	27.21292	27.21692	27.22105	27.22493	27.22892
		Stop	27.21692	27.22104	27.22493	27.22892	27.23292
	Minor computation time						
		Duration	0.077667	0.077333	0.077667	0.075	0.077333
		Start	27.21692	27.22105	27.22493	27.22892	27.23292
		Stop	27.21692	27.22105	27.22493	27.22892	27.23292
	Post-execution computation time						
		Duration	0.035	0.035	0.037667	0.035	0.037667
		Start	27.21692	27.22105	27.22493	27.22892	27.23292
		Stop	27.21692	27.22105	27.22493	27.22892	27.23292
	Execution Cycle						
		Duration	3998.498	4128.495	3880.07	3991.13	3997.003
		Start	27.21292	27.21692	27.22105	27.22493	27.22892

Stop	27.21692	27.22105	27.22493	27.22892	27.23292
Total Step Size					
Duration	3999.535	4129.55	3881.11	3992.137	3998.063
Start	27.21292	27.21692	27.22105	27.22493	27.22892
Stop	27.21692	27.22105	27.22493	27.22892	27.23292
Idle					
Duration	1898.627	1900.14	1770.065	1889.452	1898.817
Start	27.21295	27.21695	27.22108	27.22496	27.22895
Stop	27.21485	27.21885	27.22285	27.22685	27.23085
Number of Overruns					
Duration	**0**	**0**	**0**	**0**	**0**
Start	**0**	**0**	**0**	**0**	**0**
Stop	**0**	**0**	**0**	**0**	**0**
Send_RT to ss_area2					
Duration	3.847333	3.852333	4.165	4.072333	3.855
Start	27.21292	27.21692	27.22105	27.22493	27.22892
Stop	27.21293	27.21693	27.22106	27.22494	27.22893
Recv_RT from ss_area2					
Duration	1900.585	1902.087	1772.103	1891.555	1900.678
Start	27.21295	27.21695	27.22108	27.22496	27.22895
Stop	27.21485	27.21885	27.22285	27.22685	27.23085
Send_RT to ss_area3					
Duration	3.39	3.47	3.377667	3.59	3.375
Start	27.21293	27.21693	27.22106	27.22494	27.22893
Stop	27.21293	27.21693	27.22106	27.22494	27.22893
Recv_RT from ss_area3					
Duration	0.147333	0.142333	0.145	0.127333	0.137333
Start	27.21485	27.21885	27.22285	27.22685	27.23085
Stop	27.21485	27.21885	27.22285	27.22685	27.23085
Send_RT to ss_area4					
Duration	3.487667	3.495	3.522333	3.8	3.512667
Start	27.21293	27.21693	27.22106	27.22494	27.22893
Stop	27.21293	27.21693	27.22106	27.22494	27.22894
Recv_RT from ss_area4					
Duration	0.037333	0.037333	0.04	0.04	0.04
Start	27.21485	27.21885	27.22285	27.22685	27.23085
Stop	27.21485	27.21885	27.22285	27.22685	27.23085
Send_RT to ss_area5					
Duration	3.482333	3.467667	3.477333	3.52	3.442667
Start	27.21293	27.21693	27.22106	27.22494	27.22894
Stop	27.21294	27.21694	27.22107	27.22495	27.22894
Recv_RT from ss_area5					
Duration	0.037667	0.037667	0.037333	0.035	0.035
Start	27.21485	27.21885	27.22285	27.22685	27.23085
Stop	27.21485	27.21885	27.22285	27.22685	27.23085
Send_RT to ss_area6					
Duration	3.49	3.48	3.617333	3.495	3.53

		Start	27.21294	27.21694	27.22107	27.22495	27.22894
		Stop	27.21294	27.21694	27.22107	27.22495	27.22894
	Recv_RT from ss_area6						
		Duration	0.035	0.037667	0.035	0.035	0.035
		Start	27.21485	27.21885	27.22285	27.22685	27.23085
		Stop	27.21485	27.21885	27.22285	27.22685	27.23085
	Send_RT to ss_area7						
		Duration	3.755	3.687667	3.507667	3.51	3.455
		Start	27.21294	27.21694	27.22107	27.22495	27.22894
		Stop	27.21294	27.21694	27.22107	27.22495	27.22895
	Recv_RT from ss_area7						
		Duration	0.035	0.035	0.035	0.037333	0.037333
		Start	27.21485	27.21885	27.22285	27.22685	27.23085
		Stop	27.21485	27.21885	27.22285	27.22685	27.23085
	Send_RT to ss_area8						
		Duration	3.442333	3.502667	3.442333	3.472667	3.415
		Start	27.21294	27.21694	27.22107	27.22495	27.22895
		Stop	27.21295	27.21695	27.22108	27.22496	27.22895
	Recv_RT from ss_area8						
		Duration	0.037333	0.035	0.037667	0.035	0.035
		Start	27.21485	27.21885	27.22285	27.22685	27.23085
		Stop	27.21485	27.21885	27.22285	27.22685	27.23085
ss_area2	3000.000 Mhz						
	Status Update						
		Duration	0.397333	0.39	0.56	0.4	0.417667
		Start	27.20471	27.20871	27.21271	27.21672	27.22071
		Stop	27.20471	27.20871	27.21271	27.21672	27.22071
	Data Acquisition						
		Duration	2.955	3.605	3.052667	3.105	3.047333
		Start	27.20676	27.21091	27.21478	27.21877	27.22277
		Stop	27.20676	27.21091	27.21478	27.21877	27.22277
	Multi-Receive						
		Duration	1915.198	1923.462	1773.677	1903.32	1914.543
		Start	27.2028	27.20679	27.21094	27.21481	27.2188
		Stop	27.20471	27.20871	27.21271	27.21671	27.22071
	Handle target requests						
		Duration	0.192333	0.367667	0.215	0.192333	0.217667
		Start	27.20676	27.21091	27.21478	27.21877	27.22277
		Stop	27.20676	27.21091	27.21478	27.21877	27.22277
	Handle host requests						
		Duration	0.047667	0.055	0.04	0.047667	0.04
		Start	27.20676	27.21091	27.21478	27.21877	27.22277
		Stop	27.20676	27.21091	27.21478	27.21877	27.22277
	Pre-execution computation time						
		Duration	0.035	0.035	0.035	0.032667	0.032333
		Start	27.20277	27.20676	27.21091	27.21478	27.21877

Stop	27.20277	27.20676	27.21091	27.21478	27.21877
Major computation time					
Duration	2047.63	2192.67	2067.217	2052.997	2057.197
Start	27.20277	27.20676	27.21091	27.21478	27.21877
Stop	27.20676	27.21091	27.21478	27.21877	27.22277
Minor computation time					
Duration	0.082333	0.075	0.08	0.09	0.077667
Start	27.20676	27.21091	27.21478	27.21877	27.22277
Stop	27.20676	27.21091	27.21478	27.21877	27.22277
Post-execution computation time					
Duration	0.04	0.032667	0.037667	0.045	0.042667
Start	27.20676	27.21091	27.21478	27.21877	27.22277
Stop	27.20676	27.21091	27.21478	27.21877	27.22277
Execution Cycle					
Duration	3992.143	4146.345	3871.003	3986.778	4001.163
Start	27.20277	27.20676	27.21091	27.21478	27.21877
Stop	27.20676	27.21091	27.21478	27.21877	27.22277
Total Step Size					
Duration	3993.458	4147.847	3872.493	3987.94	4002.575
Start	27.20277	27.20676	27.21091	27.21478	27.21877
Stop	27.20676	27.21091	27.21478	27.21877	27.22277
Idle					
Duration	1914.25	1922.538	1772.837	1902.337	1913.72
Start	27.2028	27.20679	27.21094	27.21481	27.2188
Stop	27.20471	27.20871	27.21271	27.21671	27.22071
Number of Overruns					
Duration	**0**	**0**	**0**	**0**	**0**
Start	**0**	**0**	**0**	**0**	**0**
Stop	**0**	**0**	**0**	**0**	**0**
Send_RT to sm_area1					
Duration	4.287667	3.992333	4.405	4.105	4.13
Start	27.20277	27.20677	27.21091	27.21479	27.21877
Stop	27.20278	27.20677	27.21092	27.21479	27.21878
Recv_RT from sm_area1					
Duration	1915.917	1924.155	1774.568	1904.038	1915.275
Start	27.2028	27.20679	27.21094	27.21481	27.2188
Stop	27.20471	27.20871	27.21271	27.21672	27.22071
Send_RT to ss_area3					
Duration	3.387667	3.382333	3.377667	3.397667	3.392667
Start	27.20278	27.20677	27.21092	27.21479	27.21878
Stop	27.20278	27.20677	27.21092	27.21479	27.21878
Recv_RT from ss_area3					
Duration	0.14	0.162667	0.15	0.16	0.145
Start	27.20471	27.20871	27.21271	27.21672	27.22071
Stop	27.20471	27.20871	27.21271	27.21672	27.22071
Send_RT to ss_area4					
Duration	3.42	3.435	3.467667	3.467333	3.477333

	Start	27.20278	27.20677	27.21092	27.21479	27.21878
	Stop	27.20278	27.20678	27.21092	27.2148	27.21878
Recv_RT from ss_area4						
	Duration	0.04	0.04	0.037667	0.04	0.037667
	Start	27.20471	27.20871	27.21271	27.21672	27.22071
	Stop	27.20471	27.20871	27.21271	27.21672	27.22072
Send_RT to ss_area5						
	Duration	3.397333	3.48	3.487667	3.637667	3.485
	Start	27.20278	27.20678	27.21092	27.2148	27.21878
	Stop	27.20279	27.20678	27.21093	27.2148	27.21879
Recv_RT from ss_area5						
	Duration	0.035	0.035	0.037333	0.037667	0.037333
	Start	27.20471	27.20871	27.21271	27.21672	27.22072
	Stop	27.20471	27.20871	27.21271	27.21672	27.22072
Send_RT to ss_area6						
	Duration	3.442333	3.662667	3.46	3.547333	3.455
	Start	27.20279	27.20678	27.21093	27.2148	27.21879
	Stop	27.20279	27.20678	27.21093	27.2148	27.21879
Recv_RT from ss_area6						
	Duration	0.035	0.035	0.037333	0.035	0.037333
	Start	27.20471	27.20871	27.21271	27.21672	27.22072
	Stop	27.20471	27.20872	27.21271	27.21672	27.22072
Send_RT to ss_area7						
	Duration	3.425	3.457333	3.425	3.997667	3.412667
	Start	27.20279	27.20678	27.21093	27.2148	27.21879
	Stop	27.20279	27.20679	27.21094	27.21481	27.2188
Recv_RT from ss_area7						
	Duration	0.037667	0.035	0.035	0.035	0.035
	Start	27.20471	27.20872	27.21271	27.21672	27.22072
	Stop	27.20471	27.20872	27.21271	27.21672	27.22072
Send_RT to ss_area8						
	Duration	3.402667	3.407333	3.642333	3.565	3.397667
	Start	27.20279	27.20679	27.21094	27.21481	27.2188
	Stop	27.2028	27.20679	27.21094	27.21481	27.2188
Recv_RT from ss_area8						
	Duration	0.037667	0.037667	0.035	0.037333	0.035
	Start	27.20471	27.20872	27.21271	27.21672	27.22072
	Stop	27.20471	27.20872	27.21271	27.21672	27.22072



Source Code for the GPU-only Modeling

This is the main program that implements `GPU_Only.cpp` simulation model.

GPU_Only.cpp

```
1 #ifndef _TEST_H
2 #define _TEST_H
3
4 #include <stdio.h>
5 #include <iostream>
6 #include <time.h>
7 #include <stdlib.h>
8 #include <windows.h>
9 #include <vector>
10 #include "cublas.h"
11
12
13 #include "MATRIXonCPU.h"
14 #include "MATRIXonGPU.h"
15
16 ////////////////////////////////////////////////////////////////////////////////
17 void startTimer();
18 int getTimer();
19 /** clock frequency*/
20 __int64 mFreq;
21
22 /** initial timer count*/
23 __int64 mInitTicks;
24 ////////////////////////////////////////////////////////////////////////////////
25 // declaration of CUDA fuctions
26 extern "C" int COPY_CPU_ON_GPU(float* HOST, float* DEVICE, int ROW);
27 extern "C" void COPY_mat_ON_mat(float* DEVICE_from, float* DEVICE_to, int ROW);
28
29 extern "C" void SQRT(float* DEVICE, int ROW);
30 extern "C" void matrixAbs(float* DEVICE, int ROW);
31 extern "C" void vecSin(float* DEVICE, float* RESULT, int ROW);
32 extern "C" void vecCos(float* DEVICE, float* RESULT, int ROW);
33
34 extern "C" void vecSUMvec(float* DEVICE1, float* DEVICE2, float* RESULT, int ROW);
35 extern "C" void vecDOTvec(float* DEVICE1, float* DEVICE2, float* RESULT, int ROW);
36 extern "C" void vecDIVvec(float* DEVICE1, float* DEVICE2, float* RESULT, int ROW);
37 extern "C" void vecSUMscal(float* DEVICE, float ALPHA, int ROW);
38 extern "C" void vecMIX2(float* DEVICE1, float* DEVICE2, float* DEVICE3,
39             float* DEVICE4, float* DEVICE5, float* DEVICE6,
40             float* DEVICE7, float* DEVICE8, float* DEVICE9,
41             float* RESULT, int ROW);
42 extern "C" void vecCASCADE(float* DEVICE1, float* DEVICE2, float* DEVICE3,
43                float* DEVICE4, float* DEVICE5, float* DEVICE6,
44                float* DEVICE7, float* DEVICE8, float* DEVICE9,
45                float* RESULT, int ROW);
46 extern "C" void vecSPLIT3(float* DEVICE1, float* RESULT, int offset, int ORDER);
47
48 extern "C" void limiter(float* DEVICE1, float* DEVICE2, float* DEVICE3, int ROW);
49 extern "C" void diag(float* DEVICE_VEC, float* RESULT_MAT, int ROW);
50 extern "C" void copy_to_Jacob(float* d_J_Fi_VEC, float* d_J_MAT,
51                int Block_mod, int MIN_index, int MAX_index, int ROW);
52 extern "C" void copy_to_Jacob2(float* d_J_Fi_VEC, float* d_J_MAT, int index, int ROW);
53
54 ////////////////////////////////////////////////////////////////////////////////
55
56 const int MAX_GPU_COUNT = 1;//=N_a
57
58 //create an array of 2 event Handles
59 HANDLE hEvent[MAX_GPU_COUNT];
60 HANDLE StartEvent[MAX_GPU_COUNT];
61
62 int blockWidth=64;
63 int time2=0;
64 ////////////////////////////////////////////////////////////////////////////////
65 // Power System Components
66 ////////////////////////////////////////////////////////////////////////////////
67 #define N 10 //The total number of all generators
68 #define max_Na 10 //The maximum allowable machines in each area
69 #define N_a 1//The total number of all areas = MAX_GPU_COUNT
70
71 #define pi 3.14159265
72 #define epsilon 0.05
73 #define h 0.01
74 #define wR 2*pi*50
75
76 #define duration1 0.5/h
77 #define duration2 8
78 #define duration3 1/h
79 #define duration duration1+duration2+duration3
80
81 struct G{
82   float pref[N+1][N+1];
83     float fault[N+1][N+1];
84   float postf[N+1][N+1];
85 };
86
87 struct B{
88   float pref[N+1][N+1];
89     float fault[N+1][N+1];
90   float postf[N+1][N+1];
91 };
92
93
94 struct machine{
95     //last iteration values
96     float delta, slip, sfd, sd1, sq1, sq2, V1, V2, V3;
97     float id, iq, vd, vq, vt;
98     float ifd, efd, Efd, Vref, id1, iq1, iq2;
99     float s_ad, s_aq;
100    float Eds, Eqs;
101    float Te, Tm, Pt, Qt;
102    float u1, u2, u3, u4, u5, u6;
103  float H, D;
104    //Fundamental parameters which are calculated based on the Standard params
105    float Lad, Laq, Lfd, Ld1, Lq1, Lq2, Rfd, Rd1, Rq1, Rq2, Lad_sec, Laq_sec;
106
107    //previous time step values
108    float pre_delta, pre_slip, pre_sfd, pre_sd1, pre_sq1, pre_sq2, pre_V1, pre_V2, pre_V3;
109    float pre_Te;
110    float pre_ifd, pre_efd, pre_id1, pre_iq1, pre_iq2;
111    float pre_vt;
112
113    float KA, TR, Efd_max, Efd_min;
114    float Kstab, TW, T1, T2, vs_max, vs_min;
115
116    int EXC, PSS;
117 };
118
119 struct BUS{
120    float ID, IQ, It;
121    float VD, VQ;
122 };
123 typedef struct{
124  int gen_num;
125  int size;
126  int Gens[max_Na+1];//the ID number of each generator in each area
127  struct machine Gen[max_Na+1];
128  struct BUS bus[max_Na+1];
129  struct G G;
130  struct B B;
131  float **G_NN, **G_NM, **B_NN, **B_NM, **off_diag_G_NN, **off_diag_B_NN;
132  float r_Te[9][max_Na+1], r_ifd[9][max_Na+1], r_efd[9][max_Na+1],
133    r_id1[9][max_Na+1], r_iq1[9][max_Na+1], r_iq2[9][max_Na+1], r_Vt[9][max_Na+1];
134
135  int change1, change2, change3;
136
137  float *h_deltaX, *h_F, *h_J;
138  float *h_ID,*h_IQ,*h_U1,*h_U2,*h_U3,*h_U4,*h_U5,*h_U6;
139  float *h_A5,*h_A6,*h_A7,*h_A8;
140  float *h_sigma1,*h_sigma2,*h_sigma3,*h_sigma4;
141  float *h_diag_U1,*h_diag_U2,*h_diag_U3,*h_diag_U4;
142  float *h_RESULT;
143
144  float *d_delta,*d_slip,*d_sfd,*d_sd1,*d_sq1,*d_sq2,*d_V1,*d_V2,*d_V3;
145  float *d_id,*d_iq,*d_vd,*d_vq,*d_vt;
146  float *d_ifd,*d_efd,*d_Efd,*d_Vref,*d_id1,*d_iq1,*d_iq2,*d_s_ad,*d_s_aq;
147  float *d_Eds,*d_Eqs,*d_Te,*d_Tm,*d_Pt,*d_Qt,*d_H,*d_D;
148  float *d_Lad,*d_Laq,*d_Lfd,*d_Ld1,*d_Lq1,*d_Lq2,*d_Rfd,
149    *d_Rd1,*d_Rq1,*d_Rq2,*d_Lad_sec,*d_Laq_sec;
```

```
150   float *d_pre_delta,*d_pre_slip,*d_pre_sfd,*d_pre_sd1,
151     *d_pre_sq1,*d_pre_sq2,*d_pre_V1,*d_pre_V2,*d_pre_V3;
152   float *d_pre_Te,*d_pre_ifd,*d_pre_efd,*d_pre_id1,
153     *d_pre_iq1,*d_pre_iq2,*d_pre_vt;
154   float *d_KA,*d_TR,*d_Efd_max,*d_Efd_min,*d_Kstab,*d_TW,
155     *d_T1,*d_T2,*d_vs,*d_vs_max,*d_vs_min,
156     *d_EXC,*d_PSS;
157   float *d_F1,*d_F2,*d_F3,*d_F4,*d_F5,*d_F6,*d_F7,*d_F8,*d_F9;
158   float *d_It,*d_VD,*d_VQ;
159   float *d_r_Te_del,*d_r_Te_sf,*d_r_Te_d1,*d_r_Te_q1,*d_r_Te_q2;
160   float *d_r_ifd_del,*d_r_ifd_sf,*d_r_ifd_d1,*d_r_ifd_q1,*d_r_ifd_q2;
161   float *d_r_efd_del,*d_r_efd_sf,*d_r_efd_d1,*d_r_efd_q1,*d_r_efd_q2,
162     *d_r_efd_v1,*d_r_efd_v3;
163   float *d_r_id1_del,*d_r_id1_sf,*d_r_id1_d1,*d_r_id1_q1,*d_r_id1_q2;
164   float *d_r_iq1_del,*d_r_iq1_sf,*d_r_iq1_d1,*d_r_iq1_q1,*d_r_iq1_q2;
165   float *d_r_iq2_del,*d_r_iq2_sf,*d_r_iq2_d1,*d_r_iq2_q1,*d_r_iq2_q2;
166   float *d_r_Vt_del,*d_r_Vt_sf,*d_r_Vt_d1,*d_r_Vt_q1,*d_r_Vt_q2;
167   float *d_deltaX,*d_X;
168   float *d_F,*d_J,*d_J2,*d_J2_0;
169   float *d_G_NN, *d_B_NN, *d_diag_G_NN, *d_diag_B_NN,
170     *d_off_diag_G_NN, *d_off_diag_B_NN;
171   float *d_RESULT1, *d_RESULT2, *d_RESULT3;
172   float *d_ID,*d_IQ,*d_ID_temp,*d_IQ_temp;
173   float *d_U1,*d_U2,*d_U3,*d_U4,*d_U5,*d_U6;
174   float *d_A5,*d_A6,*d_A7,*d_A8;
175   float *d_sigma1,*d_sigma2,*d_sigma3,*d_sigma4;
176   float *d_temp_MATRIX1, *d_temp_MATRIX2;
177   float *d_diag_U1, *d_diag_U2, *d_diag_U3, *d_diag_U4;
178   int *d_idx;
179
180   void GPU_initialize();
181   void system_update(int TIME_STEP);
182   void admittance_update(int TIME_STEP);
183   void Jacobian_calc(int IsFirstInitCond);
184   void rond_calc();
185 } AREA;
186
187 AREA area[MAX_GPU_COUNT+1];
188
189 float *h_ID_g,*h_IQ_g,*h_U1_g,*h_U2_g,*h_U3_g,*h_U4_g,*h_U5_g,*h_U6_g;
190
191 void AREA::GPU_initialize()
192 {
193   int N_detail=gen_num;
194
195   float *h_temp;
196   h_temp = (float*)malloc(N_detail * 1 * sizeof(float));
197
198   cublasStatus status;
199   status = cublasInit();
200
201   status = cublasAlloc(size*1, sizeof(float), (void**)&d_deltaX);
202   status = cublasAlloc(size*1, sizeof(float), (void**)&d_X);
203   status = cublasAlloc(size*1, sizeof(float), (void**)&d_F);
204   status = cublasAlloc(size*size, sizeof(float), (void**)&d_J2);
205   status = cublasAlloc(size*size, sizeof(float), (void**)&d_J2_0);
206   status = cublasAlloc(N_detail*N_detail, sizeof(float), (void**)&d_G_NN);
207   status = cublasAlloc(N_detail*N_detail, sizeof(float), (void**)&d_B_NN);
208   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_diag_G_NN);
209   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_diag_B_NN);
210   status = cublasAlloc(N_detail*N_detail, sizeof(float), (void**)&d_off_diag_G_NN);
211   status = cublasAlloc(N_detail*N_detail, sizeof(float), (void**)&d_off_diag_B_NN);
212   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_RESULT1);
213   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_RESULT2);
214   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_RESULT3);
215   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_ID_temp);
216   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_IQ_temp);
217   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_U1);
218   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_U2);
219   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_U3);
220   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_U4);
221   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_U5);
222   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_U6);
223   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_A5);
224   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_A6);
225   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_A7);
226   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_A8);
227   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_sigma1);
228   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_sigma2);
229   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_sigma3);
230   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_sigma4);
231   status = cublasAlloc(N_detail*N_detail, sizeof(float), (void**)&d_temp_MATRIX1);
232   status = cublasAlloc(N_detail*N_detail, sizeof(float), (void**)&d_temp_MATRIX2);
233   status = cublasAlloc(N*N, sizeof(float), (void**)&d_diag_U1);
234   status = cublasAlloc(N*N, sizeof(float), (void**)&d_diag_U2);
235   status = cublasAlloc(N*N, sizeof(float), (void**)&d_diag_U3);
236   status = cublasAlloc(N*N, sizeof(float), (void**)&d_diag_U4);
237
238   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_F1);
239   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_F2);
240   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_F3);
241   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_F4);
242   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_F5);
243   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_F6);
244   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_F7);
245   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_F8);
246   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_F9);
247
248   status = cublasSetVector(size, sizeof(float), h_F, 1, d_F, 1);
249   status = cublasSetMatrix(size, size, sizeof(float), h_J, size, d_J, size);
250
251   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_Te_del);
252   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_Te_sf);
253   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_Te_d1);
254   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_Te_q1);
255   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_Te_q2);
256   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_ifd_del);
257   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_ifd_sf);
258   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_ifd_d1);
259   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_ifd_q1);
260   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_ifd_q2);
261   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_efd_del);
262   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_efd_sf);
263   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_efd_d1);
264   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_efd_q1);
265   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_efd_q2);
266   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_efd_v1);
267   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_efd_v3);
268   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_id1_del);
269   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_id1_sf);
270   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_id1_d1);
271   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_id1_q1);
272   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_id1_q2);
273   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_iq1_del);
274   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_iq1_sf);
275   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_iq1_d1);
276   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_iq1_q1);
277   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_iq1_q2);
278   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_iq2_del);
279   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_iq2_sf);
280   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_iq2_d1);
281   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_iq2_q1);
282   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_iq2_q2);
283   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_Vt_del);
284   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_Vt_sf);
285   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_Vt_d1);
286   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_Vt_q1);
287   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_Vt_q2);
288
289   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_delta);
290   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_slip);
291   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_sfd);
292   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_sd1);
293   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_sq1);
294   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_sq2);
295   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_V1);
296   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_V2);
297   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_V3);
298   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_id);
299   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_iq);
300   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_vd);
301   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_vq);
```

```
302   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_vt);
303   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_ifd);
304   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_efd);
305   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Efd);
306   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Vref);
307   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_id1);
308   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_iq1);
309   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_iq2);
310   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_s_ad);
311   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_s_aq);
312   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Eds);
313   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Eqs);
314   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Te);
315   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Tm);
316   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Pt);
317   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Qt);
318   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_H);
319   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_D);
320   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Lad);
321   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Laq);
322   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Lfd);
323   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Ld1);
324   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Lq1);
325   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Lq2);
326   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Rfd);
327   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Rd1);
328   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Rq1);
329   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Rq2);
330   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Lad_sec);
331   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Laq_sec);
332   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_delta);
333   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_slip);
334   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_sfd);
335   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_sd1);
336   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_sq1);
337   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_sq2);
338   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_V1);
339   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_V2);
340   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_V3);
341   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_Te);
342   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_ifd);
343   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_efd);
344   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_id1);
345   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_iq1);
346   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_iq2);
347   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_pre_vt);
348   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_KA);
349   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_TR);
350   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Efd_max);
351   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Efd_min);
352   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_Kstab);
353   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_TW);
354   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_T1);
355   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_T2);
356   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_vs);
357   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_vs_max);
358   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_vs_min);
359   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_EXC);
360   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_PSS);
361   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_It);
362   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_ID);
363   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_IQ);
364   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_VD);
365   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_VQ);
366
367   for(int k=1;k<N_detail+1;k++)
368     h_temp[k-1]=Gen[k].delta;
369   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_delta, 1);
370
371   for(int k=1;k<N_detail+1;k++)
372     h_temp[k-1]=Gen[k].slip;
373   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_slip, 1);
374
375   for(int k=1;k<N_detail+1;k++)
376     h_temp[k-1]=Gen[k].sfd;
377   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_sfd, 1);
378
379   for(int k=1;k<N_detail+1;k++)
380     h_temp[k-1]=Gen[k].sd1;
381   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_sd1, 1);
382
383   for(int k=1;k<N_detail+1;k++)
384     h_temp[k-1]=Gen[k].sq1;
385   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_sq1, 1);
386
387   for(int k=1;k<N_detail+1;k++)
388     h_temp[k-1]=Gen[k].sq2;
389   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_sq2, 1);
390
391   for(int k=1;k<N_detail+1;k++)
392     h_temp[k-1]=Gen[k].V1;
393   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_V1, 1);
394
395   for(int k=1;k<N_detail+1;k++)
396     h_temp[k-1]=Gen[k].V2;
397   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_V2, 1);
398
399   for(int k=1;k<N_detail+1;k++)
400     h_temp[k-1]=Gen[k].V3;
401   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_V3, 1);
402
403   for(int k=1;k<N_detail+1;k++)
404     h_temp[k-1]=Gen[k].id;
405   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_id, 1);
406
407   for(int k=1;k<N_detail+1;k++)
408     h_temp[k-1]=Gen[k].iq;
409   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_iq, 1);
410
411   for(int k=1;k<N_detail+1;k++)
412     h_temp[k-1]=Gen[k].vd;
413   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_vd, 1);
414
415   for(int k=1;k<N_detail+1;k++)
416     h_temp[k-1]=Gen[k].vq;
417   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_vq, 1);
418
419   for(int k=1;k<N_detail+1;k++)
420     h_temp[k-1]=Gen[k].vt;
421   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_vt, 1);
422
423   for(int k=1;k<N_detail+1;k++)
424     h_temp[k-1]=Gen[k].ifd;
425   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_ifd, 1);
426
427   for(int k=1;k<N_detail+1;k++)
428     h_temp[k-1]=Gen[k].efd;
429   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_efd, 1);
430
431   for(int k=1;k<N_detail+1;k++)
432     h_temp[k-1]=Gen[k].Efd;
433   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Efd, 1);
434
435   for(int k=1;k<N_detail+1;k++)
436     h_temp[k-1]=Gen[k].Vref;
437   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Vref, 1);
438
439   for(int k=1;k<N_detail+1;k++)
440     h_temp[k-1]=Gen[k].id1;
441   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_id1, 1);
442
443   for(int k=1;k<N_detail+1;k++)
444     h_temp[k-1]=Gen[k].iq1;
445   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_iq1, 1);
446
447   for(int k=1;k<N_detail+1;k++)
448     h_temp[k-1]=Gen[k].iq2;
449   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_iq2, 1);
450
451   for(int k=1;k<N_detail+1;k++)
452     h_temp[k-1]=Gen[k].s_ad;
453   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_s_ad, 1);
```

```
454
455   for(int k=1;k<N_detail+1;k++)
456     h_temp[k-1]=Gen[k].s_aq;
457   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_s_aq, 1);
458
459   for(int k=1;k<N_detail+1;k++)
460     h_temp[k-1]=Gen[k].Eds;
461   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Eds, 1);
462
463   for(int k=1;k<N_detail+1;k++)
464     h_temp[k-1]=Gen[k].Eqs;
465   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Eqs, 1);
466
467   for(int k=1;k<N_detail+1;k++)
468     h_temp[k-1]=Gen[k].Te;
469   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Te, 1);
470
471   for(int k=1;k<N_detail+1;k++)
472     h_temp[k-1]=Gen[k].Tm;
473   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Tm, 1);
474
475   for(int k=1;k<N_detail+1;k++)
476     h_temp[k-1]=Gen[k].Pt;
477   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Pt, 1);
478
479   for(int k=1;k<N_detail+1;k++)
480     h_temp[k-1]=Gen[k].Qt;
481   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Qt, 1);
482
483   for(int k=1;k<N_detail+1;k++)
484     h_temp[k-1]=Gen[k].H;
485   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_H, 1);
486
487   for(int k=1;k<N_detail+1;k++)
488     h_temp[k-1]=Gen[k].D;
489   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_D, 1);
490
491   for(int k=1;k<N_detail+1;k++)
492     h_temp[k-1]=Gen[k].Lad;
493   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Lad, 1);
494
495   for(int k=1;k<N_detail+1;k++)
496     h_temp[k-1]=Gen[k].Laq;
497   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Laq, 1);
498
499   for(int k=1;k<N_detail+1;k++)
500     h_temp[k-1]=Gen[k].Lfd;
501   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Lfd, 1);
502
503   for(int k=1;k<N_detail+1;k++)
504     h_temp[k-1]=Gen[k].Ld1;
505   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Ld1, 1);
506
507   for(int k=1;k<N_detail+1;k++)
508     h_temp[k-1]=Gen[k].Lq1;
509   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Lq1, 1);
510
511   for(int k=1;k<N_detail+1;k++)
512     h_temp[k-1]=Gen[k].Lq2;
513   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Lq2, 1);
514
515   for(int k=1;k<N_detail+1;k++)
516     h_temp[k-1]=Gen[k].Rfd;
517   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Rfd, 1);
518
519   for(int k=1;k<N_detail+1;k++)
520     h_temp[k-1]=Gen[k].Rd1;
521   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Rd1, 1);
522
523   for(int k=1;k<N_detail+1;k++)
524     h_temp[k-1]=Gen[k].Rq1;
525   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Rq1, 1);
526
527   for(int k=1;k<N_detail+1;k++)
528     h_temp[k-1]=Gen[k].Rq2;
529   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Rq2, 1);
530
531   for(int k=1;k<N_detail+1;k++)
532     h_temp[k-1]=Gen[k].Lad_sec;
533   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Lad_sec, 1);
534
535   for(int k=1;k<N_detail+1;k++)
536     h_temp[k-1]=Gen[k].Laq_sec;
537   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Laq_sec, 1);
538
539   for(int k=1;k<N_detail+1;k++)
540     h_temp[k-1]=Gen[k].pre_delta;
541   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_delta, 1);
542
543   for(int k=1;k<N_detail+1;k++)
544     h_temp[k-1]=Gen[k].pre_slip;
545   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_slip, 1);
546
547   for(int k=1;k<N_detail+1;k++)
548     h_temp[k-1]=Gen[k].pre_sfd;
549   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_sfd, 1);
550
551   for(int k=1;k<N_detail+1;k++)
552     h_temp[k-1]=Gen[k].pre_sd1;
553   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_sd1, 1);
554
555   for(int k=1;k<N_detail+1;k++)
556     h_temp[k-1]=Gen[k].pre_sq1;
557   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_sq1, 1);
558
559   for(int k=1;k<N_detail+1;k++)
560     h_temp[k-1]=Gen[k].pre_sq2;
561   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_sq2, 1);
562
563   for(int k=1;k<N_detail+1;k++)
564     h_temp[k-1]=Gen[k].pre_V1;
565   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_V1, 1);
566
567   for(int k=1;k<N_detail+1;k++)
568     h_temp[k-1]=Gen[k].pre_V2;
569   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_V2, 1);
570
571   for(int k=1;k<N_detail+1;k++)
572     h_temp[k-1]=Gen[k].pre_V3;
573   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_V3, 1);
574
575   for(int k=1;k<N_detail+1;k++)
576     h_temp[k-1]=Gen[k].pre_Te;
577   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_Te, 1);
578
579   for(int k=1;k<N_detail+1;k++)
580     h_temp[k-1]=Gen[k].pre_ifd;
581   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_ifd, 1);
582
583   for(int k=1;k<N_detail+1;k++)
584     h_temp[k-1]=Gen[k].pre_efd;
585   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_efd, 1);
586
587   for(int k=1;k<N_detail+1;k++)
588     h_temp[k-1]=Gen[k].pre_id1;
589   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_id1, 1);
590
591   for(int k=1;k<N_detail+1;k++)
592     h_temp[k-1]=Gen[k].pre_iq1;
593   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_iq1, 1);
594
595   for(int k=1;k<N_detail+1;k++)
596     h_temp[k-1]=Gen[k].pre_iq2;
597   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_iq2, 1);
598
599   for(int k=1;k<N_detail+1;k++)
600     h_temp[k-1]=Gen[k].pre_vt;
601   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_pre_vt, 1);
602
603   for(int k=1;k<N_detail+1;k++)
604     h_temp[k-1]=Gen[k].KA;
605   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_KA, 1);
```

```
606
607   for(int k=1;k<N_detail+1;k++)
608     h_temp[k-1]=Gen[k].TR;
609   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_TR, 1);
610
611   for(int k=1;k<N_detail+1;k++)
612     h_temp[k-1]=Gen[k].Efd_max;
613   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Efd_max, 1);
614
615   for(int k=1;k<N_detail+1;k++)
616     h_temp[k-1]=Gen[k].Efd_min;
617   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Efd_min, 1);
618
619   for(int k=1;k<N_detail+1;k++)
620     h_temp[k-1]=Gen[k].Kstab;
621   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_Kstab, 1);
622
623   for(int k=1;k<N_detail+1;k++)
624     h_temp[k-1]=Gen[k].TW;
625   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_TW, 1);
626
627   for(int k=1;k<N_detail+1;k++)
628     h_temp[k-1]=Gen[k].T1;
629   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_T1, 1);
630
631   for(int k=1;k<N_detail+1;k++)
632     h_temp[k-1]=Gen[k].T2;
633   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_T2, 1);
634
635   for(int k=1;k<N_detail+1;k++)
636     h_temp[k-1]=Gen[k].vs_max;
637   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_vs_max, 1);
638
639   for(int k=1;k<N_detail+1;k++)
640     h_temp[k-1]=Gen[k].vs_min;
641   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_vs_min, 1);
642
643   for(int k=1;k<N_detail+1;k++)
644     h_temp[k-1]=Gen[k].EXC;
645   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_EXC, 1);
646
647   for(int k=1;k<N_detail+1;k++)
648     h_temp[k-1]=Gen[k].PSS;
649   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_PSS, 1);
650
651   for(int k=1;k<N_detail+1;k++)
652     h_temp[k-1]=bus[k].It;
653   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_It, 1);
654
655   for(int k=1;k<N_detail+1;k++)
656     h_temp[k-1]=bus[k].ID;
657   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_ID, 1);
658
659   for(int k=1;k<N_detail+1;k++)
660     h_temp[k-1]=bus[k].IQ;
661   status = cublasSetVector(N_detail, sizeof(float), h_temp, 1, d_IQ, 1);
662
663   status = cublasSetMatrix(N, N, sizeof(float), h_diag_U1, N, d_diag_U1, N);
664   status = cublasSetMatrix(N, N, sizeof(float), h_diag_U2, N, d_diag_U2, N);
665   status = cublasSetMatrix(N, N, sizeof(float), h_diag_U3, N, d_diag_U3, N);
666   status = cublasSetMatrix(N, N, sizeof(float), h_diag_U4, N, d_diag_U4, N);
667
668   status = cublasSetVector(N_detail, sizeof(float), h_U1, 1, d_U1, 1);
669   status = cublasSetVector(N_detail, sizeof(float), h_U2, 1, d_U2, 1);
670   status = cublasSetVector(N_detail, sizeof(float), h_U3, 1, d_U3, 1);
671   status = cublasSetVector(N_detail, sizeof(float), h_U4, 1, d_U4, 1);
672   status = cublasSetVector(N_detail, sizeof(float), h_U5, 1, d_U5, 1);
673   status = cublasSetVector(N_detail, sizeof(float), h_U6, 1, d_U6, 1);
674
675   cublasSetMatrix(size, size, sizeof(float), h_J, size, d_J2_0, size);
676   COPY_mat_ON_mat(d_J2_0, d_J2, size);// set d_J2=0
677
678   free(h_J);
679 }
680 void AREA::system_update(int TIME_STEP)
681 {
682   QueryPerformanceFrequency((LARGE_INTEGER*)&mFreq);
683
684   float *max_err, *err_d, *err_q;
685   int *permute;
686   int N_detail=gen_num;
687
688   max_err = (float*)calloc(1, sizeof(float));
689   err_d = (float*)calloc(1, sizeof(float));
690   err_q = (float*)calloc(1, sizeof(float));
691   permute = (int*)malloc(size * 1 * sizeof(int));
692
693   cublasStatus status;
694   status = cublasInit();
695
696   cublasScopy(N_detail, d_delta, 1, d_pre_delta, 1);
697   cublasScopy(N_detail, d_slip, 1, d_pre_slip, 1);
698   cublasScopy(N_detail, d_sfd, 1, d_pre_sfd, 1);
699   cublasScopy(N_detail, d_sd1, 1, d_pre_sd1, 1);
700   cublasScopy(N_detail, d_sq1, 1, d_pre_sq1, 1);
701   cublasScopy(N_detail, d_sq2, 1, d_pre_sq2, 1);
702   cublasScopy(N_detail, d_V1, 1, d_pre_V1, 1);
703   cublasScopy(N_detail, d_V2, 1, d_pre_V2, 1);
704   cublasScopy(N_detail, d_V3, 1, d_pre_V3, 1);
705   cublasScopy(N_detail, d_Te, 1, d_pre_Te, 1);
706   cublasScopy(N_detail, d_ifd, 1, d_pre_ifd, 1);
707   cublasScopy(N_detail, d_efd, 1, d_pre_efd, 1);
708   cublasScopy(N_detail, d_id1, 1, d_pre_id1, 1);
709   cublasScopy(N_detail, d_iq1, 1, d_pre_iq1, 1);
710   cublasScopy(N_detail, d_iq2, 1, d_pre_iq2, 1);
711   cublasScopy(N_detail, d_vt, 1, d_pre_vt, 1);
712
713   vecMIX2(d_delta, d_slip, d_sfd, d_sd1, d_sq1, d_sq2, d_V1, d_V2, d_V3, d_X, N_detail);
714
715 //////WHILE//////////////////////////////////////////////////////
716
717   max_err[0]=1;
718   while(fabs(max_err[0])>epsilon){
719
720     cublasScopy(N_detail, d_delta, 1, d_F1, 1);
721     cublasSaxpy(N_detail, -1, d_pre_delta, 1, d_F1, 1);//d_F1=delta-delta_pre
722     cublasScopy(N_detail, d_slip, 1, d_RESULT1, 1);
723     cublasSaxpy(N_detail, 1, d_pre_slip, 1, d_RESULT1, 1);//d_RESULT1=slip+slip_pre
724     cublasSaxpy(N_detail, -0.5*h*wR, d_RESULT1, 1, d_F1, 1);
725
726     cublasScopy(N_detail, d_H, 1, d_RESULT1, 1);
727     cublasSscal(N_detail, 2, d_RESULT1, 1);
728     cublasSaxpy(N_detail, h, d_D, 1, d_RESULT1, 1);//d_RESULT1=2H+h.D
729     vecDOTvec(d_RESULT1, d_slip, d_F2, N_detail);
730     cublasScopy(N_detail, d_D, 1, d_RESULT1, 1);
731     cublasSscal(N_detail, h, d_RESULT1, 1);
732     cublasSaxpy(N_detail, -2, d_H, 1, d_RESULT1, 1);
733     vecDOTvec(d_RESULT1, d_pre_slip, d_RESULT2, N_detail);
734     cublasSaxpy(N_detail, 1, d_RESULT2, 1, d_F2, 1);
735     cublasScopy(N_detail, d_Te, 1, d_RESULT1, 1);
736     cublasSaxpy(N_detail, 1, d_pre_Te, 1, d_RESULT1, 1);
737     cublasSscal(N_detail, -0.5*h, d_RESULT1, 1);
738     cublasScopy(N_detail, d_Tm, 1, d_RESULT2, 1);
739     cublasSscal(N_detail, -h, d_RESULT2, 1);
740     cublasSaxpy(N_detail, 1, d_RESULT1, 1, d_RESULT2, 1);
741     cublasSaxpy(N_detail, 1, d_RESULT2, 1, d_F2, 1);
742
743     cublasScopy(N_detail, d_sfd, 1, d_F3, 1);
744     cublasSaxpy(N_detail, -1, d_pre_sfd, 1, d_F3, 1);//d_F3=d_sfd-d_pre_sfd
745     cublasScopy(N_detail, d_efd, 1, d_RESULT1, 1);
746     cublasSaxpy(N_detail, 1, d_pre_efd, 1, d_RESULT1, 1);
747     cublasSscal(N_detail, -0.5*h*wR, d_RESULT1, 1);//
748     cublasScopy(N_detail, d_ifd, 1, d_RESULT2, 1);
749     cublasSaxpy(N_detail, 1, d_pre_ifd, 1, d_RESULT2, 1);
750     cublasSscal(N_detail, 0.5*h*wR, d_RESULT2, 1);
751     vecDOTvec(d_RESULT2, d_Rfd, d_RESULT3, N_detail);
752     cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_RESULT1, 1);
753     cublasSaxpy(N_detail, 1, d_RESULT1, 1, d_F3, 1);
754
755     cublasScopy(N_detail, d_sd1, 1, d_F4, 1);
756     cublasSaxpy(N_detail, -1, d_pre_sd1, 1, d_F4, 1);
757     cublasScopy(N_detail, d_id1, 1, d_RESULT1, 1);
```

```
758      cublasSaxpy(N_detail, 1, d_pre_id1, 1, d_RESULT1, 1);
759      cublasSscal(N_detail, 0.5*h*wR, d_RESULT1, 1);
760      vecDOTvec(d_RESULT1, d_Rd1, d_RESULT2, N_detail);
761      cublasSaxpy(N_detail, 1, d_RESULT2, 1, d_F4, 1);
762
763      cublasScopy(N_detail, d_sq1, 1, d_F5, 1);
764      cublasSaxpy(N_detail, -1, d_pre_sq1, 1, d_F5, 1);
765      cublasScopy(N_detail, d_iq1, 1, d_RESULT1, 1);
766      cublasSaxpy(N_detail, 1, d_pre_iq1, 1, d_RESULT1, 1);
767      cublasSscal(N_detail, 0.5*h*wR, d_RESULT1, 1);
768      vecDOTvec(d_RESULT1, d_Rq1, d_RESULT2, N_detail);
769      cublasSaxpy(N_detail, 1, d_RESULT2, 1, d_F5, 1);
770
771      cublasScopy(N_detail, d_sq2, 1, d_F6, 1);
772      cublasSaxpy(N_detail, -1, d_pre_sq2, 1, d_F6, 1);
773      cublasScopy(N_detail, d_iq2, 1, d_RESULT1, 1);
774      cublasSaxpy(N_detail, 1, d_pre_iq2, 1, d_RESULT1, 1);
775      cublasSscal(N_detail, 0.5*h*wR, d_RESULT1, 1);
776      vecDOTvec(d_RESULT1, d_Rq2, d_RESULT2, N_detail);
777      cublasSaxpy(N_detail, 1, d_RESULT2, 1, d_F6, 1);
778
779      cublasScopy(N_detail, d_V1, 1, d_RESULT1, 1);
780      cublasSaxpy(N_detail, -1, d_pre_V1, 1, d_RESULT1, 1);
781      vecDOTvec(d_RESULT1, d_TR, d_F7, N_detail);
782      cublasScopy(N_detail, d_V1, 1, d_RESULT1, 1);
783      cublasSaxpy(N_detail, 1, d_pre_V1, 1, d_RESULT1, 1);
784      cublasSaxpy(N_detail, 0.5*h, d_RESULT1, 1, d_F7, 1);
785      cublasScopy(N_detail, d_vt, 1, d_RESULT1, 1);
786      cublasSaxpy(N_detail, 1, d_pre_vt, 1, d_RESULT1, 1);
787      cublasSaxpy(N_detail, -0.5*h, d_RESULT1, 1, d_F7, 1);
788
789      cublasScopy(N_detail, d_V2, 1, d_RESULT1, 1);
790      cublasSaxpy(N_detail, -1, d_pre_V2, 1, d_RESULT1, 1);
791      vecDOTvec(d_RESULT1, d_TW, d_F8, N_detail);
792      cublasScopy(N_detail, d_V2, 1, d_RESULT1, 1);
793      cublasSaxpy(N_detail, 1, d_pre_V2, 1, d_RESULT1, 1);
794      cublasSaxpy(N_detail, 0.5*h, d_RESULT1, 1, d_F8, 1);
795      cublasScopy(N_detail, d_slip, 1, d_RESULT1, 1);
796      cublasSaxpy(N_detail, -1, d_pre_slip, 1, d_RESULT1, 1);
797      vecDOTvec(d_Kstab, d_TW, d_RESULT2, N_detail);
798      vecDOTvec(d_RESULT2, d_RESULT1, d_RESULT3, N_detail);
799      cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_F8, 1);
800
801      cublasScopy(N_detail, d_V3, 1, d_RESULT1, 1);
802      cublasSaxpy(N_detail, -1, d_pre_V3, 1, d_RESULT1, 1);
803      vecDOTvec(d_RESULT1, d_T2, d_F9, N_detail);
804      cublasScopy(N_detail, d_V3, 1, d_RESULT1, 1);
805      cublasSaxpy(N_detail, 1, d_pre_V3, 1, d_RESULT1, 1);
806      cublasSaxpy(N_detail, 0.5*h, d_RESULT1, 1, d_F9, 1);
807      cublasScopy(N_detail, d_pre_V2, 1, d_RESULT1, 1);
808      cublasSaxpy(N_detail, -1, d_V2, 1, d_RESULT1, 1);
809      vecDOTvec(d_RESULT1, d_T1, d_RESULT2, N_detail);
810      cublasSaxpy(N_detail, 1, d_RESULT2, 1, d_F9, 1);
811      cublasScopy(N_detail, d_V2, 1, d_RESULT1, 1);
812      cublasSaxpy(N_detail, 1, d_pre_V2, 1, d_RESULT1, 1);
813      cublasSaxpy(N_detail, -0.5*h, d_RESULT1, 1, d_F9, 1);
814
815      vecMIX2(d_F1, d_F2, d_F3, d_F4, d_F5, d_F6, d_F7, d_F8, d_F9, d_F, N_detail);
816
817      cublasSscal(9*N_detail, -1, d_F, 1);
818
819          /**********************************
820        *Jacobian Matrix********************
821         *********************************/
822
823      Jacobian_calc(TIME_STEP);
824      permute = transformToLU(1, blockWidth, d_J2, permute, size);
825      solve_on_gpu(d_J2, d_F, permute, d_deltaX, size);
826      cublasSaxpy(size, 1, d_deltaX, 1, d_X, 1);//d_X=d_X + d_deltaX
827
828      vecSPLIT3(d_X, d_delta, 0, 9);//d_delta[i]=d_deltaX[9*i+0]
829      vecSPLIT3(d_X, d_slip, 1, 9); //d_slip[i]=d_deltaX[9*i+1]
830      vecSPLIT3(d_X, d_sfd, 2, 9);  //d_slip[i]=d_deltaX[9*i+2]
831      vecSPLIT3(d_X, d_sd1, 3, 9);  //d_sd1[i]=d_deltaX[
832      vecSPLIT3(d_X, d_sq1, 4, 9);  //d_sq1[i]=d_deltaX[
833      vecSPLIT3(d_X, d_sq2, 5, 9);  //d_sq2[i]=d_deltaX[
834      vecSPLIT3(d_X, d_V1, 6, 9);   //d_V1[i]=d_deltaX[
835      vecSPLIT3(d_X, d_V2, 7, 9);   //d_V2[i]=d_deltaX[
836      vecSPLIT3(d_X, d_V3, 8, 9);   //d_V3[i]=d_deltaX[
837
838      vecDIVvec(d_sq1, d_Lq1, d_Eds, N_detail);
839      vecDIVvec(d_sq2, d_Lq2, d_RESULT1, N_detail);
840      cublasSaxpy(N_detail, 1, d_RESULT1, 1, d_Eds, 1);
841      vecDOTvec(d_Eds, d_Laq_sec, d_Eds, N_detail);
842
843      vecDIVvec(d_sfd, d_Lfd, d_Eqs, N_detail);
844      vecDIVvec(d_sd1, d_Ld1, d_RESULT1, N_detail);
845      cublasSaxpy(N_detail, 1, d_RESULT1, 1, d_Eqs, 1);
846      vecDOTvec(d_Eqs, d_Lad_sec, d_Eqs, N_detail);
847
848      vecCos(d_delta, d_RESULT1, N_detail);
849      vecSin(d_delta, d_RESULT2, N_detail);
850
851      vecDOTvec(d_RESULT1, d_Eds, d_U5, N_detail);
852      cublasSscal(N_detail, -1, d_U5, 1);
853      vecDOTvec(d_RESULT2, d_Eqs, d_RESULT3, N_detail);
854      cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_U5, 1);
855
856      vecDOTvec(d_RESULT1, d_Eqs, d_U6, N_detail);
857      vecDOTvec(d_RESULT2, d_Eds, d_RESULT3, N_detail);
858      cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_U6, 1);
859
860      cublasScopy(N_detail, d_ID, 1, d_ID_temp, 1);
861      cublasScopy(N_detail, d_IQ, 1, d_IQ_temp, 1);
862
863      vecDOTvec(d_diag_B_NN, d_U3, d_A5, N_detail);
864      vecDOTvec(d_diag_G_NN, d_U1, d_RESULT2, N_detail);
865      cublasSaxpy(N_detail, -1, d_RESULT2, 1, d_A5, 1);
866      vecSUMscal(d_A5, 1, N_detail);
867
868      vecDOTvec(d_diag_G_NN, d_U4, d_A6, N_detail);
869      vecDOTvec(d_diag_B_NN, d_U2, d_RESULT2, N_detail);
870      cublasSaxpy(N_detail, 1.0, d_RESULT2, 1, d_A6, 1);
871      cublasSscal(N_detail, -1.0, d_A6, 1);
872      vecSUMscal(d_A6, 1, N_detail);
873
874      cublasSgemv('N', N_detail, N_detail, 1.0, d_G_NN, N_detail, d_U5, 1, 0.0, d_A7, 1);
875      cublasSgemv('N', N_detail, N_detail, -1.0, d_B_NN, N_detail, d_U6, 1, 1.0, d_A7, 1);
876
877      cublasSgemv('N', N_detail, N_detail, 1.0, d_G_NN, N_detail, d_U6, 1, 0.0, d_A8, 1);
878      cublasSgemv('N', N_detail, N_detail, 1.0, d_B_NN, N_detail, d_U5, 1, 1.0, d_A8, 1);
879
880
881      err_d[0] = 1;
882      err_q[0] = 1;
883      while((fabs(err_d[0])>0.001)||(fabs(err_q[0])>0.001)){
884        cublasSgemm('N', 'N', N, N, N, 1.0, d_off_diag_G_NN, N, d_diag_U1, N, 0.0, d_temp_MATRIX2, N);
885        cublasSgemm('N', 'N', N, N, N, -1.0, d_off_diag_B_NN, N, d_diag_U3, N, 1.0, d_temp_MATRIX2, N);
886        cublasSgemv('N', N, N, 1.0, d_temp_MATRIX2, N, d_ID, 1, 0.0, d_sigma1, 1);
887
888        cublasSgemm('N', 'N', N, N, N, 1.0, d_G_NN, N, d_diag_U2, N, 0.0, d_temp_MATRIX2, N);
889        cublasSgemm('N', 'N', N, N, N, -1.0, d_B_NN, N, d_diag_U4, N, 1.0, d_temp_MATRIX2, N);
890        cublasSgemv('N', N, N, 1.0, d_temp_MATRIX2, N, d_IQ, 1, 0.0, d_sigma2, 1);
891
892        cublasSgemm('N', 'N', N, N, N, 1.0, d_G_NN, N, d_diag_U3, N, 0.0, d_temp_MATRIX2, N);
893        cublasSgemm('N', 'N', N, N, N, -1.0, d_B_NN, N, d_diag_U1, N, 1.0, d_temp_MATRIX2, N);
894        cublasSgemv('N', N, N, 1.0, d_temp_MATRIX2, N, d_ID, 1, 0.0, d_sigma3, 1);
895
896        cublasSgemm('N', 'N', N, N, N, 1.0, d_off_diag_G_NN, N, d_diag_U4, N, 0.0, d_temp_MATRIX2, N);
897        cublasSgemm('N', 'N', N, N, N, 1.0, d_off_diag_B_NN, N, d_diag_U2, N, 1.0, d_temp_MATRIX2, N);
898        cublasSgemv('N', N, N, 1.0, d_temp_MATRIX2, N, d_IQ, 1, 0.0, d_sigma4, 1);
899
900        cublasScopy(N, d_sigma1, 1, d_RESULT1, 1);
901        cublasSaxpy(N, 1.0, d_sigma2, 1, d_RESULT1, 1);
902        cublasSaxpy(N, 1.0, d_A7, 1, d_RESULT1, 1);
903        vecDIVvec(d_RESULT1, d_A5, d_ID, N);
904
905        cublasScopy(N, d_sigma3, 1, d_RESULT1, 1);
906        cublasSaxpy(N, 1.0, d_sigma4, 1, d_RESULT1, 1);
907        cublasSaxpy(N, 1.0, d_A8, 1, d_RESULT1, 1);
908        vecDIVvec(d_RESULT1, d_A6, d_IQ, N);
909
```

```
910      cublasScopy(N, d_ID, 1, d_RESULT1, 1);
911      cublasSaxpy(N, -1.0, d_ID_temp, 1, d_RESULT1, 1);
912      int q = cublasIsamax(N, d_RESULT1, 1)-1;
913      cublasGetVector(1, sizeof(float), &d_RESULT1[q], 1, err_d, 1);
914
915      cublasScopy(N, d_IQ, 1, d_RESULT1, 1);
916      cublasSaxpy(N, -1.0, d_IQ_temp, 1, d_RESULT1, 1);
917      q = cublasIsamax(N, d_RESULT1, 1)-1;
918      cublasGetVector(1, sizeof(float), &d_RESULT1[q], 1, err_q, 1);
919
920      cublasScopy(N, d_ID, 1, d_ID_temp, 1);
921      cublasScopy(N, d_IQ, 1, d_IQ_temp, 1);
922    }
923
924    vecCos(d_delta, d_RESULT1, N_detail);
925    vecSin(d_delta, d_RESULT2, N_detail);
926
927    vecDOTvec(d_IQ, d_RESULT1, d_iq, N_detail);
928    vecDOTvec(d_ID, d_RESULT2, d_RESULT3, N_detail);
929    cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_iq, 1);
930
931    vecDOTvec(d_ID, d_RESULT1, d_id, N_detail);
932    vecDOTvec(d_IQ, d_RESULT2, d_RESULT3, N_detail);
933    cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_id, 1);
934
935
936    vecDOTvec(d_ID, d_ID, d_RESULT1, N_detail);
937    vecDOTvec(d_IQ, d_IQ, d_RESULT2, N_detail);
938    cublasSaxpy(N_detail, 1, d_RESULT1, 1, d_RESULT2, 1);
939    SQRT(d_RESULT2, N_detail);
940    cublasScopy(N_detail, d_RESULT2, 1, d_It, 1); // bus current
941
942    vecDOTvec(d_ID, d_U1, d_VD, N_detail);
943    vecDOTvec(d_IQ, d_U2, d_RESULT1, N_detail);
944    cublasSaxpy(N_detail, 1, d_RESULT1, 1, d_VD, 1);
945    cublasSaxpy(N_detail, 1, d_U5, 1, d_VD, 1);//bus voltage
946
947    vecDOTvec(d_ID, d_U3, d_VQ, N_detail);
948    vecDOTvec(d_IQ, d_U4, d_RESULT1, N_detail);
949    cublasSaxpy(N_detail, 1, d_RESULT1, 1, d_VQ, 1);
950    cublasSaxpy(N_detail, 1, d_U6, 1, d_VQ, 1);//bus voltage
951
952    vecDOTvec(d_VD, d_VD, d_RESULT1, N_detail);
953    vecDOTvec(d_VQ, d_VQ, d_RESULT2, N_detail);
954    cublasSaxpy(N_detail, 1, d_RESULT1, 1, d_RESULT2, 1);
955    SQRT(d_RESULT2, N_detail);
956    cublasScopy(N_detail, d_RESULT2, 1, d_vt, 1);
957
958    vecDIVvec(d_sd1, d_Ld1, d_s_ad, N_detail);
959    vecDIVvec(d_sfd, d_Lfd, d_RESULT1, N_detail);
960    cublasSaxpy(N_detail, 1, d_RESULT1, 1, d_s_ad, 1);
961    cublasSaxpy(N_detail, -1, d_id, 1, d_s_ad, 1);
962    vecDOTvec(d_Lad_sec, d_s_ad, d_RESULT1, N_detail);
963    cublasScopy(N_detail, d_RESULT1, 1, d_s_ad, 1);
964
965    vecDIVvec(d_sq1, d_Lq1, d_s_aq, N_detail);
966    vecDIVvec(d_sq2, d_Lq2, d_RESULT1, N_detail);
967    cublasSaxpy(N_detail, 1, d_RESULT1, 1, d_s_aq, 1);
968    cublasSaxpy(N_detail, -1, d_iq, 1, d_s_aq, 1);
969    vecDOTvec(d_Laq_sec, d_s_aq, d_RESULT1, N_detail);
970    cublasScopy(N_detail, d_RESULT1, 1, d_s_aq, 1);
971
972    cublasScopy(N_detail, d_sfd, 1, d_RESULT1, 1);
973    cublasSaxpy(N_detail, -1, d_s_ad, 1, d_RESULT1, 1);
974    vecDIVvec(d_RESULT1, d_Lfd, d_ifd, N_detail);
975
976    cublasScopy(N_detail, d_sd1, 1, d_RESULT1, 1);
977    cublasSaxpy(N_detail, -1, d_s_ad, 1, d_RESULT1, 1);
978    vecDIVvec(d_RESULT1, d_Ld1, d_id1, N_detail);
979
980    cublasScopy(N_detail, d_sq1, 1, d_RESULT1, 1);
981    cublasSaxpy(N_detail, -1, d_s_aq, 1, d_RESULT1, 1);
982    vecDIVvec(d_RESULT1, d_Lq1, d_iq1, N_detail);
983
984    cublasScopy(N_detail, d_sq2, 1, d_RESULT1, 1);
985    cublasSaxpy(N_detail, -1, d_s_aq, 1, d_RESULT1, 1);
986      vecDIVvec(d_RESULT1, d_Lq2, d_iq2, N_detail);
987
988      cublasScopy(N_detail, d_V3, 1, d_vs, 1);
989
990      limiter(d_vs, d_vs_max, d_vs_min, N_detail);
991
992      cublasScopy(N_detail, d_vs, 1, d_RESULT1, 1);
993      cublasSaxpy(N_detail, -1, d_V1, 1, d_RESULT1, 1);
994      cublasSaxpy(N_detail, 1, d_Vref, 1, d_RESULT1, 1);
995      vecDOTvec(d_KA ,d_RESULT1, d_Efd, N_detail);
996      limiter(d_Efd, d_Efd_max, d_Efd_min, N_detail);
997
998      vecDIVvec(d_Efd, d_Lad, d_RESULT1, N_detail);
999      vecDOTvec(d_Rfd, d_RESULT1, d_efd, N_detail);
1000
1001     vecDOTvec(d_s_ad, d_iq, d_Te, N_detail);
1002     vecDOTvec(d_s_aq, d_id, d_RESULT1, N_detail);
1003     cublasSaxpy(N_detail, -1, d_RESULT1, 1, d_Te, 1);
1004     cublasSscal(N_detail, -1, d_Te, 1);
1005
1006     int q = cublasIsamax(size, d_deltaX, 1)-1;
1007     cublasGetVector(1, sizeof(float), &d_deltaX[q], 1, max_err, 1);
1008     }//while
1009
1010   free(max_err);
1011   free(err_d);
1012   free(err_q);
1013
1014   free(h_F);
1015   free(h_deltaX);
1016   free(permute);
1017 }
1018 void AREA::admittance_update(int TIME_STEP)
1019 {
1020   cublasStatus status;
1021   status = cublasInit();
1022   int i, j;
1023   float *h_G_NN, *h_B_NN, *h_diag_G_NN, *h_diag_B_NN, *h_off_diag_G_NN, *h_off_diag_B_NN;
1024
1025   h_G_NN = (float*)malloc(N*N*sizeof(float));
1026   h_B_NN = (float*)malloc(N*N*sizeof(float));
1027   h_diag_G_NN = (float*)malloc(N*1*sizeof(float));
1028   h_diag_B_NN = (float*)malloc(N*1*sizeof(float));
1029   h_off_diag_G_NN = (float*)malloc(N*N*sizeof(float));
1030   h_off_diag_B_NN = (float*)malloc(N*N*sizeof(float));
1031
1032   if(TIME_STEP<duration1 && change1==1){
1033         change1=0;
1034         for(i=1;i<N+1;i++){
1035             for(j=1;j<N+1;j++){
1036                 if (j<=N){
1037                     G_NN[i][j]=G.pref[i][j];
1038                     B_NN[i][j]=B.pref[i][j];
1039           h_diag_G_NN[i-1]=G.pref[i][i];//Diagonal elements of G_NN
1040           h_diag_B_NN[i-1]=B.pref[i][i];
1041         }else{
1042                     G_NM[i][j-N]=G.pref[i][j];
1043                     B_NM[i][j-N]=B.pref[i][j];
1044                 }
1045             }
1046         }
1047     status = cublasSetVector(N, sizeof(float), h_diag_G_NN, 1, d_diag_G_NN, 1);
1048     status = cublasSetVector(N, sizeof(float), h_diag_B_NN, 1, d_diag_B_NN, 1);
1049
1050     i=0, j=0;
1051     while (i < N*N){//copying matrices on arrays with column major definition
1052       for (int k = 0; k < N; k++){
1053         h_G_NN[i] = G_NN[k+1][j+1];
1054         h_B_NN[i] = B_NN[k+1][j+1];
1055         i++;
1056       }
1057       j++;
1058     }
1059     status = cublasSetMatrix(N, N, sizeof(float), h_G_NN, N, d_G_NN, N);
1060     status = cublasSetMatrix(N, N, sizeof(float), h_B_NN, N, d_B_NN, N);
1061
```

```
1062    for(i=1;i<N+1;i++)
1063          for(j=1;j<N+1;j++){
1064        if (i==j){
1065          off_diag_G_NN[i][j]=0;
1066          off_diag_B_NN[i][j]=0;
1067        }else{
1068          off_diag_G_NN[i][j]=G_NN[i][j];
1069          off_diag_B_NN[i][j]=B_NN[i][j];
1070        }
1071      }
1072
1073    i=0, j=0;
1074    while (i < N*N){//copying matrices on arrays with column major definition
1075      for (int k = 0; k < N; k++){
1076        h_off_diag_G_NN[i] = off_diag_G_NN[k+1][j+1];
1077        h_off_diag_B_NN[i] = off_diag_B_NN[k+1][j+1];
1078        i++;
1079      }
1080      j++;
1081    }
1082    status = cublasSetMatrix(N, N, sizeof(float),
1083      h_off_diag_G_NN, N, d_off_diag_G_NN, N);
1084    status = cublasSetMatrix(N, N, sizeof(float),
1085      h_off_diag_B_NN, N, d_off_diag_B_NN, N);
1086    }//pre_fault end*********************************
1087   //fault begin*********************************
1088  else if ((TIME_STEP >= duration1) &&
1089    (TIME_STEP < duration1+duration2) && change2==1){
1090        change2 = 0;
1091        for(i=1;i<N+1;i++){
1092            for(j=1;j<N+1;j++){
1093                if (j<=N){
1094                    G_NN[i][j]=G.fault[i][j];
1095                    B_NN[i][j]=B.fault[i][j];
1096          h_diag_G_NN[i-1]=G.fault[i][i];//Diagonal elements of G_NN
1097          h_diag_B_NN[i-1]=B.fault[i][i];
1098        }else{
1099                    G_NM[i][j-N]=G.fault[i][j];
1100                    B_NM[i][j-N]=B.fault[i][j];
1101                }
1102            }
1103        }
1104    status = cublasSetVector(N, sizeof(float), h_diag_G_NN, 1, d_diag_G_NN, 1);
1105    status = cublasSetVector(N, sizeof(float), h_diag_B_NN, 1, d_diag_B_NN, 1);
1106
1107    i=0, j=0;
1108    while (i < N*N){//copying matrices on arrays with column major definition
1109      for (int k = 0; k < N; k++){
1110        h_G_NN[i] = G_NN[k+1][j+1];
1111        h_B_NN[i] = B_NN[k+1][j+1];
1112        i++;
1113      }
1114      j++;
1115    }
1116    status = cublasSetMatrix(N, N, sizeof(float), h_G_NN, N, d_G_NN, N);
1117    status = cublasSetMatrix(N, N, sizeof(float), h_B_NN, N, d_B_NN, N);
1118
1119    for(i=1;i<N+1;i++)
1120          for(j=1;j<N+1;j++){
1121        if (i==j){
1122          off_diag_G_NN[i][j]=0;
1123          off_diag_B_NN[i][j]=0;
1124        }else{
1125          off_diag_G_NN[i][j]=G_NN[i][j];
1126          off_diag_B_NN[i][j]=B_NN[i][j];
1127        }
1128      }
1129
1130    i=0, j=0;
1131    while (i < N*N){//copying matrices on arrays with column major definition
1132      for (int k = 0; k < N; k++){
1133        h_off_diag_G_NN[i] = off_diag_G_NN[k+1][j+1];
1134        h_off_diag_B_NN[i] = off_diag_B_NN[k+1][j+1];
1135        i++;
1136      }
1137      j++;
1138    }
1139    status = cublasSetMatrix(N, N, sizeof(float),
1140      h_off_diag_G_NN, N, d_off_diag_G_NN, N);
1141    status = cublasSetMatrix(N, N, sizeof(float),
1142      h_off_diag_B_NN, N, d_off_diag_B_NN, N);
1143    }//fault end*************************************
1144   //post_fault begin*****************************
1145  else if (TIME_STEP >= duration1+duration2 && change3==1){
1146        change3 = 0;
1147        for(i=1;i<N+1;i++){
1148            for(j=1;j<N+1;j++){
1149                if (j<=N){
1150                    G_NN[i][j]=G.postf[i][j];
1151                    B_NN[i][j]=B.postf[i][j];
1152          h_diag_G_NN[i-1]=G.postf[i][i];//Diagonal elements of G_NN
1153          h_diag_B_NN[i-1]=B.postf[i][i];
1154        }else{
1155                    G_NM[i][j-N]=G.postf[i][j];
1156                    B_NM[i][j-N]=B.postf[i][j];
1157                }
1158            }
1159        }
1160    status = cublasSetVector(N, sizeof(float), h_diag_G_NN, 1, d_diag_G_NN, 1);
1161    status = cublasSetVector(N, sizeof(float), h_diag_B_NN, 1, d_diag_B_NN, 1);
1162
1163    i=0, j=0;
1164    while (i < N*N){//copying matrices on arrays with column major definition
1165      for (int k = 0; k < N; k++){
1166        h_G_NN[i] = G_NN[k+1][j+1];
1167        h_B_NN[i] = B_NN[k+1][j+1];
1168        i++;
1169      }
1170      j++;
1171    }
1172    status = cublasSetMatrix(N, N, sizeof(float), h_G_NN, N, d_G_NN, N);
1173    status = cublasSetMatrix(N, N, sizeof(float), h_B_NN, N, d_B_NN, N);
1174
1175    for(i=1;i<N+1;i++)
1176          for(j=1;j<N+1;j++){
1177        if (i==j){
1178          off_diag_G_NN[i][j]=0;
1179          off_diag_B_NN[i][j]=0;
1180        }else{
1181          off_diag_G_NN[i][j]=G_NN[i][j];
1182          off_diag_B_NN[i][j]=B_NN[i][j];
1183        }
1184      }
1185
1186    i=0, j=0;
1187    while (i < N*N){//copying matrices on arrays with column major definition
1188      for (int k = 0; k < N; k++){
1189        h_off_diag_G_NN[i] = off_diag_G_NN[k+1][j+1];
1190        h_off_diag_B_NN[i] = off_diag_B_NN[k+1][j+1];
1191        i++;
1192      }
1193      j++;
1194    }
1195    status = cublasSetMatrix(N, N, sizeof(float),
1196      h_off_diag_G_NN, N, d_off_diag_G_NN, N);
1197    status = cublasSetMatrix(N, N, sizeof(float),
1198      h_off_diag_B_NN, N, d_off_diag_B_NN, N);
1199    }
1200
1201  free(h_G_NN);
1202  free(h_B_NN);
1203  free(h_diag_G_NN);
1204  free(h_diag_B_NN);
1205  free(h_off_diag_G_NN);
1206  free(h_off_diag_B_NN);
1207 }
1208
1209 void AREA::Jacobian_calc(int IsFirstInitCond)
1210 {
1211   int N_detail=gen_num;
1212
1213   float *d_J_F1i_del, *d_J_F2i_del, *d_J_F3i_del, *d_J_F4i_del, *d_J_F5i_del, *d_J_F6i_del, *d_J_F7i_del;
```

```
1214    float *d_J_F1i_sl, *d_J_F2i_sl, *d_J_F8i_sl;
1215    float *d_J_F2i_sf, *d_J_F3i_sf, *d_J_F4i_sf, *d_J_F5i_sf, *d_J_F6i_sf, *d_J_F7i_sf;
1216    float *d_J_F2i_d1, *d_J_F3i_d1, *d_J_F4i_d1, *d_J_F5i_d1, *d_J_F6i_d1, *d_J_F7i_d1;
1217    float *d_J_F2i_q1, *d_J_F3i_q1, *d_J_F4i_q1, *d_J_F5i_q1, *d_J_F6i_q1, *d_J_F7i_q1;
1218    float *d_J_F2i_q2, *d_J_F3i_q2, *d_J_F4i_q2, *d_J_F5i_q2, *d_J_F6i_q2, *d_J_F7i_q2;
1219    float *d_J_F3i_v1, *d_J_F7i_v1;
1220    float *d_J_F8i_v2, *d_J_F9i_v2;
1221    float *d_J_F3i_v3, *d_J_F9i_v3;
1222
1223    cublasStatus status;
1224    status = cublasInit();
1225
1226    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F1i_del);
1227    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F2i_del);
1228    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F3i_del);
1229    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F4i_del);
1230    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F5i_del);
1231    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F6i_del);
1232    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F7i_del);
1233    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F1i_sl);
1234    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F2i_sl);
1235    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F8i_sl);
1236    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F2i_sf);
1237    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F3i_sf);
1238    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F4i_sf);
1239    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F5i_sf);
1240    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F6i_sf);
1241    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F7i_sf);
1242    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F2i_d1);
1243    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F3i_d1);
1244    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F4i_d1);
1245    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F5i_d1);
1246    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F6i_d1);
1247    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F7i_d1);
1248    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F2i_q1);
1249    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F3i_q1);
1250    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F4i_q1);
1251    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F5i_q1);
1252    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F6i_q1);
1253    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F7i_q1);
1254    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F2i_q2);
1255    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F3i_q2);
1256    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F4i_q2);
1257    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F5i_q2);
1258    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F6i_q2);
1259    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F7i_q2);
1260    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F3i_v1);
1261    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F7i_v1);
1262    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F8i_v2);
1263    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F9i_v2);
1264    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F3i_v3);
1265    status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_J_F9i_v3);
1266
1267    COPY_mat_ON_mat(d_J2_0, d_J2, size);// set d_J2=0
1268
1269    if(IsFirstInitCond==0){
1270      cublasSscal(N_detail, 0, d_J_F1i_del, 1);
1271      vecSUMscal(d_J_F1i_del, 1, N_detail);
1272
1273      cublasSscal(N_detail, 0, d_J_F1i_sl, 1);
1274      vecSUMscal(d_J_F1i_sl, -0.5*h*wR, N_detail);
1275
1276      cublasScopy(N_detail, d_H, 1, d_J_F2i_sl, 1);
1277      cublasSscal(N_detail, 2, d_J_F2i_sl, 1);
1278      cublasSaxpy(N_detail, h, d_D, 1, d_J_F2i_sl, 1);
1279
1280      vecDOTvec(d_Kstab, d_TW, d_J_F8i_sl, N_detail);
1281      cublasSscal(N_detail, -1, d_J_F8i_sl, 1);
1282
1283      vecDIVvec(d_Rfd, d_Lfd, d_J_F3i_sf, N_detail);
1284      cublasSscal(N_detail, 0.5*h*wR, d_J_F3i_sf, 1);
1285      vecSUMscal(d_J_F3i_sf, 1, N_detail);
1286
1287      vecDIVvec(d_Rd1, d_Ld1, d_J_F4i_d1, N_detail);
1288      cublasSscal(N_detail, 0.5*h*wR, d_J_F4i_d1, 1);
1289      vecSUMscal(d_J_F4i_d1, 1, N_detail);
1290
1291      vecDIVvec(d_Rq1, d_Lq1, d_J_F5i_q1, N_detail);
1292      cublasSscal(N_detail, 0.5*h*wR, d_J_F5i_q1, 1);
1293      vecSUMscal(d_J_F5i_q1, 1, N_detail);
1294
1295      vecDIVvec(d_Rq2, d_Lq2, d_J_F6i_q2, N_detail);
1296      cublasSscal(N_detail, 0.5*h*wR, d_J_F6i_q2, 1);
1297      vecSUMscal(d_J_F6i_q2, 1, N_detail);
1298
1299      vecDOTvec(d_KA, d_Rfd, d_RESULT3, N_detail);
1300      vecDIVvec(d_RESULT3, d_Lad, d_J_F3i_v1, N_detail);
1301      cublasSscal(N_detail, 0.5*h*wR, d_J_F3i_v1, 1);
1302
1303      cublasScopy(N_detail, d_TR, 1, d_J_F7i_v1, 1);
1304      vecSUMscal(d_J_F7i_v1, 0.5*h, N_detail);
1305
1306      cublasScopy(N_detail, d_TW, 1, d_J_F8i_v2, 1);
1307      vecSUMscal(d_J_F8i_v2, 0.5*h, N_detail);
1308
1309      cublasScopy(N_detail, d_T1, 1, d_J_F9i_v2, 1);
1310      vecSUMscal(d_J_F9i_v2, 0.5*h, N_detail);
1311      cublasSscal(N_detail, -1, d_J_F9i_v2, 1);
1312
1313      vecDOTvec(d_KA, d_Rfd, d_RESULT3, N_detail);
1314      vecDIVvec(d_RESULT3, d_Lad, d_J_F3i_v3, N_detail);
1315      cublasSscal(N_detail, -0.5*h*wR, d_J_F3i_v3, 1);
1316
1317      cublasScopy(N_detail, d_T2, 1, d_J_F9i_v3, 1);
1318      vecSUMscal(d_J_F9i_v3, 0.5*h, N_detail);
1319
1320      copy_to_Jacob2(d_J_F1i_del, d_J2, 0, N_detail);
1321      copy_to_Jacob2(d_J_F1i_sl, d_J2, 9*N_detail, N_detail);
1322      copy_to_Jacob2(d_J_F2i_sl, d_J2, 9*N_detail+1, N_detail);
1323      copy_to_Jacob2(d_J_F8i_sl, d_J2, 9*N_detail+7, N_detail);
1324      copy_to_Jacob2(d_J_F3i_sf, d_J2, 2*9*N_detail+2, N_detail);
1325      copy_to_Jacob2(d_J_F4i_d1, d_J2, 3*9*N_detail+3, N_detail);
1326      copy_to_Jacob2(d_J_F5i_q1, d_J2, 4*9*N_detail+4, N_detail);
1327      copy_to_Jacob2(d_J_F6i_q2, d_J2, 5*9*N_detail+5, N_detail);
1328      copy_to_Jacob2(d_J_F3i_v1, d_J2, 6*9*N_detail+2, N_detail);
1329      copy_to_Jacob2(d_J_F7i_v1, d_J2, 6*9*N_detail+6, N_detail);
1330      copy_to_Jacob2(d_J_F8i_v2, d_J2, 7*9*N_detail+7, N_detail);
1331      copy_to_Jacob2(d_J_F9i_v2, d_J2, 7*9*N_detail+8, N_detail);
1332      copy_to_Jacob2(d_J_F3i_v3, d_J2, 8*9*N_detail+2, N_detail);
1333      copy_to_Jacob2(d_J_F9i_v3, d_J2, 8*9*N_detail+8, N_detail);
1334
1335      //*******************************
1336    }else{
1337      rond_calc();
1338
1339      cublasSscal(N_detail, 0, d_J_F1i_del, 1);
1340      vecSUMscal(d_J_F1i_del, 1, N_detail);
1341
1342      cublasScopy(N_detail, d_r_Te_del, 1, d_J_F2i_del, 1);
1343      cublasSscal(N_detail, -0.5*h, d_J_F2i_del, 1);
1344
1345      vecDOTvec(d_Rfd, d_r_ifd_del, d_J_F3i_del, N_detail);
1346      cublasSscal(N_detail, 0.5*h*wR, d_J_F3i_del, 1);
1347
1348      vecDOTvec(d_Rd1, d_r_id1_del, d_J_F4i_del, N_detail);
1349      cublasSscal(N_detail, 0.5*h*wR, d_J_F4i_del, 1);
1350
1351      vecDOTvec(d_Rq1, d_r_iq1_del, d_J_F5i_del, N_detail);
1352      cublasSscal(N_detail, 0.5*h*wR, d_J_F5i_del, 1);
1353
1354      vecDOTvec(d_Rq2, d_r_iq2_del, d_J_F6i_del, N_detail);
1355      cublasSscal(N_detail, 0.5*h*wR, d_J_F6i_del, 1);
1356
1357      cublasScopy(N_detail, d_r_Vt_del, 1, d_J_F7i_del, 1);
1358      cublasSscal(N_detail, -0.5*h, d_J_F7i_del, 1);
1359      ///////////////////////////////////////////////////////////////
1360      cublasSscal(N_detail, 0, d_J_F1i_sl, 1);
1361      vecSUMscal(d_J_F1i_sl, -0.5*h*wR, N_detail);
1362
1363      cublasScopy(N_detail, d_H, 1, d_J_F2i_sl, 1);
1364      cublasSscal(N_detail, 2, d_J_F2i_sl, 1);
1365      cublasSaxpy(N_detail, h, d_D, 1, d_J_F2i_sl, 1);
```

```
1366
1367     vecDOTvec(d_Kstab, d_TW, d_J_F8i_sl, N_detail);
1368     cublasSscal(N_detail, -1, d_J_F8i_sl, 1);
1369     ////////////////////////////////////////////////////////////////
1370     cublasScopy(N_detail, d_r_Te_sf, 1, d_J_F2i_sf, 1);
1371     cublasSscal(N_detail, -0.5*h, d_J_F2i_sf, 1);
1372
1373     vecDOTvec(d_Rfd, d_r_ifd_sf, d_J_F3i_sf, N_detail);
1374     cublasSscal(N_detail, 0.5*h*wR, d_J_F3i_sf, 1);
1375     vecSUMscal(d_J_F3i_sf, 1, N_detail);
1376
1377     vecDOTvec(d_Rd1, d_r_id1_sf, d_J_F4i_sf, N_detail);
1378     cublasSscal(N_detail, 0.5*h*wR, d_J_F4i_sf, 1);
1379
1380     vecDOTvec(d_Rq1, d_r_iq1_sf, d_J_F5i_sf, N_detail);
1381     cublasSscal(N_detail, 0.5*h*wR, d_J_F5i_sf, 1);
1382
1383     vecDOTvec(d_Rq2, d_r_iq2_sf, d_J_F6i_sf, N_detail);
1384     cublasSscal(N_detail, 0.5*h*wR, d_J_F6i_sf, 1);
1385
1386     cublasScopy(N_detail, d_r_Vt_sf, 1, d_J_F7i_sf, 1);
1387     cublasSscal(N_detail, -0.5*h, d_J_F7i_sf, 1);
1388     ////////////////////////////////////////////////////////////////
1389     cublasScopy(N_detail, d_r_Te_d1, 1, d_J_F2i_d1, 1);
1390     cublasSscal(N_detail, -0.5*h, d_J_F2i_d1, 1);
1391
1392     vecDOTvec(d_Rfd, d_r_ifd_d1, d_J_F3i_d1, N_detail);
1393     cublasSscal(N_detail, 0.5*h*wR, d_J_F3i_d1, 1);
1394
1395     vecDOTvec(d_Rd1, d_r_id1_d1, d_J_F4i_d1, N_detail);
1396     cublasSscal(N_detail, 0.5*h*wR, d_J_F4i_d1, 1);
1397     vecSUMscal(d_J_F4i_d1, 1, N_detail);
1398
1399     vecDOTvec(d_Rq1, d_r_iq1_d1, d_J_F5i_d1, N_detail);
1400     cublasSscal(N_detail, 0.5*h*wR, d_J_F5i_d1, 1);
1401
1402     vecDOTvec(d_Rq2, d_r_iq2_d1, d_J_F6i_d1, N_detail);
1403     cublasSscal(N_detail, 0.5*h*wR, d_J_F6i_d1, 1);
1404
1405     cublasScopy(N_detail, d_r_Vt_d1, 1, d_J_F7i_d1, 1);
1406     cublasSscal(N_detail, -0.5*h, d_J_F7i_d1, 1);
1407     ////////////////////////////////////////////////////////////////
1408     cublasScopy(N_detail, d_r_Te_q1, 1, d_J_F2i_q1, 1);
1409     cublasSscal(N_detail, -0.5*h, d_J_F2i_q1, 1);
1410
1411     vecDOTvec(d_Rfd, d_r_ifd_q1, d_J_F3i_q1, N_detail);
1412     cublasSscal(N_detail, 0.5*h*wR, d_J_F3i_q1, 1);
1413
1414     vecDOTvec(d_Rd1, d_r_id1_q1, d_J_F4i_q1, N_detail);
1415     cublasSscal(N_detail, 0.5*h*wR, d_J_F4i_q1, 1);
1416
1417     vecDOTvec(d_Rq1, d_r_iq1_q1, d_J_F5i_q1, N_detail);
1418     cublasSscal(N_detail, 0.5*h*wR, d_J_F5i_q1, 1);
1419     vecSUMscal(d_J_F5i_q1, 1, N_detail);
1420
1421     vecDOTvec(d_Rq2, d_r_iq2_q1, d_J_F6i_q1, N_detail);
1422     cublasSscal(N_detail, 0.5*h*wR, d_J_F6i_q1, 1);
1423
1424     cublasScopy(N_detail, d_r_Vt_q1, 1, d_J_F7i_q1, 1);
1425     cublasSscal(N_detail, -0.5*h, d_J_F7i_q1, 1);
1426     ////////////////////////////////////////////////////////////////
1427     cublasScopy(N_detail, d_r_Te_q2, 1, d_J_F2i_q2, 1);
1428     cublasSscal(N_detail, -0.5*h, d_J_F2i_q2, 1);
1429
1430     vecDOTvec(d_Rfd, d_r_ifd_q2, d_J_F3i_q2, N_detail);
1431     cublasSscal(N_detail, 0.5*h*wR, d_J_F3i_q2, 1);
1432
1433     vecDOTvec(d_Rd1, d_r_id1_q2, d_J_F4i_q2, N_detail);
1434     cublasSscal(N_detail, 0.5*h*wR, d_J_F4i_q2, 1);
1435
1436     vecDOTvec(d_Rq1, d_r_iq1_q2, d_J_F5i_q2, N_detail);
1437     cublasSscal(N_detail, 0.5*h*wR, d_J_F5i_q2, 1);
1438
1439     vecDOTvec(d_Rq2, d_r_iq2_q2, d_J_F6i_q2, N_detail);
1440     cublasSscal(N_detail, 0.5*h*wR, d_J_F6i_q2, 1);
1441     vecSUMscal(d_J_F6i_q2, 1, N_detail);
1442
1443     cublasScopy(N_detail, d_r_Vt_q2, 1, d_J_F7i_q2, 1);
1444     cublasSscal(N_detail, -0.5*h, d_J_F7i_q2, 1);
1445     ////////////////////////////////////////////////////////////////
1446     cublasScopy(N_detail, d_r_efd_v1, 1, d_J_F3i_v1, 1);
1447     cublasSscal(N_detail, -0.5*h*wR, d_J_F3i_v1, 1);
1448
1449     cublasScopy(N_detail, d_TR, 1, d_J_F7i_v1, 1);
1450     vecSUMscal(d_J_F7i_v1, 0.5*h, N_detail);
1451     ////////////////////////////////////////////////////////////////
1452     cublasScopy(N_detail, d_TW, 1, d_J_F8i_v2, 1);
1453     vecSUMscal(d_J_F8i_v2, 0.5*h, N_detail);
1454
1455     cublasScopy(N_detail, d_T1, 1, d_J_F9i_v2, 1);
1456     vecSUMscal(d_J_F9i_v2, 0.5*h, N_detail);
1457     cublasSscal(N_detail, -1, d_J_F9i_v2, 1);
1458     ////////////////////////////////////////////////////////////////
1459     cublasScopy(N_detail, d_r_efd_v3, 1, d_J_F3i_v3, 1);
1460     cublasSscal(N_detail, -0.5*h*wR, d_J_F3i_v3, 1);
1461
1462     cublasScopy(N_detail, d_T2, 1, d_J_F9i_v3, 1);
1463     vecSUMscal(d_J_F9i_v3, 0.5*h, N_detail);
1464
1465     copy_to_Jacob2(d_J_F1i_del, d_J2, 0, N_detail);
1466     copy_to_Jacob2(d_J_F2i_del, d_J2, 1, N_detail);
1467     copy_to_Jacob2(d_J_F3i_del, d_J2, 2, N_detail);
1468     copy_to_Jacob2(d_J_F4i_del, d_J2, 3, N_detail);
1469     copy_to_Jacob2(d_J_F5i_del, d_J2, 4, N_detail);
1470     copy_to_Jacob2(d_J_F6i_del, d_J2, 5, N_detail);
1471     copy_to_Jacob2(d_J_F7i_del, d_J2, 6, N_detail);
1472
1473     copy_to_Jacob2(d_J_F1i_sl, d_J2, 9*N_detail, N_detail);
1474     copy_to_Jacob2(d_J_F2i_sl, d_J2, 9*N_detail+1, N_detail);
1475     copy_to_Jacob2(d_J_F8i_sl, d_J2, 9*N_detail+7, N_detail);
1476
1477     copy_to_Jacob2(d_J_F2i_sf, d_J2, 2*9*N_detail+1, N_detail);
1478     copy_to_Jacob2(d_J_F3i_sf, d_J2, 2*9*N_detail+2, N_detail);
1479     copy_to_Jacob2(d_J_F4i_sf, d_J2, 2*9*N_detail+3, N_detail);
1480     copy_to_Jacob2(d_J_F5i_sf, d_J2, 2*9*N_detail+4, N_detail);
1481     copy_to_Jacob2(d_J_F6i_sf, d_J2, 2*9*N_detail+5, N_detail);
1482     copy_to_Jacob2(d_J_F7i_sf, d_J2, 2*9*N_detail+6, N_detail);
1483
1484     copy_to_Jacob2(d_J_F2i_d1, d_J2, 3*9*N_detail+1, N_detail);
1485     copy_to_Jacob2(d_J_F3i_d1, d_J2, 3*9*N_detail+2, N_detail);
1486     copy_to_Jacob2(d_J_F4i_d1, d_J2, 3*9*N_detail+3, N_detail);
1487     copy_to_Jacob2(d_J_F5i_d1, d_J2, 3*9*N_detail+4, N_detail);
1488     copy_to_Jacob2(d_J_F6i_d1, d_J2, 3*9*N_detail+5, N_detail);
1489     copy_to_Jacob2(d_J_F7i_d1, d_J2, 3*9*N_detail+6, N_detail);
1490
1491     copy_to_Jacob2(d_J_F2i_q1, d_J2, 4*9*N_detail+1, N_detail);
1492     copy_to_Jacob2(d_J_F3i_q1, d_J2, 4*9*N_detail+2, N_detail);
1493     copy_to_Jacob2(d_J_F4i_q1, d_J2, 4*9*N_detail+3, N_detail);
1494     copy_to_Jacob2(d_J_F5i_q1, d_J2, 4*9*N_detail+4, N_detail);
1495     copy_to_Jacob2(d_J_F6i_q1, d_J2, 4*9*N_detail+5, N_detail);
1496     copy_to_Jacob2(d_J_F7i_q1, d_J2, 4*9*N_detail+6, N_detail);
1497
1498     copy_to_Jacob2(d_J_F2i_q2, d_J2, 5*9*N_detail+1, N_detail);
1499     copy_to_Jacob2(d_J_F3i_q2, d_J2, 5*9*N_detail+2, N_detail);
1500     copy_to_Jacob2(d_J_F4i_q2, d_J2, 5*9*N_detail+3, N_detail);
1501     copy_to_Jacob2(d_J_F5i_q2, d_J2, 5*9*N_detail+4, N_detail);
1502     copy_to_Jacob2(d_J_F6i_q2, d_J2, 5*9*N_detail+5, N_detail);
1503     copy_to_Jacob2(d_J_F7i_q2, d_J2, 5*9*N_detail+6, N_detail);
1504
1505     copy_to_Jacob2(d_J_F3i_v1, d_J2, 6*9*N_detail+2, N_detail);
1506     copy_to_Jacob2(d_J_F7i_v1, d_J2, 6*9*N_detail+6, N_detail);
1507
1508     copy_to_Jacob2(d_J_F8i_v2, d_J2, 7*9*N_detail+7, N_detail);
1509     copy_to_Jacob2(d_J_F9i_v2, d_J2, 7*9*N_detail+8, N_detail);
1510
1511     copy_to_Jacob2(d_J_F3i_v3, d_J2, 8*9*N_detail+2, N_detail);
1512     copy_to_Jacob2(d_J_F9i_v3, d_J2, 8*9*N_detail+8, N_detail);
1513   }
1514
1515
1516     cublasFree(d_J_F1i_del);
1517     cublasFree(d_J_F2i_del);
```

```
1518     cublasFree(d_J_F3i_del);
1519     cublasFree(d_J_F4i_del);
1520     cublasFree(d_J_F5i_del);
1521     cublasFree(d_J_F6i_del);
1522     cublasFree(d_J_F7i_del);
1523     cublasFree(d_J_F1i_sl);
1524     cublasFree(d_J_F2i_sl);
1525     cublasFree(d_J_F8i_sl);
1526     cublasFree(d_J_F2i_sf);
1527     cublasFree(d_J_F3i_sf);
1528     cublasFree(d_J_F4i_sf);
1529     cublasFree(d_J_F5i_sf);
1530     cublasFree(d_J_F6i_sf);
1531     cublasFree(d_J_F7i_sf);
1532     cublasFree(d_J_F2i_d1);
1533     cublasFree(d_J_F3i_d1);
1534     cublasFree(d_J_F4i_d1);
1535     cublasFree(d_J_F5i_d1);
1536     cublasFree(d_J_F6i_d1);
1537     cublasFree(d_J_F7i_d1);
1538     cublasFree(d_J_F2i_q1);
1539     cublasFree(d_J_F3i_q1);
1540     cublasFree(d_J_F4i_q1);
1541     cublasFree(d_J_F5i_q1);
1542     cublasFree(d_J_F6i_q1);
1543     cublasFree(d_J_F7i_q1);
1544     cublasFree(d_J_F2i_q2);
1545     cublasFree(d_J_F3i_q2);
1546     cublasFree(d_J_F4i_q2);
1547     cublasFree(d_J_F5i_q2);
1548     cublasFree(d_J_F6i_q2);
1549     cublasFree(d_J_F7i_q2);
1550     cublasFree(d_J_F3i_v1);
1551     cublasFree(d_J_F7i_v1);
1552     cublasFree(d_J_F8i_v2);
1553     cublasFree(d_J_F9i_v2);
1554     cublasFree(d_J_F3i_v3);
1555     cublasFree(d_J_F9i_v3);
1556         /*end of Jacobian*******************/
1557 }
1558 void AREA::rond_calc()
1559 {
1560   int N_detail=gen_num;
1561 
1562   float *d_r_Eqs_d1,*d_r_Eqs_sf,*d_r_Eds_q1,*d_r_Eds_q2;
1563   float *d_r_u5_del,*d_r_u5_sf,*d_r_u5_d1,*d_r_u5_q1,*d_r_u5_q2;
1564   float *d_r_u6_del,*d_r_u6_sf,*d_r_u6_d1,*d_r_u6_q1,*d_r_u6_q2;
1565   float *d_r_A7_del,*d_r_A7_sf,*d_r_A7_d1,*d_r_A7_q1,*d_r_A7_q2;
1566   float *d_r_A8_del,*d_r_A8_sf,*d_r_A8_d1,*d_r_A8_q1,*d_r_A8_q2;
1567   float *d_r_ID_del,*d_r_ID_sf,*d_r_ID_d1,*d_r_ID_q1,*d_r_ID_q2;
1568   float *d_r_IQ_del,*d_r_IQ_sf,*d_r_IQ_d1,*d_r_IQ_q1,*d_r_IQ_q2;
1569   float *d_r_id_del,*d_r_id_sf,*d_r_id_d1,*d_r_id_q1,*d_r_id_q2;
1570   float *d_r_iq_del,*d_r_iq_sf,*d_r_iq_d1,*d_r_iq_q1,*d_r_iq_q2;
1571   float *d_r_sad_del,*d_r_sad_sf,*d_r_sad_d1,*d_r_sad_q1,*d_r_sad_q2;
1572   float *d_r_saq_del,*d_r_saq_sf,*d_r_saq_d1,*d_r_saq_q1,*d_r_saq_q2;
1573   float *d_r_VtD_del,*d_r_VtD_sf,*d_r_VtD_d1,*d_r_VtD_q1,*d_r_VtD_q2;
1574   float *d_r_VtQ_del,*d_r_VtQ_sf,*d_r_VtQ_d1,*d_r_VtQ_q1,*d_r_VtQ_q2;
1575 
1576     cublasStatus status;
1577   status = cublasInit();
1578 
1579 
1580   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_Eqs_d1);
1581   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_Eqs_sf);
1582   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_Eds_q1);
1583   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_Eds_q2);
1584   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_u5_del);
1585   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_u5_sf);
1586   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_u5_d1);
1587   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_u5_q1);
1588   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_u5_q2);
1589   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_u6_del);
1590   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_u6_sf);
1591   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_u6_d1);
1592   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_u6_q1);
1593   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_u6_q2);
1594   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_A7_del);
1595   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_A7_sf);
1596   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_A7_d1);
1597   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_A7_q1);
1598   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_A7_q2);
1599   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_A8_del);
1600   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_A8_sf);
1601   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_A8_d1);
1602   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_A8_q1);
1603   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_A8_q2);
1604   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_ID_del);
1605   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_ID_sf);
1606   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_ID_d1);
1607   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_ID_q1);
1608   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_ID_q2);
1609   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_IQ_del);
1610   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_IQ_sf);
1611   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_IQ_d1);
1612   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_IQ_q1);
1613   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_IQ_q2);
1614   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_id_del);
1615   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_id_sf);
1616   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_id_d1);
1617   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_id_q1);
1618   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_id_q2);
1619   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_iq_del);
1620   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_iq_sf);
1621   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_iq_d1);
1622   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_iq_q1);
1623   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_iq_q2);
1624   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_sad_del);
1625   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_sad_sf);
1626   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_sad_d1);
1627   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_sad_q1);
1628   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_sad_q2);
1629   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_saq_del);
1630   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_saq_sf);
1631   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_saq_d1);
1632   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_saq_q1);
1633   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_saq_q2);
1634   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_VtD_del);
1635   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_VtD_sf);
1636   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_VtD_d1);
1637   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_VtD_q1);
1638   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_VtD_q2);
1639   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_VtQ_del);
1640   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_VtQ_sf);
1641   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_VtQ_d1);
1642   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_VtQ_q1);
1643   status = cublasAlloc(N_detail*1, sizeof(float), (void**)&d_r_VtQ_q2);
1644 
1645 
1646 
1647   vecDIVvec(d_Laq_sec, d_Lq1, d_r_Eds_q1, N_detail);
1648   vecDIVvec(d_Laq_sec, d_Lq2, d_r_Eds_q2, N_detail);
1649   vecDIVvec(d_Lad_sec, d_Lfd, d_r_Eqs_sf, N_detail);
1650   vecDIVvec(d_Lad_sec, d_Ld1, d_r_Eqs_d1, N_detail);
1651   ////////////////////////////////////////////////////
1652   vecSin(d_delta, d_RESULT1, N_detail);
1653   vecDOTvec(d_RESULT1, d_Eds, d_r_u5_del, N_detail);
1654   vecCos(d_delta, d_RESULT2, N_detail);
1655   vecDOTvec(d_RESULT2, d_Eqs, d_RESULT3, N_detail);
1656   cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_u5_del, 1);
1657 
1658   vecDOTvec(d_RESULT1, d_r_Eqs_sf, d_r_u5_sf, N_detail);
1659   cublasSscal(N_detail, -1, d_r_u5_sf, 1);
1660 
1661   vecDOTvec(d_RESULT1, d_r_Eqs_d1, d_r_u5_d1, N_detail);
1662   cublasSscal(N_detail, -1, d_r_u5_d1, 1);
1663 
1664   vecDOTvec(d_RESULT2, d_r_Eds_q1, d_r_u5_q1, N_detail);
1665   cublasSscal(N_detail, -1, d_r_u5_q1, 1);
1666 
1667   vecDOTvec(d_RESULT2, d_r_Eds_q2, d_r_u5_q2, N_detail);
1668   cublasSscal(N_detail, -1, d_r_u5_q2, 1);
1669   ////////////////////////////////////////////////////
```

```
1670  vecDOTvec(d_RESULT1, d_Eqs, d_r_u6_del, N_detail);///d_RESULT1=sin(delta)
1671  vecDOTvec(d_RESULT2, d_Eds, d_RESULT3, N_detail);///d_RESULT2=cos(delta)
1672  cublasSaxpy(N_detail, +1, d_RESULT3, 1, d_r_u6_del, 1);
1673  cublasSscal(N_detail, -1, d_r_u6_del, 1);
1674
1675  vecDOTvec(d_RESULT2, d_r_Eqs_sf, d_r_u6_sf, N_detail);
1676
1677  vecDOTvec(d_RESULT2, d_r_Eqs_d1, d_r_u6_d1, N_detail);
1678
1679  vecDOTvec(d_RESULT1, d_r_Eds_q1, d_r_u6_q1, N_detail);
1680  cublasSscal(N_detail, -1, d_r_u6_q1, 1);
1681
1682  vecDOTvec(d_RESULT1, d_r_Eds_q2, d_r_u6_q2, N_detail);
1683  cublasSscal(N_detail, -1, d_r_u6_q2, 1);
1684  ////////////////////////////////////////////////////
1685  vecDOTvec(d_diag_G_NN, d_r_u5_del, d_r_A7_del, N_detail);
1686  vecDOTvec(d_diag_B_NN, d_r_u6_del, d_RESULT3, N_detail);
1687  cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_A7_del, 1);
1688
1689  vecDOTvec(d_diag_G_NN, d_r_u5_sf, d_r_A7_sf, N_detail);
1690  vecDOTvec(d_diag_B_NN, d_r_u6_sf, d_RESULT3, N_detail);
1691  cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_A7_sf, 1);
1692
1693  vecDOTvec(d_diag_G_NN, d_r_u5_d1, d_r_A7_d1, N_detail);
1694  vecDOTvec(d_diag_B_NN, d_r_u6_d1, d_RESULT3, N_detail);
1695  cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_A7_d1, 1);
1696
1697  vecDOTvec(d_diag_G_NN, d_r_u5_q1, d_r_A7_q1, N_detail);
1698  vecDOTvec(d_diag_B_NN, d_r_u6_q1, d_RESULT3, N_detail);
1699  cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_A7_q1, 1);
1700
1701  vecDOTvec(d_diag_G_NN, d_r_u5_q2, d_r_A7_q2, N_detail);
1702  vecDOTvec(d_diag_B_NN, d_r_u6_q2, d_RESULT3, N_detail);
1703  cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_A7_q2, 1);
1704  ////////////////////////////////////////////////////
1705  vecDOTvec(d_diag_G_NN, d_r_u6_del, d_r_A8_del, N_detail);
1706  vecDOTvec(d_diag_B_NN, d_r_u5_del, d_RESULT3, N_detail);
1707  cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_A8_del, 1);
1708
1709  vecDOTvec(d_diag_G_NN, d_r_u6_sf, d_r_A8_sf, N_detail);
1710  vecDOTvec(d_diag_B_NN, d_r_u5_sf, d_RESULT3, N_detail);
1711  cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_A8_sf, 1);
1712
1713  vecDOTvec(d_diag_G_NN, d_r_u6_d1, d_r_A8_d1, N_detail);
1714  vecDOTvec(d_diag_B_NN, d_r_u5_d1, d_RESULT3, N_detail);
1715  cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_A8_d1, 1);
1716
1717  vecDOTvec(d_diag_G_NN, d_r_u6_q1, d_r_A8_q1, N_detail);
1718  vecDOTvec(d_diag_B_NN, d_r_u5_q1, d_RESULT3, N_detail);
1719  cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_A8_q1, 1);
1720
1721  vecDOTvec(d_diag_G_NN, d_r_u6_q2, d_r_A8_q2, N_detail);
1722  vecDOTvec(d_diag_B_NN, d_r_u5_q2, d_RESULT3, N_detail);
1723  cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_A8_q2, 1);
1724  ////////////////////////////////////////////////////
1725  vecDOTvec(d_r_A7_del, d_A5, d_r_ID_del, N_detail);
1726  vecDOTvec(d_A5, d_A5, d_RESULT3, N_detail);
1727  vecDIVvec(d_r_ID_del, d_RESULT3, d_r_ID_del, N_detail);
1728
1729  vecDIVvec(d_r_A7_sf, d_A5, d_r_ID_sf, N_detail);
1730
1731  vecDIVvec(d_r_A7_d1, d_A5, d_r_ID_d1, N_detail);
1732
1733  vecDIVvec(d_r_A7_q1, d_A5, d_r_ID_q1, N_detail);
1734
1735  vecDIVvec(d_r_A7_q2, d_A5, d_r_ID_q2, N_detail);
1736  ////////////////////////////////////////////////////
1737  vecDOTvec(d_r_A8_del, d_A6, d_r_IQ_del, N_detail);
1738  vecDOTvec(d_A6, d_A6, d_RESULT3, N_detail);
1739  vecDIVvec(d_r_IQ_del, d_RESULT3, d_r_IQ_del, N_detail);
1740
1741  vecDIVvec(d_r_A8_sf, d_A6, d_r_IQ_sf, N_detail);
1742
1743  vecDIVvec(d_r_A8_d1, d_A6, d_r_IQ_d1, N_detail);
1744
1745  vecDIVvec(d_r_A8_q1, d_A6, d_r_IQ_q1, N_detail);
1746
1747  vecDIVvec(d_r_A8_q2, d_A6, d_r_IQ_q2, N_detail);
1748  ////////////////////////////////////////////////////
1749  ///d_RESULT1=sin(delta); d_RESULT2=cos(delta)
1750
1751  vecDOTvec(d_r_ID_del, d_RESULT2, d_r_id_del, N_detail);
1752  vecDOTvec(d_ID, d_RESULT1, d_RESULT3, N_detail);
1753  cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_id_del, 1);
1754  vecDOTvec(d_r_IQ_del, d_RESULT1, d_RESULT3, N_detail);
1755  cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_id_del, 1);
1756  vecDOTvec(d_IQ, d_RESULT2, d_RESULT3, N_detail);
1757  cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_id_del, 1);
1758
1759  vecDOTvec(d_r_ID_sf, d_RESULT2, d_r_id_sf, N_detail);
1760  vecDOTvec(d_r_IQ_sf, d_RESULT1, d_RESULT3, N_detail);
1761  cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_id_sf, 1);
1762
1763  vecDOTvec(d_r_ID_d1, d_RESULT2, d_r_id_d1, N_detail);
1764  vecDOTvec(d_r_IQ_d1, d_RESULT1, d_RESULT3, N_detail);
1765  cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_id_d1, 1);
1766
1767  vecDOTvec(d_r_ID_q1, d_RESULT2, d_r_id_q1, N_detail);
1768  vecDOTvec(d_r_IQ_q1, d_RESULT1, d_RESULT3, N_detail);
1769  cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_id_q1, 1);
1770
1771  vecDOTvec(d_r_ID_q2, d_RESULT2, d_r_id_q2, N_detail);
1772  vecDOTvec(d_r_IQ_q2, d_RESULT1, d_RESULT3, N_detail);
1773  cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_id_q2, 1);
1774  ////////////////////////////////////////////////////
1775  ///d_RESULT1=sin(delta); d_RESULT2=cos(delta)
1776  vecDOTvec(d_r_IQ_del, d_RESULT2, d_r_iq_del, N_detail);
1777  vecDOTvec(d_IQ, d_RESULT1, d_RESULT3, N_detail);
1778  cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_iq_del, 1);
1779  vecDOTvec(d_r_ID_del, d_RESULT1, d_RESULT3, N_detail);
1780  cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_iq_del, 1);
1781  vecDOTvec(d_ID, d_RESULT2, d_RESULT3, N_detail);
1782  cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_iq_del, 1);
1783
1784  vecDOTvec(d_r_IQ_sf, d_RESULT2, d_r_iq_sf, N_detail);
1785  vecDOTvec(d_r_ID_sf, d_RESULT1, d_RESULT3, N_detail);
1786  cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_iq_sf, 1);
1787
1788  vecDOTvec(d_r_IQ_d1, d_RESULT2, d_r_iq_d1, N_detail);
1789  vecDOTvec(d_r_ID_d1, d_RESULT1, d_RESULT3, N_detail);
1790  cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_iq_d1, 1);
1791
1792  vecDOTvec(d_r_IQ_q1, d_RESULT2, d_r_iq_q1, N_detail);
1793  vecDOTvec(d_r_ID_q1, d_RESULT1, d_RESULT3, N_detail);
1794  cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_iq_q1, 1);
1795
1796  vecDOTvec(d_r_IQ_q2, d_RESULT2, d_r_iq_q2, N_detail);
1797  vecDOTvec(d_r_ID_q2, d_RESULT1, d_RESULT3, N_detail);
1798  cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_iq_q2, 1);
1799  ////////////////////////////////////////////////////
1800  vecDOTvec(d_Lad_sec, d_r_id_del, d_r_sad_del, N_detail);
1801  cublasSscal(N_detail, -1, d_r_sad_del, 1);
1802
1803  vecDOTvec(d_Lfd, d_r_id_sf, d_RESULT3, N_detail);
1804  cublasSscal(N_detail, -1, d_RESULT3, 1);
1805  vecSUMscal(d_RESULT3, 1, N_detail);
1806  vecDIVvec(d_RESULT3, d_Lfd, d_r_sad_sf, N_detail);
1807  vecDOTvec(d_r_sad_sf, d_Lad_sec, d_r_sad_sf, N_detail);
1808
1809  vecDOTvec(d_Ld1, d_r_id_d1, d_RESULT3, N_detail);
1810  cublasSscal(N_detail, -1, d_RESULT3, 1);
1811  vecSUMscal(d_RESULT3, 1, N_detail);
1812  vecDIVvec(d_RESULT3, d_Ld1, d_r_sad_d1, N_detail);
1813  vecDOTvec(d_r_sad_d1, d_Lad_sec, d_r_sad_d1, N_detail);
1814
1815  vecDOTvec(d_Lad_sec, d_r_id_q1, d_r_sad_q1, N_detail);
1816  cublasSscal(N_detail, -1, d_r_sad_q1, 1);
1817
1818  vecDOTvec(d_Lad_sec, d_r_id_q2, d_r_sad_q2, N_detail);
1819  cublasSscal(N_detail, -1, d_r_sad_q2, 1);
1820  ////////////////////////////////////////////////////
1821  vecDOTvec(d_Laq_sec, d_r_iq_del, d_r_saq_del, N_detail);
```

```
1822    cublasSscal(N_detail, -1, d_r_saq_del, 1);
1823
1824    vecDOTvec(d_Laq_sec, d_r_iq_sf, d_r_saq_sf, N_detail);
1825    cublasSscal(N_detail, -1, d_r_saq_sf, 1);
1826
1827    vecDOTvec(d_Laq_sec, d_r_iq_d1, d_r_saq_d1, N_detail);
1828    cublasSscal(N_detail, -1, d_r_saq_d1, 1);
1829
1830    vecDOTvec(d_Lq1, d_r_iq_q1, d_RESULT3, N_detail);
1831    cublasSscal(N_detail, -1, d_RESULT3, 1);
1832    vecSUMscal(d_RESULT3, 1, N_detail);
1833    vecDIVvec(d_RESULT3, d_Lq1, d_r_saq_q1, N_detail);
1834    vecDOTvec(d_r_saq_q1, d_Laq_sec, d_r_saq_q1, N_detail);
1835
1836    vecDOTvec(d_Lq2, d_r_iq_q2, d_RESULT3, N_detail);
1837    cublasSscal(N_detail, -1, d_RESULT3, 1);
1838    vecSUMscal(d_RESULT3, 1, N_detail);
1839    vecDIVvec(d_RESULT3, d_Lq2, d_r_saq_q2, N_detail);
1840    vecDOTvec(d_r_saq_q2, d_Laq_sec, d_r_saq_q2, N_detail);
1841    ////////////////////////////////////////////////////
1842    vecDOTvec(d_r_sad_del, d_iq, d_RESULT3, N_detail);
1843    vecDOTvec(d_s_ad, d_r_iq_del, d_r_Te_del, N_detail);
1844    cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_Te_del, 1);
1845    vecDOTvec(d_r_saq_del, d_id, d_RESULT3, N_detail);
1846    cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_Te_del, 1);
1847    vecDOTvec(d_s_aq, d_r_id_del, d_RESULT3, N_detail);
1848    cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_Te_del, 1);
1849    cublasSscal(N_detail, -1, d_r_Te_del, 1);
1850
1851    vecDOTvec(d_r_sad_sf, d_iq, d_RESULT3, N_detail);
1852    vecDOTvec(d_s_ad, d_r_iq_sf, d_r_Te_sf, N_detail);
1853    cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_Te_sf, 1);
1854    vecDOTvec(d_r_saq_sf, d_id, d_RESULT3, N_detail);
1855    cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_Te_sf, 1);
1856    vecDOTvec(d_s_aq, d_r_id_sf, d_RESULT3, N_detail);
1857    cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_Te_sf, 1);
1858    cublasSscal(N_detail, -1, d_r_Te_sf, 1);
1859
1860    vecDOTvec(d_r_sad_d1, d_iq, d_RESULT3, N_detail);
1861    vecDOTvec(d_s_ad, d_r_iq_d1, d_r_Te_d1, N_detail);
1862    cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_Te_d1, 1);
1863    vecDOTvec(d_r_saq_d1, d_id, d_RESULT3, N_detail);
1864    cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_Te_d1, 1);
1865    vecDOTvec(d_s_aq, d_r_id_d1, d_RESULT3, N_detail);
1866    cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_Te_d1, 1);
1867    cublasSscal(N_detail, -1, d_r_Te_d1, 1);
1868
1869    vecDOTvec(d_r_sad_q1, d_iq, d_RESULT3, N_detail);
1870    vecDOTvec(d_s_ad, d_r_iq_q1, d_r_Te_q1, N_detail);
1871    cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_Te_q1, 1);
1872    vecDOTvec(d_r_saq_q1, d_id, d_RESULT3, N_detail);
1873    cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_Te_q1, 1);
1874    vecDOTvec(d_s_aq, d_r_id_q1, d_RESULT3, N_detail);
1875    cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_Te_q1, 1);
1876    cublasSscal(N_detail, -1, d_r_Te_q1, 1);
1877
1878    vecDOTvec(d_r_sad_q2, d_iq, d_RESULT3, N_detail);
1879    vecDOTvec(d_s_ad, d_r_iq_q2, d_r_Te_q2, N_detail);
1880    cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_Te_q2, 1);
1881    vecDOTvec(d_r_saq_q2, d_id, d_RESULT3, N_detail);
1882    cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_Te_q2, 1);
1883    vecDOTvec(d_s_aq, d_r_id_q2, d_RESULT3, N_detail);
1884    cublasSaxpy(N_detail, -1, d_RESULT3, 1, d_r_Te_q2, 1);
1885    cublasSscal(N_detail, -1, d_r_Te_q2, 1);
1886    ////////////////////////////////////////////////////
1887    vecDIVvec(d_r_sad_del, d_Lfd, d_r_ifd_del, N_detail);
1888    cublasSscal(N_detail, -1, d_r_ifd_del, 1);
1889
1890    cublasScopy(N_detail, d_r_sad_sf, 1, d_RESULT3, 1);
1891    cublasSscal(N_detail, -1, d_RESULT3, 1);
1892    vecSUMscal(d_RESULT3, 1, N_detail);
1893    vecDIVvec(d_RESULT3, d_Lfd, d_r_ifd_sf, N_detail);
1894
1895    vecDIVvec(d_r_sad_d1, d_Lfd, d_r_ifd_d1, N_detail);
1896    cublasSscal(N_detail, -1, d_r_ifd_d1, 1);
1897
1898    vecDIVvec(d_r_sad_q1, d_Lfd, d_r_ifd_q1, N_detail);
1899    cublasSscal(N_detail, -1, d_r_ifd_q1, 1);
1900
1901    vecDIVvec(d_r_sad_q2, d_Lfd, d_r_ifd_q2, N_detail);
1902    cublasSscal(N_detail, -1, d_r_ifd_q2, 1);
1903    ////////////////////////////////////////////////////
1904    vecDOTvec(d_Rfd, d_r_ifd_del, d_r_efd_del, N_detail);
1905
1906    vecDOTvec(d_Rfd, d_r_ifd_sf, d_r_efd_sf, N_detail);
1907
1908    vecDOTvec(d_Rfd, d_r_ifd_d1, d_r_efd_d1, N_detail);
1909
1910    vecDOTvec(d_Rfd, d_r_ifd_q1, d_r_efd_q1, N_detail);
1911
1912    vecDOTvec(d_Rfd, d_r_ifd_q2, d_r_efd_q2, N_detail);
1913    ////////////////////////////////////////////////////
1914    vecDIVvec(d_r_sad_del, d_Ld1, d_r_id1_del, N_detail);
1915    cublasSscal(N_detail, -1, d_r_id1_del, 1);
1916
1917    vecDIVvec(d_r_sad_sf, d_Ld1, d_r_id1_sf, N_detail);
1918    cublasSscal(N_detail, -1, d_r_id1_sf, 1);
1919
1920    cublasScopy(N_detail, d_r_sad_d1, 1, d_RESULT3, 1);
1921    cublasSscal(N_detail, -1, d_RESULT3, 1);
1922    vecSUMscal(d_RESULT3, 1, N_detail);
1923    vecDIVvec(d_RESULT3, d_Ld1, d_r_id1_d1, N_detail);
1924
1925    vecDIVvec(d_r_sad_q1, d_Ld1, d_r_id1_q1, N_detail);
1926    cublasSscal(N_detail, -1, d_r_id1_q1, 1);
1927
1928    vecDIVvec(d_r_sad_q2, d_Ld1, d_r_id1_q2, N_detail);
1929    cublasSscal(N_detail, -1, d_r_id1_q2, 1);
1930    ////////////////////////////////////////////////////
1931    vecDIVvec(d_r_saq_del, d_Lq1, d_r_iq1_del, N_detail);
1932    cublasSscal(N_detail, -1, d_r_iq1_del, 1);
1933
1934    vecDIVvec(d_r_saq_sf, d_Lq1, d_r_iq1_sf, N_detail);
1935    cublasSscal(N_detail, -1, d_r_iq1_sf, 1);
1936
1937    vecDIVvec(d_r_saq_d1, d_Lq1, d_r_iq1_d1, N_detail);
1938    cublasSscal(N_detail, -1, d_r_iq1_d1, 1);
1939
1940    cublasScopy(N_detail, d_r_saq_q1, 1, d_RESULT3, 1);
1941    cublasSscal(N_detail, -1, d_RESULT3, 1);
1942    vecSUMscal(d_RESULT3, 1, N_detail);
1943    vecDIVvec(d_RESULT3, d_Lq1, d_r_iq1_q1, N_detail);
1944
1945    vecDIVvec(d_r_saq_q2, d_Lq1, d_r_iq1_q2, N_detail);
1946    cublasSscal(N_detail, -1, d_r_iq1_q2, 1);
1947    ////////////////////////////////////////////////////
1948    vecDIVvec(d_r_saq_del, d_Lq2, d_r_iq2_del, N_detail);
1949    cublasSscal(N_detail, -1, d_r_iq2_del, 1);
1950
1951    vecDIVvec(d_r_saq_sf, d_Lq2, d_r_iq2_sf, N_detail);
1952    cublasSscal(N_detail, -1, d_r_iq2_sf, 1);
1953
1954    vecDIVvec(d_r_saq_d1, d_Lq2, d_r_iq2_d1, N_detail);
1955    cublasSscal(N_detail, -1, d_r_iq2_d1, 1);
1956
1957    vecDIVvec(d_r_saq_q1, d_Lq2, d_r_iq2_q1, N_detail);
1958    cublasSscal(N_detail, -1, d_r_iq2_q1, 1);
1959
1960    cublasScopy(N_detail, d_r_saq_q2, 1, d_RESULT3, 1);
1961    cublasSscal(N_detail, -1, d_RESULT3, 1);
1962    vecSUMscal(d_RESULT3, 1, N_detail);
1963    vecDIVvec(d_RESULT3, d_Lq2, d_r_iq2_q2, N_detail);
1964    ////////////////////////////////////////////////////
1965    vecDOTvec(d_r_ID_del, d_U1, d_r_VtD_del, N_detail);
1966    vecDOTvec(d_r_IQ_del, d_U2, d_RESULT3, N_detail);
1967    cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_VtD_del, 1);
1968    cublasSaxpy(N_detail, 1, d_r_u5_del, 1, d_r_VtD_del, 1);
1969
1970    vecDOTvec(d_r_ID_sf, d_U1, d_r_VtD_sf, N_detail);
1971    vecDOTvec(d_r_IQ_sf, d_U2, d_RESULT3, N_detail);
1972    cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_VtD_sf, 1);
1973    cublasSaxpy(N_detail, 1, d_r_u5_sf, 1, d_r_VtD_sf, 1);
```

```
1974
1975   vecDOTvec(d_r_ID_d1, d_U1, d_r_VtD_d1, N_detail);
1976   vecDOTvec(d_r_IQ_d1, d_U2, d_RESULT3, N_detail);
1977   cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_VtD_d1, 1);
1978   cublasSaxpy(N_detail, 1, d_r_u5_d1, 1, d_r_VtD_d1, 1);
1979
1980   vecDOTvec(d_r_ID_q1, d_U1, d_r_VtD_q1, N_detail);
1981   vecDOTvec(d_r_IQ_q1, d_U2, d_RESULT3, N_detail);
1982   cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_VtD_q1, 1);
1983   cublasSaxpy(N_detail, 1, d_r_u5_q1, 1, d_r_VtD_q1, 1);
1984
1985   vecDOTvec(d_r_ID_q2, d_U1, d_r_VtD_q2, N_detail);
1986   vecDOTvec(d_r_IQ_q2, d_U2, d_RESULT3, N_detail);
1987   cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_VtD_q2, 1);
1988   cublasSaxpy(N_detail, 1, d_r_u5_q2, 1, d_r_VtD_q2, 1);
1989   ///////////////////////////////////////////////////////
1990   vecDOTvec(d_r_ID_del, d_U3, d_r_VtQ_del, N_detail);
1991   vecDOTvec(d_r_IQ_del, d_U4, d_RESULT3, N_detail);
1992   cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_VtQ_del, 1);
1993   cublasSaxpy(N_detail, 1, d_r_u6_del, 1, d_r_VtQ_del, 1);
1994
1995   vecDOTvec(d_r_ID_sf, d_U3, d_r_VtQ_sf, N_detail);
1996   vecDOTvec(d_r_IQ_sf, d_U4, d_RESULT3, N_detail);
1997   cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_VtQ_sf, 1);
1998   cublasSaxpy(N_detail, 1, d_r_u6_sf, 1, d_r_VtQ_sf, 1);
1999
2000   vecDOTvec(d_r_ID_d1, d_U3, d_r_VtQ_d1, N_detail);
2001   vecDOTvec(d_r_IQ_d1, d_U4, d_RESULT3, N_detail);
2002   cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_VtQ_d1, 1);
2003   cublasSaxpy(N_detail, 1, d_r_u6_d1, 1, d_r_VtQ_d1, 1);
2004
2005   vecDOTvec(d_r_ID_q1, d_U3, d_r_VtQ_q1, N_detail);
2006   vecDOTvec(d_r_IQ_q1, d_U4, d_RESULT3, N_detail);
2007   cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_VtQ_q1, 1);
2008   cublasSaxpy(N_detail, 1, d_r_u6_q1, 1, d_r_VtQ_q1, 1);
2009
2010   vecDOTvec(d_r_ID_q2, d_U3, d_r_VtQ_q2, N_detail);
2011   vecDOTvec(d_r_IQ_q2, d_U4, d_RESULT3, N_detail);
2012   cublasSaxpy(N_detail, 1, d_RESULT3, 1, d_r_VtQ_q2, 1);
2013   cublasSaxpy(N_detail, 1, d_r_u6_q2, 1, d_r_VtQ_q2, 1);
2014   ///////////////////////////////////////////////////////
2015
2016   vecDOTvec(d_VQ, d_r_VtQ_del, d_RESULT1, N_detail);
2017   vecDOTvec(d_VD, d_r_VtD_del, d_RESULT2, N_detail);
2018   cublasSaxpy(N_detail, 1, d_RESULT2, 1, d_RESULT1, 1);
2019   vecDIVvec(d_RESULT1, d_vt, d_r_Vt_del, N_detail);
2020
2021   vecDOTvec(d_VQ, d_r_VtQ_sf, d_RESULT1, N_detail);
2022   vecDOTvec(d_VD, d_r_VtD_sf, d_RESULT2, N_detail);
2023   cublasSaxpy(N_detail, 1, d_RESULT2, 1, d_RESULT1, 1);
2024   vecDIVvec(d_RESULT1, d_vt, d_r_Vt_sf, N_detail);
2025
2026   vecDOTvec(d_VQ, d_r_VtQ_d1, d_RESULT1, N_detail);
2027   vecDOTvec(d_VD, d_r_VtD_d1, d_RESULT2, N_detail);
2028   cublasSaxpy(N_detail, 1, d_RESULT2, 1, d_RESULT1, 1);
2029   vecDIVvec(d_RESULT1, d_vt, d_r_Vt_d1, N_detail);
2030
2031   vecDOTvec(d_VQ, d_r_VtQ_q1, d_RESULT1, N_detail);
2032   vecDOTvec(d_VD, d_r_VtD_q1, d_RESULT2, N_detail);
2033   cublasSaxpy(N_detail, 1, d_RESULT2, 1, d_RESULT1, 1);
2034   vecDIVvec(d_RESULT1, d_vt, d_r_Vt_q1, N_detail);
2035
2036   vecDOTvec(d_VQ, d_r_VtQ_q2, d_RESULT1, N_detail);
2037   vecDOTvec(d_VD, d_r_VtD_q2, d_RESULT2, N_detail);
2038   cublasSaxpy(N_detail, 1, d_RESULT2, 1, d_RESULT1, 1);
2039   vecDIVvec(d_RESULT1, d_vt, d_r_Vt_q2, N_detail);
2040   ///////////////////////////////////////////////////////
2041   vecDOTvec(d_KA, d_Rfd, d_RESULT1, N_detail);
2042   vecDIVvec(d_RESULT1, d_Lad, d_r_efd_v1, N_detail);
2043   cublasSscal(N_detail, -1, d_r_efd_v1, 1);
2044
2045   cublasScopy(N_detail, d_RESULT1, 1, d_r_efd_v3, 1);
2046   vecDIVvec(d_RESULT1, d_Lad, d_r_efd_v3, N_detail);
2047   ///////////////////////////////////////////////////////
2048
2049   cublasFree(d_r_Eqs_d1);
2050   cublasFree(d_r_Eqs_sf);
2051   cublasFree(d_r_Eds_q1);
2052   cublasFree(d_r_Eds_q2);
2053   cublasFree(d_r_u5_del);
2054   cublasFree(d_r_u5_sf);
2055   cublasFree(d_r_u5_d1);
2056   cublasFree(d_r_u5_q1);
2057   cublasFree(d_r_u5_q2);
2058   cublasFree(d_r_u6_del);
2059   cublasFree(d_r_u6_sf);
2060   cublasFree(d_r_u6_d1);
2061   cublasFree(d_r_u6_q1);
2062   cublasFree(d_r_u6_q2);
2063   cublasFree(d_r_A7_del);
2064   cublasFree(d_r_A7_sf);
2065   cublasFree(d_r_A7_d1);
2066   cublasFree(d_r_A7_q1);
2067   cublasFree(d_r_A7_q2);
2068   cublasFree(d_r_A8_del);
2069   cublasFree(d_r_A8_sf);
2070   cublasFree(d_r_A8_d1);
2071   cublasFree(d_r_A8_q1);
2072   cublasFree(d_r_A8_q2);
2073   cublasFree(d_r_ID_del);
2074   cublasFree(d_r_ID_sf);
2075   cublasFree(d_r_ID_d1);
2076   cublasFree(d_r_ID_q1);
2077   cublasFree(d_r_ID_q2);
2078   cublasFree(d_r_IQ_del);
2079   cublasFree(d_r_IQ_sf);
2080   cublasFree(d_r_IQ_d1);
2081   cublasFree(d_r_IQ_q1);
2082   cublasFree(d_r_IQ_q2);
2083   cublasFree(d_r_id_del);
2084   cublasFree(d_r_id_sf);
2085   cublasFree(d_r_id_d1);
2086   cublasFree(d_r_id_q1);
2087   cublasFree(d_r_id_q2);
2088   cublasFree(d_r_iq_del);
2089   cublasFree(d_r_iq_sf);
2090   cublasFree(d_r_iq_d1);
2091   cublasFree(d_r_iq_q1);
2092   cublasFree(d_r_iq_q2);
2093   cublasFree(d_r_sad_del);
2094   cublasFree(d_r_sad_sf);
2095   cublasFree(d_r_sad_d1);
2096   cublasFree(d_r_sad_q1);
2097   cublasFree(d_r_sad_q2);
2098   cublasFree(d_r_saq_del);
2099   cublasFree(d_r_saq_sf);
2100   cublasFree(d_r_saq_d1);
2101   cublasFree(d_r_saq_q1);
2102   cublasFree(d_r_saq_q2);
2103   cublasFree(d_r_VtD_del);
2104   cublasFree(d_r_VtD_sf);
2105   cublasFree(d_r_VtD_d1);
2106   cublasFree(d_r_VtD_q1);
2107   cublasFree(d_r_VtD_q2);
2108   cublasFree(d_r_VtQ_del);
2109   cublasFree(d_r_VtQ_sf);
2110   cublasFree(d_r_VtQ_d1);
2111   cublasFree(d_r_VtQ_q1);
2112   cublasFree(d_r_VtQ_q2);
2113 }
2114
2115 void startTimer()
2116 {
2117   QueryPerformanceCounter((LARGE_INTEGER*)&mInitTicks);
2118 }
2119 int getTimer()
2120 {
2121   __int64 t;
2122   QueryPerformanceCounter((LARGE_INTEGER*)&t);
2123   return (t-mInitTicks)*1000/mFreq;
2124 }
2125 #endif
```

Tesla S1070 Manufacturer Data Sheet

The specifications illustrated in this section are borrowed from [91].

Configuration

There are two configurations available (Table 1) for the Tesla S1070 computing system.

Table 1. System Configuration

Specification	Description
Ordering Part Numbers	**920-20804-0001-000** (-500 configuration, Turnkey, with standard HICs and external cables included) **920-20804-0002-000** (-500 configuration, A La Carte, with no HICs and no cables so user can specify accessories) **920-20804-0006-000** (-400 configuration, Turnkey, with standard HICs and external cables included) **920-20804-0005-000** (-400 configuration, A La Carte, 1.296 GHz peak clock with no HICs and no cables so user can specify accessories)
GPU	T10 GPU
GPU Processor clock	-500 configuration: 1.44 GHz peak clock -400 configuration: 1.296 GHz peak clock
GPU Memory clock	792 MHz
Memory configuration	16.0 GB total configured as 4.0 GB per GPU
Memory I/O	512-bit per GPU
System I/O	Two PCIe connections. Each connection leads to two of the four GPUs.
PCI Express cables	→ A 0.5-meter cable is included in the "turnkey" kit → A 2.0-meter cable is available but must be ordered separately

Mechanical Specification

System Chassis

The Tesla S1070 (Figure 4) uses a 1U form factor chassis and conforms to the EIA 310E specification for 19-inch 4-post racks with 900 mm to 1000 mm depth. The chassis dimensions are 1.73 inches high × 17.5 inches wide × 28.5 inches deep.

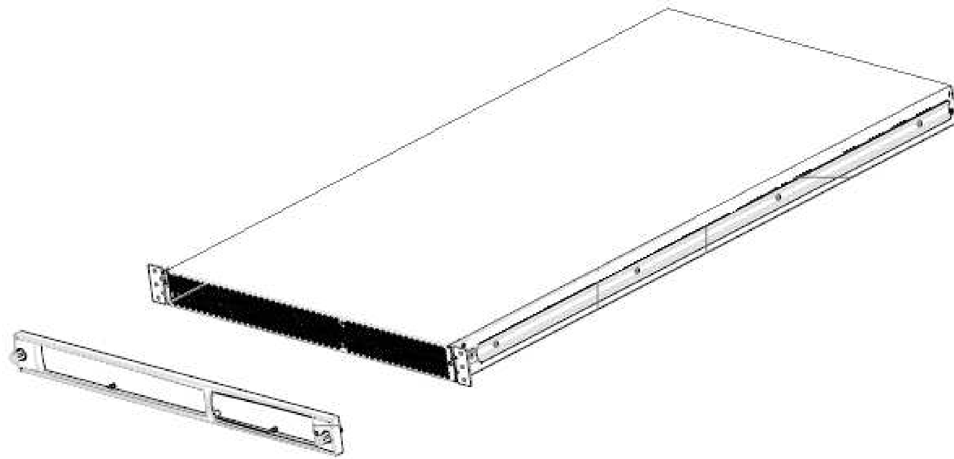


Figure 4. System Chassis Drawing

PCI Express Cable

The Tesla S1070 uses 0.5-meter PCI Express cables as the standard connection to the host system(s). Figure 6 shows the dimensions of this cable and its connectors. A 2.0-meter version of the cable is also available as a standalone accessory and uses the same connectors as the 0.5-meter cable.

Note: For Figure 6 the dimensions are in millimeters unless otherwise labeled.

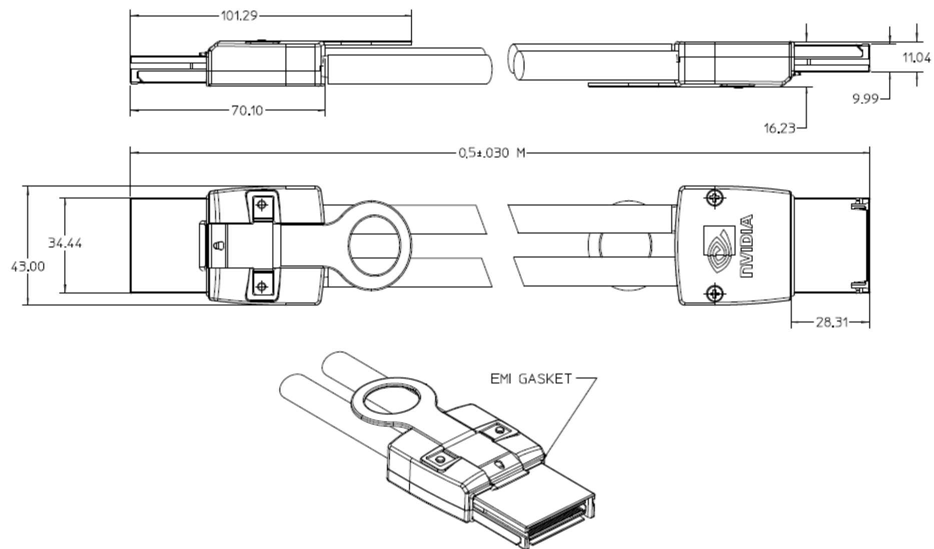
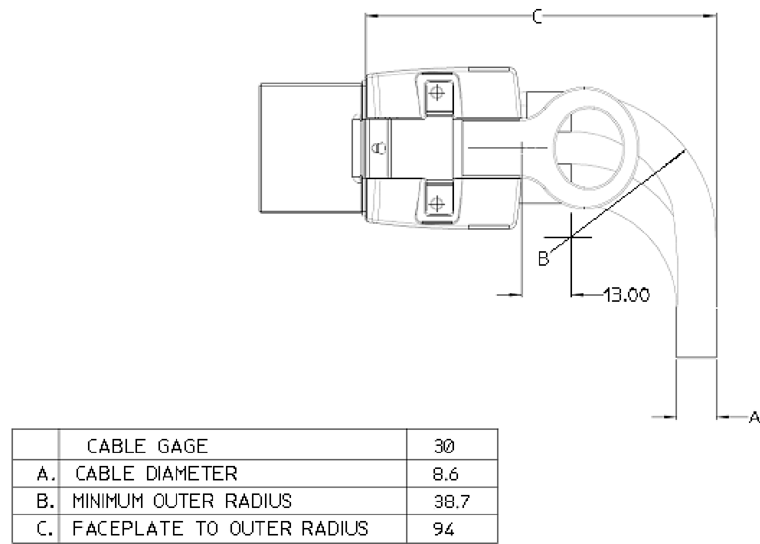


Figure 6. PCI Express Cable (0.5 Meter)

The minimum bend radius is 38.7 mm for the PCI Express cable. Figure 7 shows details of how this is measured relative to the I/O plate on the host interface card and relative to the cable/connector interface.



	CABLE GAGE	30
A.	CABLE DIAMETER	8.6
B.	MINIMUM OUTER RADIUS	38.7
C.	FACEPLATE TO OUTER RADIUS	94

Figure 7. PCI Express Cable Minimum Bend Radius

Rails for Rack Mounting

The Tesla S1070 uses a pair of rails for mounting to a 4-post, EIA rack. The rails can expand to fit a distance from 730 mm (28.74 inches) to 922 mm (36.3 inches) for the inside dimension between the front and rear posts. See Figure 8 for the exact dimension details.

Note: For Figure 8 the dimensions are in millimeters unless noted in square brackets [xx.yy +/- zz] that indicate dimensions in inches.

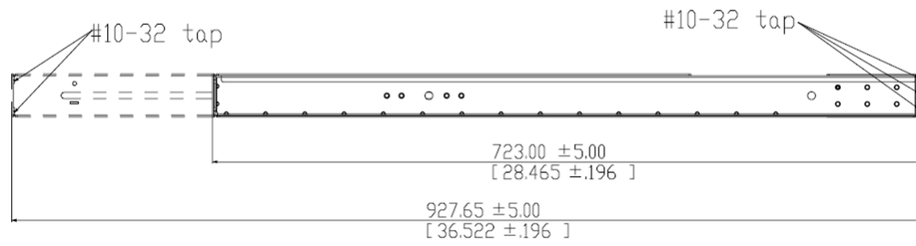


Figure 8. Rail for Rack Mounting

Environmental Specifications

Table 2. Environmental Specifications and Conditions

Specifications		Conditions
Operating	Input Power	90 to 274 VAC 50 to 60 Hz
	Temperature	10 °C to 35 °C (50 °F to 95 °F) at sea level with an altitude derating of 1.0 °C per every 1000 ft.
	Humidity	10 % to 80 % RH, 28 °C (82.4 °F) maximum wet bulb temperature, non-condensing
	Altitude	0 to 5000 feet mean sea level (MSL)
	Shock	Half sine 40g, 2 ms duration
	Vibration	Sinusoidal 0.25g, 10 to 500 Hz, 3 axis. Random 1.0 Grms, 10 to 500 Hz
	Acoustics	TBD dBa at 1 meter in front of system
	Airflow	143 cfm maximum
Non-Operating	Temperature	-40 °C to 60 °C (-40 °F to 140 °F)
	Humidity	10 % to 80 % RH, 38.7 °C (101.7 °F) maximum wet bulb temperature, non-condensing
	Altitude	0 to 10,000 feet mean sea level (MSL) with maximum allowable rate of altitude change of 2000 ft/min.
	Shock	Half-sine: 80G, 2ms Trapezoidal: 40G, 150 in/sec
	Vibration (random)	0.015-0.008G/Hz, 5-500 Hz, 10 minutes

Single Line Diagram of Test Systems

In this section the single-line diagram of the test systems used in this thesis are given. The $Scale$ 1 system is IEEEs New England test system which its complete data and load flow results are also given in the PSS/E's *.raw file format. Other test systems are made by duplicating this system.

E.1 Scale 1

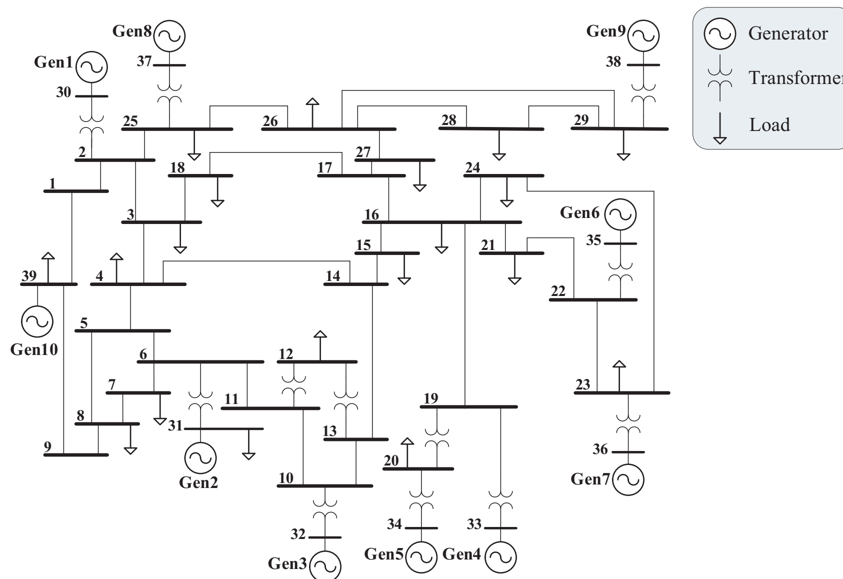


Figure E.1: Scale 1 system: 39 buses, 10 generators.

E.1.1 Load Data

```
     Bus         Pload           Qload
     Number      (MW)            (MVAR)
     1,           0.000,          0.000
     2,           0.000,          0.000
     3,         322.000,          2.400
     4,         500.000,         84.000
     5,           0.000,       -200.000
     6,           0.000,          0.000
     7,         233.800,        840.000
     8,         522.000,        176.000
     9,           0.000,          0.000
    10,           0.000,          0.000
    11,           0.000,          0.000
    12,           8.500,         88.000
    13,           0.000,          0.000
    14,           0.000,          0.000
    15,         320.000,        153.000
    16,         329.400,        323.000
    17,           0.000,          0.000
    18,         158.000,         30.000
    19,           0.000,          0.000
    20,         680.000,        103.000
    21,         274.000,        115.000
    22,           0.000,          0.000
    23,         247.500,         84.600
    24,         308.600,        -92.200
    25,         224.000,         47.200
    26,         139.000,         17.000
    27,         281.000,         75.500
    28,         206.000,         27.600
    29,         283.500,        126.900
    31,           9.200,          4.600
    39,        1104.000,        250.000
0 / END OF LOAD DATA, BEGIN GENERATOR DATA
```

E.1.2 Generator Data

```
    Bus     Pgen        Qgen        Qmax       Qmin         Vsched      Mbase        Rsource     Xsource
    Number  (MW)        (MVAR)      (MVAR)     (MVAR)       (pu)        (MVA)        (pu)        (pu)
    30,      250.000,   216.192,    800.000,   -500.000,    1.04750,    1000.000,    0.00140,    0.20000
    31,      572.930,   550.596,    800.000,   -500.000,    1.04000,    1000.000,    0.02700,    0.20000
    32,      650.000,   265.198,    800.000,   -500.000,    0.98310,    1000.000,    0.00386,    0.20000
    33,      632.000,   123.851,    800.000,   -500.000,    0.99720,    1000.000,    0.00222,    0.20000
    34,      508.000,   175.201,    400.000,   -300.000,    1.01230,    1000.000,    0.00140,    0.20000
    35,      650.000,   317.879,    800.000,   -500.000,    1.04930,    1000.000,    0.06150,    0.20000
    36,      560.000,   245.844,    800.000,   -500.000,    1.06350,    1000.000,    0.00268,    0.20000
    37,      540.000,    52.019,    800.000,   -500.000,    1.02780,    1000.000,    0.00686,    0.20000
    38,      830.000,   158.226,    800.000,   -500.000,    1.02650,    1000.000,    0.00300,    0.20000
    39,     1011.777,   342.702,   1500.000, -1000.000,    1.03000,    1000.000,    0.00100,    0.02000
0 / END OF GENERATOR DATA, BEGIN BRANCH DATA
```

E.1.3 Branch Data

From Bus Number	To Bus Number	Line R (pu)	Line X (pu)	Charging (pu)
1,	2,	0.00350,	0.04110,	0.69870
1,	39,	0.00100,	0.02500,	0.75000
2,	3,	0.00130,	0.01510,	0.25720
2,	25,	0.00700,	0.00860,	0.14600
3,	4,	0.00130,	0.02130,	0.22140
3,	18,	0.00110,	0.01330,	0.21380
4,	5,	0.00080,	0.01280,	0.13420
4,	14,	0.00080,	0.01290,	0.13820
5,	6,	0.00020,	0.00260,	0.04340
5,	8,	0.00080,	0.01120,	0.14760
6,	7,	0.00060,	0.00920,	0.11300
6,	11,	0.00070,	0.00820,	0.13890
7,	8,	0.00040,	0.00460,	0.07800
8,	9,	0.00230,	0.03630,	0.38040
9,	39,	0.00100,	0.02500,	1.20000
10,	11,	0.00040,	0.00430,	0.07290
10,	13,	0.00040,	0.00430,	0.07290
13,	14,	0.00090,	0.01010,	0.17230
14,	15,	0.00180,	0.02170,	0.36600
15,	16,	0.00090,	0.00940,	0.17100
16,	17,	0.00070,	0.00890,	0.13420
16,	19,	0.00160,	0.01950,	0.30400
16,	21,	0.00080,	0.01350,	0.25480
16,	24,	0.00030,	0.00590,	0.06800
17,	18,	0.00070,	0.00820,	0.13190
17,	27,	0.00130,	0.01730,	0.32160
21,	22,	0.00080,	0.01400,	0.25650
22,	23,	0.00060,	0.00960,	0.18460
23,	24,	0.00220,	0.03500,	0.36100
25,	26,	0.00320,	0.03230,	0.51300
26,	27,	0.00140,	0.01470,	0.23960
26,	28,	0.00430,	0.04740,	0.78020
26,	29,	0.00570,	0.06250,	1.02900
28,	29,	0.00140,	0.01510,	0.24900

0 / END OF BRANCH DATA, BEGIN TRANSFORMER DATA

E.1.4 Transformer Data

From Bus Number	To Bus Number	Specified R (pu)	Specified X (pu)	Winding (MVA)
2,	30,	0.00000,	0.01810,	100.00
6,	31,	0.00000,	0.02500,	100.00
10,	32,	0.00000,	0.02000,	100.00
11,	12,	0.00160,	0.04350,	100.00
12,	13,	0.00160,	0.04350,	100.00
19,	20,	0.00070,	0.01380,	100.00
19,	33,	0.00070,	0.01420,	100.00
20,	34,	0.00090,	0.01800,	100.00
22,	35,	0.00000,	0.01430,	100.00
23,	36,	0.00050,	0.02720,	100.00
25,	37,	0.00060,	0.02320,	100.00
29,	38,	0.00080,	0.01560,	100.00

0 / END OF TRANSFORMER DATA

E.1.5 Load-Flow Results

Bus Number	Code	G-Shunt	B-Shunt	Voltage (pu)	Angle (deg)
1,	1,	0.000,	0.000,	1.03297,	-9.3761
2,	1,	0.000,	0.000,	1.01107,	-6.5836
3,	1,	0.000,	0.000,	0.97373,	-9.6291
4,	1,	0.000,	0.000,	0.93270,	-10.4775
5,	1,	0.000,	0.000,	0.91852,	-9.0155
6,	1,	0.000,	0.000,	0.91880,	-8.1528
7,	1,	0.000,	0.000,	0.86315,	-10.6842
8,	1,	0.000,	0.000,	0.88084,	-11.4079
9,	1,	0.000,	0.000,	0.98072,	-11.2091
10,	1,	0.000,	0.000,	0.93851,	-5.4052
11,	1,	0.000,	0.000,	0.93050,	-6.3356
12,	1,	0.000,	0.000,	0.91230,	-6.3741
13,	1,	0.000,	0.000,	0.93620,	-6.2589
14,	1,	0.000,	0.000,	0.93615,	-8.2553
15,	1,	0.000,	0.000,	0.93910,	-8.7854
16,	1,	0.000,	0.000,	0.95674,	-7.1671
17,	1,	0.000,	0.000,	0.96640,	-8.3608
18,	1,	0.000,	0.000,	0.96767,	-9.3332
19,	1,	0.000,	0.000,	0.97919,	-1.8354
20,	1,	0.000,	0.000,	0.98066,	-3.2817
21,	1,	0.000,	0.000,	0.97306,	-4.4866
22,	1,	0.000,	0.000,	1.00987,	0.3478
23,	1,	0.000,	0.000,	1.00805,	0.1165
24,	1,	0.000,	0.000,	0.96783,	-7.0433
25,	1,	0.000,	0.000,	1.02018,	-5.1214
26,	1,	0.000,	0.000,	1.00002,	-6.3872
27,	1,	0.000,	0.000,	0.97808,	-8.5778
28,	1,	0.000,	0.000,	1.00181,	-2.5385
29,	1,	0.000,	0.000,	1.00379,	0.4750
30,	2,	0.000,	0.000,	1.04750,	-4.1349
31,	2,	0.000,	0.000,	1.04000,	0.3286
32,	2,	0.000,	0.000,	0.98310,	2.6947
33,	2,	0.000,	0.000,	0.99720,	3.3869
34,	2,	0.000,	0.000,	1.01230,	1.9120
35,	2,	0.000,	0.000,	1.04930,	5.3801
36,	2,	0.000,	0.000,	1.06350,	8.2185
37,	2,	0.000,	0.000,	1.02780,	1.7236
38,	2,	0.000,	0.000,	1.02650,	7.6230
39,	3,	0.000,	0.000,	1.03000,	-10.9600

E.2 Scale 2

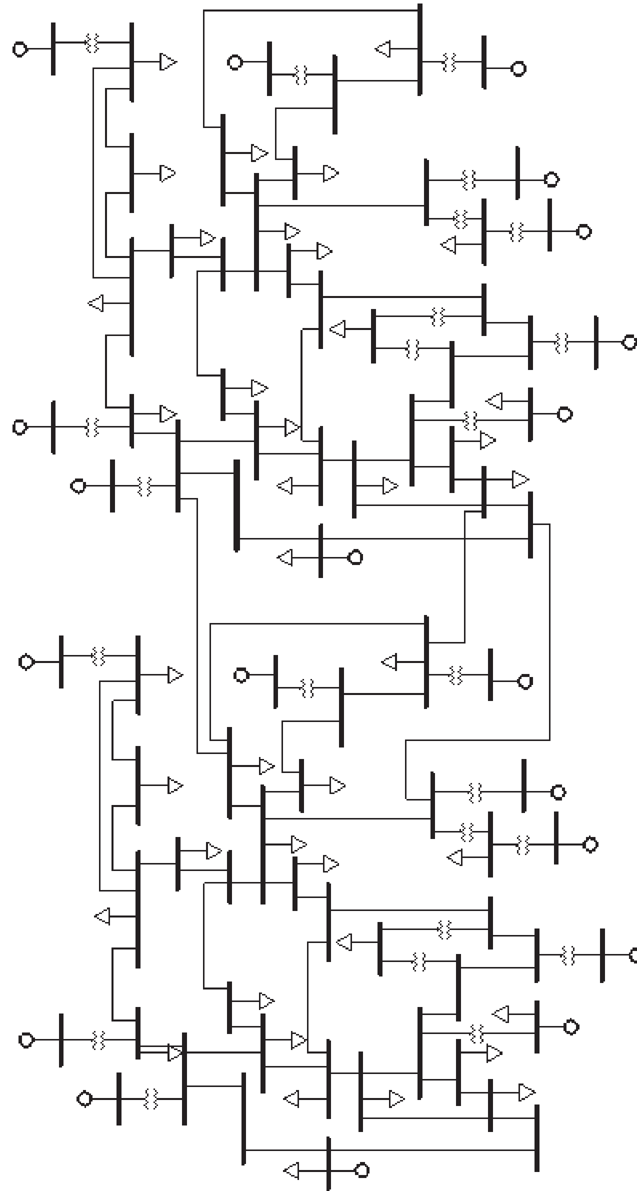


Figure E.2: Scale 2 system: 78 buses, 20 generators.

E.3 Scale 4

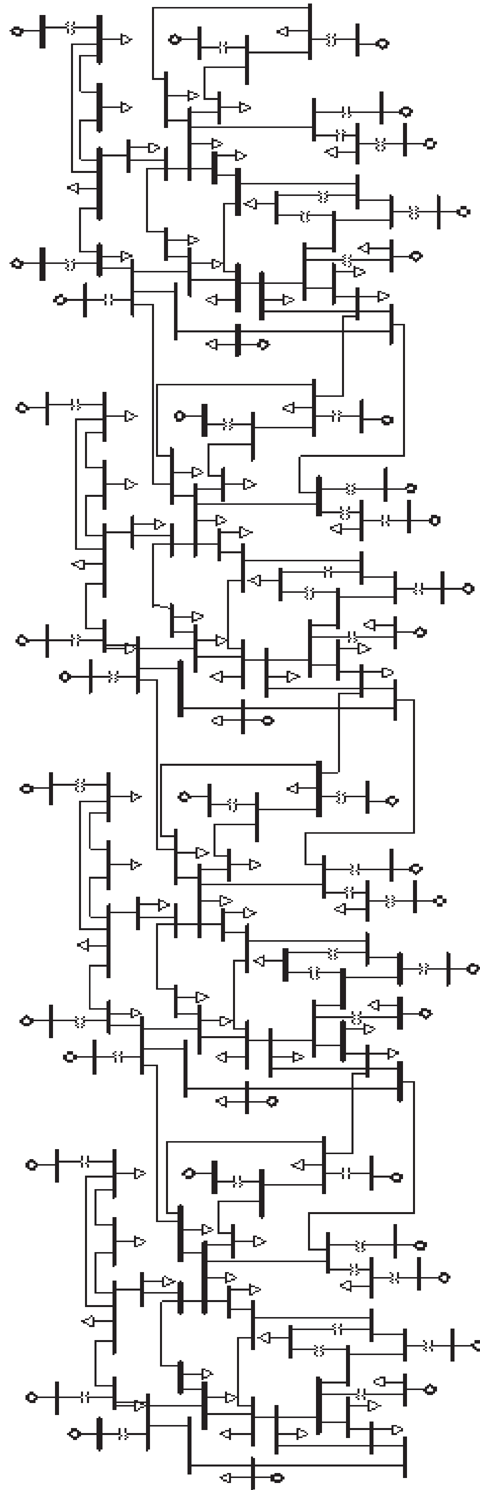


Figure E.3: Scale 4 system: 156 buses, 40 generators.

E.4 Scale 8

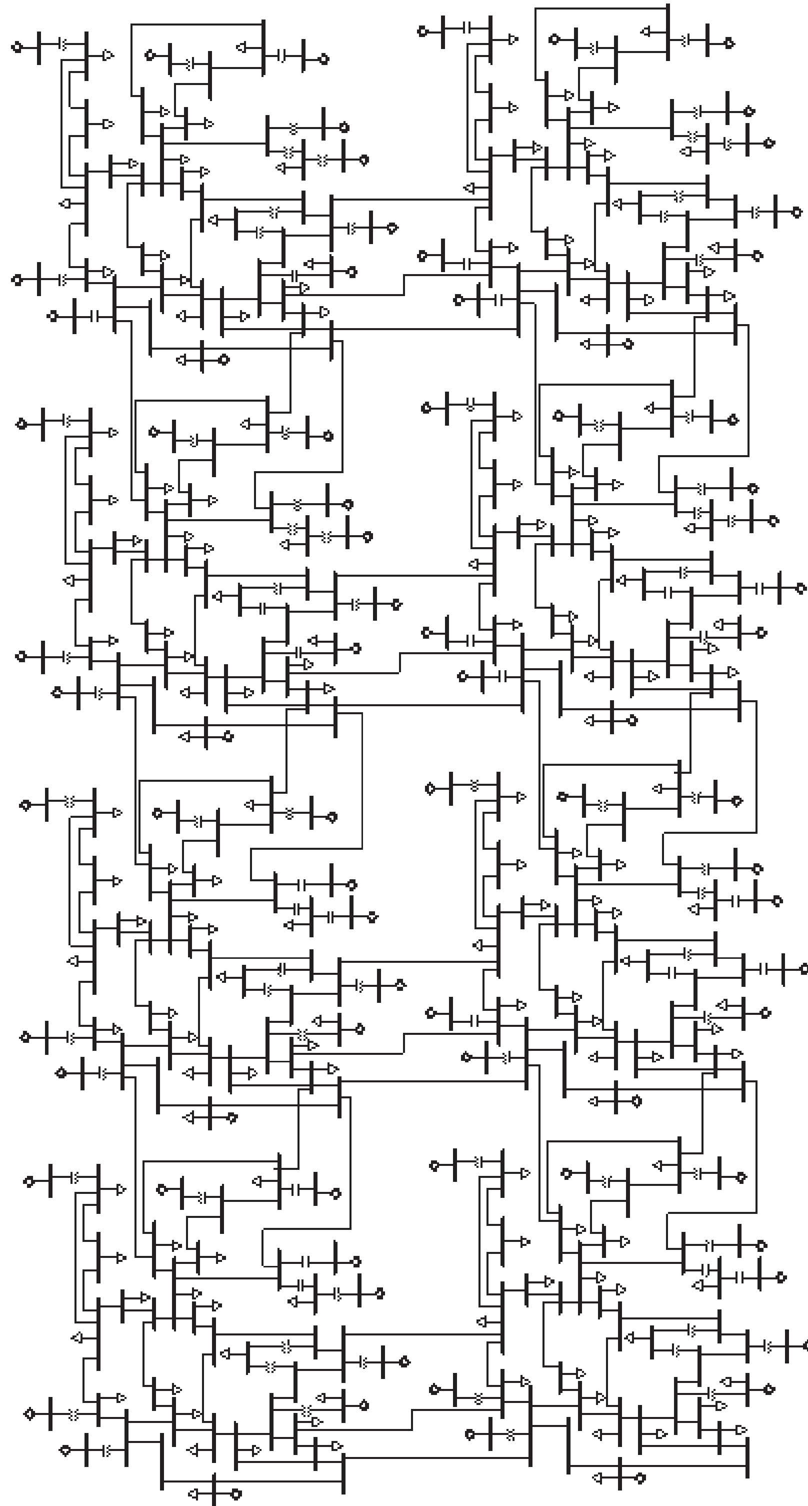


Figure E.4: Scale 8 system: 312 buses, 80 generators.

E.5 Scale 16

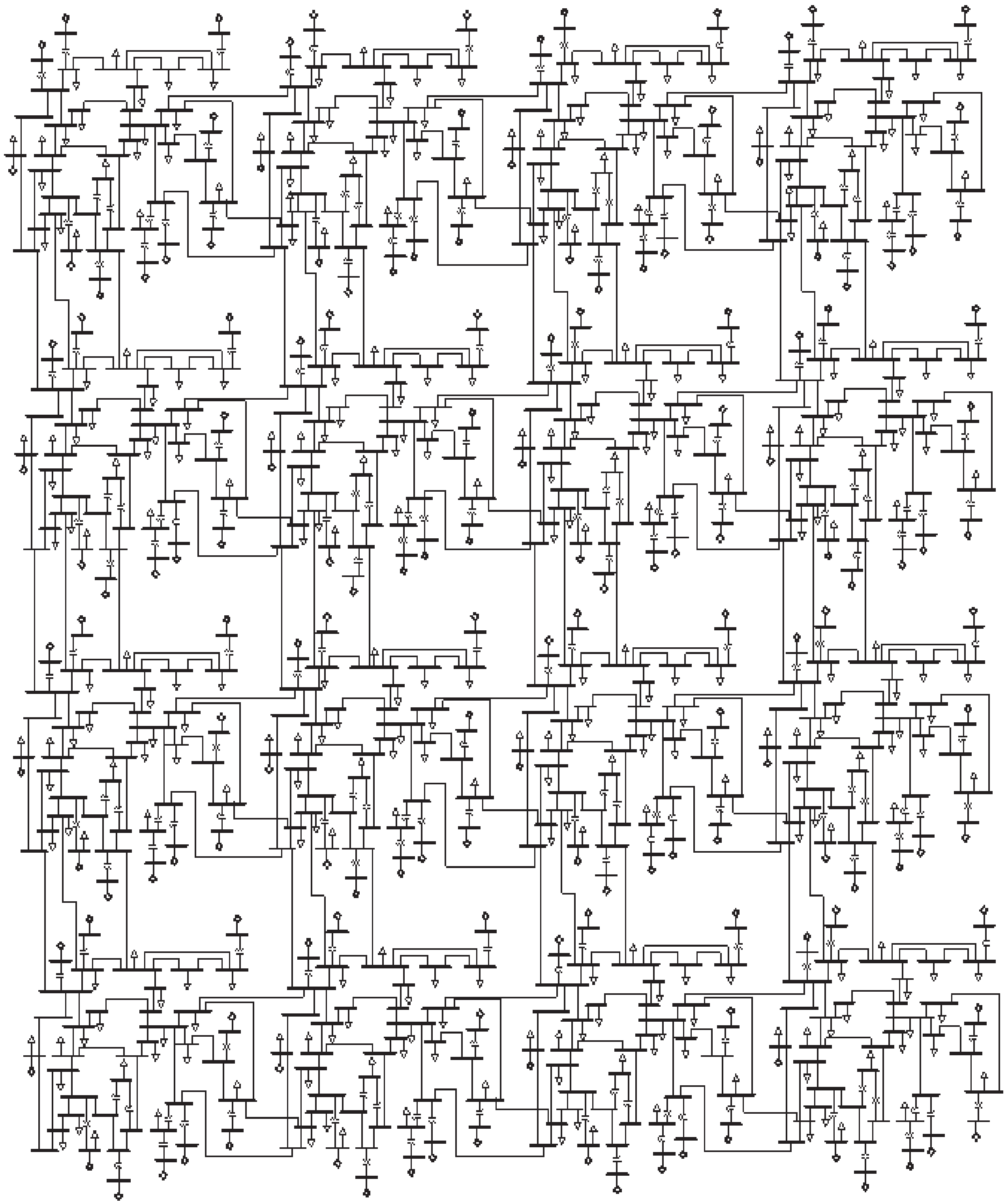


Figure E.5: Scale 16 system: 624 buses, 160 generators.

E.6 Scale 32

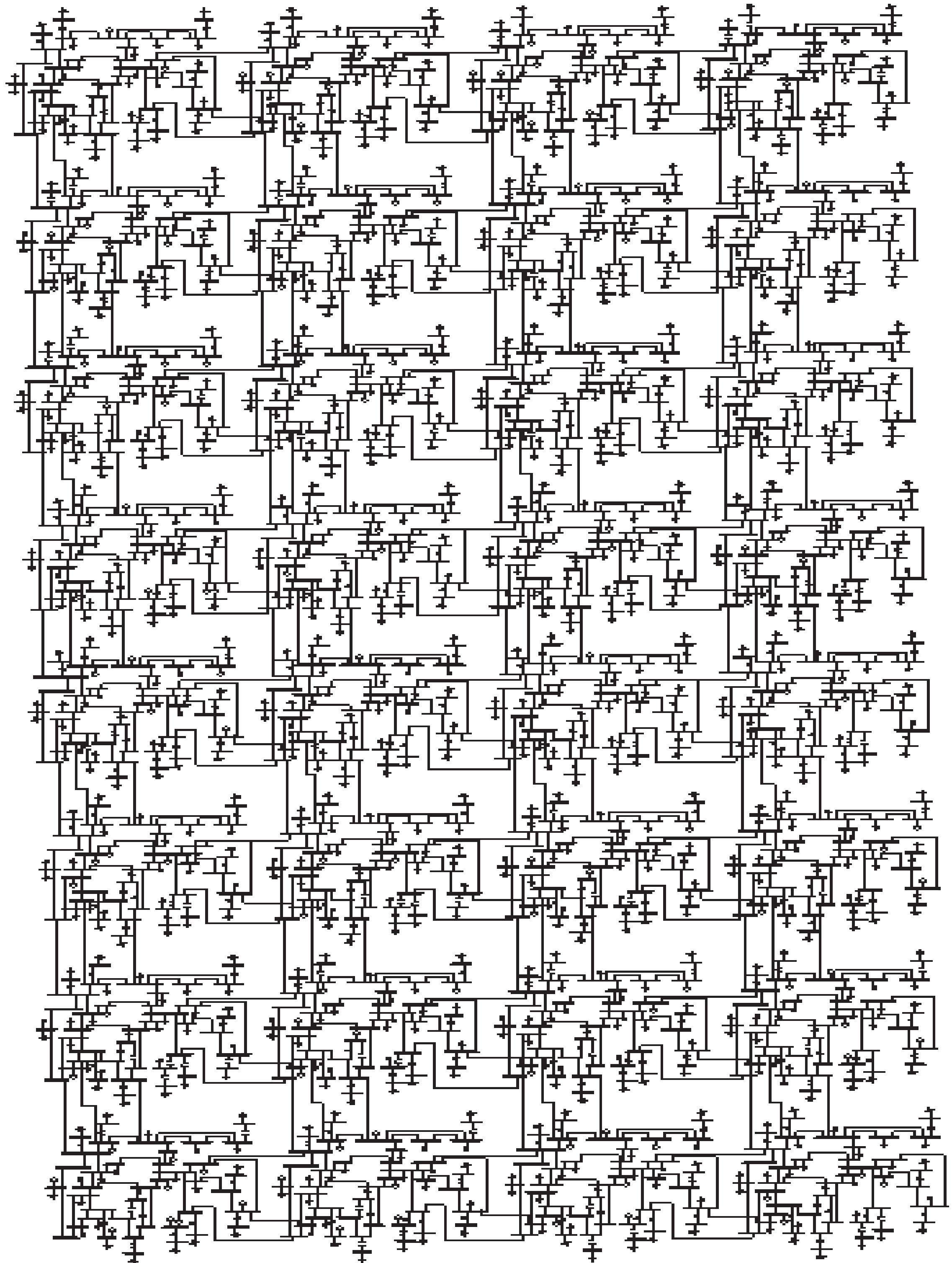


Figure E.6: Scale 32 system: 1248 buses, 320 generators.

E.7 Scale 64

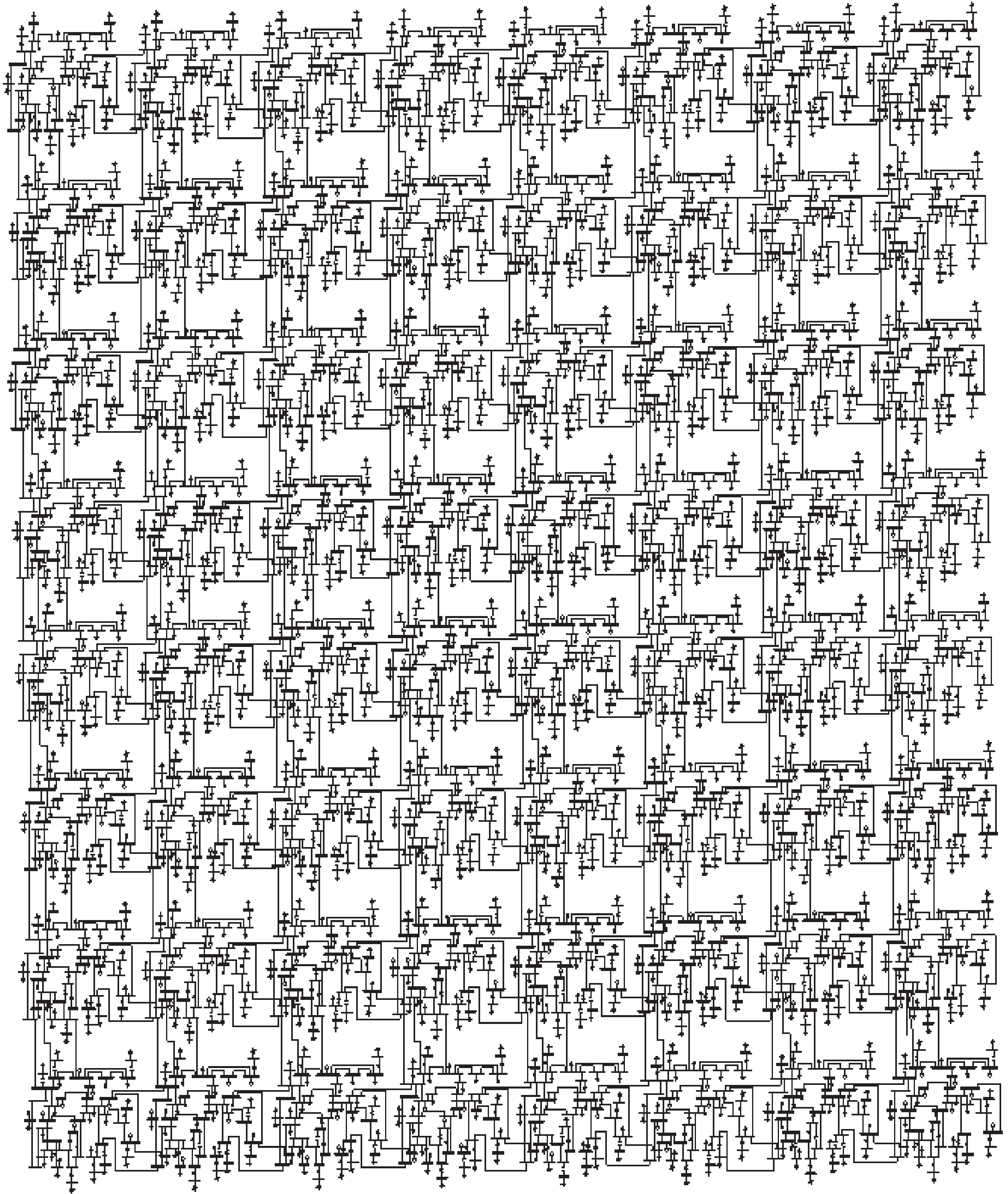


Figure E.7: Scale 64 system: 2596 buses, 640 generators.

E.8 Scale 128

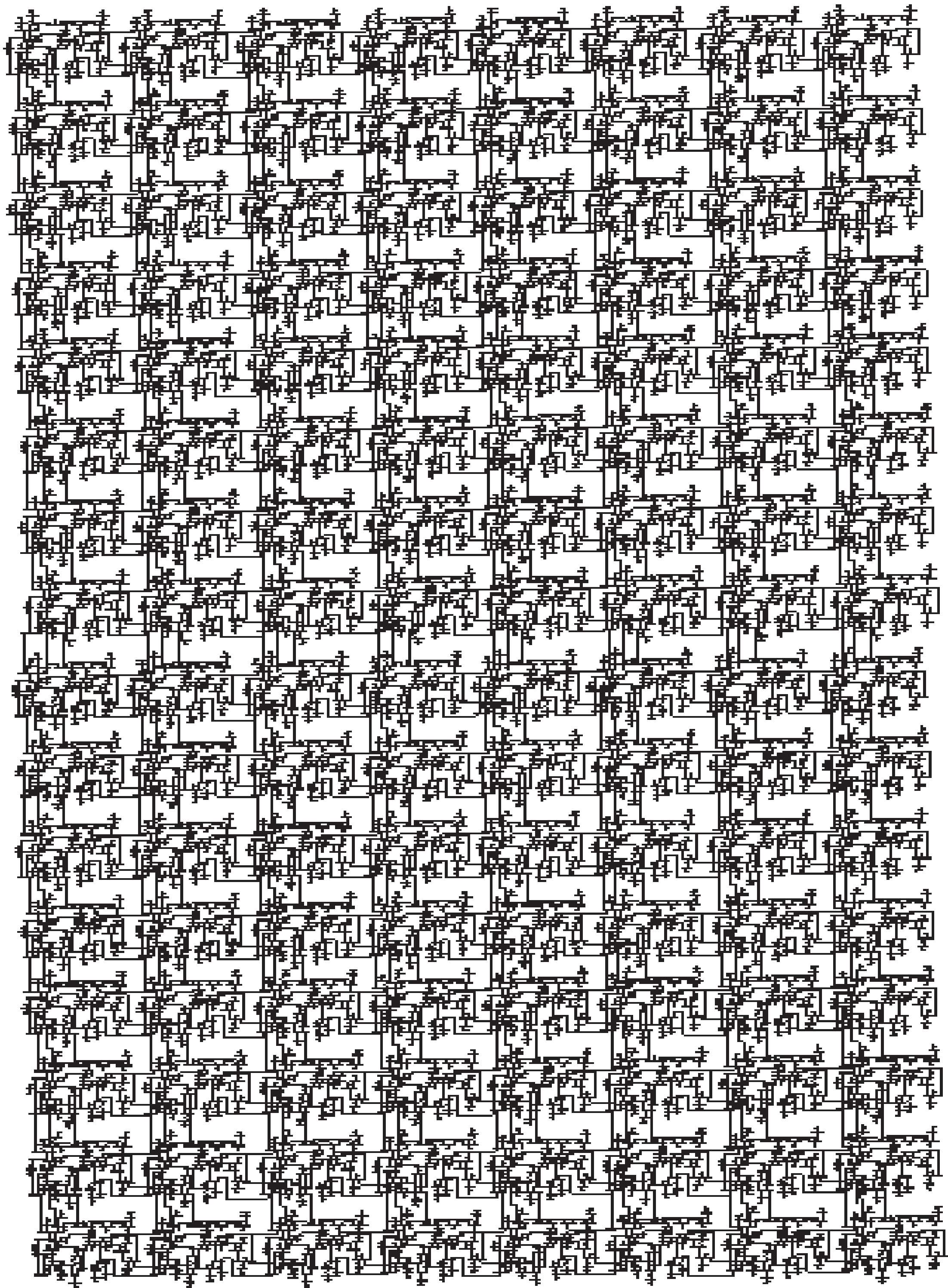


Figure E.8: Scale 128 system: 4992 buses, 1280 generators.

E.9 Scale 180

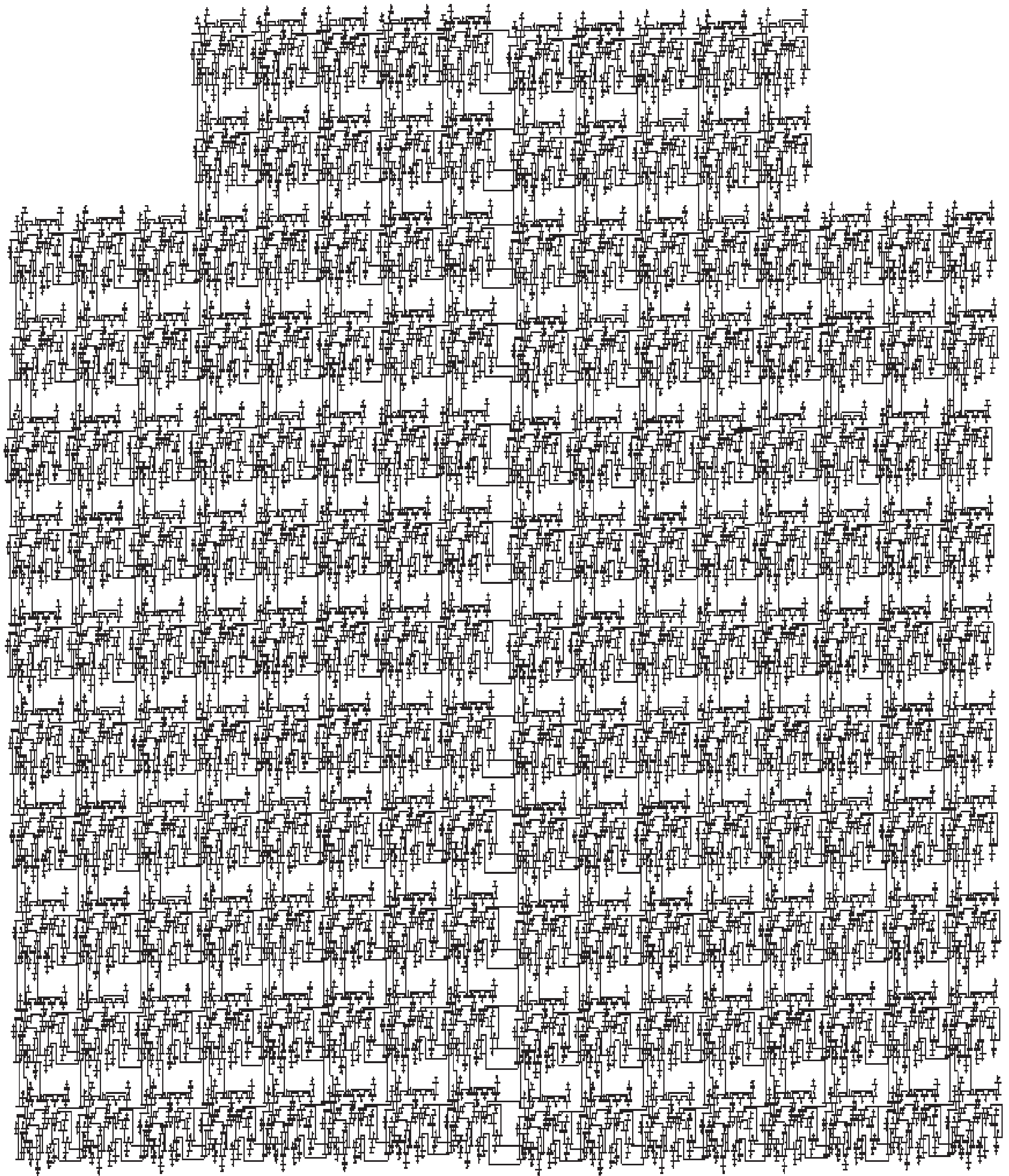


Figure E.9: Scale 180 system: 7020 buses, 1800 generators.