NAME OF AUTHOR....JAMES HINMAN VANCE.....

TITLE OF THESIS....The Effects of a Mathematics Laboratory Program in Grades 7 and 8 -- An Experimental Study

UNIVERSITY...of ALBERTA.....

DEGREE FOR WHICH THESIS WAS PRESENTED...Ph. D.........

YEAR THIS DEGREE GRANTED......1969.........

(Signed)...James H. Vance....

PERMANENT ADDRESS:

...Box 217.........
...Raymond.........
...Alberta.........

DATED...August 21...1969

THE UNIVERSITY OF ALBERTA

THE EFFECTS OF A MATHEMATICS LABORATORY PROGRAM IN GRADES 7 AND 8 -- AN EXPERIMENTAL STUDY

by

© JAMES HINMAN VANCE

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF SECONDARY EDUCATION

EDMONTON, ALBERTA

FALL, 1969

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "The Effects of a Mathematics Laboratory Program in Grades 7 and 8 -- An Experimental Study" submitted by James Hinman Vance in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Thomas E. Kieren
Supervisor

Joel W. Moody

K. A. Neufeld

R. S. Mcintosh

A. Côté

Viggo P. Hansen
External Examiner

Date July 25, 1969

ABSTRACT

The purpose of the study was to implement and to investigate the effects of a mathematics laboratory program in Grades 7 and 8. The experimental program consisted of ten activity lessons, one of which replaced a regular mathematics class one period each week for ten weeks. Each lesson was based on some type of concrete material and was designed to lead to the discovery of a new mathematical concept or relationship.

Three groups were formed from the 14 classes of Grade 7 and 8 students in an Edmonton junior high school.

The Mathematics Laboratory Group (ML). Students grouped in two's and using written instructions worked directly with the physical materials accompanying each lesson.

The Class Discovery Group (CD). The laboratory activities adapted as "discovery" lessons were presented to whole classes of students by their teachers who demonstrated with the concrete materials.

The Control Group (CON). Students in this group continued to study the regular program the full time allotted for instruction in mathematics.

Instruments were developed to compare the mathematical achievement and attitudes of the three groups and to ascertain the reaction of the students and teachers to the

two experimental settings. Two-way unweighted means analysis of variance was used to test the major hypotheses.

It was found that the use of 25% of class time for the informal exploration of new mathematical ideas did not adversely affect achievement in the regular program over the three month period. In addition, tests of immediate learning, cumulative achievement, higher level thinking and problem solving, and divergent thinking indicated that both ML and CD students had benefited mathematically from participating in the experimental program. Although test scores on the above measures of the CD students were slightly higher than those of the ML students, the reaction of the ML students to their instructional setting was more favorable than that of the CD students. The most popular feature of the laboratory method identified by the students was the opportunity it provided for working independently of the teacher. Differences among the three groups on the attitude measures favored the ML students, but the differences were not significant for three of the four tests. The laboratory students rated higher than students in the other two groups in feeling that learning mathematics is fun or enjoyable and in the view that mathematics is a subject which can be investigated experimentally using real objects.

ACKNOWLEDGEMENTS

I wish to express my appreciation to Dr. T. E. Kieren for his guidance, encouragement, and sincere interest throughout the course of the investigation and the writing of the dissertation.

I would like to thank Dr. K. A. Neufeld and Dr. R. S. Mortlock for their help and advice in construct-in the testing instruments and in preparing the final form of the report. I also thank the other members of the committee, Dr. J. W. Macki, Dr. A. Côté, and Dr. V. P. Hansen, for their valuable suggestions and comments on the study.

Appreciation is expressed to the administration and students at Westlawn Junior High School. Special thanks go to the mathematics teachers, Messrs. D. Tessari, W. Lepatski, and A. Petruk whose cooperation and extra effort made the study possible.

Finally, I wish to thank my wife Sheryl for her encouragement, support, and assistance during the writing of the dissertation.

TABLE OF CONTENTS

LIST OF TABLES

TABLE PAGE

LIST OF FIGURES

CHAPTER I

BACKGROUND AND SIGNIFICANCE OF THE STUDY

I. Introduction

In the last decade remarkable changes have occurred at all levels of school mathematics. While the initial emphasis was on determining what subject matter could and should be taught in the different grades, interest later shifted to questions related to the aims of mathematics learning and how this learning should take place. New methods were developed to allow for more individualization of instruction, to involve the student more directly in the doing of mathematics, and to make the learning process more meaningful and interesting to the pupils. Many of the new methods have been referred to as "laboratory" approaches to mathematics teaching. Mathematics laboratory projects which are now being developed in many different countries can, in general, be characterized by the following features:

1. The learner is actively engaged in the doing of mathematics; he is not a passive observer in the learning process.

2. Concrete materials, structured games or environmental tasks are used to give meaning to mathematical concepts.

3. The students work much of the time individually or in small groups from written instructions.

Proponents and developers of mathematics laboratories (Dienes, 1960; E. Biggs, 1968; Davis, 1967a; Fehr, 1968) have cited many advantages of this approach to learning.

Some of the functions and aims of the laboratory method are:

1. to permit students to learn abstract concepts in a concrete setting and thus increase their understanding of these ideas;

2. to enable students to personally experience the joy of discovering principles and relationships;

3. to arouse interest and motivate learning;

4. to cultivate favorable attitudes toward mathematics;

5. to encourage and develop creative problem solving ability;

6. to allow for individual differences in the manner and speed at which students learn;

7. to enable students to see the origin of mathematical ideas and to participate in "mathematics in the making";

8. to enrich and vary instruction.

The main purpose of laboratory programs has perhaps been best stated by the authors of one of the most well-known and successful efforts, the Nuffield Project, in their booklet <u>I Do, and I Understand</u>:

> Running through all the work is the central notion that children must be set free to make their own discoveries and think for themselves, and so achieve understanding...
> (The Nuffield Foundation, 1967, General Introduction).

II. Need for the Study

Although a great deal of work involving mathematics laboratories has been centered in England (Sealey, 1967), many other countries have also developed projects utilizing activity methods in mathematics (Williams, 1967). Work has been carried out in Australia (Golding, 1968), the Canary Islands (Caparros and Delgado, 1967), France (Picard, 1967), Hungary (Gador, 1967), and Russia (Menchinskaya, 1967). In the United States, laboratory techniques were adopted by the University of Illinois Mathematics Project (Easley, 1964) and the Madison Project (Davis, 1967a) when it was decided that greater use should be made of physical materials and small group work and less emphasis placed on class discussion of mathematical situations in symbolized form.

The use of laboratory methods requires fundamental changes in the classroom social structure, the role of the teacher, and in the nature of the curriculum. Dienes, for example, has written that the new methods will probably make it necessary to:

> abolish almost completely the present method of class teaching...and to replace this by individual learning or learning in small groups, from concrete material and written instructions with the teacher acting as guide and counsellor (1960, p. 29).

In spite of these implications, little research has been conducted either to establish the validity of the claims made by advocates of laboratory learning or to evaluate the extent to which a variety of desired objectives of mathematics teaching can be achieved in this kind of instructional setting. Regarding new methods and innovations in education, Cronbach has said:

> I have no faith in any generalization upholding one technique against another, whether that preferred method be audiovisual aids, programmed instruction, learning by doing, inductive teaching or whatever. A particular educational

> tactic is part of an instructional system; a proper educational design calls up that tactic at a certain point in the sequence for a certain period of time, following and preceding other tactics. No conclusions can be drawn about the tactic considered by itself (1966, p. 77).

Brown (1968) emphasized that, at present, we do not know what students should be exposed to what methods, and that it is unlikely that there is one answer to all problems with all students. Davis (1968) called for a series of studies to determine how mathematics laboratories should be used and to investigate possible advantages and weaknesses of laboratory methods. The SMSG Panel on Research (1968) suggested the following basic research problem: "Can mathematical concepts be learned through activities in mathematics laboratories?" The panel further pointed out the need for an evaluation model for mathematics laboratory activities.

Clearly there is need for research to determine in what ways and for what purposes mathematics laboratories can be most effectively used for students at different educational levels and of varying ability in mathematics.

III. The Problem

The main purpose of the study was to investigate various aspects of learning in an instructional setting in which students worked in small groups using physical materials and written instructions. The problem was to determine what benefits accrued to seventh and eighth grade students as a result of using 25 percent of the time allowed for instruction in mathematics to participate in a program of laboratory activities. In this context, answers to the following questions were sought: To what extent are students able to learn a new mathematical concept through performing a laboratory activity? Can students transfer or apply knowledge and experience gained in the laboratory to related but new situations? How is achievement in the regular program affected by reducing the class time devoted to it in order that students may work in the laboratory? How does laboratory experience affect student understanding of and attitude toward mathematics as a discipline of study? What is the reaction of students and teachers to laboratory learning? To what extent can objectives of the laboratory method be achieved in a regular classroom setting in which the teacher demonstrates with concrete materials?

IV. The Experimental Setting

Three experimental groups were formed from 14 classes of Grade 7 and Grade 8 students in a single junior high school.

The Mathematics Laboratory Group

The laboratory program which was developed for this investigation was designed to function as an adjunct to the modern mathematics courses used in Grade 7 and Grade 8. The program comprised ten activity lessons which were held on a once-a-week basis in place of regular mathematics classes. Each of the lessons was developed to permit the discovery of a new concept or relationship through the manipulation of some type of concrete material. In the laboratory the students worked in pairs with various physical materials and from written instructions.

The Class Discovery Group

In order to assess the unique effects of the laboratory experience on student behavior, a second experimental setting was established in which the ten laboratory activities were adapted as special "discovery" lessons and presented to classes of students by their teachers who used the concrete materials as instructional aids.

The Control Group

Students in control classes were not exposed to the experimental lessons, but continued to study the regular course for the full time allotted for instruction in mathematics.

V. Delimitations of the Study

To obtain answers to the question "How should mathematics laboratories be used?" a particular laboratory program was developed and evaluated. This program operated in conjunction with modern mathematics courses in Grade 7 and Grade 8 for the purpose of enriching, supplementing, and providing a change from textbook-based work and did not replace or compete with these programs of study.

Mathematical topics were not developed over an extended period of time. Rather ten different problems which were not related to each other or to the regular courses in any particular way were investigated in single fifty minute periods.

Each laboratory activity was designed to lead the students to an understanding of an abstract mathematical idea, the solution of a new problem or the discovery of a relationship. The activities were not real-life oriented nor were they intended to serve in a review or remedial capacity.

Only seventh and eighth grade students and mathematics teachers in a single junior high school together with the investigator participated in the study. The experimental program extended over a period of ten weeks.

VI. Outline of the Report

The present chapter has introduced the problem. Chapter II gives a review of literature related to the study. Chapter III includes a detailed description of the development of the experimental program and the design of the study. The results of the investigation are reported in Chapter IV, and Chapter V contains a summary of the study and the conclusions and implications drawn from it.

CHAPTER II

REVIEW OF THE LITERATURE

In this chapter literature relating to the aims and methods of the laboratory approach in mathematics is reviewed. The first part of the chapter is devoted to the establishment of a theoretical basis for mathematics laboratories. Following this, the development of several recent mathematics programs based on laboratory methods is discussed. In the last part of the chapter research studies on laboratory instruction in science teaching and on manipulative learning and of various attempts to enrich or to individualize instruction in mathematics are reviewed. It is the aim of this review to establish a frame of reference for the study.

I. Theoretical Basis

The methods and procedures employed in mathematics laboratories are closely associated with current aims of mathematics teaching and are supported by recent findings related to how children learn.

Concept Formation

The development of understanding and meaning is one of the principle goals of the mathematics program. The major function of mathematics laboratories is to provide a setting in which the learner can acquaint himself with mathematical ideas and begin to form abstract concepts through the manipulation of physical objects. The importance of concretely oriented experiences to later meaningful analytic symbolic learning is indicated in the literature on concept formation (Van Engen, 1953) and is further supported by recent work in the field of developmental psychology. The theories and work of Piaget, Bruner, Dienes, and other cognitive psychologists have had considerable influence on the renewed interest in activity methods and provide a psychological basis for a laboratory approach to mathematics learning.

Piaget. Piaget's work threw light on how children first form certain mathematical concepts. In the child's development he distinguished certain stages of intellectual growth, two of which concern students at the upper elementary and junior high school levels (Harrison, 1969). During the concrete operations stage which occurs from about seven to eleven years, thought structures are not yet separated from their concrete context, and concepts are formed only on the basis of experience with real objects. It is only at the formal operations stage which usually begins at the age of eleven or twelve that children start to reason by hypothesis rather than in terms of real objects only.

According to Piaget (1964), experience is one of the main factors influencing intellectual development. He distinguished two psychologically different kinds of experience, physical experience and logical-mathematical experience. Physical experience consists of actions with objects leading to some knowledge about the properties of the objects themselves. Logical-mathematical experience, however, does not come from the objects, but is drawn from the actions which are effected on them and results in knowledge about the properties of the actions. Mathematical thinking is further developed when these actions are interiorized so that they can be carried out without the use of physical objects. During the concrete operations stage, the co-ordination of actions needs the support of concrete material, but as a child reaches the stage of formal operations, physical experience is no longer essential.

This theory of mental growth supports the use of activity methods in learning situations. One implication for education (Flavell, 1963) is that the teaching or learning process should follow the developmental process of internalization of actions. The child should first investigate the concept to be learned at the concrete level by manipulating objects. Gradually the objects can be replaced by pictorial representations and then by symbols as the overt actions are transformed into mental operations.

Bruner. Bruner's work on cognitive development has been influenced by Piaget's findings. Bruner (1966) distinguished three modes of representation by which man translates experience into a model of the world. The first, called enactive representation, is a way of representing past events through appropriate motor responses (actions). The second, called iconic representation, summarizes events by the organization of images. The third, called symbolic representation, is representation by arbitrary symbols. He applied this framework to a theory of instruction with particular reference to mathematics (Bruner, 1966). Any idea, problem or body of knowledge to be taught can be represented to the student in any of the three modes. An enactive representation would indicate the actions or constructions necessary to arrive at a certain concept or result. Iconic representation would consist of a set of pictures or images that stand for the concept. A symbolic representation would be a description of the concept in words or symbols.

Although he stated that an optimum sequence of progression would move from enactive through iconic to symbolic representation, Bruner recognized that some learners are able to by-pass the first two stages. Indeed, the usual procedure, particularly at the secondary level, is for the teacher to begin by characterizing a concept by a symbol or definition. The danger in this approach is that when symbolic transformations fail, the learner will not be able to fall back on the actions and images which they represent. To most effectively teach an idea, one should first provide an opportunity for concrete manipulation leading to an association with the idea. Next, the child must be encouraged to form images of the idea in the constructed forms. The final task would be the development of a notational system which describes the construction and yet is free of the manipuation and the images. Thus the learner would not only understand the abstract idea, but he would have a stock of images which embody the abstraction and which could be used as checks on symbolic manipulation and in dealing with new problem situations.

To summarize, Bruner has said that intellectual development which is exemplified in the learning of mathematics:

> begins with instrument activity, a kind of definition of things by doing them. Such operations become represented and summarized in the form of particular images. Finally, and with the help of a symbolic notation that remains invariant across transformations in imagery, the learner comes to grasp the formal or abstract properties of the things he is dealing with. But...he nonetheless continues to rely upon the stock of imagery he has built en route to abstract mastery (1966, p. 68).

Dienes. Based upon Piaget's description of intellectual growth, the mathematician-psychologist Z. P. Dienes has identified three stages in the development of a concept (Dienes, 1960). The first stage is a preliminary, somewhat groping stage, in which activity is undirected and more or less random. In this playful exploring period, conceptualization begins to occur so that in the second, more structured stage conscious mathematical activity can take place. At this time more awareness and direction come into thinking. Activity is purposeful although the subject is not fully aware of what he is learning or looking for. A large number of experiences of different kinds and yet which lead to the same concept are necessary during this period. After sufficient information has been accumulated out of these experiences, the third stage arrives as an "insight" is achieved. This realization is usually followed by the desire to analyze what has been learned and to practise the newly-formed concept. In this way an insight becomes concrete enough to be used as an object of play and a new cycle can begin. Dienes says that we must provide experiences for students which permit and encourage the development of concepts according to these cycles.

Dienes originally formulated his theory of mathematics-learning in terms of certain principles (Dienes, 1960, 1964). According to the dynamic principle, "all abstraction and therefore all mathematics springs from experience (1960, p. 120)". The constructivity principle is based on the fact that children develop constructive thinking long before analytic thinking, and that analytic thinking is based on construction. In other words, the child builds up his ideas through his own experiences before he attempts to analyze them from a logical point of view. Analytic or logical thinking does not usually occur before the age of 12. Until this time the child thinks constructively or in terms of actions with real objects. The multiple embodiment principle requires that each concept to be learned be represented in a variety of different ways so that after a number of allied experiences the student will be able to abstract the common concept. Dienes has designed special structured materials which serve to embody some of the more important concepts.

Dienes has reported several research experiments in which his learning principles were applied and further developed. In an early study Dienes (1959) investigated the structure of the personality in relation to mathematical concept formation. The learning tasks were constructed in such a way as to permit the play-purposeful activity-insight stages in forming the concepts, to allow the subjects to develop each concept at their own speed and in their own way, and to permit the learners to correct their own mistakes through further play experiences. The results of the investigation suggested that there is a sex difference in the relationship between the ability to do the tasks and certain personality traits. He also hypothesized that:

> Girls approach tasks more from the point of view of the construction of the whole, boys more from the point of view of analysis of parts (p. 64).

An account of a study carried out by Morino-Abbele at the University of Florence was reported by Dienes. The purpose of this experiment was to investigate the stages through which children pass when presented with mathematically structured material with which they are allowed to play freely. Ten six year old children who had received no formal training in mathematics were given the multibase arithmetic blocks and simply told to do what they wanted with them. Several stages were observed and identified as follows:

> Through a 'free play' activity the child reaches a phase of 'structured activity' in which he strives for 'geometrical shapes'. He then reaches a stage of 'perception of quantity' or 'quantitative stage' which is his first approach to the concept of 'number' (Dienes, 1963, p. 200).

An Experimental Study of Mathematics Learning (Dienes, 1963) is a report of a study undertaken by Dienes in association with Bruner at the Harvard University Center for Cognitive Studies. Dienes described the study as an exploratory investigation rather than a controlled experiment in which specific hypotheses were tested. Five experimental groups were formed. Group A consisted of four third grade children. Group B was a second grade class in which the students worked individually or in groups of seven. Groups C, D, and E were composed of four Grade 4 children, six Grade 4 children, and 14 Grade 6 students respectively. In each group the children used structured materials or played mathematical games to learn abstract concepts. The method used in the research consisted of observation by one or more investigators who recorded the subjects' work with the physical materials. An attempt was then made to determine the underlying thought processes of the learners in trying to solve the problems. As a result of the work with these groups, Dienes found it necessary to modify his learning principles. He wrote: "It is no longer quite clear that cognitive organization proceeds from the unstructured to the structured, from play to construction (1963, p. 156)." Both the constructivity and the dynamic principles were incorporated into a new model in which cycles of play and construction follow each other enabling the learner to discover the regularities in the structure. This rule-structure is not necessarily analyzed but may be made operational through practice. Dienes also realized that the multiple embodiment principle did not take sufficient account of the difficulties of transfer from one structure to another. He found that the 'noise' from the multiple embodiments may impede as well as assist in the abstraction process, especially for quick learners.

Motivation

An important function of the laboratory method is to arouse student interest in learning mathematics and provide motivation for continued study.

Piaget's equilibrium model of motivation (Baldwin, 1967) holds that it is schemas in the process of organization that children tend to repeat playfully and with pleasure. Learning occurs when the schemas required for the solution of a problem are not too far removed in complexity from those schemas already available (assimilable but not completely so), and the child restructures the schemas he has (accommodates) to the conditions of the problem. A new schema, once established through practice, then becomes available as a tool for further learning. In this model concrete materials serve not only to stimulate interest and curiosity but to bridge the gap between the schemas available to the child and those demanded by the mathematical concept. In discussing the significance of Piaget's work to educational practice, Lunzar concluded:

> (1) that we can promote understanding of mathematical concepts by providing experiences of the right kind, (2) that the experience or activity provided is not an end in itself although it is planned to arouse interest; it is devised with the declared purpose of stimulating spontaneous discovery by the child (quoted by E. Biggs, 1965, p. 7).

Bruner (1966) discussed motivation in terms of first arousing in the learner some optimal level of uncertainty so that he will begin to explore the problem. To maintain problem-solving activity he must have some idea of the goal of the task and feedback as to the relevance of tested alternatives. According to Dienes (1960) self-motivating learning experiences can be created by allowing children to work individually or in small groups at their own rate and without a punishment-reward system to back up the learning. Polya (1965) wrote that the best motivation is interest in the material. Glennon and Callahan defined cognitive motivation as referring to "motives that reside in the task itself rather than those external to it (1968, p. 72)".

In a study on cognitive motivation Slavina (1967) introduced didactic games involving calculations with seven and eight year old children who exhibited "intellectual passivity". He reported that when problems were solved in play that could not be solved by ordinary means, the children's negative emotions toward mental work were replaced by positive attitudes. By the sixth day of this treatment the problem solving activity, stimulated at first by play, became permanent and transferred to other school tasks not involving play situations.

Suchman (1964) has identified three conditions for inquiry: some focus for the student's attention, freedom to reach out for information, and a response environment so that when he reaches out for the data, he gets something. Under these conditions, said Suchman, motivation becomes cognitive and takes two forms:

> One is the motivation to close the gap, to make the match between what is perceived and what is known, the motivation of closure. But here we found that beyond this...they continued to inquire, to pull out data and to process them. They were motivated it seemed by the act of inquiry itself (1964, p. 106).

Laboratory methods are designed to provide motivation according to these theoretical models. Concrete materials serve to stimulate interest and curiosity and clarify the goal of the learning task. The students are permitted to use this material freely to obtain data from which hypotheses can be formulated. These hypotheses can be evaluated, again using the physical aids.

Students are also motivated by feelings of success. A concrete approach allows learners to use their senses to see the problem and the results thus eliminating language difficulties. After the pupil has experienced the reality, symbolic representation can be introduced.

Davis (1964c) discussed two kinds of motivation operating in Madison Project group discovery lessons. First, there are intrinsic rewards which are derived from solving a problem or discovering an important relationship. Not only does a discovery result in reduction in cognitive strain, but the child has the satisfaction of having his idea or hypothesis verified. The student is also motivated by the competition arising out of the group situation, and he is rewarded in being able to tell his classmates and teacher what he has found out.

Development of Thinking Skills

One of the foremost objectives of the mathematics program and indeed of education today is to encourage and to develop independent and creative thinking skills. Piaget has been quoted as saying that the goal in education should be "to create the possibilities for a child to invent and discover himself (Duckworth, 1964, p. 497)". According to Davis we should teach "creative problem solving, the ability to discover patterns in abstract situations... (1964a, p. 11)". E. Biggs (1968) has written that one of her aims in teaching mathematics at any level is to give the students the opportunity to think for themselves. According to Lamon (1969) much of the value of mathematics lies in the thinking skills acquired through the mental manipulation of abstractions and concepts.

Morley (1967) described mathematics as a creative activity involving classification, problem formulating, generalizing, symbolizing, and problem solving. Concrete materials serve not merely as objects of play or visual aids but provide children with situations which engage them in formulating and solving problems. Dienes said that children working with his materials learn to look for patterns, to expect relationships, and know how to investigate problems. They acquire a "mental set towards suspecting certain relationships in certain situations, leading them to the formulation of hypotheses, and consequently to the desire to test such hypotheses (1967, p. 25)". Taba concluded that the ability to transfer learning is achieved:

> by emphasis on cognitive principles applied either to methods of learning or to the understanding of content, or on ways of learning that stress flexibility of approach and that develop an alertness to generalizations and their applications to new situations (1962, p. 125).

From his studies investigating the role of manipulation in creativity, Torrance concluded:

> To encourage inventiveness, it would seem desirable to stimulate and implement, and thus keep alive, the natural inclination to manipulate and experiment with objects and ideas (1963, p. 117).

It has been found that a manipulative or game approach to developing ideas seems to enlarge the child's repertoire of ideas and appears to relate to his being more creative (Sutton-Smith, 1967).

Origin and Personal Knowledge of Mathematical Ideas

Another purpose of the laboratory method is to give students a broader understanding of the origin of mathematical ideas and how relationships are discovered and new problems formulated and solved. Polya has referred to the two faces of mathematics:

> Mathematics presented in the Euclidean way appears as a systematic deductive science, but mathematics in the making appears as an experimental, inductive science. Both aspects are as old as the science of mathematics itself (1957, p. vii).

Modern mathematics programs have tended to emphasize the formal, deductive nature of mathematics and treat it as an abstract science and have played down its relationship to the real world. Fehr (1947) said that laboratory teaching is one way to give reality to mathematics without the loss of its abstract and theoretical nature. He noted that the study of mathematics had its origin in concrete objects of the physical world and advocated the use of physical objects to aid in developing concepts which originally arose as abstractions from the study of those objects. Davis (1964b) said that we should create learning situations to enable students to gain a feeling for the history of mathematics and acquire the belief that mathematics is discoverable.

Laboratory experiences in mathematics are intended in part to encourage students to be aware of and to look for underlying structure in their world and to appreciate the role of induction and analogy in forming and solving problems in mathematics. At the same time students are able to gain personal knowledge of mathematical ideas by discovering concepts and patterns for themselves through a hypothesis-testing procedure.

II. The Laboratory Method in Mathematics

The idea of a mathematics laboratory is not new. Since the turn of the century writers in the field of mathematics education have urged teachers to use laboratory lessons to supplement regular classroom instruction. In 1902, E. H. Moore (1903) called for the development of a laboratory system of instruction in mathematics to develop in students the spirit of research and an appreciation of both the theoretical and practical methods of science. Most recommendations for laboratory lessons, though, have been concerned with a small number of topics at the high school level. Goldziher (1908), for example, recommended that the laboratory be used for instruction in measuring and weighing, collecting statistics, the construction of graphs, and the preparation of geometrical models. The literature is replete with references to the teaching of geometry via a laboratory method. Jones (1905) suggested that the method of observation and experiment should be used as an introduction to geometry. The Eighteenth Yearbook of the National Council of Teachers of Mathematics (1945) described a laboratory approach to solid geometry, the purpose of which was to enable students to see spatial relationships and to train the spatial imagination and at the same time arouse interest in the subject.

McDill (1931) viewed the laboratory as a place where the pupil works as an individual and with his hands carrying out experiments. Ramseyer (1935) conceived of the mathematics laboratory as a "device for vitalizing mathematics" and as a place where students could see the part that mathematics has played in changing history. Fehr (1947) advocated that special laboratory periods be held for the purpose of creating a spirit of research and discovery.

In spite of such writings the laboratory approach failed to become popular even in the restricted sense described. Johnson (1962) has pointed out that although there has been a multitude of information concerning various learning and teaching devices, very little instruction on how to apply them has been available. As a result there is a general lack of familiarity with laboratory principles and techniques by the average teacher of mathematics. Johnson conceived of a laboratory situation in which students learn by experimentation why certain methods work and how to apply them to real and practical problems. He cautioned, however, that the laboratory should be used with restraint and only to supplement the regular classroom lecture.

Structured Materials Approach

The idea of using concrete apparatus for educational purposes has a long history (Smith, 1965). Special materials designed to represent in concrete form the elements of arithmetical operations have been developed by many different educators (Williams, 1961).

Stern (1949) was concerned that children should appreciate the structure of our number system and devised materials to introduce basic mathematical concepts through measurement. She described her method, Structural Arithmetic, as a laboratory approach for groups of children or for individual instruction.

Cuisenaire's basic philosophy of teaching was that children must learn by action (Gattegno, 1960). Gattegno (1960) has said that the purpose of the Cuisenaire rods is to facilitate the identification of numbers, to allow children to discover their groupings, and to prepare the way for related perceptions. Through manipulation of the rods the child discovers new combinations which increases his skill in calculation and at the same time increases his interest and knowledge. The rods also permit the child to check his own results and correct his mistakes as he is gradually brought to a certain level of abstraction. Both the Cuisenaire and Stern methods placed a strong emphasis on developing computational skills on the part of the students.

Mathematics Laboratory Projects

England. Work in basic curriculum reform in mathematics at the elementary level in the direction of active learning, small group work, and the use of physical materials and environmental tasks was carried out in England under the influence and direction of Dienes, Sealey, Edith Biggs, and Matthews (Sealey, 1967).

Dienes' (1963) early work was in connection with the Leicestershire Mathematics Project which began in 1958.

From its inception use was made of structured physical materials to encourage preverbal mathematical thinking by providing for adequate experiences with a concept at a pre-symbolic level before the introduction and manipulation of symbolism. The Project expanded to involve over half the schools in Leicestershire.

Another strong advocate of active learning in arithmetic was Leonard Sealey who also tried out his ideas in schools in Leicestershire. Sealey emphasized that children need to discover mathematics for themselves. He wrote:

> Because children are individuals, their learning must be individual. Mathematical needs will vary. Children will learn at different rates and in different ways -- but they should be learning by 'doing' and by using real things (1960, p. 10).

He recommended the use of structured materials but stressed the need for a variety of experiences. He was also concerned with the application of mathematical ideas in real situations through environmental tasks with money, measurement, weight, and time. Sealey suggested that work cards on these topics could be made by the children themselves as well as by the teacher.

<u>The Nuffield Foundation Mathematics Teaching Project</u>. The Nuffield Project in England began in 1964. The director of the Project, Geoffrey Matthews (1968), explained that its purpose was to re-appraise the content of elementary mathematics and to produce a course for children from 5 to 13. Materials produced by the Project writers were directed toward the teachers in the form of guides. The core of the

Project consisted of Teachers' Centres all over the country where teachers could meet regularly to examine the guides, discuss their problems, exchange new ideas, and learn new mathematics themselves using the same materials and methods as the pupils. The approach emphasized learning by doing in a free atmosphere which would permit children to work to their own capacity as members of small groups.

Much experimental work under Edith Biggs had preceded the formulation of the Project. She has stated that her aims in teaching mathematics at all levels are to give the students:

> (1) the opportunity to think for themselves, (2) the opportunity to appreciate the order and pattern which is the essence of mathematics, not only in the man-made world but in the natural world as well, and (3) the number skills (E. Biggs, 1968, pp. 406-407).

She estimated that in 1968 about 25 percent of the elementary schools in England had substantially changed the teaching of mathematics, shifting towards active learning. She also reported that an increasing number of schools at the secondary stage were experimenting with the presentation of new content to students working in small groups.

Although Piaget's ideas are at the base of the Nuffield Project, the guides it produced were concerned with suggestions and applications which could be put into immediate practice in the classroom and were not concerned with theoretical aspects of mathematical learning. Many of the materials which the children worked with were inexpensive and of the homemade variety, such as boxes containing various kinds of objects, jars of water, and colored squares. The learning tasks were largely environmentally oriented as opposed to being directed toward the formation of abstract mathematical concepts through a sequence of activities with structured materials as in the Dienes method.

Activity cards were used extensively. Children worked in groups at their own pace and often on tasks which they had selected or developed themselves.

Projects in Other Countries. The mathematics laboratory movement was reflected in curriculum developments in several other countries. Dienes went to Australia in 1961 and during the four years he spent there introduced his ideas and materials in several experimental schools. His work greatly influenced the new mathematical programs which were adopted after his departure, as many of his innovations and recommendations were included in the curriculum revisions (Golding, 1968b).

An introductory mathematics program for primary schools was established in France in 1965 (Picard, 1967). Both the principles underlying the methods and the concrete materials used in the program were derived in part from Dienes. Several experiments which were to be continued for at least three years were conducted to evaluate the program.

In Hungary (Gador, 1967), experiments in mathematics teaching at the primary stage which began in 1963 emphasized the importance of individual experiences. The tools of learning, including Cuisenaire rods and Dienes blocks, were made available to every child. To develop intrinsic motivation, emphasis was placed on learning, understanding, and problem solving rather than on personal accomplishments or individual merit.

The Canary Islands Mathematics Project (Caparros and Delgado, 1967) was developed following a careful study of ideas and existing mathematics programs in various parts of the world. The first year of the program instruction cards based on various principles laid down by Dienes, Cuisenaire, Suppes, and others were prepared on mathematical and logical topics. The Project also included laboratory work with structured materials devised by Hull, Dienes, Cuisenaire, and Stern.

The Madison Project. The Madison Project under Robert Davis in the United States began in 1956. It was not concerned with the production of textbooks but with the actual encounters children would have with mathematics in the classroom. The purpose was to shift from:

> ...'mathematics as the memorization of facts' to 'mathematics as the processes of explication, creation, description, analysis, etc.'... (Davis, 1967a, p. 47).

These experiences were to be made as pleasurable and profitable as possible and yet at the same time be mathematically sound. The construction of curriculum materials was based on certain topics such as the notion of a "variable" which could be extended to develop new mathematical concepts. The Project produced a sequence of curricula most of which were intended to be supplementary to existing programs in mathematics.

Of the five curricula developed by the Project, Curriculum β (Davis, 1967a) utilized a laboratory approach most extensively. It had been created in consequence of the Project's judgement that there needed to be provision for more individualization of instruction, more small group work and less teacher domination of the learning environment, more use of physical materials, and more diversity in the kinds of activities available for the children. The basis of this decision was a study by a clinical psychologist who spent a year interviewing children from Grades 6 and 7 (Barrett, 1967). Barrett found that:

> the children liked those subjects which involved physical activity and opportunity to talk to other children and disliked those subjects which involved sitting still, and which offered no opportunity to talk to friends (p. A-27).

This curriculum was designed to provide a foundation for relating the areas of arithmetic, algebra, geometry, and science. It included individualized "shoe box" projects (activity packages containing materials and instructions) and ideas and materials which had been developed by other projects and individuals. Units from the Nuffield Project, Edith Biggs, Dienes, Sealey, and others were used in assembling the curriculum.

Teacher Training in Laboratory Methods

In addition to the Teachers' Centres established by the Nuffield Project, other programs for acquainting teachers with laboratory approaches to the teaching of mathematics have been reported.

E. W. Golding (1968b) who worked with Dienes during his stay in Australia outlined a program for educating teachers to the new methods. An important part of his proposed training program would be concrete laboratory sessions in which the teachers would do tasks with learning materials, construct embodiments for mathematical ideas, and write instruction cards and invent games for children. He wrote:

> These sessions should be conducted as nearly as possible to the way in which a lesson leading to the abstraction of a concept is presented to a class of children. Teachers divide into groups of four or five, the necessary materials are distributed, with instruction cards, and each group is expected to proceed to follow the instructions on the card. It is not necessary that any group will be working on the same concept, or with the same learning materials as any other group. Each group will be expected to work as a co-operative unit, without a chosen leader, and the members of the group will be expected to discuss what is being done, and what each thinks can be learned from it. The teacher-educator in charge should act as advisor, when and if invited to do so, or if he thinks that the group cannot profitably proceed. Leading questions rather than precise information should be given in such cases (section 4, p. 12).

Fitzgerald (1968) described a mathematics laboratory designed to give prospective elementary school teachers personal experiences in a learning situation in mathematics that was individually oriented and activity-centered. During the weekly two hour sessions, the college students worked in small groups of one to four with physical materials. Topics studied were taken from various areas of mathematics, and a great variety of material mostly purchased through commercial sources was employed. According to Fitzgerald, evaluation of the effects of the laboratory program failed to indicate an increase in the mathematical competence of the students. However, the attitudes of the students themselves regarding their laboratory experiences were highly favorable.

III. Research Studies

Comprehensive reviews of literature on the use of instructional aids in mathematics have been written by Suddeth (1963) and Harvin (1964). The studies which are now reviewed are those which are related to the theory previously discussed or to the experimental program which was developed for the present investigation and which is described in Chapter III.

Laboratory Instruction in Science

Considerable research has been conducted to investigate the relative merits of the lecture-demonstration method and the individual laboratory method of instruction in secondary school physics, chemistry, and biology. From his review of these studies Duel (1937) concluded that the lecture-demonstration method seemed to be superior to the laboratory method with respect to immediate learning and that one method appeared to be as effective as the other with respect to the retention of information and attitudes toward science. In a later review of research comparing laboratory and demonstration methods in physics teaching Kruglak and Wall (1959) predicted that it would be unlikely that future investigations would show any significant differences between the two methods if the criterion measures were pencil and paper tests of facts, principles, and applications. They hypothesized that while the laboratory probably makes a unique contribution to the teaching of physics, conventional laboratory instruction is not designed to maximize the learning of materials commonly tested on measures of achievement. Kruglak and Wall advocated both experimentation with novel laboratory instructional methods and the development and use of performance tests and other kinds of measures in harmony with specific objectives of laboratory learning. Studies comparing the inductive-deductive or problem-solving method in science laboratory work with an approach directed toward verifying facts rather than solving a problem have shown the problem-solving method to be more effective (Rainey, 1965).

The research cited above was carried out at the high school and college levels. It could be hypothesized that direct experiences, particularly those designed for discovery rather than for verification, would have greater value for learners moving from the stage of concrete operations into the formal operations stage than they would have for older students.

Manipulative Learning in Mathematics

A large number of studies comparing classes taught by the Cuisenaire method with those taught by traditional methods have been conducted (Haynes, 1963; Passy, 1963; Brownell, 1963; Lucow, 1964; Hollis, 1965; Nasca, 1966). In analyzing the Canadian research on the effectiveness of the Cuisenaire approach, Nelson (1964) made the following observations which might well apply to the other research efforts:

1. Criterion tests based on the Cuisenaire programs or which emphasize the advanced computational skills tend to give an advantage to pupils who use the Cuisenaire materials.

2. The ability of students using Cuisenaire rods to manipulate symbols far outstrips their comprehension of the significance of the manipulations.

3. The ability of children to use the material as an aid in computation seems to reach a peak at about the Grade 3 level.

4. There is limited evidence concerning possible harmful side effects of Cuisenaire experiences.

5. There is no evidence to support the claim that the use of the Cuisenaire materials has a positive effect on the learning of subject matter in other areas.

6. There is no evidence comparing the effectiveness of the Cuisenaire materials with other structural materials such as those designed by Stern and Dienes.

Uni-model methods such as those of Stern and Cuisenaire have been criticized as being too narrow in scope. Dienes (1963) claimed that the exclusive use of one material for a certain length of time may have the effect of making other applications of the mathematical structure more difficult. John Biggs (1965) said that if one kind of blocks provides the sole concrete experiences for the learner, then assimilation may occur only at the sensori-motor and perceptual levels. He believed that multi-model procedures such as that advocated by Dienes have the advantage of specifically tailoring the child's background to provide schemata out of which the desired concepts can be abstracted and to form the data for further abstractions at a higher level.

A longitudinal experiment to compare three structural methods of teaching arithmetic -- the Cuisenaire, the Stern, and the Dienes -- with traditional methods was carried out by the National Foundation for Educational Research in England and Wales between the years of 1961 and 1963 (Williams, 1967). The three experimental methods were used with children from 7 to 9. Tests of mechanical, problem, and concept arithmetic, together with attitude questionnaires were administered at the beginning and end of each experimental period. Other information was obtained through school visits and teacher questionnaires.

J. Biggs (1965) reported the following preliminary results of the investigation described above:

1. While many traditionally taught children were good calculators, many were "number anxious" and lacked conceptual understanding.

2. Most children taught by uni-model methods differed little in attitudes or performance from traditionally taught children. However, for boys of high intelligence the uni-model group was superior.

3. Children taught by multi-model methods (Dienes) appeared to have markedly better understanding and attitudes and motivation in arithmetic. This was especially true for girls of average and low ability and for boys.

Additional findings of the same study were discussed by Dienes (1966). He reported that the children taught by the Dienes method were no different from the children taught by the other methods in verbal ability but were superior in mechanical and concept arithmetic and also had more favorable attitudes toward mathematics. Slow learners were found to derive the most benefit from the Dienes method.

Brownell (1963, 1966a) conducted a study to determine the progress of children in British infant schools toward abstraction and maturity of arithmetical concepts after being exposed for three years to either the Cuisenaire program, the Dienes program or a Conventional program. Using an interview technique, data were collected on responses to eight number combinations and twelve verbal problems. Findings revealed that in the English schools the conventional program was more effective than either the Cuisenaire or Dienes programs in moving children to more meaningful abstract ways of thinking. The Cuisenaire program was slightly more effective than the Dienes program in this respect. Little difference was found in the ability of subjects in the three groups to explain solutions to verbal problems, but the Dienes group was superior in problem solving. Brownell admitted that since the goals of the three programs varied considerably, the study did not completely evaluate the effectiveness of any one program.

Lamon's (1969) study was designed to assess the degree of difficulty encountered at several levels of conceptual operation by children between the ages of 9 and 12 years who had been exposed to the Dienes, the Cuisenaire, and a contemporary instructional mathematics program. The tasks involved learning the structure of selected mathematical groups. The findings of the study revealed no significant differences in performance among groups of students in the three programs. It was concluded that previous mathematics education had little or no effect on the ability of students to learn these group structures.

Brown (1968) cited the British Plowden Report on the evaluation of the Nuffield Project which pointed out that the use of physical materials seems to be successful with certain students and in certain areas of mathematics, but with other students it seems to slow up learning. As a result, the Report concluded, the children in the new projects do not perform as well on tests in arithmetic as the pupils in traditional classes.

An account of a study carried out by Ferrara-Mori and Morino-Abbele to evaluate the effectiveness of certain perceptive aids in learning mathematical concepts was reported by Dienes (1963). Four fourth grade classes were randomly assigned to two experimental groups. One group used geometrical materials while the control group used only numbers and traditional materials. At the end of a two month period all children took a test on the basic principles taught during the experiment. Analysis of results showed that those pupils who had worked in constructive situations handling geometric forms had a better understanding of the mathematical concepts and their logical implications and extensions.

Menchinskaya (1967) reported that research carried out in the Soviet Union has shown that the introduction of concrete material does not always help pupils to solve a certain problem. Particularly in cases where the pupils are required to find a familiar principle in the context of a new problem, the introduction of concrete materials seems to complicate the problem rather than facilitate its solution.

Swick (1960) investigated the effect of using selected multi-sensory aids in arithmetic programs in Grades 2 through 5. During the first period of the study 15 groups of children were taught by the methods in use before the experiment began. In the second period a carefully planned program making use of multi-sensory concrete teaching aids was introduced. Tests in comprehension, arithmetical reasoning, and quantitative understanding were administered at the beginning of the study and at the end of period 1 and period 2. Although no details of the analysis were given, Swick reported that the findings failed to suggest that the experimental program had special value for pupils of either high or low intelligence or for pupils high or low in achievement. Attitudes toward arithmetic of second or third grade pupils and of the teachers involved in the experiment were found to have improved, however.

Reddell and DeVault (1960) conducted a study on the relative effectiveness of three different types of aids in improving the mathematical understanding and achievement of fifth grade pupils and their teachers. Two commercial aids, the Abacounter and the Educator were compared with teacher prepared aids. Different forms of the California Arithmetic Test were used to obtain pretest and posttest scores for the 18 experimental classes. Tests were also given to the 18 teachers of these classes and to 6 control teachers at the beginning and end of the five month project. Using analysis of variance of gain scores it was found that pupils using the Abacounter or Educator made significantly greater gains in arithmetical reasoning and total arithmetic than those using teacher-made aids. Mean increases in arithmetic fundamentals significantly favored the Abacounter group over the other two groups. On the tests of understanding taken by the teachers only the gains made by those using the two commercial aids were significant at the .05 level. The investigators concluded that specific teaching aids did make a difference in the achievement of pupils.

Enrichment Programs

Several enrichment programs have been developed and implemented for the purpose of meeting individual differences and motivating students. Peeler (1962) wrote that:

> enrichment materials bring a great variety of experience to the learning situation. Their concrete and visual characteristics attract attention, and attention precedes interest (p. 273).

Brandes (1953) investigated the use of recreational mathematical materials at the secondary school level. He found that most teachers who responded to his questionnaire felt that the use of such materials in mathematics improved their classes and stimulated interest for mathematics in most of their pupils.

Hudson's (1958) study was designed to determine the value of certain enrichment materials on arithmetic achievement in the fourth grade. Pupils in the experimental classes were given enrichment problem-type materials together with other amusing problems, and the teachers were provided with puzzles, games, riddles, and other challenging materials to use with these students. Analysis of variance procedures revealed that for the whole population, while the control group had a mean intelligence significantly higher at the .01 level than the experimental group, gain in arithmetic reasoning ability was significantly higher for the experimental group at the .05 level. Similar findings were reported for students in the 100 IQ-and-above category on posttest computational scores. Hudson also reported that the teachers felt that the enrichment materials were useful for stimulating and maintaining interest and that children seemed to gain more desirable attitudes toward arithmetic through their use.

Jones (1968) reported a pilot study in which a modified programmed-lecture and mathematical-game approach was used to instruct two classes of summer school ninth grade students having a history of failure in mathematics. Games were introduced to demonstrate practical applications of learning mathematics and ways of having fun with mathematical ideas. Pretest and posttest scores on the Co-operative Arithmetic Test were analyzed using a correlated t test. Results of the analysis indicated that all students had made statistically significant grade level gains over the nine week experimental period. In addition, the percentages of students in the two classes with favorable attitudes toward mathematics increased from 11 percent and 8 percent in the spring to 77 percent and 84 percent in the summer.

Hopkins (1966) conducted a study on the use of arithemetic time in the fifth grade. The control group continued to follow the school's regular course of study in which about half the time was spent on meaningful activities and the other half on drill. The experimental group also used about 50 percent of the time for meaningful activities, but in the remaining time studied new topics usually introduced in later courses (Madison Project materials). The findings revealed no significant differences between group mean scores in computational proficiency, but the experimental group scored significantly higher than the control group on a test for understanding of arithmetic principles. There was no significant interaction between treatment effects and achievement level for either criterion measure. Hopkins concluded that a class of pupils that utilizes time usually spent on drill to explore large mathematical concepts has a better understanding of basic arithmetical principles than does a class that uses drill as a cognitive process.

Bale (1966) investigated a mathematics enrichment program for potentially talented seventh grade students. One period a week for eight weeks students in the experimental groups studied enrichment material on their own while students in the control group continued with regular work from the textbook. Bale reported no significant difference between the groups in achievement in regular classroom mathematics but found that the experimental groups scored significantly higher than the control group on an achievement test based on the enrichment materials.

In reviewing the literature on enrichment, Ebeid (1964) noted that most of the reported studies emphasized that the enrichment materials were self-motivating and that the novelty and excitement they provided were important factors in producing the motivation.

Individualized Instruction Programs

Ebeid (1964) conducted an experiment at the seventh and eighth grade levels in which student self-selected activities were combined with group instruction using the SMSG textbooks. One of four mathematics periods the first semester and two of four periods the second semester were devoted to the self-selected activities. A control group in another school used the SMSG program the full time allotted for instruction in mathematics. No significant differences at the .05 level were found between the two groups on either STEP or SMSG mathematics tests or on an attitude toward mathematics measure. Differences on the STEP test and the attitude scale favored the experimental group although not significantly. In addition, Ebeid stressed that the students became individually involved in many different activities most of which were beyond and of a different nature than the concepts and skills covered in the mathematics tests. He did not attempt to determine the extent of this learning, however. He reported that the students expressed appreciation for the opportunity to work individually and at their own level of interest and ability, and he recommended that more materials be prepared for use on an individual basis as part of instruction in mathematics.

This procedure was extended the following year to permit full time self-selection in mathematics by seventh and eighth grade students (Fitzgerald, 1967). The students were allowed to choose from a large collection of books and materials and worked at tables in small groups. When a task was completed it was handed in to the teacher who marked it and then returned it to the student. Industry ratios which varied for individual students from 12 percent to 100 percent generally tended to increase during the year. Findings indicated that the bright students using self-selection did not learn as much as those in the control classes. The seventh grade students and the slow girls in the self-selected classes developed better attitudes toward mathematics than those in the conventional classes.

In the third year of the project two treatments were designed emphasizing independent study with the teacher as a resource person. One quarter of the time was spent in teacher presentation to the whole class, and the remainder of the time was used for individual and group study. No differences in achievement were found between the experimental and control groups.

Greathouse (1966) compared group-oriented and individual-oriented meaningful methods with pupil samples at the fifth and sixth grade levels. No significant differences were found between the two groups on the criterion measures used although the results favored the students taught by the individual-oriented method. He concluded that:

> Teachers become more perceptive to individual pupil needs when using the individual-oriented method than when employing whole group methods.

May (1968) described the "learning laboratories" in the Winnetka Public Schools. The classroom teachers decided which students should attend the labs and how often. Some pupils were sent for motivation and others for remedial work. Enrichment units for use by students from Grade 2 to Grade 8 were developed, and other materials were prepared to satisfy the particular needs of students as they arose. Three modes -- concrete, computation, and abstract -- were built into the units. The pupils worked with concrete materials, recorded their observations, and then made generalizations from the data. In the labs the students worked individually or in pairs and were free to ask questions, talk, and observe other students working. Written instructions were used, and the students picked up and put away their own materials. A mathematics consultant in charge of each lab assigned the units and made an evaluation report to the classroom teachers. The consultant was assisted by non-professional aids who kept records, took care of the materials, and also helped the students. According to May, the main idea was to keep the children so motivated by their own work that they would want to learn more. She reported that independent and active participation produced an extremely high degree of motivation.

IV. Chapter Summary

The rationale for mathematics laboratories is as follows:

1. According to cognitive theory, actual active manipulation of concrete materials is important in early stages of the development of mathematical concepts.

2. Intrinsic or cognitive motivation can result when students are presented with learning tasks that permit them to collect data leading to the formation of hypotheses which they are then able to evaluate and modify themselves.

3. This kind of mathematical activity encourages the development of thinking skills on the part of the students.

4. Through mathematics laboratory experiences students gain a greater understanding and appreciation of the way mathematical concepts originate and are extended.

Many mathematics projects utilizing laboratory techniques have been developed. The use of physical materials and small group work for the purpose of providing for individual differences are characteristic features of these programs. Research on the laboratory approach to learning mathematics has been mainly exploratory and would fall into Johnson's (1966) categories of action-naturalistic or philosophical research.

Studies on the use of manipulative materials and multi-sensory aids have shown that these can be useful devices in helping children learn mathematical ideas. Motivation resulting from the novelty of using the materials seems to be an important part of such programs. Other programs designed to enrich or add variety to existing courses of study or to individualize instruction have been evaluated in terms of student learning and attitudes. It has been found that time spent on these activities has not adversely affected achievement in mathematics and that, in general, student reaction has been highly favorable.

This study has as part of its framework the theory and research cited above. The purpose of the investigation was to examine the effects of using 25 percent of the school mathematics time for informal investigation of new ideas and problems in a laboratory setting. The ability of seventh and eighth grade students to learn in a mathematics laboratory and their reaction to this kind of learning experience in mathematics were investigated.

CHAPTER III

EXPERIMENTAL DESIGN AND RESEARCH PROCEDURES

I. Introduction

How should mathematics laboratories be used? The primary purpose of this study was to introduce and evaluate a mathematics laboratory program operating as an adjunct to existing courses at the seventh and eighth grade levels. The program consisted of ten activity lessons which replaced a regular mathematics class every week for ten weeks. Each of the lessons was developed to permit the students, who worked in groups of two from written instructions, to discover a new concept or relationship through the manipulation of concrete materials.

In order to assess the effectiveness of this procedure and to determine the particular advantages and benefits of laboratory teaching as compared with teacher-directed class learning, a second experimental setting was established. In this setting the laboratory activities adapted as "discovery lessons" were presented to whole classes of students by teachers who used the physical materials as instructional aids. These lessons were also held on a weekly basis in regularly scheduled mathematics periods.

In addition to the two experimental groups, called the <u>Mathematics Laboratory</u> group and the <u>Class Discovery</u> group, a <u>Control</u> group of students continued to study the regular mathematics program the full time allotted for instruction in the subject.

Outline of the Chapter

In the last chapter the rationale and background of laboratory methods in mathematics was discussed. The present chapter contains a description of the experimental settings and the research procedures used in the study. The remainder of the chapter is organized into the following sections:

II. The Mathematics Laboratory Program

III. The Class Discovery Setting

IV. The Inservice Program

V. Design of the Study

VI. Research Questions and Corresponding Instruments for Collecting Data

VII. The Testing Program

VIII. Null Hypotheses Tested

IX. Statistical Procedures

II. The Mathematics Laboratory Program

Theoretical Basis

A review of literature pertaining to the aims and methods of laboratory teaching procedures provided guidelines for the development of the mathematics laboratory program used in this study. The program was piloted in two different junior high schools in the spring (Kieren and Vance, 1968) and fall of 1968.

At the outset it was decided that laboratory work should not replace existing courses but should supplement them to provide the students with a different kind of learning experience in mathematics. The objectives of such experiences would be to acquaint the students on an informal basis with new concepts not covered in their textbooks, to encourage the development of independent and creative thinking on the part of the students, and to provide motivation and improve attitudes in learning mathematics. Previous research (Ebeid, 1964; Hopkins, 1965) had shown that achievement in regular programs does not suffer if part of the time normally devoted to mathematics study is used for other interesting, meaningful work in the subject area.

Laboratory activities were to be centered around concrete materials with which students could work together in small groups at their own speed and in their own way. A variety of materials and activities was sought in order to include a broad range of mathematical ideas and to maintain interest from week to week. Instruction cards were written to accompany each lesson to permit the groups to work on various problems independently of the teacher.

Choice of Grade Level

The laboratory program was developed for use at the seventh and eighth grade levels. In the past activity methods have been employed mainly with elementary school children and have not been considered appropriate for older students nor in keeping with the way in which learning was to take place in higher grades. Davis (1964a), however, observed that while fifth graders seem to be "natural intellectuals", by contrast, seventh and eighth graders are closer to being "engineers" at heart. He further pointed out that adolescents want to "move around physically, to do things, to explore, to take chances, to build things, and so on (p. 149)". Barrett's (1967) study with sixth and seventh grade students confirmed that children like subjects which involve opportunity for physical activity and for talking with other pupils. The junior high years have long been regarded as a time of declining interest in school and of unfavorable attitudes toward learning.

In view of these arguments it was felt that Grade 7 and Grade 8 students would most welcome and appreciate a new approach to learning in which physical objects were manipulated and which allowed for work to be carried out independently of the teacher.

Nature and Purpose of the Program

The laboratory program was designed to function as an adjunct to the existing mathematics curriculum. It was developed not to replace the present courses of study, but rather to enrich and to supplement them. The laboratory activities provided enrichment by bringing in interesting problems and relationships not encountered in the textbooks.

By presenting basic concepts such as the notion of "set" in a concrete manner or by using them in a novel situation, the activities served to supplement and to support the regular program. Finally, the mathematics laboratory provided a change from the textbook and chalkboard approach to learning mathematics, permitting physical activity and opportunity for individual discovery of concepts and relationships.

The laboratory lessons were such that they could be taken in any order by students in these grades at any time during the year. For the purposes of this study laboratory periods were scheduled once a week in place of a regular mathematics class, thus reducing the time spent on the prescribed course.

Selection of Laboratory Activities

Activities were sought which would provide opportunities for students to develop mathematical concepts through their own experience with concrete materials. Over the years many topics have been suggested as being particularly suitable for mathematics laboratory lessons. Recently there has appeared an influx of commercially produced physical aids, manipulative devices, and mathematical games. In January, 1968, the present investigator selected from among available materials those considered to be most appropriate as laboratory activities for Grade 7 and Grade 8 students. Some of the materials included their own sets of assignment cards; in other cases, instructions were written by the investigator. A total of 16 different activities were thus assembled and consequently piloted by two Grade 7 and three Grade 8 classes over a period of two months (Kieren and Vance, 1968). Commercial materials used included Dienes' Multibase Arithmetic Blocks and Algebraic Experience Materials, The Madison Project's "Shoeboxes", a geoboard, logic blocks, and the game of TUF. Other materials were devised to enable students to investigate the properties of the mathematical group, find relationships between area and perimeter, discover Euler's rule for polyhedra, and deduce the formula for the number of subsets of a set containing n elements. Based on observations made during the two months and information gathered from a questionnaire completed by the students, it was decided that ten laboratory lessons should be written for the main study. Nine of these were based on activities used during the preliminary investigation; a tenth lesson involved experiments in probability. The latter topic was chosen partly because of student preference for those lessons which involved the greatest amount of physical activity.

The Laboratory Activities

Following is a description of the ten lessons which comprised the mathematics laboratory program. Each activity is discussed in terms of the purpose, procedure, and relation to the Grade 7 and Grade 8 courses.* References for the activities and sample lessons are contained in Appendix A.

1. Area and Perimeter. The purpose of this activity was to investigate the way in which the perimeters of rectangles having a fixed area vary. The specific problem was to arrange 36 wooden unit squares so as to form a rectangle in as many ways as possible and to determine the area and perimeter of each.

The concepts of area and perimeter are introduced and taught intuitively in Grade 5 and reviewed in Grade 6. Formulas for finding area and perimeter of a rectangle are not discussed until near the end of the Grade 8 textbook. This lesson then reviewed concepts of area and perimeter, developed their formulas in the case of a rectangle, brought in the notion and use of a function, and introduced the historic isoperimetric problem.

2. Intersecting Sets. The problem was to find the total number of elements in two intersecting sets. The generalization to be arrived at is:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Half of Dienes Logic Blocks formed the universal set for this activity. Each block has three attributes: color, shape, and size. Subsets were defined in terms of these attributes and were physically represented as blocks in

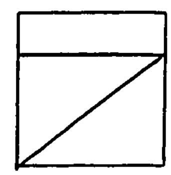
---

*H. Van Engen, *et al.*, *Seeing Through Mathematics*, Books 1 and 2, Special Edition (Toronto: W. J. Gage, 1964). Abbreviations: STM-1; STM-2.

hula-hoops. The first problem to be solved was to find a method of determining the number of blocks having two or three specified attributes; for example, the number of blocks which are both red and square.

Intersection and union of sets and Venn diagrams are taught in Grade 7, but questions relating to the number of members in sets and in their unions and intersections are not considered.

3. <u>Symmetries of a Square</u>. This activity was designed to illustrate the properties of a mathematical group in a finite system without numbers. Four moves were defined on an eight inch square of clear plastic around which two elastic bands were placed as illustrated. These four moves together with the binary relation "is followed by" provided a physical embodiment of the Klein group.



Certain of the group properties are discussed in Grade 7 in connection with the natural and rational numbers, but the notion of a group is not encountered in the <u>Seeing Through Mathematics</u> series until the ninth grade.

4. <u>Areas of Polygons</u>. The purpose of this lesson was to permit students to discover methods for finding the area of a triangle in order to be able to determine the areas of polygonal regions. The different figures were easily and quickly formed on a geo-board with rubber bands. The basic concept to be grasped was that the area of any triangular region formed on the geo-board is equal to one-

half the area of the rectangle enclosing it; that is, the rectangle on the same base and having the same altitude.

The students are taught in Grade 6 and in Grade 8 that a parallelogram has the same area as a rectangle having an equal base and of equal altitude and that the area of a triangle is one-half the area of a parallelogram having a common base and a common altitude. However, methods for determining actual areas of triangles and other polygons were new to the students. Application of the formula for finding the area of a triangle was also a new experience for these learners.

5. How Many Subsets? The purpose of this activity was to find all subsets of a given set which have a given number of elements and to discover a method of determining the total number of subsets of a set containing a given number of elements. The students worked with five sticks of different lengths and colors (Cuisenaire rods). By grouping the rods in various ways they formed and counted all possible subsets of sets with one, two, three, and four members. Using this information, they were to discover the rule relating the number of elements in a set and the number of subsets which can be formed from that set, namely, a set with n numbers has 2^n subsets.

The terms set, subset, and empty set were familiar to the students. However, neither the relationship to be found nor problems involving combinations are treated in the mathematics curriculum outlined for these students.

6. Mathematical Balance. In this activity the students used a balance arm to investigate and verify certain properties of the natural numbers and to solve simple linear equations.

The distributive law and commutative law of multiplication are discussed specifically in STM-1 after being used in earlier grades. Students are taught to use substitution to find solution sets for equations. The method of additive inverses is treated in STM-2. In this laboratory lesson the process of "subtracting equals from both sides" was explained in terms of the balance arm model.

7. Numeration in Bases 3 and 5. In this activity the students used Dienes' Multibase Arithmetic Blocks to count and to perform arithmetic operations in bases 3 and 5. The lesson included sections on expressing numbers in base 3 and 5 notation and adding, subtracting, multiplying, and dividing in these bases. It was not expected that most students would get to the work on multiplication and division in the available time.

Although a chapter on numeration in other bases is included in the Grade 7 text, it was considered optional and was not taught to the classes involved in the present study.

8. Probability. The purpose of this activity was to introduce concepts of probability through experiments involving coins, dice, and sampling urns. Students were to learn what is meant by the term probability as it is used

in mathematics and to determine probabilities in simple situations. At the same time they were to discover that in actual experiments, predicted results are only approximately obtained and that deviations from expected values are common when the number of trials is small.

A chapter on this topic is in the Grade 8 text, but is optional and was not taught to students involved in the study. There have been strong recommendations that probability be included in the mathematics curriculum beginning in the elementary grades.

9. Measurement of a Circle. The purpose of this activity was to develop formulas for finding the perimeter and area of a circle. Students discovered π by measuring the diameter and circumference of four wooden discs. The formula for the area of a circle was developed by rearranging the eight sectors of a wooden disc to form a pseudo-rectangle.

The concept of π and formulas for the area and perimeter of a circle are not taught until Grade 9 in the STM program.

10. Polyhedra. The objective of this lesson was the discovery of Euler's Rule through examination of models of various prisms, pyramids, and regular polyhedra. Students also studied these solid figures by noting the shape of each face and by constructing cardboard models.

The concepts treated in this lesson were new to the students. Solid figures are first discussed in Grade 9.

Summary of Laboratory Activities

The following is a listing of the laboratory activities and related materials:

Activity	Materials
1. Area and Perimeter	Cubical counting blocks
2. Intersecting Sets	Attribute blocks
3. Symmetries of a Square	Eight inch square of clear plastic
4. Areas of Polygons	Geoboard
5. How Many Subsets?	Colored rods
6. Mathematical Balance	Balance arm
7. Numeration in Bases 3 and 5	Multibase arithmetic blocks
8. Probability	Coins, dice, sampling urns
9. Measurement of a Circle	Wooden discs
10. Polyhedra	Models of polyhedra

Summary of Mathematical Ideas

The mathematical ideas treated in each activity did not depend on material covered in other lessons nor on the topics being studied in the regular courses. As indicated below, however, many of the significant and important basic concepts developed in the junior high school mathematics curriculum were encountered in the experimental program.

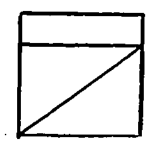
Topic \ Lesson	1	2	3	4	5	6	7	8	9	10
Set		*		*	*					
Numeration							*			
Number systems and their properties			*			*				
Conditions			*			*				
Function	*			*	*				*	*
Geometry, transformations	*		*	*					*	*
Area, perimeter	*			*					*	
Probability		*			*			*		

The Written Instructions

Sequence characteristics. Each laboratory activity was designed to provide for the discovery of a mathematical concept or relationship. The instructions were written to enable students to arrive at the desired conclusions or generalizations inductively through information obtained using concrete materials. The sequence of learning activities built into each lesson is now described. In the first stage of each lesson informal, exploratory questions and activities were provided to acquaint the students with both the physical objects to be used and the nature of the problem to be investigated. Next, the students were directed to use the concrete material to answer questions or to obtain data hopefully leading to the desired generalization.

Before the rule or method was verbalized the learner was encouraged to find and to express in his own way any relationships resulting from his observations. Finally, the students used the new rule to answer questions and do exercises, thus practising the new concept. While doing this they could check results obtained symbolically with those obtained through manipulating the concrete objects.

To illustrate this sequence of learning activities in an actual lesson, the instructions for Symmetries of a Square are now described. This activity was designed to illustrate the properties of a mathematical group in a system without numbers. The concrete material consisted of an eight inch square of clear plastic around which two elastic bands were placed as shown. By flipping and rotating the square in different ways, it can be made to assume different positions as indicated by the design.



The students were first instructed to find and sketch the different positions and describe (in their own way) the "moves" by which each position could be obtained from the standard position above. Four moves were then specified and named: a vertical flip (V), a horizontal flip (H), a rotation of 180° (R), and a slide (I). The students were led to discover that making any two of the moves consecutively is equivalent to making a single move in the set. For example, V followed by H is equivalent to R. The table of pairs of moves under the operation "is followed by" was to be completed. This was to be done at first by making two moves and noting the results. It was

First Move

Second Move

*	I	V	H	R
I				
V				
H		R		
R				

assumed that after the pupils had made a few entries in this way that they would be able to guess certain results and then verify them through manipulation. Furthermore, it was hoped that many students upon observing the pattern being established in the table would make the final entries without making any use of the square. Opportunities to work with this newly acquired information were provided by exercises in the system such as V * (H * R) = ___. Students were encouraged to check some of their answers by manipulating the square. Finally, analytic questions relating to the role of the move I, inverses of moves, and the associative property were raised.

Preparing the Instructions. In writing the instructions, both the age and mathematical background of the students were taken into consideration. Preparing suitable written instructions posed several problems. The original intent was to keep the instructions fairly loose and open-ended to encourage each group to proceed in its own way and develop its own methods. During the pilot project, however, it became evident that most of these young learners preferred and needed directions which contained specific information on how to proceed and feedback on how well they were progressing at various stages of the lesson. On the other hand, long passages or detailed explanations had to be avoided since students tended to skip over sections of this nature. Since the same instructions were to be used by all students in both grades, it was necessary to provide for the needs of learners with different capabilities. An attempt was made to structure the lessons so that everyone would be able to complete the minimal amount of work needed to arrive at the major conclusions. In addition, alternate activities and extra exercises were provided to insure that each student would be challenged for the entire laboratory period.

Sets of instructions were mimeographed so that each pair of students could write directly on them rather than keeping a laboratory notebook. Previous experience with both this type of instruction booklet and with assignment cards which were not to be written on had shown that students liked the added convenience of the former and that using them saved time.

After instructions for the ten lessons had been written, a second pilot study was undertaken to test their effectiveness. Once a week for three weeks one Grade 7 and one Grade 8 class worked on these activities. All instruction booklets were collected and examined. The lessons were then revised to clarify ambiguous sections and, when necessary, to adjust their length so that they could be completed in the time available.

Use of the Concrete Materials

Each of the lessons was centered about some type of manipulative aid or set of physical objects. The purposes of the concrete materials in each activity were:

1. to stimulate students' interest;
2. to provide a real setting for the problem to be solved or concept to be investigated;
3. to provide a means by which the learner could begin to solve the problem or explore the concept through actual manipulation;
4. to provide a means for verifying hypotheses and checking answers independently of the teacher or textbook.

Different physical materials were used in each activity to create new interest at the beginning of each laboratory period.

Role of the Teacher

In a laboratory setting the teacher's job is quite different from that in the traditional mathematics classroom where he teaches the same lesson to thirty students. Dienes (1960) said that the teacher must give up his authoritarian role as a source of power and information and instead act as guide and counsellor to groups of children working on different tasks. Davis (1967a), too, expressed concern over learning environments which had become teacher dominated. Later Madison Project curricula were developed to provide for more small group work.

Some of the duties and responsibilities of teachers under the new methods have been suggested by Golding (1968a). Before each lesson the teacher would be responsible for choosing and preparing assignment cards for the various groups. At the beginning of each lesson he would introduce the question to be considered and then send the children into their groups. As they worked he would pass from one group to another giving help when necessary, verifying the progress being made, discussing work with the children, and sometimes testing for understanding of the ideas being investigated. If necessary he could modify the composition of the groups or change the activities if the students appeared to be losing interest.

In the mathematics laboratory developed for this study the teachers' responsibilities consisted almost entirely of supervising and assisting students during the laboratory periods, since lessons and materials had been prepared by the investigator. The teachers' main duty was to give help when it was requested or obviously needed in interpreting instructions or understanding new ideas. The teachers had been cautioned not to render more assistance than was actually needed but to encourage and allow the students to work on their own in order to discover the concepts for themselves.

Review Exercise Sheets

Results of questionnaires completed following pilot studies indicated that many students were uncertain about the purpose and mathematical content of the lessons and were not sure just what they had learned during the laboratory periods. Less than 50 percent of the students felt that working in the laboratory had increased their mathematical knowledge. As a result review exercise sheets to accompany each lesson and to be completed by the students during the last few minutes of each laboratory period were prepared. The function of these questions was to draw the students' attention to the purpose of the activity just completed and to indicate to them the things they were to have learned and how well they had in fact learned them. It was felt that the latter objective could be realized even though the review sheets were not to be returned to the students since each week new groups were assigned to each activity. Another purpose of the review exercises was to indicate to the investigator and to the teachers the effectiveness of the lessons in teaching new concepts and the progress of each student in the laboratory program.

The Mathematics Laboratory Room

For both pilot studies the laboratory area was created from a standard mathematics classroom. On certain days of the week specified as "lab days", stations at which the activities were located were formed by moving flattop desks together in various locations around the room. In one school the concrete materials and written instructions were housed on shelves along one wall of the room; in the other school they were kept in an adjoining storage area when not in use. For each laboratory class materials and a set of instructions accompanying them were placed at each station before the students arrived. At the end of the period these instructions were collected and new ones put in their place for the next group.

The school at which the main study took place was newly completed, and a room had been designated solely for use as a mathematical laboratory. To accommodate classes of more than 20 students, it was necessary to establish two stations for each of the ten activities. For this purpose 18 trapazoidal-shaped desks and two tables were moved into the room. The desks had been designed to enable two people to sit and work together. The larger tables were necessary for an activity in which hula-hoops were used. The stations were numbered from 1 to 20 with Activity 1 located at Stations 1 and 11, Activity 2 at Stations 2 and 12, and so on. Physical materials for the activities were left permanently at each station and additional supplies were stored in a cupboard in one corner of the room. The written instructions and review sheets were placed on open shelves along one wall where they were readily accessible to the students. On a bulletin board above these shelves laboratory schedules for each class were posted.

Grouping of Students

Size of group. In order to permit and encourage discussion of instructions, hypotheses and conclusions, it was decided that students should perform the laboratory activities in small groups rather than individually. Golding (1968a) wrote that while there is a place for whole class teaching and for individual work, most learning should take place with students working in small groups, and Farnham (1965) noted that children generally produce a higher level of thought when working with a partner than when working individually. Many of the activities required a team approach in which one student would manipulate and another record. It had been noted in pilot studies that in groups containing more than two students, one student was often left out of the activity and the ensuing discussion. As a result of this observation it was felt that individuals would obtain maximum benefits from their laboratory experiences by working in a group of two.

Criteria for grouping. Golding (1968a) suggested a program of instruction based on small group work in which the composition of the groups continually changes. When a new topic is begun heterogeneous groups would be formed in order that less capable students could be helped by the more capable ones during the time in which the rules of the activity or game were learned. At various stages in the development of the topic, students working at about the same level in the various groups would be brought together.

leading eventually to the formation of homogeneous groups. When another question was to be introduced, heterogeneous groups would again be established.

Various other kinds of groupings have been suggested (The Nuffield Foundation, 1967). In the friendship group children work with chosen companions. The mixed ability group contains students of both high and low ability so that the more able children can sssist the less able learners. The disadvantages of this method are that "the able children tend to be understretched in such groups (The Nuffield Foundation, 1967, p. 22)" and the slower students may be unable to keep up. In the common ability group children who work at about the same level of attainment are placed together.

For the present study teachers were asked to group students who were to work in the mathematics laboratory using the following guidelines: the two were to be of the same sex, be able to work at approximately the same speed and level of comprehension, and be able to get along with each other without being too "friendly".

In classes containing an odd number of boys or girls it was necessary to establish groups containing three students. Groups of three were always scheduled to work at both stations at which the activity they were assigned to perform was carried out. Each week depending on the nature of the activity the three students could work together or one could work alone at the second station. Similarly, scheduling for the groups of two was arranged so that if a student's partner was absent he would have the option of joining another group assigned to the same activity at the other station.

Organization and Operation of the Laboratory

Each student assigned to work in the laboratory was given a schedule showing the order in which he and his partner were to rotate through the ten activities. At the beginning of each laboratory period one member of each team would pick up a set of instructions for that day's lesson from the shelves at the back of the room. Concrete materials for the activity remained permanently at the station set up for that lesson. The students would remain at that station during the fifty minute laboratory period, working with the physical materials and writing all observations and answers in the instruction booklet. Although a teacher was present and available for help when needed, the students were encouraged to work on their own as much as possible. Six minutes before the end of the period the students would stop work and while one person straightened up the station, the other deposited their instructions in a box provided for that purpose and picked up review sheets for himself and his partner. These questions were answered by each student who would then drop his sheet in a box located near the door as he left the laboratory.

Summary

The laboratory program was designed to function as an adjunct to the existing mathematics courses in Grade 7 and Grade 8. The program consisted of ten activity lessons which replaced a regular mathematics class one period a week for ten weeks. Each of the lessons was developed to permit the discovery of a new concept or relationship through the manipulation of some type of concrete material. In the laboratory the students worked in pairs from written instructions. At the conclusion of each 50 minute laboratory period, the students completed a review exercise sheet on the mathematical ideas contained in that day's lesson.

III. The Class Discovery Setting

Purpose

In discussing the problems involved in evaluating new methods, Williams said that the control group should be taught:

> under conditions which replicate all aspects of those obtaining in the experimental group (including novelty) excepting for that aspect which is under investigation (1967, p. 66).

The present study was undertaken to investigate various outcomes associated with learning in an instructional setting in which students work in groups of two from written instructions to explore new mathematical ideas through the manipulation of concrete materials. The Class Discovery setting provided a more traditional instructional setting in which the mathematical ideas in the laboratory activities could be presented to groups of students.

Description

The program consisted of the ten laboratory activities adapted for presentation to whole classes of students by a teacher using concrete materials as instructional aids. The discovery approach used in teaching the lessons was developed in terms of teacher and student behavior patterns as suggested by Sigurdson and Johnston (1968) and sequence characteristics of the learning activities (Worthen, 1967).

1. A general problem was presented by the teacher in relation to some physical objects or manipulative aid which he demonstrated.

2. Students were encouraged to suggest hypotheses or guess outcomes in connection with the concrete materials. The teacher then used the materials to test the hypotheses and gather data leading to the solution of the problem. During this initial investigation stage precise language and symbolic representation of relationships were not stressed; rather students were allowed to formulate answers and methods in their own words. Alternate methods and solutions were accepted and encouraged by the teacher.

3. The teacher then summarized what had been learned and stated precisely the rule or generalization which was sought.

4. Students were given exercises applying the new rule or method to consolidate the concepts which had been developed.

Comparison of the Experimental Settings

The Class Discovery Setting parallelled the Mathematics Laboratory Setting in the following respects:

1. The same lessons were studied.

2. The students were given an active role in discovering mathematical concepts and relationships suggested by a concrete object or in a real situation.

3. Within each lesson the same sequence of learning activities was followed: (a) informal exploratory activities with the concrete materials, (b) the use of the physical objects to gather data and test hypotheses, (c) statement of the rule or method which was sought, (d) practice exercises.

4. The lessons were taken one period a week for ten weeks during school time formerly allotted to the study of mathematics.

5. Learning was not test-oriented nor was homework assigned.

The two instructional settings used in this study differed in several fundamental respects. One important difference was in the use made of the concrete materials. In the laboratory each pair of students had direct access to the objects accompanying an activity. This permitted each learner to personally manipulate the objects and to discover and verify the mathematical concepts himself. In the class setting the students watched as the teacher demonstrated with the materials. In terms of Bruner's (1966) theory the objects used in this way served to permit the learners to form mental images; hence the concepts were presented in what might be termed a type of iconic mode. In the laboratory the learners' initial contact with each new idea was at the enactive level.

A second approach to comparing the two methods is in terms of the size of the instructional group. In the laboratory the students worked in groups of two. They were encouraged to discuss their ideas with one another as they carried out the activities. Assistance from the teacher was given, usually, only when requested and was thus geared to the particular needs of the students who had asked for help. In the large group Class Discovery setting the students were able to benefit from the ideas, questions, and enthusiasm generated by the various class members and were motivated by being able to generate and display ideas in competition with others.

The two treatments also differed in the method by which the lessons were presented to the students. In the laboratory the students worked from written instructions, while in the Class Discovery setting a teacher presented the problems and guided the class to the desired conclusions. One purpose of written instructions was to provide for individual differences by allowing pairs of learners to proceed through each lesson at their own rate and in their own way rather than as directed by a teacher who had to consider the overall ability and enthusiasm of the class. On the other hand, while the written instructions were fixed, the teacher could adapt each lesson to the level and interests of his class. Further, he could structure his presentation in such a way as to give just enough help and information to keep the lesson moving and to insure that the necessary learning experiences occurred and that the desired conclusions were in fact reached by the end of the lesson.

IV. The Inservice Program

Training the Teachers

All three mathematics teachers at the school in which the study was to take place agreed to participate. One of the teachers had been involved in the pilot laboratory project the previous year and was thus already familiar with the procedure and some of the problems to be considered. The investigator visited the school several times preceding the beginning of the study in order to set up the laboratory and to talk to the teachers. Each teacher was given a looseleaf notebook containing a complete outline of the study and the instructional materials for each of the ten activities. These included a set of laboratory instructions, statements of the purpose and the behavioral objectives of the lessons, suggestions and directions for adapting the laboratory activity to the class setting, and the review exercise sheets to be given following the lesson (Appendix A). The looseleaf also contained guidelines for teaching the Class Discovery lessons, laboratory schedules for each class, directions for laboratory students, and instructions for introducing the students to the laboratory program (Appendix B). The investigator visited the school at least once a week during the course of the study and talked informally with the teachers concerning their progress and any difficulties being encountered.

Preparing the Students

Students in those classes which had been assigned to the laboratory group were told that for the next ten weeks their mathematics course would include laboratory lessons in addition to their regular textbook work. The students were taken on a tour of the laboratory, and the procedure to be followed was explained to them. Each pupil was given information sheets containing this procedure and the schedule for each group to follow.

Students in the Class Discovery setting were informed that one day a week they would study a special lesson in which they were to solve an interesting problem or investigate a new concept not found in the textbook.

Control classes were not specifically informed of the study. It was suggested that if students from this group inquired about the mathematics laboratory, they were to be told that only certain classes would be using it for the present and that other classes would use it at a later time.

V. Design of the Study

The present study involved the students and mathematics teachers at Westlawn Junior High School, Edmonton, Alberta.

Subjects

The school population consisting entirely of Grade 7 and Grade 8 students had been organized into seven classes at each grade level. Assignment to classes in September, 1968, had not been random but was made on the basis of the non-academic options chosen by the students. The area from which the school population was drawn was made up of mainly lower-middle socioeconomic districts of the city. The mean Lorge-Thorndike Verbal and Non-verbal IQ scores were 114.3 and 113.3 respectively for the Grade 8 students and 110.0 and 110.3 for the Grade 7 students.

Teachers

The three mathematics teachers were male and had all taught the modern mathematics program used in the school, Seeing Through Mathematics (STM), Special Edition (Van Engen, et al.), for at least two years. The teachers' training and experience is indicated in Table 1.

TABLE 1

TEACHER TRAINING AND EXPERIENCE

Teacher	Degrees	Teaching Experience in Years	
		Total	STM
1	B.Ed.; M.Ed.	7	2
2	B.Ed.; B.Sc.	7	3
3	-----	35	5

Teachers 1 and 2 both taught mathematics to two classes in each grade; teacher three had three classes in each grade.

Scheduling had been arranged so that two or three of the classes met at the same time on certain days and team teaching was carried out on a regular basis with all classes.

Procedure

At each grade level the classes taught by each teacher were randomly assigned to the three experimental groups -- Mathematics Laboratory (ML), Class Discovery (CD), and Control (CON). One class in both grades taught by each teacher was placed in the ML group. Teachers 1 and 2 each had two classes in the CD and CON groups, counterbalanced so that a Grade 7 and a Grade 8 class were in each of these groups; teacher 3 had two classes in each of the groups. In order that he too would teach the discovery lessons to only one class of students, it was decided that the investigator would teach one of his classes on those days on which the discovery lessons were to be given. The assignment of classes to treatments is given in Table 2.

TABLE 2

ASSIGNMENT OF CLASSES TO TREATMENTS

Teacher	ML	CD	CON
1	7D	7A	
	8C		8E
2	7E		7G
	8B	8F	
3	7B	7C	7F
	8A	8D*	8G

*Class taught by investigator

A weekly schedule for the Mathematics Laboratory activities and Class Discovery lessons was drawn up by the investigator in consultation with the teachers. An effort was made to schedule the experimental periods for both treatment groups on different days of the week and at different times of the school day. The schedule is outlined in Table 3.

TABLE 3

SCHEDULE OF ML AND CD PERIODS

Block	Monday	Tuesday	Wednesday	Thursday	Friday
1-2	CD-7C		ML-7E		
3-4	ML-8A	ML-7B CD-7A			
5-6					ML-7D
7-8		ML-8C	ML-8B CD-8D		
9-10					
11-12			CD-8F		

During the period in which the experiment was carried out (January through March, 1969), those parts of the regular Grade 7 and Grade 8 mathematics courses which were normally covered in this time period were taught to all groups of students. In Grade 8 this consisted of Unit 10 (The rational number system) of Seeing Through Mathematics - Book 2 (STM-2). In Grade 7 the sections covered were Unit 3 (Conditions in two variables), Unit 4 (Conditions involving rate pairs), lessons 53 and 55 of Unit 5 (Numeration systems), and Unit 6 (The natural number system) of Seeing Through Mathematics - Book 1 (STM-1). Instruction in mathematics was conducted in four 50 minute periods (double modules) each week. The Control classes used all of this time studying the STM material. The Mathematics Laboratory and Class Discovery groups devoted three periods a week to the regular program; the fourth was spent investigating the special enrichment lessons.

VI. Research Questions and Corresponding Instruments for Data Collection

1. Achievement in the Regular Mathematics Programs

Students in the ML and CD groups spent 25 percent less class time studying the regular courses than students in the CON group. This time was instead used to explore new mathematical problems and concepts. How did this use of class time affect achievement in the regular program?

Comprehensive tests based on regular classroom work were administered to all students at the beginning and at the end of the study. In addition, quizzes were given periodically during the course of the experiment. These tests were prepared and given by the teachers at the school so that the measures would reflect the aims and methods used in instructing the students. On the basis of their training and experience these teachers were judged to be well qualified to construct valid and reliable tests for the material they taught to their students.

STM-1 Posttest. This test consisted of 50 multiple choice items based on the material in the Grade 7 course covered from January through March, 1969. The items were selected by the teachers from the 90 items in the Grade Seven Mathematics Spring Examination prepared by the Edmonton Public School Board for use by schools in the system. This latter test covered all work taken since the beginning of the school year. A Kuder-Richardson Formula 20 reliability coefficient of .78 was computed for the STM-1 Posttest.

STM-2 Posttest. This test consisted of 45 multiple choice items based on material from the Grade 8 mathematics course covered from January through March, 1969. A Kuder-Richardson Formula 20 reliability coefficient of .84 was found for this test.

STM-1 Pretest. This test was based on work in the Grade 7 mathematics course taken before January, 1969. It was made up of both multiple choice items and questions requiring the solution of problems. Out of a possible 67 marks a mean of 33.5 and a standard deviation of 13.6 were found for the total Grade 7 population on the test.

STM-2 Pretest. This was a 45 item multiple choice test covering work from the Grade 8 mathematics course taught before January, 1969. A Kuder-Richardson Formula 21 reliability coefficient of .83 was computed for the test.

STM-1 Quizzes. During the experimental period three quizzes for which a total of 99 marks was possible were given to the Grade 7 students. These quizzes were based on regular classroom work.

STM-2 Quizzes. Four quizzes for which there was a total of 155 marks were taken by the Grade 8 students during the course of the study.

The STM pretests and posttests had 50 minute time limits. Each quiz required only part of a 50 minute period for administration.

2. Immediate Learning in the ML and CD Instructional Settings

How well were students working in small groups with concrete materials and written instructions able to learn new mathematical ideas? How did their understanding of the topics investigated compare with that of students who studied these lessons as part of a whole class under the direction of a teacher?

Immediate Learning test (IL). This investigator-prepared instrument consisted of ten subtests corresponding to the ten experimental lessons. At the end of each ML and CD period the appropriate subtest was administered to the students as a review exercise sheet to test their comprehension of the material they had studied. The questions were based on the behavioral objectives for each activity. Five marks were allotted for each subtest, making 50 points possible for the total test.

3. Cumulative Achievement in the Experimental Materials

To what extent were the students in the ML and CD groups able to retain concepts learned during the special laboratory and discovery lessons over the ten week period of the study? How well were the CON students who had not been exposed to the experimental material able to correctly respond to questions based on this material?

Cumulative Achievement test (CA). This instrument was constructed by the investigator and consisted of 40 multiple choice items for which there were five choices. Four items were based on each of the ten experimental lessons. The questions were constructed on the basis of the stated behavioral objectives for each lesson. Many of the items were adapted from questions which appeared on the review exercise sheets accompanying each activity. A first draft of the test was administered to students who had participated in the three week pilot study. A revised form of the test was examined and constructively criticized by professors and graduate students in mathematics education. Following this, further revisions were made leading to the final form of the test which had a 50 minute time limit. For the combined group the test was found to have a Kuder-Richardson Formula 20 reliability coefficient of .75.

4. Higher Level Thinking and Problem Solving

Avital and Shettleworth (1968) distinguished three levels of mathematical thinking. At the lower levels the student reproduces a fact, recognizes material in the form in which it was previously taught or applies a procedure he has learned. At the highest level (called open search) the student must solve problems which are based on but which go behold the previously learned material; he produces something that is entirely new to him.

How well would students from the three experimental groups be able to solve problems which went beyond the material studied in the ML and CD periods?

Higher Level Thinking and Problem Solving test (HLTPS). This test which was prepared by the investigator consisted of ten questions, one corresponding to each of the ten experimental topics. Each of the questions contained two or four parts and was designed to test the student's ability to go beyond the ideas treated in one of the activities. The questions involved either an application of these concepts or procedures in a new or more complex problem situation or an extension of the concepts. This test had a possible score of 40 points and had a 50 minute time limit. Items for this test were given to students following the pilot study. The test was subsequently revised by the investigator who received valuable assistance from a professor of mathematics education.

5. Divergent Thinking in Mathematics

Davis (1964c) stated that in evaluating new programs we should be concerned with:

> the way the child explores on his own, his original 'creative' ideas, and other responses which he will not necessarily produce in response to external cues (p. 157).

One of the objectives of mathematics teaching listed by Edith Biggs (1968) is to:

> Let the children discover for themselves the mathematical patterns which are to be found everywhere in the man-made and natural environment (p. 408).

Does the presentation of a series of mathematical problems based on concrete materials assist students in becoming more perceptive to the possible uses of physical objects in representing mathematical ideas and solving mathematical problems? Can students who have investigated certain mathematical ideas in particular physical situations see these same ideas in a different physical embodiment? To obtain answers to these questions a test situation was devised in which the students were to relate a set of concrete objects to the study of mathematics.

The Cards Used in Mathematics test (CUM). In this test the students were asked to suggest as many ways as possible in which a certain set of physical materials could be used to illustrate mathematical properties or solve mathematical problems. The material to which they were to relate mathematics consisted of a set of 40 "cards" made of heavy cardboard. The cards were numbered on one side from 1 to 10 in each of four colors. The backs of the cards were black.

The teacher administering the test read the instructions over with the students. To further explain what was wanted, examples of mathematical ideas which could be investigated using a balance arm were given. (The teacher discussed these examples with the students while demonstrating with an actual balance arm.) The students were then given 10 minutes to make responses with respect to the cards. A student's score on the test was the number of different correct responses made (Prouse, 1964).

This instrument was developed with students who had participated in the pilot study and with the assistance of a graduate student working in the area of creativity in mathematics.

Using the marking scheme developed by the investigator a panel of eight judges assigned scores to eight test papers. The degree of agreement among the judges as measured by Kendall's coefficient of concordance W (Ferguson, 1966, pp. 225-227) was found to be .86 (Appendix C).

6. Attitudes Towards Mathematics

The development of favorable attitudes toward mathematics is an important objective of mathematics teaching (Johnson, 1957). What was the effect of the weekly laboratory periods and discovery lessons on the attitudes of students involved in these programs? An attitude scale developed by Remai (1965) was used to gather data relative to this question.

A Mathematical Study (AMS). This instrument consisted of 23 multiple choice items. Each item contained a stem for which five alternate completions were provided. These completions were weighted from 1 to 5, and the total score for a student was obtained by summing the weights of the alternatives he had selected. A high score on the test indicated a favorable attitude towards learning mathematics. The time required for administering the test was about 15 minutes. Remai reported a test-retest reliability coefficient of .77 and an internal consistency coefficient based on analysis of variance of .86.

7. Student View of Mathematics as a Discipline of Study

What do students think about mathematics and how it originates and is learned? Did the mathematics laboratory experiences or the discovery lessons affect student feelings and attitudes toward the study of mathematics? To obtain information concerning these and other related questions, an instrument based on the semantic differential technique developed by Osgood, *et al.* (1957) and applied to attitudes toward learning mathematics by Anttonen (1968) was constructed by the present investigator.

Learning and Doing Mathematics scale (LDM). This instrument consisted of a series of pairs of polar terms in relation to which the subject was to react to the concept "Learning and doing mathematics". A response to an item was made by placing an X in one of seven spaces between the two terms according to the way in which the terms reflected the subject's feelings toward the concept. Seventeen word pairs were chosen to assess the feeling of the students toward various aspects of mathematics. Items were scored from 1 to 7, a high score indicating a favorable or desired response.

Posttest scores on the 17 items were subjected to principle axis factor analysis. Four factors having eigenvalues greater than 1 were found. The number of factors was reduced to three and varimax orthogonal rotations were applied (Appendix C). On the basis of this analysis, three subscales of the LDM scale were formed as follows:

1. Enjoyment Subscale

interesting	-	dull
find out	-	be told
fair	-	unfair
fun	-	boring
active	-	passive
new	-	old
useful	-	useless
experimental	-	cut and dried
enjoyable	-	distasteful

2. Familiarity Subscale

familiar	-	strange
real	-	unreal
abstract	-	concrete
relaxed	-	tense

3. Situation Subscale

laboratory	-	textbook
objects	-	symbols

Two items, "noisy - quiet" and "rule - guess", were not included on any of the subscales because their high loadings on the three factors were negative. Subscale scores were obtained by summing the scores on each of the items within a subscale.

8. Student Reaction to the ML and CD Treatments

An important source of information about the effectiveness of an instructional technique or a new program is the student himself. How did the student react to learning mathematics in a laboratory setting? What did the students in the Class Discovery group think about the special lessons? To obtain student views concerning the two treatments, questionnaires were constructed and administered to all participating students following the study.

Student Questionnaire (SQ). This instrument contained items designed to elicit student responses to a variety of aspects of the experimental program. Two forms of the questionnaire were developed, one for the ML students and the other for the CD students. The first 17 items on the two forms were parallel and had been constructed in order to compare student reaction to the two experimental instructional settings. These items consisted of statements to which the students responded "Agree", "Uncertain" or "Disagree". Additional items of this type on both forms concerned questions of particular relevance to only one of the treatments. The ML questionnaire also contained items requiring the students to select one of several statements. Students in both groups were asked for further comments on the experimental program. Earlier versions of the questionnaire had been used in the previous mathematics laboratory investigations carried out by the writer.

9. Teacher Opinions

The teachers who had implemented the program of activity lessons in the Mathematics Laboratory and Class Discovery settings were in an excellent position to comment on the effectiveness and relative merits of the two treatments. A questionnaire was prepared by the investigator for the teachers to determine their reaction to the study.

Teacher Questionnaire. This instrument was designed to investigate the opinions of the teachers who had participated in the experiment regarding the effects of the program on their students. In addition, questions were asked concerning their role as teachers in the two settings and how they would propose that laboratory and discovery lessons be used in the future.

VII. The Testing Program

All tests and measures of student performance and attitudes described in the previous section of this chapter were administered by the three mathematics teachers to their own classes. Before a test originating outside the school was to be given, the investigator delivered the necessary materials to the teachers and explained to them the purpose of the instrument and the procedure to be followed in administering it. In addition, written instructions for administering the test were left with each teacher.

In late December, 1968, all Grade 7 classes wrote the STM-1 pretest and all Grade 8 classes wrote the STM-2 pretest as prepared by the teachers at the school. Also at this time the attitude test (AMS) was administered to three Grade 7 classes and four Grade 8 classes, and the Learning and Doing Mathematics scale (LDM) was administered to the other four Grade 7 classes and the other three Grade 8 classes. Half the classes in each treatment group had been chosen randomly to take one of these two scales as indicated in Table 4.

TABLE 4

CLASSES PRETESTED ON THE AMS AND LDM SCALES

Scale	Treatment		
	ML	CD	CON
AMS	7E, 8A, 8C	7A, 8F	7F, 8G
LDM	7B, 7D, 8B	7C, 8D	7G, 8E

During the course of the study (January through March, 1969) three STM-1 and four STM-2 quizzes were administered to all Grade 7 and Grade 8 classes respectively. Immediate Learning subtests were administered weekly in the form of review exercise sheets at the conclusion of each Mathematics Laboratory and Class Discovery period.

All other criterion measures were given during the last week in March and the first part of April. The final testing program required five days and was spread over a period of three weeks. The first week the Cumulative Achievement test and the Higher Level Thinking and Problem Solving test were administered to all students. The second week the AMS and LDM scales were administered one day, and the Cards Uses in Mathematics test and the Student Questionnaire (where it applied) were administered a second day. The students wrote the STM-1 and STM-2 Posttests at the beginning of the third week. While these tests were being taken by the students, the teachers were asked to complete the Teacher Questionnaire.

An effort was made to have students, who were absent when the Cumulative Achievement test, the Higher Level Thinking and Problem Solving test or the STM tests were given, write the missed test at a later time. Usually, no attempt was made to obtain scores from students who missed other tests.

VIII. Null Hypotheses Tested

The null hypotheses associated with each of the questions and criterion variables discussed in the last section of this chapter are now stated.

Achievement in the Regular Mathematics Program

Hypothesis 1. At the (a) Grade 7 level (b) Grade 8 level, there are no significant differences among group mean scores obtained by the ML, CD, and CON students on (i) the STM Posttest (ii) the combined STM Quizzes.

Immediate Learning in the ML and CD Instructional Settings

Hypothesis 2. At the (a) Grade 7 level (b) Grade 8 level, there are no significant differences between group mean scores obtained by ML and CD students on any of the ten IM subtests or total IM test.

Cumulative Achievement in the Experimental Materials

Hypothesis 3. At the (a) Grade 7 level (b) Grade 8 level, there are no significant differences among group mean scores obtained by the ML, CD, and CON students on the CA test.

Higher Level Thinking and Problem Solving

Hypothesis 4. At the (a) Grade 7 level (b) Grade 8 level, there are no significant differences among group mean scores obtained by the ML, CD, and CON students on the HLTPS test.

Divergent Thinking in Mathematics

Hypothesis 5. At the (a) Grade 7 level (b) Grade 8 level, there are no significant differences among group mean scores obtained by ML, CD, and CON students on the CUM test.

Attitude Towards Mathematics

Hypothesis 6. There are no significant differences among adjusted group mean attitude scores obtained by ML, CD, and CON students on the AMS.

Student View of Mathematics as a Discipline of Study

Hypothesis 7. There are no significant differences among adjusted group mean scores obtained by ML, CD, and CON students on the three subscales of the LDM scale.

Student Reaction to the ML and CD Treatments

Hypothesis 8 (a). For the first 17 items on the SQ there is no significant relationship between treatment group and response to the item.

Hypothesis 8 (b). Within the ML and CD groups there are no significant relationships between the responses to the SQ items and the following variables: (i) grade, (ii) teacher, (iii) sex, and (iv) achievement level.

IX. Statistical Procedures

Sample

The experimental sample consisted of all Grade 7 and Grade 8 students at Westlawn Junior High School who:

1. had written the STM Pretest;
2. if in the ML or CD groups, had been present for at least seven of the ten experimental lessons;
3. had not been absent from school more then 15 days (out of a possible 58) during January, February, and March;
4. had not missed more than two of the posttests. Because the sample included some students for whom certain posttest scores were not obtained, the number of subjects considered in testing various hypotheses varied slightly.

Since it had not been possible to randomly assign students to the three treatment groups, tests for significant differences among the groups with respect to mean STM Pretest scores were carried out at each grade level. The following hypothesis was tested using one-way analysis of variance:

> At the (a) Grade 7 level (b) Grade 8 level, there are no significant differences among the mean scores obtained by the ML, CD, and CON groups on the STM Pretest.

Table 5 gives the means and standard deviations obtained by the three groups, and Table 6 summarizes the analysis of variance carried out the scores.

TABLE 5

STM PRETEST GROUP MEANS AND STANDARD DEVIATIONS

Group	Grade 7			Grade 8		
	N	Mean	S.D.	N	Mean	S.D.
ML	67	31.39	13.36	83	23.86	8.00
CD	55	35.87	13.81	47	22.02	7.18
CON	49	31.96	13.51	49	25.69	7.76
Totals	171	32.99		179	23.88	

TABLE 6

ANALYSIS OF VARIANCE OF STM PRETEST SCORES

	Source	df	MS	F	P
Grade 7	Groups	2	340.53	1.86	.160
	Error	168	183.56		
Grade 8	Groups	2	161.84	2.71	.069
	Error	176	59.78		

As indicated in Table 5, the probability that differences did exist among the groups on the STM Pretest was .160 in Grade 7 and .069 in Grade 8. Although these probability levels are not usually considered statistically significant, it was felt that any differences existing among the groups on these tests should be controlled for in analyzing data obtained from criterion measures. To do this and also to investigate the effects of the treatments on students at different levels of prior achievement in mathematics, the experimental population at each grade level was divided into three achievement levels -- high (H), average (A), and low (L) -- on the basis of the STM Pretest scores. The two cutting points separating the three levels were determined by taking scores one-half a standard deviation on either side of the mean for the test. Thus approximately 30 percent of the subjects were in each of the high and low achievement groups and about 40 percent in the average achievement group. The range of scores defining each category and the number of students at each level are presented in Table 7.

TABLE 7

FORMATION OF ACHIEVEMENT LEVEL GROUPS

Level	STM-1 Scores	Frequency	STM-2 Scores	Frequency
High	41 - 67	55	29 - 45	52
Average	26 - 40	65	20 - 28	71
Low	0 - 25	51	0 - 19	56
		171		179

The number of subjects in each achievement level within each of the treatment groups is given in Table 8.

TABLE 8

CELL FREQUENCIES FOR TREATMENT GROUP AND ACHIEVEMENT LEVEL CLASSIFICATIONS

	Achievement Level	Treatment			Total
		ML	CD	CON	
Grade 7	High	20	22	13	55
	Average	25	20	20	65
	Low	22	13	16	51
	Total	67	55	49	171
Grade 8	High	24	8	20	52
	Average	33	20	18	71
	Low	26	19	11	56
	Total	83	47	49	179
Combined	High	44	30	33	107
	Average	59	40	38	137
	Low	47	32	27	106
	Total	150	102	98	350

<u>Procedure for Testing Hypotheses</u>

The .05 level of significance was used in this study. However, the probabilities associated with each statistical test are reported for the reader.

<u>Two-way analysis of variance</u>. Hypotheses 1 through 5 were tested at each grade level using a treatment by achievement level factorial design. Since the sample group

sizes were not related to theoretical population sizes, two-way unweighted means analysis of variance (unequal n's) was considered appropriate (Winer, 1962, p. 241).

The steps followed in using this statistical procedure are now outlined. First, the treatment by achievement level interaction was examined. If the interaction was significant (at the .05 level), the treatment effects at each achievement level were investigated considering these levels as single factor experiments (Winer, 1962, p. 210). At those levels where treatment effects were found to be significant, tests for the significance of difference between pairs of means were made following Scheffé's procedure (Winer, 1962, p. 88). If no significant interaction effect was found, the main effects due to treatments were observed. If the treatment effects were significant, tests of significance of differences between pairs of group means were made using Scheffé's method.

An assumption underlying the analysis of variance is that the error variance is homogeneous. Bartlett's test (Winer, 1962, p. 95) was used to test this assumption. For each analysis cell variances and the results of Bartlett's test are reported in Appendix E. F-tests are considered robust with respect to homogeniety of variance (Winer, 1962, p. 239), and, according to Hays:

> modern opinion holds that the analysis of variance can and should be carried on without a preliminary test of variances ... (1963, p. 381).

Solomon four-group design. To determine whether pretesting had any differential effect on the AMS and LDM scale posttest scores of the three treatment groups a Solomon four-group design (Campbell and Stanley, 1966) was utilized. In this design half of the subjects in each group are given a pretest and all the subjects are given the posttest using a particular instrument. The posttest scores are analyzed using two-way analysis of variance to determine the main effects of pretesting and the testing-treatment interaction. If these effects are not significant, group mean posttest scores are analyzed using analysis of covariance, pretest scores being the covariate.

Analysis of covariance. Hypotheses 6 and 7 were tested using analysis of covariance of posttest scores, pretest scores being the covariates. Where treatment effects were found to be significant, a multivariate analysis procedure was used to compare adjusted group means two at a time to determine which treatment groups were significantly different from each other.

An assumption underlying the analysis of covariance is that the regression coefficients within each of the treatment classes are homogeneous (Winer, 1962, p. 583). Appendix F lists the regression coefficients and also reports the results of Box's (1949) approximate F-test on the homogeniety of variance and covariance for each analysis.

Chi-square test. Hypotheses 8(a) and 8(b) were tested using the chi-square test for independence (Ferguson, 1966, p. 30). In this test the observed cell frequencies are compared with the cell frequencies expected from row and column totals if the variables were independent. In making the test, if the significance of the value of chi-square was greater than .05, the null hypothesis that the two variables were not related was rejected.

The present chapter has described the experimental setting and the design of the study. The following chapter reports the findings of the investigation.

CHAPTER IV

RESULTS OF THE STUDY

The purpose of the study was to investigate the effects of a mathematics laboratory program at the seventh and eighth grade levels. This chapter reports the data collected during the investigation and the results of testing the hypotheses of the study. The findings are reported under the headings corresponding to the research questions discussed in Section VI of Chapter III. In presenting the findings related to each question, the null hypothesis is restated, the testing procedure described, and the results of the analysis given.

Most of the statistical analyses were carried out on the IBM 360/67 computer at the University of Alberta using programs developed and tested by the Division of Educational Research at that institution.

I. Achievement in the Regular Mathematics Program

Hypothesis 1 Restated

At the (a) Grade 7 level (b) Grade 8 level, there are no significant differences among group mean scores obtained by the ML, CD, and CON students on (i) the STM Posttest (ii) the Combined STM Quizzes.

Test Procedure

The four parts of Hypothesis 1 were tested by means of a two-way analysis of variance.

Results

Tables 9, 11, 13, and 15 list the cell means for each measure. Cell frequencies and variances are contained in Appendix E. Tables 10, 12, 14, and 16 summarize the analysis of variance performed on the scores obtained from these measures. As indicated in these tables, there were no significant treatment by achievement level interactions nor were the main effects due to treatments significant.

Conclusions

Null Hypothesis 1 was not rejected. No significant differences in achivement in the regular mathematics program were found to exist among the ML, CD, and CON students on either the STM posttest or the Combined STM Quizzes at either the Grade 7 or Grade 8 level. There were no significant differential treatment effects across three levels of achievement.

TABLE 9

STM-1 POSTTEST CELL MEANS

Group	ML	CD	CON
High	30.55	29.95	30.85
Average	24.44	24.05	25.50
Low	18.77	20.15	23.13
Group Means	24.59	24.72	26.49

TABLE 10

ANALYSIS OF VARIANCE OF STM-1 POSTTEST SCORES

Source	df	MS	F	P
Achievement	2	1211.66	41.86	.000
Treatment	2	57.22	1.98	.142
Interaction	4	20.75	.72	.582
Within	160	28.94		

TABLE 11

COMBINED STM-1 QUIZZES CELL MEANS

Group	ML	CD	CON
High	78.35	77.00	81.54
Average	64.08	60.84	65.65
Low	47.09	45.92	47.87
Group Means	63.17	61.26	65.02

TABLE 12

ANALYSIS OF VARIANCE OF COMBINED STM-1 QUIZZES SCORES

Source	df	MS	F	P
Achievement	2	12676.10	90.01	.000
Treatment	2	172.34	1.22	.297
Interaction	4	14.33	.10	.982
Within	158	140.83		

TABLE 13

STM-2 POSTTEST CELL MEANS

Group	ML	CD	CON
High	29.63	32.00	30.80
Average	22.70	21.90	23.28
Low	17.42	17.84	20.30
Group Means	23.25	23.91	24.79

TABLE 14

ANALYSIS OF VARIANCE OF STM-2 POSTTEST SCORES

Source	df	MS	F	P
Achievement	2	1717.91	52.82	.000
Treatment	2	34.19	1.05	.352
Interaction	4	16.56	.51	.729
Within	169	32.52		

TABLE 15

COMBINED STM-2 QUIZZES CELL MEANS

Group	ML	CD	CON
High	104.86	113.38	108.00
Average	73.52	68.95	75.47
Low	51.35	57.24	47.22
Group Means	76.58	79.85	76.90

TABLE 16

ANALYSIS OF VARIANCE OF COMBINED STM-2 QUIZZES SCORES

Source	df	MS	F	P
Achievement	2	34162.30	103.32	.000
Treatment	2	144.84	.44	.646
Interaction	4	348.69	1.05	.381
Within	156	330.64		

II. Immediate Learning in the ML and CD Settings

Hypothesis 2 Restated

At the (a) Grade 7 level (b) Grade 8 level, there are no significant differences between group mean scores obtained by ML and CD students on any of the IL subtests or on the total IL test.

Test Procedure

Subtest and total test scores of those students who had been present for all ten experimental lessons were analyzed using two-way analysis of variance.

Results

No significant interaction effects were observed for any of the ten subtests at either grade level. Table 18 lists the IL subtest cell means and treatment effect F-ratios from the analysis of variance. There were five points possible for each of the ten subtests. As indicated in Table 18, F-ratios significant at the .05 level were observed for subtests 3, 5, 7, and 9 in Grade 7 and for subtests 3, 6, 7, 8, and 10 in Grade 8.

Table 19 gives the total IL test cell means, and Table 20 summarizes the analysis of variance of the total IL test scores. At the Grade 7 level there was a significant interaction (Figure 1), and treatment effects were examined at each of the three achievement levels. The results of these analyses for treatment effects at achievement levels in Grade 7 were as summarized below.

Achievement Level	F	P	
High	.05	.832	
Average	21.82	.000	(CD > ML)
Low	8.98	.008	(CD > ML)

TABLE 17

IL TEST CELL FREQUENCIES

Achievement Level	Grade 7		Grade 8	
	ML	CD	ML	CD
High (H)	18	11	20	7
Average (A)	13	13	24	10
Low (L)	13	6	17	13
Total	44	30	61	30

TABLE 18

IL SUBTEST CELL MEANS AND TREATMENT EFFECT F-RATIOS

Subtest		Grade 7 ML	Grade 7 CD	Grade 8 ML	Grade 8 CD
1.	H	4.06	3.73	4.45	4.86
	A	1.92	2.92	4.04	3.60
	L	1.62	1.83	3.06	2.92
		2.53	2.83	3.85	3.79
		F = .86 P = .357		F = .04 P = .840	
2.	H	3.33	2.36	3.60	3.29
	A	2.38	2.77	2.88	2.90
	L	2.31	1.67	2.41	2.77
		2.68	2.27	2.96	2.98
		F = 1.77 P = .188		F = .01 P = .932	
3.	H	3.94	4.27	4.55	4.71
	A	2.77	4.15	4.00	4.60
	L	2.92	3.83	3.18	4.08
		3.21	4.09	3.91	4.46
		F = 7.12 P = .010		F = 6.25 P = .014	
4.	H	2.72	2.45	3.25	3.29
	A	2.08	2.46	2.29	2.70
	L	1.54	1.50	2.00	2.31
		2.11	2.14	2.51	2.76
		F = .01 P = .924		F = 1.17 P = .282	
5.	H	2.44	3.45	3.65	3.42
	A	1.38	3.31	2.50	2.10
	L	1.00	3.00	2.00	2.46
		1.61	3.25	2.72	3.00
		F = 31.36 P = .000		F = .90 P = .347	

Subtest		Grade 7 ML	Grade 7 CD	Grade 8 ML	Grade 8 CD
6.	H	1.22	1.36	1.60	2.86
	A	.38	1.00	1.46	2.30
	L	.31	1.17	.71	1.23
		.64	1.18	1.25	2.13
		F = 3.32 P = .073		F = 10.77 P = .002	
7.	H	2.94	3.82	3.45	5.00
	A	1.31	3.08	3.25	3.60
	L	1.23	3.17	2.88	3.23
		1.83	3.35	3.19	3.94
		F = 16.87 P = .000		F = 5.25 P = .024	
8.	H	3.61	3.63	4.00	2.86
	A	2.38	3.46	3.50	2.30
	L	1.77	2.67	2.65	1.08
		2.59	3.25	3.38	2.08
		F = 3.94 P = .051		F = 29.52 P = .000	
9.	H	2.11	2.45	2.85	2.14
	A	1.23	2.00	2.08	2.00
	L	1.08	1.50	1.18	1.31
		1.47	1.98	2.04	1.82
		F = 4.39 P = .040		F = .58 P = .450	
10.	H	3.94	3.27	4.25	4.29
	A	2.15	3.23	3.83	3.20
	L	1.31	2.33	3.18	2.23
		2.47	2.95	3.75	3.24
		F = 1.74 P = .192		F = 4.27 P = .042	

TABLE 19

IL TOTAL TEST CELL MEANS

Achievement Level	Grade 7		Grade 8	
	ML	CD	ML	CD
High (H)	30.33	30.82	35.65	36.71
Average (A)	18.00	28.38	29.83	30.30
Low (L)	14.92	22.67	23.24	23.62
Group Mean	21.09	27.29	29.57	30.21

TABLE 20

ANALYSIS OF VARIANCE OF IL TEST SCORES

	Source	df	MS	F	P
Grade 7	Achievement	2	783.07	24.39	.000
	Treatment	1	636.97	19.84	.000
	Interaction	2	172.45	5.37	.007
	Within	68	32.10		
Grade 8	Achievement	2	1007.47	38.51	.000
	Treatment	1	7.75	.30	.588
	Interaction	2	.75	.03	.972
	Within	85	26.16		

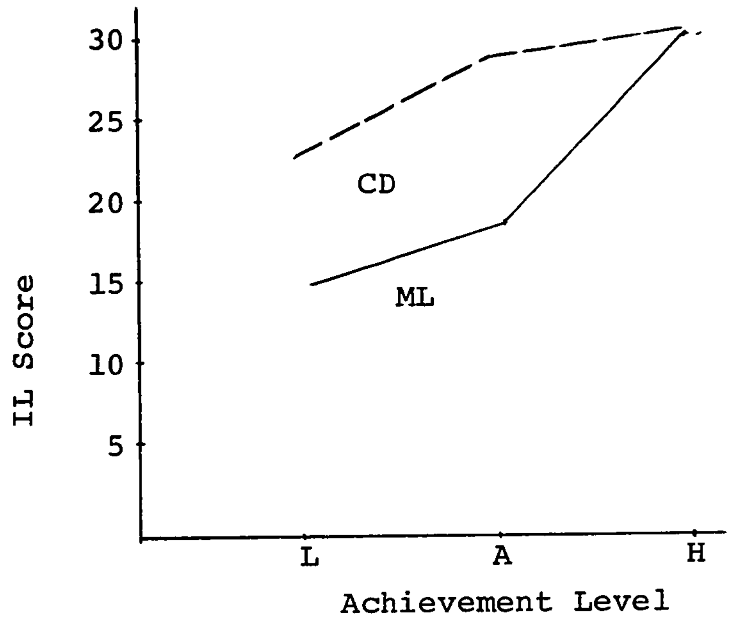


Figure 1

Profiles of Simple Effects
of Achievement Levels for Treatments
(Grade 7 IL Test)

At the Grade 8 level no significant interaction or treatment effects were found for the total test.

Conclusions

Grade 7. Null hypothesis 2(a) was rejected for subtests 3, 5, 7, and 9. On these subtests the CD students scored significantly higher than the ML students. For the total test there was no significant difference between group mean scores at the high achievement level, but at the average and low levels of achievement the CD students performed significantly better than the ML students.

Grade 8. Null hypothesis 2(b) was rejected for subtests 3, 6, 7, 8, and 10. Differences on subtests 3, 6, and 7 favored the CD group while the ML group was superior on subtests 8 and 10. No significant difference between the two groups was found on the total test scores, and there were no differential treatment effects across achievement levels.

III. Cumulative Achievement in the Experimental Program

Hypothesis 3 Restated

At the (a) Grade 7 level (b) Grade 8 level, there are no significant differences among group mean scores obtained by the ML, CD, and CON students on the CA test.

Test Procedure

At each grade level CA test scores of students in the three groups were analyzed using two-way analysis of variance.

Results

Cell means for the CA test are listed in Table 21. There were a possible 40 points on this test. The cell means for each of the subtests are presented in Appendix D.

TABLE 21

CA TEST CELL MEANS

	Grade 7			Grade 8		
	ML	CD	CON	ML	CD	CON
H	15.45	16.59	10.77	21.46	22.29	15.00
A	10.25	11.70	8.10	14.19	15.05	11.64
L	7.95	10.15	8.50	10.19	12.47	10.00
	11.22	12.81	9.12	15.28	16.60	12.22

The analysis of variance performed on these scores is summarized in Table 22.

Grade 7. There was no significant interaction, but a significant treatment effect was observed. The treatment group means were compared two at a time using Scheffé's method. The results of these pairwise comparisons were as summarized below.

	F	P
ML vs CD	2.40	.094
ML vs CON	3.88	.023 (ML > CON)
CD vs CON	10.89	.000 (CD > CON)

TABLE 22

ANALYSIS OF VARIANCE OF CA SCORES

	Source	df	MS	F	P
Grade 7	Achievement	2	419.58	27.18	.000
	Treatment	2	168.58	10.92	.000
	Interaction	4	30.91	2.00	.097
	Within	160	15.44		
Grade 8	Achievement	2	823.73	41.34	.000
	Treatment	2	212.67	10.67	.000
	Interaction	4	49.24	2.47	.047
	Within	166	19.93		

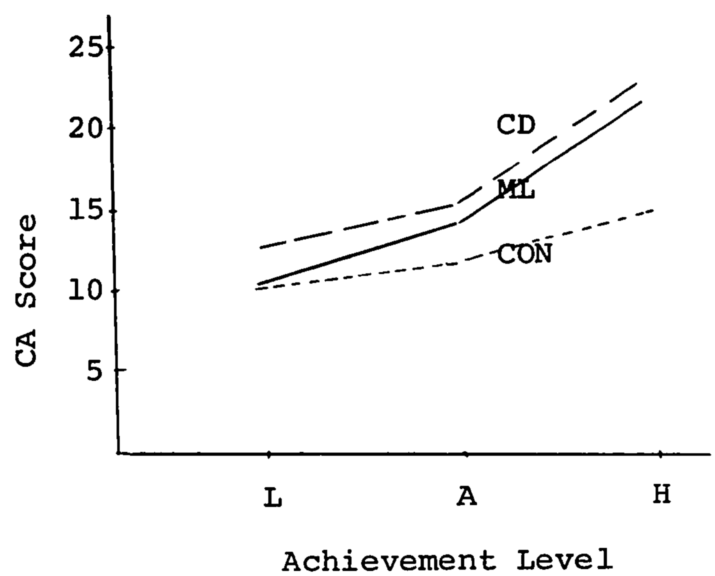


Figure 2

Profiles of Simple Effects
of Achievement Levels for Treatments
(Grade 8 CA Test)

Grade 8. Since a significant interaction effect was observed (Figure 2), treatment effects were examined at each achievement level. The results of these analyses are summarized below.

Achievement Level	F	P
High	10.24	.000
Average	3.16	.049
Low	2.10	.133

Since significant F-ratios were observed for the high and average levels of achievement, Scheffé's procedure for making multiple comparisons was carried out. The results of these analyses are summarized in Table 23.

TABLE 23

PROBABILITIES FOR SCHEFFE MULTIPLE COMPARISON OF MEANS -- GRADE 8 CA TEST

Comparison	Achievement Level	
	High	Average
ML vs CD	.932	.778
ML vs CON	.001 (ML > CON)	.147
CD vs CON	.009 (CD > CON)	.060 (CD > CON)

Conclusions

Grade 7. Hypothesis 3(a) was rejected. Significant differences were observed among the treatment groups on the CA test. There were no significant differences between the mean scores of the ML and CD groups, but both of these groups scored significantly higher than the CON group.

Grade 8. At the high achievement level the ML and CD students were statistically superior to the CON students. No significant differences were found between the ML and CD group means at this level. At the average achievement level an overall treatment effect was observed, but no significant differences between pairs of means were found. However, the trend was the same as at the high level. No significant differences among the three treatment groups were indicated at the low achievement level, although the CD students scored higher than students from the other two groups.

IV. Higher Level Thinking and Problem Solving

Hypothesis 4 Restated

At the (a) Grade 7 level (b) Grade 8 level, there are no significant differences among group mean scores obtained by the ML, CD, and CON students on the HLTPS test.

Test Procedure

Two-way analysis of variance was used to test this hypothesis at each grade level.

Results

Tables 24 and 25 contain the data which were analyzed to test the above hypothesis and a summary of the analysis of variance performed on the HLTPS test scores. This test

TABLE 24

HLTPS TEST CELL MEANS

	Grade 7			Grade 8		
	ML	CD	CON	ML	CD	CON
H	10.60	13.71	7.38	17.75	18.38	13.05
A	6.09	8.65	5.78	10.32	11.60	9.71
L	4.32	4.54	2.69	6.48	8.00	5.88
	7.00	8.97	5.28	11.52	12.66	9.54

TABLE 25

ANALYSIS OF VARIANCE OF HLTPS TEST SCORES

	Source	df	MS	F	P
Grade 7	Achievement	2	569.18	45.85	.000
	Treatment	2	166.30	13.40	.000
	Interaction	4	26.54	2.14	.079
	Within	157	12.41		
Grade 8	Achievement	2	971.14	45.90	.000
	Treatment	2	97.23	4.60	.011
	Interaction	4	29.03	1.37	.246
	Within	162	21.16		

had a possible 40 points. Subtest cell means are given in Appendix D. As indicated in Table 25, at both grade levels the treatment-achievement level interaction was not significant but significant treatment effects were found. The results of comparing group mean scores two at a time by Scheffé's method are now presented.

	Grade 7		Grade 8	
	F	P	F	P
ML vs CD	4.47	.013 (CD > ML)	.80	.450
ML vs CON	3.21	.043 (ML > CON)	2.38	.095
CD vs CON	1.33	.000 (CD > CON)	4.44	.013 (CD > CON)

Conclusions

Grade 7. The CD group was statistically superior to both the ML and CON groups on the HLTPS test. The ML students scored significantly higher than the CON students. There was no significant differential treatment effects across grade levels.

Grade 8. There was no significant difference between the ML and CD students on the HLTPS test. The CD group scored significantly higher than students in the CON classes. The ML group scored higher than the CON group but the difference was not significant at the .05 level. No significant differential treatment effects across achievement levels were found.

V. Divergent Thinking in Mathematics

The Cards Uses in Mathematics test (CUM) was used to investigate two questions:

1. Did the ML or CD students produce more correct responses than the CON students?

2. Were the responses of the ML and CD students of a different nature than those made by the CON students.

Hypothesis 5 Restated

At the (a) Grade 7 level (b) Grade 8 level, there are no significant differences among group mean scores obtained by the ML, CD, and CON students on the CUM test.

Test Procedure

Two-way analysis of variance was performed on the test scores of the three groups at each grade level.

TABLE 26

CUM TEST CELL MEANS

	Grade 7			Grade 8		
	ML	CD	CON	ML	CD	CON
H	1.95	1.95	1.23	2.54	4.13	2.55
A	1.59	1.40	1.20	1.80	2.10	1.00
L	.86	1.38	1.13	1.39	1.32	1.18
	1.47	1.58	1.19	1.91	2.51	1.58

TABLE 27

ANALYSIS OF VARIANCE OF CUM TEST SCORES

	Source	df	MS	F	P
Grade 7	Achievement	2	4.29	4.24	.016
	Treatment	2	1.97	1.95	.146
	Interaction	4	1.40	1.38	.243
	Within	157	1.01		
Grade 8	Achievement	2	38.86	17.81	.000
	Treatment	2	9.27	4.25	.016
	Interaction	4	3.50	1.60	.176
	Within	160	2.18		

Results

The data which were analyzed to test Hypothesis 5 are presented in Table 26. Table 27 summarizes the analysis of variance of the CUM scores. Interaction effects were not significant for either the Grade 7 or Grade 8 analysis.

Grade 7. Although the group mean scores of both the ML and CD students were higher than the CON group mean score, the observed F-ration was not significant. Therefore, the null hypothesis was not rejected.

Grade 8. Hypothesis 5(b) was rejected and comparisons between pairs of group means were made using Scheffé's procedure. The results were as follows:

	F	P
ML vs CD	2.16	.118
ML vs CON	.69	.501
CD vs CON	4.13	.018 (CD > CON)

Conclusions

For both the Grade 7 and Grade 8 samples the CD students obtained the highest group mean score on the CUM test. The ML students obtained the next highest group mean score and the lowest group mean score was obtained by the CON students. However, the only significant difference between group means was at the Grade 8 level where it was found that the CD students were statistically superior to the CON students.

Responses of ML, CD, and CON Students

Students' responses to the CUM test which were judged acceptable were grouped into 17 categories according to the physical properties of the cards which would be utilized and the mathematical ideas involved. The categories are described in Appendix C. Table 28 lists the number of responses in each category made by the ML, CD, and CON groups.

TABLE 28

NUMBER OF RESPONSES ON CUM TEST

Category	Grade 7			Grade 8			Combined			Total
	ML	CD	CON	ML	CD	CON	ML	CD	CON	
1	30	22	26	44	24	34	74	46	60	180
2	15	15	6	9	8	9	24	23	15	62
3	4	3	3	6	2	0	2	6	0	8
4	0	1	0	2	5	0	2	6	0	8
5	16	11	6	15	15	23	31	26	29	86
6	5	5	6	4	7	1	9	12	7	28
7	1	1	0	3	0	0	4	1	0	5
8	0	0	0	4	3	4	4	3	4	11
9	4	3	1	14	9	5	18	12	6	36
10	2	2	0	7	2	0	9	4	0	13
11	2	3	0	15	9	0	17	12	0	29
12	3	4	0	9	5	3	12	9	3	24
13	0	0	0	1	1	2	1	1	2	4
14	14	16	11	16	14	1	30	30	12	72
15	1	0	1	1	2	0	2	2	1	5
16	0	0	0	0	0	1	0	0	1	1
17	0	1	0	0	0	0	0	1	0	1
Total	97	87	60	150	106	83	247	193	143	583
n	64	54	48	77	47	45	141	101	93	335

The pattern of responses observed in certain categories is now discussed.

Category 1. This category consisted of low quality responses, mainly statements involving numerals (for example, 3 + 4 = 7) or unexplained references to mathematical ideas taken from the example provided (for example, prime, commutative law). Comparing the three groups it is seen that the percentages of acceptable responses made by the ML, CD, and CON students which fell into this category were 30, 24, and 42 respectively.

Category 3 (Numeration). Of the eight responses in this category, two were made by ML students and six by CD students. Activity 6 of the experimental program dealt with numeration in base three and base five.

Category 9 (Sets). The numbers of responses made in this category by the 141 ML students, the 101 CD students, and the 93 CON students were 18, 12, and 6 respectively. Sets were referred to in several of the experimental lessons.

Category 10. This category consisted of the problem investigated in Activity 5, that of finding all possible subsets or the number of all possible subsets of a given set. All of these responses came from students in the ML and CD groups (nine and four, respectively).

Category 11. This category corresponded to Activity 2 and concerned the problem of finding the number of elements in the union and intersection of two intersecting sets. No responses in this category were made by CON students, but about 12 percent of the students in both the experimental groups suggested this problem.

Category 12 (Area and Perimeter). These concepts were discussed in three of the experimental lessons. Three responses were made in this category by CON students as compared with 12 by the ML students and 9 by the CD students.

Category 14 (Probability). In Activity 8 students performed experiments and were introduced to theoretical ideas in probability. About 21 percent of the ML group and 30 percent of the CD group responded in this category while only 13 percent of the CON students made responses concerning probability. Of the 12 responses made by the CON group, 11 were made by students in one Grade 7 class.

Category 17. A finite mathematical group was investigated in Activity 3. One Grade 8 CD student attempted (rather unsuccessfully) to establish a mathematical system whose elements consisted of the four colors of the cards.

Conclusions

Many of the responses made by ML and CD students were in terms of problems and ideas they had encountered during the experimental lessons. They appeared to be able to relate these ideas which had been introduced to them in a variety of physical settings to the test situation.

Pretesting Effects on Attitude Measures

To determine the effects of pretesting on the AMS scale and LDM subscale posttest scores, students in half the classes in each treatment group were given the AMS pretest but did not take the LDM pretest. The other half of the students took the LDM pretest but did not take the AMS pretest. All students took both posttests. For the AMS and the three subscales of the LDM scale the following hypothesis was tested:

> There are no significant main or interactive effects due to pretesting on posttest scores.

Data for testing these hypotheses and the results of the analysis of variance performed on the posttest scores are summarized in Tables 29 and 30. Only those students for whom both pre- and post-AMS scores or both pre- and post-LDM scores were available were included in the sample for these analyses. One low-ability Grade 7 class which had taken the LDM pretest was dropped from the sample due to the relatively high responses made by these students to items on this scale during the pretest.

As indicated in Table 30 there were no significant test or treatment effects for the AMS scale or the LDM Enjoyment subscale, but interaction effects for both these measures were significant. This would indicate differential treatment effects due to pretesting on the treatment groups. However, examination of the cell means of the two groups of students suggests that the interaction was due not to the testing but to the nature of the pretested and

TABLE 29

CELL FREQUENCIES AND MEANS OF PRETESTED AND UNPRETESTED GROUPS ON AMS AND LDM SCALES

	ML	CD	CON	ML	CD	CON
	AMS			LDM-Enjoyment		
Pretested	(64) 71.23	(46) 75.76	(40) 76.15	(50) 45.28	(46) 41.37	(47) 39.70
Unpretested	(50) 77.72	(46) 71.87	(47) 71.43	(64) 40.45	(46) 45.17	(40) 46.30
	LDM-Familiarity			LDM-Situation		
Pretested	17.58	16.78	17.02	8.98	8.24	7.64
Unpretested	16.98	18.46	17.70	8.86	8.07	5.85

TABLE 30

AMS AND LDM PRETESTING EFFECTS: PROBABILITIES OF F-RATIOS

Source	AMS	LDM		
		Enjoyment	Familiarity	Situation
Testing	.652	.123	.283	.046
Treatment	.918	.960	.869	.000
Interaction	.004	.000	.215	.094

unpretested groups. It is seen in Table 29 that the test-treatment interaction for the two measures is in the opposite direction. Since the two scales should be measures of the same attribute, it was concluded that the observed testing interactions were likely a result of group characteristics.

No main or interactive effects due to testing were observed for the LDM Familiarity subscale. For the LDM Situation subscale the interaction effects were not significant, but there were significant testing effects. Examination of cell means indicates that the pretested group scored higher than the unpretested group on the posttest. The treatment effect was also significant, and this was further investigated using analysis of covariance (Section VII).

VI. Attitude Toward Mathematics

Hypothesis 6 Restated

There are no significant differences among adjusted group mean attitude scores obtained by ML, CD, and CON students on the AMS.

Test Procedure

Hypothesis 6 was tested using analysis of covariance of AMS posttest scores, pretest scores being the covariate.

Results

Table 31 shows the pretest and posttest mean AMS scores for the three groups, and Table 32 summarizes the analysis of variance performed on these scores. As seen in Table 31 the mean attitude scores of the ML and CD groups remained constant over the three month experimental period while the CON group mean attitude score decreased slightly.

TABLE 31

TREATMENT GROUP MEAN AMS PRETEST AND POSTTEST SCORES

Group	N	Pretest	Posttest	
			Unadjusted	Adjusted
ML	68	70.08	70.97	73.93
CD	46	76.39	76.35	74.81
CON	40	48.80	75.97	72.71

TABLE 32

ANALYSIS OF COVARIANCE OF AMS POSTTEST SCORES

Source	df	MS	Adj F	P
Group	2	46.84	.55	.576
Within	150	84.65		

Conclusions

Hypothesis 6 was not rejected. No significant differences among adjusted group mean attitude posttest scores as measured by the AMS were found.

VII. Student View of Mathematics as a Discipline of Study

Hypothesis 7 Restated

There are no significant differences among adjusted mean scores obtained by ML, CD, and CON students on the three subscales of the LDM scale.

Test Procedure

For each of the three subscales, posttest scores were subjected to analysis of covariance, using corresponding pretest scores as the covariate.

Results

Table 33 summarizes the data for comparing the three groups on the three subscales of the LDM scale. As shown in Table 33 the adjusted mean of the ML group was greatest, the adjusted mean of the CD group next highest, and the adjusted mean of the CON group lowest for each of the three measures. The analysis of covariance, summarized in Table 34, revealed that treatment effects were significant for the Situation subscale but were not significant for the Enjoyment subscale or the Familiarity subscale. The adjusted treatment means for the Situation subscale were compared two at a time. The results of these comparisons are now presented:

	F	P
ML vs CD	.25	.776
ML vs CON	3.61	.030 (ML > CON)
CD vs CON	1.81	.168

TABLE 33

MEAN LDM PRETEST AND POSTTEST SCORES

Group	N	Pretest	Posttest Unadjusted	Posttest Adjusted
		Enjoyment Subscale		
ML	50	43.34	45.28	44.36
CD	46	40.00	41.37	42.11
CON	47	40.98	39.70	39.96
		Familiarity Subscale		
ML	50	17.12	17.58	17.58
CD	46	16.76	16.78	16.95
CON	47	17.45	17.02	16.87
		Situation Subscale		
ML	50	7.70	8.98	8.95
CD	46	6.96	8.24	8.53
CON	47	8.21	7.64	7.39

TABLE 34

ANALYSIS OF COVARIANCE OF POSTTEST LDM SCORES

Source	df	MS	Adj F	P
		Enjoyment Subscale		
Group	2	232.14	2.82	.063
Within	139	82.23		
		Familiarity Subscale		
Group	2	7.39	.41	.666
Within	139	18.14		
		Situation Subscale		
Group	2	31.03	3.80	.025
Within	139	8.17		

Conclusions

Hypothesis 7 was not rejected for the LDM Enjoyment or Familiarity subscales but was rejected for the Situation subscale. On each of the subscales the highest adjusted group means were obtained by the ML students and the lowest group means were obtained by the CON students. The differences among adjusted group mean scores on the Enjoyment and Familiarity subscales were not significant. The ML students rated significantly higher on the Situation subscale than the CON students. No significant differences were observed on this measure between the ML and CD students or the CD and CON students.

VIII. Student Questionnaire Findings

Comparisons Between the ML and CD Responses

Seventeen items on the two forms of the student questionnaire (SQ) were designed to permit a comparison of the feelings of the ML and CD students toward the experimental program. These items related either to attitudes toward the experimental activities or to opinions concerning benefits derived through having participated in the program.

Hypothesis 8(a) restated. For the first 17 items on the student questionnaire there is no significant relationship between treatment group and response to the item.

<u>Test procedure</u>. Chi-square tests were made on the number of "Agree", "Uncertain", and "Disagree" responses made by the ML and CD students to each item.

<u>Results</u>. Table 35 gives the percentages of "Agree" (A), "Uncertain" (U), and "Disagree" (D) responses for the two groups on each of the 17 items, the value of chi-square (χ^2) testing the relationship between treatment and response, and the probability of the observed chi-square.

TABLE 35

PERCENTAGES AND SIGNIFICANCE OF RESPONSES OF ML AND CD STUDENTS TO QUESTIONNAIRE ITEMS

Item			Percentages of Responses		
			A	U	D
	Attitudes Toward the Experimental Activities				
1.	I enjoyed the ML/CD periods.	* ML	86	8	6
		** CD	70	24	6
		χ^2 = 13.7; P = .001			
5.	The ML/CD periods provided a "welcome break from routine".	ML	86	10	4
		CD	64	22	14
		χ^2 = 16.1; P = .000			
3.	During the ML/CD periods I felt that I could learn without pressure.	ML	74	16	10
		CD	55	39	6
		χ^2 = 17.1; P = .000			
8.	I liked having concrete materials to refer to in solving problems in mathematics.	ML	79	18	3
		CD	62	28	10
		χ^2 = 9.1; P = .010			
11.	I found the ML/CD periods more interesting than work from the text.	ML	85	10	5
		CD	72	19	9
		χ^2 = 5.6; P = .062			

TABLE 35 (Continued)

Item			Percentages of Responses		
			A	U	D
16.	I would rather do textbook exercises than investigate problems based on concrete materials.	ML	9	12	79
		CD	12	21	67
		χ^2 = 5.3; P = .071			
9.	I was often bored during the ML/CD periods.	ML	13	17	70
		CD	24	27	49
		χ^2 = 11.1; P = .004			
13.	I sometimes felt I was wasting my time during the ML/CD periods.	ML	20	15	65
		CD	20	28	52
		χ^2 = 6.87; P = .034			
15.	Later ML/CD lessons were not as interesting as the earlier ones.	ML	21	19	60
		CD	32	29	39
		χ^2 = 10.8; P = .005			
	Opinions Concerning Benefits from the Programs				
2.	The ML/CD periods increased my mathematical knowledge.	ML	47	49	4
		CD	52	38	10
		χ^2 = 4.6; P = .098			
4.	The ML/CD lessons helped me to better understand my regular math course.	ML	43	42	15
		CD	47	28	25
		χ^2 = 5.8; P = .056			
10.	During the ML/CD periods I learned material for later grades.	ML	33	60	7
		CD	41	50	9
		χ^2 = 2.3; P = .318			
6.	I'm not sure what I learned from the ML/CD lessons.	ML	13	35	52
		CD	11	37	52
		χ^2 = .4; P = .822			
12.	The ML/CD lessons broadened my view of mathematics.	ML	44	42	14
		CD	41	40	19
		χ^2 = 1.0; P = .599			

TABLE 35 (Continued)

Item			Percentages of Responses		
			A	U	D
14.	I learned how to investigate mathematical problems independently.	ML	58	33	9
		CD	35	50	15
		χ^2 = 12.7; P = .002			
7.	I now find math more interesting than I used to.	ML	35	45	20
		CD	37	39	24
		χ^2 = .9; P = .632			
17.	I feel that my attitude toward mathematics has improved over the past three months.	ML	55	35	10
		CD	42	41	17
		χ^2 = 4.8; P = .093			

* n = 144
** n = 102

Conclusions. Hypothesis 8(a) was rejected for items 1, 5, 3, 8, 9, 13, 14, and 15. For these items a significant relationship between treatment and student opinion was found to exist. Inspection of the percentages of students in the two groups who agreed and disagreed with these and other statements suggests that the ML students responded more favorably to their experiences in the experimental program than the CD students.

Comparisons Within the ML and CD Groups

The percentages of students who responded in the three categories to the additional SQ items for the ML CD students are given in Table 36. The relationship between the students' grade, teacher, sex,and achievement level and their responses to each of the items on the questionnaire was investigated.

Hypothesis 8(b) restated. Within the ML and CD groups there are no significant relationships between the responses to the SQ items and the following variables: (i) grade, (ii) teacher, (iii) sex, and (iv) achievement level.

Test procedure. The above hypotheses were tested using the chi-square statistic.

Results. The data for testing the above hypotheses and the probabilities of the observed chi-square values are presented in Appendix G. Of the 100 tests made for the ML students, five values of chi-square were found to be significant at the .05 level, and of the 80 tests made for the CD students, five significant values of chi-square were observed. This many significant results would be expected by chance. Only on Item 2 were significant relationships found within both ML and CD groups when the subjects were classified according to a particular variable. In this case, a significant relationship between achievement level and a belief that the experimental program had increased the student's mathematical knowledge was observed for both the ML and CD groups.

TABLE 36

RESPONSES TO ADDITIONAL STUDENT QUESTIONNAIRE ITEMS

Item		Percentages of Responses		
		A	U	D
	ML Items			
19.	I enjoyed working in a small group.	83	10	7
23.	Working with a partner helped me to better understand the activities because we discussed them together.	74	12	14
21.	I used the concrete materials to check my ideas and answers.	73	21	6
22.	In the lab I felt that I could work at my own speed.	72	9	19
20.	I liked working from written instructions rather than having a teacher give me directions.	54	29	17
18.	I think I would have learned more if the teacher had taught the class as a group and demonstrated with the concrete materials.	22	26	52
24.	It was sometimes too noisy in the lab to concentrate.	46	19	35
25.	After several lab sessions I became more confident in my abilitiy to do the activities without asking the teacher for help.	53	37	10
	CD Items			
18.	During the special lessons the same students usually made all the discoveries.	43	25	32

TABLE 36 (Continued)

Item		Percentages of Responses		
		A	U	D
19.	During the special lessons there was usually more noise and activity than during regular math lessons.	44	28	28
20.	I would have preferred studying these lessons with a partner in the math lab to taking them with the rest of the class from the teacher.	49	28	23

Conclusions: The reaction of students within the ML and CD groups to the experimental program was generally not related to grade, teacher, sex or achievement level. On the other hand, significant relationships were observed between the reaction of the students to the program and the experimental group to which they belonged.

ML Student Questionnaire: Part B

1. With respect to the use made of the concrete materials in the laboratory, 56 percent of the students said they used it throughout each lesson to find and check answers, 39 percent said they used it only to understand the problem and to begin finding answers, and 5 percent said they needed the materials only to determine the nature of the problem to be considered.

2. In regard to the kind of written instructions wanted by students, 46 percent of the respondents preferred

that directions be more specific and complete than those provided, 37 percent indicated that the instructions used were adequate, and 17 percent said they would rather not have to follow specific instructions but be allowed to work with the material as they wished.

3. With regard to grouping for laboratory work, 17 percent of the students said they would prefer to work alone and 4 percent indicated they would rather work in a teacher-directed class situation. Of the 79 percent who preferred the partner system, 45 percent said they would want to choose their own partner while 55 percent were satisfied with having the teacher assign partners.

Most and Least Liked Experimental Activities

Students in the ML and CD groups were asked to indicate which two lessons they had enjoyed the most and which two they had least enjoyed. Information obtained on this question is summarized in Table 37. Not all students gave two responses in each category.

TABLE 37

MOST AND LEAST LIKED ACTIVITIES IN THE EXPERIMENTAL PROGRAM

Lesson Number	Number of Times Selected			
	ML (n = 144)		CD (n = 102)	
	Most	Least	Most	Least
1	9	27	19	27
2	23	23	22	10
3	12	43	21	27
4	13	21	15	14
5	5	23	14	14
6	80	10	24	7
7	30	28	19	16
8	92	2	36	8
9	4	53	8	32
10	4	35	7	24
Totals	272	265	185	179

The activities which were most popular in both settings were Number 8, Probability, and Number 6, Mathematical Balance. The least popular lesson for both groups was Number 9, Measurement of a Circle.

IX. Teacher Questionnaire Findings

The purpose of the teacher questionnaire was to determine the teachers' reaction to and evaluation of various aspects of the experimental program in the two settings. Following is a summary of the findings:

1. The teachers felt that the ML setting was more successful for students of high and average ability, but that the CD setting was better for the low ability students.

2. Two of the three teachers indicated that they had preferred their role in the laboratory to their classroom role and that student behavior was easier to control in this situation than in the Class Discovery setting.

3. With respect to the laboratory program the teachers reported that:

   (a) in most classes student interest remained at a high level throughout the experiment,

   (b) students seemed to require and to request less assistance during the final weeks of the study than during the earlier laboratory periods,

   (c) many students, especially those of lesser ability, developed a tendency to hurry through certain sections of lessons without making a real effort to understand the instructions and carry out the indicated activities or exercises.

4. In regard to the class time used for the experimental program, the teachers said that time spent on review, checking assignments, and individual help was reduced and that this seemed to affect mainly students of low ability.

5. Suggestions for further use of mathematics laboratories were:

   (a) Use activities and materials in connection with topics in the regular program.

   (b) Use a laboratory program as an academic elective.

   (c) Use the laboratory once a week for special enrichment activities.

X. Chapter Summary

In this chapter the three groups were compared at the seventh and eighth grade levels with respect to textual achievement, learning associated with the experimental program, and attitudes toward mathematics learning. In addition, student and teacher reaction to the two instructional settings for the experimental program were obtained. Analysis of the data indicated the following conclusions and trends:

1. There were no differences among the three groups in achivement in the regular mathematics program.

2. On measures of learning based on the experimental materials both the ML and CD students scored higher than the CON students. The CD group generally performed slightly better on these tests than the ML group.

3. Attitudes of both the ML and CD groups toward various aspects of mathematics study were generally higher than the corresponding attitudes of the CON group, although differences in most cases were not significant.

Table 38 summarizes the results of the achievement and attitude measures.

TABLE 38

SUMMARY OF RESULTS OF POSTTEST MEASURES

Test	Grade 7			Grade 8		
	ML	CD	CON	ML	CD	CON
Achievement in Regular Program (Unweighted means)						
STM Posttest	24.59	24.72	26.49	23.25	23.91	24.79
STM Quizzes	63.17	61.26	65.02	76.58	79.85	76.90
Learning in Experimental Program (Unweighted means)						
Immediate Learning	21.09	27.29	---	29.57	30.21	---
Cumulative Achievement	11.22	12.81	9.12	15.28	16.60	12.22
Transfer	7.00	8.97	5.28	11.52	12.66	9.54
Divergent Thinking	1.47	1.58	1.19	1.91	2.51	1.58

Attitude Measures (Adjusted means)

	ML	CD	CON
A Mathematics Study (Attitude)	73.93	74.81	72.71
Learning and Doing Mathematics			
Enjoyment	44.36	42.11	39.96
Familiarity	17.58	16.95	16.87
Situation	8.95	8.53	7.39

4. The ML students' reaction to the experimental program was generally better than the reaction of the CD students, although both groups responded favorably to the program.

5. The teachers reacted positively to the experiment in general and to the laboratory instructional setting in particular.

CHAPTER V

SUMMARY, CONCLUSIONS, RECOMMENDATIONS, AND IMPLICATIONS

I. Purpose and Design of the Study

The major purpose of the study was to investigate various aspects of learning in a mathematics laboratory. A mathematics laboratory program comprising ten activity lessons and designed to operate as an adjunct to modern courses in the seventh and eighth grades was developed by the investigator. One mathematics period a week for ten weeks students assigned to the Mathematics Laboratory group went to the laboratory where they worked in pairs from written instructions on one of the lessons. Each activity was designed to lead students to the discovery of a new concept or relationship through the manipulation of some kind of concrete material.

To assess the relative effectiveness of this method of instruction the same lessons were also presented on a once-a-week basis in a teacher-directed class setting to students in the Class Discovery group. Students in the Control group continued to study the regular mathematics program, Seeing Through Mathematics (STM), the full time allotted for instruction in the subject.

The subjects for the experiment were students from seven Grade 7 and seven Grade 8 classes at Westlawn Junior High School, Edmonton, Alberta. At each grade level three classes were assigned to the Mathematics Laboratory (ML) group and two each to the Class Discovery (CD) and control (CON) groups. Each of the three mathematics teachers at the school had one class in each grade in the laboratory group and taught the discovery lessons to one other class. The investigator taught these lessons to the fourth class in the Class Discovery group.

Instruments were developed to gather data on a variety of aspects of the experimental program to determine what benefits, if any, accrued to students as a result of their laboratory experiences. The experiment was designed to obtain answers to the following questions:

1. How was achievement in the regular mathematics program affected by using 25 percent of the time normally devoted to it for informal investigation of new problems and concepts suggested by physical materials?

2. What was the relative effectiveness of the Mathematics Laboratory and Class Discovery settings in teaching new ideas?

3. To what extent were students able to retain concepts encountered during the experimental program?

4. To what extent were students able to apply techniques and concepts studied in the experimental program to new or more complex problem situations?

5. How would students from the three groups respond to a test situation in which they were to relate the study of mathematics to a set of physical materials?

6. Did student attitudes toward mathematics change as a result of participating in the experiment?

7. Were there any differences among the three groups in the feelings of the students about mathematics as a discipline of study?

8. How did the Mathematics Laboratory and Class Discovery students react to the experimental program?

9. How did the teachers feel about the two instructional settings?

On the basis of scores obtained by the subjects on STM pretests, high (H), average (A), and low (L) achievement categories were established at each grade level. Major hypotheses were tested separately for the seventh and eighth grade samples using a treatment by achievement level factorial design. Other hypotheses were tested using analysis of covariance and the chi-square statistic. The .05 level of significance was used throughout the study.

II. Summary of Results

Analysis of the data obtained during the investigation revealed the following results:

1. There were no significant differences among the three groups at either grade level in achivement in the regular mathematics program as measured by either the STM Posttest or the Combined STM Quizzes.

2. For the Grade 7 sample the CD students scored significantly higher than the ML students on four of the ten Immediate Learning (IL) subtests. No significant differences between the groups were found on the other subtests. For the total IL test there was no significant difference between the two groups at the high achievement level, but for the average and low achievement levels the CD group was statistically superior. At the Grade 8 level the CD students scored significantly higher on three

IL subtests while the ML students performed statistically better on two subtests. For the total IL test there was no significant difference between group mean scores obtained by the eighth grade ML and CD students.

3. At the seventh grade level there was no significant difference between the ML and CD groups on the Cumulative Achievement (CA) test, but both of these groups scored significantly higher than the CON group. These same results were observed for the high achievement level eighth grade students. At the average level the overall treatment effect was significant and although the same trend was observed, no significant differences between pairs of means were found. No significant treatment effect was found at the low achievement level for the Grade 8 sample.

4. For the Grade 7 students the CD group was statistically superior to both the ML and CON groups on the Higher Level Thinking and Problem Solving (HLTPS) test. The ML students also scored significantly higher than the CON students on this test. In Grade 8 there was no significant difference between the scores of the ML and CD students. The CD group scored significantly higher than the CON group. The ML students also scored higher than the CON students on this test, but the difference was not significant.

5. No significant differences on the Cards Uses in Mathematics (CUM) test were found among the three groups at the Grade 7 level. At the Grade 8 level the CD group scored significantly higher than the CON group, but no other differences between group mean scores were significant. An examination of the kinds of responses made by the three groups indicated that the ML and CD students had used many of the ideas encountered in the experimental program.

6. No significant differences were found among adjusted group mean attitude scores obtained by the ML, CD, and CON students on the AMS, although the attitude scores of the CON students declined while the scores of the students in the other two groups were stable over the three month period.

7. No significant differences were found among adjusted group mean scores obtained by the three groups on the LDM Enjoyment or Familiarity subscales. The highest group means on both of these measures were obtained by the ML group followed by the CD group and the CON group. On the Situation subscale the ML students scored significantly higher than the CON students. The CD students also scored higher than the CON students, but the difference was not significant.

8. Student Questionnaire results indicated generally highly favorable student reaction to both the ML and CD programs. Chi-square tests on the responses of the two groups of students to comparable items revealed significant relationships on attitudes toward the experimental program. Inspection of the data suggested that the ML students reacted more favorably to most aspects of the program than the CD students, although the CD students felt that they had learned more mathematics. The most popular feature of laboratory learning appeared to be the opportunity it provided for working independently of the teacher.

9. Results of the Teacher Questionnaire indicated that the three teachers involved in the experiment felt that the program had been successful. They enjoyed their role in the laboratory setting. Comparing the two programs, the teachers felt that the ML approach was better for the high ability students but that the CD setting was more effective in terms of enjoyment and of new concepts gained by students of low ability in mathematics.

III. Conclusions and Discussion

The study was carried out to investigate the effects of a mathematics laboratory program at the seventh and eighth grade levels. To evaluate this program, the laboratory students were compared with students in two other groups -- students who had studied the laboratory lessons in a teacher-directed class situation and students who were not exposed to the experimental lessons but who studied the regular program the full time allotted for mathematics instruction.

Achievement in the Regular Mathematics Program

The use of one of four mathematics periods a week for the informal investigation of new ideas and problems not encountered in the regular Grade 7 and Grade 8 courses did not adversely affect achievement in the regular program over a ten week period. These results are consistent with previous research findings (Ebeid, 1964; Hopkins, 1965). The teachers, who reported that less time was spent on review and drill in the classes involved in the experimental program than in control classes, were of the opinion that the stúdents of low ability were most affected by the reduced time. Further, it should be noted that on three of the four measures of achievement in the regular program the control students scored higher than students in the two experimental programs.

Immediate Learning in the ML and CD Settings

Except for seventh grade students of average and low achievement in mathematics, understanding of the concepts in the experimental program as measured by the review exercises was as great in the laboratory setting as in the class setting. This finding suggests that eighth grade and high ability seventh grade students were able to learn these ideas as effectively working in small groups using written instructions as they could in a teacher-directed class situation. It is not clear if most of the Grade 7 sample was not mature enough to work on their own or whether the ideas and instructions were not suitable for them. At all achievement levels the Grade 8 students scored higher than the Grade 7 students, but the differences between grade levels were greater within the ML group. Even at the Grade 8 level, certain lessons were learned better in the class situation than in the laboratory. These activities dealt with a mathematical group, solving linear equations, and numeration in different bases. It might be concluded that verbal instructions are more efficient than written instructions in explaining certain ideas and processes. Another possible explanation is that the relationships in these lessons were better learned by observing a demonstration than by manipulating objects.

Cumulative Achievement in the Experimental Program

A test based on the concepts contained in the ten experimental lessons and administered to the three groups at the conclusion of the study indicated that significant learning had occurred in both the ML and CD instructional settings and for both Grade 7 and Grade 8 samples. Test score differences favored the CD group over the ML group although not significantly. Both groups, however, performed significantly better than the CON group on this test.

Although the mean scores of the ML and CD students were low (about 12 in Grade 7 and 16 in Grade 8, out of a possible 40 points), it must be noted that the lessons had been taken, on the average, five and a half weeks prior to the administration of the test and in single 50 minute periods. Furthermore, the concepts to be learned in each lesson were not directly related to each other nor to the regular program, and no followup or review work was provided after a lesson was completed. Learning during the program was not test-oriented.

Item analysis of the CA test indicated that it may have been too long for many students. Attitudes toward the test situation should also be considered. For the ML and CD students, this was a test to see how much they had learned, but the CON students were told that the test was on material they had not studied and that they were simply to do their best. Thus, there might have been an inhibitory effect on ML and CD students. It is also recalled that ML and CD students who had missed up to three of the experimental lessons were included in the sample for this and other analyses.

Higher Level Thinking and Problem Solving

On a test of higher level thinking and problem solving based on the concepts contained in the experimental program, the ML and CD students performed better than the CON students. It was concluded that the ideas and techniques learned during the experimental program aided the ML and CD students in solving the new problems.

As was the case with the CA test, group means on the HLTPS test were lower than the investigator had expected, but again there were indications that students did not have sufficient time to attempt all the questions. Rather they chose to work on selected questions, usually the earlier ones.

An examination of cell means in Tables 21 and 24 reveals that at each achievement level in both grades the CA and HLTPS scores of the CD students were highest, the scores of the ML students were slightly lower, and the CON students' scores were lowest. The differences between the means of the two experimental groups and the mean of the control group are seen to be greatest at the high achievement level. This finding suggests that the mathematical content of the experimental program was most suitable for students of high ability.

Divergent Thinking in Mathematics

A test situation was presented to all students in which they were to relate the study of mathematics to a set of physical materials, in this case 40 "cards" numbered from 1 to 10 in four colors. The mean number of acceptable responses among the three groups did not differ significantly for the Grade 7 students, although both the CD and ML students scored higher than the CON students. In

Grade 8 both experimental groups again scored higher than the control group, with the difference between the CD and CON students being significant. Although the experimental groups did not, in general, produce significantly more correct responses, their responses did reflect their experience in the experimental program. Concepts and problems presented in the ten lessons in terms of various concrete embodiments were listed by the students as being ideas which could be investigated or demonstrated using the cards.

Response to this creativity-type test was generally poor except for students of high ability, particularly those in the CD group. Many classes seemed unable to make any non-trivial responses. Perhaps more information could have been obtained on the question of the students' ability to perceive mathematical ideas in objects of the real world if a more direct testing approach had been employed. For example, the test might have required the subjects to explain how certain concepts could be illustrated in particular concrete settings.

In any case, the laboratory and class discovery experiences appeared to have provided the students with a repertoire of responses for the particular test situation presented.

Attitudes Toward Mathematics

Two instruments were used to investigate the feelings of students in the three groups toward various aspects of mathematics study. Remai's A Mathematics Study (AMS) was designed to assess attitudes toward or interest in learning mathematics. The Learning and Doing Mathematics (LDM) semantic differential scale was used to determine student views on (a) how much they enjoyed mathematics study, (b) how familiar or real they felt its subject matter was, and (c) the situation or tools for investigating mathematical ideas. Pre- and posttest scores were obtained for half the students in each treatment group on the AMS and the other half of the students on the LDM scale.

Data from the AMS indicated that the attitude toward mathematics of students in the ML and CD groups did not change during the three month study, but that the attitude of the CON students decreased slightly. When the posttest scores were analyzed using pretest scores as covariates, no significant differences were found among the three groups. Information obtained from the student and teacher questionnaires revealed that students regarded the experimental program as being distinct from the regular mathematics course, as indeed it was intended to be. Since the same text and methods continued to be used in the regular program, it is perhaps not surprising that the introduction of the laboratory program did not have a marked effect on student attitudes toward learning mathematics.

Neale (1969) suggested that intrinsic motivation will not play a major role in learning until basic changes are made in the school to permit learning to be more flexible and individualized and more independent of teacher domination and evaluation. Evidence supporting this conjecture is suggested by data obtained on Item 17 of the student questionnaire. Percentages of students who responded "Agree", "Uncertain", and "Disagree" to the statement that their attitude toward mathematics had improved during the course of the study were 55, 35, and 10 for the ML group and 42, 41, and 17 for the CD group. Both of these programs had encouraged learning in a more relaxed and less test-oriented environment, but the laboratory setting provided for more individualized instruction and less teacher direction and control.

On the LDM Enjoyment subscale, the group mean scores of the ML and CD students increased during the experiment while the scores of the CON students decreased slightly. The adjusted group mean score of the ML students was highest, followed by those of the CD and CON students. The probability that significant differences existed among the group mean scores was .06. These findings, although by no means conclusive, do suggest that the laboratory program tended to have a positive effect on student feeling that mathematics is fun or enjoyable activity.

On the LDM Familiarity subscale, no significant differences were observed among the scores of students in the three groups although again the highest group mean was obtained by the ML group and the lowest by the CON group.

A significant treatment effect was observed on the LDM Situation subscale. It was found that the ML students rated significantly higher than the CON students on their view that mathematics is a subject which can be investigated in a laboratory situation using real objects rather than being strictly a textbook subject in which symbols are manipulated. The CD group also scored higher than the CON group on this scale, but the difference was not significant. Even though this subscale consisted of only two items and therefore may have low reliability, it appears that the students in the experimental program did obtain a greater understanding of the role of physical materials and laboratory-type investigations in mathematics study.

In summary, these results suggest that laboratory experiences such as those provided in the experimental program developed for this study can give students insights into certain aspects of mathematics and at the same time make its study more enjoyable.

Student Reaction to the Experimental Program

Analysis of student questionnaire returns indicated that while the experimental program had been well received in both settings, the students who had performed the activities in the mathematics laboratory (ML group) reacted more favorably to their experiences than those who had taken the lessons in a class from their teacher (CD group). Eighty-six percent of the students in the ML group and seventy percent in the CD group indicated that they had enjoyed the program. Similarly, a greater proportion of students in the ML group than in the CD group agreed with statements that the periods had provided a welcome break from routine, they (the students) had been able to work without pressure, they found the lessons more interesting than work from the text, and they had learned how to investigate mathematical concepts independently. Further, fewer ML students said that they were often bored or had sometimes felt that they were wasting their time during these lessons. On the other hand, more CD students agreed that these lessons had increased their mathematical knowledge.

Concerning the special features of laboratory learning, 83 percent of the students reported that they liked working in a small group, 74 percent felt that working with a partner had helped them to understand the new ideas, 73 percent said they used the concrete materials to check their ideas and answers, 72 percent felt they had been able to work at their own rate, and 54 percent indicated that they liked working from written instructions rather than taking directions from a teacher.

Comments made by the ML students proved to be both interesting and useful in evaluating the laboratory program. From the students' point of view the most popular feature appeared to be the opportunity it provided for working independently of the teacher. About 40 percent of the students mentioned this aspect. Other features referred to by the students were being able to work with a partner, to use concrete materials, and to work without pressure. Adjectives used to describe laboratory experiences included fun, interesting, enjoyable, and helpful. One low-ability student admitted that he liked it because "you could fool around a little". The following are comments made by students which reflect these feelings:

> I liked the privilege of working at your own speed and without a teacher always telling you what to do. It was fun and helped me a great deal. I think it is better than teaching from the book and is a lot more interesting.

> I liked it because there was no teacher pushing you to do more work. I think the teacher should trust you to do more work (which I did) and I think this worked out fine. Another thing I liked about the lab is being able to use solid materials.

> I liked not having the teacher dictate to you. Almost everyone looked forward to Wednesday.

> I liked where you could find out and prove things yourself so you would know for a fact that something is true.

While the feature of the mathematics laboratory that students seemed to like best was being able to work on their own rather than being taught by the teacher, almost half of them had agreed with the statement that it was sometimes too noisy in the laboratory to concentrate, and many students expressed concern that they did not learn as much as they should have. To remedy this situation they suggested longer periods, closer teacher supervision and direction, and continued work on a topic until it was mastered. But only 22 percent of the respondents to the questionnaire thought they would have learned more if the teacher had taught the lessons, and only four percent indicated that in the future they would prefer taking such lessons in a regular class setting.

Few differences were found among the responses to questionnaire items of students classified according to grade, teacher, sex or achievement level. The following comparisons are worthy of mention, however. Fifty-three percent of the Grade 7 students and 35 percent of the Grade 8 students thought that the laboratory activities had helped them with their regular course. While more boys (63 percent) than girls (45 percent) said they preferred using written instructions, 84 percent of the girls compared with 63 percent of the boys indicated that they had benefited mathematically by discussing the ideas with their partner. One rather surprising finding was that more girls (84 percent) than boys (73 percent) said they liked having concrete materials to refer to in solving problems.

The percentages of high, average, and low achievers who felt that their mathematical knowledge had increased as a result of working in the laboratory were 61, 54, and 26 respectively. The greatest differences among responses of students taught by the three teachers were on items relating to ability to work without pressure and to preference for using written instructions.

The two activities the ML students liked the most were the one involving experiments in probability and the one using a balance arm. The lesson most often listed as that least enjoyed was the one on the circumference and area of a circle. Student preference for the various lessons suggests that they liked those activities which involved using the physical materials throughout the period for collecting data or finding answers and disliked those which required very much thinking or computation.

In the class setting the students named these same three lessons as the two they had enjoyed the most and the one they had least enjoyed. It is noted that the lesson on probability was the only one in which the students in the CD classes were actually able to manipulate real objects. In one class the teacher had permitted the students to form small groups to perform the experiments and collect data. The balance arm seemed to fascinate the students even though they were not allowed to use it individually. It is rather interesting that two of the three teachers named this lesson as being one of the two least successful ones, whereas their students had chosen it as one of the best lessons. The results of the students' choices indicate very strongly that adolescents like class situations in which they can be active, and confirm Davis' (1964c) observation that seventh and eighth grade students are "engineers at heart" rather than intellectuals. An examination of the responses of students within the Class Discovery group taught by teachers 3 and 4 to Items 19, 1, 3, 11, 16, 9, and 20 (Appendix G) sheds further light on this issue. It would appear that the amount of noise and activity perceived during the lessons was directly related to how enjoyable and interesting the students found them and inversely related to preferring a laboratory setting in which to study them. It is interesting to note that while 49 percent of the students in the class setting said that they would have preferred studying these lessons with a partner in the mathematics laboratory, only four percent of the students who had worked in the laboratory indicated they would have preferred a regular class situation.

Comments representing both positive and negative views of the program of discovery lessons were made by students. Many felt that the lessons were interesting and provided an enjoyable change from textbook mathematics. Others, however, could not see that they served any useful purpose. As with ML students, concern was expressed about difficulties in learning and retaining the new concepts. The following comments were made by CD students:

> You find out about math but still have fun doing it.

> Classmates seemed to work harder than in regular class. Different people participated in the lessons rather than the same few in regular class. The lessons were babyish at first and then got harder.
>
> Interesting, but I was not able to remember what I learned each week.
>
> Relief from regular way of doing math but quite useless without notes -- the new ways of figuring out problems are otherwise forgotten.
>
> I disliked the way we just had to sit there and listen in the same desk. I would have liked to work the problems by myself or with a partner.

Teacher Reaction to the Experiment

Like the students the teachers felt that both the laboratory activities and the discovery lessons provided an interesting and worthwhile change from the regular mathematics program. Although they acknowledged that the time was well spent and that the students benefited from the program, the teachers felt that more direction and followup were needed to consolidate the learning that had taken place.

With respect to the role and responsibilities of the teacher in the mathematics laboratory, comments were favorable. Although the lessons and materials had been prepared by the investigator for this study, the teachers suggested ways in which they proposed to utilize laboratory lessons in specific situations and to achieve particular objectives in mathematics courses.

The following statements are comments made by the teachers about the program in the two settings:

Mathematics Laboratory

1. My Grade 7 group liked the lab but tended to look upon it as more of a game situation rather than a learning situation. My Grade 8 class surprised me -- even though they were a low academic group, they seemed to get as much from these lab exercises as from classroom instruction.

2. I found that motivation of the students was a more important factor than intelligence. Students who feel that school is a pain did not do well. Other less intelligent students who concentrated and asked for help did well.

3. The lab method does not make some concepts clear to all students. When the going gets tough the poorer students gave up and left the problem unanswered. Many students were content with doing this work in any way that was the quickest. Both classes enjoyed the lab.

Class Discovery

1. The lessons seemed to be well received. Good student response -- they felt the lessons were not as formal as regular lessons. Many lessons needed more time -- some required less. Students wanted to manipulate the materials when they saw them. Some materials were hard to manipulate in front of a class. I enjoyed most of them.

2. Welcome break from regular drudgery. Generally liked by the class but they did not attach as much importance to them since they were not directly connected to the regular lessons.

3. The lessons were quite stimulating, and it appears that substantial progress was made.

IV. Recommendations

On the basis of this investigation, the writer strongly recommends that mathematics laboratory experiences be provided on a regular basis in Grades 7 and 8. The purpose of such laboratory work should be to allow the students to actively and freely explore new ideas in mathematics. In addition to laboratory instruction in which the students work in small groups with physical materials and from written instructions, it is suggested that teachers make more use of concrete objects and other instructional materials to introduce and motivate new ideas in class teaching situations.

The mathematics laboratory should be used in different ways and to achieve different objectives. First, teachers should carefully examine the regular mathematics programs to determine areas in which laboratory lessons could profitably be used to introduce new concepts or to review previously taught material. Topics such as probability, numeration, measurement, and solid figures are particularly well suited for laboratory study.

Other activites should be specifically designed for the purpose of providing opportunities for students to discover new mathematical ideas and relationships by generalizing from data obtained by manipulating concrete materials. In these lessons the emphasis would be on the process and experience of discovery rather than on the end product. Activities 5 and 10 of the experimental program which lead to the discovery of a rule for determining the number of subsets of a given set and Euler's formula for polyhedra would fall into this category. Finally, it is suggested that certain laboratory periods be provided in which students are free to choose from various topics and materials according to their own level of interest and comprehension. A selection of games and puzzles should be available to create interest in doing mathematics. Although activities should be meaningful and worthwhile, learning should be relatively informal and free of teacher domination and evaluation.

Teachers who have the opportunity to offer an academic option for students who wish further study in mathematics might set up a mathematics laboratory for this course. Under these circumstances, teachers would be relatively free to experiment with a number of kinds of laboratory activities such as those described above and also be able to determine how well their students are able to learn in this kind of instructional setting and the amount of class instruction or individual help needed to obtain maximum results.

Although teachers may wish to develop many of their own lessons (and it is hoped that they will do so), it has been the experience of the writer that the production of laboratory material involves considerable time and effort. Written instructions must often be tried out and revised several times before they can be successfully used by students. Teachers who are willing to write laboratory activities should have some released time, and there should be a central pool for lessons developed within a school system. While the activities prepared by the investigator for the present study dealt mainly with abstract mathematical ideas, lessons involving less sophisticated concepts should be written, especially for students of low ability.

Although the writer feels that the mathematics laboratory can be a useful device for attaining some of the important goals of mathematics teaching, he does not wish to imply that the laboratory method should be used exclusively or that is is superior to more conventional class teaching procedures. Teachers should be familiar with many instructional techniques and know in what circumstances and for what purposes each can be most effectively used.

V. Implications for Further Research

This study was designed to ascertain some of the effects of introducing a program of mathematics laboratory activities into the Grade 7 and Grade 8 curriculum. The investigation was exploratory in many respects and represents only an initial attempt to determine how mathematics laboratories might be used in today's schools and the outcomes of learning in this kind of instructional setting. Further research is needed to explore the many aspects of laboratory approaches to mathematics teaching. The following are a few of the problems suggested by the present study which could be investigated experimentally.

The present study had many limitations and should be replicated in other schools and at other grade levels using more precise measuring instruments and tighter experimental controls. Future experiments should be carried out in settings which permit random assignment of students to groups. The treatments should also extend over a longer period of time, perhaps a full year, to investigate the long range effects of a laboratory enrichment program on achievement in the regular program, on new learning, and on attitudes and to ascertain the stability of student feeling toward the laboratory approach to mathematics.

The laboratory method which was investigated in this study was experimentally complex; that is, it combined several features which operated together to produce the reported results. A series of experiments could be conducted to assess the contribution of each of these features. Questions such as the following could be investigated: To what extent and for which topics are concrete materials useful in helping students at the seventh and eighth grade levels understand new mathematical ideas? Given a new concept to be learned, is one type of physical aid more effective than another or should a variety of materials always be used? Can students learn as effectively from written instructions as they can from a teacher? In what type of situation is small group learning more effective than individual or class learning in mathematics?

Although students reacted favorably to the laboratory program, they considered this work as being separate from their regular mathematics course. How would students feel about laboratory learning in mathematics if it involved their regular course of studies over an extended period of time? To enable students to more objectively compare the laboratory method with a teacher-directed class discovery method, the present study could be modified so that all students would be exposed to both treatments. Two groups would study five lessons in the laboratory and five in the class setting on alternate weeks. The lessons investigated by the first group in the laboratory would be taken by the other group in the class teaching situation. Thus, students would not only be in a better position to judge the merits of the two approaches, but they might be able to use the laboratory more effectively due to their experience in the class discovery lessons.

It is generally agreed that all methods of instruction are not equally effective for all learners. What factors affect a student's ability to learn by a laboratory approach? What role do personality variables play in this regard? For instance, it might be hypothesized that aggressive students like situations in which they can be active and work in their own way while non-aggressive individuals prefer more direction and control. Cognitive learning styles and thinking patterns such as those identified in a study by Dienes and Jeeves (1965) might also be related to student preference for a particular method. Future studies might be designed for the purpose of developing predictive instruments to identify students who would be most successful under different methods of instruction.

It is recognized that the laboratory program developed for this study was somewhat rigid and did not allow sufficiently for individual differences in the ability of students to learn new mathematical concepts. All subjects were assigned the same tasks, and they followed the same written instructions and spent the same amount of time working on each activity. The results of the post achievement and transfer tests indicated that students of high ability in mathematics were best able to retain and apply the new ideas explored in the experimental program. Learners of average and low ability scored only slightly higher on these tests than control students. The following procedures could be investigated as ways of permitting average and low ability students to attain the same level of understanding as that obtained by the brighter students: different activities leading to the concepts might be made available for students of varying abilities; instructions might be written at various levels for different kinds of students; students might be required to continue work on each activity until they had mastered the basic ideas contained in it before moving to a new lesson; more teacher assistance might be provided for students who experience difficulty working from written instructions. Programming the laboratory instructions for computer assistance might prove to be the most effective means of providing for individual differences. If this were done each station in the laboratory might contain a computer terminal and a variety of physical materials. Various branches could be built into the program for each activity to permit each individual (or group) to move through the object-image-symbol sequence in forming the desired concepts at his own rate and according to his own needs. This would also prevent students from omitting or "skipping over" important sections of a lesson.

Instructions provided with the laboratory activites used in the present study were fairly complete and specific due to the fact that pilot investigations had indicated that students were unable or unwilling to work on open-ended kinds of problems. Ways of educating students to be able to create and use their own methods to investigate general problems should be developed. Can students develop in independence to the stage when given materials they structure and solve problems of their own? The specific question of directed versus non-directed laboratory work should also be investigated. What are the relative effects of the two approaches on the learning and retention of concepts, attitudes, and the ability to apply the scientific method to the solution of new problems? Is there an optimal amount of direction and feedback that should be provided in laboratory lessons?

Further research is required to determine those objectives of mathematics learning for which a laboratory approach is most appropriate. What kinds of skills, information, and attitudes are best acquired in a laboratory situation? Which mathematical ideas are taught more effectively by a laboratory approach than by a teacher demonstration or discovery lesson or by direct exposition? It is likely that new concepts would best be taught using a combination of laboratory and teacher-directed classroom work. The optimal amount of time which should be spent using the two methods for a given topic would have to be determined experimentally. A study might be designed to investigate the relative merits of providing laboratory experience preceding or following classroom work. In some instances the teacher might feel that his students need concrete experiences before an abstract concept could be meaningfully discussed and developed in class; in other situations ideas introduced by the teacher would be further explored or investigated experimentally by the students in the laboratory.

Another area for consideration would concern the role of the mathematics laboratory in relation to the spiral curriculum. Should concrete experiences be provided at various levels of the spiral or only when the concept is first introduced? The purposes for which the mathematics laboratory might be used at the high school level should also be investigated.

In order to evaluate the effects of the laboratory method, tests must be found which discriminate between students who have had laboratory experience and those who have not. Future studies should develop and use performance tests in which the students must deal directly with real objects in active physical situations to formulate and to solve problems. A laboratory test might consist of a series of stations each containing a problem based on a real situation. These items could be related to problems previously investigated in the laboratory or they could be questions designed to test the students' ability to use laboratory techniques to formulate hypotheses and arrive at solutions to new problems. The writer feels that instruments such as the Cards Uses in Mathematics test can be useful devices to encourage students to be creative in mathematics and to determine how well they understand certain abstract concepts. Test situations should be devised in which the subject is to generate problems and also which require the student to explain a concept in terms of a particular physical model.

In the writer's view, extensive and well planned research is necessary for obtaining answers to the many questions relating to the role of the mathematics laboratory as a setting in which the study of mathematics can be made both meaningful and interesting.

BIBLIOGRAPHY

BIBLIOGRAPHY

Anttonen, R. G. An examination into the stability of mathematics attitude and its relationship to mathematics achievement from elementary to secondary school level. Dissertation Abstracts, 1968, 28, 3011-A.

Avital, S. M. and Shettleworth, S. J. Objectives for mathematics learning: Some ideas for the teacher. Toronto: The Ontario Institute for Studies in Education, Bulletin No. 3, 1968.

Baldwin, A. L. Theories of child development. New York: John Wiley & Sons, 1967.

Bale, D. J. A comparison of programmed and conventional mathematics enrichment materials over two grade seven mathematics achievement levels. Unpublished master's thesis, University of Alberta, 1966.

Barrett, H. Cited by R. B. Davis, The Madison Project Final Report, Project No. D-233, Contract No. OE-6-10-183. Vol. II. U.S. Office of Education, Bureau of Research, 1967. P. A-27.

Bassetti, F., Malament, D. and Ruchlis, H. Solid shapes lab. New York: Science Materials Center, 1961.

Berkeley, E. C. Probability and statistics--An Introduction through experiments. New York: Science Materials Center, 1961.

Biggs, E. E. Mathematics in primary schools. Curriculum Bulletin No. 1, Schools Council for the curriculum and examination. London: Her Majesty's Stationery Office, 1965.

Biggs, E. E. Mathematics laboratories and teacher centres--the mathematics revolution in Britain. The Arithmetic Teacher, 1968, 15, 400-408.

Biggs, J. B. Towards a psychology of educative learning. International Review of Education, 1965, 11, 77-92.

Box, G.E.P. A general distribution for a class of liklihood criteria. Biometrika, 1949, 36, 317-346.

Brandes, L. G. Using recreational mathematics materials in the classroom. The Mathematics Teacher, 1953, 46, 326-329.

Brown, K. E. Third international curriculum conference. The Arithmetic Teacher, 1968, 15, 409-412.

Brown, K. E. and Abell, T. L. Research in the teaching of elementary school mathematics. The Arithmetic Teacher, 1965, 12, 547-549.

Brownell, W. A. Arithmetical abstractions--progress toward maturity of concepts under differing programs of instruction. The Arithmetic Teacher, 1963, 10, 322-329.

Brownell, W. A. Arithmetic abstractions. Cited by Z. P. Dienes, Mathematics in primary education. Hamburg: Unesco Institure for Education, 1966. Pp. 58-60. (a)

Brownell, W. A. The evaluation of learning under dissimilar systems of instruction. The Arithmetic Teacher, 1966, 13, 267-274. (b)

Bruner, J. Toward a theory of instruction. Cambridge: Harvard University Press, 1966. Pp. 39-72.

Campbell, D. T. and Stanley, J. C. Experimental and quasi-experimental designs for research. Chicago: Rand McNally, 1966.

Caparros, J. and Delgado, M. First report on the Canary Islands Mathematics Project. International Study Group on Mathematics Learning, Centre Psycho-Mathématique, University of Sherbrooke, Quebec, 1967. (Mimeographed)

Cohen, D. Inquiry in mathematics via the geo-board. Teacher's Guide. New York: Walker, 1967.

Cronbach, L. J. The logic of experiments on discovery. In L. S. Shulman and E. R. Keislar (Eds.), Learning by discovery: A critical appraisal. Chicago: Rand McNally, 1966. Pp. 77-92.

Davidson, P. S. An annotated bibliography of suggested manipulative devices. The Arithmetic Teacher, 1968, 15, 509-524.

Davis, R. B. _Discovery in mathematics: A text for teachers_. Palo-Alto: Addison-Wesley, 1964. (a)

Davis, R. B. Some remarks on a theory of instruction. In R. E. Ripple and V. N. Rockcastle (Eds.), _Piaget rediscovered_. Ithica, New York: School of Education, Cornell University, 1964. Pp. 134-138. (b)

Davis, R. B. The Madison Project's approach to a theory of instruction. _Journal of Research in Science Teaching_, 1964, 2, 146-162. (c)

Davis, R. B. A modern mathematics program as it pertains to the interrelationship of mathematical content, teaching methods and classroom atmosphere (The Madison Project). Final Report, Project No. D-233, Contract No. OE-6-10-183. Vol. I, II. U.S. Office of Education, Bureau of Research, 1967. (a)

Davis, R. B. _The changing curriculum: Mathematics_. Washington, D.C.: Association for Supervision and Curriculum Development, NEA, 1967. (b)

Davis, R. B. In C. A. Riedesel, Topics for research studies in elementary mathematics. _The Arithmetic Teacher_, 1968, 14, p. 681.

Dienes, Z. P. _Concept formation and personality_. Leicester, England: University Press, 1959.

Dienes, Z. P. _Building up mathematics_. London: Hutchinson Educational Ltd., 1960.

Dienes, Z. P. _An experimental study of mathematics learning_. London: Hutchinson & Co., 1963.

Dienes, Z. P. _The power of mathematics_. London: Hutchinson Press, 1964.

Dienes, Z. P. _Mathematics in primary education_. Prepared by the International Study Group for Mathematics Learning, Palo Alto, California. Hamburg: Unesco Institute for Education, 1966.

Dienes, Z. P. & Golding, E. W. _Learning logic, logical games_. Pinnacles: The Educational Supply Association Limited, 1966.

Dienes, Z. P. & Golding, E. W. Geometry of congruence. New York: Herder & Herder Inc., 1967.

Dienes, Z. P. and Jeeves, M. A. Thinking in structures. London: Hutchinson Educational Press, 1965.

Duckworth, E. Piaget rediscovered. The Arithmetic Teacher, 1964, 11, 496-499.

Duel, H. W. Measurable outcomes of laboratory work in science: A review of experimental investigations. School Science and Mathematics, 1937, 37, 795-810.

Easley, J. A. Jr. The University of Illinois Mathematics Project. In R. E. Ripple and V. C. Rockcastle (Eds.), Piaget rediscovered. Ithaca, New York: School of Education, Cornell University, 1964. P. 141.

Ebeid, W. T. An experimental study of the scheduled classroom use of student self-selected materials in teaching junior high school mathematics. (Doctoral dissertation, University of Michigan), Ann Arbor, Michigan: University Microfilms, 1964. No. 64-12, 586.

Farnham, D. J. Madison Project in an English primary school. Mathematics Teaching, 1965, No. 33, 37-41.

Fehr, H. F. The place of multisensory aids in the teacher training program. The Mathematics Teacher, 1947, 40, 212-216.

Fehr, H. F. Reorientation in math education. The Mathematics Teacher, 1968, 61, 593-601.

Ferguson, G. A. Statistical analysis in psychology and education. (2nd ed.). New York: McGraw-Hill, 1966.

Fitzgerald, W. Individualizing instruction in mathematics. In W. R. Houston (Ed.), Improving mathematics education forelementary school teachers. A conference report sponsored by the Science and Mathematics Teaching Center, Michigan State University, August, 1967. Pp. 53-64.

Fitzgerald, W. M. A mathematics laboratory for prospective elementary school teachers. The Arithemtic Teacher, 1968, 15, 547-549.

Flavell, J. H. The developmental psychology of Jean Piaget. Princeton: D. Van Nostrand Company, 1963.

Gador, P. Experiments on mathematics teaching in Hungary at the primary stage. In J. D. Williams (Ed.), Mathematics reform in the primary school. Hamburg: Unesco Institute for Education, 1967. Pp. 90-96.

Gattegno, C. A teacher's introduction to the Cuisenaire-Gattegno method of teaching arithmetic. Reading, England: Lamport Gilbert & Co. Ltd., 1960.

Glennon, V. J. and Callahan, L. G. (Eds.) Elementary school mathematics: A guide to current research. (3rd ed.) Washington, D.C.: Association for Supervision and Curriculum Development, NEA, 1968.

Golding, E. W. Mathématique moderne dans les écoles élémentaires: Commentaires sur les développements de la situation d'apprentissage (Translated). International Study Group for Mathematics Learning, University of Sherbrooke, Quebec, 1968. (a) (Mimeographed)

Golding, E. W. Report on teacher education with special reference to mathematics learning. International Study Group for Mathematics Learning, Centre Psycho-Mathématique, University of Sherbrooke, Quebec, 1968. (b) (Mimeographed)

Goldziher, K. Mathematics laboratories. School Science and Mathematics, 1908, 8, 753-757.

Greathouse, J. J. An experimental investigation of relative effectiveness among three different arithmetic teaching methods. Dissertation Abstracts, 1966, 26, 5913.

Harrison, D. B. Piagetian studies and mathematics learning and instruction. Paper presented at the 47th Annual Meeting of the NCTM in Minneaplis, Minnesota, April 25, 1969.

Harvin, V. R. Analysis of the uses of instructional materials by a selected group of teachers of elementary school mathematics. (Ed.D. thesis, Indiana University), Ann Arbor, Michigan: University Microfilms, 1964. No. 65-394.

Haynes, J. O. Cuisenaire rods and the teaching of multiplication to third-grade children. (Doctoral dissertation, Florida State University), Ann Arbor, Michigan: University Microfilms, 1964. No. 64-3598.

Hays, W. L. Statistics for psychologists. Holt, Rinehart & Winston, 1963.

Hollis, L. V. A study to compare the effects of teaching first and second grade mathematics by the Cuinsenaire-Gattegno method with a traditional method. School Science and Mathematics, 1965, 65, 683-687.

Hopkins, C. D. An experiment on use of arithemtic time in the fifth grade. Dissertation Abstracts, 1966, 26, 5291.

Hudson, F. H. A study of an enrichment program in arithemtic for children in the fourth grade. Dissertation Abstracts, 1958, 18 (Part 1), 958.

International Study Group for Mathematics Learning. Mathematics in primary education. Hamburg: Unesco Institute for Education, 1966.

Johnson, D. A. Attitudes in mathematics classrooms. School Science and Mathematics, 1957, 57, 113-120.

Johnson, D. A. A pattern for research in mathematics education. The Mathematics Teacher, 1966, 59, 418-428.

Johnson, D. A. and Rising, G. R. Guidlines for teaching mathematics. Belmont, California: Wadsworth Publishing Company, 1967.

Johnson, L. K. The mathematics laboratory in today's schools. School Science and Mathematics, 1962, 62, 586-592.

Jones, F. T. Some experiences in laboratory mathematics and their results. School Science and Mathematics, 1905, 5, 406-410.

Jones, T. The effect of modified programmed lectures and mathematical games upon achievement and attitude of ninth-grade low achievers in mathematics. The Mathematics Teacher, 1968, 61, 603-607.

Kieren, T. E. and Vance, J. H. The theory of active learning: Its application in a mathematics workshop. The Manitoba Journal of Education, 1968, 4, 33-40.

Kruglak, H. and Wall, C. N. Laboratory performance tests for general physics. Kalamazoo, Michigan: Western Michigan University, 1959.

Lamon, W. E. An analysis of some structural learning characteristics of children from differing instructional background as manifested in the learning of certain mathematical group concepts. Paper presented at the AERA convention, Los Angeles, February, 1969.

Lucow, W. H. An experiment with the Cuisenaire method in grade three. American Educational Research Journal, 1964, 1, 159-167.

Lunzer, E. A. The work of Piaget and its relevance for the teacher. Unpublished manuscript, Manchester University. Cited by E. E. Biggs, Mathematics in primary schools. London: H. M. Stationery Office, 1965. P. 6.

Matthews, Geoffrey. The Nuffield Mathematics Teaching Project. The Arithmetic Teacher, 1968, 15, 101-102.

May, L. J. Learning laboratories in elementary schools in Winnetka. The Arithmetic Teacher, 1968, 15, 501-503.

McDill, R. M. Laboratory work in geometry. The Mathematics Teacher, 1931, 24, 14-21.

Menchinskaya, N. A. Some aspects of primary school mathematics teaching in Russia. In S. D. Williams (Ed.), Mathematics reform in the primary school. Hamburg: Unesco Institute for Education, 1967. Pp. 118-122.

Moise, E. E. Activity and motivation in mathematics. Mathematics Teaching, 1966, No. 34, 64-67.

Morley, A. Changes in primary school mathematics--Are they complete? Mathematics Teaching, 1967, No. 41, 20-24.

Moore, E. H. On the foundations of mathematics. Science, N.S., 1903, 17, 401-416.

Nasca, D. Comparative merits of a manipulative approach to second grade arithmetic. The Mathematics Teacher, 1966, 13, 221-226.

National Council of Teachers of Mathematics. 18th Yearbook, Multisensory aids in the teaching of mathematics. Washington, D.C.: Author, 1945.

Neale, D. C. The role of attitudes in learning mathematics. Paper presented at the 47th Annual Meeting of the NCTM in Minneapolis, Minnesota, April 24, 1969.

Nelson, L. D. Analysis of Canadian research on the effectiveness of Cuisenaire materials. In Canadian Council for Research in Education, Canadian experience with the Cuisenaire method. Toronto: Canadian Education Association, 1964. Pp. 183-187.

Osgood, C. E., Suci, G. J. and Tannenbaum, P. H. The measurement of meaning. Urbana, Illinois: University of Illinois Press, 1957.

Passy, R. A. The effect of Cuisenaire material on reasoning and computation. The Arithemtic Teacher, 1963, 20, 439-440.

Peeler, H. Enrichment materials for school mathematics. The Arithmetic Teacher, 1962, 9, 271-275.

Piaget, J. Development and learning. In R.E. Ripple and V. N. Rockcastle (Eds.), Piaget rediscovered. Ithaca, New York: School of Education, Cornell University, 1964. Pp. 7-20.

Picard, N. Curricular change in the French primary schools. In J. D. Williams (Ed.), Mathematics reform in the primary school. Hamburg: Unesco Institute for Education, 1967. Pp. 72-78.

Polya, G. How to solve it. (2nd. ed.) Garden City, N. Y.: Doubleday, 1957.

Polya, G. Mathematical discovery. Vol. 2. New York: John Wiley & Sons, 1965.

Prouse, H. L. The construction and use of a test for the measurement of certain aspects of creativity in seventh-grade mathematics. (Doctoral dissertation, State University of Iowa) Ann Arbor, Michigan: University Microfilms, 1964. No. 65-500.

Rainey, R. G. The effects of directed versus non-directed laboratory work on high school chemistry achievement. Journal of Research in Science Teaching, 1965, 3, 286-292.

Ramseyer, J. A. The mathematics laboratory--A device for vitalizing mathematics. The Mathematics Teacher, 1935, 28, 228-233.

Reddell, W. D. and DeVault, M. V. In-service research in arithmetic teaching aids. The Arithmetic Teacher, 1960, 7, 243-347.

Remai, H. An experimental investigation comparing attitudes toward mathematics of modern and traditional students at the junior high school level. Unpublished master's thesis, University of Alberta, 1965.

School Mathematics Study Group. Report of the SMSG Panel on Research, Chicago, November 15-16, 1968.

Sealey, L.G.W. The creative use of mathematics in the junior school. Oxford: Basil Blackwell, 1960.

Sealey, L.G.W. An outline of curricular changes in Great Britain. In J. D. Williams (Ed.), Mathematics reform in the primary school. Hamburg: Unesco Institute for Education, 1967. Pp. 106-114.

Sigurdson, S. E. and Johnston, R. J. A discovery unit on quadratics. The Manitoba Journal of Education, 1968, 4, 20-32.

Slavina, L. S. Specific features on the intellectual work of unsuccessful pupils. In B. Simon (Ed.), Psychology in the Soviety Union. Stanford, California: Stanford University Press, 1967. P. 205. Cited by V. J. Glennon and L. G. Callahan (Eds.), Elementary school mathematics: A guide to current research. (3rd ed.) Washington, D.C.: NEA, 1968.

Smith, M. The psychological case for using structural apparatus. Mathematics Teaching, 1965, No. 30, 7-10.

Stern, C. Children discover arithmetic. New York: Harper & Brothers, 1949.

Suchman, J. The Illinois studies in inquiry training. In R. E. Ripple and V. N. Rockcastle (Eds.), Piaget rediscovered. Ithaca, New York: School of Education, Cornell University, 1964. Pp. 105-108.

Suddeth, M. C. An historical survey of the development of the use of instructional materials for the teaching of arithmetic in the elementary grades as recorded in selected publications. (Ed.D. thesis, Indiana University) Ann Arbor, Michigan: University Microfilms, 1963. No. 63-2614.

Sutton-Smith, B. The role of play in cognitive development. In W. Hartup and N. Smothergill (Eds.) The young child, Review of research. Washington, D.C.: National Association for the Education of Young Children, 1967. Pp. 96-108.

Swick, D. F. The value of multi-sensory learning aids in the teaching of arithmetical skills and problem solving--An experimental study. Dissertation Abstracts, 1960, 20, 3669.

Taba, H. Curriculum development, Theory and practice. New York: Harcourt, Brace & World, 1962.

The Nuffield Foundation. I do, and I understand. London: Newgate Press, 1967.

Torrance, E. P. Education and the creative potential. Minneapolis: The University of Minnesota Press, 1963.

Van Engen, H. The formation of concepts. In National Council of Teachers of Mathematics 21st Yearbook, The learning of mathematics, Its theory and practice. Washington, D.C.: Author, 1953, Pp. 69-98.

Van Engen, H., Hartung, M. L., Trimble, H. C., Berger, E. J., Cleveland, R. W., and Evenson, A. R. Seeing through mathematics. Books 1 & 2, Special edition. Toronto: W. J. Gage, 1964.

Wheeler, D. H. Dienes on the learning of mathematics. Mathematics Teaching, 1964, No. 27, 40-44.

Williams, J. D. Teaching arithmetic by concrete analogy--II. Structural systems. Educational Research, 1961, 4, 163-192.

Williams, J. D. (Ed.) Mathematics reform in the primary school. Hamburg: Unesco Institute for Education, 1967.

Winer, N. J. Statistical principles in experimental design. New York: McGraw-Hill, 1962.

Worthen, B. A comparison of discovery and expository sequencing in elementary mathematics instruction. In J. M. Scandura (Ed.), Research in mathematics education. Washington, D.C.: NCTM, 1967. Pp. 44-59.

APPENDIX A

THE EXPERIMENTAL PROGRAM

Sources and references for the laboratory activities and concrete materials.

Purpose, behavioral objectives, and concrete materials for each activity.

Instructional materials for Activities 1, 8, and 9: Laboratory instructions and suggestions for adaptation as Class Discovery lessons.

SOURCES AND REFERENCES FOR LABORATORY ACTIVITIES AND CONCRETE MATERIALS

Lesson	Reference	Concrete Materials
1. Area and Perimeter	Z. P. Dienes, Building up Mathematics, 1960.	Cubical Counting Blocks, Milton Bradley Company, Springfield, Mass.
2. Intersecting Sets	Z. P. Dienes and E. W. Golding, Learning Logic, Logical Games, 1966.	Blocs logiques, Education Nouvelle, Montreal.
3. Symmetries of a Square	D. Cohen, Inquiry in Mathematics via the Geoboard, 1967.	Eight inch plastic square, Homemade.
4. Areas of Polygons	D. Cohen, Inquiry in Mathematics via the Geoboard, 1967.	Geoboard, Walker Teaching Programs and Teaching Aids, New York.
5. How Many Subsets?	C. Gattegno, Arithmetic with numbers in Colour, Book 10, 1960.	Cuisenaire Rods, Cuisenaire Company of America, Mt. Vernon, New York.
6. Mathematical Balance	Z. P. Dienes, Tasks and manual for use with the algebraic experience materials.	Invicata Plastics Mathematical Balance, The Educational Supply Association Limited, Schools Material Division, Harlow, Essex.
7. Numeration in Bases 3 and 5	Z. P. Dienes, Tasks and manual for use with the multibase arithmetic blocks.	Multibase Arithmetic Blocks, The Educational Supply Association, Limited.

Lesson	Reference	Concrete Materials
8. Probability	E. C. Berkely, Probability and Statistics - An Introduction Through Experiments, 1961.	Probability and Statistics Labs, Division of Gamco Industries, Big Spring, Texas.
9. Measurement of a Circle	Madison Project Shoebox "Discs" and D. A. Johnson and G. R. Rising, Guidlines for Teaching Mathematics, 1967.	Madison Project Shoebox "Discs", St. Louis; Wooden disc cut into sectors, Homemade.
10. Polyhedra	F. Bassetti et al., Solid Shapes Lab, 1961.	Solid Shapes Lab, Science Materials Center, New York.

Lesson 1 AREA AND PERIMETER

Purpose: To investigate the perimeters of rectangles with a fixed area.

Behavioral Objectives:

1. The students should understand what is meant by the area and perimeter of rectangles constructed of unit squares and be able to find the area and perimeter by counting.

2. The students should be able to tell when it is possible to form a rectangle from a given number of unit squares and having a given number of rows or columns.

3. The students should be able to find the area and perimeter of rectangles given the length and width.

4. The students should realize that the areas of all rectangles made up of a fixed number of unit squares are equal.

5. The students should realize that the perimeters of rectangles having the same area but different dimensions are different. They should know that the square has the least perimeter and that the perimeter increases as one dimension increases and the other decreases.

Concrete Materials:

Lab - 36 unit squares (are actually cubes)

Classroom - 36 unit squares.

Lesson 2 INTERSECTING SETS

Purpose: To investigate the problem of finding the total number of elements in two intersecting sets.

Behavioral Objectives:

1. The students should know that each of the blocks (or pictures of the blocks) has three attributes--color, shape, and size. They should be able to tell how many of blocks there are with a particular attribute. For example, how many are red, circular, large, etc.

2. Given two attributes, say blue and small, the students should be able to place each block into the correct region of a Venn diagram.

3. The students should be able to identify blocks having two given attributes, and be able to find the number of such blocks knowing the number of blocks having each attribute separately.

4. The students should be able to identify blocks having at least one of two given attributes (for example, either square or large) and be able to find the number of such blocks using (not necessarily explicitly) the relation

$$n\ (A \cup B) = n(a) + n(B) - n(A \cap B).$$

5. The students should be able to find the nuber of blocks <u>not</u> having a given attribute.

Concrete Materials:

Lab - 24 attribute blocks; 2 hoola-hoops.

Classroom - attribute blocks and chart of the 24 attribute blocks.

Lesson 3 SYMMETRIES OF A SQUARE

Purpose: To illustrate the properties of a mathematical group in a finite system without numbers.

Behavioral Objectives:

1. The students should know that any two of the moves I, V, H, and R made consecutively is equivalent to making a single move from this set and be able to find this move by manipulating the square (or seeing it manipulated).

2. The students should understand the nature of the identity move I and know that it corresponds to the number 0 in adding and to the number 1 in multiplying.

3. The students should know that each move has an inverse and that in this system each move is its own inverse.

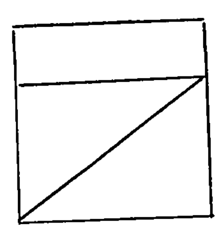
4. The students should know that the system is associative and thus be able to easily do such exercises as

V * V * R * R * H = ___.

Concrete Materials:

Lab - A clear plastic or heavy cardboard square as shown.

Classroom - A large clear plastic or cardboard square.



Lesson 4 AREAS OF POLYGONS

Purpose: To discover methods for finding the area of a triangle in order to be able to determine the areas of polygons.

Behavioral Objectives:

1. The students should know that the area of a figure formed by placing rubber bands around nails on a geoboard is the number of unit squares within that figure.

2. The students should know that a diagonal of a rectangle divides the rectangle into two triangles each having an area equal to one-half the area of the rectangle.

3. The students should be able to find the area of any triangle that can be made on the geoboard by enclosing it in a rectangle and subtracting triangular regions.

4. Given any triangle the students should be able to designate one side as the base and find the corresponding height.

5. The students should know that triangles having equal bases and equal heights are equal in area.

6. The students should be able to compute the area of a triangle given the base and height.

7. The students should be able to find the area of any polygon formed on the geoboard.

Concrete Materials:

Lab - A geoboard and rubber bands.

Classroom - A plastic geoboard to be used on the overhead projector and some prepared transparencies.

Lesson 5 HOW MANY SUBSETS?

Purpose: To determine the number of subsets of a set containing a given number of members.

Behavioral Objectives:

1. The students should be able to tell when a set is a subset of another set. They should know that the empty set and the whole set are also subsets of any set.

2. The students should know that sets with the same members are the same even though the elements may be listed in different orders. For example, $\{a,b\} = \{b,a\}$.

3. The students should be able to list all subsets of the sets $\{R\}$, $\{R,Y\}$, $\{R,Y,G\}$, and $\{R,Y,G,B\}$ and know that subsets of $\{R,Y,G,B,O\}$ include sets having 0, 1, 2, 3, 4, and 5 members.

4. The students should know that if a set with n members has x subsets, then a set with n + 1 members has 2x subsets.

5. The students should know that a set with n members has 2^n subsets.

Concrete Materials:

Lab - 5 rods of different lengths and colors.

Classroom - 5 rods of different lengths and colors.

Lesson 6 MATHEMATICAL BALANCE

Purpose: To demonstrate certain properties of the natural numbers and the solution of linear equations by means of a balance arm.

Behavioral Objectives:

1. The students should be able to express natural numbers in terms of sums and products of other natural numbers.

2. The students should know that, for example, 3 rings on hook four on one side balance 4 rings on hook three on the other side, thus enforcing the commutative law of multiplication.

3. The students should know, for example, that 3 rings on hook five and 3 rings on hook two on one side balance 3 rings on hook seven on the other side, thus enforcing the distributive law.

4. The students should be able to find solutions of linear equations of the form $ax + b = cx + d$ (where all coefficients and solutions are natural numbers) by trial and error and also by the "equal subtraction" method.

Concrete Materials:

Lab - Balance arm and rings.

Classroom - Balance arm and rings.

Lesson 7 NUMBERATION IN BASES 3 AND 5

Purpose: To count and perform arithmetic operations in Bases 3 and 5.

Behavioral Objectives:

1. The students should be able to express numbers such as fourteen and thirty-four in Base 3 and Base 5 notation.

2. The students should be able to add and subtract and perhaps multiply and divide in bases three and five.

Concrete Materials:

Lab - Base 3 and Base 5 arithmetic blocks.

Classroom - A few pieces from the sets of Base 3 and Base 5 arithmetic blocks.

Lesson 8 PROBABILITY

Purpose: To introduce the students to basic concepts in probability through experiments with coins, dice, and sampling urns.

Behavioral Objectives:

1. The student should be able to record the results of an experiment on a prepared form.

2. The students should know what is meant by the term probability as it is used in mathematics and be able to determine probabilities in simple situations.

3. The students should find out that, experimentally, results predicted from probability theory are approximately, but only approximately, obtained and that deviations from expected values are common, especially in small samples.

Concrete Materials:

Lab - A penny, a dice, four sampling urns.

Classroom - Coins, dice, four or five sampling urns.

Lesson 9 MEASUREMENT OF A CIRCLE

Purpose: To find π experimentally and to develop methods of determining the circumference and area of a circle.

Behavioral Objectives:

1. The students should know that the ratio of the circumference of any circle to its diameter is a fixed number called π and that the value of π is about 3 1/7.

2. The students should know that the area of a circle is the product of its radius and one-half its circumference.

3. The students should be able to use the formulas $C = \pi d$, $C = 2\pi r$, and $A = \pi r^2$ to find the circumference and area of a circle and the diameter and radius if the circumference or area is given.

Concrete Material:

Lab - Five wooden discs; a ruler and length of string; a wooden disc cut into sectors.

Classroom - Several wooden discs; ruler and string; a cardboard disc cut into sectors for use on overhead projector.

Lesson 10 POLYHEDRA

Purpose: To discover Euler's formula for simple polyhedra through examination of the regular polyhedra and various prisms and pyramids.

Behavioral Objectives:

1. The students should be able to recognize triangles, squares, pentagons, and hexagons as faces of solid shapes.

2. The students should be able to count the number of faces, edges, and vertices of a polyhedron.

3. The students should know Euler's rule and be able to apply it to find one of the variables, given the other two.

4. The students should be able to recognize and distinguish between prisms and pyramids and be able to name those introduced in the lesson.

5. The students should know what is meant by the term regular polyhedron; they should know that there are only five such polyhedra and be able to recognize them.

Concrete Materials:

Lab - Models of a tetrahedron, cube, octohedron, square pyramid, pentagonal pyramid, triangular prism, pentagonal prism, hexagonal prism; cardboard nets of the pentagonal prism and pentagonal pyramid; scissors and construction paper.

Classroom - Models of the polyhedra above plus models of a dodecahedron and icosahedron; transparencies of the nets of the pentagonal prism, penatagonal pyramid, dodecahedron, and icosahedron.

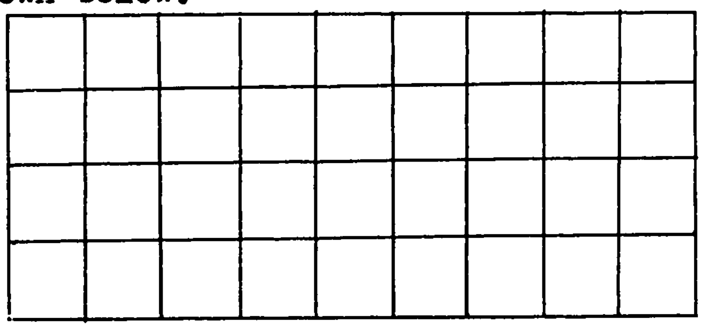
Names ______________________

Grade ___ Date ___________

LAB 1 (11) AREA AND PERIMETER

In the box you will find 36 unit squares.
Arrange the squares to form a rectangle of 4 rows and 9 columns as shown below.

4 rows



9 columns

The AREA of a figure is the number of square units needed to cover its surface.

The area of this rectangle is _____ square units.

The PERIMETER of a figure is the distance around the <u>outside</u> of the figure. You would need to know the perimeter of a field in order to determine the length of a fence needed to enclose it.

(1) The perimeter of this rectangle is ___ units.

Now rearrange the blocks to form a rectangle with 12 rows.

How many columns does this rectangle have? ________

Its area is _____ square units.

(2) Its perimeter is ____ units.

(Check your answers to (1) and (2) by looking in the table on page 2. If you did not have the correct answers, make these rectangles again to discover why.)

Using all 36 blocks, form as many other rectangles as you can. For each rectangle determine the area and perimeter. Record your answers in the table below.

No. of rows	No. of columns	Area (sq. units)	Perimeter (units)
4	9	36	26
12	3	36	30

Why can't you make a rectangle with 5 rows and still use all the blocks?

What do you notice about the area of each of these rectangles?

Why is this so?

(Note: The perimeter of the rectangle with 1 row and 36 columns or 36 rows and 1 column is 74 units. Check your answer in the table before going on. If you did not have 74 units can you now see that this is the right answer?)

How could you find the perimeter of a rectangle without counting? (For example, suppose you were given that a rectangle had 4 rows and 9 columns.)

Which of the rectangles that you made has

(a) the smallest perimeter?

_____ rows and ____ columns

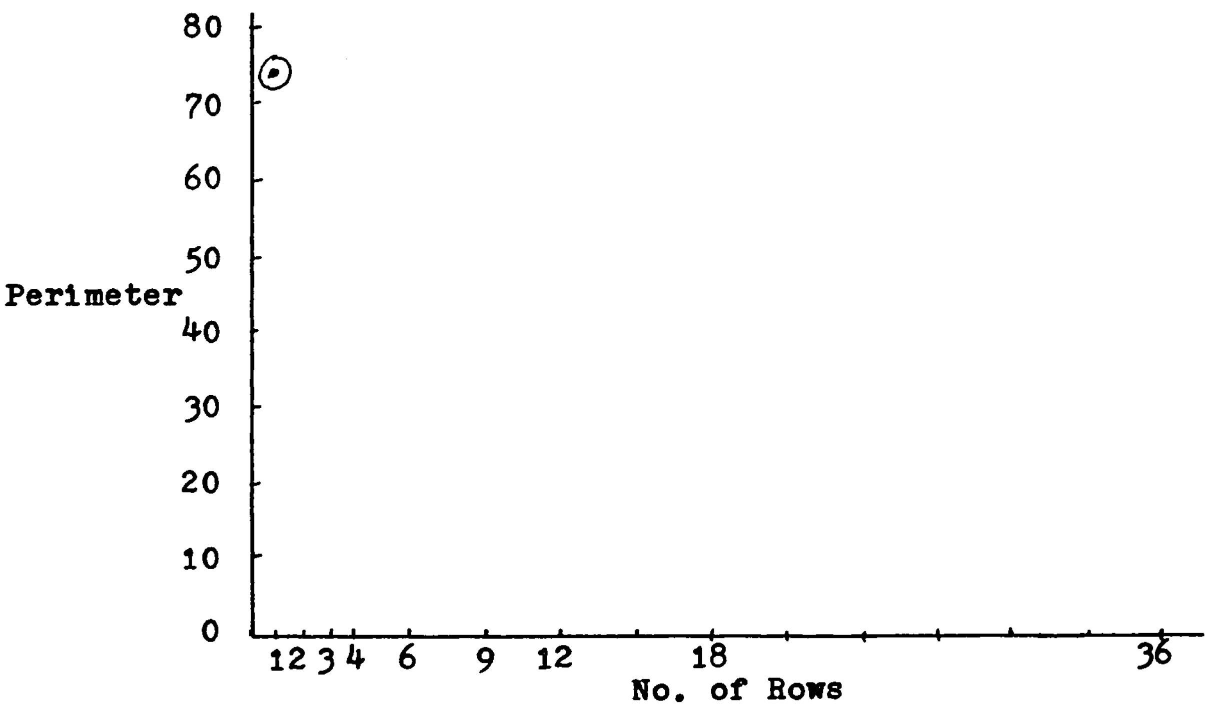
(b) the largest perimeter?

_____ rows and ____ columns

Complete the table below for the perimeters of rectangles having an area of 36 square units. (You may use information from the previous table.)

(1) No. of rows	(2) No. of columns	(3) Perimeter
1	36	74
2		
3		
4		
6		
9		
12		
18		
36		

Plot the ordered pairs from columns (1) and (3) in the table on page 3 on graph below. Then draw a smooth curve through the points plotted to complete the graph. (The first point (1, 74) is already marked.)



Use the graph to answer the following questions:

1. Which rectangle has the smallest perimeter?

    ____ rows and ____ columns

    What name is given to this special rectangle? ________________

2. What is the approximate perimeter of the rectangle having

    (a) 15 rows? _____ units.

    (b) a length of 30 units _____ units

    (c) a width of $1\frac{1}{5}$ units _____ units

3. What are the dimensions of the rectangle having a perimeter of

    (a) 50 units? width ____ units; length ____ units

    (b) 80 units? width ____ units; length ____ units

Make up more questions like these and answer them.

Imagine now that you have 64 unit squares and you are to make rectangles using all of these squares.

1. What would be the area of each of these rectangles?

   ________ square units.

2. Which rectangle would have

   (a) the smallest perimeter? ______________________________

   (b) the largest perimeter? ______________________________

3. Make a table (like the one on page 3) showing the perimeter of each rectangle which can be formed from the 64 unit squares. Put this information on a graph (as you did on page 4).

Page 1.

Show the 36 unit squares and ask the students how they could arrange them to form a rectangle. Show one of the suggestions on the overhead projector. Discuss the area and perimeter of this rectangle. (Students often count the number of blocks around the edge of the rectangle rather than the number of units around the outside.) Do not introduce formulas at this point.

Consider a rectangle of different dimensions and repeat the procedure above. Record the results in a table as on page 2.

Page 2, 3.

Have the students make the table on some paper and complete it for as many different rectangles as they can think of which could be formed from the 36 blocks. Complete the table on the board and ask for generalizations from the class. The following points should be covered:

1. Area is constant. (Why?)

2. Perimeter changes. (In what way)

    (a) rectangle with the smallest perimeter.

    (b) rectangle with the largest perimeter.

3. Methods or formula for finding the number of columns, area, and perimeter given the number of rows.

4. Rectangles that cannot be formed using unit squares.

    (a) how can you tell?

    (b) use of fractions.

Page 3, 4.

On the board complete the table. Plot the ordered pairs on the grid using the overhead projector. Draw a smooth curve.

Discuss the questions on the bottom of page 4.

Page 5.

Depending on the time, you may want to discuss the same questions concerning the rectangle that could be formed from 64 unit squares. Do this only as time permits; it is optional.

During the last five minutes, have the students do and hand in the review exercises.

Names ____________________

Grade ____ Date ____________

LAB 8 (18) PROBABILITY

Experiment 1: Flipping a coin.

If you were to flip a coin ten times, how many heads and tails would you expect to obtain? Heads ____; Tails ____.

How many heads and tails would you expect to obtain by tossing a coin 40 times? Heads ____; Tails ____.

You will need a penny. Flip it in the air in such a way that it turns over at least three or four times. Record the outcome -- H if heads, T if tails -- in the table below. Flip the coin a total of 40 times, recording each outcome.

(One person should flip the coin, the other record the outcomes.)

No. of throw	Outcome	No.	Outcome	No.	Outcome	No.	Outcome
1		11		21		31	
2		12		22		32	
3		13		23		33	
4		14		24		34	
5		15		25		35	
6		16		26		36	
7		17		27		37	
8		18		28		38	
9		19		29		39	
10		20		30		40	
H ___		H ___		H ___		H ___	
T ___		T ___		T ___		T ___	

Total no. of heads ____; Total number of tails ____

Theore t ically, when you flip a coin, it is just as likely to come down heads as it is tails.

We say that "the <u>probability</u> is $\frac{1}{2}$" that the coin will come down heads.

Since the probability is $\frac{1}{2}$, you may think that you should get five heads and five tails in ten throws and 20 heads and 20 tails in 40 throws, etc. In practice, as you have observed, this is not likely to be true where the number of throws is small, but as the number of throws increases, you should get nearer to a fifty-fifty split.

The chance of getting all 40 throws heads or tails is very small -- about one out of a thousand billion. You can be fairly sure that you will not get exactly 20 heads either. You will probably get between 15 and 25 heads.

<u>Experiment 2</u>: Rolling a die.

Roll the die (singular of dice) 30 times, keeping a record of each result on another sheet. Then total the number of one's, two's, three's, four's, five's, and six's that occurred. Enter these figures in column (2) below.

(1) Outcome	(2) First trial	(3) Second Trial
one		
two		
three		
four		
five		
six		
	30	30

We would expect that since there are six outcomes and each outcome should be "equally likely" (unless the die is loaded), each outcome should occur five times. In other words the probability is $\frac{1}{6}$, and out of every 30 rolls we would expect five one's, five two's, five three's. etc.

Ordinary experience, however, indicates that you would be very unlikely to obtain exactly five of each outcome.

In order to get more evidence, repeat this experiment, and record your results in column (3).

Did you get the same results on the first and second trials?

____

Experiment 3: Guessing the urn.

Find the four yellow urns marked A, B, C, and D. Each urn contains 20 beads, some white and some black.

Urn no.	1	contains	18 black	beads and	2 white	beads.
"	2	"	14 black	"	6 white	"
"	3	"	10 black	"	10 white	"
"	4	"	4 black	"	16 white	"

Take urn A. Without removing the lid can you guess which urn (1, 2, 3 or 4) this is? Perform the following experiment to help you guess: Shake a bead into the bubble 10 times, recording the number of times a black bead appears and the number of times a white bead appears.

A: black ___ white ___ Guess: Urn no. ____

In the same way identify B, C, and D.

B: black ___ white ___ Guess: Urn no. ___

C: black ___ white ___ Guess: Urn no. ___

D: black ___ white ___ Guess: Urn no. ___

To see how well you guessed, remove the tops from the urns and look in. (Do not remove the beads from the urns.)

Since there are 20 beads in urn no. 2 and six are white, there are six chances out of 20 that a bead appearing will be white. Therefore the probability of shaking a white bead in urn no. 2 is $\frac{6}{20}$ or $\frac{3}{10}$.

Similarly the probability of shaking a black bead in urn no. 4 is $\frac{4}{20}$ or $\frac{1}{5}$.

Now suppose an urn has 30 beads of which 10 are red, 15 are white, and 5 are black. What is the probability of shaking (a) a red bead? _____

(b) a white bead? _____

(c) a black bead? _____

If there is still time, repeat experiments 2 or 3.

Page 1.

Have each student flip a coin (some twice) and record 40 outcomes in a table as shown.

Page 2.

Discuss what is meant by probability in this case. Note the difference between the expected outcome and the experimental outcome.

Page 3.

Have one die for each row. Let each student roll the die once; then record the results of 30 rolls. Repeat this experiment and compare the results of the two trials. Discuss probability in this situation.

Page 4.

Show the urns to the students and explain their contents (write this information on the board). Ask for suggestions as to how to identify the urns. Perform the experiment by giving one urn to each row and letting each group determine (experimentally) its content. Record the results and predictions of each group. Check by removing the lids. (You may prepare extra urns for more than four groups.)

Page 5.

Be sure to have enough time to talk about finding probabilities in various situations and letting the students work the problems.

If time remains, repeat any of the experiments.

Have the students do and hand in the review exercises.

Names ______________

Grade ___ Date __________

LAB 9 (19) MEASUREMENT OF A CIRCLE

Take a disc from the box. Using only the string and ruler, what distances can you measure on the disc?

Measure the diameter (distance across the disc) and the circumference (distance around the outside) of each disc. Record these measurements in the table below.

diameter	circumference

Examine the table. Can you write a rule which approximately relates the diameter and the circumference of a circle?

____

Use your rule to predict the circumference of a circular object of diameter

(a) 1 inch ______

(b) 10 inches ______

(c) 15 inches ______

Predict the diameter of the circular object which has a circumference of

(a) 19 inches ______

(b) 44 inches ______

(c) 1 inch ______

You have found that the circumference of a circle is always a little more than 3 times its diameter.

This ratio of the circumference of a circle to its diameter is called "pi" and its symbol is π.

The numerical value of π is approximately $3\frac{1}{7}$.

So if you know the diameter of a circle, you can find its circumference by the rule

$$\text{Circumference} = \pi \times \text{diameter}$$

or

$$C = \pi d$$

How could you find the circumference of a circle if you know its <u>radius</u>?

The diameter of a circle is twice the radius so

$$\text{Circumference} = 2 \times \pi \times \text{radius}$$

or

$$C = 2\pi r$$

If a circle has a radius of 5 inches, find

(a) its diameter ________

(b) its circumference ________

Find the radius of a circle having a circumference of

(a) 25 inches ________

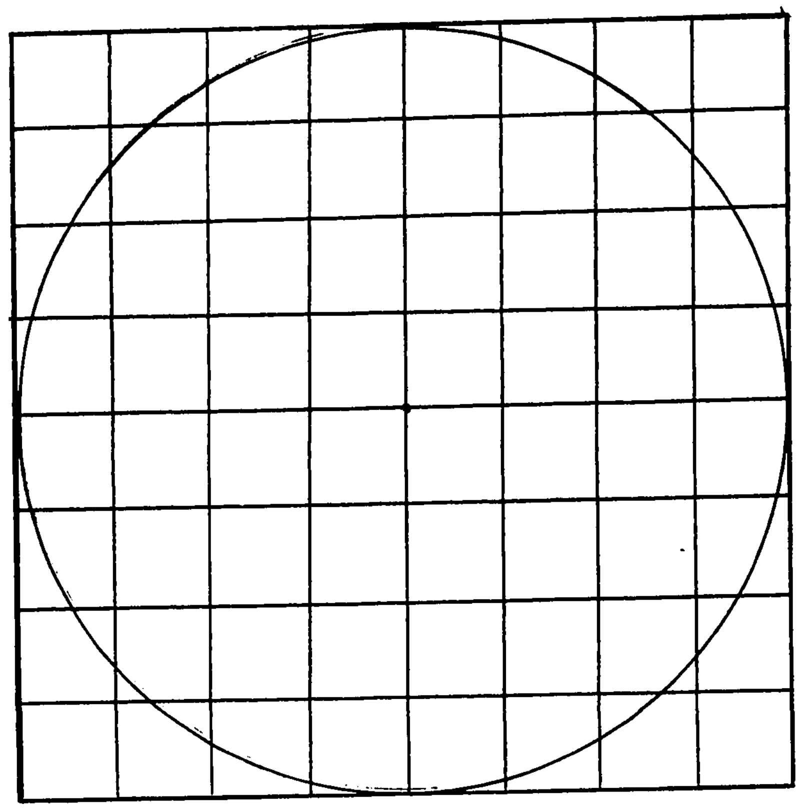
(b) 63 inches ________

You have measured the radius, the diameter, and the circumference of a disc. What other measurement might one want to have?

____

The **area** of any closed figure is the number of unit squares required to cover its surface.

Find the approximate area of the circle below by counting unit squares.

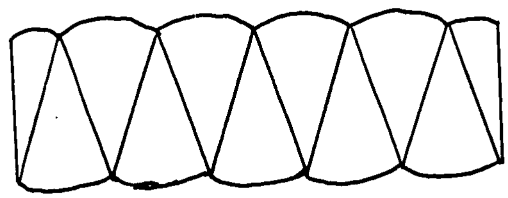


Radius = 4 units

Area = _____ square units

You have seen that you can find the perimeter of a circle if you know the radius. Is there also a rule for finding the area of a circle if the radius is known?

Take the wooden frame containing the disc that has been cut up into sectors. Remove the sectors and reassemble them as illustrated below.



Can you now see a way of finding the area?

The area of this "rectangle" is found by multiplying the length of its base by its height.

The length of the base is ½ of the circumference of the circle.

Why half? ______________________________

The height is the radius of the circle? (Can you see why?)

The circumference is $2\pi r$; therefore half the circumference is πr.

Hence

$$\text{Area of a circle} = \pi \times r \times r = \pi r^2$$

Use this formula to find the area of a circle having a radius of 4 units. ______________

Compare this answer with the one you obtained on page 3 by counting unit squares. Previous answer __________

Fill in the blanks in the table below:

Circle No.	Radius (units)	Diameter (units)	Circumference (units) $C = \pi d$ or $2\pi r$	Area (square units) ½ circumference X radius	π X r X r
1	3½				
2		10			
3			44		
4					50

Add to the table by making up your own questions and answering them.

Lesson 9 MEASUREMENT OF A CIRCLE Classroom Procedure

Page 1.

Hold up a disc. Discuss what distances could be measured and how. Let the students estimate these measurements before you measure them and record them. Repeat this procedure with other discs, having the students guess the circumference given the diameter.

Page 2.

Discuss π, formulas $C = \pi d$, $C = 2\pi r$.

Page 3.

Show the slide of the circle of radius four units on the grid. Let each student estimate the area by counting unit squares. Record various guesses.

Page 4.

Show the transparency of the circle divided into the sectors. Ask for suggestions for rearranging the pieces in order to find the area of the circle. Discuss any suggestions. On the overhead projector place the cardboard or paper circle cut into sectors and rearrange these pieces to form a "rectangle".

Discuss the area formula. Use the formula to find the area of a circle of radius four units. Compare this with previous estimations.

Page 5.

Have the students copy the table and fill in the missing numbers. Check answers. Do additional exercises if time permits.

Have students complete and hand in the sheet of Review Exercises.

APPENDIX B

TEACHER AND STUDENT INSTRUCTIONS FOR THE EXPERIMENTAL PROGRAM

Introducing the students to the laboratory program.

Student instructions for laboratory work.

Sample Mathematics Laboratory class schedule.

General guidlines for teaching the Class Discovery lessons.

Schedule of Class Discovery lessons.

INTRODUCING THE STUDENTS TO THE LABORATORY PROGRAM

Before each class has its first laboratory period, the students should be oriented to the laboratory program. The following points should be explained:

1. Laboratory work will make up part of their mathematics course between now and Easter. Laboratory periods will be held once a week in a regularly scheduled mathematics period.

2. Learning mathematics in the lab will be different from the way it is studied in the classroom in several ways.

(a) In the lab there are 20 stations instead of rows of desks. Two students work at each station.

(b) At each station there is some type of concrete material. These objects are to help the learner understand the problem being investigated and to begin to solve it.

(c) The lab lessons deal with problems and ideas not specifically taught in the textbook. There are ten different lab lessons so it will take ten weeks to do them all.

(d) A teacher cannot give ten different lessons to fifteen groups of students at the same time; instead there are sets of written instructions explaining what to do. The teacher will be present to give help when necessary, but pairs of students are to work mainly on their own.

(e) Students can work at their own speed. Most students will probably <u>not</u> get through all the pages of instruction for some of the lessons and it is not necessary to do so. However, they should work carefully and what they do complete, should be done well.

3. At the end of each lab period they will do a short review sheet so they can tell some of the things they may have learned. After they have completed all of the labs there will be some tests to see what was learned and how they may have benefited from the lab experience. Their lab mark, however, will depend not so much on test scores as on the way they work in the lab.

4. Tour of the lab -- point out the following:

(a) Twenty stations numbered 1 - 20. Stations 1 - 10 are the same as 11 - 20.

(b) Location of the instruction sheets and review sheets; boxes for depositing completed instruction and review sheets.

(c) The concrete materials at each station. Before leaving the lab each week the students should tidy up the station, leaving all materials as they found them. Missing or damaged materials should be reported.

5. Students are given lab instruction sheets and schedules of their lab periods.

STUDENT INSTRUCTIONS FOR WORKING IN THE MATH LAB

1. Before entering the lab each week, check the Lab Schedule sheet to find out which activity you are to work on.
2. One member of the team picks up one set of lab instructions for the activity.
3. Go to the appropriate station and do the activity as outlined in the instructions. Write all answers and comments on the instruction sheets.
4. About six minutes before the end of each lab period, one member of each team hands in the set of instructions and picks up a Review Exercise sheet for each person. While this is being done, the other team member should straighten up the station (return concrete materials to boxes, etc.).
5. Each person completes the review sheet. The purpose of these exercises is to indicate to you the kinds of things you have learned during the period.
6. As you leave the lab, hand in the Review Exercises sheet.

Note:

1. If your lab partner is absent, you may either work alone for that period or join the other group working on the same activity. For example, if you are to do Lab 4, go to Station 14 which has the same activity.
2. You are <u>not</u> expected to finish most sets of instruction cards. Take your time, read the instructions carefully, and before you go on to something new be sure you have completed the previous tasks.
3. Your teacher will give you help when needed but try to do as much as you can on your own.

7B

SAMPLE MATH LAB SCHEDULE

Tuesday, Mod 3-4

Names \ Week	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Date	Jan. 14	21	28	Feb. 4	11	18	25	Mar. 4	11	18
Janet B. Betty C.	7	2	5	6	4	1	9	3	8	10
Scot B. Michael H.	10	7	2	5	6	4	1	9	3	8
Peter M. Roy E.	8	10	7	2	5	6	4	1	9	3
Roxanne S. Laura M.	3	8	10	7	2	5	6	4	1	9
Laurie D. Lynette K.	17	12	15	16	14	11	19	13	18	20
Ted D. Ronald P.	20	17	12	15	16	14	11	19	13	18
Dale G. Leslie P.	18	20	17	12	15	16	14	11	19	13
Walter M. David K. Bruce M.	9 19	3 13	8 18	10 20	7 17	2 12	5 15	6 16	4 14	1 11
Sharyn K. Diane P. Sandra M.	1 11	9 19	3 13	8 18	10 20	7 17	2 12	5 15	6 16	4 14
George G. Dale K.	13	18	20	17	12	15	16	14	11	19

CLASS DISCOVERY LESSONS -- GENERAL GUIDELINES FOR TEACHERS

1. These lessons provide the students with a change or break from the regular mathematics program. The students should be made to feel at ease as they investigate interesting, new mathematical problems. It is hoped they will come to regard these periods as "fun."

2. The lessons generally follow the same pattern as the laboratory activities. Modifications and suggestions for each lesson are outlined on the Classroom Procedure sheets which follow each set of lab instructions.

3. The purposes of the concrete materials in a lesson are to:

   (a) stimulate interest,

   (b) provide a concrete or real setting for the concept to be investigated or problem to be solved,

   (c) provide a means for checking hypotheses.

4. The lessons should be taught by the "discovery" approach. Allow the students to investigate various solutions or answers before giving the desired conclusions. The students should be encouraged to make hypotheses and to test them out. Let them guess. Permit and encourage alternate solutions and methods.

5. Do not stress precise language or symbolism in the early stages of a lesson. Allow the students to formulate answers and methods in their own words.

6. Notwithstanding (4) and (5), it is the responsibility of the teacher to structure and control the development of each lesson so that all important ideas are dealt with and desired conclusions specifically and clearly stated.

7. With about six minutes remaining, give each student the Review Exercises sheet. Collect these from the students.

SCHEDULE OF CLASS DISCOVERY LESSONS

Class \ Week	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
7A	5	6	4	1	9	3	8	10	7	2
7C	6	10	1	8	4	9	5	3	7	2
8F	1	3	9	10	4	6	8	5	2	7
8D	8	2	4	10	7	5	9	6	1	3

APPENDIX C

TESTING INSTRUMENTS

STM-1 Posttest

STM-2 Posttest

Immediate Learning Subtests (Review Exercise Sheets)

Cumulative Achievement Test

Higher Level Thinking and Problem Solving Test

Cards Uses in Mathematics Test

(a) Categories of responses
(b) Sample student responses
(c) Data for computing Kendall's Coefficient of Concordance W for agreement among 8 judges on 8 student test papers

A Mathematics Study

Learning and Doing Mathematics Scale

(a) Factor analysis of posttest scores -- varimax orthogonal rotations

Mathematics Laboratory Student Questionnaire

Class Discovery Student Questionnaire

Teacher Questionnaire

STM-1 POSTTEST

INSTRUCTIONS: For each of the following exercises select the best of the four answers.

1. Find the sum of the following: 8650, 7584, 1360 and 7662

   A. 25,266   B. 25,056   C. 25,256   D. none of these

2. The sum of 679, 519, 306, 476, 493 is

   A. 2573   B. 2473   C. 2581   D. 2463

3. The product of 650 and 27 is

   A. 677   B. 17,550   C. 17,350   D. none of these

4. Subtract 5058 from 6805. The difference is

   A. 1747   B. 1682   C. 1743   D. 1862

5. Find the quotient of 129,804 and 174

   A. 740 r 0   B. 746 r 0   C. 740 r 44   D. 74 r 44

6. Subtract 708 from 51772. The difference is

   A. 50,098   B. 51,066   C. 51,064   D. 50,064

7. Multiply 88 by 12. The product is

   A. 1046   B. 1056   C. 946   D. 1052

8. Divide 171380 by 19. The quotient is

   A. 92 r 0   B. 930 r 17   C. 9020 r 0   D. 9200 r 110

9. $\{0, 3, 6 \ldots , 15000000\}$ is

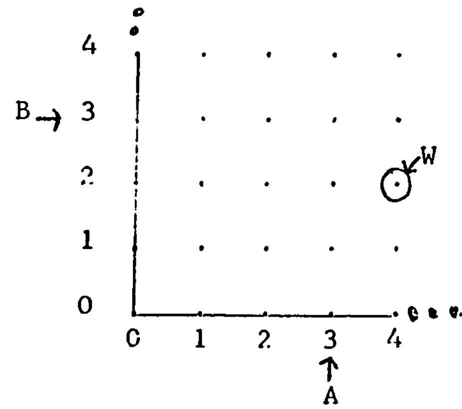
   A. a set of odd numbers

   B. a condition

   C. a finite set

   D. an infinite set

10. In the graph on the right, the second component of the ordered pair (2, 3) should be associated with ________.
(U = N X N)



A. point A only

B. point B only

C. either A or B

D. neither A nor B

11. In the graph shown in question 34 the W is associated with the point

A. (4)

B. (4, 2)

C. (2)

D. (2, 4)

12. R = {0, 1, 2, 3} and S = {2, 3, 4, 5}. The natural number associated with R X S is

A. 6 B. 8 C. 15 D. 16

13. The universe for (x, y) = N X N. Select the solution set.

$x - y = 32 \wedge y + 4 < 7$

A. {(32, 2) (33, 1) (34, 0)}

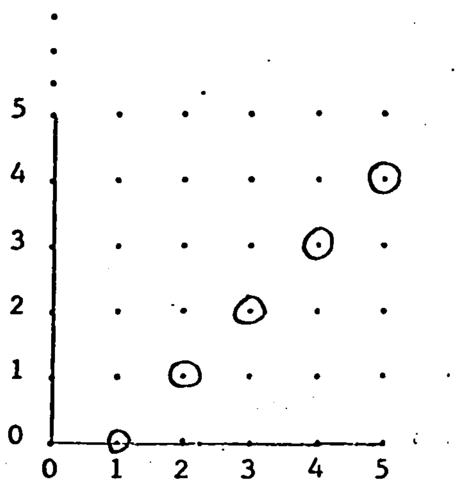
B. {(32, 0) (33, 1) (34, 2)}

C. {(2, 32) (1, 33) (0, 34)}

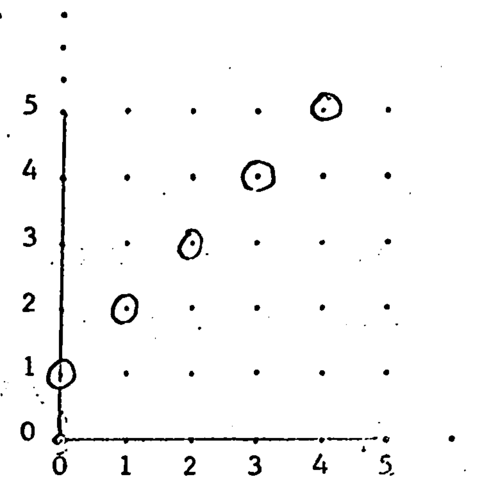
D. {(0, 32) (1, 33) (2, 34)}

14. Which of the graphs pictured below is the graph for $\{(x, y) \mid y = 5 - x\}$
The universe for (x, y) is $N \times N$

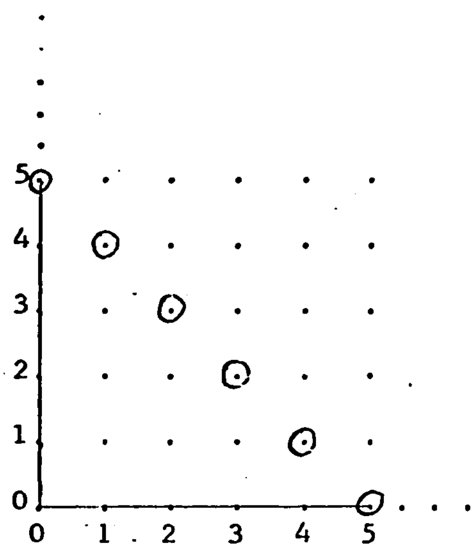
A.



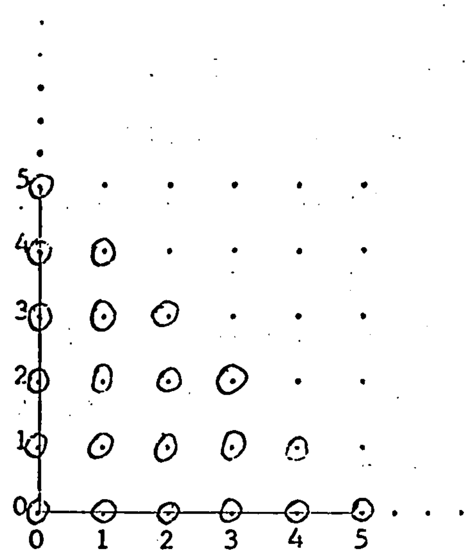
B.



C.



D.



15. Mrs. Murphy bought fewer than 5 pounds of apples. Altogether she bought 7 pounds of apples and oranges. How many pounds of apples and how many pounds of oranges could she have bought? Select the correct condition.

A. $x < 5 \wedge x + 7 > y$

B. $x < 5 \wedge x + y = 7$

C. $x < 5 \wedge 7 + x = y$

D. $x + y < 7 \wedge x < 5$

16. A farmer has 32 more steers than hogs. If he had 47 more steers, he would have 172. How many steers and how many hogs does the farmer have. $U = N \times N$

A. $\{(140, 47)\}$ B. $\{(79, 93)\}$ C. $\{(125, 93)\}$ D. $\{(93, 79)\}$

17. The sum of two numbers is less than 12. The second number is twice the first number. What can the numbers be? U = N X N

A. $\{(0, 1), (1, 3), (2, 5), (3, 6)\}$

B. $\{(0, 0), (1, 2), (2, 4), (3, 6)\}$

C. $\{(0, 2), (1, 3), (2, 4), (3, 5), (4, 6)\}$

D. $\{(1, 3), (2, 4), (3, 5), (4, 6), (5, 7)\}$

18. When the first of two numbers is subtracted from the second number, the difference is 9. The first number is less than 7. U = N X N. Which ordered pair listed below is a member of the solution set?

A. (1, 10)   B. (4, 13)   C. (5, 14)   D. all of these

19. Altogether, Mrs. Jones baked fewer than 7 pies and cakes. She baked fewer than 5 pies. Select the sentence that expresses a condition for the problem. U = N X N

A. $x + y < 7 \wedge x < 5$

B. $x > 7 \wedge x - 5 < 5$

C. $x < 5 \wedge x + y = 7$

D. $x < 5 \wedge 7 + x = y$

20. Elizabeth has finished more experiments for science class than Nancy has. When Elizabeth finishes 8 more, she will have finished 21 experiments. How many experiments can Nancy have finished? U = N X N

A. 13

B. (13, 10)

C. 0, 1, 2, ..., 13

D. 0, 1, 2, ..., 12

21. Three years from now, Bill will still be less than 21 years old. Five years ago, he was more than 9 years old. How old can Bill be now? U = N.
This problem was solved as follows. U = N

Let x,be Bill's present age in years
$x + 3 < 21 \wedge x - 5 > 9$

$\{0, 1, 2, ..., 17\} \cap \{16, 17, 18, ...\} = \{16, 17\}$
Bill is either 16 or 17 years old.

Answer one of the following:

A. The compound condition is incorrect

B. One of the simple conditions has the wrong solution set.

C. The problem was solved correctly.

D. The intersection was performed incorrectly.

22. Which of the following is <u>not</u> an explanation of why 6/8 and 12/16 are equivalent?

A. 6(16) = 12(8)

B. If each component of 6/8 is multiplied by 2, 12/16 is obtained

C. $6 < 8$ and $12 < 16$.

D. $6/8 \sim 3/4$ and $12/16 \sim 3/4$.

23. ______ does not contain the rate pair that represents two eggs to three cups of milk.

A. 2/3

B. (2, 3)

C. $\{2, 3\}$

D. 2:3

24. _______ is a member of $\{2/3 \ldots\}$ and the sum of its components is 20.

A. 6/14

B. 8/12

C. 10/15

D. 4/16

25. To find the solution of $175/200 \sim 252/n$, you should first find the solution of _____. U = C

A. 175 + n = 200 + 252

B. 200n = 175(252)

C. 175(200) = 252n

D. 175n = 252(200)

26. $\{(x, y) | x/y \sim 3/4$ does <u>not</u> contain the rate pair

A. 12/15

B. 6/8

C. 27/36

D. 9/12

27. Mrs. Adams bought 18 ears of corn priced at 6 ears for 29¢. U = C. The sentence that expresses the condition for the problem would be

A. $6/18 \sim x/29$

B. $6/29 \sim x/18$

C. $6/29 \sim 18/x$

D. none of these

28. 1536 is 32% of what number.

A. 4915.2

B. 491.52

C. 480

D. none of the above

29. _____ expresses the condition "18 is 36% of what number?"

A. $x/18 \sim 36/100$

B. $18/x \sim 36/100$

C. $36/18 \sim x/100$

D. $18/36 \sim x/100$

30. Ann typed 315 words in 7 minutes. Which condition expressed below should you use to find how many words Ann typed per minute? The universe for the variables is C.

A. $x/1 \sim 315/7$

B. $x/1 \sim 300/60$

C. $x/315 \sim 7/1$

D. $2205/5 \sim 1101/x$

31. What number is 16% of 425?

A. 92

B. 3481.25

C. 2656.25

D. 68

32. 47 is what per cent of 94?

A. 200

B. 150

C. 50

D. 100

33. A certain rate pair is equivalent to 3/5 and has a second component that is 16 more than the first component. Which condition should be used to find the rate pair? U = C X C

A. $x/y \sim 3/5 \wedge x + y = 16$

B. $x/y \sim 3/5 \wedge y - x = 16$

C. $x/y \sim 3/5 \wedge 16 > x - y$

D. $x/y \sim 3/5 \wedge y + 16 = x$

34. Mary wants to buy a phonograph that sells for $40. She has already saved 30% of the amount she needs. Which condition should you use to find how many more dollars she must save to buy the phonograph? U = C X C

A. $x/40 \sim 30/100 \wedge 40 + x = y$

B. $40/x \sim 30/100 \wedge y - x = 40$

C. $x/40 \sim 30/100 \wedge 40 + y = x$

D. $x/40 \sim 30/100 \wedge x + y = 40$

35. A store gave a 15% discount on a clothes drier that was regularly priced at $220. The net price of this drier amounted to how many dollars?

A. $33

B. $14.67

C. $187.

D. $205.33

36. Mr. Brown borrowed $1250 from his bank. At the end of one year, he paid the bank $1325. What was the rate of interest?

A. 5.66%

B. 94%

C. 6%

D. 75%

37. Mr. Howe sold a house and lot for $28,400. He received a commission of $852. What was his rate of commission?

A. 3.33%

B. 96.67%

C. 97%

D. 3%

38. On a test Bob solved 17 out of 20 problems. Jack solved 27 out of 30 problems on another test. Which of the following would help you determine which boy did better?

A. Bob missed 3 problems and Jack missed 3 problems?

B. Bob solved 85% of the problems on his test and Jack solved 90% of the problems on his test.

C. Jack solved 10 more problems than Bob solved.

D. Bob solved 17% of the problems on his test and Jack solved 27% of the problems on his test.

39. The standard name for $3^2 \cdot 10^3$ is

A. 180

B. 270

C. 6000

D. 9000

40. The number 80000 expressed in scientific notation is

A. 8×10^4

B. 8×10^5

C. 80×10^3

D. $2^3 \times 10000$

41. The expanded form of the numeral 5600 is

A. 5.6×10^3

B. 56×10^2

C. $5(10)^3 + 6(10)^2 + 0(10)^1 + 0(1)$

D. 56^{100}

42. In scientific notation we would express 3,000,000 as

A. 3×10^6

B. 6×10^3

C. $3^6 \times 10$

D. 10×6^3

43. The numbers 5^3, 2^{12}, 3^5, 12^2 arranged in order from the smallest first to the largest last would read

A. $2^{12}, 3^5, 5^3, 12^2$

B. $5^3, 12^2, 3^5, 2^{12}$

C. $12^2, 5^3, 3^5, 2^{12}$

D. $5^3, 3^5, 12^2, 2^{12}$

44. The set which is equivalent to $\{\triangle, \square, \bigcirc, \triangledown\}$ is

A. $\{4\}$

B. $\{0, 1, 2, 3, 4\}$

C. $\{2, 4, 6, 8\}$

D. $\{\triangle, \square, \bigcirc\}$

45. U = N. The set which is <u>NOT</u> a proper subset of $\{0, 1, 2, 3\}$ is

A. $\{\ \}$

B. $\{0, 3\}$

C. $\{x \mid x = 3\}$

D. $\{x \mid x < 4\}$

46. Which of the following quotients cannot be found?

A. $24 \div 0$

B. $0 \div 24$

C. $85 \div 17$

D. $182 \div 13$

47. The standard name for $600 \div 4 \times 5 + 5$ is

A. 24

B. 35

C. 755

D. 1500

48. The use of the distributive property of multiplication over addition is illustrated in the true statement.

A. $3 \times (17 + 2) = (3 \times 17) + 2$

B. $3 \times (17 + 2) = 3 \times (2 + 17)$

C. $3 \times (17 + 2) = (3 \cdot 17) + (3 \cdot 2)$

D. $3 \times (17 \cdot 2) = (3 + 17) \cdot (3 + 2)$

49. An example of the associative property of addition is

A. $m (n + p) = mn + mp$

B. $(a + b) + c = a + (b + c)$

C. $d + e$ is a member of N

D. $x + y = y + x$

50. The set of numbers which is closed under subtraction is

A. the set of even natural numbers

B. the set of odd natural numbers

C. the set of natural numbers

D. none of the above

MATHEMATICS EIGHT
(STM-2 POSTTEST)

Name______________________

INSTRUCTIONS: Four responses are listed for each exercise. Select the response that best completes the exercise.

1. The reciprocal of $^{+}\frac{2}{3}$ is

(a) $^{-}\frac{2}{3}$ (b) $\frac{1}{^{-}\frac{3}{2}}$ (c) $^{+}\frac{3}{2}$ (d) $\frac{1}{^{-}\frac{2}{3}}$ 1. ________

2. The quotient $7^{-16} \div 7^{8}$ is equal to 2. ________

(a) 7^{-2} (b) 7^{-8} (c) 7^{8} (d) 7^{-24}

3. If two rational numbers of arithmetic are reciprocals of each other, then their product is

(a) $\frac{0}{1}$ (b) $\frac{1}{0}$ (c) $\frac{1}{1}$ (d) $\frac{0}{0}$ 3. ________

4. An example of a basic fraction is

(a) $\frac{15}{54}$ (b) $\frac{11}{23}$ (c) $\frac{27}{45}$ (d) $\frac{72}{50}$ 4. ________

5. If the coordinate of point M is $\frac{2}{3}$ and the coordinate of point N is $\frac{4}{5}$, then m(MN) =

(a) $\frac{2}{3} - \frac{4}{5}$ (b) $\frac{2}{3} + \frac{4}{5}$ (c) $\frac{4}{5} - \frac{2}{3}$ (d) $\frac{4}{5}\left(\frac{2}{3}\right)$ 5. ________

6. The set of rational numbers is equal to

(a) $R_n \cup (\{0\} \cap R_p)$ (c) $(\{0\} \cup R_n) \cap R_p$

(b) $R_p \cap R_n \cap \{0\}$ (d) $R_p \cup \{0\} \cup R_n$ 6. ________

7. The rational number $\frac{-2}{5}$ is indicated by

(a) $\left(\frac{2}{5},0\right)$ (b) $\left(\frac{3}{5},1\right)$ (c) $\left(0,\frac{3}{5}\right)$ (d) $\left(1\frac{2}{5},2\right)$ 7. ________

8. Another name for 3^{5} is

(a) $\frac{5}{3}$ (b) 15 (c) 81 (d) 243 8. ________

9. The second power of $\frac{-3}{7}$ is

(a) $\frac{-9}{21}$ (b) $\frac{+9}{49}$ (c) $\frac{-9}{49}$ (d) $\frac{+6}{14}$ 9. ____________

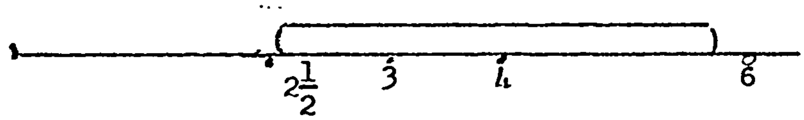
10. From the definition of equivalent ordered pairs for directed segments, (a,b) ~ (c,d) if
The universe for a, b, c, and d is R_a

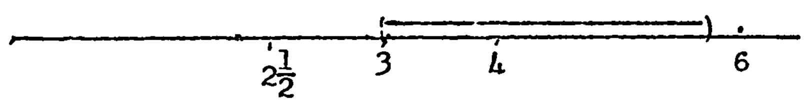
(a) ac=bd (c) ad = cb

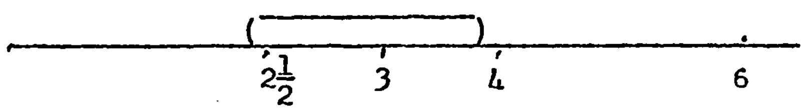
(b) a + d = c + b (d) a + c = b + d 10. ____________

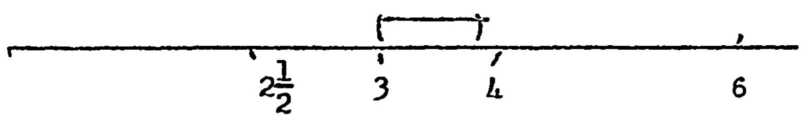
11. The graph of the solution set of

$\{x \mid 3 \leq x < 6\} \cap \{x \mid 2\frac{1}{2} < x < 4\}$ when $U = R_a$ is

A.


B.


C.


D.


12. U = N. The solution set of $\{x \mid x + 13 > 21\}$ is

A. {9, 10, 11, ...}

B. {8}

C. $\{x \mid x \geq 8\}$

D. $\{x \mid x \leq 8\}$

13. $\{x \mid \sim (x = 2) \vee (x < 2)\} =$

A. $\{x \mid x < 2 \vee x = 2\}$

B. $\{x \mid x < 2 \vee x > 2\}$

C. $\{x \mid x > 2 \wedge x > 2\}$

D. $\{x \mid x \geq 2\}$

14. $U = R_a$. The solution set of $(s + 6.4 > 9.9) \vee (s + 2 = 5.5)$ is

A. $\{3.5\}$

B. $\{s \mid s \geqq 3.5\}$

C. $\{s \mid s > 3.5\}$

D. $\{s \mid s \leqq 3.5\}$

15. Two trains travel the 180 miles between Edmonton and Calgary daily. The night train travels at an average rate of 60 m.p.h. The day train travels at an average rate of 75 m.p.h. How long can it take to go from Edmonton to Calgary. (The universe for each variable is Z).

x, the number of hours it can take to travel from Edmonton to Calgary. The best condition for the above problem is

A. $60 / 1 \sim 180 / x \vee 75 / 1 \sim 180 / x$

B. $60 / 1 \sim 180 / x \wedge 75 / 1 \sim 180 / x$

C. $60 / x \sim 180 / 1 \vee 75 / x \sim 180 / 1$

D. $1 / 60 \sim 180 / x \vee 1 / 75 \sim 180 / x$

16. If 8 sneezes is the same measure as one snort, then how many snorts is the same measure as 1.26 sneezes? $U = R_a$.

x, the number of snorts that is the same measure as 1.26 sneezes. Choose the best condition for the problem.

A. $8 / 1 \sim x / 1.26$

B. $8 / 1.26 \sim x / 1$

C. $1.26 / 8 \sim 1 / x$

D. $8 / 1 \sim 1.26 / x$

17. George spent at least six dollars for lumber. He gave the clerk twenty dollars and received more than five dollars in change. How much did George spend for lumber? $U = Z$.

x, the amount George spent for lumber. The best condition for the above problem is

A. $x > 6 \wedge x - 20 > 5$

B. $x \geqq 6 \wedge x - 20 > 5$

C. $x > 6 \wedge 20 - x > 5$

D. $x \geqq 6 \wedge 20 - x > 5$

18. The set of positive rational numbers is not closed under

A. addition

B. subtraction

C. multiplication

D. division

Mathematics Eight (Cont'd.)

19. A decimal numeral naming $\frac{^-5}{6}$ is

A. $^-.83$ C. $^-.8\overline{3}$

B. $^-.833$ D. $^-.\overline{83}$

20. $U = Z_r$. Which of the following conditions does not have a solution?

A. $3\frac{1}{2} - x = 5$

B. $\frac{3}{4} + x = \frac{6}{8}$

C. $2x = \frac{13}{12}$

D. $\frac{^-1}{2}x = 25$

21. $(\frac{6}{13} + \frac{4}{7}) + 1 = \frac{6}{13} + (\frac{4}{7} + 1)$ is a true statement because of the following property:

A. associative property of addition

B. distributive property of multiplication over addition

C. commutative property of multiplication

D. commutative property of addition

22. $\frac{2}{7}(\frac{1}{9} + \frac{3}{5}) = \frac{2}{7}(\frac{3}{5} + \frac{1}{9})$ is a true statement because of the following property:

A. associative property of addition

B. distributive property of multiplication over addition

C. commutative property of multiplication

D. commutative property of addition

23. The universe for each variable is R. The condition $xy \in R$ expresses the

A. commutative property of multiplication

B. closure property of multiplication

C. identity element property of multiplication

D. inverse property of multiplication

24. What is the quotient of $^{+}\frac{15}{16}$ and $^{-}\frac{3}{4}$? ($^{+}\frac{15}{16}$ is the dividend.)

A. $\frac{-4}{5}$
C. $^{-}\frac{5}{4}$
B. $\frac{+4}{5}$
D. $^{+}\frac{5}{4}$

25. What is the sum of $^{+}5.33$ and $^{-}1.75$?

A. $^{+}7.58$
C. $^{+}4.08$
B. $^{-}4.08$
D. $^{-}7.58$

26. What is the product of $^{+}6.4$ and $^{-}3.9$?

A. $^{+}2.5$
C. $^{-}24.36$
B. $^{+}10.3$
D. $^{-}24.96$

27. What is the difference of $^{+}4\frac{3}{4}$ and $^{+}4\frac{1}{6}$? ($^{+}4\frac{3}{4}$ is the subtrahend)

A. $\frac{-7}{12}$
B. $\frac{+7}{12}$
C. $\frac{-1}{2}$
D. $\frac{+1}{2}$

28. 9380 expressed in scientific notation is

A. 938.0×10^{1}
C. 9.380×10^{3}
B. 93.80×10^{2}
D. $.9380 \times 10^{4}$

29. The standard name for 9.89×10^{-5} is

A. .00000989
C. 989000
B. .0000989
D. .989000

30. The repetend of the decimal for $\frac{1}{33}$ is

A. 3
C. 03
B. 32
D. 30

31. The product of $(\frac{-11}{13})^{6}$ and $(\frac{-11}{13})^{4}$ expressed in exponential notation is

A. $(\frac{-11}{13})^{2}$
C. $(\frac{-11}{13})^{-10}$
B. $(\frac{-11}{13})^{10}$
D. $(\frac{-11}{13})^{-2}$

Mathematice Eight (Cont'd.)

32. I_p I_n $\{0\} =$

A. I

B. R_a

C. R

D. $\{\ \}$

33. U = R. R_p is the union of

A. R_n and R_p

B. R_p and $\{0\}$

C. R_n and $\{0\}$

D. I_n and $\{0\}$

34. The reciprocal of $^{+}\frac{2}{3}$ is

A. $^{-}\frac{2}{3}$

B. $\frac{1}{^{-}\frac{3}{2}}$

C. $^{+}\frac{3}{2}$

D. $\frac{1}{^{-}\frac{2}{3}}$

35. The additive inverse of $\frac{^{+}27}{3}$ is

A. $\frac{^{+}3}{27}$

B. $\frac{^{-}3}{27}$

C. $^{+}\frac{27}{3}$

D. $^{-}9$

36. $^{-}37.4 = 5m + \frac{1}{2}m$ is equivalent to

A. $^{-}37.4 = 5\frac{1}{2}\ (m + m)$

B. $^{-}37.4 = (5 + \frac{1}{2})\ m^2$

C. $^{-}37.4 = 5\frac{1}{2}m$

D. $^{-}37.4 = (5 + \frac{1}{2})m$

37. The multiplicative inverse of $^{-}\frac{13}{5}$ is

A. $^{+}\frac{13}{5}$

B. $\frac{^{+}5}{13}$

C. $\frac{^{-}5}{13}$

D. $^{-}2\frac{3}{5}$

38. The sum of $^{-}\frac{3}{4}$ and $^{-}3$ is

A. $^{-}3\frac{3}{4}$

B. $^{-}2\frac{1}{4}$

C. $^{+}\frac{9}{4}$

D. $^{+}\frac{15}{4}$

39. The sum of $^{+}25$, $^{+}15$, $^{-}25$, $^{-}15$ is

A. 0 B. $^{+}30$ C. $^{-}80$ D. $^{+}80$

40. The product of $^{-}2.65$ and $^{-}.32$ is

A. $^{+}8.480$ B. $^{-}.8480$ C. $^{+}.8480$ D. $^{-}84.80$

41. What is the quotient of 5 and 2? (2 is the divisor)

A. .4 B. $^{-}2.5$ C. $^{+}2.5$ D. $^{-}.4$

42. One year the lowest temperature recorded in Edmonton was 28.2° below zero. The lowest temperature recorded in Calgary that year was 10.6° higher than this. What was the lowest temperature recorded in Calgary that year. U = R.

x, the number of degrees that was the lowest temperature in Calgary that year.

The best condition for the problem is

A. $28.2 + 10.6 = x$ C. $x - {}^{-}28.2 = 10.6$

B. ${}^{-}28.2 - x = 10.6$ D. $x + 10.6 = 28.2$

43. What numbers can be subtracted from $9\frac{1}{2}$ so that it is not the case that each difference is greater than four? U = R.

The best condition for this problem is

A. $9\frac{1}{2} - x > 4 \wedge \sim (x > 4)$

B. $\sim (x - 9\frac{1}{2} > 4)$

C. $9\frac{1}{2} - x > 4 \wedge x \leqq 4$

D. $\sim(9\frac{1}{2} - x > 4)$

44. The expanded form of 50.31 is

A. $5(10^{1}) + 0(10^{0}) + 3(10^{-1}) + 1(10^{-2})$

B. $5(10^{2}) + 0(10^{1}) + 3(10^{-1}) + 1(10^{-2})$

C. $5(10^{2}) + 0(10^{1}) + {}^{-}3(10^{-1}) + {}^{-}1(10^{-2})$

D. $5(10^{2}) + 0(10^{1}) + {}^{-}3(10\ ) + {}^{-}1(10^{2})$

45. The complete factorization of 625 is

A. 6.25×10^{2} C. 5^{4}

B. 5^{5} D. $5^{2} \times 25^{1}$

Name ____________________ Grade ___ Date ____________

No. 1 (11) AREA AND PERIMETER Review Exercises (IL SUBTEST)

1. A rectangle is made from 50 unit squares.

   What is its area? ______

2. Unit squares are arranged into 8 rows and 3 columns to form a rectangle.

   (a) What is the area of the rectangle? ______

   (b) What is the perimeter of the rectangle? ______

3. Given 36 unit squares, what would be the dimensions of the rectangle you could make (using all of the squares) which would have

   (a) the largest perimeter? _____ by _____

   (b) the smallest perimeter? _____ by _____

Name ____________________ Grade ___ Date ____________

No. 2 (12) INTERSECTING SETS Review Exercises

Answer the following questions about the 24 shapes you have been investigating.

1. (a) How many are blue? _____

   (b) How many are small? _____

   (c) How many are both small and blue (small-blues)? _____

   (d) How many are either blue or small or both? _____

2. How many are either circular or triangular or both? _____

Name ______________________ Grade ___ Date _________________

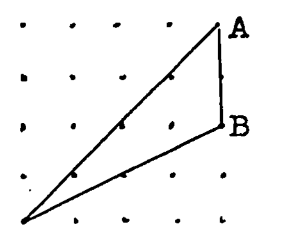
No. 3 (13) SYMMETRIES OF A SQUARE Review Exercises

The following questions are to be answered with reference to the four moves I, V, H, and R as defined in the lesson.

1. R * I = ____
2. V * ___ = R
3. H * ___ = I
4. H * (R * V) = (H * R) * ___
5. I * H * H * V * V * R * R * I * H = ___

Name ______________________ Grade ___ Date ______________

No. 4 (14) AREAS OF POLYGONS Review Exercises

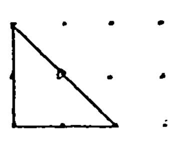


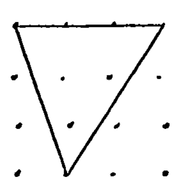
1. Taking AB as the base of the triangle at the right, what is its height?

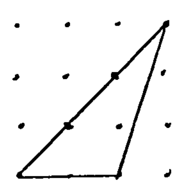
_____ units

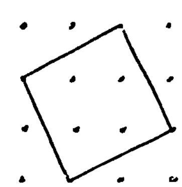
2. Let this represent 1 unit of area. 

Find the areas of the following figures.









_____ _____ _____ _____

Name ____________________ Grade ___ Date ______________

No. 5 (15) HOW MANY SUBSETS? Review Exercises

1. How many of the following sets are <u>different</u> subsets of $\{R, O, G, B, Y\}$?

   $\{R, G, O\}$, $\{X, B, Y\}$, $\{\ \}$, $\{R, O, B, Y, G\}$, $\{B, Y\}$, $\{Y, B\}$

   ______

2. How many subsets are there of sets containing

   (a) 1 member? ______

   (b) 5 members? ______

3. A set with 7 members has 128 subsets. How many subsets does a set containing 8 members have? ______

4. How many subsets of the set $\{a, b, c, d\}$ contain exactly 3 members? ______

Name ____________________ Grade ___ Date ______________

No. 6 (16) MATHEMATICAL BALANCE Review Exercises

1. The commutative property for multiplying natural numbers states that if a and b are natural numbers, $a \times b = b \times a$. How could you demonstrate this principle using a balance?

   ____

2. A prime number is a number whose only factors are 1 and itself. For example, 5 and 19 are prime numbers but 12 is not prime since 12 can be written as 3×4 etc.

   How could you use a balance to demonstrate that 7 is prime?

   ____

3. Solve the following condition. Show how you do it.

   $8n + 4 = 3n + 34$

Name ____________________ Grade ___ Date ____________

No. 7 (17) NUMERATION IN BASES 3 AND 5 Review Exercises

1. How would the number seven be written in base three? ________

2. Add in base three

$$\begin{array}{r} 1\ 2\ 1 \\ +\ 1\ 2\ 1 \\ \hline \end{array}$$

3. Subtract in base five

$$\begin{array}{r} 4\ 4\ 0 \\ -\ 3\ 1\ 2 \\ \hline \end{array}$$

Name ________________________ Grade ____ Date _______________

No. 8 (18) PROBABILITY Review Exercises

1. If you flip a coin, what is the probability it will come down heads? _________

   If you were to actually flip a coin 100 times, what would you expect the results to be?

   ____

2. If a die is thrown, what is the probability a four will appear? _________

3. An urn has 20 beads of which 5 are white and 15 are black. What is the probability of shaking

   (a) a white bead? _____

   (b) a black bead? _____

Name ______________________ Grade ___ Date ______________

No. 9 (19) MEASUREMENT OF A CIRCLE Review Exercises

1. A circle has a diameter of 7 inches. What is its circumference? ______

2. A circle has a circumference of 50 inches. What is its radius? ______

3. If you know the circumference and the radius of a circle, how could you find the area?

   Area = ________________________________

3. A circle has a radius of 5 inches. What is its area?

   ____

Name ______________________ Grade ___ Date ______________

No. 10 (20) POLYHEDRA Review Exercises

1. State Euler's Rule for polyhedra.

   ____

2. A certain polyhedron has 8 faces and 12 vertices. How many edges does it have? ________

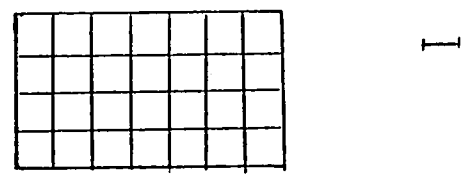
3. A pentagonal prism has 7 faces of which _____ are pentagons.
   A pentagonal pyramid has 6 faces of which ______ are pentagons.
   A regular dodecahedron has 12 faces of which ____ are pentagons.

SPECIAL MATHEMATICS ACHIEVEMENT TEST
(CA TEST)

In each of the questions choose the best answer from among the five choices. Indicate your choice on the answer sheet by making a mark under the letter corresponding to your answer. Do not write on the question booklet. There is no penalty for guessing so answer every question.

1. What is the perimeter of the given figure?

   A. 18 units
   B. 28 units
   C. 7 units
   D. 22 units
   E. 11 units



2. A rectangle is formed from twelve wooden squares each 1" by 1". What is the area of the rectangle?

   A. 24 square inches B. 12 square inches
   C. 144 square inches D. 48 square inches
   E. It depends on the length and width of the rectangle

3. Of all the rectangles that can be formed by using all of a set of 72 unit squares, what are the dimensions of the one which has the smallest perimeter?

   A. The longest rectangle which can be made.
   B. The rectangle with the greatest length and the smallest width.
   C. The rectangle which is nearest to a square.
   D. The rectangle whose length is twice its width.
   E. All rectangles which can be made have the same perimeter.

4. A table top is 10 feet long and has an area of 50 square feet. What is its perimeter?

   A. 25 feet B. 500 feet C. 15 feet
   D. 20 feet E. 30 feet

A box contains a set of 24 blocks no two of which are exactly alike. Each of the blocks is either red, blue or yellow; cirular, square, triangular or oblong in shape; and is of one of two sizes -- small or large. There are equal numbers of blocks of each color, shape and size in the box.

Answer questions 5, 6 and 7 about this set of blocks.

5. How many blocks are either red or yellow?

A. 16 B. 12 C. 0 D. 4 E. 8

6. How many blocks are both large and blue?

A. 8 B. 20 C. 12 D. 4 E. 0

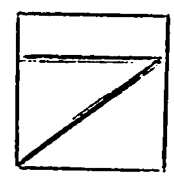
7. There are 3 small-circular blocks. How many blocks are either small or circular (or both)?

A. 18 B. 21 C. 3 D. 12 E. 15

8. A classroom contains both boys and girls. Of the 14 boys in the room, 5 have blue eyes. There are a total of 11 students in the room with blue eyes. One day the teacher announces that any student who is either a boy or who has blue eyes may leave school early. How many students are permitted to leave early?

A. 30 B. 16 C. 25 D. 5 E. 20

Four moves are defined on the transparent plastic square pictured on the right. At the beginning and end of each move the square must be lying flat on the desk at which the person moving the square is working. The moves are:



- V: the square is flipped over vertically (away from you)
- H: the square is flipped over horizontally (tp the left)
- R: the square is turned half-way around to the right
- I: the square is slid to the right without being turned or flipped

If the symbol * means "is followed by", complete the following sentences in this system (questions 9 - 12):

9. H * I = ___

A. V B. H C. R D. I E. none of these

10. H * R = ___

A. V B. H C. R D. I E. none of these

11. V * (H * R) = (V * H) * ___

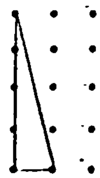
A. V B. H C. R D. I E. none of these

12. V * V * H * H * R * R = ___

A. V B. H C. R D. I E. none of these

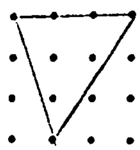
Let the shaded area represent 1 square unit. In questions 13 - 16 determine how many square units are within the given figure.

13.



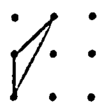
A. $2\frac{3}{4}$
B. $1\frac{1}{2}$
C. $2\frac{1}{2}$
D. $1\frac{3}{4}$
E. 2

14.



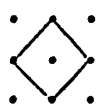
A. $5\frac{1}{2}$
B. $4\frac{1}{2}$
C. 4
D. 6
E. 5

15.



A. $\frac{1}{4}$
B. 1
C. $\frac{1}{2}$
D. $1\frac{1}{4}$
E. $\frac{1}{3}$

16.



A. $3\frac{1}{2}$
B. $2\frac{1}{2}$
C. $1\frac{1}{2}$
D. 2
E. 3

17. How many different subsets are there of the set $\{x\}$?

A. 2 B. 1 C. 0 D. 4 E. 3

18. How many different subsets does the set $\{a, b, c\}$ have?

A. 7 B. 3 C. 6 D. 9 E. 8

19. How many different subsets of $\{w, x, y, z\}$ contain exactly 2 members?

A. 2 B. 6 C. 4 D. 8 E. 5

20. A set containing 5 members has 32 subsets. How many subsets are there of a set with 6 members?

A. 34 B. 16 C. 38 D. 33 E. 64

21. When Bob who weighs 80 pounds sits 6 feet from the center of a teeter-totter, he balances Joe who is sitting 8 feet from the center on the other side. How much does Joe weigh?

A. 60 lbs. B. 70 lbs. C. 78 lbs.
D. 80 lbs. E. 48 lbs.

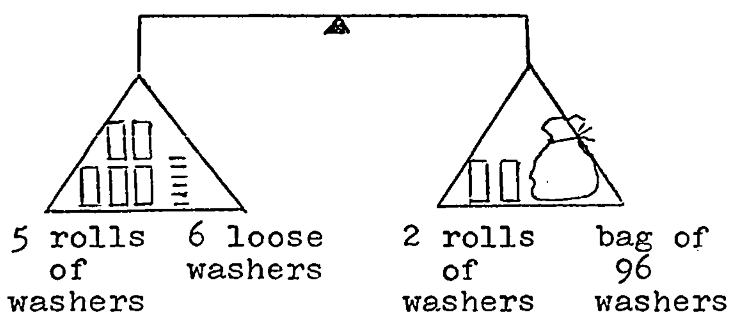
22. John, Sam and Al each weigh 75 pounds. If John sits 3 feet from the center of a teeter-totter and Sam sits 5 feet from the center on the same side as John, how far from the center on the other side must Al sit to balance John and Sam?

A. 15 ft. B. 4 ft. C. 8 ft.
D. $7\frac{1}{2}$ ft. E. 16 ft.

23. Jill and Mary sitting on one side of a teeter-totter balance Ann and Sue sitting on the other side. Jill weighs 65 pounds and is 8 feet from the center; Mary weighs 70 pounds and is 5 feet from the center; Ann is sitting 5 feet from the center and weighs 70 pounds; and Sue weighs 65 pounds. How far is Sue from the center?

A. 10 ft. B. 6.5 ft. C. 5 ft.
D. 7 ft. E. 8 ft.

24. In the diagram there are the same number of washers on each side of the balance. What mathematical condition would one solve to determine the number of washers in each roll?



A. $5 + n + 6 = 2 + n + 96$
B. $5 + n \times 6 = 2 + n \times 96$
C. $5n + 6 = 2n + 96$
D. $5n \times 6 = 2n \times 96$
E. none of these

25. How would the number eight be expressed in base five numeration?

A. 103 B. 13 C. 35 D. 31 E. 53

26. What number does the numeral 102 in base three represent?

A. five B. eleven C. twelve D. nine
E. one hundred and two

27. Add in <u>base five</u>: $\begin{array}{r} 42 \\ +\ \underline{23} \end{array}$

A. 120 B. 65 C. 70 D. 210 E. 110

28. Subtract in <u>base three</u>: $\begin{array}{r} 221 \\ -\ \underline{102} \end{array}$

A. 111 B. 12 C. 122 D. 112 E. 119

29. If a dice is rolled, what is the probability that a five will appear?

A. $\frac{1}{6}$ B. $\frac{1}{5}$ C. 5 D. $\frac{5}{6}$ E. It is not certain

30. Examine the following statements:

1. If a coin is tossed twice, it will always come down heads one time and tails the other.
2. If a coin is tossed 20 times, it would be very unusual if it did not come down heads ten times and tails ten times.
3. If a coin is tossed 100 times, it must land heads between 40 and 60 times.
4. If a coin is tossed 500 times in a row, it is impossible for it to land heads every time.

Which of the above is (are) true?

A. None of them B. Number 3 only C. Number 4 only
D. Numbers 2, 3, and 4 E. All of them

31. A box contains 4 green marbles and 7 red marbles. A marble is drawn from the box without looking in. What is the probability that the marble is green?

A. $\frac{3}{11}$ B. $\frac{7}{11}$ C. $\frac{4}{11}$ D. 4 E. $\frac{4}{7}$

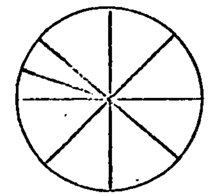
32. A sampling urn contains 30 beads, some white and some black. When the urn is inverted, a bead appears in the bubble in the lid. To determine the number of beads of each color, Mark inverts the urn 15 times. If he observed 4 white beads and 11 black beads, which of the following would be the <u>best</u> guess?

A. 4 white; 26 black
B. 14 white; 16 black
C. 5 white; 15 black
D. 10 white; 20 black
E. 19 white; 11 black

33. A circle has a diameter of 6 inches. Which of the following is the best approximation of its circumference?

A. 19 in. B. 36 in. C. 2 in. D. 17 in. E. 38 in.

34. By dividing a circle into sectors (as illustrated) and rearanging them, it can be demonstrated that the area of a circle is equal to



A. the circumference times the diamter
B. the circumference times the radius
C. $\frac{1}{2}$ the circumference times the diameter
D. $\frac{1}{2}$ the circumference times the radius
E. $\frac{1}{2}$ the diameter times π

35. A circle has a circumference of 62 inches. What, approximately, is its radius?

A. 10 in. B. 20 in. C. 18 in. D. 6.2 in. E. 12 in.

36. A circle has a radius of 3 inches. What, approximately, is its area in square inches?

A. 36 B. 6 C. 28 D. 19 E. 9

37. How many edges does a cube have?

A. 16 B. 12 C. 6 D. 8 E. 4

38. If F stands for the number of faces, V the number of vertices, and E the number of edges of a polyhedron, then

A. $F + V = E$
B. $F + V + E = 12$
C. $F + V + 2 = E$
D. $F + E = V - 2$
E. $F + V = E + 2$

39. A hexagonal prism is made up of

A. 2 hexagons and 6 rectangles
B. 1 hexagon and 6 triangles
C. 2 hexagons and 5 rectangles
D. 1 hexagon and 5 triangles
E. 2 hexagons and 6 triangles

40. A regular icosahedron has 20 faces of which

A. 6 are squares and 14 are triangles
B. 10 are pentagons and 10 are triangles
C. 20 are triangles
D. 10 are squares and 10 are triangles
E. 12 are hexagons and 8 are triangles

SPECIAL MATHEMATICS PROBLEM-SOLVING TEST

(HLTPS TEST)

The purpose of this test is to see how well you are able to solve new problems in mathematics; that is, the questions are based on situations or material you may not yet have studied directly in school. Do not be afraid to write down any answers you may think of, even if you are not sure that they are correct. Work first on those questions that you feel you can answer best, then try the others. Do not spend too much time on any one question. Good luck!

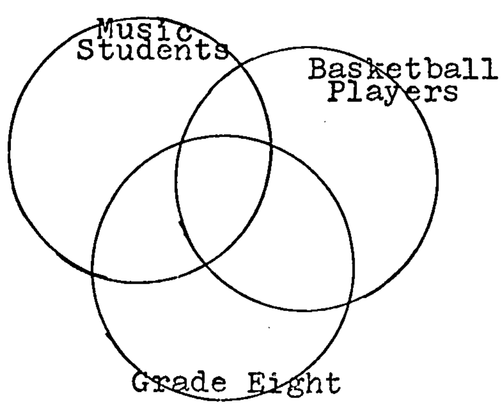
1. You are given a piece of wire 40 inches in length and asked to bend it into the shape of a rectangle.

   (a) What would the perimeter of your rectangle be?

   (b) Draw a diagram showing how you would bend the wire to make the rectangle having the greatest possible area.

   (c) Draw a diagram showing how you would bend the wire to form a rectangle having a very small area.

2. (a) Bob, Mary, Al and Ed are music students. Bill, Don, Ann and Al play basketball. Ed, Al, Ann and John are in grade eight. Write the name of each student in the appropriate section of the diagram below.



   (b) The only languages spoken by the 100 passengers on a train are English and French. If 70 speak English and 60 speak French, how many speak both languages?

3. A class is playing a mathematical game. Bob is chosen to go to the front of the room. He is permitted to move in four different ways as follows:

R: quarter turn to the right (e.g. turn from facing north to east)
L: quarter turn to the left
H: half turn to the right
W: whole turn to the left

If the symbol * stands for "is followed by", the four moves R, L, H, and W together with * make up a mathematical system. This new system has many of the properties of the set of fractions under multiplication.

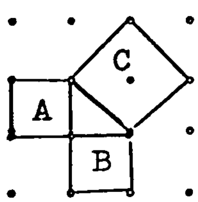
(a) When two fractions are multiplied together, the result is another fraction. What is R * H ?

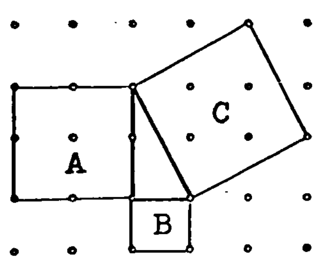
(b) The number 1 is called the <u>identity</u> for multiplying fractions because 1 times any fraction equals that same fraction again. What is the identity element in the system above?

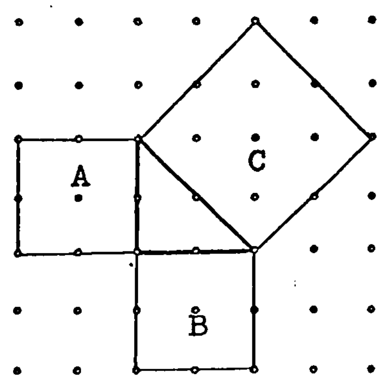
(c) The <u>inverse</u> of the fraction $\frac{2}{3}$ is $\frac{3}{2}$ since $\frac{2}{3} \times \frac{3}{2} = 1$. What is the inverse of L in the system above?

(d) Multiplying fractions is <u>commutative</u>; that is, if a and b are two fractions, a x b = b x a. Is the system above commutative? Illustrate you answer with an example.

4. For each of the rightangle triangles below, squares have been constructed on the sides of the triangle.
(a) For each triangle find the area of the three squares on its sides. Enter the results in the table below.

(1)


(2)


(3)


triangle	Area of square		
	A	B	C
1			
2			
3			

(b) Find and write a rule relating the areas A, B and C

5. (a) A club is electing its executive which consists of a president and a vice-presidient. Four people are running for the two offices. The one recieving the most votes becomes president and the one receiving the second highest number of votes becomes the vice-president.

How many different executives can result from this election?

____

(b) The pattern of numbers below is called Pascal's Triangle.

row							sum of row
0				1			1
1			1		1		2
2			1	2	1		4
3		1	3		3	1	8
4	1	4	6	4	1		16
5	_	_	_	_	_	_	

(i) Use Pascal's Triangle to answer the following question: A set S has four members. How many subsets of S have exactly three members?

____

(ii) Fill in row 5 (above) of Pascal's Triangle.

6. Solve each of the following conditions using a method that does not simply involve trying out different numbers to see if they work. Show step-by-step solutions.

(a) $5n + 2 = 3n + 20$

(b) $4n - 5 = 23$

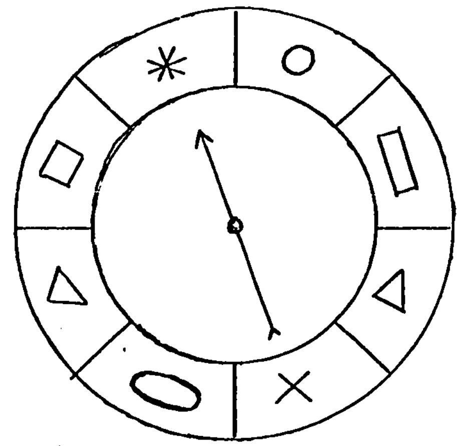
7. (a) Subtract in <u>base six</u>

$$\begin{array}{r} 4\ 3\ 5 \\ -\ 1\ 5\ 1 \\ \hline \end{array}$$

(b) Multiply in <u>base four</u>

$$\begin{array}{r} 1\ 2 \\ \times\ \ 3 \\ \hline \end{array}$$

8. (a) If you spin the arrow on the spinner pictured on the right, what is the probability the arrow will end up pointing to either a circle or a triangle? (Assume the spinner will not stop between choices.)



____

(b) If two coins are tossed at one time, what is the probability both will land heads up?

____

9. (a) A large circle is painted on the ground in the school yard. Bill started at the center of the circle (C) and walked ten steps to point A on the circumference of the circle. He then walked along the edge of the circle ten more steps reaching point B (Fig. 1). Mathematicians define the angle BCA formed in this way to be 1 radian.

Bill then continued walking around the circle back to point A. Determine as precisely as you can the number of radians in the angle moved through in walking all the way around the circle (Fig. 2). Explain how you get your answer.

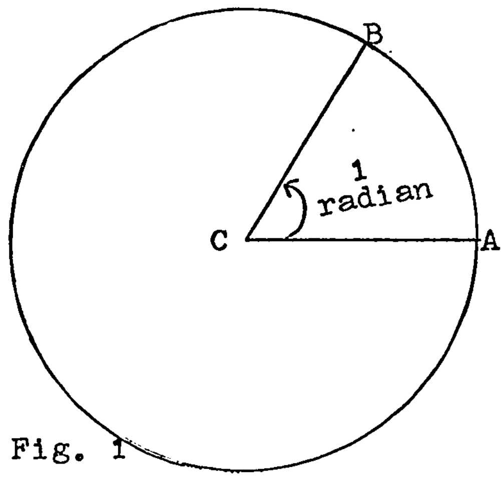


Fig. 1

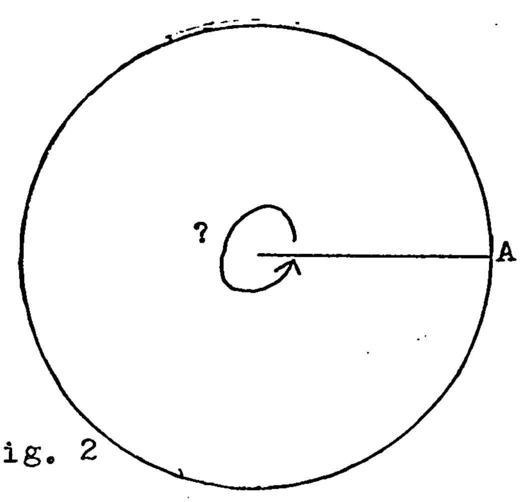
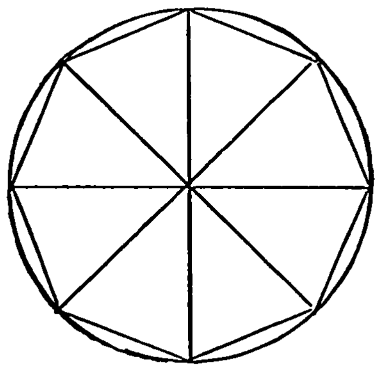


Fig. 2

9. (b) The approximate area of the circle below can be found by finding the area of one of the triangular regions and multiplying by 8. Again using only triangular regions, how could you obtain a still better approximation of the area of the circle? Explain or indicate on the circle how you would do this.



10. (a) A certain pyramid has a base in the shape of an octagon (8 sides). How many faces, vertices and edges does this pyramid have?

Faces _____ Vertices _____ Edges _____

(b) How could you check your answers without looking at a model?

(c) Euler's rule for polyhedra does not hold true for certain 3-dimensional figures. Describe or sketch what one of these might look like.

Grade ___ Name ______________________

CARDS USES IN MATHEMATICS TEST
(CUM TEST)

INSTRUCTIONS:

In this test you are to make up problems of your own which would involve certain concrete materials. You are to list as many different mathematical problems or properties as you can think of which could be investigated, solved or demonstrated using these materials.

Use you imagination. Feel free to respond in any way and in as many ways as you can. Always attempt to give correct responses, but do not worry if your answers happen to be wrong.

Your responses can be from any area of mathematics. Do not solve the problems, but describe them briefly with examples where possible.

Example: Write down all the problems or properties you can think of which could be investigated using a BALANCE ARM AND RINGS.

1. Demonstrate the commutative property of multiplication - e.g. 2 rings on hook 3 balance 3 rings on hook 2.
2. Find solution sets of conditions - e.g. $2 + n = 6$.
3. Demonstrate inequalities - e.g. $2 + 3 < 7$.
4. Factoring - e.g. 12 can be balanced by 3 rings on hook 4 or 2 rings on hook 6, etc.
5. Find prime numbers- e.g. using a single hook, 7 can be made only by putting 1 ring on hook 7 or 7 rings on hook 1.
6. (There are many more.)

QUESTION: You are given a SET OF 40 CARDS. The cards are numbered from 1 to 10 in each of four colors. The backs of the cards are all the same. List the mathematical problems you can investigate or the properties you could demonstrate using all or any of these cards in any way you wish.

Please number your responses. Use the back of this sheet if you need more room. You have 10 minutes. BEGIN.

1. ____

____

____

____

CATEGORIES OF RESPONSES FOR THE CUM TEST

1. Use of numerals without verbal explanation or mention of a topic from the example without explanation. Ex. $2 \times 3 = 6$, $3 + 4 = 4 + 3$, $5 + n < 10$, commutivity, primes.

2. Use of numerals

   (a) To teach numerals, counting, place two together.
   (b) To write fractions, ordered pairs, make ratios.
   (c) Flash cards, drill number facts.
   (d) Play arithmetic games or perform successive operations on a number.
   (e) Solve conditions, inequalities. Ex. $3 + 2 < \square$.

3. Numerals for problems in number theory

   (a) Associate with a number its factors, multiples; pick out the primes; find the multiples of 2, 3, etc.
   (b) Odd-even number questions and properties.
   (c) Find the maximum sum, product of pairs of numbers from 1 - 40.

4. Numeration

   (a) Numerals for working in different bases.
   (b) Let colors represent place value.

5. Use as counters to explain arithmetic operations

   (a) Grouping and regrouping to show adding, subtracting, multiplying, dividing, factoring.
   (b) Grouping and regrouping to show commutative, associative, distributive properties.
   (c) Solution of conditions, inequalities.
   (d) 1-1 correspondence, less than.

6. Use of set of cards for arithmetic problems

   (a) Word problems involving numbers ≤ 40.
   (b) Fractions, counting - How many are red, etc.?
   (c) Percentage - What per cent is red?

7. Concepts of algebra

   (a) Let colors have different values. For example, if red = 1 and green = 2, find 3 red + 4 green.
   (b) Rule: can add or subtract only like (by color) numbers.

8. Integers

   (a) Illustrate positive and negative numbers by different colors.
   (b) Use two different colors to make a number line.
   (c) Show adding, subtracting of negative and positive numbers on the number line or by colors.

9. Sets - basic concepts

   (a) Consider colors, numbers as sets.
   (b) Membership in sets.
   (c) Subsets.
   (d) Complement of a set.
   (e) Intersection, union of two sets.

10. Problem: Find all possible subsets or the number of all possible subsets for a given set

11. Problem: Find the number of members in the intersection and union of two subsets, either disjoint or intersecting (use the attributes of color and number).

12. Area and perimeter

   (a) Area.
   (b) Perimeter.
   (c) Use of area of rectangle to show commutivity of multiplication.

13. Geometry

   (a) Plane, line.
   (b) Make squares, polygons.

14. Probability

15. Use cards as weights on balance

16. Illustrate fractions by cutting up cards

17. Mathematical group

SAMPLE RESPONSES TO CUM TEST

Write numbers on the back and have flash cards.

Finding successors -- E g. 3, then look for the card following 3.

You have 5 red cards and 3 blue cards. How many cards do you have altogether?

If you had 18 cards in two different colors (same number of each), how many would you have of each color?

How many cards with No. 1 and No. 2 equal to ten?

Commutative property -- Use backs of cards.

□□ + □□□ = □□□□□
□□□ + □□ = □□□□□

Show bases 3, 5, 10 through addition of these cards.

Use when teaching positive and negative integers. I_N = black and blue, I_P = red and yellow, Zero = back of one card.

The cards measure approximately 2 inches wide and three inches high. What is the area? 2 x 3 = 3 x 2 (Commutative).

Let each card stand for 1 square unit. Take 28 cards and arrange them in a rectangle. What is the area and perimeter?

Are the numbers on red cards from 1 to 10 closed under addition and multiplication?

Mathematical problems. Eg. Sally had one blue card worth 4 and 2 red cards worth 3 each. How much were all the cards worth together?

Subsets -- Eg. Yellows are a subset of all the cards.

How many different subsets of two members are there in the yellow cards?

How many subsets are there in 10 cards numbered 1 to 10? First take 1 card; there are 2 subsets - 1 and the empty set. Now take cards 1 and 2 - there are 4 subsets - card 1, card 2, cards 1 and 2, and the empty set. Now take cards 1, 2 and 3. There are 8 subsets - card 1, card 2, card 3, cards 1 & 2, cards 2 & 3, cards 1 & 3, cards 1, 2, & 3, and the empty set. Do you see a pattern? For each card you add, the number of subsets doubles. Ten cards have 1024 subsets.

How many 3's or red's or both in the set?

Demonstrate intersecting sets by taking all the 5 cards out and putting them in one pile and all the red cards out and putting them into another pile. One should intersect the other with red 5's in it.

Probability can be demonstrated by turning all the cards over and selecting certain ones. 20 draws should produce 5 of each color.

Draw any card and discuss what chances there are of it being both blue and No. 3.

If you draw all the cards out of a jug, what are your chances of picking a blue card? (They are 10 out of 40.)

DATA FOR COMPUTING KENDAL'S COEFFICIENT

OF CONCORDANCE W (WITH TIED RANKS)*

Ranks Assigned to Eight Students on the
CUM Test by Eight Judges

Judge	Student A	B	C	D	E	F	G	H
1	6.5	8	1	4	5	2.5	2.5	6.5
2	5	8	2	5	7	2	2	5
3	6	8	2	4	6	2	2	6
4	7	4.5	1	7	7	2.5	2.5	4.5
5	7	8	1	4	5.5	2.5	2.5	5.5
6	8	5.5	2.5	5.5	5.5	1	2.5	5.5
7	5.5	7.5	1.5	4	7.5	3	1.5	5.5
8	5.5	8	2	4	7	3	1	5.5
Totals	50.5	57.5	13.0	37.5	50.5	18.5	16.5	44.0

*G.A. Ferguson, Statistical Analysis in Psychology and Education, 1966, pp. 225-227.

A MATHEMATICS STUDY

The best answer to each statement is your own first impression. There are no right or wrong answers. Your responses will be kept confidential (your teachers will not see them; the results will in no way affect your grade).

Think carefully, but do not spend too much time on any one question. Let your <u>own</u> personal experience guide you to choose the answer you feel about each statement.

Mark all responses on the answer sheet provided (Part 1). Do not write on the question booklet.

PLEASE MARK A RESPONSE FOR EVERY STATEMENT.

---

(1) I find most mathematics lessons:

A. extremely interesting.
B. quite interesting.
C. interesting.
D. not very interesting.
E. not interesting at all.

(2) A knowledge of mathematics for any job at all is:

A. most important.
B. very important.
C. quite important.
D. of small importance.
E. not important.

(3) If I did <u>not</u> have to take mathematics; I would like school:

A. much less.
B. a little less.
C. same as now.
D. a little better.
E. much better.

(4) Mathematics is:

A. the most important subject.
B. one of the more important subjects.
C. just as important as any other subject.
D. not as important as some of the other subjects.
E. the least important subject.

(5) I find problem solving:

A. extremely interesting.
B. quite interesting.
C. interesting.
D. not very interesting.
E. not interesting at all.

(6) When I have difficulty with a new topic in my mathematics course, I ask my teacher to clarify the section:

A. very frequently.
B. frequently.
C. sometimes.
D. hardly ever.
E. never.

(7) If books about mathematics were available, I would:

A. read most of them.
B. read some of them.
C. look at the diagrams and pictures.
D. page through some of them.
E. never look at them.

(8) If someone says mathematics classes are worthless and a waste of time, I would:

A. strongly disagree.
B. tend to disagree.
C. not take a side.
D. tend to agree.
E. strongly agree.

(9) When I do my homework, my mathematics is:

A. always done first.
B. often done first.
C. usually done first.
D. sometimes done first.
E. never done first.

(10) I find mathematical puzzles:

A. extremely interesting.
B. quite interesting.
C. sometimes interesting.
D. not very interesting.
E. not interesting at all.

(11) I would be interested in taking other subjects that make use of:

A. a great deal of mathematics.
B. quite a bit of mathematics.
C. some mathematics.
D. a little mathematics.
E. no mathematics.

(12) If given the opportunity to join one of the following clubs, I would prefer a:

A. mathematics club.
B. science club (physics).
C. science club (chemistry).
D. science club (geology).
E. literary club.

(13) If I could receive one of the following magazines for a year, I would pick:

A. a mathematics magazine for high school students.
B. a magazine combining science and mathematics for high school students.
C. a science magazine for high school students.
D. a geology magazine for high school students.
E. a literary magazine for high school students.

(14) When I study my mathematics course, I most often:

A. make written summaries of the sections covered.
B. do additional problem solving.
C. do many drill questions.
D. memorize the formulas given in the text.
E. look over some work done previously.

(15) If I listed my courses in order of preference, I would place mathematics:

A. first.
B. second.
C. third.
D. fourth.
E. fifth.

(16) Whenever mathematical problems are presented to us for solving, I get:

A. a great deal of satisfaction in working them out.
B. quite a bit of satisfaction in working them out.
C. some satisfaction in working them out.
D. very little satisfaction in working them out.
E. no satisfaction in working them out.

(17) My mathematics course has made:

A. mathematics enjoyable for me.
B. mathematics a pleasant course.
C. me feel indifferent towards mathematics.
D. mathematics classes an uncomfortable experience for me.
E. me strongly dislike mathematics.

(18) When I do my mathematics homework, I am usually:

A. extremely interested.
B. interested.
C. somewhat interested.
D. not too interested.
E. not interested at all.

(19) When we start a new topic in mathematics, I am usually:

A. keenly interested.
B. interested.
C. somewhat interested.
D. not too interested.
E. not interested at all.

(20) The average amount of time I spend on homework assignments in mathematics takes the following time per day:

A. more than one hour.
B. 3/4 hour to one hour.
C. 1/2 hour to 3/4 hour.
D. 1/4 hour to 1/2 hour.
E. 0 hour to 1/4 hour.

(21) When I get an assignment in mathematics:

A. I do it immediately.
B. I do it eventually.
C. I may get it done.
D. I put it off as long as possible.
E. I don't do it.

(22) Most of my work in this class is done:

A. to satisfy my curiosity about mathematics.
B. to gain competence in mathematics.
C. to get a good mark.
D. to just pass the class.
E. to put in the time alloted to mathematics.

(23) During mathematics lessons, I feel:

A. extremely confident in myself.
B. quite confident in myself.
C. confident in myself.
D. a little unsure of myself.
E. very unsure of myself.

At the top of the next page to be given you, you will see the statement: Learning and Doing Mathematics

Below this statement is a series of word pairs like the following:

happy _____:_____:_____:_____:_____:_____:_____ sad

You are to react to the statement by placing an X in one of the seven blank spaces between the two paired words. Mark your X in that space which best indicates the degree of your feeling toward the statement as expressed by the word pair.

For example, suppose the word pair is

happy _____:_____:_____:_____:_____:_____:_____ sad

If the statement "Learning and Doing Mathematics" suggests very strongly to you the idea "happy", place an X near "happy", thus:

happy __X__:_____:_____:_____:_____:_____:_____ sad

If you feel that the statement very strongly suggests the idea "sad", place an X near "sad", thus:

happy _____:_____:_____:_____:_____:_____:__X__ sad

The less strongly you feel that one of the two paired words expresses your reactions to the statement, the closer you will place your X to the middle space. If you are neutral about the statement for a particular word pair or if you feel the word pair is unrelated to the statement, place your X in the middle space.

IMPORTANT:

1. Be sure you mark an X for every word pair. DO NOT OMIT ANY.

2. Mark only one X for each word pair.

Work fast. It is your first feelings that we want. On the other hand please do not be careless because we want your true feelings.

NAME ______________________________ GRADE ______ DATE ______________

Learning and Doing Mathematics

dull	____:____:____:____:____:____:____	interesting
familiar	____:____:____:____:____:____:____	strange
find out	____:____:____:____:____:____:____	be told
unfair	____:____:____:____:____:____:____	fair
objects	____:____:____:____:____:____:____	symbols
quiet	____:____:____:____:____:____:____	noisy
textbook	____:____:____:____:____:____:____	laboratory
boring	____:____:____:____:____:____:____	fun
real	____:____:____:____:____:____:____	unreal
relaxed	____:____:____:____:____:____:____	tense
active	____:____:____:____:____:____:____	passive
new	____:____:____:____:____:____:____	old
abstract	____:____:____:____:____:____:____	concrete
rule	____:____:____:____:____:____:____	guess
useful	____:____:____:____:____:____:____	useless
experimental	____:____:____:____:____:____:____	"cut and dried"
enjoyable	____:____:____:____:____:____:____	distasteful

TEK/jcw
Nov./68

FACTOR ANALYSIS OF LDM POSTTEST SCORES

VARIMAX ROTATED FACTORS

	Communalities	1	2	3
1	0.745	0.775	0.363	-0.115
2	0.443	0.057	0.654	-0.109
3	0.237	0.479	-0.011	-0.087
4	0.501	0.537	0.461	0.016
5	0.406	0.002	-0.064	0.634
6	0.240	-0.056	-0.465	0.142
7	0.599	-0.009	0.004	0.774
8	0.696	0.778	0.283	-0.099
9	0.451	0.417	0.526	0.013
10	0.401	0.304	0.545	0.107
11	0.399	0.578	0.136	0.214
12	0.469	0.636	-0.199	-0.157
13	0.374	-0.076	0.593	0.127
14	0.432	-0.405	-0.443	0.267
15	0.492	0.629	0.306	0.048
16	0.517	0.659	-0.054	0.283
17	0.711	0.741	0.401	-0.045
	8.111	4.283	2.510	1.318

Transformation Matrix

0.852	0.522	-0.042
0.480	-0.746	0.462
0.210	-0.413	-0.886

STUDENT QUESTIONNAIRE ON THE MATH LAB

A. The following are opinions which might be expressed by students who worked in the math lab. For each opinion please circle whether you agree (A), are uncertain (U) or disagree (D).

1.	I enjoyed the lab periods	A	U	D
2.	Working in the lab has increased my knowledge in mathematics.	A	U	D
3.	In the lab I felt that I could work without pressure.	A	U	D
4.	The lab lessons helped me to better understand my regular math course.	A	U	D
5.	The lab lessons provided a "welcome break from routine".	A	U	D
6.	I'm not sure what I learned in the math lab.	A	U	D
7.	I now find math more interesting than I used to.	A	U	D
8.	I liked having concrete material to refer to in solving problems in mathematics.	A	U	D
9.	I was often bored during the lab periods.	A	U	D
10.	In the lab I learned material for later grades.	A	U	D
11.	I found the lab periods more interesting than work from the textbook.	A	U	D
12.	Working in the lab broadened my view of math.	A	U	D
13.	I sometimes felt I was wasting my time in the lab.	A	U	D
14.	In the lab I learned how to investigate mathematical problems independently.	A	U	D
15.	Later lab periods were not as much fun as the earlier ones.	A	U	D
16.	I would rather do textbook exercises than investigate problems based on concrete materials.	A	U	D
17.	I feel that my attitude toward mathematics has improved over the past three months.	A	U	D
18.	I think I would have learned more if the teacher had taught the class as a group and demonstrated with the concrete materials.	A	U	D

19. I enjoyed working in a small group. A U D

20. I liked working from written instructions rather than having a teacher give me directions. A U D

21. I used the concrete materials to check my ideas and answers. A U D

22. In the lab I felt I could work at my own speed. A U D

23. Working with a partner helped me to better understand the activities because we discussed them together. A U D

24. It was sometimes too noisy in the lab to concentrate. A U D

25. After several lab sessions I became more confident in my ability to do the activities without asking the teacher for help. A U D

B. In the following questions circle the letter corresponding to the statement you choose.

1. In the lab I made use of the concrete material

   A. throughout the period finding and checking answers.
   B. only to understand the problem and to begin finding answers.
   C. only to see the problem. The material was largely unnecessary as I could usually answer the questions without using it at all.

2. In future lab lessons I would prefer

   A. that the instructions be more specific and complete.
   B. that sets of instructions similar to those used be provided.
   C. working with the material in my own way without having to follow specific instructions.

3. When solving problems based on concrete materials I would

   A. prefer to work alone.
   B. like to work with a partner as before.
   C. like to work with a partner only if I could choose him (or her); otherwise work alone.
   D. rather work along with the class as a group under the direction of the teacher.

C. The ten lab lessons are written on the board. List (by number) the two lessons you liked most: ____ ____

Which two activities did you least enjoy doing? ____ ____

D. Other Comments: Comment further on any aspect of your experience in the math lab. What did you like about it most? How could it be improved?

STUDENT QUESTIONNAIRE ON THE SPECIAL (DISCOVERY) LESSONS

A. The following are opinions which might be expressed by students who had the series of special lessons based on concrete materials. For each opinion please <u>circle</u> whether you agree (A), are uncertain (U) or disagree (D).

1.	I enjoyed the special lessons.	A	U	D
2.	The special lessons increased my knowledge in mathematics.	A	U	D
3.	During the special lessons I felt that I could learn without pressure.	A	U	D
4.	The special lessons helped me to better understand my regular math course.	A	U	D
5.	The special lessons provided a "welcome break from routine".	A	U	D
6.	I'm not sure what I learned from the special lessons.	A	U	D
7.	I now find math more interesting than I used to.	A	U	D
8.	I liked having concrete material to refer to while solving problems in mathematics.	A	U	D
9.	I was often bored during the special lessons.	A	U	D
10.	During the special lessons I learned material for later grades.	A	U	D
11.	I found the special lessons more interesting than work from the textbook.	A	U	D
12.	The special lessons broadened my view of math.	A	U	D
13.	I sometimes felt I was wasting my time during the special lessons.	A	U	D
14.	Through taking the special lessons I learned how to investigate mathematical problems independently.	A	U	D
15.	I didn't enjoy the later special lessons as much as I did the earlier ones.	A	U	D

16.	I would rather do textbook exercises than investigate problems based on concrete materials.	A	U	D
17.	I feel that my attitude toward mathematics has improved over the past three months.	A	U	D
18.	During the special lessons the same students usually made most of the discoveries.	A	U	D
19.	During the special lessons there was usually more activity and noise than during regular mathematics lessons.	A	U	D
20.	I would have preferred studying these lessons with a partner in the math lab to taking them with the rest of the class from the teacher.	A	U	D

B. List (by number) the two lessons you liked most: ___ ___

Which two lessons did you least enjoy? ___ ___

C. Write any other comments you may have concerning the special lessons.

TEACHER QUESTIONNAIRE

Name ________________________

Degree(s) ____________________

Teaching Experience

Total number of years ____

Years teaching Grade 7 &/or 8 math ____

Years teaching STM-1 &/or STM-2 ____

A. The Mathematics Laboratory (ML) and Class Discovery (CD) Settings. In each of the following please circle ML, CD or S (same).

1. Which setting was more enjoyable for			
(a) high-ability students?	ML	S	CD
(b) average-ability students?	ML	S	CD
(c) low-ability students?	ML	S	CD
2. Which setting was more effective in terms of the concepts gained by			
(a) high-ability students?	ML	S	CD
(b) average-ability students?	ML	S	CD
(c) low ability-students?	ML	S	CD
3. Which setting did you prefer in terms of your role as a teacher?	ML	S	CD
4. Which setting required greater effort on your part (considering that the activities were already prepared and assuming that you were acquainted with the lessons)?	ML	S	CD
5. In which setting was student behavior easier to control (or require the least control)?	ML	S	CD

B. In the Math Lab, which two activities did you feel that the students liked

the most? ________________ ________________

the least? ________________ ________________

In the Class Discovery setting, which two lessons did you feel were

the most successful? ____________ ____________

the least successful? ____________ ____________

C. The Math Lab Setting. Comment on any trends which may have developed in the following areas over the ten weeks:

(a) Student interest in working in the lab.

Grade 7

Grade 8

(b) The amount of help required or requested by students while doing lab lessons.

Grade 7

Grade 8

(c) Tendency on the part of students to skip through sections of lessons without reading them carefully or doing the required work.

Grade 7

Grade 8

(d) Other comments on the Math Lab.

D. The Class Discovery Setting.

(a) What was the general reaction of the students to having the special lessons based on concrete materials in addition to (in place of) textbbok work?

(b) Other comments on these classroom lessons.

E. What do you feel was the effect of cutting the class time devoted to the regular mathematics course from four to three periods a week?

F. To what extent and in what way would you want to use math lab periods and special classroom discovery lessons to supplement, enrich or vary instruction in the future?

APPENDIX D

CA AND MLTPS SUBTEST CELL MEANS

CUMULATIVE ACHIEVEMENT SUBTEST CELL MEANS

		Grade 7			Grade 8		
		ML	CD	CON	ML	CD	CON
1	H	1.80	2.00	1.31	2.38	3.14	1.50
	M	1.50	1.20	.84	1.66	1.80	1.35
	L	.68	.92	1.06	.81	1.53	1.00
		1.33	1.37	1.07	1.61	2.16	1.28
2	H	1.80	1.95	1.62	2.46	2.14	1.90
	M	1.13	1.40	.79	1.50	1.30	1.06
	L	.55	.92	.56	1.27	1.16	.70
		1.16	1.43	.99	1.74	1.53	1.22
3	H	2.05	2.50	1.23	2.96	2.14	1.25
	M	1.42	1.55	1.16	1.81	2.55	1.00
	L	1.18	1.23	.94	1.50	2.00	.90
		1.55	1.76	1.11	2.09	2.23	1.05
4	H	2.70	2.23	1.46	2.79	2.86	2.50
	M	2.08	1.90	1.32	2.50	2.35	2.23
	L	1.45	1.46	1.44	1.65	2.16	1.70
		2.08	1.86	1.40	2.32	2.46	2.15
5	H	.95	.95	1.08	1.92	1.86	.85
	M	.63	1.00	.42	.91	.95	.53
	L	.41	.77	.25	.50	.42	.30
		.66	.91	.58	1.11	1.08	.56
6	H	2.25	2.82	2.46	2.96	3.00	3.00
	M	1.50	1.90	2.16	2.22	2.05	1.76
	L	1.36	1.69	1.69	2.08	2.21	1.90
		1.70	2.14	2.10	2.42	2.42	2.22
7	H	1.10	1.00	.63	1.58	3.00	.70
	M	.42	.75	.37	.75	1.55	1.00
	L	.36	1.00	.62	.31	.89	.80
		.63	.92	.54	.88	1.81	.83

		Grade 7			Grade 8		
		ML	CD	CON	ML	CD	CON
8	H	1.20	1.95	.69	1.96	1.86	2.00
	M	.79	1.20	.68	1.44	1.30	1.53
	L	.82	1.54	.94	1.04	.95	1.40
		.94	1.56	.77	1.48	1.37	1.64
9	H	.80	.41	.15	.67	.86	.70
	M	.33	.15	.11	.53	.75	.47
	L	.36	.31	.31	.38	.47	.50
		.50	.29	.19	.53	.69	.56
10	H	.80	.77	.15	1.79	1.43	.60
	M	.46	.65	.26	0.88	.45	.71
	L	.77	.31	.69	.65	.68	.80
		.68	.58	.37	1.11	.85	.70
T	H	15.45	16.59	10.77	21.46	22.29	15.00
	M	10.25	11.70	8.10	14.19	15.05	11.64
	L	7.95	10.15	8.50	10.19	12.47	10.00
		11.22	12.81	9.12	15.28	16.60	12.22

HIGHER LEVEL THINKING AND PROBLEM SOLVING

SUBTEST CELL MEANS

		Grade 7			Grade 8		
		ML	CD	CON	ML	CD	CON
1	H	1.65	1.86	1.00	2.67	2.38	1.95
	M	1.09	1.05	.78	1.74	1.65	1.71
	L	.77	.46	.31	.96	1.17	1.25
		1.17	1.12	.70	1.79	1.73	1.64
2	H	2.30	3.05	2.92	3.50	3.13	3.55
	M	2.22	1.95	2.28	2.68	2.60	2.82
	L	1.63	1.54	1.13	2.08	1.78	2.00
		2.05	2.18	2.11	2.75	2.50	2.79
3	H	1.40	1.76	.54	2.79	2.63	1.70
	M	.61	1.20	.33	1.74	2.05	1.35
	L	.32	.31	.38	1.28	1.11	.38
		.78	1.09	.42	1.94	1.93	1.14
4	H	2.45	2.00	.77	2.58	2.25	1.50
	M	.87	1.80	.78	1.42	1.50	1.35
	L	.91	1.00	.38	.68	1.28	.75
		1.41	1.60	.64	1.56	1.68	1.20
5	H	.75	.67	.54	1.29	1.75	1.15
	M	.35	.40	.61	.55	.85	1.06
	L	.05	.38	.13	.36	.33	.50
		.38	.48	.42	.73	.98	.90
6	H	.10	.90	.15	.92	1.63	1.30
	M	.22	.15	.33	.55	.55	.41
	L	.09	.00	.00	.32	.67	.00
		.14	.35	.16	.60	.95	.57
7	H	.25	1.76	.00	1.38	2.63	.20
	M	.13	.85	.00	.45	1.30	.00
	L	.00	.23	.00	.00	.83	.00
		.13	.95	.00	.61	1.59	.07

		Grade 7			Grade 8		
		ML	CD	CON	ML	CD	CON
8	H	.10	.33	.46	.75	.50	.40
	M	.09	.40	.33	.39	.30	.35
	L	.00	.15	.00	.08	.11	.25
		.06	.29	.26	.41	.30	.33
9	H	.55	.52	.54	.40	.25	.70
	M	.26	.45	.17	.26	.20	.53
	L	.14	.15	.19	.36	.39	.63
		.32	.37	.30	.35	.28	.62
10	H	1.05	.86	.46	1.54	1.25	.65
	M	.26	.40	.22	.55	.60	.12
	L	.41	.31	.19	.36	.33	.13
		.57	.52	.29	.82	.73	.30
T	H	10.60	13.71	7.38	17.75	18.38	13.05
	M	6.09	8.65	5.78	10.32	11.60	9.71
	L	4.32	4.54	2.69	6.48	8.00	5.88
		7.00	8.97	5.28	11.52	12.66	9.55

APPENDIX E

CELL FREQUENCIES AND BARTLETT'S TEST FOR HOMOGENEITY OF VARIANCE FOR THE ANALYSIS OF VARIANCE

STM Posttests
Combined STM Quizzes
IL Total Test
CA Test
HLTPS Test
CUM Test

CELL FREQUENCIES, VARIANCES, AND BARTLETT'S

TEST FOR HOMOGENIETY OF VARIANCE

	STM-1 ML	STM-1 CD	STM-1 CON	STM-2 ML	STM-2 CD	STM-2 CON
STM Posttests						
H	(20) 19.84	(22 [illegible]	(13) 40.31	(24) 19.03	(8) 12.58	(2) 74.70
A	(25) 26.51	(19) 37.50	(20) 31.27	(33) 23.53	(20) 32.83	(18) 62.92
L	(22) 26.47	(13) 14.81	(15) 75.98	(26) 11.55	(19) 18.70	(10) 52.46
	(67)	(54)	(48)	(83)	(47)	(48)
		CHISQ = 8.81			CHISQ = 32.25	
		P = .359			P = .000	
STM Combined Quizzes						
H	(20) 68.24	(21) 84.60	(13) 97.78	(22) 232.22	(8) 335.98	(19) 369.89
A	(24) 150.95	(19) 193.14	(20) 129.41	(31) 260.26	(19) 334.06	(17) 574.89
L	(22) 144.37	(13) 266.75	(15) 174.98	(23) 320.24	(17) 317.82	(9) 317.95
	(66)	(53)	(48)	(76)	(44)	(45)
		CHISQ = 11.19			CHISQ = 4.95	
		P = .191			P = .763	

	Grade 7 ML	Grade 7 CD	Grade 7 CON	Grade 8 ML	Grade 8 CD	Grade 8 CON
IL Total Test						
H	(18) 45.06	(11) 17.96		(20) 24.35	(7) 24.91	
A	(13) 43.83	(13) 20.42		(24) 22.06	(10) 20.68	
L	(13) 34.58	(6) 10.27		(17) 30.32	(13) 36.09	
	(44)	(30)		(61)	(30)	
		CHISQ = 6.29			CHISQ = 1.43	
		P = .279			P = .921	

CA Test

	Grade 7			Grade 8		
	ML	CD	CON	ML	CD	CON
H	(20)	(22)	(13)	(24)	(7)	(2)
	25.52	23.87	11.53	27.30	29.57	24.32
A	(24)	(20)	(19)	(32)	(20)	(17)
	17.93	9.69	11.99	15.19	27.94	12.24
L	(22)	(13)	(16)	(26)	(19)	(10)
	10.81	16.14	7.47	13.44	21.04	14.22
	(66)	(55)	(48)	(82)	(46)	(47)
	CHISQ = 12.30 P = .138			CHISQ = 8.09 P = .425		

HLTPS Test

	Grade 7			Grade 8		
	ML	CD	CON	ML	CD	CON
H	(20)	(21)	(13)	(24)	(8)	(20)
	14.57	23.11	15.09	28.37	28.84	21.73
A	(23)	(20)	(18)	(31)	(20)	(17)
	7.90	15.61	13.59	22.83	24.46	20.10
L	(22)	(13)	(16)	(25)	(18)	(8)
	4.13	14.94	4.10	15.93	15.65	11.27
	(65)	(54)	(47)	(80)	(46)	(45)
	CHISQ = 23.24 P = .003			CHISQ = 4.23 P = .836		

CUM Test

	Grade 7 ML	Grade 7 CD	Grade 7 CON	Grade 8 ML	Grade 8 CD	Grade 8 CON
H	(20)	(21)	(13)	(24)	(8)	(18)
	2.05	1.85	.53	2.51	4.98	3.67
A	(23)	(20)	(20)	(30)	(20)	(16)
	1.59	.46	.48	2.92	1.88	.80
L	(21)	(13)	(15)	(23)	(19)	(11)
	.43	.76	.41	1.43	1.23	.76
	(64)	(54)	(48)	(77)	(47)	(45)
	CHISQ = 33.76 P = .000			CHISQ = 21.00 P = .007		

APPENDIX F

REGRESSION COEFFICIENTS AND BOX'S F-TEST FOR HOMOGENIETY OF VARIANCE AND COVARIANCE FOR THE ANALYSIS OF COVARIANCE

Test	Regression Coefficients				Box's F-Test	
	ML	CD	CON	Pooled	F	P
AMS	.70	.66	.79	.71	2.02	.131
LDM						
Enjoyment	.13	.60	.69	.50	1.85	.158
Familiarity	.21	.47	.69	.46	.01	.994
Situation	.35	.41	.54	.43	1.28	.274

APPENDIX G

PERCENTAGES AND CHI-SQUARE TESTS OF SIGNIFICANCE OF RESPONSES TO ML AND CD STUDENT QUESTIONNAIRES OF STUDENTS CLASSIFIED BY GRADE, TEACHER, SEX AND ACHIEVEMENT LEVEL

GROUP FREQUENCIES FOR CHI-SQUARE TESTS

ML Group (N = 144)

Grade	Teacher	Sex	Ach. Level
7. 64	1. 46	Boys (B) 71	H 44
8. 80	2. 53	Girls (G) 73	A 54
	3. 45		L 46

CD Group (N = 102)

Grade	Teacher	Sex	Ach. Level
7. 55	1. 24	Boys (B) 57	H 32
8. 47	2. 25	Girls (G) 45	A 40
	3. 31		L 30
	4. 22		

ML STUDENT QUESTIONNAIRE

Item	Grade				Teacher				Sex				Ach. Level			
		A	U	D		A	U	D		A	U	D		A	U	D
1	7.	86	11	3	1.	78	9	13	B	83	10	7	H	84	7	9
	8.	86	5	9	2.	89	6	5	G	89	6	5	A	89	7	4
					3.	91	9	0					L	85	9	6
χ^2		3.44				7.712				1.192				1.322		
P		.179				.127				.551				.858		
2	7.	81	13	6	1.	82	9	9	B	86	11	3	H	82	14	4
	8.	89	9	2	2.	81	15	4	G	85	10	5	A	89	9	2
					3.	93	7	0					L	85	9	6
χ^2		1.914				.497				.714				2.105		
P		.384				.165				.700				.716		
3	7.	73	19	8	1.	63	22	15	B	76	13	11	H	75	16	9
	8.	75	14	11	2.	70	19	11	G	73	19	8	A	80	13	7
					3.	91	7	2					L	67	20	13
χ^2		1.000				10.487				1.354				2.020		
P		.606				.033				.508				.732		
8	7.	75	20	5	1.	74	24	2	B	73	20	7	H	77	21	2
	8.	81	16	3	2.	81	19	0	G	84	16	0	A	81	15	4
					3.	80	11	9					L	76	20	4
χ^2		.992				8.145				5.844				.908		
P		.609				.086				.054				.923		
11	7.	78	14	8	1.	83	11	6	B	86	10	4	H	82	9	9
	8.	90	6	4	2.	83	11	6	G	84	9	7	A	89	11	0
					3.	89	7	4					L	82	9	9
χ^2		3.880				.947				.472				5.160		
P		.144				.918				.789				.271		

Item	Grade	A	U	D	Teacher	A	U	D	Sex	A	U	D	Ach. Level	A	U	D
16	7.	11	11	78	1.	11	17	72	B	10	14	76	H	14	4	82
	8.	7	13	80	2.	9	10	81	G	8	10	82	A	4	20	76
					3.	7	9	84					L	11	9	80
χ^2		.555				2.758				.895				8.720		
P		.756				.599				.639				.068		
9	7.	9	13	78	1.	15	20	65	B	14	21	65	H	14	18	68
	8.	15	22	63	2.	13	15	72	G	11	15	74	A	9	19	72
					3.	9	20	71					L	15	18	67
χ^2		4.119				1.347				1.450				.887		
P		.128				.853				.484				.926		
13	7.	20	10	70	1.	28	13	59	B	27	11	62	H	11	18	71
	8.	19	20	61	2.	15	17	68	G	12	19	69	A	20	9	71
					3.	15	16	69					L	26	20	54
χ^2		3.119				3.410				5.564				5.859		
P		.210				.492				.062				.210		
15	7.	22	22	56	1.	22	19	59	B	24	21	55	H	25	11	64
	8.	20	16	64	2.	22	21	57	G	18	16	66	A	20	24	56
					3.	18	15	67					L	17	20	63
χ^2		.991				1.119				1.770				3.086		
P		.609				.891				.413				.544		
2	7.	47	48	5	1.	33	61	6	B	56	40	4	H	61	34	5
	8.	47	49	4	2.	56	40	4	G	38	58	4	A	54	46	0
					3.	51	47	2					L	26	65	9
χ^2		.079				6.590				4.891				15.802		
P		.961				.159				.087				.003		

Item		Grade A	Grade U	Grade D		Teacher A	Teacher U	Teacher D		Sex A	Sex U	Sex D		Ach. Level A	Ach. Level U	Ach. Level D
4	7.	53	38	9	1.	43	37	20	B	42	42	16	H	41	45	14
	8.	35	45	20	2.	36	49	15	G	44	41	15	A	42	43	15
					3.	51	38	11					L	46	37	17
χ^2		5.820				3.488				.037				.749		
P		.055				.480				.982				.945		
10	7.	39	52	9	1.	37	61	2	B	38	52	10	H	41	52	7
	8.	29	66	4	2.	30	62	8	G	29	67	4	A	33	57	9
					3.	33	56	11					L	26	70	4
χ^2		3.399				3.197				4.00				3.550		
P		.183				.525				.136				.470		
6	7.	12	27	61	1.	22	32	46	B	10	39	51	H	14	23	63
	8.	14	41	45	2.	11	34	55	G	16	30	54	A	13	35	52
					3.	7	38	55					L	13	46	41
χ^2		3.985				4.869				2.128				5.573		
P		.136				.301				.345				.233		
12	7.	45	39	16	1.	32	46	22	B	51	34	15	H	54	39	7
	8.	44	44	12	2.	45	44	11	G	39	49	12	A	44	39	17
					3.	55	36	9					L	35	48	17
χ^2		.457				6.428				3.573				4.882		
P		.796				.169				.168				.300		
14	7.	70	24	6	1.	56	33	11	B	59	28	13	H	59	34	7
	8.	49	40	11	2.	57	36	7	G	58	37	5	A	57	32	11
					3.	62	29	9					L	59	32	9
χ^2		6.807				.833				2.938				.567		
P		.003				.934				.230				.967		

Item	Grade	A	U	D	Teacher	A	U	D	Sex	A	U	D	Ach. Level	A	U	D
7	7.	39	39	22	1.	30	40	30	B	39	40	21	H	32	54	14
	8.	31	50	19	2.	32	51	17	G	30	51	19	A	39	39	22
					3.	42	45	13					L	33	43	24
x^2			1.740				5.566				1.973				3.160	
P			.419				.234				.373				.531	
17	7.	62	30	8	1.	48	41	11	B	56	33	11	H	57	34	9
	8.	49	40	11	2.	59	34	7	G	53	39	8	A	58	35	7
					3.	58	31	11					L	50	37	13
x^2			2.725				1.739				.761				1.184	
P			.256				.784				.684				.881	
19	7.	83	11	6	1.	78	11	11	B	80	11	9	H	84	11	5
	8.	84	9	7	2.	85	11	4	G	86	8	6	A	83	11	6
					3.	84	7	6					L	83	6	11
x^2			.259				2.644				.958				2.255	
P			.879				.619				.619				.689	
23	7.	75	14	11	1.	74	11	15	B	63	17	20	H	77	14	9
	8.	73	10	17	2.	76	13	11	G	84	7	9	A	76	11	13
					3.	71	11	18					L	67	11	22
x^2			1.577				.919				7.604				3.165	
P			.455				.922				.022				.531	
21	7.	77	17	6	1.	61	30	9	B	72	21	7	H	82	16	2
	8.	70	25	5	2.	81	15	4	G	74	22	4	A	76	18	6
					3.	76	20	4					L	61	30	9
x^2			1.318				5.431				.590				5.698	
P			.517				.246				.774				.223	

Item	Grade	A	U	D	Teacher	A	U	D	Sex	A	U	D	Ach. Level	A	U	D
20	7.	56	25	19	1.	35	33	32	B	63	20	17	H	61	21	18
	8.	53	31	16	2.	55	32	13	G	45	37	18	A	56	35	9
					3.	73	20	7					L	46	28	26
χ^2			.708				17.550				5.981				6.981	
P			.702				.001				.050				.137	
18	7.	17	27	56	1.	35	19	46	B	25	23	52	H	11	30	59
	8.	25	26	49	2.	13	30	57	G	18	30	52	A	20	30	50
					3.	18	29	53					L	33	19	48
χ^2			1.393				7.547				1.740				6.581	
P			.498				.110				.419				.160	
24	7.	44	19	37	1.	52	18	30	B	51	24	25	H	48	18	34
	8.	47	20	33	2.	43	21	36	G	41	15	44	A	52	20	28
					3.	42	20	38					L	37	20	43
χ^2			.394				1.138				5.724				3.083	
P			.821				.888				.057				.544	
25	7.	53	41	6	1.	48	35	17	B	55	35	10	H	50	41	9
	8.	53	35	12	2.	47	47	6	G	51	40	9	A	63	31	6
					3.	64	29	7					L	44	41	15
χ^2			1.731				8.106				.321				5.126	
P			.421				.088				.852				.275	

CD STUDENT QUESTIONNAIRE

Item	Grade	A	U	D	Teacher	A	U	D	Sex	A	U	D	Ach. Level	A	U	D
1	7.	67	26	7	1.	79	17	4	B	65	28	7	H	77	16	7
	8.	72	24	4	2.	64	28	8	G	76	20	4	A	72	25	3
					3.	58	32	10					L	59	31	10
					4.	82	18	0								
χ^2			.529				5.702				1.360				3.567	
P			.768				.458				.506				.468	
5	7.	62	18	20	1.	63	8	29	B	63	21	16	H	63	20	17
	8.	66	28	6	2.	64	24	12	G	65	24	11	A	73	17	10
					3.	61	26	13					L	53	31	16
					4.	68	32	0								
χ^2			4.501				10.520				.536				3.339	
P			.105				.104				.765				.503	
3	7.	53	36	11	1.	63	29	8	B	53	42	5	H	57	33	10
	8.	57	43	0	2.	48	52	0	G	58	35	7	A	60	35	5
					3.	45	42	13					L	47	50	3
					4.	68	32	0								
χ^2			5.478				9.345				.481				3.308	
P			.065				.115				.787				.508	
8	7.	60	29	11	1.	71	25	4	B	61	30	9	H	63	27	10
	8.	64	28	8	2.	56	28	16	G	62	27	11	A	70	23	7
					3.	52	32	16					L	50	38	12
					4.	73	27	0								
χ^2			.227				6.849				.231				3.077	
P			.893				.335				.891				.545	
11	7.	66	20	14	1.	71	21	8	B	70	19	11	H	73	17	10
	8.	81	17	2	2.	76	24	0	G	75	18	7	A	75	15	10
					3.	61	19	20					L	69	25	6
					4.	86	9	5								
χ^2			5.378				9.269				.556				1.491	
P			.068				.159				.758				.828	

Item	Grade				Teacher				Sex				Ach. Level			
		A	U	D		A	U	D		A	U	D		A	U	D
16	7.	18	20	62	1.	12	17	71	B	16	24	60	H	7	30	63
	8.	4	24	72	2.	4	32	64	G	7	18	75	A	12	15	73
					3.	22	23	55					L	16	22	62
					4.	4	14	82								
χ		4.735					9.032				3.270				3.214	
P		.094					.172				.195				.523	
9	7.	27	24	49	1.	25	8	67	B	33	26	41	H	30	17	53
	8.	19	32	49	2.	20	44	36	G	11	29	60	A	17	28	5
					3.	29	36	35					L	25	38	37
					4.	18	18	64								
χ^2		1.344					12.434				7.319				4.901	
P		.511					.053				.026				.298	
13	7.	18	29	53	1.	8	25	67	B	19	30	51	H	17	30	53
	8.	21	28	51	2.	16	24	60	G.	20	27	53	A	27	20	53
					3.	26	32	42					L	12	38	50
					4.	27	32	41								
χ^2		.156					5.936				.124				4.216	
P		.925					.430				.940				.378	
15	7.	25	31	44	1.	21	25	54	B	37	24	39	H	23	30	47
	8.	40	26	34	2.	40	28	32	G	27	33	40	A	35	25	40
					3.	29	36	35					L	38	31	31
					4.	41	23	36								
χ^2		2.608					4.791				1.498				2.333	
P		.271					.571				.473				.675	
2	7.	60	29	11	1.	67	25	8	B	55	33	12	H	63	27	10
	8.	43	49	8	2.	40	48	12	G	49	44	7	A	63	32	5
					3.	55	32	13					L	28	56	16
					4.	46	50	4								
χ^2		4.244					5.825				1.767				11.260	
P		.120					.443				.413				.024	

Item	Grade	A	U	D	Teacher	A	U	D	Sex	A	U	D	Ach. Level	A	U	D
4	7.	53	24	23	1.	62	17	21	B	51	25	24	H	37	37	26
	8.	40	34	26	2.	40	36	24	G	42	33	25	A	63	22	15
					3.	45	29	26					L	38	28	34
					4.	41	32	27								
χ^2		1.817				3.671				1.081				7.352		
P		.403				.721				.582				.118		
10	7.	44	51	5	1.	50	50	0	B	39	52	9	H	53	47	0
	8.	38	49	13	2.	32	52	16	G	44	47	9	A	33	52	15
					3.	39	52	9					L	41	50	9
					4.	46	45	9								
χ^2		1.731				4.840				.388				6.315		
P		.421				.565				.824				.177		
6	7.	7	35	58	1.	4	38	58	B	12	39	49	H	7	30	63
	8.	15	40	45	2.	16	48	36	G	9	36	55	A	8	40	52
					3.	10	32	58					L	19	41	40
					4.	14	32	54								
χ^2		2.489				4.628				.531				4.897		
P		.288				.592				.767				.298		
12	7.	40	42	18	1.	37	38	25	B	40	35	25	H	40	33	27
	8.	43	38	19	2.	28	48	24	G	42	47	11	A	50	40	10
					3.	42	45	13					L	31	47	22
					4.	59	27	14								
χ^2		.131				6.128				3.302				5.060		
P		.937				.409				.192				.281		
14	7.	35	51	14	1.	42	50	8	B	37	49	14	H	33	47	20
	8.	36	49	15	2.	28	56	16	G	33	51	16	A	37	55	8
					3.	29	52	19					L	34	47	19
					4.	45	41	14								
χ^2		.041				3.365				.147				2.765		
P		.980				.762				.929				.598		

Item	Grade	A	U	D	Teacher	A	U	D	Sex	A	U	D	Ach. Level	A	U	D
7	7.	40	38	22	1.	46	42	12	B	42	33	25	H	30	47	23
	8.	34	40	26	2.	32	44	24	G	31	47	22	A	43	40	17
					3.	36	35	29					L	38	31	31
					4.	36	37	27								
x^2		.423				2.802				2.014				3.098		
P		.810				.833				.365				.542		
17	7.	44	45	11	1.	42	46	12	B	40	42	18	H	40	40	20
	8.	41	36	23	2.	36	40	24	G	44	40	16	A	45	43	12
					3.	45	45	10					L	40	41	19
					4.	45	32	23								
x^2		2.967				3.522				.187				.857		
P		.227				.741				.911				.931		
18	7.	40	20	40	1.	33	21	46	B	47	25	28	H	53	10	37
	8.	47	30	23	2.	40	40	20	G	38	24	38	A	32	30	38
					3.	45	19	36					L	47	31	22
					4.	55	18	27								
x^2		3.420				7.357				1.269				7.124		
P		.181				.289				.530				.129		
19	7.	33	29	39	1.	46	37	17	B	40	23	37	H	47	23	30
	8.	57	26	17	2.	40	32	28	G	49	33	18	A	40	30	30
					3.	22	23	55					L	47	28	25
					4.	77	18	5								
x^2		7.618				24.069				4.645				.731		
P		.022				.001				.098				.947		
20	7.	56	18	26	1.	46	29	25	B	56	25	19	H	50	23	27
	8.	41	38	21	2.	56	28	16	G	40	31	29	A	50	30	20
					3.	64	10	26					L	47	28	25
					4.	23	50	27								
x^2		5.237				13.414				2.712				.696		
P		.073				.037				.258				.952		

APPENDIX H

RAW SCORES FOR ALL SUBJECTS

ID Numbers - The first three digits of the five digit identification number for each student indicates his class and treatment group as follows:

ID	Class	Group	ID	Class	Group
001 - 125	7D	ML	101 - 132	8C	ML
040 - 064	7E	ML	135 - 166	8B	ML
070 - 093	7B	ML	168 - 198	8A	ML
201 - 224	7A	CD	340 - 367	8F	CD
265 - 295	7C	CD	370 - 399	8D	CD
430 - 455	7G	CON	501 - 530	8E	CON
470 - 495	7F	CON	570 - 595	8G	CON

The last of the five digits indicates the student's sex: 1 - Boy; 2 - Girl.

Column Heading	Test	Column Heading	Test
1	STM Pre	9	AMS Post
2	STM Post	10	LDM Enjoyment Pre
3	STM Combined Quizzes	11	LDM Familiarity Pre
4	IL	12	LDM Situation Pre
5	CA	13	LDM Enjoyment Post
6	HLTPS	14	LDM Familiarity Post
7	CUM	25	LDM Situation Post
8	AMS Pre		

O - indicates that the test was not taken.

Ɵ - indicates a score of zero.

A line through a score indicates that the data was incomplete.

ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
102	32	30	54	16	14	4	2	0	0	0	0	0	0	0	0
201	27	26	52	14	7	4	θ	0	104	0	0	0	0	0	0
302	13	13	33	16	5	2	2	0	0	0	0	0	0	0	0
401	21	17	44	15	5	0	2	0	101	0	0	0	0	0	0
501	18	26	69	27	7	4	θ	0	85	0	0	0	0	0	0
702	25	18	39	12	6	3	θ	0	81	0	0	0	0	0	0
1002	39	21	81	17	10	4	1	0	96	0	0	0	0	0	0
1101	23	21	66	0	3	7	6	0	0	0	0	0	0	0	0
1202	15	23	57	15	11	6	1	0	98	0	0	0	0	0	0
1301	28	25	51	14	9	5	1	0	84	0	0	0	0	0	0
1401	37	31	83	0	12	5	1	0	108	0	0	0	0	0	0
1502	21	24	59	0	11	4	1	0	86	0	0	0	0	0	0
1602	31	11	38	0	10	0	θ	0	60	0	0	0	0	0	0
1701	9	16	35	10	9	3	0	0	0	0	0	0	0	0	0
1801	8	17	55	6	9	5	θ	0	84	0	0	0	0	0	0
1902	40	22	56	16	6	4	θ	0	56	0	0	0	0	0	0
2001	42	27	65	24	8	3	2	0	95	0	0	0	0	0	0
2102	13	19	40	7	4	1	1	0	65	0	0	0	0	0	0
2202	13	15	22	14	3	8	0	0	0	0	0	0	0	0	0
2302	20	23	58	0	4	6	1	0	0	0	0	0	0	0	0
2402	31	18	61	9	8	8	1	0	0	0	0	0	0	0	0
2502	28	24	67	0	5	0	0	0	0	0	0	0	0	0	0
4001	28	29	59	0	18	12	1	77	71	0	0	0	0	0	0
4101	13	11	37	0	8	2	θ	87	65	0	0	0	0	0	0
4202	12	15	45	0	7	5	1	81	72	0	0	0	0	0	0
4302	40	27	57	0	4	5	4	0	0	0	0	0	0	0	0
4402	46	30	67	32	13	12	1	85	70	0	0	0	0	0	0
4502	43	20	65	19	12	4	1	65	64	0	0	0	0	0	0
4602	41	28	81	34	11	11	1	88	85	0	0	0	0	0	0
4702	14	18	54	19	9	6	1	58	67	0	0	0	0	0	0
4901	17	25	35	0	8	5	1	0	0	0	0	0	0	0	0
5001	54	30	80	29	12	13	3	82	89	0	0	0	0	0	0
5101	31	20	75	0	11	1	1	58	69	0	0	0	0	0	0
5202	48	33	76	26	21	12	5	84	91	0	0	0	0	0	0
5301	53	30	68	18	15	13	1	88	89	0	0	0	0	0	0
5402	28	21	40	0	5	7	θ	0	0	0	0	0	0	0	0
5502	32	22	62	9	6	5	1	99	102	0	0	0	0	0	0
5602	50	31	83	39	17	10	6	82	80	0	0	0	0	0	0
5702	42	30	72	31	17	6	1	62	68	0	0	0	0	0	0
5802	54	38	86	35	15	11	1	73	82	0	0	0	0	0	0
5902	36	27	67	0	10	6	1	53	52	0	0	0	0	0	0
6002	35	26	67	19	13	7	1	80	85	0	0	0	0	0	0
6102	18	21	41	0	15	6	1	48	57	0	0	0	0	0	0
6201	11	18	35	19	10	3	1	63	64	0	0	0	0	0	0
6302	10	14	51	0	9	4	1	0	0	0	0	0	0	0	0
6401	30	27	58	17	10	5	2	87	80	0	0	0	0	0	0
7001	54	27	80	37	12	11	2	0	80	48	15	11	50	17	13
7102	58	36	90	34	20	16	2	0	65	37	18	9	41	24	13

ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7202	48	29	83	30	5	9	2	0	65	43	14	7	40	14	9
7401	32	34	76	32	17	9	1	0	67	41	10	8	54	19	14
7501	36	31	76	0	0	7	2	0	57	41	19	8	29	13	6
7601	26	24	68	22	11	10	5	0	68	0	0	0	0	0	0
7701	30	27	85	28	17	2	3	0	91	49	8	8	63	22	5
7901	41	38	92	42	24	15	1	0	60	52	21	10	40	16	10
8001	22	22	50	12	14	6	1	0	94	29	11	11	54	22	11
8102	42	28	83	32	19	13	1	0	79	43	15	7	50	16	12
8202	42	32	72	23	14	9	1	0	85	42	17	5	49	21	8
8301	38	23	72	0	14	9	2	0	64	54	19	11	50	22	14
8401	39	16	67	21	4	7	3	0	91	21	10	8	55	20	12
8501	26	21	66	0	9	3	2	0	80	51	22	6	52	22	10
8602	22	29	59	0	7	6	2	0	84	39	16	10	53	11	13
8702	48	34	86	36	24	18	3	0	82	35	20	8	41	18	8
8801	47	36	86	0	11	11	2	0	55	44	22	3	28	26	5
8902	13	8	52	22	11	3	1	0	87	46	15	5	50	21	10
9001	29	28	0	0	16	11	0	0	97	54	11	10	54	23	12
9102	48	29	72	25	16	9	3	0	53	23	14	11	38	16	11
9302	42	25	80	0	14	6	2	0	80	52	14	9	48	19	14
20102	55	35	86	35	23	20	5	84	81	0	0	0	0	0	0
20202	32	0	71	24	11	14	1	72	67	0	0	0	0	0	0
20302	12	24	38	18	8	1	2	73	71	0	0	0	0	0	0
20401	22	18	47	0	11	8	2	79	78	0	0	0	0	0	0
20501	33	26	74	0	12	11	2	91	89	0	0	0	0	0	0
20601	58	33	80	33	27	18	2	86	84	0	0	0	0	0	0
20701	28	27	53	0	11	7	2	64	67	0	0	0	0	0	0
20802	52	33	81	36	14	16	2	99	94	0	0	0	0	0	0
20901	45	29	53	0	12	14	1	62	65	0	0	0	0	0	0
21001	34	20	66	22	16	13	1	86	85	0	0	0	0	0	0
21101	31	15	63	25	9	4	θ	89	82	0	0	0	0	0	0
21201	14	19	38	20	9	0	θ	76	69	0	0	0	0	0	0
21301	48	25	76	0	21	8	2	92	96	0	0	0	0	0	0
21401	48	26	87	33	16	15	2	87	83	0	0	0	0	0	0
21502	41	20	68	29	10	9	2	89	68	0	0	0	0	0	0
21601	34	15	76	35	15	14	2	89	95	0	0	0	0	0	0
21701	46	15	70	24	10	6	θ	81	83	0	0	0	0	0	0
21801	53	34	95	0	20	21	2	89	89	0	0	0	0	0	0
21901	29	21	31	20	8	6	2	74	82	0	0	0	0	0	0
22002	22	39	72	0	21	15	6	81	80	0	0	0	0	0	0
22102	36	21	77	30	12	14	1	61	71	0	0	0	0	0	0
22201	47	28	92	23	14	12	1	84	77	0	0	0	0	0	0
22302	44	30	79	0	13	7	2	0	0	0	0	0	0	0	0
22402	62	14	87	27	18	14	2	83	72	0	0	0	0	0	0
26501	26	24	48	29	10	9	1	0	88	0	0	0	0	0	0
26601	23	23	44	0	12	2	3	0	81	0	0	0	0	0	0
26701	25	22	48	23	4	3	1	0	64	0	0	0	0	0	0
26802	34	19	46	0	7	4	2	0	0	0	0	0	0	0	0
26902	44	31	83	0	18	20	2	0	88	0	0	0	0	0	0

ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
27001	36	21	53	0	9	4	1	0	55	28	5	3	26	13	10
27102	9	18	41	0	4	5	1	0	83	52	18	9	50	19	9
27202	36	21	51	27	10	11	2	0	60	35	18	5	33	17	11
27302	55	33	73	29	12	14	1	0	81	52	21	6	52	21	7
27401	40	27	47	30	9	7	2	0	77	52	22	11	43	16	14
27501	55	38	80	0	14	10	1	0	77	44	16	8	39	14	3
27601	0	27	74	0	18	0	1	0	73	28	20	10	40	26	8
27702	17	28	62	24	11	4	1	0	86	44	16	6	48	20	10
27801	30	23	47	0	12	2	1	0	60	12	13	8	18	22	8
27901	29	23	66	31	12	5	1	0	83	43	15	10	47	16	8
28002	43	28	80	34	14	12	1	0	79	42	22	6	49	20	6
28101	58	40	[illegible]	0	26	18	0	0	70	32	14	7	50	25	8
28202	35	32	73	33	14	7	3	0	74	40	20	2	44	19	3
28301	12	20	60	0	16	4	2	0	76	34	21	6	46	15	7
28401	36	27	82	0	17	15	1	0	77	51	26	6	48	22	5
28501	24	20	32	0	9	9	1	0	79	40	16	10	41	19	6
28602	27	41	72	0	10	10	1	0	76	33	10	2	50	15	13
28702	52	26	76	0	17	14	1	0	63	52	18	6	47	20	9
28801	40	31	73	30	11	8	1	0	83	44	18	8	52	13	14
28901	25	14	31	0	14	5	2	0	69	25	18	10	48	23	7
29001	46	41	88	0	19	21	2	0	77	34	23	5	43	25	6
29101	41	23	67	31	11	11	3	0	90	45	13	2	54	17	7
29201	9	20	24	0	7	3	1	0	0	0	0	0	0	0	0
29301	50	25	67	32	15	7	2	0	65	30	9	2	23	9	3
29402	14	22	45	24	10	1	0	0	87	51	22	5	47	14	2
29501	26	23	58	33	19	8	1	0	69	40	14	7	37	16	10
43002	53	24	81	0	10	11	1	0	90	52	24	8	61	25	8
43101	13	22	40	0	11	5	1	0	71	30	12	7	43	21	8
43202	22	26	45	0	9	1	2	0	81	43	13	8	40	13	10
43302	40	32	79	0	9	13	2	0	90	51	23	7	57	25	8
43402	5	21	49	0	6	2	2	0	77	51	14	7	49	16	8
43502	36	27	68	0	10	2	2	0	95	57	20	9	51	23	7
43602	42	28	80	0	5	0	1	0	81	36	21	6	46	22	8
43701	34	22	54	0	6	2	1	0	78	49	13	3	44	16	11
43801	26	27	44	0	7	7	1	0	82	47	22	8	47	20	7
43901	33	35	75	0	8	9	1	0	85	51	26	7	41	25	7
44001	52	29	72	0	13	4	1	0	84	46	24	5	48	24	5
44102	26	18	61	0	4	0	1	0	108	0	0	0	0	0	0
44201	24	28	53	0	9	5	1	0	62	34	15	5	41	20	10
44302	28	22	61	0	1	3	1	0	89	33	11	6	45	17	5
44402	25	23	45	0	6	1	1	0	60	39	14	5	45	12	3
44501	32	19	57	0	11	8	1	0	45	41	24	11	28	12	14
44602	36	26	78	0	10	5	1	0	89	51	21	8	59	19	7
44702	27	24	60	0	6	7	1	0	70	38	11	7	46	10	6
44801	26	26	72	0	8	5	1	0	97	41	23	6	57	24	5
44901	24	21	32	0	12	3	2	0	87	50	11	12	57	8	9
45001	16	19	19	0	11	7	1	0	0	0	0	0	0	0	0
45101	62	47	96	0	18	15	3	0	96	57	17	4	49	15	2

ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
45202	30	25	60	0	7	6	1	0	78	41	16	8	47	17	5
45302	23	24	54	0	5	3	1	0	71	52	16	5	45	18	5
45402	30	22	61	0	8	5	0	0	54	52	17	6	20	16	9
45501	44	24	73	0	10	3	1	0	102	60	25	5	60	23	8
47001	11	17	49	0	5	2	0	79	59	0	0	0	0	0	0
47201	27	33	66	0	14	13	1	82	80	0	0	0	0	0	0
47301	26	16	64	0	5	5	1	57	65	0	0	0	0	0	0
47401	22	27	72	0	11	2	1	87	95	0	0	0	0	0	0
47502	45	33	96	0	10	7	1	0	0	0	0	0	0	0	0
47602	35	20	87	0	6	9	2	86	85	0	0	0	0	0	0
47702	23	16	46	0	6	1	1	0	0	0	0	0	0	0	0
47801	9	28	52	0	14	0	0	99	93	0	0	0	0	0	0
47902	39	30	77	0	0	0	0	89	80	0	0	0	0	0	0
48002	33	28	72	0	11	1	1	79	83	0	0	0	0	0	0
48101	15	21	42	0	9	0	2	81	54	0	0	0	0	0	0
48202	57	32	90	0	15	10	0	64	60	0	0	0	0	0	0
48301	41	28	65	0	12	7	1	83	89	0	0	0	0	0	0
48502	44	38	90	0	11	10	2	74	62	0	0	0	0	0	0
48602	50	35	84	0	12	7	1	78	76	0	0	0	0	0	0
48701	19	28	48	0	8	2	1	73	89	0	0	0	0	0	0
48802	9	0	33	0	6	4	0	0	0	0	0	0	0	0	0
48902	55	26	69	0	7	6	1	80	79	0	0	0	0	0	0
49002	28	24	63	0	8	0	3	79	64	0	0	0	0	0	0
49102	49	29	80	0	8	6	1	99	94	0	0	0	0	0	0
49201	35	36	80	0	7	4	2	99	102	0	0	0	0	0	0
49402	51	28	84	0	9	10	2	84	75	0	0	0	0	0	0
49502	34	24	46	0	16	5	1	86	94	0	0	0	0	0	0

ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
10102	21	13	60	25	12	9	1	60	57	0	0	0	0	0	0
10202	15	26	61	23	12	9	0	56	64	0	0	0	0	0	0
10302	16	15	23	20	10	2	1	0	0	0	0	0	0	0	0
10401	17	11	38	0	7	7	θ	58	65	0	0	0	0	0	0
10501	10	16	43	0	9	7	1	67	64	0	0	0	0	0	0
10601	15	19	47	0	8	0	1	52	57	0	0	0	0	0	0
10702	24	27	65	22	11	5	1	0	0	0	0	0	0	0	0
10802	24	19	67	0	14	7	θ	74	84	0	0	0	0	0	0
10902	8	17	56	0	7	8	1	69	72	0	0	0	0	0	0
11002	22	19	45	28	10	4	1	72	76	0	0	0	0	0	0
11102	30	23	~~41~~	0	17	13	1	63	64	0	0	0	0	0	0
11202	34	32	122	28	14	15	1	75	64	0	0	0	0	0	0
11302	22	26	56	29	9	3	1	40	94	0	0	0	0	0	0
11401	20	24	80	28	13	16	θ	64	78	0	0	0	0	0	0
11501	18	21	84	18	15	0	1	75	83	0	0	0	0	0	0
11601	22	16	~~46~~	0	10	8	1	64	57	0	0	0	0	0	0
11701	19	16	45	0	16	7	0	57	49	0	0	0	0	0	0
11801	32	27	93	34	22	13	1	71	71	0	0	0	0	0	0
12002	30	33	107	38	19	19	1	69	73	0	0	0	0	0	0
12101	22	21	61	17	9	7	1	69	91	0	0	0	0	0	0
12202	29	26	~~61~~	0	13	8	1	82	73	0	0	0	0	0	0
12502	17	21	60	0	14	13	1	80	69	0	0	0	0	0	0
12601	24	18	~~38~~	0	15	0	0	56	60	0	0	0	0	0	0
12702	27	33	109	0	10	6	0	80	79	0	0	0	0	0	0
12802	22	20	61	0	6	6	1	55	54	0	0	0	0	0	0
12902	12	14	~~37~~	23	6	7	1	57	48	0	0	0	0	0	0
13002	16	23	~~29~~	19	8	4	1	57	43	0	0	0	0	0	0
13101	20	16	44	24	13	9	1	46	60	0	0	0	0	0	0
13202	11	14	35	21	8	2	θ	0	0	0	0	0	0	0	0
13501	26	24	89	30	14	17	2	0	66	45	15	10	37	10	9
13601	30	26	103	41	23	24	3	0	84	55	20	5	48	17	7
13702	29	26	83	35	14	11	5	0	82	43	16	6	46	20	11
13802	25	27	87	33	0	5	3	0	68	54	17	14	30	14	8
13902	38	30	108	41	18	23	2	0	74	43	21	8	47	22	9
14002	28	23	90	35	15	14	7	0	83	36	22	2	53	17	7
14102	31	23	74	37	20	11	4	0	84	39	15	9	53	22	9
14201	28	17	75	0	17	8	1	0	82	45	17	5	37	12	7
14301	39	35	116	41	32	19	6	0	72	40	17	4	47	13	9
14401	28	23	74	26	17	15	1	0	77	45	24	6	34	19	6
14501	29	25	75	36	24	22	1	0	77	43	19	12	47	16	6
14601	30	25	117	33	22	29	3	0	89	49	20	4	42	19	6
14701	28	26	72	33	17	13	3	0	75	48	22	6	46	26	11
14802	31	27	98	0	16	20	3	0	81	44	20	7	50	16	5
14901	15	14	32	27	10	9	1	0	63	0	0	0	0	0	0
15001	27	24	80	0	22	17	1	0	72	54	24	7	51	22	8
15102	37	36	126	43	28	22	5	0	88	49	17	10	42	15	13
15202	37	35	127	42	25	18	5	0	79	33	16	3	40	16	4
15301	25	22	79	0	20	3	1	0	89	47	18	10	46	17	11

ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
15401	36	30	110	33	16	12	3	0	75	48	27	4	41	19	4
15501	33	32	96	27	24	14	3	0	80	43	19	7	41	19	7
15602	31	37	108	0	24	17	2	0	94	56	20	10	53	22	11
15701	36	28	101	35	30	17	1	0	89	46	18	5	55	19	7
15902	25	23	66	28	11	0	6	0	62	37	12	10	40	9	5
16001	40	28	102	40	29	21	3	0	80	53	18	4	40	10	12
16102	18	20	66	33	8	11	6	0	87	45	14	8	35	14	11
16202	32	30	100	28	22	15	2	0	74	19	14	3	44	13	8
16301	42	37	131	37	22	26	3	0	85	50	9	10	48	16	8
16401	33	33	106	35	24	23	0	0	85	41	16	9	53	15	8
16502	27	33	79	37	17	19	5	0	61	21	14	11	28	10	4
16602	17	18	58	33	15	8	2	0	76	52	24	11	51	18	8
16801	25	28	90	33	21	9	1	92	90	40	20	9	44	17	6
16901	24	23	66	35	17	6	1	47	46	59	14	9	54	20	13
17101	28	27	91	27	19	17	1	79	74	0	0	0	0	0	0
17401	10	16	50	21	5	2	1	75	77	0	0	0	0	0	0
17501	15	17	36	11	12	7	2	63	51	0	0	0	0	0	0
17601	24	19	82	31	11	13	2	0	0	0	0	0	0	0	0
17702	28	27	90	34	18	15	1	60	47	0	0	0	0	0	0
17801	16	13	48	19	13	11	2	87	77	0	0	0	0	0	0
18102	20	23	46	34	13	8	1	71	60	0	0	0	0	0	0
18201	21	15	66	32	12	8	2	57	81	0	0	0	0	0	0
18302	16	22	55	24	16	15	1	56	51	0	0	0	0	0	0
18402	16	20	89	28	14	3	2	75	75	0	0	0	0	0	0
18502	13	17	19	0	3	4	0	72	68	0	0	0	0	0	0
18602	31	27	104	29	17	14	2	75	63	0	0	0	0	0	0
18701	12	14	44	27	8	6	1	80	72	0	0	0	0	0	0
18801	17	17	77	26	6	1	1	86	96	0	0	0	0	0	0
19001	19	18	46	22	14	11	3	69	76	0	0	0	0	0	0
19101	20	17	57	0	15	15	0	59	68	0	0	0	0	0	0
19202	23	23	65	32	11	13	3	90	88	0	0	0	0	0	0
19301	10	19	69	0	12	7	1	71	75	0	0	0	0	0	0
19502	25	27	101	33	18	16	4	78	79	0	0	0	0	0	0
19701	27	26	86	30	17	9	0	89	99	0	0	0	0	0	0
19801	10	15	~~14~~	0	9	1	1	67	62	0	0	0	0	0	0
34001	17	15	52	0	11	15	5	78	80	0	0	0	0	0	0
34101	39	37	114	36	0	15	2	85	92	0	0	0	0	0	0
34201	23	30	104	0	21	23	3	84	88	0	0	0	0	0	0
34302	13	24	39	23	18	7	0	91	78	0	0	0	0	0	0
34402	18	18	85	28	16	15	0	67	58	0	0	0	0	0	0
34502	26	24	94	33	13	12	1	72	77	0	0	0	0	0	0
34602	27	25	58	28	13	14	1	54	57	0	0	0	0	0	0
34701	16	15	33	9	9	8	1	78	81	0	0	0	0	0	0
34801	15	24	77	28	19	11	2	95	82	0	0	0	0	0	0
34901	23	11	42	0	7	7	3	41	77	0	0	0	0	0	0
35001	21	18	52	29	18	14	3	69	75	0	0	0	0	0	0
35101	19	12	67	0	7	0	1	0	0	0	0	0	0	0	0
35302	29	25	94	28	17	15	2	63	55	0	0	0	0	0	0

ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
35402	19	16	69	0	11	3	0	61	66	0	0	0	0	0	0
35502	22	15	67	27	14	7	1	72	70	0	0	0	0	0	0
35602	18	18	80	30	22	12	0	44	56	0	0	0	0	0	0
35701	35	31	129	40	23	18	4	82	73	0	0	0	0	0	0
35802	14	21	90	26	9	6	1	60	61	0	0	0	0	0	0
35902	25	18	84	0	16	19	2	70	71	0	0	0	0	0	0
36001	19	16	75	21	16	9	1	77	75	0	0	0	0	0	0
36101	32	32	127	38	25	15	7	67	61	0	0	0	0	0	0
36301	32	34	114	34	23	22	6	96	96	0	0	0	0	0	0
36401	16	21	43	17	9	5	1	56	55	0	0	0	0	0	0
36501	20	15	88	0	12	9	4	81	100	0	0	0	0	0	0
36701	13	15	42	0	5	6	2	0	0	0	0	0	0	0	0
37001	42	34	141	44	27	30	7	0	80	36	11	2	44	18	6
37102	35	33	90	0	28	18	3	0	99	52	22	8	57	24	12
37201	24	21	49	27	13	8	1	0	78	33	20	5	45	14	11
37302	22	28	68	0	15	10	4	0	91	48	22	8	43	20	10
37502	23	16	44	0	11	8	1	0	41	41	12	11	15	10	8
37602	26	30	91	0	25	11	4	0	73	0	0	0	0	0	0
37702	21	18	48	27	5	5	4	0	68	36	15	8	38	17	5
37801	29	30	98	37	13	14	2	0	68	30	13	10	44	15	9
37901	17	21	69	19	10	8	2	0	61	40	11	10	32	11	11
38002	16	13	45	24	11	3	1	0	49	34	13	11	30	9	10
38202	20	24	67	27	12	4	1	0	50	48	17	11	42	14	14
38402	28	28	82	41	21	11	1	0	63	47	24	5	48	20	5
38602	27	29	56	34	21	16	2	0	83	49	16	10	52	19	10
38802	25	24	71	0	9	16	1	0	74	40	18	4	37	16	4
39002	14	26	~~36~~	26	18	4	1	0	75	0	0	0	0	0	0
39101	16	17	38	0	13	10	1	0	61	37	19	8	34	10	5
39301	21	19	~~57~~	0	19	15	4	0	65	0	0	0	0	0	0
39502	24	27	65	0	14	11	1	0	72	34	15	4	36	14	6
39602	15	18	54	0	11	15	1	0	49	40	18	4	37	14	11
39702	14	13	~~54~~	25	14	9	2	0	66	35	21	8	37	12	14
39801	11	22	53	30	8	6	1	0	51	36	10	7	21	4	2
39902	14	12	42	31	22	4	2	0	69	0	0	0	0	0	0
50101	28	13	90	0	10	12	0	0	49	21	24	11	29	23	8
50302	31	31	105	0	14	8	0	0	35	13	14	8	13	22	5
50402	23	0	65	0	13	9	1	0	50	30	10	3	22	11	3
50502	20	23	63	0	13	11	1	0	49	26	12	11	28	11	11
50601	20	23	23	0	8	8	0	0	58	36	15	4	35	16	8
50701	25	34	85	0	11	14	0	0	60	56	20	12	44	18	9
50802	15	23	51	0	8	0	1	0	79	40	16	8	50	13	8
50901	21	32	80	0	7	12	1	0	93	53	16	11	52	28	2
51001	26	30	98	0	14	3	1	0	0	0	0	0	0	0	0
51101	20	30	61	0	11	7	1	0	73	41	22	8	34	16	9
51202	26	15	78	0	12	8	1	0	70	31	18	9	46	19	8
51302	12	0	44	0	0	9	1	0	68	39	19	14	34	15	14
51401	22	36	112	0	13	11	0	0	75	59	22	12	45	17	6
51502	16	22	27	0	9	0	1	0	70	24	15	6	30	16	2

ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
51602	20	22	93	0	5	4	1	0	0	0	0	0	0	0	0
51701	29	32	95	0	12	6	1	0	73	37	25	11	33	18	14
51801	16	12	30	0	9	0	1	0	68	30	14	11	17	8	8
51902	19	24	83	0	8	10	1	0	66	50	19	11	35	17	9
52002	15	33	64	0	14	6	1	0	59	41	14	13	26	19	12
52102	13	21	42	0	6	7	2	0	52	42	15	11	25	9	8
52202	15	24	53	0	7	2	0	0	79	32	16	8	50	13	8
52302	30	24	90	0	14	6	1	0	37	27	18	8	17	20	6
52402	17	12	40	0	13	6	0	0	50	34	11	14	32	7	13
52601	33	33	92	0	8	11	1	0	74	43	18	11	21	10	8
52801	26	23	85	0	16	14	1	0	56	19	14	8	22	13	5
52901	20	12	46	0	11	14	1	0	54	0	0	0	0	0	0
53001	20	13	46	0	9	2	0	0	48	0	0	0	0	0	0
57002	37	30	123	0	21	17	0	70	71	0	0	0	0	0	0
57102	37	40	125	0	13	13	2	85	79	0	0	0	0	0	0
57202	38	29	109	0	19	18	3	99	87	0	0	0	0	0	0
57301	31	27	53	0	9	9	2	46	65	0	0	0	0	0	0
57402	19	9	21	0	18	7	2	81	82	0	0	0	0	0	0
57502	31	32	113	0	9	16	6	72	69	0	0	0	0	0	0
57602	34	35	106	0	6	8	2	80	81	0	0	0	0	0	0
57701	26	32	108	0	20	19	3	75	76	0	0	0	0	0	0
57802	36	30	115	0	13	14	3	68	63	0	0	0	0	0	0
57901	15	23	35	0	8	0	3	72	70	0	0	0	0	0	0
58002	25	26	86	0	14	11	1	92	81	0	0	0	0	0	0
58102	29	13	95	0	14	12	4	82	75	0	0	0	0	0	0
58202	34	37	109	0	17	10	2	87	89	0	0	0	0	0	0
58301	35	37	81	0	26	22	3	90	86	0	0	0	0	0	0
58402	30	31	111	0	16	13	0	0	0	0	0	0	0	0	0
58502	34	34	128	0	18	12	2	72	53	0	0	0	0	0	0
58602	34	23	102	0	21	12	1	64	62	0	0	0	0	0	0
58702	23	18	51	0	0	0	0	40	39	0	0	0	0	0	0
58901	38	37	129	0	15	17	3	60	65	0	0	0	0	0	0
59001	34	32	109	0	16	22	2	0	0	0	0	0	0	0	0
59102	39	41	143	0	19	15	8	88	86	0	0	0	0	0	0
59502	22	24	78	0	11	6	3	91	82	0	0	0	0	0	0