University of Alberta

Fault Diagnosis of Sampled Data Systems

by

Somayeh Mostafavi

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*To My **Love** Nima and My Dear Son Daniel*

Abstract

This thesis is concerned with fault detection of sampled data systems with special emphasis on the time varying characteristics of the system. A discretization method based on the norm invariant transformation is proposed for time invariant systems with aperiodic sampler and time varying systems with periodic sampler. The outcome of this transformation for both cases is a discrete time varying system.

The problem of fault detection for discrete time varying systems is studied and the proposed approach along with the norm invariant transformation is used for fault detection of sampled data system with time varying characteristics. Also, because of the time varying nature of Kalman filter, its application as an observer for fault detection of discrete time varying systems is investigated.

Finally, the fault detection method with norm invariant transformation is used for fault diagnosis of network control system with network induced delay.

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Introduction and Literature Review

1.1 Introduction

Faults in feedback control systems can result in undesirable performance or instability. Therefore they should be identified so that appropriate remedies can be applied and desirable performance is achieved. Numerous design methods are available in the literature for fault detection [1, 2, 3].

The model-based fault detection technique has been developed remarkably since early 1970's. Its efficiency in detecting faults has been successfully demonstrated by a great number of applications in the aerospace, chemical and automotive industries [1]. Generally speaking, model based fault detection is concerned with residual generation and evaluation. A fault detection filter can be constructed for this purpose to reduce the effects of external disturbance and model uncertainty, while maximizing its sensitivity to faults so that faults in the system can be detected as soon as possible. Figure 1.1 shows a schematic description of a model based fault detection approach.

On the other hand, most controllers are implemented on computers where the actual process is a continuous time system, while the controller and the fault detection system are implemented in computers or other digital devices. Fault detection in sampled data systems has received a lot of attention during the last two decades.

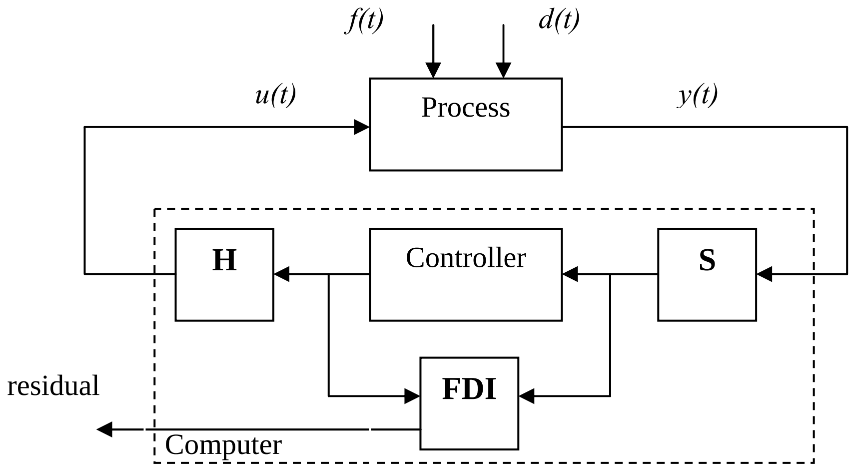


Figure 1.1: Schematic description of model based fault detection

In the development of sampled data control and fault detection (FD) theory, one of the important properties considered is periodic sampling which results in a periodic time varying closed loop system. It is reasonable to assume periodic sampling in the conventional implementation of sampled data systems. However, it is hard to perform periodic sampling in some of the more recent applications. For example, in networked control systemsresources for measurement are restricted and hence the sampling operation tends to be aperiodic and uncertain [4]. In view of the widespread use of such types of system, it is important to study the fault diagnosis for varying sampling intervals as well.

In this thesis, a general framework for fault detection of time varying sampled data systems is developed. These systems include continuous time linear time invariant systems with aperiodic sampler and linear time varying systems with periodic sampler. By defining norms of the sampled data systems and the resulted discrete time varying system, this framework allows us to extend norm based method of discrete time varying fault detection to sampled data systems.

In contrast to the existing results for fault diagnosis of linear time invariant systems, there are relatively few results on linear time varying systems. In this thesis, we introduce a method for designing the residual generator for linear discrete time varying systems. Furthermore, when studying fault detection of discrete time varying systems, it is worthwhile to investigate the possibility of using the time varying nature of Kalman filter [5] to utilize it as a fault detection tool. Part of this thesis focuses on this approach and elaborates on the idea and its application.

In summary, the proposed fault diagnosis approach together with norm invariant transformation is used for fault detection of a sample data system composed of either a time invariant system with aperiodic sampler or a time varying system with periodic sampler.

The effectiveness of the above described approach is shown by implementing it to fault detection of network control systems (NCS). Network control systems are feedback control systems wherein the control loops are closed via real time networks and are comprised of a large amount of actuators, sensors and controllers which are equipped with network interfaces, and are defined as nodes of the network.

Despite the enormous advantages of NCS, the introduction of networks also brings some new problems and challenges, such as network-induced delay, packet dropout, network scheduling and quantization problems. Induced delay in the network, either constant or time-varying, can degrade the total performance of the system and even can destabilize the system. Some constraints of NCS, such as networked induced delay and packet dropout, can influence the fault diagnosis system as well; therefore, fault diagnosis and tolerant control approaches that are used for traditional control systems cannot be used directly for this type of systems. In this thesis, a sampled data system model for NCS is proposed in which the network induced delay is also considered. Using this model, the fault detection problem can then be handled by applying the norm based time varying residual generation method.

1.2 Literature review

1.2.1 Norm invariant transformation

Many research results on fault detection of sampled data system suggest an indirect design by either

- Designing a continuous time FD for the continuous time process and then discretizing the FD and applying that to the real system, or
- Discretizing the continuous time system and designing a discrete time FD for that system and then applying that to the real system.

Approximations exist in both approaches and therefore the FD system may not perform as expected. There are also methods based on introducing appropriate operators that capture the inter sample behavior [7, 8].

Izadi et al. developed a general framework for sampled date fault detection [8]. By defining the norms of sampled data systems and the so-called norm invariant transformation, this framework allows the extension of any (H_2 or H_∞) norm based method of discrete time fault detection to sampled data systems. In their work, a linear time invariant continuous time system with a transfer function $\hat{g}(s)$ was considered as

$$\hat{g}(s) = C(SI - A)^{-1}B.$$

The sampling system was considered to have a sampling period of h. A norm invariant transformation was introduced which is a method of discretization that preservers the norms of sampled data systems. The norm invariant transformation of system G is defined as

$$\hat{g}_J(z) = \left[\begin{array}{c|c} A_J & B_J \\ \hline C & 0 \end{array}\right]$$

Where

$$A_J = e^{Ah} \quad , \quad B_J {B_J}^T = \int_0^h e^{A\tau} B B^T \, e^{A^T \tau} d\tau$$

It is shown that in order to design a norm based residual generator, one can replace the sample data system with $\hat{g}_J(z)$ so the discrete time FD design can be applied. Any optimal norm based residual generator for this discrete time system will be optimal for the original sample data system as well.

1.2.2 Fault diagnosis

Observer based fault detection has attracted a lot of attention during the past two decades [9-12]. The core of observer based fault detection is generation of the residual signal which is robust to disturbance and sensitive to faults. Some suitable criteria for both objectives based on H_∞ norm and H_2norms, have been proposed in the frequency domain to evaluate the effectiveness of robust fault detection filter design [6, 11, 13].

In Ding et al., a unified H_∞ optimization approach has been proposed for fault detection [14]. Through co-inner-outer factorization, a residual generator was given by solving a Riccati equation. In [15], linear matrix inequality (LMI) method has been applied to robust fault diagnosis problems. Both Li et al. [16] and Zhang et al. [17], extended the unified optimization results on FD in LTI systems to linear time varying systems in the framework of H_∞/H_∞. Another research has shown that the robust FD problems under different performance indices H_∞/H_∞ , H_2/H_∞ and H_-/H_∞ can be solved by a unified optimal fault detection filter solution [16].

A game detection approach has been also proposed in [18] for fault detection filter design, however, the model considered is too specific and the sensitivity of the residual to faults has not been considered.

The results shown in [14] were then extended to linear time varying systems from a time domain perspective by Liu and Zhou [19]. By applying Coprime factorization, an optimal solution to robust FD was given.

A finite frequency domain approach to fault diagnosis has been proposed in [20]. The unified approach by Ding et al. [14] has also been extended to linear periodic discrete time system by Zhang et al. [21]. Same authors also provided a Fault Detection Filter (FDF) design under the assumption of disturbance being completely decoupled [22]. Zhong et al., presented a Krein space approach to H_∞ fault estimation for linear discrete time varying systems in the framework of H_∞ filtering formulation [23]. In Zhong et al., after introducing the finite horizon H_∞ and H_- criteria of FD systems, the problem has been formulated as an H_∞/H_∞ or H_-/H_∞ maximization problem concerned with finding the optimal observer gain matrix and a post filter [12].

It should be noted that in contrast to the existing results for linear time invariant systems, there are very few results on fault detection of linear time varying systems.

1.2.3 Fault diagnosis of Network Control Systems with networked induced time delay

Introduction of networks brings some new problems and challenges such as network induced delay [30-34]. Compared with rich results in control and stability of network control systems, there are limited number of contributions on fault detection of NCS with network induced time delay [35]. A switched fault detection filter was proposed by Mao et al. to detect the faults where the network induced delay was assumed to be an integer multiple of sampling period [36]. To deal with arbitrary unknown network induced delay, a threshold was used in [37] to enhance robustness of fault detection system to network induced delay. In [38], an adaptive fault diagnosis method was proposed after the matrix of delay was obtained. Both of the previously mentioned methods made the assumption that the delay is less than one sampling period. Some considered a networked induced delay of greater than one sampling time and transformed it to a polytopic uncertainty [39]. In [40], a residual generation method was proposed based on an estimation of the uncertain delay, however the persistent excitation of the process is required.

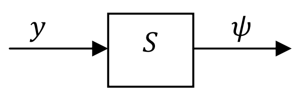
1.3 Review of Existing Results

This section provides a review of the existing results on discretization of sampled data systems with periodic sampler, basic structure of residual generator and the idea of norm based fault detection for linear discrete time invariant systems.

1.3.1 Definitions

A brief description of Sampler, Hold, signal and system spaces and the definitions of H_2 norm and H_∞ norm are given as follows:

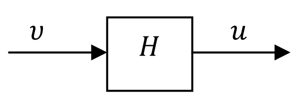
Ideal Sampler *S*: the sampling operator with sampling rate h



The sampler periodically samples $y(t)$ to yield to the discrete time signal $\psi(k)$ where

$$\psi(k) = y(kh)$$

Hold Operator *H*: the hold operator with sampling rate h



The hold operator converts the discrete time signal $v(k)$ to continuous time signal $u(t)$ by holding it constant over the sampling intervals, i.e.

$$u(t) = v(k) \text{ for } kh \leq t < (k+1)h$$

Signal and System Spaces:

For continuous time, $\mathbb{R}$,$\mathbb{R}_{+=}\{t: t \geq 0\}$ and $\mathbb{R}_- = \{t: t < 0\}$, if a signal $f(t)$ is defined as

$$f(t) = \begin{bmatrix} f_1(t) \\ \vdots \\ f_n(t) \end{bmatrix}$$

The signal spaces would be

$$\mathcal{L}(\mathbb{R}, \mathbb{R}^n) = \{f: \mathbb{R} \longrightarrow \mathbb{R}^n\}$$

$$\mathcal{L}(\mathbb{R}_+, \mathbb{R}^n) = \{f: \mathbb{R}_+ \longrightarrow \mathbb{R}^n\}$$

System $G_{m\times p}$ is a linear transformation $\mathcal{L}(\mathbb{R}, \mathbb{R}^m) \longrightarrow \mathcal{L}(\mathbb{R}, \mathbb{R}^p)$.

In discrete time, $\mathbb{Z}$,$\mathbb{Z}_+ = \{k: k \geq 0\}$ and $\mathbb{Z}_- = \{k: k < 0\}$, if a signal $\psi(k)$ is defined as

$$\psi(k) = \begin{bmatrix} \psi_1(k) \\ \vdots \\ \psi_n(k) \end{bmatrix}$$

The signal spaces would be

$$\ell(\mathbb{Z}, \mathbb{R}^n) = \{\psi: \mathbb{Z} \longrightarrow \mathbb{R}^n\}$$

$$\mathcal{L}(\mathbb{Z}_+, \mathbb{R}^n) = \{\psi: \mathbb{Z}_+ \longrightarrow \mathbb{R}^n\}$$

System $G_{d_{m\times p}}$ is a linear transformation $\ell(\mathbb{Z}, \mathbb{R}^m) \longrightarrow \ell(\mathbb{Z}, \mathbb{R}^p)$.

H_2 Norm:

For a SISO system, the H_2 norm is defined as [7]

$$\|\hat{g}(s)\|_2^2 = \|g(t)\|_2^2 = \int_{-\infty}^{\infty} g(t)^2 dt$$

This means that the H_2 norm of the transfer function $\hat{g}(s)$ equals to the $\mathcal{L}_2$ norm of its impulse response. For MIMO systems, the H_2 norm is defined as

$$\|\hat{g}(s)\|_2^2 = \sum_{i=1}^{p} \|G\delta(t)e_i\|_2^2$$

where $e_i, i = 1, \ldots, p$, denotes the standard basis vector in $\mathbb{R}^p$ and $\delta(t)$ denotes the unit impulse function. Thus, $\delta(t)e_i$ is an impulse applied to the ith input channel.

SG is a time varying but h periodic system. To generalize the definition of H_2norm to sampled data systems, we define the H_2 norm of SG as the total energy of the outputs when impulses are applied in one sampling period to the system:

$$\|SG\|_2^2 = \frac{1}{h}\sum_{i=1}^{p}\left(\int_0^h \|SG\delta(t-\tau)e_i\|_2^2 \, d\tau\right)$$

$\boldsymbol{H_\infty}$ Norm:

For both SISO and MIMO system, the H_∞ norm is defined as [7]

$$\|\hat{g}(s)\|_\infty = \sup\{\|y\|_2 \colon \|u\|_2 = 1\}$$

The major difference between $\|\hat{g}(s)\|_2$ and $\|\hat{g}(s)\|_\infty$ is that the 2-norm is an average system gain for known inputs, while H_∞ norm is the worst case system gain for unknown inputs.

The H_∞ norm of sampled data system SG (in signal space)is defined as [7]

$$\|SG\|_\infty = \sup\{\|SGu\|_2 \colon \|u\|_2 = 1\}$$

1.3.2 Discretization Transformation For Sampled Data Systems

In this section, an overview of existing discretization methods for sampled data systems is provided which can be used for control and fault detection of this type of systems. The continuous time invariant system is considered here where it is sampled using a periodic sampler. Step invariant transformation and bilinear transformation are discussed. It should be noted that norm invariant transformation discussed in the previous section, is a discretization method that only preserves the norms and it is only used for fault detection of sampled data systems.

1.3.2.1 Step Invariant Transformation

A continuous time invariant system G with the following state space realization is considered:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

The step invariant transformation maps system G to system G_d with the following state space realization:

$$\dot{x} = A_d x + B_d u$$

$$y = Cx + Du$$

where

$A_d = e^{Ah}$ and $B_d = \int_0^h e^{A\tau}\, d\tau B$

It should be noted that only matrices A and B change in this transformation. Step invariant transformation is used for both direct and indirect control of sampled data systems.

1.3.2.2 Bilinear Transformation

Another common way of discretizing sampled data systems is bilinear transformation. For the continuous time invariant system, $\hat{g}(s)$, given by

$$\hat{g}(s) = D + C(SI - A)^{-1}B,$$

the bilinear transformation maps $\hat{g}(s)$ to $\hat{g}_{bt}(\lambda)$ with the following representation:

$$\hat{g}_{bt}(\lambda) = \left[\begin{array}{c|c} A_{bt} & B_{bt} \\ \hline C_{bt} & D_{bt} \end{array}\right]$$

where

$$A_{bt} = \left(I - \frac{h}{2}A\right)^{-1} (I + \frac{h}{2}A)$$

$$B_{bt} = \frac{h}{2}\left(I - \frac{h}{2}A\right)^{-1} B$$

$$C_{bt} = C(I + A_{bt})$$

$$D_{bt} = D + CB_{bt}$$

$$s = \frac{2}{h}\frac{1-\lambda}{1+\lambda}$$

In order for the above state formula to be valid, $\left(I - \frac{h}{2}A\right)$ should be invertible which means that $\frac{2}{h}$ is not an eigenvalue of A.

1.3.3 Fault detection For Discrete Time Invariant Systems

There are basically three types of residual generators including fault detection filter, diagnostic observer and parity relation based residual generator. The fault detection approach to be used later in the thesis is based on fault detection filter, and therefore an overview of this approach is provided in this section.

1.3.3.1 Basic structure of residual generators

The basic idea of model based residual generation is to reconstruct the process outputs, therefore the input-output model

$$y(z) = G_u(z)u(z) + G_d(z)d(z) + G_f(z)f(z) \tag{1.1}$$

is a suitable tool for the description of LTI system, where $u(z)$ and $y(z)$ denote the input and output vectors, $G_u(z)$ is the known transfer function matrix that describes the modeled input/output behavior of the process under consideration. $d(z)$ is a vector representing disturbances and model uncertainty that are unknown but bounded. $f(z)$ is an unknown vector that represents all possible faults and will be zero in the fault free case. $G_f(z)$ and $G_d(z)$ are known transfer function matrices. We assume that the state space realization of $G_u(z)$ is represented by (A, B, C, D).

The following form of LTI residual generator is given

$$r(z) = Q(z)\left(\hat{M}_u(z)y(z) - \hat{N}_u(z)u(z)\right) \tag{1.2}$$

where $Q(z)$ is the parameterization matrix which can be arbitrarily selected from the set of stable systems $\mathcal{R}H_\infty$, and $\hat{M}_u(z)$ and $\hat{N}_u(z)$ are derived using coprime factorization

$$\hat{M}_u(z) = I - C(zI - A + LC)^{-1}L$$

$$\hat{N}_u(z) = D + C(zI - A + LC)^{-1}(B - LD)$$

with the so-called observer gain matrix L.

The relationship of the residual generator (1.2) to the original idea of the model based residual generator is demonstrated by the fact that

$$\hat{M}_u(z)y(z) - \hat{N}_u(z)u(z) = y(z) - \hat{y}(z)$$

where $\hat{y}(z)$ is an estimation of $y(z)$, delivered by an identity observer.

In this way, the $Q \in \mathcal{RH}_\infty$ is defined as the post filter. Moreover, it can be shown that the dynamics of the residual generator (1.2) is governed by

$$r(z) = Q(z)\hat{M}_u(z)(G_d(z)d(z) + G_f(z)f(z)) \tag{1.3}$$

1.3.3.2 Fault detection Filter (FDF)

The fault detection filter was proposed by Ding et al.[41] and is summarized here. Suppose that the following sate space model is used for process description

$$x(k+1) = Ax(k) + Bu(k) + E_f f(k) + E_d d(k) \tag{1.4}$$

$$y(k) = Cx(k) + Du(k) + F_f f(k) + F_d d(k)$$

The core of an FDF is a full order state observer of the form

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L(y(k) - C\hat{x}(k) - Du(k)) \tag{1.5}$$

Using equation (1.5), the residual vector is defined by

$$r(k) = Q(y(k) - \hat{y}(k)) = Q(y(k) - C\hat{x}(k) - Du(k))$$

whose dynamics is governed by

$$e(k+1) = (A - LC)e(k) + \left(E_f - LF_f\right)f(k) + (E_d - LF_d)d(k)$$

$$r(k) = QCe(k) + QF_f f(k) + QF_d d(k)$$

The design parameters of the FDF are the observer gain and the matrix Q which is equivalent to an algebraic post filter. The advantages of using FDF lie in its close relationship with the state observer and modern control theory.

1.3.3.3 Norm based fault detection

The dynamics of residual signal was given in equation (1.3). As mentioned, $Q \in \mathcal{RH}_\infty$ is a designable post filter. If a post filter Q can be found such that

$$Q\widehat{M}_u G_d = 0$$

$$Q\widehat{M}_u G_f \neq 0$$

then the perfect decoupling of the residual signal from the unknown inputs is achieved. If such a post filter cannot be found, an optimization is necessary to compromise between sensitivity of residual signals to faults and its robustness to the disturbance.

Norm based optimization is a widely accepted approach for this problem where the optimization problem is considered as:

$$J = min_{Q(z)\in\Re\mathcal{H}_\infty} \frac{\left\|Q(z)\widehat{M}_u(z)G_d(z)\right\|_\eta}{\left\|Q(z)\widehat{M}_u(z)G_f(z)\right\|_\eta}$$

where $\eta = 2$ or ∞. In other words, the problem is to find Q such that the above minimization problem is satisfied.

There are different methods to solve this minimization problem [14 ,41, 42]. In this section, the results of post filter design for discrete time invariant systems developed by Li et al. [43] is presented, where the post filter that they present satisfies both ∞ and 2 norm optimization problem.

Consider a discrete time invariant system with the following state space realization

$$x(k+1) = Ax(k) + Bu(k) + B_f f(k) + B_d d(k)$$

$$y(k) = Cx(k) + Du(k) + D_f f(k) + D_d d(k)$$

where $f(k), d(k)$ and $u(k)$ are fault, disturbance and input signals, respectively. Matrices $A, B, B_f, B_d, C, D, D_f$ and D_d are of appropriate dimensions. Assume D is full rank and (C, A) is detectable.

The residual signal is generated using

$$\hat{x}(k+1) = (A + L_0C)\hat{x}(k) + (B + L_0D)u(k) - L_0y(k)$$

$$r(k) = R\big(y(k) - C\hat{x}(k) - Du(k)\big)$$

The post filter Q is found such that it satisfies the optimization problem. Q is defined as $Q = {R_d}^{-1/2}$, and the following conditions hold

$$R_d = D_d {D_d}^T + CP^{-1}C^T > 0$$

$$L_0 = -(AP^{-1}C^T + B_d {D_d}^T){R_d}^{-1}$$

$$AP^{-1}A^T - P^{-1} + \left(AP^{-1}C^T + B_d {D_d}^T\right){R_d}^{-1}\left(D_d {B_d}^T + CP^{-1}A^T\right) + B_d {B_d}^T = 0$$

In chapter 3, we extend this norm based fault detection of discrete time invariant systems to fault detection of discrete time varying systems.

1.4 Outline of thesis

In chapter 2, the norm invariant transformation for a time invariant system with aperiodic sampler and a time varying system with periodic sampler is presented.

Chapter 3 provides a fault diagnosis approach for discrete time varying systems. This approach is an extension of a robust fault detection for continuous time varying systems proposed by Li et al. [6]. An example is given to illustrate the effectiveness of this approach. Also, an overview of discrete time varying Kalman filter is given and a fault detection approach using Kalman filter as an observer for discrete time varying systems is studied. Simulation results are used to illustrate the proposed method.

Chapter 4 illustrates how to present a network control system with a varying delay as a discrete time varying system using norm invariant transformation, and how to implement the fault detection method of chapter 3 for this system.

Chapter 5 is a summary of the thesis and presents possible future research directions.

Chapter 2

Discretization Transformation For Sample Data System

2.1 Introduction

One of the important properties considered in fault detection of sampled data systems is periodic sampling which results in a periodic time varying system. It is reasonable to assume periodic sampling in the conventional implementation of sampled data systems. However, it is hard to perform periodic sampling in some applications such as network control systems [4].

A framework was developed in [8] for time invariant sampled data systems with periodic sampler using norm invariant transformation. In this chapter, we develop a framework for discretization of sampled data systems which is particularly useful for fault detection of these systems. A continuous time invariant system with aperiodic sampler and continuous time varying system with periodic sampler is considered. By defining norms of sampled systems and the system resulted from norm invariant transformation, which is a discrete time varying system, it can be shown that the norms are equivalent. This framework allows us to extend norm based fault detection methods for discrete time varying systems to sampled data systems.

The purpose of this chapter is to show that the norms of sampled continuous time varying system with periodic sampler are equal to the corresponding norms of a proposed discrete time varying system. Also it is shown that the norms of a sampled LTI system with aperiodic sampler are equal to the corresponding norms of the resulted discrete time varying system.

The remaining of the chapter is organized as follows: In section 2, the norms of sampled systems and the norm invariant transformation are defined for discrete time system with aperiodic sampler. Section 3 provides the norm invariant transformation formulation for continuous time varying system with periodic sampler. The step invariant transformation is also presented to compare the results with norm invariant transformation. Some concluding remarks are presented at the end.

2.2 Norms of sampled data time invariant system with aperiodic sampler

Assume that G is a stable and strictly proper continuous time system with p inputs and m outputs, and

$$\hat{g}(s) = \left[\begin{array}{c|c} A_c & B_c \\ \hline C_c & 0 \end{array}\right]$$

The sampled system SG maps the continuous time signals to discrete time signals.

2.2.1 Norm Invariant Transformation

In this section, a norm invariant transformation for time invariant systems with aperiodic sampler is developed.

The variable sampling rate h_i is considered to be a summation of constant known h and unknown uncertainty $(\Delta(i))$ as $h_i = h + \Delta(i-1)$ where $0 < h_i \leq L$. The sampling operator and the hold operator are considered to have a sampling period h_i.

A linear time invariant continuous time system is considered as

$$\hat{g}(s) = C_c(SI - A_c)^{-1}B_c$$

where $(A_c, B_c, C_c, 0)$ is the state space realization of system G. The norm invariant transformation of G is denoted as $g_J$with the state space realization

$$x_J(k+1) = A_J(k)x_J(k) + B_J(k)u_J(k)$$

$$y_J(k) = C_J(k)x_J(k) + D_J(k)u_J(k)$$

where

$$A_J(0) = e^{A_c(h_1-h_0)}, \qquad A_J(k) = e^{A_c(h_{k+1}-h_k)}$$

$$B_J(k)B_J^T(k) = \int_0^L Me^{-A_c\tau}B_cB_c{}^Te^{-A_c{}^T\tau}M^Td\tau$$

$$C_J = \frac{1}{\sqrt{L}}C_c, D_J = 0$$

$$M = e^{A_ch_0}e^{A_ch_0}e^{A_ch_1} \dots e^{A_ch_k}$$

In the following sections we show that H_2 and H_∞ norm of system g_J is equivalent to the H_2 and H_∞ norm of sampled system SG where SG is a time varying but L periodic system.

2.2.1.1 H_2 norm of SG

The sampler is considered to be periodic. We define the signal H_2-norm of SG as the total energy of the outputs when impulses are applied in one period to the input channels, i.e.

$$\|SG\|_2^2 = \sum_{i=1}^p \frac{1}{L}\left(\int_0^L \|SG\delta(t-\tau)e_i\|_2^2\, d\tau\right) \tag{2.1}$$

where $e_i, i = 1, \dots, p$, denote the standard basis vector in $\mathbb{R}^p$ and $\delta(t)$ denotes the unit impulse function. Thus, $\delta(t)e_i$ is an impulse applied to the ith input channel.

The H_2-norm of SG is related to the H_2-norm of the discrete time varying system g_J as shown in the following lemma.

Lemma 1. The H_2 norm of a sampled data system SG with aperiodic sampler is given by

$$\|SG\|_2^2 = \|g_J\|_2^2$$

Proof:

From linear control theory, it is known that

$$G\delta(t) = C_c e^{At} B\mathbf{1}(t) \tag{2.2}$$

Where $\mathbf{1}(t)$ is the unit step function.

From the above equation, $SG\delta(t-\tau)$ with aperiodic sampler is given as

$$SG\delta(t-\tau) = \left\{0, C_c e^{A_c(h_0-\tau)} B_c, \dots, C_c e^{A_c(h_i-\tau)} B_c, \dots\right\} \tag{2.3}$$

Using equations (2.1) and (2.3) we get

$$\frac{1}{L}\sum_{i=1}^{p} \|SG\delta(t-\tau)e_i\|_2^2 = tr\left(\sum_{k=1}^{\infty} \frac{1}{L} C_c F e^{-A_c\tau} B_c {B_c}^T e^{-{A_c}^T\tau} F^T {C_c}^T\right)$$

where

$$F = e^{A_c h_0} e^{A_c h_1} \dots e^{A_c h_k} \tag{2.4}$$

Integrating equation (2.4) results in

$$\frac{1}{L}\int_0^L \sum_{i=1}^{p} \|SG\delta(t-\tau)e_i\|_2^2 \, d\tau = tr\left(\sum_{k=1}^{\infty} \frac{1}{L} C_c F\left(\int_0^L e^{-A_c\tau} B_c {B_c}^T e^{-{A_c}^T\tau} d\tau\right) F^T {C_c}^T\right)$$

$$= tr\left(\sum_{k=1}^{\infty} \frac{1}{L} C_c N\left(\int_0^L e^{-A_c\tau} B_c {B_c}^T e^{-{A_c}^T\tau} d\tau\right) N^T {C_c}^T\right)$$

where

$$N = e^{A_c h_0} e^{A_c(h_1-h_0)} e^{A_c h_0} \dots e^{A_c(h_k-h_{k-1})} e^{A_c h_{k-1}}$$

A change of variable from $(k-1)$ to (k) yields

$$\frac{1}{L}\int_0^L \sum_{i=1}^{p} \|SG\delta(t-\tau)e_i\|_2^2 \, d\tau$$

$$= tr(\sum_{k=0}^{\infty} \frac{1}{L} C_c \, E(\int_0^L M e^{-A_c\tau} B_c {B_c}^T e^{-{A_c}^T\tau} M^T d\tau \,) E^T {C_c}^T)$$

where

$$E = e^{A_c(h_1-h_0)} \dots e^{A_c(h_{k+1}-h_k)} \text{ and } M = e^{A_c h_0} e^{A_c h_0} e^{A_c h_1} \dots e^{A_c h_k}$$

and the following equality can be concluded:

$$\frac{1}{L}\int_0^L \sum_{i=1}^{p} \|SG\delta(t-\tau)e_i\|_2^2 \, d\tau$$

$$= tr(\sum_{k=0}^{\infty} C_J A_0 A_1 \dots A_k (B_J(k) B_J^T(k))(A_0 A_1 \dots A_k)^T {C_J}^T)$$

$$= \|g_J\|_2^2 \qquad (2.5)$$

where g_J is a time varying system with the state space realization defined as

$$x_J(k+1) = A_J(k)x_J(k) + B_J(k)u_J(k)$$

$$y_J(k) = C_J(k)x_J(k) + D_J(k)u_J(k)$$

and A_J, B_J, C_J and D_J are given by

$$A_J(0) = e^{A_c(h_1-h_0)}, \qquad A_J(k) = e^{A_c(h_{k+1}-h_k)}$$

$$B_J(k)B_J^T(k) = \int_0^L M e^{-A_c\tau} B_c {B_c}^T e^{-{A_c}^T\tau} M^T d\tau$$

$$M = e^{A_c h_0} e^{A_c h_0} e^{A_c h_1} \dots e^{A_c h_k}$$

$C_J = \frac{1}{\sqrt{L}} C_c, D_J = 0$

2.2.1.2 H_∞ norm of SG

H_∞ norm of SG (signal space norm) is defined as follows [7]

$$\|SG\|_\infty = sup_{\|u\|_2 \leq 1} \|SGu\|_2$$

The following lemma is helpful in calculation of $\|SG\|_\infty$.

Lemma 2. The H_∞ norm of SG is given as

$$\|g_J\|_\infty = \|SG\|_\infty$$

Proof:

The H_∞ norm of SG is defined as

$$\|SG\|_\infty = \sup\{\|SG\|_2 \colon \ \|u\|_2 = 1\}$$

Since the 2-norm is preserved using norm invariant transformation, using Lemma 1, the above definition can be written as

$$\|SG\|_\infty = \sup\{\|g_J\|_2 \colon \ \|u\|_2 = 1\} = \|g_J\|_\infty$$

which proves that the H_∞ norms of g_J and SG are equal.

2.2.2 Norm invariant transformation properties

Lemmas 1 and 2 show that the norm of the sampled system SG is equal to the norm of the discrete time varying system g_J. Since this approach preserves H_2 and H_∞ norms, the discretization method is called norm invariant transformation. Note that the inputs of the discrete time varying system g_J are not related to the actual inputs of the original continuous time varying system G. Also system g_J

introduces some fictitious inputs which are only used for design and have no physical meaning.

2.2.3 Step Invariant Transformation

Step invariant transformation for continuous time invariant system with aperiodic sampler maps the state matrices (A_c, B_c, C_c, D) to (A_d, B_d, C_c, D) where

$A_d(k) = e^{A_c(h_{k+1}-h_k)}$ and $B_d(k) = \int_0^{h_{k+1}-h_k} e^{A_c\tau} d\tau\, B$.

As can be verified by Lemma 1, matrices $A_d(k)$ derived from step invariant transformation and $A_J(k)$ derived from norm invariant transformation are the same.

2.3. Norms of sampled data time varying system with periodic sampler

2.3.1. Norm Invariant Transformation

In this section, the concept of H_2 and H_∞ norms are generalized to define appropriate norms for time varying sampled data system with periodic sampler.

The sampling rate is a constant h. A linear time varying continuous time system G is considered where

$G(t, \tau) = C(t)\Phi(t, \tau)B(\tau)$

and the continuous state transition matrix Φ holds the following equality

$$\Phi(t, t_o) = \Phi(t, t_1)\Phi(t_1, t_o) \qquad (2.6)$$

The norm invariant transformation of G is denoted by G_J with the state space realization

$x_J(k+1) = A_J(k)x_J(k) + B_J(k)u_J(k)$

$$y_J(k) = C_J(k)x_J(k) + D_J(k)u_J(k)$$

where

$$C_J[k] = \frac{1}{\sqrt{h}}C(kh)$$

$$\Phi_J[k, m+1] = \Phi(kh, mh), \quad k-1 \geq m$$

$$B_J[m]B_J^T[m] = \int_0^h \Phi(h,\tau)B(\tau)B^T(\tau)\Phi^T(h,\tau)d\tau$$

$$D_J(k) = 0$$

Φ_J is the state transition matrix for the discrete time varying system G_J.We shall prove that H_2 and H_∞ norm of system g_J is equivalent to the H_2 and H_∞ norm of sampled system SG where SG is a time varying but h periodic system.

2.3.1.1 H_2 norm of SG

The sampler is considered to be periodic. We define the H_2-norm of SG as the total energy of the outputs when impulses are applied in one period to the input channels, i.e.

$$\|SG\|_2^2 = \frac{1}{h}\sum_{i=1}^{p}(\int_0^h \|SG\delta(t-\tau)e_i\|_2^2\, d\tau$$

where $e_i, i = 1, \dots, p$, denote the standard basis vector in $\mathbb{R}^p$ and $\delta(t)$ denotes the unit impulse function. Thus, $\delta(t)e_i$ is an impulse applied to the ith input channel.

The H_2-norm of SG is related to the H_2-norm of discrete time varying system G_J as shown by the following lemma.

Lemma 3. The H_2 norm of a SG with aperiodic sampler is given by

$$\left\|G_J\right\|_2^2 = \|SG\|_2^2$$

The 2-norm here represents the signal norm.

Proof:

We know that $G(t,\tau) = C_c(t)\Phi(t,\tau)B_c(\tau)$. Therefore $SG(t,\tau)\delta(t-\tau)$ with aperiodic sampler is given as

$$SG\delta(t-\tau) = \{0, C_c[h]\Phi(h,\tau)B_c(\tau), \ldots, C_c[kh]\Phi(kh,\tau)B_c(\tau), \ldots\} \tag{2.7}$$

Then

$$\|SG\|_2^2 = \frac{1}{h}\sum_{i=1}^{p}\left(\int_0^h \|SG\delta(t-\tau)e_i\|_2^2 \, d\tau\right)$$

For continuous time system condition (2.6) holds, therefore we have

$$\Phi(t,\tau) = \Phi(t,mh)\Phi(mh,\tau) \tag{2.8}$$

which results in

$$\frac{1}{h}\sum_{i=1}^{p}\|SG\delta(t-\tau)e_i\|_2^2 =$$

$$\frac{1}{h}tr\left(\sum_{k=1}^{\infty} L(k)\Phi(mh,\tau)B_c(\tau){B_c}^T(\tau)\Phi^T(mh,\tau)L^T(k)\right)$$

where $L(k) = C_c(kh)\Phi(kh,mh)$

Therefore integrating both sides results in

$$\frac{1}{h}\int_0^h\sum_{i=1}^{p}\|SG\delta(t-\tau)e_i\|_2^2 \, d\tau$$

$$= \frac{1}{h}tr\left(\sum_{k=1}^{\infty}\int_0^h L(k)\Phi(mh,\tau)B_c(\tau){B_c}^T(\tau)\Phi^T(mh,\tau)L^T(k)d\tau\right)$$

$$\frac{1}{h}\int_0^h \sum_{i=1}^{p} \|SG\delta(t-\tau)e_i\|_2^2\, d\tau$$

$$= \frac{1}{h} tr(\sum_{k=0}^{\infty} L(k) \int_0^h \Phi(mh,\tau)B_c(\tau){B_c}^T(\tau)\Phi^T(mh,\tau)d\tau L^T(k))$$

A typical discrete time varying system is defined as:

$$G_d[k,m] = C_d[k]\Phi_d[\mathrm{k,m}+1]B_d[\mathrm{m}] \tag{2.9}$$

where

$$\Phi_d[k,k_o] = A_d[k-1]A_d[k-2] \ldots A_d[k_o] \tag{2.10}$$

Therefore defining a discrete time system G_J with the state space realization:

$$x_J(k+1) = A_J(k)x_J(k) + B_J(k)u_J(k)$$

$$y_J(k) = C_J(k)x_J(k) + D_J(k)u_J(k)$$

where

$$C_J[k] = \frac{1}{\sqrt{h}} C_c(kh)$$

$$\Phi_J[k,m+1] = \Phi(kh,mh), \qquad k-1 \geq m$$

$$B_J[m]B_J^T[m] = \int_0^h \Phi(mh,\tau)B_c(\tau){B_c}^T(\tau)\Phi^T(mh,\tau)d\tau \tag{2.11}$$

results in the conclusion that $\|G_J\|_2^2 = \|SG\|_2^2$.

Now $A_J[k]$ needs to be defined. From the definition of $\Phi_J, \Phi_J[k,k_o] = A_J[k-1]A_J[k-2] \ldots A_J[k_o]$, we have

$$\Phi_J[k,k_o] = A_J[k-1]\Phi_\mathrm{J}[k-1,k_o]$$

From equation (2.11), we know that $\Phi_J[k, k_o] = \Phi(kh, mh)$, so

$$\Phi(kh, mh) = A_J[k-1]\Phi((k-1)h, mh)$$

which results in

$$A_J[\mathrm{k}] = \Phi\big((k+1)T, mT\big) \tag{2.12}$$

2.3.1.2 $\boldsymbol{H_\infty}$ norm of $\boldsymbol{SG}$

H_∞ norm of SG (signal space) is defined to be [7]

$$\|SG\|_\infty = sup_{\|u\|_2 \leq 1}\|SGu\|_2$$

The following lemma will help in the calculation of $\|SG\|_\infty$.

Lemma 4. The H_∞ norm of SG is given as

$$\|G_J\|_\infty = \|SG\|_\infty$$

Proof:

The H_∞ norm of SG is given as

$$\|SG\|_\infty = \sup\{\|SG\|_2\text{: } \|u\|_2 = 1\}$$

Since the 2-norm is preserved using norm invariant transformation, the above definition can be written as

$$\|SG\|_\infty = \sup\{\|G_J\|_2\text{: } \|u\|_2 = 1\} = \|G_J\|_\infty$$

So it is concluded that H_∞ norm of SG is equivalent to the H_∞ norm of G_J.

2.3.2 Norm invariant transformation properties

Lemmas 3 and 4 show that the norm of the sampled system SG is equal to the norm of the resulting discrete time varying system G_J, introduced in the previous section. Since this approach preserves H_2 and H_∞ norms, the discretization method is a norm invariant transformation. Same as in Section 2.2.2., it should be noted that the inputs of the discrete time varying system G_J are not related to the actual inputs of the original continuous time varying system G and system G_J introduces some inputs which are used solely for the design purposes.

2.3.3 Step invariant transformation for time varying system

Step invariant transformation for time varying system with periodic sampler is derived in this section.

For a time varying system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$

$$y(t) = C(t)X(t) + D(t),$$

the state is calculated as

$$x(t) = \Phi(t, t_o)x(t_o) + \int_{t_o}^{t} \Phi(t, \tau)B(\tau)u(\tau)d\tau$$

For a piece wise constant input, $u(t) = u[k]$, $kT \leq t < (k+1)T$, $x(t)$can be calculated at $t = kT$ and $t = (k+1)T$ as

$$x[k] \coloneqq x(kT) = \Phi(kT, t_o)x(t_o) + \int_{t_o}^{kT} \Phi(kT, \tau)B(\tau)d\tau\, u[k] \tag{2.13}$$

$$x[k+1] \coloneqq x((k+1)T)$$

$$= \Phi((k+1)T, t_o)x(t_o) + \int_{t_o}^{(k+1)T} \Phi((k+1)T, \tau)B(\tau)d\tau\, u[k] \tag{2.14}$$

From linear control theory, we know that the following condition holds for a continuous time varying system

$$\Phi(t, t_o) = \Phi(t, t_1)\Phi(t_1, t_o)$$

which results in

$$\Phi\big((k+1)T, t_o\big) = \Phi\big((k+1)T, kT\big)\Phi(kT, t_o) \tag{2.15}$$

Substituting $\Phi\big((k+1)T, t_o\big)$ from (2.15) in equation (2.14) results in

$$x[k+1] = \Phi\big((k+1)T, kT\big)(\Phi(kT, t_o)x(t_o) + \int_{t_o}^{(k+1)T} \Phi(kT, \tau)B(\tau)d\tau\, u[k])$$

$$= \Phi\big((k+1)T, kT\big)(\Phi(kT, t_o)x(t_o) +$$

$$\int_{t_o}^{kT} \Phi(kT, \tau)B(\tau)d\tau\, u[k] + \int_{kT}^{(k+1)T} \Phi(kT, \tau)B(\tau)d\tau\, u[k])$$

$$= \Phi\big((k+1)T, kT\big)(\Phi(kT, t_o)x(t_o) +$$

$$\int_{t_o}^{kT} \Phi(kT, \tau)B(\tau)d\tau\, u[k]) + \int_{kT}^{(k+1)T} \Phi((k+1)T, \tau)B(\tau)d\tau\, u[k] \tag{2.16}$$

Now, Substituting (2.13) in (2.16) yields

$$x[k+1] = \Phi\big((k+1)T, kT\big)x[k] + \int_{kT}^{(k+1)T} \Phi((k+1)T, \tau)B(\tau)d\tau\, u[k]$$

which is state space representation for a discrete time varying system where

$$x[k+1] = A_d[k]x[k] + B_d[k]u[k]$$

$$A_d[k] = \Phi\big((k+1)T, kT\big)$$

$$B_d[k] = \int_{kT}^{(k+1)T} \Phi((k+1)T, \tau)B(\tau)d\tau \tag{2.17}$$

Using Lemma 3, one can conclude that matrices $A_d(k)$ derived from step invariant transformation and $A_J(k)$ derived from norm invariant transformation are the same.

2. 4 Conclusion

In this chapter, a norm invariant transformation was presented for time varying sample data system with periodic sampler, for which the resulting time varying discrete time system has the same H_2 and H_∞ norm as the original sampled data system. Using this norm invariant transformation, fault detection methods for discrete time varying systems can be applied to continuous time systems, which are computationally simpler than fault detection methods for continuous time varying systems.

A norm invariant transformation was also introduced for time invariant sample data system with aperiodic sampler, and it was proved that the resulting time varying discrete time system presents the same H_2 and H_∞ norm as the original time invariant sampled data system. Since there are limited numbers of fault detection approaches for sampled data systems with aperiodic samplers, the presented transformation in this chapter is useful to transform such systems to a discrete time varying system with the same norms and use fault detection approache available for time varying systems.

Chapter 3

Fault detection approach for discrete time varying systems

3.1 Introduction

In the event of faults in actuators, sensors or controllers, a feedback control design for a system may result in instability or unsatisfactory performance. In order to maintain the performance of system, fault should be promptly detected so that appropriate remedies can be applied.

Observer based fault detection has attracted a lot of attention in the past two decades [12]. The core of observer based fault detection is generation of residual signals which are sensitive to faults and robust to unknown inputs.

In contrast to existing results for fault detection of time invariant systems, there are a few results on fault detection of linear time varying systems. Li and Zhou developed an observer based fault detection approach for continuous time varying systems [6]. Several multi objective fault detection problems were given for time varying systems in time domain. The optimal solutions had a simple structure in an observer form by solving a standard differential Riccati equation. The objective of this chapter is to extend their approach to discrete time varying systems. It should be noted that Zhong et al [12], provided a similar fault detection filter for discrete time varying systems, but the approach to the derivation is different here.

Also, considering the time varying properties of Kalman filter we investigate the possible application of Kalman filter as an observer for fault detection of discrete time varying systems.

The remaining of the chapter is organized as follows: In section 2, the fault detection method for discrete time varying systems is presented and a simulation example is provided. Section 3 provides formulation of the Kalman filter along with a residual generator for discrete time varying systems [5] followed by an example to illustrate the results of fault detection using Kalman filter. The concluding remarks are given at the end.

3.2 Residual generator for discrete time varying system

Consider a discrete time varying system G with disturbance and possible faults of the following state space realization

$$x(k+1) = A(k)x(k) + B(k)u(k) + B_d(k)d(k) + B_f(k)f(k) \qquad (3.1)$$
$$y(k) = C(k)x(k) + D_d(k)d(k) + D_f(k)f(k)$$

where $y(k)$ is the vector of the plant outputs, $u(k)$ the vector of control signals, $d(k)$ the vector of unknown inputs and $f(k)$ the vector of the faults to be detected.

For all coefficient matrices in the above state space realization, the following assumptions are made.

Assumption 1. $(C(k), A(k))$ is detectable;

Assumption 2. $D_d(k)$ has full rank

3.2.1 Definitions and preliminary results

Some definitions and lemmas are given in this section to develop the theorem for fault detection of discrete time varying system.

Definition 1. The adjoint of system G is denoted by $\tilde{G}$. The adjoint system is a linear system that has the property $\langle Gu, y\rangle = \langle u, \tilde{G}y\rangle$. For system G with the state space realization given by (3.1), non-causal realization of the adjoint system $\tilde{G}$ is [12]:

$$\lambda(k) = A^T\lambda(k+1) + C^T u(k+1) \tag{3.2}$$

$$y_1(k) = B^T\lambda(k) + D^T u(k)$$

A causal realization of the adjoint system is:

$$\lambda(k+1) = A^{-T}\lambda(k) - A^{-T}C^T u(k+1) \tag{3.3}$$

$$y_1(k+1) = B^T A^{-T}\lambda(k) + (D^T - B^T A^{-T} C^T)u(k)$$

Definition 2. System G is co-isometric between two Hilbert spaces, if and only if $G\tilde{G} = I$.

Definition 3. For a linear system G, its H_∞ norm (in ℓ_2) is defined as

$$\|G\|_\infty = \sup_{\mathrm{u}\in\ell_2}\frac{\|y\|_2}{\|u\|_2} = \sup_{\mathrm{u}\in\ell_2}\frac{\|Gu\|_2}{\|u\|_2}$$

Definiton 4. For a linear system G, its H_2 norm is the expected root mean square value of the output when the input is a realization of a unit variance white noise process. The 2- norm of G is defined by [12]

$$\|G\|_{2,[0,T]} = \sqrt{E\left\{\frac{1}{T}\sum_{i=0}^{T} y(i)' y(i)\right\}} \tag{3.4}$$

Definition 5. For a linear system G, its H_{-} index is defined as

$$\|G\|_{-} = \inf_{\mathrm{u}\in\mathcal{L}_2}\frac{\|Gu\|_2}{\|u\|_2}$$

H_{-}norm is a good measurement for a system's smallest gain.

Lemma1. Let $A: y \to z$ and $B: w \to y$ be two systems with appropriate dimensions, where w, y and z are signals in $\ell_2[0, \infty)$, then

$$\|AB\|_\infty \leq \|A\|_\infty \|B\|_\infty$$

$$\|AB\|_2 \leq \|A\|_\infty \|B\|_2$$

$$\|AB\|_- \leq \|A\|_\infty \|B\|_- \qquad (3.5)$$

Definition 6. Let G be a finite dimensional linear time varying system. G admits a exponentially stable proper left coprime factorization if there exist exponentially stable finite dimensional linear time varying systems M, N, X and Y such that $G = M^{-1}N$ and $XN + YM = I$. Here, (N, M) is called the coprime pair of system G.

Lemma 2. Let G be a linear time varying system and assume $\big(C(k), A(k)\big)$ is detectable. If $L(t)$ is a matrix function with appropriate dimensions such that system $x(k+1) = [A(k) + L(k)C(k)]x(k)$ is exponentially stable [6], then G admits a left coprime factorization pair (N, M) with a realization for system N

$$x(k+1) = [A(k) + L(k)C(k)]x(k) + [B(k) + L(k)D(k)]w(k)$$

$$y(k) = C(k)x(k) + D(k)w(k)$$

and a realization for M

$$x(k+1) = [A(k) + L(k)C(k)]x(k) + L(k)w(k)$$

$$y(k) = C(k)x(k) + w(k)$$

3.2.2 Problem formulation

The state space realization given above can be written as

$$y = [G_u \quad G_d \quad G_f] \begin{bmatrix} u \\ d \\ f \end{bmatrix}$$

Since $(C(t), A(t))$ is detectable, system G has the following coprime factorization

$$G = M^{-1}N = M^{-1}[N_u \quad N_d \quad N_f]$$

To decouple the residual signal from the input signal completely, the fault detection filter has the following form:

$$r = Q(My - N_u u) = Q[M \quad -N_u]\begin{bmatrix} y \\ u \end{bmatrix}$$

where N_u and M are linear systems with appropriate dimensions given in the coprime factorization and Q is a linear bounded system to be designed.

In general, a good fault detection filter must make a tradeoff between robustness to disturbance rejection and sensitivity to faults. Therefore, the next step is to design a system Q such that the residual signal $r(t)$ meets the mentioned property. We need to choose certain performance criteria so that the fault detection filter has satisfactory fault detection sensitivity and guaranteed disturbance rejection effect.

G_{rf} and G_{rd} are the system from fault signal and disturbance signal to residual, respectively. If $f(t)$ is modeled as unknown energy or power bounded signals then $\|G_{rf}\|_{-}$ is a reasonable performance criterion for measuring fault detection sensitivity. If $d(t)$ is modeled as unknown energy or power bounded signals, then H_∞ is a widely accepted worst case measure and $\|G_{rd}\|_\infty$ is a good indicator of disturbance rejection performance. If $d(t)$ and/or $f(t)$ are white noise, the H_2 norms of G_{rf} and G_{rd} are more suitable criteria.

Based on the definitions of norm, the following three fault detection filter design can be formulated:

$\boldsymbol{\mathcal{H}_{-}/\mathcal{H}_\infty}$: For the uncertain system described above, a linear bounded system Q is to be found such that $\|QN_d\|_\infty \leq \beta$ and $\|QN_f\|_{-}$ is maximized where β is a given disturbance rejection level.

$\mathcal{H}_2/\mathcal{H}_\infty$: For the uncertain system described above, a linear bounded system Q is to be found such that $\|QN_d\|_\infty \leq \beta$ and $\|QN_f\|_2$ is maximized where β is a given disturbance rejection level.

$\mathcal{H}_\infty/\mathcal{H}_\infty$: For the uncertain system described above, a linear bounded system Q is to be found such that $\|QN_d\|_\infty \leq \beta$ and $\|QN_f\|_\infty$ is maximized where β is a given disturbance rejection level.

Lemma 3. Suppose G is a state space system, if there exists a bounded matrix $P(t)$ satisfying

$$\begin{cases} I = D(k)D^T(k) - D(k)B^T(k)A^{-T}(k)C^T(k) & (I) \\ D(k)B^T(k)A^{-T}(k)P^{-1}(k) + C(k) = 0 & (II) \\ A(k)P^{-1}(k)A^T(k) + B(k)B^T(k) - P^{-1}(k+1) = 0 & (III) \end{cases} \tag{3.6}$$

Then G is co-isometric.

Proof.

If we define system G as:

$$x(k+1) = A(k)x(k) + B(k)y_1(k) \tag{3.7}$$

$$y(k) = C(k)x(k) + D(k)y_1(k)$$

The adjoint system $\tilde{G}$ is given by

$$\lambda(k) = (k+1)\lambda(k+1) + C^T(k+1)u(k+1) \tag{3.8}$$

$$y_1(k) = B^T(k)\lambda(k) + D^T(k)u(k)$$

and a causal realization of the adjoint system is

$$\lambda(k+1) = A^{-T}(k+1)\lambda(k) - A^{-T}(k+1)C^T(k+1)u(k+1) \tag{3.9}$$

$$y_1(k+1) = B^T(k+1)A^{-T}(k+1)\lambda(k) + \\ (D^T(k+1) - B^T(k+1)A^{-T}(k+1)C^T(k+1))u(k+1)$$

The state space representation for $G\tilde{G}$ is

$$X_a(k+1) = \begin{bmatrix} A^{-T}(k+1) & 0 \\ B(k+1)B^T(k+1)A^{-T}(k) & A(k+1) \end{bmatrix} X_a(k) + \begin{bmatrix} -A^{-T}(k)C^T(k) \\ B(k+1)(D^T(k+1) - B^T(k+1)A^{-T}(k)C^T(k)) \end{bmatrix} u$$

$$y(k+1) = [D(k+1)B^T(k+1)A^{-T}(k+1) \quad C(k+1)]X_a(k) + D(k+1)(D^T(k+1) - B^T(k+1)A^{-T}(k+1)C^T(k+1))u(k+1)$$

(3.10)

By introducing $\lambda(k) = P(k+1)x(k+1)$, we get

$$y(k+1) = \big(D(k+1)B^T(k+1)A^{-T}(k+1)P(k+1) + C(k+1)\big)x(k+1) + D(k+1)(D^T(k+1) - B^T(k+1)A^{-T}(k+1)C^T(k+1))u(k+1)$$

(3.11)

In order to satisfy the co-isometric condition; $u(k+1) = y(k+1)$, the following conditions needs to be satisfied:

$$\begin{cases} I = D(k+1)D^T(k+1) - D(k+1)B^T(k+1)A^{-T}(k+1)C^T(k+1) \\ D(k+1)B^T(k+1)A^{-T}(k+1)P(k+1) + C(k+1) = 0 \end{cases}$$

(3.12)

Substituting equation (3.7) in (3.12) results in

$$\begin{aligned} \lambda(k+1) &= P(k+1)x(k+2) \\ &= P(k+1)A(k+1)x(k+1) + P(k+1)B(k+1)y_1(k+1) \end{aligned}$$

(3.13)

From the causal realization of the adjoint system given in equation (3.9), we have:

$$\begin{aligned} \lambda(k+1) &= A^{-T}(k+1)\lambda(k) - A^{-T}(k+1)C^T(k+1)u(k+1) \\ &= A^{-T}(k+1)\lambda(k) - A^{-T}(k+1)C^T(k+1)y(k+1) \end{aligned}$$

$$= A^{-T}(k+1)\lambda(k) - A^{-T}(k+1)C^T(k+1).\left(C(k+1)x(k+1) + D(k+1)y_1(k+1)\right)$$

$$= \left(A^{-T}(k+1)P(k+1) - A^{-T}(k+1)C^T(k+1)C(k+1)\right)x(k+1) - A^{-T}(k+1)C^T(k+1)D(k+1)y_1(k+1)$$

(3.14)

From the equality of (3.13) and (3.14) one can conclude the followings:

$$P(k+2)A(k+1)x(k+1) + P(k+2)B(k+1)y_1(k+1) = \left(A^{-T}(k+1)P(k+1) - A^{-T}(k+1)C^T(k+1)C(k+1)\right)x(k+1) - A^{-T}(k+1)C^T(k+1)D(k+1)y_1(k+1)$$

(3.15)

$$\left(A^{-T}(k+1)P(k+1) - A^{-T}(k+1)C^T(k+1)C(k+1) - P(k+2)A(k+1)\right)$$
$$.x(k+1) = \left(P(k+2)B(k+1) + A^{-T}(k+1)C^T(k+1)D(k+1)\right)y_1(k+1)$$

(3.16)

On the other hand, using (3.9) we have

$$y_1(k+1) = B^T(k+1)A^{-T}(k+1)\lambda(k) + \left(D^T(k+1) - B^T(k+1)A^{-T}(k+1)C^T(k+1)\right)u(k+1)$$

(3.17)

Therefore

$$y_1(k+1) = \left[I - \left(D^T(k+1) - B^T(k+1)A^{-T}(k+1)C^T(k+1)\right)D(k+1)\right]^{-1}\left[B^T(k+1)A^{-T}(k+1)P(k+1) + \left(D^T(k+1) - B^T(k+1)A^{-T}(k+1)C^T(k+1)\right)C(k+1)\right]x(k+1)$$

(3.18)

$s.t.\ I - \left(D^T(k+1) - B^T(k+1)A^{-T}(k+1)C^T(k+1)\right)D(k+1)$ is invertible.

Now, define

$$M = B^T(k+1)A^{-T}(k+1)P(k+1) + \big(D^T(k+1) - B^T(k+1)A^{-T}(k+1)C^T(k+1)\big)C(k+1)$$

The second condition in (3.12) is

$$D(k+1)B^T(k+1)A^{-T}(k+1)P(k+1) + C(k+1) = 0 \quad (3.19)$$

Multiplying both sides of the equation by $\big(D^T(k+1) - B^T(k+1)A^{-T}(k+1)C^T(k+1)\big)$ results in the following:

$$\big(D^T(k+1) - B^T(k+1)A^{-T}(k+1)C^T(k+1)\big)D(k+1)B^T(k+1) A^{-T}(k+1)P(k+1) + \big(D^T(k+1) - B^T(k+1)A^{-T}(k+1)C^T(k+1)\big) C(k+1) = 0 \quad (3.20)$$

Using equation (3.20), M is simplified as

$$\begin{aligned} M &= B^T(k+1)A^{-T}(k+1)P(k+1) - (D^T(k+1) - B^T(k+1)A^{-T}(k+1) C^T(k+1))D(k+1)B^T(k+1)A^{-T}(k+1)P(k+1) \\ &= \big[I - \big(D^T(k+1) - B^T(k+1)A^{-T}(k+1)C^T(k+1)\big)D(k+1)\big]B^T(k+1) A^{-T}(k+1)P(k+1) \end{aligned} \quad (3.21)$$

Using the above equality in equation (3.18), results in

$$y_1(k+1) = B^T(k+1)A^{-T}(k+1)P(k+1)x(k+1) \quad (3.22)$$

By substituting (3.22) in (3.16) we get

$$
\begin{aligned}
(A^{-T}(k+1)P(k+1) - A^{-T}(k+1)C^T(k+1)C(k+1) - P(k+1)A(k+1)) \\
= \big(P(k+2)B(k+1) + A^{-T}(k+1)C^T(k+1)D(k+1)\big)B^T(k \\
+ 1). A^{-T}(k+1)P(k+1)
\end{aligned}
\tag{3.23}
$$

Using the second condition of equation (3.12)

$$
D(k+1)B^T(k+1)A^{-T}(k+1)P(k+1) + C(k+1) = 0,
$$

equation (3.23) becomes

$$
\begin{aligned}
A^{-T}(k+1)P(k+1) - P(k+1)A(k+1) = \\
P(k+2)B(k+1)B^T(k+1)A^{-T}(k+1)P(k+1)
\end{aligned}
$$

So in order for a system to be co-isometric the following conditions should hold:

$$
\begin{cases}
I = D(k)D^T(k) - D(k)B^T(k)A^{-T}(k)C^T(k) & (I) \\
D(k)B^T(k)A^{-T}(k)P^{-1}(k) + C(k) = 0 & (II) \\
A(k)P^{-1}(k)A^T(k) + B(k)B^T(k) - P^{-1}(k+1) = 0 & (III)
\end{cases}
$$

Now, the co-isometric conditions for linear time varying discrete time system is available. The follwing Observer is introduced in order to find the optimal filter for residual generation.

Theorem 1.

For the linear time varying system G, an optimal filter for all $\mathcal{H}_-/\mathcal{H}_\infty$, $\mathcal{H}_2/\mathcal{H}_\infty$ and $\mathcal{H}_\infty/\mathcal{H}_\infty$ problems is the following filter [6]:

$$
\begin{aligned}
\hat{x}(k+1) &= (A(k) + L_0(k)C(k))\hat{x}(k) + (B(k) + L_0(k)D(k))u(k) - L_0 y(k) \\
r(k) &= \beta {R_d}^{-1/2}\big(y(k) - C\hat{x}(k) - Du(k)\big)
\end{aligned}
\tag{3.24}
$$

where

$$
R_d(k) = D_d(k){D_d}^T(k) + C(k)P^{-1}(k)C^T(k) > 0
$$

$$L_0(k) = -(A(k)P^{-1}(k)C^T(k) + B_d(k){D_d}^T(k)){R_d}^{-1}(k)$$

(3.25)

and $P(k)$ is the solution of the following algebraic Riccati equation:

$$A(k)P^{-1}(k)A^T(k) - P^{-1}(k+1) + \left(A(k)P^{-1}(k)C^T(k) + B_d(k){D_d}^T(k)\right).$$

$${R_d}^{-1}(k)\left(D_d(k){B_d}^T(k) + C(k)P^{-1}(k)A^T(k)\right) + B_d(k){B_d}^T(k) = 0$$

(3.26)

Proof.

Since N_d admits the following spectral factorization

$$N_d{N_d}^{\sim} = VV^{\sim},$$

By multiplying both sides of the above equation by Q we have

$$(QN_d)(QN_d)^{\sim} = (QV)(QV)^{\sim}$$

$\|QV\|_\infty = \|QN_d\|_\infty$ and $\|QV\|_2 = \|QN_d\|_2$

As mentioned before the goal is to find a linear bounded system Q such that $\|QN_d\|_\infty \leq \beta$ and $\left\|QN_f\right\|_2$ is maximized. Using the above equality, $\|QN_d\|_\infty < \beta$ and lemma 1, we have

$$\left\|QN_f\right\|_2 = \left\|QVV^{-1}N_f\right\|_2 \leq \|QV\|_\infty\left\|V^{-1}N_f\right\|_2 = \|QN_d\|_\infty\left\|V^{-1}N_f\right\|_2$$

$$\leq \beta\left\|V^{-1}N_f\right\|_2$$

The equality is also true for $\mathcal{H}_\infty$ and $\mathcal{H}_-$ norms. So now we have to find a Q such that the above equality is satisfied. When $QV = \beta I$, that is, $Q = \beta V^{-1}$, the conditions are satisfied and filter is optimal.

Now the problem narrows down to finding the conditions which need to be satisfied in order for N_d to admit spectral factorization.

Using Left Co-prime factorization, linear system N_d is introduced as:

$$x_3(k+1) = \big(A(k) + L(k)C(k)\big)x_3(k) + \big(B_d(k) + L(k)D_d(k)\big)d(k)$$

$$y_3(k) = C(k)x_3(k) + D_d(k)d(k)$$

(3.27)

If state space realization of V^{-1} is given as

$$q(k+1) = (A(k) + L_0(k)C(k))q(k) + (L_0(k) - L(k))y(k)$$

$$u(k) = R_d^{-\frac{1}{2}}(C(k)q(k) + y(k))$$

By defining a new state $x(k) = x_3(k) + q(k)$, state realization for $V^{-1}N_d$ is given as:

$$x(k+1) = (A(k) + L_0(k)C(k))x(k) + (B_d(k) + L(k)D_d(k))d(k)$$

$$r(k) = R_d^{-\frac{1}{2}}(k)(C(k)x(k) + D_d(k)d(k))$$

(3.28)

Now the problem comes down to finding the observer gains. In order for $V^{-1}N_d$ to be co-isometric, conditions (3.26) should hold. From conditions (I) and (II) we have

$$I = D(k+1)D^T(k+1) + C(k+1)P^{-1}(k+1)C^T(k+1)$$

Substituting D and C of the system $V^{-1}N_d$ given in equation (3.28), we have

$$I = R_d^{-\frac{1}{2}}(k+1)D_d(k+1)D_d^{\,T}(k+1)R_d^{-\frac{T}{2}}(k+1) + R_d^{-\frac{1}{2}}C(k+1)P^{-1}(k+1)C^T(k+1)R_d^{-\frac{T}{2}}(k+1)$$

Therefore

$$R_d(k) = D_d(k){D_d}^T(k) + C(k)P^{-1}(k)C^T(k).$$

From condition (II) we get

$$R_d^{-\frac{1}{2}}(k)D_d(k)\big(B_d(k) + L_0(k)D_d(k)\big)^T(A(k) + L_0(k)C(k))^{-T}P(k) + R_d^{-\frac{1}{2}}(k)C(k) = 0$$

$$D_d(k)\big({B_d}^T(k) + {D_d}^T(k){L_0}^T(k)\big) = -C(k)P^{-1}(k)(A^T(k) + C^T(k){L_0}^T(k))$$

$$L_0(k) = -(A(k)P^{-1}(k)C^T(k) + B_d(k){D_d}^T(k)){R_d}^{-1}(k)$$

From condition (III) we have

$$(A(k) + L_0(k)C(k))P^{-1}(k)(A(k) + L_0(k)C(k))^T(k) + (B_d(k) + L_0(k)D_d(k))(B_d(k) + L_0(k)D_d(k))^T(k) - P^{-1}(k+1) = 0$$

$$\big(A(k) + L_0(k)C(k)\big)P^{-1}(k)A^T(k) + \big(A(k) + L_0(k)C(k)\big)P^{-1}(k)C^T(k){L_0}^T(k) + (B_d(k) + L_0(k)D_d(k))\,{B_d}^T(k) + (B_d(k) + L_0(k)D_d(k)){D_d}^T(k){L_0}^T(k) - P^{-1}(k+1) = 0$$

$$\big(A(k) + L_0(k)C(k)\big)P^{-1}(k)A^T(k) + \big(B_d(k) + L_0(k)D_d(k)\big){B_d}^T(k) + \{A(k)P^{-1}(k)C^T(k) + L_0(k)C(k)P^{-1}(k)C^T(k) + B_d(k){D_d}^T(k) + L_0(k)D_d(k){D_d}^T(k)\}{L_0}^T(k) - P^{-1}(k+1) = 0$$

$$\{A(k)P^{-1}(k)A^T(k) + B_d(k){B_d}^T(k) + L_0(k)(C(k)P^{-1}(k)A^T(k) + D_d(k){B_d}^T(k)\} + \{A(k)P^{-1}(k)C^T(k) + B_d(k){D_d}^T(k) + L_0(k)\left(C(k)P^{-1}(k)C^T(k) + D_d(k){D_d}^T(k)\right)\}{L_0}^T(k) - P^{-1}(k+1) = 0$$

From conditions (I) and (II) and the definition of $R_d(k)$ the following algebric Riccati equation is concluded:

$$A(k)P^{-1}(k)A^T(k) - P^{-1}(k+1) + \left(A(k)P^{-1}(k)C^T(k) + B_d(k){D_d}^T(k)\right).$$
$$\qquad {R_d}^{-1}(k)\left(D_d(k){B_d}^T(k) + C(k)P^{-1}(k)A^T(k)\right) + B_d(k){B_d}^T(k) = 0$$

This proves that Theorem 1 provides us with the residual generator for discrete time varying systems.

3.2.3 FD for the system under norm invariant transformation

In Chapter 2, it was illustrated how to obtain a linear discrete time varying equivalent models of aperiodic time invariant sampled data system and periodic time varying sampled data system using norm invariant transformation. In this chapter, a fault detection (FD) method was introduced for the resulting linear discrete time varying systems.

In general, the information about the input to the system is known but there is no knowledge about the disturbance and fault in the system. In norm invariant transformation, it is not required to have any information about the input. Hence, in a sampled data system, one can use norm invariant transformation to find the input matrices, B_d and B_f, and use the proposed step invariant transformation to find B. After the discretization using step and norm invariant transformation, the fault detection scheme for discrete time varying systems illustrated in the previous section, can be applied to the system.

For time invariant system with aperiodic sampler, using step and norm invariant transformation, the corresponding system matrices are as follows:

$$A(k) = e^{A_c(h_{k+1}-h_k)}$$

$$B(k) = \int_0^{h_{k+1}-h_k} e^{A_c\tau}\, d\tau\, B_c$$

$$C = \frac{1}{\sqrt{L}}C_c,\ D = 0$$

$$B_f(k)B_f^T(k) = \int_0^L Me^{-A_c\tau}B_{c\_f}{B_{c\_f}}^T e^{-{A_c}^T\tau}M^T d\tau d\tau$$

$$B_d(k)B_d^T(k) = \int_0^L Me^{-A_c\tau}B_{c\_d}{B_{c\_d}}^T e^{-{A_c}^T\tau}M^T d\tau$$

$$M = e^{A_c h_0}e^{A_c h_0} \ldots e^{A_c h_k}, 0 < h_k \leq L$$

(3.29)

where B_{c_f}, B_{c_d} and B_c are known matrices in continuous time. $B_f(k)$ and $B_d(k)$ can be found using Cholesky factorization.

For time varying system with periodic sampler, using step and norm invariant transformation, the corresponding matrices are:

$$A[k] = \Phi\big((k+1)T, kT\big)$$

$$B[k] = \int_{kT}^{(k+1)T} \Phi((k+1)T, \tau)B_c(\tau)d\tau$$

$$B_d[m]B_d^T[m] = \int_0^h \Phi(mh, \tau)B_{c\_d}(\tau){B_{c\_d}}^T(\tau)\Phi^T(mh, \tau)\, d\tau$$

$$B_f[m]B_f^T[m] = \int_0^h \Phi(mh, \tau)B_{c\_f}(\tau){B_{c\_f}}^T(\tau)\Phi^T(mh, \tau)\, d\tau$$

$$C_d[k] = \frac{1}{\sqrt{h}}C_c(kh)$$

(3.30)

where B_{c_f}, B_{c_d} and B_c are known matrices in continuous time. $B_f(k)$ and $B_d(k)$ can be found using Cholesky factorization.

Having the above system descriptions from Chapter 2, one can use the fault detection method of Section 3.1 to identify the system faults.

3.2.4. Simulation

To illustrate the effectiveness of the proposed fault detection scheme for discrete time varying systems along with the norm invariant transformation, an example is given in this section. A time invariant system sample data system with aperiodic sampler with the following state space realization is considered:

$$x(k+1) = Ax(k) + Bu(k) + E_f f(k) + E_d d(k)$$

$$y(k) = Cx(k) + Du(kt) + F_f f(k) + F_d d(k)$$

where

$$A = \begin{bmatrix} -0.01 & 0 \\ 0 & -0.05 \end{bmatrix}, B_d = \begin{bmatrix} -0.1 & 0 \\ -0.1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 2 \end{bmatrix},\ D = 0$$

$$B_f = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = [0.001 \quad 0]$$

Assume that the noise $d(t)$ and the fault $f(t)$ are of the following forms, respectively,

$$f(t) = \begin{cases} 1 & 50 < k < 100 \\ -1 & 200 < k < 250 \\ 0 & otherwise \end{cases}$$

$$d = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}$$

while the input $u(t)$ is considered to be zero. The aperiodic sampling intervals are generated randomly in MATLAB. A vector of 20 different sampling instants was considered where the values changed between 0 to 1 and for each instant of time, a value was randomly chosen from this vector.

For the $\mathcal{H}_-/\mathcal{H}_\infty$, $\mathcal{H}_2/\mathcal{H}_\infty$ and $\mathcal{H}_\infty/\mathcal{H}_\infty$ problems with $\beta = 1$,the fault detection filter gain $L_0(k)$ is shown in Figure 3.1. Figure 3.2 shows the residual signal.

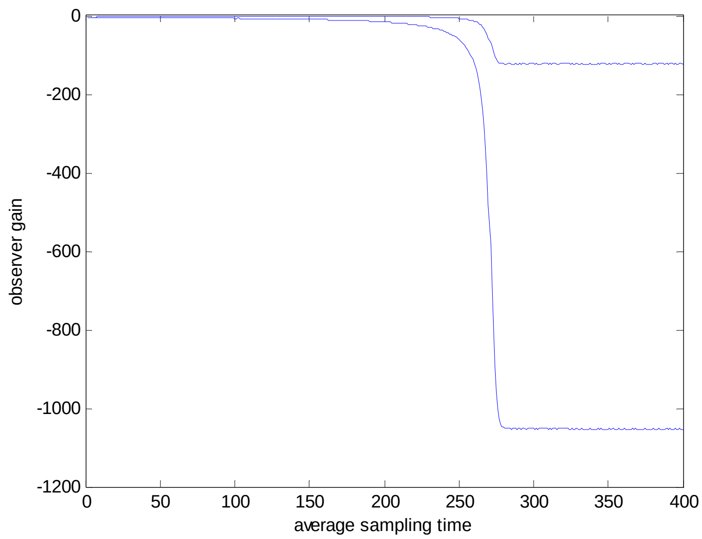


Figure3.1: Fault detection filter gain $L_0(k)$.

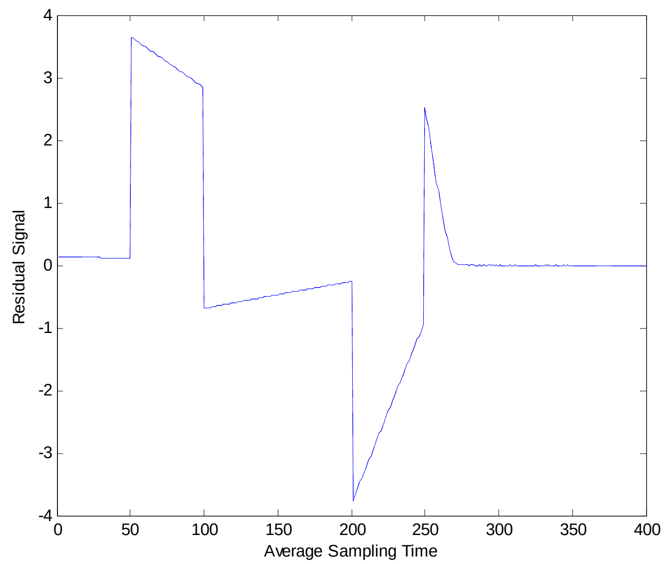


Figure 3.2: Residual signal

As can be seen from Figure 3.2, the residual signal is nonzero with relatively large amplitude for the instants where fault is present.

3.3 Application of Kalman filter in Fault Detection

In the past decades, considerable efforts have been devoted to study of the filtering problem of dynamic systems [24-27]. The Kalman filtering, which addresses the minimization of filtering error covariance, has proven to be the most representative one among varieties of filters and widely investigated [28].

Kalman filter is a set of mathematical equations that provides an efficient computational means to estimate the state of a process, in a way that minimizes the mean of the squared error. The filter is very powerful in several aspects: it supports estimation of past, present and even future states and it can do so even when the precise nature of the modeled system is unknown. But more importantly, Kalman filter formulation is available for estimating the states of a discrete time varying systems.

In this section, Kalman filter is used as an observer for fault detection of discrete time varying systems. First the formulation of discrete time varying Kalman filter is given and then its use in fault detection is discussed.

3.3.1 Discrete time varying Kalman filter

Suppose we have a linear discrete time varying system given as follows [28]:

$$x(k) = F(k-1)x(k-1) + G(k-1)u(k-1) + w(k-1)$$

$$y(k) = H(k)x(k) + v(k) \tag{3.31}$$

The noise processes $\{w(k)\}$ and $\{v(k)\}$ are white, zero-mean, uncorrelated, and have known covariance matrices $Q(k)$ and $R(k)$ repectively:

$$w(k) \sim (0, Q(k))$$

$$v(k) \sim (0, R(k))$$

$$E[w(k)w^T(k)] = Q(k)\delta_{k-j}$$

$$E[v(k)v^T(k)] = R(k)\delta_{k-j}$$

$$E[w(k)v^T(k)] = 0$$

where δ_{k-j} is the Kronecker delta function. The goal is to estimate the state $x(k)$ based on our knowledge of the system dynamics and the availability of the noisy measurements $\{y(k)\}$. The amount of the information that is available for state estimation varies depending on the particular problem that is being solved.

The term $P(k)$ is used to denote the covariance of the estimation error. $P^-(k)$ and $P^+(k)$ denote the covariance of the estimation error $\hat{x}^-(k)$ and $\hat{x}^+(k)$, repectively. $\hat{x}^+(k)$ and $\hat{x}^-(k)$ are both estimates of $x(k)$. However, $\hat{x}^-(k)$ is the estimate of $x(k)$ before the measurement $y(k)$ is taken into account, and $\hat{x}^+(k)$ is the estimate of $x(k)$ after the measurement $y(k)$ is taken into account.

The discrete time Kalman filter is summarized the following algorithm:

1- The Kalman filter is initialized as follows:

$$\hat{x}^+(0) = E\big(x(0)\big)$$

$$P^+(0) = E[\big(x(0) - \hat{x}^+(0)\big)\big(x(0) - \hat{x}^+(0)\big)^T]$$

2- The Kalman filter is given by the following equations for each time step:

$$P^-(k) = F(k-1)P^+(k-1)F^T(k-1) + Q(k-1)$$

$$K(k) = P^-(k)H^T(k)\big(H(k)P^-(k)H^T(k) + R(k)\big)^{-1} = P^+(k)H^T(k)R^{-1}(k)$$

$$\hat{x}^-(k) = F(k-1)\hat{x}^+(k-1) + G(k-1)u(k-1)$$

$$\hat{x}^+(k) = \hat{x}^-(k) + K(k)\big(y(k) - H(k)\hat{x}^-(k)\big)$$

$$P^+(k) = \big(I - K(k)H(k)\big)P^-(k)\big(I - K(k)H(k)\big)^T + K(k)R(k)K^T(k)$$

$$= \left[\left(P^{-}(k)\right)^{-1} + H^{T}(k)R^{-1}(k)H(k)\right]^{-1} = (I - K(k)H(k))P^{-}(k)$$

The first expression for $P^{+}(k)$ can be shown to be more stable and robust than the third expression. The first expression guarantees that $P^{+}(k)$ will always be symmetric positive definite, as long as $P^{-}(k)$ is symmetric positive definite [28].

It should be noted that the optimal Kalman filter is a well known application example of H_2 norm, in which the H_2 norm of the transfer function matrix from the noise to the estimation error is minimized [41].

3.3.2 Discrete time varying Kalman filter fault detection approach

As discussed earlier in chapter 1, if we have a system with the state space realization

$$x(k+1) = A(k)x(k) + Bu(k) + E_f(k)f(k) + E_d(k)d(k)$$

$$y(k) = C(k)x(k) + D(k)u(k) + F_f(k)f(k) + F_d(k)d(k)$$

The residual vector is defined by

$$r(k) = (y(k) - \hat{y}(k) = (y(k) - C\hat{x}(k) - Du(k)) \tag{3.32}$$

where the post filter is considered to be an Identity matrix.

In order to find $\hat{x}(k)$ an observer is required, which in this case we present to use Kalman filter. Using the discrete time varying equations of Kalman filter given in the previous section where the disturbance is considered to be white noise, $\hat{x}(k)$ is found and using equation (3.32), the residual signal is generated resulting in the detection of the fault. The following simulation illustrates this application of Kalman filter.

3.3.3 Simulation

Consider the following linear discrete time varying system with the state space realization [12]

$$x(k+1) = A(k)x(k) + B_u(k)u(k) + B_d(k)d(k) + B_f(k)f(k)$$

$$y(k) = C(k)x(k) + D_u(k)u(k) + D_d(k)d(k) + f_f(k)f(k)$$

where

$$A(k) = \begin{bmatrix} 0.2e^{-\frac{k}{100}} & 0.6 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.7 \end{bmatrix} \quad , \quad B_d(k) = \begin{bmatrix} 1.3 \\ 0.5 \\ 0.9^k \end{bmatrix}$$

$$B_f(k) = \begin{bmatrix} 0.7 \\ 0.4 \\ 0.5 \end{bmatrix} \quad , \quad C(k) = \begin{bmatrix} -0.5 & 1.5 & 0 \end{bmatrix}$$

$$D_d(k) = 0.5 \quad , \; D_f(k) = 0.5$$

Assume that the input $u(k)$ and the noise $d(k)$ are of the following forms, respectively,

$$u(k) = 0$$

$d(k)$ =white noise

and the fault $f(k)$ is shown in Figure 3.6.

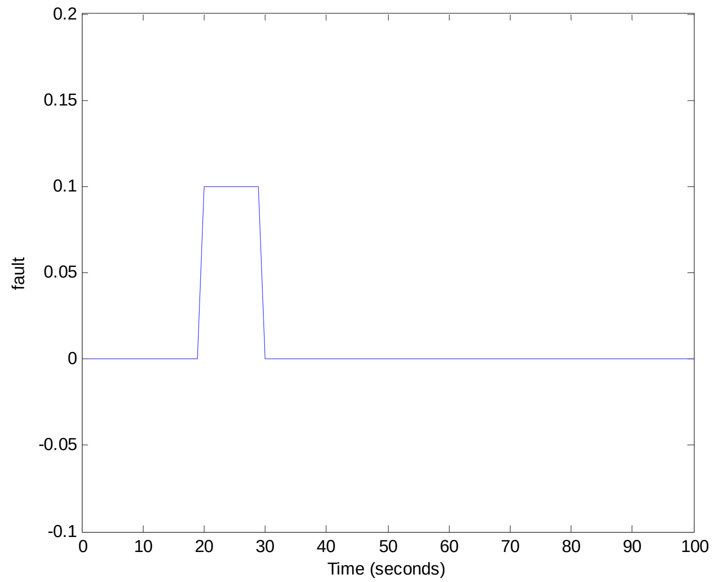


Figure 3.6: Fault signal

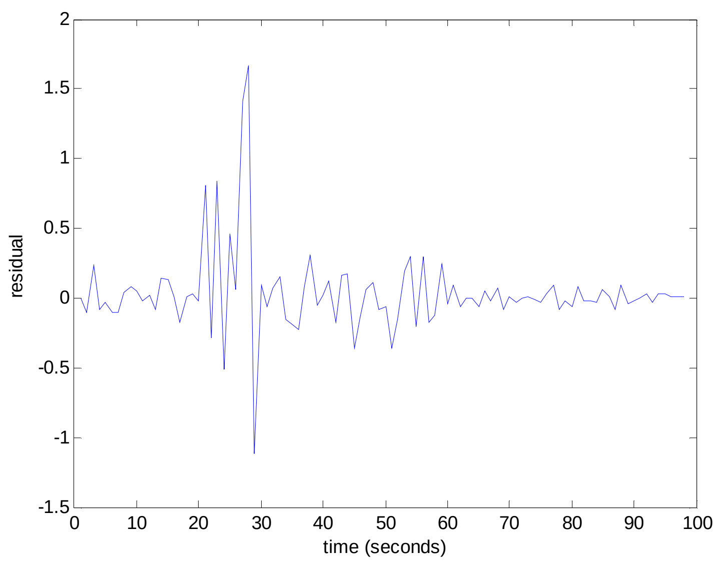


Figure 3.7: Residual signal for discrete time varying Kalman filter FDF

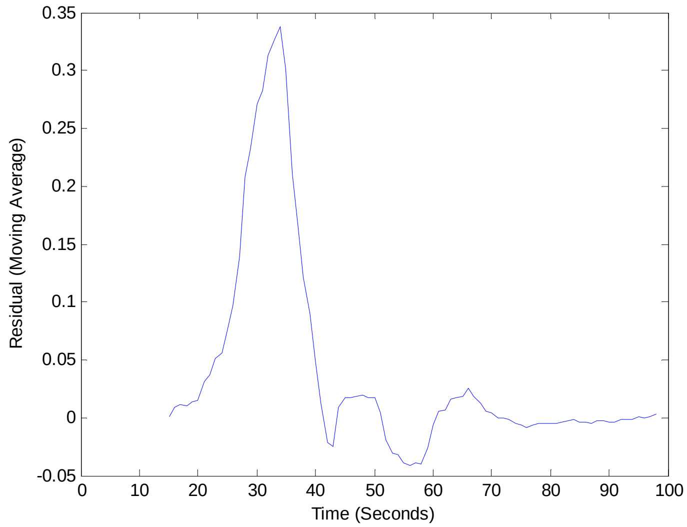


Figure 3.8: Residual signal using Moving Average

The residual signal is shown in Figure 3.7. In order to analyze the results, moving average of the residual signal has been taken and shown in Figure 3.8 which clearly indicates the presence of the fault at $20s \leq k \leq 30s$.

3. 4 Conclusion

A fault detection method was extended to use for discrete time varying systems. In comparison to applying a fault detection method to a continuous time varying method, our approach has easier computations which all are algebraic. In chapter 2, it was shown that using norm invariant transformation for time varying sample data system with periodic sampler or time invariant sample data system with aperiodic sampler, will result in a discrete time varying system where the norms of the two systems are equivalent. In this chapter, a simulation example was used to demonstrate the effectiveness of using norm invariant transformation and the proposed fault detection method for discrete time varying systems, to detect the system faults.

The Kalman filter formulation for discrete time varying systems was reviewed and its application for fault detection was investigated. A simulation example was used to illustrate the use of Kalman filter in fault detection of discrete time varying systems.

Chapter 4

Fault detection For Systems With Network Induced Delay

4.1 Introduction

Network control systems (NCS) are feedback control systems wherein the control loops are closed via real time networks and are comprised of a large amount of actuators, sensors and controllers which are equipped with network interfaces.This new type of information transmission reduces system wiring, eases maintenance and diagnosis, and increases system agility, which make NCS a promising structure for control systems.

Despite the enormous advantages of NCS, the introduction of networks also brings some new problems and challenges, such as network-induced delay, packet dropout, network scheduling and quantization problems. While NCS offers obvious benefits, the mentioned challenges increases vulnerability of the control system. The issues of safety and fault tolerance becomes more stringent in this case. Indeed, faults in one part of the system can propagate through and affect the other parts even in remote locations. In order to maintain performance of control systems, faults should be promptly detected and identified so that appropriate remedies can be applied. The problem of fault detection and isolation has been widely studied in the past decades and numerous design methods are available in the literature [1,2,3]. However, there is limited number of contributions about fault detection of NCS with networked induced time delay. Considering unknown time delay of less than one sampling period, a threshold was used to enhance robustness of fault detection to the delay [4] and an adaptive fault diagnosis method after the signature matrix of delay was used [5]. In [6], a network induced delay of greater than one sampling time was considered and transformed to a polytopic uncertainty.

In the development of fault detection for networked control systems, one of the important properties considered is periodic sampling. However, in network control systems resources for measurement are restricted and hence the sampling operation tends to be aperiodic and uncertain [4]. In view of the widespread use of such types of system, it is important to study the fault diagnosis of NCS for varying sampling intervals.

In chapter 2, a framework was developed for sampled data systems with periodic sampler using norm invariant transformation. In this chapter we develop a general framework for sampled data fault detection for a NCS with aperiodic sampler which offers a convenient tool for both design and analysis. By defining norms of sampled system and the system resulted from norm invariant transformation, this framework allows us to extend norm based method of discrete time varying fault detection to network control systems. Using the fault detection scheme proposed for discrete time varying systems in chapter 3, fault detection of network control systems with aperiodic sampler is achieved.

The remaining of the chapter is organized as follows: Section 2 describes the NCS model with time varying delay. In section 3, simulation results are presented to illustrate the efficiency of using the NCS model along with the fault detection method presented in chapter 3. Concluding remarks are provided in section 4.

4.2 NCS Model

NCS is assumed to be composed of a plant, an actuator, a sensor and a remote control station connected to the sensor and actuator through the network as shown in Figure 4.1 taken from [6].

Dynamics of the plant are given by

$$\dot{x}(t) = A_c x(t) + B_c u(t) + B_d d(t) + B_f f(t)$$

$$y(t) = C_c x(t) + D_d d(t) + D_f f(t) \tag{4.1}$$

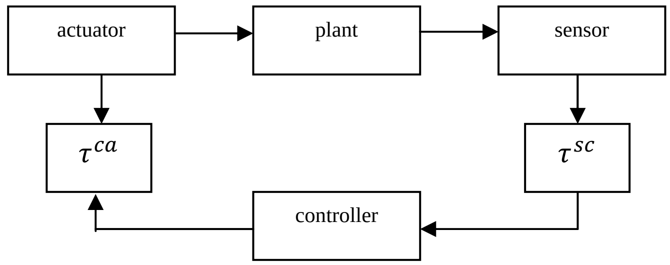


Figure 4.1: NCS Structure

Where $x(t), u(t), d(t), f(t)$ are the plant's state, input, disturbance and fault, respectively. Matrices $A_c, B_c, B_d, B_f, C_c, D_d$ and D_f are of appropriate dimensions.

Suppose the sensor is clock driven, the actuator is clock driven and the sampling period is h_k. There is a sensor to controller delay τ_k^{sc}, therefore the controller receives $y(k)$ at time instant $h_k + \tau_k^{sc}$, and there is a controller to actuator delay τ_k^{ca},so the actuator receives the signal $u(k)$ at time instant $h_k + \tau_k^{sc} + \tau_k^{ca}$, and is kept unchanged by the zero order hold until the next control signal $u(k+1)$ arrives [39];

$$u(t) = u(k), \;\; t \in [h_k + \tau_k^{sc} + \tau_k^{ca}, h_{k+1} + \tau_{k+1}^{sc} + \tau_{k+1}^{ca}] \tag{4.2}$$

If there is no packet dropout and the control law is fixed, the sensor to controller delay and the controller to actuator delay can be lumped together as $\tau_k =_k^{sc} + \tau_k^{ca}$. The delay is presented as $\tau_k = lh + \varepsilon_k$, where l is a known constant integer and ε_k is unknown but bounded by $0 < \varepsilon_k < h$.

Using the norm invariant transformation discussed in chapter 2, the following discrete time varying model can be derived

$$x(k+1) = A_d(k)x(k) + \Gamma_0^{\varepsilon_k}(k)u(k-l) + \Gamma_1^{\varepsilon_k}(k)u(k-l-1) + W_d(k)d(k) + W_f(k)f(k)$$

$$y(k) = C_d(k)x(k) + V_d(k)d(k) + V_f(k)f(k)$$

where

$$A_d(k) = e^{A_c(h_{k+1}-h_k)}$$

$$\Gamma_0^{\varepsilon_k}(k) = (\int_0^{h_{k+1}-h_k-\varepsilon_k} e^{A_c\tau} d\tau)B_c$$

$$\Gamma_1^{\varepsilon_k}(k) = (\int_{h_{k+1}-h_k-\varepsilon_k}^{h_{k+1}-h_k} e^{A_c\tau} d\tau)B_c$$

$$M = e^{A_c h_0} e^{A_c h_0} \dots e^{A_c h_k} \quad , \quad 0 < h_k \leq L$$

$$W_f(k)W_f^T(k) = \int_0^L M e^{-A_c\tau} B_f {B_f}^T e^{-{A_c}^T\tau} M^T d\tau$$

$$W_d(k)W_d^T(k) = \int_0^L M e^{-A_c\tau} B_d {B_d}^T e^{-{A_c}^T\tau} M^T d\tau$$

$$C_d = \frac{1}{\sqrt{L}} C_c$$

To overcome the effect of networked induced delay, the system is augmented as

$$x(k+1) = A_d(k)x(k) + \Gamma_0^k(k)u(k-l) + \bar{W}_d(k)\bar{d}(k) + W_f(k)f(k)$$

$$y(k) = C_d(k)x(k) + \bar{V}(k)\bar{d}(k) + V_f(k)f(k) \qquad (4.3)$$

where

$$\bar{W}_d(k) = \begin{bmatrix}\Gamma_1^{\varepsilon_k}(k) & W_d(k)\end{bmatrix}, \bar{d}(k) = \begin{bmatrix}u(k-l-1) \\ d(k)\end{bmatrix}$$

$$\bar{V}(k) = [0 \quad V_d(k)]$$

The fault detection approach introduced in chapter 3 along with the model of NCS given in (4.3) is used for fault detection of network control system with network induced delay. In general, fault detection filter must make a tradeoff between two objectives: robustness to disturbance rejection and sensitivity to faults. The objective here is to make the FD filter robust to the newly defined disturbance signal $\bar{d}(k)$ and also sensitive to the fault.

4.3 Simulation

We shall illustrate the fault detection filter design for the proposed NCS model with a simple time invariant network control system, sampled with aperiodic sampler.

Consider the time invariant network system with the following state space realization:

$$\dot{x}(t) = A_c x(t) + B_c u(t) + B_d d(t) + B_f f(t)$$

$$y(t) = C_c x(t) + D_d d(t) + D_f f(t)$$

where

$$A_c = \begin{bmatrix} -0.01 & 0 \\ 0 & -0.05 \end{bmatrix} \quad , \quad B_d = \begin{bmatrix} -0.1 & 0 \\ -0.1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad , \quad D_d = D_f = 0$$

$$B_f = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad , \quad C_c = \begin{bmatrix} 0.001 & 0 \end{bmatrix}$$

Assume that the noise $d(t)$ and the fault $f(t)$ are of the following forms, respectively,

$$f(t) = \begin{cases} 1 & 10 < k < 30 \\ -1 & 50 < k < 60 \\ 0 & 0therwise \end{cases}$$

$$d = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}$$

while the input $u(k)$ is shown in Figure 4.2. As an example, the values of sampling period for instants of 0 to 50 are shown in Figure 4.3, where they have been picked randomly in MATLAB.

For the $\mathcal{H}_-/\mathcal{H}_\infty$, $\mathcal{H}_2/\mathcal{H}_\infty$ and $\mathcal{H}_\infty/\mathcal{H}_\infty$ problems with $\beta = 1$,the fault detection filter gain $L_0(k)$ is shown in Figure 4.4. Figure 4.5 shows the residual signal.

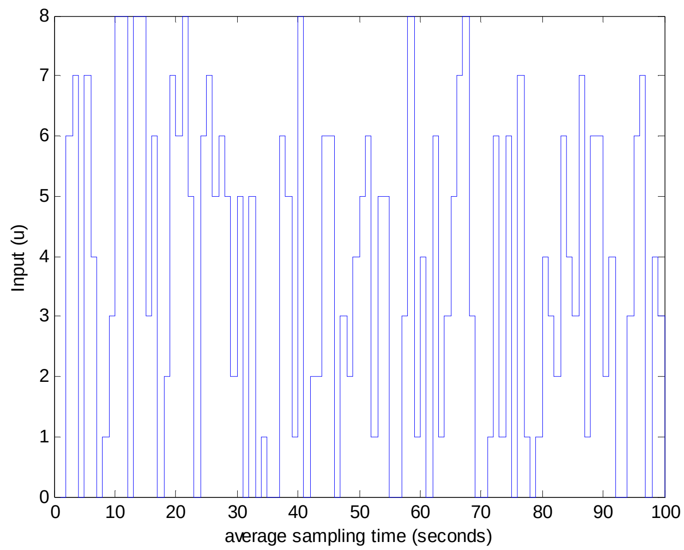


Figure 4.2: Signal input $u(k)$

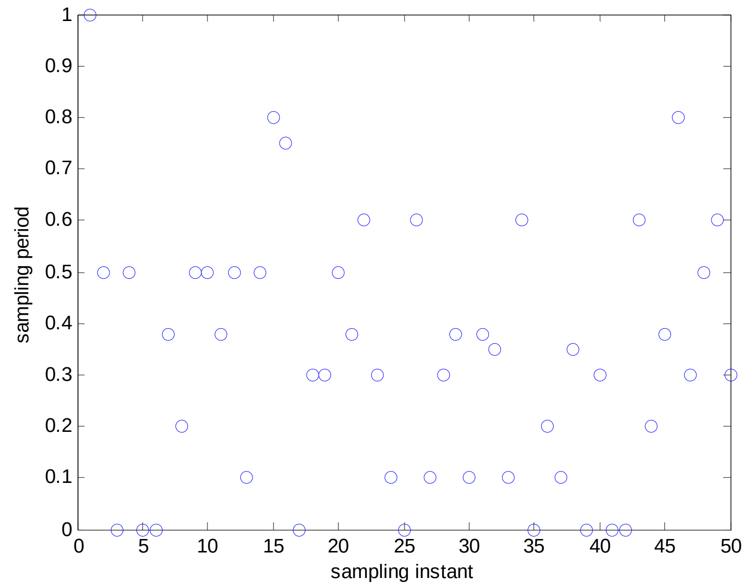


Figure 4.3: Sampling period

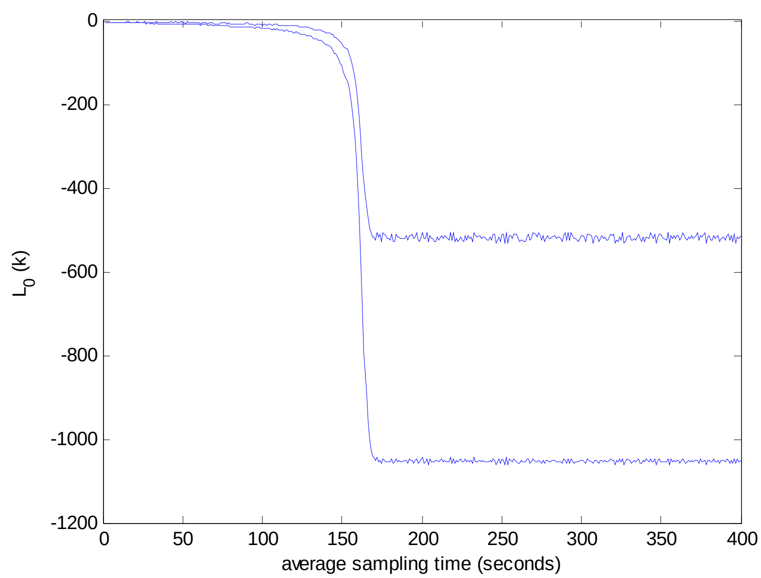


Figure 4.4: Fault detection filter gain $L_0(k)$.

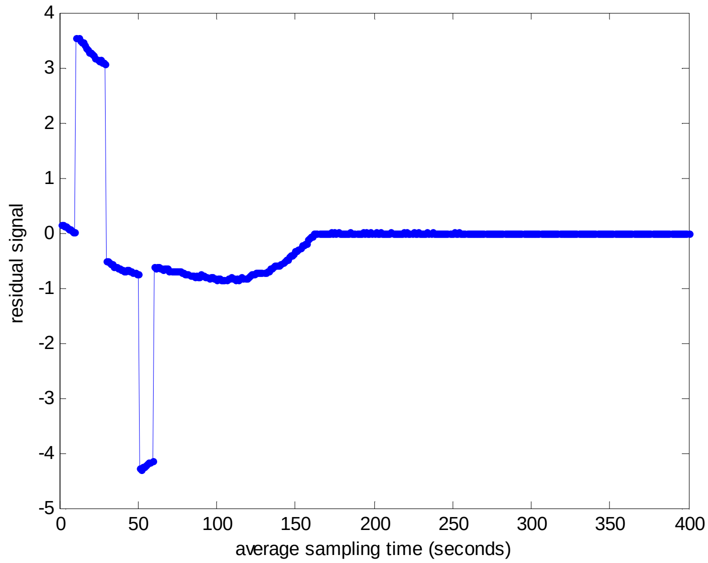


Figure 4.5: Residual signal

As can be seen from Figure 4.5, the residual signal is nonzero with relatively large amplitude for the instants where fault is present.

4. 4 Conclusion

The problem of fault diagnosis for network control systems was considered in this chapter where the time varying network induced varying delay was taken into account. The norm invariant transformation developed in chapter 2 was used to model a time invariant NCS sampled with aperiodic sampler. The resulting model which considers the aperiodic sampling and provides a more realistic model for fault detection of NCS is a time varying discrete time system. A fault detection method for discrete time varying systems, as discussed in chapter 3, was applied to the proposed model. A simulation was used to illustrate the efficiency of the proposed method.

Chapter 5

Conclusions

5. 1 Concluding remarks

In this thesis, we started with developing norm invariant transformation for a time varying sampled data system with periodic sampler and a time invariant sampled data system with aperiodic sampler. It was shown that using these norm invariant transformations, for each case, an specific discrete time varying system results where the norms of the resulting system are equivalent to the norms of original sampled data system.

A fault detection method was proposed for discrete time varying systems in Chapter 3. The proposed method is an extension of the approach of [6] to the discrete time case. It was shown that using this fault detection method and the norm invariant transformation, one can detect the faults in time varying sample data system with periodic sampler or time invariant sample data system with aperiodic sampler.

Due to the time varying nature of the Kalman filter, application of the Kalman filter as an observer for fault detection was investigated for discrete time varying systems and a simlation example was provided to illustrate this approach.

Finally, the implementation of norm invariant transformation and the fault detection method of discrete time varying systems in network control system application was studied. A network control system with the varying network induced delay was considered in this case. The sampler was assumed to be aperiodic. Using norm invariant transformation method of Chapter 2, a time varying discrete model was derived for NCS which then was used to implement the proposed fault detection method of Chapter 3. Simulation results were given to illustrate the method.

5. 2 Future work

In addition to network induced delays, the introduction of networks also brings other problems and challenges, such as packet dropout, network scheduling and quantization problems. The norm invariant transformation along with the fault detection approach for discrete time varying systems can be further studied in network control systems considering these problems. The delay could also be considered to be more than one sampling time. When packet dropout and delay are both present, it adds more complexity to the fault detection. Research on the combination of problems using our discretization method could also be considered in further studies.

The study of real applications, in laboratory or industrial scale, to apply the norm invariant transformation along with the fault diagnosis approach for time varying systems can be an important contribution.

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