Relay Selection in Wireless Networks: An Approach Inspired by the Secretary Problem

by

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Abstract

In cooperative wireless networks with multiple relays, relay selection is an important step that has attracted a lot of research interest over the years. Many variations of the relay selection problem with different assumptions, goals, and selection approaches have been studied. In most of these approaches, it is assumed that the channel-state-information (CSI) is known. However, finding the CSI requires testing all the relays, which can be costly. This thesis proposes modeling the relay selection problem as an optimal stopping rule problem in two scenarios, one when testing to find the CSI is not costly and one where there is cost.

When finding CSI has no cost, we investigate modeling the relay(s) selection problem as an optimal stopping rule problem such as a secretary problem or one of its variants. For both single relay selection (SRS) and multiple relay selection (MRS), and both known and unknown channel distributions (statistics), we show that the relay selection problem in each case can be mapped to a specific version of the secretary problem. The solution for each version differs from one another. All versions try to make the selection without testing all the channels.

When testing each relay has a cost, we solve the problem using a hybrid approach. The method is based on a combination of the secretary problem and the random selection, and maximizes the overall achievable rate of the network. Simulation results verify that our method can achieve up to 7.5 dB performance gain compared to the random approach that randomly selects the

relays and achieves 20 dB performance gain compared to a solution that tests all the relays and picks the best one(s).

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Contents

List of Tables

List of Figures

List of Acronyms

Abbreviation	Definition
SRS	single relay selection
MRS	multiple relay selection
AF	amplify-and-forward
DF	decode-and-forward
kRSn	selection of k relays among n relays
AWGN	additive-white-gaussian-noise
b/s/Hz	bits-per-second-per-hertz

Chapter 1

Motivation and Background

1.1 Thesis Motivation

In cooperative communication systems, relays facilitate the communication between the source and destination to improve the reliability of wireless communication. Relay-assisted communication is especially promising when the source-destination link is long, or when the line of sight can be obstructed [1]. There are many relaying strategies in the literature that have been utilized in wireless standards such as using fixed relays in the European Telecommunications Standards Institute/Digital Enhanced Cordless Telephony (ETSI/DECT) standard and using non-transparent (NT) relays in the IEEE 802.16 working group and 3GPP standards [2].

A variety of relaying protocols have been suggested in the literature. Two of the most common relaying protocols are amplify-and-forward (AF) and decode-and-forward (DF) [3]. The main difference between DF and AF relaying is that in DF, a relay decodes the source message and then forwards a re-encoded version to the destination, whereas in AF, the relay only amplifies the source signal. There are other relaying protocols that try to achieve diversity gains and/or spectral efficiency [4]–[9]. In [7], hybrid decode-amplify-forward (HDAF) relaying protocol is discussed. Also, in [9] an incremental relaying protocol is proposed to increase the spectral efficiency for the cooperative transmission. This protocol exploits a limited feedback from the destination to indicate when the relay is allowed to forward the message to the destination.

When more than one relay is available, deciding which relay(s) should forward the transmitted message highly impacts the performance, in terms of both energy efficiency and the network reliability. There are many different single relay selection (SRS), and multiple relay selection (MRS) strategies based on varying assumptions, optimization goals, or relay selection criteria [10]–[25].

Some examples of relay selection for SRS scenario are as follows:

- In [10], a selection scheme is given to maximize the weighted sum-rate capacity for DF relaying.
- In [11] - [13], the worst receive signal-to-noise ratio (SNR) of two users is maximized for DF relaying.
- In [14], a cross-layer relay selection metric, which depends on both the instantaneous channel conditions and the queuing status, is investigated.
- In [15], the instantaneous sum-rate of AF relaying is maximized.
- In [17], the mutual information of AF relaying is maximized.

Some of the examples of relay selection for MRS scenario are as follows:

- In [13], for DF relaying, a scheme that selects two relays out of n available relays to minimize the average bit error rate (BER) is proposed.
- In [25], for AF relaying, maximizing the worst receive SNR is considered.

One of the most intuitive solutions for relay selection is to choose the relay(s) with the highest end-to-end SNR(s). To find the end-to-end SNR for each relay, the gain of the channels of the source-relay link and the relay-destination link must be known. Therefore, one needs to test all the relays to find their end-to-end SNRs and then selects the relay(s) with the highest end-to-end SNR(s). Most of the literature dealing with the problem of relay selection based on maximum end-to-end SNR assume that the channel state information (CSI) of the source-relay and relay-destination links are known [26]–[28]. One of the drawbacks of this assumption is that it requires testing all the channels. This leads to wasting energy and time, especially when

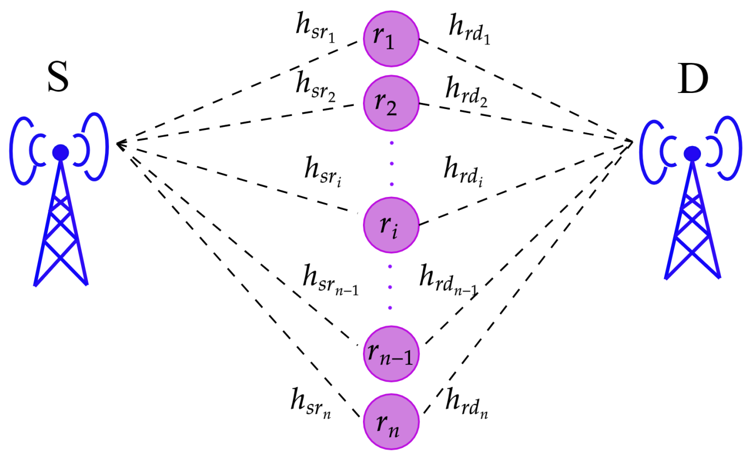


Figure 1.1: Relaying scenario model. Here n is the number of relays.

there are setups that have a large number of relays such as internet-of-things (IoT) [29]. IoT is growing large, therefore, it needs higher densities of relays, sensors, actuators to adapt to the increasing complexity and heterogeneity of advanced network devices [30]. Handling a large number of relays in IoT setup is challenging.

This problem gets even more severe for networks that have fast varying channels in which even if the CSIs are collected, they are quickly outdated. Then in this case, one may end up testing the relays one by one and deciding on the spot whether to use them or not.

From this point of view, the problem of relay selection is similar to *stopping rule problems*, a family of math problems that try to choose a time to take a particular action (e.g., relay selection) in order to maximize an expected reward (e.g., maximizing the rate of the network).

1.2 Thesis contributions

In this thesis, a system with one source node S and one destination node D and n relays $r_1, r_2, ..., r_n$ as shown in Fig. 1.1 is investigated. Note that there is no direct link between S and D in this setup, meaning that the communication between S and D is made possible by the relays. Hence, a relay or multiple relays will be selected for data transmission.

In this thesis, we have done two studies. In the first study, we assumed that collecting the CSI of the relays is not costly and in the second one, the cost of collecting CSI is also considered.

In the first study, based on the nature of the relay selection problem, we show that several different scenarios of the relay selection problem can be modeled as specific versions of stopping rule problems. Using this modeling, we develop relay selection algorithms inspired by stopping rule problems. In this way, we select the relay(s) without testing all of them. Since the CSI changes fast, the secretary approach allows us to make a good selection and achieve data rates comparable to the case that CSI of all the relays were known beforehand.

This problem is similar to one of the most popular stopping rule problems called standard secretary problem [31]. In standard secretary problem there are n candidates for a secretarial position and the employer is interviewing (testing) them one by one and should decide on the spot whether to hire or reject them (should decide when to stop interviewing the candidates). Once one candidate is selected the employer no longer proceeds with the interview of the rest of the candidates [32].

In the second study, we have modeled the relay selection problem as a modified secretary problem and proposed an algorithm to solve it. First, we propose an algorithm to solve the SRS problem and then we use that algorithm to solve the MRS problem. For the selections based on the end-to-end SNR, the CSIs of the channels need to be collected. To collect the CSIs, all the channels need to be tested which takes time. This means that testing the relays are costly. Despite the similarities of the standard secretary problem to the SRS problem, there are some assumptions for the standard secretary problem that do not fit to model the relay selection problem. For instance, one assumption is that there is no cost for testing the candidates.

Therefore, we have to modify the standard secretary problem to solve the relay selection problem. Our assumptions are as follows: (i) we have assumed that the distribution of the channel gains are known (ii) there is a cost for testing the channels, so that when the cost of testing is growing large, one can stop testing more relays and choose one relay randomly. The goal is to select some suitable relay(s) that maximize the achievable rate. We have proposed a strategy to select a suitable relay based on the modified secretary problem. For ease of discussions, we first solved this problem for SRS and then use the SRS solution to solve the MRS problem as well.

1.3 Thesis background

This section describes relaying protocols, relay selection and the basics of the secretary problem. First, two common relaying protocols are introduced. Second, the achievable rates for SRS and MRS are derived. Third, the original secretary problem, the approach to solve this problem and its applications are described in detail.

1.3.1 Relaying protocols

There are many relaying protocols available in the literature such as AF, DF, compress and forward, incremental relaying, coded cooperation [33]. Two of the most common relaying protocols are AF and DF. Our solution for the relay selection in this thesis is for the DF scenario. However, as we will discuss later, the solution can also be applied to AF subject to an accurate approximation. Therefore, let us first discuss these two major relaying protocols AF and DF for the system model shown in Fig. 1.1.

Amplify-and-forward (AF)

In this protocol, first the source sends its message. Then the selected relay will amplify its received signal and forward it to the destination node. Assuming relay i is selected, the signal received by relay r_i, $i \in 1, \dots, n$ and destination D are:

$$y_{sr_i} = \sqrt{P_s} h_{sr_i} s + n_{sr_i} \tag{1.1}$$

$$y_{rd_i} = h_{rd_i} M_{r_i} y_{sr_i} + n_{rd_i} \tag{1.2}$$

where $M_{r_i} = \sqrt{\frac{P_{r_i}}{P_s \left|h_{sr_i}\right|^2 + N_0}}$. In (1.1), s is the transmitted symbol by source S, P_s is the transmitted power of source S and P_{r_i} is the transmitted power by relay r_i. Also, $h_{sr_i} \sim \mathcal{CN}\left(0, \sigma_{sr_i}^2\right)$ and $h_{rd_i} \sim \mathcal{CN}\left(0, \sigma_{rd_i}^2\right)$ are the complex channel gain of the source-relay channels and the relays-destination channels respectively, where, $\mathcal{CN}\left(\mu, \sigma^2\right)$ denotes a complex circularly symmetric Gaussian distribution with mean μ and variance σ^2. Finally, $n_{sr_i} \sim \mathcal{CN}\left(0, \sigma_{sr_i}^2\right)$ and $n_{rd_i} \sim \mathcal{CN}\left(0, \sigma_{rd_i}^2\right)$ are the additive white Gaussian noise (AWGN) at the relay and the destination, respectively.

The end-to-end SNR can be found as:

$$\mathrm{SNR}_{eq_i} = \frac{\gamma_{sr_i}\gamma_{rd_i}}{\gamma_{sr_i} + \gamma_{rd_i} + 1}. \tag{1.3}$$

In the above equation, $\gamma_{sr_i} = |h_{sr_i}|^2 \frac{P_s}{N_0}$ and $\gamma_{rd_i} = |h_{rd_i}|^2 \frac{P_r}{N_0}$ are the SNRs of the source-relay and the relay-destination links, respectively. We can approximate the end-to-end SNR in (1.3) as follows:

$$\text{approximated } \mathrm{SNR}_{eq_i} = \min\{\gamma_{sr_i}, \gamma_{rd_i}\}. \tag{1.4}$$

The above equation is a very accurate approximation when either $\gamma_{sr_i} \gg \gamma_{rd_i}$ or $\gamma_{rd_i} \gg \gamma_{sr_i}$.

Decode-and-forward (DF)

In DF relaying, once the source S sends its message, the selected relay fully decodes that message, re-encodes it, and forwards it to the destination D. Since we have assumed that in this setup, there is no direct link from the source to destination, the end-to-end SNR can be written as follows [3]:

$$\mathrm{SNR}_{eq_i} = \min\{\gamma_{sr_i}, \gamma_{rd_i}\}. \tag{1.5}$$

1.4 Relay selection

In a dense communication system, several relays may be available to forward the transmitted message. Deciding which relay(s) assist with the communication is quite difficult and highly impacts the performance of the network in terms of the achievable rate of the system. In this part, the frequently used approaches for relay selection are discussed.

Random selection is a simple solution yet not always effective for choosing the relay(s). Selection based on the achievable rate is an important criterion that is discussed in here for relaying protocols AF and DF in both SRS and MRS scenarios.

Achievable rate

One of the most intuitive criteria for relay selection is to select the relay(s) that lead to maximum achievable rate for a communication system. The achievable rate of a system where one relay has been selected is as follows:

$$\text{achievable rate of SRS} = 1/2 \log_2(1 + \text{SNR}_{eq_i}). \tag{1.6}$$

In the above equation, the SNR_{eq_i} corresponds to Eqn. (1.3) and (1.5) for AF and DF protocols respectively.

The achievable rate for SRS can be easily extended to the case where k relays are selected. Assuming the maximum-ratio-combining (MRC) at the receiver, the end-to-end SNR of the system is the sum of the end-to-end SNRs of the k selected relays. According to [34], MRC is the optimal algorithm for maximizing the achievable rate. So the achievable rate of this system assuming that the MRC technique is employed at the destination node D can be written as follows:

$$\text{achievable rate of MRS} = 1/2 \log_2(1 + \Sigma_{i=1}^{k} \text{SNR}_{eq_i}), \quad 1 \leq k \leq n \tag{1.7}$$

To find the achievable rate of MRS for the systems with AF and DF protocols, Eqn. (1.3) and (1.5) are plugged as the SNR_{eq_i} into the above equation respectively.

Overall achievable rate

To select a relay, one may first test the relay, meaning that the CSI of its channels to source and destination is collected. This process takes some time, which we refer to as the test time. If we assume that the transmission time is one unit and testing one relay takes α units of time, we can define a cost for testing. This cost depends on the ratio of test time to the transmission time and is defined as follows:

$$\alpha = \frac{\text{test time}}{\text{transmission time}}. \tag{1.8}$$

Now let us assume that k relay(s) among n relays is (are) selected and we have done m number of tests with cost parameter α to select the relay(s). Since there is a cost for testing the relays, the achievable rate is also affected. To distinguish from the term "achievable rate", here we use the term "overall achievable rate" to refer to the rate of the system taking into account the cost for testing the relays. Given that $1 + m\alpha$ units of time are needed to achieve 1 unit of time for data transmission, the overall achievable rate of the cooperative communication system shown in Fig. 1.1, is as follows:

$$\text{overall achievable rate} = \frac{1/2 \log_2(1 + \Sigma_{i=1}^{k} \mathrm{SNR}_{eq_i})}{1 + m\alpha}, \quad 1 \leq k \leq n. \tag{1.9}$$

1.5 Secretary problem

The optimal stopping rule problems are a family of math problems that try to choose a time to take a particular action in order to maximize an expected reward [35]. These types of problems are found in statistics, mathematics, communication, business, economics, etc [36].

The secretary problem is one of the most popular stopping rule problems appeared in the late 1950's and early 1960's [31]. This problem is also called marriage problem, beauty contest, dowry problem [37].

Imagine that there is one secretarial position available among n available applicants. The candidates are interviewed sequentially and a decision for each candidate needs to be made on the spot. Once a candidate is selected, the interview process will be terminated. Once rejected, that candidate cannot be recalled. The goal is to maximize the probability of selecting the best candidate [38].

The solution to this problem is a class of stopping rules that for some integer $q \geq 1$ pass over the first $q-1$ candidates and thereafter stop and select the next candidate who is better than the first $q-1$ candidates [39].

The probability that the best candidate is the jth candidate is:

$$P(j^{th} \text{ is best}) = \frac{1}{n}. \tag{1.10}$$

If the jth candidate is the best, then the probability of selecting it is the probability that the best candidate among the first $j-1$ candidates appears in the first $q-1$ candidates:

$$P(j^{th} \text{ selected}|j^{th} \text{ is best}) = \frac{q-1}{j-1}. \tag{1.11}$$

Summing over the product of the previously mentioned probabilities gives the probability of selecting the best candidate when we pass the $q-1$ candidates and select the first candidate who is better than the previous $q-1$ candidates. Let us call this probability $\Phi(q)$:

$$\begin{aligned}\Phi(q) &= \sum_{j=q}^{n} P(j^{th} \text{ is the best and is selected}) \\ &= \sum_{j=q}^{n} P(j^{th} \text{ is best}) P(j^{th} \text{ selected}|j^{th} \text{ is best}) \\ &= \frac{1}{n}\sum_{j=q}^{n} \frac{q-1}{j-1} = \frac{q-1}{n}\sum_{j=q-1}^{n-1} \frac{1}{j}, \quad 1 < q \leq n \end{aligned} \tag{1.12}$$

The optimal q is the one that maximizes the above probability. In order to find the rule that leads to the optimal q, we can use the following:

$$\Phi(q) \;\geq\; \Phi(q+1) \implies \frac{q-1}{n}\sum_{j=q-1}^{n-1}\frac{1}{j} \;\geq\; \frac{q}{n}\sum_{j=q}^{n-1}\frac{1}{j} \implies \sum_{j=q}^{n-1}\frac{1}{j} \;\leq\; 1. \tag{1.13}$$

For small values of n, computing the optimal q is easy. Among all the values of q that satisfy Eqn. (1.13), the minimum q is the optimal. For large values of n (when $n \to \infty$):

$$x = \lim_{n\to\infty} \frac{q}{n}. \tag{1.14}$$

Then we can simply change the summation in $\Phi(q)$ to integrals as follows:

$$\Phi(q) = \frac{q-1}{n} \sum_{j=q-1}^{n-1} \frac{n}{j} \frac{1}{n} = x \int_{x}^{1} \frac{1}{x} dx = -x \log(x). \tag{1.15}$$

By taking a derivative with respect to x and solving that equation one can find that the optimal x is $1/e$ [39]. Therefore, the optimal rule for large n is to reject the first n/e candidates and select the next candidate that is better than the first n/e candidates.

1.5.1 Applications of the secretary problem

The secretary problem not only can be used to model different problems such as online auctions [40], but also can be used as a tool to study different phenomenon especially in the field of neuroscience and psychology. As an example, in [41] there was a study about the neuropsychological behavior of patients with Parkinson's disease. They used the secretary problem to assess whether Parkinson's disease patients have deficits in a sequential sampling task. In [42], the secretary problem was used to solve the airline ticket purchasing problem. For each ticket, one needs to decide to either buy that ticket or reject the current ticket price and wait for a ticket with a better price in the future. In this thesis, we apply the secretary problem to the relay selection problem.

1.6 Thesis outline

The outline of the thesis is as follows:

- Chapter 2

  This chapter investigates modeling different scenarios of relay selection problem as specific versions of the secretary problem when cost of testing relays is assumed to be zero. Also, this chapter further studies how the algorithms solving the secretary problem can be used to solve the relay selection problems. Moreover, these studies have been done for both SRS and MRS. The results show that our method can achieve up to 7.5 dB performance gain compared to the random approach that selects the relays randomly.

- Chapter 3

  This chapter proposes modeling the relay selection problem as a stopping rule problem for the case where testing relays is costly. Furthermore, this chapter proposes an algorithm to select suitable relay(s) based on the modified secretary problem to solve this problem. Simulation results verify that our method can achieve 20 dB performance gain compared to a solution that tests all the relays and picks the best one(s) and 6 dB performance gain compared to a solution that randomly picks a relay(s).

- Chapter 4

  This chapter presents the conclusions of the thesis and future research directions.

Chapter 2

Relay Selection Over Wireless Networks: An Approach Based on Secretary Problem and Its Variants

This chapter presents and analyses relay selection strategies for the system shown in Fig. 1.1 based on the secretary problem and its variants. In other words, different scenarios of relay selection have been modeled as a different version of the secretary problem. The algorithms that fit to solve each problem are derived. These algorithms try to maximize the achievable rate of the system. Further, the insights provided by the obtained results are discussed.

2.1 Problem definition

Here, the aim is to select k relays among n relays so that the achievable rate of the system given in (1.7) is maximized. Since the logarithm is an increasing function, maximizing the achievable rate is equivalent to maximizing the end-to-end SNR of the system. Once each relay is tested (the end-to-end SNR corresponding to that relay is found and its corresponding achievable rate is calculated), we need to decide whether we want to select that relay or move on to the next relay. We assume that the CSIs get outdated, meaning that if we do not select the relay and move forward, this decision cannot be revoked at a later time.

The above problem resembles an optimal stopping rule problem called the secretary problem. The standard secretary problem is defined as below: There are n candidates for a secretarial position available. The candidates are interviewed one by one in a random order, and a decision for each candidate needs to be made on the spot. Once rejected, that candidate cannot be recalled. The goal is to maximize the probability of selecting the best candidate [31].

The above secretary problem is similar to a relay selection problem in which the relays are the candidates. Let us familiarize ourselves with the general characteristics of the secretary problem in which relays are the candidates:

- The number of candidates is known.
- The candidates are tested sequentially in a random order.
- An applicant, once rejected, cannot be recalled.
- In case $n-1$ candidates are rejected, the last candidate is hired [31].

The secretary problem has various versions that can be applied to a relay selection problem. These flavors correspond to whether the distribution of the random quality of the relays is known or unknown (channel statistics are known or unknown) and whether we need to select one or more than one relay. In the next section, different scenarios of relay selection are each modeled to a version of the secretary problem and solved.

2.2 Problem solution

Here, different scenarios of relay selection have been each modeled as a different version of the secretary problem. These scenarios are grouped into two categories, SRS and MRS.

2.2.1 Various SRS problems as different scenarios of secretary problem

In this section, we assume that only one candidate is going to be selected (SRS). We have also assumed a general case that the distance of the source-relay link is not equal to the distance of relay-destination link. In other words, the links are asymmetrical.

Unknown channel statistics

In this scenario, the channels statistics are unknown. We do not have any information about the distribution of the SNR and the symmetry of the channels. Such a scenario resembles a standard secretary problem in which the distribution of the quality of the candidates is unknown. Therefore, the SRS problem in this case can easily be modeled as a standard secretary problem and be solved with the solution to the standard secretary problem. The algorithm for solving this problem is shown in Alg. 1:

Algorithm 1 Secretary problem Algorithm for relay selection

```
value = max{SNR_eq1, SNR_eq2, ..., SNR_eq⌊n/e⌋}
select = 0
for n/e + 1 ≤ i ≤ n − 1 do
    if SNR_eqi ≥ value then
        Select relay i;
        select = 1
        break
    else
        Reject relay i
    end if
end for
if select == 0 then
    Select relay n
end if
```

According to Alg. 1, one should test the first n/e of the relays and remember the value of the maximum end-to-end SNR among these n/e relays then once we test other relays we need to compare their end-to-end SNR value to the best end-to-end SNR value among first n/e relays. We select any relay whose value is better than the best value among first n/e relays. Also, in case all the relays expect the last one are tested and none of them is selected, the last relay should be selected.

We will see that Alg. 1 finds a really good relay with fewer number of CSI measurements compared to the solution that tests all the relays. Moreover, a solution based on testing all relays needs to make the unrealistic assumption that CSI does not change otherwise, it won't be able to pick the relay with the maximum end-to-end SNR among all the n relays.

Known distribution

Here, we know that the average SNR of the link s-r is η times the average SNR of the link r-d ($\gamma_{sr} = \eta\gamma_{rd}$) and the distribution of the end-to-end SNR of the relays are known. Therefore, we can no longer use the standard secretary problem solution and we need to derive another solution for this problem. Let us define X as a random variable describing the end-to-end SNR of the relays. In fact, our goal is to maximize this utility through the relay selection process. Now let us assume that $n-2$ relays have been tested and none of them were selected and the second last relay (the $(n-1)^{th}$ relay) is going to be tested. The options for this relay are as follows:

- Test the $(n-1)^{th}$ relay and find its corresponding end-to-end SNR. Let us call this end-to-end SNR as X_1.

- Another option is to test the $(n-1)^{th}$ relay and decide not to select it. Here, based on our assumption since we have tested $n-1$ relays and did not make our selection, we are supposed to select the last relay without testing it. The utility of selecting the last relay is the same as the utility of selecting one relay randomly which is $E(X)$, the expected value of the end-to-end SNRs.

It becomes clear that our selection process has to be as follows:

$$\text{If} \quad X_1 > E(X) \implies \text{The } (n-1)^{th} \text{ relay is selected.} \tag{2.1}$$

$$\text{If} \quad X_1 < E(X) \implies \text{The last relay is selected.} \tag{2.2}$$

This means a selection threshold can be defined for the $(n-1)^{th}$ relay as $E(X)$. Now, we can define the utility of the $(n-1)^{th}$ relay as:

$$Y_1 = P(X_1 > E(X))E(X_1|X_1 > E(X)) + P(X_1 < E(X))E(X). \tag{2.3}$$

Now if we do the same procedure for the $(n-2)^{nd}$ relay, the options will be as follows:

- Test the $(n-2)^{nd}$ relay and decide to select this relay. Therefore, the end-to-end SNR of the system will be X_2.

- Test the $(n-2)^{nd}$ relay and decide not to select it and move on to the $(n-1)^{th}$ relay.

$$\text{If} \quad X_2 > Y_1 \implies \text{The } (n-2)^{nd} \text{ relay is selected.} \tag{2.4}$$

$$\text{If} \quad X_2 < Y_1 \implies \text{Move on to the } (n-1)^{th} \text{ relay.} \tag{2.5}$$

By comparing the above equations, one can find the threshold corresponding to the $(n-2)^{nd}$ relay as Y_1. Also, we can summarize the above equations and find the utility of the relay $(n-2)$ as below:

$$Y_2 = P(X_2 > Y_1)E(X_2|X_2 > Y_1) + P(X_2 < Y_1)Y_1. \tag{2.6}$$

As you can see, all the $n-1$ thresholds can be found through backward induction. Now, let us generalize the backward induction and derive a recursive formula for calculating the thresholds. Without loss of generality, we assume that the end-to-end SNR of each relay and their corresponding thresholds are indexed backward. Let R_i and R_{i+1} be the thresholds corresponding to the relays having X_i and X_{i+1} as their end-to-end SNRs respectively. Therefore, we can simply derive the recursive formula as below:

$$\begin{aligned} R_{i+1} = P(X_i > R_i)E(X_i|X_i > R_i) + P(X_i < R_i)R_i, \\ R_1 = E(X),\ 1 \leq i < n-1. \end{aligned} \tag{2.7}$$

The above equation can also be simplified as:

$$\begin{aligned} R_{i+1} &= \int_{R_i}^{\infty} f(x)dx \left(\frac{\int_{R_i}^{\infty} xf(x)dx}{\int_{R_i}^{\infty} f(x)dx} \right) + \left(\int_0^{R_i} f(x)dx \right) R_i \\ &= \int_{R_i}^{\infty} xf(x)dx + R_i \int_0^{R_i} f(x)dx. \end{aligned} \tag{2.8}$$

Here, $f(x)$ is the distribution of the end-to-end SNRs of the system. The way the relays, their corresponding thresholds and end-to-end SNRs are indexed is shown in Fig. 2.1 [32].

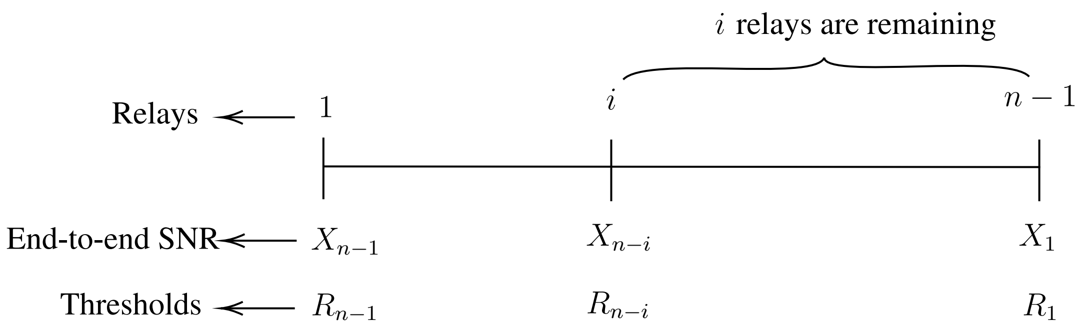


Figure 2.1: Indexing the known distribution scenario

In this scenario, if we assume that the gain of source-relay and relay-destination are Rayleigh distributed, their absolute value squared will be exponentially distributed. Therefore, the distribution of γ_{sr} and γ_{rd} are also exponential.

So according to the secretary problem in the case of known distribution, we need to find the distribution of the end-to-end SNR of the relays. For simplicity, we can approximate the SNR$_{eq}$ for the AF protocol and use Eqn. (1.4). Also, in the DF protocol, (1.5) can be used for the SNR$_{eq}$ without any approximation. Therefore, SNR$_{eq}$ for AF/DF relaying will have an exponential distribution [1] with $\lambda_{\text{SNR}_{eq}} = (\eta + 1)\lambda_{sr}$ in which $\lambda_{sr} = \frac{1}{\sigma^2 snr}$ and σ^2 is the variance of the source-relay and relay-destination gain and $SNR = \frac{P}{N}$ which is a fixed number.

By plugging in an exponential distribution with $\lambda_{\text{SNR}_{eq}}$ as $f(x)$ into (2.8), the recurrence relation for the thresholds corresponding to each relay would be as follows:

$$R_{i+1} = R_i + \frac{1}{(\eta+1)\lambda} e^{-\lambda(\eta+1)R_i}, \quad R_1 = \frac{1}{(\eta+1)\lambda} \tag{2.9}$$

This means the end-to-end SNR of each relay is compared to its corresponding threshold. And if the end-to-end SNR is larger than the correspond-

[1]The minimum of two independent exponential random variables with parameters λ_1 and λ_2 is another exponential random variable with $\lambda = \lambda_1 + \lambda_2$

ing threshold, that relay is chosen; otherwise, we move on to test the next relay.

2.2.2 Various MRS problems as different scenarios of secretary problem

In this section, the original secretary problem is extended to the selection of more than one relay.

Unknown distribution

Here we are going to select $k > 1$ relays among n relays. In this scenario, the distribution of the random quality of the candidates are unknown and we do not have any information about the distribution of the end-to-end SNR of the relays. The achievable rate of such a system is shown in Eqn. (1.7). According to (1.7), in order to maximize the achievable rate one needs to maximize the sum of the SNR_{eq}. Therefore, in order to model the MRS problem as a version of multiple selection in secretary problem, one needs to look for a version that aims to maximize the sum of the payoffs.

In [40], a secretary algorithm for choosing k elements from n items trying to maximize the sum of the payoffs is proposed. The main idea in this algorithm is to group the candidates. The problem of selecting k candidates among n number of candidates can be decomposed to two sub problems. In the first sub problem, one should pick up $\lfloor k/2 \rfloor$ of the candidates among the first candidate until the $(m = B(n, 1/2))$th [2] candidate and in the second sub problem, one should pick up the remaining ones $(k - \lfloor k/2 \rfloor)$ among the $(m+1)$th candidate until the last candidate. To solve the first sub problem, it can be further decomposed to sub problems recursively until it is reduced to a problem of size 1, where at this stage Alg. (1) can be used to select one candidate. To solve the second sub problem, the selection of the $k - \lfloor k/2 \rfloor$ candidates is done compared to the maximum score one have seen among the first candidate until the $(m = B(n, 1/2))$th candidate.

[2] m is a random sample from a binomial distribution

As an example, for selecting $k = 5$ candidates among $n = 50$ candidates, this problem reduces to two sub problems: the first one is selecting $\lfloor 5/2 \rfloor = 2$ candidates out of $m_1 = B(50, 1/2) = 21$ candidates and the second one is selecting $5 - \lfloor 5/2 \rfloor = 3$ candidates among the $(m_1 + 1 = 22)$th candidate until the last candidate ($n = 50$). In the next step, the first problem is reduced to selection of $\lfloor 2/2 \rfloor = 1$ candidates out of $m_2 = B(m_1, 1/2) = 12$. Now that the algorithm has reduced to the problem of selecting one candidate, we can apply the standard secretary solution to select one candidate among the first $m_2 = 12$ candidates. Also, out of the first $m_1 = 21$ candidates we were supposed to select two candidates and we have already selected one of them. Therefore, we have to select another one from $m_2 + 1 = 13$ to $m_1 = 21$ and this selection is done compared to the maximum score we have seen so far. Now for solving the second problem, the remaining three candidates are selected from the $(m_1 + 1)$th to nth candidates, whose test scores are greater than the maximum test score we have seen so far within the range of first m_1 candidates. The pseudo Algorithm of this method is shown in Alg. 2. The proof of this algorithm is shown in [40].

We can easily apply this algorithm to MRS scenario in which the distribution of the random quality of the relays are unknown. In this problem, the candidates are the relays and the test score of the candidates are the end-to-end SNRs of the relays.

Known distribution

Here, we know that the SNR of link s-r is η times the SNR of the link r-d and the distribution of the end-to-end SNR of the relays are known. And k relays are going to be selected. The aim is to select the relays that can maximize the achievable rate of the system. This is equal to maximizing the sum of the end-to-end SNRs of the relays.

For ease of discussions, let us first solve this problem for the case that $k = 2$. In this case, the total end-to-end SNR is the sum of the end-to-end SNRs of the two relays. Now let S_n be the utility of choosing two relays among n relays. As a simple example, when $n = 2$ and $k = 2$, then the two relays

Algorithm 2 Secretary problem Algorithm for selection of k candidates among n candidates

s $\longrightarrow$ Number of Binomial samples
f $\rightarrow$ Final step.
$M = [m_0, m_1, m_2, \ldots, m_s] \rightarrow$ Binomial samples.
$s = 0 \rightarrow s$ is initialized to zero.
while $k > 1$ **do**
  $k = \lfloor k/2 \rfloor$
  $s = s + 1$
end while
$M(0) = n \rightarrow m_0$ is initialized to n
for $1 \leq i \leq s$ **do**
  $M(i+1) = B(M(i), 1/2)$
  B is Binomial distribution
end for
1) Apply original secretary problem for candidates 1 to m_s. The test result of the selected applicant is called y_l
2)
for $m_s \leq i \leq m_{s-1}$ **do**
  if $y_i > y_l$ **then**
    Select the y_i as the second applicant $y_l = y_i$
  end if
end for
  $\vdots$
s) Find the s^{th} applicant in the range m_2 to m_1 the same as step 2
f) Within the range m_1 to n select the remaining $n - s$ applicants that their rank exceeds y_l

must be selected and their utility $S_2 = 2E(X)$, where $E(X)$ is the average of the distribution of the end-to-end SNR of the relays. The decision tree for $n = 3$ and $k = 2$ is shown in Fig. 2.2. Here X_3 is the the end-to-end SNR for the first selection opportunity and X_1 is that for the last. Once one chooses the first relay, then the problem changes to choose-one-relay scenario.

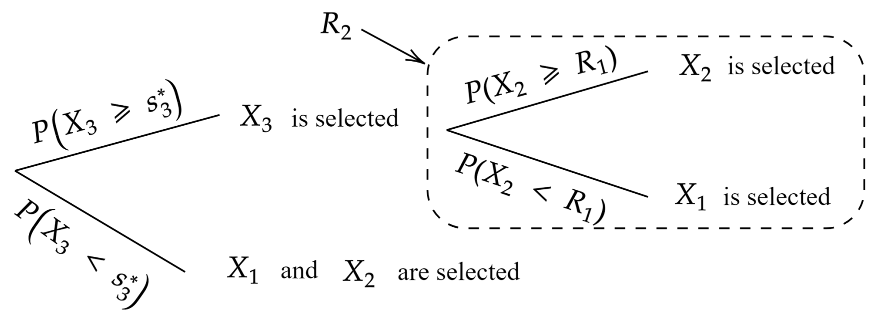


Figure 2.2: Decision tree for the case that $k = 2$ and $n = 3$.

When the end-to-end SNR X_3 is presented we accept it, if it exceeds or equals some number s_3, otherwise we reject it. We must find the optimum value s_3^* of s_3. If we take X_3, we have left with two more opportunities and one choice, and we already know that R_2 is the gain to be expected. Therefore, we can write the expected value for s_3 as below:

$$\begin{aligned} S_3(s_3) &= P(X_3 \geq s_3)[E(X_3|X_3 > s_3) + R_2] + [1 - P(X_3 \geq s_3)]S_2 \\ &= S_2 + \int_{s_3}^{\infty} x f(x)dx - (S_2 - R_2) \int_{s_3}^{\infty} f(x)dx, \end{aligned} \quad (2.10)$$

where, $f(X)$ is the distribution of the end-to-end SNRs of the relays.

Now, if we get a derivative with respect to s_3, we can maximize $S_3(s_3)$

$$\frac{dS_3(s_3)}{ds_3} = -s_3 f(s_3) + (S_2 - R_2) f(s_3). \quad (2.11)$$

Setting the derivative equal to zero, we have the optimizing value of s_3, as

$$s_3^* = S_2 - R_2, \quad (2.12)$$

$$S_3 = S_2 + \int_{s_3^*}^{\infty} x f(x)dx - (S_2 - R_2) \int_{s_3^*}^{\infty} f(x)dx. \quad (2.13)$$

If we extend this scenario to n relays, we will have

$$s_n^* = S_{n-1} - R_{n-1}, \tag{2.14}$$

$$S_n = S_{n-1} + \int_{s_n^*}^{\infty} xf(x)dx - (S_{n-1} - R_{n-1}) \int_{s_n^*}^{\infty} f(x)dx. \tag{2.15}$$

This scenario can be extended to three choice problem. Then in this scenario, we will have T_n which is the same as S_n for two choice problem. Here, $n \geq 3$ and $T_3 = 3E(X)$. t_n is the threshold for the first choice:

$$t_n^* = T_{n-1} - S_{n-1} \tag{2.16}$$

$$T_n = T_{n-1} + \int_{t_n^*}^{\infty} xf(x)dx - (T_{n-1} - S_{n-1}) \int_{t_n^*}^{\infty} f(x)dx \tag{2.17}$$

We can extend this scenario to four choice problem. Then, F_n is the utility of selecting four relays among n relays. We can easily have a recursive formula for F_n [32].

$$f_n^* = F_{n-1} - T_{n-1} \tag{2.18}$$

$$F_n = F_{n-1} + \int_{f_n^*}^{\infty} xf(x)dx - (F_{n-1} - T_{n-1}) \int_{f_n^*}^{\infty} f(x)dx \tag{2.19}$$

This scenario can be extended to k choice problem easily.

2.2.3 SRS with Recall

In all the previous parts, there was an assumption that recall is not allowed. In other words, an applicant once rejected cannot be recalled. However, in this part we want to allow limited recalls. This is a type of secretary problem that at each stage, each of the last m applicants that were interviewed are available for employment and the aim is to select the best applicant. In the relay selection problem, if CSI does not change too fast, some of the recently tested relays can still be used without worrying that their CSI is changed. Therefore, this type of secretary problem is similar to an SRS problem in which the CSI of the last m channels are feasible. The algorithm to select a relay among n relays when the memory is m, is shown in Alg. 3.

The idea to solve this problem is to stop testing in a stage in which the relatively best applicant is about to become unavailable [43]. Let us call this stage as r^*. For a problem in which one candidate needs to be selected among the n applicants and the memory is m, one needs to test the first r^* candidates. Then for any candidate k after r^*, if after testing, the best candidate so far (among all tested candidates) is the one at the leftmost position of the selection window $[k-m+1, k-m+2, ..., k]$, or equivalently, the best candidate so far is candidate $k-m+1$, we stop and pick up candidate $k-m+1$. If we do not stop until the last candidate, we pick up the last candidate. Here, k represents the number of the candidates that have been tested so far. The proof and the detailed algorithm to find the r^* is discussed in [43]. We have brought the table for the r^* corresponding to each m (Table 2.1). For other parameters of this table, refer to the Table on page 5 of [43].

Algorithm 3 SRS when recall is allowed with memory m

```
[index, value] = max{SNR_eq_1, SNR_eq_2, ..., SNR_eq_r*}
select = 0
for r* + 1 ≤ i ≤ n − 1 do
  if SNR_eq_i > value then
    value = SNR_eq_i
    index = i
  else
    if i − m + 1 == index then
      Relay i − m + 1 is selected.
      select = 1
      break
    end if
  end if
end for
if select == 0 then
  select the last relay
end if
```

m	r*	m	r*
n = 50		n =100	
5	21	10	43
10	24	20	47
15	25	30	49
20	25	40	50

Table 2.1: Recall table for different size of memory m, r^* and n.

2.3 Results

In this section we evaluate the secretary-based relay selection methods in terms of the achievable rate of the network versus SNR.

Simulation results are shown for two relaying protocols AF and DF. Channel distributions are assumed to be Reyleigh. The results have been tested for two cases of known and unknown distributions for number of relays $n = 15$ and $n = 50$ and the secretary based method is compared to two other methods: exhaustive search and random selection. The exhaustive search tests all the relays and then decides which relay(s) is/are the best. It assumes that the CSIs do not change, so when it picks the best relay, the best CSI is still available for that relay. In many practical setting, this assumption is not valid. Hence, exhaustive search results are included as a benchmark only. The random approach selects a relay(s) randomly without doing any tests. The range of the SNR that has been tested is 0-40 dB. These methods are all grouped and tested for both SRS and MRS scenarios.

2.3.1 SRS methods results

Here, all the simulations were done in order to select one relay among n relays. The simulations were done for two cases where the distribution of the channels are once unknown and once known.

Unknown distribution

The original secretary problem has been simulated for two relaying protocols. We have assumed that the relays have been placed exactly in the middle of the source and the destination, i.e.,$\gamma_{sr} = \gamma_{rd}$. The simulations are shown in Fig. 2.3 and Fig. 2.4. As depicted in Fig. 2.3, when comparing the secretary approach of part a to that of the random approach, there is a 5 dB improvement in order to achieve 4 b/s/Hz (bits per second per hertz). One can see that when the number of relays is larger the secretary approach's performance improves both in terms of the achievable rate and the percentage of relays tested.

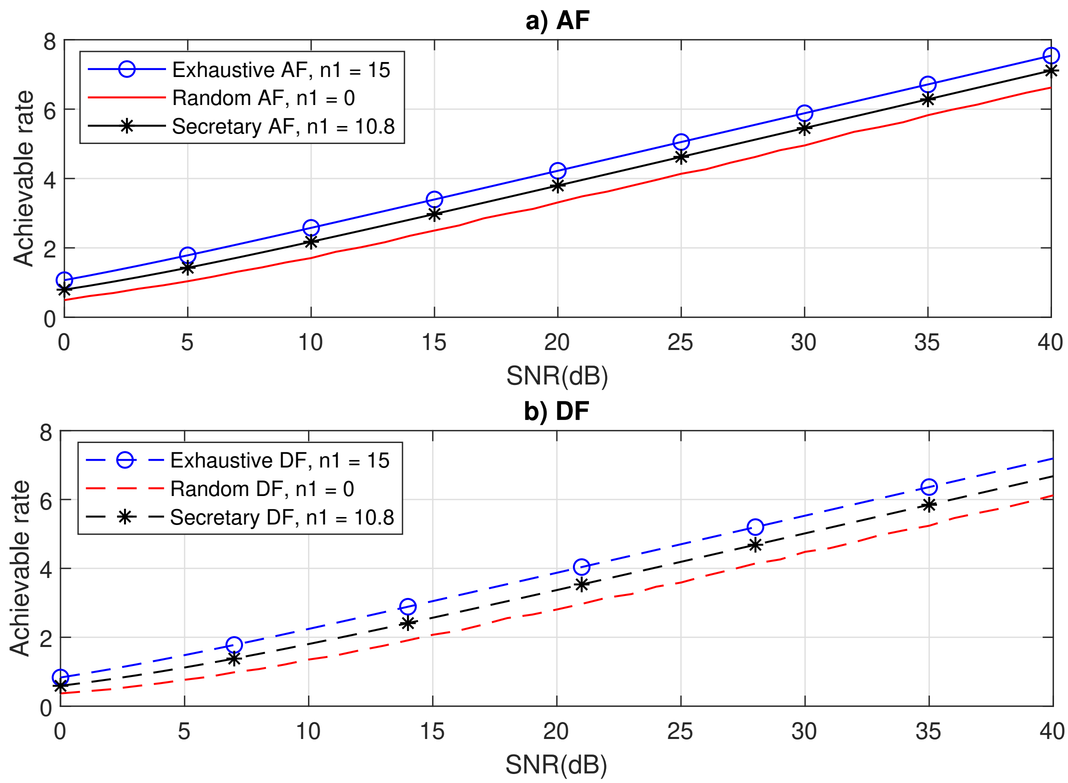


Figure 2.3: SRS scenario for the case of $n = 15$ relays when the channel distribution is unknown. a) AF relaying b) DF relaying. n_1 is the average number of tests needed to select a relay.

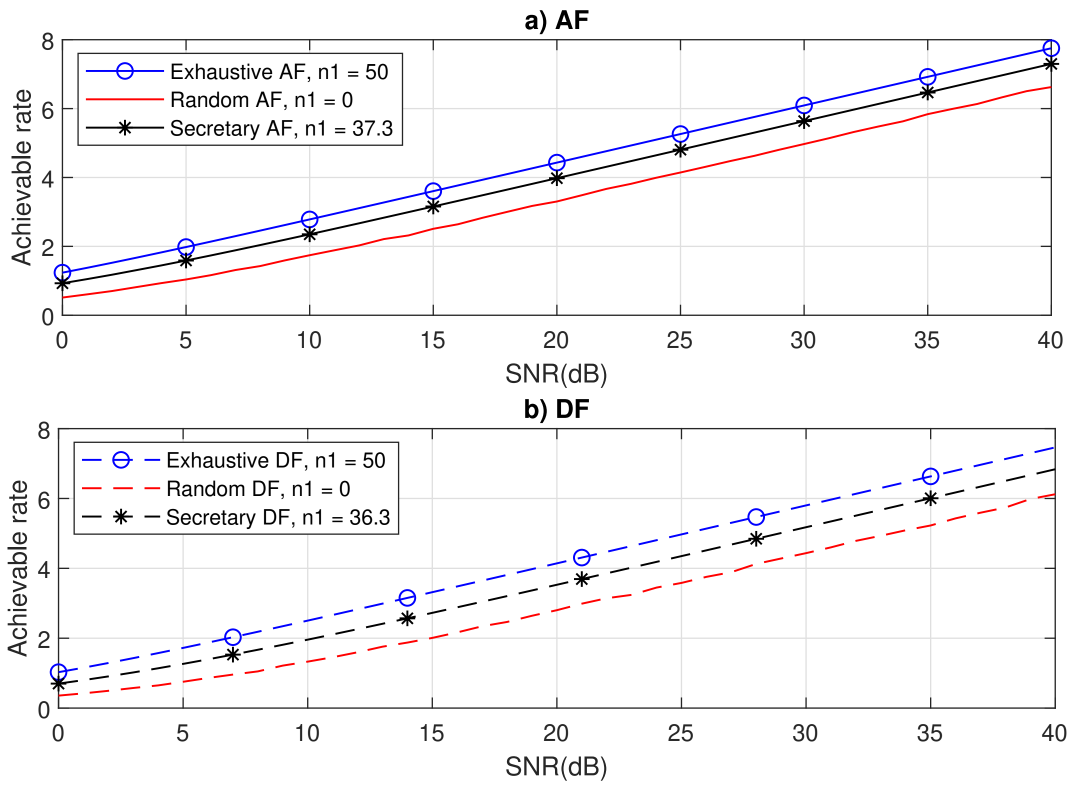


Figure 2.4: SRS scenario for the case of $n = 50$ relays when the channel distribution is unknown. a) AF relaying b) DF relaying. n_1 is is the average number of tests needed to select a relay.

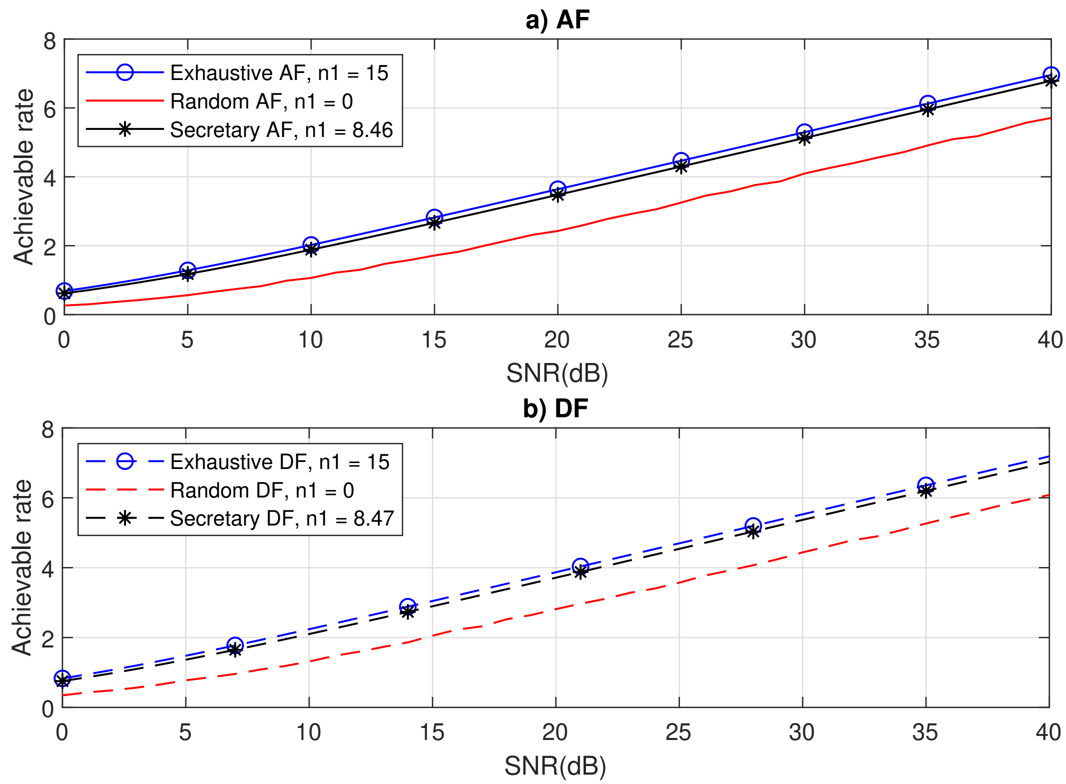


Figure 2.5: SRS scenario for the case of $n = 15$ relays that the distribution of the relays are known. a) AF relaying b) DF relaying. n_1 is is the average number of tests needed to select a relay.

Known distribution

The set up of the relays is similar to the set up of the previous part. Also, the distribution of relays are known so the secretary approach would be the same as subsection 2.2.1 that requires finding $n - 1$ thresholds according to the distribution of the channels in each setup.

The simulations were done for the case that all the links are having Reyleigh distribution. The results are depicted in Fig. 2.5 and Fig. 2.6. As shown in Fig. 2.6 part a and b, the secretary approach almost achieves the ultimate performance possible, depicted by the exhaustive search. Also, in part a of Fig. 2.6, there is almost 10 dB improvement in order to achieve rate 4 b/s/Hz when comparing the secretary approach to the random approach. By comparing the results of known channel distribution to the results of unknown

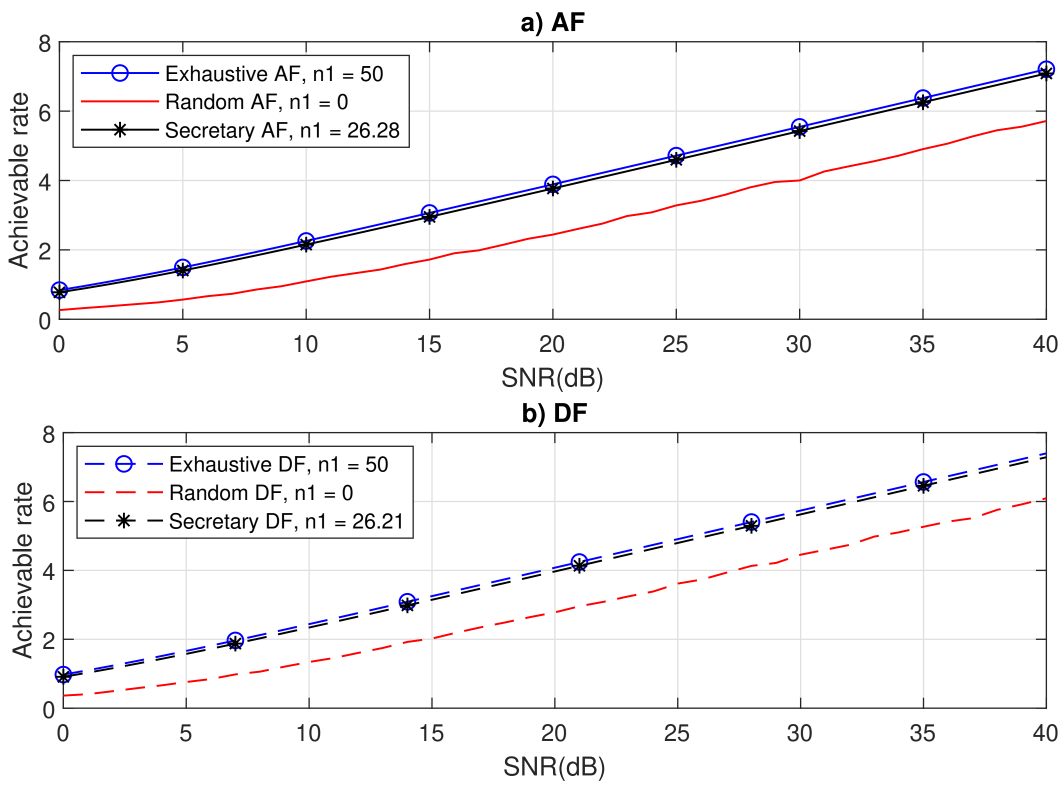


Figure 2.6: SRS scenario for the case of $n = 50$ relays that the channel distribution is known. a) AF relaying b) DF relaying. n_1 is is the average number of tests needed to select a relay.

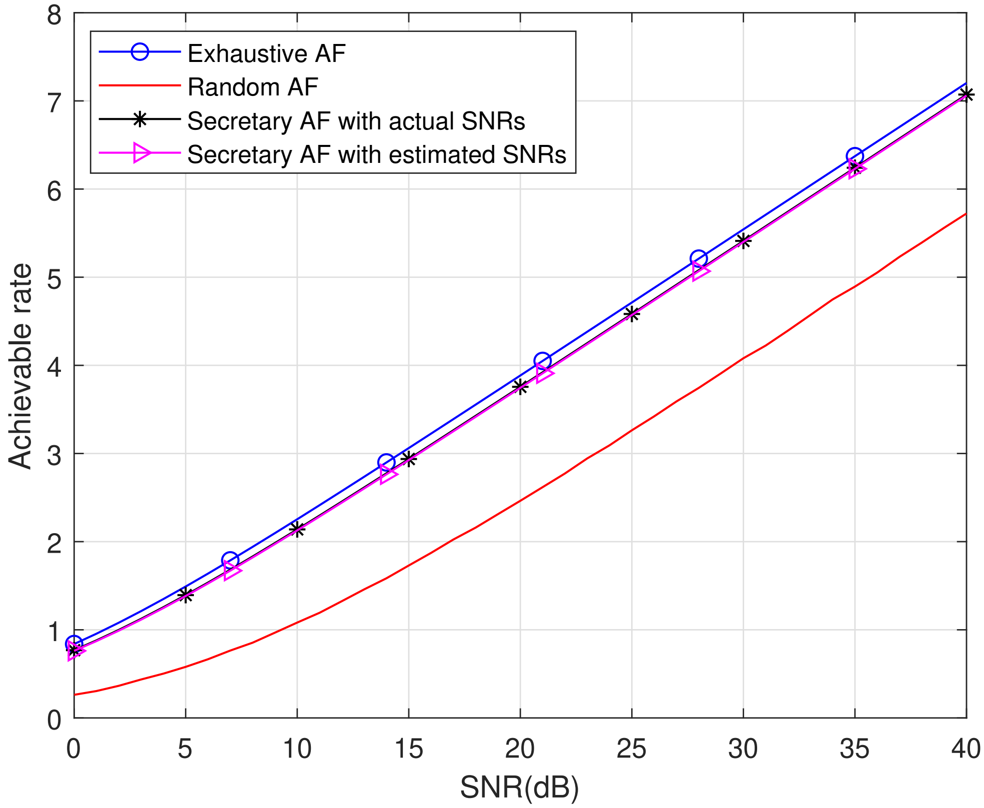


Figure 2.7: Robustness of secretary based SRS scenario for the case of $n = 50$ relays that the channel distribution is known.

channel distribution, one can see that not only the performance in terms of the achievable rate is improved but also the number of tests in order to achieve this performance is reduced. Hence, knowing the channel distribution significantly helps the secretary approach and allows it to approach the ultimate performance.

In Fig. 2.7, we have conducted another experiment to test the robustness of the SRS algorithm when the distribution of the gain of the channels is known. In this experiment, the secretary algorithm is applied on a case where there is a 3 dB mismatch between the estimated and actual SNR (pink graph). As you can see, even with 3 dB mismatch, the results are still promising.

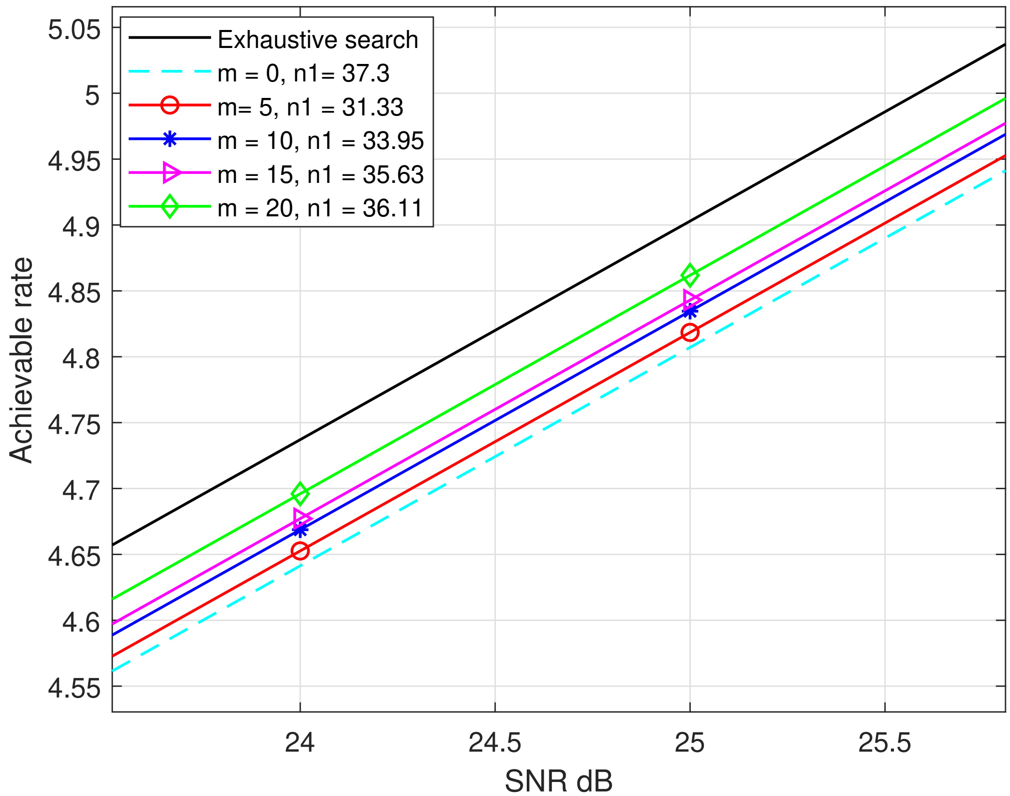


Figure 2.8: SRS scenario for the case of $n = 50$ relays where recall with memory m is allowed. n_1 is is the average number of tests needed to select a relay.

SRS with recall

Here, we have assumed that recall with memory m is allowed. In other words, at each stage of testing the relays, each of the last m relays are available to be selected. The simulation results were done for the case where there are $n = 50$ relays, the relaying protocol is AF. In order to do the simulations, we have used Table 2.1 to find the r^* corresponding to each memory size. The results are shown in Fig. 2.8. As you can see, when the memory size increases the performance gets better and the gap between to the exhaustive search gets smaller.

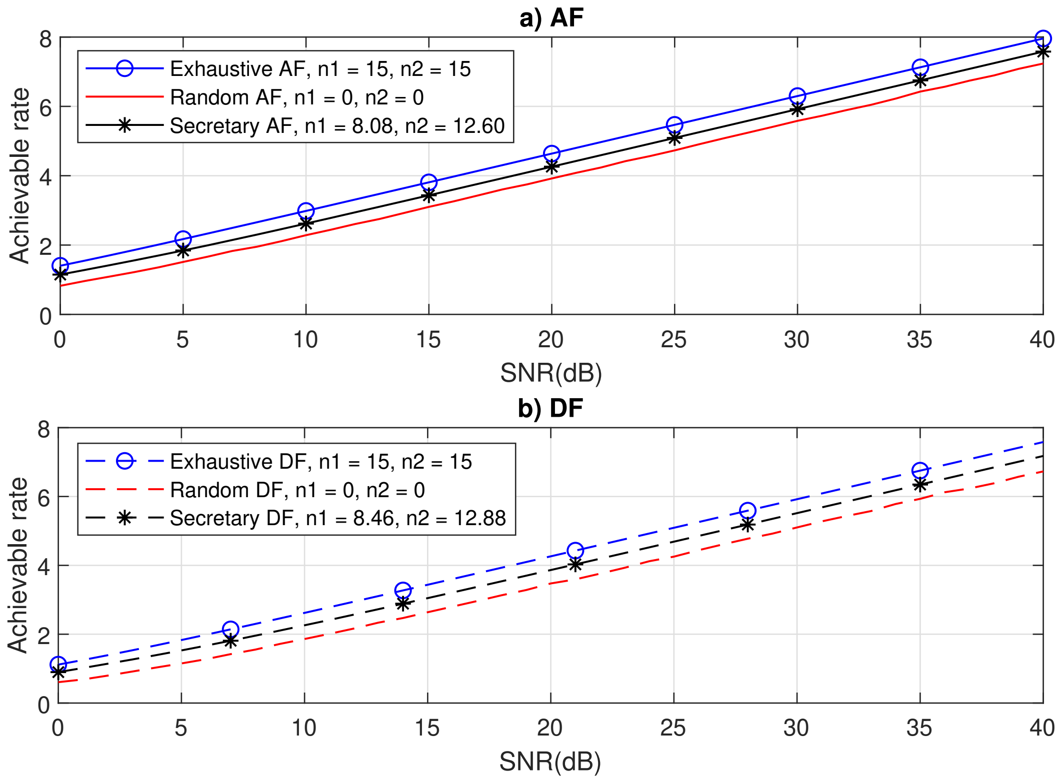


Figure 2.9: MRS scenario for the case of $n = 15$ relays with unknown channel distribution. a) AF relaying b) DF relaying. n_1 and n_2 are the average number of tests to select the best and the second best relays.

2.3.2 MRS methods results

The simulations of this section were done in order to select two or more relays. For simplicity, we have brought the simulations for selecting two cooperative relays among n relays.

Unknown distribution

The distribution of the relays are unknown. We have assumed that the relays are placed in the middle of the source and the destination. Fig. 2.9 and Fig. 2.10 show the results for $n = 15$ and $n = 50$ respectively. According to Fig. 2.10 part b, there is a 4 dB improvement in order to achieve rate 4 b/s/Hz when comparing the secretary approach to the random approach.

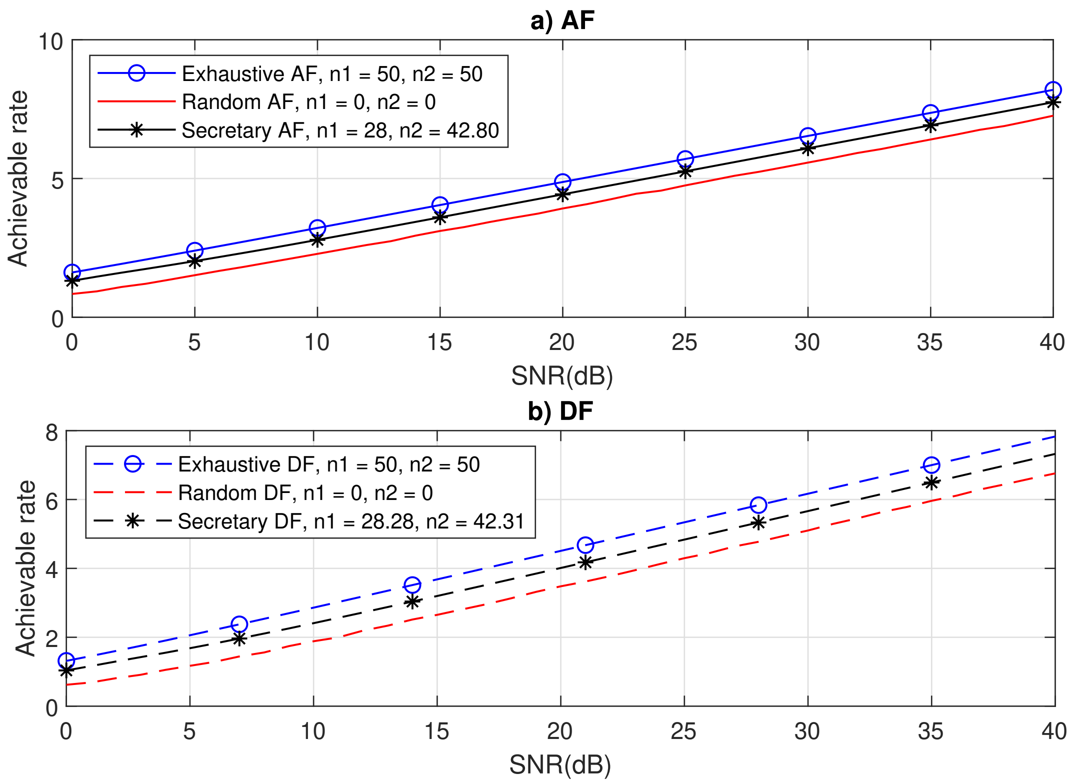


Figure 2.10: MRS scenario for the case of $n = 50$ relays with unknown channel distribution.. a) AF relaying b) DF relaying. n_1 and n_2 are the average number of tests to select the best and the second best relays.

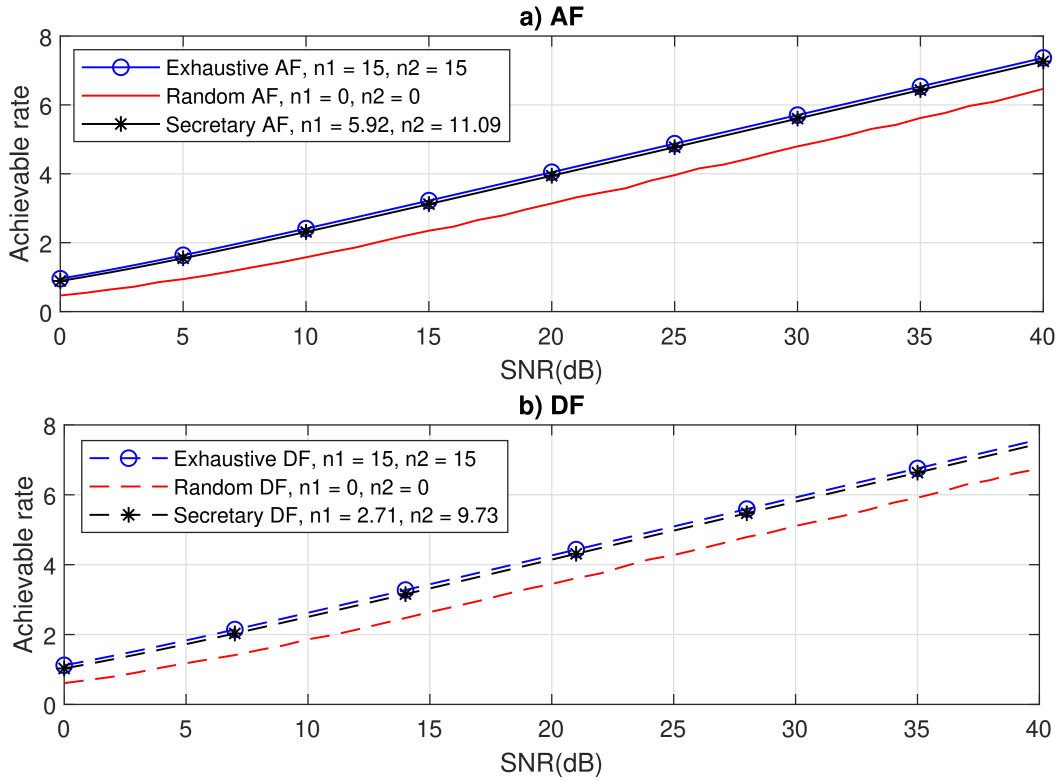


Figure 2.11: MRS scenario for the case of $n = 15$ relays with known channel distribution. a) AF relaying b) DF relaying. n_1 and n_2 are the average number of tests to select the best and the second best relays.

Known distribution

The set up is similar to the previous part. Since the distribution is known the approach is the same as section 2.2.2. The simulations were done for the case that all the links are having Reyleigh distribution. The results are depicted in Fig. 2.11 and Fig. 2.12. By comparing the results of this part to the case of unknown channel distribution, one can see that not only the performance in terms of achievable rate is better but also the number of tests in order to achieve this performance is smaller. In other words, knowing the channel distribution significantly helps the secretary approach and allows it to approach the performance of the exhaustive search.

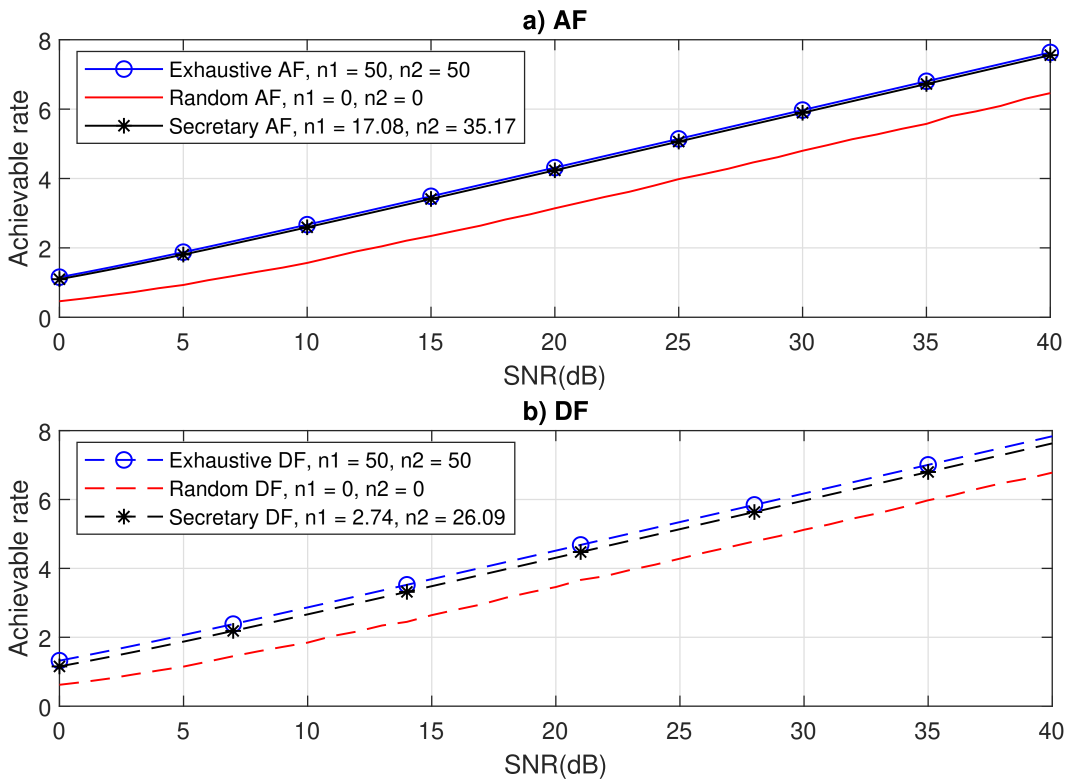


Figure 2.12: MRS scenario for the case of $n = 50$ relays when channel distribution is known. a) AF relaying b) DF relaying. n_1 and n_2 are the average number of tests to select the best and the second best relays.

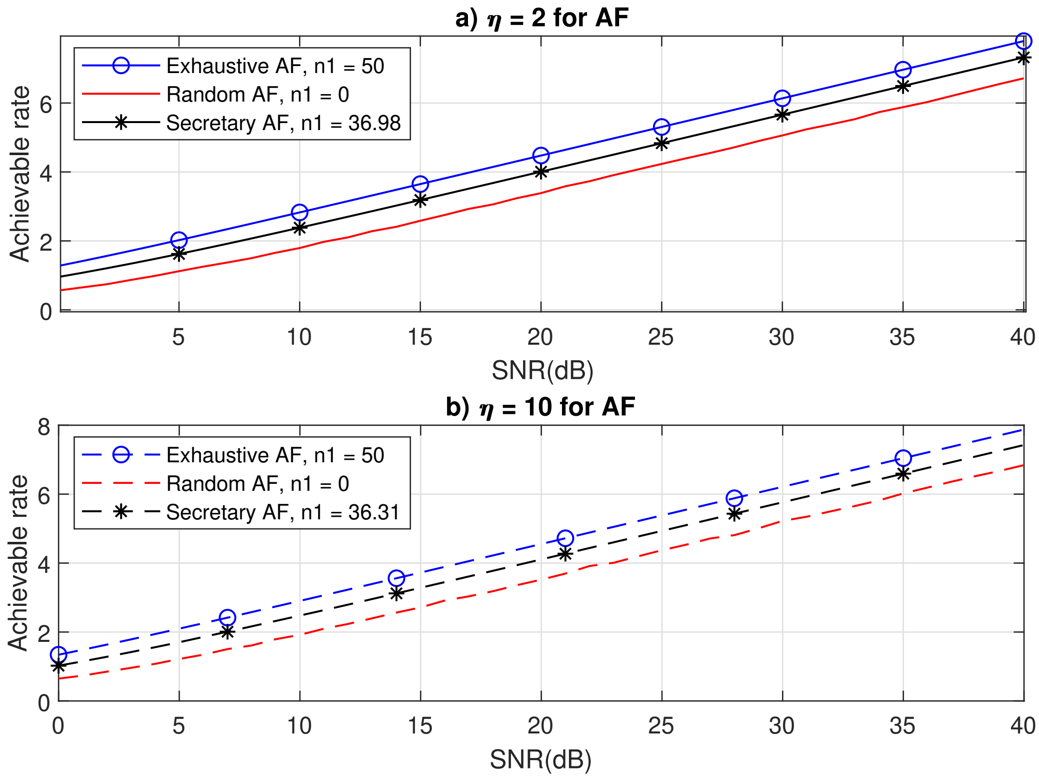


Figure 2.13: SRS scenario for $n = 50$ relays when channel distribution is unknown and the links are asymmetrical. η is the asymmetrical parameter.

2.3.3 Asymmetric links results

All the previous results are for the case that the source-relay and relay-destination links are symmetrical ($\eta = 1$). Here, we have assumed that the links are asymmetrical and ($\eta \neq 1$). Since we have discussed different scenarios in the previous parts and the results for the ASL (asymmetric link) is similar to the previous parts, here, we have brought two results that are shown in Fig. 2.13 and Fig. 2.14. The results show that secretary approach is also working for the cases that the links are asymmetrical.

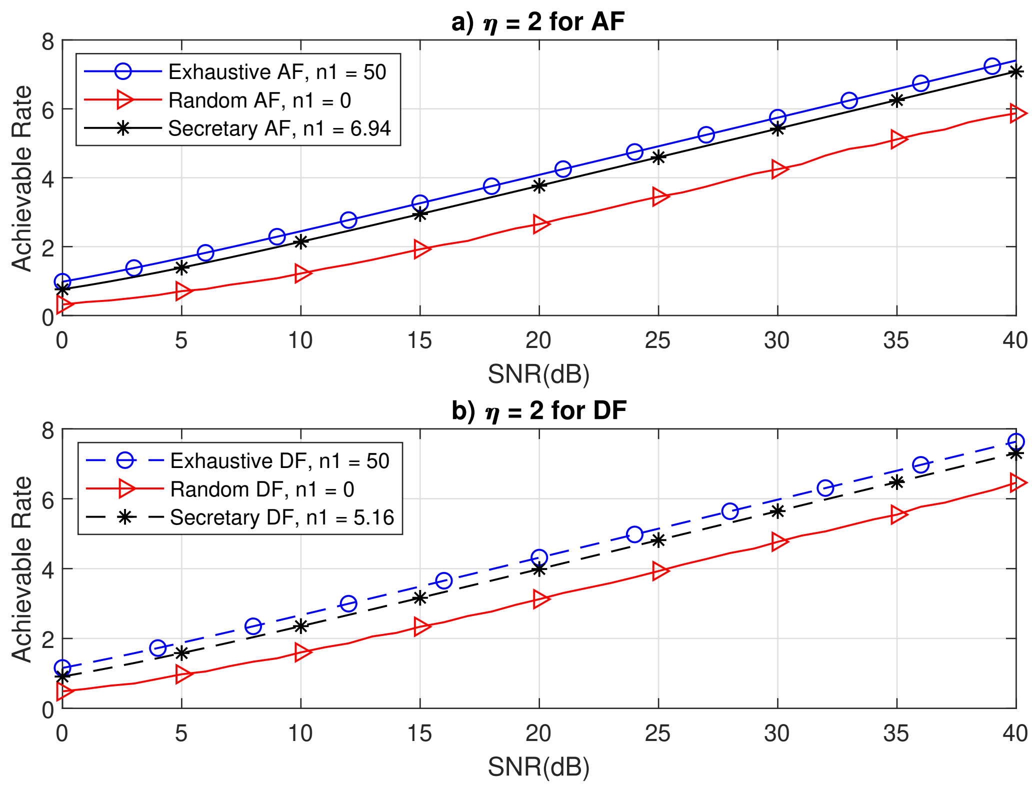


Figure 2.14: SRS scenario for $n = 50$ relays when channel distribution is known and the links are asymmetrical. η is the asymmetrical parameter.

Chapter 3

Relay Selection Over Wireless Networks: Inspired by the Secretary Problem when testing is costly

This chapter proposes modeling the relay selection problem as an stopping rule problem for the case where testing is costly. Furthermore, this chapter proposes a novel hybrid algorithm to solve this problem. These algorithms were derived in order to maximize the overall achievable rate of the system. The results were derived for both SRS and MRS scenarios and were shown based on the overall achievable rate versus SNR.

3.1 Problem definition

Here, the aim is to select k relays among n relays in a way that the overall achievable rate of the system given in (1.9) is maximized. Once each relay is tested, the end-to-end SNR corresponding to that relay is found and its corresponding achievable rate is obtained. Then, we need to decide whether we want to select that relay or move on to the next relay. Since the CSI of the channels get outdated, if we do not select the relay and move forward, this decision cannot be revoked at a later time.

The above problem resembles an optimal stopping rule problem called the secretary problem. The standard secretary problem is defined as below: There are n candidates for a secretarial position available. The candidates are interviewed one by one sequentially in random order, and a decision for each candidate needs to be made on the spot. Once rejected, that candidate cannot be recalled. The goal is to maximize the probability of selecting the best candidate [31].

As you see, the above secretary problem is similar to a relay selection problem in which the relays are the candidates. There are some assumptions for each of these problems. The similarities and the differences of these assumptions are as follows:

3.1.1 Similarities

There are some assumptions in the standard secretary problem that can be applied to the relay selection problem as well:

- The number of candidates is known.
- The candidates are tested sequentially in random order.
- An applicant, once rejected, cannot be recalled.
- In case $n-1$ candidates are tested, and all have been rejected, the last candidate is hired [31].

3.1.2 Differences

There are also some assumptions in the standard secretary problem that are not generally true for the relay selection problem.

- There is only one secretarial position available. However, in relay selection we are going to select k relay(s) ($1 \leq k \leq n$).
- Interviewing each candidate does not involve any costs. However, as defined in the overall achievable rate, testing the relays is costly (cost α).
- The secretary problem tries to maximize the chance of success. However, we need to maximize the overall achievable rate.

- Once a candidate is interviewed and been rejected, the next candidate should be interviewed until a candidate is selected. In other words, the algorithm cannot be interrupted in the middle, and the selected candidate needs to be interviewed. This assumption does not really apply to the relay selection problem. Due to the cost of testing, there may occur a situation in which, after testing some relays, the cost gets very high that it is worth choosing a relay randomly without testing rather than continuing testing. In other words, the testing may be interrupted.

Secretary problem has various versions discussed above. However, in all of them, the algorithm cannot be interrupted in the middle. Due to this reason and the reasons discussed above, we have come up with a hybrid algorithm that is a modified version of the secretary problem to solve the relay selection problem.

In the next section, we will discuss the modified secretary problem algorithm.

3.2 Problem solution

In this section, we will first develop an algorithm to solve the problem for SRS and then use this algorithm to solve the MRS problem. We have assumed that the statistics of the channel gains are known and have modified the algorithm of the secretary problem to fit a relay selection problem so that the testing cost for each relay will also be included in the scenario of selection. We will discuss this scenario for simplicity for both SRS and MRS with two common relaying protocols AF and DF.

3.2.1 Single relay selection

Here, we are going to select one relay. In the standard secretary, each relay is first tested and then a decision whether to select or reject that relay is made. In our setup, because of cost, there is a third option, i.e., selecting a relay without even testing it. Our goal is to maximize a utility function which is the overall achievable rate of the system. We have three options when we move on to each relay:

- We test that relay and select it;
- We test that relay and decide not to select it and decide to move on to the next relay;
- We do not test that relay and decide to select it.

We have also assumed that in case $n-1$ relays have been tested, and we have not made our selection yet, the last relay will be selected without testing. To develop our selection algorithm, we note that if we have rejected the first $n-2$ relays and have reached to the $(n-1)^{th}$ relay, there are three options:

- One option is to test the $(n-1)^{\text{th}}$ relay and decide to select it. Now, let us assume that the achievable rate of the relay $n-1$ is called X_1, so in case we select that relay, its overall achievable rate is $\frac{X_1}{1+(n-1)\alpha}$.
- The second option is to test that relay and decide not to select. Here, since we have tested all the $n-1$ relays and did not make our selection, we should select the last relay without testing it. Therefore, the utility of this selection is $\frac{E(X)}{1+(n-1)\alpha}$, where $E(X)$ is the expected value of the achievable rate of the selected relay.
- We do not test the $(n-1)^{\text{th}}$ relay and select it which means that $n-2$ tests have already been done to get to this stage. Therefore, the utility of this option is $\frac{E(X)}{1+(n-2)\alpha}$.

Now let us compare the achievable rates for the first and the second options above:

If $\frac{X_1}{1+(n-1)\alpha} > \frac{E(X)}{1+(n-1)\alpha}$ i.e., if $X_1 > R_1 \triangleq E(X)$, the $(n-1)^{th}$ relay is selected.
If $\frac{X_1}{1+(n-1)\alpha} < \frac{E(X)}{1+(n-1)\alpha}$ i.e., if $X_1 < R_1$, the last relay is selected.

As you can see from the two above equations, we can define a threshold for the $(n-1)^{th}$ relay as $E(X)$ and use this threshold to decide whether we should take the $(n-1)^{th}$ relay or not. Let us summarize the two above equations and define the expected reward (utility) Y_1 if we test the $(n-1)^{\text{th}}$ relay. Based on what was discussed,

$$Y_1 = E[\max\{\frac{X_1}{1+(n-1)\alpha}, \frac{E(X)}{1+(n-1)\alpha}\}]. \tag{3.1}$$

Another comparison is for the third option (the option that does not test the $(n-1)^{th}$ relay). This comparison show that the selection process must be such that:

If $\frac{E(X)}{1+(n-2)\alpha} \geq Y_1$, then we should select the $(n-1)^{th}$ relay without testing it.

If $\frac{E(X)}{1+(n-2)\alpha} < Y_1$, then we move on to test the $(n-1)^{th}$ relay.

To summarize, if the first $(n-2)$ relays are rejected, the selection process discussed above gives expected reward:

$$Z_1 = \max\{\frac{E(X)}{1+(n-2)\alpha}, Y_1\}. \tag{3.2}$$

What we did for the $(n-1)^{th}$ relay can be extended to other relays and form the optimal selection algorithm. To see how, let us do the same procedure for the $(n-2)^{nd}$ relay. This relay has also three options:

- One option is to test the $(n-2)^{nd}$ relay and select that relay. Let us denote the achievable rate of this relay as X_2, so the overall achievable rate of this relay will be $\frac{X_2}{1+(n-2)\alpha}$.

- The second option is to test that relay and decide not to select. This means we should move on to the $(n-1)^{th}$ relay, with expected reward Z_1.

- We decide to select the $(n-2)^{nd}$ relay without testing it. Since we have already tested $n-3$ relays. The utility of this option is $\frac{E(X)}{1+(n-3)\alpha}$.

Now let us compare the achievable rate for the two options where we test the $(n-2)^{th}$ relay to find a decision threshold for this relay.

If $\frac{X_2}{1+(n-2)\alpha} \geq Z_1$ i.e., if $X_2 \geq R_2 \triangleq Z_1(1+(n-2)\alpha)$, then the $(n-2)^{nd}$ relay is selected.

If $\frac{X_2}{1+(n-2)\alpha} < Z_1$ i.e., if $X_2 < R_2$, then move on to the $(n-1)^{th}$ relay.

We can summarize the two equations and define the expected reward when we move on to test the $(n-2)^{nd}$ relay as follows:

$$Y_2 = E[\max\{\frac{X_2}{1+(n-2)\alpha}, Z_1\}]. \tag{3.3}$$

Again, another comparison is with the third option (the option that does not test the $(n-2)^{th}$ relay) and the expected reward of the two other options that test the relay. Accordingly, the expected reward of the $(n-2)^{nd}$ relay is given as:

$$Z_2 = \max\{\frac{E(X)}{1+(n-3)\alpha}, Y_2\}. \tag{3.4}$$

The decision tree of the above scenario for n relays is shown in Fig. 3.1. As

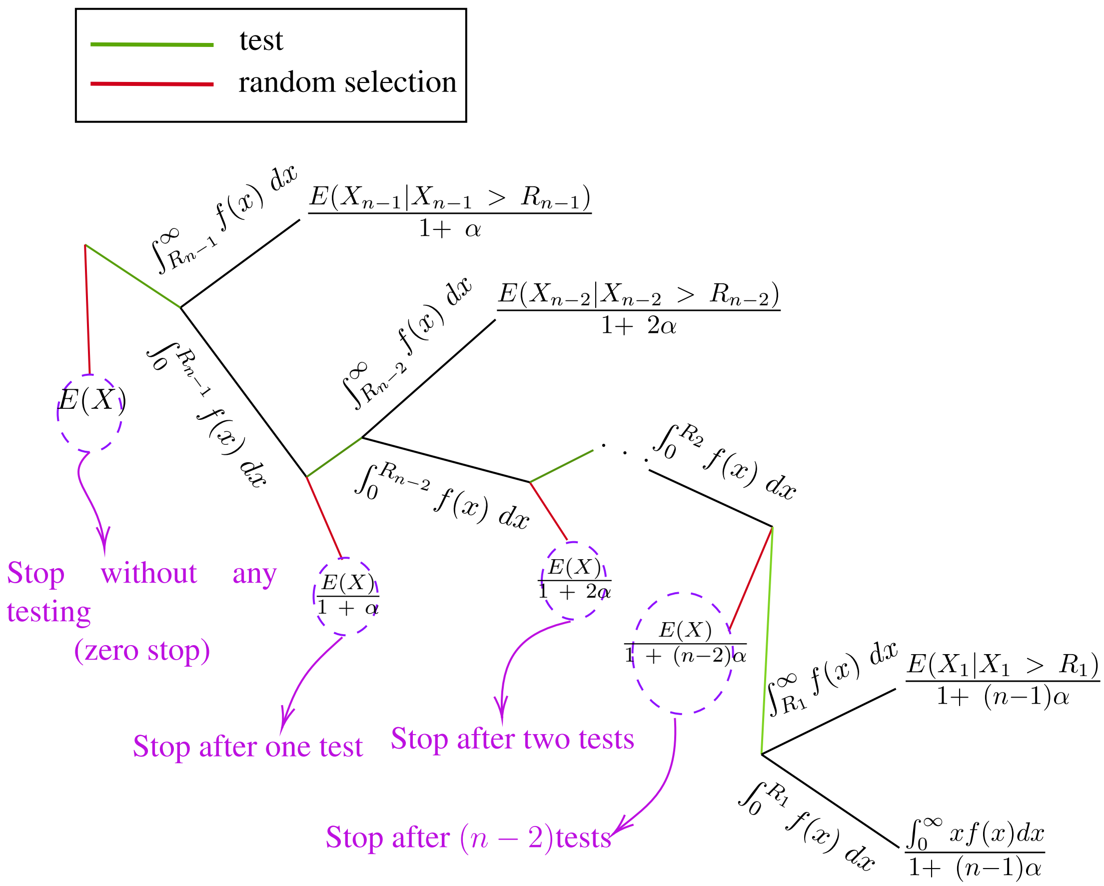


Figure 3.1: Decision tree for SRS based on secretary approach

you can see, we are dealing with two types of decisions (thresholds). One type of decision is when testing a relay, whether to select it or not. For this type of decision, we have previously shown how to find thresholds then decide whether to select a relay or not. Fig. 3.2 demonstrates the relays and their corresponding thresholds. As you can see, X_{n-i} is the achievable rate of the

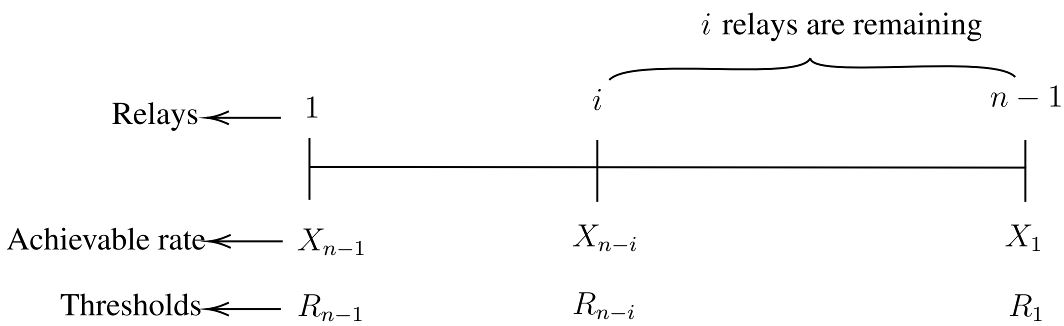


Figure 3.2: Diagram of the i^{th} relay and its corresponding achievable rate and threshold.

i^{th} relay tested. Another type of decision for a relay is whether to test it or not. As we will discuss later, this type of decision can be predetermined, which means that one can figure out the maximum number of tests needed to select a relay. In other words, one should continue testing relays unless a relay is selected or we have reached the maximum number of tests. If we reach the maximum number of tests, we need to select a relay without testing. Therefore, we have just discussed the thresholds that can be used for deciding to select a relay after testing it. These $n-1$ thresholds can be found following a similar backward induction for each stage of the selection process.

Assuming the maximum number of tests is predetermined, we can test the relays and compare their achievable rates to their corresponding thresholds. If the achievable rate is greater than its corresponding threshold, we should select that relay. Otherwise, test the next relay. We need to continue this procedure until we reach the maximum number of tests. In case up to the maximum number of tests, no relay was selected, a relay is selected randomly. The pseudo-code of the above algorithm is described in Alg. 4.

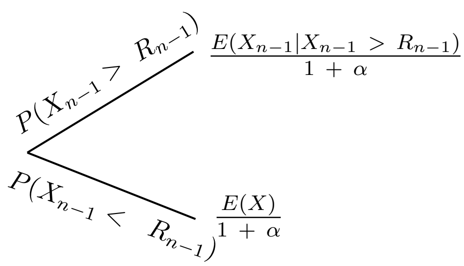
The detail on how to predetermine the maximum number of tests is mentioned below. First, let us start with two simple cases where the maximum number of tests are one and two, respectively. The decision tree of these two scenarios and the relation of the thresholds of each stop is as follows:

- **Stop after testing one relay:** To find the decision tree of this case,

Algorithm 4 SRS algorithm

Cost α is fixed.
X_{n-i} denotes the achievable rate of relay i.
R_{n-i} denotes the threshold when testing relay i.
m_{max} is the predetermined maximum number of tests.
select = 0
for $1 \leq i \leq m_{max}$ **do**
  if $X_{n-i} > R_{n-i}$ **then**
    Select relay i
    select = 1
    break;
  else
    Do not select relay i
  end if
end for
if select == 0 **then**
  Select a relay without testing.
end if

we need to cut the decision tree in Fig. 3.1 from stopping point "Stop after one test".

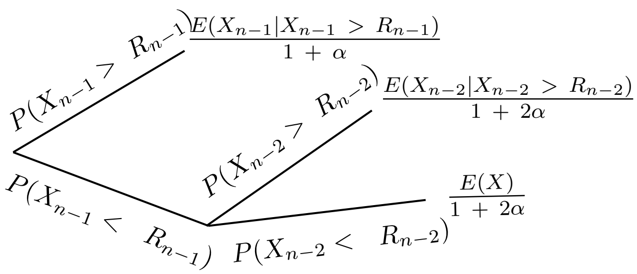


Now we can write the utility of the above decision tree and call it stop_1.

$$\text{stop}_1 = P(X_{n-1} > R_{n-1})\frac{E(X_{n-1}|X_{n-1} > R_{n-1})}{1+\alpha} +$$
$$P(X_{n-1} < R_{n-1})\frac{E(X)}{1+\alpha} = \frac{\int_{R_{n-1}}^{\infty} xf(x)dx + R_{n-1}\int_0^{R_{n-1}} f(x)dx}{1+\alpha}. \quad (3.5)$$

If we take a derivative from Eqn. (3.5) with respect to R_{n-1}, the optimal threshold will be $R_{n-1} = E(X)$. This optimal threshold is similar to the case that there are $n = 2$ relays, which means that there is just one threshold R_1, and we decide to stop after testing one relay. Therefore, the threshold of a relay selection scenario with n relays that stops testing after doing one test is equal to the threshold R_1 of a problem that there are only $n = 2$ relays.

- **Stop after testing two relays:** The decision tree for the case that two relays are tested is as follows:



The above decision tree is similar to the case that $n = 3$ and two relays have been tested. Therefore, the thresholds R_{n-1} and R_{n-2} in the above decision tree would be equal to the thresholds R_2 and R_1 for $n = 3$ respectively. Now, we can write the utility of this decision tree and call it stop_2.

$$\text{stop}_2 = \frac{\int_{R_2}^{\infty} xf(x)dx + R_2 \int_0^{R_2} f(x)dx}{1+\alpha}. \tag{3.6}$$

Now, let us generalize this scenario to the case where we decide to stop testing the relays after testing i of them (stop_i). In other words, if the maximum number of tests is i, the thresholds of this problem are as of a problem that there are $i+1$ relays, and we are going to select one of them, which means that there will be i thresholds. The recursive equation in order to find these thresholds is as follows:

$$R_{k+1} = \Big(\int_{R_k}^{\infty} xf(x)dx + R_k \int_0^{R_k} f(x)dx\Big)\frac{1+(i-k)\alpha}{1+(i-k+1)\alpha},$$
$$R_1 = E(X),\ 1 \le k \le i-1. \tag{3.7}$$

Also, stop_i is as follows:

$$\text{stop}_i = \frac{\int_{R_i}^{\infty} xf(x)dx + R_i \int_0^{R_i} f(x)dx}{1+\alpha}. \tag{3.8}$$

In the above equation, $f(x)$ is the achievable rate distribution of the relays. We can find all the $n-1$ thresholds analytically based on the achievable rate distribution. As you can see in (3.8), since the stop values are only related to thresholds and the distribution and not the previous observations, we can easily find the maximum number of tests. To do so, for a problem of size n where one relay should be selected, we need to find the values of the $n-1$ stops. Let us call the option that without any testing selects a relay as stop_0. Therefore, we will have a list of stops $\{\text{stop}_0, \text{stop}_1, \text{stop}_2, ..., \text{stop}_{n-1}\}$. These stops represent the utility of the decision tree in Fig. 3.1 with all possible maximum number of tests. Now, we need to compare the value of these stops and find the maximum one. The index of that maximum stop corresponds to the maximum number of tests required to select a relay. This approach not only simplifies the decision tree but also provides a closed-form for the thresholds and removes the effect of *max* from the equations. Therefore, in case of implementation, the algorithm runs faster.

We have found the thresholds for both AF and DF relaying protocols as below:

- **AF relaying thresholds:** In order to find the thresholds for any system, one needs to find the distribution $f(x)$ of the corresponding system, which in our case will be the achievable rate distribution for AF protocol. For simplicity, we can approximate the SNR_{eq} with Eqn. (1.4).

  By this approximation, if we assume that the gain of the channels are Rayleigh then the SNR_{eq} for AF relaying will have an exponential distribution with $\lambda_{\text{SNR}_{eq}} = 2\lambda_{sr}$ [1] in which $\lambda_{sr} = \frac{1}{\sigma^2 snr}$ and σ^2 is the variance of the source-relay and relay-destination gain and $snr = \frac{P}{N}$ which is a fixed number. The $f(x)$ for the AF protocol will be $2\lambda_{sr}\ln(2)4^x e^{-\lambda_{sr}(4^x-1)},\ x > 0$. Here, we have assumed the source-relay link and the relay-destination links are symmetrical. Therefore, the thresholds will be as follows:

$$R_{k+1} = \Big(R_k - \frac{e^{\lambda_{sr}}}{2\ln(2)} Ei(-4^{R_k}\lambda_{sr})\Big)\frac{1+(i-k)\alpha}{1+(i-k+1)\alpha},$$
$$R_1 = \frac{-e^{\lambda_{sr}}}{2\ln(2)} Ei(-\lambda_{sr}),\ 1 \le k \le i-1. \quad (3.9)$$

[1] The minimum of two independent exponential random variables with parameters λ_1 and λ_2 is another exponential random variable with $\lambda = \lambda_1 + \lambda_2$

In the above equation, $E_i(\cdot)$ is the exponential integral defined below:

$$Ei(z) = -\int_{-z}^{\infty} \frac{e^{-t}dt}{t}. \tag{3.10}$$

- **DF relaying thresholds:** For the DF relaying, the SNR$_{eq}$ is Eqn. (1.5). The SNR$_{eq}$ for the DF relaying is the same as the approximated SNR$_{eq}$ for the AF relaying protocol, then, in case of Rayleigh channel gains, the achievable rate for DF relaying will have the same distribution as the achievable rate for AF relaying protocols. Therefore, the thresholds will be the same as the previous part (Eqn. (3.9)).

3.2.2 kRSn(k relay selection among n relays)

Here, we are going to select more than one relay and maximize the overall achievable rate of the system as defined in Eqn. (1.9). In this equation, k is the number of selected relays among n relays, and m is the number of the relays that have been tested so far to choose the k relays. The cost of α is also defined as Eqn. (1.8). Also, all the assumptions from the previous part are valid, and testing is costly.

In order to solve this problem, the approach is similar to the SRS scenario since testing the relays are costly. Here, we are going to generalize the SRS scenario (1RSn) in which, based on the ratio α, the system decides to go either with a secretary based approach that tests each relay and compares it to its corresponding threshold or select a relay(s) randomly. The same as the previous section, in order to solve this problem, we need to find the distribution of the achievable rate. However, finding the distribution of the achievable rate with MRC when selection is for more than one relays is hard. Therefore, for simplicity, the selection of the relays is according to the sum of the achievable rates of the relays rather than their MRC. The sum of the achievable rates of the relays is an upper bound for the selection based on MRC. The proof of the above lemma is as follows:

Proof. We know that SNR_{eq_i} is always positive.

$$
\begin{aligned}
\mathrm{SNR}_{eq_i} \geq 0 \Rightarrow 0 \leq \prod_{i=1}^{k} \mathrm{SNR}_{eq_i} + ... \\
\Leftrightarrow 1 + \Sigma_{i=1}^{k}\mathrm{SNR}_{eq_i} \leq 1 + \Sigma_{i=1}^{k}\mathrm{SNR}_{eq_i} + \prod_{i=1}^{k} \mathrm{SNR}_{eq_i} + ... \\
\Leftrightarrow \log_2(1 + \Sigma_{i=1}^{k}\mathrm{SNR}_{eq_i}) \leq \log_2(\prod_{i=1}^{k}(1 + \mathrm{SNR}_{eq_i})) \\
\Leftrightarrow 0.5\log_2(1 + \Sigma_{i=1}^{k}\mathrm{SNR}_{eq_i}) \leq \Sigma_{i=1}^{k} 0.5\log_2(1 + \mathrm{SNR}_{eq_i}).
\end{aligned} \tag{3.11}
$$

This completes the proof that the sum of the achievable rates of the relays is an upper bound for reporting the achievable rate based on MRC. □

For ease of discussions, let us solve this problem for the case that $k = 2$ relays are going to be selected among $n = 3$ relays. The decision tree of this example is shown in Fig. 3.3. As you see when $n = 3$, first, we need to decide to either start testing relays or select the two relays randomly. In case we select the two relays randomly (without testing), the gain is $2E(X)$. Also, in case we decide to test the relays: One needs to compare X_3 (the overall achievable rate of the relay indexed as 1) with the threshold t_3^{2*} (the corresponding threshold for relay indexed 1 in the scenario that two relays are going to be selected). If X_3 is smaller than the threshold, this relay is not selected and since we have to select two relays, we should select the two remaining relays without testing. Then our gain is $\frac{2E(X)}{1+\alpha}$. Now, if X_3 is greater than that threshold, then that relay is selected. So now we need to select one more relay. In other words, the problem will shrink to the problem of SRS for $n = 2$ (1RS2)(selection of one relay among the two relays).

We can generalize the decision tree in Fig. 3.3 to the case that we are going to select k relays among n relays. In other words, we are going to solve the problem of kRSn. To do so, first, we need to decide either to choose the k relays randomly that in this case, our gain is $kE(X)$ or to compare the overall achievable rate of the first relay (relay with index 1)(X_n) to its corresponding threshold. If X_n is greater than its corresponding threshold t_n^{k*} then that relay is selected and our gain is $\frac{E(X_n|X_n>t_n^{k*})}{1+\alpha}$ and we still need to select $k-1$ relays

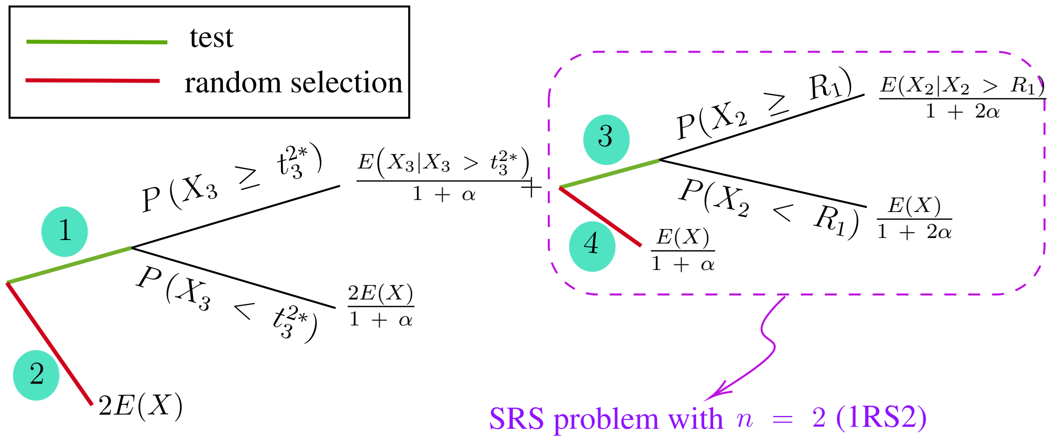


Figure 3.3: Decision tree for 2RS3

among $n-1$ remaining relays which means now we need to solve the problem $(k-1)$RS$(n-1)$. Also, in case that X_n is less than its corresponding threshold, then that relay is not selected, and since we have done one test so far, we need to solve the problem of selecting k relays among $n-1$ relays.

Therefore, the generalized decision tree of the selection of k relays among n relays (kRSn) can be defined as in Fig. 3.4. In this tree, the corresponding thresholds to each relay are named as t_i^{j*}, $n-i$ is the index of the relay that has the corresponding threshold, and j shows how many relays still need to be selected. Remember that when $j = 1$ (just one more relay needs to be selected), the thresholds will be like the thresholds of the SRS scenario.

To maximize the probability of selecting the best k relays, we need to find the maximum number of tests for the decision tree in Fig. 3.4. As you can see, there is a recursive relation between the decision trees of $(k-1)$RS$(n-1)$, (k)RS$(n-1)$, and (k)RS(n). Therefore, we can write the algorithm as of Alg. 5.

The thresholds named as t in (3.4) can be found with a similar approach used in the SRS method. However, there is no closed-form for the thresholds since they vary based on the maximum number of tests in the decision tree. As an example, let us find the thresholds for the decision tree in Fig. 3.3:

- **Path** 1 **and** 3**:** In order to find the thresholds t_3^{2*} and R_1, we need to

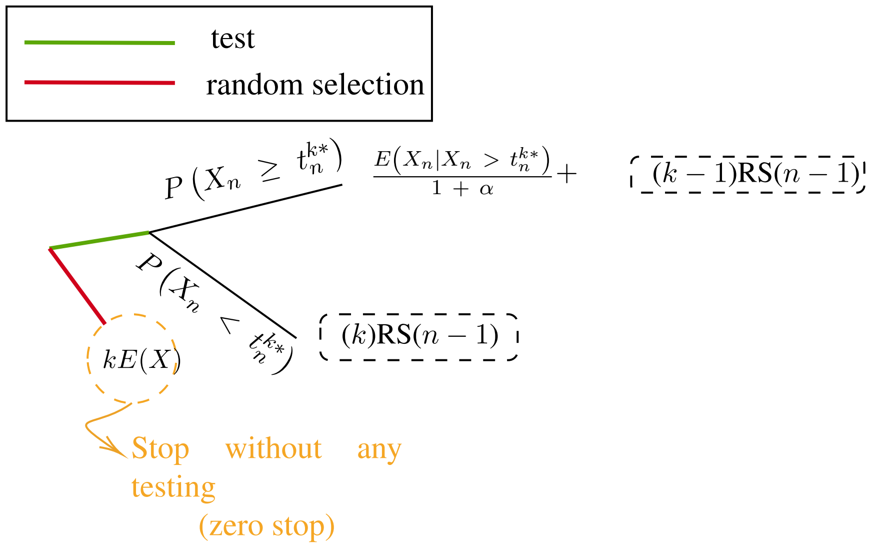


Figure 3.4: Decision tree for kRSn

Algorithm 5 MRS algorithm for the decision tree in Fig. 3.4

Cost α is fixed.
def MRS(k, n):
  if $k == 1$ **then**
    return Alg. 4
  else
    if $P(X_n \geq t_n^{k*})\left[\frac{E(X_n|X_n > t_n^{k*})}{1+\alpha} + \text{MRS}(k-1, n-1)\right] + P(X_n < t_n^{k*})\text{MRS}(k, n-1) < kE(X)$ **then**
      return Select all k relays randomly without testing.
    end if
  end if

take a derivative with respect to each of them.

$$\frac{d\Big(P(X_2 > R_1)\frac{E(X_2|X_2>R_1)}{1+2\alpha} + P(X_2 < R_1)\frac{E(X)}{1+2\alpha}\Big)}{dR_1}$$

$$= \frac{\frac{1}{1+2\alpha}d\Big(\int_{R_1}^{\infty} xf(x)dx + E(X)\int_0^{R_1} f(x)dx\Big)}{dR_1}$$

$$= \frac{1}{1+2\alpha}(-R_1 f(R_1) + E(X)f(R_1)) = 0 \Longrightarrow$$

$$R_1 = E(X). \quad (3.12)$$

Now let us find the threshold t_3^{2*}. According to Eqn. (3.7) we will have:

$$P(X_2 > R_1)\frac{E(X_2|X_2 > R_1)}{1+2\alpha} + P(X_2 < R_1)\frac{E(X)}{1+2\alpha} = \frac{R_2}{1+\alpha}. \quad (3.13)$$

Therefore, if we take a derivative with respect to t_3^{2*}, we will have:

$$\frac{d\Big((P(X_3 > t_3^{2*})\frac{E(X_3|X_3>t_3^{2*})+R_2}{1+\alpha} + P(X_3 < t_3^{2*})\frac{2E(X)}{1+\alpha}\Big)}{dt_3^{2*}}$$

$$= -t_3^{2*} f\left(t_3^{2*}\right) + (2E(X) - R_2)\, f\left(t_3^{2*}\right) = 0$$

$$\Longrightarrow t_3^{2*} = 2E(X) - R_2. \quad (3.14)$$

- **Path** 1 **and** 4**:** In this case, we just need to find the threshold for t_3^{2*} which is as follows:

$$\frac{d\Big((P(X_3 > t_3^{2*})\frac{E(X_3|X_3>t_3^{2*})+E(X)}{1+\alpha} + P(X_3 < t_3^{2*})\frac{2E(X)}{1+\alpha}\Big)}{dt_3^{2*}}$$

$$= -t_3^{2*} f(t_3^{2*}) + (2E(X) - E(X))f(t_3^{2*}) = 0$$

$$\Longrightarrow t_3^{2*} = E(X).$$

$$(3.15)$$

- **Path** 2**:** In this case there are not any thresholds and the two relays need to be selected randomly.

All the other thresholds can be found similarly to the example above. The $f(x)$ is the distribution of the achievable rate of the relays that as discussed in the SRS section is $2\lambda_{sr}\ln(2)4^x e^{-\lambda_{sr}(4^x-1)},\ x > 0$ for AF/DF.

3.3 Results

In this section, we evaluate the relay selection methods based on the hybrid algorithm in terms of the overall achievable rate of the network (the capacity of the network) versus SNR and compare them to two other methods (exhaustive search and random approach). The exhaustive search tests all the relays and then decides which relay(s) is/are the best. Also, the random approach selects a relay(s) randomly without doing any tests.

Simulation results are shown for two relaying protocols, AF and DF. n relays are placed between the source and the destination, and their channel distribution is assumed to be Rayleigh. The results have been tested for two scenarios, SRS and more than one relay selection (kRSn). These selections were made for the SNR in the range of $0-40$ dB. Also, Eqn. (1.3) and Eqn. (1.5) were used to produce the SNR_{eq} for implementing the AF and DF protocols respectively.

3.3.1 Single relay selection results

The simulations were done in order to select one relay among $n = 50$ relays. Fig. 3.5 shows the simulation results for selecting one relay among 50 relays for relaying protocols AF and DF with cost $\alpha = 0.01$. Firstly, to do this simulation, the Alg. 4 decides the best path for each SNR, and then the relays are tested based on that path. As you can see, when comparing the hybrid approach to the exhaustive search approach that tests all the relay and then selects the best one, there is an 8.5 dB improvement in DF protocol in order to achieve the overall achievable rate of 4 b/s/Hz (bits per second per hertz). Fig. 3.6 shows similar simulation results for a higher cost ($\alpha = 0.1$) for relaying protocols AF and DF. Since the cost is higher, you can see that the gap between the exhaustive search method and hybrid method is greater and the effect of the cost comes more into the picture. Now, suppose we compare the hybrid method in DF to the random approach and the exhaustive method. In that case, there is a 4 dB and 28 dB improvement in order to achieve the overall achievable rate of 1 b/s/Hz, respectively.

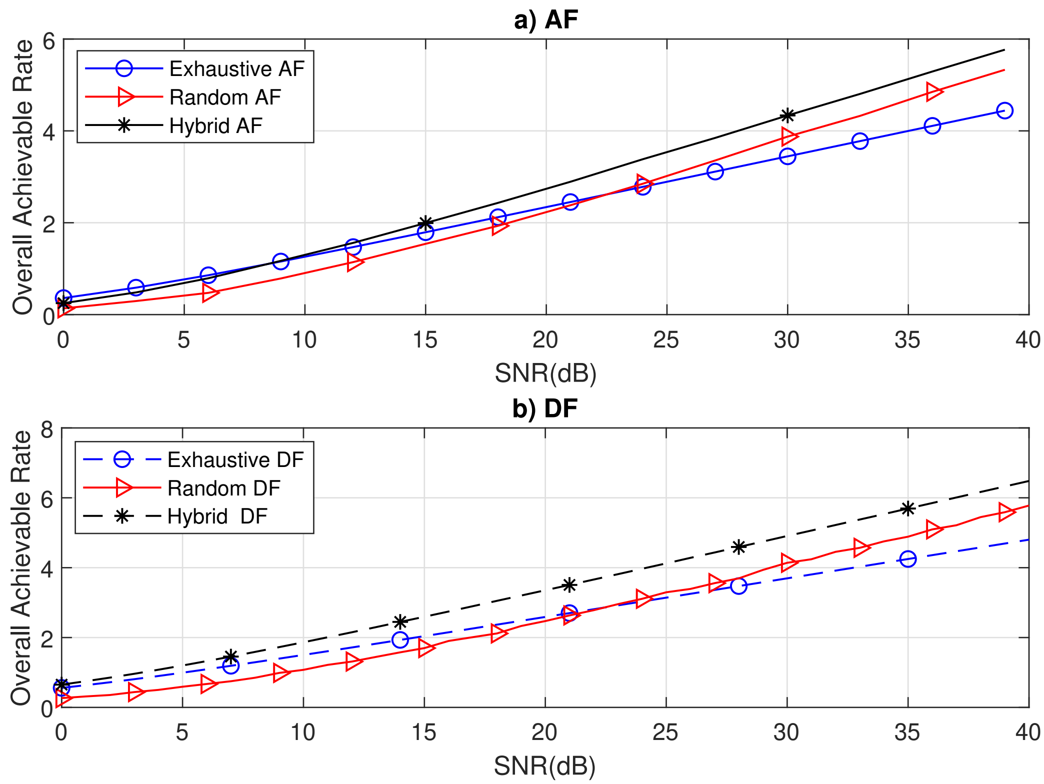


Figure 3.5: SRS for the two relaying protocols a) AF and b) DF with $\alpha = 0.01$.

3.3.2 Multiple relay selection results

The simulations in this part were done to select k relays among n relays. We have done the simulations for two cases, 2RS50 and 3RS50. These simulations were done for two relaying protocols with two costs $\alpha = 0.01$ and $\alpha = 0.1$. Fig. 3.7 and Fig. 3.8 show the simulation results for 2RS50 with cost $\alpha = 0.01$ and $\alpha = 0.1$ respectively. By comparing the hybrid method to the exhaustive approach for DF protocol, you can see that there is almost 10 dB improvement to achieve the overall achievable rate of 4 b/s/Hz. The similar analysis was done for 3RS50, and the results are shown in Fig. 3.9 and Fig. 3.10. As you can see in Fig. 3.10 since selection of three relays with this cost is very costly, the algorithm decides to move on with the random method for selection.

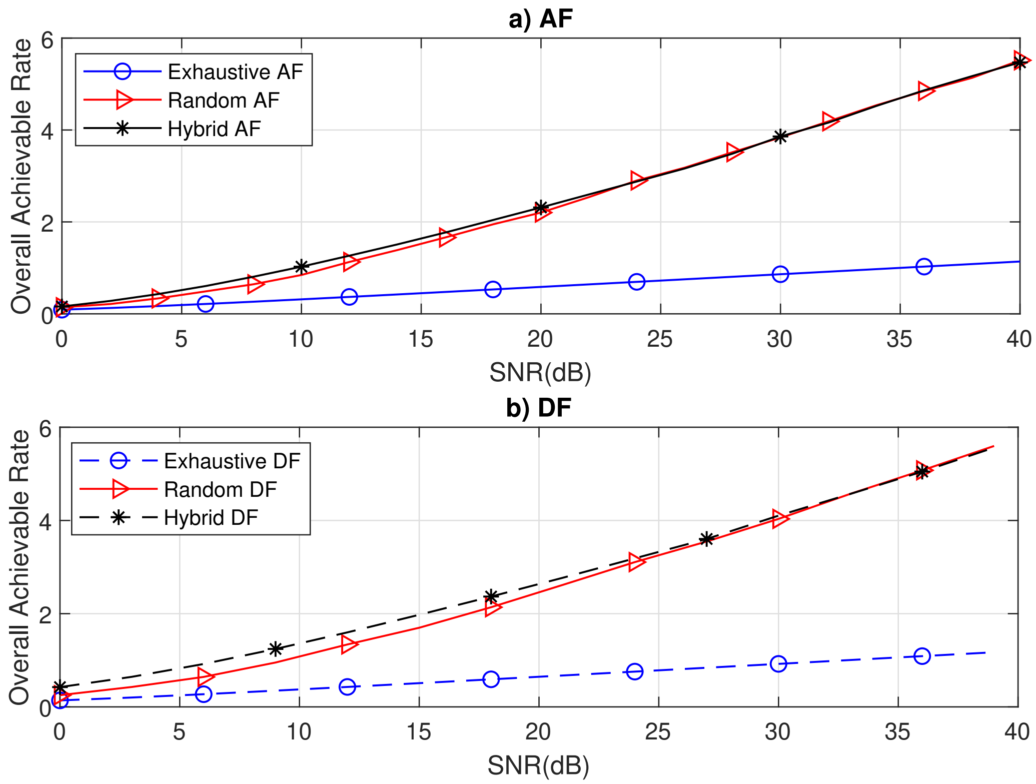


Figure 3.6: SRS for the two relaying protocols a) AF and b) DF with with $\alpha = 0.1$.

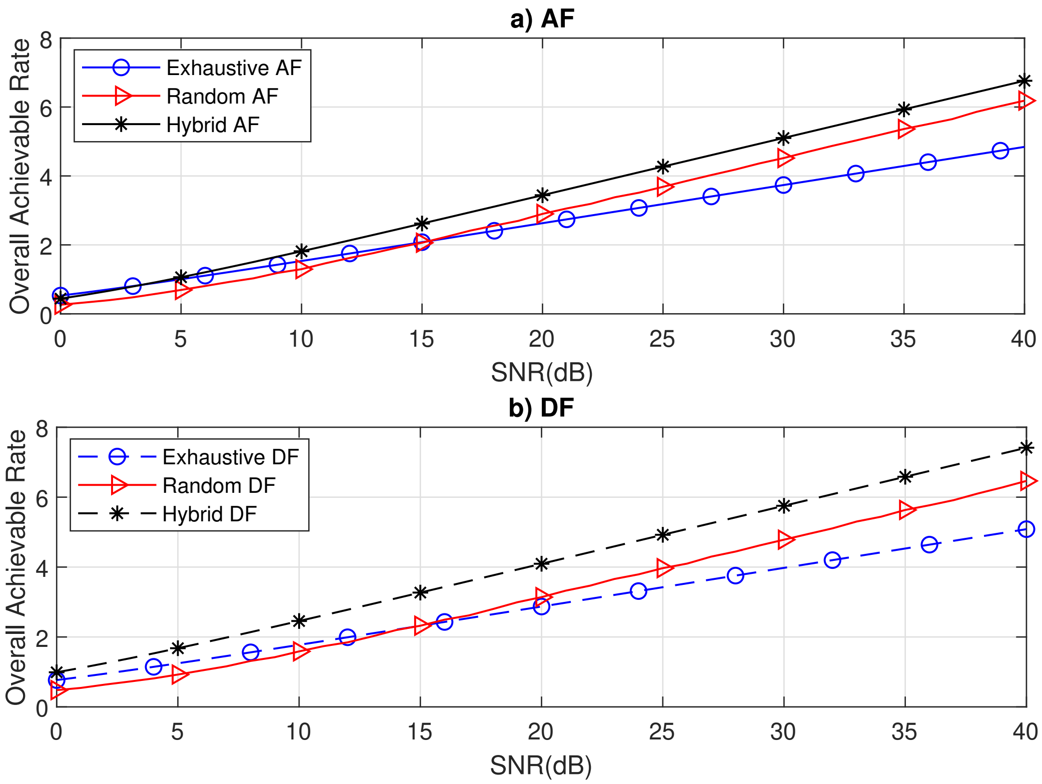


Figure 3.7: 2RS50 for the two relaying protocols a) AF and b) DF with the $\alpha = 0.01$.

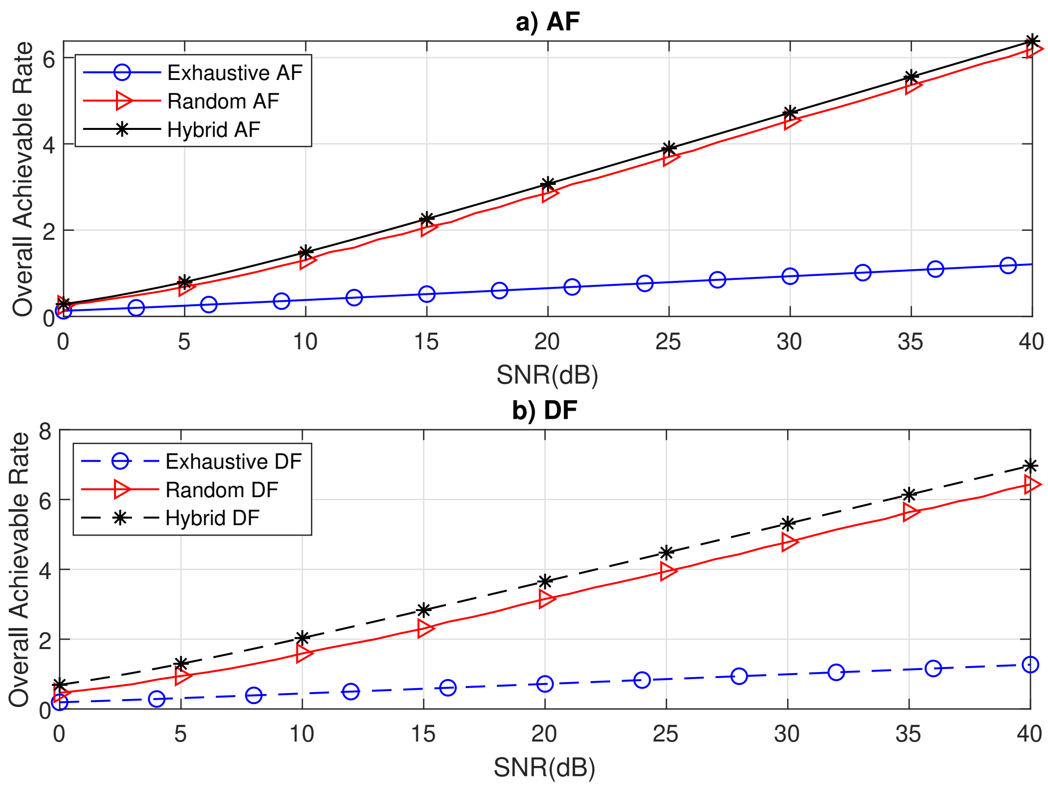


Figure 3.8: 2RS50 for the two relaying protocols a) AF and b) DF with the $\alpha = 0.1$.

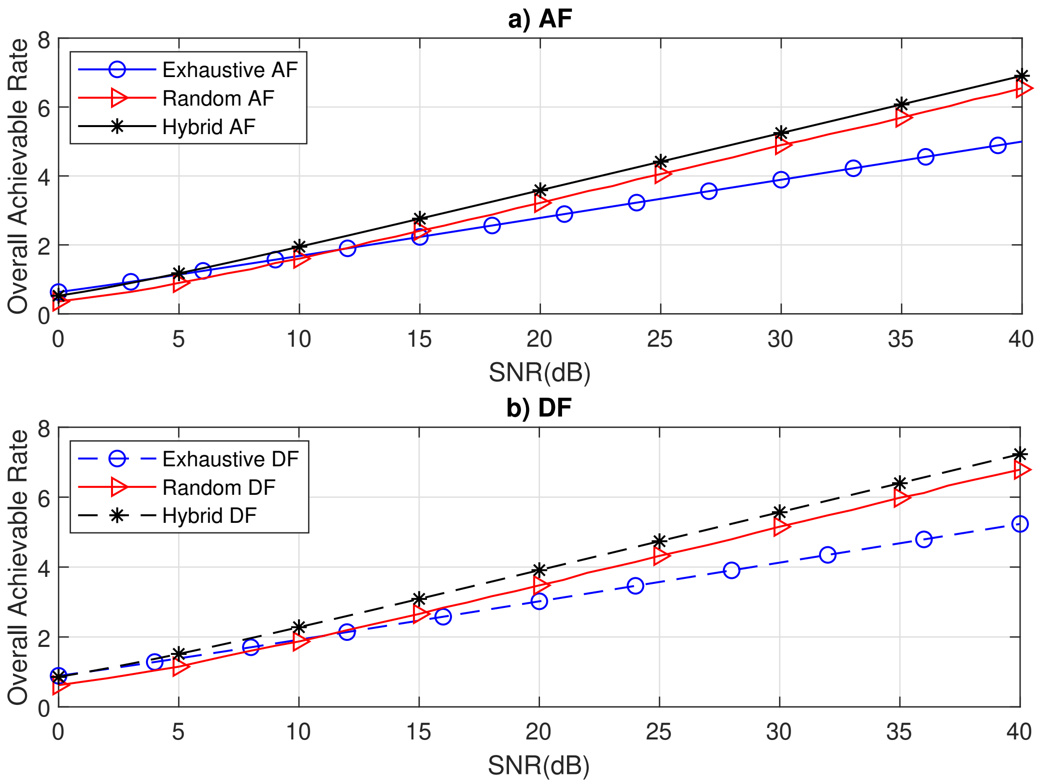


Figure 3.9: 3RS50 the two relaying protocols a) AF and b) DF with the $\alpha = 0.01$.

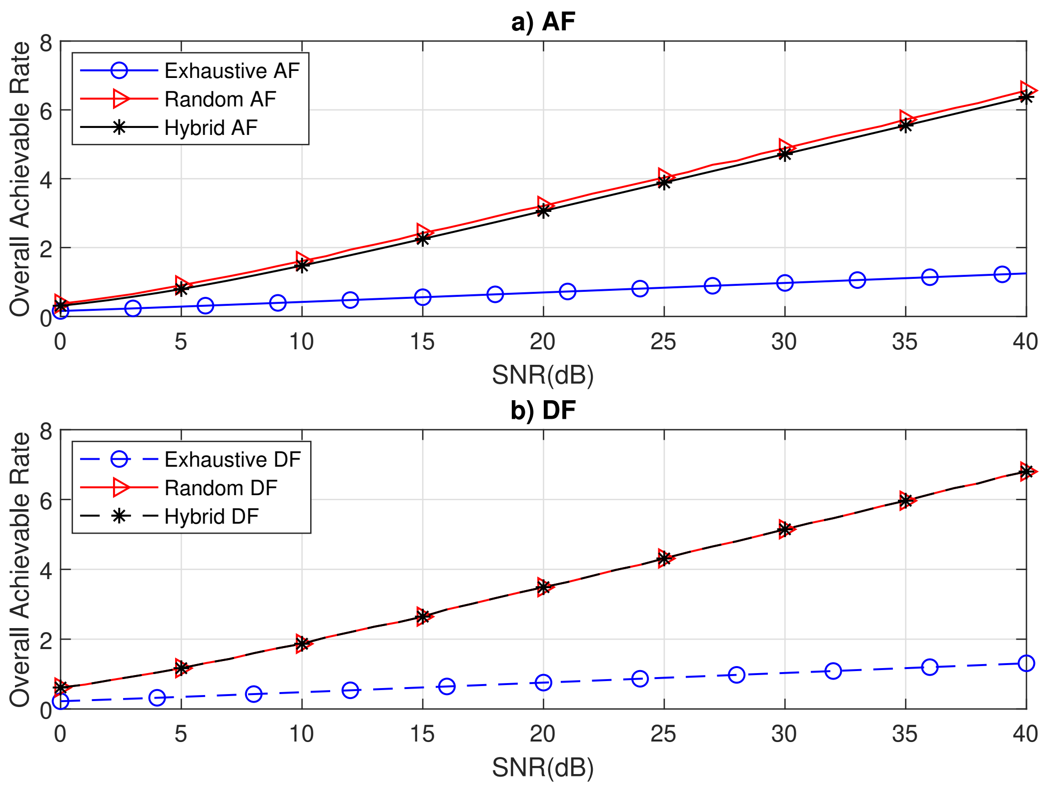


Figure 3.10: 3RS50 for the two relaying protocols a) AF and b) DF with the $\alpha = 0.1$.

Chapter 4

Conclusion and Future Research Directions

4.1 Conclusions

In this thesis, relay selection strategies inspired by the algorithm to solve the secretary problem were derived. Two separate studies were discussed in this thesis:

- The first study was on the secretary problem and its variants that we used to model the corresponding relay(s) selection problems for both SRS and MRS scenario with AF and DF relaying protocols. Different algorithms have been introduced for each case of relay selection scenario. Since the secretary method is a kind of an optimal stopping rule problem, it tries to solve the relay selection problem by testing fewer number of relays compared to the solution that tests all the relays then selects the best one(s). Also, knowing the channel distribution helps the secretary approach significantly and allows it to approach the ultimate performance where all the relays are tested and then selected. Simulation results verify that the relay selection method based on the secretary problem can achieve up to 7.5 dB performance gain compared to other approaches without testing all the relays.

- The second study was for the case where testing relays for the selection is costly. In this study, a hybrid algorithm (a combination of random and secretary approach) has been proposed to solve the corresponding relay(s) selection problems for both SRS and MRS scenarios with DF relaying protocol. This method is designed so that the testing cost for selecting the relay(s) comes into the picture and tries to solve the relay selection problem by testing quite fewer numbers of relays, which means saving more time. Simulation results verify that without testing all the relays, the relay selection method based on the hybrid method can achieve up to 20 dB performance gain compared to the method that tests all the relays and 6 dB gain compared to the method that selects the relay(s) randomly.

4.2 Future Research Directions

The possible future works can be as follows:

4.2.1 Relay selection for the recall and the known channel distributions scenario

We have discussed the cases where recall was allowed for a scenario in which the channel distributions were unknown. Also, we have separately discussed the relay selection problem when the distribution of the channels is known. The intersection of these two matters can create an interesting research area. This study can be done for both SRS and MRS problems.

4.2.2 Relay selection for a direct link from source to the destination scenario

In this thesis, we have assumed that there is no direct link from the source to the destination and solved the relay selection problem for this system. An interesting observation will be solving this problem when there is a direct link in the system as well. The possible steps toward solving this problem can be as follows:

Firstly, one needs to find the end-to-end SNR for a system with a direct link. Secondly, for the case where the channel distributions are known, one needs to find the distribution of the end-to-end SNRs of the relays and follow the next steps described in Chapter 2 and Chapter 3.

4.2.3 Relay selection for other relaying protocols rather than AF and DF

In this thesis, all the problems were solved for the two of the common relaying protocols, AF and DF. One future direction could be applying the secretary algorithms to solve relay selection problems for other cases of relaying protocols such as compress and forward. This also can be done by finding the end-to-end SNR corresponding to the relaying protocols and applying the algorithms introduced in Chapters 2 and 3.

4.2.4 Multiple relay selection when the number of relays to be selected is unknown

In this thesis, the number of relays that were going to be selected was known. Further research can investigate the case where one does not have a fixed number of relays to be selected in mind, and the goal is to achieve a certain throughput. Therefore, in such a scenario, the number of selected relays may vary.

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