# Two Essays on Firm's Marketing Strategy: Auction Overlap Decision of Online Auctioneers and Product Positioning of Disadvantaged Firms

by

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## ABSTRACT

**Essay 1: Optimal Seller Strategy in Overlapping Auctions**

Overlapping auctions for identical products are commonly observed on auction websites. Such an online setting for competitive auctions changes the consumers' bidding behaviors. Bidders are able to forward-look, cross-bid and learn during the bidding process. The change in the bidding behaviors has important implications for sellers' revenue. We adopt a game-theoretical modeling method, aiming to understand—in a single-seller auctions website where the seller is able to decide auction design and bidders are able to forward-look, cross-bid, and learn—the existence of auctions' overlapping. We also determine its value, pinpoint the factors influencing the degree of overlap, and solve the optimal degree of overlap under different conditions. We find that the degree of overlap depends on the trade-offs among four factors: bidders' forward-looking, bidders' learning, time discounting and varied demand. Hence, the combination of the impacts from these factors decides the optimal overlapping strategy of a revenue maximizing seller.



*Keywords:* overlapping auctions; cross-bidding; learning; bid shading; forward-looking



**Essay 2: Product Positioning Strategy of the Firms without a Competitive Advantage**

This essay examines the optimal product positioning strategy of asymmetric firms. Much of the theoretical discussion on firm strategies, such as technical investments and advertising, focuses on symmetric firms. Less attention is given to weak firms when they face strong companies such as IBM, Sony, GE, Apply and Wal-Mart. We find that weak firms position their products on areas which they are at advantages relative to their competitors. More importantly, they have an incentive to avoid close competition with strong firms by randomizing their product positions in the market if the advantage gap is still small, and to focus on the extreme edges of the market if the advantage gap is large. However, strong firms act in the opposite way: locating on mass market when the advantage gap is small, and randomizing their market positions and expand to on the edges of the market when the advantage gap is larger. Moreover, the strong firms make less on radical investment than on incremental on product's features. These findings show the fundamentally different product positioning strategies under asymmetric competition than under symmetric one.

*Keywords:* product positioning, location model, Hotelling model, Colonel Blotto game, asymmetric firms, marketing strategy

## PREFACE

My dissertation has received tremendous help from my advisor. He keeps giving his valuable comments and suggestions during the writing and revision.

The first essay of this dissertation is our collaborative work. The second essay of this dissertation is conducted by myself.

## DEDICATION

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CONTENTS

# CHAPTER 1: INTRODUCTION

This thesis discusses the seller's positioning strategy in competitive markets with demand uncertainty and is composed of two essays. The first essay focuses on auctioneers in the environment of multiple competitive online auctions. We use analytical modeling to study the seller's decision on the degree of overlap among competitive online auctions and the determining factors. Research on concurrent online auctions is a new and important area of study. Our exploration of the overlapping strategy for a seller contributes to the current literature theoretically, adding new knowledge to our understanding of sellers' and bidders' behaviors in concurrent online auctions. This study also provides managerial recommendations to auction sellers for a better arrangement of their auctions.

The second essay focuses on asymmetric sellers where one firm has a competitive advantage over the other in the context of product positioning for multiple products. We develop a game-theoretical model and show that, due to their heterogeneous abilities, the firms perform substantially differently from when they are symmetric, which is the case in most marketing studies. Weaker firms may deviate from their strong areas when they meet with stronger competitors. They have incentive to invest in the areas where consumers' preferences are more diverse. By doing so, weaker firms increase the vulnerability of stronger firms, thus attracting consumers without directly combating with strong firms. This study leads to an interesting insight to the reason why the marketing strategy of weaker firms differ from that of stronger firms, and provides an operational guide in positioning for multiple products in asymmetric firms.

## 1.1 Essay 1: Optimal Seller Strategy in Overlapping Auctions

A substantial number of retailers and catalogue firms, as well as individuals, are taking advantage of the boom of online auction websites, such as eBay, WebStore, and OnlineAuction, to buy or sell products. This growing use of online auctions and their spectacular commercial success have motivated many academic studies (Bapna, Goes and Gupta 2000, 2003; Roth and Ockenfels 2002; Wilcox 2000; Greenleaf et al. 2002; Ariely and Simonson 2003; Zeithammer 2006; Yao and Mela 2008; Haruvy et al. 2008).

Researchers note that, compared with the traditional (offline) auctions, online auctions differ in many aspects. One key difference is the existence of overlapping auctions for similar products on the same websites. That is due to several reasons. First, it is difficult for bidders to stay in multiple traditional auction houses at the same time. But such spatial constraints disappear in the online auction. Current internet technologies enable bidders to access multiple online auctions simultaneously. And more new bidders arrive from a larger variety of places, bidding in multiple auctions simultaneously. Second, sellers are able to list multiple auctions overlapped to different degrees, because bidders can participate in multiple auctions simultaneously and demands are not divided among auctions. If bidders do not win in a certain auction, they generally have the opportunity to move to a competing auction. Overlapping auctions are frequently observed in auction websites, such as SamsClub, MegaClub, and eBay. However, most auctions studies regard each auction separately, and the theoretical and empirical studies on different degrees of overlap among auctions are still limited in number.

The second difference is the existence of cross-bids among competing auctions. When auctions are overlapped, some bidders cross bid, switching to an auction with the lowest “standing” bid (Anwar, et al. 2006; Andersson, et al. 2012). When auctions overlap more, bidders switch more often. Therefore, the bidding history of a single auction is less likely to reveal bidders’ true valuation distribution, providing bidders with less accurate information concerning the value of other bidders. Therefore, overlapping may influence the precision of information released from previous auctions and the greater the degree of overlap, the lower the precision of the bidders’ valuation distribution. Thus, varying the degree of overlap can work as a tool for a seller to control the precision of the information released.

Third, online websites post bidding histories of competitive auctions so that bidders can learn the values of uncertain products. Bidders thus can learn from 1) their bidding interactions with others on currently competing auctions, and 2) complete bidding information from previous auctions. These two types of learning have different impacts on the dynamics of individual bidding behaviors. Information about bids from interactions (type 1) may prompt bidders to cross-bid, chasing the lowest bid among currently competing auctions; information about bids from previous auctions (type 2) helps reduce the uncertainty in their product valuation, causing more aggressive bidding in later auctions.

In type 1 learning, bidders learn through the observation of bids (behavior). Such learning belongs to observational learning. Studies about learning in single or sequential auctions, such as common value models (Milgrom and Weber 1982; Board 2009; Neugebauer and Selten 2006; Dufwenberg and Gneezy 2002) and independent value models in which a seller releases information (Ganuza and Zuasti 2004; Vagstad 2001), have been widely conducted. However, the analyses of observational learning in the environment of overlapping auctions are still new.

The first essay of this thesis considers the above differences, analyzing the seller's optimal overlapping strategy and bidders' learning behaviors in online competitive auctions in an attempt to identify the following: the impact of the degree of overlap on the highest bids in competitive auctions; the optimal overlapping strategy for seller to sell experienced products, whose values are difficult to determine for buyers; the chance and the way for bidders to learn during the bidding when bidding information is available on competitive auctions.

Overlapping auctions can provide a situation in which multiple sellers compete with each other to sell a single item, or a single seller arranges periods of overlapping time to sell multiple identical products. The first essay focuses on the latter. When selling multiple identical items, the seller needs to determine the optimal degree of overlap for his auctions. In this type of overlapping auctions, we consider bidders' three important behaviors and one factor (time discounting).

1) Forward looking. When a bidder participates in an early auction, she may know that if she wins, she will lose the opportunity to win in future auctions with a lower price. Therefore, she usually reduces her highest bid in the current auction (Engelbrecht-Wiggans 1994; Jofre-Bonet and Pesendorfer 2003; Zeithammer 2006). Such a practice is called bid shading.

2) Cross-bidding. When auctions are overlapped, bidders are able to cross bid to an auction with the lowest "standing" bid. Such actions cause more bids, which may damage

the quality of information about bidders' valuation.

3) Learning. When a bidder stays in later auctions, she is exposed to the complete bidding histories and the highest bids of early auctions. Hence, she may change her belief (the expected valuation and confidence interval about product value) if her prior belief is uncertain.

4) Time Discounting. The seller and bidders need to make decisions over time. Our study considers that they are both sensitive to the duration of the auctions. When all the other conditions remain unchanged, they prefer their outcomes realized earlier.

Our study finds that different degrees of overlap influence the seller's revenue through forward-looking, cross-bidding, learning, and time discounting. For example, the seller prefers to increase the degree of overlap so that his revenue can be realized earlier. However, when the degree of overlap is larger the bidder reduces her bid, and in equilibrium this amount of bid-shading increases with the degree of overlap, and thus decreases the seller's revenue. Moreover, when the degree of overlap is larger, the information about the bidders' valuations released from previous auctions more likely will be marred by frequent cross-bids. Consequently, bidders learn less and their degree of uncertainty is only reduced little, resulting in less aggressive bidding in later auctions. The less aggressively the bidders bid, the lower the seller's revenue is. The trade-off of these four elements decides the optimal overlapping strategy of a revenue maximizing seller.

Compared with current literature, this study differs mainly in two aspects. First, previous literature on overlapping auctions studied the consumers' bidding strategies in concurrent auctions when the degree of overlap was given. By contrast, in our study, the degree of overlap is dynamic, and is the seller's optimal strategy made after considering time discounting and bidder behaviors of forward-looking, cross-bidding and learning. Second, although previous theoretical literature predicted that overlapping auction design would reduce the seller's revenue due to the bidders' forward-looking behavior (Huang et al. 2007; Zeithammer 2006), empirical observation showed the growth in the number of overlapping auctions with different degrees of overlap (Bapna et al. 2009). We provide a theoretical explanation for such popularity. Our findings also provide managerial recommendations applicable to the real practice in online auctions.

### 1.2 Essay 2: Product Positioning Strategy of Firms without a Competitive Advantage

Weak firms need different product positioning strategies due to the former lacking competitive advantage. In this essay, we answer the following questions: when weak firms carry out their marketing strategy, such as the investment on product quality, shall they choose to invest on their strong points, or on their weak edges to compensate? How much effort shall they allocate on different features of a product? Does the relative level of advantage between firms influence their product positioning strategies in the market place?

Those questions fall into the product positioning category in the 4P marketing strategy (i.e., PRODUCT, PRICE, PLACE, and PROMOTION). Much of the analytical discussion on firm strategies for investments and advertisement focuses on symmetric firms. Less attention has been given to the strategies of weak firms when they face strong ones such as IBM, Sony, GE, Apply and Wal-Mart. This is the area still lacking theoretical understandings and suggestions on the operational level.

We develop a game-theoretical model in which two asymmetric companies, different in their competitive ability to attract consumers, make product location decisions when the consumers' tastes are uncertain. In our one-dimensional model, firms need to choose the ideal locations to maximize their profits under a same setting as Hotelling location model. In our multi-dimensional model, we apply the Colonel Blotto game as firms need to consider simultaneously their product positions in multi-dimensions. This model shows that due to their heterogeneous abilities, the firms perform substantially differently from when they are symmetric, the latter being the case in most marketing studies. Weak firms may deviate from their strong areas when they meet with strong competitors. They have incentive to invest the areas where consumers' preferences are more diverse. By doing so, the weak firms increase the vulnerability of strong firms, thus attracting consumers without directly combating with strong firms. Secondly, weak firm's level of differentiating is negatively related with the advantage gap. When the competitive advantage of his rival is small, if staying at the market niche with randomization, the weak firm can be easily knocked out by his rival, once the strong firm moves to his market location. Instead he shall differentiate his market positions, including the mass market which is the core market of his rival. By doing so, the weak firm generates a threat in the mass market so that the strong firm has to focus resources to defend, and thus leaving large shares in the market niches to the weak firm. When the competitive advantage of his rive is larger, the weak firm's strategy is less differentiated than the strong one does, totally giving up center positions and focused on the extreme points. His location strategy turns from "randomization" to "focus", and from "deviating competition" to "fighting for survival".

Our model applies to markets where firms want to decrease direct competition, they avoid the "competing-on-price-only" syndrome, such as in many oligopoly markets for telecommunication and network, airlines, steel and oil business (Porter 1980, Levitt 1991); when brains as well as muscle are important for success, such as in competitive sports and team games; when firms need to set up store locations before providing services or products to consumers, especially when prices are dynamic and can be easily adjusted.

The game is played in two stages: First, firms simultaneously choose their product positions. Second, consumers purchase a product from one of the firms which gives them the higher utility. We solve the location strategies of both firms for subgame-perfect Nash equilibrium. We start with the simplest possible model in which the firms consider the product positioning on one dimension, in order to highlight how asymmetric advantage impacts on firms' competition. Then we extend it to a multi-dimensional position model.

Our paper fills the gap by providing theoretical explanations to many observations, for example, why weak firm differentiate their product positioning as a way to mitigate competition. It also explains why firms when they introduce new products, usually position products away from the main market, looking for a niche market. Why strong firms (incumbents) would like to build upon their existing set of capabilities and bring in incremental technologies to market, instead of developing radical ones. Why McDonald's likes to locate near Burger King, but Burger King does not. Most importantly, understanding how to compete in this world is crucial. We are the first analytical paper in the marketing literature attempting to provide , although in a simple manner (consumer homogenous weight on features), a resource allocation /product positioning plan on different features of a product (in Proposition 4 and 5).

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# CHAPTER 2: OPTIMAL SELLER STRATEGY IN OVERLAPPING AUCTIONS

## 2.1 Introduction

In recent decades, auctions have captured a significant share of online purchases. This has resulted in many retailers using auctions as an alternative channel of distribution, selling products in online auctions over time. Multiple auctions, which are conducted to sell identical or substitute items on the same website with time overlapping, are referred as overlapping auctions.

Overlapping auctions can be run by multiple sellers, such as those in eBay, CQout, bid4assets and so on. For example, when we typed in “Apple iPod 4th …” on eBay, over sixty auctions popped out, selling identical Apple iPods. On such multiple-seller auction websites, overlapping is unavoidable because sellers are unable to predict the occurrence of their rivals’ auctions. However, surprisingly on single-seller’s online auction websites, such as Shopgoodwill (a pure online seller), Samsclub (a both online and offline seller), and Policeauctions (a government online site), overlapping auctions are still commonly observed. For example (see Figure 2-1), on Samsclub four auctions were selling 55’ Vizio LCD 1080p 120Hz HDTVs, each lasting 4 hours and 18 minutes with 3 hours and 36 minutes overlapped; the degree of overlap is 77%. Two auctions were selling a set of Circulon Bakewares, each lasting 7 hours with 1 hour overlapped; the degree of overlap is 14%. The above observations generate following questions: 1) why does SamsClub arrange overlapping auctions? and 2) why does the seller set a different degree of overlap among products?

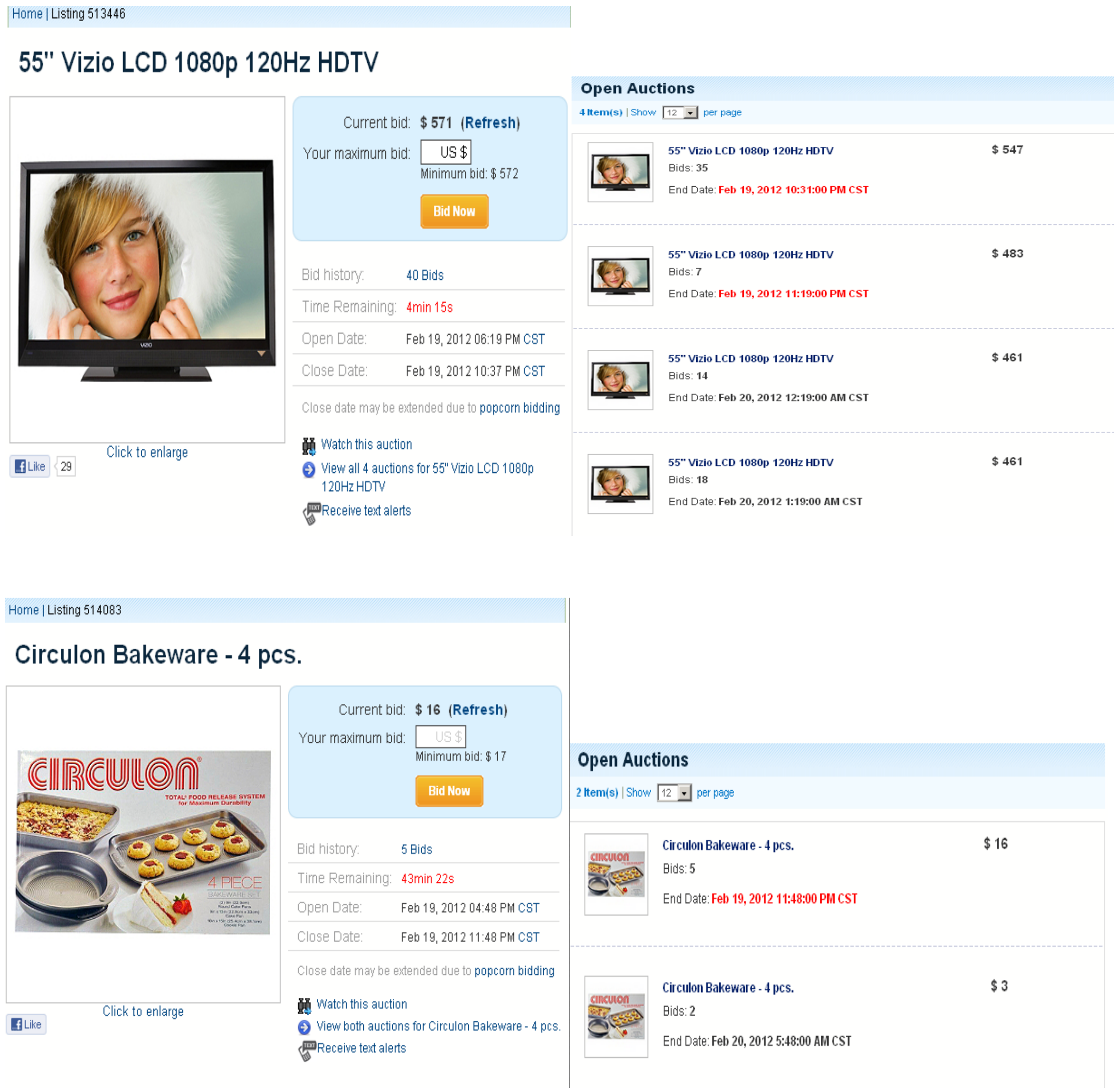


Figure 2-1. Examples of overlapping auctions in Samsclub.com

In ***simultaneous*** auctions (full overlap) bidders can cross-bid. That is, they may switch between auctions, bidding in the auction listing the lowest price[1]. In ***sequential***

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[1] Research on simultaneous auctions can be traced to McAfee (1993)'s model, focusing on bidder's cross-bidding behaviours. One group of studies has found that the proportion of cross-bids is large (Peters and Severinov 1997, 2006; Anwar et al. 2006), whereas the other

auctions (zero overlap) bidders in early auctions can forward-look; that is, they anticipate the occurrence and prices of future auctions. Moreover, bidders in later auctions can learn. That is, they observe the bidding history of earlier auctions, so the uncertainty in bidder's valuation (i.e. the assessment on the product value) is reduced. These two behaviors influence the bidding outcomes differently. Forward-looking behavior results in less aggressive bidding (and lower prices), because, foreseeing the forthcoming of future auctions, bidders are able to trade off the chance to win at the present time for the chance to win in the future (Engelbrecht-Wiggans 1994; Jofre-Bonet and Pesendorfer 2003; Zeithammer 2006, 2007a). However, the bidders' reduced uncertainty through learning will result in more aggressive bidding (and higher prices) in later auctions (Kagel and Levin 1986; Kim and Che 2004; De Silva, Dunne, Kankanamge and Kosmopoulou 2005).

Overlapping auctions are associated with more bidding behaviors than simultaneous and sequential auctions. In ***overlapping auctions*** (except the above two extreme cases), facing complex information during the bidding process, bidders can forward-look the price of the future auction, while cross-bidding in the middle of two auctions—due to overlapping. Finally, they can learn more about the product value when the first auction ends.

In this study, we set up a decision context in first-price ascending auctions. Prior to the first auction, bidders forward-look the second auction. Moreover, bidders are uncertain about the product value, but have prior beliefs (Compte and Jehiel 2004; Bergemann and

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group has found limited cross-bidding (Haruvy and Popkowski Leszczyc 2010).

Pesendorfer, 2007; Eso and Szentes 2007; Hossain 2008). Therefore, when the first auction completes, the winner leaves with the item and the remaining bidders continue to participate in the second auction.[2] They learn from signals (e.g., the bidding history of the completed auction) and update their beliefs. However, the quality of the signals is marred by cross-bids which happen when two auctions overlap.

We start with a benchmark model in which bidders are forward-looking but do not learn. Our analysis reveals that it is optimal for the seller to run the two auctions completely overlapped (i.e. run simultaneously). In this simple model, two factors are discussed: forward-looking and time discounting. 1) *Forward-looking.* Forward-looking makes bidders see an option to win in the second auction at a potentially lower price, as the winner of the first auction leaves; seeing such an option results in bidders' bid-shading (i.e. bidding less) in the first auction. Such bid shading stops when two auctions run simultaneously, therefore, for this consideration the seller prefers full overlap. 2) *Time-discounting.* The seller's discounting of future payoffs makes him favor a shorter duration of auctions, i.e. a greater degree of overlap. As a result, the seller's profit under full overlap is always optimal.

Next, we consider the combined effect from the four factors under different degrees of overlap: bidders' forward-looking, bidders' learning, time-discounting and the varied demand. Forward-looking and time-discounting have been discussed in the previous paragraph; therefore we only discuss the other two factors here. 3) *Learning.* We find that

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[2] In our extension in §2.5.1, we modify this assumption by incorporating the random arrival of bidders, that is, the number of bidders varies between the two auctions.

learning influences the final bids of two auctions in the following ways. First, when bidders are uncertain about the value of a product, learning helps them reduce the uncertainty in their assessments. This reduced uncertainty results in bidders' more aggressive biddings (and a higher final bid) in the *second* auction. Second, bidders can also predict this higher final bid in the future auction, which reduces their chances of winning there. Hence they bid more aggressively (and bid higher) in the *first* auction. The above two impacts of learning make the seller more profitable; and the seller thus would reduce the degree of overlap (i.e. the longer duration) to increase the time of learning. Although researchers have suggested that the price in the first of two overlapping auctions provided a focal point for bidders to bid in the subsequent auction (Haruvy, et al. 2014), not many papers have explicitly modeled bidders' learning in overlapping auctions. 4) *Varied demand.* The shorter degree of overlap (i.e. the longer duration) increases the demand of both overlapped auctions when bidders are allowed to entry randomly during the bidding process, and thus may increase the final bids of two auctions. Combining all those four factors, we find that the optimal overlapping strategy is the trade-off among these four factors: bidders' forward-looking, bidders' learning, time discounting, and the varied demand. We also pinpoint the conditions that partial overlapping auctions are optimal: (1) bidders' uncertainty about product value is in a middle-range and (2) the time discounting effect is not strong.

The remainder of the chapter is organized as follows. Section 2.2 discusses the existing literature. Section 2.3 sets up the framework of the analytical model, followed by the analysis of a forward-looking case as a benchmark and a full model in Section 2.4 and several model extensions in Section 2.5. Finally, Section 2.6 contains the concluding remarks.

## 2.2 Literature review

### *2.2.1 Auction type*

Auctions can be classified according to various criteria. For example, based on whether the bids are openly observed, there are open and sealed-bid auctions. Based on whether bids are rising or falling during the auctions, we can distinguish between ascending and descending auctions. We can also differentiate Auctions into Independent Private Value (IPV) and Common Value (CV) auctions according to whether bidders' valuations for the object are independent of other bidders.

#### 2.2.1.1 Independent Private Value (IPV) model

The basic auction environment is as follows: a single object is sold to *n* bidders. Each bidder knows her valuation $v_i$ (her type). Their valuations are independent. Two features make the Independent Private Value (IPV) model different from the later auctions to be discussed: (1) individual valuations are independent of other bidders'; (2) their valuations are independent of the information received from other bidders during the auctions. That is, bidders' valuations are private.  IPV models may not include the case where bidder behavior depends on one's expectation about others' valuations and bids, but it enables us to derive several important insights.

There are four main types of auctions, for a single item being sold (and many variants of these types): English and Dutch auctions (open auctions), and First- and second price sealed-bid auctions.

***English auction*** is an open ascending price auction, the most common form of auction used today, such as for selling commodities, antiques and artwork, used products, and real estates. During the auction, bidders bid against one another and bids rise sequentially. The auction ends when no participant is willing to bid further, and the last bidder pays the level of her bid. If there is a "reserve" price set for the item and the final bid does not reach that level, the item remains unsold.

***Dutch auction*** is an open descending price auction, commonly used for selling perishable commodities such as flowers, fish, and tobacco. The Dutch tulip auction is a typical example. The auction, start at a high price, which is reduced continuously until a bidder is willing to accept the bid or until the seller's reserve price is met.

***First-price sealed-bid auction*** is a type of sealed-bid auction, commonly used for B2B procurements, hosted by companies, organizations, and the government. In the auction, all bidders simultaneously submit sealed bids and the highest bidder wins and pays the level of her bid. Distinct from the previous auctions, bidders do not know others' bids and they can bid only once.

***Second-price sealed-bid auction*** (also known as a Vickrey auction) is another type of sealed auctions, commonly conducted in online environment, such as sponsored search auctions, but rarely used in other contexts. In the auction, bidders simultaneously submit sealed bids and the highest bidder wins and pays an amount equal to the second-highest bid rather than her own bid.

The most important theorem connecting the above auction mechanisms is **Revenue Equivalence Theorem.** According to Myerson (1981), Riley and Samuelson (1981) and Harris and Raviv (1981), Revenue Equivalence Theorem states that if bidders have values identically and independently distributed with a common distribution, then in equilibrium all auction mechanisms always award the object to the bidder with the highest value. Furthermore, given that a bidder with the lowest valuation receives zero in profits, all auction mechanisms generate the same revenue in expectation.

Two approaches to solving for symmetric equilibrium bidding strategies are the "First Order Conditions" approach and the "Envelope Theorem" approach.

***"First Order Conditions" approach.*** Bidder *i*'s expected utility is a function of her bid $b_i$ and her valuation $v_i$ ; that is,

$$u_i = max_{b_i}(v_i - b_i).\Pr\left[\, b(v_j) < b_i, \forall j \neq i\right] = max_{b_i}(v_i - b_i)F^{n-1}(b^{-1}(b_i)),$$

where we assume that the bidders' valuation distributions follow $F(.)$. The first order condition is

$$(v_i - b_i)(n-1)F^{n-2}(b^{-1}(b_i))f(b^{-1}(b_i))\frac{1}{b^{-1\prime}(b_i)} - F^{n-1}(b^{-1}(b_i)) = 0.$$

Then

$$b^{-1'}(b_i)) = (v_i - b_i)(n-1)\frac{f(b^{-1}(b_i))}{F(b^{-1}(b_i))}.$$

In the symmetric equilibrium, $b_i(v) = b(v)$, for all i. and $b(v_{min}) = v_{min}$. Then we can derive

$$b(v) = v - \frac{\int_{v_{min}}^{v} F^{n-1}(t)dt}{F^{n-1}(v)}$$

***The "Envelope Theorem" approach.*** Applying the envelope theorem (Milgrom and Segal 2002) on the expected utility equation of a bidder, we have

$$\frac{d(u)}{dv}|_{v=v_i} = F^{n-1}\left(b^{-1}\big(b_i(v_i)\big)\right) = F^{n-1}(v_i).$$

$u(v_i) = u(v_{min}) + \int_{v_{min}}^{v_i} F^{n-1}(t)dt$. As the bidder with the lowest valuation will never win, $u(v_{min}) = 0$. Combining with the last two equations, we can derive the same bidding equation as in "First order conditions" approach.

### 2.2.1.2 Common Value (CV) model

Common value auctions consider a situation where information about the value of the object for sale is dispersed among bidders. There are several different ways of modeling. One is to describe auctions in which the value of the goods is the same for all bidders but this value is not known at the time when auction starts, such as selling the rights of exploration of some natural resource. (Some call this auction a pure common value auction.) The other is to describe auctions in which (1) bidders have different information, (2) learning one bidder's information could cause others to re-assess her estimation about the item's value, and (3) the information of bidders are not independent. When bidder *i*'s estimation is high, bidder *j*'s is also likely to be high (Milgrom and Weber 1982). Some call it an interdependent values model, or affiliated value model (Menezes and Monteiro 2005).

In common value models, Revenue Equivalence may not hold. Common value auctions are usually associated with a feature, called "winner's curse" (Thaler 1988), because the winner tends to be the one who estimates the value higher than the other bidders, which signals that she overestimated the product value. Therefore, to avoid the winner's curse, a strategic bidder will bid a smaller fraction of her signal as the number of bidders increases (Thaler 1988; Ioannidis 2008). The explanation is as follows. Suppose each bidder doesn't know the true value of the item, but receives a signal, which is about the true value; that is,

$$s_i = v + e_i,$$

where $s_i$ is the signal, $v$ is the true value of the product, and $e_i$ is the error term and $E[e_i] = 0$ for bidder $i$.

Although the expectation of the error term is zero, but the bidder with the highest valuation is the one who receives the highest error larger than $E[e_i]$. As a result, she bids above the product value.

Cremer, et al. (2007) model a case in which bidders are uncertain about the product value, so they must pay admission fee to obtain their valuations. In their model, the duration of auction consists of discrete periods. They are able to set different reservation prices and the admission fee each period, by which they can let bidders truthfully reveal their valuations and extract fully of their surpluses. They find that the admission fees of the bidders increase if the bidders are given information simultaneously and decrease if they are given information sequentially at the beginning of each period.

### *2.2.2 Competing auctions*

Researchers have noted that bidding behavior in competing auctions differs from that in a single auction. Bidders consider future chances of winning in later auctions; they may learn from early auctions and hence update their valuations in the later auctions; they may bid across competing auctions available. Those bidding behaviors switch the demand and ensure that online competitive auctions do not act independently of one another. Since 1990s, a rapid growth in analytical and empirical literature about competitive online auctions has occurred (see the summarized list in Table 2-1). The studies can be classified into three categories according to the degree of overlap: studies on simultaneous, sequential and partially overlapping auctions. Ockenfels et al. (2006) also provide an excellent review of the earlier studies on online auctions.

#### 2.2.2.1 Simultaneous auctions

Simultaneous auctions are one special case of overlapping auctions, in which there is total overlap. The analysis of sellers selling identical products in simultaneous auctions can be traced to McAfee (1993)'s theoretical model. For our purposes, it is important to note that several studies consider cross-bidding an important bidding behavior in competitive auctions. They not only provide a theoretical proof those bidders always cross-bid to chase the lowest prices among auctions, but also empirically verify that the proportion of cross bids is large. Unlike those in our study, however, the overlapping auctions in their papers are run by multiple sellers. Moreover, in the simultaneous auctions, there is no need to consider learning and forward looking.

McAfee (1993) finds that in equilibrium auctions are set with efficient reserve prices and sellers post the efficient reserve price equal to their value of the product. His model is extended by Peters and Severinov (1997) to consider two conditions in which buyers know and do not know their own valuations before they choose the auction. Later, Peters and Severinov (2006) model the bidding behaviors and demonstrate the existence of a perfect Bayesian equilibrium at which the bidders search for the auctions with the lowest bid, always bid on an auction with the lowest "standing" bid, and bid with the minimum increment. The learning behavior of bidders (e.g., the value of the products, the number of bidders) is not discussed in their models.

Anwar et al. (2006) provide strong empirical support for the bidders' strategy prescribed by Peters and Severinov (2006) using data from eBay. In auctions with three different endings (at the same day, within an hour, within a minute of each other), they find that a significant proportion of agents bid across competing auctions (19 percent in the daily, 31 percent in the hourly and 32 percent in the minute sample). The number of bids submitted in the auction with the lowest standing bid is high (62 percent in the daily, 72 percent in the hourly and 76 percent in the minute sample). The ratio of the average price paid by cross-bidders to that by non-cross-bidders is approximately 0.93. Andersson et al. (2012) investigate simultaneous online auctions for identical tickets and show that 69.9% of the bids are cross-bids.

#### 2.2.2.2 Sequential auctions

Sequential auctions are another special case of overlapping auctions in which there is no overlap. Studies on sequential auctions date back to the 1980s. Ashenfelter (1989) observes that the prices of wines decrease over time when identical lots of items auctioned sequentially. Such a price decline is observed in many empirical studies with different explanations as follows. McAfee and Vincent (1992) regard risk premium crucial to this decline; that is, the risk-adverse bidders bid less in the prior auction due to the consideration of the risks involved in the future price in subsequent auctions. Engelbrecht-Wiggans and Kahn (1999) argue that the size of demand plays a key role. Using data from cattle auctions, they show that the price declines in sequential auctions as demand reduces. Zeithammer (2006) considers buyers' forward-looking behaviors. The bidders bid less concurrently because they expect the chance in future auctions for a lower price. Arora, et al. (2003) studies the optimal bidding strategies of a rational and risk-neutral bidder in sequential online auctions and how the volatility in the number of bidders in the second period impacts the first period bid.

In our model, we posit that prices fall in overlapping auctions due to bidders' forward-looking behaviors. But we further incorporate bidders' cross-bidding and learning in overlapping auctions, and consider that those behaviors also play important roles in bidders' bidding strategies.

**2.2.2.3 Partially overlapping auctions**

Auctions can be overlapped partially. Current studies focus on the impact of overlapping auctions after the level of overlap is given, but not on the overlapping strategy itself; that is, they compare the seller's revenue, the efficiency of auctions, or bidding prices between non-overlapped auctions with ones, which are to a certain extent overlapped. Stryszowska (2006) compares the pricing and efficiency between simultaneous and overlapping online auctions under the independent private value framework. In the overlapping condition, she sets two auctions running for three periods with one period overlapped. Overlapping auctions are more efficient than simultaneous auctions, because in overlapping auctions, even if the coordination starts late, bidders always have time to safely reallocate without engaging in risky sniping. Hung et al. (2007) construct a model of overlapping English auctions, one half of which are overlapped with an early auction and the other half overlapped with a later auction; that is, at any point, bidders always observe two auctions running. In equilibrium, the bidders, on all auctions except the last one, forward look and thus bid less than their valuations. Second, a buyer never cross-bids and participates only in one auction which ends earlier.

Hoppe (2008) designs an experiment to test bidders' behavior and auction efficiency in second-price English auctions. In the experiments, four bidders participate in three overlapping auctions. There are two treatments of the degree of overlap: 5/6 and 1/6 overlaps. He finds that the seller's revenue is significantly higher in overlapping auctions than in simultaneous auctions, but the degree of overlap has no impact on the seller's revenue.

Bapna et al. (2009) collect data from Mega Club, an online auction website, and examine the impact of many market-level factors, such as the price information of prior auctions, the degree of overlap, the auction format, and the overall market supply, on auctions' final bids. They find that overlapping has a significant negative influence on prices, and the magnitude of such negative impact increases with the degree of overlap. They separate the impact of overlap into two components: that from the preceding overlap and that from the following overlap. Information from preceding overlapping auctions gives signals about prior prices and demand (e.g. the number of bidders) that is not available in simultaneous auctions. This strongly influences the price of the focal auction. Information from subsequent overlapping auctions provides information about the supply (e.g. the number of products offered), which causes bidders to bid less than their true valuation because of the positive opportunity cost to bid in the future auction. The impact of this information is similar to forward-looking and learning behavior discussed in our model.

Haruvy et al. (2014) were first to use click-stream data to study bidders' search behavior among concurrent running auctions by manipulating three factors of product description, the number of competing auctions, and the degree of overlap. They found that the higher degree of auction overlap increased bidders' price sensitivity. It is more likely that they switch to the auction with the lower listing bid. However, they found no significant evidence of the mediating effect of bidders' search behavior on this relationship. To summarize, the study of partially overlapping auctions is relatively new both empirically and analytically. We posit that two important behaviors of bidders are likely to influence the final bids in overlapping auctions: forward-looking and learning. Our paper differs from the literature in three aspects. First, we are the first to build a theoretical model to incorporate bidders' forward looking and learning behavior to investigate the optimum overlapping strategy. Second, in our model, the seller is able to vary the degree of overlap to sell multiple products, a condition more typical of the real auction environment (Bapna et al. 2009). Third, previous theoretical literature predicts that overlapping auction design always reduces a seller's revenue, as it reduces the highest bids in auctions (Huang et al.

2007, Zeithammer 2006). In contrast, empirical observations show the growth of overlapping auctions with different degrees of overlap (Bapna et al. 2009). Therefore, by considering both forward looking and learning, we provide a theoretical explanation of such phenomenon. Predictions from our analytical studies are then expected to provide managerial recommendations more applicable to the real practice in online auctions.

Table 2-1 Summary of research on competing auctions

| Paper | Type | Design | Overlap | Research focus | Results |
|---|---|---|---|---|---|
| McAfee (1993) | Analytical | Different sellers | Total | Reserve price | There exists am equilibrium where sellers post an efficient reserve price equal to the sellers' value of the product and an auction with efficient reserve is an optimal mechanism. |
| Peters and Severinov (1997) | Analytical | Different sellers | Total | Reserve price, Information uncertainty | The authors expand McAfee (1993) model to consider two conditions in which buyers know and do not know their own valuations before they choose the auction. |
| Arora, et al. (2003) | Analytical and Empirical | One seller | Zero | The impact of the uncertainty of future bidders | The uncertainty on the number of bidders in the second period lowers the first period bid. |
| Peters and Severinov (2006) | Analytical | Different sellers | Total | Cross-bid | There exists a perfect Bayesian equilibrium with bidders cross bid and always bid on the auction with the lowest "standing" bid and bid with the minimum increment. |
| Anwar et al. (2006) | Empirical | Different sellers | Total | Cross-bid | The authors provide an empirical support to the bidders' strategy prescribed by Peters and Severinov (2006) using data from eBay. |
| Andersson, et al. (2012) | Empirical | Different sellers | Total | Cross-bid | The authors provide empirical support for the bidders' strategy prescribed by Peters and Severinov (2006). On tickets online auctions, they observe that 69.9% of the bids are cross-bids even though a majority of the bidders are never cross-bidders. |

| | | | | | |
|---|---|---|---|---|---|
| Ashenfelter (1989) | Empirical | Different sellers | None | Prices decline | Prices of wine decrease over time when identical lots of item auctioned sequentially |
| McAfee and Vincent (1992) | Empirical | Different sellers | None | Prices fall, risk premium | Prices fall in sequential auctions due to the risk premium. Risk adverse bidders bid less in the prior auction, due to a risk premium considering prices in subsequent auctions. |
| Engelbrecht-Wiggans and Kahn (1999) | Analytical | Different sellers | None | Prices fall | The authors use data from dairy cattle auctions plus independent appraisals of the cattle sold to verify the existence of the "declining price anomaly" in sequential auctions. |
| Zeithammer (2006) | Analytical and Empirical | One seller | None | Prices fall, Forward-looking | Buyers bid less due to forward-looking strategies in sequential auctions. |
| Stryszowska (2006) | Analytical | One seller | 1/3 | Auction efficiency | Overlapping auctions are more efficient than simultaneous auctions. On the other hand, the seller may prefer simultaneous auctions. |
| Ockenfels, et al.(2006) | Survey | | Not mention ed | | A comprehensive survey on theoretical, empirical, and experimental research on bidder and seller strategies in online auctions, including online auction design. |
| Hung, et al. (2007) | Analytical | One seller | 1/2 | Forward looking, cross-bid | In equilibrium, bidders forward look, therefore, bid their last bid less than their valuations, except for the last auction. The expected equilibrium prices are identical among all auctions, except for the last. Secondly, the buyers never cross-bid. |
| Hoppe (2008) | Experimen tal | One seller | 1/6 and 5/6 | Seller's revenue | Seller's revenue is significantly higher in overlapping multiple auctions than in simultaneous auctions and the degree of overlap does not impact seller revenue. |
| Bapna, et al. (2009) | Empirical | One seller | Data-based | Information impact | Overlap of an auction with other competing auctions has a significant negative influence on bids, and the impact from information about following auctions is stronger than that from information about prior closing auctions. |

| | | | | | |
|---|---|---|---|---|---|
| Haruvy, et al. (2014) | Experimental | One seller | Full and 1/2 | Bidders' search and choice | Using click-stream data, the authors designed experiments and found that the bidders' search and choice of auctions effected by information transparency, the number of simultaneous auctions and the degree of overlap. |

This paper contributes to the literature in several ways. First, it adds to the growing online auction literature. Online auctions do not constrain bidders from participating at multiple auctions simultaneously as the traditional auctions. With the growth and the popularity of online auction, multiple competing auctions for identical products become common phenomena. However, most of the literature focused on simultaneous or sequential auctions, and only a few studies considered overlapping online auctions (see Haruvy et al. 2008 for a review). Those, which did study overlapping online auctions, analyzed bidders' cross-bidding (Hung et al. 2007) and bidders' forward-looking strategies (Zeithammer 2006, 2007a). Hung et al. (2007) found that forward-looking bidders bid less than their valuations in all auctions but the last one and bidders never cross-bid during the bidding process. Hoppe (2008) in his laboratory experiments found that the seller's revenue was significantly higher in overlapping auctions than in simultaneous ones because the bidder with the second highest valuation had a chance to win in the other auction through cross-bidding when she was outbid in one auction. However, Hoppe did not find the impact of the changes in the degree of overlap on the seller's revenue. In a series of studies of forward-looking, Zeithammer (2006) modeled identical sequential eBay auctions and found that bidders took information about competing auctions into account, and thus reduced their bids when they knew about the future availability of products. Zeithammer (2007a, b) also showed in his analytical model that sellers could influence bidders' bid-shades by the time he announced the opening of future auctions: at the start of the auction or after the end of the earlier auction. His analysis showed that commitment reduced bid shading, while waiting reduced demand uncertainty. Bapna et al. (2009) empirically studied the impact of information flow on final bids. On one hand, they found that information from preceding auctions provided signals about prior prices and demand, which might influence the price of the current auction; on the other hand, they found that information about following auctions provided the number of products offered, which caused bidders to shade their bid. Such impacts of information are analogous to the notion of bidder learning and forward-looking in our model. However, our paper takes a different approach. We study the seller's decision on the degree of overlap between auctions, and the factors that influence this decision. We also provide the theoretical explanation for the popularity of overlapping online auctions.

Second, this paper contributes to the existing literature on information release. The benefits to sellers of offering information are well documented in the literature. It is generally considered that it is optimal for sellers to fully disclose information (Ganuza 2003; Bergemann and Pesendorfer 2007; Eso and Szentes, 2007). The overlapping strategy can be viewed as one type of information release, where before auctions starts a seller posts the number of overlapping auctions and also selects the degree of overlap to influence the precision of the information on the bidders' valuations (about the product value).

Concerning bidders' learning, the literature essentially focused on exogenous signals. For example, Hossain (2008) considered a model of when to bid, in which one bidder was informed and the other was uninformed about the product value. The uninformed bidder received a random signal and learned whether her valuation was above or below the current price. He found that in equilibrium the informed bidder bid early if her valuation was low and bid late otherwise, while the uninformed bidder bid at any time during the course of the auction. Compte and Jehiel (2004) modelled the bidding strategy in the context where bidders learned their valuations at a random time. The learned value can be positive (larger than average) or negative (equal to average); therefore, he found that waiting was always a weakly dominating strategy in equilibrium. In the above papers, the learned value is not related to the bidding context, but is exogenously given by nature. Our study extends the scope of the literature on learning to include endogenous signals. In our paper, the information a bidder learns is influenced by the overlapping conditions. Furthermore, we model bidders' learning in three possible types: 1) bidders learn at the end of the first auction about other bidders' valuations; 2) bidders learn at the end of the first auction about the ending price of the first auction, and 3) bidders learn about the drop-out bids of other bidders during the bidding process.

We find that the seller's optimal strategy differs in these three types of learning. When bidders learn at the end of the first auction about the other bidders' valuations (type 1), there exist some conditions in which partial overlapping strategies are optimal, regardless whether demand is fixed or varies during the auctions. In the other two types of learning, there exist some conditions in which partial overlapping strategies are optimal when demand varies. Otherwise the seller's optimal strategy is to run almost simultaneous auctions (i.e. a small time gap is needed between two auctions).

### 2.3 Model setup

We consider a context where a seller sells two homogeneous products to $n$ bidders in two ascending bid auctions. The two auctions overlap to $a$ degree, where $a \in [0,1]$ (see Figure 2-2). When $a = 0$, the seller holds *sequential* auctions; when $a = 1$, he holds *simultaneous* auctions; when $1>a>0$, he holds *partially overlapping* auctions. For simplicity, the duration of each auction is set equal to 1, and thus the total duration is $2-a$.

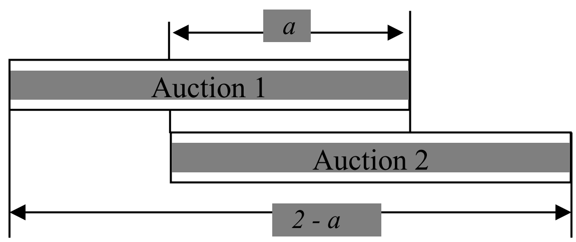


Figure 2-2. Overlapping auctions, where *a* is the degree of overlap

The number of bidders is assumed to be greater than 3 to avoid strategic interactions among bidders.[3] Bidders are assumed to be risk averse, and want at most one product. Bidders are uncertain about the product value, which is assumed to fall within a range. After the start of the auction and new information about product value becomes available, bidders will update their valuations. We denote the prior belief about the product valuation for bidder *i* as $v_i$, which is randomly drawn from a common distribution $F(v)$. Hence we

[3]When there are two bidders, each will win an auction and pay the minimum bid increment. When there are three bidders, the bidders with the first and second highest valuation will each win an auction, each paying an amount equal to the value of the third highest bidder. Hence at least 4 bidders are needed.

assume valuations to be affiliated (Milgrom and Weber 2002). $F(v)$ is assumed to follow a uniform distribution in the range from [0, 1]. Let $E[.]$ denote the expected value of bidders' valuations and let the superscript denote the ranking (of valuation) of $n$ bidders, then $v^{[k]}(n)$ represents the $k^{th}$ highest valuation among $n$ bidders and $E[v^{[k]}(n)]$ represents the expected value of the $k^{th}$ highest valuation among $n$ bidders.

Both the seller and bidders are assumed to discount future revenues or rewards. Discount rates may vary between bidders and the seller, and may also differ across product categories (e.g. discount rates tend to be higher for high-tech or seasonable products). For simplicity, we assume discount rates (denoted as $\beta$) to be the same for bidders and the seller, ranging within [0, 1].

***Forward-looking bidders.*** Forward-looking bidders anticipate the prices of future auctions and take that into account when placing a bid. In particular, anticipating an opportunity to win an identical item in a future auction at a lower price, a forward-looking bidder reduces her final bid in the current auction (Jofre-Bonet and Pesendorfer 2003; Zeithammer 2006, 2007a). This reduction is called bid-shading. The amount of bid-shading at $a$ degree of overlap is denoted as $\Delta(a)$.

***Bidders' learning.*** Learning reduces bidders' uncertainty and is modeled in two ways: 1) bidders only learn at the end of the first auction and update their valuations based on the information released (i.e., the valuation distribution of other bidders), or 2) they learn during the bidding process and update their valuations based on the final bids of other bidders. The first type of learning tends to be more applicable when bidders' valuations are not readily observable during the auction (e.g., due to a significant amount of snipe bidding).[4] We adopt the first way of learning in the full model and leave the second type for the extension.

***The sequence of the game.*** We model the game in four stages: 1. The seller decides the degree of overlap $a$ between the two auctions; 2. All bidders participate in the first auction, and after time period $1-a$, the second auction starts. After this time, bidders are allowed to cross-bid between auctions. When the first auction ends, the winner pays the amount of her bid and leaves with the item; 3. The remaining bidders update their valuations; 4. The remaining bidders participate in the second auction. At the conclusion of the second auction, the winner pays the amount of her bid.

The sequence of the game and bidders' prior value distribution are assumed to be common knowledge. We are looking for the subgame perfect equilibrium of the game.

### 2.4 Analysis and results

We start with a benchmark model, in which bidders are certain about the product value (i.e. no learning), and next consider the full model, in which uncertain bidders learn about product value.

#### *2.4.1 Benchmark model: bidders forward-look only*

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[4] Over one-third of bids arrive in the last few minutes and many bidders reveal their maximum willingness to pay until the closure of the auction (Roth and Ockenfels 2002; Bajari and Hortascu 2003).

The benchmark model is a private value model, where bidders know their own valuation with certainty and thus do not learn. However, bidders are forward-looking and consider potential prices in future auctions when $0 \le a < 1$ (i.e. two auctions are partially overlapped).[5]

The model is solved through backward induction. We start at the **last stage**, where $n-1$ bidders bid in the second auction. Since no further auctions follow, this is a standard ascending bid auction, in which the bidder with the highest valuation wins the item and pays the value she bids. Therefore, the expected highest bid in the second auction, denoted as $b_2$, is equal to the expected second-highest valuation among the $n-1$ remaining bidders (see Step 2 of Appendix 2-B for technical details), hence:

$$b_2 = E[v^{[2]}(n-1)]. \tag{2-1}$$

Given that bidders are forward-looking, the bidder with the highest valuation can select to win either in the first or in the second auction. If she wins the second auction, her expected utility is: $u_2^{[1]} = E[v^{[1]}(n)] - E[v^{[2]}(n-1)]$, where $u_2^{[1]}$ denotes the expected utility for winning in the second auction and is equal to the expected valuation minus the expected highest bid in the second auction. Using $E[v^{[2]}(n-1)] = E[v^{[3]}(n)]$[6], $u_2^{[1]}$ can be further expressed as:

---

[5] We consider the case where the degree of overlap is smaller than 1, such that the ending times of two auctions are sufficiently different, that bidders who lose in the first auction, can still bid in the second one.

[6] As bidders do not learn, valuations remain unchanged, and the expected second-highest

$$u_2^{[1]} = E[v^{[1]}(n)] - E[v^{[3]}(n)] \quad . \tag{2-2}$$

Since there is no bidder learning, we skip the **third stage** and consider the **second stage**, in which $n$ bidders bid in the first auction. The high value bidder places her maximum bid ($b_1$) up to the level which makes her indifferent as to winning in the first or the second auction.

$$E[v^{[1]}(n)] - b_1 = \beta^{1-a} u_2^{[1]}, \tag{2-3}$$

where the left side of Equation (2-3) is the expected utility of winning in the first auction, and the right side is the expected time-discounted utility of winning in the second auction. Substituting Equation (2-2) into (2-3), we obtain her expected highest bid in the first auction as follows:

$$b_1 = E[v^{[1]}(n)] - \beta^{1-a} E[v^{[1]}(n) - v^{[3]}(n)] . \tag{2-4}$$

Now we calculate bid-shading $\Delta(a)$. Without forward-looking, the winner needs to pay $E[v^{[2]}(n)]$ (see Step 2 of Appendix 2-B for details); with forward-looking, the winner needs to pay $b_1$ (see Equation 2-4). $\Delta(a)$ is the difference between the above two values, i.e., $\Delta(a) = -E[v^{[1]}(n) - v^{[2]}(n)] + \beta^{1-a} E[v^{[1]}(n) - v^{[3]}(n)]$, and is simplified as (see Appendix 2-B for the proof):

---

valuation among $n-1$ bidders in the second auction equals to the expected third highest valuation among $n$ bidders in the first auction.

$$\Delta(a) = (2\beta^{1-a} - 1)/(n+1), \quad (2\text{-}5)$$

where the time-discount rate shall satisfy the condition that $\beta^{1-a} \geq 1/2$. Equation (2-5) implies the following: 1) $\partial\Delta(a)/\partial a > 0$ .The amount of bid-shading increases, as the degree of overlap becomes larger. Therefore, forward-looking behavior benefits bidders but leaves the seller worse off; 2) $\partial\Delta(a)/\partial n < 0$ . The amount of bid-shading decreases, as the number of bidders increases. This suggests that as competition among bidders intensifies, the chance to win in the second auction decreases, and as such, bidders shade their bids less in the first auction.

We last look at the **first stage**, in which the seller decides the degree of auction overlap. The seller's revenue is the sum of the highest bids in two auctions. Based on (2-1) and (2-4), the seller is expected to gain $R = E[v^{[1]}(n)] - \beta^{1-a} E[v^{[1]}(n) - 2v^{[3]}(n)]$, which is further simplified as:

$$R = \frac{n}{n+1} - \beta^{1-a}(\frac{4-n}{n+1}).$$

When $a = 1$ ( i.e. two auctions run simultaneously), bidders cannot forward-look, resulting in seller's expected revenue being two times the third-highest bidders' expected valuations: $2E[v^3(n)]$. (Because the two simultaneous auctions act as a single auction selling two identical products, where bidding stops when only two bidders remain.) This can be further simplified as $2(n-1)/(n+1)$ (see Appendix 2-A for technical details). Then the seller`s revenue and bid-shading are:

$$R = \begin{cases} \frac{n}{n+1} + \beta^{1-a}(\frac{n-4}{n+1}) & 0 \le a < 1 \\ \frac{2(n-1)}{n+1} & a = 1 \end{cases} . \quad (2\text{-}6)$$

$$\Delta(a) = \begin{cases} \frac{2\beta^{1-a} - 1}{n+1} & 0 \le a < 1 \\ 0 & a = 1 \end{cases} . \quad (2\text{-}7)$$

Forward-looking results in bid-shading in the first auction (by the winning bidder), and thus reduces the seller's profit. Equation (2-7) shows the trajectory of the amount of bid shade as the degree of overlap changes. As the degree of overlap *a* becomes larger, the amount of bid-shading gradually increases, because the total auction duration decreases, resulting in less discounting of future pay-offs. In simultaneous auctions (*a* = 1), bidders cannot forward-look and hence do not reduce their bid. We summarize our finding in Proposition 2-1.

**PROPOSITION 2-1.** *When bidders are certain about product value, the seller's optimal selling strategy is to run auctions simultaneously (* $a^* = 1$ *).*

It is straightforward to show that, given a certain number of bidders, the revenue with total overlap is larger than that with any partial overlap (see Figure 2-3). Hence, it is optimal for a seller to adopt simultaneous auctions, since this strategy eliminates the loss due to forward-looking and time-discounting. This result is consistent with the finding in Zeithammer (2006).

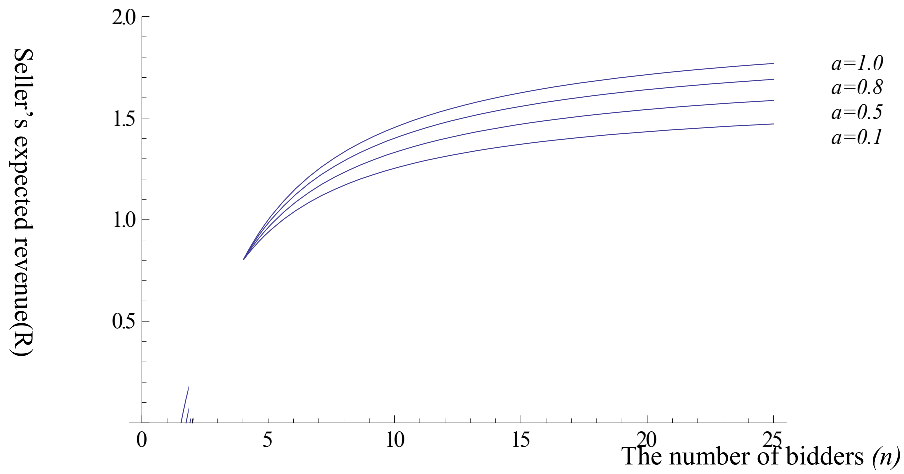


Figure 2-3. Seller's revenue for different degree of overlap (for $\beta = 0.6$ )

### *2.4.2 Full model: bidders forward-look and learning*

Next we add bidder learning about the product value from the bids in the first auction.

#### *2.4.2.1 Bidder's learning*

***Bidders' valuations.*** Bidders are uncertain about product value. Bidder *i*'s valuation, denoted as $v_i$, has the form:

$$v_i = w_i + \varepsilon_v , \qquad (2\text{-}8)$$

where $w_i$ is the bidder *i*'s expected valuation, and $\varepsilon_v$ is the error term, assumed to follow a uniform distribution $b(.)$ with mean 0 and variance $\sigma_v^2$ in support of $[-\lambda, \lambda]$ and with CDF $B(.)$. Bidder *i*'s valuation $v_i$ falls in the range from $[w_i - \lambda, w_i + \lambda]$. When $\lambda = 0$ (i.e., bidders have certain valuations), then the model reduces to our benchmark model.

Additionally, bidders' expected valuations are heterogeneous, and bidder *i*'s expected valuation $w_i$ equals:

$$w_i = E[w] + \varepsilon_w, \qquad (2\text{-}9)$$

where $E[w]$ is the mean value across all bidders and $\varepsilon_w$ is the error tem, which is assumed to follow distribution, $f(.)$ with mean $0$ and variance $\sigma_w^2$ and with corresponding CDF $F(.)$.

Figure 2-4 provides a graphical illustration, where *f(w)* depicts the aggregate distribution of valuation across bidders, and the three small curves (*b(.)*), superimposed on *f(w)*, show the variation in individual bidders' valuations.

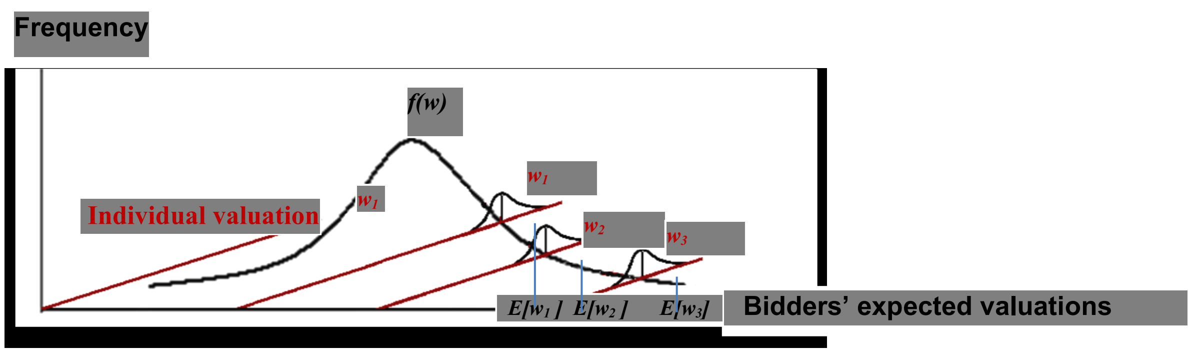


Figure 2-4. The distributions of bidders' valuations

***Highest bid.*** All bidders in an ascending bid auction, except for the winner, bid up to their maximum willingness to pay (MWTP). When bidders' valuations are certain, the MWTP is equal to their valuations. When valuations are uncertain, risk-averse bidders will bid up to their expected valuation minus a risk premium ($r\sigma_v^2/2$). Mathematically,

$$MWTP_i = w_i - r\sigma_v^2/2, \qquad (2\text{-}10)$$

where $r$ is the risk coefficient. It is positive to guarantee that bidders are risk averse. This mean-variance formulation has been widely used in finance studies (Pulley 1983; Aivazian, et al. 1983), where it has been demonstrated to be a valid approximation of the Von Neumann- Morgenstern utility function. These existing studies have also shown that decision makers can effectively maximize expected utility when knowing only the mean and the variance of their valuation distributions. A higher level of uncertainty will result in a lower MWTP.

***Updating valuations.*** The first type of learning we consider occurs at the end of the first auction. At the end of the first auction, bidders receive a signal *s* on the product value, and then update their valuations. Their posterior beliefs can be expressed as a weighted average of the prior mean and the signal in the form of a linear approximation, following Erbenova and Vagstad (1999) and Vagstad (2007): $E[v_i \mid s_i] = \tau s_i + (1-\tau) w_i$,[7] where

---

[7] This simple learning equation retains general properties of Bayesian updating. If we use Bayesian updating, the posterior distribution of valuation can be expressed as: $b(v_i) = f(v_i|S_i) = \frac{f(v_i)\ p(S_i)}{\int_0^1 f(v)\ p(S_i)dv}$. First, we show that the martingale property of the learning is remained, i.e.,

$$E(v_i|S_i) = \int_0^1 \frac{f(v_i)\ p(S_i)}{\int_0^1 f(v)\ p(S_i)dv} g(S_i)dS_i = f(v_i)\frac{\int_0^1 p(S_i)g(S_i)dS_i}{\int_0^1 f(v)\ p(S_i)dv} = f(v_i)\frac{p(S_i)\int_0^1 g(S_i)dS_i}{p(S_i)\int_0^1 f(v)\ dv} = f(v_i).$$

Second, if both the signal and the bidders' initial valuation follow normal distributions, the result is the same; that is, $E(v_i|S_i) = \frac{(1/\sigma_{prior}^2)E(v_i)+(1/\sigma_{signal}^2)E(S_i)}{(1/\sigma_{prior}^2)+(1/\sigma_{signal}^2)} = (1-\tau)E(v_i) +$

$\tau \in [0,1]$ denotes the precision of the signal. Let the distribution of bidder *i*'s posterior belief be $BB(v_i)$ with mean $w_i'$ and variance $\sigma_{v'}^2$ in the range of $[w_i' - \lambda', w_i' + \lambda']$, then

$$BB(x_i) = \int_{\{s:E[v_i|s]<x_i\}} b(s)ds = \int_{\{s:s<(x_i-(1-\tau)w_i)/\tau\}} b(s)ds = B(\frac{x_i-(1-\tau)w_i}{\tau}), \tag{2-11}$$

Using (2-11), we obtain the mean and variance of bidder *i*'s posterior belief as:

$$w_i' = w_i, \tag{2-12}$$

$$\sigma_{v'}^2 = (1-\tau)^2 \sigma_v^2. \tag{2-13}$$

Given the distribution of prior beliefs $B(v) \sim U[w-\lambda, w+\lambda]$ and the above results for the posterior belief, we obtain $\sigma_v^2 = \lambda^2/3$ and $\sigma_{v'}^2 = (1-\tau)^2 \lambda^2/3$, respectively.

After bidders update, the expected value remains the same. However, the range of the expected value distribution shrinks, so bidders are more certain about their valuations. When the signal is more accurate (i.e. a larger value of $\tau$), the bidders place less weight on their prior beliefs. Both the updating and the accuracy of signals decrease bidders' uncertainty, resulting in more aggressive bidding in the second auction.

***Degree of overlap.*** The degree of overlap may also influence the precision of the released signal. In sequential auctions bidders bid up to their MWTP in the first auction, but in overlapping auctions they may switch to the second auction before they bid up to their MWTPs in the first auction. Therefore, the greater the degree of overlap, the less precise

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$\tau E(S_i)= (1-\tau)w_i + \tau s_i$, where we let $\tau = \frac{1/\sigma_{signal}^2}{(1/\sigma_{prior}^2)+(1/\sigma_{signal}^2)}$, and $s_i = E(S_i)$.

the signal received from the first auction. Mathematically, $d\tau / da < 0$. Hence by varying the degree of overlap, the seller is able to influence the precision of the released information.

In the extreme case of simultaneous auctions ($a = 1$), no new information is revealed and hence bidders do not update, so let $\tau(1) = 0$. In the other extreme case of sequential auctions ($a$=0), the maximum amount of information is released, so we let $\tau(0) = 1$. To show the relationship between updating and the degree of overlap analytically, we specify $\tau = 1 - a^{\kappa}$. This functional form ensures $d\tau / da < 0$, and it also provides sufficient flexibility to capture possible negative relationships between $\tau$ and $a$. Parameter $\kappa > 0$, measures the ease of learning, given a fixed amount of information. For example, if $\kappa$ is large, it is more difficult to reduce uncertainty due to the nature of the product (e.g. art work).

***2.4.2.2 Seller's optimal overlapping strategy***

The game is modelled through four stages as specified in Section 2.2 and solved through backward induction. The solution considers cases of $a = 1$ and $a \neq 1$. When $a = 1$, two auctions run simultaneously and bidders cannot forward-look, and $\Delta(1) = 0$ and $\tau(1) = 0$. Based on the MWTP function (10) in Section 2.3.2.1, the seller's expected revenue is $E[v^{[2]}(n)] + E[v^{[3]}(n)] - r(\sigma_v^2)$, which can be simplified as $2(n-1)/(n+1) - r\lambda^2/3$. When $a \neq 1$, bidders are forward-looking and learn. The optimal overlapping strategy is summarized as follows (see Appendix 2-C for the proof).

**PROPOSITION 2-2.** *When bidders are uncertain about their valuations,*

1. *The amount of bid shading is* $\Delta(a)=\begin{cases}\frac{2\beta^{1-a}-1}{n+1}+[(1-\tau)^2\beta^{1-a}-1]\frac{\lambda^2 r}{6} & 0\le a<1\\ 0 & a=1\end{cases}$, *and the seller's revenue is* $R=\begin{cases}\frac{n}{n+1}+\beta^{1-a}(\frac{n-4}{n+1}-\frac{r(1-\tau)^2\lambda^2}{3}) & 0\le a<1\\ \frac{2(n-1)}{n+1}-\frac{r\lambda^2}{3} & a=1\end{cases}$.

2. *There exists a unique degree of overlap* $a^*\in[0,1]$ *which maximizes the seller's expected revenue in two auctions selling identical products. In particular, the optimal degree of overlap is as follows:*

   - *When* $\beta\ne 1$ *(time discounting),*

   *1) If* $\kappa>\max((\frac{3(n-4)}{(n+1)r\lambda^2}-1)\frac{(-\ln\beta)}{2},\frac{1+\tilde{a}\ln\beta}{2})$*, then* $a^*=\tilde{a}$.

   *2) If* $\kappa\le\min((\frac{3(n-4)}{(n+1)r\lambda^2}-1)\frac{(-\ln\beta)}{2},\frac{1+\tilde{a}\ln\beta}{2})$*, then if* $\beta>\tilde{\beta}$*,* $a^*=0$*; else* $a^*=1$.

   *3) If* $\min((\frac{3(n-4)}{(n+1)r\lambda^2}-1)\frac{(-\ln\beta)}{2},\frac{1+\tilde{a}\ln\beta}{2})<\kappa<\max((\frac{3(n-4)}{(n+1)r\lambda^2}-1)\frac{(-\ln\beta)}{2},\frac{1+\tilde{a}\ln\beta}{2})$*,*

*then*

   *a) If* $(\frac{3(n-4)}{(n+1)r\lambda^2}-1)\frac{(-\ln\beta)}{2}>\frac{1+\tilde{a}\ln\beta}{2}$*, then* $a^*=1$.

   *b) If* $(\frac{3(n-4)}{(n+1)r\lambda^2}-1)\frac{(-\ln\beta)}{2}<\frac{1+\tilde{a}\ln\beta}{2}$*, then if* $\beta>\tilde{\beta}$*,* $a^*=0$ *and otherwise,* $a^*=1$.

   - *When* $\beta=1$ *(no time discounting), if* $r\lambda^2>6/(n+1)$*, then* $a^*=0$*; else* $a^*=1$.

*where* $\tilde{a}$ *satisfies* $(\tilde{a}^{2\kappa}-\frac{3(n-4)}{(n+1)r\lambda^2})\ln\beta=2\kappa\tilde{a}^{2\kappa-1}$ *and* $\tilde{\beta}=\frac{n-2}{n-4}-\frac{r\lambda^2(n+1)}{3(n-4)}$.

Proposition 2-2 shows the optimal degree of overlap under different market conditions: the extent of valuation uncertainty (or the perceived risk due to valuation uncertainty ( $r\lambda^2/2$ )), the discount rate ( $\beta$ ), and the ease of learning ( $\kappa$ ). As illustrated in Figure 5, the seller's revenue increases and then decreases as the degree of overlap becomes larger. The maximum revenue (1.2451) is reached at 0.8285 degree of overlap.

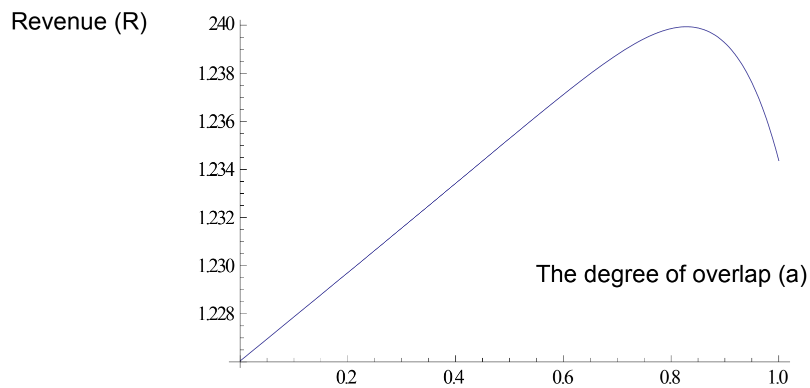
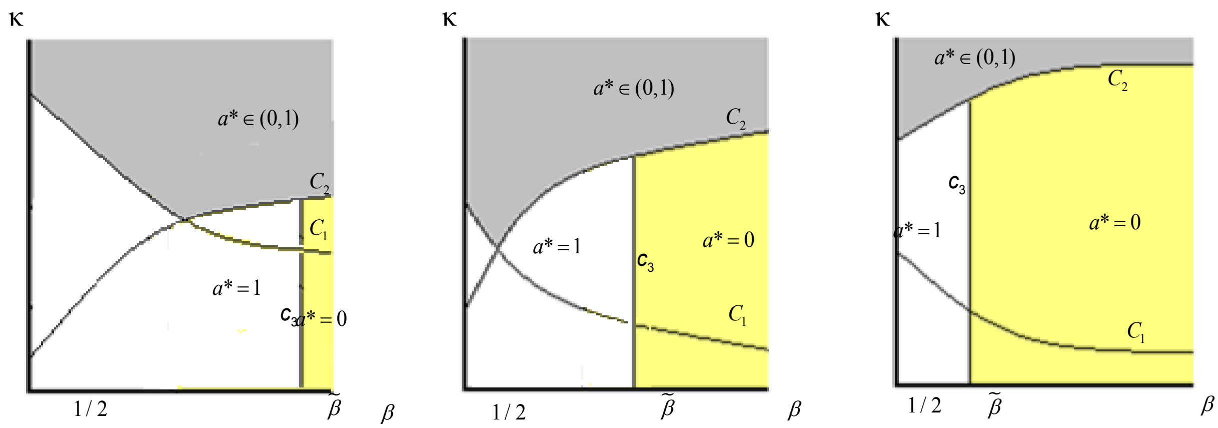



Figure 2-5. Seller's revenue for different degrees of overlap

( $\lambda = 0.25,\ n = 7,\ \kappa = 5,\ r = 0.5,$ and $\beta = 0.95$ ).

***Graphic illustration.*** Figure 2-6 helps us visualize the optimal overlapping strategies via three graphs. The x-axis denotes the time-discount rate $\beta$ and the y-axis the ease of learning $\kappa$ . From left to right, bidders' valuation uncertainty ( $r\lambda^2/2$ ) (when $\beta \neq 1$ ) increases from low to high.

In each graph, the strategy space is divided by three lines, named $c_1$, $c_2$ *and* $c_3$, into three regions, where either simultaneous, sequential or partial overlapping strategies are optimal (see Step 2 of Appendix 2-C for technical details). We find that the partial overlapping strategy tends to be more profitable than simultaneous and sequential selling strategies when (1) bidders' uncertainty about product value is at a mid-range, (2) bidders' valuation uncertainty is easy to reduce via learning (a high level of $\kappa$ ), and (3) the effect of time-discounting is not strong. Changing model parameters affects the locations of these three lines, but the relative configuration remains the same.




(a) Low valuation uncertainty (b) Medium valuation uncertainty (c) High valuation uncertainty

Figure 2-6. A graphical presentation of the findings in Proposition 2-2

***Time-discounting***. Taking the derivative of the optimal degree of overlap with respect to the time-discounting, we obtain a negative relationship between time-discounting and the degree of overlap when the time-discounting rate is sufficient large (i.e., there is limited time-discounting). There exists a unique value for $\hat{\beta}$ such that $\partial a^* / \partial \beta < 0$ when $\beta > \hat{\beta}$, and $\partial a^* / \partial \beta = 0$ otherwise. This is because time-discounting reduces seller's profit, thus it is optimum to shorten the duration (i.e. to increase $a^*$). For a numeral illustration, solving the optimal degree of overlap, we apply the same parameter values used in the example for Figure 5 except for $\beta$. We find that $\hat{\beta} = 0.7554$, and when $\beta \in [0, 0.7754]$, $a^* = 1$; when $\beta \in [0.7754, 1]$, $a^*$ decreases as $\beta$ increases. As a comparison in the example from Figure 2-5, when $\beta = 0.95$, $a^* = 0.8285$.

When there is no discounting ($\beta = 1$), it is optimal for a seller to run sequential auctions when the valuation uncertainty satisfies $r\lambda^2 > 6/(n+1)$, and run simultaneous auction otherwise. Without accounting for time-discounting, the seller is better off to let bidders' learn by running sequential auctions when bidders' valuation uncertainty is high, however, when uncertainty is low it is better to run simultaneous auctions, to stop bidders from forward-looking.

***Valuation uncertainty.*** This is measured by $\sigma_v^2$, where $\sigma_v^2 = \lambda^2/3$. From Figure 6 we see that as valuation uncertainty increases (from left to right), the region where sequential auctions are optimal increases, while the regions for simultaneous and partial overlapping auctions decreases. From Proposition 2-2, we also derive $\partial a^* / \partial \lambda^2 < 0$, suggesting that high bidders' valuation uncertainty prompts a seller to reduce the degree of overlap to facilitate learning.

When bidder's valuation uncertainty is low, benefits from learning are limited. (In the extreme when $\sigma_v^2 = 0$, i.e. the benchmark model, the optimal overlapping strategy is $a = 1$.) Thus the seller increases the degree of overlap to eliminate time-discounting. When uncertainty is high, benefits from learning are considerable. First, bidders in the second auction will bid more aggressively due to the reduced uncertainty. Second, this first consequence makes the bidders in the first auction also bid more aggressively by foreseeing the more intense future competition. Thus it is optimum for the seller to decrease the degree of overlap to enhance learning.

### 2.5 Extensions

We next consider two extensions. In Section 2.5.1, we allow random entry of bidders in auctions, while in Section 2.5.2, bidders learn from the bids of other bidders during the bidding process.

### *2.5.1 Random arrival of bidders*

Following Shmueli, Russo and Jank (2007), we assume that bidders arrive randomly during the bidding process according to a Poisson distribution. Thus, the total number of bidders is determined by the average arrival rate *η* and auction duration. Bidders who arrive during the first auction are named "initial" bidders and those who arrive after "new" bidders. Given durations of *1* and *2-a* for the first and second auctions, the expected number of "initial" bidders is: $N_1 = \sum_{n=0}^{\infty} \frac{\eta^n e^{-\eta}}{n!} = \eta$, and the expected number of "new" bidders is: $N_2 = \sum_{n=0}^{\infty} \frac{(\eta(1-a))^n e^{-\eta(1-a)}}{n!} = \eta(1-a)$. The two types of bidders have different distributions for their valuations, as "initial" bidders' valuations are updated at the end of the first auction, while "new" bidders' are not.

We rank bidders based on their expected valuations and denote $Initial^{[i]}$ as the $i^{th}$ highest MWTP among "initial" bidders, and $New^{[i]}$ as the $i^{th}$ highest MWTP among "new" ones. When $0 \le a < 1$, we have three cases to consider:

1) If $Initial^{[3]} > New^{[1]}$, then a "initial" bidder wins and pays $Initial^{[3]}$;

2) If $Initial^{[2]} > New^{[1]} > Initial^{[3]}$, then an "initial" bidder wins and pays $New^{[1]}$; and

3) If $New^{[1]} > Initial^{[2]}$, then a "new" bidder wins and pays $Initial^{[2]}$. [8]

In the first case, the expected highest valuation of "new" bidders is low enough (less than $Initial^{[3]}$) that the "new" bidders' entry does not influence the final price of the second auction. Then the results remain the same as in the full model. In the other two cases, the arrival of "new" bidders increases the final price of the second auction; in particular, when $New^{[1]} > Initial^{[3]}$, which is a common term satisfying the conditions of both Case 2 and 3. As a result, the expected final bid is $New^{[1]}$ or $Initial^{[2]}$, which is higher than $Initial^{[3]}$, the expected final bid without entry.

**PROPOSITION 2-3.** *Extending the main model to allow for bidders' random entry during the bidding process, 1) there exists a unique optimal degree of overlap* $a^* \in [0,1]$*, and 2) this optimal degree of overlap is no less than in the main model, ceteris paribus (see Appendix D for the proof).*

This result extends the finding in Proposition 2-2, allowing for new bidders to enter during the bidding process, and finds that partial overlapping auctions are optimal under similar conditions as in Proposition 2-2. So it shows the robustness of our results.

Entry of new bidders tends to favor the seller, as higher demand increases the likelihood of new bidders with higher valuations. Therefore, the seller's expected revenue will increase.

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[8] The case $New^{[2]} > Initial^{[2]}$ never happens, as simple calculation shows that the 2nd-highest MWTP among the "initial" bidders is always larger than that among the "new" bidders.

The result shows that the optimal degree of overlap is no less than in the full model. This is the case, since "new" bidders do not learn, reducing the benefit of longer auctions, and hence, a seller has an incentive to increase the degree of overlap. And this effect is greater than the benefits from increased demand due to reduced overlap.

### *2.5.2 Bidder learning during the bidding process*

While in the main model we assumed that bidders do not learn until the first auction is completed (e.g. due to a large extent of snipe bidding), we now develop a model in which bidders learn and update their valuations during the bidding process based on the bids by other bidders. Following previous research, we assume that updates are based on the drop-out points of other bidders (Milgrom and Weber 1982; Levin, Kagel, and Richard 1996; Hong and Shum 2013). This will be more applicable in cases where there is less snipe bidding (e.g. high stake auctions, or B2B auctions) where bidders are more likely to bid up to their value during the auction. We consider models both with and without bidders' entry.

#### *2.5.2.1 without bidder entry*

As noted in Equation 2-8, bidder *i*'s initial valuation is $v_i = w_i + \varepsilon_v$, where $w_i$ is the initial expected valuation, following distribution *f(.)*. Since *f(.)* is common knowledge, bidders are able to infer $w^{[1]}, w^{[2]}, \ldots, w^{[k]}, \ldots$, where $w^{[k]}$ is the $k^{th}$ highest expected valuation among all bidders.

***Updating valuations.*** During the course of the auctions, the bidder with the lowest valuation drops out first, then the one with the second lowest valuation and so on. The $k^{th}$

drop-out value, denoted as $s^{[n-k]}$ (the reversed order of bidders' valuations is because the bidder with the lowest valuation drops out first), can be expressed as:

$$s^{[n-k]} = v^{[n-k]} + \varepsilon, \tag{2-14}$$

where $\varepsilon$ follows a distribution with mean 0 and variance $\sigma_\varepsilon^2$ and reflects the distance between the bidder's drop-out value and her valuation. [9]

By Equation (2-8) and (2-14), we have $s^{[k]} = w^{[k]} + \varepsilon_v + \varepsilon$. Drop-out value $s^{[k]}$ can be either the same, higher or lower than the expected value $w^{[k]}$. The discrepancy between $s^{[k]}$ and $w^{[k]}$ triggers remaining bidders to update, identical to the procedure in Equation (2-11). As verified in (2-12) and (2-13), the posterior distributions have the Martingale property; that is, the means of the updated valuations are unchanged, but the variance distributions shrinks by $(1-\tau_k)^2$ at the $k^{th}$ update, where $\tau_k$ is the weight the bidders put on the signal for the $k^{th}$ update. To illustrate, assume that $\varepsilon_v$ (Equation 2-8) and $\varepsilon$ (Equation 2-14) are normally distributed[10].

In *Round 1*, knowing the distributions of $\varepsilon_v$ and $\varepsilon$, the weight the bidders put on the signal is:

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[9] This distribution can be any symmetric distribution, such as normal or uniform.

[10] Results hold for other symmetric distributions because the precision of the signal during updating is decided by two exogenous variables—the variance of the signal and the variance of the initial valuation distribution—not by the degree of the overlap.

$$\tau_1 = \sigma_v^2 / (\sigma_v^2 + \sigma_s^2) \quad . \tag{2-15}$$

Then, using Equation (2-15), we obtain the updated variance: $\sigma_{v,1}^2 = \sigma_v^2 \sigma_s^2 / (\sigma_v^2 + \sigma_s^2)^2$.

In *Round 2* after observing the value of the second drop-out $s^{[2]}$, the posterior distributions of Round 1 become the priors of this round, and the $n-2$ remaining bidders update their valuations: $\tau_2 = \sigma_{v,1}^2 / (\sigma_{v,1}^2 + \sigma_s^2)$, $\sigma_{v,2}^2 = \sigma_{v,1}^2 \sigma_s^2 / (\sigma_{v,1}^2 + \sigma_s^2)^2$. In *Round k*, we obtain: $\tau_k = \sigma_{v,k-1}^2 / (\sigma_{v,k-1}^2 + \sigma_s^2)$, $\sigma_{v,k}^2 = \sigma_{v,k-1}^2 \sigma_s^2 / (\sigma_{v,k-1}^2 + \sigma_s^2)^2$. We have a recursive system, where the value of the $k^{th}$ drop-out is decided by the *k-1*$^{th}$ updated valuation distributions; the *k-1*$^{th}$ updated distributions are derived based on the value of the *k-1*$^{th}$ drop-out; the value of the *k-1*$^{th}$ drop-out is decided by the *k-2*$^{th}$ updated valuation distributions and so on. Updates repeat *n-1* rounds until the next-to-last bidder drops out.

As the number of updates increases, the variance in the bidders' valuation distributions and the weights the bidders put on the signals decrease, i.e. $\sigma_{v,1}^2 > \sigma_{v,2}^2 > ... > \sigma_{v,k}^2 > ...$ and $\tau_1 > \tau_2 > ... > \tau_k > ...$. As bidders gradually become more knowledgeable and thus learn less.

***The seller's optimal overlapping strategy.*** *In the first auction,* the bidder with the highest valuation bids up to a level which makes her indifferent between winning the first or the second auction. Thus, $E[V^{[1]}(n)] - b_1 = \beta^{(1-a)} u_2^{[1]}$. Rearranging this equation yields $b_1 = E[V^{[1]}(n)] - \beta^{(1-a)} u_2^{[1]}$. This shows that although the last bidder learns, her last bid is decided by bid-shading due to her forward-looking, and not due to the reduced uncertainty after learning. *In the second auction,* the last bidder updates $n-1$ times; hence, her final bid is related to the number of bidders and not to the degree of overlap. In sum, the degree of overlap does not affect bidders' learning.

Seller's revenue is the discounted sum of the two final bids (see Appendix 2-E for the proof):

$$R = \begin{cases} \frac{n}{n+1} + \beta^{1-a}\left(\frac{n-4}{n+1} + \frac{r\lambda^2 \prod_{i=1}^{n-1}(1-\tau_i)^2}{3}\right) & 0 \le a < 1 \\ \frac{2(n-1)}{n+1} - \frac{r\lambda^2}{3} & a = 1 \end{cases} \quad (2\text{-}16)$$

The revenue under total overlap is larger than under partial overlap, for any $a$ in the region of [0, 1). Therefore, the optimal degree of overlap is $a^* = 1$.This is because the degree of overlap does not affect bidders' learning, so the seller will let the auctions overlap totally to eliminate time-discounting and bid-shading.

### *2.5.2.2 with bidder entry*

As in the previous section, bidders learn their valuations during the bidding process. Also they arrive randomly during the auction according to a Poisson process, as specified in Section 2.5.1.

Again the ending price in the first auction is dependent on forward-looking and not on learning. However, in the second auction, the degree of overlap influences learning, which impacts the ending price. With bidder entry, the degree of overlap impacts the demand (i.e. the number of bidders), which influences (1) learning (the number of times the final bidder updates her valuations), and (2) the likelihood of entry by a bidder with a higher expected valuation. Reducing the degree of overlap (i.e. a longer duration) increases the above two chances. As a result, the seller needs to trade this off against the opposing effects due to time-discounting and forward-looking, resulting in conditions where partial overlapping is optimal. Based on the same argument as in Proposition 2-3, we find that the optimal degree of overlap is no less than without bidders' entry. We summarize this finding in Proposition 5, together with the model without bidders' entry.

**PROPOSITION 5.** *Extending the main model to allow for bidder learning during the auction from the drop-out point of other bidders, 1) if the number of bidders is fixed, then the optimal degree of overlap is* $a^* = 1$*; 2) if bidders arrive randomly during the bidding process according to a Poisson process, there exists a unique solution* $a^* \in [0,1)$.

## 2.6 Discussion and conclusion

As more and more sellers use online auctions as an alternative or main channel of distribution, the question of how to best sell these products over time has become important. Our study determines the optimal way to sell multiple identical product auctions over time (i.e., simultaneously, sequentially or partially overlapping), and the factors that influence this.

This essay contributes to auction theory by providing a theoretical explanation for the popularity of overlapping auctions in the real online auction environment. Most studies have focused on simultaneous or sequential auctions, and only a few considered overlapping auctions. Those that have analyzed such auctions have taken the degree of overlap as an exogenous variable.

Different from previous research, we focus on the joint impact of four different factors (bidders' forward-looking, learning, time discounting and varied demand) that influence the optimal selling strategy (degree of overlap). Our findings are summarized in Table 2-1.

Table 2-1 The optimal selling strategy of different models

| Models / Factor | No Learning | Type 1 Learning | | Type 2 Learning | |
|---|---|---|---|---|---|
| | **Benchmark model** | **Full model** | **Extension 1** | **Extension 2** | **Extension 3** |
| | **M1 without entry** | **M2 without entry** | **M3 with entry** | **M4 without entry** | **M5 with entry** |
| Forward-looking | + | + | + | + | + |
| Time-discounting | + | + | + | + | + |
| Varied-demand | NA | NA | - | NA | - |
| Learning | NA | - | - | 0 | - |
| Optimal Strategy | Total overlap | Partial overlapping exists. | Partial overlapping exists. | Total overlap | Partial overlapping exists. |

Note: "+" ("-"): It is optimal for a seller is to increase (decrease) the degree of overlap due to the specific factor.

We find that **forward-looking** bidders foresee an option to win in the second auction at a potentially lower price, resulting in bid-shading in the first auction. Therefore, a seller should increase the degree of overlap to reduce bid-shading. Seller's **time-discounting** of future payoffs also has a positive effect on the degree of overlap. As a result, with forward-looking and time-discounting (the benchmark model), the seller's profit under full overlap is always optimal.

The degree of overlap directly influences **demand** (the number of bidders). Therefore it is optimal for the seller to reduce the degree of overlap (i.e., a longer total duration), such that more bidders can enter the auction. **Learning** plays an important role in our model. When bidders are uncertain about the product value, learning helps to reduce their uncertainty. Therefore, learning results in more aggressive bidding and a higher price in the second auction. Additionally, forward-looking bidders, who are able to predict this higher future price, due to learning, will bid more aggressive in the first auction, resulting in a higher selling price.

The relationship between learning and the degree of overlap depends on the type of learning. 1) Bidders learn at the end of the first auction about other bidders' valuation, which is related to the degree of overlap (see models ***M2 and M3***); therefore, it is optimal for the seller to reduce the degree of overlap, since the longer duration will enhance bidders' learning. 2) If bidders learn during the bidding process – which is unrelated to the degree of overlap in the model without bidder entry (model ***M4***), since bidders update based on the highest bid by other bidders regardless of the degree of overlap. Therefore, learning increases the seller's profit, but the degree of overlap has no impact on learning. The optimal degree of overlap is identical to that in the benchmark model without learning (i.e., total overlap). However, with bidder entry (model ***M5***), less overlap will lead to more bidders, and increased learning resulting in higher prices. Therefore, the seller will want to reduce the degree of overlap.

We also pinpoint the conditions when partial overlapping auctions are optimal: a) bidders' uncertainty about product value is at a mid-range, b) it is easy to reduce value uncertainty through learning, and c) the effect of time-discounting is not strong. However, without uncertainty, it is optimum to run simultaneous auctions, and without time discounting it is optimal for a seller to run sequential auctions when the valuation uncertainty satisfies $r\lambda^2 > 6/(n+1)$, and run simultaneous auction otherwise.

Our model of overlapping can be extended in different directions and integrated with other research. For example, the present research can be extended by relaxing the assumption of symmetry in bidders' responses to informative signals. Bidders may face either positive or negative signals (e.g., a selling price that is lower than expected) when updating their valuations. We may expect that a bidder's response (update) to a negative signal may be stronger than to a positive signal (e.g., Kahneman and Tversky 1979). Future research may also integrate overlapping strategies for auctions selling complementary products with bundling across auctions (Popkowski Leszczyc and Häubl 2010).

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**Appendices**

## Appendix 2-A

The reason of assuming that the number of bidders is no less than 4 is to avoid strategic interaction among bidders. When the number of bidders is low enough, bidders can identify each other's moves during the bidding, and thus perform strategically. For example, when *n=2*, there are two bidders (e.g. A and B). Each bidder wants to bid first in the first auction due to the time-discounting effect. Then if A is able to bid first, B chooses to win in the second auction. (We assume that the loss due to time-discounting is less than a minimal-incremental-amount.) Therefore, each bidder bids at a minimal-incremental-amount. The same result holds if B is able to bid first.

When *n=3*, there are three bidders (e.g. A, B and C). Suppose that Bidder A's valuation is the highest, B's is the second-highest, and C's is the lowest. As a result, C drops out first in the first auction. If A's bid is currently the highest there, then B goes for the second auction, in which B and C compete and B wins when the bid is up to C's expected valuation and C drops out. As a results, both A and B pay the bid which equals C's expected valuation. The same result holds if B's bid is currently highest in the first auction when C drops out.

In sum, at least four bidders are needed to avoid bidders' strategic interactions.

## Appendix 2-B: Derivation of the Seller's Revenue in Case 1 when $\boldsymbol{a = 1}$ in benchmark model

The derivation proceeds in two steps.

**Step 1: Deriving the $k^{th}$ -highest expected valuation.** Bidders' private valuations are drawn independently from a known distribution $F(.)$. Let us denote $v^{[k]}(n)$ as the $k^{th}$ -highest valuation among $n$ bidders, and $g^{[k]}(.)$ and $G^{[k]}(.)$ be its corresponding PDF and CDF. Then $G^{[1]}(v,n)$, the CDF of the 1st-highest valuation, is the probability that all bidders' valuations are no higher than $v$, mathematically, $G^{[1]}(v,n) = F(v)^n$. As a result, we are able to derive its density $g^{[1]}(v,n) = nF(v)^{n-1} f(v)$. Similarly, we derive, $g^{[2]}(v,n) = (n-1)nF(v)^{n-2}(1-F(v))f(v)$ and $G^{[2]}(v,n) = F(v)^n + nF(v)^{n-1}(1-F(v))$. Therefore we obtain their expected valuations:

$E[v^{[1]}(n)] = \int_0^{\bar{v}} vg^{[1]}(v,n) = \int_0^{\bar{v}} vnF(v)^{n-1} f(v)dv$ and $E[v^{[2]}(n)] = \int_0^{\bar{v}} vg^{[2]}(v,n) = \int_0^{\bar{v}} v(F(v)^n + nF(v)^{n-1}(1-F(v))f(v)dv$.

**Step 2: Deriving the seller's revenue.** In the single auction, the proof follows the way in Menesez and Monteiro's book (2005). For example, bidder 1 needs to determine her best final bid, given that she knows only her valuation and the distribution of others' valuations. Bidder 1's profit will only be positive when her bid is the largest among all others' bids. Therefore, her expected payoff by bidding $b = b_1$ is

$$\pi(b_1) = (v-b_1)\Pr(b_1 > \max\{b(v_2),...b(v_n)\}) = (v-b_1)Pr(b_1 > b(v_2),...,b_1 > b(v_n)) = (v-b_1)Pr(b_1 > b(v_2))...Pr(b_1 > b(v_n))$$
.

Because bidders do not bid above their own valuations (i.e., $b_i = b(v_i) \le v_i$), Bidder 1's expected payoff becomes $\pi(b_1) = (v-b_1)\Pr(b_1 > v_1)...\Pr(b_1 > v_n) = (v-b_1)F(b_1)^{n-1}$. In a symmetric equilibrium, it further becomes $\pi(b) = (v-b)F(b)^{n-1}$. Taking the first order condition and letting $\pi'(b)=0$, we obtain $b^* = \frac{(n-1)\int_0^v xf(x)F(x)^{n-2}dx}{F(v)^{n-1}}$ for $v>0$.

Next we calculate the expected revenue for the seller:

$$R = E[\max\{b^*(v_1),...,b^*(v_n)\}] = E[\max\{b^*(v_1,...,v_n)\}].$$

The probably that all valuations are below a given value *v* is $F(v)^n$ and thus its density is $nF(v)^{n-1}f(v)$. As a result, $R=\int_0^{\bar{v}} nb^*(v)F(v)^{n-1}f(v)dv$. Substituting the best bid $b^*$ obtained earlier into the above revenue function, we have

$$\begin{aligned} R &= \int_0^{\bar{v}} nb^*(v)F(v)^{n-1}f(v)dv \\ &= \int_0^{\bar{v}} n\frac{(n-1)\int_0^v xf(x)F(x)^{n-2}dx}{F(v)^{n-1}}F(v)^{n-1}f(v)dv \\ &= \int_0^{\bar{v}} (\int_0^x n(n-1)yF(y)^{n-2}f(y)dy)f(x)dx \end{aligned}$$

Changing the order of integration in the last integral (given that $0<y<x$, and $0<x<\bar{v}$), we obtain $\int_y^{\bar{v}} f(x)dx = 1-F(y)$), and the expected revenue becomes

$$R = n(n-1)\int_0^{\bar{v}} y(1-F(y))F(y)^{n-2}f(y)dy.$$

Checking the expected 2nd-highest valuation in Step 1, we obtain $R=E[v^{[2]}(n)]$. The understanding is that in the single auction, the next-to-last bidder drops out when the bid goes up to her valuation, and the last bidder stops at that price; therefore $R=E[v^{[2]}(n)]$. Similarly, in two simultaneous auctions the last two bidders stop bidding when the bid is up to the third-to-last bidder's valuation; therefore the seller's expected revenue is $2E[v^{[3]}(n)]$.

Assuming $F(v) \sim [0,1]$, we obtain $E[v^{[1]}(n)] = \int_0^1 vnF^{(n-1)}(v)f(v)dv = n/(n+1)$, $E[v^{[1]}(n-1)] = (n-1)/n$,

$E[v^{[2]}(n)] = \int_0^1 vn(n-1)[F^{(n-2)}(v) - F^{(n-1)}(v)]f(v)dv = (n-1)/(n+1)$, $E[v^{[2]}(n-1)] = (n-2)/n$, and

$E[v^{[3]}(n)] = (n-2)/(n+1)$. Then, the seller's expected revenue equals $2(n-1)/(n+1)$.■

### Appendix 2-C: Derivation of Equation 2-5

If the bidder does not forward-look, her highest bid in the first auction is expected to be $E[v^{[2]}(n)]$; if she forward-looks, $b_1 = E[v^{[1]}(n)] - \beta^{1-a}E[v^{[1]}(n) - v^{[3]}(n)]$ (Equation 2-4). The level of bid shading is the difference between the highest bids with and without forward-looking. Therefore, bid shading $\Delta(a) = -E[v^{[1]}(n) - v^{[2]}(n)] + \beta^{1-a}E[v^{[1]}(n) - v^{[3]}(n)]$, given $F(v) \sim U[0,1]$, $\Delta(a) = (2\beta^{1-a} - 1)/(n+1)$, and , $\beta^{1-a} \geq 1/2$, which is the condition that ensures that the level of bid shading is positive.■

### Appendix 2-D: Proof of Proposition 2-2

The proof proceeds in two steps.

**Step 1: deriving the seller's revenue,** solving for both cases; for $a = 1$ and $a \neq 1$. When $a = 1$, bidders neither forward-look nor learn; $\Delta(1) = 0$ and $\tau(1) = 0$. By the bidding function (10), the seller's expected revenue is $E[v^{[2]}(n)] + E[v^{[3]}(n)] - r(\sigma_v^2) = 2(n-1)/(n+1) - r\lambda^2/3$.

When $a \neq 1$, the game is played in four stages (see section 2.3). By backward induction, we first look at the **last stage** — $n-1$ bidders bid in the *2nd* auction. The one with the 2nd highest valuation wins, because the bidder with the highest valuation has won and left after the first auction. Then the expected highest bid is:

$$b_2 = E[V^{[3]}(n)] - r\sigma_v^2 / 2, \quad \text{(A1)}$$

where $\sigma_v^2$ and $\sigma_{v'}^2$ are the variances of prior and posterior distributions of valuations, respectively.

In the **third stage,** the first auction ends, and bidders update their beliefs and form the posterior distributions $BB(v,a)$, as specified in Eq. (2-11).

In the **second stage,** the bidder with the highest valuation wins in the first auction. She bids up to the level which makes her indifferent as to whether she wins in the first auction or in the second. That is, $E[v^{[1]}(n)] - b_1 = \beta^{1-a} E[v^{[1]}(n) - v^{[3]}(n) + r\sigma_{v'}^2 / 2]$, where LHS is the expected utility if she wins in the *1*$^{st}$ auction and RHS is the utility if she wins in the *2*$^{nd}$ auction, therefore,

$$b_1 = E[v^{[1]}(n)] - \beta^{1-a} E[v^{[1]}(n) - v^{[3]}(n) + r\sigma_{v'}^2 / 2]. \quad \text{(A2)}$$

We also derive $\Delta(a) = E[v^{[2]}(n) - v^{[1]}(n) - r\sigma_v^2 / 2] + \beta^{1-a} E[v^{[1]}(n) - v^{[3]}(n) + r\sigma_{v'}^2 / 2]$.

Given $F(v) \sim U[0,1]$, $\Delta(a) = \dfrac{2\beta^{1-a} - 1}{n+1} + [(1-\tau)^2 \beta^{1-a} - 1]\dfrac{\lambda^2 r}{6}$.

In the **first stage**, the seller decides the optimal degree of overlap to maximize his expected revenue, i.e. $R = b_1 + \beta^{1-a} b_2$. By (A1) and (A2),

$R = E[v^{[1]}(n)] - \beta^{1-a}(E[v^{[1]}(n)] - E[v^{[3]}(n)] + r\sigma_{v'}^2 / 2) + \beta^{1-a}(E[v^{[3]}(n)] - r\sigma_{v'}^2 / 2)$. Given $F(v) \sim U[0,1]$ and $B(v) \sim U[w-\lambda, w+\lambda]$, $\sigma_v^2 = \lambda^2 / 3$, and $\sigma_{v'}^2 = (1-\tau)^2 \lambda^2 / 3$. Substituting these variances into the revenue equation, we obtain $R = \frac{n}{n+1} + \beta^{1-a}(\frac{n-4}{n+1} - \frac{r(1-\tau)^2\lambda^2}{3})$ .

In summary, we derive the amount of bid shading as

$$\Delta(a) = \begin{cases} \frac{2\beta^{1-a}-1}{n+1} + [(1-\tau)^2\beta^{1-a}-1]\frac{\lambda^2 r}{6} & 0 \le a < 1 \\ 0 & a = 1 \end{cases} \quad \text{(A3)}$$

The seller's expected revenue is:

$$R = \begin{cases} \frac{n}{n+1} + \beta^{1-a}(\frac{n-4}{n+1} - \frac{r(1-\tau)^2\lambda^2}{3}) & 0 \le a < 1 \\ \frac{2(n-1)}{n+1} - \frac{r\lambda^2}{3} & a = 1 \end{cases} \quad \text{(A4)}$$

**Step 2: deriving the seller's optimal overlapping strategy.** When $0 \le a < 1$, substituting $\tau = 1 - a^{\kappa}$ into (A4) and taking the first order condition on $a$, we obtain

$$\frac{\partial R}{\partial a} = -\beta^{1-a}\ln\beta(\frac{n-4}{n+1} - \frac{ra^{2\kappa}\lambda^2}{3}) - \beta^{1-a}\frac{2\kappa ra^{2\kappa-1}\lambda^2}{3} = 0$$

From this we obtain the equality

$$-\frac{3(n-4)}{(n+1)r\lambda^2}\ln\beta + a^{2\kappa}\ln\beta = 2\kappa a^{2\kappa-1} \quad \text{(A5)}$$

Note that Line $c_1$ in Figure 2-6 is drawn by the above function. Further arrangement of (A5) derives $\kappa^* = (\frac{3(n-4)}{(n+1)r\lambda^2} - 1)\frac{-\ln\beta}{2}$, where $\kappa^*$ is the minimum value of $\kappa$ to ensure that the solution of $a^*$ is in [0, 1]; otherwise $a^*$ exists only on either of the corners (either, 0 or 1).

Taking the second order derivative of $a$, we obtain

$$\frac{\partial^2 R}{\partial a^2}|_{a^*} = \beta^{1-a}\frac{2\kappa r a^{2\kappa-1}\lambda^2}{3}[a\ln\beta-(2b-1)]|_{a^*}.$$

When $\partial^2 R/\partial a^2|_{a^*} < 0$, we obtain the inequality

$$\kappa > (1 + a^* \ln\beta)/2. \qquad \text{(A6)}$$

Note that ***Line $c_2$*** in Figure 2-6 is drawn by the function $\kappa^{**} = (1 + \tilde{a}\ln\beta)/2$, where $\kappa^{**}$ is the minimum value of $\kappa$ to ensure that the solution of $a^*$ is a maximum point, at which the seller's revenue is optimal; otherwise $a^*$ exists only on one of the corners (either 0 or 1).

Equation (A5) is the first order condition to solve $a^*$. Equation (A6) is the second order condition in which the solution from (A5) is a maximum.

The seller's optimal overlapping strategy is summarized in Lemma 2D1 and 2D2.

***Lemma 2D1 :*** *when* $\beta = 1$.

*1) if* $r\lambda^2 > 6/(n+1)$, *then* $a^* = 0$ *and* $R^* = (2n-4)/(n+1)$;

*2) if* $r\lambda^2 \le 6/(n+1)$, *then* $a^* = 1$ *and* $R^* = 2(n-1)/(n+1) - r\lambda^2/3$.

*Proof:* First, we substitute $\beta = 1$ and $\tau = 1 - a^\kappa$ into (A4). We then take derivative of the revenue function (under $0 \le a < 1$), and obtain $\partial R/\partial a < 0$. This result shows that under the conditions of $\beta = 1$ and $0 \le a < 1$, the optimal degree of overlap is $a^* = 0$. We also derive the corresponding revenue $R(a=0) = (2n-4)/(n+1)$.

The revenue function (A4) is discontinuous at $a = 1$, where $R(a=1) = 2(n-1)/(n+1)$. We then compare these two revenues

$R(a=0)-R(a=1)=\frac{2n-4}{n+1}-\frac{2(n-1)}{n+1}+\frac{r\lambda^2}{3}=\frac{-2}{n+1}+\frac{r\lambda^2}{3}$. This equation is positive if $r\lambda^2>6/(n+1)$. Therefore, we conclude that if $r\lambda^2>6/(n+1)$, the optimal degree of overlap $a^*=0$; otherwise, $a^*=1$ and its corresponding revenue is $R^*=2(n-1)/(n+1)-r\lambda^2/3$. ■

***Lemma 2D2:*** *when* $\beta\neq 1$.

*1) If* $\kappa>\max(\frac{3(n-4)}{(n+1)r\lambda^2}-1)\frac{(-\ln\beta)}{2},\frac{1+\tilde{a}\ln\beta}{2})$, *then* $a^*=\tilde{a}$.

*2) If* $\kappa\leq\min(\frac{3(n-4)}{(n+1)r\lambda^2}-1)\frac{(-\ln\beta)}{2},\frac{1+\tilde{a}\ln\beta}{2})$, *then*

*a ) if* $\beta>\tilde{\beta}$ *, then* $a^*=0$*;*

*b) if* $\beta\leq\tilde{\beta}$ *, then* $a^*=1$.

*3) If* $\min(\frac{3(n-4)}{(n+1)r\lambda^2}-1)\frac{(-\ln\beta)}{2},\frac{1+\tilde{a}\ln\beta}{2})<\kappa<\max(\frac{3(n-4)}{(n+1)r\lambda^2}-1)\frac{(-\ln\beta)}{2},\frac{1+\tilde{a}\ln\beta}{2})$, *then*

*a) If* $(\frac{3(n-4)}{(n+1)r\lambda^2}-1)\frac{(-\ln\beta)}{2}>\frac{1+\tilde{a}\ln\beta}{2}$, $a^*=1$.

*b) If* $(\frac{3(n-4)}{(n+1)r\lambda^2}-1)\frac{(-\ln\beta)}{2}<\frac{1+\tilde{a}\ln\beta}{2}$ *,then when* $\beta>\tilde{\beta}$, $a^*=0$ *and when* $\beta\leq\tilde{\beta}$, $a^*=1$,

where $\tilde{a}$ satisfies $(\tilde{a}^{2\kappa}-\frac{3(n-4)}{(n+1)r\lambda^2})\ln\beta=2\kappa\tilde{a}^{2\kappa-1}$ and $\tilde{\beta}=\frac{n-2}{n-4}-\frac{r\lambda^2(n+1)}{3(n-4)}$.

*Proof:* We discuss three cases as follows.

***Case 1.*** The optimal overlapping strategy is obtained within [0, 1] under two conditions:

1) First Order Condition (FOC): $a^*$ satisfies Equation (A5).We plot the LHS and RHS functions of Equation (A5) in Figure A1. These two lines intersect in (0, 1) when $k > (\frac{3(n-4)}{(n+1)r\lambda^2} - 1)(\frac{-\ln\beta}{2})$. Thus, FOC is equivalent to $k > (\frac{3(n-4)}{(n+1)r\lambda^2} - 1)(\frac{-\ln\beta}{2})$.

2) Second Order Condition (SOC): $a^*$ satisfies $\kappa > (1 + a^* \ln\beta)/2$ in Equation (A6).

Combining the above two conditions, we conclude that if $\kappa > \max(\frac{3(n-4)}{(n+1)r\lambda^2} - 1)\frac{(-\ln\beta)}{2}, \frac{1+\tilde{a}\ln\beta}{2})$, there exists a unique solution $a^* = \tilde{a}$, where $\tilde{a}$ satisfies $(\tilde{a}^{2\kappa} - \frac{3(n-4)}{(n+1)r\lambda^2})\ln\beta = 2\kappa\tilde{a}^{2\kappa-1}$.

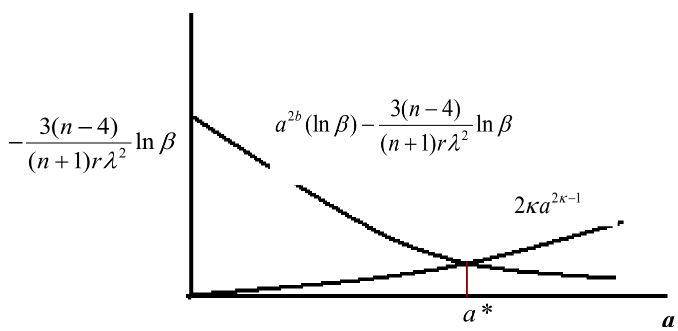


Figure A1: LHS and RHS of Equation (A5)

***Case 2.*** Based on Equation (A6), we know that when $\kappa \le \min(\frac{3(n-4)}{(n+1)r\lambda^2} - 1)\frac{(-\ln\beta)}{2}, \frac{1+\tilde{a}\ln\beta}{2})$, there is no $a^*$ found within (0,1). Therefore, the solution exists only at one of the corners (boundaries) of the region of *a*.

We thus calculate the revenues at these two corners to check which corner provides the highest revenue. Based on the revenue equation (A4), we obtain $R|_{a=1} = \frac{2(n-1)}{n+1} - \frac{r\lambda^2}{3}$ and

$R|_{a=0} = \frac{n}{n+1} + \beta\frac{n-4}{n+1}$. The difference

$R|_{a=0} - R|_{a=1} = \frac{n}{n+1} + \beta\frac{n-4}{n+1} - \frac{2(n-1)}{n+1} + \frac{r\lambda^2}{3} = \beta\frac{n-4}{n+1} - \frac{n-2}{n+1} + \frac{r\lambda^2}{3}$. Letting $\tilde{\beta} = \frac{n-2}{n-4} - \frac{r\lambda^2(n+1)}{3(n-4)}$, we

conclude that if $\beta > \tilde{\beta}$, then $a^* = 0$; if $\beta \le \tilde{\beta}$, then $a^* = 1$.

***Case 3.*** When $\min(\frac{3(n-4)}{(n+1)r\lambda^2} - 1)\frac{(-\ln\beta)}{2}, \frac{1+\tilde{a}\ln\beta}{2}) < \kappa < \max(\frac{3(n-4)}{(n+1)r\lambda^2} - 1)\frac{(-\ln\beta)}{2}, \frac{1+\tilde{a}\ln\beta}{2})$,

based on (A5), we know that no $a^*$ exists within (0,1). So the solution exists only at one of the corners. As in Case 2, we conclude that when $\beta > \tilde{\beta}$, $a^* = 0$ and when $\beta \le \tilde{\beta}$, $a^* = 1$, where

$\tilde{\beta} = \frac{n-2}{n-4} - \frac{r\lambda^2(n+1)}{3(n-4)}$.

Note that Line $\underline{c_3}$ in Figure 2-6 is drawn by $\tilde{\beta} = \frac{n-2}{n-4} - \frac{r\lambda^2(n+1)}{3(n-4)}$. ■

## Appendix 2-E: Proof of Proposition 2-3

The proof follows two steps.

**Step1: deriving the seller's optimal overlapping strategy**. When $a = 1$, bidders neither forward-look nor learn; so $\Delta(1) = 0$ and $\tau(1) = 0$. The seller's revenue is

$E[v^{[2]}(N_1)] + E[v^{[3]}(N_1)] - r\sigma_v^2 = 2(\eta-1)/((\eta+1) - r\lambda^2/3$, the same result as in Proposition 2-2.

When $0 \le a < 1$, using backward induction, we first look at the **last stage.** There are two groups of bidders in the second auction: $\eta - 1$ "initial" bidders from the first auction, and $\eta(1-a)$ "new" bidders. Because "initial" bidders, unlike "new" bidders, learn information from previous experiences, the two groups' valuations are distributed differently. The expected highest bid in the second auction thus depends on the highest

MWTPs in these two groups. Let us denote $Initial^{[i]}$ as the $i^{th}$ highest MWTP among "initial" bidders, and $New^{[i]}$ as the $i^{th}$ highest MWTP among "new" bidders. Given $F(v) \sim U[0,1]$, we derive:

$$Initial^{[1]} = E[v^{[1]}(\eta) - r\sigma_{v'}^2 / 2] = \frac{\eta}{\eta+1} - r\sigma_{v'}^2 / 2$$

$$Initial^{[2]} = E[v^{[2]}(\eta) - r\sigma_{v'}^2 / 2] = \frac{\eta-1}{\eta+1} - r\sigma_{v'}^2 / 2$$

$$Initial^{[3]} = E[v^{[3]}(\eta) - r\sigma_{v'}^2 / 2] = \frac{\eta-2}{\eta+1} - r\sigma_{v'}^2 / 2$$

$$New^{[1]} = E[v^{[1]}(\eta(1-a)) - r\sigma_{v'}^2 / 2] = \frac{\eta(1-a)}{\eta(1-a)+1} - r\sigma_{v'}^2 / 2$$

$$New^{[2]} = E[v^{[2]}(\eta(1-a)) - r\sigma_{v'}^2 / 2] = \frac{\eta(1-a)-1}{\eta(1-a)+1} - r\sigma_{v'}^2 / 2 \cdot$$

***Lemma 2D3.*** The highest bid in the second auction is

$$b_2 = \begin{cases} \max\{\frac{n-2}{n+1} - r(\sigma_{v'}^2)/2, \frac{n(1-a)}{n(1-a)+1} - r(\sigma_v^2)/2\} & \text{if an "initial" bidder wins} \\ \frac{n-1}{n+1} - r(\sigma_{v'}^2)/2 & \text{if a "new" bidder wins} \end{cases} .$$

Proof: Two cases are discussed.

1) If an *"initial"* bidder wins, that is, $Initial^{[2]} \geq New^{[1]}$ and $\frac{\eta-1}{\eta+1} - r(\sigma_{v'}^2)/2 > \frac{\eta(1-a)}{\eta(1-a)+1} - r(\sigma_v^2)/2$, then the expected highest bid is $\max\{Initial^{[3]}, New^{[1]}\}$, that is,

$$b_2 = \max\{\frac{n-2}{n+1} - r(\sigma_{v'}^2)/2, \frac{n(1-a)}{n(1-a)+1} - r(\sigma_v^2)/2\} \cdot$$

2) If a *"new"* bidder wins, that is, $New^{[1]} \geq Initial^{[2]}$ and $\frac{\eta-1}{\eta+1} - r(\sigma_{v'}^2)/2 < \frac{\eta(1-a)}{\eta(1-a)+1} - r(\sigma_v^2)/2$,

then the expected highest bid is $Initial^{[2]}$, that is, $b_2 = \frac{n-1}{n+1} - r(\sigma_{v'}^2)/2$.

The expected highest bid is $Initial^{[2]}$, because $Initial^{[2]} > New^{[2]}$, given that the "initial" bidders' uncertainties have reduced. Merging the results from the above cases, we obtain the equation in Lemma 2D3.■

In the **third stage**, the first auction ends, and bidders update their beliefs and form the posterior distributions $BB(v,a)$.

In the **second stage**, $\eta$ bidders bid in the first auction. The bidder with the highest valuation wins the auction and bids her price at $b_1$, which make her indifferent as to whether she wins in the first or second auction; that is, $E[v^{[1]}(\eta)] - b_1 = \beta^{1-a} u_2^{[1]}$. We derive

$$b_1 = E[v^{[1]}(\eta)] - \beta^{1-a} u_2^{[1]}. \tag{A7}$$

We next consider the level of bid shading. This is the amount that makes the bidder with the highest valuations indifferent as to whether she wins now or postpones her winning. If she postpones her winning, her expected value is

$$u_2^{[1]} = \frac{\eta}{\eta+1} - b_2 \ . \tag{A8}$$

So submitting Equation (A8) to Equation (A7), and comparing the difference between the bid with and without forward-looking, we derive $\Delta(a) = E[v^{[2]}(\eta)] - r(\sigma_v^2)/2) - E[v^{[1]}(\eta)] + \beta^{1-a} u_2^{[1]}$.

In the **first stage**, the seller's expected revenue is

$$
\begin{aligned}
R &= b_1 + \beta^{1-a} b_2 \\
&= E[v^{[1]}(\eta)] - \beta^{1-a}(E[v^{[1]}(\eta)] - b_2) + \beta^{1-a} b_2 \\
&= (1-\beta^{1-a})(E[v^{[1]}(\eta)] + 2\beta^{1-a} b_2 \\
&= (1-\beta^{1-a})(\frac{\eta}{\eta+1}) + 2\beta^{1-a} b_2
\end{aligned}
$$
.

$b_2$ takes different values under conditions (see Lemma 2D3), so we discuss the seller's revenue as follows.

1) If an *"initial"* bidder wins and bids up to the expected *3rd* highest valuation among "initial" bidders, $b_2 = Initial^{[3]}$, then the seller's revenue

$$R = (1-\beta^{1-a})\frac{n}{n+1} + 2\beta^{1-a}[\frac{n-2}{n+1} - \frac{r(1-\tau)^2 \lambda^2}{6}].$$

This happens when the order of MWTPs follows $Initial^{[2]} > Initial^{[3]} > \underline{New^{[1]}}$.

2) If an *"initial"* bidder wins and bids up to the expected highest valuation among "new" bidders, $b_2 = New^{[1]}$, then the seller's revenue $R = (1-\beta^{1-a})\frac{n}{n+1} + 2\beta^{1-a}[\frac{n(1-a)}{n(1-a)+1} - \frac{r\lambda^2}{6}].$

This happens when the order of MWTPs follows $Initial^{[2]} > \underline{New^{[1]}} > Initial^{[3]}$.

$New^{[1]} > Initial^{[3]}$ is equivalent to $\frac{\eta(1-a)}{\eta(1-a)+1} > \frac{\eta-2}{\eta+1}$, and that means $a < 2(\eta+1)/3\eta$.

3) If a *"new"* bidder wins and bids up to the expected *2nd* highest valuation among $\eta$ "initial" bidders, $b_2^{[1]} = Initial^{[2]}$, then the seller's revenue

$$R = (1-\beta^{1-a})\frac{n}{n+1} + 2\beta^{1-a}[\frac{n-1}{n+1} - \frac{r(1-\tau)^2 \lambda^2}{6}].$$

This happens when the order of MWTPs follows $\underline{New^{[1]}} > Initial^{[2]} > Initial^{[3]}$.

$New^{[1]} > Initial^{[2]}$ is equivalent to $\frac{\eta(1-a)}{\eta(1-a)+1} > \frac{\eta-1}{\eta+1}$, and that means $a < (\eta+1)/2\eta$.

In Case 1, the result remains the same as that in the full model, because the MWTPs of all "new" bidders are lower than the *3^rd^* highest MWTP of the "initial" bidders; therefore, these "new" bidders have no chance to win in the second auction, and thus have no impact on the seller's revenue.

Next, based on the revenue functions in each condition and following a process similar to that in step 2 of Proposition 2-2, we derive the seller's optimal overlapping strategy.

**Step 2: comparing with the optimal overlap strategies.** The rank of the highest valuations among the "initial" and "new" bidders plays a critical role in the final price in the second auction. In cases except for Case 1, the seller's revenues increase. When the new arrivals have a higher highest valuation, that is, $New^{[1]} > Initial^{[3]}$, the winner in the second auction bids higher. In addition, the winner in the first auction anticipates this and then reduces the level of bid shading. Hence the final bids on both auctions are higher.

$New^{[1]} > Initial^{[3]}$ means that $\frac{\eta(1-a)}{\eta(1-a)+1} - \frac{r\lambda^2}{6} > \frac{\eta-2}{\eta+1} - \frac{a^{2b} r\lambda^2}{6}$. Let

$LL(a) := \frac{2\eta - 3\eta a + 2}{\eta(\eta+1)(1-a)+(\eta+1)} - \frac{r\lambda^2}{6}(1-a^{2b})$. We notice that $LL(a)$ decreases on *a* and when *a=1,* it is negative. Let $\hat{a}$ be the solution of $LL(a)$, satisfying $New^{[1]} = Initial^{[3]}$. This threshold $\hat{a}$ is unique if it exists. The arrival of "new" bidders affects the final outcome in the second auction if $a < \hat{a}$, and does not otherwise.

Below we provide two examples. First, we solve the problem for a same example in Proposition 2-2, i.e. $\lambda = 0.25, \eta = 7, \kappa = 5, r = 0.5, \beta = 0.95$. We obtain $\hat{a}$ =0.7568, $a^* = 0.83$, and $R^* = 1.2451$. This is the case for $a^* > \hat{a}$, in which "new" bidders have no impact on the ending price of the second auction and the optimal strategy and its corresponding revenue are the same as in the full model.

Second, we solve the problem for the different values of the parameters, i.e. $\lambda = 0.8, \eta = 7, \kappa = 2, r = 0.5, \beta = 0.95$. We obtain $\hat{a}$ =0.4227, $a^* = 0.4212$ and $R^* = 1.4784$. We also calculate the solution in the full model for $n = \eta$ and keeping all the remaining parameter values equal, then we obtain $a^* = 0.3554, R^* = 1.1828$. This is the case for $a^* < \hat{a}$, in which "new" bidders have an impact on the ending price of the second auction.

■

### Appendix 2-F: Derivation of Equation 2-14

We solve the model in 2.5.2.2 for both cases when $a = 1$ and $a \neq 1$. When $a = 1$, bidders are not forward-looking and do not learn, $\Delta(1) = 0$ and $\tau(1) = 0$. Using the bidding function (2-10), the seller's expected revenue is

$E[v^{[2]}(n)] + E[v^{[3]}(n)] - r(\sigma_v^2) = 2(n-1)/(n+1) - r\lambda^2/3$.

When $a \neq 1$, the game is played in four stages (see Section 2.3). Let us denote the variance of the prior belief, the posterior belief at the end of the first auction, and the posterior belief at the end of the second auction as $\sigma_v^2, \sigma_{v'}^2, \sigma_{v''}^2$ respectively. Through backward induction, we first look at the **last stage** — $n-1$ bidders bid in the second auction.

The last bidder updates $n-1$ times, which is the number of bidders without counting herself. Therefore, her final bid is related to the number of bidders, not to the degree of overlap. The degree of overlap does not affect bidders' learning. The highest bid is expected to be

$$b_2 = E[v^{[3]}(n)] - r\sigma_{v''}^2 / 2 . \tag{A9}$$

**The third stage** is omitted as learning is considered during the bidding process.

In the **second stage,** the bidder with the highest valuation wins in the first auction. As in section 2.4.1, she bids up to the level, which makes her indifferent as to whether she wins in the first auction or in the second; that is, $E[v^{[1]}(n)] - b_1 = \beta^{1-a} E[v^{[1]}(n) - v^{[3]}(n) + r\sigma_{v''}^2 / 2]$, where LHS is the expected utility if she wins in the first auction and RHS is the utility if she wins in the second auction. Therefore,

$$b_1 = E[v^{[1]}(n)] - \beta^{1-a} E[v^{[1]}(n) - v^{[3]}(n) + r\sigma_{v''}^2 / 2] . \tag{A10}$$

The equation shows that although the last bidder learns during the bidding process, her last bid is decided by the amount of bid-shading from the expected valuation due to her forward-looking behavior, not due to learning.

At the **first stage**, the seller decides the optimal degree of overlap to maximize his expected revenue — $R = b_1 + \beta^{1-a} b_2$. By (A9) and (A10), $R = E[v^{[1]}(n)] - \beta^{1-a}(E[v^{[1]}(n)] - E[v^{[3]}(n)] + r\sigma_{v''}^2 / 2) + \beta^{1-a} E[v^{[3]}(n)] - r\sigma_{v''}^2 / 2$. Given $F(v) \sim U[0,1]$ and $B(v) \sim U[w-\lambda, w+\lambda]$, $\sigma_v^2 = \lambda^2 / 3$ and $\sigma_{v''}^2 = \prod_{i=1}^{n-1}(1-\tau_i)^2 \lambda^2 / 3$. Substituting these variances into the revenue equation, we obtain $R = \frac{n}{n+1} + \beta^{1-\alpha}\left(\frac{n-4}{n+1} - \frac{r\lambda^2 \prod_{i=1}^{n-1}(1-\tau_i)^2}{3}\right)$.

In summary, the seller's expected revenue is

$$R = \begin{cases} \frac{n}{n+1} + \beta^{1-\alpha}\left(\frac{n-4}{n+1} - \frac{r\lambda^2 \prod_{i=1}^{n-1}(1-\tau_i)^2}{3}\right) & 0 \le \alpha < 1 \\ \frac{2(n-1)}{n+1} - \frac{r\lambda^2}{3} & a = 1 \end{cases}.$$

The revenue under total overlap is larger than under partial overlap for any *a* in the region of [0, 1). Therefore, we conclude that the optimal overlapping strategy is total overlap. ■

# CHAPTER 3: PRODUCT POSITIONING STRATEGY OF FIRMS WITHOUT A COMPETITIVE ADVANTAGE

## 3.1 Introduction

What type of product positioning strategies should small and medium firms use? Should they carry out the best practices of large firms? The notion that small companies adopt similar strategies as big ones, which many case studies have focused on, may be wrong. Directly copying big firms' operations may not help small firms break through the market clutter, especially for those firms with disadvantaged positions. Instead, they need to use different strategies than the large firms.

To elaborate, take the product positioning strategy of Pepsi in India as an example. Pepsi entered the Indian market in the 80s, unavoidably met its strong competitor, Coca-Cola, which set up two brands there: Coca-Cola and Sprite. Coca-Cola was the company's core brand, focusing on the general market, and Sprite played the role of a cover brand and later became a core brand. Coca-Cola's advertising was targeted to the mass market.[11]

In the Indian Cola war, Pepsi is surely a weaker player compared to Coca-Cola. Coca-Cola is widely viewed as the origin of the cola drink. In 2012, Coca-Cola's share was up to 61%, but Pepsi was only 36% in the Indian market.[12] Instead of directly fighting with Coca-

[11] Brands are categorized into three types: the core brand (acting as the flagship brand), the cover brand (acts as a cushion to the core brand to soak up competition), and the stand-alone brand neither (acting independently from core nor cover brands).

[12] http://www.euromonitor.com/soft-drinks-in-india/report.

Cola, Pepsi marketed itself as the “Choice of a New Generation” with three brands—Pepsi, 7UP, and Mountain Dew—aiming for young adults. Among them, Pepsi was a core brand; Mountain Dew and 7 UP were stand-alone brands (See Figure 3-1). Besides, Pepsi constantly responded to threats from Coca-Cola by changing its advertising strategy.


Figure 3-1: Product Positioning of Pepsi and Coca-Cola in Indian Market (Gupta et al. 2010).[13]

The rule under which such big businesses as IBM, Amazon, and Coca-Cola operate is not applicable to small firms. Large enterprises, regardless of industry, have an advantage due to many factors: incumbency, better name recognition, greater advertising funds, etc. So, if other firms sell a commodity similar to the best ones, it’s impossible for them to win consumers. Notice here that Pepsi adopted a different product positioning strategy, differentiating itself from Coca-Cola, offering in the youth market, which Coca-Cola didn’t offer.

Qualitative approaches used to study positioning strategies have been numerous.

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[13] http://tejas.iimb.ac.in/articles/58.php, Image Advertising: the Advertising Strategies of Pepsi and Coca Cola in India, Seema Gupta, K Naganand and Avneesh Singh Narang, 2010, August.

Their approaches include focus groups, role-play, association techniques, depth interviews, and so on (Calder 1994; Hooley, Piercy and Nicoulaud 2013). These researches try to use projective techniques and/or a number of stimuli to uncover how the firm's product is positioned in the mind of consumers. Different from these, this paper adopts an analytical approach to the positioning research. Using a game-theoretical modelling approach, not only do we explain the niche marketing strategies by SMEs, but we also provide a suitable positioning strategy for smaller firms based on their competitive strength in the marketplace. In the model, two firms are asymmetric in their competitive advantages. The model shows that under heterogeneous abilities, firms perform substantially different from when they are symmetric: small firms may deviate from their strong areas when they meet with stronger competitors; they tend to invest the areas where consumers' preferences are more diverse; they may not totally give up the mass markets when the relative strength is small. By doing so, they increase the vulnerability of stronger firms in those areas, thus attracting those consumers without directly combating with strong firms.

Except for having a general concept of a product positioning strategy, we lack a theoretical (quantitative) understanding and operational guide on how to position products for weaker firms when facing stronger competitors. The result—that stronger firms focus more on the mass market and weaker firms differentiate—has been manifested in many contexts such as firms' choice on the number of versions of a product (the length of product line). For example, Apple only provides two basic series of laptops: Apple Macbook Air (11" and 13") and Macbook Pro (13" with or without retina display), and in total only offer four types of laptops on its websites. Lenovo has T Series, X Series, ThinkPad Yoga, E Series, ThinkPad Helix, L Series, W Series, Y Series, Z Series, Flex Series, U Series, Yoga Series, and G series, offering over thirty versions of laptops.

The implications of this result are actually more general than simply an investigation of a firm's product positioning strategy. It also sheds light on areas such as political election, university positions and sports. In the empirical study of the public candidate elections, many scholars found that the weaker candidates would move away from its political center (left wing or right wing), and the advantaged ones would do the opposite, and that challengers tend to adopt more extreme positions than incumbents (E.E Schattschneider 1960; Fiorina 1973; James M. Snyder, Jr. and Groseclose 2001). Aragones (2002, p.132) effectively says

"Candidates diverge, and this divergence occurs in predicable ways. Candidates with charisma end up reinforcing their advantage by adopting relatively more centrist platforms on average, while the ugly, clumsy, and inarticulate flounder on the periphery of the policy space."

When competitors in the markets differ in their capacities and reputations (leading to both real and perceived differences in their advantages), we naturally think that weak ones will evade the competition of big firms, seeking the niches. It is like falling in love: it seems natural and predictable, but the underneath transient hormone changes are complex, such as the change of Cortisol, FSH, and testosterone in our bodies (Marazziti and Canale 2004). Similarly, this seemingly trivial strategy of weak firms is never simple when we ask the following questions:

- Product positioning of disadvantaged firms
  - Do they always choose a niche strategy to differentiate themselves from the strong one?
  - If not always, under what conditions do they or don't they?

— Does the relative strength between firms matter in this decision?

- Resource allocation

  — Under what conditions shall a firm invest on its strong aspects?

  — Under what conditions shall a firm invest on its weak aspects?

  — How shall its efforts/resources be allocated among different product features?

- Which type of firms contributes more to innovative R&Ds?

  — The strong firm or the weak one? Why?

In many cases, we may need to consider pricing together with the positioning decision. This non-pricing strategy fits many situations as well. For example,

- When firms want to decrease direct competition, they avoid the "competing-price-only" syndrome, such as in many oligopoly markets of telecommunication and network, airlines, steel, and oil businesses (Porter 1980; Levitt 1991).
- Many situations exist when brains as well as muscle are important for success, such as in competitive sports and team games.
- In political platforms, the location needs to be decided without price competition.
- Due to certain legal or technical reasons in some markets, the scope of price competition is limited. The prices of air tickets in the United States before deregulation were determined exogenously by the prices of gas and mileage.
- In some cases firms need to set up store locations before providing services or products to consumers, especially when the price is very dynamic and easily changed later.
- In some markets the products' demand, such as agriculture products, is very inelastic. The price of the product is not determined by a single firm, but by the

interaction of all consumers and sellers. Nichols (1951) studied the cigarette industry and showed that since the 1920s, cigarette manufacturers competed through advertising and brand proliferation rather than through price cuts.

- Competition in advertising, a non-negligible industry including television, radio, newspapers, magazines, and direct mail, is a typical non-price competition.

Hotelling (1929) and Tirole (1988, pp 287) provided an analytical one-dimensional model of symmetric firms competing for locations with fixed prices, besides their standard models.

**3.2 Literature Review**

Based on market feature, the organizational target, and resources, the firm (re)positions its products by changing its specifications. Whan et al. (1986) suggested six stages: 1) identify competitors, 2) determine how competitors are perceived and evaluated, 3) determine competitors' positions, 4) analyze customers, 5) select position, and 6) monitor the position.

Physical location choice is one of the most important elements in the positioning decision in many industries. It has been broadly studied both empirically and theoretically. Empirical work generally adopts the discrete choice model to estimate the determining factors in firms' physical location choices (Berry 1992; Mazzeo 2002; Seim 2006; Watson 2009). The following three empirical papers provide empirical support for this essay.

Thomadsen (2007) examined the product positioning strategies of McDonald's and Burger King outlets (McDonald's is stronger than Burger King). He observed that McDonald's more aggressively located his outlets close to Burger King's if Burger King's outlets were located in ideal market places, especially in small markets. On the contrary, Burger King's outlets always moved away to avoid direct competition. Thomas and Weigelt (2000) used the data from the U.S. automobile industry to test the relationship between firms' heterogeneous capabilities and their product differentiation. By checking where managers locate new car models in physical attribute spaces, they found that that strong firms (i.e., large market share firms, incumbents, and domestic firms) were more likely than weak firms (i.e., smaller share firms, entrants, and foreign firms) to locate new car models near existing ones and away from those of rivals. Netz and Taylor (2002) tested the location theory using empirical data of the gasoline stations in the Los Angeles area. They found that gasoline stations showed more differentiation in physical space and the space of product attributes (e.g., repair services, a convenience store, a car wash, and so on) to mitigate price competition. Both papers mentioned that price was an important factor in the firms' decision, but they did not include it in their positioning decisions due to varied reasons. In Thomadsen's (2007) model, price is not included because he assumed that franchisees set prices at their outlets the same due to a static Bertrand game (pp 793).[14] He said, "Bertrand competition is a reasonable assumption in this industry because firms offer to sell as many meals as demanded at posted prices, and because the firms can change their

[14] The features of the static Bertrand game are as follows: 1) two firms have same marginal cost of production; 2) they face a downward sloping demand curve $q = D(p)$; 3) firms can set any price; and 4) the unique equilibrium is that both firms set price equal to the marginal cost of production. Consumers can be very price sensitive, but whether a firm responds to the consumer's price sensitivity depends on his pricing strategy, which considers other factors such as marketing objective, cost analysis, and competitors.

prices quickly and easily." In Thomas and Weigelt's (2000) paper, the pricing decision was not included in their positioning stage for a different reason. They modeled the entry decision as the functions of sales, fixed costs, models, number of firms, and year. Additionally, they modeled the firm's pricing decision as a function of the firm's product features and the entry of competitors (pp 901). Such a separate model setup made the entry decision uncorrelated with its own and competitors' pricing decisions directly. Netz and Taylor (2002) did not include pricing of the competitors into their models, probably because of similar gasoline prices across the stations.

Theoretical work adopts the pioneer work of Hotelling (1929). In the Hotelling model, where market is fixed in the line of [0, 1] and the customers are evenly distributed along the market, two sellers would choose to locate at the median point and split the market into halves. Another standard model of spatial differentiation is the number of consumers locating uniformly in a circular city. Two conflicting forces decide the location in the models: 1) firms like to differentiate due to competition (strategic effect); and 2) firms like to be located close to where the demand is, such as the center of the linear market (demand effect). Those models are related to the horizontal differentiation model, and the result shows the Principle of Minimum Differentiation. That is, the demand effect is stronger than the strategic effect. At equal prices, competing firms choose the same product location—at the center of the market—when products are allowed to be differentiated on a single horizontal dimension. Tirole (1988) provides a summary about how product differentiation along a single dimension (horizontal, vertical, or informational) softens price competition. However, the differentiation at the multidimensional level was still unexplored.

Some researchers extended the one-dimensional market competition model to multi-dimensional ones by adding variations in product characteristics. The usual variation is to consider product quality and feature variety, typically named as product horizontal and vertical differentiations. Economides (1986) considered a model in which the firms made decisions first about quality and then about product variety, solving each decision sequentially as if doubling Hotelling markets. He found that firms tended to maximize variety differentiation but minimized quality differentiation. de Palma et al. (1985) applied a Logit model to capture the consumer's heterogeneity towards product features and incorporated it into their analytical model, because simultaneously solving multiple decisions analytically is technically difficult. Gabszewicz and Thisse (1986) found that more stability in price and product competition was to be expected under vertical rather than under horizontal product differentiation. Vandenbosh and Weinberg (1995) considered a sequential game in which the firm first chose two product characteristics simultaneously and then chose their prices. To solve the decisions of product features simultaneously, they introduced the method of angle-competition and used the angle to show the relative strength of two firms on two product features (e.g., 45 degree refers to the case when firms are equal in advantages on two product dimensions; any other degree refers to the cases in which one firm is stronger than the others on one dimension). They found that firms tended to choose positions to maximize one dimension but minimize the other—a MaxMin product differentiation. Our work relates to this school of literature, but we also consider asymmetric firms: one is stronger than the other due to various factors, such as incumbency, reputation, or size. As such, this asymmetric advantage may lead to different results.

Wernerfelt and Karnani (1987) provide suggestions to firms when they need to make decisions—when to invest and whether to focus or be flexible in the resource allocation on projects—under uncertainty. They said that "a weaker competitor shall go for a small chance of a big profit rather than for a big chance of a very small profit, and a small firm should try to find a niche rather than competing head-on against a big firm in the major market segment." However, they did not provide any rigorous testing or analytical explanations. Most analytical models of asymmetric firms are entry models, in which an incumbent is usually considered stronger than an entrant. Hauser and Shugan (1983) discussed the product positioning strategy for the incumbent: how it adjusted its pricing (increase or decrease), the quality of product attributes (on the attribute attacked or the one not attacked), and the budget on the awareness advertisement and the distribution when facing a competitive new product. However, their model focused on the incumbent's positioning strategies only, by assuming that everything related to the entrant such as price and the locations of the attributes was exogenously given. Later, Moorthy (1988) also discussed asymmetric firms (incumbent and entrant) and showed that if one firm could enter the market first, it could gain a first-mover advantage and defend itself from later entrants, for it was given the chance to pre-empt the most desirable product position. Hoch, Raju, and Seyman (2002) considered a model where a retailer introduces a Store Brand (SB) when he already sells two National Brands (NB). Via game-theoretical modeling, they analyzed the positioning of SB; that is, the appropriate perceptual market location: either close to the leading brand (NB1), a second brand (NB2), or in the middle. They found that when two NBs were symmetric, targeting one of the NBs would be better; when two NBs were asymmetric, targeting NB1 would lead to higher profits. However, the result from Du, Lee, and Staelin (2005) differed: it is better to position the store brand close to the weak or medium national brands instead of the strong one. Tyagi (2000) examined the product positioning of the first- and second-movers. He found that if the first-mover predicted that the cost of the second-mover was lower, the first-mover should leave the most attractive location in the market and move to a market niche. The larger the second-mover's cost advantage, the farther the first-mover should locate. Budd, Harris et al. (1993) analyzed the evolution of market structure between strong and weak firms. A stochastic model was adopted to capture the dynamic change in the gap of strengths between the strong and the weak firms due to their different annual input of efforts. They found an asymptotic expansion between strong and weak firms and identified certain conditions in which the weak firm put in more effort.

Most empirical work on asymmetric firms' product positioning is also related with entry. For example, Carpenter and Nakamoto (1990) studied the optimal positioning, advertising, and pricing strategies for an entrant when the market has already been dominant by a strong incumbent. Due to consumers' asymmetric preferences, they found that "me-too" strategies should not be adopted as entry strategies.

Our multi-dimensional model, especially its solving method, belongs to the theme of the Colonel Blotto game. This is a two-person zero-sum game in which two players are tasked to simultaneously send their troops over several battlefields, and the player with more troops in a battlefield wins that field, and the payoff is equal to the number of fields won. (See the appendix for a detailed description of the Colonel Blotto game). This game is used to solve resource allocation problems with multi-dimensional conflicts and has been applied in many areas, such as wars, electoral competitions, tournaments (Groserclose

2001; James M. Snyder, Jr.and Groseclose 2001; Krasa and Polborn 2010; Dragu and Fan 2010), and auctions. For example, Bayes, Kovenock, and Viries (1996) described a first-price all-pay auction, in which bidders simultaneously submitted bids for an item and all players forfeited their bids. Szentes and Rosenthal (2003) constructed a Colonel Blotto model to find symmetric equilibria for a specific sealed-bid auction, where two bidders bid for three identical objects and the objects' marginal valuations decrease. The first paper applying this game-theoretic model in marketing is Friedman (1958). In his model, two firms competed and made their advertisement allocations on multiple areas with budget constraints. He showed that both the stronger and weaker firms allocated the amount of investment chosen randomly from a uniform distribution in the same interval, but the weaker firm advertised to each consumer only at a certain probability (e.g., the ratio of strength). This model did not include the niche marketing strategies adopted by SMEs.

The technical difficulty in solving this problem has restricted its wide application. For centuries, economists were dedicated to seeking efficient solutions. In their various versions and extensions, the first solution is provided by Borel and Ville (1938). Later by using the properties of regular *n*-gons, Grosee and Wagner (1950) generalized Borel's two solutions to the case of two players with symmetric forces. Recently, Roberson (2006) provided a feasible method for constructing a mixed equilibrium of n-variate distributions.

Table 3-1: Summary of research on product location models

| Author | Research Focus | Firms' Strength | Dimension |
|---|---|---|---|
| ***Empirical*** | | | |
| Berry (1992) | Market entry of airlines | NA | Mult. |
| Mazzero | Product choice on quality in motel markets | Asym. | Mult. |

| | | | |
|---|---|---|---|
| (2002) | | | |
| Seim (2006] | Location choices in the video retail industry | NA. | Mult. |
| Watson (2009) | Product variety and competition for eyeglasses retail market | NA. | Mult. |
| Thomadsen (2007) | Market locations of McDonald's and Burger King's outlet market locations | Asym | Mult. |
| Thomas and Weigelt (2000) | Location of new car models in physical attribute space | Asym. | Mult. |
| Carpenter and Nakamoto (1990) | Optimal positioning, advertising and pricing strategies for entries | Asym. | Mult. |
| Netz and Taylor (2002) | Gasoline stations showed more differentiation in product positioning in two dimensions: in physical space and the space of product attributes to mitigate price competition | NA | Mult. |
| ***Analytical*** | | | |
| Hotelling (1929) | Competing firms choose the same product location | Sym. | One |
| d'Aspremont, et al. (1979) | Competing firms choose the most differentiated locations | Sym. | One |
| Tirole (1988) | Product differentiation in one dimension (horizontal, vertical, or info.) | Sym. | One |
| Economides (1986) | Product differentiation in two dimensions | Sym. | Two |
| de Palma, et al. (1985) | Minimum differentiation holds under sufficient heterogeneity | Sym. | Two |
| Gabszewicz and Thisse (1986) | The more stability in price and product competition was to be expected under vertical rather than horizontal product differentiation | Asym. | Two |
| Vandenbosh, Weinberg (1995) | MaxMin product differentiation in a vertical differentiation model | Sym. | Two |
| Hauser and Shugan (1983) | Positioning for the incumbent on pricing, attributes, ad., and distribution | Asym. | One |

| | | | |
|---|---|---|---|
| Moorthy (1988); Tyagi (2000) | Incumbent and entrant and the first-move advantage | Asym. | One |
| Hoch, et al. (2002); Du, et. al (2005) | Positioning of a Store Brand among two National Brands | Asym. | One |
| Friedman (1958) | Firms' advertisement allocations on multiple areas with budget constraint | Asym. | Mult. |
| Budd, Harris et al. (1993) | The evolution of the gap between strong and weak firms | Asym. | NA |
| ***Methodology*** | | | |
| Bayes, et.al (1996) | Asymmetric equilibria for a first-price all-pay auction | Asym. | Mult. |
| Szentes and Rosenthal (2003) | Symmetric equilibria for a specific sealed-bid auction | Asym. | Mult. |
| Roberson (2006) | Mixed equilibrium of n-variant distributions | Asym. | Mult. |

In this essay we explain why weaker firms differentiate their locations and how they allocate their limited budgets to different areas, both in a one-dimensional model as Hotelling's and in a multi-dimensional model as Colonel Blotto's. Despite the numerous empirical analyses on how weak firms position their products in the market, most theoretical models focus on the competition among symmetric firms. A rigorous theoretical analysis on small or weaker firms' marketing strategies is still missing.

Second, the study on the marketing strategy of weaker firms is much too important to ignore. In the United States in 2010, small businesses made up 99.7% of U.S. employer firms, generating 64% of net new private-sector jobs and 49.2% of private-sector employment. Furthermore, not only do small firms spend almost twice as much of their R&D budget on fundamental research as do large firms, but also small companies are roughly thirteen times more innovative per employee than large firms.[15] The National Science Foundation estimates that 98% of "radical" product developments result from research done in the labs of small companies. Therefore, research on product positioning strategy holds a number of valuable benefits not only for small businesses but also for national economic growth and human welfare.

This essay is organized as follows: In Sections 3.3 to 3.4, we describe the model setup and provide the analysis of the subgame perfect equilibria of the game, and include the main result of the paper in both one-dimensional and multiple-dimensional models. In Sections 3.5 and 3.6, we discuss our results and provide further research, concluding the result in Section 3.7.

### 3.3 One-dimensional location decision

We start by analyzing a one-dimensional location model, to highlight the impact of the asymmetric advantage between firms on their product locations.

**Firms**. Consider two asymmetric firms, indexed by $j$, where $j = \{A, B\}$, selling homogenous products. We allow for the asymmetry in the firms' capability of attracting consumers: Firm A has an advantage over Firm B, allowing generation of greater sales. There can be many factors such as better distribution network, consumer services, or reputation. We assume firms' product prices to be the same, and their marginal costs of production to be constant and equal to zero. Firms need to choose ideal market locations, $(x_A, x_B)$, to maximize their profits.

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[15] Source: www.sba.gov/advocacy/7540/42371, "Small Business GDP: Update 2002-2010."

**Consumers.** The market is comprised of $m$ consumers, each requiring at most one unit of the product. They are uniformly located along a linear market of [0, 1]. So, the market has $m$ mutually exclusive locations of $\{0, \frac{1}{m-1}, \frac{2}{m-1}, \ldots, \frac{m-2}{m-1}, 1\}$, with one consumer at each location.

Consumer $i's$ utility is $u_i = r + a - t(x_i - x_A)^2 - p_A$ if she buys from Firm A, and $u_i = r - t(x_i - x_B)^2 - p_B$ if she buys from Firm B, where $x_i$ is consumer $i's$ location. The consumer's travel cost is measured by Euclidean distance between her and the firm's location. Another interpretation of travel cost is the disutility caused by a mismatch between consumers' tastes and product features. $t$ is the unit transportation cost and we normalize it by letting $t$=1. $a$ is the advantage gap between the strong and the weak firm and is positive. $r$ is the reservation price for the consumer. Finally, we assume $p_A = p_B$. Consumers have the option not to buy: when u is low enough such that when u reaches $\underline{U}$ the consumer will stop buying. Currently, we assume that r is high enough that the consumer always chooses to buy.

The sequence of the game is as follows: In Stage 1, firms simultaneously choose their locations $(x_A, x_B)$ in the linear market of [0, 1]. In Stage 2, consumers choose from which firm to buy. We solve for the subgame-perfect Nash equilibrium of the game; that is, the location strategies chosen by both firms. We look for the symmetric location distribution in the equilibrium because firms' profits remain unchanged after both flip their locations along the central point of the market.

### 3.3.1 The advantage gap is small

When the advantage gap $a$ is zero, two firms are symmetric, and then the model reduces to the standard Hotelling model: two symmetric firms locate at the center of the market in equilibrium. At the other extreme, when the advantage gap $a$ is very large, the stronger firm captures the whole market by staying at the center. Thus it will not respond to its rival's deployment, only to the demand condition. In both cases, the results are trivial. However, if the advantage gap is small, the strategic interaction between firms occurs. There are many such cases, for example, between Pepsi and Coca-Cola, between United Airlines and American Airlines, between McDonald's and Burger King, etc. In the rest of this paper, we investigate those cases where the advantage gap between firms is positive but small.

We start with the case of $a < (\frac{1}{m-1})^2$ , where the competitive advantage between firms is noticeable but very small. Specifically, consumers choose the firm that costs them less in travel, and only when the travel costs to firms are equal, consumers choose to buy from Firm A.

**Proposition 3-1.** *When the advantage gap between two firms is sufficiently small and* $m > 1$*, a pure strategy equilibrium does not exist.*

*Proof.* Suppose the claim in the proposition is wrong. That is, there exists a pure strategy of firms' locations: $(x_{A*}, x_{B*})$. Then $x_{A*}$ and $x_{B*}$ are also fixed points, which can be predicted by both firms.

1. If two points are at the same location, that is, $x_{A*} = x_{B*}$, then Firm A gets all the consumers and Firm B gets zero due to Firm A's competitive advantage. If Firm B moves

one location away from the current position, it can obtain positive market share due to the smallness in the advantage gap. Thus it has an incentive to deviate and $(x_{A*}, x_{B*})$ is not stable.

2. If two points are different, that is, $x_{A*} \neq x_{B*}$, then Firm A has the incentive to move to $x_{B*}$ . By doing so, it can capture all consumers and becomes more profitable. So, $(x_{A*}, x_{B*})$ is not stable.

In either case at least one firm has an incentive to deviate from $(x_{A*}, x_{B*})$. This is contradictory to the statement that $(x_{A*}, x_{B*})$ is the equilibrium solution. ■

The intuition of Proposition 3-1 is that if firms' locations are fixed and predictable, the advantaged firm can copy the strategy of the disadvantaged one and win over all consumers for certain. Therefore, the disadvantaged firm must randomize among a set of locations, adopting a kind of "Guerrilla-warfare" strategy.

Let firm $j$'s mixed strategies in equilibrium be $\sigma^j = (\sigma_1^j, \dots, \sigma_m^j)$. Next we derive them under the case of a very small advantage gap, i.e., $a < (\frac{1}{m-1})^2$ . Firms' profits, when they choose their location *a* and *b* respectively, can be expressed as follows:[16]

$$\pi_A(a,b) = \begin{cases} m - a + 1 + [\frac{a-b-1}{2}] & a > b \\ m & a = b \\ a + \left[\frac{b-a-1}{2}\right] & b > a \end{cases} , \qquad (3\text{-}1)$$

[16] In the equation, "[$x$]" denotes the smallest integer larger than or equal to $x$. For subscripts that fall outside the range, the values of those terms are set to 0.

$$\pi_B(a, b) = m - \pi_A(a, b). \tag{3-2}$$

From the above expressions, we notice a natural symmetry in the profits in the location-strategy space: firms' profits remain unchanged after both locations flip along the central point of the market. Consequently, we expect the symmetry in the distribution of each firm's mixed strategy in equilibrium. That is, $\sigma_1^j = \sigma_m^j$, $\sigma_2^j = \sigma_{m-1}^j, \dots, \sigma_{[\frac{m}{2}]}^j = \sigma_{[\frac{m}{2}]+1}^j$ for $m > 3$.

#### *3.3.1.1 An example*

Market demand is exogenously determined. When $m$ increases, the market is bigger in terms of population. We provide a simple example of $m = 6$, and calculate the mixed strategy quilibrium for both firms before deriving the general solution.[17]

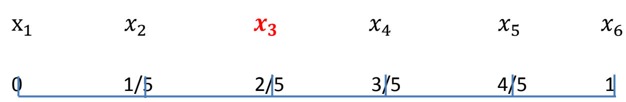


Figure 3-2: The distribution of consumers in the market when *m=6*.

**Firms' profits.** Figure 3-2 shows six consumers locating at one of {0, 1/5, 2/5, …, 1}. Suppose Firm A chose Location $x_3$, we recall the firms' profits function in (3-1) and (3-

[17] In the example, I chose the even number for *m* because it is easier to divide the market share between two firms. The natural result holds when *m* is odd. Especially when *m* is greater, the mixed strategies when *m* is odd and when *m* is even shall converge to the same.

2), then Firm A's profit, given Firm B's location strategy of $\sigma^B = (\sigma_1^B, \dots, \sigma_6^B)$, can be written as:

$$\pi_A(x_3, \sigma^B) = 5\sigma_1^B + 4\sigma_2^B + 6\sigma_3^B + 3\sigma_4^B + 4\sigma_5^B + 4\sigma_6^B. \qquad (3\text{-}3)$$

The *first* term of (3-3) is its profit if Firm B stays at Location $x_1$: it surely wins over five consumers (i.e., three consumers locate on its right, one consumer between two firms, and one consumer on its location); this demand is multiplied by $\sigma_1^B$ , which is the probability that Firm B chooses to stay at Location $x_1$. The *second* term is its profits if Firm B stays at Location $x_2$: it wins over four consumers (i.e., three consumers on its right, and one consumer on its location); this demand is multiplied by $\sigma_2^B$ , which is the probability that Firm B chooses to stay at Location $x_2$. The *third* term is its profits if Firm B stays at Location $x_3$: it wins over all consumers; this demand is multiplied by $\sigma_3^B$, which is the probability that Firm B chooses to stay at Location $x_3$. The *fourth* term is its profits if Firm B stays at Location $x_4$: it wins over three consumers on its left; this demand is multiplied by $\sigma_4^B$ , which is the probability that Firm B chooses to stay at Location $x_4$. The *fifth* term is its profits if Firm B stays at Location $x_5$: it wins over four consumers who locate at its left and on its location; this demand is multiplied by $\sigma_5^B$, which is the probability that Firm B chooses to stay at Location $x_5$. The *last* term is its profit if Firm B stays at $x_6$: Firm A wins over four consumers who locate at $x_1$, $x_2, x_3$ and $x_4$; this demand is multiplied by $\sigma_6^B$, which is the probability that Firm B chooses to stay at Location $x_6$. Similarly, Firm A's profit of choosing other locations, given Firm B's strategy of $\sigma^B$, can be written as

$$\pi_A(x_1, \sigma^B) = 6\sigma_1^B + \sigma_2^B + 2\sigma_3^B + 2\sigma_4^B + 3\sigma_5^B + 3\sigma_6^B,$$

$$\pi_A(x_2, \sigma^B) = 5\sigma_1^B + 6\sigma_2^B + 2\sigma_3^B + 3\sigma_4^B + 3\sigma_5^B + 4\sigma_6^B.$$

Due to the symmetric property in the distribution of the mixed strategy, we only need to consider the situations on half locations.

**Mixed strategies**. Although there are six locations for firms to choose, they may choose fewer locations. That is, the support of the location distribution in the mixed strategy Nash equilibrium may be smaller. Let $x_{k*}^j$ be the starting location in the support of Firm $j$ 's strategy. Therefore, profits on locations farther left from $x_{k*}^j$ and farther right from $x_{m-k+1*}^j$ are zero, and within $\{x_{k*}^j, \ldots, x_{m-k+1*}^j\}$ are equal, which is the important feature of any mixed strategy equilibrium that given the strategies chosen by the other players, each player is indifferent among all the actions that they select with positive probability.

If $x_{k*}^A$ starts at the first place, then the payoffs of the two farthest adjacent locations, $x_1 and\ x_2$, shall be equal, i.e., $\pi_A(x_2, \sigma^B) = \pi_A(x_1, \sigma^B)$. This leads to $-5\sigma_2^B - \sigma_3^B = 0$, which is impossible because it further leads to $\sigma_2^B = \sigma_3^B$ =0 and finally leads to $\sigma_1^B = \cdots = \sigma_6^B = 0$. Therefore we know that $x_1$ is not in the support of Firm A's strategy. Going through a similar procedure, we can exclude $x_1$ from the support of Firm B's strategy as well.

Next let $\pi_A(x_3, \sigma^B) = \pi_A(x_2, \sigma^B)$, and this leads to $-\sigma_2^B + 4\sigma_3^B = 0$. Because $\sigma_1^B + \sigma_2^B + \sigma_3^B + \sigma_4^B + \sigma_5^B + \sigma_6^B = 1$ and the mixed strategy is symmetric, we obtain the mixed strategy of Firm B as *(0, 0.4, 0.1, 0.1, 0.4, 0).* Similarly, the mixed strategy of Firm A is *(0, 0.1, 0.4, 0.4, 0.1, 0),* and the support of both firms is $(x_2, x_3, x_4, x_5)$.

Picking any location in the support of strategies, for example, Location $x_2$, we can calculate the profits of both firms as the following:

$$\pi_A(x_2, \sigma^B) = 5\sigma_1^B + 6\sigma_2^B + 2\sigma_3^B + 3\sigma_4^B + 3\sigma_5^B + 4\sigma_6^B = 4.1,$$

$$\pi_B(x_2, \sigma^A) = 5\sigma_1^A + 0\sigma_2^A + 2\sigma_3^A + 2\sigma_4^A + 3\sigma_5^A + 3\sigma_6^A = 1.9,$$

$$\pi_A + \pi_B = 6.$$

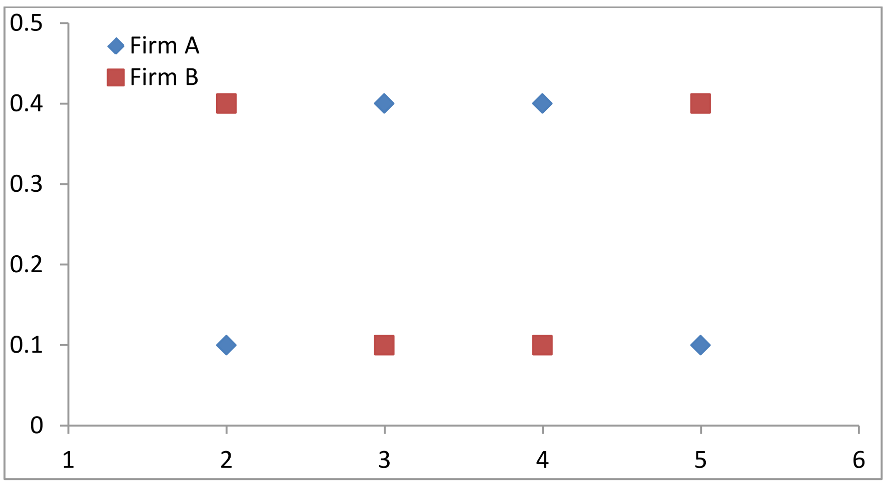


Figure 3-3: Weaker firm's "Guerrilla-warfare" strategy

when the advantage gap is very small

As illustrated in Figure 3-3, both firms differentiate their locations. The mixed strategy equilibrium involves Firm A staying at four locations with corresponding probabilities *{0.1, 0.4, 0.4, 0.1},* four times more staying on the center. However, Firm B stays at four locations with corresponding probabilities *{0.1, 0.4, 0.4, 0.1}*, four times more staying on the edges of the market. Both firms do not choose to stay at $x_1$ *and* $x_6$. This kind of market structure is caused by the strategic interactions between firms. One firm has a competitive advantage over the other. Any consumer will strictly prefer the advantaged firm if both firms are at the same location. As a result, the weak firm must differentiate. On the other hand, because the advantage gap is small, the strong firm ends up reinforcing its advantage by adopting a relatively more central market, in a probabilistic sense, because the weak firm visits there with positive probability.

### *3.3.1.2 General case*

In this section, we present the general form of the firm's location strategy. Given Firm A at $x_i$ and Firm B's strategy, $\sigma^B$, it can be written as follows:

$$\pi_A(x_i, \sigma^B) = \sum_{j=1}^{[(i-1)/2]} (m - i + j + 1)(\sigma_{i-2j-1}^B + \sigma_{i-2j}^B) + (m - i + 1)\,\sigma_{i-1}^B + m\sigma_i^B$$
$$+ i\sigma_{i+1}^B + \sum_{j=1}^{[(m-i)/2]} (i + j)\big(\sigma_{i+2j}^B + \sigma_{i+2j+1}^B\big).$$

Within the support of the mixed strategy equilibrium, firms' profits at each location are equal. Besides, the sum of the probabilities of visiting each location is 1 and the distributions of mixed strategies are symmetric. With all those properties, we are able to obtain Firm B's mixed strategy in equilibrium, satisfying

$$(k-1)\sigma_k^B = (n-k)\sigma_{K+1}^B + \sum_{j=k+2}^{[\frac{m}{2}]} \sigma_j^B \quad for\ k^{A*} \le k < [\tfrac{m}{2}]. \tag{3-4}$$

Similarly, we can obtain Firm A's mixed strategy in equilibrium, satisfying

$$(k-1)\sigma_k^A = (m-k)\sigma_{k+1}^A + \sum_{i=k+2}^{[\frac{m}{2}]} \sigma_i^A \quad for\ k^{B*} \le k < [\tfrac{m}{2}]. \tag{3-5}$$

***Lemma 3-1***. *The starting location in the support of the mixed strategies in equilibrium satisfies* $k^{A*} = k^{B*} > [\frac{m}{4}]$. *(See the proof in the appendix.)*

Now we state the main results of this section, which describes the unique mixed equilibrium of our game in the following proposition:

**Proposition 3-2.** *When* $0 < t < \frac{1}{m-1}$, *and* $m > 3$, *there exists a unique symmetric location strategy for both firms. Mathematically,*

- *for the disadvantaged Firm B, it is (* $\sigma^B_{k*}, \sigma^B_{k*+1}, \dots, \sigma^B_{m-k*+1}$*), which satisfies*

$(k-1)\sigma^B_k = (n-k)\sigma^B_{K+1} + \sum_{j=k+2}^{[\frac{m}{2}]} \sigma^B_j \quad for\ k^{A*} \le k < [\frac{m}{2}].$ *and*

- *for the advantaged Firm A, it is (* $\sigma^A_{k*}, \sigma^A_{k*+1}, \dots, \sigma^A_{m-k*+1}$*), which satisfies*

$(k-1)\sigma^A_k = (m-k)\sigma^A_{k+1} + \sum_{i=k+2}^{[\frac{m}{2}]} \sigma^A_i \quad for\ k^{B*} \le k < [\frac{m}{2}].$

- $k^{A*} = k^{B*} > [\frac{m}{4}]$.

The equations of the mixed strategy in Proposition 3-2 is comprised of a list of $\left[\frac{m}{2}\right] - k^{B*}$ *and* $[\frac{m}{2}] - k^{A*}$ equations for Firm A and B, respectively, solvable recursively with the additional conditions that $\sum_{i=1}^{m} \sigma^A_i = \sum_{i=1}^{m} \sigma^B_i = 1$ and the symmetric property of the strategy distributions, as shown in the example of *m=6.*

Next a natural question raised is "what do those mixed strategies of firms look like? Do they show the similar pattern as illustrated in Figure 3-3?" Proposition 3-3 states the answer.

**Proposition 3-3.** *In equilibrium, Firm A's mixed strategy has a reversed u shape while Firm B's has a u shape. Mathematically, they satisfy*

- *Within the support of Firm A's strategy,* $\sigma^A_{K+1} \ge \sigma^A_K$ *for* $k < [\frac{m}{2}]$; $\sigma^A_{K+1} \le \sigma^A_K$ *for*

$k > [\frac{m}{2}]$ .

- *Within the support of Firm B's strategy,* $\sigma_{K+1}^B \leq \sigma_K^B$ *for* $k < [\frac{m}{2}]$; $\sigma_{K+1}^B \geq \sigma_K^B$ *for*

$k > [\frac{m}{2}]$ . *(Please see the proof in the appendix.)*

In the general setting, we verify the U-shaped location strategy of the disadvantaged firm and the reversed U-shaped location strategy of the advantaged firm, showing that the larger firms tend to focus while the smaller firms tend to randomize, exactly as illustrated in Figure 3-3.

**3.3.2 The advantage gap is a little larger**

Mentioned in Section 3.3.1, firms choose locations differently when the advantage gap is zero compared to when the gap is very large. In this section, we analyze how a small change in the scale of the advantage gap causes firms to react differently in their product location strategies. We assume $(\frac{1}{m-1})^2 \leq a < 3(\frac{1}{m-1})^2$, under which the advantage gap is still small, but becomes a little bit larger than in Section 3.3.1. Specifically, Firm A not only wins over the consumer whose travel costs of visiting two firms are the same, but also wins over the consumer whose travel cost of visiting Firm A is no $3(\frac{1}{m-1})^2$ larger than that of visiting Firm B. This time, Firm A may win over consumers even when the cost of visiting it is larger than that of visiting the other.

In the following, we present the result of $m = 6$, and omit the general solution due to its complexity in the form. The model setting is the same as the previous case. And six consumers locate at one of {0, 1/5,…, 1} as in Figure 3-2. Due to the symmetric property in the distribution of the mixed strategy, we consider only half of the locations. A firm's profit when it chooses one of those locations is as follows, given the other's location strategy (in the same manner to derive 3-3):

$$\pi_A(x_1, \sigma^B) = 6\sigma_1^B + 2\sigma_2^B + 2\sigma_3^B + 3\sigma_4^B + 3\sigma_5^B + 4\sigma_6^B,$$

$$\pi_A(x_2, \sigma^B) = 6\sigma_1^B + 6\sigma_2^B + 6\sigma_3^B + 3\sigma_4^B + 4\sigma_5^B + 4\sigma_6^B,$$

$$\pi_A(x_3, \sigma^B) = 5\sigma_1^B + 6\sigma_2^B + 6\sigma_3^B + 6\sigma_4^B + 4\sigma_5^B + 5\sigma_6^B,$$

$$\pi_B(x_1, \sigma^A) = 0\sigma_1^B + 0\sigma_2^B + 1\sigma_3^B + 1\sigma_4^B + 2\sigma_5^B + 2\sigma_6^B,$$

$$\pi_B(x_2, \sigma^A) = 0\sigma_1^B + 0\sigma_2^B + 0\sigma_3^B + 2\sigma_4^B + 2\sigma_5^B + 3\sigma_6^B,$$

$$\pi_B(x_3, \sigma^A) = 4\sigma_1^B + 0\sigma_2^B + 0\sigma_3^B + 0\sigma_4^B + 3\sigma_5^B + 3\sigma_6^B.$$

We are looking for the symmetric distribution of the mixed strategies and obtain the equilibrium as the following:

A: *(0, 1/3, 1/6, 1/6, 1/3, 0),*

*B: (0, ½, 0, 0, ½, 0),*

$$\pi_A(x_2, \sigma^B) = 6\sigma_1^B + 6\sigma_2^B + 6\sigma_3^B + 3\sigma_4^B + 4\sigma_5^B + 4\sigma_6^B = 5,$$

$$\pi_A(x_2, \sigma^A) = 0\sigma_1^A + 0\sigma_2^A + 0\sigma_3^A + 2\sigma_4^A + 2\sigma_5^A + 3\sigma_6^A = 1,$$

$$\pi_A + \pi_B = 6.$$

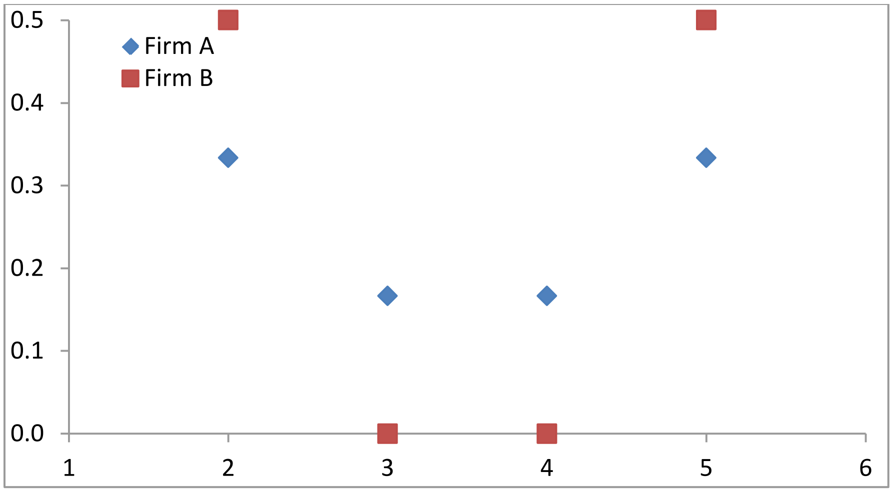


Figure 3-4: Weaker firm's "Flee and defend" strategy

when the advantage gap is a little larger

We plot the optimal location strategies of the firms in Figure 3-4. Both firms choose to randomize in locations. Firm A randomizes on four locations, staying more on the edges by putting double weight on the edges rather than on the center. Firm B randomizes only on two edges with equal probability. Compared with the previous case, we notice several changes as the advantage gap increases: 1) The profits of the weak firm reduces (i.e., from the previous 1.9 to 1) while that of the strong firm increases (i.e., from 4.1 to 5). However, the weak firm can still earn profits with the correct strategy; 2) The weak firm's strategy is less differentiated while the strong firm's is more differentiated. There are also some gaps in the weak firm's location strategy, which means that it has totally given up center positions and focused on the extreme points.

**3.4 Multi-dimensional location decision**

It is useful to think of a product as a bundle of characteristics: physical attributes, quality, location, time, availability, etc., as in many cases, consumers have different tastes and prefer one firm's product over the others, even if both firms sell the products at the same price. As a result, marketing managers regularly face product positioning decisions on multiple dimensions (e.g., speed, ease of use, capacity, etc.). Figure 3-5 illustrates eight dimensions on which computer companies positioned their laptops in 2013. For example, Apple's laptop stood at the top of four features: *'Design,' 'Keyboard & Touchpad,' 'Display & Audio,' and 'Software.'* Lenovo's laptop stood at the top of two features: *'Value & Selection' and 'Innovation.'* More investment usually brings in better quality. Early in 2013, Intel invested $300 million to do research on the thinness and lightness of a laptop, and introduced a new category of laptop: Ultrabook with its thinness less than 20mm/0.8 inches and its lightness less than 4 pounds.[18]

| 2013 BEST & WORST NOTEBOOK BRANDS FULL SCORECARD | DESIGN | KEYBOARD & TOUCHPAD | TECH SUPPORT | DISPLAYS & AUDIO | VALUE & SELECTION | INNOVATION | SOFTWARE |
|---|---|---|---|---|---|---|---|
| Apple | 14 | 15 | 13 | 9 | 5 | 8 | 5 |
| Lenovo | 13 | 14 | 10 | 7 | 9 | 10 | 4 |
| Asus | 12 | 11 | 9 | 8 | 8 | 10 | 3 |
| HP | 12 | 13 | 10 | 7 | 8 | 4 | 4 |
| Samsung | 13 | 10 | 13 | 8 | 7 | 6 | 3 |
| Sony | 13 | 9 | 14 | 6 | 7 | 4 | 4 |
| Dell | 11 | 13 | 5 | 7 | 7 | 6 | 3 |
| Acer | 10 | 11 | 9 | 7 | 7 | 6 | 3 |
| Toshiba | 8 | 9 | 8 | 7 | 8 | 4 | 3 |

[18]http://techcrunch.com/2011/08/10/intel-capital-launches-300m-ultrabook-fund-to-invest-invest-in-lightweight-personal-computing-technologies.

Figure 3-5: Consumer evaluations on multiple features of laptop in 2013[19]

### 3.4.1 Model setup

In this section, we extend the model to multi-dimensions. In the following, we list the differences from the one-dimensional model in the setup.

**Firms.** Two asymmetric firms sell a product with multiple features. Firms need to simultaneously choose ideal market positions for $n$ features of the product. Let $(x_i^A, x_i^B)$, where $i = 1, \ldots, n$, be the choice of location in Feature $i$. In the one-dimension model, location means the relative physical position of the firm, and product quality remains the same if firms move along the linear market of [0,1].

Here in the multi-dimensional case, we refer to the location in each dimension as the relative quality level on that dimension, so quality level increases as firms move from zero to one. This setting dramatically eases the computational complexity in solving a multi-dimensional problem. This is a simplified assumption, because there exist product features where the concept of location extends easily to any choice of a product characteristic. For example, firms often "locate" along a single dimension in choosing product durability and quality (sudsiness, softness, cleaning power, absorbency, etc.), but not always, e.g., color, screen size.

Firm A is more competent than Firm B. In multi-dimensions, measuring competency is difficult. Firms win over consumers due to their combined satisfaction from

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[19] http://blog.laptopmag.com/best-worst-notebook-brands-2013/3.

several dimensions, and firms, whose products are low quality in several dimensions, may still be considered stronger due to their product's good performance in other dimensions. Therefore, we redefine the advantage gap and let it refer to firms' capabilities, i.e., $\eta_A > \eta_B$, which is normalized within (0, 1]. This capability determines the efficiency of a firm to carry out research and/or marketing plans, and thus a strong firm has a lower cost to obtain or stay at high quality in product features. Therefore, a firm's capability is linked to its choice of product positioning in the following way:

$$\frac{x_1^j \ldots + x_n^j}{\eta_j} = B, \tag{3-6}$$

where $B$ is the budget constraint. Equation (3-6) shows a positive relationship between the firm's capability and its total location choices under budget $B$. An advantaged firm is able to choose the higher location than the disadvantaged one under similar investment.

As a result, with a lower capability, a weak firm is usually unable to provide higher quality in all dimensions, and has to strategically choose certain dimensions with the consideration of the moves of the strong firms.

**Consumers.** We assume the total market size to be one and consumers put the same weights on their preference for each dimension. (This is the simplest case. There is another case in which consumers put more weight on certain features than others in their purchase decisions.) In each feature/dimension, consumers prefer the firm that can provide them a higher utility. For example, the probability that a consumer prefers Firm $B$ to A in Dimension $i$, given two firms location of ( $x_i^A$, $x_i^B$), is

$$Pr_i(\text{Consumers prefer Firm } B\ ) = \begin{cases} 0 & if\ x_i^A > x_i^B \\ 1/2 & if\ x_i^A = x_i^B \\ 1 & if\ x_i^A < x_i^B \end{cases}. \tag{3-7}$$ [20]

### 3.4.2 Analysis

The firms' advantage gap can be measured by $\frac{\eta_A}{\eta_B}$. When $\frac{\eta_A}{\eta_B} = 1$, two firms are symmetric, and then our model is similar to Vandenbosh and Weinberg's model (1995), in which firms adopt a MaxMin product differentiation strategy. That is, firms tend to maximize the differentiation in some dimensions but minimize differentiation in others. In this section, we consider the case of $\frac{\eta_A}{\eta_B} > 1$, where firms are asymmetric in the competitive advantage. Furthermore, this competitive advantage gap is small.

If a strong firm is highly competitive, it can outperform a weak one in all dimensions, thus resulting in a pure strategy that the strong one takes the whole market. However, when the advantage gap is small, there exists a mixed strategy, because with budget constraints, a strong firm can't guarantee to win in all dimensions, thus a weak one stands a chance of taking a bite occasionally by acting strategically. Let us denote the mixed strategies as a vector of *n*- univariate marginal distributions, i.e., $(F_i^j, \dots, F_n^j)$.

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[20] There are also other ways to model the probability for a firm to win over a consumer in one dimension; for example, $Pr_i(\text{Consumers prefer Firm } B) = \frac{x_i^B}{x_i^A + x_i^B}$.

**Firms' profits**. In each dimension, consumers prefer the firm that brings them a higher value. The overall expected market share can be measured by the summed market share in each dimension divided by *n*, and firms' profits are proportional to their market share across all dimensions. For example,

$$\text{Market share}^B = \sum_{i=1}^{n} \frac{Pr_i(\text{Consumers prefer Firm } B)}{n}.$$

Substituting Equation (3-7) into the above equation, Firm B's problem can be expressed as:

$$\pi^B \propto max_{(F_i^B,\ldots,F_n^B)} \sum_{i=1}^{n} \int_0^1 [\frac{1}{n} F_i^A(x)] dF_i^B, \tag{3-8}$$

$$\text{S.T.: } E[x_1^B \ldots + x_n^B] = B\eta_B .$$

Therefore, we obtain the Lagrangian form of the firms' maximization problem as follows:

$$L((F_i^j, \ldots, F_n^j), \lambda_j) \propto max_{(F_i^j,\ldots,F_n^j)} \sum_{i=1}^{n} \int_0^\infty (\frac{1}{n} F_i^{-j}(x)] dF_i^j + \lambda_j (B\eta_j - E[x_1^j \ldots + x_n^j]),$$

Because $\lambda_j B\eta_j$ and $\lambda_A B\eta_A$ are constant, the best location strategy for Firm *j* is equivalent to the solution of the following problem (3-9).

$$L((F_i^j, \ldots, F_n^j), \lambda_j) \propto max_{(F_i^j,\ldots,F_n^j)} \sum_{i=1}^{n} \int_0^\infty [\frac{1}{n\lambda_j} F_i^{-j}(x) - x] dF_i^j \tag{3-9}$$

Next we derive properties of equilibrium mixed strategies in Lemma 3-2 to Lemma 3-5.

***Lemma 3-2.*** *In every dimension and in each firm,* $\frac{1}{n\lambda_A} F_i^B(x) - x$ *is constant for x* *in the support of Firm A's strategy of (0,* $s_i$]; $\frac{1}{n\lambda_B}F_i^A(x) - x$ *is constant for x in the support of Firm B's strategy of [0,* $s_i$].

Solving problem (3-9) is the same as solving the bidder's bidding strategy in the simultaneous two-bidder all-pay auctions with complete information in Bay, Kovenock, and Vries (1996). Lemma 3-2 is one of their results, so we omit the proof. Note that the upper boundaries of both firms are the same in any dimension *i*.

***Lemma 3-3.*** *In equilibrium,* $\frac{\lambda_B}{\lambda_A} = \frac{\eta_A}{\eta_B}$. *(The proof is in the appendix.)*

***Lemma 3-4.*** *In equilibrium,* $s_i = \frac{1}{n\lambda_B}$. *(The proof is in the appendix.)*

***Lemma 3-5.*** *In equilibrium,* $\lambda_B = \frac{1}{2\eta_A B}$. *(The proof is in the appendix.)*

Noticing that different scales of advantage gap cause opposite interactions between asymmetric firms in the one-dimensional model, here we also separate our discussion under two cases: 1) under a small advantage gap of $1 \leq \frac{\eta_A}{\eta_B} < \frac{n}{2}$, and 2) under a little larger advantage gap of $\frac{n}{2} \leq \frac{\eta_A}{\eta_B} < (n-1)$, to test the robustness of the result found previously.

***3.4.2.1 The advantage gap is small***

We begin with the case of $1 \leq \frac{\eta_A}{\eta_B} < \frac{n}{2}$, where the competitive advantage between firms is very small. Specifically, this condition guarantees that both firms differentiate their positions and do not exhaust all resources on one dimension. From the property derived in Lemma 3-2, we know that in equilibrium $\frac{1}{n\lambda_A}F_i^B(x) - x$ is constant for $x = s_i$ and *x*. That is,

$$\frac{1}{n\lambda_A}F_i^B(x) - x = \frac{1}{n\lambda_A} - s_i \ .$$

Simplifying the above equation, we derived:

$$F_i^B(x) = 1 - s_i n\lambda_A + xn\lambda_A \ .$$

Substituting the results from Lemma 3-2 to Lemma 3-4 into the above equation, we obtain Firm B's mixed strategy as the following:

$$F_i^B(x) = (1 - \frac{\eta_B}{\eta_A}) + (\frac{\eta_B}{\eta_A})\frac{xn}{2\eta_A B} \ , x \in (0, \frac{2}{n}B\eta_A]. \tag{3-10}$$

Similarly, we know from Lemma 2 that in equilibrium $\frac{1}{n\lambda_B}F_i^A(x) - x$ is constant for *x=0* and *x*. That is,

$$\frac{1}{n\lambda_B}F_i^A(0) - 0 = \frac{1}{n\lambda_B}F_i^A(x) - x.$$

Simplifying the above equation, we derive:

$$F_i^A(x) = xn\lambda_B.$$

Substituting the result of $\lambda_B = \frac{1}{2\eta_A B}$ in Lemma 4 into the above equation, we obtain Firm A's mixed strategy as the following:

$$F_i^A(x) = \frac{nx}{2B\eta_A}, \ x \in (0, \frac{2}{n}B\eta_A]. \tag{3-11}$$

We are able to calculate Firm A's expected evaluation after normalizing product price to 1:

$$\pi^A = max_{(F_i^A,\ldots,F_n^A)} \sum_{i=1}^n \int_0^\infty [\frac{1}{n} F_i^B(x)] dF_i^A,$$

Substituting Equation (3-10) into the above equation, we obtain the following:

$$\pi^A = \sum_{i=1}^n \int_0^{\frac{2}{n}B\eta_A} [\,(1-\frac{\eta_B}{\eta_A}) + (\frac{\eta_B}{\eta_A})\frac{xn}{2\eta_A B}\,]\frac{1}{2B\eta_A}dx,$$

then we derive:

$$\pi^A = 1 - \frac{\eta_B}{2\eta_A}. \quad (3\text{-}12)$$

and

$$\pi^B = 1 - \pi^A = \frac{\eta_B}{2\eta_A}. \quad (3\text{-}13)$$

Firms' mixed strategies and expected payoffs in equilibrium under a small advantage of $1 \le \frac{\eta_A}{\eta_B} < \frac{n}{2}$ are summarized in Proposition 3-4.

**Proposition 3-4.** *When* $1 \le \frac{\eta_A}{\eta_B} < \frac{n}{2}$ *and* $n \ge 3$*, in equilibrium,*

*1) the disadvantaged firm's positioning strategy is* $F_i^B(x) = \left(1 - \frac{\eta_B}{\eta_A}\right) + \left(\frac{\eta_B}{\eta_A}\right)\frac{xn}{2\eta_A B}$ , $x \in \left[0, \frac{2}{n}B\eta_A\right]$. The expected profit is $\frac{\eta_B}{2\eta_A}$.

*2) the advantaged firm's positioning strategy is* $F_i^A(x) = \frac{nx}{2B\eta_A}$, $x \in (0, \frac{2}{n}B\eta_A]$. The expected profit is $1 - \frac{\eta_B}{2\eta_A}$.

Knowing the distributions of the location model in individual dimensions, we calculate the degree of dispersion of their location strategies in equilibrium.

$$var(x_i^B) = \int_0^{\frac{2}{n}B\eta_A} x^2 dF_i^B = \frac{2B(\eta_A - \eta_B)}{n} + \frac{4B^2\eta_A\eta_B}{3n^2},$$

$$var(x_i^A) = \int_0^{\frac{2}{n}B\eta_A} x^2 dF_i^A = \frac{4B^2(\eta_A)^2}{3n^2}.$$

Because $n > 3, then\ \ 4B\eta_A = 4\eta_A \frac{x_1^A \ldots + x_n^A}{\eta_A} \leq 4n < 3n^2$, therefore

$$var(x_i^B) > var(x_i^A).$$

Thus, we uncover a remarkable degree of robustness of asymmetric firms' product positioning strategy in the multi-dimensional setting; that is, a strong firm tends to focus, while a weak one tends to differentiate.

### *3.4.2.2 The advantage gap is a little larger*

Next we check whether firms' strategies change under different scales of advantage gap, and whether the pattern of change is the same as in the one-dimensional model if they change. Thus we look at the case of a larger advantage, i.e., $\frac{n}{2} \leq \frac{\eta_A}{\eta_B} < n$.

**Proposition 3-5.** *When* $\frac{n}{2} \leq \frac{\eta_A}{\eta_B} < n$ *and* $n \geq 3$*, in equilibrium,*

*1) the disadvantaged firm's positioning strategy is* $F_i^B(x) = \left(1 - \frac{n}{2}\right) + \left(\frac{n}{2}\right)\frac{x}{\eta_B B}$ , $x \in [0, B\eta_B]$. *The expected profit is* $\frac{2}{n} - \frac{2}{n^2}\left(\frac{\eta_A}{\eta_B}\right)$.

*2) the advantaged firm's positioning strategy is* $F_i^A(x) = \begin{cases} \frac{2x(\eta_B - \frac{\eta_A}{n})}{B(\eta_B)^2} & x \in [0, B\eta_B) \\ 1 & x \in [B\eta_B, B\eta_A] \end{cases}$.

*The expected profit is* $\left(1 - \frac{2}{n}\right) + \frac{2}{n^2}\left(\frac{\eta_A}{\eta_B}\right)$.

The proof of Proposition 3-5 is similar to that in Proposition 3-4, and thus is omitted. By comparing firms' mixed strategies under different scales of the advantage gap, we find similar patterns: 1) Firm B's profit decreases and Firm A's increases; for example, Firm B's profit of $\frac{2}{n} - \frac{2}{n^2}\left(\frac{\eta_A}{\eta_B}\right)$ is less than $\frac{1}{n}$ under the larger advantage gap of $\frac{n}{2} \leq \frac{\eta_A}{\eta_B} < (n-1)$ while his profit of $\frac{\eta_B}{2\eta_A}$ is larger than $\frac{1}{n}$ under the small advantage gap of $1 \leq \frac{\eta_A}{\eta_B} < \frac{n}{2}$ . 2) Firm B may give up some locations/dimensions, and invest all his resources on one or two other dimensions, because his product location *x* in individual dimension reaches up to the capability-adjusted budget $B\eta_B$ under the larger advantage gap, but does not under the small advantage gap. His strategy becomes relatively focused.

Thus we verify that in multi-dimensional location choice game, the strong firm tends to focus and the weak firm tends to differentiate when the advantage gap is small and vice-versa when the advantage gap is a little larger.

**3.5 Discussion**

The successful competitive strategy of a firm amounts to combining, attacking, and defending moves to build a position in the chosen marketplace, using an analogy of military warfare and market competition. Those strategies vary depending on the nature of the market and the relative strength of firms (Kotler and Singh 1981; Ries and Trout 1986).

As summarized in Table 3-1, Firm A's location strategy turns from "focus on core competencies" to "expend aggressively to edges" and from "investing on the strength" to "compensate on the weakness" after noticing his rival has retreated from the mass market. When the relative strength is not very obvious, firms already strong in the market may pursue essentially defensive strategies to enable them to hold and win the current position against their potential attackers, especially when their product is mature. If the relative strength is obvious, firms may turn from defensive to building strategies; for example, aggressively expanding its domestic and/or international markets geographically and taking sales and customers from competitors.

| Advantage gap | **Strategy** | **Firm A** (stronger) | **Firm B** (weaker) |
|---|---|---|---|
| | A little Larger | Randomization | Focus |
| | Very Small | Focus | Randomization |

Table 3-1: Firms' location strategies under different levels of advantage gaps

On the contrary, Firm B's location strategy turns from "randomization" to "focus," and from "deviating competition" to "fighting for survival." Weaker firms' level of randomization is negatively related with the advantage gap.

Simply avoiding the clutter of mass markets to stay in the niches isn't enough. A common understanding that *"firms need to differentiate and that weak firms should go to the market niche to get a larger share of a smaller market rather than compete with a stronger firm to get a smaller share of a large market*" may not always hold. In this paper, we find that such a strategy is inapplicable under certain conditions. When the competitive advantage of the rival is small, if staying at the market niches with a fixed strategy, the weaker firm can be easily knocked out by its rival, as the stronger firm would choose that location by predicting the weaker firm's move. Instead, the weaker firm shall randomize its positions, including randomly stepping into the mass market, which is the core market of its rivals. By doing so, the weaker firm generates a threat in the mass market so that the stronger firm has to focus resources to defend, thus leaving large shares in the market niches to the weaker firm. This result can be illustrated by the locations of McDonald's and Burger King outlets in areas of Santa Clara by Thomadsen (2007). In his presented map along a stretch of El Camino Real in Santa Clara, California, we can find a random pattern on the nearest distances between two sellers' outlets — some were very close and others were very far.

When the relative strength is large, market niche strategies, focusing on a limited sector of the total market, shall be adopted by weaker firms; for example, when major automobile manufacturers concentrate their attention on the large-scale segments of the car markets in an attempt to keep costs down through the standardization of product parts and the economics of scales of assembly lines. Smaller firms can focus on many small markets, such as the market for high-quality and hand-crafted cars (Guerzoni 2014). Additionally, that provides some insight into why the National Science Foundation reported that small firms invested double the budget amount on fundamental research than big firms did, and 98% of "radical" product developments resulted from research done by small firms.

When firms carry out their marketing strategy, such as advertisements or quality investments, some people may think that strong/large firms contribute more to the innovation and radical invention. In this essay, we find the opposite result: small/weak firms are the sources of most "radical products." The strong firms enhance their core competency when the competition is fierce, and invest on radical products only when the competition is moderate. Such a phenomenon is supported by the flush of radical innovations in Silicon Valley in a highly competitive high-technology industry. Under the intensified competition, weak/small firms increase their investment on radical or fundamental products for survival, or alter/achieve a new form of competitive advantage.

### 3.6 Limitations and future research

The equilibrium results from both one- and multi-dimensional vertical models provide some important insights into the optimal behavior of the weaker firm competing with a stronger one. However, the results should be viewed in light of the model's assumptions. For example, the firms may enter the market sequentially, the non-price assumption may be limiting, and what about the heterogeneity of competitive strength in different dimensions? We discuss limitations and future research in the following pages.

#### 3.6.1 Continuous consumers' preferences

This essay considers the discrete distribution of consumers in the marketplace in the Region of [0, 1]. That is, for $m$ consumers, there are $m$ mutually exclusive locations of $\{0, \frac{1}{m-1}, \frac{2}{m-1}, \ldots, \frac{m-2}{m-1}, 1\}$. There are pros and cons of this assumption.

1) By discretely setting consumers' locations, we are able to analyze discretely for the firms' location strategy within a different range of advantage gap with an example. For this, continuous assumption may not be able to achieve this effect so easily.
2) When the number of consumer increases, this distribution approximates a continuous uniform distribution.

However, its limitation is also obvious. For each different advantage gap, we need to derive different demand equations. Also, the equation becomes very cumbersome as the advantage gap increases. It will make it impossible to solve analytically without turning to the simulation when the advantage gap is very large. As a result, here I explore an alternative way, in which the consumer's preference is continuous. The current model is simple and used to test the robustness of our findings.

Let us assume that the total number of consumers is normalized to be 1 and they are uniformly distributed on the market place of [0, 1]. As in Proposition 3-1 in Section 3.3.1, we first prove the non-existence of pure strategies.

***Proposition 3-6.*** *Pure strategy equilibrium does not exist when the advantage gap between firms is sufficiently small.*

*Proof.* Suppose the claim in the proposition is wrong. That is, there exists a pure strategy of firms' locations $(x_{A*}, x_{B*})$.

We first derive the demands for both firms. Let $x$ be the location of a consumer who feels indifference to purchasing from Firm A or Firm B, and then her utility function satisfies:

$$r + a - (x - x_A)^2 - p_A = r - (x - x_B)^2 - p_B.$$

This gives $x = \frac{{x_B}^2 + a - {x_A}^2 - (p_A - p_B)}{2(x_B - x_A)}$.

Let Firm A stay on the left of Firm B, then the demands of both firms are as follows:

$$D_A = x,\ D_B = 1 - x.$$

Next we start with the simple case when $p_A = p_B$. Also we check the amount of the sufficient small advantage gap: $x$ shall be less than 1, otherwise, Firm A is too strong to drive Firm B out of the market. As such,

$$a < (x_B - x_A)(2 - x_B - x_A).$$

Both firms maximize their profits by choosing their locations.

$$\pi_A = max_{x_A} p_A D_A, \pi_B = max_{x_B} p_B D_B.$$

Taking the first order of Firm A's profit function, $\frac{d\pi_A}{dx_A} = 0$, we obtain:

$$x_B{}^2 + x_A{}^2 = 2x_B x_A - a. \tag{3-14}$$

Similarly, taking the first order of Firm B's profit function, $\frac{d\pi_B}{dx_B} = 0$, we obtain:

$$x_B{}^2 + x_A{}^2 = 2x_B x_A + a. \tag{3-15}$$

Equations (3-14) and (3-15) are contradictory, so a pure strategy equilibrium does not exist. ■

Next we look for a mixed strategy equilibrium to the game. Let firm $j'$s mixed strategies in equilibrium be $\sigma^j$, and the boundaries of strategic equilibrium support be $[\underline{x}, \overline{x}]$. Note that the demand functions differ when Firm A stands on the left or right of Firm B.

$$\pi_A(x_A = \underline{x}, \sigma^B) = \int_{\underline{x}}^{\overline{x}} \frac{x_B{}^2 + a - \underline{x}^2}{2(x_B - \underline{x})} \sigma^B \, dx_B, \tag{3-16}$$

$$\pi_A\left(x_A=\frac{1}{2},\sigma^B\right)=\int_{\underline{x}}^{\frac{1}{2}}\frac{x_B{}^2+a-\frac{1}{2}^2}{2\left(x_B-\frac{1}{2}\right)}\sigma^B\,dx_B+\int_{\frac{1}{2}}^{\overline{x}}(1-\frac{x_B{}^2+a-\frac{1}{2}^2}{2\left(x_B-\frac{1}{2}\right)})\sigma^B\,dx_B, \quad (3\text{-}17)$$

$$\pi_B\left(\sigma^A,x_B=\underline{x}\right)=\int_{\underline{x}}^{\overline{x}}\left[1-\frac{\underline{x}^2+a-x_A{}^2}{2(\underline{x}-x_A)}\right]\sigma^A\,dx_A, \quad (3\text{-}18)$$

$$\pi_B\left(\sigma^A,x_B=\frac{1}{2}\right)=\int_{\underline{x}}^{\frac{1}{2}}[1-\frac{(\frac{1}{2})^2+a-x_A{}^2}{2\left(\frac{1}{2}-x_A\right)}]\sigma^A\,dx_A+\int_{\frac{1}{2}}^{\overline{x}}\frac{(\frac{1}{2})^2+a-x_A{}^2}{2\left(\frac{1}{2}-x_A\right)}\sigma^A\,dx_A, \quad (3\text{-}19)$$

A mixed strategy consists of a pair of probability distributions over the respective strategy space with the property that for each player any strategy chosen with positive probability must be optimal against the other player's probability mixture. Then we drive

$$\int_{\underline{x}}^{\overline{x}}\frac{x_B{}^2+a-\underline{x}^2}{2(x_B-\underline{x})}\sigma^B\,dx_B=\int_{\underline{x}}^{\frac{1}{2}}\frac{x_B{}^2+a-\frac{1}{2}^2}{2\left(x_B-\frac{1}{2}\right)}\sigma^B\,dx_B+\int_{\frac{1}{2}}^{\overline{x}}(1-\frac{x_B{}^2+a-\frac{1}{2}^2}{2\left(x_B-\frac{1}{2}\right)})\sigma^B\,dx_B, \quad (3\text{-}20)$$

$$\int_{\underline{x}}^{\overline{x}}\left[1-\frac{\underline{x}^2+a-x_A{}^2}{2(\underline{x}-x_A)}\right]\sigma^A\,dx_A=\int_{\underline{x}}^{\frac{1}{2}}[1-\frac{(\frac{1}{2})^2+a-x_A{}^2}{2\left(\frac{1}{2}-x_A\right)}]\sigma^A\,dx_A+\int_{\frac{1}{2}}^{\overline{x}}\frac{(\frac{1}{2})^2+a-x_A{}^2}{2\left(\frac{1}{2}-x_A\right)}\sigma^A\,dx_A, \quad (3\text{-}21)$$

$$\overline{x}=1-\underline{x}. \quad (3\text{-}22)$$

The optimal mixed strategy $(\sigma^A,\sigma^B)$ satisfying the conditions in Equation (3-20) to (3-22).

The complexity of the equations forbids me from deriving a clean solution. However, the mixed strategy exists because the game is compact, Hausdorff, and zero sum (i.e., the game is reciprocally semi-continuous), and payoff secure,[21] satisfying the existing conditions of mixed strategy (Reny 1999; Ragh and Jofre 2006; Carmona 2009).

### 3.6.2 Competitive strength

**Heterogeneity.** Competitive strength can be heterogeneity in locations/spaces. Some firms are strong in certain areas, such as management, marketing, and operations, but weak in others, such as R&D and innovation. This heterogeneity provides rich theological scopes for further studies. One way is to view it from firms' aspects; for example, allowing for different investment costs in different product features or providing different capacity constraints on different locations. The other is to analyze it from the consumer's aspect; for example, allowing for the difference in the firm's ability to differentiate or to shrink the distance of the product features toward consumers' ideal points.

**Measure of competitive strength.** In the multi-dimensional model, I measure this competency directly in the firm's side, denoted by $\eta$, i.e. $\eta_A > \eta_B$. This capability determines the efficiency of a firm to carry out research and/or marketing plans, and thus a strong firm has a lower cost to obtain or stay at high quality in product features.

However, in the one-dimension model, one limitation is to specify the strength of the firms on the consumers' utility function. This may lead to confusion, because that added

[21] Reny (1999, pp. 1030) defines it as follows: "A game is payoff secure if for very joint strategy, x, each player has a strategy that virtually guarantees the payoff he receives at x, even if the others play slightly differently than at x."

parameter (the advantage gap) is equivalent to the measure of the product quality. As such, it may not capture the competitive advantage of a firm, because in some cases, weaker firms staying in the niche markets may provide products with much higher quality. Therefore, further study will focus on finding a better way to model the competitive advantage.

Many directions shall be considered. First, the strength of the firms can be expressed by such factors as lower production costs, larger resources (less budget constraints), and the size of firms. In this case, it shall be reflected on the firms' profit function.

Second, the strength of the firms can also refer to higher brand equity, better services the consumer received (shorter shipping time), more convenience provided (such as a consumer can rent a car in this city and return it at any location worldwide), and so on. Then it may be represented by a parameter in the consumer's utility function.

Third, the strength of the firms can also be reflected in the market share. For example, Hoch, Raju, and Seyman (2002), in their model of the marketplace for a store-brand product (from a relatively weaker firm) facing two national-brand products (from relatively stronger firms), their difference is represented through the demand functions as follows (on page 381):

$$q_1 = \frac{1}{a_1+a_2+a_s}[a_1 - p_1 + \tfrac{1}{2}\{\theta(p_1 - p_2)\} + \delta_1(p_s - p_1)],$$

$$q_2 = \frac{1}{a_1+a_2+a_s}[a_2 - p_2 + \tfrac{1}{2}\{\theta(p_1 - p_2)\} + \delta_1(p_2 - p_s)],$$

$$q_s = \frac{1}{a_1+a_2+a_s}[a_s - p_s + \frac{1}{2}\{\delta_1(p_1 - p_s)\} + \delta_2(p_2 - p_s)],$$

where $q_1$, $q_2$ and $q_s$ are the demands for the firm offering National Brand 1, the firm offering National Brand 2, and the store offering its private brand; $p_1, p_2$ *and* $p_s$ are the prices charged by those firms; $\theta$ *and* $\delta$ are the cross-price sensibility representing the competition between firms. They use the base levels of consumers ($a_1, a_2$ *and* $a_s$) to represent the market power of firms. For example, $a_1 > a_s$ and $a_2 > a_s$. (National brands are stronger than store brands.)

Another example is Budd, Harris, and Vickers (1993). In analyzing the driving effects on competition between duopoly over time, they consider the firm with market share over ½ as the leader and the other the follower.

### 3.6.3 Sequential versus simultaneous moves

In real life, the market is dynamic, so the consideration of sequential moves is important and seems preferable to a simultaneous location game. Simultaneous entries are commonly found in reality. For example, on sealed B2B reverse auctions, the sellers submit their bids simultaneously to obtain business from the buyer (e.g., government). Another example is the airlines' entry decisions to serve some new airline routes after the deregulation of the airline industry. Before 1978, the Civil Aeronautics Board (CAB) severely limited the entry of airlines into interstate trunk, international, or territorial scheduled passenger services. Over 25 years have passed since 1978, and a great deal of entry has occurred.[22] A natural starting point is to examine what happens when the firms are simultaneous Nash competitors. Next we look at sequential moves of the firms.

When a firm chooses to move first, it enjoys the advantage gained by the initial significant occupant of a market segment, gaining control of market locations, larger market shares as a monopoly, scale of cost if successful, or pre-empt brand image, all of which the followers may not be able to match. Procter & Gamble's success in introducing disposable diapers to China is an example. It launched "*Pampers*" in China in 1998. Today after years of research and experiences, *Pampers* is the No. 1 selling diaper in China, standing at $1.4 billion in 2010—roughly a quarter the size of the U.S. market.[23]

Sometimes, the first-mover is not able to capitalize on its advantage, leaving later entrants to compete more effectively and efficiently, which is called a second-mover advantage; for example, Charles Stack Online Bookstore vs. Amazon. BookStacks launched online in 1992, the very first online bookstore known to date. It was set up by Bezos; it began advertising on over 28,000 other Internet sites and dominated the business. In 1994, Jeff Bezos founded Amazon.com as an online bookstore in 1995. The product lines were quickly expanded to computer software, video games, furniture, toys, and many others. Now we know Amazon instead of Bookstacks. Similarly, Apple is surely not the first company entering the MP3 player market, but now dominates the market. There had been over 50 companies selling portable MP3 players in the U.S. before Apple's entry in

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[22] http://www.gao.gov/archive/1996/rc96079.pdf.

[23] http://www.chinadaily.com.cn/bizchina/2009-06/08/content_8259758.htm.

November 2001.

Moorthy (1988) considered that if one firm entered the market first, it could gain a first-mover advantage, for it was given the chance to pre-empt the most desirable product position. For symmetric firms, after the first firm has located, the profit-maximizing decision rule of the second firm is clearly to choose the larger interval and to locate as closely as possible to the position of the first firm to capture the trade of consumers in the "hinterland" from it to the end of the market. Recognizing this, the first firm can do no better than to locate in the market center. Results become complicated for oligopoly firms, or for firms with asymmetric strength. For example, Hoch, Raju, and Seyman (2002) considered three firms, a Store Brand (SB) and two National Brands (NB), and suggested that the store brand shall be either located close to the leading brand (NB1), a second brand (NB2), or in the middle. They found that when two NBs were symmetric, targeting one of the NBs would be better; when two NBs were asymmetric, targeting a stronger NB would lead to higher profits. On the contrary, Du, Lee, and Staelin (2005) found that the store brand should locate close to the weak or medium national brand instead of the strong one.

Either a stronger or weaker firm can enter first, being a location leader. However, the result differs if the moving sequence changes. Tyagi (2000) found that if the weaker firm moves first and the product cost of the second-mover was predicted to be lower, it would leave the most attractive location in the market and move to a market niche. The larger the second-mover's cost advantage, the farther away the first-mover locates. Hoch, Raju, and Seyman (2002), and Du, Lee, and Staelin (2005) found that if the weak firm moves later, it chose to close the strong firm.

Besides the capability of the firms, whether the market rewards the first-mover or the second-mover also depends on other factors: the tastes of consumers change, sources outside the industry, such as supporting companies, universities, government, and research centers (Kessler et al. 2000), and the market structure (first-cycle, slow-cycle, or standard-cycle of turnover). Both firms' internal and external factors decide the success of the first-mover or second-mover. We will consider this interesting timing issue of market entry, analyzing how first-move and second-move impact on the location strategies of asymmetric firms.

### 3.6.4 Price decision

In our model, price is taken exogenously. It fits for the following markets; for example, in the markets where price competition is so dynamic that the retailers are able to make the location decisions if they take prices into consideration, or in the markets where prices are less involved. The prices of agriculture products are determined not by a single firm but joined by all consumers and sellers. In the communication industry including television, radio, newspapers, magazines, direct mail, and advertisements, their positioning strategies do not involve price competition. Rust and Donthu (1988) analyzed the positioning and repositioning of the cable networks and observed that, given that a perceptual space exists in the viewers, all movie channels (HBO, SHOWTIME, and CINEMAX) were perceived to be similar and hence were located near each other. ESPN and CNN were located in two different and relatively isolated areas due to their unique programming contents; LIFETIME, USA, and WTBS programed broadly and did not focus on any one type of show, so they were perceived to be at the center and did not own a unique target audience. As such, new cable channels should adopt the locations that were highly associated with investigative news reporting programs or action/mystery shows.

On the other hand, there are markets in which price is a major consideration. Numerous analytical models considered pricing competition when analyzing product positioning and they modelled the problem as a two-stage sequential game, in which firms decide their product positioning first and then make price decisions. In such games, the firm needs to think about the pricing when considering product positioning. In particular they should be aware that if they locate closely, they are able to make incursions of their competitors' market; on the other hand, they would not move too close as the competition becomes too intense. The necessity of balancing these two opposite strengths helps derive equilibrium for optimal product positioning and pricing choices (d'Aspremont, Gabszewicz, and Thisse 1986). However, some practical difficulties exist, as outlined below.

1) Pricing equilibrium for any chosen location made in the first stage does not always exist. d'Aspremont, Gabszewicz, and Thisse (1979) showed that the pure location decision of the Hotelling model did not exist. Anderson (1987 and 1988) showed that the existence of pure location equilibrium of the Hotelling model depended on the assumption of travel cost and firms' profit functions. Moreover, a pure equilibrium strategy may not exist for the Hotelling model for the two-stage game for different travel cost functions.

2) Price and some product features are two different dimensions. For the price, the lower the better, therefore, as the price increases, the consumers' preference is monotonically decreasing. This difference is traditionally named vertical differentiation, in

which all consumers opt for the same variant. However, in the product features, sometimes the concept of location cannot be extended easily to any choice of a product characteristic. For some product features, such as color and screen size, the consumers' preferences are dispersed. This differentiation is traditionally named horizontal differentiation, which means that consumers' tastes are distributed randomly in any location in the product features.

Integrating these two different distributions of differentiations is interesting but adds complicity to the analysis. Current researches are only able to extend the model to cover two product features. Even in those researches, Economides (1986) had to treat these two types of differentiation (quality and product variety) similarly, solving sequentially as if doubling Hotelling markets. de Palma et al. (1985) had to use the Logit model and incorporated it into their analytical model. Vandenbosh and Weinberg (1995) introduced a complicated method: angle-competition, to show the relative strength of firms on two product features.

### 3.7 Conclusion

Understanding how to compete is crucial. This essay provides a theoretical explanation for some observations: why weaker firms differentiate their product positioning as a way to mitigate competition; why McDonald's likes to be located near Burger King, but Burger King doesn't; and why stronger firms (incumbents) like to build upon their existing set of capabilities and bring in incremental technologies to market, instead of developing radical ones. This is a starting point to understand the role of competitive strength in shaping firms' product positioning strategies. Despite numerous empirical analyses, most theoretical models focused on the competition among symmetric firms, except for entry models. A rigorous theoretical analysis on product positioning of weaker firms is still missing.

This study suggests an operational guide for marketing managers on product positioning/resource allocation issues. As such, the insights gained from the paper contribute to the theory of marketing strategy of asymmetric firms, especially small firms. The study on the marketing strategy of weaker firms is much too important to ignore. Over 99.9% of employer firms are SMEs, generating over 50% of employment. Furthermore, not only do small firms spend almost twice as much of their R&D budget on fundamental research as large firms do, but also are roughly thirteen times more innovative per employee than large firms. Therefore, our study holds huge benefits, not only for small business, but also for the national economic growth and human welfare.

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## Appendices

### Appendix 3-A: A Colonel Blotto game

**Battlefield.** Two generals are to capture 2 locations next day. Colonel Blotto has 4 regiments of troops. The opposing commander, Colonel Lotso, has 3 regiments. If both Colonels play strategically, how should they distribute their troops?

**Payoffs.** In the field, the army with more troops in the site wins the site. If one sends $x$ troops, and the other sends y, and $y<x$, then the Colonel who sent $x$ wins and gets payout of $y + 1$: the number of y enemies captured, plus 1 point for securing the site. If each sent all of their troops to different sites, although both secure the site, but do not defeat the enemy, both payoffs are 0.

**Solution:** Colonel Blotto has 5 strategies at his disposal: (4, 0), (0, 4), (3, 1), (1, 3), and (2, 2). The set (4, 0) means that he sends all his 4 troops to one of the sites. Colonel Lotso has 4 strategies at his disposal: (3, 0), (0, 3), (2, 1), and (1, 2).

**Lotso**

| **Blotto** — Payoff | (3,0) | (0,3) | (2,1) | (1,2) |
|---|---|---|---|---|
| (4,0) | (4,-4) | (0,0) | (2,-2) | (1,-1) |
| (0,4) | (0,0) | (4,-4) | (1,-1) | (2,-2) |
| (3,1) | (1,-1) | (-1,1) | (3,-3) | (0,0) |
| (1,3) | (-1,1) | (1,-1) | (0,0) | (3,-3) |
| (2,2) | (-2,2) | (-2,2) | (2,-2) | (2,-2) |

There is a mixed strategy solved for this game:

| **Blotto** | Strategy | (4,0) | (0,4) | (3,1) | (1,3) | (2,2) |
|---|---|---|---|---|---|---|
| | Prob | 4/9 | 4/9 | 0 | 0 | 1/9 |

| **Lotso** | Strategy | (3,0) | (0,3) | (2,1) | (1,2) |
|---|---|---|---|---|---|
| | Prob | 1/18 | 1/18 | 4/9 | 4/9 |

We write out the 5 x 4 matrix for the set of strategies and calculate the payout for each battle. The first number is the payoff of Colonel Blotto, and the second is Colonel Lotso's. For example, (1,-1) is the payoff out of the battle of the strategy (4,0) versus (1,2). Blotto wins the first site and loses the second, therefore, his payoff is *1 (defeat one troop)+1(secure the first site)-1(lose the second site)=1*. Lotso loses in the first site but wins in the second, so his total payoff is *-1(lose one troop)-1(lose the first site) +1(win the second site)=-1*. Their total payoffs are zero, a zero-sum game.

Now we provide the proof for the above strategy. Let us denote the strategy for Blotto to choose each row as follows: *p, p, q, q, 1-2p-2q*; and the strategy for Lotso to choose each column as follows: *r, r, ½-r, ½-r.*

**Step 1: Payoff function**

*Blotto*'s payoff of choosing each row is as follows:

Row 1: $4r + 0 + 2(1/2 - r) + (½ - r) = 3/2 + r$

Row 2: the same due to the symmetry in the payoffs

Row 3: $r – r + 3 (1/2 – r ) + 0 = 3/2 – 3 r$

Row 4: the same as above

Row 5: $-4 r + 4 (1/2 – r ) = 2 – 8 r$

Then *Row 1=Row 3* yields $3/2 + r = 3/2 – 3 r$, and then further yields $r = 0$. *Row 1=Row5* yields $3/2 + r = 2 – 8 r$, and then further yields $r=1/18$. Therefore, $r =0$, *or* $r=1/18$.

*Lotso*'s payoff of choosing each column is as follows:

Column 1: $-4p + 0 - q + q + 2(1 - 2p - 2q) = 2 - 8p - 4q$

Column 2: the same as above

Column 3: $-2p - p - 3q - 0 - 2(1 - 2p - 2q) = -2 + p + q$

Column 4: the same as above

<u>*Column 1= Column 3*</u> yields $2 - 8p - 4q = -2 + p + q$, and then further yields $9p + 5q = 4$.

**<u>Step 2: Equilibrium strategy</u>**

**We show that r=0 is not possible**. Suppose it is the equilibrium strategy of Blotto, then it means that Row 5 can't be in the support of Blotto's equilibrium strategy. As such, the probability of visiting Row 5 is zero, i.e., $1 - 2p - 2q = 0$. Therefore, with the knowledge of $9p + 5q = 4$, derived previously, we derive that $p=3/8$ *and* $q=1/8$.

We submit the result of p and q back to Lotso's payoff function and find that his payoff is negative. Therefore, r=0 is impossible.

Then we derive $r = 1/18$. It means the probability of visiting Row 3 and 4 is zero, i.e., $q=0$. Therefore, with the knowledge of $9p + 5q = 4$ derived previously, we derive $p=4/9$. ■

**Interpretation.** When both play their optimal strategy, Blotto can expect a payout of 14/9 due to his one extra regiment. Blotto should concentrate his troops to specific sites and occasionally split his troops, just to make sure that Lotso cannot steal a location easily. Lotso has many mixed strategies. The shown now is the symmetric one. He is in the weak position, not able to win against Blotto in the numbers, so he has to spread his troops out and hope to secure an undefended location with 1 regiment. Occasionally Lotso will deploy all his troops to one site or the other, just so that Blotto cannot win by spreading his troops evenly all the time.

**Appendix 3-B: Proofs**

***Proof of Lemma 3-1***

***Lemma 3-1***. *The range of strategy supports of* $k^{A*} = k^{B*} > \frac{m}{4}$ .

*Proof*. Suppose not: when $k \leq \frac{m}{4}$ , $\sigma_k^A > 0$.

$$k \leq \frac{m}{4} \Longrightarrow 2k \leq \frac{m}{2}$$

$$\Longrightarrow k - 1 \leq \frac{m}{2} - k - 1$$

$$\Longrightarrow (k-1)\sigma_k^A \leq \left(\frac{m}{2} - k - 1\right)\sigma_k^A = \sum_{i=k+2}^{\frac{m}{2}} \sigma_k^A$$

$$\Longrightarrow (k-1)\sigma_k^A \leq \sum_{i=k+2}^{\frac{m}{2}} \sigma_j^A$$

In Equation (3-3), the mixed strategy for Firm A is $(k-1)\sigma_k^A = (m-k)\sigma_{k+1}^A + \sum_{i=k+2}^{\frac{m}{2}} \sigma_i^A$. Compare with LHS and RHS of the mixed strategy for Firm A, we know that $\sigma_k^A = 0$ for $k \leq \frac{m}{4}$ . It is contradictory.

In the end, from (3-2) and (3-3), we derive that $k^{B*} \leq k^{A*}$ and $k^{A*} \leq k^{B*}$, therefore, $k^{A*} = k^{B*}$. ■

***Proof of Proposition 3-3***

***Proposition 3-3.*** *In equilibrium, Firm A is in the reversed u shape and Firm B in the U shape. Mathematically, they satisfy*

• *Within the support of Firm A's strategy,* $\sigma_{K+1}^{A} \geq \sigma_{K}^{A}$ *for* $k < \frac{m}{2}$; $\sigma_{K+1}^{A} \leq \sigma_{K}^{A}$ *for* $k > \frac{m}{2}$.

• *Within the support of Firm B's strategy,* $\sigma_{K+1}^{B} \leq \sigma_{K}^{B}$ *for* $k < \frac{m}{2}$; $\sigma_{K+1}^{B} \geq \sigma_{K}^{B}$ *for* $k > \frac{m}{2}$.

*Proof.* $k < \frac{m}{2}$ means that $k < \frac{m+1}{2}$, then we derive $k-1 < m-k$.

$$k-1 < m-k$$

$$\Longrightarrow (k-1)\sigma_{k}^{B} < (m-k)\sigma_{k}^{B}.$$

From (3-2) in Proposition 2, we know that $(k-1)\sigma_{k}^{B} = (m-k)\sigma_{k+1}^{B} + \sum_{i=k+2}^{n/2} \sigma_{i}^{B}$ for $k^{A*} \leq k < \frac{m}{2}$, then

$$(k-1)\sigma_{k}^{B} < (m-k)\sigma_{k}^{B} \quad \Longrightarrow (m-k)\sigma_{k+1}^{B} + \sum_{i=k+2}^{m/2} \sigma_{i}^{B} < (m-k)\sigma_{k}^{B}.$$

For this inequality to be held, $\sigma_{K+1}^{B} \leq \sigma_{K}^{B}$ for $k < \frac{m}{2}$.

The second part in Proposition can be proved in the same way. ■

***Proof of Lemma 3-3***

***Lemma 3-3.*** *In equilibrium,* $\frac{\lambda_B}{\lambda_A} = \frac{\eta_A}{\eta_B}$.

*Proof:* Because each firm uses up all the resources, then $\mathrm{E}[\frac{x_1^j \ldots + x_n^j}{\eta_j}] = B$.

$$\Longrightarrow \frac{E[x_1^A \ldots + x_n^A\}}{\eta_A} = \frac{E[x_1^B \ldots + x_n^B]}{\eta_B},$$

$$\Longrightarrow \frac{\sum_{i=1}^{n} \int_0^\infty x dF_i^A(x)}{\eta_A} = \frac{\sum_{i=1}^{n} \int_0^\infty x dF_i^B(x)]}{\eta_B}. \qquad \text{(B-1)}$$

Lemma 2 show that $\frac{1}{n\lambda_B} F_i^A(x) - x$=constant, which means

$$dF_i^{-j}(x) = n\lambda_j \, dx. \qquad \text{(B-2)}$$

Substituting (3-9) to (3-8), we obtain

$$\frac{\sum_{i=1}^{n}\int_0^{\infty} x\, n\lambda_B \, dx}{\eta_A} = \frac{\sum_{i=1}^{n}\int_0^{\infty} x\, n\lambda_A \, dx}{\eta_B}$$

$$\Rightarrow \frac{\lambda_B}{\eta_A} = \frac{\lambda_A}{\eta_B}. \blacksquare$$

### ***Proof of Lemma 3-4***

*Lemma 3-4. In equilibrium,* $s_i = \frac{1}{n\lambda_B}$.

*Proof:* Lemma 3-2 show that $\frac{1}{n\lambda_B} F_i^A(x) - x$ is constant for x=0 and x= $s_i$.

$$\frac{1}{n\lambda_B} F_i^A(0) - 0 = \frac{1}{n\lambda_B} F_i^A(s_i) - s_i$$

$$\Rightarrow s_i = \frac{1}{n\lambda_B}. \blacksquare$$

### ***Proof of Lemma 5***

*Lemma 3-5. In equilibrium,* $\lambda_B = \frac{1}{2\eta_A B}$.

*Proof:* Firm B's budget constrain can be expressed as

$$\sum_{i=1}^{n}\int_0^{\frac{1}{n\lambda_B}} xd\, F_i^A(x) = \eta_A B. \qquad \text{(B-3)}$$

Equation (B-2) shows that

$$dF_i^A(x) = n\lambda_B \, dx. \qquad \text{(B-4)}$$

Substituting (B-4) to (B-3), we obtain

$$\sum_{i=1}^{n} \int_{0}^{\frac{1}{n\lambda_B}} x\, n\lambda_B \, dx = \eta_A B. \qquad \text{(B-5)}$$

Solving for $\lambda_B$, we obtain $\lambda_B = \frac{1}{2\eta_A B}$. ■

# CHAPTER 4: CONCLUSION

Product positioning is an important strategic marketing decision. Effective product positioning ensures that marketing messages and products are affectively communicated to target consumers, avoiding purely price competition. This dissertation uses the knowledge of product positioning to explain the overlapping phenomenon in the single seller online auctions and niche marketing of the weak firms and integrates them together, although two essays sit in two totally different markets:  a seller in the online auction, and the disadvantaged sellers in a general market.

*In the first essay,* overlapping design is just a special way of product positioning among many potential repositioning strategies. A seller needs to best position his auctions online overtime, deciding whether to set them simultaneously, partial-overlapping, or sequentially, taking into account of bidders' forward- looking, cross-bidding and learning behaviors. In online auctions where bidders, not the seller, decide the final prices of the products, the level of overlap can be used to influence the final bids. *In the second essay*, disadvantaged firms need to best position their products in the market, deciding whether to fight, defend, or flee, taking into account the relative competitive advantage and the dimensions of a product.

Our findings in the two essays show that product positioning decisions of firms can convey valuable information on the demand and the competition conditions in the market.

To this extent, *the first essay* addresses the optimal overlapping strategy for a retailer selling two identical items in an online auction. Five models, from the benchmark one without learning, to the full model with type 1 learning, to the model with entry and type 2 learning, are analyzed and we find that:

1. The optimal overlapping is the trade-off among four factors: forward-looking, learning, time discounting, and the varied demand. **Forward-looking** makes bidders see an option to win in the second auction by a potentially lower price as the winner of the first auction will leave, resulting in bidding less in the first auction. Such bid shading is not present when two auctions run simultaneously, therefore, for this consideration the seller prefers full overlap. **Time-discounting** makes the seller favor a shorter duration of auctions, i.e. a greater degree of overlap. **Learning** helps reduce bidders' uncertainty in valuation, resulting in bidders' more aggressive biddings in both auctions. It causes the final bid in the second auction higher). Additionally, forward-looking bidders, who are able to predict this less lucrative future chance due to learning, bid more aggressive in the first auction (i.e. it causes the final bid in the first action higher.). The change of overlap impacts on **Demand.** The shorter degree of overlap (i.e. the longer duration) means higher demand and thus higher final bid, and more updates when learning happens during the bidding process.
2. Under different conditions, different overlapping strategies shall be adopted. For example, the partial overlapping strategy tends to be more profitable than simultaneous and sequential selling strategies when (1) bidders' uncertainty about product value is a mid-range, (2) value uncertainty is easy to reduce through learning, and (3) the time-discounting effect is not strong.

Those provide a theoretical explanation for the popularity of overlapping auctions in the real online auction environment. The essay contributes to the auction theory literature, as most current studies have focused on simultaneous or sequential auctions, and only a few have considered overlapping auctions.

*In the second essay*, we explore the product positioning strategies for the asymmetric firms. Two models, one-dimensional and multi-dimensional, are analyzed via game-theoretical modeling. We find that:

1. A successful competitive strategy needs to consider an attack or defense based on the nature of the market and their relative competitive strength. The weaker firm chooses to use a niche strategy; however, simply avoiding the clutter of mass markets to stay in the niches isn't enough. They need to randomize to attack the core markets of the stronger firms when the differences in strength are small. This strategy is similar as Guerrilla tactics, and its effectiveness lies in the difficulty for the stronger firms to predict the attacks and defence adequately, so that the stronger firms have to strengthen their cores and leave the niche markets to the weaker firms.
2. Among the stronger and weaker firms, small/weak firms are more likely the sources of most "radical products". Successful innovation provides the chance for smaller firms to better serve their niche markets, but also breaks the trend of "an increasing division emerging between the winners and losers".

This research provides a theoretical explanation to many observations: the focus and differentiation strategies of asymmetric firms under different levels of advantage gaps and the flush of radical innovations of small firms, and also provide an operational guide to marketing managers on the product positioning/resource allocation issues. This essay also provides a starting point to understand the role of competitive strength in shaping firms' product positioning strategies.