Mineral Resource Classification and Drill Hole Optimization Using Novel Geostatistical Algorithms with a Comparison to Traditional Techniques

by

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A thesis submitted in partial fulfillment of the requirements for the degree of

Master of Science

in

Mining Engineering

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Abstract

The classification of mineral resources must follow standards that were created to regulate the public disclosure of projects assisting investors and their advisers in making investment decisions and preventing the publication of erroneous, misleading and fraudulent information. The definition of classification categories are subjective and based on the degree of confidence in geologic continuity, granting the choice of an adequate technique for classification to an expert professional, commonly referred as a competent or qualified person. Many techniques have been developed for resource classification in recent years and to understand the state of practice of resource classification, a survey of Canadian NI 43-101 reports was conducted. The survey revealed that geometric techniques dominate the techniques used for classification and that, although geostatistical techniques are not commonly used in practice, kriging variance appeared as criteria for classification more often than expected.

Geostatistical techniques have the potential to introduce relevant information to the classification paradigm, such as, accounting for the spatial correlation of attributes of interest or even allowing the assessment of local distributions that enable the use of meaningful probabilistic classification criteria. Kriging variance is known to generate undesirable artifacts (bullseyes) and often requires post processing. A novel cross-validation variance technique that keeps the advantages of variance based techniques while reducing artifacts is proposed in this thesis. The classification is

performed by (1) removing one or more drill holes with highest kriging weight (2) calculating KV using the surrounding data and (3) applying a threshold for classification. The thresholds applied are naturally higher than those originally used for regular kriging variance due to the removal of nearby drill holes.

A second technique based on a moving window classification applied to conditionally simulated realizations is also proposed. This addresses the problem of the scale of classification and artifact generation leading to a high resolution classification with reduced artifacts. Moreover, simulation uses meaningful probabilistic criteria for the classification such as precision and confidence (e.g. a block is classified as measured if its grade falls within ±15% of the mean 95% of the times).

The optimum location of infill drill holes is also addressed in this thesis. An objective function that maximizes classified resources while minimizing the kriging variance is proposed. The optimization algorithm based on an intelligent random search with a random restart and local refinement. Although the proposed technique is not guaranteed to find a global optimum, the proposed methodology is capable of finding reasonable solutions that lead to improved resources. All techniques developed in this thesis are applied to synthetic examples and a case study. The case study is a Cu-Mo deposit located in northern Chile.

To my wife, Marina, for your love, support and patience.

To my parents, Marden and Neide, and to my brothers,

André and Thiago.

Acknowledgements

I would like to express my gratitude to my supervisor, Dr. Jeff Boisvert, for his support, encouragement and patient guidance without whom this thesis would not have been possible.

I would like also to thank the Centre for Computational Geostatistics (CCG) for the financial support and also thank the sponsors from industry for supporting our research. I thank all fellow colleagues at CCG for their assistance and friendship.

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List of Abbreviations and Symbols

ACVV	average cross-validation variance
ADHS	average drill hole spacing
AKV	average kriging variance
BRS	blind random search
BV	theoretical variance of blocks within the domain
CIM	Canadian Institute of Mining
CVV	cross-validation variance
CRIRSCO	Committee for Mineral Reserves International Reporting Standards
CV	coefficient of variation
DHS	drill hole spacing
GA	genetic algorithm
GR	gradient based technique
GSLIB	Geostatistical Software Library
JORC	Joint Ore Reserves Committee
KE	kriging efficiency
KV	kriging variance
LRS	localized random search
MRS	modified random search
NAEN	National Association for Subsoil Examination
NR	neighborhood restrictions
NROs	National Reporting Organizations
PERC	Pan-European Reserves and Resources Reporting
REE	Rare Earth Elements
RS	regression slope
SAMCODES	South African Mineral Codes
SERNAGEOMIN	Chilean Geological Survey

SEDAR	System for Electronic Document Analysis and Retrieval
SGS	sequential Gaussian simulation
SME	Society for Mining, Metallurgy & Exploration
SMU	selective mining unit
WAKV	weighted average kriging variance

ah	range in horizontal plane
av	range in vertical direction
c_1	weight given to total resource tonnage $[0, 1]$ (g^{-1})
c_2	weight given to the KV $[0, 1]$ (t^2/g^2)
c_1'	intended weight given to total resource tonnage
c_2'	intended weight given to the KV
c	sample spacing along the drill hole
$CCOF_i$	cumulated contribution to the objective function of the i$^{\text{th}}$ drill hole
$Exp(\mathbf{h})$	exponential variogram model
$\gamma(\mathbf{h})$	variogram function
$\mathbf{h}$	lag vector
$I_i(\mathbf{x})$	binary variable: 1 when the i$^{\text{th}}$ block meet the classification criteria and 0 otherwise
M	number of random perturbations to estimate standardization factors
maj	range in major direction of anisotropy
med	range in medium direction of anisotropy
min	range in minor direction of anisotropy
μ	Lagrange multipliers
n	number of drill holes being optimized
nb	number of blocks in the model
$n_v(\mathbf{u})$	number of samples found within the volume $V(\mathbf{u})$
$O^{(0)}$	objective function initial value
O_1	objective function component linked to maximization of resources
O_2	objective function component linked to minimization of KV

$O_j^{(m)}$	objective function component j value after perturbation m
p_i	probability of selecting the i^{th} drill hole
ρ_i	density of the i^{th} block (t/m^3)
$sph(\mathbf{h})$	spherical variogram model
$s(\mathbf{u})$	data spacing at location **u**
$\sigma^2_{iK}(\mathbf{x})$	KV of the i^{th} block considering the infill drilling (g^2/t^2)
$\mathbf{u}$	point in space
V	volume of the block (m^3)
$V(\mathbf{u})$	search volume
$\mathbf{x}$	set of drill hole parameters
Z_i^K	kriging estimate of the i^{th} block (g/t)
w_i	weight given to the i^{th} block

Chapter 1: Introduction

Mining projects are capital intensive and commonly listed in stock exchange to raise capital for development. As with any investment, understanding the risks involved is crucial. The quantity of mineral resources is a critical asset of a mining project and the degree of confidence in its estimation must be clearly reported to investors and their advisers. Because the mining sector has a large impact in many countries economies, there was a movement towards the standardization and regulation of the public disclosure of mineral projects.

Mineral resource classification standards were created in order to define rules for public disclosure of mineral projects providing investors with reliable information to assist in making investment decisions and preventing the disclosure of misleading, erroneous or fraudulent information. The idea of creating codes and guidelines for the regulation of the public disclosure of exploration results, mineral resources and mineral reserves is not new. The first published was the JORC Code in 1989, which is now in its sixth edition. Due to the globalization of the mining industry, there was a need for the development of an international standard (Weatherstone, 2008).

The Committee for Mineral Reserves International Reporting Standards (CRIRSCO) composed of representatives of the major National Reporting Organizations (NROs) developed the Template mostly based on the JORC Code with the purpose of defining a minimum international standard for public disclosure of mineral projects that could be used by countries that want to create or update their own codes with international best practices. The NROs are: Australia (JORC), Canada (CIM Standing Committee on Reserve Definitions), Chile (National Committee), Europe (PERC), Russia (NAEN), South Africa (SAMCODES) and United States (SME). (CRIRSCO, 2013)

The estimation of mineral resources is based on samples from the deposit, which are acquired from different sources such as drill cores, trenches, channels, random chips, among others. The estimation quality and geological confidence are not only dependent on the quantity of available data but also on its quality. The quality of these samples have a direct effect on the resources and must be ensured by quality control and quality assurance programs. A number of different quality parameters are discussed by CRIRSCO (2013), Yeates and Hodson (2006), Postle et al. (2000) and Dominy et al. (2002). According to the Canadian Institute of Mining (CIM) standards on mineral resources and reserves, the classification of mineral resources is dependent on *"... nature, quality, quantity and distribution of data..."* (Postle et al., 2000). Based on the quality of their estimates the resources are classified in one of three possible categories: inferred, indicated, and measured with increasing degree of geological confidence.

1.1 Problem statement

Although classification standards exist, they are subjective and mostly rely on the judgement of the qualified/competent person. The existing standards do not specify the methodology to be applied for the definition of a resource category. Geometric techniques that use the quantity of data as the criteria for classification have been applied for many years and are the most used techniques in practice. In the past decades, many other more sophisticated techniques were proposed, but are rarely used in practice because of increased complexity, dependency on parameter selection, and artifact generation.

Geometric techniques such as drill hole spacing (DHS) and neighbourhood restrictions (NR) are the most commonly used in practice. These methods are very simple to apply and have understandable parameters that make them transparent to most interested parties and generate reproducible classification results. On the other hand the results cannot be easily translated to a quantitative measure of accuracy/confidence on resource estimation and any statement regarding accuracy is qualitative. These methodologies are described in Section 1.2.2.

Geostatistics provides tools for the quantitative measure of accuracy/confidence in resource estimation, representing an improvement when compared to geometric techniques. The use of kriging variance (KV) for classification generates artifacts close to sample locations, the same happens when simulation is used to classify at a selective mining unit (SMU) scale. Two new techniques for classification are proposed in this thesis, the first is based on the KV and cross-validation and the second is based on simulation. These techniques retain the improvements of using geostatistics for classification while reducing artifacts, enhancing the accuracy of classification models. Even in cases that the direct use of geostatistical techniques is deemed inappropriate, it can be used to check the classification results of standard techniques.

A second area of interest related to the classification of resources, is the optimization of drill holes to maximize classified resources. Current drill hole optimization techniques are mostly 2D, while 3D methods are limited as they consider some parameters constant or they have sub-optimal parameter definition. An ideal optimization algorithm should allow for 3D drill holes; however, allowing for arbitrary collar, strike and dip parameters, coupled with the optimization of N drill holes results in a difficult, non-convex optimization problem. Moreover, the objective function to optimize depends on the particular deposit, but is often linked to minimizing a local uncertainty measurement, commonly the KV. The goal of locating infill drill holes also changes during the project life. Perhaps the minimization of the KV is deemed important in the early stages of a project. In later stages, the location of infill drill holes may be targeted to increase classified resources to increase reserves listed in the stock exchange. An optimization strategy that simultaneously maximizes classified resources while minimizing the KV is proposed to deal with different optimization objectives. In addition to enhanced parameter flexibility, the consideration of resources classification in the objective function allows for improved resources.

Thesis Statement

The proposed classification techniques improve classification accuracy, while incorporating resource classification into infill drilling optimization allows for the design of drilling campaigns that maximize classified resources, while minimizing *local uncertainty.*

1.2 Literature review

The idea of classifying resources based on the confidence level of estimation was introduced by the national standards for classification. The most recognized national codes are those from countries that constitute the NROs and the main features of their codes are introduced in the Template proposed by CRIRSCO. A brief discussion regarding the key aspects of the Template is presented here (Section 1.2.1).

Geometric techniques such as DHS and NR are the simplest and most popular among practitioners. Due to their relevance, a detailed description of these techniques is given (Section 1.2.2).

The KV is not as popular as geometric techniques for resource classification, but it has been used in practice. It has the advantage of accounting for data redundancy and grade continuity. The use of conditional simulation has been recommended by a number of authors due to its potential to introduce valid measures of accuracy and confidence to the classification paradigm, but it is still not used in practice. An overview of different geostatistical based techniques is provided (Section 1.2.3).

The optimization of infill drill holes has been investigated by different authors in recent years. A brief description of these works is given (Section 1.2.4).

1.2.1 Classification standards

The Template published by CRIRSCO is very similar to The JORC Code 2012 Edition, but it is compatible with the codes of the NROs (CRIRSCO, 2013). Although the Template itself does not constitute a 'code', it was chosen due to its compatibility with the well known national codes avoiding the need to describe them separately.

The classification standards are guided by three main principles, which are transparency, materiality and competence. A public report provides all necessary information for decision making and it has to be based on the work of a competent/qualified person. A competent/qualified person is a skilled and experienced professional, a member of a recognised professional organization and is responsible and accountable for part of or the whole content of the report. (CRIRSCO, 2013)

The motivation behind developing classification standards is to provide a general definition of different categories based on a quantified level of geological confidence so that a qualified/competent person or persons can judge the uncertainty based on their past experience with similar deposits. The Template defines three main categories: exploration results, mineral resources and mineral reserves (Figure 1.1).

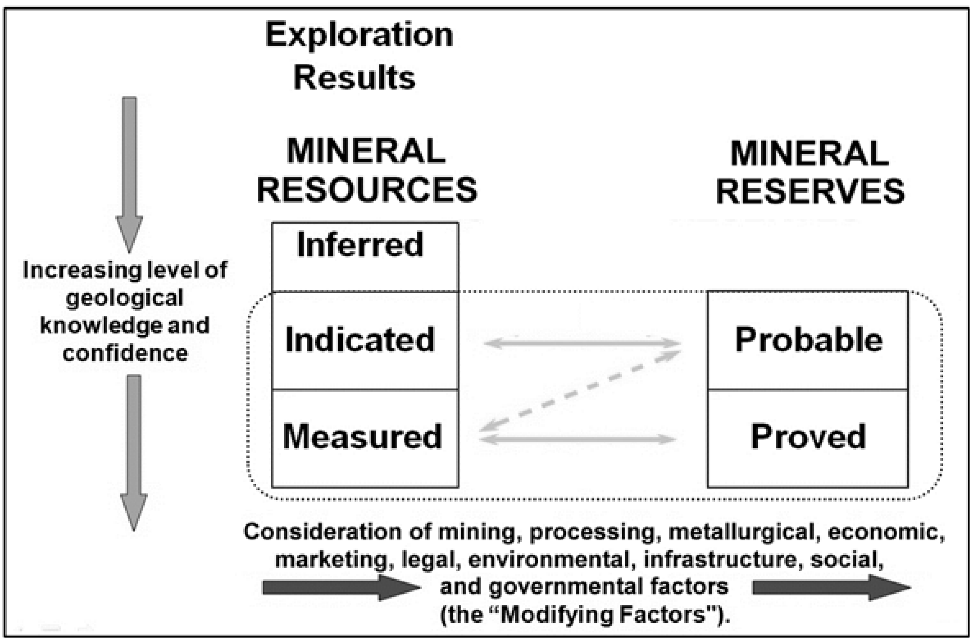


Figure 1.1: General relationship between exploration results, mineral resources and mineral reserves. (CRIRSCO, 2013)

Exploration results are used for reporting data and information generated in the early stages of exploration but are not sufficiently reliable for calculation of reasonable estimates of tonnage and grade. The category type and classification criteria must be made clear in the report. (CRIRSCO, 2013)

Mineral resource, which is the main focus of this thesis, is defined as *"... a concentration or occurrence of solid material of economic interest in or on the Earth's crust in such form, grade or quality and quantity that there are reasonable prospects for eventual economic extraction. The location, quantity, grade or quality, continuity and other geological characteristics of a mineral resource are known, estimated or interpreted from specific geological evidence and knowledge, including sampling"* (CRIRSCO, 2013, p. 10). This means that in addition to the confidence in geologic and grade continuity, reasonable expected technical and economic factors based on previous experience on similar deposits must be considered in order to define mineral resources.

Mineral reserve is defined as *"... the economically mineable part of a measured and/or indicated mineral resource"* (CRIRSCO, 2013, p. 15). In order to classify as mineral reserves the technical and economical viability of extraction must be demonstrated. Mineral reserves are subdivided into proved and probable according to the confidence on the technical, economic, environmental, social and governmental factors (modifying factors, see Figure 1.1) used to convert the mineral resources into mineral reserves.

Mineral resources are subdivided into three categories: inferred, indicated and measured with increasing level of confidence in the geologic and grade continuity. The definition of each category in the Template is given as follows (CRIRSCO, 2013, p. 11-13):

> *"An inferred mineral resource is that part of a mineral resource for which quantity and grade or quality are estimated on the basis of limited geological evidence and sampling. Geological evidence is sufficient to imply but not verify geological and grade or quality continuity. ...*
>
> *An indicated mineral resource is that part of a mineral resource for which quantity, grade or quality, densities, shape and physical characteristics are estimated with sufficient confidence to allow the application of Modifying Factors in sufficient detail to support mine planning and evaluation of the economic viability of the deposit. Geological evidence is derived from adequately detailed and reliable exploration, sampling and testing and is sufficient to assume geological and grade*

or quality continuity between points of observation. ...

A measured mineral resource is that part of a mineral resource for which quantity, grade or quality, densities, shape, and physical characteristics are estimated with confidence sufficient to allow the application of modifying factors to support detailed mine planning and final evaluation of the economic viability of the deposit. Geological evidence is derived from detailed and reliable exploration, sampling and testing and is sufficient to confirm geological and grade or quality continuity between points of observation."

1.2.2 Geometric techniques

The most commonly used classification techniques are geometric. These methods are preferred due to their simplicity and transparency, which make them easily understandable for all stakeholders (Deutsch et al., 2006). There are a variety of geometric measures used for classification, but the most popular are DHS and NR.

Drill hole spacing (DHS)

This technique classifies blocks based on the spacing between drill holes near the block location under consideration. The application of this technique is straightforward when drill holes are vertical and regularly spaced with minimal deviation. In this situation the classification can be reduced to two dimensions and easily done by hand with the use of polygons.

In cases where the drill holes are irregularly spaced, drilled in different directions or with significant deviations, the DHS may be calculated locally. There is no unique way to calculate DHS. A methodology for the unbiased calculation of data spacing/density based on Delaunay triangulation and Voronoi polygons is proposed by Naus (2008). Wilde (2010) proposed a program for calculation of DHS for non-vertical drill holes that uses Equation 1.1.

$$s(\mathbf{u}) = \left(\frac{V(\mathbf{u})}{c \cdot n_v(\mathbf{u})}\right)^{\frac{1}{2}} \tag{1.1}$$

where:

$s(\mathbf{u})$	- data spacing at location **u**;
$V(\mathbf{u})$	- search volume;
c	- sample spacing along the drill hole;
$n_v(\mathbf{u})$	- number of samples found within the volume $V(\mathbf{u})$;

Thresholds on DHS are often selected based on past experience with similar deposits at the discretion of the qualified person. The DHS accounts only for the quantity of data. Deutsch et al. (2006) suggests the use of DHS for resource classification while using conditional simulation only to support the selection of input parameters. The calculation of DHS in this thesis is performed using the approach presented in Appendix A that was proposed as an improvement to the methodology proposed by Wilde (2010).

Neighbourhood restrictions (NR)

The NR technique of classifying blocks is based on a distance to nearby samples and constraints related to the number and configuration of the data within a search radius (Figure 1.2). This technique is most commonly applied by defining estimation passes with different search parameters. Blocks that are estimated by less restrictive passes are classified as inferred, an intermediate restrictive pass defines the indicated category and the most restrictive pass defines the measured blocks. (Emery et al., 2006)

1.2.3 Geostatistical approaches

The increasing popularity of geostatistical methods for classification is because of the potential to introduce additional relevant factors such as grade continuity, data redundancy, and statistically valid measures of accuracy and confidence. There have been

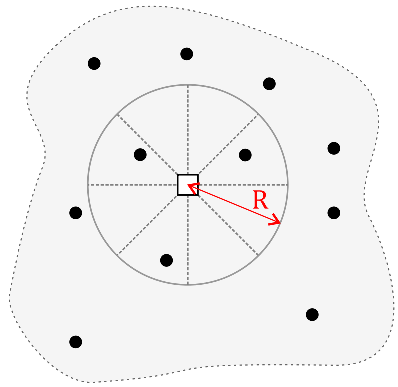


Figure 1.2: Illustration of the NR technique with three informed octants and three drill holes found within a search radius R.

a number of different techniques for classification, mostly based on KV. The use of conditional simulation has been suggested by a number of authors as a better approach to access uncertainty, but is not common industry practice.

Kriging variance (KV)

Kriging is an interpolation technique that minimizes the squared error between the estimated value and the unknown true value. The resultant error variance, also known as the KV, is only dependent on the estimation location, the position of samples and the variogram. The interested reader is referred to Journel and Huijbregts (1978) for a detailed derivation/explanation of kriging and the KV.

Typically, the KV is used as a classification criteria by applying thresholds based on the variogram. The application of these thresholds to the KV in order to define the categories was recommended by Royle (1977), Sabourin (1984) and Froidevaux, et al. (1986) (as cited in Sinclair and Blackwell, 2002).

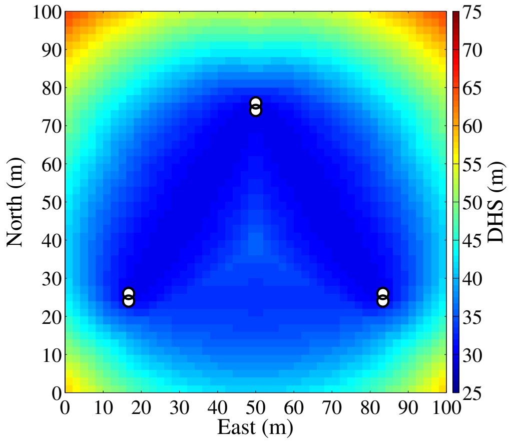
More sophisticated techniques based on KV were proposed by a number of authors. David (1988) proposed the use of a relative kriging standard deviation defined as the ratio between kriging standard deviation and the estimated value of a block for classification. Arik (1999) proposed a classification based on a combination of the ordinary kriging variance and the weighted average of the squared difference between the estimated value of a block and the data values used in its estimation. This combined variance is also used in the calculation of a resource classification index proposed later by the same author. The resource classification index includes the estimated value of the block and a calibration factor (Arik, 2002).

Yamamoto (2000) proposed a classification technique based on interpolation variance that is the weighted average of the squared difference between the estimated value of a block and the data values used in estimation, the weights used are the ordinary kriging weights. Mwasinga (2001) gives a brief description of some other geostatistical classification approaches such as variogram range; kriging variance pdf; confidence limits based on normal and lognormal models; block efficiency; Isobel Clark's classification index and linear regression slope.

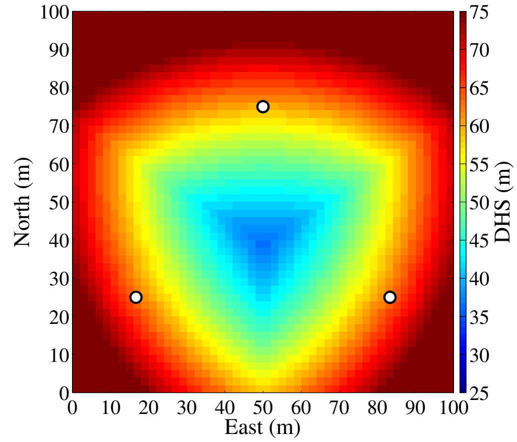
The advantage of using KV as the criteria for classification is the consideration of the spatial structure of the variable and the redundancy between samples (Figure 1.3); however, it often produces classification maps with undesirable artifacts (Figure 1.4). Artifacts are common near sample locations as the KV is very low, resulting in patches of measured blocks in indicated zones. Moreover, the KV does not account for the proportional effect, which is a common characteristic of earth sciences data and may be important in the high grade zones where the variance is often high.

Kriging efficiency (KE) and regression slope (RS) were proposed by Krige (1996) to evaluate the quality of estimation and avoid conditional bias (Equations 1.2 and 1.3 respectively). These techniques have been used for resource classification as shown in Section 1.2.1. The use of KE or RS for classification will result in classification maps similar to KV in that artifacts persist as both indexes are close to one near data locations.

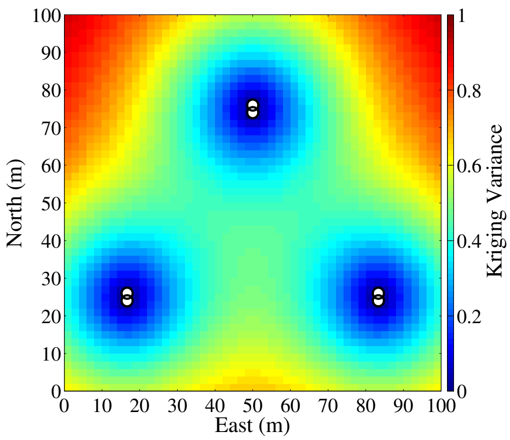
$$KE = \frac{BV - KV}{BV} \tag{1.2}$$



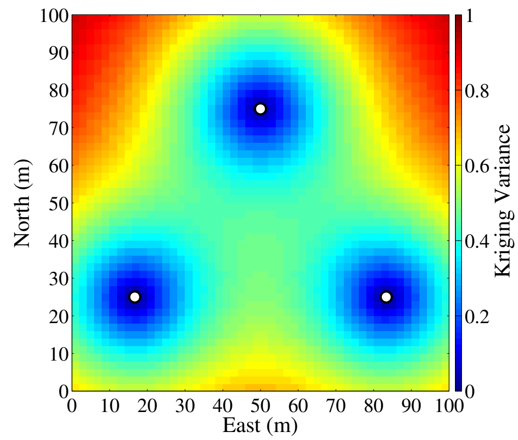
(a) DHS with redundant data



(b) DHS without redundant data



(c) KV with redundant data



(d) KV without redundant data

Figure 1.3: Sensitivity of DHS and KV to redundant data. DHS is very sentitive to redundant data while KV accounts for it properly. The white markers are data locations.

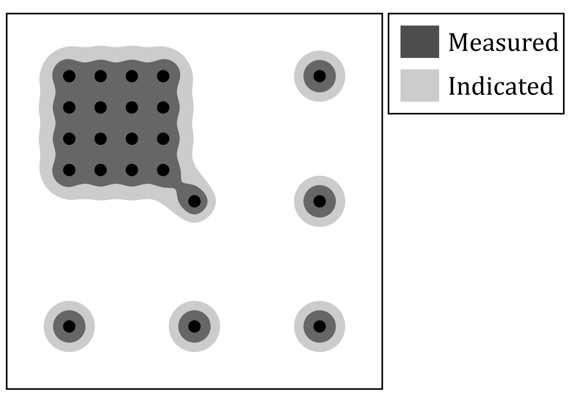


Figure 1.4: Illustrative example of artifacts generated by the use of KV in resource classification.

$$RS = \frac{BV - KV + |\mu|}{BV - KV + 2|\mu|} \tag{1.3}$$

where:

KE	- kriging efficiency;
RS	- regression slope;
BV	- theoretical variance of blocks within the domain;
KV	- kriging variance;
μ	- Lagrange multipliers;

A new classification technique based on the KV and cross-validation is proposed in order to keep the advantages of variance based techniques while reducing the artifacts from conventional methods improving the accuracy of classification results and reducing the need to manually adjust KV based techniques.

Conditional simulation

The KV generates smooth maps that do not consider the proportional effect (Manchuk et al., 2009) and the true variability of the data. Conditional simulation corrects for this at the cost of generating multiple realizations that must be processed simultaneously. The mining industry is hesitant to consider conditional simulation as the processing of multiple realizations for mine design is difficult (Dominy et al., 2002) however, it is becoming more common (Snowden, 2001). Each realization generated by simulation is an equally probable representation of the mineral grades and the full set of realizations must be treated as an ensemble, which allows for the ability to quantify the uncertainty in the variable under consideration.

The realizations can be scaled to any volume of interest, which is often an SMU or a production volume over some time period of interest. The scaled models can be used to evaluate the distribution of grades at a specific support allowing for a meaningful utilization of probabilistic criteria for resource classification. It is up to the qualified person to determine the criteria that would define each category. There are at least three critical parameters to be defined: volume under consideration; precision; and confidence interval (e.g. the values of a quarterly production volume must fall within ±15% of the mean 95% of the time in order to be classified as measured).

A further advantage of using simulation based techniques is the possibility of considering many other important factors that should be considered for resource classification such the incorporation of all identified sources of error (Dominy et al., 2002). Moreover, a significant proportion of current geostatistical research is focused on generating better conditional simulations; using simulation for classification allows practitioners to take advantage of the numerous advances being made in this field of study.

The use of conditional simulation for resource classification is suggested by many authors such as Wawruch and Betzhold (2005), Dohm (2005), Dominy et al. (2002) and Snowden (2001), which suggest it is a better approach to access uncertainty when compared to the KV. Other authors such as Deutsch et al. (2006) recommends its use only as a supporting tool while the final classification criteria should remain geometric. The hesitation to use simulation stems from the concern that the results of classification are highly dependent on the modeller assumptions and the parameters chosen, making resource disclosure less transparent to investors when advanced and complex methodologies are used.

Dohm (2005) proposed a methodology that uses conditional simulation to estimate the coefficient of variation (CV) of different production volumes: local (SMU), monthly and annual. The estimated CVs are later used to define change of support factors that account for the correlation between the blocks. These factors are used to determine the threshold between classification categories. A block (SMU) with a CV (given by its kriging standard deviation and kriging estimate) small enough to support a monthly production volume with a precision of ±15% with 90% confidence (assuming Gaussian distributions) is classified as measured. The annual production volume is used to define the indicated category and the remaining blocks are assigned to the inferred category. The main drawback of this methodology is that conditional simulation is only used to make a global estimate of the coefficient of variation for different production volumes not taking advantage of additional local information contained in the realizations. It also assumes the block distribution to be Gaussian while the simulation can be used to provide the local distribution at any location.

Similar artifacts as observed when using KV are also observed while using conditional simulation results to classify SMU blocks. The classification at a larger scale is often suggested when considering simulation because it allows for the use of meaningful confidence/precision parameters and avoids the generation of artifacts in the final classification map, but the classification results may vary depending on the grid definition due to the coarse resolution of panels. The technique proposed in this thesis combines the advantages of using a larger volume for classification with the desired SMU resolution for the classification maps, but reduces artifacts.

1.2.4 Infill drilling optimization

Determining the optimum location of infill drilling has been a constant focus of research because (1) the high cost of drilling suggests that using fewer drill holes is preferred and (2) targeting locations of the deposit that are locally uncertain should result in better mine planning decisions and increased profit of the operation. Existing methodologies for optimizing the location of infill drill holes consider minimizing the KV as the objective and are mostly restricted to two dimensions (Soltani and Hezarkhani, 2013).

The use of the KV to assess local uncertainty is attractive for a number of reasons (1) the KV can be calculated before the drilling is executed (2) KV can consider anisotropies in the deposit (3) spatial relationships between locations can be correctly accounted for and (4) the KV is independent of the grade. The simplest optimization scheme is to optimize a single drilling location by selecting the location with the highest KV, drill holes are placed in a one-at-a-time manner after recalculating the KV; however, this is unlikely to yield an optimum reduction in the overall variance as it does not consider the interaction between the new drill holes and the interaction with existing drill holes (Gershon, 1987).

The optimization of infill drilling locations must be done considering all drill holes simultaneously in order to be able to find the optimal solution. The potentially large number of drill holes to optimize and the number variables required for drill hole parameterization, coupled with the relatively few constraints on these variables, leads to a high dimensional non-convex optimization problem; an exhaustive search for an optimum solution is infeasible, motivating the use of advanced optimization algorithms.

Scheck and Chou (1983) proposed a method based on fixed point theory. The optimization is achieved by an iterative gradient based technique (GR) that is highly dependent on the starting locations and quickly converges to a local minimum. Getting a good starting location for this optimization problem constitutes itself a difficult optimization exercise. Other drawbacks of this methodology are (1) the consideration of only two dimensions and (2) the assumption that kriging weights are constant within a certain neighborhood to simplify the partial derivatives.

Gershon (1987) proposed the use of integer programming for selecting a set of optimum locations from previously selected candidates using the branch and bound procedure. This methodology is capable of providing an optimum within the previously selected locations but cannot provide the best solution among all possible locations, especially in three dimensions where the strike and dip of the drill holes would provide additional complexity. Moreover, when too many sites are considered this technique becomes unpractical.

Soltani et al. (2011) use a binary genetic algorithm (GA) in which the objective function is the minimization of the average kriging variance (AKV) for an industrial mineral deposit. They apply the algorithm in 3D and a 2D simplification of the same deposit in order to highlight the importance of considering the third dimension; however, the drill holes are still considered vertical. Mohammadi et al. (2012) applies simulated annealing to find the optimal locations for infill drill holes for a 3D case study using the weighted average kriging variance (WAKV) as the objective function, where the weights are the estimated grade of each block. In this work, block grades are introduced as relevant factors in the objective function, but again the drill holes are considered vertical. Soltani and Hezarkhani (2013) use direct search simulated annealing to optimize the location of 3D directional drill holes. To be able to optimize directional drill holes, the azimuth is considered constant while the dip is optimized to maximize the intersection of the drill hole with the ore body and minimize the proportion of the drill hole within the overburden material. Although some of the presented works consider the optimization of 3D drill holes they are still limited in that they often fix some of the parameters that define the drill holes, such as strike and dip. There is an apparent hole in the available techniques, the optimization of an arbitrary number of drill holes with unconstrained parameterization (collar location, strike and dip) to minimize KV and/or maximize resources is proposed.

1.3 Thesis organization

The subjective nature of the regulatory codes regarding the classification procedure allows the practitioners to use any technique deemed adequate. In order to evaluate the current state of practice regarding resource classification, a survey of 120 recent Canadian NI 43-101 technical reports was conducted. The results are shown and discussed in Chapter 2.

The most commonly used classification techniques are reviewed and two new techniques are proposed. The first is based on KV and involves removing one or more drill holes with the highest weights while performing kriging and using the resultant KV for classification. This technique has the advantages of variance based techniques and reduces artifacts (see Chapter 3). The second is based on conditional simulation and uses a moving window approach for classification at the desired selective mining unit (SMU) resolution based on larger production volume criteria. This technique has the advantage of accounting for heteroscedasticity, which is a common characteristic in mineral deposits and also reduces artifacts since a production volume scale is considered for the actual classification (see Chapter 4). In order to demonstrate the applicability of the proposed techniques for resource classification a case study is developed in Chapter 5.

The possibility of using some of the aforementioned classification techniques in the optimization of infill drilling for maximizing the classified resources is also explored. In recent years there has been many works related to infill drilling optimization and most of these works are based on minimizing local uncertainty (i.e KV). Many techniques have been used for this purpose, such as: random search, modified random search (MRS), GA, GR and simulated annealing. Most of these works are restricted to 2D examples but some authors have proposed methodologies for optimization in 3D. Accounting not only for uncertainty (KV) but also for grades has been considered by using the estimated grade of each block as a weight in the objective function. The main contribution of this work on this topic is to introduce a different way to formulate the objective function that is not limited to minimizing the KV, but also focusing on maximizing the classified tonnage. This subject is developed in Chapter 6 along with a case study.

Chapter 2: Current State of Practice Regarding Resource Classification

A number of different techniques have been proposed for resource classification, but only few of them are used in practice. It is important to understand the factors that motivate the use of certain techniques over others, as well as the limitations of the most common techniques. This information can be used to assist in the development of new techniques that would provide significant improvements in resource classification.

The public disclosure of mineral projects by companies listed on Canadian exchanges must follow the Canadian Institute of Mining (CIM) standards for mineral resources. The documents that contain this disclosure are known as NI 43-101 and are publicly available through the SEDAR website (SEDAR, 2013). The SEDAR database constitutes a great source of information and a survey on its database was performed for the evaluation of the current state of practice regarding resource classification.

2.1 Methodology

This study was conducted in 2013 with the objective of evaluating the common techniques currently used for resource classification and for this reason it includes Canadian NI 43-101 technical reports issued mostly in 2012 (125 reports) and 2011 (27 reports).

The main focus of this survey was to understand the current practice in resource classification but additional useful information was also retained from the surveyed reports. The database consists of the following information:

- project location
- principal commodities

- classification technique
- classification criteria
- estimation technique

Only reports with sufficient information to determine the technique used for classification were considered. The reports that were not considered in this study are: reports without resource classification; reports with only Inferred resources; reports with classified resources but without clear explanation of the methodology applied; and, reports on the same deposit that were already included in the database and that did not present major changes.

2.2 Results

The locations of the projects are shown in Table 2.1. Only 20.8% of the projects are located in Canada, followed by United States (16.8%), Africa (13.9%), and South America (11.9%).

Within Canada the provinces that had most number of projects were: Québec (44.8%), British Columbia (34.5%), and Yukon (27.6%) (Table2.2).

Location	% of reports
Canada	20.8
United States	16.8
Central America	7.9
South America	11.9
Africa	13.9
Europe	9.9
Asia	8.9
Australia	9.9

Table 2.1: Geographic distribution of surveyed NI 43-101 reports in World

The percentage of reports by commodity is given in Table 2.3. Gold is most common with 53.7% of the reports. Copper, Iron and Rare Earth Elements (REE) also appear in the top of the list. All these commodities experienced an increase in price during the years preceding the year of the surveyed reports, which explains their prevalence.

Province	% of reports
Québec	31.0
British Columbia	23.8
Yukon	19.1
Ontario	7.1
Labrador	4.8
Manitoba	4.8
Nunavut	4.8
Northwest Territories	2.4
Saskatchewan	2.4

Table 2.2: Geographic distribution of surveyed NI 43-101 reports in Canada

Commodity	% of reports
Gold	28.9
Gold-Copper	12.8
Gold-Silver	12.1
Iron	11.4
Copper	5.4
REE	4.7
Silver	3.4
Lead-Zinc	3.4
Copper-Nickel	2.7
Nickel	2.7
Uranium	2.4
Other	10.1

Table 2.3: Frequency of each commodity on the surveyed NI 43-101 reports

The most common classification techniques are NR and DHS, accounting for more than 75% of the reports. The KV also appeared in a surprising number of reports and was often combined with a geometric technique. The prevalence of KV (8.6%) was expected to be lower as most practitioners are reluctant to use non-geometric techniques that require many additional subjective modeling decisions.

It is common to find the combined use of different techniques. Some techniques that appear in combination with the most common techniques are: RS, KE and manual classification.

Type	% of reports	Regular drilling (%)	Irregular drilling (%)
NR	50.7	6.5	93.5
NR + DHS	2.0	66.7	33.3
NR + MANUAL	2.0	0.0	100.0
DHS	25.0	34.2	65.8
KV	2.6	0.0	100.0
KV + NR	5.3	0.0	100.0
KV + DHS	0.7	0.0	100.0
RS + NR	2.0	0.0	100.0
RS + DHS	1.3	100.0	0.0
RS + NR + DHS	0.7	100.0	0.0
RS + NR + MANUAL	0.7	0.0	100.0
KE + NR	0.7	0.0	100.0
MANUAL	2.0	0.0	100.0
OTHER	4.6	0.0	100.0

Table 2.4: Summary of classification methods used in NI 43-101 technical reports published in Canada in 2012 and 2011 (152 reports were considered)

As expected, geometric techniques are the most used in practice. DHS seems to be preferred in cases in which the drill holes are fairly regularly spaced, which simplifies the calculation of spacing and the classification maps can be defined by manually drawing polygons. Note that reports classified manually with DHS as the criteria for classification were considered in the DHS category. The reports considered in the manual category were those in which the classification was performed by hand, but considering different criteria that may or may not include DHS.

The NR technique seems to be preferred when the drill holes are irregularly spaced, which is commonly the case in mining. The use of NR for irregular drilling could be due to the fact that most commercial software do not offer an option for DHS calculation for irregularly spaced drill holes. Moreover, NR can be easily applied by modifying the search parameters of the estimation functionality in most available software. The most common constraints considered while applying NR are: minimum number of data, minimum number of drill holes and minimum number of informed octants. Appropriate thresholds are decided based upon the experience of the qualified person. The neighbourhood range is sometimes associated to the variogram ranges, but it is not a rule.

The KV was used in 8.6% of the surveyed reports and in most of these cases KV was applied in combination with other techniques with the purpose of avoiding artifacts. In the cases where KV was used alone, the classified maps were reviewed bench by bench and artifacts were removed by hand. Automated techniques, such as dilatation and erosion were also used to remove artifacts from classification maps. The variance thresholds for different classification categories are often chosen based on an equivalent DHS that would support that category.

It is interesting to note that two other techniques based on KV were also found in this survey. Slope of regression and kriging efficiency are values derived from the KV and produces results that are similar to the results of classification based solely on KV.

Another interesting information retrieved from the surveyed reports is the estimation technique applied in each case. The most commonly used estimation techniques, according to the survey, are given in Table 2.5. There is a prevalence of geostatistical techniques with ordinary kriging accounting for over 50% of the surveyed reports. Inverse distance is widely used with 29.61% of the reports and polygonal is surprisingly high accounting for 7.24% of the reports. In some cases, different techniques are considered appropriated for different domains for the same project, which explains the use of more than one estimation technique for some of the reports.

Technique	% of reports
Polygonal	7.3
Inverse Distance	29.6
Inverse Distance and Ordinary Kriging	2.6
Ordinary Kriging	54.0
Ordinary Kriging and Multiple Indicator Kriging	1.3
Ordinary Kriging and Indicator Kriging	1.3
Multiple Indicator Kriging	2.6
Sequential Gaussian Simulation	0.7
Median Indicator Kriging	0.7

Table 2.5: Techniques used for estimation in surveyed NI 43-101

2.3 Conclusion

The geographic distribution of projects listed in the Canadian Stock Exchange is diverse with the majority of the projects located in the Americas (57.4%). Within Canada, the provinces and/or territories that lead in the number of projects are: Québec, British Columbia and Yukon accounting for 73.8% of reports.

As expected, geometric techniques are the most common in practice. DHS is used mostly when the drill holes are regularly spaced while NR is preferred for irregularly spaced drill holes. The lack of software that supports drill hole spacing calculation for irregularly sampled deposits might be the reason for the prevalence of NR.

The KV has been used in several reports, mostly combined with other techniques for artifacts reduction. Even in cases in which the KV was used alone, the final results were treated for artifact reduction. KV is closely linked to DHS but with appealing advantages such as the ability to account for redundancy between data and the use of a measure of spatial continuity (variogram). Despite the benefits of KV, the artifact generation seems to be a major issue. These facts motivated the development of the cross-validation variance (CVV), which is presented in Chapter 3. This technique preserves the advantages of KV classification, but with reduced artifacts.

More advanced geostatistical techniques, such as conditional simulation, were not used for resource classification. Simulation could bring a number of benefits to resource classification such as the use of confidence intervals that account for the proportional effect, which is commonly observed in mining data. KV is a good measure of the spatial distribution of data, but it is not a good measure of uncertainty as it does not account for data values or shape of local distributions. KV is able to provide the local distribution of uncertainty in Gaussian space, but an appropriate measure of local uncertainty would require the assessment of the local distributions in the original units, which is achieved by using simulation. The lack of its use in the surveyed reports together with its potential benefits motivated the proposal of a technique based on simulation, which is detailed in Chapter 4.

Chapter 3: Cross Validation Variance

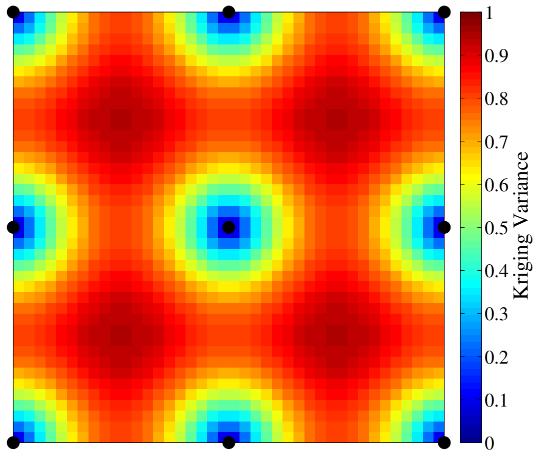
The survey presented in Chapter 2 showed that geostatistical techniques are the most used in practice for resource estimation, however, this is not true for classification. Even though geostatistical techniques are not commonly applied for classification, the survey results revealed a higher usage than expected. There are interesting properties of KV that motivates its use for classification, such as accounting for data redundancy and spatial structure of data, but the artifact generation and the need for additional subjective modeling parameters prevents widespread use. The main issue with KV are the artifacts that require post-processing for their removal. CVV was developed to overcome the limitations of the existing techniques while keeping the advantages of variance based techniques (Silva and Boisvert, 2014).

This chapter is organized as follows. Section 3.1 gives a detailed description of the CVV technique while Section 3.2 presents a small example that was built in order to compare the proposed technique with the most used in practice (NR, DHS and KV).

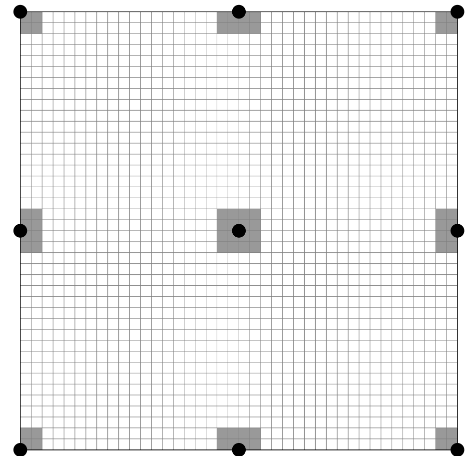
3.1 Methodology

Artifacts (bullseyes and holes) are generated because the KV is very low near data locations. An example of artifacts generated by using KV for classification is shown in Figure 3.1. Blocks very close to a drill hole that are located in areas with low drilling density are classified with a resource category that is higher than it should be (Figure 3.1b). This could result in measured blocks in zones that are expected to be indicated, or indicated and measured blocks in inferred zones. Figure 3.1c and 3.1d shows a proper classification using KV in a densely sampled area that is expected to be classified. The threshold for classification are the same for both cases (Figure 3.1b and 3.1d).

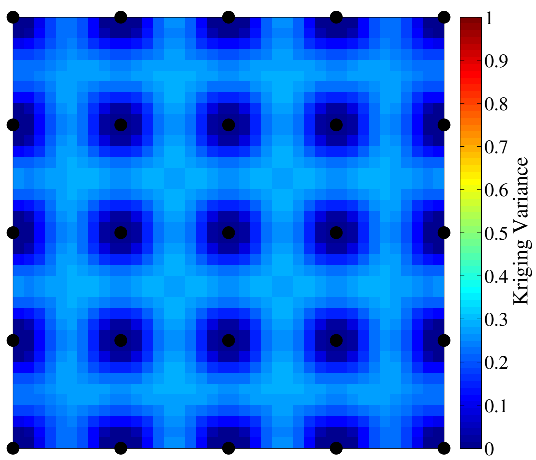
The impact of removing one drill hole on the KV of nearby blocks is expected to be higher



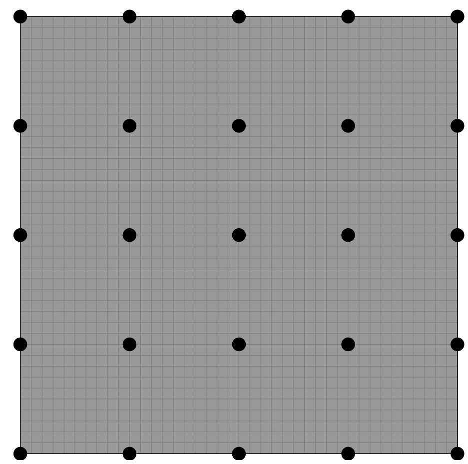
(a) KV value - low data density



(b) KV classification - low data density



(c) KV value - high data density



(d) KV classification - high data density

Figure 3.1: KV classification artifacts. (Classified blocks are colored in gray in *b* and *d*; black dots are data locations)

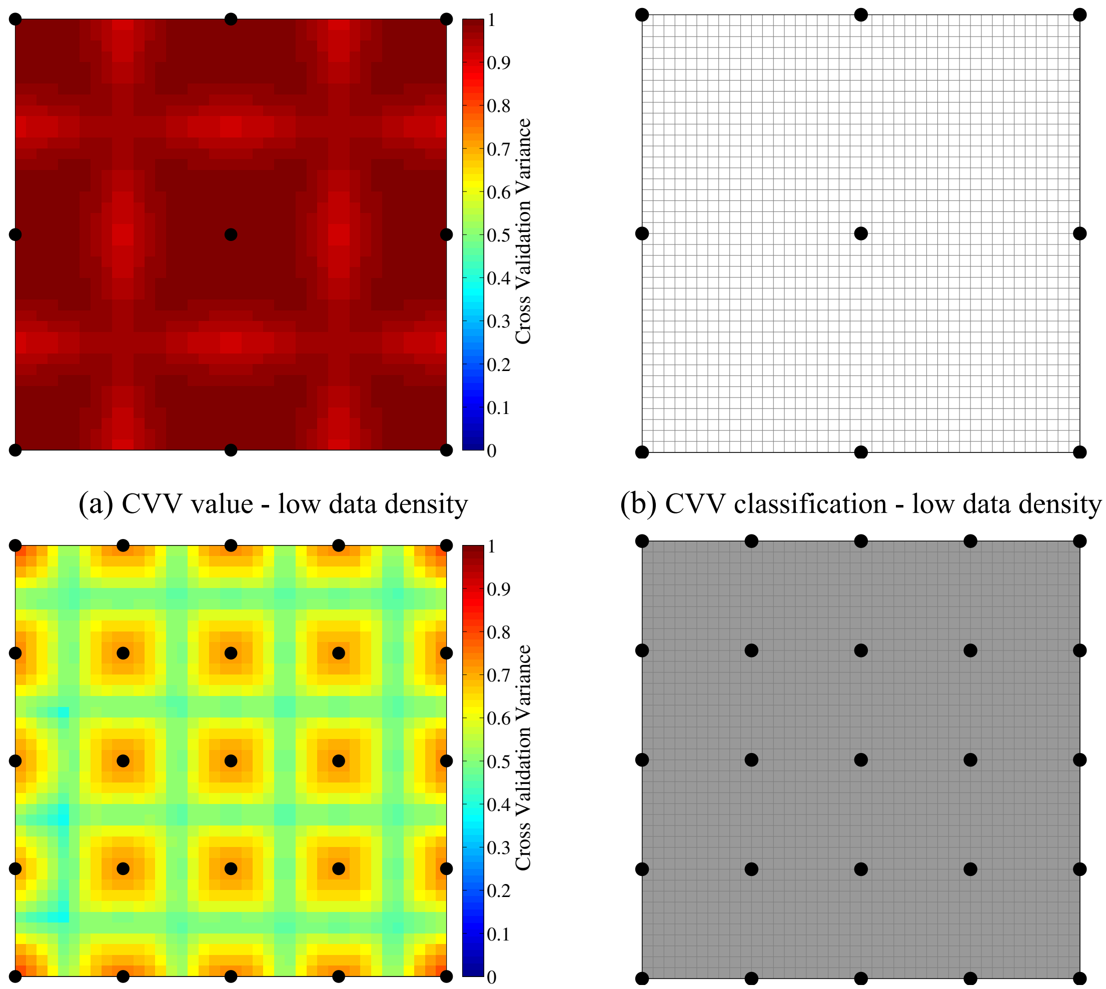
in low drilling density zones than it is in high drilling density zones. Based on this fact a methodology based on cross-validation was used to reduce artifacts, resulting in more accurate classification maps that retain the advantages of variance based techniques.

The CVV is calculated by removing one or more drill holes with the highest weights while performing kriging and using the resultant KV to classify the blocks. This technique: is suitable for regular and irregular drilling patterns; accounts for spatial structure and redundancy between data; and reduces artifacts caused by using the KV alone. Classification is done by (1) removing the drill hole with highest kriging weight (2) calculating KV using the surrounding data and (3) applying a threshold for classification.

The number of drill holes to be removed and thresholds are defined by the user in order to minimize the undesirable ‘holes’ and ‘patches’ that are created with conventional KV classification. An improved reduction of artifacts can be achieved by using the average cross-validation variance (ACVV) resulting from removing different numbers of drill holes and averaging all the CVV's.

The same exercise presented for KV in Figure 3.1 was used to calculate the CVV removing a single sample for each block to be classified. The result is shown in Figure 3.2. As expected, the variance of each block is higher than the original KV for all blocks in both high and low data density case and, as result, the classification threshold must also increase. The thresholds for KV are usually decided to match a certain DHS, the same can be done for CVV. For this example the classification threshold was increased from 0.30 to 0.80. As observed in Figure 3.2b the misclassified blocks were removed while the high data density zone remained classified.

A block must be informed by at least two drill holes in order to be classified when CVV is calculated by removing one drill hole. A general purpose software for kriging in 3D (`kt3d`) that is part of the Geostatistical Software Library (GSLIB) (Deutsch and Journel, 1998) was modified for the calculation of CVV in this work.



(a) CVV value - low data density

(b) CVV classification - low data density

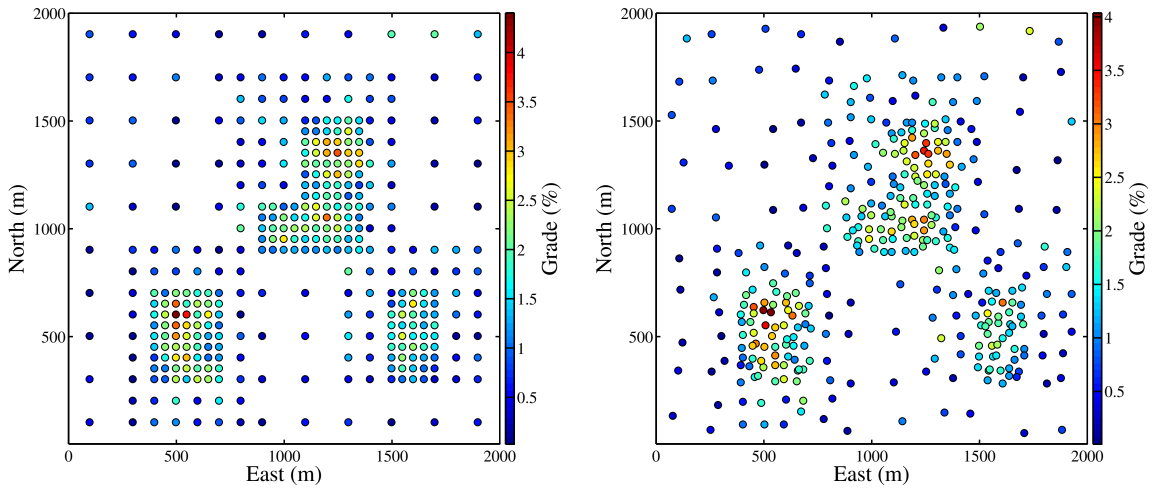
(c) CVV value - high data density

(d) CVV classification - high data density

Figure 3.2: CVV classification. (Classified blocks are colored in gray in *b* and *d*; black dots are data locations)

3.2 Comparison with common classification techniques

The NR, DHS, KV and the proposed CVV are compared for 2D and 3D examples with regular and irregular drilling patterns to highlight the advantages and disadvantages of each. Using `sgsim` (Deutsch and Journel, 1998), the 2D model was generated by an unconditional sequential Gaussian simulation (SGS) and sampled on a regular and irregular grid (Figure 3.3). The 3D example uses data from drill holes on a porphyry copper-gold deposit (Figure 3.4).



(a) 2D model - regular drilling

(b) 2D model - irregular drilling

Figure 3.3: 2D example generated by `sgsim` and sampled in a regular and irregular pattern

Classification for the 2D regular grid is trivial, but is included as a bench mark for the techniques. The model is created to resemble a constant thickness (10 m) tabular deposit in which the modeling block size is 25m by 25m. For the regular 2D example the model is sampled by three regular grids: 200x200m; 100x100m; and, 50x50m (Figure 3.3a). For the irregular 2D example a random component is added to the coordinates of the regular grid before sampling (Figure 3.3b). The variogram of the data is composed of two isotropic spherical models with ranges of 200 and 300 meters with 25% and 75% of contribution to the sill respectively.

For the 3D example the variogram of the data is composed of three spherical models and a nugget effect of 15% (Equation 3.1). The 3D example have two nominal DHS of 50x50 and 25x25 meters. The modeling block size for the 3D example is 15x15x10m.

$$\begin{aligned}\gamma(\mathbf{h}) = 0.15 + 0.18 \times sph_{\substack{av=15m\\ah=23m}}(\mathbf{h}) + 0.17 \times sph_{\substack{av=180m\\ah=23m}}(\mathbf{h}) \\ +0.50 \times sph_{\substack{av=180m\\ah=100m}}(\mathbf{h})\end{aligned} \tag{3.1}$$

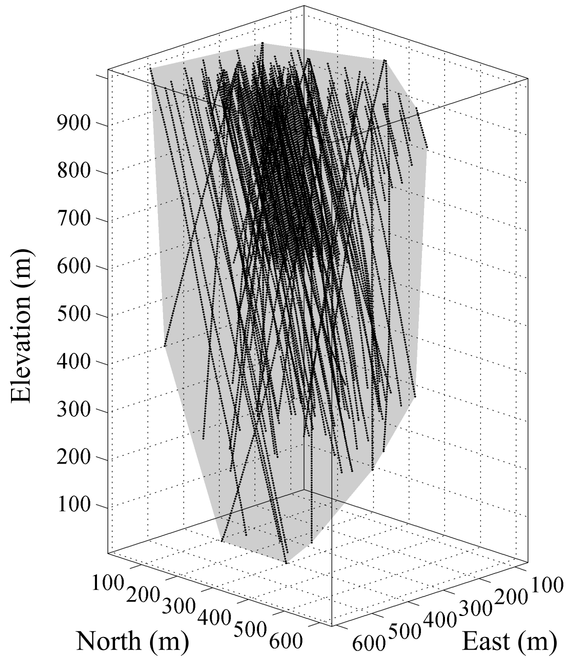


Figure 3.4: 3D example: porphyry copper-gold deposit.

3.2.1 2D regular

The synthetic 2D example with a regular drilling pattern is considered first to visualize the results of each technique (Figure 3.3a and Figure 3.5). For DHS the measured blocks are those within the area drilled at 50x50m grid with extrapolation of half a spacing (25m), indicated blocks are those within the area drilled at 100x100m with extrapolation of 50m and inferred blocks are those within the area drilled at 200x200m.

An additional contribution of this work is that the use of the average drill hole spacing (ADHS) removes the reliance on selecting a single value of n. The methodology for the calculation of ADHS is described at Appendix A. In this example the method for calculation of ADHS is the search for a number of data and the parameters used are $n = 2$ to 8 with steps of one unit. The thresholds for classification are 50m for measured, 100m for indicated and 200m for inferred.

For the NR classification the parameters are chosen by a visual sensitivity analysis in order capture the areas considered measured, indicated and inferred. Blocks with at least 8 drill

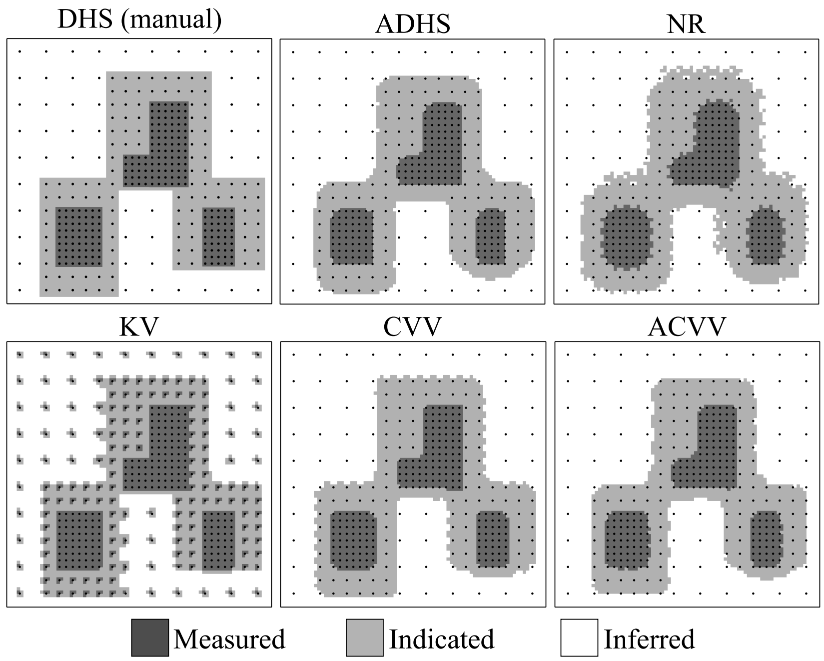


Figure 3.5: Classification results for the 2D regular grid. Axes dimensions: 2000m by 2000m.

holes within 100m are considered measured, indicated blocks are those with at least 8 drill holes within 200m.

For the KV classification the thresholds are defined based on same drill hole spacing used for DHS classification. The threshold between measured and indicated is 13% of the sill and the threshold between indicated and inferred is 31% of the sill.

The number of drill holes removed for the CVV method is one and the thresholds are chosen by a visual sensitivity analysis in order to reduce artifacts. The removal of drill holes increases the KV for each block, which leads to higher thresholds when compared to the KV technique. The thresholds used are 20% and 50% of the sill. For the ACVV classification, the number of drill holes removed is one and two and the thresholds are 25% of the sill and 60% of the sill, again selected based on visual inspection.

For this synthetic example the DHS zones defined by hand (titled DHS in Figure 3.5) are matched well by the majority of the techniques as this is a fairly easy set of drill holes to classify. As expected, the KV perform well in classifying different zones but with the problem of artifacts (patches) close to drilling locations that are successfully removed using the proposed CVV methodology. In this case there is no anisotropy and the proportional effect is not considered.

3.2.2 2D irregular

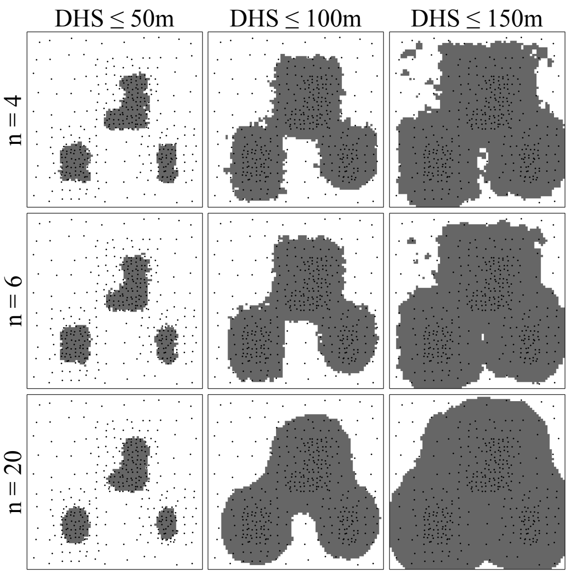
The 2D example with irregular drilling is used to visualize the effect of parameters for each technique and to visualize the adequateness of each technique to situations in which classification is not straightforward.

Drill Hole Spacing (DHS)

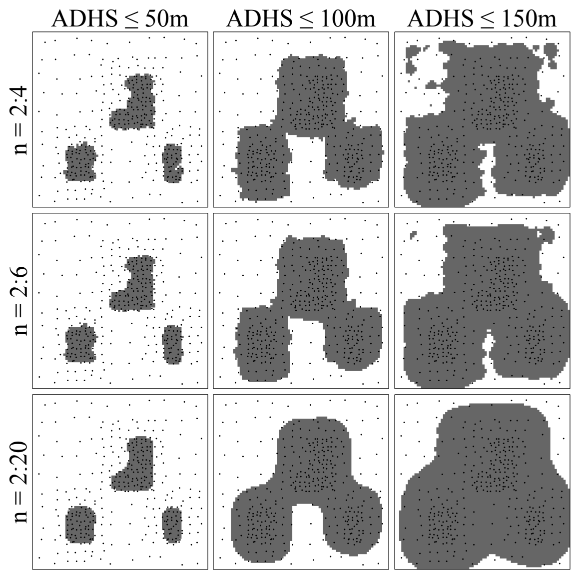
A visual analysis of the parameters for DHS is shown in Figure 3.6a. Increasing the number of data used in calculation reduces the artifacts but also increases misclassified blocks. There is no control on the search radius considered as it is a function of the block location and number of data searched (n). Data far from a block may inadvertently assign a higher category for a block; a small number of drill holes is recommended to avoid this problem. More accurate (closer to the known 'by hand' technique) and smoother (fewer holes and patches) maps can be achieved using the proposed ADHS technique (Figure 3.6b)

Neighborhood restrictions (NR)

A visual analysis of the parameters for the NR technique is shown in Figure 3.7. Classification based on NR requires two parameters (search radius and minimum number of drill holes) and performs similarly to DHS for irregular drilling patterns. The classification maps may require post processing to reduce noise in the classification borders.



(a) DHS calculated with a single parameter



(b) ADHS searching 2 to 4, 2 to 6 and 2 to 20 drill holes

Figure 3.6: Sensitivity on DHS parameters. Axes dimensions: 2000m by 2000m.

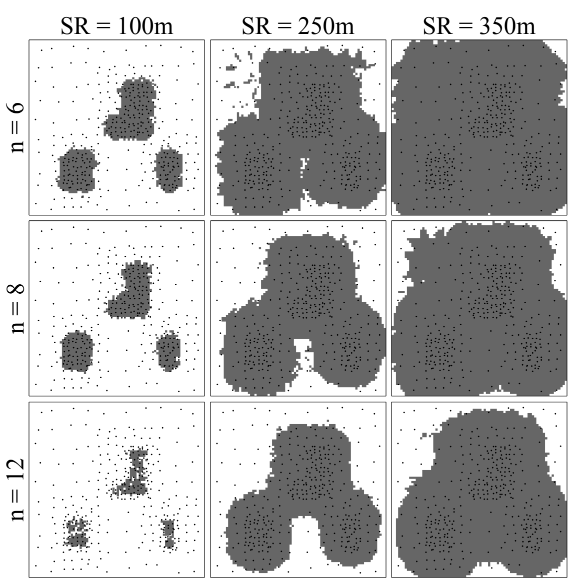
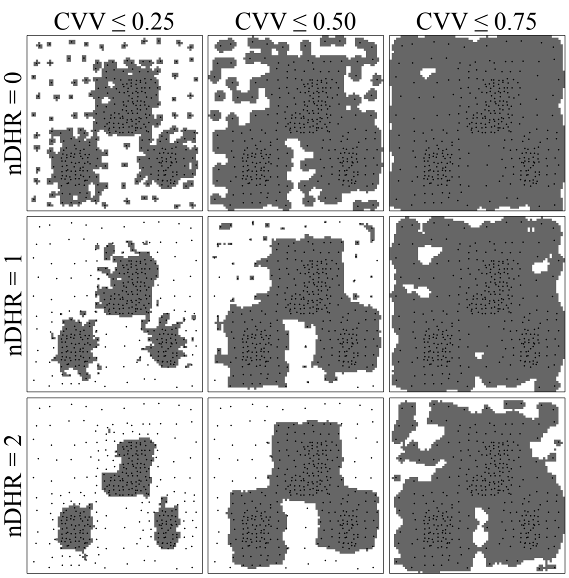


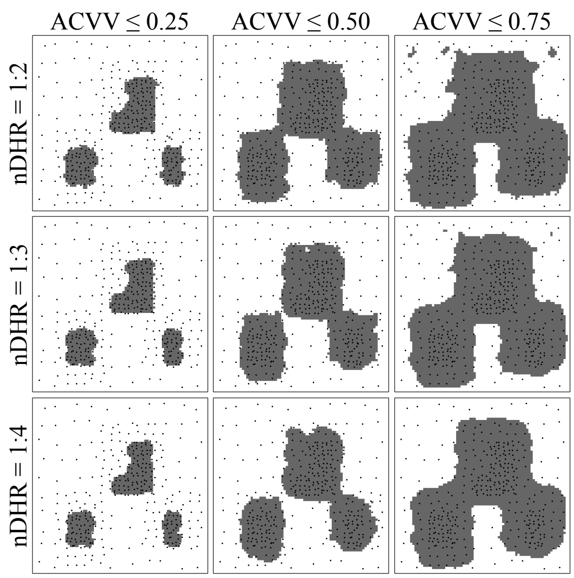
Figure 3.7: Sensitivity on NR parameters: search radius (SR) and minimum number of drill holes (n). Axes dimensions: 2000m by 2000m.

Kriging variance (KV) and Cross-validation variance (CVV)

A visual analysis of parameters for the CVV technique is shown in Figure 3.8. Blocks that are close to redundant drill holes tend to stay in the same category as with the conventional KV method; blocks that are located close to isolated drill holes tend to be downgraded. This is a desirable characteristic but a balance must be made between removing 'patches' and creating new 'holes'. In general, the technique reduces the artifacts compared to using the KV alone (Figure 3.5, Figure 3.9 and Figure 3.10). If the removal of one drill hole is not sufficient for removing artifacts the ACVV may be considered.



(a) CVV calculated removing a single drill hole



(b) ACVV calculated removing 1 to 2, 1 to 3 and 1 to 4 drill holes

Figure 3.8: Sensitivity on CVV parameters: threshold and number of drill holes removed (nDHR). Axes dimensions: 2000m by 2000m. Removing 0 drill holes, nDHR=0, is equivalent to the traditional KV technique.

Classification results

The result of classification for the 2D irregular case is shown in Figure 3.9 for all techniques and illustrates how the different techniques considered perform in a non-straightforward way.

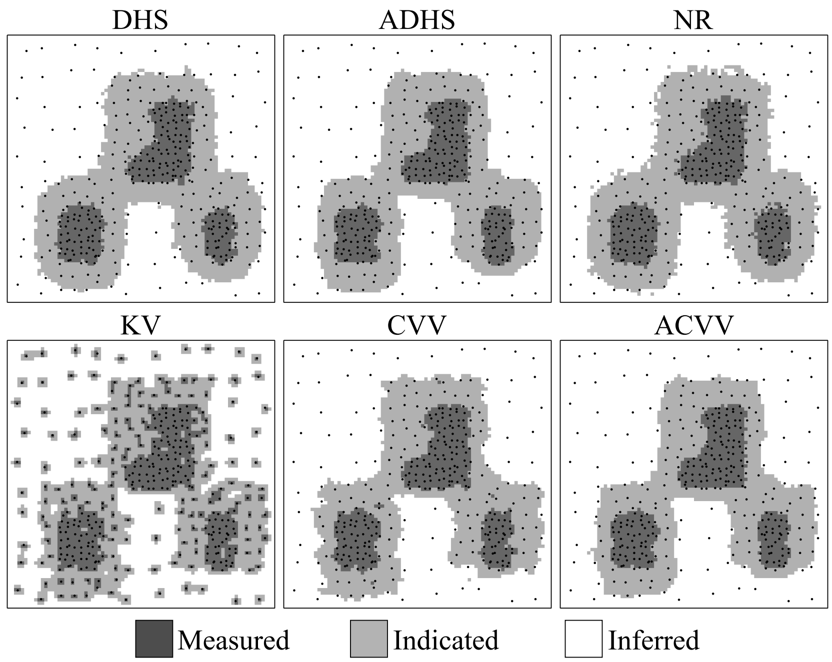


Figure 3.9: Classification results for the 2D irregular grid. Axes dimensions: 2000m by 2000m.

The DHS and ADHS are calculated using the methodology presented in Appendix A with n = 8 and n = 2 to 8 respectively. Blocks with DHS/ADHS less or equal to 50m are measured, blocks with DHS/ADHS less or equal to 100m are indicated, remaining blocks are inferred.

For NR classification the parameters are chosen by a visual sensitivity analysis in order to take the best combination that captured the areas considered measured, indicated and inferred. Blocks with at least 8 drill holes within 100m are considered measured, indicated blocks are those with at least 8 drill holes within 200m.

For the KV classification the thresholds are defined based on a regular grid of 50x50m for measured and 100x100m for indicated. The threshold between measured and indicated is 13% of the sill and the threshold between indicated and inferred is 31% of the sill based on an equivalent DHS.

The number of drill holes removed for the CVV method is one and the thresholds are chosen by a visual sensitivity analysis in order to minimize artifacts. The thresholds used are 20% and 50% of sill. For the ACVV the number of drill holes removed are one and two and the thresholds are 25% and 60% of the sill.

3.2.3 3D example

The 2D examples are appropriate for vertically drilled holes, but often mineral classification problems are three dimensional with a significant proportion being irregularly drilled as a high degree of geological confidence requires drill holes intersecting the ore body in different directions (Yeates and Hodson, 2006). For the 3D example (Figure 3.4), a sensitivity analysis similar to the 2D irregular case is performed in order to select the parameters for various classifiers. The classification models are shown in Figure 3.10.

For this example the grade values are estimated by ordinary kriging (Figure 3.11) and the resources are calculated and classified with each technique. The results of resource calculation and classification are given in Figure 3.12.

The quantitative results for geometric methods and proposed technique are similar with a slight increase in the indicated category for the proposed technique (CVV). There is a considerable increase in the measured category while using KV mainly due to the ‘patches’ artifacts that are common with this classification technique. Ignoring the KV technique, it is interesting to note that the measured and indicated results are surprisingly consistent across all techniques. Of course, the benefit of using KV in CVV is that local classification can be more accurate as data redundancy and anisotropy can be incorporated.

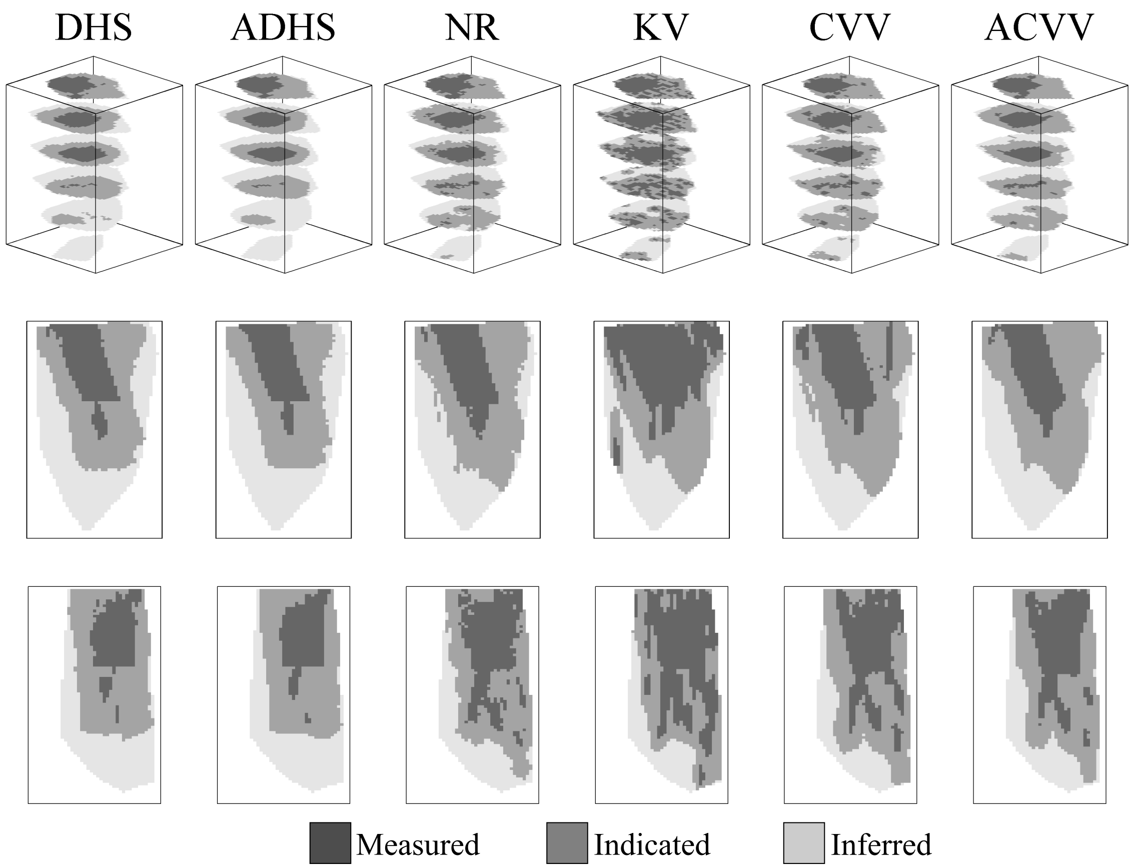


Figure 3.10: Classification results for the 3D example. Axes sizes: 1000m (vertical); 600m (east); and 560m (north). Horizontal slices at elevations 150m, 318m 486m, 654m, 822m, and 990m (first row). Vertical slices at 352.5m east (second row) and 352.5m north (third row).

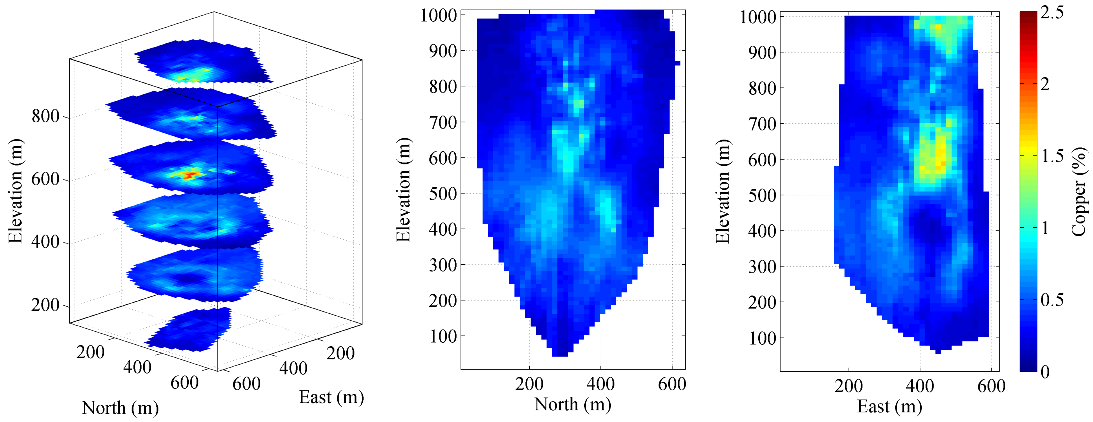


Figure 3.11: Estimated grades for the 3D example. Horizontal slices at elevations 150m, 318m 486m, 654m, 822m, and 990m (left). Vertical slices at 352.5m east (center) and 352.5m north (right).

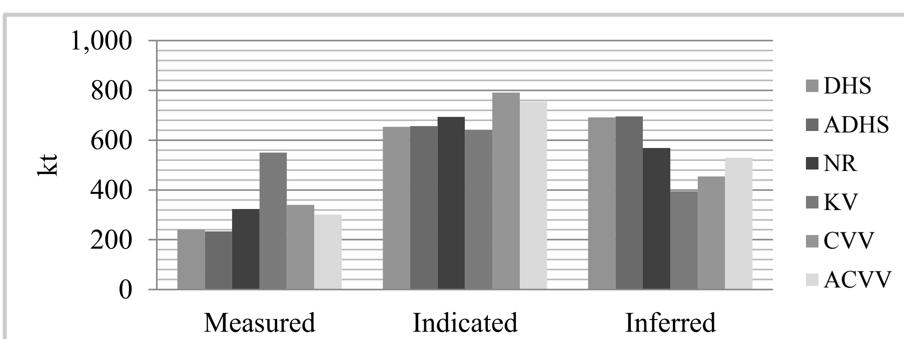


Figure 3.12: Resource classification results, showing the metal tonnage.

3.3 Conclusion

Geometric techniques, which are most commonly used, do not account for the spatial continuity of the variables nor redundancy between data but typically result in classification maps that have less artifacts and are less sensitive to modeling parameters (i.e. kriging and simulation parameters).

When the anisotropy of the deposit is significant and known, it is important that this be incorporated into classification in some way in order to improve the local classification. The KV captures this information but often results in artifacts when used in classification. The combination of cross validation with the KV is able to reduce these undesirable features and incorporate known information on spatial continuity into classification.

The proposed technique represents a viable alternative for resource classification. As with all resource classification techniques, it is the responsibility of the practitioner to assess the appropriateness of the final result based on knowledge of the deposit.

Chapter 4: Simulation Approach for Classification at SMU Scale

Classification of mineral resources plays an important role in the economic assessment of any mining project. There have been many proposed methodologies for applying geostatistical techniques to classification, mostly based on KV; however geometric techniques are the most used in practice due to ease of implementation. KV considers the spatial structure of the variable under study and deals with redundancy, but does not consider heteroscedasticity, which is a common characteristic in mineral deposits. The use of conditional simulation has the potential to overcome this limitation and its use for classification is attractive, although it is not often applied in practice because of its complexity and sensitivity to key parameters such as the covariance function and trend model, which are very dependent modeling assumptions, making resource disclosure less transparent to investors (Silva and Boisvert, 2014). A new methodology for the application of simulation to classification is proposed here in order to classify at the SMU resolution based on a larger production volume criteria.

4.1 Methodology

It is desirable to have a classification model at the SMU scale (Wawruch and Betzhold, 2005), but at this scale the variability is often too high leading to difficulties of using probabilistic criteria (i.e. the values must fall within ±15% of the mean 95% of the time) for resources classification. In order to classify at the SMU scale, the probabilistic criteria has to be less restrictive for allowing measured or indicated resources, but the uncertainty at this scale is only reasonable during the production stage when dense data from blast drill holes is available. Moreover, artifacts are often generated close to drilling locations where SMU blocks are classified as measured even in sparsely sampled areas. These artifacts are undesirable (Deutsch et al., 2006) as they lead to small disconnected volumes that would not be considered during the definition of reserves. These problems are often remedied by classifying resources based on larger volumes, which may represent monthly, quarterly or yearly production (Figure 4.1). In this case the probabilistic criteria can be more restrictive leading to more control at a meaningful scale with fewer artifacts.

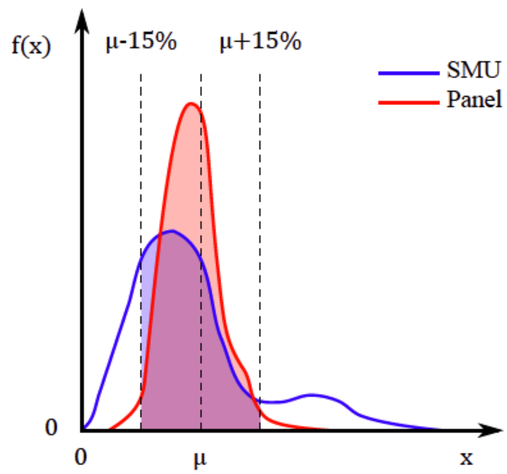


Figure 4.1: SMU grade distribution vs panel grade distribution.

The quarterly production volume is much larger than the SMU size and its shape, volume and position are often unknown as it depends on a detailed mine plan that is certain to change as more data is collected; however, the shape of these larger panels can be determined by previous experience in similar deposits in conjunction with relevant information such as a grade variability model (Wawruch and Betzhold, 2005). Different grid definitions for this large scale block model leads to different classification models (Figure 4.2).

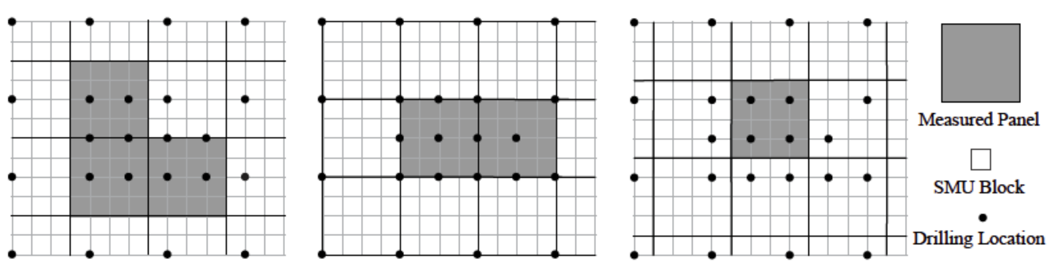


Figure 4.2: Illustrative example of different classification results for different grid origins based on a large production volume scale.

In order to have the desired SMU scale classification resolution while minimizing artifacts, a large production volume is required but the exact panel positioning is not known at the stage of classification. A local classification is proposed that considers a window representing the production panel, which is centered at each SMU block that is classified according to the classification of the panel (Figure 4.3).

The proposed methodology requires the definition of the classification scale (panel) and resolution (SMU) as well as multiple realizations of the truth generated with adequate simulation technique. The steps for classification of a SMU block are: (1) center the panel volume at the center of the SMU to be classified (2) assess the grades of the panel over multiple realizations (3) calculate the average grade and therefore the lower and upper thresholds for each category accordingly with the required precision for each category (4) count the number of blocks that falls within the thresholds of each category (5) assign the category according the required confidence interval of each category.

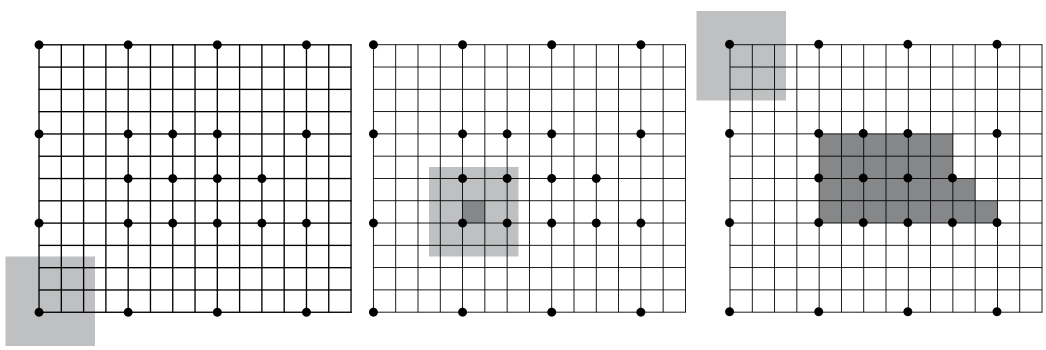


Figure 4.3: Illustrative example of moving window classification. Left: The SMU block is not considered measured as the uncertainty in the larger production volume (light grey) is large. Center: The SMU block is considered measured as there is low uncertainty in the larger production volume (light grey) due to the denser data. Right: SMU blocks considered measured.

4.1.1 Probabilistic criteria

Although simulation is highly dependent on a modeler's parameter selection during its application (covariance function, trend model, etc.), the actual classification parameters are much less subjective than the parameters used for geometric based classification techniques. The probabilistic criteria have a clear meaning and are easily understood. It is easier to understand and justify the classification of a block as measured when its grade falls within ±15% of the mean 95% of the time, than when there are eight drill holes within 100m range. The meaning of eight drill holes within 100m range is not clear, although it may come from previous experience with similar deposits. Moreover a standard probabilistic statement for resources could be made (but is beyond the scope of this work); a standard number of drill holes within a distance is impossible because of the vastly differing geologies of different deposits.

The specific values for the probabilistic criteria to be used is out of the scope of this work, it is certainly case specific and requires expert judgment, as with all classification approaches. The parameters usually range between ±10% to ±30% for precision and between 95% and 80% for confidence intervals (Dohm, 2005; Dominy et al., 2002; Wawruch and Betzhold, 2005; Yeates and Hodson, 2006). The criteria used in the following section and case study (Chapter 5) are within this range.

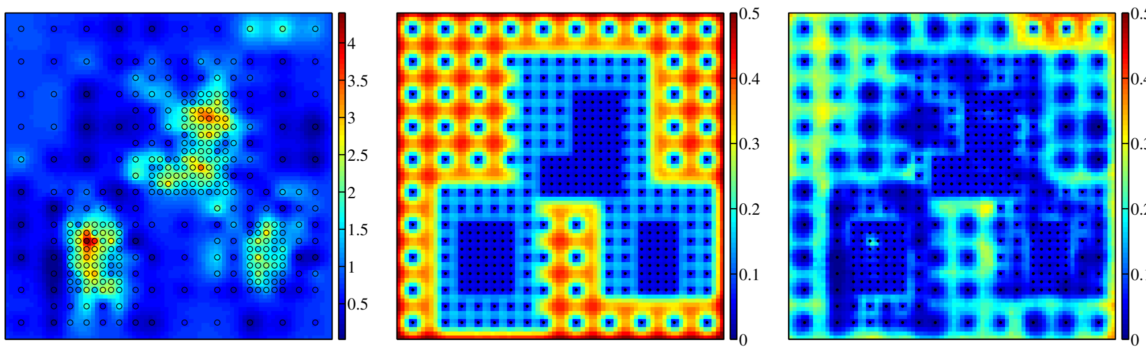
4.1.2 Synthetic examples

For the sake of comparison, the proposed technique is applied to the same 2D and 3D examples presented in Chapter 3 for CVV (Figures 3.4 and 3.4). For the 2D example the SMU size is 25m by 25m and the quarterly production is given by a panel size of 150m by 150m. The SMU size for the 3D example is 15m by 15m by 10m and the quarterly production is given by a panel size of 150m by 150m by 60m.

Local uncertainty

KV accounts for data redundancy and spatial correlation, but it is not a good measure of local uncertainty as it cannot capture properties such as heteroscedasticity that often appears in form of proportional effect as the distribution of geological variables tends to be positively-skewed. In this context, geostatistical simulation will provide a better assessment of local uncertainty (Figure 4.4). The example shown in Figure 4.4 displays the difference between the assessment of local uncertainty with kriging (KV) and conditional simulation (conditional variance) highlighting how simulation captures the dependency of local uncertainty on grade values.

The contribution of proportional effect is mitigated as the volume under consideration increases due to averaging, but the evaluation of the importance the of proportional effect is not possible until simulation is used for assessing the local distribution at the required volume.



(a) Average grade (%) (b) KV ($\%^2$) (c) Conditional variance ($\%^2$)

Figure 4.4: Difference between the assessment of local uncertainty with kriging (KV) and simulation (conditional variance) at SMU scale. Axes dimensions: 2000m by 2000m.

Sensitivity analysis

For a better visualization of the effect of the probabilistic criteria on classification results of the proposed methodology, a visual sensitivity analysis is shown in Figure 4.5. The classification results are not only a function of data location as with KV, but also depend on grade values.

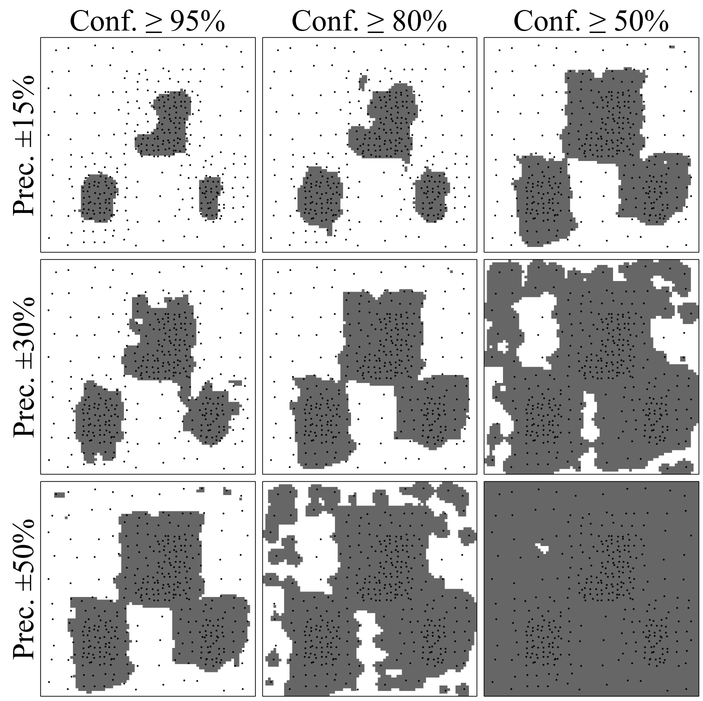


Figure 4.5: Sensitivity on conditional simulation parameters. Axes dimensions: 2000m by 2000m.

When the proportional effect is present, high grade zones will display higher variability than low grade zones for the same data configuration, but it does not mean that lower grade zones are more likely to be classified as a higher category (i.g. measured as opposed to indicated). That is because when precision and confidence intervals are used as criteria for classification, they are also dependent on grade, but in a non-intuitive way. As the grade increases the precision interval, which is relative to the mean, also increases, which means that more variability is allowed for high grade zones while a very small mean grade will lead to small precision interval allowing less variability in low grade values (Figure 4.6). The final classification will depend on the balance between the probabilistic criteria and proportional effect.

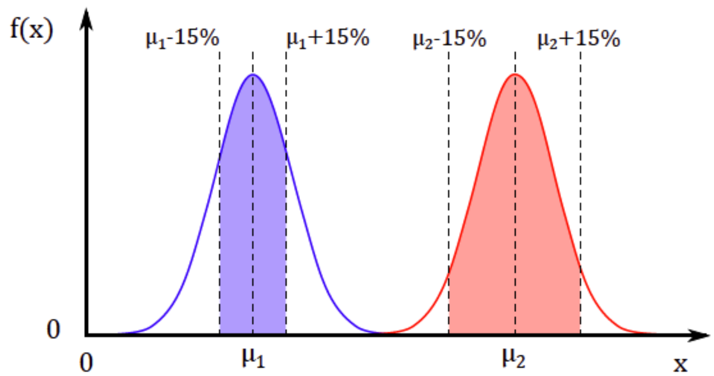
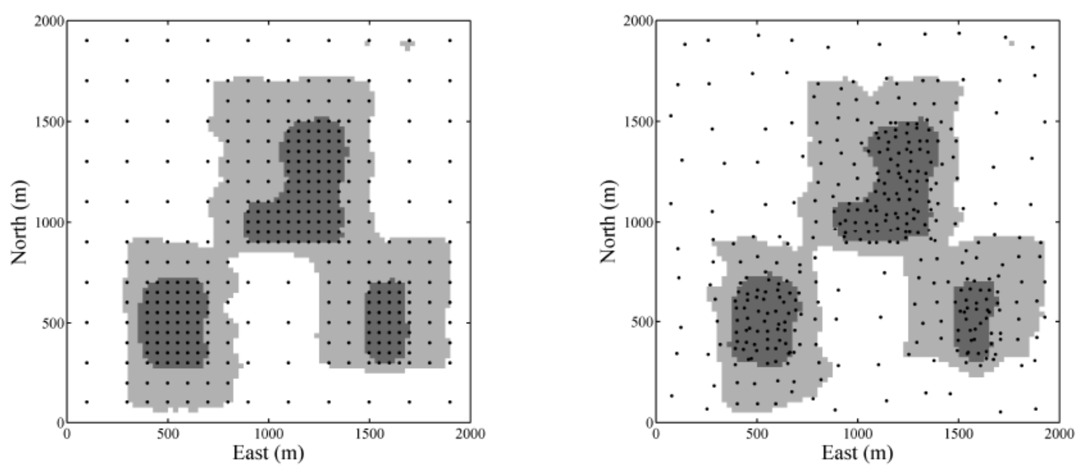


Figure 4.6: Illustration of dependency of the precision interval on the average grade.

Classification results for the 2D examples

The classification based on conditional simulation is performed with the proposed technique for regular and irregular drilling patterns. In order to define measured blocks the quarterly production panel must have a precision of at least ±15% with 95% of confidence while indicated must have a precision of ±30% at 80% confidence interval. The result is shown in Figure 4.7. The classification result accounts for heteroscedasticity, has minimal artifacts and is at SMU scale.



(a) Regular drilling.　(b) Irregular drilling.

Measured　Indicated　Inferred

Figure 4.7: Classification result for the 2D examples.

Scale of classification

The conventional classification for small (SMU) and large scale (panel) is compared with the proposed methodology for classifying at a local scale using a large scale criteria (Figure 4.8). The proposed technique of centering a production volume on each SMU (Figure 4.8 right) reduces artifacts and does not have the undesirable reliance on a fixed large scale grid, where panels clearly contain part measured and part inferred SMU blocks (Figure 4.8 center). In this comparison, the chosen criteria for SMU scale classification is precision of ±30% with 90% confidence for measured and ±30% with 50% confidence for indicated. For the large scale the criteria is precision of ±15% with 95% confidence for measured and ±30% with 80% confidence for indicated.

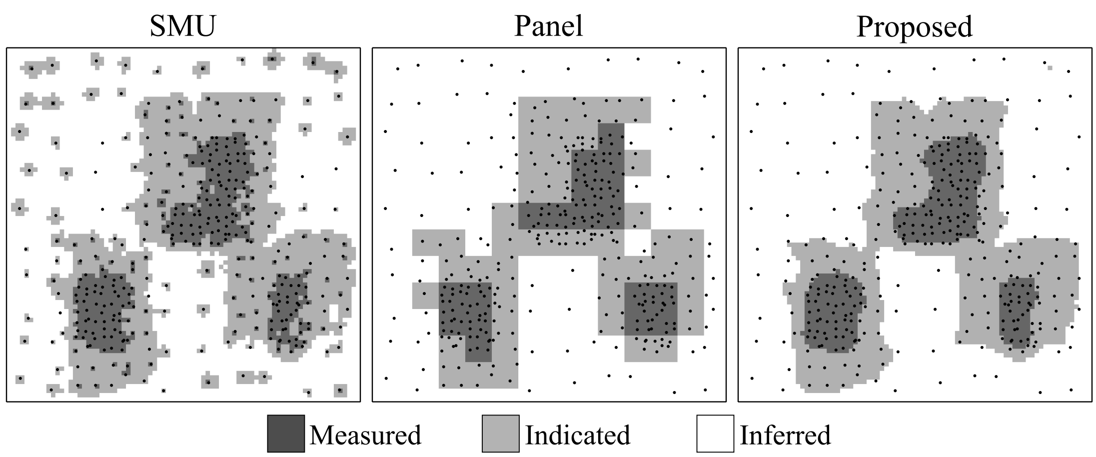


Figure 4.8: The classification based on conditional simulation for conventional SMU scale, conventional panel scale and the proposed SMU scale classification methodology. Axes dimensions: 2000m by 2000m.

Classification results for the 3D example

For the 3D example, the probabilistic criteria is a precision of ±15% with 95% confidence for measured and ±30% with same confidence interval for indicated. The classification models are shown in Figure 4.9. Again, the classification was performed at the SMU scale with quarterly production volume criteria resulting in very few artifacts.

The group of measured blocks that seem to be disconnected from the main measured mass is caused by a number of directional drill holes that cross that volume (Figure 4.9). Of course the benefit of incorporating simulation into classification is that local classification can be more accurate as data redundancy, anisotropy and proportional effect can be incorporated.

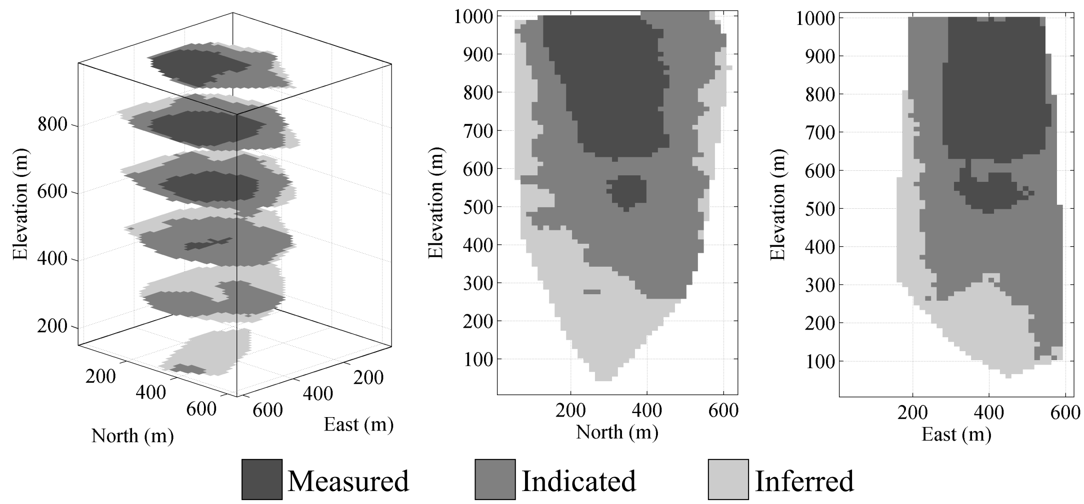


Figure 4.9: Classification with the proposed moving window applied to conditionally simulated realizations for the 3D example. Horizontal slices at elevations 150m, 318m 486m, 654m, 822m, and 990m (left). Vertical slices at 352.5m east (center) and 352.5m north (right).

4.2 Conclusion

When the proportional effect is deemed relevant and/or the consideration of other sources of error is needed, simulation based techniques are useful for resource classification. The proposed methodology is capable of performing classification at a typical block modeling scale (often SMU) but with reduced artifacts as a production volume scale is considered for the actual classification.

The proposed technique represents a viable alternative for resource classification. As with all resource classification techniques, it is the responsibility of the practitioner to assess the appropriateness of the final result based on knowledge of the deposit.

Chapter 5: A Case Study on Resource Classification

Since the creation of classification standards for resource classification, a number of different techniques for classification have been developed, however, only few of them are actually used in practice. In this chapter, the most popular techniques for resource classification (DHS, NR and KV) and the two techniques proposed in Chapters 3 and 4 (CVV and a moving window classification based on conditionally simulated realizations) are applied to the resource classification of a Cu-Mo porphyry deposit located in northern Chile. The obtained results revealed the dissimilarity among different classification techniques especially when anisotropy and the proportional effect are present.

5.1 Methodology

In practice, the parameters for classification are selected by an experienced professional based on his knowledge of the deposit. In order to perform the comparison of different classification techniques, the parameters and criteria for each technique (with the exception of KV) are selected with the intention of achieving similar classified volumes; this allows for a comparison between the techniques rather than a comparison of how to parameterize. The KV classification is an exception to the previous statement due to the low variance value close to data locations that causes the volume of measured and indicated to be naturally higher due to these artifacts. A description of the data and a brief description the parameters used by each classification method follows.

5.1.1 Data set

This case study is based on a data set from a Cu-Mo porphyry deposit in northern Chile, located in a granite-granodiorite complex of the lower Paleocene age that has been dated by the Chilean Geological Survey (SERNAGEOMIN) at 64 ±2 Ma (K-Ar isotopes). The batholith has been tentatively interpreted to be situated along a north-east trending job in a regional north-south trending reverse fault.

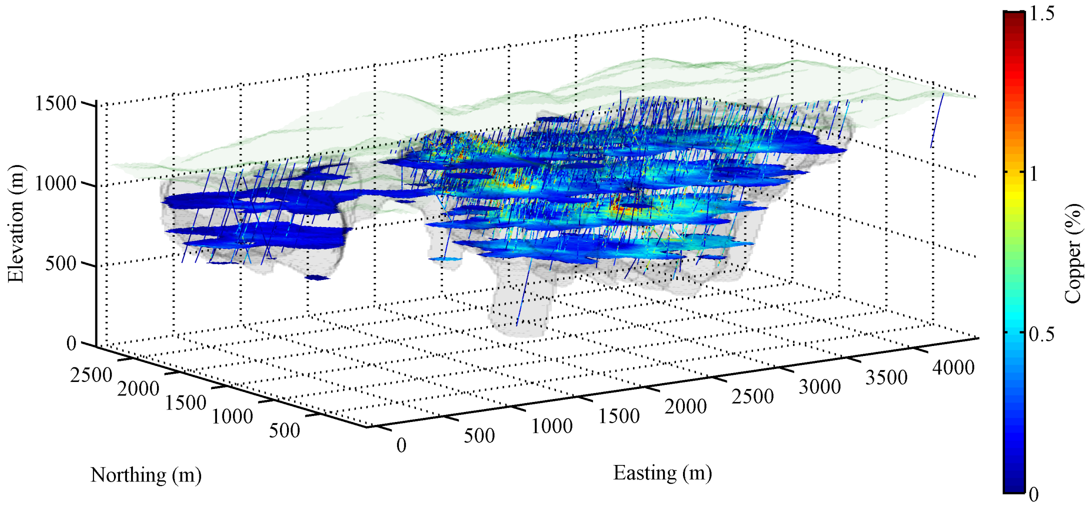
The data consists of a set of drill holes, a geologic model and a surface model (Figure 5.1). The size of the 3D model is 201x124x101 blocks of 20x20x15 meters in x, y and z.

The declustered histogram of the data is shown in Figure 5.2. The distribution of grades is positively skewed with mean of 0.34% of copper and standard deviation of 0.28% of copper leading to a coefficient of variation (CV) of 0.82. There are 36,373 informed composites with length of 5.0 meters each.The variogram model consists of a nugget effect of 0.10 and 4 nested structures (Equation 5.1 and Figure 5.3). The direction of major continuity has an azimuth of 135° from north to east and a dip of 0°.

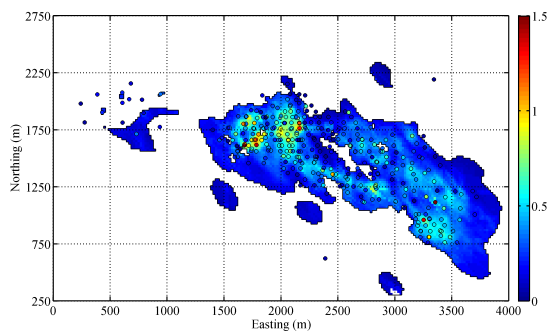
$$\begin{aligned}\gamma(\mathbf{h}) = 0.1 + 0.29 \times Exp_{\substack{maj=40\\min=40\\med=40}}(\mathbf{h}) + 0.28 \times Sph_{\substack{maj=120\\min=150\\med=160}}(\mathbf{h}) \\ +0.20 \times Sph_{\substack{maj=420\\min=240\\med=500}}(\mathbf{h}) + 0.13 \times Sph_{\substack{maj=1000\\min=240\\med=500}}(\mathbf{h})\end{aligned} \tag{5.1}$$

5.1.2 Drill hole spacing (DHS)

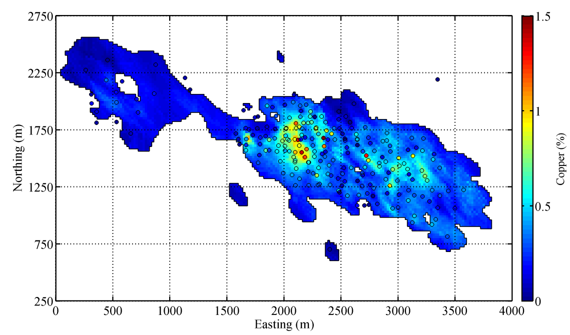
There is no unique way to calculate the DHS of irregularly spaced drill holes. In this case study, the methodology presented in Appendix A is used. The calculation is made based on a given number of data. For smother and more accurate results the average of multiple input parameters is used with the number of data searched ranging from two to eleven with a step size of one unit. The thresholds used for DHS are 27 meters for measured and 58 meters for indicated.



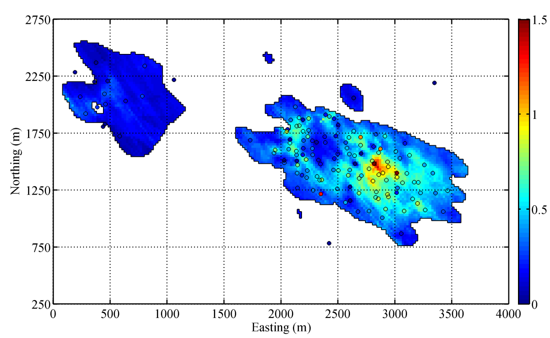
(a) 3D view



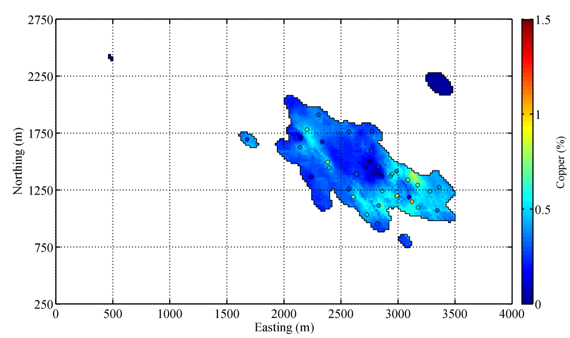
(b) Horizontal slice, elevation 1170 m



(c) Horizontal slice, elevation 960 m



(d) Horizontal slice, elevation 750 m



(e) Horizontal slice, elevation 540 m

Figure 5.1: Data set (surface model, geologic boundary and drill holes) and estimated model.

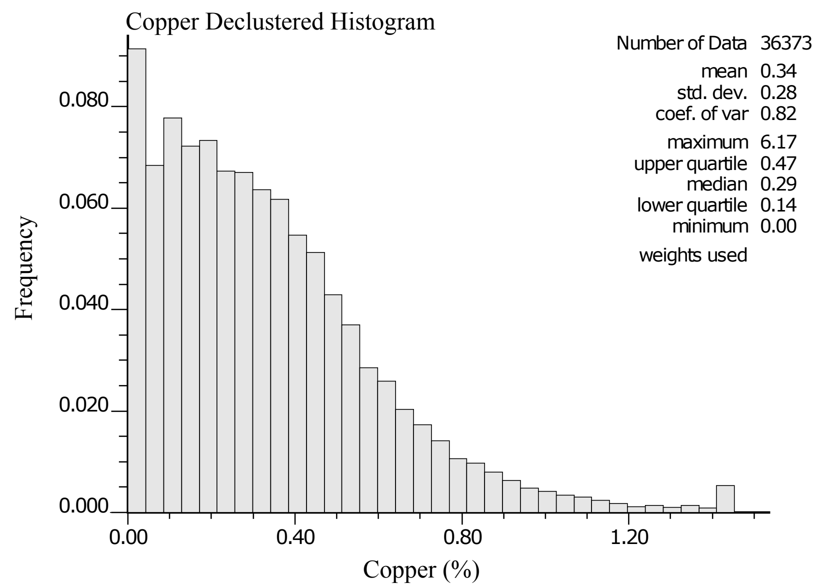


Figure 5.2: Declustered histogram.

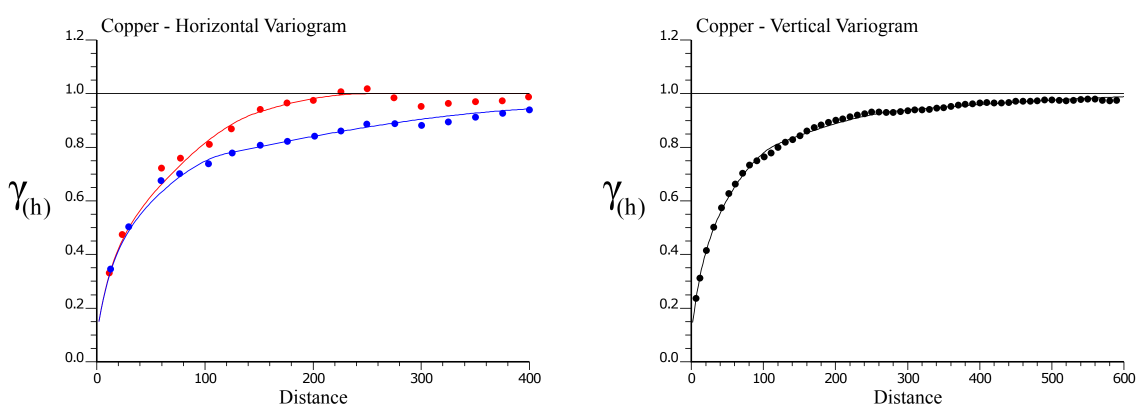


Figure 5.3: Modeled experimental variogram. Left horizontal (blue line and dots: azimuth of 135°; red line and dots: azimuth of 45°, right vertical (dip of 90°).

5.1.3 Neighborhood restrictions (NR)

The NR consists of defining a search range within which a certain number of constraints must be met in order for a block to be classified. In this work the only constraint used is a minimum number of drill holes. In order to be classified as measured a block is required to be informed by 11 different drill holes within 114 meters range, while to be defined as inferred there must be 5 different drill holes within 135 meters from the center of the block.

5.1.4 Kriging variance (KV)

The classification based on KV is performed by defining thresholds for each category. In this work the blocks with KV below 35% of the block variance are classified as measured while the blocks with KV below 65% of block variance, but higher than 35% are classified as indicated.

5.1.5 Cross validation variance (CVV)

The CVV methodology involves the calculation of the KV after the removal of one or more drill holes with the highest kriging weights in order to reduce artifacts from the KV classification. Up to seven drill holes are removed for the calculation of CVV for this case. Blocks with CVV below 66% of the block variance are classified as measured while the blocks with CVV below 83% of block variance, but higher than 66% are classified as indicated. As the CVV is calculated by removing drill holes with highest kriging weights the thresholds used for classification are higher for CVV than for KV. The difference between thresholds used for CVV and KV increases when more drill holes are removed for CVV calculation.

5.1.6 Moving window classification based on conditionally simulated realizations

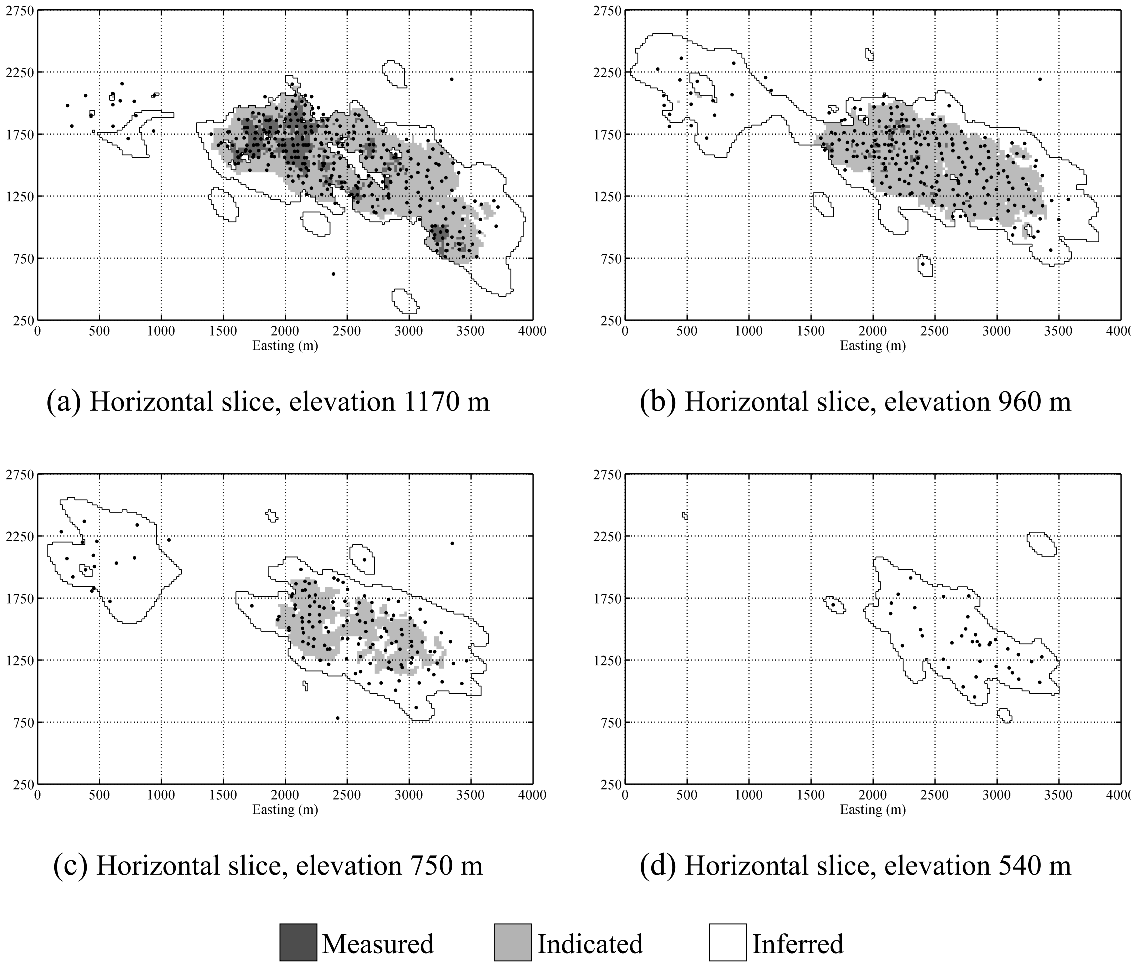
This methodology was developed in order to use meaningful classification criteria applied to large volumes (panels, quarterly or yearly production volumes) but to classify blocks at the SMU resolution. The size of the panel considered for classification is 200x200x60 meters in x, y and z directions, representing a quarterly production volume. The probabilistic criteria considered for classification are a precision of ±15% for measured and ±30% for indicated with 95% of confidence for both measured and indicated.

5.2 Results and Discussion

The classification results for the DHS technique are shown in Figure 5.4. The use of the average DHS from calculation using multiple parameters results in a smooth classification map that does not require post processing. The classification only depends on the density of data regardless of the local configuration as data redundancy is not captured by the DHS classification. Although the presented results do not consider anisotropy, the DHS can be calculated with anisotropy by performing the appropriate change of coordinates before the DHS calculation and converting the results back to the original coordinate system after the calculations.

The classification results for the NR technique are shown in Figure 5.5. The classification maps for NR are noisy and similar to the DHS with a single parameter. This technique is the most popular among the practitioners as it can be easily applied by changing the search parameters of the estimation functionality of most commercial software while calculating DHS in these software is more challenging.

The classification results for the KV technique are shown in Figure 5.6. This example illustrates the possible reason for hesitancy in using KV for classification. The artifacts close to data location causes the presence of measured and indicated blocks within



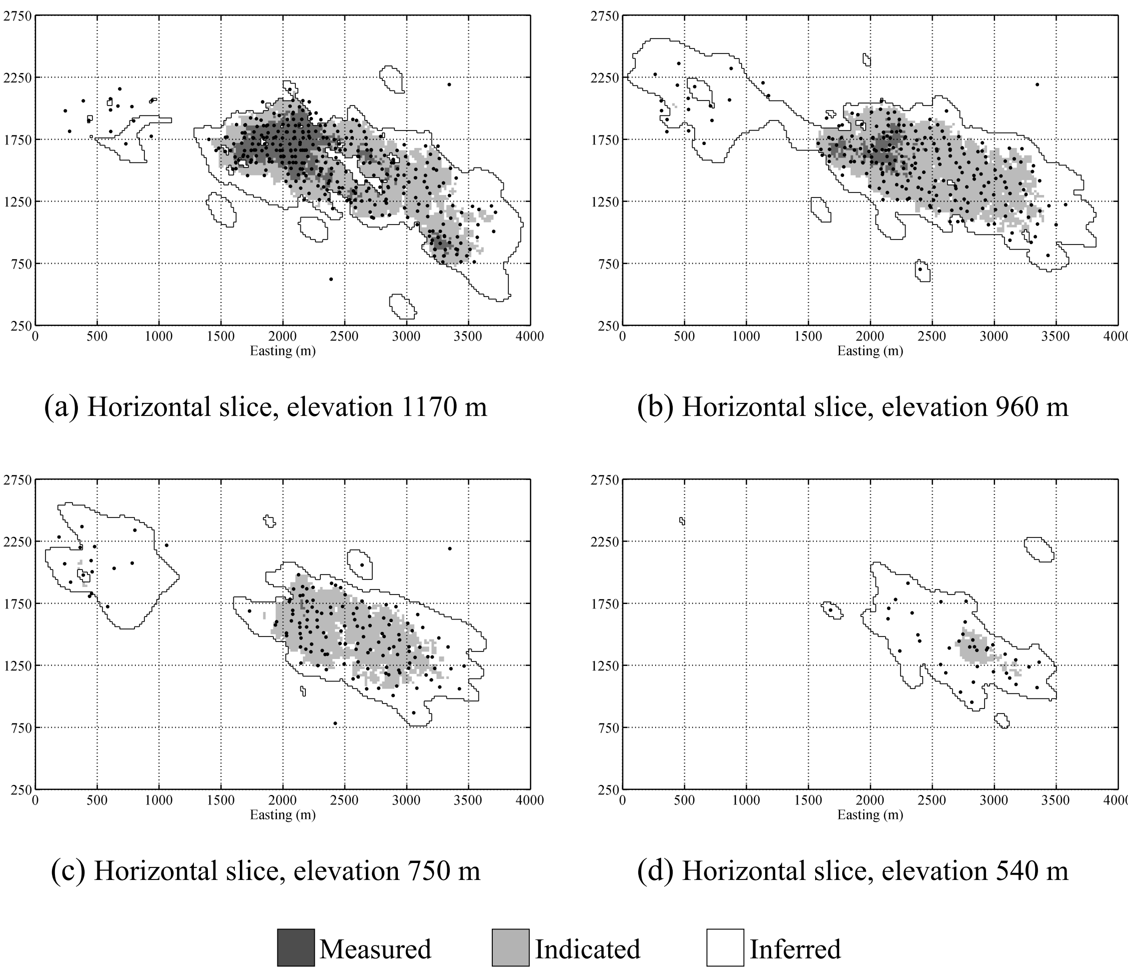
(a) Horizontal slice, elevation 1170 m

(b) Horizontal slice, elevation 960 m

(c) Horizontal slice, elevation 750 m

(d) Horizontal slice, elevation 540 m

Figure 5.4: Classification results for DHS.



(a) Horizontal slice, elevation 1170 m

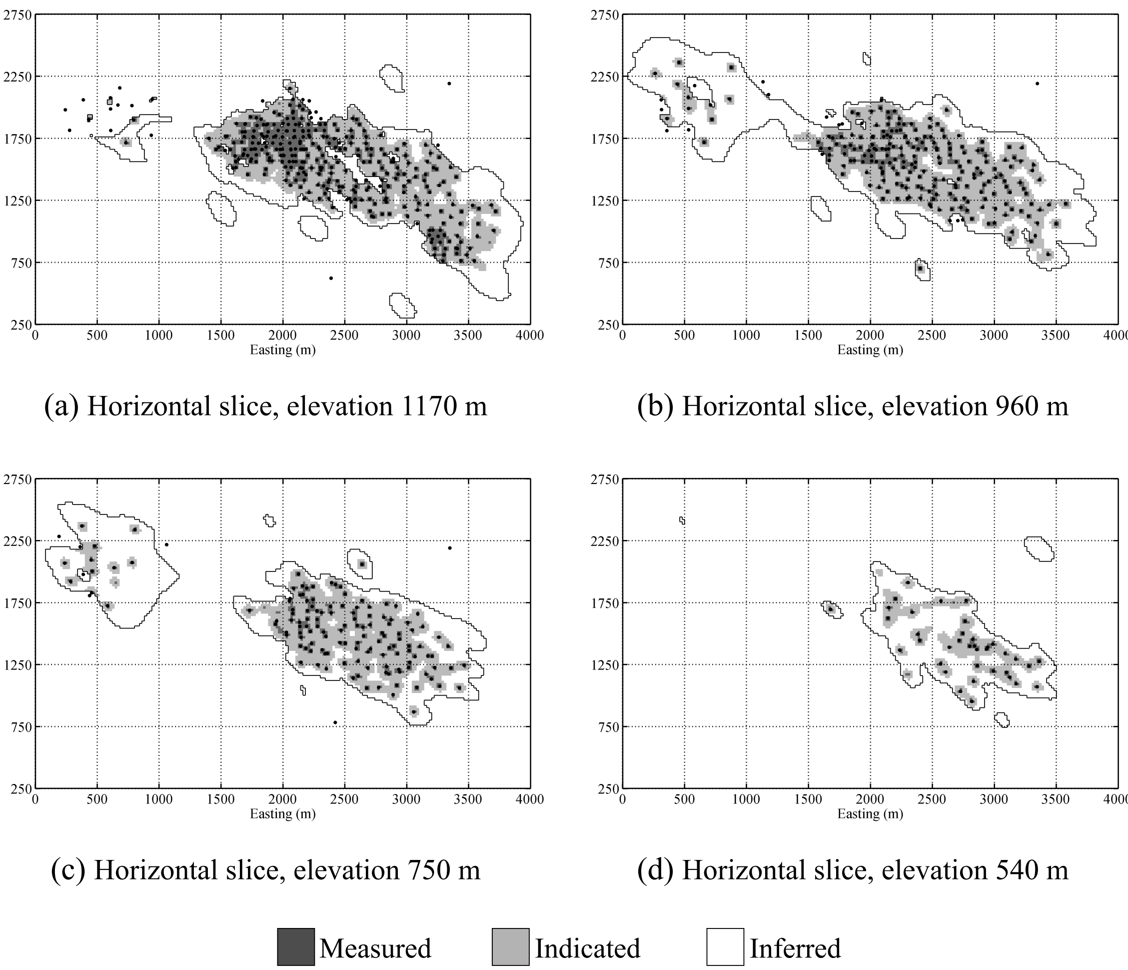
(b) Horizontal slice, elevation 960 m

(c) Horizontal slice, elevation 750 m

(d) Horizontal slice, elevation 540 m

Figure 5.5: Classification results for NR.

sparsely sampled areas that would not be classified by any other technique. Although KV accounts for important factors such as the spatial structure of the variable (variogram) and redundancy between data locations, its use is not reasonable without further processing of results for artifact removal. In fact, the review of recent technical reports presented in Chapter 2 revealed that post processing of KV classification results is always required when it is used alone as criteria for classification.



(a) Horizontal slice, elevation 1170 m

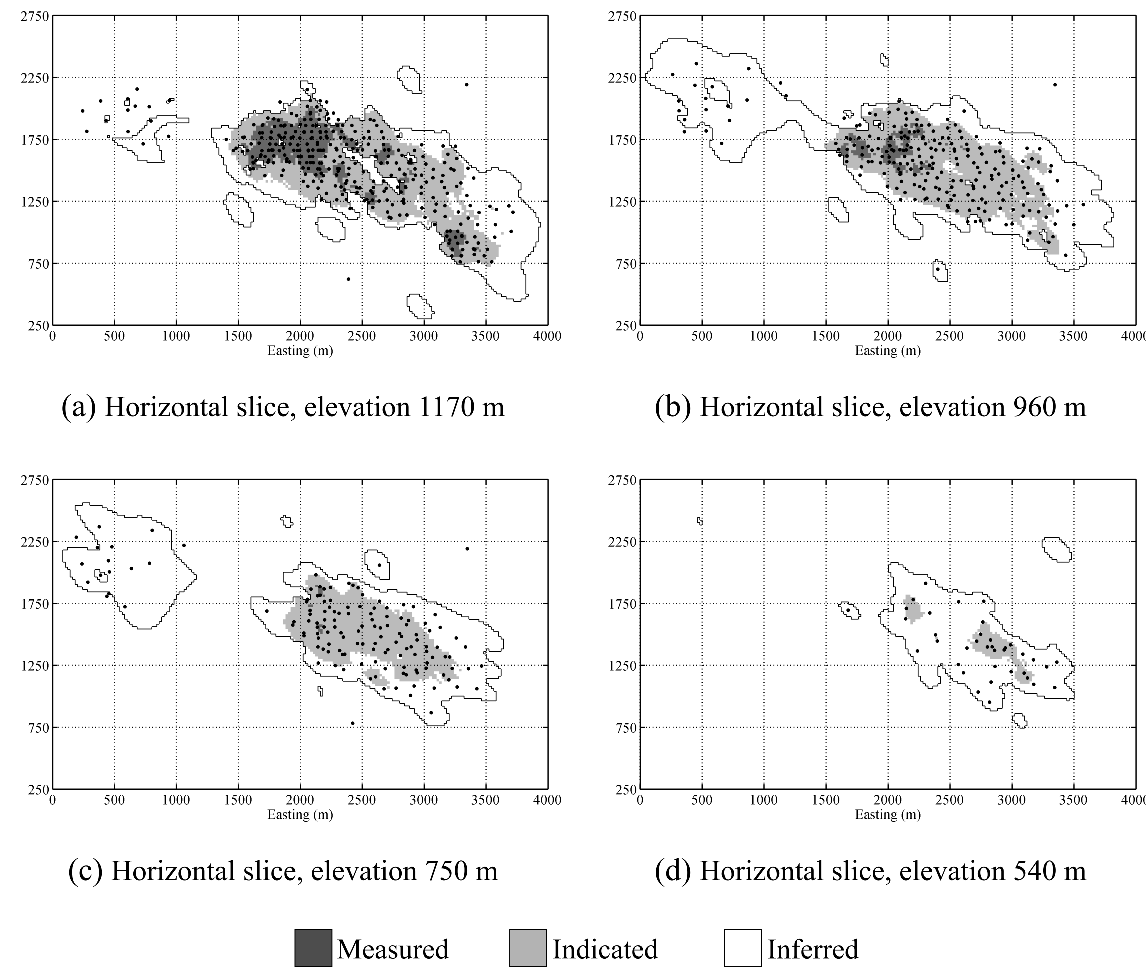
(b) Horizontal slice, elevation 960 m

(c) Horizontal slice, elevation 750 m

(d) Horizontal slice, elevation 540 m

Figure 5.6: Classification results for KV.

The CVV technique was developed in order to reduce artifacts from KV classification while keeping the desired features. The classification results for the CVV technique are shown in Figure 5.7. The technique greatly reduced the artifacts from conventional KV classification. The anisotropy from the spatial correlation structure (variogram) is observed in the classification maps. Considering that the CVV technique makes use of additional information regarding the spatial structure of the variable, the results for CVV are likely more accurate if compared to geometric techniques.



(a) Horizontal slice, elevation 1170 m

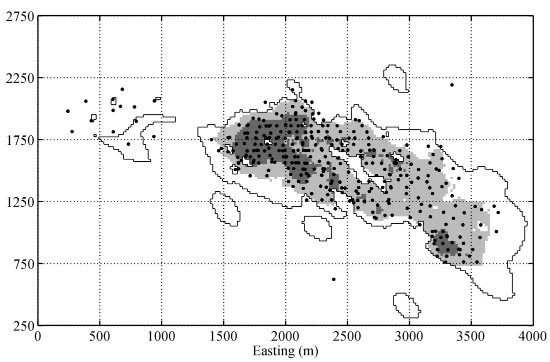
(b) Horizontal slice, elevation 960 m

(c) Horizontal slice, elevation 750 m

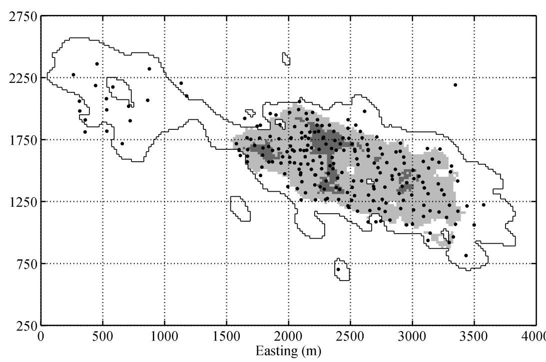
(d) Horizontal slice, elevation 540 m

Figure 5.7: Classification results for CVV.

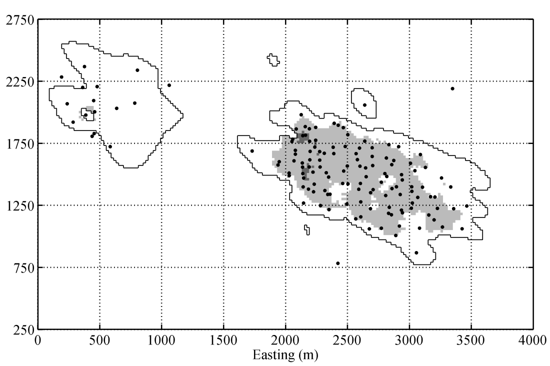
The classification results for the simulation based technique are shown in Figure 5.8. The classification map differs from the previous results from other classification methods. This is mostly due to the proportional effect. The simulation of grades conditional to the data allows for the assessment of the grade distribution at the desired scale, which enables the consideration of the proportional effect. This characteristic can be clearly observed in the horizontal section at an elevation of 750 m (Figure 5.8c) close to the coordinates 2800 m (easting) and 1400 m (northing) where there is unclassified (inferred) blocks that are considered indicated by all other techniques. These blocks that are not classified by the simulation technique are in a relatively high grade zone (Figure 5.1d) and consequently have a higher uncertainty due to the dependency of the variance on the grades (proportional effect).



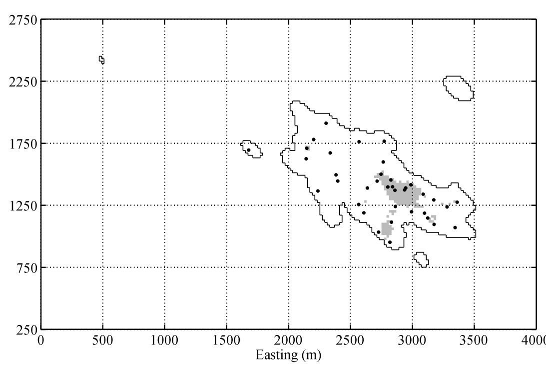
(a) Horizontal slice, elevation 1170 m



(b) Horizontal slice, elevation 960 m



(c) Horizontal slice, elevation 750 m



(d) Horizontal slice, elevation 540 m

Measured Indicated Inferred

Figure 5.8: Classification results for moving window classification based on conditionally simulated realizations.

A summary of the classification results is shown in Table 5.1. The average grade of classified blocks are consistent among the geometric techniques (DHS and NR) and CVV while for KV and the proposed simulation based technique the average grade was consistently lower for measured and indicated category. The reason for KV to result in lower grades is due to artifacts that classifies blocks at sparsely sampled locations that are usually low grade zones. For the simulation based technique the reduction in grade is related to the proportional effect that lead to more variability at high grade zones decreasing the number of blocks classified at these areas, for the same reason more blocks are classified at low grade zones.

Technique	Measured (Mt)	Measured (% of copper)	Indicated (Mt)	Indicated (% of copper)
DHS	150	0.50	1226	0.38
NR	202	0.50	1257	0.39
KV	579	0.41	1445	0.36
CVV	204	0.50	1303	0.38
Simulation	199	0.44	1311	0.37

Table 5.1: Summary of classification results. (Tonnage obtained assuming a density of 2.30 t/m^3)

5.3 Conclusion

The obtained results reveals the dissimilarity among different classification techniques especially when anisotropy and proportional effect are present. The recently proposed techniques are successfully applied and performed as expected. The CVV technique is able to remove the artifacts from the conventional KV classification maps while preserving the gains from using the available information regarding the spatial correlation structure of the variable. The simulation based technique generated an artifact free classification map using meaningful probabilistic parameters such as required precision and confidence. The proportional effect has significant impact on the final results for this case study leading to a lower average grade within the measured and indicated categories.

When the spatial correlation structure and proportional effect are deemed important the geostatistical methods can be considered for resource classification for improved accuracy of final classification models.

Chapter 6: Maximizing Resources with Optimum Infill Drilling

In the mineral industry the information available for modeling is limited and represents a very small fraction of the domain of interest. Fortunately geological data are often spatially correlated, which enables the inference of attributes at unsampled locations with a quantifiable degree of uncertainty. The uncertainty in the estimates is related to the amount of information available and will always be present as it is unpractical to sample the entire domain. Uncertainty can only be managed not eliminated. The definition of an acceptable level of uncertainty is not straightforward and varies for different commodities and mineralization types.

Regardless of the commodity or mineralization type, gathering information is necessary. In the mining industry, cores from diamond drill holes are a common source of information for modeling and are often executed in phases. The costs of acquiring data is high and for this reason all available information must be used for the planning of infill drill holes including the information regarding the spatial continuity of the attributes of interest.

The planning of infill drill holes should consider the spatial continuity of the attributes of interest and the definition of an acceptable level of overall uncertainty in order to avoid unnecessary costs. Often, a reasonable level of uncertainty is linked to the requirements for resource classification.

The objective of infill drilling may vary during the project life. In the early stages it may be important to focus on exploration and delineation of prospective areas. In more advanced stages the infill drilling might be focused on generating indicated and measured resources that can be converted into reserves and used to increase the value of the property.

In this case, the reduction in local uncertainty (KV) is not the only desirable characteristic of the objective function; the grade of each block as well as the reachability of the areas is also important. Shallow high grade zones are areas of higher economic interest and are often first to be developed; therefore, these areas are frequently the first to be converted into reserves. An objective function that considers resource classification is very relevant. The proposed objective function considers both the KV for local uncertainty reduction and the maximization of the tonnage of classified resources, also, potentially weighted by extraction order to give preference to increasing classified resources early in the project life.

6.1 Background

Random search methods are simple to apply and if applied well can lead to reasonable solutions in a reasonable period of time. Essentially, this family of algorithms explores the solution space randomly. The algorithm used in this work for optimization of the proposed objective function is stochastic in nature and based on a random search with some improvements to enhance convergence and local refinement of the final solution. A brief description of relevant random search methods is given.

6.1.1 Blind random search (RBS)

A blind random search (BRS) is the simplest implementation of a random search. It is called 'blind' because each iteration dismisses the information acquired in previous iterations. This algorithm is initialized using a randomly defined variable set or any other pre-defined feasible solution. In each iteration, the solution space is randomly sampled and the objective function is evaluated. The new set of variables is kept if it results in a better solution than the current optimum, otherwise it is discarded. This process is repeated until a maximum number of iterations is reached or a certain objective function target is met (Spall, 2005). As the dimensionality of the problem increases (i.e. by increasing the number of drill holes to be optimized or considering the strike and dip for each drill hole) this methodology quickly loses its efficiency because it cannot sufficiently explore the solution space and does not result in a reasonable optimum solution (Spall, 2005).

6.1.2 Localized random search (LRS)

Each iteration of the BRS algorithm explores the entire solution space and does not keep record of previous solutions. More sophisticated random search algorithms have been proposed, including the "localized" random search (LRS) that uses the current best solution to help propose a new, better, solution. The algorithm is similar to the BRS algorithm, but instead of generating a variable set that is independent of previous sets, in each iteration, a step size is generated and added to the current best solution before the evaluation of the objective function (Spall, 2005).

This improvement keeps the information acquired in previous iterations and permits the search size to be reduced as optimization proceeds, essentially performing a localized search for the solution and improving the ability to find a locally optimum solution. Matyas (1965) proposed a LRS algorithm and proved its convergence to a global optimum for a sufficient number of iterations, this proof was later revised by Baba et al. (1977).

6.1.3 Modified random search (MRS)

Another variation of the random search is the MRS. Rather than changing all variables in each iteration, only one variable or a subset of variables is perturbed and the objective function is reevaluated; the change is accepted if an improvement is observed in the objective value (Wilde, 2009). Wilde (2009) showed that for a small 2D example this technique outperforms other common optimization strategies including the BRS, GA, GR, Nelder-Mead simplex, and Hooke-Jeeves pattern search. MRS outperformed the BRS and yielded good results with a reasonable number of iterations when compared to the other tested techniques, suggesting its potential for the optimum placement of drill holes.

6.2 Methodology

The optimization algorithm chosen for this work is a mix between the LRS and MRS algorithms. Two major improvements are proposed to enhance efficiency. The first improvement is the use of weighted probabilities for drill hole selection and the second improvement is the use of a search restriction schedule to perform local refinement of the results. Details regarding the objective function are described first and is followed by the proposed optimization algorithm. A small synthetic example is used to illustrate and support some of the assumptions and choices made.

6.2.1 Objective function

The proposed objective function is a combination of maximizing the tonnage of metal that meets a classification criteria and minimizing the overall KV of the blocks (Equation 6.1). Each term of the objective function receives a weight to allow for tuning of the maximization of resources and minimizing of the overall variance.

$$max : f(\mathbf{x}) = \sum_{i=1}^{nb} w_i \Big\{ c_1 \left[Z_i^K \times \rho_i \times V \times I_i(\mathbf{x}) \right] - c_2 \left[\sigma_{iK}^2(\mathbf{x}) \right] \Big\} \tag{6.1}$$

where:

nb	- number of blocks in the model;
c_1	- weight given to total resource tonnage $[0, 1]$ (g^{-1});
c_2	- weight given to the KV $[0, 1]$ (t^2/g^2);
$I_i(\mathbf{x})$	- binary variable: 1 when the i$^{\text{th}}$ block meet the classification criteria and 0 otherwise;
Z_i^K	- kriging estimate of the i$^{\text{th}}$ block (g/t);
ρ_i	- density of the i$^{\text{th}}$ block (t/m^3);
V	- volume of the block (m^3);

$\sigma^2_{iK}(\mathbf{x})$ - KV of the i^th block considering the infill drilling (g^2/t^2);

w_i - weight given to the i^th block;

$\mathbf{x}$ - set of drill hole parameters;

The weight w_i is considered in order to allow for different weights for different regions of the deposit according to the mining schedule. This allows the user to give priority to areas that are likely to produce earlier or to avoid including blocks that are outside the expected final pit limits of the mine. Setting w_i to 1 for all blocks ignores this factor; however, even a simplistic set of w_i where w_i decreases linearly with depth would improve the final optimization results, resulting in optimized drill holes that prefer the reduction of uncertainty and increase of resources in areas closer to the surface.

Weighting and magnitude of the components of objective function

The proposed objective function have two components. The first is linked to the amount of classified resources while the second is related to the overall KV. The magnitude of each component depends on the nature of the deposit, number and location of available drill holes, number of infill drill holes to be optimized, among other factors. If the magnitude of each component is very different, then using $c_1 = 0.5$ and $c_2 = 0.5$ does not guarantee that equal importance is being given to each term of the objective function, making the choice of weighting very difficult.

A straightforward solution is to calculate the weights c_1 and c_2 based on the intended weights (c'_1 and c'_1) and standardization factors so that they account for the different magnitude of each component of the objective function. In this case $c'_1 = 0.5$ and $c'_2 = 0.5$ will give approximately equal importance to each factor of the objective function.

The factors can be defined by the average variation resultant from a number of independent perturbations using the same mechanism that the optimization algorithm uses (Deutsch and Cockerham, 1994). The calculation can be done prior to the start of the optimization algorithm. The objective function can be rewritten as in Equation 6.2 and the components can be simplified as O_1 and O_2 (Equation 6.3). In this case the standardized weights (c_1 and c_1) are calculated as shown in Equation 6.4, where M is the number of random perturbations, $O_j^{(m)}$ is the value of the component j after perturbation m and $O^{(0)}$ is the initial value of the objective function.

$$max : f(\mathbf{x}) = c_1 \sum_{i=1}^{nb} w_i \left[Z_i^K \times \rho_i \times V \times I_i(\mathbf{x})\right] - c_2 \sum_{i=1}^{nb} w_i \left[\sigma_{iK}^2(\mathbf{x})\right] \tag{6.2}$$

$$max : f(\mathbf{x}) = c_1 O_1 - c_2 O_2 \tag{6.3}$$

$$c_j = c_j' \frac{M}{\sum_{m=1}^{M} |O_j^{(m)} - O^{(0)}|} \ , \qquad j = (1\, and\, 2) \tag{6.4}$$

6.2.2 Drill hole parameterization

Drill holes are defined by four parameters: (1) X location of the collar, (2) Y location of the collar (3) azimuth and (4) dip. The composite size for the drill holes is an input parameter and is constant during the optimization process. The length of a hole is calculated to maximize the length of intersection with the ore body. The x and y coordinates of the collar are optimized while the z coordinate is interpolated using a topographic surface. The dip and azimuth are optimized within a user defined range, which could be the full range [0º,90º] for dip and [0º,360º] for strike, or practical constraints can be applied to limit the range.

6.2.3 Proposed algorithm

The proposed algorithm for the optimization of n infill drill holes is defined by the following steps:

1. Initialization: the algorithm starts with a random set of infill drill holes generated by randomly drawing licit values for each of the 4n parameters, where n is the number of infill drill holes to optimize.
2. Iteration: for each iteration, a drill hole is randomly chosen according to its selection probability and its parameters are changed by a step value randomly selected within a search range defined by the search restriction schedule curve. Both the search restriction curve and selection probabilities are discussed below. If the change results in a better objective function value it is kept otherwise it is rejected.
3. Stopping criteria: step 2 is repeated until the maximum number of iterations is reached.

6.2.4 Weighted probabilities

In the MRS algorithm (Wilde, 2009) one drill hole is moved at a time, but the drill hole to be moved is selected with equal probability. A drill hole that is already in a reasonable location should have a smaller probability to be moved while a drill hole that is not significantly contributing to the objective function should receive higher priority. Thus, the probability to move a drill hole is related to the cumulated improvement on the objective function and speeds up convergence.

When the objective function is reevaluated, the amount of improvement is stored, and this value is used to calculate the probability of selecting this drill hole in the next iteration (Equation 6.5).

$$p_i = \frac{max(CCOF) + min(CCOF) - CCOF_i}{\sum_{j=1}^{n} [max(CCOF) + min(CCOF) - CCOF_j]} \tag{6.5}$$

where:

n - number of drill holes being optimized;

p_i - probability of selecting the i^{th} drill hole;

$CCOF_i$ - cumulated contribution to the objective function of the i^{th} drill hole;

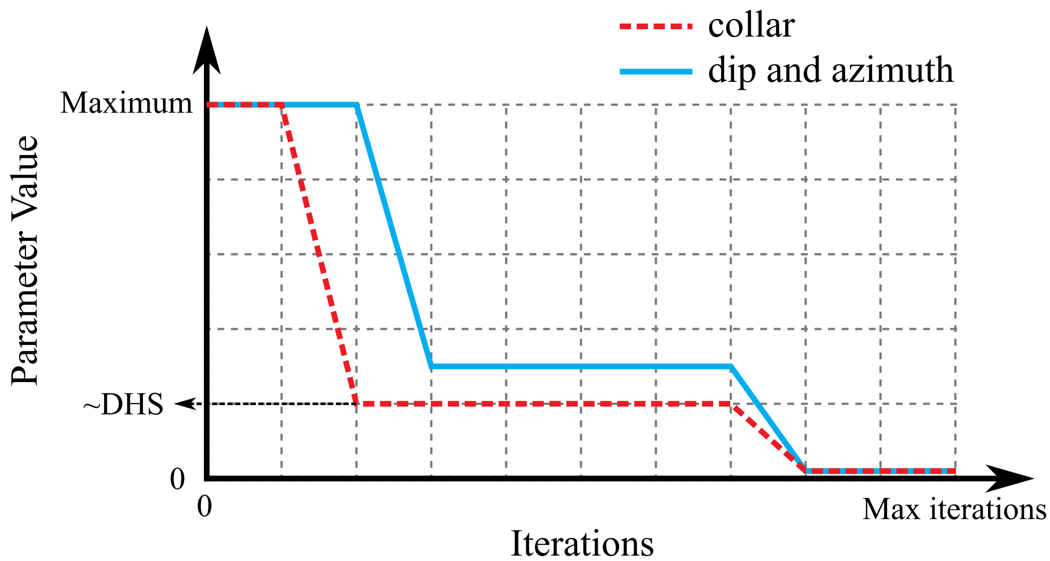


Figure 6.1: Search restriction schedule

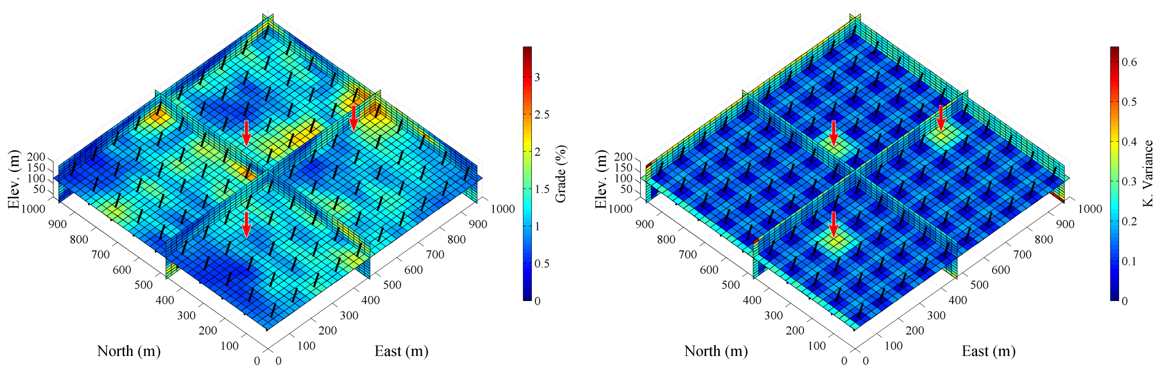
6.2.5 Search restriction schedule

When an infill drill hole is moved randomly during the search for an optimum result it can move to any position in the domain. This is required to explore the entire solution space for prospective regions, but it makes it very unlikely that a local optimum can be found because of the high dimensionality of the problem. Gradient based techniques for local refinement of the final solution of the randomized search algorithm were attempted without success due to the non-convex characteristics of the objective function, even at a local scale. A stochastic method is used for the local refinement. A search restriction schedule is used to define the parameter range to optimize and changes as iterations progress (Figure 6.1).

The search restriction schedule controls how the drill hole parameters are perturbed over the course of n iterations and is an input parameter for the algorithm. After a number of attempts with different curves (not shown here), some interesting characteristics were observed and allows for some recommendations for the selection of the schedule. It was observed that the three step curve as shown in Figure 1 performs well. The starting search size is given by the full range of values for each variable. The average spacing between existing drill holes represents a good search size for the second intermediate step of the collar parameters. The use of a delay between collar scheduling and angles scheduling (azimuth and dip) is recommended so that the angles can have better local refinement once a reasonable collar location has been selected. The best middle step for azimuth and dip is not clear and depends on the dimensions of the model and shape of the ore body. The last step is defined based on the degree of accuracy needed and should be consistent with the block model specifications.

6.2.6 Synthetic example

A small synthetic example is used to benchmark the different optimization techniques, to support the proposed improvements, and to demonstrate that the proposed algorithm can effectively find optimum locations in a case where the optimum solution is known. This example consists of a 3D model with 40x40x14 blocks of size 25x25x12.5 meters in x, y and z respectively. A regular drilling pattern (100x100 meters) with 3 missing drill holes in high and low grade zones is considered. The starting drill holes have a common azimuth of zero degrees measured from north and a dip of 75 degrees (Figure 6.2).



(a) Estimated grades (b) Initial KV

Figure 6.2: Initial conditions for the synthetic example (the red arrows are the locations of removed drill holes)

For this synthetic example the intuitive best location for the infill drill holes are the locations where there they are missing from the regular pattern, the locations with the highest grades are preferred in the case of optimizing fewer than three infill drill holes. In this work classification is based on the KV and the threshold used to classify was 19% of

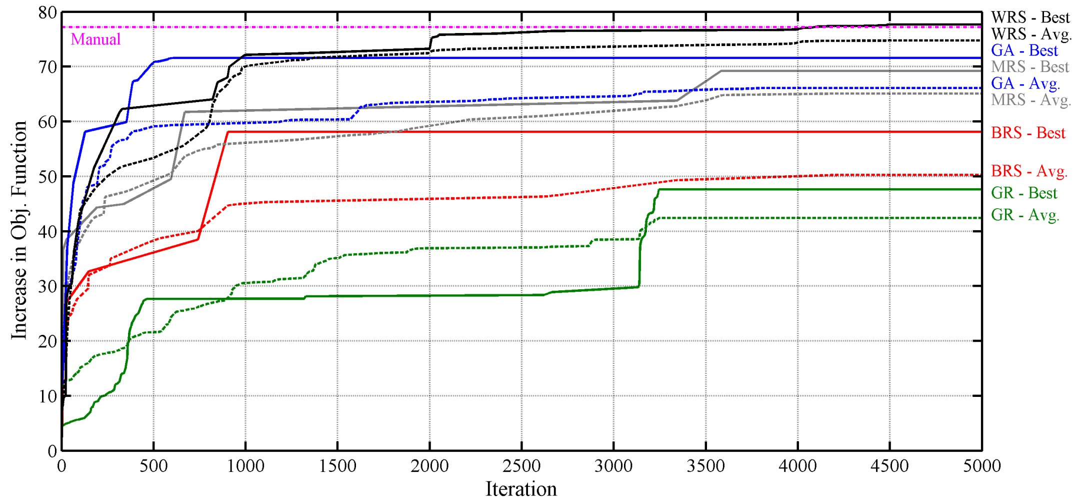


Figure 6.3: Results from different techniques for the synthetic example. The solid lines show the best of 5 different random starts and the dashed lines show the average for the technique.

the variance of the data; however, any method of classification could be used. The variogram is a spherical model with no nugget effect and a range of 270 meters in the horizontal direction and 50 meters in the vertical direction.

Among the three sites with the drill holes removed, two are in a high grade zone and one in a low grade zone (Figure 6.2a). When n is considered to be 2, the algorithm is expected to be able to find the two high grade locations; high grade locations are preferred because they contribute more to the classification component of the objective function.

The optimization techniques applied to this synthetic example are BRS, MRS, GA, GR and the proposed weighted random search with search restriction schedule (WRS). Each algorithm is run for 5,000 iterations and randomly restarted for five different runs. For GR, random restarts were allowed within the 5,000 iterations after stabilization of the objective function. Full weight was given to the classified resource ($c_1 = 1.0$ and $c_2 = 0.0$). The best run, the average of five runs and objective using the manual choice consistent with the known drill hole pattern, are shown in Figure 6.3.

The proposed algorithm not only found these two locations, but it also found a better result (objective function = 77.7) than the manual choice (objective function = 77.2). The solution with the optimization is better because the grade distribution around the drill holes is not homogeneous and the effect on classification is slightly better with the optimized drill hole rather than the regular pattern.

All tested techniques perform worse than WRS. GA does well in exploring the solution space and converges to a solution quickly but it fails to perform a local refinement of the results and is not able to better the manual choice. The MRS converges more slowly to a result, but its final results are similar, the selection of one drill hole per iteration improves in relation to the BSR, which changes all drill holes in each iteration. The GR algorithm did not perform well mostly due to the non-convex nature of the problem and the limited number of random restarts.

The search restriction schedule permits the local refinement of the solution, but it also may cause the algorithm to be trapped in a local minima. The results from the five runs for the WRS algorithm were 77.7, 77.2, 76.6, 76.2, and 66.3. Two out of five runs outperformed the manual choice, while two other runs were reasonably close and better than any other tested technique. There is still a chance that the algorithm finds a local minimum, random restarts are important.

6.2.7 Real case example

The real case example is based on the same Cu-Mo porphyry deposit presented in Chapter 5. As the original database is densely drilled, only subset of original drill holes are used here to allow for a better visualization of the proposed algorithm.

The data consists of a set of drill holes, geologic model and surface model (Figure 6.4). The size of the 3D model is 100x57x48 blocks of 40x40x30 meters in x, y and z. The variogram model consists of a nugget effect of 0.10 and 4 nested structures (Equation 6.6). The direction of major continuity has an azimuth of 135^o from north to east and a dip of 0^o.

Classification is performed by applying a threshold to the KV of 40%. A geometric method,

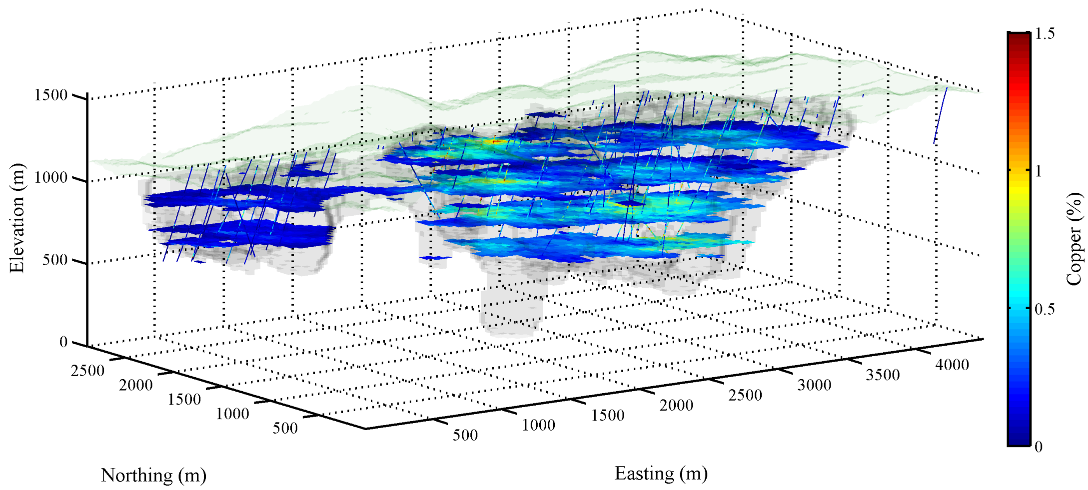


Figure 6.4: Real case study. Grade at levels 1170, 960, 750 and 540 are shown. The topography and ore body extents are shown transparent.

such as number of nearest drill holes to a given location, could also be used and would in fact improve the results as the calculation of the objective function would be quicker.

$$\begin{aligned}\gamma(\mathbf{h}) = 0.1 + 0.29 \times Exp_{\substack{maj=40\\min=40\\med=40}}(\mathbf{h}) + 0.28 \times Sph_{\substack{maj=120\\min=150\\med=160}}(\mathbf{h}) \\ +0.20 \times Sph_{\substack{maj=420\\min=240\\med=500}}(\mathbf{h}) + 0.13 \times Sph_{\substack{maj=1000\\min=240\\med=500}}(\mathbf{h})\end{aligned} \tag{6.6}$$

The mining schedule for this example is not available, but weights are assigned to the blocks in order to avoid the preferred location of infill drill holes where the deposit is thicker. Weights are assigned according to the vertical distance to the surface; blocks with easy access are likely to be reached in early stages of mining and are assigned high weights, deep blocks received lower weights. A linearly decrease in the weights is used with the weights ranging from 1.00 for blocks on the surface to 0.05 in deepest parts of the deposit.

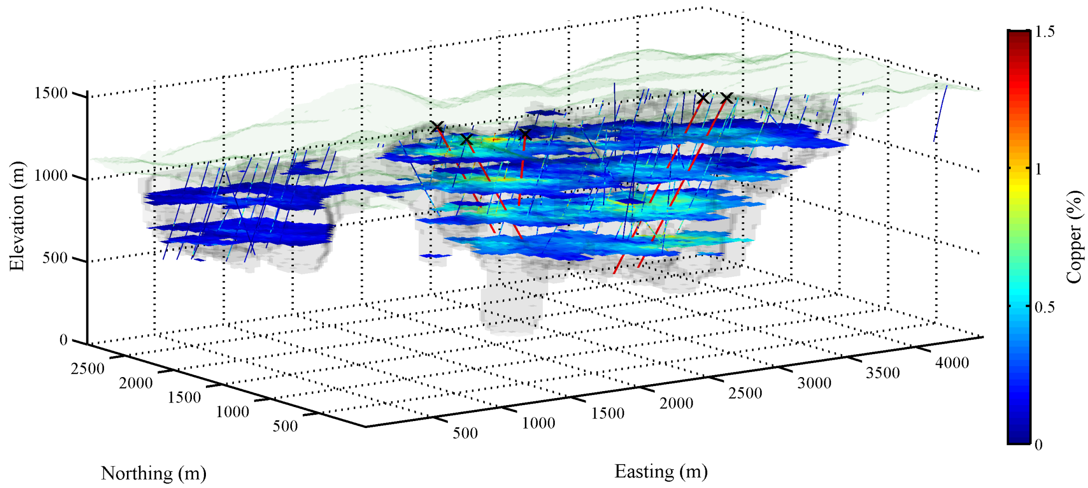
6.3 Results and discussion

There is no guarantee of a global solution due to the non-convex and high dimensional nature of drill hole optimization. Ideally, the results would be compared to an expert's interpretation of the deposit where the infill drilling locations would be selected based on a high degree of familiarity with the deposit. In this instance, such an expert interpretation is not available. Moreover, the manual choice of locations would vary among different experts taking into account different factors such as exploration potential, past knowledge with similar deposits, etc. Rather, it is suggested that the results of this optimization be used as a starting point for such an expert to manually adjust; giving them potential drill hole configurations that are preferentially located in reasonable areas.

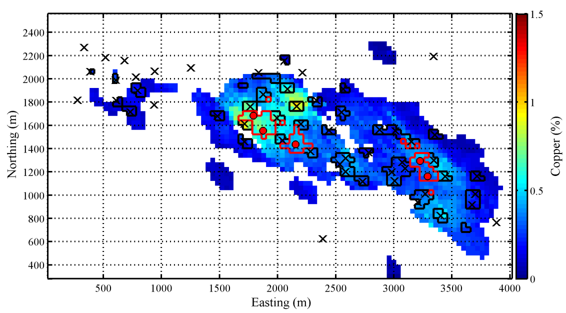
To visualize the performance of the proposed optimization algorithm on the real deposit, two cases are run for the optimization of 5 and 15 infill drill holes (Figures 6.5 and 6.6 respectively). In the 3D views the optimized drill holes are represented by a red line while their collars are represented by black markers. In plan view, the black markers are the locations where an existing drill hole crosses that section and the black lines delineate the blocks that are already classified. The red markers are the locations where the optimized drill holes cross the section and the red lines delineate the blocks added to classified resources due to the optimized drilling.

The results presented in Figure 6.5 show that the optimized infill drilling tends to follow the high grade zones, this is particularly evident for greater elevations where the blocks have received higher weights. New holes tend to be located near existing drill holes when the interaction between them improves the reduction in KV to the point where more blocks become classified above the 40% KV threshold. This behaviour is more pronounced when optimizing a larger number of drill holes (Figure 6.6).

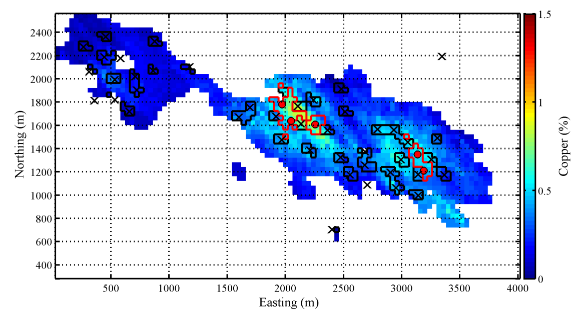
The results of the optimization of 15 infill drill holes reveals additional interesting features. Again, priority is given to the shallow high grade zones. Moreover, there is a link between the KV threshold and the DHS of the final configuration. The optimized drill holes tend to



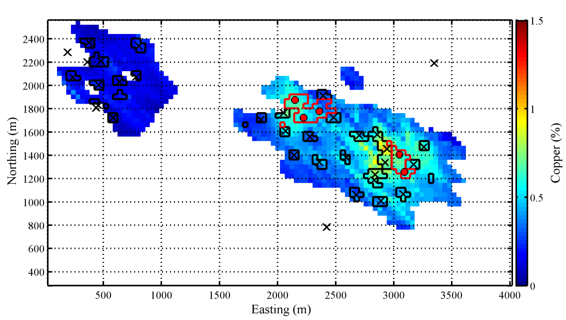
(a) 3D view



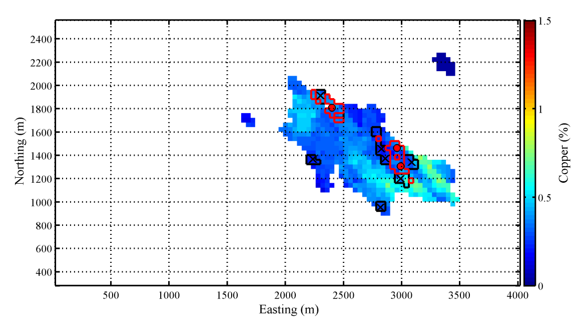
(b) Horizontal slice, elevation 1170 m



(c) Horizontal slice, elevation 960 m



(d) Horizontal slice, elevation 750 m



(e) Horizontal slice, elevation 540 m

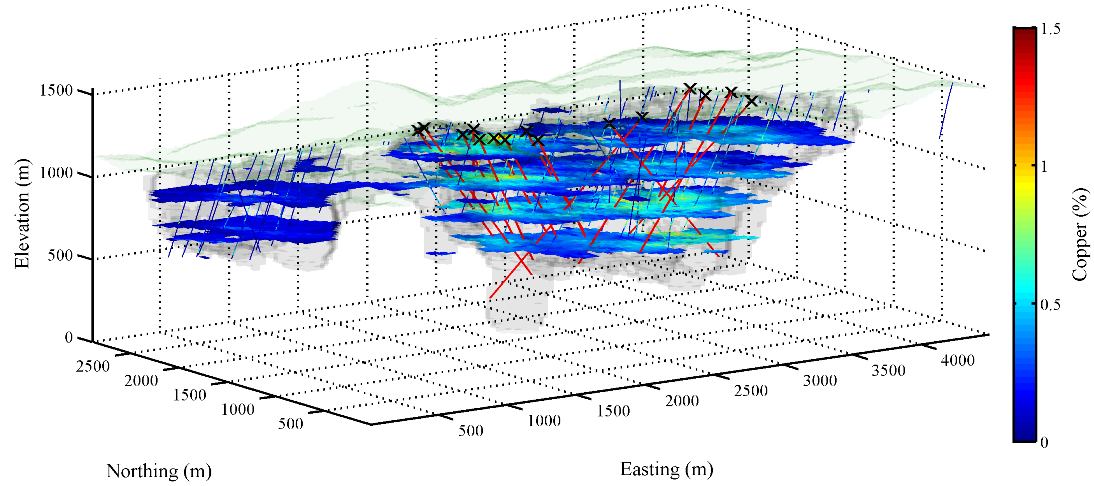
Figure 6.5: Optimization of 5 infill drill holes. (a) Existing drill holes are black and new infill drilling is red. (b) through (e) different slices showing blocks with existing drill hole data (black x) and new infill drilling (red dots) showing classified resources without new drilling (black outline) and classified resources with new infill drilling (red outline).

be located with a fairly regular spacing, which is desirable. as an expert would likely plan an infill campaign with regular spacing in the less informed areas.

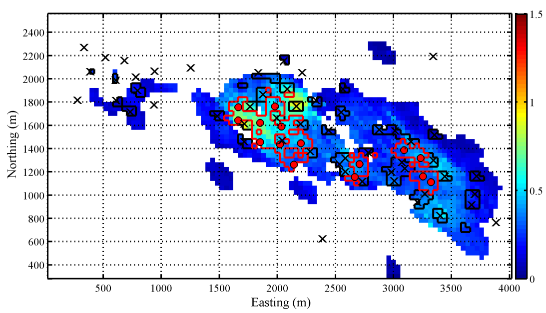
The proposed algorithm can be used to evaluate the gains in resources with an increasing number of optimized drill holes. This would assist in the planning of future drilling campaigns in a cost-benefit analysis. This possibility is investigated with multiple runs of the proposed algorithm with increasing number of optimized drill holes (Figure 6.7). The weights are also varied in order to evaluate the impact of the objective function weights on the amount of classified resources and on the reduction in KV (Figure 6.7). In this example the weights were not standardized to account for the magnitude of the components.

As expected the increase in resources per infill drill hole decreases as the number of optimized drill holes increases. The slope change is more evident for the increase in the resources than it is for the decrease in KV. This is related to the dependence of the resources on the grades as high grade zones are selected first while this has less effect on the KV that is independent of the grade. The change of slope for the reduction in KV is linked to other factors such as the thickness of geologic model and the complex interactions between drill holes. If the cost of drilling is known, this analysis could easily be adjusted to a maximization of profit by selecting the most appropriate number of holes for the infill campaign.

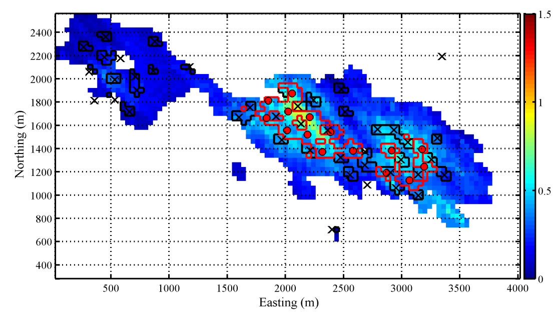
As expected, the amount of classified resources reduces with reducing the weight (c_1). The impact of the weights decreases as the number of infill drill holes increase, which is, again, a result of the priority given to the high grade zones. The use of equal weights did not decrease considerably the amount of classified resources especially for more than 100 infill drill holes. The result of the optimization of 150 infill drill holes with full weight given to resources was slightly worse than for equal weights indicating a suboptimal solution and again stressing the importance of multiple runs (random restart).



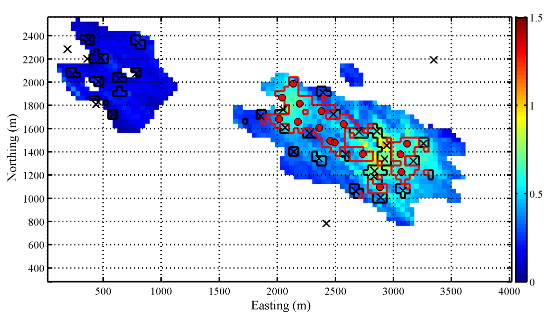
(a) 3D view



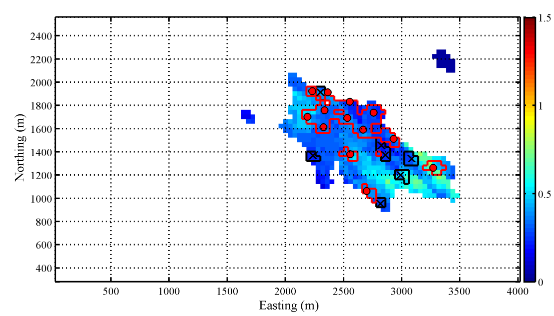
(b) Horizontal slice, elevation 1170 m



(c) Horizontal slice, elevation 960 m



(d) Horizontal slice, elevation 750 m



(e) Horizontal slice, elevation 540 m

Figure 6.6: Optimization of 15 infill drill holes. (a) Existing drill holes are black and new infill drilling is red. (b) through (e) different slices showing blocks with existing drill hole data (black x) and new infill drilling (red dots) showing classified resources without new drilling (black outline) and classified resources with new infill drilling (red outline).

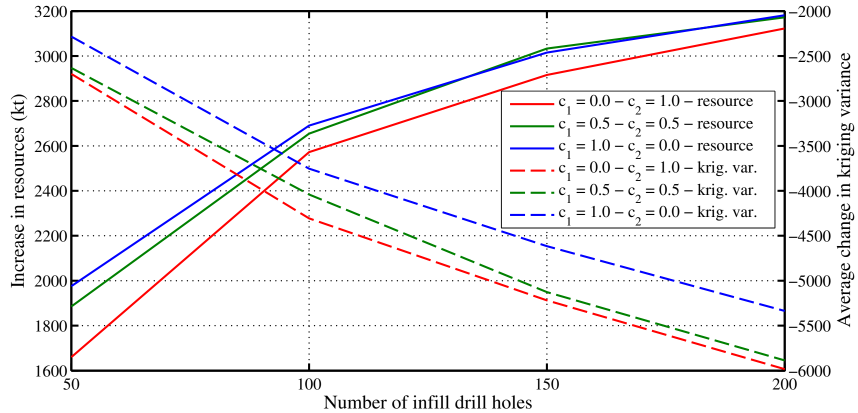


Figure 6.7: Sensitivity of classified resources and KV varying the weights.

6.4 Conclusion

There have been many algorithms proposed for infill drill hole optimization. Despite the importance of this problem, the third dimension is often ignored and very few techniques exist to simultaneously optimize n holes with arbitrary strike and dip. The proposed drill hole parameterization gives more flexibility in the optimization algorithm and the proposed improvements to the existing techniques resulted in enhanced efficiency and better objective function results as demonstrated on the synthetic example. Improvements were even seen when compared to the manual choice where regular drilling is usually expected to be optimum.

Chapter 7: Conclusion and Future Work

7.1 Conclusion

Mineral resource classification standards were recently developed to define rules for the public disclosure of mineral projects and prevent the disclosure of erroneous, misleading or fraudulent information. The classification is performed accordingly to the degree of confidence in the geologic continuity. There are a number of factors that influences the confidence in the geologic models, which includes quality, quantity and distribution of data, among others. Classification standards do not define the appropriate techniques to be used for classification leaving the decision for an experienced qualified/competent person. For this reason, since the creation of the standards, a number of different techniques for classification were developed. The main contributions of this thesis are:

1. A review of current state of practice regarding resource classification based on a survey on Canadian NI 43-101 reports;
2. A novel technique for classification based on cross-validation and KV that was developed to address some of the limitations of existing techniques variance based techniques while keeping its desirable features;
3. A novel technique based on conditionally simulated realizations that uses a moving window to classify SMU blocks accordingly to probabilistic criteria applied to a larger volume, which allows for reduced artifacts and meaningful classification parameters;
4. A methodology for the optimization of infill drill holes location that uses an intelligent random search algorithm with local refinement to minimize the variance while maximizing resources.

The survey on Canadian NI 43-101 reports revealed that although there are many different classification techniques, only few are actually used in practice. The geometric techniques are preferred among practitioners, mostly because of the ease of their application. NR is the most common in practice and this may be attributed to the possibility of using the tools available in commercial software for its calculation while the calculation of DHS, the second most used technique, is not commonly available in commercial software and its application mostly consists of drawing polygons by hand, bench by bench. As result, DHS is mostly applied to deposits with regular drilling patterns. Methods involving KV are not commonly used in practice, however, the results from the survey revealed a higher usage than expected. There are a number of factors that may lead to the lack of use of more advanced methods such as KV or simulation. The increase in complexity (number of parameters) and the sensitivity to the modeler's choice makes these methods less transparent to the parts involved. Other characteristics that lead to the generation of artifacts also discourages the use of geostatistical techniques. In this thesis, two techniques were proposed in order to address the weaknesses of current techniques in order to produce more accurate classification maps.

The increase in the amount of data often leads to a decrease in the uncertainty and improves the confidence on the estimates, however, geologic data are often spatially correlated and the reduction in uncertainty caused by the increased amount of data is not a simple function of the number of data, but it is also function of the spatial distribution of this data and the spatial continuity of the attribute under study. KV itself may not be a good measure of uncertainty because it does not capture important characteristics of the data, such as the shape of the distribution and heteroscedasticity; KV accounts for data redundancy, uses information regarding the spatial correlation of the data and better captures the complex relationships between data availability and confidence on estimates.

A number of methods based on KV have been developed, but few are actually used in practice and when they are used, post processing is often required due to artifact generation. Dilatation and erosion techniques, manual post processing or combination with other techniques are commonly used for removing the undesirable artifacts. The CVV technique proposed in this thesis is able to generate classification maps considerably reducing the artifacts while maintaining the advantages of the variance based methods;

however, CVV is still limited in terms of being homoscedastic.

Conditional simulation is a powerful tool for the assessment of uncertainty. It accounts data quantity, spatial correlation and the distribution of the attributes of interest. By generating multiple equally probably realizations of the truth, it allows for the assessment of the local distribution of the attributes at any scale. Meaningful probabilistic criteria that satisfies the needed degree of confidence can be applied to the multiple realizations in order to assign the blocks with an appropriate category (measured, indicated or inferred). The main concerns with using geostatistical simulation in classification mode is the increased complexity and dependency on modeler's parameters. Little can be done to address these concerns as simulation will always be more complex than geometric techniques and will require more expertise from the modeler. When deemed appropriate, conditional simulation can contribute greatly to the understanding and management of uncertainty, thus providing valuable information that can be used for resource classification.

Other more practical concerns regarding the application of conditional simulation, can be addressed. The support or volume in which the classification is performed is of great importance to final results. If simulation is used for the classification of small SMU blocks, the probabilistic criteria must be relaxed and it loses its meaning. For this reason, the consideration of larger volumes (monthly, quarterly or annual production volumes) is often recommended; however, these larger production volumes are not always well defined at early stages of a mining project leading to the uncertainty in the grid definition that can impact the results for such coarse grid. In addition, classification at a small scale often generates artifacts similar to those generated with the variance based methods near data locations. The proposed moving window classification applied to conditionally simulated realizations addresses these scale issues, as wells as, artifact generation by using a moving window that represents the larger production volume (panel) centered at SMU block and performing the classification at the desired resolution (SMU) based on a meaningful probabilistic criteria applied at the larger scales.

The application of the proposed techniques to synthetic examples and a case study showed that these techniques are able to perform as intended, generating more accurate classification models that have fewer artifacts.

Gathering information is expensive and, as mentioned before, uncertainty is not only a function of the number of data, but also of its spatial distribution and correlation. It is important to consider all available information while planning a new drilling campaign in order to avoid wasting resources. In early stages of projects, infill drilling campaigns are focused on exploration while in later stages it is focused on local uncertainty reduction aimed at the generation of classified resources (measured and indicated) that are eligible to be upgraded to proven and probable reserves. The incorporation of resource classification into the optimization of infill drill holes is then very relevant in mining.

In this thesis an optimization methodology that accounts for the different goals of infill drilling is proposed and successfully applied to a synthetic example and case study. The results for the synthetic example outperformed the optimum 'by hand' demonstrating its capacity for improving resources. The results of the application to the case study cannot be quantitatively evaluated as the comparison would require an expert's choice, which is not available, but the results were consistent with the proposed objective function and demonstrated desirable characteristics such as fairly regular spacing along horizontal sections and optimum interaction with existing drill holes and among new ones for optimum reduction of variance and maximization of resources. In addition, the proposed methodology is useful for analyzing the relationship between the gain in resources and reduction in overall variance with the increasing number of drill holes. At times, expert inputs that are deemed important are not captured by the algorithm, in these cases, the results of this methodology still have a great potential to serve as starting point for the planning of infill drilling campaigns.

Two new methodologies for resource classification are proposed throughout this thesis. These techniques address some of the limitations of existing techniques and improve the quality of classification models by reducing artifacts and by introducing relevant factors to the classification paradigm such as spatial correlation of variables, heteroscedasticity and shape of distributions. The consideration of these factors leads to a better accounting of data redundancy and proportional effect. In the case of simulation the use of probabilistic criteria allows for a better standardization of classification criteria as probabilistic criteria have clear meaning and can be similarly applied to different deposits. Also, a methodology for optimizing the location of infill drill holes aiming the maximization of resources is proposed, which allows for improved resources.

7.2 Future Work

Two new techniques were developed for the classification of mineral resources, which present qualitative improvements to the existing techniques. A quantitative evaluation of the goodness of classification results is a difficult task that should be addressed. The development of a methodology that can quantify and demonstrate the appropriateness of each methodology to a specific site would be very valuable for this subject.

The proposed methodology for the optimization of infill drill holes is suitable for drilling performed from surface, which makes the current implementation unsuitable for underground drilling. In order to allow underground drilling, a different parameterization of drill holes would be required. As underground drilling is often executed in fans, the paramterization of these fans could be added to the current methodology in order to consider underground drilling enhancing the flexibility to the algorithm.

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Appendix A: DHS Calculation

The DHS calculation in this thesis was performed using the software named `dhs3d` that was developed for smooth DHS calculation and improved accuracy, which are often required for applications such as mineral resources classification, uncertainty management and data spacing studies. The software is suitable for the calculation of DHS in 3D. A small example is used to demonstrate its features and improvements.

A.1 Methodology

The methodology implemented in the software is similar to the one proposed by Wilde (2010) for calculation of data spacing/density, but several changes were made as the objective here is the calculation of DHS. For the calculation of DHS the problem can be reduced to a 2D calculation that uses a single datum from each drill hole. The selected location is the closest to the block under consideration within a tolerance in the vertical direction equal to the size of the block (Figure A.1).

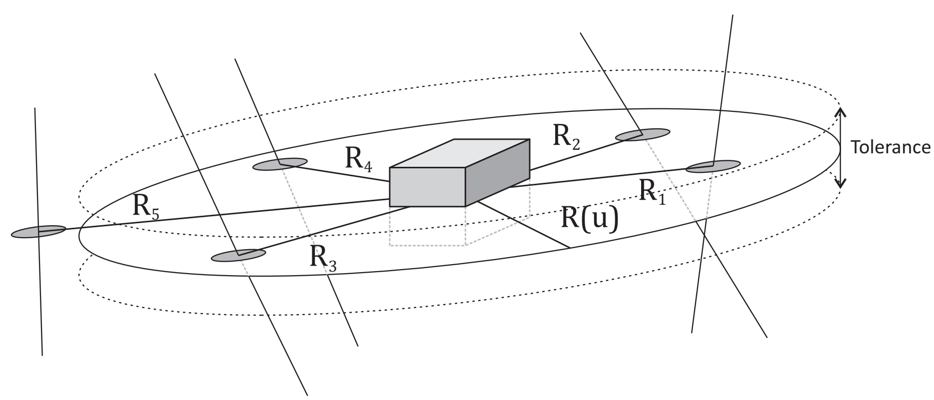
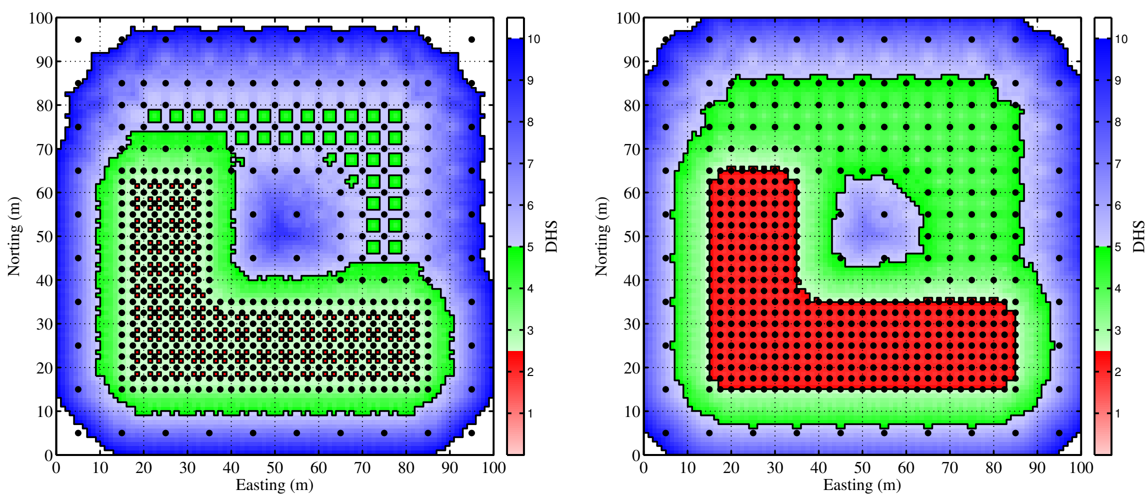


Figure A.1: Search scheme at location u when number of drill holes searched is equal to 4. The tolerance in vertical direction is fixed and equals to the block vertical dimension.

There are three search options for the DHS calculation. The DHS can be calculated based on a fixed number of data searched (n) or by a given search geometry (circle or square). When a squared search is used the DHS is calculated using Equation A.1, which is Equation 1.1 as proposed by Wilde (2010), reduced to two dimensions. The concept of DHS is often linked to equally spaced cases (i.e. 50x50m or 100x100m) and, for this reason, when a circular search is used or when the number of data is specified by the user, Equation A.2 is used to calculate the DHS for improved accuracy (Figure A.2).

$$DHS(\mathbf{u}) = \left(\frac{R^2(\mathbf{u})}{n(\mathbf{u})} \right)^{\frac{1}{2}} \tag{A.1}$$

$$DHS(\mathbf{u}) = R(\mathbf{u}) \left(\frac{2}{n(\mathbf{u})} \right)^{\frac{1}{2}} \tag{A.2}$$



(a) DHS calculation with Equation A.1

(b) DHS calculation with Equation A.2

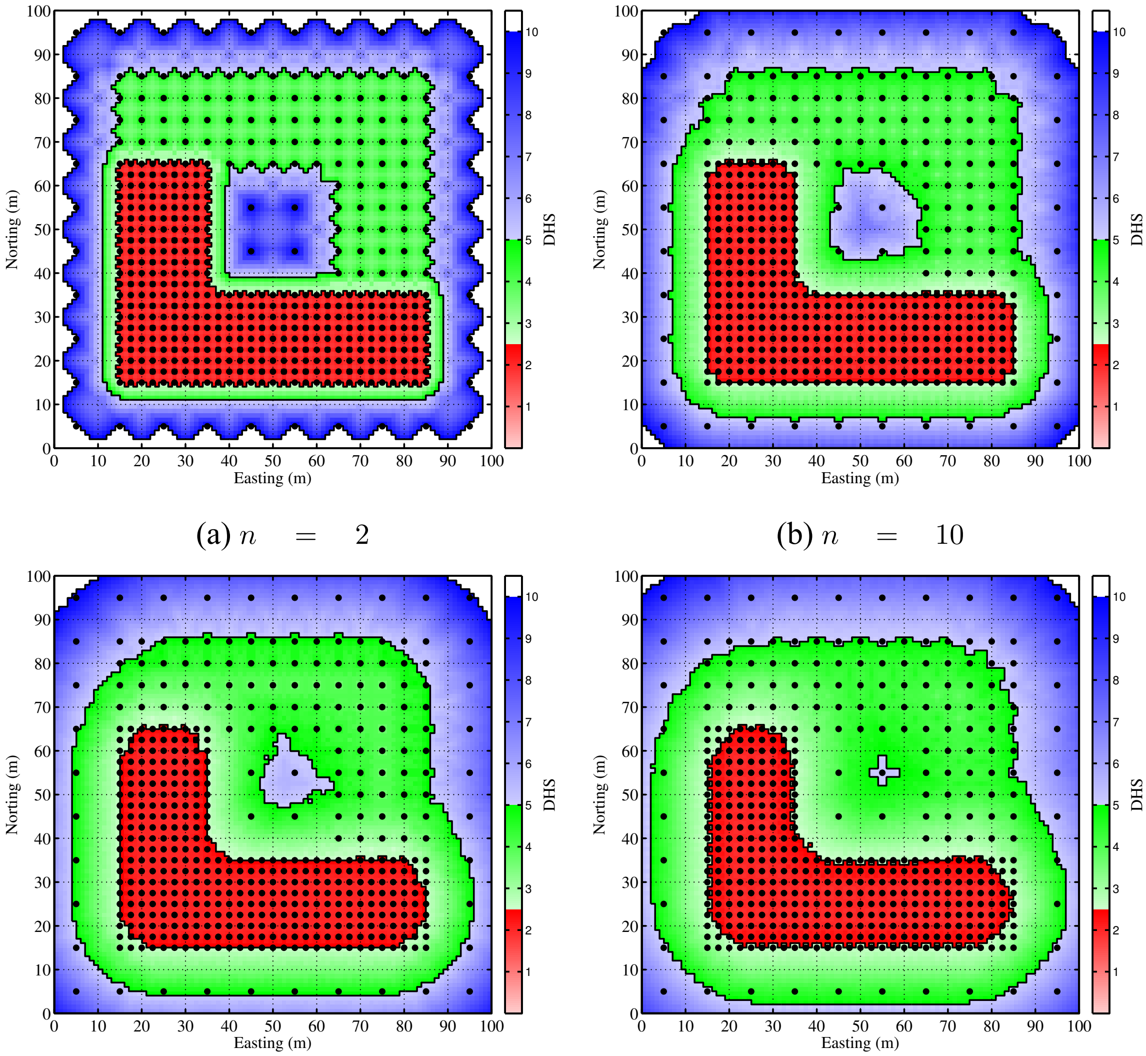
Figure A.2: DHS calculation with different equations for $n = 10$. Three different regular spacing are present: 10x10m, 5x5m and 2.5x2.5m.

$R(\mathbf{u})$ is calculated using Equation A.3. When the user selects a search geometry, the input parameter is used to find $n(\mathbf{u})$ and the data found is used to calculate the final $R(\mathbf{u})$ that is used in the calculation. For the squared search, $R_n(\mathbf{u})$ is not the Euclidian distance from the block to the sample, rather Equation A.4 is used.

$$R(\mathbf{u}) = \frac{R_n(\mathbf{u}) + R_{n+1}(\mathbf{u})}{2} \tag{A.3}$$

$$R_n(\mathbf{u}) = \mid X_n - X(\mathbf{u}) \mid + \mid Y_n - Y(\mathbf{u}) \mid \tag{A.4}$$

The calculation of DHS with a single parameter tends to be noisy if the number of data searched is too low or the search size is too small. In order to get smoother models with a single parameter a larger search is needed, but the increase in the search leads to a decrease in accuracy (Figure A.3). In order to obtain smoother and more accurate DHS models the implemented software allows the definition of multiple parameters and the resultant DHS is the average of all calculations. Figure A.4 shows an example of calculation of DHS using n equals two to twelve with a step size of one unit.



(a) $n = 2$

(b) $n = 10$

(c) $n = 20$

(d) $n = 30$

Figure A.3: DHS calculation with single parameter.

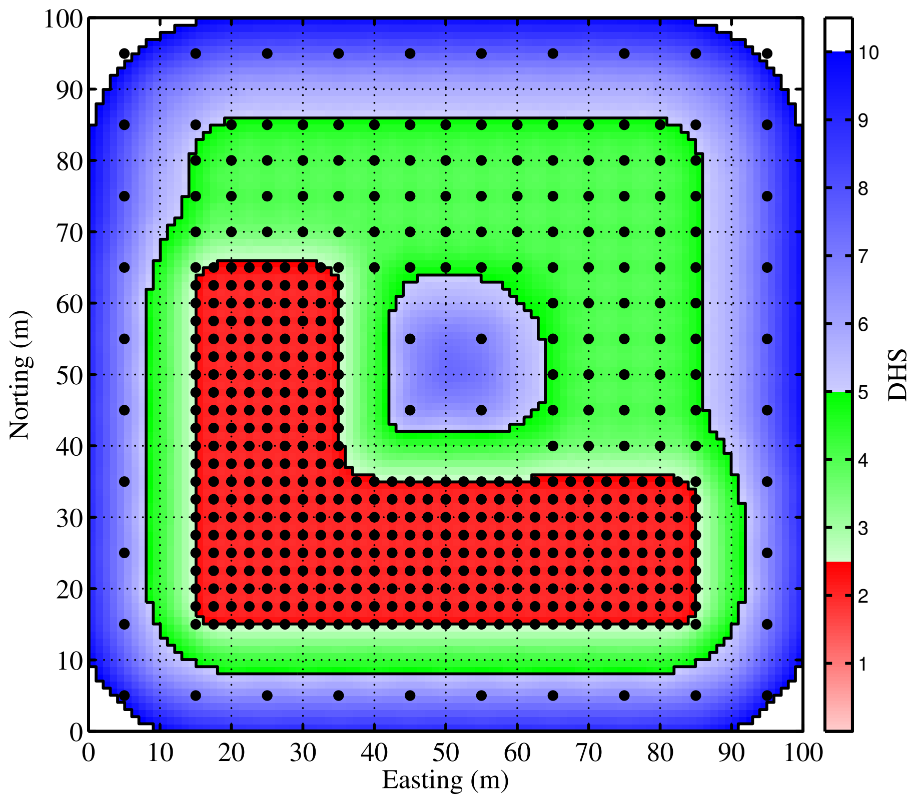


Figure A.4: Average DSH calculated n equals two to twelve with step size of one unit.