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ON THE ELEMENT-SOLUTION OF ELECTRICAL NETWORKS

by

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ABSTRACT

In this thesis, three measurement methods for the explicit solution of the constituent elements of an electrical network, from a set of measurements made at certain points of the network without dismantling it, are presented. The methods are based on the frequency-response testing of the network at certain nodes and require that at least one of the network elements be known or at least one known element be added to the network.

The first method computes all the elements of the network in one computation. The second method solves the network elements one node-set a time and is especially suitable for the solution of large networks. The third method computes the element variations, from a set of transfer function measurements, by an iterative Jacobian method. For each method, a suitable computational scheme is presented. Various sensible conjectures associated with the measurement methods are also included.

In all the methods, the computational experiments with a number of networks showed excellent agreement with the actual element values. The methods may be used in the testing of a working electro-mechanical system and in automatic test applications.

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LIST OF PRINCIPAL SYMBOLS

β	The convergence constant
δb	The column vector representing the variations in the element values
δT	The column vector representing the variations in the inverse transfer function
A	Node-branch incidence submatrix of a network without any added element
A_1	Node-branch incidence submatrix of a network with the added elements
b_k	Value of the element in the branch k
$b]_i$	Vector representing the element values obtained in the i-th iteration
C	The component vector
e	Total number of elements of a network
E_k	Voltage source in the branch k
F	The forcing function vector
G	Node admittance matrix of a network
J	The Jacobian matrix
q	Number of external elements added to a network
T_i	Inverse transfer function of a network at a frequency s_i
V	The node voltage matrix

v'	Voltage at the node i with respect to reference node
v_{ij}	Voltage at the node i at a frequency j
v_{b_i}	Voltage across the element b_i
x_i	Weighting factor of the element b_i
x_{ij}	Weighting factor of the element b_i at a frequency j

CHAPTER 1

INTRODUCTION

The measurement of the constituent elements of an electrical network of known configuration from the data measured at certain points of the network where connections may be made without dismantling the network is of practical importance for the maintenance of the network concerned. As an example, in the servicing of electronic equipment of known circuit configuration, the measurement of the network elements of the equipment detects any fault associated with it. In such measurements, however, the number of unknown elements, in general, exceeds that of the measuring points and as a consequence, for example, the frequency response measurements at some measuring points have to be carried out at more than one frequency. The problem of diagnosis of the network thus resolves into finding suitable measurement criteria for the unique solution of the unknown elements. It is also the problem of ascertaining the minimum number of measurements that is necessary and sufficient for the solvability of the network under consideration. The attractive feature of such method of measurement is that it would facilitate the evaluation of the network elements without the necessity of dismantling the network and as a result, it would provide an effective way of network element identification of a working system.

Though such a method of network identification appears to be quite attractive from the practical point of view, no considerable attention has so far been paid to the problem. Until now, only a few studies have appeared in literature. These studies are, however, limited to the solvability of the single-element-kind networks and have been triggered by Berkowitz [1] in 1962. In his work, Berkowitz derived necessary and sufficient conditions for the solvability of linear, passive, lumped-parameter networks relating the number of elements capable of being evaluated to the number of available and partly-available terminals [1]. The problem of solvability of single-element-kind networks has later been studied by Bedrosian and Berkowitz [2] and Bedrosian [3]. In their work, Bedrosian and Berkowitz [2] presented further conditions on the solvability of and a general solution procedure for single-element-kind networks based on a specially partitioned node-node admittance matrix. The statistical considerations in the solvability of such networks with some partly-available and some non-available nodes based on the maximum-likelihood estimation procedure has later been reported by Berkowitz and Wexelblat [4]. Recently, Even and Lempel [5] have attempted to find the minimum subset of network nodes with respect to which the network elements of a single-element-kind network may be uniquely determined by indicating the necessary and sufficient conditions for a node to be essential as a measuring terminal. Though their work is revealing, it furnishes only a partial answer to the necessary and sufficient conditions for the unique solution of the elements of a passive single-element-kind network.

The purpose of this thesis is to formulate measurement methods for the explicit solution of the unknown elements of an LCRM network from a set of measurements made at certain points of the network, without dismantling the system. Three such measurement methods have been presented and the appropriate computational schemes have been developed for the calculation of the element values from the measured data. Various sensible conjectures associated with the measurement methods have also been investigated by computational experiments. Such measurement methods may be applied in the testing of electromechanical systems (including electrical networks and control systems) and have wide application possibilities in automatic test equipments.

Each measurement method needs at least one of the network elements known. If none of the elements is known, then a known, external element is to be added between two suitable nodes of the network. In the case of a network having no frequency-dependent element, known frequency-dependent element or elements must be embedded to confer ~~the~~ network the necessary frequency dependence.

In Chapter 2 a measurement procedure for the simultaneous solution of all the unknown elements from a set of node voltage measurements is discussed. A computer program for the computation of the element values from the measured data is developed. A few theorems associated with the measurement method are presented. Computational experiments with a number of examples are made to investigate the measurement procedure. A special kind of network, known as "singular networks", are also found to be solvable with the present method by the addition of more than one known, external frequency-dependent element.

An alternative method for the explicit solution of the network elements is presented in Chapter 3. In this method, the unknown elements at a particular node are computed one node-set at a time by considering the node equation of that node at a sufficient number of frequencies. The attractive feature of this method is that it avoids the complexity of handling big matrices and is especially attractive when the solution of only a specific portion of a network is of interest. A computer program is developed with the property that once the node sequence, the known element or elements, and the node voltages at the required number of frequencies are specified, the unknown element values of the nodes will be computed in succession along that node sequence. The measurement method is verified with a few examples.

Chapter 4 describes a method for the measurement of the variations of the network elements. The method requires the measurement of the transfer function of the network at a certain number of frequencies and involves the measurements at the input and the output nodes only. The method may be used to obtain the element variations of a wide range of networks including those having a few or all of the nodes, except the input and the output nodes, inaccessible. In order to achieve greater accuracy, an iterative method is presented. A computer program that calculates the element variations from a set of transfer function measurements is developed and the experimental results with a few networks are provided. Concluding remarks on the measurement methods are presented in Chapter 5.

CHAPTER 2

EXPLICIT SOLUTION OF NETWORK ELEMENTS BY A SET OF NODE VOLTAGE MEASUREMENTS

2.1 The measurement method

This consists of measuring the node voltages of the network at a sufficient number of frequencies and then to evaluate the elements from such measurements. The method needs one known element to be added between two suitable nodes. In some cases, it requires more than one known element to be added between suitable points. As the number of elements, in general, exceeds the number of test points, measurement of the voltages at more than one frequency is required. If the network has no frequency-dependent element, known frequency-dependent element/elements at suitable points must be embedded to confer the network the necessary frequency-dependence.

Consider the network shown in figure 2.1 having e elements, and $(m+1)$ nodes (the $(m+1)$th node being the reference or datum node). Let q be the number of known elements being added between certain points so that the resulting network has $(e+q)$ elements.

The Kirchhoff's Current Law matrix equation for the network is given by [6]

$$A_1 \; I = 0 \qquad (2.1)$$

where A_1 is the node-branch incidence submatrix of order

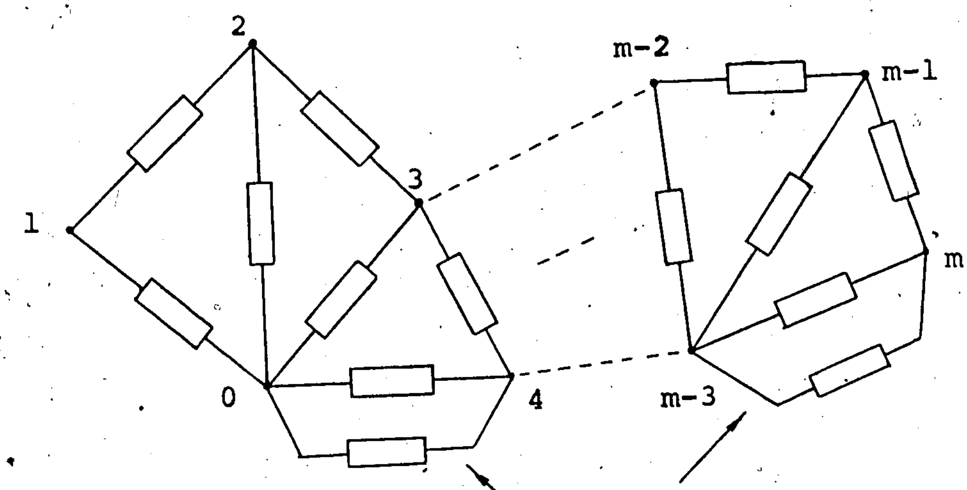

Figure 2.1 A basic electrical network with $m+1$ nodes.

$m \times (e+q)$ formed from the node-branch incidence matrix of the network by deleting the row due to the reference node, and I is a column vector given by

$$I = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_e \\ I_{e+1} \\ \vdots \\ I_{e+q} \end{bmatrix} \tag{2.2}$$

where I_i is the current flowing through the branch i with an element of value b_i. Denoting v_{b_i} as the voltage across the element b_i, I may be written as

$$I = x_i \, v_{b_i} \, b_i \tag{2.3}$$

where

$b_i = g_i = 1/R_i$, for a resistive branch with a resistance R_i ;

$= C_i$, for a capacitive branch with a capacitance C_i ;

$= 1/L_i$, for an inductive branch with an inductance L_i;

and

$x_i = 1$, for a resistive branch;

$= s$, for a capacitive branch;

$= 1/s$, for an inductive branch;

where s is the Laplace variable.

From equations (2.1), (2.2), and (2.3), we get

$$A_1 \begin{bmatrix} x_1 v_{b_1} b_1 \\ x_2 v_{b_2} b_2 \\ \vdots \\ x_e v_{b_e} b_e \\ x_{e+1} v_{b_{e+1}} b_{e+1} \\ \vdots \\ x_{e+q} v_{b_{e+q}} b_{e+q} \end{bmatrix} = 0 \qquad (2.4)$$

Let A be the node-branch incidence submatrix (obtained by deleting the row due to the reference node) of the network without the added elements. A is, thus, a matrix of order $m \times e$ and is a proper subset of the matrix A_1. Partitioning the matrix A_1, equation (2.4) may be written as

$$\left[\begin{array}{c|c} \overset{\leftarrow e \rightarrow}{A} & \overset{\leftarrow q \rightarrow}{A_2} \end{array} \right] \begin{bmatrix} x_1 v_{b_1} b_1 \\ x_2 v_{b_2} b_2 \\ \vdots \\ x_e v_{b_e} b_e \\ \hline x_{e+1} v_{b_{e+1}} b_{e+1} \\ \vdots \\ x_{e+q} v_{b_{e+q}} b_{e+q} \end{bmatrix} = 0 \quad , \qquad (2.5)$$

whence

$$A \begin{bmatrix} x_1 & v_{b_1} & b_1 \\ x_2 & v_{b_2} & b_2 \\ \vdots & & \\ x_e & v_{b_e} & b_e \end{bmatrix} = -A_2 \begin{bmatrix} x_{e+1} & v_{b_{e+1}} & b_{e+1} \\ x_{e+2} & v_{b_{e+2}} & b_{e+2} \\ \vdots & & \\ x_{e+q} & v_{b_{e+q}} & b_{e+q} \end{bmatrix} \qquad (2.6)$$

A_2 is a matrix of order $m \times q$.

Equation (2.6) may be rearranged as

$$\begin{bmatrix} A_{11}x_1 v_{b_1} & A_{12}x_2 v_{b_2} & \cdot\ \cdot & A_{1e}x_e v_{b_e} \\ A_{21}x_1 v_{b_1} & A_{22}x_2 v_{b_2} & \cdot\ \cdot & A_{2e}x_e v_{b_e} \\ & \vdots & & \\ A_{m1}x_1 v_{b_1} & A_{m2}x_2 v_{b_2} & \cdot\ \cdot & A_{me}x_e v_{b_e} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_e \end{bmatrix}$$

$$= -A_2 \begin{bmatrix} x_{e+1} v_{b_{e+1}} b_{e+1} \\ x_{e+2} v_{b_{e+2}} b_{e+2} \\ \vdots \\ x_{e+q} v_{b_{e+q}} b_{e+q} \end{bmatrix}, \qquad (2.7)$$

or

$$P\,C = f \quad , \qquad (2.8)$$

where

$$P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_m \end{bmatrix} = \begin{bmatrix} A_{11}x_1v_{b_1} & A_{12}x_2v_{b_2} & \cdots & A_{1e}x_ev_{b_e} \\ A_{21}x_1v_{b_1} & A_{22}x_2v_{b_2} & \cdots & A_{2e}x_ev_{b_e} \\ & \vdots & & \\ A_{m1}x_1v_{b_1} & A_{m2}x_2v_{b_2} & \cdots & A_{me}x_ev_{b_e} \end{bmatrix}$$

$$= A\, V_b \ , \qquad (2.9a)$$

V_b is a diagonal matrix of the form

$$V_b = \begin{bmatrix} x_1v_{b_1} & & & 0 \\ & x_2v_{b_2} & & \\ & & \ddots & \\ 0 & & & x_ev_{b_e} \end{bmatrix} , \qquad (2.9b)$$

P_i is the row vector due to node i , C is the component vector given by

$$C = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_e \end{bmatrix} , \qquad (2.9c)$$

and f is the forcing function vector expressed as

$$f = \begin{bmatrix} f_1 \\ f_2 \\ \cdot \\ \cdot \\ f_e \end{bmatrix} = -A_2 \begin{bmatrix} x_{e+1} v_{b_{e+1}} b_{e+1} \\ x_{e+2} v_{b_{e+2}} b_{e+2} \\ \cdot \\ \cdot \\ x_{e+q} v_{b_{e+q}} b_{e+q} \end{bmatrix} \quad (2.9d)$$

f_i is the element of the vector f due to node i.

Equation (2.8) is an important relationship. It represents m simulteneous node equations in e unknowns, with, for a connected network, e being greater or equal to m. For the obtainment of the component values, we must have e simulteneous equations which can be conveniently obtained by considering the m node equations of equation (2.8) at a sufficient number of frequencies.

It may be seen from equation (2.8) that the node equation due to a node i is given by

$$P_i = [\ A_{i1} \quad A_{i2} \quad \cdot \quad \cdot \quad A_{ie}\ ]\ V_b \quad (2.10)$$

Using subscript j to represent the frequency j at which such node equation has been considered, we write

$$P_{ij} = [\ A_{i1} \quad A_{i2} \quad \cdot \quad \cdot \quad A_{ie}\ ]\ V_{bj} \quad (2.11)$$

The corresponding element of the forcing function vector f is

$$f_{ij} = -\left[A_{i(e+1)} \; \cdot \cdot \; A_{i(e+q)} \right] \begin{bmatrix} x_{e+1} v_{b_{e+1}} j^{b_{e+1}} \\ x_{e+2} v_{b_{e+2}} j^{b_{e+2}} \\ \vdots \\ x_{e+q} v_{b_{e+q}} j^{b_{e+q}} \end{bmatrix} \quad (2.12)$$

Assume that the m node equations of equation (2.8) have been measured with some or all of the node equations considered at more than one frequency so that we get a set of e node equations which may be expressed, from equations (2.8), (2.11), and (2.12), as

$$V C = F \; . \quad (2.13)$$

Here, V is a matrix of order $e \times e$ and F is a column matrix given by

$$V = \begin{bmatrix} V_1 \\ --- \\ V_2 \\ --- \\ \vdots \\ --- \\ V_e \end{bmatrix} \; , \; = \begin{bmatrix} P_{11} \\ \vdots \\ P_{1j_1} \\ P_{21} \\ \vdots \\ P_{2j_2} \\ \vdots \\ P_{m1} \\ \vdots \\ P_{mj_m} \end{bmatrix} \quad (2.14)$$

and

$$F = \begin{bmatrix} f_{11} \\ \cdot \\ \cdot \\ f_{1j_1} \\ f_{21} \\ \cdot \\ \cdot \\ f_{2j_2} \\ \cdot \\ \cdot \\ f_{m1} \\ \cdot \\ \cdot \\ f_{mj_m} \end{bmatrix} \qquad (2.15)$$

where the node equation at the node 1 has been taken up at j_1 frequencies, that at the node 2 at j_2 frequencies, and so on. Note that

$$j_1 + j_2 + \cdot \cdot \cdot + j_m = e \qquad (2.16)$$

From equation (2.13), the component vector may be obtained as

$$C = V^{-1} F \qquad (2.17)$$

provided the matrix V is nonsingular.

It can be seen that the minimum number of frequencies at which the network must be measured is given by the ratio e/m, provided the ratio be rounded off to the next higher integer in case it is a fraction.

2.2 The algorithm for the computer solution

A digital computer algorithm for the solution of the network elements will now be presented.

2.2.1 Convention of the branch voltage and the source voltage

The voltage v_{b_k} across the branch-element b_k is related to the branch voltage v'_k of the branch k of figure 2.2(a) by [7]

$$v_{b_k} = v'_k - E_k \qquad (2.18)$$

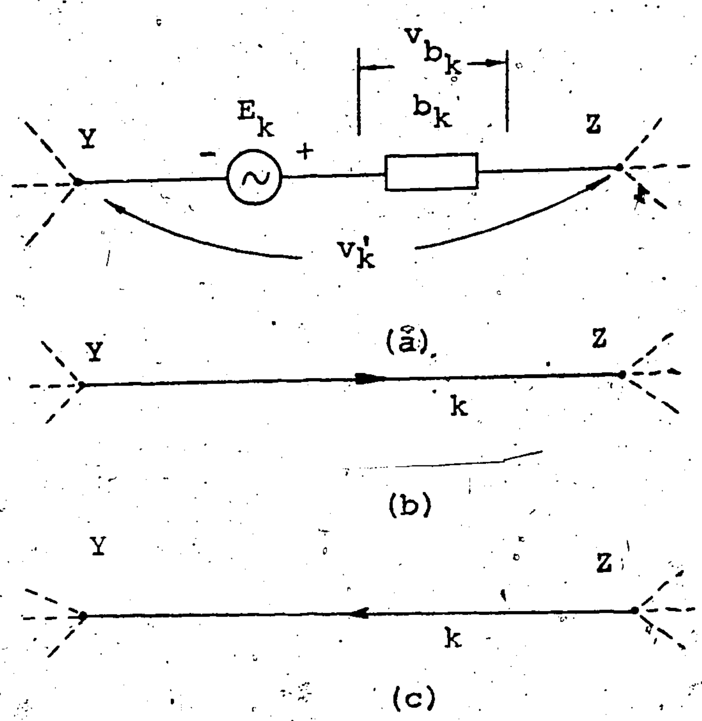

Figure 2.2 Conventions of the branch voltage and the source voltage.

The following conventions are to be followed for the use of this equation:

Once a particular direction (which is arbitrary) has been assigned for the branch, the head of the arrow represents the reference point for that branch so far as the voltage across the branch and that across the element of the branch are concerned. Thus, for figure 2.2:

Voltage across the branch k

$$= v_y - v_z \quad \text{for figure 2.2(b)},$$

$$= v_z - v_y \quad \text{for figure 2.2(c)}.$$

The sign convention for the voltage source E_k in the branch k is as follows:

If the voltage source has a polarity such that it should deliver current into the branch in the same direction as the arrow, E_i is assumed positive, otherwise it is assumed negative.

Thus, for the branch notation of figure 2.2(b), $E_k = |E_k|$, whereas for figure 2.2(c), $E_k = -|E_k|$.

Hence

$$v_{b_k} = (v_y - v_z) + |E_k| \quad \text{for figure 2.2(b)},$$

$$= (v_z - v_y) - |E_k| \quad \text{for figure 2.2(c)}.$$

2.2.2 Voltage across the branch element from the node voltage

The analysis in Section 2.1 has been carried out in terms of the voltage across the branch elements, whereas in the practical situations, only the node voltages will be measured. In order to use the results in Section 2.1, one must calculate the voltage across the branch elements from the node voltages.

Let v_j be the voltage at the node j with respect to the reference node. With our sign convention, the relationship between the node voltage and the branch voltage is given by [8],[9]

$$v' = A_1^T v \qquad (2.19)$$

Equations (2.18) and (2.19) may be combined to get

$$\begin{bmatrix} v_{b_1} \\ v_{b_2} \\ \vdots \\ v_{b_e} \\ \vdots \\ v_{b_{e+q}} \end{bmatrix} = A_1^T \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix} - \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_e \\ \vdots \\ E_{e+q} \end{bmatrix} \qquad (2.20)$$

Equation (2.20) gives the relationship between the branch voltages and the node voltages of a network.

2.2.3 Calculation of the node voltage

In order to calculate the node voltages of the network, the following relationships may be conveniently used [10],[11]

$$G\ v = A_1\ Y\ E \qquad (2.21)$$

where G is the node admittance matrix given by

$$G = A_1\ Y\ A_1^T \qquad (2.22a)$$

where

$$Y = Z^{-1} \qquad (2.22b)$$

$$Z = R + sL + (1/s)D \qquad (2.22c)$$

R is a diagonal matrix of order (e+q) with the diagonal element $r_{ii} = R_i$, the resistance of the branch i ($r_{ii} = 0$ if the branch does not contain a resistance); D is a diagonal matrix of order (e+q) with the diagonal element $d_{ii} = 1/C_i$, where C_i is the capacitance of the branch i ($d_{ii} = 0$ if the branch does not contain a capacitance); L is a square matrix of order (e+q) with the diagonal element $l_{ii} = L_i$, the inductance of the branch i ($l_{ii} = 0$ if the branch is noninductive), and

$l_{ij} = M_{ij}$, where M_{ij} is the mutual inductance between the branch i and the branch j .

For such calculations, current source ,if any, must be converted into its equivalent voltage source . It may be pointed out that in order to use these equations, the matrix Z must be nonsingular which requires that [12]

(i) each of the current sources must be connected across a single branch so that it can be transformed into an equivalent voltage source with a series impedance;

(ii) each of the voltage sources must be in series with some impedance .

If the above conditions are not satisfied, then I-shift transformation [12] in the case of current source, and E-shift transformation [12] in the case of voltage source ,must be made in order to make the matrix Z non-singular.

2.3 Flow chart of the computer program

The flow chart may be obtained from the discussion in the previous sections.The computer program must be such that once the node-branch incidence submatrix A_1 , the E-matrix , and a sufficient number of node voltage measurements are given, it will calculate the forcing function vector F at different frequencies and then form the V-matrix to use equation (2.17) to find the component values. However, for the purpose of the computational experiments, the computer program has been written with the additional feature that it calculates the node voltages that would have been obtained through measurements in practice, and with this set of node voltages, calculates the element values. For the purpose of greater accuracy, the program must be written in double precision. In writing the program, we shall assume, for clarity and without restriction, that there is no mutual inductance present in the network.

The flow chart will have the following major steps:

1. Read m, n, n_r, n_l, n_c, and the frequencies at which measurements are to be made. m is the total number of nodes excluding the reference node; n is the total number of elements including the added elements; n_r, n_l, and n_c are the total number of resistances, inductances, and capacitances respectively.
2. Read the elements of the matrix A_1, the elements of the matrix E, and the values of the resistances, inductances and capacitances. In order to ease the formation of the Y-matrix, element values are assigned these locations:

Resistances: $R(1), R(2), \ldots, R(n_r)$;

Inductances: $L(n_r+1), L(n_r+2), \ldots, L(n_r+n_l)$;

Capacitances: $C(n_r+n_l+1), C(n_r+n_l+2), \ldots, C(n)$.

In assigning locations for the added element, care must be taken so that if such element is capacitive, it is assigned the location C(n). For two capacitive elements, assign them in locations C(n-1) and C(n), etc. If the added element is a resistance, assign

Resistances: $R(1), R(2), \ldots, R(n_r-1)$;

Inductances: $L(n_r), L(n_r+1), \ldots, L(n_r+n_l-1)$;

Capacitances: $C(n_r+n_l), C(n_r+n_l+1), \ldots, C(n-1)$;

Added resistance: R(n).

For two added resistances, assign such that the added elements occupy the locations R(n-1) and R(n). Similar procedure is to be adopted in case the added element is an inductance.

3. Form the matrix Y. Since there is no mutual inductance, the matrix Z of the network will be diagonal and as a result, $Y_{ii} = 1/Z_{ii}$. The matrix Y will thus be a diagonal matrix and have the following diagonal elements:

$$Y_{ii} = 1/R(i) , \quad i = 1, 2, \ldots, n_r ;$$

$= 1/[sL(i)]$, $i = n_r+1, n_r+2, . . . , n_r+n_l$;

$= sC(i)$, $i = n_r+n_l+1, n_r+n_l+2, . . . , n.$

These are the diagonal elements when the added element is a capacitance. For a resistive or an inductive added element, Y_{ii} is to be formed in accordance with the locations assigned in the step 2.

4. Premultiply the matrix Y by the incidence matrix A_l to form the matrix D given by

$$D = A_l Y \qquad (2.23)$$

5. Postmultiply the matrix D by A_l^T to get the node-admittance matrix G.

6. Postmultiply the matrix D by the matrix E to form the matrix H given by

$$H = A_l Y E \qquad (2.24)$$

7. Invert the matrix G and premultiply the inverted matrix by the matrix H to get the node voltage matrix V.

8. Obtain the branch voltages from the node voltages by the use of equation (2.20) by the procedure as follows:

   (i) postmultiply the matrix A_l^T by the node voltage matrix to get the matrix VX such that

$$VX = A_1^T v \quad (2.25)$$

(ii) subtract the matrix E from the matrix VX to obtain the branch voltage matrix.

9. Multiply the elements of the branch voltage matrix by s for a capacitive branch, by 1/s for an inductive branch, and keep the elements unchanged for a resistive branch. Thus, the term $x_i v_{b_i}$ at the frequency s is obtained. These are the diagonal elements of the matrix V_b.

10. Form the element f_{ij} of the forcing function vector by the use of equation (2.12). In the case of only one added element, the element f_{ij} is given by $A_{in} v_{b_n j}\ b_n$, where $v_{b_n j}$ is the element of the matrix V_b due to the added element b_n at a frequency j.

11. Form the matrix P of equation (2.9a) from the A-matrix and the matrix V_b by postmultiplying the A-matrix with the matrix V_b.

12. Return to step 3 to repeat the subsequent steps at a new frequency until all the frequencies have been considered.

13. Form the matrix V from the P-matrices by the use of equation (2.14).

14. Invert the matrix V.

15. From the element f_{ij}, form the forcing function vector F by the use of equation (2.15).

16. Obtain the component matrix by premultiplying the vector F with the inverse of the V-matrix.

The details of the above steps are illustrated in the flow chart given in figure 2.3.

2.4 A few theorems

A few theorems which are relevant to the present measurement method will be presented in this Section.

2.4.1 Some definitions

Directly connected node: The nodes X and Y are called "directly connected" if there is at least one single network element (either a resistance, a capacitance, or an inductance) with or without a series-voltage source or a parallel-current source, connecting them. This is illustrated in figure 2.4.

Primary node: A node is called a primary node if it is directly connected to the reference node.

2.4.2 Theorem 1: The node equation of each primary node must be taken at least at one frequency for the V-matrix

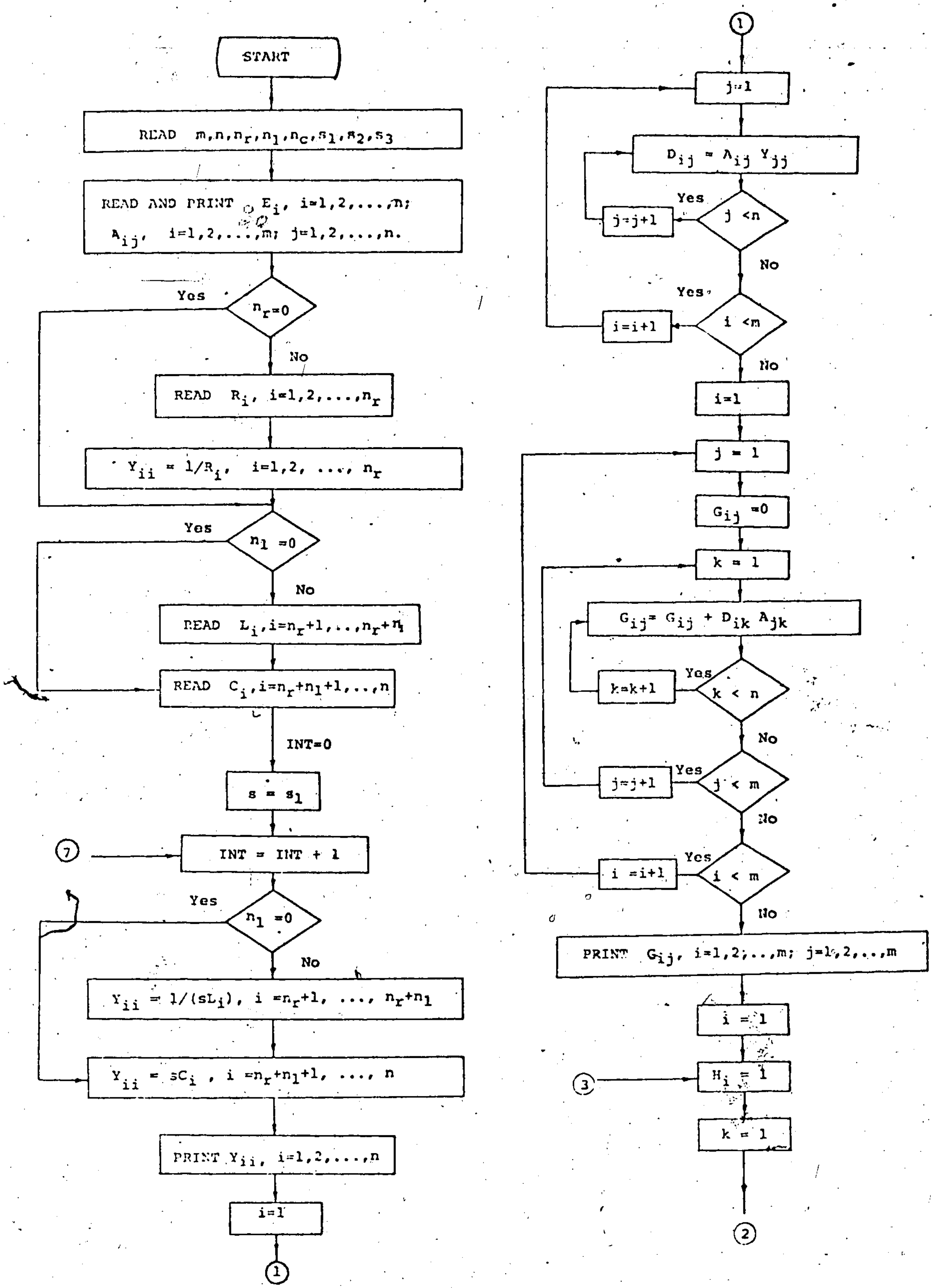

Figure 2.3 The flow chart.

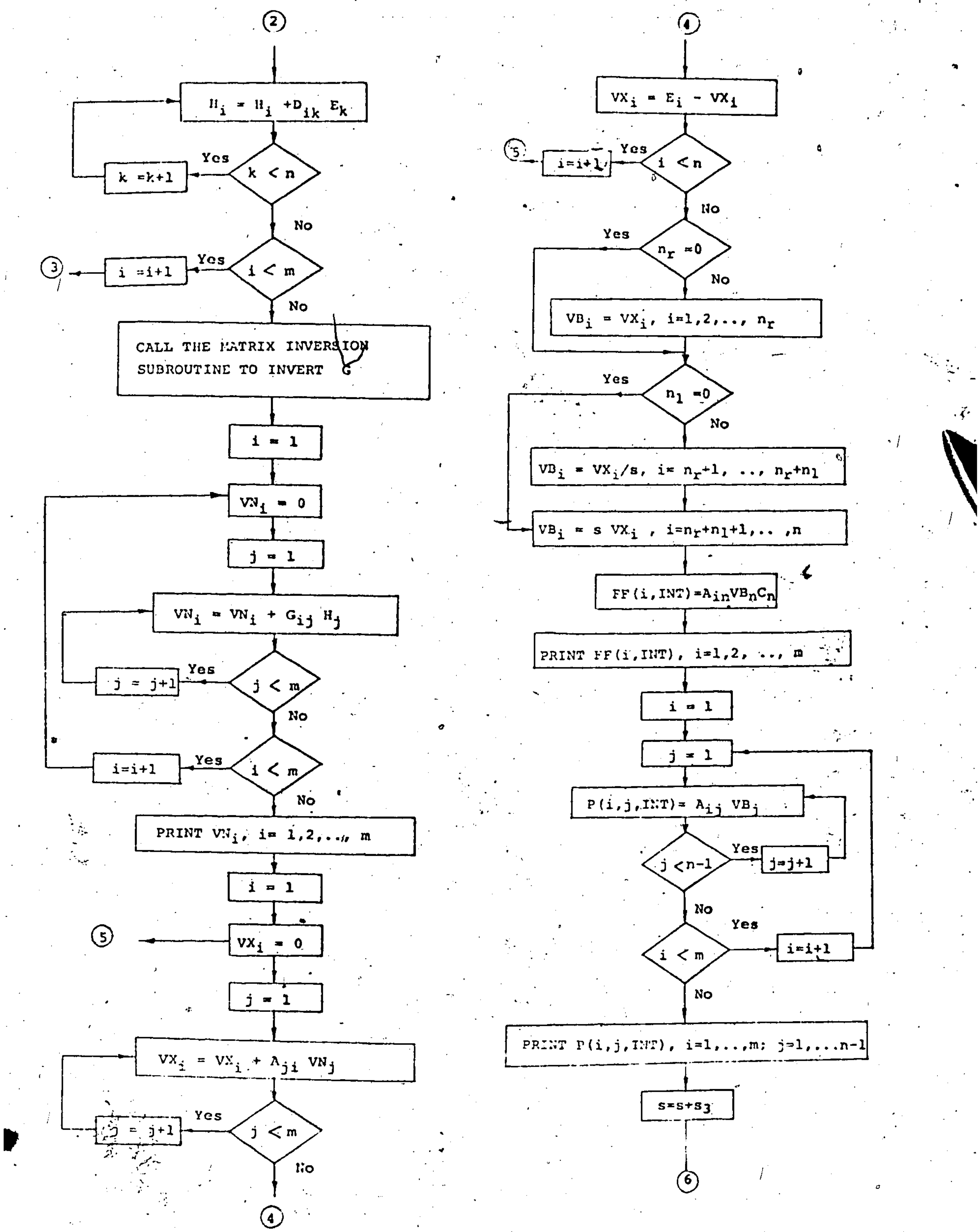

Figure 2.3 The flow chart(Continued) .

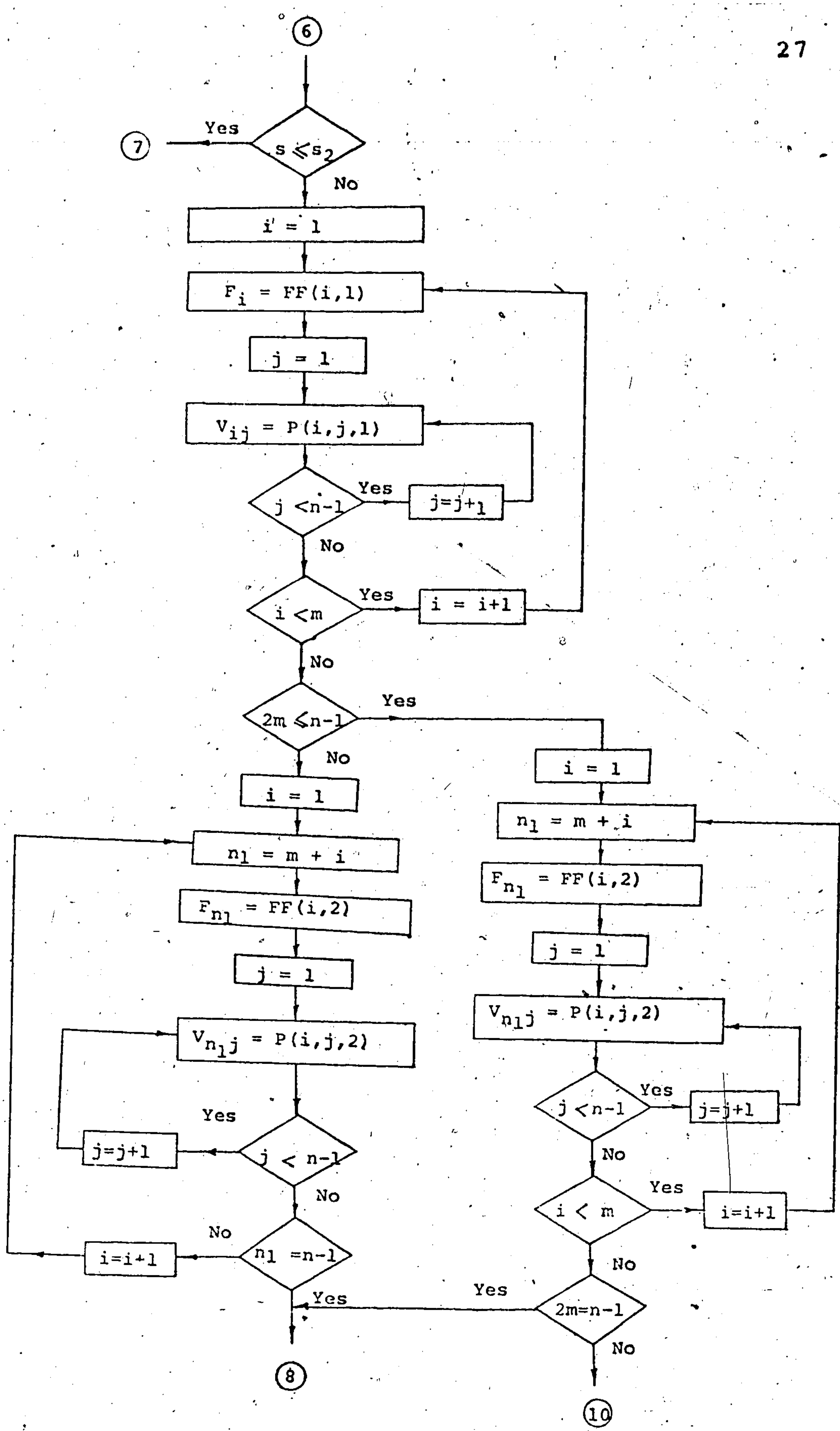

Figure 2.3 The flow chart(Continued).

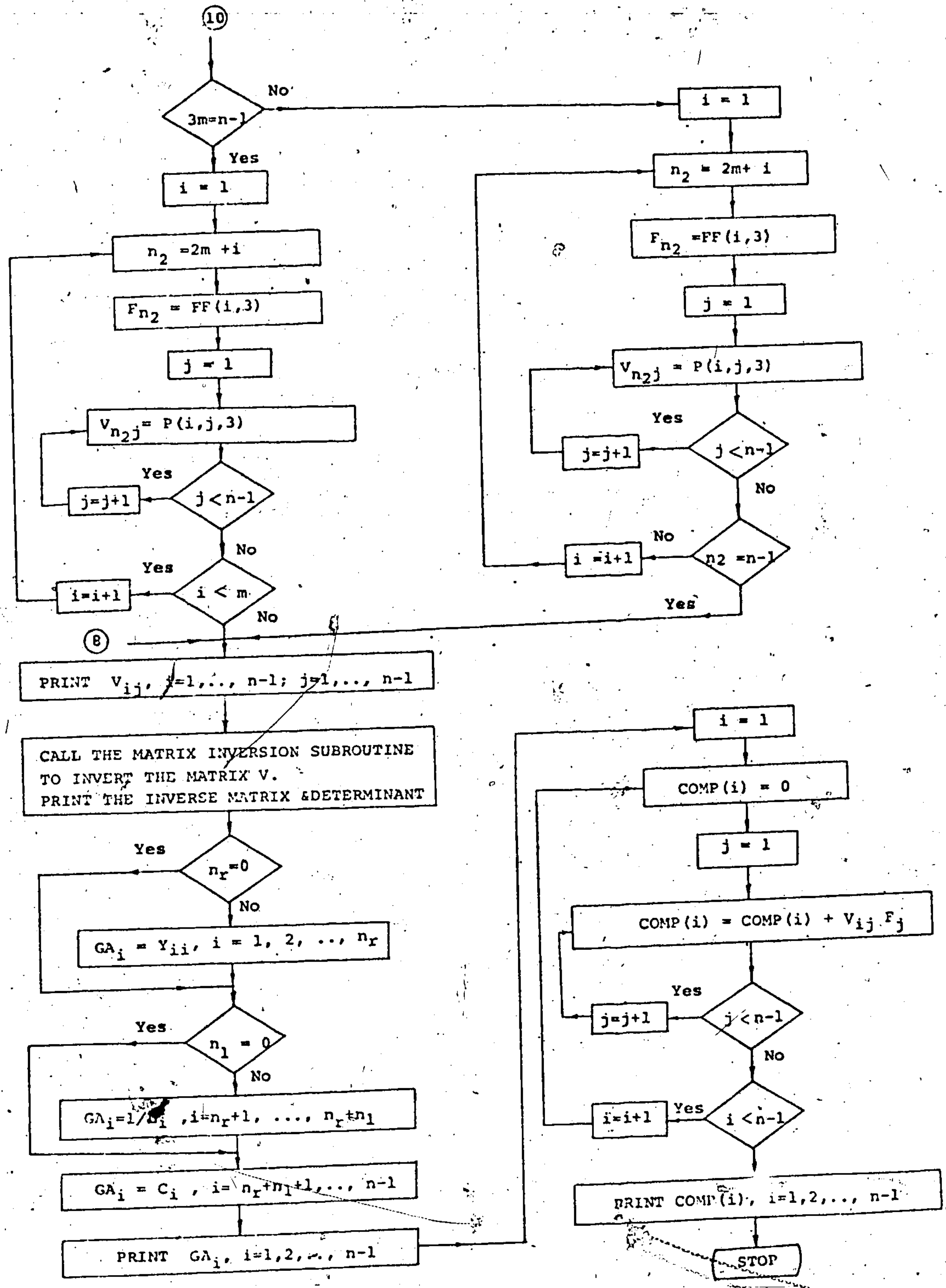

Figure 2.3 The flow chart(Continued).

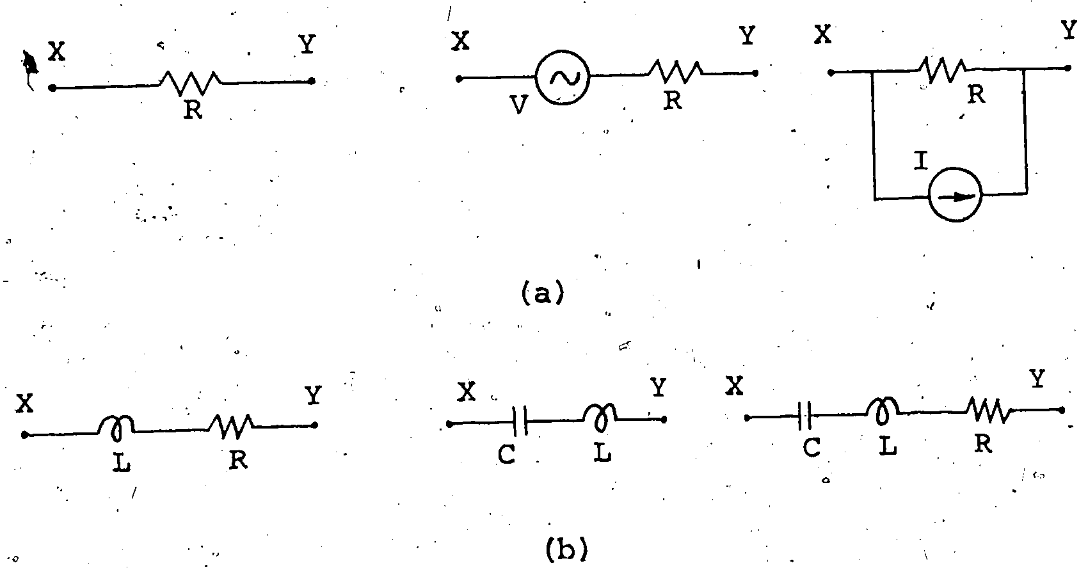

Figure 2.4 (a) Nodes X and Y are directly connected,
(b) Nodes X and Y are not directly connected.

to be nonsingular.

Proof:

The node-branch incidence matrix A_a of a network has two non-zero elements in each column. The matrix A is obtained from the matrix A_a by deleting the row due to the reference node which has non-zero elements in the columns due to those branches which are incident to the reference node. As a result, the matrix A will have a single non-zero element (either +1 or -1) in each of the columns due to those branches which are incident on the reference node. The single non-zero element for a particular column appears in the row due to the primary node which is directly connected to the reference node through that branch. If the node equation at that primary node is not taken, the V-matrix will have a zero column corresponding to that branch, making the V-matrix singular.

<u>Corollary 1.1</u>: The nodes which are directly connected to the primay node must be measured at least at one frequency.

<u>Proof</u>:

By Theorem 1, the node equations due to the primary nodes must be taken at least at one frequency. In writing such node equations, one must know the voltages across all the branches which are incident at that node. As a consequence, one must measure the node voltages of all the nodes which are directly connected to the node under consideration.

<u>Corollary 1.2</u>: For the node equation of any particular node at any frequency, the node voltages of all the nodes, directly connected to that node, must be measured at that frequency.

2.4.3 <u>Theorem 2</u>: Each node voltage of a connected network must be measured at some frequency.

<u>Proof</u>:

It has been shown that the primary nodes as well as the nodes which are directly connected to the primary nodes must be measured at least once. Let us now consider the remaining nodes which are neither primay node nor directly connected with them. They are either of the following two types:

(a) Nodes which are directly connected to a directly-connected node

(b) Nodes which are not directly connected to any directly-connected node.

For the type (a) node, let such a node X be connected to a directly-connected node n_d through an unknown element i . Since the columns of the node-branch incidence submatrix A_1 due to the element i has non-zero element in the row due to the node n_d, and the node X only, either one or both of the node equations must be taken at least at one frequency. As a consequence, the voltages at both the nodes n_d and X must be measured at least once. For the type (b) node, let Y and Z be two such nodes which are not directly connected to any directly-connected node. Let the branch i connect them. Now, in the matrix A, the elements in the columns due to the branch i are zero everywhere except in the row due to the node Y and Z. Thus, for the matrix V to be nonsingular, either one or both of the node equations must be included. The voltages of the nodes X and Y, therefore, must be measured at least at one frequency.

2.4.4 <u>Theorem 3</u>: For the V-matrix to be non-singular, the number of different frequencies at which the node equation of a node may be taken to form the V-matrix, cannot exceed (d-1), where d is the degree of that node.

<u>Proof</u>:

Consider a node X with d branches incident at it as shown in figure 2.5. The Kirchhoff's Current Law at the node is given by

$$x_1 v_{b_1 b_1} + x_2 v_{b_2 b_2} + \; . \; . \; . \; + x_d v_{b_d b_d} = 0 \qquad (2.26)$$

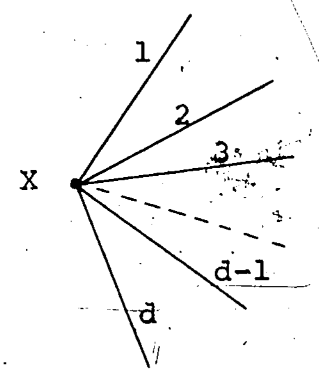

Figure 2.5 A node with d branches incident on it.

Writing this equation in d different frequencies, we get

$$\begin{aligned} x_{11} v_{b_1 1 b_1} + x_{21} v_{b_2 1 b_2} + \; . \; . \; . \; + x_{d1} v_{b_d 1 b_d} &= 0 \\ x_{12} v_{b_1 2 b_1} + x_{22} v_{b_2 2 b_2} + \; . \; . \; . \; + x_{d2} v_{b_d 2 b_d} &= 0 \\ &\vdots \\ x_{1d} v_{b_1 d b_1} + x_{2d} v_{b_2 d b_2} + \; . \; . \; . \; + x_{dd} v_{b_d d b_d} &= 0 \end{aligned} \qquad (2.27)$$

Equation (2.27) is a set of linear homogeneous equations with d unknown variables $b_1, b_2, \ldots, b_d$. Since the variables are non-zero, the solution of this set of equations must be non-trivial necessiating that the coefficient matrix

$$\begin{bmatrix} x_{11}v_{b_1 1} & x_{21}v_{b_2 1} & \cdots & x_{d1}v_{b_d 1} \\ x_{12}v_{b_1 2} & x_{22}v_{b_2 2} & \cdots & x_{d2}v_{b_d 2} \\ & \vdots & & \\ x_{1d}v_{b_1 d} & x_{2d}v_{b_2 d} & \cdots & x_{dd}v_{b_d d} \end{bmatrix}$$

must be singular. The set of equations are, thus, linearly dependent. As a consequence, the matrix V with this set of equations as a subset will be singular. For this set of homogeneous equations to have non-trivial solution, the rank of the coefficient matrix cannot exceed $(d-1)$. Therefore, $(d-1)$ is an upper bound for the number of independent equations. Hence the theorem.

<u>Corollary 3.1</u>: Theorem 3 is also true for the nodes where known elements have been added. The degree d at these nodes includes the known, added elements.

Proof:

The node, where some known elements have been added, also satisfies equation (2.27), where d represents the total number of elements including the added ones. The node, thus, must satisfy Theorem 3.

2.4.5 Theorem 4: If two or more elements of same kind (either resistances, capacitances, or inductances) are connected in parallel between two nodes, the matrix V will be singular.

Proof:

Let two branches i and j, having the same kind of elements, be connected in parallel between two nodes X and Y as shown in figure 2.6. The node

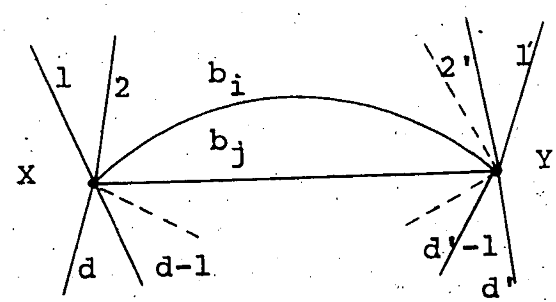

Figure 2.6 Nodes X and Y are connected by two parallel branches b_i and b_j.

equations of the node X at m different frequencies are given by

$$x_{11}v_{b_11}b_1 + \cdot \cdot +x_{i1}v_{b_i1}b_i + \cdot \cdot +x_{j1}v_{b_j1}b_j + \cdot \cdot +x_{d1}v_{b_d1}b_d = 0$$

$$x_{12}v_{b_12}b_1 + \cdot \cdot +x_{i2}v_{b_i2}b_i + \cdot \cdot +x_{j2}v_{b_j2}b_j + \cdot \cdot +x_{d2}v_{b_d2}b_d = 0$$

$$\vdots$$

$$x_{1m}v_{b_1m}b_1 + \cdot \cdot +x_{im}v_{b_im}b_i + \cdot \cdot +x_{jm}v_{b_jm}b_j + \cdot \cdot +x_{dm}v_{b_dm}b_d = 0 \qquad (2.28)$$

The node equations of the node Y at p different frequencies are

$$x_{1'1}v_{b_{1'}1}b_{1'} + \cdot \cdot +x_{i1}v_{b_i1}b_i + \cdot \cdot +x_{j1}v_{b_j1}b_j + \cdot \cdot +x_{d'1}v_{b_{d'}1}b_{d'} = 0$$

$$x_{1'2}v_{b_{1'}2}b_{1'} + \cdot \cdot +x_{i2}v_{b_i2}b_i + \cdot \cdot +x_{j2}v_{b_j2}b_j + \cdot \cdot +x_{d'2}v_{b_{d'}2}b_{d'} = 0$$

$$\vdots$$

$$x_{1'p}v_{b_{1'}p}b_{1'} + \cdot \cdot +x_{ip}v_{b_ip}b_i + \cdot \cdot +x_{jp}v_{b_jp}b_j + \cdot \cdot +x_{d'p}v_{b_{d'}p}b_{d'} = 0 \qquad (2.29)$$

Since none of the remaining nodes of the network has the branches i and j incident at it, it is clear that the entries of the V-matrix in the column corresponding to branch i and branch j will be zero except for the nodes X and Y. Again, since the elements b_i and b_j are of same type, and since they are parallel,

$$x_{ik} v_{b_i k} = x_{jk} v_{b_j k}$$

indicating that the columns of the matrix V corresponding to the elements b_i and b_j are the same. The matrix V is, thus, singular.

2.4.6 Theorem 5: For a ladder network with a single element between the nodes, the matrix V is always nonsingular.

Proof:

Consider the ladder network of figure 2.7.

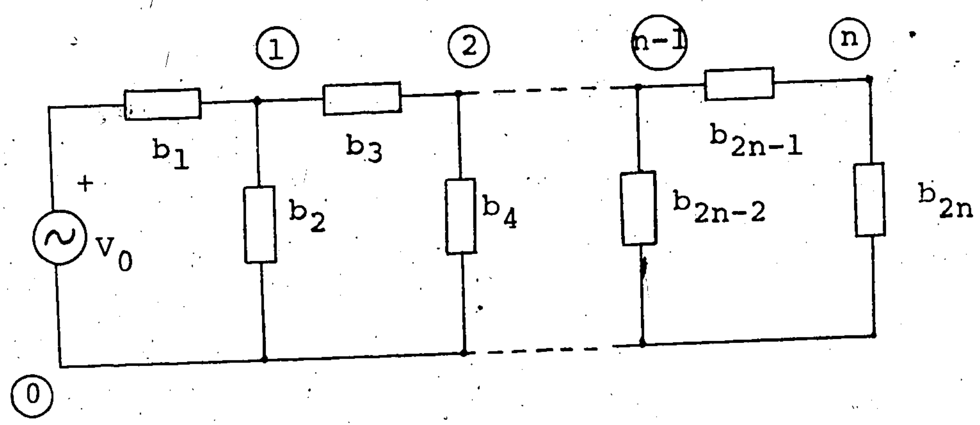

Figure 2.7 A ladder network with n nodes.

In terms of the node voltages, the V-matrix of the network may be obtained as

$$
V = \begin{bmatrix}
x_{11}(v_{01}-v_{11}) & -x_{21}v_{11} & x_{31}(v_{21}-v_{11}) & 0 & \cdots & & & 0 \\
x_{12}(v_{02}-v_{12}) & -x_{22}v_{12} & x_{32}(v_{22}-v_{12}) & 0 & \cdots & & & 0 \\
0 & 0 & -x_{31}(v_{21}-v_{11}) & -x_{41}v_{21} & x_{51}(v_{31}-v_{21}) & 0 & \cdots & 0 \\
0 & 0 & -x_{32}(v_{22}-v_{12}) & -x_{42}v_{22} & x_{52}(v_{32}-v_{22}) & 0 & \cdots & 0 \\
 & & & \vdots & & & & \\
0 & \cdots & 0 & & x_{(2n-3)1}[v_{(n-2)1}-v_{(n-1)1}] & & \begin{array}{c} -x_{(2n-2)1}v_{(n-1)1} \\ x_{(2n-1)1}[v_{n1}-v_{(n-1)1}] \end{array} & 0 \\
0 & \cdots & 0 & & x_{(2n-3)2}[v_{(n-2)2}-v_{(n-1)2}] & & \begin{array}{c} -x_{(2n-2)2}v_{(n-1)2} \\ x_{(2n-1)2}[v_{n2}-v_{(n-1)2}] \end{array} & 0 \\
0 & \cdots & & \cdots & 0 & & -x_{(2n-1)1}[v_{n1}-v_{(n-1)1}] & -x_{2n\,1}v_{n1} \\
0 & \cdots & & \cdots & 0 & & -x_{(2n-1)2}[v_{n2}-v_{(n-1)2}] & -x_{2n\,2}v_{n2}
\end{bmatrix}
\qquad (2.30)
$$

By elementary transformation, the determinant of the matrix V may be evaluates as

$$\Delta_n = K \begin{vmatrix} x_{11}(v_{01}-v_{11}) & -x_{21}v_{11} & x_{31}(v_{21}-v_{11}) & 0 & \cdots & & 0 \\ x_{12}(v_{02}-v_{12}) & -x_{22}v_{12} & x_{32}(v_{22}-v_{12}) & 0 & \cdots & & 0 \\ 0 & 0 & -x_{31}(v_{21}-v_{11}) & -x_{41}v_{21} & x_{51}(v_{31}-v_{21}) & 0 \cdots & 0 \\ 0 & 0 & -x_{32}(v_{22}-v_{12}) & -x_{42}v_{22} & x_{52}(v_{32}-v_{22}) & 0 \cdots & 0 \\ & & & \vdots & & & \\ 0 \cdots & 0 & & x_{(2n-3)1}[v_{(n-2)1}-v_{(n-1)1}] & & & -x_{(2n-2)1}v_{(n-1)1} \\ 0 \cdots & 0 & & x_{(2n-3)2}[v_{(n-2)2}-v_{(n-1)2}] & & & -x_{(2n-2)2}v_{(n-1)2} \end{vmatrix} \qquad (2.31)$$

$$= K \ \Delta_{n-1}$$

where

$$K = x_{2n\ 2}v_{n2}x_{(2n-1)1}[v_{(n-1)1} - v_{n1}] - x_{2n\ 1}v_{n1}x_{(n-1)2}[v_{(n-1)2} - v_{n2}] \qquad (2.32)$$

Equation (2.31) may be used to derive

$$\Delta_n = [x_{2n\,2}v_{n2}x_{(2n-1)1}\{v_{(n-1)1}-v_{n1}\} - x_{2n\,1}v_{n1}x_{(2n-1)2}\{v_{(n-1)2}-v_{n2}\}]$$

$$\times [x_{2(n-1)2}v_{(n-1)2}x_{(2n-3)1}\{v_{(n-2)1}-v_{(n-1)1}\} - x_{2(n-1)1}v_{(n-1)1}x_{(2n-3)2}\{v_{(n-2)2}-v_{(n-1)2}\}]$$

$$\times \cdots \times [x_{42}v_{22}x_{31}(v_{11}-v_{21}) - x_{41}v_{21}x_{32}(v_{21}-v_{22})]$$

$$\times [x_{22}v_{12}x_{11}(v_{01}-v_{11}) - x_{21}v_{11}x_{12}(v_{02}-v_{12})] \quad (2.33)$$

A sufficient condition for the matrix V to be nonsingular may be easily derived from equation (2.33) as

$$\frac{x_{2a-1}}{x_{2a}}\left[\frac{v_{a-1}}{v_a} - 1\right]\Bigg|_1 \neq \frac{x_{2a-1}}{x_{2a}}\left[\frac{v_{a-1}}{v_a} - 1\right]\Bigg|_2 \quad (2.34)$$

for $a = 1, 2, \ldots, n$.

If this condition is not satisfied at one or more nodes, the matrix V will be singular.

The node equation at the output node n may be written as

$$x_{2n-1}b_{2n-1}(v_{n-1} - v_n) - x_{2n}b_{2n}v_n = x_{2n+1}b_{2n+1}v_n \quad (2.35)$$

whence

$$\frac{v_{n-1}}{v_n} - 1 = \frac{x_{2n+1}\, b_{2n+1} + x_{2n}\, b_{2n}}{x_{2n-1}\, b_{2n-1}} \qquad (2.36)$$

The node equation at any other node 'a' is given by

$$x_{2a-1}\, b_{2a-1}\,(v_{a-1} - v_a) + x_{2a+1}\, b_{2a+1}\,(v_{a+1} - v_a) - x_{2a}\, b_{2a}\, v_a = 0 \qquad (2.37)$$

from which, we get

$$\frac{x_{2a-1}}{x_{2a}} \left(\frac{v_{a-1}}{v_a} - 1\right) = \frac{b_{2a}}{b_{2a-1}} - \frac{x_{2a+1}}{x_{2a}}\, \frac{b_{2a+1}}{b_{2a-1}} \left(\frac{v_{a+1}}{v_a} - 1\right) \qquad (2.38)$$

We now consider several cases of interest:

2.4.6.1 Resistive ladder

For the resistive ladder, the nonsingularity conditions reduce to the condition that the ratio $\left[\frac{v_{a-1}}{v_a} - 1\right]$ should remain same at two different frequencies. For a resistive ladder, equation (2.36) and equation (2.38) may be used to obtain

$$\frac{v_{n-1}}{v_n} - 1 = \frac{g_{2n} + sC}{g_{2n-1}} \qquad (2.39a)$$

and

$$\frac{v_{a-1}}{v_a} - 1 = \frac{g_{2a}}{g_{2a-1}} - \left[\frac{v_{a+1}}{v_a} - 1\right]\frac{g_{2a+1}}{g_{2a-1}} \qquad (2.39b)$$

It may be seen from equations (2:39a) and (2.39b) that for the node 'a' to satisfy the nonsingularity condition, we must necessarily have the term $[(v_{a-1}/v_a)-1]$ or the term $[(v_{a+1}/v_a)-1]$ frequency-dependent.

Now, from equations (2.39a) and (2.39b), we get

for node n-1:

$$\frac{v_{n-2}}{v_{n-1}} - 1 = \frac{g_{2n-2}(g_{2n}+g_{2n-1})+g_{2n-1} + sC(g_{2n-1}+1)}{g_{2n-3}(g_{2n}+g_{2n-1}+sC)}$$

$$= \frac{c_1 + d_1 s}{a_1 + b_1 s} \quad ;$$

for node n-2:

$$\frac{v_{n-3}}{v_{n-2}} - 1 = \frac{g_{2n-4}}{g_{2n-5}} - \frac{g_{2n-3}}{g_{2n-5}} \left(\frac{v_{n-1}}{v_{n-2}} - 1 \right) = \frac{c_2 + d_2 s}{a_2 + b_2 s} ;$$

In general, for the node i:

$$\frac{v_{(n-i)-1}}{v_{n-i}} - 1 = \frac{c_i + d_i s}{a_i + b_i s} \qquad (2.40)$$

where a_i, b_i, c_i, and d_i are non-zero positive quantities. It is clear from the above analysis that the non-singularity conditions are satisfied at each node of such a ladder.

2.4.6.2 Resistance-capacitance ladder

2.4.6.2.1 R-C ladder:

For the ladder network of figure 2.8, the non-singularity condition is that the term $[(v_{a-1}/v_a)-1]/s$ must be frequency-dependent, which is equivalent to the condition that the term $[(v_{a+1}/v_a)-1]/s$ must be frequency-dependent [see equation (2.39b)].

For this ladder, equations (2. 36) and (2.38)

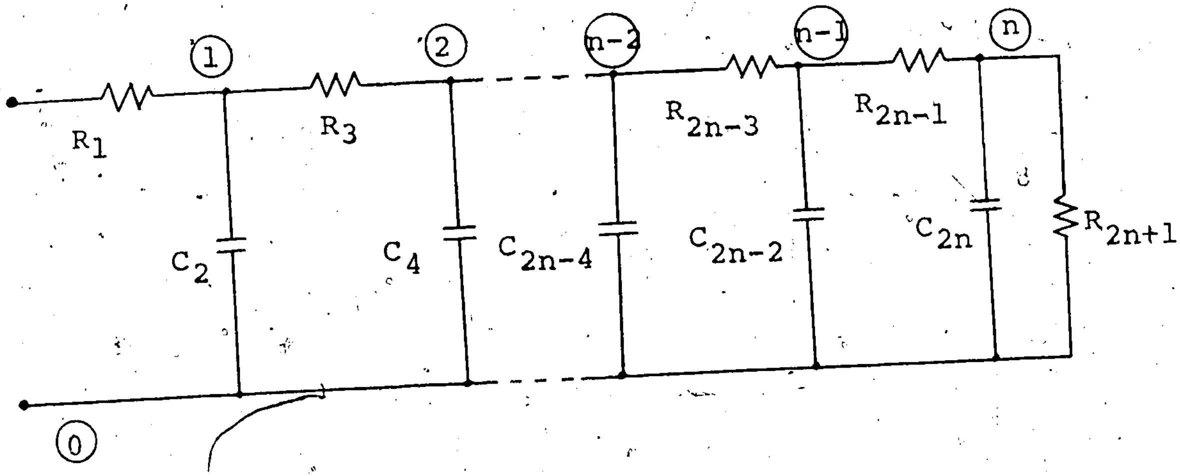

Figure 2.8 A resistance-capacitance ladder.

may be used to obtain

for node n:

$$\left(\frac{V_{n-1}}{V_n} - 1 \right) \frac{1}{s} = \frac{C_{2n} + g_{2n+1}/s}{g_{2n-1}} \qquad (2.41)$$

The nonsingularity condition is thus satisfied by the node n.

<u>for node n-1:</u>

$$\frac{v_n}{v_{n-1}} - 1 = - \frac{g_{2n+1} + sC_{2n}}{g_{2n+1} + g_{2n-1} + sC_{2n}} \tag{2.42}$$

<u>for node n-2:</u>

$$\frac{v_{n-1}}{v_{n-2}} - 1 = - \frac{a_1 s^2 + (b_1 - C_{2n})s + d_1}{a_1 s^2 + b_1 s + c_1} \tag{2.43}$$

where

$$a_1 = \frac{C_{2n} C_{2n-2}}{g_{2n-3}}$$

$$b_1 = \frac{(g_{2n+1} + g_{2n-1})C_{2n-2}}{g_{2n-3}} + \left(\frac{g_{2n-1}}{g_{2n-3}} + 1\right)C_{2n}$$

$$c_1 = d_1 + g_{2n-1} + g_{2n+1}$$

and

$$= \frac{g_{2n-1}\, g_{2n+1}}{2n-[illegible]}$$

It may be shown that for any arbitrary node n-i ($i \neq 0$),

$$\frac{v_{(n-i)+1}}{v_{n-i}} - 1 = - \frac{A_i s^i + A_{i-1} s^{i-1} + \ldots + A_1 s + A_0}{A_i s^i + B_{i-1} s^{i-1} + \ldots + B_1 s + B_0} \tag{2.44}$$

where A_k and B_k , ($k = 0, 1, 2, \ldots, i$), are all non-zero positive constants. From equation (2.44), it may be shown that

$$\frac{v_{(n-i)+1}}{v_{n-i}} \neq K s ,$$

where K is an arbitrary constant, indicating that all the nodes satisfy the nonsingularity condition.

2.4.6.2.2 C-R ladder:

The nonsingularity conditions for the C-R ladder

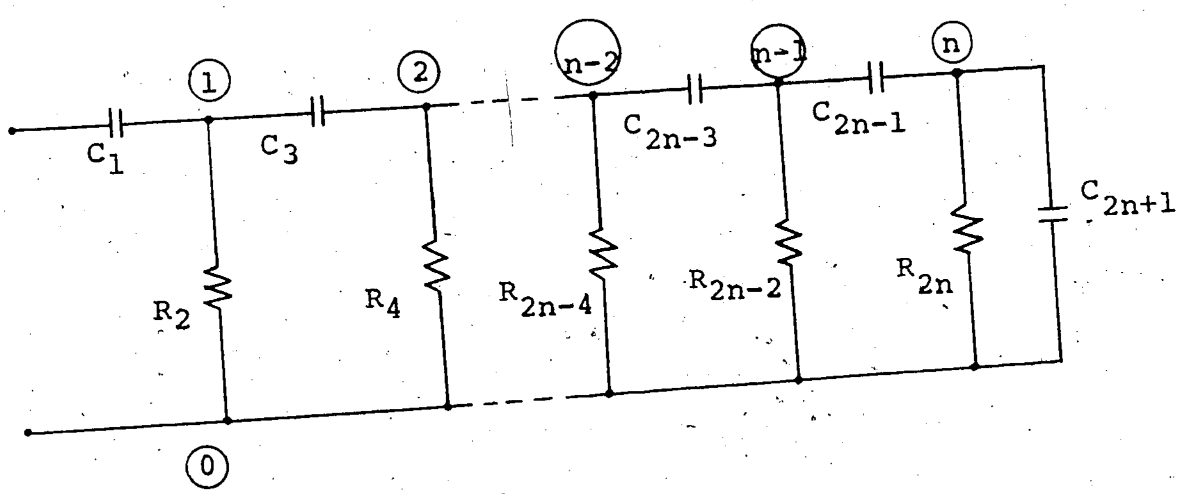

Figure 2.9 A capacitance-resistance ladder.

shown in figure 2.9 are that the term $s[(v_{a-1}/v_a)-1]$ and hence the term $s[(v_{a+1}/v_a)-1]$ must be frequency-dependent for all values of a.

As in the case of R-C ladder,

for node n:

$$s\left(\frac{v_{n-1}}{v_n} - 1\right) = \frac{C_{2n+1}s^2 + g_{2n}s}{C_{2n-1}s} \qquad (2.45)$$

node n-1:

$$s\left(\frac{v_n}{v_{n-1}} - 1\right) = -\frac{C_{2n+1}s^2 + g_{2n}s}{(C_{2n+1}+C_{2n-1})s + g_2} \qquad (2.46)$$

for node n-2:

$$s\left(\frac{v_{n-1}}{v_{n-2}} - 1\right) = -\frac{a_2 s^3 + b_2 s^2 + c_2 s}{(d_2 + a_2)\, s^2 + (b_2+1)s + c_2} \qquad (2.47)$$

where

$$a_2 = C_{2n+1}C_{2n-1}$$

$$b_2 = g_{2n-2}(C_{2n+1}+C_{2n-1}) + g_{2n}C_{2n-1}$$

$$c_2 = g_{2n}g_{2n-2}$$

and $d_2 = (C_{2n+1}+C_{2n-1})/g_{2n}$

In general, for the node n-i ($i \neq 0$):

$$s\left(\frac{v_{(n-i)+1}}{v_{n-i}} - 1 \right) = - \frac{a_i s^{i+1} + b_i s^{i} + \ldots + z_i s}{\alpha_i s^{i} + \beta_i s^{i-1} + \ldots + \gamma_i s + \delta_i} \qquad (2.48)$$

where a_i, b_i, . . . , z_i, α_i, β_i, . . . , δ_i are all non-zero positive quantities. It is clear from the above equation that the term

$$s\left(\frac{v_{(n-i)+1}}{v_{n-i}} - 1 \right)$$

is always frequency-dependent. From equation (2.45) , it is seen that the term for the output node is also frequency-dependent. The matrix V with such a ladder is, thus, non-singular.

With the same approach , we have been able to show, by routine procedures, that the V-matrix for an L-C ladder, an R-L ladder, as well as for any combination of L-C, R-C, C-R, R-L, and resistive ladders is always nonsingular.

2.5 Examples and experimental results

For the verification of the measurement method by computational experiments, the examples considered are given below.

2.5.1 Example 1

(a) The network:

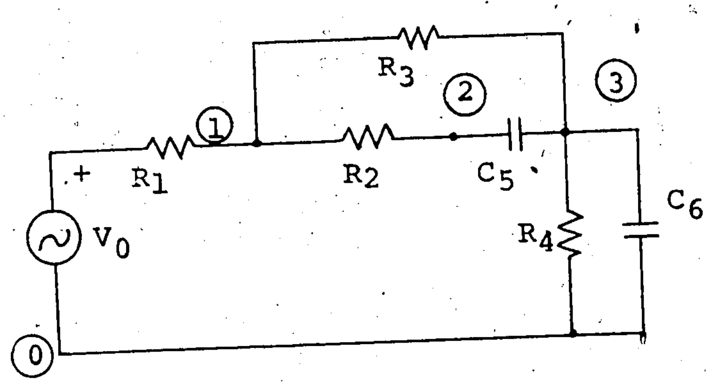

Figure 2.10 The network of Example 1.

(b) The diagraph:

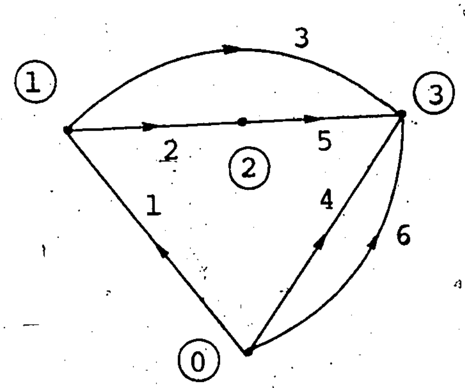

Figure 2.11 Diagraph of the network of Example 1.

(c) The matrix A_1:

$$\begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 & -1 \end{bmatrix}$$

(d) The matrix E:

With $R_1 = R_2 = R_3 = R_4 = 1\ \Omega$, $C_5 = C_6 = 1$ F., the network was solved for 10 different combinations of the node equations. It was found that, so long as the number of node equations taken at a particular node does not exceed the (degree-1) of that node, the element values calculated by the present method are, in fact, the same as the actual element values. However, when the number of node equations at any node exceeds the (degree-1) of that node, the V-matrix was found to be singular as expected.

2.5.2 Example 2

(a) The network: (A twin-T network.)

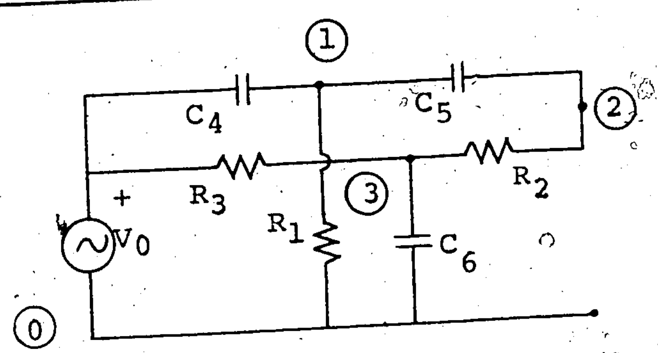

Figure 2.12 Twin-T network.

The network after the E-shift:

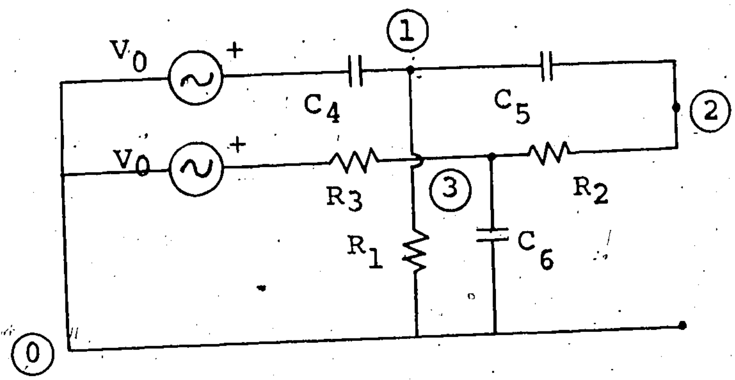

Figure 2.13 The network of Example 2 after the E-shift.

(b) The diagraph:

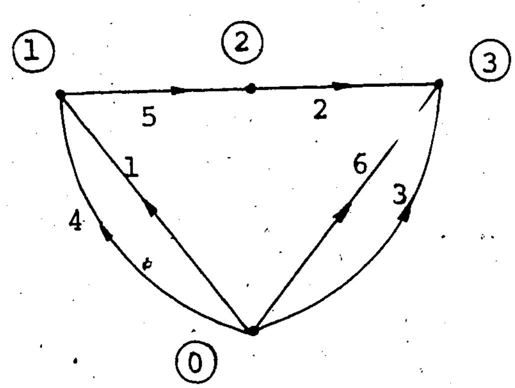

Figure 2.14 Diagraph of the Twin-T network.

(c) The A_1-matrix:

$$\begin{bmatrix} -1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 & -1 \end{bmatrix}$$

(d) <u>The E-matrix</u>:

$$[\ 0 \quad 0 \quad V_0 \quad V_0 \quad 0 \quad 0\ ]^T$$

(e) <u>The results</u>:

Assuming C_6 as the known element, the values of the remaining elements were calculated with 40 different combinations of element values and node equations. The calculated element values were found to be the same as the actual element values in all but the following cases:

(i) The V-matrix was found to be singular when the node equation of any node was taken at a number of frequencies greater than the (degree - 1) of the node.

(ii) When the node equation of node 2 was taken at the critical frequency of the network (the critical frequency of the twin-T network is the frequency at which a pole and a zero of the network coincide resulting in their cancellation). At the critical frequency, all the node voltages were found to be same. Thus, with the node equation of node 2 taken at this frequency, the row of the resulting V-matrix corresponding to node 2 became zero with the consequence that the V-matrix became singular.

For the measurement of the twin-T network, the node equation of node 2 must not be taken at the critical frequency.

2.5.3 Example 3

(a) The network:

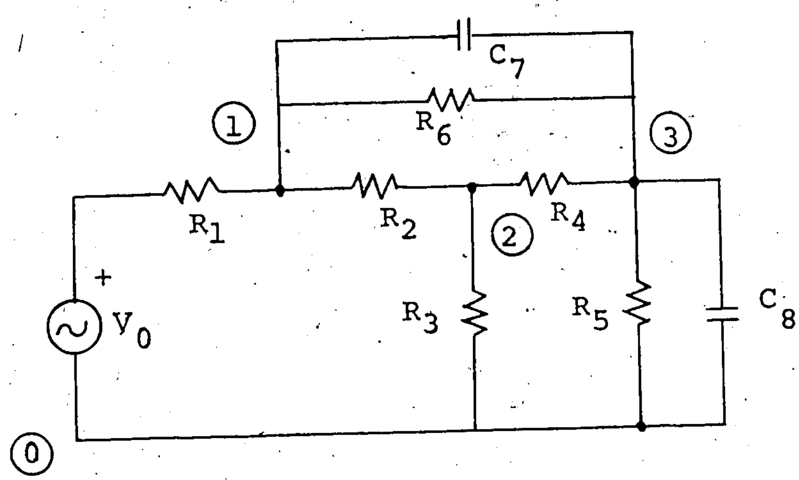

Figure 2.15 The network of Example 3.

(b) The diagraph:

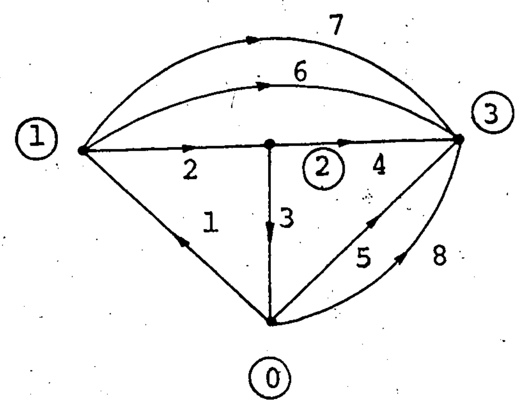

Figure 2.16 Diagraph of the network of Example 3.

(c) <u>The A_1-matrix:</u>

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & \text{[illegible]} & \text{[illegible]} & 1 & -1 & -1 & -1 \end{bmatrix}$$

(d) <u>The E-matrix:</u>

$$[\, V_0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,]^T$$

(e) <u>The results:</u>

With $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 3\ \Omega$, $R_4 = 2\ \Omega$, $R_5 = 1\ \Omega$, $R_6 = 7\ \Omega$, $C_7 = 7$ F., and $C_8 = 2$ F., the element values were computed for 8 different combinations of the node equations and of the measuring frequencies. The calculated element values coincided with the actual ones in all the combinations except when the node equation at node 3 was taken at four frequencies when the V-matrix became singular. The V-matrix with the node equation of node 3 taken at four different frequencies and with $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 1\,\Omega$, $C_7 = C_8 = 2$F., the V-matrix was seen to be singular irrespective of the combination of the node equations and the measuring frequencies. This is due to the fact that since with these element values, $R_5\, C_8 = R_6\, C_7$, the voltage of the node 3 does not change with frequency making the V-matrix singular.

It may also be mentioned that, although the degree of node 3 is five, the node equation at that node cannot be taken at more than three frequencies.

2.5.4 Example 4

(a) The network:

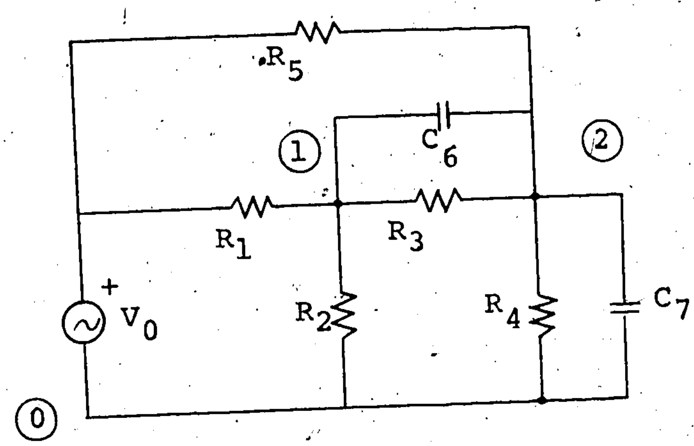

Figure 2.17 The network of Example 4.

The network after the E-shift:

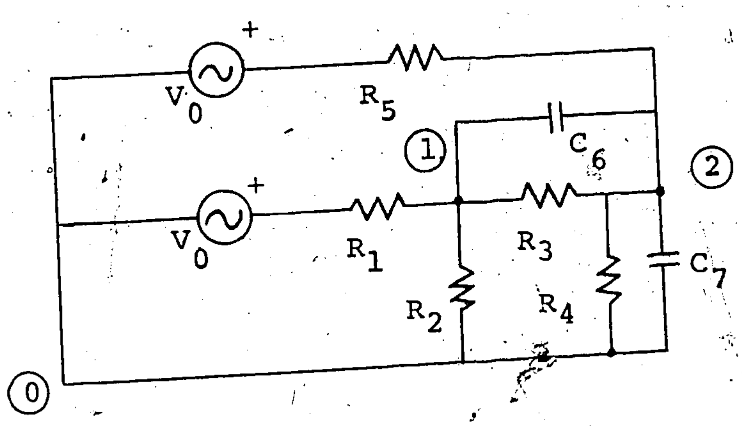

Figure 2.18 The network of Example 4 after the E-shift.

(b) The diagraph:

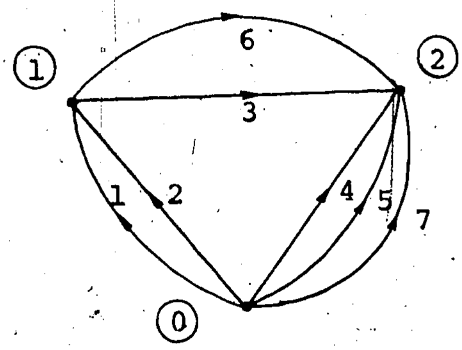

Figure 2.19 Diagraph of the network of Example 4.

(c) The A_1-matrix:

$$\begin{bmatrix} -1 & -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

(d) The E-matrix:

$$[\ V_0\ 0\ 0\ 0\ V_0\ 0\ 0\ ]^T$$

(e) The results:

The elements of the network were computed with $R_1 = 1\ \Omega$, $R_2 = 2\Omega$, $R_3 = R_4 = 1\Omega$, $C_6 = 1F.$, and $C_7 = 7F.$ 12 different combinations of the node equations and measuring frequencies were taken. The calculated results showed close agreement with the actual element values in all but the following cases:

(i) When the number of node equations at the node 1 exceeds the (degree-1) of that node,

(ii) when the number of node equations at the node 2 exceeds three, though its degree is five.

2.5.5 Example 5

2.5.5.1 Example 5.1

(a) The network:

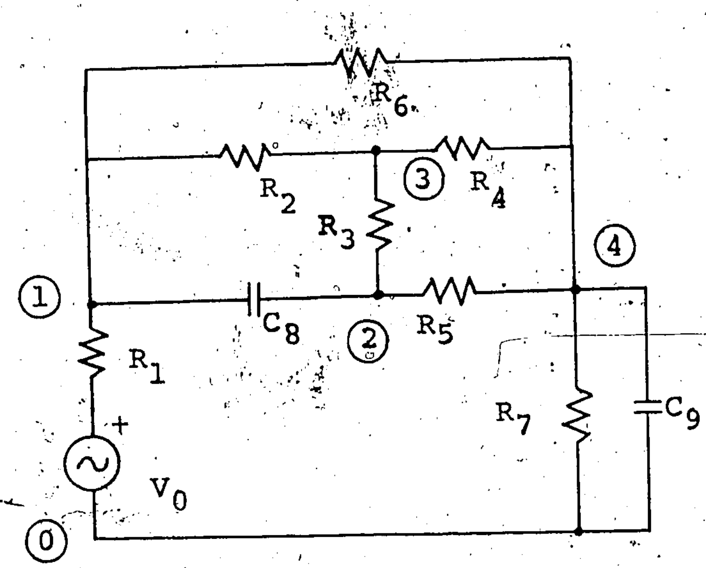

Figure 2.20 The network of Example 5.1.

(b) The diagraph:

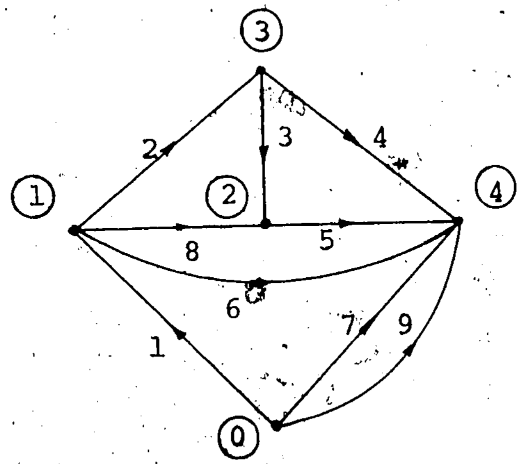

Figure 2.21 Diagraph of the network of Example 5.1.

(c) The A_1-matrix:

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 & -1 & 0 & -1 \end{bmatrix}$$

(d) The E-matrix:

$$[\, V_0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,]^T$$

(e) The results:

With $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = 1\ \Omega$, $C_8 = 2$ F., $C_9 = 1$ F., and with all the node equations taken at two frequencies $s_1 = 2$ rad/sec, and $s_2 = 5$ rad/sec, the computed element values were accurately the same as the actual ones. Next, with $R_1 = 1\ \Omega$, $R_2 = 2\Omega$, $R_3 = 5\Omega$, $R_4 = 3\ \Omega$, $R_5 = 7\Omega$, $R_6 = 4\Omega$, $R_7 = 2\Omega$, $C_8 = 3$ F., and $C_9 = 8$ F., 8 different combinations of node equations and measuring frequencies were considered. Another 8 combinations were taken with the same resistance values and with $C_8 = 0.2$ F., and $C_9 = 0.25$ F. The element values computed agreed accurately in all the combinations except the following cases where the V-matrix was found to be singular:

(i) when the node equation of node 1 was taken at three or more frequencies,

(ii) when the number of equations of node 4 exceeded three.

It is, thus, seen that the node equation of node 1 cannot be considered at more than two, and that of node 4 at more than three frequencies, though the degrees of the nodes are three and four respectively.

2.5.5.2 <u>Example 5.2</u>

(a) The network: The network is the same as that of Example 5.1 except that the positions of the resistance R_5 and the capacitance C_8 have been interchanged.

(b) The results:

With $R_1 = 1\Omega$, $R_2 = 3.1\Omega$, $R_3 = 6\ \Omega$, $R_4 = 2.2\ \Omega$, $R_5 = 5.4\ \Omega$, $R_6 = 1.4\Omega$, $R_7 = 4\ \Omega$, $C_8 = 1.5$ F., and $C_9 = 2.7$ F., six different combinations of node equations and the frequencies were taken. The calculated results were found to be correct in all combinations except when the node equation of node 1 and that of node 4 were taken at more than two and three frequencies respectively.

2.5.6 Example 6

(a) The network:

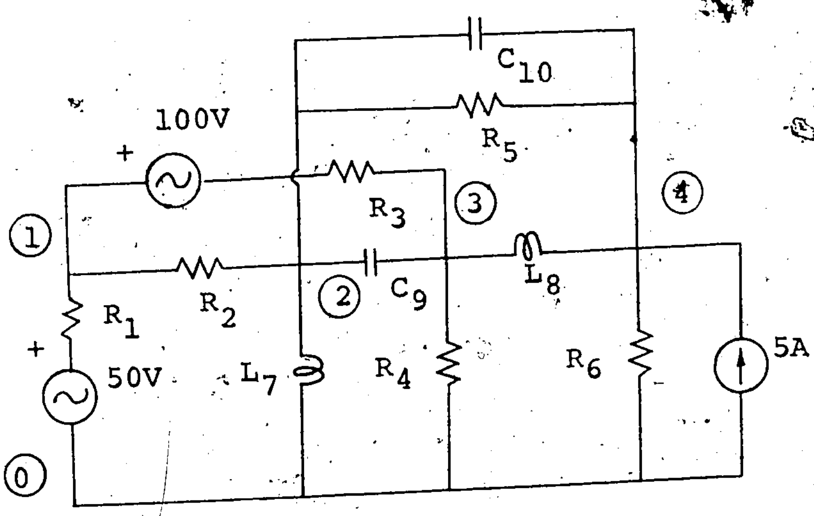

Figure 2.22 The network of Example 6.

(b) The diagraph: (After converting the current source into the equivalent voltage source.)

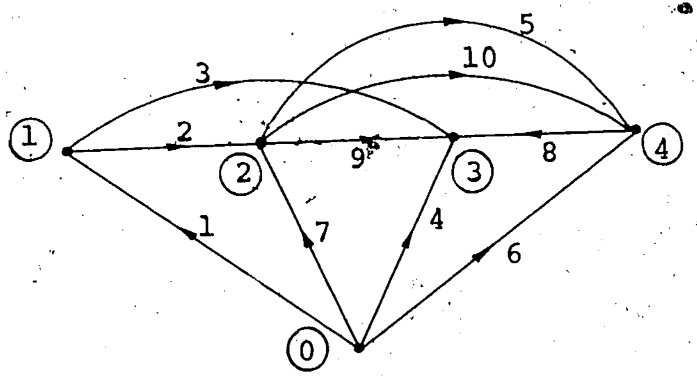

Figure 2.23 Diagraph of the network of Example [illegible].

(c) <u>The A_1-matrix:</u>

$$\begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & 0 & -1 \end{bmatrix}$$

(d) <u>The E-matrix:</u>

$$[\; 50 \quad 0 \quad -100 \quad 0 \quad 0 \quad 50 \quad 0 \quad 0 \quad 0 \quad 0 \;]^T$$

(e) <u>The results:</u>

The network was investigated with $R_1 = 2\ \Omega$,

```
V MATRIX:      S1=  1   S2=  3   S3=  5
                                                          0.000 00 0.000 00 0.000 00 0.000 00
-0.970 01-0.600 02 0.390 02 0.000 00 0.000 00 0.000 00 0.110 00 0.000 00-0.140 01
 0.000 00 0.000 02 0.000 00 0.000 00 0.200 00 0.000 00 0.000 00 0.160 01 0.140 01
 0.000 00 0.000 00-0.700 02 0.110 01 0.000 00 0.500 02 0.300 00-0.160 01 0.000 00
 0.000 00 0.000 00 0.700 00 0.000 00-0.260 00 0.500 00-0.750-02 0.000 00-0.340 01
 0.000 00 0.000 00 0.700 00 0.000 00 0.000 00 0.000 00-0.200-01 0.000 00-0.270 01
 0.000 00 0.000 02 0.000 00 0.000 00 0.500 00 0.500 00-0.290-01 0.000 00 0.270 01
 0.000 00 0.000 02 0.000 00 0.000 00 0.000 00 0.000 00 0.000 00 0.630 00 0.000 00
 0.000 00 0.000 00-0.390 02 0.830 00 0.000 00 0.400 02 0.000 00-0.630 00 0.000 00
 0.000 00 0.000 00 0.000 00 0.000 00-0.900 00 0.000 00-0.130-01 0.000 00-0.320 01
 0.000 00-0.600 02 0.000 00 0.000 00 0.800 00 0.000

INVERSE OF THE MATRIX V:
                                                    0.270 01  0.200 01 -0.230 00 -0.130 00 -0.470 01
-0.100 00 -0.140 00  0.190 00  0.120 00  0.110 01  0.740 00 -0.730-16 -0.730-16 -0.170 01
-0.000 00 -0.550-01  0.730-16  0.970-16  0.230 01  0.160 01 -0.710-01 -0.310-01 -0.380 01
-0.000 00 -0.120 00  0.460-01  0.310-01  0.450 02  0.410 02 -0.320 01 -0.230 01 -0.100 03
-0.000 00 -0.270 01  0.270 01  0.220 01 -0.110 02 -0.450 01  0.470-15  0.420-15  0.150 02
-0.000 00 -0.620-01 -0.420-15 -0.610-15 -0.330 00 -0.130 00  0.150-16  0.140-01  0.460 00
-0.000 00  0.190-01 -0.150-15 -0.110-01  0.370 03  0.250 01 -0.240-13 -0.240-13 -0.590 03
-0.000 00 -0.780 00  0.240-13  0.330-13 -0.830 01 -0.340 01  0.200-15  0.100 01  0.120 02
 0.000 00  0.470-01 -0.480-15 -0.100 01  0.160 02  0.120 02 -0.110-14 -0.110-14 -0.270 02
-0.000 00 -0.390 00  0.110-14  0.150-14

DET AV=  -0.82178800 03

COMPONENT VALUES(ACTUAL)                     COMPONENT VALUES    (CALCULATED)

S1=  1   S2=  3   S3=  5   S4= 7

         0.50000000                           0.50000000
         0.05000000                           0.05000000
         0.20000000                           0.20000000
         0.06666667                           0.06666667
         0.10000000                           0.10000000
         0.10000000                           0.10000000
         2.00000000                           2.00000000
         2.85714286                           2.85714286
         2.20000000                           2.20000000
```

Figure 2.24 The computed element values of Example 6 with the node equation of node 1 taken at one, those of nodes 2, 3, and 4 taken at four, two, and two frequencies respectively.

$R_2 = 20\ \Omega$, $R_3 = 5\ \Omega$, $R_4 = 15\ \Omega$, $R_5 = 10\ \Omega$, $R_6 = 10\ \Omega$, $L_7 = 0.5$ H., $L_8 = 0.35$ H., $C_9 = 2.2$ F., and $C_{10} = .1$ F. The measuring frequencies in rad/sec were $s_1 = 1$, $s_2 = 3$, $s_3 = 5$, and $s_4 = 7$. Five different combinations of node equations were taken and it was found that the calculated element values agreed accurately with the actual ones, provided the number of node equations of any node does not exceed the (degree -1) of that node. The computed result with the node equation of node 1 taken at one, those of nodes 2, 3, and 4 taken at four, two, and two frequencies respectively are shown in figure 2.24.

2.5.7 Example 7

(a) The network:

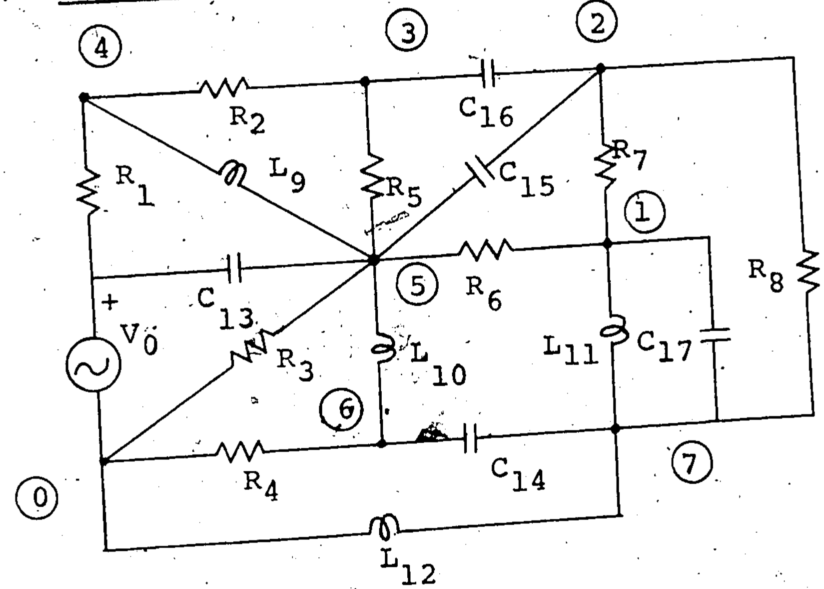

Figure 2.25 The network of Example 7.

The network after the E-shift:

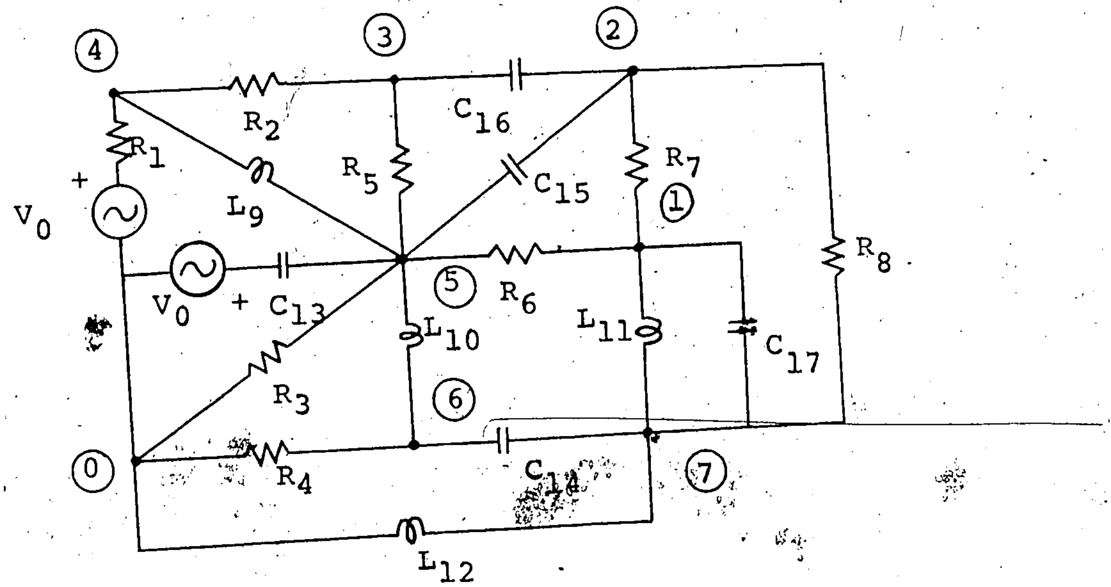

Figure 2.26 The network of Example 7 after the E-shift.

(b) The diagraph:

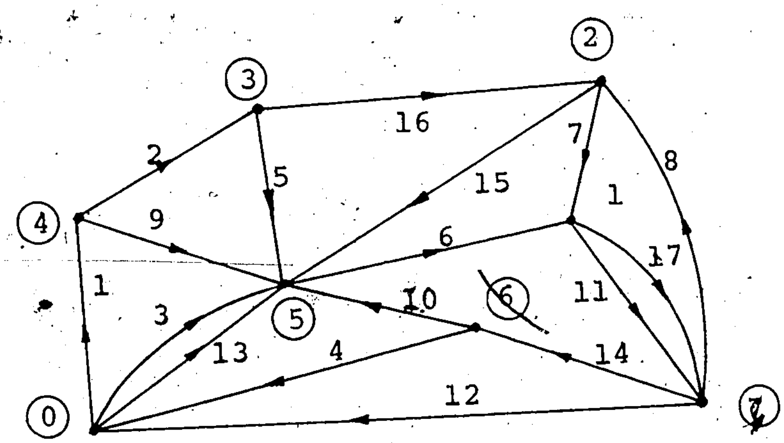

Figure 2.27 Diagraph of the network of Example 7.

(c) The results:

Eight different combinations of the node equations and the measuring frequencies were considered for the network with $R_1 = 1\ \Omega$, $R_2 = 3.1\ \Omega$, $R_3 = 1.9\Omega$, $R_4 = 2.5\ \Omega$, $R_5 = 0.5\ \Omega$, $R_6 = 1.6\ \Omega$, $R_7 = 1.24\Omega$, $R_8 = 4.1\Omega$, $L_9 = 2$ H., $L_{10} = 1.2$ H., $L_{11} = 3.4$ H., $L_{12} = 0.6$ H., $C_{13} = 0.24$ F., $C_{14} = 6.1$ F., $C_{15} = 4.3$ F., $C_{16} = 0.71$ F., and $C_{17} = 1.7$ F. The element values obtained by the method were accurately the same as the actual element values. It was found that for all the nodes, the number of node equations that may be taken without making the V-matrix singular is the (degree - 1) of the node.

2.5.8 Example 8

(a) The network:

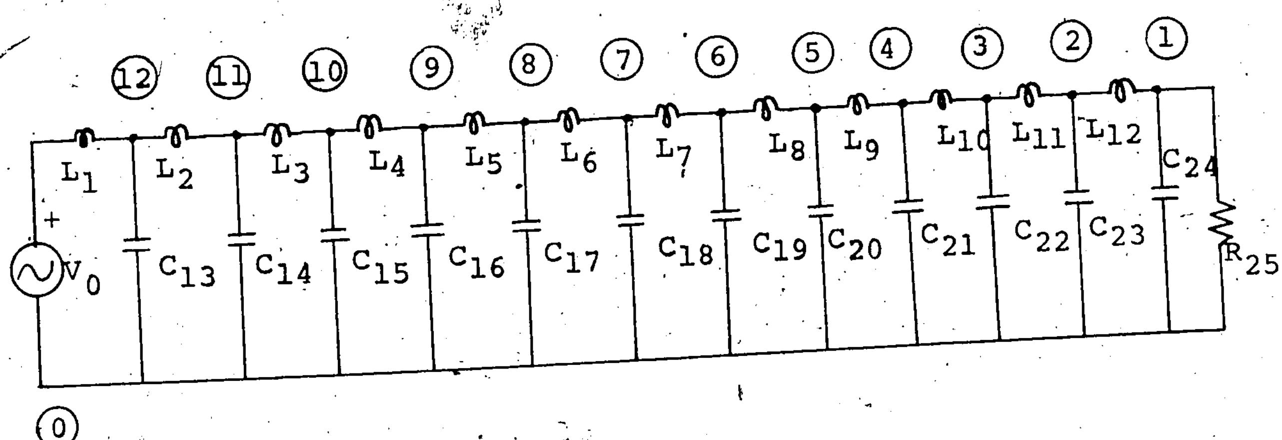

Figure 2.28 The ladder network of Example 8.

(b) The diagraph:

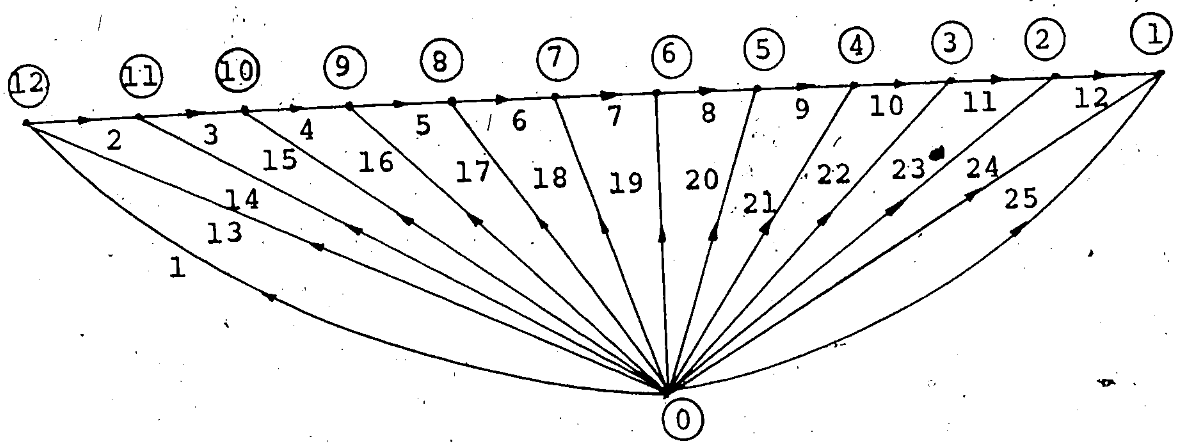

Figure 2.29 Diagraph of the network of Example 8.

(c) The results:

With $L_1 = 1$ H., $L_2 = 1.05$ H., $L_3 = 1.1$ H., $L_4 = 1.15$ H., $L_5 = 1.2$ H., $L_6 = 1.25$ H., $L_7 = 1.3$ H., $L_8 = 1.35$ H., $L_9 = 1.4$ H., $L_{10} = 1.45$ H., $L_{11} = 1.5$ H., $L_{12} = 1.55$ H., $C_{13} = 1$ F., $C_{14} = 0.95$ F., $C_{15} = 0.9$ F., $C_{16} = 0.85$ F., $C_{17} = 0.8$ F., $C_{18} = 0.75$ F., $C_{19} = 0.7$ F., $C_{20} = 0.65$ F., $C_{21} = 0.6$ F., $C_{22} = 0.55$ F., $C_{23} = 0.5$ F., $C_{24} = 0.45$ F., $R_{25} = 0.125\ \Omega$, and with the measuring frequencies in rad/sec as $s_1 = 1$, and $s_2 = 2$, the element values computed are shown in figure 2.30, which are, in effect, the same as the actual element values.

COMPONENT VALUES(ACTUAL) COMPONENT VALUES (CALCULATED)

COMPONENT MATRIX(CALCULATED): S1= 1 S2= 2

Component values (actual)	Component values (calculated)
1.00000000	1.00000000
0.95238095	0.95238095
0.90909091	0.90909091
0.86956522	0.86956522
0.83333333	0.83333333
0.80000000	0.80000000
0.76923077	0.76923077
0.74074074	0.74074074
0.71428571	0.71428571
0.68965517	0.68965517
0.66666667	0.66666667
0.64516129	0.64516129
1.00000000	1.00000000
0.95000000	0.95000000
0.90000000	0.90000000
0.85000000	0.85000000
0.80000000	0.80000000
0.75000000	0.75000000
0.70000000	0.70000000
0.65000000	0.65000000
0.60000000	0.60000000
0.55000000	0.55000000
0.50000000	0.50000000
0.45000000	0.45000000

Figure 2.30 The computed element values of Example 8 with the node equations of all the nodes taken at two frequencies each.

2.5.9 Example 9

(a) The network:

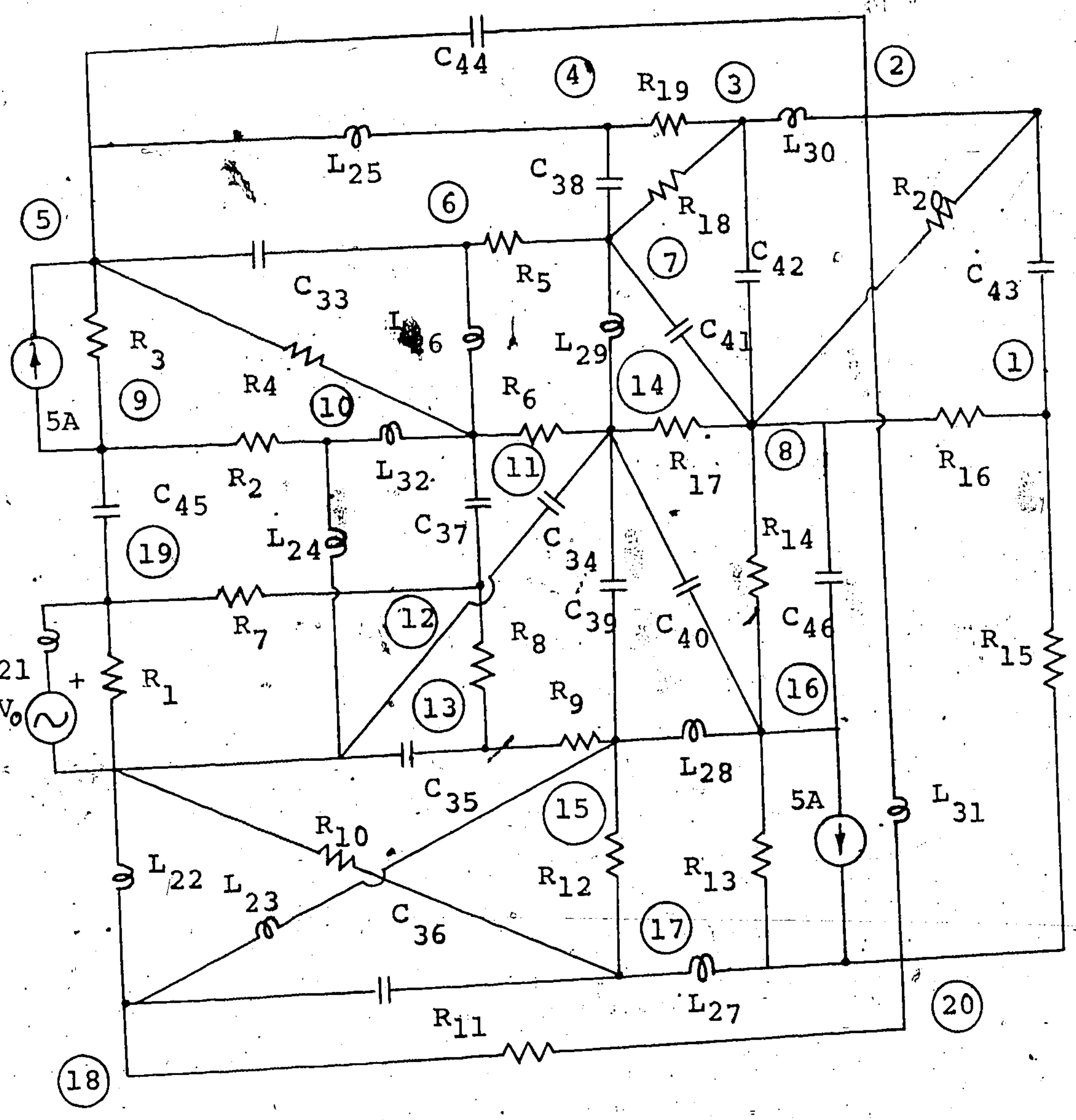

Figure 2.31 The network of Example 9.

(b) The diagraph: The diagraph, after converting the current sources into equivalent voltage sources, are shown in figure 2.32.

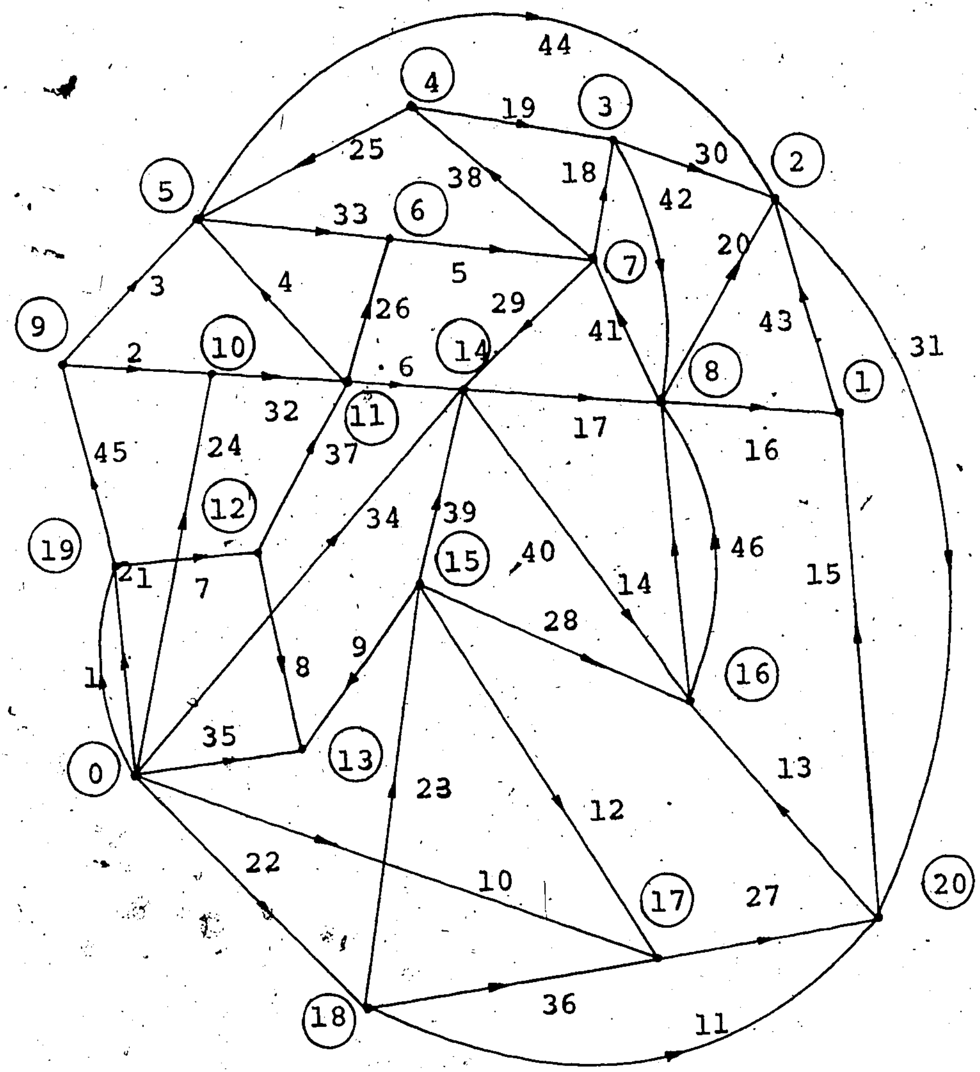

Figure 2.32 Diagraph of the network of Example 9.

(c) <u>The results:</u>

Twelve different combinations of node equations and measuring frequencies were considered to investigate the network with the following element values:

$R_1 = 12\ \Omega$, $R_2 = 1.2\ \Omega$, $R_3 = 10\ \Omega$, $R_4 = 2.1\Omega$, $R_5 = 3\ \Omega$, $R_6 = 5\ \Omega$, $R_7 = 7.1\Omega$, $R_8 = 1\Omega$, $R_9 = 4\Omega$, $R_{10} = 3.2\Omega$, $R_{11} = 1\Omega$, $R_{12} = 5\Omega$, $R_{13} = 5\Omega$, $R_{14} = 1.5\ \Omega$, $R_{16} = 20\Omega$, $R_{17} = 2.5\Omega$, $R_{18} = 6\Omega$, $R_{19} = 1.6\Omega$, $R_{20} = 4\Omega$, $L_{21} = 1$ H., $L_{22} = 0.8$ H., $L_{23} = 0.5$ H., $L_{24} = 0.32$ H., $L_{25} = 0.75$ H., $L_{26} =$ [illegible] H., $L_{27} = 0.5$ H., $L_{28} = 0.4$ H., $L_{29} = 0.6$ H., $L_{30} = 1$ H., $L_{31} = 0.2$ H., $L_{32} = 0.25$ H., $C_{33} = 1$ F., $C_{34} = 5$ F., $C_{35} = 2.5$ F., $C_{36} = 1.2$ F., $C_{37} = 6$ F., $C_{38} = 2.2$ F., $C_{39} = 1.25$ F., $C_{40} = 3.2$ F., $C_{41} = 1$ F., $C_{42} = 2.5$ F., $C_{43} = 3$ F., $C_{44} = 1.8$ F., $C_{45} = 5.2$ F., and $C_{46} = 1$ F.

It was found that when the equation of node 19 was taken at more than two frequencies, the V-matrix became singular, although the degree of the node is four. For all other cases, the calculated element values were found to be accurately the same as the actual values. It was also found that ,except for node 19, the node equations of all the nodes may be taken at a maximum of the (degree - 1) different frequencies without making the V-matrix singular.

2.6 Singular network

In the examples of the previous Section, we have seen that all the networks can be solved by the addition of one known, external element to the network. But there are some networks which cannot be solved by the addition of a single external element because the V-matrix of the network is singular irrespective of the element values and the measuring frequencies. Such a network may be termed a "singular network". For the solution of such a network, more than one known, external element are required to be added to the network. Such a network is shown in figure 2.33.

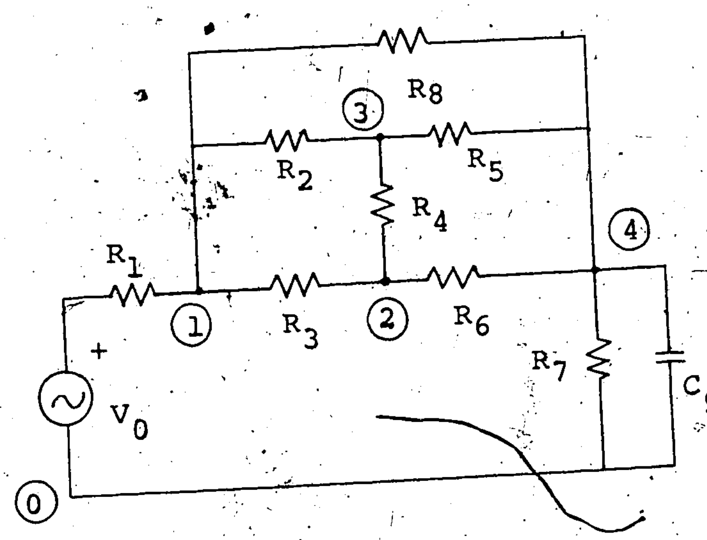

Figure 2.33 A singular network.

With the capacitance C_9 as the added element, an attempt was made to solve the network by the different sets of element values and with different combinations of the measuring frequencies. In all the cases, the V-matrix was found to be singular. However, the network became solvable as soon as a second external capacitance C_{10} was added in between the datum node and the node 2 or node 3, when the V-matrix became nonsingular.

A second example of a singular network is shown in figure 2.34. With the capacitance C_{18} as the added element, the V-matrix of the network was found to be

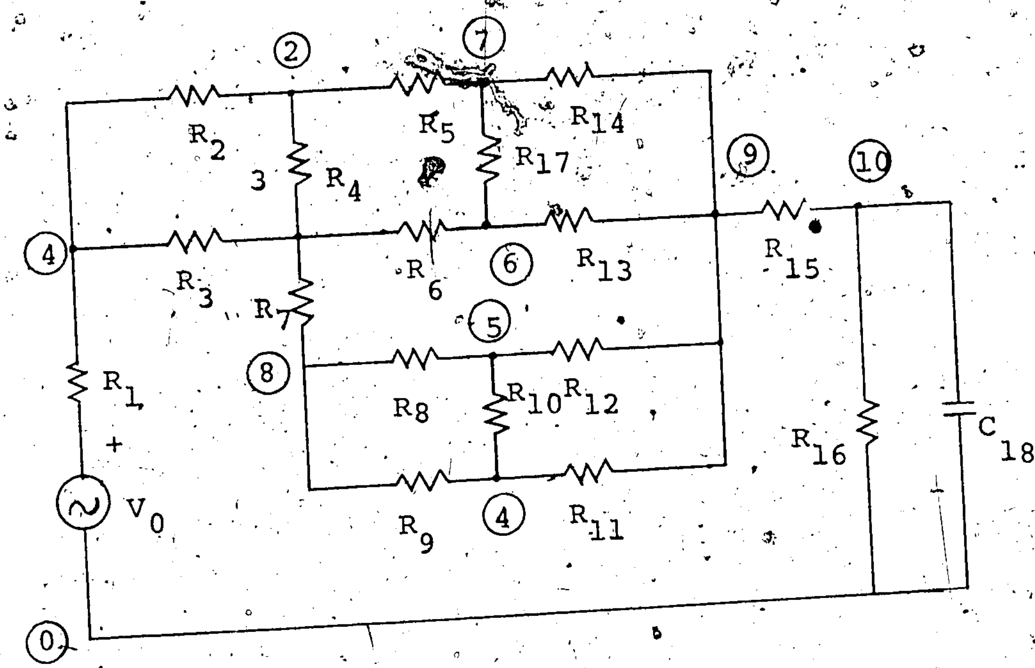

Figure 2.34 A singular network.

singular irrespective of the element values and the measuring frequency. However, with an additional capacitance, C_{19} connected between the node 4 and the datum node, the V-matrix became nonsingular and the network became solvable.

The above examples demonstrate that a singular network may be made solvable by the incorporation of some external elements between suitable nodes.

CHAPTER 3

EXPLICIT SOLUTION OF NETWORK ELEMENTS BY THE METHOD OF PARTITIONING

The method for the solution of network elements described in Chapter 2 requires that n simulteneous equations be solved, where n is the total number of network elements. As a consequence, the method appears to be inconvenient for the solution of large networks, which involves the inversion of a large V-matrix. It is the purpose of this Chapter to present an alternative method that avoids the complexity of handling big matrices. In the present method, only one node of the network is considered at a time and consequently the method involves the inversion of matrices the order of none of which exceeds the (degree - 1) of the node concerned. In addition to the simplicity of this measurement procedure, the method has the advantage that it may be used to solve only a part of a network without solving the whole network and is especially attractive to the cases where only a specific portion of a big network is required to solve. The measurement method and the necessary algorithm for the computer program will be described with a few examples.

3.1 The solution method

The solution method consists of considering a particular node at a time and writing the node equation of that node at a number of frequencies equal to the number of unknown elements at that node. The solution of this set of node equations gives the unknown elements of that node. Let d be the degree of a node and x be the number of known elements incident on it. For the solution of the $(d-x)$ unknown elements of the node, the node equation at that node must be considered at $(d-x)$ different frequencies so that the set of equations at that node is given by

$$\begin{bmatrix} x_{11}v_{b_1 1} & x_{21}v_{b_2 1} & \cdots & x_{k1}v_{b_k 1} \\ x_{12}v_{b_1 2} & x_{22}v_{b_2 2} & \cdots & x_{k2}v_{b_k 2} \\ \vdots & & \vdots & \\ x_{1k}v_{b_1 k} & x_{2k}v_{b_2 k} & \cdots & x_{kk}v_{b_k k} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix} = - \begin{bmatrix} \sum_{i=1}^{x} x_{i1}v'_{b_i 1} \\ \sum_{i=1}^{x} x_{i2}v'_{b_i 2} \\ \vdots \\ \sum_{i=1}^{x} x_{ik}v'_{b_i k} \end{bmatrix} \qquad (3.1)$$

where $k = (d-x)$, b_i is the value of the i-th unknown element with a voltage $v_{b_i j}$ across it occuring at a frequency j, and $v'_{b_i j}$ is the voltage across the i-th known element at the frequency j.

This set of equations may be written, in compact form, as

$$V_b C = F_b \tag{3.2}$$

where V_b is the branch voltage matrix, F_b is the forcing function vector of the node and these are given by

$$V_b = \begin{bmatrix} x_{11}v_{b_1 1} & x_{21}v_{b_2 1} & \cdots & x_{k1}v_{b_k 1} \\ x_{12}v_{b_1 2} & x_{22}v_{b_2 2} & \cdots & x_{k2}v_{b_k 2} \\ \vdots & & & \vdots \\ x_{1k}v_{b_1 k} & x_{2k}v_{b_2 k} & \cdots & x_{kk}v_{b_k k} \end{bmatrix} \tag{3.3a}$$

$$F_b = - \begin{bmatrix} \sum_{i=1}^{x} x_{i1}v'_{b_i 1} \\ \sum_{i=1}^{x} x_{i2}v'_{b_i 2} \\ \vdots \\ \sum_{i=1}^{x} x_{ik}v'_{b_i k} \end{bmatrix} \tag{3.3b}$$

and C is the component vector to be determined.

From equation (3.2), the component vector may be obtained as

$$C = V_b^{-1} \; F_b \qquad (3.[illegible])$$

provided that the branch voltage matrix V_b is nonsingular.

Since the number of different frequencies at which the node equation may be considered at any node cannot exceed the (degree-1) of that node, it is essential that at least one of the elements incident on the node must be known. For this purpose one known element is added to the node that is to be considered first. Once the elements of that node have been solved, a new node, which is directly connected to the first node, is then considered. The elements of such a node can now be solved since one or more elements connecting this node with the previously-solved node are now known. Following this, a new node, which is directly connected to this node or to any previously-solved node is considered since it has one or more elements known. This process may be continued until all the elements have been found out.

To illustrate the measurement procedure, consider the diagraph of an electrical network shown in figure 3.1. Node 1 is assumed such that when a known, external element is embedded between it and another suitable node,

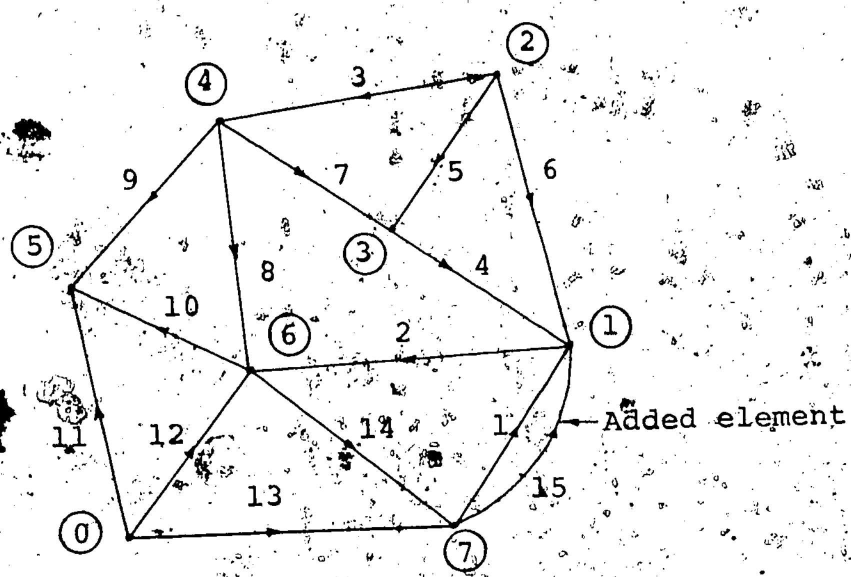

Figure 3.1 The diagraph of an electrical network.

the node equations of this node at four different frequencies are linearly independent. The known, added element is represented by the branch 15. The node equation of node 1 at four different frequencies may be expressed as

$$x_{11}{}^{v}b_{1}1^{b_{1}}+x_{21}{}^{v}b_{2}1^{b_{2}}+x_{41}{}^{v}b_{4}1^{b_{4}}+x_{61}{}^{v}b_{6}1^{b_{6}} = -x_{15,1}{}^{v}b_{15}1^{b_{15}}$$

$$x_{12}{}^{v}b_{1}2^{b_{1}}+x_{22}{}^{v}b_{2}2^{b_{2}}+x_{42}{}^{v}b_{4}2^{b_{4}}+x_{62}{}^{v}b_{6}2^{b_{6}} = -x_{15,2}{}^{v}b_{15}2^{b_{15}}$$

$$x_{13}{}^{v}b_{1}3^{b_{1}}+x_{23}{}^{v}b_{2}3^{b_{2}}+x_{43}{}^{v}b_{4}3^{b_{4}}+x_{63}{}^{v}b_{6}3^{b_{6}} = -x_{15,3}{}^{v}b_{15}3^{b_{15}}$$

$$x_{14}{}^{v}b_{1}4^{b_{1}}+x_{24}{}^{v}b_{2}4^{b_{2}}+x_{44}{}^{v}b_{4}4^{b_{4}}+x_{64}{}^{v}b_{6}4^{b_{6}} = -x_{15,4}{}^{v}b_{15}4^{b_{15}}$$

(3.5)

Since the element b_{15} is known, this set of equations can be used to solve the values of the elements b_1, b_2, b_4, and b_6. Once these elements are known, any of the nodes 2, 3, 6, and 7 may next be considered. It may be seen that for the node 7, two elements, elements 1 and 5, are known and as a result, the number of different frequencies at which the node equation at the node is to be considered is two. For each of the nodes 2, 3, and 6, only one element is known and as a consequence, the number of different frequencies at which the node equation is to be considered is 2, 2, and 4 respectively. Assuming that the node equation of node 2 may be considered at two different frequencies, node 2 is considered next. This gives the values of the elements 3 and 5. Node 3 is then considered which needs measurement at one frequency only and the value of the element 7 is known. In this way, we consider the nodes 4, 5, and 6 respectively. It may be seen that in that sequence of nodes, the number of different frequencies at which the node equations are to be considered at these nodes is two in each case.

3.2 Criteria for the choice of the node sequence

A network may be solved using a different sequence of nodes. As for example, an alternative node sequence for the network of figure 3.1 is 1, 7, 6, 5, 4, and 2 (node 3 not taken at all) with the number of frequencies at these nodes being 4, 2, 3, 2, 2, and 1 respectively. The number of frequencies at which the node equation of any node is to be taken depends on the node sequence and by the suitable choice of the node sequence, the number of different frequencies that is to be used for the solution of a network may be minimized. The following are the criteria for the choice of the node sequence:

(i) The starting node must be one at which the node equation may be taken at a number of frequencies equal to the (degree - 1) of that node.

(ii) The nodes, which are to be considered subsequently, must be directly connected to the starting node or to a node (nodes) which has (have) already been considered.

(iii) If x is the maximum number of frequencies at which the node equation of a node with a degree of d may be taken; then for that node to be considered, at least (d-x) elements incident on it must be known. If the number of known elements at that node is less than (d-x), then before considering this node, other nodes, directly-connected to it, must be considered until the number of known elements incident on this node equals or exceeds (d-x).

(iv) The node sequence should be chosen such that the number of different frequencies to be used for the solution of the network elements, is minimum.

3.3 The Flow chart

The computer program should be written such that once a specific node sequence has been indicated, and the node voltages at the required number of frequencies have been measured, the unknown elements of the network can be found along that node sequence. The following are the steps to be taken for such computations:

1. Assign the node sequence by numbering the starting node as node 1, the node to be considered next as node 2, the node to be taken immediately after node 2 as node 3, and so on.
2. Assign the network elements the locations according to the step 2 of Section 2.3 of the previous Chapter. It may be mentioned that the branch voltage representing the added element is to be labelled as the n-th branch, where n is the total number of network elements including the added one.
3. All the branch voltages of the network at the required number of different frequencies are then calculated by the method suggested in the step 3 to step 8 of Section 2.3 of the previous Chapter.
4. The P-matrix and the F-matrix of the network are then obtained by the step 9 to step 12 of Section

2.3 of Chapter 2.

5. The V_b-matrix of the starting node is then formed from the P-matrix and the row of the A-matrix due to the starting node.

6. The forcing function vector for the starting node is then obtained by the use of equation (3.3b).

7. The component matrix is then obtained by inverting the V_b-matrix and multiplying the inverted matrix with the forcing function vector as indicated by the equation (3.4).

8. Modify the A-matrix by reducing the columns corresponding to the elements which are just obtained in the previous step, to zero.

9. Consider the next node, say node k . Form the forcing function vector for this node. The element of the forcing function vector due to a particular frequency is obtained by multiplying all the elements, known so far, by the corresponding elements of the k-th row of the P-matrix at that frequency and then subtracting the product from the element of the k-th row of the F-matrix at that frequency.

10. Form the V_b-matrix of this node from the P-matrix and the k-th row of the modified A-matrix.

11. Repeat the steps 7 to 10 until all the unknown elements have been found.

The details of the above steps are given in the flow chart of figure 3.2.

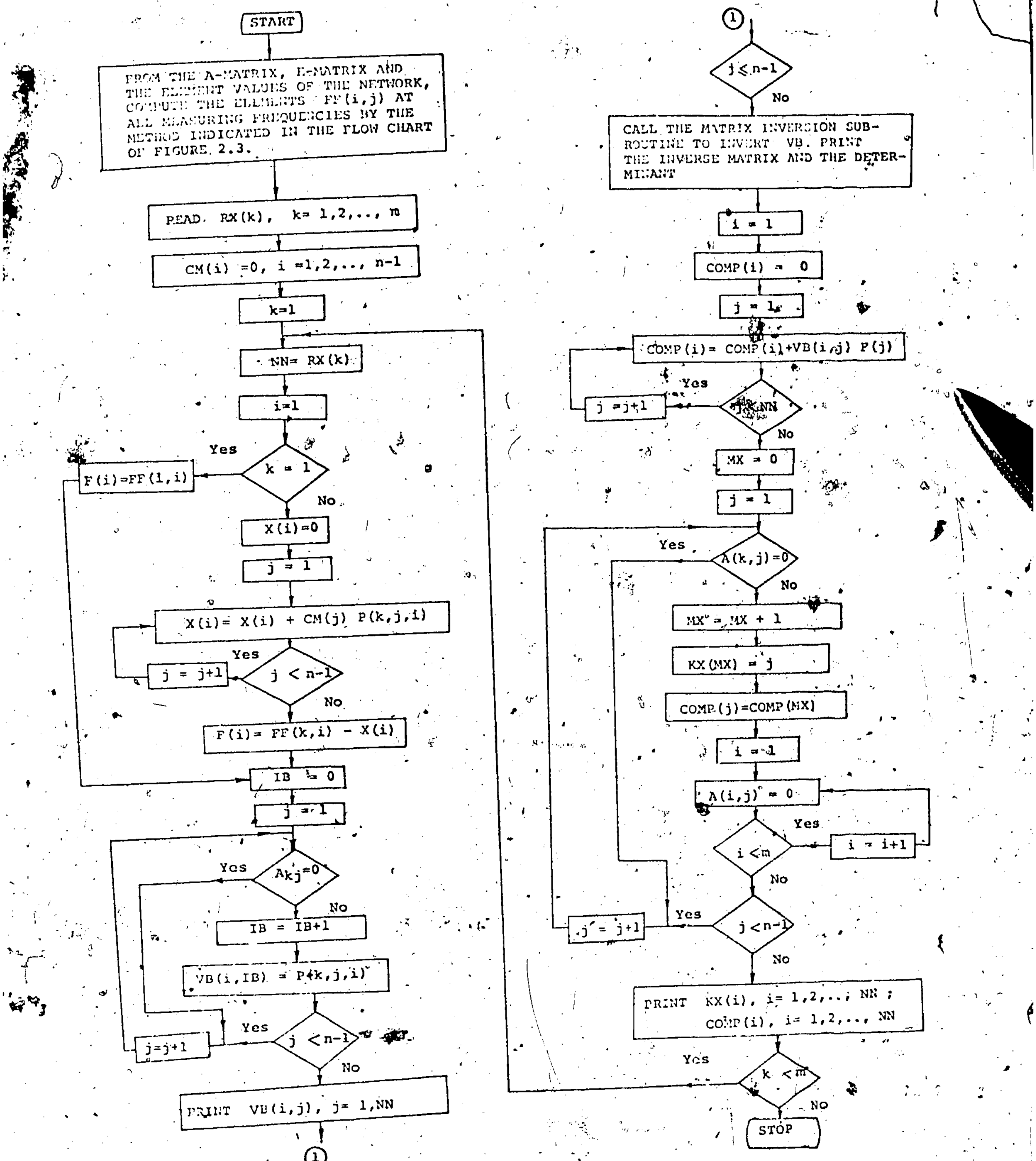

Figure 3.2 The flow chart.

3.4 Examples and experimental results

In order to verify the measurement method, the networks of Examples 7, 8, and 9 of Chapter 2 have been considered and in all the cases, the calculated element values agree with the actual values. Whilst the same network may be solved along different node sequences (except for the ladder network which may be solved by only one node sequence), only one node sequence per network has been chosen for the purpose of the verification of the measurement procedure.

3.4.1 Example 1

(a) The network: The network is the same as that of Example 7 of Chapter 2. C_{17} is the known, added element.

The node sequence: The node sequence followed by the measurement procedure is indicated by the diagraph of figure 3.3.

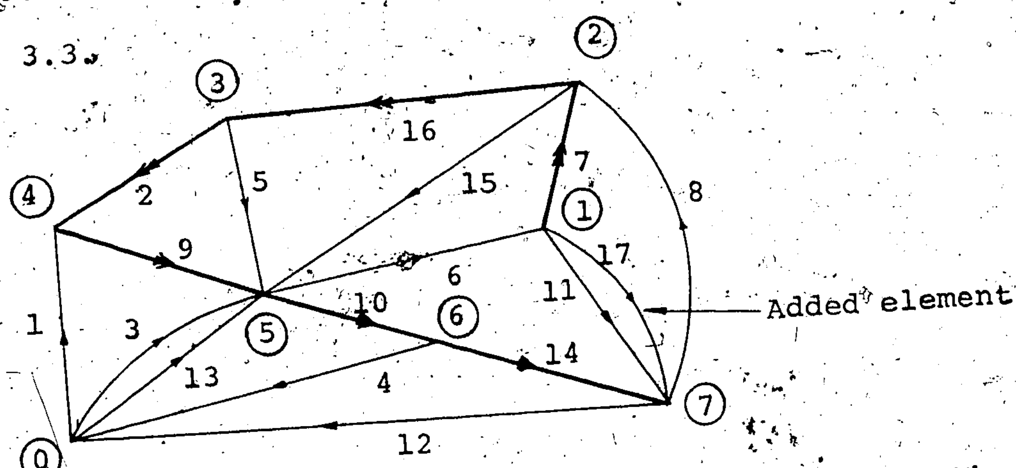

Figure 3.3 The diagraph of the network of Example 1. The thicker line indicates the node sequence.

METHOD OF PARTITIONING: EXAMPLE 1:

COMPONENT VALUES: RESISTANCES: 1.[illegible] 1.10 1.90 2.20 0.50 1.40 1.24 4.10

INDUCTANCES: 2.0[illegible] 1.[illegible] 3.40 0.[illegible]

CAPACITANCES: 0.24 0.10 4.10 0.71 1.70

E MATRIX: -1.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 0.0 0.0

VB MATRIX AT THE NODE 1

18.33045	-21.07126	1.93415
[illegible]	-23.21642	0.24917
21.7[illegible]	-23.90[illegible]	0.08745

COMPONENT VALUES AT THE NODE: 1

	ACTUAL	CALCULATED
ELEMENT: 6	0.71428571	0.71428571
ELEMENT: 7	0.80645161	0.80645161
ELEMENT: 11	0.23411765	0.23411765

VB MATRIX AT THE NODE 2

23.01541	-13.51028	31.28675
23.[illegible]	-17.77711	73.03554
34.[illegible]	-21.67103	95.7253[illegible]

COMPONENT VALUES AT THE NODE: 2

	ACTUAL	CALCULATED
ELEMENT: 8	0.[illegible]44	0.24390244
ELEMENT: 15	4.30000000	4.30000000
ELEMENT: 16	[illegible]1000000	2.71000000

VB MATRIX AT THE NODE 3

24.05804	7.47497
38.42037	19.73078

COMPONENT VALUES AT THE NODE: 3

	ACTUAL	CALCULATED
ELEMENT: 2	-0.32258065	0.32258065
ELEMENT: 5	2.00000000	2.00000000

VB MATRIX AT THE NODE 4

16.05219	-16.58307
15.90060	-6.22986

COMPONENT VALUES AT THE NODE: 4

	ACTUAL	CALCULATED
ELEMENT: 1	1.00000000	1.00000000
ELEMENT: 9	0.50000000	0.50000000

VB MATRIX AT THE NODE 5

-17.36474	-14.65537	-17.36474
-15.10181	-6.57569	-47.40543
-13.35[illegible]	-4.21766	-66.79991

COMPONENT VALUES AT THE NODE: 5

	ACTUAL	CALCULATED
ELEMENT: 3	0.52631579	0.52631579
ELEMENT: 10	0.93333333	0.93333333
ELEMENT: 13	0.24000000	0.24000000

VB MATRIX AT THE NODE 6

-2.70177	-1.00034
3.9[illegible]27	-1.19081

COMPONENT VALUES AT THE NODE: 6

	ACTUAL	CALCULATED
ELEMENT: 4	0.45454545	0.45454545
ELEMENT: 14	0.10000000	0.10000000

VB MATRIX AT THE NODE 7

-0.[illegible]0843

COMPONENT VALUES AT THE NODE: 7

	ACTUAL	CALCULATED
ELEMENT: 12	1.[illegible]6667	1.66666667

Figure 3.4 The calculated element values of Example 1.

(c) Number of frequencies at which the node equation is to be considered:

Two : Nodes 3, 4, 6, and 7;

Three : Nodes 1, 2, and 5 .

(d) The results:

The calculated values of the network elements at different nodes are shown in figure 3.4. It is seen that the calculated values are the same as the actual values.

3.4.2 Example 2

(a) The network: Same as the network of Example 8 of the previous Chapter. R_{25} is the added element.

(b) The node sequence: The network has only one node

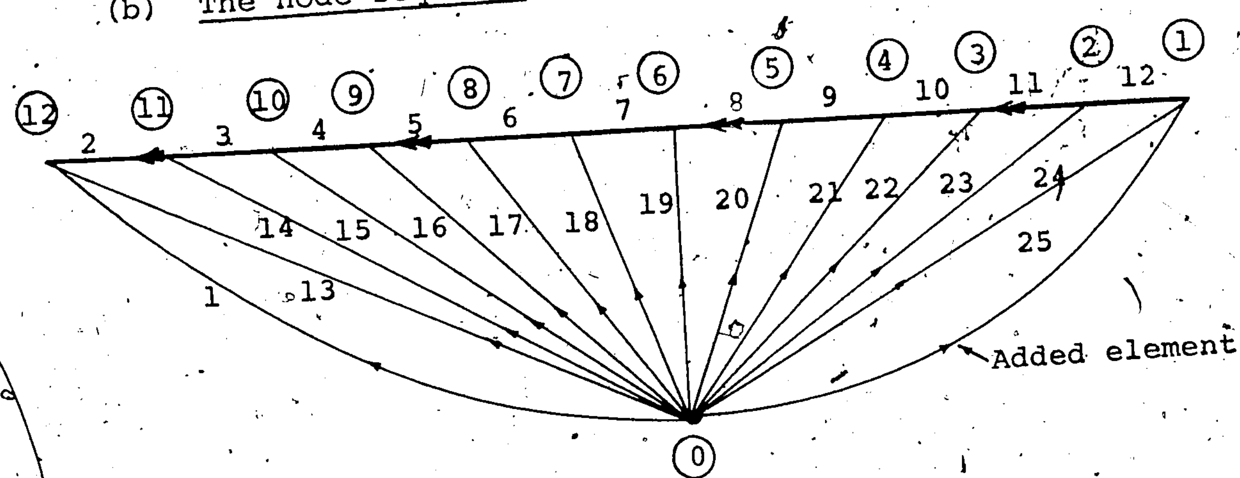

Figure 3.5 The diagraph of the ladder network of Example 2. The node sequence is indicated by the thicker line.

sequence as shown in figure 3.5.

(c) <u>Number of frequencies at which the node equation is to be considered:</u>

Two at each node.

(d) <u>The results:</u>

With the element values same as those of Example 8 of the previous Chapter, the calculated values of the elements agreed accurately with the actual values.

3.4.3 <u>Example 3</u>

(a) <u>The network:</u> The network is the same as that of Example 9 of Chapter 2 with the exception that the added element C_{46} has been embedded between the node 1 and node 20.

(b) <u>The node sequence:</u> As indicated in figure 3.6.

(c) <u>Number of frequencies at which the node equation is to be considered:</u>

One : Node 16 ;

Two : Nodes 6,7,8,9,10,11,12,13,18, and 19;

Three : Nodes 1,3,5,14,15, and 17;

Four : Node 2.

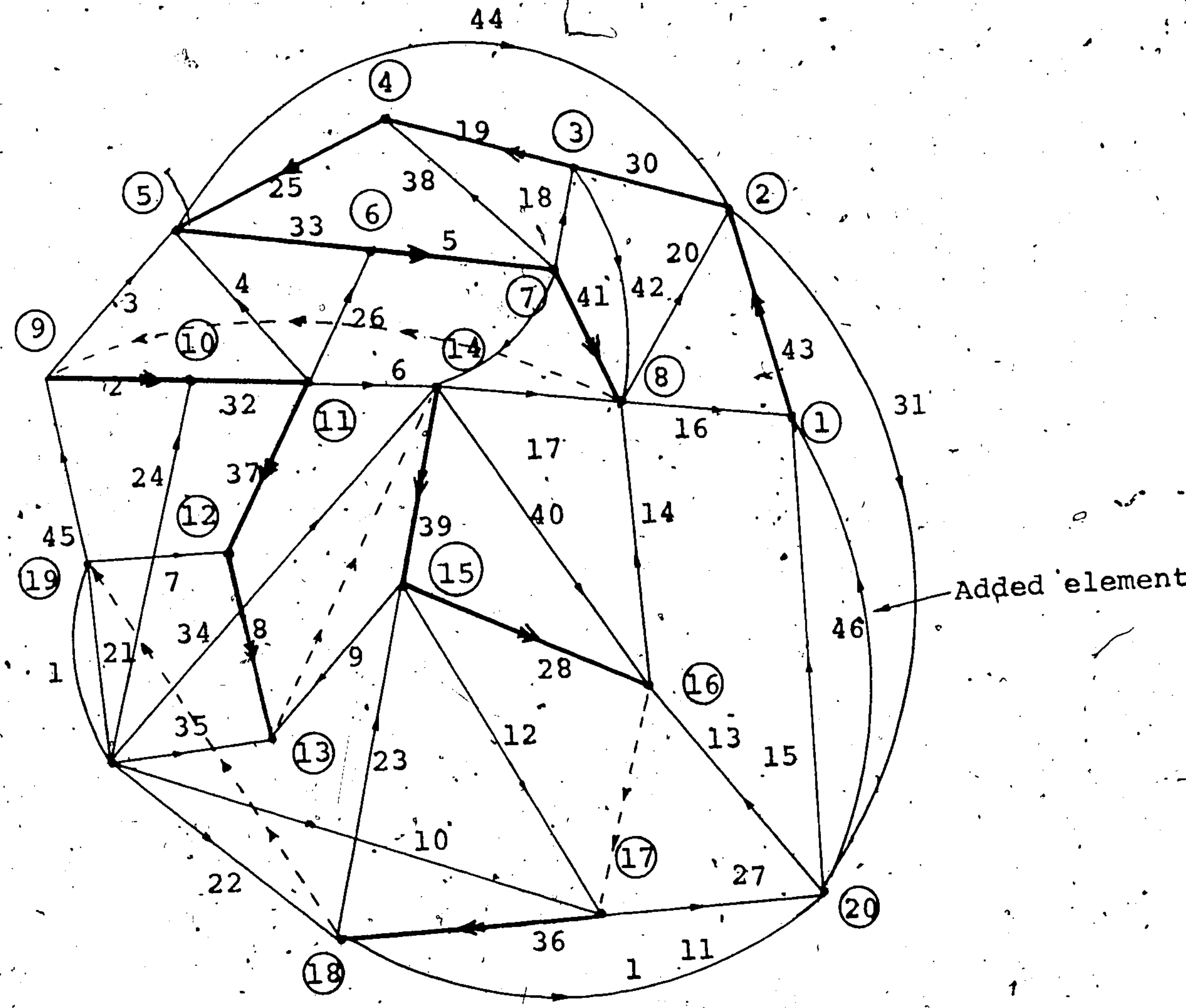

Figure 3.6 The diagraph of the network indicating the node sequence by the thicker line(the dotted lines show the continuation of the sequence).

(d). The results:

With the element values same as those of Example 9 of Chapter 2, the computed element values agreed accurately with the actual values.

CHAPTER 4

DETERMINATION OF THE VARIATIONS OF NETWORK ELEMENTS

In an electrical network, the network elements are subject to variations due to the change of the ambient temperature, aging, etc. Such variations in the element values alter the behaviour of the network. It is the purpose of this Chapter to present a method by which such variations may be calculated. The method requires the measurement of the transfer function of the network at a certain number of frequencies and hence involves, for example, the measurement of the voltages of the input and the output terminals only. As a consequence, the method may be applied to a variety of networks including those having a few or all of the nodes, except the input and the output ones, inaccessible.

4.1 The measurement method

The measurement method consists of measuring the variations in the transfer function, due to the variations in the element values, at a sufficient number of frequencies and then finding the element variations by the use of the Jacobian matrix of the network evaluated at the original element values. For a passive network with n elements, T_i, the inverse of the transfer function H_i (the ratio of the output voltage to the input voltage) at a frequency s_i may be expressed as

$$T_i = 1/H_i = f_i(b_1, b_2, . ., b_n) \tag{4.1}$$

where b_k is the k-th element of the network.

Let the elements b_j, $j = 1, 2, .., n$ be subject to variations so that the element values are $b_j + \delta b_j$, $j = 1, 2, .., n$. The resulting transfer function T_i' is given by

$$T_i' = T_i + \delta T_i = f_i(b_1+\delta b_1, b_2+\delta b_2, . ., b_n+\delta b_n) \tag{4.2}$$

Assuming that the partial derivatives $\partial f_i/ \partial b_j$ exist at the original element values b_j, $j = 1,2,..,n$; equation (4.2) may be expanded in a Taylor series about the original element values to get

$$T_i + \delta T_i = f_i(b_1, b_2, . ., b_n)$$
$$+ \frac{\partial f_i(b_1, b_2, . ., b_n)}{\partial b_1} \delta b_1 + \frac{\partial f_i(b_1, b_2, . ., b_n)}{\partial b_2} \delta b_2$$
$$+ \; . \; . \; . \; + \frac{\partial f_i(b_1, b_2, . ., b_n)}{\partial b_n} \delta b_n$$

$$+ \frac{1}{2} \left[\frac{\partial^2 f_i(b_1, b_2, \ldots, b_n)}{\partial b_1^2} \delta b_1^2 + \frac{\partial^2 f_i(b_1, b_2, \ldots, b_n)}{\partial b_2^2} \delta b_2^2 \right.$$

$$\left. + \ldots + \frac{\partial^2 f_i(b_1, b_2, \ldots, b_n)}{\partial b_n^2} \delta b_n^2 \right] + \ldots \quad (4.3)$$

Dropping all the terms of the order of δb^2 or higher, and recalling equation (4.1), equation (4.3) may be used to obtain

$$\delta T_i = \frac{\partial T_i}{\partial b_1} \delta b_1 + \frac{\partial T_i}{\partial b_2} \delta b_2 + \ldots + \frac{\partial T_i}{\partial b_n} \delta b_n \quad (4.4)$$

With the variation in the transfer function measured at n different frequencies (i.e. at s_1, s_2, . . ., s_n), we get a set of n equations expressed in the matrix form as

$$\begin{bmatrix} \delta T_1 \\ \delta T_2 \\ \vdots \\ \delta T_n \end{bmatrix} = \begin{bmatrix} \frac{\partial T_1}{\partial b_1} & \frac{\partial T_1}{\partial b_2} & \ldots & \frac{\partial T_1}{\partial b_n} \\ \frac{\partial T_2}{\partial b_1} & \frac{\partial T_2}{\partial b_2} & \ldots & \frac{\partial T_2}{\partial b_n} \\ & \vdots & & \\ \frac{\partial T_n}{\partial b_1} & \frac{\partial T_n}{\partial b_2} & \ldots & \frac{\partial T_n}{\partial b_n} \end{bmatrix}_{b=b_j} \begin{bmatrix} \delta b_1 \\ \delta b_2 \\ \vdots \\ \delta b_n \end{bmatrix} \quad (4.5)$$

Equation (4.5) may be written as

$$\delta T = J \ \delta b \tag{4.6}$$

where δT and δb are two column vectors representing the variations in the transfer function and in the element values respectively, and J is the Jacobian matrix evaluated at the original element values and is given by

$$J = \begin{bmatrix} \frac{\partial T_1}{\partial b_1} & \frac{\partial T_1}{\partial b_2} & \cdots & \frac{\partial T_1}{\partial b_n} \\ \frac{\partial T_2}{\partial b_1} & \frac{\partial T_2}{\partial b_2} & \cdots & \frac{\partial T_2}{\partial b_n} \\ & \vdots & & \\ \frac{\partial T_n}{\partial b_1} & \frac{\partial T_n}{\partial b_2} & \cdots & \frac{\partial T_n}{\partial b_n} \end{bmatrix}_{b=b_j} \tag{4.7}$$

The subscript b_j on the Jacobian matrix indicates that it must be evaluated at the original element values.

From equation (4.6), the variations in the element values, represented by the column matrix δb, is obtained as

$$\delta b = J^{-1} \ \delta T \tag{4.8}$$

The new element values are given by

$$b' = b + \delta b \qquad (4.9)$$

where the column matrices b and b' correspond to the original and the changed element values respectively.

An electrical network may have some of its elements with no appreciable variations, while the remaining elements are subject to appreciable variations. As for example, a network with high-quality resistors with extremely low drift, the measurement of the variations in the values of its capacitors and inductors is of practical importance. The measurement method can be readily applied in this case. Let m be the number of such elements the variations of which are to be determined. For the remaining (n-m) elements b_k, $k = m+1, m+2, \ldots, n$, the variations δb_k are zero so that equation (4.4) takes the form

$$\delta T_i = \frac{\partial T_i}{\partial b_1}\delta b_1 + \frac{\partial T_i}{\partial b_2}\delta b_2 + \cdots + \frac{\partial T_i}{\partial b_m}\delta b_m \qquad (4.10)$$

and consequently, the matrices δT, δb, and J of equation (4.6) reduce to

$$\delta T = [\ \delta T_1,\ \delta T_2,\ \ldots,\ \delta T_m\ ]^T \qquad (4.11a)$$

$$\delta b = [\ \delta b_1,\ \delta b_2,\ \ldots,\ \delta b_m\ ]^T \qquad (4.11b)$$

and

$$J = \begin{bmatrix} \frac{\partial T_1}{\partial b_1} & \frac{\partial T_1}{\partial b_2} & \cdots & \frac{\partial T_1}{\partial b_m} \\ \frac{\partial T_2}{\partial b_1} & \frac{\partial T_2}{\partial b_2} & \cdots & \frac{\partial T_2}{\partial b_m} \\ & \vdots & & \\ \frac{\partial T_m}{\partial b_1} & \frac{\partial T_m}{\partial b_2} & \cdots & \frac{\partial T_m}{\partial b_m} \end{bmatrix}_{b = b_j} \qquad (4.11c)$$

where $b_1, b_2, \ldots, b_m$ are the elements subjected to variations.

From equation (4.8), it is seen that the variations in the element values may be obtained by measuring the variations in the inverse transfer function at a number of frequencies equal to the number of elements the variations of which are being calculated, and then premultiplying the resulting column matrix δT with the inverse of the Jacobian matrix of the network evaluated at the original element values.

It may be noted that when the input and the output variables are of same kind (either voltage or current), there may be several combinations of the element variations that will result in the same change in the transfer function and as a result, at least one element, either an existing one or an added element, must be known for the unique solution of the element variations.

4.2 An iterative Jacobian method for the accurate determination of network element variations

Since all the terms of the order of δb^2 or higher in equation (4.3) have been dropped to derive equation (4.8), the variations in the element values obtained by the method of the previous Section are only approximate, though fairly accurate, especially when such variations are small. However, by repeating the method of the previous Section in an iterative manner, the element variations may be accurately determined even when such variations are quite high. In the present iterative method, the vector $\delta b]_1$ is first obtained from the measured vector $\delta T]_0$ by the method of the previous Section. A fraction β ($0 < \beta \leqslant 1$) of the vector $\delta b]_1$ is now taken as the effective element variation vector so that the resulting element values are given by

$$b]_1 = b]_0 + \beta \ \delta b]_1$$

$$= b]_0 + \beta \ J_0^{-1} \ \delta T]_0 \qquad (4.12)$$

where $b]_0$ is the vector representing the original element values and the vector $\delta T]_0$ represents the variations in the inverse transfer function measured and J_0 is the Jacobian matrix evaluated at the original element values. Instead of $b]_0$, the vector $b]_1$ is now assumed as the original element vector. The Jacobian matrix J_1 and the vector $\delta T]_1$ are calculated with this element vector $b]_1$. The vector $\delta b]_2$ is then calculated. A fraction β of the vector $\delta b]_2$ is then added to the vector $b]_1$ to get the element vector $b]_2$ given by

$$
\begin{aligned}
b]_2 &= b]_1 + \beta \quad \delta b]_2 \\
&= b]_1 + \beta \quad J_1^{-1} \quad \delta T]_1 .
\end{aligned}
\qquad (4.13)
$$

The vector $b]_2$ is now taken as the original element vector. This procedure is repeated say p times until the vector $\delta b]_p$ becomes negligibly small. The element vector $b]_p$ is then taken as the actual element vector, i.e.

$$b]_p \approx b]_a \qquad (4.14)$$

where $b]_a$ is the actual element vector after variations.

To illustrate the measurement procedure, consider a network in which only one element , say the i-th element b_i is subject to variations. The plot of the inverse transfer function T against b_i is shown in figure 4.1 where b_{i0} and T_0 represent the original values of the element and the inverse transfer function respectively and b_{ia} and T_a represent their respective values after variations. Here, the slope of the curve at the point O (representing the original values) gives the inverse of the

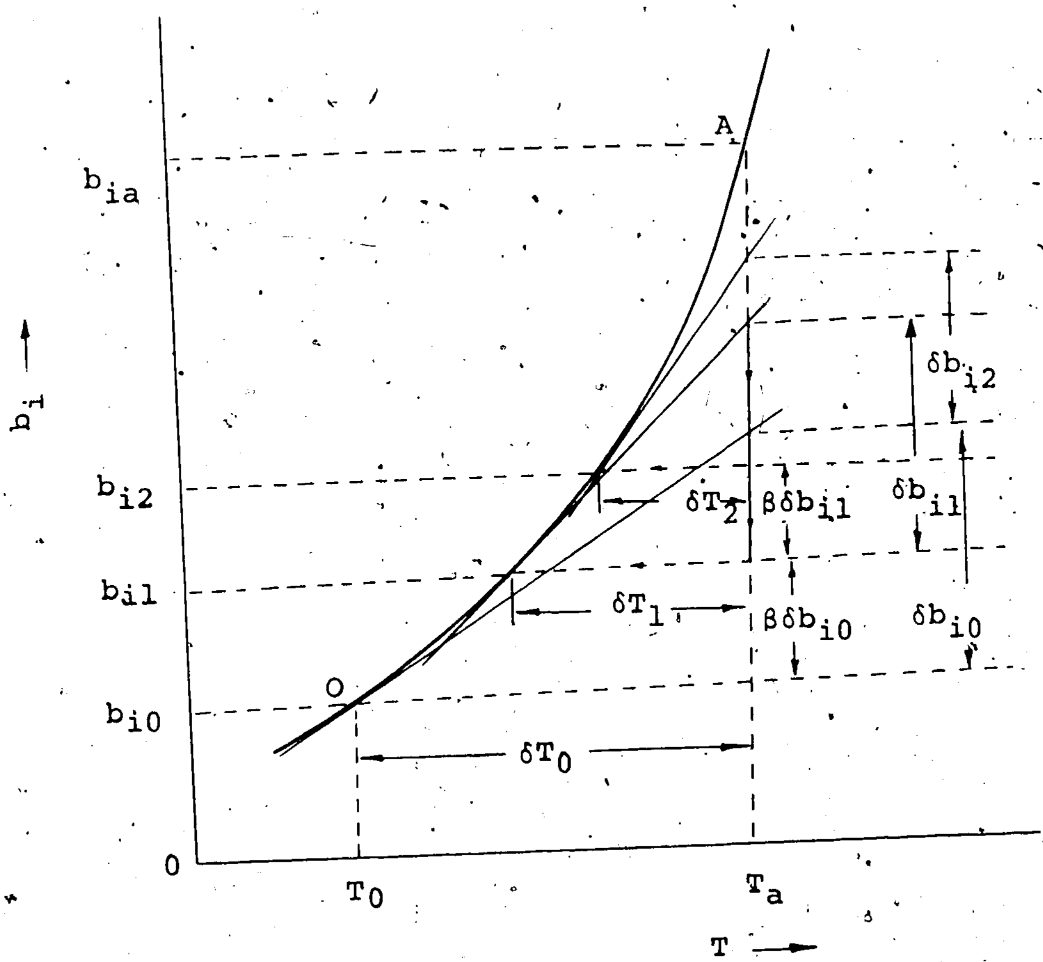

Figure 4.1 The iterative method with the element b_i subjected to variations.

Jacobian matrix J_0. From the value of b_{i0} and the inverse of the matrix J_0, the variation δb_{i0} is determined according to the method suggested in the previous Section. The new element b_{i1}, after the first iteration, is

$$b_{i1} = b_{i0} + \beta \ \delta b_{i0} \qquad (4.15)$$

Now, assuming b_{i1} as the original element value, the Jacobian matrix J_1 and the variation δT_1 are calculated and the resulting variation δb_{i1} is obtained. A fraction β of this variation δb_{i1} is again taken as the effective element variation which is added to previously-assumed original value b_{i1} to form b_{i2} which is now taken as the original value. This procedure is repeated and in the p-th iteration, the value b_{ip} will be close to the actual value b_{ia}.

It may be mentioned that the calculated element value after the j-th iteration is given by

$$b_{ij} = b_{i(j-1)} + \beta \ J_{j-1}^{-1} \ \delta T_{j-1} \qquad (4.16)$$

The factor β is called the convergence constant and lies in the rangle $0 < \beta \leq 1$.

Observe that greater the value of the constant β, smaller is the number of iterations required by the method. The value of β, therefore, should be chosen as large as possible. However, in some cases, especially when the element variations are quite large, a large value of β may make the iterative process divergent with the calculated element variations increasing in successive iterations. In such cases, β must be chosen small enough to ensure convergence.

4.3 The flow chart

From the measurement method indicated in the ealier Sections, a computer program may be written that will find the new element values from the variations in the transfer function measured at an appropriate number of frequencies. The steps to be taken for such computations are given below:

1. Assign the output node as the N-th node, where N is the total number of nodes excluding the reference node. Read the values β, KK(i), DA, and m. KK(i) is the percentage variation of the i-th element subject to variations, DA is a factor by which the elements are to be perturbed to calculate the Jacobian matrix, and m is the total number of elements the variations of which are to be determined.
2. From the node-branch incidence submatrix A_1 and the vector E, and with the original element values of the network, calculate the admittance matrix G and the vector H at the starting frequency by the steps 1 to 6 of Section 2.3 of Chapter 2. Invert the matrix G.
3. Assign VN(1,j) as the output voltage with the original element values and VN(2,j) as the output voltage with the actual element values (after variations), both occuring at a frequency j. Cal-

culate the voltage VN(1,1) at the output node by premultiplying the column vector H by the row vector formed by the N-th row of the inverse of the G-matrix.

4. Repeat the procedure suggested in the steps 2 and 3 at (m-1) different frequencies to obtain the output voltages. VN(1,2),VN(1,3),. . . , VN(1,m).

5. With the actual element values (after variations), compute the output voltages VN(2,1),VN(2,2), . . . VN(2,m) by the steps 2 and 4.

Calculate the matrix $\delta T]_1$ by the use of the expression

$$\delta T(i) = \text{element of the i-th row of the matrix } \delta T]_1$$

$$= V_0 \left[\frac{1}{VN(2,1)} - \frac{1}{VN(1,i)} \right] \qquad (4.17)$$

where V_0 is the input voltage.

7. Assign V(1,j) as the output voltage when the i-th network element of value b_i is changed to $(1+DA)b_i$; ~~and~~ V(2,j) as the output voltage when the element value is changed to $(1-DA)b_i$; both calculated at a frequency j. Take i = 1. With the element value b_i changed by an increment amount to $(1+DA)b_i$, and with the remaining elements at their original values, calculate the out-

put voltages $V(1,j)$, $j = 1, 2, \ldots, m$ at m different frequencies following the steps 2 and 4.

8. Change the element value to $(1-DA)b_i$ and compute the output voltage $V(2,j)$, $j = 1, 2, \ldots, m$ following the preceeding step.

9. Calculate all the elements of the i-th column of the Jacobian matrix by the use of the expression

$$\frac{\partial T_j}{\partial b_i} = \left[\frac{1}{V(1,j)} - \frac{1}{V(2,j)} \right] \frac{V_0}{2\, DA \cdot b_i} \tag{4.18}$$

10. Increase the value of i by 1. Go back to step 7 until the value of i exceeds m . All the elements of the Jacobian matrix is now obtained.

11. Invert the resulting Jacobian matrix and postmultiply the inverted matrix by the column matrix $\delta T]_1$ calculated in the step 6, to obtain the incremental matrix δb.

12. With the original element vector, add the matrix $\beta\ \delta b$ to get the new element vector.

13. Putting th element vector ,obtained in the step 12, as the original element vector, repeat the steps 2 to 12 a number of times until the elements of the incremental matrix δb are less than some pre-assigned values or until the number

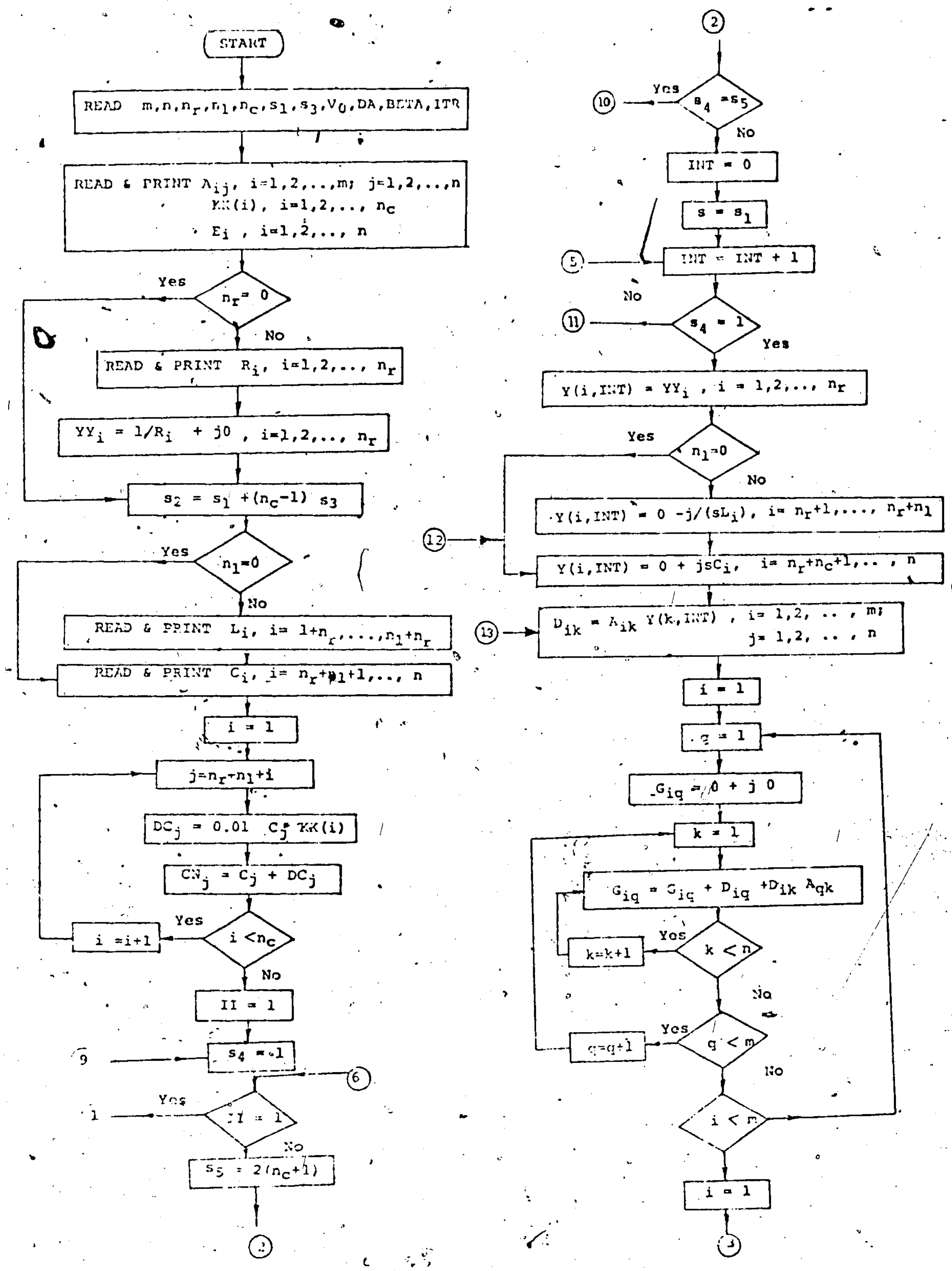

Figure 4.2 The flow chart.

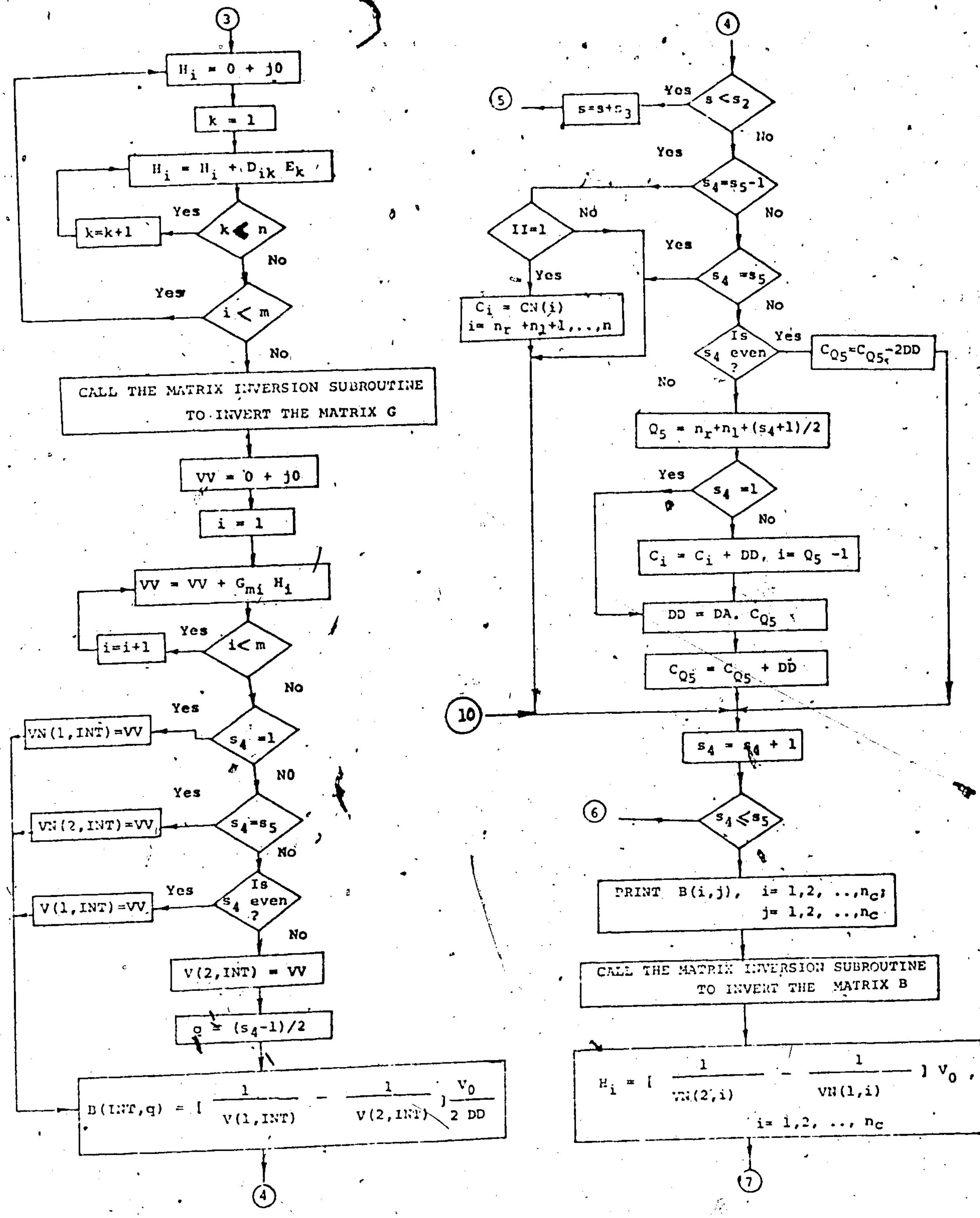

Figure 4.2 The flow chart(Continued).

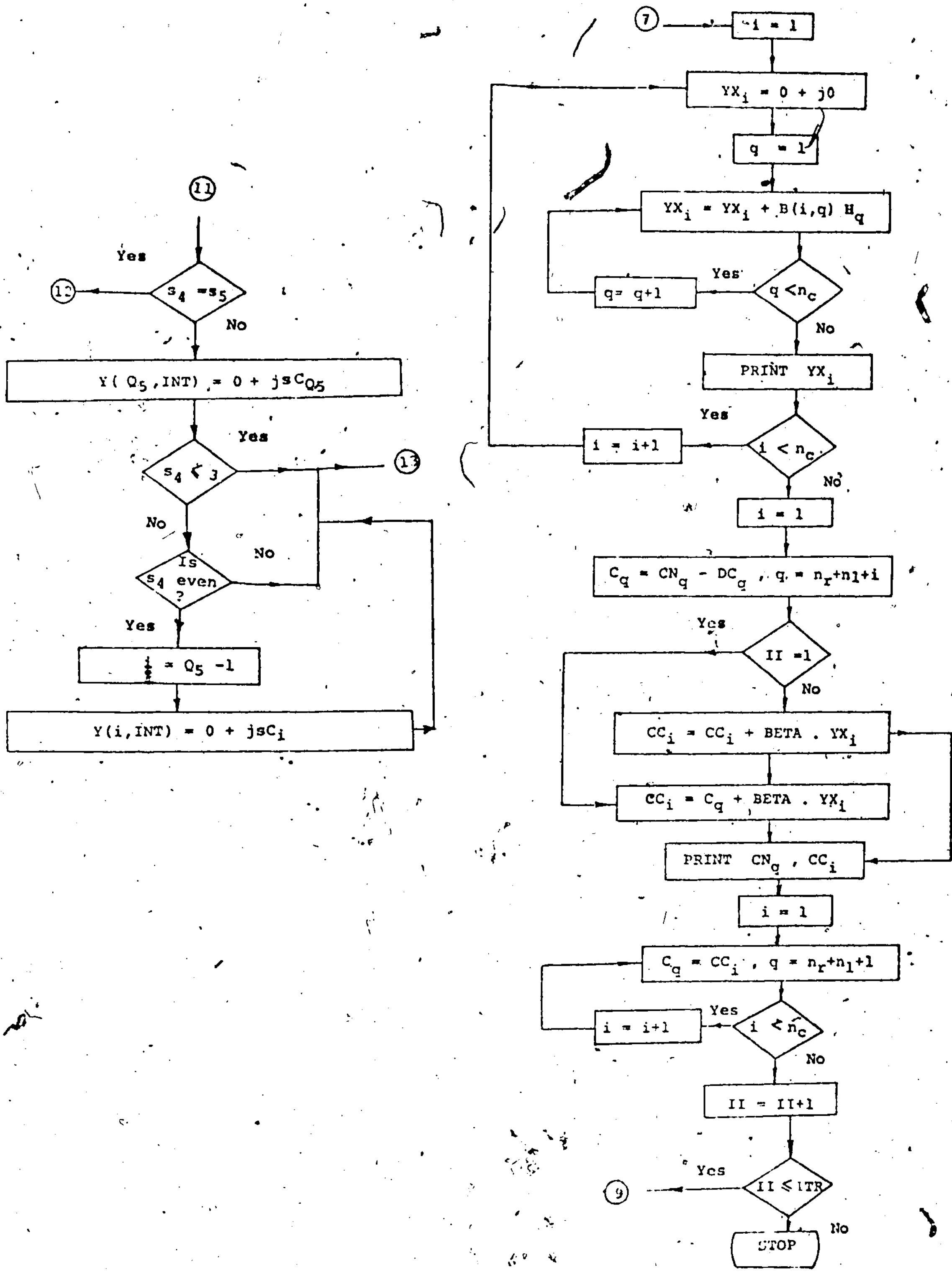

Figure 4.2 The flow chart(Continued).

of iterations equals a certain pre-assigned value. The element vector obtained in the final iteration is then assumed as the new element vector if the process has converged.

The details of the above steps are illustrated by the flow diagram of figure 4.2.

4.4 Examples and experimental results

4.4.1 Example 1

(a) The network:

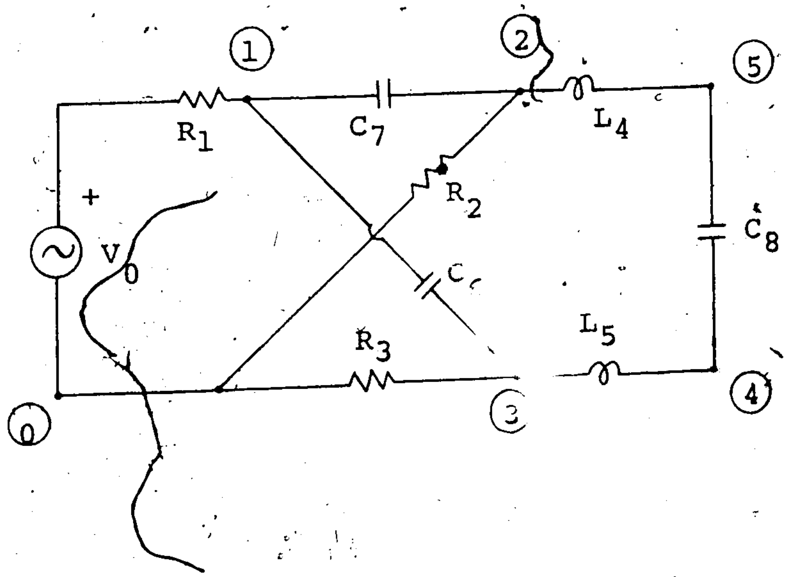

Figure 4.3 The network of Example 1.

(b) The diagraph:

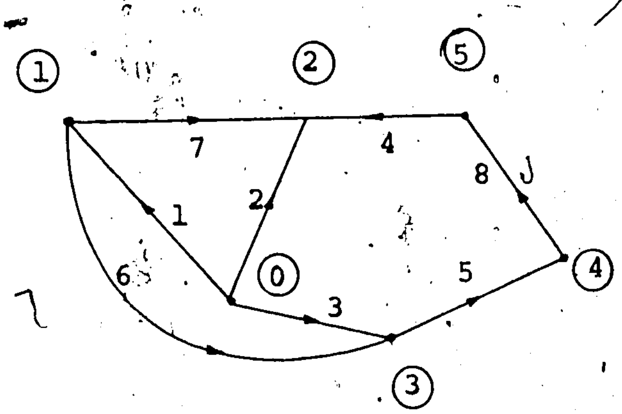

Figure 4.4 Diagraph of the network of Example 1.

(c) The results:

The network was investigated with $R_1 = 1\ \Omega$, $R_2 = 8.1\ \Omega$, $R_3 = 5.2\ \Omega$, $L_4 = 1.2$ H., and $L_5 = 0.5$ H. Only the capacitors were assumed to undergo variations. The original values of the capacitors were $C_6 = 1.2$ F., $C_7 = 2.5$ F., and $C_8 = 0.5$ F. With the convergence constant $\beta = 1$, the variations in the capacitor values were calculated when they were subjected to a wide range of variations. The calculated variations in the network elements agreed accurately with the actual variations even when the capacitors had a variation of 70% each. The method gave an accurate result even when C_6 was varied

```
ELEMENT VARIATIONS BY JACOBIAN ITERATION: EXAMPLE 1:

COMPONENT VALUES:
RESISTANCES:   1.00 0.10 5.20
INDUCTANCES:   1.20 0.50
CAPACITANCES:  1.[illegible] 2.50 0.50
      CONVERGENT CONSTANT =   1.0000

      PERCENTAGE VARIATION OF THE CAPACITORS:
      C 6:      30  %
      C 7:      70  %
      C 8:      90  %
****************************
      ITERATION NO:  1
      JACOBIAN MATRIX:
( 0.1175D-01-0.6530D-01)(-0.7107D-02 0.5556D-01)( 0.1760D-01 0.1793D 00)(
(-0.9013D-03 0.4367D-01)( 0.7245D-04 0.4109D-03)( 0.1702D-02 0.1370D-01)(
(-0.2346D-03 0.2110D-01)( 0.7494D-04 0.7940D-03)( 0.9700D-05 0.2234D-02)(
      DETERMINANT OF THE JACOBIAN MATRIX= ( 0.1602628D-05, -0.1630194D-03)
      NEW ELEMENT VALUES(ACTUAL)              NEW ELEMENT VALUES(CALCULATED):
                                                  1.2618220,  0.1823D-02
              1.5599999                           7.9320364,  0.1335D 00
              4.2499986                           1.2121315,  0.3007D-01
              0.9499999
      VALUE OF THE NORM =    5.4797803
****************************
      ITERATION NO:  2
      JACOBIAN MATRIX:
(-0.3144D 02 0.1217D 02)(-0.1223D 01 0.2119D 00)(-0.4504D 02 0.1974D 02)(
(-0.2027D-03 0.3580D-01)( 0.1661D-04 0.1061D-03)( 0.2076D-04 0.2332D-02)(
(-0.3011D-03 0.1982D-01)( 0.4217D-05 0.9832D-04)( 0.[illegible]D-05 0.4242D-03)(
      DETERMINANT OF THE JACOBIAN MATRIX= ( 0.8553135D-04, -0.2360791D-04)
      NEW ELEMENT VALUES(ACTUAL)              NEW ELEMENT VALUES(CALCULATED):

                                                  1.4341512, -0.7443D-04
              1.5599999                           4.3511709,  0.5083D 00
              4.2499996                           1.0565329,  0.4645D-01
              0.9499999
      VALUE OF THE NORM =    3.5514111
****************************
      ITERATION NO:  3
      JACOBIAN MATRIX:
(-0.3991D 00 0.1680D 01)(-0.1362D 00 0.3684D 00)(-0.7471D 00 0.6639D 01)(
(-0.4862D-03 0.2570D-01)( 0.2694D-04 0.2769D-03)( 0.1659D-04 0.2308D-02)(
(-0.1621D-03 0.1360D-01)( 0.1053D-04 0.2341D-03)( 0.1847D-05 0.4283D-03)(
      DETERMINANT OF THE JACOBIAN MATRIX= ( 0.5612260D-05, -0.1327946D-03)
      NEW ELEMENT VALUES(ACTUAL)              NEW ELEMENT VALUES(CALCULATED):
                                                  1.5326343,  0.3570D-03
              1.5599999                           4.3877475,  0.4950D 00
              4.2499996                           0.9630262,  0.1348D-01
              0.9499999
      VALUE OF THE NORM =    0.1182280
****************************
      ITERATION NO:  4
      JACOBIAN MATRIX:
(-0.1364D 00 [illegible])([illegible])([illegible])(
(-0.4125D-03 [illegible])([illegible])([illegible])(
(-0.1107D-03 [illegible])([illegible])([illegible])(
      DETERMINANT OF THE JACOBIAN MATRIX= ( [illegible]D-05, -0.1072075D-03)
      NEW ELEMENT VALUES(ACTUAL)              NEW ELEMENT VALUES(CALCULATED):
                                                  1.5597284,  0.3403D-03
              1.5599999                           4.2519558,  0.4761D 00
              4.2499996                           0.9502920,  0.1557D-01
              0.9499999
      VALUE OF THE NORM =    0.1044240
```

Figure 4.5 The computer output showing the calculated element values against the actual ones when element values of Example 1 vary by 30%, 70%, and 90% respectively.

```
ELEMENT VARIATIONS BY JACOBIAN ITERATION: EXAMPLE  1

   COMPONENT VALUES:
RESISTANCES:   1.00 0.10 5.20
INDUCTANCES:   1.20 0.50
CAPACITANCES:   1.20 2.50 0.50
         CONVERGENT CONSTANT =   1.0000

         PERCENTAGE VARIATION OF THE CAPACITORS:
         C 6:      50  %
         C 7:      90  %
         C 8:      90  %
******************************
         ITERATION NO:  1
         JACOBIAN MATRIX:
( 0.1135D-01-0.6530D-01)(-0.7197D-02 0.5556D-01)(-0.1769D-00 0.1793D-01)(
(-0.8033D-03 0.4767D-01)( 0.7245D-04 0.4100D-03)(-0.1592D-03 0.1779D-01)(
(-0.2746D-03 0.2119D-01)( 0.3434D-04 0.7849D-03)( 0.9760D-05 0.2234D-02)(
         DETERMINANT OF THE JACOBIAN MATRIX= ( 0.1602628D-05, -0.1670184D-04)
         NEW ELEMENT VALUES(ACTUAL)                NEW ELEMENT VALUES(CALCULATED):
                                                        1.1918700, -0.3775D-01
                    1.7999999                          12.1642713,  0.1091D 01
                    4.7499995                           1.5918305,  0.1523D 00
                    0.9499999
         VALUE OF THE NORM =   9.7375012
******************************
         ITERATION NO:  2
         JACOBIAN MATRIX:
( 0.2652D 00 0.5260D 01)(-0.3915D-02 0.2995D-01)( 0.6236D 00 0.7139D 01)(
(-0.1170D-02 0.3977D-01)( 0.5157D-05 0.6554D-04)( 0.1912D-04 0.1736D-02)(
(-0.3942D-03 0.2145D-01)( 0.2004D-05 0.3810D-04)( 0.1958D-05 0.2490D-03)(
         DETERMINANT OF THE JACOBIAN MATRIX= ( -0.1738423D-07, -0.1518029D-05)
         NEW ELEMENT VALUES(ACTUAL)                NEW ELEMENT VALUES(CALCULATED):

                                                        1.5222549, -0.3752D-01
                    1.7999999                          23.9997925,  0.1793D 01
                    4.7499995                           1.5355935,  0.2133D 00
                    0.9499999
         VALUE OF THE NORM =  11.8407697
******************************
         ITERATION NO:  3
         JACOBIAN MATRIX:
( 0.6480D-01 0.8475D 00)(-0.1306D-03 0.1027D-02)( 0.1070D 00 0.5397D 00)(
(-0.5305D-03 0.2350D-01)( 0.1042D-05 0.1241D-04)( 0.1477D-04 0.1263D-02)(
(-0.1904D-03 0.1265D-01)( 0.4067D-06 0.9979D-05)( 0.1516D-05 0.2223D-03)(
         DETERMINANT OF THE JACOBIAN MATRIX= ( -0.1786334D-08, -0.8336091D-07)
         NEW ELEMENT VALUES(ACTUAL)                NEW ELEMENT VALUES(CALCULATED):
                                                        1.5317994, -0.5151D-01
                    1.7999999                         283.3904261,  0.3935D 02
                    4.7499995                           3.1072426,  0.7373D 00
                    0.9499999
         VALUE OF THE NORM = 259.9976632
******************************
         ITERATION NO:  4
         JACOBIAN MATRIX:
(-0.2773D-01 0.3476D 00)(-0.4753D-06 0.6533D-06)( 0.1516D-02 0.3763D-01)(
[illegible]
[illegible]
         DETERMINANT OF THE JACOBIAN MATRIX= ( 0.2419007D-11, -0.1283102D-10)
         NEW ELEMENT VALUES(ACTUAL)                NEW ELEMENT VALUES(CALCULATED):
                                                        0.1672085,  0.3742D-01
                    1.7999999                       *************, -0.3073D 04
                    4.7499995                          30.7305566,  0.5397D 01
                    0.9495999
         VALUE OF THE NORM =*************
```

Figure 4.6 The actual and calculated element values when C_6, C_7, and C_8 of Example 1 vary by 50%, 90%, and 90% respectively.

by 30%, C_7 and C_8 by 90% each. However, with the element variations greater than this range, the calculated element values diverged away from the actual values as illustrated by figure 4.5 and figure 4.6. However, with a smaller value of the convergence constant, the element variations may be calculated beyond this range. With

$\beta = 0.25$, the computed element variations coincided with the actual variations even when the capacitors were varied by 120% each. However, a smaller value of β necessiates a greater number of iterations. As for example, with $\beta = 0.25$ and with the capacitor variations of 90% each, the calculated results were fairly accurate at about the 30th iteration compared to the fact that with $\beta = 1$ and with capacitor variations of 30% each, an accurate result was obtained at the third iteration.

4.4.2 Example 2

(a) The network:

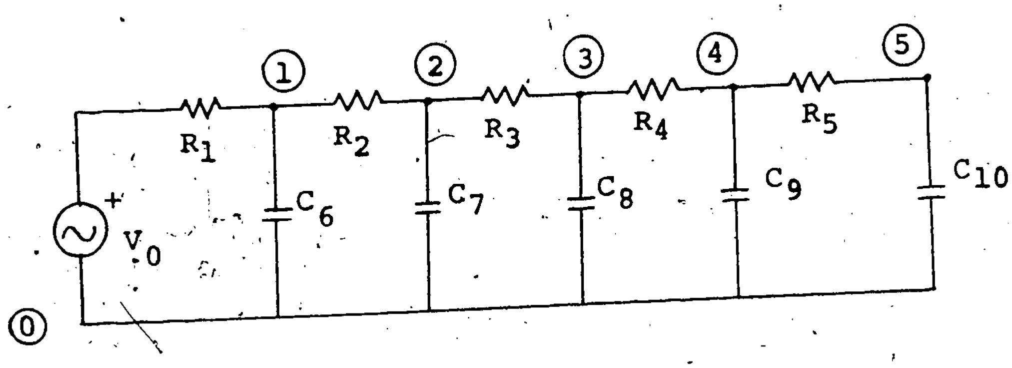

Figure 4.7 The ladder network of Example 2.

(b) The diagraph:

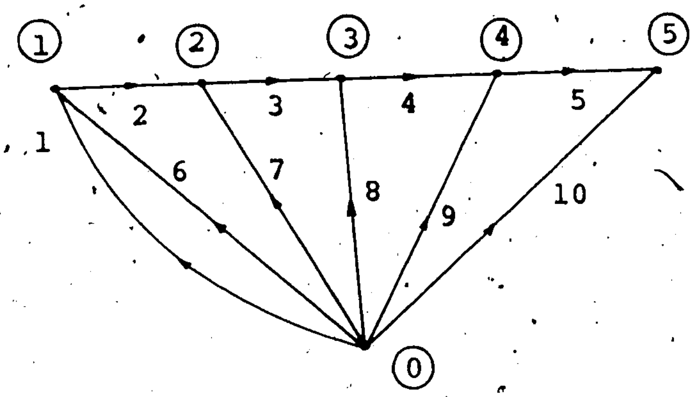

Figure 4.8 Diagraph of the network of Example 2.

(c) The results:

The network was studied with $R_1 = 1\Omega$, $R_2=2.1\Omega$, $R_3 = 3.2\ \Omega$, $R_4 = 1.2\Omega$, $R_5 = 5\ \Omega$, and with the original values of the capacitors as $C_6 = 1F.$, $C_7 = 1.5\ F.$, $C_8 = 2.4\ F.$, $C_9 = 0.5\ F.$ and $C_{10} = 2.1\ F.$ Only the capacitors were assumed to vary. With $\beta = 1$, the calculated element values were accurately the same as the actual values at the fourth iteration so long as the variations of the capacitors did not exceed ±15% each. Beyond this limit, the computed values diverged away from the actual element values. To calculate the element variations beyond this limit, the convergence constant β was taken as 0.25 and the element variations were found to agree accurately with the capacitor variations up to ± 35% each, although the number of iterations required to reach the final value was about 35 in each case.

4.5 Discussions

By choosing the convergence constant sufficiently small, it is possible, as has been found in the above examples, to find the element values subjected to a wide range of variations. In each iteration, the distance, in the m-dimensional element space, of the solution point from the solution point obtained in the previous iteration was calculated by the use of the equation

$$d(i) = [(b_{1i} - b_{1(i-1)})^2 + (b_{2i} - b_{2(i-1)})^2 + \ldots + (b_{mi} - b_{m(i-1)})^2]^{1/2} \qquad (4.19)$$

where $d(i)$ is the distance calculated in the i-th iteration, and b_{kj} is the value of the element b_k obtained in the j-th iteration. From equation (4.16), for the i-th iteration:

$$b]_i = f(b]_{i-1}) = b]_{i-1} + \beta \; J^{-1}_{i-1} \; \delta T]_{i-1} \qquad (4.20)$$

whence

$$\left\| f(b]_i) - f(b]_{i-1}) \right\| = \left\| b]_{i+1} - b]_i \right\| \qquad (4.21)$$

For the function f in equation (4.20) to define a contraction mappring, we require [13]

$$\left\| f(b]_i) - f(b]_{i-1}) \right\| \leqslant K \left\| b]_i - b]_{i-1} \right\|$$

i.e.

$$\left\| b]_{i+1} - b]_i \right\| \leqslant K \left\| b]_i - b]_{i-1} \right\| \qquad (4.22)$$

where $K < 1$. Since

$$\left\| b]_{i+1} - b]_i \right\| = d(i+1)$$

the condition of expression (4.22) may be written as

$$d(i+1) \leqslant K \; d(i) \qquad (4.23)$$

It has been observed that in the above examples, for all the cases where the calculated element vector coincided with the actual element vector,

$$d(i+1) < d(i) \qquad (4.24)$$

Condition (4.23) is thus satisfied indicating that the function f representing the present iterative method is, possibly, a contraction mapping for which

$$\lim_{i \to \infty} f(b]_i) = b \qquad (4.25)$$

where b is the actual solution vector.

CHAPTER 5

CONCLUSIONS

In this thesis, measurement methods for the explicit solution of the element values of an electrical network from a set of node voltage measurements were considered. Three such methods were presented. All the methods require that at least one of the network elements must be known, otherwise at least one known,external element must be added to the network. In each of the measurement method, an efficient computer program was developed that computes the unknown element values from the appropriate set of measured data.

In the first method, the node equations are considered at a sufficient number of frequencies and all the unknown elements are calculated in one computation. The method may be used to solve a network of arbitrary size , though with a large network, the method involves the handling of inconveniently large matrices.

The second method of the element value solution considers one node at a time and solves for its unknown elements before considering the next node. A computer program for such a method was presented. This measurement method is especially attractive in the case when the network is very large or when only a portion of a network is required to be solved.

The third method computes the variations of the network elements by the use of the Jacobian matrix. An iterative method for the accurate solution of the element variations and the computer program for such solutions were provided. Experiments with a number of networks indicates that the method may be successfully used for a wide range of element variations. For a particular network, such range of element variations may be increased by choosing a smaller value of the convergence constant, though the number of iterations for the accurate solution increases with such decrease in the value of the convergence constant.

The computational experiments with a variety of networks,provided in this thesis,indicated close agreement with the actual values and no network was found where the above measurement methods failed.

The measurement procedures may be extended for the solution of the element values of active networks. The methods may be used in the testing of electromechanical systems and in automatic testing applications.

REFERENCES

1. Berkowitz, R. S., "Conditions for Network Element Value Solvability", IRE Transactions on Circuit Theory, Vol. CT-9, pp. 24-29, March 1962.

2. Bedrosian, S. D.. and Berkowitz, R. S., "Solution Procedure for Single-Element-Kind Networks", 1962 International Convention Records, Part 2, pp. 16-24.

3. Bedrosian, S. D., " A Simple Explicit Solution of Networks with Two Internal Nodes", IEEE Transactions on Communication and Electronics, Vol. 71, pp. 219-224, March 1964.

4. Berkowitz, R. S. and Wexelblat, R. L., "Statistical Considerations on Network Element Value Solution", IRE Transactions on Military Electronics, Vol. MIL-6, pp. 282-288, July 1962.

5. Even, S. and Lempel, A., "On a Problem of Diagnosis", IEEE Transactions on Circuit Theory, Vol. CT-14, pp. 361-364, September 1967.

6. Seshu, S. and Reed, M. B., "Linear Graphs and Electrical Networks", Addision-Wesley, Reading, p. 121; 1961.

7. Chan, S. P., "Introductory Topological Analysis of Electrical Networks", Holt, Rinehart and Winston Inc., N.Y., p.105; 1969.

8. Seshu, S. and Reed, M. B., "Linear Graphs and Electrical Networks", Addision-Wesley, Reading, p.132; 1961.

9. Chan, S. P., "Introductory Topological Analysis of Electrical Networks", Holt, Rinehart and Winston Inc., N.Y., p.114; 1969.

10. Chan, S. P., "Introductory Topological Analysis of Electrical Networks", Holt, Rinehart and Winston Inc., N.Y., p. 123; 1969.

11. Seshu, S. and Reed, M. B., "Linear Graphs and Electrical Networks", Addision-Wesley, Reading, p. 133; 1961.

12. Chan, S. P., Chan, S. Y. and Chan, S. G., "Analysis

of Linear Networks and Systems", Addision-Wesley, Reading, pp. 63-68; 1972.

13. Kolmogorov, A. N. and Fomin, S. V., "Elements of the Theory of Functions and Functional Analysis", Vol. 1, Graylock Press, N.Y., pp.43-46; 1957.

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(2) Amplitude Stabilised Transistorised Low Frequency Oscillator. Accepted for publication in the International Journal of Electronics, London.

(3) Grounded Inductor Simulation Using a Non-Inverting Voltage Controlled Voltage Source. Paper presented and published in a Symposium on "Recent Advances in Electronics Engineering in India" held in Benaras Hindu University in June, 1971.

(4) Amplitude Stabilized Low Frequency Oscillator. To be published, International Journal of Electronics, London.

(5) Active R.C. All-Pass networks Using a Summing or a Differential Amplifier. Accepted for publication in the International Journal of Electronics, London.