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Full Name of Author — Nom complet de l'auteur
SHU-MING ALBERT LAI

Date of Birth — Date de naissance
7th AUG. 1956

Country of Birth — Lieu de naissance
HONG KONG

Permanent Address — Résidence fixe
20B, NASSAU STREET, 17th FLOOR, MEI FOO SUN CHUEN, KOWLOON, HONG KONG

Title of Thesis — Titre de la thèse
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Name of Supervisor — Nom du directeur de thèse
JAMES G. MACGREGOR

Date
December 23, 1980

Signature
[signature]

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Monte Carlo Study of Pattern Loadings on Continuous Beams and Slabs

by

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

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(SIGNED) ....

PERMANENT ADDRESS:

20B, Nassau Street, 17th Fl.

Mei Foo Sun Chuen

Kowloon, Hong Kong

DATED December 22 1980

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled Monte Carlo Study of Pattern Loadings on Continuous Beams and Slabs submitted by Shu-Ming Albert LAI in partial fulfilment of the requirements for the degree of Master of Science.

....

Supervisor

....

....

C. James Montgomery

Date...December 15, 1980........

Abstract

A design load pattern is proposed to represent the 0.999 fractiles of the maximum lifetime live load effects. The proposed load pattern is different from the traditional ACI load pattern, which produces maximum load effects with inconsistent probability of being exceeded. Monte Carlo simulations have been used to generate families of the lifetime maximum loading effects in a series of continuous beam-column frames. Linear programming is then used to compute the magnitudes of the pattern loads for design. The effect of pattern loadings for the dead load is found to be insignificant.

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Notation

A : floor area in square meters

A_I : influence area in square meters

A_T : tributary area in square meters

a : sum of the variances of γ_b and γ_f

: load factor in design loading

a : an element of an influence coefficient matrix at row i and column j

b : experimental constant used in the description of the sustained load

: load factor in design loading

d_i : penalty for load effect i

E : transient EUDL

E(Z) : expected value of variable Z

EUDL : equivalent uniformly distributed load

I(x,y) : coordinate of influence surface at (x,y)

k : parameter defined by Eq. 2.7b

K_B : beam stiffness

K_C : column stiffness

L : sustained EUDL

L_n : nominal live load

M_{mi} : absolute value of a high fractile of the distribution of the lifetime minimum or maximum load effect i from the Monte Carlo analysis

M_{pi} : absolute value of the minimum or maximum factored load effect i which can be obtained at a critical section from a design load pattern

m : mean survey load in kN/sq. meter

m_Z : mean of variable Z

n : total number of load effects to be fitted by a design load pattern

: total number of spans in a frame

Q : weight of a single concentrated load in the transient load model

R : number of loads per cell in the transient load model

r : correlation coefficient between the dead loads on adjacent spans

V_X : coefficient of variation of the dead load

w(x,y): load intensity at (x,y)

X_j : uniformly distributed dead load on span j

Y_i : dead load effect i

γ_b, γ_f : zero mean random variables of building and floor effects

$\epsilon(x,y)$: local load intensity variation from the floor mean at (x,y)

λ : mean number of load cells in the influence area

σ_Z : standard deviation of variable Z

Definitions

Monte Carlo Analysis

Case MC1 : a loading situation in the Monte Carlo analysis in which the influence area of each span is assumed to be occupied by one room

Case MC2 : a loading situation in the Monte Carlo analysis similar to Case MC1 except that the influence area of two particular adjacent spans is assumed to be occupied by one room

Case MC3 : a conservative assumption that the larger of the Monte Carlo values from Cases MC1 and MC2 are used in the modelling of pattern loadings

Design Loadings

alternate span loading : heavy factored design load, $a_1 L_n$, on alternate spans, and light factored design load, $b_1 L_n$, on all other spans where $a_1 > b_1$

single span loading : heavy factored design load, $a_2 L_n$, on one span, and light factored design load, $b_2 L_n$, on all other spans where $a_2 > b_2$

adjacent span loading : heavy factored design load, $a_3 L_n$, on two adjacent spans, and light factored design load, $b_3 L_n$, on all other spans where $a_3 > b_3$

Pattern A : a combination of alternate span loading plus adjacent span loading as appropriate, with $a_1 = a_3$ and $b_1 = b_3$

Pattern B : single span loading

Pattern C : a combination of alternate span loading plus adjacent span loading as appropriate, with a_1 and b_1 independent of a_3 and b_3 respectively

Pattern D : a combination of single span loading plus adjacent span loading as appropriate, with a_2 and b_2 independent of a_3 and b_3 respectively

1. INTRODUCTION

1.1 Pattern loadings

1.1.1 Pattern Loadings of Live Load

In structural design, it is not the design load but the effect of the load (moment, axial force, etc), which is of ultimate interest. Design codes in the United States (1) and Canada (19) require that structural elements shall be designed for the maximum effect of the design live load. The distribution of loads causing the maximum load effect in a particular structure can be determined by drawing an influence line for the structure and applying loads on those parts of the structure where their effects will be additive. In a continuous beam or a one way slab, the influence lines of all major load effects (8) indicate that the loadings required for maxima can be reduced to two types:

a. full loading on alternate spans,

b. full loading on two adjacent spans and on alternate spans beyond these.

These two are used interchangeably as required to give the maximum load effect at each section.

Section 8.9 of the ACI code (1) follows a similar load pattern for the live load effect except that the second type is simplified to full loading on two adjacent spans only. The change has little effect on the corresponding maximum load effect because the influence ordinates in the alternate spans are relatively very small. The ACI load pattern is referred to in this report as the "traditional load pattern".

The probability of occurrence of the above extreme loading cases is not known, however. Moreover, this traditional pattern may not produce the maximum load effects with a consistent probability of being exceeded.

1.1.2 Pattern Loadings of Dead Load

The common practice of structural design in the United States and Canada does not take into account the pattern loading effects of the dead load. In other words, the factored dead load is considered constant on all spans in a structure. Section 2.3.3.1 of the British concrete design code (4), however, requires that a pattern loading of dead loads should be considered with the minimum design load equal to the nominal dead load and the maximum equal to the factored dead load to account for the most unfavorable condition. The load pattern considered is similar to the traditional load pattern (Sect. 3.2.2.1 of Ref. 4).

1.2 Purpose and Scope of this Study

The purpose of this study is to find a simple design live load pattern which will produce all load effects of interest with a consistent probability of being exceeded for the extreme cases. As a part of this study, the traditional load pattern is evaluated statistically. Simple analyses of a series of elastic frames are carried out to compute the load effects correponding to randomly generated distributions of live loads. A Monte Carlo process is used to generate the loadings according to the probabilistic live load model, which has been described in the literature (5, 6, 11, 16, 20, 21). Linear programming is used to develop an equivalent design loading pattern for the extreme loading cases. Although the Monte Carlo study is based on office live loads, the methodology can be extrapolated to other types of occupancy.

For the dead load, the purpose of the study is to investigate the effect of pattern loadings. Because of the simplicity of the probabilistic dead load model, direct statistical calculation of dead load effects for the extreme cases is performed. Again, linear programming is used to develop an equivalent design dead load pattern, which is compared with North American and British practice.

1.3 Outline of Contents

Chapter 2 contains statistical descriptions of the live load and the dead load, which have been developed by several investigators (11,16,20). The analytical study is described in Chapter 3, which describes the Monte Carlo analysis and linear programming for the live load effects, and the statistical calculations of the dead load effects. The details of the structures studied are also presented. In Chapter 4, the results of the study are discussed together with the development of a most desirable design load pattern for the live load and the dead load. Finally, a summary and conclusions are presented in Chapter 5.

2. LOADING MODELS

2.1 Live Load Model

2.1.1 Nature of Live Load

The gravity live load on a floor area can be represented by the two components: a sustained load and a transient load. The sustained load acts continuously in time and remains relatively constant between distinct changes which are generally considered to occur when there are changes in tenants or occupancy. The sustained load consists of furniture, moveable property and personnel normally present. Such loads are usually measured in live load surveys. For certain periods, referred to in this report as "vacant periods", the sustained load may be entirely absent.

The transient load happens infrequently with a relatively high intensity and short duration in the order of hours. It is caused by unusual events such as crowding of people and stacking of furniture on a certain area. These special loading situations are not usually observed in live load surveys.

2.1.2 Live Load Surveys

A major live load survey (18) comprising thirty two buildings was carried out in England in 1965 to 1967 by the Building Research Station (BRS). More recently an extensive live load survey (9) of office buildings in the United States of America was conducted in 1974 to 1975 by the

National Bureau of Standards (NBS). Twenty three buildings were surveyed. Statistical analysis (9) of the NBS survey data indicates that the mean sustained live load is strongly correlated to room use, but is independent of room area. In order to enhance the flexibility of room usage, the mean load of all office buildings surveyed is used in the live load model presented in Section 2.1.3. The mean unit load of all the randomly selected buildings from the NBS survey is 0.555 kN/square meter (11), which is very close to 0.565 kN/square meter (17) obtained from the BRS survey.

The variance of unit load, Var(u), appears to decrease with increasing floor area, A (see Table 7 of Ref. 18, Figs 29,30 of Ref. 9 and Fig. 2 of Ref. 11). The analysis (11) of the NBS survey data yields the following equation, discussed more fully in Section 2.1.3(a):

$$\mathrm{Var}(u) = 0.0601 + 1.407/A \qquad (2.1)$$

In comparison, Var(u) obtained by McGuire and Cornell (17) from the BRS survey data is:

$$\mathrm{Var}(u) = 0.0466 + 1.782/A \qquad (2.2)$$

The values of Var(u) computed from these equations have been shown to be very close (see Fig. 3 of Ref. 11).

The results of other surveys associated with a room area of about 18.6 square meters (200 square feet) are listed in Table 2.1(5). The survey area weighted average of the mean unit load is 0.566 kN/square meter. Values ranging from 0.555 to 0.575 have been presented in Refs. 11, 12 and 17. The survey area weighted average of the standard deviation based on an area of 18.6 square meters is 0.35 kN/square meter.

2.1.3 Sustained Live Load Model

(a) Sustained Load

The instantaneous sustained live load intensity, w(x,y), at any point in a building can be represented by a linear relation (21):

$$w(x,y) = m + \gamma_b + \gamma_f + \epsilon(x,y) \tag{2.3}$$

In this relation, m is the mean survey load for the type of occupancy considered. Two independent zero-mean random variables, γ_b and γ_f ,represent the deviation of the building average unit load from m and the deviation of the floor average unit load from the building average, $m + \gamma_b$, respectively. The zero mean property of γ_b and γ_f is justified as the mean unit load is independent of the floor area (Sect. 2.1.2). The term $\epsilon(x,y)$ is a stochastic process representing the deviation from the floor average, $m + \gamma_b + \gamma_f$. It has a zero mean and is independent of γ terms.

In general, the values of $\epsilon(x,y)$ at two points are correlated because if the load intensity is higher than the

TABLE 2.1 SUMMARY OF LIVE LOAD SURVEY RESULTS FOR OFFICES (From Ref. 5)

Year	Survey	Place	Surveyed area (m^2)	Survey load	
				Mean (KN/m^2)	Std. dev.* (KN/m^2)
1893	Blackall	U.S.A.	7100	0.780	-
1923	Coley	U.S.A.	3700	0.555	-
	Blackall	U.S.A.	1100	0.302	0.13
	McIntyre	U.S.A.	3470	0.436	-
1931	White	England	13300	0.517	-
1947	Dunham	U.S.A.	46370	0.685	-
1952	Dunham et al	U.S.A	1270	0.484	-
1968	Bryson & Gross	U.S.A.	4990	0.570	0.25
			4500	0.464	0.21
1969	Karman	Hungary	13900	0.603	0.33
1970	Mitchell & Woodgate	England	160,000	0.565	0.38**
1973	Paloheimo	Finland	3000	0.330	-
1974	Culver	U.S.A.	53900	0.555	0.37***
	Dayeh	Australia	28000	0.412	0.25
	Schwartz	U.S.A.	400	0.776	0.52
			Area-weighted average	0.566	0.35

* Based on an office area of approximately 18.6 m^2 (200 sq.ft)

** Derived from Eq. 2.1

*** Derived from Eq. 2.2

floor average at a certain point, then it is likely that the load at a nearby point is also high. Nevertheless, Hauser (13) has shown that this correlation decays rapidly as the distance between the loads increases. The analysis was based on the BRS survey results (18); the correlation coefficient was shown to be less than 0.25 for distances greater than 3 meters. Therefore, if the floor area is not too small, $\epsilon(x,y)$ can be reasonably assumed to be an uncorrelated process (17).

Based on Eq. 2.3 and the preceding assumptions, the mean and standard deviation of the instantaneuos unit load, u, on a floor area, A, are (17):

$$E(u) = m \tag{2.4}$$

$$Var(u) = a + b/A \tag{2.5}$$

in which a is the sum of the variance of γ_b and γ_f, and b is an experimental constant.

Eq. 2.5 suggests that when A is sufficiently large, Var(u) approaches a. Hence a is determined from survey data for a large area. The value for b is selected such that Eq. 2.5 gives a good fit to survey data for other areas. The BRS survey of office live loads shows a standard deviation of 0.216 kN/square meter at A ≐ 192 square meters for floors other than basements and grounds (Table 7 of Ref. 18). This yields a = 0.0466 kN^2/m^4. In Ref. 17 the BRS data, which are presented in Table 7 of Ref. 18, are approximated with a curve based on Eq. 2.5 with b = 1.782 kN^2/m^2. The analysis (11) of the NBS survey data of office live loads presented in Figs. 29, 30 of Ref. 9 yields a = 0.0601 kN^2/m^4 and b = 1.407 kN^2/m^2. The two curves are very similar as shown in Fig. 3 of Ref. 11. Using the standard deviation of 0.35 kN/square meter for A = 18.6 square meters presented in Section 2.1.2 and setting a equal to 3.0 % of b, compared to 2.6% from the BRS survey and 4.3% from the NBS survey, yields a = 0.044 kN^2/m^4 and b = 1.460 kN^2/m^2. These values are used for analysis.

(b) Sustained Load Effect

To relate a random set of loads distributed over an area to a particular load effect (moment, shear, etc.), the sum of the products of the magnitude of the loads at each point times the height of the influence surface for that load effect at that point is used. The mean and standard deviation of the sustained equivalent uniformly distributed load (EUDL), L, which will produce the same load effect as the actual random set of loads are (17):

$$E(L) = m \tag{2.6}$$

$$Var(L) = a + kb/A_I \tag{2.7a}$$

in which

$$k = \int_0^1 \int_0^1 I^2(x,y) \Big/ \left[\int_0^1 \int_0^1 I(x,y)\right]^2 \tag{2.7b}$$

The function I(x,y) is the normalized influence surface of the load effect of interest over an influence area, A_I.

The influence area, A_I, is generally assumed to be equal to the floor area over which the influence surface of the load effect considered is significantly different from zero (17). For beam load effects in a conventional one way slab or beam-column frame building, the significant influence area is generally considered as twice the conventional tributary area of one span. For the column load effects, the significant influence area is 4 times the tributary area.

The values of k have been shown to be insensitive to different end conditions of a single beam and number of spans in a frame. Using approximate polynomial influence surfaces, k is found to be 2.04 for end moments, 2.76 for mid-span moments, 2.20 for column loads and similar values for other load effects (17). Thus the variance Var(L) computed using Eq. 2.7(a) for A_I = 37.2 square meters (400 sq. ft.) ranges from 0.144 to 0.179 for the extreme cases. Because of the relative insensitivity of k to load effect types, k = 2.2 for all load effect types has been suggested (11) and it is used in this study.

(c) Statistical Distribution of Sustained Load

Five major live load surveys have been analysed in Ref. 6 using three probability models: normal, lognormal, and gamma. The results indicate that the gamma distribution is generally better in the overall fit and gives the best fit in two-thirds of the cases. Furthermore, the gamma model is in good agreement with the survey data of most occupancy types in upper tail region and even at the 99.9% level for offices (see Table 2 of Ref. 6). As the EUDL differs from the actual unit load only by the weighting function I(x,y), the instantaneous EUDL is also considered as gamma distributed.

(d) Statistical Distribution of Sustained Load Duration

Throughout the life of a floor area, the sustained load is likely to be influenced primarily by the change in occupancy while minor fluctuations during the same occupancy can be ignored. Therefore the sustained load is assumed to be constant during the time between occupancy changes. The load during any such constant period will be represented by the instantaneous sustained EUDL.

The NBS office live load survey data for occupancy duration exhibit an exponential distribution as shown in Fig. 18 of Ref. 9. The mean duration is 8.0 years, which is close to the mean value of 8.8 years obtained by the BRS survey (18). The BRS survey data for occupancy duration also agree fairly well with an exponential model as shown in Fig. 5 of Ref. 21. Figure 18 of Ref. 9 suggests a minimum duration of 1 year. In this study the the duration of occupancies is modelled with an exponential distribution with a minimum duration of 1 year and a mean of 8 years.

2.1.4 Vacant Period

Vacancy or unloading events in an area are generally neglected in deriving the lifetime maximum EUDL as a conservative assumption (17). In this study, however, unloading events in a span are significant and should be considered. Although no data are available about office vacancy, it is likely that unloading occurs during periods of occupancy changes, repainting or remodelling of furniture. In this study, an area is arbitrarily assumed to be entirely unloaded during each change of occupancy. Within the same occupancy, additional unloading events are assumed to occur every 2 years.

The duration of an unloading event is assumed from 1 day to 2 months and is approximated by a uniform distribution. The sensitivity of load effects to the above-mentioned parameters is examined in Sect. 4.1.1(b).

2.1.5 Transient Live Load Model

(a) Transient Load

Very few data are available on the transient live load. Nevertheless, a model has been proposed (20) assuming a series of randomly distributed load cells, each of which contains a cluster of concentrated loads. Similar to the sustained load, the load effect is also considered to obtain the EUDL. The mean, m_E, and variance, σ_E^2, of the EUDL, E, associated with one transient load event are (17):

$$m_E = \lambda m_R m_Q / A_I \qquad (2.8)$$

$$\sigma_E^2 = \lambda k (m_R \sigma_Q^2 + m_Q^2 \sigma_R^2 + m_R^2 m_Q^2) / A_I^2 \qquad (2.9)$$

in which λ = mean number of load cells in the influence area, A_I . Q and R are random variables representing the weight of of a single concentrated load in the cell and the number of loads per cell respectvely. k is equal to 2.2 (Sect. 2.1.3(b)).

There are no data available with which Eqs. 2.8, 2.9 can be estimated with confidence. McGuire and Cornell (17) have estimated the values of the following parameters (Table 1 of Ref. 17): (m_Q, σ_Q) = (0.65, 0.13)kN; (m_R, σ_R) = (5, 2). In estimating λ, which is area-dependent, it is reasonable to assume that the average number of load cells per unit area (and therefore the mean and standard deviation of unit load) decreases with increasing area. McGuire and Cornell (17) have proposed the following values of λ, which are a function of influence area, A_I :

$$\begin{aligned} A_I &> 37.2\ m^2 & \lambda &= \sqrt{(A_I - 15.3)/0.84} \\ A_I &= 27.9\ m^2 & \lambda &= 3.99 \\ A_I &= 18.6\ m^2 & \lambda &= 2.7 \\ A_I &= 9.3\ m^2 & \lambda &= 1.4 \end{aligned} \qquad (2.10)$$

The mean and standard deviation of the unit transient load from Eqs. 2.8 and 2.9 based on McGuire and Cornell's assumptions are plotted as a function of influence area in

Fig. 2.1. The figure shows the decrease of the mean and standard deviation with increasing area.

Ellingwood and Culver (11) estimated that $(m_Q, \sigma_Q) =$ (0.67, 0.11)kN, $(m_R, \sigma_R) = (4, 2)$ and

$$\lambda = \sqrt{(A_I - 14.4)/0.58} \tag{2.11}$$

Values of the mean and standard deviation from Eqs. 2.8 and 2.9 based on these assumptions are plotted in Fig. 2.1 for comparison with McGuire and Cornell's curves. The figure shows that the two sets of assumptions give curves that are very similar for $A_I > 30\ m^2$. For $A_I < 30\ m^2$, Ellingwood's curve appears unreasonable as the mean unit load decreases very rapidly with decreasing area (the same unreasonable trend occurs in standard deviation as $A_I < 20\ m^2$) In personal communication with Ellingwood, it has been suggested that the mean unit load, and therefore the average number of load cells per unit area, should be assumed to be constant when the influence area is less than 30 m^2, ie,

$$A_I < 30\ m^2 \qquad \lambda = 0.173A_I \tag{2.12}$$

The mean and standard deviation based on Eq 2.12 are also plotted in Fig. 2.1, which shows a reasonable trend in the standard deviation. In this study, Eqs. 2.11 (for $A_I > 30\ m^2$) and 2.12 are used to calculate the mean and standard deviation of the transient EUDL.

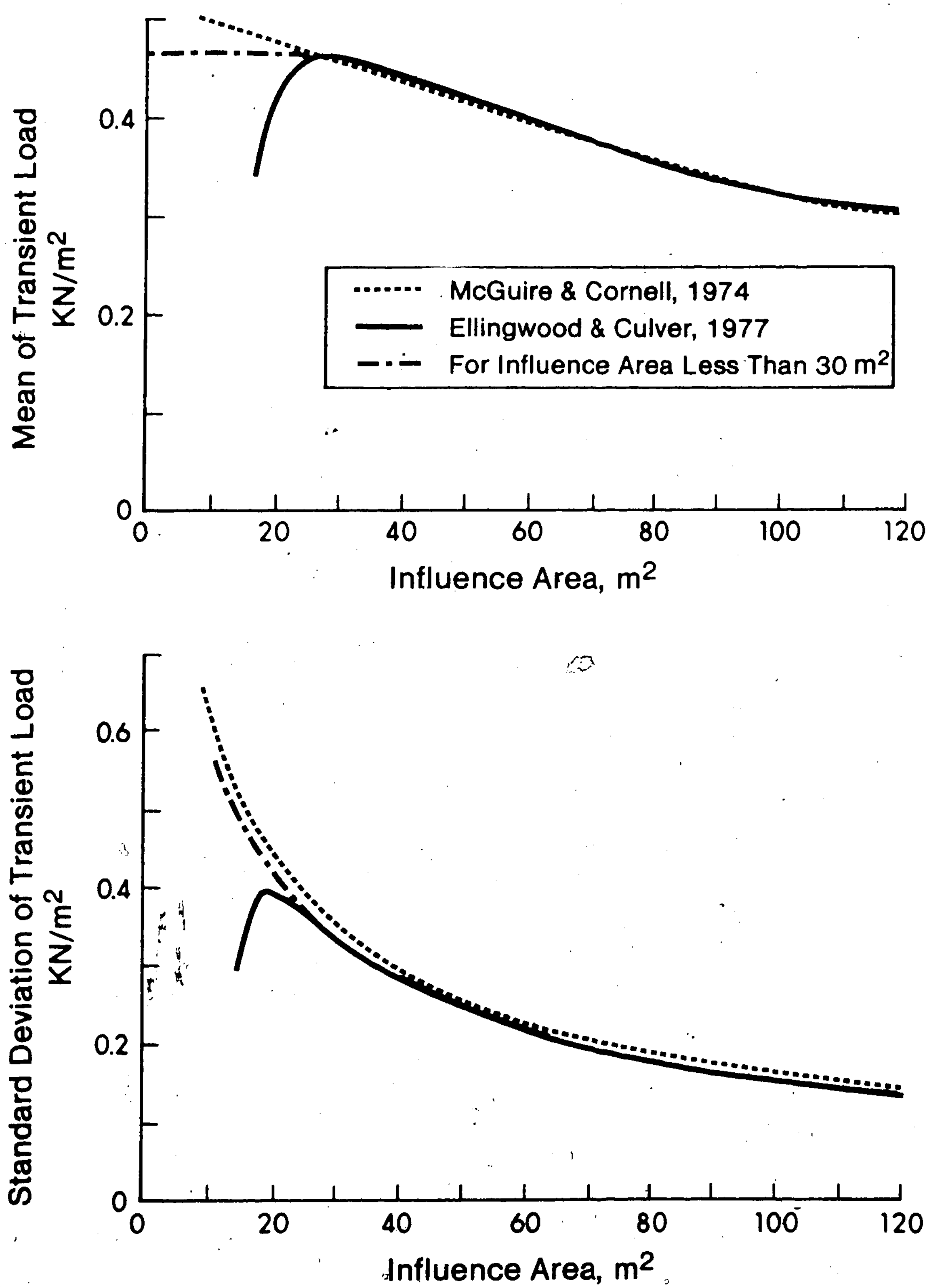


Fig.2.1 Mean and Standard Deviation of Transient Load

(b) **Statistical Distribution of Transient Load Event**

Peir (20) has shown by means of a numerical example that the distribution of the transient load can be represented by a gamma distribution. The occurrence of the transient load is generally assumed to follow a Poisson process (17) based on the assumption that transient load events are independent in space and time. The time between two consecutive transient load events is assumed to conform to an exponential distribution with a mean of 1 year (11,17) for offices.

The duration of each transient load event is assumed to be 8 hours (15). Moreover, transient load events are assumed not to overlap, although a transient load event can be followed immediately by another transient load event so that the minimum time between occurrences is 8 hours.

2.1.6 Live Load Model Used in this Study - Summary

The sustained EUDL is gamma-distributed with a mean of 0.566 kN/square meter and a variance equal to

$$Var(L) = 0.044 + 3.212/A_I \tag{2.13}$$

The duration of a sustained load event is modelled with an exponential distribution with a minimum duration of 1 year and a mean of 8 years.

One unloading event occurs at the start of each sustained load event and additional unloading events occur every 2 years during that sustained load event. The duration of a vacant period is approximated by a uniform distribution with a minimum duration of 1 day to a maximum of 2 months.

The transient EUDL is assumed to be gamma-distributed with a mean equal to

$$m_E = 2.68\lambda/A_I \qquad (2.14)$$

and a variance equal to

$$\sigma_E^2 = 19.9\lambda/A_I^2 \qquad (2.15)$$

where

$$\lambda = \sqrt{(A_I - 14.4)/0.58} \qquad \text{for } A_I > 30 \text{ m}^2 \qquad (2.16)$$

$$\lambda = 0.173\ A_I \qquad \text{for } A_I \leqslant 30 \text{ m}^2 \qquad (2.16)$$

The duration of each transient load event is 8 hours. The time between two consecutive transient load events is modelled with an exponential distribution with a minimum time of 8 hours and a mean of 1 year.

2.2 Dead Load Model

The dead load includes the weight of structural members, permanent equipment and permanently supported non-structural elements such as partitions, roofing and installations. During the life of a structure, the dead load maintains a relatively constant magnitude though slight changes may occur.

Magnitudes of dead loads may vary from those assumed due to variations in size of members, density of material and weight of non-structural items. Generally, the weights of non-structural elements have a strong effect on the variability in the dead load (12).

The probability distribution of the dead load is generally assumed to be normal. The ratio of mean load to nominal dead load is commonly assumed to be unity and the coefficient of variation (12) from 0.06 to 0.15. In this study, the mean load is assumed to be 1.05 times the nominal load, and the coefficient of variation of the dead load is taken as 0.10, based on Ref. 12.

3. ANALYTICAL STUDY

3.1 Live Load Study

A Monte Carlo analysis is used to generate distributions of the maximum lifetime loadings at various points in a structure. A linear programming technique is then used to select a pattern loading to represent a particular fractile of the maximum lifetime loadings.

3.1.1 Monte Carlo Analysis

(a) Description of Monte Carlo Technique

The Monte Carlo technique is a method to generate a large number of hypothetical samples by computer simulations. This method requires a mathematical relationship between the variable being studied and each basic variable which affects it, plus a statistical description of each basic variable. A set of values of the basic variables is randomly selected according to their statistical properties and the final variable is determined using the relationship between the basic variables and the final variable. This procedure is repeated many times to get a large sample of the final variable for statistical analysis.

In this study, the Monte Carlo technique is used to generate large samples of lifetime minimum and maximum live load effects at various points in a given frame. During the lifetime of the frame, random values of the sustained load, vacant period and transient load are continuously generated, and load effects are calculated accordingly. The minimum and maximum values of each load effect during the lifetime are eventually selected. This procedure is repeated enough times (500 to 5000 times) to give usable statistical distributions of the lifetime minimum or maximum values.

(b) Assumptions in the Computer Program

The computer program studies a given one-story frame that is randomly loaded according to the live load models described in Chapter 2. The life of the frame is assumed to be 50 years.

During the lifetime of a given floor area, successive loading events are assumed to be independent. Based on the insignificant correlation between the sustained live loads of two rooms found in the analysis (7) of loading data, it is reasonable to assume no correlation for the sustained load before or after an occupancy change. Consecutive transient load events (Sect. 2.1.5(b)), are also assumed to be independent, and so are unloading events.

The number of rooms and room arrangement on the floor area of a frame give rise to different loading changes occuring over the floor. It has been shown (16) that it is conservative to assume the entire 'significant' influence area to be occupied by one room. As mentioned in Section 2.1.3(b), the significant influence area is the area over which the influence surface of the load effect considered is 'significantly' different from zero. For most beam load effects, the influence surface in the spans other than the span where the load effect is considered is relatively small. For this reason McGuire and Cornell (17) have taken the significant influence area for all beam load effects to be the influence area of the span considered (ie twice the tributary area of the span considered).

For the interior support moments, however, the influence surface is usually significant in the two spans adjacent to the support. Conservatively, the significant influence area equal to twice the tributary area of two adjacent spans should also be investigated. This matches the situation in which one room occupies the entire influence area of two adjacent spans so that the same loads occur over two spans at any time. For axial forces in interior columns, the significant influence area is also equal to twice the tributary area of two spans (ie 4 times the tributary area of the column) because the influence surface is significant in two adjacent spans. McGuire and Cornell (17) have suggested this influence area in the case of interior columns but not in the case of negative moments at interior supports.

In the Monte Carlo analysis, two loading distributions are assumed:

Case MC1 - the influence area of each span is occupied by one room.

Case MC2 - similar to Case MC1 except that the influence area of two particular adjacent spans are occupied by one room.

Because Case MC1 is a conservative assumption for most load effects as discussed above and appears most likely to happen, it is investigated for all load effects of interest. For comparison Case MC2 is also examined for interior negative support moments and interior column loads.

Case MC1 implies that all sustained load, transient load and unloading events on adjacent spans are independent (Sect. 2.1.5(b)), and the influence area used in Eqs. 2.7, 2.9 to calculate the standard deviation of load magnitudes is twice the tributary area of the loaded span. For Case MC2, the loads are identical on two adjacent spans where the interior support moments and the column load are considered; the influence area used in Eqs. 2.7, 2.9 for the loads on the two spans is twice the tributary area of the two spans. The loadings on other spans are independent as in Case MC1.

The computer program based on Case MC1 is described in the next section. For Case MC2, the computer program is only slightly modified to apply identical loads on two particular adjacent spans. The pattern loading models developed later will be based on Case MC1 loadings for all load effects. For comparison, the models will also be based on Case MC2 loadings for load effects which are more critical in

Case MC2 loadings (interior column loads, etc.) and based on Case MC1 loadings for all other load effects. The latter case is later referred as Case MC3.

(c) **Description of the Computer Program**

The computer program is capable of generating estimates of the lifetime minimum and maximum values of a number of load effects in a population of 500 to 5000 identical single storey frames. The 50 year life of the frame is divided into 8-hour intervals selected so that the length of one interval is the same as the duration of a transient load with the result that during any interval the loads on any span do not change. The loads on each span are randomly chosen from the loading distributions and the load effects caused by the loads occuring in each interval are determined. When this has been done for each load interval in the 50 year life, minimum and maximum values for each load effect during this period are selected and denoted as the lifetime minimum and maximum values, respectively. The above steps are denoted as one run in the computer program. In most cases studied, 5000 runs for one particular frame are used to obtain 5000 lifetime minimum and maximum values of each load effect considered.

The structural analysis in the program is simplified by using an influence coefficient matrix. The matrix for a particular frame is multiplied by the vector consisting of values of the uniformly distributed load on each span at a particular instant to obtain the corresponding load effects. The formulation of the influence coefficient matrix is shown in Appendix A.

Condensed flow diagrams of the computing program are shown in Figs. 3.1 to 3.3 (the program listing is in Appendix B). All the required data such as the statistical parameters, the influence coefficient matrix for the frame studied, the number of spans and the number of runs are first read in. Then the random sustained loads, unloading events and transient loads are generated for each span in the 50 year lifetime. For efficiency, actual calculation of the total load on each span is done only when the load on that span changes; the interval number at the time of change is also recorded. Similarly the load effects are calculated using the influence coefficient matrix whenever the total load changes on any span. At the final stage of each run, lifetime minimum and maximum values of load effects are selected. Finally, the lifetime minimum and maximum values of each load effect from each run are sorted and printed out.

3.1.2 Linear Programming

(a) Description of Linear Programming

Linear programming is a technique to provide optimum solutions to problems with a number of possible solutions. The problem has to be formulated into a linear program which is a mathematical model consisting of an objective function

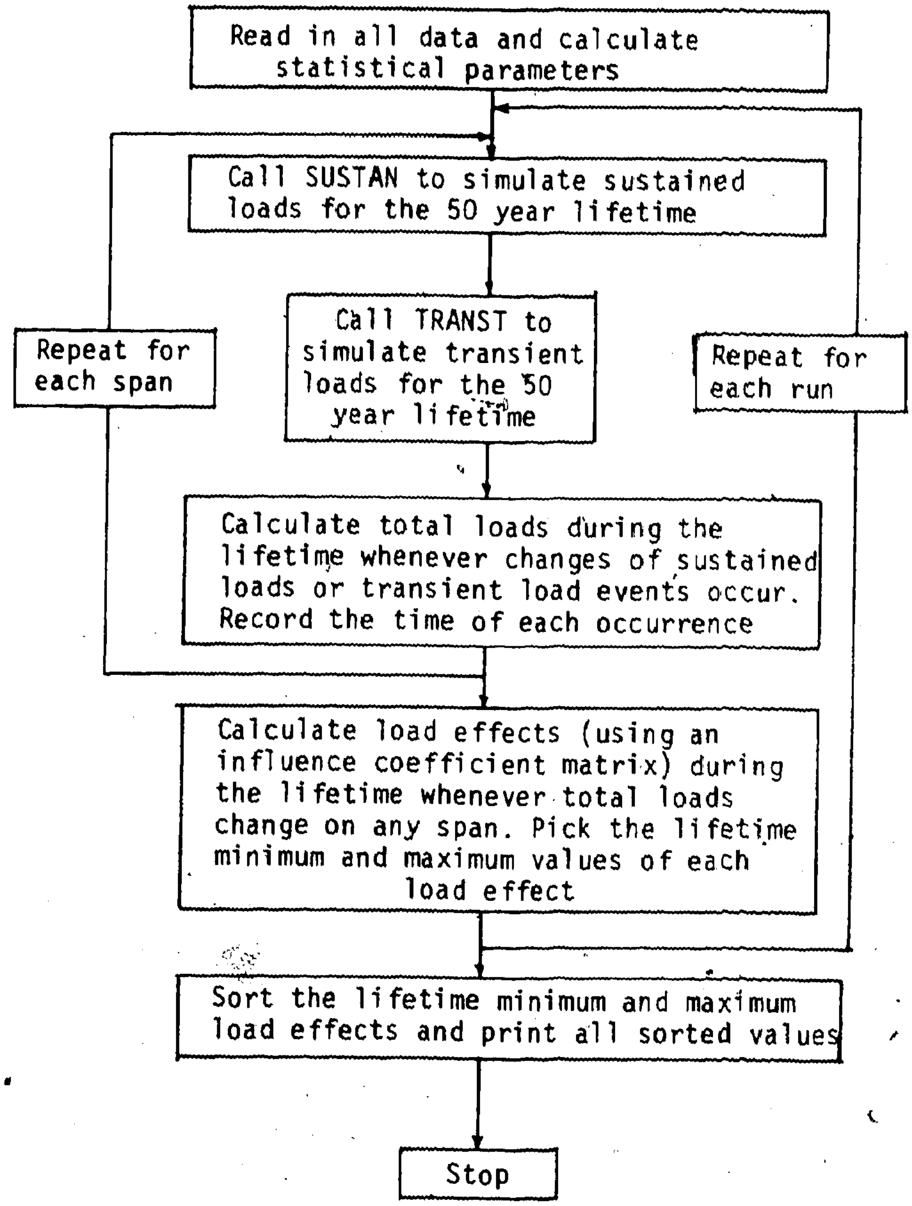


FIG. 3.1 CONDENSED FLOW DIAGRAM OF THE MONTE CARLO PROGRAM

SUBROUTINE SUSTAN

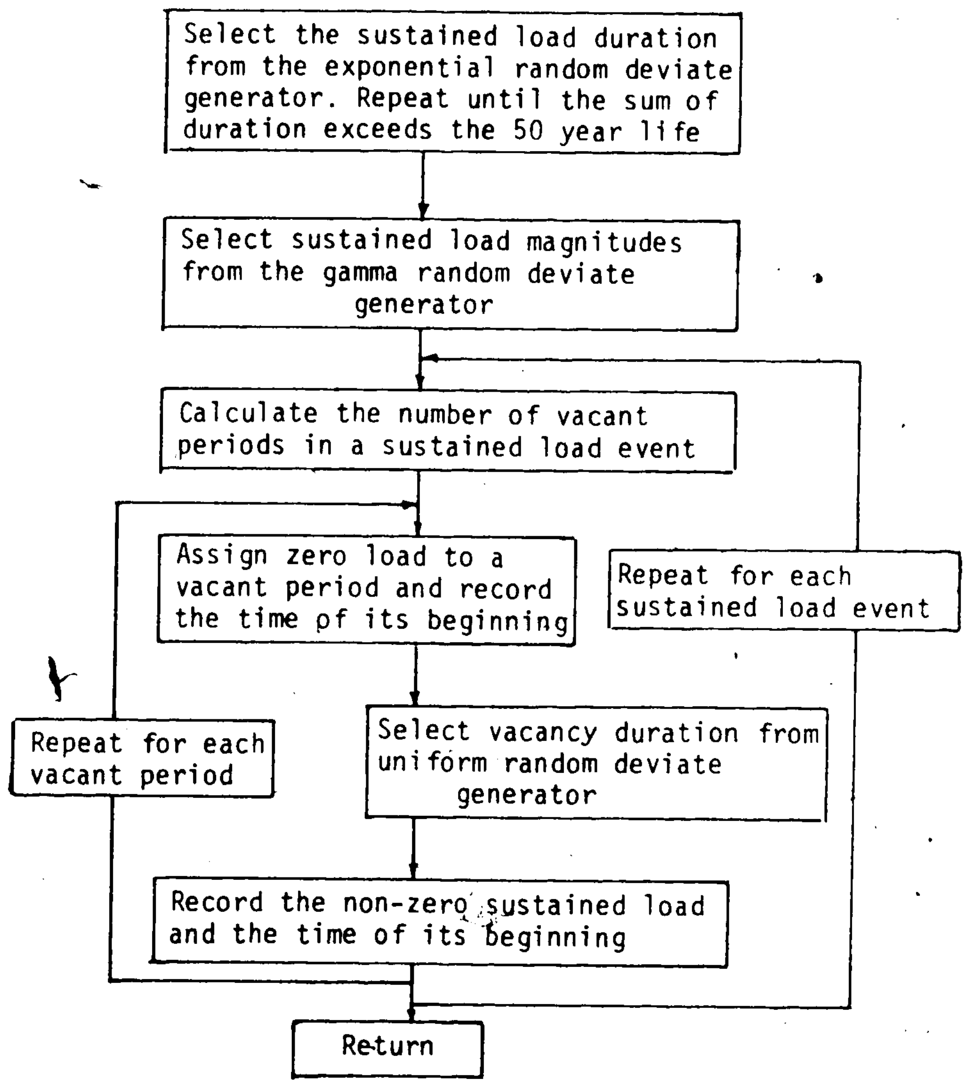


FIG. 3.2 SUBROUTINE SUSTAN OF THE MONTE CARLO PROGRAM

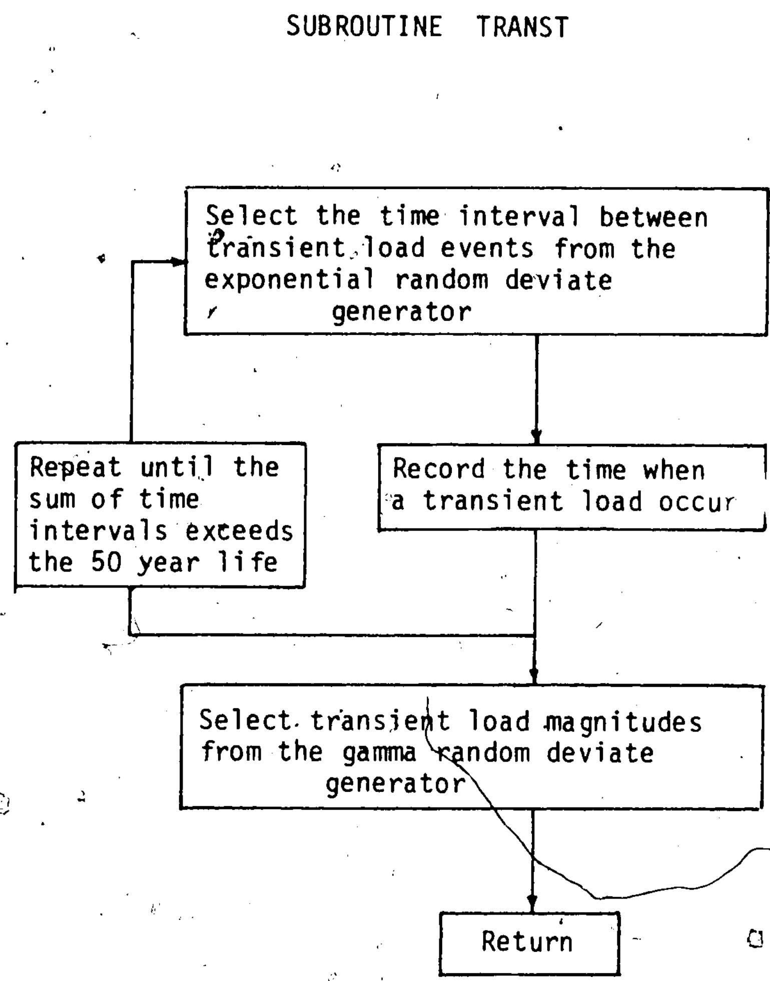


FIG. 3.3 SUBROUTINE TRANST OF THE MONTE CARLO PROGRAM

and constraints, both in terms of linear expressions of variables. The objective function is the factor to be optimized such as maximization of profits or minimization of costs. The optimization is restricted by the constraints on the values of the variables. The constraints are in linear forms of inequality expressions or equations.

A linear program with two variables can be solved graphically. For more than two variables the Simplex method (10) has been developed. The method is an iterative technique performed using a computer. Theoretically speaking, the method is capable of giving the true optimal solution to the problem solved. In practice, however, the solution may be inaccurate due to the rounding errors accumulated in repeated computations. For these and other reasons, the so called Revised Simplex method has been developed to reduce the errors and thus improve the solution. This method is described more fully in Ref. 10. In this study the Revised Simplex method is used to solve linear programs; the computer program for this method is available from the IMSL library (14).

In this study the Monte Carlo results for the extreme loading cases are approximated with design load patterns. Linear programming is used to find the load factors corresponding to a particular load pattern to best approximate the Monte Carlo results. The objective is to select load factors to minimize the differences between the factored load effects from a particular load pattern and the corresponding load effects from the Monte Carlo results. A series of different load pattern schemes are considered to select the most desirable model.

(b) **Design Loadings and Load Fractiles Used in the Study**

In any design load pattern to be discussed, the load on each span is the product of a load factor and the nominal live load. The nominal live load, L_n, is a defined quantity based on Section 4.1.6 of National Building Code of Canada (19):

$$L_n = (0.3 + \sqrt{9.8/A_T})L_o \qquad \text{for } A_T > 20 \text{ m}^2$$
$$L_n = L_o \qquad \text{for } A_T \leqslant 20 \text{ m}^2 \qquad (3.1)$$

in which A_T is the tributary area in square meters. Since consideration is given to office buildings in this study, L_o is equal to 2.4 kN/square meter.

Basically two types of design load patterns should be considered. The first type is similar to the traditional load pattern discussed in Section 1.1.1, except that there may be a small load on the unloaded spans. The second type is a simpler pattern suggested by Beeby (2), consisting of heavy loading on only one span and light loading on all other spans. Other patterns to be considered will be derived from these two basic types. The design load patterns will be described more fully in Chapter 4.

In order that the factored load pattern selected in this part of the study should have a probability of occurrence equivalent to that implied by the National Building Code (19) or ANSI (12) load factors for live load on a simple span, the probability of occurrence of various factored live loads has been studied. The probabilty that the lifetime maximum load will be less than or equal to the factored live loads based on current NBC, proposed ANSI and current ANSI are listed in Table 3.1 for three different influence areas. The values in the table have been determined by using a Monte Carlo program based on the one described in Section 3.1.1, except that the number of spans has been set equal to one and the output is expressed in terms of lifetime maximum values of the live load instead of load effects. The results indicate that the factored live load values are close to the 0.999 fractile except for small areas. For this reason, the 0.999 fractiles of the lifetime minimum or maximum load effects are used in the modelling of pattern loadings.

(c) Formulation of the Linear Program

For convenience in the following discussion, M_{pi} is used to represent the absolute value of the algebraic minimum or maximum factored load effect i which can be obtained at a critical section from a design load pattern, and M_{mi} is the corresponding absolute value of a high fractile of the distribution of the minimum or maximum load effect i from the Monte Carlo analysis. Note that the

TABLE 3.1 PROBABILITY OF THE LIFETIME MAXIMUM LOAD BEING LESS THAN OR EQUAL TO THE FACTORED LIVE LOAD

A_I	$1.5L_n^*$ (NBC)	$1.6L_n^{**}$ (Proposed ANSI)	$1.7L_n^{***}$ (Current ANSI)
20 m^2	0.925	0.956	0.973
50 m^2	0.9982	0.9986	0.9954
100 m^2	0.9984	0.9988	0.9912 to 0.9970

* Assume $A_T = 0.5A_I$ in Eq. 3.1

** $L_n = (0.25 + \frac{15}{\sqrt{A_I}})L_o$ for $A_I > 400$ sq.ft

$L_n = L_o$ for $A_I \leqslant 400$ sq.ft

*** $L_n = [\,1 - \min\{0.0008A_T,\ 0.6,\ 0.23(1 + \frac{D_n}{L_o})\}\,]L_o$

Assume $A_T = 0.5A_I$

$\frac{D_n}{L_o} = 0.67$ to 2.0

subscript p refers to pattern and m refers to Monte Carlo.

In the optimization, the penalty for values of M_{mi} less than M_{pi} (an unconservative case) is twice that for M_{mi} greater than M_{pi}. The objective is to minimize the sum of penalties for all load effects to be fitted by a load pattern. In this way the results tend to be conservative.

As discussed in the previous section, M_{mi} are taken equal to absolute values of the 0.999 fractiles from the Monte Carlo results. Two cases are assumed in the modelling with a load pattern. The first case is that the Monte Carlo results based on Case MC1 discussed in Section 3.1.1(b) are used for M_{mi} in the linear programming. The second case is that the Monte Carlo results based on Case MC3 discussed in Section 3.1.1(b) are also used for M_{mi}.

The load effects, M_{pi}, for a given load pattern and frame are determined in the linear function of load factors using the influence coefficient matrix (Appendix A) as follows:

$$M_{pi} = aL_nC_1 + bL_nC_2 \qquad (3.2)$$

where a and b are load factors, which will be defined more fully in Chapter 4. The nominal live load, L_n, has been defined by Eq. 3.1. The term C_1 is the sum of the influence coefficients of load effect i for the spans which are loaded by the large uniform load, aL_n, and C_2 is the sum of the remaining influence coefficients of load effect i. Because

M_{pi} are defined as absolute values in the linear program, the algebraic sign of each influence coefficient for negative load effects (negative moments, negative shears) has to be converted to the opposite sign when formulating Eq. 3.2.

The linear program is formulated as follows: The objective function is

$$\text{minimize } z = \sum_{i=1}^{n} d \qquad (3.3)$$

where n is the total number of load effects to be fitted, and d_i is the penalty for load effect i. The penalty, d_i, is defined in the following:

If $M_{pi} > M_{mi}$

$$d_i = M_{pi} - M_{mi} \qquad (3.4)$$

If $M_{pi} < M_{mi}$

$$d_i = 2(M_{mi} - M_{pi}) \qquad (3.5)$$

Note that both M_{pi} and M_{mi} are absolute values. The parameters d_i and M_{pi} (in terms of a and b) are variables, while M_{mi} are known values as defined previously. Eqs. 3.4, 3.5 are transformed into constraints in the linear program as shown in Eq. 3.6. The inequality expressions are used instead of equations because eqs. 3.4 and 3.5 are never satisfied simultaneously. This penalty expression is arbitrarily chosen such that unconservative estimates of load effects will receive twice the penalty as an equal conservative estimate. No significant difference in load factors has resulted from using multipliers of 3 or 4 in Eq. 3.5.

The constraints are:

$$\left.\begin{aligned} M_{pi} - M_{mi} &\leq d_i \\ M_{mi} - M_{pi} &\leq 0.5d_i \end{aligned}\right\} \quad \text{for } i=1,n \qquad (3.6)$$

The variables, a, b and d_i, are also defined as non-negative variables in the linear program. In addition to the above constraints, the load factors a and b are required to be less than 2.0 (except for tributary areas less than 20 m^2).

3.1.3 Structures Studied

An infinite number of variations in types of frames could be studied. To limit the problem, symmetrical two, three and five span frames based on Section 8.9.1 of the ACI code (1) are studied. These structures are single-story sections of frames with the far ends of columns above and below the floor assumed to be fixed.

Symmetrical frames with variations in the following variables are studied:

a. the tributary area,

b. the number of spans,

c. the span length ratio,

d. the column to beam stiffness ratio.

Variable a affects the loads on a span; variables b, c and d affect the influence coefficient matrix. The frames studied are listed in Table 3.2. The values of the variables are chosen in order to get the effects of each variable.

The load effects of interest in this study are moments at the ends, quarter points and mid points of the spans, end shears, column moments and axial forces. So as to be able to construct moment envelopes, lifetime minimum and maximum values are required. For the others, only absolute lifetime maximum values are needed. Since the frames considered herein are symmetrical, only the load effects on one half of a frame are required for analysis. Axial deformations of members are ignored in the structural analysis.

3.2 Dead Load Study

3.2.1 Statistical Calculation

Direct statistical calculation of dead load effects is based on the following theorem (3): If a random variable, Y, is the sum of normally distributed variables, X_i; namely,

$$Y = \sum_{i=1}^{n} a_i X_i \tag{3.7}$$

where a_i is the coefficient associatec with X_i, then Y is also normally distributed. The mean of Y is (Eq. 2.4.81a of Ref. 3):

$$E(Y) = \sum_{i=1}^{n} a_i E(X_i) \tag{3.8}$$

TABLE 3.2 FRAMES STUDIED

Frame	Number of Spans	Span Length Ratio	A_T^* (m^2)	$\frac{K_c}{K_B}^*$
1	2	1:1	25	0.2
2	3	1:1:1	25	0.2
3	5	1:1:1:1:1	25	0.2
4	3	1:1:1	25	2.0
5	3	1:1.5:1	25	0.2
6	3	1.5:1:1.5	25	0.2
7	3	1:1:1	50	0.2
8	3	1:1:1	10	0.2

* Based on the (longer) span length of 5 m

and the variance of Y is (Eq. 2.4.81b of Ref. 3):

$$\mathrm{Var}(Y) = \sum_{i=1}^{n} a_i^2 \mathrm{Var}(X_i) + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} a_i a_j \mathrm{Cov}(X_i, X_j) \qquad (3.9)$$

where $\mathrm{Cov}(X_i, X_j)$ represents the covariance of variables X_i and X_j. The results are valid whether X_i are correlated or independent. In the latter case $\mathrm{Cov}(X_i, X_j)$ equals zero and the second term in Eq. 3.9 disappears.

A particular dead load effect, Y_i, is a linear function of the uniformly distributed dead load on each span. The relation is represented by

$$Y_i = \sum_{j=1}^{n} a_{ij} X_j \qquad (3.10)$$

where a_{ij} is an element of the influence coefficient matrix at row i and column j for a particular frame (derivation of the matrix is shown in Appendix A). The term X_j is the uniformly distributed dead load on span j, and n is the total number of spans in the frame. Since the dead loads, X_j, are normally distributed variables (sect. 2.2), Y_i is also normally distributed. The mean and the variance of X_j is the same for j = 1 to n, therefore, using Eqs 3.8 and 3.9, the mean and variance of Y_i become

$$E(Y_i) = m_X \sum_{j=1}^{n} a_{ij} \qquad (3.11)$$

$$\mathrm{Var}(Y_i) = V_X^2 m_X^2 \left(\sum_{j=1}^{n} a_{ij}^2 + 2r \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} a_{ij} a_{ik} \right) \qquad (3.12)$$

where r is the correlation coefficient between X_j. As mentioned in the dead load model in Section 2.2, the mean unit dead load (or the mean uniformly distributed load per unit width), m_X, is assumed to be 1.05 times the nominal unit dead load which is taken equal to unity for analysis, and the coefficient of variation, V_X, is assumed to be 0.10. No data are available to estimate the value of the correlation coefficient, r, between X_j. However, it is most likely that the dead load on one span is highly correlated with the dead load on adjacent spans, therefore, r = 0.75 has been arbitrarily assumed for analysis. The extreme situations of r = 0.0 (completely independent loads on all spans) are also examined for comparison. Perfectly correlated loads (r = 1.0) have not been considered since this corresponds to constant loads on all spans. Based on the normal distribution of the dead load effect with its mean and variance given by Eqs. 3.11 and 3.12, any fractile of a dead load effect can be calculated.

3.2.2 Linear Programming

In the similar manner as for the live load, the dead load effects obtained from the statistical calculation for the extreme cases are modelled with simple load patterns. The formulation of the linear program for the dead load is similar to that discussed in Section 3.1.2(c) for the live load. The term M_{mi} in Section 3.1.2(c) is redefined here as the absolute values of the 0.999 fractiles determined from the statistical calculation. For comparison, the load pattern based on 0.99 fractiles of the dead load effects is also examined.

3.2.3 Structures Studied

The type of structures studied for the dead load is the same as for the live load (Sect. 3.1.3). Since the results of the analysis of live loads presented in Chapter 4 show that the type of frames has an insignificant influence on the final results of analysis, Frame 2 in Table 3.2 has been arbitrarily chosen to be studied for the dead load.

4. DISCUSSION OF RESULTS

4.1 Live Load

4.1.1 Results of Monte Carlo Analysis

(a) Overview of Monte Carlo Results

Eight different frames (see Table 3.2) have been studied in the Monte Carlo analysis. The Monte Carlo results of a typical frame with 3 spans of constant span length, column to beam stiffness ratio equal to 0.2 and tributary area of one span equal to 25 sq. meters (Frame 2) are shown in Figs. 4.1-4.3. Each figure shows the moment envelopes obtained from the 0.999, 0.99 or 0.5 fractiles for loading Cases MC1 and MC3, and the moment envelopes from the ACI load pattern (the traditional pattern).

In Fig. 4.1, the 0.999 fractiles of the negative moments at the interior support for loading Case MC3 are about 10% higher than those for Case MC1, while moment envelopes for both cases are the same at all other sections. The comparison of loading Cases MC1, MC2 and MC3 for different frames will be discussed in Section 4.1.1(c).

Figure 4.1 also shows that the factored moment envelope from the ACI load pattern based on a load factor of 1.7 and a nominal live load given by Eq. 3.1 is close to the 0.999 fractiles from the Monte Carlo results, except for negative moments at the interior support and positive moments at the interior support. Since the latter are offset by dead load moments, they will be disregarded.

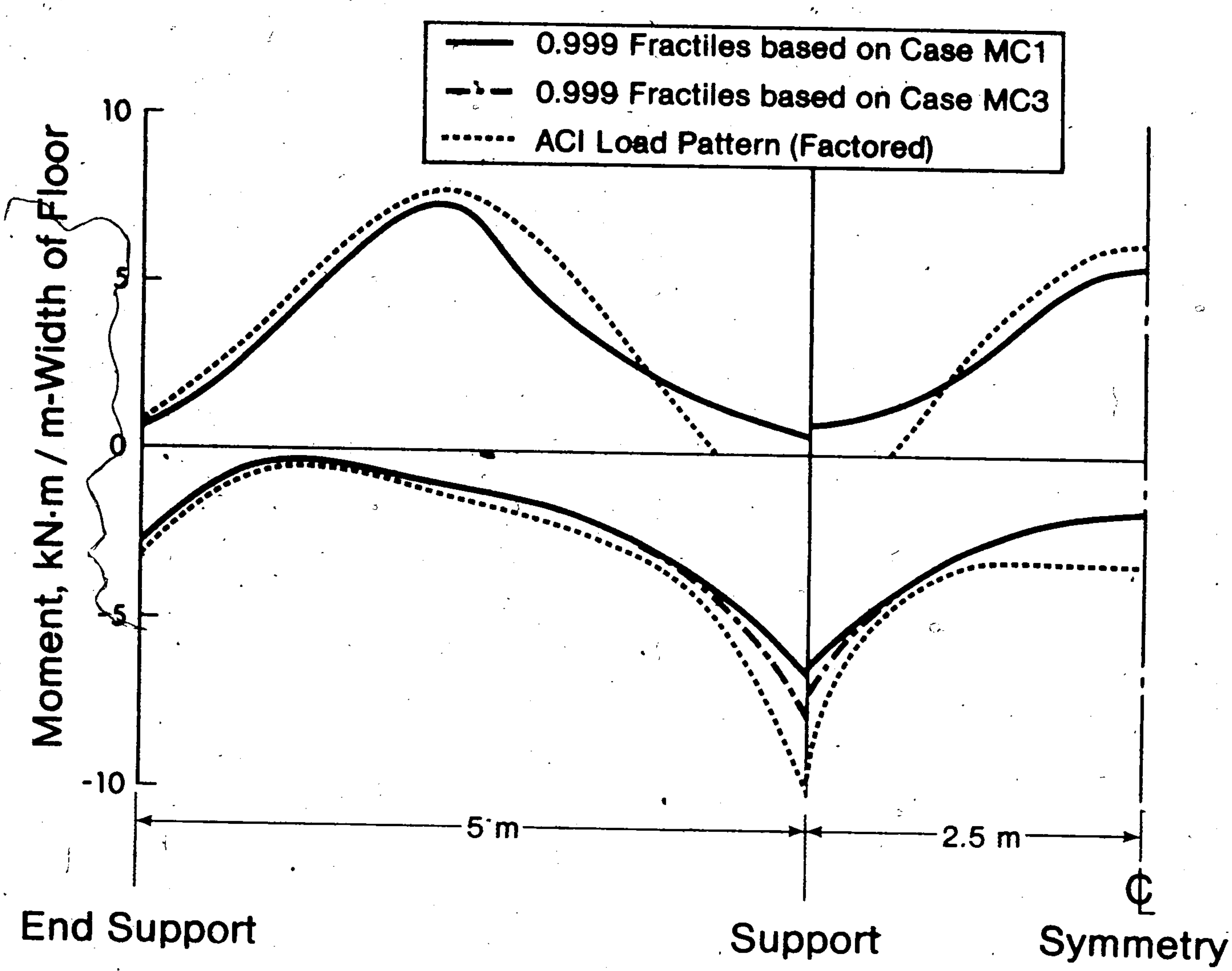


Fig.4.1 Comparison of 0.999 fractiles from Monte Carlo results with the moment envelope from ACI load pattern - Frame 2.

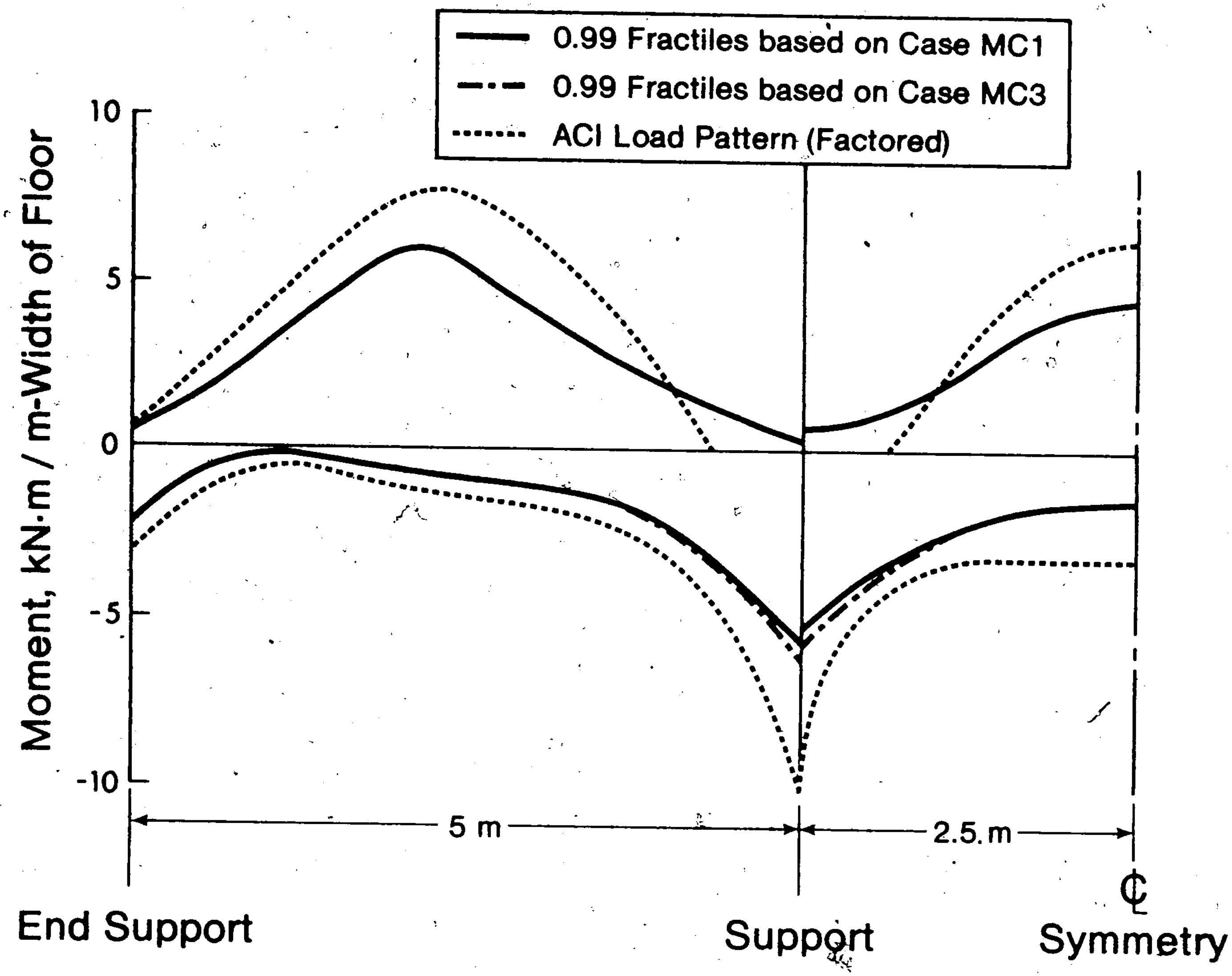


Fig.4.2 Comparison of 0.99 fractiles from Monte Carlo results with the moment envelope from ACI load pattern-Frame 2.

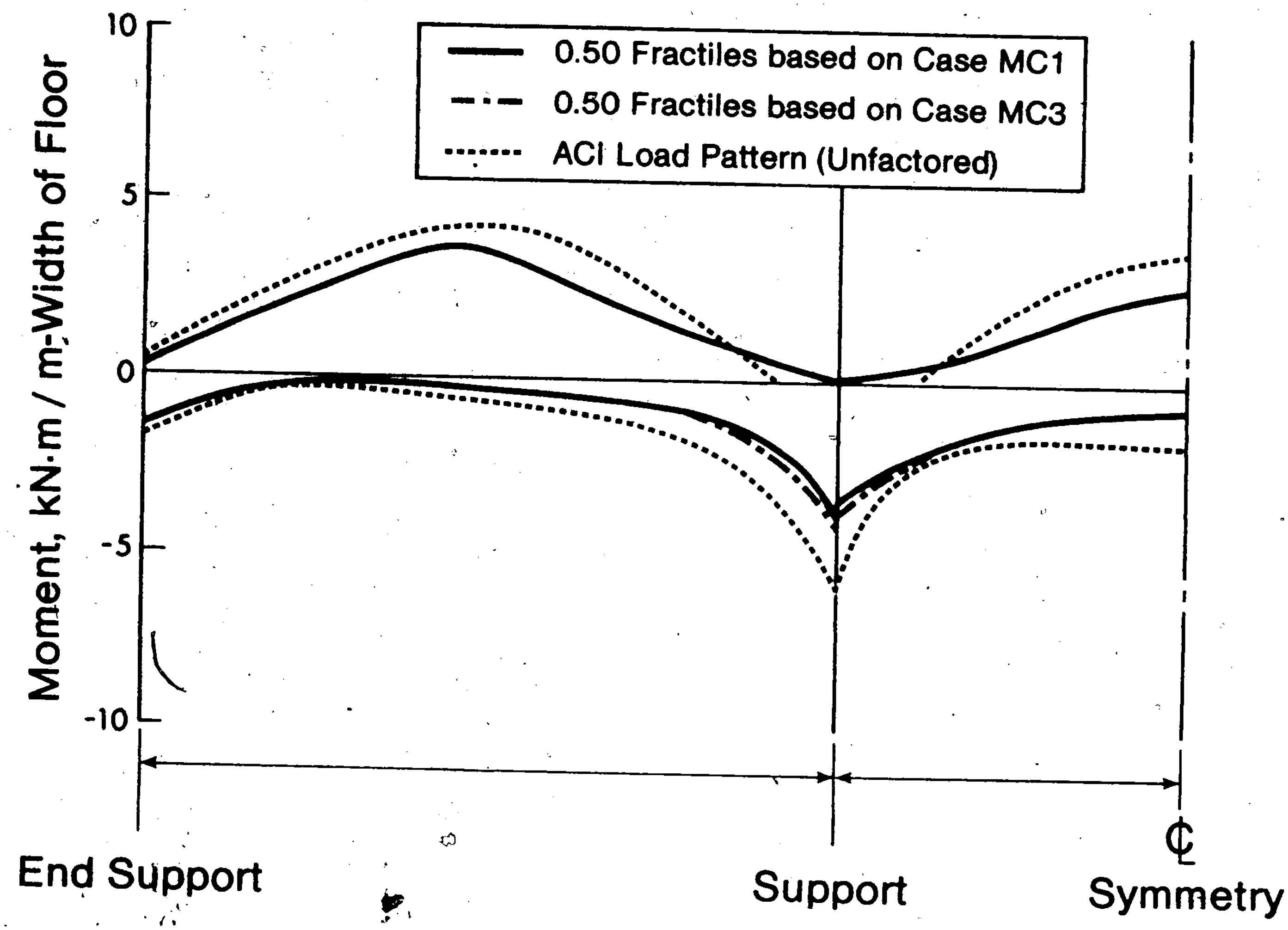


Fig.4.3 Comparison of 0.50 fractiles from Monte Carlo results with the moment envelope from ACI load pattern - Frame 2.

The interior negative support moments from the ACI load pattern are about 40% larger than the 0.999 fractiles for Case MC3. This large discrepancy suggests that the design load pattern of full factored load on the two spans adjacent to the support is too severe. This is confirmed by the results of linear programming discussed in Section 4.1.2.

In a similar manner to Fig. 4.1, Fig. 4.2 shows that the 0.99 fractiles of the negative moments at the interior support for loading Case MC3 are larger than those for Case MC1, and the discrepancy between the 0.99 fractiles of the interior negative support moments and those from the factored ACI load pattern is much larger than the discrepancy at the other sections. In Fig. 4.3, the moment envelope from 0.5 fractiles is compared to that from the unfactored ACI load pattern. The unfactored moment envelope from the ACI load pattern is seen to be more severe than that from the 0.5 fractiles at all critical sections.

With respect to the end shears, the 0.999 fractiles from the Monte Carlo results are slightly less than the factored shears from the ACI load pattern. For the interior column axial load, the 0.999 fractile based on loading Case MC3 is about 10% higher than that based on loading Case MC1, while the exterior column load remains the same in both loading cases. The different loading cases will be discussed more fully in Section 4.1.1(c). In the same way as the negative moment at the interior support, the interior column axial load obtained from the ACI load pattern based on a load factor of 1.7 is much larger than the 0.999 fractile. Again, this is confirmed by the results of linear programming discussed in Section 4.1.2.

(b) **Effects of Variables on Monte Carlo Results**

A number of parameters have been varied in the Monte Carlo Analysis. They can be divided into two categories: the parameters of the vacant period, and those of frame properties which include the tributary area, number of spans, span length ratio and column to beam stiffness ratio.

In the description of the live load model, it has been arbitrarily assumed that no live loads are present during "vacant periods" representing tenant changes, etc. Although it seems reasonable that they should occur, no data are available on the characteristics of such periods. In the analysis, the vacant period has been described in terms of its duration and the time between successive vacant periods. To study the effects of the assumed values, a parametric study has been made. Table 4.1 shows the effects of varying the parameters of the vacant period on the Monte Carlo results. The 0.99 fractiles from the 500-run computer results using Frame 8 (see Table 3.2) based on the assumed parameters in Section 2.1.4 are compared with the results based on other sets of parameters. The sums of the values, while meaningless in themselves, show a small increase in results when the time between periods increases or the duration of the period decreases. These differences are of

TABLE 4.1 EFFECTS OF VARYING THE PARAMETERS DESCRIBING THE VACANT PERIOD ON THE MONTE CARLO RESULTS (Based on Case MC1) - Frame 8 (See Table 3.2)

Location (Fig.4.4)	0.99 Fractiles of Lifetime Minimum				0.99 Fractiles of Lifetime Maximum			
	P1*	P2*	P3*	P4*	P1*	P2*	P3*	P4*
1	-3.466	-3.618	-3.358	-3.672	0.614	0.572	0.557	0.619
2	-0.401	-0.375	-0.381	-0.396	5.848	6.167	5.804	6.141
3	-1.457	-1.359	-1.323	-1.470	8.232	8.592	8.000	8.721
4	-2.762	-2.588	-2.783	-2.568	3.667	3.612	3.586	3.946
5	-8.329	-8.887	-8.383	-8.406	0.682	0.721	0.790	0.686
6	-7.680	-7.714	-7.308	-7.300	1.066	1.195	1.187	1.086
7	-3.297	-3.176	-3.242	-3.552	3.573	3.315	3.516	3.268
8	-2.046	-2.166	-2.000	-2.036	6.987	6.816	6.742	6.305
9	-	-	-	-	10.224	10.786	10.124	10.743
10	-12.090	-12.512	-12.269	-12.406	-	-	-	-
11	-	-	-	-	11.235	11.098	11.035	10.468
12	-	-	-	-	1.733	1.809	1.679	1.836
13	-	-	-	-	1.151	1.146	1.125	1.231
14	-	-	-	-	5.112	5.393	5.062	5.372
15	-	-	-	-	8.186	8.618	8.106	8.304
Sum	-41.53	-42.39	-41.05	-41.61	68.31	69.84	67.32	68.72

*	P1	P2	P3	P4
Duration of a vacant period	1-60 days	1-60 days	1-120 days	1-120 days
Time between two periods	2 years	3 years	2 years	3 years

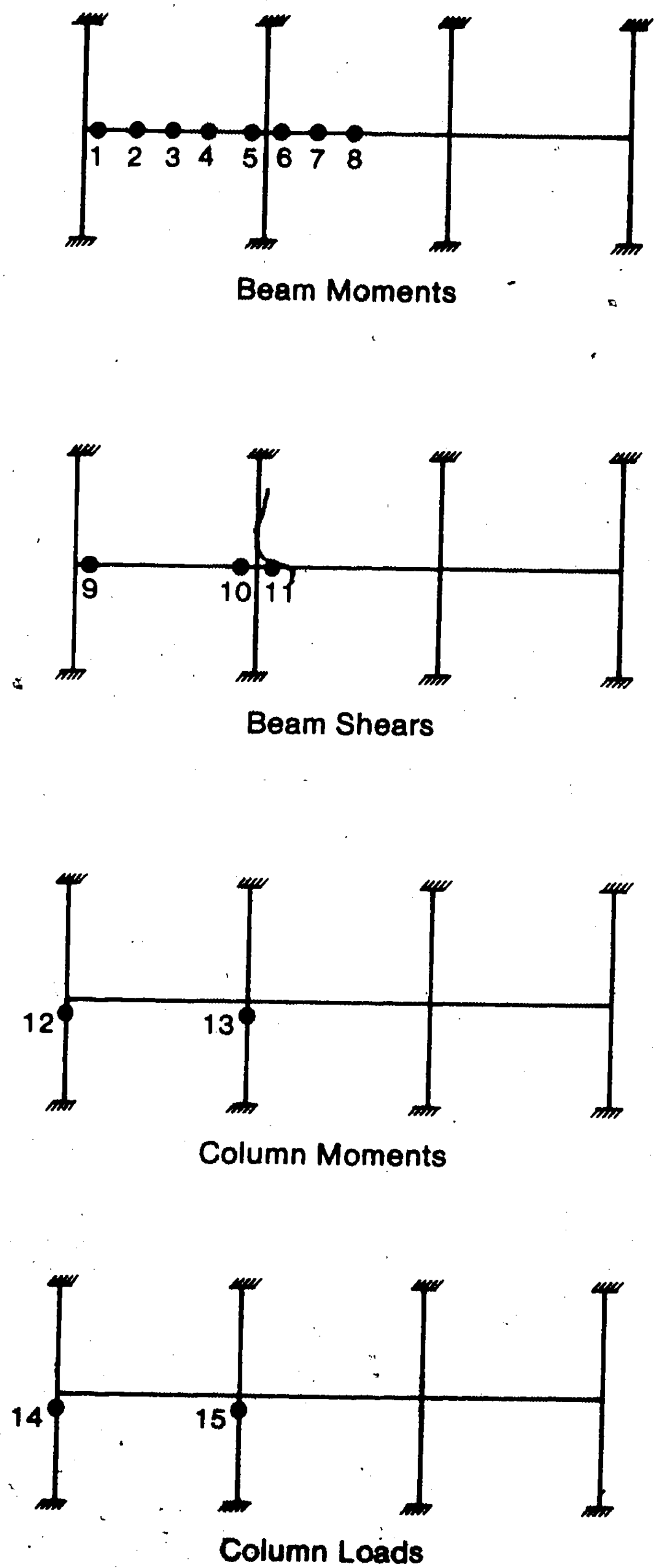


Fig.4.4 Locations of Calculated Load Effects

the same magnitude as the errors introduced by other assumptions and the values of these parameters as chosen in Section 2.1.4 are assumed to be good enough for the final analysis.

The tributary area of a frame affects the loads on each span, while other frame properties affect the influence coefficients, ie, the transformation of the loads to load effects. In the description of the live load model, the magnitude of the live load is a function of tributary area (or influence area); the larger is the tributary area, the smaller is the live load. As a result, for comparable frames, the frame with a larger tributary area obtains smaller values of load effects per unit width from the Monte Carlo analysis than one with a smaller tributary area does. For the other frame properties, the longer span of two adjacent uneven spans gets larger moments and shears than the shorter span does; when the column to beam stiffness ratio is higher, the beam moments become smaller while the column moments are larger. The effects of different number of spans on the Monte Carlo results, however, are not sufficiently consistent to draw any conclusion. To account for the effects of a series of frame variables, a number of different frames have been considered in the linear programming analysis presented in Section 4.1.2(b).

(c) Comparison of Monte Carlo Results for Loading Cases MC1, MC2 and MC3

The 0.999 fractiles of the interior negative support moments and the interior column load based on Cases MC1 and MC2 are compared in Table 4.2 for four different frames. The results indicate that Case MC2 (two adjacent spans occupied by one room) always govern the interior column load but may or may not govern the interior moments. This can be explained by observing the influence coefficients in Table 4.2. For the interior column load, the influence coefficients are always significant in the first two adjacent spans for any frame. For the support moments, Case MC1 governs when the influence coefficient in one of the first two spans is relatively small, as shown by Frame 4 in which the column to beam stiffness ratio is 2.0.

The results of the comparison, therefore, verify the assumption discussed in Section 3.1.1(b) that, conservatively, the span(s) where the influence surface is significant should be occupied by one room in the Monte Carlo analysis.

A third loading case, Case MC3, has been derived using the larger of the Monte Carlo values from Case MC1 and Case MC2 for the interior negative support moments, the interior column loads from Case MC2, and all other moments, shears and exterior column loads from Case MC1. While this appears to be an extreme case, it does allow for the possibilities that the loads on any two adjacent spans may

TABLE 4.2 COMPARISON OF MONTE CARLO RESULTS BASED ON CASES MC1 AND MC2

Load Effect	Frame (See Table 3.2)	0.999 Fractiles Case MC1	0.999 Fractiles Case MC2	Influence Coefficients Span 1	Influence Coefficients Span 2	Influence Coefficients Span 3
Interior Support Moment of First Span (KN.m/m-width)	2	-6.911	-7.326	-1.726	-0.994	0.248
	4	-7.032	-6.083	-1.923	-0.363	0.031
	5	-4.650	-5.329	-0.671	-1.192	0.119
	6	-6.964	-6.703	-1.904	-0.386	0.255
First Support Moment of Second Span (KN.m/m-width)	2	-6.437	-7.107	-1.190	-1.478	0.369
	4	-7.066	-5.979	-0.372	-1.892	0.161
	5	-5.783	-5.873	-0.465	-1.588	0.159
	6	-5.405	-5.936	-1.454	-0.574	0.379
Interior Column Load (KN/m-width)	2	6.821	7.759	1.500	1.367	-0.185
	4	6.236	6.950	1.313	1.302	-0.058
	5	5.940	6.884	0.955	1.452	-0.083
	6	6.434	7.372	1.640	0.879	-0.305

be correlated or that the loads on all spans are independent.

4.1.2 Results of Linear Programming

(a) Comparison of Different Design Load Patterns

The design load patterns considered in this study are ba[illegible]ee types of design loading illustrated in Fig[illegible]se have been selected because of their [illegible] and their ability to represent extreme loading cases. The three types of loading are:

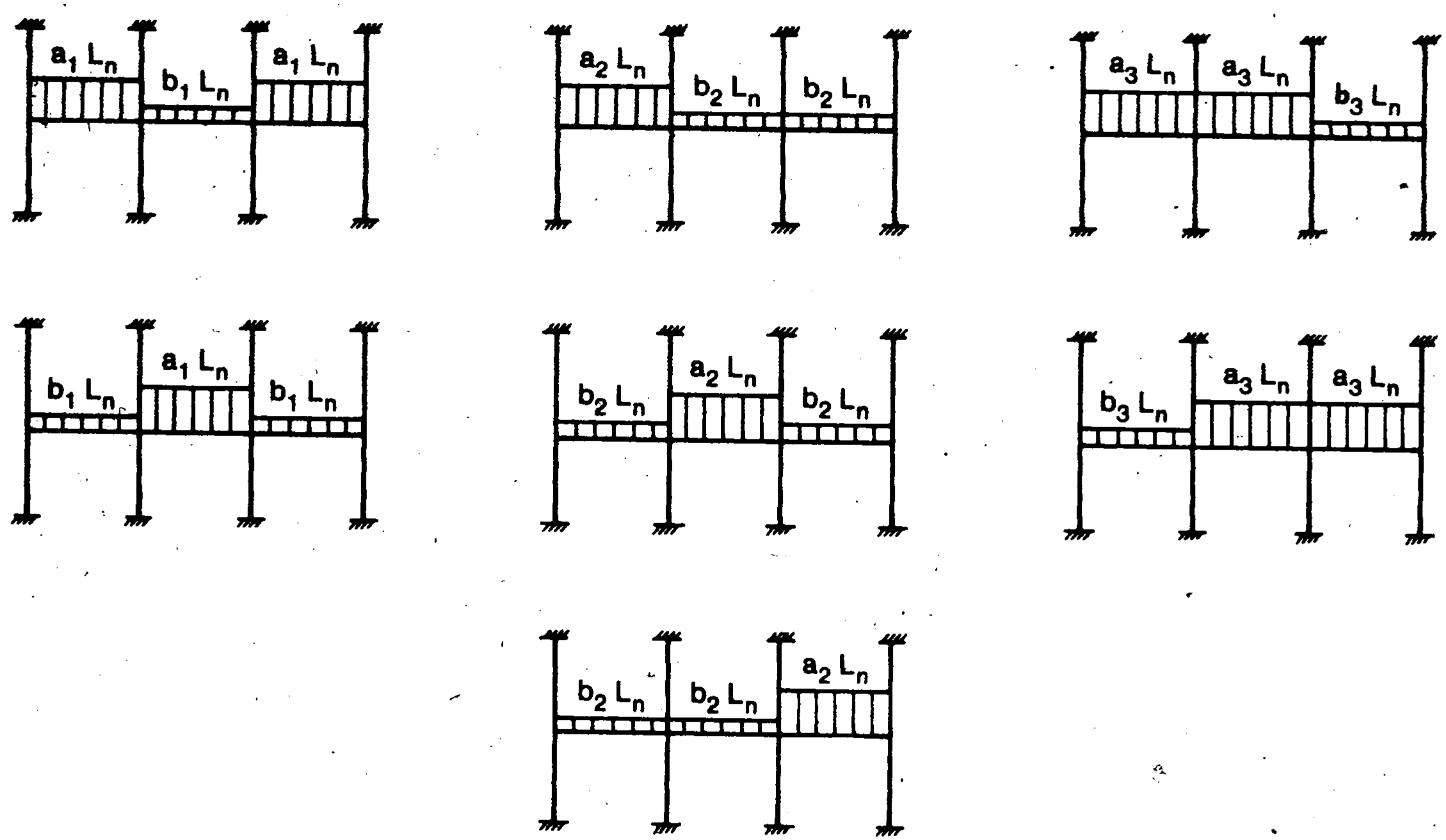
alternate span loading - heavy factored design load, $a_1 L_n$, on alternate spans, and light factored design load, $b_1 L_n$, on all other spans where $a_1 > b_1$,

single span loading - heavy factored design load, $a_2 L_n$, on one span and light factored design load, $b_2 L_n$, on all other spans where $a_2 > b_2$,

adjacent span loading - heavy factored design load, $a_3 L_n$, on two adjacent spans and light factored design load, $b_3 L_n$, on all other spans where $a_3 > b_3$.

In each type of loading, the load on a span is the product of a load factor, a or b, and the nominal live load, L_n, which is given by Eq. 3.1.

From the three basic types of loading, four design load



(a) Alternate Span Loading (b) Single Span Loading (c) Adjacent Span Loading

Fig. 4.5 Types of Design Loading

patterns are considered:

Pattern A - a combination of alternate span loading (Fig. 4.5a) plus adjacent span loading (Fig. 4.5c) as appropriate, with $a_1 = a_3$ and $b_1 = b_3$,

Pattern B - single span loading (Fig. 4.5b)

Pattern C - a combination of alternate span loading (Fig, 4.5a) plus adjacent span loading (Fig. 4.5c) as appropriate, with a_1 and b_1 independent of a_3 and b_3 respectively,

Pattern D - a combination of single span loading (Fig. 4.5b) plus adjacent span loading (Fig. 4.5c) as appropriate, with a_2 and b_2 independent of a_3 and b_3 respectively.

Pattern A is similar to the traditional design load pattern except that there may be a small load on the unloaded spans. Pattern B is a simpler model suggested by Beeby (2). The consideration of Patterns C and D results from the lack of consistency in the results of Patterns A and B as discussed later. A possible load pattern similar to Pattern D except $a_2 = a_3$ and $b_2 = b_3$ is not considered because the results of Pattern D indicates that a_2 is much different from a_3 as shown later.

The 0.999 fractiles of the lifetime minimum and maximum load effects from the Monte Carlo analysis of Frame 2 (with constant span length ratio and column to beam stiffness ratio equal to 0.2, see Table 3.2) have been used to investigate the validity of different design load patterns. The results are presented in Table 4.3.

Pattern A

Based on loading Case MC1 (loads on all spans independent), load factors of 1.56 and 0.02 are obtained for heavily and lightly loaded spans. The penalty obtained is very high because the design load pattern overestimates the interior negative support moments by more than 40%. In the design load pattern, those load effects are modelled with the adjacent span loading. The other load effects which are modelled with the alternate span loading are relatively well fitted.

Loading Case MC3 gives higher values for the interior support moments and the interior column load than Case MC1 does (see Table 4.2). When these load effects are modelled with Pattern A, better agreement is obtained. Nevertheless, they are still significantly overestimated, as shown by the high penalty. These results, however, infer that the penalty can be minimized by treating load factors of the alternate span loading and those of the adjacent span loading independently, leading to consideration of Pattern C.

TABLE 4.3 COMPARISON OF DIFFERENT DESIGN LOAD PATTERNS

Load Factor	Pattern A		Pattern B		Pattern C		Pattern D	
	Case MC1	Case MC3	Case MC1	Case MC3	Case MC1	Case MC3	Case MC1	Case MC3
$a_{1,2}$	1.56	1.56	1.66	1.66	1.63	1.63	1.66	1.66
$b_{1,2}$	0.02	0.02	0.07	0.07	0.06	0.06	0.03	0.03
a_3	-	-	-	-	1.26	1.24	1.26	1.24
b_3	-	-	-	-	1.23	0.28	1.23	0.28
Total Penalty	12.6	10.5	7.3	11.4	4.7	4.0	2.9	2.2

Pattern B

When load factors are derived for Pattern B based on loading Case MC1, the penalty obtained is less than for Pattern A. For Case MC1, Pattern B underestimates the interior support moments and the interior column load by more than 10%, while other load effects are well fitted. For Case MC3, the penalty is even higher because the interior support moments and the interor column load are more severely underestimated. This is conceivable because for those load effects with significant influence surface in adjacent spans, the single span loading pattern (Pattern B) appears less representative of the extreme loading cases than the adjacent span loading does. Therefore, it is reasonable to also consider a Pattern D, which includes the adjacent span loading in addition to the single span loading.

Patterns C and D

The load factors for the Pattern C loading are given in Table 4.3. These suggest that the 0.999 fractiles of the load effects from the Monte Carlo analysis based on Case MC1 can best be represented by an alternate span loading with 1.63 or 0.06 times the nominal load placed to give maximum values of load effects normally predicted by alternate span values, plus a constant load of 1.23 to 1.26 times the nominal load when predicting interior column loads or negative moments at interior supports. For Case MC8, the load factors of the alternate span loading are the same as those of Case MC1 because the alternate span loading fits the same load effects in both loading cases. The adjacent span loading, however, is different, with 1.24 times the nominal load on two adjacent spans and a small load on the other spans. The penalty for Pattern C, shown in Table 4.3, is considerably reduced. As anticipated, the load factor a_3 of the adjacent span loading, which models the interior support moments and the interior column load, is significantly less than the load factor a_1 of the alternate span loading.

In Pattern D, the load factors for the adjacent span loading are the same as those in Pattern C because they both model the same load effects, while the load factors for the single span loading are slightly higher than those in Pattern D. The penalty for Pattern D is less than for Pattern C, which indicates that the single span loading may be more representative of the extreme loading cases for 0.999 fractiles of the load effects concerned than the alternate span loading.

Patterns C and D will be examined in the next section for different frames to investigate the effects of other variables and also as a continuous comparison of the two load patterns. Because the adjacent span loading models the same load effects in both patterns, the real comparison is between the alternate span and single span loadings, which model the load effects other than those modelled by the adjacent span loading. The significance of the penalty as related to the degree of fit of a pattern will be illustrated as well in the next section.

(b) Effects of Variables

The load factors of Patterns C and D for different frames based on Cases MC1 and MC3 are presented in Table 4.4. As mentioned previously, the load factors of the adjacent span loading are the same in both Patterns C and D. With two exceptions discussed later, the adjacent span loading has been used to fit the interior negative support moments and the interior column loads. The other load effects have been modelled with the single span or alternate span loading. The load factors for the single span or alternate span loading are essentially the same in Cases MC1 and MC3, because in both cases the single span or alternate span loading has been used to fit the same load effects.

It has been shown in Table 4.2 that Case MC1 governs the interior negative support moments in Frame 4 (with column to beam stiffness ratio equal to 2.0, see Table 3.2) and also governs the interior negative support moment of the longer (exterior) span in Frame 6 (with span length ratio equal to 1.5:1.0:1.5, see Table 3.2). This is because the influence surface in one of the adjacent spans is relatively small, as explained in Section 4.1.1(c). When considering Frame 4, the interior support moments and the interior column load have been poorly fitted by the adjacent span

TABLE 4.4 EFFECTS OF VARIABLES ON DESIGN LOAD PATTERNS

Frame (See Table 3.2)	Load Effects Governed by Adjacent Span Loading						All Other Load Effects					
	Case MC1			Case MC3			Alternate Span Loading (Cases MC1, MC3)			Single Span Loading (Cases MC1, MC3)		
	a_3	b_3	Penalty	a_3	b_3	Penalty	a_1	b_1	Penalty	a_2	b_2	Penalty
1	1.08	0.0	0.3	1.23	0.0	0.0	1.58	0.03	1.3	1.58	0.03	1.3
2	1.26	1.23	0.7	1.24	0.28	0.0	1.63	0.06	4.0	1.66	0.03	2.2
3	1.07	0.11	1.2	1.22	0.10	0.1	1.56	0.07	13.4	1.60	0.03	6.4
4	1.07*	0.0*	0.0	1.20*	0.0*	0.0	1.63	0.03	2.6	1.62	0.02	2.0
5	1.07	0.0	0.1	1.22	0.0	0.3	1.57	0.14	2.8	1.60	0.08	2.5
6	1.15*	0.0*	0.3	1.26	0.0	0.1	1.66	0.05	8.5	1.63	0.02	3.5
7	1.18	1.18	0.6	1.25	0.14	0.0	1.43	0.04	4.3	1.46	0.02	1.9
8	1.65	1.65	1.[illegible]	1.63	0.36	0.1	2.28	0.09	6.5	2.33	0.06	4.0

* For this frame the interior negative support moment has been included in the single span or alternate span loading (see text).

loading; the penalty is severe because the interior column load is much overestimated while the negative moments are underestimated if a constant load factor a_3 is used. The load factor a_3 is about 1.35, which is higher than the value obtained by Frame 2 presented in Table 4.3. To reduce the penalty and hence increase the accuracy, the interior negative support moments have been included in the single span or alternate span loading, which gives much better fit, especially the single span loading. The adjacent span loading, which fits the interior column load only, gives the load factor a_3 equal to 1.07 for Case MC1 and 1.20 for Case MC3, as presented in Table 4.4.

In a similar manner, the interior support moment of the longer span in Frame 6 is also included in the single span or alternate span loading. The above special arrangement of the interior support moments, however, leads to the consideration of both single span (or alternate span) loading and adjacent span loading even when calculating the interior negative support moments.

Table 4.4 shows that the load factors of the alternate span loading or the single span loading appear insensitive to most variables except the tributary area (Frames 7, 8). For a tributary area, A_T, of 10 m^2 (Frame 8), the load factor a_1 or a_2 is much higher than the load factor for A_T = 25 m^2 (Frames 1-6). This is because the nominal live load (Eq. 3.1) becomes constant for A_T < 20 m^2, while the loads determined from the live load model as used in this study keeps increasing for A_T < 20 m^2(see Eq. 2.7, Fig. 2.1). For A_T = 50 m^2, the load factor a is smaller than that for A_T = 25 m^2, but the effect can be considered relatively insignificant. In all cases, the load factor b_1 or b_2 is close to zero.

The variables have some significant effects on the penalties in the alternate span loading as shown in Table 4.4. The penalties in the single span loading, however, consistenly remain small and less than the penalties in the alternate span loading for all different frames. Note that the penalty for the five span frame (Frame 3) is higher than the others because more load effects have been fitted. This indicates that the single span loading is consistently a better representation of the extreme loading cases for the load effects with a significant influence surface in one span and relatively small influence surfaces in all other spans. The average value of the load factor a_2 of the single span loading for Frames 1-7 is 1.60 (excluding the large load factor obtained by Frame 8), and b_2 is practically equal to zero.

In a similar manner as for the alternate span or single span loadings, most variables have an insignificant effect on the load factors for the adjacent span loading case except the effect of small tributary area. The load factor a_3 is similar for Cases MC1 and MC3 ranging from 1.07 to 1.26 for MC1 and 1.20 to 1.26 for MC3 for frames with large tributary areas (Frames 1-7). Since Case MC3 represents a more conservative situation, it is considered more desirable as the basis for a design recommendation. The values of the load factor a_3 for different frames (except Frame 8) based on Case MC3 are consistent and the average is 1.23, which is 0.77 of the load factor a_2 of the single span loading. For simplicity, it is proposed that in design the load factor a_3 should be taken equal to 3/4 of the load factor of the single span loading. The load factor b_3, which ranges from 0 to 0.28 for Frames 1 to 7 with an average of 0.07, shall be set equal to zero for design purposes.

The degree of fit of Pattern D is illustrated in Tables 4.5 and 4.6 by Frame 2 (Table 3.2) for loading Case MC3 with load factors presented in Table 4.4. Column 5 in these tables illustrates the relative significance of the penalty as functions of a single beam moment, shear, etc. Pattern D, based on Case MC3, is taken as a design recommendation which will be presented more fully in the next section.

4.1.3 Design Recommendation

It is recommended that the design value of a live load effect in a continuous beam or a one way slab be determined using whichever of the following types of loading that produces the largest value:

a. factored live load on one span,

b. 3/4 of the factored live load on two adjacent spans.

The load factor corresponding to 0.999 fractiles of the

TABLE 4.5 COMPARISON OF 0.999 FRACTILES OF LIFETIME MAXIMUM LIVE LOAD EFFECTS WITH THOSE FROM LOAD PATTERN D - FRAME 2 (See Table 3.2)

Location (Fig.4.4) [1]	0.999 Fractiles [2]	Design Value [3]	Deviation =Abs[3]-Abs[2] [4]	$\frac{[4]}{F} \times 100\%$** [5]
1	0.505	0.575	0.070	1.0
2	4.864	4.863	-0.001	-0.01
3	6.847	6.859	0.012	0.2
4	3.100	3.100	0.0	0.0
5	-	-	-	-
6	-	-	-	-
7	2.862	3.127	0.265	3.8
8	5.642	6.004	0.362	5.2
9	8.503	8.503	0.0	0.0
10	-	-	-	-
11	9.078	9.208	0.130	2.3
12	1.442	1.444	0.002	0.04
13	0.977	0.974	-0.003	-0.06
14	8.502	8.504	0.002	0.04
15*	15.518	15.536	0.018	0.16

* Fitted by the adjacent span loading (see Sect. 4.1.2(a))

** For beam moments, $F = \frac{L_n S^2}{8}$ where L_n is the nominal live load given by Eq.3.1 and S is the span length (5 m).

For end shears, $F = \frac{L_n S}{2}$

For column moments, $F = \frac{L_n S^2}{12}$

For column loads, $F = \frac{L_n S}{2}$ (exterior)

$F = L_n S$ (interior)

TABLE 4.6 COMPARISON OF 0.999 FRACTILES OF LIFETIME MINIMUM LIVE LOAD EFFECTS WITH THOSE FROM LOAD PATTERN D - FRAME 2 (See Table 3.2)

[1]*	[2]*	[3]*	[4]*	[5]*
1	-2.883	-2.888	0.005	0.07
2	-0.347	-0.337	-0.010	-0.1
3	-1.200	-1.365	0.165	2.4
4	-2.255	-2.510	0.255	3.7
5**	-7.326	-7.326	0.0	0.0
6**	-7.107	-7.107	0.0	0.0
7	-2.771	-2.884	0.113	1.6
8	-1.787	-1.420	-0.367	-5.3
9	-	-	-	-
10	-9.941	-9.913	-0.028	-0.5
11	-	-	-	-

* Refer to Table 4.5

** Fitted by the adjacent span loading (see Sect. 4.1.2(a)) lifetime minimum and maximum live load effects for office buildings is 1.6. This design load pattern is valid for calculating moment envelopes, end shears, column moments and column loads.

The one span loading gives larger values for most load effects than the adjacent span loading does. The adjacent span loading, however, gives larger values for load effects with significant influence surfaces in two adjacent spans such as the interior column loads and the negative moments at interior supports. Nevertheless, for the frame with a high ratio of column to beam stiffness ratio such as 2.0, the one span loading also governs the negative moments at interior supports.

4.2 Dead Load

The British Standard Code of Practice (4) requires pattern application of dead load to a structure as shown in Table 4.7. This is not required by the ACI Code (1) or NBC Code (19). The 0.999 and 0.99 fractiles of the dead load effects have been found to be well modelled by Load Pattern A (see Sect. 4.1.2(a)), which is similar to the traditional load pattern. The load factors, a and b, corresponding to the 0.999 and 0,99 fractiles are presented in Tables 4.8 and 4.9 for three values of the coefficient of correlation, r, between the dead loads on adjacent spans.

Table 4.8 shows that the British dead load pattern closely approaches the load pattern which produces 0.999

TABLE 4.7 CURRENT DESIGN DEAD LOAD FACTORS STIPULATED BY DIFFERENT DESIGN CODES

Code	British	NBC	ACI
a*	1.4	1.25	1.4
b**	1.0	1.25	1.4

TABLE 4.8 LOAD FACTORS CORRESPONDING TO 0.999 FRACTILES OF DEAD LOAD EFFECTS

Correlation Coefficient	0.0	0.75	1.0
a*	1.35	1.37	1.37
b**	0.96	1.22	1.37
Total Penalty	0.86	0.35	-

TABLE 4.9 LOAD FACTORS CORRESPONDING TO 0.99 FRACTILES OF DEAD LOAD EFFECTS

Correlation Coefficient	0.0	0.75	1.0
a*	1.27	1.29	1.29
b**	0.98	1.18	1.29
Total Penalty	0.65	0.26	-

* Large load factor in Pattern A (see Sect. 4.1.2(a))

** Small load factor in Pattern A (see Sect. 4.1.2(a)).

fractiles of load effects for r = 0.0 or completely uncorrelated dead loads on adjacent spans. On the other hand, the NBC and ACI assumptions of constant factored dead loads on all spans approach the case of strongly or completely correlated dead loads on all spans. The dead load factors in the NBC and ACI Codes correspond to the 0.99 and 0.999 fractiles, respectively, as shown in Tables 4.8 and 4.9.

The extreme situation of r = 0.0 is unlikely in a real situation since the same source of concrete will generally be used for all spans and the same framing crew will probably build the forms for all spans. A more realistic situation of r = 0.75, however, indicates in Tables 4.8 and 4.9 that the dead loads are approximately constant on all spans for 0.99 and even for 0.999 fractiles of load effects. For computational simplicity, it is reasonable to consider that the factored dead loads are constant on all spans.

5. SUMMARY AND CONCLUSIONS

5.1 Live Load

A Monte Carlo process was used to generate extreme values of moments and other major load effects for a series of frames subjected to varying live loads during their lifetime. The live load model used in this study was developed by several investigators (11,16,20) based on the office live loads. The analyses generated load effects for loadings assumed to be independent on each span and for loadings which were correlated on two adjacent spans but independent on all others.

Linear programming was used to investigate the validity of several possible design load patterns for approximatig the 0.999 fractiles of lifetime minimum and maximum load effects from the Monte Carlo results. The results indicate that the design load pattern which best represents the 0.999 fractiles is either the factored live load on one span, or 3/4 of the factored live load on any two adjacent spans, as appropriate to give maximum load effects. The corresponding load factor was found to be 1.6 to represent the 0.999 fractile of the maximum loading effects. This design load pattern represents a conservative loading assumption that the loads may be independent on all spans or correlated on any two adjacent spans.

This design load pattern was shown to be valid for different frame properties although when the tributary area of one span is less than 20 square meters, the load factors should be increased. This is because the nominal live load (Eq. 3.1) is unconservative for tributary areas less than 20 square meters.

5.2 Dead Load

Direct statistical calculation of dead load effects for the extreme cases was performed due to the simplicity of the dead load model. Again, linear programming was used to develop an equivalent design dead dead load pattern. The results indicate that the effect of pattern loadings is insignificant for a correlation coefficient between dead loads on adjacent spans equal to 0.75. Hence it is recommended that pattern dead loads not be used in design.

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APPENDIX A

Formulation of an Influence Coefficient Matrix

Load effects caused by any uniformly distributed load on each span are represented by the following relation:

$$Y_{i=1,m} = \sum_{j=1}^{n} a_{ij} X_j \qquad \text{(A.1)}$$

where

Y_i = value of load effect i

a_{ij} = an element of the influence coefficient matrix at row i and column j for a particular frame

X_j = uniformly distributed load on span

m = total number of load effects considered

n = total number of spans in the frame

Accordingly, a_{ij} are determined by the following method:

$$\text{Let } X_{j=k} = 1 \text{ and } X_{j\neq k} = 0 \qquad \text{(A.2)}$$

Eq. A.2 is then applied in Eq. A.1, which results in

$$a_{ik} = Y_i \quad \text{for } i = 1,m \qquad \text{(A.3)}$$

because $Y_{i=1,m}$ can be determined (when $X_{j=k} = 1$ and $X_{j\neq k} = 0$) using a simple frame analysis computer program, the influence coefficient matrix can be formulated by repeating the analysis for k = 1 to n.

APPENDIX B

Listing of the Monte Carlo Program

```
C************************************************************************
C**                                                                    **
C**                           MONTELIVE                                **
C**                                                                    **
C************************************************************************
C**                                                                    **
C**   THIS PROGRAM STATISTICALLY COMPUTES THE LIFETIME MAXIMUM AND     **
C**   MINIMUM LIVE LOAD (GRAVITY LOAD) EFFECTS (MOMENTS,SHEARS AND     **
C**   AXIAL FORCES) IN A CONTINUOUS FRAME USING THE MONTE-CARLO        **
C**   TECHNIQUE                                                        **
C**                                                                    **
C************************************************************************
C
C     BASIC ASSUMPTIONS:
C             LIVE LOADS ARE COMPOSED OF SUSTAINED LOADS AND TRANSIENT
C             LOADS. PERIODS OF UNOCCUPANCY ARE ALSO CONSIDERED. THE
C             LOADING ON ONE SPAN IS UNIFORM (OVER AN AREA EQUAL TO
C             TWICE THE TRIBUTARY AREA OF ONE SPAN) AND STATISTICALLY
C             INDEPENDENT OF THE LOADING ON ANY OTHER SPAN. SUCCESSIVE
C             LOADING EVENTS ARE ALSO INDEPENDENT.
C
C     REMARKS:
C             THE NUMBER OF RUNS (NURUN) MUST BE GREATER THAN 1.
C             THE NUMBER OF SPANS MUST NOT BE LESS THAN 2.
C             IF VALUES OF 0.01 FRACTILE TO 0.99 FRACTILE WITH INCREMENTS
C             OF 0.01 FRACTILE ARE DESIRED IN THE OUTPUT,NURUN MUST BE
C             MULTIPLES OF 100.
C
C
      DOUBLE PRECISION SSEED,SDSEED,UNSEED,TLSEED,TSEED
C
      COMMON SSEED,SDSEED,UNSEED,TLSEED,TSEED
      COMMON XMAXM(5000,23),XMINM(5000,18),Q(500,5),INT(500,5),
     1       TRANL(200),INTR(200),SUSLN(100),INTSN(100),XMCOEF(23,5),
     1       PS(5),PT(5),SLENDA(5),TLENDA(5),ISLIFE,NURUN,NUSPAN,NUROW,
     1       ISMEAN,ISDMIN,NDMIN,NDMAX,NTIME,NFMIN,ITMEAN,ITRMIN
      COMMON ISPAN,IRUN,NU01,NFRAME,NUROW2
C
      CALL DATA
C
      DO 100 IRUN=1,NURUN
C
      DO 200 ISPAN=1,NUSPAN
C
      CALL SUSTAN
C
      CALL TRANST
C
      CALL COMBND
C
  200 CONTINUE
C
      CALL EFFECT
C
  100 CONTINUE
C
      CALL OUTPUT
C
      STOP
C
      END
C
C
C                           *****************
C
C                           SUBROUTINE DATA
C
C                           *****************
C
C
C    THIS SUBROUTINE READS AND WRITES ALL THE INPUT DATA AND THEN
C    CALCULATES THE REQUIRED STATISTICAL PARAMETERS. ALL THE TIME
C    RELATED VALUES ARE CONVERTED INTO UNITS OF TRANSIENT LOAD
C    DURATION,THAT IS,ONE TIME UNIT IS EQUAL TO THE TIME OF THE
C    DURATION OF ONE TRANSIENT LOAD. ALL SUBSEQUENT CALCULATIONS
C    WILL BE BASED ON THIS UNIT.
C
      SUBROUTINE DATA
C
      DOUBLE PRECISION SSEED,SDSEED,UNSEED,TLSEED,TSEED
C
      COMMON SSEED,SDSEED,UNSEED,TLSEED,TSEED
      COMMON XMAXM(5000,23),XMINM(5000,18),Q(500,5),INT(500,5),
     1       TRANL(200),INTR(200),SUSLN(100),INTSN(100),XMCOEF(22,5),
     1       PS(5),PT(5),SLENDA(5),TLENDA(5),ISLIFE,NURUN,NUSPAN,NUROW,
     1       ISMEAN,ISDMIN,NDMIN,NDMAX,NTIME,NFMIN,ITMEAN,ITRMIN
      COMMON ISPAN,IRUN,NU01,NFRAME,NUROW2
C
      DIMENSION SPANL(5)
C
C    READ ALL INPUT DATA
C
      READ(5,101)NURUN,SLIFE,NFRAME,ICHECK
      READ(5,102)NUSPAN,WIDTH,RATIO
      READ(5,103)(SPANL(I),I=1,NUSPAN)
      READ(5,111)NROWM1,NROWM2
      READ(5,111)NROWS1,NROWS2
      READ(5,111)NROWC1,NROWC2
      READ(5,111)NROWA1,NROWA2
      READ(5,104)NUROW,NUROW2
C
      DO 11 I=1,NUROW
      READ(5,105)(XMCOEF(I,J),J=1,NUSPAN)
   11 CONTINUE
C
      READ(5,106)SUMEAN,SUSTD,AREA,SSEED
      READ(5,107)SMEAN,SDMIN,SDSEED
      READ(5,108)DMIN,DMAX,UNTIME,PWIN,UNSEED
```

```
      READ(5,109)MR,MRSTD,XMO,OSTD,TLSEED
      READ(5,110)TMEAN,TRMIN,TRDUR,TSEED
C
C   WRITE ALL INPUT DATA
C
      WRITE(6,1001)
      WRITE(6,1002)NFRAME
      WRITE(6,1003)
      WRITE(6,1004)NURUN,SLIFE
      WRITE(6,1005)
      WRITE(6,1006)NUSPAN,WIDTH,RATIO
      WRITE(6,1007)(SPANL(I),I=1,NUSPAN)
      WRITE(6,1008)
      WRITE(6,1020)NROWM1,NROWM2
      WRITE(6,1021)NROWS1,NROWS2
      WRITE(6,1051)NROWC1,NROWC2
      WRITE(6,1022)NROWA1,NROWA2
      WRITE(6,1009)NUROW,NUROW2
C
      DO 12 I=1,NUROW
      WRITE(6,1010)(XMCOEF(I,J),J=1,NUSPAN)
   12 CONTINUE
C
      WRITE(6,1011)
      WRITE(6,1012)SUMEAN,SUSTD,AREA,SSEED
      WRITE(6,1013)SMEAN,SDMIN,SDSEED
      WRITE(6,1014)
      WRITE(6,1015)DMIN,DMAX,UNTIME,FMIN,UNSEED
      WRITE(6,1016)
      WRITE(6,1017)MR,MRSTD,XMO,OSTD,TLSEED
      WRITE(6,1018)TMEAN,TRMIN,TRDUR,TSEED
      WRITE(6,1019)
C
      IF(NURUN.LT.2)CALL EXIT
      IF(NUSPAN.LT.2)CALL EXIT
      IF(ICHECK.EQ.1)CALL EXIT
C
C   CONVERT THE DESIGN LIFETIME INTO THE PREVIOUSLY DEFINED UNITS
C
      ISLIFE=SLIFE*365.*24./TRDUR+0.5
C
C   DETERMINE THE VARIANCE OF SUSTAINED EUDL FOR EACH SPAN
C
      WRITE(6,1031)
      SUVAR=SUSTD**2
      SIES2=SUVAR/(0.03+1.0/AREA)
      SIG2=0.03*SIES2
      WIDTH=WIDTH*2.
      SIES2=SIES2*2.2/WIDTH
      SOMEAN=SUMEAN*SUMEAN
C
      DO 21 ISPAN=1,NUSPAN
      VARSU=SIG2+SIES2/SPANL(ISPAN)
C
C   DETERMINE THE PARAMETERS FOR THE GAMMA DISTRIBUTION OF THE
C   SUSTAINED EUDL
C
      PS(ISPAN)=SOMEAN/VARSU
      SLENDA(ISPAN)=SUMEAN/VARSU
C
C   WRITE THE STANDARD DEVIATION OF THE SUSTAINED EUDL
C
      STDSU=SQRT(VARSU)
      WRITE(6,1032)ISPAN,STDSU
C
   21 CONTINUE
C
C   CONVERT THE MEAN AND MINIMUM SUSTAINED LOAD DURATION INTO THE
C   PREVIOUSLY DEFINED UNITS
C
      ISMEAN=SMEAN*365.*24./TRDUR+0.5
      ISDMIN=SDMIN*365.*24./TRDUR+0.5
C
C   CONVERT THE DATA ON UNOCCUPIED PERIODS
C
      NDMIN=DMIN*24./TRDUR+0.5
      NDMAX=DMAX*24./TRDUR+0.5
      NTIME=UNTIME*365.*24./TRDUR+0.5
      NFMIN=FMIN*365.*24./TRDUR+0.5
C
C   DETERMINE THE MEAN AND VARIANCE OF TRANSIENT EUDL FOR EACH SPAN
C
      WRITE(6,1033)
      XMOR=XMO*FLOAT(MR)
      XRSTD2=FLOAT(MRSTD)**2
      ORSTD=2.2*(OSTD*OSTD*FLOAT(MR)+XMO*XMO*XRSTD2+XMOR**2)
C
      DO 22 ISPAN=1,NUSPAN
      AREA=SPANL(ISPAN)*WIDTH
      IF(AREA.LT.30.0)GO TO 31
      X=(AREA-14.4)/0.58
      BLENDA=SQRT(X)
      GO TO 32
   31 BLENDA=0.173*AREA
   32 TRMEAN=BLENDA*XMOR/AREA
      TRVAR=BLENDA*ORSTD/(AREA*AREA)
C
C   DETERMINE THE PARAMETERS FOR THE GAMMA DISTRIBUTION OF TRANSIENT
C   EUDL
C
      PT(ISPAN)=TRMEAN*TRMEAN/TRVAR
      TLENDA(ISPAN)=TRMEAN/TRVAR
C
C   WRITE THE MEAN AND STANDARD DEVIATION OF THE TRANSIENT LOAD
C
      TRSTD=SQRT(TRVAR)
      WRITE(6,1034)ISPAN,TRMEAN,TRSTD
C
```

```
   22  CONTINUE
C
C     CONVERT THE TIME BETWEEN OCCURRENCES OF TRANSIENT LOADS INTO THE
C     PREVIOUSLY DEFINED UNITS
C
      ITMEAN=TMEAN*24./TRDUR+0.5
      ITRMIN=TRMIN*24./TRDUR+0.5
C
      WRITE(6,1051)
      WRITE(6,1052)NROWM1,NROWM2
      WRITE(6,1053)NROWS1,NROWS2
      WRITE(6,1082)NROWC1,NROWC2
      WRITE(6,1054)NROWA1,NROWA2
C
      RETURN
C
C     FORMAT STATEMENTS
C
  101 FORMAT(I10,F10.2,2I10)
  102 FORMAT(I10,2F10.2)
  103 FORMAT(5F10.2)
  104 FORMAT(2I10)
  105 FORMAT(5F10.2)
  106 FORMAT(3F10.2,D10.2)
  107 FORMAT(2F10.2,D10.2)
  108 FORMAT(4F10.2,D10.2)
  109 FORMAT(2I10,2F10.2,D10.2)
  110 FORMAT(2F10.2,D10.2)
  111 FORMAT(2I10)
 1001 FORMAT('1',////10X,'**************************************************
     1***'        /10X,' MONTE-CARLO STUDY OF PATTERN LOADINGS(LIVE LOAD)'
     1          /10X,'*****************************************************')
 1002 FORMAT(////5X,'ECHOCHECK OF INPUT DATA - FRAME',I4
     1              /5X,'**************************')
 1003 FORMAT(//5X,'GENERAL DATA'/5X,'------------')
 1004 FORMAT(10X,'NUMBER OF RUNS                          =',I10
     1          /10X,'DESIGN LIFE OF THE STRUCTURE       =',F10.1,' YEARS')
 1005 FORMAT(//5X,'FRAME PROPERTIES'/5X,'----------------')
 1006 FORMAT(10X,'NUMBER OF SPANS                         =',I10
     1          /10X,'TRIBUTARY WIDTH                    =',F10.2,' METERS'
     1          /10X,'RATIO OF COLUMN TO BEAM STIFFNESS=',F10.2)
 1007 FORMAT(/10X,'SPAN LENGTH IN METERS'/15X,5F10.2)
 1008 FORMAT(/10X,'LOAD EFFECT INFLUENCE COEFFICIENT MATRIX:'
     1          /10X,'-----------------------------------------')
 1009 FORMAT(/10X,'TOTAL NUMBER OF ROWS =',I4
     1/20X,'(THE LAST',I3,' ROWS DO NOT REQUIRE LIFETIME MINIMUM VALUES)
     1')
 1010 FORMAT(15X,5E10.4)
 1011 FORMAT(////5X,'SUS[illegible] LOAD'/5X,'---------------')
 1012 [illegible]10X,'MEAN SU[illegible] LOAD                =',F10.3,' KN/SQ.METE
     [illegible]'STANDARD [illegible]ATION OF SURVEY LOAD=',F10.3,' KN/SQ.METE
     [illegible]'THE ASSOCI[illegible]AREA                =',F10.2,' SQ.METER'
     1  [illegible]/1[illegible]'INITIAL MAGNITUDE SEED NUMBER    =',D18.10)
 1013 [illegible]10[illegible]'MEAN LOAD DURATION               =',F10.2,' YEARS'
     1   [illegible]'MINIMUM LOAD DURATION           =',F10.2,' YEARS'
     1          /10X,'INITIAL DURATION SEED NUMBER     =',D18.10)
 1014 FORMAT(//5X,'UNOCCUPIED PERIOD'/5X,'-----------------')
 1015 FORMAT(10X,'MINIMUM DURATION                      =',F10.2,' DAYS'
     1          /10X,'MAXIMUM DURATION                 =',F10.2,' DAYS'
     1          /10X,'TIME BETWEEN TWO PERIODS         =',F10.2,' YEARS'
     1          /10X,'MINIMUM TIME BETWEEN TWO PERIODS =',F10.2,' YEARS'
     1          /10X,'INITIAL SEED NUMBER              =',D18.10)
 1016 FORMAT(//5X,'TRANSIENT LOAD'/5X,'--------------')
 1017 FORMAT(10X,'MEAN NUMBER OF LOADS PER CELL         =',I5
     1          /10X,'STANDARD DEVIATION OF LOADS PER CELL=',I5
     1          /10X,'MEAN WEIGHT OF SINGLE LOAD       =',F10.3,' KN'
     1          /10X,'STANDARD DEVIATION OF SINGLE LOAD =',F10.3,' KN'
     1          /10X,'INITIAL MAGNITUDE SEED NUMBER    =',D18.10)
 1018 FORMAT(10X,'MEAN TIME BETWEEN TWO LOADS           =',F10.2,' DAYS'
     1          /10X,'MINIMUM TIME BETWEEN TWO LOADS   =',F10.2,' DAYS'
     1          /10X,'DURATION OF ONE LOAD             =',F10.2,' HOURS'
     1          /10X,'INITIAL TIME SEED NUMBER         =',D18.10)
 1019 FORMAT(//5X,'*****END OF INPUT DATA*****')
 1020 FORMAT(/10X,'ROW',I3,' TO',I3,' CORRESPONDS TO BEAM MOMENTS')
 1021 FORMAT(10X,'ROW',I3,[illegible],I3,' CORRESPONDS TO SHEARS')
 1081 FORMAT(10X,'ROW',I3,' TO',I3,' CORRESPONDS TO COLUMN MOMENTS')
 1022 FORMAT(10X,'ROW',I3,' TO',I3,' CORRESPONDS TO AXIAL FORCES')
 1031 FORMAT(///10X,' SPAN      STANDARD DEVIATION OF SUSTAINED EUDL'
     1          /30X,'--KN/SQ.METER--')
 1032 FORMAT(5X,I5,17X,F10.3)
 1033 FORMAT(//10X,' SPAN        MEAN OF TRANSIENT EUDL      STD DEVIATION'
     1          /25X,'KN/SQ.METER                KN/SQ.METER')
 1034 FORMAT(6X,I5,F20.3,F23.3)
 1051 FORMAT('1',///////////////50X,'*************************'
     1                  //50X,'  RESULTS OF ANALYSIS'
     1                   //50X,'*************************')
 1052 FORMAT(////37X,'FROM LOCATION',I3,' TO',I3,' - BEAM MOMENTS IN KN-
     1METER/METER-WIDTH')
 1053 FORMAT(////37X,'FROM LOCATION',I3,' TO',I3,' - SHEARS IN KN/METER-
     1WIDTH')
 1082 FORMAT(////37X,'FROM LOCATION',I3,' TO',I3,' - COLUMN MOMENTS IN K
     1N-METER/METER-WIDTH')
 1054 FORMAT(////37X,'FROM LOCATION',I3,' TO',I3,' - AXIAL FORCES IN KN/
     1METER-WIDTH')
C
      END
C
C
C
C
C                         *******************
C
C                         SUBROUTINE SUSTAN
C
C                         *******************
C
C
C     THIS SUBROUTINE SELECTS THE NUMBER AND MAGNITUDES OF SUSTAINED LOADS
C     OCCURRING IN ONE SPAN IN THE LIFETIME OF THE STRUCTURE. THE DURATION
```

```
C     AND NUMBER OF UNOCCUPIED PERIODS ARE ALSO SELECTED. LOAD MAGNITUDES
C     (KN/METER/METER-WIDTH)AND THE TIME AT THE BEGINNING OF EACH DIFFERENT
C     LOAD ARE STORED IN 'SUSLN' AND 'INTSN' RESPECTIVELY.
C
      SUBROUTINE SUSTAN
C
      DOUBLE PRECISION SSEED,SDSEED,UNSEED,TLSEED,TSEED
C
      COMMON SSEED,SDSEED,UNSEED,TLSEED,TSEED
      COMMON XMAXM(5000,22),XMINM(5000,16),Q(500,5),INT(500,5),
     1       TRANL(200),INTR(200),SUSLN(100),INTSN(100),XMCOEF(22,5),
     1       PS(5),PT(5),SLENDA(5),TLENDA(5),ISLIFE,NURUN,NUSPAN,NUROW,
     1       ISMEAN,ISDMIN,NDMIN,NDMAX,NTIME,NFMIN,ITMEAN,ITRMIN
      COMMON ISPAN,IRUN,NU01,NFRAME,NUROW2
C
      DIMENSION WK(152),ISDUR(50),SUSL(50),R(1)
C
      ISTOT=0
      NUSUS=1
      ISF=ISLIFE-ISDMIN
      SMEAN=FLOAT(ISMEAN)
      SDMIN=FLOAT(ISDMIN)
C
C     SELECT SUSTAINED LOAD DURATION FROM THE EXPONENTIAL DISTRIBUTION
C
      NUR=1
   11 CALL GGEXN(SDSEED,SMEAN,NUR,R)
      IF(R(1).LT.SDMIN)GO TO 11
C
C     DETERMINE NUMBER OF SUSTAINED LOADS IN THE DESIGN LIFETIME
C
      ISDUR(NUSUS)=R(1)+0.5
      ISTOT=ISTOT+ISDUR(NUSUS)
      IF(ISTOT.GT.ISF)GO TO 12
      NUSUS=NUSUS+1
      GO TO 11
   12 ISDUR(NUSUS)=ISLIFE-(ISTOT-ISDUR(NUSUS))
C
C     SELECT THE MAGNITUDE OF EACH SUSTAINED LOAD FROM THE GAMMA
C     DISTRIBUTION
C
      CALL GGAMR(SSEED,PS(ISPAN),NUSUS,WK,SUSL)
      SLEND=SLENDA(ISPAN)
      DO 13 I=1,NUSUS
      SUSL(I)=SUSL(I)/SLEND
   13 CONTINUE
C
      NP=1
      ISN=0
      NPSD=1
      TIME=FLOAT(NTIME)
      DMIN=FLOAT(NDMIN)
      DIFF=FLOAT(NDMAX)-DMIN
C
      DO 21 ISUS=1,NUSUS
C
C     DETERMINE NUMBER OF UNOCCUPIED PERIODS IN EACH SUSTAINED LOAD
C     ASSUMING THAT AN UNOCCUPIED PERIOD OCCURS AT THE BEGINNING OF EACH
C     SUSTAINED LOAD. THE MINIMUM TIME (NFMIN) BETWEEN THE OCCURRENCE OF
C     THE LAST UNOCCUPIED PERIOD AND THE OCCURRENCE OF NEXT SUSTAINED
C     LOAD MUST BE GREATER THAN THE MAXIMUM DURATION OF AN UNOCCUPIED
C     PERIOD (NDMAX). NFMIN MUST BE LESS THAN THE MINIMUM SUSTAINED
C     LOAD DURATION(SDMIN).
C
      ISD=ISDUR(ISUS)
      XNUNG=FLOAT(ISD)/TIME+1.0
      NUNG=XNUNG
      ITIME=TIME*(XNUNG-FLOAT(NUNG))+0.5
      IF(ITIME.LT.NFMIN)NUNG=NUNG-1
C
      DO 22 J=1,NUNG
C
C     ASSIGN ZERO LOAD TO EACH UNOCCUPIED PERIOD AND RECORD THE TIME AT
C     THE BEGINNING OF EACH UNOCCUPIED PERIOD
C
      ISN=ISN+1
      SUSLN(ISN)=0.0
      INTSN(ISN)=NP
C
C     SELECT DURATION OF EACH UNOCCUPIED PERIOD FROM UNIFORM DISTRIBUTION
C     (THE MINIMUM DURATION NDMIN MUST BE GREATER THAN OR EQUAL TO THE
C     DURATION OF A TRANSIENT LOAD) .
C
      INEG=DMIN+DIFF*GGUBFS(UNSEED)+0.5
C
C     RECORD THE NON-ZERO SUSTAINED LOAD AND THE TIME AT ITS BEGINNING
C     IT IS ASSUMED THAT THE MINIMUM DURATION OF A SUSTAINED LOAD ISDMIN
C     MUST BE GREATER THAN THE MAXIMUM DURATION OF AN UNOCCUPIED PERIOD
C     (NDMAX).
C
      ISN=ISN+1
      SUSLN(ISN)=SUSL(ISUS)
      INTSN(ISN)=NP+INEG
C
C     IT IS ASSUMED THAT THE TIME BETWEEN TWO UNOCCUPIED PERIODS (NTIME)
C     MUST BE GREATER THAN THE MAXIMUM DURATION OF AN UNOCCUPIED PERIOD
C     (NDMAX)
C
      NP=NP+NTIME
   22 CONTINUE
C
      NPSD=NPSD+ISD
      NP=NPSD
   21 CONTINUE
C
C     STORE THE LAST SUSTAINED LOAD AT THE END OF THE LIFETIME
C
      ISN=ISN+1
```

```
      SUSLN(ISN)=SUSL(NUSUS)
      INTSN(ISN)=ISLIFE
C
      RETURN
C
      END
C
C
C
C                      *******************
C
C                      SUBROUTINE TRANST
C
C                      *******************
C
C     THIS SUBROUTINE SELECTS THE MAGNITUDES AND TIME BETWEEN OCCURRENCES
C     OF TRANSIENT LOADS IN ONE SPAN IN THE LIFETIME OF THE STRUCTURE. THE
C     LOAD MAGNITUDES(KN/METER/METER-WIDTH) AND THE TIME AT THE OCCURRENCE
C     OF TRANSIENT LOADS ARE STORED IN 'TRANL' AND 'INTR' RESPECTIVELY.
C     IT IS ASSUMED THAT THE OCCURRENCE OF TRANSIENT LOADS IS INDEPENDENT
C     OF THE OCCURRENCE OF UNOCCUPIED PERIODS,THEREFORE,A TRANSIENT LOAD
C     IS PROBABLE TO OCCUR IN AN UNOCCUPIED PERIOD
C
      SUBROUTINE TRANST
C
      DOUBLE PRECISION SSEED,SDSEED,UNSEED,TLSEED,TSEED
C
      COMMON SSEED,SDSEED,UNSEED,TLSEED,TSEED
      COMMON XMAXM(5000,23),XMINM(5000,18),Q(500,5),INT(500,5),
     1       TRANL(200),INTR(200),SUSLN(100),INTSN(100),XMCOEF(23,5),
     1       PS(5),PT(5),SLENDA(5),TLENDA(5),ISLIFE,NURUN,NUSPAN,NUROW,
     1       ISMEAN,ISDMIN,NDMIN,NDMAX,NTIME,NFMIN,ITMEAN,ITRMIN
      COMMON ISPAN,IRUN,NUD1,NFRAME,NUROW2
C
      DIMENSION WK(500), R(1)
C
C     ASSUME THE FIRST TRANSIENT LOAD OCCURS AT THE BEGINNING
C
      INTR(1)=1
C
      ITOT=1
      NUTR=2
      ISP=ISLIFE-ITRMIN
      TMEAN=FLOAT(ITMEAN)
      TRMIN=FLOAT(ITRMIN)
C
C     SELECT THE TIME BETWEEN OCCURRENCES OF TRANSIENT LOADS FROM
C     EXPONENTIAL DISTRIBUTION. THE MINIMUM TIME(ITRMIN)MUST BE GREATER
C     THAN OR EQUAL TO THE DURATION OF ONE TRANSIENT LOAD(1)
C
      NUR=1
   11 CALL GGEXN(TSEED,TMEAN,NUR,R)
      IF(R(1).LT.TRMIN)GO TO 11
C
C     DETERMINE NUMBER OF TRANSIENT LOADS IN THE LIFETIME
C
      INTIME=R(1)+0.5
      ITOT=ITOT+INTIME
      IF(ITOT.GT.ISP)GO TO 12
C
C     RECORD THE TIME AT EACH TRANSIENT LOAD OCCURRENCE
C
      INTR(NUTR)=ITOT
C
      NUTR=NUTR+1
C
C     CONTROL THAT THE NUMBER OF TRANSIENT LOADS WILL NOT EXCEED THE
C     STORAGE SPACE
C
      IF(NUTR.EQ.200)GO TO 12
C
      GO TO 11
C
C     THE LAST TRANSIENT LOAD WHICH IS ASSIGNED ZERO MAGNITUDE OCCURS AT
C     THE END OF THE LIFETIME
C
   12 INTR(NUTR)=ISLIFE
      TRANL(NUTR)=0.0
C
C     SELECT THE MAGNITUDE OF EACH TRANSIENT LOAD FROM GAMMA DISTRIBUTION
C
      ITR=NUTR-1
      CALL GGAMR(TLSEED,PT(ISPAN),ITR,WK,TRANL)
      TLEND=TLENDA(ISPAN)
      DO 13 I=1,ITR
      TRANL(I)=TRANL(I)/TLEND
   13 CONTINUE
C
      RETURN
C
      END
C
C
C
C
C
C
C
C
C                      *******************
C
C                      SUBROUTINE COMBND
C
C                      *******************
C
C     THIS SUBROUTINE COMBINES THE SUSTAINED LOADS AND TRANSIENT LOADS.
C     THE LOAD MAGNITUDES AND THE TIME AT THE BEGINNING OF EACH DIFFERENT
```

```
C   COMBINED LOAD ARE STORED IN 'Q' AND 'INT' RESPECTIVELY.
C
      SUBROUTINE COMBND
C
      DOUBLE PRECISION SSEED,SDSEED,UNSEED,TLSEED,TSEED
C
      COMMON SSEED,SDSEED,UNSEED,TLSEED,TSEED
      COMMON XMAXM(5000,23),XMINM(5000,18),Q(500,5),INT(500,5),
     1       TRANL(200),INTR(200),SUSLN(100),INTSN(100),XMCOEF(23,5),
     1       PS(5),PT(5),SLENDA(5),TLENDA(5),ISLIFE,NURUN,NUSPAN,NUROW,
     1       ISMEAN,ISDMIN,NDMIN,NDMAX,NTIME,NFMIN,ITMEAN,ITRMIN
      COMMON ISPAN,IRUN,NUQ1,NFRAME,NUROW2
C
      NO=1
      ISN=1
      ITR=1
C
   10 CONTINUE
C
      LNTSN=INTSN(ISN)
      LNTR=INTR(ITR)
      IF(LNTSN-LNTR)11,12,13
C
C
   11 Q(NO,ISPAN)=SUSLN(ISN)
      INT(NO,ISPAN)=LNTSN
      ISN=ISN+1
      NO=NO+1
      GO TO 10
C
C
C   IT IS ASSUMED IN THE FOLLOWING THAT THE MINIMUM DURATION OF A
C   SUSTAINED LOAD MUST BE GREATER THAN OR EQUAL TO TWICE THE DURATION
C   OF A TRANSIENT LOAD.
C
   12 Q(NO,ISPAN)=SUSLN(ISN)+TRANL(ITR)
      INT(NO,ISPAN)=LNTSN
      IF(LNTSN.EQ.ISLIFE)GO TO 14
      INTR2=LNTR+1
      ITR=ITR+1
      ISN=ISN+1
      NO=NO+1
      IF(INTR(ITR).EQ.INTR2)GO TO 10
      Q(NO,ISPAN)=SUSLN(ISN-1)
      INT(NO,ISPAN)=INTR2
      NO=NO+1
      GO TO 10
C
C
   13 Q(NO,ISPAN)=SUSLN(ISN-1)+TRANL(ITR)
      INT(NO,ISPAN)=LNTR
      INTR2=LNTR+1
      ITR=ITR+1
      NO=NO+1
      IF(INTR(ITR).EQ.INTR2)GO TO 10
      IF(INTR2.EQ.LNTSN)ISN=ISN+1
      Q(NO,ISPAN)=SUSLN(ISN-1)
      INT(NO,ISPAN)=INTR2
      NO=NO+1
      GO TO 10
C
C
   14 IF(ISPAN.EQ.1)NUQ1=NO
C
      RETURN
C
      END
C
C
C                               ******************
C
C                               SUBROUTINE EFFECT
C
C                               ******************
C
C
C   THIS SUBROUTINE CALCULATES LOAD EFFECTS IN EACH SPAN WHEN THERE ARE
C   CHANGES OF LOADS IN ANY SPAN. THE LIFETIME (ALGEBRAIC) MAXIMUM AND
C   MINIMUM VALUES PER UNIT WIDTH ARE SELECTED AND STORED IN 'XMAXM'
C   AND 'XMINM' RESPECTIVELY
C
      SUBROUTINE EFFECT
C
      DOUBLE PRECISION SSEED,SDSEED,UNSEED,TLSEED,TSEED
C
      COMMON SSEED,SDSEED,UNSEED,TLSEED,TSEED
      COMMON XMAXM(5000,23),XMINM(5000,18),Q(500,5),INT(500,5),
     1       TRANL(200),INTR(200),SUSLN(100),INTSN(100),XMCOEF(23,5),
     1       PS(5),PT(5),SLENDA(5),TLENDA(5),ISLIFE,NURUN,NUSPAN,NUROW,
     1       ISMEAN,ISDMIN,NDMIN,NDMAX,NTIME,NFMIN,ITMEAN,ITRMIN
      COMMON ISPAN,IRUN,NUQ1,NFRAME,NUROW2
C
      DIMENSION XMAX(23),XMIN(18),W(5),KOUNT(5)
C
      DO 12 IROW=1,NUROW
      XMAX(IROW)=0.0
   12 CONTINUE
C
C   BECAUSE INT(1,ISPAN)=1 FOR ISPAN=1,NUSPAN, THEREFORE THE FIRST XMIN
C   AND XMAX CAN BE ASSUMED AS THE FOLLOWING:
C
      DO 21 IROW=1,NUROW
      DO 22 ISPAN=1,NUSPAN
      XMAX(IROW)=XMAX(IROW)+XMCOEF(IROW,ISPAN)*Q(1,ISPAN)
   22 CONTINUE
   21 CONTINUE
C
      NUROW1=NUROW-NUROW2
```

```
      IF(NUROW1.EQ.0)GO TO 41
      DO 33 IROW=1,NUROW1
      XMIN(IROW)=XMAX(IROW)
   33 CONTINUE
C
   41 CONTINUE
      DO 11 ISPAN=1,NUSPAN
      KOUNT(ISPAN)=2
   11 CONTINUE
C
   10 K=KOUNT(1)
      IF(K.GT.NUO1)GO TO 13
C
C     DETERMINE THE EARLIEST OCCURRENCE OF LOAD CHANGES AMONG THE SPANS
C
      MIN=INT(K,1)
      DO 14 ISPAN=2,NUSPAN
      NO=KOUNT(ISPAN)
      IF(INT(NO,ISPAN).LT.MIN)MIN=INT(NO,IBPAN)
   14 CONTINUE
C
C     SELECT THE APPROPRIATE LOADINGS TO CALCULATE LOAD EFFECTS
C     USING THE LOAD EFFECT INFLUENCE COEFFICIENTS
C
      DO 15 ISPAN=1,NUSPAN
      NO=KOUNT(ISPAN)
      IF(INT(NO,ISPAN).EQ.MIN)GO TO 16
      W(ISPAN)=Q(NO-1,ISPAN)
      GO TO 15
   16 W(ISPAN)=Q(NO,ISPAN)
      KOUNT(ISPAN)=KOUNT(ISPAN)+1
   15 CONTINUE
C
      DO 17 IROW=1,NUROW
      XMOMT=0.0
C
      DO 18 ISPAN=1,NUSPAN
      XMOMT=XMOMT+XMCOEF(IROW,ISPAN)*W(ISPAN)
   18 CONTINUE
C
C     PICK THE GREATER AND SMALLER LOAD EFFECT. THEN FINALLY DETERMINE
C     THE LIFETIME (ALGEBRAIC) MAXIMUM AND MINIMUM VALUES.
C
      IF(IROW.GT.NUROW1)GO TO 25
      IF(XMOMT.LT.XMIN(IROW))GO TO 34
   25 IF(XMOMT.GT.XMAX(IROW))XMAX(IROW)=XMOMT
      GO TO 17
   34 XMIN(IROW)=XMOMT
C
   17 CONTINUE
C
      GO TO 10
C
   13 CONTINUE
C
C     STORE THE LIFETIME MAXIMUM AND MINIMUM VALUES
C
      DO 20 IROW=1,NUROW
      XMAXM(IRUN,IROW)=XMAX(IROW)
   20 CONTINUE
C
      IF(NUROW1.EQ.0)GO TO 42
      DO 35 IROW=1,NUROW1
      XMINM(IRUN,IROW)=XMIN(IROW)
   35 CONTINUE
C
   42 RETURN
C
      END
C
C
C                          ********************
C
C                          SUBROUTINE OUTPUT
C
C                          ********************
C
C
C     THIS SUBROUTINE SORTS THE CIFETIME MAXIMUM AND MINIMUM LOAD
C     EFFECTS AND COMPUTES STATISTICAL PARAMETERS. FINALLY WRITES
C     THE RESULTS AND THE SORTED VALUES.
C
      SUBROUTINE OUTPUT
C
      DOUBLE PRECISION SSEED,SDSEED,UNSE[illegible]LSEED,TSEED
C
      COMMON SSEED,SDSEED,UNSEED,TLSEED,TSEED
      COMMON XMAXM(5000,23),XMINM(5000,18),Q(500,5),INT(500,5),
     1       TRANL(200),INTR(200),SUSLN(100),INTSN(100),XMCOEF(23,5),
     1       PS(5),PT(5),SLENDA(5),TLENDA(5),ISLIFE,NURUN,NUSPAN,NUROW,
     1       ISMEAN,ISDMIN,NDMIN,NDMAX,NTIME,NFMIN,ITMEAN,ITRMIN
      COMMON ISPAN,IRUN,NUO1,NFRAME,NUROW2
C
      DIMENSION RMOMT(5000),RM(98)
C
      DOUBLE PRECISION STDV
C
      XNURUN=FLOAT(NURUN)
      XMULT=XNURUN/100.
      MULT=NURUN/100
      DIFF=XMULT-FLOAT(MULT)
C
      INC=XNURUN*0.01+0.5
      MAX=XNURUN*0.99+0.5
      NR=NURUN+1
      NUROW1=NUROW-NUROW2
C
      DO 11 IROW=1,NUROW
```

```
      C
            IF(IROW.GT.NUROW)GO TO 22
      C
            DO 12 IRUN=1,NURUN
            RMOMT(IRUN)=XMINM(IRUN,IROW)
       12   CONTINUE
      C
      C    SORT THE MINIMUM VALUES BY ALGEBRAIC VALUE
      C
            CALL VSRTA(RMOMT,NURUN)
      C
      C    CALCULATE THE MEAN,STANDARD DEVIATION,COEFFICIENT OF VARIATION
      C    COEFFICIENT OF SKEWNESS AND COEFFICIENT OF KURTOSIS
      C
            CALL STAT(RMOMT,NURUN,RMEAN,STDV,COVAR,COSKEW,COKUR)
      C
      C    WRITE THE RESULTS AND THE SORTED LIFETIME MINIMUM VALUES
      C
            WRITE(6,102)IROW,NFRAME
            WRITE(6,1001)
            WRITE(6,103)RMEAN,STDV,COVAR,COSKEW,COKUR
      C
      C    IF NUMBER OF RUNS IS MULTIPLES OF 100,VALUES OF 0.01 FRACTILE TO
      C    0.99 FRACTILE WITH INCREMENTS OF 0.01 FRACTILE WILL BE PRINTED
      C
            IF(DIFF.GE.0.009)GO TO 90
      C
            DO 14 I=1,99
            J=NR-INC*I
            RM(I)=RMOMT(J)
       14   CONTINUE
      C
            WRITE(6,1003)
            WRITE(6,104)(RM(I),I=1,99)
      C
       90   WRITE(6,1005)
            WRITE(6,104)(RMOMT(I),I=1,NURUN)
      C
       22   CONTINUE
            DO 13 IRUN=1,NURUN
            RMOMT(IRUN)=XMAXM(IRUN,IROW)
       13   CONTINUE
      C
      C    SORT THE MAXIMUM VALUES AND CALCULATE THE STATISTICAL PARAMETERS
      C    THEN WRITE THE RESULTS AND THE SORTED VALUES
      C
            CALL VSRTA(RMOMT,NURUN)
            CALL STAT(RMOMT,NURUN,RMEAN,STDV,COVAR,COSKEW,COKUR)
      C
            WRITE(6,102)IROW,NFRAME
            WRITE(6,1002)
            WRITE(6,103)RMEAN,STDV,COVAR,COSKEW,COKUR
      C
            IF(DIFF.GE.0.009)GO TO 91
            WRITE(6,1003)
            WRITE(6,104)(RMOMT(I),I=INC,MAX,INC)
      C
       91   WRITE(6,1005)
            WRITE(6,104)(RMOMT(I),I=1,NURUN)
      C
      102   FORMAT('1',//5X,'AT LOCATION',I3,' (REFERRING TO THE ROW NUMBER OF
           1 THE LOAD EFFECT INFLUENCE COEFFICIENT MATRIX) - FRAME',I4
           1        /5X,'****************')
      103   FORMAT(///10X,'MEAN                          =',F10.3
           1        //10X,'STANDARD DEVIATION            =',F10.3
           1        //10X,'COEFFICIENT OF VARIATION      =',F10.3
           1        //10X,'COEFFICIENT OF SKEWNESS       =',F10.3
           1        //10X,'COEFFICIENT OF KURTOSIS       =',F10.3//)
      104   FORMAT(5(2X,10F13.3/))
      1001  FORMAT(//5X,'LIFETIME (ALGEBRAIC) MINIMUM VALUES PER UNIT WIDTH:'
           1       /5X,'--------------------------------------------------')
      1002  FORMAT(//5X,'LIFETIME (ALGEBRAIC) MAXIMUM VALUES PER UNIT WIDTH:'
           1       /5X,'--------------------------------------------------')
      1003  FORMAT(10X,'VALUES OF 0.01 FRACTILE TO 0.99 FRACTILE WITH INCREMEN
           1TS OF 0.01 FRACTILE '/10X,'----------------------------------------
           1--------------------------------------'/)
      1005  FORMAT(///10X,'COMPLETE DATA'/10X,'-------------'/)
      C
       11   CONTINUE
      C
      C    WRITE THE FINAL SEED NUMBERS
      C
            WRITE(6,105)
            WRITE(6,106)SSEED
            WRITE(6,107)SDSEED
            WRITE(6,108)UNSEED
            WRITE(6,109)TLSEED
            WRITE(6,110)TSEED
      C
      105   FORMAT(//5X,'FINAL SEED NUMBERS'/5X,'------------------')
      106   FORMAT(/10X,'SSEED    =',D18.11)
      107   FORMAT(10X,'SDSEED   =',D18.11)
      108   FORMAT(10X,'UNSEED   =',D18.11)
      109   FORMAT(10X,'TLSEED   =',D18.11)
      110   FORMAT(10X,'TSEED    =',D18.11)
      C
            RETURN
      C
            END
      C
      C
      C                         ****************
      C
      C                         SUBROUTINE STAT
      C
      C                         ****************
      C
      C
```

```
C     THIS SUBROUTINE CALCULATES THE MEAN,STANDARD DEVIATION,COEFFICIENT
C     OF VARIATION,COEFFICIENT OF SKEWNESS AND COEFFICIENT OF KURTOSIS.
C
      SUBROUTINE STAT(X,N,XMEAN,STDV,COVAR,COSKEW,COKUR)
C
      DIMENSION X(N)
C
      DOUBLE PRECISION VAR,STDV
C
      COVAR=0.0
      COSKEW=0.0
      COKUR=0.0
      XN=FLOAT(N)
C
      SUM1=0.0
      DO 11 I=1,N
      SUM1=SUM1+X(I)
   11 CONTINUE
C
      XMEAN=SUM1/XN
C
      SUM2=0.0
      DO 12 I=1,N
      SUM2=SUM2+X(I)**2
   12 CONTINUE
C
      VAR=(SUM2-XMEAN*SUM1)/FLOAT(N-1)
      STDV=DSQRT(VAR)
C
      IF(XMEAN.EQ.0.0)GO TO 15
      COVAR=STDV/XMEAN
C
   15 IF(STDV.EQ.0.0)GO TO 16
C
      SUM3=0.0
      DO 13 I=1,N
      SUM3=SUM3+X(I)**3
   13 CONTINUE
C
      SKEW=(SUM3-3.0*XMEAN*SUM2)/XN+2.0*(XMEAN**3)
      COSKEW=SKEW/(STDV**3)
C
      SUM4=0.0
      DO 14 I=1,N
      SUM4=SUM4+X(I)**4
   14 CONTINUE
C
      XKUR=(SUM4-4.0*XMEAN*SUM3+6.0*XMEAN*XMEAN*SUM2)/XN-3.0*(XMEAN**4)
      COKUR=XKUR/(STDV**4)
C
   16 RETURN
C
C
C
C
C                                 ***************
C
C                                  IMSL ROUTINES
C
C                                 ***************
C
C
C   THE FOLLOWING IS A BRIEF DESCRIPTION OF THE IMSL SUBROUTINES AND
C   FUNCTIONS WHICH ARE USED IN THIS PROGRAM.  THIS INFORMATION IS
C   AVAILABLE IN THE IMSL LIBRARY REFERENCE MANUAL EDITION 7 JAN., 1979
C   THE IMSL ROUTINES ARE STORED IN THE PUBLIC FILE *IMSLLIB
C
C
C                          *****
C   IMSL ROUTINE NAME  -   GGEXN
C                          *****
C
C   PURPOSE            -   EXPONENTIAL RANDOM DEVIATE GENERATOR
C
C   USAGE              -   CALL GGEXN(DSEED,XM,N,R)
C
C   ARGUMENTS    DSEED -   INPUT.  AN INTEGER VALUE IN THE EXCLUSIVE RANGE
C                          (1,2147483647). DSEED IS REPLACED BY A NEW DSEED
C                          TO BE USED IN SUBSEQUENT CALLS.  DSEED MUST BE
C                          TYPED DOUBLE PRECISION IN THE CALLING PROGRAM.
C
C                   XM -   INPUT MEAN VALUE
C
C                    N -   INPUT NUMBER OF DEVIATES TO BE GENERATED
C
C                    R -   OUTPUT VECTOR OF LENGTH N CONTAINING THE
C                          EXPONENTIAL DEVIATES
C
C
C
C                          *****
C   IMSL ROUTINE NAME  -   GGAMR
C                          *****
C
C
C   PURPOSE            -   ONE PARAMETER GAMMA RANDOM DEVIATE GENERATOR,AND
C                          USABLE AS THE BASIS FOR TWO-PARAMETER GAMMA
C                          DEVIATE GENERATOR
C
C   USAGE              -   CALL GGAMR(DSEED,A,N,WK,R)
C
C   ARGUMENTS    DSEED -   INPUT.  AN INTEGER VALUE IN THE EXCLUSIVE RANGE
C                          (1,2147483647). DSEED IS REPLACED BY A NEW DSEED
C                          TO BE USED IN SUBSEQUENT CALLS.  DSEED MUST BE
C                          TYPED DOUBLE PRECISION IN THE CALLING PROGRAM.
C
C                    A -   INPUT SHAPE PARAMETER FOR THE DESIRED GAMMA
C                          FUNCTION. A MUST BE GREATER THAN 0.
C
C                    N -   INPUT NUMBER OF DEVIATES TO BE GENERATED
C
```

```
C                              WK - VECTOR OF DIMENSION 3*(N+1) USED AS WORK AREA
C
C                               R - OUTPUT VECTOR OF LENGTH N CONTAINING THE GAMMA
C                                   DEVIATES
C
C
C                                   ******
C   IMSL ROUTINE NAME             - GGUBFS
C                                   ******
C
C   PURPOSE                       - BASIC UNIFORM (0,1) RANDOM NUMBER GENERATOR -
C                                   FUNCTION FORM OF GGUBS
C
C   USAGE                         - FUNCTION GGUBFS(DSEED)
C
C   ARGUMENTS    GGUBFS           - RESULTANT DEVIATE
C
C                DSEED            - INPUT   AN INTEGER VALUE IN THE EXCLUSIVE RANGE
C                                   (1,2147483647)  DSEED IS REPLACED BY A NEW DSEED
C                                   TO BE USED IN SUBSEQUENT CALLS   DSEED MUST BE
C                                   TYPED DOUBLE PRECISION IN THE CALLING PROGRAM
C
C
C                                   *****
C   IMSL ROUTINE NAME             - VSRTA
C                                   *****
C
C   PURPOSE                       - SORTING OF ARRAYS BY ALGEBRAIC VALUE
C
C   USAGE                         - CALL VSRTA(A,LA)
C
C   ARGUMENTS                   A - ON INPUT, A CONTAINS THE ARRAY TO BE SORTED
C                                   ON OUTPUT, A CONTAINS THE SORTED ARRAY
C
C                              LA - INPUT VARIABLE CONTAINING THE NUMBER OF ELEMENTS
C                                   IN THE ARRAY TO BE SORTED
C
C
C
C
      END                           **** END OF THIS PROGRAM ****
END OF FILE
```