Essays on Information and Transaction Quantities

by

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Abstract

This thesis presents two essays. I (1) study the effect of short selling regulation SHO on informational efficiency of naked short selling activity; (2) analyze the role of reference demands in traders' decision-making. In these essays, I highlight the informative role of transaction quantities.

Chapter 1 studies the impact of the 2005 short selling regulation (regulation SHO) and its more restrictive version of 2008. Regulation SHO was put in place to curb potentially manipulative naked short selling. However, this regulation has been criticized in the literature for reducing market quality. Contrary to other findings, I show that this regulation deters uninformed traders, and improves the informativeness of naked short sellers. In particular, after 2008, the aggregate naked short selling activity has increased in information content and has become significantly connected to the percentage of net short positions in the E-Mini stock index futures markets. Consistent with the increased informativeness of naked short sellers, I find that the market views excessive and persistent naked short selling activity as a bearish signal only after 2008.

In Chapter 2, I analyze an economy where each trader demonstrates the behaviour of assessing other traders' average opinion of his demand. Each trader forms his unique expectation about such an average opinion to obtain his reference demand. Traders attempt to find their optimal demands so that they do not substantially deviate from their reference demands. I find that the standard differences of opinion models suggest that traders take positions exactly equal to their own reference demands, and further, volume and social welfare increase once traders pay attention to their reference demands. However, I propose a novel model indicating that social welfare does not necessarily increase. In this suggested model, when traders pay more attention to their reference demands, numerical instances show that volume increases and their optimal demands get closer to their reference demands. The models explain one component of the information in equilibrium price that comes from demand.

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Introduction

This thesis consists of two essays about transaction quantities, as they play an important role in informational efficiency and forecasting.

In the first essay, chapter 1, I study the effect of recent short selling regulation SHO which was implemented in 2005 to curb potentially abusive manipulative naked short sellers. When executing short sales, naked short sellers do not borrow or arrange to borrow shares to be delivered within the standard three-day settlement period. When shares are delivered with a delay, i.e. not delivered on time, the so-called failure-to-deliver (FTD) occurs. The literature indicates that most FTD occurrences are due to naked short selling activity.

The following incident demonstrates the underlying problems related to large FTDs. In February of 2005, Robert Simpson, an investor, purchased 1,158,209 shares of Global Links Corporation for $\$5,205.00$ from the OTC bulletin board. Global Links Corporation was a small real estate company, and its CEO Frank Dobrucki mentioned in 2005 that his company had millions of dollars in real estate assets. The total shares available for purchase at that time was 1,158,064. Therefore, Simpson purchased 145 shares more than the total shares outstanding, a small but significant difference. Simpson properly completed this transaction, filed the appropriate paperwork with the U.S. Securities and Exchange Commission (SEC), and did not trade a single share in the next two days. However, shares continued to be traded at high volumes: 37,044,500 shares the day after Simpson's trade, and 22,471,000 shares on the following trading day. Although the dollar amounts of the company's trades were not high, it is important to remember that in the national stock exchanges cash and stock transfer are done separately. My focus in chapter 1 is on whether stock transfers are done properly in and of themselves. Therefore, the above serves as an example showing how the stock market might not function the way we would expect.

When FTDs are large and persistent, they create an artificial supply of shares, may affect market stability, make investors lose confidence in the markets, and distort prices. To reduce FTDs, the SEC further strengthened the regulation SHO in September 2008 (2008 regulation). This update was first supported by some companies whose shares are publicly traded. The existing literature, however, asks for relaxing the 2008 regulation and going back to the original regulation SHO implemented in 2005. For instance, Fotak et al. (2014) argue that large FTDs can lead to pricing efficiency and liquidity, and the impact of naked short selling is similar to the impact of covered short selling on the markets. Liu et al. (2015) also find that FTDs are higher when accounting fundamentals contain negative information about company's performance, and hence that FTDs are informative. This literature questions whether the 2008 regulation is overall beneficial for the markets.

The goal of this regulation is to reduce manipulative trading. However, we cannot identify manipulative traders, and therefore I ask whether the 2008 regulation has been in part successful in deterring relatively uninformed naked short sellers. I implement three empirical methods to answer this question.

My findings suggest that after the implementation of the 2008 regulation, both uninformed and informed short sellers have been reduced in numbers from the pool of short sellers. More importantly, uninformed naked short sellers may have been deterred more so than informed ones. In my first empirical test, I measure the informativeness of aggregate naked short selling using net short activities of five trader categories in electronically traded, small denomination stock index futures contracts (E-Minis). For the sample after September 2008, I find a strong relationship between the aggregate naked short selling in the stock market and the percentage of the net short positions of several trader categories in E-Mini stock index futures, such as dealers and leveraged funds. This relationship is not evident prior to September 2008. Although there are a variety of measures which have been used to calculate the informational efficiency of traders, I rely on the correlation among traders' activities to capture the informativeness of trades. One reason that this method is useful is because statistical methods have some limitations and cannot capture all sources of information. In my method, since some trader categories in stock index futures are found to be informed, their activities should be tied to naked short selling activity if the latter is found to be informative.

In my second empirical test, I look at the excessive and persistent FTD stocks which are publicly released by the NYSE and Nasdaq on each business day in the so-called threshold lists. I find that these stocks, which are under pressure from naked short sellers, experience near-future negative abnormal returns. What this suggests is that the market views the threshold listing as a bearish signal. This is only true after September 2008, while prior to this date the market views threshold listing as a bullish signal, i.e. stocks experience near-future positive abnormal returns. However, consistent with the findings for prior September 2008, I find that naked short sellers are contrarians and that they do not systematically profit from their short positions in the near future after September 2008.

Finally, the third empirical test to further support my findings is an investment strategy using weekly information frequency. This frequency is more appropriate than monthly or quarterly frequency as it captures most of the FTD information which happens only days after the settlement dates. To test whether FTDs contain information that is not fully incorporated in price, Liu et al., (2015) proposed a zero-investment strategy using quarterly frequency: long stocks with low short interest and low FTD stocks, and short stocks with high short interest and high FTD stocks. I find that this strategy's performance is more pronounced with weekly frequency and that this performance is more evident in the small Nasdaq firms.

In the second essay, chapter 2, I ask whether traders are fully certain about the size of their demand before submitting it. I model an economy where each trader adjusts their demand in a way which is not substantially different (larger) from the average of all other traders' opinions about their own demand. If the adjustments do not conform with the average opinion of their demand, they might experience a loss. For example, in 2012 a branch of JPMorgan Chase & Co. was selling larger than expected derivative positions despite some criticism. Other institutions started learning more about those derivatives, took opposite bets and made profits; JPMorgan Chase & Co. lost over six billion dollars.

An example of optimal demand is given by Kelly criterion, which gives the percentage of wealth that should be invested in order to maximize the long-term growth in a series of bets. Assume that a and b are positive numbers, and there is an investment opportunity with two outcomes. Let p be the probability of success where the value of an investment increase from 1 to $1 + b$, and $q = 1 - p$ be the probability of failure where the value of the investment decreases from 1 to $1 - a$. Kelly criterion suggests that the percentage of wealth that should be invested is

$$x^* = \frac{p}{a} - \frac{q}{b}$$

Although there are many underlying assumptions in this formula, it gives an (upper) estimation of what percentage of capital should be invested, and it helps to determine the optimal demand. This criterion was first introduced by Kelly (1956), and later on has been widely used in gambling and investment (well-known investors such as Warren Buffett and Bill Gross have also used this criterion; see Pabrai, 2007, and, Thorp, 2008). This criterion can serve as a tool for investors to estimate others' optimal demand. In this case, each investor can have an estimation of the average opinion of their own demand so that they do not deviate from it.

In the literature, the transaction quantities were found to contain information about future trading. For instance, Evans and Lyons (2007) find that transaction flows in foreign exchange markets forecast future macro variables significantly better than the exchange rate. Love and Payne (2008) find that public news is impounded into prices through order flows. Furthermore, Hong and Yogo (2012) find that in predicting future returns, open interest could be more informative than price in futures markets.

The market participants need to make strategic decisions with imperfect information and the more information they use, the more precise demand they would get. For example, price setters need to predict pricing decisions of other companies, producers in oligopolistic markets need to predict the output of their competitors, and speculators need to predict whether other speculators plan to attack a currency. In all these instances, the participants should pay some attention to the average opinion about their demand before trading.

To be close to the average opinion, the trader must have an estimation of the average opinion of their own demand. In my utility function, I assume that each trader reflects the behaviour that they would *not* be happier if they submit a demand which is substantially deviated from the average opinion of their own demand. I present three models which emphasize the ability of traders to estimate their demand from the average opinion about their own demand. This ability may arise due to the fact that traders have common models and have some estimations of each others' beliefs. For simplicity, in the presented models, I assume that all traders use the same utility maximization procedure (i.e. use the same model).

I define the *reference demand* to be a trader's expectation of the average of all other traders' opinion about his demand. In my models, I assume that traders try not to be too far from their own reference demands. I analyze the role of reference demand by employing three models; first, a classical differences of opinion model where traders disagree about the mean of the final payoff; second, a similar model where traders also disagree about the precision of the signals they receive; third, I propose a generalized version of the first model, where traders do not necessarily know the other traders' beliefs but receive noisy signals about those beliefs.

I find that in the classical differences of opinion models, the first two models, traders' optimal demands are equal to their own reference demands. Furthermore, the social welfare increases when traders pay more attention to the average opinion about their demand. However, in the proposed model, these conclusions are not necessarily hold. For all models, I find that volume increases once traders pay more attention to the average opinion about their demands which might explain some part of the empirically observed patterns. The foreign exchange markets often show the combination of relatively low volatility and high liquidity environments (Mangram, 2012), and institutional investors prefer stocks with large size, low volatility and high liquidity (Pinnuck, 2004). The presented models can therefore be further investigated to explain these high volume-low volatility environments.

Chapter 1

The Effect of Short Selling Regulation SHO on Informational Efficiency

1.1 Introduction

Non-manipulative short selling can play a valuable role in efficient price discovery, liquidity, and risk management. After nearly seventy years of no new regulation,[1] the short selling regulation of 2005, known as regulation SHO, was introduced to curb potentially manipulative naked short selling - selling short without borrowing or arranging to borrow shares. Naked short selling contributes to most failure-to-deliver (FTD) occurrences, i.e. situations when buyers do not receive the shares within the standard three-day settlement period. An FTD leads to a conversion of a securities contract into an undated futures-type contract, without the buyer's consent. Large and persistent FTDs may affect market stability, distort prices, and make investors lose confidence. The regulation was strengthened in September 2008 (2008 regulation) primarily to reduce FTDs. However, the current literature argues for relaxing the 2008 regulation because higher FTDs can lead to pricing efficiency and liquidity (Fotak et al., 2014), facilitate price discovery (Liu

[1] See appendix 1.8 for a brief U.S. short selling regulation history.

et al., 2015), and expedite market making in a fast-moving market. As a result, regulating naked short selling creates a conundrum. Since we cannot identify manipulative traders, I ask the empirically important question whether the 2008 regulation has been successful in, at least, deterring relatively uninformed naked short sellers.

My findings suggest that after the 2008 regulation, uninformed naked short sellers may have been deterred more so than informed ones. I measure the informativeness of the naked short sellers with the activities of five trader categories in electronically traded, small denomination stock index futures contracts (E-Minis).[2] I find that after the 2008 regulation there is a stronger relationship between the aggregate naked short selling in the stock market and the percentage of the net short positions of several trader categories in E-Mini futures, such as dealers and leveraged funds. This relationship is not evident prior to September 2008.

Contrary to past findings, I also find that after 2008, excessive and persistent FTD stocks, from the publicly released threshold lists, are associated with near future negative abnormal returns. This means that the market views threshold listing as a bearish signal only after 2008; it is consistent with the informativeness of naked short selling activity. However, consistent with the past findings, I find that naked short sellers are contrarians, and that they do not systematically profit from their short positions in the near future.

One would expect the 2008 regulation to be successful in deterring uninformed traders because of the following supporting research. First, Finnerty (2005) models market equilibrium in which naked short selling (NSS) is likely to be used for manipulation, which suggests that fewer uninformed traders may reduce manipulation. Diamond and Verrecchia (1987)'s model predicts that more uninformed traders can potentially drop out of

[2]Schwarz (2012) finds that stock index futures' speculators are informed. For discussions on how price discovery, small pricing errors, informational efficiency, and special attractions for arbitrageurs and informed traders are related to E-Mini futures see Hasbrouck (2003), Kurov (2004, 2008), Chung et al., (2010). Futures markets can also incorporate new information more quickly than cash markets given their inherent leverage, low transaction costs, and lack of short-sale restrictions (Chan, 1992; Tse, 1999).

short pools than informed traders due to higher short selling restriction costs. Boehmer et al. (2015) also show that recent NSS bans strengthen the informativeness of short selling. Second, an alternative to regulation is a judicial process. However, in dealing with the NSS complaints, the courts have mostly failed because the existing US federal or state rules make plaintiffs unwilling to pursue and unable to win an NSS case. For example, plaintiffs cannot prove any market manipulations, they often do not know whom to sue, they are unable to calculate the recoverable damages, and they cannot win against the brokers and the clearing house (Stokes, 2009).

Although leaving securities markets unregulated might be optimal given that contracting is an option, the rationale behind the benefits of a more restrictive short selling regulation is as follows. First, relaxing the regulation could result in large FTDs which create problems through excessive artificial supply regarding ownership benefits (dividends, lending, voting rights).[3] Regulation SHO aims to reduce FTDs. The short selling regulation SHO which was implemented in January of 2005 made some restrictions on the locate and close out requirements. It requires brokers and dealers to "locate" prior to effecting the short sale, i.e. borrowed or arranged to borrow or has reasonable grounds that the security can be borrowed to be delivered on the settlement date. Furthermore, they should "close-out", i.e. deliver the shares in time[4]. According to this regulation, hedgers and arbitrageurs are not exempted from the locate requirement but market makers still are. In regards to fulfilling the close-out requirement, only options market makers are still exempted. Later on, the 2008 regulation increases traders' responsibility to deliver the shares within specified time frames, including the previously exempted options market makers. What is new in the 2008 regulation is that brokers and dealers should close-out before the beginning of regular trading hours on day $T + 4$ for trades on day T. However, if it can be demonstrated that the FTDs are from long sale fails and fails attributable to bona fide market making, the open failed positions should be closed out before the beginning of regular trading hours on day $T + 6$. Furthermore, the options

[3] It is difficult to seek compensation in courts without securities' certificates.

[4] See appendix 1.9 for an example of short selling mechanism.

market makers are no longer exempted from the close-out requirement and a naked short selling anti-fraud rule was passed. Therefore, a restrictive regulation may reduce noise and improve informational efficiency at the cost of reduced liquidity. Second, it may prevent ex-post cheating and externalities. Third, it can reduce the cost associated with delayed delivery of securities' certificates and help maintain long-term relations. Finally, it can minimize the cost of the SEC's manipulation monitoring and the legal proceedings.

To test whether FTDs contain information that is not fully incorporated in price, Liu et al., (2015) proposed a zero-investment strategy: long stocks with low short interest and low FTD stocks, and short stocks with high short interest and high FTD stocks. By using quarterly data, they find that this strategy outperforms a strategy that longs low short interest stocks and shorts high short interest stocks. I find that the former strategy's superior performance is even more pronounced with weekly frequency and that this performance is more evident in the small Nasdaq firms. For the former strategy, I also find that dealers' and asset managers' trading activity in index futures is associated with returns on short and long legs.

The NSS activity is related to E-Mini index futures net shorting activity, which consequently helps predict the FTDs. I use FTDs as a proxy for NSS activity (e.g. see Fotak et al., 2014). I find that for speculators, i.e. traders other than dealers or intermediaries, the higher the net short positions, the fewer FTDs will occur in the stock market from all traders. For dealers or intermediaries, the greater the net short positions they take, the more FTDs will occur in the stock market from all traders. These results hold for the sample after September of 2008 but are not significant before that date.

To ensure that the FTD-index futures relationship is not driven by the cost of borrowing, I control for some proxies such as institutional ownership, book-to-market, and market capitalization. I also control for volume, short interest, return, and put open interest as their variations are associated with FTDs. The results hold when I use similar samples before or after September 2008. Furthermore, results hold when using small windows before or after this date; thus, results are not driven by some pre-existing patterns prior to this date and the 2008 regulation is likely to be responsible for the change in the informativeness of FTDs. Notice that one of the benefits of stock index futures is that they give an opportunity for investors to short the entire market with low margins and transaction costs. In my sample, I focus on E-Mini index futures contracts due to data availability and their trading benefits; I also find that a specific trader category is not always informed across all contracts.

I perform further robustness tests to better understand the FTD-index futures relationship. First, I change my measure to newly created FTDs instead of total FTDs and find that the relationship strongly holds. This is important because the index futures' open interest for each trader category changes every week, which suggests that their positions are aligned with the FTD trades. Second, I find that stock order imbalance and price pressure cannot explain the relationship. Third, the FTDs of Exchange Traded Funds (ETFs) are not related to index futures, hence the relationship is not caused by the ETFs. Fourth, I also test the relationship across NYSE and Nasdaq exchanges and find that results hold with some minor differences.

The rest of this chapter is organized as follows. Section 1.2 reviews the literature. Section 1.3 describes the sample and variables, and section 1.4 presents the main results. Section 1.5 discusses some additional tests to better understand the main results. Section 1.6 presents the robustness checks and section 1.7 concludes the chapter.

1.2 Related Literature

Naked short selling means selling shares short without borrowing or arranging to borrow shares first, and therefore failing to deliver the shares on the settlement date. Naked short sellers can, theoretically, sell unlimited shares of stock and drive down the price. This is in contrast to covered short sellers who are limited to the amount of stock they can borrow. Small firms might be a target of naked short sellers in different situations. For instance, when the stock price of a small firm drops, usually it is more difficult for small firms to negotiate financing. They might also fall into the convertible debt trap where lenders have an incentive to put downward pressure on price in order to receive more shares on conversion. The restrictive regulation of 2005, and its updated version of 2008, tries to address problems associated with the NSS complaints and reduce FTDs.

Interestingly, the literature hardly finds any evidence on any price manipulation by naked short sellers. For instance, Stone (2010) shows that on the day of the trade, stocks subject to FTDs outperform other stocks. Boulton and Braga-Alves (2012) find that the NSS activity does not exacerbate downward price momentum and that naked short sellers do not systematically make profits, and, Fotak et al. (2014) argue that the NSS activity is similar to that of covered short sellers with the same beneficial impact on pricing efficiency and liquidity. Fotak et al. (2014) also show that during the 2008 financial crisis, the FTDs in stocks of financial institutions were in response to publicly available information, such as credit rating downgrades, and naked short sellers were not responsible for downgrades. Boni (2006) finds that the FTDs declined after regulation SHO, and Evans et al., (2009) argue that FTDs are linked to rebate rates and therefore FTDs originate largely from the NSS activity. Evans et al., (2009) also find that the relation between borrowing costs and options prices is significantly weaker when failing is cheaper than borrowing. Furthermore, the FTD open interest is explained by the information about accounting fundamentals, and undelivered short interest contains more predictive information than delivered short interest (Liu et al., 2015). Liu et al. (2015) also argue that majority of FTDs are not inherently manipulative.

As a departure from the current literature, I argue that a significant change has happened after September 2008 when the regulation SHO was updated. The quality of markets might have been reduced, as in Fotak et al. (2014), however, this is the cost of the effort to deter manipulative naked short sellers. I find that the change in regulation increases the informativeness of the NSS activity by deterring the uninformed naked short sellers. This finding is consistent with Diamond and Verrecchia (1987)'s model which predicts that high short selling constraints deter relatively uninformed traders out of short pools more so than relatively informed traders. Boulton and Braga-Alves (2012) show that market views excessive FTD-stocks, which are publicly released in threshold lists, as bullish signals. In contrast, I find that market views threshold listing as a bearish signal after 2008. This is consistent with Stratmann and Welborn (2014) who find stocks with high FTDs, top 1%, experience subsequent abnormal negative returns. However, they used the sample before September 2008. Stratmann and Welborn (2013) find that eliminating the options market makers exception in September 2008 led to fewer FTD. They argue that short sellers were perhaps taking advantage of loopholes to synthetically build short positions though options market. These papers are consistent with the increased informativeness of naked short sellers which has happened by deterring the uninformed traders. Jain and Jain (2015) show that the relation between naked short selling and some borrowing cost proxies is weaker after the implementing the 2008 regulation. However, I show that, after controlling for all of their borrowing cost proxies, the relationship between naked short selling and E-Mini futures activity got stronger after 2008. The 2008 regulation might also help reduce predatory short sellers, especially when they engage in the NSS. Brunnermeier and Pedersen (2005) show that if a distressed large trader is forced to sell, the predatory traders will initially sell alongside the predictable liquidation which increases order imbalance and causes excess volatility. The predatory traders can engage in the NSS activity which might disrupt market stability.

To measure the informativeness of naked short sellers, I use traders activities in E-Mini stock index futures. The E-Mini index futures contracts are electronically-traded small-sized contracts, and for which the disaggregated trader category data are publicly available. The index futures markets have some interesting features. Chan (1992) suggests that if informed traders possess market-wide information, they are more likely to use futures contracts instead of component stocks. Schwarz (2012) investigates the effect of stock index futures positions' public announcements on prices and concludes that large speculators' positions are informative to investors. Gagnon (2014) shows that introduction of single stock futures relaxes short-selling constraints.

Compared to regular outcry futures contracts, E-Minis are characterized by superior liquidity (Wang et al., 2007), they have successfully attracted smaller investors (Ates and Wang, 2003), improved the information flow in futures markets (Tu and Wang, 2007), have had smaller pricing errors and are of special interest for informed traders (Chung, 2010). Previous studies find that most of the price discovery occurs in the E-mini markets (TSE, 1999; Hasbrouck, 2003). However, contrary to these studies, Ivanov et al., (2013) find that spot Exchange Traded Fund (ETF) markets, rather than stock index futures markets, lead price discovery. Stratmann and Welborn (2012) argue that market makers failing to deliver to avoid paying borrowing costs and the ETF FTDs could have consequences for market stability. However, as a robustness check, I find that E-Mini futures cannot explain the ETF FTDs.

Put options can also act as a substitute for short selling (Lamont and Thaler, 2003; Ofek et al., 2004). However, optionable stocks have a higher short interest (Figlewski and Webb, 1993), and bid-ask spreads of put options increase for banned short-sale stocks (Grundy et al., 2012), which suggests complementarity between put options trading and short selling. To reconcile these conflicting views, Lin and Lu (2015) argue that, on one hand, when investors are not allowed to sell short, put options trading volume will drop and the hedging costs for options market makers will increase, which drives the complementarity effect; on the other hand, when investors are allowed to sell short, informed traders' speculative demand drives the substitution effect, which causes an increase in both put volume and put bid-ask spreads with equity lending fees. My main result for speculators (dealers) is consistent with Lin and Lu (2015) that speculative (hedging) demand drives the substitution (complementarity) effect. However, I use index futures because first, index futures contracts in my sample are less costly and less complex than options; second, there is some evidence that traders prefer futures markets to options markets (Hsin et al., 1994); third, the index futures data has been disaggregated by trader types in contrast to the put option data; last, I control for put options' open interest and the results still hold.

My results highlight the benefits of regulations which are consistent with several previous studies. Given the failure of courts and the SEC enforcement office in dealing with the NSS complaints, the additional regulation of 2005 could be a better substitute for courts (Shleifer, 2010). On the other hand, the 2008 regulation makes all traders equally responsible in dealing with the abusive NSS activity. Therefore, the 2008 regulation might work better than the courts or the SEC, in that regulations are most effective when traders obey them because of the fear of lawsuits, rather than because of the fear of government enforcement (La Porta et al., 2006). Notice that the SEC Complaint Center calls the NSS complaints *problematic*, and only 2.5% of the complaints received prior to September 2008, were forwarded for further review, as spending additional resources might reveal no illegal NSS activity.[5]

The results do not necessarily distinguish between informed and uninformed trader categories across different futures contracts. Conventional wisdom holds that small traders are uninformed and institutional traders are informed, but the empirical evidence is mixed unless subsets of these two groups are considered. Examples of trader categories who have been found to be informed are: select mutual fund managers (Cohen et al., (2005); Kosowski et al., 2006), institutional nonprogram short sellers (Boehmer et

[5]See `http://www.sec.gov/about/offices/oig/reports/audits/2009/450.pdf`. Note that furthermore, the clearing house, Depository Trust Clearing Corporation, through its subsidiary, the National Securities Clearing Corporation (NSCC), is unwilling to close out all fails by force because this action might increase the risk of settlement and the interference in securities pricing. The last option for the plaintiffs is to file a complaint regarding the Stock Borrow Program, in which the NSCC borrows shares from the lending members for delivery on the settlement date. However, all NSS cases against the program have been dismissed so far, and the program was discontinued March 14, 2014, see `http://www.dtcc.com/~/media/Files/pdf/2014/2/7/a7676.ashx`.

al., 2008), retail short sellers (Kelley and Tetlock, 2014), and intermediaries (Anand and Subrahmanyam, 2008). For each trader category, I use its transaction quantity to uncover the NSS activity. Transaction quantities are found to be informative in the literature; for instance, Hong and Yogo (2012) find that futures open interest could be more informative than prices.

1.3 Data and Variables

1.3.1 Sample Selection

The sample includes data from January 3, 2005 to December 30, 2014 with all U.S. firms in the CRSP database whose share codes are 10 or 11 and listed on NYSE, NASDAQ, and AMEX. The data has been divided into two subsamples: before and after September 17th, 2008. The reason for this separation is that the SEC implemented the Rule 204T on September 17th, 2008, and also the SEC's website has a full sample of failure to deliver data only after September 16th, 2008. I exclude financial firms with SIC codes between 6,000 to 6,999 and utility firms with SIC codes between 4900 and 4999. Observations with a stock price equal to or less than $1 are removed. I obtain the number of outstanding fails to deliver from the SEC's website.[6] I eliminated those firms which had no failure to deliver data during the period, and for any daily missing observation, I assume that failure to deliver is zero.

For the multiple issue firms, I only keep the most liquid securities with the highest volume. I remove the missing values of return, shares outstanding and short interest, and trim price and shares outstanding at the top and bottom 1%. Finally, I winsorize all continuous variables at the 1% and 99% percentiles to reduce the influence of outliers. Short Interest and Institutional Ownership data are from COMPUSTAT and these variables are deflated by shares outstanding (short interest data are in the Supplemental Short

[6] `http://www.sec.gov/foia/docs/failsdata-archive.htm`

Interest File). If the deflated Institutional Ownership is below 0.00001 or above 0.99999, the values are replaced with 0.00001 and 0.99999, respectively. Turnover is defined as the daily volume divided by shares outstanding. Weekly returns are the compounded daily of dividend-included returns. Failure-to-deliver outstanding is deflated by shares outstanding. I follow Fama and French's (1993) methodology to calculate the Book-to-market and size, using data from the merged CRSP-Compustat database. The Book-to-market ratio is the positive book common equity for the fiscal year ending in $t-1$ calendar year, divided by the market equity at the end of December of $t-1$, matched with the returns for July of year t to June of $t+1$. From June of year t, size is computed as price times shares outstanding, matched with returns from July of year t to June of year $t+1$.

I download the financial futures data for the above sample period from the Commodity Futures Trading Commission (CFTC)'s website.[7] The CFTC has published data on the level and direction of positions held by pre-defined groups of traders in futures markets through the Commitments of Traders reports. The trader classification has been disaggregated into five groups: Dealer/Intermediary, Asset Manager/Institutional, Leveraged Funds, Other Reportable, and Non-Reportable traders. The data indicates the report date, not the release date, and provide a breakdown of each Tuesday's traders hold positions. There are only six stock index futures contracts that have full-time series observations during the sample where only one of them is a regular contract, $S\&P$ 500. I drop this regular contract because Kurov (2004) shows that price discovery initiated in the more liquid E-Mini $S\&P$ 500 contract. Note that the Emini $S\&P$ 500 futures contract maintains a very tight bid-ask spread of just a single tick which is \$12.50 per contract. These contracts are E-Mini NASDAQ 100, E-Mini $S\&P$ 400, E-Mini $S\&P$ 500 which are traded at Chicago Mercantile Exchange (CME), E-Mini RUSSELL 2000 traded at CME prior to September 17 of 2008 and afterwards at ICE FUTURES U.S., and E-Mini Dow Jones Industrial Average traded at Chicago Board of Trade.

[7] I only download "futures only" data as I get similar results with "futures and options combined" data. To download the data visit `http://www.cftc.gov/MarketReports/CommitmentsofTraders/HistoricalCompressed/index.htm`

All of these futures contracts are financially settled and trade on the March quarterly expiration cycle (March, June, September, and December). They are traded on CME's GLOBEX electronic platform, except for E-Mini Russell 2000 which after September 17 of 2008 trades at ICE, but prior to that date was traded on CME. E-Mini Trading also occurs overnight while the underlying stocks in the cash market are not open for trading. This overnight trading is possible because many large cap stocks on $S\&P$ 500 and Nasdaq 100 are traded on the overseas markets, and because foreign stock indexes trading around the globe, it gives traders an additional glimpse of the overnight trends.

The CFTC defines the five trader categories as follows. *Dealer/Intermediary* are traders that design and sell different financial assets and typically are on the "sell side" of the market. This category includes large U.S. and non-U.S. banks and dealers in securities, swaps and other derivatives. The rest of the market comprises the "buy-side" of the market. *Asset Manager/Institutional* are institutional investors whose clients are predominantly institutional, such as pension funds, endowments, insurance companies, mutual funds and those portfolio/investment managers.

Leveraged Funds are typically hedge funds and a variety of money managers such as registered commodity trading advisors, registered commodity pool operators or unregistered funds identified by CFTC. *Other Reportables* are reportable traders which are not placed into one of the above three categories. The traders in this category mostly use these markets to hedge their business risk, examples of these traders are corporate treasuries, central banks, smaller banks, mortgage originators, credit unions. Finally, *Non-Reportables* refers to small traders.

1.3.2 Weekly Time Series Variables

Due to the availability of futures data, I assume that "weeks" start on Wednesdays and end on Tuesdays. I take the weekly average of deflated failure to delivery ratios for each security to find its weekly variable, then I calculate FTD_t as the equally weighted average across securities. These values are multiplied by 10^4. Short interest frequency in the sample is biweekly and monthly. I assign the reported number to every day prior to reporting days and deflate it by shares outstanding and take a weekly average of these ratios to calculate its weekly variable. The equally weighted average of weekly short interest is denoted as SI_t.

The weekly return, $Return_t$, is the equally weighted average across the securities. From the weekly average of volume deflated by shares outstanding for each security, I calculate the equally weighted average across all firms to find $Turnover_t$. Turnover controls for the possibility that FTDs may increase with volume. TED_t is the weekly average of the TED spread which is the weekly average of the difference between Eurodollar Deposit (London) 3-month and TBills Secondary Market 3-month. I use TED spread as a proxy for funding illiquidity in the time series regressions.

1.3.3 Monthly Panel Variables

The monthly FTD_{it} for security i is the monthly average of failure to deliver outstanding divided by the total shares outstanding of that month. BM_{it} is the monthly book-to-market defined in section 1.3.1. $MCAP_{it}$ is the log of market capitalization. $Turnover_{it}$ is the monthly volume scaled by shares outstanding. SI_{it} is the average short interest scaled by shares outstanding. POI_{it} is the monthly sum of put open interest. $Return_{it}$ is the monthly return. IO_{it} is the quarterly values of institutional ownership divided by the shares outstanding in month t. The summary statistics are shown in Table 1.1.

{*Table* 1.1 *about here.*}

1.3.4 Futures Pressure

I follow Schwarz (2012) to construct the futures pressure (an excess short ratio) as follows:

$$FP_{it}^{c} = \frac{Short_{it}^{c} - Long_{it}^{c}}{\sum_j (Short_{jt}^{c} + Long_{jt}^{c})} \tag{1.1}$$

where $Short_{it}^{c}$ and $Long_{it}^{c}$ are the number of short and long positions held by the trader category i on contract c at the end of week t. Note that $FP_{it}^{c} \in [-\frac{1}{2}, \frac{1}{2}]$ and $\sum_i FP_{it}^{c} = 0$, therefore, for every long there must be a short and it is impossible for the whole market to change position in the same direction.

For Dealers, the futures pressure is denoted as the hedging pressure in the literature, and for other trader categories, it is called speculative pressure. Hedging pressure is motivated by the theory of backwardation which is found in the literature (Carter, Rausser, and Schmitz, 1983; Chang, 1985; Bessembinder, 1992; De Roon et al., 2000; Sanders et al., 2004; Hong and Yogo, 2012).[8]

Since Dealers are typically on the sell side of the market, the futures pressure for dealers is expected to be positive and for other traders is expected to be negative. However, in my sample, only for one contract the futures pressure for Dealers is negative. Figures 1.2,1.3,1.4,1.5, and 1.6 show that the Dealers' futures pressure is mostly positive, except for E-Mini Russell 2000 where Dealers are on the buy side of trades (Figure 1.4). To find the monthly futures pressure, I take the monthly average of weekly futures pressure.

[8] In the definition of futures pressure, the result qualitatively do not change once we include the spread positions in the denominator; the spread positions are the number of short positions that are offset by the number of long positions in other maturities.

1.4 Main Results

1.4.1 Time Series Regressions

I run the following time series regression for each trader category and each index futures contract.

$$FTD_t = \alpha + \eta FP^c_{it-1} + Controls_t + \beta_1 FTD_{t-1} + \beta_2 FTD_{t-2} + \epsilon_t \tag{1.2}$$

Where the control variables are: short interest, SI_t, return, $Return_t$, turnover, $Turnover_t$ and TED spread, TED_t; all defined in section 1.3.2. The variable of interest, FP^c_{it-1}, is the futures pressure (FP, hereafter) of trader category i on contract c at the end of the week $t-1$ defined by equation 1.1. The results will be qualitatively the same if I use the lag of control variables. The reason I use the lag of futures pressure is the following: since any naked short selling will result in a possible increase of the FTDs three days later or more, the NSS activity is most likely to have happened the week before or the index futures will lead the NSS activity. In either case, the lag of futures pressure, if it contains similar information, should be related to the contemporaneous FTDs. Figure 1.1 shows the time series of the aggregate FTDs and, as we expect, there is a structural break on September 17, 2008, where the FTD_t has substantially decreased and remained at the same level afterwards. Figure 1.1 also illustrates that the FTDs are higher for small stocks and also that their drop is more apparent.

{*Figure* 1.1 *about here.*}

In equation 1.2, I use two lags of FTD_t to control for autocorrelations, and also the use of higher order lags results in less significant slopes on those lags, which qualitatively gives similar results. In relation to the control variables, notice that higher shorting activity and higher trading activity should result in higher FTDs (D'Avolio, 2002); therefore, we expect positive slopes on short interest and turnover. To make sure the results are not driven by the variation in aggregate return, I also control for return. Finally, the TED spread is a proxy for funding illiquidity that can explain aggregate market level FTD (Anand, Irvine, Puckett, and Venkataraman, 2013).

The coefficient of interest, η, reveals the connection between the FTDs and futures pressure. If the naked short selling activity has become more informative after September 17, 2008, we should expect to have more non-zero η's for some trader categories on each contract, and also for some contracts within each trader category, compared to pre-September 17, 2008. Furthermore, if η is significantly different from zero, I hypothesize that it is positive for dealers or intermediaries, but negative for other trader categories. Table 1.2 shows the dealers and asset managers' η for E-Mini Nasdaq 100. As we see, η is positive for dealers and negative for asset managers in the presence of some of all of the control variables. Next, I focus on each individual futures contract as these contracts can be used for different purposes and have their own markets.[9]

{*Table* 1.2 *about here.*}

E-Mini DJIA. Figure 1.2 shows the FP time series for different trader categories on this contract. Dealers' FP is mostly above the horizontal axis which is expected as the dealers tend to be net short. Leveraged funds are mostly net long and seem to actively bet against dealers. The FP's correlations among speculators are low, and the correlation between dealers' FP and leveraged funds' FP is -91.67%. In Table 1.3, I only report the values of η from equation 1.2 where all the control variables are used. When some control variables are dropped and η is not significant, I marked the values of η, with "s", to distinguish them from others. As we see, the dealers' η is positive and significant for all models. Due to high negative correlation, the leveraged funds' η is negative and significant for all models, as expected. Interestingly, the non-reportables (NR) are significant with the predicted negative sign. This might be because of informed small short sellers where

[9] In my regression analysis, if I use the full sample and add a regulation dummy variable and its interaction with the futures pressure, I get similar results with the sub-sample results of before and after September 17, 2008. In this framework, the difference between before and after the regulation is significant.

the use of E-Mini DJIA can reflect their NSS activity. Small traders are found to be informed, such as in Kelley and Tetlock (2014). The other two trader categories, asset managers (AM) and other reportables (OR), do not have a significant association with the aggregate FTDs. Although E-Mini $S\&P$ 500 is the most liquid index futures contract in the world with over two million average annual volume, the E-Mini DJIA is the leader among my sample E-Minis based on their average annual tick movements[10] which is attracted by leveraged funds and small traders.

$\{Figure$ 1.2 *about here.*$\}$

$\{Table$ 1.3 *about here.*$\}$

E-Mini Nasdaq 100. This contract, which contains the top 100 non-financial stocks in the NASDAQ composite index, has the FP time series shown in Figure 1.3. The pair FP correlations are as follows: −94.23% for dealers and asset managers, −83.94% for dealers and leveraged funds, and 66.43% for asset managers and leveraged funds. The other pair correlations are small. Table 1.3 reports the slope η for all models given in Table 1.2 for E-Mini Nasdaq 100. As we see, η is positive for dealers and negative for asset managers and leveraged funds. Other reportables and non-reportables' FP are insignificantly related to the FTDs.

$\{Figure$ 1.3 *about here.*$\}$

E-Mini Russell 2000. For stocks in the bottom two-thirds of Russell 3000 which form this index, the FP time series for each trader category is shown in Figure 1.4. Contrary to all other contracts in my sample, dealers are mostly net long, and leveraged funds are mostly net short. Despite having high correlations between dealers' FP and other traders' FP (−62% to −76%), only leveraged funds, other reportables, and non-reportables are associated with the FTDs according to Table 1.3. This index has the highest annual

[10] See `https://www.daytradingcourse.com/articles/Emini_Tick_Comparison.pdf`

average movement in dollar amounts among E-Minis in my sample[10].

{*Figure* 1.4 *about here.*}

E-Mini S&P 400. As the benchmark for mid-sized firms, the FP for each trader category on this index has the time series illustrated in Figure 1.5. The FPs for dealers and leveraged funds, and dealers and asset managers have high negative correlations, −80.45% and −72.07%, respectively, and other pair FP correlations are low. Table 1.3 shows that dealers and leveraged funds are significantly associated with the FTDs with predicted signs (positive for dealers and negative for leveraged funds). For asset managers, in all the models described in Table 1.2, η is not significantly less than zero so we can not make a conclusive interpretation about this trader category.

{*Figure* 1.5 *about here.*}

E-Mini S&P 500. As the most liquid index futures in the world, the FP for different trade categories for this contract are shown in Figure 1.6. Dealers and leveraged funds are mostly net short and asset managers are net long. The pair FP correlations of asset managers and leveraged funds with dealers are about −50%; the FP correlations between asset managers and non-reportables is 43%, and other pair FP correlations are low. Table 1.9 shows no strong association between traders' FP and the FTDs. Perhaps due to noise, excessive usage of this contract and its underlying, or limited observations, we cannot document any connection in the time series framework, however, in the panel regressions presented below we find some strong associations.

{*Figure* 1.6 *about here.*}

{*Table* 1.9 *about here.*}

We can also look at the informativeness of each trader category's FP across all futures contracts. Table 1.3 shows that dealers (leveraged funds) have a positive (negative) slope of η, which reveals strong association to the next week's FTDs, for all contracts except for E-Mini S&P 500. This is consistent with the findings elsewhere suggesting that, for instance, intermediaries and hedge funds are informed but mutual funds, a subset of asset managers, are not well informed (e.g. see Fama and French, 2010). The table shows that the asset managers' trading activity on E-Mini Nasdaq 100, and the non-reportables' trading activity on E-Mini DJIA are strongly related to the NSS activity.

1.4.2 Portfolio Approach

I ask whether firm characteristics can explain the futures pressure effect. This is important because D'Avolio (2002) finds that the borrowing costs are higher for stocks with small market capitalization, high turnover, low institutional ownership, and high short interest. Furthermore, Jones and Lamont (2002) find that when shorting demand could not be met through traditional channels, market-to-book ratios increase and then fell as the overvaluation subsided. Since leverage-constrained traders, on average, hold portfolios with betas above one and other traders hold portfolios with betas less than one (Frazzini and Pedersen, 2014), I also separate firms based on their market beta. Therefore, I rank firms into five portfolios based on their size, book-to-market, institutional ownership, turnover, short interest, and rolling beta. I also divide the firms into two groups: optionables and non-optionables, as put options could also be a substitute for shorting the stocks. Boni (2006) finds that stocks with put option have a higher probability of large and persistent fails.

For each portfolio, I run regression 1.2 for all trader categories and contracts. In Tables 1.4, 1.5, 1.6, 1.7, and 1.8, I find that the results are generally stronger for small size, low book-to-market, high turnover, high short interest, low beta, and non-optionable firms. The results for institutional ownership portfolios are not strongly significant in most cases. The overall results are qualitatively similar with the time series and panel regression results.

$\{Tables\ 1.4 - 1.8\ \textit{about here.}\}$

1.4.3 Panel Regressions

Consider the following panel regression of FTDs over the futures pressure

$$FTD_{jt} = \alpha_j + \eta FP^c_{it-1} + Controls_{jt} + \epsilon_{jt} \tag{1.3}$$

where the control variables, $Controls_{jt}$ are: Institutional ownership, IO_{jt}, Short Interest, SI_{jt}, Market Capitalization, $MCAP_{jt}$, Book-to-Market, BM_{jt}, Turnover, $Turnover_{jt}$, Put Open Interest, POI_{jt}, Return, $Return_{jt}$, all defined in section 1.3.3, and FP^c_{it-1} is the FP of trader category i on contract c in month $t-1$. The α_j is the firm fixed effect and standard errors are clustered by both firm and time. Note that in equation 1.3, the time fixed effect has been dropped because of two reasons. First, there is no apparent time effect in the FTD's after September 2008 as also apparent in Figure 1.1. Second, for some trader categories the slope of η is significantly different from zero and for others it is not. Therefore, the estimated non-zero η in equation 1.3 uncovers a cross-firm invariant activity which is related to future failure-to-delivery[9].

Table 1.9 shows the FP for all trader categories on E-Mini Nasdaq 100. As we expect from the time series regressions, dealers have positive η and asset managers and leveraged funds have negative η when estimating equation 1.3. The R^2 is about 15% for all the trader categories and I get similar results when I drop put open interest. The control variables have expected signs with the firm characteristics as explained in section 1.4.1, and have the following signs: book-to-market (negative), institutional ownership (negative), put open interest (positive), short interest (positive), size (negative) and turnover (positive). The signs are consistent with the prior literature.

I repeat the same procedure and report the slope of FP for all the E-Minis in Table

1.10. As shown in this table, the *E-Mini DJIA*'s FP's are also matched with the time series analysis where dealers have positive η and leveraged funds and non-reportables have negative η. There is a weak evidence, t-stat$=-1.64$, that asset managers' FP could also be associated with the FTDs. *E-Mini Russell 2000*'s FP's are significantly associated to FTDs for leveraged funds and non-reportables. However, the other-reportables do not show up to be associated with the FTDs in contrast to the time series analysis.

{*Table* 1.10 *about here.*}

In that table, Table 1.10, *E-Mini S&P 400* has the futures pressure for dealers, leveraged funds and non-reportables corresponded to the FTDs with predicted signs. The non-reportables show up to be strongly associated with the FTDs in contrast to the time series analysis. Finally, the *E-Mini S&P 500*'s FP's are strongly related to the FTDs for leveraged funds and other-reportables and at 10% level with the non-reportables. This is interesting as none of the traders are significantly associated with the FTDs in time series regressions. All in all, with the exception of *E-Mini S&P 500*, panel and time series regressions give us a similar relationships between different trader categories and the FTDs. In the next sections, I will mostly rely on the results based on the panel regressions to fully control for firm characteristics.

The informativeness of trader categories' FP across contracts is similar to the corresponding time series regression results, with some exceptions. The FP from dealers (leveraged funds) is positively (negatively) associated with the next month's FTDs. Similar to Table 1.3, asset managers' trading activity on only E-Mini Nasdaq 100, but not other contracts, has a strong connection to the next month's FTDs. However, the leveraged funds' trading activity on E-Mini *S&P* 500 is reversed, and they act as dealers while other-reportables and non-reportables on this contract act as speculators, as can be seen from their negative slopes of η. Non-reportables in all contracts, except E-Mini Nasdaq 100, have negative slopes which reveal their informativeness, and that is consistent with the retail traders being informed (Kelley and Tetlock, 2014).

1.4.4 Regulation SHO Prior to September 17, 2008

One might argue that the futures pressure effect mostly originates after implementing Regulation SHO in January 2005 prior to the rules implemented on September 17 of 2008. To test this, I repeat the same analysis for the period of June 13 of 2006 to August 30 of 2008. Note that June 13 of 2006 is the beginning of the disaggregated futures data. Table 1.11 reports the slope η in equation 1.3 for the E-Mini futures contracts. Except for Asset Managers' futures pressure on E-Mini Russell 2000, all other traders' futures pressures of all contracts are insignificant. Therefore, there is not enough evidence supporting the futures pressure effect right after implementing regulation SHO. As an important robustness check, I run my analysis for the sample after September 17 of 2008, where the FTDs are above 10,000 number of shares, similar to the reported data prior to September 17 of 2008. Table 1.12 shows the panel regression of equation 1.3 using this sample, and it confirms the same results as with the full sample which are presented in Table 1.10. Notice that the SEC reports the FTD open interest prior to September 17 of 2008 only when it is above 10,000. This might cause information loss as I assume the missing FTDs are zero which serves as an FTD lower bound. However, the above 10,000 of FTDs reported by the SEC have the advantage to be large enough which cannot entirely be created by processing or human errors and should be mostly originated from naked short sellers (NSS). The difference between Tables 1.11 and 1.12 is the effect of September 17 of 2008 modification in regulation SHO strengthening the connection between stock index futures and FTDs.

{*Table* 1.11 *about here.*}

{*Table* 1.12 *about here.*}

According to Diamond and Verrechia (1987), the tighter short constraints makes short selling be more informative as the proportion of informed traders increases. Therefore, if the same theory applies to the NSS, we should be able to see a more pronounced futures pressure effect after September 17 of 2008 due to removing noise. I repeat the time series analysis using the CFTC aggregated reports where the trader categories are Commercials (Dealers and Asset Managers), Non-Commercials (Leveraged Funds and Other Reportables) and Non-Reportables. With the aggregated futures data, I extend the sample starting from January 2 of 2005 to September 17 of 2008. I find that only for a few portfolios, formed by firm characteristics as described in section 1.4.2., there are some signs of futures pressure effect which are not comparable to the many portfolios revealing the futures effect after September 17 of 2008.

Perhaps we cannot make a complete and robust argument about the presence of futures pressure effect prior to September 17 of 2008 due to the data availability in the time series framework. The time span for the panel regressions is 27 months and for the time series regression is 195 weeks. As mentioned earlier, I rely mostly on the panel regression results as the existence of many firms reduces the statistical problems associated with a short time span. Once the panel regressions are used, the results suggest strong connection only after September 17 of 2008. To make sure that the regulation effect is the only important effect, I repeat the panel regression tests for six months before and after September 17 of 2008. I find that only after this date the futures pressure effect holds. What this tells us is that the futures pressure effect is not driven by some pre-existing patterns prior to September 17 of 2008. When I use similar time duration for the period after, for instance, any window with the duration equal to June 2006-August 2008, I still find that the results hold only for those samples chosen after September 17 of 2008.

1.5 Additional Tests

1.5.1 Threshold Securities

To better understand the results, I further consider the relation between FTDs and returns. One especial way is to consider the stocks with excessive FTDs, for instance, those stocks which are listed on threshold lists released by the national stock exchanges. The three major exchanges in the U.S. release daily threshold lists for those securities with aggregate open failure-to-deliver equal to or greater than 10,000 shares and 0.5% of the total shares outstanding for five consecutive settlement days. A security will be removed from the threshold list if it does not exceed the specified level of fails for five consecutive settlement days. I collect these lists from the NYSE [11] and NASDAQ websites.[12]

In the previous sections, I found that there is a negative association between speculators' futures pressure and the FTDs. In other words, the more net long positions these traders take, the higher FTDs will occur in the stock market. One might argue that this phenomenon is related to the short selling constraints and overvaluation hypothesis. Consistent with the overvaluation hypothesis, Autore et al. (2015), by using data for only before 2009, find positive (negative) abnormal returns around threshold additions (removals). In that case, failure to delivery indicates short sale constraints and when short sale constraints are tight, there is a price appreciation. Where FTD is high, stocks then are overvalued and when short sale constraints are easing, price declines. Since stock index futures' returns and the underlying stock returns are closely related, and mostly E-Mini index futures markets lead the stock market, the overvaluation could be related to the long pressure from speculators in E-Mini markets which therefore will be associated with high FTDs.

Using a sample prior to September 17 of 2008, Boulton and Braga-Alves (2012) show that naked short sellers are contrarians and systematically do not profit from their short positions. Furthermore, they find that new threshold listings are associated with the positive returns. For the sample post-September 17 of 2008, I confirm these conclusions with one exception, that is, the new listing corresponds to the negative returns for stocks added to threshold lists. I use an event-study around threshold listings, (day=0), to find the Cumulative Abnormal Return (CAR) for several windows based on the CAPM. The

[11] `https://www.nyse.com/regulation/threshold-securities`
[12] `http://www.nasdaqtrader.com/trader.aspx?id=regshothreshold`

results are qualitatively the same with Fama-French three-factor or Carhart four-factor models. I report the equally weighted CARs over several windows. Since it is required to have high FTDs for five consecutive settlement dates to be on the threshold lists, day -4 is the first day with high FTDs and the trade should be initiated three days prior, due to the U.S. three day cycle. Therefore, to be on the threshold list, day -7 is the date when the trade is initiated (Boulton and Braga-Alves, 2012).

Figure 1.7 shows the mean and median of FTDs around threshold listing. There is a jump on day -4 which is the first settlement date after the naked short sellers initiated their trades on day -7. The mean and median increase further from -4 to -3 and remain steady over $-2, -1, 0$ and decrease immediately after the security added to the threshold list. This decline continues for about two weeks; in other words, it takes two weeks for the FTDs to drop substantially. In unreported tables, the mean and median of FTDs for firms on the threshold list are higher after September 17 of 2008 than prior to that date.

{*Figure* 1.7 *about here.*}

Table 1.13 shows the event study results. As we see, the windows $[-12, -7]$ and $[-8, -7]$ have positive and significant CARs, which suggests that naked short sellers are contrarians by trading against a positive price increase. To find out if they would profit from their trades, I look at the windows $[-7, 0], [-6, 0], [-4, 0], [-3, 0]$ and we see that all of them have positive and significant CARs which translate to no systematic profit gain after initiating the trade. Notice that the CARs are positive for a large window prior to threshold listing, $[-20, 0], [-10, 0]$. Hence, positive returns occur over the two weeks preceding a stock's appearance on the threshold list (in unreported tables these returns are higher after September 17 of 2008). Since prior to September 17 of 2008, day $+13$ was the mandatory close out, I also look at $[-6, +13]$ but the corresponding CAR is insignificant.

{*Table* 1.13 *about here.*}

Day +5 is the first day a security can be removed from the lists. Windows such as $[-10, +5]$ and $[-7, +5]$ have positive CARs and windows $[-20, +5]$ and $[-4, +5]$ have insignificant CARs. This translates into no systematic profit from when the trade is initiated until the security is removed from the lists. I also look at the market reaction to listing in a short window around threshold listings, $[-1, +1]$, but I get insignificant CAR.

The last important test is to see whether threshold listing is a bearish or bullish signal for the market participants. I look at several windows, such as $[+1, +5], [+6, +13]$ and $[+1, +20]$, and I get negative and significant CARs. This important result is only true after September 17 of 2008 (CARs are positive for these windows prior to that date). This suggests that after September 17 of 2008, the market interprets naked short sellers to be informative.

1.5.2 FTD-Short Interest Strategy

To find out whether FTDs contain information that is not fully incorporated in prices, we can calculate abnormal returns from a zero-investment strategy. Boehmer et al. (2010) find positive abnormal returns from a strategy which sells high short interest stocks and buys low short interest (SI-strategy). Liu et al (2015) find 16.5% annualized abnormal returns (much higher than the SI-strategy), with quarterly frequency, on the zero-investment strategy which longs low short interest and low FTD stocks, and shorts high short interest and high FTD stocks (FTD-SI strategy). I sort firms based on their lag of short interest into five quantiles, then I sort each quantile into five portfolios according to their previous week's FTD. To calculate the average abnormal return on my sample with weekly frequency, I first calculate the weekly Fama-French three factors and then find the intercept from the following regression

$$r_{pt} - r_{ft} = \alpha + \beta(r_{Mt} - r_{ft}) + sSMB_t + hHML_t + \epsilon_t \tag{1.4}$$

where r_{pt} is the return on a portfolio from the nested sorting explained in the above. The results are shown in Table 1.14. The long leg is the portfolio with low short interest and low FTD (portfolio $EW11/VW11$ for equally weighted/value weighted, respectively). The short leg is $EW55/VW55$ (high short interest and high FTD). When considering all firms, the annualized average abnormal return from the FTD-SI strategy is 44% for equally weighted and 25% for the value-weighted portfolios (the dependent variable in the above equation is $r_{pt}^{11} - r_{pt}^{55}$). Equally-weighted method makes the small firms more pronounced where the FTD is higher, and therefore the average abnormal return is higher. These numbers are 10% and 11% for the NYSE and 54% 32% for NASDAQ. Since FTDs are lower on NYSE stocks and NYSE might be more efficient, these numbers are relatively expected.

{*Table* 1.14 *about here.*}

To investigate the relationship between futures pressure and the information content in FTDs which affect returns, I test the effect of futures pressure on the FTD-SI strategy. Since on every futures contract, each trader category monotonically affects the FTDs, it is interesting to see how the lag of futures pressure affects the next period excess risk-adjusted returns on the long leg and short leg portfolios of the FTD-SI strategy. To understand this, I estimate η from the following regression

$$r_{pt} - r_{ft} = \alpha + \eta FP_{it-1}^{c} + \beta(r_{Mt} - r_{ft}) + sSMB_t + hHML_t + \epsilon_t \tag{1.5}$$

where the long leg and short leg portfolios are $VW11$ and $VW55$, respectively. Table 1.15 shows the coefficient η for dealers and asset managers of the futures contracts in my sample. As we can see, for asset managers on the long leg, the futures pressure of two out of five contracts are significant and negatively related to future return: E-Mini Russell 2000 and E-Mini $S\&P$ 500. This indicates that the higher net short positions taken by asset managers translate in lower average abnormal returns in the next period. When I separate the firms by their stock exchanges, NYSE and NASDAQ, these two contracts appear to mostly predict NASDAQ returns. The asset managers' futures pressure on E-

Mini DJIA does also weakly, at 10% level, predict the negative average abnormal returns within the exchanges.

{*Table* 1.15 *about here.*}

On the short leg, in addition to E-Mini Russell 2000 and E-Mini $S\&P$ 500, the asset managers on E-Mini $S\&P$ 400 also predict the abnormal returns correctly. Note that the η from the long-short strategy is only significant for NYSE firms for these three contracts and it is with positive sign. This indicates that the more net short positions taken by asset managers, the higher return from the FTD-SI strategy will be obtained. This is not happening on NASDAQ stocks as the futures pressure affects the long and short legs in the same direction which will be neutralized in the long-short strategy. The other two contracts in my sample, E-Mini DJIA and E-Mini NASDAQ 100, negatively predict abnormal returns from the FTD-SI strategy. This suggests that the more net long positions taken by asset managers on these two contracts positively affect the next period return from the FTD-SI strategy. For other trader categories, the futures pressure effect is rather weak in my sample; perhaps, short sales restrictions weaken the contemporaneous relationship between futures and the cash market (Jang et al., 2001). In sum, as we see from the above analysis and regardless of the signs, the lag of futures pressure is related to abnormal returns.

1.6 Robustness Checks

1.6.1 New FTDs

One might argue that futures pressure is associated with the old FTDs rather than being related to the newly created FTDs. To test this, I use Fotak et al (2014)'s method to construct a proxy for the new FTD shares. For each day d and firm i, the difference $FTD_{id} - FTD_{id-1}$ is equal to the number of new FTD shares minus the number of previously outstanding FTDs closed on day d. Since we do not have data on the number of previously outstanding settled FTDs, we assume it is equal to γFTD_{id-1} where γ is the delivery rate of the FTD shares. This method might give us negative values which I replace them with zeros. Therefore, the new FTD shares are estimated as following

$$FTD_{id}^{new} = Max\{FTD_{id} - FTD_{id-1} + \gamma FTD_{id-1}, 0\} \tag{1.6}$$

To find the monthly values, I take an average of this new FTD measure. With the assumption of constant γ, I find that the results are qualitatively similar to the main results when the total open interest of failure-to-delivery is used. I report the results for $\gamma = 3\%$ as in Fotak et al (2014) where they estimate it based on an SEC memorandum.[13] The results, however, hold for a wide range of constant γ and gives qualitatively similar results as the one with total FTDs (see Table 1.16).

{*Table* 1.16 *about here.*}

1.6.2 Stock Order Imbalance

Another concern is that the futures pressure effect might be driven by stock order imbalance. To test this, I identify Buyer/Seller initiated trades using Lee and Ready (1991) methodology. I follow Peterson and Sirri (2003) and Bessembinder (2003) recommendation to use a zero second delay for the reporting time for trades. I use WRDS-derived Trades files (WCT datasets) to find the daily sell and buy volume for each security in my sample. The data is available until the end of September 2014. I then calculate the following ratio

$$OIB_{it} = \frac{\sum_d (Sell_{id} - Buy_{id})}{\sum_d (Sell_{id} + Buy_{id})} \tag{1.7}$$

where $Sell_{id}$ and Buy_{id} are the sell and buy volume of security i on day d of week t and the summation is over all days in week t. For monthly OIB_{it}, I repeat the above procedure for months instead of weeks. To calculate the weekly time series stock order imbalance, I take an equally weighted average of OIB_{it} across firms to find OIB_t. Alternatively I

[13] `https://www.sec.gov/spotlight/failstodeliver082106.pdf`

work with the following order imbalance measure. On each day, I calculate the dollar order imbalance as

$$DOIB_d = \frac{\sum_i (Price_{id} \times Sell_{id} - Price_{id} \times Buy_{id})}{\sum_i (Price_{id} \times Sell_{id} + Price_{id} \times Buy_{id})} \tag{1.8}$$

where $Price_{id}$ is the closing price of security i on day d. The advantage of this method is to distinguish the heavily traded securities. This measure is similar to Hong and Yogo (2012)'s imbalance measure. I define $DOIB_t$ as the weekly average of $DOIB_d$. The results are robust with either measure, OIB_t or $DOIB_t$, in the weekly time series, and with OIB_{it}, in monthly panel regressions when they are used as control variables. The results are qualitatively the same as the presented main results (see Table 1.17).

{*Table* 1.17 *about here.*}

1.6.3 Futures Pressure and Price Pressure

Price pressure hypothesis claims that an increase in demand (supply) causes an upward (downward) bias in price which is temporary and will be subsequently reversed. If futures pressure induces price pressure, the change in futures price generates price pressure. Therefore, if futures pressure affects future FTDs through price pressure, then the change in futures pressure could also be related to future FTDs. To test this, I follow de'Roon et al. (2000) and add price pressure in the presence of futures pressure to find out whether the results can be explained by price pressure. I therefore run

$$FTD_t = \alpha + \eta FP^c_{it-1} + \gamma PP^c_{it-1} + Controls_t + \beta_1 FTD_{t-1} + \beta_2 FTD_{t-2} + \epsilon_t \tag{1.9}$$

where PP^c_{it-1} is the price pressure of its own futures contract defined as $PP^c_{it-1} = FP^c_{it-1} - FP^c_{it-2}$. For all contracts and stock exchanges, NYSE and NASDAQ, I get similar results as without the price pressure in the control variable set. Therefore, the sign and statistical significance of η is what we expect and price pressure cannot remove the futures pressure effect (see Table 1.18).

{*Table* 1.18 *about here.*}

1.6.4 Exchange Traded Funds

One might wonder whether the futures pressure effect arises from the ETFs' activities, and therefore, futures pressure predicts stock FTDs through the ETF FTDs. Due to creation and redemption of shares at the ETFs, some traders might use stock index futures to hedge or take the opposite direction of trades which makes their futures activity to be related to the ETF FTDs. On the other hand, the SEC's documents show that the ETF FTDs have increased as opposed to stock FTDs after September 17, 2008; this might have some relations to our results for the period after September 17, 2008.

To test the relationship between futures pressure and the ETF FTDs, I repeat the panel regression given by equation 1.3 to estimate η. I follow the same procedure to construct my sample as the main sample with the exception that I choose the ETFs with the share code of 73 and work at the security level with those of more than 30 observations after September 17, 2008. I drop one of the control variables, the book to market. The results show that there is no significant evidence showing any connections between the ETF FTDs and futures pressure. The only exception is leveraged funds futures pressure from E-Mini $S\&P$ 400 which is significant at 5% with the positive sign. This suggests that higher long positions taken by leveraged funds increase the ETF FTDs which has the potential to further explain the recent increase in the ETF FTDs after September 17 of 2008 (see Table 1.19).

{*Table* 1.19 *about here.*}

1.6.5 NYSE vs NASDAQ

I ask whether the predictability of the FTDs through index futures is different across the two main stock exchanges, NYSE and NASDAQ. This might be true as each E-Mini contract in my sample follows an index with constituents predominantly from one exchange than the other. Prior studies have examined the market microstructure differences between an auction and a dealer market such as NYSE and NASDAQ. These studies suggest that NASDAQ has significantly higher execution costs, stronger liquidity premium, and higher volatility than NYSE (Bessembinder and Kaufman, 1997; Eleswarapu, 1997; Weston, 2000; Jiang, Kim and Wood, 2011; Chung et al., 2003). Theissen (2000) reports that prices are more informationally efficient on NYSE than NASDAQ, thus attracting more short-selling activity on NYSE. Diether et al. (2009) and Edwards and Hanley (2008) find that short selling of NASDAQ stocks is significantly greater than the short selling of NYSE stocks. To reconcile these two views, using a matched sample of NYSE and NASDAQ stocks, Blau et al. (2011) find that NASDAQ has a greater level of short selling than the NYSE has; however, NASDAQ has less relative short activity than the NYSE when considering short selling that executes on NYSE.

I investigate whether the E-Mini index futures in my sample have different effects between NYSE and NASDAQ stocks. I first look at the slope η in equation 1.2, the time series regression, for all contracts and trader categories with all the controls variables. When η is significantly different from zero but it is insignificant after dropping some control variables, I call the slope spurious. The results show that the E-Mini DJIA has a similar effect on NASDAQ and NYSE, but the slopes for dealers and leveraged funds on NYSE are spurious. E-Mini NASDAQ 100 and E-Mini Russell 2000 hold their effects on NASDAQ, but on NYSE their effects are spurious. E-Mini $S\&P$ 400 keeps the effect on NASDAQ but looses on NYSE, in contrast to E-Mini $S\&P$ 500 which has no predictive power on NASDAQ but the slope of its asset managers is significant on NYSE. The results from monthly panel regressions using equation 1.3 are mostly consistent with the main results except for the E-Mini $S\&P$ 500, where other-reportables have negative and significant η and leveraged funds have positive η. The panel regression results might suggest that even with the presence of minor differences, the characteristics of stocks play a more important role than the differences between exchanges. Although we confirm the results using monthly rather than weekly frequency, the differences between the exchanges might be more pronounced when data with a higher frequency is used (see Tables 1.20 and 1.21).

$\{Tables\ 1.20 - 1.21\ \textit{about here.}\}$

1.7 Conclusion

The existing literature argues that the short selling regulation of 2005 (regulation SHO), which was updated in 2008, reduces market quality. However, in this chapter, I show that the 2008 regulation improves the informativeness of naked short sellers by deterring some uninformed naked short sellers.

The updated regulation of 2008 made traders strictly responsible for delivering the shares sold short within specified time frames. I find the following empirical results for the sample period following the 2008 regulation. First, the higher net short positions are taken by speculators, traders other than dealers or intermediaries, in the E-Mini index futures markets, are associated with fewer FTDs in the stock market. Second, the greater the net short positions taken by dealers, the higher will be the FTDs in the stock market. Third, the publicly released excessive and persistent FTD stocks earn negative abnormal returns. These results do not hold prior to September 17, 2008. This suggests that the uninformed, and perhaps manipulative, traders are deterred, which consequently improves the informativeness of the NSS activity.

To further support my results, by using an investment strategy, I show that FTDs contain some information which is not incorporated in price. The outcome of this chapter is important in an environment where courts and the SEC have failed in dealing with the NSS complaints. The results also hold for the newly-created FTDs; and price pressure, stock order imbalance, and the Exchange Traded Funds' FTDs cannot explain the observed relationship between E-Mini index futures and stock FTDs. Using monthly horizon, I find similar relationship across NYSE and Nasdaq exchanges.

This chapter opens a window to research on trading activities around predictable order flows. Bessembinder (2014) suggests that predictable order flows in the presence of only a few strategic traders in a less resilient market might be disruptive to the market quality. Since strategic traders might also engage in the NSS activity, investigating the FTDs around predictable order flows might be useful for future papers.

1.8 Appendix A: Short Selling Regulation History in the U.S.

In this section, I briefly review the short selling regulation history. In September of 1931, NYSE banned short selling. In October of 1931, the NYSE prohibited the short selling on a downtick. In 1938, the SEC implemented the Rule 10a-1[14] or the uptick test for exchange traded securities.

In 2004, the SEC passed a new regulation called the Regulation SHO. Under this regulation, the uptick rule was temporarily suspended for selected joint stock companies. Further, this regulation also established the requirement of marking the orders to see if selling is long, short, or is numbered as short exempt (for example, short selling transactions made by market makers). Additionally, the regulation SHO almost entirely reduced the possibility of excessive naked short selling[15] by introducing the need for locating securities which is crucial for borrowing in advance in an efficient short selling. On January 3rd of 2005, the SEC implemented the regulation SHO which restricted short selling

[14] "The SEC: rule 10a-1 under the Securities Exchange Act, which prohibited short sales of exchange-listed securities at levels below the last sale price, or at the last sale price unless that price was above the next preceding different price (a zero-plus tick)."

[15] "The SEC: naked short selling, while not defined in the federal securities laws or self-regulatory organization rules, generally refers to the one in which the investor does not own the securities at the time of the sale and has not made arrangements to borrow them in time to make delivery to the buyer within the standard 3-day settlement period."

through the locate[16] and close-out[17] requirements. However, the SEC exempted options market makers (OMMs) from the close-out requirement to allow registered OMMs to sell short threshold[18] securities, due to concerns regarding liquidity, the pricing of options, and the argument that no OMM would make markets without the ability to hedge by selling short the underlying securities.[19] The SEC decided to suspend the uptick rule during the closed session on July 6th, 2007. This decision was implemented on August 14th, 2007. From July 21st, 2008 to August 20th, 2008, naked short selling was banned on 19 financial stocks.

On September 17th, 2008, the SEC updated the regulation by eliminating the OMM exemption and implemented a "hard $T+3$ close-out", i.e. the rule requires that "short sellers and their broker-dealers deliver securities by the close of business on the settlement date (three days after the sale transaction date, or T+3) and imposing penalties for failure to do so."; The SEC mentions that 'as a result, OMMs will be treated in the same way as all other market participants, and required to abide by the hard T+3 close-out requirements that effectively ban naked short selling." If the participants can demonstrate that the FTDs are from long sale fails and fails attributable to bona fide market making, the failed positions should be closed out before the beginning of regular trading hours on day $T+6$. These rules on July 31st, 2009, became permanent. On September 18 of 2008, the SCE adopted the naked short selling anti-fraud rule. For some

[16] "The SEC: the rule prohibits a broker-dealer from accepting a short sale order in any equity security from another person, or effecting a short sale order for the broker-dealer's own account unless the broker-dealer has (*i*) borrowed the security, or entered into an arrangement to borrow the security, or (*ii*) has reasonable grounds to believe that the security can be borrowed so that it can be delivered on the date delivery is due".

[17] "The SEC: the rule requires a participant of a clearing agency registered with the SEC to take action to close out the fail to deliver that has remained for 13 consecutive settlement days by purchasing securities of like kind and quantity."

[18] "The SEC: a threshold security is defined in as any equity security of an issuer where for 5 consecutive settlement days: there are aggregate fails to deliver at a registered clearing agency, (e.g. NSCC, see Putnins (2010) for more information about clearing system in the U.S.), of 10,000 shares or more per security; that the level of fails is equal to at least 0.5% of the issuer's total shares outstanding and the security is included on the list published by a self-regulatory organization."

[19] "The SEC: OMMs are allowed to sell short threshold securities in order to hedge options positions, or to adjust such hedges, if the options positions were created prior to the time that the underlying security became a threshold security. Any fails to deliver from short sales that are not effected to hedge pre-existing options positions, and that remain for thirteen consecutive settlement days, are subject to the mandatory close out requirement."

specified financial stocks, short selling was banned from September 19th, 2008 to October 8th, 2008. On February 24th, 2010 the alternative uptick rule was implemented.

1.9 Appendix B: Short Selling Structure

The mechanism of short selling is illustrated in Figure 1.8. According to this figure, assume that a buyer submits a buy order of 300 shares to broker B on day T. On the same day, a short seller submits an order of 100 short sales to broker A, and a market maker involves in naked short selling and sells 200 shares. Since broker A has only 40 shares available from all of its customers that can be borrowed, it needs to borrow 60 shares from a securities lender. After doing so on day T, broker A sells 100 shares in the stock market, and on the same day, broker B buys 300 shares. On day T, the stock exchange electronically transmits details of traders to the clearing house.

In the U.S., clearing and settlement are provided by the subsidiaries of Depository Trust and Clearing Corporation (DTCC), which are National Securities Clearing Corporation (NSCC) and the Depository Trust Company (DTC). The NSCC does the computation of the obligations to be settled (i.e. fulfilling the clearing role), and the DTC transfers securities and cash between buyers and sellers (fulfilling the settlement role). Putnins (2010) documents a comprehensive study on clearing and settlement in the U.S.

Through NSCC's Continuous Net Settlement (CNS) system, most US equity trades are cleared and settled. The NSCC multilaterally nets trades by stock and on day $T+2$ notifies participants, such as brokers, broker-dealers, banks and insurance and investment companies, of their net positions (net short or net long), and summaries of all their trades. On day $T+3$, the NSS sends the instructions to the DTC containing net securities positions to be settled, and the DTC makes the transfers of stock.

The cash transfer is done separately through the Federal Reserve's money transfer system (Fedwire). If the DTC is able to transfer all the stock, and there are no open failed positions from previous days, then all participants receive their shares. A mechanism that the NSCC has in place in order to reduce the number of failed to receive, is the Stock Borrow Program. This program helps those DTCC members with net short positions to borrow shares from the other DTCC members with excess shares that are willing to lend. This program, however, has been dismantled since March 14, 2014. The borrowing party pays interest on the value of the loan and repays the loan by purchasing the equivalent shares.

Chapter 2

The Role of Reference Demand in Decision Making

2.1 Introduction

Is there a situation where traders are absolutely certain about their optimal demands? In this chapter, I model an economy where each trader adjusts his demand before submitting his order in a way which is not too far from his *reference demand*. I define the reference demand to be a trader's expectation of the average of all other traders' opinions about his optimal demand.

I use three models to analyze the role of the reference demand. I find that for two standard differences of opinion models, as traders pay more attention to their reference demands, the trading volume and social welfare increase. However, I build a third model indicating that the social welfare does not necessarily increase in such a case. In this proposed model, consistent with the other two models, when traders pay more attention to their reference demands, numerical instances show that volume increases.

There are several reasons that traders might pay attention to the others' opinions about their demand. First, transaction quantities contain information about future trading as empirically documented by previous studies. Evans and Lyons (2007) find that transaction flows in foreign exchange markets forecast future macro variables significantly better than the exchange rate. Love and Payne (2008) find that public news impounded into prices via order flow. Furthermore, Hong and Yogo (2012) find that in predicting future returns, open interest could be more informative than price in futures markets. Since the transaction quantities for aggregate traders are informative, and traders are aware of it, therefore in my model they try to use as much as information as possible before submitting their demands. By doing so, each individual becomes more informed and if there is a commonality in the average opinion about demands, it could be incorporated into price.

The second reason for using the reference demand is that traders frequently use similar models and have estimations of each others' beliefs, therefore they can hypothesize an economy with estimations of other traders' demands. Since it is important that what opinion all traders collectively have about a specific trader's demand, that same trader could use that opinion in his decision making process. The use of common models could also indirectly point to traders also paying attention to others' opinions about their demands. When using a common model framework, traders might follow some type of coordinated behaviour with other traders and therefore some non-fundamental factors may be part of the decision making. Allen, Morris and Shin (2006) argue that in a dynamic model, higher order beliefs become relevant and are a key determinant of traders' ability to coordinate with each other. Therefore, the average opinion influences a trader's own decision when they submit their demand. As an example of this case, Rangvid et al., (2013), find that the average of all forecasters' beliefs affects each forecaster's belief.

The third reason is that sometimes traders may anticipate the possibility that their decisions may turn out to be wrong in hindsight. As a result, according to the theory of regret aversion, they try to reduce this possibility. This is consistent with Zeelenberg et al. (1996) where they find that individuals tend to make regret-minimizing decisions rather than risk-minimizing decisions. In my proposed model, traders are happier if they adjust their demand with a reference demand which is the expectation of an average opinion. Consequently, they can make a regret-minimizing decision.

In my analysis, I employ the following three models. First, a differences of opinion model where traders know each others' exact beliefs about the mean of the final payoff and simply agree to disagree. Second, a similar model whereby in addition to disagreement about the mean of final payoff, traders have heterogeneous signal precisions. This heterogeneity can be interpreted as non-uniform confidence levels about the signals they receive. The third model is a generalized version of the first model where traders do not necessarily know other traders' specific beliefs about the mean of the final payoff, but have noisy estimations about those beliefs. All of these models use a common utility function. The models are in a dynamic setting as some differences are evident between the last trading date, i.e. a static model, and other trading dates.

This utility function is constructed with two components: one term is the classic constant absolute risk aversion (CARA) utility and the other is an adjusting factor. The adjusting factor makes a trader less happy when his demand is too far from the average opinion about it. With this design, traders also learn from others' views about their trading positions. I find that when traders know each others' specific beliefs, their optimal demands are equal to their reference demands. However, if they only have noisy estimations about each others' beliefs, their optimal demands are different from their reference demands. In the latter case, the more they pay attention to the average opinion about their demands, their optimal demands become closer to their reference demands. At limit, once traders pay full attention to the average opinion about their demand, their optimal demand will be equal to their reference demands, as in the case when their beliefs are common knowledge.

Once traders have heterogeneous signal precisions, then the correlations between prices and positions are not necessarily zero which is consistent with empirical findings.

In this case, I show that the magnitude of the price-position correlation increases when traders pay more attention to the average opinion about their demand. Fishe et al (2014) find that in commodity futures markets, money managers' positions are positively correlated with prices. They conclude that money manager signals are larger and therefore their associated price changes are larger.

An entity might end up in a loss if no adjustment of demand with the average opinion about demand is implemented. In 2012, a branch of JPMorgan Chase & Co. was selling large derivative position which substantially deviated from the average opinion about that position. Other institutions started learning about this trade and some criticized the bank. It is important to point out that those traders with opposite bets made profits as a result of the banks' lack of attention to the average opinion to its demand. Note that the lack of attention to the average opinion in a subsidiary can potentially drive the company into distress.

There could be some scenarios where traders have approximations of each others' demands as highlighted in the following examples. In sunshine trading, liquidity traders pre-announce the size of their orders (Admati and Pfleiderer (1991). Commodity firms that face similar spot price uncertainty and use similar models, might have approximations of each others' futures positions. Traders who are pessimistic about an asset might be attracted to buy rather than sell, in order to coordinate with others on the direction of trades. Since the demand for securities drives prices up or down, some issuers might adjust prices according to demands. Ruf (2011) finds that issuers are able to anticipate demand in short term and preemptively adjust prices for warrants upwards (downwards) on days when investors are net buyers (sellers).

In my model, I assume that all traders are rational, but in addition to fundamental motives, they are also motivated to not stray too far from the average opinion about their demands. Keynes (1936) established the notion of *beauty contests* where traders need to forecast the forecast of others. To formulate this Keynesian metaphor, Morris and Shin (2002) introduced a utility function which has two components: one is negative of a loss function which measures how an action is far from the fundamental; the other is the beauty contest term which measures how well an action is coordinated with others' actions. I construct my utility function inspired by their introduced utility function. However, my departure from their paper is that each trader does not compete to second guess other traders' actions, but submits an optimal demand which is not too far from the average opinion about his demand.

Note that standard or noisy rational expectations equilibrium models cannot generate public disagreements among traders and cannot unify with the empirically observed volume patterns. However, the classic models of differences of opinions, e.g. Harrison and Kreps (1978), where traders' beliefs are common knowledge, can generate public disagreements (Banerjee et al., 2010). Similarly, in my models, I assume that traders have differences of opinions, and for the purpose of tractability, they do not condition on price. Since traders use the same utility maximization procedure, they determine their reference demand via their signals about the other traders' beliefs. They next solve for the optimal demand now knowing the reference demand. While it might be possible that different traders have different degrees of paying attention to reference demands, in this chapter I assume that the traders have uniform degrees of paying attention to the average opinion about their demands. Lastly, since the empirical evidence I cite rests in the derivatives markets, in my models, I assume that the net supply is zero.

The rest of this chapter is organized as follows. Section 2.2 reviews the related literature and section 2.3 presents a dynamic model with three subsections. Subsection 2.3.1 presents a classic differences of opinions model where traders submit their demands that are not substantially deviated from the average opinion of their demand. Subsection 2.3.2 gives a generalization of the model presented in subsection 2.3.1 and illustrates a case when traders' beliefs are not common knowledge. Section 2.3.3 introduces disagreement in traders' signals precisions. Section 2.4 shows the role of reference demands on social welfare and volume. Section 2.5 illustrates a numerical example of the model in section 2.3.2. Section 2.6 concludes the chapter.

2.2 Related Literature

In this chapter, I use two strands of the literature, one on informative demand and the other on the beauty contest metaphor; I try to explain the role of reference demand by employing various theoretical models. The acceptable models in economics assume traders are rational and self-regarding, despite much evidence that challenges this view. Traders not only care about choosing an action which is appropriate to the underlying state of fundamentals, but also about coordination with the actions of other traders. Therefore, beliefs about other traders' strategies are vital in determining outcomes.

For the traders to coordinate on an efficient equilibrium, they must believe that other traders will coordinate with them. In my models I assume they do coordinate with each other in the sense that they all use the reference demand with same degree of paying attention to the average opinion about their demand. In these situations, traders' decisions as well as the average opinions will be reflected in their demand functions. In the next two subsections, I explain the two strands of the literature I use in greater detail.

2.2.1 Keynesian Beauty Contest

Predicting actions of other traders are crucial for many economic decisions. For instance, in an oligopoly, firms may need to predict how much their competitors will invest in production capacity; traders in financial markets may need to predict how much other traders are willing to pay for their assets.

In 1936, John Maynard Keynes paid significant attention to the factors that relate an asset price to its fundamental value based on expected future payoffs. He described markets as a beauty contest, where people second guess the action of others. In the real world, only a few people might follow this strategy, but Keynes's beauty contest metaphor offers a simplified setting to study the behaviour of traders. At that time, a newspaper in London was running a beauty contest in which readers were asked to select a set of the six most beautiful photos from 100 photographs; whoever picked the most popular pictures was entitled to a prize.

Keynes observed that participants in the financial markets shared the essence of the above newspaper competition, where investors in short-run are rational and similarly are governed by their expectations about other investors beliefs; this could potentially deviate investors' expectations about the true value of an asset. In other words, Keynes argued that investors are involved in picking the most beautiful, i.e. the most popular asset, because traders pay attention to what other investors think about the asset rather than its fundamental value.

Keynes (1936, chapter 12, p. 156) described his metaphor of the beauty contest as: "... professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees. "

Thus in forming a portfolio, Keynes suggests that what matters the most is how market investors in aggregate perceive the stocks' values rather than the fundamental value of those stocks. This metaphor might lead to the idea that investors should revise their beliefs based on what they think the market population believes as a whole. One meaning of the metaphor is that investors forecast stock prices rather than the company earnings. Precisely, today's price is a forecast of what the market population expects the tomorrow's price to be, rather than an estimation of the present value of future dividends.

Notice that in order to predict investors' behaviour that form expectations about the actions of others, each investor should predict other investors' expectations about the action of others, and so on. This leads to the well-known infinite regress of expectations. Therefore, in order to form the demand for an asset, investors should calculate two components. First, forecast the future payoffs and second to try and guess other market participants' forecast and others' forecasts of others' forecast, and higher order forecasts. In this scenario, investors are said to have higher order expectations (HOE hereafter). Therefore, the two factors of mass psychology and higher order opinions, drive a wedge between an asset's fundamental value and its price. Market psychology has been largely ignored for a long time but is growing in the field of behavioural finance (Barberis and Thaler, 2003, Hirshleifer, 2001). Similarly, the impact of HOE on asset prices is not well understood and has received little attention. Notice that according to Keynes, investors are concerned about what the market will value an asset, rather than what it is really worth.

Moulin (1986) was the first to numerically discuss Keynesian beauty contest theory. In its simplest form, in a beauty contest game, players choose a natural number between zero and one hundred. The winner is the one with a number which is the closest to a given fraction less than one, for instance $\frac{2}{3}$, of the average of all players' entries. The Nash equilibrium of this game is where everyone chooses 0. Using this simple game, a survey result published by Camerer et al. (2004) shows that in their sample, CEOs do guess the other players' actions, but they are not concerned about HOEs. Since Moulin (1986), all kinds of beauty-contest games have been widely used and investigated.

One of the most interesting articles in the beauty contest literature is the one in Morris and Shin (2002), which formalizes the basic ideas in a static game. In the game, a continuum number of players receive public and private signals about an underlying fundamental. The players have a trade-off between being accordant with the fundamental value and being accordant with others' actions which are reflected in their two-term utility function. They use the model to explain why the price might not fully reveal the fundamentals due to noise in the public information. This is the case where players rely heavily on public information as they are clueless about others' private information. This eventually leads to the deviation of price from the fundamental due to noise in the public signal. For example, public news with no support from the fundamental might cause significant price changes such as bubbles and crashes. Therefore, in Morris and Shin (2002)'s model, public news play two roles. One conveys information about the unknown fundamental value, information role, and the second is common information across all investors' information sets, commonality role. A strong support for Morris and Shin (2002)'s conclusions is found in James and Lawler (2011). Notice that in addition to private understandings of the underlying fundamental, which are part of the private signals, investors consider previous winners' action as noisy public signals. For instance, investors learn and react regarding the actions taken by Warren Buffett and George Soros or other famous investment publications.

Morris and Shin (2002)'s conclusion has also been criticized in different dimensions. For instance, their result that transparency, more precise public information, might damage social welfare drives from the nature of their "assumed" utility function with the so-called beauty contest term to incorporate beauty contest motive. This term creates incentives for traders to attempt to coordinate with the actions of others despite having no social value nor being related to the fundamental. Given this incentive, the beauty contest term highlights the commonality role of the public signal, which also makes the intensity of this commonality role to be fixed. This fixed role keeps the investors' actions closer to the average action of other investors.

Morris and Shin (2002)'s model gives a unique optimal action which is a weighted sum of the public signal and the player's private signal. Since public news is a better estimate of the actions of others than the idiosyncratic private signal, excessive weight is placed on the public signal. Therefore, the noise in a public signal will be more pronounced and it can do more harm than good. For example, in the context of bank runs or currency attacks, increasing transparency by a central banker can give rise to inefficiencies in equity markets. This happens because investors rely less on their private information. In contrary, articles such as Angeletos and Pavan (2004), Hellwig (2005), Roca (2010), find that improvements in the quality of public information are beneficial to social welfare. With more transparency, there might be an overreaction to public information in the financial markets. To reduce this overreaction, central banks can disclose information to only a fraction of market participants or can disclose information with ambiguity to all participants. Furthermore, Baeriswyl and Cornand (2012) show that in theory both communication strategies are equivalent.

The conclusion of Morris and Shin (2002)' model does not necessarily hold if the information is a choice for the players. When the information structure is exogenously given, a number of studies reveal inefficiencies in the collection and use of information. For instance, Angeletos and Pavan (2007) study a model with a continuum of players, each player observes a private and a public signal, the payoff function is quadratic, and the information structure is Gaussian. They use an efficiency benchmark: a society is the best where it could achieve keeping information decentralized. They show that when the social value with coordination and the incentives to coordinate actions are different, the use of information can be inefficient.

Keynesian metaphor has recently formalized and applied to stock prices by Allen, Morris, and Shin (2006) and Gao (2008). According to this metaphor, traders' decisions are driven by anticipation of others' actions, and not fully by the actual knowledge of the companies they trade. Therefore, informed traders do not necessarily compete to neutralize the actions of uninformed traders. In this situation, traders tend to put a disproportionally high weight on public information in their forecast of prices which is claimed to be informationally inefficient (Allen, Morris, and Shin, 2006).

In their model, Allen, Morris, and Shin (2006) assume that investors are short-horizon, and therefore they exist before a firm's fundamental value is known. Hence, each investor's payoff depends on how much other investors would like to pay, rather than the present value of future cash flows of the asset. In this case, they rationalize the beauty contest metaphor as a consequence of investors' short horizons. Similar to Morris and Shin (2002), public news play two roles, the information role, and the commonality role. Therefore, investors put extra weight on public information. Since the noise in private signals cancel out when demands are aggregated, but the noise in the public signal remains in the aggregate demand, hence the infusion of additional public information might be harmful as all investors share the same noise term in the public information. This highlights the second role of public information, the commonality role, which pushes stock prices away from the fundamental value towards the public news. As a result, the public signal has an impact on price beyond its informational value.

In the models presented in this chapter, I emphasize the idea that by adding the beauty contest metaphor to our analysis about demand, the financial markets are better characterized. While any overreaction is not socially desirable, the efficiency requires that prices reflect fundamentals, but this might not be the case when investors gain from predicting the average opinion of other investors, where public information might be very useful. In contrast to Morris and Shin (2002)'s conclusion, Svensson (2006) suggests that the relative precision of public news versus private news, which is used as a condition under which an increasing transparency reduces welfare, seems unreasonable given empirically plausible parameter values. For instance, one can employ recent evaluations of public versus private-sector forecasts on GNP growth or inflation (Romerand Romer, 2000). Therefore, Svensson (2006) claims that Morris and Shin (2002)'s model makes a good case "in favour of" transparency.

One of the most interesting models besides Allen, Morris, and Shin [2006]'s model is the one presented by Gao (2008). He argues that greater public information, such as accounting disclosure, improves market efficiency even in the presence of the beauty contest effect. He measures market efficiency by price efficiency. As we know, information about the future a firm's cash flows are provided in accounting disclosure, which might be overused by short-horizon investors as public information that has two roles as explained above. However, Gao (2008) argues that public information should not be withdrawn. This is because more financial reporting improves the overall price informativeness.

Gao (2008) also claims that with more precise public information, the Keynesian beauty contest effect intensifies. The intuition is that there is an endogenous link between the aforementioned dual-role that public news play, i.e. the link between the information role and the commonality role. As public news become more precise, the information value of it will increase, and consequently, the short horizon investors overuse it further because of its commonality role of forecasting average opinion. However, despite modelling differences, Morris and Shin (2002)'s conclusion is perhaps due to the assumed utility function which divorces the link between the dual role of public news.

I follow Morris and Shin (2002)'s beauty contest utility function. In Morris and Shin (2002), speculators benefit from forecasting the average opinion better than others, however, in my model players suffer if their demands substantially get deviated from the average opinion about their demands. I, therefore, employ a utility function with two components, one reflecting the fundamentals and the other the beauty contest term. Consequently, the separation of fundamental and beauty contest term is done through demand, and, the aforementioned two roles of public news are endogenously related through demand and average opinion on demand. The average opinion about an investor's demand, and consequently the investor's demand, are driven using the public and private signals about the fundamental. Therefore, my proposed models have both the dual role of public news and the separation of fundamental and beauty contest.

Some players might make profits by taking advantage of Keynesian beauty contest metaphor. In a theoretical model, Kudoh and Ishikawa (2012) show that only informed traders speculate on the future public opinion about the liquidation value of the asset. Therefore beauty contest should be considered as part of their speculation activity. Bagnoli et al. (2014) examine analysts' stock recommendations and find that when recent and future investor sentiment is bullish, analysts make more favourable stock recommendations. Therefore, analysts recommend stocks not only based on stocks' intrinsic values, but also because of other signals that affect prices. This suggests that analysts might consider issuing their recommendations as a Keynesian beauty contest.

Allen, Morris, and Shin (2006) was one of the early papers that formally analyzed the role of HOE in asset pricing. In a model, the authors show that HOE differs from the first order average expectations because the general law of iterated expectations does not hold for "average expectations" of an asset's payoff. In their model, the asset under consideration has a single terminal payoff and they find the equilibrium price as a function of HOE of the asset, where public news biasing stock prices toward public information and plays the aforementioned dual role. In contrast to Allen, Morris and Shin (2006), the paper Banerjee et al. (2009) argue that to have the empirically observed price drift in a model, higher order differences of opinion is necessary for heterogeneous beliefs to generate price drift. Kondor (2012) develops a framework where investors have heterogeneous trading horizons. He claims that public news can reduce disagreements about fundamentals while increasing higher order disagreements about price; this happens when the correlation of private information across investors is sufficiently low. In such an environment, after a public announcement, investors' beliefs get closer to each other while trading happens at high volumes, and this situation is empirically observed.

In this chapter, I employ differences of opinion models. A genuine disagreement seems to be a plausible description of a speculative market. Shiller (1995), for example, studied opinion polls among U.S. and Japanese investors about likely price development on Japanese equity markets and found a mutually recognized disagreement.

Modeling in financial markets has been evolved, and rational expectation hypothesis is still the basis of most contemporary asset pricing models despite its weak empirical support. Shiller (1981) and LeRoy and Porter (1981) argue that the observed price volatility is too large to be justified by rational expectations models, and these papers are the origin of a literature trying to explain the excess volatility puzzle in the framework of rational expectations. One strand in this literature is trying to build an asset pricing model with additional variables which generate a time-varying discount factor, see, e.g., Campbell and Schiller (1987) and (1988). However, Shiller (2003) argues that none of these factors seems to explain all the excess volatility.

Rather than the classical volatility test, some other evidence against rational expectation models exit (Zhong, Darrat and Anderson 2003). With these apparent failures, many researchers tried to explain the origin of price movements using non-fundamental factors, such as those market irregularities described in behavioural finance. For instance, using a model with habit formation, Campbell and Cochrane (1999) through changes in risk aversion in a theoretical model, generate excess volatility. By introducing loss aversion, Barberis, Huang and Santos (2001) try to explain the puzzle as well. Both of these models, replicate several distinctive features such as the observed volatility. I, however, focus on differences of opinion models including one in the presence of HOEs.

In one of the models presented below, I employ HOE or higher order beliefs. The notion of HOE might only be a mathematical concept, but it has some support. In theoretical papers such Townsend (1983) and Basak (2000), in the context of stock markets, authors show that HOE induces higher price volatility than the rational expectation models do so. The fact is that since investors make decisions based upon the variations generated by the (noisy) decisions of others, it causes an additional volatility. Kurz (1974) calls this phenomenon an endogenous uncertainty. Higher-order beliefs can also induce a disconnection between and the fundamental value and price (Bacchetta and van Wincoop, 2004). Lastly, higher order beliefs generate a level of volatility which is in line with the observed price volatility (Pierre Monnin, 2004).

Notice that the HOEs produced by the Keynesian beauty contest are an infinite hierarchy of beliefs which are quite complicated to analyze, however, they can be simplified. To simplify beliefs hierarchies, Harsanyi (1967) imposes a restriction on the first order beliefs - the beliefs of each investor about the private signal of the other. The restriction is that there must exist a join prior, common to all investors, such that given an investor's private valuation, his first order beliefs can be interpreted as the Bayesian update of this prior. This restriction ensures that between an investor's private valuation and his first order belief, there is a common knowledge link. Biais and Bossaerts (1998) further showed that Harsanyi (1967)'s restrictions simplify the beliefs hierarchy while allowing for disagreement among investors. In regard to an approximation of the infinite regress of expectations with a finite dimensional representation, Nimark (2011) shows that the approximation is possible to an arbitrary accuracy under quite general conditions.

Lastly, in regards to asset bubbles, which is one of the most unpredictable events in the financial markets, beauty contest models might be useful to be employed. Rational speculative trading is one explanation for stock bubbles and when investors use HOEs about others' stock valuations to anticipate profits, the resulting speculative trading sustains a bubble. Balakrishnan et al. (2012) studies the technology bubble in 2000-2001 and show that there is a strong positive correlation between bubble continuation and concentration in a technological company's analyst buy recommendations. Therefore, they document that analysts' buy recommendations are observable signals associated with traders' beliefs about anticipated speculative profits. My models can also be used to investigate bubbles. Note that speculative rational trading dates back to Keynesian beauty contest and since then HOE's in differences of opinion models have been used to explain stock bubbles (see Harrison and Kreps, 1978). There is also a recent trend in the literature trying to rationalize differences of opinion by identifying specific market frictions (Daniel et al., 2001; Abreu and Brunnermeier, 2002; and Hong et al., 2008). The presented models below try to add a new element to differences of opinion models with HOEs.

2.2.2 Informative Demand

In this section, I highlight some of the previous studies that show trading transactions are informative about the future prices. Llorente and Wang (2015) study the positions taken by the four classes of the CME's Customer Type Indicator which are: a local trader who trades for his own account (CTI1), a commercial clearing member for his proprietary accounts (CTI2), an exchange member for his own account through a local trader (CTI3), and the general public (non-members) (CTI4). They find that the CTI3 group might possess private information which is not fully reflected in market prices due to the following reasons. First, the daily changes in the positions they take can forecast future price movements. Second, changes in CTI3's overnight positions can also predict higher moments of the price change in the following day. For example, a decrease in their overnight position predicts negative skewness and lower kurtosis, while an increase predicts a positive skewness and higher kurtosis. Finally, other CTI groups' overnight positions have mixed correlations across maturities but CTI3's positions are significantly positively correlated across maturities.

Chen et al. (2014) employ an option volume variable, the public net buying-to-open volume, which is defined as the total open-buy orders of all the deep out-of-the-money $S\&P$ 500 puts by public investors minus their open-sell orders on the same set of options in each month. Since options are in zero net supply, the amount of net selling by firm investors and market-makers is equal to the amount of net buying by the public investors. They find that the net amount of deep out-of-the-money $S\&P$ 500 put options that public investors purchase in a month (or equivalently, the amount that financial intermediaries sell) strong predictors of future market returns and the returns on many other assets.

As I mentioned in the previous section, Keynesian beauty contest, through HOEs, can explain the additional volatility observed in data. To see a connection between HOEs and volatility, notice that an important aspect of volatility is its relation to some liquidity variables such as trading volume and open interest (Martinez and Tse, 2008). Measured as the number of transactions in a futures contract during a specified period of time, trading volume has been used as a measure for the rate of information arrival (Sutcliffe, 2006). Trading volume also measures speculative demand for futures (Lucia and Pardo, 2010). Trading volume is also viewed as a proxy for new information (Copeland, 1976, Clark, 1973). Wang and Yau (2000), among others, predict a positive relationship between volume and volatility.

Further, open interest, measured as the total number of futures contracts which have not been closed out, is an important determinant of volume (Mougoue and Aggarwal, 2011), is regarded as a proxy for dispersion of beliefs (Bessembinder et al., 1996; Mougoue and Aggarwal, 2011), and is an indicator of sentiment in futures markets (Aguenaou et al., 2011). Bessembinder and Seguin (1992) argue that there may be a correlation between the number of informed traders and open interest. Ferris et al. (2002) argue that given pricing error information shocks, open interest in $S\&P$ 500 index futures is a useful proxy for examining the flow of capital into or out of the market.

Previous research has also explained some other informational aspects of trading transactions. Lillo et al. (2005) show that order flow is a long memory process because large orders to buy or sell can incrementally be traded over periods of time. Garleanu et al. (2009) find that demand pressure helps explain the well known option-pricing puzzles. Schneider (2009) argues that trading volume helps traders to evaluate the precision of the aggregate information in the price. Kehrle and Puhan (2013) find that the informed option demand which is not driven by liquidity or hedging motives is profitable.

Another related subject is the issue of excess comovement of prices, the situation where commodity prices remain correlated even after the impact of common factors is adjusted. This phenomenon appeared to be significant during crises and recent years when trading volume had a large increase. However, large increase alone might not be responsible for the excess movement. Pindyck and Rotemberg (1990) showed that hedging and speculative pressure in commodity futures markets can be related to excess comovement in commodity prices. Gospodinov and Jamali (2013) collect 187 real and nominal variables and use factor models to approximate the fundamentals driving commodity prices. They find that hedging and speculative measures explain the estimated excess comovement. This shows a strong impact of the financialization process, and more importantly, the impact from some trader categories on prices which show supply and demand are not the only factors determining price movements. The strong connection between excess comovement and hedging and speculative pressure is interesting as it shows the positions taken by categories of investors should contain some information which is not necessarily incorporated in an aggregate price number.

Gospodinov and Jamali (2013) also suggest that the positions of futures traders in some commodities, such as metals and energy, strongly respond to monetary policy shocks. These shocks appears to be propagated to commodity prices through net long, or shot, of positions taken by trader categories in futures markets. Perhaps the authors' explanations relies on Hong and Yogo (2012)'s findings that trading activity variables such as open interest has a strong impact on future prices, and this happens when traders have limited absorption capacity towards large order flows. In situations when large trades are invested in commodity indices, they are simultaneously invested in many commodity futures, and therefore, the aggregate positions taken by trader categories may partly explain the excess comovement.

2.3 Model

I begin with the common setup of the three finite period models presented in the following subsections. Assume that trading dates are $t = 1, ..., T-1$. There is a continuum of traders, indexed by the unit interval $[0, 1]$, who trade two assets, one risk-free and one risky. The riskless interest rate is zero and the risky asset is assumed to have *zero* net supply. Let F denote the final payoff of the risky asset at time $t = T$, denote its price on date t as P_t, and let $P_T = F$. An example of this economy is a futures contract on a risky asset with the final spot price of F. Before trading starts on date $t = 1$, traders share a common prior on the distribution of the final payoff, given by

$$F \sim N(v_0, \rho_0) \tag{2.1}$$

where v_0 and ρ_0 are exogenously given. For each date $1 \leq t \leq T-1$ and before the trading session on date t, each trader observes a private signal given by

$$s_{it} = F + \epsilon_t + \epsilon_{it} \tag{2.2}$$

where

$$\epsilon_t \sim N(0, \lambda_t), \quad \epsilon_{it} \sim N(e_{it}, q_t), \quad \int e_{it} di = 0 \tag{2.3}$$

and ϵ_{it} and ϵ_t are i.i.d for all t and i, $\int \epsilon_{it} di = 0$, and q_t, λ_t, e_{it} are known to all traders on all dates. If $\int e_{it} di$ is not zero, redefine the private signals as $\tilde{s}_{it} = s_{it} - \int e_{it} di =$

$F + \epsilon_t + \tilde{\epsilon}_{it}$, where $\tilde{\epsilon}_{it} \sim N(e_{it} - \int e_{it} di, q_t)$. In this setting, trader i disregards other traders' signals and sets their posterior to $F|s_{it}$. This assumption is common to models based on D.O., in contrast to the noisy rational expectation equilibrium models. In the following subsections, I assume that this disregarding behaviour exists, i.e. traders agree to disagree, even when traders have noisy signals about other traders' beliefs. Denote trader i's conditional belief on date t about F as

$$v_{it} = E_{it}[F], \quad \rho_t = Var_{it}(F) \tag{2.4}$$

where the subscript t in the conditional moments means given all the available information before trading on day t. I assume that ρ_t is invariant across traders, unless otherwise specified, hence traders disagree about the mean of F but agree on its precision. Conditional computations give

$$v_{it} = (1-\pi_{t-1})v_{it-1} + \pi_{t-1}(s_{it} - e_{it}), \quad \rho_t = \rho_{t-1}(1-\pi_{t-1}), \quad \pi_{t-1} = \frac{\rho_{t-1}}{\rho_{t-1} + q_t + \lambda_t} \tag{2.5}$$

for $t = 1, 2, ..., T-1$. Note that those traders who value the asset more than the current asset holders, and therefore purchase it, have larger private valuations. The equilibrium price will be presented is given in terms of the average valuation, and perhaps one other component, which is denoted by

$$\overline{v}_t = \overline{E}_t[F] = \int E_{it}[F]di = \int v_{it}di \tag{2.6}$$

From equation 2.5, the dynamic equation for the average valuation is given by

$$\overline{v}_t = (1 - \pi_{t-1})\overline{v}_{t-1} + \pi_{t-1}(F + \epsilon_t) \tag{2.7}$$

Note that if $\overline{v}_t$ and $\overline{v}_{t-1}$ are known to all traders from equilibrium prices of dates t and $t-1$, then each trader knowns $(F + \epsilon_t)$ from equation 2.7, and can find their own ϵ_{it} from $\epsilon_{it} = s_{it} - (F + \epsilon_t)$. Let x_{it} denote trader i's demand on date t. Trader i may take either a long position ($x_{it} > 0$), a short position ($x_{it} < 0$) or stay out of the market

($x_{it} = 0$). Notice that the traders face no position limits and there is no requirement to post margin. The special feature of this model is that traders also coordinate with the average opinion belief about their positions. Let x^e_{it} be the trader i's expectation of the average opinion about trader i's position on day t, i.e.

$$x^e_{it} = E_{it}[\overline{E}_t[x_{it}]] = E_{it}[\int E_{jt}[x_{it}]dj] \tag{2.8}$$

where $E_{jt}[x_{it}]$ is the trader j's expectation of x_{it} based on his information about trader i's beliefs on date t. I call x^e_{it} the *reference demand*, and notice that trader i knows x^e_{it} before trading on day t. The model is designed in a way that trader i knows his reference demand, x^e_{it}, but trader $j \neq i$ does not necessarily know x^e_{it}. Since all traders use the same model, they can estimate each others' demands according to their understanding of each others' beliefs, hence they can form $E_{jt}[x_{it}]$. I will define the utility function in a way that each trader pays attention for the average opinion about his demand. To reflect paying attention to x^e_{it}, I work with a utility function inspired by the utility function introduced in Morris and Shin (2002) which highlights the role of coordination among traders. Without loss of generality, I assume that all traders start with zero wealth on day $t = 1$, and end up with the sum of the capital gains in their portfolios across days until day $t = T$. With the final wealth given by

$$wealth_i = \sum_{t=1}^{T-1} x_{it}(P_{t+1} - P_t) \tag{2.9}$$

trader i, on any date t, seeks to maximize his expected utility over the sum of the capital gains from time t to T, given by

$$EU_{it} = E_{it}[-(e^{-W_{it}})^{1-r} \times (e^{\frac{1}{2}(x_{it}-x^e_{it})^2})^r] \tag{2.10}$$

where the trader's risk aversion is normalized to one, and

$$W_{it} = x_{it}(P_{t+1} - P_t) + ... + x_{iT-1}(F - P_{T-1}) \tag{2.11}$$

is the trader i's profit from trading after date $t-1$. The first term in the expected utility is the fundamental component and with a higher level of W_{it}, traders i is happier. The second term is the reference adjusting component, and the parameter r gives the weight on the guessing about the average opinion on trader i's position. In such a setting, each trader is increasingly unhappy when taking a position which deviates substantially from the average opinion about his position. Therefore, the second term adjusts the utility function for the purpose of paying attention, even small, to the average opinion about demand.

When using the above utility function, there is an externality where each trader tries to guess the average opinion about his demand, and the larger r makes this externality more severe. In this scenario, speculators might gain from forecasting the average opinion better than others when adjusting their demands. The parameter r is the strength of the strategic motive of not being too far from the average opinion about demands. This motive might have become more pronounced in recent years due to higher volume, financialization, higher assets' cross correlations, transparency and an increase in the amount of public news that biases prices away from fundamentals.

Information about traders' expected demands can be carried through networks within groups or organizations. The expected demands can be calculated by using public news, historical trends, and information acquisition within network groups. Since acquiring private information might be costly, each trader tries to obtain the best estimate of their demand using public information including an approximation of the average opinion about their demand. In this scenario, traders learn about the average opinion about their demand over time and they revise their approximation of such an average. The information choice, however, should be optimal and some traders might not use the average opinion of others about their demand. Jimenez-Martinez (2012) develops a model where traders make decisions about their information acquisition. He argues that traders anticipate the expected utility that they will have based upon their given acquired information.

In my model, traders try to improve their knowledge from all the available information including the average opinion about their demand. This sort of decision making, when incorporating the anticipation of others' perceptions, can affect the optimal demand, and consequently the liquidity.

Notice that on day t, each trader maximizes his future profit for the purpose of finding his optimal demand on day t, and furthermore, only adjusts his demand on day t. To keep the utility function simple and eliminate extra cumbersome calculations, I assume that traders on day t do *not* adjust their current demand when incorporating the differences between their future demands and the corresponding reference demands, i.e. $(x_{it'} - x^e_{it'})^2$ terms are dropped, where $t' > t$.

For the value of $r = 0$, the above utility function reduces to the classic CARA utility which only highlights the information content of signals about the final payoff. When $r = 1$, the traders ignore their information about the final payoff and solely act based upon the average opinion about their demands. In practice, r might be a small positive number and changes across time and traders. I assume that $r \in (0, 1)$ and it is constant for all traders at any point in time.

To find the optimal demand, each trader maximizes his expected utility and finds his optimal demand as a function of the equilibrium price, his beliefs and his reference demand. In the spirit of differences of opinions (D.O.) models, and for tractability, I assume that traders do not condition on price when they maximize their optimal demand. Once the optimal demand is found in terms of the reference demand, traders are able to find their reference demands by using equation 2.8, hence they can find their optimal demand adjusted by reference demand. Finally, the equilibrium price is found from the market clearing condition.

2.3.1 Standard D.O. with Reference Demand

In the standard differences of opinions (D.O.) models, traders' beliefs are common knowledge but they agree to disagree. In this section, I assume that each trader's expected utility is given by equation 2.10, and further on each date t, each trader knows other traders' beliefs, hence trader i's expectation of any other trader j is given by

$$E_{it}[v_{jt}] = v_{jt} \quad \forall i, j \tag{2.12}$$

Let the trader i's state vector on day t be

$$Z_{it} = \left(\begin{smallmatrix} P_t \\ v_{it} \end{smallmatrix}\right) \tag{2.13}$$

hence, trader i's expectation of the average expectation of Z_{it} is

$$E_{it}[\overline{E}_t[Z_{it}]] = E_{it}\left(\begin{smallmatrix} P_t \\ v_{it} \end{smallmatrix}\right) = Z_{it} \tag{2.14}$$

I use the following lemma to find the optimal demand.

Lemma 1. *Assume that the reference demand for trader i on day t is given by*

$$x^e_{it} = \Delta'_t Z_{it} + d_t E_{it}[\overline{E}_t[x^e_{it}]]$$

where Δ_t and d_t are known to all traders on day t, and $|d_t| < 1$. In this case, the reference demand is

$$x^e_{it} = \frac{1}{1 - d_t} \Delta'_t Z_{it} \tag{2.15}$$

The proof is in the appendix. To find the optimal demand on date $t = T - 1$, trader i

finds his demand from

$$
\begin{aligned}
x_{iT-1} &= arg\max_x E_{iT-1}[-exp\{-(1-r)x(F-P_{T-1})+\frac{1}{2}r(x-x^e_{iT-1})^2\}] \\
&= arg\max_x -exp\{-(1-r)x(v_{iT-1}-P_{T-1})+\frac{1}{2}\rho_{T-1}(1-r)^2x^2+\frac{1}{2}r(x-x^e_{iT-1})^2\} \\
&= arg\min_x\{-(1-r)x(v_{iT-1}-P_{T-1})+\frac{1}{2}\rho_{T-1}(1-r)^2x^2+\frac{1}{2}r(x-x^e_{iT-1})^2\}
\end{aligned}
$$

where $\frac{1}{2}r(x-x^e_{iT-1})^2$ is constant according to the assumption that x^e_{iT-1} is known to trader i on day $T-1$ before trading. From the first order condition, we find

$$x_{iT-1} = \Delta'_{T-1}Z_{iT-1} + d_{T-1}x^e_{iT-1}$$

where

$$\Delta_{T-1} = \frac{1-r}{r}d_{T-1}h, \qquad d_{T-1} = \frac{r}{(1-r)^2\rho_{T-1}+r}, \qquad h = \left(\begin{smallmatrix}-1\\1\end{smallmatrix}\right) \tag{2.16}$$

Therefore, the reference demand is given by

$$
\begin{aligned}
x^e_{iT-1} &= E_{iT-1}[\overline{E}_{T-1}[x_{iT-1}]] \\
&= \Delta'_{T-1}E_{iT-1}[\overline{E}_{T-1}[Z_{iT-1}]] + d_{T-1}E_{iT-1}[\overline{E}_{T-1}[x^e_{iT-1}]] \\
&= \Delta'_{T-1}Z_{iT-1} + d_{T-1}E_{iT-1}[\overline{E}_{T-1}[x^e_{iT-1}]]
\end{aligned}
$$

which according to Lemma 1 gives

$$x^e_{iT-1} = \frac{1}{1-d_{T-1}}\Delta'_{T-1}Z_{iT-1}$$

Hence,

$$
\begin{aligned}
x_{iT-1} =& \Delta'_{T-1}Z_{iT-1} + d_{T-1}x^e_{iT-1} \\
=& \Delta'_{T-1}Z_{T-1} + d_{T-1}\frac{1}{1-d_{T-1}}\Delta'_{T-1}Z_{iT-1} \\
=& \frac{1}{1-d_{T-1}}\Delta'_{T-1}Z_{iT-1}
\end{aligned}
$$

that can be simplified further to

$$x_{iT-1} = x^e_{iT-1} = \frac{1}{(1-r)\rho_{T-1}}(v_{iT-1} - P_{T-1}) \tag{2.17}$$

From the market clearing condition, $\int x_{iT-1}di = 0$, we find the equilibrium price as

$$P_{T-1} = \overline{v}_{T-1} \tag{2.18}$$

similarly, we can find the optimal demand and the equilibrium price for any date $t < T-1$ as stated below.

Proposition 1. *For any date $t < T$ and any parameter $r \in (0,1)$, trader i's demand, reference demand, and the equilibrium price are given by*

$$x_{it} = x^e_{it} = \alpha_t(v_{it} - P_t), \qquad P_t = \overline{v}_t \tag{2.19}$$

where $\alpha_{T-1} = \frac{1}{(1-r)\rho_{T-1}}$, and for any $t < T-1$, $\alpha_t = \frac{1}{(1-r)\pi_t(\rho_t+\lambda_{t+1})}$.

The proof is in the appendix. As shown, α_t is a function of r, which shows the traders behaviour of paying attention to the average opinion about their demand is a component of the equilibrium, among others. I analyze the properties of this model in section 2.4. In the next section, I present a generalized version of this model.

2.3.2 A Generalization

Since in reality traders are not aware of other traders' exact beliefs, i.e. v_{jt}, in this section I generalize the standard D.O. with Reference Demand model in section 2.3.1, to which traders do not know other traders' beliefs, but receive noisy signals about those beliefs. Hence, traders can form expectations of other traders' beliefs. In such a setting, traders are also able to construct a signal about the average valuation, i.e. $\overline{v}_t$, by averaging their signals about other traders' beliefs. Therefore, traders have different beliefs about the average valuation. Assume that on each day t, trader i in addition to receiving the private signal about the final payoff, i.e. s_{it}, has the following priors on trader j's belief and the average valuation

$$v_{jt} \overset{i}{\sim} N(v_{it}, \zeta_t), \quad \overline{v}_t \overset{i}{\sim} N(v_{it}, \zeta_t) \tag{2.20}$$

where ζ_t is exogenously given and known to all traders. Further, trader i receives the second private signal on day t about trader j's belief which is given by

$$v^i_{jt} = v_{jt} + \delta_{it} + \omega_t, \quad \delta_{it} \sim N(0, \iota_t), \quad \omega_t \sim N(0, \kappa_t) \tag{2.21}$$

where $F, \overline{v}_t, \epsilon_t, \epsilon_{it}, \delta_{it}, \omega_t$ are all uncorrelated from each other except for F and $\overline{v}_t$. With an additional assumption, the covariance between F and $\overline{v}_t$ is calculated in equation 2.94. Assume that $v^i_{it} = v_{it}$ and $\{\delta_{it}\}_{i\in I}$ are i.i.d. In this model traders also reflect differences of opinion, therefore they put

$$v_{it} = E_{it}[F|s_{it}, \{v^i_{jt}\}_{j\in I}] = E_{it}[F|s_{it}], \quad \rho_t = Var_{it}(F|s_{it}, \{v^i_{jt}\}_{j\in I}) = Var_{it}(F|s_{it}) \tag{2.22}$$

where I is the set of all traders. What equation 2.22 tells us is that traders on day t update their beliefs about the final payoff only by using their private signals s_{it}, and they do not revise those beliefs after receiving the second set of private signals about other traders' beliefs. This highlights the differences of opinion metaphor that traders disregard others' beliefs when updating their beliefs. Furthermore, as will be given below in detail, traders use their second private signals about other traders' beliefs to update their opinion about the average valuation and consequently find their reference demands. This assumption is specific for this model and implies that although traders disregard other traders' opinions when updating their beliefs about the final payoff, they only use the information content of other traders' beliefs to find their beliefs about the average opinion about their own optimal demands, i.e. their reference demands.

In this setting, trader i's expectation of trader j's belief is given by

$$E_{it}[v_{jt}] = (1-\eta_t)v_{it} + \eta_t v_{jt}^i, \quad \eta_t = \frac{\zeta_t}{\zeta_t + \iota_t + \kappa_t} \tag{2.23}$$

Trader i can simply construct a signal about the average valuation by averaging the signals about all traders' beliefs given by

$$\overline{v}_t^i := \int v_{jt}^i dj = \int v_{jt} + \delta_{it} + \omega_t dj = \overline{v}_t + \delta_{it} + \omega_t \tag{2.24}$$

Therefore, trader i's expectation of the average valuation is

$$E_{it}[\overline{v}_t] = E_{it}[\overline{v}_t|\overline{v}_t^i] = (1-\eta_t)v_{it} + \eta_t \overline{v}_t^i \tag{2.25}$$

and the average valuation's updated variance is $Var_{it}(\overline{v}_t|\overline{v}_t^i) = \zeta_t(1-\eta_t)$. Notice that the above expectation can also be computed by averaging trader i's expectation of all traders' beliefs as the following

$$E_{it}[\overline{v}_t] = E_{it}[\int v_{jt} dj] = \int E_{it}[v_{jt}]dj = \int (1-\eta_t)v_{it} + \eta_t v_{jt}^i dj = (1-\eta_t)v_{it} + \eta_t \overline{v}_t^i$$

Now we can calculate the average opinion about the average valuation. First, note that the average of traders' signals about the average valuation from the social planner's perspective is

$$\int \overline{v}_t^i di = \int \overline{v}_t + \delta_{it} + \omega_t di = \overline{v}_t + \omega_t \tag{2.26}$$

To find the average of average valuations, $\overline{E}_t[\overline{v}_t]$, we compute that

$$\overline{\overline{v}}_t := \overline{E}_t[\overline{v}_t] = \int E_{it}[\overline{v}_t]di = \int (1-\eta_t)v_{it} + \eta_t \overline{v}_t^i di = \overline{v}_t + \eta_t \omega_t \tag{2.27}$$

Notice that we can rewrite the average of all traders' signals about the average valuation as

$$\int \overline{v}_t^i di = \overline{v}_t + \omega_t = (1 - \frac{1}{\eta_t})\overline{v}_t + \frac{1}{\eta_t}\overline{\overline{v}}_t \tag{2.28}$$

Trader i's expectation about the average of average valuation is given by

$$\begin{aligned} E_{it}[\overline{\overline{v}}_t] &= E_{it}[\overline{v}_t + \eta_t \omega_t | \overline{v}_t^i] \\ &= (1 - \eta_t) v_{it} + \eta_t \overline{v}_t^i + \eta_t \frac{\kappa_t}{\zeta_t + \iota_t + \kappa_t}(\overline{v}_t^i - v_{it}) \\ &= (1 - \eta_t - \frac{\kappa_t}{\zeta_t}\eta_t^2) v_{it} + (\eta_t + \frac{\kappa_t}{\zeta_t}\eta_t^2)\overline{v}_t^i \end{aligned} \tag{2.29}$$

where I assume that trader i only uses his signal about the average valuation, $\overline{v}_t^i$, to update his beliefs about $\overline{\overline{v}}_t$. Since $\int v_{it}^j dj = \overline{v}_t + \omega_t$, the average expectation of trader i's belief is given by

$$\overline{E}_t[v_{it}] = \int E_{jt}[v_{it}] dj = \int (1 - \eta_t) v_{jt} + \eta_t v_{it}^j dj = \overline{\overline{v}}_t \tag{2.30}$$

To find the average expectation of trader i's signal about the average valuation, $\overline{E}_t[\overline{v}_t^i]$, we first need to find trader j's expectation about $\overline{v}_t^i$, which is given by

$$\begin{aligned} E_{jt}[\overline{v}_t^i] &= E_{jt}[\overline{v}_t + \delta_{it} + \omega_t | \overline{v}_t^j] = E_{jt}[\overline{v}_t + \omega_t | \overline{v}_t^j] \\ &= (1 - \frac{\zeta_t + \kappa_t}{\zeta_t + \iota_t + \kappa_t}) v_{jt} + \frac{\zeta_t + \kappa_t}{\zeta_t + \iota_t + \kappa_t}\overline{v}_t^j \end{aligned}$$

where I assume trader j only uses his signal about the average valuation, $\overline{v}_t^j$ to update his beliefs about trader i's signal about the average valuation. Therefore, we have

$$\begin{aligned} \overline{E}_t[\overline{v}_t^i] &= \int (1 - \frac{\zeta_t + \kappa_t}{\zeta_t + \iota_t + \kappa_t}) v_{jt} + \frac{\zeta_t + \kappa_t}{\zeta_t + \iota_t + \kappa_t}\overline{v}_t^j dj = \overline{v}_t + \frac{\zeta_t + \kappa_t}{\zeta_t + \iota_t + \kappa_t}\omega_t \\ &= -\frac{\kappa_t}{\zeta_t}\overline{v}_t + (1 + \frac{\kappa_t}{\zeta_t})\overline{\overline{v}}_t \end{aligned} \tag{2.31}$$

To find the optimal demand, I introduce some additional notations. Let the trader i's state vector on day t be

$$Z_{it} = \begin{pmatrix} P_t \\ \overline{v}_t^i \\ v_{it} \end{pmatrix} \tag{2.32}$$

Trader i's expectation about the average expectation of Z_{it} is

$$\begin{aligned} E_{it}[\overline{E}_t[Z_{it}]] =& E_{it}\begin{pmatrix} P_t \\ \overline{E}_t[\overline{v}_t^i] \\ \overline{E}_t[v_{it}] \end{pmatrix} = E_{it}\begin{pmatrix} P_t \\ -\frac{\kappa_t}{\zeta_t}\overline{v}_t+(1+\frac{\kappa_t}{\zeta_t})\overline{\overline{v}}_t \\ \overline{\overline{v}}_t \end{pmatrix} \\ =& \begin{pmatrix} P_t \\ -\frac{\kappa_t}{\zeta_t}((1-\eta_t)v_{it}+\eta_t\overline{v}_t^i)+(1+\frac{\kappa_t}{\zeta_t})((1-\eta_t-\frac{\kappa_t}{\zeta_t}\eta_t^2)v_{it}+(\eta_t+\frac{\kappa_t}{\zeta_t}\eta_t^2)\overline{v}_t^i) \\ (1-\eta_t-\frac{\kappa_t}{\zeta_t}\eta_t^2)v_{it}+(\eta_t+\frac{\kappa_t}{\zeta_t}\eta_t^2)\overline{v}_t^i \end{pmatrix} \\ =& G_t' Z_{it} \end{aligned} \tag{2.33}$$

where

$$G_t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \eta_t+\frac{\kappa_t}{\zeta_t}\eta_t^2+(\frac{\kappa_t}{\zeta_t}\eta_t)^2 & \eta_t+\frac{\kappa_t}{\zeta_t}\eta_t^2 \\ 0 & 1-\eta_t-\frac{\kappa_t}{\zeta_t}\eta_t^2-(\frac{\kappa_t}{\zeta_t}\eta_t)^2 & 1-\eta_t-\frac{\kappa_t}{\zeta_t}\eta_t^2 \end{pmatrix} \tag{2.34}$$

By using the eigen-decomposition method, we can rewrite G_t as

$$G_t = V_t D_t V_t^{-1}, \quad D_t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (\frac{\kappa_t}{\zeta_t}\eta_t)^2 \end{pmatrix}, \quad V_t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\eta_t+\frac{\kappa_t}{\zeta_t}\eta_t^2}{1-\eta_t-\frac{\kappa_t}{\zeta_t}\eta_t^2-(\frac{\kappa_t}{\zeta_t}\eta_t)^2} & -1 \\ 0 & 1 & 1 \end{pmatrix} \tag{2.35}$$

where D_t is the diagonal matrix whose diagonal elements are eigenvalues of G_t, and the columns of V_t are the eigenvectors of G_t. Next, I bring the following lemma which is a generalized version of Lemma 1.

Lemma 2. *Assume that the reference demand for trader i on day t is given by*

$$x_{it}^e = \Delta_t' Z_{it} + d_t E_{it}[\overline{E}_t[x_{it}^e]]$$

where the scalar d_t and the 3×1 vector Δ_t are known to all traders on day t, and $|d_t| < 1$.

In this case, the reference demand is

$$x_{it}^e = \Delta_t'\{I + V_t \Lambda_t V_t^{-1}\} Z_{it} \tag{2.36}$$

where I is the identity matrix, and Λ_t is defined in equation 2.80.

The proof is in the appendix. Now we are at the stage to calculate the optimal demand. To find the optimal position on date $t = T-1$, trader i finds his demand from

$$\begin{aligned}
x_{iT-1} &= arg\max_x E_{iT-1}[-exp\{-(1-r)x(F - P_{T-1}) + \frac{1}{2}r(x - x_{iT-1}^e)^2\}] \\
&= arg\max_x -exp\{-(1-r)x(v_{iT-1} - P_{T-1}) + \frac{1}{2}\rho_{T-1}(1-r)^2x^2 + \frac{1}{2}r(x - x_{iT-1}^e)^2\} \\
&= arg\min_x \{-(1-r)x(v_{iT-1} - P_{T-1}) + \frac{1}{2}\rho_{T-1}(1-r)^2x^2 + \frac{1}{2}r(x - x_{iT-1}^e)^2\}
\end{aligned}$$

where $\frac{1}{2}r(x - x_{iT-1}^e)^2$ is constant since x_{iT-1}^e is known to trader i on day $T-1$ before trading, and hence from the first order condition we have

$$\begin{aligned}
x_{iT-1} &= \frac{1-r}{(1-r)^2\rho_{T-1} + r}(v_{iT-1} - P_{T-1}) + \frac{r}{(1-r)^2\rho_{T-1} + r} x_{iT-1}^e \\
&= \frac{1-r}{r} d_{T-1} h' Z_{iT-1} + d_{T-1} x_{iT-1}^e
\end{aligned} \tag{2.37}$$

where

$$d_{T-1} = \frac{r}{r + (1-r)^2\rho_{T-1}}, \qquad h = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \tag{2.38}$$

To find the reference demand from the above equation, we compute that

$$\begin{aligned}
x_{iT-1}^e =& E_{iT-1}[\overline{E}_{T-1}[x_{iT-1}]] \\
=& \frac{1-r}{r} d_{T-1} h' E_{iT-1}[\overline{E}_{T-1}[Z_{iT-1}]] + d_{T-1} E_{iT-1}[\overline{E}_{T-1}[x_{iT-1}^e]] \\
=& \frac{1-r}{r} d_{T-1} h' G_{T-1}' Z_{iT-1} + d_{T-1} E_{iT-1}[\overline{E}_{T-1}[x_{iT-1}^e]] \\
=& \Delta_{T-1}' Z_{iT-1} + d_{T-1} E_{iT-1}[\overline{E}_{T-1}[x_{iT-1}^e]]
\end{aligned}$$

where

$$\Delta_{T-1} = \frac{1-r}{r} d_{T-1} G_{T-1} h \tag{2.39}$$

hence, from Lemma 2 we have

$$x^e_{iT-1} = \Delta'_{T-1}\{I + V_{T-1}\Lambda_{T-1}V^{-1}_{T-1}\}Z_{iT-1} \tag{2.40}$$

Therefore, the optimal demand is given by

$$\begin{aligned} x_{iT-1} &= \frac{1-r}{r} d_{T-1} h' Z_{iT-1} + d_{T-1}\Delta'_{T-1}(I + V_{T-1}\Lambda_{T-1}V^{-1}_{T-1})Z_{iT-1} \\ &= \Omega'_{T-1} Z_{iT-1} \end{aligned} \tag{2.41}$$

where

$$\Omega_{T-1} = \frac{1-r}{r} d_{T-1} h + d_{T-1}(I + (V'_{T-1})^{-1}\Lambda_{T-1}V'_{T-1})\Delta_{T-1} \tag{2.42}$$

Define the variables $\alpha_{T-1}, \phi_{1T-1}$, and ϕ_{2T-1} such that

$$\Omega_{T-1} = \begin{pmatrix} -\alpha_{T-1} \\ \alpha_{T-1}\phi_{2T-1} \\ \alpha_{T-1}\phi_{1T-1} \end{pmatrix} \tag{2.43}$$

With this notation, the optimal demand is given by

$$x_{iT-1} = \alpha_{T-1}(\phi_{1T-1} v_{iT-1} + \phi_{2T-1}\overline{v}^i_{T-1} - P_{T-1})$$

From the market clearing condition, $\int x_{iT-1} di = 0$, we compute the equilibrium price as

$$P_{T-1} = (\phi_{1T-1} + \phi_{2T-1} - \frac{\phi_{2T-1}}{\eta_{T-1}})\overline{v}_{T-1} + \frac{\phi_{2T-1}}{\eta_{T-1}}\overline{\overline{v}}_{T-1} \tag{2.44}$$

Therefore, the equilibrium price is a linear function of the average valuation and the average of average valuation. For other dates, similar equations hold in equilibrium as stated below.

Proposition 2. *For any date $t < T$, any reference parameter $r \in (0,1)$, and for the matrices Σ_t and A_{t+1} defined in the proof, if $\Sigma_t - 2\Sigma_t A_{t+1}\Sigma_t$ is positive definite and $e_1'\Sigma_t(I - 2A_{t+1}\Sigma_t)^{-1}e_1$ does not belong to the internal $[\frac{-2r}{(1-r)^2}, 0]$, then trader i's demand and reference demand, and the equilibrium price are given by*

$$x_{it} = \alpha_t(\phi_{1t}v_{it} + \phi_{2t}\overline{v}_t^i - P_t), \qquad P_t = (\phi_{1t} + \phi_{2t} - \frac{\phi_{2t}}{\eta_t})\overline{v}_t + \frac{\phi_{2t}}{\eta_t}\overline{\overline{v}}_t \tag{2.45}$$

$$x_{it}^e = \alpha_t^e(\phi_{1t}^e v_{it} + \phi_{2t}^e\overline{v}_t^i - P_t) \tag{2.46}$$

where $\alpha_t, \phi_{1t}, \phi_{2t}, \alpha_t^e, \phi_{1t}^e, \phi_{2t}^e$ are given in equation 2.111, and are invariant across traders. Furthermore, if the vector F_t defined in equation 2.121 is bounded when $r \to 1$, then

$$lim_{r\to 1}(x_{it} - x_{it}^e) = 0 \tag{2.47}$$

The proof is in the appendix. Notice that this proposition simply claims that the equilibrium exists given its stated conditions. According to this proposition, in addition to other components affecting equilibrium price and demand such as disagreement about the mean of future payoff, the behaviour of paying attention to the average opinion about demand is also one component of the equilibrium. Therefore, when traders pay attention to the average opinion about their demand, this behaviour will be incorporated in equilibrium price and demand and they change with r. In this case, each trader will be happier as they have compared their demand driven solely from fundamentals with a references point, an average opinion, and by doing this they have made a regret-minimizing decision. When traders pay full attention to the average opinion about their demands, in limit their optimal demands will be equal to their own reference demands. This model has been further numerically analyzed in section 2.5. As will be shown in that section, there are some differences between the last trading date and other dates which makes a dynamic setting more favourable than a static setting.

As we will see in section 2.4, this type of decision making of adjusting demands with reference demands affects the trading volume and social welfare. In the following section, I remove the assumption of uniformity in traders' signal precisions.

2.3.3 A Model with Heterogenous Signal Precisions

In this section, I present another theoretical model which highlights the outcomes of the standard D.O. with Reference Demand model, model in section 2.3.1, one step closer to reality. In the cited empirical papers in section 2.2, it is stated that positions taken by traders are strongly correlated with prices. This is however cannot be observed in my suggested two models in the previous subsections. The reason is that all traders in those models have the same signal precisions on each date which leads to

$$Cov_{it}(x_{it+1}, P_{t+1}) = 0$$

Cao and Ou-Yang (2009) claim that trading volume can be divided into four components; those from the current and next period disagreements about the mean of the public signals, and those from the disagreements about the precisions of the current and all past public signals. I follow Cao and Ou-Yang (2009)'s model's setting where traders not only disagree about the mean of public signals but also disagree about the precision of the signals. The latter disagreement helps the positions taken by traders to be correlated with prices. I employ their model alongside with my proposed utility function.

Similar to the common setting at the beginning of section 2.3, assume that trading dates are $t = 1, ..., T-1$, and there is a continuum of traders, indexed by the unit interval $[0, 1]$, who trade two assets, one risk-free and one risky. The riskless interest rate is zero and the risky asset is assumed to have *zero* net supply. The traders receive the following public signals

$$y_t = F + \epsilon_t, \tag{2.48}$$

where $F \sim N(v_0, 1/\rho_0)$ is the final payoff, and trader i believes that $\epsilon_t \sim N(e_{it}, 1/h_{it})$. Let the average precision to be $h_t = \int h_{it}di$, and assume that $e_{iT} = 0$. Without loss of generality assume that $\int e_{it}h_{it}di = 0$ (otherwise redefine the public signal as $\tilde{y}_t = y_t - \frac{1}{h_t}\int e_{it}h_{it}di = F + \tilde{\eta}_t$ where $\tilde{\epsilon}_t \sim N(e_{it} - \frac{1}{h_t}\int e_{it}h_{it}di, 1/h_{it})$). The expected utility function is given by equation 2.10. Note that the conditional expectations and conditional precision of F for trader i and the average trader are given by

$$\rho_{it} = (Var_{it}(F))^{-1} = \rho_0 + \sum_{j=0}^{t} h_{ij}, \quad \rho_t = \int \rho_{it}di = \rho_0 + \sum_{j=0}^{t} h_j \tag{2.49}$$

$$v_{it} = \frac{1}{\rho_{it}}(\rho_0 v_0 + \sum_{j=0}^{t} h_{ij}(y_j + e_{ij})), \quad \overline{v}_t = \frac{1}{\rho_t}(\rho_0 v_0 + \sum_{j=0}^{t} h_j y_j) \tag{2.50}$$

Trader i's valuation and the average valuation can also be written recursively as

$$v_{it} = \frac{\rho_{it-1}}{\rho_{it}}v_{it-1} + \frac{h_{it}}{\rho_{it}}(y_t + e_{it}), \quad \overline{v}_t = \frac{\rho_{t-1}}{\rho_t}\overline{v}_{t-1} + \frac{h_t}{\rho_t}y_t \tag{2.51}$$

Trader i's wealth is defined as

$$\tilde{W}_{it} = \tilde{W}_{it-1} + x_{it}(P_t - P_{t-1}) \tag{2.52}$$

Assume that traders start with zero initial wealth, hence on each day t, the aggregate wealth is zero, i.e. $\int \tilde{W}_{it}di = 0$. Once traders try to optimize the expected utility, they get a similar set of equations as in Cao and Ou-Yang (2008); the corresponding equation to their equation (A11), which solves for the optimal demand, is given by

$$\begin{aligned}&(1-r)x_{it} + r(x_{it} - x^e_{it}) + (\rho_{it+1}\rho_{it}\rho_{t+1}/(\rho_{t+1} - \rho_t) - 1)\rho_{it}(v_{it} - P_t)\\ &- \rho^2_{it}\rho_{it+1}\rho_{t+1}/(\rho_{t+1} - \rho_t)(v_{it} - P_t + e_{it+1}) - (\rho_{it}\rho_{it+1}\rho_{t+1}/(\rho_{t+1} - \rho_t) - 1)(\rho_{it+1} - \rho_{it})e_{it+1} = 0\end{aligned}$$

which simplifies to

$$x_{it} = \rho_{it}(v_{it} - P_t) + \frac{\rho_t h_{it+1} e_{it+1}}{h_{t+1}} + r x^e_{it} \tag{2.53}$$

Similar to the standard D.O. model presented in section 2.3.1, for each trader j we have

$$E_{jt}[v_{it}] = v_{it} \tag{2.54}$$

hence,

$$x^e_{it} = E_{it}[\overline{E}_t[[x_{it}]] = \rho_{it}(v_{it} - P_t) + \frac{\rho_t h_{it+1} e_{it+1}}{h_{t+1}} + r x^e_{it} = x_{it} \tag{2.55}$$

From the market clearing condition, $\int x_{it} di = 0$, and the property $\int h_{it+1} e_{it+1} di = 0$, we can also compute the equilibrium price as stated below.

Proposition 3. *For any date $t < T$, and any parameter $r \in (0, 1)$, trader i's demand, reference demand, and the equilibrium price are given by*

$$x_{it} = x^e_{it} = \frac{1}{1-r}\rho_{it}(v_{it} - P_t) + \frac{\rho_t h_{it+1} e_{it+1}}{(1-r)h_{t+1}}, \qquad P_t = \overline{v}_t \tag{2.56}$$

Furthermore, the covariance between equilibrium prices and positions is given by

$$Cov_{it}(x_{it+1}, P_{t+1}) = \frac{h^2_{t+1}(h_{it+1} + \rho_{it+1})}{(1-r)h_{it+1}\rho_{t+1}\rho_{it+1}}(\frac{h_{it+1}}{h_{t+1}} - \frac{\rho_{it+1}}{\rho_{t+1}}) \tag{2.57}$$

The proof can be completed by using backward induction similar to the one presented in Cao and Ou-Yang (2009). Notice that when $\frac{h_{it+1}}{h_{t+1}} > \frac{\rho_{it+1}}{\rho_{t+1}}$, the covariance is positive for trader i. The fraction $\frac{h_{it+1}}{h_{t+1}}$ can be regarded as a measure of confidence for trader i about the public signal, and $\frac{\rho_{it+1}}{\rho_{t+1}}$ is the relative conditional precision of F to the average trader. Similarly, in a static model with two traders, Fishe et al (2014) show that the covariance between positions and prices are positive for the trader with the largest signal variance. They also empirically show that in commodity futures markets, money managers (hedgers)' positions are positively (negatively) correlated with prices, suggesting that they receive large (small) signals and thus their associated price change should be larger (smaller). Note that this does not suggest that which category is better at forecasting future returns. The interesting theoretical contribution that I make here is that the magnitude of correlation between price and position increases with r. This suggests that for those traders with positive price-position correlations, if they pay more attention to the average opinion about their own demand, their positions will have stronger correlations with prices. In the next section I highlight the outcomes of this model on social welfare and volume.

2.4 Social Welfare and Volume

I this section I analyze the effect of introducing the reference demand on social welfare and trading volume using the three models presented in the previous section. For the classical D.O. with Reference Demand models, models in sections 2.3.1 and 2.3.3, I find that when traders pay more attention to the average opinion about their demands, i.e. r increases, the social welfare function also increases. However, this is not necessarily true for the generalized model presented in section 2.3.2. All models seem to uniformly agree on volume and with an increase in r, volume also increases.

2.4.1 Social Welfare

Define the social welfare function on day t to be the following normalized average traders' expected utilities.

$$Welfare_t = \frac{-1}{1-r} \int Log(-EU_{it}) di \tag{2.58}$$

where traders' expected utilities are maximized and the equilibrium prices are given. For the standard D.O. model with Reference Demand, section 2.3.1, the social welfare function is

$$\begin{aligned}
Welfare_t &= \frac{-1}{1-r}\int Log(-EU_{it})di \\
&= \frac{-1}{1-r}\int Log(\gamma_t exp\{Z'_{it}A_tZ_{it} + \frac{1}{2}r(x_{it}-x^e_{it})^2\})di \\
&= \frac{-1}{1-r}\{log(\gamma_t) + \int Z'_{it}A_tZ_{it} + \frac{1}{2}r(x_{it}-x^e_{it})^2 di\} \\
&= \frac{-1}{1-r}\{log(\gamma_t) + \int Z'_{it}A_tZ_{it}di\} \\
&= \frac{-1}{1-r}log(\gamma_t) + \frac{-1}{1-r}\int Z'_{it}\theta_t\left(\begin{smallmatrix}1 & -1\\ -1 & 1\end{smallmatrix}\right)Z_{it}di \\
&= \frac{-1}{1-r}log(\gamma_t) + \frac{-\theta_t}{1-r}\int (v_{it}-P_t)^2 di \\
&= \frac{-1}{1-r}log(\gamma_t) + \frac{-\theta_t}{1-r}\int Z'_{it}hh'Z_{it}di
\end{aligned} \tag{2.59}$$

Since $\gamma_t \leq 1$ and $\theta_t < 0$ do not depend on r, we conclude that when r increases, social welfare also increases.

Similarly, for the model with heterogenous signal precisions, section 2.3.3, first notice that the expected utility is given by

$$EU_{it} = -exp\{-(1-r)\tilde{W}_{it} - \frac{1}{2}\rho_{it}(v_{it}-P_t)^2\} \tag{2.60}$$

Therefore, the social welfare function is given as

$$\begin{aligned}
Welfare_t &= \frac{-1}{1-r}\int Log(-EU_{it})di \\
&= \frac{-1}{1-r}\int -(1-r)\tilde{W}_{it} - \frac{1}{2}\rho_{it}(v_{it}-P_t)^2 di \\
&= \int \tilde{W}_{it}di + \frac{1}{2(1-r)}\int \rho_{it}(v_{it}-P_t)^2 di \\
&= \frac{1}{2(1-r)}\int \rho_{it}(v_{it}-P_t)^2 di
\end{aligned} \tag{2.61}$$

We conclude that when r increases, the social welfare function also increases.

We now turn to the social welfare function for the generalized model in section 2.3.2. First, I calculate this function for the last trading date, $t = T-1$, which is the corresponding static model when $T = 2$. We have

$$\begin{aligned} Welfare_{T-1} &= \frac{-1}{1-r}\int Log(-EU_{iT-1})di \\ &= \frac{-1}{1-r}\int log(\gamma_{T-1}) + Z_{iT-1}^{'}A_{T-1}Z_{iT-1} + \frac{1}{2}r(x_{iT-1} - x_{iT-1}^{e})^2 di \end{aligned} \tag{2.62}$$

where

$$A_{T-1} = -(1-r)\frac{\Omega_{T-1}h^{'} + h\Omega'_{T-1}}{2} + \frac{1}{2}(1-r)^2\rho_{T-1}\Omega_{T-1}\Omega_{T-1}^{'}$$

and $\gamma_{T-1} = 1$, from the proof of Proposition 2. Since we have

$$\begin{aligned} x_{iT-1} - x_{iT-1}^{e} &= \frac{1-r}{r}d_{T-1}h'Z_{iT-1} + (d_{T-1}-1)\Delta_{T-1}^{\prime}(I + V_{T-1}\Lambda_{T-1}V_{T-1}^{-1})Z_{iT-1} \\ &= \frac{1-r}{r+(1-r)^2\rho_{T-1}}F_{T-1}^{\prime}Z_{iT-1} \end{aligned} \tag{2.63}$$

where

$$F_{T-1} = h - (1-r)\rho_{T-1}(I + (V_{T-1}^{-1})'\Lambda_{T-1}V_{T-1}^{\prime})\Delta_{T-1} \tag{2.64}$$

hence

$$\frac{1}{2}r(x_{iT-1} - x_{iT-1}^{e})^2 = \frac{r(1-r)^2}{2(r+(1-r)^2\rho_{T-1})^2}Z_{iT-1}^{\prime}F_{T-1}F_{T-1}^{\prime}Z_{iT-1} \tag{2.65}$$

Therefore, we can rewrite the social welfare function as

$$\begin{aligned} Welfare_{T-1} &= \frac{-1}{1-r}\int Z_{iT-1}^{'}A_{T-1}Z_{iT-1} + \frac{r(1-r)^2}{2(r+(1-r)^2\rho_{T-1})^2}Z_{iT-1}^{\prime}F_{T-1}F_{T-1}^{\prime}Z_{iT-1}di \\ &= \frac{-1}{1-r}\int Z_{iT-1}^{'}\Psi_{T-1}Z_{iT-1}di \end{aligned} \tag{2.66}$$

where

$$\Psi_{T-1} = A_{T-1} + \frac{r(1-r)^2}{2(r+(1-r)^2\rho_{T-1})^2} F_{T-1}F'_{T-1} \tag{2.67}$$

In general, when $T > 2$, we can find the social welfare function as follows. According to the proof of proposition 2, we can write

$$\frac{1}{2}r(x_{it} - x_{it}^e)^2 = \frac{1}{2}r(\frac{1-r}{r+(1-r)^2 a_t})^2 F'_t Z_{it} F'_t Z_{it} = \frac{r(1-r)^2}{2(r+(1-r)^2 a_t)^2} Z'_{it} F_t F'_t Z_{it} \tag{2.68}$$

Hence, the social welfare is

$$\begin{aligned} Welfare_t &= \frac{-1}{1-r}\int Log(-EU_{it})di \\ &= \frac{-1}{1-r}\int Log(\gamma_t exp\{Z'_{it}A_t Z_{it} + \frac{1}{2}r(x_{it}-x_{it}^e)^2\})di \\ &= \frac{-1}{1-r}\{log(\gamma_t) + \int Z'_{it}A_t Z_{it} + \frac{1}{2}r(x_{it}-x_{it}^e)^2 di\} \\ &= \frac{-1}{1-r}log(\gamma_t) - \frac{1}{1-r}\int Z'_{it}\Psi_t Z_{it} di \end{aligned} \tag{2.69}$$

where

$$\Psi_t = A_t + \frac{r(1-r)^2}{2(r+(1-r)^2 a_t)^2} F_t F'_t \tag{2.70}$$

It is difficult to analyze the eigenvalues of Ψ_t for $1 \leq t \leq T-1$, therefore, I examine these eigenvalues numerically. With the initial values of the model given in section 2.5, we see that γ_t could be different than 1 except for $t = T-1$, and Ψ_t is not necessarily a definite matrix (neither positive not negative definite). This suggests that we cannot be conclusive about the effect of r on social welfare for this generalized model.

2.4.2 Trading Volume

With higher levels of demand, we expect the volume to be at the higher levels when r increases. This resembles an environment where traders trade more often if they pay more attention to other traders' opinions. In this case, demand, volume and open interest all depend on the parameter r. Notice that trader i's trading size on day t is $x_{it} - x_{it-1}$, therefore the trading volume is

$$Volume_t = \frac{1}{2} \int |x_{it} - x_{it-1}| di \tag{2.71}$$

Once traders reach the equilibrium on day t, I ask whether the trading volume could have been at a higher level had they paid more attention to the average opinion about their demands, i.e. r is higher. For the standard D.O. with Reference Demand model in section 2.3.1, we compute that

$$\begin{aligned} Volume_t &= \frac{1}{2} \int |x_{it} - x_{it-1}| di \\ &= \frac{1}{2} \int |\alpha_t(v_{it} - P_t) - \alpha_{t-1}(v_{it-1} - P_{t-1})| di \\ &= \frac{1}{2(1-r)} \int |\alpha'_t(v_{it} - P_t) - \alpha'_{t-1}(v_{it-1} - P_{t-1})| di \end{aligned} \tag{2.72}$$

where $\alpha'_t = (1-r)\alpha_t$ does not depend on r. We conclude that an increase in r leads to an increase in trading volume. For the model with the heterogenous signal precisions, section 2.3.3, we calculate that

$$\begin{aligned} Volume_t &= \frac{1}{2} \int |x_{it} - x_{it-1}| di \\ &= \frac{1}{2} \int |\frac{1}{1-r}\rho_{it}(v_{it} - P_t) + \frac{\rho_t h_{it+1} e_{it+1}}{(1-r)h_{t+1}} - \frac{1}{1-r}\rho_{it-1}(v_{it-1} - P_{t-1}) - \frac{\rho_{t-1} h_{it} e_{it}}{(1-r)h_t}| di \\ &= \frac{1}{2(1-r)} \int |\rho_{it}(v_{it} - P_t) + \frac{\rho_t h_{it+1} e_{it+1}}{h_{t+1}} - \rho_{it-1}(v_{it-1} - P_{t-1}) - \frac{\rho_{t-1} h_{it} e_{it}}{h_t}| di \end{aligned} \tag{2.73}$$

therefore, an increase in r also leads to an increase in the trading volume. For the generalized model of section 2.3.2, I investigate the effect of r on volume by analyzing the expected volume in the next section.

2.4.3 Expected Volume

To investigate the role of r across trading dates, I calculate the expected volume, i.e. $E[Volume_t]$ before trading starts on date $t = 1$. We have

$$E[Volume_t] = E[\frac{1}{2}\int |x_{it} - x_{it-1}|di] = \sqrt{\frac{1}{2\pi}Var(x_{it} - x_{it-1})} \tag{2.74}$$

For the standard D.O. with Reference Demand model in section 2.3.1, first note that

$$\begin{aligned} Var(v_{it} - P_t) =& Var((1-\pi_{t-1})(v_{it-1} - \overline{v}_{t-1}) + \pi_t(s_{it} - e_{it} - F - \epsilon_t)) \\ =& (1-\pi_{t-1})^2 Var(v_{it-1} - P_{t-1}) + \pi_t^2 q_{t+1} \end{aligned}$$

Thus, $Var(v_{it} - P_t), t \geq 1$, can be recursively found, starting with $Var(v_{i1} - P_1) = \pi_1^2 q_2$, and they do not dependent on r. We compute that

$$\begin{aligned} Var(x_{it} - x_{it-1}) =& Var(\alpha_t(v_{it} - P_t) - \alpha_{t-1}(v_{it-1} - P_{t-1})) \\ =& Var((1-\pi_{t-1})(v_{it-1} - \overline{v}_{t-1}) + \pi_t(s_{it} - e_{it} - F - \epsilon_t) - \alpha_{t-1}(v_{it-1} - P_{t-1})) \\ =& Var(((1-\pi_{t-1})\alpha_t - \alpha_{t-1})(v_{it-1} - P_{t-1}) + \pi_t(\epsilon_{it} - e_{it})) \\ =& ((1-\pi_{t-1})\alpha_t - \alpha_{t-1})^2 Var(v_{it-1} - P_{t-1}) + \pi_t^2 q_t \end{aligned}$$

Therefore the expected volume is

$$\begin{aligned} E[Volume_t] &= \sqrt{\frac{((1-\pi_{t-1})\alpha_t - \alpha_{t-1})^2 Var(v_{it-1} - P_{t-1}) + \pi_t^2 q_t}{2\pi}} \\ &= \sqrt{\frac{((1-\pi_{t-1})\alpha'_t - \alpha'_{t-1})^2 Var(v_{it-1} - P_{t-1})}{2\pi(1-r)^2} + \frac{\pi_t^2 q_t}{2\pi}} \end{aligned} \tag{2.75}$$

where, as previously stated, $\alpha'_t = (1-r)\alpha_t$ does not depend on r. Therefore, for the higher levels of r, the expected volume increases.

We now turn to the generalized model in section 2.3.2. I use the following proposition.

Proposition 4. *For any date t, the expected volume for the generalized model in section 2.3.2 is given by*

$$E[Volume_t] = \sqrt{\frac{1}{2\pi} M_{t-1}^{\widehat{Y}} \Sigma_{t-1}^{\widehat{Y}} (M_{t-1}^{\widehat{Y}})'} \tag{2.76}$$

where $M_{t-1}^{\widehat{Y}}$ and $\Sigma_{t-1}^{\widehat{Y}}$ are defined in the proof.

The proof is in the appendix. Since it is difficult to analyze the expected volume for this model, I refer to the numerical example in section 2.5. As shown in that section, the expected volume increases for all dates when r increases. The monotonically increasing expected volume could be interpreted as follows. Once traders pay more attention to the average opinion about their own demand, an extra level of volume will be added to the market. This phenomenon could explain the rise in volume in sunshine trading, where some traders pre-announce their trading size before they submit their orders (Admati and Pfleiderer, 1991), or in predatory trading, where predators sell alongside a distressed trader who is in need to reduce their position (Brunnermeier and Pedersen, 2005).

2.5 A Numerical Example

In this section, I present a numerical example for the generalized model in section 2.3.2. when the model is in equilibrium according to Proposition 2 assumptions. Let $T = 6$ and $t = 1, 2, 3, 4, 5$ be the trading dates. I choose the following parameter values:

$$\rho_0 = 1, \quad q_t = \lambda_t = 2 \quad \zeta_t = \iota_t = \kappa_t = 1 \tag{2.77}$$

I let the reference parameter change from 1 basis point to just below 99.9%, in the following set of size N

$$r \in \{r_0 + k\Delta r\}_{k=0}^{N-1}, \quad r_0 = 0.0001, \quad \Delta r = 0.03, \quad r_0 + N\Delta r \leq 0.999 \tag{2.78}$$

where $N = 34$. This selection serves as an example and it indicates that for all values of r in the interval $(0, 1)$, the equilibrium exists. For any realization of the private signals and beliefs, the optimal demands and equilibrium prices are given by Proposition 2. The selection of parameters' initial values should be aligned with eventually meeting the conditions stated in Proposition 2.

In equilibrium, the coefficients of price, P_t, i.e. coefficients of $\overline{v}_t$ and $\overline{\overline{v}}_t$, and their summation are shown in Figure 2.1. Interestingly, when r approaches 1, the coefficient of $\overline{v}_t$ uniformly decreases to a negative value, but the coefficient of $\overline{\overline{v}}_t$ uniformly increases to a positive value, for all trading dates. The sum of these two coefficients as was shown in section 2.3.2 is equal to 1 for the last trading day, and for other dates, it is very close to 1. Notice that if the equilibirum price is given by $P_t = -\phi_t\overline{v}_t + (1 + \phi_t)\overline{\overline{v}}_t$, for a positive value of ϕ_t, then P_t is positive because $P_t = \overline{v}_t + (1 + \phi_t)\eta_t\omega_t$.

The values of γ_t against r are plotted in Figure 2.2. Due to the conditions in Proposition 2, these values should be positive for all trading dates. More importantly, for smaller values of r, we see that γ_t is less than 1, and for bigger values of r, γ_t is greater than one. This indicates that their logs, $Log(\gamma_t)$, are negative and positive for smaller and bigger values of r, respectively. Figure 2.2 also shows that analyzing a dynamic model, rather than a static model with only one trading session of $t = T - 1$, is more beneficial. Figure 2.3 shows the norm of the vector of coefficients of $x_{it} - x_{it}^e$, and as expected, it goes to zero when r approaches to 1. This indicates that at the limit, traders take positions exactly equal to their reference demands. Similar to Figure 2.2, this figure also shows that the last trading session behaves differently and the speed of convergence of $x_{it} - x_{it}^e$ when r approaches to 1 is faster for other dates than the last trading date. This makes a dynamic setup of the model more favourable.

$$\{Figures\ 2.1,\ 2.2,\ and\ 2.3\ about\ here.\}$$

In Figure 2.4, the eigenvalues of Ψ_t are plotted. The main point from this figure is that for most values of r and for each trading date, there is at least one positive eigenvalue and at least one negative eigenvalue. For the third eigenvalue, we see that for all values of r this eigenvalue is nonnegative for the last trading date in contrast to other dates. This will indicate that the social welfare function is not necessarily increasing when r goes to 1. Figure 2.5 shows the expected volume for each day, and as we see, it is an increasing function of r, fairly moderate for smaller value of r, however, when r approaches to 1, it increases at a larger rate.

$$\{Figures\ 2.4\ and\ 2.5\ about\ here.\}$$

2.6 Conclusion

In this chapter, I theoretically highlight the role of transaction quantities in decision making. I study an economy where all traders have the same utility maximization model, learn from others' view about their demands, and do not deviate substantially from the average opinion of their demand. Traders form expectations of the average opinion about their demands to determine their reference demands. Each trader finds his optimal demand based on the information about the final payoff with the adjustment that it is not too far from his reference demand. I find that in classical differences of opinion models, traders take positions exactly equal to their reference demand, and the more they pay attention to the average opinion about their demand, the trading volume and social welfare increase.

I propose a new model and find that traders do not necessarily take positions exactly equal to their reference demands and the social welfare does not necessarily increase when they pay more attention to the average opinion about their demands. Consistent with the classical differences of opinion model, numerical instances show that volume increases in my proposed model, when traders pay more attention to the average opinion about their demand.

This chapter contributes to the empirical literature that finds that the transaction quantities are informative (Hong and Yogo (2012), Evans and Lyons (2007), Llorente and Wang (2015). It also highlights that prices contain some information coming from positions. For future research, by employing the models presented in this chapter, one might investigate the excess volatility puzzle or price drift which are the features of dynamic models.

2.7 Appendix: Proofs

Proof of Lemma 1 and Lemma 2. We only need to prove Lemma 2 as Lemma 1 is the special case with $G_t = I$ and removing the corresponding dimension for $\overline{v}_t^i$. Define the operator $\mathcal{L}_{it}^1 = E_{it}[\overline{E}_t[.]]$. First, notice that $\mathcal{L}_{it}^1 Z_{it} = G_t' Z_{it}$, and $G_t^n = V_t D_t^n V_t^{-1}$. Now, we compute that

$$\mathcal{L}_{it}^1 x_{it}^e = \Delta_t' \mathcal{L}_{it}^1 Z_{it} + d_t \mathcal{L}_{it}^2 x_{it}^e = \Delta_t' G_t' Z_{it} + d_t \mathcal{L}_{it}^2 x_{it}^e$$

Hence, we can see that if $\mathcal{L}_{it}^n x_{it}^e = \Delta_t'(G_t')^n Z_{it} + d_t \mathcal{L}_{it}^{n+1} x_{it}^e$ for some natural number n, then

$$\begin{aligned}
\mathcal{L}_{it}^{n+1} x_{it}^e &= \Delta_t'(G_t')^n \mathcal{L}_{it}^1 Z_{it} + d_t \mathcal{L}_{it}^{n+2} x_{it}^e \\
&= \Delta_t'(G_t')^{n+1} Z_{it} + d_t \mathcal{L}_{it}^{n+2} x_{it}^e
\end{aligned}$$

Since $\{|\mathcal{L}_{it}^n x_{it}^e|\}_{n=1}^{\infty}$ is bounded by the sum of trading volumes before day $t+1$, we have $\lim_{n\to\infty} d_t^n \mathcal{L}_{it}^n x_{it}^e = 0$, and therefore

$$\begin{aligned}
x_{it}^e &= \Delta_t' Z_{it} + d_t \mathcal{L}^1 x_{it}^e \\
&= \Delta_t' Z_{it} + d_t \Delta_t' G_t' Z_{it} + d_t^2 \mathcal{L}^2 x_{it}^e \\
&= ... = \Delta_t' Z_{it} + \Delta_t' \sum_{n=1}^{\infty} d_t^n (G_t')^n Z_{it} \\
&= \Delta_t' \{I + V_t \Lambda_t V_t^{-1}\} Z_{it} \qquad (2.79)
\end{aligned}$$

where

$$\Lambda_t = \begin{pmatrix} \frac{d_t}{1-d_t} & 0 & 0 \\ 0 & \frac{d_t}{1-d_t} & 0 \\ 0 & 0 & \frac{d_t(\frac{\kappa_t}{\zeta_t}\eta_t)^2}{1-d_t(\frac{\kappa_t}{\zeta_t}\eta_t)^2} \end{pmatrix} \qquad (2.80)$$

Proof of Proposition 1 and Proposition 2. I will prove proposition 2 by backward induction. Proposition 1 is a special case and at the end I highlight the differences. Let $A_T = 0$, and assume that the expected fundamental part of the utility at time $t+1$, where $1 \leq t+1 \leq T-1$ is exponential quadratic, i.e.

$$E_{it+1}[-(e^{-W_{it+1}})^{1-r}] = -E_{it+1}[exp\{-(1-r)W_{it+1}\} = -\gamma_{t+1} exp\{Z'_{it+1} A_{t+1} Z_{it+1}\} \quad (2.81)$$

where $\gamma_{t+1} = \Pi_{s=t+1}^{T-1} \frac{1}{\sqrt{det(I-2A_{s+1}\Sigma_s)}}$ is positive, and A_{t+1} is symmetric. Note that $\gamma_{T-1} = 1$, and since

$$Z'_{iT-1}\Omega_{T-1}h'Z_{iT-1} = \Omega'_{T-1}Z_{iT-1}h'Z_{iT-1} = \Omega'_{T-1}Z_{iT-1}Z'_{iT-1}h = Z'_{iT-1}h\Omega'_{T-1}Z_{iT-1}$$

we compute that

$$\begin{aligned}
&E_{iT-1}[-(e^{-W_{iT-1}})^{1-r}] \\
&\quad = E_{iT-1}[-exp\{-(1-r)W_{iT-1}\}] = E_{iT-1}[-exp\{-(1-r)x_{iT-1}(F-P_{T-1})\}] \\
&\quad = -exp\{-(1-r)x_{iT-1}(v_{iT-1}-P_{T-1}) + \frac{1}{2}(1-r)^2 x_{iT-1}^2 \rho_{T-1}\} \\
&\quad = -exp\{-(1-r)\Omega'_{T-1}Z_{iT-1}h'Z_{iT-1} + \frac{1}{2}(1-r)^2\rho_{T-1}\Omega'_{T-1}Z_{iT-1}\Omega'_{T-1}Z_{iT-1}\} \\
&\quad = -exp\{-(1-r)Z'_{iT-1}\frac{\Omega_{T-1}h' + h\Omega'_{T-1}}{2}Z_{iT-1} + \frac{1}{2}(1-r)^2\rho_{T-1}Z'_{iT-1}\Omega_{T-1}\Omega'_{T-1}Z_{iT-1}\} \\
&\quad = -exp\{Z'_{iT-1}A_{T-1}Z_{iT-1}\} \qquad (2.82)
\end{aligned}$$

where

$$A_{T-1} = -(1-r)\frac{\Omega_{T-1}h' + h\Omega'_{T-1}}{2} + \frac{1}{2}(1-r)^2\rho_{T-1}\Omega_{T-1}\Omega'_{T-1} \quad (2.83)$$

Next, we need to find the moments of

$$Z_{it+1} = \begin{pmatrix} P_{t+1} \\ \bar{v}^i_{t+1} \\ v_{it+1} \end{pmatrix} \sim N(\mu_{it}, \Sigma_t) \quad (2.84)$$

Assume that

$$P_{t+1} = (\phi_{1t+1} + \phi_{2t+1} - \frac{\phi_{2t+1}}{\eta_{t+1}})\overline{v}_{t+1} + \frac{\phi_{2t+1}}{\eta_{t+1}}\overline{\overline{v}}_{t+1} \tag{2.85}$$

therefore,

$$\begin{aligned} P_{t+1} &= (\phi_{1t+1} + \phi_{2t+1} - \frac{\phi_{2t+1}}{\eta_{t+1}})\overline{v}_{t+1} + \frac{\phi_{2t+1}}{\eta_{t+1}}(\overline{v}_{t+1} + \eta_{t+1}\omega_{t+1}) \\ &= (\phi_{1t+1} + \phi_{2t+1})((1-\pi_t)\overline{v}_t + \pi_t(F + \epsilon_{t+1})) + \phi_{2t+1}\omega_{t+1} \end{aligned} \tag{2.86}$$

we also have

$$v_{it+1} = (1-\pi_t)v_{it} + \pi_t(F + \epsilon_{t+1} + \epsilon_{it+1} - e_{it+1}) \tag{2.87}$$

$$\begin{aligned} \overline{v}^i_{t+1} &= \overline{v}_{t+1} + \delta_{it+1} + \omega_{t+1} \\ &= (1-\pi_t)\overline{v}_t + \pi_t(F + \epsilon_{t+1}) + \delta_{it+1} + \omega_{t+1} \end{aligned} \tag{2.88}$$

therefore if we denote

$$Y_{it+1} = (F, \overline{v}_t, \epsilon_{t+1}, \epsilon_{it+1}, \delta_{it+1}, \omega_{t+1})' \tag{2.89}$$

and

$$M_t = \begin{pmatrix} (\phi_{1t+1}+\phi_{2t+1})\pi_t & (\phi_{1t+1}+\phi_{2t+1})(1-\pi_t) & (\phi_{1t+1}+\phi_{2t+1})\pi_t & 0 & 0 & \phi_{2t+1} \\ \pi_t & 1-\pi_t & \pi_t & 0 & 1 & 1 \\ \pi_t & 0 & \pi_t & \pi_t & 0 & 0 \end{pmatrix}, \qquad b_{it} = (0, 0, (1-\pi_t)v_{it} - \pi_t e_{it+1})' \tag{2.90}$$

where M_t is known to all traders and b_{it} is a constant for trader i, then we have

$$Z_{it+1} = M_t Y_{it+1} + b_{it} \tag{2.91}$$

Since the moments of Y_{it+1} are

$$E_{it}[Y_{it+1}] = \mu_{it}^Y = \begin{pmatrix} v_{it} \\ (1-\eta_t)v_{it}+\eta_t\overline{v}_t^i \\ 0 \\ e_{it+1} \\ 0 \\ 0 \end{pmatrix}, \quad Var_{it}(Y_{it+1}) = \Sigma_t^Y = \begin{pmatrix} \rho_t & \xi_t & 0 & 0 & 0 & 0 \\ \xi_t & \zeta_t(1-\eta_t) & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_{t+1} & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{t+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \iota_{t+1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa_{t+1} \end{pmatrix} \tag{2.92}$$

hence, the moments of Z_{it+1} are

$$E_{it}[Z_{it+1}] = \mu_{it} = M_t\mu_{it}^Y + b_{it}, \quad Var_{it}(Z_{it+1}) = \Sigma_t = M_t\Sigma_t^Y M_t' \tag{2.93}$$

Next, I find $\xi_t = Cov_{it}(F, \overline{v}_t)$. Notice that

$$\begin{aligned} \xi_{t+1} &= Cov_{it+1}(F, \overline{v}_{t+1}) = Cov_{it}(F, (1-\pi_t)\overline{v}_t + \pi_t(F+\epsilon_{t+1})|s_{it+1}, \overline{v}_{t+1}^i) \\ &= (1-\pi_t)Cov_{it}(F, \overline{v}_t|s_{it+1}, \overline{v}_{t+1}^i) + \pi_t Cov_{it}(F, F|s_{it+1}) \\ &= (1-\pi_t)\xi_t + \pi_t\rho_{t+1} \end{aligned} \tag{2.94}$$

where $\xi_0 = Cov_{i0}(F, v_0) = 0$, and I assume that traders drop future information when calculating the covariance between the final payoff and the current average valaution, and therefore they put $Cov_{it}(F, \overline{v}_t|s_{it+1}, \overline{v}_{t+1}^i) = Cov_{it}(F, \overline{v}_t)$. Notice that $Cov_{it}(F, F|s_{it+1}, \overline{v}_{t+1}^i) = Var_t(F|s_{it+1}) = \rho_{t+1}$ because traders have differences of opinion and when calculating their beliefs, they disregard other traderes' belief about the final payoff. For simplicity in our calculations, I introduce a new notation and rewrite μ_{it} as

$$\mu_{it} = Q_t'Z_{it}, \quad Q_t = \begin{pmatrix} 0 & 0 & 0 \\ (\phi_{1t+1}+\phi_{2t+1})(1-\pi_t)\eta_t & (1-\pi_t)\eta_t & 0 \\ (\phi_{1t+1}+\phi_{2t+1})((1-\pi_t)(1-\eta_t)+\pi_t) & (1-\pi_t)(1-\eta_t)+\pi_t & 1 \end{pmatrix} \tag{2.95}$$

Next, note that

$$E_{it}[\overline{E}_t[\mu_{it}]] = E_{it}[\overline{E}_t[Q_t'Z_{it}]] = Q_t'G_t'Z_{it} \tag{2.96}$$

Finally, denote the unit base vectors as

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.97}$$

To find the optimal demand, I use the following Lemma.

Lemma 3. *Assume that A is a real symmetric matrix and $Z \sim N(\mu, \Sigma)$, then $Q = c + B'Z + Z'AZ$ has the following moment generating function*

$$M_Q(t) = \frac{1}{\sqrt{det(I - 2tA\Sigma)}} exp\{tc - \frac{1}{2}\mu'\Sigma^{-1}\mu + \frac{1}{2}(\mu + t\Sigma B)'(I - 2tA\Sigma)^{-1}\Sigma^{-1}(\mu + t\Sigma B)\}$$

This Lemma is proved by Mathai and Provost (1992)[1]. Since based on our assumption $\Sigma_t - 2\Sigma_t A_{t+1}\Sigma_t$ is positive definite, hence $I - 2A_{t+1}\Sigma_t = \Sigma_t^{-1}(\Sigma_t - 2\Sigma_t A_{t+1}\Sigma_t)$ is invertible and its determinant is positive. Therefore, according to Lemma 3, the optimal demand on date t is

$$\begin{aligned}
x_{it} &= arg \max_x E_{it}[-exp\{-(1-r)W_{it} + \frac{1}{2}r(x - x^e_{it})^2\}] \\
&= arg \max_x E_{it}[-exp\{-(1-r)x(P_{t+1} - P_t) + \frac{1}{2}r(x - x^e_{it})^2\}E_{it+1}[exp\{-(1-r)W_{it+1}\}]] \\
&= arg \max_x E_{it}[-\gamma_{t+1} exp\{C_t + B_t' Z_{it+1} + Z'_{it+1} A_{t+1} Z_{it+1}\}] \\
&= arg \max_x \frac{-\gamma_{t+1}}{\sqrt{det(I - 2A_{t+1}\Sigma_t)}} \times \\
&\qquad exp\{C_t - \frac{1}{2}\mu_{it}'\Sigma_t^{-1}\mu_{it} + \frac{1}{2}(\mu_{it} + \Sigma_t B_t)'(I - 2A_{t+1}\Sigma_t)^{-1}\Sigma_t^{-1}(\mu_{it} + \Sigma_t B_t)\} \\
&= arg \min_x \{C_t - \frac{1}{2}\mu_{it}'\Sigma_t^{-1}\mu_{it} + \frac{1}{2}(\mu_{it} + \Sigma_t B_t)'(I - 2A_{t+1}\Sigma_t)^{-1}\Sigma_t^{-1}(\mu_{it} + \Sigma_t B_t)\}
\end{aligned} \tag{2.98}$$

where

$$C_t = x(1-r)P_t + \frac{1}{2}r(x - x^e_{it})^2, \quad B_t = -x(1-r)e_1 \tag{2.99}$$

According to the following lemma, the minimum exists and it is unique.

[1]Mathai A. M. and Provost S. B. (1992). Quadratic Forms in Random Variables, Theory and Applications, p. 40, Theorem 3.2a.1

Lemma 4. *Let*

$$g(x) = C_t - \frac{1}{2}\mu_{it}'\Sigma_t^{-1}\mu_{it} + \frac{1}{2}(\mu_{it} + \Sigma_t B_t)'(I - 2A_{t+1}\Sigma_t)^{-1}\Sigma_t^{-1}(\mu_{it} + \Sigma_t B_t)$$

then $g(x)$ has a unique minimum.

Proof of Lemma 4. Write $g(x)$ as $g(x) = D + \frac{1}{2}K'LK$, where $L = (I - 2A_{t+1}\Sigma_t)^{-1}\Sigma_t^{-1}$, and

$$D = x(1-r)P_t + \frac{1}{2}r(x - x_{it}^e)^2 - \frac{1}{2}\mu_{it}'\Sigma_t^{-1}\mu_{it}, \quad K = \mu_{it} + \Sigma_t B_{t+1}$$

To have a unique minimum, the second derivative should be positive, $\frac{\partial^2 g}{\partial x^2} > 0$ for all x. We calculate that

$$\frac{\partial^2 g}{\partial x^2} = \frac{\partial^2 D}{\partial x^2} + \frac{\partial K'}{\partial x} L \frac{\partial K}{\partial x} + \frac{1}{2}\{\frac{\partial^2 K'}{\partial x^2} LK + K'L\frac{\partial^2 K}{\partial x^2}\}$$

Since $-\frac{1}{2}\mu_{it}'\Sigma_t^{-1}\mu_{it}$ does not depend on x, and x_{it}^e is known to trader i on day t, the second derivative of D is $\frac{\partial^2 D}{\partial x^2} = r > 0$. We can also rewrite $K = \mu_{it} - (1-r)x\Sigma_t e_1$; hence, $\frac{\partial K}{\partial x} = -(1-r)\Sigma_t e_1$, and $\frac{\partial^2 K}{\partial x^2} = 0$. We therefore compute the second derivative of g as

$$\frac{\partial^2 g}{\partial x^2} = r + (1-r)^2 e_1'\Sigma_t' L \Sigma_t e_1$$

which is positive because L is positive definite. **Q.E.D.**

For the unique minimum, x_{it} solves the first order condition given by

$$\frac{\partial C_t}{\partial x} - \mu_{it}'\Sigma_t^{-1}\frac{\partial \mu_{it}}{\partial x} + (\mu_{it} + \Sigma_t B_{t+1})'(I - 2A_{t+1}\Sigma_t)^{-1}\frac{\partial B_{t+1}}{\partial x} = 0 \qquad (2.100)$$

To solve this equation, notice that

$$\frac{\partial C_t}{\partial x} = (1-r)P_t + rx - rx^e_{it}, \qquad \frac{\partial \mu_{it}}{\partial x} = 0, \qquad \frac{\partial B_{t+1}}{\partial x} = (r-1)e_1 \tag{2.101}$$

Hence, we can rewrite equation 2.100 as

$$(1-r)P_t + rx - rx^e_{it} + (\mu'_{it} + x(r-1)e'_1\Sigma_t)(I - 2A_{t+1}\Sigma_t)^{-1}(r-1)e_1 = 0 \tag{2.102}$$

therefore,

$$\begin{aligned} x_{it} =& \frac{-(1-r)P_t + rx^e_{it} - \mu'_{it}(I-2A_{t+1}\Sigma_t)^{-1}(r-1)e_1}{r + (r-1)e'_1\Sigma_t(I-2A_{t+1}\Sigma_t)^{-1}(r-1)e_1} \\ =& \frac{-(1-r)P_t + rx^e_{it} + (1-r)Z'_{it}Q_tL_t}{r+(1-r)^2a_t} \end{aligned} \tag{2.103}$$

where

$$L_t = (I - 2A_{t+1}\Sigma_t)^{-1}e_1, \qquad a_t = e'_1\Sigma_t(I-2A_{t+1}\Sigma_t)^{-1}e_1 \tag{2.104}$$

To find x^e_{it}, we compute that

$$\begin{aligned} x^e_{it} = E_{it}[\overline{E}_t[x_{it}]] &= \frac{-(1-r)P_t + rE_{it}[\overline{E}_t[x^e_{it}]] + (1-r)E_{it}[\overline{E}_t[Z'_{it}Q_tL_t]]}{r+(1-r)^2a_t} \\ &= \frac{-(1-r)e'_1Z_{it} + rE_{it}[\overline{E}_t[x^e_{it}]] + (1-r)L'_tQ'_tG'_tZ_{it}}{r+(1-r)^2a_t} \\ &= \frac{-(1-r)e'_1 + (1-r)L'_tQ'_tG'_t}{r+(1-r)^2a_t}Z_{it} + \frac{r}{r+(1-r)^2a_t}E_{it}[\overline{E}_t[x^e_{it}]] \\ &= \Delta'_tZ_{it} + d_tE_{it}[\overline{E}_t[x^e_{it}]] \end{aligned} \tag{2.105}$$

where

$$\Delta_t = \frac{1-r}{r}d_t(G_tQ_tL_t - e_1), \qquad d_t = \frac{r}{r+(1-r)^2a_t} \tag{2.106}$$

hence, if $|d_t| < 1$, which is equivalent to

$$a_t > 0 \quad or \quad a_t < \frac{-2r}{(1-r)^2} \tag{2.107}$$

then according to Lemma 2, we have

$$x_{it}^e = \Delta_t'(I + V_t\Lambda_t V_t^{-1})Z_{it} \tag{2.108}$$

Therefore, the optimal demand is

$$\begin{aligned} x_{it} &= \frac{-(1-r)e_1'Z_{it} + r\Delta_t'(I + V_t\Lambda_t V_t^{-1})Z_{it} + (1-r)L_t'Q_t'Z_{it}}{r + (1-r)^2 a_t} \\ &= \Omega_t' Z_{it} \end{aligned} \tag{2.109}$$

where

$$\Omega_t = \frac{-(1-r)e_1 + r(I + (V_t')^{-1}\Lambda_t V_t')\Delta_t + (1-r)Q_t L_t}{r + (1-r)^2 a_t} \tag{2.110}$$

Denote the variables $\alpha_t, \phi_{1t}, \phi_{2t}, \alpha_t^e, \phi_{1t}^e$, and ϕ_{2t}^e such that

$$\Omega_t = \begin{pmatrix} -\alpha_t \\ \alpha_t\phi_{2t} \\ \alpha_t\phi_{1t} \end{pmatrix}, \quad (I + (V_t')^{-1}\Lambda_t V_t')\Delta_t = \begin{pmatrix} -\alpha_t^e \\ \alpha_t^e\phi_{2t}^e \\ \alpha_t^e\phi_{1t}^e \end{pmatrix} \tag{2.111}$$

Thus, the optimal demand is given by

$$x_{it} = \Omega_t' Z_{it} = \alpha_t(\phi_{1t}v_{it} + \phi_{2t}\overline{v}_t^i - P_t) \tag{2.112}$$

and the reference demand is given by

$$x_{it}^e = \alpha_t^e(\phi_{1t}^e v_{it} + \phi_{2t}^e \overline{v}_t^i - P_t) \tag{2.113}$$

The equilibrium price can be found from the market clearing condition, $\int x_{it} di = 0$, as

$$P_t = (\phi_{1t} + \phi_{2t} - \frac{\phi_{2t}}{\eta_t})\overline{v}_t + \frac{\phi_{2t}}{\eta_t}\overline{\overline{v}}_t, \tag{2.114}$$

To complete the induction, we need to show that

$$
\begin{aligned}
&E_{it}[-(e^{-W_{it}})^{1-r}] = -E_{it}[exp\{-(1-r)W_{it}\} \\
&= \frac{-\gamma_{t+1}}{\sqrt{det(I-2A_{t+1}\Sigma_t)}} exp\{\widehat{c}_t - \frac{1}{2}\mu_{it}'\Sigma_t^{-1}\mu_{it} + \frac{1}{2}(\mu_{it}+\Sigma_t B_t)'(I-2A_{t+1}\Sigma_t)^{-1}\Sigma_t^{-1}(\mu_{it}+\Sigma_t B_t)\} \\
&= -\gamma_t exp\{Z_{it}' A_t Z_{it}\} \qquad (2.115)
\end{aligned}
$$

where $\gamma_t = \frac{\gamma_{t+1}}{\sqrt{det(I-2A_{t+1}\Sigma_t)}}$ is positive, and A_t is symmetric, for $\widehat{c}_t = x_{it}(1-r)P_t$. First, notice that

$$
B_t = -x_{it}(1-r)e_1 = -(1-r)e_1\Omega_t' Z_{it} =: E_t Z_{it} \qquad (2.116)
$$

$$
\begin{aligned}
\widehat{c}_t &= (1-r)x_{it}P_t = (1-r)\Omega_t' Z_{it} e_1' Z_{it} = \\
&= Z_{it}'(1-r)\Omega_t e_1' Z_{it} = Z_{it}'\frac{1-r}{2}(e_1\Omega_t' + \Omega_t e_1')Z_{it} =: Z_{it}' H_t Z_{it} \qquad (2.117)
\end{aligned}
$$

Hence, we have

$$
\begin{aligned}
&E_{it}[-(e^{-W_{it}})^{1-r}] \\
&= -\gamma_t exp\{\widehat{c}_t - \frac{1}{2}\mu_{it}'\Sigma_t^{-1}\mu_{it} + \frac{1}{2}(\mu_{it}+\Sigma_t B_t)'(I-2A_{t+1}\Sigma_t)^{-1}\Sigma_t^{-1}(\mu_{it}+\Sigma_t B_t)\} \\
&= -\gamma_t exp\{Z_{it}' H_t Z_{it} - \frac{1}{2}Z_{it}' Q_t \Sigma_t^{-1} Q_t' Z_{it} + \frac{1}{2}(Q_t' Z_{it} + \Sigma_t E_t Z_{it})'(I-2A_{t+1}\Sigma_t)^{-1}\Sigma_t^{-1}(Q_t' Z_{it} + \Sigma_t E_t Z_{it})\} \\
&= -\gamma_t exp\{Z_{it}' A_t Z_{it}\} \qquad (2.118)
\end{aligned}
$$

where

$$
A_t = H_t - \frac{1}{2}Q_t\Sigma_t^{-1}Q_t' + \frac{1}{2}(Q_t + E_t'\Sigma_t)(I-2A_{t+1}\Sigma_t)^{-1}\Sigma_t^{-1}(Q_t' + \Sigma_t E_t) \qquad (2.119)
$$

Therefore A_t is symmetric and is recursively given in terms of A_{t+1}. This completes the induction. Finally, we show that $x_{it} - x_{it}^e$ goes to zero when r approaches to one. We compute that

$$x_{it} = \Omega_t' Z_{it} = \frac{-(1-r)e_1' Z_{it} + r\Delta_t'(I + V_t\Lambda_t V_t^{-1})Z_{it} + (1-r)L_t'Q_t'Z_{it}}{r + (1-r)^2 a_t}$$
$$= x_{it}^e + \frac{1-r}{r+(1-r)^2 a_t} F_t' Z_{it} \tag{2.120}$$

where

$$F_t = Q_t L_t - e_1 - (1-r)a_t(I + (V_t')^{-1}\Lambda_t V_t')\Delta_t \tag{2.121}$$

Therefore if F_t is bounded when $r \to 1$, we have

$$\lim_{r\to 1}(x_{it} - x_{it}^e) = \lim_{r\to 1} \frac{1-r}{r+(1-r)^2 a_t} F_t' Z_{it} = 0 \tag{2.122}$$

This completes the proof. **Q.E.D.**

Now, I turn to the proof of proposition 1 which is a special case. We need to eliminate the corresponding dimension for $\overline{v}_t^i$, hence $Z_{it+1} = \binom{P_{t+1}}{v_{it+1}} \sim N(\mu_{it}, \Sigma_t)$. Notice that $P_{T-1} = \overline{v}_{T-1}$, and the expression for A_{T-1} by using $\alpha_{T-1} = \frac{1}{(1-r)\rho_{T-1}}$ can be further simplified to

$$A_{T-1} = -\frac{1}{2}(1-r)\alpha_{T-1}\{\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\} + \frac{1}{2}(1-r)^2\rho_{T-1}\alpha_{T-1}^2\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$
$$= \frac{-1}{2\rho_{T-1}}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \tag{2.123}$$

The vector Y_{it+1} becomes

$$Y_{it+1} = (F, \overline{v}_t, \epsilon_{t+1}, \epsilon_{it+1})' \tag{2.124}$$

and if $P_{t+1} = \overline{v}_{t+1}$, then

$$M_t = \begin{pmatrix} \pi_t & 1-\pi_t & \pi_t & 0 \\ \pi_t & 0 & \pi_t & \pi_t \end{pmatrix}, \quad b_{it} = (0, (1-\pi_t)v_{it} - \pi_t e_{it+1})' \tag{2.125}$$

hence $Z_{it+1} = M_t Y_{it+1} + b_{it}$, and the moments of Y_{it+1} are

$$E_{it}[Y_{it+1}] = \mu_{it}^Y = \begin{pmatrix} v_{it} \\ \overline{v}_t \\ 0 \\ e_{it+1} \end{pmatrix}, \quad Var_{it}(Y_{it+1}) = \Sigma_t^Y = \begin{pmatrix} \rho_t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_{t+1} & 0 \\ 0 & 0 & 0 & q_{t+1} \end{pmatrix} \tag{2.126}$$

thus, the moments of Z_{it+1} are

$$E_{it}[Z_{it+1}] = \mu_{it} = M_t \mu_{it}^Y + b_{it}, \quad Var_{it}(Z_{it+1}) = \Sigma_t = M_t \Sigma_t^Y M_t' = \begin{pmatrix} \pi_t^2(\rho_t+\lambda_{t+1}) & \pi_t^2(\rho_t+\lambda_{t+1}) \\ \pi_t^2(\rho_t+\lambda_{t+1}) & \pi_t \rho_t \end{pmatrix} \tag{2.127}$$

Now, we can rewrite μ_{it} as

$$\mu_{it} = Q_t' \tilde{Z}_{it}, \quad \tilde{Z}_{it} = \begin{pmatrix} P_t \\ \overline{v}_t \\ v_{it} \end{pmatrix}, \quad Q_t = \begin{pmatrix} 0 & 0 \\ 1-\pi_t & 0 \\ \pi_t & 1 \end{pmatrix} \tag{2.128}$$

therefore $E_{it}[\overline{E}_t[\mu_{it}]] = Q_t' \tilde{Z}_{it}$. The optimal demand becomes

$$\begin{aligned} x_{it} =& \frac{-(1-r)P_t + r x_{it}^e - \mu_{it}'(I - 2A_{t+1}\Sigma_t)^{-1}(r-1)e_1}{r + (r-1)e_1'\Sigma_t(I - 2A_{t+1}\Sigma_t)^{-1}(r-1)e_1} \\ =& \frac{-(1-r)P_t + r x_{it}^e + (1-r)\tilde{Z}_{it}' Q_t L_t}{r + (1-r)^2 a_t} \end{aligned} \tag{2.129}$$

Note that in the second equality of the above equation $\tilde{Z}_{it}$ has been introduced only as a notation. We find x_{it}^e as

$$\begin{aligned} x_{it}^e = E_{it}[\overline{E}_t[x_{it}]] &= \frac{-(1-r)P_t + r E_{it}[\overline{E}_t[x_{it}^e]] + (1-r)E_{it}[\overline{E}_t[\tilde{Z}_{it}' Q_t L_t]]}{r + (1-r)^2 a_t} \\ &= \frac{-(1-r)e_1'\tilde{Z}_{it} + r E_{it}[\overline{E}_t[x_{it}^e]] + (1-r)L_t' Q_t' \tilde{Z}_{it}}{r + (1-r)^2 a_t} \\ &= \frac{-(1-r)e_1' + (1-r)L_t' Q_t'}{r + (1-r)^2 a_t}\tilde{Z}_{it} + \frac{r}{r + (1-r)^2 a_t} E_{it}[\overline{E}_t[x_{it}^e]] \\ &= \Delta_t' \tilde{Z}_{it} + d_t E_{it}[\overline{E}_t[x_{it}^e]] \end{aligned} \tag{2.130}$$

where

$$\Delta_t = \frac{1-r}{r} d_t (Q_t L_t - e_1), \quad d_t = \frac{r}{r + (1-r)^2 a_t} \tag{2.131}$$

Thus, according to Lemma 1, since $E_{it}[\overline{E}_t[\tilde{Z}_{it}]] = \tilde{Z}_{it}$, we have

$$x^e_{it} = \frac{1}{1-d_t}\Delta'_t\tilde{Z}_{it} \tag{2.132}$$

Therefore, the optimal demand is

$$\begin{aligned} x_{it} &= \frac{-(1-r)e'_1\tilde{Z}_{it} + r\frac{1}{1-d_t}\Delta'_t\tilde{Z}_{it} + (1-r)L'_tQ'_t\tilde{Z}_{it}}{r+(1-r)^2a_t} \\ &= \Omega'_{0t}\tilde{Z}_{it} \end{aligned} \tag{2.133}$$

where

$$\Omega_{0t} = \frac{-(1-r)e_1 + \frac{r}{1-d_t}\Delta_t + (1-r)Q_tL_t}{r+(1-r)^2a_t} =: \begin{pmatrix} -\alpha_{1t} \\ \alpha_{1t}\phi_{2t} \\ \alpha_{1t}\phi_{1t} \end{pmatrix}$$

Thus, the optimal demand and the equilibrium price are given by

$$x_{it} = \Omega'_{0t}\tilde{Z}_{it} = \alpha_{1t}(\phi_{1t}v_{it} + \phi_{2t}\overline{v}_t - P_t), \quad P_t = (\phi_{1t}+\phi_{2t})\overline{v}_t \tag{2.134}$$

hence by denoting $\alpha_t = \frac{\alpha_{1t}\phi_{1t}}{\phi_{1t}+\phi_{2t}}$ and $\phi_t = \phi_{1t}+\phi_{2t}$, we have

$$x_{it} = \Omega'_tZ_{it} = \alpha_t(\phi_tv_{it} - P_t), \quad P_t = \phi_t\overline{v}_t \tag{2.135}$$

where

$$\Omega_t = \begin{pmatrix} -\alpha_t \\ \alpha_{1t}\phi_t \end{pmatrix} \tag{2.136}$$

which gives

$$x^e_{it} = E_{it}[\overline{E}_t[x_{it}]] = E_{it}[\overline{E}_t[\Omega'_tZ_{it}]] = \Omega'_tZ_{it} = x_{it} \tag{2.137}$$

Since $x_{it} = x^e_{it}$, equation 2.129 solves for

$$x_{it} = \frac{L'_t\begin{pmatrix} 1-\pi_t & \pi_t \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \overline{v}_t \\ v_{it} \end{pmatrix} - P_t}{(1-r)a_t} = \frac{L'_t\begin{pmatrix} \frac{1}{\phi_t}(1-\pi_t) & \pi_t \\ 0 & 1 \end{pmatrix} - e'_1}{(1-r)a_t}Z_{it} \tag{2.138}$$

Therefore the equilibrium price is

$$P_t = -\frac{e_2'\begin{pmatrix} \frac{1}{\phi_t}(1-\pi_t) & 0 \\ \pi_t & 1 \end{pmatrix} L_t - e_2' e_1}{e_1'\begin{pmatrix} \frac{1}{\phi_t}(1-\pi_t) & 0 \\ \pi_t & 1 \end{pmatrix} L_t - e_1' e_1} \overline{v}_t = \phi_t \overline{v}_t \tag{2.139}$$

where

$$\phi_t = -\frac{\binom{\pi_t}{1}' L_t}{\begin{pmatrix} \frac{1}{\phi_t}(1-\pi_t) \\ 1 \end{pmatrix}' L_t - 1} \tag{2.140}$$

Let $\Sigma_t = \begin{pmatrix} \sigma_{1t} & \sigma_{1t} \\ \sigma_{1t} & \sigma_{2t} \end{pmatrix}$, where $\sigma_{1t} = \pi_t^2(\rho_t + \lambda_{t+1})$ and $\sigma_{2t} = \pi_t \rho_t$. we compute that

$$(I - 2A_{t+1}\Sigma_t)^{-1} = \{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 2\theta_{t+1}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} \sigma_{1t} & \sigma_{1t} \\ \sigma_{1t} & \sigma_{2t} \end{pmatrix}\}^{-1} = \frac{1}{1-f_t}\begin{pmatrix} 1-f_t & -f_t \\ 0 & 1 \end{pmatrix} \tag{2.141}$$

where $f_t = -2\theta_{t+1}(\sigma_{1t} - \sigma_{2t})$. Therefore

$$L_t = (I - 2A_{t+1}\Sigma_t)^{-1} e_1 = \frac{1}{1-f_t}\begin{pmatrix} 1-f_t & -f_t \\ 0 & 1 \end{pmatrix} e_1 = e_1 \tag{2.142}$$

Hence $\phi_t = -\frac{\pi_t}{\frac{1}{\phi_t}(1-\pi_t)-1}$, or $\phi_t = 1$. We can also find α_t as

$$\alpha_t = -e_1' \frac{\begin{pmatrix} \frac{1}{\phi_t}(1-\pi_t) & 0 \\ \pi_t & 1 \end{pmatrix} L_t - e_1}{(1-r)a_t} = -\frac{\begin{pmatrix} \frac{1}{\phi_t}(1-\pi_t) \\ 0 \end{pmatrix}' e_1 - 1}{(1-r)a_t} = \frac{\pi_t}{(1-r)a_t} \tag{2.143}$$

However, we have

$$a_t = e_1' \Sigma_t (I - 2A_{t+1}\Sigma_t)^{-1} e_1 = e_1' \begin{pmatrix} \sigma_{1t} & \sigma_{1t} \\ \sigma_{1t} & \sigma_{2t} \end{pmatrix} \frac{1}{1-f_t}\begin{pmatrix} 1-f_t & -f_t \\ 0 & 1 \end{pmatrix} e_1 = \sigma_{1t} \tag{2.144}$$

Therefore $\alpha_t = \frac{\pi_t}{(1-r)\sigma_{1t}}$ for $t < T-1$. Now assume that $A_{t+1} = \theta_{t+1}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, where $\theta_{T-1} = \frac{-1}{2\rho_{T-1}}$. I show that $A_t = \theta_t \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ for some negative value of θ_t. Equation 2.119

can be rewritten by using the updating formula[2] as follows. We have

$$
\begin{aligned}
(I-2A_{t+1}\Sigma_t)^{-1}\Sigma_t^{-1} &= (\Sigma_t-\Sigma_t(2A_{t+1})\Sigma_t)^{-1} = \Sigma_t^{-1}+\Sigma_t^{-1}\Sigma_t[\frac{1}{2}A_{t+1}^{-1}-\Sigma_t\Sigma_t^{-1}\Sigma_t]^{-1}\Sigma_t\Sigma_t^{-1} \\
&= \Sigma_t^{-1}+[\frac{1}{2}A_{t+1}^{-1}-\Sigma_t]^{-1}
\end{aligned}
$$

hence

$$
\begin{aligned}
A_t &= H_t-\frac{1}{2}Q_t\Sigma_t^{-1}Q_t'+\frac{1}{2}(Q_t+E_t'\Sigma_t)(\Sigma_t^{-1}+[\frac{1}{2}A_{t+1}^{-1}-\Sigma_t]^{-1})(Q_t'+\Sigma_tE_t) \\
&= H_t+\frac{1}{2}(Q_tE_t+E_t'Q_t'+E_t'\Sigma_tE_t)+(Q_t+E_t'\Sigma_t)(I-2A_{t+1}\Sigma_t)^{-1}A_{t+1}(Q_t'+\Sigma_tE_t)
\end{aligned}
\tag{2.145}
$$

Now we compute that

$$
\begin{aligned}
\frac{1}{2}E_t'\Sigma_tE_t &= \frac{1}{2}(1-r)\alpha_t\begin{pmatrix}1&-1\\0&0\end{pmatrix}\begin{pmatrix}\sigma_{1t}&\sigma_{1t}\\\sigma_{1t}&\sigma_{3t}\end{pmatrix}(1-r)\alpha_t\begin{pmatrix}1&0\\-1&0\end{pmatrix} = \frac{1}{2}(1-r)^2\alpha_t^2\sigma_{1t}\begin{pmatrix}1&-1\\-1&1\end{pmatrix} \\
H_t+\frac{1}{2}(Q_tE_t+E_t'Q_t') &= \frac{1}{2}(1-r)\alpha_t\begin{pmatrix}-2&1\\1&0\end{pmatrix}+\frac{1}{2}(1-r)\alpha_t\{\begin{pmatrix}1-\pi_t&\pi_t-1\\\pi_t&-\pi_t\end{pmatrix}+\begin{pmatrix}1-\pi_t&\pi_t\\\pi_t-1&-\pi_t\end{pmatrix}\} \\
&= -(1-r)\alpha_t\pi_t\begin{pmatrix}1&-1\\-1&1\end{pmatrix}
\end{aligned}
$$

Finally, we have

$$
\begin{aligned}
Q_t'+\Sigma_tE_t &= \begin{pmatrix}1-\pi_t&\pi_t\\0&1\end{pmatrix}+\begin{pmatrix}\sigma_{1t}&\sigma_{1t}\\\sigma_{1t}&\sigma_{3t}\end{pmatrix}(1-r)\alpha_t\begin{pmatrix}1&-1\\0&0\end{pmatrix} = \begin{pmatrix}1-\pi_t+(1-r)\alpha_t\sigma_{1t}&\pi_t-(1-r)\alpha_t\sigma_{1t}\\(1-r)\alpha_t\sigma_{1t}&1-(1-r)\alpha_t\sigma_{1t}\end{pmatrix} \\
A_{t+1}(Q_t'+\Sigma_tE_t) &= \theta_{t+1}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}\begin{pmatrix}1-\pi_t+(1-r)\alpha_t\sigma_{1t}&\pi_t-(1-r)\alpha_t\sigma_{1t}\\(1-r)\alpha_t\sigma_{1t}&1-(1-r)\alpha_t\sigma_{1t}\end{pmatrix} = \theta_{t+1}(1-\pi_t)\begin{pmatrix}1&-1\\-1&1\end{pmatrix} \\
(I-2A_{t+1}\Sigma_t)^{-1}A_{t+1}(Q_t'+\Sigma_tE_t) &= \frac{1}{1-f_t}\begin{pmatrix}1-f_t&-f_t\\0&1\end{pmatrix}\theta_{t+1}(1-\pi_t)\begin{pmatrix}1&-1\\-1&1\end{pmatrix} = \frac{\theta_{t+1}(1-\pi_t)}{1-f_t}\begin{pmatrix}1&-1\\-1&1\end{pmatrix} \\
(Q_t+E_t'\Sigma_t)(I-2A_{t+1}\Sigma_t)^{-1}A_{t+1}(Q_t'+\Sigma_tE_t) &= \begin{pmatrix}1-\pi_t+(1-r)\alpha_t\sigma_{1t}&(1-r)\alpha_t\sigma_{1t}\\\pi_t-(1-r)\alpha_t\sigma_{1t}&1-(1-r)\alpha_t\sigma_{1t}\end{pmatrix}\frac{\theta_{t+1}(1-\pi_t)}{1-f_t}\begin{pmatrix}1&-1\\-1&1\end{pmatrix} \\
&= \frac{\theta_{t+1}(1-\pi_t)^2}{1-f_t}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}
\end{aligned}
$$

[2]Equation (A-66b) in page 822 of "Greene, William H., 2002, Econometric Analysis, 5th Edition, New York, Prentice Hall", which is given by $[A\pm BCB']^{-1}=A^{-1}\mp A^{-1}B[C^{-1}\pm B'A^{-1}B]^{-1}B'A^{-1}$.

Therefore

$$A_t = -(1-r)\alpha_t\pi_t\left(\begin{smallmatrix} 1 & -1 \\ -1 & 1 \end{smallmatrix}\right) + \frac{1}{2}(1-r)^2\alpha_t^2\sigma_{1t}\left(\begin{smallmatrix} 1 & -1 \\ -1 & 1 \end{smallmatrix}\right) + \frac{\theta_{t+1}(1-\pi_t)^2}{1-f_t}\left(\begin{smallmatrix} 1 & -1 \\ -1 & 1 \end{smallmatrix}\right) = \theta_t\left(\begin{smallmatrix} 1 & -1 \\ -1 & 1 \end{smallmatrix}\right) \tag{2.146}$$

where

$$\begin{aligned}\theta_t =& -(1-r)\alpha_t\pi_t + \frac{1}{2}(1-r)^2\alpha_t^2\sigma_{1t} + \frac{\theta_{t+1}(1-\pi_t)^2}{1-f_t} = \frac{-1}{2}(1-r)\alpha_t\pi_t + \frac{\theta_{t+1}(1-\pi_t)^2}{1-f_t} \\ =& \frac{-1}{2}\frac{\pi_t^2}{\sigma_{1t}} + \frac{\theta_{t+1}(1-\pi_t)^2}{1+2\theta_{t+1}(\sigma_{1t}-\sigma_{2t})}\end{aligned} \tag{2.147}$$

Notice that if $\theta_{t+1} < 0$, then $\theta_t < 0$ because

$$\sigma_{1t}-\sigma_{2t} = \pi_t^2(\rho_t+\lambda_{t+1})-\pi_t\rho_t = \pi_t\left(\frac{\rho_t}{\rho_t+q_t+\lambda_{t+1}}(\rho_t+\lambda_{t+1})-\rho_t\right) = \pi_t\rho_t\left(\frac{\rho_t+\lambda_{t+1}}{\rho_t+q_t+\lambda_{t+1}}-1\right) < 0$$

Since $\theta_{T-1} = \frac{-1}{2\rho_{T-1}}$, we conclude that for any date t, A_t is semi-negative definite and does not depend on r.

Proof of Proposition 4. Denote

$$\widehat{Y}_{it} = (F, \overline{v}_{t-1}, \epsilon_t, \epsilon_{it}, \delta_{it}, \omega_t, \delta_{it-1}, \omega_{t-1}, v_{it-1})' \tag{2.148}$$

and

$$J_1 = \left(I_{6\times 6} \; 0_{6\times 3} \right), \quad J_{2t-1} = \left(\begin{smallmatrix} 0_{2\times 8} & 0_{2\times 1} \\ 0_{1\times 8} & 1-\pi_{t-1} \end{smallmatrix}\right), \quad J_3 = \begin{pmatrix} 0 & 1 & 0_{1\times 4} & 0 & 0 & 0 \\ 0 & 0 & 0_{1\times 4} & 1 & 0 & 0 \\ 0 & 0 & 0_{1\times 4} & 0 & 1 & 0 \\ 0 & 0 & 0_{1\times 4} & 0 & 0 & 1 \end{pmatrix} \tag{2.149}$$

Thus, we have

$$Y_{it} = J_1\widehat{Y}_{it}, \quad b_{it-1} = J_{2t-1}\widehat{Y}_{it} - \begin{pmatrix} 0 \\ 0 \\ \pi_{t-1}e_{it} \end{pmatrix} \tag{2.150}$$

and

$$Z_{it-1} = R_{t-1}\left(\begin{smallmatrix} \overline{v}_{t-1} \\ \delta_{it-1} \\ \omega_{t-1} \\ v_{it-1} \end{smallmatrix}\right) = R_{t-1}J_3\widehat{Y}_{it}, \quad R_{t-1} = \left(\begin{smallmatrix} \phi_{1t-1}+\phi_{2t-1} & 0 & \phi_{2t-1} & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{smallmatrix}\right) \tag{2.151}$$

The variance-covariance matrix of $\widehat{Y}_{it}$ is

$$\Sigma^{\widehat{Y}}_{t-1} = \left(\begin{smallmatrix} \Sigma^{Y}_{t-1} & 0 & 0 & \xi_{t-1} \\ 0 & \iota_t & 0 & 0 \\ 0 & 0 & \kappa_t & 0 \\ \xi_{t-1} & 0 & 0 & \sigma^2_{v_{it-1}} \end{smallmatrix}\right) \tag{2.152}$$

where $Cov_t(F, v_{it}) = \xi_t$, and $\sigma^2_{v_{it}}$ is recursively given by

$$\begin{aligned}
\sigma^2_{v_{it}} =& Var((1-\pi_{t-1})v_{it-1} + \pi_{t-1}(F + \epsilon_t + \epsilon_{it} - e_{it})) \\
=& (1-\pi_{t-1})^2\sigma^2_{v_{it-1}} + \pi^2_{t-1}(\rho_t + \lambda_t + q_t) + 2\pi_{t-1}(1-\pi_{t-1})Cov(v_{it-1}, F) \\
=& (1-\pi_{t-1})^2\sigma^2_{v_{it-1}} + \pi_{t-1}\rho_{t-1} + 2\pi_{t-1}(1-\pi_{t-1})\xi_{t-1}
\end{aligned} \tag{2.153}$$

starting with $\sigma^2_{v_{i0}} = 0$. Now we compute that

$$\begin{aligned}
x_{it} - x_{it-1} &= \Omega'_t Z_{it} - \Omega'_{t-1} Z_{it-1} \\
&= \Omega'_t(M_{t-1}Y_{it} + b_{it-1}) - \Omega'_{t-1}R_{t-1}J_3\widehat{Y}_{it} \\
&= \{\Omega'_t M_{t-1}J_1 + \Omega'_t J_{2t-1} - \Omega'_{t-1}R_{t-1}J_3\}\widehat{Y}_{it} - \Omega'_t\left(\begin{smallmatrix} 0 \\ 0 \\ \pi_{t-1}e_{it} \end{smallmatrix}\right) \\
&= M^{\widehat{Y}}_{t-1}\widehat{Y}_{it} - \Omega'_t\left(\begin{smallmatrix} 0 \\ 0 \\ \pi_{t-1}e_{it} \end{smallmatrix}\right)
\end{aligned} \tag{2.154}$$

where

$$M^{\widehat{Y}}_{t-1} = \Omega'_t M_{t-1}J_1 + \Omega'_t J_{2t-1} - \Omega'_{t-1}R_{t-1}J_3 \tag{2.155}$$

Therefore, $Var(x_{it} - x_{it-1}) = M^{\widehat{Y}}_{t-1}\Sigma^{\widehat{Y}}_{t-1}(M^{\widehat{Y}}_{t-1})'$, and the expected volume will be given as

$$E[Volume_t] = \sqrt{\frac{1}{2\pi}M^{\widehat{Y}}_{t-1}\Sigma^{\widehat{Y}}_{t-1}(M^{\widehat{Y}}_{t-1})'}$$

Conclusion

In chapter 1 of the thesis, I study the effect of recent short selling regulation, regulation SHO, on informational efficiency of naked short sellers. Regulation SHO was implemented in January 2005 and was strengthened in September 2008 (the 2008 regulation). The existing literature argues that the updated 2008 regulation reduces market quality and should be relaxed. The 2008 regulation makes traders strictly responsible for delivering the shares sold short within the specified time frames so that the Failure-to-delivery (FTD) will be reduced in the stock market.

In contrast to the literature, I find that the updated regulation of 2008 has had positive impacts. By performing three empirical tests, I show that the 2008 regulation improves the informativeness of naked short sellers by deterring some uninformed naked short sellers. Since the goal of the regulation was to curb potentially abusive manipulative naked short sellers, my findings suggest that the 2008 regulation is beneficial in helping to reduce manipulation.

The first test is to find the association between naked short sellers and traders in the E-Mini stock index futures markets. Some traders in E-Mini index futures are found to be informed. I find that, first, the higher net short positions taken by speculators, traders other than dealers or intermediaries, in the E-Mini index futures markets are associated with fewer FTDs in the stock market. Second, the greater the net short positions taken by dealers, the higher will be the FTDs in the stock market. Third, the publicly released excessive-FTD stocks earn negative abnormal returns. These results do not hold prior to

September 17, 2008. Fourth, to test whether naked short selling activity contains some information which is not incorporated in price, I implement a zero-investment strategy and I obtain sizable abnormal returns. This suggests that the uninformed, and perhaps manipulative, traders are deterred, which consequently improves the informativeness of the NSS activity. This outcome is important in an environment where courts and the SEC have failed in dealing with the NSS complaints. The results hold for the newly-created FTDs, and price pressure, stock order imbalance, and Exchange Traded Funds' FTDs cannot explain the observed relationship between E-Mini index futures and stock FTDs. Using monthly horizon, I find a similar relationship across NYSE and Nasdaq exchanges.

This chapter might help research on trading activities around predictable order flows. Bessembinder (2014) suggests that predictable order flows might be disruptive to the market quality in the presence of only a few strategic traders in a less resilient market. Since strategic traders might also engage in the NSS activity, investigating the FTDs around predictable order flows might shed light on this topic.

In chapter 2 of the thesis, I study the role of reference demands in traders' decision making to determine their optimal demands. I define a trader's reference demand to be his expectation of the average of all other traders' opinion about his demand. John Maynard Keynes' influential Beauty Contest metaphor of financial markets shows that market participants not only pay attention to fundamentals, but also to the actions of others. I employ this metaphor to an economy where traders pay attention to the average opinion about their own demand and adjust their demand with their reference demand to find their optimal demand. In this case, traders' demands depend not only on the fundamentals but also the degree of paying attention to the average opinion about their demands. Such decision making could be regret-minimizing as well.

I present three models in which traders are happier if their optimal demands are not too far from their reference demands. In classical differences of opinion models, with or without heterogenous signal precisions, I find that traders' optimal demand is exactly equal to their reference demands. Furthermore, the social welfare increases if traders pay more attention to the average opinion about their demands. However, I propose a novel model where the social welfare does not necessarily increase when traders pay more attention to the average opinion about their demand. All three models seem to agree on volume, in the sense that the more traders pay attention to the average opinion about their demands, volume increases. In my proposed model, when traders pay full attention to the average opinion about their demands, numerical instances show that their optimal demand is equal to their reference demand in limit.

For future research, one might consider explaining some empirical observations using the idea that traders do not want to submit demands which are substantially different from the average opinion on their demands. For instance, the excess volatility puzzle or price drift might be further explained using the use of reference demand.

Table 1.1: Sample Summary Statistics

	N	Mean	SD	Minimum	25th Pctl	50th Pctl	75th Pctl	Maximum
Before September 17, 2008								
FTD_{it}	111536	0.000486194	0.0015068	0	0	0.000021251	0.00019	0.0107558
$MCAP_{it}$	111536	2.178 B	5.162 B	.014 B	0.154 B	0.475 B	1.564 B	34.255 B
$Turnover_{it}$	111536	0.0019454	0.0018192	0.000065485	0.00071	0.001432	0.002543	0.0100468
BM_{it}	86767	0.4689257	0.319755	0.0407502	0.245924	0.395326	0.607292	1.7690107
POI_{it}	64758	18897.38	55698.79	0	555	2524	12504	1487584
SI_{it}	111536	0.0551605	0.0593906	0.000061152	0.011783	0.036784	0.077464	0.2993971
IO_{it}	111536	0.6228667	0.2969748	0.0036468	0.390042	0.690301	0.874697	0.99999
$Return_{it}$	111536	0.0056312	0.1241817	-0.3206713	-0.064874	0.001493	0.068522	0.4255582
$\#Firms$		3418						
$\#Months$		44						
After September 17, 2008								
FTD_{it}	179338	0.000094765	0.000262802	0	0.000007911	0.000022107	0.00005984	0.0019835
$MCAP_{it}$	179338	2.578 B	6.034 B	.011 B	0.151 B	0.532 B	1.935 B	39.697 B
$Turnover_{it}$	179338	0.0019592	0.0019718	0.000065236	0.000701	0.001386	0.002484	0.0115354
BM_{it}	162129	0.6924387	0.6236567	0.0369028	0.300956	0.525019	0.87209	3.8459907
POI_{it}	124356	16653.22	49832.63	0	304	1692	10510	1398690
SI_{it}	179338	0.0504644	0.0543292	0.000154616	0.014343	0.032567	0.067332	0.2761397
IO_{it}	179338	0.5647552	0.3185032	0.00001	0.301555	0.650352	0.832917	0.99999
$Return_{it}$	179338	0.0147941	0.1420443	-0.3604061	-0.061856	0.008742	0.080809	0.53125
$\#Firms$		3857						
$\#Months$		75						

The monthly FTD_{it}, for security i and month t, is the monthly average of failure to deliver outstanding divided by the total shares outstanding of that month. BM_{it} is the monthly book-to-market defined in section 1.2.1. $MCAP_{it}$ is the log of market capitalization. $Turnover_{it}$ is the monthly volume scaled by shares outstanding. SI_{it} is the average short interest scaled by shares outstanding. POI_{it} is the monthly sum of put open interest. $Return_{it}$ is the monthly return. IO_{it} is the quarterly values of institutional ownership divided by the shares outstanding in month t.

Table 1.2: FTD and E-Mini NASDAQ 100's Futures Pressure

	D	AM	D	AM	D	AM	D	AM	D	AM	D	AM
FP^c_{it-1}	0.19	-0.24	0.19	-0.24	0.20	-0.25	0.18	-0.21	0.24	-0.33	0.20	-0.23
	(4.02)	(-3.09)	(4.03)	(-3.09)	(3.59)	(-2.53)	(3.73)	(-2.59)	(4.78)	(-3.72)	(3.84)	(-2.41)
$Return_t$			0.00	0.00							0.18	0.17
			(0.01)	(0.01)							(0.72)	(0.66)
TED_t					0.01	0.00					-0.02	-0.02
					(0.37)	(0.09)					(-1.06)	(-1.33)
SI_t							8.32	7.86			8.58	8.20
							(4.15)	(3.57)			(4.50)	(3.80)
$Turnover_t$									17.63	16.81	19.78	18.92
									(3.25)	(3.11)	(3.56)	(3.42)
Intercept	0.32	0.32	0.32	0.32	0.32	0.32	-0.06	-0.04	0.16	0.17	-0.26	-0.23
	(15.46)	(14.66)	(14.46)	(13.99)	(15.43)	(14.35)	(-0.70)	(-0.41)	(3.08)	(3.19)	(-2.50)	(-2.13)
FTD_{t-1}	0.37	0.38	0.37	0.38	0.37	0.37	0.34	0.35	0.36	0.37	0.34	0.35
	(7.27)	(7.12)	(7.25)	(7.16)	(7.13)	(6.95)	(6.93)	(6.87)	(7.22)	(7.01)	(7.01)	(7.08)
FTD_{t-2}	0.19	0.19	0.19	0.19	0.18	0.19	0.17	0.17	0.18	0.18	0.18	0.19
	(4.03)	(4.02)	(4.05)	(4.04)	(4.12)	(4.37)	(3.61)	(3.62)	(4.03)	(4.03)	(4.17)	(4.52)

The estimated slopes from equation 1.2 where the dependent variable is FTD_t, the equally weighted average of failure to delivery deflated by shares outstanding. The independent variable of interest is futures pressure (FP_{it-1}), given by equation 1.1, from Dealers (D) and Asset Managers (AM) on E-Mini NASDAQ 100. $Return_t$ is the average weekly return across firms, TED_t is the TED spread, SI_t is the average short interest, and $Turnover_t$ is average turnover, all defined in section 1.3.2. Note that there are six models given in this table named from left to right as follows: No Control, Return Only, TED only, SI only, Turn only and All Controls. The standard errors are corrected with the original method of Newey West (1987) with five lags. The sample is from September 17 of 2008 to December 30, 2014.

Table 1.3: Time Series Regression of FTD on Futures Pressure

	D	AM	LF	OR	NR
E-Mini Dow Jones	0.24***	-0.18	-0.32**	-0.31	-0.81***
	(2.95)	(-0.72)	(-2.54)	(-0.97)	(-3.60)
E-Mini Nasdaq 100	0.20***	-0.23**	-0.44***	-0.59	-0.34
	(3.84)	(-2.41)	(-4.74)	(-1.06)	(-1.61)
E-Mini Russell 2000	$0.21^{s,**}$	-0.08	-0.35***	-2.12***	$-3.53^{s,***}$
	(2.56)	(-0.51)	(-2.60)	(-3.78)	(-3.45)
E-Mini S&P 400	0.25***	$-0.37^{s,**}$	-0.23*	-0.87	-0.41
	(2.70)	(-2.37)	(-1.81)	(-0.94)	(-0.95)
E-Mini S&P 500	0.25	-0.03	-0.13	-0.81	-0.32
	(1.00)	(-0.11)	(-0.40)	(-1.13)	(-1.02)

The weekly time series regression which is given by equation 1.2, where the dependent variable is FTD_t, the equally weighted average of failure to delivery deflated by shares outstanding. The independent variable of interest is futures pressure (FP_{it-1}), given by equation 1.1, for five E-Mini contracts, five trader categories and the All Controls" model given in Table 1.2. The slopes with s mean that one of the models in Table 1.2 is not significant even at 10%. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The standard errors are corrected with the original method of Newey West (1987) with five lags. The sample is from September 17, 2008 to December 30, 2014. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 1.4: FTD-FP relationship across portfolios for E-Mini Dow Jones

	D	tstat	AM	tstat	LF	tstat	OR	tstat	NR	tstat
All Firms	0.24	2.95	-0.18	-0.72	-0.32	-2.54	-0.31	-0.97	-0.81	-3.60
Put Firms	0.43	3.65	-0.04	-0.08	-0.51	-2.75	-1.25	-3.19	-1.12	-4.03
Non-Put Firms	0.30	3.10	-0.61	-2.41	-0.33	-2.23	-0.59	-1.48	-0.78	-2.65
BM1 Firms	0.31	3.99	-0.28	-1.01	-0.45	-3.71	0.04	0.09	-0.81	-3.26
BM3 Firms	0.12	1.52	0.02	0.09	-0.11	-0.87	-0.58	-2.06	-0.56	-2.80
BM5 Firms	0.31	3.99	-0.28	-1.01	-0.45	-3.71	0.04	0.09	-0.81	-3.26
Beta1 Firms	0.50	3.53	-1.11	-2.19	-0.51	-2.45	-0.83	-1.48	-1.43	-3.06
Beta3 Firms	0.11	1.26	0.22	0.69	-0.12	-0.92	-0.30	-0.86	-0.70	-2.71
Beta5 Firms	0.25	1.92	-0.46	-1.10	-0.35	-1.74	-0.54	-0.82	-0.39	-1.02
IO1 Firms	-0.04	-0.24	-0.31	-0.64	0.07	0.30	2.10	2.34	-0.39	-0.60
IO3 Firms	0.22	2.70	0.00	0.00	-0.40	-2.87	-0.02	-0.05	-0.57	-2.68
IO5 Firms	0.32	1.55	0.25	0.40	-0.43	-1.22	-1.09	-1.90	-1.49	-4.33
SI1 Firms	0.04	0.72	-0.43	-2.37	-0.05	-0.59	0.87	2.50	-0.12	-0.53
SI3 Firms	0.07	0.87	0.23	0.72	-0.04	-0.34	-0.33	-1.00	-0.50	-3.08
SI5 Firms	0.66	2.21	0.40	0.60	-0.80	-1.84	-0.36	-0.29	-2.76	-3.04
Size1 Firms	0.27	2.97	-0.32	-1.15	-0.32	-2.27	-0.24	-0.53	-0.98	-3.69
Size3 Firms	0.30	2.97	0.37	0.98	-0.29	-2.05	-1.45	-3.33	-1.10	-4.31
Size5 Firms	-0.02	-0.46	0.42	2.90	0.03	0.55	-0.09	-0.60	-0.29	-2.51
Std1 Firms	-0.03	-0.49	0.72	3.41	0.06	0.77	-0.20	-0.87	-0.43	-3.61
Std3 Firms	0.01	0.14	0.75	2.02	0.03	0.22	-0.21	-0.73	-0.62	-4.14
Std5 Firms	0.03	0.09	1.70	2.24	-0.02	-0.05	0.89	0.70	-1.99	-2.43
Turn1 Firms	-0.01	-0.14	0.09	0.61	0.04	0.83	0.00	0.02	-0.31	-2.46
Turn3 Firms	0.07	1.02	0.38	1.33	-0.10	-0.77	-0.35	-1.26	-0.52	-3.29
Turn5 Firms	1.30	3.55	-1.20	-1.19	-1.73	-3.58	-0.62	-0.35	-2.71	-2.48

The weekly time series regression which is given by equation 1.2 for different portfolios (see section 1.4.2 for portfolio constructions). The dependent variable is FTD_t, the equally weighted average of failure to delivery deflated by shares outstanding. The independent variable of interest is futures pressure (FP_{it-1}), given by equation 1.1, for E-Mini Dow Jones contract and its five trader categories and the All Controls" model given in Table 1.1. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The entries of the first row, All Firms, are also given in Table 1.3. *Put Firms* are those stocks with available put options. *Non-Put Firms* are those non-optionable stocks. *BM, Beta, IO, SI, Size, Std,* and *Turn* stand for book-to-market, rolling beta, institutional ownership, short interest, market capital, standard deviation, and turnover. Portfolios are updated weekly and by using these characteristics ranked from low to high and numbered from 1 to 5, respectively. The standard errors are corrected with the original method of Newey West (1987) with five lags. The sample is from September 17, 2008 to December 30, 2014.

Table 1.5: FTD-FP relationship across portfolios for E-Mini Nasdaq 100

	D	tstat	AM	tstat	LF	tstat	OR	tstat	NR	tstat
All Firms	0.20	3.84	-0.23	-2.41	-0.44	-4.74	-0.59	-1.06	-0.34	-1.61
Put Firms	0.36	5.11	-0.44	-3.36	-0.67	-5.41	-0.72	-1.01	-0.12	-0.53
Non-Put Firms	0.25	4.13	-0.36	-3.33	-0.43	-3.65	-0.39	-0.78	-0.16	-0.65
BM1 Firms	0.28	4.37	-0.46	-3.29	-0.42	-3.84	-0.08	-0.12	-0.21	-0.83
BM3 Firms	0.09	1.97	-0.07	-0.81	-0.30	-3.64	-0.04	-0.09	-0.26	-1.46
BM5 Firms	0.28	4.37	-0.46	-3.29	-0.42	-3.84	-0.08	-0.12	-0.21	-0.83
Beta1 Firms	0.41	4.04	-0.44	-2.63	-0.94	-5.10	-1.76	-1.97	0.18	0.49
Beta3 Firms	0.12	2.10	-0.08	-0.75	-0.33	-3.46	-0.25	-0.40	-0.46	-2.06
Beta5 Firms	0.24	2.62	-0.38	-2.20	-0.33	-1.82	-1.65	-1.92	-0.28	-0.62
IO1 Firms	0.11	0.76	0.08	0.30	-0.22	-0.84	-2.85	-2.16	-0.22	-0.42
IO3 Firms	0.17	2.76	-0.24	-2.20	-0.30	-3.48	0.05	0.10	-0.23	-1.08
IO5 Firms	0.31	1.86	-0.47	-1.67	-0.63	-2.34	1.37	1.29	-0.57	-1.12
SI1 Firms	0.06	1.10	-0.05	-0.56	-0.06	-0.70	-0.40	-1.19	-0.23	-1.30
SI3 Firms	0.10	1.95	-0.05	-0.55	-0.27	-3.31	-0.13	-0.27	-0.38	-2.23
SI5 Firms	0.48	2.51	-0.36	-0.97	-1.11	-2.95	-2.18	-1.15	-0.43	-0.63
Size1 Firms	0.19	3.04	-0.13	-1.18	-0.45	-3.87	-0.72	-1.13	-0.34	-1.29
Size3 Firms	0.32	4.21	-0.48	-3.14	-0.54	-4.98	0.60	0.90	0.13	0.54
Size5 Firms	0.04	1.31	-0.08	-1.41	-0.06	-1.32	0.48	1.80	0.05	0.42
Std1 Firms	0.03	0.70	0.06	0.62	-0.11	-1.86	0.25	0.87	-0.11	-0.90
Std3 Firms	0.01	0.17	0.09	0.77	-0.22	-2.57	0.73	1.33	-0.18	-0.90
Std5 Firms	0.21	0.83	0.60	1.22	-0.71	-1.62	-3.20	-1.88	-0.44	-0.78
Turn1 Firms	0.00	0.19	0.06	1.40	-0.13	-2.37	-0.15	-0.59	-0.02	-0.19
Turn3 Firms	0.09	2.01	-0.06	-0.77	-0.24	-3.27	-0.02	-0.04	-0.47	-2.63
Turn5 Firms	1.06	4.21	-1.49	-2.75	-1.78	-3.79	-2.71	-1.28	-1.09	-1.08

The weekly time series regression which is given by equation 1.2 for different portfolios (see section 1.4.2 for portfolio constructions). The dependent variable is FTD_t, the equally weighted average of failure to delivery deflated by shares outstanding. The independent variable of interest is futures pressure (FP_{it-1}), given by equation 1.1, for E-Mini Nasdaq 100 contract and its five trader categories and the All Controls" model given in Table 1.1. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The entries of the first row, All Firms, are also given in Table 1.3. *Put Firms* are those stocks with available put options. *Non-Put Firms* are those non-optionable stocks. *BM, Beta, IO, SI, Size, Std,* and *Turn* stand for book-to-market, rolling beta, institutional ownership, short interest, market capital, standard deviation, and turnover. Portfolios are updated weekly and by using these characteristics ranked from low to high and numbered from 1 to 5, respectively. The standard errors are corrected with the original method of Newey West (1987) with five lags. The sample is from September 17, 2008 to December 30, 2014.

Table 1.6: FTD-FP relationship across portfolios for E-Mini Russell 2000

	D	tstat	AM	tstat	LF	tstat	OR	tstat	NR	tstat
All Firms	0.21	2.56	-0.08	-0.51	-0.35	-2.60	-2.12	-3.78	-3.53	-3.45
Put Firms	0.02	0.19	0.41	2.23	-0.70	-3.03	-2.91	-3.87	-4.44	-2.77
Non-Put Firms	0.27	2.42	0.11	0.70	-0.65	-3.76	-3.39	-4.05	-2.57	-1.78
BM1 Firms	0.20	2.32	0.01	0.06	-0.53	-3.63	-2.17	-3.62	-3.39	-2.95
BM3 Firms	0.00	0.02	0.07	0.65	-0.08	-0.62	-0.52	-1.01	-1.28	-1.50
BM5 Firms	0.20	2.32	0.01	0.06	-0.53	-3.63	-2.17	-3.62	-3.39	-2.95
Beta1 Firms	0.19	1.24	0.27	1.24	-0.94	-3.48	-2.05	-1.70	-4.28	-2.23
Beta3 Firms	-0.13	-1.08	0.33	2.06	-0.08	-0.44	-0.90	-1.50	-0.63	-0.57
Beta5 Firms	0.43	2.83	-0.56	-2.24	-0.43	-1.74	-5.15	-4.64	-5.16	-2.76
IO1 Firms	-0.03	-0.13	0.08	0.23	0.06	0.15	-0.32	-0.15	-1.64	-0.60
IO3 Firms	0.35	2.91	-0.12	-0.78	-0.30	-1.81	-2.69	-4.48	-3.00	-2.67
IO5 Firms	-0.03	-0.14	0.46	1.80	-0.42	-1.00	-4.04	-2.18	-0.30	-0.13
SI1 Firms	0.30	3.76	-0.55	-3.09	-0.27	-2.35	-0.94	-1.90	-2.91	-3.12
SI3 Firms	-0.01	-0.04	0.07	0.58	-0.03	-0.16	-1.06	-1.61	-1.79	-1.57
SI5 Firms	-0.12	-0.54	0.39	0.98	-0.04	-0.09	-0.62	-0.25	-5.12	-1.74
Size1 Firms	0.11	1.14	0.11	0.59	-0.34	-2.35	-0.49	-0.56	-3.87	-3.23
Size3 Firms	-0.22	-1.80	0.59	3.55	-0.36	-1.90	-2.81	-3.56	0.50	0.37
Size5 Firms	-0.09	-2.50	0.13	2.86	0.09	1.12	-0.60	-2.04	1.27	2.47
Std1 Firms	-0.17	-3.07	0.16	2.77	0.38	2.80	-0.07	-0.20	1.64	2.55
Std3 Firms	-0.14	-1.81	0.14	1.48	0.22	1.06	0.35	0.67	-0.46	-0.38
Std5 Firms	-0.54	-1.76	0.62	1.35	1.48	2.12	-0.13	-0.04	2.19	0.63
Turn1 Firms	-0.13	-3.50	0.21	3.48	0.05	0.73	1.03	3.17	0.40	0.74
Turn3 Firms	0.02	0.24	0.11	0.79	-0.09	-0.71	-0.83	-1.59	-1.63	-1.63
Turn5 Firms	0.75	2.31	-0.62	-1.05	-1.39	-1.97	-11.14	-3.48	-11.71	-2.43

The weekly time series regression which is given by equation 1.2 for different portfolios (see section 1.4.2 for portfolio constructions). The dependent variable is FTD_t, the equally weighted average of failure to delivery deflated by shares outstanding. The independent variable of interest is futures pressure (FP_{it-1}), given by equation 1.1, for E-Mini Russell 2000 contract and its five trader categories and the All Controls" model given in Table 1.1. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The entries of the first row, All Firms, are also given in Table 1.3. *Put Firms* are those stocks with available put options. *Non-Put Firms* are those non-optionable stocks. *BM, Beta, IO, SI, Size, Std,* and *Turn* stand for book-to-market, rolling beta, institutional ownership, short interest, market capital, standard deviation, and turnover. Portfolios are updated weekly and by using these characteristics ranked from low to high and numbered from 1 to 5, respectively. The standard errors are corrected with the original method of Newey West (1987) with five lags. The sample is from September 17, 2008 to December 30, 2014.

Table 1.7: FTD-FP relationship across portfolios for E-Mini S&P 400

	D	tstat	AM	tstat	LF	tstat	OR	tstat	NR	tstat
All Firms	0.25	2.70	-0.37	-2.37	-0.23	-1.81	-0.87	-0.94	-0.41	-0.95
Put Firms	0.28	2.31	-0.13	-0.67	-0.40	-2.29	0.66	0.70	0.06	0.08
Non-Put Firms	0.34	2.45	0.00	0.00	-0.54	-3.73	1.84	2.35	-0.20	-0.36
BM1 Firms	0.23	2.15	-0.09	-0.57	-0.27	-1.63	-1.51	-1.65	-1.61	-2.76
BM3 Firms	0.13	1.95	-0.40	-3.84	-0.05	-0.55	-0.02	-0.03	0.41	1.26
BM5 Firms	0.23	2.15	-0.09	-0.57	-0.27	-1.63	-1.51	-1.65	-1.61	-2.76
Beta1 Firms	0.46	2.13	-0.35	-1.15	-0.50	-1.93	0.51	0.43	0.30	0.43
Beta3 Firms	0.12	1.20	-0.08	-0.52	-0.11	-0.79	-0.30	-0.37	-0.11	-0.26
Beta5 Firms	0.45	2.90	-0.57	-2.11	-0.47	-2.43	-1.77	-1.02	-0.83	-1.09
IO1 Firms	0.28	1.27	-1.39	-2.68	0.47	1.45	-4.99	-2.13	-1.13	-0.87
IO3 Firms	0.24	2.68	-0.06	-0.36	-0.27	-2.62	-1.12	-1.21	-1.12	-3.14
IO5 Firms	0.29	0.96	-0.03	-0.08	-0.53	-1.60	0.52	0.38	-1.11	-0.56
SI1 Firms	0.30	3.44	-0.47	-3.05	-0.13	-0.83	-2.02	-2.25	-0.40	-0.79
SI3 Firms	0.18	1.91	-0.37	-2.81	-0.05	-0.46	-0.44	-0.60	0.33	0.90
SI5 Firms	0.09	0.26	-0.52	-0.99	0.35	0.67	-1.77	-0.58	-0.08	-0.06
Size1 Firms	0.21	1.68	-0.49	-2.39	-0.11	-0.71	-1.38	-1.18	0.39	0.76
Size3 Firms	-0.03	-0.25	0.52	2.96	-0.33	-2.61	1.59	1.83	-0.31	-0.50
Size5 Firms	-0.09	-2.15	0.17	2.91	0.01	0.13	0.16	0.50	-0.10	-0.42
Std1 Firms	-0.05	-1.30	0.03	0.43	0.08	0.99	-0.01	-0.01	0.17	0.80
Std3 Firms	-0.05	-0.66	-0.04	-0.35	0.11	1.05	-0.35	-0.60	0.55	1.68
Std5 Firms	-0.17	-0.52	-0.70	-1.27	1.37	2.96	-2.93	-0.93	0.17	0.15
Turn1 Firms	-0.05	-1.21	-0.03	-0.38	0.07	1.09	0.15	0.34	0.57	3.01
Turn3 Firms	0.12	1.48	-0.21	-1.58	-0.06	-0.61	-0.27	-0.36	-0.07	-0.18
Turn5 Firms	1.01	2.26	-1.42	-1.93	-0.94	-1.50	-3.24	-0.83	-2.48	-1.40

The weekly time series regression which is given by equation 1.2 for different portfolios (see section 1.4.2 for portfolio constructions). The dependent variable is FTD_t, the equally weighted average of failure to delivery deflated by shares outstanding. The independent variable of interest is futures pressure (FP_{it-1}), given by equation 1.1, for E-Mini S&P 400 contract and its five trader categories and the All Controls" model given in Table 1.1. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The entries of the first row, All Firms, are also given in Table 1.3. *Put Firms* are those stocks with available put options. *Non-Put Firms* are those non-optionable stocks. *BM, Beta, IO, SI, Size, Std,* and *Turn* stand for book-to-market, rolling beta, institutional ownership, short interest, market capital, standard deviation, and turnover. Portfolios are updated weekly and by using these characteristics ranked from low to high and numbered from 1 to 5, respectively. The standard errors are corrected with the original method of Newey West (1987) with five lags. The sample is from September 17, 2008 to December 30, 2014.

Table 1.8: FTD-FP relationship across portfolios for E-Mini S&P 500

	D	tstat	AM	tstat	LF	tstat	OR	tstat	NR	tstat
All Firms	0.25	1.00	-0.03	-0.11	-0.13	-0.40	-0.81	-1.13	-0.32	-1.02
Put Firms	-0.27	-0.81	0.68	2.52	-0.11	-0.23	-1.07	-1.19	-0.70	-1.79
Non-Put Firms	0.17	0.99	0.46	2.40	-0.42	-1.66	-1.22	-1.47	-0.80	-2.80
BM1 Firms	0.06	0.21	0.08	0.30	0.09	0.25	-0.44	-0.57	-0.43	-1.40
BM3 Firms	-0.11	-0.69	0.16	1.02	0.14	0.48	0.26	0.48	-0.28	-1.10
BM5 Firms	0.06	0.21	0.08	0.30	0.09	0.25	-0.44	-0.57	-0.43	-1.40
Beta1 Firms	0.27	1.07	0.51	1.62	-0.79	-2.06	-1.44	-1.39	-0.51	-1.11
Beta3 Firms	-0.19	-0.70	0.24	1.01	0.20	0.57	-0.41	-0.65	-0.28	-0.86
Beta5 Firms	0.79	2.29	-0.91	-2.23	-0.30	-0.71	-3.04	-2.27	0.17	0.33
IO1 Firms	0.05	0.14	-1.67	-3.24	0.54	0.95	-0.14	-0.09	2.24	2.67
IO3 Firms	0.20	0.61	0.03	0.15	-0.16	-0.46	-0.62	-0.84	-0.13	-0.49
IO5 Firms	-0.32	-0.49	0.99	2.21	0.69	1.13	-1.44	-0.85	-1.95	-2.51
SI1 Firms	0.62	4.14	-0.90	-3.39	-0.21	-1.25	-0.58	-0.91	0.25	0.93
SI3 Firms	-0.05	-0.21	0.12	0.74	0.01	0.03	0.43	0.75	-0.23	-0.88
SI5 Firms	-0.41	-0.63	-0.18	-0.27	0.71	0.62	-1.69	-0.84	1.22	1.02
Size1 Firms	0.09	0.37	-0.15	-0.46	-0.13	-0.38	-0.94	-1.21	0.38	1.08
Size3 Firms	-0.62	-2.22	1.12	4.42	-0.01	-0.03	-0.54	-0.60	-1.11	-2.98
Size5 Firms	-0.28	-2.88	0.22	3.12	0.17	1.23	0.47	1.60	-0.21	-1.59
Std1 Firms	-0.21	-1.82	0.20	2.03	0.12	0.73	0.45	1.33	-0.19	-1.15
Std3 Firms	-0.43	-1.93	0.23	1.51	0.29	0.90	0.81	1.37	-0.03	-0.10
Std5 Firms	-0.54	-0.98	-0.72	-1.07	0.95	1.15	1.75	0.83	1.96	1.72
Turn1 Firms	-0.17	-1.97	0.15	1.71	0.03	0.20	0.18	0.60	0.07	0.43
Turn3 Firms	-0.13	-0.45	0.23	1.18	0.09	0.25	0.08	0.14	-0.27	-0.96
Turn5 Firms	0.91	0.98	-0.88	-0.76	0.24	0.21	-2.91	-0.86	-1.03	-0.73

The weekly time series regression which is given by equation 1.2 for different portfolios (see section 1.4.2 for portfolio constructions). The dependent variable is FTD_t, the equally weighted average of failure to delivery deflated by shares outstanding. The independent variable of interest is futures pressure (FP_{it-1}), given by equation 1.1, for E-Mini S&P 500 contract and its five trader categories and the All Controls" model given in Table 1.1. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The entries of the first row, All Firms, are also given in Table 1.3. *Put Firms* are those stocks with available put options. *Non-Put Firms* are those non-optionable stocks. *BM, Beta, IO, SI, Size, Std,* and *Turn* stand for book-to-market, rolling beta, institutional ownership, short interest, market capital, standard deviation, and turnover. Portfolios are updated weekly and by using these characteristics ranked from low to high and numbered from 1 to 5, respectively. The standard errors are corrected with the original method of Newey West (1987) with five lags. The sample is from September 17, 2008 to December 30, 2014.

Table 1.9: Panel regression of FTD and E-Mini NASDAQ 100's Futures Pressure

	D	AM	LF	OR	NR
FP_{it-1}	1.09***	-1.75***	-1.98***	0.75	-1.45
	(3.34)	(-2.64)	(-4.49)	(0.37)	(-1.47)
BM_{jt}	-0.11	-0.12	-0.09	-0.11	-0.12
	(-1.51)	(-1.55)	(-1.28)	(-1.45)	(-1.63)
IO_{jt}	-0.36*	-0.26	-0.49***	-0.47**	-0.46**
	(-1.75)	(-1.14)	(-2.57)	(-2.01)	(-2.02)
POI_{jt}	2.75***	2.76***	2.79***	2.85***	2.84***
	(3.29)	(3.28)	(3.29)	(3.26)	(3.26)
$Return_{jt}$	0.15	0.13	0.09	0.01	0.04
	(0.76)	(0.69)	(0.4)	(0.05)	(0.16)
SI_{jt}	14.52***	14.41***	14.67***	14.67***	14.67***
	(8.2)	(8.39)	(8.01)	(7.95)	(7.93)
$MCAP_{jt}$	-0.41***	-0.42***	-0.35***	-0.25***	-0.27***
	(-4.66)	(-4.12)	(-5.06)	(-4.14)	(-4.37)
$Turnover_{jt}$	343.44***	344.26***	337.75***	328.35***	328.62***
	(14.39)	(13.88)	(14.88)	(14.74)	(14.82)
Intercept	-0.04	-0.04	-0.03	-0.03	-0.03
	(-0.89)	(-0.91)	(-0.77)	(-0.64)	(-0.65)
R^2	15%	15%	15%	14%	14%

The monthly panel regression which is given by equation 1.3, where the dependent variable is FTD_{jt}, the failure to delivery deflated by shares outstanding of firm j in month t. The independent variable of interest is futures pressure (FP_{it-1}) for trader i in month $t-1$, given by equation 1.1, for E-Mini NASDAQ 100 and five trader categories. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The control variables are Institutional ownership, IO, Short Interest, SI, Market Capitalization, $MCAP$, Book-to-Market, BM, Turnover, $Turnover$, Put Open Interest, POI, Return, *Return*, defined in section 1.3.3. The standard errors are clustered by both firm and time and only firm fixed effect has been used. The sample is from September 17, 2008 to December 30, 2014. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 1.10: Panel regressions of FTD and Futures Pressure

	D	AM	LF	OR	NR
E-Mini Dow Jones	1.26*** (4.13)	-1.6* (-1.64)	-1.99*** (-3.02)	0.38 (0.19)	-2.51** (-2.25)
E-Mini Nasdaq 100	1.09*** (3.34)	-1.75*** (-2.64)	-1.98*** (-4.49)	0.75 (0.37)	-1.45 (-1.47)
E-Mini Russell 2000	0.47 (1.46)	0.44 (1.37)	-2.22*** (-2.69)	-5.52 (-1.43)	-13.16*** (-4.25)
E-Mini S&P 400	0.87*** (2.65)	0.04 (0.07)	-1.03** (-2.08)	-2.51 (-0.73)	-17.41** (-2.19)
E-Mini S&P 500	-0.33 (-0.51)	0.27 (0.46)	1.98** (2.39)	-4.78** (-2.42)	-1.8* (-1.75)

The estimated slope of futures pressure, η, in monthly panel regression which is given by equation 1.3, where the dependent variable is FTD_{jt}, the failure to delivery deflated by shares outstanding of firm j in month t. The independent variable of interest is futures pressure (FP_{it-1}) for trader i in month $t-1$, given by equation 1.1, for five E-Mini contracts and five trader categories. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The control variables are Institutional ownership, IO, Short Interest, SI, Market Capitalization, $MCAP$, Book-to-Market, BM, Turnover, $Turnover$, Put Open Interest, POI, Return, *Return*, defined in section 1.3.3. The standard errors are clustered by both firm and time and only firm fixed effect has been used. The sample is from September 17, 2008 to December 30, 2014. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 1.11: Pre September 2008 Panel regressions of FTD and Futures Pressure

	D	AM	LF	OR	NR
E-Mini Dow Jones	0.42 (0.22)	-0.24 (-0.17)	-1.06 (-0.34)	1.78 (0.67)	0.34 (0.17)
E-Mini Nasdaq 100	0.43 (0.27)	1.73 (0.46)	-1.78 (-1.02)	3.19 (0.76)	0.77 (0.22)
E-Mini Russell 2000	2.77 (1.01)	-11.06** (-2.17)	1.88 (0.32)	11.99 (0.75)	-5.03 (-1.06)
E-Mini S&P 400	0.44 (0.19)	0.64 (0.2)	0.38 (0.05)	4.48 (0.17)	-7.61 (-1.31)
E-Mini S&P 500	1.42 (0.58)	3.09 (0.59)	-3.15 (-1.1)	-2.45 (-0.14)	-2.13 (-0.4)

The estimated slope of futures pressure, η, in monthly panel regression which is given by equation 1.3, where the dependent variable is FTD_{jt}, the failure to delivery deflated by shares outstanding of firm j in month t. The independent variable of interest is futures pressure (FP_{it-1}) for trader i in month $t-1$, given by equation 1.1, for five E-Mini contracts and five trader categories. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The control variables are Institutional ownership, IO, Short Interest, SI, Market Capitalization, $MCAP$, Book-to-Market, BM, Turnover, $Turnover$, Put Open Interest, POI, Return, *Return*, defined in section 1.3.3. The standard errors are clustered by both firm and time and only firm fixed effect has been used. The sample is from June 2006 to August 2008. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 1.12: Post September 2008 Panel regressions of FTDs Above 10,000

	D	AM	LF	OR	NR
E-Mini Dow Jones	1.38*** (4.63)	-2.7*** (-2.96)	-2.12*** (-3.17)	-0.08 (-0.04)	-1.96* (-1.79)
E-Mini Nasdaq 100	1.14*** (3.48)	-1.9*** (-2.84)	-1.94*** (-4.28)	-0.36 (-0.2)	-1.23 (-1.23)
E-Mini Russell 2000	0.78** (2.34)	0.19 (0.61)	-2.82*** (-3.43)	-7.96** (-2.05)	-15.23*** (-4.72)
E-Mini S&P 400	1.06*** (3.22)	0.1 (0.16)	-1.47*** (-3.07)	-1.37 (-0.4)	-18.32** (-2.36)
E-Mini S&P 500	0.23 (0.38)	0.15 (0.28)	1.16 (1.35)	-6.69*** (-3.34)	-1.84* (-1.92)

The estimated slope of futures pressure, η, in monthly panel regression which is given by equation 1.3, where the dependent variable is FTD_{jt}, the failure to delivery deflated by shares outstanding of firm j in month t. The independent variable of interest is futures pressure (FP_{it-1}) for trader category i in month $t-1$, given by equation 1.1, for five E-Mini contracts and five trader categories. Only those stocks with the quantity of shares failed to deliver of above 10,000 shares have been considered and otherwise the quantity is set to zero. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The control variables are Institutional ownership, IO, Short Interest, SI, Market Capitalization, $MCAP$, Book-to-Market, BM, Turnover, $Turnover$, Put Open Interest, POI, Return, $Return$, defined in section 1.3.3. The standard errors are clustered by both firm and time and only firm fixed effect has been used. The sample is from September 17, 2008 to December 30, 2014. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 1.13: Cumulative Abnormal Retursn Around Threshold Listing

Window	CAR	t-stat
$[-20, 0]$	0.038*	1.74
$[-12, -7]$	0.036**	2.54
$[-10, 0]$	0.063***	3.93
$[-8, -7]$	0.035***	3.16
$[-7, 0]$	0.05***	3.9
$[-6, 0]$	0.026***	2.79
$[-4, 0]$	0.027***	3.54
$[-3, 0]$	0.023***	3.09
$[-20, +5]$	0.017	0.7
$[-10, +5]$	0.042**	2.35
$[-7, +5]$	0.028*	1.93
$[-6, +13]$	-0.01	-0.67
$[-4, +5]$	0.005	0.51
$[-1, +1]$	0.003	0.37
$[+1, +5]$	-0.022***	-3.45
$[+6, +13]$	-0.015*	-1.8
$[+1, +20]$	-0.055***	-3.67

The cumulative abnormal returns against the CAPM for several windows around threshold listing of day=0. The sample is from September 17, 2008 to December 30, 2014 with the first year as the base year. The t-statistics are adjusted by the Newey-West (1987) method with five lags. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 1.14: FTD-Short Interest Average Abnormal Return

	All Firms		NYSE		NASDAQ	
	Ab Ret	Annualized	Ab Ret	Annualized	Ab Ret	Annualized
EW	0.0070	44%	0.0018	10%	0.0083	54%
	(5.40)		(1.48)		(5.00)	
$EW11$	0.0062	38%	0.0018	10%	0.0067	41%
	(5.02)		(2.53)		(4.43)	
$EW55$	−0.0008	−4%	0.0000	0%	−0.0016	−8%
	(-0.83)		(-0.02)		(-1.59)	
VW	0.0043	25%	0.0020	11%	0.0053	32%
	(3.50)		(1.66)		(3.50)	
$VW11$	0.0027	15%	0.0001	0%	0.0035	20%
	(2.71)		(0.16)		(3.30)	
$VW55$	−0.0017	−8%	−0.0019	−10%	−0.0018	−9%
	(-1.95)		(-1.83)		(-1.59)	

The average abnormal return for all firms and the two exchanges, NYSE and NASDAQ, by using equation 1.4. The frequency is weekly and the sample is from September 17, 2008 to December 30, 2014. $EW11$ and $EW55$ are long leg and short leg equally weighted portfolios of the FTD-SI strategy explained in section 1.5.2., and EW represents the AAR for the long-short strategy with equally weighted averaging. $VW11$, $VW55$ and VW are correspondingly defined for value weighted averaging. The t-statistics are corrected by the original method of Newey West (1987) with five lags.

Table 1.15: Futures Pressure on FTD-Short Interest Strategy

Firms	Contract	Long Leg				Short Leg			
		D	t-stat	AM	t-stat	D	t-stat	AM	t-stat
All Firms	E-Mini Dow Jones	-0.01	-0.99	-0.03	-0.73	0.00	0.51	0.00	-0.05
	E-Mini Nasdaq 100	-0.01	-0.98	0.01	1.08	0.00	0.24	0.01	0.76
	E-Mini Russell 2000	0.02	1.67	-0.03	-2.32	0.02	1.92	-0.03	-2.64
	E-Mini S&P 400	0.00	-0.19	-0.02	-1.38	0.02	2.34	-0.04	-2.32
	E-Mini S&P 500	0.02	1.10	-0.06	-2.50	0.05	2.38	-0.05	-2.60
NYSE	E-Mini Dow Jones	0.01	2.11	-0.04	-1.99	0.00	0.15	0.05	1.34
	E-Mini Nasdaq 100	0.00	1.07	-0.01	-0.93	-0.01	-0.83	0.01	1.37
	E-Mini Russell 2000	0.00	0.05	0.01	1.65	0.01	1.40	-0.03	-2.20
	E-Mini S&P 400	0.00	0.23	0.01	0.68	0.01	1.23	-0.03	-1.63
	E-Mini S&P 500	0.00	0.20	0.02	1.42	0.01	0.54	-0.05	-1.51
NASDAQ	E-Mini Dow Jones	-0.01	-0.52	-0.07	-1.71	0.01	1.08	-0.04	-0.90
	E-Mini Nasdaq 100	-0.01	-1.45	0.01	1.37	0.01	1.16	0.00	-0.40
	E-Mini Russell 2000	0.02	2.04	-0.04	-2.61	0.02	1.69	-0.03	-1.63
	E-Mini S&P 400	0.00	-0.33	-0.02	-0.95	0.03	2.23	-0.04	-1.79
	E-Mini S&P 500	0.03	1.16	-0.10	-3.97	0.06	2.14	-0.06	-1.88

The slope of futures pressure in equation 1.5 for Dealers (D) and Asset Managers (AM) on the long leg, short leg and long-short leg strategy explained in section 1.5.2. The sample is from September 17, 2008 to December 30, 2014. The t-statistics are corrected by the original method of Newey West (1987) with five lags.

Table 1.16: Panel regressions of new FTDs and Futures Pressure

	D	AM	LF	OR	NR
E-Mini Dow Jones	0.56***	-0.71	-0.79***	-0.22	-1.63***
	(4.47)	(-1.65)	(-3.62)	(-0.29)	(-4.49)
E-Mini Nasdaq 100	0.45***	-0.7***	-0.8***	-0.5	-0.68*
	(4.48)	(-3.54)	(-5.39)	(-0.58)	(-1.73)
E-Mini Russell 2000	0.18	0.19	-0.88***	-2.44*	-5.95***
	(1.59)	(1.42)	(-3.03)	(-1.8)	(-3.51)
E-Mini S&P 400	0.37***	-0.1	-0.49**	-0.41	-5.31**
	(3.19)	(-0.45)	(-2.25)	(-0.33)	(-2.44)
E-Mini S&P 500	-0.04	0.06	0.61	-1.85**	-0.65*
	(-0.14)	(0.24)	(1.54)	(-2.04)	(-1.81)

The estimated slope of futures pressure, η, in monthly panel regression which is given by equation 1.3, where the dependent variable is newly created FTD_{jt}, defined by equation 1.6 with $\gamma = 0.03$, for firm j in month t. The independent variable of interest is futures pressure (FP_{it-1}) for trader i in month $t-1$, given by equation 1.1, for five E-Mini contracts and five trader categories. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The control variables are Institutional ownership, IO, Short Interest, SI, Market Capitalization, $MCAP$, Book-to-Market, BM, Turnover, $Turnover$, Put Open Interest, POI, Return, *Return*, defined in section 1.3.3. The standard errors are clustered by both firm and time and only firm fixed effect has been used. The sample is from September 17, 2008 to December 30, 2014. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 1.17: Panel regressions of FTDs and Futures Pressure with Stock Order Imbalance

	η, γ	D	AM	LF	OR	NR
E-Mini Dow Jones	η	1.27***	-1.6	-2.01***	0.38	-2.52**
		(4.12)	(-1.64)	(-3.02)	(0.19)	(-2.28)
	γ	-0.49**	-0.45**	-0.51**	-0.45**	-0.46**
		(-2.18)	(-2.04)	(-2.23)	(-2.06)	(-2.08)
E-Mini Nasdaq 100	η	1.09***	-1.76***	-1.99***	0.75	-1.47
		(3.35)	(-2.66)	(-4.48)	(0.37)	(-1.48)
	γ	-0.49**	-0.48**	-0.48**	-0.45**	-0.46**
		(-2.21)	(-2.21)	(-2.14)	(-2.05)	(-2.07)
E-Mini Russell 2000	η	0.48	0.43	-2.22**	-5.49	-13.3***
		(1.47)	(1.33)	(-2.7)	(-1.43)	(-4.25)
	γ	-0.46**	-0.44**	-0.44**	-0.44**	-0.49**
		(-2.06)	(-2.01)	(-2.05)	(-2.08)	(-2.2)
E-Mini S&P 400	η	0.88***	0.01	-1.05**	-2.43	-17.4**
		(2.67)	(0.02)	(-2.09)	(-0.72)	(-2.2)
	γ	-0.48**	-0.45**	-0.46**	-0.45**	-0.41**
		(-2.13)	(-2.16)	(-2.08)	(-2.07)	(-2.58)
E-Mini S&P 500	η	-0.34	0.25	2.03**	-4.78**	-1.79*
		(-0.52)	(0.43)	(2.43)	(-2.41)	(-1.73)
	γ	-0.45**	-0.45**	-0.48**	-0.45**	-0.44**
		(-2.06)	(-2.04)	(-2.17)	(-2.03)	(-2.03)

The estimated slope of futures pressure, η, in monthly panel regression which is given by equation 1.3, controlled for Stock Order Imbalance, with estimated slope of γ, defined by equation 1.7. The dependent variable is FTD_{jt} for firm j in month t. The independent variable of interest is futures pressure (FP_{it-1}) for trader i in month $t-1$, given by equation 1.1, for five E-Mini contracts and five trader categories. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The control variables are Institutional ownership, IO, Short Interest, SI, Market Capitalization, $MCAP$, Book-to-Market, BM, Turnover, $Turnover$, Put Open Interest, POI, Return, *Return*, defined in section 1.3.3. The standard errors are clustered by both firm and time and only firm fixed effect has been used. The sample is from September 17, 2008 to December 30, 2014. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 1.18: Time Series regressions of FTDs and Futures Pressure with Price Pressure

	η, γ	D	AM	LF	OR	NR
E-Mini Dow Jones	η	0.24***	-0.21	-0.31**	-0.44	-0.92***
		(2.75)	(-0.87)	(-2.26)	(-1.38)	(-3.22)
	γ	-0.03	0.48	-0.05	1.01	0.49
		(-0.15)	(0.62)	(-0.24)	(0.91)	(0.92)
E-Mini Nasdaq 100	η	0.20***	-0.23**	-0.45***	-0.54	-0.38*
		(3.63)	(-2.33)	(-4.35)	(-0.95)	(-1.67)
	γ	0.15	-0.18	0.03	-0.43	0.27
		(0.60)	(-0.37)	(0.07)	(-0.41)	(0.53)
E-Mini Russell 2000	η	0.21***	-0.08	-0.38***	-1.92***	-4.49***
		(2.65)	(-0.55)	(-2.81)	(-3.42)	(-4.15)
	γ	-0.33	0.29	0.52	-4.29**	3.97**
		(-0.76)	(0.48)	(0.99)	(-2.35)	(2.05)
E-Mini S&P 400	η	0.27***	-0.44***	-0.24*	-0.98	-0.50
		(2.94)	(-2.74)	(-1.86)	(-1.06)	(-1.09)
	γ	-0.70*	0.73*	0.17	1.43	0.66
		(-1.83)	(1.73)	(0.39)	(0.58)	(0.64)
E-Mini S&P 500	η	0.21	0.08	-0.13	-0.72	-0.41
		(0.83)	(0.31)	(-0.42)	(-0.97)	(-1.30)
	γ	0.72	-0.91*	0.04	-0.76	0.95
		(1.21)	(-1.77)	(0.07)	(-0.49)	(1.28)

The estimated slope of futures pressure, η, in weekly time series regression which is given by equation 1.2, controlled for price pressure, with estimated slope of γ, defined by equation 1.9. The dependent variable is FTD_{jt} for firm j in month t. The independent variable of interest is futures pressure (FP_{it-1}) for trader i in month $t-1$, given by equation 1.1, for five E-Mini contracts and five trader categories. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The control variables are Institutional ownership, IO, Short Interest, SI, Market Capitalization, $MCAP$, Book-to-Market, BM, Turnover, $Turnover$, Put Open Interest, POI, Return, $Return$, defined in section 1.3.3. The standard errors are clustered by both firm and time and only firm fixed effect has been used. The sample is from September 17, 2008 to December 30, 2014. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 1.19: Panel regressions of ETF FTDs and Futures Pressure

	D	AM	LF	OR	NR
E-Mini Dow Jones	-548.8	1843.2	737.67*	1688.1	-189.9
	(-1.57)	(1.24)	(1.68)	(0.57)	(-0.17)
E-Mini Nasdaq 100	-263.3	555.15	147.04	1862.7	48.6
	(-1.48)	(1.49)	(0.49)	(1.05)	(0.07)
E-Mini Russell 2000	-116.4	-284.3	962.58	1996.6	6854.9*
	(-0.49)	(-0.66)	(1.52)	(0.59)	(1.73)
E-Mini S&P 400	-388.8	-219.4	1524.2**	-8382	-2780
	(-1.38)	(-0.84)	(2.02)	(-1.35)	(-1.12)
E-Mini S&P 500	-74.09	-762.8	1244.3	3132	-126.1
	(-0.12)	(-0.84)	(1.27)	(1)	(-0.1)

The estimated slope of futures pressure, η, in monthly panel regression which is given by equation 1.3, where the dependent variable is FTD_{jt} for ETF j in month t. The independent variable of interest is futures pressure (FP_{it-1}) for trader i in month $t-1$, given by equation 1.1, for five E-Mini contracts and five trader categories. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The control variables are Institutional ownership, IO, Short Interest, SI, Market Capitalization, $MCAP$, Book-to-Market, BM, Turnover, $Turnover$, Put Open Interest, POI, Return, *Return*, defined in section 1.3.3. The standard errors are clustered by both firm and time and only firm fixed effect has been used. The sample is from September 17, 2008 to December 30, 2014. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 1.20: Panel regressions of NYSE FTDs and Futures Pressure

	D	AM	LF	OR	NR
E-Mini Dow Jones	1.33***	-1.92*	-1.9***	-1.26	-2.62**
	(4.43)	(-1.94)	(-3)	(-0.69)	(-2.19)
E-Mini Nasdaq 100	1.14***	-1.91***	-1.87***	1.29	-1.58
	(3.81)	(-3.11)	(-5.15)	(0.75)	(-1.63)
E-Mini Russell 2000	0.5	0.53*	-2.44***	-8.56**	-11.9***
	(1.54)	(1.79)	(-2.9)	(-2.15)	(-3.56)
E-Mini S&P 400	0.85**	0.37	-1.31****	-0.77	-15.1**
	(2.53)	(0.6)	(-2.68)	(-0.25)	(-2.15)
E-Mini S&P 500	-0.23	0.95*	1.42	-4.54**	-2.98***
	(-0.36)	(1.77)	(1.63)	(-2.56)	(-3.34)

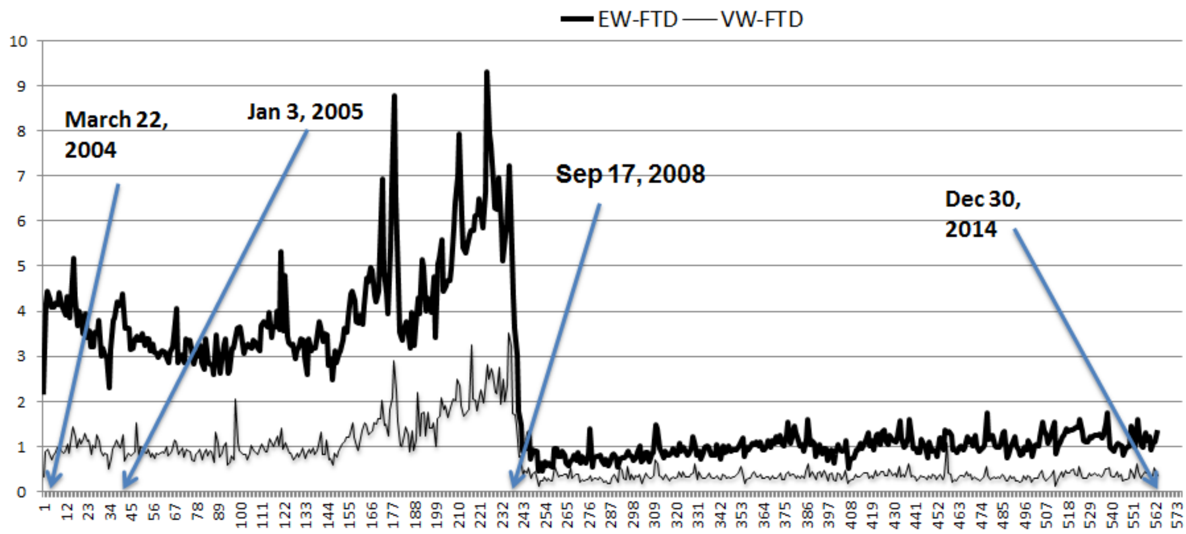
The estimated slope of futures pressure, η, in monthly panel regression which is given by equation 1.3, where the dependent variable is FTD_{jt} for NYSE firm j in month t. The independent variable of interest is futures pressure (FP_{it-1}) for trader i in month $t-1$, given by equation 1.1, for five E-Mini contracts and five trader categories. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The control variables are Institutional ownership, IO, Short Interest, SI, Market Capitalization, $MCAP$, Book-to-Market, BM, Turnover, $Turnover$, Put Open Interest, POI, Return, *Return*, defined in section 1.3.3. The standard errors are clustered by both firm and time and only firm fixed effect has been used. The sample is from September 17, 2008 to December 30, 2014. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Table 1.21: Panel regressions of NASDAQ FTDs and Futures Pressure

	D	AM	LF	OR	NR
E-Mini Dow Jones	1.25***	-1.41	-2.04***	1.06	-2.52**
	(3.32)	(-1.24)	(-2.79)	(0.51)	(-2.16)
E-Mini Nasdaq 100	1.07***	-1.68**	-2.05***	0.27	-1.37
	(2.77)	(-2.23)	(-3.53)	(0.12)	(-1.33)
E-Mini Russell 2000	0.44	0.42	-2.16**	-3.66	-13.2***
	(1.21)	(1.1)	(-2.27)	(-0.82)	(-3.71)
E-Mini S&P 400	0.86**	0.01	-0.98*	-3.17	-18.3**
	(2.27)	(0.02)	(-1.67)	(-0.78)	(-2.1)
E-Mini S&P 500	-0.39	0	2.12**	-4.86**	-1.16
	(-0.56)	(0)	(2.38)	(-2.22)	(-0.85)

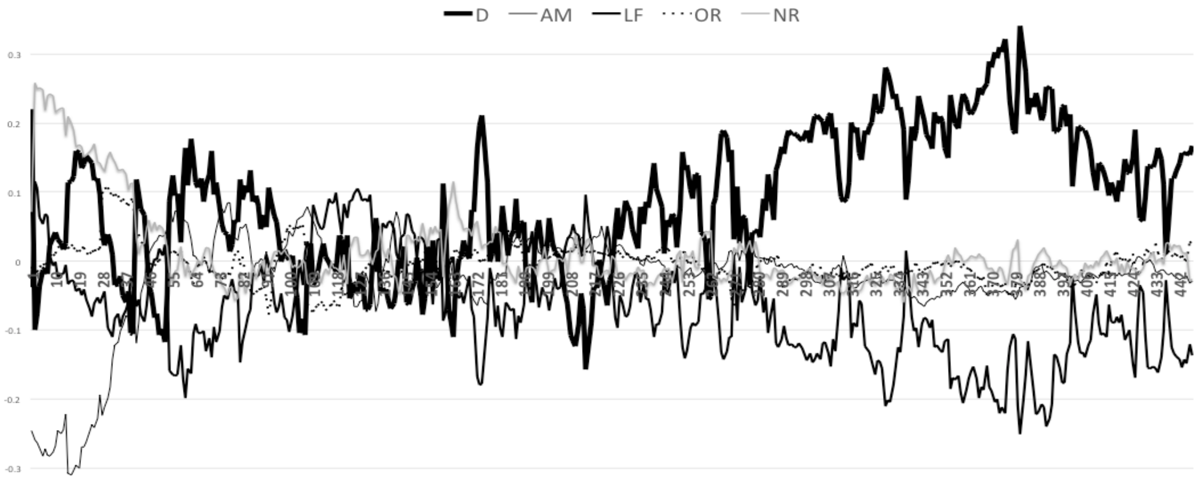
Th estimated slope of futures pressure, η, in monthly panel regression which is given by equation 1.3, where the dependent variable is FTD_{jt} for NASDAQ firm j in month t. The independent variable of interest is futures pressure (FP_{it-1}) for trader i in month $t-1$, given by equation 1.1, for five E-Mini contracts and five trader categories. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1. The control variables are Institutional ownership, IO, Short Interest, SI, Market Capitalization, $MCAP$, Book-to-Market, BM, Turnover, $Turnover$, Put Open Interest, POI, Return, *Return*, defined in section 1.3.3. The standard errors are clustered by both firm and time and only firm fixed effect has been used. The sample is from September 17, 2008 to December 30, 2014. * Significant at 10%; ** significant at 5%; *** significant at 1%.

Figure 1.1: Weighted average of failure to delivery as a percentage of shares outstanding



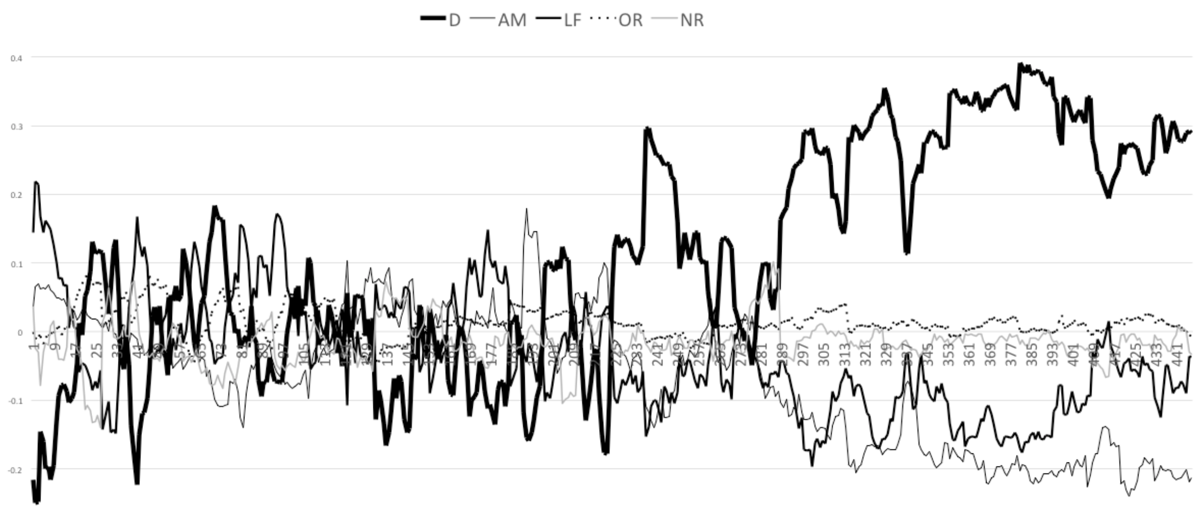
The value-weighted and equally-weighted average of failure to delivery as a percentage of shares outstanding shown in basis points. There are 563 weekly observations from March 22, 2004= $week\#1$, to December 30, 2014= $week\#562$. The change in regulation SHO happened on September 17, 2008= $week\#235$.

Figure 1.2: E-Mini DJIA's Futures Pressure for Five Trader Categories



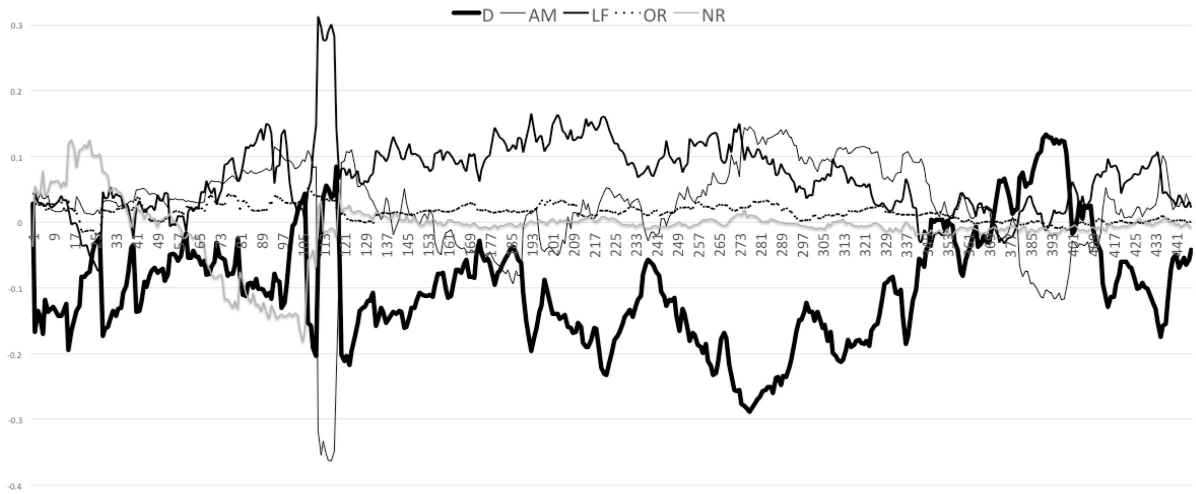
E-Mini DJIA's futures pressure defined in equation 1.1 for different trader categories from June 13, 2006= *week*#1 to December 30, 2014= *week*#447. September 17, 2008 corresponds to *week*#120. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1.

Figure 1.3: E-Mini NASDAQ 100's Futures Pressure for Five Trader Categories



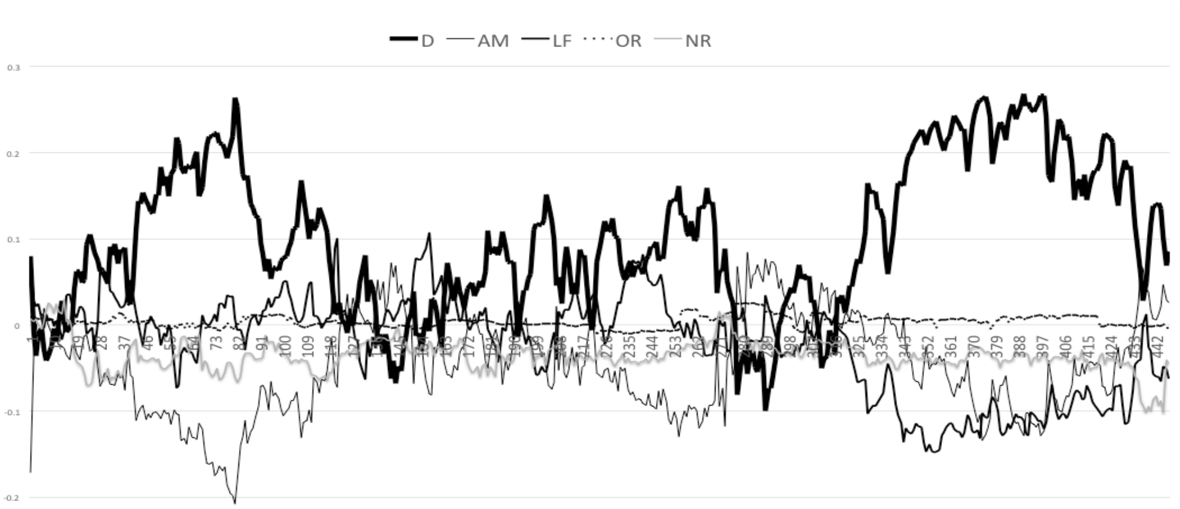
E-Mini NASDAQ 100's futures pressure defined in equation 1.1 for different trader categories from June 13, 2006= *week*#1 to December 30, 2014= *week*#447. September 17, 2008 corresponds to *week*#120. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1.

Figure 1.4: E-Mini Russell 2000's Futures Pressure for Five Trader Categories



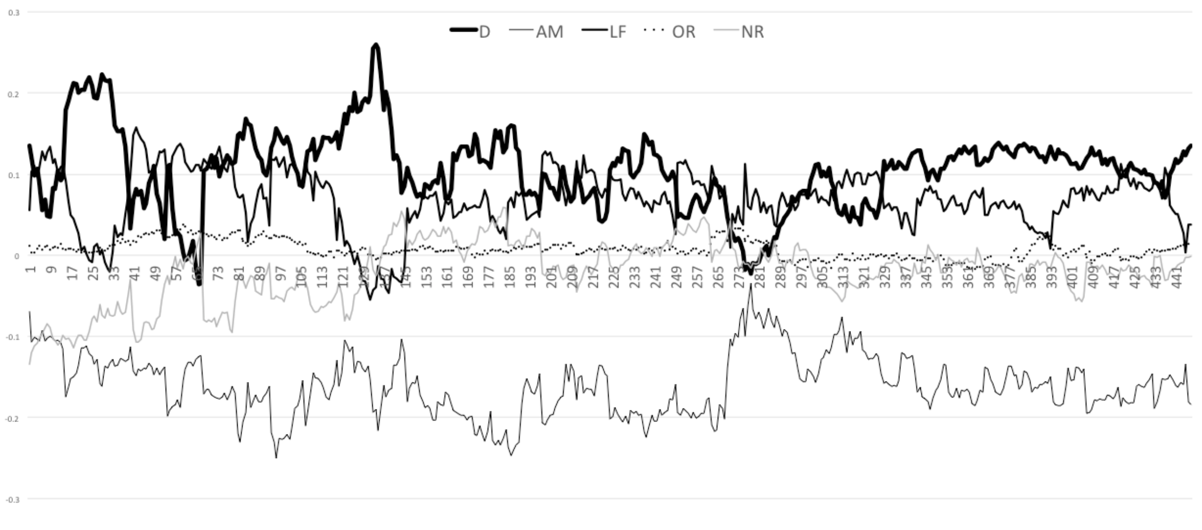
E-Mini Russell 2000's futures pressure defined in equation 1.1 for different trader categories from June 13, 2006= *week*#1 to December 30, 2014= *week*#447. September 17, 2008 corresponds to *week*#120. In September 2008, this contract was moved from CME to be traded on ICE. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1.

Figure 1.5: E-Mini $S\&P$ 400's Futures Pressure for Five Trader Categories



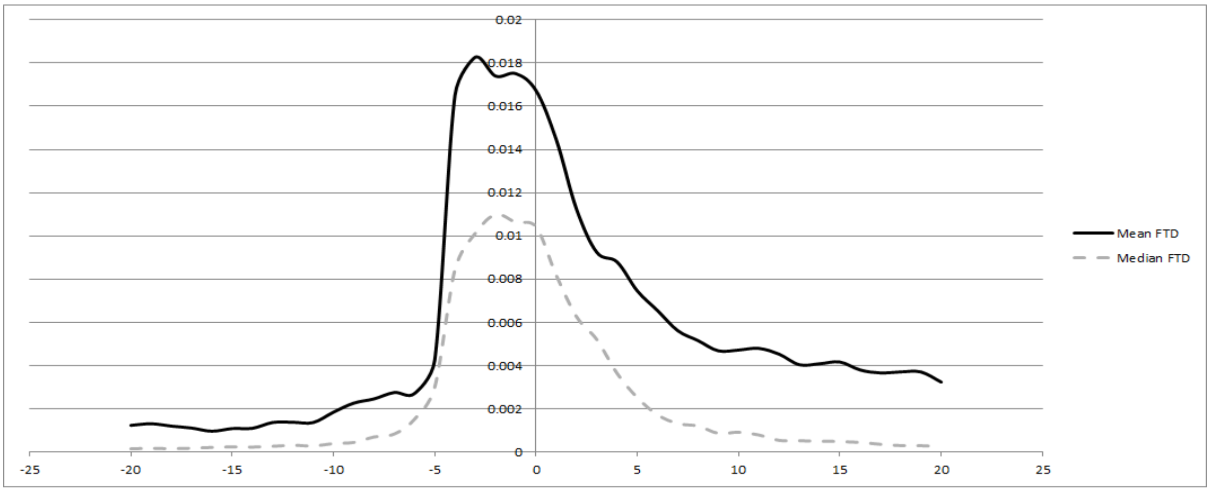
E-Mini $S\&P$ 400's futures pressure defined in equation 1.1 for different trader categories from June 13, 2006= *week*#1 to December 30, 2014= *week*#447. September 17, 2008 corresponds to *week*#120. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1.

Figure 1.6: E-Mini $S\&P$ 500's Futures Pressure for Five Trader Categories



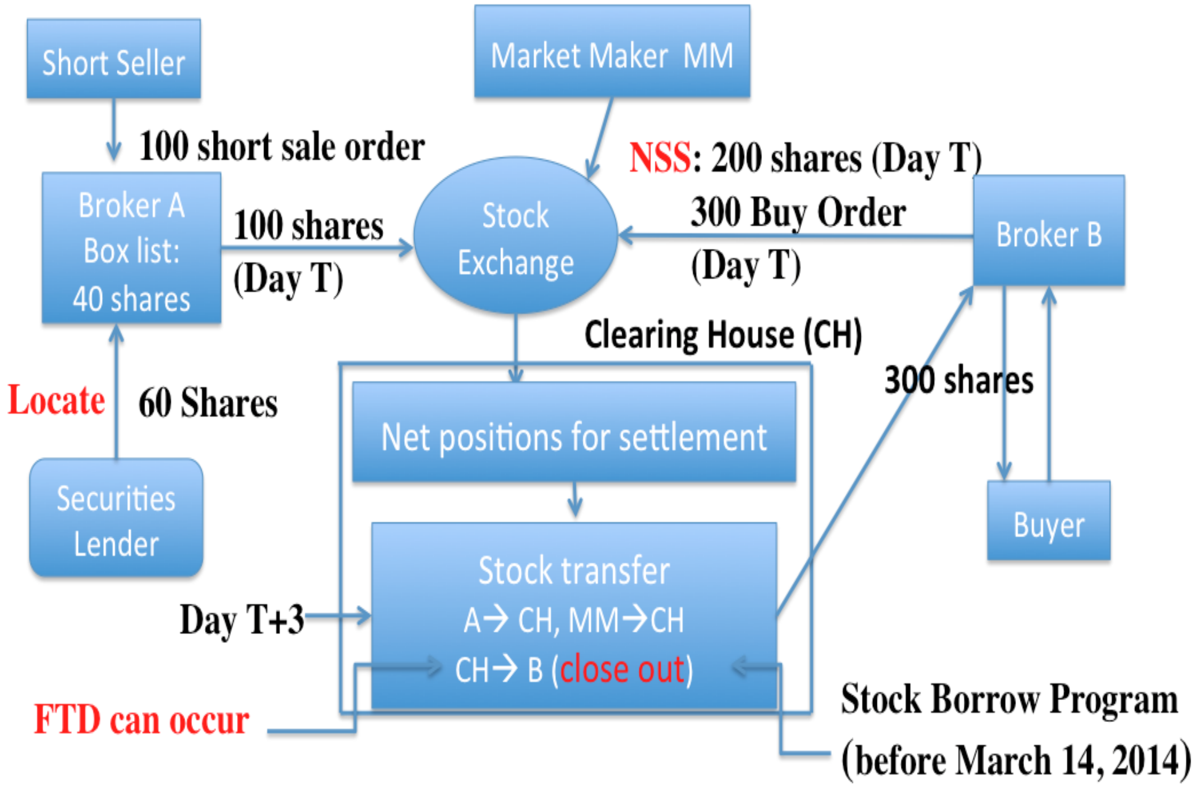
E-Mini $S\&P$ 500's futures pressure defined in equation 1.1 for different trader categories from June 13, 2006= $week\#1$ to December 30, 2014= $week\#447$. September 17, 2008 corresponds to $week\#120$. *D, AM, LF, OR* and *NR* stand for Dealers, Asset Managers, Leveraged Funds, Other Reportables and Non-Reportables which are defined in section 1.3.1.

Figure 1.7: FTD's Mean and Median Around Threshold Listing



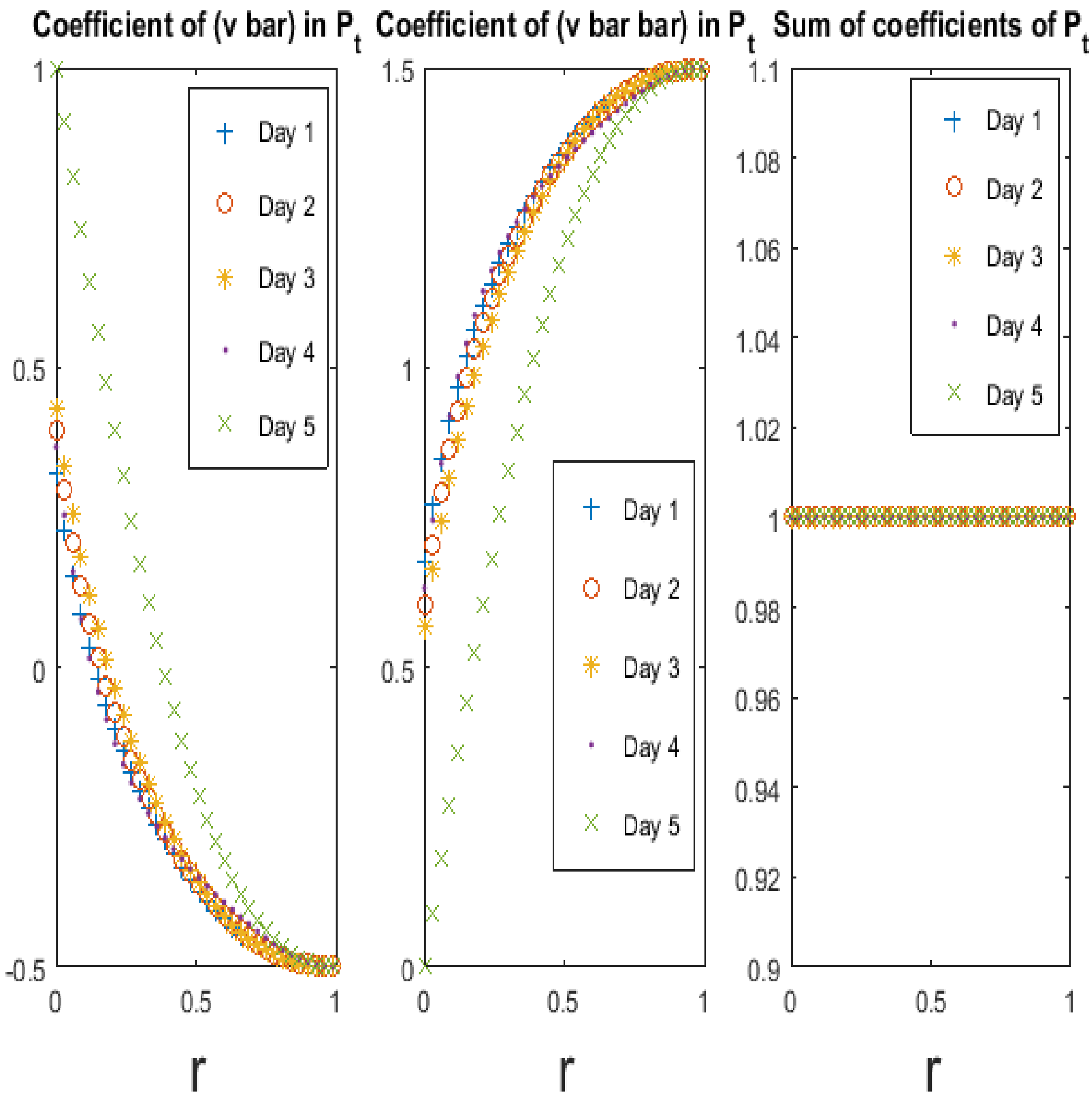
The mean and median values of failure to deliver as a percentage of shares outstanding, FTD, around threshold listing day (=0) from day -20 to day +20. The sample is from September 17, 2008 to December 30, 2014. The sample of securities and threshold listings are explained in sections 1.3.1 and 1.5.1.

Figure 1.8: Short Selling Structure



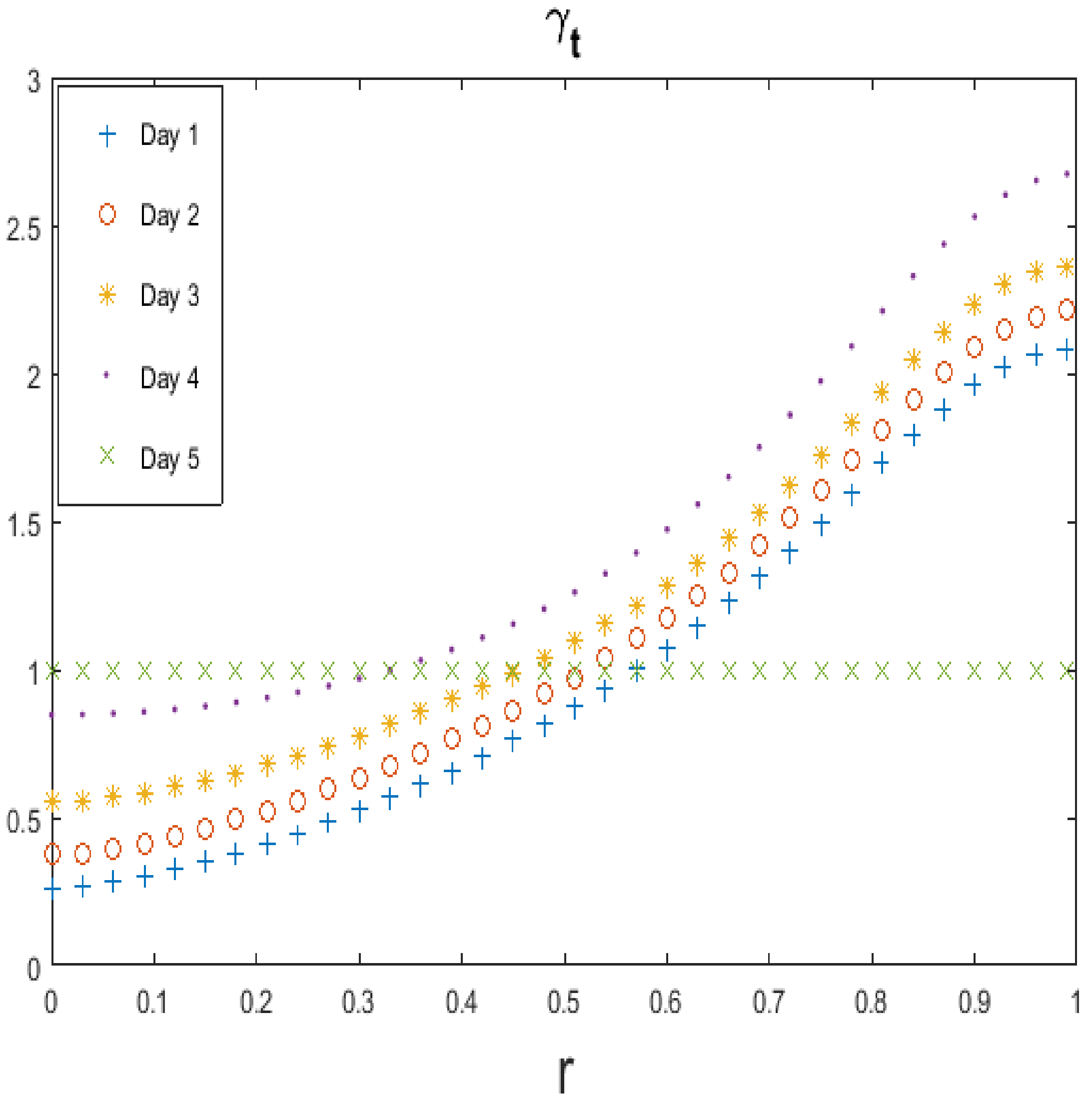
The mechanism of short selling. Refer to Appendix 1.9 for details.

Figure 2.1: Coefficients of equilibrium price and their sum as functions of r



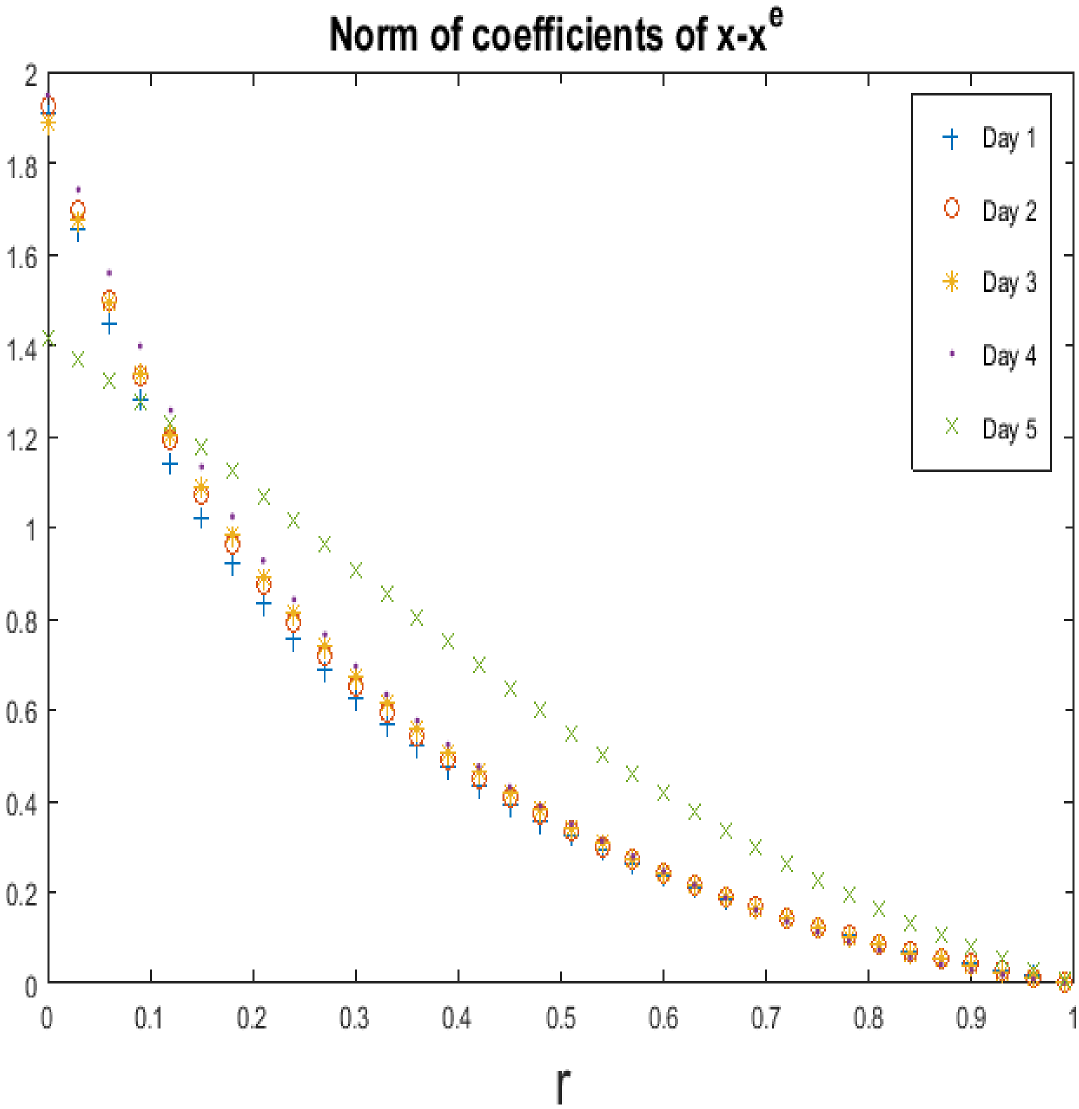
The coefficients of equilibrium price, P_t, in equation 2.45 and their sum for the model presented in section 2.3.2 are plotted against the parameter r. I assume that $T = 6$ and $t = 1, 2, 3, 4, 5$ are the trading dates. The chosen parameter values are: $\rho_0 = 1$, $q_t = \lambda_t = 2$, $\zeta_t = \iota_t = \kappa_t = 1$. The r values are from $\{r_0 + k\Delta r\}_{k=0}^{N-1}$ with $r_0 = 0.0001$, $\Delta r = 0.03$, and $N = 34$.

Figure 2.2: γ_t as a function of r



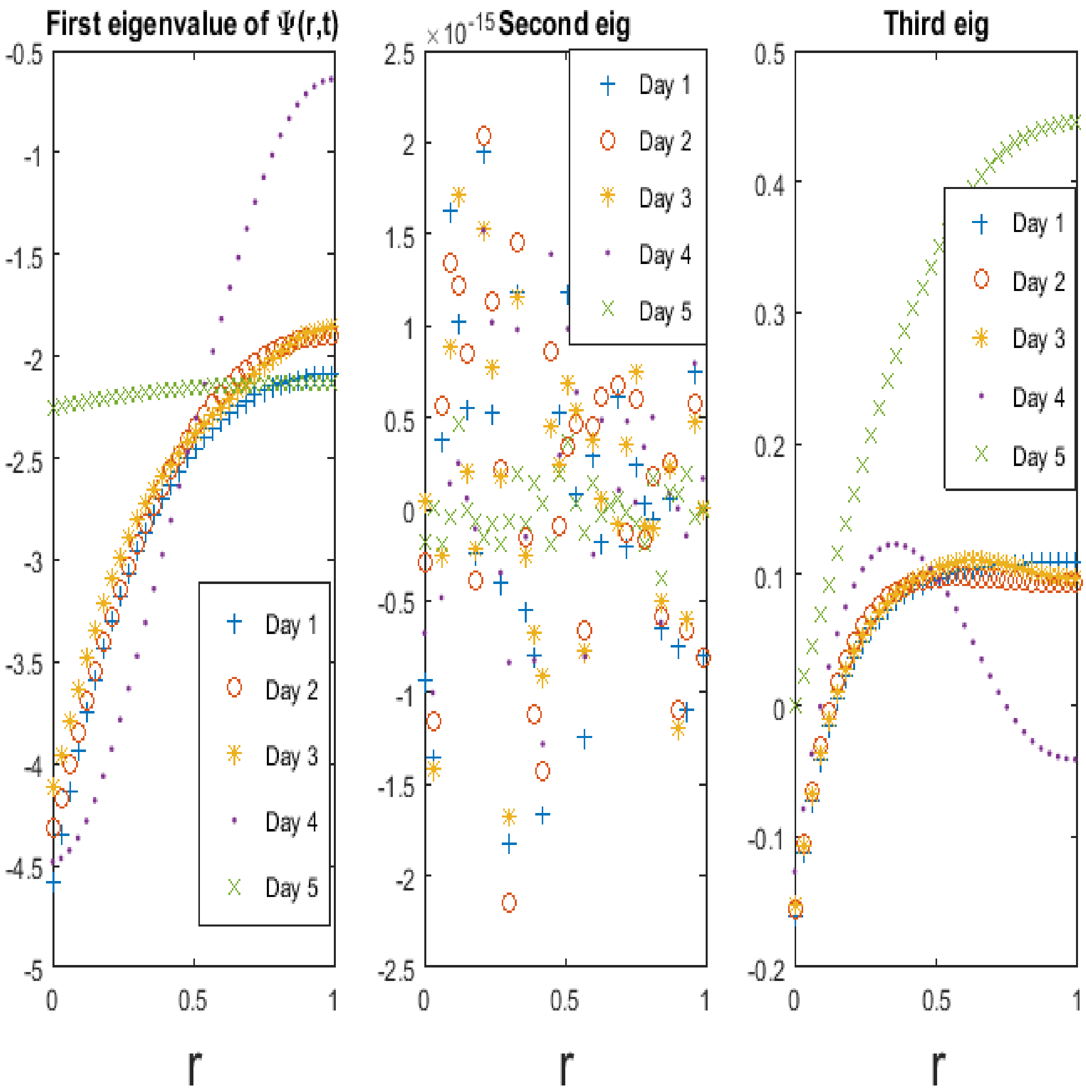
The parameter γ_t in equation 2.81 for the model presented in section 2.3.2 is plotted against the parameter r. I assume that $T = 6$ and $t = 1, 2, 3, 4, 5$ are the trading dates. The chosen parameter values are: $\rho_0 = 1$, $q_t = \lambda_t = 2$, $\zeta_t = \iota_t = \kappa_t = 1$. The r values are from $\{r_0 + k\Delta r\}_{k=0}^{N-1}$ with $r_0 = 0.0001$, $\Delta r = 0.03$, and $N = 34$.

Figure 2.3: Norm of the coefficients of $x_{it} - x_{it}^e$ as a function of r



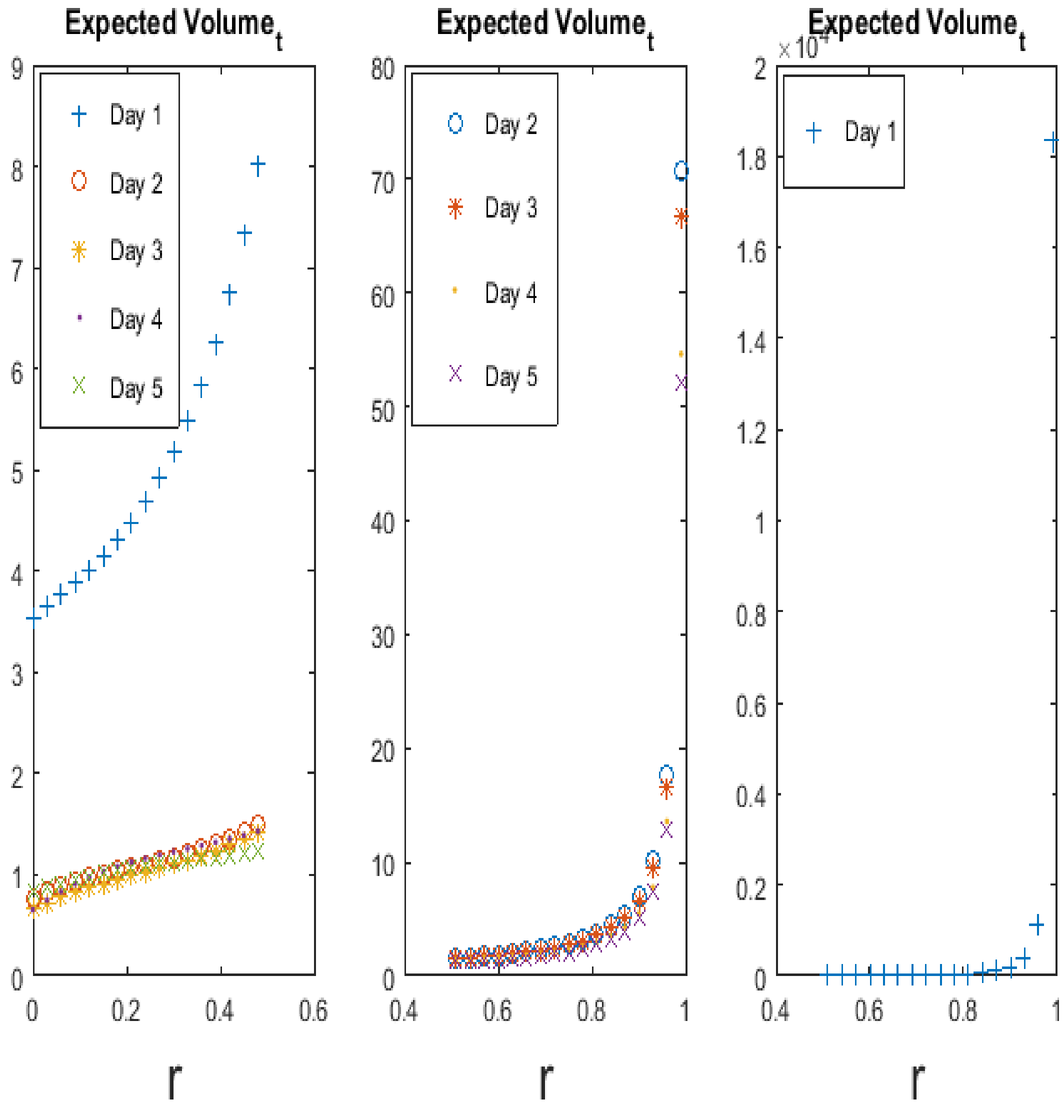
The norm of the coefficients of $x_{it} - x_{it}^e$ in equation 2.120 for the model presented in section 2.3.2 is plotted against the parameter r. I assume that $T = 6$ and $t = 1, 2, 3, 4, 5$ are the trading dates. The chosen parameter values are: $\rho_0 = 1$, $q_t = \lambda_t = 2$, $\zeta_t = \iota_t = \kappa_t = 1$. The r values are from $\{r_0 + k\Delta r\}_{k=0}^{N-1}$ with $r_0 = 0.0001$, $\Delta r = 0.03$, and $N = 34$.

Figure 2.4: Eigenvalues of Ψ_t as functions of r



The eigenvalues of Ψ_t defined in equation 2.70 for the model presented in section 2.3.2 are plotted against the parameter r. I assume that $T = 6$ and $t = 1, 2, 3, 4, 5$ are the trading dates. The chosen parameter values are: $\rho_0 = 1$, $q_t = \lambda_t = 2$, $\zeta_t = \iota_t = \kappa_t = 1$. The r values are from $\{r_0 + k\Delta r\}_{k=0}^{N-1}$ with $r_0 = 0.0001$, $\Delta r = 0.03$, and $N = 34$.

Figure 2.5: Expected Volume as a function of r



The expected volume given in equation 2.76 for the model presented in section 2.3.2 is plotted against the parameter r. I assume that $T = 6$ and $t = 1, 2, 3, 4, 5$ are the trading dates. The chosen parameter values are: $\rho_0 = 1$, $q_t = \lambda_t = 2$, $\zeta_t = \iota_t = \kappa_t = 1$. The r values are from $\{r_0 + k\Delta r\}_{k=0}^{N-1}$ with $r_0 = 0.0001$, $\Delta r = 0.03$, and $N = 34$.

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