Variable Speed Limit Compliance Impact on Bottleneck Lane Changing Pattern

by

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Abstract

Variable Speed Limit (VSL), an active traffic demand management (ATDM) strategy, is widely exercised to improved freeway operations by influencing collective driving pattern during congested condition. Since high traffic flow coupled with elevated lane changing rate can onset congestion on freeway bottleneck locations, VSL benefits should be correlated with its' proficiency in modifying lane changing patterns of traffic stream. Additionally, different compliance level (CL) of VSL should have different impact on congestion reduction and thus on lane changing pattern. Existing macroscopic lane changing models are proved to be inadequate to address the influence of two major aspects of traffic flow variables (speed difference and density difference) on lane changing decisions. Since most of the earlier lane changing models considered either one of them as lane changing incentive, their estimation were found to be impressionable by roadway traffic condition. This study introduces a model framework to approach the problem using simple microscopic lane-changing concept within a macroscopic model. The proposed model considers both speed and density difference as lane-changing incentive based on prevailing roadway condition. The resulting macroscopic model was calibrated and validated with empirical data. The results showed that the proposed model was able to competently reproduce the expected lane changing flows at different traffic state. The proposed lane changing model was applied to estimate lane changing from field observations at a recurrent bottleneck section for 'without' and 'with' VSL control cases. The primary objective of this research is to study the effect of optimal VSL control on collective lane changing by modeling several compliance level to VSL and also to study field observations in order to identify the transformation in lane changing behavior due to VSL Control. In this study, several CL to VSL strategies have been modeled to quantify the transformation in lane changing pattern at different compliance level. The model data analysis have shown definite correlations between CL and lane changing flow. Model output also demonstrated that mobility benefits can be achieved through VSL control with improved compliance. To investigate this conjecture, the field test data of Advisory VSL control on Whitemud Drive, Edmonton, Alberta, Canada have been analyzed. The research findings demonstrated that voluntary compliance to VSL have insignificant implications on compliance behavior and consequently collective lane changing pattern. To obtain substantial advantage, the VSL compliance improvement will be necessary which will eventually bring favorable change in collective lane changing pattern.

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Chapter 1
Introduction

1.1 INTRODUCTION

Traffic congestion is considered to be one of the major issue of traffic operations. The continuously increasing vehicles and traffic demands during peak hours often lead to congested roadway condition. Since roadway capacity is limited, the additional demand are accommodated with added delay for road users. A research study of Transport Canada stated that, a conservation estimation of total annual cost of congestion for the major cities in Canada ranges from $2.3 billion to $3.7 billion by consuming extra time and fuel, and increased emission of greenhouse gases. Moreover, the unstable traffic states arise due to congested roadways increase the collision probability and reduce road users' safety.

The urban freeway users are the predominant victims of congestion. Since most of the urban commuters use freeways in morning peaks to go to the offices and evening peaks to return homes, congestions occur regularly during peak hours at freeway bottlenecks, on-ramps and off-ramps. Non-recurrent congestions can take place due to traffic incident, roadway maintenance and construction, adverse weather conditions etc. One of the solution of this problem can be expanding road infrastructure which is constrained by space availability and high capital investment. More economically viable solution of managing the excessive traffic demand is to make efficient use of existing road networks. Some of these Active Traffic Demand Management (ATDM) strategies include: Ramp Metering (RM), Variable Speed Limits (VSL), Route Guidance (RG) etc., which are intended to increase the efficiency of existing roadways.

The question of efficient use of roadways arise due to the fact that traffic congestions reduce the capacity of the roadways. During high traffic demand the vehicles keep moving maintaining a close gap and often look for more comfortable driving condition. As a result, the drives tend to change lane whenever they find some incentive and feel safe to complete the maneuver without causing any collision. As the traffic demand increases the demand of lane changing increases too and the effects of these maneuvers are far more severe than that of low traffic demand condition.

These sort of lane changing during high traffic demand can initiate shockwave and consequently reduce the traffic throughput at bottleneck sections.

The purpose of VSL is to adjust freeway speed limits when the traffic state is approaching congestion due to high traffic demand. A suitably operated and enforced VSL control can avoid the constraints mentioned above by providing a practically viable speed restriction and help moving traffic in an organized manner through the bottleneck without reducing the capacity. Given the enormous cost of congestion to society and urgent need for solutions, this research investigates the VSL strategy effectiveness since it supposed to improve the collective driver behavior of freeway mainline traffic.

1.2 RESEARCH PROBLEM STATEMENT

Lane changing is a basic driving behavior and an integral part of freeway driving. Every driver take part in this maneuver at some point of their driving experience and route choice decision. From research standpoint, this routine maneuver must be thoroughly understood since they have significant implications on traffic flow. Due to disruptive nature of this maneuver, it is important to perceive their importance on triggering traffic breakdown phenomenon on freeways. Therefore, lane changing in freeways demand comprehensive research. Understating overall operation of freeways include apprehending the factors involved in lane changing decision making process. Only proper perception of freeway operation can assist in employing suitable traffic management strategies to increase the efficiency of freeway.

Over the past few decades, numerous research efforts had been made based on observation of freeway traffic characteristics and dynamics. These researches lead to the development of strategies and systems for dynamic traffic management. However, considering the limitations of this approach, the most recent researches are focused on developing model-based perception of traffic dynamics and make use of these models on operational control and dynamic traffic management. For instance, Cell Transmission Model (CTM) based Ramp Metering (RM) was utilized in various circumstances ***(1-3)***. Use of second-order model for Variable Speed Limit (VSL) on freeway operation control is another example of utilization of analytical model ***(4-9)***.

VSL is a freeway traffic control strategy that aims at restricting traffic throughput on bottleneck section to the capacity of the bottleneck section by reducing speed limits at an upstream location of the bottleneck. The potency of this strategy depends on regulating the traffic throughput. Since lane changing traffic create voids in traffic streams and these voids can effectively reduce traffic flow, therefore, the capability of VSL control is also dependent on reforming collective lane changing patter. Again, frequent lane changing maneuver at high traffic demand can arise congested traffic condition. Consequently, the perception of mechanism of lane changing decision making process can provide insights on the effectiveness of VSL control. Therefore the quest of this research lies on understanding the answers of two principal detail: (a) the lane changing behavior of freeway traffic and (b) the influence of VSL compliance on reforming this behavior.

1.3 RESEARCH OBJECTIVES AND SCOPES

Considering the issues discussed in previous section, this research is devoted to provide a comprehensive perception of lane changing behavior and factors influencing lane changing by developing a macroscopic lane changing model. Additionally, this research is intended to explore the influence of VSL compliance on aggregated lane changing pattern of road users. Reflecting the major focus, this research intends to provide fundamental support for macroscopic lane changing model and employ that model to identify the implications VSL control on collective lane changing pattern of freeway traffic. The overall objective of the research can be broken down into two major objectives:

(a) Exploring the lane changing decision making process in macroscopic level of detail. In contrast to earlier macroscopic lane changing model, this model will consider varying aspects of speed and density difference in lane changing decision at different traffic state. Successful implementation of the model in estimating lane changing flow will provide us with the opportunity to identify potential transformation in collective lane changing pattern due to different traffic control and management strategies. Additionally, this study will promote traffic management strategies involving speed and density control to restrict lane changing maneuvers in order to increase operational efficiency of freeways.

(b) Study the shift in lane changing behavior of road users due to imposed VSL control on recurrent bottleneck locations of freeways. VSL controls on freeway are supposed to restrict mainline traffic throughput in bottleneck which can be achieved by changing lane changing behavior. This study will inspect the potential of VSL control with regard to reforming collective lane changing pattern from microscopic model simulation and field observations. Varying compliance to VSL will also be considered in model simulation to identify potential impact of compliance on collective lane changing behavior.

The research scope will be restricted to freeways. The test site will be approximately 7-kilometers (km) long in between east of 122 Street on-ramp and west of 159 Street off-ramp on Whitemud Drive (WMD), Edmonton, Alberta, Canada. The studied roadway has a static speed limit of 80 kilometers per hour (kph) and experiences a directional average annual daily traffic (AADT) of about 100 thousands vehicle. The freeway is outfitted with 9 loop detector stations on the mainline, each consisting of dual-loop detector groups in each travel lane. Each of the on-ramp and off-ramps are equipped with loop detectors. An identical microscopic WMD model is coded and calibrated to obtain measured traffic data and feedback to model predictive control (MPC) based VSL control. A real field test of MPC based VSL control took place on the studied corridor from August 11, 2015 to September 4, 2015 in order to reduce the detrimental effects of recurrent evening peak congestions on WMD. The data collected during the field test will also be used to identify the true influence of VSL compliance on lane changing.

1.4 RESEARCH CONTRIBUTIONS

The work performed in this research explored some fundamental aspects of traffic dynamics and make a link among them. The major contributions of this research includes:

a) Proposing a simple and efficient macroscopic lane changing model by acknowledging varied influence of speed and density at different traffic state. Earlier macroscopic lane changing models considered individual influence of fundamental traffic flow parameters for estimating lane changing flow and disregarding other influencing parameters often lead to erroneous estimation. Furthermore, the distinction in lane changing behavior at different traffic state was ignored in the process. This study compared and acknowledged the effects of two fundamental traffic flow parameter in lane changing at different traffic state.

b) Examining the impact of VSL control compliance on lane changing practice of road users using model simulations and field observations. This study can contribute to our knowledge base by providing insights about VSL control implications in transforming lane changing characteristics of road users. Microscopic model simulation of real-world traffic enabled us to apprehend possible impacts of VSL compliance on lane changing. Moreover, field observations of VSL control provide sapience of compliance characteristics at different speed limits as well as on collective lane changing.

1.5 ORGANIZATION OF THE THESIS

As stated before, this research is focused on two major issues and the thesis is formed keeping those issues in mind. Chapter 2 briefly reviews the previous macroscopic models and their shortcomings in explaining freeway lane changing behavior. This chapter also discuss lane changing maneuver from microscopic perspective. A macroscopic lane changing model is proposed in this chapter that is responsive at different traffic condition. Afterwards, the calibration and validation of the proposed model is presented in this chapter. Chapter 3 starts by discussing the previous studies on effectiveness of VSL control. Followed by a simulation study of MPC based VSL control on the studied roadway at varying compliance level. The result analysis identifies the varied extent of transformation in collective lane changing behavior at different level of compliance. The results are supported by lane changing estimation using the proposed model (in Chapter 2) from the field test data. The research conclusions, limitations and future research directions are presented in Chapter 4.

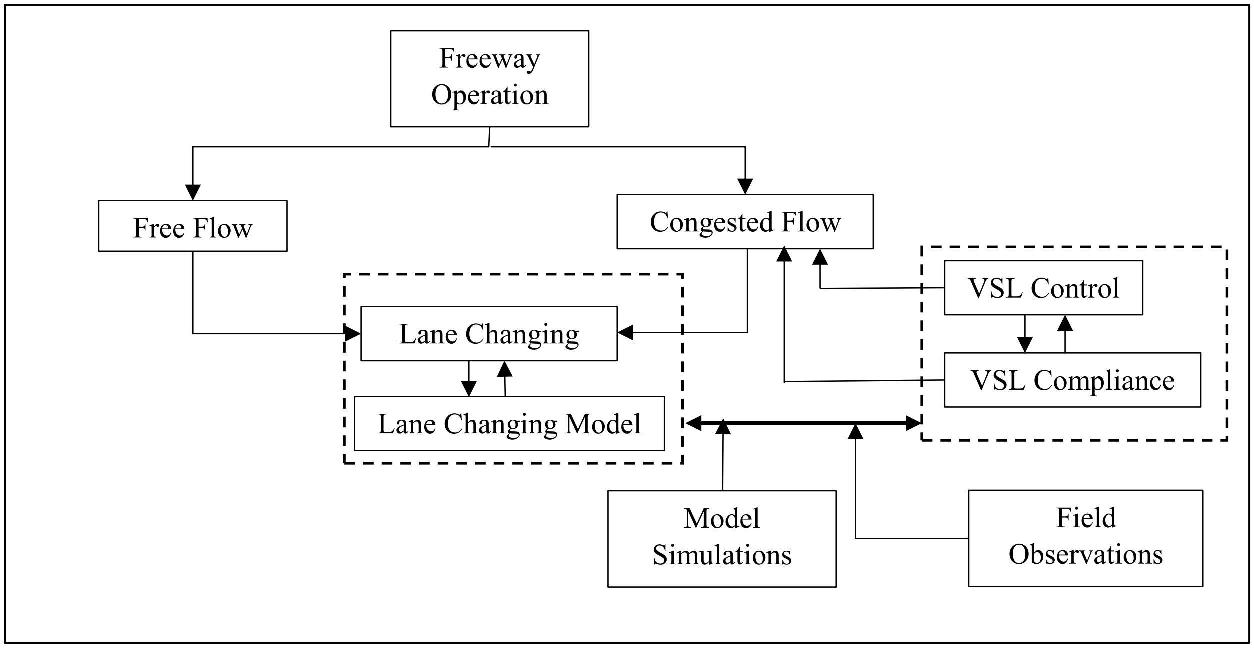

FIGURE 1.1 Research Components Correlation Flow Chart

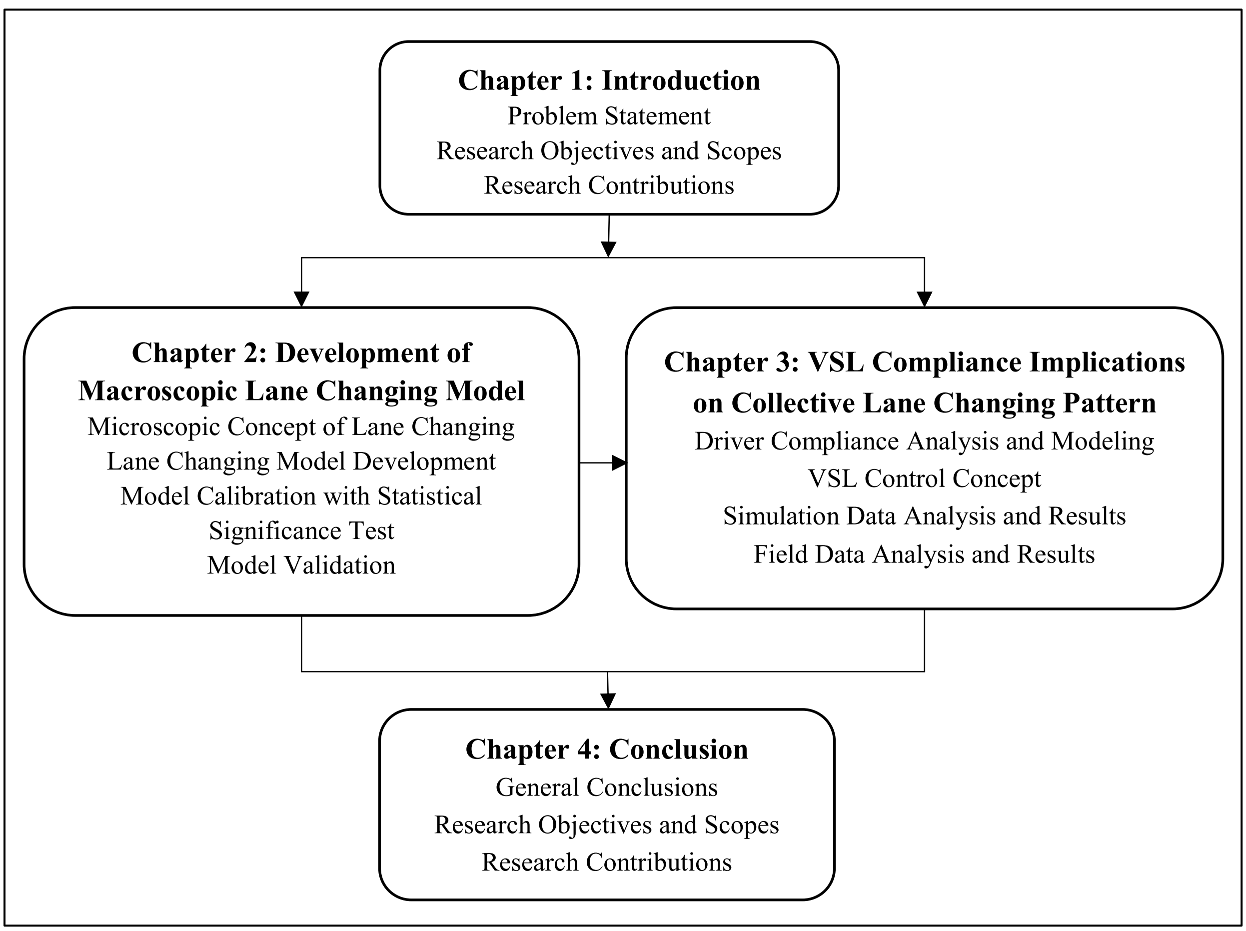

FIGURE 1.2 Flow Chart of Main Contents of the Thesis

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Chapter 2
Development of a Macroscopic Lane Changing Model

2.1 INTRODUCTION

Lane changing maneuver in roadways are considered to have a disruptive effect on continuous traffic flow which can cause congestion and even worse, lead to collisions. Since these two are major problems of modern motorway system, the vehicular lane changing behavior has generated enormous research attention. Numerous studies have attempted to capture its qualitative behaviors and quantitative understanding which resulted into a number of lane changing models. Most of the macroscopic lane changing model proposed so far are established upon some microscopic considerations (car following, gap acceptance etc.) while considering homogeneous groups of vehicle in the roadway. To date, macroscopic lane changing studies were focused mainly on two aspects: (a) the relation of the number of lane changes with other traffic state variables, such as traffic flow, mean speed, density etc. ***(1-3)*** and (b) lateral and longitudinal distribution of lane changing ***(4-7)***. The prior aspect of lane changing studies considered change or difference in traffic flow variables as the source of lane changing. These models considered that the imbalance of different traffic variables among adjacent lanes caused the lane changing in order to bring about balance in traffic state. Macroscopic lane changing models proposed by Laval ***(8)*** considered speed difference between adjacent lanes as a source of lane changing, while in Zhu's model ***(9)***, the density difference between adjacent lanes had impact on lane changing. A new concept of lane changing intensity was introduced in Jin's model ***(10-12)*** which was a ratio of total lane changing time of lane changing vehicles to total travel time of all vehicles during a time interval. In this model, it was considered that the lane changing were incurred to achieve uniform density among lanes. Roncoli et al. ***(13)*** proposed a model to quantify the lateral and longitudinal flow among adjacent lanes which considered density difference as a motivation which was based on the notion of "density equilibrium" proposed by some previous researches ***(14-16)***. Helbing ***(17)*** introduced a macroscopic traffic flow model to account for acceleration, deceleration, overtaking and lane changing maneuvers. The model derived lane changing between interacting lanes considering queuing effects on the roadway. Knoop et al. ***(18, 19)*** had studied the total densities and lane densities at different traffic state and reported that lane changing flows were influenced by several incentives and the densities of the origin and the target lane were considered to be most substantial. Although these models considered different stimulus to estimate lane changing flow, they hardly pondered the idea of including two traffic state variables as source terms. Most of the previous models regarded either speed difference or density difference for estimating lane changing flow which could cause ambiguity when these source terms reach in an equilibrium state (either speed or density equilibrium). However, in reality, the lane changing flow did not stop when the traffic state reached one of those equilibrium.

To address this conflict, it is the main objective of this part of the study to consider disparity of both traffic state variables (speed, density) to estimate lane changing flow and to identify significance of these variables in generating lane changing flow at different traffic state. The simplicity of the model structure is the primary requirement of this study for enabling this model to use in large-scale road network. Having said that, the accuracy of this model is needed to be evaluated by real traffic data for employing it in practical purpose. The rest of the chapter is organized as follows: First, the concept of lane changing is discussed from microscopic point of view which gives an indication of influencing factors for different lane changing types at different traffic conditions. Grounded on microscopic concept the macroscopic model is developed on Lane changing model development section. This section also describes feasibility criterion of different lane changing pattern based on triangular fundamental diagram. Then the model parameters are calibrated in Model calibration section with real traffic data for different model scenarios. Followed by, model validation section that identify different lane changing pattern at different traffic state and estimate them accordingly. Conclusions are then presented with suggestion for future research directions.

2.2 LANE CHANGING MODELS

With the realization about the significance of lane changing in traffic congestion formation and traffic safety, numerous effort was made by researchers to model this aspect of traffic behavior. These models consider different decision making process of lane changing and different impacts on surrounding traffic due to lane changes. Although, there had been significant progress in modeling traffic lane changing behavior, a complete understanding of lane changing remained elusive. For the sake of clarity and focus, this section concentrated on few representative lane changing models in the literature, rather than trying to comprehensively cover all existing models.

2.2.1 Gipps Lane Changing Model

Gipps ***(20)*** was among the first to propose a lane changing model that incorporated car following theory and introduced a structure for lane changing decision for drivers facing conflicting goals. Although trade-offs among choices and variations of driving behavior were ignored, this deterministic model provided a structure for lane changing model based on drivers tendency to maintain the desired speed or prepare for turning maneuver. More precisely, as the driver approaches a desired turn, the priority shifts from maintaining desired speed to staying in correct route. The lane changing model proposed by Gipps was a generic model that can used with any car-following model if the following vehicle's speed is bounded by appropriate condition as the following equation:

$$v_n(t+T) = b_nT + \{\mathrm{b_n^2T^2} - \mathrm{b_n}\left[2\left(\mathrm{x_{n-1}(t)} - \mathrm{s_{n-1}} - \mathrm{x_n(t)}\right) - \mathrm{v_n(t)T} - \frac{\mathrm{v_{n-1}^2(t)}}{\hat{\mathrm{b}}}\right]\}^{1/2}$$

where, $v_n(t+T)$ is maximum safe speed for vehicle *n* with respect to leader *n-1* at time *t+T*, b_n is the most severe braking of driver at vehicle *n*,$\hat{\mathrm{b}}$ is an estimate of braking employed on leading vehicle, n-1 by driver of vehicle n, T is the time step of updating vehicle speed and position, $\mathrm{x_n(t)}$ is the location of vehicle n at time T and $\mathrm{s_{n-1}}$ is the effective length of vehicle n-1. By extending Gipps model, Yang and Koutsopoulos ***(21)*** developed and implemented mandatory and discretionary lane changing model in microscopic simulator, MITSIM. A distinct feature of Yang and Koutsopoulos ***(21)*** model was to make the model more realistic by considering lane changing probability.

2.2.2 Utility Theory based Lane Changing Model

Ahmed ***(22)*** had adopted utility theory to model the decision making process of lane changing. The proposed model consisted four unobservable decision hierarchy level, similar to Yang and Koutsopoulos ***(21)***. Driver heterogeneity and traffic state dependence were also considered in this model. According to this model, the utility of lane changing at time t for driver n would be

$$U_{tn} = \gamma^T X_{tn} + v_n + \varepsilon_{tn}$$

Where U_{tn} is the utility got driver n at time t; X_{tn} is a vector of explanatory variables; γ is a vector of unknown parameters; v_n is an individual-specific random term; ε_{tn} is a random term that varies across different time period for a given individual as well as across individuals. The conditional probability of observing a lane changing pattern for driver n was expressed as

$$P(\{J_{1n}, J_{2n}, \dots \dots, J_{tn}, \dots \dots, J_{T_n n}\} | v_n) = \prod_{t=1}^{T_n} P(J_{tn}|v_n) = \prod_{t=1}^{T_n} P(L|v_n)^{\delta_{tn}^L} P(R|v_n)^{\delta_{tn}^R} P(C|v_n)^{\delta_{tn}^C}$$

Where, $J_{tn} \in \{L, R, C\}$; L: change to the left lane; R: change to the right lane; C: continuing in the current lane.

$$\delta_{tn}^L = \begin{cases} 1 & \text{if driver n changes to the left lane at time t} \\ 0 & \text{otherwise} \end{cases}$$

δ_{tn}^R, δ_{tn}^C are similarly defined

The authors estimated the parameters of the model for a special case using vehicle trajectories; merging to the left lane from a freeway on-ramp, which requires both gap acceptance and lane changing decisions. From their study they found that, critical follow gap is an important factor for lane changing maneuver which is sensitive to traffic conditions. The study also exploited the forced merging behavior in heavily congested traffic. In this model, the mandatory and discretionary lane changing were estimated separately. However, this model failed to capture the conditions that triggers a mandatory lane changing and therefore unable to identify the trade-offs between mandatory and discretionary lane changing conditions.

2.2.3 Cellular Automata Based Lane Changing Model

Cellular automata were traditionally used to replicate macroscopic behavior of a complex system with minimal microscopic description. Several notable traffic cellular automate (TCA) models were developed to reproduce car following and lane changing behavior of traffic stream. Similar to traditional lane changing model, TCA based lane changing model considered desirability and necessity of lane changing and gap acceptance. According to TCA, a roadway segment with L number of lanes and K number of cells per lane can accommodate lane changing by satisfying following constraints:

$$g_{Si}^{l} \geq 0 \wedge g_{Si}^{f} \geq 0$$

where g_{Si}^{l} and g_{Si}^{f} are the lane changer's net space gaps with the leader and the follower in the target lane. One of the first TCA based lane changing model was porposed by Nagatani ***(23, 24)*** which was further improved by Wagner ***(25)*** and Nagel ***(26)***. Maerivoet and Moor ***(27)*** proposed two schemes to address the conflicts of occupying the same lane from two neighboring lanes in roads with more than two lanes. One of the major drawback of TCA based lane changing models are its rigidness in lane changing duration. These models considered the lane changing duration to be one time stamp (usually one second) which is unrealistic and inconsistent with observations.

2.2.4 Fuzzy Logic based Lane Changing Model

Fuzzy logics have been frequently applied in various fields, including lane changing modeling ***(28-32)***. The lane changing decision in fuzzy logic is divided into mandatory and discretionary lane changing and depends on necessity of lane changing and gap acceptance. The lane changing decisions are fuzzified as IF-THEN rules. The following is a typical If-THEN lane changing rule:

IF: (vehicle i is eligible for using the left lane) and (the gap between vehicle i and the leader in the left lane is large) and (the gap between vehicle i and the follower in the left lane is large) and (the speed in the current lane is low) and (the speed in the left lane is high)

THEN: (vehicle I changes to the left lane)

Usually fuzzy logics are considered to be capable of representing drivers' lane changing decision making process more accurately. However, defining fuzzy sets and their associated membership are challenging which makes calibrating and validating fuzzy logic based lane changing models extremely difficult.

2.2.5 Markov Process Based Lane Changing Model

The first Markov Chain based stochastic lane changing model was proposed by Worrall ***(2)*** which considered homogenous traffic flow and calibrated using lane changing data of a six lane Chicago freeway. Markov process based lane changing model proposed by Pentalnd and Liu ***(33)*** divided the lane changing decision making process into few continuous steps, which are: (a) centering the car in the initial lane, (2) observe clearance on target lane, (3) initiating lane changing, (4) executing lane changing decision, (5) terminating lane changing and (6) centering the car in target lane. Results obtained from the study stated that the lane changing actions are best defined as a sequence based control steps rather than as a sequence of raw position and velocities. Although this model can reproduce lane changing frequency, it was unable to explain the lane changing decision process which made them unfit for microscopic simulations. Toledo and Katz ***(34)*** proposed a utility theory based hidden Markov Model to overcome this limitation.

2.2.6 Daganzo's Lane Changing Model

Laval and Daganzo ***(8)*** have developed a kinematic wave theory based hybrid lane changing model where each lane is connected to its' neighboring lanes traffic by lane changing vehicles. The lane changing vehicles are considered as dimensionless moving bottlenecks that blocks traffic behind them. Assuming a triangular fundamental diagram, the net lane changing rate is defined by this model as

$$\varphi_{ji} = \frac{\mu_i}{T_i + \sum_{i \neq j} \Delta x L_{ji}} L_{ji}$$

Where μ_i is the available capacity of the lane i; T_i is the desired through flow of lane i; L_{ji} is the desired lane changing rate from lane j to lane i. Both T_i and L_{ji} were determined by density, time and space measurements. In discretized time-space dimension, the limit of lane changing rate was expressed as

$$\lim_{\Delta x \to o} \varphi_{ji} = \frac{\mu_i \pi_{ji} S_j}{u S_i}$$

Where, π_{ji} is the fraction of choice-makers per unit time with intension of changing lanes from lane j to i and assumed to be proportional to the speed difference between lanes j and I, which can be approximated as $\frac{\Delta v_{ji}}{u\tau}$; τ is the time a driver takes to decide and execute lane changing; S is the desired amount of advancing vehicle in discrete time stamp and can be approximated as $\Delta t \times \min\{uk, Q\}$; u is the free flow speed. Although, the proposed hybrid model provided consistent results in macroscopic level, its' compatibility with microscopic lane changing and car-following models are yet to be evaluated. Furthermore, this model did not differentiated between lane changing types (i.e. mandatory, discretionary) and scenarios (i.e. free, cooperative, forced).

2.2.4 Jin's Lane Changing Model

Jin ***(11, 35)*** proposed a novel approach to extend kinematic wave theory for modeling lane changing traffic flow. In his model, he introduced lane changing intensity, ε(x,t) that was denied as the ratio of the total lane changing time over the total travel time. Lane changing intensity was considered as a time and space dependent variable which could be determined by drivers' lane changing decision and their characteristics at the microscopic level. For uniform traffic, where traffic density is same in lanes and all vehicles travel at the same speed, the lane changing intensity is defined as

$$\varepsilon = \alpha \frac{\rho_{LC} t_{LC}}{\rho T}$$

Where ρ_{LC} is the density of lane changing traffic; t_{LC} is the lane changing duration; $\alpha = \frac{N_{LC}}{\rho_{lc} L}$ and $T = \frac{L}{v}$; L is the length of the lane changing region; N_{LC} is the total number of lane changing maneuvers in lane changing region during T. Jin calibrated lane changing intensities and corresponding fundamental diagrams as well as demonstrated that several traffic phenomenon related to lane changing (i.e. capacity drop, jam densities) could be captured by the proposed modeling framework. The analysis showed that systematic lane changes cause capacity drop and that depends on the proportion of the weaving traffic. Jin's model assumed that the number of lane change was proportional to the weaving flow rate. Therefore, this model was considered more suitable for modeling mandatory lane changing phenomenon.

2.3 MICROSCOPIC CONCEPT OF LANE CHANGING

Microscopic traffic models describe longitudinal car-following and lateral lane-changing behavior of individual vehicle. Most fundamental car following models were based on desired or safe distance criterion which usually assumed to maintain a linear relationship with speed. These car following models stated that the subject vehicle intends to maintain a safe distance from leading vehicle in the same lane and that distance was a function of subject vehicle's speed. Although, this model performed satisfactorily in most situations, problems were found when a lane changing occurs. In such situations, insertion of a vehicle from adjacent lane causes a distinct reduction in following distance. As a result, a sudden deceleration would generate from car-following model which in turn cause a shockwave upstream of the lane changing location. However, in real traffic, several lane changes can occur at a relatively short road span without causing any shockwave in upstream. This can be explained by the fact that drivers are usually ready to endure much shorter spacing (which is derived from speed function of car following model) when they can anticipate the action of the lane changing drivers and instead of braking abruptly, they moderately decelerate to ensure that spacing with the new leading vehicle increase gradually to desired spacing. This process created the link between car-following and lane changing model in microscopic level. Therefore, the role of lane changing model is to determine under prevailing condition whether it is feasible for the subject vehicle to move into the target lane considering the gap between leading and following vehicle in target lane as well as potential speed incentive for subject vehicle. A lane changing model, irrespective of microscopic or macroscopic, should always consider two essential criteria: (a) The Motivation Criterion; there should be a substantial reason for lane changing, such as desired speed increment or getting rid of congested state and (b) The Safety Criterion; the lane changing should only be permissible if the maneuver is safe, i.e. the gap available is adequate for a vehicle to get into the in target lane without colliding with adjacent vehicles.

The lane changing maneuver of subject vehicle and feasibility can be classified into three distinct types in regard to the space gap and speed incentive at target lane during the lane changing process ***(36)***.

a) Free Lane Changing: This type of lane changing happens when the gap between leader and follower is relatively large and entry of the subject vehicle does not cause observable hindrance in upstream. This type of lane changing occurs at free-flow condition when vehicles have numerous opportunities to change lane and the drivers wait for desired space gap in target lane to execute lane changing. According to Hidas ***(36)***, the subject vehicle can change lane if the gaps in front and behind the vehicle are within a minimum acceptable gap:

$$g_{l,min} = g_l(v_l) \quad \text{and} \quad g_{f,min} = g_f(v_s)$$

b) Cooperative Lane Changing: This type of lane changing is characterized by an increasing change in gap between leader and follower prior to lane change of the subject vehicle, indicating that the follower decelerate cooperatively to facilitate the lane changing. This type of lane change has dependency on both gap availability and anticipated speed reduction of the following vehicle in the target lane which Hidas ***(36)*** portrayed as following equations:

$$g_{l,min} = g_{min} + c_l(v_s - v_l) \quad \text{and} \quad g_{f,min} = g_{min} + c_f(v_f - v_s)$$

c) Forced Lane Changing: This type of lane changing is indicated by sharp increase in gap between leader and follower due to lane changing. This type of lane changing are perceivable during congested condition when vehicles are searching for an opportunity to sneak into neighboring lane to obtain speed or density benefits. The deceleration of following vehicles due to forced lane change is higher than that of Cooperative lane change. As a result of these sever deceleration, shockwaves generate at the upstream of lane changing location. The feasibility criterion for forced lane changing are similar to cooperative lane changes, except the fact that speed reduction anticipation are higher and gap between leading and following vehicles tends to decrease prior to lane change in this case. The following figure (Figure 2.1) showed the observable gaps prior to a lane change: $g_{f/l}^{S/T}$ is the gap between subject vehicle and following (f) or leading vehicle (l) in subject lane(S) or target lane (T), $g^{S/T}$ is the gap between leading vehicle and following vehicle in subject lane or target lane, $V_{f/l}^{S/T}$ is the speed of leading or following vehicle in subject or target lane. V^S is the speed of the subject vehicle.

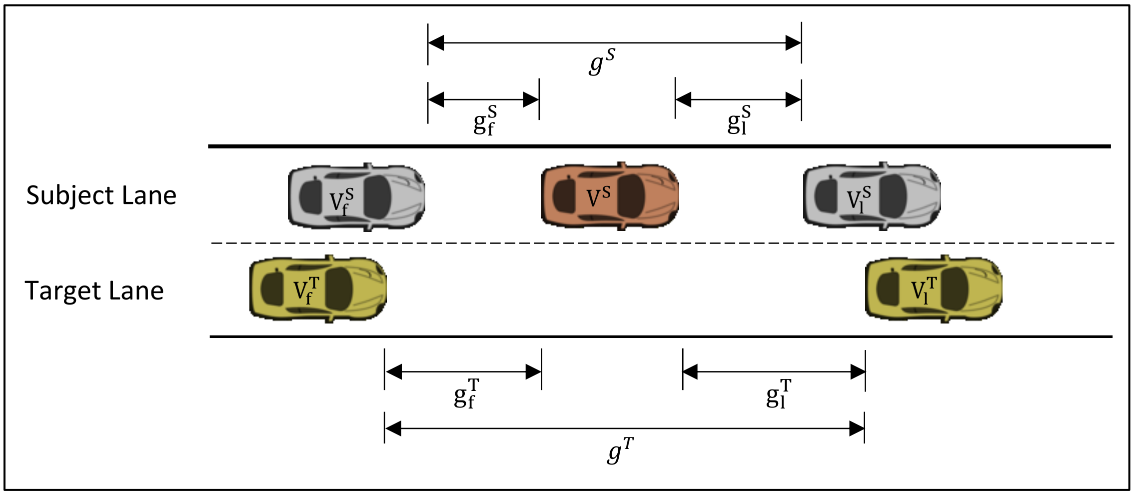


FIGURE 2.1 Lane changing parameters in Microscopic level of detailing

In macroscopic level of detailing, the models study the aggregated behavior considering groups of homogenous vehicles assessing traffic density, speed and flow rate as variables with respect to location and time. As both microscopic and macroscopic models characterize the motion of vehicles in roadways, it is possible to explain one with another through insightful understanding and simplification of comparable variables. As was proposed by Laval ***(37)*** during his attempt to formulate a link between macroscopic lane changing models with microscopic lane changing behavior. That study took a simplified approach of explaining microscopic parameters with macroscopic variables. Comparable approach with certain modification is taken to develop the macroscopic lane changing model in this study which will be described elaborately in next section.

2.4 LANE CHANGING MODEL DEVELOPMENT

For the purpose of this study, a multi-lane roadway network is divided into number of segments. The index i=1, 2, ….. , I is initiated for segments and the index j= 1, 2, ….., J for lanes. The discretized lane entities are describe by the following variables:

- The density $\rho_{i,j}(k)$ [vpkpl] is the number of vehicles per unit length of lane j in segment i at time step k
- The mean speed $v_{i,j}(k)$ [kph] of vehicles in lane j of segment i at time step k

- The flow $q_{i,j}(k)$ [vphpl] is the traffic volume that are passing through lane j of segment i during time interval $(k,k+1)$
- The net lane-changing flow, $\varphi_{i,j}(k)$ [veh/hr] is the total number of vehicles that change lane from $\hat{j}$ lane to j lane minus the vehicles that moved out from j to $\hat{j}$ at segment i during time interval $(k,k+1)$. Here, $\hat{j} = j \pm 1$.

The proposed lane changing considers the competition between lane changing demands due to varied intensive with the available capacity in the target lane. To bring about a balance between these two factors, a set of function was specified in this model: (a) Lane changing demand flow between lane j and ĵ, $\mathrm{D_j(k)}$ [vphpl], (b) Available capacity of the target lane j, $\mathrm{C_j(k)}$[vphpl] and (c) Vehicles passing through lane j, $\mathrm{T_j(k)}$ [vphpl]. Equation (2.1), (2.2) and (2.3) were used to estimate these three elements. At each iteration one calculates $\mathrm{D_j(k), C_j(k)}$ and $\mathrm{T_j(k)}$ to compute lane changing flow using incremental transfer (IT) principle (Equation 2.5).

$$\mathrm{D_{i,j}(k)}=\mathrm{w^v_{i,j}}\left[\mathrm{q_{i-1,\hat{j}}(k)} \times \max\left(0, \mathrm{v_{i,j}(k)} - \mathrm{v_{i,\hat{j}}(k)}\right) + \mathrm{q_{i-1,j}(k)} \times \min\left(0, \mathrm{v_{i,j}(k)} - \mathrm{v_{i,\hat{j}}(k)}\right)\right] + \mathrm{w^\rho_{i,j}}\left[\mathrm{q_{i-1,\hat{j}}(k)} \times \max\left(0, \rho_{\mathrm{i,\hat{j}}}(\mathrm{k}) - \rho_{\mathrm{i,j}}(\mathrm{k})\right) + \mathrm{q_{i-1,j}(k)} \times \min\left(0, \rho_{\mathrm{i,\hat{j}}}(\mathrm{k}) - \rho_{\mathrm{i,j}}(\mathrm{k})\right)\right] \quad \ldots\ldots\ldots\ldots\ldots (2.1)$$

$$\mathrm{C_{i,j}(k)} = \min\left\{\mathrm{w_{i,j}}\left(\rho^{\mathrm{jam}}_{\mathrm{i,j}} - \rho_{\mathrm{i,j}}(\mathrm{k})\right), \mathrm{Q_{max}}\right\} \quad \ldots\ldots\ldots\ldots\ldots (2.2)$$

$$\mathrm{T_{i,j}(k)} = \begin{cases} \mathrm{v^{free}_{i,j}} \times \rho_{\mathrm{i,j}}(\mathrm{k}) & \text{when } \rho_{\mathrm{i,j}}(\mathrm{k}) \le \rho^{\mathrm{cr}}_{\mathrm{i,j}} \\ \mathrm{w_{i,j}}\left(\rho^{\mathrm{jam}}_{\mathrm{i,j}} - \rho_{\mathrm{i,j}}(\mathrm{k})\right) & \text{when } \rho_{\mathrm{i,j}}(\mathrm{k}) > \rho^{\mathrm{cr}}_{\mathrm{i,j}} \end{cases} \quad \ldots\ldots\ldots\ldots\ldots (2.3)$$

Total Demand on Target Lane = $\mathrm{T_{i,j}(k)} + \sum \mathrm{D_{i,j}(k)}$ ………… (2.4)

Therefore, Net-lane changing flow, $\emptyset_{\mathrm{i,j}}(\mathrm{k}) = \min\left[1, \frac{\mathrm{C_{i,j}(k)}}{\mathrm{T_{i,j}(k)} + \sum \mathrm{D_{i,j}(k)}}\right] \mathrm{D_{i,j}(k)}$ ………… (2.5)

Here, $v_{i,j}^{free}$, $\rho_{i,j}^{cr}$ and $\rho_{i,j}^{jam}$ are free flow speed, critical density and jam density of lane j at segment i. $w^{v}_{i,j}$ and $w^{\rho}_{i,j}$ are segment and lane specific parameters that symbolizes the fraction of choice-makers wishing to change lane due to imparted speed and density benefits by the neighboring lane. Both these factors are affiliated with the position and location of lane as well traffic state. The inclusion of these factors and their dependency on traffic state make this model different than other lane changing model.

The lane changing demand generating function in Equation 2.1 considered that the vehicles approaching the studied roadway stretch ($q_{i-1,\hat{j}}, q_{i-1,j}$) would consider changing lane based on the speed and density imbalance between two adjacent lanes (target lane, j and neighbor lane, ĵ) and for maximum simplicity it was assumed that the lane changing decisions hold a linear relation to these imbalances. Employing Equation 2.1 total lane changing demand for target lane was measured. For instance, if the speed of target lane (j) at an time stamp was found to be higher and density was lower than neighbor lane (ĵ), then a fraction of vehicles travelling through neighbor lane (($q_{i-1,\hat{j}}$) would consider changing lane towards target lane. The proportion of vehicles that would consider lane changing would be linearly related to the difference in speed and density between two lanes [$v_{i,j}(k) - v_{i,\hat{j}}(k)$, $\rho_{i,\hat{j}}(k) - \rho_{i,j}(k)$]. The fraction of vehicle, that would consider changing lane due to these two inequality, would be decided by calibrating the model with lane changing field observation data.

Since there was demand for lane changing, there should be supply too. In this specific case, the supply would be the capacity of target lane. Flow through a lane at an instant is restricted by predominant lane flow capacity which is specified by Equation 2.2. Unlike other models, this model considered varied capacity of lane which depended on individual lane density at an instance. This consideration would give an opportunity to regard for capacity drop phenomenon during congested state. In addition to the lane changing demand, there was another demand which was desire of vehicles to pass though studied stretch of the road without changing lane. Considering a triangular fundamental diagram, the through flow is calculated using Equation 2.3 for target lane. To complete the lane changing maneuver, it is important to consider a balance between total demand generate by target lane (Equation 2.4) and the capacity of the target lane since the available capacity may not be sufficient for accepting full lane changing demand flow. The model, therefore, considered the incremental-transfer principle to estimate proportion of successful net-lane changing flow (Equation 2.5). For example, if the total demand at an instant was higher than the capacity of the target lane, only a portion of the lane changing demand would be fulfilled due to capacity constraints of the target lane.

As mentioned in previous section of the chapter, different types of lane changes are activated at different traffic state and lane changing feasibility depends on different factors. Therefore, this model calibrates these factors for different types lane change at different traffic state. Majority of the earlier macroscopic lane changing model considered some form of fixed empirical parameter to estimate the demand for lane changing. However, these models often neglected the transformation of lane changing behavior at different traffic state and their impact on those empirical parameters. In addition to that, the dependency on a single incentive for lane changing in all type of lane changing and traffic state reduced the competency of the model to be equivalently precise for roadway traffic condition. In light of different types of lane changing considered in microscopic detailing, this model has classified the lane changing into three types based on traffic state, which are as follows (Figure 2.2):

- Type I: This type of lane changing take place at free flow traffic state and at relatively low traffic density. Since the mean speed of lanes remain identical at low traffic density, speed incentive plays insignificant role in this type of lane change. The feasibility criterion for this type of lane changes are

$$\mathrm{v_j(k) \geq v_j^{free}} \qquad \text{and} \qquad \mathrm{\rho_j(k) \leq K\rho_j^{cr}}$$

  Here, K is an empirical factor that defines the extent of this type of lane change. An appropriate value of K can be determined by educated judgement of lane changing behavior and comparing the significance of $w_{i,j}^{v}$ for Type I lane changing at different K values.

- Type II: Lane changing of this type are influenced by both speed and density incentives. This type of lane changing initiates at a higher density when drivers start to sense restrained and try to move into a relatively unconfined lane. The feasibility criterion for Type II lane changing are:

$$\mathrm{v_j(k) \geq v_j^{free}} \qquad \text{and} \qquad \mathrm{K\rho_j^{cr} < \rho_j(k) < \rho_j^{cr}}$$

- Type III: With further increase in density, drivers are continuously looking for opportunities to change into lane with higher benefit and make the maneuver with a slightest convenience. To accommodate this, the following vehicles have to substantially reduce their speed which lead to lower mean speed. Therefore, the feasibility criterion for this type of lane changing are:

$$v_j(k) < v_j^{free} \quad \text{and} \quad \rho_j(k) \geq \rho_j^{cr}$$

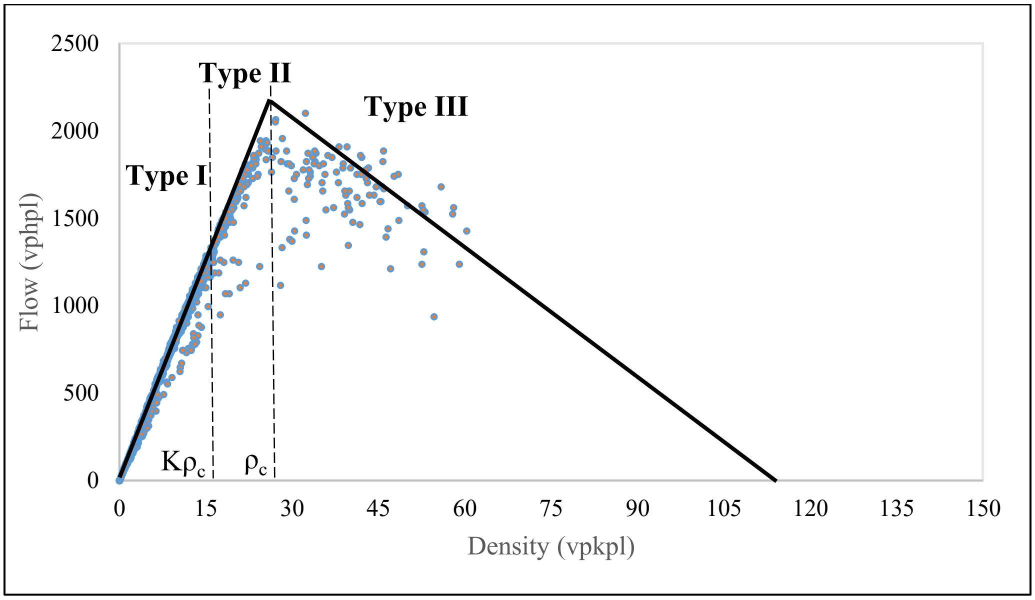


FIGURE 2.2 Classifying Lane Changing Type in Flow-Density Diagram

2.5 MODEL CALIBRATION PROCESS

The proposed macroscopic lane changing model introduced in pervious section was applied to a particular motorway in order to calibrate its parameters and validate the model formation. The chosen roadway is a stretch of Whitemud Drive located in Edmonton, Alberta, Canada. It is an urban freeway characterized by traffic demand pattern driving to and from the city. The studied stretch, sketched in Fig. 2.3, is 700 meter long and contains four lanes in westbound direction. It starts immediately upstream of the on-ramp connected to Fox Drive where it composed by three lanes and it terminates about 300 meter upstream of the off-ramp connecting 149 Street. Due to high traffic demand during peak hours, especially evening peak, this stretch of the road experience some congestion which influenced to select this segment of the road for lane changing study. Data was collected through two loop detector stations to deliver measurements of flow and mean speed of each lane in every 20 seconds. A traffic video camera is also located on the roadside that can capture traffic movements of the selected area (Fig 2.3). In the rest of the chapter, the lanes are numbered 1,…., 4 from the shoulder lane (close to shoulder) to the median lane (close to the road median). This study only considered the lane changing of Lane 4 (Median Lane) and Lane 3 (Outer Middle Lane) in order to maintain a lane changing behavior free from the influence of on-ramp and off-ramp flow.

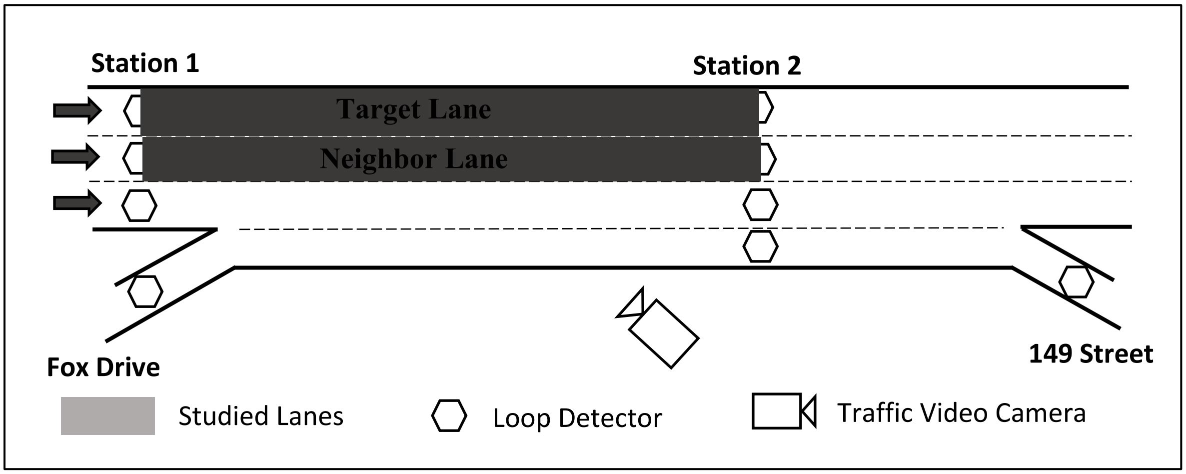


FIGURE 2.3 Studied Roadway Stretch for Lane Changing Model

Initially the lane changing flow were counted manually for each lane from the recorded video. Although the quality of the measured lane changing data was excellent, the procedure was long, time consuming and sensitive to the visibility. To eliminate these issues with satisfactory ground truth data, the flow conservation law between two loop detectors was used to calculate lane changing flow and then compared with the measured lane changing flow from the recorded video to justify the applicability of the method. The flow conservation equation used to measure the lane changing flow is as following:

$$\rho_{2,j}(k+1) = \rho_{2,j}(k) + \frac{T}{L}\left[q_{1,j}(k) - q_{2,j}(k) + \emptyset_{2,j}(k)\right] \qquad \ldots\ldots\ldots\ldots (2.6)$$

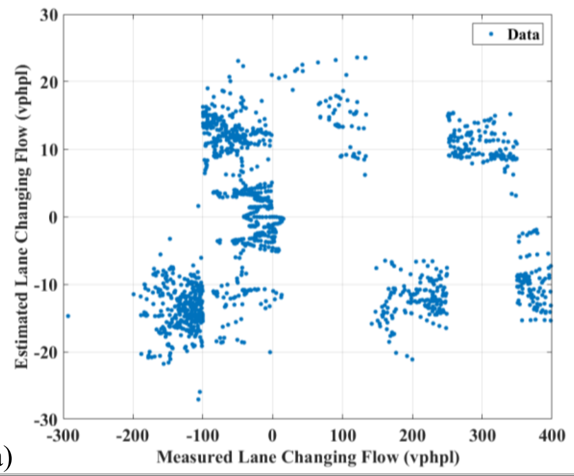
Here $q_{1,j}$, $q_{2,j}$ are the traffic flow through the studied lanes obtained from loop detector Station 1 and Station 2 respectively, $\rho_{2,j}$ is the density obtained from loop detector data at Station 2 and $\emptyset_{2,j}$ is the net-lane changing flow at the same location. All the parameters were measured at 5 minutes moving average time window that updates in every 20 seconds. The video was recorded during afternoon peak and mid-day off-peak hours for 30 minutes duration in each cases. The results were compared with the obtained lane changing flow data from video recordings. The comparison showed that the net-lane changing flow calculated using flow conservation equation were within ±10% of the measured lane changing from video recordings. That enabled us to apply Equation 2.6 to calculate net-lane changing flow using loop detector data and associate it with the model parameters calibration and validation.

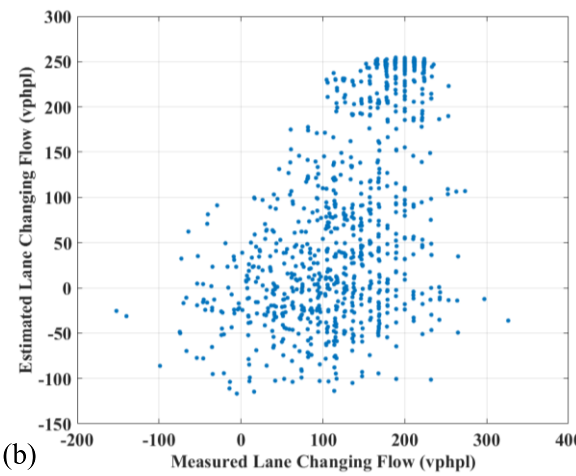
At the beginning, the triangular fundamental diagram was plotted for the studied lane to obtain free-flow speed ($v_{i,j}^{free}$), critical density ($\rho_{i,j}^{cr}$), jam density ($\rho_{i,j}^{jam}$) and congestion speed ($w_{i,j}$) for each studied lane. Identification of $v_{i,j}^{free}$ and $\rho_{i,j}^{cr}$ are essential to distinguish between different types of lane changing at different traffic state. In this study, the value of K was assumed as 0.5 for Lane 4 which suggested that lane changing of Type I would be observable up to half of critical density of that lane. Afterwards, the net-lane changing flow was measured from Station 1 and Station 2 loop detector data for three different types of lane changing. Station 2 loop detector were then used as an input of the model to calibrate the model parameters ($w_{i,j}^{v}, w_{i,j}^{\rho}$) using measured lane changing flow of different types. The calibration of the model parameters to best match the calculated lane changing flow was performed for the validation of the proposed lane changing model. Linear regression method was exercised to calibrate the model parameters. Three different case was considered for each type of lane change which are:

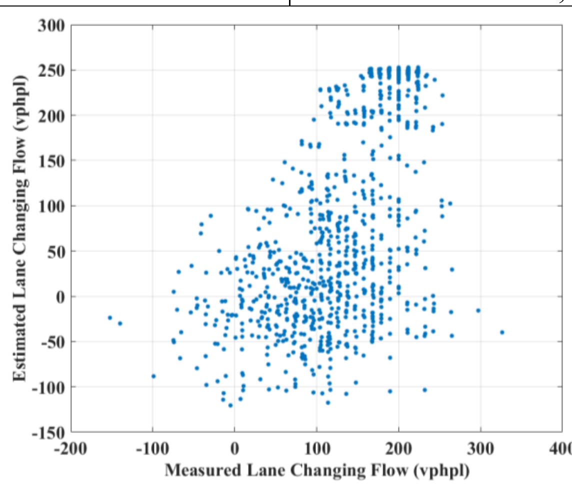
i. Case A: Lane changing take place for speed difference only ($w_{i,j}^{\rho} = 0$)
ii. Case B: Lane changing take place for density difference only ($w_{i,j}^{v} = 0$)
iii. Case C: Lane changing take place for both speed and density difference

For Type I lane changing, the net-lane changing flow was measured for the instances when the density at Station 2 were less than or equal to half of critical density of that segment (K=0.5). Altogether, 1976 time stamp were selected for parameter calibration which includes 691 instances of vehicles coming in Lane 4 and 1285 cases of flow towards Lane 3. As presented in Table 1, Case A could not accurately estimate Type I lane changing. On the other hand, both Case B and Case C showed identical compatibility with measured lane changing flow. The parameter of speed difference ($w^{v}_{i,j}$) for Case C was found to be insignificant which indicated negligible effect of speed incentive on Type I lane changing. The analysis of Type I lane changing insinuated that considering density incentive alone was enough for this type of lane changing in free–flow traffic state. The comparison between calculated lane changing flow and estimated lane changing flow obtained from the calibrated model parameters as well the calibration precision representing parameters are presented in Table 2.1.

TABLE 2.1 Calibration output of Model Parameters for Lane Change Type I

<table>
<tr><td>


(a)
</td><td>


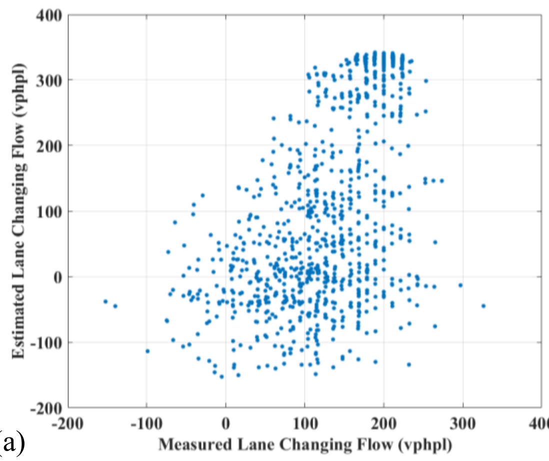
(b)
</td></tr>
<tr><td>$w^{v}_{2,4}$= -0.0021 , $w^{\rho}_{2,4}$= 0 ($w^{v}_{2,4}$ Sig. < 0.001)</td><td>$w^{v}_{2,4}$= 0 , $w^{\rho}_{2,4}$= 0.0415 ($w^{\rho}_{2,4}$ Sig. < 0.001)</td></tr>
<tr><td>R^2 = 0.004, RMSE = 163.023</td><td>R^2 = 0.652, RMSE = 45.509</td></tr>
<tr><td colspan="2">
(c)


</td></tr>
<tr><td colspan="2">$w^{v}_{2,4}$= -0.0002 , $w^{\rho}_{2,4}$= 0.0410 ($w^{v}_{2,4}$ Sig.=0.23>0.05, $w^{\rho}_{2,4}$ Sig. < 0.001)</td></tr>
<tr><td colspan="2">R^2 = 0.654, RMSE = 45.007</td></tr>
</table>

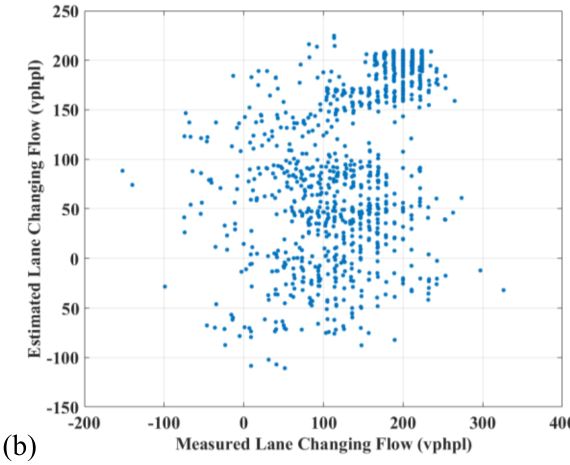
The analysis of the Type II lane changing was based on the assumption that this type of lane changing could be influenced by both speed and density difference which essentially suggest that both of the model parameters would play significant roles to estimate lane changing flow. Via careful observation of the analysis results, it was noticed that Case C provided the most accurate estimation of net lane changing flow with two significant parameters. Since both of these parameter were significant in decision making of lane changing manuever, the level of accuracy considering each influence individually showed comperatively high R^2 values. Considering speed and density difference not only put emphasis on both aspects of decision making but also can explain esitmation dilemma arose at instances with zero speed or density differences. Case A and Case B failed to explain the lane changing phenomenon among neghboring lane during eqaul speed or equal density. Since Case C considered both speed and density difference as source term, the error due to this unexplained phenomenon had been reduced. For the analysis of Type II lane changing pattern, 1031 samples were selected among which 638 samples demonstrated lane changing flow towards Lane 4 and the rest showed flow headed for opposite lane. Table 2.2 showed the comparison between calculated lane chaging flow from field and estimated lane changing flow from calibrated model.

TABLE 2.2 Calibration output of Model Parameters for Lane Change Type II

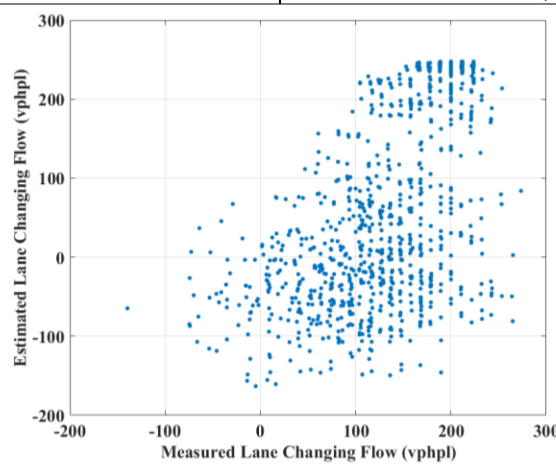
<table>
<tr><td>



(a)</td><td>



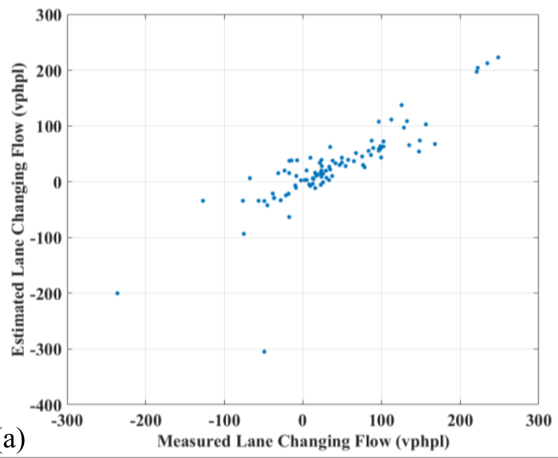
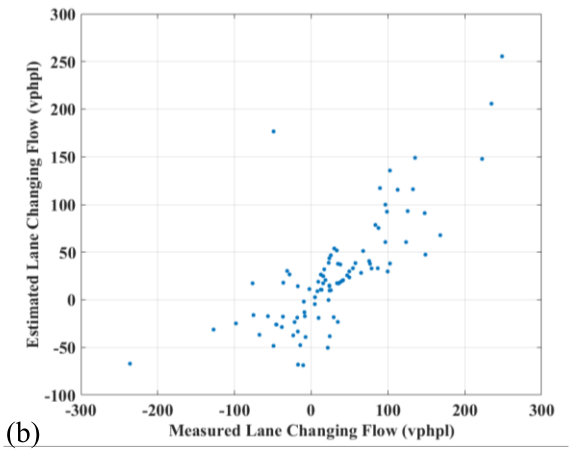
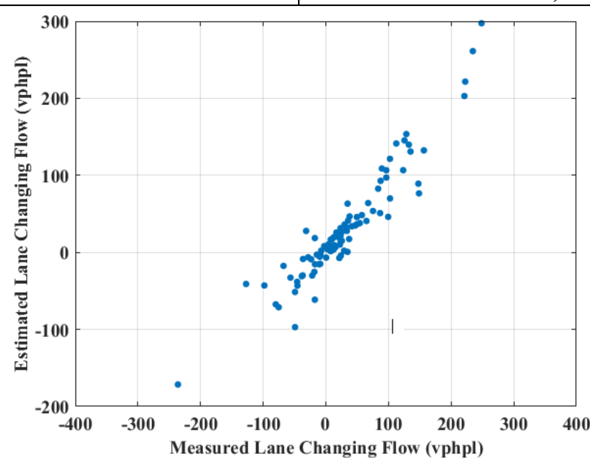
(b)</td></tr>
<tr><td>$w^{v}_{2,4}$= 0.0207 , $w^{\rho}_{2,4}$= 0 ($w^{v}_{2,4}$ Sig. < 0.001)</td><td>$w^{v}_{2,4}$= 0 , $w^{\rho}_{2,4}$= 0.0575 ($w^{\rho}_{2,4}$ Sig. < 0.001)</td></tr>
<tr><td>R^2 = 0.639, RMSE = 47.782</td><td>R^2 = 0.537, RMSE = 53.338</td></tr>
<tr><td colspan="2">



(c)</td></tr>
<tr><td colspan="2">$w^{v}_{2,4}$= 0.0147 , $w^{\rho}_{2,4}$= 0.0274 ($w^{v}_{2,4}$Sig. < 0.001, $w^{\rho}_{2,4}$ Sig. < 0.001)</td></tr>
<tr><td colspan="2">R^2 = 0.674, RMSE = 45.2</td></tr>
</table>

Third type of lane changing pattern was considered during congested roadway condition. Since the samples were considered for analysis only if the feasibility criterion for each lane changing type sustained for over five minutes, a relatively small number of samples were obtained for parameter calibration of Type II lane change. Altogether 103 samples were procured for Type III lane changing analysis. As expected, the analysis showed better accuracy with considering both speed and density difference compared to other two cases. The key feature of the calibrated parameter were the higher value of $w^{v}_{2,4}$ and lower value of $w^{\rho}_{2,4}$, compared to parameters for Type II which implied increased sensitivity of speed difference over density difference between neighboring lanes during congested traffic state.

TABLE 2.3 Calibration output of Model Parameters for Lane Change Type III

 (a)	 (b)
$w^{v}_{2,4}$= 0.0268 , $w^{\rho}_{2,4}$= 0 ($w^{v}_{2,4}$Sig. < 0.001)	$w^{v}_{2,4}$= 0 , $w^{\rho}_{2,4}$= 0.048 ($w^{\rho}_{2,4}$ Sig. < 0.001)
R^2 = 0.848, RMSE = 46.706	R^2 = 0.741, RMSE = 47.827
(c) 	
$w^{v}_{2,4}$= 0.0185 , $w^{\rho}_{2,4}$= 0.0298 ($w^{v}_{2,4}$ Sig. < 0.001, $w^{\rho}_{2,4}$Sig. < 0.001)	
R^2 = 0.88, RMSE = 30.59	

2.6 STATISTICAL TESTS FOR MODEL MODIFICATION

The calibrated model parameters identified in previous section were further modified based on assumed lane changing direction ('To' or 'From' target lane) which would increase accuracy of the model. To modify model parameters, different sets of sample were used for each types of lane change to present unbiased nature of the proposed model. A series of statistical tests were ran through the models with modified parameters. At first, Type I lane change data were taken for analysis and divided into two groups based on the sign of net lane changing flow measurements. '–ve' sign of net lane changing flow indicated that total number of vehicles entering the target lane was lower than total number of vehicles leaving the lane within a time interval and vice versa.

Independent variables of respective time stamp were accompanied with the classified flows. Then, linear regression along with some statistical tests were exercised on those classified samples to identify the model parameters and identify model accuracy since ‘goodness of fit’ was not sole criterion for testing the credibility of model. Similar work flow was followed for Type II and Type III lane changes. Table 2.4 provided the summary of the analysis preformed on all lane changing types.

TABLE 2.4 Modified Model Parameters and Statistical Test Results

<table>
<tr><td colspan="6">Model Parameters</td></tr>
<tr><td>Lane Change Type</td><td colspan="3">$w^{v}_{2,4}$</td><td colspan="2">$w^{\rho}_{2,4}$</td></tr>
<tr><td>Type I (-ve flow)</td><td colspan="3">0</td><td colspan="2">0.0278</td></tr>
<tr><td>Type I (+ve flow)</td><td colspan="3">0</td><td colspan="2">0.0525</td></tr>
<tr><td>Type II (-ve flow)</td><td colspan="3">0.0159</td><td colspan="2">0.0284</td></tr>
<tr><td>Type II (+ve flow)</td><td colspan="3">0.0122</td><td colspan="2">0.0339</td></tr>
<tr><td>Type III (-ve flow)</td><td colspan="3">0.0201</td><td colspan="2">0.0429</td></tr>
<tr><td>Type III (+ve flow)</td><td colspan="3">0.0229</td><td colspan="2">0.0209</td></tr>
<tr><td colspan="6">Goodness of Fit Test</td></tr>
<tr><td>Lane Change Type</td><td colspan="3">R^2</td><td colspan="2">Adjusted R^2</td></tr>
<tr><td>Type I (-ve flow)</td><td colspan="3">0.533</td><td colspan="2">0.533</td></tr>
<tr><td>Type I (+ve flow)</td><td colspan="3">0.708</td><td colspan="2">0.708</td></tr>
<tr><td>Type II (-ve flow)</td><td colspan="3">0.628</td><td colspan="2">0.627</td></tr>
<tr><td>Type II (+ve flow)</td><td colspan="3">0.792</td><td colspan="2">0.791</td></tr>
<tr><td>Type III (-ve flow)</td><td colspan="3">0.77</td><td colspan="2">0.753</td></tr>
<tr><td>Type III (+ve flow)</td><td colspan="3">0.932</td><td colspan="2">0.93</td></tr>
<tr><td colspan="6">Significance and Collinearity Test of Model Parameters</td></tr>
<tr><td rowspan="2">Lane Change Type</td><td rowspan="2">parameters</td><td rowspan="2">t</td><td rowspan="2">Sig.</td><td colspan="2">Collinearity Statistics</td></tr>
<tr><td>Tolerance</td><td>VIF</td></tr>
<tr><td>Type I (-ve flow)</td><td>$w^{\rho}_{2,4}$</td><td>39.339</td><td>0.000</td><td rowspan="2">-</td><td rowspan="2">-</td></tr>
<tr><td>Type I (+ve flow)</td><td>$w^{\rho}_{2,4}$</td><td>52.509</td><td>0.000</td></tr>
<tr><td rowspan="2">Type II (-ve flow)</td><td>$w^{v}_{2,4}$</td><td>17.632</td><td>0.000</td><td rowspan="2">0.658</td><td rowspan="2">1.519</td></tr>
<tr><td>$w^{\rho}_{2,4}$</td><td>11.731</td><td>0.000</td></tr>
<tr><td rowspan="2">Type II (+ve flow)</td><td>$w^{v}_{2,4}$</td><td>23.005</td><td>0.000</td><td rowspan="2">0.319</td><td rowspan="2">3.136</td></tr>
<tr><td>$w^{\rho}_{2,4}$</td><td>19.705</td><td>0.000</td></tr>
<tr><td rowspan="2">Type III (-ve flow)</td><td>$w^{v}_{2,4}$</td><td>8.818</td><td>0.000</td><td rowspan="2">0.886</td><td rowspan="2">1.129</td></tr>
<tr><td>$w^{\rho}_{2,4}$</td><td>6.551</td><td>0.000</td></tr>
<tr><td rowspan="2">Type III (+ve flow)</td><td>$w^{v}_{2,4}$</td><td>10.212</td><td>0.000</td><td rowspan="2">0.23</td><td rowspan="2">4.357</td></tr>
<tr><td>$w^{\rho}_{2,4}$</td><td>5.194</td><td>0.000</td></tr>
</table>

As the model structure was grounded on only two variables (speed difference, density difference between adjacent lanes) without considering any intercept, the significance test of the model was deemed obsolete. The model parameters presented on Table 2.4 was determined by calibrating the models of respective lane changing type with measured data. The R^2 values and Adjusted R^2 values for Type I models were equal as the model considered single variable for estimation. The remaining lane changing types' R^2 value were different from adjusted R^2 values. The reduced values of adjusted R^2 indicated that variable inclusion improved the model by less than expected. Although, higher values of R2 showed implied the prediction certainty of the model. For instance, the R^2 value for Type II lane change with +ve flow (net flow toward target lane) was 0.792, thus 79.2% of the data could be explained by the regression equation, implying that the fitting results were reliable.

The significance and collinearity test of model parameters were designed to examine whether considered variables were significant in estimating lane changing flow and whether any significant correlation existed between the variables. The null hypothesis H_0 and alternative hypothesis H_1 for significance test were developed as follows:

H_0: $w_{2,4}^{v}, w_{2,4}^{\rho} = 0$

H_1: $w_{2,4}^{v}, w_{2,4}^{\rho} \neq 0$

T-statistic was chosen in significance test of regression coefficients and significance level α was set as 0.05. Coefficients with p-value less than 0.05 would prove the significance in estimating lane changing flow. As found from Table 2.4, all the coefficients were significant for model formation. The collinearity of the considered variables were measured through Tolerance and Variance Inflation Factor (VIF) of the variables. Usually, significant collinearity were considered to be present between variables when the tolerance and VIF values were less than 0.1 and higher than 5.0 respectively **(22)**. However, all the tolerance value were higher than 0.1 and VIF values were below 5.0, thus the inexistence of collinearity between variables were verified.

2.7 MODEL VALIDATION

In this section, we validated the proposed model with identified model format for different lane changing type. In order to test and demonstrate the validity of the proposed model, the calibrated parameters were applied on the same motorway stretch but for different days. The direction of lane changing at a certain time stamp was presumed based on speed and density differences between neighboring lanes. Afterwards, the proposed model was used to estimate the net-lane changing flow of the selected lane for five days (July 20, 2015 to July 24, 2015). The following three steps were followed to execute proposed model with suitable empirical parameters:

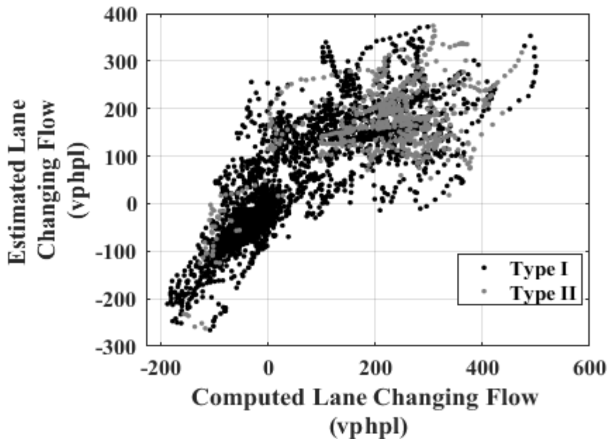
Step 1. Model Input: Recorded loop detector data were used as model input after processing zero-flow data and data smoothing process. In this study, the mean speed were assumed to be equal to speed limit (80khp) at time stamps with zero traffic flow in a lane. Then the traffic state variables ((Flow, Speed and Density) were smoothened by averaging them over a 5 minutes moving time window that updated in every 20 seconds. The smoothing of those parameter would assist in excluding random fluctuation, traffic impulse etc.

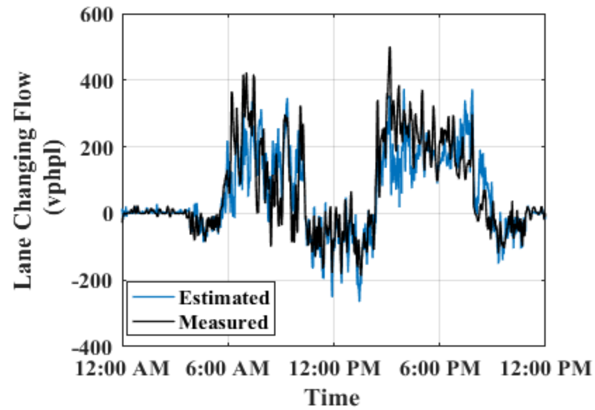
Step 2. Determining the type of Lane Change: The model input were passed through two-dimensional feasibility criterion for each lane change type. Based on mean speed and density at a time stamp, the lane changing type was classified which was important to select whether both of the incentives should be considered or not.

Step 3. Assigning proper model parameters: After selecting lane changing type, the difference of lane changing incentives (speed, density) were measured between adjacent lanes. Based on these measurements, the direction of lane changing flow was detected and appropriate sets of model parameters were applied to estimate net-lane changing flow.

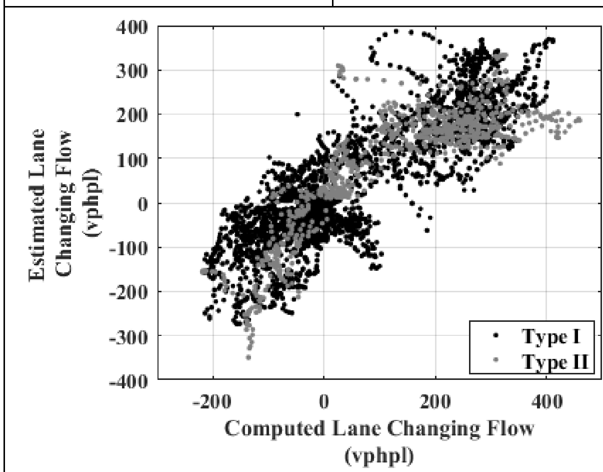
The results of the validation process showed acceptable accuracy in estimating lane changing flow while comparing with the calculated lane changing flow. As showed in Table 2.4, the R^2 value were within 0.7-0.8 which indicated satisfactory accuracy in estimating lane changing flow. Although RMSE values were higher than the calibration RMSE values, they were plausible for such large samples. Via careful inspection of the graphs, it was noticeable that the model provided fairly accurate estimation of net lane changing flow when actual lane changing between lanes were small. Estimation precision were reduced to some extent for comparatively high lane changing flow instances. This can be explained by the fact that lane changing characteristics of individual vehicles were dependent on a number of interdependent factors including the factors considered in the model. Divergent driving behavior of drivers raised the uncertainty and variation. Although, it was considered that the lanes would be free from lane changing effects due to on-ramp and off-ramp, identifying this type of lane changing was beyond the scope of this model. Given this diversity and complexity, it appeared that the proposed model was dependable enough to provide reliable lane changing estimation.

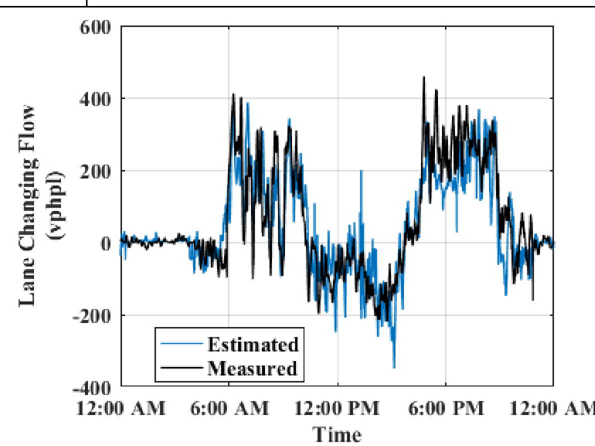
TABLE 2.5 Validation output of Proposed Model



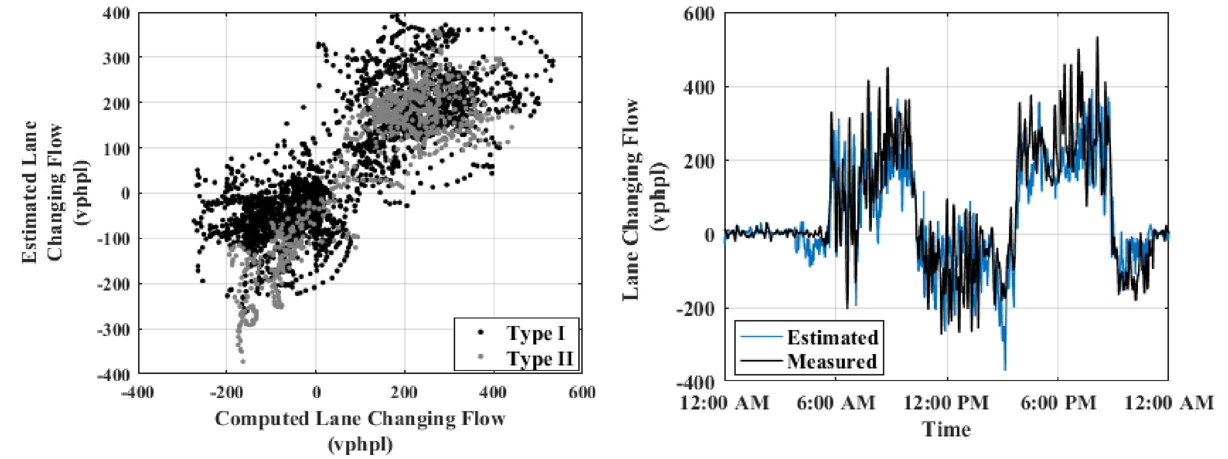


R^2= 0.745	MAE = 49.98	RMSE=76.18

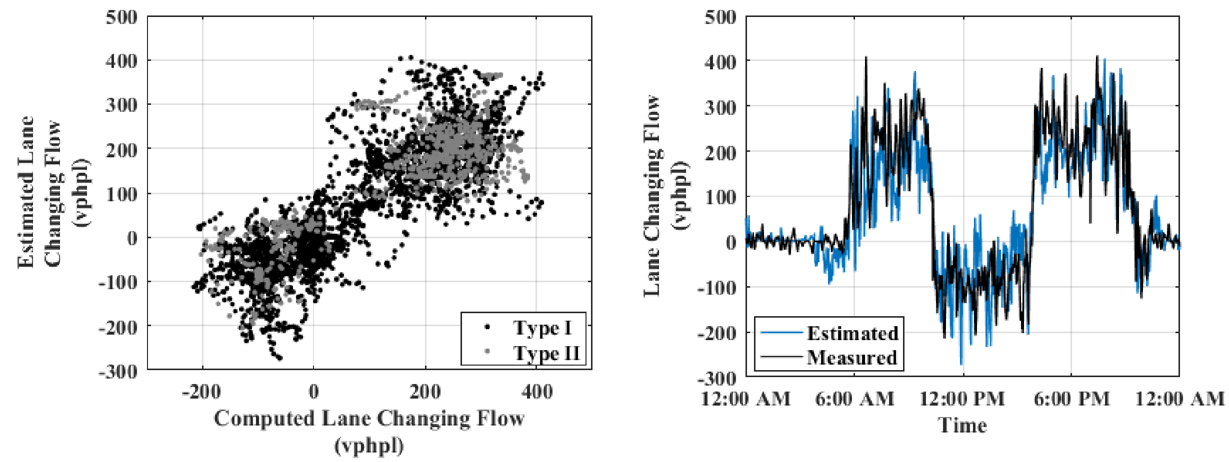




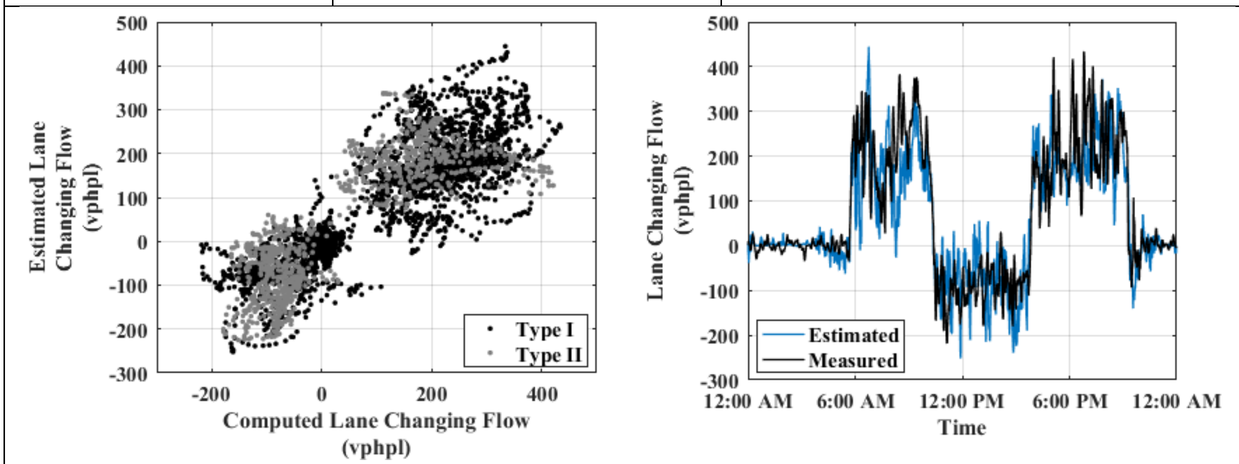
R^2= 0.762	MAE = 54.94	RMSE=75.42



R^2= 0.716	MAE = 62.84	RMSE=88.3



R^2= 0.745	MAE = 57.87	RMSE=76.87



R^2= 0.77	MAE = 53.72	RMSE= 74.92

The outputs of the validation process indicated that two types of previously defined lane changing patterns (Type I and Type II) were witnessed. The absence of Type III lane changing pattern was due to the lack of stable congested state on those days. The observed lane changing pattern throughout the day were very similar in the studied days. High inflow (vehicles coming from neighboring lane) towards the target lane (Median Lane) were observed during the extended morning and evening peak hours which was expected due high number of vehicles on the roadway. In between the extended peak hours, the vehicles tend move right towards neighboring lane (Outer Middle Lane) since driving through that lane would provide them more freedom and flexibility in choosing their route in response to the prevailing traffic conditions.

2.8 CONCLUSION

In this chapter, we presented a model for lane changing traffic dynamics considering the microscopic lane changing concept. A key assumption was that the lane changing pattern of traffic were largely influenced by the speed as well as density advantage offered by the neighboring lane and the net-lane changing flows maintain a linear association with these two incentives. Based on this notion, we proposed a lane changing model different from earlier models which explored the influences of speed and density stimulus at different traffic states. By incorporating triangular fundamental diagram in different lane changing pattern, this study provides a simple framework to look into lane changing traffic dynamics at different roadway conditions. In order to test and demonstrate the authenticity of the proposed model, a calibration and validation procedure, based on field data collected by a set of loop-detectors in freeway, has been conducted. Despite the numerous complex motivators of lane changing maneuvers, the model is capable to satisfactorily replicate the lane changing patterns observed in the field.

The model studied here endures certain limitations by assuming the lane changing flows were free from the influence of the existing on-ramps and off-ramps. By considering flow conservation of individual lanes to compute the lane changing on field, the study provides a continuous and reliable source of lane changing flow with minimum effort. However, the rapid change in traffic state might provide some errors in lane changing flow since flow conservation law holds for steady traffic state. Additionally, the proposed model considers the lane changing pattern in aggregated level of detail and more detailed analysis of lane changing maneuvers could give us more insights on a lane changing traffic.

Currently, the study is limited to freeway lanes containing one neighboring lane and vehicle movements were assumed to be free from mandatory lane changings. But in reality, the roadways that has more than two lanes in each direction as well as contain lanes that are effected by lane changing traffic from two neighboring lanes. Again, there are continuous access opportunities in freeways and the lanes closer to the on-ramps and off-ramps are most influenced by mandatory lane changing to seize these opportunities. Therefore, it is essential to provide a general approach to address these issues in future.

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Chapter 3
VSL Compliance Implications on Collective Lane changing Pattern

3.1 INTRODUCTION AND BACKGROUND

Variable Speed Limit (VSL) is an active traffic demand management (ATDM) strategy that makes use of traffic state variables (i.e. traffic flow, speed, density etc.) to estimate optimal speed limits at which drivers should be driving, given prevalent roadway and traffic condition. VSL has been exercised for last fifty years in numerous North American and European cities as a part of congestion management, incident management, weather advisory with an intention to bring about mobility and safety benefit of the road users. These benefits are based on presumption that reducing speed limit on upstream of bottleneck prior to the onset of congestion has the potential of reducing traffic delay and increasing throughput of bottleneck. In a situation of high traffic demand, speeds inhomogeneity between lanes influence drivers to change lane and cause disturbance to the flow which can lead to bottleneck situations and even worse, collisions. It essentially implied that the advantages of VSL is subjected to the extent of transformation in collective lane changing behavior induced by control and also on the CL of the VSL.

Although the positive impacts of VSL have been theoretically established, the empirical evaluations from numerous field tests produced inconsistent results. In 2003, Park and Yadlepati ***(1)*** performed a simulation based study with varying compliance rate at a work zone that showed increase travel time with increasing compliance rate without providing any indication of overall change in driving behavior due to VSL. A different study using model simulation by Heygi et al. ***(2)*** concluded that VSL could reduce travel time by 21%. Similar conclusion was drawn by Long et al. ***(3)***that showed model predictive control (MPC) based VSL could provide homogeneity in traffic speed in addition to the reduced travel time. However, both study considered 100% compliance to the speed limits. Piao and McDonald ***(4)*** conducted a microscopic simulation study which indicated higher reduction in lane changing frequency when VSL was enforced compared to VSL implementation without enforcement. Although, the study did not account for defensive and aggressive drivers on the road. Uniformity in driving behavior was achieved by VSL which was reported by Mirshahi et al. ***(5)*** which eventually increased trip reliability, traffic throughput during congestion and delayed onset of traffic breakdown. An increasing CL with time was reported in few studies ***(6, 7)*** with higher compliance to VSL than static speed limits. Another study on advisory VSL field test was failed to find any significant impact on traffic operation due to low compliance on VSL ***(8)***. Although, the simulation based study suggested better traffic operations considering higher compliance. The study concluded that mandatory VSL would be superior to advisory VSL in affecting collective driving behavior. A driving simulator study on drivers' response to VSLs found that VSL had very little effect on driving behavior of the road users and the drivers' tend to drive at uniform speed with smaller variations in speed ***(9)***. Habtemichael and Picado Santos ***(10)*** studied safety and operation benefits of VSL at differnet traffic condtions and different CL. The study found that VSL provided highest safety benefits during highly congested traffic conditions and highest operational benefits during lightly congested conditions. Su et al. ***(11)*** performed a similar study using an MPC based VSL control on AIMSUN traffic simulator which showed similar mobility benefits at 100% and 30% compliance rate. However, no indication was provided to specify which CL was most achievable in practice. The VSL system along I-270/I-255 corridor in St. Louis was evaluated by Long et al ***(3)***. VSL compliance of an average of 94% of higher for SL of 70 mph, 60 mph, 50 mph and 84% or higher at 40 mph was observed in this study. McMurtry et al. ***(12)*** conducted a study on a six mile test site with a long distance work zone on I-80 north of Wanship, Utah which tested drivers' response to VSL signs. The comparison between the static and VSL showed higher compliance to VSL with lower speed variations. Hadiuzzaman et al. ***(13)*** evaluated in VISSIM an MPC-based VSL control strategy from relieving congestion caused increased demand and higher lane changing flow at recurrent bottleneck. The study showed significant improvement in travel time with 90% compliance rate. With the similar setup, Hadiuzzaman and Qiu ***(14)*** proposed a VSL control based on cell transmission model and used the MPC to dynamically change the SL in real-time. Again, this study showed 10-15% reduction in travel time with 90% compliance rate. Another model simulation based study by Hadiuzzaman et al. ***(15)*** showed that the mobility and safety benefits were positively correlated with increasing compliance rate. Bhowmick et al. ***(16)*** analyzed the effects of drivers' compliance to VSL on traffic operation. The study findings suggested that even with lower level of compliance, VSL could effectively reduce speed variation of traffic stream.

Neither empirical observations nor simulation studies provided any conclusive outcome on compliance to VSLs and their implications on collective lane changing pattern of the bottlenecks. Considerable disparity and lack of exertion exist in the literature regarding the impacts of VSLs. The reasons for such diverse findings would be unstandardized methodology of study, inappropriate simulation approach as well as dissimilar nature of bottleneck formation and traffic composition. Therefore, the objectives of this study include, (1) assessing the impacts of different compliance level of VSLs on previously mentioned parameters in a simulated environment and (2) verifying the expected outcomes of VSL implementation from the field observations. The remainder of the chapter is organized into six sections: the "Driver Compliance Analysis and Modeling Methods" section presents the approach of modeling speed distributions with VSL as a function of compliance rate. The "Lane Changing Flow Estimation Method" section briefly described the methodology of measuring lane changing flow from field and model simulations. Followed by, "VSL Control on Model Simulation and Field Implementation" section that describes the MPC based VSL control strategy which is used for traffic state prediction and VSL control on model simulation and field test. Next, "Simulation Data Analysis and Results" section presents the simulation results and the relative impacts of CLs on performance parameters. "Field Data Analysis and Results" section analyzes the CL, speed and lane changing flow data from VSL field test and identify their implications on traffic stream. Finally, the last section offers conclusions and suggestions for future research.

3.2 DRIVER COMPLIANCE ANALYSIS AND MODELING METHODS

To apprehend the impacts of drivers' compliance to VSL, analyzing drivers' responses to the static speed limit in the studied location was necessary. The embedded loop detectors at predefined locations on the studied roadways was used as the source of information in order to apprehend aggregated driving behavior at static speed limit. The analysis of compliance to the static speed limit was based on loop detector data (mean speed, flow, density) obtained during off-peak hours. Only off-peak hour data were selected for the analysis of static speed limit compliance to ensure that traffic was freely flowing. Loop detector data from different locations of the studied corridor were collected for the purpose of this analysis. The following speed parameters were computed from the chosen data for the posted speed limit (80 kph) using commercial software called Traffic Operations and Planning Software (TOPS): (a) mean speed= 85.6 kph (1.075 x speed limit), (b) Standard Deviation (σ) = 10.3 kph and (c) Coefficient of variation (CV) = 0.12. These are the fundamental traffic speed behavior representing parameters which illustrated the driving behavior of the road users of the studied corridor.

Regarding observations of compliance characteristics, this study considered vehicles as compliant when the average speed varied within ±5 kph of the posted speed limit. For instance, the static speed limit of the studied corridk4or was 80kph. According to the stated consideration the vehicles that drove within ±5 kph of the posted speed limit (75 kph- 85 kph) were considered as vehicles conforming the speed restrictions. This consideration provided the scope to study the impacts of defensive and aggressive drivers in the traffic to be modeled and identify their impact on overall traffic speed. Another important parameter considered in the compliance analysis was the mean headway of vehicles. The mean headways of the vehicles were derived from obtained density values for each time instant. Assuming the headways of vehicles were normally distributed, the percentage of vehicles selected for the compliance study were those that maintained an average headway ≥ 2 seconds. The samples consisting less than 5% vehicles with 2 seconds or higher mean headways were removed from the analysis, as they were considered rare instances. For example, if the mean headways at a time stamp was calculated as 2.46 with a standard deviation of 0.85, then it could be stated according to the properties of standard normal distribution that 70% of the vehicle drove over that location had maintained over 2 second headways. Similarly, mean speed at a time stamp was also considered as a categorizing parameters for classifying compliant and non-compliant vehicles. Considering the mean speeds were also normally distributed, ***(17, 18)***, the percentage of selected vehicles that drove within ±5 kph of the posted speed limit were regarded as compliant vehicles and the remaining vehicles were divided into defensive vehciles(below 5kph of posted speed limits) or aggressive vehicles (above 5kph posted speed limits). The analysis of the loop detector data using the mentioned process showed that the compliance level for the static speed limit (80 kph) of the studied roadway was approximately 48%. Following the aforementioned categorization of compliant and non-compliant drivers as well as considerations regarding mean headways and mean speed of vehicles at a time stamp, it was also measured that approximately 20% of the drivers drove defensively and approximately 32% drove aggressively.

For the purpose of studying compliance, a microscopic model was used to configure several CLs with different desired speed distribution curves in VISSIM. This study considered four levels of compliance in the model, which included a compliance rate of 20% (Low CL), 50% (Moderate CL), 80% (High CL) and 100% (Ideal CL). Furthermore, to imitate a real-world scenario and maintain consistency with the field considerations in assessing compliance, it was assumed that compliant vehicles were those traveling within ±5 kph of the speed limits. The remaining non-compliant drivers were divided into two classes: defensive and aggressive. Those driving more than 5kph below the speed limit were considered defensive drivers. In contrast, drivers traveling at a speed more than 5kph above the speed limit were considered to be aggressive. Table 3.1 presents the speed distributions that were implemented in models. Earlier studies based on real-world speed observation revealed that the percentage of aggressive drivers increased as the posted speed limit decreased ***(3, 18-21)***. Considering such driving behavior, the desired speed distributions were provided with a higher percentage of aggressive drivers associated with low SLs compared to high SLs. For example, for a low CL and a speed limit of 70 kph, out of 80% non-compliant drivers, 40% were considered to be defensive and the rest aggressive. Whereas, for the same CL and a speed limit of 20 kph, the percentage of defensive and aggressive drivers were 30% and 50% respectively. The resulting desired speed distribution curves are shown in Figure 3.1 as a function of CL. In the figure, the lower, middle and upper diagonal lines for each CL were representing defensive, compliant and aggressive driver respectively. Based on the observations by Giles et al. ***(19)***, the lower and upper bound of the desired speed distribution curves were bound by ±2σ of mean speed related to the speed limit, where σ= mean speed multiplied by COV. Considering a fixed COV, obtained from field data, upper and lower bounds of the curves for each speed limit were computed and distributed among defensive and aggressive drivers. To imitate real-world scenario, it was assumed that COV was fixed at 0.12 and the mean speed was 1.075 times higher than the posted speed limit. These considerations were made to construct speed distribution curve for model simulation at different compliance level and at different speed limits.

TABLE 3.3 Non-compliant Drivers Distribution at Different Speed Limits

Speed Limit (kph)	Low Compliance (20%)[a]		Moderate Compliance (50%)[a]		High Compliance (80%)[a]		Ideal Compliance (100%)[a]	
	Non-Compliance (80%)		Non-compliance (50%)		Non-compliance (20%)		Non-compliance (0%)	
	Defensive (%)	Aggressive (%)	Defensive (%)	Aggressive (%)	Defensive (%)	Aggressive (%)	Defensive (%)	Aggressive (%)
20	30	50	15	35	5	15	0	0
30	30	50	15	35	5	15	0	0
40	30	50	15	35	5	15	0	0
50	30	50	15	35	5	15	0	0
60	40	40	20	30	10	10	0	0
70	40	40	20	30	10	10	0	0
80	40	40	20[b]	30[b]	10	10	0	0

[a] Compliance within ±5 kph of the posted speed limit

[b] Observed speed limit compliance pattern in WMD with 80kph static speed limit

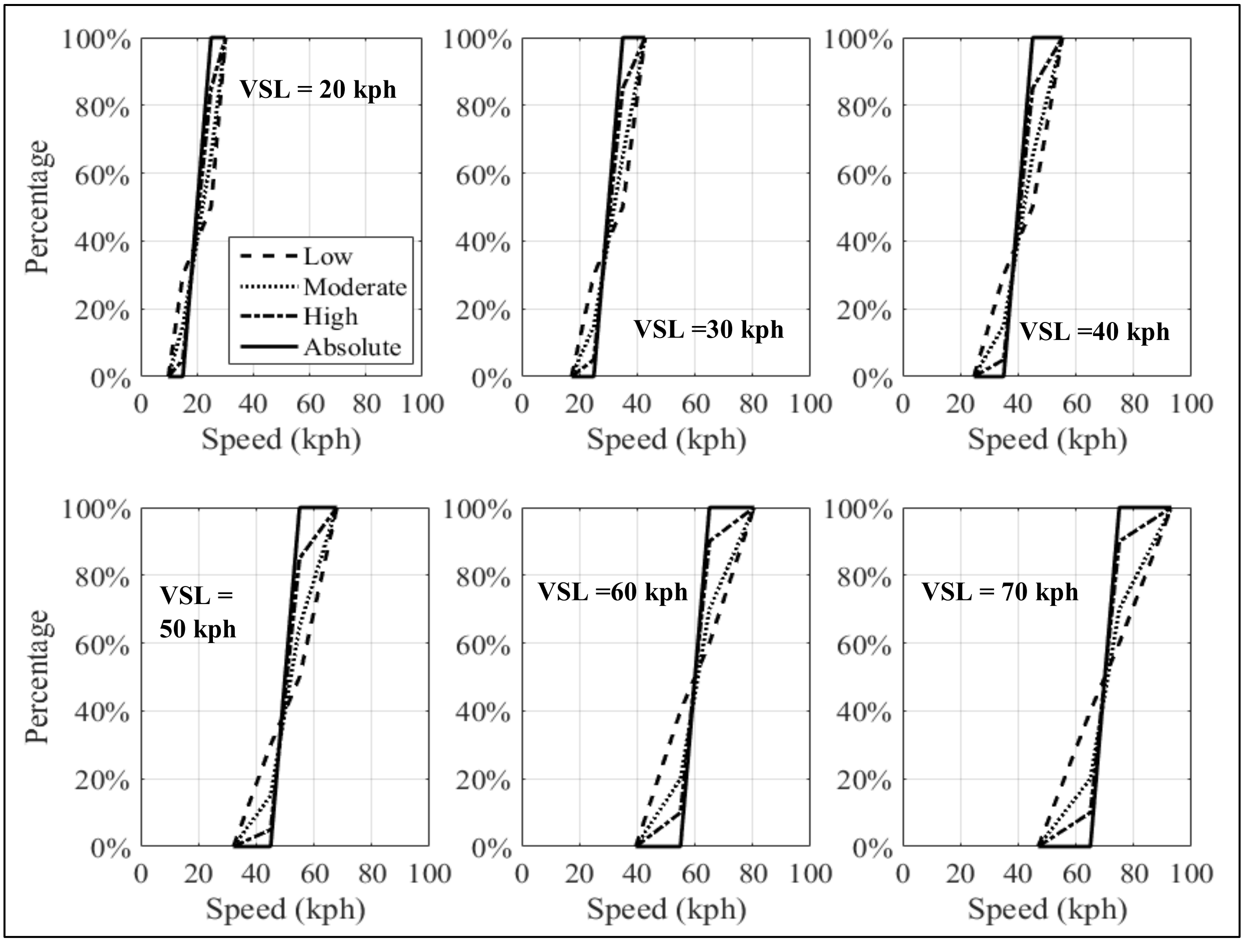

FIGURE 3.1 Speed Distribution Curve as a function of Compliance Levels for Different Speed Limits

3.3 LANE CHANGING FLOW ESTIMATION PROCESS

Two different models were used for calculate lane changing flow in the studied roadway. One was applied to calculate the lane changing flow from the model which was built within VISSIM. Another one to estimate the lane changing flow from obtained field observations. Since this research was concerned about relative change in lane changing flow due to imposed speed restrictions on bottleneck, the lane changing models considered for model simulations was exercised without calibrating field observation based lane changing model. The lane changing model employed in VISSIM was developed by Willmann and Sparmann (1978). This model considered two types of lane changing which are: lane changing to a faster lane and lane changing to a slower lane. The vehicles consider either type of these lane changing by considering three different situations: whether there is a desire to change lane, whether the present driving situation in the adjacent lane is favorable and whether the movement to a neighboring lane is possible. In addition to this classification, the lane changing in VISSIM is also classified into necessary lane change and free lane change. If the vehicles needed to change lane to reach the next connector of a route, they would change lane following necessary lane changing model. The parameters of this type of lane changing contained the maximum acceptable lane changing by subject vehicle and the following vehicle in target lane. The free lane change model considered speed and density incentive to change lane. No matter which type of lane change it was, the vehicles would not change lane without finding a 'safe gap'. For the purpose of this study, the lane changing parameter were kept on their default values for the bottleneck section. Since, the purpose of this study was to identify the relative change in lane changing patterns due to VSL, the calibration of lane changing model was disregarded in this study.

To estimate lane changing flow from obtained loop detector data, a macroscopic lane changing model was developed. This model considered speed as well as density differences between adjacent lanes on a roadway segment as lane changing incentives and presumed a linear relationship of lane changing frequency with these incentives. The detail description of this lane changing model was provided in previous chapter of this thesis (Section 2.3 Lane Changing Model Development). This model took three types of lane changing into considerations based on lane density and estimated different model parameter values of the aforementioned incentives for different lane changing types. The feasibility criterion of those lane changing types are provided below:

Type I: $\mathrm{v_j(k) \geq v_j^{free}}$ and $\mathrm{\rho_j(k) \leq K\rho_j^{cr}}$

Type II: $\mathrm{v_j(k) \geq v_j^{free}}$ and $\mathrm{K\rho_j^{cr} < \rho_j(k) < \rho_j^{cr}}$

Type III: $\mathrm{v_j(k) < v_j^{free}}$ and $\mathrm{\rho_j(k) \geq \rho_j^{cr}}$

Here, K is also a model parameter that defines the extent of Type I lane changing with regard to critical density. An appropriate value of K was determined from the observations of lane changing behavior and comparing the significance of $w_{i,j}^{v}$ for Type I lane changing for different K values.

Type I lane changing essentially considered the lane changing practice of the road users that were free from the influence of potential speed incentive due to lane changes. This type of lane changing behavior is predominant during free flow condition with relatively low density. Lane changing of type I was regarded as free lane changing decision from road users' perspective which eventually showed inclination towards density variance among lanes. Type II and Type III lane changing considered the lane changing for road users that were influence by both speed and density difference between adjacent lanes. However, type II lane changes took place at free flow traffic state, where as type III occurred at congested state. All of the model parameters for different lane changing types were calibrated and evaluated based on field observations. The estimated lane changing flow provided satisfactory results that made this model suitable to estimate field lane changing for this study.

3.4 VSL CONTROL CONCEPT

The idea behind VSL control strategy is to limit mainline traffic flow at the upstream of bottleneck in the range of bottleneck capacity to prevent traffic breakdown and subsequent capacity drop. It is assumed that by imposing relatively low speed, the VSL can effectively reduce the mainstream traffic flow. It is an inherent assumption in VSL control that lower speed limit can serve the same capacity at higher occupancy level. During this transition from lower to higher occupancy a flow reduction takes place to uphold the flow conservation law and this in turn restricts flow of mainline traffic.

In order to clearly explain the VSL control system the following example is provided. Here, the freeway is divided into two consecutive cell based on their homogenous roadway and geometric characteristics (Figure 3.2). VSL control strategy considered traffic flow characteristics of all these cells. During peak period, there is high traffic flow approaches from upstream cell to bottleneck cell. This high traffic flow results in high density. Different lane changing maneuvers i.e. merging, diverging, weaving at the bottleneck section disrupts this dense traffic flow. As a result speed drop occurs and queue formation begins at bottleneck cell which are the signs of traffic breakdown. Since queue means high density of traffic, it generates lower discharge flow from that cell (Q'_b)

according to the law of flow conservation. This reduction in flow (Q_b - Q'_b) is termed as 'Capacity drop'. Here, Q_b and Q'_b represents bottleneck capacity and reduced bottleneck capacity due to capacity drop respectively.

To this end, VSL control strategy can be adopted to avert capacity drop by limiting feeding flow to bottleneck cell. To maximize bottleneck flow, the upstream discharge flow should be maintained to bottleneck capacity (Q_b) level. In order to attain this objective, reduced speed limits are provided in the control cell which essentially increases traffic density ($\rho'_c > \rho_c$). Here, ρ_c and ρ'_c represents control cell critical density at free flow speed and at reduced speed respectively. This high traffic density triggers lower traffic flow according to flow conservation law. Therefore, the control strategy aims at assigning an optimal speed at control cell which can effectively restrict the mainline discharge flow to bottleneck capacity. The effectiveness of VSL as a control strategy rests upon its ability to restrict mainline flow.

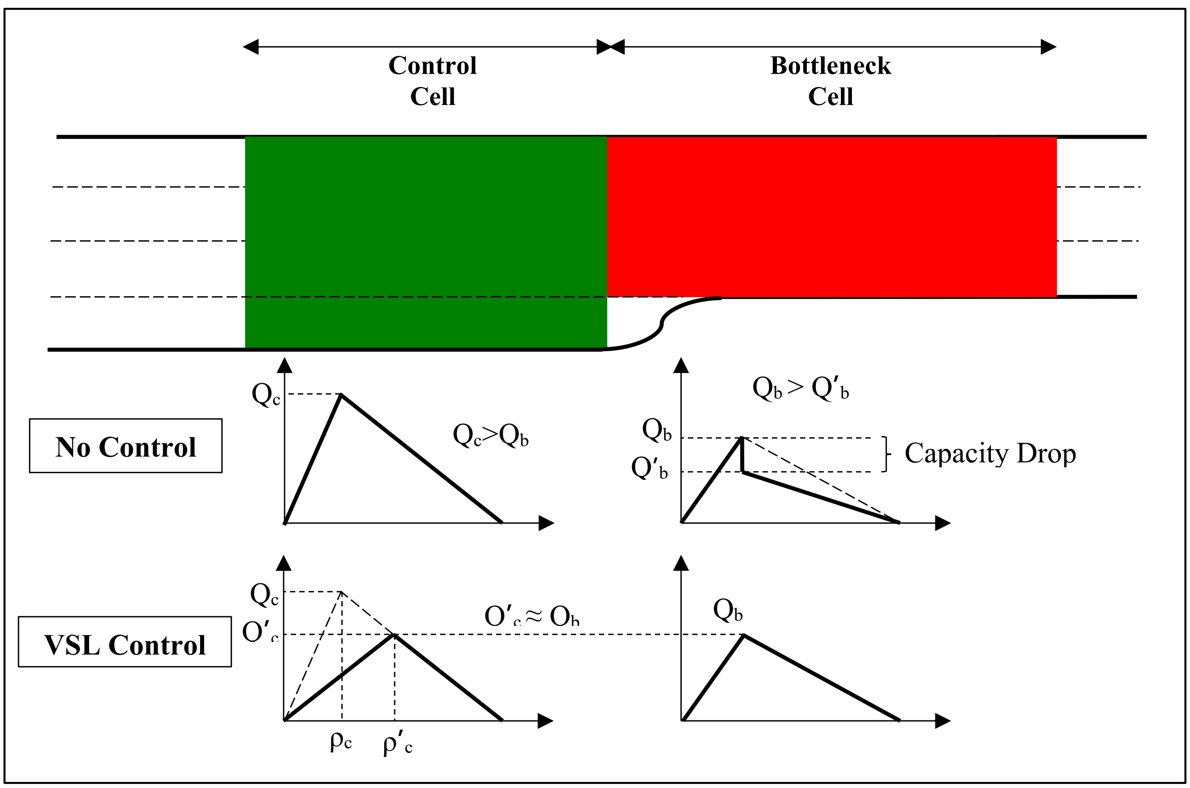

FIGURE 3.2 Potential Impacts of VSL Control on Bottleneck Traffic Operation

3.5 VSL CONTROL ON MODEL SIMULATION AND FIELD IMPLEMENTATION

A proactive VSL control strategy proposed by Hadiuzzaman et al. ***(22)*** was applied both in model and field that aimed at optimizing network mobility. This MPC framework was constructed by multi-module structure which includes collecting traffic state information from filed, predicting traffic states, optimizing network mobility parameter and applying control variables. A METANET-based dynamic traffic model, DynaTAM-VSL (Dynamici Analysis Tool for Active Traffic and Demand Management-Variable Speed Limit) was used for traffic state prediction from obtained real-time traffic information. Brief description of the prediction model and control algorithm is provided below.

Initially the studied roadway stretch was divided into several homogenous segemnts (i= 1,2……I) of length L_i and lanes λ_i. The predictions of traffic density, $\rho_i(k)$ and velocity, $v_i(k)$ at each time index *t (t=kT,* T= discrete time step= 20s, k= present time step) were calculated by modifying the second-order traffic state estimation model METANET. The evolution of traffic density and speed were predicted using the following two equations:

$$\rho_i(k+1) = p_i(k) + \frac{T}{L_i\lambda_i}[\lambda_{i-1}q_{i-1}(k) - \lambda_i q_i(k) + r_i(k) - s_i(k)] \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

$$v_i(k+1) = v_i(k) + \frac{T}{\tau}\{u_i(k) - v_i(k)\} + \frac{T}{L_i}v_i(k)[v_{i-1}(k) - v_i(k)] - \frac{T\nu}{\tau L_i}\frac{\rho_{i+1}(k) - \rho_i(k)}{\rho_i(k) + \varsigma} \quad \ldots (3.2)$$

Where, q = boundary flow between segment (vph), r = on ramp flow (vph), s = off-ramp flow (vph), τ = reaction time parameter (hr), ν = anticipation parameter and ς = positive constant. The last three mentioned parameters are global parameters that were calibrated from obtained traffic data. The speed dynamics equation differs from METANET model by replacing desired speed, $V_{e,i}[\rho_i(k)]$ with VSL speed value, $u_i(k)$ on the control segments of the corridor. The boundary flows among successive links were estimated by following equation considering link specific constraints and capacity drop concept:

$$q_i(k) = \min\left[v_i(k)p_i(k) + r_{i+1}(k) - s_{i+1}(k), Q_{max,i+1,u_{i+1}}, w_{i+1,u_{i+1}}\left(\rho_{jam,i+1} - \rho_{i+1}(k)\right)\right] (3.3)$$

The control algorithm also constrained VSL rates based on the inequality constraints. There were three constraints considered to ensure steady VSL rates. First two constraints bound the optimal speed limit between maximum and minimum values and the third constraint limited the SLs changing rate to maintain safe operation. The constraints are describes as follows:

Constraint 1: Road users safety and roadway design compliance was ensured by limiting the VSL, u_i to the maximum posted speed limit of the roadway, V_{max}. Therefore, $u_i \leq V_{max}$.

Constraint 2: To maintain a stable and continuous flow at all times, the VSL was limited to the minimum allowable speed, V_{min} of the roadway, such that $u_i \geq V_{min}$.

Constraint 3: To make the road users speed transition safe, a threshold value was set for the change in speed limits in consecutive time stamps. The threshold value for this control case was 10 kph, since the posted speed limits on the freeways in Canada were multiples of 10 kph. Therefore, $|u_i(k) - u_i(k+1)| \leq 10$ km/h.

The objective function of the control algorithm included weighted function of mainline Total Travel Time (TTT) and Total Travel Distance (TTD) to ensure maximum mobility benefits from VSL control. The weighting factors ($\propto_{TTT} = 80$ and $\propto_{TTD} = 1$) were selected beforehand to maintain a prudent relationship between the chosen mobility parameters. The optimal VSL values were calculate for a control horizon (N_c) of 3 min and the traffic state was predicted for a prediction horizon (N_p) of 5 min.The objective function aimed at minimizing the value of J to ensure a balance between traffic travel time and throughput. Since, minimizing travel time only could lead to reduced mainline traffic flow, total travel distance of the traffic along the corridor was also considered in the optimization to bring about a balance between these two factors.

$$J = T \sum_{j=1}^{N_p} \sum_{i=1}^{I} L_i \lambda_i [\propto_{TTT} \rho_i(k+j) - \propto_{TTD} q_i(k+j)] \quad \dots\dots\dots\dots\dots\dots (3.4)$$

The proposed VSL control algorithm was implemented on westbound direction of 6.5 kilometers stretch of an urban freeway corridor, Whitemud Drive in Edmonton, Alberta, Canada. The studied corridor contained a bottleneck segment between 122 Street on-ramp and Terwillegar Drive off-ramp. The performance of this bottleneck segment was evaluated in this study by simulation and field data. Calibration of the model was conducted from both system and operational perspective to replicate real-life traffic operation. The system calibration was performed using a TFlowFuzzy procedure in VISUM to generate a synthetic origin-destination trip demand matrix for the base model. The operational calibration of the model was exercised to reproduce bottleneck on the model by adjusting the Safety Distance Reduction Factor (SDRF) and Headway Time (CC1) of the bottleneck links. The calibrated microscopic model was used as a platform for executing model predictive VSL control. Based on the previously mentioned control algorithm, a field test of VSL control took place on this bottleneck section from August 11, 2015 to September 4, 2015. The test was conducted on weekdays during afternoon peak hours (4:30 PM to 6:30 PM) [Figure 3.4]. Two dynamic message signs (DMS) were placed to provide VSL information to road users. During the field test on the studied location, the VSL were neither mandatory nor enforceable. Rather it was announced beforehand to the road users that the SLs showed on DMS's were advisory SLs. Therefore, the compliances of the VSL were entirely voluntary. On the other hand, as mentioned before, the CLs for different SLs were predefined and the impacts of VSLs were quantified comparing performance parameter among base condition without VSL control (moderate CL with 80 kph static SL) with different CLs. To eliminate the randomness of the model results, 15-simulations, each with a set of random seeds, were conducted for each CL. The simulation duration was 2 hour excluding an additional warm-up period of 5 minutes. The process of VSL compliance analysis in microscopic model platform is presented in Figure 3.3. For VSL performance analysis through model, traffic speed and volume information were imported from loop detectors coded in the simulation model similar to the position of loop detector locations in field. To analyzed lane changing pattern from model, the bottleneck link was selected to collect lane changing information of each lane throughout the simulation period. For field test performance analysis, a set of without VSL control days were selected comparing the statistical significance of the demand profiles of the VSL control days during evening peak hours. The nearby days with significantly similar demand over the analysis period were chosen.

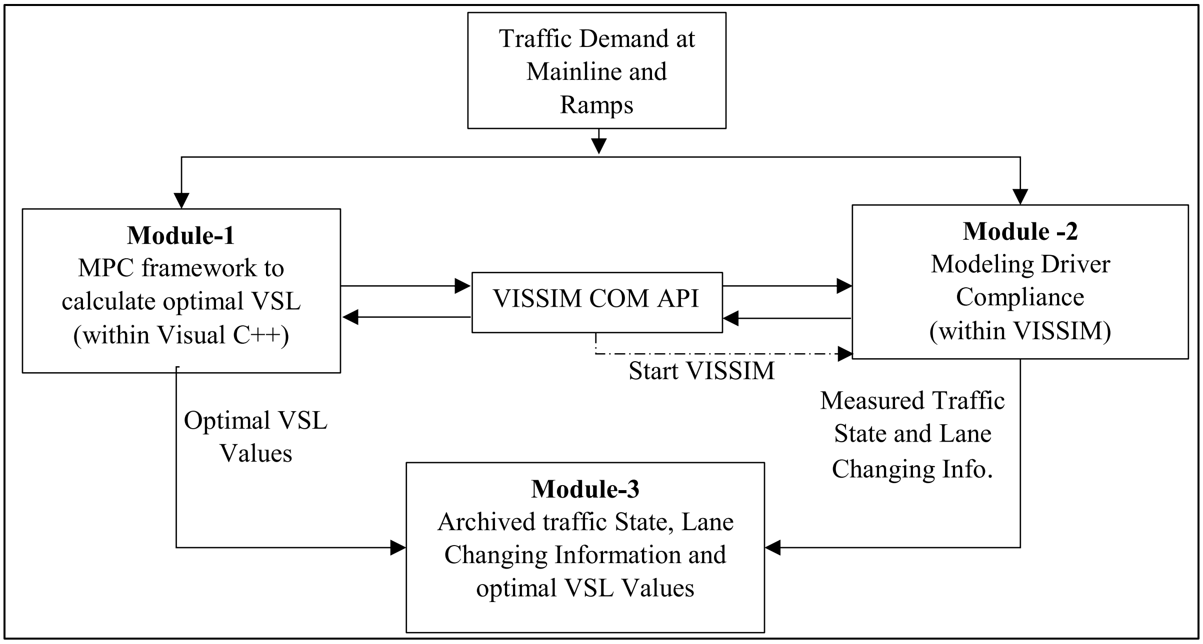

FIGURE 3.3 Integrated Model Simulation Platform for VSL Compliance Analysis

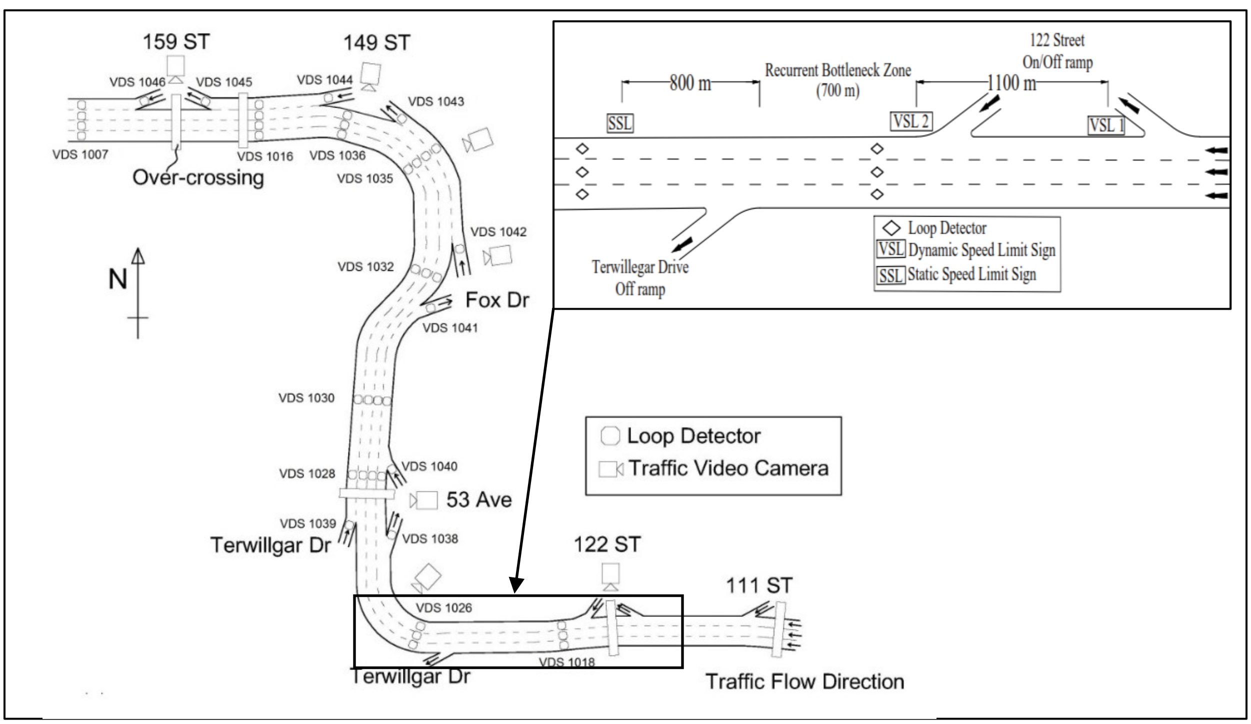

FIGURE 3.4 Studied Freeway Stretch and Bottleneck Segment

3.6 SIMULATION DATA ANALYSIS AND RESULTS

The constructed microscopic traffic model of the studied corridor was run with a fixed SL of 80kph to obtain traffic data that represent field traffic condition without VSL control. During the first hour of operation, the demand provided on the model simulation was very high to produce congestion on the predefined bottleneck location. As a result, a sharp decrease in operating speed was observed in the bottleneck location within a very short time. The compliance condition for the simulation run was selected as moderate CL to the speed limit and the provided traffic demand of the model was equivalent to average evening peak demand in the corridor that caused congestion in the recurrent bottleneck location. Afterwards, the simulation was performed with VISSIM COM support to execute VSL control on the model and to obtain traffic data at different CL. In this analysis, mean speed and lane changing data of the recurrent bottleneck section of the corridor was collected from coded loop detectors and integrated lane changing counter in VISSIM. Considering the geometry, location of on-ramp, off-ramp and most importantly perceiving the influence of lane changing in congestion formation, it was settled to calculate the total lane changing (number of vehicles going out of the subject lane and number of vehicles coming in from adjacent lanes) of middle lane on the bottleneck segment of corridor. The exclusive impacts of CLs were identified by comparing the performance parameters (mean speed, lane changing) among Non-control case with moderate compliance with VSL control scenario considering different compliance cases (Absolute CL, High CL, Moderate CL, Low CL). The lane changing flow and mean speed profiles are presented in Figure 3.5 for a sample simulation (random seed number = 43) at different CLs.

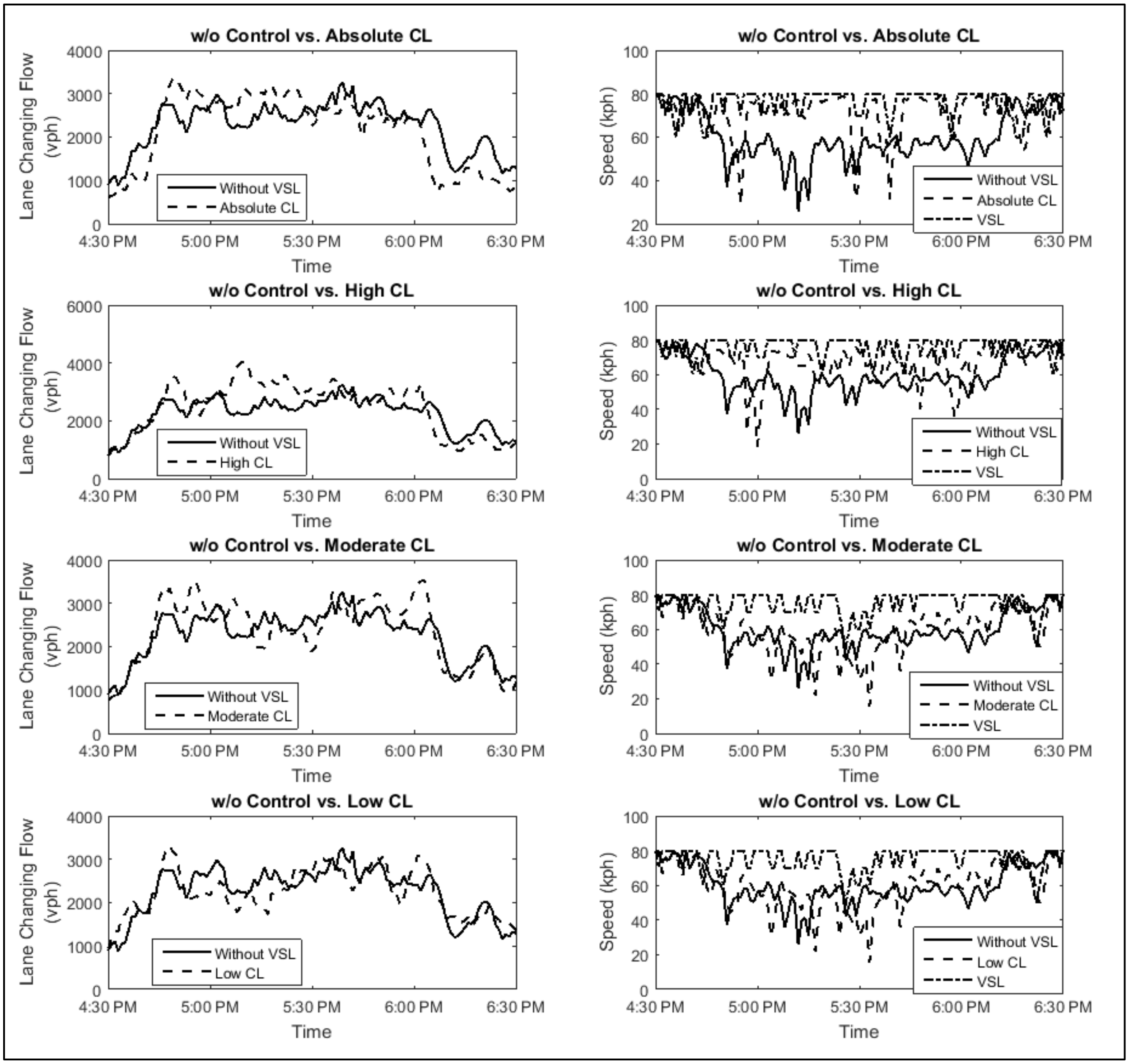

FIGURE 3.5 Performance Parameters Comparison between Non-control and Control Scenario at different Compliance Level

Table 3.2 and Figure 3.6 demonstrated the identified effects of VSL control on lane changing flow and mean speed from the model simulation at different CLs. From Table 3.2 and Figure 3.6, it was apparent that CLs of VSL impacted the lane changing behavior at the bottleneck section. The average number of lane changing was reduced by distributing the lane changing over a larger spread. Also the comparison of total number of lane changing between Absolute CL and without VSL control showed overall reduction in lane changing which indicated that it was possible to establish a stable traffic state when all the drivers abide by the VSL. An abrupt increment of average as well as standard deviation of lane changing and total lane changing was observed while comparing the lane changing behavior with Absolute CL (100%) and High CL (80%). This unexpected change was due to overall increase in speed restriction with lower compliance. The VSL control imposed reduced SL for 33-35% of the peak hours at High CL compared to 26-29% at Absolute CL. The imposed speed restriction at comparatively lower compliance level resulted into a reduced the average speed during the study period. Despite the reduction of average speed, the minute change in standard deviation of speed suggested that the vehicles were still driving at a relatively higher speed which made the lane changing maneuver easier. Therefore, it was quite evident that, the increased speed restriction supplemented with compliance reduction increased total number of lane changing maneuver. However, relatively high compliance of VSL restricted the speed reduction to a level that was favorable to change lane without causing substantial hindrance to the traffic flow that can cause severe speed reduction of overall traffic flow.

Under the circumstances with reduced compliance to VSL from 80% to 20%, a gradual reduction in lane changing flow and speed was observed from the analysis. Due to reduced compliance of road users, there was increased number of defensive and aggressive drivers on the road. The aggressive drivers would look for opportunity of lane changing to gain speed and density incentive and caused impedance on traffic stream. On the other hand, the defensive drivers would increase obstruction in the natural pattern of traffic flow due to their slow movement. The conceivable conflict of interest between these two groups of road users and comparatively minor involvement of compliant road users would generate lower operating speed of traffic. In consequence, it would become more difficult for the drivers to change lanes and the operating speed was also reduced which induced higher speed restrictions. It could be concluded from Figure 3.6(c) that the lane changing flow started to reduce with reduced SLs irrespective of CL.

TABLE 3.4 Descriptive Statistical Analysis of Performance Evaluation Parameters at different Compliance Level

	w/o VSL	Absolute CL	High CL	Moderate CL	Low CL
Lane Changing Flow (vph)					
Average	2236	2104	2426	2319	2208
Standard Deviation	578	875	910	762	544
25th Percentile	1776	1050	1424	1680	1776
Median	2412	2460	2784	2496	2160
75th Percentile	2670	2862	3138	2904	2694
LC Increase (%)	-	- 5.9	8.51	3.71	-1.22
Mean Speed (kph)					
Average	60.51	71.17	67.01	60.24	53.5
Standard Deviation	11.49	10.47	10.76	13.13	14.87
Maximum	80.26	80.21	82.36	80.30	79.31
Minimum	25.87	28.67	18.48	15.72	23.63

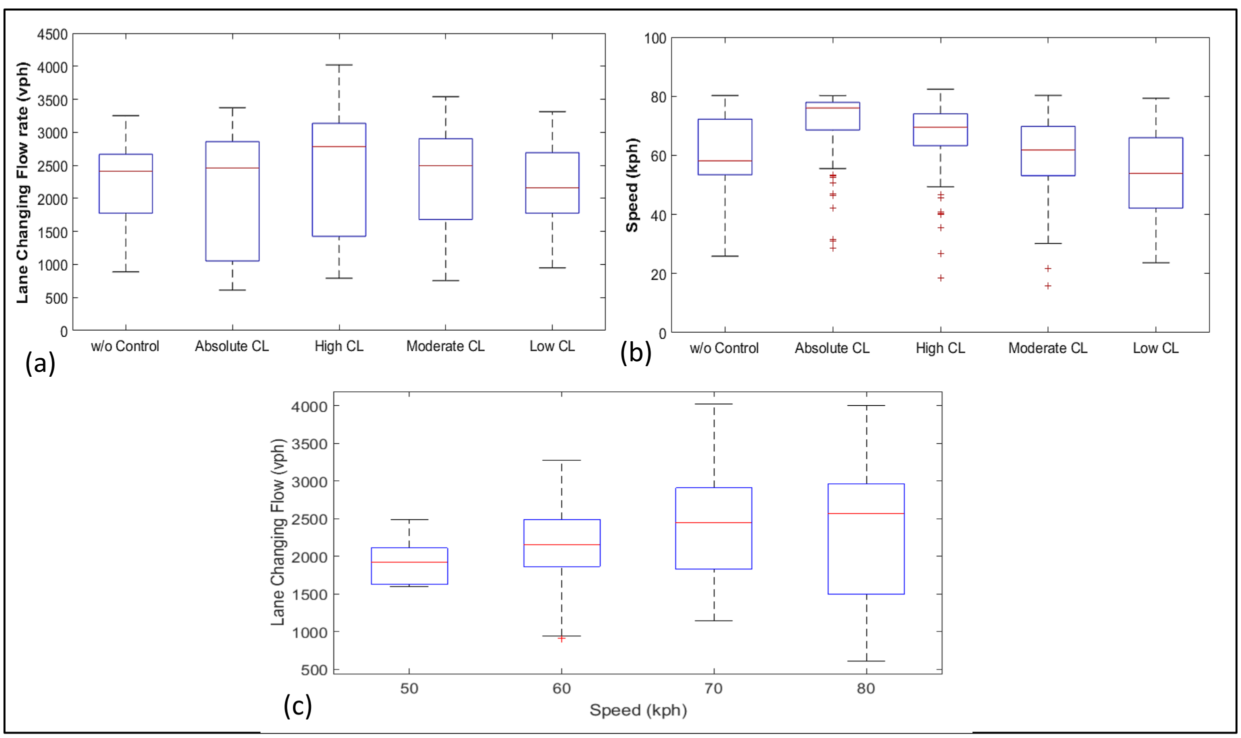

FIGURE 3.6 Simulation Model Data Analysis Resulting (a) Lane Changing Flow Rate Distribution Pattern as a function of Compliance Level, (b) Speed Distribution Pattern as function of Compliance Level and (c) Lane Changing Flow Distribution Pattern as a function of Speed

3.7 FIELD DATA ANALYSIS AND RESULTS

To analyze the impacts of VSL on compliance and lane changing behavior, the mean speed and flow data were collected from the loop detectors located on the bottleneck segment of the studied corridor. The information was collected for 10 days with imposed speed restrictions during evening peak hours and compared with days prior to the initiation of VSL field test with 80kph static SL in effect. Without VSL control days were selected comparing the traffic demand profile during evening peak hours and days with highest correlations were chosen for analysis. Since, the SL was fixed at 80kph at days without VSL control, it was only possible to obtain CL at 80kph which was, as stated before in driver compliance and modeling methods section, approximately 48% with 20% defensive and 32% aggressive drivers. On the other hand, VSL control provided the opportunity to identify the CL at different posted SL which was presented in Figure 3.6. This figure revealed that, the CL with VSL control did not showed any significant change at 80 kph SL in comparison to without control scenario. Unavailability of CLs for lower SLs compelled the authors to examine other performance parameters to identify any plausible change in compliance behavior regarding SLs lower than 80kph. Figure 3.7 (a) and (b) showed similarity in lane changing flow and speed distributions respectively between VSL control and non-control days. There were no significant difference in either of the performance parameters. Therefore, it would be a reasonable conjecture that the VSL field test with voluntary compliance had any significant effect on collective compliance behavior. From Figure 3.6, it was also apparent that the compliance rate varies at different SLs and in order to perceive the effects VSL controls on overall bottleneck performance, different compliance rate for different SL should be taken into considerations. As presented in Figure 3.7, a substantial increase was observed in compliance as well as aggressive driver percentage at lower SL (40 kph and 50 kph). In contrast, an increase in defensive drivers and reduction in compliance drivers with a stable percentage of aggressive drivers were witnessed at lower speed restrictions (70 kph and 60 kph).

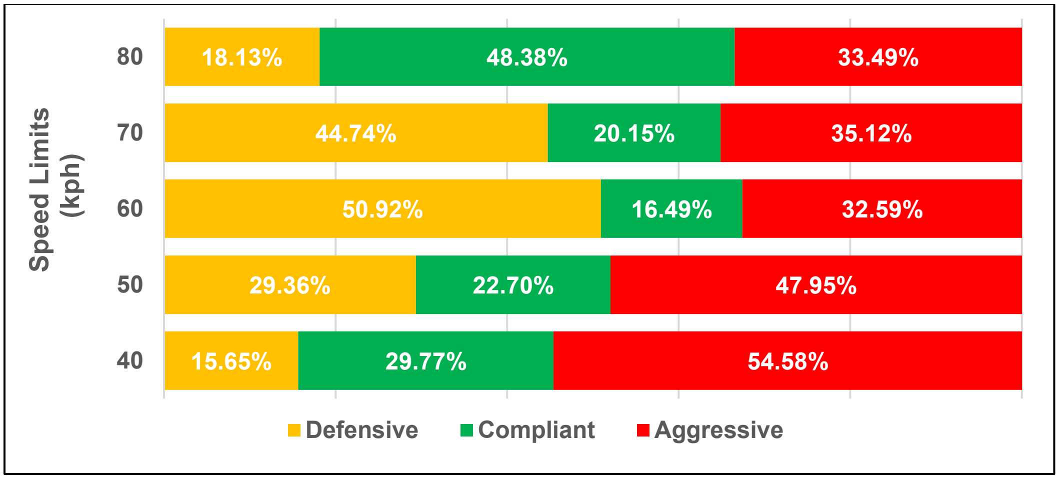

FIGURE 3.7 Identified Speed Limit Compliance at Different Speed Limits from Field Data

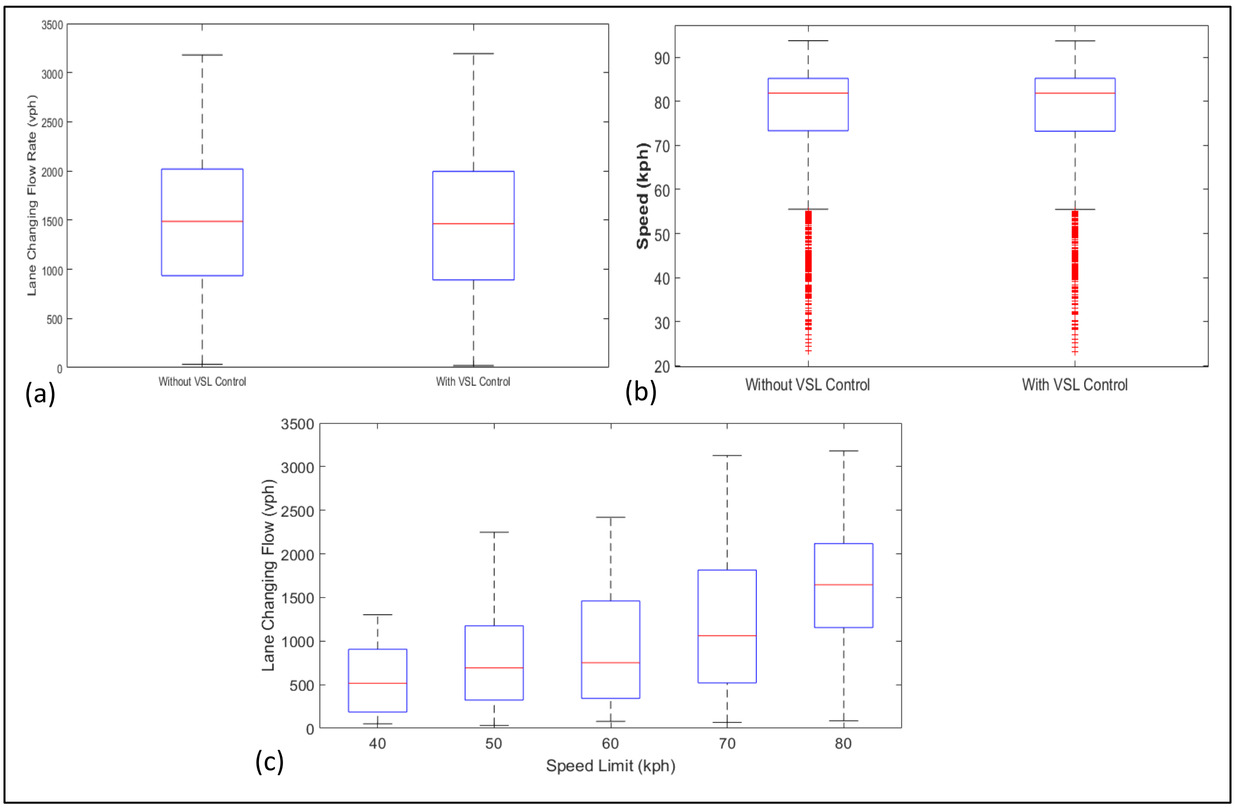

FIGURE 3.8 Field Data Analysis (a) Comparing Lane Changing Flow Rate Distribution Pattern, (b) Comparing Speed Distribution Pattern between with and without VSL control and (c) Lane Changing Flow Distribution Pattern as a function of Speed Limits

The analysis of lane changing flow at different SLs showed gradual reduction with the reductions of SLs as expected from the earlier analysis with model data [Figure 3.8(c) and Figure 3.6(c)]. The extent of lane changing flows increased initially with SLs of 70 kph. This was due to the fact that SLs reduction was incorporated with the mean speed reduction of the bottleneck segment and as the mean speed reduced the drivers began to change lanes for better driving opportunity. On the contrary, with larger speed reduction, the drivers found it difficult to change lanes at low speed and high density. This outcome was also observed by Wei et al. ***(23)***.

3.8 CONCLUSION

The primary objective of this chapter was to perceive not only the contribution of compliance to VSL on mean speed of the bottleneck section but also on lane changing behavior by developing simulation model representing real-world roadway conditions. The secondary objective was to assess the actual VSL field test data to identify the field compliance and also their implications on aforementioned performance parameters. Different compliance rate to VSL would generate different outcome on congestion mitigation of the bottleneck. Thus, several CL to VSL control were modeled with the speed distribution parameters obtained from static SLs of the studied road. The modeled speed distribution take both defensive and aggressive drivers into consideration along with the compliant drivers to mimic realistic traffic behavior. Afterwards, a MPC based multi-module VSL control was executed on microsimulation model at different CLs. The data of the simulation study found that mobility benefits were positively related with increasing CLs. The study also revealed that a notable benefits could not be expected without increasing compliance to VSL. Furthermore, the bottleneck performance could be worsened if VSL control would reduce SLs abidance. As for collective lane changing behavior, the VSL control with improved compliance would make the traffic stream more stable with higher lane changing flow at high speed. Lane changing maneuver could cause disruption to natural flow of traffic at low CL which would eventually resulted into congestion formation. The study findings of real-world VSL field test revealed important insight regarding SLs compliance behavior of traffic. Analysis of the speed data identified the existence different compliance rate for different SLs. Although a pattern of increasing defensive drivers with higher SLs were observed, the assumption of increased percentage of aggressive driver at lower SLs during model simulation was proved to be legitimate from the findings of the field data analysis. The evaluation of performance parameters showed identical response to VSL control in comparison to without control scenario. It is important to note that, the VSL implemented during the field test was advisory to the road users. The study of lane changing behavior at different SLs from the field test data manifested comparable pattern to the results of identical analysis of model data.

The study observations provide significant implications for VSL implementation on freeways with recurrent bottleneck segments. Based on the SLs compliance behavior in the field, the city transportation authority can work on policies to improve compliance in order to obtain benefits from the control. Transportation research community can be benefited from the findings by apprehending the gravity of SL compliance on collective lane changing behavior as well as speed and on VSL control in general.

Based on the conclusions derived from the study, the authors will work on imitating real-world speed distributions for different SLs and implication of their alteration on VSL control. Additionally, the findings of the study influenced the authors to study and modify the MPC based control algorithm to account for SL compliance. The modification will help to increase mobility benefits even without improving SL compliance.

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Chapter 4
Conclusions and Recommendations

4.1 GENERAL CONCLUSIONS

This thesis is primarily concerned with identifying lane changing flow from macroscopic traffic flow parameters and how this lane changing flows are influenced due to varying compliance of VSL control. The general conclusions obtained from the research are as follows

- In chapter 2, a location and traffic state dependent lane changing model was proposed that considered both speed and density difference as lane changing influences. The purpose behind developing a macroscopic lane changing model was to capture the comparative significance of traffic parameters in the lane changing decision making process. Thereby, eliminate possible equilibrium conundrum encountered by earlier macroscopic lane changing models and reduce conceivable sources of error.
- By incorporating lane changing flow with fundamental diagram, this model provided a simple framework to resemble longitudinal traffic flow dynamics on lane changing traffic dynamics.
- Classifying lane changing flow based on fundamental diagram enabled the model to provide more accurate estimation considering varying influence of decision making parameters.
- In chapter 3, the performance of MPC based VSL control was assessed in regard to reformation in lane changing pattern and resulting speed in bottleneck. In order to constitute a realistic evaluation, several compliance levels were modeled based on speed distribution parameters obtained from field observations.
- The model simulation demonstrated that increasing compliance of VSLs would lead to stable traffic flow with lower speed variations and higher lane changing at free flow state. On the contrary, lower CL would increase speed variations with extended speed restrictions and restrict lane changing maneuver due to cramped traffic state.
- Field observations of VSL field test disclosed unchanged compliance to static SL (80 kph). Although, the shift in compliance due to speed restrictions (SL<80 kph) were inconclusive due to inadequate compliance information on reduced SL, a distinctive increment in defensive drivers at higher SL (70 kph, 60 kph) and aggressive drivers at lower SL (50 kph, 40 kph) was observed.

4.2 LIMITATIONS OF THE STUDY

The extent of the study was limited due to various factors. For lane changing estimation, the study included the lanes least impressionable by lane changes due to on-ramps and off-ramps. Additionally, the studied lane for lane changing estimation had one neighbor lane. Therefore, vehicles could change lanes in one side of the lane. The consideration of a middle lane with two neighboring lane would make the study more captivating and complex. In order to apprehend the diversity of lane changing types, only congested stretch of the freeway was considered of the study, assuming segments without flow disruption would show less diversity and similar lane changing pattern.

The VSL compliance study findings provided important implications on lane changing pattern of traffic. However, the study was limited to the field observations of advisory VSL which displayed insignificant change in overall lane changing behavior. Also, the compliance behavior of traffic stream at maximum speed limit (80 kph) was found to be unchanged. On the other hand, the model simulation imposed VSL as a mandatory restriction with predefined compliance level. The results of model simulation showed promising improvements with higher compliance. Therefore, it would be improper to expect substantial benefits from VSL with voluntary compliance from road users. Since the field test of VSL did not enforced mandatory VSL compliance, the study was limited to the observations of voluntary compliance. Regardless of the fact that the field observations at different speed limits during VSL field test provided insight of compliance, unavailability of prior compliance information at different speed limits without control cases restricted the comparison of potential change due to VSL control.

4.3 RECOMMENDATIONS FOR FUTURE RESEARCH

This research provided significant understanding of lane changing behavior at freeway as well as VSL compliance impact on collective lane changing behavior. In order to utilize the gathered information, the studies related to this topic are needed to be continued in different directions. For instance, the lane changing flow estimation study can be improved by considering subject lanes with two neighboring lanes and their relative involvement in lane changing flow. Furthermore, the effects lane changing flow with regard to lane position as well as relative location of on-ramp, off-ramps are required to be investigated. The current study on lane changing flow estimation considered two fundamental traffic flow parameters for estimation. In future, the lane changing flow estimation should also considered different exogenous factors for more precise estimation as well as the variations those exogenous factors over time and space will require research attention.

The speed limit compliance study rendered a comprehensive perception of VSL compliance impact on collective lane changing pattern of traffic. In future, the study focusing improving the VSL control algorithm should be considered. At present, the control algorithm considered posted VSL as the desired speed of the traffic which is not realistic due to varying compliance of speed limits at different speed limits. Moreover, the existence of defensive and aggressive driver in real-traffic make the assumption of desired speed being equal to speed limit obsolete. Therefore, further research is required to apprehend the impacts of speed limit compliance on desired speed which will consequently help to modify the control algorithm. Furthermore, additional research resource should be invested on generating increased benefit from VSL with a minimal change in compliance.

4.4 CONCLUSION

This research explored a branch of traffic operation which was previously uncharted and provided with consequential insights regarding lane changing traffic flow and speed limit compliance behavior of road users. The findings of this research will influence future studies of researchers with similar research interest as well as assist in decision making process of freeway traffic operation authority. The research findings have greater application and traceability in future transportation research and innovative technologies. Traffic state control with the help of connected vehicle technology can be enhanced with the understanding of speed and density influence on lane changing behavior. It will be remarkable to integrate proposed lane changing model with autonomous vehicle technology and assist in lane changing decision making from macroscopic perspective which will eventually help achieving maximum capacity of the freeways. Furthermore, the VSL compliance study will provide insights about potential impacts of different penetration rates of connected vehicles and autonomous vehicles on traffic flow. In short, the connection of this research with future transportation research and technology can promote new research interests and enhance our knowledge and understating about traffic operation and management.

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