University of Alberta

Three Essays on Joint decisions in Business

by

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Dedication

To my mother, Xiue Geng,

My husband, Hai Li Wang,

My Children, Alexander Yisu and Caroline Yifan Wang

and im memory of my father

Zhongshui Niu

Abstract

Managers regularly make various decisions within various functional areas in a business environment. The interaction among decisions exists not only within functional areas but also at the interfaces of different functional areas. In today's competitive business world, it is critical for companies to realize the importance of making integrated decisions because making decisions separately generally result in sub-optimal solutions. This dissertation includes three independent papers on the theme of joint decisions in business. Joint decision making is both a current managerial concern and an important research issue. This dissertation applies management science techniques and game theory approaches to help make joint decisions in the areas of operations management and marketing.

The first paper presents a state-of-the-art survey on joint pricing and inventory/production decisions in supply chains. I classified related research into four categories based on the various assumptions on supply chain structures, demand patterns, and interaction among chain members. Directions for future research are suggested.

The second paper models and analyzes joint pricing and inventory/production decisions in a two-stage dual-channel retail supply chain. The paper analyzes joint decisions problems under three scenarios by incorporating intra-product line price interaction in the EOQ model and shows that a unique equilibrium exists under certain realistic conditions. It also provides numerical results that offer insights for pricing strategies for the dual-channel retailer supply chain and for product design for different channels.

The third paper is about a different set of joint decisions: Joint pricing and referral reward programs. Pricing has been well known as an effective way of managing demand. Referral reward programs are an increasingly popular way to manage consumer referrals with incentives. This paper examines joint optimal pricing and referral reward programs for two competing firms selling substitutable products/services to a common customer pool. The paper provides managerial insights to help managers to determine optimal price and the timing strategy and the level of a reward.

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To all of you, thanks.

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This dissertation includes three independent papers on the theme of joint decisions in business. Chapter 2 and Chapter 3 are two papers on joint pricing and inventory/production decisions in supply chains. Chapter 4 is a paper on joint pricing and referral reward programs under competition. Chapters 2, 3, and 4 are written as independent and self-contained papers. Chapter 5 provides a conclusion for the dissertation and points out other potential joint decisions in a business environment for future research.

The dissertation applies management science techniques and game theory approaches to help make joint decisions in the areas of operations management and marketing. I address inventory/production issues in operations management and pricing and promotion issues in marketing. Research in this dissertation has the potential to bring together concepts from operations management and marketing. A considerable amount of literature describes advances in research and management practices in the area of joint pricing and inventory/production decisions in a single firm. The idea of joint pricing and inventory/production decisions can be extended to multi-firm supply chains. The main objective of supply chain management is to minimize the chain-wide costs, while satisfying service requirements and focusing on how to coordinate supply chain members in an efficient manner. It is intuitive to think that supply chain members could leverage chain-wide profits if their pricing and inventory/production decisions are made jointly. Thus, I investigate joint inventory and pricing problems in a multi-firm supply chain setting in Chapters 2 and 3. Chapter 4 focuses on two marketing issues, pricing and promotion, under competition in a duopoly environment.

Joint decision making is both a current managerial concern and an important research issue. Managers regularly make strategic, tactical, and operational decisions within various functional areas in a business environment. The interaction among decisions exists not only within functional areas but also at the interfaces of different functional areas. In today's competitive business world, it is critical for companies to realize the importance of making integrated decisions because making decisions separately generally result in sub-optimal solutions.

The joint decisions that I am interested in are decisions involving a single firm, i.e., the firm makes two or more decisions simultaneously to improve its profitability. It is possible for two or more firms make efforts to jointly make their decisions. However, making joint decisions among firms is out of the scope of this dissertation. In this dissertation, I consider simultaneous decisions regarding pricing strategy and other functional decisions in a single firm. Price is perhaps the most important of the four Ps of the marketing mix: Price, product, promotion, and place, because it is the only one that generates revenue for a company. Pricing strategy, a key marketing tool to directly influence a firm's demand, is a critical decision that is made jointly with other decisions in this dissertation. Joint pricing and inventory/production decisions, one of the two sets of joint decisions in this dissertation, lie at the interface of marketing and operations management. The other set of joint decisions, joint pricing and referral reward programs, involves the 2Ps, price and promotion, in the marketing mix.

Joint pricing and inventory/production decisions in a single firm have been extensively studied over past 50 years with more than 100 articles published. A broad review can be found in Eliashberg and Steinberg (1993) and Yano and Gilbert (2002). An emerging body of literature deals with such joint decisions in a multi-firm supply chain setting. Efforts have focused on developing coordination mechanisms to achieve integrated performance without real vertical integration in supply chains. Cachon (2003) and Chan et al. (2004) provide intensive reviews on supply chain coordination. The first papers dealing with joint pricing and inventory decisions in a multi-firm supply chain were published about 20 years ago, and there is a growing interest in this area both in the academic community and in the business world. Although the time span of this body of literature is still relatively modest compared with some other areas in supply chain management and operations management, I feel that the academic and practical importance of the topic merits a survey paper.

Therefore, Chapter 2 presents a state-of-the-art survey on joint pricing and inventory/production decisions in supply chains. The main feature of these reviewed articles is that they address both retail price-sensitive demand processes that are seldom considered in inventory control research and inventory issues that many marketing and economics researchers have ignored. A significant amount of research in this area is concerned with the coordination mechanism among chain members and characteristics of optimal decisions. The trend of coordinating a supply chain is accelerated by competitive pressures and the realization that the overall supply chain performance can be dramatically improved by designing commercial relationships that can achieve coordination. The research is classified into four categories based on the various assumptions on supply chain structures, demand patterns, and interaction among chain members.

Although the survey shows that the body of literature is very diverse and has great academic and practical relevance, the surveyed research simplifies joint decision problems in the real world with various (sometimes restrictive) assumptions. Therefore, there are some important gaps between what has been done in research and the problems that arise in industrial settings. More work can be done to examine joint decision problems to consider multiple products and production capacity, information asymmetry, time-dependent demand processes, coordination and benefit sharing, and more complicated and practical supply chain structures.

In Chapter 3, I model and analyze joint pricing and inventory/production decisions in a two-stage dual-channel retail supply chain. Since the Internet is becoming increasingly important as a sales channel, most large retail firms have adopted a multi-channel strategy that includes both web-based channels (online stores) and pre-existing off-line channels (brick-and-mortar stores). Competition between two retail channels is easy to ignore in such a system because they belong to one owner. I focus on competition between the two retail channels and examine its impact on joint decisions.

For a dual-channel retailer, pricing in one channel will affect the demand in the other channel. The demand subsequently affects the retailer's replenishment (ordering) decisions, which have an impact on the producer's inventory/production plans and wholesale price decisions. Thus, pricing decisions and inventory/production decisions interact within each firm and between the firms in the supply chain. Pricing and inventory decisions for the channels under competition are important issues that management teams have to deal with. In Chapter 3, I analyze joint pricing and inventory/production problems under three scenarios by incorporating intra-product line price interaction in the EOQ model. It is shown that a unique equilibrium exists under certain realistic conditions. I obtain several insights. One of the main findings is a price convergence effect between the online and the off-line channels as the degree of substitution of the products increases. This finding implies that pricing for different product categories for the online store and the off-line store must be done strategically. Furthermore, when products sold in different channels have different configurations, the difference in the production costs should be taken into consideration when making pricing decisions.

Chapter 4 is about a different set of joint decisions: Joint pricing and referral reward programs. Referral reward programs, which are firm-promoted Word-of-Mouth campaigns, are an increasingly popular way to manage consumer referrals with incentives. Firms have introduced incentive programs with various types of rewards (cash back, vouchers, free gifts, free minutes, airmiles, future purchase discounts) that are designed to encourage existing customers to recommend products or services to others. The use of referral reward programs has been growing steadily because reward programs are a cost-effective way to recruit customers and increase sales (Biyalogorsky et al., 2001). Although both pricing and referral reward programs are effective tools to manage demand, it is also true that pricing strategies and referral reward programs can be wasteful when not designed properly. Furthermore, price affects the demand for a product or service and probably the customers' referral intentions as well. Because of the limited research on joint pricing and referral reward programs, the literature review is provided with my research questions and my analytical model in Chapter 4.

Chapter 4 examines joint optimal pricing and referral reward programs for two competing firms selling substitutable products/services to a common customer pool. It is common to see that an absolute reward or a certain percentage of the recommended purchases is offered to referrers. I model the reward both as an absolute value and a percentage of the retail price and determine what are the optimal pricing and reward decisions in each scenario.

I classify referral reward programs as being either *easy access* or *restricted access*. There are many examples of easy access programs, which make customers aware of the reward before a purchase. Other companies inform customers of their referral reward programs only after customers have made a purchase. Since customers' expected net prices are different for different strategies, firms can choose different strategies influence the timing of the awareness of the referral programs, which in turn affect customers' purchasing decisions. Firms might have various reasons to choose different strategies. Restricted access programs might be chosen for attracting referrals only, while easy access program might be chosen for attracting both self-search customers and referrals. In addition, the set-up costs for restricted access programs are generally lower than easy access programs. Therefore, in addition to making decisions on prices and referral rewards, both firms also choose a reward timing strategy: Making customers aware of the program before purchase by offering an easy access program or making customers aware of the program after purchase by offering a restricted access program.

I first analyze the interaction between the two firms as a simultaneous game. Then, I extend the model to a sequential game setting. We find that both firms benefit from offering referral rewards in the simultaneous game and in the sequential game. When both firms offer easy access programs, they benefit most from offering the best possible reward, no matter whether they provide absolute rewards or percentage rewards and no matter whether they compete in simultaneous games or sequential games. In a simultaneous game with absolute rewards, both firms choose to the same program type (easy access or restricted access.) However, which strategy is better depends on the sizes of the firms' potential markets, customer preferences over the firms, and customer responsiveness to the referral rewards. When percentage rewards are offered, it is in both firms' interest to offer easy access programs.

Chapter 5 concludes the dissertation and discusses related potential research questions in business environments.

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Chapter 2: Joint Pricing and Inventory/Production Decisions in a Supply Chain: A state-of-the-art Survey

1 Introduction

There is a considerable amount of literature that describes advances in research and management practices in the area of joint pricing and inventory/production decisions (from now on, joint decisions) in a single firm. These papers deal with the interface between marketing and manufacturing decisions, specifically the simultaneous determination of pricing and inventory/production planning decisions. Research on joint decisions in a single firm has spanned over 50 years, and over 100 articles have been published. Eliashberg and Steinberg (1993) and Yano and Gilbert (2002) provide a broad review on this topic.

The idea of joint decisions can be extended to multi-firm supply chains. The main objective of supply chain management is to minimize the chain-wide costs, while satisfying service requirements and focusing on how to coordinate supply chain members in an efficient manner. Cachon (2003) and Chan et al. (2004) provide intensive reviews on supply chain coordination. It is intuitive to think that supply chain members could leverage chain-wide profits if their pricing and inventory/production decisions are made jointly. There is an emerging body of literature that deals with joint decisions in a multi-firm supply chain setting. The first papers were published over 20 years ago, and there is a growing interest in the area. Although the time span of this body of literature is still relatively modest compared to some other areas in supply chain management, we feel that the academic and practical importance of the topic merits a survey paper.

This paper surveys the current status of research as well as directions for future research in the area of joint decisions in a supply chain. The rest of the paper is organized as follows. Section 2 discusses joint decisions. Section 3 summarizes modeling assumptions in the research and categorizes existing models into four categories. Section 4 presents our survey based on the categories identified in Section 3. Section 5 discusses our findings and directions for future research.

2 Joint Pricing and Production/Inventory Decisions

In practice, pricing and inventory/production decisions have traditionally been determined sequentially. Even in a single firm with multiple functional departments, the marketing department determines prices to maximize the firm's profit, based on the demand forecast and estimated average product costs. The production department then takes these prices as given and makes production/inventory decisions in order to minimize total operational costs while satisfying forecasted demand. Usually, no mechanisms are adopted to coordinate these decisions. If we consider different departments in a single firm as the members of a supply chain, the marketing department will be regarded as the buyer and the production department as the supplier. Thus, the average production costs estimated by the marketing department can be regarded as the transfer prices from the production department to the marketing department.

In a supply chain with multiple firms, prices, production plans, and purchase plans have to be determined by each firm member. For example, on the supplier's side, wholesale prices, production plans, and raw material purchase plans have to be determined. The retailer's prices and replenishment decisions according to the contract arrangements between chain members also have to be determined. However, the decision models that chain members can implement depend on the structure of the supply chain. In a decentralized supply chain, each member is an independent interest entity and makes its decisions only to maximize its own profits. In contrast, in a vertically integrated (centralized) supply chain, all decisions are made by a central decision maker who considers maximizing the chain-wide profit. Since each independent interest entity considers only its own profits in its decision making, double marginalization exists in a decentralized supply chain. Thus, a vertically integrated supply chain is more efficient than a decentralized supply chain (Spengler, 1950).

Both practical and theoretical efforts have focused on the development of coordination mechanisms that can help achieve integrated performance without real vertical integration. In recent years, mechanisms such as quantity discounts, buy-back contracts, and two-part tariff contracts have been applied in many industries as well as research to improve coordination in decentralized supply chains. Reviews on supply chain coordination and contracting can be found in Cachon (2003) and Tsay et al. (1999). If a decentralized supply chain performs as efficiently as a centralized supply chain, perfect coordination is achieved. Thus, the optimal solution of a centralized supply chain provides a benchmark to evaluate the performance of these coordination mechanisms.

In addition to the development of coordination mechanisms, many practitioners have realized that sequential pricing and inventory/production decisions are problematic and have made efforts to improve efficiencies by making joint decisions both in single firms and among chain members. In a single firm, teams across multiple functional departments are formed to make better decisions. Similarly, firms forming a supply chain have been establishing strategic relationships to improve the performance of the supply chain to enhance their competitive advantages.

It is evident to us that research on pricing is separate from research on inventory/production decisions. On the one hand, there is plenty of research on inventory management and on supply chain management that almost invariably assumes that the demand processes are determined exogenously; thus, they are uncontrollable. This line of research mainly focuses on capturing the inventory holding and stock out penalty costs at different stages of a supply chain. Thus, in this exogenous demand setting, the main objective is to minimize either the firm's or the chain-wide total inventory and production costs. In reality, however, demand processes can often be controlled by varying the price structure. For example, in a supply chain, the inventory decisions of each member can be affected by changes in retail prices. On the other hand, in the marketing and economics research on pricing and channel coordination, price, which is determined by considering marginal costs while ignoring inventory/production issues, is the main instrument available to coordinate a channel.

To summarize, we can see that marketing and economics researchers have been concerned, for the most part, with the effect of price changes on demand without much regard to inventory and production problems; whereas, operations management researchers have started with the assumption that retail prices are given and attempted to minimize total inventory and production costs subject to some exogenous demand pattern. The artificial separation of marketing and operational decisions reduces efficiency for both single firms and multiple-firm supply chains. Kunreuther and Richard (1971) indicate that the sequential procedure is likely to be very costly.

There is a growing body of literature that studies joint decision making problems in decentralized supply chains. In this paper, we review articles that focus on joint decision making in these types of supply chains and identify future key research topics within this area. The main feature of these reviewed articles is that they address both retail price-sensitive demand processes that are seldom considered in inventory control research and inventory issues that most marketing and economic researchers have ignored. A significant amount of research in this area is concerned with how to achieve supply chain coordination in a decentralized supply chain. The trend of coordinating a supply chain is accelerated by competitive pressures and the realization that the overall supply chain performance can be dramatically improved by designing commercial relationships that can achieve coordination.

We now turn our attention to the modeling considerations and categories that will provide structure to our survey.

3 Model Categories

As discussed in the previous section, the main motivation behind joint decision making is the price-sensitive nature of demand that creates an interface between pricing and production. While all the literature surveyed in this paper shares the paradigm of joint decision making, there are other aspects in which the approaches differ.

In this section, we discuss the role of supply chain structure, demand pattern, and some other aspects in choosing and developing joint decision making models. Furthermore, we categorize the research in joint decision making in multi-firm supply chains into four categories based on supply chain structures, demand patterns, and interaction among chain members.

3.1 Supply Chain Structures

Most reviewed articles consider two-level supply chains in which the suppliers can be referred to as the upstream firms, the manufacturers, or the distributors; and the buyers as the downstream firms, the distributors, or the retailers.

The supply chain structure with a single supplier and a single buyer has been the most widely studied structure in research on joint decision problems. We use 1-to-1 to denote this structure. Some articles consider supply chains with a single supplier and multiple buyers. If the supplier sells to multiple geographically dispersed buyers who face identical demand processes and ignores transportation and lead time differences, this type of supply chain can be treated as a 1-to-1 case. We use 1-to-M to denote supply chains with one single supplier and multiple non-identical buyers.

When there is more than one buyer in a supply chain, the mechanism used to coordinate firms can be different. In particular, competition among buyers can affect the coordination mechanisms. We use 1-to-M/w to denote supply chains with a single supplier and multiple competitive buyers. We use 1-to-M/n to denote supply chains with a single supplier and multiple non-competitive buyers. Thus, in the following sections, supply chains with a single supplier and a single buyer, a single supplier and multiple non-competitive buyers, or a single supplier and multiple competitive buyers are referred to as 1-to-1, 1-to-M/n, or 1-to-M/w, respectively.

3.2 Demand Patterns

This survey considers research with price-sensitive demand that can be modeled as either deterministic or stochastic processes. Most articles have addressed demand as a deterministic demand (Dd) function of a retail price when there is a single buyer, or of a retail price vector in cases of multiple buyers that charge different prices. Deterministic price-sensitive demand is generally formulated as a downward sloping curve of retail prices. Typically, a linear demand function has been employed. Some articles model demand as constant elasticity functions. Most research considers demand functions to be invertible; therefore, a corresponding price curve exists, and demand and price have a one to one corresponding relationship, which is quite critical to most analyses.

Joint decision problems with stochastic demand (Sd) are usually seen as price-sensitive newsvendor problems. Price-sensitive stochastic demand can be formulated as an addition or a multiplication of a price sensitive-deterministic component and a random component. Petruzzi and Dada (1999) point out that different demand forms cause significantly different conclusions for joint decision problems in Mills (1959) and Karlin and Carr (1962). A fundamental difference between these two different demand forms is the role that price plays in demand uncertainty. With additive demand, the variance of demand is independent of price, while the demand coefficient of variation increases with price. With multiplicative demand, the variance of demand decreases with price, while the demand coefficient of variation is independent of price. Although pricing provide an opportunity to reduce the risk of overstocking or understocking, it cannot ideally decrease both demand variance and coefficient of variation (two most commonly used measure of variation) for either additive or multiplicative demand. However, Petruzzi and Dada (1999) identify that in the additive demand case, a lower price can decrease the coefficient of variation without adversely affecting the demand variance; and in the multiplicative case, a higher price can decrease the demand variance without adversely affecting the coefficient of variation. Thus, different pricing decisions are reached for different demand forms. In Section 4.4, we discuss how joint decision problems can be transformed to incorporate both demand forms.

3.3 Other Modeling Assumptions

Static or dynamic decisions. Static decisions refer to pricing and production/inventory decisions that remain constant along the planning horizon. If decisions can be changed during the planning horizon, they are referred to as dynamic decisions. Dynamic decisions are becoming more and more widely applied both in the research world and in practice. Revenue management, for example, is an intensive application area of dynamic pricing decisions. Costs associated with changing decisions over time have to be factored in when making and applying dynamic decisions. For traditional industries with mature products and stable markets, a static decision strategy is generally a good choice that is easy to manage. In some industries or new business models, such as e-commerce, dynamic decisions are more common because sellers can change pricing quickly; thus, it is less costly to obtain more profits than with static decisions. Although there is a rich body of research on dynamic pricing and inventory control in a single firm (Elmaghraby and Keskinocak, 2003, for a review), we found only a few papers considering dynamic pricing and inventory control in a supply chain setting, which we will review in Sections 4.1.2 and 4.4.

Cost structures. In most cases, a supplier's costs include fixed ordering costs, production costs (if manufacturing), inventory holding costs, backlogging costs, and order processing costs; while a buyer's costs include fixed ordering costs, processing costs (if value is added to the product), inventory holding costs, and purchasing costs.

Production costs can be modeled as convex or concave functions of the quantity produced. The selection of the form of the cost function depends on the characteristics of the production situation and can be critical to the analysis. Eliashberg and Steinberg (1987) justify the use of convex production costs by quoting Johnson (1974): "the convex production cost model often results from situations where there are multiple production sources in a period and it is assumed that production costs are proportional to the quantity produced by a resource. By assigning production first to the resource with the lowest unit cost until its capacity is reached, then proceeding to use the next cheapest resource to capacity, etc., one develops a total production cost that is convex in the total amount scheduled for the period." The production cost structure is not always convex in the quantity produced. Setup charges in batch production, fixed charges with order quantities, and economies of scale in production processes can cause concave costs in quantity.

Usually, unit holding costs incorporate the opportunity cost of capital tied in inventory along with other carrying costs. Inventory holding costs are usually modeled as linear functions of quantity with unit holding cost proportional to unit cost.

The basic and well-known economic order quantity (EOQ) model seeks the optimal order quantity under consideration of ordering costs, purchasing costs, and inventory holding costs. Research that considers setup costs or fixed ordering costs usually models joint decision problems using extended price-sensitive EOQ models, where total costs are convex in quantity at any given price.

Replenishment policies. The chain members replenish their inventory by ordering from their suppliers. In the surveyed articles, constant replenishment lead times are generally assumed. The lead times are scaled to zero for simplicity since the time horizon can always be shifted to accommodate non-zero lead times. Section 5 discusses one article that considers stochastic lead times.

Products. Most research considers a single product that is manufactured or purchased by the supplier and then distributed to the buyers. In Section 5, we discuss articles that consider multiple products that are partially substitutable or complementary.

Modeling approaches. Game theory is a powerful tool for analyzing problems with entities that interact. Supply chain management that considers interaction among members is a field where game theory can make a substantial contribution. In the reviewed papers, we notice that game theory concepts such as Stackelberg, Nash, and Pareto solutions are usually employed. Stackelberg and Nash games are played when the players in a game do not cooperate. A Stackelberg game, known as a leader/follower game, is applied to joint decision problems in a decentralized supply chain in which the dominant party in the chain, usually the manufacturer (sometimes the retailer), the leader who makes decisions first then the followers, usually the buyers (sometimes the supplier), make their decisions. If the actions of the members in a chain are taken simultaneously, a Nash game is played, and the solution identifies a point from which none of the players has an incentive to deviate. If the members in a supply chain cooperate to make decisions, concepts such as Nash Bargaining or Pareto solutions are applied. Readers are referred, for example, to Fudenberg and Tirole (1991) for an introduction of various game theory concepts.

Next, we categorize the related articles into four categories based on supply chain structures, demand patterns, and interaction among chain members.

3.4 Categorization of Research

The bulk of the existing research can be partitioned into four categories based on supply chain structures, demand patterns, and interaction among chain members that we have just discussed.

- 1-to-1-Dd settings in which supply chains with a single supplier and a single buyer face a deterministic demand
- 1-to M/w-Dd settings in which supply chains with a single supplier and multiple competitive buyers face deterministic demand
- 1-to-M/n-Dd settings in which supply chains with a single supplier and multiple non-competitive buyers face deterministic demand
- Sd settings in which supply chains face stochastic demand. We consider research on stochastic demand under different chain structures in one category because of the limited number of publications focusing on stochastic demand

4 Survey

4.1 Single supplier, Single buyer, Deterministic Demand: 1-to-1-Dd

Most of the research on joint decisions belongs to this category. Monahan (1984) and Stuckey and White (1993) justify the consideration of 1-to-1 channels and specify some special conditions in which this type of channel exists. Although Ertek and Griffin (2002) point out channels that meet these conditions include a small portion of the economy, Tsay et al. (1999) believe that more firms can benefit from forming such a 1-to-1 long term relationship because it can reduce ordering costs, facilitate more information sharing, and promote collaborative product design/redesign.

According to different decision patterns, research in these 1-to-1-Dd settings can be divided into two subcategories: Models with static decisions and models with dynamic decisions. We do a detailed survey of these two subcategories in the next subsections.

4.1.1 Static joint decisions in 1-to-1-Dd settings

Most of the models formulated in 1-to-1-Dd settings extend the basic EOQ model to include a price-sensitive demand process. The objective becomes profit maximization instead of cost minimization. In a 1-to-1 supply chain, both members have to solve their own price-sensitive EOQ problem. This means that in general the supplier controls its wholesale price and production rate or replenishment decisions according to its optimal EOQ model solution; the buyer controls its retail price and replenishment and according to its EOQ model solution.

Since the supplier and the buyer in this type of supply chain are independent entities, interaction between them can usually be formulated as a non-cooperative game. Various instruments such as quantity discounts and buy-back contracts have been studied to induce parties in decentralized systems to make decisions to achieve coordination. Since the dominant party usually benefits most from channel coordination, it interests to introduce these instruments to induce coordination. Channel coordination is an important research area in supply chain management. A basic principle of coordination is that each member in the channel should be no worse off than with no coordination.

Although a well-known conclusion in Jeuland and Shugan (1983) indicates that quantity discounts can achieve perfect coordination in a 1-to-1 channel when inventory issues are ignored, Weng (1995) concludes that quantity discounts alone are not sufficient to guarantee perfect coordination when solving joint decision problems, while quantity discounts and franchise fees can. He also identifies the equivalence of all-unit quantity discount and incremental quantity discount policies in the optimal coordinated decisions.

However, whether quantity discount can coordinate a channel depends on whether the ownership of inventory is transferred from the supplier to the retailer (Boyaci and Gallego, 2002). The authors show that quantity discounts can coordinate a 1-to-1 channel only with the assumption that the retailer owns the inventory; in consignment selling and with Vendor Managed Inventory (VMI), where the retailer does not own inventory and only pays the wholesaler the unit wholesale price when the items are sold, quantity discounts cannot coordinate. They show that a channel that employs consignment selling or VMI can be coordinated by charging end customers the channel-optimal retail price. The authors also show that the level of demand can determine whether sequential decisions are good approximations of jointly optimal decisions, and joint decisions are needed only for supply chains with relatively low demand rates. Furthermore, the authors extend their work to include multiple geographically dispersed retailers. We classify this paper into the 1-to-1 settings because most of this paper is about 1-to-1 supply chains and only a small portion is the extension to multiple retailers.

Chakravarty and Martin (1991) show that a discount to a regular wholesale price can be used to achieve coordination between a supplier and a buyer. The authors define the retailer's gain and the manufacturer's gain as increases in their profits due to the wholesale price discount. Chakravarty and Martin (1991) outline a heuristic to simultaneously determine the optimal price discount, retail price, and inventory decisions that maximize the sum of the retailer's gain and the manufacturer's gain with constraints that the gains are strictly positive and have a fixed ratio. However, coordination achieved by these optimal joint decisions is not a perfect one. The authors also investigate the impact of setup costs, fixed profit ratios, integration, and other factors on joint decisions.

A common business practice is for a supplier to offer a temporary price discount to motivate its retailers to increase their order quantities and offer discounts to end customers so that the supplier's demand and profit are increased. Ardalan (1994) studies this practice and develops models for determining joint decisions for the retailer to maximize its present value of the profit from the one-time special order due to the temporary price discount and from all future normal orders. The numerical examples in the paper show that a temporarily discounted wholesale price is beneficial to the whole system (although the author does not show that this result holds in general.) Ardalan (1995) develops an average profit model for the same problem, instead of the present value approach, and compares the results. The comparison indicates that if the level of a discount is low, the two approaches provide similar solutions. Thus, although the average profit approach is less accurate, a supplier should choose it when offering a low discount because the approach is easy to apply.

Abad (1994) shows that coordination between a supplier and a buyer can be achieved by formulating the problem as a cooperative game and following the Pareto efficient and the Nash bargaining solutions. However, the paper does not mention whether the coordination solution can achieve system optimal profit. The author also extends the problem to include a group of buyers and proposes two pricing schemes to coordinate the supplier and the group of buyers.

Spengler (1950) indicates that vertical integration in a 1-to-1 channel can result in a lower retail price and hence increase welfare. However, Reyniers (2001) questions this well-established result and shows that it holds only if inventory related costs are sufficiently small. The author finds that when the inventory related costs are high, the optimal price is likely higher than that in the decentralized channel. Furthermore, it is found that the delivery frequency is also higher because the manufacturer takes the retailer's holding cost into consideration.

Supplier-driven channels, in which suppliers make take it or leave it decisions such as wholesale prices, are common both in practice and in research. Although we can see that many retailers have strong bargaining power, research on buyer-driven problems is rare. One of the two models in Ertek and Griffin (2002) studies buyer-driven problems. The other model is for a supplier-driven supply chain. In each of the models, the dominant party is developed as the first mover in a Stackelberg game. In the buyer-driven channel, the buyer always sets the selling price as a linear function of the wholesale price. It is shown that the buyer only uses a multiplier and sets the mark-up at zero. The numerical results in the paper show that the total profit is the highest in the centralized channel, followed by the buyer-driven channel, then the supplier-driven channel (although the authors do not prove that this ordering holds in general.) In addition, the authors show that it is possible for the buyer to set a high multiplier to force the supplier to choose a low wholesale price in the buyer-driven channel. However, it is important to note that only the buyer incurs logistics costs in these models.

Qi et al. (2004) consider consequences of demand disruption that have not been addressed in other research on joint decision problems. The authors classify their work as *disruption management*, which analyzes the costs associated with changes to a predetermined plan. The authors propose a two-period supply chain model to study joint decision problems for a seasonal product. Their objective is to develop a supply chain coordination scheme when a production plan has to be revised when there is demand disruption. It is shown that under certain wholesale quantity discount policies, the chain can be coordinated with both members better off.

4.1.2 Dynamic joint decisions in 1-to-1-Dd settings

Revenue Management has promoted the adoption of dynamic pricing strategies in many industries. With the ease of making price changes on the Internet, dynamic decisions are now frequently used in business-to-customer and business-to-business commerce (Elmaghraby and Keskinocak, 2003). We expect that dynamic decisions will continue to obtain more attention.

Jorgensen (1986) investigates dynamic optimal production, purchasing, and pricing policies over finite horizon as a continuous-time problem in a 1-to-1 supply chain. By assuming a linear demand and quadratic holding (shortage) costs and ordering costs, the joint decision problem is modeled as a two-player nonzero-sum differential game. The author models the inventory levels as the state variables and the rates of production and purchasing and prices as the control variables. Assuming the game is played with open-loop controls, the author finds a Nash equilibrium and characterizes the optimal dynamic pricing and inventory decisions for both the supplier and the retailer.

Eliashberg and Steinberg (1987) study a 1-to-1 supply chain in a continuous-time model under a Stackelberg framework with a constant wholesale price over the selling season. The product is produced and delivered continuously to the buyer who processes the product further. The deterministic demand has a quadratic seasonality effect, and the processing cost functions of both parties are assumed to be quadratic as well. Using optimal control theory, the authors characterize the pattern of the supplier's production decision and the patterns of the buyer's pricing, processing, and inventory decisions.

Desai (1996) examines the same setting as Jorgensen (1986) except that he analyzes two more contract types between the manufacturer and the retailer in addition to the fixed wholesale price contract. One contract type is based on a constant retailer processing rate, and the other is a general contract. The authors assume quadratic total production processing and holding functions. After analyzing the optimal decisions for the three contract types, Desai (1996) shows that retail price is the same for different contract types. However, the manufacturer's price, the production rate, and the retailer's processing rate are different. The author emphasizes the importance of coordination issues in joint decisions models because they find that optimal decisions are different in a centralized channel and in a decentralized channel.

It is intuitive to infer that integration in a firm's functional departments can benefit the firm because of double marginalization theory. However, Kumar et al. (2000) analyze the effects of the buyer's functional decentralization on the performance of a 1-to-1 channel. Using the modeling framework in Jorgensen (1986), Kumar et al. (2000) allow the buyer to have sequential decisions in its marketing and production functions. By comparing the optimal policies and profits in the decentralized buyer case to those in the centralized case, the authors find a counter-intuitive result that both members benefit from the buyer's functional decentralization.

The supply chain structure investigated in Zhao and Wang (2002) consists of a manufacturer and a value-added buyer making discretely dynamic pricing and inventory decisions over a finite selling season. The authors formulate the problem as a three-stage leader/follower game in which the decision sequence is the wholesale price, joint decisions for the buyer, then the production plan for the manufacturer. The production costs of the manufacturer and value-added processing work done by the retailer are convex in quantity. Their main conclusion is that a certain pricing scheme charged by the manufacturer can coordinate the channel; however, the retailer needs to pay the manufacturer a lump sum, and this lump sum depends on the buyer's reservation profit. The authors then show that it is in the buyer's interest to not only keep his reservation profit secret but also "lead (or mislead) the manufacturer to believe that he has a higher reservation profit."

4.2 Single Supplier, Multiple Non-competing Retailers, Deterministic Demand: 1-to-M/n-Dd

The single buyer assumption is quite restrictive because, in practice, suppliers typically sell through many retailers. In this category, we survey research analyzing joint decision problems in 1-to-M/n-Dd supply chains in which a single supplier and multiple non-competitive buyers face deterministic demand.

Weng (1995, discussed in section 4.1.1) shows that the conclusion in Jeuland and Shugan (1983), that a single quantity discount wholesale price schedule can fully coordinate a 1-to-1 channel, does not hold when inventory issues are considered. Ingene and Parry (1995a) show that it does not hold in a 1-to-M/n channel either, even if inventory issues are ignored. However, Wang and Wang (2005) study a supplier's optimal quantity discount policy for multiple independent and non-identical retailers. The authors show that a common discrete quantity discount schedule with a specific designed discounting policy for each buyer can coordinate the channel and is better than all-units and incremental quantity discounts.

Chen et al. (2001b) examine the channel coordination problem in the same setting as in Ingene and Parry (1995a) except that inventory issues are considered. In addition to the setup cost, the holding cost, and the purchasing cost that are usually included in an EOQ model, an additional cost component, management costs, is modeled. Management costs, modeled as a concave function of the buyer's annual sales volume, is incurred by the supplier to deal with each buyer's needs and transactions. We mentioned in section 4.1.1 that Weng (1995) concludes that the coordination scheme using an order quantity discount and a periodic franchise fee can achieve perfect coordination in 1-to-1 settings. Chen et al. (2001b) show that this scheme cannot perfectly coordinate a channel with multiple non-identical buyers. The authors prove that perfect coordination can be achieved through fixed fees and a nontraditional discount pricing scheme including three discount components based on the retailer's annual purchase, order quantity, and order frequency, respectively. Chen et al. (2001a) develop algorithms to determine optimal joint decisions and optimal sequential decisions for an integrated channel and optimal joint decisions for a decentralized channel.

4.3 Single Supplier, Multiple Competing Retailers, Deterministic Demand: 1-to-M/w-Dd

The retailers in a 1-to-M supply chain can compete with each other in the end market. In this case, one retailer's decisions for its ordering polices and retail price might influence the decisions of other retailers. The following research considers this relationship when modeling joint decision problems. Demand functions with price substitution factors between the retailers are generally assumed.

Ingene and Parry (1995b) investigate coordination issues in a channel with one manufacturer selling its product to two competing retailers facing linear demand. The authors assume a simple cost structure similar to that in Ingene and Parry (1995a), but each party incurs a fixed cost in addition to the unit variable cost. The authors show that a linear quantity discount schedule can perfectly coordinate the channel, but no general two-part tariff contracts with a constant per-unit wholesale price can. As in Ingene and Parry (1995a), it is not always in the manufacturer's interest to coordinate the channel when there is more than one retailer.

Bernstein and Federgruen (2003) investigate pricing and replenishment decisions in a 1-to-M supply chain facing linear demand. The authors show that lower and upper bounds on optimal profits and the corresponding pairs of prices and order intervals in the centralized system can be computed based on the power of two replenishment policies identified in Roundy (1985). In the decentralized system, if the wholesale price is a constant vector, a Nash equilibrium is proved to exist under both price and quantity competitions under a condition that they argue will usually hold in practice. In addition, the Nash equilibrium is proved to be unique under another condition that they claim will typically hold in practice. Although no linear wholesale pricing scheme can induce perfect coordination, the authors show that a nonlinear wholesale pricing scheme similar to that in Chen et al. (2001a) can coordinate the supply chain.

Bernstein et al. (2006) study coordination issues in a 1-to-M/w supply chain that works under VMI. Under VMI, the supplier sets not only a wholesale price scheme and its own replenishment but also the retailers' replenishment. Only retail prices are left for the retailers to decide. The author show that although a constant wholesale price or discount pricing cannot achieve perfect coordination in a traditional 1-to-M/w supply chain where retailers manage their own inventories (RMI), it can perfectly coordinate the chain under VMI when the retailers engage in both price and quantity competitions. They also show that the difference between the wholesale prices for different retailers depends on their competition advantages.

4.4 Stochastic Demand: Sd

We are not aware of much work regarding joint decisions in supply chains to meet price-sensitive stochastic demand. Therefore, we cast a wider net and include papers where suppliers do not consider production decisions, or retailers do not make joint decisions, or both. The articles that fall in this category generally do not consider setup costs and holding costs and model only average unit production or purchasing cost.

Most models of joint decisions with stochastic demand can be seen as price-sensitive newsvendor problems in a supply chain setting. Petruzzi and Dada (1999) offer an excellent review on joint decision problems in price-sensitive newsvendor settings in one firm. The authors transform joint pricing and inventory decision problems into joint pricing and stocking factor decision problems. The transformation incorporates both additive and multiplicative stochastic demand. It provides analytical tractability and is employed by many researchers (for example Arcelus et al., 2008a; Boyaci et al., 2008).

While Li (2008) quantifies the inefficiency of uncoordinated pricing decisions for the supplier and the retailer in a 1-to-1 supply chain, some work focuses on various contracts to coordinate supply chains, especially buy-back (return) contracts. Buy-back contracts are a common practice to attract more orders from buyers because suppliers are willing to share the risk of overstocking caused by demand uncertainty. A buy-back price is an additional decision that needs to be made in this line of work. Arcelus et al. (2008a) mainly show that the existence of a secondary market for unsold items is beneficial to both a manufacturer and a retailer with a buy-back policy. Without a secondary market, the supplier is always better off, but the retailer may not be. Song et al. (2008) identify necessary and sufficient conditions under which optimal wholesale and buy-back prices are independent of stochastic components in demand. Then, they show that the relevant profit ratios for a supplier and a retailer when facing stochastic demand are identical to the ratios for those using price-only contracts when facing deterministic demand. Lau et al. (2007b) show that a buy-back policy plus a manufacturer-imposed maximum retail price can coordinate a 1-to-1 newsvendor setting. Emmons and Gilbert (1998) conclude that both the supplier and the retailer in the same setting can be better off with a wholesale price in a certain range and a buy-back policy.

However, Granot and Yin (2005) prove that a manufacturer in a price-sensitive newsvendor setting may not want to introduce buy-backs in equilibrium under certain conditions. These conditions include that unsold items have no salvage value and the expected demand is a negative power function of price. When the expected demand is a linear or exponential function of price, it is shown that buy-backs can significantly improve the channel efficiency and shift profit to the manufacturer. Granot and Yin (2007, 2008) consider the influence of sequential commitments in all members' decisions, as well as the retailer's commitment postponement, on the performance of a 1-to-1 supply chain. The authors show that the performance of the supply chain is highly dependent on how these commitments are sequenced. Tang and Yin (2007) present a two-stage stochastic model for two retailers using two pricing strategies: Joint pricing and inventory decisions and price postponement to manage uncertain supply. It is found that the retailers benefit from price postponement.

Other mechanisms that can be used to improve the performance of supply chains have also been examined. Yao et al. (2008) investigate a revenue-sharing contract offered by a manufacturer to two competing retailers as a coordination mechanism. They find that a revenue-sharing contract improves the chain performance compared with a price-only contract. However, the retailers are shown to be worse off, while the manufacturer enjoys first mover advantages. In addition, the optimal solutions, which provide too little profit to the retailers, may drive the retailers away from the market. Hsieh et al. (2008) consider a decentralized model and three coordinated models for joint decisions in a 1-to-1 newsvendor setting with asymmetric demand information. Return policy and quantity discounts are considered in these models. Surprisingly, the authors find that a coordinated model with quantity discount, return policy, and no information sharing is the best. Arcelus et al. (2008b) examine the effect of direct rebates on the manufacturer's optimal decisions in a price-sensitive newsvendor setting. The article also considers the impact of demand functions on the retailer's profitability and identifies the conditions under which the retailer is willing to share its private demand information. Webster and Weng (2008) develop a model of manufacturing and distribution supply chains facing price-sensitive newsvendor problems under VMI and RMI. It is found that VMI is against the interest of the buyer in such a setting. The whole chain is shown to perform significantly better under RMI, which is counterintuitive.

Weng (1997) proposes a model to address the problem of coordinating the decisions of the manufacturer and the distributor in a 1-to-1 channel to meet the stochastic demand of a seasonal product. The author shows that a two-part tariff policy with a fixed fee plus the manufacturer's unit cost can coordinate such a channel. Ha (2001) focuses on designing a contract to maximize the supplier's profit in a price-sensitive newsvendor setting. The author finds that if the supplier knows the buyer's marginal cost, a buy-back policy combined with either franchising or quantity and price fixing can coordinate the channel and maximizes the manufacturer's profit; however, when the buyer is in a favorable position due to its private marginal cost information, the supplier can design a menu of incentive compatible contracts including a cutoff level to achieve profit maximization.

Federgruen and Heching (2002) consider joint dynamic pricing and inventory control in a centralized 1-to-M channel over an infinite horizon. Because the supplier (a distribution center) does not hold inventory, when the order arrives at the distribution center, it is allocated to the retailers immediately. The ordering decisions for the distributor, the allocation decisions, and the dynamic retail pricing decisions are made by one decision maker to maximize the system profit. Their numerical study of a national retail company shows that joint decisions significantly improve the performance of the channel compared to traditional sequential planning approach. Bernstein and Federgruen (2005) consider single period models for a 1-to-M channel with competing or non-competing retailers facing stochastic demand. The authors show that when the retailers do not compete, a linear Price Discount Sharing (PDS) scheme combined with a traditional buy-back contract can result in perfect coordination. Although revenue sharing or profit sharing can also achieve coordination, PDS is easy to monitor and can avoid moral hazard. When retailers compete, the authors show that a Nash equilibrium definitely exists under certain conditions, and multiple Nash equilibria may exist. They further show that perfect coordination can still be induced by a PDS with a non-linear component under price competition. Bernstein and Federgruen (2004) consider a periodic review, infinite horizon model in the same setting as Bernstein and Federgruen (2005). However, Bernstein and Federgruen (2004) mainly characterize the equilibrium behavior of the retailers under a simple wholesale pricing scheme.

Boyaci et al. (2008) study a two-period model in a 1-to-1 supply chain where the manufacturer sells to a price-sensitive newsvendor retailer under wholesale price only contracts. This paper captures the demand correlation between the two periods, which is generally ignored by most works. Hsieh and Wu (2008) investigate coordinated decisions in a three-level supply chain that consists of one Original Equipment Manufacturer (OEM), one manufacturer, and one distributor. Both demand and supply for the chain are uncertain.

In the final section, we discuss the assumptions of the surveyed articles, gaps between practice and research, and suggestions for future research.

5 Discussion and Directions for Future Research

Joint decision problems in real world settings can be very complicated. A supply chain usually has multiple levels, multiple suppliers and multiple buyers at each level, and multiple product types. The production capacity and inventory capacity are limited. A supply chain is also a dynamic system, in which participating members and available resources change continuously. Uncertainty exists in every aspect of a supply chain. Business relationships between suppliers and buyers are dynamic. Supplies are uncertain due to breakdown of production systems, workforce strikes, or infrastructure problems on shipping routes. Demand is uncertain as well due to seasonal demand fluctuations, changes of customer preferences, pricing decisions, or stock levels. Therefore, we need robust, dynamic, efficient, effective, and responsive models to address these issues and to obtain realistic insights about managerial concerns. Although this survey shows that the body of literature, although not vast, is very diverse and has great academic and practical relevance, the surveyed research simplifies joint decision problems in the real world with various (sometimes restrictive) assumptions. There are important gaps between what has been done in research and the problems that arise in industrial settings.

Multiple products and production capacity. Although we have seen research on joint decisions with multiple products in a single firm (see Chapter 4.1 in

Chan et al., 2004, for a review), most research on joint decision problems in supply chains focuses on a single product. In the real world, supply chains seldom involve only a single product.

In the case of multiple products, the pricing decisions of one product can affect the demand of others when the products are (partially) substitutable or complementary. Furthermore, interaction exists in the manufacturing of products such as sharing production resources and setups. Chen and Chen (2007) mention that coordinating the replenishment of various items in a multi-product situation can reduce the total cost, mainly due to economics of scales and the simultaneous setup and delivery of distinct items. Ingene and Parry (1995a) propose future research that extends their single-product model to include manufacturers selling multiple substitutable products through a common retailer, or to include retailers carrying substitutable products of multiple manufacturers. In Section 4.4, we noted that Chen and Chen (2007) believe that joint replenishment program in conjunction with channel coordination can realize additional cost savings in the supply-side in a multi-item distribution channel. Wang (2006) examines the joint decisions for a special case of multiple perfectly complementary products, in the sense that these products have to be sold in sets of one unit of each. Research considering multiple competing retailers in the previous section such as Bernstein and Federgruen (2005) can be seen as work dealing with substitutable products on the retailer's side of the supply chain, but each retailer deals with only one product. Not much work deals with multiple products on the supplier's side.

Production capacity becomes an important issue when multiple products in a supply chain share the same resources. Thus, it is interesting to incorporate capacity constraints and to see their effect on joint decisions and how buyers react to the product capacity limit of suppliers. Ongoing work in Yano and Steinberg (2004) addresses decisions on pricing, ordering, and capacitated production when two suppliers provide two partially substitutable products to one-retailer. A simple cost structure for all members is assumed. Hsieh and Wu (2008) incorporate outsourcing decisions when the limited capacity cannot meet the demand in their model.

Qi (2007) investigates the order splitting strategy when a manufacturer deals with multiple capacitated suppliers in a centralized setting.

Incorporating the interaction of multiple products in joint decision models in supply chains is an important but challenging direction for future research. With multiple substitutable or complementary products, pricing of one product will affect the demand and/or production for other products. All the suppliers and buyers have to determine their inventory and pricing decisions for all the products. Even if only two products and two members are involved, eight decision variables have to be considered.

Information Asymmetry. Most of the surveyed articles assume that complete information is shared between the members in a supply chain. Although Zhao and Wang (2002) argue that such an assumption is a good approximation to reality, this is not true in many cases. It is sometimes difficult or even impossible to get private information about other parties. It can even be in one party's interest to mislead others about its private information. Zhao and Wang (2002) demonstrate that a buyer is better off to keep its reservation profit a secret or even to mislead its supplier to believe that it has a higher reservation profit. Considering asymmetric information in joint decision problems can be useful and interesting. In Section 4, we mentioned that in the models of Ha (2001) and Yao et al. (2008), each retailer has private cost information. Lau et al. (2007a) assume asymmetrical manufacturing cost information in a 1-to-1 system without inventory planning. Burnetas et al. (2007) investigate quantity discounts in single-period supply contracts with asymmetric demand information and an exogenous retail price. Hsieh et al. (2008) also consider asymmetric demand information.

More work needs to be done to examine joint decision models with full information sharing in an asymmetric information setting. A line of research on pricing in Marketing and a line of research on inventory decisions in Inventory Management have considered asymmetric information in demand and inventory related costs. Joint pricing and inventory decision problems under asymmetric information can benefit from those lines of research.

Demand processes. Accurate estimation of demand input of a supply chain system is very important to the success of a joint decision model. Most research applies a down sloping linear function possibly with adjustment for seasonal fluctuations. In the numerical analyses, the parameters in the demand functions are usually given as constants. Characterizing and justifying joint decision models and conclusions from real data is an important future research direction. We also discussed in 3.2 that modeling stochastic demand as additive or multiplicative can have significant impact on solutions. However, most earlier work assumes either additive or multiplicative demand with little justification (Lau et al., 2007b). Empirical work is needed to justify the use of additive or multiplicative demand.

One critical assumption about the demand processes in current research is that they are independent among different periods in periodic review models. Time-independent demand does exist for most non-durable goods such as milk and bread or for situations where the selling season is normally too short to give buyers enough information to affect future purchases. However, it is expected that customers can delay their purchase of durable goods if they estimate a lower price in the future; thus demand in one period is correlated with demand in other periods. Zhao et al. (2007) analyze cost savings of an early order commitment in a supply chain when demand in two periods is autocorrelated. However, inventory decisions are ignored.

Yano and Gilbert (2002) indicate that the effect of price changes on customers' behavior of durable goods consumption over a time horizon has attracted little attention in research. In recent years, more researchers have started to examine the impact of strategic customers on demand and on optimal pricing decisions (Aviv and Pazgal, 2005; Su, 2006; Levin et al., 2007; Elmaghraby et al., 2008). Ongoing work by Su and Zhang (2007) considers the impact of strategic customer behaviour on supply chain performance. The authors find that a wholesale price only contract can coordinate the supply chain when considering strategic customer behaviour. However, the supplier in the chain do not make inventory decisions.

Joint decision problems in a supply chain facing time-dependent demand caused by strategic customer behaviors need more attention.

Coordination and benefit sharing. Inducing coordination among supply chain members is currently one of the major managerial concerns among practitioners. It is also an intensive research area because of the belief that coordination can improve the efficiency of the whole chain. We have seen that some traditional channel coordination mechanisms such as quantity discounts, two-part tariff contracts, and buy-back agreements have been studied to align the actions of members to achieve coordination. The impact of other marketing related practices on this issue is worth considering.

One intriguing issue about supply chain coordination is the ways in which the members in a supply chain can benefit from the improved chain performance due to coordination. In a fair commercial relationship, each party should be better off in order to be willing to take part in the coordination. Even if one party is dominant in the chain and uses certain instruments to induce its partners to coordinate for its own benefits, the other participating parties should at least be no worse off. A number of articles focus on how to achieve coordination while ignoring the issue of benefit sharing in designing a coordination scheme. Research such as Weng (1997), Bernstein and Federgruen (2005), and Chakravarty and Martin (1991) assumes that the sharing scheme is represented by a given fraction of the chain-wide profit, the revenue, or the gain that the supplier receives. A sharing scheme reflects each channel member's bargaining power, which is affected by a number of operational and business parameters. Bernstein and Marx (2006) examine the role of retailers' reservation profit levels, which depend on their bargaining power, on the allocation of chain-wide profit. Although only ordering decisions are considered and retail prices are exogenous, Bernstein and Marx (2006) provide a way to examine the division of supply chain profit. Nonetheless, not all sharing schemes are easy to monitor or to implement in practice.

Most articles that we surveyed assume risk-neutral chain members. However, attitudes toward risk should probably be taken into account when designing coordination mechanisms, especially when demand is stochastic, because the risk-taking attitude of the members can affect their decisions and their profit sharing. Wang and Webster (2007) investigate the coordination issue in a supply chain with a risk-neutral manufacturer and what they refer to as a "loss-averse retailer."

The design of various benefit sharing schemes and their impact on joint decisions need to be investigated.

Supply chain structures. Most of the research considers only two-level supply chains with one supplier. Supply chains in practical problems usually include more than two levels and more members in each level, which increases the complexity of the problems. Examining whether some conclusions in existing two-level supply chain models will still hold in supply chains with more than two levels would be worthwhile. A stream of research in production and inventory management has been done in multi-level supply chains (see, for example, Federgruen, 1993, for a review). This stream can be combined with the joint decision problems to see the performance of the extended chain and whether some coordination mechanisms still work in new settings.

Hsieh and Wu (2008) is one study on joint decisions in a decentralized three-level supply chain except that the demand is purchase price-dependent instead of retail price-sensitive. With the increasing popularity of the Internet, Internet Marketing is gaining more and more attention both in practice and in research. Many retail firms such as Barnes and Noble adopt a multi-channel strategy that includes both web-based channels and pre-existing off-line channels. In addition, more and more firms in a variety of industries have added direct channels (e.g., Nike and IBM) to their retail channels. Boyaci (2005) studies such a channel. The study finds that "aligning the incentives of the manufacturer and the retailer to achieve supply chain coordination is a challenging task in multi-channel distribution systems. Most of the contracts proposed in the vertical competition literature (linear price-only, buy-back or holding cost subsidy, Vendor-Managed-Inventory (VMI), revenue sharing, rebate contracts) fail to achieve coordination."

A line of work in supply chain management has focused on multiple sourcing problems from multiple suppliers. Having multiple suppliers can reduce the risk of production or sales being disrupted from any problems in one supplier. However, most models that we surveyed assume one single supplier. Joint decisions problems in a multiple supplier supply chain needs attention. Qi (2007) investigates order splitting strategies when a manufacturer deals with multiple capacitated suppliers without considering any of the suppliers' decisions.

Several articles study joint decision problems in supply chains with one supplier and multiple buyers. The buyers usually are modeled as the follower in the game except in the buyer-driven model in Ertek and Griffin (2002). Messinger and Narasimhan (1995) believe that the bargaining power in grocery channels has shifted to powerful buyers such as Wal-Mart. A future research topic would be joint decision problems in a buyer-driven supply chain: What are the characteristics of the pricing and replenishment decisions? How does the supply chain perform when multiple manufacturers supply a large retailer?

We realize that subtle changes in modeling assumptions or the focus of analysis can often lead to dramatically different mathematical structures because joint decision problems in supply chains are complex. For example, Ray et al. (2005) incorporate a critical business characteristic, delivery time variability, into their joint decision model. Stochastic delivery time has not been considered in the models that we surveyed. Ray et al. (2005) argue that ignoring the randomness of delivery time trivializes the interaction between pricing and stocking decisions. Another interesting approach related to joint decision making is presented in Kachani and Perakis (2002). The authors propose a fluid model to investigate how price and levels of inventory affect inventory behavior at each member in a supply chain. However, their goal is not to make pricing and inventory decisions, but to describe inventory behavior. In addition to the aspects that we suggested for future work, we believe there can be other new research streams in joint decision problems.

This survey not only considered the state of the art of research in joint decision making but also classified the research into categories based on the approach and assumptions used. The survey also identified the main trends in the current literature and promising areas for future research. We strongly believe that joint pricing and inventory/production decisions can result in substantially increased profits for supply chains. Research in this area has the potential to bring together concepts from operations management, economics, and marketing. Significant pioneering work has been done, and it provides motivation and suggests promising directions for future research.

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Chapter 3: Joint Pricing and Inventory/Production Decisions in a Two-stage Dual-channel Retailer Supply Chain

1 Introduction

With the increasing popularity of the Internet, "online retailing revenue is expected to reach $211.4 billions in 2006, a 20% gain over the 2005 revenues of $176.4 billions" according to "The 2006 State of Retailing Online," the ninth annual report published by `shop.org` and conducted by Forrester Research. Since the Internet is becoming increasingly important as a sales channel, it is not surprising that web-based channels are fast being integrated into the channel strategy of traditional off-line retailers. Most large retail firms have adopted a multi-channel strategy that includes both web-based channels and traditional brick-and-mortar channels. For example, Chapters Inc. has both on-line and off-line book stores. Despite its advantages, such as reaching more potential customers, a retailer's dual-channel system introduces new management concerns. Logistics systems, for example, usually need to be redesigned when a traditional retailer becomes a dual-channel retailer. If the frequency of Internet orders for a dual-channel retailer becomes large, order fulfilment from its existing infrastructure is not desirable anymore. de Koster (2003) suggests that an Internet delivery distribution center with specially designed warehouses is established to help a dual-channel retailer obtain economies of scale. In addition, two channels serving common market segments compete. Thus, pricing and inventory decisions for the channels under competition are important issues that the management team has to deal with. In this paper, we consider a two-level supply chain with a dual-channel retailer and model the joint pricing and inventory/production decisions (from now on, joint decisions) at both the supplier and the retailer level.

There is a considerable amount of literature that describes advances in research and management practices in the area of joint decisions in a single firm. The literature deals with the decisions at the interface between marketing and manufacturing, specifically the simultaneous determination of pricing and inventory/production decisions. This line of research has spanned over 50 years with over 100 published articles. A broad review on this topic can be found in Eliashberg and Steinberg (1993) and Yano and Gilbert (2002). Research on joint decisions has drawn a great deal of attention because joint decisions can improve a firm's profitability over traditional sequential decisions. An example in Kunreuther and Richard (1971) indicates that profits can be increased by 12.5% if pricing and inventory/production decisions are made jointly rather than sequentially.

The idea of joint decisions can be extended to multi-firm supply chains. The main objective of supply chain management is often to maximize chain-wide profits, while satisfying service requirements and focusing on how to coordinate supply chain members in an efficient manner. Reviews on supply chain coordination can be found in Cachon (2003) and Chan et al. (2004). It is intuitive to think that supply chain members could further leverage chain-wide profits if their pricing and inventory/production decisions are made jointly.

There is indeed an emerging body of literature that deals with joint decisions in multi-firm supply chain settings. The first papers were published over 20 years ago, and there is a growing interest in the area. Most joint decisions research focuses on a single product through a supply chain (see, for example, Weng, 1995). However, when the retailer in a supply chain has dual channels that serve common market segments, the products sold in one channel can be seen as substitutes of the products sold in the other channel; thus creating channel competition. Real-world pricing and inventory/production decisions are not made in isolation for each channel in this type of supply chain. In a dual-channel retailer supply chain, pricing of the product in one channel will affect the demand in the other channel. This subsequently affects the retailer's replenishment (ordering) decisions, which have an impact on the supplier's inventory/production plans and wholesale price decisions. Thus, not only do each member's pricing decisions interact with its inventory/production decisions but also the supplier's decisions interact with the retailer's decisions.

We believe that it is important to further investigate joint decision problems in a dual-channel retailer supply chain. Given that research on joint decisions in a supply chain lies at the interface between marketing and operations management, we also believe that this line of research will foster collaboration between and contribute to these two functional areas of business.

We note that in a supply chain network with demand substitution, joint decisions can potentially become complicated. The interaction between demand for different products in addition to other interactions such as resource sharing, makes the problem difficult because substitution prevents the separate analysis of each product. Furthermore, the large number of decision variables makes the problem difficult to solve. A two-stage supply chain with one supplier and a dual-channel retailer has seven decisions variables: Two retail prices (different prices in different channels), two wholesale prices (if product configurations for the two channels are different), two replenishment decisions for the retailer, and the product replenishment decision for the supplier.

In most cases, the members in a decentralized supply chain make their own pricing decisions. Regarding inventory decisions, traditionally, in a decentralized supply chain each member owns and manages its own inventory. We refer to this as Retailer Managed Inventory (RMI) because the retailer controls its inventory. However, Vendor Managed Inventory (VMI) has become increasingly popular since the 1980s. The popularity of VMI is possibly encouraged by VMI arrangements between Wal-Mart and some of its suppliers, such as Procter and Gamble (P&G) and Rubbermaid. One key feature of VMI is that the supplier is responsible not only for its own replenishment but also for the retailers. VMI is mainly adopted to coordinate the following key operations: inventory replenishment, transportation scheduling, and production capacity allocation among chain members. Although the processes of establishing and executing VMI practices and the agreements among supply chain members in VMI are critical to the success of the practice, in this paper we assume that a VMI partnership is already in place and consider joint decision problems under VMI.

Although many types of contracts, such as buy-back and discount contracts, can be employed by the members in decentralized chains, we limit our attention to wholesale price contracts because this type of contract is one of the most popular in practice. We ask the following key questions:

1. Does a pure strategy equilibrium exist for the two-player game under RMI and VMI, respectively? How do the supplier and the dual-channel retailer in a supply chain make their pricing and inventory/production decisions in equilibrium?

2. How does each member perform under RMI and VMI? Which supply chain structure provides solutions closer to a centralized supply chain (CSC)?

3. How do the cross-price effect between the products in a dual-channel retailer supply chain and other system parameters affect the supplier's and the retailer's decisions and profits?

We perform our investigation in the following two-stage supply chain setting: The supplier in the chain purchases or manufactures the product according to the Economic Order Quantity (EOQ) model and sells the product through a retailer with an on-line and an off-line retail store. Under RMI, the retailer makes decisions for the inventory replenishment and retail prices for the two stores, while the supplier makes its own wholesale price and production or purchase decisions. Under VMI, the supplier makes the production/procurement decision for itself, as well as the inventory replenishment decisions for the retail stores. The retailer only decides retail prices. Price-sensitive EOQ joint decision supply chain models are proposed to determine and understand the interactions between pricing and production/inventory decisions across products at both the supplier and the retailer levels. The models are formulated as Stackelberg games in which the supplier, as a leader, has the economic power and managerial ability to take into account the reactions of the downstream retailer.

We assume a general linear demand function in which demand for each retailer channel is a linear function of retail prices in both channels. The parameters of the demand functions for the online store and the off-line stores may take different values. Thus, the two channels are not symmetric in our model. We first analytically prove that a unique pure strategy Nash equilibrium exists under both RMI and VMI without assuming any specific relationship between the paraments for two stores in our mathematical analysis.

Then, we use numerical experiments to show the impact of the substitution of the stores and products, i.e., the cross-price effect, on the supplier's and the retailer's decisions in the two channels. In the numerical experiments, we mainly assume that customers are more sensitive to price changes in the online store than the off-line store, and are equally sensitive to price difference between two channels. The analysis results in several insights. We find a price convergence effect between the online and the off-line stores as the degree of substitution between the stores and the products increases. The same observation holds for wholesale prices. This finding provides managerial implications on pricing for different product categories in online and off-line stores. For products with strong cross-price effect such as CDs and books, the prices in the two chancels should be close. The opposite pricing strategies should be applied to products with lower cross-price effects.

The EOQ of the off-line store decreases with the degree of substitution; the EOQ of the online store, on the other hand, increases with the degree of substitution. This result provides some guidance on inventory replenishment policies when intra-channel competition exists. It is also interesting to see that the retailer's and the supplier's profits either increase or decrease with the degree of substitution depending on the production costs and self-price sensitivities of demands. Whether the firms can benefit by considering the cross-price effect depends on the production costs and self-price sensitivity of demands as well. Both the supplier and the retailer benefit from a reduction in production costs. Regarding setup (ordering) costs, the chain-wide profit increases more with the reduction in the supplier's setup (ordering) costs than with the retailer's setup costs when both are reduced by the same magnitude. Interestingly, the retail price decisions are quite insensitive to the inventory cost parameters. The supply chain-wide profit is the highest in a centralized structure followed by VMI. The decentralized chain under RMI is less desirable. Furthermore, an unexpected result is that the retailer benefits from developing VMI, while the supplier does not.

The rest of the paper is organized as follows: Section 2 reviews related literature. Section 3 discusses our assumptions and presents our models. Section 4 analyze the models. Section 5 presents numerical examples. Finally, Section 6 offers concluding comments. Mathematical proofs that are not in the main body can be found in the Appendix.

2 Literature Review

The popularity of online shopping has forced traditional brick-and-mortar companies to redesign their distribution channels (Kopczak, 2001). Many manufacturers have added direct internet-enabled distribution channels to distribute their products in addition to their traditional retail stores. Netessine and Rudi (2006) and Chiang and Monahan (2005) investigate inventory management strategies in such a supply chain setting. At the same time, we can also see that many traditional retail stores have added an online channel to their existing traditional brick-and-mortar channels. However, companies should be cautious when deciding to have multiple channels because of the possible competition among the channels. King et al. (2004) examine the conditions under which traditional retailers should incorporate online channels. In this paper, we assume that the supply chain is designed and in place to accommodate both web-based channels and pre-existing off-line channels for a given retailer. We investigate the operational decisions under this structure.

Research on joint decisions in supply chains combines research on supply chain management and research on joint decision problems in single firms. Research shows that joint decisions can often improve chain-wide performance by incorporating pricing into the system, which is ignored in most supply chain management research. Most models in research on joint decisions are formulated to solve pricing and inventory/production decisions for each member in a two-stage supply chain with a single supplier and a single buyer. A few papers, such as Bernstein and Federgruen (2003), Chen et al. (2001), and Weng and Zeng (2001), consider single-supplier and multi-buyer supply chains. Multiple competing retailers selling the same product, such as in Bernstein and Federgruen (2005), can be seen as equivalent to selling multiple substitutable products. Competition among retailers has the same effect as competition between substitutable products, which is what we consider in the retailer's two channels. However, we consider the retailer's interest in both channels as a whole. Mechanisms to achieve coordination among members without commercial integration, which is the focus of many papers, are out of the scope of this paper.

Another line of related work considers pricing strategies in different supply chain structures. Choi (1991), Choi (1996), and Lee and Staelin (1997) examine channel pricing strategies for substitutable products under various supply chain models. However, this line of work does not consider any member's inventory/production decisions.

There is a rich body of literature on VMI. Cachon and Fisher (1997) and Clark and Hammond (1997) are two examples of empirical work on Campbell Soup's VMI implementation. Clark and Hammond (1997) find that retailers under VMI perform much better than non-VMI retailers. Another stream of work on VMI studies cost savings under given VMI structures (see, for example, Cachon, 2001). Our work that concerns joint decisions under a given VMI structure falls in this category. However, our optimal decisions are based on a more realistic business setting: Profit maximization, instead of cost minimization that is often applied in VMI scenarios.

Next, we discuss the model assumptions and introduce the joint decision models.

3 Assumptions and Problem Formulation

We consider a two-stage supply chain with one supplier and one retailer who has two distribution channels. In practice, a retailer may have one online store and more than one retail outlet. In our stylized model, we consider one off-line outlet store that competes with the online store; thus, products in one channel are seen as substitutes for the products in the other channel. We refer to the online store as store 1 and to the off-line store as store 2. Each store carries the product manufactured or distributed by the supplier to meet the end market demand $\boldsymbol{D} = (D_1, D_2)$. The demand for the online store and the demand for the off-line store depend on the prices in the two channels. The supplier may charge different wholesale prices for orders from the online store and from the off-line store due to different product configurations or processing costs.

We assume that the off-line store has its own inventory to supply the demand of local customers. There are many possible ways for an online store to distribute its products to its customers (de Koster, 2003). We assume that the demand for the online store is satisfied by a warehouse, which is strategically located for convenient delivery to the online customers. Thus, we have two separate inventory storage locations fulfilling orders from the online channel and the off-line channel, respectively. This assumption is realistic for many retail situations and enables us to focus on the impact of price interaction between products on inventories and performance. The orders from the two channels are processed and fulfilled separately by the supplier. In this paper, we use stores interchangeably with distribution centers regarding inventory management because we consider only two stores in the model and each store corresponds to a different distribution center.

3.1 Modeling Assumptions

We model demand as the following linear function of prices:

$$D_i = k_i - \alpha_i p_i + \theta_{ij}(p_j - p_i) \tag{1}$$

with $k_i > 0$, $\alpha_i > 0$, $i = 1, 2$ and $j = 3 - i$, where $\theta_{ij} \geq 0$ represents the competition (substitution or cross-price) effect between products in the online store (store 1) and the off-line store (store 2). This effect is caused by channel differentiation and product differentiation. The higher θ_{ij}, the more substitutable the product in store j is for the product in store i; that is, competition is more severe between the two stores.

In addition, we have the following assumption about demand:

$$\alpha_i + \theta_{ij} > \theta_{ji} \tag{2}$$

for $i = 1, 2$ and $j = 3 - i$. This assumption states that an increase in the price in either store results in a decrease of total sales in the market. It also implies that a price change in store i has a greater effect on its own demand than on that of the other store. This is an intuitive condition and can be seen in research that adopts a linear demand function such as Bernstein and Federgruen (2003) and Choi (1991).

Next, we introduce our models under three scenarios: RMI, VMI, and CSC.

3.2 Model Formulation under RMI

Each retail store operates under a deterministic price-sensitive EOQ model. The retailer seeks the optimal retail prices $\boldsymbol{p} = (p_1, p_2)$ and the optimal order quantities $\boldsymbol{q} = (q_1, q_2)$ from the supplier for both retail stores. The total ordering cost of each product consists of a fixed cost per order s_i plus a cost per unit w_i. The unit inventory holding cost is h_i. All other assumptions follow those of the classic EOQ model except that demand is price-sensitive. The retailer's yearly profit under any given wholesale prices $\boldsymbol{w} = (w_1, w_2)$, which equals the gross revenue minus the ordering and the inventory holding costs of the two stores, is a function of $\boldsymbol{p}$ and $\boldsymbol{q}$:

$$\pi^r_{RMI}(\boldsymbol{p}, \boldsymbol{q}) = \sum_{i=1}^{2} (p_i - w_i) D_i(\boldsymbol{p}) - s_i D_i(\boldsymbol{p})/q_i - h_i q_i/2 \tag{3}$$

We assume that the supplier purchases the product from an outside supplier or manufacturers the product under a deterministic EOQ model and then sells the product to the retail stores at $\boldsymbol{w}$. If the demands from the two retail stores can be approximated as a constant demand rate $D_1 + D_2$, the supplier's economic order quantity can be modeled as $Q = \sqrt{2S(D_1 + D_2)/H}$, where S and H are the supplier's setup cost and unit holding cost, respectively. The approximation considers the integrated impact of the demands from the two stores on the supplier's inventory decisions. In addition, it is possible that the supplier provides the products to other retailers in addition to these two retail stores. The approximation is more accurate when the supplier supply a large number of retailers, and the demand from the two retail stores is approximately a fixed percentage of the total demand for the product from all retailers.

The resulting ordering and holding cost for the supplier is $\sqrt{2SH(D_1 + D_2)}$. We denote by c_i the unit cost of acquiring or manufacturing the product if the supplier has different product configurations or different order processing costs for the online store and the off-line store which result in different unit costs. The supplier's yearly profit for two stores is equal to the sum of yearly gross revenue from each store minus the ordering and holding costs.

$$\pi^s_{RMI}(\boldsymbol{p}, \boldsymbol{w}) = \sum_{i=1}^{2}(w_i - c_i)D_i(\boldsymbol{p}) - \sqrt{2SH(D_1(\boldsymbol{p}) + D_2(\boldsymbol{p}))} \tag{4}$$

Both the supplier and the retailer aim to maximize their own profits.

3.3 Model Formulation under VMI

One of the key features of VMI is that the supplier is responsible for replenishing the inventories both at the retailer stores and at the supplier's site. Although the supplier owns the product until sold, the retailer may still incur costs associated with inventory maintenance such as inspection, shipping from the distribution center to the outlets, repacking the product, shelf maintenance, as well as other costs associated with the retail store's sales efforts such as promotions. These costs are modeled as γD^δ in several marketing studies (see, for example, Corstjens and Doyle, 1981), where γ and δ are constant, and D is the demand rate. We note that $\delta \leq 1$ due to economies of scale. Bernstein et al. (2006) model inventory related costs on the retailer's side in the same way when the supplier is in charge of all replenishment.

The supplier is responsible for determining its own inventory decision Q, the replenishment quantity $\boldsymbol{q}$ for the two retail stores, as well as the wholesale price $\boldsymbol{w}$. We model the supplier's problem under the EOQ framework. The supplier incurs a fixed cost S and unit holding cost H at its own site, either manufacturing or purchasing at unit cost c_i. For simplicity, we assume that the supplier incurs an inventory related set up cost s_i and a unit holding cost h_i at the retailer's distribution center i that are the same as what the retailer incurs under RMI (this assumption is easily relaxed).

It is thus straightforward to obtain the optimal delivery quantities $q_i = \sqrt{2s_iD_i/h_i}$. The supplier's yearly profit is as follows:

$$\pi^s_{VMI}(\boldsymbol{p}, \boldsymbol{w}) = \sum_{i=1}^{2}\left[(w_i - c_i)D_i(\boldsymbol{p}) - \sqrt{2s_ih_iD_i(\boldsymbol{p})}\right] - \sqrt{2SH(D_1(\boldsymbol{p}) + D_2(\boldsymbol{p}))} \quad (5)$$

The retailer determines the retail price $\boldsymbol{p}$. The retailer's problem can be formulated as follows:

$$\pi^r_{VMI}(\boldsymbol{p}) = \sum_{i=1}^{N}(p_i - w_i)D_i(\boldsymbol{p}) - \gamma_iD_i^{\delta_i} \quad (6)$$

Both the supplier and the retailer aim to maximize their own profits by determining the decisions under their control.

3.4 Model Formulation under CSC

In this scenario, there is a central planner who makes decisions for the supplier and the retailer. The planner's objective is to maximize the profit of the entire supply chain. The chain-wide profit function is defined as the sum of the suppliers's profit and the retailer's profit, and, after some simplification, is given by the following equation:

$$\pi(\boldsymbol{p})_{CSC} = \sum_{i=1}^{2}\left[(p_i - c_i)D_i(\boldsymbol{p}) - \sqrt{2s_ih_iD_i(\boldsymbol{p})}\right] - \sqrt{2SH(D_1(\boldsymbol{p}) + D_2(\boldsymbol{p}))}, \quad (7)$$

where the notation is the same as in the RMI and VMI settings.

4 Model Analysis

In Section 3, we proposed the models for the two-stage dual-retailer channel supply chain under three scenarios: RMI, VMI, and CSC. In this section, we analyze the models under these three scenarios. The model under CSC is a non-linear optimization problem controlled by one decision maker. Under VMI and RMI, the interaction between the supplier and the retailer are analyzed as a Stackelberg game with the supplier as the leader. A backward induction procedure is applied to solve the problem. In the backward induction procedure, the retailer makes decisions first, then the supplier makes decisions based on the retailer's decisions.

4.1 Model Analysis under RMI

For any $\boldsymbol{p}$, which, in turn, determines the annual demand rate $\boldsymbol{D}$, the retail store i's optimal order size is the EOQ order quantity $q_i = \sqrt{2s_iD_i(\boldsymbol{p})/h_i}$. The resulting ordering and holding cost is $\sqrt{2s_ih_iD_i(\boldsymbol{p})}$. Thus, the retailer's yearly profit function (3) can be rewritten as

$$\pi^r_{RMI}(\boldsymbol{p}) = \sum_{i=1}^{2}(p_i - w_i)D_i(\boldsymbol{p}) - \sqrt{2s_ih_iD_i(\boldsymbol{p})} \tag{8}$$

For any $\boldsymbol{w}$ charged by the supplier, the retailer's objective is to choose $\boldsymbol{p}$ in order to maximize its yearly profit. In general, this profit function fails to exhibit any known structural properties to ensure a unique optimal solution. However, we can show that this function is strictly concave in $\boldsymbol{p}$ in most, if not all, realistic markets in which sales-to-inventory ratios are not excessively low and demand elasticities are not excessively large (in absolute value). More specifically, we introduce the following conditions: Let $I_i = p_iD_i$, which is the total gross income for selling the product in store i at price p_i, and let $V_i = \sqrt{2h_is_iD_i}$ denote the optimal total inventory and setup cost for the product selling in store i. We use e_i to denote the absolute price elasticity of store i's demand. In the linear demand example, $e_i = (\alpha_i + \theta_{ij})p_i/D_i$. We assume that

$$e_i \leq 4\frac{I_i}{V_i}, \tag{9}$$

This condition has been discussed in Bernstein and Federgruen (2003, 2004), which show that this condition is satisfied in virtually all realistic markets. According to the empirical data that the authors refer to, we assume the following strengthened condition:

$$e_i \leq 0.4\frac{I_i}{V_i}. \tag{10}$$

We can get condition (10) from the data, originally published in Tellis (1988), in Bernstein and Federgruen (2003). We believe this condition (10) is also satisfied in most realistic markets. However, Bernstein and Federgruen (2003) assume only $e_i \leq 4I_i/V_i$ because it is sufficient to prove their results.

Theorem 1. *When (9) applies, the retailer's problem is strictly jointly concave in* $\boldsymbol{p}$*.*

A closed-form solution for $\boldsymbol{p}$ in terms of $\boldsymbol{w}$ cannot be obtained from solving the first order conditions of the retailer's problem. However, the strict concavity of the retailer's problem guarantees a one-to-one mapping between $\boldsymbol{p}$ and $\boldsymbol{w}$. Therefore, we can obtain $\boldsymbol{w}$ represented by (11) in terms of $\boldsymbol{p}$ by solving the first order conditions of the retailer's problem:

$$\begin{aligned} w_i(\boldsymbol{p}) = & -\frac{\sqrt{h_i s_i}}{\sqrt{2D_i}} - \frac{(\alpha_j+\theta_{ji})\theta_{ij} - (\alpha_j+\theta_{ji})\theta_{ji}}{(\alpha_i+\theta_{ij})(\alpha_j+\theta_{ji}) - \theta_{ij}\theta_{ji}} p_j - \frac{\theta_{ji}^2 + \theta_{ij}\theta_{ji} - 2(\alpha_i+\theta_{ij})(\alpha_j+\theta_{ji})}{(\alpha_i+\theta_{ij})(\alpha_j+\theta_{ji}) - \theta_{ij}\theta_{ji}} p_i \\ & - \frac{\theta_{ji}k_j + (\alpha_j+\theta_{ji})k_i}{(\alpha_i+\theta_{ij})(\alpha_j+\theta_{ji}) - \theta_{ij}\theta_{ji}}, \text{ for } i,j = 1,2 \text{ and } i \neq j \end{aligned} \tag{11}$$

From (11), we obtain

$$\frac{\partial w_i(\boldsymbol{p})}{\partial p_i} = -\frac{\theta_{ji}^2 + \theta_{ij}\theta_{ji} - 2(\alpha_i+\theta_{ij})(\alpha_j+\theta_{ji})}{(\alpha_i+\theta_{ij})(\alpha_j+\theta_{ji}) - \theta_{ij}\theta_{ji}} + \frac{\sqrt{h_i s_i}(\alpha_i+\theta_{ij})}{(2D_i)^{\frac{3}{2}}} \tag{12}$$

and

$$\frac{\partial w_i(\boldsymbol{p})}{\partial p_j} = -\frac{(\alpha_j+\theta_{ji})(\theta_{ij}-\theta_{ji})}{(\alpha_i+\theta_{ij})(\alpha_j+\theta_{ji}) - \theta_{ij}\theta_{ji}} - \frac{\sqrt{h_i s_i}\theta_{ij}}{(2D_i)^{\frac{3}{2}}} \tag{13}$$

From (12), we see that p_i increases with w_i, which is intuitive; if the product has a higher purchase cost, it will have a higher retail price. However, the retail price in one store may increase or decrease with the wholesale price of the other store. That is, $\partial w_i/\partial p_j$ can be positive or negative. It depends on the difference between the cross-price effect parameters θ_{ij} and θ_{ji}. If $\theta_{ij} \geq \theta_{ji}$, which shows that demand of store j is less sensitive to the price change in store i than store i to that of store j, p_j decreases as w_i increases. However, if $\theta_{ij} < \theta_{ji}$, $\partial w_i/\partial p_j$ can be positive or negative.

Substituting $w_i(\boldsymbol{p})$ into the supplier's problem, we have the following:

$$\pi^s_{RMI}(\boldsymbol{p}) = -\sqrt{2SH[D_1(\boldsymbol{p}) + D_2(\boldsymbol{p})]} - \left(\frac{\theta_{ji}k_j + (\alpha_j + \theta_{ji})k_i}{(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji}) - \theta_{ij}\theta_{ji}} + c_i\right) D_i(\boldsymbol{p}) - \frac{1}{2}\sqrt{2s_i h_i D_i(\boldsymbol{p})}$$
$$+ \sum_{i=1}^{2} \left(-\frac{(\alpha_j + \theta_{ji})\theta_{ij} - (\alpha_j + \theta_{ji})\theta_{ji}}{(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji}) - \theta_{ij}\theta_{ji}} p_j - \frac{\theta_{ji}^2 + \theta_{ij}\theta_{ji} - 2(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji})}{(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji}) - \theta_{ij}\theta_{ji}} p_i \right) D_i(\boldsymbol{p}). \tag{14}$$

If we assume the following condition:

$$\max\left(\frac{(\alpha_1 + \alpha_2)(\alpha_{\max} + |\theta_{ij} - \theta_{ji}|)}{(\alpha_{\min} + \theta_{\min})(\alpha_{\min} - \theta_{\max})}, \frac{(\alpha_{\max} + |\theta_{ij} - \theta_{ji}|)^2}{(\theta_{\min} + \alpha_{\min})^2}\right) \leq \frac{39\sqrt{2}}{2} \approx 27, \tag{15}$$

where $\alpha_{\max} = \max(\alpha_1, \alpha_2)$, $\alpha_{\min} = \min(\alpha_1, \alpha_2)$, $\theta_{\max} = \max(\theta_{12}, \theta_{21})$, and $\theta_{\min} = \min(\theta_{12}, \theta_{21})$, we have Theorem 2.

Theorem 2. *When conditions (10) and (15) apply, the supplier's objective function is strictly jointly concave in* $\boldsymbol{p}$.

From the previous discussion, we know that condition (10) holds for almost all industries. Although conditions (10) and (15) are sufficient to guarantee strict concavity, our numerical experiments will show that they are not necessary.

Theorem 3. *When conditions (2), (10), and (15) apply, a unique equilibrium exists for the Stackelberg game.*

Proof of Theorem 3. Because the retailer's objective function is strictly concave, there is one unique optimal retail price $\boldsymbol{p}$ and one order quantity $\boldsymbol{q}$ for any given wholesale price $\boldsymbol{w}$. The strict concavity of the supplier's objective function guarantees that there is only one unique optimal solution for $\boldsymbol{w}$ and Q. □

The unique equilibrium, however, does not have a closed-form solution. The numerical analysis in Section 5 will provide insights about the properties of the solution.

Next, we analyze the VMI model.

4.2 Model Analysis under VMI

The retailer's objective function under VMI is represented by (6). Note that the cost component, $\phi_i = \gamma_i D_i^{\delta_i}$, is similar to the total cost in the EOQ model under RMI, $\sqrt{2h_i s_i D_i}$, when $\delta_i = 1/2$. Since $\delta_i \leq 1$ because of economics of scale, we can show $(\delta_i(1-\delta_i))^{-1} \geq 4$. Thus, condition (10) can be rewritten as the following condition:

$$e_i \leq 4\frac{I_i}{\phi_i}, i = 1, 2 \tag{16}$$

We note that in Bernstein et al. (2006), condition (16) is shown to be invariably satisfied.

Theorem 4. *When condition (16) applies, the retailer's objective function under VMI is strictly jointly concave in* $\boldsymbol{p}$.

As discussed in Section 4.1, $\boldsymbol{w}$ can be expressed in terms of $\boldsymbol{p}$ by solving the first order conditions of the retailer's problem.

The supplier is responsible for determining the replenishment quantity $\boldsymbol{q}$ for each of the deliveries to the retail stores and its own replenishment or production quantity Q, which is accommodated in objective function (5). Substituting $w_i(\boldsymbol{p})$ from the first order conditions of the retailer's problem into (5), we have

$$\begin{aligned}\pi^s_{VMI}(\mathbf{p}) = &- \sqrt{2SH(D_1(\mathbf{p}) + D_2(\mathbf{p}))} \\ &+ \sum_{i=1}^{2} \left(-\frac{(\alpha_j + \theta_{ji})\theta_{ij} - (\alpha_j + \theta_{ji})\theta_{ji}}{(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji}) - \theta_{ij}\theta_{ji}} p_j - \frac{\theta_{ji}^2 + \theta_{ij}\theta_{ji} - 2(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji})}{(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji}) - \theta_{ij}\theta_{ji}} p_i \right) D_i \\ &- \left(\frac{\theta_{ji}k_j + (\alpha_j + \theta_{ji})k_i}{(\alpha_i + \theta_{ij})(\alpha_j + \theta_{ji}) - \theta_{ij}\theta_{ji}} + c_i \right) D_i - \sqrt{2s_i h_i D_i} - \gamma_i \delta_i D_i^{\delta_i}\end{aligned} \tag{17}$$

In order to prove that (17) is strictly concave over $\boldsymbol{p}$, we assume the following condition:

$$\max\left(\frac{(\alpha_1 + \alpha_2)(\alpha_{\max} + |\theta_{ij} - \theta_{ji}|)}{(\alpha_{\min} + \theta_{\min})(\alpha_{\min} - \theta_{\max})}, \frac{(\alpha_{\max} + |\theta_{ij} - \theta_{ji}|)^2}{(\theta_{\min} + \alpha_{\min})^2} \right) \leq (19 - 10\delta_{\max})\sqrt{2}, \tag{18}$$

where $\delta_{\max} = \max(\delta_1, \delta_2)$. Because $\delta_i < 1$, $(19 - 10\delta_{\max})\sqrt{2} < 11$. This is a tighter condition than (15).

Theorem 5. *When conditions (10), (16), and (18) apply, the supplier's objective function under VMI is strictly jointly concave over* $\boldsymbol{p}$.

We note that these conditions are sufficient to guarantee strict concavity, but as our numerical experiments will show, they are not necessary.

Theorem 6. *When conditions (10), (16), and (18) apply, one unique equilibrium exists for the Stackelberg game under VMI.*

Proof of Theorem 6. The proof is straightforward from Theorem 4 and Theorem 5. □

4.3 Model Analysis under CSC

In this scenario, one decision maker determines all the decisions at the same time. The objective function (7) is as follows:

$$\pi_{CSC}(\boldsymbol{p}) = \sum_{i=1}^{2}(p_i - c_i)D_i(\boldsymbol{p}) - \sqrt{2s_ih_iD_i(\boldsymbol{p})} - \sqrt{2SH(D_1(\boldsymbol{p}) + D_2(\boldsymbol{p}))} \tag{19}$$

We can see that the objective function is similar to the supplier's problem under RMI except that the revenue comes directly from the end market. We assume the following condition:

$$\max\left(\frac{(\alpha_1+\alpha_2)(\alpha_{\max}+|\theta_{ij}-\theta_{ji}|)}{(\alpha_{\min}+\theta_{\min})(\alpha_{\min}-\theta_{\max})}, \frac{(\alpha_{\max}+|\theta_{ij}-\theta_{ji}|)^2}{(\theta_{\min}+\alpha_{\min})^2}\right) \leq \frac{18}{5}\sqrt{2} \approx 13.5. \tag{20}$$

This condition is stricter than (15) but less strict than (18).

Theorem 7. *When conditions (10) and (20) apply, one unique set of optimal* $\boldsymbol{p}$, $\boldsymbol{q}$, *and* Q *exists for the centralized supply chain.*

Next, we turn our attention to the numerical analysis in order to gain more insights under different business scenarios.

5 Numerical Analysis

In Section 4, we demonstrated the existence of a unique set of optimal joint decisions, under certain conditions, for RMI, VMI, and CSC. However, the complexity of the models prevented us from finding closed-form solutions. Instead, we carry out numerical analysis to investigate the impact of cross-price effect on joint decisions, which is one of the main concerns of this paper, as well as the impact of self-price sensitivities and production and inventory costs on joint decisions.

5.1 Parameter Settings

Suppose that the products sold at the two retail stores have different configurations, therefore, their unit costs are different. We assume that the product in the online store, store 1, has a lower cost and a higher self-price sensitivity than the product in the off-line store, store 2. It is common for products in off-line stores to be customized, thus, have higher unit costs. Online customers have higher price sensitivity because it is easy for them to compare prices and switch to other sellers. In addition, the empirical studies in Degeratu et al. (2000) suggest that online customers are more price sensitive than off-line customers. The authors also examined the combined effect of price and promotion on price-sensitivity. When the combined effect is considered, online customers are less price-sensitive. In our experiments, we only consider the effect of price, thus, it is reasonable to assume a higher price sensitivity for the online store.

The substitutability between the products in the two stores is caused by both product differentiation and store differentiation. We do not identify these effects separately. If the products sold are exactly the same in both stores, the substitutability is only caused by store differentiation. In this paper, the combined substitutability is represented by θ_{ij} in the demand function. A common approach is to assume that the product sold in one store has the same cross-price effect to the price changes of the product as in the other store (see, for example, Choi, 1996). Thus, we assume $\theta_{12} = \theta_{21} = \theta$.

Table 1 shows the first two cases that we will analyze. The base case is chosen

Table 3-1: The parameter sets

	Set 1 (base case)		Set 2 (high cost difference)	
	Store 1	Store 2	Store 1	Store 2
Self-price sensitivity, α	15	11	15	11
Cross-price effect, θ	2	2	2	2
Potential market size, k	1000	1000	1000	1000
Production cost, c	25	35	5	35
Supplier's setup cost, S	1000	1000	1000	1000
Supplier's unit holding cost, H	1	1	1	1
Retailer's setup cost, s	100	200	100	200
Retailer's unit holding cost, h	1.5	2	1.5	2

to be reasonably realistic. The high cost difference case is identical to the base case, except that store 1 has a much lower unit production cost. By comparing these two cases, we will highlight the impact of the production cost difference on optimal joint decisions. Observe that the unit production costs range from \$5 to \$35. Based on this we will restrict the numerical analysis to retail prices in the range from \$1 to \$150 for both products.

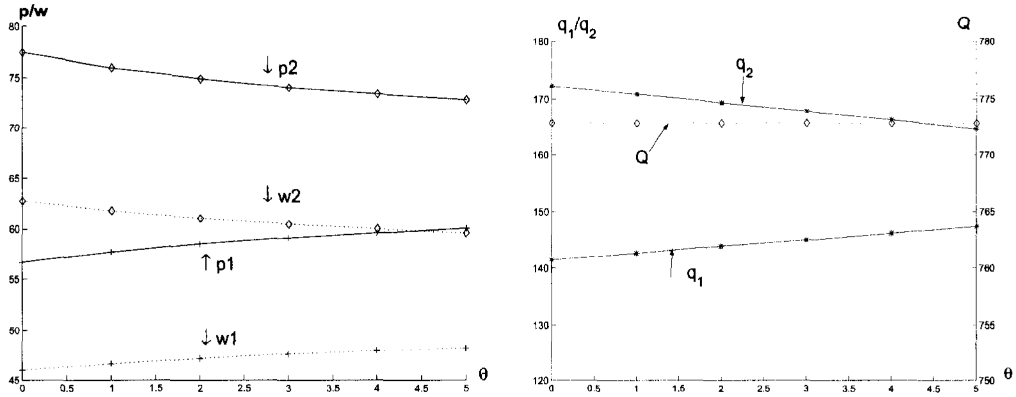
Next, we turn to the numerical analysis and investigate the impact of the different parameters.

5.2 Cross-price Effect

In a multi-retailer channel supply chain, understanding the degree to which the products in different channels are substitutable is crucial for both the manufacturer and the retailer. The cross-price effect has impacts on the pricing, the production, and the profitability of both stores and products, and we expect considering it will benefit the supply chain members.

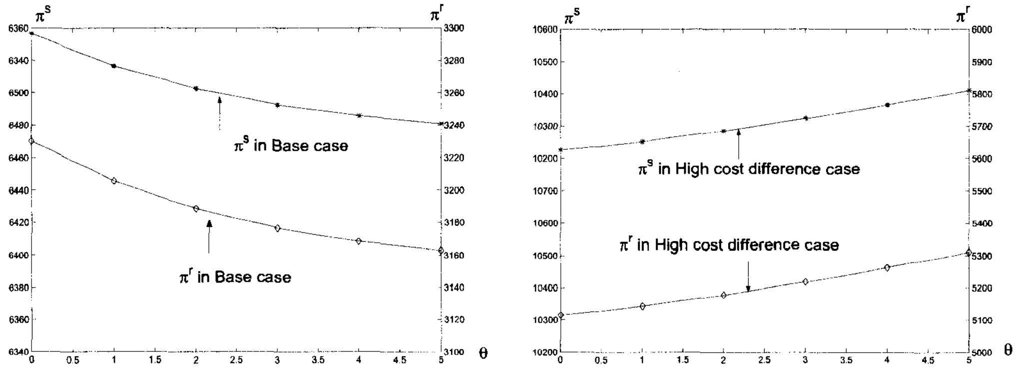
5.2.1 Impact of cross-price effect

The cross-price effect is represented by θ. A higher θ indicates that the consumers consider products sold in the two retail stores as more and more substitutable (less and less differentiated), which can indicate that the consumers have weaker pref-



(a) (b)

Figure 3-1: The impact of cross-price effect on optimal decisions



(a) (b)

Figure 3-2: The impact of cross-price effect on optimal profits

erence for one retailer channel over the other (less store differentiation). Product category is one factor that affects retail store differentiation. For example, store differentiation is low for books but high for clothes. The optimal decisions for each members under different cross-price effects are shown in Figure 3-1 using the base case under RMI.

We observe a clear price convergence as shown in Figure 3-1a. As two products become more substitutable (θ increases), their retail prices become closer to each other, and so do their wholesale prices. The retail price in the off-line store which is higher at $\theta = 0$ decreases, while the retail price in the online store, which is lower at $\theta = 0$, increases with θ. The wholesale prices demonstrate the same trend. The retail margin, the difference between retailer price and wholesale price, increases for products in the online store and decreases for products in the off-line store. These are intuitively appealing results. If the consumers have weaker preference to purchase from one store rather than the other store (larger θ), the prices in different stores get closer.

The EOQ for the off-line store (q_1) decreases and that for online store (q_2) increases with θ as shown in Figure 3-1b. In an EOQ model, demand changes in the same direction as order quantity. Thus, this result indicates that more consumers choose to buy online if the two products become more substitutable. The supplier's EOQ (Q) is relatively stable with respect to θ, so does demand for the supplier as the sum of the demand from the two stores.

The supplier's and the retailer's profits decrease with θ in the base case as shown in Figure 3-2a. They increase with θ in the high cost difference case as shown in Figure 3-2b. We find that the production costs c_i and the self-price sensitivities α_i play a role. In the base case, the production costs c_1 and c_2 are close, the difference between self-price sensitivities α_1 and α_2 play a key role in the impact of substitution (store substitution in this scenario) on the profits. Increased θ indicates that the customers have weaker preference for one store over the other. Thus, the impact of self-price sensitivities is reduced by the increased substitution of products (θ). Therefore, the low production cost difference leads to fiercer inter-product competition when θ increases, which damages both the supplier's and the retailer's profits.

In the high cost difference case, the impact of a larger difference in production costs c_1 and c_2 dominates that of the self-price sensitivity on the profits. When the impact of different self-price sensitivities is reduced by the increased substitution, the high production cost difference allows the members to take advantage of the fact that customers treat the products as more similar and more substitutable; thus, both the retailer and the supplier earn higher profits.

The above observations about the price convergence effect and the profit trends have implications for the pricing strategies of products sold in online and off-line stores. For products that are highly substitutable, such as books, the price difference in the two channels should be small. In addition, when a retailer has different product configurations for different retail channels, it should take the design of the products into consideration. The larger difference between production costs benefits both members.

We have analyzed the impact of the cross-price effect for the base case, the high cost difference case, and many other cases, under both RMI and VMI. The general pattern of price convergence and the qualitative behavior of the optimal decisions and optimal profit is the same in all cases that we have analyzed. In the remainder of this section, we will report additional numerical results for the base case. Unless otherwise noted, the observations we make hold for all other cases that we have analyzed, under both VMI and RMI.

5.2.2 Benefit from considering cross-price effect

Researchers such as Zhu and Thonemann (2003) show that firms benefit from considering the cross-price effect. However, in our model firms cannot always benefit from this consideration. When making decisions on the prices and the inventory replenishment without considering the cross-pricing effect, the firms take the demands as $D_i^w = k_i - \alpha_i p_i$ instead of the true demands $D_i = k_i - \alpha_i p_i + \theta_i(p_j - p_i)$. We use the superscript w to distinguish the demands without considering the cross-price effect from the true demands. The profits without considering θ are calculated by the firms' decisions on prices and the true demands in the market. From Figure 3-3, we can see that both members benefit from considering θ in the base case, and both suffer slightly from considering θ in the high cost difference case.

In the base case, the firms benefit from considering θ because the joint decisions are made taking the fierce competition between two channels into consideration, which we have discussed in Section 5.2.1. The competition between two channels is subtle in the high cost difference case, which makes no big difference in profitability of the supply chain members.

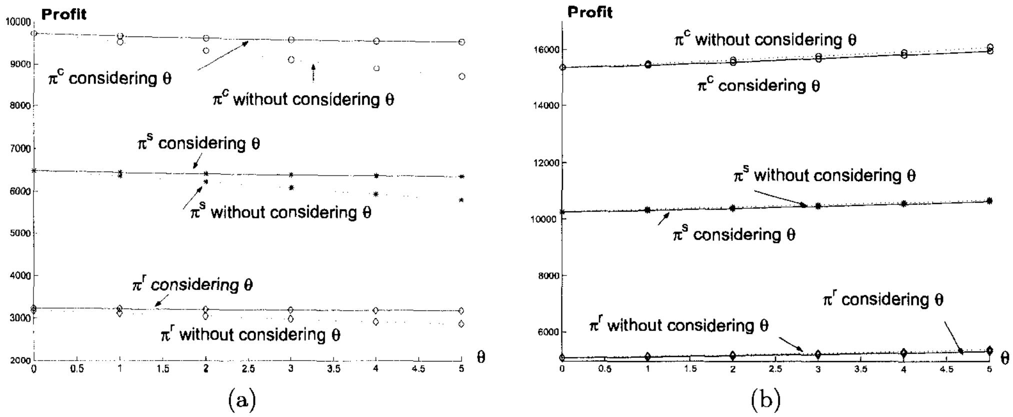


Figure 3-3: The profits with or without considering the cross-price effect

Table 3-2: Summary of the impact of cross-price effect on optimal decisions and profits

	The online store ($i = 1$)	The off-line store ($i = 2$)	The supplier
Retail price (p_i)	↑	↓	N/A
Demand and EOQ (D_i and q_i)	↑	↓	N/A
Wholesale price (w_i)	↑	↓	N/A
Total demand ($D_1 + D_2$) and Q	N/A	N/A	−−
Profits	↓ or ↑		↓ or ↑

5.2.3 Summary of Our Key Findings

We summarize our findings about the impact of the cross-price effect on each member's optimal decisions and performance in Table 3-2, where a notation of ↑ (↓) indicates an increase (decrease) of a quantity, and a notation of −− indicates no change.

5.3 Impact of Self-price Sensitivity

Recall that the self-price sensitivity is represented by α_i in the demand function. For the dual-retail channel supply chain, α_1 and α_2 have a symmetric impact on optimal decisions. When α_i increases, so that the demand for store i becomes more sensitive to its own price, the retail price and the wholesale price in this store decrease, which is intuitive. The retail price and the wholesale price of the other store are less sensitive, although they do decrease as well because of the price convergence effect.

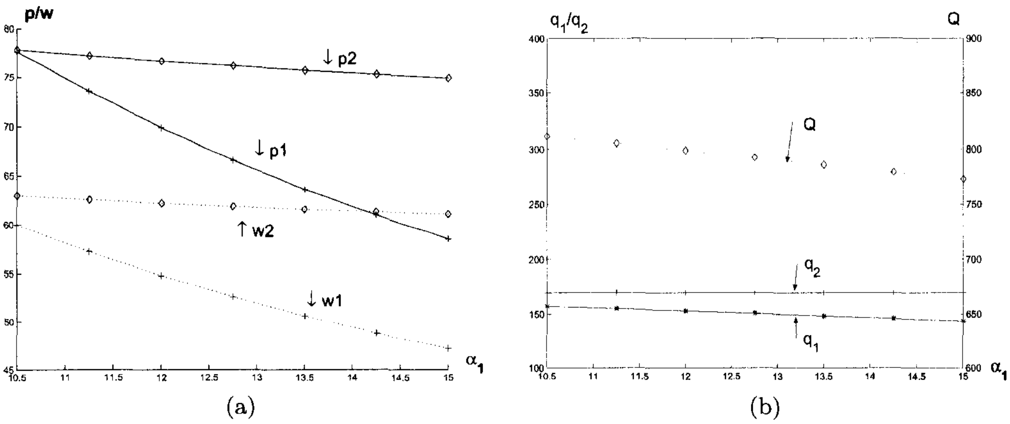


Figure 3-4: The impact of α_1 on optimal decisions

These observations are shown in Figure 3-4a.

From Figure 3-4b, we see that the EOQ of store i decreases with α_i because D_i decreases with α_i but is insensitive to α_j. The total demand and the supplier's EOQ decrease with respect to both α_i and α_j.

Both the supplier's and retailer's profits decrease with α_i. See Figure 3-5. This result is intuitive because when one product becomes more price-sensitive, the firms that manufacture and sell this product can only achieve a lower profit. This observation holds in a supply chain environment.

5.4 Impact of Production and Inventory Costs

In this section, we examine the impact of production costs, inventory holding costs, and setup (ordering) costs on optimal solutions, as well as the performance of the members and the entire supply chain.

5.4.1 Impact of production costs

As we can see in Figure 3-6, when the production cost for store i is reduced, the optimal retail price and wholesale price for store i decrease as expected. The retail price and the wholesale price for one store are not sensitive to the production cost for the other store. The demand for store i increases with a reduction in c_i because of the decreased prices. Thus, the retailer's EOQ of store i increases with a decrease

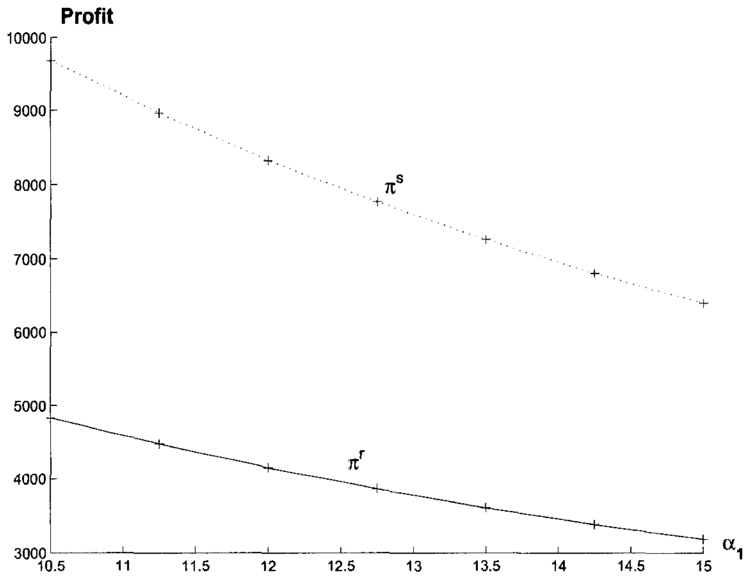


Figure 3-5: The impact of α_1 on optimal profits

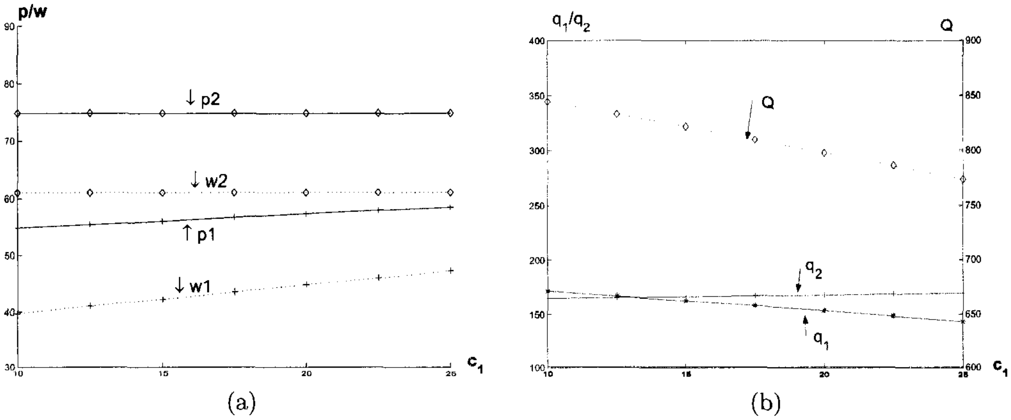


Figure 3-6: The impact of production cost c_1 on optimal decisions

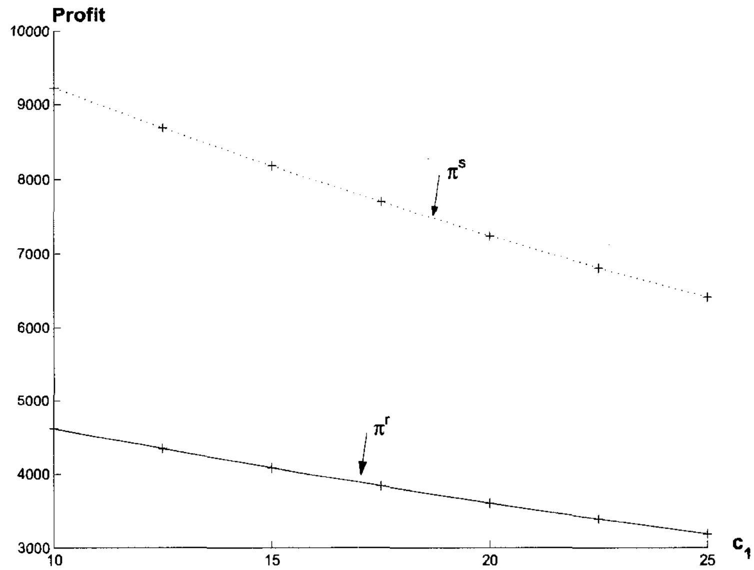


Figure 3-7: The impact of production cost c_1 on optimal profits

of production cost c_i. The demand for product j is not sensitive to c_i and slightly decreases because of the decrease in p_i. The supplier's order size decreases with production costs c_i because the total demand decreases. Both supplier and retailer benefit from a reduction in c_i.

5.4.2 Impact of inventory costs

From the models, we can see that the unit holding costs, H and h_i, and the setup costs, S and s_i, can have an impact on the optimal decisions and profits. However, because the total costs $\sqrt{2SH(D_1 + D_2)}$ and $\sqrt{2s_ih_iD_i}$ depend on the multiplications of the setup costs and the unit holding costs, we examine only the effect of the supplier's setup cost, S, and the buyer's setup cost, s_i.

Impact of the supplier's setup cost. We illustrate the impact in Figures 3-8 and 3-9. We can see that only the supplier's order size (Q) increases with the supplier's setup cost as expected. All other decisions remain approximately the same, even when the setup cost S is doubled.

Impact of the retailer's ordering costs. We illustrate the impact in Figures 3-10 and 3-11. All the optimal solutions are stable to the changes in s_i except the

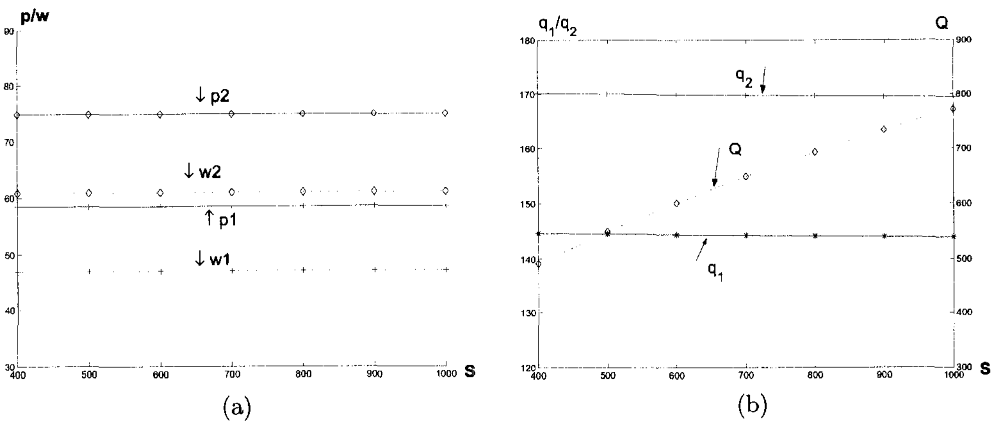


Figure 3-8: The impact of the supplier's setup cost on optimal decisions

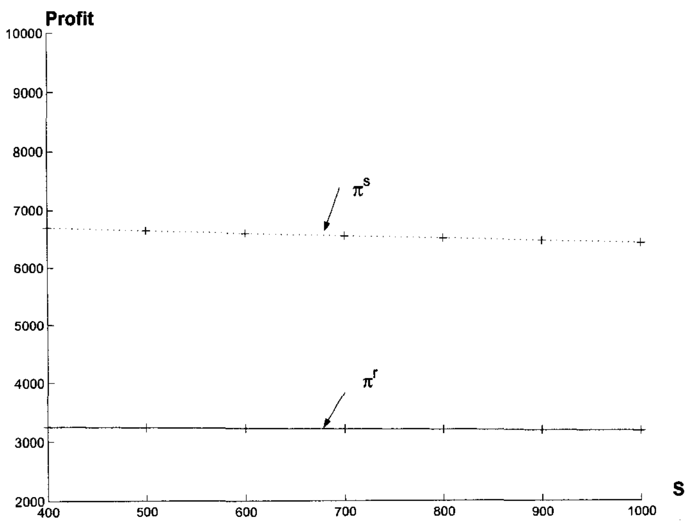


Figure 3-9: The impact of the supplier's setup cost on optimal profits

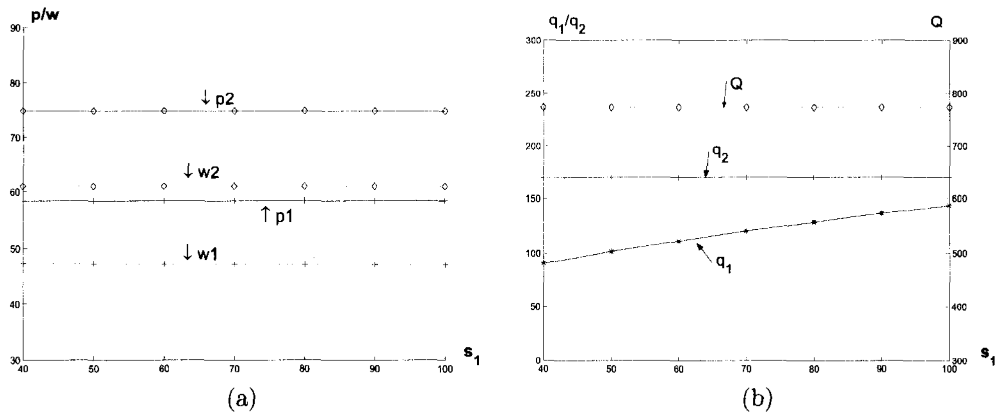


Figure 3-10: The impact of the retailer's setup cost on optimal decisions

EOQ of store i, q_i, as expected. Because $q_i = \sqrt{2s_iD_i/h_i}$, q_i increases with s_i.

5.5 Impact of Inventory Management Mode

In this section, we compare each member's performance under VMI and RMI and compare the supply chain performance under VMI, RMI, and CSC. We assume that the retail stores do not incur any inventory cost under VMI, that is to say, $\gamma_i D_i^{\delta_i} = 0$. The supplier incurs the same inventory related cost s_i and h_i in retail store i as the retail store does under RMI; thus, we use the same cost parameters for VMI, RMI, and CSC, therefore, any differences in optimal profit between the three scenarios will be caused only by differences in supply chain structure or inventory management partnerships.

The supplier's profit is less, while the retailer's profit increases with VMI compared to RMI. The retailer clearly benefits from VMI. This result justifies the fact that powerful retailers such as Wal-Mart have been promoting the use of VMI in practice.

The experimental results are shown in Figure 3-12. The supply chain can achieve a significantly higher profit in the centralized scenario, while it achieves a better profit under VMI than under RMI. We believe that a supply chain performs better under VMI than RMI because of centralized inventory control under VMI, which makes inventory management more efficient under VMI. However, we notice that the performance of the chain does not increase significantly when only inventory

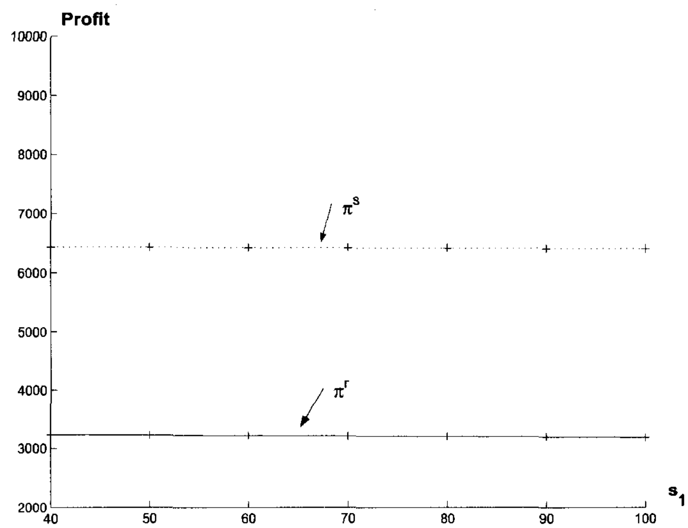
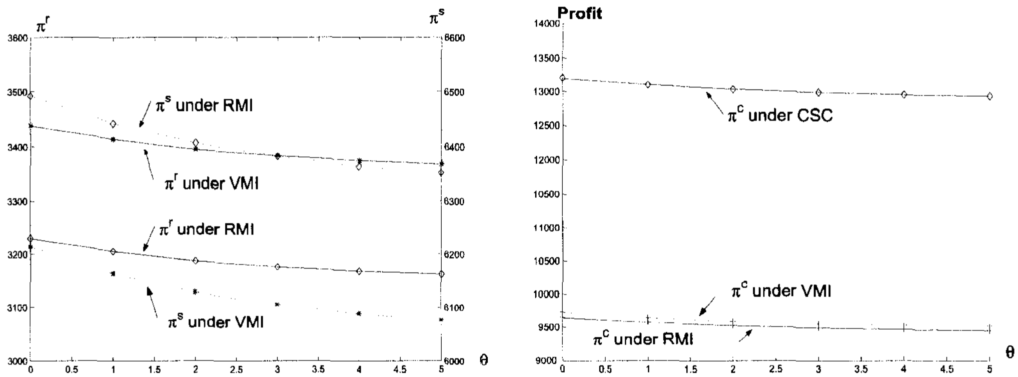


Figure 3-11: The impact of the retailer's setup cost on optimal profits

decisions are centralized (under VMI.) The performance of the chain increases significantly when both pricing and inventory decisions are centralized (under CSC.) We also notice that the supplier as the Stackelberg leader does not benefit from the VMI. That is to say, the supplier actually obtains more profit under RMI than under VMI. All the profit gain from VMI goes to the retailer. Of course, the supplier might gain from customer satisfaction in the long run, which is not captured in our models. Here, our main focus is how the pricing and replenishment strategies interact and are affected by competition and inventory factors under different inventory management scenarios.

6 Concluding Remarks

In this paper, we consider joint pricing and inventory/production decision problems for the members in a two-stage dual-channel retailer supply chain. The supplier in the chain purchases or manufactures its product under an EOQ model and sells the product through an exclusive dual-channel retailer. A typical dual-retailer channel in the e-commerce era includes an online store and an off-line store. Because of economies of scale, the Internet delivery distribution center is established specially



(a) (b)

Figure 3-12: The impact of inventory management mode

to supply online customers. Thus, each retailer channel has its own warehouse. We model the inventory management in each of them using an EOQ model with a cross-pricing effect. The products sold in the online store and the off-line store might have different configurations and be seen as substitutes by consumers, which can be considered as a multiple product problem on the retailer's side. However, on the supplier side, the product is from one supplier or from the same product line; thus, we consider a joint setup cost in the supplier's problem.

We study three supply chain structures: RMI, VMI, and CSC. Under VMI, both the online store and the off-line store, owned by the same entity, carry the product and determine the replenishment and retail price decisions. Under VMI, the retailer only decides retail prices, whereas the supplier makes the inventory replenishment/production lot size decisions for the whole supply chain. Our main purpose is to investigate how the supplier and the retailer in such a supply chain system make their joint pricing and inventory/production decisions in equilibrium and to determine the impact of the cross-price effect on the decisions and on each member's and the supply chain's performance under VMI and RMI. We build price-sensitive EOQ joint decision supply chain models in order to understand the interactions between pricing and production/inventory decisions at both the supplier and the retailer levels. We formulate Stackelberg games that enable the supplier as the leader to take into account the reactions of the downstream retail stores when making decisions.

We show that a unique Nash equilibrium exists for the Stackelberg games when certain conditions apply under RMI and VMI. Under CSC, there is a unique optimal solution. One critical condition concerns the relationship between the demand price elasticity and the sales-to-inventory value of the product. The condition is shown to be virtually satisfied in most, if not all, realistic markets in which sales-to-inventory ratios are not excessively low and demand elasticities are not excessively large (in absolute value). Furthermore, numerical experiments show that a unique Nash equilibrium appears to exist even if some of or all the conditions do not apply.

The impact of the cross-price effect between products sold in the dual-retailer channel is one of the main issues in this paper. The cross-price effect in such a channel represents impacts of both store differentiation (consumer preference on store types) and product differentiation (different product configurations). We observe a price convergence effect when the cross-price effect becomes stronger. The observation implies that pricing for different product categories for the online store and the off-line store must be done strategically. For products such as books, where consumers may not have a strong preference for either online or off-line shopping, the prices converge; that is, prices are getting closer as the cross-price effect increases; while for products such as apparel, where consumers may strongly prefer to shop off-line because of the additional information from the shopping experience, the prices might be quite far apart. Furthermore, when the products sold in different channels have different configurations, for example, a hard-cover edition of a book is available only in the off-line store, this should be taken into consideration when making pricing decisions.

Demand for the store with the lower price (usually the online store) increases, but the demand for the off-line store decreases when there is a strong cross-price effect (a larger substitution factor). The inventory decisions in the retail stores follow the same trend that demand has. Demand for the supplier (the total demand) is relatively insensitive to the cross-price effect, and the same is true for the EOQ for the supplier. The difference between the production (purchasing) costs and the difference between self-price sensitivities play an important role when examining the impact of cross-price effect on the supplier's and the retailer's profits. If the difference between production costs is large and dominates, the chain can take advantage of the fact that customers treat the products in the two stores as more alike and more substitutable; thus, the chain can obtain more profit. If the difference between production costs is small and the difference between the self-price sensitivities dominates, the competition among products increases with substitution, which causes the supplier, the retailer, and the chain as well to lose profit. These observations shed light on a supplier's product configuration for different retailer channels and the retailer's store management decisions. In their decision-making, it is important to examine the mixed controlling effect of product costs and price sensitivities.

The impact of self-price sensitivities and production and inventory costs on optimal decisions and profits is straightforward. It is worth noting that the effort to reduce the supplier's setup cost is more rewarding than reducing the retailer's setup costs, which provides a recommendation for the initial steps to improve supply chain performance, especially under VMI. In the setting of this paper, we found that the supply chain performance is best under CSC followed by VMI, and RMI is least desirable. This result is in tune with earlier supply chain studies with different settings. An unexpected result, that of the retailer benefiting from developing VMI, while the supplier does not, may be due to the fact that some benefits such as customer satisfaction are not captured in our models.

In this paper, we have shown that unique optimal decisions exist in a two-stage dual-retailer channel supply chain without coordination efforts. We further offer numerical insights on the impact of the cross-price effect and other parameters on optimal decisions, each member's performance, and the chain-wide performance without coordination efforts. Channels with different contract types and coordination efforts offer an interesting area for further research, as do models with more products in all supply chain stages. However, more elaborate analytical methods are needed to solve these supply chain models with added complexity.

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Chapter 4: Optimal Pricing and Referral Reward Programs under Competition

1 Introduction

Word-of-Mouth (hereafter WOM) refers to the spread of information, especially recommendations, in an informal, person-to-person manner. Individuals are more inclined to believe WOM marketing, also known as buzz marketing and viral advertising, than more formal forms of promotion because of personal connections involved (Grewal et al., 2003). Thus, marketers have increasingly considered WOM as a marketing tool (Rosen, 2000). The initial efforts to manage WOM focused on encouraging customers to pass positive rather than negative information and on targeting influential customers such as opinion leaders (Ryu and Feick, 2007). Work such as Dellarocas (2003) emphasizes the importance of understanding, measuring, and managing WOM.

Although WOM may work with or without incentives, firms are increasingly aware of the need to manage customer referrals with incentives. Firms have introduced incentive programs such as various types of rewards (cash back, vouchers, free minutes, airmiles, future purchase discounts) to stimulate existing customers to recommend a product or service to others. The use of referral reward programs has been growing steadily because reward programs are often a cost-effective way to recruit customers and increase sales (Biyalogorsky et al., 2001). The 601 Deli and Catering Referral Reward program says "Instead of spending big bucks on radio and television advertising, we've decided to use our best advertisement—our satisfied customers, like you—and reward you in the process" (601Deli, 2008). Competition Headwear believes that "the most effective way of increasing our business is through WOM advertising." (Headwear, 2008)

In addition to traditional industries, high-tech businesses also use WOM to promote their products or services. With the popularity of Internet, a new form of WOM, digital WOM, is created to increase the scale and scope of WOM. Digital WOM passes information easily to a large population of geographically dispensed customers, which may not know easy other. Many online stores such as *Massage-Chair-Relief.com* and *airfilterstore.com* provide referral reward programs. If a referral to *airfilterstore.com* leads to a sale, the company will reward the customer who made the referral with a gift certificate of 10% of the referred purchase. Since 1996, the leading online retailer *Amazon.com* has been using its Associates Program (over 1,000,000 members) to reach more customers. By putting links in personal webpages, social networks, review sites, online communities, or searching engines, Associates can drive internet traffic to Amazon and earn referral fees of up to 10% on all recommended sale. Although we see more examples of referral reward programs for services than for products, referral reward programs can be successfully used to promote both services and tangible products. The following are some examples of referral reward programs from a brief Internet search:

1. Sprint PCS and Nextel customers may encourage their friends to purchase Sprint PCS or Nextel phones and services from *sprint.com* or from participating Sprint or Nextel retail stores. Both the referred customer and the referring customer can earn a $25 Visa debit card.

2. MDS (Managed Data Solutions) offers a special referral rewards program exclusively for current MDS customers. When a current customer refers a new customer who signs up for hosting services, MDS will credit a month's service fee to the current customer.

3. *Massage-Chair-Relief.com* will send a current customer a check for $100.00 if a referred customer buys a massage chair from the company online.

Referral reward programs can be wasteful when not designed properly. Therefore, managing a reward program is critical to its success. Furthermore, pricing as an important market tool affects the demand for a product or service and probably affects the customers' referral intentions as well. However, Biyalogorsky et al. (2001) believe that lowing price itself is not the best mechanism to increase demand. Although lowering price will increase customers' buying intentions, it may cause customers who have no referral intentions to enjoy the low price but do not refer. Hence, referrals will not increase. Therefore, it seems appropriate to manage both pricing and reward programs to effectively harness the power of both. This will be the focus of our analysis in this paper.

In this paper, we model and analyze optimal pricing and referral reward programs for two competing firms. The two competing firms can be two online stores, two brick-and-mortar stores, or one online store and one brick-and-mortar store. However, in this paper, we do not differentiate competition under these three circumstances. We present a model and analyze two general competing firms selling substitutable products or services to a common pool of potential customers.

Depending the timing of customers' awareness of referral reward programs, there are two types programs commonly offered in practice. Some programs are easy for customers to find and access, thus, customers know about the rewards before making a purchase. Some programs are restricted only to current customers, so customers know about the rewards only after making a purchase. Firms can choose their timing strategy to achieve which type of program to offer. A firm choosing to make their customers aware of reward programs before purchase usually advertises its reward program to make it easy to access. A firm also can inform only their existing customers about the program by offering a restricted access reward program. Firms might have various reasons to choose different timing strategies. Restricted access programs might be chosen for attracting referrals only and avoiding possible relationship damage, while easy access programs might be chosen for attracting both initial customers and referrals. In addition, the set-up costs for restricted access programs are generally lower than easy access programs.

A reward can take the form of an absolute value or of a percentage of purchase value. An example of the first form, absolute reward, is that Shaw cable offers $50 credit on the referrer's bill for any referred sale. An example of the other form, percentage reward, is that Amazon gives up to 10% of its sales from recommendations to the referrers. In this paper, we will investigate what the differences are between offering absolute rewards and percentage rewards.

The following are our research questions:

1. In a competitive market, two firms can compete on pricing strategies and referral reward programs. What are the optimal pricing strategies and referral reward programs in such situations?

2. Firms can advertise their referral reward programs to make them easy to access. Should firms do this? If they should, does that change the optimal pricing and referral reward decisions?

3. Are there any differences between offering absolute rewards and percentage rewards?

4. What factors influence optimal pricing and referral reward decisions? Possibilities include customer preferences for firms and the probability that a referral results in a sale, which might vary between companies.

5. If the competitor of a firm has a referral reward program, what is the best strategy regarding pricing and referral reward programs for this firm?

To address these questions, we formulate both simultaneous games between two firms with relatively equal power and sequential games for two firms with unequal power. Interaction between two firms are modeled as three-stage games in which the two firms choose their timing strategies: When to tell their customers about the rewards (easy access or restricted access) in the first stage, levels of rewards in the second stage, and retail prices in the third stage. We will provide further discussion on this order of decisions in Section 3.1. If the firms make simultaneous decisions in each stage, we formulate a three-stage simultaneous game and find that the more rewards the firms offer, the more they benefit if they offer easy access programs. This finding holds both for absolute rewards and percentage rewards. Regarding the choice between easy access and restricted access programs, the type of referral reward plays a role. On the one hand, with absolute rewards, the best strategy for both firms is to offer the same type of program: Either easy access or restricted access. However, which strategy is better depends on demand functions, successful referral rates, and levels of rewards. One the other hand, when offering percentage rewards, it is always best for both firms to offer easy access programs.

If the firms make sequential decisions in each stage, we formulate a three-stage sequential game. However, we limit our attention to easy access programs and reduce the three-stage game to a two-stage game. We find that both firms benefit more from higher referral rewards including absolute rewards and percentage rewards. This conclusion is the same as in a simultaneous three-stage game. In addition, we find that both firms perform better in a sequential game than in a simultaneous game.

The rest of the paper is organized as follows: Section 2 provides a brief review of the related literature. Section 3 discusses our modeling assumptions. Section 4 presents the demand formulation, which is a critical part of our model formulation. Section 5 introduces the model and analysis in a simultaneous game. Sections 6 provides the extension of the model to a sequential game. Section 7 offers concluding comments.

2 Literature Review

Although referral reward programs may be effective at increasing sales, there is limited research on these programs. Some empirical research studies the effects of incentives on WOM. For example, Blodgett et al. (1997) examine the impact of customers's experience in returning products on their negative WOM intentions. Wirtz and Chew (2002) investigate how incentives would work to promote WOM with other factors such as satisfaction and tie strength. Their findings suggest that "incentives are an effective catalyst for increasing the likelihood of WOM being generated by satisfied customers, and that tie strength is an important variable in explaining WOM behaviour." Ryu and Feick (2007) find a positive link between referral rewards and referral likelihood. The referral likelihood is stronger when referring to weaker ties and to weaker brands. Jumar et al. (2007) emphasize identifying and capitalizing on customers who bring in the most referrals by calculating a customer's lifetime value.

To our best knowledge, only three papers have investigated referral reward programs analytically. Biyalogorsky et al. (2001) examine managing pricing and referral reward programs based on the maximization of one customer's life time value. The customer's life time value is his contribution from his initial purchase and the possible following referrals. The authors find that referral reward should be offered with pricing strategy when the customer's delight threshold is at an intermediate level. Pricing strategy itself can maximize the customer's contribution when his threshold is either low or high. However, competition is not considered because the authors investigate one customer's life long value. We model the competition of two firms maximizing their profits and find that the optimal decision is to increase prices and provide referral rewards.

Lobler and Welk (2004) focus on joint pricing and referral reward for one firm. The optimal decision is obtained based on maximizing the firm's profit from initial demand, autonomous referrals from delighted customers who refer without any reward, and referrals from customers who are encouraged to refer by reward. Customers who would refer even there is no reward but take the reward when it is offered are free riders in Lobler and Welk (2004). If the product sold by the firm can delight customers easily, the autonomous referral rate is high. The authors examine the impact of an autonomous referral rate and the number of free riders on joint pricing and reward decisions. The authors show that both optimal decisions decrease with an autonomous referral rate when free riding problems are not considered. However, when free riding problems are considered, the optimal price may increase or decrease under certain conditions, but the optimal reward always increases with the number of free riders. Lobler and Welk (2004) consider managing pricing and referral reward programs are for one firm, thus, they do not consider competition.

Chen and Shi (2001) study customer recommendation programs under various industry structures (monopoly, duopoly, and competitive market) and consider two types of reward (cash and discounted future purchase.) Although a monopolist and duopoly firms would choose cash rewards, and future discounts, respectively, firms in a competitive market face little difference when choosing different types of reward. The competition in their paper is symmetric, which may not be true for many scenarios such as competition between online stores and brick-and-mortar stores. We consider asymmetric competition in a duopoly setting.

The interest of this paper is in managing pricing and referral reward programs when there is competition in the market. We analyze the optimal mix of prices and rewards in a duopoly environment, where two firms compete not only on price but also on referral reward. Both retail price and reward influence profit by having an impact on demand and profit margin. A higher reward generally attracts more demand, so does a lower price. However, when a reward is offered, a firm needs to increase its retail price to have a higher profit margin to compensate the costs of the reward. However, retail price cannot increase too much because higher retail price will decrease the demand and hence has a negative effect on profit. Thus, the firm needs to find an optimal mix of price and reward to obtain a balance between demand and profit margin to maximize its profit.

3 Modeling Assumptions

We state, discuss, and justify our modeling assumptions on the order of decisions, the buying and referring processes, and other related aspects in this section.

3.1 Order of decisions

We consider two firms, firm 1 and firm 2, selling two substitutable products. The firms need to make their pricing and reward decisions before the products are launched. Although customers know the retail prices and the referral reward programs before they make purchase and referral decisions, for the firms, these three decisions, easy or restricted access, level and type of reward, and price are not likely determined at the same time due to certain institutional considerations.

First of all, the marketing department of a firm normally does not have a team for reward programs like for pricing strategies. A marketing company is usually needed to design a reward program and advertise it. The firm needs to contact the marketing company at least a few months before the product is launched. For example, the firm has to start to get in touch with some advertising companies in the summer if a reward program is considered to be launched with the product in December. Thus, we think that it is reasonable to assume that the firm needs to choose its timing strategy first to consider whether external help on the reward program is needed. Then, when designing the details of the reward program, the firm determines the level of the reward after it has chosen its timing strategy. Pricing decision is usually made when it is close to the selling season. So it the last decision to make.

We consider that the two competing firms follow the same order of decisions. Therefore, we formulate the competition between the two firms as a three-stage game. In the first stage, the two firms choose their timing strategies. In the second stage, they determine levels of rewards. In the third stage, they choose retail prices.

In addition, we believe that reward programs involve relatively longer-term decisions than pricing strategies. A reward program generally lasts more than half a year. However, pricing decisions change frequently due to the popularity of dynamic pricing techniques. Thus, it is reasonable for us to formulate a three-stage game with pricing decisions determined in the last stage. Although pricing is a static decision in this paper, it can be changed when needed. Under such circumstances, the game in the third stage has to be replayed with updated information on realized market.

Next, we discuss model assumptions in customers' buying and referring processes after they know the firm's decisions.

3.2 Buying and Referring Processes

We consider two types of customers, initial customers and referrals. Initial customers are the buyers whose purchase decisions are a result of self search, while referrals consider purchasing because of recommendation.

The two firms sell substitutable products over an infinite selling horizon consisting of discrete time periods. In the first period, some initial customers decide to buy from either firm 1 or firm 2. It takes one period for a referrer to make his referral decision and to search for an appropriate person to recommend. If the referral decides to buy, he will buy in the same period. In each following period, there will be both initial customers and referrals for each firm. We can group the initial customers in different periods and model them as a linear demand function of prices charged and rewards offered by the two firms if we do not consider discount factors. Next, we introduce our assumptions about buying and referring, some of which are similar to the assumptions in Biyalogorsky et al. (2001).

Assumption 1 (Timing of the awareness of the referral reward programs): *There are two possibilities for the timing of the awareness for initial customers: Customers know about the reward program **after** their purchases (restricted access) or **before** their purchases (easy access.)*

Both possibilities are common in reality. We can search on Internet to see many commercials for referral reward programs, which are perfect examples of easy access programs that make customers aware of the reward **before** their purchases. However, some companies inform customers of their referral reward programs only after they have made a purchase. Since some music clubs notify their new customers about their referral rewards after customers subscribe for a while (Biyalogorsky et al., 2001), these clubs are examples of firms offering restricted access programs. These music clubs may want to build a relatively strong bond first. In Section 1, we have discussed the considerations that firms may choose different timing strategies to influence the timing of the awareness of the referral programs, which in turn affect customers' purchasing decisions. We use timing strategies: *before* and *after* and *easy access* and *restricted access* interchangeably in this paper.

Biyalogorsky et al. (2001) and Lobler and Welk (2004) assume that a customer is informed of a referral reward program only after his purchase. In Chen and Shi (2001), a customer is aware of referral rewards before making a purchase. In our model, we examine the impact of both timing strategies on managing pricing and reward decisions and analyze which strategy is preferred by firms under competition.

Assumption 2 (Initial customers): *The demand from initial customers is a generalized linear demand function of expected net prices of both firms.*

As in Chapter 3, we model the initial demand for firm i ($i = 1, 2$) as $D_{i0} = k_i - m_i\bar{p}_i + \beta_i(\bar{p}_j - \bar{p}_i)$ with $k_i > 0$, $m_i > 0$, $\beta_i \geq 0$, and $j = 3 - i$. k_i represents the potential market size; m_i measures the self-price sensitivity of firm i; β_i represents competition between the products of the two firms; $\bar{p}_i$, expected net price, represents the difference between a retail price p_i and an expected reward $\alpha_i r_i$. As what we will discuss in Assumption 4 in this section, not every recommendation will result in a sale. By assuming a successful referral rate α_i, a risk-neutral customer expects to obtain $\alpha_i r_i$ as the reward for referring one person if the customer knows the reward before purchase. So the customer expects to pay $\bar{p}_i = p_i - \alpha_i r_i$ for the product with an easy access reward program. The expected net price is the same as the retail price when restricted access programs are offered. We will provide the details of our demand formulation in Section 4.

Assumption 3 (Number of referrals): *Each buyer refers at most one customer.*

Biyalogorsky et al. (2001) and Lobler and Welk (2004) have the same assumption. Biyalogorsky et al. (2001) argue that if the reward is linear in the number of referrals, assuming one referral has the same effect as assuming a constant number of referrals. The number of referrals can be normalized to one, and the reward can be normalized as well. Chen and Shi (2001) assume that each customer can recommend a constant number of customers. We assume that one customer can make at most one recommendation.

The referral process includes three steps: A referrer first decides if he wants to recommend a friend; he searches his network to find a friend to refer; the friend makes the purchase decision in the last step.

Assumption 4 (Successful referral rate): *The possibility that the recommendation of a customer of firm i results in a sale is an increasing function of reward.*

The probability, also referred as successful referral rate, is the joint probability of the event that a referrer decides to refer, the event that he can find a friend to refer, and the event that the friend buys. In this paper, we focus on the joint probability. We model the impact of one factor, the level of a reward, on a successful referral rate. We believe that referrers will be more likely to refer and will put more efforts in searching for a friend to recommend if rewards are higher. We denote a successful referral rate as α_i $(0 \leq \alpha_i < 1)$, and α_i increases with r_i.

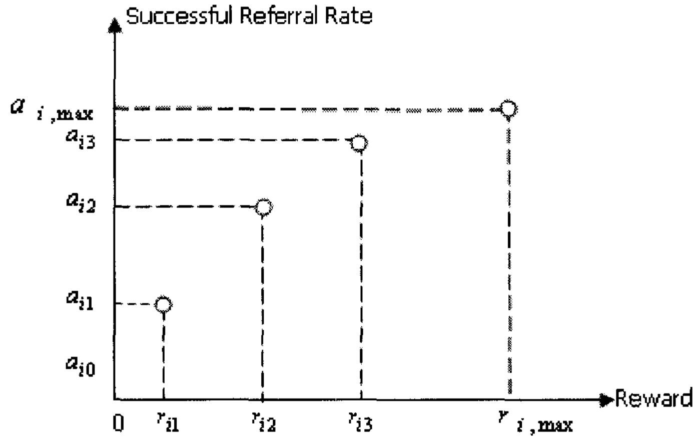

Figure 4-1: An illustration of successful referral rate and reward

This assumption is adopted by Lobler and Welk (2004) and Biyalogorsky et al. (2001). In Biyalogorsky et al. (2001), α_i is a parameter. Although Chen and Shi (2001) assume that each referrer can recommend a constant number of customers, these referrals can be referred by different customers to different firms. Thus, there is also a successful referral rate for each recommendation due to social network overlaps.

We further assume that only a finite number of r_i are possible, and α_i is defined only for this set in the interest of tractability. That is to say, firm i can choose to offer a reward from the following set of n_i elements, $\{0, r_{i,1}, r_{i,2}, \cdots, r_{i,n_i-1}\}$, where $0 < r_{i,1} < r_{i,2} < \cdots < r_{i,n_i-1} \equiv r_{i,\max}$. The corresponding successful referral rates are also a set of n_i elements, $\{\alpha_{i,0}, \alpha_{i,1}, \alpha_{i,2}, \cdots, \alpha_{i,n_i-1}\}$, where $\alpha_{i,0} < \alpha_{i,1} < \alpha_{i,2} < \cdots < \alpha_{i,n_i-1} \equiv \alpha_{i,\max}$. Please see Figure 4-1 for an illustration. $\alpha_{i,0}$ can be seen as a autonomous successful referral rate. It is common for a firm to offer a reward in round numbers, for example, \$10.00, \$25.00, and \$50.00. In addition, customers may not be sensitive to a small change in a reward. For example, the successful rate could be the same for rewards between \$25.00 and \$49.00. Thus, the firms are better off offering a \$25.00 and a \$50.00 rewards, which result in different successful referral rates.

We will discuss more about the modeling of successful referral rate in Section 7.

Assumption 5 (Referrals refer). *A referred customer who buys the product has the same successful referral rate as the person who referred him.*

Next, we discuss other considerations in formulating the problem.

3.3 Other Considerations

There are some other considerations worth mentioning before we formulate our model.

1. *Customer's recommendation cost.* This cost is seen as the investment in recommendation and possible damage to a relationship because of the possible negative effect of a recommendable purchase. Generally, a customer's recommendation cost depends on referring to weak ties or strong ties and on referring weak brands or strong brands. A recommendation cost can be ignored when referring to strong brands and to strong ties because not much effort is needed, and the negative effect of purchase from recommendation is small. When referring to weak ties, especially online recommendation to the general public, it takes effort to build the referee's reputation and recommend the products. However, a recommendation cost can be treated as a reduction to referral rewards. Thus, rewards are seen as net rewards. Based on these considerations, we ignore the recommendation cost in this paper.

2. *Future purchase discount vs. cash rewards (free gifts).* Biyalogorsky et al. (2001) and Lobler and Welk (2004) consider cash rewards. Chen and Shi (2001) investigate both. We consider only cash rewards in this paper.

3. *Absolute rewards vs. percentage rewards.* In this paper, we model a reward both as an absolute value and a percentage of a retail price to see what are optimal pricing and reward decisions in each scenario. Percentage rewards can be seen as percentage discounts to prices. When percentage rewards are offered, customers will receive a high reward if price is high. Higher reward results in higher successful referral rate. Thus, we expect that customers are more likely to recommend products/services with higher prices as long as the net prices is lower.

4. *One time purchase vs. repeated purchase.* Chen and Shi (2001) consider repeated purchases, which are important to one of their referral reward types, future

purchase discount. We follow Biyalogorsky et al. (2001) and Lobler and Welk (2004) and only model one-time purchases.

5. *Reward program costs for firms.* It is true that setup costs for easy access programs might be higher than restricted access programs because of possible advertising expenditures. However, these setup costs are relatively low compared with the expected total rewards. It is reasonable to assume that fixed costs for a referral reward program are ignored. We only model the variable cost, r (the reward), for each actual purchase from recommendation.

Next, we introduce our demand formulation.

4 Demand Formulation

Pricing strategy and referral reward programs are seen as effective tools to attract demand in this paper. Thus, demand modeling is a critical part of the problem. Demand for each firm depends on its own pricing and reward decisions and the other's as well. We first model the initial demand and referrals for each firm under various cases when they choose different timing strategies, rewards, and prices. If we use a and b to represent when a firm makes customers aware of its reward, **after** and **before**, we have four possible strategy sets (cases): aa, ab, ba, and bb, where the first letter denotes firm 1's timing strategy, and the second letter denotes firm 2's timing strategy.

Next, we formulate demand for cases aa, ba,ba, and bb.

4.1 Case aa

Since initial customers do not know about the referral rewards when they make purchase decisions, we formulate the initial demand as $D_{i0} = k_i - m_i p_i + \beta_i(p_j - p_i)$, where $k_i > 0$, $m_i > 0$, $i = 1, 2$ and $j = 3 - i$.

The expected referred demand D_{ir} for firm i is the geometric series, $\sum_{l=1}^{\infty} \alpha_i^l D_{i0}$, where l represents time period, which equals

$$D_{ir} = \frac{\alpha_i}{1 - \alpha_i}[k_i - m_i p_i + \beta_i(p_j - p_i)].$$

4.2 Case *ab*

Here, customers, whom we implicitly assume to be risk-neutral, will anticipate that they receive $\alpha_2 r_2$ if they purchase from firm 2 and recommend to a friend. Thus, their expected payment is $\bar{p_2}$. Therefore, we model the initial demands as $D_{10} = k_1 - m_1 p_1 + \beta_1(\bar{p_2} - p_1)$ and $D_{20} = k_2 - m_2\bar{p_2} + \beta_2(p_1 - \bar{p_2})$.

Following the formulation of referrals in case *aa*, the expected referred demand D_{1r} and D_{2r} for the two firms are given by

$$\begin{aligned} D_{1r} &= \frac{\alpha_1}{1-\alpha_1}\{k_1 - m_1 p_1 + \beta_1(\bar{p_2} - p_1)\} \text{ and} \\ D_{2r} &= \frac{\alpha_2}{1-\alpha_2}\{k_2 - m_2\bar{p_2} + \beta_2(p_1 - \bar{p_2})\}. \end{aligned}$$

4.3 Case *ba*

Case *ba* is symmetric with case *ab*.

4.4 Case *bb*

In this case, customers will anticipate receiving $\alpha_i r_i$ if they purchase from firm i and recommend to a friend. Thus, we model the initial demand as $D_{i0} = k_i - m_i\bar{p_i} + \beta_i(\bar{p_j} - \bar{p_i})$. The expected referred demand D_{ri} for firm i is given by

$$D_{ir} = \frac{\alpha_i}{1-\alpha_i}\{k_i - m_i\bar{p_i} + \beta_i(\bar{p_j} - \bar{p_i})\}$$

Next, we analyze the problem in a three-stage simultaneous game.

5 Model Formulation and Analysis: Simultaneous games

According to the discussion in Section 3.1, we model our problem as a three-stage game. The decisions for each firm are when to make customers aware of the reward (**before** or **after**) in the first stage, level of reward in the second stage, and retail price in the third stage. The competition in each stage can be modeled as a simultaneous game or a sequential game. When the market power is relatively equal for the two firms, it is more appropriate to model the competition as a simultaneous game, i.e., two firms make decisions at the same time in each stage. A sequential game is formulated for each stage when one firm makes its decision first as a leader. In Section 6, we will offer further discussion on sequential decisions in each stage and extend the model to a three-stage sequential game. In this section, we model a three-stage simultaneous game under the following three scenarios.

- Scenario 1: *No referral rewards.* Each firm chooses its retail price to maximize its profit facing the competition from the other firm.
- Scenario 2: *Absolute rewards.* Each firm chooses its optimal mix of timing strategy, retail price, and an absolute reward to maximize its profit when facing the competition from the other firm.
- Scenario 3: *Percentage rewards.* Each firm chooses its optimal mix of timing strategy, retail price, and a percentage of price as a referral reward to maximize its profit when facing the competition from the other firm.

Scenario 1 is a special case of the other two scenarios when both firms choose to offer zero rewards. Since the solution in this scenario is a benchmark to measure the benefit of offering referral rewards, we discuss it as our first scenario.

We use backward induction to analyze the problems in scenario 2 and scenario 3. That is to say, first, we solve for optimal prices in the third stage for each case and for a certain pair of rewards offered by two firms. Then, optimal rewards in the second stage are chosen for each case. Finally, the two firms face a simultaneous game represented in Table 4-1 in the first stage. They can identify the possible pure Nash equilibrium according to the optimal profits (payoffs) in each case (strategy set). We use superscripts to distinguish the profits and decisions in the four cases. Please note that the optimal pricing and reward decisions which are chosen in the second and the third stages for the four strategy sets could be different. The optimal decisions have the denotations of the four cases as superscripts to show their differences. We also illustrate the optimal pricing and reward decisions in Table 4-1.

Now, we are ready to analyze the problem in the three scenarios.

Table 4-1: The normal form representation of a joint price and reward game

Firm 1 \ Firm 2	a(fter)	b(efore)
a(fter)	$(\pi_1^{aa*}, \pi_2^{aa*})(p_1^{aa*}, p_2^{aa*}, r_1^{aa*}, r_2^{aa*})$	$(\pi_1^{ab*}, \pi_2^{ab*})(p_1^{ab*}, p_2^{ab*}, r_1^{ab*}, r_2^{ab*})$
b(efore)	$(\pi_1^{ba*}, \pi_2^{ba*})(p_1^{ba*}, p_2^{ba*}, r_1^{ba*}, r_2^{ba*})$	$(\pi_1^{bb*}, \pi_2^{bb*})(p_1^{bb*}, p_2^{bb*}, r_1^{bb*}, r_2^{bb*})$

5.1 *N*o Referral Rewards

When both firms offer no referral rewards, they compete on retail prices to maximize their profits taking autonomous referrals into consideration. The problem is modeled as follows:

$$\pi_i^N(p_i) = [k_i - m_i p_i + \beta_i(p_j - p_i)]p_i + \frac{a_{i0}}{1-a_{i0}}[k_i - m_i p_i + \beta_i(p_j - p_i)]p_i, \qquad (1)$$

where a_{i0} is the successful rate of autonomous referrals. The superscript N denotes the no referral rewards scenario.

After rearranging the profit function as $\pi_i^N(p_i) = \frac{1}{(1-\alpha_{i0})}[k_i - (m_i+\beta_i)p_i + \beta_i p_j]p_i$, the optimal prices and profits can be obtained as follows:

$$\begin{aligned} p_i^{N*} &= \frac{2(m_j+\beta_j)k_i + \beta_i k_j}{4(m_i+\beta_i)(m_j+\beta_j) - \beta_i\beta_j} \text{ and} \\ \pi_i^{N*} &= \frac{m_i+\beta_i}{1-\alpha_{i0}}\left(\frac{2(m_j+\beta_j)k_i + \beta_i k_j}{4(m_i+\beta_i)(m_j+\beta_j) - \beta_i\beta_j}\right)^2. \end{aligned}$$

5.2 *A*bsolute Rewards

In this scenario, both firms offer absolute rewards. After we model and analyze the four cases in this scenario, we can find what the pure Nash equilibrium is in a three-stage simultaneous game with absolute rewards.

5.2.1 Case aa

The problem for each firm is given by

$$\pi_i^{Aaa}(p_i) = [k_i - m_i p_i + \beta_i(p_j - p_i)]p_i + \frac{\alpha_i}{1-\alpha_i}[k_i - m_i p_i + \beta_i(p_j - p_i)](p_i - r_i).$$

The first letter (A) of the superscript denotes this absolute reward scenario, and the other two letters (aa) denote that both firms inform customers after their purchases.

The objective functions can be rearranged as follows:

$$\pi_i^{Aaa}(p_i) = \frac{1}{(1-\alpha_i)}[k_i - (m_i + \beta_i)p_i + \beta_i p_j](p_i - \alpha_i r_i).$$

For any pair of (r_1, r_2) from the reward sets, we can solve the first order conditions of the problems and obtain $p_i^*(p_j) = [k_i + \beta_i p_j + (m_i + \beta_i)\alpha_i r_i]/[2(m_i + \beta_i)]$.

Thus, given any pair of (r_1, r_2), we have

$$\begin{aligned} p_i^{Aaa*} &= \frac{2(m_j+\beta_j)k_i + \beta_i k_j + \beta_i(m_j+\beta_j)\alpha_j r_j + 2(m_i+\beta_i)(m_j+\beta_j)\alpha_i r_i}{4(m_i+\beta_i)(m_j+\beta_j) - \beta_i\beta_j} \text{ and} \\ \pi_i^{Aaa*} &= \frac{m_i+\beta_i}{1-\alpha_i}\left(\frac{2(m_j+\beta_j)k_i + \beta_i k_j + \beta_i(m_j+\beta_j)\alpha_j r_j - [2(m_i+\beta_i)(m_j+\beta_j) - \beta_i\beta_j]\alpha_i r_i}{4(m_i+\beta_i)(m_j+\beta_j) - \beta_i\beta_j}\right)^2. \end{aligned}$$

Compared with the no referral reward scenario, the prices are higher to compensate for the cost of the rewards. The higher retail prices result in less initial customers for both firms, while the rewards attract more referrals. Examining π_i^{Aaa*}, we find that the second factor of it decreases with r_i, while the first factor increases with r_i. Thus, given r_j, firm i chooses a reward that optimizes its profit. A higher reward causes the price to be high, and then initial demand will be low. However, a lower reward reduces the referrals. Thus, we expect that the reward will not be too high or too low.

A pair of (r_1, r_2) that maximizes the two firms' profits at the same time will be chosen as the optimal rewards by examining Table 4-2. This part can only be done with specific demand functions and reward sets. It is possible that no pure-strategy equilibrium can be obtained for this case.

Table 4-2: Find the best rewards in each case

$r_1^* \backslash r_2^*$	$r_{2,0}$	$r_{2,1}$	$r_{2,2}$	$\cdots$	r_{2,n_2-1}
$r_{1,0}$	(π_1^*,π_2^*)	(π_1^*,π_2^*)	(π_1^*,π_2^*)	$\cdots$	(π_1^*, π_2^*)
$r_{1,1}$	(π_1^*, π_2^*)	(π_1^*, π_2^*)	(π_1^*,π_2^*)	$\cdots$	(π_1^*,π_2^*)
$r_{1,2}$	(π_1^*,π_2^*)	(π_1^*,π_2^*)	(π_1^*,π_2^*)	$\cdots$	(π_1^*,π_2^*)
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
r_{1,n_1-1}	(π_1^*,π_2^*)	(π_1^*,π_2^*)	(π_1^*,π_2^*)	$\cdots$	(π_1^*, π_2^*)

5.2.2 Case *ab*

The two objectives are formulated as follows:

$$\begin{aligned}
\pi_1^{Aab}(p_1) &= [k_1 - m_1 p_1 + \beta_1(\bar{p}_2 - p_1)]p_1 \\
&+ \frac{\alpha_1}{1-\alpha_1}[k_1 - m_1 p_1 + \beta_1(\bar{p}_2 - p_1)](p_1 - r_1) \text{ and} \\
\pi_2^{Aab}(p_2) &= [k_2 - m_2 \bar{p}_2 + \beta_2(p_1 - \bar{p}_2)]p_2 \\
&+ \frac{\alpha_2}{1-\alpha_2}[k_2 - m_2 \bar{p}_2 + \beta_2(p_1 - \bar{p}_2)](p_2 - r_2).
\end{aligned}$$

The objective functions can be rearranged as follows:

$$\begin{aligned}
\pi_1^{Aab}(p_1) &= \frac{1}{(1-\alpha_1)}[k_1 - m_1 p_1 + \beta_1(\bar{p}_2 - p_1)]\bar{p}_2 \text{ and} \\
\pi_2^{Aab}(\bar{p}_2) &= \frac{1}{(1-\alpha_2)}[k_2 - m_2 \bar{p}_2 + \beta_2(p_1 - \bar{p}_2)]\bar{p}_2.
\end{aligned}$$

For any pair of (r_1, r_2) from the reward sets, we solve the first order conditions of the problems and obtain

$$\begin{aligned}
p_1^{Aab*} &= \frac{2(m_2+\beta_2)k_1 + \beta_1 k_2 + 2(m_1+\beta_1)(m_2+\beta_2)\alpha_1 r_1}{4(m_1+\beta_1)(m_2+\beta_2) - \beta_1\beta_2} \text{ and} \\
\bar{p}_2^{Aab*} &= p_2^{Aab*} - \alpha_2 r_2 = \frac{2(m_1+\beta_1)k_2 + \beta_2 k_1 + \beta_2(m_1+\beta_1)\alpha_1 r_1}{4(m_1+\beta_1)(m_2+\beta_2) - \beta_1\beta_2}.
\end{aligned}$$

The optimal profits for the two firms are given by

$$\begin{aligned}\pi_1^{Aab*} &= \frac{m_1+\beta_1}{1-\alpha_1}\left(\frac{2(m_2+\beta_2)k_1+\beta_1 k_2-[2(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2]\alpha_1 r_1}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}\right)^2 \text{ and}\\ \pi_2^{Aab*} &= \frac{m_2+\beta_2}{1-\alpha_2}\left(\frac{2(m_1+\beta_1)k_2+\beta_2 k_1+\beta_2(m_1+\beta_1)\alpha_1 r_1}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}\right)^2.\end{aligned}$$

Since π_2^{Aab} increases with α_2, firm 2 always chooses $r_{2,\max}$ to reach its highest successful referral rate $\alpha_{2,\max}$. Because customers know about the referral reward program of firm 2 before their purchases, they will make purchase decisions based on firm 2's expected net price. Thus, firm 1 uses only its expected net price to compete with firm 1's retail price and reward. The optimal expected net price is not related to firm 2's reward as we can see from $\bar{p}_2^{Aab*}$. Therefore, firm 2's initial demand does not change with its own reward. But the referrals increase with its level of reward. Therefore, firm 2 would offer its highest referral reward, $r_{2,\max}$. An implicit assumption for this conclusion is that customers are risk-neutral, who make purchasing decisions based on expected net price. Thus, offering the highest possible reward in an easy access program is the most profitable way by attracting referrals without losing initial costumers.

Firm 1 does not follow the same strategy because a higher reward would result in a higher retail price, thus a lower initial demand. Thus, firm 1 would find a balance between lowering the initial demand and increasing the referrals. Because π_1^{Aab*} depends only on r_1, the best reward pair can be chosen by two firms in this case.

5.2.3 Case *ba*

Case *ba* is symmetric with case *ab*. Thus, the optimal prices and profits are

$$
\begin{aligned}
\bar{p}_1^{Aba*} &= p_1^{Aba*} - \alpha_1 r_1 = \frac{2(m_2+\beta_2)k_1 + \beta_1 k_2 + \beta_1(m_2+\beta_2)\alpha_2 r_2}{4(m_1+\beta_1)(m_2+\beta_2) - \beta_1\beta_2}, \\
p_2^{Aba*} &= \frac{2(m_1+\beta_1)k_2 + \beta_2 k_1 + 2(m_1+\beta_1)(m_2+\beta_2)\alpha_2 r_2}{4(m_1+\beta_1)(m_2+\beta_2) - \beta_1\beta_2}, \\
\pi_1^{Aba*} &= \frac{m_1+\beta_1}{1-\alpha_1}\left(\frac{2(m_2+\beta_2)k_1 + \beta_1 k_2 + \beta_1(m_2+\beta_2)\alpha_2 r_2}{4(m_1+\beta_1)(m_2+\beta_2) - \beta_1\beta_2}\right)^2, \text{ and} \\
\pi_2^{Aba*} &= \frac{m_2+\beta_2}{1-\alpha_2}\left(\frac{2(m_1+\beta_1)k_2 + \beta_2 k_1 - [2(m_2+\beta_2)(m_1+\beta_1) - \beta_1\beta_2]\alpha_2 r_2}{4(m_1+\beta_1)(m_2+\beta_2) - \beta_1\beta_2}\right)^2.
\end{aligned}
$$

In this case, firm 1 chooses its $r_{1,\max}$, while firm 2 finds a reward that maximizes its profit π_2^{Aba*}. Because π_2^{Aab*} depends only on r_2, the best reward pair can be chosen by two firms in this case.

5.2.4 Case *bb*

The objective of firm i is formulated as follows:

$$
\begin{aligned}
\pi_i^{Abb}(p_i) &= [k_i - m_i\bar{p}_i + \beta_i(\bar{p}_j - \bar{p}_i)]p_i \\
&+ \frac{\alpha_i}{1-\alpha_i}[k_i - m_i\bar{p}_i + \beta_i(\bar{p}_j - \bar{p}_i)](p_i - r_i).
\end{aligned}
$$

The objective function can be rearranged as the following:

$$
\pi_i^{Abb}(\bar{p}_i) = \frac{1}{(1-\alpha_i)}[k_i - m_i\bar{p}_i + \beta_i(\bar{p}_j - \bar{p}_i)]\bar{p}_i.
$$

For any pair of (r_1, r_2) from the reward sets, we solve the first order conditions of the problems and obtain the optimal prices and profits

$$
\begin{aligned}
\bar{p}_i^{Abb*} &= p_i^{Abb*} - \alpha_i r_i = \frac{2(m_j+\beta_j)k_i + \beta_i k_j}{4(m_i+\beta_i)(m_j+\beta_j) - \beta_i\beta_j} \text{ and} \\
\pi_i^{Abb*} &= \frac{m_i+\beta_i}{1-\alpha_i}\left(\frac{2(m_j+\beta_j)k_i + \beta_i k_j}{4(m_i+\beta_i)(m_j+\beta_j) - \beta_i\beta_j}\right)^2.
\end{aligned}
$$

We can see that the expected net prices are the same as in the no referral reward scenario. That is to say that both initial demand and the profit from each sale are the same as in no referral reward scenario. Since the referrals increase with the level of reward, firm i chooses $r_{i,\max}$ to have the maximum successful referral rate $\alpha_{i,\max}$ to get the maximum referrals.

What the pure Nash equilibrium could be in such a simultaneous game is discussed in the next section.

5.2.5 Finding the pure Nash equlibrium

We have discussed the reward decisions in the second stage and the pricing decisions in third stage for each case using backward induction. In case aa, we use r_i^{Aaa*} to denote the level of reward that maximizes the profit and α_i^{Aaa*} to denote the corresponding successful referral rate. For any reward that firm j chooses, firm i would choose a reward that maximizes its profit. We can see that the reward is not going to be too low because the low α caused by a low reward decreases referrals. It would not be too high because a higher reward will cause higher price which decreases the initial demand. The optimal reward depends on the demand functions and the referral functions of the two firms. There is no clear pattern in what the firms will choose for their reward programs. In addition, there is no guarantee that the two firms can find an optimal (r_1, r_2) that can maximize the two firms' profits at the same time.

In cases ab, ba, and bb, an optimal pair of rewards exists. The firm that offers the easy access referral reward program ("b") always chooses the highest reward in its reward set to get its maximum referral rate. The other firm will find a reward to maximize its own profit. So $r_2^{Aab*} = r_{2,\max}$ and $r_1^{Aba*} = r_{1,\max}$. Similarly, we have $r_2^{Abb*} = r_{2,\max}$ and $r_1^{Abb*} = r_{1,\max}$.

The optimal prices and payoffs for each strategy set are shown in Tables 4-3 and 4-4. We can examine Table 4-4 to find the pure Nash equilibrium for the three-stage simultaneous game.

From the formulation of the problems and the analysis, we can see that when one firm offers an easy access program, this firm actually competes with the other firm on its expected net retail price. Thus, its reward decision can be separated from its decision on the expected net retail price. Because referrals increase with rewards, it is in the firm's interest to offer its highest reward. For the firm that offers a restricted access program, it competes with the other firm on both price and reward. The decision on reward cannot be separated from the decision on price. Therefore, we have the following theorems:

Theorem 1. *When a firm offers an easily accessed referral reward program, it always chooses its highest possible reward in its reward set when competing in a duopoly simultaneous game.*

Proof of Theorem 1. When firm 1 offers an easy access program in cases ba and bb, its optimal profits are given by $\frac{m_1+\beta_1}{1-\alpha_1}\left(\frac{2(m_2+\beta_2)k_1+\beta_1 k_2+\beta_1(m_2+\beta_2)\alpha_2^{Aba*}r_2^{Aba*}}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}\right)^2$ and $\frac{m_1+\beta_1}{1-\alpha_1}\left(\frac{2(m_2+\beta_2)k_1+\beta_1 k_2}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}\right)^2$, respectively. Since the second factor for the profits does not depend on firm 1's reward, the best reward for firm 1 when it offers an easy access program is $r_{1,\max}$, which results in $\alpha_{1,\max}$. Similarly, the best reward for firm 2 when it offers an easy access program in cases ab and bb is $r_{2,\max}$, which results in $\alpha_{2,\max}$. □

Theorem 2. *If a pure Nash equilibrium of a simultaneous game where two firms compete on joint price and absolute reward exists, the two firms will make the same timing decision (easy access or restricted access.)*

Proof of Theorem 2. We have discussed that existence of the best rewards is guaranteed in cases ab, ba, and bb. Let us assume that the best rewards can also be obtained from the reward sets in case aa. By examining the profits in Table 4-4, we can see that bb dominates ab and ba because $\pi_2^{Abb} \geq \pi_2^{Aba}$ and $\pi_1^{Abb} \geq \pi_1^{Aab}$. However, depending on the demand functions and the reward sets, bb and/or aa can be the pure Nash equilibrium(a) because not one of the firms is always better off to choose a or b. □

Table 4-3: The optimal prices of the simultaneous game

	a	b
a	$\frac{K_1+2(m_1+\beta_1)(m_2+\beta_2)\alpha_1^{Aaa*}r_1^{Aaa*}+\beta_1(m_2+\beta_2)\alpha_2^{Naa*}r_2^{Aaa*}}{M_1}$, $\frac{K_2+2(m_1+\beta_1)(m_2+\beta_2)\alpha_2^{Aaa*}r_2^{Aaa*}+\beta_2(m_1+\beta_1)\alpha_1^{Aaa*}r_1^{Aaa*}}{M_1}$	$\frac{K_1+2(m_1+\beta_1)(m_2+\beta_2)\alpha_1^{Aab*}r_1^{Aab*}}{M_1}$, $\frac{K_2+\beta_2(m_1+\beta_1)\alpha_1^{Aab*}r_1^{Aab*}}{M_1}+\alpha_{2,\max}r_{2,\max}$
b	$\frac{K_1+\beta_1(m_2+\beta_2)\alpha_2^{Aba*}r_2^{Aba*}}{M_1}+\alpha_{1,\max}r_{1,\max}$, $\frac{K_2+2(m_1+\beta_1)(m_2+\beta_2)\alpha_2^{Aba*}r_2^{Aba*}}{M_1}$	$\frac{K_1}{M_1}+\alpha_{1,\max}r_{1,\max}$ $\frac{K_2}{M_1}+\alpha_{2,\max}r_{2,\max}$

Table 4-4: The normal form of the simultaneous game with absolute rewards

	a	b
a	$\frac{m_1+\beta_1}{1-\alpha_1^{Aaa*}}\left(\frac{K_1+\beta_1(m_2+\beta_2)\alpha_2^{Aaa*}r_2^{Aaa*}-0.5M_2\alpha_1^{Aa*}r_1^{Aaa*}}{M_1}\right)^2$, $\frac{m_2+\beta_2}{1-\alpha_2^{Aaa*}}\left(\frac{K_2+\beta_2(m_1+\beta_1)\alpha_1^{Aaa*}r_1^{Aaa*}-0.5M_2\alpha_2^{Aaa*}r_2^{Aaa*}}{M_1}\right)^2$	$\frac{m_1+\beta_1}{1-\alpha_1^{Aab*}}\left(\frac{K_1-0.5M_2\alpha_1^{Aab*}r_1^{Aab*}}{M_1}\right)^2$ $\frac{m_2+\beta_2}{1-\alpha_{2,\max}}\left(\frac{K_2+\beta_2(m_1+\beta_1)\alpha_1^{Aab*}r_1^{Aab*}}{M_1}\right)^2$
b	$\frac{m_1+\beta_1}{1-\alpha_{1,\max}}\left(\frac{K_1+\beta_1(m_2+\beta_2)\alpha_2^{Aba*}r_2^{Aba*}}{M_1}\right)^2$, $\frac{m_2+\beta_2}{1-\alpha_2^{Aba*}}\left(\frac{K_2-0.5M_2\alpha_2^{Aba*}r_2^{Aba*}}{M_1}\right)^2$	$\frac{m_1+\beta_1}{1-\alpha_{1,\max}}\left(\frac{K_1}{M_1}\right)^2$ $\frac{m_2+\beta_2}{1-\alpha_{2,\max}}\left(\frac{K_2}{M_1}\right)^2$

$M_1 = 4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2$; $M_2 = 4(m_1+\beta_1)(m_2+\beta_2)-2\beta_1\beta_2$ $K_1 = 2(m_2+\beta_2)k_1+\beta_1 k_2$; $K_2 = 2(m_1+\beta_1)k_2+\beta_2 k_1$

When the two firms are symmetric, that is to say that they have a same demand function, a same reward set, and a same successful referral rate set, bb dominates aa.

Theorem 3. *For two firms with a same demand function, a same reward set, and a same successful referral rate set, both firms offer easy access programs in the pure Nash equilibrium.*

Proof of Theorem 3. With a same demand function, a same reward set, and a same successful referral rate set, suppose r^{Aaa*} and corresponding α^{Aaa*} has been chosen in case aa. Then the profit for each firm is

$$\pi^{Aaa*} = \frac{m+\beta}{1-\alpha^{Aaa*}} \left(\frac{2(m+\beta)k + \beta k - [(m+\beta)(2m+\beta) - \beta^2]\alpha^{Aaa*} r^{Aaa*}}{4(m_1+\beta_1)(m_2+\beta_2) - \beta_1\beta_2} \right)^2,$$

which is no more than

$$\pi^{Abb*} = \frac{m+\beta}{1-\alpha_{\max}} \left(\frac{2(m+\beta)k + \beta k}{4(m_1+\beta_1)(m_2+\beta_2) - \beta_1\beta_2} \right)^2.$$

□

Next, we analyze this simultaneous game when percentage rewards are offered.

5.3 *P*ercentage Rewards

In the following analysis, we assume that the reward $r_i = \gamma_i p_i$, where $0 \le \gamma_i \le 1$. So the decisions are the price p_i, the percentage of the price γ_i, and the timing strategy for firm i. Since we assume that only a finite number of rewards are defined, firm i choose γ_i from a finite set.

5.3.1 Case aa

The objective function can be rearranged as follows:

$$\pi_i^{Paa}(p_i) = \frac{1-\alpha_i\gamma_i}{1-\alpha_i}[k_i - (m_i+\beta_i)p_i + \beta_i p_j]p_i,$$

where the superscript letter P denotes the percentage reward scenario. For any given pair of (γ_1, γ_2) from the percentage reward sets, we solve the first order condition of the problems and obtain

$$\begin{aligned}
p_i^{Paa*} &= \frac{2(m_j+\beta_j)k_i+\beta_i k_j}{4(m_i+\beta_i)(m_j+\beta_j)-\beta_i\beta_j}, \text{ and} \\
\pi_i^{Paa*} &= \frac{(m_i+\beta_i)(1-\alpha_i\gamma_i)}{1-\alpha_i}\left(\frac{2(m_j+\beta_j)k_i+\beta_i k_j}{4(m_i+\beta_i)(m_j+\beta_j)-\beta_i\beta_j}\right)^2.
\end{aligned}$$

It is worth noticing that the optimal γ_i depends only on firm i's own percentage reward set, which is different from the absolute rewards scenario. This is because for any firm, the profit from the referrals is a multiplication of the profit from the initial demand when the reward is a percentage of price. So the total profit is the profit without rewards multiplied by $\frac{1-\alpha_i\gamma_i}{1-\alpha_i}$, which depends only on the reward of firm i. Thus, the two firms essentially compete only on prices. While in the absolute reward scenario, the two firms compete on both prices and rewards.

Therefore, in this case, each firm can separately decide its own reward. We can also see this from the profit function π_i^{Paa*}. Firm i will choose the γ_i that maximizes $\frac{1-\alpha_i\gamma_i}{1-\alpha_i}$. The pure Nash equilibrium exists for this case because each firm can chooses its decisions separately.

5.3.2 Case *ab*

The objective functions can be rearranged as follows:

$$\begin{aligned}
\pi_1^{Pab}(p_1) &= \frac{1-\alpha_1\gamma_1}{(1-\alpha_1)}\{k_1-(m_1+\beta_1)p_1+\beta_1\bar{p}_2\}p_1 \text{ and} \\
\pi_2^{Pab}(p_2) &= \frac{1}{(1-\alpha_2)}\{k_2-(m_2+\beta_2)\bar{p}_2+\beta_2 p_1\}\bar{p}_2.
\end{aligned}$$

For any given pair of (γ_1, γ_2), we can solve the problem and obtain

$$\begin{aligned}
p_1^{Pab*} &= \frac{2(m_2+\beta_2)k_1+\beta_1 k_2}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}, \\
\bar{p}_2^{Pab*} &= (1-\alpha_2\gamma_2)p_2^{Pab*} = \frac{2(m_1+\beta_1)k_2+\beta_2 k_1}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}, \\
\pi_1^{Pab*} &= \frac{(m_1+\beta_1)(1-\alpha_1\gamma_1)}{1-\alpha_1}\left(\frac{2(m_2+\beta_2)k_1+\beta_1 k_2}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}\right)^2, \text{ and} \\
\pi_2^{Pab*} &= \frac{(m_2+\beta_2)}{1-\alpha_2}\left(\frac{2(m_1+\beta_1)k_2+\beta_2 k_1}{[4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2]}\right)^2.
\end{aligned}$$

The expected net prices are the same as when no referral reward programs are offered. Firm 2 will choose $\gamma_{2,\max}$ and have $\alpha_{2,\max}$. Firm 1 will choose the γ_1 that maximizes $\frac{1-\alpha_1\gamma_1}{1-\alpha_1}$.

5.3.3 Case *ba*

This case is symmetric with case *ab*. Thus,

$$\begin{aligned}
\bar{p}_1^{Pba*} &= (1-\alpha_1\gamma_1)p_1^{Pba*} = \frac{2(m_2+\beta_2)k_1+\beta_1 k_2}{[4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2](1-\alpha_2\gamma_2)}, \\
p_2^{Pba*} &= \frac{2(m_1+\beta_1)k_2+\beta_2 k_1}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}, \\
\pi_1^{Pba*} &= \frac{m_1+\beta_1}{1-\alpha_1}\left(\frac{2(m_2+\beta_2)k_1+\beta_1 k_2}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}\right)^2, \text{ and} \\
\pi_2^{Pba*} &= \frac{(m_2+\beta_2)(1-\alpha_2\gamma_2)}{1-\alpha_2}\left(\frac{2(m_1+\beta_1)k_2+\beta_2 k_1}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}\right)^2.
\end{aligned}$$

Firm 1 will choose $\gamma_{1,\max}$ and have $\alpha_{1,\max}$. Firm 2 will choose the γ_2 that maximizes $\frac{1-\alpha_2\gamma_2}{1-\alpha_2}$.

5.3.4 Case *bb*

The objective function can be rearranged as follows:

$$\pi_i^{Pbb}(\bar{p}_i) = \frac{1}{(1-\alpha_i)}[k_i - m_i\bar{p}_i + \beta_i(\bar{p}_j - \bar{p}_i)]\bar{p}_i.$$

For any pair of (γ_1, γ_2), we solve the first order conditions of the problems and obtain

$$\begin{aligned} \bar{p}_i^{Pbb*} &= (1-\alpha_i\gamma_i)p_i^{Pba*} = \frac{2(m_j+\beta_j)k_i+\beta_i k_j}{4(m_i+\beta_i)(m_j+\beta_j)-\beta_i\beta_j} \text{ and} \\ \pi_i^{Pbb*} &= \frac{m_i+\beta_i}{1-\alpha_i}\left(\frac{2(m_j+\beta_j)k_i+\beta_i k_j}{4(m_i+\beta_i)(m_j+\beta_j)-\beta_i\beta_j}\right)^2. \end{aligned}$$

Firm i will choose $\gamma_{i,\max}$ and have $\alpha_{i,\max}$. The expected net prices are the same as the prices when no referral reward programs are offered. The optimal retail price charged by firm i is $\frac{p_i^{N*}}{1-\alpha_{i,\max}\gamma_{i,\max}}$. Thus, when $\gamma_{i,\max}$ is high, the price will be high enough to obtain the same expected net price.

5.3.5 Finding the pure Nash equilibrium

Before we identify the pure Nash equilibrium for the simultaneous game with percentage rewards, we want to mention that the optimal expected net prices in all the cases are the same as the prices when referral rewards are not offered. The net prices in the simultaneous game with absolute rewards are not. The reason is that the profit from the referrals is a multiplication of the profit from the initial demand for any firm when percentage rewards are offered. Thus, the total profit for firm i is its initial profit multiplied by $\frac{1-\alpha_i\gamma_i}{1-\alpha_i}$, which depends only on the reward of firm i. Essentially, the two firms compete only on the expected net prices when percentage rewards are offered.

We have discussed the process to find the optimal prices and the optimal percentages of reward for each case. In case aa, the optimal reward plan for firm i is to choose γ_i^{a*} that maximizes $\frac{1-\alpha_i\gamma_i}{1-\alpha_i}$. In case ab, firm 2 chooses the largest possible percentage $\gamma_{2,\max}$ to get $\alpha_{2,\max}$ to maximize the profit while firm 1 chooses γ_1^{a*} that maximizes $\frac{1-\alpha_1\gamma_1}{1-\alpha_1}$. In case ba, firm 1 chooses the largest possible percentage $\gamma_{1,\max}$ to get $\alpha_{1,\max}$ to maximize the profit while firm 2 chooses γ_2^{a*} that maximizes $\frac{1-\alpha_2\gamma_2}{1-\alpha_2}$. In case bb, both firms choose $\gamma_{i,\max}$ to get $\alpha_{i,\max}$. The optimal payoffs are shown in Table 4-5. Then, we can compare the optimal profits for the four cases in the table to find the pure Nash equilibrium for the simultaneous three-stage game.

Table 4-5: The payoffs in the simultaneous game with percentage rewards

	a	b
a	$\frac{(m_1+\beta_1)(1-\alpha_1^{a*}\gamma_1^{a*})}{1-\alpha_1^{a*}}\left(\frac{2(m_2+\beta_2)k_1+\beta_1k_2}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}\right)^2$, $\frac{(m_2+\beta_2)(1-\alpha_2^{a*}\gamma_2^{a*})}{1-\alpha_2^{a*}}\left(\frac{2(m_1+\beta_1)k_2+\beta_2k_1}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}\right)^2$	$\frac{(m_1+\beta_1)(1-\alpha_1^{a*}\gamma_1^{a*})}{1-\alpha_1^{a*}}\left(\frac{2(m_2+\beta_2)k_1+\beta_1k_2}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}\right)^2$ $\frac{(m_2+\beta_2)}{1-\alpha_2^{max}}\left(\frac{2(m_1+\beta_1)k_2+\beta_2k_1}{[4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2]}\right)^2$
b	$\frac{m_1+\beta_1}{1-\alpha_1^{max}}\left(\frac{2(m_2+\beta_2)k_1+\beta_1k_2}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}\right)^2$, $\frac{(m_2+\beta_2)(1-\alpha_2^{a*}\gamma_2^{a*})}{1-\alpha_2^{a*}}\left(\frac{2(m_1+\beta_1)k_2+\beta_2k_1}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}\right)^2$	$\frac{m_1+\beta_1}{1-\alpha_1^{max}}\left(\frac{2(m_2+\beta_2)k_1+\beta_1k_2}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}\right)^2$ $\frac{m_2+\beta_2}{1-\alpha_2^{max}}\left(\frac{2(m_1+\beta_1)k_2+\beta_2k_1}{4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2}\right)^2$

Theorem 4. *In a simultaneous game with two firms offering easily accessed percentage referral rewards, the pure Nash equilibrium is that both firms offer their highest possible rewards and set their expected net retail prices at* $\frac{2(m_j+\beta_j)k_i+\beta_ik_j2}{4(m_i+\beta_i)(m_j+\beta_j)-\beta_i\beta_j}$ *when the initial demands are given by* $D_{i0} = k_i - m_i\bar{p}_i + \beta_i(\bar{p}_j - \bar{p}_i)$.

Proof of Theorem 4. In Table 4-5, strategy set bb dominates the other three sets because $\alpha_i^{a*} \leq \alpha_{i,\max}$ and $\frac{1-\alpha_1^{a*}\gamma_i^{a*}}{1-\alpha_1^{a*}} \leq \frac{1}{1-\alpha_{i,\max}}$. Thus, both firms choose to offer the best possible $\gamma_{i,\max}$ to have $\alpha_{i,\max}$. The expected net prices can be obtained from the analysis of case bb. □

In addition to analytically prove that case bb is the best strategy set, we can also intuitively explain it as follows: Since the net expected prices the same in the four cases, the initial demand is the same for all the cases. The number of referrals in case bb is higher than in any other case because firms choose to offer their highest rewards. Plus, the unit profit that firms can achieve from any initial customer is higher in case bb, while the unit profit from any referrals are the same for all cases. Thus, in total, firms are most profitable in case bb.

5.4 Discussion of the Optimal Decisions

We have discussed the optimal decisions in three scenarios: no referral rewards, absolute referral rewards, and percentage referral rewards. We found that the pure Nash equilibrium in the absolute referral reward scenario could be to choose either easy access or restricted access programs for two firms at the same time, while the pure Nash equilibrium is definitely to choose easy access programs in the percentage referral reward scenario.

When two firms offer percentage rewards, the two firms essentially compete on the prices no matter whether easy access programs (bb) or restricted access ones (aa) are provided. Competing only on the prices is caused by the effects that percentage rewards have on the demand and on the profit. One firm can make its own reward decision separately from the other firm's reward decisions. And it is in the firms' interest to offer easy access programs. This argument is also true when two firms offer easily accessed absolute reward programs. However, when two firms offer absolute reward programs with restrictions, they compete not only on prices but also on rewards. One firm's reward and price decisions interact with the other firm's reward and price decisions. Whether easy access programs are better than restricted access ones depend on the demand functions, the levels of the rewards offered, and the successful referral rates.

If the pure Nash equilibrium in the absolute referral reward scenario is to offer easy access programs, the firms' payoffs are no less than no referral rewards because the no referral reward scenario is a special case of the absolute referral reward scenario when rewards are zero. However, we have to realize that we have this result based on the assumption that the fixed costs of offering reward programs are negligible.

Offering easy access programs (bb) is one of the possible pure Nash equilibria in the absolute referral reward scenario and is the only pure Nash equilibrium in the percentage referral reward scenario. The expected net prices are the same in case bb in these two scenarios, which are also the same with the prices when no referral rewards are offered. Thus, when easy access programs are offered, initial demand is the same in all scenarios. The number of referrals is larger because $\alpha_{i,\max}$, the maximum successful referral rate, is larger than the autonomous successful referral rate. Furthermore, the unit profit that firms can achieve from any initial customer is higher because the retail price is higher, while the unit profit from referrals are the same for all cases. In total, firms are more profitable when referral programs are offered.

6 Analysis of Sequential Game

From the previous discussion, offering an easily accessed referral program is a dominant strategy when percentage referral rewards are offered. And it is also one of the best strategies when absolute referral rewards are offered. Thus, we extend our model and analysis to a sequential game setting in this section assuming that the timing strategies have been chosen in the first stage of the three-stage game that we have discussed. We solve only reward decisions in the second stage and pricing decisions in the third stage. In each stage, we assume that firm 1 is the leader, and firm 2 is the follower without loss of generality.

In order to distinguish this sequential game from a simultaneous game, we use superscripts ABB and PBB in our models in this section, where A and P represent the types of the rewards and BB represents the strategy set.

6.1 No Referral Rewards

The objective function is the same as in the simultaneous game. We solve firm 2's problem first and obtain $p_2 = [k_2 + \beta_2 p_1]/[2(m_2 + \beta_2)]$. Substituting p_2 into firm 1's problem and solving it, we have

$$
\begin{aligned}
p_1^{N*} &= \frac{2(m_2+\beta_2)k_1 + \beta_1 k_2}{4(m_1+\beta_1)(m_2+\beta_2) - 2\beta_1\beta_2}, \\
p_2^{N*} &= \frac{2(m_1+\beta_1)k_2 + \beta_2 k_1}{4(m_1+\beta_1)(m_2+\beta_2) - 2\beta_1\beta_2} - \frac{\beta_1\beta_2 k_2}{[4(m_1+\beta_1)(m_2+\beta_2) - 2\beta_1\beta_2][2(m_2+\beta_2)]}, \\
\pi_1^{N*} &= \frac{2(m_1+\beta_1)(m_2+\beta_2) - \beta_1\beta_2}{2(1-a_{10})(m_2+\beta_2)}(p_1^{N*})^2, \text{ and} \\
\pi_2^{N*} &= \frac{m_2+\beta_2}{1-a_{20}}(p_2^{N*})^2.
\end{aligned}
$$

Subscript N is used to represent the no referral rewards scenario as in the analysis of the simultaneous game.

6.2 *A*bsolute Rewards

The objective function is the same as in the simultaneous game and it can be formulated as the following:

$$\pi_i^{ABB}(\bar{p}_i, r_i) = \frac{1}{(1-\alpha_i)}[k_i - m_i\bar{p}_i + \beta_i(\bar{p}_j - \bar{p}_i)]\bar{p}_i.$$

Solving firm 2's problem and substituting it into firm 1's problem, we obtain the follows for any pair of (r_1, r_2):

$$\begin{aligned}
\bar{p}_1^{ABB*} &= p_1^{ABB*} - \alpha_1 r_1 = \frac{2(m_2+\beta_2)k_1 + \beta_1 k_2}{4(m_1+\beta_1)(m_2+\beta_2) - 2\beta_1\beta_2}, \\
\bar{p}_2^{ABB*} &= p_2^{ABB*} - \alpha_2 r_2 = \frac{2(m_1+\beta_1)k_2 + \beta_2 k_1}{4(m_1+\beta_1)(m_2+\beta_2) - 2\beta_1\beta_2} \\
&\quad - \frac{\beta_1\beta_2 k_2}{[4(m_1+\beta_1)(m_2+\beta_2) - 2\beta_1\beta_2][2(m_2+\beta_2)]}, \\
\pi_1^{ABB*} &= \frac{2(m_1+\beta_1)(m_2+\beta_2) - \beta_1\beta_2}{2(1-\alpha_2)(m_2+\beta_2)}\left(\frac{2(m_2+\beta_2)k_1 + \beta_1 k_2}{4(m_1+\beta_1)(m_2+\beta_2) - 2\beta_1\beta_2}\right)^2, \text{ and} \\
\pi_2^{ABB*} &= \frac{m_2+\beta_2}{1-\alpha_1}\left(\frac{2(m_1+\beta_1)k_2 + \beta_2 k_1}{4(m_1+\beta_1)(m_2+\beta_2) - 2\beta_1\beta_2} - \frac{\beta_1\beta_2 k_2}{[4(m_1+\beta_1)(m_2+\beta_2) - 2\beta_1\beta_2][2(m_2+\beta_2)]}\right)^2.
\end{aligned}$$

In order to maximize the profits, firm i chooses $r_{i,\max}$ to achieve $\alpha_{i,\max}$.

6.3 *P*ercentage Rewards

The objective function is the same as in the simultaneous game, and it can be formulated as the following:

$$\pi_i^{Pbb}(\bar{p}_i, \gamma_i) = \frac{1}{(1-\alpha_i)}[k_i - m_i\bar{p}_i + \beta_i(\bar{p}_j - \bar{p}_i)]\bar{p}_i.$$

For any pair of (γ_1, γ_2), we obtain the following solutions:

$$\begin{aligned}
\bar{p}_1^{ABB*} &= (1-\alpha_1\gamma_1)p_1^{PBB*} = \frac{2(m_2+\beta_2)k_1 + \beta_1 k_2}{4(m_1+\beta_1)(m_2+\beta_2) - 2\beta_1\beta_2}, \\
\bar{p}_2^{ABB*} &= (1-\alpha_2\gamma_2)p_2^{PBB*} = \frac{2(m_1+\beta_1)k_2 + \beta_2 k_1}{4(m_1+\beta_1)(m_2+\beta_2) - 2\beta_1\beta_2} \\
&\quad - \frac{\beta_1\beta_2 k_2}{[4(m_1+\beta_1)(m_2+\beta_2) - 2\beta_1\beta_2][2(m_2+\beta_2)]}, \\
\pi_1^{PBB*} &= \frac{2(m_1+\beta_1)(m_2+\beta_2) - \beta_1\beta_2}{2(1-\alpha_2)(m_2+\beta_2)}\left(\frac{2(m_2+\beta_2)k_1 + \beta_1 k_2}{4(m_1+\beta_1)(m_2+\beta_2) - 2\beta_1\beta_2}\right)^2, \text{ and} \\
\pi_2^{PBB*} &= \frac{m_2+\beta_2}{1-\alpha_1}\left(\frac{2(m_1+\beta_1)k_2 + \beta_2 k_1}{4(m_1+\beta_1)(m_2+\beta_2) - 2\beta_1\beta_2} - \frac{\beta_1\beta_2 k_2}{[4(m_1+\beta_1)(m_2+\beta_2) - 2\beta_1\beta_2][2(m_2+\beta_2)]}\right)^2.
\end{aligned}$$

In order to maximize the profits, firm i chooses $\gamma_{i,\max}$ to achieve $\alpha_{i,\max}$.

6.4 Discussion of the Optimal Decisions

We have considered case *bb* for three scenarios in a sequential game. The net prices $\bar{p}_1$ and $\bar{p}_2$ in the absolute reward scenario and in the percentage reward scenario are the same as the prices in the no referral scenario. The profits in these two scenarios are higher than in the no referral reward scenario. The optimal prices and profits and their comparison with those in the simultaneous game are shown in Table 4-6. The firms will choose to offer their highest referral rewards. The reason is the same with what we have discussed for case *bb* in the simultaneous game: The firms essentially compete on prices rather than on prices and rewards.

In addition, both retail prices in the sequential game are higher than the prices in the simultaneous game. Both firms are better off in a sequential game than in a simultaneous game. Thus, if two firms choose to compete in a sequential game or in a simultaneous game, they prefer the sequential game. However, the customer welfare is worse in a sequential game than in a simultaneous game because the retail prices are higher.

In Section 1, we have asked the question: If the competitor of a firm has a referral reward program, what is the best strategy regarding pricing and referral reward programs for this firm? We have also mentioned Amazon's Associate Program. Thus, as a competitor of Amazon, should Chapters offer a referral reward program? In the light of our analysis, Chapters should offer an easy access program if most of the referrers of Chapters and Amazon buy the product. In our model, a customers has to buy to refer. In Amazon's Associate Program, an associate is not necessary to buy. Thus, our model apply to competition between Amazon and Chapters when most of the referrers buy the product. Chapters does not currently offer a referral reward program. It is possible that it did not make a correct decision or that it considered factors that are not in our model, such as initial investments, management of referral reward programs, and technical support.

Table 4-6: The comparison of prices and profits for simultaneous and sequential games (case *bb*)

Scenarios	Decisions	Sequential Game	Simultaneous Game	Seq-simu
No rewards	p_1	$\frac{K_1}{M_2}$	$\frac{K_1}{M_1}$	>0
	p_2	$\frac{K_2}{M_2} - \frac{\beta_1\beta_2 k_2}{2M_2(m_2+\beta_2)}$	$\frac{K_2}{M_1}$	>0
	π_1	$\frac{0.5M_2}{2(1-\alpha_{01})(m_2+\beta_2)}\left(\frac{K_1}{M_2}\right)^2$	$\frac{m_1+\beta_1}{1-\alpha_{01}}\left(\frac{K_1}{M_1}\right)^2$	>0
	π_2	$\frac{m_2+\beta_2}{1-\alpha_{02}}\left(\frac{K_2}{M_2} - \frac{\beta_1\beta_2 k_2}{2M_2(m_2+\beta_2)}\right)^2$	$\frac{m_2+\beta_2}{1-\alpha_{02}}\left(\frac{K_2}{M_1}\right)^2$	>0
Absolute rewards	p_1	$\frac{K_1}{M_2} + \alpha_{1,\max} r_{1,\max}$	$\frac{K_1}{M_1} + \alpha_{1,\max} r_{1,\max}$	>0
	p_2	$\frac{K_2}{M_2} - \frac{\beta_1\beta_2 k_2}{2M_2(m_2+\beta_2)} + \alpha_{2,\max} r_{2,\max}$	$\frac{K_2}{M_1} + \alpha_{2,\max} r_{2,\max}$	>0
	π_1	$\frac{0.5M_2}{2(1-\alpha_{1,\max})(m_2+\beta_2)}\left(\frac{K_1}{M_2}\right)^2$	$\frac{m_1+\beta_1}{1-\alpha_{1,\max}}\left(\frac{K_1}{M_1}\right)^2$	>0
	π_2	$\frac{m_2+\beta_2}{1-\alpha_{2,\max}}\left(\frac{K_2}{M_2} - \frac{\beta_1\beta_2 k_2}{2M_2(m_2+\beta_2)}\right)^2$	$\frac{m_2+\beta_2}{1-\alpha_{2,\max}}\left(\frac{K_2}{M_1}\right)^2$	>0
Percentage rewards	p_1	$\frac{K_1}{M_2(1-\alpha_{1,\max}\gamma_{1,\max})}$	$\frac{K_1}{M_1(1-\alpha_{1,\max}\gamma_{1,\max})}$	>0
	p_2	$\left[\frac{K_2}{M_2} - \frac{\beta_1\beta_2 k_2}{2M_2(m_2+\beta_2)}\right]\frac{1}{1-\alpha_{2,\max}\gamma_{2,\max}}$	$\frac{K_2}{M_1(1-\alpha_{2,\max}\gamma_{2,\max})}$	>0
	π_1	$\frac{0.5M_2}{2(1-\alpha_{1,\max})(m_2+\beta_2)}\left(\frac{K_1}{M_2}\right)^2$	$\frac{m_1+\beta_1}{1-\alpha_{1,\max}}\left(\frac{K_1}{M_1}\right)^2$	>0
	π_2	$\frac{m_2+\beta_2}{1-\alpha_{2,\max}}\left(\frac{K_2}{M_2} - \frac{\beta_1\beta_2 k_2}{2M_2(m_2+\beta_2)}\right)^2$	$\frac{m_2+\beta_2}{1-\alpha_{2,\max}}\left(\frac{K_2}{M_1}\right)^2$	>0

$M_1 = 4(m_1+\beta_1)(m_2+\beta_2)-\beta_1\beta_2$; $M_2 = 4(m_1+\beta_1)(m_2+\beta_2)-2\beta_1\beta_2$; $K_1 = 2(m_2+\beta_2)k_1+\beta_1 k_2$; $K_2 = 2(m_1+\beta_1)k_2+\beta_2 k_1$

7 Concluding Remarks

In this paper, we examined the management of pricing and referral reward programs for two competing firms selling substitutable products or services to a same customer pool. Two types of referral rewards, absolute rewards and percentage rewards, are investigated. In addition to making decisions on prices and levels of referral rewards, the firms also decide their reward timing strategy: Making the customers aware of the program before their purchasing decisions by offering an easy access program or making them aware of the program after their purchases by offering a restricted access program. We formulate three-stage games with time strategies of the reward programs chosen in the first stage, levels of rewards chosen in the second stage, and pricing decisions chosen in the third stage. We first analyze the interaction between the two firms as a simultaneous three-stage game where they make decisions in each stage simultaneously. Then, we extend it to a sequential game setting where the firms make decisions in each stage sequentially.

We find that both firms benefit from offering referral rewards in simultaneous and sequential games. In a simultaneous game with absolute rewards, both firms should choose the same program type (easy access or restricted access.) However, which strategy is better depends on the sizes of the firms' potential markets, customer preferences over the two firms (these two factors are represented by the demand functions), and customers' responsiveness to the referral rewards (represented by the successful referral rates). When percentage rewards are offered, it is in both firms' interest to offer easy access programs. When the firms offer easy access programs, they should always offer their highest possible rewards, no matter whether they provide absolute rewards or percentage rewards and no matter whether they compete in simultaneous games or sequential games.

An extreme example of a reward is to offer the retail price. We believe this rarely happens. Generally speaking, consumers are not sensitive to a reward when it reaches a certain level compared to the price. Thus, firms should be able to get their maximum successful rates at lower rewards. Even if the extreme case happens, because not every consumer can successful recommend a friend ($\alpha_i < 1$), firms still can make profits.

We acknowledge that our analysis is based on the assumption that the successful referral rate is the same for all customers who recommend. We have discussed three possible steps in the referral process: To decide to refer, to find a friend to refer, and the friend's decision to buy the product. The successful referral rate in this paper is the joint probability of the event that the referrer decides to refer, the event that the referrer can find a friend to refer, and the event that the friend buys. Any difference in the probability of any of the steps could result in different successful referral rates for different customers. Biyalogorsky et al. (2001) investigate the probabilities of the first step and the third step of the process. They assume that the probability of recommendation depends on whether the customer is delighted. If the customer is delighted, the probability of the friend's decision to buy is α_i. So the successful referral rate for each customer is different. We expect that examining each step separately would provide more insights into our model and analysis, and we suggest this as a future research topic.

We also assume that a successful referral rate is a discrete function of reward for the interest of tractability. It is possible to release this assumption to consider continues functions as an extension to this paper. A successful referral rate can also depend on retail price. When retail price is low, customers might not consider that referring is worth of the efforts involved. When retail price is higher, customers might not anticipate that the many recommendations will turn into sales. Thus, referral intention and successful referral rate might be low when retail price is either low or high, which is a different trend as the impact of reward.

In our model, we consider the dynamics of the effect of a referral reward program only in the demand formulation and formulate static games between two firms. Dynamic games can be modeled in a multiple-period setting to further examine the representation of referral reward advertisement among different periods. Firms can manage their prices and rewards in different periods. Customers can make their purchasing and referral decisions anticipating the price and reward changes.

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Chapter 5: Closing Remarks

My main research interest is to apply quantitative methods and analytical approaches to business problems in order to help researchers and practitioners better understand real-world operations, especially the operations in operations management and marketing. Interaction exists among strategic, tactical, and operational decisions that managers have to decide regularly within various functional areas in a business environment. It is critical for managers to realize the importance of making integrated decisions because making decisions separately generally result in sub-optimal solutions. Since pricing strategy is a critical factor influencing the demand for products and services, I focus on joint pricing and inventory/production decision problems at the interface of marketing and operations and joint pricing and referral reward programs in marketing in this dissertation.

Chapter 2 presented a state-of-the-art survey on joint pricing and inventory and production decisions in supply chains. I classified the related research into four categories based on the various assumptions on supply chain structures, demand patterns, and interaction among chain members. I found that a significant amount of research in this area is concerned with the coordination mechanism among chain members and characteristics of optimal decisions. My survey showed that the related research simplifies joint decision problems in the real world with various (sometimes restrictive) assumptions. Therefore, I pointed out some directions for future research, such as to consider multiple products and production capacity, information asymmetry, time-dependent demand processes, coordination and benefit sharing, and more complicated and practical supply chain structures.

Chapter 3 modeled and analyzed joint pricing and inventory/production decisions in a two-stage dual-channel retail supply chain under three scenarios: Retailer Managed Inventory (RMI), Vendor Managed Inventory (VMI), and Centralized Supply Chain (CSC). I showed that a unique equilibrium exists under certain realistic conditions for each of the three scenarios. One of my main findings was a price convergence effect between the online and the off-line channels as the degree of substitution for the product and the retail stores increases. This finding implies that pricing for different product categories for the online store and the off-line store must be done strategically. Furthermore, when products sold in different channels have different configurations, the difference in the production costs should be taken into consideration when making pricing decisions.

In Chapter 3, I also found that demand for the store with a lower price increases, but the demand for the off-line store decreases when there is a strong cross-price effect (a larger substitution factor). The inventory decisions in the retail stores follow the same trend that demand has. Demand for the supplier does not have a significant change with the cross-price effect, nor does the EOQ for the supplier. Whether the supplier and the retailer benefit from the consideration of the cross-price effect depends on the production costs and the self-price sensitivities. If the difference between production costs is large and dominates, the monopoly chain can take advantage of the fact that customers treat the products in the two stores as more alike and more substitutable; thus, the monopolist can obtain more profit. If the difference between production costs is small and the difference between the self-price sensitivities dominates, the competition among products increases with substitution, which causes the supplier, the retailer, and the chain as well to lose profit. These observations shed light on a supplier's product configuration for different retailer channels and the retailer's store management decisions. In their decision-making, it is important to examine the mixed controlling effect of product costs and price sensitivities.

The impact of self-price sensitivities and production and inventory costs on optimal decisions and profits is straightforward. It is worth noting that the effort to reduce the supplier's setup cost is more rewarding than reducing the retailer's setup costs, which provides a recommendation for the initial steps to improve supply chain performance, especially under VMI. In the setting of this paper, we found that the supply chain performance is best under CSC followed by VMI, and RMI is least desirable. This result is in tune with earlier supply chain studies with different settings. An unexpected result, that of the retailer benefiting from developing VMI while the supplier does not, may be due to the fact that some benefits such as customer satisfaction are not captured in our models.

Chapter 4 investigated joint pricing and referral reward programs under competition. We analyzed interaction between two firms as a simultaneous game. Then, we extend it to a sequential game setting. Two types of referral rewards, absolute rewards and percentage rewards, are investigated. In addition to making decisions on prices and the levels of referral rewards, the firms also decide their reward timing strategy: Making the customers aware of the program before their purchasing decisions by offering an easy access program or making them aware of the program after their purchases by offering a restricted access program. We found that both firms benefit from offering referral rewards in simultaneous and sequential games. In a simultaneous game with absolute rewards, both firms should choose the same program type (easy access or restricted access.) However, which strategy is better depends on the sizes of the firms' potential markets, customer preferences over the two firms (these two factors are represented by the demand functions), and customers' responsiveness to the referral rewards (represented by the successful referral rates). When percentage rewards are offered, it is in both firms' interest to offer easy access programs. When the firms offer easy access programs, they should always offer their highest possible rewards, no matter whether they provide absolute rewards or percentage rewards and no matter whether they compete in simultaneous games or sequential games.

Businesses can benefit by considering many other decisions in manufacturing, marketing, and other functional areas jointly. For example, a rich body of research has focused on joint pricing and horizontal differentiation (for example, location and product differentiation) under competition since Hotelling (1929), which also lies at the interface of marketing and operations. In addition, a stream of work in inventory management develops various inventory policies to satisfy demand from different customer classes (Arslan et al., 2007; Frank et al., 2003; Benjaafar and Hafsi, 2006). However, the demand rate for each class is exogenous, which in general is not realistic. If a pricing strategy is applied to influence the demand rate for each demand class, the inventory decisions and rationing policies could be different from previous research. Especially when the production capacity is limited, pricing can be used to adjust demand to satisfy the consumers that are most profitable.

For a variety of reasons, most of the US firms surveyed in Presutti (1992) prefer multiple suppliers. Firms make their best purchase decisions by dealing with suppliers with various lead times, contracting prices, and flexibilities. Thus, from a supplier's perspective, it could optimize its pricing, lead time quotes, and service levels at the same time to maximize profit. Joint pricing and lead-time planning is another set of joint decisions that can be explored at the interface of operations and marketing. Palaka et al. (1998), Easton and Moodie (1999), and Lederer and Li (1997) are a few examples of pioneering work on this topic that provides guidelines for further research.

A referral reward program is one of the tools to promote products and services to increase sales. Referral reward programs interact not only with pricing strategies but also with other promotion mechanisms such as advertising and cumulative purchase rewards. Chen and Shi (2001) briefly examine the relation between advertising and referral reward programs and find that they are complements in a competitive market. Thus, I believe firms could benefit by considering their advertising strategies and referral reward programs simultaneously. Some businesses such as Petopia (2008) offer both referral reward programs and cumulative purchase rewards. Kim et al. (2002, 2004) are two examples investigating optimal pricing and cumulative rewards in a competitive environment. It would be interesting to see how a referral reward program interacts with a cumulative purchase reward in one decision set.

In addition to these examples of possible joint decision problems mentioned in operations management and marketing, we can also apply the concept and the approaches to other business functions, such as finance, to help firms prosper in today's increasingly severe competitive business environment.

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Appendix: Proofs of Chapter 3

Proof of Theorem 1. We use D_i to denote $D_i = k_i - \alpha_i p_i + \theta_{ij}(p_j - p_i)$. The second derivatives of function (8) are given by

$$\begin{aligned}
\frac{\partial \pi_{RMI}{}^{r2}}{\partial p_1{}^2} &= -2(\alpha_1 + \theta_{12}) + \frac{(\alpha_1 + \theta_{12})^2\sqrt{2h_1 s_1 D_1}}{4D_1^2} + \frac{\theta_{21}^2\sqrt{2h_2 s_2 D_2}}{4D_2^2} \\
\frac{\partial \pi_{RMI}{}^{r2}}{\partial p_2{}^2} &= -2(\alpha_2 + \theta_{21}) + \frac{\theta_{21}^2\sqrt{2h_1 s_1 D_1}}{4D_1^2} + \frac{(\alpha_2 + \theta_{21})^2\sqrt{2h_2 s_2 D_2}}{4D_2^2} \\
\frac{\partial \pi_{RMI}{}^{r2}}{\partial p_1 \partial p_2} &= \theta_{12} + \theta_{21} - \frac{\theta_{12}(\alpha_1 + \theta_{12})\sqrt{2h_1 s_1 D_1}}{4D_1^2} - \frac{\theta_{21}(\alpha_2 + \theta_{21})\sqrt{2h_2 s_2 D_2}}{4D_2^2}
\end{aligned}$$

From assumption (10), we can verify $1/10(\alpha_1 + \theta_{12}) \geq (\alpha_1 + \theta_{12})^2\sqrt{2h_1 s_1 D_1}/4D_1^2$ and $1/10\theta_{21} \geq \theta_{21}^2\sqrt{2h_2 s_2 D_2}/4D_2^2$. Therefore, we have

$$\begin{aligned}
\alpha_1 + \theta_{12} + \theta_{21} &> \frac{(\alpha_1 + \theta_{12})^2\sqrt{2h_1 s_1 D_1}}{4D_1^2} + \frac{\theta_{21}^2\sqrt{2h_2 s_2 D_2}}{4D_2^2} \\
\Longrightarrow 2(\alpha_1 + \theta_{12}) &> \frac{(\alpha_1 + \theta_{12})^2\sqrt{2h_1 s_1 D_1}}{4D_1^2} + \frac{\theta_{21}^2\sqrt{2h_2 s_2 D_2}}{4D_2^2} \\
\Longrightarrow \frac{\partial \pi_{RMI}{}^{r2}}{\partial p_1{}^2} &< 0
\end{aligned}$$

By applying the same reasoning, we can show $\partial \pi_{RMI}{}^{r2}/\partial p_2{}^2 < 0$ as well.

Next, we verify that the determinant of the Hessian matrix is positive, or equivalently

$$\frac{\partial \pi_{RMI}{}^{r2}}{\partial p_1{}^2}\frac{\partial \pi_{RMI}{}^{r2}}{\partial p_2{}^2} > (\frac{\partial \pi_{RMI}{}^{r2}}{\partial p_1 \partial p_2})^2$$

$$\Longleftrightarrow \left[(\alpha_1 + \theta_{12})(1 - \frac{(\alpha_1 + \theta_{12})V_1}{4D_1^2}) + \theta_{21}(1 - \frac{\theta_{21}V_2}{4D_2^2})\right]\left[\theta_{12}(1 - \frac{\theta_{12}V_1}{4D_1^2}) + (\alpha_2 + \theta_{21})(1 - \frac{(\alpha_2 + \theta_{21})V_2}{4D_2^2})\right]$$

$$> \left[\theta_{12}(1 - \frac{(\alpha_1 + \theta_{12})V_1}{4D_1^2}) + \theta_{21}(1 - \frac{(\alpha_2 + \theta_{21})V_2}{4D_2^2})\right]\left[\theta_{12}(1 - \frac{(\alpha_1 + \theta_{12})V_1}{4D_1^2}) + \theta_{21}(1 - \frac{(\alpha_2 + \theta_{21})V_2}{4D_2^2})\right]$$

Since the objective function is shown to be strictly concave in $\boldsymbol{p}$ under conditions (2) and (9). □

Proof of Theorem 2. The second derivatives of the supplier's objective function are

$$\frac{\partial \pi_{RMI}{}^{s2}}{\partial p_1^2} = -4(\alpha_1+\theta_{12}) + \frac{1}{2}\frac{(\alpha_1+\theta_{12})^2\sqrt{2s_1h_1D_1}}{4D_1^2} + \frac{1}{2}\frac{\theta_{21}^2\sqrt{2s_2h_2D_2}}{4D_2^2} + \frac{S^2H^2[-(\alpha_1+\theta_{12})+\theta_{21}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}} \tag{21}$$

$$\frac{\partial \pi_{RMI}{}^{s2}}{\partial p_2^2} = -4(\alpha_2+\theta_{21}) + \frac{1}{2}\frac{(\alpha_2+\theta_{21})^2\sqrt{2s_2h_2D_2}}{4D_2^2} + \frac{1}{2}\frac{\theta_{12}^2\sqrt{2s_1h_2D_1}}{4D_1^2} + \frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}} \tag{22}$$

$$\frac{\partial \pi_{RMI}{}^{s2}}{\partial p_1 \partial p_2} = 2(\theta_{12}+\theta_{21}) - \frac{1}{2}\frac{(\alpha_2+\theta_{21})\theta_{21}\sqrt{2s_2h_2D_2}}{4D_2^2} - \frac{1}{2}\frac{(\alpha_1+\theta_{12})\theta_{12}\sqrt{2s_1h_2D_1}}{4D_1^2} + \frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}][-(\alpha_1+\theta_{12})+\theta_{21}]}{[2SH(D_1+D_2)]^{\frac{3}{2}}} \tag{23}$$

Let us first prove $\frac{\partial \pi_{RMI}{}^{s2}}{\partial p_1^2} < 0$. From the retailer's problem, we know

$$-2(\alpha_1+\theta_{12}) + (\alpha_1+\theta_{12})^2\sqrt{2h_1s_1D_1}/0.4D_1^2 + \theta_{21}^2\sqrt{2h_2s_2D_2}/0.4D_2^2 < 0,$$

that is to say,

$$-\frac{1}{10}(\alpha_1+\theta_{12}) + \frac{1}{2}(\alpha_1+\theta_{12})^2\sqrt{2h_1s_1D_1}/4D_1^2 + \frac{1}{2}\theta_{21}^2\sqrt{2h_2s_2D_2}/4D_2^2 < 0$$

.

Thus, as long as we can show $-\frac{39}{10}(\alpha_1+\theta_{12}) + \frac{S^2H^2[-(\alpha_1+\theta_{12})+\theta_{21}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}} \leq 0$, we will have $\frac{\partial \pi_{RMI}{}^{s2}}{\partial p_1^2} < 0$.

$$\begin{aligned}
-\frac{39}{10}(\alpha_1+\theta_{12}) + \frac{S^2H^2[-(\alpha_1+\theta_{12})+\theta_{21}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}} &\leq 0 \\
\Longleftrightarrow \frac{39}{10}(\alpha_1+\theta_{12})[2SH(D_1+D_2)]^{\frac{3}{2}} &\geq S^2H^2[-(\alpha_1+\theta_{12})+\theta_{21}]^2 \\
\Longleftrightarrow (D_1+D_2)^{\frac{3}{2}} &\geq \frac{\sqrt{SH}}{\frac{39}{5}\sqrt{2}(\alpha_1+\theta_{12})}[-(\alpha_1+\theta_{12})+\theta_{21}]^2.
\end{aligned}$$

$$\Longleftrightarrow [k_1+k_2-(\alpha_1+\theta_{12}-\theta_{21})p_1-(\alpha_2+\theta_{21}-\theta_{12})p_2]^{\frac{3}{2}} \geq \frac{\sqrt{SH}}{\frac{39}{5}\sqrt{2}(\alpha_1+\theta_{12})}[-(\alpha_1+\theta_{12})+\theta_{21}]^2$$

The assumption, $e_i \leq 0.4D_ip_i/\sqrt{2S_iH_iD_i}$ where $e_i = (\alpha_i + \theta_{ij})p_i/D_i$, connects parameters and prices together. Since we are considering shared setup cost in the supplier's problem, we can apply the assumption in the form $p_1/D_1 \leq 0.4D_1p_1/\sqrt{HSD_1}$ assuming that each product takes half of the setup cost.

$$\frac{(\alpha_1+\theta_{12})p_1}{D_1} \leq \frac{0.4D_1p_1}{\sqrt{HSD_1}}$$
$$\Longleftrightarrow (\alpha_1+\theta_{12})\sqrt{HSD_1} \leq 0.4D_1^2$$
$$\Longleftrightarrow (\alpha_1+\theta_{12})^2 HSD_1 \leq 0.4^2D_1^4$$
$$\Longleftrightarrow (\alpha_2+\theta_{21})^2 HSD_2 \leq 0.4^2D_2^4$$

If we denote $\theta_{\min} = \min(\theta_{21}, \theta_{12})$ and $\alpha_{\min} = \min(\alpha_2, \alpha_1)$, we have

$$\begin{aligned} HS(D_1+D_2)(\alpha_{\min}+\theta_{\min})^2 &\leq 0.4^2(D_1^4+D_2^4) \\ \Longrightarrow HS(D_1+D_2)(\alpha_{\min}+\theta_{\min})^2 &\leq 0.4^2(D_1+D_2)^4 \\ \Longrightarrow \sqrt{HS(D_1+D_2)}(\alpha_{\min}+\theta_{\min}) &\leq 0.4(D_1+D_2)^2 \\ \Longrightarrow \frac{\sqrt{HS}}{\frac{0.4}{(\alpha_{\min}+\theta_{\min})}} &\leq (D_1+D_2)^{\frac{3}{2}} \end{aligned}$$

In order to have $\frac{\partial \pi_{RMI}{}^{s2}}{\partial p_2^2} < 0$, we need $(D_1+D_2)^{\frac{3}{2}} \geq \frac{\sqrt{SH}}{\frac{39}{5}\sqrt{2}(\alpha_2+\theta_{21})}[-(\alpha_2+\theta_{21})+\theta_{12}]^2$. So if $\frac{(\alpha_{\max}+|\theta_{ij}-\theta_{ji}|)^2}{(\theta_{\min}+\alpha_{\min})^2} \leq \frac{39\sqrt{2}}{2} \approx 27$, $\frac{\partial \pi^{r2}}{\partial {p_1}^2} \leq 0$ and $\frac{\partial \pi^{r2}}{\partial {p_2}^2} \leq 0$. So it means for some industries and some parameters, the inequality may not hold.

In order to prove $\frac{\partial \pi_{RMI}{}^{s2}}{\partial {p_1}^2}\frac{\partial \pi_{RMI}{}^{s2}}{\partial {p_2}^2} > (\frac{\partial \pi_{RMI}{}^{s2}}{\partial p_1 \partial p_2})^2$, we have to show

$$[-4(\alpha_1+\theta_{12}) + \frac{1}{2}\frac{(\alpha_1+\theta_{12})^2\sqrt{2s_1h_1D_1}}{4D_1^2} + \frac{1}{2}\frac{\theta_{21}^2\sqrt{2s_2h_2D_2}}{4D_2^2} + \frac{S^2H^2[-(\alpha_1+\theta_{12})+\theta_{21}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}}]*$$

$$[-4(\alpha_2+\theta_{21}) + \frac{1}{2}\frac{(\alpha_2+\theta_{21})^2\sqrt{2s_2h_2D_2}}{4D_2^2} + \frac{1}{2}\frac{\theta_{12}^2\sqrt{2s_1h_2D_1}}{4D_1^2} + \frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}}]$$

$$>$$

$$[2(\theta_{12}+\theta_{21}) - \frac{1}{2}\frac{(\alpha_2+\theta_{21})\theta_{21}\sqrt{2s_2h_2D_2}}{4D_2^2} - \frac{1}{2}\frac{(\alpha_1+\theta_{12})\theta_{12}\sqrt{2s_1h_2D_1}}{4D_1^2}$$
$$+\frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}][-(\alpha_1+\theta_{12})+\theta_{21}]}{[2SH(D_1+D_2)]^{\frac{3}{2}}}]^2$$

This can be written as $(A+B)(C+D) > (E+F)^2$, where

$$\begin{aligned}
A &= -\frac{1}{10}(\alpha_1+\theta_{12}) + \frac{1}{2}\frac{(\alpha_1+\theta_{12})^2\sqrt{2s_1h_1D_1}}{4D_1^2} + \frac{1}{2}\frac{\theta_{21}^2\sqrt{2s_2h_2D_2}}{4D_2^2} \\
B &= -\frac{39}{10}(\alpha_1+\theta_{12}) + \frac{S^2H^2[-(\alpha_1+\theta_{12})+\theta_{21}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}} \\
C &= -\frac{1}{10}(\alpha_2+\theta_{21}) + \frac{1}{2}\frac{(\alpha_2+\theta_{21})^2\sqrt{2s_2h_2D_2}}{4D_2^2} + \frac{1}{2}\frac{\theta_{12}^2\sqrt{2s_1h_2D_1}}{4D_1^2} \\
D &= -\frac{39}{10}(\alpha_2+\theta_{21}) + \frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}} \\
E &= \frac{1}{20}(\theta_{12}+\theta_{21}) - \frac{1}{2}\frac{(\alpha_2+\theta_{21})\theta_{21}\sqrt{2s_2h_2D_2}}{4D_2^2} - \frac{1}{2}\frac{(\alpha_1+\theta_{12})\theta_{12}\sqrt{2s_1h_2D_1}}{4D_1^2} \\
F &= \frac{19}{20}(\theta_{12}+\theta_{21}) + \frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}][-(\alpha_1+\theta_{12})+\theta_{21}]}{[2SH(D_1+D_2)]^{\frac{3}{2}}}.
\end{aligned}$$

In the retailer's problem, it is shown that $-A > E$ and $-C > E$ hold for all industries. Next, we show the condition under which $-B > F$ and $-D > F$ hold.

$$(D_1+D_2)^{\frac{3}{2}} \geq \frac{\sqrt{SH}(\alpha_1+\theta_{12}-\theta_{21})(\alpha_1+\alpha_2)}{\frac{39\sqrt{2}}{5}(\alpha_1-\theta_{21})}$$

$$\Longrightarrow \frac{39\sqrt{2}}{5}(\alpha_1-\theta_{21})(D_1+D_2)^{\frac{3}{2}} \geq \sqrt{SH}(\alpha_1+\theta_{12}-\theta_{21})(\alpha_1+\alpha_2)$$

$$\Longrightarrow \frac{39\sqrt{2}}{5}(\alpha_1+\theta_{12})(D_1+D_2)^{\frac{3}{2}} - \sqrt{SH}[\alpha_1+\theta_{12}-\theta_{21}]^2$$

$$\geq \frac{39\sqrt{2}}{5}(\theta_{12}+\theta_{21})(D_1+D_2)^{\frac{3}{2}} + \sqrt{SH}(\alpha_2+\theta_{21}-\theta_{12})(\alpha_1+\theta_{12}-\theta_{21})$$

$$\Longrightarrow \frac{39\sqrt{2}}{5}(\alpha_1+\theta_{12})(D_1+D_2)^{\frac{3}{2}} - \sqrt{SH}[\alpha_1+\theta_{12}-\theta_{21}]^2$$

$$> \frac{19\sqrt{2}}{10}(\theta_{12}+\theta_{21})(D_1+D_2)^{\frac{3}{2}} + \sqrt{SH}(\alpha_2+\theta_{21}-\theta_{12})(\alpha_1+\theta_{12}-\theta_{21})$$

$$\Longrightarrow \frac{39}{10}(\alpha_1+\theta_{12}) - \frac{S^2H^2[-(\alpha_1+\theta_{12})+\theta_{21}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}}$$

$$> \frac{19}{20}(\theta_{12}+\theta_{21}) + \frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}][-(\alpha_1+\theta_{12})+\theta_{21}]}{[2SH(D_1+D_2)]^{\frac{3}{2}}}$$

$$\Longrightarrow -B > F$$

Thus, we need $(D_1+D_2)^{\frac{3}{2}} \geq \frac{\sqrt{SH}(\alpha_1+\theta_{12}-\theta_{21})(\alpha_1+\alpha_2)}{\frac{39\sqrt{2}}{5}(\alpha_1-\theta_{21})}$ for $-B > F$ and $(D_1+D_2)^{\frac{3}{2}} \geq \frac{\sqrt{SH}(\alpha_2+\theta_{21}-\theta_{12})(\alpha_1+\alpha_2)}{\frac{39\sqrt{2}}{5}(\alpha_2-\theta_{12})}$ for $-D > F$. We have $\frac{\sqrt{HS}}{\frac{0.4}{(\alpha_{\min}+\theta_{\min})}} \leq (D_1+D_2)^{\frac{3}{2}}$ from the assumption. Therefore, as long as $\frac{(\alpha_1+\alpha_2)(\alpha_{\max}+|\theta_{ij}-\theta_{ji}|)}{(\alpha_{\min}+\theta_{\min})(\alpha_{\min}-\theta_{\max})} \leq 27$, $\frac{\partial \pi_{RMI}{}^{s2}}{\partial p_1{}^2}\frac{\partial \pi_{RMI}{}^{s2}}{\partial p_2{}^2} > (\frac{\partial \pi_{RMI}{}^{s2}}{\partial p_1 \partial p_2})^2$ because $(A+B)(C+D) > (E+F)^2$.

In summary, the supplier's problem can be proved to be strictly concave when

$$\max(\frac{(\alpha_1+\alpha_2)(\alpha_{\max}+|\theta_{ij}-\theta_{ji}|)}{(\alpha_{\min}+\theta_{\min})(\alpha_{\min}-\theta_{\max})}, \frac{(\alpha_{\max}+|\theta_{ij}-\theta_{ji}|)^2}{(\theta_{\min}+\alpha_{\min})^2}) \leq \frac{39\sqrt{2}}{2} \approx 27$$

.

□

Proof of Theorem 4. The second derivatives of the retailer's problem under VMI are given by

$$\frac{\partial \pi_{VMI}{}^{r2}}{\partial p_1{}^2} = -2(\alpha_1+\theta_{12}) + (\alpha_1+\theta_{12})^2\gamma_1\delta_1(1-\delta_1)D_1^{(\delta_1-2)} + \theta_{21}^2\gamma_2\delta_2(1-\delta_2)D_2^{(\delta_2-2)} \tag{24}$$

$$\frac{\partial \pi_{VMI}{}^{r2}}{\partial p_2{}^2} = -2(\alpha_2+\theta_{21}) + (\alpha_2+\theta_{21})^2\gamma_2\delta_2(1-\delta_2)D_2^{(\delta_2-2)} + \theta_{12}^2\gamma_1\delta_1(1-\delta_1)D_1^{(\delta_1-2)} \tag{25}$$

$$\frac{\partial \pi_{VMI}{}^{r2}}{\partial p_1 \partial p_2} = \theta_{12}+\theta_{21}-(\alpha_1+\theta_{12})\theta_{12}\gamma_1\delta_1(1-\delta_1)D_1^{(\delta_1-2)}-(\alpha_2+\theta_{21})\theta_{21}^2\gamma_2\delta_2(1-\delta_2)D_2^{(\delta_2-2)} \tag{26}$$

From conditions (2) and (16), it is straightforward to show

$$(\alpha_i+\theta_{ij}) \geq \gamma_i\delta_i(1-\delta_i)(\alpha_i+\theta_{ij})^2 D_i^{(\delta_i-2)}$$

$$(\alpha_i+\theta_{ij}) > \theta_{ji} > \gamma_j\delta_j(1-\delta_j)\theta_{ji}^2 D_j^{(\delta_j-2)}$$

for $i = 1, 2$ and $i \neq j$. By the same reasoning given in the proof of Theorem 1, we can verify

$$\frac{\partial \pi_{VMI}{}^{r2}}{\partial p_1{}^2} < 0,$$

$$\frac{\partial \pi_{VMI}{}^{r2}}{\partial p_1{}^2} < 0,$$

$$\frac{\partial \pi_{VMI}{}^{r2}}{\partial p_1{}^2}\frac{\partial \pi_{VMI}{}^{r2}}{\partial p_2{}^2} > (\frac{\partial \pi_{VMI}{}^{r2}}{\partial p_1 \partial p_2})^2.$$

Therefore, the objective function is strictly concave in $\boldsymbol{p}$.

□

Proof of Theorem 5. The second derivatives of the supplier's objective function are

$$\begin{aligned}\frac{\partial \pi_{VMI}{}^{s2}}{\partial p_1^2} =& -4(\alpha_1+\theta_{12}) + \frac{(\alpha_1+\theta_{12})^2\sqrt{2s_1h_1D_1}}{4D_1^2} + \frac{\theta_{21}^2\sqrt{2s_2h_2D_2}}{4D_2^2} \\ & - \omega_1(\alpha_1+\theta_{12})^2 D_1^{(\delta_1-2)} + \omega_2\theta_{21}^2 D_2^{(\delta_2-2)} + \frac{S^2H^2[-(\alpha_1+\theta_{12})+\theta_{21}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}}\end{aligned} \tag{27}$$

$$\begin{aligned}\frac{\partial \pi_{VMI}{}^{s2}}{\partial p_2^2} =& -4(\alpha_2+\theta_{21}) + \frac{(\alpha_2+\theta_{21})^2\sqrt{2s_2h_2D_2}}{4D_2^2} + \frac{\theta_{12}^2\sqrt{2s_1h_2D_1}}{4D_1^2} \\ & - \omega_1 b_{12}^2 D_1^{(\delta_1-2)} + \omega_2(\alpha_2+\theta_{21})^2 D_2^{(\delta_2-2)} + \frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}}\end{aligned} \tag{28}$$

$$\begin{aligned}\frac{\partial \pi_{VMI}{}^{s2}}{\partial p_1 \partial p_2} =& 2(\theta_{12}+\theta_{21}) - \frac{(\alpha_2+\theta_{21})\theta_{21}\sqrt{2s_2h_2D_2}}{4D_2^2} - \frac{(\alpha_1+\theta_{12})\theta_{12}\sqrt{2s_1h_2D_1}}{4D_1^2} \\ & - \omega_1(\alpha_1+\theta_{12})\theta_{12}D_1^{(\delta_1-2)} - \omega_2\theta_{21}(\alpha_2+\theta_{21})D_2^{(\delta_2-2)} \\ & + \frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}][-(\alpha_1+\theta_{12})+\theta_{21}]}{[2SH(D_1+D_2)]^{\frac{3}{2}}}\end{aligned} \tag{29}$$

where $\omega_i = \gamma_i\delta_i^2(1-\delta_i)$. From the previous proof, we have $-\frac{1}{5}(\alpha_1+\theta_{12}) + (\alpha_1+\theta_{12})^2\sqrt{2h_1s_1D_1}/4D_1^2 + \theta_{21}^2\sqrt{2h_2s_2D_2}/4D_2^2 < 0$ and $-2\delta(\alpha_1+\theta_{12}) + \omega_1(\alpha_1+\theta_{12})^2D_1^{(\delta_1-2)} + \omega_2\theta_{21}^2D_2^{(\delta_2-2)} < 0$, we have to show that

$$(-4+\frac{1}{5}+2\delta)(\alpha_1+\theta_{12}) + \frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}][-(\alpha_1+\theta_{12})+\theta_{21}]}{[2SH(D_1+D_2)]^{\frac{3}{2}}} \le 0$$

to have $\frac{\partial \pi_{RMI}{}^{s2}}{\partial p_1^2} < 0$. Following the same reasoning in the proof of the supplier's problem under RMI, as long as $\frac{(\alpha_{\max}+|\theta_{ij}-\theta_{ji}|)^2}{\theta_{\min}+\alpha_{\min}} \le (19-\delta_{\max})\sqrt{2}$, $\frac{\partial \pi_{RMI}{}^{r2}}{\partial p_1{}^2} \le 0$ and $\frac{\partial \pi_{RMI}{}^{r2}}{\partial p_2{}^2} \le 0$.

Assume that $\delta_{\max} = \max(\delta_1, \delta_2)$. To verify if $\frac{\partial \pi_{VMI}{}^{s2}}{\partial p_1^2}\frac{\partial \pi_{VMI}{}^{s2}}{\partial p_2^2} > (\frac{\partial \pi_{VMI}{}^{s2}}{\partial p_1 \partial p_2})^2$, we rewrite it in the form of $(\overline{A}+\overline{B}+\overline{C})(\overline{D}+\overline{E}+\overline{F}) > (\overline{HIJ})^2$, where

$$\begin{aligned}
\overline{A} &= -\frac{1}{5}(\alpha_1+\theta_{12}) + \frac{(\alpha_1+\theta_{12})^2\sqrt{2s_1h_1D_1}}{4D_1^2} + \frac{\theta_{21}^2\sqrt{2s_2h_2D_2}}{4D_2^2} \\
\overline{B} &= -2\delta_{\max}(\alpha_1+\theta_{12}) + \omega_1(\alpha_1+\theta_{12})^2 D_1^{(\delta_1-2)} + \omega_2\theta_{21}^2 D_2^{(\delta_2-2)} \\
\overline{C} &= -(\frac{19}{5}-2\delta_{\max})(\alpha_1+\theta_{12}) + \frac{S^2H^2[-(\alpha_1+\theta_{12})+\theta_{21}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}} \\
\overline{D} &= -\frac{1}{5}(\alpha_2+\theta_{21}) + \frac{(\theta_{21})^2\sqrt{2s_2h_2D_2}}{4D_2^2} + \frac{\theta_{12}^2\sqrt{2s_1h_2D_1}}{4D_1^2} \\
\overline{E} &= -2\delta_{\max}(\alpha_2+\theta_{21}) + \omega_1 b_{12}^2 D_1^{(\delta_1-2)} + \omega_2(\alpha_2+\theta_{21})^2 D_2^{(\delta_2-2)} \\
\overline{F} &= -(\frac{19}{5}-2\delta_{\max})(\alpha_2+\theta_{21}) + \frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}} \\
\overline{G} &= \frac{1}{10}(\theta_{12}+\theta_{21}) - \frac{(\alpha_2+\theta_{21})\theta_{21}\sqrt{2s_2h_2D_2}}{4D_2^2} - \frac{(\alpha_1+\theta_{12})\theta_{12}\sqrt{2s_1h_2D_1}}{4D_1^2} \\
\overline{H} &= \delta_{\max}(\theta_{12}+\theta_{21}) + \omega_1(\alpha_1+\theta_{12})\theta_{12}D_1^{(\delta_1-2)} - \omega_2\theta_{21}(\alpha_2+\theta_{21})D_2^{(\delta_2-2)} \\
\overline{I} &= (\frac{19}{10}-\delta_{\max})(\theta_{12}+\theta_{21}) + \frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}][-(\alpha_1+\theta_{12})+\theta_{21}]}{[2SH(D_1+D_2)]^{\frac{3}{2}}}
\end{aligned}$$

It has been shown that $-\overline{A} > \overline{G}$, $-\overline{B} > \overline{H}$, $-\overline{D} > \overline{G}$, and $-\overline{E} > \overline{G}$ hold. We want to verify if $-\overline{C} > \overline{I}$ and $-\overline{F} > \overline{I}$.

$$(D_1+D_2)^{\frac{3}{2}} \geq \frac{\sqrt{SH}(\alpha_1+\theta_{12}-\theta_{21})(\alpha_1+\alpha_2)}{\frac{(19-10\delta)2\sqrt{2}}{5}(\alpha_1-\theta_{21})}$$

$$\Longrightarrow (\frac{19}{5}-2\delta_{\max})(\alpha_1+\theta_{12}) - \frac{S^2H^2[-(\alpha_1+\theta_{12})-\theta_{21}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}}$$

$$\geq (\frac{19}{5}-2\delta_{\max})(\theta_{12}+\theta_{21}) + \frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}][-(\alpha_1+\theta_{12})+\theta_{21}]}{[2SH(D_1+D_2)]^{\frac{3}{2}}}$$

$$\Longrightarrow (\frac{19}{5}-2\delta_{\max})(\alpha_1+\theta_{12}) - \frac{S^2H^2[-(\alpha_1+\theta_{12})-\theta_{21}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}}$$

$$\geq (\frac{19}{5}-2\delta_{\max})(\theta_{12}+\theta_{21}) + \frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}][-(\alpha_1+\theta_{12})+\theta_{21}]}{[2SH(D_1+D_2)]^{\frac{3}{2}}}$$

$$\Longrightarrow -(\frac{19}{5}-2\delta_{\max})(\alpha_1+\theta_{12}) + \frac{S^2H^2[-(\alpha_1+\theta_{12})+\theta_{21}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}}$$

$$> (\frac{19}{10} - \delta_{\max})(\theta_{12} + \theta_{21}) + \frac{S^2H^2[-(\alpha_2 + \theta_{21}) + \theta_{12}][-(\alpha_1 + \theta_{12}) + \theta_{21}]}{[2SH(D_1 + D_2)]^{\frac{3}{2}}}$$

$$\Longrightarrow -\overline{C} > \overline{I}$$

Thus, we need $(D_1 + D_2)^{\frac{3}{2}} \geq \frac{\sqrt{SH}(\alpha_1+\theta_{12}-\theta_{21})(\alpha_1+\alpha_2)}{\frac{(19-10\delta_{\max})2\sqrt{2}}{5}(\alpha_1-\theta_{21})}$ for $-\overline{C} > \overline{I}$ and $(D_1 + D_2)^{\frac{3}{2}} \geq \frac{\sqrt{SH}(\alpha_2+\theta_{21}-\theta_{12})(\alpha_1+\alpha_2)}{\frac{(19-10\delta_{\max})2\sqrt{2}}{5}(\alpha_2-\theta_{12})}$ for $-\overline{F} > \overline{I}$. We have $\frac{\sqrt{HS}}{\frac{0.4}{(\alpha_{\min}+\theta_{\min})}} \leq$ $(D_1 + D_2)^{\frac{3}{2}}$ from the assumption (10). Therefore, as long as

$$\frac{(\alpha_1 + \alpha_2)(\alpha_{\max} + |\theta_{ij} - \theta_{ji}|)}{(\alpha_{\min} + \theta_{\min})(\alpha_{\min} - \theta_{\max})} \leq (19 - 10\delta_{\max})\sqrt{2},$$

$\frac{\partial \pi_{VMI}{}^{s2}}{\partial p_1{}^2} \frac{\partial \pi_{VMI}{}^{s2}}{\partial p_2{}^2} > (\frac{\partial \pi_{VMI}{}^{s2}}{\partial p_1 \partial p_2})^2.$

In summary, the supplier's problem can be proved to be strictly concave in $\boldsymbol{q}$ under VMI when

$$\max(\frac{(\alpha_1 + \alpha_2)(\alpha_{\max} + |\theta_{ij} - \theta_{ji}|)}{(\alpha_{\min} + \theta_{\min})(\alpha_{\min} - \theta_{\max})}, \frac{(\alpha_{\max} + |\theta_{ij} - \theta_{ji}|)^2}{(\theta_{\min} + \alpha_{\min})^2} \leq (19 - 2\delta_{\max})\sqrt{(2)}$$

□

Proof of Theorem 7. The second derivatives of the objective function are

$$\begin{aligned}\frac{\partial \pi_{CSC}^{s2}}{\partial p_1^2} =& -2(\alpha_1 + \theta_{12}) + \frac{(\alpha_1 + \theta_{12})^2\sqrt{2s_1h_1D_1}}{4D_1^2} + \frac{\theta_{21}^2\sqrt{2s_2h_2D_2}}{4D_2^2} + \\ & \frac{S^2H^2[-(\alpha_1 + \theta_{12}) + \theta_{21}]^2}{[2SH(D_1 + D_2)]^{\frac{3}{2}}}\end{aligned} \tag{30}$$

$$\begin{aligned}\frac{\partial \pi_{CSC}^{s2}}{\partial p_2^2} =& -2(\alpha_2 + \theta_{21}) + \frac{(\alpha_2 + \theta_{21})^2\sqrt{2s_2h_2D_2}}{4D_2^2} + \frac{\theta_{12}^2\sqrt{2s_1h_2D_1}}{4D_1^2} + \\ & + \frac{S^2H^2[-(\alpha_2 + \theta_{21}) + \theta_{12}]^2}{[2SH(D_1 + D_2)]^{\frac{3}{2}}}\end{aligned} \tag{31}$$

$$\begin{aligned}\frac{\partial \pi_{CSC}^{s2}}{\partial p_1 \partial p_2} =& (\theta_{12} + \theta_{21}) - \frac{(\alpha_2 + \theta_{21})\theta_{21}\sqrt{2s_2h_2D_2}}{4D_2^2} - \frac{(\alpha_1 + \theta_{12})\theta_{12}\sqrt{2s_1h_2D_1}}{4D_1^2} + \\ & \frac{S^2H^2[-(\alpha_2 + \theta_{21}) + \theta_{12}][-(\alpha_1 + \theta_{12}) + \theta_{21}]}{[2SH(D_1 + D_2)]^{\frac{3}{2}}}\end{aligned} \tag{32}$$

From the previous proof, we have

$$-\frac{1}{5}(\alpha_1+\theta_{12})+(\alpha_1+\theta_{12})^2\sqrt{2h_1s_1D_1}/4D_1^2+\theta_{21}^2\sqrt{2h_2s_2D_2}/4D_2^2<0,$$

we have to show

$$(-2+\frac{1}{5})+\frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}][-(\alpha_1+\theta_{12})+\theta_{21}]}{[2SH(D_1+D_2)]^{\frac{3}{2}}}\leq 0$$

to have $\frac{\partial\pi_{CSC}^{s2}}{\partial p_1^2}<0$.

Following the same reasoning in the proof of the supplier's problem under RMI, as long as $\frac{(\alpha_{\max}+|\theta_{ij}-\theta_{ji}|)^2}{(\theta_{\min}+\alpha_{\min})^2}\leq\frac{18}{5}\sqrt{2}\approx 13.5$, $\frac{\partial\pi_{CSC}^{r2}}{\partial {p_1}^2}\leq 0$ and $\frac{\partial\pi_{CSC}^{r2}}{\partial {p_2}^2}\leq 0$.

To verify if $\frac{\partial\pi_{CSC}^{s2}}{\partial {p_1}^2}\frac{\partial\pi_{CSC}^{s2}}{\partial {p_2}^2}>(\frac{\partial\pi_{CSC}^{s2}}{\partial p_1\partial p_2})^2$, we rewrite it in the form of $(\underline{A}+\underline{B})(\underline{C}+\underline{D})>(\underline{E}+\underline{F})^2$, where

$$\begin{aligned}
\underline{A} &= -\frac{1}{5}(\alpha_1+\theta_{12})+\frac{(\alpha_1+\theta_{12})^2\sqrt{2s_1h_1D_1}}{4D_1^2}+\frac{\theta_{21}^2\sqrt{2s_2h_2D_2}}{4D_2^2}\\
\underline{B} &= -\frac{9}{5}(\alpha_1+\theta_{12})+\frac{S^2H^2[-(\alpha_1+\theta_{12})+\theta_{21}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}}\\
\underline{C} &= -\frac{1}{5}(\alpha_2+\theta_{21})+\frac{(\theta_{21})^2\sqrt{2s_2h_2D_2}}{4D_2^2}+\frac{\theta_{12}^2\sqrt{2s_1h_2D_1}}{4D_1^2}\\
\underline{D} &= -\frac{9}{5}(\alpha_2+\theta_{21})+\frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}}\\
\underline{E} &= \frac{1}{10}(\theta_{12}+\theta_{21})-\frac{(\alpha_2+\theta_{21})\theta_{21}\sqrt{2s_2h_2D_2}}{4D_2^2}-\frac{(\alpha_1+\theta_{12})\theta_{12}\sqrt{2s_1h_2D_1}}{4D_1^2}\\
\underline{F} &= \frac{9}{10}(\theta_{12}+\theta_{21})+\frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}][-(\alpha_1+\theta_{12})+\theta_{21}]}{[2SH(D_1+D_2)]^{\frac{3}{2}}}
\end{aligned}$$

It has been shown that $-\underline{A}>\underline{E}$, $-\underline{C}>\underline{E}$ hold for almost all industries. We want to verify if $-\underline{B} > \underline{F}$ and $-\underline{D} > \underline{F}$ hold.

$$\begin{aligned}
(D_1+D_2)^{\frac{3}{2}} &\geq \frac{\sqrt{SH}(\alpha_1+\theta_{12}-\theta_{21})(\alpha_1+\alpha_2)}{\frac{9}{5}2\sqrt{2}(\alpha_1-\theta_{21})} \\
\Longrightarrow -\frac{9}{5}(\alpha_1+\theta_{12}) - \frac{S^2H^2[-(\alpha_1+\theta_{12})+\theta_{21}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}} &\geq \frac{9}{5}(\theta_{12}+\theta_{21}) \\
&+ \frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}][-(\alpha_1+\theta_{12})+\theta_{21}]}{[2SH(D_1+D_2)]^{\frac{3}{2}}} \\
\Longrightarrow -\frac{9}{5}(\alpha_1+\theta_{12}) + \frac{S^2H^2[-(\alpha_1+\theta_{12})+\theta_{21}]^2}{[2SH(D_1+D_2)]^{\frac{3}{2}}} &> \frac{9}{10}(\theta_{12}+\theta_{21}) \\
&+ \frac{S^2H^2[-(\alpha_2+\theta_{21})+\theta_{12}][-(\alpha_1+\theta_{12})+\theta_{21}]}{[2SH(D_1+D_2)]^{\frac{3}{2}}} \\
\Longrightarrow -\underline{B} &> \underline{F}
\end{aligned}$$

$(D_1+D_2)^{\frac{3}{2}} \geq \frac{\sqrt{SH}(\alpha_2+\theta_{21}-\theta_{12})(\alpha_1+\alpha_2)}{\frac{9}{5}2\sqrt{2}(\alpha_1-\theta_{21})}$ is needed for $-\underline{D} > \underline{F}$. Therefore, as long as

$$\frac{(\alpha_1+\alpha_2)(\alpha_{\max}+|\theta_{ij}-\theta_{ji}|)}{(\alpha_{\min}+\theta_{\min})(\alpha_{\min}-\theta_{\max})} \leq \frac{18}{5}\sqrt{2} \approx 13.5,$$

$\frac{\partial \pi_{CSC}{}^{s2}}{\partial p_1{}^2}\frac{\partial \pi_{CSC}{}^{s2}}{\partial p_2{}^2} > (\frac{\partial \pi_{CSC}{}^{s2}}{\partial p_1 \partial p_2})^2$.

In summary, the centralized joint decision problem can be proved to be strictly concave in $\boldsymbol{q}$ when

$$\max(\frac{(\alpha_1+\alpha_2)(\alpha_{\max}+|\theta_{ij}-\theta_{ji}|)}{(\alpha_{\min}+\theta_{\min})(\alpha_{\min}-\theta_{\max})}, \frac{(\alpha_{\max}+|\theta_{ij}-\theta_{ji}|)^2}{(\theta_{\min}+\alpha_{min})^2} \leq \frac{18}{5}\sqrt{2}$$

. $\square$