A stochastic grid filter for multi-target tracking

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ABSTRACT

In this paper, we discuss multi-target tracking for a submarine model based on incomplete observations. The submarine model is a weakly interacting stochastic dynamic system with several submarines in the underlying region. Observations are obtained at discrete times from a number of sonobuoys equipped with hydrophones and consist of a nonlinear function of the current locations of submarines corrupted by additive noise. We use filtering methods to find the best estimation for the locations of the submarines. Our signal is a measure-valued process, resulting in filtering equations that can not be readily implemented. We develop Markov chain approximation approach to solve the filtering equation for our model. Our Markov chains are constructed by dividing the multi-target state space into cells, evolving particles in these cells, and employing a random time change approach. These approximations converge to the unnormalized conditional distribution of the signal process based on the back observaions. Finally we present some simulation results by using the refining stochastic grid (REST) filter (developed from our Markov chain approximation method).

Keywords: multi-target tracking, measure-valued process, filtering equations, Markov chain approximations, REST

1. INTRODUCTION

There is an increasing interest in the area of multi-target tracking due to its wide applications to defense, surveillance, search and rescue, financial markets, and communication networks. Here, we just list some recent research papers in this area: Agate and Sullivan,[1] Ballantyne, Chan, and Kouritzin,[2] Ballantyne, Hailes, Kouritzin, Long and Wiersma,[3] Coraluppi and Grimmett,[4] Hue, Le Cadre, and Pérez,[6] Mahler and Zajic,[11] Zajic and Mahler.[13] Multi-target tracking is a very difficult and challenging problem, especially when the involved targets are weakly interacting. Many existing methods for multi-target tracking are valid only for independent targets. But, in reality, we often encounter situations in which targets have correlation and interaction. In this paper, we would like to develop a new filtering method to solve the interacting multi-target tracking problem. In our simulation study, we consider the submarine model with several submarines and the observation data are collected from certain number of sonobuoys equipped with hydrophones. From our simulation results, it shows that our new Markov chain approximation method performs well.

Here, we mention some existing works for multitarget tracking by Mahler and Zajic,[11] Orton and Fitzgerald,[12] Hue, Le Cadre and Pérez[6] and Ballantyne, Chan and Kouritzin.[2] Mahler and Zajic[11] were interested in finding the unknown number of targets in the region of state space rather than the exact location of each target, and used the *probability hypothesis density* (PHD) to design an approximate multitarget filter to estimate the expected

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number of targets. Orton and Fitzgerald[12] applied independent partition particle filter method for tracking the direction-of-arrival (DOA) of multiple moving targets. In the resampling step, particles with high weights are copied many times, and those with low weights only a few, or not at all. Hue, Le Cadre and Pérez[6] proposed the multitarget particle filter (MTPF), which combines the two major steps (prediction and weighting) of the classical particle filter with a Gibbs sampler-based estimation of the assignment probabilities. They also added two statistical tests to decide if a target has appeared or disappeared from the surveillance area.

We consider an interacting multi-target tracking problem, where our targets are constrained to live within the closure $\bar{D}$ of a d- dimensional rectangular region $D = (0, L_1) \times \cdots \times (0, L_d)$. Indeed, we assume that the states of our m-target system satisfy the following model:

$$dX_t^k = b(X_t^k, S_t)dt + \sigma(X_t^k, S_t)dW_t^k \tag{1}$$
$$X_0^k = x_0^k, \quad k = 1, \cdots m, \quad S_t = \sum_{k=1}^{m} \delta_{X_t^k},$$

where b is the drift coefficient, σ is the diffusion coefficient matrix, δ_x is the Dirac measure at x, $\{W^k, k = 1, \cdots, m\}$ are independent d-dimensional Brownian motions, and $a = \sigma\sigma^T$. The coefficients b and σ are given in certain ways so that all the targets are kept inside the region D.

We assume that the r-dimensional observation process Y is given by

$$Y_i = h(S_{t_i}) + V_i, \qquad t_i = i\varepsilon, \tag{2}$$

where h is bounded and continuous, $\varepsilon > 0$, and V_i' are mutually independent $(\mathbf{0}, \tau^2\mathbf{I})$ - Gaussian random variables which are independent of $W^k (k = 1, \cdots, m)$.

We now fix some notations. For a Polish space E, we denote the space of bounded continuous functions on E by $C_b(E)$. Let $D([0,T], E)$ denote the set of all cadlag functions from $[0,T]$ into E, and take $\mathcal{P}(E)$ and $\mathcal{M}_f(E)$ to denote the spaces of probability measures and positive finite measures on E, respectively.

For $\mu_1, \mu_2 \in \mathcal{M}_f(D)$, the Wasserstein metric is defined by

$$\rho(\mu_1, \mu_2) = sup\{|\langle\phi, \mu_1\rangle - \langle\phi, \mu_2\rangle| : \phi \in B\},$$

where

$$B = \{\phi : |\phi(x) - \phi(y)| \le |x - y|, |\phi(x)| \le 1, \forall x, y \in \bar{D}\}.$$

We set

$$\mathcal{M}_c^m(\bar{D}) = \{\mu \in \mathcal{M}_f(\bar{D}) : \mu = \sum_{k=1}^{m} \delta_{x_k}, x_k \in \bar{D}, i = 1, \cdots, m\}.$$

Then $\mathcal{M}_c^m(\bar{D})$ is the state space of the stochastic process $\{S_t, t \ge 0\}$, and $(\mathcal{M}_f(\bar{D}), \rho)$ and $(\mathcal{M}_c^m(\bar{D}), \rho)$ are complete and separable compact metric spaces. Our goal is to find the best estimate for the location of each target based on the observations Y by filtering methods.

This paper is organized as follows. In Section 2, we first list some fundamental theoretical results, such as the Kallianpur-Striebel formula, the Zakai equation (cf. Zakai[14]) for weakly interacting multi-target systems and its asymptotic equation. In Section 3, Markov chain approximations are studied as means to implement the filter. In Section 4, a submarine model and observation model are described in detail. In Section 5, simulation data is presented by calculating the mean-square errors (the distance between our filter and the signal state). Lastly, in Section 6, conclusions are presented.

2. FILTERING EQUATIONS FOR MULTI-TARGET SYSTEMS

Let us fix a complete probability space $(\Omega, \mathcal{F}, P)$ on which all stochastic processes are defined.

We assume that the coefficients in (1) $b : \bar{D} \times \mathcal{M}_f(\bar{D}) \to R^d$ and $\sigma : \bar{D} \times \mathcal{M}_f(\bar{D}) \to R^d \otimes R^d$ satisfy the following Lipschitz conditions:

For each $x_1, x_2 \in \bar{D}$, $\mu_1, \mu_2 \in \mathcal{M}_f(\bar{D})$,

$$|b(x_1,\mu_1) - b(x_2,\mu_2)| \le K(|x_1 - x_2| + \rho(\mu_1,\mu_2))$$

and

$$||\sigma(x_1,\mu_1) - \sigma(x_2,\mu_2)|| \le K(|x_1 - x_2| + \rho(\mu_1,\mu_2))$$

for some constant K, where $||\cdot||$ denotes the matrix norm. Then from the arguments used by Kotelenez,[8] the equation (1) has a unique solution $X = (X^1, \cdots, X^m)^T \in C([0,T]; \bar{D}^m)$ a.s., which is an $\bar{D}^m$-valued Markov process.

We define

$$L(\mu)f(x) = \frac{1}{2}\sum_{i,j=1}^{d} a_{ij}(x,\mu)\frac{\partial^2 f(x)}{\partial x_i \partial x_j} + \sum_{i=1}^{d} b_i(x,\mu)\frac{\partial f(x)}{\partial x_i}, \quad f \in \mathcal{D}(L) = C_b^2(\bar{D}).$$

Let

$$\mathcal{D}(A) = \{F : F(\mu) = \Phi(\langle f_1,\mu\rangle, \cdots, \langle f_n,\mu\rangle), \Phi \in C_0^2(R^n), f_1, \cdots, f_n \in \mathcal{D}(L), n \ge 1\},$$

where $\langle f,\mu\rangle = \int f d\mu$. Then, for any $F(\cdot) = \Phi(\langle f_1,\cdot\rangle, \cdots, \langle f_n,\cdot\rangle) \in \mathcal{D}(A)$, we define

$$\begin{aligned} AF(\mu) \quad = \quad & \sum_{p=1}^{n} \frac{\partial \Phi}{\partial z_p}\langle L(\mu)f_p(\cdot),\mu\rangle \\ & + \frac{1}{2}\sum_{p,q=1}^{n}\frac{\partial^2 \Phi}{\partial z_p \partial z_q}\sum_{i,j=1}^{d}\left\langle a_{i,j}(\cdot,\mu)\frac{\partial f_p(\cdot)}{\partial x_i}\frac{\partial f_q(\cdot)}{\partial x_j},\mu\right\rangle, \end{aligned} \tag{3}$$

where $\Phi = \Phi(z_1, \cdots, z_n)$ and $f_p = f_p(x_1, \cdots, x_d)$.

An application of Itô's formula to $F(S_t)$ for $F \in \mathcal{D}(A)$ yields that

$$F(S_t) - \int_0^t AF(S_u)du$$

is a martingale. Hence A is the generator of S_t. For a real number a, we denote by $\lfloor a \rfloor$ the greatest integer not more than a. We let $\mathcal{F}_t^S = \sigma(S_s, 0 \le s \le t)$, $\mathcal{F}_t^Y = \sigma(Y_{t_i}, i = 1, \cdots, \lfloor t/\varepsilon \rfloor)$, and $\mathcal{F}_t = \mathcal{F}_t^S \vee \mathcal{F}_t^Y$. We define the optimal filter by

$$\pi_t(F) = E[F(S_t)|\mathcal{F}_t^Y], \qquad \forall F \in C_b(\mathcal{M}_c^m(\bar{D})), \tag{4}$$

which is the least mean-square estimate of $F(S_t)$ given all the observations up to time t. Since the optimal filtering process $\{\pi_t\}$ satisfies a nonliner stochastic evolution equation, we introduce a new probability measure to get a linear stochastic equation which is much easier to solve numerically. For this, we define

$$\xi_i(\mu) = \exp\left\{\frac{\langle h(\mu), Y_i\rangle - \frac{1}{2}|h(\mu)|^2}{\tau^2}\right\},\ \mu \in \mathcal{M}_c^m(\bar{D}),$$

$$\bar{\xi}_i = \xi_i - 1,$$

and

$$\eta_t = \prod_{i=1}^{\lfloor t/\varepsilon \rfloor} \xi_i(S_{t_i}).$$

We define a new probability measure Q by

$$\left.\frac{dQ}{dP}\right|_{\mathcal{F}_t} = \eta_t^{-1}.$$

Then, under Q, $Y_{t_i}, i = 1, \cdots, \lfloor t/\varepsilon \rfloor$ are mutually independent $(\mathbf{0}, \tau^2 \mathbf{I})-$Gaussian random variables which are independent of $\{S_s, 0 \le s \le t\}$, and $\{S_s, 0 \le s \le t\}$ has the same distribution as under P. Then, we have the Kallianpur-Striebel formula

$$\pi_t(F) = \frac{E^Q[F(S_t)\eta_t|\mathcal{F}_t^Y]}{E^Q[\eta_t|\mathcal{F}_t^Y]} := \frac{\sigma_t(F)}{\sigma_t(1)}, \quad \forall F \in C_b(\mathcal{M}_c^m(\bar{D})), \tag{5}$$

where $\sigma_t(F) = E^Q[F(S_t)\eta_t|\mathcal{F}_t^Y]$ is an unnormalized filter. The process σ_t satisfies the following weak form of Zakai equation: for all $F \in \mathcal{D}(A)$,

$$\sigma_t(F) = \pi_0(F) + \int_0^t \sigma_s(AF)ds + \sum_{i=1}^{\lfloor t/\varepsilon \rfloor} \sigma_{t_i}\left(F \cdot \frac{\bar{\xi}_i}{\xi_i}\right), \quad a.s. \;\; Q. \tag{6}$$

In next section, we are going to discuss Markov chain approximations to (6).

3. MARKOV CHAIN APPROXIMATIONS

The Markov chain approximation discussed in this paper is motivated by the stochastic particle models of chemical reaction with diffusion. We refer to Kouritzin, Long and Sun[9] as well as Jha, Kouritzin and Kurtz[7] for some historical comments and recent results.

Let $D = (0, L_1] \times \cdots \times (0, L_d]$, and $D_N = \{j = (j_1, \cdots, j_d) \in \mathbf{N}^d : 1 \le j_i \le N \text{ for each } 1 \le i \le d\}$. For $j = (j_1, \cdots, j_d) \in D_N$, we take $C_j^N = (\frac{(j_1-1)L_1}{N}, \frac{j_1 L_1}{N}] \times \cdots \times (\frac{(j_d-1)L_d}{N}, \frac{j_d L_d}{N}]$ and $C^N = \{C_j^N, j \in D_N\}$. For $J = (J_1, \cdots, J_m) \in D_N^m$ satisfying $J_k \ne J_i$ for $k \ne i$, we set

$$M_J^N = \{\mu \in \mathcal{M}_c^m(D); \mu = \sum_{k=1}^m \delta_{x_k}, x_k \in C_{J_k}^N, k = 1, \cdots, m\}.$$

so $M_J^N = M_{J'}^N$ if $J' = (J_1', \cdots, J_m')$ is any permutation of $J = (J_1, \cdots, J_m)$. Let $[J_1, \cdots, J_m]$ denote the set of all permutations of $J_1, \cdots, J_m$, and let

$$\mathbb{J}_N = \{J = [J_1, \cdots, J_m]; J_k \in D_N, k = 1, \cdots, m, J_k \ne J_i \quad for \quad k \ne i\},$$

then $\mathcal{M}_c^m(D) = \bigcup_{J \in \mathbb{J}_N} M_J^N$. For $J = [J_1, \cdots, J_m] \in \mathbb{J}_N$, $J_k = (J_k^{(1)}, \cdots, J_k^{(d)}) \in D_N$, $k = 1, \cdots, m$, let $Z_{J_k}^N = \left(\frac{J_k^{(1)} L_1}{N}, \cdots, \frac{J_k^{(d)} L_d}{N}\right)$ and $\nu_J^N = \sum_{k=1}^m \delta_{Z_{J_k}^N}$. For $1 \le i \le d$, let $e_i = (0, \cdots, 0, 1, 0, \cdots, 0)$ with 1 in the i-th coordinate.

Let $\mathcal{D}_0 = \{F : F(\mu) = \prod_{i=1}^n \langle f_i, \mu \rangle, f_i \in \mathcal{D}(L), i = 1, \cdots, n, n \ge 1\}$. For $F(\cdot) = \prod_{i=1}^n \langle f_i, \cdot \rangle \in \mathcal{D}_0$, we have

$$\begin{aligned} AF(\mu) \;\; &= \;\; \sum_{p=1}^n \prod_{q \ne p} \langle f_q, \mu \rangle \langle L(\mu) f_p(\cdot), \mu \rangle \\ &+ \frac{1}{2} \sum_{\substack{p,q=1 \\ p \ne q}}^n \prod_{k \ne p,q} \langle f_k, \mu \rangle \sum_{i,j=1}^d \left\langle a_{i,j}(\cdot, \mu) \frac{\partial f_p}{\partial x_i} \frac{\partial f_q}{\partial x_j}, \mu \right\rangle. \end{aligned}$$

Set $D_N^0 = \{j = (j_1, \cdots, j_d) \in D_N; 2 \le j_1, \cdots, j_d \le N-1\}$. We introduce difference operators as follows:

$$\triangledown_i^N f(x) \equiv \frac{N}{L_i}[f(x + \frac{L_i}{N} e_i) - f(x)] \approx \frac{\partial f(x)}{\partial x_i},$$

and

$$\triangledown_{ij}^N f(x) \equiv \frac{N^2}{L_i L_j}[f(x + \frac{L_i}{N} e_i) - f(x + \frac{L_i}{N} e_i - \frac{L_j}{N} e_j) - f(x) + f(x - \frac{L_j}{N} e_j)] \approx \frac{\partial^2 f(x)}{\partial x_i \partial x_j}.$$

Let

$$\langle f, \mu\rangle_N \equiv \sum_{j\in D_N^0} f(Z_j^N)\mu(C_j^N) \approx \langle f, \mu\rangle,$$

where $Z_j^N = (\frac{j_1 L_1}{N}, \cdots, \frac{j_d L_d}{N})$. Now, we define

$$L^N(\mu)f(\cdot) = \frac{1}{2}\sum_{i,j=1}^{d} a_{ij}(\cdot,\mu) \bigtriangledown_{ij}^N f(\cdot) + \sum_{i=1}^{d} b_i(\cdot,\mu) \bigtriangledown_i^N f(\cdot) \tag{7}$$

and

$$\begin{aligned} A^N F(\mu) &= \sum_{p=1}^{n}\prod_{q\neq p}\langle f_q,\mu\rangle_N \langle L^N(\mu) f_p, \mu\rangle_N \\ &+ \frac{1}{2}\sum_{\substack{p,q=1\\ p\neq q}}^{n} \prod_{k\neq p,q}\langle f_k,\mu\rangle_N \times \sum_{i,j=1}^{d}\langle a_{ij}(\cdot,\mu) \bigtriangledown_i^N f_p \bigtriangledown_j^N f_q, \mu\rangle_N \end{aligned} \tag{8}$$

We want to find numerical solution to the Zakai equation (6) by using Markov chain approximation approach. For each $J = [J_1, \cdots, J_m] \in \mathbb{J}_N$, we take $F(\cdot) = \prod_{k=1}^{m}\langle 1_{C_{J_k}^N}, \cdot\rangle$ to implement the algorithm. Let l_N be a function of N such that $l_N \to \infty$ as $N \to \infty$. We set

$$n_J^N(0) = \lfloor l_N \mu_0(M_J^N)\rfloor = \lfloor l_N P(S_0 \in M_J^N)\rfloor. \tag{9}$$

We distribute $n_J^N(0)$ particles into cell J according to the initial distribution of the signal process. Particles in each cell will evolve by undergoing births or deaths from observations, diffusion, and drift. Then, our Markov chain approximation is given by

$$\sigma_t^N F = \sum_{J\in\mathbb{J}_N} \frac{n_J^N(t)}{l_N} F(\nu_J^N), \qquad \forall F \in \mathcal{D}_0 \tag{10}$$

where $1/l_N$ denotes the mass of each particle and $n_J^N(t)/l_N$ can be considered as the concentration or density of particles in cell J. We can prove that $\sigma_t^N F$ converges to the solution of (6) as $N \to \infty$. The convergence analysis will be presented in a seperate paper.

4. SUBMARINE MODEL

4.1. Signal model

We assume that there are m targets (submarines) randomly moving in a certain ocean region. We denote by (x, y) the location, v the velocity, and θ the orientation of each target. The motion model of the i-th target satisfies the following SDE's:

$$dx_t^i = v_t^i cos\theta_t^i dt + \sigma_x dW_t^{x,i}, \tag{11}$$

$$dy_t^i = v_t^i sin\theta_t^i dt + \sigma_y dW_t^{y,i}, \tag{12}$$

$$dv_t^i = (avg(v) - v_t^i)dt + \sqrt{(max(v) - v_t^i)(v_t^i - min(v))}dW_t^{v,i}, \tag{13}$$

$$d\theta_t^i = \left[f_y^i(t)cos(\theta_t^i) - f_x^i(t)sin(\theta_t^i)\right]\theta_f dt + \sigma_\theta dW_t^{\theta,i}, \tag{14}$$

where σ_x, σ_y, σ_θ and θ_f are constants, $W_t^{x,i}, W_t^{y,i}, W_t^{v,i}$ and $W_t^{\theta,i}$ $(i = 1, \cdots, m)$ are independent Brownian motions, and

$$f_x^i(t) = f_x^{i,S}(t) + f_x^{i,A}(t) + f_x^{i,C^1}(t) + f_x^{i,C^2}(t),$$

provided that

$$f_x^{i,S}(t) = \sum_{\substack{k=1 \\ k\neq i}}^{m} \frac{S_f(x_t^i - x_t^k)}{(x_t^i - x_t^k)^2 + (y_t^i - y_t^k)^2},$$

$$f_x^{i,A}(t) = \sum_{k=1}^{m} \frac{A_f}{\sqrt{m}\left[1 + (x_t^i - x_t^k)^2 + (y_t^i - y_t^k)^2\right]} cos(\theta_t^k),$$

$$f_x^{i,C^1}(t) = \frac{C_f^1}{\sqrt{m}} \cdot \frac{(\bar{x}_t - x_t^i)}{\sqrt{(\bar{x}_t - x_t^i)^2 + (\bar{y}_t - y_t^i)^2},}$$

where

$$\bar{x}_t = \frac{1}{m}\sum_{k=1}^{m} x_t^k, \quad \bar{y}_t = \frac{1}{m}\sum_{k=1}^{m} y_t^k,$$

$$f_x^{i,C^2}(t) = \frac{max(x)}{x_t^i} - \frac{max(x)}{max(x) - x_t^i} \cdot C_f^2,$$

where $max(x)$ is the maximum value of x in the domain, S_f, A_f, C_f^1, and C_f^2 are constants.

Similarly,

$$f_y^i(t) = f_y^{i,S}(t) + f_y^{i,A}(t) + f_y^{i,C^1}(t) + f_y^{i,C^2}(t),$$

provided that

$$f_y^{i,S}(t) = \sum_{\substack{k=1 \\ k\neq i}}^{m} \frac{S_f(y_t^i - y_t^k)}{(x_t^i - x_t^k)^2 + (y_t^i - y_t^k)^2},$$

$$f_y^{i,A}(t) = \sum_{k=1}^{m} \frac{A_f}{\sqrt{m}\left[1 + (x_t^i - x_t^k)^2 + (y_t^i - y_t^k)^2\right]} sin(\theta_t^k),$$

$$f_y^{i,C^1}(t) = \frac{C_f^1}{\sqrt{m}} \cdot \frac{(\bar{y}_t - y_t^i)}{\sqrt{(\bar{x}_t - x_t^i)^2 + (\bar{y}_t - y_t^i)^2},}$$

and

$$f_y^{i,C^2}(t) = \frac{max(y)}{y_t^i} - \frac{max(y)}{max(y) - y_t^i} \cdot C_f^2,$$

This motion model characterizes the following steering behaviors of multiple targets: separation, alignment, cohesion, and containment. Separation means that each target should steer to avoid collision with other targets. Alignment means that all the targets should steer towards the average heading. Cohesion means that each target should move toward to average position of all the targets. Containment is the force to keep all of our targets inside the region.

For simplicity, we assume that $x \in [0,1], y \in [0,1], v \in [min(v), max(v)]$, and $\theta \in [0, 2\pi]$. We can easily rewrite the above submarine model as in the form of SDE (1).

4.2. Observation model

We assume that there are p sonobuoys (equipped with hydrophones) deployed in the underlying ocean region to track m targets (submarines). From the sonar contact data, we can get sensor measurements about the distance r_{ij} from the i-th sonobuoy to the j-th target, and the counter-clockwise orientation α_{ij} of j-th target relative to the i-th sonobuoy. For convenience, we denote by (x_R^i, y_R^i) the location of the i-th sonobuoy, and by $(x_{t_k}^j, y_{t_k}^j)$ the location of the j-th target at time t_k. The observation model is given by:

$$Y_k^{i,j} = h(x_R^i, y_R^i; x_{t_k}^j, y_{t_k}^j) + v_k^{i,j}, \ i = 1, \cdots, p, \quad j = 1, 2, \cdots, m, \tag{15}$$

where $h(\cdot) = (h_1(\cdot), h_2(\cdot))^T$ with

$$h_1(x_R^i, y_R^i; x_{t_k}^j, y_{t_k}^j) = \sqrt{(x_R^i - x_{t_k}^j)^2 + (y_R^i - y_{t_k}^j)^2},$$

and

$$h_2(x_R^i, y_R^i; x_{t_k}^j, y_{t_k}^j) = \tan^{-1}\left(y_{t_k}^j - y_R^i, x_{t_k}^j - x_R^i\right),$$

and $v_k^{i,j}$ are independent Gaussian random variables with variance τ_r^2 for the distance and τ_α^2 for the orientation. In our simulation, we set $m = 2$ and $p = 4$.

5. SIMULATIONS

5.1. Simulation description

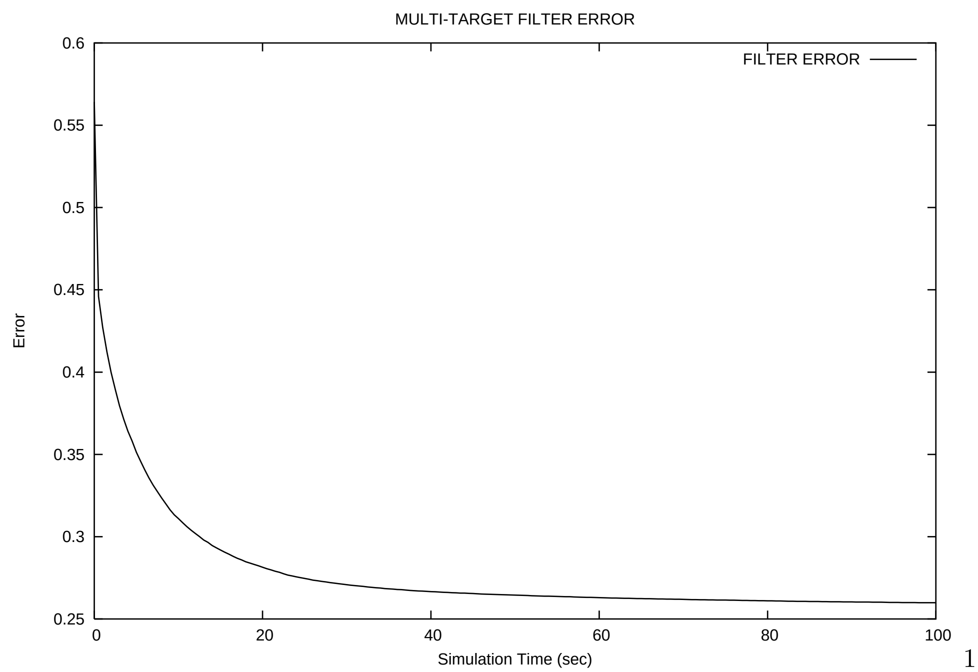

Figure 1. 2-target simulation with the REST filter

We ran our REST filter with a simulated signal with two submarines. Due to limited computational resources, only 10 runs of 200 iterations were performed. For each iteration we measure the filter error. These error values are presented in Figure. 1.

Each simulation starts at time $t_0 = 0$ time units and progresses to $t_{100} = 200$ time units, with iterations of 0.5 time units each.

Observations have four sonabuoys and are constructed at time intervals $t_{k+1} - t_k = 0.5$ time units.

The results showed that the filter effectively localized the signal fairly quickly. With further refinements we expect the filter will be able to use a finer grid, resulting in even lower error.

5.2. Comparison method

After every observation update the filter is evaluated in terms of an estimate error value. Normally, the error of a filter approximation would be measured with a MSE defined by:

$$\text{MSE}(t_k) = \frac{1}{r_{\max}} \sum_{r=1}^{r_{\max}} \text{d}(S_{t_k}^r, E[S_{t_k}^r | Y_1^r, \ldots, Y_k^r])^2, \tag{16}$$

where S^r, Y_k^r are the signal path and observations from run number r, $r_{\max}$ is the total number of simulation runs, and d is some distance function defined on the signal domain. However, in the case of multiple targets in which the signal is a counting measure rather than a single point, and in which a distance between two counting measures or the mean of a set of counting measures which may contain differing number of points has no usual definition, no standard MSE calculation is possible and a different value for filter error must be defined.

We simply match each target in the signal, to the closest target in the filter estimate, and use a standard distance function to determine the error.

6. CONCLUSIONS

In this paper, we develop a novel Markov chain approximation method for tracking weakly interacting multiple targets. Due to interaction between targets, our signal is an infinite dimensional empirical measure-valued process which makes the tracking very difficult. Many existing methods are inapplicable to this case. Our simulation result shows that our new method works well even though the computation is rather intensive. In the near future, we will try to simulate for signal model with more targets and compare computational efficiency of our method with other recently developed methods such as refining branching paricle filter and hybrid weighted interacting particle filter methods.

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