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An inquiry into collaborative learning in middle school mathematics
by

Sherry Talbot

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ABSTRACT

This study looks at how middle school students constructed understanding by working in collaborative groups in mathematics classes. Collaborative learning supports students' needs to work together to exchange and refine ideas and to socialize with their peers.

During the study a sixth grade mathematics class was given a variety of tasks designed to develop their understanding of multiplication and division, and their responses and work on three collaborative tasks were looked at. It emerged that the qualities of the tasks had an effect on the learning that was occurring. These tasks were analysed within a frame of complexity science, looking at the components of redundancy, internal diversity, organized randomness or liberating constraints, decentralized control, and neighbour interactions.

The story of learning told by the students' interactions while working together on the tasks supports the assertion that collaborative learning is beneficial in helping students construct understanding of mathematical concepts.

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CHAPTER ONE: AN INTRODUCTION TO MY INQUIRY

My area of interest in math education is two-fold. First, it is about how students construct understanding of mathematics concepts through interactive projects and problem solving. The other aspect is how learning situations affect students' attitudes toward mathematics. This study tells the story of my inquiry into collaborative learning in a sixth-grade mathematics class, focussing on how activities and students' interactions create opportunities for constructing understanding and fulfilling socialization needs.

As a secondary mathematics teacher I could see that learning mathematics has many dimensions, and the statistics on achievement being presented to me did not seem to be matching what my experience told me was happening in mathematics classrooms. Upon returning to post-secondary education following many years of teaching, I found that there was a large gap between what researchers at the university understand about student achievement and learning, and in what I was hearing. Most of the research that I had to work with in the schools was statistics centred on improving standardized exam marks, or was offered as support for some new kind of educational program that the administration had decided was important for teachers to implement. I was growing tired of hearing the phrase "Studies show..." and had come to see that meaningful research was not making a connection with teachers, and was not making the kind of impact on learning that it should.

It seemed to me that investigating the inherent significance of classroom situations could illuminate the triumphs and frustrations of educational practice.

Listening to teachers and students in math classes is important because it illustrates how people are able to know and love or hate mathematics. Their actions and interactions often tell us what we have not heard them say. I have seen many statistics on math achievement that indicate students can do math well, but when I talked to them I could "see" that they were confused about concepts and were cynical about mathematical knowledge. By investigating students' experiences while working on mathematics projects and problems we can "see" further into their understandings and feelings.

As a mathematics teacher in secondary schools I kept abreast of the changes in curriculum, taking part in teacher in-services and implementing the use of manipulatives, group projects and criterion-based assessment tools. Upon returning to post-secondary education and studying the research that underpins the changes in our understanding of how students learn mathematics, I began to see that there is more to students constructing understanding than putting them in groups or giving them tools. I could see that the intention and structure of the activity and the interaction and reaction of both students and teachers were integral to not only individual meaning-making but also to whole group learning.

The purpose of this study is to investigate the assumption that collaborative learning is beneficial in helping students construct understanding of mathematical concepts. As part of the study I consider how collaborative work is supported by adolescents' need to socialize and to feel that they are a part of a positive learning environment.

My study is structured in this way. I begin in Chapter Two by looking at what research has revealed about the changes in our understanding of how students learn mathematics and how this has, or should have, impacted what happens in classrooms. I look at what research shows us about the influence of sociocultural aspects of mathematics classrooms and what they can contribute to learning. These aspects are brought together in collaborative learning and I pose the questions guiding my research. In Chapter Three I consider aspects of implementing collaborative learning and the influence of complexity science. I explain the methodology that I used and the class that I worked with in my study in Chapter Four. I describe the work that I did with the class in Chapter Five, starting with the plan and moving through the tasks and activity that made up the study. In Chapter Six I look at how the students worked to understand the mathematics in three main collaborative tasks. I do this through a frame of complexity science, connecting the class' activities to the conditions of complexity theory. In Chapter Seven I conclude my investigation and look at how activities provided opportunities for socialization and collaboration, thereby promoting understanding in mathematics classes. Chapter Eight provides a look at how my study evolved and what the implications are for teaching practice.

CHAPTER TWO: THE SIGNIFICANCE OF RESEARCH IN MATHEMATICS EDUCATION

Skill and understanding in mathematics is regarded as important to success in our increasingly technological world. Therefore research into how we learn mathematics is needed to inform curriculum and standards in our schools. In 1989 the National Council of Teachers of Mathematics (NCTM), which claims to be the world's largest mathematics education organization, published what has become an influential document, *Curriculum and Evaluation Standards for School Mathematics*, emphasizing a paradigm shift in the way that mathematics is taught in North American schools. The foundation of this shift emphasizes the importance of student understanding and thinking processes, focussing on problem-solving, reasoning and communication rather than the structure of mathematical knowledge (http://www.nctm.org/standards/default.aspx?id=58).

Research in mathematics education has supported the move from traditional teaching methods to current social constructivist learning discourses. Ben-Hur (2006) states that "a plethora of research has established that concepts are mental structures of intellectual relationships, not simply a subject matter... [with] the attention of research in education... shifting from the content (e.g., mathematical concepts) to the mental predicates, language, and preconcepts" (p. 4).

My education in mathematics began in the fifties and was very traditional. Although I became a secondary mathematics teacher in the seventies, I was away from the field for some time, teaching language to adults. I did not return to teaching mathematics to children until just after the reforms were introduced in the late eighties. The curriculum had changed and I found myself in a situation where I needed to construct a new understanding of the spirit of mathematics education and learning, and of how I might structure my lessons to encompass these changes. For me this learning process involved coming to know how the focus of mathematics education had shifted, how this shift was changing classrooms, how the sociocultural characteristics of adolescents influenced and were impacted by these changes, and how I could provide effective learning environments for my students.

A Paradigm Shift in Understanding How Students Learn Mathematics

The paradigm shift in mathematics is part of an overall shift in awareness of how people learn by constructing understanding of their world, often referred to as constructivism. In psychology, constructivism is associated with the work of Jean Piaget, and refers to the process by which the cognitive structures that shape our knowledge of our world evolve through the interaction of environment and subject (Marshall, 1998, p. 609). Constructivist theory in education indicates that learning is an active process in which learners construct new ideas or concepts based upon their current/past knowledge. The learner selects and transforms information, constructs hypotheses, and makes decisions, relying on a cognitive structure to do so. Cognitive structure, for example schema and mental models, provides meaning and organization to experiences, and allows the individual to "go beyond the information given". In the classroom the teacher would encourage students to discover principles by engaging in an active dialog, and translate information to be learned into a format appropriate to the learner's current state of understanding.

In the NCTM's *Journal for Research in Mathematics Education*, published in December 1994, Kieran looked at the movements in math education over the past twenty-five year period, presenting an interview with Thomas Kieren from the University of Alberta and Thomas Romberg from the University of Wisconsin, both of whom had been extensively involved for many years in mathematics education research. Their comments exemplify how research has evolved from looking empirically at students' behaviour to trying to understand how learners are thinking. "This evolution has been accompanied by the development of research approaches that emphasize the observation of the processes of learning rather than the measurement of their products" (p. 605). They discussed how research on constructivist learning has shown that understanding is an ongoing activity, not an achievement, and how this activity takes place in a social context. Kieran also investigated the newest shift in research on constructivist perspectives "from an individual-cognitive Piagetian framework to a social-interactionist Vygotskian one" (p.605). The major theme of Vygotsky's Social Development Theory is that social interaction plays a fundamental role in the development of cognition with every function in the child's development, appearing first between people and then inside the child. This theory of learning is referred to as social constructivism. Paul Ernest (1996) maintains "that mathematics is corrigible, fallible and a changing social product... like any other branch of knowledge" (p. 3) establishing the basis for a social constructivist philosophy of mathematics education.

In mathematics education in Western Canada we saw the influence of constructivist learning theory when in 1993 Alberta became partners with Manitoba, Saskatchewan, British Columbia, Yukon Territory and the Northwest Territories to develop the *Western Canadian Protocol for Collaboration in Basic Education Kindergarten to Grade 12* with the aim of establishing a common framework for education in western Canada. The foundation for the framework's philosophy in mathematics came directly from the NCTM *Standards*, and in 1995 the *Common Curriculum Framework for K-12 Mathematics* was implemented. (Western Canadian Protocol, 1995, p. 4-15) In my work as a secondary math teacher in Alberta I became very familiar with the *Common Curriculum Framework for K-12 Mathematics* and with the work of the NCTM. These works – the *Protocol* and the *Standards* – became the foundation for the development of my understanding of how students learn mathematics.

The Changes in Mathematics Classrooms

Progress in understanding how students think about and learn mathematics has changed research in mathematics education as well as curriculum and standards. This, then, should lead to changes in teaching practice and in classrooms. Whether these changes are making an impact in classrooms is not as clearly indicated. From my own experience here in Alberta, the *Western Canadian Protocol [WCP]* has certainly been the basis for professional development in math, but, other than using a variety of new textbooks that aligned lessons with the objectives in the *WCP*, work in math classrooms has, in my experience, continued to be very much the same as it has always been, with teacher-led explanations and student practice. My impressions of the sluggish adaptation of classroom practice are supported by mathematics education research.

In 1999 James Hiebert addressed the role that research can and should play in shaping standards. Although he found that the *Standards* are consistent with research evidence on teaching and learning, that in itself is not enough. There needs to be an understanding of the process of how that conclusion has been reached. Hiebert discussed the limitations of research and revealed what research has shown about the current state of classroom teaching by stating that "we have a quite consistent, predictable way of teaching mathematics in the United States and that we have used the same basic methods for nearly a century… 'Teachers are essentially teaching the same way they were taught in school'… And, in the midst of current reforms, the average classroom shows little change" (p. 11). Traditional methods emphasize teaching procedures with little attention to helping students construct conceptual ideas, and, as a result, they are learning simple calculation procedures, terms and definitions rather than how to solve new problems or engage in other mathematical processes. Research shows that instructional programs can be designed to facilitate constructive learning. However, often these alternative programs are seen to be "experimental" while traditional programs are seen to be "proven". According to Hiebert, the main reason that there has been a lack of implementation of the

Standards is that it is difficult to change the way we teach. The reforms require substantial changes that require teachers to learn new methods, and teachers have few opportunities to do so. Not only do students need to construct understanding, teachers must as well.

In Alberta we do not face as many obstacles with implementing changes in mathematics education as many teachers do in the U.S. We have a consistent program of studies for all schools with the *WCP,* and because of our different tax structure there may be more professional development opportunities for teachers. For example, over the past several years there have been grants for some school initiatives from the Department of Education to support professional development for math teachers (AISI)[1]. However, students' success is still measured through a provincial standardized testing program, encouraging many schools to focus their teaching strategies toward doing well on those tests rather than improving learning situations in classrooms. It also seems to me that other factors, the intensification of teachers' work, inconsistent support, and the inaccessibility of research information, have been detrimental to any significant movement toward change. Mathematics teachers have been teaching and assessing mathematics concepts and processes in a traditional way for a long time, and, in Alberta, they have been reassured that their work is effective through the publication of the high scores that their students earn in national and international exams.

[1] AISI, Alberta Initiative for School Improvement, is a funding program developed by the Alberta Education department. The goal of AISI is to improve student learning and performance through professional development projects in schools. Money for these projects, including several in mathematics, is additional to regular school funding.

In my experience in secondary schools I have found that many teachers remain unaffected by mathematics education reform and do not understand how important it is for student learning. It had not been at all clear that the changes in curriculum were based on solid results from research. It was reassuring for me to find that Battista (1999) has asserted that:

> All current major scientific theories describing students' mathematics learning agree that mathematical ideas must be personally constructed by students as they try to make sense of situations... More than two decades of scientific research in mathematics education have refined the constructivist view of mathematics learning to provide detailed explanations of how students construct increasingly sophisticated ideas about particular mathematical topics, of what students' mathematical experiences are like, of what mental operations give rise to those experiences, and of the sociocultural factors that affect students' construction of mathematical meaning. (p. 429)

I could see that not only did the students' experiences with mathematics in the classroom make a difference, but that social experiences affected their learning as well.

Socialization Aspects of Mathematics Classrooms

There are sociocultural aspects of the educational experiences that we provide for secondary students that impact learning during adolescence. In examining how we structure learning situations that provide the opportunities for students to construct understanding by collaborating, we must take into consideration changes that students experience during adolescence and how they influence the structure of mathematics classrooms.

An investigation into classroom peer factors and their role in adolescents' sense of belonging in mathematics classrooms by Hamm and Faircloth (2005) found that developing a sense of belonging is not only central to healthy adjustment, it is foundational to students' motivation and achievement (p.345). "Mathematics classrooms, in particular, appear to present a challenging social context for early adolescents ... students seek but struggle to find a sense of belonging within their mathematics classrooms because of the emphasis on individualism common to traditional mathematics instruction" (p.346). Students indicated that dislike of their math classes stemmed from the "lack of opportunity to support, and be supported by, their classmates, and to interact with one another on mathematical tasks" (p.346). Hamm and Faircloth contend that the individualism and competition found in traditional mathematics classes caused students to become disengaged, and that it is particularly important for middle-years students to experience support, encouragement and acknowledgement within the classroom community. This study indicates that developing classrooms which encourage constructivist learning in middle school requires more than a variety of problem solving activities that encourage students to interact. It also requires a classroom culture that accommodates the special attributes of adolescents, and teachers who listen carefully to what the students have to say about their views and experiences, inviting them to identify aspects of schooling that get in the way of learning.

Considering Research into Collaborative Learning

Having students collaborate and interact with each other to discover and apply mathematics concepts requires attention to several aspects of classroom dynamics. We must look at the types of tasks and activity that students can engage in; how teachers structure activities and provide opportunities for interaction, how teachers interact with student groups, how students' social development and needs impact learning activity, how students' ideas develop through contact with other ideas, and how the class as a whole changes as ideas emerge. Stories of learning experiences and researchers' observations have underlined the importance of providing these collaborative learning opportunities in our mathematics classes.

The Development of Effective Learning Environments in Mathematics

Research on constructing understanding in mathematics underscores the importance of moving away from traditional practices that stifle learning and turn students away from mathematics. Schifter and Fosnot (1993) explain that constructivist learning is "primarily a process of concept construction and active interpretation – as opposed to the absorption and accumulation of received items of information" (p.8). This understanding of how people learn has impacted education because "no matter how lucidly and patiently teachers explain *to* their students, they cannot understand for their students" (Schifter and Fosnot, 1993, p.9). They have also indicated what mathematics classrooms that encourage constructivist learning should look like saying that "teaching mathematics must reconceived as the provision of meaningful problems designed to encourage and facilitate the constructive process" (Schifter and Fosnot, 1993, p.9).

Leone Burton (1999) provides us with an explanation of the responsibilities of students and teachers:

> The purpose of schooling in mathematics, then, shifts from the acquisition of knowledge 'objects' to the acquisition and usage of a reflective process of coming to know within a learning community where discourse is prominent (p.31)... I have pointed out that in some areas, at some times, for some pupils, a match will be made between pupils' knowings and the body of knowledge deemed socially desirable - knowing of knowledge. The teacher's responsibility, then, is to facilitate this match where appropriate while, at the same time, ensuring that the energy, confidence and enthusiasm to enquire is nurtured in all learners and that the process is fed by the strengths of the learning community in breadth, depth and heterogeneity. (p.33)

Burton also draws our attention to the fork in the road of mathematics education that we have come to.

> We can continue to demand that all those who are proficient in mathematics provide evidence of one style of thinking and of one validated social practice or we can begin to recognize the realities of learning communities and move to maximize on their potentialities and minimize their disadvantages. For me, the choice is clear. (p.33)

It is for me as well.

The broad question that guided my research was:

Is collaborative learning beneficial in helping students construct understanding of mathematical concepts?

Within this question I explored the questions:

How can students in middle school mathematics construct understanding by investigating problems and concepts collaboratively?

How can the use of materials designed to encourage collaboration and discussion support understanding in middle school mathematics classes?

What is the story of mathematics learning told by students' interactions?

CHAPTER THREE: IMPLEMENTING COLLABORATIVE LEARNING

The effective implementation of constructivist learning opportunities in math classrooms requires an understanding of how students interact with each other and with the concepts. Cobb, Yackel, and Wood (1992) present their approach to classroom practice that treats mathematics learning as a constructive activity within a necessary communal, social milieu and argue that the traditional, representational method of teaching math is not effective for significant learning. They offer support for the definition of teaching as "an activity in which we guide students' constructive efforts, thereby initiating them into taken-as-shared mathematical ways of knowing. Concomitantly, learning would be viewed as an active, constructive process in which students attempt to resolve problems that arise as they participate in the mathematical practices of the classroom" (p. 10). They emphasize that constructivism should not be interpreted as "a process of spontaneous, unguided, independent invention", and that as part of social practice, understanding mathematics evolves from both the student's constructive activities and the community's "taken-as-shared meanings and practices" (p. 27). Cobb et al. firmly promote constructivist learning as a carefully understood process with chosen and guided investigative activities.

The basis of much collaborative learning in mathematics classes is grounded in having students work together to solve problems. Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier and Wearne (1996) apply Dewey's idea of reflective inquiry to facilitate students' understanding. Their basic principle is that students should be allowed to make their math problematic, having students work to solve the problems through peer interactions, and present and discuss solutions. Hiebert et al. assert that "the culture of classrooms will need to change" (p.19). They find that students who engage in reflective inquiry "develop deeper structural understandings of the number system than their peers who move through a more traditional skills-based curriculum" (p.17). Hiebert et al. provide support for the reform of mathematics with a theoretical framework for reflective inquiry, descriptions of problem-solving activities, and evidence of the effectiveness of having students work together on math tasks.

Constructivist learning may be, and has been, interpreted as merely allowing students to go their own way and make interpretations that are not beneficial to their learning. It may also be seen as the teacher allowing the students to just having fun without direction for their learning. In the early nineties there was some reaction to how the new paradigm of constructivist learning was being interpreted by teachers in the classroom. Heaton (1992) presents research demonstrating how a teacher who, by focussing on developing a positive classroom atmosphere and teaching style rather than promoting understanding, taught her students incorrect mathematics concepts. Heaton's purpose was to raise questions "about the role of subject matter and the responsibilities of teachers and inservice programs in reforming mathematics teaching and learning" (Heaton, 1992, p. 162). She emphasizes that teachers must guide students through the development of their understanding.

Teachers must be aware of how students come to understand mathematics concepts when organizing activities promoting investigation and interaction. Fawcett and Garton (2005) investigated the effect of collaborative learning on children's problem-solving ability from both a Vygotskian framework and a Piagetian perspective. They contended that although both of these methods are grounded in having students construct meaning through interacting with others, the Vygotskian framework requires interaction with a more competent partner and the Piagetian perspective requires conflict arising from peer interaction. This has implications for how teachers structure group work in the classroom. Fawcett and Garton studied year two (ages six and seven) children, and found that the two theoretical positions are not as mutually exclusive as they are often portrayed. They did find that "simply assigning students to groups and telling them to work together will not necessarily promote cooperation or achievement ...training children in interactive skills... may be a prerequisite of successful peer collaboration" (p.163).

Looking at how cooperative learning groups assist students with problem-solving, Duren and Cherrington (1992) found that "students who worked cooperatively were able to remember and apply the problem-solving strategies better than those students from the independent practice classes" (p.81). They saw that students working cooperatively were more willing to persevere with problems, verbalized possible strategies and justified solutions, were open to alternative strategies, and received more corrective feedback from peers. They argue that this approach seeks "to minimize student anxiety and competition by creating an environment where students feel safe to make and learn from mistake... giv[ing] students an opportunity to talk aloud" (p.80). Duren and Cherrington's work takes into consideration the feelings and attitudes of middle school students and the importance of the culture of the class.

Complexity Science and Learning

Complexity theory[2] can help us see how to create collaborative situations in the classroom so that learning can emerge. Complexity science helps us to see our world and everything in it as interconnected, "nested" systems and it helps us to realize that everything that happens affects everything else in some way or another. Complexity science takes notice of social interactions as well as physical situations, and through it we can investigate how complex our learning institutions really are. Davis and Simmt (2003) assert that complexity science is best defined in terms of its diverse objects of study which are identified by two key qualities. Firstly, each complex phenomenon, or system, is adaptive, changing its own structure "and as such is better described as in terms of Darwinian evolution rather than Newtonian mechanics". Secondly, it is emergent, "composed of and arises in the co-implicated activities of individual agents... not just the sum of its parts but the product of the parts and their interactions" (p. 138). Because complex systems are continually adapting they are referred to by complexivists as learning systems. Davis and Simmt maintain that mathematics classrooms can be adaptive, self-organizing complex systems, characterized by

[2] I am using the terms "complexity science" and "complexity theory" interchangeably. Much of the literature refers to the idea as complexity science, but in a lecture that I attended in 2007 Brent Davis explained that complexity theory is becoming the term of preference.

conditions that must be present for complex systems to arise and that "understanding the *collective* as a cognizing agent (as opposed to a *collection* of cognizing agents) presents some important advantages" (p. 144).

Any class is a complex system that is adaptive and self-organizing. Although we would like to think it is organized around the curriculum, the learning that is taking place in many classrooms is not that which is intended. There are five conditions that are required in order for a complex system to exist and flourish – redundancy, internal diversity, organized randomness or liberating constraints, decentralized control and neighbour interactions. By understanding the conditions needed for a complex system to thrive and by making sure that they nurture the learning that we want to take place, we can provide the fertile ground for the growth of a mathematical community, a collective learning system. Complexity science can provide the opportunity for us to observe mathematics learning in a social constructivist manner. Furthermore, by attending to these necessary conditions we can affect the transformation of the learning community in ways that promote learning as a socially and culturally situated activity. Davis and Simmt (2003) assert that complexity science has "moved from a focus on description to something more prescriptive… [moved] beyond the question, 'What's happening?' to include the question, 'How can it be made to happen?'… become not just a valuable means to interpret, but a source of practical advice to mathematics teachers" (p.144).

In 1989 William E. Doll Jr. wrote that

> complexity theory may well have as strong an influence on our views about teaching and learning as it is now having on our understanding of

> the physical sciences and mathematics… we are in the midst of an epochal or megaparadigm change, one many label as a move from the abstract formality of modernism to the eclectic creativity of postmodernism. (p. 65)

He goes on to describe how he and a sixth grade mathematics teacher, Ron, used ideas founded in complexity theory to inspire students to solve problems. Doll recounts how they decided to give their students "flexibility in their intellectual and social organization" to solve the problems in their own way and time. He describes how the patterns that emerged were both "disorderly and coherent", how they saw both "randomness and progressive order" in the students' approaches to the work, and how a new type of order emerged – "progressive, constructive, personal, interactive" (p. 66). Doll and Ron strayed from the traditional textbook that they saw as linear, overly simple and encouraging memorization rather than understanding, choosing to construct problems that allowed the students many points of entry and many opportunities to be creative and interact. In conclusion Doll comments that complexity theory gave them "along with challenges and complications, insights into teaching and learning… [to] see how complexity can merge with simplicity requires only that we study and teach our subjects – at any level – with depth, not superficially" (p. 69-70). Doll's study (1989) illustrates that looking at the curriculum through the lens of complexity theory shows us not only how to adjust the curriculum to make it more dynamic, but also how to use the language of complexity to describe and understand the learning that emerges when conditions for complex learning systems are established.

Doll (1989) described how complexity science can help us to go beyond the aim of reproducing mathematical skill and knowledge based capacity to address many other aspects - developing creative capabilities in mathematics, developing empowering mathematical capabilities and critical appreciation of the social applications and uses of mathematics, and developing an inner appreciation of mathematics' big ideas and nature. Elaine Simmt and Brent Davis (2003) explain how complexity theory can help us deliberately create and nurture complex systems in mathematics classrooms by attending to the five conditions that are necessary for complex emergence. The conditions are: means to enable the expression of the diversity that is present; redundancy among agents relative to the issue at hand; mechanisms to prompt ideas to interact; decentralized control; and organized randomness.

Davis and Sumara (2005) explain that:

> learning is an emergent event... the understandings and interpretations that are generated cannot be completely pre-stated, but must be allowed to unfold. Control of outcomes, that is, must be decentralized. They must to some extent emerge and be sustained through shared projects, not through prescribed learning objectives... Complexity cannot be scripted (p. 460).

Considering Adolescent Culture in Constructing Understanding

One of the key factors in setting the stage for collaborative learning is the culture of the classroom – a culture that recognizes the unique features of adolescent learners and the need for opportunities for students to construct understanding.

In discussing the suitability of project problems Wiest (2000) asserts that "[m]iddle-grade students crave independence and individuality at the same time that they become increasingly social" (p.286). Georgakis (1999) discusses how much her students love to solve problems in groups and asserts that "[t]he students are active learners, relying on one another for ideas" (p.224).

In order to have situations in middle school mathematics classrooms that kindle effective learning opportunities there needs to be both a classroom culture that promotes interaction and stimulating mathematics activities. In her work Cathy Humphreys worked with researcher Jo Boaler to develop and film lessons (Boaler and Humphreys, 2005). Humphreys illustrated her beliefs in how the classroom culture and mathematics activities build the foundation for her mathematics teaching. Firstly she believes "that *learning mathematics* means making sense of mathematical relationships... *looking* for patterns, *conjecturing, justifying, analyzing, wondering*, and so on", and secondly "that *teaching mathematics* means... setting up situations that give every student the opportunity to engage in sense making" (p. 11) (author's emphasis). She elaborated with several points to explain how she implements these beliefs in her classroom concluding with the point that "talking and listening to each other (not just the teacher) about mathematical ideas help us understand mathematical ideas in different ways" (p. 12). As teachers strive to change and improve their mathematics teaching they often need support and others to talk to about the challenges they encounter.

The direction for building stimulating learning experiences in mathematics can come from those who are most directly affected by the change. Angier and Povey (1999), tell the story of mathematics reform from the perspective of a teacher and her students. "The intention here is to privilege the voices of the participants, particularly those of young people themselves, whose views and experiences are sometimes absent from educational studies." (p.147). The voices in this article speak in a way that help me feel the effects of the changes they experienced, and of the emotions that arose as they worked and learned together. They also take me back to my own experiences with middle school math students, and help me to understand the importance of developing a culture of learning and living together. The story describes the culture of a classroom that reflects the views and needs of young people in contemporary society, and tells what it means to be a teacher in a secondary mathematics classroom while holding "together curriculum, pedagogy, epistemology and classroom practices and relationships" (p.147). Angier and Povey use the metaphor of "spaciousness" to represent this type of classroom culture.

Angier and Povey's (1999) representation of the classroom as "shifting, contested and problematic", as opposed to uniform and determinate, recognises the reality that students and teachers live with every day. By listening carefully to what the students have to say about their views and experiences, and inviting them to identify aspects of schooling that get in the way of their learning, Angier and Povey discover that their students wish for more democratic classrooms. "[I]n the context of the classroom relationships they valued, their talk about the nature of mathematics is democratic in tone: they speak about the subject as though they own it" (p.148). This is certainly a departure from traditional math classes where we find the analytic, empirical model of lecture, skill drills, and the teacher as the driving authority. It also acknowledges the power that students seek and need in becoming effective members of a learning situation. "Participants in the classroom need to renegotiate, in ways that acknowledge the need to shift the distribution of power, the relationships upon which their classroom is predicated" (p.154).

Examples of social constructivist learning programs illustrate that the important tasks of reforming mathematics classes are similar across the levels of schooling in many aspects. There are, however, two issues that emerge as students move from elementary school into middle school mathematics. As students enter adolescence their social roles gain heightened importance, and peer relationships and a sense of belonging have a strong impact on how well they interact in learning situations. Also, adolescents are becoming more critically aware of their surroundings, thereby needing to understand the importance and applicability of the knowledge with which they are engaging.

Recent advances in understanding how the human brain develops have revealed that there is a growth spurt during adolescence. In 2004 I attended a European League for Middle Level Education conference where Dr. Robert Sylwester from Eugene, Oregon explained how adolescence is the time when people learn how to interact socially in much the same way people learn how to develop their survival skills during early childhood. Adolescents are driven to develop connections with others and interact socially just as children are driven to learn to walk and manipulate their physical surroundings. We are obligated, and privileged, to provide opportunities and guidance for adolescents to develop and improve their social skills in appropriate ways. Peer relationships and how these play out in the classroom take on key importance. Middle school students will strive to interact in every situation so, rather than try to suppress their impulses, we can use their enthusiasm and energy in positive ways to stimulate their learning. (Appendix A)

In my experience with middle school students, I have found that not only do they tend to react with frustration and disengagement when the work is presented in the traditional instructional manner, they also need to understand and accept the necessity of knowing and understanding the concepts. As they mature, students develop a sense of discrimination, becoming more and more able to evaluate whether tasks and concepts are important and valuable or inconsequential and disconnected from their lives. Of course, in many cases they don't have the experience to judge accurately, so it is important to establish a learning community that gives them the feeling that their work is stimulating and important. When students have input into the learning, they see that the tasks are worthy.

Schoenfeld (1994) states that "the mathematics speaks through all who have learned to employ it properly, and not just through the authority figure in front of the classroom... the class becomes a community of mathematical judgement" (p.62).

Teachers Constructing Understanding of Collaborative Learning

Many teachers have learned mathematics in a traditional manner and must, in essence, reconceptualize their teaching to implement collaborative learning in their classrooms. In a case study by Wood, Cobb and Yackel (1991), researchers studied the process of how a teacher changed her beliefs about learning and teaching as she reorganized her practice to implement constructivist learning in her mathematics classroom. As the teacher implemented reform practices it was necessary for her to adjust her understanding of how to teach mathematics, and how to provide support for students as they worked toward constructing meaningful understanding. Wood et al. documented how the teacher had to make changes in thinking and practice (teacher reconceptualizations) and emphasized that the teacher needed support in making the changes. Firstly, the teacher had to reconstruct classroom social norms to promote social constructivist learning. At the same time, researchers provided the conditions she needed to construct understanding of the process that she had to implement. Secondly, the teacher had to come to see teaching as a process of negotiation rather than imposition, guiding students' learning. The researchers, then, had to provide guidance in helping her to develop her understanding of this process and to implement effective procedures. Thirdly, the teacher had to guide students' work toward the construction of meanings and procedures compatible with those of the wider society. Concurrently, the researchers saw that the teacher needed to be able to share experiences and develop understanding with a larger community of teachers.

The same types of conditions and support that need to be provided for students to construct their understanding of mathematics concepts need to be provided for teachers to construct their understanding of how to teach in constructivist classrooms. It follows that the same would hold to be true for administrators, supervisors, consultants and curriculum developers. Just as one cannot expect teachers to change their teaching practice merely because they have been told to do so, one cannot expect those who guide and evaluate teachers to change their practice just because they have been told to do so. They will need support and guidance from researchers, teacher educators and policy makers in order to construct their understanding of effective mathematics reform. Teachers are also learners in collaborative classroom situations.

As well as needing community and professional support when moving away from the traditional classroom model, teachers require innovative classroom materials to implement. I have cited articles that give examples of projects and problems, providing a flavour of the types of lessons with which teachers and students can engage. However, teachers need a series of problems and activities that can propel constructive learning throughout the middle school years. Clarke (1997) found that "the provision of innovative curriculum materials and the opportunity for reflection on students' work ... [were] major factors in teacher's finding that their previous practice was problematic" (p.297).

The literature on constructing understanding in mathematics through collaborative inquiry consistently confirms the effectiveness of the approach and the supportive classroom culture that it produces. The literature also underscores the importance of moving away from traditional practices that stifle learning and turn students away from mathematics. Writing and research that addresses these concerns in a way that is meaningful and useful to teachers and school administrators is important for our learning institutions. Reform has been slow and inconsistent for a number of reasons, especially at the secondary level, and it is important that educators, students and administrators see that learning mathematics constructively is the means to success.

CHAPTER FOUR: THE RESEARCH STUDY

Traditionally, much of the evaluation of the effectiveness of mathematics teaching and programs has been based on quantitative research and testing. Quantitative research has dominated educational research for more than a century, and is based on the methods of scientific inquiry. The assumption is that patterns of teachers' and students' behaviour are subject to predictable laws and axioms (Creswell, 2005, p.41). In 1973 Stanley Erlwanger published "Benny's Conception of Rules and Answers in IPI Mathematics" detailing research he had conducted by asking a student about the rules for doing mathematics. Benny was successfully working in an individualized instruction program, one of many that had sprung up as part of the back-to-the-basics movement. Although Benny had been able to achieve well on the tests of the program, when Erlwanger questioned him about his understanding of the concepts, Benny's rules for doing math were illogical and clearly wrong. (Erlwanger, 1973, p. 49) The publication of this study addressed two big issues in math research – how students learn, and how mathematics learning is researched. Erlwanger opened the possibility of researching how students learn using as evidence records of students' thinking processes and using interviews for gathering data.

Case Study

My study focussed on how students in a mathematics class were able to construct understanding by investigating problems and concepts collaboratively. This focus on the students' interactions and thinking produced a story of mathematics learning. My qualitative study into collaborative learning in middle school took the form of an interpretive case study in which I recorded evidence of students' thinking processes and interactions through the use of observations, interviews, journal writing and samples of work. According to Merriam (1998) "a researcher could study [a case] to achieve as full an understanding of the phenomenon as possible" (p. 28). I studied the actions and interactions of students in a middle school class (the case) to understand learning in mathematics (the phenomenon). "By concentrating on a single phenomenon or entity (the case), the researcher aims to uncover the interaction of significant factors characteristic of the phenomenon... Qualitative case studies can be characterized as being particularistic, descriptive, and heuristic" (Merriam, 1998, p. 29).

I investigated the theoretical assumption that collaborative learning is beneficial in helping students construct understanding of mathematical concepts. In being heuristic – illuminating the reader's understanding of collaborative learning in mathematics – my study became evaluative as my objective was "to develop a better understanding of the dynamics" of this type of mathematics program (Merriam, 1998, p. 39). I also intend for my study to be of use to teachers in the classroom because, according to Merriam (1998), by "[using] common language, as opposed to scientific or educational jargon, allows the results of a study to be communicated more easily to nonresearchers" (p. 39).

I offered to support a teacher as a participant observer, making my study also an intervention. I was able to find a teacher who was willing to try collaborative learning in her classroom and to plan together with me a unit of study that incorporated situations for students to construct understanding by working together. By planning together we could make sure that the teacher was comfortable with the activities, that the activities were engaging and rich with opportunities for students to collaborate, that I as researcher was present for support, and that there were occasions for me as researcher to observe and collect data. During the learning activities I could observe the students' actions and interactions. The focus of my observations were "a specific issue, with a case (or cases) used to illustrate the issue... an instrumental case, because it serves the purpose of illuminating a particular issue" (Creswell, 2005, p. 439), the issue of whether or not it is valuable to students to provide collaborative learning opportunities in mathematics.

The Class for the Study

The class that I observed and worked with was a grade six class at Smith Meadows School[3]. Smith Meadows School is an elementary school, kindergarten to grade six, in a middle income suburb of a large city. When I began this research project the class had twenty-eight students. During the course of the project two of the students moved to other schools. Half of the students were female and they were all about the same age – eleven to thirteen years. There was a mixture of nationalities and of abilities. The class spent their day in a classroom with individual desks that could be moved around. The teacher that I worked with, Ellen Peterson, taught them for half the day in Mathematics, Social Studies and Physical Education. For the other half of the

[3] All names used in the study are pseudonyms in order to preserve anonymity.

day a different teacher taught them Language Arts and Science. They had been working together since the beginning of September and would be together until the end of June.

I had the opportunity to work with the grade six class from the middle of January to the middle of February. The class was working on the number strand focussing on operations and the unit that the teacher, Ellen Peterson, and I planned aimed at providing a conceptual foundation for multiplying and dividing whole and decimal numbers. I was able to observe the students working on a variety of collaborative tasks and was able to obtain a wide variety of student work samples, and written and oral responses to questions about student experiences.

Over the period that I worked with the students I was in class twelve times with each class period ranging from forty minutes to two hours. During the classes I took field notes whenever possible and after leaving the school I wrote them into a diary as well as recording the events that had occurred that I hadn't been able to write about during class. In the diary I also collated the results of students' written reflections.

After my school visits ended I interpreted my data by writing about what I saw and then about what I made of what I saw. I realized that the collaborative tasks that the students had worked on had different qualities and that made a difference on how the students took them up. I then did analysis of the three main tasks by looking at how the components of these tasks fit with the conditions of complexity science.

CHAPTER FIVE: WORKING WITH THE CLASS

This story of how the students at Smith Meadows interacted, both as individuals and as a whole class, to bring meaning to mathematics illustrates how collaboration on tasks can be the basis for learning. Christiansen and Walther (1986) discuss the assertion by Davydov and Markova (1981) that an educational task is a means to recognize that "... '(learning) is primarily a process of change, reorganization, and enrichment of the child himself'." (p. 263). Christiansen and Walther emphasize how "... learning cannot take place through activity performed by an individual in isolation, but must unfold in relation to activity mediated by other persons... and often by activity performed by a group including the individual in question" (1986, p.267).

My research plan focussed on collaborative learning situations for students led by their regular teacher. This would require that I support the teacher in any way that she needed. Not only was my focus on collaborative learning, it was also necessary to respond to the needs of the students educationally and socially.

A few weeks before I was to begin the project I met with Ellen Peterson at Smith Meadows Elementary School to discuss the research plan for my inquiry into collaborative learning in mathematics. Her grade six class was working on the addition and subtraction part of a unit on number operations and we agreed that I could work on some activities that would require the students to work in groups for the multiplying and dividing section of the unit. Ellen had been using photocopied booklets from the Quest 2000 series with the students. She also had manipulative materials available for use with her students that she indicated she would like her students to experience using. I proposed that I would send her a list of activities that she could choose from during the first week back from the winter break and we could discuss what she wanted to do with them.

The Unit Plan

My focus in developing a plan was to give the students the opportunity to develop a connection between number operations and manipulatives leading to a foundation for the algorithms that they had been using. In order for the students to expand their understanding and to work collaboratively, I wanted them to interact with manipulatives, each other, the teacher and the symbolic representations.

I developed a mini-unit plan that was based on the outcomes from the Alberta Education program of studies that focused on exploring multiplication and division in a variety of ways (Appendix B). The unit involved various kinds of activities, I included work with base ten block manipulatives and a number of activities requiring students to collaborate in groups and to share their understanding with the whole class.

Ellen and I discussed how we should proceed with the unit. She felt it would be best to focus on using the base ten blocks and I showed her a little of how to do multiplication with the blocks as she was unfamiliar with using them to model multiplication and division. We agreed to start by having each group of students make a web or poster showing everything that they knew about multiplication so that we could do a formative assessment of their knowledge.

We could then move on to working with base ten blocks to build their relational understanding which is "knowing both what to do and why" (Skemp, 1987, p. 153).

We also agreed that Ellen would arrange the students' seating plans and the groupings throughout the study. Since the students were in the classroom for all their subjects, she preferred that the groupings were made to be suitable for projects in other subject areas. The teacher always put the students into pairs and groupings according to what she thought were best for them. Her priorities in grouping students were on having students together who would not clash and would not cause disruptions (behaviour), those who would be able to help each other (personality), and who were at different, but not too different, levels of understanding (ability).

On the day that we were to begin the unit I arrived to find that Ellen was very concerned about the students' work on the worksheet she had given them the day before. She showed me how even her best students had been making mistakes in multiplying two digit whole numbers, which was a concept that she felt they should have been able to do consistently. An example she gave me was when multiplying 16 x 23 a student did this:

23		23		23
x16		x16		x16
	(6x3)	18	and (1x2)	2

to give 218 for the final answer.

We discussed how the students were applying parts of the algorithm they had been taught, but weren't seeing the reasoning behind it. Ellen felt that it would be more beneficial for the students to start the concept development work with manipulatives right away. According to the plan I would circulate among the students and help them with the blocks as she led them through the activities that we had briefly discussed. However, she preferred that I lead them through the activities as I knew them better; she knew the students better and could keep them focussed during the task.

Developing Understanding with Manipulatives

As the students entered the classroom and settled into their places I heard a student say, "Oh good we get to play with blocks". Ellen moved the students into pairs so that one student could have the bucket of base ten blocks on his or her desktop and the other could have the "staging area" where they would display the pattern. This provided a means for the students to cooperate and discuss the pieces needed and how to lay them out.

I asked the students if any of them had worked with base ten blocks before. Only a handful of them said they had used them for counting and making numbers years before in early grades. I then led the students through naming the pieces, making numbers, adding and subtracting. I demonstrated on the overhead and wrote the symbols on the whiteboard so the students could connect the patterns they were making with the written numbers and operations. As I led the students through the activities Ellen circulated among the class, helping the students with their work and keeping them on task.

From my point of view, mostly at the front of the classroom, I heard almost entirely math talk from the students. Occasionally students would use the blocks to "build" things rather than to do math tasks, but were either brought back on task by Ellen or their partner. The students worked very well at their tasks. Ellen and I discussed what we had been able to accomplish, and being pleased with their progress and concentration, decided to continue working for an extended time with the manipulatives and the ideas around them. We then worked on writing rules for adding and subtracting with written numbers, calling them symbols, and then for adding and subtracting with base ten blocks. I focussed attention on what was happening with exchanging the blocks when "carrying" and "borrowing". We also renamed the pieces to show decimal numbers and add and subtract them. The students continued to work very well together and appeared to be doing a lot of math.

At the end of the first session, Ellen was very pleased with what the students had been able to accomplish. She said that the class as a whole had had a reputation for being unfocussed and had been a challenge to work with. She was pleased with how well they worked with their partners, how willing they were to work for fairly long periods of time with the blocks, and how much they seemed to be learning about how the numbers and operations worked. We agreed that we would continue to work with the base ten blocks in the next days.

We began the next session by reviewing what we had done previously and then began to do multiplication, first by grouping and then by representing the operation with the area of a rectangle. I demonstrated and challenged them with questions such as 16x23. I repeatedly connected the design of the blocks to the steps of the symbolic process with various algorithms. After making the rectangle I would write the numbers out as usual and write each of the parts as numbers pointing out on the design where each amount came from, adding up the four parts. I would then have them check the product with the algorithm as I led them through it on the whiteboard. By using grid charts connected to the sections of the base ten block rectangle, students could see a connection between the algorithm and the area model. For some students the grid chart became the algorithm that they could use, and be confident of having a sensible product. We worked together through double the regular time allotted to math.

Using Base Ten Block Manipulatives

During activity and tasks with base ten blocks, I saw students collaborating to bring meaning to what they were doing. Having students work in pairs with one set of blocks and one staging area between them, they were able to support each other, give suggestions and communicate to determine the best way to display the grouping or shape required. In doing this the students displayed interest in working with the blocks. By having the students trade roles of staging and bringing out blocks, they became more familiar with the process and developed confidence.

Students who were reluctant to try this new type of activity became more adventurous. For example, during the first session with the blocks I walked toward the back of the class and asked one student, Evan, if he could explain the sum that he and his partner had just made. He had his flats stacked in a circular pattern and he asked me if I liked his design. I asked him again if he knew the math and he told me that his partner figured out the math but he was a "builder" and he wanted to make good designs. During the next session when we were doing multiplication I again went to Evan's desk and asked him if he knew the math. This time he knew precisely what multiplication problem he and his partner had made and did not mention his building.

There were also students who had struggled with written mathematics who were focussed and very successful with the blocks. Sandra was always very quiet and cooperative and sat at the back of the room. When working with the base ten blocks she took control in her pair. She concentrated fully and seemed to be engrossed in showing the answers. Eventually I began to call her the Queen of the blocks because she always finished up quickly and accurately and looked up with a look of satisfaction for her work. When I talked to Ellen about Sandra, Ellen said that she loved puzzles. She also said that Sandra was very weak in her academic work and had many problems with understanding and communicating. Yet she was excellent with understanding and calculating with the blocks and it was clear that she was really feeling good about what she could do. However, before my research project was over Sandra's family moved and she went to another school. I did not get the opportunity to see how well she might have been able to transfer her accomplishments with the blocks to other types of understanding.

There were students who developed an understanding of multiplication, moving from the blocks to chart patterns, who still struggled with the standard algorithm. This was clear on the test at the end of the unit where students made mistakes with multiplying when using the algorithm but were all able to multiply by drawing the solution with blocks or in a chart. Corbin was a very vocal and social student who liked to respond to questions when he knew the answers. He also liked to talk and be involved when doing the activities, but wasn't always so anxious to participate when there was work to be done. During the first project he was talking to another student so adeptly that I had the impression that he was clever with math ideas. I was surprised when Ellen said that he really struggled with math. However, I soon saw that it was the case and he was so confused when using algorithms that much of his work made very little sense. When attempting to explain the work he had written down it was obvious that he didn't understand what he had done. At the beginning of the unit he didn't appear to have any understanding of the meaning of multiplication. Corbin was adept at avoiding situations where he had to show his math work and indicated that he felt that he was bad at it. With plenty of prompting and monitoring he began to work with the tiles and appeared to be getting the idea of how to multiply with them. Working with the tiles was the beginning of a new understanding of multiplication for Corbin while working collaboratively compelled him to be honest with his attempts, communicating understanding of the process. He could use the chart to multiply large numbers but had to be careful because he wasn't always accurate with his multiplication tables. On the unit test he did poorly, quite possibly because he resorted to using the algorithm rather than the chart when calculating. He did the question correctly where he showed multiplication with tiles. It was clear he could use them and the grid chart by the end of the unit but didn't consider them options when doing multiplication. I talked to him about it after the test and told him that he must always use the chart for multiplying because he was good at it. The next time I saw him he made a point of telling me how he always used the chart after that.

As a whole the class was enthusiastic about the blocks, worked well and remained focussed for extended periods of time. At the end of the unit the students wrote a test with a question on multiplying two two-digit numbers by making a tile diagram or filling in a chart. All but one of the students who attempted it were able to draw a correct answer, and five students who were still unable to provide a correct product when asked to multiply two two-digit numbers on other questions on the test were able to correctly draw the diagram or chart to find the product. Before leaving Smith Meadows I asked nine of the students which of the several tasks and activities we had worked on were the best and why. Seven of them included the blocks in their answers and five of them felt that it taught them about multiplication.

Once the students had developed their skills with the base ten blocks and could demonstrate multiplication and division consistently, Ellen and I decided that the next step was to have them work in larger groups with these concepts. I would prepare a group task for them to work on next. They could work in seven groups of four, and I could circulate to observe how they would collaborate.

The Pizza Task – Multiplying and Dividing Whole Numbers

The ideas for the group task that I developed for the class arose from a casual conversation at the end of the class. We had just built a product with the blocks and I gave the students examples of types of word or story problems with it as a solution. Of course, one of the examples used pizza pieces, as middle school teachers are wont to do. Then to make it connect to a size they could visualize I blurted out something about pieces of pizza one metre by one metre in size and how the pizza that was represented would cover the playground. Most of the students began to get ready to leave the room and a couple of the students at the front of the classroom said a couple more things about the huge pizza slices.

In making a task for the students I wanted to have something that would connect to their interests and have the possibility of involving the work with modelling multiplication and division that we had just done. I looked up world records for the largest pizza and made a task that required the students to make a plan for a pizza that would be larger than the largest recorded pizza and would win the Guiness World Record for the school. I concocted a method for making the pizza on the rectangular gymnasium floor. The students would also be required to divide the record pizza up amongst the students at the school.

Collaborative Work

Before the students began their group work I talked to them about expectations. I gave them a rubric that indicated how they were expected to produce quality work, cooperate and concentrate (Figure 5.1). I explained how the rubric worked and gave examples of behaviors that would fit each category. This set boundaries for how students were to work together but did not prescribe how they were to approach the problem or what type of mathematics they could use. This chart was referred to every time the students began work on collaborative tasks during the study.

Group Work Scoring Guide

	Work Quality	Cooperation	Concentration
Wow	Does excellent work.	Shares ideas and helps others. Helps solve group problems. Helps organize and set group goals. Takes on extra tasks.	Brings group back on task
Yes	Does good work.	Compromises and cooperates. Shares ideas. Discusses problems.	Stays on topic.
Yes, but ...	Does satisfactory work	Cooperates most of the time. Listens to others.	Is mostly on topic.
No, but ...	Does minimum work.	Does not cooperate well. Does not listen to others.	Is often off topic.
No	Does not produce work.	Argues with other group members. Does not participate.	Distracts and distu others.

Figure 5.1. Collaborative work rubric for students

Record Pizza

The students were given a handout for this project (Figure 5.2). I read it to the whole class and explained the task. I drew a one metre square on the whiteboard to illustrate the size of the pieces. Ellen put the students in groups of four.

The students began discussing the idea of making a big pizza. Some of the groups took buckets of base ten blocks to their desks. They began to focus on finding the size of the giant rectangular pizza built in Iowa. A few students asked to go to the office and find out the size of the gym. Then a couple of students wanted to use the computers to search for solutions. Ellen and I redirected these students toward the problem.

Record Pizza

Grade Six Class Goes for the Record

Mrs McMillan, the principal at Smith Meadows School, has decided to have her school try to make the largest pizza ever and get into the Guiness Book of World Records. She has chosen Mrs Peterson's grade six class, as leaders in the school, to coordinate the project. Fortunately, an anonymous donor has offered to provide the funds required to rent sheet metal to cover the floor of the school gymnasium (to serve as a giant pizza pan) and massive heat lamps to shine from the gymnasium ceiling to bake the pizza, as well as all the ingredients. Mrs McMillan is confident that her grade sixes will be able to calculate the amount of pizza they will need to make in order to achieve the record, and that they will be able to come up with a plan to share the pizza equitably. The students of the sixth grade will be presenting their plans on this exciting quest.

Current Data on Large Pizzas

1990 – In South Africa a pizza with a diameter of 37.4 metres was made. This size gives **1098.52 square metres** of pizza. (This is the record holder according to Wikipedia).

2005 – A school in Iowa organized the creation of a rectangular pizza that was **39.32 m x 30.05 m** in size.

THE TASK FOR YOUR GROUP

Your group will make a **poster** illustrating how the class can make a pizza big enough to win the Guiness Book of World Records prize for the largest pizza.

On the poster you will show with **a diagram, numbers**, and **words** how big the pizza will be. (Make sure that it is bigger than the two pizzas mentioned above).

You must also indicate how you will **share** out the pizza when it is done. You need to decide how many people or families will get a piece and how big each piece of pizza will be. (Be sure that this part of your plan makes sense).

When you have finished your poster you will present it to the class, and you will have to prove that your calculations will work to achieve this amazing task. (Perhaps there has never been such a delicious use of a school gymnasium ever attempted).

Figure 5.2. Student handout for Record Pizza task

Many of the students expressed confusion about what to do. The groups who had taken buckets of base ten blocks to their desks didn't seem to know quite what to do with them. Gradually they seemed to realize that they needed to make their pizza bigger than the one built in Iowa. This idea took some groups quite a long time and some prompting to begin to look at the size they needed. During this part of the task the atmosphere in the class seemed to be chaotic and somewhat unfocussed. All of the groups, however, were attempting to do something with the task and the class as a whole was active.

While listening to and interacting with the groups I saw a variety of methods that students used to help each other understand the problem and work toward solutions.

One group used the base ten blocks to model the largest pizza on record, using whole numbers near the size rather than decimals. Then they added one more row of blocks to make their pizza larger and counted the blocks rather than calculating. When counting two of the members of the group pointed and counted out loud, showing the others how the total was determined. They then drew the block pattern on their poster. They had some difficulty seeing the connection between the blocks and the one-metre drawing at the front of the room, eventually reasoning out how each block represented one of the large pieces of pizza.

Another group used a scale factor of 75cm = 300cm to draw out their pizza. When I asked them how they had decided on that one student said she had figured it out. The rest of the group let her do the calculations with the calculator. When she started measuring she recognized that her drawing was too big.

Every group had a poster with a labelled drawing of their pizza. Most of the pizzas were thirty metres by forty metres or larger. Three of the seven groups represented the pizzas with rectangles made of squares that indicated that they connected the area to the base ten block flats in developing the size. There was one group that struggled with the task ending up with a drawing of a pizza labelled with a very large size. All the divisions of pizza among the students at the school were attempted but not all were well explained.

At the end of the project all the groups presented their posters to the class and explained their representations. Most of the students were very pleased with their posters. There was very little discussion about the ideas on the posters. Although the students had been quite excited about the prospect of making a record pizza at the beginning, they were quite ready to move on by the time they finished their posters as they were quite large and took quite a bit of time to finish.

Students Reflecting on Their Work

I spoke to the students about thinking about and writing about their work. I put these questions on the board and asked them to respond to them in writing:

What did you like about working on this math project?

What did you like about working with a group on this project?

Did someone help you understand something?

Did you help someone understand something? What method (how) did they use to help?

You can write anything else you would like about the project.

Twenty-six students wrote their reflections.

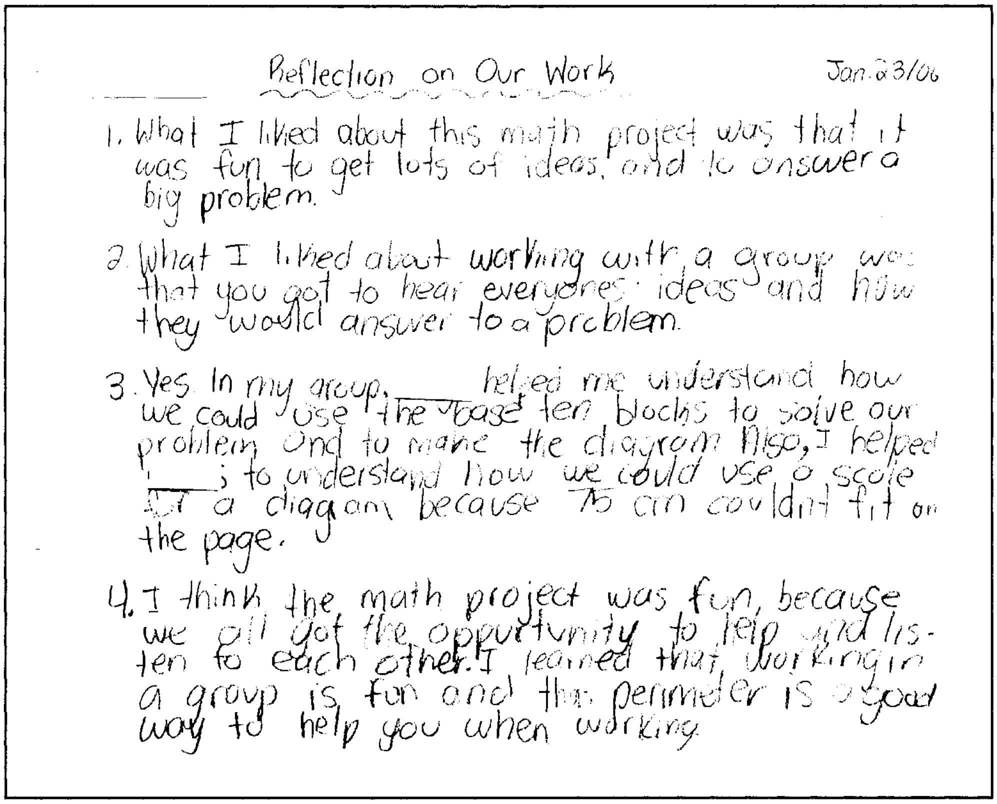
One student, Richard, wrote "I don't really like thesse (sic) kind of math projects I like math booklets." He said that he preferred to work on photocopied booklets from the Quest 2000 series. Another student, Eddie, wrote that he liked the pizza project. All the other responses cited things that the students liked, as the questions had directed them.

Ten students wrote that they liked that the project was about pizza, or drawing, or the world record, and five students wrote that the project was fun, interesting, or exciting and unique. Six students indicated that they like some aspect of the mathematics of the project and seven students liked the group work.

In response to what they liked about working with a group, nine students indicated that they like sharing ideas, two students liked working together, nine students liked that they did not have to do all the work or that the work was easier with a group to work with. Six students indicated they like group work because of other reasons – it was fun, faster, less stress, easier, people help you, people teach you, you can learn new things, you can correct other people's mistakes.

In response to questions about students helping each other to understand something, twelve students indicated specific help with math concepts – calculating, base ten blocks, scale, measurement, perimeter, area, operations. Five students stated that they were helped through the use of calculators, and five students indicated that they were helped by someone showing them, explaining to them or writing something that helped them understand.

The students' responses to the reflection questions displayed their feelings about how they felt about being involved in this project and working with others. Andrea is a quiet student who tries her best. During the group work she emerged as a leader and was showing her group how to use a scale to draw the diagram so that their diagram would fit on the poster sheet. This was knowledge that she had acquired previously as I had never mentioned scale diagrams and it had the group very focussed on how to best use it. Andrea also used the term "perimeter" to describe what they were using to measure the diagram (Figure 5.3). She meant "area" and although she knew that she was working with square units she had confused the terms. There were others in the class who picked up on this during the class work and used the term "perimeter" to describe their diagrams. I didn't pick up on this error until the students presented their posters and missed an opportunity to make a valuable connection and correction in their terminology.



Reflection on Our Work Jan. 23/06

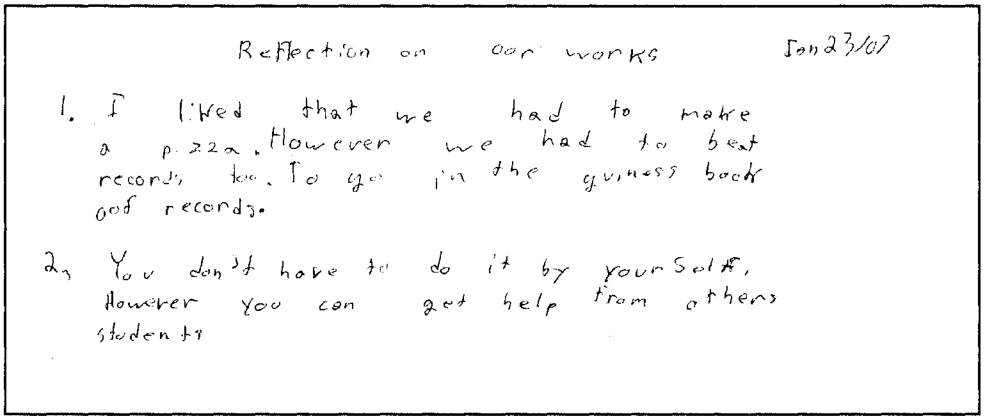
1. What I liked about this math project was that it was fun to get lots of ideas, and to answer a big problem.
2. What I liked about working with a group was that you got to hear everyones' ideas and how they would answer to a problem.
3. Yes In my group, ____ helped me understand how we could use the base ten blocks to solve our problem and to make the diagram. Also, I helped ____ to understand how we could use a scale for a diagram because 75 cm couldn't fit on the page.
4. I think the math project was fun, because we all got the oppurtunity to help and listen to each other. I learned that working in a group is fun and that perimeter is a good way to help you when working.

Figure 5.3. Andrea's reflections on working on the Record Pizza task

One of the students, Mitch, had really been caught up in the idea of the class getting a world record. Mitch was often kept in for recess detention because of disruptive behaviour, and during the project when the students were working on their posters he continued working on it during one of his detentions. I was sitting at the teacher's desk and he was talking to me about his work. Suddenly he flung himself across one of the student desks dramatically calling out,

"Ohh… I really hope that we can make the pizza in the gym and win the world record."

It was obvious that he really believed that it was a possibility and was immersed in the fantasy that we had created. I was a bit worried at this point that I was going to have to seriously disappoint some of the students at the end of the project. Students at this age, however, seem to be able to throw themselves into a situation with enthusiasm and then move on to another with just as much enthusiasm. Mitch's reflection (Figure 5.4) focussed on what he liked about the project and was positive, but he didn't have anything to say about the mathematics of it.



Reflection on our works Jan 23/07

1. I liked that we had to make a pizza. However we had to beat records too. To go in the guiness book oof records.

2. You don't have to do it by yourSelf, However you can get help from others students

Figure 5.4. Mitch's reflections on working on the Record Pizza task

At the end of my work with the students at Smith Meadows School I went back to these written reflections to look at the responses of two students – Lindsay and Jennifer (Figure 5.5 and Figure 5.6). These two were not in the same group for the Pizza Project but were paired up for a later task on division of decimals. They had much difficulty working together on the division task and essentially did not communicate at all. Their responses on the reflections indicate that they enjoyed working with a group and felt positive about the project and their contributions.

Jan.23/01

Reflection on ourWork.

1) This math project was really exciting and unique.

2) I liked working in a group because you have people to help you do most of the work so you don't have do all of it.

3) Yes. Everyone helped each other understand most of the work and helped to work it out if it was complicated.

4) They worked it with the calculator wrote it down or they would talk it out and if I still didn't understand they would find a way to let me understand.

I learnt that you can use base Ten blocks to help you figure stuff out. I also learned how to play other fascinating games.

Thanks Mrs. Talbot

Figure 5.5. Lindsay's reflections on working on the Record Pizza task

Jan 23, 2001

Reflection on our work

1. What I liked about this project was that we worked in groups.

2. I liked working in a group because not just one person did everything, everyone worked as a whole group.

3. I don't think anyone didn't understand things but sometimes there were little thing people didn't understand. If someone had something wrong the method the person would use was just explaining with words.

4. I never really cared about world records before but now that We've done this project, I might try it in the future.

Figure 5.6. Jennifer's reflections on working on the Record Pizza task

Error Correction with Multiplication

Because Ellen had been very concerned about the student's mistakes in using the multiplication algorithm, we decided to give the students an error correction task to see if they would use some of the new strategies we had introduced (see Appendix B for an elaboration of this task).

Division

After having students do another task to connect their understanding to the multiplication algorithms we began to move on to whole number division. We gave the students a review of whole number division and introduced division with base ten blocks, moving from repeated subtraction to grouping and then to an area model. With the students working in pairs again, we worked through using the blocks to show repeated subtraction and grouping with questions like 72÷8 and 96÷12. I also gave an example of a story problem, involving a length of ribbon that needed to be cut into pieces that required division for solving. I also referred to the division that they had used for sharing the pizza. I then showed the students how to use the base ten blocks to make a rectangle of a determined area with a given side length in order to show division, for example 156÷13. This activity took some time and many students appeared to struggle with providing acceptable solutions.

Division Party Task

In order to have students collaborate to investigate division as a grouping we decided to give the students a task that had them dividing up pieces of candy for a party to determine basic divisibility rules. I adjusted an activity developed by students in a university curriculum and methodology class (Figure 5.7).

The students, in groups of four, used unit cubes to represent the candies and worked well on this task, for the most part. They did not seem to be as enthusiastic about the problem as they had been for the pizza task. Quite a few of the students used the multiplication chart on the wall to fill in the chart rather than count the cubes.

One of the groups asked me what it meant to make a rule. Once I got them started with the first few words (A number that is divisible by .....) and talked to them about it, they were able to construct some rules. Other groups nearby listened and were able to use the same pattern of words for their rules.

Another group had made their groups of candies and filled in their charts and discussed the rules. When they had finished most of the rules, I asked them what would happen if they added the digits of the numbers that were divisible by three. Two members of the group discovered the rule fairly quickly and were eager to share it with the whole class at the end of the class when we discussed their findings as a whole class.

This task did not appear to be as challenging as the pizza task. The students worked well together and helped each other out, but did not need to cooperate as much. Their explanations and actions were not as intricate. This task, however, gave them the opportunity to approach division in a different context and to practice using grouping in division.

The Valentines Day Party **Name** ____________________
Math 6 Assignment **Date**____________________

You are having a Valentines day party. Three things must happen for you to have a successful party:
1) no less than 2, and no more than 10 people can come
2) each person at your party has to have the same amount of candy
3) there can not be any leftover candy

The only problem is that you don't know how many people are actually going to come. Each person in your group can be invited to your party, but only if there is enough candy for them to come. Start in row 2 of the chart below with 30 pieces of candy. As a group, divide up the provided candy in a way that the party will be successful. See how many different numbers of your friends can come so that you still have a successful party. Fill in the chart below indicating ALL of the possible solutions.

Pieces of Candy	# of Friends that Can Come to the Party
ex. 4	**2, 4**
30	
18	
36	
27	
20	
15	
8	
22	
10	
33	
16	
21	
35	

Answer the questions on another sheet of paper
1) Is there a pattern when you have an even number of candies? If so, what can we tell about these numbers? Can you create a rule for dividing numbers like this?
2) Are there any patterns related to having 30, 20, or 10 pieces of candy? If so, what does this tell us about these numbers? Can you create a rule for dividing numbers like these?
3) Are there any patterns related to having 30, 20, 15, 10 and 35 pieces of candy? If so what can we say about these numbers? Can you create a rule for dividing numbers like these?
4) Are there any patterns related to having 18, 27, 15, 33, and 21 pieces of candy? If so, what do these numbers have in common that create the pattern? Can you create a rule for dividing numbers like these?
Bonus: Are there any patterns specifically related to the having 18, 27 and 36 pieces of candy? If so, what do these numbers have in common that create the pattern? Can you create a rule for dividing numbers like these?

Figure 5.7. Student handout for Valentines Day Party task

Building Understanding of Operations with Decimal Numbers

We then moved from whole numbers to decimal numbers. We reviewed ways of thinking about multiplication as repeated addition, groups of things, hops along a number line (using metre sticks), and using the area of rectangles with the base ten blocks. Once again the students worked with the blocks in pairs, communicating with their partners to choose the blocks from the bucket and place the configuration on one desk. I showed them how to re-value the blocks, giving the flat (100) the value of one, the rod (10) a value of one tenth and the cube a value of one hundredth. At a couple of points the students became confused about the values – for example, 5 rods being 5 tenths and 50 hundredths – but mostly were able to work with the blocks to multiply decimals. As we multiplied a variety of numbers, I wrote them on the whiteboard and counted the number of decimal places in the factors and the products. We were then able to work together to establish the rule for determining the number of decimal places in the product. The students practiced multiplying with blocks, charts and with the algorithm and rule for decimal places in the product.

Division with Decimals

The clear progression was to now work with division and decimals. We had provided a variety of experiences in the prerequisite understanding – multiplication with whole numbers, division with whole numbers, representing decimal numbers and multiplication with decimal numbers. The students had developed the knowledge and skills to be able to derive the answer to a question about dividing a decimal number. I decided to give them a task without posing any possibilities for using representations or algorithms.

The task that I developed was modelled after the task for multiplying integers in Davis and Simmt's "Understanding Learning Systems: Mathematics Education and Complexity Science" (2003). They describe how Simmt had worked with her students in adding and subtracting integers. The task she gave the students was "to consider the statement '3×-4=?' They were paired off by the teacher and each pair was given 10 minutes to agree on a product and to 'Show how you know' – that is, to prepare an explanation on chart paper for presentation to their classmates" (p. 158).

For my task pairs of students within their groups of four would work on an answer to the question:

What is 1007.5 ÷ 26? Show how you know.

Bonus: Write a story problem that matches the numbers and operation. Check with the other pair in your group and be prepared to share your solution and story problem with the class.

My intention was for the students to have to try to represent and calculate something with which they were not entirely familiar so that they had to try new things and communicate with each other. I had used the divisor of 26 because there were twenty-six students in the class at that time. I also used a decimal number that could be interpreted as an amount of money that would give a two decimal answer, perhaps interpreted as a monetary amount, when divided by 26.

By requiring students to share their solution they would have to communicate their thinking.

The first part of the lesson was devoted to reviewing the rules for multiplication, going over the homework worksheet and reviewing all the ways that we could think about multiplication as a whole class. Then I talked a little about division, and students responded with suggestions that division was the 'opposite' of multiplication, repeated subtraction and grouping. I asked students how they could show that something was true or prove their answer was correct. Responses were given that included showing their work, writing down the steps, and writing the rules. I again reminded them of diagrams, number lines and pictures. I then gave them the task above to work on in pairs and a large sheet of paper on which to display their work.

Ellen had organized the students' desks into groups of four and wanted them to stay with this configuration. Within each set of four she had males and females and had grouped them according to whom she thought would help each other and would not be disruptive when working together. They were instructed to work with a partner in their group and most chose another of the same sex to work with. The first part of the task seemed a bit chaotic with students seeming to be unsure of what was expected and how to proceed. Almost all students took out their calculators to get the answer. A couple of groups got blocks but didn't know what to do with them because the number 1007.5 was too large for them to start to lay them out according to their past experience.

Two of the groups saw that there was a bonus question and went straight to designing a story problem after using the calculator to find the answer. It seemed as though they thought that the idea of a bonus meant that that was a more important aspect than the solution. Almost every group sought help from Ellen and me. We answered their questions with more specific questions or, if they seemed totally frustrated, with suggestions for possible tracks to try. I was genuinely worried that the task had been too difficult and we would end up with frustrated, disillusioned students. I told the students that there would be a limit to the time they would have and that seemed to get most of them going with trying some things.

Group Responses to the Division Task

The students were working in pairs within their groups of four. There were eleven pairs and one group of three students.

All groups had the answer from using a calculator and most of the groups started to work on writing out the algorithm, which prompted some struggles because of the decimal that they were not familiar with in the division algorithm. The groups working with an algorithm used either the traditional one, with multiples of the divisor and subtraction, or the one with repeated subtraction that I had shown them when working with whole number division.

Mark and Eddie, who comprised a working pair, both often struggle in class – Mark because of behaviour and Eddie because of difficulty with basic concepts. These two saw that the story problem was a bonus task and went straight to that part of the task indicating that they were going to do really well because a bonus must be more important. They liked bikes so they made the story about bikes, using money and saying that Jason wanted to buy bikes worth $26 for his friends and figuring out that he could buy 38 bikes with 75 cents left over. It took a bit of prompting for them to see that he would have more than that left. They then wanted to try to draw a diagram to show it. I suggested that they draw the bikes rather than trying to draw all the blocks or ticks for each dollar. They were very pleased with the idea and were interested in drawing the bikes with coloured markers. When I prompted them to draw ("Just draw what you wrote"), they drew the 38 bikes and said they had 0.75 left. When I asked if they would draw 0.75 of a bike they said nobody would want that and laughed a little ("You couldn't do that") and wanted to write with numbers .75. I suggested that they draw the money and they were able to figure out the exact change left and drew it.

Mark and Eddie were very pleased with their poster, especially the coloured bikes that they had drawn. They began to show the groups around them. This appeared to really motivate the other groups to think about the division problem in ways different than an algorithm and an answer. Several of the groups began to think of different things that could be bought and made into pictures.

All but one of the groups that had not gone directly to the bonus story problem put one of the two types of division algorithms we had worked with on their paper. They then started working out how to follow the process through to show the work. A few students began to look over to others groups' work. This seemed to help them figure out how to proceed with theirs.

Lindsay and Jennifer were paired up for this task. Lindsay is older than the other class members and more mature. She is ahead of most of the others, having come from outside Canada and having been put a year behind. She is also more into adolescent social behaviours. Jennifer is quiet and almost always seems withdrawn. She has long hair and usually has her head down with her hair covering her face. When they worked together they sat across from each other rather than side by side as most of the other pairs sat. Lindsay took over the paper, pen and calculator. I never once saw Jennifer look up from her fingers in her lap. Lindsay struggled and struggled with trying to figure out how to do the algorithm with decimals. She crossed out her first attempt on the paper and I had to give her some direction just to get it done by the time she had to present. She was very displeased with what she had done, but would not try to include Jennifer.

After working through more than half the morning Ellen and I told the students that they would have only a few minutes left and then they would have to present their work to the class. They really worked hard to get them done. Some had to stop without totally finishing.

One pair of students was not able to find a problem that showed division as equal sized groups. They used the idea of buying and had "Sam" buy a variety of items for skiing, subtracting the different prices from $1007.50. They then had some money left for him to bring his friends on a ski trip but did not find how much it would cost for each of the twenty-six students. Two of the pairs represented numbers by drawing blocks but struggled with the representation of the decimal. Four of the pairs had only the algorithm to prove that their answer was correct. Everyone had come up with something that they could show to the class even though not everyone had demonstrated an understanding of division as equal shares.

Involving Students in Group Projects

During group projects - record pizza, the party and the decimal division - the students again were able to work well together to demonstrate or build understanding of multiplying and dividing. Not only did the group projects give students the opportunity to demonstrate that they were engaging in activity that involved mathematics, they provided a connection between the students' world and mathematics and they gave these young people a context for social interaction.

In designing and choosing group tasks for the students I focussed on the importance of having them become engaged in the activities. Students must feel that they belong in the classroom or group in order to feel motivated and safe in taking risks with mathematical ideas. My experience with adolescents has underlined the fact that they also are becoming very aware of their social status and need to feel included and important. This is supported by Hamm and Faircloth (2005) and Angier and Povey (1999).

As the students worked on the group projects they developed their communication and cooperation skills. Although every group activity began with a feeling of chaos and disorganization, the class as a whole became more quickly focussed as they became used to working on mathematics in groups. With the Record Pizza Project and the Valentines Day Party task the students were connecting ideas and practicing concepts that we had worked on in class. When we got to the Division of Decimal Numbers Group Task the students were asked to work on a concept that went beyond their current understanding.

The students had not been accustomed to doing collaborative work in mathematics before the project. In every part of the project I saw students working well and engaging in mathematics activity when they were given the opportunity to collaborate. They were able to develop strategies to communicate mathematically and involve each other while working together on the tasks and activities.

During every session that I had with the students at Smith Meadows the students worked through longer periods of math classes than they had been accustomed to. Collaborative work and project work takes more time than direct teaching and practice drills, but more students were able to focus for longer periods of time. Ellen Peterson commented several times on how pleased she was that the students had attended well to mathematics for extended periods. Not every student was always involved in every activity, nor did every student react positively at all times. However, most students responded to the work with interest and sometimes enthusiasm.

CHAPTER SIX: HOW STUDENTS CONSTRUCTED UNDERSTANDING BY INVESTIGATING COLLABORATIVELY

At the beginning of the project, my intention was to provide the students with a variety of means for understanding multiplication and division with whole numbers and decimal numbers, and to give them opportunities to work together in social constructivist situations. As we progressed through the activities I began to see how the students were becoming a complex community of learners with the class as a whole displaying understanding that may not have arisen had students not been collaborating during activities.

Davis and Simmt (2002) demonstrated how a mathematics class became a complex learning entity. They explain that "Complexity science is interested in questions of emergence – that is, in those instances when coherent collectives arise through the ongoing effort of individuals to maintain their fit within evolving circumstances" (p. 831). Davis and Simmt's use of the components of complexity science to frame learning instances illustrates ways in which we can interpret collaborative activity in mathematics classrooms. They contend that beyond analyzing learning situations, the conditions necessary for complex emergence "can be put to instrumental use in the preparation of classroom tasks" (p. 839).

Within a mathematics class where student collaborate, individuals are allowed to interact and they have proximity, common goals and common tasks. Although this situation provides part of the basis for complexity to emerge, something more is needed. According to Davis and Simmt (2002) "... a shift in interpretive focus is required... away from what *must* or *should* happen toward what *might* or *can* happen… on proscription rather than prescription" (2002, p.833) (authors' italics).

During my study the class engaged with three main collaborative tasks – the Pizza Project, the Valentines Day Party and the division of decimal numbers task. In each of these tasks there was a different prescriptive/proscriptive process and a varying emergence of components of complexity. The effectiveness of these tasks in stimulating explorative mathematics learning may be interpreted as a consequence of the degree of emergence of the class as complex learning entity.

Establishing a Frame of Complexity Theory

By investigating these three tasks through the analysis of the conditions of complexity we can gain a perspective on how the class could come to develop a deeper understanding of aspects of multiplying and dividing whole and decimal numbers. We can also discern how the development of tasks that promote activity which provides a foundation for complexity theory can become more impelling by focusing on establishing and encouraging the components of complexity – redundancy, internal diversity, organized randomness or liberating constraints, decentralized control, and neighbour interactions.

Unchanging Conditions

Because the components of redundancy and internal diversity are concerned with the composition of the class and its experience, they did not vary for any of the tasks and activity. Although each of these conditions could be affected by being able to choose to work only with selected students, this is not usually an option for teachers. Most often teachers take the students they are given and teach them within the system that is established in their schools and school districts. However, by being aware of how redundancy and internal diversity impacts the students' learning experiences, teachers can develop the possibilities for them to contribute to providing rich opportunities for the class.

Redundancy

The component of redundancy addresses the "sameness" among the members of the group. It allows interactions between members and for individual understandings to compensate for others' failings hence enabling the emergence of collective understandings. "Sameness among agents – in background, purpose, and so on – is essential in triggering a transition from a collections of *me's* to a collective of *us*" (Davis and Simmt, 2003, p. 150) (authors' italics).

Any class in a school such as Smith Meadows has "sameness" because of what it is and has been. The students mostly come from the community, are in the same age range, and many have been together at the school for some time experiencing many similar learning situations. In this project redundancy was further established through the base ten block activities with which the whole class became engaged. I began each new concept with having students show understanding of how to display numbers with base ten blocks. Because many of the students had been struggling with the algorithms for multiplying and dividing before this research project began, I wanted to increase the ways that they could refer to them. I provided them with opportunities to use manipulatives and visualize multiplication as both grouping of objects and as area. Division was presented as the opposite of multiplication, and they also used grouping and area to determine the answers to division problems. In tandem with the base ten blocks we used written symbols, charts and diagrams. The students were also asked to explain verbally and in writing how they determined their answers. The multiplication algorithm was used alongside the base ten block modelling. I also asked them to physically point to parts of the model that was represented in the algorithm. We worked through multiplying and dividing with whole numbers, and then reviewed multiplying with whole numbers and extended it to multiplying with decimals. As we worked with the blocks students were encouraged to collaborate while preparing and demonstrating their models.

This work not only extended the students' connections between concrete and symbolic representations, it also gave them an experience that they could refer to when encountering new ideas. Because the students were working together, it was necessary for them to communicate and share ideas. This collaboration encouraged them to make connections to new ways of doing things and thinking about the concepts. They also had to think carefully about their work and be prepared to change or defend their representations if others thought it was wrong.

Internal Diversity

Internal diversity occurs when the class has a mixture of students with a range of abilities and experiences allowing the students to work in mixed ability groupings. Because of internal diversity, members of the collective can contribute in very different yet specialized ways. Its presence provides a source of possible responses to emerging situations.

The students at Smith Meadows School are from a middle income suburb and come from a range of nationalities, types of homes, and abilities. Students are not streamed into ability levels and the classes are integrated. This grade six class of twenty-six students had a full range of abilities, though, in general, the teacher's impression was that they were weak, and could be easily distracted, needing consistent controls on their behaviour. This impression formed part of the rationale for our responsibilities, mine as the researcher, and of the teacher. Because the students needed to work in collaborative groups for the research project, the teacher preferred that I present the lessons while she circulated amongst the class helping them and prompting them to stay on task. This, in a sense, contributed to the diversity of the class. Because the teacher had had very little experience with base ten blocks, modelling and the variety of approaches to representing multiplying and dividing, as I interacted with the class during lessons, the teacher's interpretations and interactions with the class helped me to adapt the lesson to both the students' and teachers' needs.

The teacher was responsible for the students' seating plans and the groupings throughout the project, putting them into pairs and groupings according to what she thought would be best for them. She chose students to sit together who had different, but not too different, abilities so that they could help each other. She also felt that it was important that students who tended to be disruptive when near each other were separated. The teacher's controls on grouping put some limits on internal diversity as not everyone could respond to others' ideas at all times. They may have helped in other ways by making sure that disruptive behaviour did not interfere with connections to others' ideas.

Changing Conditions for the Three Tasks

The conditions of organized randomness or liberating constraints, decentralized control, and neighbour interactions changed for each of the three tasks - the Pizza Project, the Valentines Day Party and the division of decimal numbers.

Organized Randomness or Liberating Constraints

During the activity it is necessary to maintain a balance between redundancy and diversity so that the activities "are matters of neither 'everyone does the same thing' nor 'everybody does their own thing' but of everyone participating in a joint project" (Davis and Simmt, 2003, p. 155). In order to have students benefit from others' activity there needs to be the opportunity for them to interact or even to see the range of ideas and responses that emerge as they construct understanding. However, if students don't have a common purpose there may be too many different things happening to be able to build that understanding.

To set up the situation for this condition to emerge, the teacher must put some restrictions on the work with time restraints. She must be careful not to give answers or provide rules or conditions that have to be followed while working on the concept or problem. Proscriptive rather than prescriptive ways of negotiating the classroom norms will form the foundation for organized randomness. Students should be informed of what they cannot do rather than what they must or should do. It is important that even the most unusual ideas should be considered as they may stimulate divergent, creative responses by others in the group.

This gives all members opportunities to contribute to the collective project of generating knowledge, making the group more knowledgeable than any one member of it.

Decentralized Control

Decentralized control requires that students are free to use any means to determine the solution to the problem or investigate the concept. Therefore the role of leadership is not explicitly assigned to an individual but emerges as the collective organizes itself. As students take charge, different learning styles and ideas become dominant and unusual events can occur. This allows individuals who may not be chosen as leaders to contribute, changing the flow of ideas and approaches to understanding.

Decentralized control changes the way that roles are assigned to individuals. Traditionally the teacher assigns roles – such as recorder, timer, reporter – to group members or asks the groups to assign roles before work begins on the task. The procedure of allowing roles to emerge as the work progresses challenges the debates over learner-centred versus teacher-centred approaches to teaching. Davis and Simmt (2003) contend that “the notion of decentralized control... compels us to question an assumption... that the locus of learning is the individual. Learning occurs on other levels as well" (p. 152).

Neighbour Interactions

The condition of neighbour interactions allows for ideas to "bump up" against each other. Students should discuss solutions within their groups and should not be stopped from looking over to others' work or from showing their work to others. There must be opportunities for ideas, questions or comments to interact with one another. Focus is placed on the representation and interpretation of diverse and emergent ideas. "Without these neighboring interactions, the mathematics classroom cannot become a mathematics community" (Davis and Simmt, 2003 p. 156).

Investigating the Changing Conditions for the Three Tasks

The differences in these conditions induced a variety of responses among the individuals in the class, causing the learning experience to transform the class as a whole in its understanding. These differences came from a multiplicity of factors – type of task, groupings of students, interactions of class members and teachers, materials used, timing of work – that were imbedded in the situation.

Complexity and the Pizza Task

Some restrictions were put on the work to balance redundancy and diversity. Students were instructed that each group had to produce a poster that had to have a diagram, numbers and words. No restrictions were put on the type of diagram or how the numbers and words were used. Time restraints were not initially given, but were instituted as students carried out the task. If groups were not moving forward with their work they were told to begin work on the poster or to divide up tasks, or sometimes told to focus on the task at hand.

As required for decentralized control, the teachers did not give answers, but at times suggested means of attempting a solution or posed questions about the work in order to prompt another method of looking at it. Leadership within the groups (of four students) was not assigned and in all of the groups one or two leaders emerged. Students could look at others' work, and at times teachers suggested that members of a group look to see what others were doing. Groups could decide how to approach and solve the problem without restrictions on mathematical actions. This allowed students to communicate their ideas and connect them to their own experiences giving rise to much discussion. Adolescents look for and enjoy opportunities to interact and impress each other, and giving control over decisions to students helps them feel that their ideas are important.

Neighbour interactions were encouraged with students discussing solutions within their groups and looking over to others' work or showing their work to others. There were no organized or specified opportunities for ideas, questions or comments to interact with one another during the activity but the students were able to formally see what others had done during the final presentation when each group presented their poster and plan to the class.

Looking at the Pizza Task

This task was designed to have students collaborate to find ways to display their knowledge of the area model of multiplication and their understanding of grouping. It gave them an opportunity to consolidate and practice the work we had done with base ten blocks in a practical or applied context that was relevant to their experience. The students used talk and written representations to show their understanding. They were able to help each other and every group had an acceptable product – though one group did not have an efficient model (the pizza was much bigger than it needed to be). There was a range of solutions for dividing the pizza amongst all the people in the school, and sometimes amongst the families in the community. The posters, however, were all very similar as shown in the sample poster in Figure 6.1.

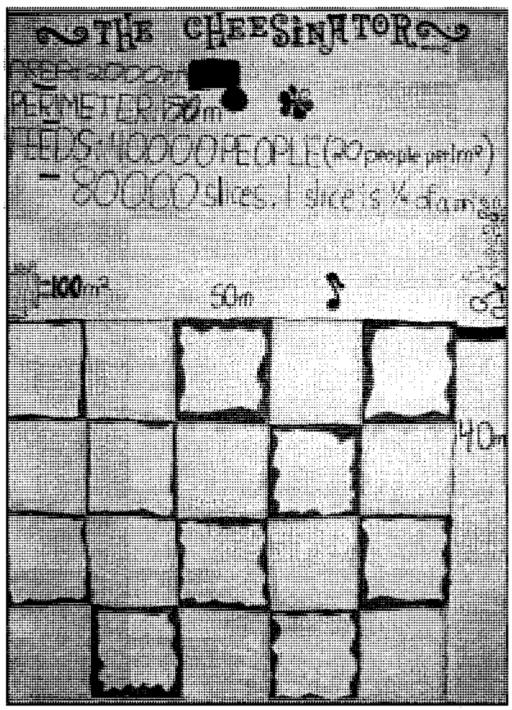


Figure 6.1. Poster for Record Pizza Task

The students were able to demonstrate a good understanding and visualization of multiplying and of area. When dividing they were able to group pieces, but not all solutions were practical. The survey showed that some students felt they had learned by doing the project. However, the project did not extend the thinking or understanding of pure math concepts beyond what we had covered in class. This was a result of there being a specific product required that was not necessarily open. Making a poster was very time consuming and, once students had determined the representation, did not provide opportunity for extension of ideas. Students liked showing their solutions and seeing others' solutions, but because it came at the end of the project they were restricted in refining their ideas or building on others' ideas. Having neighbour interactions somewhat confined to the group work and having not built wider opportunities to interact into the project restricted the opportunity for representation and interpretation of diverse and emergent ideas.

Complexity and the Valentines Day Party Task

In this task organized randomness was limited by the format of the task. Many restrictions were put on the work as the handout led the students through a series of questions and actions. Students, in their groups, had to each fill out their own chart and answer the questions. Time restraints were not initially given, but were instituted as the talk within the groups diverged from the task.

As in the previous group task, decentralized control meant that the teachers did not give answers. At times they suggested means of attempting a solution or drew students' attention to how the rules could be worded.

Leadership within the groups was not assigned but did not seem to be necessary as each student had his or her own worksheet. Students could look at others' work, and at times teachers suggested that individuals see what others in their groups were doing.

Neighbour interactions consisted of students discussing possible answers within their groups. They were not stopped from looking over to others' work or from showing their work to others. However, there were no organized or specified opportunities for ideas, questions or comments to interact with one another during the activity.

Looking at the Valentines Day Party Task

This task was designed to have students investigate and exercise their knowledge and understanding of grouping. It gave them an opportunity to practice the work we had done with base ten blocks in a practical or applied context that was relevant to their experience. It also gave them the chance to manipulate objects. The students were able to help each other and every group developed rules with some prompting from the teachers.

The students were able to visualize and demonstrate grouping with manipulatives. This task was good for visualization and practice of the concept of grouping. According to Ben-Hur (2006) students "lose 'premature' concepts over time if they do not continue to practice and reflect upon them" (p. 14). However, the task did not extend the students' thinking or understanding of math concepts. It was not especially effective for prompting students to interact in a manner that could induce meaningful collaboration. This was a result of the task being very controlled. The handout, with a chart and directed questions, did not present the opportunity for representation and interpretation of diverse and emergent ideas.

Complexity and the Division of Decimal Numbers Group Task

For this task students worked in pairs within groups of four. They were given very open instruction to find the answer and show how they knew it was right. Almost no restrictions were put on the work. Each pair had a large sheet of paper on which to write their work. No restrictions were put on how the answer could be found or how they should represent it. Time restraints were not initially given, but were instituted as students carried out the task and the end of the available time loomed.

The teachers did not give answers, but at times suggested or asked questions in an attempt to have students investigate a new tactic. Leadership within the groups was not assigned and, since the students were working in pairs, t was not practical. Students could look at others' work, and at times suggestions were given to see what others were doing. Because there were so many pairs and the display papers were so large, students could easily see what others around them were doing. Groups could decide how to approach and solve the problem without restrictions on mathematical actions.

Students could discuss solutions within their pairs and groups and were not stopped from looking over to others' work or from showing their work to others. Students were expected to interact with one another during the activity and were encouraged to check their work with the other pair in their group. The students were able to formally see what others had done at the end of the task when each group presented their solution and story problem to the class.

Looking at the Division of Decimals Task

This task was designed to have students work on something that was new – division with decimals. The students had experience with division with whole numbers and with multiplication with decimals. The intention was for the students to have to attempt to represent and calculate something that they had not formally encountered so that they would have to build on acquired experience and knowledge in trying new things and communicating with each other.

The nature of this task required students to extend their thinking and understanding of math concepts beyond what we had covered in class. Perhaps because the task was at once both familiar and unfamiliar, it prompted much discussion and variety of responses.

The divisions of decimals task was certainly the most stimulating of the three tasks. Many more ideas were bumping into each other and the interactions between the students were more meaningful. Many students were trying unfamiliar things and discussing how they would work. Many students seemed to be proud of their representations, were taking ownership of their work and wanted to show it to others and explain it. Even though the ideas spread basically from one pair, the others could change them enough to make them their own. During the activity the class changed as a whole – from a diverse group of searchers to a mostly unified body that knew something new in its own way.

The emergence of this learning entity appeared to get underway with the first pair of students, Mark and Eddie, who went directly to the bonus problem rather than working on the algorithm. These two were known for needing a lot of direction during class, and very often were off topic. They were accustomed to neither doing well nor getting positive feedback for their efforts. It was obvious that they thought that they were going to "beat the system" by writing a story problem and getting the bonus. Because of the decentralized control built into the task through the lack of prescriptive controls, no one made them change their tack. It was also clear that these two were comfortable working with money amounts. They liked the idea of being able to buy things – especially bikes. They became involved with drawing the solution in colour and were so pleased with what they were doing that they began to show it off to others around them.

The representation of division in Mark and Eddie's poster (Figure 6.2) showed how the amount could be depicted as groupings, though not all equal ones. The amount left over, $19.50, is a remainder. Their design also made a connection between the decimal number 1007.5 and things the students could relate to – money, buying items, sharing with friends and family, and colourful portrayals of objects that kids like.

Mark and Eddie had used a calculator to determine the answer to the division question, to find the total for all the bikes, and to get the remainder amount. They communicated with each other consistently, checking that their calculations and their answers made sense. At no time did they attempt to use an algorithm.

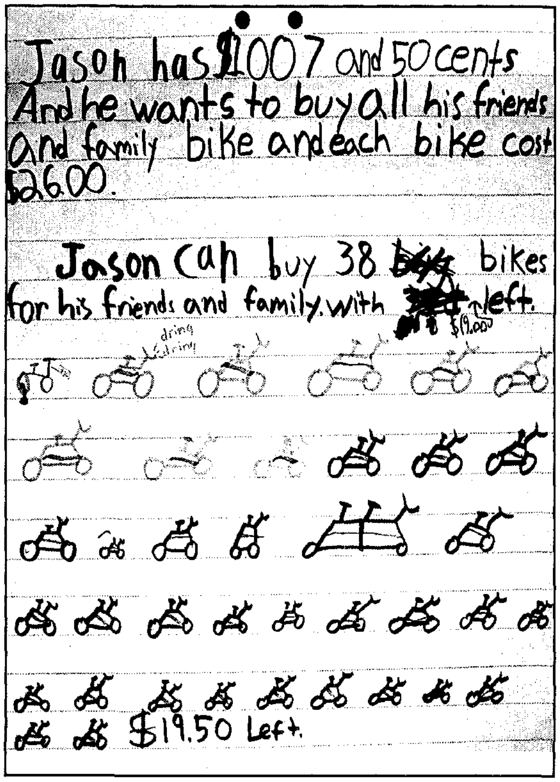


Figure 6.2. Mark and Eddie's poster for the division of decimals task

Sophie and Richard, who were the pair that were sitting next to Mark and Eddie on one side, had used a calculator to find the answer and had begun to work with the repeated subtraction algorithm on their poster. Neither Sophie nor Richard had appeared to be especially astute when working on mathematics problems. Sophie, however, always strove to delve into the tasks and to complete the work with as much detail as possible in order to do as well as she could. As Mark and Eddie were showing off their drawing Sophie looked intently over at their poster. She even got out of her desk to go over to listen to them explain it. Then she and Richard began to discuss how they could represent their answer as purchases. Perhaps because they had already written out the algorithm, they were able to find a way to represent the division as 26 groups of 38.75. Their story problem was about buying 26 desks for $38.75, and they had four depictions of the division using numbers, pictures, groupings and words. This was the most comprehensive depiction of the division task that evolved from the collaborative activity (Figure 6.3).

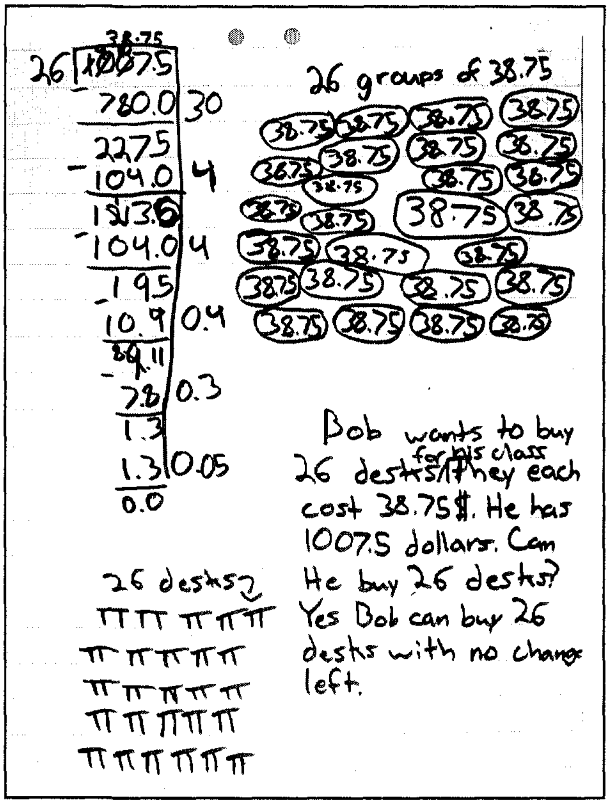


Figure 6.3. Sophie and Richard's poster for the division of decimals task

Other groups had also taken notice of the ideas that Mark and Eddie and Sophie and Richard had displayed. Some began to show groupings with pictures and circled numbers, and wrote story problems. Two groups attempted to draw base ten block representations of 1007.5 and one group used a detailed written description of the steps they took when dividing with the algorithm.

There were, however, groups that never went beyond attempting to use the algorithm to show proof of the answer. Lindsay and Jennifer, for example, never were able to come up with a poster that Lindsay was satisfied with (Figure 6.4).

Lindsay was accustomed to being in control and having lots of success in her school work. She appeared to be bound to using an algorithm and never allowed any input from Jennifer. Lindsay worked hard but couldn't make sense of how this type of grouping would work. She crossed out her first attempt but continued along the same tack. The lack of collaboration separated them from connecting to the development of ideas. Those students who did not collaborate and communicate with others were the ones who struggled with the task.

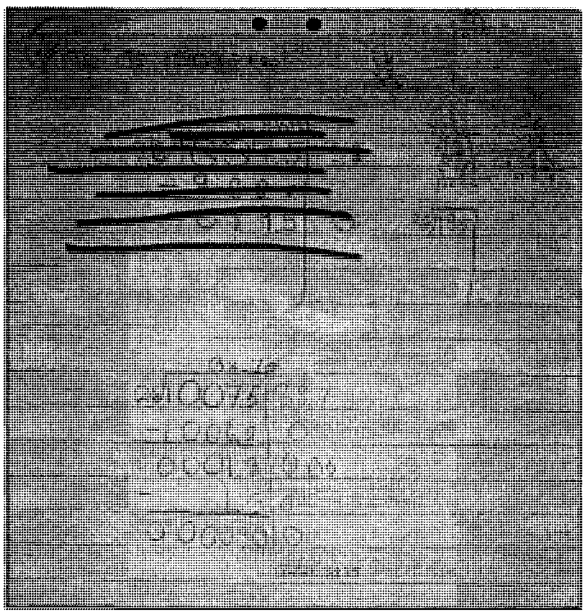


Figure 6.4. Lindsay and Jennifer's poster for the division of decimals task

The status of students within the learning community appeared to affect the possibilities for learning to occur. For example, Mark and Eddie were considered by some of the other students to be “roguish”. Although many students avoided becoming involved with them, Mark and Eddie were seen to be street-wise. Their unusual style was not necessarily seen as something to emulate, but was seen as something worth taking note of. In this way, although they were not regarded as leaders, they directed the thinking of some others onto paths that may not have been taken if they were totally outsiders. Conversely, the widely different status of Lindsay and Jennifer certainly hindered their ability to collaborate and develop their understanding of the concept. Lindsay had worked and collaborated well in other groups, but clearly felt that she had to take control and that Jennifer had nothing to contribute. Jennifer clearly thought that she had no power to say or do anything within the situation even though she had worked well with other students, and indicated that she had enjoyed doing so.

Both of these situations reveal how the social dynamics of the groupings affected learning. Not only did the students’ sense of being an important part of the action influence the exchange and development of ideas, it also appeared to affect the students’ motivation to collaborate. This is supported by Hamm and Faircloth (2005) in their study of the impact classroom peer factors on achievement.

Actions Within the Entire Group

The class as a whole evolved through collaboration in order to make sense of the concept of dividing a decimal number based on the experiences they had had in the previous days. Before we started the project they had worked with one algorithm for dividing and had not used drawings, manipulatives or words to describe their thinking about grouping. The redundancy that those experiences gave the class helped to form a learning entity that knew what division meant in a much broader way than it had before the collaborative learning began.

The diversity of the class was essential for the development of the range of ideas that gave depth to the understanding of the concept for the group. The students who diverged from the traditional algorithm spurred thinking that expanded the ways in which everyone could see how division of a decimal number could be comprehended. The unusual responses that built the understanding arose because of the freedom that allowed all ideas the opportunity to develop.

The scope of neighbour interactions and decentralized control produced a community of learners that essentially acted as a whole. The class was responsible for its production and depiction of an answer that made sense, and in doing so made progress toward developing a notion based on its experience. The class as a whole had come to make sense of a new concept without being told how it worked.

Reinforcing the Concept of Division of Decimals

After the students had presented their posters I presented to the class how division with a decimal number might work with a long division algorithm. I followed up with this instruction for several reasons.

6.13 Estimate the solution to calculations involving whole numbers and decimals (2-digit whole number multipliers and divisors). (p. 21)[4]

Although outcome 6.12 indicates that students may use appropriate technology (that I interpret as a calculator), my experience with teachers, administrators and parents has led me to think that most people expect students to know how to use long division. Looking at a past Provincial Achievement Test for grade six mathematics I found this calculation question "What is 63.27 ÷ 3?" I was not certain whether students would think that they could answer the question by keying it into their calculators and writing the answer. If any of them did use the algorithm they would have had more experience with it.

I did not spend much time on teaching "how" to divide a decimal number nor did I give the students a practice sheet on it. (Ellen may have given them division worksheets when I was not there or for review later in the year.) Most of their experience with division came from their work on the problem task.

[4] These excerpts were taken from one of the programs of study that the Government of Alberta makes available to teachers - the Kindergarten to Grade 6 mathematics program of studies. All of the versions are derived from *The common curriculum framework for K-12 mathematics: Western Canadian protocol for collaboration in basic education*, 1995.

CHAPTER SEVEN: CONSTRUCTING UNDERSTANDING COLLABORATIVELY

In this case study I investigated how sixth grade students in a standard class could learn mathematics through interacting with each other over the course of a unit on number operations focussing on multiplication and division. During the study I observed students constructing understanding while working collaboratively in a variety of ways. They collaborated to develop a range of representations, to practice methods, to demonstrate understanding, and to expand knowledge. I observed individual students developing their abilities through collaboration in ways that they would not have been able to while working individually. I also observed the class become a complex learning community that was able to delve into expanding areas of knowledge. These events were dependent on opportunities presented to the class that were based on accommodating the students' academic and social development, and on the non-prescriptive aspects of the tasks and activity.

Building on the Need to Socialize

In order to collaborate in ways that promote learning students need to be able to socialize. During adolescence students begin to need to develop peer relationships and are driven to socialize, providing the ideal context to build upon with social constructivist learning activities. The students in this study enjoyed working together and found it to be a useful and meaningful experience. This was indicated by the quality and amount of the work they did, the collegial atmosphere of the classroom, and their oral and written responses. In every instance that they worked collectively the activity went beyond the allotted time for mathematics. The students' eagerness to work together was the foundation for effective collaborative learning.

The means to having the students use the social context to enhance their learning was based on helping them develop effective ways to work together without controlling their actions, and on giving them activities that they could relate to – that is, that most of them found useful and interesting. To do this it was important to give them the opportunity to develop a range of connections to mathematical concepts and to promote interactions. Students' needs to socialize can, however, become distracting rather than empowering, so a balance needed to be established and maintained. This required that the tasks and behavioural expectations be proscriptive rather than prescriptive.

Designing Opportunities for Collaboration

It became clear that the degree of prescription in the tasks limited the amount of exploration. The party task had more controlled activity and, though it provided the opportunity for students to work together well, it only presented them with practice. The pizza task provided more freedom but had some controls that restricted expression. The division of decimals task was proscriptive and produced the opportunity for the class to evolve into a complex learning community. This type of opportunity cannot occur for every lesson. The class had developed pre-requisite understanding in a variety of ways and had practiced procedures so that they were familiar and the class could refer to and use them. The group was at a juncture where they could go beyond what they knew, to discover something new. It was important for me to recognize and capitalize on this moment by designing an activity that gave the class the stimulus and autonomy to do so – an opportunity with direction and freedom. It was also important that I follow up this activity with the sharing of ideas, the development and practice of this concept so that it could become part of the students' understanding of mathematics.

When developing materials and tasks we need to look at where the students are and what they need in their quest for understanding mathematics. Sometimes it will be visualization, sometimes practice, sometimes reinforcement, sometimes consolidation, sometimes exploration, but in this case I was able to implement collaborative learning and the opportunity to interact into most of what the students did. This produced a very positive experience for the whole class.

Structure and Freedom in Learning

Freedom to explore socially and academically, produced opportunities for developing effective techniques for collaborating and for new ideas to emerge in learning mathematics. I discovered that as I worked with the students I needed to respond to their needs with tasks that differed in the amount of control and prescription. Sometimes as the students interacted it looked as though they were off-topic when, in fact, their diversions were producing support for the development of ideas. Other times they seemed to be working on the task but the interaction was non-productive and veered toward merely being social. For these reasons it was imperative for the teacher to be involved with the class activity. Indeed, I came to understand that I needed to be learning with the class and responding to the directions that they took. In this way we built our story of learning mathematics together. Not every individual responded to every task or activity, but as a whole we moved toward a more comprehensive and fulfilling understanding of the mathematics we took on.

Students' Impressions of the Tasks

Before leaving Smith Meadows School I was able to interview ten of the students to find out which of the math activities we had done that they thought were the "best" and why. I included individual as well as collaborative activities. At the time we had started a new unit on ratios by measuring a giant footprint and determining the size of the giant that had left the print. Again the students had thrown themselves into the fantasy of the work and were noisily helping each other measure their body parts. Two of the ten students thought that this project – the Giant's Footprint – was the best because of all the calculations and the challenge or mystery. Their responses may have been influenced by the timing of the interview as they responded with the work that they were currently doing.

Five of the students thought that the base ten blocks and charts activity was the best and one other student mentioned it as a second choice. Their comments included:

"It's easy and I learned it fast. It's faster and it helps me the most."

"You start learning really quick... you get the answer really easy... and they'd be able to multiply better. The charts give more ways to show it."

"Because it was the most challenging and it teaches us the most."

"Because it shows how you can do multiplication in different ways."

"Because it is the easiest and that would be the easiest one to teach them first. With the charts you can teach them to line up the decimals and stuff."

"I don't know why. It just makes your work easier."

Two students chose the Record Pizza task as the best. One of the students who chose this task spoke about working with others saying,

"It was fun working in groups and seeing what the pizza was like with the cubes. I got help from people… when I got stuck on things."

The other student who chose the pizza task said,

"It seems pretty accurate – the world record. It shows them how to measure. It takes a lot of math. It solves a problem."

Lindsay chose the error correction task as the best because,

"I love multiplication and it is good to show people where they are right and wrong."

The responses indicated to me that the students felt that the work we had done was interesting and that it helped them learn mathematics. Not every student was absorbed in the work at every moment, but there was the feeling that we were all involved in working together to understand meaningful mathematics in a way that was not too difficult.

Perhaps one of the most memorable moments for me was when I had come to the class after being away for a couple of days. The students were out for recess and when they came into the classroom one of the girls saw me, ran up to me and gave me a hug saying, *"Oh good – you're back."* This was quite unexpected and spontaneous, but very nice and encouraging.

CHAPTER EIGHT: HOW THE STUDY EVOLVED

I began this study with an interest in the meaning that students make when they collaborate to solve problems or work on projects in mathematics. The situation in the classroom that promoted the construction of understanding in this way required that I pay attention to the students' needs to interact socially and the opportunities that the work provided for them to engage with mathematics. As the students and teachers worked together I began to realize that the dynamics of the tasks that I presented to the class were critical for generating situations for students to create understanding.

The focus of my study then evolved. As the activities that the class engaged in developed, it was clear that the structure and intention of the tasks were producing a variety of opportunities for the students and the class as a whole to learn. The tasks needed to connect to the needs of the class to collaborate and to generate knowledge.

It became clear that I had to pay attention to more than the apparent purpose of the activity when designing and reflecting on the tasks. Not only did they have to provide genuine opportunities for students to collaborate and share meaning, each one had to serve to provide some aspect(s) of learning – diversity of connections to concepts, developing redundancy within the group, practice in order to develop a new idea, reflection on understanding, demonstration and sharing representations, freedom to explore, boundaries to focus investigation, reason to exchange ideas, different people to direct investigation – for the class as a whole to come to a new understanding.

Implications of the Study

Clearly all these tasks were needed at different stages of the class's development. One type of task repeated at each level would not have resulted in the students being able to progress individually or as a learning entity in the way that it did. It was also necessary for the teacher to become involved in the activity generated by the tasks, not only to spur the ideas by interacting but also to respond to the development of understanding with subsequent tasks and activities that address students' interests and understanding and promote deeper understanding.

Without reflecting on the deeper currents of the students' interactions, I may have been disappointed with how collaborative work had helped the students to progress. Without the development of my understanding of complexity theory and how a learning entity evolves, I might have gotten the impression that students needed more direction rather than freedom to create reactions to ideas. One could have looked at the results of the division task and been disappointed in how well each of the students had developed an understanding of the concept. Or, one might have thought that since students were able to come up with correct answers that every lesson should be based on that same model and that they could effectively "teach" themselves. It became clear to me that neither of these would be a satisfactory consequence of my investigation and that a balance is needed.

The students' reactions told me that they enjoyed the collaborative work we were doing. I certainly enjoyed it and found it to be a very satisfying way to teach. I will certainly continue to include collaborative work in my teaching, and now I will be able to consider how the students are evolving individually and collectively both with their understanding of and attitudes toward mathematics. I found this experience to be much more effective than much of what I had done in the past in my mathematics classes.

John P. Smith III (1996) addressed the issue of teachers' need to feel efficacious in their work and how lack of efficacy may limit reform in mathematics teaching. Traditionally teachers have taught math by "telling" students, that is stating facts and demonstrating procedures, and this has formed the basis of their feelings of efficacy. Reforms in math change this focus, and unless teachers develop different means of feeling efficacious, they may not continue to attempt to align their practices. Smith elaborated on the many aspects of this – prior experience and beliefs, limits of past mathematical experiences, range of adjustments, resistance within school culture, and how change is unsettling. However the components of teaching that build efficacy are also practices that are consistent with reform principles. Once teachers have successful experiences teaching and working with students to help them construct understanding they will begin to feel that it is more rewarding and will be willing to risk changing their practice.

This experience will certainly change the way that I teach mathematics and I will share my practices with my colleagues. I will look for ways to implement the types of collaborative activity that worked in this study in other areas of mathematics and will continue to try a variety of ways of having students work together looking for more types of tasks that will inspire social constructivist learning.

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APPENDIX A

Talbot's report on a presentation by Rober Sylwester

Robert Sylwester

"Robert Sylwester is an Emeritus Professor of Education at the University of Oregon who focuses on the educational implications of new developments in science and technology. He has written several books and more than 150 journal articles. His most recent books are *Student Brains, School Issues* and *A Biological Brain in a Cultural Classroom*. The Education Press Association of America gave him Distinguished Achievement awards for his 1993 and 1994 syntheses of cognitive science research, published in Educational Leadership. He has made more than 1300 conference and in-service presentations on educationally significant developments in brain/stress theory and research" (http://technologysource.org/author/robert_sylwester/).

Robert Sylwester writes a monthly column for the acclaimed Internet magazine Brain Connection (www.brainconnection.com). He was the keynote speaker, and gave presentations at the ELMLE (European League for Middle Level Education) Conference in Barcelona, Spain January 23-25, 2004. He also recommends the book *The Primal Teen* by Barbara Strauch for parents and teachers living and working with middle school children.

Brain Development

Every living organism has two purposes – to stay alive and to reproduce. According to Sylwester, first we learn the skills we need to survive as human beings – from the age of 0 to 10 years – and then we learn how to be a productive/reproductive being – from about 11 to 20. Each of these 10-year periods is composed of about 4 years of general incompetence and 6 years of maturation. The concrete operations (of survival) occur with the development of the back of the brain, and the formal/abstract operations occur with the development of the front of the brain.

When a child is born all kinds of biological events occur to make a child love its parents, and to make the parents love the child. During the next ten years the child's parents make the decisions for him – what to eat, when to go to bed, where to live, etc.

During the first four years the child's sensory lobes (back of the brain) develop and the child learns how to walk, how to talk, how to get food, and all the other skills necessary for survival. Not only is he ready to learn these things, he is driven to learn them. For example, he will struggle to get down from an adult's arms once he is ready to learn to walk. At this time, adults expect the child to be incompetent at these skills. He will stumble and fall, and his parents will be there to help him up and support him till he becomes competent at walking. We expect children to learn from their mistakes and eventually become proficient.

After about four years, children have the basic survival skills they need, and they spend the next six years perfecting them. This is when most go to school, join clubs, and take lessons. They are learning how to do things, but rely on adults to solve their problems and tell them whether they should do things.

At the age of 11 a child's frontal lobes begin to develop. Now the child begins to learn how to socialize and how to make decisions. The ability to attach himself to a peer group, to socialize, to make decisions and to solve problems sets the stage for him to become independent, find a mate and reproduce.

Again the 10-year period is divided into a 4-year part of general incompetence followed by 6 years of general competence, and most people move into high school at the age of about 14. During the time period of 11 to 14 the child moves from childhood to puberty with the onset of reproductive capabilities, from concrete to formal operations with the maturation of intelligent thought/behaviour, and from an authoritarian to a peer morality.

Once again, the child is not only ready to learn these things; he is driven to learn them. Just as he strove to get down and walk before he was a year old, he will strive to become part of a peer group. Also, at this time children must, in a sense, fall out of love with their parents, because one cannot fall in love with someone else when he still worships his parents.

Once again, for the first 4 years the child begins slowly and awkwardly, and during the next 6 years moves toward confident competence. Although we expect children to fall when learning to walk, we often have difficulty dealing with children when they are awkward with social skills and decision-making. Just as the child needed love and support when learning survival skills, he needs love and support when learning these skills. He also needs support from adults who are not his parents, such as teachers.

During this exploratory time we need to expect children to be incompetent, to accept their efforts to learn, and to give them opportunities to practice decision-making in a safe environment. We need to expect that our children's friends/peer group is becoming a key influence in their lives.

At about the age of 14, children have developed some competence and spend the next 6 years developing their abilities.

Age 11 to 20 Tasks

Children learn about:

How to become a productive reproductive human being

Sexuality and Commitment

Vocation

Moral and Reflective Thought

(Move towards delayed, reflective responses)

APPENDIX B

Mini-Unit Plan for Grade Six Starting January 15, 2007

Strand: Number
Number Operations: Students will demonstrate an understanding of and proficiency with calculations, and will decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.
General Outcome: To apply arithmetic operations on whole numbers and decimals in solving problems.
The Specific Outcomes for this mini-unit are:
6.12 Solve problems that involve arithmetic operations on decimals to thousandths, using appropriate technology (2-digit whole number multipliers and divisors).
6.13 Estimate the solution to calculations involving whole numbers and decimals (2-digit whole number multipliers and divisors).
6.14 Use a variety of methods to solve problems with multiple solutions.

Exploring Multiplication and Division

Multiplication:
As repeated addition
As groups
As "of"
As area
As "hops" along a number line (distance)

Division:
As repeated subtraction
As putting "into groups"
As separation "by" amounts
As dimensions (of area)
As the number of "hops" along a number line

Multiplication Activities

1. Formative assessment of students' perceptions of multiplication.

a. With a partner write a story problem that uses multiplication in the solution. Solve the problem. Show how you can prove that your solution is correct.

b. Together with your group make a web or poster showing everything that you know about multiplication. Include diagrams, words, numbers, etc. Be prepared to explain the poster to the class.

2. Develop a variety of ways of thinking about multiplication

a. Create a number line (could use metre sticks or tape measures). Using the millimetre marks as numbers, show the multiplication of whole numbers, such as 7x15, 13x11, etc. Then use the number line to determine the multiplication of decimal numbers using the millimetre marks as thousandths, such as 16x0.15, etc.

b. Using base-10 blocks show the multiplication of several whole numbers. Then make the flat into a whole representing 10 tenths x 10 tenths. Using rods show multiplication of tenths such as 3x0.7, etc. Next, make the large block into a whole – with rods being hundredths and flats being tenths. Demonstrate the multiplication of tenths, hundredths and thousandths.

3. Bring together ideas and demonstrate understanding.

a. Together with your group determine a rule or rules that students can use when multiplying decimal numbers. Make sure your rule is clear and can be used in any problem.

b. With your group make a poster showing the multiplication of 17x2.45. On the poster illustrate how you know that your product is correct. Be prepared to explain your justification.

Division Activities

Review
Do a version of The Halloween Party lesson plan as a way of reviewing division rules and of looking at division as grouping and the inverse of multiplication.

1. Develop a variety of ways of thinking about division. With each activity write the divisions with both symbols ÷ √

a. Look at the number line and do the inverse of the multiplication problems done previously, such as 7x15=105 so 105÷7=15 by showing that 105 centimetres broken into 7 pieces gives pieces of 15 centimetres; and that 3x0.7=2.1 so 2.1÷3 gives pieces of 0.7.

b. Arrange base ten blocks in rectangles to show the dimensions as factors thereby giving the quotient in division. For example, take blocks that equal 1.2 and arrange them in a rectangle with one side being 0.3 showing that 1.2÷0.3 gives a rectangle with a side of 4. After showing several examples, challenge the groups to come up with some examples of their own.

2. Bring together ideas and demonstrate understanding.

a. Together with your group/partner write a story problem that uses division in the solution. Illustrate your solution.

b. Together with your group determine a rule or rules that students can use when dividing decimal numbers. Make sure your rule is clear and can be used in any problem.

c. With your group make a poster showing the division of 27.3÷4. On the poster illustrate how you know that your product is correct. Be prepared to explain your justification.

APPENDIX C

Error Correction Task

The error correction task was modelled after the mistakes that some of the students had made with the multiplication algorithm before we began the study. I gave the students this story problem and told them that the method was wrong so they had to look for how to correct it. I wrote it on the white board, read it aloud, and instructed them to write a response to it.

Leopold is a grade six student. Recently he did this multiplication question in this way:

36	36	36	36
x23	x23	x23	x23
	18	6 18	618
	(3x6=18)	(2x3=6)	

Leopold did not like it when his teacher, Mrs Beauty, marked it wrong. He insisted that he had done the multiplication correctly. Your task is to show him how he was mistaken. Prove to him what the correct answer is.

The students were told they could use whatever they wanted to prove the answer, including the base ten blocks. Twenty-five students wrote answers. All but one student described how to do the algorithm correctly – some in great detail. Five students got the answer wrong and three more were not conclusive about the answer. Rosalee, who seemed to have difficulty writing down her thoughts, was the only student who used the base ten blocks and drew the solution, as seen in Figure C-1. Figures C-2, C-3 and C-4 show sample responses.

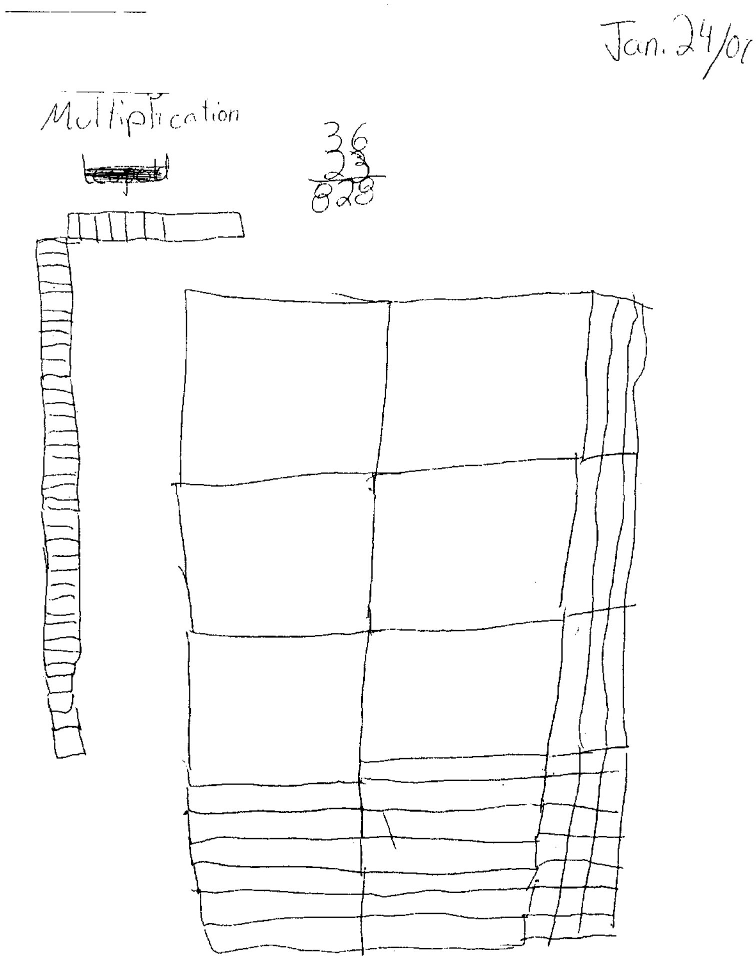
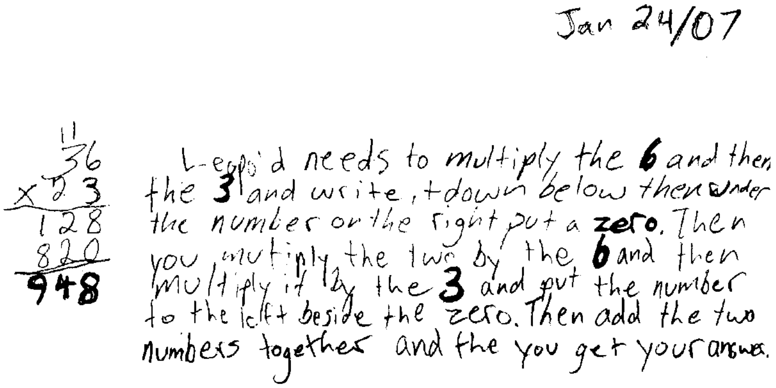


Figure C.1. Rosalee's representation of multiplication using base ten blocks on the error correction task.

First we have to multiply the 6 and the 3 and it makes 18 then we multiply 3 and t2 and make 6 and the correct answer is 68

$$\begin{array}{r} 36 \\ \times\, 23 \\ \hline 68 \end{array}$$

Figure C.2. Sandra's response on the error correction task. Sandra had been "Queen of the Blocks.



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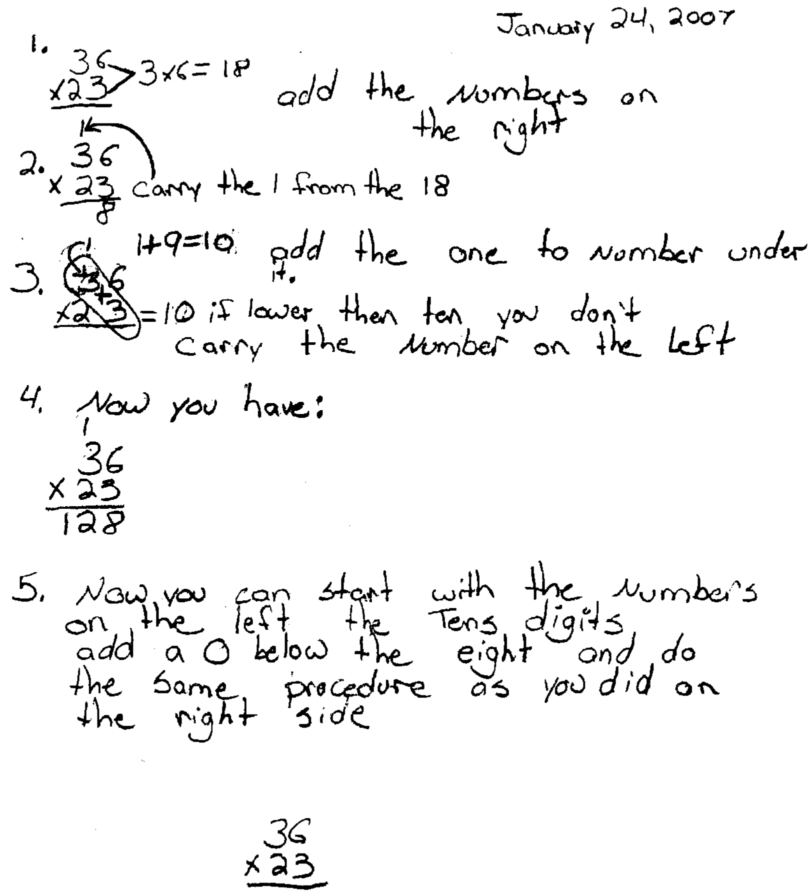
11
36
× 23
128
820
948

L-eopoid needs to multiply the 6 and then the 3 and write it down below then under the number on the right put a zero. Then you mutiply the two by the 6 and then multiply it by the 3 and put the number to the left beside the zero. Then add the two numbers together and the you get your answer.

Figure C.3. Gordie's response on the error correction task.



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1. 36 × 23 > 3×6 = 18 add the numbers on the right

2. 36 × 23, 8 carry the 1 from the 18

3. 1+9=10 add the one to number under it. 36 × 23 = 10 if lower then ten you don't carry the number on the left

4. Now you have:

36
× 23
128

5. Now you can start with the numbers on the left the Tens digits add a 0 below the eight and do the same procedure as you did on the right side

36
× 23

Figure C.4. John's response on the error correction task.