THE UNIVERSITY OF ALBERTA

AN INDIVIDUALIZED APPROACH TO THE OPTIMIZATION OF A HUMAN MOTION

by

PIERRE LEO GERVAIS

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF PHYSICAL EDUCATION AND SPORT STUDIES

EDMONTON, ALBERTA

FALL 1986

ISBN 0-315-32468-6

THE UNIVERSITY OF ALBERTA

RELEASE FORM

NAME OF AUTHOR PIERRE LEO GERVAIS

TITLE OF THESIS AN INDIVIDUALIZED APPROACH TO THE OPTIMIZATION OF A HUMAN MOTION

DEGREE FOR WHICH THESIS WAS PRESENTED DOCTOR OF PHILOSOPHY

YEAR THIS DEGREE GRANTED FALL 1986

(SIGNED) ....

PERMANENT ADDRESS:

10832 - 84 Avenue

Edmonton, Alberta

T6E 2J2

DATED October 9, 1986

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled AN INDIVIDUALIZED APPROACH TO THE OPTIMIZATION OF A HUMAN MOTION submitted by PIERRE LEO GERVAIS in partial fulfilment of the requirements for the degree of DOCTOR OF PHILOSOPHY.

Supervisor

....

....

....

....

....

External Examiner

Date: October 9, 1986

DEDICATION

This thesis is dedicated to the memories of my grandparents, Celestine and Eulysse Ovide Gervais.

ABSTRACT

The purpose of this study was to develop an approach to assess an individual's performance and then to predict that individual's optimum performance. The skill choosen for this task was the handspring 1 1/2 front salto vault in men's artistic gymnastics. This study was delimited to the study of the preflight, push-off and postfight phases for the purposes of the performance assessment. The prediction of an optimal performance was delimited to the prediction of the movement in the push-off and postflight phases. The performance assessment consisted of first developing a deterministic model of the task's performance objective of maximizing the points awarded for the execution of the skill. Measurement of the performance variables, determined from the model, was carried out using standard high speed cinematography. These measures indicated quantitatively that the performance of the skill was a good typical high level performance. Based on the performance result of *points awarded*, the performer's objective function was composed of those performance variables that if maximised, would minimize the point deductions. Postflight height and distance were identified as those variables. Angular momentum was included in a penalty function form to assure that sufficient angular momentum was present for successful completion of the skill. A Lagrangian approach was used to derive the equations of motion and a *Ritz* procedure, using fifth degree polynomials was used to represent and

discretize the state variables (the generalized coordinates). A Complex algorithm was used to solve the optimization problem. Simul[illegible] the postflight's predicted results was achieved usi[illegible] interacti[illegible] program which made use of an optimization scheme. The [illegible]ost function used in the program was the difference between the simulated coordinates for the center of mass and the predicted values. Adjoined to this function was the difference between the simulated and predicted postflight angular momentum quantity. The predicted optimum performance of the skill displayed greater virtuosity in both postflight height and distance. Angular momentum was also greater. A comparison of this study's results with previously published data on the handspring 1 1/2 front salto vault support the conclusion that the optimum solution predicted valid results and a feasible optimal performance for the individual investigated.

ACKNOWLEDGEMENTS

The author wishes to thank Dr. E. Wendy Bedingfield for her supervision, advice, friendship and faith to allow me the freedom to pursue this study.

Thank-you Dr. Robert B. Wilberg for your assistance and continued interest. I am indebted to Dr. John R. Colbourne for his contributions and participation on my supervisory committee. Appreciation is extended to Drs. Zoly J. Koles and H. Arthur Quinney for their assistance as my thesis committee members and to Dr. Marion Alexander for her valued input as external examiner.

Sincere appreciation is extended to Drs. J. B. Haddow and H. Wolkowicz for their valued suggestions and assistance during the early stages of this research. I wish to acknowledge and thank Gordon Marchiori and Iraklis Kollias, friends, who assisted in the filming. I would also like to thank Reeve Martin and Francis Tally who participated as subject and judge respectively.

A very special thank-you is extended to Wendy Gervais for her persistent support, patience, love and Emilienne. To my parents, Paul and Gisèle who set the standard of achievement, thank-you. To Carl and Pat Schell appreciation is extended to you for your support and interest throughout my studies.

Table of Contents

List of Tables

List of Figures

I. INTRODUCTION

One of the objectives of biomechanics research has been the optimization of human movement performance (Hatze, 1973). Through quantitative measurements of movement, biomechanists employing the methods of mechanics have attempted to assess human motion in order to better understand and ultimately improve performance.

In the realm of sports, the practitioner often modelled performance of the successful champion. One of the major drawbacks of such practices was that very often not only were the good performance techniques copied but so were many of the faults. Hay (1973) pointed out that the athlete and coach is faced with the task of determining those features of the champion's technique which contribute to the successful performance and are worth copying. Hay suggested that the logical basis for solving this problem could be found through biomechanical investigation. Thus, using biomechanics, evaluation of those techniques conducive to an excellent performance by the elite athlete could be obtained.

Biomechanics research of sport movements have taken two basic approaches, kinematic and kinetic. The kinematic approach, involving displacement-time histories, has provided qualitative data from which descriptions of movement patterns and techniques have been possible. A more comprehensive understanding of movement is provided through the study of those forces causing and modifying the motion.

Kinetic analysis usually involves some form of modelling of the human body as an open chain system of rigid bodies. The rigid body dynamics problem has further been categorized into two types, the direct and inverse dynamics problems. The direct rigid body dynamics problem is such that the forces acting on the system are known *a priori* and the resulting motion of the system is sought. The more common type of problem in analysis of sport skills is the inverse problem. This form of investigation attempts to derive the forces acting on the system from known motion.

Investigation of the elite athlete, whether through kinematic or kinetic analysis, has provided quantitative information for the practitioner. Caution must be exercised when using information derived from the elite athlete when coaching the less skilled performer. The difficulty lies in the fact that data which is an explicit function of technique is also an implicit function of the anthropometric and physiological properties of the particular elite athlete studied. Very often there will exist a great deal of variance between the elite athlete and the less skilled athlete with respect to such quantities as strength, flexibility and somatotype.

Even with the knowledge acquired from the biomechanical analysis of elite performances, the coach is still faced with the problem of how to guide an athlete from the present performance level to a higher yet feasible level of performance. Thus one of the primary objectives of this study was to provide a strategy for the optimization of an individual's performance of a sport skill consistent with the performer's anthropometric characteristics and the specific features of the sport itself.

In the learning process of a skill, it has been postulated that the individual will unconsciously choose one of many ways to reach their goal, so as to ease the burden of fatigue (Hellebrandt, 1958). Nubar and Contini in 1961 considered that the extremization of a performance index might be man's adaptive process in movement. The authors postulated a *minimal principle* in biomechanics that "in all likelihood the individual will, consciously or otherwise, determine his motion (or his posture, if at rest) in such a manner as to reduce his total muscular effort to a minimum consistent with imposed conditions, or constraints" (p. 377).

In more recent years, the concept that one moves under an optimality criterion has been used to study human locomotion. Researchers such as Chow and Jacobson (1971), and Hatze (1980) have used optimal control theory in their studies whereas Seireg and Arvikar (1973) and Gruver and Sachs (1980) have used linear and nonlinear optimization respectively. Those using these methods are still very much in the minority.

The classical dynamics approach, used most often in biomechanical investigations, is not amenable to the handlings of a great number and variety of constraints. Yet, optimization theory is well equiped to handle magnitude constraints. Therefore it would appear logical that optimization theory be employed in the development of a protocol for the prediction of an optimal skill performance.

A. The Handspring One and One Half Front Salto Vault

The handspring 1 1/2 front salto vault as performed in men's artistic gymnastics is a common vault seen at the senior level of competition. In the tuck position the vault is categorizied in the family of vaults having the highest base score 9.8 (FIG Code of Points, 1980).

This skill is confined to general motion in the sagittal plane with rotational movement about axes perpendicular to this plane. In judging, a vault is divided into 2 parts, preflight and second flight. The performance of the handspring 1 1/2 front salto vault proceeds from a support phase to landing in which the trunk goes through approximately 3π radians of rotation. A schematic representation of the vault is depicted in figure I.1.

B. Statement of the Problem

The purpose of this study was to develop an approach to the analysis of human motion which will allow individual assessment and prediction of an optimal performance of a movement and which further considers both anthropometric and environmental constraints.

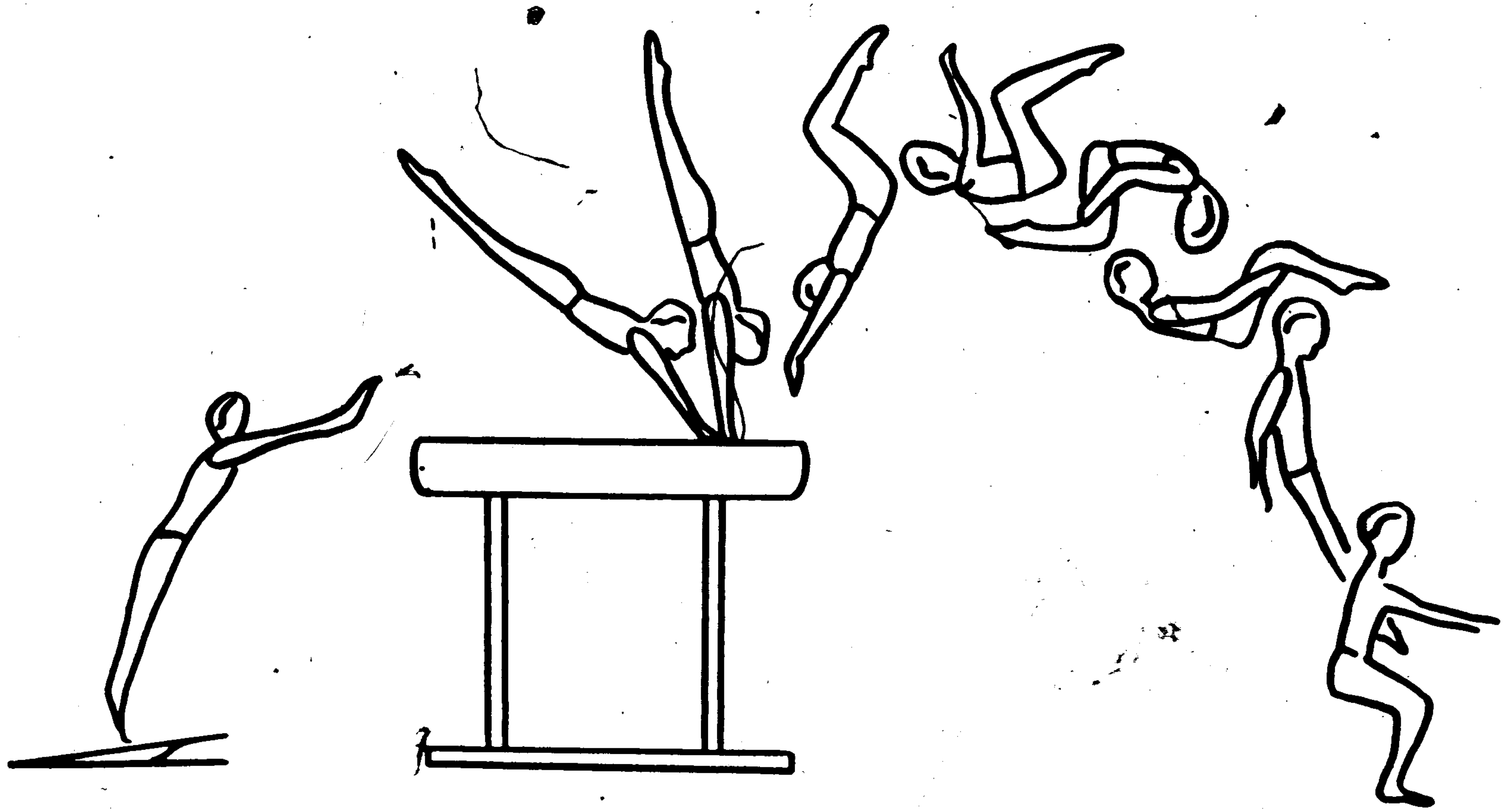

Figure I.1 Handspring One and One Half Front Salto Vault

The development, specifically, will proceed through the study of an individual's performance of a handspring one and one half front salto long horse vault in men's artistic gymnastics.

Limitations

The limitations within this study involved the use of existing body segment parameter data, the data reduction system and approach available, and the athlete investigated.

Dempster (1955) conducted his original research on male cadavers ranging in age from 52 to 83 years of age. Clauser, McConville, and Young's (1969) data, on segmental center of mass, was based in large part on Dempster's data. The anthropometric characteristics of these cadavers cannot be correlated highly with a 22 year old male gymnast but these data are the most complete and most appropriate that are available nonetheless.

The data collection from high speed cinematography is subject to errors associated with the physical imperfections due to camera and projector lens optics and possible camera/projector misalignments. In addition to these errors random error can be attributed to the researcher's ability to locate defined points during the digitizing process. These errors were controlled for as much as possible by strict adherence to standard filming protocol. Data smoothing was also used to attenuate the experimental error.

The athlete investigated, after the performance assessment data was collected, ceased to be an active gymnast. This prevented the study's practical recommendations from being implemented by the subject.

Delimitations

The study was delimited to the study of the preflight, push-off (on-horse), and postflight phases for the performance assessment and the push-off and postflight phases for the prediction of an optimal performance. Three repetitions (trials) were used for this investigation. The film transport rate was at 100 frames per second.

Definition of Terms

Accuracy the maximum amount by which the result differs from the true value - measurement with small systematic error (Beckwith and Buck, 1973, p. 94).

Postflight Phase that part of the second flight from the moment the hands lose contact with the horse up to the moment before the feet contact the landing mat in the handspring 1 1/2 front salto vault.

Precision the degree of agreement between repeated results - measurement with small random error (Beckwith and Buck, 1973, p. 94).

Preflight includes the run, board contact, preflight phase, and horse contact up to the moment the hands lose contact with the horse in the handspring 1 1/2 front salto vault.

Preflight Phase that part of the preflight from the moment the feet leave the board up to the moment before the hands make contact with the horse in the handspring 1 1/2 front salto vault.

Push-off Phase that part of the preflight from the moment the hands make contact with the horse up to the moment before the hands lose contact with the horse in the handspring 1 1/2 front salto vault.

Second Flight from the moment the hands leave the horse, up to and including the stand in the handspring 1 1/2 front salto vault.

II. REVIEW OF LITERATURE

The review of the related literature for the purposes of this study was divided into the literature pertaining to the handspring 1 1/2 front salto vault and the literature related to the use of optimization in the study of human movement.

A. Handspring One and One Half Front Salto Vault

The handspring 1 1/2 front salto vault can be categorized in the family of handspring vaults. The handspring vault is a much simpler vault requiring less angular momentum than is required in the second flight of the handspring 1 1/2 front salto vault. In spite of many biomechanical differences the results obtained in a number of handspring investigations are worthwhile reviewing here.

Dianis (1979) studied the handspring vault as performed by 10 female gymnasts. He used cinematography to measure kinematic variables. In correlational analysis between the judges' scores and kinematic variables, Dianis found that the preflight angular velocity appeared to be related to the contact time on the horse and the second flight variables such as height and distance. However the angular velocity was not found to correlate significantly with the judged scores based on the kinematic variables. Dianis (1980,1981) further developed his 3 segment model of the handspring vault into a mathematical model which allowed the manipulation of kinematic variables. In his model, Dianis assumed that the wrist to center of mass distance was a constant during the time the gymnast was in contact with the horse. In describing the repulsion phase Dianis (1981) wrote that the gymnast is "capable of voluntarily exerting a force upon the horse. The reaction to this force, in conjunction with the kinematic variables of the body at the conclusion of the compression phase, determines the afterflight characteristics of the vault" (p. 36). Dianis, however concluded that for the handspring vault this force during repulsion had a minimal effect on the after-flight characteristics. Bruggeman (1979) reported similar findings and concluded that for well executed handspring vaults the gymnast possesses enough angular momentum in the preflight that "during support on the horse a further increase of rotatary impulse is not necessary"(p. 19). As will be mentioned later, these results were not echoed for the more demanding after-flight of the handspring 1 1/2 front salto vault. Dianis (1980) disclosed that at least for the handspring vault, a long after-flight is probably the most important kinematic aspect of the vault. This variable may be considered to be its performance criterion.

Bajin (1979) evaluated the temporal and angular kinematics of the push-off phase of the handspring 1 1/2 front salto vault as performed by 4 of the world's top gymnasts. When talking about the innovations being made in vaulting, Bajin pointed out that the "push-off has become the most important phase of the total vault. It is also the most difficult" (p. 1). He contended that an excellent push-off was characterized by a complete stretch during this phase. The purpose of Bajin's study was to assess the extension achieved in the the active joints by these gymnasts. He found that the near end vaults had a greater contact time on the horse than those vaults in which the hands contacted the far end of the horse (further away from the board). Bajin reported that for near end push-offs the shoulders were dominant whereas for the far end the lower back and hips dominated. The vaults assessed in this study were peformed in competition between 1974 and 1976. At that time the vaulting horse was divided into two zones, near and far end, thus the distinction between the two vaults. In 1982 Cheetham studied 23 handspring 1 1/2 front salto vault performed by accomplished university gymnasts. He did not distinguish between near and far end vaults and his data does not allow for interpretation of such a distinction. It is this author's opinion, based on observation, that most vaults perfomed today contact approximately in the center of the horse closer to the near end. Dillman, Cheetham and Smith (1985) found that for the 8 finalists in the 1984 Olympics the average distance from the front of the horse to contact for the handspring 1 1/2 front salto vault was 0.71 meters. The range was between 0.45 and 1.0 meters. The horse is approximately 1.6 meters in length. One can then speculate that on the average the shoulders are the predominant joint during the push-off from the horse when performing a handspring front salto type long horse vault.

Bajin (1980) used cinematography to evaluate the handspring 1 1/2 and 2 1/2 front salto vaults as performed by the then World Champion, Roche from Cuba. Bajin found that both vaults had similar center of mass displacement profiles. He suggested that the weakest component of the vaults was the repulsions technique. The gymnast did not have complete body extension immediately after push-off from the horse.

Cheetham (1982) attempted the identification of those preflight variables that affect the three main postflight variables of the handspring 1 1/2 front salto vault. These postflight variables were height, distance of the landing from the end of the horse and the angular velocity. Cheetham revealed that for his subjects, there were no significant correlations found between any of the preflight kinematic variables and postflight height and angular velocity. Cheetham listed that the postflight distance was:

1. directly related to the distance and time of preflight
2. inversely related to the angle of take-off
3. inversely related to the change in horizontal velocity on the board,
4. directly related to the horizontal take-off velocity
5. inversely related to the vertical velocity on horse contact (Cheetham, 1982, p.246).

Cheetham also pointed out that all but one gymnast exhibited an impulse on the horse. Dillman et al. (1985) in their descriptive analysis of the handspring front salto vaults also observed a vertical velocity increase to almost 3.0 meters per second (p. 110).

B. OPTIMIZATION

Luenberger (1973) described optimization and its potential with the following:

> It offers a certain degree of philosophical elegance that is hard to dispute, and it often offers an indispensible degree of operational simplicity. Using this optimization philosophy, one approaches a complex problem, involving the selection of values of a number of interrelated variables, by focusing attention on a single objective designed to quantify performance and measure the quality of the decision. This one objective is maximized (or miminized, depending on the formulation) subject to the constraints that may limit the selection of decision variable values. If a suitable single aspect of a problem can be isolated and charaterizied by an objective, be it profit or loss in a business setting, speed or distance in a physical problem, optimization may provide a suitable framework for analysis.
>
> It is of course, a rare situation in which it is possible to fully represent all the complexities of variable interactions, constraints, and appropriate objectives when faced with a complex decision problem. Thus, as with all quantitative techniques of analysis, a particular optimization formulation should only be regarded as an approximation. (p. 1)

The use of mathematical programming or optimization is still a relatively young subdiscipline in mathematics and as such its use in the study of human movement is just emerging. The first mention of the extremization of a performance index was in 1961 by Nubar and Contini. There was no other reported research in the area until 1968

(Beckett and Chang). Yet it was not until 1971 that Chow and Jacobson actually attempted an optimal control problem approach to the study of a motion. It is interesting to note that since Nubar and Contini's publication there have been approximately 50 research publications to date but 50 percent of those have been reported in the literature since 1980.

The study that sparked much of the research in the area was that of Nubar and Contini (1961), in which they postulated that the extremization of a performance index might be man's adaptive process in movement. Intuitively the authors suggested that "little trace of uncertainty is apparent in the movements of the individual. Rather, his pattern of motion is generally precise at each instant, as though it were in obedience to some strong inner rule of conduct. This rule seems concerned with the reduction of exertion to a minimum at all times consistent with the task assigned" (p. 380, 381). Nubar and Contini stated their minimization problem as the individual's selection of a set of joint movements such that an effort function was minimized consistent with imposed constraints. They defined their effort function as exertion or muscular effort as related to energy expenditure in all muscular activity, as a product of a constant, the joint moment squared and the time of its application. The authors illustrated their theory's use in eliminating the indeterminacy in the equation for a static case equilibrium example of a 5 segment body representation in 2 dimensional space. They also suggested that a dynamic case may be handled using the techniques of the calculus of variations.

The next team of reseachers to consider studying human motion under a minimum principle was that of Beckett and Chang (1968, 1969). They hypothesized that in well-learned activities such as gait, man would move in such a way as to minimize the amount of mechanical work done. To assess their hypothesis, the authors investigated the swing leg in normal walking. The leg was modelled as a rigid body chain, and the equations of motion were derived. They then imposed forces and moments at the joints of the leg to produce motion that was consistent with the geometric constraints and which resulted in minimum energy expenditure. The simulated kinematic profile was compared with experimental data reported in the literature. The results appeared to compare reasonably well, thus supporting their hypothesis. They further investigated the amount of work done as a function of cadence. They found that there was a sharp decline in energy expended as the speed was reduced to a critical or ideal cadence after which continued slower walking speeds resulted in a slow increase in energy requirements. They concluded stating that, "This would seem to indicate that for a given individual there is a given distance with a minimum effort. Given the parameters of the body one can determine this gait by analysis" (Beckett and Chang, 1968, p. 153).

Chow and Jacobson used mechanical energy expenditure, as did Nubar and Contini, as the basis for the derivation of their performance criterion. They attempted to improve on one of the shortcomings they associated with Nubar and Contini's study, ie. "that although the effort function being chosen is selected on the basis of mathematical tractability and physical appeal, there is no apparent connection to the physiology of the muscle activating system" (Chow and Jacobson, 1971, p. 263). Therefore, Chow and Jacobson based their performance criterion on external characteristics and on experimental characteristics of certain functional relationships of skeletal muscles. Using the tension-velocity and length-tension relationships and EMG characteristics they stated mathematically that muscle force was a function of neural stimulation, muscle length and velocity. Confining their discussion to one-joint agonist/antagonist muscles, the authors assumed constant moment arms. Utilizing these assumptions and characteristics they formed a cost function which related mechanical energy expenditure proportional to mechanical work ie. the integral of the square of the net moments:

$$W \cong \text{constant} \int_{t_0}^{t_f} u^2(t)\, dt.$$

Chow and Jacobson described the programming approach that they used to study the human movement of gait. Their description of the approach can be considered the standard format taken by most researchers that followed and is therefore quoted here:

> To formulate the problem structure, specification is required of: (a) an appropriate mathematical model for simulating the functional behavior of the locomotor system; (b) kinematic and dynamic constraints on the basis of gait information; (c) initial, terminal, and "inflight",conditions; (d) an optimality criterion that can be extremized to yield the actuating moments and other quantities of motion. (1971, p. 246)

Chow and Jacobson used a 2 link representation of the lower extremities and introduced appropriate constraints to ensure a realistic simulation of the gait pattern. The optimal control problem consisted of three sub-optimal problems corresponding to each of the three phases of the study. State constraints were imposed on the problem which were phase specific. The constrained problem was reformulated into an unconstrained one using a penalty function technique. The problem was then solved using a first-order algorithm for the search procedure and a fixed integration step size fourth-order Runge-Kutta for the integration of the state and adjoint equations. According to Chow and Jacobson, the results of the simulation agreed well with experimental studies reported in the literature. A draw back of this study was that its results were not investigated experimentally with a living subject. Recently Hatze (1980) has critized this work for making the assumption of constant moment arms when he reported published results indicating variances of up to 200 percent.

Researchers in biomechanics have made extensive use of mathematical optimization when solving and simulating indeterminate systems as is often encountered when invesigating individual muscle forces about a joint. Due to the number of muscles spanning a joint, those that are synergistic and those that are in an agonist/antagonist relation usually result in a system with far fewer governing equations then there are unkowns. Making an objective decision as to sequencing and load sharing for each muscle can only be addressed as an optimization problem even if external force data is provided by, for example, force platforms (Vaughan, 1984).

Seireg and his colleagues have utilized the tools of optimization to solve the redundancy problem in biomechanics since about 1973. In their first published research, Seireg and Arvikar developed a model to evaluate muscle forces and joint forces in the lower extremities for different static positions. The authors estimated origin and insertion coordinates from anatomical literature. They represented the 29 muscles on the 3 link model by straight line tensile force representations directed along the lines joining the points of insertion and origin. A linear system of 21 equations was used to describe the model with far more unknowns representing the 29 muscle forces, joint reaction forces and ligament moments. To solve this indeterminacy, the authors proposed 4 objective functions:

1. minimum forces in the muscles,

2. minimum work done by muscles,
3. minimum vertical reaction at the three joints,
4. minimum moments carried by the ligaments at the three joints.

To evaluate their model and determine the appropriate cost function, the authors investigated two leaning tasks and a stooping posture. The *SIMPLEX* algorithm was used to solve these linear programming problems.

Selection and examination of the performance criterion were based on the comparison made between the results and recorded EMG data for 6 superficial muscles. The authors concluded that the performance criterion was a "weighted sum of the muscle forces and joint moments. A weighting factor between 4 and infinity would be applicable to all the investigated postures" (Seireg and Arvikar, 1973, p. 325).

In 1975, Seireg and Arvikar extended their work from the static approach to the investigation of gait by means of a quasi-static approach. Here again, an indeterminate problem was formed. Slow walking was investigated with the purpose of determining the muscles load sharing as well as the joint reaction forces in the lower extremties. The objective function used was the minimization of the sum of the muscle forces plus four times the sum of the moments at all the joints. The *SIMPLEX* algorithm was again used to find the 146 unknown variables. To show the versatility of their method the authors modified the objective function to include joint reaction forces thus providing a gait pattern with the aim of minimizing pain. Again the authors stated that "in general the theoretical results for every muscle of the lower extremities showed good correlation with the reported averages of the EMG pattern obtained experimentally from different subjects" (Seireg and Arvikar,1975a, p. 94).

Seireg and others used these models in a number of quasi-static investigations. The linear programming problems all consisted of the minimization of a weighted sum of all the muscle forces and ligament forces at all the joints. In addition to the lower extremities, the redundancy problem was solved for the upper extremities (Arvikar and Seireg , 1978), the spinal column (Seireg and Arvikar, 1975b) and the jaw, hand and foot as reported by Seireg (1982) in his review paper "Optimum Control of Human Movement". Much of their work culminated in a general purpose interactive computer program which incorporated the models, allowed for various objective functions and would solve for the desired muscle and joint reaction forces (Williams and Seireg, 1979). To use this program, the motion of the system must be known apriori and anatomical data must be provided. The program sets up the dynamic equations using the principle of virtual work and uses a linear programming optimization scheme to handle the redundant force actuators.

Another area of research that has evolved from Seireg and his colleagues, has been in the area of synthesis and optimization of movement (Townsend and Seireg, 1972, 1973 and Seireg and Baz 1971).

Townsend and Seireg (1972) developed a general theory for the analysis and synthesis of bipedal locomotion. Their 6 degree of freedom model consisted of a rigid body with massless extensible legs. The authors approached the analysis of bipedal locomotion in two phases, the analysis phase and synthesis phase. Using motion data, taken from the literature in a set of dynamic equations, the resulting controls (forces and moments) were found. In the synthesis phase, both trajectories and controls were synthesized concurrently to produce an optimal performance of the model with respect to each of the 3 motion criteria used. The cost functions used were maximum stability (as characterized by minimizing the footprint) and combinations of stability plus minimum energy expenditure with different weighting factors (Townsend & Seireg, 1972, p. 82). Townsend and Seireg concluded that the synthesized trajectories did not resemble normal human gait but suggested their use in investigating concepts of support, stability and energy expenditure. In a subsequent study, Townsend and Seireg (1973), looked at the optimal programming approach with 3 varying models of complexity. Their first model was that used in their first study. Models 2 and 3 were derived from a system of two rigid bodies supported by massless extensible legs. The three models had 6, 7 and 9 degrees of freedom respectively. Controls were much the same as in their previous study except that they reflected the additional requirements of control due to the greater number of mechanical degrees of freedom. Similar conclusions to their first study were drawn by Townsend and Seireg when they used two motion criteria based on the minimization of a weighted combination of the size of the base of support (footprint), energy expenditure and magnitude of the system's external and internal angular motions.

Seireg and Baz (1971) also developed a simple model for the analysis and optimization of swimming. Modelling the fluid dynamics of the swimmer, optimal parameters producing maximum swimming speed could be sought for any specific body power. The authors also suggested that their model could be used to determine the effects of arm and leg patterns on swimming performance (Seireg, 1982, p. 164).

A large part of the research in biomechanics utilizing optimization has involved solving the redundancy problem in order to predict muscle load sharing and sequencing. Crowninshield (1978a, 1978b) used a similar [illegible]near programming approach as did Seireg's group to pred[illegible] forces about the elbow. In an attempt to [illegible] synergistic muscle action that was phys[illegible] reasonable, Crowninshield proposed a different [illegible] function and constrained muscle strengths. The linear objective function proposed was the sum of the muscle stress which was equal to the ratio of muscle force divided by physiological cross sectional area. The author supported his choice by stating that unlike Seireg's choice of objective function (sum of muscle forces), this one did not impart a unique advantage to the muscles of the largest moment arms. Crowninshield reported a good correlation between his results and recorded EMG activity.

Crowninshield and Brand in 1981 stated that all optimization procedures required the assumption that the body selects muscles for a given activity under some criterion. Here the authors provided a heuristic proof, supported in part by the literature, for a selection of muscle sequencing and load sharing based on maximizing endurance of musculoskeletal function. They used elbow flexion (3 muscle model) and gait (47 musculo-tendinous elements) for examples of their nonlinear optimization approach with an objective to minimize the summation of muscle stress to the n^{th} power. Using n=3, asserting that the objective function chosen had good convexity and was subject to linear constraints only, the authors stated that their solution was a global or absolute minimum. Crowninshield and Brand reported substantial agreement between the temporal aspects of muscle activity prediction and EMG patterns.

Pedotti et al. in 1978 and in 1982, reported on their research into muscle force sequencing in human locomotion. Using a 2 dimensional, 3 linked representation of the lower extremities, 11 muscles were considered for analysis. Kinematic data and ground reaction forces for two subjects were found experimentally. Constraint equations in the form of equalities between the measured force arms times muscle forces and observed torques about the knee and ankle were used as constraint equations. Four differing objective functions were investigated:

1. $J_1 = \sum_i^{11} F_i$, related to total muscular force required to produce an observed torque;
2. $J_2 = \sum_i^{11} F_i^2$, minimized total muscular force but penalized large individual muscles;
3. $J_3 = \sum_i^{11} F_i/F_{max_i}$, similar to J_1 but demanded larger force production from the larger muscles, and took into account the instantaneous state of each muscle since F_{max_i} depended upon the instantaneous lengths of the muscles as well as their velocities;
4. $J_4 = \sum_i^{11} (F_i/F_{max_i})^2$, used muscles most efficiently while keeping their level of activation as low as possible (Pedotti et al., 1982, p. 150).

These four separate performance indexes were minimized at each instant of time. A linear programming approach was used for J_1 and J_3 whereas for J_2 and J_4 the constraints were adjoined to the performance indexes using the method of Lagrangian Multipliers. A unique feature of Pedotti's approach, and thus a criticism of previous works described, is that predicted results were compared to torque data, kinematic data and EMG data from the subject being investigated and not to data reported in the literature perhaps from different subjects. The results revealed good agreement between simulated force patterns and EMG data when using J.. Even though the gait pattern of the two subjects were similar, there were differing EMG and torque data. Pedotti (1977) also found similar results in another study: "despite the similar kinematics, the torque time courses of different subjects present significant differences in agreement with different temporal sequences of muscle activation" (p. 53). The other three objective functions indicated shorter muscle force durations when compared to the EMG signals.

Hardt (1978) addressed the problem of determining muscle forces in the leg during level walking from methods similar to those of previous authors (Seireg & Arvikar, 1975a). He modelled the lower extremity three dimensionally and modelled 3 muscles solely as tension sources with no time varying quantities. He quantized the gait cycle into 50 equidistant time intervals. His linear optimization problem was to minimize the sum of the muscle forces subject to the joint angle and torque trajectories. These trajectories were available measures taken from the literature. The *SIMPLEX* algorithm was used as the method of solution. Results were compared to available EMG and force data for gait. It was found that impossible force requirements were placed on the tensor fascia lata and that there was lack of activity in 7 muscles. One of Hardt's objectives in this study was to evaluate the optimization approach. He was highly critical of the minimum force criterion since it did not reflect many of the properties of muscles. He then formulated an objective function which incorporated some of the known muscle properties and minimized the instantaneous energy requirements of the muscles. The results of this second optimization did not produce any significant improvement over the first.

Based on his findings, Hardt was also very critical of modelling muscle force-limb movement problems completely from a mechanical point of view. Hardt's criticism appears somewhat conflicting or inconsistent if based solely on his findings. It is unfortunate that Hardt did not have the benefit of evaluating the techniques of Crowninshield and Pedotti et al. who also reported their studies in 1978. Hardt complained that the use of the minimum force criterion precluded a solution set favouring those muscles with the greatest moment arms. This was the rationale used by Crowninshield for selecting his different objective functions (1978a, p. 90). Hardt questioned the use of unidirectional force actuators and suggested that among other things the nonlinear maximum force-velocity and maximal force-length relationship be modelled, features he did not incorporate in the second objective function he evaluated. Although one must criticize Pedotti et al. (1978, 1982) for not justifying their selection of objective functions, they do incorporate some of these muscle characteristics in J_3 and J_4 (Pedotti et al., 1978, pp.

p. 62-65).

Another source of confusion was the use of a linear programming scheme. Hardt stated that nonlinear characteristics should be incorporated into the model and also gave the shortcomings of linear programming. He warned that the use of a linear objective function and constraint space

> limited the number of nonzero variables in a particular solution to a range between the number of equality constraints and the total number of constraints (equality + inequality), the so called basic feasible solution. Since the demonstrated solution involved seven equality and no inequality constraints, only seven of the 31 muscles were active at any one time. This is an artificial restriction imposed on muscle use before the optimization is performed. (Hardt, 1978, p. 77)

Hardt still formulated a linear programming problem in his second optimization. This limitation associated with linear programming may partially explain why Pedotti's J_4, a nonlinear objective function, was found to be the best and why Crowninshield and Brand in 1981 resorted to a nonlinear objective function and programming approach.

Another limitation pointed out by Hardt of the methods he used is that of static optimization. One may also question whether Hardt's conclusions are warranted in view of the quality of his data if one considers Pedotti's criticism of research in which evaluations are based on data from different sources. Hardt does however conclude his paper by endorsing the use of an optimization scheme to solve the indeterminacy problem, with the provision that "the proper solution will require more input as to the physiology of the system since the optimization process itself must be viewed and therefore formulated as an analog to the real system rather than as solution convenience" (Hardt, 1978, p. 77).

Patriarco, Mann, Simon and Mansour (1981) continued with Hardt's objective of evaluation of the optimization approaches for predicted muscle forces in gait. These authors evaluated the significance of various factors which contribute to the formulation of a muscle force optimization solution. They used Hardt's (1978) 31 muscle model. As did Hardt, they evaluated two cost functions, a minimum sum of the muscle forces (Seireg and Arvikar, 1975) and an energy cost function similar to that proposed in Hardt's 1978 paper. This energy function was derived from a relative rather than an absolute measure of energy consumption of the muscles. Patriarco et al. (1981) also made corrections to Hardt's model to account for tendon lengths. To determine individual muscle forces, the joint torques and kinematics were required as input into the model. To assess the accuracy of these data, the authors derived these values independently using MIT's and the Boston Children's Hospital Gait Laboratory's data reduction systems on the same raw data taken from the same subjects. In addition to the two objective functions, evaluation was carried out using physiologically based constraints on the muscle forces. Using the *SIMPLEX* algorithm the problems were solved.

The authors found that the results (patterns and quantities of force) were crucially dependent on accurate determination of kinematic data and calculation of joint torques rather than on the performance index selected. The physiologically based information in the form of constraints on the individual muscles supplemented the optimization procedure, meaning that it gave better results. They admitted that the joint torques actually dominated the muscle force distribution solution and that they were more influential than the mathematical techniques and assumptions used to compensate for muscle redundancy. Probably the most interesting finding of this was that

> of all criteria for judging the validity of muscle force solutions, the temporal pattern was least sensitive to errors. Consequently, the choice among different models and approaches solely on the basis of general qualitative behavior of the muscle gait pattern should be viewed with caution because of the ease with which the general pattern of flexion-extension can be realized. (Patriarco et al., 1981, p. 520)

This is the most widely used validation technique employed by those authors reviewed to this point.

In yet another 'New Technique to Solve the Indeterminate Problem', An, Kevak, Chao and Morrey (1984) proposed a new objective function and used a static, linear programming approach to solve the redundancy problem about the elbow joint. An's objective function differed from Crowninshield's in that instead of minimizing the sum of the stresses, the upper bound, a simple variable for all of the muscle stress, was minimized. The authors compared their approach to the solution used in previous studies: a) the sum of the muscle forces, b) the sum of the muscle stresses, c) the square of 'a)' and d) the square of 'b)'. A nonlinear programming algorithm was used to solve c) and d). The authors reported that their approach compared favourably with results obtained in d). They defended the use of their method over others by stating that it has the benefit of mathematical efficiency (a linear programming approach) and that it "allows a solution which considers more even distribution of muscle stress among all 'synergistic' muscles" (An et al., 1984, p. 367).

A possible problem associated with these previous indeterminacy studies was alluded to in Hardt's (1978) paper, that of static or quasi-static approaches. The moments being investigated are continuous in time yet the model, and approaches, are discrete with respect to time. The question natura. arises as to how these static approaches truly simulate dynamic systems, a question that as of yet has not been addressed in the literature. Two possible reasons might explain the lack of continuous time models. First, to include the dynamic aspects of the system, the objective function would be extremized subject to ordinary or partial differential equations. This would commonly involve an optimal control problem with both state and control variables. The only example of this form cited so far has been Chow and Jacobson's (1971) study in which the state variables were the generalized coordinates i.e.

joint angles and their first derivatives, and the controls were the muscle forces. Most of these problems are nonlinear and involve greater mathematical complexity than those static case examples discussed so far. Secondly, and probably the most significant, is that biomechanists very seldom have the luxury of having state or control trajectories stated analytically as parametric functions. What is more common is sampled data at discrete time points such as is available experimentally from high speed cinematography. This usually adds additional mathematical and computer demands since all processing such as integration must be done numerically. However, these potential problems have not prevented other researchers from looking at optimal control of human movement.

Morel, Bourassa and Marcos (1985) looked at human locomotion using an optimum control problem approach. Their approach and problem was very similar to the study presented by Chow and Jacobson (1971) described earlier. The state variables were the two generalized coordinates (hips and knee angle) and their first time derivatives. The control variables were the joint torques. The objective function, as with Chow and Jacobson, related to energy expenditure through an expression of mechanical work. Adjoined to this cost function was a penalty function to handle the constraints. There were a few differences between the two studies, but these appear to be minor. For example, a Newton Raphson instead of a Runge-Kutta integration scheme was used in Morel's study. No novel technique or new findings about gait were provided as a result of this paper.

Ghosh and Boykin (1975 and 1976) studied the Kip-up on the horizontal bar using an optimum control approach under a minimum time strategy. The authors represented the human subject as a three segment link model. They used Hanavan's (1966) mathematical model to measure the subject's inertial properties. The control variables were the voluntary muscle torques at the shoulder and hips. Controlled constraints, described as functions of the state variables, were obtained from direct experimental measures of the subject. The state variables were the generalized coordinates representing angular position and their first time derivatives at each rigid body link. Defining a Lagrangian and then the Hamiltonian for the system, Ghosh and Boykin derived Hamilton's equations of motion. This analytical approach is seldom used by others since most studies have more than three degrees of freedom (Hatze, 1981).

A skilled gymnast was filmed while performing a kip-up in which he was asked to complete the skill in the shortest time possible. The lack of agreement between measured and simulated results in the state trajectories, was attributed to the large scale deformation observed in the trunk during the actual performance. The authors suggested that the difference between a measured and simulated minimum time might be explained by the following:

1. the actual motion was not the gymnast's minimum,

2. the rate of change of the torques was not considered,
3. the spring at the shoulder and hip was too simplistic, and
4. there may have been a significant difference between the extrapolated portion of the control limit functions used and the gymnast's actual torque limits (Ghosh and Boykin, 1975, p. 199).

Even though Ghosh and Boykin admitted to having numerical difficulties with their approach they do conclude by saying that the approach is viable. They surmised that more accurate solutions could be found if improvements were made with respect to the methods for determining average joint centers and muscle torque limits. They also recommended that more efficient numerical schemes for constrained nonlinear optimization problems be found. Their approach required 136 x 10^3 bytes of storage, and about 6 seconds per iteration. Ghosh and Boykin reported that one solution required 125 iterations.

Campbell and Reid (1985) presented a simplified model of a golf swing to illustrate the applicability of an optimum control theory approach to the study of complex human skills. The authors employed the resultant torques at each hinge as the controls for the triple pendulum representation of the downswing. A Lagrangian formulation was used in the equations of motion which were stated in canonical form. Campbell and Reid used a Cybex II to collect joint torque values for the shoulder and the wrist through a range of velocity values from 0 to 300°/sec. Although this was an improvement over Ghosh and Boykin's (1976) method to arrive at torque constraint bounds two limitations were encountered. First, no values could be found for the upper torso. A constant value, equal to the maximum observed upper torso torque was used for the minimization problem. Secondly, the predicted solutions were beyond the range of velocities measured so a linear predicter equation was required. A similar problem was encountered by Ghosh and Boykin.

Campbell and Reid (1985) investigated two objective functions. The maximization problem was identified as the club head velocity at impact. The amount of mechanical work required to drive the ball 250 yards was chosen as the cost function for the minimization problem. Constraints were augmented to the performance criteria using a penalty function and a steepest descent-ascent algorithm was used to solve the problems. The authors submitted that,

> in optimum control studies of complex human motions, a simplification that is made is to use the joint actuator's torques as the system controls. This assumption will produce feasible solutions only when the controls are properly constrained ... The results of this study indicate that the methods and equipment for determining torque constraints in simplified models be improved. (Campbell & Reid, 1985, p. 531)

The preceding discussion of the literature has dealt with studies in which the state and control histories were discrete data. Some authors posed their models as being static or quasi-static to compensate for the available data which in most cases came from cinematographical analysis. In the dynamic cases Chow and Jacobson (1971), Morel et al. (1985), Ghosh and Boykin (1975), and Campbell and Reid (1985), most of the differentiation and integration had to be handled numerically. Chow and Jacobson were the only authors to comment on the consequences due to the limitation of using discrete experimental data: "in each integration step, the control histories $u(t)$, state trajectories $x(t)$, and ground reactions ($X(t)$, $Y(t)$) are represented by their values at the end points. Presumably such a 'discretization' could have an effect on the quality and convergence rate of the computation" (1971, p. 281).

Campbell and Reid (1985) and Ghosh and Boykin (1975) attempted to express their control constraints analytically as functions of the state variables, position and velocity. However, this still left the authors to contend with solving the state and co-state equations at n number of discrete points, enough to give an adequate sample to display the control and state trajectories.

Gruver, Ayoub and Muth (1979), and Gruver and Sach (1980) designed an approach to determine an optimum lift. "A lift that would be performed by an individual optimizing his performance consistent with task constraints" (Gruver and Sachs, 1980, p. 191). These authors modelled their lifter as a five rigid body linkage moving in a two dimensional plane with feet stationary. The constraints were anatomical limitations, state rates, and task specific constraints, i.e. lifting materials, destination and possible path. Mechanical energy was used as the objective function. The constraint variables were the net voluntary torques at the joints. Two lifting methods were used: a) straight knees and bent back and b) bent knees and straight back. Three different lifting tasks were constrained, 1. foot to shoulder, 2. waist to shoulder and 3. foot to waist. These authors were now faced with solving an optimum control problem of minimizing a functional subject to differential equations, boundary conditions, and inequality constraints. The authors use a Ritz approach to approximate the states thereby converting the original minimization problem into a finite dimensional nonlinear programming problem subject to linear constraints. The states were approximated by special functions[1] that were twice continuously differentiable. The problem was solved using a modified gradient projection algorithm. One can only speculate as to how great the computer demands might have been for the original problem when Gruver et al. reported an average of 300 Kbytes of storage and 5 to 10 minutes of CPU time per optimization using this fairly efficient optimization algorithm.

Hatze (1976, 1980, 1981, 1983) has repeatedly criticized the approaches taken by the previous researchers to solve the redundancy problem in human movement. His main areas of contention, some echoed by the same researchers he

---

[1] the approximating functions were due to Slots and Stone (1969) who specified the displacement-time curve of the free joints of the body.

has criticized, are the use of voluntary torques as the controls, lack of adequate muscle models, the over simplification of the system's dynamics by the linear models, the accuracy of the body segment parameter data and kinematic data, the appropriateness of the objective functions used on the models, and the use of EMG data as a means of model validation. Since it is not the intent of this review to cite specific opposition for each individual work, Hatze's research and his approaches to resolve the above shortcomings will be given.

In 1973, Hatze proposed his concept of optimal control of human movement. As he stated "if an individual repeats a specific motion under similar environmental conditions a certain number of times, the motion will change in a particular way. For the healthy individual we may assume that this adaptation goes in the direction of optimizing the motion in question" (1973, p.138). Hatze also submitted that a mathematical approach to motion optimization would be highly beneficial to the practitioner in terms of economies of time and effort. He believed that the usual trial and error approach was very time consuming and required favorable conditions and sophisticated training methods to be highly successful for motion optimization. He acknowledged Beckett and Chang's principle of optimality and suggested that for well learned cyclic motion, the performance criterion is probably the minimization of the total energy expenditure. In his concluding statement he set the stage for the next step of his research. To tackle the problem of optimal control, he contended, the first step "is to build a general model of the human muscle which allows for all possible states of different types of muscles in the body which predicts correctly the energetic situation for each possible state" (Hatze, 1973, P.141).

Hatze described his skeletal model in a 1974 publication and gave a fairly thorough examination of its use in the time optimum control problem of a constrained kicking motion (1976). In this project Hatze used his model to simulate the optimum motion in a kicking task under the performance criterion of minimum time. The task consisted of kicking a target in the shortest time, starting from a set initial condition, hanging down relaxed, and ending at a predefined terminal state space. Hatze's modelling differed from others in that he had formulated two interdependent systems to describe the dynamic behavior of the musculoskeletal system. The complete set of differential equations governing the motion of the system was divided into the *link-mechanical* set and the *musculo-mechanical* set of equations of motion.

The link-mechanical equations of motion were set up using a Lagrangian formulation where the two generalized coordinates were the hip and knee angles. Taking the first derivative of these coordinates as two additional state variables the equations were expressed in canonical form. The right hand side of these equations, the nonconservative generalized forces, were treated as being the sum of the passive torque (due to tendons, connective tissue, etc.) and muscular torques across the respective joints. Hatze (1977) in a generalized formulation and discussion of his control equations and model, extended this right hand side to include external forces or torques such as wind resistance, external friction, etc. (p. 801). Of these forces, the most important were the muscular torques, as these were the connection between the link-mechanical part and the musculo-mechanical part. "These moments are created internally and can be controlled in a highly organized fashion. Indeed, it is these quantities which (indirectly) contain the control parameters of all the muscle groups involved and which enable the biosystem to form a rich variety of coordinated motions" (Hatze, 1976, p. 105).

The total muscular torque for a given joint is equal to the sum of the moments due to each muscle group spanning the joint in question. Hatze held that there were only five muscle groups of significance in this two dimensional kicking action. Hatze further defined that these moments were equal to the muscle force times its moment arm. Hatze had assumed a stationary axis of rotation for the hip and knee in deriving the equations of motion in the linked mechanical system. He does however point out that this could not be extended into the formulation within the musculo-mechanical part because of the influence of the instantaneous axis of rotation on the magnitude of the moment arms.

Hatze made the distinction between his muscle model and those used in the optimum control problems previously referenced by writing that: "none of these models account for the peculiar behavior of the stretched and stimulated muscle, and possibly even more important, they all exclude the two physiological control parameters; motor-unit recruitment and stimulation-frequency change" (Hatze, 1976, P.109). Hatze gave a set of four differential equations to represent the dynamic behavior of each muscle which contained the two neuromuscular control parameters, motor unit recruitment and stimulation frequency. Some of these phenomenological behaviors represented in these equations are the force - velocity - length - time - active state relationships existing in the contractile element of muscle (Hatze 1976, p. 112).

Hatze described the equation formulation and measurement of required parameters in a series of papers (1977b, 1977c, 1978, 1981) and in his book devoted to the topic; "Myocybernetic control models of skeletal muscle" (1981). These details are beyond the scope of this report. One of the major criticisms one can direct towards Hatze in his reporting is the lack of detail, usually veiled in complexity. Even in his most detailed descriptions one is left wondering how to obtain crucial parameters, for instance his control parameters for motor unit recruitment and stimulation frequency. Hatze (1981) wrote in his book, his most detailed thesis, "a detailed description of the experimental and computational procedures for estimating the neural input control functions $z_i(t)$ and $v_i(x,t)$, $i=1\ldots, m'$ from electromyographic recordings is far beyond the scope of this monograph. Indeed, these models and techniques are so complicated that their detailed description requires a separate treatise" (p. 90). It is unfortunate that the seemingly very important techniques have not been reported in the literature. This information derived from EMG signals will also be discussed later.

Having defined the equations of motion for the link-mechanical system and the musculo-mechanical system and having found the required parameters, Hatze's optimum control problem was to find the admissable controls, stimulation frequency and motor unit recruitment, which transferred the state from its given initial position to a final position in the shortest time possible. This time optimal problem in which the right hand end point of the state trajectories was variable was solved using a differential dynamic programming algorithm. The results revealed two significant findings. The solution was not a global minimum as a similar minimum was reached with different controls. Hatze conluded that the "definition of an 'optimum solution' has to be somewhat arbitrary for this complex system and should, in fact, encompass a certain range of solutions. In other words there appears to be a cluster of near-optimum solutions in the vicinity of the (practically never obtainable) exact optimum solution" (1976, p.125). The second finding related to the control behavior of musculo-skeletal systems, was that for a *maximum effort* motion such as was tested, the controls were of bang-bang form.

Hatze compa[illegible] the optimum model solution to the performance of [illegible] subject and found that those state trajectories wh[illegible]ffered from the predicted, were from motions that took longer. For those living system motions near-optimal, Hatze found that they were close to the theoretical optimal process. In addition to the congruency between the state trajectories as measured with electro goniometers, validity for his model was demonstrated by the control variable similarities.

Hatze (1975) demonstrated the practicality of his method in terms of performance training. In this paper the author related the way he trained his subject to perform his predicted optimal. The initial training, in a period of two weeks, had the subject repeating the activity with feedback consisting of time only. The optimum training period used visual feedback in the form of the current state trajectories superimposed on the predicted optimum state trajectories. The optimal training proved to be far more effective than the initial natural adaptation. Hatze also observed that when the subject fatigued there was a change from a time optimality to that of an energy optimality.

Hatze (1977) extended his musculo-skeletal control model to include a complete set of control equations for general three dimensional motion with n number of links and m number of muscle groups. Hatze's research to date has culminated in a very impressive and substantial optimum control simulation of the take-off phase in the long jump. (Hatze, 1981). Here, he simulated 0.16 seconds of the planar motion of a 17 segment hominoid in a state space consisting of 42 link-mechanical variables and 46 muscle groups resulting in an additional 230 state variables. The performance criterion was maximum distance jumped. Further description of the methods of solution and means of arriving at the body segment parameters were described in additional treatises (Hatze, 1981, 1982, 1983).

Hatze must be commended on his work. There are no authors that have attempted a more comprehensive modelling of the musculo-skeletal control of human movement as is evident by the quality and proliferation of his research publications. Although Hatze's models appear to have great potential in studying human movement, one must also describe his models as being a bit esoteric due in part to computer hardware and software limitations. Hatze (1981) pointed out that a total of some 2,300 man hours were required to complete the implementation and simulation of the long jump. In 1983 he suggested that in the realm of software development for optimization "only a team of experts including system scientists, numerical mathematicians and fully experienced programmers can master this task satisfactorily" (p.10). It would appear therefore that although possessing great potential his models may not have much practical appeal for the average laboratory and sport biomechanist. This sentiment was also shared by Audu and Davy (1985) when they stated the following:

> Although the elegant muscle models proposed by Hatze hold promise for much more accurately describing many of the recognized characteristics of muscles, their use in even moderately sized problems appear to lead to enormous computational costs. As a consequence, these models probably preclude their use on moderately sized computing machinery which might be available to most of the biomechanics research community. (p. 147)

One may speculate that the resource demands might partially account for the apparent lack of simulations reported in the literature using Hatze's approach. Hatze has reported his kick and long jump and Audu and Davy (1985) replicated Hatze's kick in a study assessing muscle model complexity.

Audu and Davy investigated the significance of muscle model complexity in terms of computational results and computational cost. Using Hatze's (1976) muscle model as their basis for comparison, they investigated three other models of lesser complexity and a fifth model in which the net torques were used as the controls for the kicking tasks. Models 1 and 2 were linear models. Model 1 was a simple force generator in parallel with a linear spring and dashpot. Model 2 had an additional contractile and series elastic element replacing model 1's simple force generator. Model 3 was structured in the same manner as model 2 but assumed Hill's force-velocity relationship in the contractile element and the series elastic element was modelled as a nonlinear spring. The primary difference between Hatze's model (model 4) and model 3 were with respect to the force velocity, activation-stimulation frequency, and force-length relationship. The authors use all of the muscle parameters reported by Hatze (1976) but used a different optimization approach, amenable to faster convergence with a large scale problem. The authors found that there was close agreement between the results from model 3 and 4. They concluded that the optimum motion-histories could not be found using the simpler models 1 and 2. Interestingly enough the optimization which had the torques as control variables yielded optimum state trajectories that agreed favorably with the results from models 3 and 4. However, the envelope of the moments did not follow the same pattern as in models 3 and 4. Audu and Davy (1985) concluded by stating that "model 3 which is somewhat less complex than Hatze's provided appreciable gains in efficiency and stability while giving very similar results" (p. 156). These authors also investigated the model's sensitivities to muscle parameters. Using perturbed muscle lengths for the iliopsoas group resulted in different optimal state trajectories for the hip and knee. "These results, as well as others we obtained illustrate the strong influence of the model parameters on solutions using these more complex muscle models; and suggest that a formal sensitivity analysis would be fruitful" (Audu & Davy, 1985, p. 156).

Finally, comparing the controls for model 4 with those of 1, 2 and 3, the first 3 model controls are not exactly *bang-bang* controls. Audu and Davy attribute this to the use of a gradient type optimization algorithm. It is unfortunate that the authors did not replicate model 4 using their optimization algorithm. A run of model 4 may have produced controls that were not exactly *bang-bang*. In talking about the *bang-bang* form of the controls, Hatze (1976) pointed out that "recent experimental results (Hatze and Hayes, to be published) seem to confirm this prediction although only approximations to the actual bang-bang controls could be observed" (p. 804).

One of the greatest potentials in using mathematical optimization in sport biomechanics is the chance of predicting optimal sports performances. Hatze's (1981) long jump take-off simulation, was an example of the use of optimal control theory in maximizing the athlete's performance objective of distance jumped. Although less numerous than those studies of gait there has been a more frequent occurrence of optimal programming in the study of sport in recent years.

Hubbard and Barlow (1980), in their study of pole vaulting, examined the question: "what is the set of inputs which the vaulter should choose (given a set of initial conditions) in order to maximize the height of the center of mass while simultaneously clearing the bar?" (p. 34). These authors formed a three body, five degree of freedom model of the vaulter, a static model of the pole and included the torque dynamics into their overall model of the pole and vaulter system. The objective function was the maximum height cleared. Constraints were set up to assure successful passage over the bar and to prohibit excessively large applied joint torques. This nonlinear, free end time, 2 point boundary value problem was solved numerically using a penalty function approach. Hubbard and Barlow (1980) drew a number of practical observations for the practitioner with respect to this type of performance analysis. They signified that the optimal torque profiles can provide information to the athlete as to where and when to do work and which muscle groups are important. The authors contended that, "it is better to know when to push and pull (i.e. the optimal torque profiles) than to attempt to match one's own trajectories to it" (Hubbard & Barlow, 1980, p. 46). They also suggested that repeated solutions with different initial conditions, muscle constraints, etc. can give information with respect to these quantities (Hubbard & Barlow, 1980).

Hubbard and Trinkle used a simpler model to investigate the effects of initial conditions alone, (without the additional complications connected with muscle torques) on the maximum height cleared for the Eastern Roll (1982) and the Fosbury-Flop (1985) high jumps. Here Hubbard and Trinkle modelled the jumper as a single rigid body and assumed a horizontal position of the body at the top of its trajectory. The question investigated was that "given an initial kinetic energy, what is the maximum height that a jumper can clear (or equivalently), in order to clear a given height, what is the minimum initial kinetic energy required) and what are the take-off conditions which specify the solution completely?" (Hubbard & Trinkle, 1985, p. 448). The objective function height cleared, was defined as a function of the free variables: the initial conditions take-off height, take-off angle, take-off velocity and take-off angular velocity. These constrained nonlinear programming problems were solved numerically. For the Eastern Roll, Hubbard and Trinkle (1982) asserted "that it is advantageous for the jumper to lean back at take-off sacrificing some potential energy mgdsinθ, so that less rotational kinetic energy will be required to become horizontal at the instant the bar is cleared" (p. 173). For the Fosbury-Flop, the authors stated the "results show that the optimal jump always consists of two brushes with the bar, near but not at the jumper's center, so that the zenith of the jumper c.m. does not coincide with the crossbar position (Hubbard & Trinkle, 1985, p. 452).

Hubbard (1984) in his study of 'Optimum Javelin Trajectories' used an optimization technique to search for the optimum initial conditions over a meaningful subset of the entire initial condition space. From the review of the literature, Hubbard found that the range was the function of five initial conditions: take-off velocity, height of release, angular velocity, attitude angle and attack angle. Assuming that in any one throw, take-off height and velocity are maximum, Hubbard looked at two sub-optimum problems in which two variables were free and a sub-sub-optimum problem, in which only one variable was free. He also investigated the global optimum with the three variables being free. For this last problem, he attempted an unconstrained solution, but found that it had to be constrained to ensure a legal throw. Finally, design and environmental parameters were investigated i.e. minimum allowed javelin mass, headwind and tailwind. In this study, Hubbard demonstrated the practicality of an optimization approach. For the global optimum problem, had he attempted a *'brute-force'* approach (in which all the possibilities are simulated) for say 30 values of the three free variables, a total of 30^3 = 27,000 simulations would have had to be performed. Hubbard cautioned that "this number of simulations would require more than 4 solid days on PDP-11/44 and was judged unfeasible." (p. 782).

Vaughan (1985) introduced a simple general purpose nonlinear optimization program with an example of a jump shot in which initial conditions were the state variables. The gradient projection algorithm was written in *Basic* for micro-computer use. The objective function was the minimum clearing distance over the opponent's hands subject to the constraints which would ensure a successful completion of the shot. It was the author's intent in this paper to demonstrate the practicality and to promote the use of mathematical programming in sports biomechanics. Bauer (1982) discussed the possibility of optimization and demonstrated its use with an investigation of a giant swing using a two-link representation of the giant swing. An optimal control problem was posed in which the total energy was to be minimum. The authors used a series expansion to represent the control function, thus reducing the dynamic optimization problem to that of an ordinary optimization problem. In this treatise, Bauer revealed a perplexing aspect of modelling; "it is difficult to improve the model through comparison with experimental results because one cannot tell whether deviation from reality comes from inaccurate model or inaccurate performance index formulation." (1982, p. 142). Other authors have proposed the use of optimization techniques for purposes of improving technique. Borysiewicz, Bucka, and Konar (1981) proposed an optimum control model and procedure as an alternative to the traditional form of training. They used the snatch in weight lifting as their specific example. Remizov (1984) used maximum distance jumped as his objective function in his investigation of the 'Biomechanics of Optimal Flight in Ski Jumping'. The control function in this study was the attack angle time history.

Optimization and optimum control have been used in a number of diverse areas and are recorded here to illustrate the versatility of this logical, systematic approach to decision-making. Soong (1973) used optimization to present a theoretical optimal design for an archery bow. Vaughan, Andrews and Hay (1982) use optimization in selecting the most appropriate body segment parameters to use for specific studies. They chose the minimization of the difference between the sum of the measured distal forces and torques and the optimized calculated distal forces and torques, squared, as their cost function. Chao and Rim (1973) used optimization as the basis of an alternate approach to data smoothing. In their study, they determined the applied moments at the joints from displacement data by minimizing the difference between the observed displacement and that calculated by solving the equations of motion. Penrod, Davy, and Singh (1974) used an optimization scheme to solve the redundancy problem when determining force distribution in the tendons of the wrist. Lastly Vaughan, Hay and Andrew (1982) used an optimization approach to solve a redundancy problem associated with a closed loop problem such as the double support phase in walking. In this study Vaughan et al. investigated two objective functions, the sum of the forces and torques and just the sum of the torques in the joints of the closed loop. They found that the latter cost function gave better results.

In reviewing the preceding studies it appears that we are confronted with a rather confusing situation with respect to the objective functions chosen by man in the performance of motor skill. There have been a number of different performance criteria put forth and most authors have held they were valid as was evidenced by the interpretation of their results. As has been alluded to in the previous discussion the performance objective of a skill may be the task determinants such as time or distance, but for those activities of a well learned cyclic nature, energy may be the cost. Some additional answers may lie in the comments given by researchers in two separate sets of experiments. Keller theorized and used an optimization approach to formulate optimal strategies in running (1973, 1974, 1977). Keller used the obvious performance criterion for foot racing; minimize time to cover a specific distance. Keller's results prompted him to propose that for races under a critical distance of 291 meters that the runner should run at maximum acceleration. He suggested that for longer races the athletes accelerate for a time t, maintain velocity for a period of time and that it would be observed that velocity actually decreased beyond that period until the finish of the race. Keller, noted that "runners at distances greater than 291 meters often finish with a kick rather than with the negative kick of the optimum solution. This discrepancy indicates either that they are not doing as well as they could, or that the theory is inadequate.

Presumably their goal is to beat competitors rather than to achieve the shortest time and that goal influences their strategy" (Keller, 1977, p. 172). Therefore, it may be said that the performance objective may be very different from the seemingly logical or obvious.

Nelson (1983) compared movements which are optimal under different objectives. The specific performance criteria he considered related to movement time, distance, velocity, energy, acceleration and jerk (rate of change of acceleration). In his paper, Nelson discussed the concept of performance trade-offs between competing objectives. He defined an 'economical movement' as one "which is not optimal in any single criterion sense (not minimum-time or maximum-distance, or minimum-energy, etc.) but rather one which represents a reasonable trade-off between the competing physical costs, while meeting the primary requirements of the movement task" (Nelson, 1983, p. 141). Nelson submitted that for skilled movement involving distance and time, the governing motor control strategies may be designed to meet the primary objectives of the task, independent of considerations of physical economy. Therefore, as Nelson contends, a single performance criterion may be insufficient to describe an optimum performance, "especially when the process of optimizing one criterion generally pushes the solution to the limits of one or more of the criteria which were included as constraints" (1983, p. 140).

In summary, optimization has been used as a tool in the study of human movement and has brought objectivity and economy to the decision-making processes. It has facilitated the pursuit of the solution of the indeterminacy problem of muscle redundancy at the joints. It has given some researchers the means to investigate and model human control of skeletal action. And finally, mathematical programming has been demonstrated to be a viable approach to the prediction of optimum performance in sport.

III. METHODS

The dual purpose of this study was the development and implementation of a protocol which allowed individualized assessment of a performance and prediction of an optimal performance of a sport skill which is consistent with anthropometric and environmental constraints.

The sequence taken in this study was the assessment of the performance of the handspring one and one half front salto vault, the optimization and prediction, and finally the simulation of the predicted optimal performance.

A. The Subject

The subject for this study was a 22 year old senior level national competitor who has used the handspring one and one half front salto vault in competition.

B. Performance Assessment

The handspring one and one half front salto long horse vault as performed in men's artistic gymnastics is a common vault seen at the senior level of competition. In the tucked or piked position the vault is categorized in the family of vaults which has a base score of 9.8 (FIG 1979). A performance of the vault is assessed subjectively in competition. The performer starts with a base score from which deductions are made for performance faults. The gymnast may receive up to 0.2 points as a bonus for virtuosity which is normally given for amplitude (height) in the second flight. Ideally the gymnast strives for a maximum score out of a possible 10. This is achieved by maximizing amplitude and minimizing performance faults while adhering to the movement constraints required in the skill.

The FIG code of points divides the vault into two parts, the *pre-flight*, up to the moment the hands leave the horse, and the *second flight*, after the hands leave the horse up to and including the stand. General deductions are made for form violations such as poor position of feet, legs, bent arms, etc when the vault does not require it. Specific deductions for the pre-flight have to do with the body position with respect to the horizontal when the hands first make contact with the horse. The major deductions for the second flight are for insufficient height and distance, figure III.1.

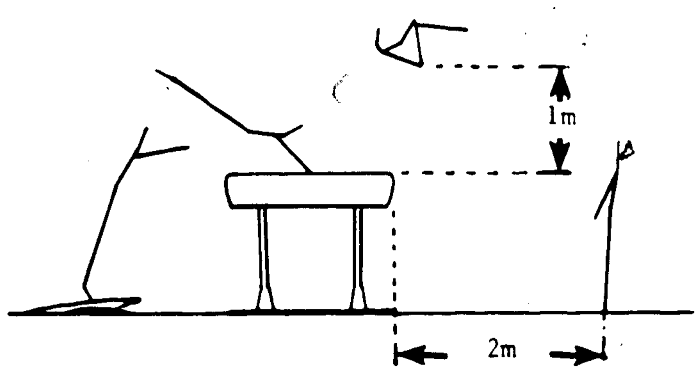

Figure III.1 Minimum Second Flight requirements

To determine the performance variables that should be measured in order to assess a performance, a deterministic type mechanical model was developed which translated the subjective evaluation of the skill into quantifiable variables.

The Deterministic Model

The development of the deterministic model for the handspring one and one half front salto vault consisted of 2 steps. The first consisted of identifying the result of the skill and subdividing it into its point deductions per part. The second step was the translation of these subjective movement evaluation variables (deductions) into mechanical quantities. A further subdivision of these quantities into those quantifiable kinematic and kinetic factors which produced those mechanical quantities was also performed. The approach was based in part on Hay's deterministic models for qualitative analysis (Hay, 1984). Hay described this model as being,

> made up of mechanical quantities, or appropriate combinations of mechanical quantities, and it is so arranged that all of the factors included at one level of the model completely determine those included at the next highest level. It is this second feature which leads us to refer to the model as a deterministic model. (1984, p.71)

The result, as previously stated, is the points awarded. The final score the gymnast receives is dependent on the base score, the amplitude (virtuosity) and the deductions given (figure III.2).

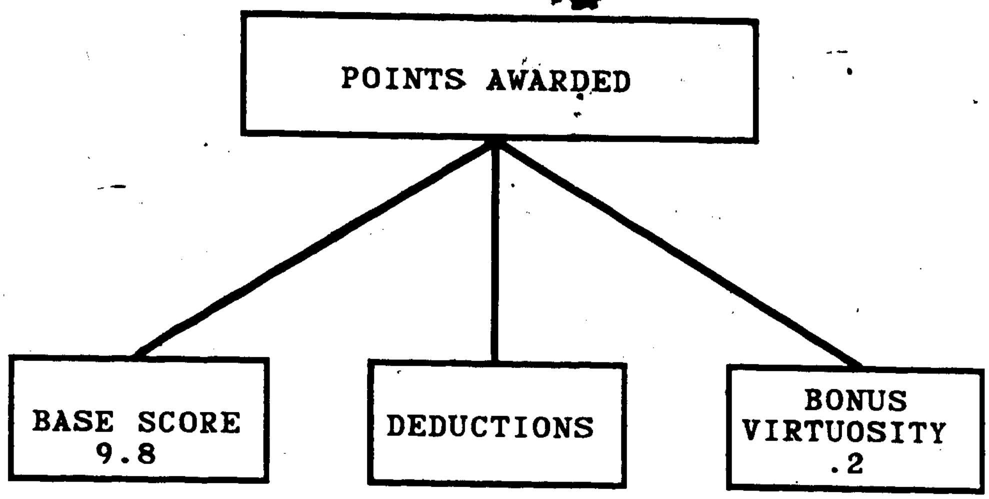

Figure III.2 Step 1: Results and Division of the Results

The skill, for evaluative purposes, is divided into a pre-flight and second flight. Deductions consist of general form deductions and specific ones for body position, amplitude and range (figure III.3).

In the second step, these deductions are expressed in mechanical terms. In describing the procedure for this step Hay and Reid (1982) suggested the factors included at this stage should be mechanical quantities. However they stated "this rule should be disregarded only when the use of another term conveys the same meaning in a more concise way. Thus, for example, it would be more appropriate to use the term body position at touchdown than the more precise but much more long-winded terms coordinates of the heel at touchdown, lengths of body segments...(Hay & Reid, 1983, p. 272). The 'general body position' will be the term used for

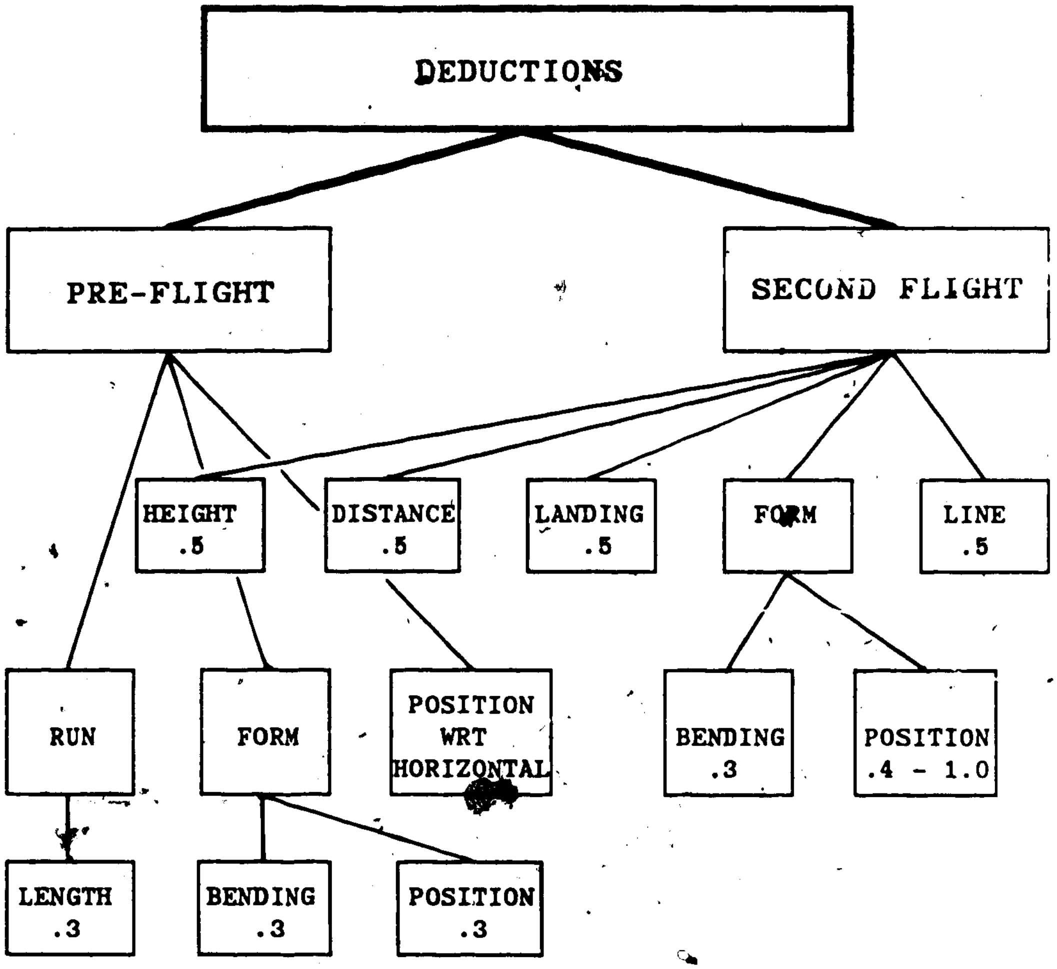

Figure III.3 Deductions

example as a simplification for the specific deduction of bending a particular limb.

The height and distance achieved by the gymnast are dependent upon those mechanical relationships that influence projectile motion. These factors, for uniformly accelerated motion, are the height of the center of mass at take-off from the horse, the take-off velocity and angle of take-off. The height at take-off is the cumulative height of the horse and the height of the center of mass with respect to the horse. The height of the center of mass is dependent upon the athlete's physique and body position at the time of take-off.

The take-off velocity was modelled using the impulse-momentum relationship. According to Newton's second law, Force **F** can be expressed in the form

$$\mathbf{F} = \frac{d}{dt}(mv) \qquad [1]$$

where mv is the linear momentum of the athlete's center of mass. Letting t_1 be the time when contact is made with the horse and t_2 the time the athlete leaves the horse equation [1] can be rewritten as

$$\mathbf{F}\,dt = d\,(mv)$$
$$\int_{t_1}^{t_2} \mathbf{F}\,dt = mv_2 - mv_1$$
$$mv_2 = \int_{t_1}^{t_2} \mathbf{F}\,dt + mv_1 \qquad [2]$$

then the velocity at take-off can be expressed as

$$v_2 = \frac{1}{m}\int_{t_1}^{t_2} \mathbf{F}\,dt + v_1 \qquad [3]$$

where $\int_{t_1}^{t_2} \mathbf{F}\,dt$ is the impulse. These relationships are expressed in block form in figure III.4. The last major deduction can occur during the landing depending on how the athlete comes to a stand. If steps are taken or the hand touches the ground, appropriate point losses will ensue. The complete deterministic model is given in figure III.5. Note that there has been no modelling of the angular characteristics of the skill. The skill requires that the trunk during the second flight complete approximately 3π radians of rotation, however, there are no specific

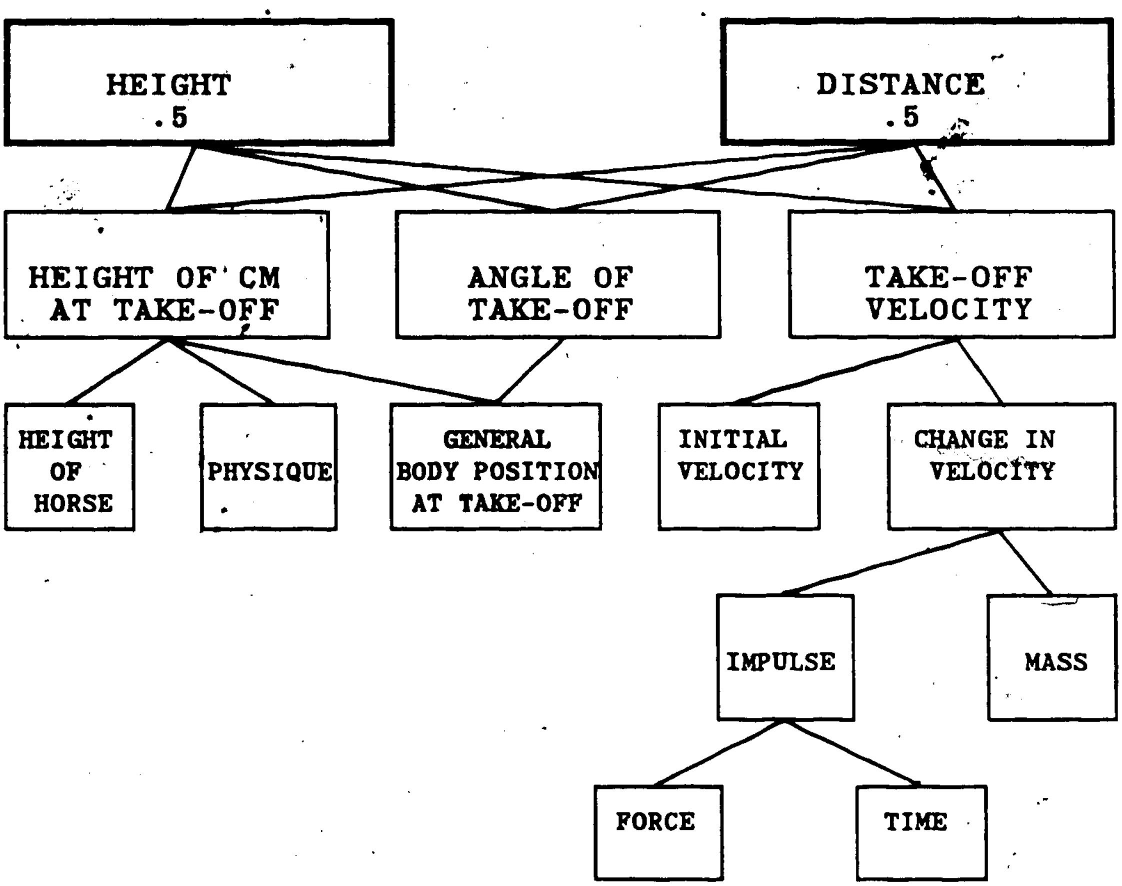

Figure III.4 Second Flight Projectile Model

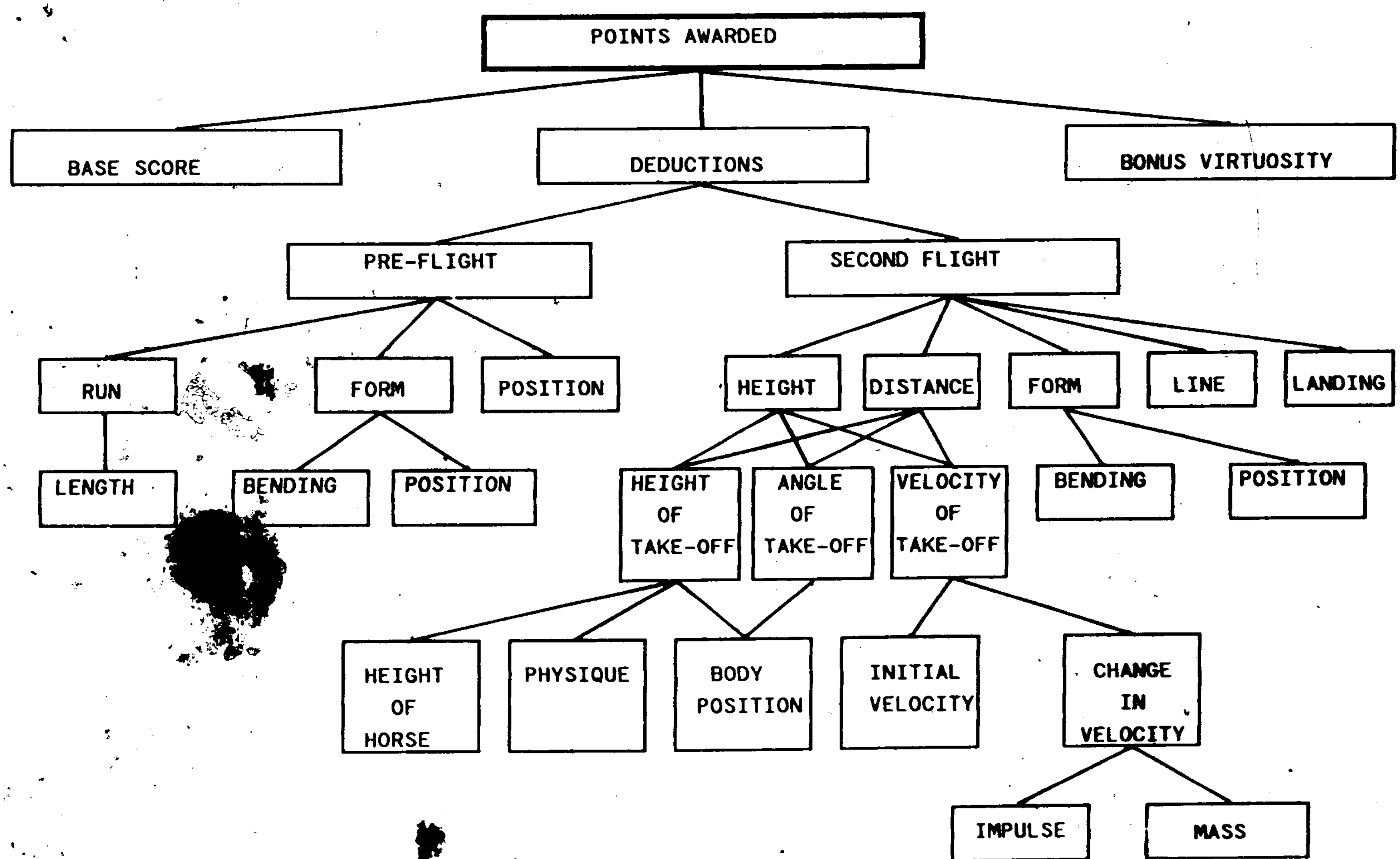

Figure III.5 Deterministic Model of the Handspring 1 1/2 Front Salto Long Horse Vault

deductions related to this aspect. The angular displacement and manner in which it is achieved characterize the skill and thus establishes the base score from which the skill is judged.

For the purposes of this study the performance assessment centered on the second flight of the skill. The performance variables selected for the assessment are those variables found in the deterministic model under the height and distance deductions. Even though the second flight was used for the performance assessment, some performance variables are found in the pre-flight of the skill, for example the impulse. The variables used to assess the performance of the handspring 1 1/2 front salto vault are defined in figure III.6 and listed in table III.1.

Table III.1 Performance Variables

<u>**TIME**</u>(seconds)	
THC	-duration of horse contac[illegible]
TPSF	-duration of postflight [illegible]
<u>**DISPLACEMENT**</u>(meters)	
HCMTO	-height of CM on leavin[illegible]
HCM	-maximum height of CM in postflight
HJGD	-maximum height of hip in postflight
DPSF	-range of CM in postflight
DJGD	-distance from end of horse to landing

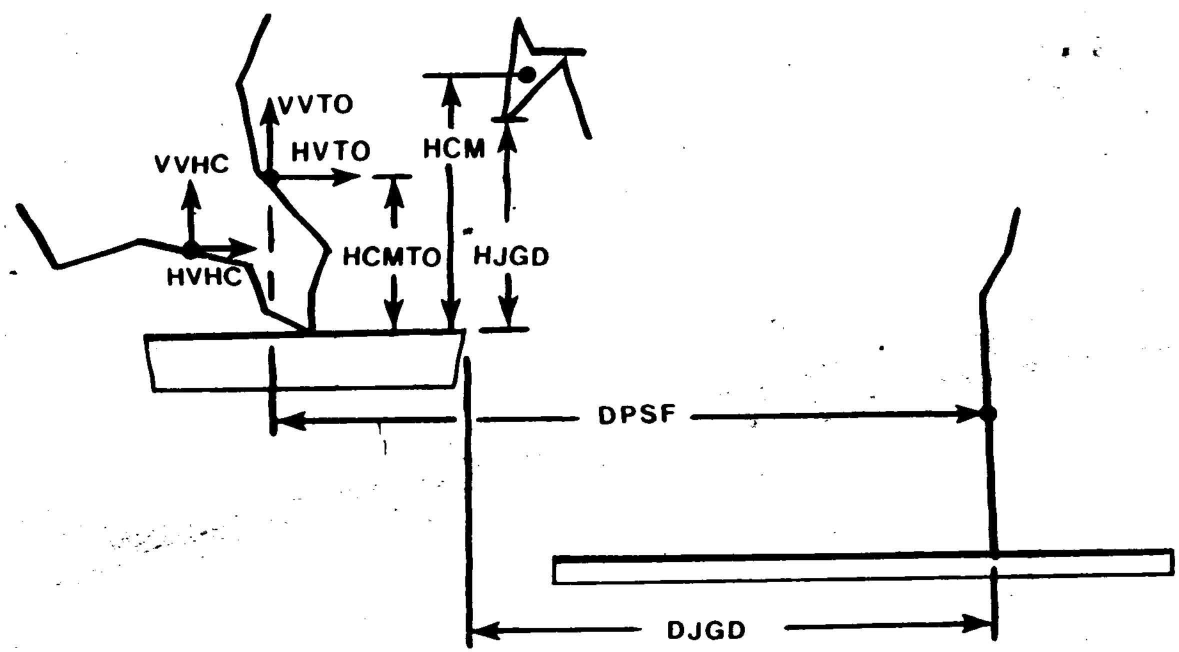

Figure III.6 Performance Variables

VELOCITIES(meters/s)

HVHC	-horizontal velocity of CM at horse contact
VVHC	-vertical velocity of CM at horse contact
HVTO	-horizontal velocity of CM on leaving horse
VVTO	-vertical velocity of CM on leaving horse
ΔHVH	-change in horizontal velocity on the horse
ΔVVH	-change in vertical velocity on the horse

ANGLES(radians)

AHC	-angle between left horizontal and line through hands and CM, at horse contact
AHD	-angle between left horizontal and line through hands and CM, at horse departure
AHTO	-angle of take-off from the horse

ANGULAR MOMENTUM($Kg\text{-}m^2/s$)

AMPF	-angular momentum pre-flight
AMPSF	-angular momentum postflight

The number of variables selected for the performance assessment was less exhaustive than those measured in Dillman, Cheetham, Smith (1985) and Cheetham (1982). The purpose of Dillman et al.'s report was that of a descriptive analysis of long horse vaulting thus the reason for the large number of parameters. The relative importance of one parameter over another was not investigated. Cheetham's study, "The men's handspring front one and one half somersault vault: Relationship of early phase to postflight", measured essentially the same parameters as did Dillman et al. The only variable from the pre-flight not included in this present study that Cheetham found correlated significantly with a postflight variable, judged distance, was the change in horizontal velocity on the reuther board. It was felt that due to the distribution of point deductions[3] and the co-purposes of optimization and simulation, the early phases of the skill, the run and board contact, would not be assessed directly.

As mentioned earlier the present study was designed to ultimately predict an optimal performance. The evaluation of the performance was necessary to assess the level of competency and provide valued feedback to the performer and coach and also to provide the initial input data into the optimization solution. Having developed a deterministic model of the skill and having determined those performance variables which could best evaluate the performance the next step was the actual data collection. The measurement of the performance variables was acccomplished using standard two dimensional high speed cinematography.

----

[3]The major deduction in the preflight, body position with respect to horizontal, is virtually never given in handspring type vaults as the body invariably gets higher than 20°.

Data Collection

Data was collected at the University of Alberta. The subject was filmed during one of his practice sessions. A Photo Sonics 1PL 16mm high speed camera was positioned 15 meters from the vaulting horse such that its optical axis was perpendicular to the long axis of the horse. At a running speed setting of 100 frames per second and an internal LED pulse setting of 10 Hz, a permanent record of 3 performances of the skill in both space and time, was obtained. The film used was 7250 (ASA 400) Kodak Video News Film which allowed for a 1/800 of a second exposure time using a 45° shutter angle. The center of the horse coincided with the center of the film plane. The field of view covered by this camera configuration was 7.8 meters in the horizontal dimension. The running speed of 100 frames per second was later verified when the film was developed. The actual film speed, of 100 frames per second, was determined by comparing the number of frames exposed to the number of light marks put on the film by the internal LED.

The film was later projected, by a Traid V/R - 100 mirror projector, onto a Bendix digitizing board which was interfaced to a Hewlett Packard 9825B desktop computer. A conversion factor relating projected image size to real life size was determined from the ratio of board units for a marked reference length on the horse and the actual measured length of this reference. This conversion factor was used for the subsequent spatial determination of the digitized

Cartesian coordinates of selected points taken from the film.

The segmental end points on the subject that were digitized were the wrist, elbow, shoulder, hip, knee and ankle joint centers for the left side of the body. These points were digitized for the complete movement starting 10 frames prior to board contact up to and including 10 frames past the stand at landing. Due to the symmetry of this skill only one side of the body was digitized and the appropriate changes were made to the body segmental parameters to reflect this feature.

Dianis (1979) used a 3 segment linked representation for his model of the handspring 1 1/2 front salto vault. Cheetham (1982) on the other hand, used an 8 segment linked representation for his study of the handspring 1 1/2 front salto vault. Cheetham included the head, feet and hands as separate segments in calculations of the subjects center of mass displacement-time histories. Body segment parameter data (Dempster, 1955 and Clauser, McConville, and Young, 1969) is available such that the distal segments of the head and neck, hand and foot can be represented as single units with their respective adjoining proximal segments, the trunk, forearm and shank respectively. For the purposes of this study and for mathematical expediency, the body of the subject was represented as a system of 5 linked rigid body segments. These segments were the forearms plus hands, arms, trunk plus head and neck, thighs and legs plus feet. The segmental linked rigid body representation was accomplished under the following assumptions: (Hanavan 1964, Winter 1979, Miller 1979):

1. The body segments were considered rigid and of constant density,
2. The center of mass remained in a fixed position within the segment during movement,
3. The mass moment of inertia for each segment remained constant during the movement,
4. The rigid body links rotate about fixed transverse axes through the joint centers,
5. The joints were considered frictionless.

The system, the 5 segment representation for the athlete performing the skill, has 7 degrees of freedom. In conjuction with the definition of the degrees of freedom (Wells, 1967) and the Lagrangian formulation for deriving the equations of motion[3], 7 generalized coordinates were used to completely describe the spatial configuration of the body at any one time with respect to an inertial reference frame. The x_i (i=1,...,7) were defined as in figure III.7. The coordinates x_1 and x_2 represented the wrist's horizontal and vertical coordinates with respect to the origin of a Cartesian reference frame located at the reuther board. x_3 is the angle measured with respect to a left horizontal line through the wrist joint center. The remaining generalized coordinates are relative angles measured in the counter

---

[3] The Lagrangian formulation for the equations of motion was used in the optimization portion of this study.

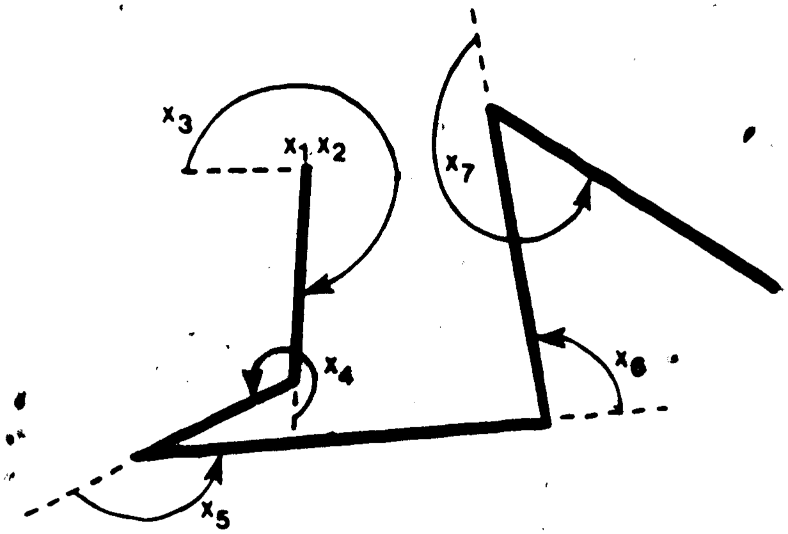

Figure III.7 The Generalized Coordinates

clockwise direction. The *dot product identity* was used in the calculation of the angular coordinates. These generalized coordinates were used to assess the general body configuration at critical times in the performance. Prior to the calculation of the center of mass coordinates, body segment parameter data was selected and anthropometric measurements made.

Body Segment Parameter Data

To describe and quantify the inertial properties of the athlete and to locate the center of mass, Dempster's (1955) and Clauser, McConville and Young's (1969) body segment data were used. These data consisted of the percentage mass of the 5 segments, the segments' center of mass locations expressed as a percent of the total length of the segment and the radius of gyration expressed as a percent of the total length of the segment. The body segment parameters used for the study are listed in table III.2.

The segmental lengths were measured from the film record. The assumption previously made for the model with respect to fixed joint center location was not adhered to during the complete performance of the skill. Therefore, it was decided that an average length value be used under the assumption it would better accommodate the assumption made previously for the rigid body segments. The frames of film digitized to calculate the segmental lengths were those deemed to provide the best compliance to pure planar motion of all body parts. The preflight, after the gymnast left the board up to and including the last instant of contact with the horse prior to the second flight was selected because it best exemplified minimum out of plane motion of any segment. All three trials were used to calculate segmental lengths. Once the segmental lengths were determined, the body segment parameter data was used to determine the location of the segmental centers of mass and moments of inertia.

Center of Mass

The segmental method, (Hay, 1985) was used to determine the two dimensional location of the center of mass. The following were used to calculate the Cartesian coordinates for the center of mass, using the seven generalized

Table III.2 Body Segment Parameter Data

Segment	Definition	Seg mass	Seg CM	Rad gyration
forearm + hand	ulnar styloid/elbow axis	.044	.318	.468
upper arm	elbow axis/glenohumeral axis	.056	.564	.322
trunk,head + neck	glenohumeral axis/greater trochanter	.578	.340	.503
thigh	greater trochanter/femoral condyles	.20	.433	.323
shank + foot	femoral condyles/medial malleolus	.122	.606	.416

Coordinates:

-Horizontal Coordinate (Xcm)

$$Xcm = \sum_{i=1}^{5} m_i (x_1 - (\sum_{j=1}^{i} \zeta_{ij} l_j \cos(\theta_j)) - d_i \cos(\theta_i)) \qquad [4]$$

-Vertical Coordinate (Ycm)

$$Ycm = \sum_{i=1}^{5} m_i (x_2 + (\sum_{j=1}^{i} \zeta_{ij} l_j \sin(\theta_j)) + d_i \cos(\theta_i)) \qquad [5]$$

where:

m_i = segmental mass

d_i = distance between segment *i's* distal end and its center of mass

l_i = length of segment *i*

$$\zeta_{ij} = \begin{cases} 0 \text{ for } i=j \\ 1 \text{ for } i \neq j \end{cases}$$

$\theta_1 = x_3$

$\theta_{i=2,...,5} = \theta_{i-1} - x_{i+2}$.

The first time derivative, ie. velocity, of the center of mass was found using central differences (Miller and Nelson, 1973)[4].

---

[4] Central differences were used through out this study for the determination of displacement-time histories of the performance variables unless stated otherwise.

Angular Momentum

The last two performance assessment variables measured were the angular momentum prior to horse contact and during postflight. Using the generalized coordinates and the center of mass data the subject's angular momentum (**H**) was found using the following:

$$\mathbf{H} = \sum_{i=1}^{5} [\bar{r}_i \times m_i \bar{v}_i + I_i \bar{\omega}_i] \qquad [6]$$

where:

$\bar{r}_i$ is the position vector from the body's center of mass to segment *i*'s center of mass,

m_i is the mass of segment *i*,

$\bar{v}_i$ is the linear velocity of segment *i*'s mass center,

I_i is the moment of inertia of segment *i* about its mass center

$\bar{\omega}_i$ is the angular velocity of segment *i* which in terms of the generalized coordinates was the following

a. $\omega_1 = \dot{x}_3$

b. $\omega_i = \omega_{i-1} - \dot{x}_{i+2}$ for i=2,3,4,5.

Data Smoothing and Error Assessment

In addition to adhering to sound cinematographical protocol during the actual filming to minimize experimental error, data smoothing was used to further reduce the noise inherent in the data. A second order low pass Butterworth digital filter (Walton, 1981) was used to attenuate the error in the generalized coordinates. To eliminate the phase shift, which results from a unidirectional pass of the filter, a second pass in the reverse direction was performed on the filtered data. In effect, this resulted in a fourth-order, zero phase shift filter. First and second time derivatives were found using central differences.

The degree of smoothing is dependent upon the cut-off frequency selected. Selection of the cut-off frequency was based on a comparison between various velocity-time plots derived using first central differences with raw displacement data and displacement data filtered at cut-off frequencies between 5 Hz and 9 Hz. The plot or plots which best depicted the trend in the data were later selected for further evaluation. Final selection of a cut-off was made by an evaluation of the residuals between the filtered and raw displacement data for the differing cut-off frequencies.

McLaughlin, Dillman, and Lardner (1977) suggested that if one adhered to good filming protocol,

> the error associated with obtaining measures from film can be reduced to errors of distortion and measurement. Relative to distortion, it is desired to ascertain whether the image recorded and projected on a screen represents the actual motion. With respect to measurement, estimates are needed of

the degree t[illegible] which an object projected on the analyzer can be measured. (p. 573)

To estimate the total error associated with the filming and analysis system known measures wer[illegible] McLaughlin et al. (1977) used known distances o[illegible] in., and 1 in. in their assessment of digitizing [illegible]racy. Distances of 0.35 meters and [illegible]ters were used in this study. An estimate of the [illegible]ty of the measuring procedure was achieved by redigit[illegible]g randomly selected frames of film.

C. Prediction of an Optimal Performance

The main objective of this research was the prediction of an optimal performance of the handspring 1 1/2 front salto vault for the subject being assessed. To accomplish this task an optimization approach was taken.

A human performance optimization problem may be stated generally as:

Identify the optimal state trajectories $x(t) \in U$, U the set of admissible states, that will minimize (maximize) an objective function

$$J = F[x(t),t] \quad [7]$$

subject to:

1. physique, and inertial characteristics of the athlete; environmental restrictions and sport regulations.
2. state constraints

$$s_1 \le x(t) \le s_2 \quad [8]$$

due mainly to physical characteristics ie. *range of motion*

The cost function is akin to the result for a skill. The cost function or result then for the handspring 1 1/2 front salto vault, as found in the deterministic model, was the points awarded for a performance. The objective can be achieved by minimizing the point deductions or performance faults. As was stated in previous sections on the performance assessment, the major point deductions are found in the second flight. Excluding form deductions, the major deductions are associated with the height and distance achieved in the postflight. The problem of minimizing the point deductions can be restated as a problem with the objective of maximizing the height and distance subject to the appropriate constraints.

Prior to a more thorough discussion of the objective function's explicit form, the mathematical model must be specified.

Mathematical Model

For the purposes of this study a mathematical model was derived for the push-off phase and postflight phase of the handspring 1 1/2 front salto vault.

Chow and Jacobson (1971) in their mathematical modelling of bipedal gait stated:

> The objective of mathematical modelling is to describe the angular motions and relate them to the overall translatory process of locomotion. Such a mechanical description is separated from the action of the muscles and other physiological considerations. (p. 247)

Utilizing the 5 segment body representation of the athlete, figure III.8, a Lagrangian approach was used to derive the equations of motion.

To formulate the Lagrangian (L) an expression for the system's kinetic energy and potential energy was needed. The center of mass Cartesian coordinates for the respective links are; (ζ_i = i^{th} horizontal coordinate; η_i = i^{th} vertical coordinate)

$$
\begin{aligned}
\zeta_1 &= x_1 - d_1 C_1 \\
\eta_1 &= x_2 + d_1 S_1 \\
\zeta_2 &= x_1 - l_1 C_1 - d_2 C_2 \\
\eta_2 &= x_2 + l_1 S_1 + d_2 S_2 \\
\zeta_3 &= x_1 - l_1 C_1 - l_2 C_2 - d_3 C_3 \\
\eta_3 &= x_2 + l_1 S_1 + l_2 S_2 + d_3 S_3 \\
\zeta_4 &= x_1 - l_1 C_1 - l_2 C_2 - l_3 C_3 - d_4 C_4 \\
\eta_4 &= x_2 + l_1 S_1 + l_2 S_2 + l_3 S_3 + d_4 S_4 \\
\zeta_5 &= x_1 - l_1 C_1 - l_2 C_2 - l_3 C_3 - l_4 C_4 - d_5 C_5 \\
\eta_5 &= x_2 + l_1 S_1 + l_2 S_2 + l_3 S_3 + l_4 S_4 + d_5 S_5
\end{aligned}
\qquad [9]
$$

where:

$C_i = \cos(\theta)$

$S_i = \sin(\theta)$

$\theta_1 = x_3$

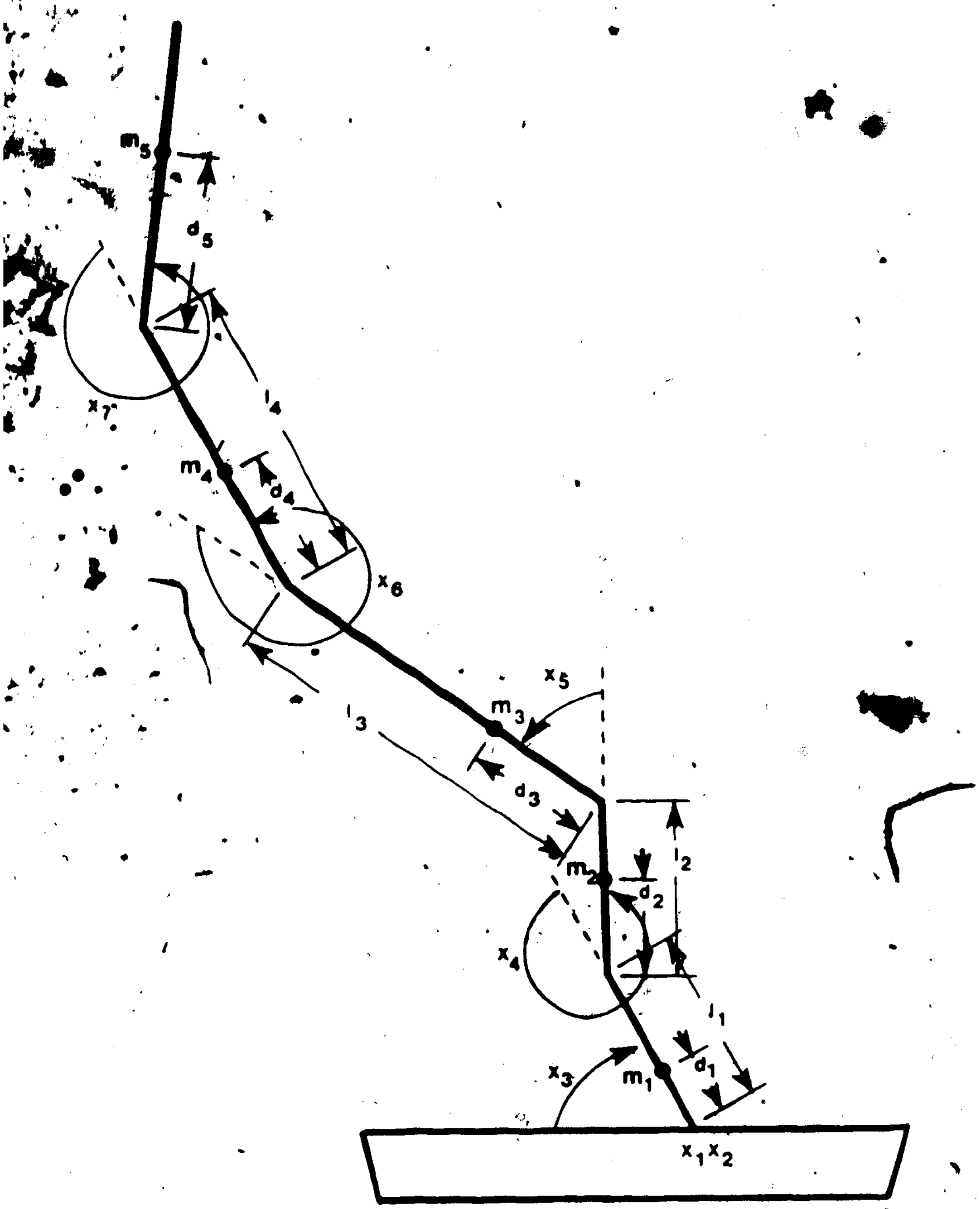

Figure III.8 Second Flight Model

$$\theta_{i=2,\ldots,5} = \theta_{i-1} - x_{i+2}$$

The kinetic energy for the i^{th} segment is

$$T_i = \frac{1}{2}m_i(\dot{\zeta}_i^{\,2} + \dot{\eta}_i^{\,2}) + \frac{1}{2}I_i\omega_i^{\,2} \qquad [10]$$

and the system's kinetic energy is

$$T = \sum_{i=1}^{5} T_i \qquad [11]$$

The potential energy for the i^{th} segment is

$$V_i = m_i g\, h_i \qquad [12]$$

where g is the acceleration constant due to gravity and h_i is the vertical Cartesian coordinate of the i^{th} segment's center of mass. The total potential energy of the system (V) is the sum of the potential energies of the five segments.

After differentiation and manipulation the *Lagrangian* is expressed as:

$$
\begin{aligned}
L &= T - V \\
&= \frac{1}{2}M(\dot{x}_1^2 + \dot{x}_2^2) + A_1\{\dot{x}_1\dot{x}_3 S_1 + \dot{x}_2\dot{x}_3 C_1\} \\
&+ A_2\{\dot{x}_1(\dot{x}_3 S_2 - \dot{x}_4 S_2) + \dot{x}_2(\dot{x}_3 C_2 - \dot{x}_4 C_2)\} \\
&+ A_3\{(\dot{x}_1(\dot{x}_3 - \dot{x}_4 - \dot{x}_5)S_3 + (\dot{x}_2(\dot{x}_3 - \dot{x}_4 - \dot{x}_5)C_3)\} \\
&+ A_4\{(\dot{x}_1(\dot{x}_3 - \dot{x}_4 - \dot{x}_5 - \dot{x}_6)S_4) + (\dot{x}_2(\dot{x}_3 - \dot{x}_4 - \dot{x}_5 - \dot{x}_6)C_4)\}
\end{aligned}
$$

$$+A_5\{(\dot{x}_1(\dot{x}_3-\dot{x}_4-\dot{x}_5-\dot{x}_6-\dot{x}_7)S_5)+(\dot{x}_2(\dot{x}_3-\dot{x}_4-\dot{x}_5-\dot{x}_6-\dot{x}_7)C_5)\}$$
$$+B_1(\dot{x}_3^2-\dot{x}_3\dot{x}_4)C_6+B_2(\dot{x}_3^2-\dot{x}_3\dot{x}_4-\dot{x}_3\dot{x}_5)C_7$$
$$+B_3(\dot{x}_3^2+\dot{x}_4^2-2\dot{x}_3\dot{x}_4-\dot{x}_3\dot{x}_5+\dot{x}_4\dot{x}_5)C_8$$
$$+B_4(\dot{x}_3^2-\dot{x}_3\dot{x}_4-\dot{x}_3\dot{x}_5-\dot{x}_3\dot{x}_6)C_9$$
$$+B_5(\dot{x}_3^2+\dot{x}_4^2-2\dot{x}_3\dot{x}_4-\dot{x}_3\dot{x}_5-\dot{x}_3\dot{x}_6+\dot{x}_4\dot{x}_5+\dot{x}_4\dot{x}_6)C_{10}$$
$$+B_6(\dot{x}_3^2-\dot{x}_4^2+\dot{x}_5^2-2\dot{x}_3\dot{x}_4-2\dot{x}_3\dot{x}_5-\dot{x}_3\dot{x}_6+2\dot{x}_4\dot{x}_5+\dot{x}_4\dot{x}_6+\dot{x}_5\dot{x}_6)C_{11}$$
$$+B_7(\dot{x}_3^2-\dot{x}_3\dot{x}_4-\dot{x}_3\dot{x}_5-\dot{x}_3\dot{x}_6-\dot{x}_3\dot{x}_7)C_{12}$$
$$+B_8(\dot{x}_3^2-2\dot{x}_3\dot{x}_4-\dot{x}_3\dot{x}_5-\dot{x}_3\dot{x}_6-\dot{x}\ [illegible]\ \dot{x}_4\dot{x}_5+\dot{x}_4\dot{x}_6+\dot{x}_4\dot{x}_7)C_{13}$$
$$+B_9(\dot{x}_3^2-2\dot{x}_3\dot{x}_4-2\dot{x}_3\dot{x}_5-\dot{x}_3\dot{x}_6-\dot{x}\ [illegible]\ \dot{x}_4+2\dot{x}_4\dot{x}_5+\dot{x}_4\dot{x}_6$$
$$+\dot{x}_4\dot{x}_7+\dot{x}_5^2+\dot{x}_5\dot{x}_6+\dot{x}_5\dot{x}_7)C_{14}$$
$$+B_{10}(\dot{x}_3^2-2\dot{x}_3\dot{x}_4-2\dot{x}_3\dot{x}_5-2\dot{x}_3\dot{x}_6-\dot{x}_3\dot{x}_7+\dot{x}_4^2+2\dot{x}_4\dot{x}_5+2\dot{x}_4\dot{x}_6$$
$$+\dot{x}_4\dot{x}_7+\dot{x}_5^2+2\dot{x}_5\dot{x}_6+\dot{x}_5\dot{x}_7+\dot{x}_6^2+\dot{x}_6\dot{x}_7)C_{15} + \frac{1}{2}F_1\dot{x}_3^2 +$$
$$\frac{1}{2}F_2(\dot{x}_3^2-2\dot{x}_3\dot{x}_4+\dot{x}_4^2) + \frac{1}{2}F_3(\dot{x}_3^2-2\dot{x}_3\dot{x}_4-2\dot{x}_3\dot{x}_5+\dot{x}_4^2+2\dot{x}_4\dot{x}_5+\dot{x}_5^2)$$
$$+ \frac{1}{2}F_4(\dot{x}_3^2-2\dot{x}_3\dot{x}_4-2\dot{x}_3\dot{x}_5-2\dot{x}_3\dot{x}_6+$$
$$\dot{x}_4^2+2\dot{x}_4\dot{x}_5+2\dot{x}_4\dot{x}_6+\dot{x}_5^2+2\dot{x}_5\dot{x}_6+\dot{x}_6^2)$$
$$+ \frac{1}{2}F_5(\dot{x}_3^2-2\dot{x}_3\dot{x}_4-2\dot{x}_3\dot{x}_5-2\dot{x}_3\dot{x}_6-2\dot{x}_3\dot{x}_7+\dot{x}_4^2+2\dot{x}_4\dot{x}_5+$$
$$2\dot{x}_4\dot{x}_6+2\dot{x}_4\dot{x}_7+\dot{x}_5^2+2\dot{x}_5\dot{x}_6+2\dot{x}_5\dot{x}_7+\dot{x}_6^2+2\dot{x}_6\dot{x}_7+\dot{x}_7^2)$$
$$-Mgx_2-A_1gS_1-A_2gS_2-A_3gS_3$$
$$-A_4gS_4-A_5gS_5. \qquad [13]$$

where:

$C_6=\cos(x_4)$

$C_7=\cos(x_4+x_5)$

$C_8=\cos(x_5)$

$C_9=\cos(x_4+x_5+x_6)$

$C_{10}=\cos(x_5+x_6)$

$C_{11}=\cos(x_6)$

$C_{12}=\cos(x_4+x_5+x_6+x_7)$

$C_{13}=\cos(x_5+x_6+x_7)$

$C_{14}=\cos(x_6+x_7)$

$C_{15}=\cos(x_7)$

$M=m_1+m_2+m_3+m_4+m_5$

$A_1=m_1d_1+m_2l_1+m_3l_1+m_4l_1+m_5l_1$

$A_2=m_2d_2+m_3l_2+m_4l_2+m_5l_2$

$A_3=m_3d_3+m_4l_3+m_5l_3$

$A_4=m_4d_4+m_5l_4$

$A_5=m_5d_5$

$B_1=m_2l_1d_2+m_3l_1l_2+m_4l_1l_2+m_5l_1l_2$

$B_2=m_3l_1d_3+m_4l_1l_3+m_5l_1l_3$

$B_3=m_3l_2d_3+m_4l_2l_3+m_5l_2l_3$

$B_4=m_4l_1d_4+m_5l_1l_4$

$B_5=m_4l_2d_4+m_5l_2l_4$

$B_6=m_4l_3d_4+m_5l_3l_4$

$B_7=m_5l_1d_5$

$B_8=m_5l_2d_5$

$B_9=m_5l_3d_5$

$B_{10}=m_5l_4d_5$

$F_1=m_1d_1^2+m_2l_1^2+m_3l_1^2+m_4l_1^2+m_5l_1^2+I_1$

$F_2=m_2d_2^2+m_3l_2^2+m_4l_2^2+m_5l_2^2+I_2$

$F_3=m_3d_3^2+m_4l_3^2+m_5l_3^2+I_3$

$F_4=m_4d_4^2+m_5l_4^2+I_4$

$F_5=m_5d_5^2+I_5$

Using the above Lagrangian the equations of motion were found using the following formula

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{x}_i} - \frac{\partial L}{\partial x_i}\right] = Fx_i \qquad [14]$$

where Fx_i is that part of the generalized force, not derived from a potential, corresponding to the generalized coordinate x_i. To optimize height and distance in the postflight and thus predict the optimal performance only the external forces and moments were formulated for the push-off phase. For i=1 and 2 the generalized forces are forces whereas for i=3 the generalized force is a moment. Using Equation [14] these generalized forces are:

$$\begin{aligned} Fx_1 = {} & M\ddot{x}_1 + A_1\{\ddot{x}_3S_1 + \dot{x}_3{}^2C_1\} + A_2\{(\ddot{x}_3-\ddot{x}_4)S_2 + (\dot{x}_3-\dot{x}_4)^2C_2\} \\ & + A_3\{(\ddot{x}_3-\ddot{x}_4-\ddot{x}_5)S_3 + (\dot{x}_3-\dot{x}_4-\dot{x}_5)^2C_3\} \\ & + A_4\{(\ddot{x}_3-\ddot{x}_4-\ddot{x}_5-\ddot{x}_6)S_4 + (\dot{x}_3-\dot{x}_4-\dot{x}_5-\dot{x}_6)^2C_4\} \\ & + A_5\{(\ddot{x}_3-\ddot{x}_4-\ddot{x}_5-\ddot{x}_6-\ddot{x}_7)S_5 + (\dot{x}_3-\dot{x}_4-\dot{x}_5-\dot{x}_6-\dot{x}_7)^2C_5\} \qquad [15] \end{aligned}$$

$$\begin{aligned} Fx_2 = {} & M\ddot{x}_2 + A_1\{\ddot{x}_3C_1 - \dot{x}_3{}^2S_1\} + A_2\{(\ddot{x}_3-\ddot{x}_4)C_2 - (\dot{x}_3-\dot{x}_4)^2S_2\} \\ & + A_3\{(\ddot{x}_3-\ddot{x}_4-\ddot{x}_5)C_3 - (\dot{x}_3-\dot{x}_4-\dot{x}_5)^2S_3\} \\ & + A_4\{(\ddot{x}_3-\ddot{x}_4-\ddot{x}_5-\ddot{x}_6)C_4 - (\dot{x}_3-\dot{x}_4-\dot{x}_5-\dot{x}_6)^2S_4\} \\ & + A_5\{(\ddot{x}_3-\ddot{x}_4-\ddot{x}_5-\ddot{x}_6-\ddot{x}_7)C_5 - (\dot{x}_3-\dot{x}_4-\dot{x}_5-\dot{x}_6-\dot{x}_7)S_5\} + Mg \qquad [16] \end{aligned}$$

$$\begin{aligned} Fx_3 = {} & A_1\{\ddot{x}_1S_1 + \ddot{x}_2C_1\} + A_2\{\ddot{x}_1S_2 + \ddot{x}_2C_2\} + A_3\{\ddot{x}_1S_3 + \ddot{x}_2C_3\} \\ & + A_4\{\ddot{x}_1S_4 + \ddot{x}_2C_4\} + A_5\{\ddot{x}_1S_5 + \ddot{x}_2C_5\} \\ & + B_1\{(2\ddot{x}_3-\ddot{x}_4)C_6 - (2\dot{x}_3-\dot{x}_4)\dot{x}_4S_6\} + B_2\{(2\ddot{x}_3-\ddot{x}_4-\ddot{x}_5)C_7 \\ & - (2\dot{x}_3-\dot{x}_4-\dot{x}_5)(\dot{x}_4+\dot{x}_5)S_7\} + B_3\{(2\ddot{x}_3-2\ddot{x}_4-\ddot{x}_5)C_8 \\ & - (2\dot{x}_3-2\dot{x}_4-\dot{x}_5)\dot{x}_5S_8\} + B_4\{(2\ddot{x}_3-\ddot{x}_4-\ddot{x}_5-\ddot{x}_6)C_9 \end{aligned}$$

$$-(2\dot{x}_3-\dot{x}_4-\dot{x}_5-\dot{x}_6)(\dot{x}_4+\dot{x}_5+\dot{x}_6)S_9\} + B_5\{(2\ddot{x}_3-2\ddot{x}_4-\ddot{x}_5-\ddot{x}_6)C_{10}$$
$$-(2\dot{x}_3-2\dot{x}_4-\dot{x}_5-\dot{x}_6)(\dot{x}_5+\dot{x}_6)S_{10}\} + B_6\{(2\ddot{x}_3-2\ddot{x}_4-2\ddot{x}_5-\ddot{x}_6)C_{11}$$
$$-(2\dot{x}_3-2\dot{x}_4-2\dot{x}_5-\dot{x}_6)\dot{x}_6S_{11}\} + B_7\{(2\ddot{x}_3-\ddot{x}_4-\ddot{x}_5-\ddot{x}_6-\ddot{x}_7)C_{12}$$
$$-(2\dot{x}_3-\dot{x}_4-\dot{x}_5-\dot{x}_6-\dot{x}_7)(\dot{x}_4+\dot{x}_5+\dot{x}_6+\dot{x}_7)S_{12}\}$$
$$+ B_8\{(2\ddot{x}_3-2\ddot{x}_4-\ddot{x}_5-\ddot{x}_6-\ddot{x}_7)C_{13}$$
$$-(2\dot{x}_3-2\dot{x}_4-\dot{x}_5-\dot{x}_6-\dot{x}_7)(\dot{x}_5+\dot{x}_6+\dot{x}_7)S_{13}$$
$$+ B_9\{(2\ddot{x}_3-2\ddot{x}_4-2\ddot{x}_5-\ddot{x}_6-\ddot{x}_7)C_{14}$$
$$-(2\dot{x}_3-2\dot{x}_4-2\dot{x}_5-\dot{x}_6-\dot{x}_7)(\dot{x}_6+\dot{x}_7)S_{14}\}$$
$$+ B_{10}\{(2\ddot{x}_3-2\ddot{x}_4-2\ddot{x}_5-2\ddot{x}_6-\ddot{x}_7)C_{15}$$
$$-(2\dot{x}_3-2\dot{x}_4-2\dot{x}_5-2\dot{x}_6-\dot{x}_7)\dot{x}_7S_{15}\} + F_1\ddot{x}_3 + F_2(\ddot{x}_3-\ddot{x}_4)$$
$$+ F_3(\ddot{x}_3-\ddot{x}_4-\ddot{x}_5) + F_4(\ddot{x}_3-\ddot{x}_4-\ddot{x}_5-\ddot{x}_6)$$
$$+ F_5(\ddot{x}_3-\ddot{x}_4-\ddot{x}_5-\ddot{x}_6-\ddot{x}_7)$$
$$+ A_1gC_1 + A_2gC_2 + A_3gC_3 + A_4gC_4 + A_5gC_5 \qquad [17]$$

where

$S_6=\sin(x_4)$

$S_7=\sin(x_4+x_5)$

$S_8=\sin(x_5)$

$S_9=\sin(x_4+x_5+x_6)$

$S_{10}=\sin(x_5+x_6)$

$S_{11}=\sin(x_6)$

$S_{12}=\sin(x_4+x_5+x_6+x_7)$

$S_{13}=\sin(x_5+x_6+x_7)$

$S_{14}=\sin(x_6+x_7)$

$S_{15}=\sin(x_7)$

Having derived the expression for Fx_1, Fx_2 and Fx_3 the objective function can be expressed mathematically.

The Objective Function

Generally, the optimization problem was to maximize the height and distance of the postflight phase of the skill. From the deterministic model it was shown that height was dependent on the vertical take-off velocity from the horse. Using the impulse-momentum relationship we have

$$Vv_f = \frac{1}{M}\int_{t_o}^{t_f} Fx_2 dt + Vv_o \qquad [18]$$

where:

Vv_f = Take-off, vertical velocity of body's CM

Vv_o = initial, vertical velocity of body's CM (determined during the performance assessment)

$t_f - t_o$ = time of push-off phase.

This take-off velocity (Vv_f) was used in the following relationship to find the vertical displacement of the center of mass:

$$\text{Height} = Ycm + \frac{Vv_f^2}{2g} \qquad [19]$$

where Ycm is the vertical Cartesian coordinate for the body's center of mass at the point of take-off from the horse.

In the judging of height in the postflight the hips must rise at least 1 meter above the horse so that no deductions are given. If we use the body's center of mass this translates into a performance objective which states that the vertex of the parabolic path of the center of mass must be greater than or equal to 1 meter plus b above the horse.

$$Ycm + \frac{Vv_t^2}{2g} \geq 1.0 + b + \text{ht. of horse} \qquad [20]$$

where b is the difference between CM and hip height. b was found during the performance assessment from the film records of the vaults. The height element of the objective function was

$$J_1 = Ycm + \frac{Vv_t^2}{2g} - (1.0 + b + 1.35) \qquad [21]$$

The distance component of the objective function is dependent upon both Vv_t and the horizontal take-off velocity from the horse(Vh_t). From the impulse-momentum relationship we have

$$Vh_t = \frac{1}{M} \int_{t_0}^{t_t} Fx_1 + Vh_0 \qquad [22]$$

which is used in the projectile equation to determine the distance achieved in the postflight

$$\text{Distance} = Vh_t(Vv_t + \sqrt{Vv_t^2 + 2gh})/g \qquad [23]$$

where h = Ycm at take-off minus Ycm at landing.

If the gymnast is to receive the maximum points for this aspect of the vault he must land 2 meters past the end of the horse. This would mean that from the position at take-off relative to the end of the horse (r) the range in the postflight must be greater than or equal to (2.0+r) meters. Therefore, the distance element of the objective function was

$$J_2 = Vh_f(Vv_f + \sqrt{Vv_f^{\,2} + 2gh})/g - (2.0+r) \qquad [24]$$

The explicit form of the objective function is then

$$J = \{Ycm + \frac{Vv_f^{\,2}}{2g} - (1.0+b+1.35)\} +$$

$$\{Vh_f(Vv_f + \sqrt{Vv_f^{\,2} + 2gh})/g - (2.0+r)\} \qquad [25]$$

Preliminary observations during the optimization process revealed that nonexplicit constraints on Fv_f and Fh_f produced unrealistic forecasts of the height and distance achieved. To rectify this problem, without introducing encumbering complexity or making drastic changes to the proposed approach of this study, penalty functions were introduced into the objective function.

Dillman et al. (1985), in their study found a range of between 2.74 and 3.01 meters for the height of the center of mass in the postflight for the handspring 1 1/2 front salto vault. Substituting Dillman et. al.'s results into the expression for height yielded

$$2.74 \le Ycm + \frac{Vv_f^2}{2g} \le 3.01 \qquad [26]$$

which when expressed for the final velocity is

$$\sqrt{2g(2.74-Ycm)} \le Vv_f \le \sqrt{2g(3.01-Ycm)} \qquad [27]$$

Dillman et al. also found a range of between 2.24 and 3.38 m/sec for the take-off velocity from the horse. Using this range and taking into consideration the athlete's physique and body position (Ycm) the bounds used in this study for Vv_f were

lower bound(VYLB) = maximum between $(2.24, \sqrt{2g(2.74-Ycm)})$,

upper bound(VYUB) = minimum between $(3.38, \sqrt{2g(3.01-Ycm)})$.

The penalty function was expressed in terms of parabolas. If Vv_f was calculated to be between its lower and upper bound the penalty, $P(Vv_f)$ was equal to zero. If Vv_f was greater than its upper bound, the penalty for this constraint violation was

$$P(Vv_f) = (Vv_f - VYUB)^2 \qquad [28]$$

Similarily for Vv_f less than its lower bound

$$P(Vv_t) = (Vv_t - VYLB)^2 \qquad [29]$$

This penalty function is depicted graphically in figure III.9. It can be seen that the farther away from the feasible region [YVLB,VYUB] the greater the penalty.

The distance component of the objective function was a more complex expression dependent upon both Vv_t and Vh_t. In addition to the dependence on velocity, distance was also dependent on the athlete's physique and body position at take-off which determined the h value which directly affected time of flight. Under these circumstances a penalty function for horizontal velocity and distance itself was

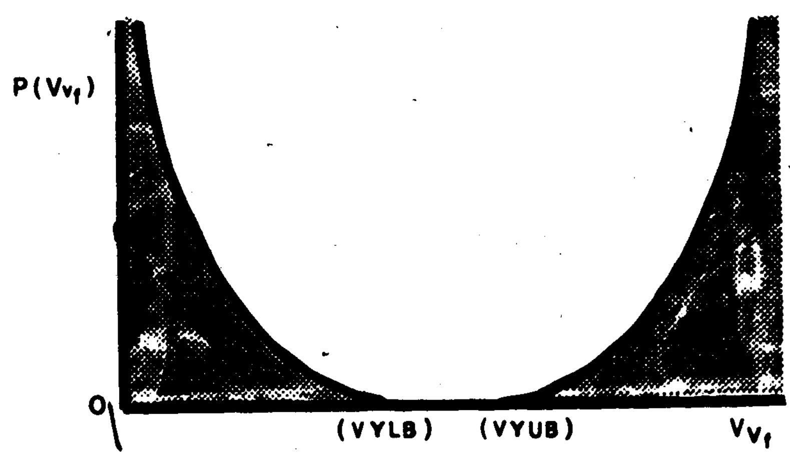

Figure III.9 Vertical Velocity Penalty Function introduced into the objective function.

Dillman found the range for Vh_t to be between 3.01 and 3.97 m/sec. Using a similar form as for $P(Vv_t)$ the penalty function $P(Vh_t)$ was as follows

$$P(Vh_t) = \begin{cases} (Vh_t - 3.01)^2 & :\text{for } Vh_t < 3.01 \\ 0 & :\text{for } 3.01 \le Vh_t \le 3.97 \\ (Vh_t - 3.97)^2 & :\text{for } Vh_t > 3.97 \end{cases} \quad [30]$$

Dillman et al. (1985) also reported postflight times of between 0.87 and 0.97 seconds. Ignoring air resistance, possible postflight distances were easily calculated[a] to lie between 2.619 and 3.851 meters beyond the take-off position. The penalty function associated with the postflight distance was

$$P(Dst) = \begin{cases} (Dst - 2.619)^2 & :\text{for } Dst < 2.619 \\ 0 & :\text{for } 2.619 \le Dst \le 3.851 \\ (Dst - 3.851)^2 & :\text{for } Dst > 3.851 \end{cases} \quad [31]$$

A penalty function approach appeared to be the most logical means of incorporating the implicit requirement of sufficient angular momentum for the postflight. The minimum absolute postflight angular momentum from the performance

----

[a] Distance = horizontal velocity times time.

assessment was used as the penalty function's upper bound (AM_u). The lower bound (AM_l) was calculated using the subject's inertial properties and the average maximum angular velocity for the postflight given in Cheetham's study (1982). The following was the penalty function used

$$P(AM) = \begin{cases} (AM - AM_l)^2 & : \text{for } AM < AM_l \\ 0 & : \text{for } AM_l \leq AM \leq AM_u \\ (AM - AM_u) & : \text{for } AM > AM_u \end{cases} \qquad [32]$$

where:

$$AM = \int_{t_o}^{t_f} Fx_3 dt + AM_{pf} \qquad [33]$$

and AM_{pf} is the average preflight angular momentum value for the 3 trials.

In summary, the *OBJECTIVE FUNCTION* which was a maximization of height and distance, also contained penalty functions for Vv_f, Vh_f, distance and angular momentum. The final form of the objective function was

$$\text{minimize } J = -\{(Ycm + \frac{Vv_f^2}{2g} - (2.35 + b)) + (Vh_f(Vv_f + \sqrt{Vv_f^2 + 2gh})/g - (2.0 + r))\} + P(Vv_f) + P(Vh_f) + P(Dst) + P(AM) \qquad [34]$$

D. Method of Solution

The Rayleigh-Ritz Process

Implicitly the optimization problem was to find the $\hat{x}(t)$ which extremizes the functional

$$J(x(t)) = F(t, x(t), \dot{x}(t), \ddot{x}(t)) \qquad [35]$$

subject to specific constraints. Assuming that $x(t) \in C^2[t_o, t_f]$, where $t_o = 0$ and t_f is equal to the time spent on the horse, a functional form of the generalized coordinates $x(t)$ was not available.

In the assessment of the performance, the film, from which the generalized coordinates are derived, is a series of still pictures taken of a continuous event. The electronic storage and processing of the generalized coordinates on a digital computer implies a finite data set. As Gill and Murray stated "clearly it would be impractical to store the finite, but enormous, set of values of $x(t)$ at each machine-representable point in the interval. Instead, we must be content with storing a reasonable amount of information, from which a satisfactory approximation to $x(t)$ can be constructed" (Gill and Murray, 1981, p. 272). Utilizing the Rayleigh-Ritz approach the discretization of the optimization problem was accomplished by approximating the generalized coordinates $x(t)$ by the functions $\hat{x}(t)$ which are finite linear combinations of basis functions (Simpson, 1969 and Gelfand & Fomin, 1963).

$$\hat{x}(t) = \sum_{j=1}^{q} c_j w_j(t) \qquad [36]$$

where c_j are a set of coefficients and $w_j(t)$ are a set of basis functions. The basis functions used were polynomials: $w_j = t^{j-1}$. Specifically the generalized coordinates were approximated by fifth degree polynomials*:

$$x(t) \cong \hat{x}(t) = c_0+c_1+c_2t^2+c_3t^3+c_4t^4+c_5t^5 \qquad [37]$$

Instead of a problem in which the independent variables are continuous functions, substitution of [37] into the objective function produced a finite dimensional problem where the polynomial coefficients were the unknown variables. To acquire the initial guess for the optimization problem, a fifth degree polynomial least square fit was used for each of the seven generalized coordinates from one of the three trials.

The integrals within the objective function were solved using a four-point *Gaussian Quadrature* scheme (Gerald, 1973,

----

*fifth degree ploynomials were chosen since they best represented the trend in the data when compared to those fits found using third degree, Chebyshev and Legendre polynomials.

p. 72). Use of this integration scheme with the Ritz approximation was suggested by Gill and Murray (1973, p. 107). This was based on earlier work by Herbold, Schultz and Varga (1969) who showed that the quadrature errors are consistent with the errors of the Ritz approximation.

Time of Horse Contact

In the impulse-momentum relationship, the change in momentum can be affected by an increase in the force applied or in the time of its application or both. Since the prediction of the optimum performance is directly related to the impulse on the horse, both components making up the impulse were considered in the optimization problem formulation. Therefore, in addition to the 42 unknowns, associated with the generalized coordinates, t_f was introduced as an independent variable. However, the use of this variable set a restriction on the source of the initial guess for the 42 polynomial coefficients.

The time of horse contact was constrained between upper and lower bounds. The maximum and minimum times for the push-off phase found in the three trials were used for the upper and lower bounds for horse contact time t_f. Due to the poor extrapolation properties of polynomials (Conte & de Boor, 1980, p. 54), the initial guess for the generalized coordinates was determined from the least square fit of the data from that trial from which the upper bound for t_f was taken.

Numerical Computation

The optimization problem of this study, was the minimization of a nonlinear objective function subject to simple inequality bounds on the variables and nonlinear inequality constraints. The constrained optimization technique employed was the *Complex* method, a direct search method. The subroutine COMPLX/DCOM, supported by Computing Services at the University of Alberta was used. This subroutine is based on Box's 1965 constrained version of the *Simplex* (or Polytope[7]) method.

Generally, the *Complex* method finds the local minimum[8] of a constrained function of n independent variables. The constraints as described in the description of the subroutine COMPLX are "inequalities of the form

$$g_i \leq x_i \leq h_i, \quad i=1,\ldots,m \qquad [38]$$

where the implicit variables $x_{n+1},\ldots,x_m$ are functions of the explicit variables $x_1,\ldots,x_n$. The first n constraints are termed explicit and the last m-n are called implicit" (University of Alberta write up R247.0977)

The basic operation in the program is that of *over-reflection*. Since this is a constrained problem two operations were used to assure a point's feasibility and to ensure that progress was made. These operations were :

---

[7]Gill and Murray (1981) prefered to use the term Polytope to avoid confusion with the Simplex method for linear programming.
[8]to find the maximum the negative of the minimum is sought.

a. moving halfway towards the centroid in the event that a trial point violated an implicit constraint or in the event that a trial point proves still to be the worst, and

b. moving inside an explicit constraint by an amount 0.000001 in the event that a trial point violates it (COMPLX write up R247.0977).

The iterative procedure as described by Box (1965) and later by Rao (1979) is as follows

step 1 *Generating the K ≥ n+1 Vertices of the Complex*

assuming an initial feasible point $x^{\circ} = x^{\circ}_1, \ldots, x^{\circ}_n$ is available, the remaining (k-1) points are found one at a time using a random number $r_i \in (0,1)$ and

$$x_i = g_i + r_i(h_i - g_i) \qquad [39]$$

Equation [39] ensures that this point satisfies the explicit constraints. If this trial point violates an implicit constraint, operation a. is used, and repeated if necessary, to make the point feasible. Proceeding in this manner the k-1 permissible points are generated.

step 2 *Over-reflection*

The objective function is evaluated at each trial point. The point, X_h corresponding to the largest function value is replaced by a new trial point X_r:

$$X_r = (1+\alpha)X_c - \alpha X_h, \; \alpha \geq 1. \quad [40]$$

where X_c = centroid of the remaining points. (Box (1965) recommended a reflection factor α = 1.3)

step 3 *Test for Feasibility*

operations a. and b. are used and repeated if necessary, ultimately a permissible point is found.

step 2 and step 3 are repeated as long as progress is made ie. the objective function continues to converge towards a minimum. The program will stop when five consecutive equal function evaluations have occurred.

The determination of a global minimum versus a local minimum can be inferred by restarting the program from different points and observing if they all converge to the same solution. A rough check was made, as to whether the solution was global, by restarting the program using different random number initiators.

The COMPLX program requires that reasonable bounds be provided for all the variables. The smallest and largest polynomial coefficients, found with the least square fit on the data for the push-off phase of the three trials, were selected as the bounds for the explicit constraints. The 43^{rd} explicit constraint bounds corresponded to the measured minimum and maximum time spent on the horse in the three trials. As there were no data available in the literature, the implicit constraint bounds were determined from the displacement time-histories of the generalized coordinates.

In the performance assessment, after the generalized coordinate data were smoothed and first and second derivatives calculated, the range for each coordinate and its derivative for each push-off phase of each trial were found. The bounds used for the implicit constraints were the averages of these values, with the exception of the limits for $x_4(t)$ and $x_7(t)$. Bending of the arms and poor position of the legs can result in deductions of between 0.3 to 1.0 or up to 0.3 respectively (FIG Code of Points, p. 36). The implicit variables for the elbows and knees were restricted to an interval between -0.3 and 0.0 radians.

The implicit variables representing the displacement of the generalized coordinates were expressed as fifth degree polynomials. These variables were functions of the explicit variables: the polynomial coefficients and the time spent in the push-off phase. The implicit variables designated for the velocity and acceleration constraints were the appropriate time derivatives of the polynomial expressions. For example the implicit variables associated with the generalized coordinate $x_5(t)$ were:

for displacement,

$$x_{59+i} = \sum_{j=1}^{6} x_{24+j} t_i^{\,j-1} \qquad [41]$$

for velocity,

$$x_{87+i} = \sum_{j=2}^{6} (j-1) x_{24+j} t_i^{\,j-2} \qquad [42]$$

and

for acceleration,

$$x_{115+i} = \sum_{j=3}^{6} (j-2)(j-1) x_{24+j} t_i^{\,j-3} \qquad [43]$$

where

$i = 1,2,3,4$

and

$t_i = (i \times .25) x_{24}$

These values for time in the polynomials were chosen so that there was a constraint for each generalized coordinate in each integration interval used in the four point Gaussian quadrature.

E. Simulation of the Predicted Optimal Performance

This portion of the study was designed to provide a visual display of the predicted movement. The approach used was similar to other studies in which the simulation was conducted using interactive computer programming in association with computer graphics (Ramey, 1973; Dapena, 1983). Previous studies were highly dependent on the operators ability to interact with the program while meeting the movement's anatomical requirements. In this study an attempt was made to make the simulation process as objective as possible. Although the method did provide for some interaction between the operator and the computer program, an optimization scheme was the primary method in the decision making process. This was the approach taken for the postflight phase of the skill. Simulation of the push-off phase's movement was a much simpler task.

Movement on the Horse

The optimization of the skill in the previous section provided the polynomial coefficients for the generalized coordinates and time of contact with the horse. Using the polynomial coefficients and selected times, $t_i \in [0, t_f]$, the generalized coordinates were calculated for those times. A stick figure graphics program written on the Hewlett Packard 9825B desktop computer was used to plot the rigid body representation of the athlete's performance during the push-off phase.

Movement During the Postflight

To simulate the predicted postflight, an optimization scheme was used. The optimization problem consisted of minimizing the differences between the predicted CM trajectories and those calculated from the manipulated generalized coordinates. The difference between predicted and calculated angular momentum was also included in the cost function. Constraint bounds were used on the generalized coordinates and their first time derivatives to assure realistic movement in the simulation. The cost function used was the minimization of J_{psf}:

$$J_{psf} = |\ XCMOPT - XCMCAL\ | + |\ YCMOPT - YCMCAL\ | + |\ AMOPT - AMCAL\ | \quad [44]$$

where XCMOPT and YCMOPT are the predicted horizontal and vertical Cartesian coordinates for the center of mass calculated from the projectile equations. The equations were

$$XCMOPT = Xcm_o + Vh_o t \quad [45]$$

$$YCMOPT = Ycm_o + Vv_o t - \tfrac{1}{2}gt^2 \quad [46]$$

where Xcm_o, Ycm_o, and Vh_o, Vv_o, are the initial postflight center of mass coordinates and their respective velocities. These values were obtained from the performance optimization. Time t, $t \in [0, T_{psf}]$ where T_{psf} was the predicted time of the postflight, was found using Ycm_o and

$$T_{psf} = \frac{Vv_o + \sqrt{Vv_o^2 + 2gh}}{g} \quad [47]$$

In the objective function ([44]), XCMCAL and YCMCAL were the calculated center of mass values found using equation [4]. AMOPT was the predicted angular momentum determined from the impulse momentum equation and the optimum performance states. AMCAL was the calculated angular momentum found using equation [6].

The *Complex* method was used for the extremization of the objective function yet a Rayleigh-Ritz procedure to approximate the generalized coordinates was not used. The actual data from the postflight phase of a trial was used for the simulation.

The simulation proceeded by first dividing the predicted postflight time by ten'. This provided 9 specific times for which the body state's were sought. This translated into 9 sequential optimization problems.

Three consecutive points in time were used in each optimization. The first point was determined in the previous optimization and was not manipulated in the current optimization''. Point 2 was from the previous optimization except in the first sequence where it was selected in the same fashion as point 3. Point 3 was chosen from the generalized coordinate data from trial 2. The point selection was based on the point's approximation to the predicted value. Central differences (Miller & Nelson, 1973)

'ten was selected because it was felt that 10 equally spaced points was sufficient to display graphically the movement of the body through the postflight.

''for the first sequence this point was the final state predicted for the push-off phase.

were used to calculate the required velocities in the angular momentum algorithm. Upon the completion of this interval optimization, point 2 was retained as the optimal state and was used as point 1 in the subsequent interval. Similarily point 3 became point 2, and point 3 was selected for the next optimization as before. This point selection process was repeated for each interval throughout the optimization sequence.

The terminal state, the last postflight position, was selected on the basis of the following:

1. the last optimization interval,
2. the three final states observed in the trials, and
3. the mechanical objectives of a safe controlled landing.

To dissipate the force of landing and maintain control, the body should maximize the time in the impulse-momentum relationship thus reducing the experienced forces. This can be accomplished by a stretched body position prior to landing. In order to satisfy this final state requirement, the constraints in the final optimization interval were set up with narrow bounds to assure the appropriate landing position.

The constraints bounds, except for the terminal state, were chosen as in the push-off optimization. Selection was based on the ranges for the generalized coordinates taken from the data on the postflight phases of the three trials. The first time derivative was the highest used in the simulation calculations therefore there were constraints for displacement and velocity only.

Accompanying each of the 9 optimizations was a graphical display of the movement in stick-figure form. This was done to monitor the progress and realism of the movement simulation. In addition to the monitoring, the interactive format of the computer program allowed the restart of the COMPLX with slightly modified variables. The modification of the variables was based on the values of the calculated variables in the objective function and the stick-figure plots. The restarting of the COMPLX optimization algorithm from different points served two purposes. First it protected against possible unrealistic or unproductive searches thus reducing CPU cost and secondly, it assisted in the search for a possible global minimum versus local minima.

IV. Results and Discussion

The purpose of this study was to assess the performance of the handspring 1 1/2 front salto vault by an individual and then to predict his optimal performance.

A. Performance Assessment

Error and Reliability of Data

Data were collected for the performance assessment using high speed cinematography. The data reduction process is subject to experimental error. To estimate the experimental error in the film analysis procedure known distances were digitized. The average absolute error in measuring known distances (0.35m and 0.69m) was 17.5mm (s=5.8mm). McLaughlin et al. (1977) suggested that the total error was equally distributed between the two points digitized for the distances and therefore the error band for each point could be estimated as ± 8.75mm (p. 573). Based on these results any point could be located in real life to within ± 8.75mm.

McLaughlin et al.'s (1977) guidelines were used to estimate that part of the total error which can be assumed to be due to distortion and measurement precision. The Bendix digitizing board has a coordinate system capable of measuring a point to within 0.01 inches. Twenty repeated measures of a well defined point resulted in a standard deviation of less than 0.01 inches. Multiplying this estimate of precision by the conversion factor (0.2586 in/m), used in translating the board units to real life measures, indicated that the accuracy to which any point could be located on the digitizing board was within ± 2.59mm. Since the total error was ± 8.75mm and the precision was ± 2.59mm, it can be assumed that the remainder ± 6.16mm was an estimate of the error due to various sources of distortion.

An estimate of the study's measuring reliability was found by redigitizing seven randomly selected frames of film. The mean error was 1.305cm (s=0.659).

Data Smoothing

Since the digitized data were subject to experimental error, a second order Butterworth digital filter was used to reduce the amount of error. The cut-off frequency was selected by first comparing the highest derivative-time profiles of the unprocessed data and the smoothed data at various cut-off frequencies. Figures IV.1 and IV.2 are examples of some of these comparisons using the velocities for the generalized coordinates X_2 and X_8. It can be seen that, in both figures, the velocity calculated using central differences with the raw data has a noise component. In figure IV.1, the 4 Hz cut-off produced much more pronounced smoothing when compared to the 6 Hz cut-off frequency. The 6 Hz cut-off more fully represented the trend in the data. Selection between 6 Hz and 8 Hz was not as obvious a

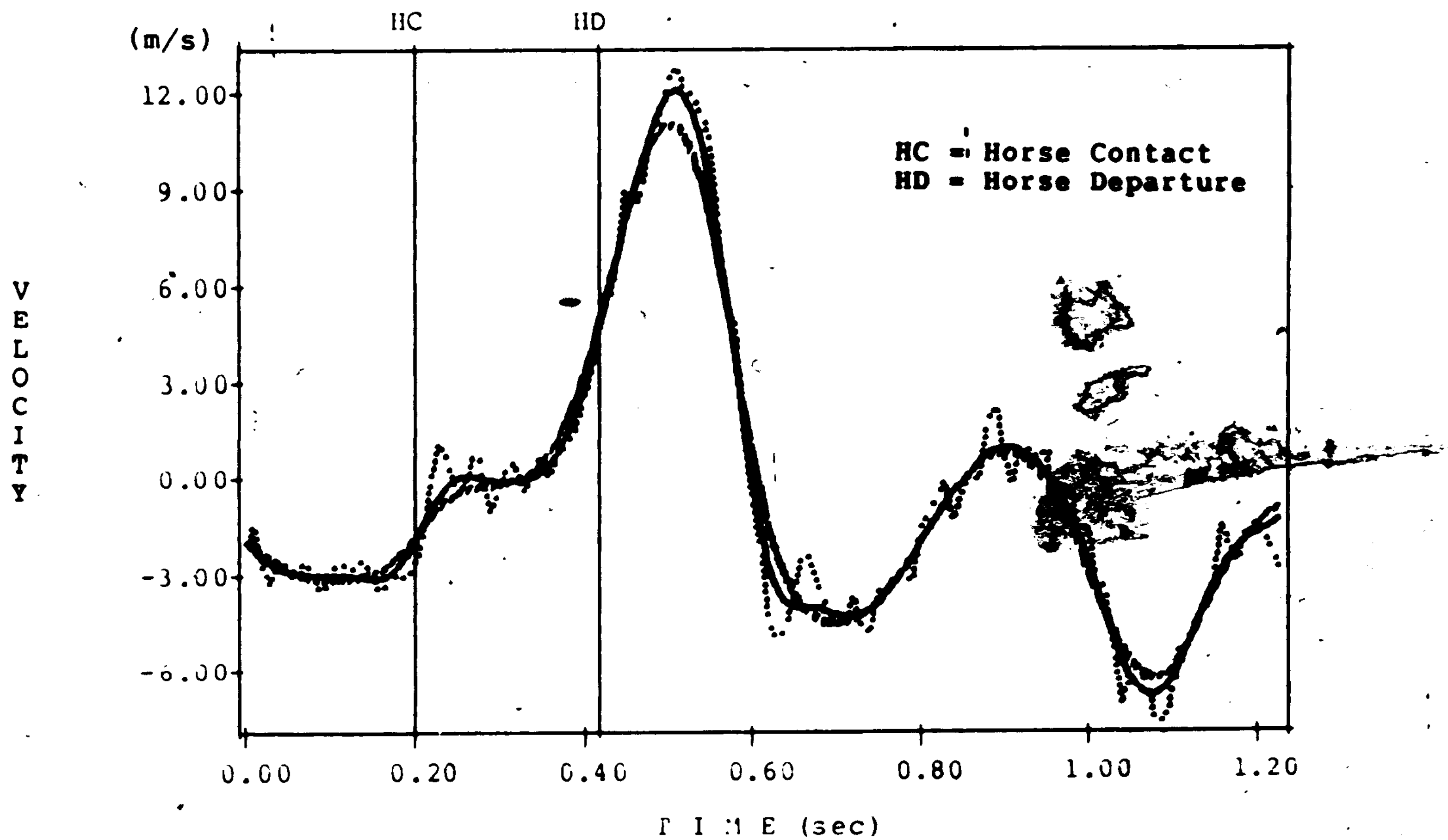

Figure IV.1 X_2 Velocities Raw *VS.* Smoothed
[... → Raw : ---- → 6 Hz : ,,,,,, → 4 Hz]

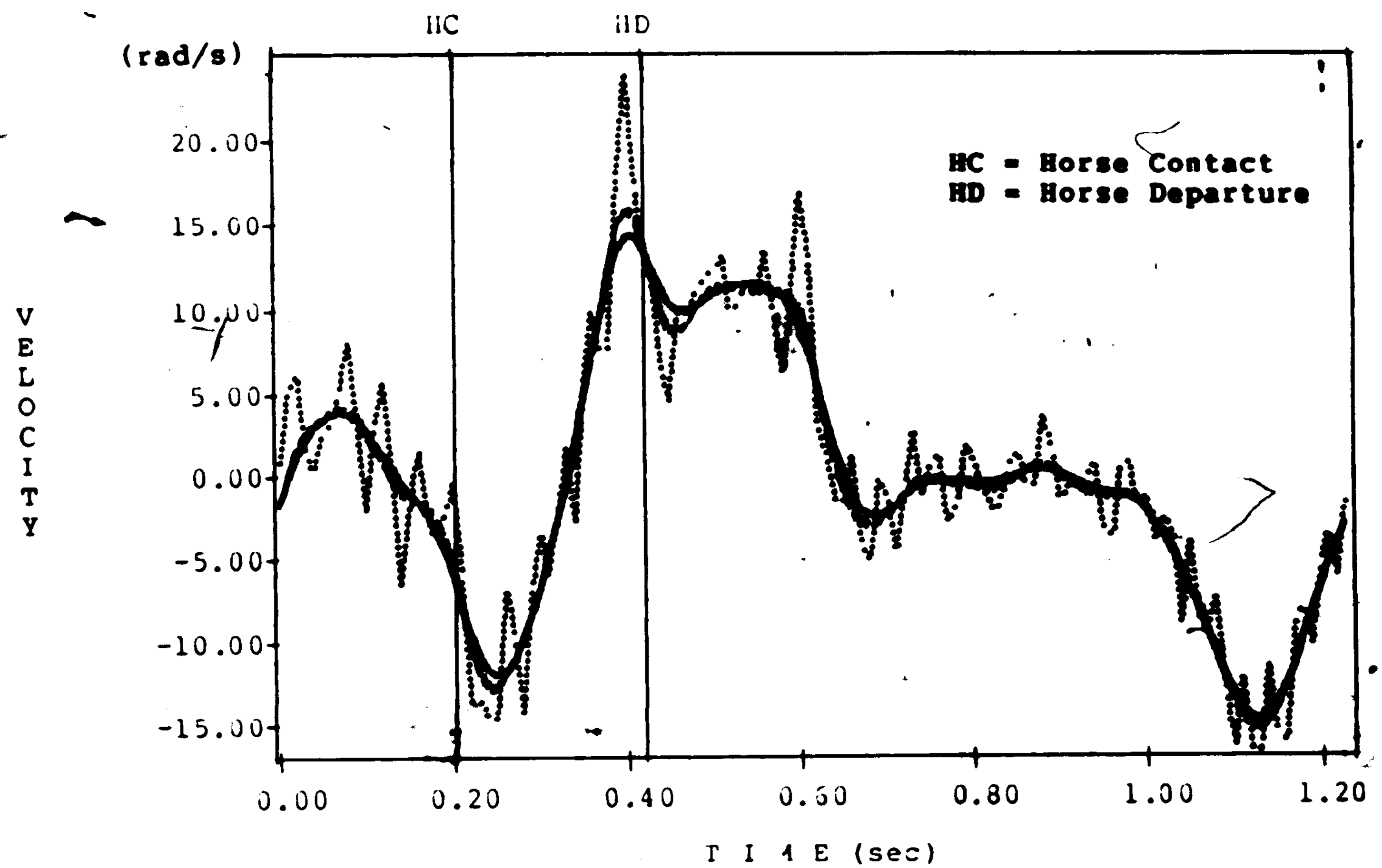

Figure IV.2 X_6 Velocities Raw VS. Smoothed
[... → Raw : ―― → 6 Hz : ,,,,,, → 8 Hz]

decision as between the 4 Hz and 6 Hz cut-offs (figure IV.1). The 6 Hz and 8 Hz curves appear to follow the trend in the data equally as well. A major difference between the two cut-offs, a function of the degree of smoothing, is the amplitude prior to departure from the horse. It has been suggested that cut-off frequency selection can be made by investigating the power spectrum of the data produced at various cut-off frequencies (Winter & Wells, 1978 and Conte & de Boor, 1980). Winter & Wells (1978), used this harmonic analysis to determine a 6 Hz cut-off when 99% of the signal power was below the 7th harmonic. Even though this form of cut-off selection has been demonstrated to be effective, harmonic analysis was not pursued here.

Conte and de Boor (1980) in their discussion on the selection of the degree of polynomial to use when approximating data suggested that a comparison between the residuals would assist in the selection process (p. 267). They contended that if the residuals (errors) behaved irregularly then one could assume that the degree of polynomial was sufficient. If the error behaved in a somewhat regular fashion not all of the information was contained in the approximating function. This selection concept was tried here. Unfortunately, as can be seen in figure IV.3 this analysis revealed only that the residuals behaved in an irregular fashion. These displays did not provide any further information for the selection of the appropriate cut-off.

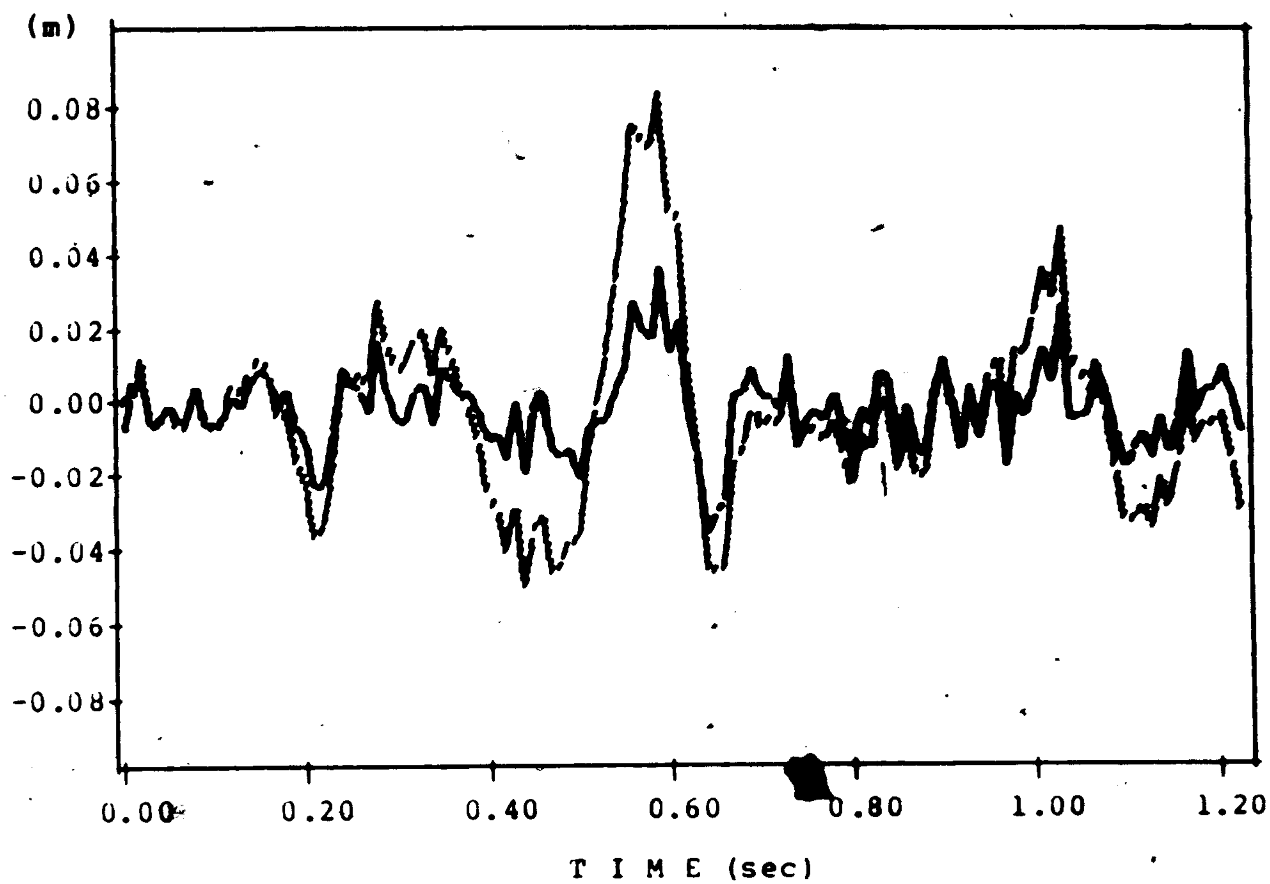

Figure IV.3 X_2 Residuals
[...... → 6 Hz : ,,,,,, → 4 Hz]

Hamming (1973) gave another possible approach to the selection of the appropriate degree of polynomial to use in least-square polynomial approximations. He suggested that if only noise was left in the residuals, by simple probability, one would expect that a given residual would be followed by one of the same sign one half of the time and by one of an opposite sign half of the time. Using this description of following the trend in the data, the number of sign changes in the residuals were determined for the different cut-off frequencies investigated. On the average it was found that 4 Hz produced too few sign changes and 8 Hz produced too many sign changes. For example in Trial 2, X_6, there were 37, 67, 77 sign changes for a cut-off of 4 Hz, 6 Hz and 8 Hz respectively for 124 data points. Based on these three forms of cut-off frequency selection over the three trials, with cut-off frequencies between 4 Hz and 9 Hz, a 6 Hz cut-off frequency was selected to smooth the generalized coordinates.

Performance Variables

Data were collected for three trials of the handspring 1 1/2 front salto vault. The three trials received point scores of 9.4, 9.5 and 9.0 respectively when judged. An illustration of the performance of the handspring 1 1/2 front salto vault is given in stick figure form in figure IV.4. table IV.1 lists all the performance variables measured.

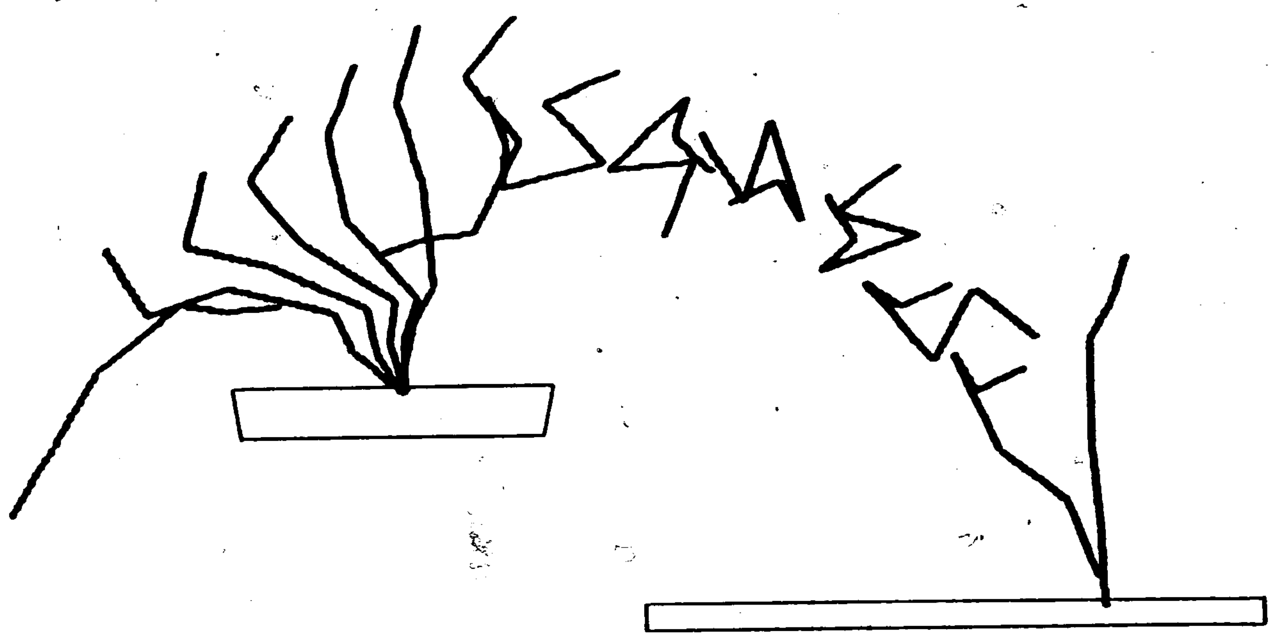

Figure IV.4 Performance Assessment Trial 2

The mean time for horse contact (THC) was identical with Dillman et al.'s (1985) mean time yet trial two was beyond the range reported by that study. The 0.22 seconds value coincided more with that reported for near end vaults by Bajin (1979). The postflight time (TPSF) was slightly smaller than those reported by Dillman et al. and Cheetham (1982). The gymnast in all three trials did not receive a point deduction for postflight range as indicated by the judged postflight variable (DJ[illegible]). He was well beyond the two meters required. These values were within those reported by Dillman et al. (1985). The gymnast also met the requirements for vertical displacement in the postflight (HJGD). The hip, in all three trials, reached a height greater than the required 2.35 meters''. The maximum height that the center of mass attained was slightly lower than the mean values reported by Dillman et al. (1985) and Cheetham (1982) but within both reported ranges.

The horizontal velocity of the center of mass at horse contact was less than those values reported in the literature. However the horizontal velocity from the horse (HVTO) was larger than that reported by either Dillman et al. or Cheetham. These differences were reflected in the comparison between the change in horizontal velocity during horse contact for the present study and Dillman et al.'s study. There was less of a change for the individual assessed in this study. The vertical velocity of the center

----

'' 2.35 meters = height of horse (1.35m) +1 meter requirement.

Table IV.1 Performance Variables

VARIABLE	TRIAL 1	TRIAL 2	TRIAL 3	MEAN
Time(sec)				
THC	0.19	0.22	0.17	0.18
TPSF	0.83	0.81	0.79	0.81
Displacement(m)				
HCMTO	2.368	2.338	2.358	2.355
HJGD	2.663	2.631	2.583	2.626
HCM	2.720	2.712	2.658	2.697
DJGD	2.723	2.918	2.763	2.801
DPSF	2.917	2.992	2.877	2.927
Velocities(m/sec)				
HVHC	4.395	4.717	3.953	4.355
VVHC	1.417	1.725	1.756	1.633
HVTO	3.728	3.401	4.027	3.719
VVTO	2.760	3.068	3.104	2.978
ΔHVH	-0.667	-1.316	0.074	-0.636
ΔVVH	1.343	1.343	1.348	1.345
Angles(rad)				
AHC	0.776	0.612	0.839	0.742
AHD	1.664	1.726	1.714	1.701
AHTO	0.644	0.721	0.656	0.674
Angular Momentum(kg-m^2/s)				
AMPF	-109.866	-100.890	-101.570	-104.107
AMPSF	-80.929	-75.869	-79.244	-78.681

of mass at horse contact was also found to be less than the values reported in the two previous studies. Nevertheless, the change in vertical velocity during horse contact was greater than that reported by Dillman resulting in almost identical mean velocity values between this study (2.978 m/s) and Dillman et al.'s (2.970 m/s).

In assessing the body position at horse contact and take-off, Dillman et al. and Cheetham used a line drawn through the hands and center of mass to represent the body. The relative position of this line with respect to the left horizontal was chosen as the descriptor for the two positions. The average values measured on the performer were slightly greater than the average reported by Dillman et al. but well within their reported ranges. In trial three the gymnast contacted the horse at the greatest angle, slightly larger than the maximum found by Dillman et al. In conjunction with a poorer landing, this contact angle might partially explain why trial 3 was judged the poorer of the three vaults. The greater contact angle found in this study compared to those found in the 8 Olympians of Dillman et al.'s study may also be partially explained by the form error that was common to all three trials. This form deduction is not likely seen at the level of competition from which Dillman et al. collected their data. The subject in this study contacted the horse with varying degrees of bend in his knees. This body configuration raised the center of mass at contact and thus increased the contact angle value.

To conclude, based on the judge's scores and the comparison with two previous studies, the performance of the handspring 1 1/2 front salto vault, by this subject was deemed a typical vault as performed at a high level of competition. There were no consistent differences with the results found by Dillman et al. (1985) or Cheetham (1982). Bajin (1979) defined the push-off to be the most important phase in the handspring 1 1/2 front vault. This vault then, evaluated by the degree of extension at take-off (AHD and HCMTO) and the vertical impulse on the horse, reflected in ΔVVH, would be considered a good vault by Bajin's criteria.

Other performance variables measured did not have comparisons as these were not measured in previous studies. These variables, however, were measured to provide input variables for the optimization, were found to be deterministic quantities[12], and were used for subsequent discussion and comparison with the predicted optimum performance.

Anthropometric Data

The anthropometric data collected for the subject is given in table IV.2. Segmental lengths were measured from the film data and averaged over the three trials for the pre-flight and on horse phases. These data were used in the calculation of the body center of mass coordinates and the

----

[12] see deterministic model Chapter III.

angular momentum.

Table IV.2 Anthropometric Data

Parameter	Forearms	Arms	Trunk	Thighs	Shanks
Lengths(m)					
$\bar{x}$(n=123)	0.288	0.255	0.563	0.450	0.446
s	0.021	0.026	0.037	0.014	0.021
Mass					
total=68.92 kg	3.032	3.860	39.836	13.784	8.408
CM location					
(m)	0.092	0.144	0.191	0.195	0.271
Moment of Inertia					
(kg·m^2)	0.055	0.026	3.195	0.291	0.290

Angular Momentum

In table IV.3 data are presented for the angular momentum calculated for the pre-flight and postflight phases. Comparison between table IV.3's values and reported values is made difficult because of the lack of data presented in the literature on angular momentum values for airborne skills and specifically for the handspring 1 1/2 front salto vault. Secondly angular momentum is dependent upon the subject's inertial properties.

Table IV.3 Angular Momentum

	Trial 1	Trial 2	Trial 3	Mean
Pre-flight				
(kg-m²/s)	-109.86	-100.89	-101.57	-104.11
Postflight				
(kg-m²/s)	-80.93	-75.87	-79.24	-78.68

Hay, Wilson and Dapena (1977) found the angular momentum for a front salto and a Yamashita long horse vault''. The angular momentum was -65.39 $kg.m^2/s$ for the front salto. Average values of -60.28 and -30.35 $kg.m^2/s$ were found for the Yamashita's pre-flight and postflight phases respectively. Miller and Morrison (1975) reported angular momenta of 93.98 $kg\text{-}m^2/s$ and 105.95 $kg\text{-}m^2/s$ for a 1 1/2 and 2 1/2 piked dives respectively. Although these skills are different from the handspring 1 1/2 front salto long horse vault there is general agreement with what might be expected. The preflight and postflight values were larger than the Yamashita's since 3π radians as opposed to π radians of displacement are expected in the postflight phase of the handspring 1 1/2 front salto vault. The time of

----

''the Yamashita is similar to the handspring vault during the preflight but is characterized by a pike/extension action of the hips in the postflight.

flight for the postflight would lie somewhere between the time taken to complete a standing front salto and the 1 1/2 piked dive. More angular momentum would be required to complete 1 1/2 rotations then a single rotation in tuck (front salto) but less would be required than in the piked position.

B. Prediction and Simulation of the Optimum Performance

Prior to the presentation of the results for the predicted optimum solution, the constraint bounds on the state variables and input variables required for the optimization solution will be listed.

Constraints and Input Parameters

Listed in table IV.4 are the anthropometric and environmental constraints used in the optimization. The constraint bounds were selected from the performance analysis of the three trials. For any particular bound the average value was chosen if the standard deviation was small, otherwise the extrema were [illegible] al 2 was judged as the best of the three vaults so [illegible] derived data was used as the initial guess in the [illegible] computations. Therefore, in addition to the above [illegible] for constraint bound selection, to meet the requirement of a feasible initial guess, trial 2 values where chosen over the previous selections if the value was outside of the selected range.

Table IV.4 Anthropometric and Environmental Constraints

Variable	Minimum	Maximum
Range of Motion		
x_3 (rad)	0.751	1.806
x_4	-0.300	0.000
x_5	0.000	1.200
x_6	-0.669	0.582
x_7	-0.300	0.000
Environmental		
x_1 (m)	2.600	2.850
x_2	1.300	1.454
Velocities		
$\dot{x}_3$ (rad/s)	1.533	8.564
$\dot{x}_4$	-2.929	7.199
$\dot{x}_5$	-8.500	5.500
$\dot{x}_6$	-11.700	15.824
$\dot{x}_7$	-2.700	5.153
$\dot{x}_1$ (m/s)	-1.861	2.200
$\dot{x}_2$	-3.000	3.876
Accelerations		
$\ddot{x}_3$ (rad/s²)	-201.500	221.800
$\ddot{x}_4$	-165.100	39.800
$\ddot{x}_5$	-158.200	345.400
$\ddot{x}_6$	-273.550	393.000
$\ddot{x}_7$	-181.500	469.700
$\ddot{x}_1$ (m/s²)	-57.500	15.200
$\ddot{x}_2$	-5.500	103.100
Time (s)		
	0.170	0.220

To facilitate a fast and efficient convergence, the range of motion constraint bound for the generalized coordinates corresponding to the elbow and knee joints (x_4 and x_7 respectively) were chosen from different data. In the actual trials there was considerable flexion/extension at these two joints. A form requirement was that little or no bending occur at these specific joints. Therefore, the trial 2 data for x_6 and x_7 used for the initial guess were modified so that less bending was possible. The velocity and acceleration bounds were chosen with the original data from the 3 trials.

The initial guess was found by fitting a 5th degree polynomial, using a least square method, to trial 2's push-off phase data and the modified x_6 and x_7 data. This resulted in 42 state variables. It is the nature of the complex algorithm, used in the optimization, that the explicit variables have constraint bounds. To accommodate this requirement, a 5th degree polynomial was fitted to the data for trials 1 and 3 as well. The minimum and maximum values from the trials, for each polynomial coefficient, were used as the constraint bounds. These values are listed in table IV.5.

Listed in table IV.6 are the input parameters used in the indicated equations. The preflight angular momentum, horizontal velocity and vertical velocity were the average values found in the performance assessment of the three trials. The maximum penalty function value for angular momentum was the maximum value observed in the three trials. The minimum value was found using the subject's smallest postflight moment of inertia[14] (5.01 kg.m^2) times Cheetham's (1982) reported average maximum postflight angular velocity

---

[14] this occurred during the tight tuck position in the salto.

Table IV.5 Constraint Bounds for Polynomial Coefficients

Generalized Coordinates	a_0	a_1	a_2	a_3	a_4	a_5
x_1 min	2.730	0.887	-26.500	45.247	-1297.290	-3835.180
max	2.776	2.200	-20.800	295.100	825.312	124.170
x_2 min	1.376	-2.750	39.140	-320.000	388.410	-164.000
max	1.393	-2.027	46.900	-256.000	803.000	1969.710
x_3 min	0.749	7.900	-207.700	-25.420	-21727.700	7626.540
max	0.855	11.820	-29.500	3183.090	-999.830	50808.238
x_4 min	-0.284	1.143	-123.801	459.729	-5677.328	-2605.835
max	0.000	3.429	-41.269	1379.186	-1892.443	7817.505
x_5 min	0.870	-7.080	96.379	-3411.889	1230.000	-22000.000
max	0.922	-3.600	272.294	-1150.000	15410.047	38970.000
x_6 min	0.169	-7.311	-248.460	710.000	-14845.300	-9456.000
max	0.582	-3.190	-64.812	3417.348	951.000	22954.860
x_7 min	-0.230	-0.435	-136.108	989.826	-18900.777	12576.104
max	0.000	-0.145	-45.370	2969.478	-6300.259	37728.312

(-17.52 rad/sec). The average difference between the maximum postflight center of mass height and the corresponding hip height was used for b in equation [20]. The height of the center of mass at landing was chosen from trial 2. Trial 2's landing was judged to be the most effective technique.

Table IV.6 Input to Optimization Program

Variables	Values	Equations Used
Angular Momentum (kg-m^2/s)		
Pre-flight (AM_{pf})	-100.89	[33]
Penalty Function min (AM_l)	-87.75	[32]
max (AM_u)	-75.87	
Pre-flight Velocity (m/s)		
Horizontal (Vh_o)	4.355	[22]
Vertical (Vv_o)	1.633	[18]
CM - Hip Height (m)		
b	0.071	[21]
Height of Center of Mass at Landing (m)	1.184	[23]

The Optimal Solution

The initial objective function value was 50278.5900, after 30 iterations and 1.1 sec this value was 2.3124. Three complete optimizations[11] were performed using the best results obtained from the previous optimization as input into the current optimization run. Only three were performed since there was no significant difference between the second and the third optimization. The final function value was 0.0999. This required approximately 14 minutes of CPU time and approximately 4500 iterations.

Listed in table IV.7 are the optimal performance variables. The optimal solution produced a penalty violation for the horizontal take-off velocity which resulted in a predicted postflight range (DPSF) that was 6.1 millimeters greater than the maximum bound set in the distance penalty function. The prediction produced a greater impulse as reflected in the larger and smaller change in vertical and horizontal velocities respectively during horse contact. With a smaller predicted horse contact time than trial 2's time, the force components of the impulse must have been proportionally greater. This would translate into a more forceful push-off on the horse.

----

[11]Repeated optimizations were performed to assure fruitful searches and a global minimum.

Table IV.7 Predicted Optimal Performance Variables

VARIABLE	VALUE
Time(sec)	
THC	0.200
TPSF	0.900
Displacement(m)	
HCMTO	2.321
HJGD	2.756
HCM	2.827
DJGD	3.65
DPSF	3.857
Velocities(m/s)	
HVTO	4.286
VVTO	3.150
ΔHVH	-0.069
ΔVVH	1.517
Position(radians)	
AHD	1.461
AHTO	0.715
Angular Momentum(kg-m^2/s)	
AMPSF	-87.696

Simulation of the Results

The constraint bounds used in the postflight optimization simulation program are given in table IV.8. These values represent the average minimum and maximum values for the generalized coordinates found over the three trials's postflight phases. The following discussion, unless otherwise indicated, will compare the handspring 1 1/2 front salto vault as performed in trial 2 with the predicted optimal. Trial 2 was chosen for comparison because it was judged the best of the three performed vaults and was used as the initial guess in the optimization computations.

Table IV.8 Constraints on Generalized Coordinates Postflight

Variables	Minimum	Maximum
Range of Motion		
x_1	2.558	7.076
x_2	1.098	3.188
x_3	1.514	12.450
x_4	-1.766	-0.200
x_5	0.447	3.366
x_6	-0.229	2.621
x_7	-2.320	-0.049
Velocities		
$\dot{x}_1$	-3.460	12.470

$\dot{x}_2$	-6.605	12.825
$\dot{x}_3$	-7.798	22.594
$\dot{x}_4$	-9.371	15.610
$\dot{x}_5$	-25.36[illegible]	14.119
$\dot{x}_6$	-15.3[illegible]	25.412
$\dot{x}_7$	-19.000	15.653

In figure IV.5 the schematic representation of an actual performance and the simulated optimal are depicted. Both kinegrams were drawn to the same scale. The individual positions represented compare approximately, as they were selected by first normalizing the time interval for both results. These diagrams aptly display the results of the optimization. The greater postflight angular momentum and time of flight resulted in a body position that was not as tightly tucked in the predicted performance. The smaller angular momentum for trial 2 necessitated a tight tucked position for the salto to minimize the moment of inertia and thus increase the angular velocity. The optimal performance would receive less form deduction during contact with the horse. The larger take-off velocities in the optimization solutions resulted in a greater landing distance and as demonstrated in figure IV.6 a greater height attained for the center of mass in the postflight.

The actual performance curve is not a smooth parabolic curve. This curve does not represent a violation of the

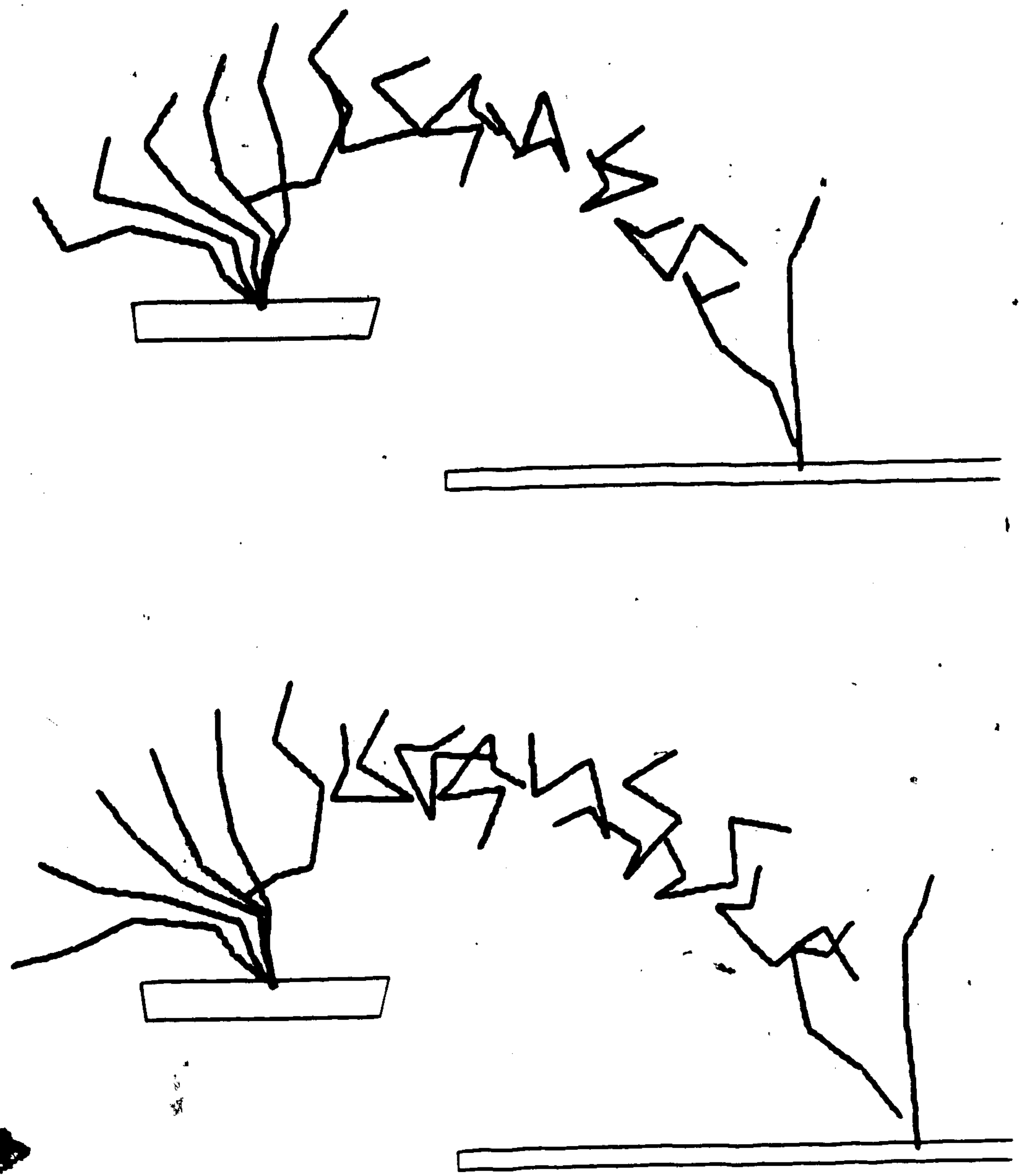

Figure IV.5 Trial and Simulated Performances

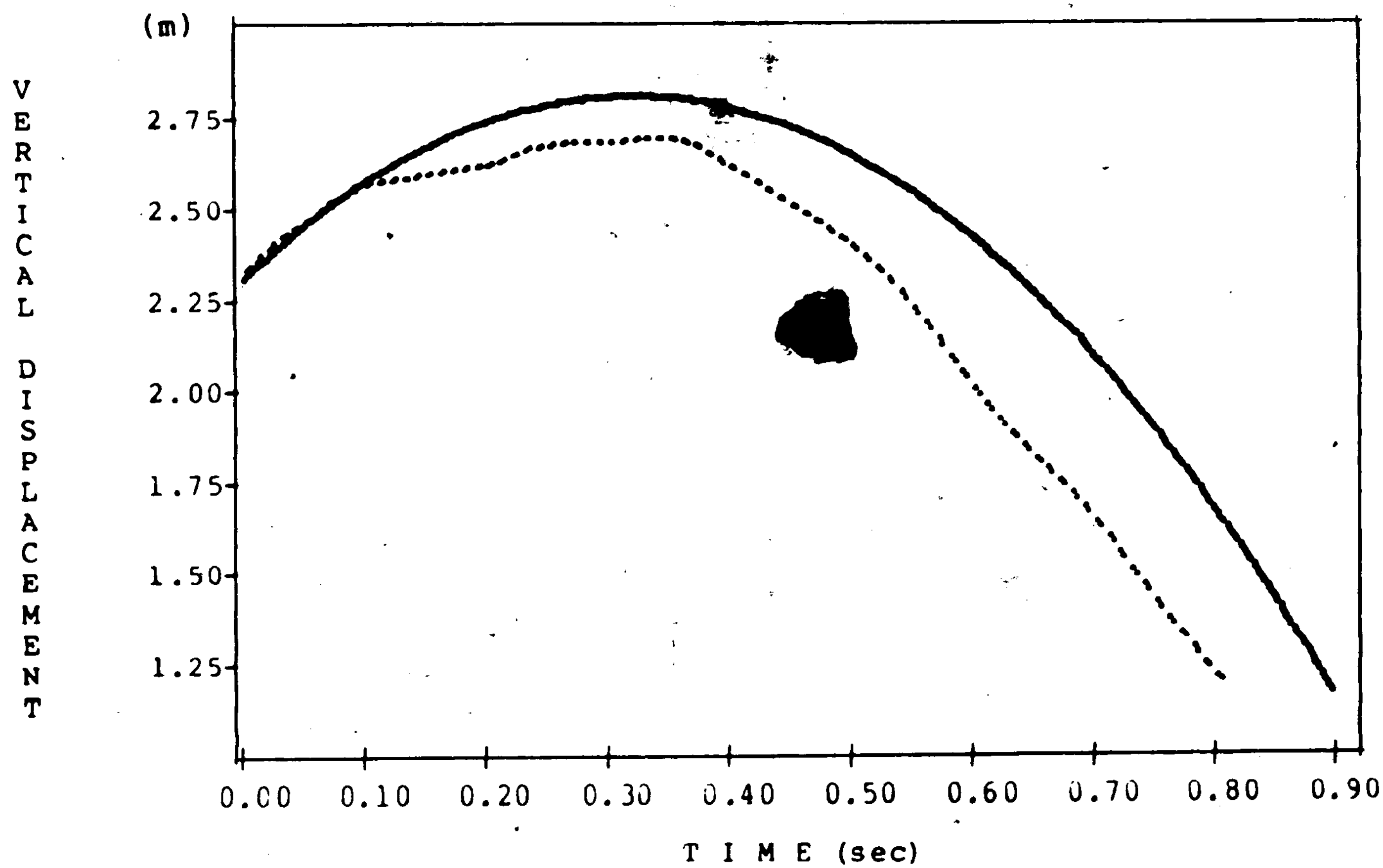

Figure IV.6 Center of Mass, Trial and Simulated
[.... → trial : ―― → simulated]

principles of uniformly accelerated motion but depicts the compounding error associated with cinematographical data. Even though the data for each individual generalized coordinate were smoothed, complete elimination of the error without altering the true signal is impossible. A compromise is made in the smoothing process between the amount of error existing in the signal and the true signal that may be eliminated. Therefore the curve shows how the existing errors, within 7 separate coordinates may accumulate when used in the same algorithms. Errors of a similar magnitude for these curves were also observed for the other two trials.

Bajin (1979) indicated that the push-off phase was the most important phase of the handspring 1 1/2 front salto vault. At the time of this study there was still a distinction between the near and far end vaults. In figure IV.5 it can be seen that this individual contacted the horse in its approximate center. Bajin indicated a distinction between the use of the hips and shoulders for the results he studied. If these joints are the prominent ones the question arises as to which may be the dominant joint in this study of the vault and if there is a distinction between the actual and predicted with respect to the joint actions at the hips and shoulders.

The displacement profiles for the shoulders and hips for the push-off phase are given in figures IV.7 and IV.8 respectively. The velocities are graphed in figures IV.9 and

IV.10. (For comparison purposes the abscissa values were normalized.) There was greater shoulder extension in the predicted performance prior to the shoulder flexion in the second half of the push-off phase. The shoulder action, between the trial and the simulation, appear to coincide with respect to when they occurred. There was greater shoulder flexion in trial 2 at horse departure yet in both cases there was incomplete shoulder flexion at take-off. When comparing the velocities for the shoulder it is apparent that the simulation's clockwise angular velocity (flexion) was smaller than that in trial two and that its peak was reached at about 80% of the phase as opposed to approximately 90% for trial 2.

The hip displacements revealed a difference in timing as well as in the range of motion. Maximum hip extension (actually hyper-extension) occurred earlier and was slightly larger in the simulation than in the actual performance. At take-off the hips were already flexed a little in the simulation. This hip flexion and smaller shoulder flexion accounted for the smaller center of mass height at take-off found in the predicted performance (2.321 meters) in comparison to the height measured for trial 2 (2.338). Figure IV.10 also revealed that the velocity at take-off was larger for the simulated performance.

It is difficult to discern which of the two joints may be predominant in the performance of this vault and its predicted optimal. Both appear to demonstrate what Gluck

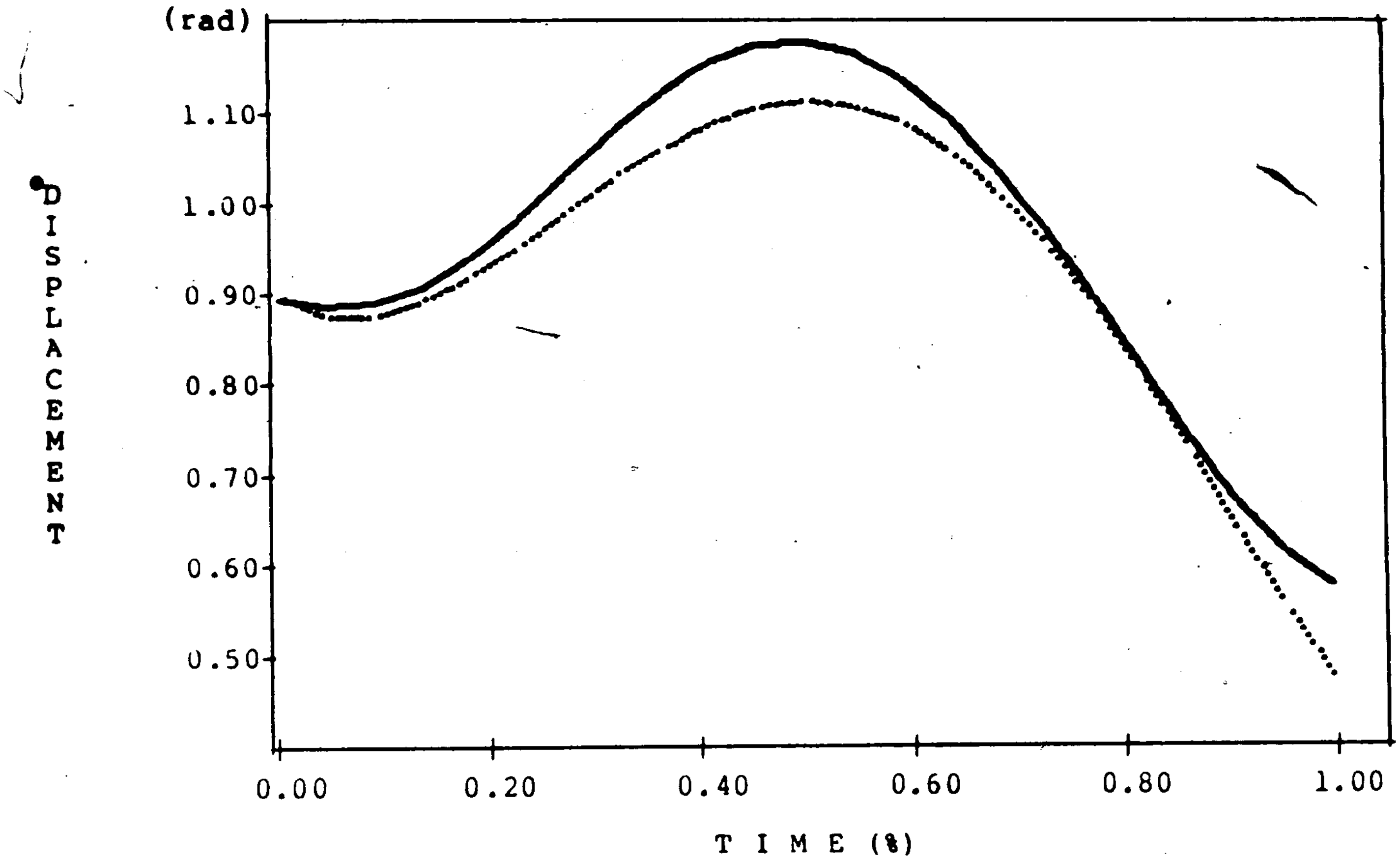

Figure IV.7 Shoulder Displacement Trial and Simulated
[.... → trial : —— → simulated]

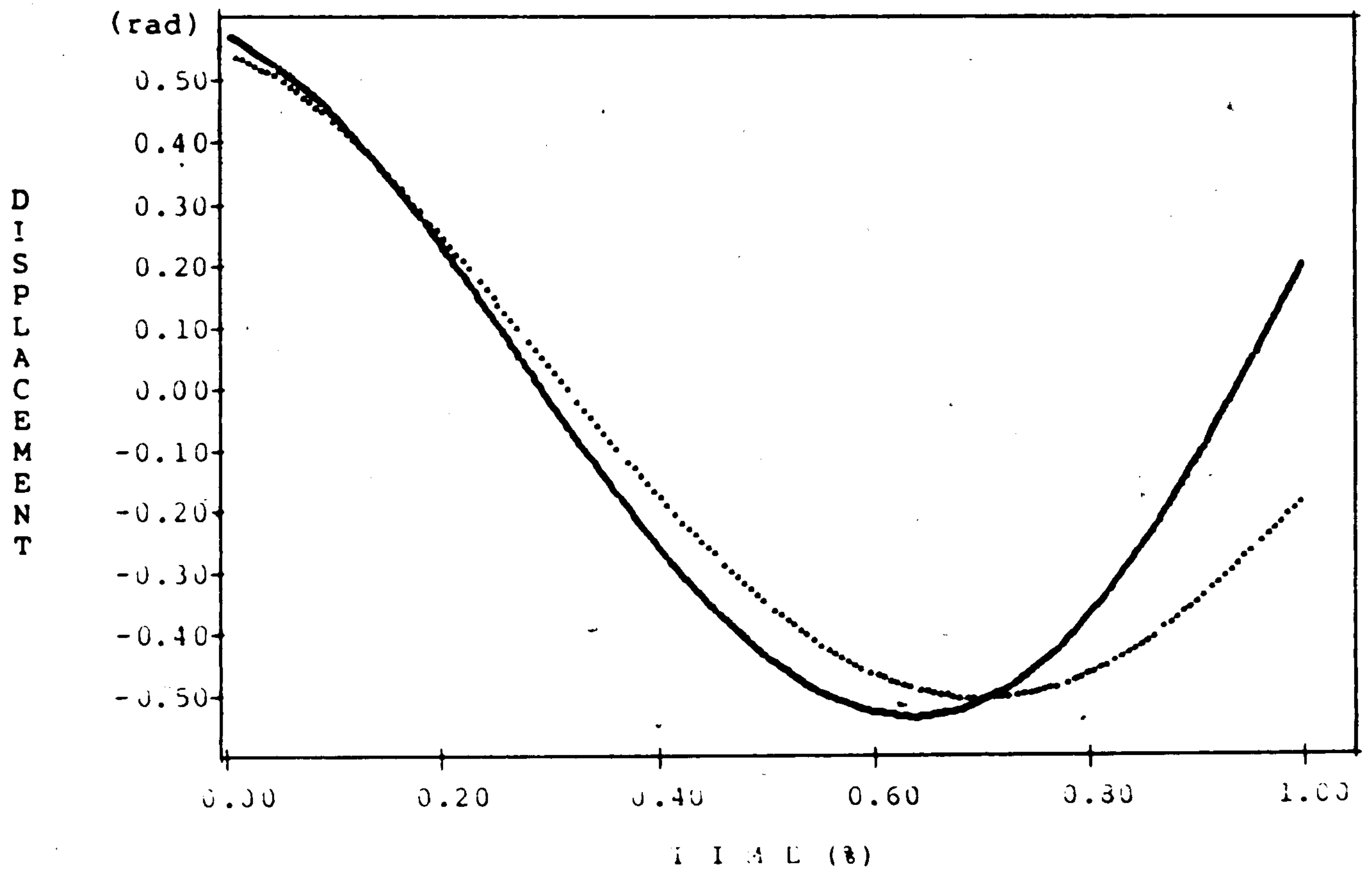

Figure IV.8 Hips Displacement Trial and Simulated
[.... → trial : —— → simulated]

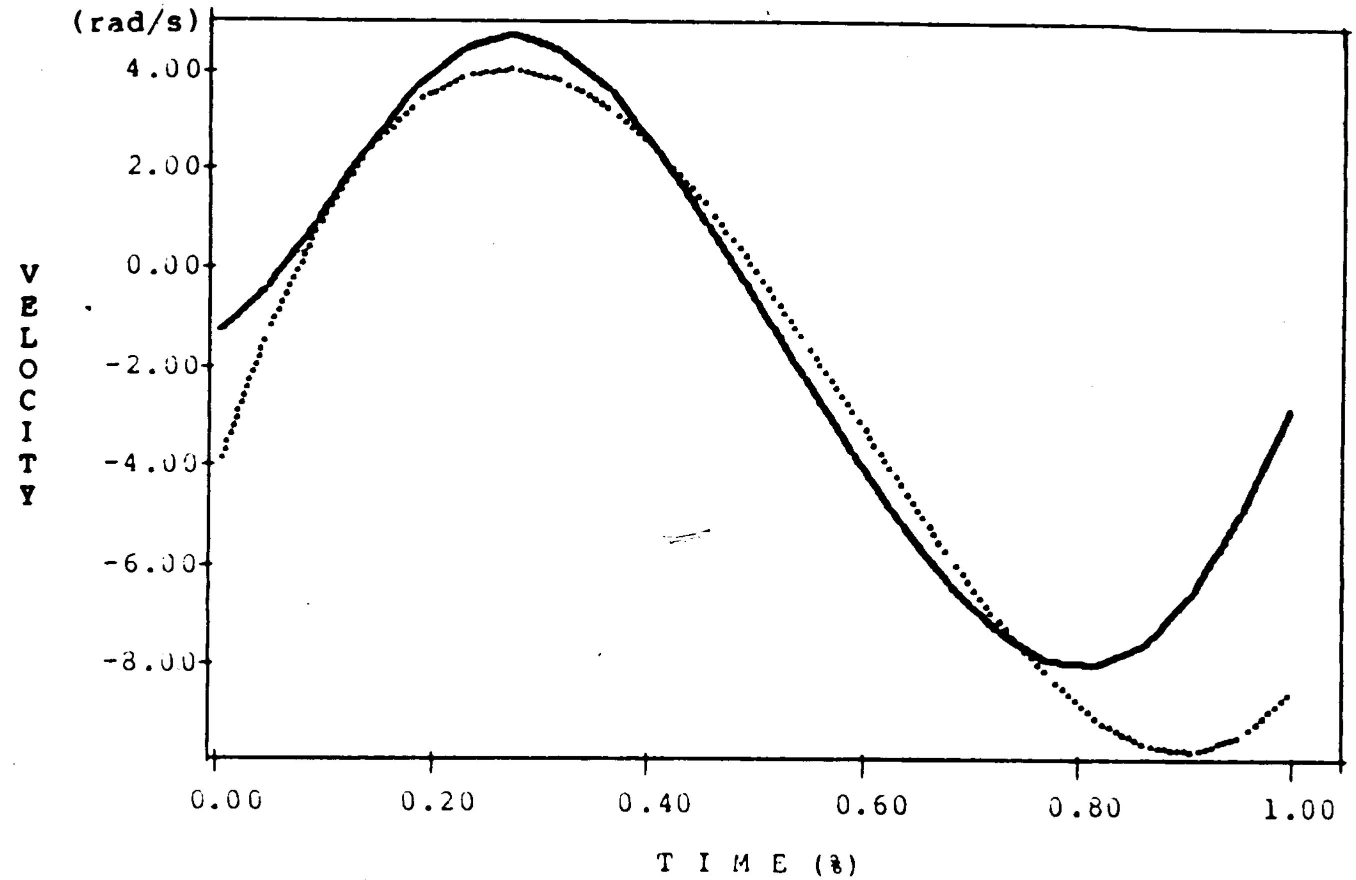

Figure IV.9 Shoulder Velocities Trial and Simulated
[.... → trial : ___ → simulated]

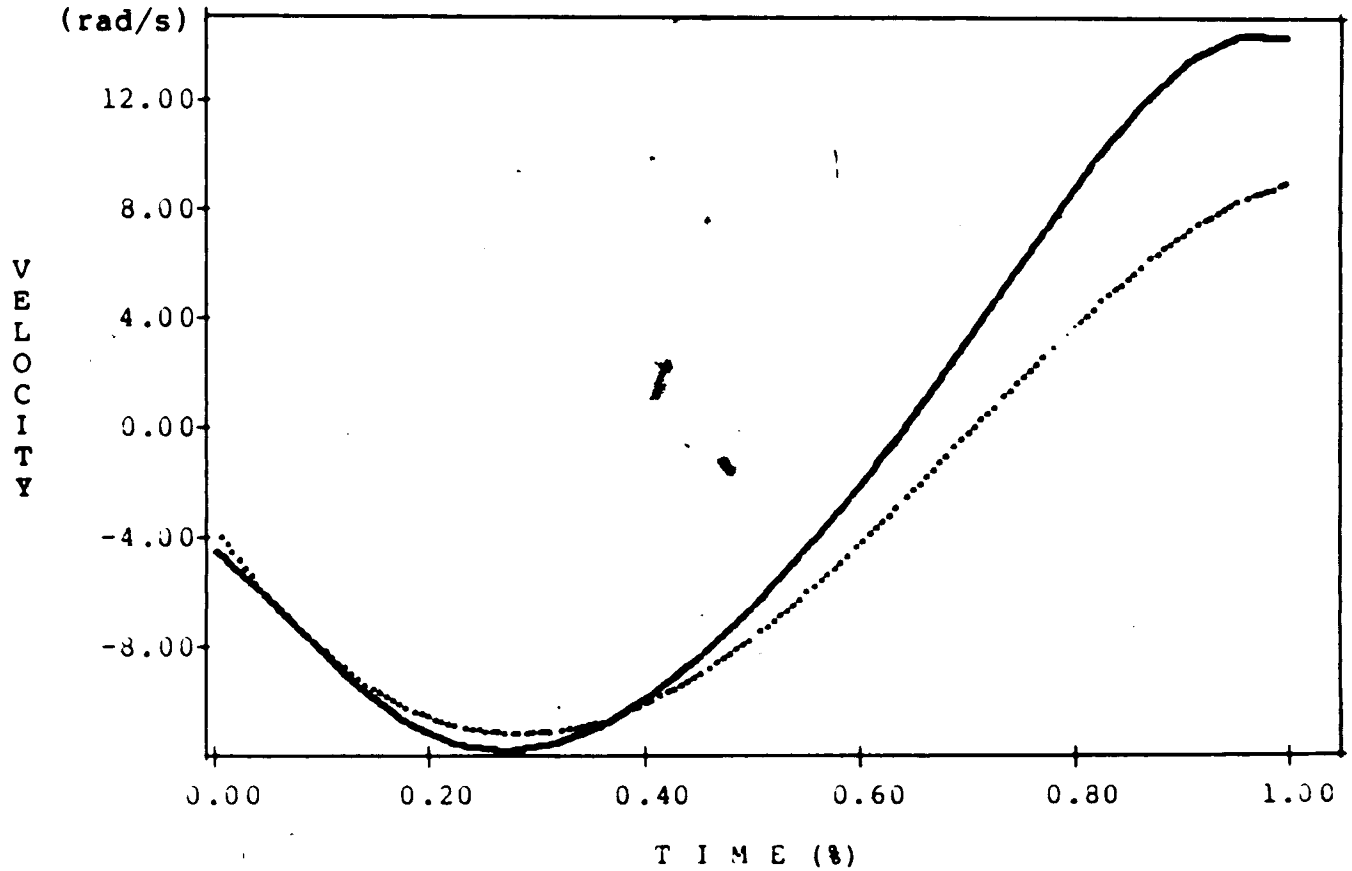

Figure IV.10 Hips Velocities Trial and Simulated
[.... → trial : ___ → simulated]

(1982) has termed the *arch-hollow* handspring vault technique (p. 91). A gymnast would contact the horse with the shoulders extended and hips hyper-extended using this style. To summarize then, both performances demonstrated shoulder extension followed by flexion. The major distinction between the two was that the hip flexion commenced earlier and with a greater rate of change in velocity in the optimum performance when compared to the actual performance. Therefore the recommendation to the gymnast might take the form of suggesting that he start his hip flexion earlier and more forcefully.

C. Comparison of Predicted Optimal with Other Studies

Panjabi (1979) in a letter to the editors in the Journal of Biomechanics on the topic of validation of mathematical models wrote:

> The basic dilemma in the process of validation may be stated in the following manner: A mathematical analogue can be validated only in a given number of known situations. Yet the main purpose of an analogue is to predict behavior in unknown situations. Thus, no perfect validation is possible. (p. 238)

Although a perfect validation was not possible, an estimate of the validity of the simulated results was attempted.

The check of the validity of this study's results consisted of a comparison between the data from the predicted optimum, with trial 2 and two previous studies of the handspring 1 1/2 front salto vault, Dillman et al. (1985) and Cheetham (1982). An inspection of the data in table IV.9 revealed good agreement between the predicted optimal data and the published data. However, differences did exist between the predicted and published data for the center of mass velocities. The change in vertical velocity reflected a consistently smaller trial initial velocity than the previously published values. Since the final take-off value was proportionally similar, a larger impluse was the consequence. Admittedly the horizontal velocity is larger than the reported averages yet it did remain below the maximum reported by Cheetham (1982). In making the comparison between the predicted optimized data and Dillman et al.'s (1985) data, for the *top 8 Olympians*, it must be kept in mind that as Bajin indicated "even with the world's best gymnasts, there is still room for improvement" (1979, p. 8).

According to the data presented in table IV.9, recognizing possible differences due to sampling techniques and body segment parameters used, it is concluded that the results in this study are reasonable and represent a valid prediction of an optimal performance.

Table IV.9 Comparison with Other Studies

<u>**Variables**</u>	**Predicted** Optimal	**Trial 2**	**Dillman et al.(1985)** $\bar{x}$ range[min,max]	**Cheetham (1982)** $\bar{x}$ range[min,max]
Time(s)				
THC	0.200	0.22	0.18 [0.15,0.19]	0.13 [0.09,0.16]
TPSF	0.900	0.81	0.92 [0.87,0.97]	1.04 [0.86,1.87]
CM Displacement(m)				
HCM	2.827	2.712	2.86 [2.74,3.01]	2.81 [2.63,2.98]
CM Velocity(m/s)				
HVTO	4.286	3.401	3.57 [3.01,3.97]	3.56 [3.03,4.32]
VVTO	3.150	3.068	2.97 [2.24,3.38]	2.86 [2.40,3.39]
ΔHVH	-0.069	-1.316	-1.54 [-2.37,-0.96]	-0.93 [-1.53,-0.40]
ΔVVH	1.517	1.343	0.22 [-0.23,0.69]	0.23 [0.00,0.91]
Position(rad)				
AHD	1.461	1.726	1.645 [1.518,1.763]	1.336 [1.082,1.536]

V. Summary and Conclusions

The purpose of this study was to develop an approach to assess an individual's performance and then to predict that individual's optimum performance. The skill chosen for this task was the handspring 1 1/2 front salto vault in men's artistic gymnastics. The individual assessed was an accomplished gymnast experienced in the performance of the particular skill. This study was delimited to the study of th[illegible] preflight, push-off and postfight phases for the pu[illegible]ses of the performance assessment. The prediction of an [illegible] performance was delimited to the prediction of the mov[illegible]ent in the push-off and postflight phases.

The performance assessment consisted of first developing a deterministic model of the task's perfo[illegible]nce objective of maximizing the points awarded for the execut[illegible]on of the skill. The approach taken was based on Hay and Reid's (1982) proposed qualitative modelling procedure. Utilizing the subjective measurement *judging code of points*, execution faults were translated into measurable mechanical quantities. This deterministic model provided a systematic and logical means of identifying the skill's performance variables.

Measurement of these performance variables for the three trials of the handspring 1 1/2 front salto vault was carried out using standard high speed cinematography. The derived generalized coordinates (Cartesian coordinates and relative angles for the 5 segment representation of the gymnast) were smoothed at a 6 Hz cut-off frequency using a second order Butterworth Digital Filter. The average time for horse contact and postflight were 0.18 and 0.81 seconds respectively. The height and range of the center of mass in the postflight were found to be 2.697 and 2.921 meters respectively. When these values were converted to their judged lengths it was determined quantitatively that the subject received no execution faults on the height and distance parameters. The mean body position at take-off from the horse indicated that the subject did not maximize the height of the center of mass. This body extension has been suggested by Bajin (1979) as being an important deterministic characteristic of a successful vault. Angular momentum values measured for the preflight and postflight phases were -104.107 and -78.681 $Kg \cdot m^2/s$ respectively. These values compared reasonably with published values for other airborne somersaulting skills.

Once having assessed the performance as being a good typical high level performance, a prediction of an optimal performance for this individual was made. Having previously identified the performance result as being the *points awarded*, those performance variables which if maximized, would minimize the point deductions, were selected in the formation of the performer's objective function. Postflight height and distance were identified as those variables. Angular momentum was included in a penalty function form to assure that sufficient angular momentum was present for successful completion of the skill. Penalty constraints for height and distance were also adjoined to the objective function to assure a realistic solution. The penalty function bounds were determined from published data on the handspring 1 1/2 front salto vault (Dillman et al., 1985 and Cheetham, 1982).

A Lagrangian approach was used to derive the equations of motion. A *Ritz* procedure, using fifth degree polynomials was used to represent and discretize the state variables (the generalized coordinates). A **Complex** algorithm was used to solve the optimization problem. The constraint bounds for the state variables were the ranges found for the polynomial coefficients from the least square fits on the three trials. The constraint bounds for the generalized coordinate time-histories came from the ranges determined in the performance assessment of the three trials. Trial 2's data was selected as the initial guess in the numerical solution of the problem.

Simulation of the postflight's predicted results was achieved using an interactive computer program which made use of an optimization scheme. The cost function used in the program was the difference between the simulated coordinates for the center of mass and the predicted values. Adjoined to this function was the difference between the simulated and predicted postflight angular momentum quantity.

The predicted optimum displayed greater virtuosity in that a greater height and distance was achieved in the postflight. Angular momentum was also greater. The suggested recommendation for the athlete, as indicated by the optimal results, is that the hips should be flexed sooner and more forcefully in the push-off phase of the skill. Comparison of the results of this study with previously published data on the handspring 1 1/2 front salto vault support the conclusion that the optimum solution predicted valid results and a feasible optimal performance for the individual investigated.

Based on the results obtained in this study, within the limitations and delimitations of this research, the following conclusions are warranted.

1. The deterministic model provides a systematic, logical approach to define the performance variables required in a performance assessment.
2. The optimization approach used in this study is a viable and practical means of predicting an optimal performance subject to the individual's anthropometric characteristics, environmental constraints and imposed initial and terminal conditions.

Recommendations

Based on the pertinent literature and on the research for and in this study the following are recommended as future research for human performance optimization.

1. The means to acquire human movement state constraints be further studied and a data base established.

2. Research into data approximation for data acquired on human movement be continued.
3. Nonlinear optimization algorithms, that are easier to implement, are more economical, efficient and capable of being supported on micro-computer should be made more accessible to the general sport biomechanics community.

Bibliography

An, K. N., Kevak, B.M., Chao, E.Y., & Morrey, B.F. Determination of muscle and joint forces: A new technique to solve the indeterrminate problem. Transactions of the ASME: Journal of Biomechanical Engineering, November 1984, 106, 364-367.

Arvikar, R. J., & Seireg, A. Evaluation of upper extremity joint forces during exercise Advances in Bioengineering, ASME December 1978, 71-73.

Audu, M. L., & Davy, D. T. The influence of muscle model complexity in musculoskeletal motion modeling Transactions of the ASME: Journal of Biomechanical Engineering. May 1985, 107, 147-157.

Bajin, B. Goniometric analysis of the push-off phase during a 1 1/2 somersault in men's gymnastic vaulting. In J. Terauds (Ed.), Science in Gymnastics. Del Mar, CA: Academic Publishers, 1979.

Bajin, B. Two and one and half front salto vault International Gymnast: Technical Supplement(No.3). July 1980, pp. ts13-ts14.

Bauer, W. L. Mathematical modelling and optimization and their influence on sport movements - Possibilities and limitations. In W. Baumann (Ed.) Biomechanics and Performance in Sport. Schorndorf: Hofmann-Verlag, 1982, 129-143.

Beckett, R., & Chang, K. An evaluation of the kinematics of gait by minimum energy. Journal of Biomechanics 1968, 1, 147-159.

Beckett, R., & Chang, K. An evaluation of the kinematics of gait by minimum energy. In D. Bootzen & H. C. Muffley (Eds.) Biomechanics, New York, Plenum Press, 1969.

Beckwith, T. G., & Buck, N. L. Mechanical measurements (2nd ed.). Reading, Mass.:Addison-Wesley, 1973.

Borysiewicz, M., Bucka, J., & Komor, A. Optimization of sport techniques using the example of weight lifting. In A. Morecki, K. Fidelus, K. Kedior, & A. Wit Biomechanics VII-B Warsaw: Polish Scientific Publishers, 1981, pp. 305-312.

Box, M. J. A new method of constrained optimization and a comparision with other methods. The Computer Journal

1965, 8, 42-52.

Brüggeman, P. Biomechanical analysis of selected vaults on the longhorse. In J. Terauds (Ed.), Science in Gymnastics. Del Mar, CA: Academic Publishers, 1979, pp.9-25.

Campbell, K. R., & Reid, R. E. The application of optimal control theory to simplified models of complex human motions: The golf swing. In D. A. Winter, R. W. Norman, R. P. Wells, K. C. Hayes, & A. E. Patla, Biomechanics IX-B. Champaign, Illinios: Human Kinetics, 1985, pp. 527-532.

Chao, E. Y-S., Rim, K. Application of optimization principles in determining the applied moments in human leg joints during gait. Journal of Biomechanics, 1973, 6, 487-510.

Cheetham, P. J. The men's handspring front one and one half somersault vault: Relationship of early phase to postflight. In J. Treuads (Ed.), Biomechanics in Sport. Del Mar: Research Center for Sport, 1982, pp. 231-247.

Chow, C. K., & Jacobson, D. H. Studies of human locomotion via optimal programming. Mathematical Bioscience, 1971, 15, 93-108.

Clauser, C. E., McConville, J. T., & Young, J. W. Weight, volume, and center of mass of segments of the human body. (AMRL-TR-69-70) Wright Patterson Air Force Base, 1969.

COMPLX/DCOM, University of Alberta, Computing Services, Subroutine Write Up R247.0977, October 1977.

Conte, S. D., & de Boor, C. Elementary Numerical Analysis: An Algorthmic Approach. New York: McGraw-Hill, 1980.

Crowninshield, R. D. Use of optimization techniques to predict muscle forces. Transactions of the ASME: Journal of Biomechanical Engineering, 1978a, 100, 88-92.

Crowninshield, R. D. Use of optimization techniques to predict muscle forces. Advances in Bioengineering, ASME, December 1978b, 57-78.

Crowninshield, R. D.,& Brand, R. A. A physiological based criterion on muscle force prediction in locomotion. Journal of Biomechanics, 1981, 14(11), 793-801.

Dapena, J. Simulation of modified human airborne movements. Journal of Biomechanics, 1983, 14, 81-89.

Dempster, W. T. Space requirements of the seated operator (WADC-TR-55-159). Wright Patterson Air Force Base, 1955.

Dianis, A. Cinematographic analysis of the handspring vault. Research Quarterly, 1979, 50(3), 341-349.

Dianis, A. A model and analysis of vaulting. International Gymnast: Techanical Supplement(No.3), July 1980, pp. ts1-ts12.

Dianis, A. A model for gymnastics of vaulting. Medicine and Science in Sport and Exercise, 1981, 13(1), 34-43.

Dillman, C., Cheetham, P. J., & Smith, S. L. A kinematic analysis of men's olympic long horse vaulting. International Journal of Sport Biomechanics, 1985, 1, 96-110.

FIG, International Gymanstics Federation, Code of Points, 1980.

Gelfand, I. M., & Fomin, S. V. Calculus of variations. Englewood Cliffs, N. J.: Prentice-Hall, 1963.

Gerald, C. F. Applied numerical analysis. Reading, Mass.: Addison-Wesley, 1973.

Gill, P. E., & Murray, W. The numerical solution of a problem in the calculus of variations, In D. J. Bell (Ed.), Recent Mathematical Developments in Control, London: Academic Press, 1973, 97-122.

Gill, P. E., Murray, W., & Wright, M. Practical optimization , London: Academic Press, 1981.

Gluck, M. Mechanics for gymnastics coaching tools for skill analysis. Springfield: Charles C Thomas, 1982.

Gruver, W. A., Ayoub, M. A., & Muth, M. B. A model for optimal evaluation of manual lifting tasks. Journal of Safety Research, Summer 1979, 11(2), 61-71.

Gruver, W. A., & Sachs, E. Algorithmic Methods in Optimal Control. Boston: Pitman Advanced Publishing Program, 1[illegible], 182-199.

Hamming, R. W. Numerical Methods for Scientists and Engineers (2nd ed.). New York: McGraw-Hill, 1973.

Hanavan, E. P. A mathematical model of the human body (AMRL-TR-64-102). Aerospace Medical Research Laboratories, Wright-Patterson Air Force Base, Ohio, 1964.

Hardt, D. E. Determining muscle forces in the leg during normal walking - An application and evaluation of optimization methods. Transactions of the ASME: Journal of Biomechanical Engineering, May 1978, 100, 72-78.

Hatze, H. Optimization of human motions. In S. Cerquiglini, A. Venerando, & J. Wartenweiler (Eds.), Medicine and Sport: Biomechanics III. Baltimore: University Park Press & S. Karger, Basel, 1973, 138-142.

Hatze, H. Biomechanical aspects of a successful motion optimization. in P. V. Komi (Ed.), Biomecahnics V-B, Baltimore: University Park Press, 1975, 5-12.

Hatze, H. The complete optimization of a human motion. Mathematical Biosciences , 1976, 28, 99-135.

Hatze, H. A complete set of control equations for the human musculo-skeletal system. Journal of Biomechanics, 1977a, 10, 799-805.

Hatze, H. A myocybernetic control model of skeletal muscle. Biological Cybernetics, 1977b, 25, 103-119.

Hatze, H. The relative contribution of motor unit recruitment and rate coding to the production of static isometric muscle force. Biological Cybernetics, 1977c, 27, 21-25.

Hatze, H. A general myocybernetic control model of skeletal muscle. Biological Cybernetics, 1978, 28, 143-157.

Hatze, H. Neuromusculoskeletal control systems modeling - A critical survey of recent developments. IEEE Transactions on Automatic Control, June 1980, AC-25(3),375-385.

Hatze, H. A comprehensive model for human motion simulation and its application to the take-off phase of the long jump. Journal of Biomechanic, 1981, 14(3), 135-142.

Hatze, H. Myocybernetic control models of skeletal muscle Characteristics and applications Pretoria: University of South Africa, 19[illegible]1.

Hatze, H. Myocybernetic optimization of sports motions - Model, methods for parameter identification, practical implementation. In W. Baumann (Ed.). Biomechanics and Performance in Sport. Schorndorf: Hofmann-Verlag, 1982, 173-182.

Hatze, H. Computerized optimization of sports motions: An overview of possibilities, methods and recent developments. Journal of Sports Sciences, 1983, 1, 3-12.

Hay, J. G. The biomechanics of sport techniques (1st ed.). Englewood Cliffs, N.J.: Prentice-Hall, 1973.

Hay, J. G. The development of deterministic models for qualitative analysis. In R. Shapiro & J. R. Marett (Eds.), Proceedings Second National Symposium on Teaching Kinesiology and Biomechanics in Sports. Colorado Springs, Colorado: 1984, pp. 71-83.

Hay, J. G. The biomechanics of sport techniques (3rd ed.). Englewood Cliffs, N.J.: Prentice-Hall, 1985.

Hay, J. G., & Reid, J. G. The anatomical and mechanical bases of human motion. Englewood Cliffs, N.J.: Prentice-Hall, 1982.

Hay, J. G., Wilson, B. D., & Dapena, J. A computational technique to determine the angular momentum of a human body. Journal of Biomechanics, 1977, 10, pp. 269-277.

Hellebrandt, F. A. The physiology of motor learning. Cerebal Palsy Review, 1958, 10, 4-13.

Herbold, R. J., Schultz, M. H., & Varga, R. S. The effect of quadrature errors in the numerical solution of boundary value problems by variational techniques. Aequationes Mathematicae, 1969, 3, 247-270.

Hubbard, M. Optimal javelin trajectories. Journal of Biomechanics, 1984, 17(10), 777-787.

Hubbard, M., & Barlow, D. A. A multiple segment pole vault model with optimization of vaulter inputs and its use in coaching. In J. M. Cooper, & B. Haven (Eds.), Proceedings of the Biomechanics Symposium Indiana University. Bloomington: The Indiana State Board of Health, 1980, pp. 33-48.

Hubbard, M., & Trinkle, J. C. Optimal initial conditions for the eastern roll high jump. In R. Huiskes, D. Van Campen, & J. De Wign (Eds.), Biomechanics: Principles and Applications. The Hague: Martinus Nijhoff Pub., 1982, pp. 169-174.

Hubbard, M., & Trinkle, J. C. Optimal fosbury-flop high jumping. In D. A. Winter, R. W. Norman, R. P. Wells, K. C. Hayes, and A. E. Patla (Eds.), Biomechanics IX-B. Champaign, Illinios: Human Kinetics, 1985, pp. 448-453.

Keller, J. B. A theory of competitive running. Physics Today, 1973, 26(9), 42-47.

Keller, J. B. Optimal velocity in a race. American

Mathematical Monthly, 1974, 81, 474-480.

Keller, J. B. A theory of competitive running. In S. P. Ladany, & R. E. Machol (Eds.), Optimal Strategies in Sport. New York: North-Holland, 1977.

Luenberger, D. G. Introduction to Linear and Nonlinear Programming. Reading, Mass.: Addison-Wesley, 1973.

McLaughlin, T., Dillman, C., & Lardner, T. Biomechanical analysis with cubic spline functions. Research Quarterly. 1977, 48(3), 569-582.

Miller, D. I. Modelling in Biomechanics: An overview. Medicine and Science in Sport, 1979, 11(2), 115-122.

Miller, D. I., & Morrison, W. E. Prediction of segmental parameters using the Hanavan human body model. Medicine and Science in Sport, 1975, 7, 207-212.

Miller, D. I., & Nelson, R. C. Biomechanics of Sport. Philadelphia: Lea & Febiger, 1973.

Morel, Y., Bourassa, P. & Marcos, B. Human locomotion via optimal programming. In D. A. Winter, R. W. Norman, R. P. Wells, K. C. Hayes, and A. E. Patla, Biomechanics IX-A. Champaign, Illinios: Human Kinetics, 1985, pp. 473-477.

Nelson, W. L. Physical principles for economies of skilled movements. Biological Cybernetics, 1983, 46, 135-147.

Nubar, Y., & Contini, R. A Minimal principle in biomechanics. Bulletin of Mathematical Biophysics, 1961, 23, 377-391.

Panjabi, M. Validation of mathematical models. Journal of Biomechanics, 1979, 12, 238.

Patriarco, A. G., Mann, R. W., Simon, S. R. & Mansour, J. M. An evaluation of the approaches of optimization models in the prediction of muscle forces during human gait. Journal of Biomechanics, 1981, 14(8),513-525.

Pedotti, A. A study of motor coordination and neuromuscular activities in human locomotion. Biological Cybernetics, 1977, 26, 53-62.

Pedotti, A., Krishnan, V. V., & Stark, L. Optimization of muscle-force sequencing in human locomotion. Mathematical Biosciences, 1978, 38, 57-76.

Pedotti, A., Rodano, R., & Frigo, C. Optimization of motor coordination in sport: An analytical and experimantal approach. In W. Baumann (Ed.), Biomechanics and Performance in Sport. Schorndorf: Hofmann-Verlag, 1982, 145-160.

Penrod, D. D., Davy, D. T., & Singh, D. P. An optimization approach to tendon force analysis. Journal of Biomechanics, 1974, 7, 123-129.

Ramey, M. R. A simulation of the running long jump. In J. L. Bleustein (Ed.), Mechanics and Sport. New York: The American Society of Mechanical Engineers, 1973, 101-112.

Rao, S. S. Optimization: Theory and Application. New Delhi: Wiley Eastern Ltd., 1979.

Remizov, L. P. Biomechanics of optimal flight in ski-jumping. Journal of Biomechanics, 1984, 17(3), 167-171.

Seireg, A. Optimum control of human movement. In W. Baumann (Ed.), Biomechanics and Performance in Sport. Schorndorf: Hofmann-Verlag, 1982, 161-171.

Seireg, A., & Arvikar, R. J. A mathematical model for evaluation of forces in the lower extremities of the musculo-skeletal system. Journal of Biomechanics, 1973, 6, 313-326.

Seireg, A., & Arvikar, R. J. The prediction of muscular load sharing in the lower extremities. Journal of Biomechanics, 1975a, 8, 89-102.

Seireg, A., & Arvikar, R. J. A comprehensive musculoskeletal model for the human vertebral column. Advances in Bioengineering ASME, 1975b, 74-75.

Seireg, A., & Baz, A. A mathematical model for swimming mechanics. In L. Lewillie, & J. P. Clarys (Eds.), Biomechanics of Swimming. Brussels: Universite Libre de Bruxelles, 1971, 81-103.

Simpson, R. B. The Rayleigh-ritz process for the simplest problem in the calculus of variations. SIAM Journal of Numerical Analysis, 1969, 6(2), 258-271.

Slots, L., & Stone, G. Biomechanical power generated by forearm flexion. Human Factors, 1967, 5, 443-452.

Soong, T.-C. An optimally designed archery. In J. L. Bleustein (Ed.), Mechanics and Sport. New York: The American Society of Mechanical Engineers, 1973, 85-100.

Townsend, M. A., & Seireg, A. The synthesis of bipedal locomotion. Journal of Biomechanics, 1972, 5, 71-83.

Townsend, M. A., & Seireg, A. Effect of model complexity and gait criteria on the synthesis of bipedal locomotion. IEEE Transactions on Biomedical Engineering, November 1973, BME-20(6), 433-444.

Vaughan, C. L. Computer simulation of human motion in sport biomechanics. In R. L. Terjung (Ed.), Exercise and Sport Sciences Reviews (vol. 12). Toronto: The Collamore Press, D. C. Heath & Co., 1984, 373-416.

Vaughan, C. L. A basic program for nonlinear optimization studies in sport biomechanics. International Journal of Sport Biomechanics, 1985, 1, 63-72.

Vaughan, C. L., Andrew, J. G., & Hay, J. G. Selection of body segment paramters by optimization methods. Journal of Biomechanical Engineering, February 1982, 104, 38-44.

Vaughan, C. L., Hay, J. G., & Andrew, J. G. Closed loop problems in biomechanics part II - An optimization approach. Journal of Biomechanics, 1982, 15(3), 201-210.

Walton, J. S. Close-Range Ciné-Photogrammetry. Unpublished doctoral dissertation, The Pennsylvania State University, 1981.

Wells, D. A. Lagrangian dynamics. New York: Schaum's Outline Series McGraw-Hill, 1967.

Williams, R. J., & Seireg, A. Interactive modelling and analysis of open or closed loop dynamic systems with redundant actuators. Transactions of the ASME, Journal of Mechanical Design, 1979, 101, 407-416.

Winter, D. A. Biomechanics of human movement. New York: John Wiley & Sons, 1979.

Winter, D. A., & Wells, R. P. Proper sampling and filtering frequencies in kinematics of human gait. Proceedings of the Seventh Canadian Medical and Biological Engineering Conference. Vancouver: August 1978.