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Modeling Risk of a Multi-State Repairable Component

by

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DEDICATION

This thesis work is dedicated with love to my family and sincere friends:

My parents: Roberto and Hilda.

My grand parents: Nino and Nina, may rest in peace.

My sisters: Claudia, Luz and Lorena.

My nephews and nieces: Ricardo, Andrea, Ana Sofia, Juan Pablo and Natalia.

My sincere friends located all over the world.

Even though most of you were thousands of miles away, your encouragement and support always kept me going.

ABSTRACT

This thesis focuses on the use of computer simulation for modeling risk of a multi-state repairable component.

In production processes, maintenance decisions are often made based on uncertain assessment of risk, not only in the probability when a process component goes into a state of failure but also in the cost of lost production and preventive maintenance. In this thesis work, preventive maintenance of a component is modeled and simulated, in order to minimize risk (cost), as:

- a Markov process with multiple states and fixed transition probabilities, under the assumption that with a sufficient number of states the Markovian property is valid,
- a non Markov process with two possible states and non-fixed transition probabilities for a periodically decreasing reliability component, and
- a non Markov process with two possible states and non-fixed transition probabilities for a continuously decreasing reliability component.

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CHAPTER 1

INTRODUCTION

1.1 MOTIVATION

In industry, many machines, systems and components are subject to failures due to damage or deterioration. These failures can produce economic losses, reduced availability, and even safety issues. The main objective of maintenance is to avoid or minimize these potential problems by performing the actions (technical or administrative) that keep the device in a working state in which it can perform its required function (maintenance) or prevent trouble from arising (preventive maintenance). In general, Maintenance is considered a fundamental part of effective and profitable industrial systems.

Maintenance optimization is an area of operations research that deals with the efficiency of maintenance activities in terms of company profits and incurred costs. If the main purpose of a mathematical model is to maximize the benefits of maintenance, then this mathematical model is called maintenance optimization model [1].

Maintenance models that consider stochastic processes are usually difficult to comprehend and interpret by technicians, professionals and managers who are traditionally educated on deterministic models. In general, risk, which is a measure of the probability and severity of undesired effects (in our case, cost), is a concept difficult to understand due to its composition of two components:

1) The consequence or cost (tangible component), and
2) The probability (intangible mathematical number) [20].

The purpose of this thesis is to address a simple Maintenance Optimization Model which deals with stochastic processes affecting a repairable component. The intention is to use a simulation tool when addressing the problem, which is easy to understand and interpret by the technicians, administrative and managers of a company, and which includes a measure of risk (cost) in its objective function.

The use of simulation would offer, once the problem has been modeled satisfactorily, a lot of flexibility to adequate and adapt to new changes in the process, by making small changes in the simulation model. Simulation would also offer the possibility of analyzing different hypothetical case scenarios of the problem, by making virtual changes in the simulation model.

1.2 BACKGROUND AND HISTORY

In the industrial world, it is critical to take maintenance decisions on:

- whether or not to stop production in order to perform preventive maintenance or to sacrifice a PM (Preventive Maintenance) in order to keep producing,
- when it is the right moment to allocate those maintenance resources, and
- selecting the areas in which the maintenance should be done,

in order to optimize processes in terms of minimizing risk. This study is focused in the first two points described above.

Becker [2] states that the objective of Preventive Maintenance (PM) is to service a equipment unit before, and not after, it causes damage in quality, production output, safety or cost. Similarly, Kececioglu [3] describes the objectives of PM as increasing a component's reliability, decreasing the number of failures, decreasing the downtime of a product or a system, decreasing requirements of spare parts, decreasing the maintenance man hours, and decreasing its life cycle cost. This work conforms to these two PM concepts, by considering components "as good as new" after PM is performed, in order to potentially minimize risk (cost).

The first scientific approaches to maintenance management date from the 1950s and 1960s [19]. Originally maintenance was focused only on minimizing failures and downtime, and so preventive maintenance programs started to be implemented. With the peak of operation research studies, new models were created in order to optimize the PM programs. Condition monitoring and techniques such as Vibration Measurement and Analysis, Infrared Thermography, Oil Analysis, Tribology, Ultrasonics, Motor Current Signature Analysis, etc. were developed to determine equipment condition in the 70's and proved to be more effective than PM programs. Computers were not used formally for maintenance purposes until the 1980s [19]. This study considers of high importance the use of computers to address PM allocation problems, and more specifically, the use of computer simulations as another important handy tool in the maintenance decision making.

Modarres, Kaminskiy and Krivtsov described how Reliability Centered Maintenance (RCM) had its roots in the 1960s when the North American civil aviation industry realized that their maintenance practices were unsafe and implied a high cost. Before the 1960s, the maintenance aviation policy was basically restoring almost everything in an airplane every certain period of time [43]. At the start, the RCM approach was only used and oriented to airplane maintenance and did not break through into other industries until the late 1980s [4], [32]. In general, RCM is an industrial improvement approach focused on the necessary processes to ensure that any component continues to do its functions.

Since then, many studies have been attempted to determine or find the best way to decide how and when to conduct maintenance activities. The main idea of Reliability Centered Maintenance (RCM) is to guarantee that a specific process or system will continue performing its intended functions; however, it is not only important to ensure the reliability and availability of a system, but also to consider the cost associated with that availability assurance in order to make right decisions. Then, RCM lacks a consistent decision analysis framework and consistent approaches to estimate risk associated with maintenance actions.

An effective framework for evaluating risk in maintenance can lead to effective algorithms to assist with decisions about how to allocate maintenance resources, avoiding the cost of wrong decisions and optimizing the frequency of maintenance and the availability of the components and the system.

In production processes, the maintenance decisions are usually based on the idea of minimizing the costs of the maintenance. In order to do this, it is common industrial practice to assess the risk of a component or system failure and its associated cost for lost production and the repair cost, and then to compare these costs with the ones associated with scheduling periodic preventive maintenance to try to avoid the failure-associated costs. This study, perform this cost comparison through the same simulation in order to determine whether or not is advisable to perform PM, and whit which periodicity this PM should be carried out.

CHAPTER 2

LITERATURE REVIEW

2.1 REPAIRABLE COMPONENTS

The traditional concept of Reliability, which is the ability of a system to perform its intended functions for a designated period of time under specified operating conditions, considers only two possible options to describe a system or a component state: Good or Failed. In this manner, the traditional roots of reliability do not consider the possibility of a repair of the system. In general, components are considered non repairable in the basic Reliability vision [4]-[11]. This study considers the situation where a component may be repairable.

In some cases, a repairable component or system that can be rehabilitated to fulfill all its required functions satisfactorily after failing to perform at least one of its required functions, by some method other than replacement of the entire component or system. This is called a repairable component or a repairable system [13]. Lindqvist [15] suggested extending this definition to include the possibility of supplementary maintenance actions with the objective of servicing the component or system for better performance. Or more briefly: A repairable component or system can be defined as one that is repaired rather than replaced after a failure [16].

The types of maintenance activities that a component might undergo have been classified in [34]:

Perfect PM or Replacement . The component or system is considered to be as good as new after the repair.

Minimal PM or Minimal Repair. This kind of repair does not change the hazard rate nor the effective age of the component or system. The repair only restores the component or system to the prior functioning state at the time of failure (this type of repair is also called: as bad as old). The hazard rate function is the probability that a component will fail in the next instant of time unit given that it is presently working [27], [34].

Imperfect Repair or Imperfect PM. This activity partially restores the state of the component or system to between as good as new (after perfect PM), and as bad as old (after minimal repair) [34].

For simulation purposes only, this study will assume that the component is going to be as goo as new after PM.

2.2 MULTI-STATE JUSTIFICATION

Virtanen [14] proposed to extend the traditional concepts of reliability arguing that when we consider a system, it may not be in one of the two states: up-state or (fully working) or down-state (failed), but rather it may be operating in a degraded state (many levels of reduced service). In that case, the conventional concepts of binary reliability systems are found to be unsuitable and inadequate. There may be situations where the system is neither in perfect condition nor fully failed, so the calculation of the reliability of the system gets compromised, or it gets a value that leads to wrong conclusions: either overestimated or underestimated reliability. In this thesis, the components subject to study will be considered “multi-state” components for the case study “model A”, which means that they would have more than the two classic reliability states: good and failed. In the case studies B and C, the components would be considered under the classic view of reliability: two states, in order to reduce computation and simulation times.

2.3 STOCHASTIC PROCESSES AND MARKOV CHAINS

A stochastic process is a random process evolving with time. It is a collection of random variables X_t indexed by time. If time is a subset of nonnegative integers numbers {0, 1, 2, 3, …}, the process is a discrete time process. A stochastic process is continuous time if time is a subset of nonnegative real numbers [0, ∞).

Since costs and failure rate may be probabilistic in production and maintenance, it is appropriate to consider these processes as stochastic in order to address the problem. In maintenance, costs are generally probabilistic because one part of these costs is the corrective maintenance cost, and this corrective maintenance cost can have different values, depending on whether or not a component or system fails.

Many stochastic processes have the property that the change at time t is determined only by the state value at the same time *t* and not by any state at previous time *t*. Such processes are called Markov processes and such property is called Markov property. In general, the Markov property states that the past has no influence on the state of the system in the future, but it suffices to consider only the present state [19].

A Markov chain is a sequence of random variables X_1, X_2, X_3, …, with the Markov property, mathematically described by:

$$\Pr(X_{n+1} = x \mid X_n = x_n, \ldots, X_1 = x_1, X_0 = x_0) = \Pr(X_{n+1} = x \mid X_n = x_n) \quad (1)$$

The possible values of X_i are called states of the system, and the conditional probabilities in equation (1) are called transition probabilities. A Markov process allows calculation of the transitions probabilities in *n* steps, The probability of going from state *i* to state *j* in n steps is:

$$p_{ij}^{(n)} = \Pr(X_n = j \mid X_0 = i) \quad . \quad (2)$$

A single step transition is obtained by:

$$p_{ij} = \Pr(X_1 = j \mid X_0 = i) \quad , \quad (3)$$

and the *n*-step transition satisfies the Chapman-Kolmogorov equation, that for any 0<k<n,

$$p_{ij}^{(n)} = \sum_{r \in S} p_{ir}^{(k)} p_{rj}^{(n-k)} \quad (4)$$

where S is the state space of the chain, in other words, S is the set of possible X_i values. [31]

The first model of this studied, also called Model A, consider that the probability of the component subject to study changing from one state to another follows this Markov property. In other words, the probability of this component changing its condition or state depends only in the current state and not in the history or the time that this component has been in a certain state.

2.4 SYSTEM RELIABILITY

This study mainly focuses on single components, but a bigger prospect would consider systems, instead of only components. It is defined here, that a system is a set of components interacting or working together as a coherent entity or more complex whole.

When studying or analyzing reliability systems, graphical models such as block diagrams and fault trees are commonly used. These two symbolic methods are briefly described below.

2.4.1 RELIABILITY BLOCK DIAGRAMS

Reliability block diagrams are commonly used to represent the effect of component failures on the overall system performance, or in other words, to describe the relationship between the performance of the system and its components [4], [27].

Figure 1 show a simple example of a Reliability Block Diagram (RBD) representing a system that contains only three components: two Hard Drives (connected in parallel) and a Circuit Board.

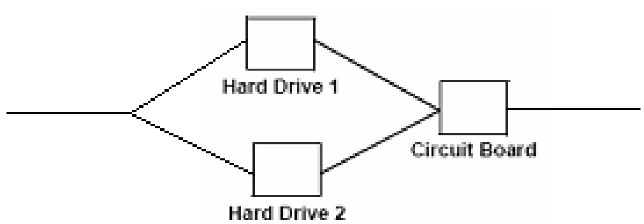


Figure 1. Reliability Block Diagram of a Simple Circuit Board [28].

The block diagram in Figure 2 represents a system comprising for two identical power generators, three identical pumps and a valve.

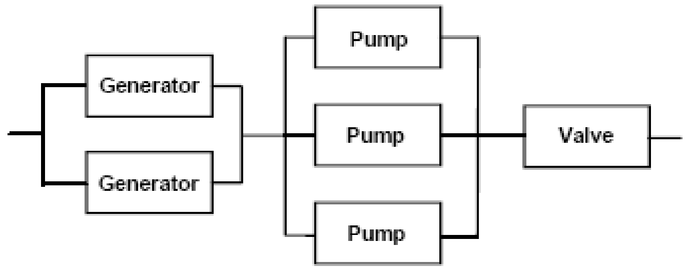
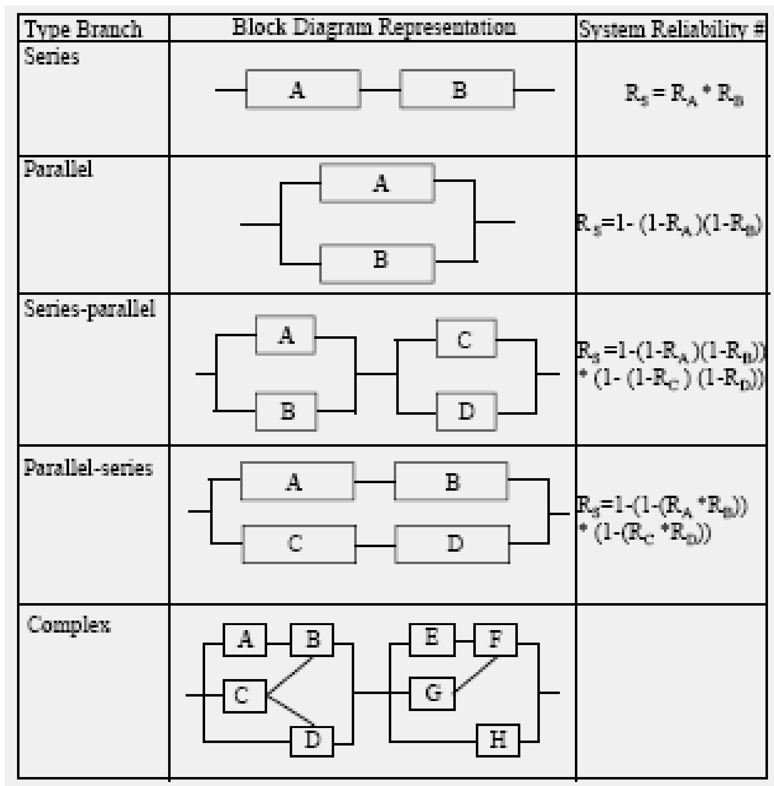


Figure 2. Example of a Reliability Block Diagram of a small system [28].

Table 1 shows some common block diagram classifications, their name and their system reliability calculation.



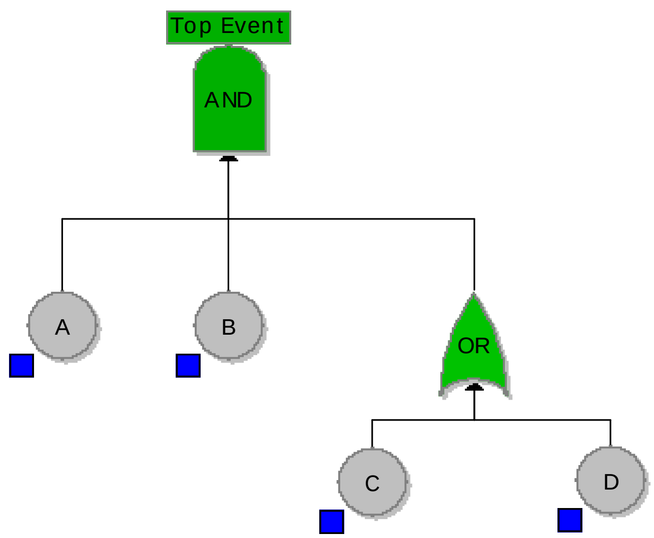
Type Branch	Block Diagram Representation	System Reliability #
Series	A B	$R_S = R_A * R_B$
Parallel	A B	$R_S=1-(1-R_A)(1-R_B)$
Series-parallel	A B C D	$R_S=1-(1-R_A)(1-R_B))$ $*(1-(1-R_C)(1-R_D))$
Parallel-series	A B C D	$R_S=1-(1-(R_A*R_B))$ $*(1-(R_C*R_D))$
Complex	A B C D E F G H	

Table 1: Reliability Block Diagram Classification [29].

2.4.2 FAULT TREE ANALYSIS

A fault tree analysis (FTA) is a system reliability analysis method developed at Bell Laboratories in 1962. This approach is a logical process with a tree form structure, in which a top event is defined as an undesirable event (usually failure). The probabilities or possible ways that the top event may occur are systematically and logically deduced. Once having the tree graphical structure, a combination of Minimal Cut Sets (MCS) and Boolean algebra is then commonly used to quantify and evaluate fault trees. A MCS is a set of components in which the failure of every component is absolutely necessary for the system to fail. For complex systems with multiple failure modes and interactions among components where the use of RBD becomes difficult, it is preferred to use FTA. A fault tree could easily be transformed to a RBD [4],[12],[27].

A simple example of a Fault Tree diagram and its evaluation using MCS and Boolean algebra is shown in Figure 3.



MC1={A,B,C}, MC2 = {A, B, D} then Pr (Top event) = Pr (A·B·C + A·B·D)

Figure 3: Fault Tree Diagram and Boolean Algebra example [18].

The logic of which this example follow is: the Minimal Cut Sets of the system are MC1={A,B,C} and MC2 = {A, B, D}, because in order for the system to fail it is absolutely necessary that there be either a simultaneous failure of components A, B and C or a simultaneous failure of components A, B and D. The probability of the top event (failure) to happen is then calculated by the use of Boolean algebra.

The same logic procedure could be used in maintenance decision practices by evaluating the probability of a system failure, given the actual composition of the system and the failure probabilities of the components of the system.

2.5 APPROPRIATENESS OF MARKOV MODELS FOR MAINTENANCE PROCESSES

Combinatorial models (fault-trees and reliability block diagrams) are available common methods for the prediction of reliability and availability of systems. Even though they can be applied to many particular cases, they don't easily permit different types of dependency, in our case of interest: repairs, spare or multi-state dependency. In this matter, Markov models are preferable due to its capability of including these dependencies and system behaviors. The size of a Markov model may grow exponentially with the number of components in the system, but the changes between states can be represented in a single transition [17]. The first case study (Model A), will consider Markov models for its survey.

The Markov assumption could be appropriate for the case of interest, if some assumptions are made, such as: no change in the failure rate of the component and no change of the operating, maintenance and repair practices. These assumptions and the conditions under which, the Markov assumption is valid will be described deeply in Chapter 3.

2.6 SEMI MARKOV AND MARKOV REWARD MODELS

In maintenance practices, some assumptions, such as, the Markovian property or more specific: the memoryless property or transitions probabilities following negative exponential distributions; may be either mistaken or violated due to changes in maintenance practices or human decisions. Another challenge could also be the possibility of including uncertainty in the transition costs. In such cases, different Markov approaches such as Semi-Markov Models (SMM), or Markov Reward Models (MRM) may be necessary [26]. These two different models that may be necessary for different case scenarios are herewith briefly explained.

When the transition time between states of a component is a random variable that does not follow an exponential distribution, the use of discrete Markov Chains for describing the system is inappropriate, and a Semi-markov processes may be more representative, since the transition probabilities in a Semi-Markov process are functions of the duration of time spent on a state of the system [22].

In general, a semi-Markov process is the process that chooses its next state according to a Markov chain but the transition time spent in this state it is a random amount of time. In general, in a Semi-Markov model the transition rates in a particular state depend on the time already spent in that state, but they do not depend on the path taken to get to the present state. The transition times between states for a component do not necessarily follow an exponential distribution.

An increased or extended Markov chain that includes a reward assignment function is called a Markov reward model (MRM). These models describe the behavior of the system with a Markov chain, where a reward rate is associated with each state such that the time spent in a state contributes to the accumulation of overall reward gained based on the reward rate [23], [24].

Markov Reward Models allow the capture of instantaneous rewards as well as cumulative rewards in a Markov process. Some applications of MRM include: reliability/availability, queuing and performance models where the reward rates associated with each state in this model could be: either 0 (downstate) or 1 (upstate) for reliability models, the number of jobs in a given state for a queuing model, and the computational capacity or a related performance measure for a combined model of performance and reliability. The advantage of a MRM is the ability to calculate steady-state reward rate and expected instantaneous reward rate measures [23], [24], [48].

In general it is assumed that if the process stays in any state i during the time unit, a certain cost r_{ii} is paid. It is also assumed that each time the process transits from state i to state j a cost r_{ij} should be paid.
These costs r_{ii} and r_{ij} are called *rewards*. A reward may also be negative when it characterizes a loss or penalty. Such a reward process associated with its states or/and transitions is called a Markov process with rewards [23], [24], [48].

A general MRM would include a reward rate r_i for each state i of a Markov chain. Then the instantaneous reward rate $Z(t)$ at time t is[48]:

$$Z(t) = r_{X(t)} \quad , \tag{5}$$

and the accumulated reward $Y(t)$ in the interval $(0, t]$ is [48]

$$Y(t) = \int_0^t Z(T)dT \quad . \tag{6}$$

The expected instantaneous reward rate $E[Z(t)]$ at time t and the expected steady-state reward rate can also be calculated as

$$E[Z(t)] = \sum_i r_i \pi_i(t) \quad , \quad \text{and} \tag{7}$$

$$\lim_{t\to\infty} E[Z(T)] = E[Z] = \sum_i r_i \pi_i \quad , \quad \text{respectively [48].} \tag{8}$$

Where $\pi_i(t)$ is the steady state probability that the system is in state i at time t and π_i is the steady state probability that the system is in state i.

Finally, the expected accumulated reward in interval $(0, t]$ *is* [48]

$$E[Y(t)] = \sum_i r_i \int_0^t \pi_i(x)dx = \sum_i r_i L_i(t) \tag{9}$$

In Markov Models, a model state usually represents a system state. In maintenance, the reward rate of the Markov Reward Model assigned to each state, is commonly, a non-negative reward rate. Often this reward rate is either 0 or 1, and indicates the level of performance related to the operational components [23], [24], [48].

2.7 MAINTENANCE SCHEDULING, FORECASTING AND MODELING

With the use of simulation, this research expects to offer additional tools for the maintenance decision making. In the past, many authors have studied and described the importance of preventive maintenance and, in general, maintenance decisions.

Heintzelman [5], discusses the importance of planning a periodic (preventive) maintenance and insists that this should be a continual process. He assures that reviewing the maintenance history periodically to determine if their operations and schedules are still efficient, could result in cost savings. The models proposed in this study offer the capability of reviewing if the PM schedules for a certain component are cost efficient, and by small changes in the simulation models, analyzing new possible scenarios.

Shue and Kuo [6] used grey theory to generate a Preventive Maintenance (PM) forecast model for prediction of PM timing of various machines in the semiconductors industry, with the main objective of minimizing equipment downtimes and optimizing production efficiency. Grey theory is used when there is a mixture of precise information (white) and lack of information (black). In other words, the information is poor or incomplete and it is used as a starting point of investigation and to seek the intrinsic structure of the system [25] . In this study, it is assumed that information is precise, especially in terms of preventive maintenance costs, probability of changing between states, and transition costs.

Lipsett [7] discusses the importance of understanding the flow of information and the cost of wrong decisions in order to make better and informed business decisions. Lipsett also relates the benefits of maintenance modeling, the existing structured analysis techniques and the value of information for maintenance decision making. This research study did not go deeply in the concepts of information management, but it offers a framework for control and distribution of maintenance information. In this work, once the simulation models have been designed, properly understood, and results have been obtained, the information in a model can be integrated into maintenance decision-making, particularly for scheduling.

A preventive maintenance (PM) policy is a guideline that indicates how a PM should be scheduled. Usually, PM policies are classified as Periodic or Sequential. Periodic policies indicate that PM activities are scheduled at integer multiples of a fixed period of time, and Sequential Policies schedule PM activities in a sequence of unequal intervals. A Sequential PM schedule is used when the system or component requires preventive maintenance more often as it ages, which is often a more realistic schedule [34].

Reliability-based maintenance method is a common approach to determine PM sequential intervals by designating a predetermined level of maximum reliability or minimum hazard rate of the system. Once the system reaches this level, a PM is performed. Another common method for determining PM sequential intervals is the optimization method, in which the main goal is either to minimize cost, maximize reliability, or a combination of both [34].

Under these points of view, it would be considered that this thesis research includes PM policies for different models, in terms of indicating PM schedule, and more precise, periodic PM policy, since the simulations results will give the optimal fixed periodic PM interval in order to minimize risk-cost. In some way, the models called Model B and Model C of this study, can be considered reliability-based, since both of this models perform PM periodically, and the reliability of the component for these models decreases constantly. These models will be discussed deeply later in Chapter 3.

Sequential imperfect models may consider changes in the hazard rate and effective age of a component or system once Preventive Maintenance is performed. Such models are called Hazard Rate Adjustment Models and Age Reduction Models, respectively. Hazard Rate Adjustment models usually consider that the hazard rate decreases to zero when the PM is performed, but then increases faster than in previous intervals due to imperfect repairs. The hazard rate function (denoted *h(t)*) is the probability of the first and only failure of a component in the next instant of time given that it has been working satisfactorily up to time t [27], [34]. Age reduction models consider an effective age reduction of the component or system after a PM and its hazard rate as a function of the effective age [34] .

Lin, Zuo and Yam introduced two new sequential imperfect PM models. These new models consider a combination of the hazard rate adjustment models and the age reduction models (also called hybrid model), in which a PM reduces the affective age of the system and increases the slope of the hazard rate function (illustrated in Figure 4). One of their models considers PM intervals as decision variables in order to optimize (minimize) cost. Their other model assumes that a PM is performed once the hazard rate of the component or system reaches a predetermined level in order to determine the PM intervals [34].

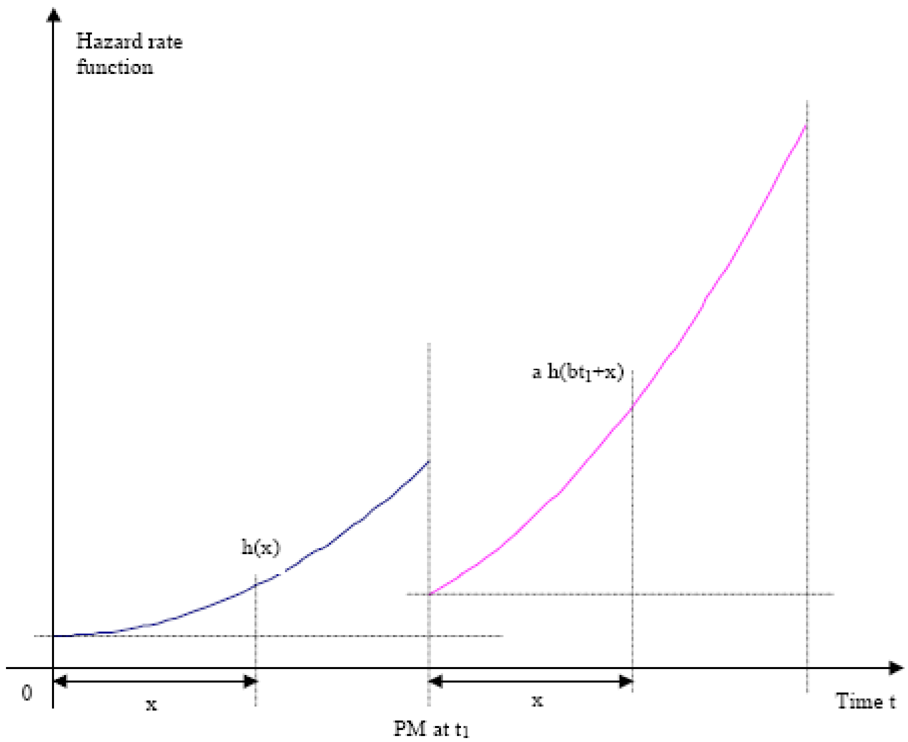


Figure 4. Hazard rate function of the proposed model by Lin et al. [34]

Of the three case studies included in this thesis research, one model (model A) does not consider changes in hazard rate through time, nor after PM, in contrast to the other two models, which consider changes on hazard rate with time and after performing PM. None of these three models considers reduction on effective age after PM, neither hazard rate as a function of the effective age of the component. All the three models consider PM intervals as the decision variables of the cost optimization objective function. In general, it can be said that of the three proposed models, one considers components with constant hazard rate, other with discretely increasing hazard rate, and the third one with linearly increasing hazard rate; this is illustrated in Figure 5.

Lin, Zuo and Yam [35] also categorized the failure modes in two groups: maintainable failure modes and non-maintainable failure modes. The hazard rates of maintainable failure modes can be reduced with PM, but the hazards rates of non-maintainable failure modes can not be modified by PM. They applied these concepts of failure modes to their hybrid model for PM activities. In this research study, all failures modes are contemplated as maintainable.

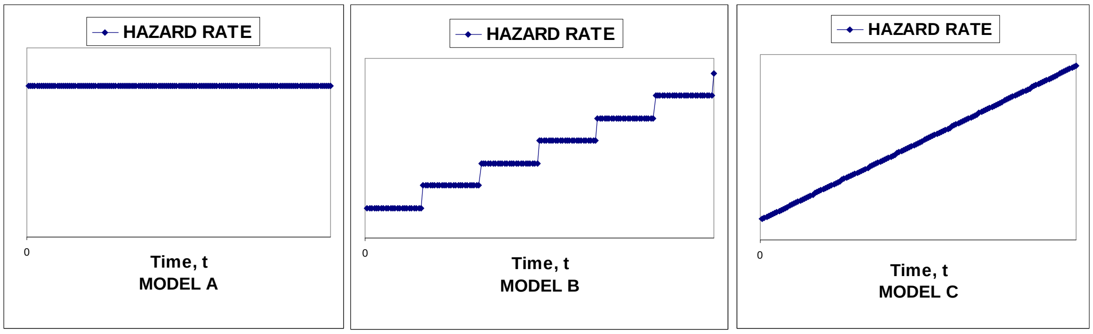


Figure 5. Time-hazard rate graphs for each model.

Caldeira, Taborda and Trigo [8] proposed an algorithm to determine the optimum frequency at which to perform preventive maintenance (PM) in equipment that exhibits linearly increasing hazard rate and constant repair rate, in order to ensure its availability. They also developed another algorithm to optimize maintenance management of a series system based on preventive maintenance over the different system components. This algorithm has the following objective function, conditions and constraints:

The objective function (defined here as a cost function per unit time) is

$$c(A_1,A_2,\ldots,A_n) = \sum_{i-1}^{n}\left[\frac{cmp_i}{\frac{2}{a_i}\,\frac{\mu(1-A_i)}{A_i}} + \frac{cmc_i}{\frac{\mu(1-A_i)}{A_i}}\right] \Rightarrow c(A_1,A_2,\ldots,A_n)$$

$$= \sum_{i-1}^{n}\left[\frac{a_i A_i cmp_i}{2\mu(1-A_i)} + \frac{A_i \times cmc_i}{\mu(1-A_i)}\right]$$

under the following initial conditions,

Initial conditions

Components	a_i	TTR	TTP	PM Cost	CM Cost	τ_i
1	1×10^{-7}	100	10	2000	4000	2000
2	5.7×10^{-7}	50	40	2500	5000	1500
3	7.97×10^{-6}	80	10	1000	2000	250

and subject to

$$\left\{ \begin{array}{l} \prod_{i=1}^{n} A_i \geq A, \\ 0 < A_i < 1, i = 1,2,\ldots,n. \end{array} \right\}$$

where A_i is the availability of component *i*, µ is the constant repair rate, a_i is the coefficient of the linearly-increasing hazard rate function $(h_i(t) = a_i t)$, ô$_i$ is the time units between preventive maintenance tasks on component *i*, cmp_i and cmc_i are the costs of each preventive and corrective maintenance task, respectively, TTR is the Mean Time to Repair (corrective maintenance), and TTP the time of one preventive action.

Their goal was to goal is to calculate the vector $[\tau_{1,}\ \tau_{2,}\ \tau_{3,}\ \ldots, \tau_{n,}]^T$ that would keep a predetermined availability level while minimizing maintenance costs. They show a numerical example and use MATLAB and Excel Solver for its solution.

Barlow and Hunter [9] studied and compared two preventive maintenance policies, one named "Perform preventive maintenance after t_0 hours of continuing operation without failure" or Policy I, and the other defined as "Perform preventive maintenance on the system after it has been operating a total of *t** hours regardless of the number of intervening failures" or Policy II. Elementary renewal theory was used to obtain these optimum policies and included the mathematical criteria for their evaluation, in order to compare them for those cases where both policies are feasible. Renewal Theory is a branch of probability theory describing problems related with the renewal of the elements of some system [46]. The Renewal Process (a generalization of the Poisson process) and the Renewal Equation are its main concepts. Policy I is more useful in maintaining simple equipment where repairs at the time of failure may occur; and Policy II is useful in large, complex systems in which is more common to schedule PM after some specific accumulated hours of operation. Under some circumstances, their formulation can also give minimum cost solutions when replacing times to repair by costs to repair.

Zhang [36] combined the replacement policy T, created by Barlow and Proschan, in which the replacement is performed when the system or component reaches the working age T or a failure occurs, and the policy N proposed by Park [40], in which the system or component has a replacement once N number of failures has been reached by the system or component. The combined bivariate replacement policy (T, N) minimizes the average cost rate (long-run average cost per unit time). Under this (T, N) policy, the system is replaced when the working time T has passed or the number of failures N is reached, whichever occurs first. In contrast to common replacement policies that assume the system as good as new after repair, Zhang's (T, N) policy considers the system after repair not to be as good as new, and due to deterioration, it is also considers that after each repair, the operating time becomes shorter and the repair time becomes longer. Eventually, the working time becomes so short and the repair time becomes so long that the system or component must be replaced. This kind of model, in which operation time becomes shorter and shorter and the repair times longer and longer after repairs are called "geometric process" models.

Zhang, Yam and Zuo [37], introduced a new policy N for a geometric process, in which a PM is incorporated to the model and the objective function is termed by the cost efficiency, which is defined as the long-run average cost per unit of working time, instead of the average cost rate, which is the long-run average cost per unit of time.

Zhang, Yam and Zuo [38], also introduced a replacement policy N, for a multistate system, with a single component, stochastic deterioration, imperfect repair and $k+1$ states, where one state is considered a working state and the other k states are considered failure states. The objective function for this policy is the long-run expected profit per unit time. They use analytical methods to determine the solutions of their optimization problem and the proofs of their theorems.

Noonan and Fain [10] developed a preventive maintenance schedule that optimizes system performance when the assumption of immediate detection or immediate repair of failures is not valid. They showed that, under certain assumptions, an optimal preventive maintenance schedule exists for every possible failure rate. Availability is their objective function and they find analytical optimal solutions for the cases where the probability of detection and repair of component failure is less than or equal than 1 ($p \leq 1$).

Zhang [11] developed a model for PM for a deteriorating system with a bathtub-shaped failure rate function (as a sum of the major failure rate and the minor failure rate), incorporating the concepts of Condition-Based Maintenance (CBM), Random Failure Corrective Repair (CR), and Block Replacement (BR); in order to reduce unexpected failures and maintenance costs. Zhang also created a combined periodic preventive replacement and spare parts provisioning policy, considering the salvage value of the used spare parts. Monte Carlo simulation was used to demonstrate the effectiveness and feasibility of his models.

Reliasoft Corporation [12] mentions the importance of maintenance in the life of a system and how maintenance affects, among others, the reliability, availability, downtime and cost of operation of a system. It also describes how costs it is always a factor in scheduling PM. Reliasoft [12] also describes the basic conditions in which PM intervals schedule should be set and its main objective:

- Condition 1: The component in question has an increasing failure rate.
- Condition 2: The overall cost of the preventive maintenance action must be less than the overall cost of a corrective action.
- Objective: Minimize Cost Per Unit Time.

In the System Analysis Reference publication of Reliasoft Corporation [12] a cost/unit time vs. time graph was presented, as shown in figure 6.

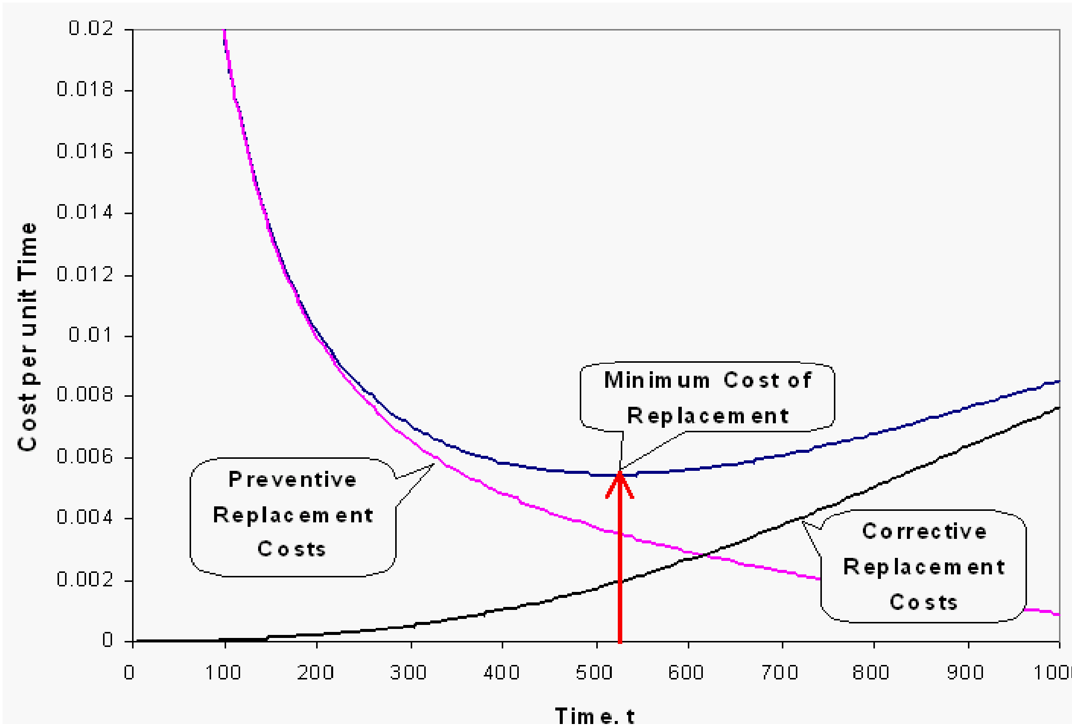


Figure 6: Preventive and Corrective Replacement Costs

This plot shows how, if the PM intervals increase, the corrective costs increase and the preventive costs decrease. Therefore, there must be an optimal PM interval that optimizes the sum of those two costs; in other words, there is a PM schedule that minimizes the total cost associated with maintenance:

$$\min F(t) = Pc + Cc \tag{10}$$

where *Pc* and *Cc* are the costs associated with Preventive and Corrective Maintenance, respectively.

This research proposes simulation models to obtain the optimum frequency at which to perform preventive maintenance (PM), for each of the three different models subject to study, in order to minimize overall cost. Once PM is performed, the components are considered as good as new, for all the models contemplated here.

2.8 RISK AND COST ANALYSIS

Risk has usually two connotations: Qualitative Risk and Quantitative Risk [4]. The Qualitative aspect of risk is the possibility of loss or injury due to an existing hazard and is usually expressed using an ordinal rating such as low, medium or high. The Quantitative aspect of Risk is expressed using numeric data, and it is the estimation of the loss likelihood [4] . For the purpose of this study, only the quantitative aspect of risk will be considered. More specifically, in this work, risk is evaluated as the cost of a certain event times the probability of this event to occur.

In Li and Zuo [33], a k-out-of-n events risk analysis is conducted. They proposed an algorithm, by enumerating all combinations of n events k at a time, to calculate the estimate risk (average total cost of k events out of n events occurring multiplied by the probability of k events out of n events occurring) and concluded that the estimate risk can be adequate for decision making since it is pretty close to the accurate risk.

An approach to improve the cost-effectiveness of a maintenance program is proposed by Kunttu et al [30], by focusing on the estimation of economic costs of faults. The novelty of this approach is its simplicity and applicability to real cases without the use of sophisticated software tools. They explain the Reliability Centered Maintenance methods to develop maintenance programs usually have three common and main stages:

1. Definition and measurement of the maintenance objectives,
2. Identification of the most critical and significant components (fault modes and failure mechanisms), and
3. Propose pertinent maintenance tasks for those critical components identified

They classified maintenance strategies in four classes:

- Condition-based maintenance

- Use-based maintenance
- Failure-based maintenance also called corrective maintenance (CM), and
- Modification

In their approach, once defined the applicable PM tasks for the identified fault modes in stage two, the cost effectiveness of each of these tasks is estimated by comparing their annual cost.

In order to calculate the annual cost incurred by each maintenance task, Kunttu et al [30] states that it is necessary to know their detailed description and frequency. In cases in which the maintenance task has been implemented in the past and therefore operational experience has been obtained, historical, statistical and expert judgment data can be used to get a good estimation of the annual cost of the maintenance strategy. In the cases in which there is no existing operational experience, the estimation numbers tend to be less accurate, but a rough number would offer the right order of magnitude, good enough for the cost-effectiveness comparison.

The cost factors for each maintenance task vary; but in general, the following cost factors should be considered: execution of the planned task, maintenance overhead costs, immediate repairs, monitoring, training, early failure repairs, income losses due to production losses, labor, spare parts, etc. These factors mentioned by Kunttu et al [30], should be considered in mind when the transition costs explained in Chapter 3 of this thesis work are estimated.

This research study relies in the possibility of estimating good cost numbers, close to reality, in order to find the optimal PM intervals for each model. Historical, statistical and expert judgment data are the key to obtain reliable transitions costs considered in this study. This transition costs will be explained in Chapter 3.

2.9 MAINTENANCE OPTIMIZATION

According to Dekker [1], maintenance optimization models, in general, cover four aspects:

1. A description of a technical system, its function and its importance.
2. A modeling of the deterioration of the system in time and possible consequences for the system
3. A description of the available information about the system and the actions open to management and
4. An objective function and an optimization technique which helps in finding the best balance.

This study follows these 4 aspects for 3 different models, but focus in the last aspect mentioned by Dekker [1]. In these cases, the objective function is to minimize cost, and simulation is the technique used for the optimization process.

2.9.1 BAYESIAN THEORY IN MAINTENANCE OPTIMIZATION

In general, Bayesian Theory uses Bayes' theorem as a rule to infer or update the degree of belief in light of new information. Bayes' Theorem follows from the concept of conditional probability and relates the conditional and marginal probabilities of stochastic events *A* and *B* as follows [4]:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{11}$$

Bayesian Decision Theory provides theoretical foundation for inference under uncertainty about the state of world, based on a probability distribution over a set of possible states.

Procaccia et al [21] presented an application of Bayesian and statistical decision theories on the maintenance of a diesel generator component. Their main idea is to match the expert judgments with the available operating data or feedback through a Bayesian approach and combine it with the risk and cost of failure. Procaccia et al [21] stated that Reliability centered maintenance has some limitations, and for this reason, they proposed a different approach, using Bayesian statistical theory, for a maintenance optimization problem.

They consider the use of expert judgment, arguing that field and historical data can offer information from which it is possible to calculate failure rates or probabilities of starting on demand, but not always show performance degradation without failure and it is important to consider the risk from possible aggravation of the degradation.

In this way, they obtain a posterior probability density by combining historic field data (prior probability density) with estimates from experts (likelihood). This probability density is richer than the prior probability density one. For questioning the experts, several different perspectives (manufacturers, staff, operators, economists and maintenance departments among others) are considered; each of these perspectives could be represented by several experts. The first step for the experts is to consider the different types of possible degradations; then they have to answer whether or not the system would perform certain missions, i.e., if the component is going to function for a specified duration or the probability of failure when required for its mission. The answers of these questions have to be binary (yes and no, 1 and 0 respectively). From here, a model expertise is created by defining a discrete distribution on X=[1,0], with

$p = Pr[X=x_1=1]$ and $1\text{-}p = Pr[X=x_2=1]$

with p being the probability that the component would perform its mission satisfactorily.

Since the opinions of the experts may differ, Bayesian modeling is applied in order to deal with this uncertainty. When field data is limited, expert opinion can be treated as observations and used in statistical inference.

Applying Bayes's theorem under the assumption that the experts are independent, Procaccia et al [21] obtained the following prior probability density distribution:

$$f(p/E)_{r_i}^{t_i} = \frac{p^a \cdot (1-p)^b}{\int_0^1 p^a \cdot (1-p)^b \, dp} = f_0(p)_{r_i}^{t_i} \qquad , \qquad (12)$$

the posterior probability density distribution:

$$f(p/\alpha,\beta)_{r_i}^{t_i} = \frac{(a+\alpha+b+\beta+1)!}{(a+\alpha)!(b+\beta)!} p^{a+\alpha}(1-p)^{b+\beta} \qquad , \qquad (13)$$

and a Bayesian estimator of this distribution is its expectation:

$$[\hat{p}]_{r_i}^{t_i} = E[f(p/\alpha,\beta)]_{r_i}^{t_i} = \frac{a+\alpha+1}{a+\alpha+b+\beta+2} \qquad , \qquad (14)$$

where a is the number of "yes" answers and b the number of "no" answers and α and β are the observed cases of proper operation and failure, respectively.

Then, a decision tree is constructed with the probability calculations for each type of degradation. This decision tree could be used to optimize the replacement time of the component studied in terms of minimizing risk (lowest expectation of cost), by estimating and including in the tree the loss functions or consequence costs that might include down time, repairs and replacement costs among other costs that may apply [21].

This study considers Procaccia's application of high value and as a good example of the use of expert judgment and historical data, when obtaining the transition probabilities and the transition costs between states of the three models studied in this thesis.

Duda [18] gives a brief outline for Bayesian Decision Analysis applied to Risk analysis, where the conditional risk is defined as:

$$R(\alpha_i|X) = \sum_{j=1}^{c} \lambda(\alpha_i|w_j)P(w_j|X) \tag{15}$$

and the risk is minimum when:

$$R* = \min\{R(\alpha_i|X), i = 1,...,a\} \tag{16}$$

CHAPTER 3

A RISK MODEL FOR A DISCRETE MULTI-STATE REPAIRABLE COMPONENT

3.1 INTRODUCTION

In this Chapter, a Markov Model is proposed to describe risk for a Multi-State Repairable Component. Typically, maintenance analyses for a repairable component consider only two states of the component: Good and Failed. In real general practice, the component may have different levels of efficiency and performance. In some cases, such as low efficiency or partial duty, reliability may be compromised but performance may still be acceptable for a reduced duration or at a reduced level of performance. Such repairable components with several levels of efficiency or performance (also called states) are considered in this study. This chapter focuses only in this Multi-State Repairable Risk Markov Model, which will be referred as Model A in future chapters to simplify and differentiate from the other two models that will be introduced and study in later chapters. All the components described in all the models of this study are considered to be repairable.

3.2 PROPOSED STATES OF A SINGLE REPAIRABLE COMPONENT

For a single repairable component, the number of states may vary according to the operation and maintenance practices associated to the component, and its different levels of performance. Lipsett, Gallardo and Zuo [26] proposed a model with eight possible states for a single repairable component. As shown in Figure 7, the eight possible states are: spare, standby, derated duty, full normal duty, minor fault, major fault, failed, and in repair.

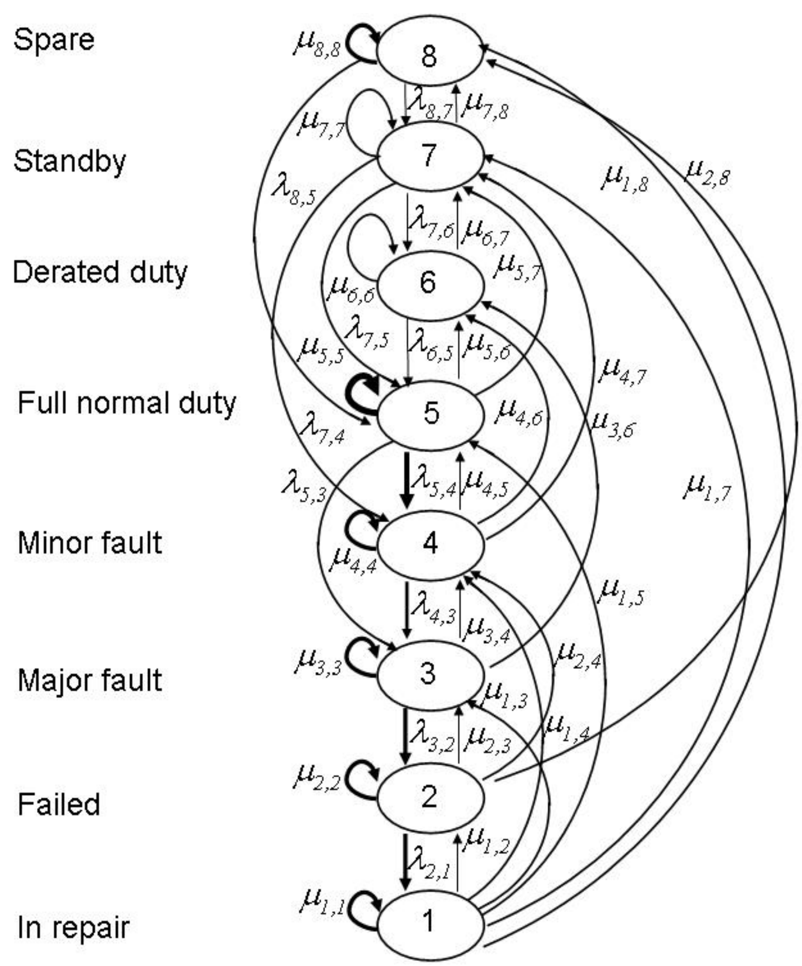


Figure 7: Multi-State Discrete Reliability Model for a Repairable Component [26]

Where μ_{ij} and λ_{ij} represent transition rates. Transition rates are commonly used instead of transition probabilities for continuous-time Markov chains. The relationship between transition rates and transition probabilities is described in Kuo [27] as the limit:

$$\lambda_{ij} = \lim_{\Delta t \to 0} \frac{p_{ij}}{\Delta t} = \left. \frac{dp_{ij}(x)}{dx} \right|_{x=0} \tag{17}$$

In this model, transition probabilities of changing from state *i* to state *j* are generally designated as P_{ij} , and more specifically μ_{ij} is the transition rate of changing to a different state *j* if the transition increases reliability of the component, λ_{ij} is the transition rate that makes the component less reliable when going from state *i* to state *j* and μ_{ii} represents the transition of remaining in the current state *i*.

Lipsett, Gallardo and Zuo described that there are only a sub set of the transitions that have nonzero probabilities due to the nature of how the component is damaged or repaired. Theoretically, any transition may occur between any two states; but in reality, for the type of component described by Lipsett et al [26], only some transitions, which describe the operating and maintenance practices, are possible.

In practice, a large number of states will be difficult to track and determine the transition probabilities. From this eight-state model, a four-state model is adapted and considered in this study (Figure 8).

The idea is that, by making a generalization and combination of one state for every two states of the eight-state model, the four-state model will simplify the computation and tracking of every transition; but at the same time, it will include several different states that would still represent and describe the operating and maintenance practices. The four states considered here are: Failed (which includes states of “failed and “in repair”), Fault (enclosing the major and minor faults concepts), On Duty (comprising the “derated duty” and “full normal duty” states) and Spare (combining the concepts of “spare” and “standby”).

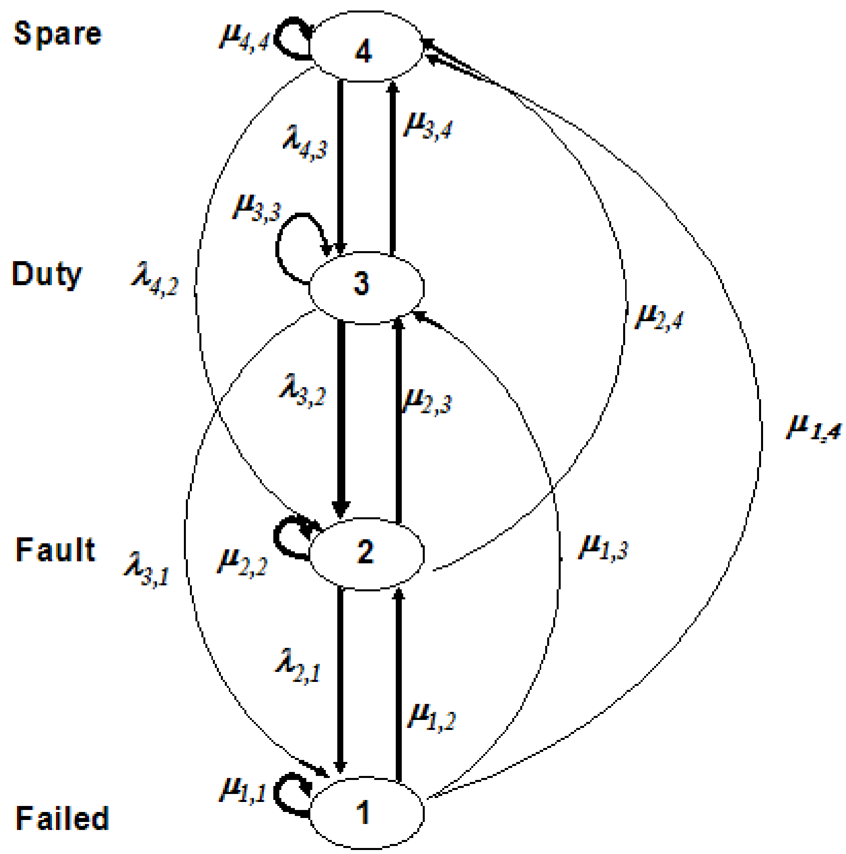


Figure 8: Proposed Multi-State Discrete Reliability Model for a Repairable Component.

A spare is a component that is not operating at the moment but is available for operation. In general, the reliability of the component is high, it might have had some of its reliability consumed in a previous state; but it may have a small probability of consuming its reliability while in this state.

An on Duty component is operating and accumulating damage, thus consuming reliability. The consumption of reliability depends on the level of service and demand of the component. Generally, it is assumed that the longer the time of operation, the more the reliability consumption.

A component in a fault state has become damaged. This happens when an on duty component has consumed its reliability and may not be able to fully perform its intended functions for the required duration of time. Depending on the level of the damage the component may be operating and integrated to the system if the system can still operate with the damaged component. For the purpose of study of the four-state model, when the component is in a fault state, there are not major production losses since it would be considered that the component can still operate. The duty and fault states are considered operating states because these are the only states during which a component might be operating.

A failed component can not perform at any level its intended function. It may be in the process of having some level of reliability restored (going to Spare state).

Due to production processes and maintenance activities, transitions between these states may occur. The probabilities of going from one state to another are named transition probabilities. The sojourn time in a state is considered to be negative exponential distributions that don't change over time in order to meet the Markovian property; this is a reasonable assumption if the maintenance and operating practices are mature and unchanging.

These transition probabilities define and help us to understand the reliability of the process, the design of the components, and the effectiveness of operating and maintenance practices.

In some ideal cases, transitions between any two states might be possible, but in reality, only some transitions between states are likely.
Table 2 shows these possible transition states considered for the proposed Multi-State Discrete Reliability Model of a Repairable Component.

TRANSITION	DESCRIPTION
SPARE TO SPARE	Component does not change state, and there is no consumption of reliability over time.
SPARE TO DUTY	Component goes into operating service in the system.
SPARE TO FAULT	Component goes into service, with an incipient failure, because reliability has been consumed over time in the spare state.
SPARE TO FAIL	Sudden catastrophic failure of unit right after installation, bad unit or bad repair prior to storing as a spare.
DUTY TO SPARE	Change in operating conditions.
DUTY TO DUTY	No change in operating conditions, component does not change state
DUTY TO FAULT	Incipient failure and degradation in performance.
DUTY TO FAIL	Unexpected and sudden severe failure that causes unacceptable performance of the component. There is a small probability of this to happen since it is more frequent the transition of the component to the fault state before getting to Failed state.
FAULT TO SPARE	Questionable component goes into spare, as a precaution.
FAULT TO DUTY	Reliability is restored without having a repair, either through field service (condition-based), misdiagnosis of fault and reclassification, or spontaneous self-repair. It is assumed that there is no change in the severity of duty.
FAULT TO FAULT	Component remains in service even though not performing adequately, affecting system performance.
FAULT TO FAIL	Loss of reliability and function to the point of unacceptable performance.
FAIL TO SPARE	The component has been repaired and put into spares inventory, or there is poor maintenance practice; either a misdiagnosis of a fault condition, or putting a failed component into spares inventory.
FAIL TO DUTY	Component goes back into service after is repaired.
FAIL TO FAULT	Component has only part of its reliability restored without being removed from the operating system, either through a partial servicing repair or a spontaneous self-correction of an intermittent fault.
FAIL TO FAIL	Component has not changed or is in repair, and system reliability has not changed.

Table 2. Transitions between states.

3.3 DEFINITION OF TRANSITION COSTS

Similar to the transition probabilities between states there are also transition costs associated with going from one state to another. In practice, these transition costs are easier to calculate than the transition probabilities if the system and process is well known and understood. The transition cost of changing from state *i* to state *j* is denoted C_{ij} . The risk associated with the transition between states is the probability of this transition or change occurrence times the cost associated to this change of states. Estimates of the transition costs depend on different factors.

Spare to Spare. There is a very small cost associated with this transition since there is no a change in state and there are not costs associated with handling or shipping; the only cost associated is related with storage. There is no change in reliability within a state.

Spare to Duty and Spare to Fault. From the maintenance cost point of view, the costs related with these transitions are mainly handling and installation of the component into the system. This cost does not include the opportunity cost of lost production during the change, if the system has to be down for the change-out.

Spare to Fail. There can be a large cost associated with this transition mainly due to production losses. This transition cost is where the opportunity cost of lost production is reflected.

Duty to Spare. This transition has costs related to handling, relocation of the component, and storage.

Duty to Duty. There is a small cost incurred in this transition, since there is no change in state, and the cost is only related to the component working and performing its intended functions.

Duty to Fault. There is a small cost associated with this transition, since the component remains operating and performing its function, this small cost is only related to operation. If the fault condition is a big decrease in functional performance, there could be a high opportunity cost of lost production from wasted product or high cost of consequential damage to other components, the component will be considered to transition to the Fail state in this 4 state model, and the costs associated with this transition would be reflected in the Duty to Fail transition cost.

Duty to Fail. There can be a large cost related to this transition due to production losses. The MTTR (Mean time to repair) costs, also called lost production, is counted only in this transition and not in other transitions.

Fault to Spare. This transition has costs associated with handling, relocation of the component, and restoration of reliability (repair). Storage cost it is only incurred when passing from spare to spare.

Fault to Duty. Costs incurred in this transition are for restoration of reliability and reparations.

Fault to Fault. This transition has a cost associated for remaining in a fault state, due to compromised process performance.

Fault to Fail. Like any other transition cost that goes to Fail state, the cost is high due to production losses.

Fail to Spare, Duty or Fault: These transitions have relatively high costs to change from Fail to any operation state, due to the cost of restoring reliability, repairs, and relocation of the component into the system.

Fail to Fail. This is the only non changing state transition that implies a high cost, due to the possibility of continuous production losses or cost of poor repairs that don't fix the problem.

3.4 ANALYTICAL RISK MODEL

The Initial Probability Vector $P^{(0)}$ that represents the probability of being in state *i* as the initial state, for a component with *n* possible states is defined as:

$$P^{(0)} = \left(P_1^{(0)}, P_2^{(0)}, P_3^{(0)}, P_4^{(0)}, P_5^{(0)}, P_6^{(0)}, P_7^{(0)} \ldots . P_n^{(0)}\right), \qquad (18)$$

where $\sum_{i=1}^{n} P_i^{(0)} = 1$

and $P_i^{(0)}$ is the *i*th element of the Initial Probability Vector $P^{(0)}$. The risk after one transition step is calculated using the following formulation:

$$Risk = \sum_{i=1}^{n} P_i^{(0)} \sum_{j=1}^{n} C_{ij} P_{ij} \qquad (19)$$

If the current state *i* is known, then $P^{(0)}$ is a vector with only zeros and one non-zero element, the *i*th element that is equal to 1, and the risk equation can be simplified as:

$$Risk_i = \sum_{j=1}^{n} C_{ij} P_{ij} \qquad (20)$$

where $Risk_i$ represents the one step risk associated of changing from the current state *i* to any state, including staying in the same state.

Analogously, by using Markov chains and considering the interactions, the Risk after *k* transition steps can be calculated, as follows.

Let the Probability Matrix P be the matrix that shows the transition probabilities between all the n possible states:

$$P = \begin{Bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} & P_{16} & \ldots & P_{1n} \\ P_{21} & P_{22} & P_{23} & P_{24} & P_{25} & P_{26} & \ldots & P_{2n} \\ P_{31} & P_{32} & P_{33} & P_{34} & P_{35} & P_{36} & \ldots & P_{3n} \\ P_{41} & P_{42} & P_{43} & P_{44} & P_{45} & P_{46} & \ldots & P_{4n} \\ P_{51} & P_{52} & P_{53} & P_{54} & P_{55} & P_{56} & \ldots & P_{5n} \\ P_{61} & P_{62} & P_{63} & P_{64} & P_{65} & P_{66} & \ldots & P_{6n} \\ \ldots & \ldots & \ldots & \ldots & \ldots & .. & \ldots & \ldots \\ P_{n1} & P_{n2} & P_{n3} & P_{n4} & P_{n5} & P_{n6} & \ldots & P_{nn} \end{Bmatrix} \tag{21}$$

where $\sum_{j=1}^{n} P_{ij} = 1, \forall i$.

In the context of transition rates, if n is the more reliable state and i is the less reliable one, then we define the Transition Rate matrix T:

$$T = \begin{Bmatrix} \mu_{11} & \mu_{12} & \mu_{13} & \mu_{14} & \mu_{15} & \ldots & \mu_{1(n-1)} & \mu_{1n} \\ \lambda_{21} & \mu_{22} & \mu_{23} & \mu_{24} & \mu_{25} & \ldots & \mu_{2(n-1)} & \mu_{2n} \\ \lambda_{31} & \lambda_{32} & \mu_{33} & \mu_{34} & \mu_{35} & \ldots & \mu_{3(n-1)} & \mu_{3n} \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & \mu_{44} & \mu_{45} & \ldots & \mu_{4(n-1)} & \mu_{4n} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ & & & & & \ldots & & \\ \lambda_{(n-1)1} & \lambda_{(n-1)2} & \lambda_{(n-1)3} & \lambda_{(n-1)4} & \lambda_{(n-1)5} & \ldots & \mu_{(n-1)(n-1)} & \mu_{(n-1)n} \\ \lambda_{n1} & \lambda_{n2} & \lambda_{n3} & \lambda_{n4} & \lambda_{n5} & \ldots & \lambda_{n(n-1)} & \mu_{nn} \end{Bmatrix} \tag{22}$$

Note that the matrix diagonal is formed by μ_{ii} values, representing no changes in state; the upper diagonal of the matrix has values μ_{ij} representing transition rates between states with increasing reliability; and the lower diagonal has only λ_{ij} representing decreasing reliability when changing states.

The Cost Matrix *C*, which includes all the cost values associated with the transitions between state *i* and state *j*:

$$C = \left\{ \begin{matrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & \cdots & \cdots & C_{1n} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & \cdots & \cdots & C_{2n} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & \cdots & \cdots & C_{3n} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & \cdots & \cdots & C_{4n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & .. \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ C_{n1} & C_{n2} & C_{n3} & C_{n4} & C_{n5} & \cdots & \cdots & C_{nn} \end{matrix} \right\} \qquad (23)$$

The Risk of changing from state *i* to state *j* is defined as the product of the probability of moving from one state to another times the cost associated to this transition between states: $R_{ij} = P_{ij}\ C_{ij}$. Then, the Risk Transition Matrix R or simply Risk Matrix R can be introduced:

$$R = \left\{ \begin{matrix} P_{11}C_{11} & P_{12}C_{12} & P_{13}C_{13} & P_{14}C_{14} & . & . & . & P_{1n}C_{1n} \\ P_{21}C_{21} & P_{22}C_{22} & P_{23}C_{23} & P_{24}C_{24} & . & . & . & P_{2n}C_{2n} \\ P_{31}C_{31} & P_{32}C_{32} & P_{33}C_{33} & P_{34}C_{34} & . & . & . & P_{3n}C_{3n} \\ P_{41}C_{41} & P_{42}C_{42} & P_{43}C_{43} & P_{44}C_{44} & . & . & . & P_{4n}C_{4n} \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ P_{n1}C_{n1} & P_{n2}C_{n2} & P_{n3}C_{n3} & P_{n4}C_{n4} & . & . & . & P_{nn}C_{nn} \end{matrix} \right\} \qquad (24)$$

which can be expressed more simply as:

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & \cdots & \cdots & R_{1n} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & \cdots & \cdots & R_{2n} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & \cdots & \cdots & R_{3n} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & \cdots & \cdots & R_{4n} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & \cdots & \cdots & R_{5n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ R_{n1} & R_{n2} & R_{n3} & R_{n4} & R_{n5} & \cdots & \cdots & R_{nn} \end{pmatrix} .. \qquad (25)$$

The risk of a process changing from state i to state j after k steps is represented as $R_{ij}^{(k)}$.

Then, for a Markov process with a constant set of one-step transition probabilities, the transition Risk matrix after k steps $R^{(k)}$ is equal to the Risk Matrix to the power of k:

$$R^{(k)} = R^k \tag{26}$$

The risk of being at state i after k steps ($R_i^{(k)}$) is called the Absolute or Total Risk; and for every k steps there is an stochastic vector formed by all the Total Risks of this step:

$$R^{(k)} = \left(R_1^{(k)}, R_2^{(k)}, R_3^{(k)}, R_4^{(k)}, R_5^{(k)}, R_6^{(k)}, R_7^{(k)} ....R_n^{(k)}\right), \tag{27}$$

where $R_i^{(k)}$ is the Risk of being at state i after k steps and $R^{(k)}$ is also known as the Risk Distribution after k steps. Following a Markov process, with a Risk Transition Matrix R its is obtained,

$$\begin{aligned} R^{(1)} &= P^{(0)}R \\ R^{(2)} &= R^{(1)}R = P^{(0)}R^2 \\ R^{(k)} &= R^{(k-1)}R = P^{(0)}R^k \end{aligned} \tag{28}$$

After k steps the risk is:

$$R^{(k)} = \left(P_1^{(0)}, P_2^{(0)}, P_3^{(0)}, P_4^{(0)}, P_5^{(0)}, P_6^{(0)} ...P_n^{(0)}\right) \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} & ... & R_{18} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} & ... & R_{27} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} & ... & R_{38} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} & ... & R_{48} \\ ... & ... & ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & ... & ... & ... \\ R_{n1} & R_{n2} & R_{n3} & R_{n4} & R_{n5} & Rn_{86} & & R_{n8} \end{pmatrix}^k . \tag{29}$$

or

$$R^{(k)} = P^{(0)} R^k = \left(R_1^{(k)}, R_2^{(k)}, R_3^{(k)}, R_4^{(k)}, R_5^{(k)}, R_6^{(k)}, R_7^{(k)} ... R_n^{(k)} \right). \qquad (30)$$

The transformation vector V_1 is defined as a column vector with *n* values all equal to one.

$$V_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ . \\ . \\ 1 \end{pmatrix}. \qquad (31)$$

The Risk after *k* steps can be calculated as:

$$Risk = \sum_{i=1}^{n} R_i^{(K)} = R^{(k)} V_1 = P^{(0)} R^k V_1, \qquad (32)$$

which in its expanded form is:

$$Risk = \left(R_1^{(k)}, R_2^{(k)}, R_3^{(k)}, R_4^{(k)}, R_5^{(k)}, R_6^{(k)}, R_7^{(k)} ... R_n^{(k)} \right) \times \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ . \\ . \\ 1 \end{pmatrix}, \qquad (33)$$

or

$$Risk = R_1^{(k)} + R_2^{(k)} + R_3^{(k)} + R_4^{(k)} + R_5^{(k)} + R_6^{(k)} + ... + R_n^{(k)}. \qquad (34)$$

This equation includes the One step case, so the equation (19) is equivalent to equation (32) when k =1:

$$Risk = \sum_{i=1}^{n} P_i^{(0)} \sum_{j=1}^{n} C_{ij} P_{ij} = P^{(0)} R V_1 \tag{35}$$

In general, the transition probabilities between states can be found empirically from historical data, including as many observations as possible. Once a component is determined to be in a certain state, then the state at the next step is computed and the transition recorded. The current state is determined by expert judgment. Alternatively, times between state transitions are tracked and then normalized to yield the transition probabilities.

In the case of spare state, the state is pretty obvious, but in the operational states duty and fault, a major inspection (condition monitoring) of the component and its functions may be required to classify the state. The observations have to be made periodically at a fixed period of time, which represents the "steps" in the markovian model. This period of time (step length) has to be as short as possible, in order to register all the transitions between states without missing any possible transition between the observations, and a low error in the estimated time at which a transition occurs, but also it has to be long enough so the observation (inspection of the component) can be realistically carried out in a regular work environment by the maintenance department. Thus, the definition of the step length becomes a critical aspect of the model implementation.

One way to define the step length is by the determining the shortest expected period of time between a state change, which can be estimated through experience and expertise judgment or through an exhaustive initial period of observation, in which the shortest period of transition time is recorded. Then the step length should be shorter than this value to be conservative.

After the "*N*" number of desired observations are registered the empirical probabilities are calculated as follows.

Let a component go through the state *i* "*m*" number of times, and "*l*" out of those "*m*" times the component changed from state *i* to state *j*. The probability of changing from state *i* to state *j* is:

$$P_{ij} = \frac{l}{m} \quad , \tag{36}$$

including the special case of remaining in the same state *i* with transition probability P_{ii}.

The same procedure would be applied for all the possible component states, and of course the equality $\sum_{j=1}^{n} P_{ij} = 1, \forall i$ would still be valid.

The larger the "*N*" number of observations the more accurate the calculation of the transition probability and the smaller the error "*e*" associated to the probability calculations. In the limit, when $N \to \infty$ the error $e = 0$. In other words, the longer the historical data is recorded the more accurate the model would be. Larger samples would minimize the impact of accidental or unlikely events. The collection of data becomes a continuous process that would continuously, while the process of recollection of new data lasts, update the transition probabilities P_{ij} to improve the accuracy of the model.

3.5 SIMULATIONS

A computer simulation is a technique used to model and imitate a real-life or hypothetical situation, using a computer program, based on a conceptual and mathematical understanding of the behavior of a system. With this technique a system and its internal processes can be studied. Predictions about the behavior, operation, and outputs of the system can be made by changing input variables. Computer simulation has become a useful tool for modeling many natural systems in physics, chemistry, and biology, in human systems such as economics and social science, and in engineering to gain insight into the design operation of technological systems. [42]

A computer simulation software package is used in this work to represent the proposed multi-state repairable component risk model. A computer-based simulation of the problem provides flexibility for analyzing a range of possible solutions and adapting for new problems. Once the model can be simulated by an adequate algorithm, it is fairly easy to change the transition probabilities, transition costs, number of steps, or any other variable in the problem to get new results or to perform a sensitivity analysis of the system variables.

In this work, the tool selected to simulate and model the problem is ReliaSoft's RENO stochastic event simulation package [47].

3.6 MARKOV PROCESS SIMULATION

A simple model was created in RENO to simulate the Markov process of the four state model of this study. In this model, the transition probability matrix that was arbitrarily considered to test the modeling and approach the flow chart was the following:

$$P = \begin{Bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.40 & 0.30 & 0.15 & 0.15 \\ 0.10 & 0.40 & 0.40 & 0.10 \\ 0 & 0.1 & 0.3 & 0.6 \end{Bmatrix}$$

The limiting probabilities for these four states, given the transition probabilities above, were found to be 0.1776, 0.2632, 0.2796 and 0.2796 for states 1, 2, 3 and 4, respectively, since:

$$P^{(k)} = \begin{Bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.40 & 0.30 & 0.15 & 0.15 \\ 0.10 & 0.40 & 0.40 & 0.10 \\ 0 & 0.1 & 0.3 & 0.6 \end{Bmatrix}^{(k)} = \begin{Bmatrix} 0.1776 & 0.2632 & 0.2796 & 0.2796 \\ 0.1776 & 0.2632 & 0.2796 & 0.2796 \\ 0.1776 & 0.2632 & 0.2796 & 0.2796 \\ 0.1776 & 0.2632 & 0.2796 & 0.2796 \end{Bmatrix}$$

as $k \rightarrow \infty$

In other words, for this specific transition matrix, the component would be 17.76% of the time in “fail” (state 1), 26.32% of the time in “fault”, 27.96% in “duty” and 27.96% of the time in “spare”. This case was then run with the RENO simulation, and the same results were obtained when the number of steps (k) and the number of simulations was sufficiently large. Very good numbers (close convergence between limiting probabilities and values obtained with RENO simulation) were reached with combinations of 5,000 steps and 5,000 simulations; and 10,000 steps and 10,000 simulations, as shown in Table 3. Very good numbers were also obtained for a combination of 1,000 steps and 1,000 simulations. The flowchart and some of the results obtained with RENO are showed in Figure 9.

In this work, the term simulation is used to describe a single pass through a flowchart or process. In the example of 5,000 steps and 5,000 simulations, a complete pass through the flowchart (simulation) was only completed when 5,000 steps were reached. This process was done 5,000 times in order to complete the 5,000 simulations. More than one simulation is done in order to represent the randomness of the process appropriately and minimize the effects of outliers. For this reason, the larger the number of the simulations, the better results that can be obtained. An average of the 5,000 set of results is calculated. It can be noticed that the number of steps has to be sufficiently large in order to imitate the infinite number of steps ($k \rightarrow \infty$), in other words, the larger the number of steps, the closer the numbers of the simulation will be to the limiting probabilities.

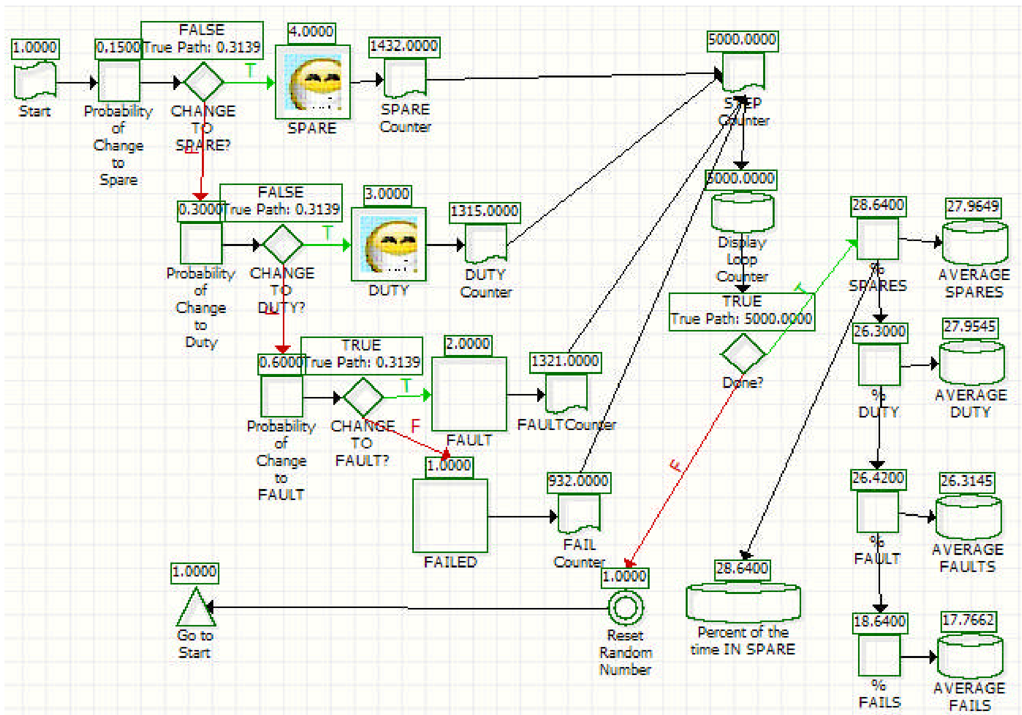


Figure 9. RENO simulation flowchart of the four state Markov process for 5,000 steps and 5,000 simulations.

Figure 10 shows a general Block diagram of this Markov simulation process.

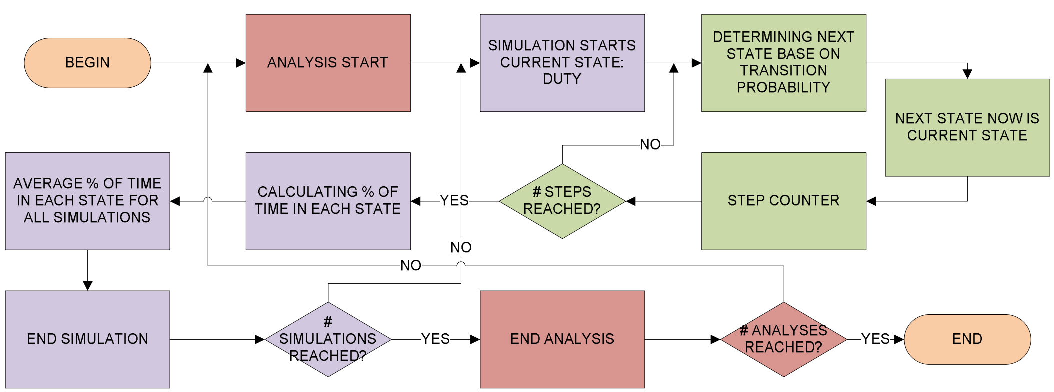


Figure 10 General Block Diagram of the Markov simulation process.

Once the flowchart was created in RENO many different analyses were run to sense the best combination of number of steps and simulations necessary to obtain acceptable results. Table 3 shows these different runs and their results. It can be confirmed that the larger the number of simulations and steps, the more accurate the numbers obtained, in other words, the obtained values approach to the limiting probabilities expected for the discrete Markov model.

# STEPS	5,000	10	100	1,000	100	1,000	5,000	10,000	
# SIMULATIONS	10	5,000	1,000	100	5,000	1,000	5,000	10,000	
STATE	**% OF TIME BEING IN A CERTAIN STATE, OBTAINED BY SIMULATION**								**GOAL**
SPARE	27.75	24.73	27.46	27.73	27.73	27.98	27.9649	27.9614	27.96
VS GOAL	0.21	3.23	0.5	0.23	0.23	0.02	0.0049	0.0014	
DUTY	17.5	28.3	27.86	27.83	27.94	27.88	27.9545	27.9592	27.96
VS GOAL	10.46	0.34	0.1	0.13	0.02	0.08	0.0055	0.0008	
FAULT	26.7	28.73	26.84	26.61	26.5	26.3	26.3145	26.315	26.32
VS GOAL	0.38	2.41	0.52	0.29	0.18	0.02	0.0055	0.005	
FAIL	18.03	18.22	17.84	17.82	17.84	17.83	17.7662	17.7644	17.76
VS GOAL	0.27	0.46	0.08	0.06	0.08	0.07	0.0062	0.0044	
SUM DIFFERENCES	**11.32**	**6.44**	**1.2**	**0.71**	**0.51**	**0.19**	**0.0221**	**0.0116**	

Table 3. Simulation results for the four state Markov process.

Among the different analyses tested with an Intel Pentium 4 CPU 2.40GHz, the best results (closest numbers to limiting probabilities) were obtained with 10,000 steps in each simulation and 10,000 simulations followed by the test with 5,000 steps in each simulation and 5,000 simulations; but considering that the run for the 10,000 steps in each simulation and 10,000 simulations took 4 days and 30 minutes, and the 5,000 steps in each simulation and 5,000 simulations one took only 5 hours and 43 minutes and that both set of results are very accurate, it is considered that a run with 5,000 simulations with 5,000 steps in each simulation would be sufficient, with an error of less than 0.025%, with respect of the limiting probabilities. For future or more complicated scenarios, where computation time becomes relevant to the process, a combination of 1,000 steps in each simulation and 1,000 simulations should also be acceptable, since in this exercise model this combination gave an error of less than 0.2%.

Some of these results are also shown in Figure 11 for one of the possible states, just as an example, spare state was selected. This figure shows only the results of the cases where the number of simulations and steps were equal or greater than 100.

The simulations were always run with a seed, which means that the software was forced to use the same sequence of random numbers to start each simulation, in order to compare the results. Specifying the use of the same seed for each simulation run allows you to obtain same value results, in other words, the simulation can be duplicated. A seed also help when tracking changes in simulation results when changing the program. Without a seed, in some computer simulation scenarios, it would be hard to realize if changes in the outcome were due to the changes in the code or due to different random numbers.

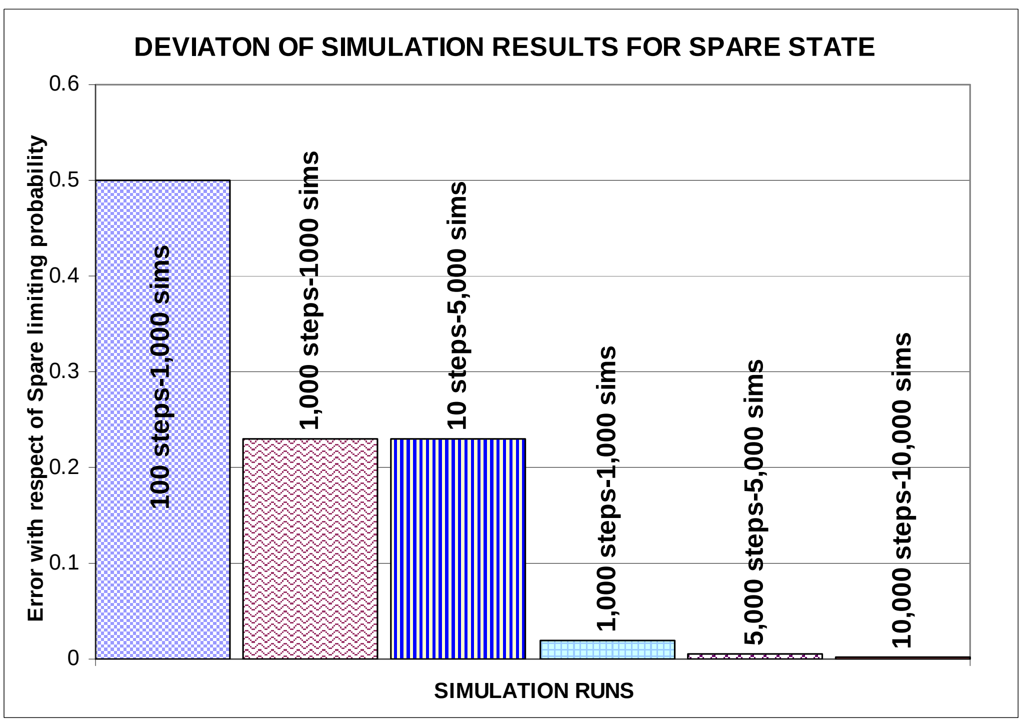


Figure 11. Comparison of limiting probability vs. Markov simulation results for Spare state.

3.6.1 SIMULATION DEVELOPMENT

Setting up the simulation involves several preparatory steps. The constants Steps, Spare, Duty, Fault and Fail were created and the assigned values were 5,000, 4, 3, 2 and 1 respectively. This constant named "steps" was changed in the different runs in order to obtain the analyses of Table 3. The transition probability matrix was created as a table as shown in Figure 12. RENO does not have a command or function for creating a matrix, but its flexibility allows you to treat a table as a matrix. One probability number with uniform distribution and values between 0 and 1, identified as "RN", was also created for this flowchart. This random number will represent the probability of going from one state to another.

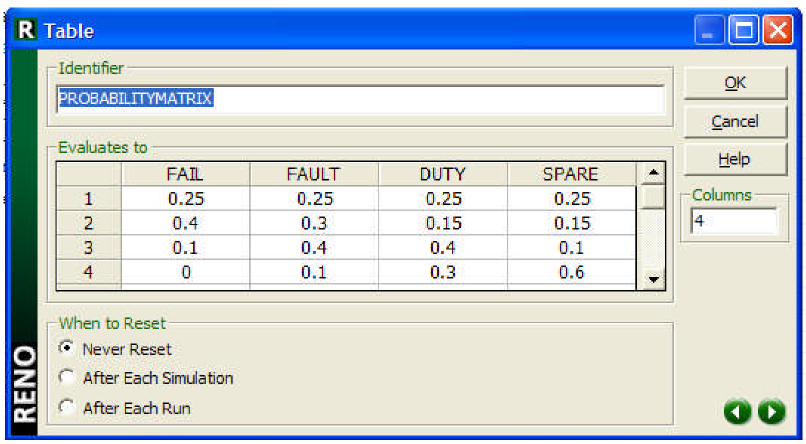


Figure 12. Probability matrix created in RENO.

Finally, the following storage variables were also defined in order to finish the flowchart:

Storage Variables	
Name	**Start Value**
NUMBEROFDUTY	0
NUMBEROFFAILS	0
NUMBEROFFAULTS	0
NUMBEROFSPARE	0
NUMBEROFSTEPS	0
PreviousState	3

The first four storage variables are intended to be used to capture the number of times when the component is at every possible state. The storage variable "number of steps" is created to record the number of steps in the Markov chain and finally, the variable "PreviousState" captures the current state of the component at any time in the process. As it can be noticed, it is assumed that the initial state of the component is State 3 (Duty) for this excercise. Thanks to the flexibility of simulation, this can always be changed easily in the definitions if this assumption is not correct. If the current state is unknown, this current state can be also simulated if the initial probabilities (Initial Probability Vector) are known or defined.

3.6.2 FLOWCHART BLOCKS

The first construct created was a "Flag Marker" named "Start". This Construct was created to be used in conjunction with the "Go to Flag" Construct named "Go to Start". The Go to Start construct returns the simulation point to the Start of the Flowchart.

Since the whole simulation was created assuming that at the beginning of the process the component is in state 3 (Duty), the first step is to determine whether the component remains in the same state or change to any of the other three states. In order to determine the next state, the program first evaluates if this new state is Spare; if not, then it assesses whether the new state is Duty, then Fault, and finally (if it was not any of these three states) by elimination, Fail would be the new state. The way the simulation actually does this evaluation is described below.

The construction block that allows us to capture the probability of going from the current state, whichever state that is, to the next state, by recalling the table "Probability Matrix" is shown in Figure 13.

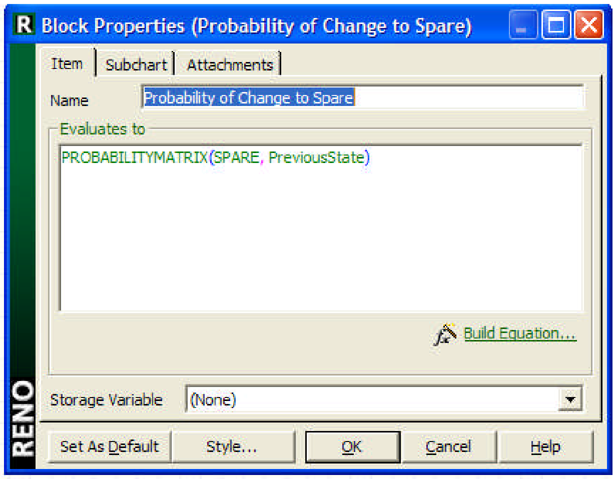


Figure 13. Change to Spare Probability Block definition.

A conditional block that tests whether the probability of changing to Spare is greater than or equal to the probability RN, with the two possible outcomes TRUE and FALSE, is included in the flowchart. If true, then the variable "PreviousState" now changes to the value "Spare", which is equal to state 4, the Spare counter is incremented by 1 and recorded in the storage variable "Numberofspares". If false, then the probability of changing to duty (Figure 14) is compared to the probability of RN. If true, a similar path to the true path for the spare state is followed. If false, then the process is repeated for fault state. Finally, if the component doesn't change to Spare, Duty, or Fault state, then the new state would be the Fail state. This new state is stored as the new value of the variable "PreviousState".

Up to this part of the flow chart, all the simulation has done is to evaluate which is the next state and count this transition. In other words, whether or not the component changes to a different state or not, if so, which state is the new one, and to number this transition. Once the new state for the component has been decided through the simulation, the step counter is incremented by one and displayed, and another conditional block, named "Done?", decides whether the number of desired (k) steps has been reached. If the number of steps has not yet been reached, then the RN probability is reset and the "Go to Start" block sets the current simulation point to the “start” flag. The whole process is repeated in order to simulate another step until the number of steps previously defined is reached.

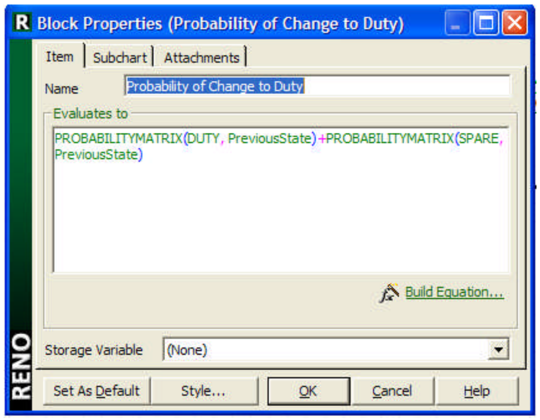


Figure 14. Change to Duty Probability Block definition.

Once the number of steps has been reached, then the percentage of times that the component was in each of the possible states is calculated and saved; and one simulation is completed. This whole process is considered a single simulation of the Markov process for k steps. To get more accurate results, many different simulations are run and the percentage of time in each state is averaged by Result Storage blocks. In this way, the results obtained at the end of the analysis are actually the average of the results obtained for every simulation. A Simulation is a single pass through a flowchart or process. A run is a set of simulations. An analysis is a set of runs. In our example, an Analysis comprises one run, a run is composed by 5,000 simulations and a simulation was only completed when 5,000 steps were reached (see Table 3 for different runs and k values).

3.7 SIMULATIONS OF COST OPTIMIZATION AND PM ALLOCATION FOR A NON-DYNAMIC PROBABILITY MATRIX MODEL (MODEL A)

Once the Markov process simulation was verified to reflect the PM process, a new flow chart, which includes cost and the allocation of preventive maintenance, was created. This new model, named model A, considers transition costs when changes between states occur and a preventive maintenance allocation after a specified number of steps. In this first PM model, once the PM is applied, it is considered that the component goes automatically to its most reliable operational state: Duty.

The flow chart for this model is shown in Figure 15, and it is similar to the one created for the Markov process (Figure 9), but with a number of additions and modifications made in order to adequate to the include costs of transitions and PM.

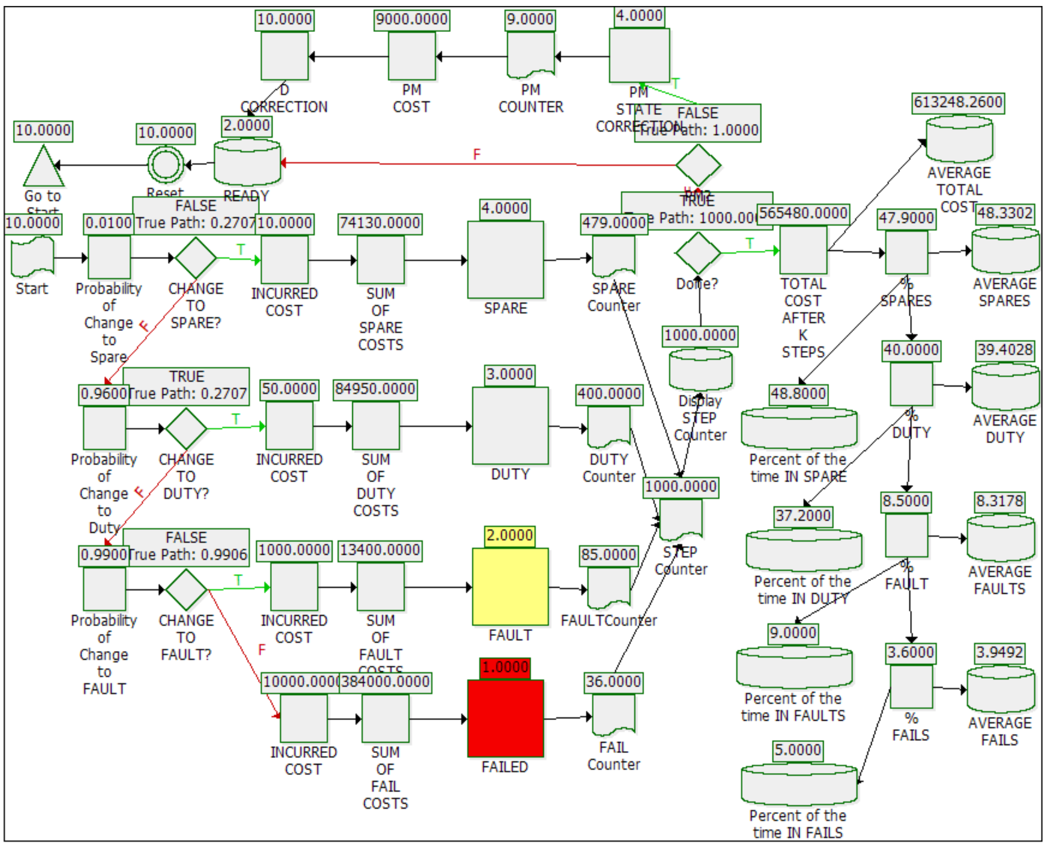


Figure 15. RENO simulation flowchart that includes cost and PM allocation.

3.7.1 NEW FLOWCHART DEFINITIONS

Two new constants named “PM” and “PMCOST” were included in the model. The constant PM was assigned a value of 10, that represents the periodic number of steps in which the PM is going to be allocated. The value 10, is an arbitrary number, since the idea of this model is to find the optimal number of steps (PM interval) for which the cost of the process is minimized; therefore, this variable PM is varied during simulation in order to determine the optimum PM interval value for the scenario.

The PMCOST constant was created to capture the costs associated with preventive maintenance. It is assumed that every time PM is performed, the cost of this PM remains constant.

Six new storage variables were also created. Four of these storage variables capture the sum of the transition costs for going to a certain state and their named "TOTAL*state*COST"; an example of one of these storage variables is shown in Figure 16. Similar storage variables were created for each of the four possible states.

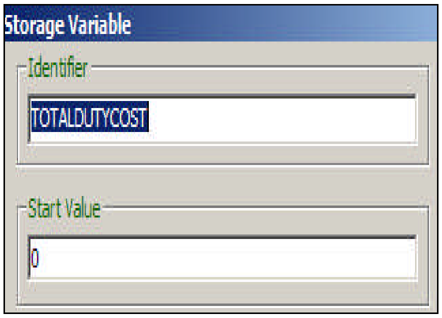


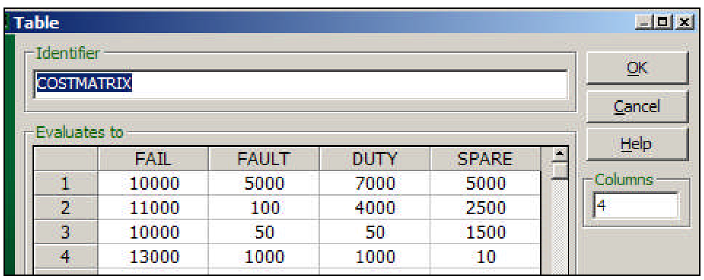
Figure 16. TOTALDUTYCOST storage variable definition.

Another storage variable, named "TOTALPMCOST" was created in order to store the total cost associated with preventive maintenance. Every time PM is performed, the TOTALPMCOST is increased by the PMcost value.

The remaining new storage variable was simply called "D". This is a dummy variable that is used to determine, in the simulation, when the PM was allocated.

Similar to the Table “PROBABILITYMATRIX”, a table named “COSTMATRIX” was added to the RENO program. This table represents the Transition Cost Matrix (Figure 17).

Lastly, the equation variable “PMTIMING” was also added to the model. This equation evaluates to: D times PM (D*PM). This variable PMTIMING will be used to determine whether it is time to perform PM or not. For example, if the PM schedule is every 100 steps, the value of the variable PM is 100, and PM should be performed at D*PM steps, which are: 1*100steps, 2*100steps, 3*100steps, 4*100steps, and so on. This will be explained in more detail when describing the PM subchart.



	FAIL	FAULT	DUTY	SPARE
1	10000	5000	7000	5000
2	11000	100	4000	2500
3	10000	50	50	1500
4	13000	1000	1000	10

Figure 17. Cost transition matrix created in RENO.

3.7.2 NEW FLOWCHART BLOCKS

Between the conditional block (which determines whether the component state changes to Spare, Duty, Fault or Fail), and the standard block (which assigns the new state of the component in the previous Markov model), two standard blocks were added: “Incurred cost” and “Sum of *state* Costs”.

The “Incurred Cost” block evaluates the transition cost incurred for the simulated change between states by recalling the cost matrix (Figure 18).

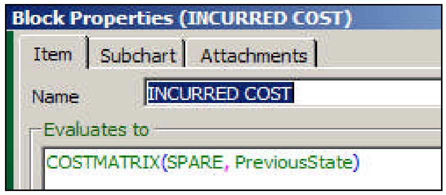


Figure 18. Incurred Cost standard block created in RENO.

The "Sum of *state* Costs" standard block for each state evaluates at "Incurred Cost" plus "TOTALstateCOST", and captures this value in the storage variable "TOTALstateCOST". In other words, the cost incurred for changing to a certain state, say, Fail state, is added and captured in the storage variable TOTALFAILSTATE. This block definition is shown in Figure 19.

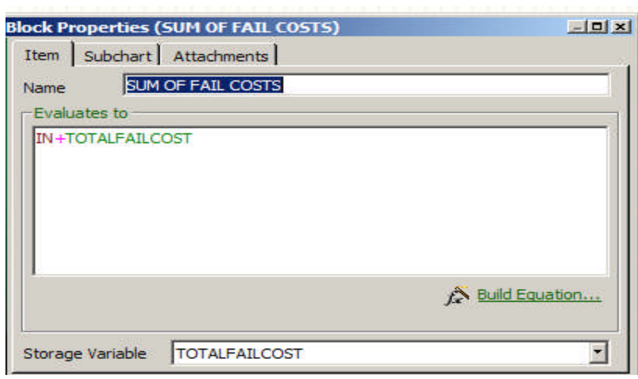


Figure 19. "Sum of State Costs" block example.

After the "True" branch of the conditional block (which determines in the Markov process flow chart whether or not the number of k steps has been reached), the standard block "TOTAL COST AFTER K STEPS", and the result storage block "AVERAGE TOTAL COST" are also included. This standard block calculates the sum of all costs, as show in Figure 20, and the result storage block averages the total cost obtained after each run.

Lastly, between the "false" branch of the conditional block mentioned before and the reset block, the subchart shown in Figure 21 was also added to the flow chart to represent PM performance.

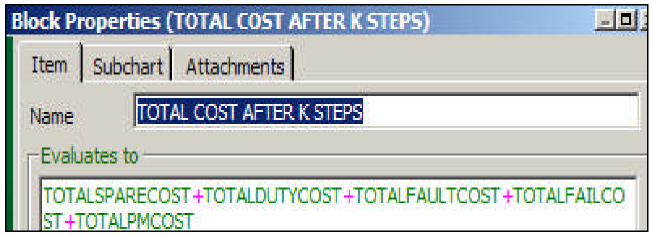


Figure 20. TOTAL COST AFTER K STEPS standard block.

The conditional block “PM?” decides whether or not a PM is allocated by comparing the number of steps with the equation variable “PMTIMING”. If the number of steps is equal to PMTIMING then the process follows the ‘true’ path, which indicates that PM is performed, otherwise the process follows the ‘false’ path. This is a threshold decision, where the PM interval is the threshold.

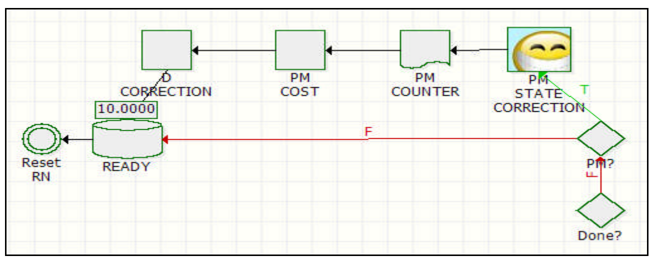


Figure 21. Subchart that represents PM performance.

When PM is performed, the standard block PM STATE CORRECTION changes the current state, whether the state may be, to Spare state, since it is assumed that after PM the component will be sent to its most reliable state, which is Spare. Then, a counter block counts the number of times PM has been performed, and stores that value in the storage variable “D”. The standard block “PM COST” calculates and stores the PM total cost by multiplying the PMCOST times D, which is the number of times PM has been performed since the simulation start time. The standard block “D CORRECTION” evaluates to D+1. This correction is made in order to ensure that the equation variable PMTIMING is still valid.

In general, when starting the simulation, the value of D defaults to 1, so the equation variable evaluates to PMTIMING=D*PM=PM. Once a PM is performed, D still evaluates at 1, because takes its value from the PM counter. The correction D=D+1 is performed so D=2, and now the next time PM is performed the number of steps reached has to be equal to PMTIMING=D*PM=2PM, and so on.

The result storage block "READY" does not have any other function than allowing the continuity of the process in both paths. This is because a Reset block can not have more than one incoming path in RENO software.

Since now PM is performed, and this PM activity is affecting the current state of the component, this process is no longer considered a Markov process. It behaves as a Markov process only until PM is performed. Once PM has changed the current state of the component, another mini Markov process is executed only to be interrupted again by PM. If PM is carried out frequently, the Markov limiting probabilities of the original probability matrix can never be reached. If PM maintenance is executed within long step intervals, then the percentage of times of the component being in every possible state would be similar to the ones indicated by the Markov process (limiting probabilities). This will be demonstrated in the results of numerical experiments **(**Section 3.7.4, Table 8).

3.7.3 PM SIMULATION SETTINGS

The first simulations for the PM model were run with the following transition probability matrix *P* and transition cost matrix *C*:

$$P = \begin{Bmatrix} 0.400 & 0.010 & 0.090 & 0.500 \\ 0.180 & 0.800 & 0.015 & 0.005 \\ 0.010 & 0.030 & 0.950 & 0.010 \\ 0.010 & 0.010 & 0.030 & 0.950 \end{Bmatrix} \quad C = \begin{Bmatrix} 10000 & 5000 & 7000 & 5000 \\ 11000 & 100 & 4000 & 2500 \\ 10000 & 50 & 50 & 1500 \\ 13000 & 1000 & 1000 & 10 \end{Bmatrix}$$

The limiting probabilities for these four states, given the transition probabilities above, were found to be 0.4863, 0.3890, 0.8.47 and 0.4 for states Fail, Fault, Duty and Spare, respectively; since:

$$P^{(k)} = \begin{Bmatrix} 0.4863 & 0.3890 & 0.0847 & 0.0040 \\ 0.4863 & 0.3890 & 0.0847 & 0.0040 \\ 0.4863 & 0.3890 & 0.0847 & 0.0040 \\ 0.4863 & 0.3890 & 0.0847 & 0.0040 \end{Bmatrix}$$

when $k \to \infty$

In this model, *P* is considered a non-dynamic probability matrix, due to the fact that the transition probabilities are fixed and don't change over time. These are artificial values with no application to any specific real system. The values were selected based on engineering expertise judgment with the intention to be representative of the transitions between states for a generic repairable component in a system.

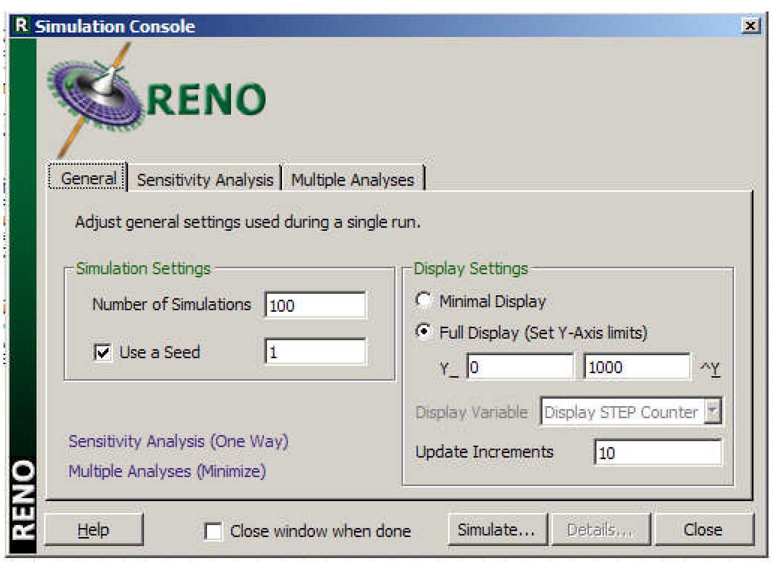


Figure 22. General simulations settings for the first PM simulation.

The general settings for this first PM simulation are shown in Figure 22. Note that only 1000 simulations were set for this run. While this number does not appear to be excessively high, when we consider this number of simulations combined with the settings of Sensitivity Analysis and Multiple Analyses would actually produce more than 25 thousand simulations.

The settings for Sensitivity Analysis and Multiple Analyses are shown in Figure 23 and Figure 24, respectively. The sensitivity analysis settings indicate that the parameter PM (time interval between maintenance activities) is no longer a constant, but now it is a variable during the simulations to calculate the effect on results due to varying this value. The start value for PM for the first set of simulations was set to be 100 time steps; this variable was linearly increased by 100 during the simulations until reaching the maximum PM value of 500. These fields determine how many runs will be performed.

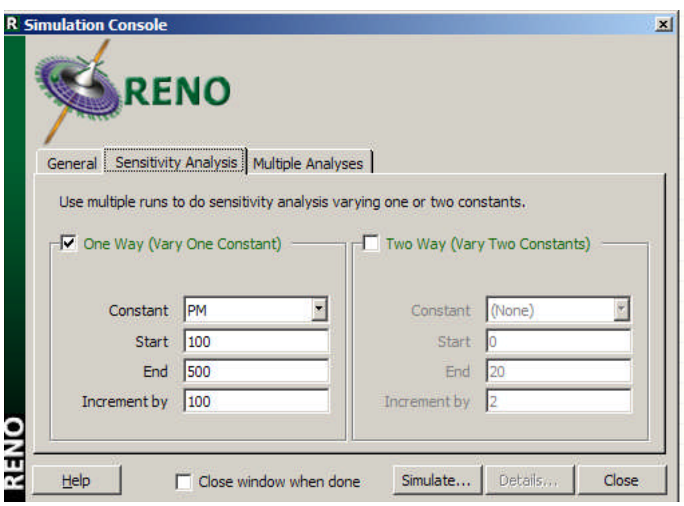


Figure 23. Sensitivity Analysis settings for the first PM simulation.

In our example, with a start value of 100, end value of 500, an increment value of 100, and the number of simulations specified on the general settings as 1000, the 1000 simulations are run with the PM constant value at 100, again with it valued at 200, again at 300 and so on, until they have been run with the PM constant at its end value of 500. In this case, that would mean that the analysis will consist of 5 runs, with 1000 simulations performed for each run.

As shown in Figure 24, the multiple analyses settings are chosen so that the optimization goal is to minimize the “average total cost” of the process after 1,000 steps. The program allows us to run multiple sensitivity analyses in order to examine whether an optimal PM length is found.

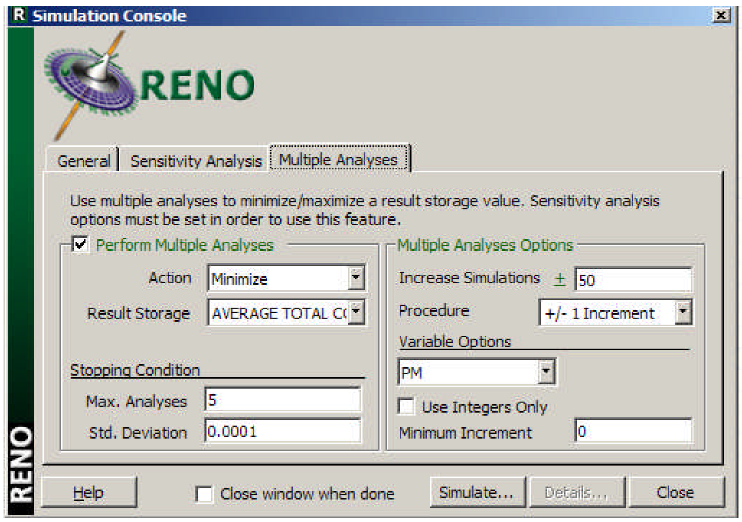


Figure 24. Multiple Analyses settings for the first PM simulation.

In this way, RENO performs the first sensitivity analysis and finds the value of the varied PM Constant at which the “average total cost” is lowest. It then takes an interval surrounding that value and divides that according to the increment specified for sensitivity analysis and performs the specified number of runs on that interval. This process is repeated over the five specified numbers of analyses, or until the designated 0.0001 standard deviation is reached, whatever occurs first, refining the interval and the minimum result with each analysis. The standard deviation setting allows specifying a convergence threshold for which future analyses are not necessary, if this value is low enough. In this case, the 0.0001 value is low in order to warranty the five analyses to be performed, unless this standard deviation is reached, in which case, future results will be practically identical and unnecessary to be preformed.

The ±50 Increase Simulations option allows us to increase the number of simulations for each consecutive analysis. In this way, RENO will perform more simulations at each consecutive analysis so that early analyses over a broad range take less time, while final analyses are performed in detail. In this case, recall that it has been already said that the software would run 1,000 simulations for each PM value in first analysis, but for the second analysis, due to this "±50 Increase" setting, 1,050 simulations will be run for each PM value, and so forth, so in the fifth analysis 1,200 simulations would be run for each PM value.

The field "Procedure" allows us to specify how RENO selects the interval for each consecutive analysis. In this case, the +/- 1 increment indicates that the interval will have a range from one increment below the value of the varied PM Constant at which the "average total cost" is lowest, to one increment above that value. The size of each increment is defined on the Sensitivity Analysis page. This can be observed in an example showed in Table 4, since the first minimum "average total cost" value was found at PM = 400, the next analysis should be between 400-1 interval (300) and 400+1 interval (500). Then, the next analysis is run between 300 and 500. In the second analysis, the minimum cost was found to be again at PM=450, therefore, the next analysis is run between the interval 400-500, and so on, until the five analyses are run.

Analysis 1	COST	Analysis 2	COST	Analysis 3	COST
100	619853.18	300	617426.8727	400	619430.6667
200	620199.74	350	624739.2	425	614853.4167
300	618453.54	400	624530.4545	450	622693.7
400	616812.3	450	613793.9455	475	624211.4167
500	619586.72	500	615812.9455	500	624713.7833

Table 4. PM simulation results example.

The 0 value on the field “Min. Increment” allows us to specify that there is not a limit for the smallest increment that can be used to determine the values of the varied PM Constant for the analyses, although realistically there will be some lower limit on PM interval.

3.7.4 MODEL A SIMULATIONS RESULTS

The simulations are run in order to find the optimal PM interval for a component with a certain probability and cost matrix. The way the simulation works is by simulating the changes of states in the component, based on its transition probabilities. Every time there is a transition between states a cost associated with this transition is computed and stored in the overall cost tally. If there was no PM activity performed, then there would be an associated cost of the whole process after k steps. If PM is done with a certain periodicity then the process gets affected by these activities, and the actual state of the component is not subject only to the probability of the state change. Therefore, the overall or total cost after k steps gets also affected by the PM schedule. The cost associated with a PM is also included in the simulations.

Given the settings described above, the first simulation was run, for a fixed PM cost of $100,000. The results obtained with this simulation are shown in Table 5. The PM interval was set to be tested for every 100 steps, starting at 100 steps and finishing at 1,100 steps. The optimal PM interval was found to be every 1,080 steps, for a run with 500 simulations. Since the analysis of the simulations was designed for only 1,000 steps, any result of the PM interval above 1,000 steps would represent not PM performed at all. This was the case for the first simulation that was run. This first run indicates that PM should not be performed.

Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST
100	1.53E+06	800	719234.1636
200	1.02E+06	830	715174.7091
300	919223.6	860	725085.8
400	817729.64	890	723818.2909
500	719943.1	920	727221.1636
600	718917.1	950	725726.8364
700	726975.9	980	725823.5455
800	720252.08	1010	617016.4364
900	718151.62	1040	617407.6909
1000	613846.14	1070	614882.9818
1100	616655.74	1100	612676.3273
Analysis 3 - PM	**AVERAGE TOTAL COST**	**Analysis 4 - PM**	**AVERAGE TOTAL COST**
1040	618147.0667	1064	617773.1077
1046	621497.4833	1066	617575.8769
1052	621693.4333	1068	620316.3231
1058	618824.0833	1070	620823.3231
1064	625625.8	1072	620354.4769
1070	615321.1667	1074	622586.6769
1076	612407.0333	1076	627297.6154
1082	615110.2167	1078	607701.8462
1088	620984.65	1080	619291.0923
1094	622769.2333	1082	621782.8154
1100	616664.4667	1084	621088.6
		1086	623859.3846
		1088	618330.2154
Analysis 5 - PM	**AVERAGE TOTAL COST**		
1074	617337.0143		
1075	620974.5		
1076	622779.9714		
1077	618314.5429		
1078	616409.0286		
1079	622151.5286		
1080	613821.1429		
1081	619289.6857		
1082	615947.5714		

Table 5. Simulation results of cost optimization for model A, with a PM cost of $100,000, each analysis set to 500 simulations.

Figure 25 shows plot of the first 3 analyses of Table 5. This figure is also a good representation of how the optimization process works during the different simulation analyses. The first analysis test a big range of different PM intervals, while further analyses focus only in a small range of PM interval values that are suspected to be close to the optimal PM interval.

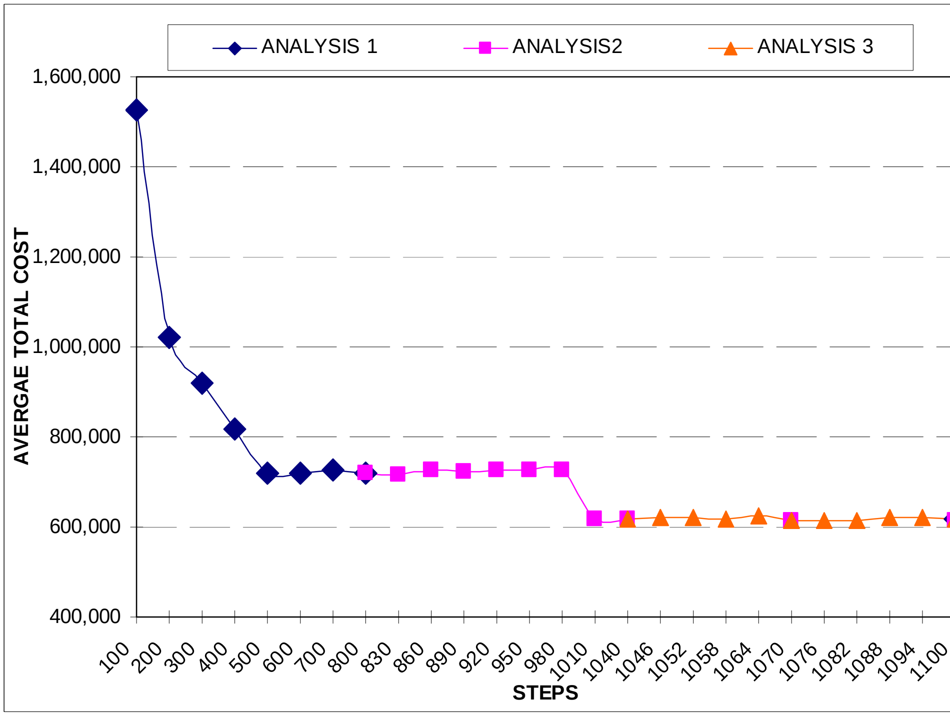


Figure 25. Simulation results sketch of the first 3 cost optimization analyses of model A, with a PM cost of $100,000 and 500 simulations per analysis.

The settings of the number of simulations per analysis were changed from 500 to 1,000 simulations, for a second run with the same PM cost, in order to analyze the sensitivity of this model to this variable; in other words, if different results would be obtained with a more detailed simulation. Increasing the number of simulations from 500 to 1,000 didn't give any real advantage in the results. The results obtained for this second run (shown in Table 6 and Figure 26) were similar to the ones obtained with fewer simulations per analysis: PM maintenance should not be performed in order to minimize cost.

As it can be seen in Table 5 and Table 6, optimal PM interval was always greater than 1,000 steps for every analysis, recommending no PM at all. The fact that there different values for the optimal Total Cost, is due to the randomness of the simulations, but it can be noticed that these values are pretty similar for every case, as expected if PM is not performed for any of those cases.

Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST
100	1.53E+06	900	715488.16
200	1.02E+06	920	722946.1
300	920453.77	940	720030.6
400	824038.74	960	718535.3
500	716033.23	980	714849.11
600	718502.8	1000	620563.34
700	720129.7	1020	619921.26
800	726390.17	1040	616511.08
900	725081.39	1060	618836.44
1000	618930.08	1080	623947.34
1100	616916.46	1100	619636.16
Analysis 3 - PM	**AVERAGE TOTAL COST**	**Analysis 4 - PM**	**AVERAGE TOTAL COST**
1000	619061.77	1064	625552.41
1008	617456.1	1065	620252.88
1016	621549.03	1066	614530.71
1024	622567.15	1067	626056.06
1032	616448.47	1068	622511.31
1040	621543.07	1069	616279.08
1048	622926.25	1070	623578.15
1056	616585.2	1071	621083.11
1064	617473.47	1072	621123.38
1072	619988.6	1073	619970.21
1080	614072.91	1074	617398.62
Analysis 5 - PM	**AVERAGE TOTAL COST**	1075	620718.7
1064	613849.66	1076	619137.71
1065	615039.64	1077	617922.05
1066	616749.89	1078	619958.94
1067	617867.19	1079	623452.1
1068	616523.11	1080	614658.1

Table 6. Simulation results of cost optimization for model A, with a PM cost of $100,000, each analysis set to 1,000 simulations.

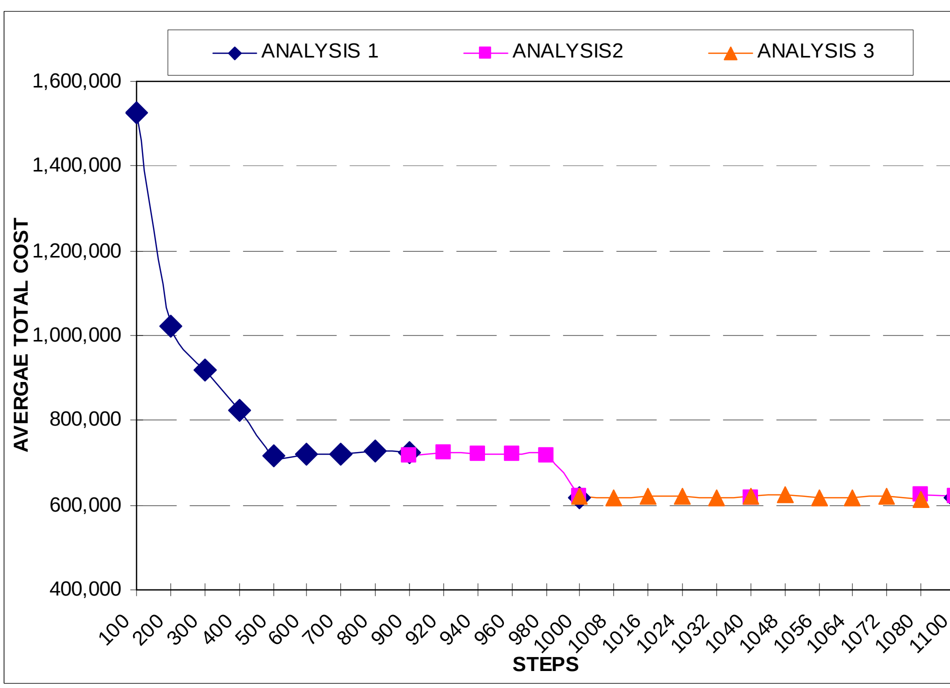


Figure 26. Sketch of simulation results of Table 6 .

Additional simulations for different PM costs were run. The PM costs were decreased to $50,000, $10,000, $5,000 and $100, in that order, trying to find a PM cost for which optimal PM interval would be less than 1,000 steps. The number of simulations was set at 1,000 simulations/analysis for a $50,000 PM cost, and at 500 simulations/analysis for the rest of the PM costs. These new settings and simulations were run in order to analyze the sensitivity of this model to those two variables, PM cost and number of simulations per analysis. Results obtained for these simulations are shown in Table 7 and Figure 27.

PM COST: $50,000			
Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST
100	1075700	900	665488
200	820687	920	672946
300	770454	940	670031
400	724039	960	668535
500	666033	980	664849
600	668503	1000	620563
700	670130	1020	619921
800	676390	1040	616511
900	675081	1060	618836
1000	618930	1080	623947
1100	616916	1100	619636
PM COST: $10,000			
Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST
100	715750	800	627952
200	661376	830	629319
300	649224	860	631823
400	637730	890	635943
500	629943	920	634918
600	628917	950	638863
700	636976	980	631292
800	630252	1010	619711
900	628152	1040	618150
1000	613846	1070	619503
1100	616656	1100	614330
PM COST: $5,000			
Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST
100	670750	800	622952
200	641376	830	624319
300	634224	860	626823
400	627730	890	630943
500	624943	920	629918
600	623917	950	633863
700	631976	980	626292
800	625252	1010	619711
900	623152	1040	618150
1000	613846	1070	619503
1100	616656	1100	614330
PM COST: $100			
Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST
100	626650	800	618052
200	621776	830	619419
300	619524	860	621923
400	617930	890	626043
500	620043	920	625018
600	619017	950	628963
700	627076	980	621392
800	620352	1010	619711
900	618252	1040	618150
1000	613846	1070	619503
1100	616656	1100	614330

Table 7. Simulation results of cost optimization for model A, different PM costs.

Decreasing the PM cost, gave the same results of not performing any PM in order to minimize cost, but it helped us to validate the model by comparing the results for this new PM cost. For example, since the average total cost obtained for a PM interval of 100 steps and a $100,000 PM cost was in the order of 1.53e+06 (Table 6), and the average total cost obtained for the same number of steps but with a PM cost of $50,000 was $1.08e+06, we can verify that these results were expected, when using a seed. If the PM interval it is 100 steps, this means that in the 1,000 steps simulation model, PM was performed 9 times (at step number 1,000 the model exits the simulation before PM is performed). When the PM cost is $100,000 then the PM total cost is $900,000, and for the $50,000 PM cost the total cost is $450,000. Therefore, a decrease in the average total cost of $450,000 for a 100 steps PM interval should be expected, and that was the one obtained in the simulations ($1,530,000-$1,080,000=$450,000).

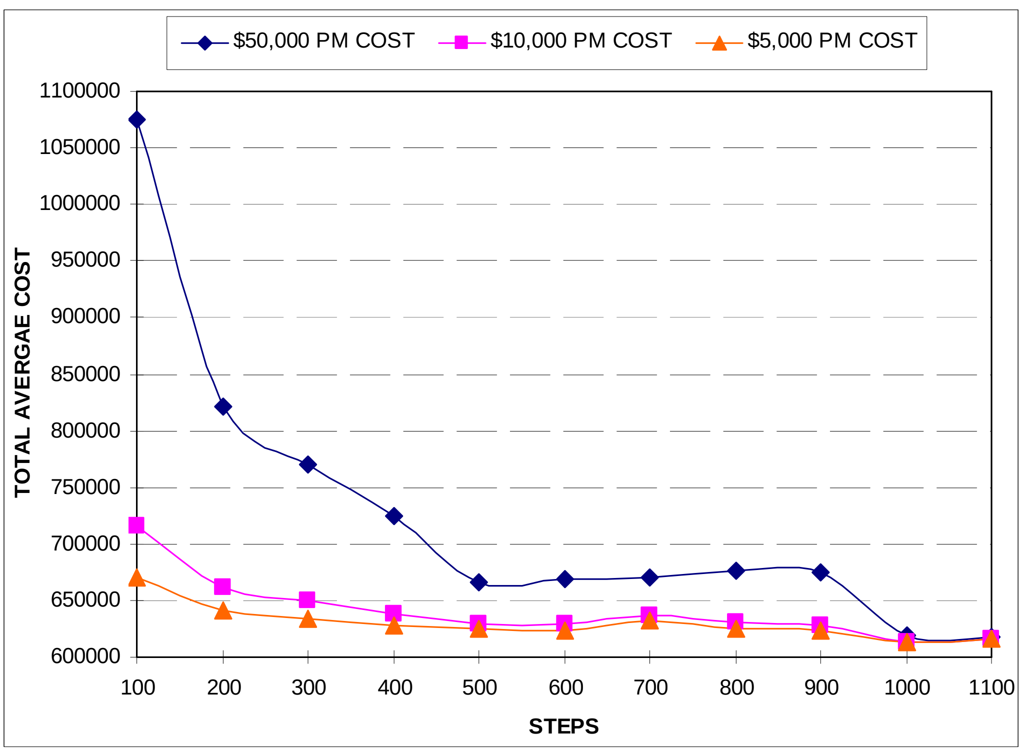


Figure 27. Sketch of costs optimization results for different PM cost of model A.

It is expected that when PM cost is pretty low, the optimal PM interval is harder to obtain due to the small changes in cost when performing PM cost, in other words, if PM cost is very small, then performing PM does not have much effect on Average Total Cost, and the results would not be sensible to PM interval anymore. Total average Cost should be similar for any PM interval. In these cases, where PM cost is low, it is expected that increasing the number of simulations would actually offer some improvement to the result process. The problem with increasing the number of simulations for any run, and not only for those where the PM cost is low, is the computation time.

As a conclusion, of the sets of simulations that were run, we can say that PM it is affecting and changing the steady state of the component, since it has been assumed that every time that PM is performed, the state of the component would be On Duty, regardless the previous and actual state prior to the PM and the transition probabilities. The steady states of the component can be compared with the actual percentage of times in which the component was in a certain state during the simulations. As expected, when PM was performed frequently, the percentage of time that the component spent in every different state was far from the one expected in the limiting probabilities; but when PM was scheduled only once or not scheduled at all, the percentage of time that the component spent in each state during simulation was similar to the respective limiting probability. This confirmed the idea that when PM is performed, the Markov process is interrupted by this PM and the limiting probabilities can never be reached, but if the PM is scheduled with long step intervals, the percentage of time spent in every state is similar to the expected from a Markov process. As it can be noticed in Table 8, the longer the PM interval the closer to the limiting probabilities (denoted as LP in Table 8). PM intervals of 500 steps or longer give close value to the LP due to the fact that these intervals represent that PM is performed only once or not all, during the 1,000 steps subject of study, and therefore, the short Markov process has been interrupted only once or not interrupted at all. Opposite to smaller PM intervals, a 100 PM interval would represent 9 PM activities and 9 interruptions of the short Markov process. In the other hand, the simulations also show that PM intervention it is not giving any benefit to the cost optimization, but only increasing the overall cost for the set of transition probabilities used.

PM	% IN FAULT	% IN FAIL	% IN DUTY	% IN SPARE
100	9.0307	4.0588	45.9106	40.9999
200	8.718	4.018	42.0589	45.2051
300	8.6692	4.0139	42.0717	45.2452
400	8.6539	4.0408	40.9967	46.3086
500	8.4243	3.9788	40.5511	47.0458
600	8.484	4.0004	40.26	47.2556
700	8.5188	4.0078	40.2493	47.2241
800	8.6936	4.063	40.2827	46.9607
900	8.5926	4.0423	40.2488	47.1163
1000	8.4787	3.9969	40.1746	47.3498
1100	8.4922	3.9975	39.7019	47.8084
LP(%)	8.47	4.00	38.90	48.63

Table 8. Comparison of percentage of time spent in every state with the limiting probabilities. .

3.7.5 SOME LIMITATIONS AND INTERPRETATIONS OF MODEL A

The accuracy of the model is limited by the data provided by the probability and cost matrices. Therefore, the definition of these matrices becomes a key part of the model. Each particular case study or system is going to have its own set of probability and cost matrices. An incorrect estimation of the transition probabilities and the transition costs would lead to an erroneous calculation of the optimal PM interval. Reliable and realistic numbers are essential in the process of decision making. For all these reasons, future studies on the best method to calculate these matrices for a particular scenario should be performed.

When defining the cost transition matrix, all possible costs associated with these transitions should be included in the transition cost between states, such as downtime, managing, transportation, opportunity cost of lost production, administrative and support activities, equipment and facilities, labor, and so forth. The only cost that is not considered in the cost matrix is the cost of performing a PM activity, which is considered separately, because it is a deterministic activity, not a random event.

When deciding the number of possible states for a specific component or system, we should have in mind that there are advantages and disadvantages of having a large number of states in a model. It has been mentioned that two state models for a component may be unsuitable or inadequate, since there might be many different levels of performance. In the model proposed by this study, the larger the number of possible states the better the adaptation of the model; having a large number of possible states, would allow you to consider many particular scenarios, characteristics and conditions of the component, including aging. The fact that aging it could somehow be included when considering the possible state of a component is a key point as to whether the markovian assumption is valid or not.

Aging it wouldn't be included as a state per se, but it would be considered in the several different levels of performance of the component. In general, it could be said that when the number of states of the component tends to infinity, the error of the model tends to zero. In other words, if it could have a large number of states, let say and infinite number of levels of performance, then all possible cases and properties of the component would be potentially included. In this way it can be assured that the markovian property is fulfilled due to the fact that, by perfectly describing the component state and the probabilities of the change of this state to a different one, the past or history of the component becomes unimportant and insignificant and all the information related to the component is included in its state and transition probabilities

In the other hand, having a huge number of possible states for the component would make almost impossible the tracking of these states, and would also increase the difficulty of the already challenging task of estimating the transition probabilities. Computational problems would also be present because of the large number of states, and in the case of the simulation tool of this study, excessively long computation time may be required. The optimal number of states for a component should then be defined. This ideal number of states may vary according with each particular case or system, but it should always be considered that this number has to be large enough to realistically represent levels of component performance, but small enough so the tracking of the states could be realistically carried out and simulations performed.

In the previous section it was discussed how PM interval affects the Markov process. When PM is performed frequently, the Markov limiting probabilities can never be reached; but when PM is carried out with long-duration intervals the numbers obtained with simulation are close to the ones indicated by the limiting probabilities. At least, at a long time scale, a new set of transitions probabilities that accounts for the transitions driven by the PM interval is obtained. The analysis and estimation of these new transition probabilities could also be an area for further research.

CHAPTER 4

SIMULATIONS OF COST OPTIMIZATION AND PM ALLOCATION FOR NON-FIXED PROBABILITY MATRIX MODELS

4.1 INTRODUCTION

The simulations for the "Model A" were run and results for this model were obtained, and interpreted. Two new models are created, which consider a non-fixed probability matrix that would change over time. The two new models allow reliability to be consumed over time, changing the transition probability of going from duty to a failure state to increase with time, and the probability of staying in a duty state to decrease. The probabilities of going from failure to any other state don not change over time. Since some of the transition probabilities change over time, the new models are not Markov processes.

4.2 NON-FIXED PROBABILITY MATRIX MODELS

The difference between the new models (Model B and model C), is that Model B considers a decrease of reliability after every certain number of steps (periodically), and Model C considers a continuous decrease of reliability at every step.

These two new models only consider two possible states: Duty and Fail. The states Spare and Fault are removed from the model to simplify calculations, and minimize computation time.

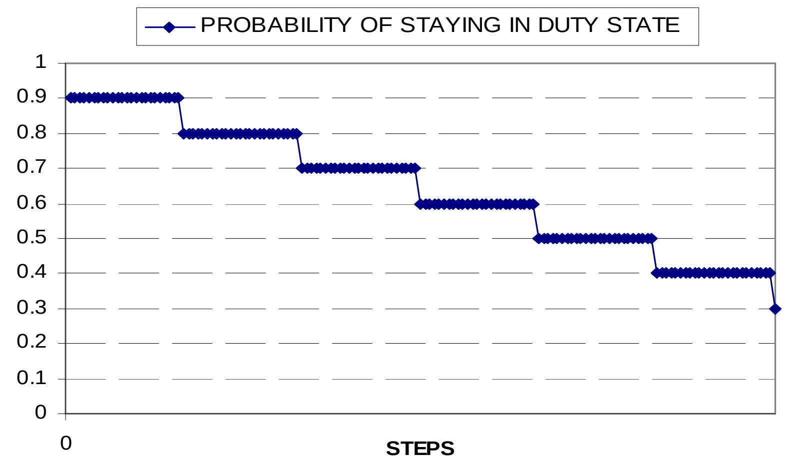


Figure 28. Model B without PM, Schematic Representation.

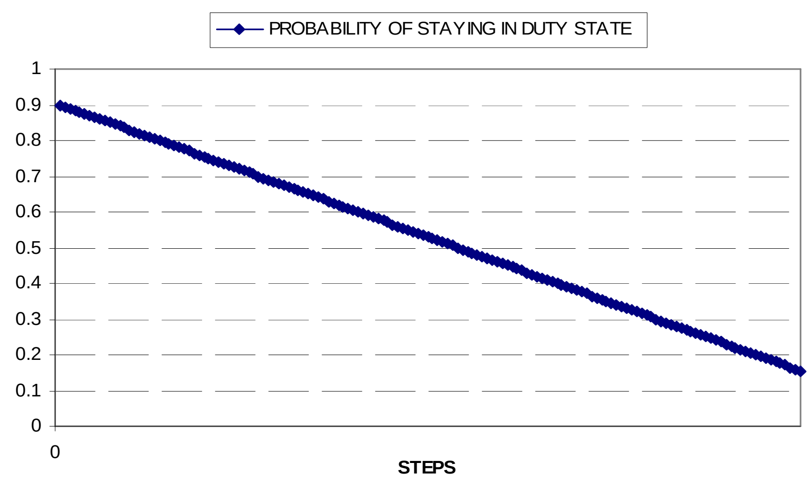


Figure 29. Model C without PM, Schematic Representation.

If preventive maintenance is performed, then the component would be considered as good as new after PM, and the probabilities of going from Duty to Fail and remaining in Duty would be restored to their original values.

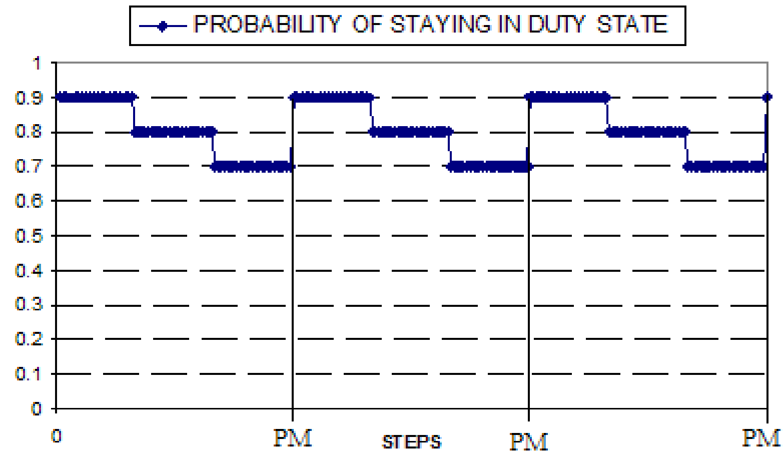


Figure 30 . Model B with PM, Schematic Representation.

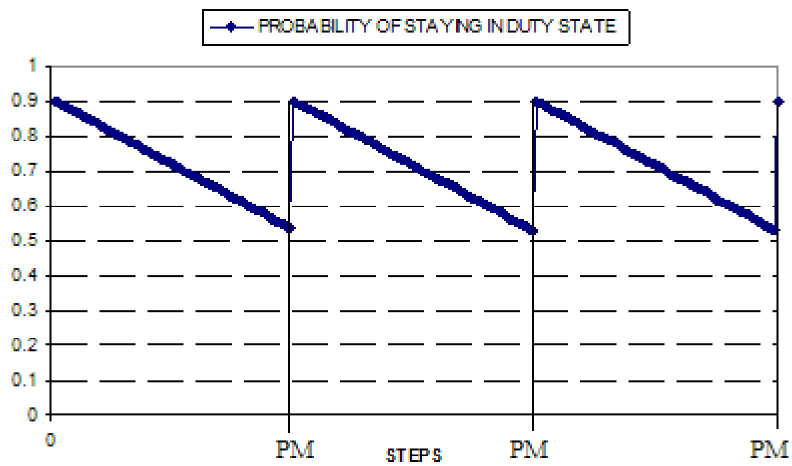


Figure 31 . Model C with PM, Schematic Representation.

The simulations for the new models considered the following transition probability matrix and transition cost matrix for the numerical experiments:

$$P = \begin{Bmatrix} 0.25 & 0.75 \\ 0.05 & 0.95 \end{Bmatrix} \qquad C = \begin{Bmatrix} 10000 & 6000 \\ 11000 & 50 \end{Bmatrix} ,$$

where the first row and first column represent Fail, and the second row and column represent Duty in both matrixes, in other words, Fail is state 1 and Duty is state 2:

$$P = \begin{Bmatrix} P_{ff} & P_{fd} \\ P_{df} & P_{dd} \end{Bmatrix} \qquad C = \begin{Bmatrix} C_{ff} & C_{fd} \\ C_{df} & C_{dd} \end{Bmatrix} \quad .$$

In these models, P is considered a dynamic probability matrix, since the transition probabilities (P_{ij}) are no longer fixed, and they change over time.

The cost of going from Fail to Fail (C_{ff}), is the cost of lost production per step. The cost of going from Fail to Duty (C_{fd}) is the cost of repairing the component, including all maintenance corrective cost in that step. The C_{df} cost is the cost of failure step, which can include lost inventory, damage to the system, lost production, etc.

Again, these are representative artificial values that were selected based on engineering expertise judgment, with no application to any specific real system.

The Model B considers that, after every 25 steps, the probability of remaining in a good state (duty) will decrease 0.01; and so the probability matrix after 25, 50 and 75 steps would be respectively:

$$P_{25} = \begin{Bmatrix} 0.25 & 0.75 \\ 0.06 & 0.94 \end{Bmatrix} \quad P_{50} = \begin{Bmatrix} 0.25 & 0.75 \\ 0.07 & 0.93 \end{Bmatrix} \quad P_{75} = \begin{Bmatrix} 0.25 & 0.75 \\ 0.08 & 0.92 \end{Bmatrix}$$

and so on. If there is not preventive maintenance, the probability matrix at the end of the simulation (1,000 steps), predicted numerically is:

$$P_{1000} = \begin{Bmatrix} 0.25 & 0.75 \\ 0.45 & 0.55 \end{Bmatrix}$$

The simulations of Model C consider a decrease in the duty to duty transition probability of 0.0005, every step. After 1,000 steps the probability matrix for this model, without preventive maintenance, would be:

$$P_{1000} = \begin{Bmatrix} 0.25 & 0.75 \\ 0.55 & 0.45 \end{Bmatrix}$$

Modifications to the flow chart of model A were necessary in order to adjust to the new models. The blocks related to the Spare and Fault states were removed, and the following changes or additions were made.

Since the probability matrix now changes over time, zero values were assigned as the original values of the "probabilitymatrix" table. This was performed, in order to be able to assign different transition probability values during the simulation. The blocks "ASSIGNFF", "ASSIGNDF", "ASSIGNFD" and "ASSIGNDD" were added to at the beginning of the flow chart. These new blocks will assign the changeable transition probabilities to the "probabilitymatrix" table, through the also new created variables "FF", "DF", "FD", and "DD". This will allow us to determine the right unsettles transition probabilities for every simulation. The original values of the variables "FF", "DF", "FD", and "DD" are, as guessed, 0.25, 0.05, 0.75, 0.95 respectively.

Also the subchart shown in Figure 32 was created and added to the two new models, to decrease or increase, as appropriate, the transition probabilities.

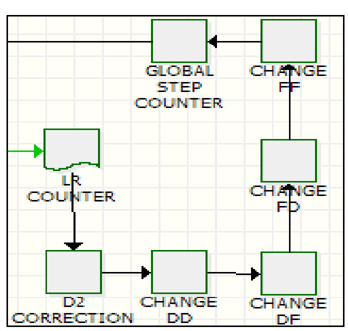


Figure 32. Decrease of reliability subchart.

This subchart includes the following blocks:

- A block counter named “LR COUNTER” and a standard block named “D2 CORRECTION”. These two blocks together help the conditional block to decide if the subchart is executed or not, (only every 25 steps the path is executed).

- Four standard blocks that allows you to decrease or increase the transition probabilities. The new transition probability values are stored in the storage variables DD, DF, FD, and FF. For our case, only the transition probabilities DD and DF are modified; DD decreases 0.01 and DF decreases 0.01 every time this path is executed. Even though the probabilities FD and FF are not modifies in our model, the blocks were created to have the flexibility and possibility to do so.

- Another standard block named “GLOBAL STEP COUNTER”, which recalls the number of steps. This block, along with the result block named “STORAGE”, just helps to give continuity to the simulation.

Finally, the PM performance subchart was also modified for both new models. Another four standard blocks were added to this section. Each of these four blocks evaluates to the new constants AANDD, AANDF, AANFD, and AANFF, and stores these values in the variables DD, DF, FD, and FF, respectively. The constants AANDD, AANDF, AANFD, and AANFF are the “as good as new” transition probabilities. In other words, these changes in the PM subchart allow us to restore the transition probabilities to their original or “as good as new” values.

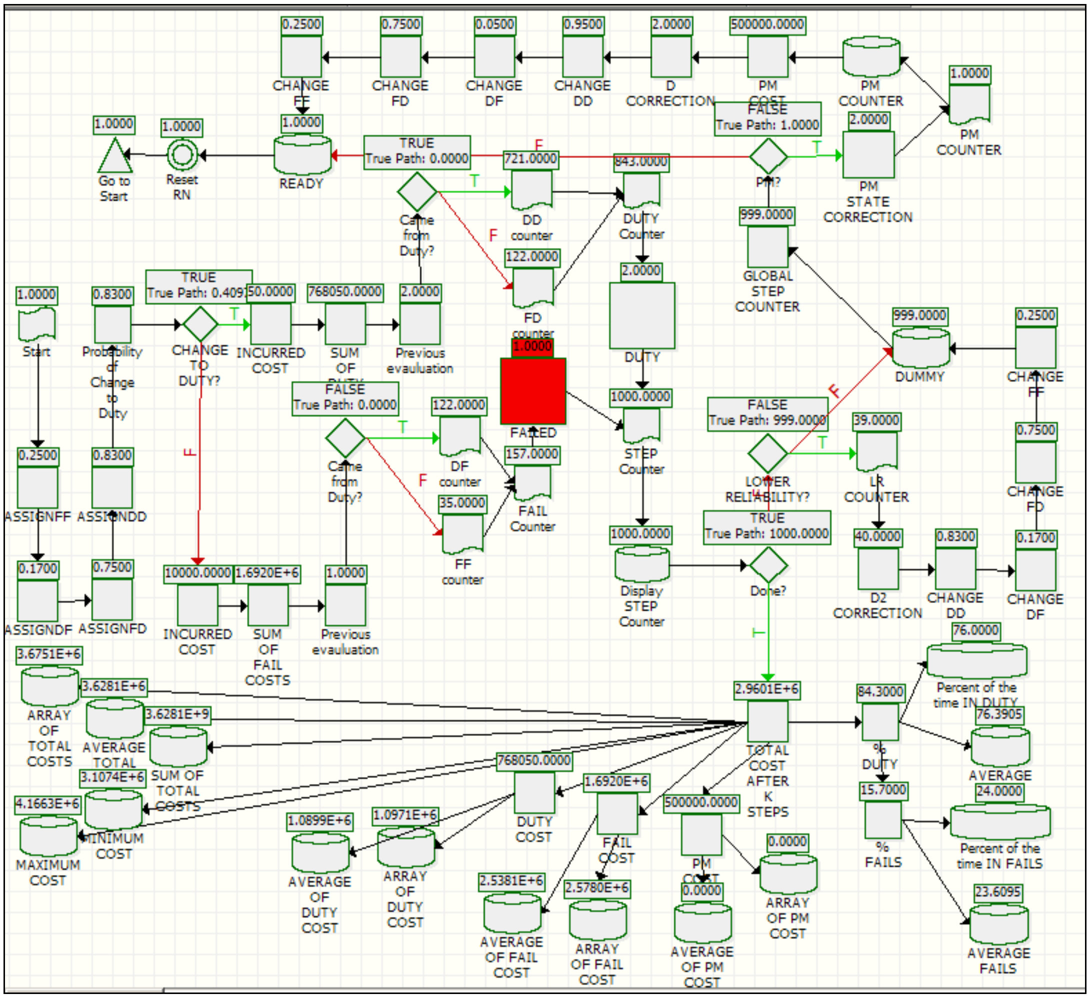


Figure 33. Model B flowchart.

Besides the changes mentioned above, additional changes were made to the model B. Another conditional block was added with the label “lower reliability?” This block will asses if the decrease of reliability subchart should be performed or not. Also the block counter was added to model B. this counter, counts the times that the transition probabilities are modified due to time. Figure 33 and Figure 34 show the flowcharts of the Model B and Model C, respectively. Figure 35 and Figure 36 are simplified block diagrams of Model B and Model C simulation processes, respectively.

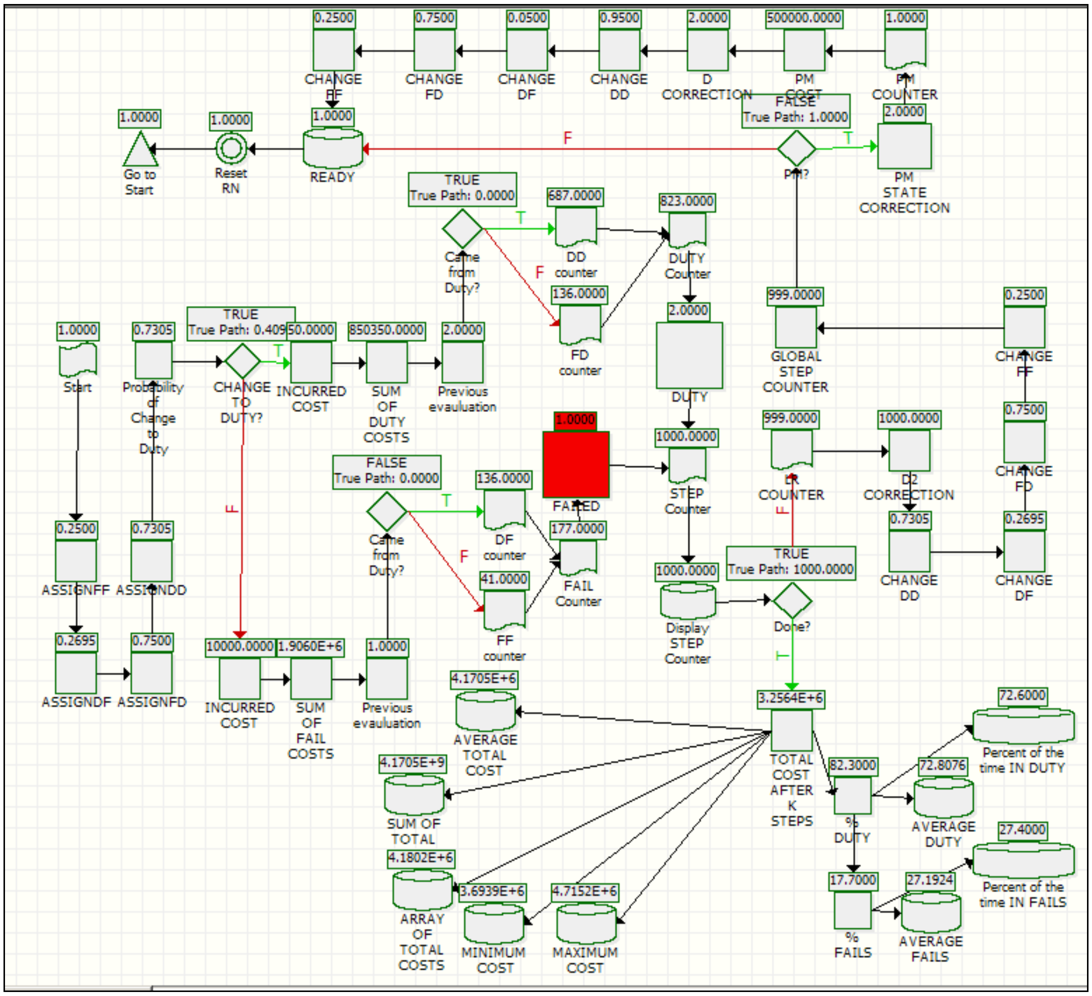


Figure 34. Model C flowchart.

Besides the modifications made in order to adjust to the new models, a couple of changes were also made just to improve the quality and quantity of the data. These extra modifications were made with the finality of record not only the number of times that the component went to a certain state, but to also record how many of these came from a certain state and how many came from the other. These modifications basically consisted in the rearrangement of the order of certain blocks and the addition of a couple of decision blocks, and some extra counter blocks.

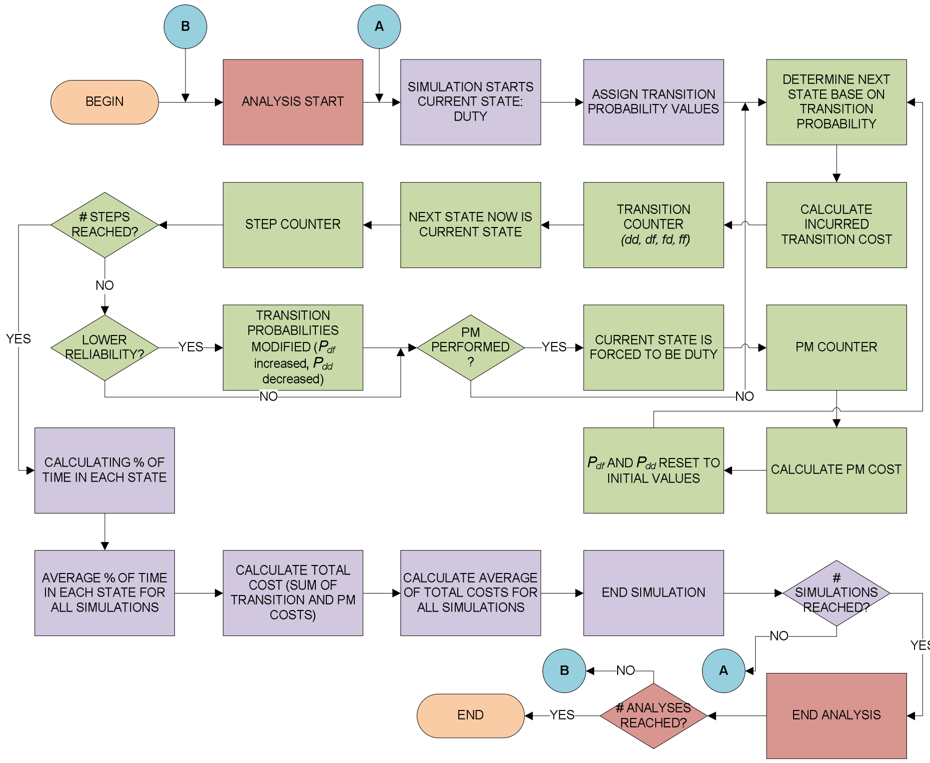


Figure 35. Simplified Block Diagram of Model B simulation process.

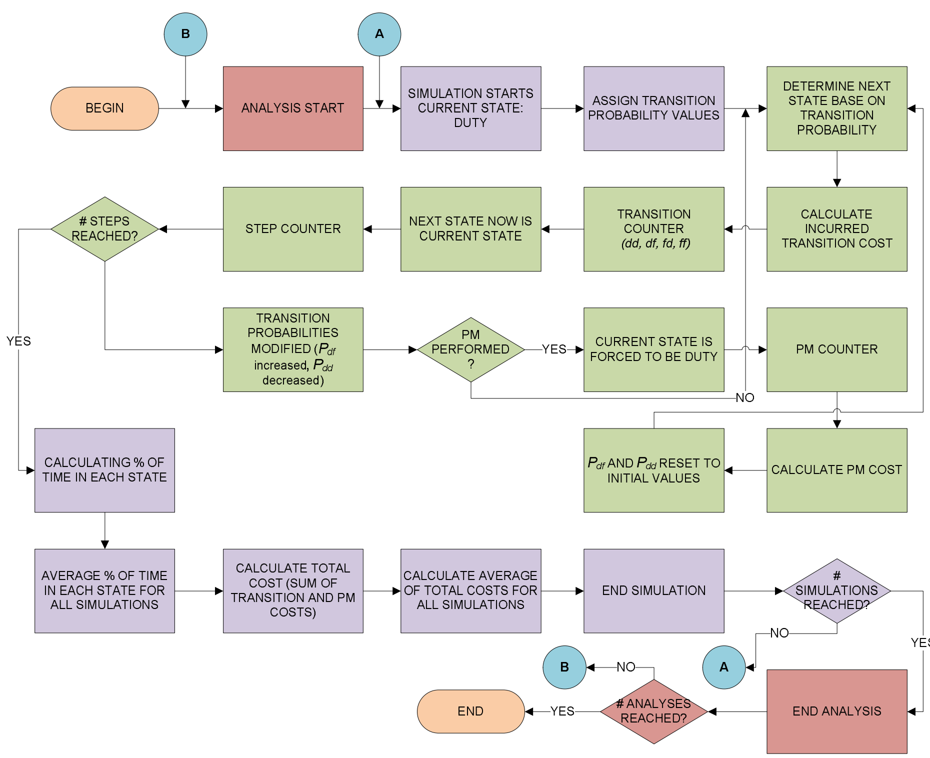


Figure 36. Simplified Block Diagram of Model C simulation process.

Figure 37 shows the modifications made to the going to duty section. As it can be observed, now the simulations divide the number of times that the component went to Duty state in two parts: the times that changed from duty to duty, and the times that went from fail to duty. Similar changes were made to the going to fail section.

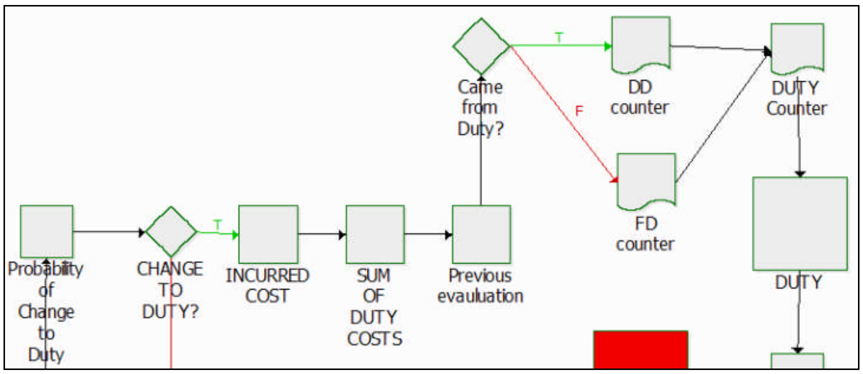


Figure 37. States transition counters modifications .

4.3 SIMULATION RESULTS OF MODEL B AND MODEL C

Multiple simulations of model B were run for different PM costs, in order to analyze the sensitivity of the model, and cover the different possible scenarios that a PM decision maker might come across. In reality, once the PM cost has been calculated for a specific component, multiple runs wouldn't be necessary, and the use of simulation as a tool for maintenance decisions should be faster and more efficient.

Table 9 shows the results of the multiple runs for model B. PM COST column represents the PM cost incurred every time PM is performed. OPTIMAL PM INTERVAL is the interval, in numbers of steps, for which the total average cost is minimum, obtained trough the simulation. NUMBER OF TIMES OF PM indicates how many times PM is performed during the whole simulation process of 1,000 steps. This number is related to the PM interval. i.e. for an optimal PM interval of 100 steps, the simulation performed PM 9 times, since at step 1,000 the run exits the simulation without going trough the PM decision sub chart.

MODEL B RESULTS			
PM COST	OPTIMAL PM INTERVAL (steps)	OPTIMAL AVERAGE TOTAL COST	NUMBER OF TIMES OF PM
$1,000	100	$1.26E+06	9
$20,000	100	$1.43E+06	9
$50,000	100	$1.70E+06	9
$60,000	150	$1.77E+06	6
$70,000	150	$1.83E+06	6
$80,000	150	$1.89E+06	6
$90,000	200	$1.94E+06	4
$100,000	200	$1.98E+06	4
$150,000	260	$2.24E+06	3
$200,000	260	$2.39E+06	3
$300,000	340	$2.63E+06	2
$400,000	380	$2.85E+06	2
$500,000	500	$2.95E+06	1
$1,000,000	500	$3.45E+06	1
$2,000,000	>1000	$3.63E+06	0

Table 9. Simulation results of cost optimization for model B.

Originally, simulations were run only for the following PM costs: $1,000, $20,000, $50,000, $100,000, $150,000, $200,000, $300,000, $400,000, $500,000, $1,000,000 and $2,000,000, in order to find the PM costs in which the optimal PM interval would be minimum (100 steps), maximum (>1000 steps, which means no PM), and some numbers in between. The problem solving method of trial and error was used to find these numbers. Then, it was noticed that there was a big gap between the $50,000 optimal PM interval, and the $100,000 optimal PM interval. As it can be seen in Table 9, for a $50,000 PM cost, PM would be performed every 100 steps which represents 9 PM events; and for a $100,000 PM cost, the optimal PM interval is 200 steps which means only 4 PM events.

Due to this gap between those to PM costs, additional simulations runs were performed for $60,000, $70,000, $80,000 and $90,000 PM costs. These are also included in Table 9.

As expected, the smallest the PM cost is, the more frequent the PM can be performed without increasing the total average cost. For a PM cost, less than or equal to $50,000, the ideal PM interval is 100 steps, which is the minimum interval possible according with the settings of the simulation.

As the PM cost is increased, the PM interval also increased up to the point where PM should not be performed at all, due to the enormous cost incurred when it is performed. Figure 38 shows the relation between PM cost and PM interval of Model B.

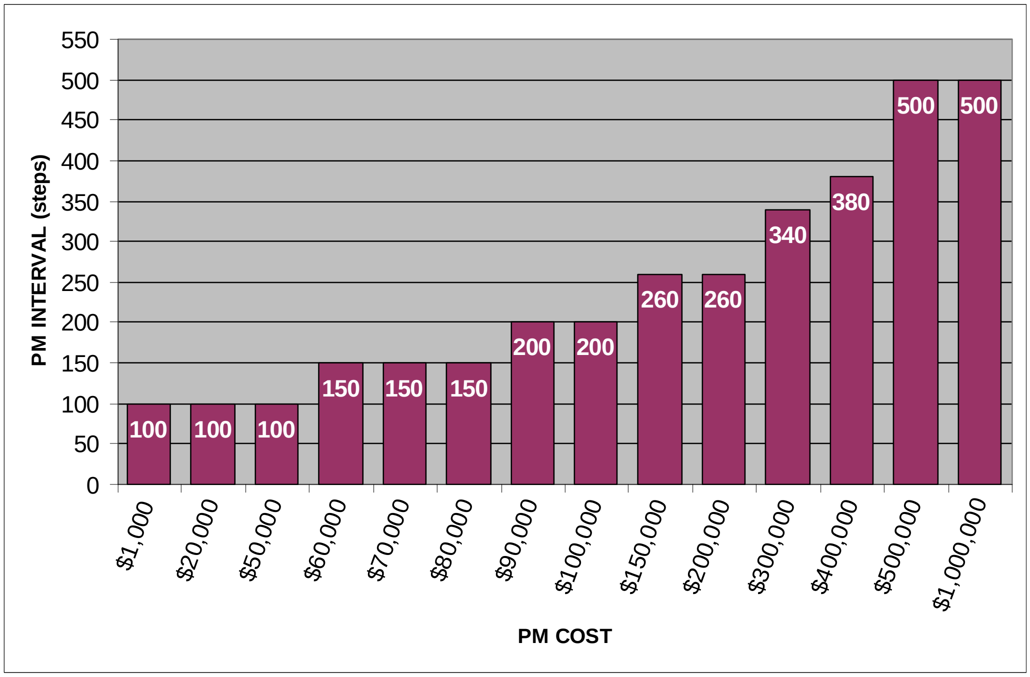


Figure 38. Optimal PM intervals for different PM costs.

For a PM cost of more than or equal to $2,000,000, the simulation results show that PM should be performed more than every 1,000 steps, which in our model actually means, no PM at all, in terms of time, since the simulations are run for 1,000 steps.

It was noticed that for a PM cost of $1,000,000 PM should be performed only once (every 500 steps), but for a $2,000,000 PM cost simulations suggest not to perform PM at all. Between those two PM costs, there should be a range of PM costs for which PM should be performed once, some other range for which PM shouldn't be performed at all, and a specific PM cost for which the optimal PM interval could be either 500 steps (one PM event), or 1,000 steps (no PM). With help of the Reno simulations for a PM cost of $1,000,000 the spreadsheet showed on Table 10 was created in order to find some numbers between the range of one PM event, and none PM events.

PM COST	$1.0M	$1.1M	$1.15M	$1.17M	$1.2M	$1.3M	$1.4M	$1.5M	$2.0M
Analysis 1	AVG TOTAL COST								
100	$10,247,000	$11,147,000	$11,597,000	$11,777,000	$12,047,000	$12,947,000	$13,847,000	$14,747,000	$19,247,000
200	$5,574,700	$5,974,700	$6,174,700	$6,254,700	$6,374,700	$6,774,700	$7,174,700	$7,574,700	$9,574,700
300	$4,823,400	$5,123,400	$5,273,400	$5,333,400	$5,423,400	$5,723,400	$6,023,400	$6,323,400	$7,823,400
400	$4,056,000	$4,256,000	$4,356,000	$4,396,000	$4,456,000	$4,656,000	$4,856,000	$5,056,000	$6,056,000
500	$3,451,300	$3,551,300	$3,601,300	$3,621,300	$3,651,300	$3,751,300	$3,851,300	$3,951,300	$4,451,300
600	$3,496,900	$3,596,900	$3,646,900	$3,666,900	$3,696,900	$3,796,900	$3,896,900	$3,996,900	$4,496,900
700	$3,635,500	$3,735,500	$3,785,500	$3,805,500	$3,835,500	$3,935,500	$4,035,500	$4,135,500	$4,635,500
800	$3,869,000	$3,969,000	$4,019,000	$4,039,000	$4,069,000	$4,169,000	$4,269,000	$4,369,000	$4,869,000
900	$4,197,700	$4,297,700	$4,347,700	$4,367,700	$4,397,700	$4,497,700	$4,597,700	$4,697,700	$5,197,700
1000	$3,622,100	$3,622,100	$3,622,100	$3,622,100	$3,622,100	$3,622,100	$3,622,100	$3,622,100	$3,622,100
MIN COST:	3,451,300	3,551,300	3,601,300	3,621,300	3,622,100	3,622,100	3,622,100	3,622,100	3,622,100

Table 10. Optimal PM interval for PM costs between $1Million and $2Million.

In order to create Table 10, a RENO simulation for a PM cost value of $1,000,000 was performed. With this simulation, the average total cost values shown in the second column of Table 10 for different PM intervals were obtained. The rest of the values for different PM Costs were obtained by numerical methods. Once the simulation has given the average total costs for different PM intervals with certain PM costs, it could be possible to extrapolate and obtain new values for different PM costs. If using an identical seed value, the way the simulation is going to behave for any analysis with the PM interval will be identical, regardless of the

PM cost value. In other words, for this numerical example, the PM cost does not affect the transition between the component states, only the PM interval affects these transitions. For this reason, a different PM cost for a fixed PM interval would only change the Total Average Cost, but not the randomness of the component states, and extrapolated values of different PM costs can be obtained. For instance, the Average Total Cost ($5,333,400) for a 300 PM interval and a $1,170,000 PM cost was obtained by adding the Average Total Cost ($4,823,400) of 300 steps PM interval with a $1,000,000 PM cost plus the additional cost of the new different PM cost. This additional cost is calculated by multiplying the difference between the two different PM costs ($1,170,000-$1,000,000) times the number of times this preventive maintenance would be performed; in the example described above, three times because when the PM interval is 300 steps, PM is performed three times in the scenario described in the simulations. Numerically:

$$ATC(1.17M,300s) = \$4{,}823{,}400 + \left[3 \times \left(\$1{,}170{,}000 - \$1{,}000{,}000\right)\right] = \$5{,}333{,}400 \tag{37}$$

where *ATC(1.17M,300s)* is the Average Total Cost for a $1,170,000 PM cost with a 300 steps PM interval. In general, *ATC(xM,ns)* is the Average Total Cost for a "*x*" PM cost with a "*n*" steps PM interval.

The rest of the values obtained in Table 10 were calculated with the same procedure. It is important to notice that only extrapolations for different values of PM cost can be obtained once a simulation PM interval of interest has been performed. In other words, extrapolations or new values for different PM costs can not be obtained numerically for any PM interval values. In the cases shown in Table 10, it would be impossible to calculate, numerically, the Average Total Cost for PM interval values of 150, 210, 780, or any other value that has not been previously obtained with RENO or any other method.

Once simulations for different PM intervals of interest have been run, tables that are similar to Table 10 can be created by a maintenance department for each particular case, or component subject to be studied. These tables can also be used in the decision making of the maintenance department by analyzing different possible PM cost scenarios, and minimizing their computation times. First, simulations for a vast range of possible PM intervals should be run in order to obtain the Average Total Costs that will be used as a base. Then, the rest of the table for different PM cost scenarios can be created with similar equations to equation (36). This could be done very fast with the use of any spreadsheet application program, instead of repeating the long simulations process for the same PM cost values that have been already run. If more details for a particular new PM cost scenario are needed, computation times for the simulation to be run at this particular case can be minimized with the use of these look-up tables. For instance, assuming that the maintenance department has created Table 11 based on the transition probability and cost matrices of a certain component, and they realized that PM cost is going to change due to external factors. The new PM cost it is calculated to be $220,000, and an extrapolation for this PM cost can also be calculated for this new PM cost value, in order to give to the maintenance department an idea, in rough numbers, of the expected optimal PM interval within the range of the table. (Table 12).

PM COST	$50,000	$100,000	$200,000	$250,000	$300,000	$400,000
Analysis 1	AVG TOTAL COST					
100	$1,697,000	$2,147,000	$3,047,000	$3,497,000	$3,947,000	$4,847,000
200	$1,774,700	$1,974,700	$2,374,700	$2,574,700	$2,774,700	$3,174,700
300	$1,973,400	$2,123,400	$2,423,400	$2,573,400	$2,723,400	$3,023,400
400	$2,156,000	$2,256,000	$2,456,000	$2,556,000	$2,656,000	$2,856,000
500	$2,501,300	$2,551,300	$2,651,300	$2,701,300	$2,751,300	$2,851,300
600	$2,546,900	$2,596,900	$2,696,900	$2,746,900	$2,796,900	$2,896,900
700	$2,685,500	$2,735,500	$2,835,500	$2,885,500	$2,935,500	$3,035,500
800	$2,919,000	$2,969,000	$3,069,000	$3,119,000	$3,169,000	$3,269,000
900	$3,247,700	$3,297,700	$3,397,700	$3,447,700	$3,497,700	$3,597,700
1000	$2,672,100	$2,722,100	$2,822,100	$2,872,100	$2,922,100	$3,022,100
MIN COST:	$1,697,000	$1,974,700	$2,374,700	$2,556,000	$2,656,000	$2,851,300

Table 11. Example of an optimal PM interval look-up table.

Based on Table 11, the exact optimal PM interval can not be obtained for this new PM cost, but the computation and simulation time for obtaining this value with RENO can be minimized to less than one third of the time for the first analysis of a regular simulation. Instead of simulating for PM intervals of 100, 200, 300, 400, 500, 600, 700, 800, 900, and 1,000 steps, simulations settings can be adapted to only evaluate for 200, 300 and 400 steps, since it can be observed in the tables created by the maintenance department that the optimal PM interval should be a number between 200 and 400 steps. Another acceptable option to reduce simulation time would be to set the first simulation analysis for every 20 steps for PM intervals between 200 and 400 (200, 220, 240,…,400). This would not reduce the simulation time of the first analysis but it would reduce the overall simulation time by reducing the number of analyses necessary to find the near optimal PM interval.

PM COST	$220,000
Analysis 1	ATC
100	$3,227,000
200	$2,454,700
300	$2,483,400
400	$2,496,000
500	$2,671,300
600	$2,716,900
700	$2,855,500
800	$3,089,000
900	$3,417,700
1000	$2,842,100
MIN COST:	$2,454,700

Table 12. Extrapolation of Average Total Costs for a PM cost of $220,000.

In Table 10, it can be noticed that the optimal average total cost for PM costs of $1.2M or more, is constant. This makes sense since the optimal PM interval for these costs is more than 1,000 steps, which means no maintenance, and therefore no additional cost due to PM added to the process. Opposite to the optimal total average costs between $1M and $1.2M, which even though they have the same optimal PM interval of 500 steps, they are different, due to the PM cost variation.

It can be shown that the limit between the one PM events and none PM events is when PM cost is $1,170,800 (breakeven cost). The optimal costs associated to this PM cost can be seen in Table 13. This breakeven cost value can be obtained by solving the equation:

$$ATC(x,500s) = \$3,451,300 + \left[1 \times (\$x - \$1,000,000)\right] = \$3,622,100$$

$$\text{then, } x = \left\{ \frac{[\$3,622,100 - \$3,451,300]}{1} + \$1,000,000 \right\} = \$1,170,800$$

PM COST	$1.1708M
Analysis 1	AVG TOTAL COST
100	$11,784,200
200	$6,257,900
300	$5,335,800
400	$4,397,600
500	$3,622,100
600	$3,667,700
700	$3,806,300
800	$4,039,800
900	$4,368,500
1000	$3,622,100
MIN COST:	$3,622,100

Table 13. Optimal average total costs for a $1,170,800 PM cost obtained by numeric method.

In this case, the optimal average total cost can be reached by performing PM only once at 500 steps or by not performing PM at all. Either way, with one PM event or none PM, the expected cost is the same, and in real practice PM would probably not be performed. For this particular numerical experiment, the PM breakeven cost value of $1,170,800 becomes a key number for the maintenance department. If the whole process remains unchangeable, including the transition probabilities and transition costs, PM would be performed only if the cost is less than $1,170,800.

A simulation for this PM breakeven cost was run in Reno, to see what the software would do in the case of having 2 different optimal costs, in other words, which decision the software makes when finding two possible identical optimal solutions. Only two analyses were included in this simulation and the results are shown in Table 14. It can be noticed that, even when there were two exact optimal values for two different PM intervals (500 and 1,000 steps), the program decided to keep evaluating and searching only for numbers close to 500 steps interval value. In other words, RENO assumes that the optimal PM interval after the first Analysis is 500 steps.

Theoretically, for this particular case, any decision of either 500 or 1,000 steps PM interval, should be acceptable, since it would offer the same results in terms of incurred cost; but in practice, it is more likely that if the same cost is going to be incurred regardless PM, the maintenance department would rather not perform PM to this component for obvious reasons.

B MODEL SIMULATION RESULTS FOR A $1,170,800 PM COST			
Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST
100	1.18E+07	200	6.25E+06
200	6.26E+06	260	5.31E+06
300	5.34E+06	320	5.49E+06
400	4.40E+06	380	4.38E+06
500	3.62E+06	440	4.55E+06
600	3.67E+06	500	3.62E+06
700	3.81E+06	560	3.66E+06
800	4.04E+06	620	3.72E+06
900	4.37E+06	680	3.77E+06
1000	3.62E+06	740	3.92E+06
1100	3.63E+06	800	4.03E+06

Table 14. B model simulation results for a PM breakeven cost obtained with RENO.

4.3.1 SOME CONSIDERATIONS AND INTERPRETATIONS ABOUT MODEL B RESULTS

The fact that the optimal PM intervals for first three PM cost in Table 9 are the same (100steps), should not be a concern in terms of having the same identical

PM interval for three complete different PM costs. All the results are suggesting is that PM should be performed as soon as possible, since PM cost is too low to actually affect the total average cost significantly.

SIMULATION RESULTS OF MODEL B FOR A PM COST OF $60,000					
Analysis 1	AVG TOTAL COST	Analysis 2	AVG TOTAL COST	Analysis 3	AVG TOTAL COST
100	1.79E+06	120	1.85E+06	132	1.84E+06
110	1.88E+06	126	1.78E+06	135	1.82E+06
120	1.86E+06	132	1.84E+06	138	1.82E+06
130	1.81E+06	138	1.83E+06	141	1.89E+06
140	1.85E+06	144	1.82E+06	144	1.82E+06
150	1.76E+06	150	1.76E+06	147	1.84E+06
160	1.85E+06	156	1.84E+06	150	1.77E+06
170	1.83E+06	162	1.88E+06	153	1.82E+06
180	1.85E+06	168	1.85E+06	156	1.85E+06
190	1.88E+06	174	1.90E+06	159	1.85E+06
200	1.81E+06	180	1.84E+06	162	1.89E+06
				165	1.88E+06
				168	1.85E+06
SIMULATION RESULTS OF MODEL B FOR A PM COST OF $70,000					
Analysis 1	AVG TOTAL COST	Analysis 2	AVG TOTAL COST	Analysis 3	AVG TOTAL COST
100	1.88E+06	120	1.93E+06	132	1.91E+06
110	1.97E+06	126	1.85E+06	135	1.89E+06
120	1.94E+06	132	1.91E+06	138	1.89E+06
130	1.88E+06	138	1.90E+06	141	1.96E+06
140	1.92E+06	144	1.88E+06	144	1.88E+06
150	1.82E+06	150	1.82E+06	147	1.90E+06
160	1.91E+06	156	1.90E+06	150	1.83E+06
170	1.88E+06	162	1.94E+06	153	1.88E+06
180	1.90E+06	168	1.90E+06	156	1.91E+06
190	1.93E+06	174	1.95E+06	159	1.91E+06
200	1.85E+06	180	1.89E+06	162	1.95E+06
				165	1.94E+06
				168	1.90E+06
SIMULATION RESULTS OF MODEL B FOR A PM COST OF $80,000					
Analysis 1	AVG TOTAL COST	Analysis 2	AVG TOTAL COST	Analysis 3	AVG TOTAL COST
100	1.97E+06	120	2.01E+06	132	1.98E+06
110	2.06E+06	126	1.92E+06	135	1.96E+06
120	2.02E+06	132	1.98E+06	138	1.96E+06
130	1.95E+06	138	1.97E+06	141	2.03E+06
140	1.99E+06	144	1.94E+06	144	1.94E+06
150	1.88E+06	150	1.88E+06	147	1.96E+06
160	1.97E+06	156	1.96E+06	150	1.89E+06
170	1.93E+06	162	2.00E+06	153	1.94E+06
180	1.95E+06	168	1.95E+06	156	1.97E+06
190	1.98E+06	174	2.00E+06	159	1.97E+06
200	1.89E+06	180	1.94E+06	162	2.01E+06
				165	2.00E+06
				168	1.95E+06

Table 15. Model B simulations results for $60,000, $70,000 and $80,000 PM costs.

For the next three PM costs it was found that the optimal PM interval it was also the same number (150 steps). This doesn't necessarily mean that they have exactly the same PM optimal interval, even though they have different PM costs, but as can be noticed in Table 15 and Figure 39, the optimal PM interval for any of these three PM costs can be any number between 147 and 152 steps. This is due to the number of analyses run for each PM cost. For model B, three analyses were run for every PM cost. If a more precise number it is need to be found, then the settings for each simulation should be increase to 5 analyses.

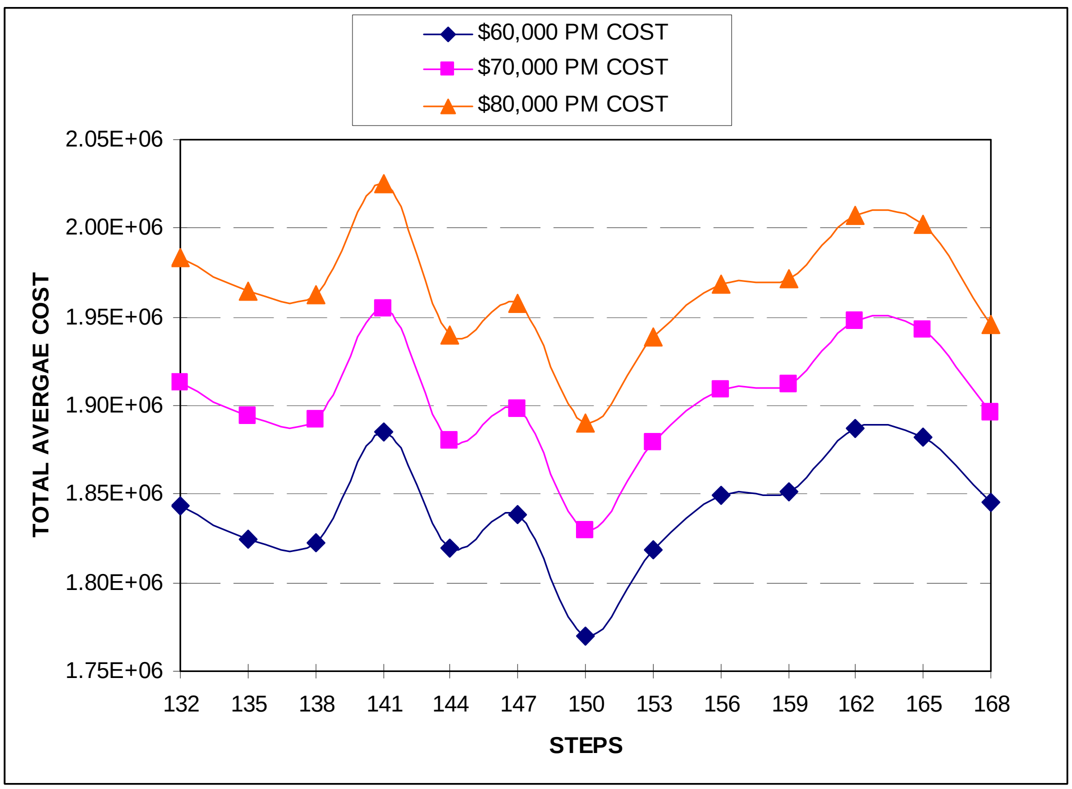


Figure 39. Sketch of Model B simulations results for $60,000, $70,000 and $80,000 PM costs.

More likely, the optimal PM interval for a PM cost of $60,000, $70,000, and $80,000 could be a number lower than 150 steps, a number closer to 150 steps and a number higher than 150 steps, respectively, but between the range of [148-152 steps] mentioned above.

Table 15 and Figure 39 show the sensitivity of Total Average Cost for $60,000, $70,000 and $80,000 PM costs. It should be noticed that, in Figure 39, only the values with a marker (triangles, squares, and rhombuses) are actual values obtained with the simulation. The lines connecting these points are drawn with the only intention to show a possible trend, but they don't represent actual values.

Also, it should be kept in mind that any of these PM intervals in the range of [148-152] steps, are basically suggesting to perform PM only 6 times during the period of study, which is 1,000 steps. Similar analogies can be made for the other cases where PM interval is the same for different PM costs.

The total average costs obtained with the simulations are probabilistic since they are the sum of the cost associated with PM activities plus the total costs associated with the transition between states. The cost associated with PM is deterministic, but the transition costs are stochastic since they depend in the transition between states, which are also probabilistic. Figure 39 shows the randomness of these stochastic costs. It should also be noticed that Figure 39 is only showing results between 132 and 168steps PM intervals, when usually simulations are run for a range of 100 to 1,000 steps intervals. In other words, Figure 39 is a "zoom in" picture of the whole simulation process. A bigger picture of the results would show a smoother line.

It can be noticed in Figure 38 that there are some discontinuities between the different PM costs. These discontinuities can be explained by two main reasons:

1. Only some different PM cost values were used to test the sensitivity of the model. If more PM costs were to be tested, fewer discontinuities would be shown in Figure 38. For instance, between the PM costs of $80,000 and

$90,000 with PM optimal intervals of 150 and 200 steps, respectively, there should be a range of PM costs for which optimal PM interval would be between 150 and 200 steps.

2. A certain range of PM intervals indicates the same number of PM activities. Even if it were possible to simulate a very large number of different PM costs, and the results were shown in a figure similar to Figure 38Figure 38, there would still be discontinuities between the PM costs. In the example mentioned above for PM costs between $80,000 and $90,000, the optimal PM intervals of 150 and 200 steps indicate 6 and 4 PM activities in the simulation model, respectively. If different PM costs were to be tested between the PM cost range of $80,000 and $90,000, a different PM interval that would indicate 5 PM activities would be obtained; this number could be any number between the range of 167-199 steps, since this is the PM interval range that indicates only 5 PM activities. More likely, the optimal PM interval that would be obtained between this range by the simulation, would be 175 steps, since this PM interval would be the one that the simulation would test within that range. If several analyses are run, then any other number between that range can be obtained. For this hypothetical case, the expected figure that would still show discontinuities is shown in Figure 40.

A wide range of different PM costs were tested during the simulations with the only intention of analyzing the sensitivity of the model. It is not expected a maintenance department would ever face such a range of possible PM cost values. In the models of this study, numerical experiments were run for PM costs in the range of $1,000 and $2,000,000. In a regular basis, the maintenance department knows the actual PM cost for the component of interest, or at least, a small range of possible PM costs.

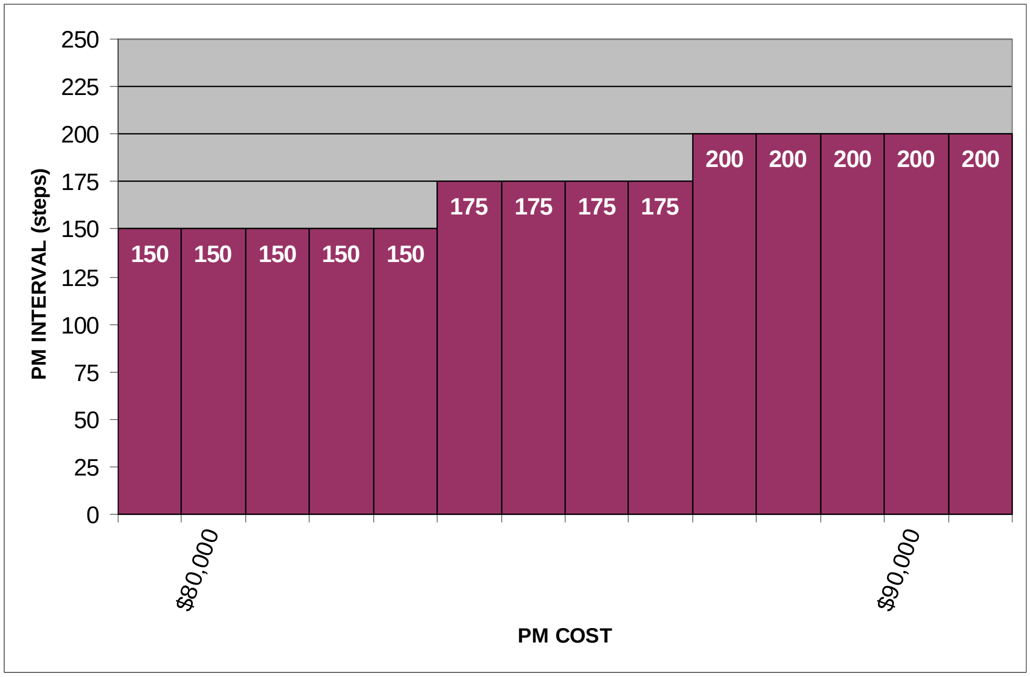


Figure 40. Expected optimal PM intervals for different values between the range of $80,000 and $90,000 PM costs.

It is important to keep in mind that PM costs should be scheduled only if the overall cost of this PM action is smaller than the overall cost of a corrective action. In the numerical experiments of this study, it is assumed that Corrective Cost is always larger than the PM cost values that were tested. Otherwise, PM should not even be considered or analyzed. In other words, the ratio between PM cost and failure replacement (corrective cost) it is always smaller than 1. Mathematically:

$$\frac{PM\ \text{cost}}{CM\ \text{cost}} < 1 \tag{38}$$

Similar results were obtained for Model C. Results of Model C simulations for different PM cost values are included in Appendix A.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

The most important conclusions and a summary of the results obtained in the research for this thesis work are highlighted in this chapter. Future work is suggested based on the work and models included in this research.

5.1 CONCLUSIONS

1. Computer simulation can be an effective tool to represent maintenance systems, and help with maintenance decision making. This research study used computer simulation for minimizing risk of multi-state repairable components with fixed and non-fixed transition probabilities models. The models proposed were imitated through simulation, and based on the results obtained, maintenance decisions could be inferred. The use of look-up tables, by the maintenance department, is suggested for forecasting and prediction of model behavior.

2. Inadequate maintenance decisions can lead to incremental overall costs. On the other hand, adequate maintenance scheduling can help to minimize costs in maintenance. The simulations results in this study suggested that for certain models and variable values, decisions like whether or not to schedule periodic PM

activities, and the periodicity of these PM activities could represent a significant increase or decrease of the overall costs of process subject to study.

3. Programming periodical Preventive Maintenance for a device that follows a negative exponential lifetime distribution does not provide any economic benefits, it actually increases the cost of maintaining the unit. The results obtained for the modeling of the Markov process of a multi-state component with non-dynamic probability matrix showed that PM activities only increase the overall cost of a process that follows exponential lifetime distribution. Several PM costs were tested; all of them offered the same results: PM should not be scheduled for this model.

4. Programming adequate periodical Preventive Maintenance for components with non fixed transition probability matrices and increasing failure rate, can lead to overall costs reduction, and a near optimal PM schedule can be obtained with the help of simulation. Two new models with periodically and continuously increasing failure rate were proposed in this study. Predictions about the overall costs of the process described by these two models were possible with the help of computer simulation.

5.2 FUTURE WORK

Future work and research can be focused in the following directions:

1. This work suggested expertise judgment, historical data and empirical probabilities to obtain transition probabilities and transition costs. Further research can focus methods for estimating these transition values for repairable components and systems, and for assessing confidence in the estimates.

2. All the models proposed in this thesis work are based in single components. Future work may consider more complex scenarios, for systems with two or more components.

3. The computer simulation models proposed in this work may be tested for a real component operating in the industry. Once a component with transition probabilities, increasing failure rate, and similar characteristics to the ones described in this work for the single component subject to study has been tested, the approaches can be developed to generalize for a range of specific components, including generating specific look-up tables for PM scheduling.

4. A general approach can be pursued for applying computer simulation models, similar to the ones proposed in this study, for decision support in different maintenance applications. Similar models may be applied to any area where transition probabilities of certain events can be calculated, and where costs associated to these events can also been estimated. As an example, in medicine, certain medical conditions can degenerate into many different (worst or better) conditions. If a transition probability matrix can be realistically formulated, and a cost or risk associated for these transitions, simulations for better possible treatments and interventions may be performed in order to minimize the impact or risk of a higher illness condition of the patient. Other applications areas likely exist.

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APPENDIX A: MODEL C SIMULATION RESULTS

MODEL C SIMULATION RESULTS, $1,000 PMCOST					
Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST	Analysis 3 - PM	AVERAGE TOTAL COST
100	1,420,600	100	1,415,500	100	1,417,600
200	1,807,600	130	1,527,500	109	1,440,700
300	2,100,200	160	1,644,000	118	1,471,900
400	2,379,100	190	1,744,300	127	1,524,900
500	2,837,900	220	1,831,100	136	1,541,600
600	2,888,500	250	1,988,900	145	1,591,100
700	3,045,500	280	2,032,300	154	1,617,600
800	3,305,600	310	2,138,900	163	1,658,300
900	3,679,000	340	2,290,700	172	1,685,900
1,000	4,164,400	370	2,325,000	181	1,704,900
1,100	4,170,500	400	2,371,800	190	1,745,800

MODEL C SIMULATION RESULTS, $20,000 PMCOST					
Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST	Analysis 3 - PM	AVERAGE TOTAL COST
100	1,591,600	100	1,586,500	100	1,588,600
200	1,883,600	130	1,660,500	109	1,611,700
300	2,157,200	160	1,758,000	118	1,623,900
400	2,417,100	190	1,839,300	127	1,657,900
500	2,856,900	220	1,907,100	136	1,674,600
600	2,907,500	250	2,045,900	145	1,705,100
700	3,064,500	280	2,089,300	154	1,731,600
800	3,324,600	310	2,195,900	163	1,772,300
900	3,698,000	340	2,328,700	172	1,780,900
1,000	4,164,400	370	2,363,000	181	1,799,900
1,100	4,170,500	400	2,409,800	190	1,840,800

MODEL C SIMULATION RESULTS, $50,000 PMCOST					
Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST	Analysis 3 - PM	AVERAGE TOTAL COST
100	1,861,600	100	1,856,500	100	1,858,600
200	2,003,600	130	1,870,500	109	1,881,700
300	2,247,200	160	1,938,000	118	1,863,900
400	2,477,100	190	1,989,300	127	1,867,900
500	2,886,900	220	2,027,100	136	1,884,600
600	2,937,500	250	2,135,900	145	1,885,100
700	3,094,500	280	2,179,300	154	1,911,600
800	3,354,600	310	2,285,900	163	1,952,300
900	3,728,000	340	2,388,700	172	1,930,900
1,000	4,164,400	370	2,423,000	181	1,949,900
1,100	4,170,500	400	2,469,800	190	1,990,800

MODEL C SIMULATION RESULTS, $60,000 PMCOST					
Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST	Analysis 3 - PM	AVERAGE TOTAL COST
100	1,951,600	100	1,946,500	100	1,948,600
200	2,043,600	130	1,940,500	112	1,928,600
300	2,277,200	160	1,998,000	124	1,979,500
400	2,497,100	190	2,039,300	136	1,963,000
500	2,896,900	220	2,067,100	148	1,944,800
600	2,947,500	250	2,165,900	160	1,994,000
700	3,104,500	280	2,209,300	172	1,990,100
800	3,364,600	310	2,315,900	184	2,014,800
900	3,738,000	340	2,408,700	196	2,076,600
1,000	4,164,400	370	2,443,000	208	2,051,600
1,100	4,170,500	400	2,489,800	220	2,078,500

MODEL C SIMULATION RESULTS, $70,000 PMCOST					
Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST	Analysis 3 - PM	AVERAGE TOTAL COST
100	2,041,600	100	2,036,500	130	2,012,700
110	2,073,600	106	2,056,300	133	2,009,600
120	2,039,900	112	2,019,800	136	2,021,400
130	2,009,700	118	2,029,000	139	2,051,300
140	2,051,900	124	2,054,900	142	2,063,300
150	2,012,200	130	2,008,500	145	2,005,100
160	2,052,500	136	2,027,500	148	2,015,700
170	2,027,400	142	2,059,000	151	2,014,800
180	2,054,800	148	2,002,800	154	2,025,000
190	2,088,400	154	2,028,500	157	2,035,200
200	2,086,600	160	2,051,000	160	2,049,500

MODEL C SIMULATION RESULTS, $80,000 PMCOST					
Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST	Analysis 3 - PM	AVERAGE TOTAL COST
100	2,131,600	100	2,126,500	100	2,128,600
200	2,123,600	140	2,128,700	120	2,111,800
300	2,337,200	180	2,096,100	140	2,115,500
400	2,537,100	220	2,153,600	160	2,114,600
500	2,916,900	260	2,231,400	180	2,099,100
600	2,967,500	300	2,333,300	200	2,127,500
700	3,124,500	340	2,450,200	220	2,162,900
800	3,384,600	380	2,478,300	240	2,250,400
900	3,758,000	420	2,595,400	260	2,241,800
1,000	4,164,400	460	2,761,200	280	2,270,500
1,100	4,170,500	500	2,915,200	300	2,338,600

MODEL C SIMULATION RESULTS, $90,000 PMCOST					
Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST	Analysis 3 - PM	AVERAGE TOTAL COST
100	2,221,600	100	2,216,500	100	2,218,600
200	2,163,600	140	2,198,700	120	2,191,800
300	2,367,200	180	2,146,100	140	2,185,500
400	2,557,100	220	2,193,600	160	2,174,600
500	2,926,900	260	2,261,400	180	2,149,100
600	2,977,500	300	2,363,300	200	2,167,500
700	3,134,500	340	2,470,200	220	2,202,900
800	3,394,600	380	2,498,300	240	2,290,400
900	3,768,000	420	2,615,400	260	2,271,800
1,000	4,164,400	460	2,781,200	280	2,300,500
1,100	4,170,500	500	2,925,200	300	2,368,600

MODEL C SIMULATION RESULTS, $100,000 PMCOST					
Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST	Analysis 3 - PM	AVERAGE TOTAL COST
100	2,311,600	100	2,306,500	100	2,308,600
200	2,203,600	140	2,268,700	120	2,271,800
300	2,397,200	180	2,196,100	140	2,255,500
400	2,577,100	220	2,233,600	160	2,234,600
500	2,936,900	260	2,291,400	180	2,199,100
600	2,987,500	300	2,393,300	200	2,207,500
700	3,144,500	340	2,490,200	220	2,242,900
800	3,404,600	380	2,518,300	240	2,330,400
900	3,778,000	420	2,635,400	260	2,301,800
1,000	4,164,400	460	2,801,200	280	2,330,500
1,100	4,170,500	500	2,935,200	300	2,398,600

MODEL C SIMULATION RESULTS, $150,000 PMCOST					
Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST	Analysis 3 - PM	AVERAGE TOTAL COST
100	2,761,600	100	2,756,500	100	2,758,600
200	2,403,600	140	2,618,700	124	2,701,200
300	2,547,200	180	2,446,100	148	2,481,900
400	2,677,100	220	2,433,600	172	2,434,800
500	2,986,900	260	2,441,400	196	2,525,500
600	3,037,500	300	2,543,300	220	2,436,800
700	3,194,500	340	2,590,200	244	2,559,500
800	3,454,600	380	2,618,300	268	2,460,000
900	3,828,000	420	2,735,400	292	2,517,900
1,000	4,164,400	460	2,901,200	316	2,629,000
1,100	4,170,500	500	2,985,200	340	2,592,100

MODEL C SIMULATION RESULTS, $200,000 PMCOST					
Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST	Analysis 3 - PM	AVERAGE TOTAL COST
100	3,211,600	100	3,206,500	140	2,966,300
200	2,603,600	140	2,968,700	164	2,851,000
300	2,697,200	180	2,696,100	188	2,725,000
400	2,777,100	220	2,633,600	212	2,622,000
500	3,036,900	260	2,591,400	236	2,693,200
600	3,087,500	300	2,693,300	260	2,598,700
700	3,244,500	340	2,690,200	284	2,645,500
800	3,504,600	380	2,718,300	308	2,736,700
900	3,878,000	420	2,835,400	332	2,879,400
1,000	4,164,400	460	3,001,200	356	2,702,200
1,100	4,170,500	500	3,035,200	380	2,733,600

MODEL C SIMULATION RESULTS, $300,000 PMCOST

Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST	Analysis 3 - PM	AVERAGE TOTAL COST
100	4,111,600	100	4,106,500	160	3,434,700
200	3,003,600	160	3,433,200	196	3,267,200
300	2,997,200	220	3,042,100	232	3,075,600
400	2,977,100	280	2,935,400	268	2,911,400
500	3,136,900	340	2,883,300	304	3,013,500
600	3,187,500	400	2,973,600	340	2,892,600
700	3,344,500	460	3,199,800	376	2,934,200
800	3,604,600	520	3,129,600	412	3,013,300
900	3,978,000	580	3,163,300	448	3,146,900
1,000	4,164,400	640	3,241,600	484	3,332,100
1,100	4,170,500	700	3,337,700	520	3,140,900

MODEL C SIMULATION RESULTS, $400,000 PMCOST

Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST	Analysis 3 - PM	AVERAGE TOTAL COST
100	5,011,600	100	5,006,500	160	4,034,700
200	3,403,600	160	4,033,200	196	3,767,200
300	3,297,200	220	3,442,100	232	3,475,600
400	3,177,100	280	3,235,400	268	3,211,400
500	3,236,900	340	3,083,300	304	3,313,500
600	3,287,500	400	3,173,600	340	3,092,600
700	3,444,500	460	3,399,800	376	3,134,200
800	3,704,600	520	3,229,600	412	3,213,300
900	4,078,000	580	3,263,300	448	3,346,900
1,000	4,164,400	640	3,341,600	484	3,532,100
1,100	4,170,500	700	3,437,700	520	3,240,900

MODEL C SIMULATION RESULTS, $500,000 PMCOST

Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST	Analysis 3 - PM	AVERAGE TOTAL COST
100	5,911,600	200	3,801,400	200	3,808,700
200	3,803,600	260	3,498,800	236	3,894,100
300	3,597,200	320	3,706,800	272	3,511,900
400	3,377,100	380	3,330,400	308	3,640,300
500	3,336,900	440	3,510,300	344	3,292,300
600	3,387,500	500	3,332,600	380	3,333,500
700	3,544,500	560	3,351,800	416	3,428,500
800	3,804,600	620	3,399,900	452	3,562,500
900	4,178,000	680	3,499,600	488	3,760,800
1,000	4,164,400	740	3,640,800	524	3,338,100
1,100	4,170,500	800	3,801,000	560	3,355,200

MODEL C SIMULATION RESULTS, $1,000,000 PMCOST

Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST	Analysis 3 - PM	AVERAGE TOTAL COST
100	10,412,000	200	5,801,400	320	5,200,300
200	5,803,600	260	4,998,800	356	4,294,500
300	5,097,200	320	5,206,800	392	4,348,800
400	4,377,100	380	4,330,400	428	4,468,500
500	3,836,900	440	4,510,300	464	4,614,400
600	3,887,500	500	3,832,600	500	3,837,200
700	4,044,500	560	3,851,800	536	3,847,100
800	4,304,600	620	3,899,900	572	3,863,500
900	4,678,000	680	3,999,600	608	3,894,300
1,000	4,164,400	740	4,140,800	644	3,943,900
1,100	4,170,500	800	4,301,000	680	4,004,500

MODEL C SIMULATION RESULTS, $2,000,000 PMCOST

Analysis 1 - PM	AVERAGE TOTAL COST	Analysis 2 - PM	AVERAGE TOTAL COST	Analysis 3 - PM	AVERAGE TOTAL COST
100	19,412,000	700	5,033,800	900	5,677,200
200	9,803,600	740	5,138,100	920	5,755,600
300	8,097,200	780	5,252,700	940	5,860,200
400	6,377,100	820	5,373,000	960	5,959,200
500	4,836,900	860	5,512,100	980	6,051,100
600	4,887,500	900	5,670,800	1,000	4,171,100
700	5,044,500	940	5,860,800	1,020	4,171,400
800	5,304,600	980	6,049,500	1,040	4,171,800
900	5,678,000	1,020	4,162,500	1,060	4,166,000
1,000	4,164,400	1,060	4,171,300	1,080	4,169,500
1,100	4,170,500	1,100	4,162,500	1,100	4,169,800