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Solution techniques for transient stability-constrained optimal power flow – Part I

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Abstract: This series of studies present the state-of-the-art for the solution of the transient stability constrained optimal power flow problem (TSC-OPF). Three different classes of solution techniques: dynamic optimisation-based, SIME method, and computational intelligence, are discussed in detail. Moreover, discussed are issues to consider while solving such problems, various application areas, and future directions in this research area. A comprehensive resource of the available literature, publicly available test systems, and relevant numerical libraries is also provided. This study presents the TSC-OPF formulation and discusses various dynamic optimisation-based approaches. Two optimisation techniques, full-space and reduced-space method, are presented for solving the resulting non-linear optimisation problem.

1 Introduction

The economic and secure operation of electric power systems is of paramount importance to the utilities and regulatory authorities around the world. Deregulation of vertically integrated utilities, while ensuring an optimal operation of the power grid, has pushed the operation to its limits. Moreover, in recent years an effort has been made to utilise clean renewable generation sources and more efficient units such as combined-cycle units. The introduction of these units and their proliferation in the future implies a reduction in the system inertia that has been regarded as a concern [1]. This situation presents new technical challenges, particularly in the reduction of system inertia through the displacement of conventional generation resources during light load periods [1].

The security analysis of electrical power systems is done by a steady state optimisation technique, referred to as security constrained optimal power flow (SCOPF) in power system parlance. An SCOPF determines a secure and optimal operating point given the different steady-state security constraints such as power balance, line flows, voltage, and capacity constraints. However, such a solution is valid only for steady-state operation since SCOPF ignores transient stability constraints. As a result, the system may undergo instability on the inception of credible transient events even though the SCOPF solution is secure. The existing industry practice to maintain transient stability is by planning mitigation schemes based on off-line stability studies. Since these schemes are primarily targeted at maintaining system reliability, they may not be optimal. Moreover, they may cause discrimination in the market players, particularly when the system is stressed [2]. Thus, ensuring dynamic security while maintaining cost-effective operation is an important emerging problem.

This series of papers present the state-of-the-art for optimal power flow (OPF) solution while considering transient stability constraints. This problem has been given acronyms such as SOPF (stability constrained OPF), OTS (OPF with transient stability constraints) or TSOPF or TSC-OPF or TSC-OPF (transient stability-constrained OPF). In this series, we refer to the problem as TSCOPF. This series of papers offers a rich single resource to the readers by (i) providing a comprehensive survey of TSC-OPF techniques including explanation of their key concepts, (ii) identifying the challenges in solving such problems, (iii) providing an extensive and comprehensive list of literature on TSC-OPF methods, (iv) including information on test data systems for simulation and analysis, and (v) providing directions of future research work.

The taxonomy of methods for solving the TSC-OPF problem is shown in Fig. 1. This paper focuses on the dynamic optimisation methods for solving TSC-OPF. The SIME method and computationally intelligent approaches are described in Part II of this series of papers.

We note that among all the TSC-OPF approaches surveyed in this two-part paper, the selection of a proper one for a given research topic or practical application is highly dependent on end-user requirement. There is no optimal method that is able to solve all the problems. One has to select a proper approach from the

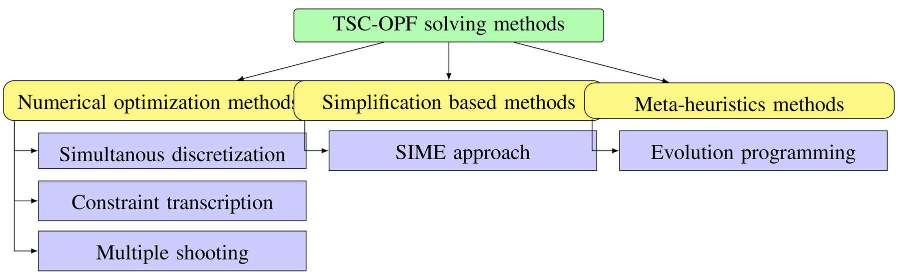

Fig. 1 *Taxonomy of dynamic constraints incorporating methods for TSC-OPF problems*

trade-off between model complexity and computational efficiency, according to its own need.

A TSC-OPF problem with very detailed model is able to capture realistic system dynamics but commonly suffers from low-computational efficiency. For example, one may interface an external time-domain simulator (e.g. PSS/E) with a meta-heuristics-based searching algorithm, as described in Section 3 of Part II. Then, the solution is guaranteed to be consistent with the simulator and able to describe detailed system dynamics. However, the computational burden may be unacceptable due to large number of simulation requests and possible unsatisfied convergence performance. Therefore, this setup is only useful for the application scenarios with no or less computational time requirement, e.g. system planning or day-ahead decision-making.

On the other hand, in the context of fast decision-making, such as real-time or hour-ahead operation, one has to tailor the TSC-OPF formulation so as to ensure its computational performance. In this circumstance, simplification-based method such as SIME is preferred, which is discussed in Section 2 of Part II. Even though it is unable to provide a very accurate description of system dynamics, it is still capable enough to provide a rough estimation on certain stability constraints, e.g. first swing of rotor-angle stability. Similar comparison is provided in Table 2 of Part I, in the framework of numerical optimisation.

In summary, as a general recommendation from the authors, one is suggested to choose a proper TSC-OPF approach according to the request of computational time frame and system dynamic model. For planning stage problems where long computing time is permitted, it is recommended to use computational intelligence or constraint transcription approaches, so as to interface with external time-domain simulator with detailed dynamic model. For operation stage problems where computing time is a concern, SIME and simultaneous discretisation approaches is recommended as they are commonly fast and robust, despite of some limitations in describing complicated system dynamic behaviour. Multiple shooting approach is right in the middle of the aforementioned trade-off, it is able to be gain additional acceleration through the parallel processing of multiple shooting intervals.

2 TSC-OPF formulation

TSC-OPF is an optimal control or dynamic optimisation problem that extends the OPF problem to include system dynamics equations and transient stability constraints. In compact form, the TSC-OPF can be described by the following set of equations

$$\text{min or max} \quad C(p) \tag{1}$$

$$\text{s.t.} \quad g_s(p) = 0 \tag{2}$$

$$h_s^- \le h_s(p) \le h_s^+ \tag{3}$$

$$p^- \le p \le p^+ \tag{4}$$

$$\dot{x} = f(x, y, p), \quad x(t_0) = I_{x0}(p) \tag{5}$$

$$0 = g(x, y, p), \quad y(t_0) = I_{y0}(p) \tag{6}$$

$$h(x(t), y(t)) \le 0, \quad \forall (t) \tag{7}$$

In the following subsections we describe the different components in (1)–(7) that are also used in the description of the different dynamic optimisation methods. Contingencies can be also included in this formulation for both steady state and transient state. For notational ease, we have avoided incorporating contingencies in the formulation.

In the following subsections we describe the different components of this formulation.

Table 1 TSC-OPF objective function

Objective function	References
minimise generation cost	[2–14]
minimise transmission losses	[8, 9]
maximise transfer capability	[4, 14–20]

2.1 Objective function

The objective function (1), depending on the study of interest, can involve minimising the generation cost, minimising the transmission line losses or maximising the available transfer capability.

Here, $p \equiv [P_g, Q_g, \theta, V]^\mathrm{T} \in \mathbb{R}^{n_p}$ are the optimising variables inherited from OPF, namely, the real and reactive power generation and the bus voltage angles and magnitudes. Table 1 lists the different objective functions used in the literature.

2.2 Steady-state equality constraints

The steady state equality constraints (2) consist of nodal power balance or current balance equations. As an example, the nodal power balance equality constraints with the voltage expressed in polar form are given by

$$P_i^{\text{inj}} - \sum_{k=1}^{n} V_i V_k (G_{ik} \cos(\theta_{ik}) + B_{ik} \sin(\theta_{ik})) = 0 \tag{8}$$

$$Q_i^{\text{inj}} - \sum_{k=1}^{n} V_i V_k (G_{ik} \sin(\theta_{ik}) - B_{ik} \cos(\theta_{ik})) = 0 \tag{9}$$

2.3 Steady state inequality constraints

The apparent power, real power or current flow along transmission lines typically constitute the steady-state inequality constraints (3) as given by

$$\Psi_l^- \le \Psi_l \le \Psi_l^+, \quad l = 1 \ldots nlines \tag{10}$$

Here, Ψ_l represents the constrained flow on the transmission line l.

2.4 Steady-state voltage limits and capacity limits

Equations (11)–(13) represent the constraints on voltages and the generator real and reactive powers that constitute the box constraints in (4)

$$P_{\mathrm{G}k}^- \le P_{\mathrm{G}k} \le P_{\mathrm{G}k}^+, \quad k = 1 \ldots ngen \tag{11}$$

$$Q_{\mathrm{G}k}^- \le Q_{\mathrm{G}k} \le Q_{\mathrm{G}k}^+, \quad k = 1 \ldots ngen \tag{12}$$

$$V_i^- \le V_i \le V_i^+, \quad i = 1 \ldots nbus \tag{13}$$

$P_{\mathrm{G}k}$ and $Q_{\mathrm{G}k}$ are the real and reactive power of generator k, respectively, and V_i is the voltage magnitude of bus i.

2.5 System dynamics equations [21, 22]

The analysis of power systems when subjected to large disturbances such as faults or equipment outages, is done using a set of differential-algebraic equations (DAE) (5) and (6). $x \in \mathbb{R}^{nx}$ represents the dynamic states that have power system models described by differential equations, and $y \in \mathbb{R}^{ny}$ represents the algebraic states. Typically, the differential equations describe the electromechanical machine dynamics, while the network equations form the algebraic equations. I_{x0} and I_{y0} are functions that describe the relation between the initial condition, $(x(t_0), y(t_0))$, of the dynamic states and the optimisation variables p.

The most widely used generator model described in the literature is the second-order classical generator model that describes the dynamics for the machine rotor angle and speed

$$\frac{d\delta}{dt} = \Delta\omega$$
$$\frac{2H}{\omega_s}\frac{d\Delta\omega}{dt} = P_m - P_e - D\Delta\omega \tag{14}$$

Here δ and $\Delta\omega$ are the machine rotor angle and the speed deviation, respectively. The algebraic network equations in the current balance form are given by:

$$\begin{bmatrix} Y_R & -Y_I \\ Y_I & Y_R \end{bmatrix}\begin{bmatrix} V_R \\ V_I \end{bmatrix} = \begin{bmatrix} I_R \\ I_I \end{bmatrix} \tag{15}$$

In (15), subscripts R and I denote the real and imaginary parts, $\boldsymbol{Y}$ is the complex admittance matrix; $\boldsymbol{V}$ is the vector of complex bus voltages; and $\boldsymbol{I}$ is the vector of complex current injections from the generators and loads. The action of discrete events, such as fault incidence/removal, transmission line switching, load loss or generation tripping, is incorporated by modifying the differential or algebraic equations at the time of the discrete event. This involves computing a predisturbance and postdisturbance solution. The mechanism of incorporating such discrete events in a transient stability simulation is described in detail in [21].

The representation of such discrete events and the trajectory sensitivity analysis were investigated in the vision of hybrid system, discussed in [23]. By describing the jump conditions, discrete events observed in system dynamics, including switching and state resetting, are modelled and analysed. This enables one to model such discrete events in a trajectory sensitivity enabled TSC-OPF approach, including constraint transcription and multiple shooting. However, their convergence property and other performance issues remain to be investigated.

2.6 Transient stability constraints

The transient stability constraints (7) yield a measure of the security and stability of the system. These could express a measure of the instability of the system or the violation of the dynamic states. When satisfied, the constraints ensure that the system is transiently secure for the given generation dispatch. The transient stability could be assessed by different measures. We highlight a few as used in the literature.

- A common measure used in the literature is the deviation of the machine rotor angle from its centre of inertia reference [2, 12, 18]

$$\delta_i(t) - \frac{1}{\sum_{i=1}^{ngen} H_i}\sum_{i=1}^{ngen} H_i\delta_i(t) \leq \delta^+ \tag{16}$$

  Here, H_i is the inertia constant of the ith generator, and $\delta_i(t)$ is its machine rotor angle at time t. The maximum rotor angle deviation δ^+ is the maximum allowed rotor angle deviation. The typical values used for δ^+ can be found in [24].
- Literature [3, 25, 7] use a dot product test based on potential energy boundary surface

$$\sum_{i}^{ngen} P_{ai}(\delta_i - \delta_{s.e.p.}) \tag{17}$$

  Here, P_{ai} is the accelerating power of the ith generator with corresponding machine angle δ_i, and $\delta_{s.e.p.}$ is its machine angle at the stable equilibrium point.
- Constraints can be also imposed on the network voltages and the transmission line flows through the postdisturbance period as considered in [7, 22]. Paramsivam *et al.* [26] define a voltage performance criterion that takes into account transient voltage dips and delayed voltage recovery phenomena.
- There are approaches to transient stability-constrained optimal power flow, in which the SIME method provides stability constraints that are directly expressed in terms of the power limit of critical generators or in terms of the power limits in lines, that do not need to include the sets of stability constraints and of dynamic constraints in the formulation, and that can use a conventional OPF for the system optimisation [15, 27]. These methods were able from the beginning to analyse large power systems using detailed modelling, as shown in the references [14, 15, 20, 27].

From the viewpoint of formulating and solving TSC-OPF problems, the stability criteria are essentially functions of post-disturbance state variables of the studied system. Basically, there are two approaches to integrate the criterion into the optimisation formulation.

(i) Individual constraints on the post-disturbance state variables, e.g. rotor angle constraints 16 used in [2, 12, 18]. This approach results in a large amount of constraints that may slow down the solving process. It is able to provide better convergence property by considering explicit constraints for various time steps.
(ii) Aggregated constraints on the post-disturbance state variables, e.g. [4, 7, 28, 29]. Instead of enforcing a large number of state variables in a given region, a stability index in the form of the integral of state variables exceeding given thresholds over time is established, then an aggregated constraint is set up to make sure the value of the stability index equal to zero. This approach is able to significantly reduce the dimension of resultant non-linear programming (NLP) model but may encounter convergence issue.

In its existing form, as given by (1)–(7), the TSC-OPF problem cannot be solved by using a non-linear optimisation solver because of the presence of differential equations. In the following sections we discuss different solution approaches such as simultaneous discretisation or discretise-then-optimise, constraint transcription or semi-infinite programming, and the multiple shooting.

3 Dynamic optimisation methods

TSC-OPF is a non-linear programming problem with DAE constraints. Mathematically, such problems can be solved by either indirect or direct dynamic optimisation methods [30]. Indirect methods, also known as variational methods, are based on the calculus of variations and Pontryagin's minimum principle in optimal control. On the other hand, direct methods use NLP algorithms after incorporating dynamic constraints into the optimisation formulation. Various TSC-OPF problems in the literature have been solved by direct methods because of their robustness in convergence, and capability to treat inequality constraints. Three main variants of direct methods for solving dynamic optimisation problems are simultaneous discretisation [31], constraint transcription [32], and multiple shooting [33]. These are described in Sections 3.1, 3.2, and 3.3, respectively. A comprehensive comparison among direct methods-based TSC-OPF algorithms from a qualitative and quantitative perspective is given in Table 2.

3.1 Simultaneous discretisation [2, 8, 22, 35]

In the direct simultaneous or discretise-then-optimise approach, the differential equations for all time steps are discretised to non-linear algebraic equations by using a numerical integration scheme. These non-linear algebraic equations are then incorporated as equality constraints in the TSC-OPF formulation. Note that any implicit/explicit single/multiple-step discretising method can be used in this approach. For example, using the second order implicit trapezoidal method, a common choice for the numerical discretisation, the resultant discretised non-linear equations for each integration step with time step Δt are given as follows

$$\begin{aligned} x(t) - \frac{\Delta t}{2}f(x(t), y(t), p) - x(t-\Delta t) \\ - \frac{\Delta t}{2}f(x(t-\Delta t), y(t-\Delta t), p) = 0 \\ g(x(t), y(t)) = 0 \end{aligned} \tag{18}$$

Table 2 Comparison of dynamic optimisation approaches

Attribute	Simultaneous discretisation	Constraint transcription	Multiple shooting
problem dimension	very large	same as OPF	moderate (depends on number of shooting steps)
floating-point operations	most	least	moderate
convergence	fast	slow	moderate
time-step adaptivity	no	yes	yes
computational bottleneck	linear solve update	sensitivity calculation	sensitivity calculation
sparsity	very sparse	similar to the OPF problem with dense blocks	moderate
Hessian	exact Hessian can be calculated	BFGS (Broyden-Fletcher-Goldfarb-Shanno) algorithm [34] approximated Hessian	BFGS (Broyden-Fletcher-Goldfarb-Shanno) algorithm [34]

The converted constraints are considered in the NLP formulation (19)–(25)

$$\min \text{ or } \max \quad C(p) \tag{19}$$

$$\text{s.t.} \quad g_s(p) = 0 \tag{20}$$

$$h_s^- \le h_s(p) \le h_s^+ \tag{21}$$

$$p^- \le p \le p^+ \tag{22}$$

$$F(x(t), x(t-\Delta t), y(t), y(t-\Delta t), p) = 0, \forall(t) \tag{23}$$

$$0 = g(x(t), y(t), p) \quad \forall(t) \tag{24}$$

$$h(x(t), y(t)) \le 0, \quad \forall(t) \tag{25}$$

The resultant NLP can be solved by using any standard NLP solver such as interior-point, reduced-space or active-set method. When the NLP solver reaches an optimal solution, all the resultant algebraic constraints are satisfied. Thus, a set of continuous trajectories for system state variables is achieved. That is, the simulation problem is solved simultaneously with the optimisation problem. The advantage of the simultaneous discretisation approach is that it can treat unstable systems and handle different constraints robustly [22]. Another advantage is that obtaining the analytical Hessian matrix is relatively easy. The availability of the Hessian greatly improves the convergence of the NLP solver.

However, the discretise-then-optimise approach leads to a large number of equality constraints to be incorporated. If N is the number of time steps considered for the length of the time horizon, $x \in \mathbb{R}^{n_x}$, $y \in \mathbb{R}^{n_y}$, and transient stability criteria inequality constraints $h \in \mathbb{R}^{n_h}$, then this approach has the approximate dimension $\mathbb{R}^{N(n_x+n_y+n_h)}$. Thus, this formulation quickly becomes computationally intractable having a very large dimension even for medium-sized systems with a moderate time horizon. Another issue with the simultaneous discretisation approach is the difficulty in using adaptive time-steps because of the embedding of the discretised differential equations, since the value of the state variables is unknown during discretisation.

3.2 Constraint transcription [26, 36]

Constraint transcription is an algorithmic framework that decouples optimisation algorithms and simulation tools. Differential

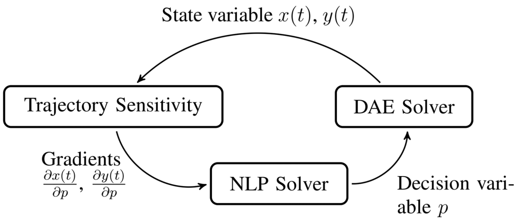

Fig. 2 *Algorithmic framework for constraint transcription*

equations are integrated outside the optimisation process and interfaced with NLP solvers. Many investigations in dynamic optimisation, including the direct sequential approach, direct single-shooting method, and control vector parameterisation, can be classified into this framework.

The general procedure of constraint transcription is expressed as the iterative process shown in Fig. 2. The goal is to find a proper value of decision variable p to minimise a given objective defined in NLP formulation and satisfy a number of constraints over not only decision variable p but also state variable $x(t)$ and $y(t)$. The procedure starts with an initial guess of p determined by user's preference, then the external DAE solver (i.e. time-domain simulator) is called to obtain $x(t)$ and $y(t)$ based on the provided p. Moreover, trajectory sensitivity analysis is performed to find the gradients $(\partial x(t)/\partial p)$ and $(\partial y(t)/\partial p)$. Based on the obtained state variable values and its sensitivity w.r.t. decision variables, the NLP algorithm is able to find out a proper way to adjust p in a new iteration so as to satisfy all the constraints while minimising the value of objective function. After a new p is obtained, DAE solver is called again to compute the updated state variables. This loop iterates until the optimality and feasibility criteria are satisfied.

Since the TSC-OPF problem is an infinite-dimensional problem with infinite dynamic constraints, semi-infinite programming, which is a typical approach of constraint transcription methods, is able to reduce the dimensionality of the problem by considering the dynamic states x, y as implicit functions of the optimisation variables p. Thus, the optimisation problem becomes finite dimensional.

In semi-infinite programming, only the path constraints are incorporated, not the DAE equations. The DAE equations are solved separately in order to obtain path constraints. This process leads to an optimisation problem that has a finite number of variables and an infinite number of constraints. Further, in order to reduce the dimensionality and to allow for smooth approaches, a constraint aggregating procedure, based on smoothing the minmax constraint, is used [36]. Instead of enforcing the constraints at each time step, the evolution of the constraint surface at some final time is used as a constraint, as given by

$$H(x(p,t), y(p,t)) = \sigma \int_0^{\mathrm{T}} [\max(0, h(x(p,t), y(p,t))]^{\eta} \mathrm{d}t = 0 \tag{26}$$

Here, η is an exponent to ensure sufficient smoothness of (26), and σ is a multiplier, similar to a penalty cost term, to ensure a decent progress of the optimisation. Since the equality constraint in (26) cannot be easily handled by optimisation solvers [4], an inequality constraint $H(x, y) \le \rho$ is used instead, where ρ is a positive small number.

With this formulation, replacing the path constraints by $H(x, y) \equiv H(p)$ with H defined by (26) completely defines a smooth problem in p only that can be solved with smooth optimisation tools

$$\begin{aligned} \min \text{ or } \max \quad & C(p) \\ \text{s.t.} \quad & g_s(p) = 0 \\ & h_s^- \le h_s(p) \le h_s^+ \\ & p^- \le p \le p^+ \\ & H(p) \le \rho \end{aligned} \tag{27}$$

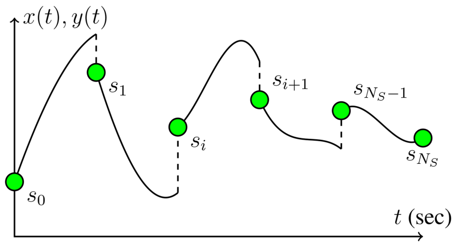

Fig. 3 *Principle of multiple shooting method*

With finite constraints, (27) represents a finite-dimensional problem in p. The major advantage of the constraint transcription approach is that the DAE system can be solved separately, thus utilising the merits of numerical integration schemes such as local truncation error control and adaptive time-stepping.

However, optimisation solvers require the derivative of $H(x(p), y(p))$ with respect to optimisation variables p, that is, the gradient $\nabla_p H$. Computing this gradient is a non-trivial task; different trajectory sensitivity analysis approaches are given in Section 4. With the gradient thus computed, any standard non-linear optimisation solver can be used to solve the TSC-OPF problem.

3.3 Multiple shooting [22, 37]

Multiple shooting, also known as direct multiple shooting, is a hybridisation of simultaneous discretisation and constraint transcription. This combination inherits the advantages of its two predecessors and avoids their drawbacks. This method was well developed for chemical engineering [37, 38] and was recently introduced to solve TSC-OPF problems [22].

In the multiple-shooting approach, a partial discretisation is performed on a coarse, fixed grid on the simulating time window. The time horizon is divided into N_S time intervals, which are called shooting intervals in the context of multiple shooting. State variables at the end of ith shooting interval are defined as ith shooting node s_i and incorporated as the variables to be optimised in the optimisation formulation.

The idea of multiple shooting is that the final value of the DAE integration on ith shooting interval (i.e. use s_{i-1} as initial value) should be consistent with the shooting node s_i at the optimal point; then a set of continuous, bounded, and optimised trajectories can be achieved. This desired property is described as the boundary continuity condition (28) and is incorporated as equality constraints in the optimisation formulation

$$s_i = S_i(s_{i-1}, p), \quad \forall i \in [1, \ldots, N_S] \tag{28}$$

Here, S_i is actually an implicit function that can be evaluated by interfacing with external time-domain simulation tools. Its returning value is the state variables at the end of simulation on the ith shooting interval, initiated from control variable p and initial value s_{i-1}. Corresponding first-order sensitivities $(\partial S_i / s_{i-1})$, $(\partial S_i / p)$ can be obtained by using trajectory sensitivity analysis.

The principle of multiple shooting is demonstrated in Fig. 3. During the iterations of the NLP solver, the piece-wise trajectories on different shooting intervals will gradually connect as the boundary continuity condition (28) forces the continuity of the piecewise trajectories.

After incorporating shooting nodes and boundary continuity conditions into the TSC-OPF problem, the following multiple-shooting-based optimisation formulation given is obtained

$$\begin{array}{ll} \text{min or max} & C(p) \\ \text{s.t.} & g_s(p) = 0 \\ & h_s^- \le h_s(p) \le h_s^+ \\ & p^- \le p \le p^+ \\ & s_i = S_i(s_{i-1}, p), \forall i \in [1 \ldots N_S] \\ & h(s_i) \le 0, \forall i \in [1 \ldots N_S] \end{array} \tag{29}$$

The selection of N_s is dependent on the required length of simulation time window and the available parallel processing units. According to the results reported in [24], the setting of N_s up to 60 is able to provide sufficient benefit of parallel acceleration for TSC-OPF problems.

The gradients (i.e. sensitivities $(\partial S_i / \partial s_{i-1})$ and $(\partial S_i / \partial p)$) used in multiple shooting method can be obtained in a similar way as constraint transcription method, i.e. using any of the trajectory sensitivity analysis approaches discussed in Section 4. The benefit of gradient computation in multiple shooting is the parallelism enabled by this method, gradient matrices of different shooting nodes can be concurrently computed as they are completely decoupled. However, one of possible drawbacks of this method is that one has to compute the sensitivity w.r.t. a complete set of state variables at previous shooting node (i.e. $(\partial S_i / \partial s_{i-1})$) along with $(\partial S_i / \partial p)$, while most of constraint transcription methods only require $(\partial S_i / \partial p)$. That means more total computing time and storage are required in terms of gradient computation for multiple shooting approach, compared with constraint transcription.

Note that initial value for system DAE s_0 is decided by steady-state operating condition p, their relationship is not explicitly stated in this formulation. The multiple-shooting method combines algorithm features from both simultaneous discretisation and constraint transcription. It can interface with external, adaptive time-step DAE solvers with an error control mechanism to solve time-domain simulation precisely, just like constraint transcription (it follows the same framework as Fig. 2). Moreover, the DAE is partially discretised into intervals and implicitly solved in the optimisation layer, which is similar to the simultaneous approach. Unlike its predecessors, the multiple-shooting approach can balance overall algorithm performance between computational efficiency and convergence property, by choosing N_S appropriately. Moreover, the multiple-shooting approach offers the potential of parallel computation. Time-domain simulation and trajectory sensitivity analysis on different shooting intervals utilise only localised information (i.e. control variable p and initial value s_i), hence these computing tasks can be distributed on different processing units and processed concurrently.

4 Trajectory sensitivity analysis [22, 39–43]

Trajectory sensitivity analysis provides valuable information about the change in the system variables as a result of small perturbations in initial conditions and/or control variables. Hiskens and Pai [39] highlight several applications of trajectory sensitivity analysis in the context of power systems. Gradient calculation using trajectory sensitivities is an essential component of the constraint transcription and multiple-shooting methods. In the following subsections, we discuss different approaches for computing the trajectory sensitivities.

4.1 Forward method

Forward sensitivity approaches are so termed because the sensitivities are computed by using a forward integration. The sensitivity equations are integrated along with the original DAE equations. Forward sensitivity analysis is most economical to use when the number of parameters is small, since the computational complexity of forward sensitivity analysis is $O(n_p)$, that is, n_p forward integrations need to be performed to compute the sensitivities. Readers are referred to [41] for a detailed description of the different methods of computing the sensitivity vectors.

Consider a general DAE

$$\boldsymbol{F}(\dot{\boldsymbol{x}}(t), \boldsymbol{x}(t), \boldsymbol{p}) = \boldsymbol{0} \quad (30)$$

where $\boldsymbol{p}$ are the parameters to be analysed.

Forward sensitivity analysis uses the following variational DAE is integrated along with original DAE (30) to generate sensitivity

$$\frac{\partial \boldsymbol{F}}{\partial \dot{\boldsymbol{x}}}\dot{\boldsymbol{s}} + \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{x}}\boldsymbol{s} + \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{p}} = \boldsymbol{0} \quad (31)$$

with initial condition $\boldsymbol{s}(t_0) = (\partial \boldsymbol{x}(t_0)/\partial \boldsymbol{p})$.

Note that the variational DAE (31) is essentially a time-variant linear system, it shares the same Jacobian matrix with the original DAE system, which makes the computational procedure faster by reusing intermediate results.

4.2 Adjoint method

Computing the derivative information by using forward sensitivity analysis requires n_p forward integration and thus is computationally intensive in the presence of large number of parameters. In such a case, adjoint sensitivity analysis is more economical since it needs only one integration in order to compute the sensitivities. To obtain the sensitivities, an adjoint DAE is formulated and is integrated backwards in time. A detailed description of the adjoint method is given in [42].

Different from forward method, which calculates the sensitivity of all the state variables w.r.t. to p, adjoint sensitivity analysis is focusing on the sensitivity of a relatively few functionals of the state variables. This property enables this method to work with constraints transcription approaches such as [44] to save computing time, compared with forward method. Consider the same DAE system (30), assume the gradient of $(\partial G/\partial p)$ is desired, where

$$G(\boldsymbol{p}) = \int_{t_0}^{\mathrm{T}} g(\boldsymbol{x}, \boldsymbol{p})\mathrm{d}t\,. \quad (32)$$

The desired gradient $(\partial G/\partial \boldsymbol{p})$ is able to be computed as

$$\frac{\partial G}{\partial \boldsymbol{p}} = \int_{t_0}^{\mathrm{T}} \left(\frac{\partial g}{\partial \boldsymbol{p}} - \boldsymbol{\lambda}^{\mathrm{T}}\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{p}}\right)\mathrm{d}t \quad (33)$$

Multiplier $\boldsymbol{\lambda}(t)$ is the solution of the following co-state equations:

$$\dot{\boldsymbol{\lambda}} = -\frac{\partial g}{\partial \boldsymbol{p}} + \boldsymbol{\lambda}^{\mathrm{T}}\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{x}} \quad (34)$$

where $\boldsymbol{\lambda}(T) = \boldsymbol{0}$.

Note that the solution of $\boldsymbol{\lambda}(t)$ requires integration from T to t_0, this backward integration is based on the solution of state variables $\boldsymbol{x}(t)$. Therefore, additional procedure is required to store the value of $\boldsymbol{x}(t)$ during forward integration of the original DAE (30), which leads to more memory consumption compared with forward method.

4.3 Finite differences

Derivative calculation with finite differencing is an easy, yet powerful, approach used in various fields where the derivative is unavailable or difficult to obtain. It is based on Taylor series truncated at various orders of expansion. The sensitivities are computed by integrating the original DAE (30) with a small perturbation of each parameter p [13].

4.4 Automatic differentiation

Automatic differentiation (AD) [43] is able to efficiently generate derivatives of a given function without additional hand coding. AD applies the chain rule on the functions described by elementary arithmetic operations (+, * etc) and functions (exp, sin etc.) in the source code, so that the computing path from input variables to output variables can be traced and analysed. The use of AD for solving TSC-OPF has been investigated by Geng *et al.* [22]. AD is essentially a tool to find derivatives for an analytical expression, but not a computational procedure such as time-domain simulation. Therefore, AD has to be used along with either forward of adjoint sensitivity analysis method, so as to complete the computation of a trajectory sensitivity analysis.

5 Optimisation algorithms

After incorporating dynamic constraints into the optimisation, TSC-OPF problem is transformed into an NLP problem. We note here that the NLP transformation assumes that the discrete variables, such as tap changers and shunt elements, are treated as continuous or fixed. Incorporation of discrete variables in TSC-OPF is an open research question. NLP problems are typically solved by interior point method (IPM) and active set methods with the former being more popular for solving OPF. IPMs have proven advantageous in solving NLP problems since the 1990s [35, 45] and have been applied to a large variety of optimal power flow problems [46]. Another interesting area of research for TSC-OPF is the use of convex relaxation [47–50], that yields global optimality for OPF. This needs development of the theoretical foundation of convexification of the TSC-OPF problem. For an in-depth discussion on advanced optimisation algorithms for solving OPF, the reader is referred to [51].

In this section, we present the algorithmic description of two variants of the IPM that have been used for solving TSC-OPF.

5.1 Full-space interior point method [6]

A general form of NLP (35) can be used to describe the resultant TSC-OPF problems. The optimising variable v includes the parameter p and discretised variables. Steady-state constraints and converted constraints from the DAE are included in equality constraints g and inequality constraints h.

$$\begin{array}{ll} \min & C(v) \\ \text{s.t.} & g(v) = 0 \\ & h^{-} \le h(v) \le h^{+} \end{array} \quad (35)$$

Inequality constraints are transformed into equality constraints by adding slack variables. Then a Lagrange function is built by using barrier functions. Newton's method is applied on the optimal condition of Lagrange function. The process of Newton iterations comprises two major phases of algorithm procedures: obtaining searching directions and determining searching step length. The former occupies most of the computing time; the latter affects the convergence of IPM. Assume v and λ are primal and dual variables in the NLP formulation. In phase 1, the search directions are obtained by solving the primal-dual system (36), which is a symmetric indefinite sparse linear system

$$\begin{bmatrix} A & J_{\mathrm{eq}}^{\mathrm{T}} \\ J_{\mathrm{eq}} & 0 \end{bmatrix} \begin{bmatrix} \Delta v \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} L_v \\ L_\lambda \end{bmatrix} \quad (36)$$

Here, $\boldsymbol{A}$ and $\boldsymbol{J}_{\mathrm{eq}}$ are Hessian and Jacobian matrix, respectively; L_v and L_λ are residuals for first-order optimality conditions; and Δv and $\Delta\lambda$ represent the searching directions of primal and dual variables. In the conventional full-space IPM algorithm, (36) is solved by a general-purpose direct sparse linear solver.

Phase 2 consists of computing the step lengths, α_v and α_λ, along the search direction in order to move closer to optimum solution

$$\begin{cases} v := \alpha_v \Delta v + v \\ \lambda := \alpha_\lambda \Delta \lambda + \lambda \end{cases} \quad (37)$$

Different strategies are available to determine the step length for better optimum searching performance in the framework of IPM. Basic primal-dual path-following IPM [52], predictor-corrector IPM [53], multiple predictor-corrector IPM [54], multiple centrality corrections (MCC) IPM [55], and weighted MCC IPM [56] were developed in order to improve robustness and reduce the number of IPM iterations. In practical implementations, Mehrotra's predictor-corrector algorithm [53] provides the basis for the major applications of this class of methods; most of power system optimisation research follow this predictor-corrector algorithm [57]. MCC on IPM-based OPF has been applied by Torres and Quintana [58], and weighted MCC has been applied by Huang and Jiang [59].

Full-space IPM algorithm is comprehensively described in [60]. Matrix structure of primal-dual systems for TSC-OPF problem was investigated in [6].

5.2 Reduced-space interior point method (RIPM) [9, 61, 62]

RIPM is an extension of conventional full-space IPM, specially designed for NLP with relatively few degrees of freedom (DOF). It was originally developed to solve optimisation problems in chemical engineering with low DOF [61, 62]. DOF is defined as (38) as follows:

$$\mathrm{DOF} = n - m \tag{38}$$

Here, n and m are the dimension of variables and equality constraints in the NLP optimisation formulation, respectively.

The TSC-OPF problem after discretisation is a typical NLP with low DOF; the reason is the process of discretisation does not add new DOF into the optimisation formulation. The converted NLP may have a large number of inequality constraints and variables, but the DOF will be constant before and after DAE discretisation or constraint transcription. RIPM is able to accelerate the solution of the primal-dual system, which is the most computational-intensive algorithmic procedure in IPM. Jiang and Geng [9] introduced RIPM to solve the TSC-OPF problems and reported significant computational performance improvement. We present a brief algorithm description of RIPM next.

First, the Jacobian matrix of equality constraints is partitioned as (39) as follows

$$J_{\mathrm{eq}} = [N \quad C] \tag{39}$$

Here, $C \in \Re^{N_{\mathrm{eq}} \times N_{\mathrm{eq}}}$ and $N \in \Re^{N_{\mathrm{eq}} \times N_{\mathrm{dof}}}$. N_{eq} and N_{dof} are the dimensions of equality constraints and DOF, respectively. The range-subspace basis matrix $\boldsymbol{Y}$ and null-subspace basis matrix Z can be calculated with coordinate decomposition

$$\boldsymbol{Y} = \begin{bmatrix} 0 \\ I_{N_{\mathrm{eq}}} \end{bmatrix}, \quad \boldsymbol{Z} = \begin{bmatrix} I \\ -C^{-1}N \end{bmatrix} \tag{40}$$

Then, search directions on range- and null-subspace are determined

$$\begin{cases} p_Y = -C^{-1}L_\lambda \\ p_Z = (\mathrm{Z^T HZ})^{-1}\mathrm{Z^T}(L_v - \mathrm{HY}p_Y) = B^{-1}\omega \end{cases} \tag{41}$$

Here, the reduced Hessian R and extended cross-term ω are defined as $Z^{\mathrm{T}}\mathrm{HZ}$ and $Z^{\mathrm{T}}(L_v - \mathrm{HY}p_Y)$, respectively.

Based on the obtained search directions in the subspaces, the search direction of the primal variables Δv can be determined by linear combination from the subspaces

$$\Delta v = Yp_Y + Zp_Z \tag{42}$$

Then, $\Delta\lambda$ can be solved by backward-substitution in (36). Instead of solving (36) directly, RIPM follows the procedures (39)–(42) to gain better computational performance while sharing the same solution results as with the full-space approach.

6 Implementation issues for dynamic optimisation methods

6.1 Initialisation [22, 27]

The initialisation of the transient stability-constrained optimal power flow is an important issue because it influences the convergence of the algorithm. Most transient stability programs use a power flow solver to initialise the machine dynamic variables. For the TSC-OPF problem, the solution from a steady-state optimal power flow has been typically used as a starting point for TSC-OPF. While considering TSC-OPF with multiple contingencies, a filtering procedure can be adopted to consider only credible contingencies [27]. In the framework of multiple shooting, a DAE relaxation technique was proposed in [22] to avoid initialisation failure when external simulation tools were requested to start from an infeasible point generated by IPM-based optimisation algorithm.

6.2 Hessian calculation for constraint transcription approaches [16, 28, 63]

Optimisation methods such as IPMs also need an accurate Hessian for fast convergence. Unfortunately, analytical Hessian calculation for constraint transcription approaches is extremely hard. Instead, the Hessian matrix can be approximated using the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm [34] that uses rank-one updates specified by gradient evaluations. Because of the approximated Hessian, the constraint transcription approaches suffer from slow convergence of the optimisation. Another point to note is that the BFGS approximated Hessian is dense due to the low-rank updates leading to storage problems. In such cases, the limited-memory BFGS algorithm can be used to directly obtain the approximate of the inverse Hessian [63].

6.3 Contingencies [22, 27, 64]

TSC-OPF with the consideration of multiple contingencies is a form of dynamic security assessment problem. Selection of credible contingencies to be considered with TSC-OPF is a challenging task given the plethora of ways (fault types, strength, location, load loss, generator loss, clearing time etc.) that a contingency can occur. Thus, a proper selection of credible contingencies is important in order to avoid overwhelming the computational problem.

Bettiol *et al.* [27] use a filtering SIME scheme to obtain the set of credible contingencies. Jiang *et al.* [64, 65] have proposed preliminary approaches to filter contingencies and identify the active set, with and without transient stability constraints. Geng *et al.* [22] have investigated different types of faults, as well as clearing times, while solving the TSC-OPF problem.

In the context of SCOPF, several interesting approaches have been developed that may be utilised for TSC-OPF for contingency filtering. Contingency filtering techniques based on severity index (SI) that rank the contingencies have been developed in [44, 66, 67]. However, the selection of the top-ranked contingencies and the selection of SI weights are heuristic measures that need to be tuned. Capitanescu *et al*. [68, 69] presented a non-dominated contingency filtering approach for the preventive and corrective security-constrained optimal power flow. The Bender's decomposition approach [46] successively adds or eliminates contingencies as the solution process progresses and is favourable for a distributed computing approach. An approach based on compression of the network that retains the sub-network of interest in detail and approximates the remainder was proposed in [70] reducing the number of contingencies to be considered.

7 Conclusions

In this paper, we presented state-of-the-art techniques for solving the TSC-OPF problem by simultaneous discretisation, constraint transcription, and multiple-shooting approaches. A brief description of trajectory sensitivity analysis was given. Two variants of the IPM to solve the resultant optimisation problem were presented. We also discussed issues that should be considered while solving such problems.

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