Kim, Amy, Hansen, Mark.

A framework for the assessment of collaborative en route resource allocation strategies.

**AUTHOR POST PRINT VERSION**

Kim, A., & Hansen, M. (2013). A framework for the assessment of collaborative en route resource allocation strategies. Transportation Research Part C: Emerging Technologies, 33, 324-339. https://doi.org/10.1016/j.trc.2012.04.015

*Manuscript
Click here to view linked References

# A framework for the assessment of collaborative en route resource allocation strategies

Amy Kim*
Department of Civil and Environmental Engineering
University of Alberta

Mark Hansen
Department of Civil and Environmental Engineering
University of California, Berkeley

* Corresponding author. Address: 3-007 CNRL/Markin Natural Resources Engineering Facility, University of Alberta, Edmonton, AB, T6G 2W2, Canada. Tel.: +17804929203, Fax.: +17804920249. The majority of this work was performed when the corresponding author was at the University of California, Berkeley.
E-mail addresses: amy.kim@ualberta.ca (A.M.Kim), mhansen@ce.berkeley.edu (M. Hansen).

## Abstract

Airspace Flow Programs (AFPs) assign ground delays to flights in order to limit flow through capacity constrained airspace regions. AFPs have been successful in controlling traffic with reasonable delays, but a new program called the Combined Trajectory Options Program, or CTOP, is being explored to further accommodate projected increases in traffic demand. In CTOP, centrally managed rerouting and user preference inputs are also incorporated into initial en route resource allocations. We investigate four alternative versions of resource allocation within CTOP in this research, under differing assumptions about the degree of random variability in airline flight assignment costs when measured against a simple model based upon the flight specific, but otherwise fixed, ratio of airborne flight time and ground delay unit cost. Two en route resource allocation schemes are based on ordered assignments that are similar to those used currently, and the other two are system-optimal assignment schemes. One of these system-optimal schemes is based on complete preference information, which is ideal but not realistic, and the other is based on partial information that may be feasible to implement but yields less efficient assignments. The main contribution of this research is a methodological framework to evaluate and compare these alternative en route resource allocation schemes, under varying assumptions about the information traffic managers have been provided about the flight operators' route preferences. The framework allows us to evaluate these various schemes under differing assumptions about how well the traffic managers' flight cost model captures flight costs. A numerical example demonstrates that a sequential resource allocation scheme – where flights are assigned resources in the order in which preference information is submitted – can be more efficient than a scheme that offers a cost minimizing allocation based on less complete preference information, and may at the same time be perceived as equitable. We also find that assigning resources in the order flights are scheduled results in less efficient allocations, but more equitable ones.

**Keywords**

Air traffic flow management (ATFM); Collaborative Trajectory Options Program (CTOP); Airspace Flow Program (AFP); en route resource allocation; user cost model; strategic planning.

## 1. Introduction

Adverse weather and heavy traffic demands frequently and severely impact flight operations in the National Airspace System (NAS). In 2008, about 88% of all delays in the NAS were attributed to these two phenomena (Bureau of Transportation Statistics, 2008). When weather disruptions and heavy traffic demands are anticipated in the en route airspace, the Federal Aviation Administration (FAA) will typically attempt to reduce the scale and cost of these impacts by employing an Airspace Flow Program (AFP). The AFP is an air traffic flow management (ATFM) initiative that facilitates decisions regarding when flights are permitted to use airspace anticipated to experience capacity/demand imbalances, several hours in advance of the problem. It plans for flights to be delayed on the ground (rather than in the air, which is much costlier), to meter their flow through constrained airspace. Although the AFP has proven to be successful in reducing the cost of flight delays since it was implemented in 2006 (FAA, 2007), a new program is being explored to better accommodate projected increases in traffic demand. In this new program, called the Combined Trajectory Options Program, or CTOP, the resources to be administered by FAA traffic managers in times of capacity shortfalls include both ground delays and reroutes. Flight operator preferences regarding the available en route resource options are also collected to better inform the resource allocation process, and in turn, minimize impacts to affected flight operators.

In this paper we investigate several alternative versions of the CTOP, which differ from one another with respect to the preference information requested of flight operators as well as the resource assignment mechanism in which the collected preferences are used. We consider two schemes based on ordered assignment that are similar to those used currently, and compare their efficiency and equity against two system-optimal assignment schemes. One of these system-optimal schemes is based on complete preference information, which is ideal but not realistic, and the other is based on partial information that may be feasible to implement but yields less efficient assignments. The main contribution of this research is a methodological framework to evaluate these alternative en route resource allocation schemes under varying assumptions about the information traffic managers have been provided about the flight operators' route preferences. The framework allows us to evaluate these various schemes under differing assumptions about how well the traffic managers' flight cost model captures flight costs.

This paper is organized as follows: Section 2 describes the current practices for en route resource allocation and provides a review of the literature. Section 3 introduces the analysis framework and flight cost model. Section 4 introduces the resource allocation schemes, while Section 5 compares the schemes analytically in a highly stylized setting. Section 6 employs numerical simulation to compare the schemes in a more realistic setting. This is followed in Section 7 by a conclusion and discussion of future research.

In this paper, "operator" or "user" refers to NAS customers such as commercial airlines and general aviation aircraft. "Traffic manager" refers to the agent responsible for allocating resources. In the U.S. these would be the traffic management specialists at the FAA's Air Traffic Control System Command Center.

## 2. Background

When en route airspace regions experience severe weather and/or traffic congestion events, flight reroutes and delays are used to address the problem both strategically and tactically. Rerouting is a manually intensive process as it requires close coordination between several traffic management units. Consequently, FAA traffic managers typically select reroutes from a standard set, employing a "one size fits all" approach (Wilmouth & Taber, 2005) without input from the flight operators. Airlines do have the option of rerouting their own flights both before and after departure, but this is subject to traffic managers' approval. Concepts that propose a more collaborative approach to rerouting have existed since the early 2000s (Ball, et al., 2002); these concepts describe a more structured approach to coordination between traffic managers and operators. There has been some research on rerouting decisions made by individual flights due to en route constraints by Ganji, et al. (2009) and Yoon, et al. (2011). Each present stochastic models where a flight can either depart as scheduled but on an alternate route that avoids the problematic airspace, or take the original route with ground delay. Their models are based on uncertainty regarding the capacity constraint duration, and allows for recourse options as a second-stage decision.

There are several programs that are used to meter traffic flow into constrained en route areas, including Miles-In-Trail, ground stops, the Ground Delay Program (GDP), and the Airspace Flow Program (AFP). In the AFP, the constrained airspace region and the flights filed into this region during the time of reduced capacity are first identified. The reduced capacity is then allocated by assigning each impacted flight a delayed departure time on the original filed route. A flight can either accept the assigned departure time, or reject it and reroute around the constrained airspace (subject to traffic managers' approval). Slots to fly through the constrained region are vacated as flights are canceled or routed out, and the schedule is compressed such that remaining flights are moved up into earlier slots as available. Currently, the assignment of delayed departure times combined with airline-initiated rerouting and cancellation has proven to be adequate for handling capacity constraints. However, with growing demand, greater rerouting coordination may be required. The Collaborative Trajectory Options Program (CTOP) is a proposed concept that builds on the AFP; it is designed to offer flight operators airspace resources that combine route options with delayed departure slots, and allow them to communicate their preferences regarding the available resources.

There is no program currently in place that applies reroutes and ground delays simultaneously – existing reroute programs cannot assign delays, and vice versa (FAA, 2010). Optimization models that consider both rerouting and delay (on the ground and en route) decisions for constrained resource allocation have, however, been studied in the literature. One of the most well-known ATFM models was proposed by Bertsimas & Stock-Patterson (1998); it requires the input of flight-specific air and ground-hold cost ratios to consider ground holding and air holding in a static deterministic setting. They also illustrate how the model can be extended to account for flight rerouting. In a following paper, they propose an aircraft rerouting model in dynamic weather conditions (Bertsimas & Stock Patterson, 2000). Hoffman, et al. (2005) develop an algorithm that allows for simultaneous rationing of ground and en route resources, as an alternative to using GDPs to handle en route constraints. Jakobovits, et al. (2005) formulated an algorithm to schedule, reroute and airhold flights flying into and around constrained airspace. Mukherjee & Hansen (2009) consider a variant of the single airport ground hold problem that considers reroutes for terminal airspace using a dynamic stochastic approach. The model by Yoon, et al. (2011) mentioned above allows for decisions regarding route hedging and ground delay, as well as recourse, when facing an uncertain weather clearance time. Considering individual flights, they show that ground holding should only be applied when the flight intends to remain on the original route; otherwise, it is optimal to take an alternate route and depart at the original scheduled departure time. The objective of many ATFM models is to minimize the system-wide cost of delay, i.e. maximize efficiency; however, providing equity between flights and/or flight operators is another important objective (Vossen & Ball, 2005) (Pourtaklo & Ball, 2009).

For traffic managers to make resource assignment decisions that are of good value to flight operators, they require information about flights operators' resource preferences. Existing resource allocation programs such as GDPs and AFPs benefit from Collaborative Decision Making (CDM) (Ball, et al., 2003), a joint government and industry initiative that improves air traffic management by encouraging the exchange of up-to-date information between traffic managers and flight operators. However, operators' preferences are not explicitly communicated through CDM. ATFM concepts in which airlines do provide preference information to the FAA's resource allocation process have been studied (Goodhart, 2000) (Hoffman, et al., 2004) but have yet to be implemented.

This paper builds on the above literature by providing a modeling framework to evaluate the performances of different en route resource allocation schemes that require different user cost inputs and resource allocation rules. It addresses a gap in the ATFM literature by explicitly considering the impacts of different levels of user preference inputs to the resource allocation, at different levels of information quality used in the allocation.

## 3. Model set-up

Here we describe our model set-up, including the geometry of our modeling framework and a function to represent flight costs. Figure 1 depicts the geometry. Two points, or fixes, in en route airspace are connected by a nominal route, designated as such because it is the lowest cost path between the two points. Flights enter the nominal route at entry fix "A" and leave at exit fix "B". Under good conditions, all aircraft that are scheduled to use the nominal route can do so at their scheduled time without experiencing delay, meaning that the nominal route has sufficient capacity to serve the pre-constraint scheduled flight demand $D_0(t)$, in units of flights per hour. Suppose now that a constraint develops at some point along the nominal route, reducing its capacity such that the flights demanding to use this route cannot be accommodated without some queuing delay. The $N$ flights originally scheduled to use this route (at a demand rate $D_0$) during the constrained period must either be rescheduled or re-routed to observe the reduced capacity. Flights are either given delayed departure times on the nominal route, or rerouted to one of $R-1$ alternate routes and possibly assigned a delayed departure time on that route. Each alternate route $r$ is characterized by its capacity and travel time. The nominal route is assumed to have the lowest travel cost. We assume that fixes A and B are not bottlenecks, and for the purpose of this analysis they can be considered the flights' origin and destination. Flight trajectories upstream of Fix A and downstream of Fix B are not considered in this analysis.

*Place Figure 1 about here.*

This research focuses on evaluating the added costs associated with greater en route time and ground delay due to the en route constraint. It is not concerned with the costs of the airlines' original scheduled flight plans, because we assume that these flight plans were those most preferred under ideal conditions. The flight cost function, $c_{n,j,r(j)}$, represents the added cost of flight $n$ taking departure slot $j$ belonging to route $r$, due to constrained operating conditions. Over all the available routes $r = 1,2,\ldots,R$, there are a total of $J$ departure slots, where $r(j)$ indicates the route that slot $j$ belongs to. $c_{n,j,r(j)}$ is a function of the additional travel time of route $r$ compared to the nominal route (assuming that aircraft fly at ideal speeds), time spent waiting on the ground for their assigned slot $j$ on route $r$, and other factors. We assume it is the sum of the above components, and quantified in units of ground delay minutes. Thus,

$$c_{n,j,r(j)} = c^a_{n,r(j)} + c^g_{n,j,r(j)} + \varepsilon_{n,r(j)}, \quad \varepsilon_{n,r(j)} \sim P \tag{1}$$

Where

- $c_{n,j,r(j)}$ represents the added cost of flight $n$ taking departure slot $j$ (which belongs to route $r$);
- $c^a_{n,r(j)}$ is the cost of the additional en route time for $n$ on route $r$;

- $c^{g}_{n,j,r(j)}$ is the cost of the departure delay due to $n$ departing in slot $j$ at fix A instead of its original scheduled departure time, and
- $\varepsilon_{n,r(j)}$ is a random term that follows some predefined distribution $P$. It represents flight $n$'s route ($r$) and situation-specific cost factors that are not captured in the deterministic part of the model. It therefore represents the differences between the true cost $c_{n,j,r(j)}$ and the deterministic cost $c^{a}_{n,r(j)} + c^{g}_{n,j,r(j)}$. $\varepsilon_{n,r(j)}$ may be positive or negative; in the latter case it represents unknown cost-mitigating factors.

$c_{n,j,r(j)}$ accounts for direct costs including additional fuel, crew time, and equipment maintenance, and indirect costs such as passenger satisfaction, gate time, flight coordination, costs related to other airline internal business objectives, and others. Air holding is not included in the model because we assume that traffic managers have perfect information about the capacity constraint location and duration, scheduled demand $D_0(t)$, and all route capacities $S_1(t), \dots, S_R(t)$ during the constrained period. Therefore, all anticipated delay is taken on the ground.

The cost function can be further identified as follows:

$$c_{n,j,r(j)} = \alpha_n \rho_{r(j)} + d_{j,r(j)} - g_{0,n} + \varepsilon_{n,r(j)}, \qquad \varepsilon_{n,r(j)} \sim P \tag{2}$$

Where

- $\alpha_n$ is the ratio of flight $n$'s unit airborne time and ground delay costs;
- $\rho_{r(j)}$ is the additional en route time on route $r$ compared to the nominal route ($\rho_{r(j)} \geq 0$);
- $d_{j,r(j)}$ is the departure time on slot $j$, belonging to route $r$, at fix A, and
- $g_{0,n}$ is flight $n$'s original pre-CTOP scheduled departure time at fix A.

The unit cost of airborne delay exceeds that of ground delay such that $\alpha_n \geq 1$. If, for instance, $\alpha_n = 2$, every one minute flight $n$ spends in the air is equivalent in cost to $n$ spending two minutes on the ground. $\rho_{r(j)}$ is non-negative, assuming the nominal route has the shortest flying time. Ground delay, or $(d_{j,r(j)} - g_{0,n})$, is non-negative as well because aircraft cannot depart before their original scheduled time. Recall that $d_{j,r(j)}$ is a departure time *from Fix A*, since we are concerned with capacity restrictions that occur between Fix A and B.

As noted above, the random term in equations (1) and (2) represents the part of airlines' route-specific flight costs about which traffic managers have little to no information. The specification of the random term, and its role in the allocation process, are key determinants of the performance of each allocation scheme. As elaborated below, in some schemes the preference information provided to the traffic manager includes the random term, while in others it does not. This determines whether the objective function used by the traffic manager fully reflects flight operator costs, or does so only partially. We specify that the random term is independent and identically distributed (iid) normal, with mean 0 and variance $\sigma^2$. The strategies are evaluated over different values of $\sigma^2$ in Sections 5 and 6.

The assumption of iid normality for the random term is made primarily for modeling convenience. Although the (deterministic and stochastic) costs of flights operated by the same airline may be correlated, we assume here that intra-airline flight differences are so pronounced that this correlation can be ignored. By assuming independence of the random term we further assume that it is not a function of the total (anticipated) delay; in fact, $\alpha_n$ is also independent of total delay according to the cost model structure. Also, the random term in equations (1) and (2) depends on the flight $n$ and the route $r$ only, and not the slot $j$. These assumptions should be revisited in future research.

## 4. Allocation schemes

En route resource allocation decisions are shaped by system capacity constraints and the allocation and equity principles chosen for implementation. Furthermore, the perceived quality of an allocation will depend on the metrics used to assess its performance. Typically, traffic managers aim to provide as much efficiency as possible while maintaining equity between flights and/or operators. If we measure performance from a user cost perspective, allocation quality is likely to increase when users' resource preferences are incorporated into the decision-making process.

The mechanisms presented here incorporate flight operators' preference information in an en route resource allocation process that combines rerouting decisions with delayed departure times. It gives operators flexibility in expressing their flights' route cost/preference information to traffic managers. The allocation cost calculation is best shown graphically as done by Hoffman, et al. (2004) to illustrate the Flow Constrained Area Rerouting Decision Support Tool concept developed at Metron Aviation. An illustration is shown in Figure 2. Suppose a flight $n$ has three route options ($R = 3$), and the operator of flight $n$ submits their inputs about the available options to traffic managers. These inputs may differ from one resource allocation scheme to another. They are used to construct $\Delta_{n,r}$, which is the cost before ground delay cost is added, of flight $n$ traveling on route $r$, measured in units of ground delay minutes. Thus, if $\Delta_{n,1} = \Delta_{n,2} + k$, the flight operator would be indifferent between having flight $n$ assigned to route 1 with no ground delay and route 2 with a ground delay of $k$ minutes. $\Delta_{n,r}$ values contain all the flight operator cost information made available to the traffic managers; traffic managers use this information to assign constrained resources to flights through the adopted allocation mechanism. The $\Delta_{n,r}$ values ensure that with any route and ground delay slot assigned to a flight, traffic managers have some indication regarding the relative value of that route/slot combination to the flight operator. The total cost (as perceived by traffic managers) for flight $n$ on route $r$ is a function of the departure slot, and resulting ground delay, assigned to $n$ on $r$. These functions are linear, as our specification assumes that each additional minute of ground delay incurred by flight $n$ in waiting for a slot on a given route increases $n$'s cost to take that route by one minute. Suppose that, based on flight availability, the ground delay flight $n$ must take is $(d_{j,1} - g_{0,n})$ on route 1, $(d_{j,2} - g_{0,n})$ on route 2, and $(d_{j,3} - g_{0,n})$ on route 3. It could then be determined that the cost for flight $n$ to take route 1 is $c_{n,j,1}$, route 2 is $c_{n,j,2}$, and route 3 is $c_{n,j,3}$. The resource that flight $n$ ultimately receives will depend on the mechanism used to assign resources to all flights in the CTOP.

*Place Figure 2 about here.*

There are many resource allocation rules that could be considered. To demonstrate two possibilities, consider the example in Figure 3. Under normal operating conditions (top left table), flights A and B prefer Route 1 with scheduled departure times 0 and 5 minutes. However, convective weather develops such that flows on both routes are metered heavily and only certain slots are available (top right table). In this situation, A and B may not be able to depart at their desired scheduled departure times. The traffic managers allocate the available resources to A and B using en route cost inputs $\Delta_{n,r}$. Flight cost is calculated based on the difference between the original departure time and the assigned departure time, plus $\Delta_{n,r}$. If traffic managers are obliged to serve Flight A first (Allocation 1), then Flight A would be given Route 1 Slot 1 as it is the cheapest option available to it. Flight B would be given Route 1 Slot 2 as its best available option. The total cost of this allocation is 250. If the goal is to minimize total cost (Allocation 2) they would assign Flight A to Route 2 Slot 1 and Flight B to Route 1 Slot 1. The cost of this allocation is 240.

*Place Figure 3 about here.*

There any many other possible allocation schemes; some are identified in Ball, Futer, Hoffman, & Sherry (2002). Traffic managers may be instructed to minimize some chosen system-wide cost metric, with or without consideration of flight/operator equity. Allocation could follow a "first-come first-serve" process where the ordering is based on the time of resource requests, the original schedule, or some other criterion. Airlines could also be assigned a proportion of the total available resources based on the number of flights they have scheduled, perhaps with some additional weighting based on aircraft size. In this research, we consider several schemes that feature different resource allocation processes and user preference inputs. In both the full information system-optimal (FISO) and parametric system-optimal (PASO) models, cost submissions from all flights regarding all available routes are considered simultaneously to perform a system optimal assignment of routes and ground delay slots to flights. The difference between FISO and PASO lies in the information contained in the preference inputs submitted by flight operators. We also consider two other schemes: first-submitted, first-assigned (FSFA) and ration-by-schedule (RBS). In both FSFA and RBS, resources are assigned to flights sequentially. Each flight is assigned the lowest cost resources available to it, based on the submitted cost information and route/slot availability, at the time of allocation. Sections 4.1 through 4.4 introduce each of these four schemes in greater detail.

## *4.1 Full information, optimal (FISO)*

In the full information system-optimal (FISO) scheme, when the CTOP is announced, traffic managers provide all operators of impacted flights with information about the constrained airspace (start time, duration, location, etc.) and the reroute options available. Operators are then asked to submit the requested cost information inputs to traffic managers by some pre-specified deadline. By this time, for each route $r$ available in the CTOP, the operator of flight $n$ submits $\Delta_{n,r}$. Traffic managers then allocate all resources simultaneously using the submitted information, with the objective of minimizing the total flight cost of the program, without explicit consideration for equity between flights and/or operators. This allocation scheme does not have any mechanism to reward or penalize flight operators for submitting their inputs. It is highly idealized, in that flight operators are not likely to be capable of providing this highly detailed and specific information in a convenient or timely manner, and in the absence of any incentives (resource or equity guarantees). However unrealistic it is, in principal the FISO model yields the most efficient system performance that can be achieved from any CTOP allocation scheme. We thus use it as a benchmark against which other schemes are evaluated and compared.

In FISO, operators do not know which route and slot the traffic managers will assign their flight(s); although operators do know what routes are available, they have no information about the ground delay that will be assigned to their flight on a given route. We assume that each operator would calculate the additional cost of a flight reassignment option using the flight cost model of equation (2). As a result, because operators submit complete information about their flights in FISO, they submit $\Delta_{n,r} \forall r$ that consist of the following parts of the flight cost model:

$$\Delta_{n,r} = \alpha_n \cdot \rho_r + \varepsilon_{n,r} \tag{3}$$

Where the random term is distributed iid normal, $\varepsilon_{n,r} \sim N(0, \sigma^2)$. Based on the illustration of Figure 2, flight $n$ could be assigned any one of the three routes using FISO. Traffic managers will identify the total minimum cost assignment based on all $\Delta_{n,r}$ submitted by all the flights for all routes, plus the ground delay associated with departure time slots.

Due to the fact that the flight operators' complete route preference information is available for use in traffic managers' decision making through the information the flight operators offer, the random term of the flight cost model (representing proprietary airline route preferences) is included in the resource allocation process, and therefore in the objective function. We randomly draw these values for our numerical examples, and formulate the FISO model as an assignment problem where flights are assigned to slots on routes. The decision variable is a binary indicator of whether a flight is assigned to a given slot $j \in J$, where $J$ is the entire set of slots available over all the available routes $r = 1,2,\dots,R$. For instance, if there are two routes with three slots each, $j = 1,2,3$ belong to route 1, and $j = 4,5,6$ belong to route 2. The model that would be solved as part of the FISO allocation scheme is:

<u>Decision variables:</u> $x_{n,j,r(j)} \in \{0,1\}$; $x_{n,j,r(j)} = 1$ if flight $n$ is assigned to slot $j$ on route $r$ and $x_{n,j,r(j)} = 0$ otherwise.

<u>Objective function:</u>

$$\min_{x_{n,j,r(j)},\forall n,j} C = \sum_{n\in F, j\in J: d_j \geq g_{0,n}} c_{n,j,r(j)} \cdot x_{n,j,r(j)} \tag{4}$$

Where

- $C$ is the total operator cost of allocating resources to flights due to the en route constraint;
- $F$ is the set of impacted flights, $|F| = N$;
- $J$ is the set of slots available over the available routes, and each slot $j \in J$ is associated with some route $r$;
- $d_{j,r(j)}$ is the departure time associated with slot $j$, which belongs to route $r$.
- $c_{n,j,r(j)} = \Delta_{n,r(j)} + d_{j,r(j)} - g_{0,n}$, and is the total cost to flight $n$ when it is assigned to slot $j$, and
- $\Delta_{n,r(j)} = \alpha_n \rho_{r(j)} + \varepsilon_{n,r(j)}$, where $\varepsilon_{n,r(j)} \sim N(0,\sigma^2)$.

<u>Constraints:</u> $d_{j,r(j)} \geq g_{0,n}$, $\sum_{j\in J} x_{n,j,r(j)} = 1, \forall n$; $\sum_{n\in F} x_{n,j,r(j)} \leq 1, \forall j$; $x_{n,j,r(j)} \in \{0,1\}, \forall n,j$.

The constraints ensure that each flight $n$'s ground delay is non-negative, each flight is assigned to one slot, each slot is assigned at most one flight, and $x_{n,j,r(j)}$ can take values of zero or one.

For modeling purposes, random sampling is used to generate $\varepsilon_{n,r(j)}$ values; as a result we solve equation (4) 5,000 times to generate an average result for $C$, for each $\sigma$ value, in the numerical examples of Section 6. We find $C$ for increasing values of $\sigma$, starting at $\sigma = 0$, in Section 6.

### *4.2 First-submitted, first-assigned (FSFA)*

In the first-submitted, first-assigned (FSFA) model, operators with impacted flights are provided information about the constrained airspace as in the full information optimal (FISO) model. However, the operators are instructed to submit their cost inputs $\Delta_{n,r}$ sometime within a planning period beginning a few hours prior to the start of the CTOP. At the time an operator submits their flight cost input(s), the FSFA algorithm assigns the lowest cost resources (route/departure slot combination) available at that time, based on the submitted flight cost information and route/slot availability. Future requests are not considered (or assumed to be known) when a flight is assigned a resource. As a result, FSFA is a greedy allocation algorithm in that it makes a myopically optimal choice for each flight in sequential order. In the case presented in Figure 2, if flight $n$ has submitted their inputs and $J^A$ is the set of available slots at the time of submission, the lowest cost route option is

$$r_n^*(j) = \operatorname*{argmin}_{j\in J^A}(\Delta_{n,r(j)} + (d_{j,r(j)} - g_{0,n})) \tag{5}$$

Where $r_n^*(j)$ is the lowest cost route option for $n$, given the set of available slots $J^A$. In the Figure 2 example, $r_n^*(j)$ is route 3.

The FSFA scheme was chosen amongst many possible allocation schemes because its prioritization logic is easily understood. The incentive to provide accurate inputs is clear – once a flight operator has submitted its inputs, it is only competing against itself for an allocation because they are processed one at a time. It also incentivizes flight operators to supply their complete route preferences as soon as possible, as they are competing on the basis of input times against other operators. The rationing logic resembles the well-established ration-by-schedule (RBS) allocation algorithm, where flights are assigned delayed departure times in the order by which they are scheduled to arrive at the capacity-constrained airport.

In order to numerically model the performance of the FSFA scheme, we employ the recursive algorithm below.

1. Assign $\alpha_n$, and randomly drawn $\varepsilon_{n,r}$ values to flights $n = 1,2,\dots,N$ to construct $\Delta_{n,r}$ values. Assign scheduled departure times $g_{0,n} \forall n$, as well. Note that this step is necessary for modeling purposes; in reality, flight operators would simply offer $\Delta_{n,r}$ to traffic managers, and the original flight schedule would be constructed from both flight schedules and submitted flight plans.

2. Order flights by increasing input submission times. We assume here that the order of submission times is random, and independent of other flight characteristics such as schedule departure time or $\alpha$ value. We index flights by their submission order $m = 1, \dots, N$.

3. Set $J^A = J$.

4. For $m = 1, 2, \dots, N$:

   a) Find the <u>feasible</u> and <u>available</u> slot $j \in J^A$ that minimizes $c_{m,j}$. A slot $j$ is <u>feasible</u> to flight $m$ if $d_j \geq g_{0,m}$.
   b) Assign $m$ to $j$, resulting in flight assignment cost $c_{m,j}$. Set $J^A = J^A - j$.
   c) Repeat (a) and (b) until $m = N$.

5. Find $\sum_{m=1}^{N} c_{m,j}$.

Repeat (1) through (5) 5,000 times for a selected value of $\sigma$.

### *4.3 Ration-by-Schedule (RBS)*

Ration-By-Schedule (RBS) is, like FSFA, a sequential allocation scheme. In this case, however, flights are assigned resources in the order by which they are scheduled to be at some reference point in the airspace (Fix A, in this research). RBS is a well-documented and widely-accepted allocation algorithm that has been in use for many years in ground delay programs (Hoffman, et al., 2005), to assign flights controlled departure times so that they arrive in the scheduled order at an airport where a GDP is in effect. RBS has been shown to be in the set of system-optimal allocations with respect to ground delay (Vossen & Ball, 2005).

To model RBS, we follow the steps of the recursive algorithm used for FSFA; however, Step 2 would be altered such that flights are ordered by increasing scheduled departure times $g_{0,n}$.

### *4.4 Parametric optimal (PASO)*

In the parametric system-optimal (PASO) scheme, we envision that flight operators provide flight cost parameter inputs to a centrally-managed FAA database. Operators would be encouraged to update their parameters as necessary. At the time that resource allocation decisions must be made (after the CTOP is announced, typically several hours prior to its actual start time (Hoffman, et al., 2004)), the parameter values contained in the database at that time will be used to determine route and ground delay assignments for all impacted flights. If we assume that traffic managers have adopted the flight cost model of equation (2), the requested parameter would be the air-to-ground cost ratio $\alpha$. Therefore traffic managers do not receive complete information (as per equation (2)) about the operators' flight costs. Rather, for the PASO scheme, $\Delta_{n,r} = \alpha_n \rho_r$. Similar to the FISO scheme, these inputs are used to perform a system-optimal resource allocation, albeit one based on incomplete information because the random term is not included in the input (recall that in FISO and FSFA, the private route preference information represented by the random term is included in the inputs $\Delta_{n,r}$).

PASO has two main features of interest. Firstly, the parametric input is very flexible in that it can be used to estimate the cost of any route option. In the first submitted, first assignment (FSFA) and full information optimal (FISO) models, $\Delta_{n,r}$ are submitted specifically for the available route options because they contain the additional, route specific information of the random term. The advantage of PASO is that even if a flight operator does not have complete information about all the routes available in CTOP, traffic managers can still use the operator's parametric input to identify a desirable option they might not have considered. Secondly, in traffic management programs like the AFP and CTOP, decisions must be made very quickly, and operators may not be able to provide highly detailed information about their flights (as represented by the random term) in a convenient or timely manner. By providing $\alpha$ values to the database, operators are ensured that the FAA has at least some generic information – not necessarily particular to a specific traffic constraint situation – about their flights and cost structure.

If the random term in the cost equation (2) has a low variance (i.e. $\sigma^2$ is small), the PASO resource allocation will be efficient, because the deterministic portion of the flight cost model is a good reflection of actual costs. If, however, the random term has a high variance, PASO resource allocations will be less efficient. We would like to ascertain how PASO performs in comparison to FSFA as the variance of the random term – and hence the incompleteness of the traffic managers' information about flight operators' route preferences – increases. PASO will always be less efficient compared to the full information optimal (FISO) model unless $\sigma = 0$. PASO is formulated identically to FISO, except that the objective function consists only of the deterministic part of the flight cost function, because the only information that traffic managers have received from flight operators in PASO are $\alpha_n$ values. As a result, $\Delta_{n,r(j)} = \alpha_n \rho_{r(j)}$ as stated earlier, and the objective in PASO becomes the minimization of the total deterministic operator cost of allocating resources to flights, or min $\hat{C}$, instead of the total (deterministic + stochastic) cost, or min $C$, in equation (4). The PASO model is otherwise identical to that of FISO.

Once allocations are made, we can calculate the expected "true" cost of the allocation $E[C]$ by adding the random term representing flight operators' private route preferences.

$$E[C] = \hat{C} + E\left[\sum_{j\in J} x_{n,j,r(j)} \cdot \varepsilon_{n,r(j)}\right] = \hat{C}, \quad \varepsilon_{n,r(j)} \sim iid\ N(0,\sigma^2) \tag{6}$$

where $\varepsilon_{n,r(j)}$ represents the private route preference for flight $n$ assigned to slot $j$, which in turn is associated with route $r$. The expectation term in (6) is zero because the $x_{n,j,r(j)}$'s are determined without regard to the random variables $\varepsilon_{n,r(j)}$.

In each allocation scheme presented, the constraint $d_{j,r(j)} \geq g_{0,n}$ was included to ensure that a flight is never assigned a CTOP departure time that precedes their original scheduled departure time at Fix A. It was felt that this constraint was appropriate in our en route resource allocation problem because flights typically do not depart airports prior to their scheduled departure times – there is little advantage to doing so, and "negative" delays are not counted towards an airline's on-time statistics. In fact, relaxing the constraint gives PASO, FSFA, and RBS some convenient properties that allow for analytic and pseudo-analytic formulations of these models (Kim, 2011). Also, under certain conditions, it was found that the constraint is rarely violated in these analytic models. A future extension of this work could remove the constraint and allow the assignment of penalties to flights arriving early to Fix A, due to the fact that early arrivals would add workload to the traffic managers' resource assignment process.

## 5. Properties of FISO, FSFA, and PASO

We construct a stylized example to investigate the stochastic properties of the FISO, FSFA, and PASO allocation schemes. In this example, two flights ($N = 2$) can be assigned to one of two routes ($R = 2$) with one slot each. The cost of flight $n$ taking route $r$ is $c_{nr} = w_{nr} + \varepsilon_{nr}$, where $w_{nr}$ represents deterministic costs and $\varepsilon_{nr}$ is the random term. The deterministic costs of a flight taking any route are equal such that $w_{11} = w_{12} = w_{21} = w_{22} = w$, and the total deterministic cost of any allocation is $W$. It follows that the random terms will dictate how resources are rationed in each allocation scheme. Table 1 below shows the deterministic costs of the two possible allocations of this example.

*Place Table 1 about here.*

Assume that the random term $\varepsilon_{n,r}$ is distributed iid normal with mean 0 and standard deviation $\sigma$. Now say that Flight 1 and Flight 2 have different optimal slots, such that if Flight 1 prefers slot 1, then Flight 2 prefers slot 2, and vice versa. The expected costs of the FISO and FSFA allocations will then be identical, $E[C_{FISO}] = E[C_{FSFA}]$. If Flights 1 and 2 have the same optimal slot (such that both flights prefer slot 1 or both prefer slot 2), then $E[C_{FISO}] < E[C_{FSFA}]$. Unless $\varepsilon_{n,r} = 0\ \forall n, r$, we cannot say how $E[C_{PASO}]$ will compare to the results of the other two schemes. We can express the expected costs resulting from the application of each resource allocation scheme as follows:

$$E[C_{FISO}] = E[min(C_1, C_2)] = W + E[min(\varepsilon_{11} + \varepsilon_{22}, \varepsilon_{12} + \varepsilon_{21})] \tag{7}$$

$$\begin{aligned} E[C_{FSFA}] = \frac{1}{2}&\begin{cases} W + E[min(\varepsilon_{11}, \varepsilon_{12}) + \varepsilon_{22}] & if\ min(\varepsilon_{11}, \varepsilon_{12}) = \varepsilon_{11} \\ W + E[min(\varepsilon_{11}, \varepsilon_{12}) + \varepsilon_{21}] & if\ min(\varepsilon_{11}, \varepsilon_{12}) = \varepsilon_{12} \end{cases} \\ &+ \frac{1}{2}\begin{cases} W + E[min(\varepsilon_{21}, \varepsilon_{22}) + \varepsilon_{12}] & if\ min(\varepsilon_{21}, \varepsilon_{22}) = \varepsilon_{21} \\ W + E[min(\varepsilon_{21}, \varepsilon_{22}) + \varepsilon_{11}] & if\ min(\varepsilon_{21}, \varepsilon_{22}) = \varepsilon_{22} \end{cases} \end{aligned} \tag{8}$$

$$E[C_{PASO}] = W + E[\varepsilon_{1r}] + E\left[\varepsilon_{1(-r)}\right] = W \tag{9}$$

Equation (8) results from having a one-half probability that preference information for each of the two flights is submitted first. Since $\varepsilon_{nr}$ are iid normal, the moments of the maximum of $N$ random variables are easily calculated (Bose & Gupta, 1959). The equations are summarized in Clark (1961); if $\varepsilon_1, \varepsilon_2 \sim iid\ N(0, \sigma^2)$, the expected value of the minimum of $\varepsilon_1$ and $\varepsilon_2$ can be expressed as:

$$E[\min(\varepsilon_1, \varepsilon_2)] = -\sigma/\sqrt{\pi} \tag{10}$$

and we can rewrite equations (7) and (8) such that

$$E[C_{FISO}] = W - \sqrt{2}\sigma/\sqrt{\pi} \tag{11}$$

$$E[C_{FSFA}] = W - \sigma/\sqrt{\pi} \tag{12}$$

See Appendix B.1 for detailed calculations.

We are interested in understanding how well an allocation scheme performs relative to the FISO scheme, which yields the most efficient total user cost solution possible under any given situation. As a result, we express the total cost results of the FSFA and PASO schemes as ratios of FISO:

$$C'_{FSFA} = \frac{E[C_{FSFA}]}{E[C_{FISO}]} = \frac{W\sqrt{\pi} - \sigma}{W\sqrt{\pi} - \sqrt{2}\sigma} \quad (13)$$

$$C'_{PASO} = \frac{E[C_{PASO}]}{E[C_{FISO}]} = \frac{W\sqrt{\pi}}{W\sqrt{\pi} - \sqrt{2}\sigma} \quad (14)$$

Due to the fact that the total deterministic cost of any allocation is always $W$ in this simple example, when traffic managers have perfect information about flight operators (represented by $\sigma = 0$, and therefore, $\varepsilon_{n,r} = 0\ \forall n, r$) the FISO and FSFA models yield identical resource allocations and total costs for a given set of parameters. Therefore, $C'_{FSFA} = 1$ when $\sigma = 0$. Clearly in more realistic scenarios where deterministic allocation costs differ, it will be the case that the deterministic results of system-optimal and greedy (such as FSFA) assignment algorithms will differ as well. As such, even when $\sigma = 0$, a FSFA allocation will be less cost efficient than a system-optimal allocation, and the entire $C'_{FSFA}$ curve would be shifted up because FSFA does not offer a system-optimal resource assignment. In any case, as $\sigma$ increases, equation (13) increases as well.

It can be observed from equation (14) that $C'_{PASO} = 1$ when $\sigma = 0$; the FISO and PASO schemes yield identical resource allocations and total operator costs not only in this simple example but under any scenario at $\sigma = 0$. Recall that PASO resource allocations do not utilize the operators' private route preference information provided through the random term. As a result, as traffic managers' uncertainty about operators' private route preferences increases (represented by increasing $\sigma$), $C'_{PASO}$ will also increase, and more rapidly than $C'_{FSFA}$.

Figure 4 displays $C'_{FSFA}$ and $C'_{PASO}$ with respect to $\sigma$. The $x$-axis represents increasing values of $\sigma$ as a proportion of $w$, where $\sigma$ is the standard deviation of the random term in the flight cost model and $w$ is the deterministic cost of any flight $n$ taking any route $r$ (identical for all $n$ and $r$). For instance, the point "0.10" on the $x$-axis indicates that $\sigma$ is 10% of $w$, or $\sigma/w = 0.10$. Increasing $\sigma$ represents greater variations in the flights' routing preferences.

*Place Figure 4 about here.*

Figure 4 shows that both $C'_{FSFA}$ and $C'_{PASO}$ are increasing convex functions of $\sigma$. $C'_{PASO}$ increases at a faster rate than $C'_{FSFA}$, and as both equal one at $\sigma = 0$, $C'_{PASO}$ is greater than $C'_{FSFA}$ at any positive value of $\sigma$.

Appendix B.2 contains the calculations pertaining to the discussion of the results above.

## 6. Numerical examples

We perform numerical simulations to further compare the performance of the allocation schemes. First, for one set of supply and demand values, we compare the overall efficiency (based on total cost) and equity (based on standard deviation of cost) of all the schemes introduced in Section 4. We then perform sensitivity tests on the relationship between the FSFA and PASO total cost results, with respect to the demand parameters of a CTOP.

Suppose $N$ flights are to be reassigned routes and departure times due to an en route constraint. The nominal route remains open, but at a greatly reduced capacity. All supply parameters are listed below in Table 2. The CTOP will have a total of five routing options, one of which includes the nominal route under a decreased capacity. Each route has a capacity (column 2) which we assume results in departure slots that are spaced at constant headways (column 3). $C_{CTOP}$ is the total capacity of all routes available in the CTOP.

*Place Table 2 about here.*

Now we introduce the flight demand characteristics. First, we assume that the flights' air-to-ground cost ratios $\alpha_n$ follow a uniform distribution in (1.5,2.5] across all flights. It is commonly cited in the literature that one unit of en route delay is equal in cost to about two units of ground delay; as a result, $\alpha$ is often assumed to equal two in existing ATFM models (Mukherjee & Hansen, 2009). The uniform distribution and its upper and lower bounds were chosen arbitrarily in the absence of any empirical information about this characteristic. In addition, it facilitated the construction of analytic and pseudo-analytic approximations to the models (Kim, 2011). Secondly, we assume that the original scheduled departure times $g_{0,n}$ are spaced at constant headways as well. Finally, we assume that the $\Delta_{n,r}$ submission order in FSFA is random and independent of other flight and flight operator characteristics. It is also possible to model FSFA input submission as a competitive process (see (Kim, 2011)).

Figure 5 presents results for the scenario described above, with $N = 75$ flights and $D_0 = 75$ flights per hour. The axes are the same as those of Figure 4, where the $x$-axis represents increasing values of the standard deviation of the random term in the flight cost model. Specifically, each point on the $x$-axis represents the value of $\sigma$ as a proportion of the average flight cost in FISO under perfect information conditions, $\hat{c}_{FISO}$. For instance, the point "0.10" means that $\sigma$ is 10% of $\hat{c}_{FISO}$. The $y$-axis again represents the total cost result of each model as a ratio of the full information system-optimal (FISO) total cost, or $y = C'_{scheme} = C_{scheme}/C_{FISO}$; $scheme \in$ {PASO, FSFA, RBS}. Recall that we compare the performances of the other models against the FISO solution ($C_{FISO}$) because FISO uses complete information (in the context of our flight cost model) to perform a system-optimal allocation, yielding the most efficient solution in any situation. Each point on Figure 5 represents the average $C'_{scheme}(= C_{scheme}/C_{FISO})$ ratio based on 5,000 simulation runs of each scheme (including FISO). The same set of 5,000 samples used to construct a point for $C'_{FSFA}$ was used to construct a corresponding point for $C'_{RBS}$ and $C'_{PASO}$ as well. A sample refers to a flight set with particular cost characteristics as determined by random draws and resulting combinations of $\alpha_n$ and $\varepsilon_{n,r}$. Evaluating each scheme's performance on the same sample set ensures that our comparison is meaningful. The faint dotted lines represent the sample standard deviation of the results represented by each point. Also, the standard error of the mean was computed for each set of simulations represented by each point in Figure 5, and were found to be insignificant, indicating that 5,000 simulation runs is a sufficient sample size.

*Place Figure 5 about here.*

Recall that in the parametric system-optimal (PASO) scheme, traffic is assigned without considering the random component of the cost function. As a result, as the dispersion of this random component increases, $C_{PASO}$ increases relative to $C_{FISO}$. Conversely, because the first-submitted, first-assigned (FSFA) scheme employs a greedy allocation algorithm, when $\sigma$ (and therefore, the traffic manager's uncertainty about the flights) is small, it does not offer a total cost solution that is as efficient as the PASO scheme, except in very contrived examples like that of Section 5. Then, as $\sigma$ increases, at some point it becomes more efficient to allocate resources employing a suboptimal allocation mechanism with complete information (FSFA), as opposed to using a system-optimal allocation with an increasingly incomplete information picture (PASO). The PASO solution is thus superior to that of FSFA only when the deterministic cost dominates; in this case, when $x\ (= \sigma/\hat{c}_{FISO})$ is less than approximately 0.18. At larger $x$ values, the FSFA model yields a more efficient solution. In other words, if one can estimate the true cost of an individual flight assignment (relative to the baseline situation in which no CTOP is required) to within an accuracy of about 20% using the simple parametric cost model, PASO is more efficient than FSFA.

Turning now to the RBS results, Figure 5 demonstrates that the total flight cost of that scheme, in which ordering is based on the original schedule, is greater than the FSFA solutions generated from a random ordering of submissions. The efficiency advantages of FSFA over RBS stems from the fact that in RBS the flights that "choose" last also have the least number of feasible choices, since they cannot choose slots earlier than their schedule departure times. Kim (2011) shows inductively that this leads to reduced efficiency for RBS. On the other hand, even in the case of RBS, at some $\sigma$ value it will always be more cost efficient to allocate resources with better information compared to using a system-optimal assignment with incomplete information (PASO).

The results also demonstrate that, while the total costs of both the FSFA and RBS schemes are far less sensitive to $\sigma$ than the total cost results of PASO, there is still a positive relationship. As $\sigma$ increases, so do inter-flight cost differences, resulting in greater efficiency losses from assigning flights sequentially rather than in a system-optimal fashion. Nonetheless, the results reveal that the loss from an incremental assignment is fairly insensitive to $\sigma$, implying one need not have precise information about $\sigma$ in order to assess these schemes. However, it also appears that the FSFA and RBS results approach one another as $\sigma$ increases; it is possible that the difference in results between two schemes may not be statistically significant at some point $x > 0.40$. These results were not explored because $x > 0.40$ represents a situation where it would be highly appropriate for traffic managers to revisit their (deterministic) cost model specification. The standard deviations on the simulated total cost results for each allocation scheme are in the order of 0.8-4.8%, with the higher end of the range observed for higher values of $x\ (= \sigma/\hat{c}_{FISO})$.

We now compare the standard deviation of individual flight costs, a measure of equity, under the various CTOP schemes. Figure 6 shows the average standard deviation of flight costs (over all simulation runs, in units of ground delay minutes) for each scheme. The $x$-axis again represents $\sigma$ for the random cost term. It can be observed that FISO, FSFA, and RBS have standard deviations of individual flight costs that are greatest at the upper end of the $\sigma$ range. For FISO the standard deviation appears to increase with a constant slope, while for FSFA and RBS the standard deviations decrease slightly until $\sigma/\hat{c}_{FISO} = 0.15$ and then begin to increase. In the lower end of the range of $\sigma/\hat{c}_{FISO}$, the sequential algorithms are able to offset the increase in cost variability by assigning flights to their preferred routes. Eventually, however, this mitigation capacity is overwhelmed the inherent variability in route specific costs, combined with the fact that the FSFA and RBS schemes "pick favorites" by ordering flights. On the other hand, the standard deviation of flight costs for PASO is constant, which is to be expected since the PASO allocation scheme does not consider the random cost component and does not involve a priori flight ordering. Thus, from an equity point of view, the PASO allocation scheme is the least dependent upon the quality of cost information provided to the allocation process.

*Place Figure 6 about here.*

A key result shown in Figure 6 is that RBS has smaller average standard deviations of individual flight costs than any of the other schemes. This is due to the fact that in FISO, PASO, and FSFA, some flights are forced to incur very high costs. In FISO and PASO, this can occur because the objective is simply to minimize the total cost, and a minimum total cost solution may be one where some flights have extremely low allocation costs while others are penalized greatly. In FSFA, flights with later departure times that submit early effectively "jump the RBS queue", and obtain extremely low cost resources that would not be possible with RBS, while delaying other flights with earlier scheduled departure times. Lower variation in flight costs could be interpreted as a potential advantage of the RBS scheme; although it is inefficient in terms of total cost, it offers the smallest overall variation in individual flight allocation costs compared to the other schemes, and can therefore be considered most equitable.

In the remainder of this section, we present a sensitivity analysis on the total cost ratio of the FSFA and PASO allocations, focusing on the effects that demand-side characteristics have on $C_{PASO}/C_{FISO}$ (henceforth referred to as $R$). We retain the supply-side characteristics introduced above in Table 2. In the first investigation (Model 1) we study the effects of changing the original scheduled flight demand rate $D_0$, the upper bound value of $\alpha$ ($\alpha_{max}$), as well as the standard deviation of the random term ($\sigma$). The duration of the CTOP is fixed at one hour. We assume that $R$ is a function of these inputs:

$$\frac{C_{PASO}}{C_{FSFA}} = R = f(x_1, x_2, x_3) \tag{15}$$

Where $x_1 = D_0, x_2 = \alpha_{max}, x_3 = \sigma/\hat{c}_{FISO}$.

We fit a response surface model to the data in order to gain some insight into the effects of the input values. We employ the translog specification for this model:

$$\left[\ln(R) - \overline{\ln(R)}\right] = \phi + \sum_{j=1}^{3} \beta_j \left[\ln(x_j) - \overline{\ln(x_j)}\right] + \sum_{j=1}^{3} \sum_{l \geq j}^{3} \gamma_{jl} \left[\ln(x_j) - \overline{\ln(x_j)}\right]\left[\ln(x_l) - \overline{\ln(x_l)}\right] + \varepsilon \tag{16}$$

The general behavior of $R$ with respect to the input variables can be observed through plots of the numerical results. However, the additional information we gain from fitting the model above is that the estimated first-order parameters, or $\beta_j \forall j$, are the elasticities of the dependent variable with respect to the explanatory variables (at average values of the explanatory variables). The parameters $\gamma_{jl}$ tell us how the elasticities changes as the input variable $x_j$ increases (when $j = l$), or the cross elasticities (when $j \neq l$).

The values assumed for the demand side variables $D_0$, $\alpha_{max}$, and $\sigma/\hat{c}_{FISO}$ are shown in Table 3. 4,000 simulation runs were performed for each combination of the indicated values for $D_0$, $\alpha_{max}$, and $\sigma/\hat{c}_{FISO}$. Recall that the supply side variables are those introduced in Table 2.

*Place Table 3 about here.*

The results of the model fitting are shown in Table 4. All estimated parameters are significant at the 1% level. We abbreviate $\sigma/\hat{c}_{FISO}$ to $\omega$ in the table.

*Place Table 4 about here.*

The first-order coefficient for $D_0$ indicates that, at the sample mean, if $D_0$ increases 10%, $R$ will increase 1.5%. Higher demand thus favors the use of incremental assignment based on complete information over system optimal assignment based on partial information. The second-order coefficient on $D_0$ indicates that the elasticity of $R$ with respect to $D_0$ is increasing with $D_0$. The first-order coefficient on the standard deviation of the random term $\sigma/\hat{c}_{FISO}$ (represented by $\omega$ in the tables) indicate a similar behavior at the mean, but the small second order coefficient implies that the elasticity is essentially constant. This reflects the trends observed in Figure 5. The first- and second-order coefficient on $\alpha_{max}$ indicates that if $\alpha_{max}$ increases at 10%, $R$ will decrease 0.6%, and the elasticity becomes more negative as $\alpha_{max}$ increases. This is to be expected, because although information about $\alpha$ is provided to both FSFA and PASO, the latter makes more efficient use of it. The interactions between the variables are quite weak, with the exception of that between $D_0$ and $\alpha_{max}$.

We now look at a second model (Model 2) where we investigate the effects that CTOP duration $T$ and $\sigma/\hat{c}_{FISO}$ ($\omega$) have on $R$. We fix the original scheduled demand rate and $\alpha$ upper bound values at $D_0 = 80$ flights per hour and $\alpha_{max} = 2.5$, respectively.

*Place Table 5 about here.*

*Place Table 6 about here.*

From the results in Table 6 we can make a similar conclusion regarding the effect of $\sigma/\hat{c}_{FISO}$ ($\omega$) on the ratio between $C_{PASO}$ and $C_{FSFA}$ ($R$). We also observe that the CTOP duration has a modest negative effect on $R$, and the second order terms are all of very small magnitude. The main conclusion is that program duration matters very little in determining the relative efficiency of FSFA and PASO.

A key observation made from these numerical investigations is that the total cost results of the FSFA scheme – which is based on an equity principle – are consistent and reasonably close to optimal over the examples presented, while the results of PASO – which is based on efficiency – deteriorate quite significantly and dramatically beyond some level of uncertainty about flight operator preferences. If traffic managers knew that a simple parametric flight cost model captured the predominant information about flight operator resource preferences, and sought an efficient, workable, and reasonably equitable allocation scheme, then PASO is a good alternative. In reality, however, it is unlikely that traffic managers would have such confidence, especially in the face of rapidly changing conditions. An allocation scheme such as FSFA or RBS would then be more prudent, with the choice between the two depending on the relative importance of efficiency and equity criteria.

## 7. Conclusions and future work

We have proposed a modeling framework through which we can evaluate and compare en route resource allocation schemes, and investigate the issues involved with incorporating user inputs in allocating constrained capacity. We specify four resource assignment schemes that feature different user preference inputs and allocation mechanisms. These schemes are designed to offer users flexibility and ease in providing general preference information, or clear incentives to make the effort required to develop and provide timelier and richer information. We evaluate and compare these schemes under changing assumptions about the magnitude of "stochastic" variation in flight assignment costs. Numerical examples illustrate situations where sacrificing a system-optimal allocation rule to obtain more detailed information about flight routing preferences will result in greater total user cost efficiency, and vice versa. If a comparatively simple user flight cost model accurately predicts flight assignment costs, the PASO assignment method, which requires simple parametric input and aims for a system optimal solution, appears promising. Conversely, if route- and situation-specific factors that are not captured by a simple parametric model hold great sway, the FSFA allocation scheme would work better, as it provides flight operators incentives to submit detailed preference information in a timely manner. If the traffic managers have little knowledge about the accuracy of their cost model specification, the FSFA method remains the more prudent choice, because it is less likely to result in severe losses in efficiency even when PASO is better. A key finding of this research is that the total cost results of the FSFA scheme – which is based on an equity principle – are reasonably consistent over the examples presented, while the results of PASO – which is based on efficiency – deteriorate quite significantly and dramatically beyond some level of uncertainty about flight operator preferences. Another interesting finding is that although RBS offers inferior total cost efficiency within our modeling framework, it does result in less variation in individual flight allocation costs, and hence in greater equity. Also, we find the performances of the assignment methods to be relatively insensitive to increasing program durations (and flight populations), but more sensitive to relative cost of airborne and ground delay, as well as the degree of imbalance between demand and capacity.

Several of the models studied in this paper assume that flight operators offer "complete" preference submissions to traffic managers. In reality, flight operators may not be able to offer this information because they are not sufficiently incentivized to provide it in such a competitive environment, and because it may be too difficult or costly to obtain this information when conditions are rapidly changing. This paper also assumes that submissions in FSFA are in a random order that is independent of flight cost characteristics. This assumption is unlikely to be true, and should be relaxed in future work. It would also be beneficial to investigate how a non-linear flight cost function specification affects model results.

## Acknowledgments

This work was sponsored by the FAA Office of Air Traffic Organization. We would like to thank Bob Hoffman, Dennis Gallus, and Nathaniel Gaertner at Metron Aviation for their great assistance, insights, and suggestions. We also thank Megan S. Ryerson of the University of Tennessee, Knoxville, and three anonymous reviewers for their comments and critique.

## Appendix

### A.1 Tables of allocation scheme acronyms and variables

Acronyms

| | |
|---|---|
| FISO | Full information system-optimal |
| FSFA | First submitted, first assigned |
| RBS | Ration-by-Schedule |
| PASO | Parametric system-optimal |

Variables

| | |
|---|---|
| $N$ | Total number of flights caught in the CTOP; $n = 1,2, \ldots, N$. |
| $R$ | Total number of routes available in CTOP; $r = 1,2, \ldots, R$. |
| $J$ | Total number of slots available in CTOP, across all routes, $j \in J$. Each slot $j$ maps to a route. |
| $J^A$ | Set of slots available to some flight $n$ at the time of submission. |
| $S_r(t)$ | Capacity on route $r$ at some time $t$ *(flights per hour)*. |
| $D_0(t)$ | Demand for nominal route, prior to constraint *(flights per hour)*. |
| $T$ | CTOP duration *(minutes)*. |
| $C$ | "True" total cost of allocation *(ground delay minutes)*. |

$\hat{C}$ Deterministic total cost of allocation *(ground delay minutes)*.

$c_{n,j,r(j)}$ "True" cost for flight $n$ to take slot $j$, which is associated with route $r$ *(ground delay minutes)*.

$\alpha_n$ En route to ground delay ratio; $n$'s airborne time cost converted to units of ground delay minutes. In the numerical examples, we assume $\alpha_n$ is uniformly distributed across all flights between $\alpha_{min}$ and $\alpha_{max}$.

$\rho_{r(j)}$ Additional en route time on route $r$ compared to nominal route *(en route minutes)*.

$d_{j,r(j)}$ Departure time on slot $j$, which belongs to route $r$, at fix A *(ground delay minutes).*

$g_{0,n}$ $n$'s original scheduled departure time at fix A *(ground delay minutes)*.

$\varepsilon_{n,r(j)}$ Random term representing $n$'s other (private) costs for flying route $r$; $\varepsilon_{n,r} \sim N(0, \sigma^2)$.

$\Delta_{n,r}$ Operator submitted cost information about flight $n$ flying route $r$ before ground delay is assigned by traffic managers as part of the resource allocation.

## B.1 Expected value of the minimum of two iid normal random variables

Since $\varepsilon_{nr}$ are iid normal, the moments of the maximum of $N$ random variables can be calculated (Bose & Gupta, 1959) (Teichroew, 1956). The equations are summarized in Clark (1961); the first moment of two independent normal random variables, $\varepsilon_1 \sim N(\mu_1, \sigma_1)$ and $\varepsilon_2 \sim N(\mu_2, \sigma_2)$ is as follows:

$$E[\max(\varepsilon_1, \varepsilon_2)] = \mu_1 \Phi(\alpha) + \mu_2 \Phi(-\alpha) + a\varphi(\alpha) \tag{B.1}$$

Where

$$a = \sigma_1^2 + \sigma_2^2 - 2\sigma^2 \rho_{1,2}$$

$$\varphi(\alpha) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

$$x^2 = \alpha^2 = (\mu_1 - \mu_2)^2 / a^2 = 0$$

Recall that $\varepsilon_{n,r} \sim iid\ N(0, \sigma)\ \forall n, r$. It then follows that $\rho_{1,2} = 0$, $\mu_1 = \mu_2 = 0$, and $\sigma_1 = \sigma_2 = \sigma$. Equation (B.1) reduces to

$$E[\max(\varepsilon_1, \varepsilon_2)] = \sigma / \sqrt{\pi} \tag{B.2}$$

Since the normal distribution is symmetric around the mean,

$$E[\min(\varepsilon_1, \varepsilon_2)] = -E[\max(\varepsilon_1, \varepsilon_2)] \tag{B.3}$$

Recall $\varepsilon_{11}, \varepsilon_{12}, \varepsilon_{21}, \varepsilon_{22} \sim N(0, \sigma)$. Equation (8) therefore reduces to

$$E[C_{FSFA}] = W + E[\min(\varepsilon_1, \varepsilon_2)] + E[\varepsilon_3] = W - \sigma/\sqrt{\pi} \tag{B.4}$$

Next, set $\varepsilon_A = \varepsilon_{11} + \varepsilon_{22}$ and $\varepsilon_B = \varepsilon_{12} + \varepsilon_{21}$. Then, we know that $var(\varepsilon_A) = var(\varepsilon_B) = var(\sum_{i=1}^{2} \varepsilon_i) = \sum_{i=1}^{2} var(\varepsilon_i) = 2\sigma^2$, and $\varepsilon_A, \varepsilon_B \sim N(0, \sqrt{2}\sigma)$ according to the properties of normal iid random variables. Equation (7) becomes

$$E[C_{FISO}] = W + E[\min(\varepsilon_A, \varepsilon_B)] = W - \sqrt{2}\sigma/\sqrt{\pi} \quad \square \tag{B.5}$$

## B.2 Some relationship properties

Define

$$E[C'_{FSFA}] = \frac{E[C_{FSFA}]}{E[C_{FISO}]} = \frac{W - \sigma/\sqrt{\pi}}{W - \sqrt{2}\sigma/\sqrt{\pi}} = \frac{W\sqrt{\pi} - \sigma}{W\sqrt{\pi} - \sqrt{2}\sigma} \tag{B.6}$$

$$E[C'_{PASO}] = \frac{E[C_{PASO}]}{E[C_{FISO}]} = \frac{W}{W - \sqrt{2}\sigma/\sqrt{\pi}} = \frac{W\sqrt{\pi}}{W\sqrt{\pi} - \sqrt{2}\sigma} \tag{B.7}$$

We also know that $lim_{\sigma \to 0} E[C'_{FSFA}] = 1$, $lim_{\sigma \to \infty} E[C'_{FSFA}] = \infty$, and $lim_{\sigma \to 0} E[C'_{PASO}] = 1$, $lim_{\sigma \to \infty} E[C'_{PASO}] = \infty$.

It is clear that as $\sigma \to \infty$, the denominators of both $E[C'_{FSFA}]$ and $E[C'_{PO}]$ decrease at faster rates than the numerators, indicating that $E[C'_{FSFA}]$ and $E[C'_{PO}]$ increase non-linearly. To verify,

$$\frac{\partial(E[C'_{FSFA}])}{\partial\sigma} = \frac{W(\sqrt{2} - 1)\sqrt{\pi}}{(W\sqrt{\pi} - \sqrt{2}\sigma)^2} \in \mathbb{R}^+, \frac{\partial^2(E[C'_{FSFA}])}{\partial\sigma^2} \in \mathbb{R}^+;\ \sigma \geq 0 \tag{B.8}$$

$$\frac{\partial(E[C'_{PASO}])}{\partial\sigma} = \frac{W\sqrt{2\pi}}{(W\sqrt{\pi} - \sqrt{2}\sigma)^2} \in \mathbb{R}^+, \frac{\partial^2(E[C'_{PASO}])}{\partial\sigma^2} \in \mathbb{R}^+;\ \sigma \geq 0 \tag{B.9}$$

Both $E[C'_{FSFA}]$ and $E[C'_{PASO}]$ are non-linearly increasing functions of $\sigma$. Their slopes increase with respect to $\sigma$ as well. It then follows that, due to the construct of the stylized model where the total deterministic cost of any allocation is always $W$, $E[C'_{FSFA}]_\sigma > E[C'_{PASO}]_\sigma, \sigma \geq 0$. Also, $E[C'_{PASO}]'/E[C'_{FSFA}]' = \frac{\sqrt{2}}{\sqrt{2}-1} = 3.414$. □

Say the deterministic cost of the FSFA allocation is $W_{FSFA}$ and that of the FISO allocation is $W_{FISO}$ when $\sigma = 0$. Then $lim_{\sigma\to 0}\, E[C'_{FSFA}] = W_{FSFA}/W_{FISO}$. □

## References


Ball, M., Futer, A., Hoffman, R. & Sherry, J., 2002. *Rationing Schemes for En-route Air Traffic Management (CDM Memorandum).* s.l.:s.n.

Ball, M. O., Hoffman, R., Odoni, A. & Rifkin, R., 2003. A Stochastic Integer Program with Dual Network Structure and its Application to the Ground-Holding Problem. *Operations Research,* 51(1), pp. 167-171.

Bertsimas, D. & Stock Patterson, S., 1998. The Air Traffic Flow Management Problem with Enroute Capacities. *Operations Research,* 46(3), pp. 406-422.

Bertsimas, D. & Stock Patterson, S., 2000. The Traffic Flow Management Rerouting Problem in Air Traffic Control: A Dynamic Network Flow Approach. *Transportation Science,* 34(3), p. 239–255.

Bose, R. & Gupta, S. S., 1959. Moments of Order Statistics from a Normal Population. *Biometrika,* Volume 46, pp. 433-440.

Bureau of Transportation Statistics, 2008. *Airline On-Time Statistics and Delay Causes.* [Online] Available at: http://www.transtats.bts.gov/OT_Delay/OT_DelayCause1.asp [Accessed 11 April 2009].

Clark, C. E., 1961. The Greatest of a Finite Set of Random Variables. *Operations Research,* 9(2), p. 18.

FAA, 2007. *FAA Greatly Expands Air Traffic Program to Minimize Summer Delays.* [Online] Available at: http://www.faa.gov/news/press_releases/news_story.cfm?newsID=8889 [Accessed August 2010].

FAA, 2010. *CTOP Industry Day.* [Online] Available at: http://cdm.fly.faa.gov/ad/R7docs/CTOP%20Industry%20Day%20presentations%2010132010.pdf [Accessed 22 08 2011].

Ganji, M., Lovell, D. J., Ball, M. O. & Nguyen, A., 2009. Resource Allocation in Flow-Constrained Areas with Stochastic Termination Times. *Transportation Research Record,* Volume 2106, pp. 90-99.

Goodhart, J., 2000. *Increasing Airline Operational Control in a Constrained Air Traffic System,* s.l.: University of California, Berkeley (PhD Dissertation).

Hoffman, R. et al., 2005. *Resource Allocation Principles for Airspace Flow Control.* San Francisco, AIAA Guidance, Navigation, and Control Conference and Exhibit.

Hoffman, R., Lewis, T. & Jakobovits, R., 2004. *Flow Constrained Area Rerouting Decision Support Tool, Phase I SBIR: Final Report,* s.l.: Metron Aviation.

Jakobovits, R., Krozel, J. & Penny, S., 2005. *Ground Delay Programs to Address Weather within En Route Flow Constrained Areas.* San Francisco, CA, AIAA Guidance, Navigation and Control Conference and Exhibit.

Kim, A. M., 2011. *Collaborative Resource Allocation Strategies for Air Traffic Flow Management,* s.l.: University of California, Berkeley (PhD dissertation).

Mukherjee, A. & Hansen, M., 2009. A dynamic rerouting model for air traffic management. *Transportation Research Part B: Methodological,* Volume 43, pp. 159-171.

Pourtaklo, N. V. & Ball, M., 2009. *Equitable Allocation of Enroute Airspace Resources.* Napa, Eighth USA/Europe Air Traffic Management Research and Development Seminar.

Teichroew, D., 1956. Tables of Expected Values of Order Statistics and Products of Order Statistics for Samples of Size Twenty of Less from the Normal Distribution. *The Annals of Mathematical Statistics,* Volume 27, pp. 410-426.

Vossen, T. & Ball, M., 2005. Optimization and Mediated Bartering Models for Ground Delay Program. *Naval Research Logistics,* 53(1), pp. 75-90.

Wilmouth, G. & Taber, N. J., 2005. *Operational Concept for Integrated Collaborative Rerouting (ICR),* McLean, VA: The MITRE Corporation, MTR05W000053.

Yoon, Y., Hansen, M. & Ball, M. O., 2011. *Optimal Route Generation with Geometric Recourse Model under Weather Uncertainty.* Berkeley, Elsevier, p. 551–571.

## List of Figures

## List of Tables

Figure 1

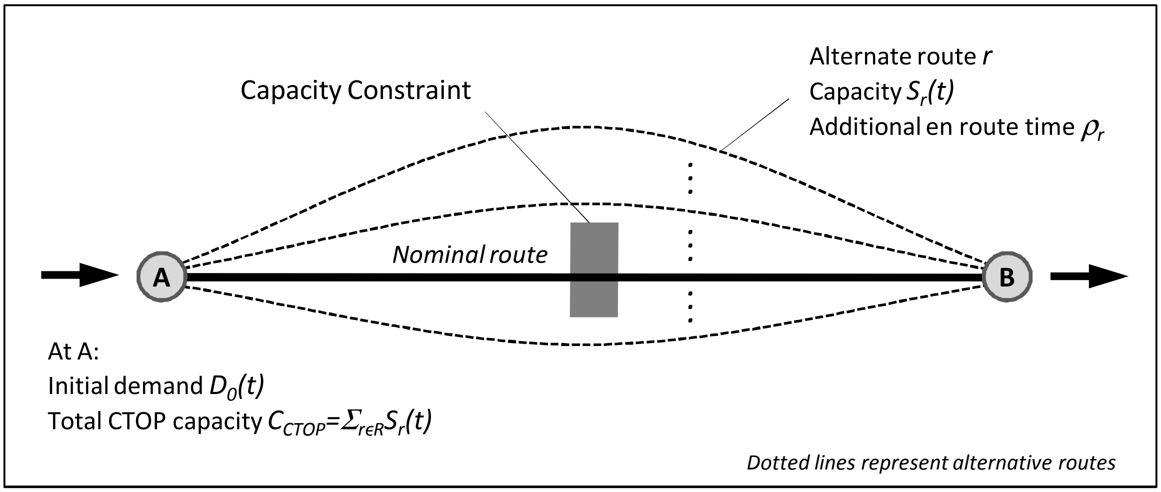


Figure 1. Model airspace geometry and select parameters.

Figure 2

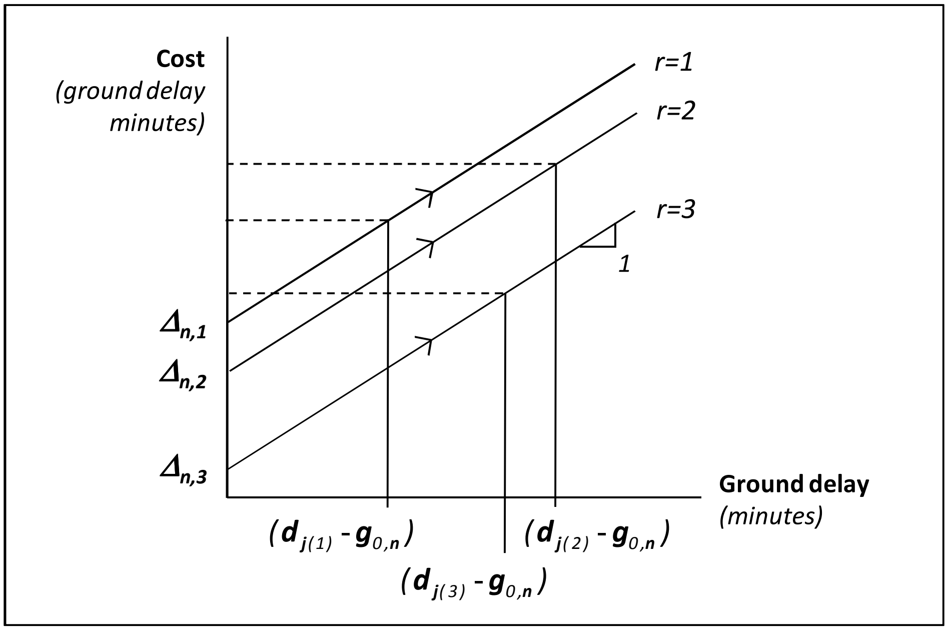


Figure 2. Illustrating the use of inputs $\Delta_{n,r}$ to determine flight costs.

**Figure 3**

**Original Demand**
*(normal conditions)*

| Flight | Original Departure Times ($g_{0,n}$) |
|---|---|
| A | $g_{0,A}=0$ |
| B | $g_{0,B}=5$ |

**System Capacity & User Preferences**
*(constrained conditions)*

| Flight | Route 1 | | | Route 2 | | |
|---|---|---|---|---|---|---|
| | Depart. time for | | Cost to | Depart. time for | | Cost to |
| | Slot 1 | Slot 2 | fly ($\Delta_{n,1}$) | Slot 1 | Slot 2 | fly ($\Delta_{n,2}$) |
| A | **5** | **60** | 100 | **0** | **20** | 150 |
| B | | | 90 | | | 140 |

| Allocation 1 | | |
|---|---|---|
| Flight | Route/Slot | Cost ($\Delta_{n,r}+d_{n,j}-g_{0,n}$) |
| A | 1,1 | *100*+(**5**-0)=105 |
| B | 1,2 | *90*+(**60**-5)=145 |
| | *Total cost* | *250* |

*OR*

| Allocation 2 | | |
|---|---|---|
| Flight | Route/Slot | Cost ($\Delta_{n,r}+d_{n,j}-g_{0,n}$) |
| A | 2,1 | *150*+(**0**-0)=150 |
| B | 1,1 | *90*+(**5**-5)=90 |
| | *Total cost* | *240* |

Figure 3. Illustration of possible allocation outcomes.

Figure 4

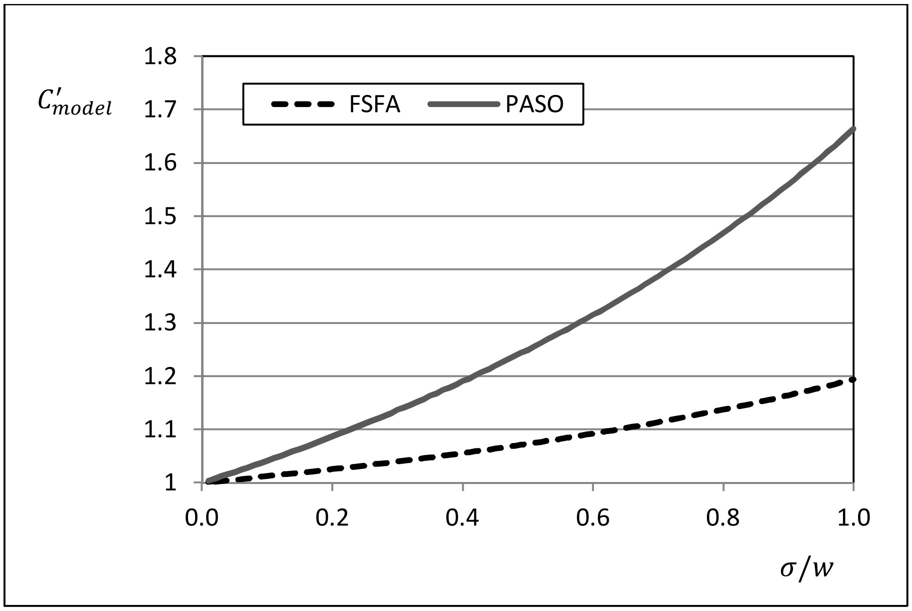


Figure 4. Total cost efficiency relationship of the allocation models.

**Figure 5**

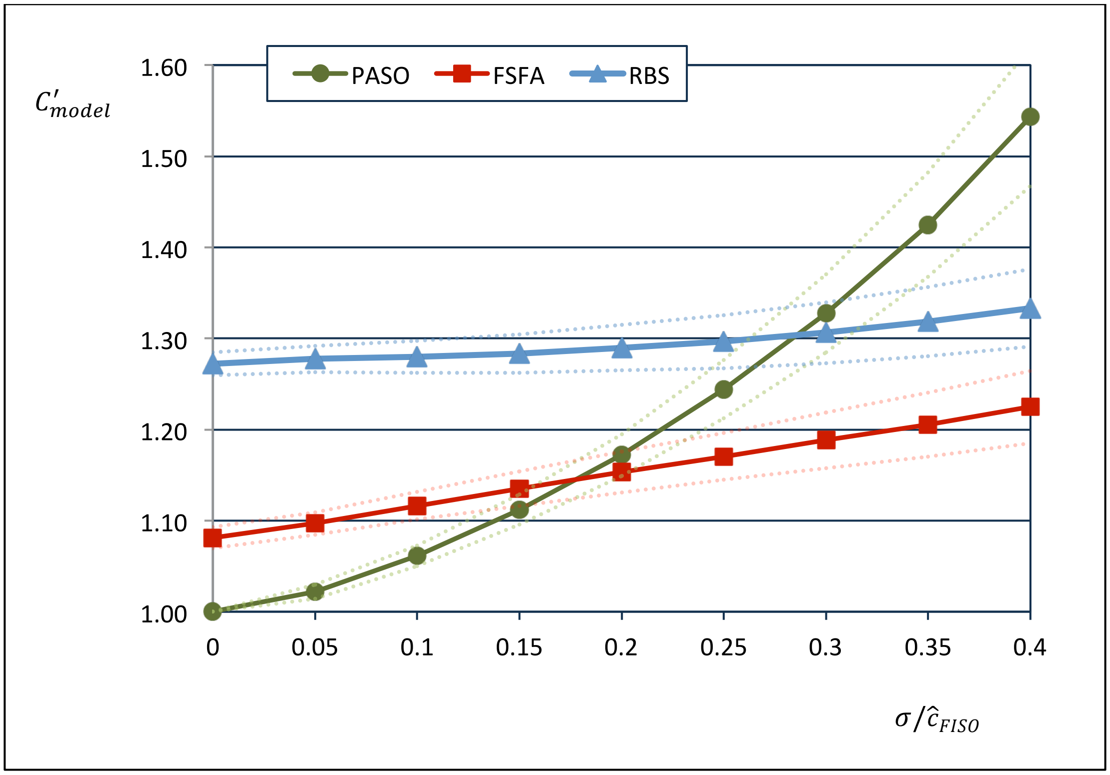


Figure 5. Total cost results, $N = 75$ flights.

Figure 5 b&w

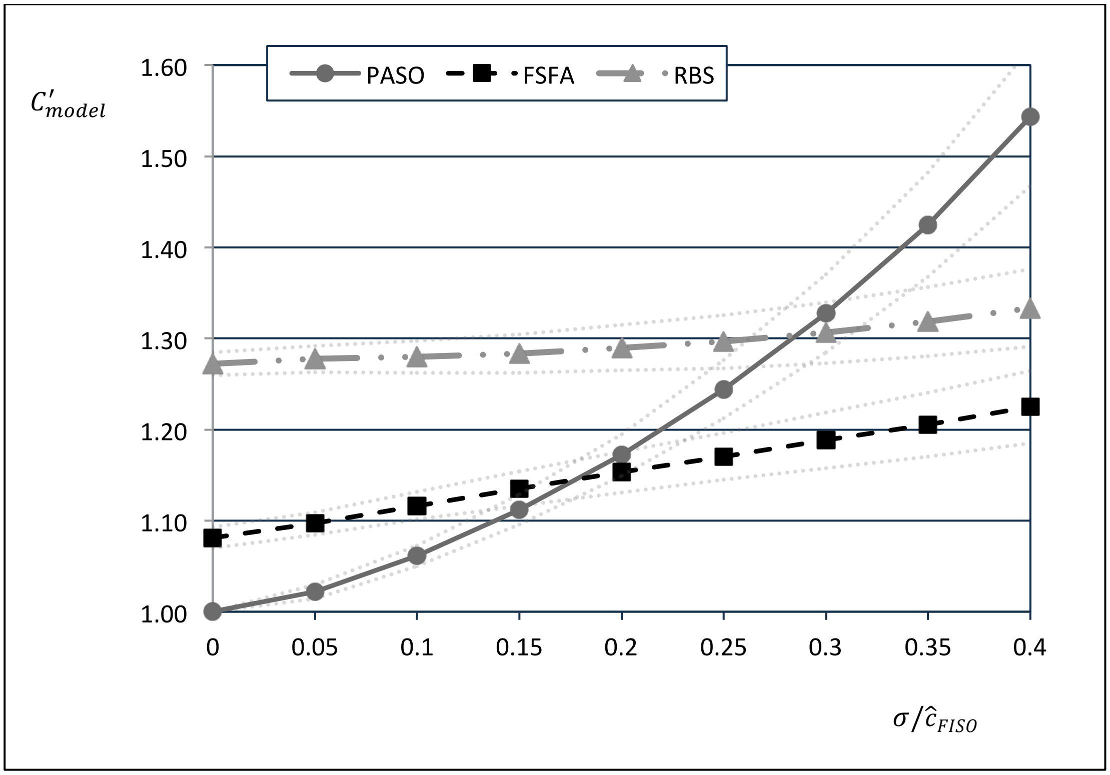


Figure 5. Total cost results, $N = 75$ flights.

**Figure 6**

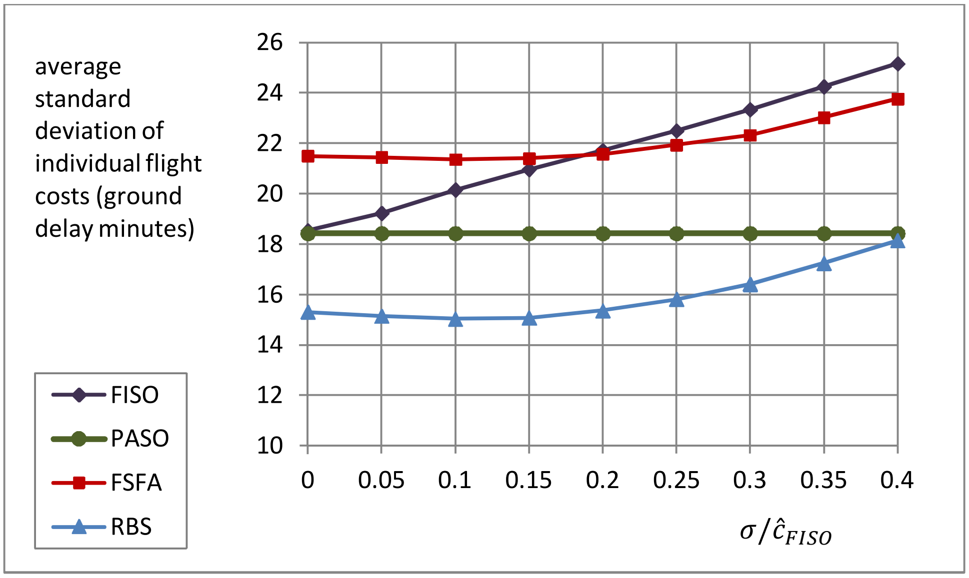


Figure 6. Standard deviation of individual flight costs.

**Figure 6 b&w**

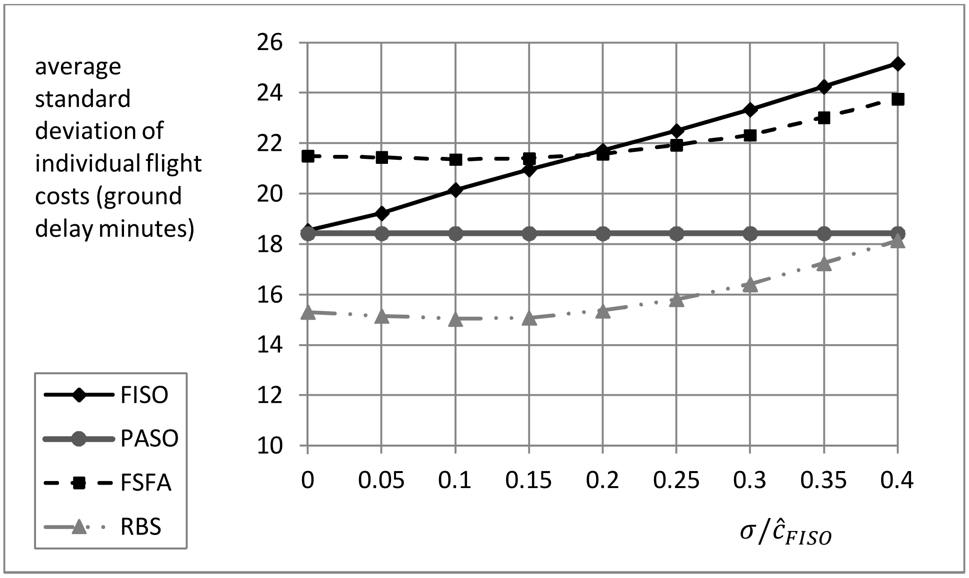


Figure 6. Standard deviation of individual flight costs.

**Table 1**

Table 1.

Stylized model allocation.

| Allocation | Flight 1 takes route: | Flight 2 takes route: | Total Deterministic Allocation Cost | Total Allocation Cost |
|---|---|---|---|---|
| 1 | 1 | 2 | $w_1 = w_{11} + w_{22} = W$ | $C_1 = W + \varepsilon_{11} + \varepsilon_{22}$ |
| 2 | 2 | 1 | $w_2 = w_{12} + w_{21} = W$ | $C_2 = W + \varepsilon_{12} + \varepsilon_{21}$ |

**Table 2**

Table 2.

Example supply parameters.

| Route | Capacity (flights per hour) | Headways (min)[a] | En Route Time (min) | $\rho_r$ (min) |
|---|---|---|---|---|
| 1 | 24 | 2.5 | 135 | 35 |
| 2 | 20 | 3 | 130 | 30 |
| 3 | 10 | 6 | 120 | 20 |
| 4 | 12 | 5 | 115 | 15 |
| 5 (nominal) | 7.5[b] | 8[c] | 100 | 0 |
| $C_{CTOP} =$ | **73.5** | | | |

a Arrival (and departure) headways at Fix A.
b Capacity after capacity reduction.
c Headways after capacity reduction.

**Table 3**

Table 3.

Inputs for sensitivity test, Model 1 (fixed CTOP duration).

| **Input variable** | **Min** | **Max** | **Mean** | **Increments** |
|---|---|---|---|---|
| $D_0$ (flights per hour) | 50 | 100 | 75 | 5 |
| $\alpha_{max}$ | 2.5 | 5 | 3.75 | Investigated at 2.5,3.5,5 |
| $\sigma/\hat{c}_{FISO}$ | 0 | 0.40 | 0.20 | 0.05 |
| $T$ | 60 | 60 | 60 | - |

**Table 4**

Table 4.

Parameter estimates, Model 1 (fixed CTOP duration).

| Variable | Parameter estimate | t-stat |
|---|---|---|
| intercept | -4.132 | -1677.2 |
| $D_0$ | 0.155 | 612.2 |
| $\alpha_{max}$ | -0.060 | -107.0 |
| $\omega$* | 0.109 | 1683.5 |
| $D_0 \cdot D_0$ | 0.122 | 106.8 |
| $\alpha_{max} \cdot \alpha_{max}$ | -0.189 | -46.3 |
| $\omega \cdot \omega$ | 0.0004 | 1676.4 |
| $D_0 \cdot \omega$ | 0.0003 | 118.0 |
| $D_0 \cdot \alpha_{max}$ | -0.077 | -39.1 |
| $\omega \cdot \alpha_{max}$ | -0.00005 | -9.2 |

* $\omega = \sigma / \hat{c}_{FISO}$

Table 5

Table 5.

Inputs for sensitivity test, Model 2 (fixed demand rate $D_0$).

| **Input variable** | **Min** | **Max** | **Mean** | **Increments** |
|---|---|---|---|---|
| $D_0$ (flights per hour) | 80 | 80 | 80 | - |
| $\alpha_{max}$ | 2.5 | 2.5 | 2.5 | - |
| $\sigma/\hat{c}_{FISO}$ | 0 | 0.40 | 0.20 | 0.05 |
| $T$ (min) | 37.5 | 75 | 56.25 | 3.75 |

**Table 6**

Table 6.

Parameter estimates, Model 2 (increasing CTOP duration).

| **Variable** | **Parameter estimate** | **t-stat** |
|---|---|---|
| Intercept | -4.291 | -869.7 |
| $\omega$* | 0.114 | 873.8 |
| $T$ | -0.023 | -46.8 |
| $\omega \cdot \omega$ | 0.0005 | 870.1 |
| $T \cdot T$ | 0.009 | 3.6 |
| $T \cdot \omega$ | -0.00005 | -11.4 |

* $\omega = \sigma/\hat{c}_{FISO}$