UNIVERSITY OF ALBERTA

MATHEMATICS AS COMMUNICATION

BY

AUDREY ANNE HODGSON-WARD ©

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of MASTER OF EDUCATION.

ELEMENTARY EDUCATION

Edmonton, Alberta

FALL 1993

ISBN 0-315-88444-4

FAX to AUDREY HODGSON-WARD
From Jean Richardson

I would appreciate your earliest possible attention in this matter as I must submit the thesis by 15 September, 1993. A copy of the completed thesis could be forwarded to you after 30 October, 1993, if you require it.

It would help me greatly if you could fax your permission to the number given below.

Fax (Area Code 403) 922-3787
The fax machine is at Ardrossan Elementary School where I teach.

Home Address 10453-82 Street
Edmonton, Alberta
Canada
T6A 3M6

Home Telephone 403-465-3972

Thank you for your consideration in the above request.

Yours sincerely,
Audrey Hodgson-Ward
(Audrey Hodgson-Ward)

THANK YOU FOR YOUR REQUEST OF ——
PERMISSION IS GRANTED SUBJECT TO ACKNOW-LEDGEMENT TO AUTHOR/S, TITLE OF BOOK, & Routledge AS PUBLISHERS.

Not for commercial publication

10453--82 Street
Edmonton, Alberta
Canada T6A 3M6

Ms. Jean Richardson
Routledge
11 New Fetter Lane
London
EC4P 4EE

Dear Ms. Richardson,

I am presently writing my Master's in Education thesis entitled "Communication in Mathematics" at the University of Alberta in Edmonton. My study involves some of the major concepts of Richard R. Skemp.

Several members of my thesis committee are unfamiliar with Professor Skemp's work and have asked that descriptions of the Skemp activities used in my study be included in the thesis for the committee's information.

I thus wish to obtain permission to reproduce the following copyrighted material to be included as an appendix to the thesis:

Author: Richard R. Skemp

Books: Structured Activities for Primary Mathematics:How To Enjoy Real Mathematics Volumes 1 and 2

Publication Date: 1989

Information to be reproduced: a) Volume 1 Pages 163-173 and 175-177
Numbers 5.1/1
5..1/2
5..1/3
5.2/1
5.2/2
5..3/1
5.4/1

b) Volume 2 Pages 113-117
Numbers 5.5/1
5.5/2

I would appreciate your earliest possible attention in this matter as I must submit the thesis by 15 September, 1993. A copy of the completed thesis could be forwarded to you after 30 October, 1993, if you require it.

It would help me greatly if you could fax your permission to the number given below.

Fax (Area Code 403) 922-5787
The fax machine is at Ardrossan Elementary School where I teach.

Home Address 10453-82 Street
Edmonton, Alberta
Canada
T6A 3M6

Home Telephone 403-465-2972

Thank you for your consideration in the above request.

Yours sincerely,
Audrey Hodgson-Ward
(Audrey Hodgson-Ward)

UNIVERSITY OF ALBERTA

RELEASE FORM

NAME OF AUTHOR: Audrey Anne Hodgson-Ward

TITLE OF PROJECT: Mathematics as Communication

DEGREE: Master of Education

YEAR THIS DEGREE GRANTED: 1993

Audrey Anne Hodgson-Ward

10453 - 82 Street
Edmonton, Alberta
T6A 3M6

October 5, 1993

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled Mathematics as Communication submitted by Audrey Anne Hodgson-Ward in partial fulfilment of the requirements for the degree of Master of Education.

Dr. Daiyo Sawada

Dr. Katherine Willson

Dr. Sol Sigurdson

October 01, 1993

Dedication

To George

who never doubted

Abstract

The purpose of the study was to analyze the communication and group interaction among Grade Three students involved in an introductory multiplication unit. The children worked in a mathematics laboratory, primarily using activities designed by Richard Skemp.

These structured activities were part of Skemp's multiplication "network", devised to assist students in creating a relational understanding of the concepts.

The research questions were:

1) What communication patterns appear
   - between adults and children?
   - among the children?
2) How are the assumptions underlying the Skemp programme reflected in the communication patterns?
3) How do the forms of organizing for learning affect the communication patterns?

The five children in the study group were chosen to represent a cross-section of the verbal, social and mathematical abilities of the class. This classroom, with its mathematics laboratory, might be considered unusual because of the shared teaching responsibilities: both the Grade Three teacher and an intern teacher taught the mathematics. The students were instructed as a whole class, and in small, cooperative learning groups. Students had been taught the cooperative skills from the Johnson, Johnson and Holubec model.

Data about the cooperation and the verbal and non-verbal communication were gathered through observations in the mathematics laboratory, journals, a questionnaire and through individual and group interviews. Data were analyzed for information outlined in the research questions.

Through analyzing the data, the following conclusions were reached. A variety of communication patterns were evident: the initial role assumed by the two teachers was highly directive and students' responses were short and unelaborated. The teachers gradually asked more open-ended questions and encouraged increased student interaction.

The children's responses became lengthier and more detailed. On days when the children worked alone, one or more students assumed the procedural/director role. All the children prompted each other verbally and non-verbally.

Their conversations supported some of Skemp's assumptions about the structured activities and the discussions which should occur.

The tasks and seating arrangements were designed to facilitate communication. The students' interaction did not always meet the expectations of the cooperative learning model, the mathematics laboratory or Skemp's mathematics programme.

Acknowledgements

I am indebted to the following people and wish to thank them.

- Dr. Dayio Sawada - for his patience and suggestions throughout the study and the writing process
- Dr. Katherine Willson - for her advice and editing assistance
- Dr. Sol Sigurdson - for his valuable time
- Mrs. J. Theander - for her excellent typing
- To the classroom teacher, intern teacher and the Grade Three students who must unfortunately remain anonymous
- To my family - for their support, patience and faith in me

TABLE OF CONTENTS

CHAPTER		PAGE

CHAPTER PAGE

CHAPTER I

STATEMENT OF THE PROBLEM

INTRODUCTION

In 1989 the National Council of Teachers of Mathematics (NCTM) published a document entitled the Curriculum and Evaluation Standards for School Mathematics (commonly referred to as "the Standards"). This document begins with a statement referring to the "call for reform in the teaching and learning of mathematics". It continues "Inherent in this document is a concensus that all students need to learn more, and often different mathematics and that instruction in mathematics must be significantly revised". The Standards outline a set of five general goals and thirteen more specific standards to improve mathematics education for the future. The five general goals for all K-12 students are:

1) that they learn to value mathematics
2) that they become confident in their ability to do mathematics
3) that they become mathematical problem solvers
4) that they learn to communicate mathematically, and
5) that they learn to reason mathematically (p. 5).

> These goals imply that students should be exposed to numerous and varied interrelated experiences that encourage them to value the mathematical enterprise, to develop mathematical habits of mind, and to understand and appreciate the role of mathematics in human affairs; that they should be encouraged to explore, to guess, and even to make and correct errors so that they gain confidence in their ability to solve complex problems; that they should read, write and discuss mathematics; and that they should conjecture, test, and build arguments about a conjecture's validity (p. 5).

In a section entitled, "The Need for Change", the authors point that the past emphasis has tended to be on computation and other traditional skills. Change is necessary because the current curriculum:

> is narrow in scope; fails to foster mathematical insight, reasoning and problem solving; and emphasizes rote activities. Even more significant is that children begin to lose their belief that learning mathematics is a sense-making experience. They become passive receivers of rules and procedures rather than active participants in creating knowledge (Standards, p. 15).

These statements from the NCTM imply that the central focus of a mathematics programme should be problem solving; students must develop the ability to think, reason and communicate mathematically (NCTM, 1989). Without communication, they could not solve problems, tell time, make connections among diverse concepts, program or use a computer, or explain the difference between one-half and one-third of an apple. Communication in mathematics has usually centered around the use of formal, abstract symbols. Mathematics teachers, researchers and writers must give increased attention to the more informal concrete language of children and the everyday math involved in it. (NCTM, 1989). Children come to know mathematics through their use of informal language. Developing a true understanding of mathematics is virtually impossible without opportunities to discuss and explore the concepts. Through written, oral and non-verbal communication, pupils may help each other by supplying vital connections between ideas for which other children were ready, but connections they had not yet made alone.

Communication is not merely conversing. The process of communication implies interaction among the communicators. This notion does not mean several people speaking at each other but with each other, taking careful note of what is said and what is meant.

Communication, cooperation, problem solving and small group work are highly interdependent. It is difficult to discuss any of these aspects in isolation from each other. Without a means to communicate, students may be unable to accomplish any task requiring them to work cooperatively. Without some sort of problem to solve or common content to use, there might be nothing to communicate. Without some students cooperation, might be unable to effectively accomplish many of their goals.

One programme which the researcher feels incorporates all of the above mentioned components was designed by R. Skemp, a mathematician and learning theorist.

Based on his own experiences and research by others (1989a, p. 1) Skemp perceived a need for a new way to teach children mathematics. In response, he devised what he describes as "a fully structured collection of more than 300 activities, covering a core curriculum for children aged 5 to 11 years old, which uses practical work extensively at all stages". He explains (ibid., p. 4) that "the concepts embodied in [the activities] fit together in ways which help learners to build good mathematical structures in their own minds".

The activities are organized into "networks"; each "network" focuses on an area such as numbers (naming and properties), fractions, shape, the number line and the number track, set based organization and the four operations. The games are meant to be used with small groups of children to teach or consolidate the concepts of a network. The activities or games are structured not only to promote communication between teacher and students, but amongst students as well. This communication takes place primarily through discussions.

Skemp (1986, pp. 114-115) outlines some of the benefits of using discussions in a mathematics classroom: interrelating ideas with those of other learners; flexibility and open-mindedness; stimulation of new ideas; and "cross-fertilization" of ideas where listening to someone else sparks new ideas or connections.

PURPOSE OF THE STUDY

The purpose of this study was to analyze the communication and group interaction which occurred among Grade Three students as they learned the skills and concepts of an introductory multiplication unit based upon the Skemp materials.

RESEARCH QUESTIONS

The Skemp materials which introduce new concepts have been designed to involve teachers directly; those which consolidate ideas are to be done by groups of children working independently. Using the materials from the multiplication unit in these two situations, it is the intent of the researcher to explore the following questions:

1) What communication patterns appear in this classroom:
   - between adults and children?
   - among the children?
2) How are the assumptions underlying the Skemp programme reflected in the communication patterns?
3) How do the forms of organizing for learning affect the communication patterns?

SIGNIFICANCE

1) Although other studies (Barnes, 1992; Brissenden, 1988) have been done which examine communication in mathematics, long-term studies involving a single topic and group of subjects appear not to have been done. One significance of this study is that it provides a concentrated focus on a single unit of mathematics and one group of learners. Thus, patterns of communication and understanding may be discerned which would not be available through shorter studies.

2) Skemp's work in the psychology and theory of teaching and learning mathematics has gained wide acceptance (Brissenden, 1988; Davis, 1990; Pimm, 1987). These authors appear to have generally accepted Skemp's assumptions about children's mathematical learning and discussion. The present study seems to be alone in attempting to test those assumptions in a school setting.

3) This study has amassed a large amount of data in the form of videotapes, audiotapes and transcripts which other researchers could use for background, further interpretation or to assess the present researcher's analysis and conclusions.

DELIMITATIONS

The 'boundaries' for this study are set out in short form below:

1) One group of four students (later five) from one Grade Three class were selected for the study by the researcher and classroom teacher. The main criteria for the selection of these specific students were the students' varying general ability, mathematical ability and social skills.

2) One teacher was an experienced mathematics teacher who was familiar with the Skemp constructivist approach to teaching elementary mathematics, not only in her classroom but in the mathematics laboratory. The second teacher was a teacher intern who, at the time of the study, had six months experience in teaching mathematics in Grade Three.

3) The study did not involve a study group and a control group matched in order to find out which group made superior gains in

five weeks. While the researcher was greatly interested in pupil growth in multiplication skills, the major foci of the study were determining by qualitative analysis, any correspondence between Skemp's suppositions and the teachers' and pupils' actual behaviour and the communication which occurred in the mathematics laboratory sessions during the study.

LIMITATIONS

Because of the delimitations set out above, there are limits to the numbers and kinds of findings which one can extract from the study:

1) The study was essentially a case study of a small group of children.
2) It would not be possible to generalize beyond this small group to larger numbers of students who might be taught through a constructivist approach. Some tentative conclusions may be drawn, but the researcher will not offer any major assertions or firm forecasts.
3) Since there was no use made of a control group, it will not be possible to state that the children learned subject matter more quickly or thoroughly than children being taught by some other method and activities.

OVERVIEW OF THE STUDY

The study focused on a group of Grade Three students during an introductory multiplication unit. The children were observed as they worked in cooperative learning groups in a programme utilizing the philosophy and activities of R. Skemp. The study was an attempt to assess:

i) what communication occurred between teachers and children and among the children;
ii) how the communication related to Skemp's assumptions about learning; and
iii) how that communication related to the structure of the mathematics programme.

Limitations may be placed on the interpretation of the data because of the size of the sample and the relatively short duration of the unit studied. Thi; class situati ı was distinguished from most others because of the regular use of a mathematics laboratory with its emphasis on cooperative learning groups, activity-based tasks and the opportunity for children to discuss the activities while learning. The background and constructivist attitude of the classroom teacher also influenced the mathematics programme.

CHAPTER II

BACKGROUND TO THE STUDY

COMMUNICATION

Verbal Communication

Verbal communication has become an increasingly important topic among mathematics educators. Many authors have recently stressed the inclusion of meaningful classroom discourse in mathematics classrooms at all levels (NCTM, 1989; NCTM, 1991; Mathematical Association, 1992; Brissenden, 1988; Barnes, 1992; Pimm, 1987). Discourse is "central to what students learn about mathematics; [it] is both the way ideas are exchanged and what the ideas entail" (NCTM, 1991, p. 34). How teachers encourage dialogue among students demonstrates the teachers' attitudes about language and about knowledge -- what it is, who owns it and how it is created by each learner.

Children learn a great deal about the world through verbal communication. They come to school with vast knowledge gained through talking, listening, questioning and playing with language. They also have intuitive mathematical knowledge. School mathematics has traditionally focused on formal representations of mathematical ideas, many of which children are not initially mentally capable of grasping. Educators must place greater value on the children's way of explaining what they already know.

> When small groups of children discuss and solve problems, they are able to connect the language they know with the mathematical terms that might be unfamiliar to them. They make sense of those problems. The use of concrete materials is particularly appropriate because they give the children an initial basis for conversation (NCTM, 1989, p. 27).

The Standards also indicate that "teachers facilitate this process when they pose probing questions and invite children to explain their thinking" (NCTM, 1989, p. 26). If children are asked to explain their thinking orally, it will help them to clarify their ideas.

Barnes expresses a similar view when he states:

> If in the classroom we limit spoken language to the teacher telling and the pupils replying to cross-examination, we ignore and reject the function of speech as an instrument of shaping experiences, that is, as a means of learning.... But it takes

> time: children need time to assimilate what they are learning by talking about it in relation to what they know already. Too many classroom discussions ask children to relate strange information only to other strange information: the conversation is carried out in terms of what the teacher knows, while the child's other experience -- in and out of school -- is excluded (1992, pp. 84-85).

The use of reflective, exploratory, and therefore informal, language makes the learner an active participant in his/her own education, not a passive recipient of other people's knowledge. Pimm (1987), Dean (1992), and Barnes (1992) all stress the need to discuss ideas, regardless of their source, in order to clarify them for one's own understanding. This process involves putting forth a conjecture, listening to others' reactions, answering questions and then refining the idea based on how it was perceived by the listeners. Such a dialogue helps the speaker and the listeners to form and redefine concepts and strategies.

Explaining Through Communication

In order to provide situations which will facilitate the discussion and development of mathematical concepts, teachers must be aware of their pupils' verbal skills. The means used to assess students' oral communication and mathematical reasoning must focus on the specific language used as well as the intent. The Curriculum and Evaluation Standards recommend that teachers:

> should pay attention to the clarity, precision and appropriateness of the language used. In addition, students' ability to understand the oral communication of others is an important component of instruction and assessment (NCTM, 1989, p. 214).

Listening to students' conversations is a less threatening means of assessing the language used and concepts grasped than requesting that individual students provide explicit explanations of concepts or procedures in a discussion with the teacher. Barnes (1992, p. 96) cautions against expecting pupils to "spell out for outsiders something they are only beginning to make sense of." The terminology and phrasing used in discussions between students will likely reveal their understanding more thoroughly because its flow is natural. Even allowing for the stops and false starts and hesitations or normal exploratory conversations, the teacher can achieve a clearer picture of students. The pupils' willingness to participate in class discussions should also be noted:

> because the degree to which children are comfortable expressing their mathematical thinking and their flexibility in using various forms of communication are primary aspects of communication (NCTM, 1989, p. 215).

Between Adults and Children

The degree to which students willingly participate in discussions is determined in part by the role taken by the teacher. Teachers in mathematics may assume roles which range from directive and authoritarian to facilitators helping children to develop their own knowledge in a constructivist atmosphere.

An authoritarian instructor tends to give orders which must be obeyed; gives information, ideas and opinions which he/she expects to be accepted uncritically; and structures the learning to create student dependence (Brissenden, 1980, p. 142). This teaching style depends on a lecture method, often leaving children with the impression that mathematics is nothing more than guessing the one correct answer which the teacher already knows. The pupils' limited role is to locate the one answer. Teachers are viewed as the ultimate judge of students' work and thinking; there is little room for discussion. Traditional, authoritarian teachers may use "cueing"; they provide a sentence in which there is a pause into which children put the appropriate word (Mathematical Association, 1992, p. 15). Pupils may also use "cueing" to encourage teachers to provide more clues about the expected answer. In contrast, "cueing" may be used in a more open-ended manner in which the instructor cues the child by asking a question or making a noncommittal response to the pupil's remark. This teacher comment is followed by a pause during which the child may elaborate on the original thought.

This more open form of responding could identify a teacher who is a facilitator. This teaching style can be characterized by a teacher who encourages discussions and mathematical reasoning by frequently asking "Why?" Such a technique, used whether or not the student's statement is correct, plays an important role in establishing classroom discourse. In addition to asking "Why?", the Professional Standards recommend that "cultivating a tone of interest when asking a student to explain or elaborate on an idea helps to establish norms of civility and respect rather than criticism and doubt" (p. 35).

A facilitator might also take the part of a learner/participant. By assuming the role of a fellow learner the teacher sends the message that teachers do not always know the answers and are willing to learn from the students. This provides the chance for students to do the talking and explaining and to discuss the difficulties that they experienced with a task or in their private conversations. This creates "a different quality of discussion and it begins to approach more nearly the type of discussion which, in ordinary life, leads to learning" (Mathematical Association, 1992, pp. 21-22). Children sharing their successes and failures help to reinforce that mathematical authority "does not reside solely with the teacher, but with the teacher and the children as an intellectual community" (Yackel et al., 1990, p. 118).

Even as a participant/learner, however, the teacher may still need to use exposition in order to introduce new ideas or skills: a teacher's explanations often help children to focus on the major points of the new idea (Mathematical Association, 1992, p. 28). This dual function of the teacher, that of facilitator and of instructor, is consistent with the teacher's role in the Skemp programme and therefore in this study.

The teacher has a specific role to play whether the classroom learning occurs in groups with the teacher assisting or without his/her constant presence. First, if the teacher is working directly with the group, he/she may play a:

> strong social role in managing the discussion: ensuring fair turns at talking, encouraging the shy, helping the diffident to give voice to their opinions, and giving a focus to the perceptions of the less fluent (Mathematical Association, 1992, p. 35).

If students have been used to a more traditional style of classroom where discussion was not valued, they may be passive and require guidance and encouragement. An environment where mathmatical discourse occurs requires that everyone's thinking be respected (NCTM, 1991, p. 35). Second, teachers must learn to listen as students do the talking, modeling and explaining. After listening to the children, the teacher should orchestrate the classroom discourse by focusing on some student contributions and not pursuing others (NCTM, 1991, p. 36). Third, teachers must avoid telling pupils that they are wrong, leaving that role to the group instead. Fourth, the teacher should provoke discussion without telling everything to the group. Fifth, he/she must notice when learning is not taking place and intervene. And sixth, the teacher needs to be aware of any problems in the group's interaction and find creative ways to resolve them (Weissglass, 1990, p. 307).

Whether they are working with individuals, small groups or the entire class, educators may also assume two more roles. Brissenden (1988) defines these additional roles as "mathematical" and "procedural". In a mathematical role, the teacher asks helpful or challenging questions, supplies mathematical vocabulary, directs attention to salient ideas and suggests ways of testing developing knowledge. In a procedural role, the teacher questions students about their progress, draws out pupils' ideas and extends their responses, ensures the smooth running of the group and encourages and supports pupils (p. 55). Brissenden cautions teachers to keep mathematical interventions to a minimum and to focus on the procedural interventions in the early stages of group work or with the whole class (1988, p. 40). Skemp's recommendation of more direct teacher involvement in the early part of concept development seems to concur with Brissenden.

A teacher who has taken all of these responsibilities while working directly with his/her pupils will have equipped them to work in small groups without constant supervision. According to the Professional Standards, with the proper training, students ought to be able to learn with and from others to clarify terminology, to consider one another's ideas and to argue about the reasonableness of alternative approaches and answers (NCTM, 1991, p. 58).

Among the Children

In their discussions about concepts and terminology, when children realize that other people's views differ, they have an opportunity to learn from each other and to reassess the validity of their own thinking. Whether they are conducting discussions in large or small groups, students serve as audience and judges. Thus (NCTM, 1991 p. 58), they need to learn how to demonstrate respect as they question ideas and solutions and to define their own thinking without becoming hostile or defensive (NCTM, 1991, p. 58). In order to make discussions effective and worthwhile for all participants, pupils need to learn how to: defend their own ideas when challenged; speculate puclicly and try to convince themselves and others of the worth of their solutions; use mathematical language; acknowledge when they are confused; and build on one another's contributions (NCTM, 1991, pp. 47-49).

The quality of the verbal interaction is a vital factor in the success of the group (Artzt and Newman, 1990, p. 17). If students have learned the skills listed above, they have a good chance of communicating effectively. To work together for everyone's benefit pupils must be aware of both the verbal and non-verbal messages they send to other children. Verbal signals are conveyed through the tone of voice, the choice of vocabulary and the emphasis placed on certain words. Non-verbal, but equally important messages are sent through the posture, body movements and facial expressions of the speaker and listener. All of these signals can assist or interfere with effective communication between adults and children and among children.

COLLABORATIVE RESEARCH

Collaborative research involves two or more participants who work together as equals to design and implement an investigation of a topic of mutual concern.

While a collaborative model may sometimes involve both students and teachers, for the purposes of this study, the collaboration occurred only among the teachers. Many of the discussions about the content and structure of the multiplication unit were among the school's two Grade Three teachers, the intern teacher and the researcher. The final decision making rested with the classroom teacher and the researcher. The researcher assumed the responsibility for the final design of the study while the classroom teacher assumed responsibility for the specific details of the multiplication unit. Despite this apparent separation of their roles, both the teacher and the researcher could propose changes or delete aspects of the study; if the teacher felt uncomfortable or did not think a suggestion would suit her class, she indicated that fact to the researcher. They would then discuss alternatives and reach an agreement. Such frequent conferences are a normal feature of collaborative research. Before undertaking a collaborative research project, the participants should be aware of the need for constant meetings and discussions. Hord (1986) lists this need among her "ten demands of collaboration":

1) mutual sense of gain
2) greater time commitment than for cooperation
3) greater effort to reach out and take action
4) frequent interaction and sharing needed
5) worthwhile rewards or outcomes for both parties
6) greater promotion of other activities between parties
7) willingness to relinquish personal control and assume more risk will create a more flexible environment
8) empathy for others' points of view
9) strong enthusiastic leadership and
10) simple patience, persistence and willingness to share (p. 26).

She also states that collaboration "requires more effort [than cooperation], but ideally yields more" (cited by Compton, pp. 2-3).

A further benefit of collaborative research is that it provides a valuable means of interacting with peers who have a similar commitment to personal growth and excellence. Schaffer and Bryant (cited by Hord, 1986, p. 24) describe this shared commitment as "working with rather than on a person."

COOPERATIVE LEARNING

Definitions

Cooperative learning also involves working with others but with a different intent than collaboration. In collaborative research, the participants create a study through joint decision making; cooperative learning involves the participants in small groups carrying out tasks which they have been assigned. Johnson, Johnson and Holubec (1990) suggest a cooperation is "working together to accomplish shared goals" and that cooperative learning is "the instructional use of small groups so that students work together to maximize their own and each other's learning" (p. 4).

In this study the students had for some months received cooperative learning training based on the Johnson, Johnson and Holubec model. These authors outline five essential components which must be included if small group learning is to be truly cooperative. These five components are:

1) positive interdependence;
2) face-to-face interaction among students;
3) individual accountability/personal responsibility;
4) the use of interpersonal skills; and
5) the processing of how well the learning groups functions (ibid., pp. 10-15; p. 122).

Johnson, Johnson and Holubec give some further detail on one of the above components, that of interpersonal skills: students must get to know and trust each other, communicate accurately and unambiguously, accept and support each other and resolve conflicts constructively (ibid., p. 15).

Benefits

If all of the five essential components from Johnson, Johnson and Holubec are included, cooperative learning can have many benefits. Johnson and Johnson (1985) summarized some of the research findings about the advantages of cooperative learning:

- more learning than in a competitive classroom;
- greater academic growth of children, especially those children in the lowest third of the class;
- higher intrinsic motivation to learn;
- more positive attitudes towards instructional experiences and the instructors;
- higher self-esteem;
- stronger perceptions that others care and want to help everyone to learn; and
- better understanding and greater acceptance of other students, including minorities and special needs peers (pp. 23-24).

The same authors (1990, p. 120) have identified further benefits including discussing procedures, sharing of knowledge and reasoning, questioning to encourage oral rehearsing and rethinking, and encouraging peers. Wiessglass (1990, pp. 305-306) outlines another set of advantages to using the small-group laboratory approach.

1) The small-group laboratory approach provides an opportunity for students to talk about mathematics instead of being passive listeners.

2) Students are more likely to ask questions of peers than they are of teachers. The approach encourages students to construct their own understanding through dialogue instead of accepting the explanations of an authority.
3) In a traditional classroom, students have little opportunity to talk about their feelings. Students working in small groups will develop friendships, discuss mathematics and talk about their feelings.
4) The combination of laboratory equipment and a cooperative approach can make the mathematics classroom a more relaxed place. The students are free to ask questions and admit they do not understand. This approach can also help students to develop confidence in their ability to evaluate their own learning and to learn self-discipline.
5) The experience of manipulating concrete models is relevant to students and helps lay the foundation for more abstract learning in the future.

Finally, Artzt and Newman (1990, p. 5) state that "cooperative learning strategies have been credited with the promotion of critical thinking, higher-level thinking, and improved problem-solving ability of students".

Application

The children in this study worked in cooperative learning groups in the mathematics laboratory. They had been instructed in the Johnson, Johnson and Holubec model with its five essential components. The Skemp mathematics programme is structured so that it includes the first four of these five components. The groups of students were provided with a set of materials and a defined task which they had to complete together (positive interdependence). The children were organized into small groups seated at rectangular or octagonal tables (face-to-face interaction). Each student was expected to take his/her turn and offer explanations or comments to the group (individual accountability). The directions for the activities contained the expectation that students would discuss their work constructively (interpersonal skills). Skemp's programme does not include any indication of group processing either during or after the children are discussing and manipulating the concrete materials for the activities.

THE SKEMPIAN APPROACH

Active Involvement

In the Skempian approach, activity-based mathematics means providing children with manipulative materials and carefully structured games or activities. These materials follow a clear progression through the development of the mathematical concepts and should be used as a central focus for the mathematical programme. Active involvement using discussions and concrete materials can assist pupils in reaching a deeper understanding and help them to internalize concepts so that the students can generalize to other mathematical situations. If children learn mathematics in a meaningful way, it may prevent them from trying "to slide through courses by externally manipulating the symbols" (Rose, 1989, p. 1). Skemp's Structured Activities for Primary Mathematics provide activities which require children to discuss their understanding of the underlying concepts and not just the symbols.

Constructivism

Skemp designed activities for primary mathematics based on his belief that students must build their own structures (or schemas) and mathmatical knowledge. Skemp is convinced that all students must understand how and why mathematics works. This comprehension provides them with virtually infinite mathematical power. This point of view about students' needs is known as constructivism. Skemp explains further, saying:

> Each individual learner has to put together [the] structures in his own mind. No one can do it for him. But this mental activity can be greatly helped by good teaching, an important part of which is providing good learning situations (1989b, p. 4).

These activities, which include active participation of all the students in the discussions and playing the games, were created to provide these "good learning situations".

Elements of Skemp's Philosophy

Skemp's constructivist philosophy of mathematics learning is based on a number of key, contrasting concepts. These include "deep" and "surface" structures, relational versus instrumental understanding and habit versus intelligent learning (1989a, 1989b).

According to Skemp, mathematics has often consisted of:

a) instrumental learning - "Rules without reasons" (1989a);

b) habit learning - rote memorization of situation-specific rules and procedures; and

c) surface structure - the mental or physical symbols used to convey mathematical meaning.

Whereas, mathematics should consist of:

d) relational understanding - understanding how individual ideas relate to each other and the overall body of knowledge;

e) intelligent learning - generating the structures of schemas from which future knowledge may be constructed and into which new ideas may be fit; and

f) deep structures - the actual mathematical ideas and their relationships.

Skemp emphasizes that it is essential to develop both the "surface" and "deep" structures because without these, mathematical communication would be impossible. Too often, students have learned only the symbols ("surface" structures) at the expense of the underlying concepts ("deep" structures).

Benefits

The advantages in acquiring the "deep" structures of relational understanding and intelligent learning lie in the degree to which they provide an intellectual framework which allows the learner to integrate and adapt newly acquired knowledge. Relational understanding may be more time-consuming to achieve but it will persist in students' memories longer than the "invert and multiply" or "borrowing" type of rules characteristic of instrumental understanding. The more mathematics a child learns, the more obvious the need for an organized, long-term method for remembering and organizing information because habit learning is effective only in limited, short-term situations. Intelligent learning allows pupils to broaden their repertoire of strategies for achieving their goals in specific situations. It also increases their independence; they do not need to rely on a teacher to confirm their thinking. They can confirm their own thinking through conversations with themselves or other learners.

ORGANIZATION FOR LEARNING

The primary form of organizing for learning in this study was the mathematics laboratory where the children cooperated verbally and non-verbally to construct their mathematical understanding.

Designs and Definitions

There are many different definitions of mathematics laboratories which contain elements of small groups of students, manipulative materials and oral discussions. Cathcart (1977, p. 1) indicates that any definition of a mathematics laboratory should consider the place, the process and the attitude. He describes the "place" as somewhere "children can handle materials, perform mathematical experiments, play mathematical games, and become involved in other activities" (ibid., p. 1). The location may be a decentralized/classroom laboratory, a centralized area, a team-room, or a roving/movable laboratory (Barson, 1977, p. 43). When setting up any type of mathematics laboratory, consideration must be given to the size and layout of the space, the furniture, and the available resources; these are all elements of the "place". In this study, the location was a centralized laboratory available to all students and teachers.

The "process", Cathcart explains, means a "procedure for teaching and learning mathematics.... Group discussions, individual projects, even lectures can be part of the process" (1977, p. 1). A further reflection of the process is seen in the ways that teachers structure the laboratory learning time and the ways in which they encourage independent thinking.

The "attitude" aspect of a laboratory description must include both the teacher and the students. Pupils must be willing to "think for themselves, to ask questions, to look for patterns -- in short to adopt an attitude of inquiry" (ibid., p. 1). The teachers must be willing to encourage and demonstrate that they value discussion, to resist the temptation to judge or even comment on every student's contribution, and to allow the group and the individuals to assess the correctness and sense of their own work. The teacher shows that he/she values a cooperative attitude and the benefits of verbal interaction by the questions he/she asks, the tone of voice used, and the non-verbal messages which are conveyed along with the verbal information.

All three factors of place, process and attitude are echoed by Barton (1977, p. 43) in his list of the similarities among many mathematics laboratories. He includes:

1) stations of activities for children working individually, in small groups or as a class;
2) rich varieties of materials;
3) teachers working with various groupings in a child-centered atmosphere;
4) open-ended activities for children to explore and extend;
5) flexible organization so children can move freely according to their interests or needs;
6) multimedia or multisensory approach to learning; and
7) textbooks and pamphlets to be used as references.

The mathematics laboratory in this study included many of these characteristics. However, children were not allowed to set their own tasks or to circulate freely; the seating arrangements and activities were prescribed by the teacher, not chosen by the pupils.

Although they did not satisfy all the above requirements, this mathematics laboratory and the Skemp programme both satisfied the terms of Weaver's definition. He suggests that mathematics laboratories must include "promising activities and experiences that are appropriate for the attainment of particular mathematical goals or objectives within thoughtfully planned, structured programs of instruction" (1977, p. 223).

Materials

In order to attain the mathematical goals, the mathematics laboratory needs to be well equipped with manipulative materials. The laboratory in the study contained a wide variety of commercial and teacher-made materials including the entire set of Skemp activities. This assortment was advantageous because if children are exposed to only one set of materials, they may focus on an attribute of the materials as though it were part of the mathematical concept. Having a variety of manipulatives ensures that pupils see multiple representations of concepts; this increases the likelihood that children will be able to separate the concepts from the materials.

Another benefit of providing different types of materials is that children may be more attentive and motivated because their curiousity will be aroused (Fennema, 1977, p. 99). The curious child may use the materials in ways that help teachers to gain insight into the child's thinking. Two further advantages of manipulatives are that they provide a common basis for understanding and discussion among students and "diversity of learning environments" (ibid., p. 99).

Discussions among students are likely if they are using manipulatives. These conversations may focus on the mathematical ideas or the more mundane aspects of the steps and procedures involved in an activity. The opportunity to engage in discussions and being able to share thoughts, observations and ideas cannot be overemphasized (Reys, 1977, p. 103).

In the same article, Reys sets out some pedagogical considerations for choosing manipulatives which will encourage this sharing described above. These considerations are that the materials should provide: a true embodiment of the concept; clearly represent the concept; motivate students; be multipurpose if possible; provide a basis for abstraction; and provide for individual manipulation by each learner.

The manipulatives supplied to the children in this study met Reys' requirements. When these or any students are profitably engaged through the use of well chosen materials, they will use their time productively, use resources appropriately, and ask for the teacher's assistance only when other sources have proven inadequate (Webb, 1986, p. 121). When the manipulatives are well chosen and matched to the task, students are able to function more independently, leaving the teacher free to observe and learn about the students.

Roles

The unobtrusive examination of students' learning is one role which might be assumed by the teacher in the mathematics laboratory. By listening to conversations and acting as an observer, not a judge, the teacher conveys the message that students are responsible for assessing their own work. The teacher, "while remaining the authority, ceases to take an authoritarian position" (Ewbank, 1977, p. 216). If children are to judge their own successes, the teacher must ensure that there is something worthwhile for students to do and discuss. Having supplied the learning situation, the educator then places the responsibility for the correction of errors on the learners. The teacher assumes a neutral stance so that students become self-reliant.

Part of this self-reliance is shown through self-control, helping others, showing tolerance, asking for assistance when necessary, thinking for oneself and engaging in on-going self and group assessment. This responsible attitude from students also needs to include acceptance of everyone's contributions. Children learn this last aspect of group work when the teacher demonstrates respect for the children, listens to them and requires that they listen to one another and provides explicit acknowledgment of every child's ideas. Many of these are traits which are beneficial to cooperative learning as well and were identified in the above section entitled "Cooperative Learning".

CHAPTER III

METHODOLOGY

This chapter will outline the design of the study and how it was negotiated, the selection of the study site and of the students involved and the roles played by the pupils, teachers and researcher. The chapter will also include information about the types of data collected and how they were analyzed.

SELECTION AND DESCRIPTION OF THE STUDY SITE

Selection

The study site was chosen because of its combination of a mathematics laboratory, the frequent use of Skemp's structured activities and the attitude and philosophy of the classroom teacher. The researcher was acquainted with the Grade Three teacher prior to the study through several in-service presentations given by the teacher and through University of Alberta courses focusing on the work of Richard Skemp. The researcher visited this school initially because of her interest in seeing the Skemp materials in use. After observing the Grade Three class in the mathematics laboratory, the researcher and teacher decided that the class could provide worthwhile data for a study. They also both felt they could benefit from the opportunity to collaborate. The school was one of a very few in the Edmonton region where the Skemp programme was being implemented in more than just an occasional classroom. The school's official mathematics philosophy was based on constructivism. All of the teachers and students in the school had access to the mathematics laboratory and, thus, to the Skemp activities. The supportive attitude of the principal was also a factor in the selection of the site.

Description

The Grade Three class in this study was a heterogeneous group of 20 children in an urban elementary school. This class might be considered unusual because of the number of English as a Second Language children, the use of a wide variety of mathematical activities and manipulative materials, the existence of the mathematics laboratory and the two teachers who worked together during mathematics classes.

There were two Grade Three classes in the school, both taught by teachers who had no previous experience at this grade level. These teachers had taught Grade Two in the previous year so some of the children had been with the same teacher for two years. The frequent collaborative planning sessions also included the school's intern teacher who was in his first year of teaching. He assisted both classes in the mathematics laboratory and instructed some other subject areas independent of the classroom teachers' direct supervision. The teacher in whose class the study took place usually provided leadership during these meetings because of her specialized mathematics knowledge and her familiarity with a wide range of materials. This teacher served as the school's mathematics coordinator and had organized the mathematics laboratory several years before. She had also helped to instruct in-service mathematics courses in her school district. The discussions occasionally included the school's principal because of his expertise and interest in mathematics.

The mathematics laboratory was organized with the pupils seated in five groups of four each. This arrangement allowed students to assist each other but also provided additional distractions for some children. As the tasks on each day changed after fifteen minutes, an adult usually moved the equipment from one table to another so the students changed their locations only when they went to the computer station.

Mathematics instruction took place every morning for 30 minutes. Monday and Friday the children were taught in the classroom; Tuesday through Thursday they proceeded to the mathematics laboratory where they were engaged in activities designed to teach and review concepts. The class spent one half-hour in the laboratory; the time was split into two 15-minute segments so the children had two different tasks to complete. On Thursdays there was only one 15-minute activity and then the children wrote in their mathematics journals for the remaining 15 minutes. The children in the class were divided into five groups so the time arrangement allowed each group to complete all five "stations" scheduled for the week's laboratory work. Many of the children had attended the school since Kindergarten or Grade One so they were familiar with the format and expectations of working in groups in the laboratory. A few of the students were new to the school during the year when the study was done but were used to group work by the time the study occurred.

The work in the mathematics laboratory included a computer "station", games and activities from several mathematics series, teacher-designed jobs as well as activities designed by Richard Skemp (1989a, 1989b). There was a strong emphasis on using manipulative materials to teach and extend the concepts.

SELECTION AND DESCRIPTION OF ROLES

Students

During their work in the mathematics laboratory, the class was divided into five groups. Originally eight of the students were placed in two groups of four each. After gathering the data and beginning the analysis, it became clear to the researcher that the data regarding the Research Questions for both research groups were so much alike that an analysis of the second group would be redundant.

The four students in each of the two study groups were chosen by the teacher and the researcher to represent a cross-section of the mathematical, cooperative and verbal skills of the class. Further references to "the study group" will include only the four students who first formed the group and then, after Day Ten, when one boy returned to the class after a lengthy family holiday, all five students.

At the beginning of the multiplication unit, the study group consisted of two girls and two boys. As mentioned above, one additional boy was added to this group on Day Ten of the study. The first day he was included in the videotaping was Day Eleven.

The names of the following students have been changed; no attempt has been made to ensure that the pseudonyms reflect the students' ethnic backgrounds.

Theresa (female) - average mathematical ability, cooperative skills varied, verbal abilities quite good;

Peter (male) - average mathematical ability, had some difficulty expressing his knowledge, tended to become flustered, good cooperative ability;

Fred (male) - high mathematical ability, good verbal skills, good cooperative skills;

Jenny (female) - high mathematical ability, good cooperative skills, good verbal skills but rarely volunteered information;

Alex (male) - high mathematical ability, excellent verbal skills, good cooperative skills. This was the student who returned to class on Day Ten of the study. His initial experiences indicated that he had some grasp of the multiplication concepts studied by the other children during his absence. The teacher and researcher felt that observing the other children teaching this boy what they had been learning might yield some valuable insights into how well they had learned so far and what type of understanding they possessed.

The children's role could be described as cooperative because they needed to work together to accomplish the goals set for them. The teachers rarely included the children in the planning. One exception to this was a class discussion at the conclusion of the unit. On this occasion the children played a collaborative role. The questions and tasks used for the individual interviews were developed in part from this discussion.

The study sought a combination of formal and informal types of cooperation. To help them with their work in the mathematics laboratory, the Grade Three children had received cooperative learning training based primarily on the Johnson and Johnson and Holubec model. The classroom teacher indicated that students had been taught the skills of mutual encouragement, group processing of information and tasks and the behaviour necessary to work well in groups. While working in the mathematics laboratory, they should also have exhibited the characteristics of informal cooperation such as taking turns and using courteous ways of speaking.

Although the distinctions between "cooperation" and "collaboration" may not be universally agreed upon, this study will accept that there are some differences. Therefore, the terms will be used to describe two style of working together. "Cooperation" will be used to designate simply working together to accomplish a goal. This style does not include any means of controlling the goal or, usually, the manner by which it will be reached. "Collaboration" will be used to designate situations in which the participants have equal decision-making status and are able to jointly influence the goal and/or how it will be accomplished.

Teachers

The classroom teacher, Ms. C, taught all of the "whole class" lessons and she and the intern, Mr. G, shared the responsibility for working with the groups. Students usually indicated through their tone of voice and phrasing of questions that they considered the intern as a "regular" teacher. The teacher was usually responsible for deciding what specific role the intern would fulfill within her classroom, particularly in the mathematics programme. Following the consultations with the intern and the other Grade Three teacher, the classroom teacher would assign specific tasks to the intern to do during the laboratory time.

Neither the classroom teacher nor the intern had ever taught an introductory multiplication unit. However, the researcher was familiar with the content, having taught Grade Three for several years. The planning meetings frequently consisted of short conversations at recess or while pupils were working in their groups. More formal meetings were also held during which the content and sequence of the material was discussed. Issues such as how to teach the notation and the meaning of the multiplication factors were discussed among the two Grade Three teachers, the intern and the researcher based on the Alberta Programme of Studies and the recommendations of the Structured Activities for Primary Mathematics (Skemp, 1989a, 1989b). Occasionally these conversations included the school's principal because of his interest in the study and his extensive mathematical knowledge.

The teachers' and researcher's roles may be described as collaborative because of the sharing of ideas and responsibilities. The adults were equal participants. At any given time in the study, one of them could take a more dominant role, based on that person's expertise in a given area.

In her five months of observations prior to the study, the researcher found it useful to elicit and accept the Grade Three teacher's informed judgements based on her more extensive knowledge of, and experience with, the Skemp materials. These judgements also included suggestions about what might be appropriate to attempt by way of communication between the teachers and pupils and among the pupils. For the five weeks of the study, the researcher was essentially an observer of students and teachers. There were several exceptions to this observer status and these will be noted at appropriate places in the study.

In the collaborative planning meetings, the researcher assumed a consultative role. Based on her nine years' experience teaching Grade Three, the researcher was able to offer advice to help the Grade Three teachers and the intern anticipate aspects of multiplication which the pupils might find relatively difficult. These observations were general: no attempt was made to impose a particular approach. The planning discussions did not involve the researcher in the structure and development of individual lesson plans or the final "shaping" of the unit. The intern played an active role in the discussions but a minor role in the final decision making. Lesson planning was the responsibility of the teacher. The collaborative planning determined the outline of the unit but its specific implementation was determined by the teacher.

The researcher's efforts to avoid influencing the specific form and content of the lessons resulted from a concern that the natural flow of the communication should continue as it had in the first six months of the school year. The researcher did not interfere with the programme or the group interaction (except as noted elsewhere) in order to attempt to obtain an accurate set of data.

The teacher and the researcher agreed prior to the start of the study that its design was the responsibility of the researcher. The teacher acted as the consultant in this case; she offered suggestions but the researcher was ultimately responsible for the decisions made. The teacher expected the researcher to familiarize herself with the children and the mathematics programme so that they could select the topic and children together. The researcher was also expected to remain aloof from the everyday activities as much as possible.

Researcher

The researcher intended to act as an observer, not an instructor, during this unit. However, due to the ongoing collaborative nature of the study, the role could be redefined. The researcher had the primary responsibility for analyzing the videotapes and audiotapes. In order to assess the degree and forms of cooperation and communication which occurred, the teachers and researcher discussed their individual observations. This ongoing consultation helped to shape the unit and the study.

Participation by the researcher in discussions with teachers, and in the questioning of children, did not destroy the quality of the data gathered. That is, while complete objectivity on the part of the researcher might be required in some kinds of studies, it was not necessary in this study. On occasion, the researcher felt it was necessary to put questions to the children to test the students' reasoning beyond what the presiding teacher or intern had just done at a particular moment in a class or group, or at a stage of the students' group interaction when it was useful to test more explicitly one of Skemp's concepts which seemed to be just "under the surface" of the students' remarks.

DATA COLLECTION PROCEDURES

Data were gathered through:

- observations in the classroom and mathematics laboratory
- videotapes and audiotapes of mathematics lessons in the classrooms and laboratory
- a journal kept by the researcher
- journals kept by the children (available to the researcher)
- conversations among the researcher, the Grade Three classroom teacher and the intern teacher
- videotaped interviews with the study group and with the individual children from this group
- a questionnaire regarding students' attitudes administered at the finish of the unit

DATA ANALYSIS PROCEDURES

The analysis was centered on three complementary components listed below: the communication patterns among the students and between teachers and students, Skemp's eight assumptions about how children probably best learn mathematical concepts and the extent to which the students' training in cooperative social skills was reflected in their verbal and non-verbal communication.

a) Teachers and students assumed many different roles. The analysis focused on the roles taken, which participants took these roles and how the communication patterns were affected.

b) Skemp's practical activities for learning mathematics are based on a series of assumptions about how children can develop mathematical knowledge. The researcher has paraphrased these assumptions and listed them. (For Skemp's original wording, see Appendix D). His assumptions include:

   1) New concepts should usually be introduced through a teacher-led discussion;
   2) Children can play the games together without direct adult supervision once they have learned the procedures;
   3) The structure of the games will encourage discussion;
   4) The discussions will be mathematical because the rules and strategies are largely mathematical;
   5) Children will question each other's moves and justify their own moves;
   6) These discussions will consolidate the students' comprehension;
   7) Explaining an idea to someone else is an effective means of improving one's own understanding; and
   8) Children can benefit just as much as adults from the opportunity to discuss and explain their ideas.

c) The cooperative social skills desired included courtesy, taking turns, effective listening skills, and willingness to help each other grasp and develop concepts.

Because of the numerous sources of information used, it is possible that some relevant information was omitted. The conditions under which the videotaping occurred were sometimes quite noisy because of the linoleum floor, the stools on which students sat, the unmuffled sound of materials

materials on the tables and the number of simultaneous conversations. It was occasionally difficult to obtain an accurate transcript. The addition of the tape recorder ensured that the conversations of the teachers and students were not missed. Every effort has been made to make the data as complete as possible.

Daily Descriptions

As the researcher observed the "live" work in the mathematics laboratory and reviewed the videotapes, she sought examples of the types and the effectiveness of the communication which took place between the adults and children and in the student groups. The researcher made note of what sorts of questions were asked, by whom, and the wording and attitude with which the questions were answered, as well as the tone of voice and non-verbal aspects of the interaction. The researcher also watched and listened for evidence of whether the discussions were mathematical and how the children reacted to requests for help from their peers. The researcher sought evidence about how the activity-based materials and the physical and temporal organization of the mathematics laboratory encouraged or curtailed the communication. Remarks were considered to be particularly significant when they were made spontaneously and not in response to an adult's prompting.

Other Sources of Data

The researcher videotaped all of the activities in the classroom and the mathematics laboratory. The classroom work usually utilized groupings of students different from those used in the mathematics laboratory. Therefore, data gathered during the classroom work were of limited usefulness and were excluded except for Day 21 when the study group worked together in the classroom. Some of the laboratory activities included review of addition and subtraction; consequently they have been excluded from the data.

After some initial self-consciousness, the children ignored the video camera most of the time. They would occasionally glance at it during their group work. They were most aware of the camera during the week when each group in the class was interviewed about the mathematics laboratory, group work and cooperative learning. In order to have a quiet atmosphere for these interviews, it was necessary to use a small room attached to the mathematics laboratory. The size of the room required students to be close and face directly into the camera. The questions asked in the interviews (see Appendix C) focused on the cooperative learning groups, the content and structure of the activities, the use of a mathematics laboratory versus a textbook and the need to communicate while learning.

While the researcher asked the students to keep journals in which they could record their understanding of multiplication, she concluded at the end of the data-gathering phase that the student notebooks did not offer additional insights into student behaviour than were obtained through videotapes and individual interviews. Thus the journals were discarded as sources of information for the study.

Individual interviews were conducted at the end of the study as a means of assessing the students' grasp of the unit's concepts. These interviews (see Appendix F) provided a more detailed picture of the understanding acquired by each student.

CHAPTER 4

DAILY DESCRIPTIONS

DAILY DESCRIPTIONS

The names in the parentheses following the number of the day in the laboratory indicate which children were present. In total, 21 days of lessons and laboratory time were taped; but since Group A was dropped from the study for reasons previously stated in the report, information was used for only 11 days of taping of Group B. The data utilized (see Appendix A) was from Days 1, 2, 4, 7, 8, 11, 12, 13, 17, 18 and 21; all other videotapes and audiotapes were of Group A. The multiplication unit extended several days beyond Day 21; these days included individuals working or the classroom groups and were not taped. During the last week of the unit, the group interviews (see Appendix C) were done as one "station" in the mathematics laboratory.

Throughout the daily descriptions the researcher has included comments intended to clarify the students' remarks and behaviour. These interpretive comments appear in an an alternative style of print.

DAY 1: (Theresa, Fred, Peter)

Skemp's Num. 5/1/1 "Make a Set, Make Others Which Match" (1989a, pp. 164-165) provided the structure for Day 1 laboratory activity. In this activity one child made a set (e.g., of four objects). All the other group members then made sets of the same size on small cardboard circles and placed them together into a "set loop". This loop was a circle of yarn meant to show that the different materials had been combined into one larger set. Each child in the group used a separate type of material (i.e., shells, buttons, plastic hearts). In one round of the game, the classroom teacher, Ms. C, asked only the pupils wearing blue to build the matching sets. Skemp suggests this procedure of using a portion of the groups (1989a, p. 165).

The class was introduced to the game (see Appendix A) which each group then played. The teacher, Ms. C, asked questions of the individual groups and the whole class. She frequently requested that each group describe its work because, although the rules of the game were the same, the groups worked with different sized sets. The teacher assumed a highly directive role because this was an introductory activity.

Approximately half way through the laboratory work, the teacher reviewed the two actions of making a set and making matching sets. She described these sets as ones "that have the same number of objects in the set". Theresa responded, "I suppose you can't use [sic] if someone put down five and then someone put down four and then someone else put down five and then someone else put down five". When asked whether she thought that would work, Theresa said she didn't know. Ms. C told her that it would not be multiplying because the sets must be the same size. *It seemed to the researcher that Theresa was trying to sort out the concepts of groups and matching groups by offering a possible non-example. It would have been interesting to ask Theresa to continue speculating or to ask why she thought her description would not work. This might have provided an opportunity for Theresa to reflect on her current understanding and how to accommodate the equal groups idea. It might also have introduced a mathematical discussion beyond the strictly procedural one which took place in Theresa's group. However, since this was an introductory game, it might have been too soon for this open sort of discussion to have much meaning.*

Theresa's group was observed during one round of the game. Theresa spoke more frequently than the two boys; she argued with Fred about procedures, answered several questions and made two mathematical comments. When instructed to create a set with nothing in it, Theresa said, "Ze number zero. Zero is nothing" and a moment later, after another student said, "Zero times four is zero", Theresa commented, "Zero times four, four times zero." *This seemed to indicate that she had some grasp of commutativity.* However, the class period ended just after she made this remark so there was no opportunity to explore her understanding. *Theresa seemed focused on the mathematics of the game as much as the procedures. The responses and explanations by the boys were limited solely to the structure of the game and did not go beyond to "play" with the ideas the way Theresa did.*

The explanations provided by members of the group were limited to how large the original set was, how many sets were put into the set loop and what the result was.

DAY 2: (Fred, Theresa, Peter, Jenny)

The class played "Make a Set, Make Others which Match" with different materials and more explanations than on Day 1. Two rounds were played using the materials from Day 1; then the groups were given Unifix cubes to use for making "trains" of cubes. Each child received a different colour of Unifix cubes so the groups could be distinguished from each other. In addition to explaining the size and number of the sets, children were asked to clarify how they had determined the result of their combined sets. Many of the children in the class were more distracted than on Day 1. The teacher indicated in a later conversation with the researcher that this lack of focus might have resulted from the two days' activities being too similar. The mathematics laboratory work usually meant that each group had a different task and they proceeded at their own pace. This time they were working step by step and waiting to listen to the other groups' explanations or to explain their work to the teacher. The students in the study group played a great deal with their manipulatives as they waited for the teacher's directions to the whole class. The children were using Unifix cubes to make "trains" showing their combined sets. Since these materials fasten together and contain a small hole in the end, the students were using them as telescopes; they were also holding them horizontally like handguns.

Theresa dominated the conversation. She made 42 comments in contrast to the 37 from the other three children combined. She instigated the telescope game but later told the others, "Come on you guys, we're not playing." She also showed off her set to a student at an adjacent table. Theresa argued with both Fred and Peter about how the game was to be played. Theresa also answered a great many of the teacher's questions and made predictions.

During one round of the "Make a Set" game, Fred made a "train" of six. Theresa predicted the result of combining all the sets would be 24. She repeated, "It's going to be 24" twice and then chanted, "Twenty-four" eight times as Fred put the individual "trains" together and counted them by ones. Peter agreed with Theresa's prediction and when Fred reached

24, Peter announced to Theresa, "See, I told you I was right."

Theresa provided the majority of the day's "smart" mathematical ideas. She modelled the use of partial sums (which Fred used again later), made correct and unsolicited predictions; and provided clear explanations. One of her explanations was wrong; this was the point at which Fred used partial sums to tell why he disagreed with Theresa's answer.

Although she participated in the day's laboratory activities and the off-task activities, Jenny spoke only twice in this half hour. Both of her comments were about the off-task behaviour. Part of the reason for her lack of speaking was her assigned role; she never had a turn to make a "train" or do the explaining. She was often quiet during the laboratory activities and rarely spoke as much as the other children but this was an exceptionally quiet day for Jenny.

Fred modelled a means of determining equal sized "trains". Rather than counting each one, he stood the "trains" side by side. After he had assembled the longer "train" using everyone's Unifix, Fred again stood it on end. Theresa immediately criticized, saying, "Trains don't stand up." Fred merely shrugged and left it as it was. At this point Peter suggested, "Let's do it with fives and see how long it is. Twenty." His answer followed almost immediately after his original idea. *He evidently did not require the Unifix cubes.* He did not have an opportunity to elaborate because the teacher arrived to ask about the results of Fred's work. When she asked how they knew what the right answer was, Theresa responded, "'Cause we just did it." *Her tone of voice was off-hand and somewhat sarcastic.* Peter offered partial sums as his method. Theresa repeated, "And plus, we just did it."

The final class activity in the laboratory was to describe the game using Skemp's "First action, second action, result" model. Each group was asked to tell what it had done and Ms. C wrote the information under the appropriate headings on the blackboard.

Some of the group's conversation focused on the necessary mathematics but much of it dealt with the materials they were playing with.

The children did question each other's moves but with a rather negative and critical attitude. Theresa was particularly quick to *criticize Fred and Peter and to support Jenny.*

The social skills which the children were expected to use were often lacking on Day 2. Ms. C's idea of asking all of the groups to describe their work for everyone could have been beneficial but the study group was inattentive.

DAY 4: (Theresa, Fred, Peter)

On the 4th day in the mathematics laboratory students used Unifix cubes once again to make "trains" with the focus on describing the "First action, second action, result". The children had "First action" cards telling them how long the rods or "trains" should be and "Second action" cards telling how many rods to join. Skemp designed the activity using a number track on which to place the rods but this was not supplied to the pupils on this day. *One advantage of not supplying the number track was that the children had to discover the results for themselves. The disadvantage of the missing number track was that they tended to use confusing and inefficient counting strategies.*

Ms. C began by informing the pupils that they were going to repeat the previous day's activity and asking why they thought she had made this decision. Students replied that repetition would help them remember or learn if they had not the first time and would teach those who had been absent. The need to explain clearly and, in particular, to listen carefully to other students' talking was strongly emphasized by the teacher. *This admonition resulted from a conversation among the adults about the lack of good listening skills used the previous day. Important ideas were ignored in most of the class groups because students were inattentive during the explanations.*

The study group started by arguing about the roles they had been assigned and how many cubes each of them should take out of the container on the table. The intern teacher intervened and helped the group organize. Their work began with two rods of ten joined together. Theresa tended to direct the work; she asked procedural questions and quizzed Fred about how he found the result. Fred responded by saying, "It's twenty. Ten ... times ... ten times two is twenty." He pointed to each section of ten cubes as he spoke. Theresa asked how he knew and Fred indicated that, "Two groups of ten is twenty." He did not specify how he had computed the answer nor did he refer to having counted the groups. He appeared to know this combination without any figuring. Next it was Theresa's turn to do the First action and Second action. She was to create rods of nine cubes. To do this, she demonstrated how to take the ten-rods, put them side by side and remove one cube from each. The Second action required four rods of nine to be joined. Fred added two groups of nine for a result of 17. Peter repeated the mistake and Theresa corrected it. She then started to combine two groups of 18. A conflict arose between Theresa and tne boys about whose turn it was to do the Result step. As Theresa and Fred were arguing, Peter recounted the rod from the 19th cube. Just then the intern returned and wanted to know what was happening. During this conversation Fred realized that Peter's answer was erroneous. The intern, Mr. G, asked a series of questions designed to prompt the use of partial products. Theresa told him about their attempt to use 18 plus 18; as she explained, she was recounting the long rod. To keep track of her counting, she was tapping each cube. She became confused and Fred placed a finger on the 19th cube. Theresa merely pushed aside his finger and resumed her counting. Fred counted along with her, using his finger again to keep his place. The result was 35. Theresa disagreed and attempted to explain how she had used the first nine cubes plus one from the second section to equal ten. Fred had begun to count once more. As Fred and Peter tried again, Theresa put her head on the table and said nothing. Peter finally indicated that he was mixed up and Theresa said, "How did you do it? You're supposed to tell us." *Her frustration and disgust were obvious from her tone of voice and body language.* Peter indicated that he had counted by ones; Theresa's response was to repeat, "ones" in a negative tone. Theresa's job was to explain what had occurred. She did this in a monotone. Fred disagreed with Theresa's account and they argued. Her defense was that she had described exactly what Peter had said he had done. Fred countered with an explanation that he had counted nine and nine make eighteen and then he counted the remaining eighteen to equal thirty-six.

The intern had returned and the children tried to tell him about their problems. *Mr. G seemed to think that Theresa was making too much of the situation; his tone of voice dismissed Theresa's concern and she* *looked unhappy after they spoke.*

During the second round, Theresa again used the partial sum strategy to find the sum of the first two groups and then counted on. She checked by counting by ones from the first cube. Peter thought the total was 39 but Theresa and Fred agreed that it was 40. Theresa supported her conclusion by pointing out that the rods were all even numbers (8) so the total also had to be an even number. Neither of the boys commented. The day's laboratory work concluded with Ms. C's stating that the group's work had actually shown multiplication. She said, "When you're multiplying, it's in two actions. You make a group and then you stick five of those groups together to find out what you get." The pupils then wrote in their mathematics journals about the week's activities.

The atmosphere in the group was more argumentative than on the previous days. All three pupils appeared frustrated at some point during this day's work. Peter was annoyed with his inability to find the answers easily. Fred had a number of disagreements with Theresa and also had difficulty finding consistent answers. He counted and recounted and achieved several different results for the same question. Theresa was generally annoyed with the boys not listening to her. She was providing some valuable mathematical suggestions. The only one of these which the boys adopted was the idea of partial sums. This was used by all three students. Theresa conveyed her frustration through her body language, tone of voice and by pushing Fred's hand aside. Her overall attitude was negative. The children contributed almost equal numbers of comments but their purposes differed. Peter appeared most focused on the multiplication task. He organized the others and initiated the first two rounds. A number of Theresa's remarks were attempts to point the boys toward appropriate strategies and the necessary mathematics. She frequently prompted the others as they worked. Fred tended to focus on explaining the procedures and answering questions from the other students and the intern teacher.

Theresa's mathematical understanding seemed more developed than the boys'. She verbalized her thinking more frequently and more clearly and had a wider range of more sophisticated strategies.

The adult-to-child communication on Day 4 was not effective. Rather than justifying their ideas to other students, the focus was on *appealing to the intern teacher to judge who was correct. If an adult had remained with the group and encouraged better listening skills, perhaps more of Theresa's ideas would have been heard and the boys would have benefitted.*

DAY 7: (Fred, Theresa, Peter, Jenny)

On Day 7 the students were continuing to play "Giant Strides on a Number Track" (1989a, pp. 165-167). Ms. C had introduced this activity on Day 5 in the classroom by having pupils role-play "giants" striding on a large number track laid on the floor. Number cards designated how long each "stride" should be and how many "strides" should be taken. These two numbers were the multiplication factors. Students were asked, as a whole class and later in their groups, to predict where the giant would finish striding. As the groups played independently using number tracks, they would use blobs of clay to represent the footsteps and confirm their predictions.

The conversation in the study group on Day 7 was divided almost equally with all four children assuming similar roles. At the start, Fred and Jenny worked together to review the procedures, prompted by Peter. The two boys and Jenny had to teach Theresa the game since she had missed its introduction on Day 5. Theresa asked why a guess was necessary; this led to a difference of opinion. Peter did not know and Fred thought all they had to do was place the clay on the number track and tell where the last stride fell. He thought that, since there were no giants, no prediction was called for. *It may have been that he thought that an actual person had to play the giant's part before the gameboard was used.* Theresa responded, "We are the giants. Well look, it's so small no-one could be the giant" and she then asked again about estimates. *In spite of the fact that this issue did not appear to be resolved, the children did make predictions as they went on.*

Peter took the first turn at being the giant. His task was to make five strides of ten spaces each. He began by "striding" with his fingers. Jenny, her finger marking the ten space, took some clay from Peter and placed it on the track. As she did, she also verbally prompted Peter. Following this, Jenny speculated that the answer would be 50. Peter replied, "Ten times five." Theresa interjected, "Is it a times thing?" *It was difficult to tell from her tone of voice whether she* *was puzzled, attempting to help or being sarcastic.* Then she asked whether "it had to be right". Peter was in the process of skip counting by tens. At this question, he gave an answer of 25 and then immediately "re-estimated" a product of 50. *It was unclear whether he felt he was truly predicting or if he was trying to find the exact product. He did not place the clay to show the strides until Fred prompted him. Seeing the clay on the number track evidently clarified Theresa's thinking.* She commented, "Oh, so every ten is a stride. Now I get it."

Fred proceeded with the next round. As Fred turned over the number cards, Peter tried to guess what they would be. As he saw a four and a two, Peter counted four groups of two on his fingers. Fred quickly put down the clay and wrote the answer. No discussion of an estimate occurred.

When Jenny turned over four and nine for the third round, Theresa said, "Four times nine. Oh, man!" Peter, meanwhile, had used his fingers to estimate 28. Jenny rapidly put down two blobs to show the first strides. *This appeared easy. However, she had to count by ones to figure out the third stride.* Theresa reminded her that four blobs were needed. Fred pointed to where he thought the next step would end. Jenny placed the clay to show 27 and then 36. Theresa prompted, "Is it right? Was it right?" Peter checked by once again counting on his fingers and confirmed the answer.

Peter had developed a system of counting groups on his fingers and he used this strategy frequently throughout the multiplication unit. For example, if he was figuring out four times three, he would tap his one finger three times, at the top, middle and bottom, with the index finger of the other hand and then repeat this to create the other three groups. This method required him to keep track simultaneously of the number of groups, the number in each group and the running total.

The children did a lot of negotiating of meaning during Day 7. They discussed the need for estimates, clarified the procedures and made predictions. Three aspects of this day were unique. First, Peter, Fred and Jenny all provided non-verbal prompts which helped their classmates. This was in contrast to Day 4 when Theresa rejected Fred's attempts to help her. Second, Theresa experienced an "aha" when she realized the significance of placing the clay at the ten space intervals. She remarked twice on this discovery. Third, Peter displayed an interest in predicting both the number cards and the products. It is questionable whether he was actually estimating: his idea of "predicting" appeared to be trying to figure out the answer and presenting it as an estimate. This is consistent with his approach throughout the study: he seemed to value being right and was disinclined to take chances.

All of the children assumed the roles of explainer, director, prompter and predictor. There was no off-task conversation or behaviour: they remained focused for the entire 15 minutes they were allotted to play "Giant Strides".

The communication and cooperative behaviour on Day 7 were commendable. The assistance which the students provided and the mathematical content of the entire discussion appear to support all of Skemp's assertions about why students ought to work together on practical activities. Theresa's "Oh, so every ten is a stride. Now I get it!" comment clearly shows how the verbal and non-verbal prompts of her groupmates enabled her to make a connection. There was no real indication that the others benefitted as much as Theresa; they seemed to be content to play the game and make and confirm predictions. The attitude of all four students was courteous and attentive to their assigned task.

DAY 8: (Theresa, Fred, Peter)

On Day 8 the children worked with Ms. C to learn Skemp's Num. 5.2/1 "I Predict - Here". They created stacks of two, three, five and then four Unifix cubes which they linked together to find the products. The number of stacks which students combined were determined by "set cards" with various arrangements of squares on which to place the stacks of cubes. Students then were required to predict how far the whole "train" would extend on a number track. They used different sized sets for each round of the activity.

The communication patterns this day were similar to Day 2 when the students focused on the procedures rather than the mathematics. Responses to the teacher's questions were often only one or two words. The children seemed to interpret the questions as "closed" rather than open-ended so little of their thinking was verbalized. This was particularly true of Fred. On four occasions he made a prediction or other spontaneous remark; all 16 of his other comments were in response to direct procedural questions.

Peter and Theresa talked more freely than Fred but the range of their conversation was restricted as well. They discussed how to figure out the answers and they made predictions. Theresa seemed to compute the answers quickly. Almost as soon as the number card was displayed she predicted the answer or indicated that she knew it. Theresa also made connections that the others did not. Even before the teacher had explained the activity, Theresa compared it to the "blob game" ("Giant Strides"). She later talked about how her task of making five stacks of two was like Fred's four stacks but she had to add two more cubes. She thought that Peter would make six stacks during his turn.

All of the children found the groups of two and five very easy to calculate by skip counting. They also utilized partial sums.

After playing the game with twos, fives and fours, Ms. C talked about the activities completed since Day 1 and said, "What we're going to do now is multiply". Theresa's reaction was negative. She repeatedly commented, "Now you tell me". *She sounded annoyed at the thought of multiplying and not just "doing times".* She even asked, "Why didn't they tell us we were doing multiplying?"

As Fred was beginning, "We took some numbers ...", Theresa interrupted with, "So you have to have the same number". When the researcher asked her to repeat and clarify her remark, Theresa responded, "You have to have a set of even numbers" and demonstrated by holding up two four-rods side by side. She was evidently indicating that multiplication needs equal sized groups. This echoed her question on Day 1 about whether the groups needed to be the same.

This day's communication contrasted strongly with the preceding day. On Day 7 the students worked together; on Day 8 the pupil-to-pupil interaction was much less. The students focused largely on the teacher's questions and the conversation was from the teacher to individual students and back again. The students' conversation among themselves took place as they were assembling the stacks of Unifix cubes; it consisted of comments about the length and colours of the stacks.

The teacher's questions tended to be directive and quite closed;

they generally required short answers. Once the children had played several rounds of the game, Ms. C asked Theresa to compare her method of determining the product to Peter's. As Theresa was attempting to explain, and she was unable to reproduce her earlier thinking, Fred quickly outlined his partial sums strategy.

Day 8 was one of the days when the researcher stepped out of her "observer only" role. As the children were cleaning up, the researcher spoke to Fred to reinforce Theresa's observation about having equal groups.

The teacher introduced the pupils to "I Predict - Here" and reinforced the procedures and concepts as Skemp recommends (Assumption #1). During the first playing, it was difficult to determine whether the children were attending to the meaning of the mathematics or concentrating almost exclusively on the steps involved.

The speed and ease with which the students could make predictions and verify them seems to indicate a development of some multiplication skills. Even if they did not know their basic facts, these students were all able to use fairly efficient strategies for determining products. Theresa, in particular, seemed at ease with the skills/facts and the concepts from the unit so far.

Days 9 and 10 dealt with Study Group A.

DAY 11: (Theresa, Fred, Peter, Jenny, Alex)

This was the first day on which Alex joined the group's work in the mathematics laboratory.

Day 11's activity consisted of the teacher working with the group on Skemp's "Number Stories" (Num. 5.4/1). This was a series of number and situation cards combined to make stories for the students to figure out. The teacher turned over the story card and two number cards representing the factors, read them aloud and then instructed the group to find the answer by using manipulatives or drawing pictures. An example of a story might be, "Giles is collecting fir cones in a wood. He gets 5 cones into each pocket and he has 6 pockets. When he gets home and empties his pockets, how many fir cones will he have?" (1989a, p. 178).

During the first round (factors of six and three), the children seemed to require repetition of the situation and the numbers to be used.

This information was written on the cards displayed on the table. They also asked questions about whether they could use whole pieces of paper and if they should include a "math sentence". Fred supplied the answer even before he began to "figure". The students used the manipulatives and sketches which they explained to the group. Methods used to calculate the product included counting by ones and by sixes and partial sums.

In round two, as soon as Ms. C read the situation and number cards (five groups of four), Theresa asked, "So we keep adding groups of four?" She was simply reminded to show how she discovered the product. The children asked a lot of procedural questions at the start of round two.

Peter's utterances as he explained his method were quite limited in length. Ms. C asked him four questions; his responses were short and to the point. He did not elaborate of any of his answers. Aside from his initial, fairly detailed statement, Fred's responses were even more limited than Peter's. His answers to most of the teacher's questions were only one word long.

When Theresa completed her explanation about using repeated addition, the teacher drew the group's attention to the method. Theresa had written out a series of five fours and joined them in a multiple addend equation. After questioning Theresa and reviewing her method for the group, Ms. C said, "This is what multiplying is. It's adding the same number over and over again." Fred appeared startled. Theresa replied, "Thank you for telling me." *It was difficult to assess her attitude. Her tone of voice did not sound really sarcastic but it was not sincere sounding either.* Fred then began a comment with, "So if it's the other way around it's ..." *The implication appeared to be that he was trying to sort out the concept of commutativity.*

As they began round three, the number cards were six and three, the same factors as the first round. Fred noticed and commented so the teacher altered the number cards to read six and four. She then read the story card. Again, the pupils asked repeatedly about the facts needed. Fred seemed quite confused by this problem. He put a "train" together and then separated it into sections again. Ms. C asked him how many groups and how many in each. Part of his confusion was evidently caused when he reversed the factors. His responses to questions were, again, quite short. To figure out their answers, Peter counted by ones; Fred and Jenny used partial sums (6 + 6 = 12, 6 + 6 = 12, 12 + 12 = 24) and Theresa indicated that she had started with 4 x 5 and added another four. This showed more advanced connections being made by Theresa. She was able to incorporate her previous knowledge about four times five as an aid to determining something new.

It was interesting to note that the way the pupils used the cubes to make their "trains" was somewhat altered from the emphasis in previous activities. In earlier games, the children had been asked to differentiate among the groups in the "train" by making them separate colours. On Day 11, several students used the cubes without grouping by colour. In one case each, Peter and Alex counted the entire "train" by ones rather than utilizing the group "chunks" to count. Yet, for the most part, the students seemed to be able to keep track of the fact they had groups and how many were in each. Several of the pupils used partial sums which demonstrated that they were comfortable with combining smaller groups and using a more efficient, advanced counting system.

There was some indication of students prompting each other's thinking; in the first round Alex was uncertain about his answer and Jenny asked him about the number of nests and how many birds were in each. This seemed to be all Alex needed to correct his answer. Direct student to student communication was rare; most of the students' remarks were "thinking out loud". This might have provided clues that other children needed.

Day 11 was dominated by question and answer exchanges between the teacher and individual students. The children's attention tended to be focused on understanding directions and making certain they were doing things properly. Comments like Theresa's, "So we keep adding fours?" or Peter's, "Can I just draw sticks?" seem to indicate that the students needed confirmation before proceeding. More than 50 student utterances were questions about procedures or answers to the teacher's questions. Significant numbers of pupil responses were limited in content and consisted of only one or two words. This might have been because the teacher was asking questions which did not seem to encourage the children to elaborate. The students made few efforts to explain their thinking to Ms. C or the others. Any explanations provided occurred because of the teacher's questioning. During the explanation phase of round two, the following exchange occurred:

Ms. C - Fred, what did you do?

Fred - I took the cubes. I made four ... five groups with four in them. Then I stuck them together and then I took off eight and then I took four and counted them.

Ms. C - And what was your answer?

Fred - Twenty.

Ms. C - Okay, so you took off eight and eight. Did you add those two together then?

Fred - Yeah.

Ms. C - And how many did you get?

Fred - Sixteen.

Ms. C - And then did you count on?

Fred - Yeah.

DAY 12: (Jenny, Theresa, Fred, Peter, Alex)

The behaviour, actions and tone of voice used by the pupils were in marked contrast to Day 11. The task on Day 12 was to play an independent game of "Giant Strides". This game had been taught to the whole class several days earlier in the classroom. Alex and Theresa were absent the day the game was learned and the classroom groups were different from the laboratory groups. By Day 12, Theresa had learned the game but Alex had not. The objective of "Giant Strides" was to show the number and size of a "giant's" steps on a number track. The strides represented factors in a multiplication equation.

As the five students began working, Alex asked how to play. He received a very brief, somewhat vague explanation from all four other students. He indicated that he understood (despite the confusing directions) and the game proceeded. As the children were playing the first round, it seemed clear that they were going to omit the estimation step emphasized by the teacher. This step had been left out of the group's explanation. They knew that they were supposed to estimate because as the researcher asked, "What's the first thing you need to do", two students said, "Guess". Theresa evidently thought that any guess would suffice so the researcher defined an estimate as a "smart guess, not just any old guess". As Jenny began to place blobs of clay representing the giant's strides (6 x 4), Fred stopped her to point out that she had skipped the 24. Jenny indicated that 24 had been her guess; however, she had not told any other child her estimate.

The second round was 5 x 10. Theresa received prompting from Peter and Fred as she tried to estimate. Fred held up his fingers to show the other boys his estimate and then he whispered, "Five tens ... five tens ... or ten fives". Theresa repeated, "Ten fives?" and the three boys all agreed that the answer was 50. Theresa accepted their estimate. *Fred's comments appear to indicate his increasing awareness of commutativity. Theresa questioned him ("Ten fives?") but he did not elaborate and she did not pursue the matter.* Theresa placed her blob of clay at 50 and then worked backwards by tens. She seemed more concerned about whether the blobs were "too skinny" than about the accuracy of her mathematics.

Throughout Fred's turn, round three, the group's behaviour and comments were off-task. As Fred made a remark, Theresa pointed at the tape recorder on the floor near the group. Peter moved to observe the machine more closely. *The children were willing to view the researcher as a teacher during the game itself but it was obvious that she did not inhibit their behaviour. They were more conscious of the tape recorder than the researcher and/or the video camera.*

Alex took his turn as the others continued to chat. The researcher drew their attention to Alex's method because no-one had been watching him. However, Alex was unable to describe what he had done. There was a pause after which Fred asked, "Did you count backwards?" When Alex said yes, Peter responded, "See, I told you it was minus." The researcher indicated that it was not "minus", to which Theresa replied, "Close enough." Alex had put his clay at 12 and counted backwards. He had ended up with two strides of four and one stride of three. No other child had noticed that Alex's strides were uneven. Theresa asked the researcher whether Alex's work was correct but the researcher told the group that they had to decide for themselves. Fred was the one who noticed the unequal groups and said that Alex could not use them.

The rest of the conversation consisted of a discussion about the fact that Alex's approach was in fact division because he had partitioned the 12 into groups. The researcher labelled Alex's method as division and then asked if it was the same as the multiplication they had been doing. Theresa and Fred both questioned whether the "Giant Strides" was actually multiplication. Then the pupils wanted to figure out more division questions; Peter and Fred both gave accurate examples of division.

The researcher asked Alex if he could do the same question as before using the "Giant Strides" instructions but he was uncertain. Fred, Theresa and the researcher provided some prompts and Alex successfully did three strides of four with a result of twelve. Theresa observed that he still landed in the same place and Jenny remarked, "But he had one more blob."

In general, the students were extremely talkative as others took their turns and it appeared that little attention was being paid to the mathematics. Another factor which had some influence on how closely the group followed the prescribed procedure was the number of comments and questions from the researcher.

About one-third of what was said on Day 12 was the answers to questions or making predictions. Twenty-four of the questions were asked by the researcher. This was in contrast to Day 11 when the teacher posed 42 questions but was still too many interventions for the researcher to have made. The researcher asked mainly procedural questions designed to make pupils more aware of what they were doing. Because four of the five students had played this game before, the researcher was able to focus on the mathematics as well.

The explanation with which Alex was provided at the start was almost entirely procedural. There was little specific mathematical detail given; the students gave instructions like, "And then you count three and you put a blob. ... so you do it once more and you go to nine and then you write it down". No-one explained that the cards were meant to represent the number and length of the "giant's" strides or what the purpose of the game was. It is possible that the purpose was not considered to be important because it was just one in a series of activities and because this was the third time the children had played it.

Theresa and Fred dominated the day's conversation. Fred acted as the guide for most of the work; he prompted and directed Alex and made predictions to which the others reacted. Theresa appeared reluctant to make predictions, preferring to adopt those made by the boys. Theresa snapped at Fred at one point when he made a prediction but Alex defended Fred by saying, "That's 'cause he's so smart." At this point in the day, Theresa was argumentative and impatient. Theresa's behaviour and attitude were silly; she sang and made frequent off-topic comments.

Fred provided more than twice as many directions and prompts as any other group member. He also stayed on-task better than the others. Fred responded most quickly to the number cards being turned over, "played" with the mathematics and indicated some understanding of commutativity. He also realized the error Alex had made in using unequal groups.

Peter's contributions to the group consisted mainly of providing directions about the necessary procedures and answering questions or making predictions. His comments were short, usually limited to two to five words.

Jenny was quiet as usual this day. She was attentive when the others took turns and promptly took her turn but said little.

Alex's contributions were mainly questions and predictions. He indicated that he understood the others' instructions at the beginning but subsequent behaviour showed misunderstanding instead. He was also unable to explain his work without prompting from the researcher and his fellow students. His final turn, putting the clay at 12 and working backward, seems to show only partial comprehension. The other students had not provided very clear models or directions. When he joined the class on Day 10, Alex already appeared to have memorized many of his multiplication facts. He said that his parents had taught him. Based on his lack of clear explanations on Day 12 and observations of his behaviour during group activities, it appeared that his knowledge was primarily instrumental. When he counted backwards from 12 in his last turn, he did not seem aware that his blobs of clay were incorrectly placed. After being asked questions and given hints by the other children, Alex was able to work forwards like the others.

The adult-to-child communication involved the researcher providing reminders about the required procedures (i.e., estimating) and attempting to clarify the strategies students utilized. She was particularly concerned that the other children notice Alex's unusual "backwards" method and that they tell him how it was different from their methods.

Theresa used a "backwards" approach during her 5 x 10 turns; this also elicited no reaction from her peers. The researcher was interested in how the four children who were familiar with the game would clarify the procedures and/or mathematics for Alex. When Fred discovered where Alex's problem lay, he addressed his explanation to the researcher and not to Alex. This focus was consistent with the teacher's role assumed by the researcher, but, in effect, excluded Alex from the conversation: Fred was pointing out the error to the adult, not to the child who had made it.

DAY 13: (Fred, Theresa, Peter, Alex)

Day 13 involved the students working with the intern teacher, Mr. G, on Skemp's "I Predict ... Here" (Num. 5.2/1). They had learned this activity on Day 6; the added component on Day 13 was a written record of their work. A lengthy discussion had occurred among the classroom teacher, the principal, the intern teacher and the researcher regarding how to introduce multiplication notation. Skemp recommends (1989a, pp. 171-172) that teachers and children use whatever notation they are "happiest with until Num. 5.5". Num. 5.5 "Number Sentences" introduces commutativity. Regardless of the form preferred, Skemp suggests that an equation such as 3 x 5 = 15 be interpreted as "5, 3 times, equals 15" (ibid., p. 171). This is more consistent with his "first action, second action" method in which the "first action" is making a set and the "second action is making matching sets. He also acknowledges that 3 x 5 = 15 can be read, as it frequently is in Canadian textbooks, as 3 sets of 5 equals 15. Skemp states that this "reverses the order of the operations, but corresponds well to the diagram below which shows the combined result" (ibid., p. 172).

3(5)

When Ms. C introduced written notation to the pupils, she acknowledged that several notations and interpretations exist but said that the class would read 3 x 5 as Skemp did in his second example. This was, she explained, because of the texts and other teachers which the children *would encounter. However, she made occasional reference during the rest of the unit to the alternatives. Once the students reach Num. 5.5, Skemp introduces computer notation (3 * 5) because it is compatible with commutativity and because students will need to be familiar with the computer representation of multiplication.*

The intern teacher emphasized the "how many groups times how many in each" interpretation in his introduction and then again frequently as the children worked. Each child used his/her own set of materials and number cards so after the initial discussion, the first number of verbal exchanges were predominantly between the intern and individual pupils.

The students organized their work in two ways. Alex and Fred made groups of Unifix cubes and then placed them on the table to determine the product; Theresa and Peter made groups and placed them on the "set" card to confirm the number of groups. Theresa, Fred and Alex chatted as they found their products. Peter spoke only in response to questions from the intern and had one brief exchange with the researcher as she asked him to demonstrate and verbalize his method of counting the groups.

The intern watched the progress of each group member carefully and asked the children to review what they had done. His frequent "closed" queries tended to produce mainly short mechanical responses.

On Day 13, several interesting behaviours happened spontaneously. Shortly after beginning, Alex was required to compute 9 x 6 but he did not have enough cubes. Theresa assisted by offering him some of her stacks of six and he thanked her. This sharing of materials and the thank-you were unusual. Alex struggled to find the product; he was using partial products and counting on but he kept getting lost. He was interrupted twice by the intern. On the first occasion, Alex was adding groups of nine ("I know that's 18. Eighteen plus nine --- twenty-seven plus nine ... thirty-six ... thirty-six plus nine ...). At this point, the intern referred to Alex's previous work (3 x 5) and asked the significance of the factors. He then inquired what would happen if Alex reversed the order of the digits. Theresa then interrupted and Mr. G helped her to do her next question. *Perhaps the intern was attempting to make Alex aware that, even though he had begun with groups of six, and was now counting by nines, he could still achieve the same result. However, after assisting Theresa, Mr. G went on to talk to Peter so Alex* *continued to work alone.* When Alex exclaimed, "Oh, God! Now I've got to start again! Mr. G, I counted them all and now I forgot the answer", the intern's attention returned to Alex. When questioned, Alex indicated that he was now using groups of six to do partial products. This was when another unusual event occurred. Fred suggested that Alex consider two groups of six at a time. He said, "Adding six and six is twelve and then two more is twenty-four and then counting on." He demonstrated his method. The intern then asked, "Is there another 24 there somewhere?" Fred replied yes. Mr. G demonstrated that Alex had two twenty-fours and one six more. Alex was able to easily add these numbers to reach a sum of 54. *This was a rare occurrence of "scaffolding" between children. Even with organizational help from Mr. G, the ideas were Fred's.*

The pupils' attitude was pleasant and cooperative. They asked to share materials instead of just taking them and they spoke politely. They were on-task throughout the 15 minutes and seemed to really be concentrating on the mathematics of the activity. There were examples of "self-talk", especially from Theresa and Alex. As the end of the working time approached, three of the students began to make comments to no-one in particular about their progress.

Aside from asking Fred to tell Alex his method, the intern did not encourage interaction among the children; however, because they were figuring out different problems simultaneously there was no real need for them to share information. Any help offered by one student to another was initiated by the pupils. On most other occasions when the study group worked with a teacher, they were required to explain their own solutions or explain someone else's. On this day, the children worked on parallel rather than cooperative tasks. It was especially interesting to note, therefore, that this was a day on which "scaffolding" and cooperative behaviour happened spontaneously.

DAY 17: (Theresa, Alex, Fred, Peter, Jenny)

The pupils were engaged in two tasks on Day 17. First, they worked independently on replicating "Books of Two" and then they played a Concentration game with the intern teacher. To make "Books of Two", the children stapled together sheets of paper, placed stickers in groups of two and wrote equations. The students were concerned about the paper being different sizes; Theresa and Alex seemed particularly bothered. There was a sample book in the tub that held the materials and the pupils referred to it frequently. *They appeared to understand the assignment: the constant examinations of the sample were evidently to confirm the number of groups or dots needed.* The book was organized with the two times table in sequential order so it was curious that the children were relying on its examples so carefully. They had not yet encountered all of the two times table in sequence but were all able to use the facts easily in their laboratory activities.

Much of the conversation centered around the procedures for assembling the books, on what colours of stickers to use and occasionally on off-task topics like pictures in the newspaper. There was relatively little conversation during the first task. The reproduction of the sample book was a fairly mechanical job. Therefore, the off-task and non-mathematical comments were not unexpected.

One of the most noticeable aspects of the talk during the "Arrays Concentration" game was the students' confusion about the objective. The intern began by introducing arrays as a means of telling "math stories". The children quickly comprehended the idea of arrays displaying two multiplication facts at once. As soon as Mr. G showed the six groups of three array, Jenny said, "Or three groups of six". As the game began, the children were told to look for "matching number sentences". The teacher provided an array card for each child; they were to find three other cards as the teacher turned over cards from the deck which he held. Pupils were also warned that they would have to be prepared to explain how the cards matched. Fred was the only child who seemed to know what to do. As they started the first round, Theresa, Alex and Jenny all expressed uncertainty. Theresa asked, "Which one comes first? Does this number come first or does this number come first?" She indicated the rows and columns of the array. Mr. G asked her to decide for herself to which she responded, "All makes the same number anyways." *This was one of several comments that Theresa made during the unit showing that she had a good grasp of commutativity. However, Theresa appeared to experience the greatest degree of confusion about the Concentration game and how to describe the arrays. She was uncooperative at times, raising* *her voice and trying to take cards out of the intern's hands while he talked to Peter.*

Jenny provided clear explanations of her matching cards despite her initial protestations that she did not comprehend the rules. Theresa had difficulty telling why she thought she had matching cards. When Mr. G asked whether one of her cards was the same as the one she had just described, Theresa stated, "No, but the answer's the same." Fred tried to help by counting five groups of six on Theresa's card but she disagreed, turning the card sideways and pointing out the six groups instead. Although she understood commutativity, Theresa did not seem to see the connection with arrays. Peter, Alex, and Fred could all describe their arrays quite easily.

Despite their initial uncertainty and numerous questions, the pupils were all able to associate the multiplication facts with the arrays and choose cards which showed related facts. Because they were engaged in playing a game in which they sought different sorts of cards, there was no cooperation during the playing phase. Once the children were asked to explain their answers, several examples of prompting or assistance occurred between pupils.

Many of the verbal exchanges were between the intern and individual children rather than involving the whole group.

The intern's questions were generally focused on having the children explain their own answers or confirm someone else's. He also asked pupils to demonstrate how they had found the products. The children's verbal reaction to the new concept of arrays was inconsistent with their choosing the correct cards laid down by the intern. If, as the children claimed, they truly did not understand, they should have been unable to find the matching sets of cards. The researcher found it difficult to reconcile this difference between the verbal and non-verbal actions of the group. The students, except Theresa, were able to match and explain the sets of cards.

DAY 18: (Theresa, Peter, Alex, Jenny, Fred)

On Day 18 the classroom teacher integrated the idea of arrays with the "Little Giant explains why" activity from Structured Activities, Vol 2 . The focus was on the similarities and differences between the trips the Big Giant and the Little Giant made. The number track and blobs of clay from "Big Giant, Little Giant" were replaced by stacks of Unifix cubes to show the giant's strides. *Skemp uses this activity to introduce the term "commutativity" and to shift from physical materials to a paper and pencil* (1989b, pp. 115-117). *His version includes the students drawing the journeys on squared paper after they are comfortable with using the rods of cubes to represent each giant.*

Students were able to easily identify the mathematical similarities and differences between the giants' journeys in each turn. After discussing the trains of cubes, the teacher asked for them to be broken apart and made into rectangles. Theresa described the two rectangles as being the same length and width but neither she nor Fred thought that the rectangles contained the same number of cubes. Fred justified his answer by noting that one rectangle contained groups of five and the other had groups of six. He interpreted this as two different totals. Once one rectangle was stacked atop the other, all of the children agreed that they were equal. Jenny knew that the rectangles were equal because the trains of cubes had been equal. Ms. C summarized the essential aspects of this activity. She pointed out that although five groups of six and six groups of five look different, they "end up at the same place". Theresa was inclined to focus on the differences in the length of the groups and Ms. C needed to reiterate that both rectangles contained the same number even though they did not appear the same. After Theresa and Jenny had been the Big Giant and the Little Giant in the first turn, Ms. C asked Fred and Alex to confirm the girls' work. Jenny had described the Little Giant's trip as "Six strides with five in each stride". Alex exclaimed, "That's backwards." Ms. C corrected him and began to show him the strides taken. Alex then clarified his statement; he had meant that the trip was "backwards from what the Big Giant did."

As Alex was answering questions about the second round, (8 x 2), he described the trains as looking different. Ms. C asked him how but Alex changed his mind. Ms. C told him that he was correct when he thought the trains did not look the same. Alex claimed the difference was, "You can't count them. 'Cause it's all one colour (Big Giant) but here

(Little Giant), you can." Both Theresa and Jenny predicted that the Big Giant's rectangle would be longer. Ms. C requested that Theresa describe the Little Giant's rectangle and provide the equation that would match it. Theresa responded with, "Oh, that's easy. Two times eight or eight times two." Ms. C insisted that Theresa choose only one equation, based on the "how many groups first" rule. As the game was being put away, Theresa complained to Jenny that she was confused. *This probably arose from the emphasis on the rectangle being described by only one equation rather than showing both as it had in the arrays game. Theresa's interpretation appeared to reflect the concept of commutativity; Ms. C's interpretation was in terms of the "first factor names the number of groups" rule.*

Students' attitudes were positive except for the occasional interjection from Theresa. Pupils' certainty about their work wavered several times. As the five groups of six and six groups of five were being reviewed for Theresa's benefit, students were asked to point to the "five times six". Alex, Peter, Fred and Theresa all indicated the same rectangle. Ms. C asked Alex, "This one?" which prompted Alex to withdraw his hand and look uncertain. The teacher chided Alex, saying, "Don't change your mind just because I asked again." Alex's action seemed to demonstrate that he thought he was being corrected or told to try again. The teacher's purpose, though, was to confirm Alex's response.

The communication pattern on Day 18 involved more student interaction than usual on a "teacher-led" day. The children discussed their interpretations of the giants' journeys instead of limiting their comments to conversations with the teacher and virtually ignoring their peers. The teacher's questions facilitated these student-to-student exchanges by asking predominantly open-ended questions. She used strategies such as asking children to compare answers with a partner and then with the whole group in order to encourage the pupils to share their thinking.

Rather than introducing a completely new concept, Ms. C was providing an activity to reinforce the commutative principle which had been used in the other Big Giant, Little Giant games and in the Concentration Arrays. The manner in which she phrased the questions helped to generate mathematical discussions; she assisted the students in drawing connections among concepts. She also reinforced the value of students helping each other and listening to the others' ideas. This served to remind the children of the cooperative social skills which were expected of them.

DAY 21: (Jenny, Fred, Peter, Theresa, Alex)

Day 21's activities took place in the classroom but since the study group was seated together, the data have been included. Except for the physical setting, the day was no different from a laboratory day.

The children in the study group were filling in the answers in duplicated multiplying booklets. The pages used clocks and coins to represent the five times tables. The students did not have any difficulty in completing the questions because they used skip counting to verify the products. Several children commented how easy they found the assignment. They did ask for help as they began new pages; however, their "What do I do here?" inquiries appeared to be almost rhetorical. The first time Alex asked this question, Fred volunteered an explanation. When the teacher arrived to check the group's progress, Theresa and Fred asked whether Ms. C wanted to mark their booklets. She responded by asking Fred whether he knew that his products were correct and then praised him when he said he was sure of his answer. This served to reinforce his self-reliance. A moment later, Fred inquired how to do the page of clocks and the teacher gave him some hints about calculating 9 x 5. When Alex had a question about the same page, Ms. C referred him to Fred for assistance. Alex's observations demonstrated good mathematical connections. He reasoned that since he knew 7 x 5 = 35, all he had to do was add two more fives to find the product of 9 x 5.

Fred was the only one who prompted a fellow student on Day 21. Alex was the sole person who explained his mathematical thinking spontaneously. Jenny and Peter were present but did not contribute anything to this short conversation.

Filling the equations in the duplicated multiplication booklet helped to reinforce the skills of the basic facts but did not require that the children cooperate because each child had his/her own copy. Two *of the students did provide assistance to each other, answering "What do I do here?" type questions and providing hints about using known facts to figure out the unknown ones.*

Communication with the teacher was quite limited and consisted of a few procedural matters (i.e., the order in which the pages had to be completed) and some mathematical advice as the children worked out the products.

GROUP INTERVIEW

The study group was interviewed in the "quiet room" of the mathematics laboratory the week following Day 21. In the interval between these two days, the students had worked on individual games to reinforce their basic facts or in their classroom groups. There was no need to videotape these activities since doing so would not add to the study data.

Students were asked whether they liked the group arrangement of the mathematics laboratory and why. They were generally positive about the group work because they could obtain and provide help to figure out their problems; have company while they worked; listen to explanations: and "learn a little bit faster so you don't have to sit there all day trying to get one or two [questions] done". The pupils disliked the group work some days because other people talked too much or gave answers instead of helping. Jenny commented that she found the group activities difficult because sometimes she could not find the words to explain.

The children were quite negative about having to provide explanations to other group members but accepted that explaining was necessary. They felt that they could help and be helped through talking. Fred identified the need to be prepared for the future. The children disliked providing explanations because they did not know how or became nervous.

If they were having difficulty, four of the five students said they would ask another child rather than a teacher. Their reasons included the fact that the teacher would just tell you to ask another child. Theresa said, "And the teacher would say, 'Ask someone in the group, they're just as good as a teacher'." Jenny felt that the teachers encouraged children's interaction and wanted the children in the groups to get along with each other. *The tone of the comments appeared to indicate that, despite Ms. C's efforts to convince them otherwise, these students did not feel that the children would be "just as good as a teacher".* Fred preferred to ask teachers for help because he was certain that they would know the expected answer.

Consistent with their answers to the previous topic, the students felt that they would receive a more understandable answer by asking a peer. They identified a common vocabulary, simpler examples and being able to understand the children better as reasons to ask another student.

The next focus of the interview was on the laboratory games versus using a textbook. Three of the five students would have preferred to learn multiplication from a textbook. Theresa first said that she liked the laboratory, then changed her mind. Fred chose the laboratory because the activities were fun and he felt that he would learn just as much as from a book. The children who wanted to use a text did so because it would prepare them for future mathematics classes. Jenny said that a text would help "so you get used to the questions. How they look when you grow up and do math, you would only be knowing the game. You won't be knowing how the math looks."

Many of the attitudes about the mathematics laboratory and the use of texts were unexpected given to the activity-based, cooperative learning and communication emphases of the school and particularly Ms. C. These attitudes appear to be contradicted, however, by some of the answers these students gave on the questionnaire.

QUESTIONNAIRE

After the multiplication unit was completed, an individual written questionnaire was administered. The researcher read the questions aloud one at a time to the whole class and children were free to ask for clarification. Questions dealt with using a textbook versus an activity-based approach, the need for manipulatives, feelings about their abilities in mathematics, their perceptions of the teacher's attitudes, seeking help and writing in the mathematics journals.

CHAPTER V

ANALYSIS OF THE RESEARCH QUESTIONS

COMMUNICATION

BETWEEN ADULTS AND CHILDREN

The study examined the roles assumed by participants, who assumed them and what influence the roles had on the communication patterns. Skemp provides little guidance regarding the roles of teachers and children or how to facilitate the type of discussion which will most benefit the pupils. He appears to feel that the materials and directions for playing the games will provide the appropriate degree of guidance. On page 9 of Structured Activities (vols. 1 and 2) Skemp states:

> While with a group, it is very difficult, if one is a teacher, not to keep actively teaching when it might be better to wait for children to do their own thinking. The kind of teaching involved here includes ways of managing children's learning experiences which are less direct, and more sophisticated than more traditional approaches. They are also more powerful.

All three adults working with the study group made the same error; they frequently asked "closed" questions rather than encouraging or allowing students to struggle through constructing their own explanations. As children seemed uncertain, the teachers tended to ask a structured series of "Well, didn't you do ...?" sorts of queries so that the child's contribution was reduced to one or two words at a time. The pattern of "closed" questions was greater at the start of the unit when pupils were first becoming familiar with the concepts. As the unit progressed, the type of questions asked by the teacher and intern altered to include more open-ended queries. These later questions sought to consolidate facts learned earlier (equal groups, the arrangement of factors, commutativity, various ways to represent multiplication such as arrays). These more open-ended inquiries elicited lengthier and more complex answers from the pupils. The teachers asked students to make connections among the tasks or concepts. On several occasions, children identified tnese connections without adult prompting.

The researcher found it challenging to remain an observer and not become involved in teaching. She was not always successful as can be seen from the transcript of Day 7 (Appendix A). The researcher's journal from Days 2 and 3 reflected her frustration and struggle to remain uninvolved in the problems experienced by Theresa, Fred and Peter. Day 3's journal entry read:

> Very difficult not to become involved in helping group with understanding the task and also had to really resist the temptation to try to improve their listening and explanations. Peter gave an interesting explanation of how he figured out 4 x 8 = 32 but Theresa and Fred were not attending.

The researcher was in the unique position of being an observer and also a peripheral member of the group. Prior to the commencement of the study, the researcher had assisted groups in the mathematics laboratory in the same manner as the teacher, intern teacher and the classroom aide who was occasionally present. At that time the students appeared to view her as just another teacher. At the beginning of the multiplication unit the children had little interaction with the researcher: they were generally able to ignore the video camera and its operator. On Day 4, though, when Theresa became particularly frustrated with Fred, she acknowledged the researcher when she complained that Fred refused to listen to her suggestions. By Day 7 when the group played Giant Strides in the "quiet room", the researcher was unable to stay completely separate from the group's work. She asked whether the pupils had made predictions and whether those predictions were accurate. When the game was taught to the class, there was an expectation that they would estimate the answers first but on Day 7 the children seemed to be ignoring that requirement. The room in which this activity took place was very small and the researcher was in closer proximity to the children than was usual. There was also little likelihood that the classroom teacher or the intern would come in to check on the group. It seemed natural for the researcher to step in and remind them of the procedures. The researcher was aware that this interference was contrary to her intentions; on later days she attempted to restrict her interaction with the group to asking for clarification of individuals' strategies. These requests usually took place in private conversations as the rest of the group worked and this procedure was consistent with the design of the study. Another reason for the researcher assuming the teaching role resulted from the classroom teacher's stated expectation that the researcher would provide "another pair of eyes and a helping hand" (private conversation, December, 1990). It was agreed that, if the classroom teacher or intern were unavailable, the researcher could assume the teacher's role temporarily.

Early in the unit Ms. C emphasized the necessity of explaining clearly and listening carefully because an adult could not always be with the groups. On Day 12 the researcher again became the teacher. The children had a curious view of the researcher on this day. They accepted her questions as if she were the teacher but also were able to ignore her sufficiently to engage in considerable off-task behaviour. Their singing and making silly comments seemed to indicate that they accepted the researcher as another group member, not as an adult whose presence should inhibit their behaviour. Their attitude appeared quite different from those days when they were working with the teacher or intern.

Among the problems on Day 4 were those caused by the intern's method of interacting with the group. As the three pupils were attempting to reach a concensus about the product of 4 x 9, the intern arrived to check their progress. He interrupted Peter's description by asking a series of three questions in a row. Peter started to respond to the first question but was cut off. It was clear that the children were struggling so the intern attempted to introduce a new means of using partial sums. Almost immediately after this, he suggested counting the train of Unifix cubes to confirm an answer. This appeared to contradict his earlier suggestion. The next time Mr. G came to listen to the group's deliberations, Theresa and Fred were arguing about Theresa's description of Peter's method. He inquired about Theresa's "problem". When she told him, the intern dismissed the incident.

The intern's interjections tended to interrupt the flow of the communication. Had he waited and listened when he first arrived at the table, he might have learned what was happening without questioning the students. This might also have altered the students' tendency to appeal to the adult to judge the accuracy of a pupil's ideas and reinforced the need to communicate effectively within the group.

The poor communication and uncooperative atmosphere on Day 4 arose from lack of effective listening by the intern teacher and children. Various strategies for finding products were being discussed because the children were investigating which method best suited them. Had they been allowed to continue their conversation, Fred and Theresa might have reached an understanding about their different views of what had happened during the 4 x 9 round. As was obvious from the questions that Fred and the others put to the teachers and from the accompanying tone of voice, the students still regarded the adults as the final authority. If a dispute arose, pupils would often appeal to an adult to resolve it: they evidently did not believe in trying to reach a group consensus instead.

On several occasions after the students had played a game, the teacher, Ms. C, made summary statements about multiplication. She drew parallels between activities, pointed out some of the similarities, and highlighted the important aspects. Skemp suggests (1989c, pp. 165-166) that asking the children put a new concept into their own words will help them to develop a clearer formulation than merely listening to the teacher. The children did note some similarities among the activities. For example, on Day 8 Theresa compared "I Predict - Here" and what she called "the blob game" ("Giant Strides") and on Day 18 Fred related the Unifix cubes arranged in rectangles to the arrays game played with Mr. G. Neither teacher requested that students explain how the activities were related or asked students to verbalize their emerging understanding of multiplication. To do so would have been entirely consistent with a constructivist approach.

The roles and expectations of the children and adults varied throughout the unit. When students worked with the two teachers, they often changed the group dynamics from interaction among themselves to a one-to-one relationship with the teacher.

AMONG THE CHILDREN

The communication patterns among the children were different from those on days when they worked with the teachers. On the days when the group worked along or with limited adult supervision, the remarks they made tended to be longer and often more detailed than on days when an adult was present.

Roles

One or more of the students would assume a directive role, either telling the others what to do or offering hints to establish the correct interpretation. This role might be taken by one child throughout a day's work or the role might be assumed by several children in turn. Fred and Theresa tended to assume the leader's role more often than the others. These directions from one child to another focused on procedures more often than mathematics. Theresa's manner was frequently abrupt as she gave her directions; on Day 4 she was upset so her manner was particularly impatient. The other four children seemed able to offer directions in a gentler manner.

The pupils also offered verbal and non-verbal prompts throughout the group work. These were given in a helpful way rather than with a superior attitude. The prompts were often made to assist another child to find a product or make an estimate. There was usually a difference in the tone of voice which distinguished a directive from a prompt. The non-verbal prompts took forms such as Fred holding up a number of fingers to show Alex and Peter how many groups or Jenny placing blobs of clay to keep track of Peter's "strides". These non-verbal assists were, like the verbal variety, usually gratefully received. Theresa's rejection of Fred's help with counting the Unifix "train" on Day 4 was a rare occurrence.

Cooperative Behaviours

The degree of cooperation among the students varied. On some days there was evidence of children asking for and offering to share materials and helping others to find solutions to the mathematical aspects of the work. Fred's "scaffolding" behaviour on Day 13 and Alex and Theresa's sharing materials on the same day were the most striking examples of this cooperative attitude. Some students from the group cooperated well with each other but not effectively with the rest of the group. Theresa offered numerous mathematical hints and strategies; she was often not listened to by the other pupils. There were indications that the children heard and adopted some of the ideas suggested. Her strategy of partial sums was used by all the group members; the others might also have suggested this method had she not mentioned it first. They all appeared quite comfortable with the strategy. On most of the independent working days the group cooperated relatively well on the mechanics of the activity but less effectively on the mathematical concepts.

Students' Views

In the group interview (see Appendix C) students generally agreed that they could request and receive help from their groupmates and that an explanation from another student was often preferable to an adult's explanation. This positive assessment of the merits of group work appears to be contradicted on numerous days by the attitude of the study group. Perhaps this point should have been clarified at the time of the interview; the researcher's view of the group interaction was not as rosy as the students'. There is evidence of students helping each other, as noted above, but there were also numerous conflicts over relatively unimportant things such as whose turn was next and who was to have a specific colour of Unifix cubes or stickers.

Off-task Behaviours and Conversations

Some of the disputes related to the assigned tasks while others arose as the children were off-task. The degree of off-task behaviour which occurred often seemed related to either the students' familiarity with the game or the amount of time they spent waiting for their turns. Days 1 and 2 contained almost as much off-task as on-task behaviour. As previously mentioned, the classroom teacher indicated to the researcher that she felt that Day 2's misbehaviour was caused by too much similarity to Day 1. On both days, the study group was expected to listen to other groups' descriptions; rather than attending carefully, they chose to play with their materials. Part of this, according to the teacher, might have been because they were not usually expected to work as a total class in the mathematics laboratory. This, plus the attractiveness of the materials, may have caused the lack of attention which occurred. Day 12 also contained many inappropriate behaviours and comments. The students were meant to be playing "Giant Strides"; this was the day when they were instructing Alex. The group seemed rushed and restless as they gave Alex a sketchy account of the rules. They were talkative and inattentive throughout the day's work; this might be explained by the lack of direct teacher supervision. Since the researcher was present in the "quiet room" where the group was playing, neither teacher checked on whether the pupils were doing what they had been assigned.

On-task Behaviours and Conversations

The length and content of the pupils' on-task comments seemed to be related to their familiarity with the task and its underlying concepts. One noticeable exception to this generalization happened on Day 4. Mr. G and Ms. C occasionally watched the pupils and offered guidance but for the most part, the students played the multiplication game unsupervised. Even when they were alone there was a concentrated focus on learning the concepts and playing the game. Unlike Days 1 and 2, students were working independently and were not distracted by the materials or their neighbours. Despite the conflicts of the day's activity, all three children stayed focused on the task.

Familiarity with the Activity

Being familiar with the routine of a game allowed the pupils to discuss the mathematics more easily. On the days when the group replayed a game, there was a greater likelihood that they would focus on its mathematical aspects. This pattern was not true for all the work; the students also became entangled in procedural disputes about the games.

The practical activities provided a focus for the mathematics and the communication; they supplied the structure and the content of the discussions. The children did discuss the mathematical concepts such as commutativity, the need for equal groups and combining groups, but their focus frequently appeared to be more of the "how" than the "what" of the tasks.

Temporal Considerations

The degree of verbal and physical interaction among the pupils was determined in part by the amount of time allotted to any one activity. The mathematics laboratory time was divided into two fifteen-minute blocks per day. There were always two tasks to accomplish in that time. One complaint voiced by the children during "clean-up" on several days was that they would only have begun a game and would be expected to move on to another assignment. They found it frustrating that they would often not have enough time to complete their games. There was at least one day, though, on which Peter sighed with relief when the working time was over and he said that he had thought the game would never end.

Synthesizing and Explaining

Encouraging children to provide "rich", complete explanations of their thinking is not easy. This is more likely to happen in a situation where the students are familiar with the task and the required mathematics than when they are learning something new. On days when pupils worked without constant adult supervision there were more examples of prompts and of students assisting each other. However, the focus was not always on the concepts. Students' patterns of speech and verbal and non-verbal interaction differed on these days as well.

The pupils were often not yet able to synthesize and comment on their learning. Their knowledge was not yet sufficiently well-formed or sophisticated enough to allow them to "think about their thinking". Skemp recommended (private conversation, May, 1991) that children should not be asked to explain their developing knowledge too soon. To do so runs the risk of confusing the children and slowing their progress toward relational understanding of the concepts.

ASSUMPTIONS UNDERLYING THE MATHEMATICS PROGRAMME

The multiplication unit in this study was primarily based on Professor Skemp's Structured Activities. Therefore, one might reasonably assume that the communication and mathematics should occur as he suggested. This assumption, see below, requires testing.

Skemp bases his practical activities for learning mathematics on a series of assumptions about the learning and teaching of "real mathematics" (1989a, p. 2). His assumptions include:

1) New concepts should usually be introduced through a teacher-led discussion;
2) Children can play the games together without direct adult supervision once they have learned the procedures;
3) The structure of the games will encourage discussion;
4) The discussions will be mathematical because the rules and strategies are largely mathematical;
5) Children will question each other's moves and justify their own moves;
6) These discussions will consolidate the students' comprehension;
7) Explaining an idea to someone else is an effective means of

improving one's own understanding; and

8) Children can benefit just as much as adults can from the opportunity to discuss and explain their ideas (ibid., p. 1; Paraphrased by the researcher; for Skemp's original wording see Appendix D).

Most of his assumptions were accurate but the study group did not always conform to the teacher's or researcher's expectations resulting from Skemp's work.

Assumption 1

Skemp explains that his activities

> fall into two main groups: those which introduce new concepts and those which consolidate these and provide variety of applications. Activities in the first group always need to be introduced by a teacher, to ensure that the right concepts are learnt. Once they have understood the concepts, children can go on to do the second kind together with relatively little supervision (ibid., p. 9).

Ms. C and Mr. G introduced the new aspects of the multiplication unit either to the individual groups or to the whole class. When new material was taught to the entire class at once, the practice was usually followed by the chance to share a description of the group's results. Ms. C's questions were intended to help the children discern the important mathematical ideas and the common parts of their work. She asked for details about the number and size of the sets used and the result when the sets were combined. She was careful to follow Skemp's "First Action, Second Action, Result" model (ibid., pp. 165-166) from the start of the unit. As the students took their turns describing their actions to their groups or the class, they followed the same pattern.

The teachers were more successful in holding the children's attention when the concepts were taught to smaller groupings of students. During a whole class discussion on Day 2, the pupils in the study group created telescopes, trains and guns with their Unifix cubes while they were meant to be listening to other groups' explanations. Even though they could all tell the teacher how they had put the groups of cubes together and what the result was, the conversation within the group was largely off-task. The only mathematical comments were made when the spokesperson for the group described their work to the teacher. The fact that they could explain the mathematics indicated that they did understand the task but the off-topic discussion appears to contradict Skemp's assertion that the conversation will be about the mathematics.

Skemp's first assumption held true in large part because, as has been shown through many years of pupil-teacher exposure, a teacher-led introduction of a new topic or concept in mathematics is an example of "scaffolding". The more experienced person, in this case the teacher, is able to provide information about the salient points and provides a framework into which the less experienced child may begin to fit his/her emerging knowledge. Some students are capable of identifying and using relevant information with little or no guidance but the result may not be as efficient or effective as having a "live" person to help remove any small but stubborn elements of confusion in the student's mind.

Teachers are usually very good, as they were in this study of Skemp and communication, in moving from the concrete to the abstract. The movement in this case was aided by the children's familiarity with the manipulative materials and routine of the mathematics laboratory. Their reasoning skills were not as good as their oral-aural skills. Thus the teachers' exposition and direct questioning were a great assistance to students' reasoning as they moved from the familiar to the unfamiliar and from the simple notion to the complex notion.

Assumption 2

Skemp's second assumption was true for this group. Once they had learned the games from one of the teachers, they were able to cope on their own.

Assumptions 3 and 4

Theresa's use of the partial sums strategy and Fred's subsequent use of the same strategy appear to demonstrate Skemp's third assumption and, in part, supports the fourth assumption. In general, his third and fourth assumptions were not as valid for these students. This is evident from the poor listening, lack of effective communication and adversarial atmosphere in the study group on Day 4. On this day, the children were quite capable of following the procedures and there was constant discussion about the mathematics; however, the atmosphere in the group was not a productive one. Fred and Peter assisted each other as they tried to find the product of 4 x 9 but their work excluded Theresa. There were sincere attempts made, mostly by Theresa, to institute a mathematical discussion but she was unsuccessful. At the beginning, Theresa assumed the role of leader, trying to persuade the boys to follow the prescribed routine. She also shared her strategy for making a ten-rod into a nine-rod. This technique, like her partial sums method, was an example of what Skemp terms "an elegant solution" (1989c, p. 79). She made two attempts to make Fred listen but he and Peter were involved in the next step and ignored her. Theresa was repeatedly rebuffed as she tried to introduce her mathematical ideas to assist in calculating the products. Fred and Peter did engage in a discussion as they worked and they talked aloud much of the time so the others in the group could hear their thinking; however, there was little cooperation among all three members of the group.

In the early part of the unit on multiplication, when the students played some of the games independently, the conversation did not always extend beyond the mechanical repetition of the steps. On Day 4, Peter began his explaining by saying, "I put down first action ..." He was interrupted by Ms. C who requested that he provide a more exact description.

By Day 7, three of the students had learned "Giant Strides" (1989a, pp. 165 and 167) from the teacher and the group was asked to play it independently. Their conversation and interaction seem contrary to Skemp's conviction that children will naturally engage in meaningful discussions. Theresa, who was just learning the game, appeared to focus on the mathematics while the others were concerned about the rules. She made comments like, "Oh, so every ten is a stride", by which she demonstrated her growing grasp of the content and the procedures.

Assumptions 5 and 6

Assumption 5 is most relevant to Day 12's work; the children did question each other and justify their own. The mathematical focus of the discussion occurred partly as interaction among the pupils and partly because of the researcher's prompts.

Theresa's remarks on Day 7 offer some proof of the sixth assumption. By talking about what was going on in the game, Theresa seemed to develop her understanding of the activity and the mathematics involved. The children did sometimes question each other's moves or strategies and justify their own assumptions but the focus was mainly on whether the rules had been followed.

Day 18's work is best reflected in Assumptions 6 and 7.

Assumptions 7 and 8

While the last two assumptions seem logical, it is difficult to see Assumption 8 demonstrated in the group's interactions. When the children explained an idea to the others, their understanding did not seem to alter visibly. If any changes occurred in their level or type of comprehension, the evidence is hard to pinpoint exactly.

Skemp's Assumption 8 (Children can benefit just as much as adults from the opportunity to discuss and explain their ideas) is irrelevant to this study. In the first place, the study did not seek to compare students and adults. In the second, the only adults present were teachers and the researcher and there are no data recorded concerning their degree of learning from each other as they discussed the pedagogy included in the multiplication unit. And last, it would not be valid to test adults' learning about general pedagogy against children's learning about specific mathematical concepts.

Of all of Skemp's assumptions, the two which least fit the study group are that the structure of the games will stimulate discussions and that these discussions will be mostly mathematical. Once the proper concepts have been learned, students ought to be able to consolidate their knowledge by re-playing the games. In this study the mathematical conversations which did occur tended to be rather limited in scope. For the most part they included details about using one of the three common strategies (partial sums or products, counting on, or counting all). Peter used an additional strategy of counting the groups on his fingers. The other pupils apparently were aware of Peter's method through prior exposure; therefore, he explained the procedure to the researcher but not to his fellow students. The group's conversations were more often about the procedures for doing the activity than strictly mathematical topics. The groups in the laboratory were rarely left unsupervised so it is difficult to speculate on the direction a discussion might have taken if the children were left completely on their own.

The multiplication activities in this unit were designed to function as mode 1 and mode 2 schema builders (1989c, p. 74). Mode 1 focuses on structured practical activities which provide a foundastion for further, more abstract learning. One advantage of mode 1 is that children may experience mathematics in ways that allow them to formulate and test ideas without relying on an adult's authority. They are also able to have a greater degree of control of their own learning processes than in a situation where they are dependent on a teacher to tell them whether they are correct. The children make predictions and through testing these, they are able to determine the validity of their thinking. Mode 2 focuses on communication in which students explain and compare their understanding. In order to do this, children must put their thoughts into words so that they can discuss whether a move in a game is mathematically allowable, and why. Skemp contends that by arguing about the correctness of a move, the children will criticize each other's thinking in "a less threatening manner" than the teacher would (1989c, p. 76. He states,

> Trying to justify, or disagree with, a move on mathematical grounds means explaining oneself clearly, and this requires on to get these ideas clear in one's own mind. Simply speaking one's thoughts aloud takes one a step in that direction (ibid., p. 76)

By completing these mode 1 and 2 activities, Skemp anticipates that pupils will be able to engage in mode 3 level thinking which involves using previous knowledge to create new ideas and connections. This is when mathematics becomes creative.

ORGANIZING FOR LEARNING

A dilemma arises in attempting to find a better way to arrange the learning atmosphere and experiences so that a truly constructivist approach occurs and relational learning takes place through all three of Skemp's modes. Issues which must be addressed include the materials, activities, content of the mathematics, control, roles played by teachers and pupils, differing perspectives about those roles and the availability of time and resources.

Skemp addresses the issue of organization:

> Consideration also needs to be given to classroom management. I am assuming that your children are already sitting in small groups, and not in rows of desks facing the front. Even so, they may be more used to working individually than co-operatively, and if this is the case then their social learning will also need to be considered. Such things as listening to each other, taking turns, discussing sensibly and giving reasons rather than just arguing, which we may take for granted, may need to be learnt. The ways of learning mathematics which are embodied in this scheme both depend on, and also contribute to, social learning and clear speech (1989a, pp. 8-9).

The children in the study were, as suggested by Skemp, used to working in cooperative groups in the mathematics laboratory but they were not consistent in listening, discussing sensibly or giving reasons instead of arguing. Their ability to conform to Skemp's and the teacher's expectations varied according to the task, the materials, the children's attitudes and whether an adult was present. The students had been taught the five essential skills of the Johnson, Johnson and Holubec cooperative learning model but neither this training nor the structure of the Skemp programme seemed to keep them consistently on-task.

On some days the physical proximity of sitting around a table assisted the children to stay on-task because they could all see the materials easily and hear each other's comments and strategies. They could also share the manipulatives so that they could demonstrate to reinforce their ideas.

The tasks and manipulative materials also had a varied effect on the on-task behaviour and the conversation. The early activities, especially those played in groups and discussed with the whole class, did not appear to interest the children very much. Pupils focused instead on using the materials as toys. On Day 4, despite their argumentative attitudes and lack of group togetherness, the students did discuss some strategies and share the needed materials. Activities later in the unit appeared to engage the children more; their vocabulary and understanding of the concepts aided them in discussing the activities.

When working without an adult, control of the group usually rested with Fred and Theresa. Fred tended to prompt the others to keep them on-task; Theresa's tone of voice and manners were often less tolerant than Fred's. The other three children also prompted the group's work occasionally. Their manner was more like Fred's; they rarely raised their voices or betrayed any impatience.

The pupils' attitudes were only partly the sort that were hoped for in the study. They did display some of the characteristics of helping others, asking for assistance when necessary, and thinking for oneself. They had some difficulty with showing acceptance for others' ideas and did not appear to do much self-evaluations.

STUDENT ATTITUDES

The opinions of the children in the study group were fairly representative of the whole class in most areas on the questionnaire. These five pupils were, however, more positive about their own mathematical abilities, about writing in mathematics, about asking others for help and about conversation as an aid to learning. In response to the statements, "I would like to learn all of my math from a textbook" and "I could learn more from a math textbook than I do in the math lab",the study group indicated that they would prefer working in the mathematics laboratory.

DISCUSSION

Developing Communication Skills

Worthwhile communication in mathematics requires that students have activities which pique their curiousity and provide material worth discussing. If children perceive that explaining the mechanics of an activity or answering a limited range of questions is all that is expected, then that is all that they will produce. The conversation is much "richer" when pupils justify their answers in order to help another child comprehend instead of talking their way through the prescribed steps. Some students may be able to explain easily but others find themselves at a loss for the right words. Teaching children how to explain or how to ask the questions so that they will receive the needed information is a vital component of a communicative classroom. If children do not understand an activity or explanation, they need to be able to isolate what parts they do or do not comprehend. This requires a somewhat more precise vocabulary than "You put that thing here" or "You just count three and put a blob" types of comments. Caution must be exercised, though, not to require such exact explanations that children become tongue-tied.

The challenge lies in developing enough communication skills to help the students explore the materials and the mathematics concepts and language without making them feel that there is a "right" way to do everything. The attitude that there is only one way to do mathematics is precisely what documents like the Standards seek to counteract.

There are benefits to be realized by helping pupils develop metacognitive skills. Reys, et al., suggest (1989, p. 39) that "the development of metacognition requires children to observe what they know and what they do, and to reflect on what they observe".

Teachers and parents can begin by asking questions like, "How do you know?" and then encourage children to ask themselves, "How should I do this? Would brainstorming help me to get started on this problem? Would a drawing help? Would it help me to talk about this problem? Is this similar to anything else I have done? How is this the same or different? What is the easiest/hardest part of this for me"?

Teachers' Verbal Behaviours

Any teacher working with students on these or any foundational mathematics must be cautious about the comments he/she makes and the way that questions are phrased. It may be appropriate to ask more focused or limited sorts of questions as a new concept is introduced; these must emphasize the important mathematics and not be limited to the procedures. To focus on "how do you play this?" may leave children with the erroneous idea that the steps are the most important part of an activity, and not the underlying concepts.

Teachers also need to avoid carrying on conversations with one individual at a time while working with a group. Obviously, it is sometimes necessary to address a comment or question to a specific child. The students will benefit from being encouraged to assume the "teacher's role" and ask each other questions and justify their own work. This is more likely to occur if the discussions include all of the students at once and the teacher "turns the tables" on children who frequently seek reassurance. If the other students are asked what they think or encouraged to debate whether answers are acceptable, they will be less likely to treat the teacher as an absolute authority.

Values

Teachers' questions reflect their attitudes about how and what students learn and about the teachers' own feelings of competence and willingness to explore mathematics. If teachers experienced instrumental intruction in their lives as students, they may view mathematics as threatening and limited. Their focus as mathematics instructors may be on the "correct" way to approach a problem or the "one right answer"; the textbook may become a lifeline instead of a guideline. Alternatively, educators who were taught by or who have discovered relational methods may be more inclined to work towards helping students understand the concepts instead of just developing the skills.

The degree to which teachers' inquiries and comments accept the child's ideas as important also conveys a message; if the questions are ones to which the adult obviously knows the answer and simply seeks confirmation from the child, then that child has no incentive. The game becomes one of guessing exactly what the teacher is aiming at and there is no invitation for the child to "play" with or investigate the mathematics.

Whatever the physical arrangements of the mathematics classroom, the atmosphere must be an open and trusting one. Students need to feel that they can request assistance from their peers and the help received is as valid as an adult's. Children need to see errors as learning experiences, not reasons for feeling ashamed. Teachers and students can all benefit from their interactions.

Thinking and the Skemp Materials

Because the Skemp materials are organized into small, sequential steps designed to build multiplication knowledge gradually, the mathematics is predetermined and so, to some extent, is the possible degree of student involvement. The tasks did not encourage these children to use higher level thinking skills, to speculate or "play" with the mathematics or to use metacognitive skills to analyze and explain their thinking to themselves or other group members. The limited role which students were apparently allowed by the tasks might have fostered the students' over developed concern about following the procedures, and not on learning the ideas. The mathematics might have seemed secondary to the rules. Many of the children in this Grade Three class had attended this school since Grade One. The mathematics philosophy of the school and the structure of the mathematics laboratory were based mainly on the work of Richard Skemp. Although it is impossible to state for certain, it seems fair to assume that the children had been taught with an emphasis on relational understanding. This ought to have included aspects of effective communication and cooperation in order to facilitate learning. Those traits appeared to be missing from much of the study group's work.

CHAPTER VI

SUMMARY

PURPOSE

The study concerned a group of Grade Three students engaged in an introductory multiplication unit. The research focused on the cooperative and communicative skills of the children as they worked in the school's mathematics laboratory.

RESEARCH QUESTIONS

The research questions dealt with three complementary aspects of the classroom structure and the mathematics programme.

1) What communication patterns appear in this classroom:
   - between adults and children?
   - among the children?
2) How are the assumptions underlying the Skemp programme reflected in the communication patterns?
3) How do the forms of organizing for learning affect the communication patterns?

RESEARCH DESIGN

Five students were chosen from among the twenty in the class. These five pupils were selected for their varying competence in mathematical, verbal and social skills. An attempt was made to identify and choose pupils who represented high, average and lower levels in each cf these skills. The Grade Three class included a number of English as a Second Language students; these children were not considered for inclusion when the study group was chosen. The classroom was unusual because of the shared teaching responsibilities. In addition to the Grade Three teacher, an intern teacher also taught some of the mathematics. During this unit, the classroom teacher did all of the whole-class instruction; she and the intern both worked with the mathematics laboratory groups on introduction and consolidation activities.

While working in the mathematics laboratory, the Grade Threes were arranged into five groups seated at small tables. There were five tasks to be completed each week; students were allowed approximately 15 minutes per task over a three day span. The children were instructed in the laboratory on Tuesdays, Wednesdays and Thursdays; the Monday and Friday lessons took place in the classroom.

To assist with their work in the mathematics laboratory and in the classroom, the pupils had received cooperative learning group training. They were expected to be able to share materials, listen effectively, speak courteously, take turns and provide each other with any necessary or requested help to accomplish the learning. Some of the concepts were introduced through whole-class lessons and some by small groups working with one of the teachers. After they were familiar with an activity, students were usually asked to play it independently. Their cooperative learning training was expected to help make their independent working time productive.

The children were taught the prescribed multiplication skills and concepts from the Alberta Progam of Studies through concrete, sequenced activities in the school's mathematics laboratory. Most of these tasks were from R. Skemp's Structured Activities for Primary Mathematics: how to enjoy real mathematics. The classroom teacher devised some variations on the Skemp activities and also included ideas from other sources. These additional materials also reflected the constructivist philosophy of the teacher and the school.

Data in the study were gathered through:

- the teachers' and researcher's observations in the classroom and the mathematics laboratory;
- videotapes and audiotapes of all the lessons and groupwork;
- the researcher's journal;
- children's journals;
- videotaped interviews with the study group together and individually; and
- a questionnaire.

Information from the journals was not used for the final analysis because more than sufficient detail could be found from the other sources.

Data were analyzed for information about the patterns of communication between adults and children and among the children, the cooperative skills expected and the assumptions upon which Skemp's programme is based.

CONCLUSIONS

This study began with three Research Questions about the cooperative and communicative interaction in the study groups.

Question 1. What communication patterns appear in this classroom:
- between adults and children?

The initial role assumed by the adults in the study was highly directive; they asked structured, closed questions to which the students made short responses. The students made few attempts to elaborate or to outline their strategies. As the unit continued, the teachers' questions became more open-ended and encouraged students to provide lengthier and more detailed answers. When the group worked with the teachers, the children spoke more frequently to the teacher than to other students unless the adult specifically requested that the children discuss and compare answers. This tendency, along with the manner in which students posed their questions to the teachers, demonstrated that the students looked on the adults as the authority on the mathematics.

- among the children?

The communication patterns among the children varied according to their knowledge about the mathematics and activity. When they worked independent of an adult's supervision, they took turns assuming the leader's role and provided verbal and non-verbal prompts to assist each other. Much of their conversations focused on the procedures although they did discuss strategies for finding products and other related mathematics.

Question 2. How are the assumptions underlying the Skemp programme reflected in the communication patterns?

Assumption 1 was confirmed in this study because of the way the classroom teacher structured the multiplication unit. Assumption 2 was valid; the children could play the games independently. Assumptions 5 and 6 were partially supported. Assumptions 3 and 4 were not entirely valid for these students; discussions did take place during the activities but were not necessarily mathematical.

Question 3. How do the forms of organizing for learning affect the communication patterns?

The tasks and the physical arrangement of the mathematics laboratory were deliberately structured to facilitate and encourage discussions among students. These discussions ought to have, but did not always, follow the procedures taught through the cooperative learning model and expected in the ideal mathematics laboratory. Children did not always take turns, share materials and provide help to each other.

RECOMMENDATIONS FOR FURTHER RESEARCH

Although this study was limited to one grade level and mathematical topic, its results suggest several areas for possible research.

It appears that Grade Three students can use discussions with their peers to assist their mathematical understanding. A suggestion for further research would be to observe students at other levels, engaged in introduction or consolidation activities in order to determine whether communication affects their learning. Part of that research might focus on various means to encourage the desired communication.

Another area of possible future research might be the effect on students' discussions of concentrated instruction in the area of metacognition. This additional tool for learning might enable students to better express their doubts and confusions as well as their successes.

Pupils in this study appeared to focus on mechanical aspects of the tasks. Further research into the Skempian approach might investigate the degree to which this is true in other groups and how this tendency might be overcome.

A final suggested research focus might be to refine the techniques of assessing and classifying meaningful communication in mathematics.

A FINAL WORD

Rose's vision of the best possible future of mathematics is powerful.

> Classrooms need to be places where the learning of mathematics is seen as the posing and investigating of questions; where errors are not viewed as faults to be remediated, but as springboards for learning; where it is acceptable for both students and teachers not to know the answer, but continue the quest; where paradoxes and ambiguities are seen as interesting challenges to be pursued; where struggling for understanding is the norm; where mathematical knowledge is seen as constructed, tentative, pluralistic, and subject to revision; where mathematics is contextual, formed by

there are many ways to solve the same problem; where induction is used as well as deduction; where the process is as important as the product; and where new mathematics can be created and enjoyed (1989, pp. 3-4).

BIBLIOGRAPHY

BIBLIOGRAPHY

Alberta Education (1991). Programme of studies: Mathematics component (revised).

Applebee, A., & Linger, J. (1983). Instructional scaffolding: Reading and writing as natural language activities. Language Arts, 60. (2), 168-175.

Artzt, A. & Newman, C. (1990). How to use cooperative learning in the mathematics class. Reston, Virginia: National Council of Teachers of Mathematics.

Barnes, Douglas (1992). From communication to curriculum. Portsmouth, New Hampshire: Boynton-Cook.

Baroody, A. (1993). Problem solving, reasoning, and communicating, K-F: Helping children think mathematically. United States: MacMillan.

Bell, A.W., Costello, J. & Kuchermann, D. (1983). Research on learning and teaching. England: NFER-NELSON.

BeMiller, S. (1987). The mathematics workbook. In T. Fulweiler (Ed.), The journal book (pp. 359-366). Portsmouth, New Hampshire: Heinemann Educational Books.

Beuthel, D. & Meyer, P. (1977). A regular classroom plus a mathematics laboratory. In W.G. Cathcart (Ed.), The Mathematics Laboratory pp. 147-150). Reston, Virginia: NCTM.

Botstein, L. (1989). Foreward: The ordinary experience of writing. In Connelly, P., & Vilardi, T. (Eds.). Writing to learn in mathematics and science (pp. xi-xviii). New York: Teachers College Press.

Brissenden, T. (1980). Mathematics teaching: Theory in practice. London: Harper & Row.

Brissenden, T. (1988). Talking about mathematics. Great Britain, Basil Blackwell.

Brooks, J.G. (1990). Teachers and students: Constructivists forging new connections. Educational Leadership 47(5), 68-71.

Brun, J. (1970). An observational method for studying classroom cognitive processes. Unpublished doctoral dissertation. Iowa State University of Science and Technology.

Burgoyne, M. & Burgoyne S. (1990). A teacher's concern: Communicating in math class. (Unpublished article).

Burns, M. (1986). Teaching "what to do" in arithmetic vs. teaching "what to do and why". Educational Leadership, 43 (7), 34.38

Cathcart, W.G. (Ed.) (1977). The mathematics laboratory. Reston, Virginia: NCTM.

Compton, J. (no date). Cooperation and collaboration. Lecture.

Davidson, N. (1990). Small-group cooperative learning in mathematics. In T. Cooney (Ed.), Teaching and Learning Mathematics in the 1990s. Reston, Virginia: NCTM.

Davidson, N. (1990). Introduction and overview. In Davidson, J. (Ed.), Cooperative learning in mathematics (pp. 1-20). United States: Addison-Wesley.

Davidson, N. & Kroll, D.L. (1991). An overview of research on cooperative learning related to mathematics. Journal for Research in Mathematics Education, 22(5), 362-365.

Davis, B. (1990). Promoting meaningful talk in the mathematics classroom. Unpublished master's thesis, University of Alberta.

Dawson, S. & Trivett, J. (1981). And now for something different: Teaching by not teaching. In Floyd, A. (Ed.). Developing Mathematical Thinking. (pp. 124-134). London: Addison-Wesley.

Dean, J. (1987). Managing the primary school. Great Britain: Croom Helm.

Dean, Joan (1992). Organizing learning in the primary school classroom (2nd edition). Great Britain: Routledge.

d'Entremont, Y. (1988). Action research as educational research. In Carson, T. & Coutre, J.C. (Eds.). Collaborative action research: Experience and reflections (pp. 47-53). Edmonton: Alberta Teachers' Association.

Ewbank, W. (1977) The mathematics laboratory: What? why? when? how? In W.G. Cathcart (Ed.), The mathematics laboratory (pp. 214-219). Reston, Virginia: NCTM.

Gilles, C. & VanDover, M. (1988). The power of collaboration. In Golub, J. (Ed.). Focus on collaborative learning (pp. 29-34). United States: National Council of Teachers of English.

Hord, S. (1986). A synthesis of research on organizational collaboration. Educational Leadership, 43 (5), 22-26.

Johnson, D. & Johnson, R. (1985). Student-student interaction: Ignored but powerful. Journal of Teacher Education, 86 (4), 22-26.

Johnson, R. & Johnson, D. (1990). Using cooperative learning in math. In N. Davidson (Ed.), Cooperative learning in mathematics. (pp. 103-125). U.S.A.: Addison-Wesley.

Johnson, D., Johnson, R. & Holubec, E. (1990). Circles of learning: Cooperation in the classroom (3rd ed.). Minnesota: Interaction Book Company.

Kutz, R. (1991). Teaching elementary mathematics: An active approach. Needham Heights, Massachusetts: Allyn and Bacon.

Mathematical Association (1992). Maths Talk. (2nd edition). Hong Kong: Stanley Thornes and the Mathematical Association.

Mersmeth, K. (1993). How old is the shepherd? An essay about mathematics education. Phi Delta Kappan, 74 (7), 548-554.

National Council of Teachers of Mathematics (1989). Curriculum and evaluation standards for elementary school mathematics. United States: NCTM.

National Council of Teachers of Mathematics (1991). Professional standards for teaching mathematics. Reston, Virginia: NCTM.

Nolder, Rita (1988). Responding to change. In D. Pimm (Ed.), Mathematics, teachers and children. (pp. 156-163). Great Britain: The Open University.

Pa, N. (1986). Meaning in arithmetic from four different perspectives. For the Learning of Mathematics, 6 (1), 11-16.

Pimm, D. (1991). Why are we doing this: Reporting back on mathematical investigations. Unpublished article.

Pimm, D. (1987). Speaking Mathematically. Great Britain: Routledge and Kegan Paul Ltd.

Reeves, N. (no date). Children writing maths. (From the Australian Reading Association).

Reid, J., Forrestal, P. & Cook, J. (1989). Small group learning in the classroom. Portsmouth, NH: Heinemann Educational Books.

Reys, R., Suydam, M. & Lindquist, M.M. (1989). Helping children learn mathematics. (2nd ed.). Englewood Cliffs, New Jersey: Prentice-Hall.

Rose, B. (1989). Using expressive writing to support the learning of mathematics. Unpublished doctoral dissertation, University of Rochester, New York.

Russell, S. & Corwin, R. (1993). Talking mathematics: 'Going slow' and 'letting go'. Phi Delta Kappan, 74(7), 555-558.

Schnell, J. & Klein, L. (1977). Development of a mathematics laboratory. In W.G. Cathcart (Ed.), The mathematics laboratory. (pp. 154-158).

Reston. Virginia: NCTM.

Sharan, S. (1980). Cooperative learning in small groups: Recent methods and effects on achievment, attitudes and ethnic relations. Review of Educational Research, 50. (2), 241-271.

Shuard, Hilary (1984). Mathematics in English primary schools. The Elementary School Journal, 84(5), 583-594.

Skemp, R. (1979). Intelligence, learning and action. Great Britain: Unwin Brothers, The Gresham Press.

Skemp, R. (1986). The psychology of learning Mathematics (new edition). Harmondsworth, Middlesex: Penguin.

Skemp, R. (1989a). Structured activities for primary mathematics: How to enjoy real mathematics: Vol. 1. London: Routledge.

Skemp, R. (1989b). Structured activities for primary mathematics: How to enjoy real mathematics: Vol. 2. London: Routledge.

Skemp, R. (1989c). Mathematics in the primary school. London: Routledge.

Sobray Evans, C. (1984). Writing to learn in math. Language Arts, 61, (8), 828-835.

Staton, J. (1984). Thinking together: Interaction in children's reasoning. In Thaiss, C. (Ed.). Speaking and writing K-12: Classroom strategies and the new research. United States: NationalCouncil of Teachers of English.

Speaking and writing, K-12. (pp. 144-187). United States: National Council of Teachers of English.

Stempien, M. & Borasi, R. (1985). Students' writing in mathematics: Some ideas and experiences. For the Learning of Mathematics, 5, (3), 14-17.

Van de Walle, J. (1990). Elementary school mathematics: Teaching developmentally. White Plains, New York: Longman.

Ward, Geoff (1988). I've got a project On .... Australia: Primary English Teaching Association, 1988.

Warden, J. (1981). Making space for doing and talking with groups in a primary classroom. In Floyd, A. (Ed.), Developing Mathematical Thinking (pp. 248-257). London: Addison-Wesley.

Weissglass, J. (1990). Cooperative learning using a small-group laboratory approach. In N. Davidson (Ed.), Cooperative Learning in Mathematics. (pp. 295-334). U.S.A.: Addison-Wesley.

Wells, Gordon (1986). The meaning Makers: Children learning language and using language to learn. Portsmouth, New Hampshire: Heinemann.

Yackel, E., Cobb, P., Wood, T., Wheatley, G., & Merkel, G. (1990). The importance of social interaction in children's construction of mathematical knowledge. In Cooney, T. (Ed.), Teaching and learning in mathematics in the 1990s (pp. 12-21). Reston, Virginia: NCTM.

APPENDIX A

Transcripts of Daily Work

APPENDIX A

The first section of this transcript deals with Group A (Ab, An, L, Ik) which was eliminated from the data. The information has been included here because it demonstrates the manner in which the multiplication unit was introduced.

DAY ONE - "Make a set, make others which match" Num. 5.1/1.

Ab - (Made set; then was trying to figure out whether L or An should do next step (person 2); Ms. C. came over and asked An: How many are you going to put on yours?

An - Four.

Ms. C - Please put it on your circle. L, how many are you going to put on your circle? (L just sitting--not making set).

L - Four.

Ms. C - Four. Do it.

- (All children made matching sets (each with different materials); while waited for next direction, L. watched T's group and Ab was watching someone else in room. Some conversation among group as waited).

Ms. C - What was the first thing that we did? (Asked a member of a non-study group). Number one made a set of things. What was the second thing that we did? Fred?

Fred - Everybody.

Theresa - In the group.

Fred - In the group put the same amount of numbers.

Ms. C - Made a matching number. Okay. Now very carefully put your yellow circles into the loop.

- (Done by Group A).

Ms. C - Then I want you to be able to talk about what you did in this way.... Okay, who was number one? Ab, what did you do?

Ab - I put four things on the yellow.... My group put four the same, and we put it into the middle.

Ms. C - How many people are in your group?

Alex - Four.

Ms. C - Ab put four objects in her circle. Everybody else in her group made a matching set in their circle and then they put them out all together. How many things do you have in the big blue loop?

Ab - Sixteen. (Had not been asked to figure this out beforehand; but replied immediately when asked).

Ms. C - Sixteen. Okay, good. Theresa, what happened at your group?

Theresa - I set a number of ...

Ms. C - How many?

Theresa - Three. And the rest of the group put three.

Ms. C - And how many people are in your group?

Theresa - Four.

Ms. C - Good, and how many things do you have in your set loop now?

Theresa - Altogether?

Ms. C - Altogether.

Peter - Twelve.

Theresa - Twelve?

Ms. C - Twelve.

- [As Ms. C asked another group to explain, Theresa and Peter had a short conversation. Could not pick up exact words but Theresa didn't look as though she had appreciated Peter's interjection].

Ms. C - (Asked for explanations from each of the other three groups. Study groups not very attentive - could hear rustling on tape, at one point Theresa said "Subtraction").

Ms. C - (Instructed children to put away their counters in their medicine cups to get ready for the next example).

An (Put up hand).

Ms. C - Number two, please put some objects on your circle. Everybody else watch number two.

An - (Put out two counters).

Ik - Two?

Ms. C - Numbers twos all put objects on their circles. Everyone else in your group make a matching set. Make a group of objects that has the same number of objects as number two's does.

- (All three in An's group had finished making groups before Ms. C had completed her directions).

Ms. C - Please put your sets together into the loop. Number three, please tell the others in your group what you just did.

Ab - (Pointed to Ik) - Number three.

Ik - We put it all in the loop. All the circle things into the circle.

An - But what did we just do?

Ab - What did we do?

L - Yeah, what did we do before?

Ab - We did the same thing that number two did. We put the same in.

L - How much does it equal?

An - You don't? ...

Ik - Eight.

L - Eight? Okay.

Ab - How much everything together?

L - Eight.

Ik - (Counted aloud by ones at same time L was giving him the answer).

L - Eight. That's what he said.

An - Two times ...

Ab - We don't do times yet.

- (Some conversation about non-math).

Ms. C - Okay, did Ik explain it right?

L/Ab - Yup.

Ms. C - Excellent, All right, please take your objects out of your loop. And take your circles out of your loop. We don't want them to roll all over the floor. Person number three put up your hand. Person number three make a set of objects on your circle.

- (Ik made set of five).

Ms. C - Everybody else in your group watch. Everyone else make a matching set. A set that has the same number in and when you've done that, put them all together in the loop.

An - (Checked Ab's set by counting by ones, then checked that her's matched).

An - All together in the loop? (Looked at T's group). Put them all together in the loop.

Ms. C - Numbers fours put up your hands. Numbers fours explain to the rest of your group what you all did together.

L - We ... Ik put down five, Ms. C told us to put all of our things in the loop.

Ik - The same.

Ab - How much does it make?

L - Um ... (looked at set loop).

An - I know how much.

Ab - She can count.

An - I know.

Ik - (Leaned forward and started to speak).

L - Don't tell me, Ik!

Ab - How much?

L - Twenty! (Sounded annoyed).

Ab - Ik, it's not fun for the other person.

L - No, it's not fun. Ms. C asked me, she didn't ask you. I can answer by myself. It's math, you don't (help?) me.

Ms. C - Okay, clear off your circles. Numbers fours please put up your hand. Number four please build a set.

L - (Made set).

Ms. C - Now listen carefully. The rest of you shouldn't have made a set yet. If you are wearing blue, you may build a matching set. If you are not wearing blue, you may only watch.

- (All checked clothing).

Ms. C - If you're wearing blue, build a matching set. If not, just watch.

Ab - I'm wearing blue.

Ms. C - If everyone at your table has blue, that's fine. Children wearing blue, please put your sets into the loop.

- (Number four and two other people put sets into the loop).

Ms. C - If you are not wearing blue at your table, please describe what happened. Number two describe what happened.

An - L made a set, then you did the same as her.

Ab - Yup.

L - Don't fool around.

Ab - Ik, did you hear An?

Ik - Yeah.

Ab - What did she say?

Ik - (??).

An - I didn't say the same thing. You all put the same amount in ... three ... and you put the same amount in as number four.

Ab - How much does it make all together?

An - Nine.

Ms. C - Please empty out your set loops.

An - 'Cause I know how to make nine.

Ms. C - You are all doing a very good job. This is all about multiplying and it's not that hard so if you're paying attention, this'll be a very easy thing for you to learn. And for those of you who already know about it, now you'll know some new things.

Ik - This is multiplying?

Ms. C - Pretty easy, huh, Ik?

An - That's times. It's the same thing as times.

Ms. C - Okay. We do two things. The first thing we do is we make a set. And the second thing we do is to make sets that match ... that have the same number of objects in the set.

T - I suppose you can't use if someone put down five and then someone else put down four and then someone else put down five and someone else put down five.

Ms. C - Would that work?

T - I don't know.

Ms. C - No, that wouldn't be multiplying. In multiplying, all the sets have to be the same. If they're not, then it doesn't work.

Ik - Do we get to use these things up there too? When we multiply?

Ms. C - Up where?

Ik - Up in the classroom.

Ms. C - When we're learning we'll use things to help us. When we get smart we won't have to use things anymore.

Theresa - Do we have to know multiplying before the end of the year?

Ms. C - Yes, we have to know multiplying before the end of the year.

An - 'Cause grade four does that too.

Ms. C - Okay. Number ones put up your hand. Number ones please build a set in your circle.

(Film now switches to Theresa's group (the study group of Peter, Theresa, Fred, Jenny)

Peter - One that we never did before.

Theresa - Why?

Peter - I don't know.

- (Discussion started with Fred but can't hear the first part; Theresa replied - Who cares ... you guys don't have to....)

Ms. C - Everybody else build a matching set.

- (Theresa and Peter built sets of one and put them into the set loop).

Ms. C - All right, put your sets together.

Theresa - Put your sets together (directed at Jenny and Fred).

Fred - You put four.

Theresa - No, I put one.

Fred - No, you put four.

Theresa - I did?

Ms. C - In a really loud voice, explain what happened. Instead oe me going to each table, explain so everyone can hear. Fred, explain what happened at your table.

Fred - Theresa put....

Peter - (Put up one finger to show how many objects Theresa had put).

Fred - One bean on her circle and then we did the same.

Ms. C - So how many sets do you have that have one thing in it?

Fred - Four.

Ms. C - And then what did you do?

Fred - We added it up and then we got four.

Ms. C - Okay, good.

- (Ms. C asked for explanations from the other three groups; fewer prompts used for the other groups).

Ms. C - Take out your circles please. Person number two, please make a set that has nothing in it.

Theresa - That's kind of easy.

- (Students picked up set circles and played with them).

Ms. C - Everyone else make a set that matches that. And then put them together in the loop. (Done). Fred, how many things do you have?

Theresa - None.

- (Ms. C asked An's group and while she was talking to them, Group B (the study group) was playing with circles).

Theresa - Ze number zero. Zero is nothing. Zero times four, four times zero....

**** Clean up ****

DAY TWO: IN LAB: Num. 5.1/1 "Make a set, make others which match" (with added explanations about how results were obtained); (Peter, Fred, Theresa, Jenny).

Ms. C - Number ones put up your hands. Number twos put up your hands. Number threes. Number fours. Number three, please make a set of objects on your circle.

Jenny - (Put three objects down).

Theresa - She puts three and she's number three.

- (Study group began making matching sets).

Theresa - I don't want to use my chubby ones (held up one of her objects).

Ms. C - Wait, don't do anything else. Okay, everyone else make a set of matching objects so that on your circle you have the same number.

- (Done by group).

Theresa - (Showed Fred how she could bend the edge of her medicine cup with the objects in it so only one at a time would come out). Here's how I do mine, Fred.

Ms. C - Okay, put your sets together in the loop. (Done). Good, figure out how many you have all together in the loop.

Theresa - (To Fred) Twelve!

Fred - What if we did it already?

Theresa - Then we know it easier.

Ms. C - Did what?

Fred - Figured it out.

Ms. C - Good. I would like the number ones to tell me how you figured out how many you have in your loop.

- (Explanations from one other group).

Ms. C - Okay, who's the number three at this table? (Study group).

Theresa - Jenny laid down three and then Peter and Fred and me laid down three and it added up to twelve.

Ms. C - How did you add it up to get twelve? What did you do?

Theresa - I added those two up (indicated two sets) and then I added them two up (other two sets) and then I added them together (the pairs of sets grouped together earlier).

Ms. C - When you added these two together, how many did you get? (Two sets).

Theresa - Six.

Ms. C - And then when you added these two up you got....

Theresa - Six.

Ms. C - And then when you added them together, you got twelve?

Theresa - Yup.

Ms. C - (Asked two other groups to explain). Please clear out your circles. Number two, please make a set of objects on your circle.

Fred - Number one?

Theresa - (??). It was probably number two (??). She did it first. (Indicated J).

Ms. C - Everyone make a matching set and put them together in the loop.

Fred - (Made set of two).

Theresa - (Made set of two). It's going to make six ... no, it's going to make eight.

Ms. C - Why do you think it's going to make eight, Theresa?

Theresa - 'Cause I did the same as I did with the other one. Put these two (sets) together and these two (sets) together and then (gestured to show combining all four sets).

Ms. C - Okay. I want number four to explain to the group what you just did and how you figured out the answer.

Peter - Fred put two and then we copied that and put them in the circle and then we added four and four and so it's eight ... eight.

Fred - We put two.

Peter - Thank you for talking for me.

Theresa - We added these together (two sets of two) and then we added these together (two sets of two) and then we added them all together and we got eight.

- (Fred and Theresa picked up two of their objects and put them up to look like eyes).

Theresa - Yeah, well I've got hearts for eyes. I've got four eyes.

Ms. C - (Asked for explanation from another group).

Switched to "Making Trains" (activity devised by Ms. C based on "Make a Set, Make Others Which Match").

Ms. C - Same idea, different activity. Person number three, make a train. Use your Unifix cubes and make a train of a certain number.

- (Study group - began to make trains).

Theresa - Yeah, like if you want two, you just ... (separated two from long rod and put them in center of table).

Peter - I'll just do this so I don't have to do nothing. (Put whole rod into center).

Theresa - No, she can make it the way she wants to.

Ms. C - Good.

Jenny - (Made a train one cube long).

Theresa - A train of one?

- (Then Jenny made train two cubes long; Theresa picked it up from table).

Ms. C - Everyone else make a train that matches number three's. Check that yours all match, that they're all the same. Then put them together into one long train and find out how long your train is.

- (Fred compared length of his rod and Jenny's rod by placing them side by side; then put them together).

Fred - It's two.

Peter - Two? I thought it had to be four.

Theresa - I want to look through it. (Picked it up and looked through like a telescope).

Peter - Put this together and look through it. It looks cool. (Combined the remaining parts of his rod and Jenny's).

Jenny - (Looked through the long rod). Neat.

Ms. C - Figure it out how long your train is.

Theresa - He (Fred) just wants to look through it.

Fred - (Picked up the train and stood it on end).

Theresa - Its supposed to be a train. Trains don't stand up, Fred.

Fred - (Shrugged).

Peter - Let's do it in five and see how long it is. Twenty. [No train made to help him figure out that would be twenty].

Theresa - Gee, Fred. Yours is like a pile. [Reference to train or to number of cubes left in Fred's hand??].

Ms. C - (Came to talk to group). How long is this train?

Jenny/ Fred - Eight.

Ms. C - Eight. How do you know that?

Theresa - 'Cause we just did it.

Peter - (Pointed to parts of train). Four and four.

Theresa - And plus we just did it.

Ms. C - It was the same one?

Theresa - Yeah. (Reference to "Make a Set; Make Others Which Match It" question done just before).

- (Ms. C moved to another group).

- (Children played with the cubes; combined them and looked through).

Theresa - Come on you guys, we're not playing.

Peter - Yes we are.

Jenny - We could put this here. (Broke apart train and put more blue cubes into middle section).

- (Children continued to play with combining trains in different lengths; Peter took train from Fred, "Give me my black": took back his Unifix).

Ms. C - Okay. Everybody get your own colours back.

Theresa - (Grabbed train from Fred). Gimme mine back. Oh you didn't have to take them.

- Children reassembled their own colours into trains).

Theresa - (Comment to student in adjacent group: held up one of Fred's blue cubes). L! She has the same colour as you.

Ms. C - Person number two, make a train. Make your train. Please make it less than seven cubes long. Please stand it up and hide the rest of the cubes so they don't get in our way. Everybody else look at person number two's train and make your train that long.

Fred - (Made train).

Fred - Six?

Theresa - (To Ms. C). He's putting six.

Ms. C - That's okay.... Then you have to show them so they can make theirs exactly the same.

Fred/ Peter - (Compared lengths; children played with the trains like guns).

Ms. C - Show them to each other so you make sure the trains are the same length and height.

- (Children did).

Theresa - Me and Fred like each other. (References to their trains being side by side).

Ms. C - Excellent. Person number two, make them into one train.

Fred - (Put together).

Theresa - It's going to make twenty-four. Twenty-four.

Peter - Okay. Twenty-four. It's going to be twenty-four.

Theresa - It's going to be twenty-four.

Fred - (Stood train up on end).

Theresa - It's going to be twenty-four. It's going to be twenty-four.

Six plus....

Fred - (Began to count by ones).

Theresa - (Kept chanting as Fred counted). Twenty-four, twenty-four ... (repeated eight times).

Fred - (Counted aloud to twenty-four).

Peter - I told you I was right (to Theresa).

Ms. C - Okay. (Asked for explanation from other groups).

- (Study group playing with trains while others explained).

Ms. C - Fred, explain what your group did.

Fred - I made a train and then Theresa and Peter and Jenny made trains. (Long train had been disassembled and individual trains were standing upright).

Ms. C - Can I see your train?

Fred - (Reassembled their long train).

Ms. C - And you made a train and then what?

Fred - I counted them.

Ms. C - And how many are in your train?

F - Twenty-four.

Ms. C - (Asked for explanation from other group; then said): everybody get your own colour back.

Theresa - I have to start that one.

Ms. C - Person number four, please make a train.

Peter - Okay. (Made train of four).

- (Other children made trains to match and they put Jenny's, Fred's, Peter's together: Theresa tossed hers into centre: Fred added it to train).

Ms. C - Everyone else make a train the same length.

Peter - (Worked on assembling their train).

Theresa - Ms. C, number ones didn't get a turn.

Ms. C - All right. Number ones, please put the train together.

Peter - (Handed train over to Theresa). Okay, you do it. (Conversational tone only).

Theresa - (Assembled train).

Peter - You could bend that (end of train) and make it a thing. That what's M (child in another group) did. You can bend it up like that. It turned up.

Theresa - All of a sudden you take them all apart again.

Peter - (To Theresa). The blue doesn't stick.

Theresa - (Held train up so blue was hanging at bottom; cubes didn't detach; laid it down).

Ms. C - Hands off the train. Hands off the rest of the cubes. Eyes looking at me.

- (Children complied).

Ms C - Okay, we're going to talk about it like this. (Headings "First Action, Second Action, Result" on blackboard). It says "First Action". That's the first thing that people did. Then I want you to tell me the second action. And what does "Result" mean? Fred?

Fred - What happened?

Ms. C - Good. The first action in your group was ... what, Peter?

Peter - We each put cubes down.

Ms. C - How many?

Peter - Four.

Ms. C - All right. (Wrote "train of 4"). Then what was the second action your group did? T?

Theresa - We put them into a train.

Ms. C - How many? How many trains did you make that were the same?

Theresa - Four.

Ms. C - You made one train of four ... and then you made....

Theresa - A big train. A long train.

Ms. C - You made one train of four and then you made four trains that were all the same, right? (Wrote "four trains" under heading "Second Action") What's the result? How long is the train you have now?

Theresa - Eight.

Ms. C - Eight cubes long? Count it.

Fred - Sixteen.

Theresa - Sixteen.

Ms. C - Sixteen? How did you figure out sixteen, Fred?

Fred - We had four and four and that made eight. And then I added eight and eight.

Ms. C - Okay. Their first action was to make a train of four. They made one train that was four trains long. They made four trains that are the exact same thing. All together they have sixteen cubes in their long train. (Repeated same telling and recording procedure with other four groups). Good work. At your tables tomorrow we're going to do an activity where you'll have cards where your first action will say to make a train of this many cubes long and the second action is to make this many trains and put them together. Then you'll find out what the result is. Please make your stacks into one colour. Don't make them into long trains. Just put yours together and put them into the center of the table.

**** Clean up ****

DAY 4: IN LAB: "Multiplying on a Number Track" Num. 5.1/2; (PV, F, T).

Ms. C - We're going to do the same thing we did again yesterday. Why do you think I would make you do the same thing twice?

Student - So we can learn.

Ms. C - Very good. Any other ideas?

Peter - So we can remember?

Ms. C - So we can remember, good.

Theresa - In case we didn't get the hang of it, we can learn.

Ms. C - Yeah, you need to get the hang of it.

An - Maybe you weren't here.

Ms. C - Right. Matthew, you weren't here. Jennifer wasn't here. Okay. Remember that there are four jobs. Remember that you should not be playing with the cubes, Theresa. Okay. First action ... second action....

Ab - Are we going to write in our math book today?

Ms. C - Yes, we're going to write.... Result (writing on board as she said the headings)....

Fred - Explain.

Ms. C - And explain. Now. We have a lot of grown-ups in the room. It's important that the grown-ups are around and are able to listen to the children. But a grown-up can't be at your table all the time to remind you that the explaining part is really important and that you need to listen to the person explaining because listening to them is going to help you.

Theresa - To understand.

Ms. C - If you're not watching for all these things ... maybe not watching how the person figured out the result, then you won't be able to explain. You need to listen and you need to make sure that everybody in your group is understanding what we're doing. Otherwise, you won't get it.

Theresa - And Jenny wasn't here.

Ms. C - Okay, now. The number thing got a little confusing yesterday but it's really very simple. You're all sitting, and you all remember that when we're playing games and doing things, we go clockwise. So if An does first action, you (pointed to child) do second action and you (pointed) do result and you (pointed to next child) explain. And then the next time, you start one person over so L would start and you (pointed) do second action, result and explain, and it just keeps going around. It's really not that complicated. If An did first action the first time, then L will do it the next time and you just keep going around. At your table (study group) if PV did the first action, who would do it the next time?

Fred/ Peter - Theresa?

Ms. C - (Checked for comprehension with each group). So that's how it goes, it's really quite simple.

An - I understand.

Ms. C - An understands. Does everyone else understand? Good. All right, set up your boards, decide who is going to do first action first and then get started.

Fred - I'll be first.

Theresa - You were first last time.

Fred - I know.

Theresa - Ms. C....

Fred - What if we have three people?

Ms. C - Then you know that person one and person four....

Theresa - He got the first last time.

Ms. C - If you do the first action you also do the explaining if there are only three people in your group.

Theresa - You guys ... we need the result....

Mr. G - You guys are wasting time.

- (Children took rods and board out of tub; setting up).

Fred - I get an extra one (extra rod).

Theresa - The (??) are all mixed up now.

Mr. G - Well, there you have it. Life's like that sometimes.

Peter - (Counted kids to show order; Theresa=1, Peter=2, Fred=3). One, two, three four.

Mr. G - One, two, three, four. (Counted children in opposite direction; observed for a moment, then left).

Peter - (Turned over first action card).

- (Children started to make rods to match).

Theresa - (To Fred). Oh, you have three. Better put them back.

Fred - No.

Theresa - Yes you do. 'Cause it's not going to be fair to us.

Fred - (Reply inaudible).

Peter - (Continued through the discussion). Second action ... (indicating Theresa) see how much you take down. High, high, high, high.

Theresa - (Turned over card; didn't read aloud; card said make two rods) Which one is yours, Fred? (Meaning rods).

Mr. G - (Returned to observe). Have you guys started yet?

Theresa - Yeah.

Peter - But these people are arguing.

Fred - Join them. (Rods).

Mr. G - What do you do with the rest of them? (Took one rod and set it aside: then took the three from Fred's hand and put them into the tub holding the Unifix cubes).

Fred - Why?

Mr. G - You don't need them.

Peter - Ten ... ten. We've done results.

Theresa - No, you figure out....

Fred - (Wrote answer down).

Theresa - How did you figure it out? You didn't tell us how you figured it out.

Fred - It's twenty. Ten ... times ... ten times two is twenty. (Pointed to rods as said numbers).

Theresa - How? How did you figure it out?

Fred - Two groups of ten is twenty.

Peter - (Explanation). I put down first action....

Ms. C - (Had been standing watching the group). What was the first action? Just saying I did the first action, that's not helpful. We need to know what the first action was.

Peter - The card said, make a rod with ten cubes.

Ms. C - And the second action was...?

Peter - Theresa turned over, make two rods.

Ms. C - Two rods. And how many cubes were in each rod?

Peter - Ten. Then Fred wrote down the answer.

Ms. C - Which was...?

Peter - Twenty.

Ms. C - Good. All right, whose job is it to do the first action now?

- (Peter and Fred pointed to T).

- (Kids got out all of their rods from the pile).

Theresa - Okay. Keep them there this time, Fred. (Turned over first action card). I turn over the card. Each make a rod of nine cubes. Here's the easy way to do it. You put two of them together and then you just rip one, two off. (Put two rods side by side and took one cube off top of each).

Peter - Now, look at one.

Theresa - Fred, look. You just put them together and....

Peter - Somebody put down the second action.

Fred - I will. Join four rods.

Theresa - Fred joins four rods. (Handed them to him).

Fred - (Joined rods).

Theresa - What's the best colour? (To Ms. C as she went by). We got four rods of nine.

Ms. C - Four rods of nine? How much does that make all together?

Fred - Nine, nine, seventeen (pointed to sections of rod as said numbers).

Peter - Nine and nine is seventeen.

Theresa - Nine and nine is eighteen.

Fred - Seven....

Theresa - Eighteen. Eighteen and eighteen is....

Peter - I know, it's ... oooh! (Paused, then hit himself on forehead).

Theresa - It's supposed to be him answering it. (Indicated Fred).

Peter - It's me.

Fred - He has to write the result.

Peter - (Counted rods by starting after the eighteenth cube; counted by ones; wrote result).

Theresa - (As Peter was writing his answer). Now tell us how you did that.

Fred - Two rods....

Mr. G - (Returned). What's happening here? Theresa, do you need those right now? (Indicated spare rods; indicated they should go into tub). What is your job now?

Peter - I'm just....

Mr. G - What are you guys doing?

Fred - Peter was counting.

Peter - I finished that. (Picked up result card to prove he had finished).

Mr. G - So each make a rod of nine. (Reading from gameboard). So you made rods with nine in each. And what did you come up with, Peter?

Peter - Twenty-six.

Mr. G - Is everone working together to make sure that Peter is doing his job right? Four nines equal twenty-six.

Theresa - We tried to help him with the answer but he wouldn't let us.

Fred - Nine and nine is eighteen and (during conversation among group members and Mr. G, Fred was recounting to check answer).

Peter - You were getting me mixed up.

Fred - Oh, oh. (When he reached twenty-seven realized that answer was wrong).

Mr. G - Check and double-check.

Fred - 'Cause it was eighteen here....

Mr. G - I have a question for you. How do you know that this is eighteen? (Put hand masking half of long rod so children would focus on half only).

Fred - Nine and nine.

Peter - Or it's seventeen.

Mr. G - How big is this here? (Other half of long rod).

Peter - Nine. (Looked as if Mr. G might mean only one nine section as he asked this question).

Mr. G - So if you know that this is nine and this is nine and this is eighteen (on nine section at a time and then the two nine sections together), how long do you know that this is? (The other half of the long rod).

Theresa - Eighteen. But we were trying to figure out eighteen plus eighteen.

Mr. G - Okay. Did you figure it out?

Theresa - No. Can we have a piece of paper?

Mr. G. - Okay, count them.

Theresa - (Began counting at nineteen; lost track). Oppsie, I'm going too fast.

Fred - (Put finger on nineteen as Theresa restarted her count).

Theresa - (Pushed Fred's finger aside and continued counting). Nineteen ... (up to) twenty-five, twenty-six....

Fred - (Began counting along, putting finger on cubes to keep track).

Theresa - All right, everyone else take my place.

Fred - Thirty-five.

Peter - (Took paper to write new result).

Theresa - It can't be thirty-five. Okay, Fred.

Fred - (Was recounting again).

Theresa - And then that's ten ... (indicated third nine-rod plus the first cube in the fourth nine-rod). No-one listens to me. (Facing the video camera).

Fred - (Counted from one this time). Thirty-eight.

Peter - Ahhh.

Theresa - (Sitting and watching boys try to figure out total number of cubes; said nothing even when they were wrong).

Peter - (Counted again, starting at one).

Theresa - (Put head down; looked frustrated).

Peter - I'm mixed up. It's thirty-five.

Theresa - Well, how did you do it? You're suppoed to tell us. (Impatient tone).

Peter - I counted it.

Theresa - But how?

Peter - By ones.

Theresa - Ones (looked disgusted).

Fred - (Recounted rod one more time). Thirty-six.

Peter - (Wrote new result on paper; put on pile on gameboard).

Theresa - (Picked it up and moved it). I turned over first action and it said to make a rod with nine cubes, and then Fred picked up second action and it said join four so we took my rod and three of his and put them together. And then Peter counted them all by ones.

Fred - No.

Theresa - That's what he told me. I'm going by what he told me.

Fred - I was counting (??).

Theresa - Well then, how did you get it?

Fred - I added. It was eighteen. Nine and nine was eighteen and then I counted all these and it was thirty-six.

Mr. G - (Returned). And what's your problem, Theresa?

Theresa - Well he (Peter) told me that he counted by ones and he (Fred) said, no. Then I asked him how he does it and he says something different and I just asked him how he does it.

Fred - Because Peter's answer was wrong.

Mr. G - Peter's answer was....

Fred - I counted to make sure if it was right or wrong. And it was wrong.

Peter - (Counted cubes from nineteen by ones). Thirty-six exactly.

Mr. G - Okay. So what's your problem, Theresa? (Put his hand on her arm and used soothing tone).

Theresa - He said....

Mr. G - (??).

Theresa - I asked him what he (Peter) 'cause he (Fred) was the person who was supposed to do the result and I'm supposed to ask him.

Mr. G - Peter made a mistake and Fred caught it. All right? So there's no problem.

- (Set up for new round of activity).

Fred - (Turned over first action card). And it says ... each make a rod of eight.

Peter - How come it's first ten, nine, eight?

Theresa - We haven't had a ten.

Fred - Yes, we did.

Peter - (Started to turn over first action cards to find the "ten" card from earlier round).

Mr. G - Don't worry about it.

Peter - (Turned over second action card). Five rods.

Theresa - We can't join them together.

Mr. G - Now how many rods are you supposed to have?

Peter - Five.

Mr. G - Did someone turn this over? (Checked second action pile).

Peter - I did.

Mr. G - Go ahead. Do your jobs.

Peter - (Joined five eight-rods).

Fred - Okay, find the result then.

Theresa - Eight and eight is sixteen plus sixteen ... (indicating the two eight-rods to make the sixteens). Eight plus eight equals sixteen ... (began counting from seventeen; reached nineteen).

Peter - Twenty-six.

Theresa - (Stopped; looked at Peter then continued from where she had left off; counted by ones up to forty). Forty.

Fred - Forty.

Theresa - (Picked up paper). I'll just count again. (Recounted by ones from beginning).

Peter - Thirty-nine. Oh, I counted....

Fred - It's forty. I counted in my head.

Theresa - Yeah, it is. 'Cause it has to be an even number 'cause all of these (rods) are even. So it couldn't be thirty-nine. How shall I make my four? Like this or just...? (Described two alternatives in the air).

Fred - It doesn't matter.

Theresa - (Wrote result).

Fred - I flipped over the first card and it said make a rod with eight cubes. And then Peter flipped over the second card and it said join five rods. So we joined five rods and then Theresa....

Theresa - Counted all the ones.

Ms. C - Do you know what this is here? (Long rod).

Peter - A rod.

Ms. C - Multiplying. This is all multiplying is. Doing one action, and then doing the second action. When you're multiplying, it's in two actions. You make a group and then you stick five of those groups together to find out what you get.

Fred - (Speaking to Ms. C, but Group A, the other study group, too noisy so can't hear on tape).

Peter - It's forty.

Theresa - I don't get....

Fred - (??; too noisy).

Theresa - I figured it out.

Peter - We done all the people.

Fred - Okay, one, two, three. (Numbered group again).

Peter - (Turned over the first action card). I'll bet you anything that it's going to be seven.

Theresa - 'Cause it's ten, nine, eight, seven.... (Counted in unison with Peter).

Peter - I suppose it's a six next.

- (discussion about bets on what the next number would be).

Fred - Okay, we each make a rod with seven.

**** Clean up ****

DAY 7: IN LAB: (in "quiet room") Num. 5.1/3 "Giant Strides on a Number Track"; (Fred, Peter, Theresa, Jenny).

Fred - Make a stride of.... (Reading gameboard).

Peter - Tell us what a stride is.

- (Peter's suggestion is to provide a review of the rules and procedures).

Fred - A stride is a step. You flip over this card and it tells you how many strides to make. And we have to make twenty-seven strides.

Theresa - Twenty-seven.

Peter - Turn it over. (Cards were still face down).

AHW - What it says on the back is "LS" and that means "length of strides". That's just so you know which space to put the cards in.

Jenny - (Turned over LS card).

Fred - You flip over the card and it tells you how many strides to make.

Jenny - Then you make a guess and put a blob of clay up to where you think the giant will end up and then you take the strides.

Fred - And then you write the answer here. (On paper).

Theresa - Nice.... Why do you have to make a guess?

Peter - I don't know.

Fred - You don't have to make a guess. You just put it on the spots and where you end up....

AHW - Peter and Jenny just said you have to make a guess. But you don't think you have to.

Fred - No because there are no giants. [Misunderstanding from previous day has interferred--Fred thinks people have to act out part of giants].

Theresa - We're the giants. Well look, it's so small no-one could be the giant. Well, what are we going to do? Do we have to make a guess or don't we have to?

Jenny - I don't think we have to.

Peter - We'll use this for a giant. (Picked up eraser).

Theresa - We'll use this (paper).

Fred - That's the answer. Write the answer on here and we're done and then it's the next person.

Theresa - Okay. Everybody goes first.

Peter - (Turned over card). Take a stride of ten.

Jenny - Spaces.

Peter - Spaces. And take five. Let's see that's ... (took blob of clay and began counting strides of ten but put no markers to show the strides).

Jenny - (Put her finger on ten; took clay from Peter and put some on ten). Just take a piece of clay and put it down here.

AHW - Have you made your guess yet?

Peter - Now let's see. Ten spaces and it's five.

Jenny - I think it's fifty.

Peter - Ten times five.

Theresa - Is it a times thing?

Peter - (Counted groups on fingers). Ten, twenty, thirty.

Theresa - Does it have to be right?

Peter - Twenty-five then, No, no. (Recounted on fingers). Ten, twenty, thirty, forty, fifty!

Peter - (Took clay; began at zero and took one stride; matched with where he had placed clay).

Jenny - Twenty.

Peter - Twenty.... (Didn't put clay marker; continued to figure out the next stride).

Fred - Put the clay down.

Peter - (Put the clay on twenty; proceeded to do rest of task without assistance). Fifty.

Fred - One, two, three, four, five. (Counting strides).

AHW - So was your prediction right?

Theresa - Oh, so every ten is a stride.

Fred - And then write your answer (to Peter).

Theresa - Oh, now I get it. So all the tens are just one stride.

Peter - (Wrote answer).

Theresa - That was just a practice.

AHW - Before you go on to the next one, can you explain what you did and what the numbers mean?

Jenny - Okay. Peter took ... Peter took a card ... flipped a card over and it said go ten spaces so we went to ten (she and Theresa both gestured to show the ten space) and he flipped another card over and it said to do five strides.

AHW - Five strides and each one is...?

Jenny - Ten.

Peter - This goes up to sixty. Giant steps need more than sixty. (Reference to number track). Okay, your turn (to Fred).

Fred - (Turned over "make a stride of" card). Four. (Turned over the 'number of strides' card).

Peter - Seven. (Guess about what card was going to be).

Fred - Two.

Peter - Okay. I'll just guess. (Counted out four groups of two on fingers). Eight.

Fred - (Put clay to show strides). Eight.

AHW - (To Fred). Did you make your prediction first?

Fred - Yes.

AHW - Did anyone predict anything that was different?

Jenny/ Peter - No.

Fred - (Wrote answer).

Theresa - I want to be after Jenny. So Jenny's next.

Jenny - (Turned over strides cards). Four. Nine.

Theresa - So nine times four ... oh man.

Peter - (Figuring out on fingers). I say it's twenty-eight.

Theresa - Well, it's her guess.

Fred - Try it.

Jenny - (Did; could do first two nines without counting; had to count to twenty-seven; stopped but still had clay in hand).

Theresa - You have to put down four blobs.

Fred - (Pointed on track to where he thought next stride would be).

Jenny - (Counted). Thirty-six.

Theresa - Is that right? Was it right?

Peter - (Recounted on fingers).

Theresa - This is hard.

Peter - I guess it is right.

**** Clean up ****

DAY 8: IN LAB: Num. 5.2/1 "I Predict--Here": Group B - with Ms. C; (Peter, Theresa, Fred).

Children began by creating rods of two cubes each.

Ms. C - Okay, that should be enough.

Theresa/Fred - Over here. (Motioned for him to include his two rods with pile).

Ms. C - And here's the number track. This is the way this activity works....

Theresa - This is like the blob game.

Ms. C - This is sort of like the blob game. Same kind of idea.

Fred - Stride game?

Ms. C - Not exactly.

Theresa - Good.

Ms. C - What, you don't like it with strides?

Theresa - No.

Peter - I liked it better with steps.

Ms. C - All right, Fred, turn over the top card please. On each one of the squares in this set, I would like you to put one of the stacks. Try and pick the different colours.

Peter - (Began to reach for rods).

Ms. C - No, Fred is doing this and you're watching. In this set, you have stacks that are how many high?

Fred - Two.

Ms. C - And you have how many stacks?

Fred - Four.

Ms. C - And if you stick them together to make a train, how long will your train be?

Fred - Eight.

Theresa - Eight.

Ms. C - Okay, if you will stack them together, we will put them on the number line to see if you are correct.

Fred - (Did).

Ms. C - And were you?

Fred - Yes.

Ms. C - All right. Excellent. Theresa, you turn over the next one. Put a stack on each square.

Theresa - (Put stacks on squares).

Ms. C - How many cubes do you have in each stack?

Theresa - Two.

Ms. C - And how many stacks do you have?

Theresa - Five.

Ms. C - And if you stick them all together, how long a train will you have?

Theresa - Ten.

Ms. C - Okay, why don't you put them together and we'll check whether you're right. How did you figure out how many you'd have?

Theresa - He (Fred) had eight and then I added two more.

Ms. C - All right.

Theresa - (Put rod on number track and it matched her prediction). I was right.

Ms. C - Peter, would you turn over the next one?

Theresa - (As she disassembled the ten-rod). These will never run out.

Peter - (Turned over card with spaces for six stacks).

Theresa - Hey, Ms. C, that was cool. He (Fred) had eight and then I had to add on two to his and that would make ten and now he (Peter) has to add on two to make whatever that is. [**Probably had good idea what product would be but Peter had not made his prediction yet; ?? reluctant to do his job/take away his chance].

Peter - I've got six.

Ms. C - Six? And how many cubes in each stack?

Peter - Two.

Ms. C - And how long a train is it going to be?

Peter - (Pause). Eleven. Twelve, I mean.

Ms. C - Twelve? How did you figure that out?

Peter - All I had to do was add two instead of five.

Theresa - It looked like you were counting one, two, three, four....

Peter - I know. Just to make sure.

Theresa - That was easy 'cause he just had to lay down eight and then I had to lay down five and then ... aaah ... (shrugged and quit talking).

Peter - (Finished his checking). Yeah, I was right.

Ms. C - Well, that seems too easy for you. Why don't you make some stacks that are five?

Theresa - All the same colour? I need a red.

Peter - Five.

Theresa - I need two more of this colour. (Held up incomplete rod).

Fred - There are only going to be two.

Theresa/Peter - Two of each colour!

Theresa - This is just like the doughnuts only it's not stopping. (Meaning????). Oh, you took mine (to Peter as he took one of same colour she was using).

Ms. C - I think we have enough. We don't want those (set aside stacks of four) ... we only want stacks of five. Okay. Same job. Fred, turn over one (card).

Theresa - (Looked at Fred's card). Oh, this is going to be easy. Easier.

Ms. C - Those five sets are a little wobbly. (Fred having trouble keeping five rods upright). Okay, Fred, how many cubes do you have in each stack?

F - Five.

Ms. C - And how many stacks do you have on your card?

Fred - Four.

Ms. C - And if you made a train, how long would it be?

Fred - Twenty.

Peter - His guess is right.

Ms. C - How did you figure that out, Fred?

Fred - I counted by fives.

Theresa - Yeah, that's why I said it's so easy.

Ms. C - (Fred checked rod on number track). Okay, Theresa, you can break that apart.

Theresa - Don't tell me I'm getting a five. (As she turned over the card). No, three. Okay, I'll use this ... and I'll need this ... (selecting rods).

Ms. C - How many in each stack?

Theresa - Five.

Ms. C - And how many stacks?

Theresa - Three.

Ms. C - And how long will the train be?

Theresa - Fifteen. (Answered before Ms. C had finished question).

Ms. C - And how did you figure that out?

Theresa - I counted by fives. I added these two together and then added this one. (Two five-rods and then other five-rod). (Put them together on track).

Peter - (Turned over card). Six again.

Theresa - (Checked length of rod on number track; sang). I'm so right ... I'm so right.

Peter - (Began putting stacks on squares).

Theresa - Oh, that's easy.

Peter - Oh man. Just enough (rods).

Ms. C - How many on each stack?

Peter - Five.

Ms. C - And how many stacks?

Peter - Six.

Ms. C - And how long a train will it be?

Peter - (Counted stacks by fives). Thirty.

Theresa - He keeps on getting the highest. Why don't we use them? Ones?

Ms. C - We will.

Peter - (Had rod on number track).

Ms. C - And where does it end up?

Peter - Thirty.

Ms. C - And were you right?

Peter - Yes.

Ms. C - Let's make stacks of three now.

Theresa - Okay. I need a green.

- (Conversation about forming the rods and about their colours).

Ms. C - Would you give me all the twos and all the ones that are left over? (Took and set aside). Fred, turn over another one.

Fred - (Turned over card with five spaces).

Ms. C - Put on the stacks of three.

Fred - (Did).

Ms. C - How many in each stack?

Fred - Three.

Ms. C - And how many stacks?

Fred - Five.

Ms. C - And when you put them together, how long will it be?

Fred - Twenty-seven (no hesitation).

Ms. C - Twenty-seven? Okay, put them together and let's see. (To Theresa and Peter). Do you think he's right?

Theresa - (Looked at stacks on card). Nope.

Peter - My guess ... I don't know.

Fred - (Put rod on track).

Theresa - I was right!

Ms. C - Now, Fred, how did you figure out twenty-seven?

Fred - I put those two together to make six ... (first two sections of rod).

Ms. C - Oh ... okay.

Theresa - They are six.

Ms. C - Okay, so six ... (put finger on sixth cube in rod).

Theresa - Oh ... six and six and ... (seemed to assume Fred had counted in groups of six).

Fred - (No further explanation asked for or given).

Ms. C - All right, Theresa. It's your turn.

Peter - Our last card (in stack).

Theresa - Two (squares). Oh, that's going to be easy.

Peter - My guess is six.

Theresa - (Put together two blue rods; took apart and used different colours; only reaction to Peter's guess was to look at him briefly; put rod on number track). Yeah, you shouldn't have answered it for me. The answer was mine.

Peter - Now everyone can ... (??).

Ms. C - Okay, now we'll have a new card for PV.

Theresa - Yeah, just 'cause I get the answer right.

Peter - (Card with ten squares; counted squares by ones). One, two, three ... (up to) ten.

Theresa - Oh, that's going to be easy.

Ms. C - You'll probably need to use some colours over again (to Peter).

Peter - (Counted ten stacks).

Ms. C - Okay, put one on each square. Fred, can you hold onto the other end? (Card wouldn't lie flat on table).

Peter - (Counted some squares aloud as placed stacks).

Ms. C - Okay, so you have how many?

Peter - Ten.

Ms. C - And how many in each stack?

Peter - Three.

Ms. C - And when you put them all together, how long will it be?

Peter - (Counted stacks by ones aloud). Thirty.

Theresa - Yes, I was right.

Ms. C Peter, you stick it together into a train. Theresa, did you figure it out the same way that Peter did?

Theresa - No.

Ms. C - By counting?

Theresa - No.

Ms. C - How did you do it?

Theresa - I put two of them together. I was counting by sixes.

Ms. C - Can you count by six for me?

Theresa - Six, twelve ... (appeared to be counting by ones; without direct reference to train) eighteen ... twenty-four. I keep on forgetting how. (As Fred gave explanation, Theresa counted the last group of six on her fingers).

Fred - I know. I added those two and they were a six (pointed to two parts of train) and then I added those two and it was twelve and then I added those two and it was eighteen and then I added those two and it was twenty-four and then I counted the last ones to make thirty.

Theresa - (At the same time as Fred's talking). And then it was thirty.

Ms. C - From twenty-four, you counted on?

Fred - Yeah.

Ms. C - All right, then.

Theresa - Can we play one more?

Ms. C - One more? All right. Make these into stacks of four. (Began to make fours).

Theresa - (Took chunk of train). I just break off four. There's a four, lady.

AHW - Fred, could you explain to me how you got your answer? I didn't hear the first part because I was listening to An. (Child in another group).

Fred - I took three and three to make six.

AHW - So you have two groups of six?

Fred - Yeah, I put them together.

AHW - And you made twelve. Okay.

Fred - And then I (???).

AHW - So you had twelve and two more groups of three is six and then you had six more. Did you add those together?

Fred - (???). Twenty-four....(??)

- Too much noise from dice rolling on Group A's table.

AHW - It's a really smart way to do it.

Theresa - Smart and complicated.

Ms. C - Okay, Fred. (Handed him pile of cards). What is the first action that you did? (Indicated approximate length of four rod with her fingers).

Theresa - We ... um ... um ... pick up a card. ("Obvious" tone of voice).

Ms. C - The first action is to make.... (Motioned to four rods).

Theresa - Oh, yeah. Sets of four.

Ms. C - Stacks of four. The second action is....

Peter - Pick a card.

Ms. C - You pick a card and a certain number of stacks, right? And each stack has? How many in each?

Peter/ Fred - Four.

Fred - (Had chosen card with four squares; had placed his four stacks).

Ms. C - And how long will the train be? (Motioned to Fred's card).

Fred - Sixteen.

Peter - Sixteen! At least, that's what my guess is.

Ms. C - And how did you figure it out?

Fred - I added those two together and got eight and then I added them all together to get sixteen.

Ms. C - And how did you do it, Peter?

Peter - I added these two (stacks) and then I counted on.

Ms. C - All right. Did you have another way, Theresa?

Peter - Eight plus eight is sixteen.

Theresa - (No response).

Ms. C - Do you think they're right, Theresa?

Theresa - Yeah.

Peter - We were right (as Fred put train on number track).

Ms. C - Okay, break it apart.

Theresa - (Turned over card). I got it again (card with two squares).

Ms. C - Okay, how many in each stack?

Theresa - Four.

Ms. C - And....

Theresa - Eight. (Interrupted Ms. C).

Ms. C - Eight?

Theresa - (Put train on number track).

Peter - Eight. (Turned over five square card).

Fred - I got that one last time.

Theresa - I got this one(???) (meaning the two square card).

Peter - (Set up stacks).

Ms. C - Okay, how many in each stack?

Peter - Four.

Ms. C - And how many stacks?

Peter - (Counted number). Uh, five.

Theresa - I know what it is.

Ms. C - Okay, and ... hang on (to Theresa) when you make a train how long will it be?

Peter - (Counted part of stacks). Twenty. (Assembled train).

Theresa - (As Peter doing train). It's going to be twenty. Twenty.

Theresa - (As Peter finished train on track). Twenty. I told you twenty. (Last comment in low voice). I told you it was twenty.

Ms. C - Well, there you go. Peter was right. What we're going to do now is multiply.

Theresa - Do we have to?

Ms. C - This...

Theresa - Now you tell me.

Ms. C - ... and the giant strides game where you put the Plasticine and the game you played with Mr. G with the sets?

Theresa - We know that.

Ms. C - That's multiplying too. Do you think multiplying is a hard thing?

Theresa - Now you tell me that it's done.

Peter - Is it times?

Ms. C - Yes. Multiplying and times are the same thing.

Fred - We took some number....

Theresa - So you have to have the same number....

AHW - Theresa just made a very smart comment. Did you hear what she said?

Theresa - What?

AHW - What was it that you just said?

Theresa - I don't know.

AHW - Fred was talking about having groups and you have some in each group. What did you say?

Theresa - You have to have a set of even numbers.

AHW - Can you multiply by three? In groups of three?

Theresa - (Picked up four rods). Four. By four.

Ms. C - Can you put together groups with three each?

Theresa - Yeah.

AHW - Yeah. They don't have to be even numbers. That wasn't what you said. You said they have to be....

Theresa - The same.

AHW - The same. That's exactly what you said. That's important.

Theresa - No, I meant even when you put them like this (held two rods side by side).

**** Clean up ****

(During clean up, AHW talked to Fred; Peter not listening??)

AHW - When Theresa was talking earlier about three times four, she was talking about three groups of four. So she was talking about groups that have the same number. When you think about all the things you've done in the past two weeks, when you made groups, all the groups had to have the same number in them, right? That's a very important idea in multiplying (left group).

Theresa - (To Fred). I got no wrong answers. I got (??) in seven times.

?? - Do we have writing?

Ms. C - Yes. We have writing.

Theresa - (To no-one in particular). Why didn't they tell us we were multiplying?

Ms. C - (To class). The date today is March the seventh.

Theresa - (To Peter who had not yet taken his journal from the middle of table). Would you pay attention. (Hit him on head with journal).

Fred What do we do with the extra one?

Ms. C - Is it yours?

Fred - No.

Ms. C - Are you going to write in it?

Fred - (Set Jenny's journal aside: opened own).

Theresa - (Looking through journal). That's how much I wrote on this page (showed Fred).

(Class continued to write for approximately five more minutes).

DAY 11: IN LAB: Study group (with Ms C) Num. 5.4/1 Number Stories.

Study Group - (Some information missed or very hard to hear on tape; laboratory really noisy and children in study group did lots of chatting as they worked).

- (Theresa, Peter, Fred, Jenny and Alex).

Ms. C - What we have here are some word stories and we want to figure out the multiplication sentences that go with them. So we'll be able to use these (manipulatives) to help us figure them out and we can use these too if we want.

Alex - They're stuck together.

Theresa - Should we take 'em apart?

Ms. C - No, just leave them there for now. I'm going to take some number cards and put them down.

Alex - (Began to read card aloud).

Ms. C - Each must have ... (from directions; as read to self, children kept talking).

Theresa - Oh, this is adding.

Fred/ Alex - No, multiplying.

Ms. C - Here, I'll give everyone a piece of paper, I changed my mind. I need to get some pencils (went to get).

Theresa - I already have a pencil.

Alex - Whoopee (sarcastically).

Ms. C - (Returned). All right.

Alex - I'm going to write my name on here. (As Ms. C began to give directions).

Ms. C - Read from story card). There are three blackbird nests. Each blackbird nest has ... (turned over number cards to fit into blanks on story card).

Fred - Four?

Alex - Three?

Ms. C - There are six blackbirds in it. If you want you can show this with the materials, or you can draw it on your paper or you can write it down to help you figure it out. But I want to see how you would figure it out.

Theresa - Do we have to do it right?

Ms. C - You're drawing a picture. How are you going to do it wrong? I don't understand. There's a tree and there are three blackbird nests in it and each nest has six blackbirds.

Theresa - There are three blackbirds and you like to draw ...(said to whom??).

Ms. C - So if you want you can draw a tree and put the three nests in it and put six birds in each nest or you could pretend these are nests. (Yellow circles used in "Make a set").

Theresa - Can we use the whole piece of paper?

Ms. C - Yeah. We're not going to take forever. Take two minutes to really quickly do that.

Alex - There are three nests....

Ms. C - Three nests, six birds in each nest.

Theresa - Three nests ... put them together....

Fred - (?) Eighteen!

Alex - Six birds in each thingamajigee.

Theresa - There are six black boards ... (??) eighteen.

Ms. C - Read what is says, Theresa.

Fred - (To Alex). Did you put it altogether?

Alex - (Counting by ones; appeared to be counting birds in individual nests). One, two ... (up to) eighteen, nineteen.

Jenny - It's eighteen. How many nests have you got?

Alex - Three.

Jenny - How many birds in each?

Alex - Oh.

Ms. C - Peter, how many birds are in each nest?

Peter - Six.

Fred - Can we write the answer now?

Mr. C - Yeah, you can write the answer.

Jenny - Could I put the math sentence down?

Ms. C - You can put the math sentence down if you want to.

Alex - Oh, yeah. (Said sentence aloud as wrote). Three multiplied by six equals eighteen.

Ms. C - Okay, super. I see all of you have written down a math sentence. What does the three stand for in the sentence?

Jenny - How many groups there are.

Ms. C - And what does the six stand for?

Jenny/ Theresa/ Alex - How many in the group?

Ms. C - How many in each group. I see that these three children (Peter Theresa, Jenny) drew a picture of the tree and then they showed that there are three nests in it and there are six birds in each nest. And then what did you...?

Jenny - (Interrupted). And we did it....

Ms. C - Hang on, Jenny, you had a good idea. We'll talk about that in a minute. What did you guys do to find the answer? (To Alex and Fred).

Fred - I added six and six and then I counted six from there.

Theresa Peter - Yeah, same.

Ms. C - Is that what you did? Okay, good. And Jenny and Alex and I would like you to listen 'cause it might give you another idea. (To Theresa and Peter), how did you guys figure out the answer?

Jenny - We put three nests with six in each ... three groups with six in each ... three groups with six of these in them. And then we put them together and added. (Had put together a train of cubes).

Ms. C - Okay. And you got eighteen both ways? Okay, excellent. Now if you have space on your paper, you can do this next picture.

Fred/ Theresa - I have space.

Ms C - We'll put this ... and this underneath.... (Arranging number and story cards). Now this has space for a name so we'll put in Jenny's name. It says, Jenny was not feeling well. Five of her friends each gave her ... four peppermints. When she counted them all, she found she had ... some peppermints. You may use the things or you may use pictures to help you figure out how many peppermints J got. Five of her friends gave her four peppermints.

Theresa - So we keep on adding fours?

Ms. C - You write it down. Show me how you'd figure out the answer.

- (Fred, Alex, Jenny used cubes to make train; Theresa drew; Peter ?).

Fred - (Kept repeating) four (as he put cubes together).

Alex - Five! (To whom??).

Theresa - Should we write the math sentence?

Ms. C - Yes, please.

Peter - Can I just draw sticks?

Ms. C - Yes.

?? - Stick them all together?

Ms. C - (To Fred as he built train). So how many groups do you have?

Fred - Five.

Ms. C - How many in each group?

Fred - Four.

Alex - And did you say each of her friends?

Ms. C - Yeah, each of her friends gave her four peppermints and she had five friends.

Theresa - I have to write J over this or else (???). (Minute later). Why don't you move that pencil?

Ms. C - Okay, do you have five of Jenny's friends? (To Theresa).

Theresa - Yeah.

Ms. C - And they each had given her how many peppermints?

Theresa - Four.

Jenny - Twenty. (To ??).

Ms. C - (Watching Fred counting train; confirmed that Peter had five friends/groups; back to Fred).

Fred - (Appeared to be having trouble finding answer; began to take apart the train; seemed to be counting one section by ones, next section??). Eight ... eight....

Ms. C - Okay, Alex, are you ready? And if you could write the math sentence to go with it that'd be great.

Theresa - Oh, I know. Now I've found it (Reference to answer??).

Ms. C - If I could have everyone's attention so we can look at how each person solved the problem. Peter, can you explain to the kids what you did?

Peter - I made circles (indicated on paper).

Ms. C - How many circles did you make?

Peter - Five.

Ms. C - And why did you make five circles?

Peter - 'Cause that's the friends and they each gave her four peppermints.

Ms. C - So how did you figure that out? What was the answer to that?

Peter - I counted. It was twenty.

Ms. C - Fred, how did you do it?

Fred - (Showed train). I took the cubes. I made four ... five groups with four in them. Then I stuck them together and then I took off eight and eight and then I took four and counted them.

Ms. C - And what was your answer?

Fred - Twenty.

Ms. C - Okay, so you took off eight and eight. Did you add those two together? Then?

Fred - Yeah.

Ms. C - And how many did you get?

Fred - Sixteen.

Ms. C - And then did you count on?

Fred - Uh huh.

Ms. C - And what did you do, Alex?

Alex - I got five of these ... what do you call them ... cubes and I got four of these, five times and I stuck them together. And I got twenty (showed train).

[**Interesting that neither child using trains had done the groups in the same colours to make them easier to distinguish**]

Ms. C - And did you do a math sentence to go with that?

Alex - Yup.

Ms. C - And it was?

Alex - Five multiplied by four equals twenty.

Ms. C - Jenny?

Jenny - I just did what I did last time.

Ms. C - Same idea? And Theresa?

Theresa - I drew five stick men ... stick girls ... and then I wrote five fours and then I put pluses and added them up.

Ms. C - Now this is interesting. I'd like everyone to look at this.

Fred, I'd like you to look at this and you, too, Alex. This is the way that Theresa figured out her answer. She knew that each person gave four and that's four plus four plus four plus four plus four and that's what she wrote down here. So then you used that to help you figure out what five times, five multiplied by four means?

Theresa - (Nodded).

Ms. C - Good. That's one way to do it. Multiplying is ... this is what multiplying is.

Fred - (Looked startled).

Ms. C - It's adding the same number over and over and over again.

Theresa - Thank you for telling me. [Odd tone; not really sarcastic but not sincere either].

Fred - So if you have four times four....

Theresa - Then that'd be ... (gestured to paper; hidden from the video camera by her body).

Ms. C - Right. So that's all you need to think about. If you forget how to figure it out, you know that it's five multiplied by four means five, or four, added together five times. Pretty simple, huh?

?? - Yeah.

Fred - So if it's the other way, it's four ... (comment lost as Theresa interrupted; can't even lip read well enough from video).

Ms. C - Okay, let's do one more and then it's time to move on. This'll be a little tricky 'cause there are five (??).

Fred - (Pointed to all three boys). Me, him, him.

Ms. C - The boys in one? Okay. This one is a little bit different so pay attention. It says at each palace gate there must be six guardsmen on duty. The palace has three gates.

Alex - There must be six.

Ms. C - Here, I'm going to change that.

Fred - We already did that.

[** Good observation, same factors as first number story**]

Ms. C - The palace has four gates and at each gate there are six guardsmen on duty. Okay, figure that out.

Theresa - I'll use stick guys.

Ms. C - Now you need to think what the problem is saying. Four gates. At each gate, there are six guardsmen.

Jenny? - Would it be like six times four?

Ms. C - How many gates are there?

Peter - Four.

Ms. C - Four gates.

Fred - These yellow ones are not that good 'cause they don't stick together.

Ms. C - You're right.

Theresa - Okay, then the answer would be....

Ms. C - (??). Six....

Fred - (Putting train together; separated into sections again; uncertain about how to proceed??).

Ms. C - Fred, could you just ... ?? ... separate. It's too hard...?

Theresa?- I got six and then I just added (??).

Ms. C - How many guardsmen at each gate?

Fred - Four.

Ms. C - No.

Alex - Six.

Fred - Six.

Theresa - (Twelve seconds later). There's six on each gate?

Ms. C - (Nodded).

Fred - (Looked around, especially as Alex wrote 6 x 4 = 24; talked with Alex).

Ms. C - Jenny, are you almost ready?

Jenny - (Affirmative reply).

Ms. C - How many groups do we have?

Fred - Four.

Ms. C - So which number should come first in your multiplying sentence?

Fred - Four.

Jenny - (Showed four fingers).

Ms. C - Four. Multiplied by.... Now, you get the same answer. Do you remember we talked about that yesterday?

Group - (Nodded).

Ms. C - You have four groups with how many in each group?

Group - Six.

Ms. C - How many guards are on duty?

Fred - Six.

Ms. C - How many guards are on duty in the whole palace?

Alex - Four. I mean, twenty-four.

Ms. C - How did you figure that out? I saw Peter counting. How did you figure it out?

Fred - I added six and six make twelve and then I added six and six to make twelve and then I added them together and it was twenty-four.

Ms. C - Alex, what did you do?

Alex - Counted.

Ms. C - Jenny?

Jenny - I added six and six together and then I did it like Fred.

Ms. C - Theresa?

Theresa - At first I had four multiplied by six. (??) ... then I added four more to this (???). (Indicated 4 x 5 equation).

Ms. C - That was a very smart way to do that. You knew that if five multiplied by four was twenty, six multiplied by four was just four more?

Theresa - (Nodded).

**** Clean up ****

DAY 12: IN LAB: Giant strides (Study group - independently): (Peter, Fred,Alex, Theresa, Jenny.

[hard to hear much dialogue from video; had to use audio cassette; some information still unclear**]**

- (Set up number track and game board; mostly done by Theresa)/

Alex - How do you play this?

Theresa - That's what I asked the first time 'cause I missed the day they did it.

Alex - Well, now you've got to tell me.

Peter - Okay, well like....

Theresa - We can use the (??).

- (Group began playing game without really providing an explanation; began turning over cards).

?? - Seven.

Fred - And two strides. Fourteen. Simple as pie. Fourteen (picked up pencil and wrote result).

Peter - I knew it.

Alex - I knew it too even before....

Theresa - (Interrupted). Nice four. Looks like an (??).

Fred - Okay. Your turn. (To Peter). Then yours. (To Alex).

Alex - What do I do?

Fred - Flip over one.

Theresa - I don't want to do nothing.

Peter - Hey, it's his turn. He's first (reference to Alex).

Jenny - I'll go. I'll go. I'll go.

Peter - 'Cause it's his first time.

Jenny - One of each card.

Fred - Flip over a card.

Alex - (To F). Moi?

Fred - (Nodded).

Theresa - Yeah.

Alex - (Turned over length and number of strides cards). Three ... and three.

Theresa - Three times three.

Fred - You make .... Now you put a thing here.

Theresa - Blob!

Fred - And then you count three and you put a blob.

Jenny - And then another one.

Fred - Until you get to three of these things.

Alex - (Looked puzzled).

Theresa - So you do it once more and you go to nine and then you do it?

Fred - And you write it down.

Theresa - You have to have three blobs.

Peter - Turn it around. (Reference to piece of paper??).

Theresa - There's twelve on it. (On piece of paper??).

Alex - I don't get this game.

Theresa - Neither do I. I only did it once.

Alex - So you've got to count three three times?

Fred - Yeah.

Alex - Got it!

[**Remarkable that he caught on so quickly considering how scrambled the directions were and that he hadn't had a model**]

Fred - And then whatever ... four three times or five three times.

Alex - Gotcha. Gotcha.

Jenny - (Turned over next cards). Six. Four.

Theresa - Nice four.

Fred - Twenty-four.

Theresa - (To Fred). You always want to answer it, don't you?

Alex - That's 'cause he's so smart.

Jenny - (About to put down blob on twenty-four).

AHW - Jenny , before you figure out what the answer is, what's the step you're supposed to do?

Fred - Guess.

Peter - Guess.

Jenny - I have to make six spaces and four strides.

AHW - Before you do that, did you hear what Fred just told you?

Theresa - (Whispered). Guess.

Jenny - Oh, guess.

AHW - What is it you're supposed to guess?

Jenny - Where we'll end up? Where the giant will land?

AHW - Where the giant will land. So before you put the blobs, what you need to do is estimate. You need to make a guess.

Theresa - So she can just say any old number?

AHW - An estimate is a smart guess. It's as close as you think you can make it. It's not just any old guess.

Theresa - Oh.

Jenny - (Made estimate; began to put blobs).

Fred - Wait. She skipped one. She went over the twenty-four.

Jenny - That's my guess.

Peter - I think it's this one (pointed to twenty-five).

Jenny - (Figured out with strides and blobs).

[Lots of comments as she worked; hard to understand on either tape]

AHW - So J thought it was twenty-four. Was it twenty-four?

?? - Yeah.

AHW - Peter, you said you thought it was twenty-five. Was there any...?

Peter - I counted wrong.

AHW - You were counting along with Jenny and you just counted wrong?

Theresa - Me too.

AHW - Okay.

Theresa - (Turned over cards with 5 and 10).

Fred - That'll be easy.

Peter - Make an estimate.

Fred - (Held up fingers to show his guess to Peter and Alex).

Theresa - (Looked at his hands/guess). Okay, um ... um....

Fred - (Whispered). Five tens ... five tens ... or ten fives.

Theresa - Ten fives?

Peter - My guess is right here. (Pointed to number early on the number track).

Fred - Fifty.

Peter - Mine too.

Alex - Mine too.

Theresa - Okay, I'll go by yours.

AHW - So, Theresa, are you going to agree with everyone else's guess?

Theresa - Yeah.

AHW - Okay.

Theresa - (Put blobs on track beginning at fifty and working backwards by tens; boys counting aloud for her). Oh, shut up. (Went back and added extra clay). I'll add a little bit extra to each one. They're skinny.

Fred - Okay, moi turn. Moi turn.

Theresa - (Wrote answer; showed to Jenny). Like my fifty?

Alex - It's fifty.

Fred - (Took turn).

[**Throughout Fred's turn, was a great deal of chatter; some effort (??not very sincere) to estimate answer: almost impossible to distinguish meaningful individual comments; Alex and Theresa singing**]

Peter - (Took turn).

- (At one point in turn, Peter said, "What if I am wrong?" (Estimate); Theresa's reply, "Then the giants will step on you."

- (Estimates - on Peter's turn). Fifty, two, eight.

[**Same chatter and silliness; at one point, Fred made comment--Theresa pointed at tape recorder**]

[**Interesting that she was more conscious of the tape recorder than of the video camera right beside her**]

Peter - (Moved places to watch tape recorder).

Theresa - (Especially silly comments and behaviour; lots of singing and nonsense sayings).

Alex - (Took second turn).

AHW - (At end of Alex's second turn). Alex, would you explain what you did? Because it was different from the other kids and it was interesting.

Peter - What did he do?

AHW - Did you see what he did?

Peter - No.

AHW - It's part of your job to be watching, right?

Peter - He stole my chair.

AHW - Well, you could have asked him to move.

Peter - I did (angry).

AHW - Well, solve the problem later. Theresa, are you watching what he's doing? (Theresa was playing with clay). Alex, would you explain how you found the answer?

Alex - (Seven second pause; looked embarrassed).

Peter - What did you do?

AHW - You started out with what? And what did you do? With the clay?

Alex - (No response; playing with clay; unable to put into words??; didn't remember??).

Fred - Did you count backwards?

Alex - Yeah, I counted backwards.

Peter - I told you it was minus.

AHW - It isn't minus.

Peter - Well, almost.

Theresa?- Close enough.

AHW - Alex, can you explain what it was you did to figure this out? Would the rest of you keep your hands off the number track please? It's important to hear Alex's explanation. (Pause; no response from Alex). You started by putting the clay at twelve. Why did you do that?

Alex - 'Cause that was my guess.

AHW - Did the rest of you think it was a good guess? Did it make sense?

? - Uh huh.

AHW - What did you do to figure out? This (number cards) says...?

Alex - Four.

AHW - And...?

Alex - Three.

AHW - And what do those numbers mean?

Alex - Four spaces.

AHW - Okay, four spaces and how many times do you do that?

Alex - Three.

AHW - Four spaces three times. So you went to the twelve and you counted backwards? How many spaces did you count?

Alex - Four

AHW - And then you did...?

Alex - Four.

AHW - And then you did three. Why is this one three? These two are four spaces, why is this one three?

Alex - (Inaudible).

AHW - What do the rest of you think about the way Alex figured this out?

Fred/ Theresa - (Comments made--hard to hear but appear to agree with A).

Jenny - (Counted the strides; needed three strides, had taken three).

Theresa - Is it right?

AHW - Why are you asking me? You take a look at it and as a group, you decide whether it's right.

Fred - Mrs. HW, that couldn't be right because it would have to be the same number of spaces in each group. He said four spaces and four spaces and then he put it here. But you can't do that. It's not four spaces (indicating last space = three).

AHW - This group had....

Fred - Three.

AHW - And this group had....

Fred - Four..

AHW - And this group had....

Fred - Four.

AHW - So this group was a different size from the other two.

Fred - Yeah.

AHW - (To group). So, what do you think? Did it work? (Pause).

Jenny?

Jenny - Not when you have the one down.

AHW - Okay, not when you have the one down. Alex did this a different way from the rest of you. Theresa, can you explain how Alex did this a different way?

Theresa - Backwards.

AHW - Pardon me?

Theresa - (Somewhat sarcastically). He went backwards and we went front, forward. Or whatever.

AHW - Okay, so he started at twelve and went back. What were the rest of you doing, Theresa?

Theresa - Going up.

AHW - Going?

Jenny - Up.

AHW - Okay. Good answer.

Alex - Am I (??).

AHW - Yeah. Does one way of doing it make more sense than the other way? (Eight second pause; no response). Do you know what Alex actually did?

Fred - No.

AHW - He did a division question.

Alex - You mean a divide?

AHW - Yeah. You did a division question. You started with a whole part, with the twelve and then you made it into groups to find out how many groups there were. In multiplication is that what we're doing?

Fred - Multiplication?

Theresa - This is multiplication?

AHW - This is multiplication. And in multiplication, you....

Jenny - Go up.

AHW - You go up. In division, it's more like going down. So Alex just showed you a division answer.

Fred - So it's twelve divided by three is four?

AHW - Right.

Peter - And ten divided by two is five?

AHW - Exactly. So Alex is way ahead of you on this one. Would you explain going up instead of backwards? Can you do it that way, Alex?

Alex - (Took clay; sort of laughed).

AHW - The other kids have done a couple for you. Can you remember what they did and explain it that way?

Fred - Isn't he supposed to get a guess down?

AHW - Uh huh.

Theresa - That was his guess.

AHW - Your guess was the twelve, right?

Alex - (Counted spaces). How many spaces? Four spaces. (Put down blobs on 4, 8, 12).

Fred? - Four times three is twelve.

AHW - How big are the groups?

Alex - Four.

AHW - And how many groups do you have?

Alex - Three.

AHW - And where do you end up?

Alex - Twelve.

Theresa - He still landed on the same one.

AHW - He still landed on the same one and it still looks the same.

Jenny - But he had one more blob.

AHW - But it's important to know that he did it in the opposite way to what you had done before.

**** Clean up **** (Conversation during clean up -- Fred/Alex/Theresa discussing where group headed for next lab activity; Peter-"I thought this would never end!"; Alex-"I made a division."; Fred-(to Ms. C) 'Ms. C, Twelve divided by three is four"; Peter-"Alex made a division").

DAY 13: IN LAB: Study Group - (with Mr. G) Num. 5.2/1 "I predict - Here" (with written record); (Peter, Fred, Alex, Theresa).

Mr. G - What your job is here, is when I say 'Go' is to take one of these cards (for putting stacks on) and we're going to look at this card and make the multiplication story. When we look at this card. T, how many groups are we going to have to make?

Theresa - (Counted squares). Nine.

Mr. G - Then what we have to do ... is to take a card and it will tell us ... we have to make nine groups.

Theresa - Of two!

Mr. G - With two in each group. (Took out stacks of two cubes; he and Theresa counted out nine stacks). So we have how many groups there?

Fred - Nine.

Mr. G - And how many are in each group?

Fred - Two.

Theresa - (Began putting stacks of cubes onto card as they had done with Ms. C).

Mr. G - (Put out hand to stop Theresa). So we don't actually have to put them on there. We can just count them to make sure there's the same number. No, just leave them, Theresa.

Theresa - Oh boy. (Pouting tone of voice).

Fred - (Counted groups to confirm number).

Mr. G - So there are nine groups here?

Fred - Two, four, six, eight ... (counting groups by twos).

Theresa - (Watching Fred counting). I know.

Fred - Fourteen or sixteen?

Alex - (Counting cubes by twos).

Mr. G - I see nine groups with two in each group.

Theresa - Eighteen.

Mr. G - So I count it two, four ... (up to) eighteen. And then if I want to write this down into a number sentence....

Alex - A number sentence?

Mr. G - ... the first thing I write down is how many groups do I have.

Peter - Nine.

Mr. G - Nine groups ... multiplied by the number in each group.

Theresa - Nine multiplied by two.

Peter - (Together with Theresa). Equals eighteen.

Mr. G - Nine multiplied by two.

Theresa - I need an eraser.

Mr. G - No, just write over ... that's right, Theresa, eighteen (pointing to her 'mistake'). What does the nine tell us?

Peter/ Theresa - How much groups?

Mr. G - What does the two tell us?

Alex/ Peter/ Theresa - How much in each one.

Mr. G - And it will always tell us, the first number will always tell us how many groups we have. And the second number will always tell us how many are in each group.

Theresa - And then the last number?

Mr. G - So what does ... without saying nine multiplied by two equals eighteen. What does that number sentence tell you? What does it say?

Theresa - Nine groups with....

Alex - Two in each group.

Theresa - ... with two in each group equals eighteen.

[** Theresa's tone of voice quite flip this day.]

Mr. G - (Gave each child a set of number tiles). Okay, what I want you to do when I say go is to take a card from here and do it, use it at least two times before you get another card. So use this one with at least two of these numbers here, okay? (Indicated tiles with numbers). You can build the number if you want to or if you can figure it out without building the number, that's okay too. It's a good thing to build it if you have to. Okay, begin.

Theresa - Do they have to be the same colour? (Cubes in groups).

Mr. G - No.

Fred - Can we pick one? (Card with diagram of stacks).

Mr. G - Yeah. So, Fred, how many groups are you going to make?

Fred - Four.

Mr. G - And how many in each group?

Fred - Four.

Mr. G - So four groups of four.

Theresa - Is this five? Is this five? (Held up card with stack on top?

Mr. G - (Referring to stack). Is it five? (The number tile she had drawn).

Theresa - Yeah.

Mr. G - How many groups do you make?

Theresa - One.

Mr. G - How many are in each?

Theresa - Five.

Mr. G - So that's your job. How do you write that out as a number sentence?

Alex - I got it already! (Reference to own work).

Theresa - One multiplied by five equals five.

Mr. G - Okay, what does this tell you?

Theresa - One group with five in each group equals five! Can I pick another one? (Number tile).

Mr. G - Yeah.

Alex - That was pretty easy. (Own task).

[**Relatively little interaction because each child had own set of cards/tiles. Occasional comments from Alex to Fred.

[**Alex/Fred making groups and laying them on table to figure out answer; Theresa/Peter-putting groups onto card to confirm right number of groups.]

Mr. G - What did you find out, Peter?

Peter - Five ... I just gotta add one to each one.

Mr. G - Oh ... no. How many groups do you have?

Peter - Two.

Mr. G - And how many in each group?

Peter - Two.

Mr. G - So what's your multiplication sentence?

Peter - Two multiplied by two.

Mr. G - Two multiplied by two. You want to write that down?

Peter - (Wrote).

Alex - Two groups of five ... that was easy. I just had to take off one.

Theresa - Look, I need to make five groups of six.

Mr. G - How many groups do you have, Peter?

Peter - Two.

Mr. G - (Pointing to equation). Which number in your number sentence tells you how many groups you have?

Peter - (?? hard to hear but sounded like) Second.

Mr. G - Point to it.

Peter - (Pointed to paper).

Mr. G - What does that number tell you?

Peter - How many in the group.

Mr. G - How many in the group. Which number tells you how many groups?

Peter - First.

Mr. G - The first. Good for you. (Turned to talk to Alex). Okay, Alex, what are you up to?

Alex - I'm doing the card. (Turned over new card). Oh, baby! One, two ... (up to nine). (Turned over number tile). Oh! Six in each group!

- (Alex checked with Theresa to see whether he could use some of the cubes in front of her).

Fred - (Turned over card with ten stacks; number tile was five; made five or six groups then put them back into the general cubes pile).

Mr. G - What did you find out?

Fred - I didn't find out anything. I just ... (??hard to hear because was seated away from the tape recorder).

Mr. G - How many groups did you make?

Fred - Ten. I need to make five groups of six.

Mr. G - So what's your first number going to be?

Fred - Ten.

Alex - (Still gathering cubes and making stacks of six).

Theresa - (Gave him some six stacks). These are all sixes. There's a six. There's a six and there's a six.

Alex - Seven, eight, nine. I just need these. Thank you very much.

Mr. G - Okay, Peter, how many are you choosing? You're only choosing four.

Peter - (Had been working on 2 x 4; had two four stacks on card; appeared to figure out on fingers; but had two number tiles in front of him; when Mr. G looked, Peter appeared to be looking at the two number cards. (Made motion between cards as if multiplying them?) and Mr. G evidently thought he was multiplying the number tiles; Mr. G put away one number tile).

Peter - (Looked at stacks). Two times four. (Wrote equation; figured out answer on fingers; wrote answer).

Mr. G - (With Fred). Ten groups with five in each group equals fifty in total. (Reading as Fred wrote).

Alex - Nine multiplied by six? I know that's eighteen. Eighteen plus nine ... twenty-seven plus nine ... thirty-six....

Theresa - All of them are six?

Alex - Thirty-six plus nine....

Mr. G - All right, what does this number tell you?

Alex - Aaah! I got confused now!

Mr. G - What number does this tell you? When you look at this number, what does it tell you? (Pointed to equation already finished on paper).

Alex - Three groups of five.

Mr. G - Equals fifteen. What would happen if ... different problem. What would happen if you put the five here and the three here? (Reversed order).

Alex - You'd get the same answer.

Mr. G - You'd get the same answer but would it look the same?

Alex - No.

Mr. G - Why not?

Alex - Numbers are switched around.

Mr. G - Three groups with five in each group. What would happen if I had the five here and the three here? How many groups would I have?

Alex - Five.

Mr. G - And how many would be in each group?

Alex - Three.

Mr. G - And so they would look different but you would still get the same answer.

Theresa - Mr. G, I'm all out of cards.

Mr. G - Okay.

Theresa - (As Mr. G sat down beside her). I did more than this but I can't find (??).

Mr. G - Okay, do this one (pointed to page).

[**More conversation now between Alex and Fred; Peter working independently, no conversation with anyone.]

Theresa - This one?

[**Conversation hard to hear because of Alex's counting out loud.]

Theresa - Six multiplied by five. Oh, that's easy. It's thirty!

Mr. G - How did you know that?

Theresa - 'Cause I had it the other way.

Mr. G - Oh. What does this number sentence tell us?

Theresa - Five multiplied by six. Five groups with six in each group ... equals thirty. And if it had another six in it and then that could be six. (Equation would be 6 x 6?).

Mr. G - I hope you're not talking like a baby. I hope you're talking like a grade three girl. (Theresa's tone of voice had become quite babyish). So this one tells you you have how many groups?

Theresa - Six.

Mr. G - And this one tells you how many groups?

Theresa - Five.

Mr. G - So you get the same answer.

Theresa - Thank you. (Very babyish voice).

Mr. G - How are you doing, Peter?

Fred - I'm on my third one.

Alex - (Still on 9 x 6; counted to fifty-four by nines). Nine multiplied by six equals.... Oh, God! Now I've got to start again! Mr. G, I counted all of them and now I forgot the answer.

Mr. G - How did you count them?

Alex - Like this. That's twelve.

Mr. G - You counted them by how?

Alex - Like this. Twelve, thirteen ... (up to) twenty-four.

Mr. G - How much is in each group? What would be a good way of counting those?

Fred - I think I know. Adding six and six is twelve and then two more is twenty-four and then counting on.

Mr. G - Good, so show that to Alex.

Fred - Okay. Six and six ... and then you add twelve more. That's twenty-four. (Demonstrated with stacks).

Alex - Oh, now I get you!

Fred - And then count on upwards.

Mr. G - What would happen.... Is there another twenty-four there anywhere?

Fred - Yeah.

Mr. G - Oh. So show me the other twenty-four. So you've got two groups of twenty-four and a group of six, right?

Alex - Twenty-four plus twenty-four....

Mr. G - Is?

Alex - Forty-eight!

Mr. G - And then you have another group of six.

Alex - (Pause while counted). Fifty-four! (Looked at Fred's work). Are you using them like that?

Fred - Yeah, I'm using one, two (up to) seven.

Alex - All right! Awesome!

Theresa - Can I use three of them?

Alex - Don't take them apart.

Mr. G - Okay, how you doing, Peter? (No audible response).

Peter - (Working on 10 x 4).

Mr. G - How many groups are you making?

Peter - Four.

Mr. G - How many groups?

Peter - Ten.

Mr. G - Ten groups, and how many in each group?

Peter - Four.

Fred - I need one cube.

Theresa - But I need that.

Alex - Oh, man. Now I need to use this other card again.

Theresa - I need some more.

Alex - Nine multiplied by three, that's going to be easy.

Fred - Hey, there's no more cubes. Who robbed all the cubes?

Theresa - I got forty-two!

Fred - Eight times four. Thirty-two?

Mr. G - Eight multiplied by four is thirty-two.

Theresa - Is that right?

Mr. G - I don't know. Is it right? How would you figure it out?

Theresa - Add these together, then these together and that's ... who cares ... up to twenty-four and then I'd add this and then I'd count all of these.

Mr. G - So you have seven groups of six. This is twenty-four, and what's this?

Theresa - (Counted from twelve to eighteen).

Mr. G - Twenty-four and eighteen.

Peter - (Counted number of stacks; counted number of squares again on card; wrote 10 x 4; figured out answer on fingers and wrote it).

AHW - Peter, when you count on your fingers, can you explain to me what you do? (Close-up of camera had been on him counting fingers).

Peter - They're in groups. (Indicated hands).

AHW - How does counting on your fingers help you to figure it out?

Peter - Okay, four (pointed to 10 x 4 on page ... you have four on one hand....

AHW - So each one of your fingers is the same as a group of four?

Peter - (Showed by counting groups of four on each finger). So the answer is forty.

AHW - Could you repeat what you just did and this time count out loud?

Peter - (Repeated with counting more audible this time).

AHW - Suppose this was ten times five ... no, ten times three. How would you do that?

Peter - Same thing! (Began counting each finger as group of three).

AHW - (Interrupted at twelve). Okay, so every time you're doing groups ... this group (picked up four stack) is the same as what you count on this finger and the next group is the same as you count on this finger?

Peter - Yeah.

AHW - What about if you had three times four?

Peter - (Put up three fingers; touched two but didn't appear to be counting).

AHW - Okay, so you knew that four plus four is eight and then you just counted on from there?

Peter - Yeah.

AHW - Who taught you that?

Peter - (??) (**Think he said his dad).

Mr. G - (Still working with Theresa on 7 x 6).

Theresa - I know how I could figure it out! (Rest inaudible).

Mr. G - Could you do that in your head? Six plus six is....

Theresa - Twelve.

Mr. G - Plus six.

Theresa - Eighteen.

Mr. G - Plus six.

Theresa - Twenty-four.

Mr. G - Plus six.

Theresa - Twenty-nine. Stopped and counted by ones with some help from Mr. G). Thirty. (Rest of conversation too hard to pick up).

Alex - I'm done with this stinking card.

Theresa - I'm done all of these cards.

Mr. G - Okay, do another card.

Theresa - I'm done all of these cards.

Alex - Could I have one? Not that one. I want this. Two! Ha! Ha!

Mr. G - Didn't you do twos already?

Fred - Yes.

Theresa - No, he didn't do twos.

Alex - What do you mean?

Fred - Twos. You didn't do two groups.

Alex - No, I didn't.

Theresa - I know this one. That's why I picked it.

Fred - Eight. Is there an eight on these cards? I keep getting (??).

Alex - Oh, wow! (To Fred).

Fred - Which cards haven't I done? Is someone done with this? (Picked up card from centre of table).

Theresa - Yes.

Alex - I made a mistake. Anyone got an eraser? Peter, do you got an eraser?

**Camera close-up on A as wrote; erased part of work.

AHW - Alex, could you just cross it out and write it again? Write the part that you fixed separately? Okay?

Mr. G/ Theresa - (Working on two times six).

Fred - Mr. G? Two multiplied by five is ten.

**** Clean up ****

Theresa - I need to do this! I just started it!

Alex - Well, too late now, my dear.

Theresa - I can't do it now. I don't know what eight times five is.

DAY 17: IN LAB: Study Group - 'Books of Two' (independently); Arrays game (with Mr. G); (Theresa, Fred, Alex, Peter, Jenny).

[**Little conversation took place; mostly watching children assemble and write their books.]

Ms. C - The group at that table needs to make the books of two like the one in the tub.

Theresa - Ones with ten pages in them?

Ms. C - Yes.

- (Children counted out sheets to use for book).

Theresa - They aren't all the same size!

AHW - Why are you telling me?

Theresa - Well, they're not!

AHW - Well?

- (Children put together and stapled sheets; conversation was children counting sheets; complaints that pages were uneven).

Theresa - Now what do we do? (No response).

Peter - Yeah, I got an even one, an even pair.

Fred - (Looked at Theresa's book). You have a big one!

Theresa - (Reply inaudible).

- (Theresa took pencils from tub).

Peter - Have you ever been in the newspapers?

Alex - Yeah.

Fred - (To Ms. C). Do we write groups of two?

Peter - Me too. Me and my brother and my other brother playing hockey. I got a picture and they put it in the Journal.

Alex - Why?

Peter - Well, I didn't see it but my brother said he saw it. Playing hockey at the (??).

Alex - (To Ms. C). These papers aren't even even.

Ms. C - Don't worry about it.

Alex - Oh, just grab ten. Oh, okay.

Theresa - (To no-one in particular). How much staples do we put in?

Jenny/ Fred - (Working without talking to anyone).

[**Took about five minutes for children to get books put together; A took longest time.]

Fred - (Had sample book in front of him; Thersa-flipped it back to the cover).

Theresa - Come on!

Fred - Come on, Theresa!

Theresa - I'm trying to see what to put on it.

Fred - (Opened book so cover was visible to Theresa and he could see the interior).

Fred - You're picking fluorescent colours?

Jenny - One times two equals two. Ow, it poked me!

Peter - All of the greens are fluorescent.

Fred - This is weird. This is weird.

Alex - These papers keep on messing up when I'm putting them in the stapler.

Fred - My finger just got jabbed. (Showed to Jenny).

[**Three minutes of working; occasional comments such as next three.]

Fred - Three times two is six.

Alex - Where's the pencils?

Peter - In the tub. (Alex got pencil).

Alex - What do we do? Where's the book? (Took book from in front of Fred; had brief look then J took it away; about one minute later Fred took from in front of Jenny).

Theresa - One times two equals two, okay?

Fred? - You're only using fluorescent colours?

?? - Four times two equals....

Theresa/Jenny - What are you doing?

Fred? - Two times two.

Peter - We have to do this.

[**Fred kept checking with the sample book; others looked less.]

- (Discussion about what colours were preferred; "I have to pick up the purple before it's too late"; "I guess I'll have to use dark colours, too. 'Cause I don't want to use the same colours over."; "I got the last two fluorescent").

Fred - Two times four.... I need the book, please.

Jenny? - Where's the red? Is there any more red?

? - Ms. C used all the red.

Jenny - I'm on five times two.

Fred - I'm on four times two.

Alex - I'm on three times two.

[**Working time about one minute.]

Theresa - What's the next one in the book? Where's the book, Alex?

Jenny - (Picked book up from center of table).

Theresa - What's the third one?

Jenny - The third one? One, two, three. Three times two.

Ms. C - How are you guys doing? I would like to be able to see them in groups of two, Fred.

Fred - About to start taking out stickers).

Ms. C - No, I wouldn't take them out. Just draw circles.

Jenny - I put them in lines.

Alex - So did I.

Ms. C - If I can see the groups of two, that's all right, but I need to able to see them in terms of groups of two. You kids probably won't have time to finish. You can take them up to the room and finish them later.

Theresa - What did you say, Jenny?

Jenny - Three times two.

Theresa - Equals six?

Jenny - Is there any more blues left?

Peter - Yeah. Right there.

Theresa - I need one blue sticker.

Alex - Where's the blue?

Theresa - I need one more blue, Jenny.

Alex - Give the girl a blue! (Like carnival barker).

Theresa - I know what I'm going to write down. Eight times two equals....

Jenny - Mr. G, do we do up to twenty times?

Mr. G - Up to ten.

**Clean up and change of activities.

- (With Mr. G).

Mr. G - I'm going to start to explain this game before Fred gets back. (Put down card). Have you ever seen this before?

Theresa - Yeah.

Mr. G - What is it, Theresa?

Theresa - I don't know. (Shrugged).

Mr. G - Does anyone know what it is? (To Alex). Does anyone know the name for this? Peter, if you could sit down. Fred could probably sit right here. Peter, if you sit down we can make sure the camera can see. Now ... have you ever seen one of these?

Fred - No.

Theresa - Yeah, in Art.

Mr. G - A picture like that with the dots?

Theresa - Yeah.

Mr. G - This picture has a special name. It's called an array. It's spelled a-r-r-a-y. And what it does is it tells us a whole bunch of stories. It tells how many groups we have.

Theresa - Three times six (silly grin).

Mr. G - ... a couple different ways and how many we have in total. (Motioned to one side of array).

Fred - There's six groups of three.

Mr. G - Six groups with three in each group. (Counted and pointed to groups).

Jenny - Or ... three groups of six.

Mr. G - Or three groups with six in each. One, two, three. One, two, three, four, five, six. So that's one thing it tells us. It also tells us one more thing.

Theresa - Okay. It's either three times six or six times three.

Mr. G - Three multiplied by six or six multiplied by three. What's the other thing that it tells us? One more thing?

Theresa - It's a card. With dots on it.

Alex - Theresa, quit being mouthy.

Theresa - Well, it is!

Mr. G - Thanks, Alex. ... Six multiplied by three or three multiplied by six. What does that tell us?

Peter - The answer!

Mr. G - All right, Peter! What is the answer?

Peter - I don't know.

Jenny - Eighteen?

Mr. G - How do you figure out the answer? Fred?

Fred - Six (??). Three.

Alex - It's eighteen.

Theresa - Twenty-three.

Fred - Three and three is six ... and three and three (pointing to rows of threes).

Alex - It's twelve.

Fred - So it's twelve.

Peter - Twenty-four.

Alex - (Put up fingers one by one; appeared to be skip counting). It's eighteen.

Mr. G - Eighteen, right. So it tells us the two different number sentences and ... the ... answer.

Alex - Answer. (Same time as Mr. G).

Mr. G - What we're going to do is play a concentration game.

Theresa - Oh no.

Alex - Cool.

Mr. G - Now, what you have to do is to figure out your number sentences, the two kinds of number sentences you're looking for.

Alex - Oh, and we have to get them the same.

Mr. G - And when I flip the cards over, you have to find the matching ones.

Fred - So we have to get three matching cards,

Mr. G - Three matching sentences. So, I'm going to give you a card. Be figuring out in your mind what you're going to be looking for. I don't want you to say it to anybody, just figure it out in your mind. (Distributed one card to each). And you're going to have to be prepared to explain it to get your points.

Theresa - Yeah?

Alex - What? I don't get it.

Jenny - Me neither.

Mr. G - You don't get it? Let me explain it again. I'm going to flip these cards over. You pick up the cards and you're going to have to explain whether you have the right cards or not.

Theresa - (Comment about planning to cheat; hard to make out anything other than "cheat").

Mr. G - Well then, I'll be watching for cheating.

Theresa - I won't show you my cards then. (Making faces at Mr. G).

Mr. G - And you're going to have to explain. You get three points. One point for each card you get right. But you have to explain them to get the point. Are you ready?

? - Yes, I am!

Mr. G - Have you figured out the three things you're looking for?

Fred - (Slapped hand on table). Are we going like this?

Mr. G - No, just pick up the cards.

Theresa - I got one more question. The one on this side (pointed to left) ... which one comes first? (Sounded genuinely confused--not smart aleck).

Mr. G - Okay, let's see what Theresa needs. Show us on this card. (Mr. G showed new card).

Theresa - Does this number come first or does this number come first? (Indicated numbers on top and side of array).

Mr. G - Ahhh. What do you think? You decide.

Theresa - All makes the same answer anyways.

Mr. G - (Put first card down; Fred took). Once I put the card down, if you miss it, you miss it. (Put down second card).

Jenny - (Waved card in air). I don't know what I have to do! I have to explain it?

Mr. G - Once we're finished. Then you need to explain if this matches your array.

Jenny - Yes, it does.

Mr. G - Well, take it then. (Put down next card; Alex took).

- (Game proceeded with little conversation).

Jenny - I think I got all three.

Alex - And so do I!

Mr. G - Okay, sit patiently. Are you watching each time I turn?

Fred? - I got four.

Peter - I missed one.

Fred - (Had three cards in hand; was counting the fots in the arrays).

Mr. G - Okay. (Moved back as though had finished the game).

Theresa - (Grabbed for card in J's hand). That's mine!! (Shouted).

Mr. G - Theresa, that's (??).

Theresa - That's mine. (Stated this time in calm voice).

Mr. G - All right, let's start with Jenny. You guys, it's your job to check her out and make sure that she's done her job correctly. You put them (cards) right down here and if Theresa moves her arm back it would probably help.

Jenny - (Spread out four cards). What do I explain?

Mr. G - Explain how each of the cards fits your array. Fred, do you want to check her out?

Jenny - Five times three is ... there's three groups of five.

Mr. G - Five groups with three in each group.

Jenny - Yeah, five groups with three in each group. And then three groups of five. And then they all add up to fifteen.

Mr. G - Is she right?

Group - (Agreed).

Mr. G - Good, three points for Jenny. Theresa, explain your arrays, please.

Theresa - (Set out four cards). Explain them all?

Mr. G - Just like Jenny did. Whether you have the right ones or not.

Theresa - Oooo. What do you mean, explain?

Mr. G - How do I know that these match your array? How do I know you've picked the right ones?

Theresa - (??); comment inaudible).

Mr. G - Yes, but explain that.

Theresa - Six times five equals thirty. And there's six in each of these (rows/columns) and there's five of these (rows/columns) and that equals thirty. And then it's the same with this one!

Mr. G - Is this the same? (Picked up one of her cards).

Theresa - No, but the answer's the same.

Fred - Um ... there's six groups ... one, two, three, four, five.

Theresa - No, there's six. (Turned card sideways; counted and pointed to columns).

Mr. G - There are six groups. (Demonstrated by running finger down columns).

Alex - Fred! Fred! Nice stringy thing. (Fred had string hanging from his sweatshirt).

Mr. G - All right, Fred, are you ready to explain? Check him up, Jenny.

Jenny - (Looking around room; not paying attention to group).

Fred - I don't know the answers.

Mr. G - Okay.

Fred - Six groups ... one, two ... (up to) six ... with four in each group.

Theresa - I know what the answer is.

Fred - Four groups of six. (Next card).

Mr. G - What is your answer, Fred?

Fred - Twenty-four?

Mr. G - Twenty-four. How did you figure it out?

Fred - Counting.

Mr. G - Besides counting them out, how can you figure them out with the help of these (arrays)?

Theresa - I know!

Mr. G - Let Fred tell us.

Fred - Four and four is eight. Eight and eight is sixteen and then I added the ... no ... I added eight and I counted.

[**During Fred's explanation, Theresa stroking Mr. G's arm.]

Mr. G - Good. All right, Peter, Let's check Peter out.

Alex - (Comment about one card which Mr. G held being twenty-four).

Mr. G - Right, twenty-four. All right, go Peter.

Peter - (??).

Mr. G - All right, how does your card fit your array?

Peter - Five, five.... (Indicated columns).

[**Theresa trying to get cards out from under Mr. G's arm while Mr. G and PV talking.]

Mr. G - So what does this mean? Five groups with three in each group.

Peter - Four.

Mr. G - Show that to me on your card.

Peter - Five and four. (Ran finger down one row and one column).

Mr. G - And last but not least....

Alex - It's true too.

Mr. G - Theresa, please. (Theresa still trying to get card from under his forearm). Just sit back. The camera's trying to get this.

Alex - Explain.

Mr. G - (Nodded).

Theresa - Explain what?

Alex - How these cards match.

Mr. G - How do these cards match your array?

Alex - There's four groups right here with three in them. So this card matches this card. Same with this one.

Mr. G - But how? The card's different. How can they match?

Alex - 'Cause there's three groups with four in each group! ("Obvious" tone of voice).

Mr. G - Okay.

Alex - And the answer is twelve so this card matches that.

Mr. G - Got it up here? (Nods from children). They we're ready for another round. Three points (Jenny), three points (Theresa), two points (Fred), one point (Peter), three points (Alex). All right, keep your arrays.

Fred - How come I only got two points?

Mr. G - 'Cause you didn't give us the answer.

Alex - 'Cause you missed one.

Theresa - (Comment to Fred??).

Mr. G - Keep your arrays please, and pass me your cards.

Fred - What arrays?

Alex - This one. (Held up own card).

Mr. G - (Pointed to array card in Fred's hand).

Theresa - I got this one! I kept this one.

Mr. G - Pass your array card to the person on your left. (Done).

Jenny - That's right. (Not left).

Mr. G - And one more time (done). Pass one more time.

Theresa - One more time. Hey, I remember that one.

Mr. G - Are you ready? (No response). Are you ready? Have you figured out what you're looking for?

Group - Yes.

Theresa - Yes, I'm looking for Alex's.

Alex - (Counting by ones; ??counting array).

Theresa - I got Alex's!

**Clean up signal given; Group B-kept playing; Mr. G assigned points for number of cards and whether they matched.

Mr. G - Good. You guys did a lot better that time. Maybe next time you'll have time to explain. Put them out and let's see who had them right.

[**As playing last round were comments like (Alex-"Yeah. That's my baby.

Welcome to the family.") (Fred-" Mine have names, Homer, Marge, Bart and ?").]

Mr. G - Asked three times for children to hand in array cards; (Alex/Fred-joking about having aces and jacks and betting money.

Theresa - Here's my allowance. Twelve dollars. (Amount of array).

Alex - Twenty-four.

Theresa - Oh, shut up.

**Finish of classtime.

DAY 18: IN LAB (With Ms. C): "Big Giant, Little Giant" Num. 5.5/1.

[**As setting up, Theresa made comment about wanting to be little giant.]

Ms. C - All right. What we're going to do is to do Big Giant, Little Giant again and then we're going to talk about how they're the same and how they're different.

Alex - Again?

Ms. C - Yeah, again. It's really important and we have to make certain everyone understands.

Alex - Oh, really important.

Ms. C - Really important.

Alex - Really important?

Ms. C - Really important. This number stands for ... it says, "My stride is".

Peter - Six.

Ms. C - Six spaces. And I take....

Theresa - How many strides? (As Ms. C turning over number card).

Ms. C - Five strides. (Theresa - said along with Ms. C). Fred, would you be the big giant and make that with the cubes?

Theresa - Are you playing?

Ms. C - Yes, I'm playing.

Theresa - Are you playing?

Ms. C - I'll play.

Theresa - (Counting around group to see what job she'd have on next round). Big giant, little giant, big giant, little giant.

Ms. C - And while Fred is doing that, the rest of you could be figuring out in your heads how many spaces long the big giant's track is going to be.

Jenny - How many spaces long the big giant's train will be?

Theresa - I know.

Ms. C - Yeah, if the giant takes five strides and every stride covers six spaces, how long will he go?

Theresa - Uh oh. Five strides.

Alex - That's pretty easy.

Theresa - How? What is it?

Alex - Thirty.

Theresa - Oh, that's easy.

Jenny - Do you know it?

Ms. C - Okay, Alex and Jenny, you whisper to each other. Peter and Theresa, did you come up with an answer?

Peter - Yeah.

Ms. C - Did you guys come up with an answer? (Yeah). Is it the same answer as them? (Theresa/Jenny looked at each other and laughed).

Fred - I think I know too.

Ms. C - (Instructed Fred to put train together). Do you two girls have the same answer, too?

Theresa - Uh huh. And that means that those two (boys) have the same answer.

Ms. C - That's very smart. Do you have the same answer, Fred? (Nodded). So how long do you think this train is?

Group - Thirty.

Ms. C - I would like someone to describe to me this trip that the giant took. Okay, Theresa.

Theresa - I changed my mind.

Ms. C - About what?

Theresa - About answering.

Ms. C - No, I would like you to do it now, please.

Theresa - The biggie giant took five strides with six little cubes in each stride and it added up to thirty.

Ms. C - Okay. Does that sound like a good way to describe what the big giant did?

Alex - (Nodded).

Jenny - (Comment?).

Ms. C - Now, Peter, I would like you to do the little giant's trip. You know that the little giant is going to have to take strides that are five spaces long and you need to make his trip the same as the big giant's trip. (Indicated length with hands). And I need the rest of you to think about how many strides the little giant will take.

Jenny - How many?

Ms. C - How many strides? Each stride the little giant takes is five spaces long, how many strides will he have to take?

Alex - How many strides ... how many?

Ms. C - Steps. Yeah, each one of these is a stride, a step. (Indicated five rods Peter was making; Peter had put two blue rods together, Ms. C put a white on from the end into the middle to separate the blues).

Alex - I know.

Theresa - I know! (In imitation).

Ms. C - Okay, you guys whisper to each other. (Alex and Fred). Did you guys agree?

Alex - Uh huh.

Ms. C - All right, so you think this is the same length? (To group). How many strides did you think it would take the little giant?

Alex - Six.

Ms. C - And were you right?

Alex - Yeah.

Ms. C - Jenny, can you explain the little giant's trip?

Jenny - Six strides with five in each stride.

Ms. C - Is that a good way to describe it?

Alex - That's backwards.

Ms. C - No, that's what the little giant did. He took one, two....

Alex - No, it's backwards for what the big giant did.

Ms. C - It's backwards? Good. Okay.... (Began looking in number cards).

Theresa - You got to find a six.

AHW - Before you go on, it looked like Alex and Fred had a different idea. (Fred nodded). Can you talk a little bit about how your ideas were different and how you figured it out?

Fred - Mine were six. It says (on gameboard). This says six fives and he said ... five.... I don't know what.

Alex - (Laughed).

Ms. C - What was yours?

Alex - Six fives. (Aside to Ms. C--I counted wrong).

Ms. C - Oh, you counted wrong? (Laughed).

AHW - Fred, how did you explain to Alex how you came up with six?

Fred - I went five, ten ... (up to) twenty-five and then I went thirty. (Put up fingers to show how had kept track). And my answer was thirty.

Ms. C - Okay, I want you to look at the trip the big giant took and the trip the little giant took, and I want you to tell me what's the same about the two trips. J? (Hand up immediately).

Jenny - They ... they ... are the same length.

Ms. C - Good. Can someone else tell us something the same about them? Another thing that's the same about them, Theresa?

Theresa - That's not the same. Mine's different.

Peter - They're both the same numbers.

Ms. C - The same length? They both end at what number?

Peter - Thirty.

Ms. C - Jenny, could you break the steps apart (little giant) and we're going to try to make a rectangle out of it.

Jenny - I know something else. They get to the place at different times.

Ms. C - Okay, they get to the place a different way. Alex, could you break this apart and make a rectangle out of it? Break up the steps and put them beside each other. (Demonstrated).

Alex - Oh.

Jenny - It's going to be shorter.

Ms. C - Okay, I want them out here where we can take a look at them. Now, I want us to look at these and tell what's the same about the two rectangles. Theresa?

Theresa - They're the same length going this way.

Ms. C - What else could we say was the same about them?

Theresa - They're the same the other way.

Ms. C - The same width? Do you think there are the same number of cubes in both rectangles?

Fred/ Theresa - No.

Jenny - Yes. Thirty cubes in each rectangle.

Ms. C - How do you know that there are the same number of cubes in both rectangles?

Jenny - 'Cause when it was over there (in trains) they both were thirty.

Ms. C - Fred, did we add any or take any away when we broke them up and made the rectangles?

Fred - No.

Ms. C - Why did you think they would have different numbers in them?

Fred - Because that one is bigger than this one. It has one, two, three, four, five ... and this has one, two ... (up to) six.

Ms. C - If we put them on top of each other and they're the same size, will we know that they have the same number in each?

Fred - (Nodded).

Theresa - That has five and that has five groups of six.

Fred - This is just like that game we played with Mr. G. It has six down here and five down here.

Ms. C - Exactly and that's why we're playing these games together. You guys are very smart.

Alex - That looks like a jelly-belly.

Ms. C - That colour? Very exciting. This is an important idea to know about multiplying. We talked about it in class but I wanted to show it to you here in the math lab. The idea is that five groups of six and six groups of five look a little different. They don't look exactly the same, do they?

Theresa - Put them up beside each other and....

Ms. C - But you get to the same place. They have the same number in them.

Theresa - Same number.

Ms. C - So this is one, two, three, four, five groups of six and this is six groups of five, but they both have how many cubes in them?

Peter - Thirty.

Jenny - Thirty.

Theresa - That was easy.

Ms. C - This is an important thing in multiplying. Now, are five groups of six and six groups of five exactly the same?

Jenny - No.

Theresa - That has one more.

Ms. C - One more what?

Theresa - Cube on each one.

Ms. C - Are these the same number of cubes?

Theresa - No, when you put them down like this. (One on top of other).

Ms. C - Yes, but we're not.... Are there the same number of cubes in both rectangles?

Theresa - Yes ... except for one(??).

Ms. C - Are there the same number of cubes in both rectangles, Theresa?

Theresa - (Rolled eyes).

Ms. C - Listen to what I'm asking. I know that the stacks are different. This has one less stack but one more in each stack, right? That's how they're different. And it's important. When you're drawing pictures of multiplying and people say, "Show me five times six" or if someone was to say, "Which one of these is five times six", which one of these would you point to? Which one is five times six?

- (All but Jenny pointed to five groups of six).

Ms. C - (To Alex). This one?

Alex - (Withdrew hand and looked at Ms. C).

Ms. C - Don't change your mind just because I asked again. Which one is five times six? (Children all pointed). What does five times six mean?

Jenny - Five groups of six.

Ms. C - Five groups of six. One, two, three, four, five, six. This is the one that shows five times six.

Fred - (Pointed to other rectangle). This is six times five.

Ms. C - This is six times five. Remember the first number tells you the number of groups. Okay, let's do one more.

Jenny - Can you ever tell how many groups ... how many (??).

Ms. C - That's not usually what we mean by it. You know that you can figure out the answer the same way.

Theresa - 'Kay then, there's sixty going to be all together. (Smart aleck tone).

Ms. C - Yeah. If we put those together.

Theresa - Okay, put 'em together, it's sixty.

Ms. C - All right. Theresa, you be the big giant. Take eight strides, no eight spaces ... two strides with eight spaces in each stride. Can you do that? And Jenny, if you're the little giant, how long is each stride you take going to be?

Jenny - How long?

Ms. C - Yeah, how long is each stride that the little giant would take?

Jenny - (Looked at gameboard; no response).

Peter - (Put up two fingers; not sure that Jenny saw).

Ms. C - The big giant is going to take two steps with eight spaces in each step. And you're going to do what?

Jenny - The little giant is going to take eight steps with two in each step.

Ms. C - Okay, great. (To Theresa). Will you let her work out of that? You both can work out of the same thing. (Same container of cubes).

Fred - I figured out the answer.

Alex - I already got it.

Theresa - I already got it and I....

Ms. C - (Interrupted because Theresa had stopped working). Could you get another group of eight, please?

Alex - What is this, eight? Eight multiplied by two?

Ms. C - (Nodded). Two steps, eight in each step. She's doing two times eight (Theresa), she's doing eight times two. (Jenny).

Alex - I know the answer.

Theresa - So do I. It's so cinchy ... I knew that was going to be a two.

Ms. C - J, you need them in groups of two, right? (Jenny--making slow progress).

Alex/ Fred - (Comments about how to find answer).

Fred - Or just say eight plus eight.

Alex - Yeah, just two times.

Jenny - Should I put them in a train?

Ms. C - Yes. And Theresa, can you make a train? How many steps is your giant taking?

Theresa - Two.

Ms. C - Thank you. Good, please put it in the middle. Peter, could you please describe this train?

Peter - The giant made eight strides.

Ms. C - No. Each one of these is a stride. (One stack).

Peter - Eight ... spaces ... and two strides.

Ms. C - Eight spaces two times. Good. And what did the little giant do?

Peter - He took two spaces and eight strides.

Ms. C - (Put trains side by side). Did both giants get to the same place?

Peter/ Theresa - Yeah.

Ms. C - What's the same about the two trips that they are taking? Fred?

Fred - They're the same length.

Ms. C - They're the same length. Good, what else, Theresa?

Theresa - They have the same number. That has two and I had two strides and that has two in each stride and it takes eight strides and I have eight in each stride.

Ms. C - Alex, what can you say that is different about them?

Alex - They look different.

Ms. C - How do they look different?

Alex - They don't look different.

Ms. C - They do look different. What looks different about them? How does this one look different from this one?

Alex - You can't ... you can't count them. 'Cause it's all one colour (big giant) but here (little giant) you can.

Ms. C - Okay, Fred and Peter, make them into rectangles, please.

Jenny - I know what this one....

Fred - Hey, this one can go together. (Trains would link together side by side).

Ms. C - Will the rectangles take up the same space?

Fred - Yeah.

Theresa - At least it looks.... It doesn't look like it.

Jenny - That's a longer rectangle. (Pointing to big giant).

Ms. C - You think that one will be longer?

Alex - No, because he put it that way. (Oriented up and down, not sideways).

Theresa - That's wider. (Little giant).

Jenny - It's taller.

Ms. C - Okay. Let's match them. Compare them and see if they're the same. Are they the same?

Peter - Yes.

Ms. C - Do they take up the same amount of space?

Theresa - What about beside each other? Like that?

Ms. C - Describe this rectangle. (Little giant). What's the multiplying sentence that would go with this rectangle?

Theresa - Oh, that's easy. Two times eight or eight times two.

Ms. C - Which one? Decide.

Jenny - I know.

Theresa - Eight times two.

Jenny - Eight times two.

Ms. C - Eight times two. And this one?

Theresa - I got it (to Jenny).

Fred/ Peter - Two times eight.

Theresa - I'm so confused. That's confusing (to Jenny).

Ms. C - Okay. And now we're going upstairs to write.

**** Clean up ****

DAY 21: IN LAB:

This work was done in the classroom, not the mathematics laboratory. However, the study group was seated together and interacting as if they were in the mathematics laboratory so the information has been included.

Ms. C - Today we're going to do math activities in the classroom. There are four different jobs. This is one of them, a book about multiplying and dividing. (Continued to introduce activities with occasional student interruptions). What you will do in this book, they want to give you some ideas to help you remember your multiplying facts. The children working on this book will be working here. (At table). That way they can help each other. One group of children will be over here on the floor. I have some activities that you can do by yourself or with a friend in your group. This is a mutiplying puzzle so you have to find the pieces that go together. There's a picture and a card that has a multiplying sentence and those need to go together. So that's one thing that's over here on the floor. The other thing is pencil pokes. (Demonstrated with a child)

AHW - (Explained that if a person who can see questions reads them aloud, both partners get practice).

Ms. C - And another thing. This would be a job that you would do by yourself. You would go over here and get a piece of scrap paper and put it down. Now one side has answers on it and one side doesn't. You go through and write down the answers and then when you're done, you turn it over and check the answers to see if you were right. We're not worried about you getting it wrong. We're not worried about if you get it wrong, that you learn the right thing to do. And so those are the things on the floor. Then Mr. G is going to play a multiplying Bingo game with you. And the last job is everyone will get ... everyone will get one of these papers. And what I want you to do is to be able to draw for me how you can figure out the answers. I drew one on here for you. This shows three groups of five and these are supposed to be hands so they have five fingers. You don't have to draw hands. You can draw whatever you like. (Sheet had five times questions) ... what the multiplying answer would be. Here's six multiplied by three so I drew six groups and I put three in each group. So when I know the answer I can put it down. When you're finished all three pages, you can start to work to get it in your head so you don't have to think about it.

- (Children asked administsrative questions about finishing).

Ms. C - (Assigned children to groups).

- (Study group - Jenny, Fred, Peter, Theresa, Alex-- working on multiplying and dividing books).

Fred - They're all so easy.

Alex - Yeah, 'cause it says two times six and six times two, I just put them backwards.

AHW - Is it easier to figure out two times six then six times two?

Alex - Uh huh. If it's six times two, you've got to do two six times.

AHW - Is it easier to add the groups when they're big ones instead of little ones?

Alex - Yeah.

AHW - Smart thinking.

Ms. C - Are the two times ones easy?

Theresa - Yeah.

AHW - (Explained A's thinking to Ms. C).

AHW - It looks like a lot of people have been working on their two times.

Theresa - I'm just looking at these. They have the same answers.

- (Some conversation between Fred and Alex).

AHW - (To Fred). Tell me what you just did.

Fred - (Counted aloud on fingers to keep track of groups of five).

Alex - Times? That scared me!

AHW - What scared you?

Alex - I thought it was plus.

Fred - (Continued to count on fingers).

AHW - (Asked Alex how had gotten one wrong answer; some comments from Theresa but too hard to hear).

Alex - (Counted aloud using fingers to keep track; found proper answer and fixed mistake).

AHW - Can you think of another way to find the answer? Instead of counting on your fingers?

Alex - No.

Ms. C - (Clarifying procedures for Peter; had evidently done matching quetions wrong way ... talked to Fred about the matching questions).

Fred - Are we supposed to do this first? (One side of page).

Ms. C - Doesn't matter.

Alex - Here it's seven times five and here it's nine times five. This is thirty-five (7 x 5) and so you just add two more fives.

Ms. C - Very smart.

Alex/ Theresa - (Conversation didn't want to have overheard?; very quiet voices and looking at Ms. C).

Ms. C - Don't just look at it to get the answer. Look at it and try to remember, okay? (To Fred as flipped back to previous page to see answer).

Fred - Okay.

Ms. C - It's got to be in here (tapped her head) and you've got to know them really fast.

Theresa - (Comment about learning facts?).

- (Continued work with some conversation; mostly silly remarks).

Alex - (New page). What do I do here?

Fred - (Looked to try to help). Tell how many ... count the groups. These are all groups with five in each group. So seven times five ... you have to look for the seven times five in here (on page). Then write the answer.

Alex - Just write the answer and (??)!

Fred - (Nodded).

Ms. C - (Came back). I'm going to do a book so when you get to a certain place, you can check your answers to make sure they're all right.

Fred - I already know one of these is right. That one. (Pointed to page) (Comment to Ms. C - about checking with her??).

Ms. C - Do you need me to tell you, though?

Fred - No.

Ms. C - Did you know that it was right?

Fred - Yeah.

Ms. C - All right then.

Fred - (New page). Ms. C, what do you do on this page? (Didn't read or look very long before asking). Do you just write the answer?

Ms. C - Yeah. If you were counting, you'd have four times five minutes. Count around the clock, five, ten, fifteen....

Fred - Twenty. And then you just put the answer?

Ms. C - Yes.

[**Especially hard to hear Theresa because her head was either down or turned away from camera; made a number of comments to other kids and Ms. C which couldn't pick up with video camera microphone.]

Ms. C - (Helped Fred figure out nine times five).

Alex - (At same page as Fred). I don't get what we do here.

Fred - Same thing.

Ms. C - Fred has figured it out.

Fred - (To Alex). You count by fives till you ... you count by fives until you get to the arrow. And whatever the arrow ... every one of these numbers is five so you jump by fives until where the arrow's pointing at it? And then you write the answer.

Theresa - That's how you get the answer? (Sarcastic).

Alex - So each one of these are five? Five, ten....

Fred - Start at the one.

Theresa - (Asked question about page).

Ms. C - You're counting nickels and so each one is five cents and the first number tells you how many groups.

Alex - How come they (repeat the same questions?).

Ms. C - 'Cause they want you to learn your five times tables.

Theresa - (Pointed to two times chart facing her on bulletin board). We could just look up there!

**** Clean up activities ****

APPENDIX B

Skemp Materials

[Num 5] MULTIPLICATION
Combining two operations

Num 5.1 ACTIONS ON SETS: COMBINING ACTIONS

Concepts
(i) The action of making a set.
(ii) The action of making a set of sets.
(iii) Starting and resulting numbers.

Abilities
(i) To make a given number of matching sets.
(ii) To state the number of a single set.
(iii) To state the number of matching sets.
(iv) To state the total number of elements.

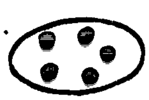
Discussion of concepts

Multiplication is sometimes introduced as repeated addition. This works well for the counting numbers, but it does not apply to multiplication of the other kinds of number which children will subsequently encounter; so to teach it this way is making difficulties for the future. This is one of the reasons why so many children have problems with multiplying fractions, and with multiplying negative numbers. The concept of multiplication which is introduced in the present topic is that of combining two operations, and this continues to apply throughout secondary school and university mathematics. And as a bonus, the correct concept is no harder to learn when properly taught.

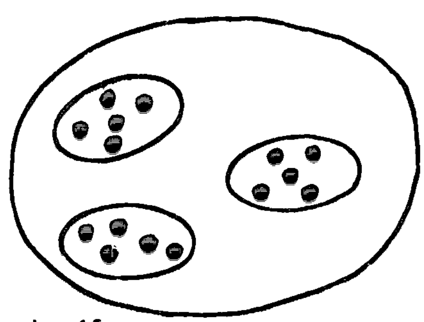
In the present case, we are going to multiply natural numbers. A natural number is the number of objects in a set, and we start with the concept as embodied in physical actions.

First action: make a set of number 5.
Second action: make a set of number 3.

To combine these, we do the first action



and then apply the second action to the result (make a set of 3 sets of 5).



This is equivalent to making a set of number 15.

At this stage, there is not a lot of difference between this and adding together 5 threes, just as near their starting points two diverging paths are only a little way apart. But in the present case one of these paths leads towards future understanding, while the other is a dead end.

So instead of the sequence 'Start, Action, Result' used in the addition and subtraction networks, we shall be using the sequence 'First action, Second action, Combined result'. Later in this network we shall discuss notations for this.

Activity 1 Make a set. Make others which match

An activity for up to 6 children. Its purpose is to introduce the concept of multiplication in a physical embodiment.

Materials

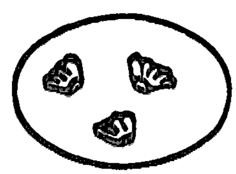
- 5 small objects for each child. These should be different for each child, e.g. shells, acorns, bottle tops. . .
- 6 small set ovals.*
- Large set loop.

* Oval cards, about 6 cm by 7.5 cm.

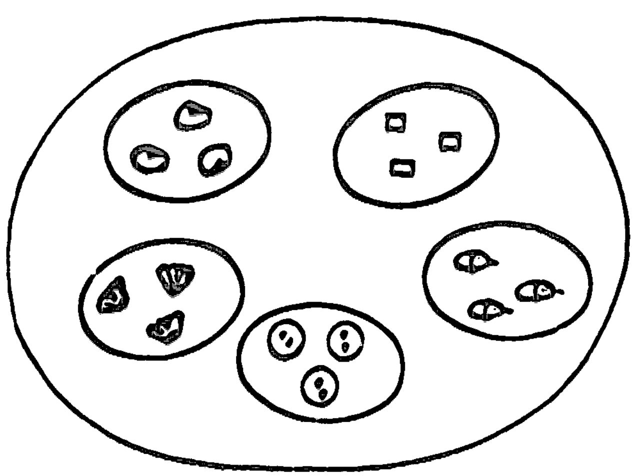
See illustrations for steps 1 and 3.

What they do

1. The first child makes a set, using some or all of his objects. A small set oval is used for this. It is best to start with a set of fairly small number, say 3.



2. Everyone makes a set which matches this, i.e. has the same number. They too use set ovals and then check with each other.
3. All the sets are put in the set loop to make one combined set, which is counted.



4. With your help, they say (in their own words) what they have done. E.g. 'Vicky made a set of 3 shells. We all made matching sets, so we made 5 sets of 3. When we put these together, there were 15 things altogether.' Or 'We made 5 sets of 3, making 15 altogether.' Or '5 sets, 3 in each, makes 15.'
5. The children take back their objects and steps 1 to 4 are repeated.
6. To give variety of numbers, sometimes only some of the children should make matching sets. E.g. everyone on this side of the table, or all the boys, or all the girls.

Activity 2 Multiplying on a number track

An activity for up to 6 children. Its purpose is to expand the concept of multiplication to include larger numbers. The use of a number track saves the time and trouble of counting the resulting sets.

Materials

- A number track 1 to 60 (2 cm spaces suggested).
- 10 cubes of one colour for each child (2 cm cubes suggested).
- Actions board, see figure 8.
- First action cards 2 to 10, e.g. 'Each make a rod with 7 cubes'
- Second action cards 1 to 6, e.g. 'Join 4 rods'
- Slips of paper to fit Combined Result space on Actions board.
- Pencil.

What they do

1. The pack of first action cards and second action cards are shuffled and put face down.
2. The top first action card is turned over, and put face up on top of the pack.
3. Each child makes a rod as instructed. The rods are then pooled for general use.
4. The top second action card is turned over, and put face up on top of the pack.
5. The number of rods indicated is taken, and joined together on the number track.
6. The result is recorded on a slip of paper, which is put in the space on the combined result card.
7. The second action card used is put face up at the bottom on the pile. Steps 3 to 6 are then repeated using the same first action card. This saves re-making the rods every time.
8. When a face-up second action card is reached, this means that all of this pack have been used once. The pack is then shuffled and put face down again.
9. Steps 2 to 7 are then repeated with the next first action card.

Activity 3 Giant strides on a number track

An activity for up to 6 children. Its purpose is to begin the process of freeing children's concept of multiplication from dependence on physical objects.

Num 5.1 Actions on sets: combining actions (cont.)

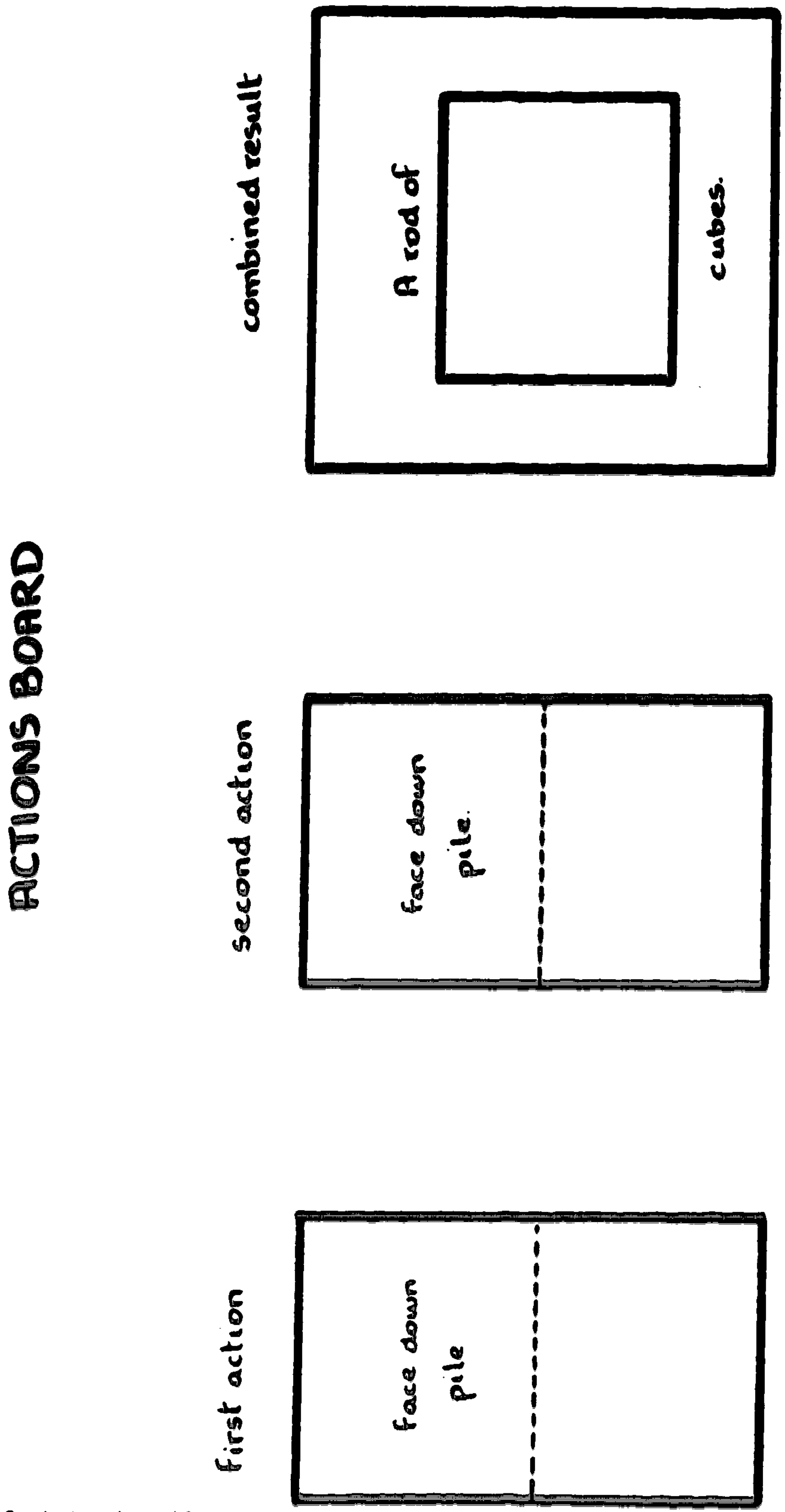


Figure 8   Actions board for multiplying on an number track.

Materials

- A card number track 1 to 50.
- Activity board, see figure 9.
- Length-of-stride cards 2 to 5.*
- Number-of-strides cards 2 to 10.*
- Blu-Tack.
- Pencil and paper.

* Single-headed to fit the dotted spaces on the activity board. Each pack should be of a different colour.

What they do

1. The two packs are shuffled and put face down in their respective dotted spaces on the activity board.
2. The top card in each pile is turned over.
3. Suppose that each stride is 3 spaces, and they take 7 strides.
4. One child puts down 'footprints' on the number track (small blobs of Blu-Tack) at spaces 3, 6, 9 and so on, representing strides each of 3 spaces. The others help by making the blobs for him, and also checking that they are put in the right spaces.
5. This continues until (in this case) 7 strides have been taken. The last footprint will be in space 21.
6. Another child records 21 on a strip of paper, which is put in the last space on the board.
7. The cards are then replaced face down at the bottom of the pile, and steps 2 to 6 are repeated.

Discussion of activities

The first two activities embody, as physical actions, the concept of multiplication as described at the beginning of this topic. In activity 1, the first action is making a set of (say) shells, and the second action is making a set of matching sets. In activity 2, the first action is making a rod out of unit cubes, and the second action is making a rod using a given number of these rods. In activity 3, this is repeated at a slightly more abstract level. The last activity also makes a start with relating multiplication to number stories. The first action is making a stride of a given number of spaces, and the second action is making a given number of strides.

All of the activities in the present topic use mode 1 schema building. No mental calculations, and no predictions, are yet involved. Very simple recording is introduced in activities 2 and 3, in which the children do not have to do any writing themselves.

OBSERVE AND LISTEN **REFLECT** **DISCUSS**

Num 5.1 Actions on sets: combining actions (cont.)

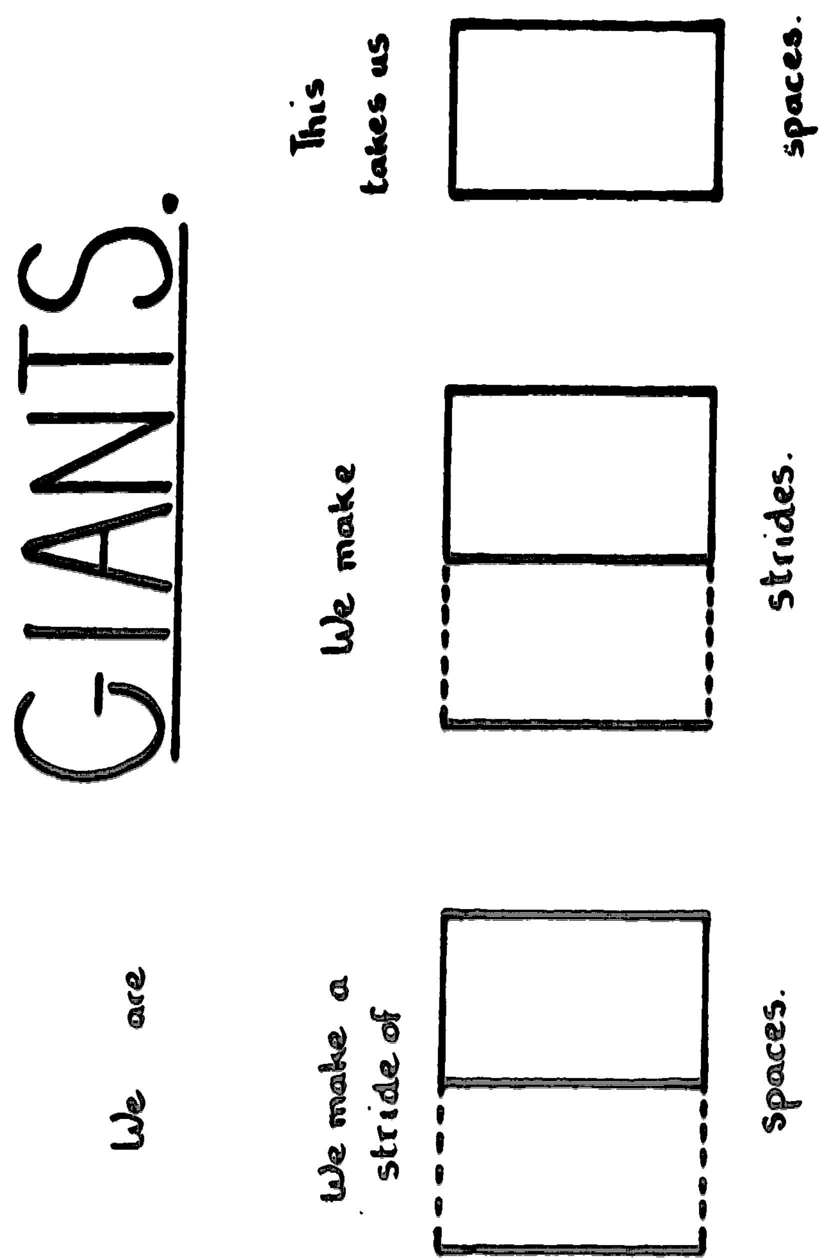


Figure 9 Giant strides on a number track.

Num 5.2 MULTIPLICATION AS A MATHEMATICAL OPERATION

Concept Multiplication as a mathematical operation.

Ability To do this mentally, independently of its physical embodiments.

Discussion of concept

Multiplication becomes a mathematical operation when it can be done mentally with numbers, independently of actions on sets or other physical embodiments. At this stage we concentrate on forming the concept, using the easiest possible numbers as operands. These are first 2, then 5, since the children have already learnt to count in twos and fives. Afterwards they will expand the concept to include multiplication of 4 and of 3.

Note that this corresponds to subitising 2-sets and 5-sets, etc. It does not start us down the path of repeated addition. Note also that in this topic the children are already multiplying these numbers by numbers up to ten.

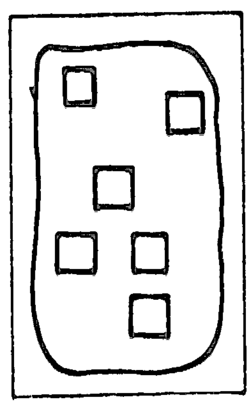
Activity 1 'I predict – here' using rods

An activity for up to 6 children. Its purpose is to introduce multiplication used predictively, as a mental operation followed by testing. It is a development of 'I predict – here' (NuSp 1.1/1).

Materials

- Set cards 1 to 10, as further described below.
- Number track 1 to 50 (2 cm suggested).
- 50 cubes (2 cm suggested).

Set cards
On each is drawn a set loop, and within the loop are drawn squares the size of a cube, in number from 1 to 10. These squares should be randomly placed.



Set card.

What they do

1. The set cards are shuffled and put in a pile face down. (Use only cards 2 to 6 to begin with.)
2. Together the children make 10 2-rods which are pooled for communal use. Each rod must be of a single colour.
3. The top activity card is turned face up, and a 2-rod is stood on each square.
4. One of the children then predicts where these will come to when put end to end on the number track. This may be done by counting in twos. He makes his prediction, e.g. with a piece of Blu-Tack. Adjacent rods should be of different colours.
5. His prediction is tested physically.
6. The track is cleared, the long rod broken up into 2-rods, and steps 3, 4, 5 are repeated with another child making the prediction.
7. When the children can do this well, the activity is repeated using 5-rods. The predictions are now made by counting in 5's.
8. After that, the activity is repeated using 3-rods and 4-rods. For these, the predictions may be made by pointing to each rod in turn and counting (for 3-rods): '1, 2, 3; 4, 5, 6; 7, 8, 9' etc.

Activity 2 Sets under our hands

An activity for up to 6 children. Its purpose is to give further practice in the operation of multiplication.

Materials

- Five small objects for each child.*
- Number cards 2 to 6.
- 6 small set ovals.*
- Large set loop.*
- Pencil and paper for each child.

* As for Num 5.1/1.

What they do

1. The first child makes a set, using some or all of her objects. As before, a small set oval is used.
2. A number card is put out to remind them what is the number of this set.
3. All the children then make matching sets, using set ovals.
4. They cover the sets with their hands.
5. They try to predict how many objects there will be when they combine all these sets into a big set. This can be done by pointing to each hand in turn and mentally counting on. E.g. if there are 4 in each set: (pointing to first hand) '1, 2, 3, 4'; (pointing to second hand) '5, 6, 7, 8'; etc.
6. They speak or write their predictions individually.
7. The sets are combined and the predictions tested.
8. Steps 1 to 7 are repeated, with a different child beginning.
9. As in Num 5.1/1, the number of sets made should be varied, by involving only some of the children. All however, should make and test their predictions.

Discussion of activities

In topic 1, the physical activities were used for schema building. The activities came first, and the thoughts arose from the activities. In the present topic it is the other way about: thinking first, and then the actions to test the correctness of the thinking. First mode 1 building, then mode 1 testing. By this process we help children first to form concepts, and then to develop them into independent objects of thought.

Activity 1 gives visual support for the mental activity of counting on, by which the children are predicting. They can see the number of cubes in each rod, as well as the number of rods. In activity 2, this visual support is partly withdrawn. They can see how many hands there are, but they have to imagine how many objects there are under each hand. In this way we take them gently along the path towards purely mental operations.

OBSERVE AND LISTEN **REFLECT** **DISCUSS**

Num 5.3 NOTATION FOR MULTIPLICATION: NUMBER SENTENCES

Concept The use of number sentences for representing the operation of multiplication and its result.

Abilities
(i) To write number sentences recording multiplication as embodied in physical materials.
(ii) To use number sentences for making predictions.

Discussion of concept

Several notations for multiplication are currently in use. All the following can be read aloud in ways which fit the meaning of multiplication which we are using.

$$5 \xrightarrow{\times 3} 15$$

means 'Make a set of 5.
Make it 3 times.
Combined result, a set of 15.'

This may be shortened to

$$5 \times 3 = 15$$

and read as '5, 3 times, equals (or makes) 15.'

This is easier to say than '5 multiplied by 3 equals 15.' (I do not recommend 'timesed by', as one sometimes hears. It is not grammar, and no easier for the children.) The above notation has the advantage that the order 'First action, Second action' is preserved.

Other notations for the same operation use parentheses, with an equals sign or an arrow.

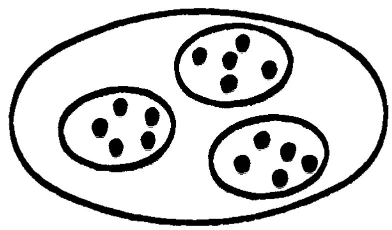
Num 5.3 Notation for multiplication: number sentences (cont.)

$3(5) = 15$ or $3(5) \rightarrow 15$

'3 sets of 5 equal (or make) 15', or '3 fives are 15'.

This reverses the order of the operations, but corresponds well to the diagram below which shows the combined result.

3(5)



I suggest that you use whichever notation you and the children are happiest with, until topic 5.5. Here I introduce a notation for binary multiplication which combines both, and when properly understood makes everything much simpler.

Until then, the activities can be used with either notation. You might think it useful for the children to understand both notations, since they will certainly meet both in their future work.

Activity 1 Number sentences for multiplication

An activity for up to 6 children. Its purpose is to introduce number sentences for recording multiplication as embodied in physical materials.

Materials

- Actions board.*
- First action cards 2 to 5.*
- Second action cards 2 to 6.*
- Five small objects for each child.**
- 6 small set ovals.**
- 1 set loop.**
- Pencil and paper for each child.

* See figure 10. This shows the actions board for ( ) notation with two cards in position (see step 2). An alternative board for × notation is provided in Vol. 1a.

** As for Num 5.1/1 and Num 5.2/2.

What they do

1. The action cards are shuffled and put in 2 piles face down on the Actions board.
2. The first child turns over the first action card. He does what it says, using some or all of his objects and a small oval set card.
3. The next child turns over the top second action card, and puts it beside the first. The board will now look like one or other of the illustrations opposite.
4. The children make as many of these sets as it says on the second card.
5. They combine the sets by putting them in the large set loop, and count the result.

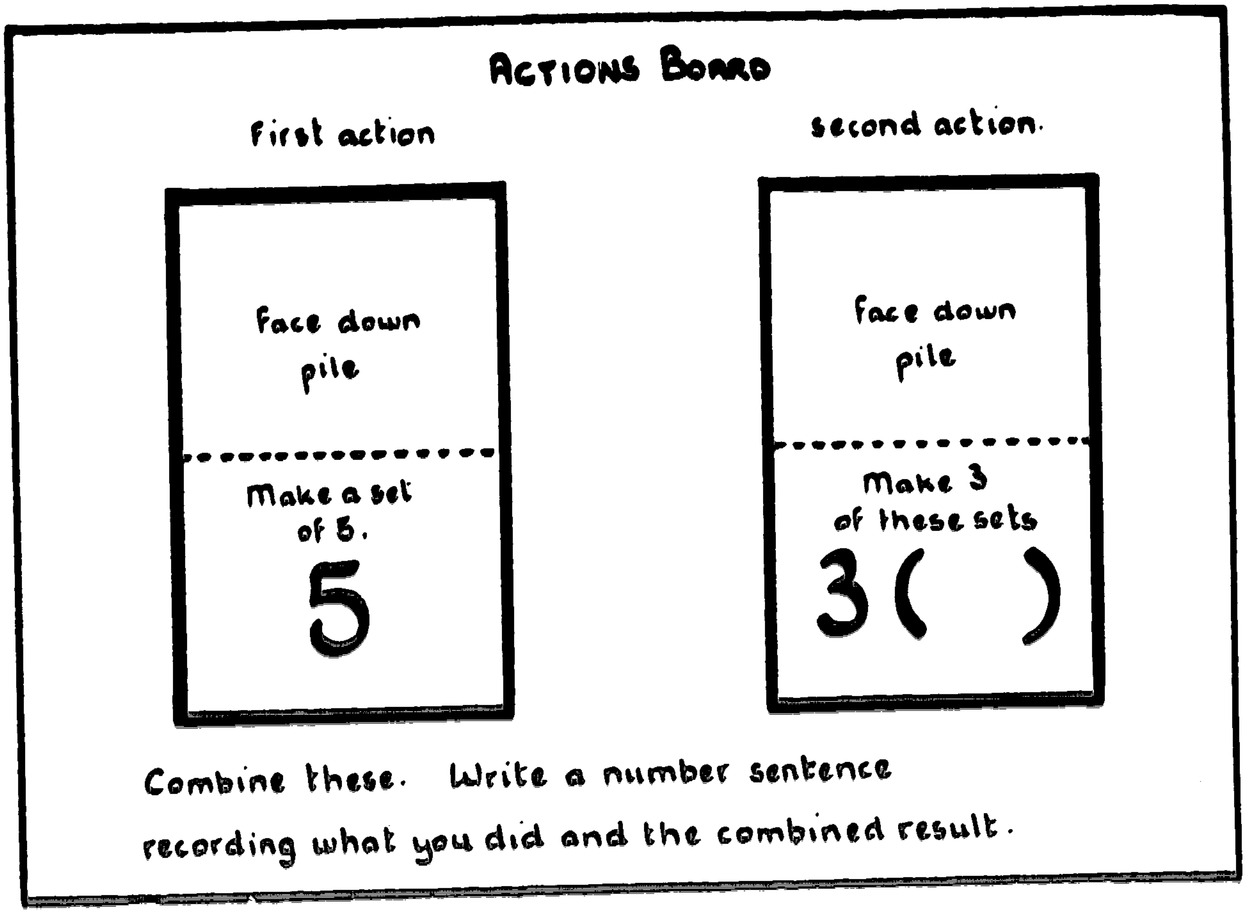


Figure 10 Actions board.

6. They write number sentences in whichever notation you show them. The meaning of these should be carefully explained. In the example shown, either this:

$$3(5) = 15$$

'Three sets of five make a set of fifteen', or
'Three fives make (or equal) fifteen'.

or this:

$$5 \times 3 = 15$$

'A set of five, made three times, makes a set of fifteen', or
'Five, three times, make (or equals) fifteen'.

7. They should learn how to read their number sentence aloud, as in these examples. It is good to be able to say these in several ways. In the process of step 6 they are comparing their results. Any discrepancies offer opportunity for discussion.
8. The objects are taken back, and steps 1 to 6 are repeated, beginning with a different child.

Activity 2 Predicting from number sentences

An activity for up to 6 children. Its purpose is to teach children to write number sentences which predict the results of multiplying.

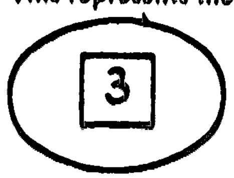
Materials

- Number cards 2 to 5.
- 6 small set ovals.*
- Large set loop.*
- Die (1 to 6, then 1 to 9).
- 5 small objects for each child.
- Pencil and paper for each child.

* As used in Num 5.1/1.

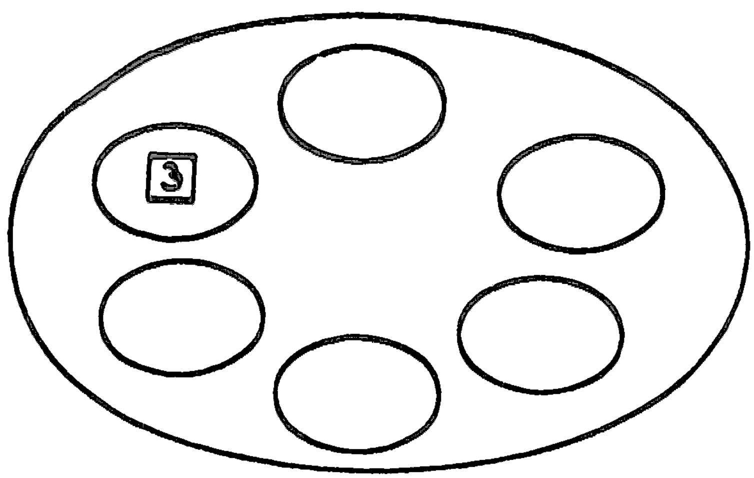
What they do

1. The number cards are shuffled and put in a pile face down.
2. The top card is turned over and put face up on one of the oval small-set cards.
   This represents the first action. E.g.,



Make a set of 3

3. The die is thrown, and that total number of small-set ovals are put out (counting the first). The set loop is put round them all, to make a big set. This represents the second action. E.g.,



4. Each child writes the beginning of a number sentence for the above. In this case,

   either 6(3)
   or 3 × 6

5. They then complete their number sentences to predict the combined result when they actually make the sets represented. To do this, they

may use the method for predicting learnt in topic 2. In this case the completed sentences would be

either $6(3) = 18$
or $3 \times 6 = 18$

6. Finally they test their predictions by putting 3 objects on each small-set card, and counting the combined set.
7. The objects are taken back, and steps 2 to 6 are repeated.

Discussion of activities

The children have already learnt to use multiplication predictively, in topic 2. The new factor here is the use of notation.

In this topic and the one before they progress from 'Do and say', to 'Do and record', and then to 'Predict and test'. In activity 1 they use number sentences to record past events; in activity 2 they use number sentences to predict future events. This parallels the progression in topic 2, in which thought begins to become independent of action. Here, we begin to link thinking with notation.

Initially, writing number sentences is an extra task rather than a help, so it needs to be learnt in a situation where the rest is familiar. For more difficult calculations, written notation is no longer an extra chore but a valuable support. It means that we do not have to 'keep everything in our head' at the same time. Pencil and paper give us an external, easily accessible, extra memory store.

Another step has been taken here. Until now, the number symbols have stood for sets of single objects. Now, some of them stand for sets of sets. So we are now handling more information at a time – one might say, in set-sized packages. This is another of the sources of the power of mathematics.

OBSERVE AND LISTEN REFLECT DISCUSS

Num 5.4 NUMBER STORIES: ABSTRACTING NUMBER SENTENCES

Concept Numbers and numerical operations as models for actual happenings, or for verbal descriptions of these.

Abilities
(i) To produce numerical models in physical materials corresponding to given number stories, to manipulate these appropriately, and to interpret the result in the context of the number story: first verbally, then recording in the form of a number sentence.
(ii) To use number sentences predictively to solve verbally given problems.

Discussion of concepts	The concept of abstracting number sentences is that already discussed in Num 3.4 (page 110), and it will be worth reading this again. In Num 4.4 it was expanded to include subtraction, and here we expand it further to include multiplication. In some applications of multiplication, we need to place less emphasis on the first action and the second action, and more on their results: namely a small set (of single objects) and a large set (a set of these sets). The use of small set ovals and a large set loop from the beginning provides continuity here.

Activity 1 Number stories

An activity for 2 to 6 children. Its purpose is to connect simple verbal problems with physical events, linked with the idea that we can use objects to represent other objects.

Materials

- Number stories on cards, of the kind shown in step 5. Some of these should be personalised, as in Num 3.4/1, but now there should also be some which do not relate to the children, more like the kind they will meet in textbooks. Also, about half of these should have the number corresponding to the small set coming first, and about half the other way about.
- Name cards for use with the personalised number stories.
- Number cards 2 to 6.
- 30 small objects to manipulate: e.g. bottle tops, shells, counters.
- 6 small set ovals.*
- A large set loop.*
- Slips of blank paper.

* As for Num 5.1/1.

What they do *(apportioned according to how many children there are)*

1. A number story is chosen. The name, cards and number cards are shuffled and put face down.
2. If it is a personalised number story; the top name card is turned over and put in the number story. Otherwise, explain 'Some of these stories are about you, and some are about imaginary people'.
3. The top number card is turned over and put in the first blank space on the story card.
4. The next number card is turned over and put in the second blank space.
5. The number story now looks like this:

> There are [3] children in each rowing boat and [4] rowing boats on the lake.
> So altogether [ ] children are boating on the lake.

6. Using their small objects (e.g. shells) to represent children, the oval cards for boats, and the set loop for the lake, the children together make a physical representation of the number story. If it is a personalised number story, this should be done by the named child.
7. The total number of shells is counted, and the result written on a slip of paper. This is put in the space on the card to complete the story.
8. While this is being done, one of the children then says aloud what they are doing. E.g. 'We haven't any boats, so we'll pretend these cards are boats, and put shells on them for children. We need 3 children in each boat, and 4 boats inside this loop which we're using for the lake. Counting the shells, we have 12 children boating on the lake.'
9. The materials are restored to their starting positions, and steps 1 to 8 are repeated.

Activity 2 **Abstracting number sentences**

An extension to activity 1 which may be included fairly soon. Its purpose is to teach children to abstract a number sentence from a verbal description.

Materials
- As for activity 1, and also:
- Pencil and paper for each child.

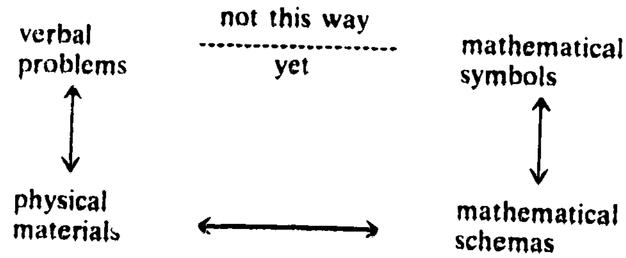
What they do As for activity 1, up to step 8.

9. Each child then writes a number sentence, as in step 6 of Num 5.3/1. They read their number sentences aloud.
10. The materials are restored to their starting positions, and steps 1 to 8 are repeated.

Activity 3 **Number stories, and predicting from number sentences**

An activity for 2 to 6 children. It combines activities 1 and 2, but in this case completing the number sentence is used to make a prediction as in Num 3.5/2.

physical materials, since these correspond well both to the imaginary events in the verbally-stated problem, and to the mathematical schemas required to solve the problem. The route shown below may look longer, but the connections are much easier to make at the present stage of learning.



Since these physical materials have already been used in the earlier stages for schema building, they lead naturally to the appropriate mathematical operations. Side by side with this, they learn the mathematical symbolism which will in due course take the place of the physical materials.

OBSERVE AND LISTEN **REFLECT** **DISCUSS**

Num 5.5 MULTIPLICATION IS COMMUTATIVE: ALTERNATIVE NOTATIONS: BINARY MULTIPLICATION

Concepts

(i) The commutative property of multiplication.
(ii) Alternative notations for multiplication.
(iii) Multiplication as a binary operation.

Abilities

(i) To understand why the result is still the same if the two numbers in a multiplication sentence are interchanged.
(ii) To recognise and write multiplication statements in different notations with the same meanings.
(iii) To multiply a pair of numbers.

Discussion of concepts

(i) Particularly when considered in a physical embodiment, the commutative property of multiplication is interesting, and surprising if we come to it with fresh eyes. Why should 5 cars with 3 passengers in each convey the same number of persons as 3 cars with 5 passengers in each? And likewise whatever the numbers? If you think that the answers are obvious, try to explain both of these before reading further. I have tried to introduce this element of surprise in the first two activities.
(ii) So far the children may have used only one notation, for multiplication but there are several in common use.

5(3) which may be read as 'Five times three' or 'Five threes'.
3×5 read as 'Three, five times'.
3×5 (note the equal spacing) which may be taken either to mean the same as the one above, or as representing binary multiplication.
$3 \overset{\times}{\rightarrow} 5$ read as 'Three multiplied by five'.

There is also this vertical notation

$$\begin{array}{r} 3 \\ \underline{\times 5} \end{array}$$

which will be needed later for calculations like this

$$\begin{array}{r} 473 \\ \underline{\times 5} \end{array}$$

We are more likely to read the lower one as 'five threes . . . five sevens . . . five fours . . .' than as 'three, five times . . . seven, five times . . . four, five times'. So here is an inconsistency.

This inconsistency is neatly removed by using the notation for binary multiplication, which is explained in the next section.

(iii) These are 3 different multiplications, shown in 2 notations.

5(3) $3 \overset{\times}{\rightarrow} 5$

6(3) $3 \overset{\times}{\rightarrow} 6$

7(3) $3 \overset{\times}{\rightarrow} 7$

The operand in every case is 3, but there are three different operations. These are, unary multiplication by 5, by 6, and by 7.

In contrast, binary multiplication has a *pair* of numbers as operand; and there is just one operation, multiply, for all pairs of numbers.

In some school textbooks, binary multiplication is represented like this.

(3,5) $\overset{\times}{\rightarrow}$

(3,6) $\overset{\times}{\rightarrow}$

(3,7) $\overset{\times}{\rightarrow}$

The above is correct mathematically, but I think it is difficult and inconvenient for children.

All the foregoing can be taken care of at a single stroke, by introducing computer notation for multiplication.

$3 * 5 = 15$ read as 'Three star five equals fifteen'

means the binary product of 3 and 5. Because multiplication is commutative (see activity 2), it includes and replaces all the following:

$5(3) = 15$	$3 \times 5 = 15$
$3(5) = 15$	$5 \times 3 = 15$
$3 \xrightarrow{\times 5} 15$	$\begin{array}{r} 3 \\ \times 5 \\ \hline 15 \end{array}$ $\begin{array}{r} 5 \\ \times 3 \\ \hline 15 \end{array}$
$5 \xrightarrow{\times 3} 15$	
$(3,5) \xrightarrow{\times} 15$	
$(5,3) \xrightarrow{\times} 15$	

This seems good value to me, particularly since children need to learn computer notation anyway. The computer books do not specifically mention binary multiplication: I am including this meaning as a bonus.

All the above will become clearer as you work through the activities in this topic.

Activity 1 Big Giant and Little Giant

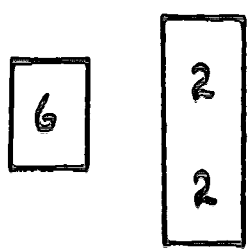
An activity for up to 6 children. It is a sequel to 'Giant strides on a number track' (Num 5.1/3), and its purpose is to introduce the commutative property of multiplication in a way which does not make it seem obvious (which it is not).

Materials

- Activity board, see figure 5.
- Two sets of number cards 6 to 9 (single).*
- One set of number cards 2 to 5 (double).*
- Double width number track 1 to 50. (Card, 1 cm squares.**)
- Blu-Tack.

* To fit the spaces in the activities board. See illustration below.
** These need to be accurate in size, since the track will also be used with 1 cm cubes in activity 2.

Number cards



What they do

1. Two children act the parts of Big Giant and Little Giant, respectively. Big Giant is large and stupid, Little Giant is smaller and cleverer.
2. Big Giant has the set of double number cards and one of the sets of single number cards. Little Giant has the other set of single cards.
3. Big Giant turns over the top cards in each of his packs, and puts these face up in their respective spaces on the activity board.
4. Big Giant puts into action the statement which he has completed. He

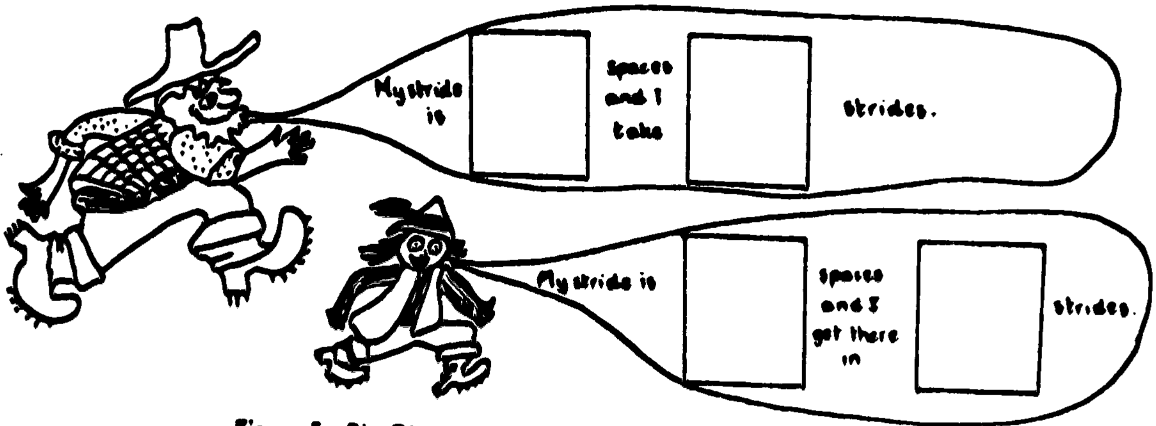


Figure 5 Big Giant and Little Giant.

does this by putting footprints (small blobs of Blu-Tack) on the number track, one for each stride of the given length. He uses the upper part of the number track.

5. Little Giant's stride is of a different length. He has to find out how many of these strides he must take to arrive at the same place as Big Giant. He does this in the same way as Big Giant, using the lower part of the number track.
6. He then puts the appropriate number card in his second space to complete his own statement.
7. The children then read aloud from the board what they have done.
8. Steps 3 to 6 are now repeated, with other children taking the parts of Big Giant and Little Giant. Say to Little Giant: 'If you think you know how many strides you need, to get to the same place as Big Giant, put the number in the space first and then see if you were right. If you don't know, find out first and then put in the number.'
9. Eventually one of the children acting as Little Giant will realise that his own numbers are always the same as Big Giant's the other way round. He will then be able to predict successfully every time. He keeps this discovery to himself for the time being.
10. This continues, with children taking turns at Little Giant, until all have discovered *how* to predict, though probably not yet *why*.
11. When this stage is reached, ask: 'But can you explain *why* your method always works?'
12. If any Little Giant can give a good explanation before having done activity 2, then he is really clever!

Activity 2 Little Giant explains why

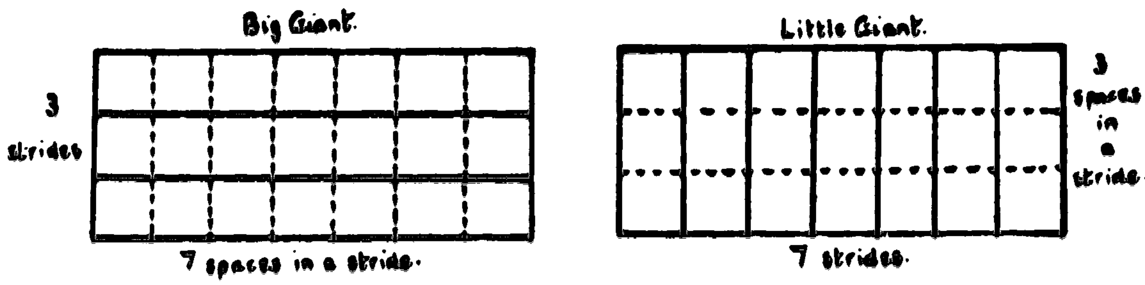
A teacher-led discussion for up to 6 children. Its purpose is to provide the explanation asked for in step 9 of activity 1. For this, it may be necessary for you to take the part of Little Giant.

Materials

- The same as for activity 1, and also
- Squared paper and pencil for each child.
- 100 1 cm cubes, if possible 20 each in 5 different colours.

What they do

1. Start with a set of cards in position on the activity board, as in steps 3, 4, 5 of activity 1. This time, Little Giant's card should also be face up. So the board will read (e.g.)
   Big Giant: 'My stride is 7 spaces and I shall take 3 strides.'
   Little Giant: 'My stride is 3 spaces and I shall get there in 7 strides.'
2. Instead of footmarks on the number track, the giants represent their strides by rods. So in this example, Big Giant uses a 7-rod to represent one of his strides, and puts together 3 of these on the number track. Little Giant uses a 3-rod for one of his strides, and joins together 7 of these alongside those of Big Giant. Adjacent strides should be of different colours, to keep them distinguishable.
3. At this stage it is still not obvious why both are of the same length, since the two journeys look different.
4. 'Now,' says Little Giant, 'we arrange the rods like this.' The long rods are then separated again into single strides, and arranged in rectangles as shown below.



5. From this it can be seen why the two journeys are of the same length. 3 strides, each of 7 spaces makes the same rectangle as 7 strides, each of 3 spaces. Both rectangles have the same number of cubes.
6. Big Giant asks, 'Will it always happen like this, whatever the numbers?'
7. The group as a whole discusses this. One or two other examples might be done using the cubes.
8. This should now be continued as a pencil and paper activity, using squared paper, as follows.
9. Two more numbers are shown on the activity board: e.g. 6 and 4.
10. All the children draw rectangles 6 squares long and 4 squares wide.
11. The children make these into diagrams for the two different ways of getting to the same place, as shown in step 4. About half the children make diagrams representing Big Giant's journey (6 spaces in a stride, 4 strides), and the rest make diagrams representing Little Giant's journey (4 spaces in a stride, 6 strides).
12. They interchange, and check each other's diagrams.
13. Finally, Little Giant tells the others: 'There is a word for what we have just learnt. Multiplication is *commutative*. It means that if we change the two numbers round, we still get the same result. Always.'

APPENDIX C

Transcript of Group Interview

APPENDIX C

Group interview in math lab; (Peter, Fred, Alex, Jenny, Theresa).

AHW - What we're going to talk about is working in the math lab and working in groups and how you feel about those kinds of things. The first thing I want you to think about is, what's the very best thing about working in groups in the math lab? What do you enjoy most?

Fred - Computers.

AHW - The subject, not the game or something but the fact that you're working in groups. What do you like about that?

Peter - They tell you the questions.

Jenny - They tell you the answers.

Theresa - They give you the answers.

Fred - They help you figure it out.

Theresa - They help you figure it out. That gives you an idea what the answer is so you can learn a little bit faster so you don't have to sit there all day trying to get about one or two done.

Alex - Sometimes if they tell you nine times four, some people forget that they can have nine four times so they start adding four....

Theresa - That's you.

Alex - ... nine times and all of a sudden they hear someone in the group goes "eighteen!" and then, and then I get the idea to add nine times....

AHW - So are the people in your group giving you the answer or are they helping you to figure out the answer?

Theresa - Sometimes they're giving it to you, though.

Alex - Helping you.

AHW - They're helping you to figure it out, not just giving you the answer, right?

Theresa - Sometimes they give you the answer by accident. They think they're just helping you, but they're giving you the answer.

AHW - Does that help?

Fred - No.

Theresa - Well, it can. If they tell how they figured it out.

AHW - Jenny, what's the best thing about working in groups?

Jenny - Playing activities with your friends.

AHW - Why?

Jenny - 'Cause there's other kids to help you.

AHW - Peter?

Peter - I've got no answer.

Peter - They tell you answers and stuff. Make you learn.

AHW - There's nothing that you like about working in groups?

Theresa - Make you?

Peter - Make them learn.

AHW - Fred?

Fred - Same as Jenny. That they help you figure out the answer. And if they tell you the answer, you can remember the answer.

AHW - So then, everyone thinks that the best thing about working in groups is being able to talk to other people and get help. Is there anything else you like about working in a group? (Pause). That's it?

Jenny - You have company.

AHW - Why is that important, Jenny?

Jenny - You'd get bored.

AHW - So if you were working by yourself you'd get bored?

Jenny - (Nodded).

Alex - I always get bored when I do math.

AHW - Even working in a group?

Alex - Yeah.

Theresa - Yeah.

AHW - Why?

Theresa - If you get the answers wrong, that's the boring part.

Alex - No, the boringest part is when you're adding up the numbers.

Theresa - No, when you have to explain.

Peter - Have to explain.

AHW - You don't like having to explain your answers?

Theresa - No.

Alex - No.

Theresa - We just like, we just sit there, duh, duh, duh, duh.

Jenny - I sometimes don't know....

Theresa - And sometimes when someone else says it, then we just say what they say and then when they ask us how we figured it out, we're just going to say, "Hey, he said it out first. I got it from him."

AHW - Which is kind of what you were doing with my first question, right? Jenny, what were you going to say?

Jenny - I forgot.

Peter - It's a thing ... oh whatever.

AHW - What's the worst thing about working in a group?

Peter - They talk a lot.

Alex - Some people get you mad.

Theresa - Sometimes they give you answers when you want to figure it out.

Jenny - Oh, I know what I wanted to say.

AHW - What were you going to say, J?

Jenny - Sometimes I don't know what to explain ... what to say.

AHW - So you sort of know how you got the answer but you can't explain it?

Jenny - I don't know what words to use.

Theresa - And when you're trying to write it down on paper, you sit there all day trying to figure it out and then suddenly you go "Yes, it's time to go clean up".

Alex - That's what I do.

Theresa - I know I just sat there, how could I ... and then I only get about one word down.

Fred - This is better than being in math.

Alex - No kidding.

Theresa - All you have to do is talk about it.

AHW - In a way, this is doing math. Talking about it is an important part.

Theresa - You're not asking us no questions though, it's easier than math.

AHW - I want you to talk a little bit more about the explaining. When you're working in a group and Ms. C or Mr. G say, "Okay, now tell what you did" and you have to explain. Can you talk a little bit more about that? About you feel about it. About how you think it's useful. Do you learn something by having to explain it?

Theresa - Other kids can.

AHW - Okay.

Theresa - I did. I need other people to explain to me.

Fred - That's what you have to do....

Theresa - The first day we came in here and we were playing that giant thingamadooie. I had to have someone to explain to me.

Fred - You have to learn how to do it now, or you won't know how to do it in university. Or when you get a job.

Theresa - You won't be able to get a job. You have to be....

AHW - Do you think that it's important to be able to explain what you're doing?

Theresa - Yeah.

AHW - Why?

Theresa - Oh, oh. I'm explaining this.

AHW - Yeah, you are. And I think that it's important.

Theresa - I think it's important but I don't know how.

AHW - Okay, that's fair.

Theresa - So I don't have to explain.

AHW - This is a really, really important idea for me, though, and for Ms. C. whether you learn by explaining to other people, because the activities that you're doing in the math lab are made up so that you have to explain. The man who made them up, Dr. Skemp, thinks that's how children learn math best.

Fred - Even computers?

AHW - No, he didn't do that computer stuff. But a lot of what you do....

Alex - Who's Mr. Skemp?

AHW - Dr. Skemp is the man who made up most of your math materials. And he thinks it's important for people to explain to other people what they're doing. That's why I want you to talk about it. Jenny?

Jenny - I don't know how to explain how to explain.

Alex - I get nervous.

AHW - Then what I want you to do is just think about this one question. If you had to do it by yourself, without anybody to talk to, without anybody to explain it to you or you could explain it to them, you're by yourself, would you learn as much math? And would you learn it as well, do you think?

Theresa - Yeah.

AHW - I'm going to start with Theresa and I'm going to ask each one of you. Do you think you learn more math if you work by yourself or if you have somebody to explain the math?

Theresa - Yeah.

AHW - You have to choose one. You can't just say yes.

Theresa - To have someone be there so you can talk to them and try and ... to have them try and ... 'cause it's the same as trying to explain. They can help you to try and explain.

AHW - Alex, by yourself or with somebody to talk to?

Alex - With somebody to talk to.

AHW - Can you tell me why?

Alex - Oh. Because sometimes you may get the answer wrong and they can help you out there. (Ten second pause).

AHW - Okay, you think about it and if you want to add something more, you can do it after we talk to the other three. Jenny, would you rather work by yourself or with someone else?

Jenny - With somebody else.

AHW - Can you tell me why?

Jenny - Because if you get an answer wrong, they can help you.

AHW - Peter?

Peter - With friends. Because when you get a wrong answer, they say, "Oh, that's the wrong question" and they tell you the answer.

AHW - Fred?

Fred - With a friend.

AHW - Because?

Fred - They help you with the answers. They help you figure out the answers.

AHW - What if you have a problem with a question? A problem figuring out how to do it or what the answer is, who's the first person you ask, someone in your group or one of the teachers?

Theresa - Teacher.

Jenny - Somebody in....

Fred - Teacher.

Peter - Group.

Jenny - Group.

Alex - Group.

Theresa - Group.

AHW - Fred, why would you ask your teacher?

Fred - 'Cause they know the answer.

Peter - But they'll tell you to figure it out.

Theresa - They tell you to ask somebody in your group.

Alex - Yeah, that's what they do.

AHW - Why do you think they do that?

Peter - So you can learn.

Theresa - So that they want the kids to learn, they already know that, the kids know that they know it so the kids think that they can ask the teachers for the answer. They don't have to ask for the answer, they just have to ask the group, "How do you do this?" It's like someone going up to you and saying, "What's the answer to this?" You wouldn't give them the answer. You'd just say, "I'll help you with it". And the teacher would say, "Ask someone in the group, they're just as good as a teacher."

Jenny - The teacher would say, "Go ask someone in your group" because the teacher wants you to get along with the people in your group.

AHW - Who do you think would give you an explanation you would understand better?

Theresa - Kid in your group.

AHW - An adult or a kid?

Theresa - A kid.

Alex - A kid because the grown-ups always give you hard ones to understand. Something like my dad explaining me something, it's like, okay, yeah, bye.

AHW - And do you have to think more about an explanation from an adult? Jenny? Who gives you harder explanations?

Jenny - An adult.

AHW - Peter?

Peter - Teacher.

AHW - Fred?

Fred - An adult.

AHW - So, given the choice between an adult and a kid....

Theresa - I'd take the kid.

AHW - Most of you would still take the kid?

Theresa - Because we're kids and the kids can....

Alex - 'Cause they use the words like cowabunga and words like that.

Theresa - They know our language better.

AHW - I've known some teachers who use words like that too. Do you have an answer, Fred?

Fred - And we understand the kids.

AHW - If you had a choice between working on the things in the lab, and working in a textbook, copying questions from a book into your journal, which would you choose to do? They're both math. They're just two different ways of doing the math.

Theresa - I'd say doing it.

AHW - With learning multiplication, would you rather do Big Giant, Little Giant and Giant Strides or would you rather have used a textbook and learned the information from the text? Theresa?

Theresa - From a textbook.

Alex - Textbook.

Peter - Textbook.

Jenny - Textbook.

Fred - Math lab.

AHW - Okay. Fred has different answers from the rest of you on almost everything here. Why a textbook?

Theresa - Uh oh. Explaining.

AHW - Yeah, explain. That's why you're here, Theresa. Jenny, why a math textbook?

Jenny - So you can get used to the math questions, how they look.

Alex - Yeah.

AHW - Why is that important?

Jenny - Because when you grow up and do math, you would only be knowing the game. You won't be knowing how the math looks.

Alex - And because if you only play games to figure out the answers in multiplication, and if you use a textbook you know what the question looks like, so if you just play games and then they ask you on a piece of paper maybe you might get screwed up inside your head. And put out the wrong answer.

Theresa - (Pointed to Alex).

AHW - Same sort of answer?

Theresa - (Nodded).

AHW - Why would you rather work in the math lab, Fred?

Fred - Because you're learning math the fun way.

AHW - Is that important to you?

Fred - It's easier.

AHW - Do you think you learn as much math in the math lab as you would if you used a textbook?

Fred - Yeah.

AHW - Which way do you think you understand math better? Doing Giant Strides and the games or by using the textbook and writing out answers? How would you understand it better?

Theresa - Textbook.

AHW - You still think a textbook, Alex?

Alex - (Nodded).

AHW - Jenny?

Jenny - Textbook.

AHW - Peter?

Peter - Same.

AHW - And Fred is going to say "Math lab!"

Fred - Math lab!

Theresa - The fun way. It's always the fun way. I love the fun way.

AHW - But your answer doesn't tell me "the fun way" because a lot of people don't think that textbooks are fun.

Jenny - I like the work better if it's fun.

Theresa - Who cares if it's fun?

Alex - You like work better than fun?

Fred - It's boring.

[**End of interview**]

APPENDIX D

Skemp's Assumptions

APPENDIX D

Activities for introducing new concepts often take the form of a teacher-led discussion. Many of the other activities take the form of games which children can play together without direct supervision, once they know how to play. These games give rise to discussion; and since the rules and strategies of the games are largely mathematical, this is a mathematical discussion. Children question each other's moves, and justify their own, thereby articulating and consolidating their own understanding. Often they explain things to each other, and when teaching I emphasize that 'When we are learning it is good to help each other.' Most of us have found that trying to explain something to someone else is one of the best ways to improve one's own understanding, and this works equally well for children.

(1989a, 1989b, p. 1)

Activities (which introduce new concepts) always need be introduced by a teacher, to ensure that the right concepts are learnt (ibid., p. 9)

APPENDIX E

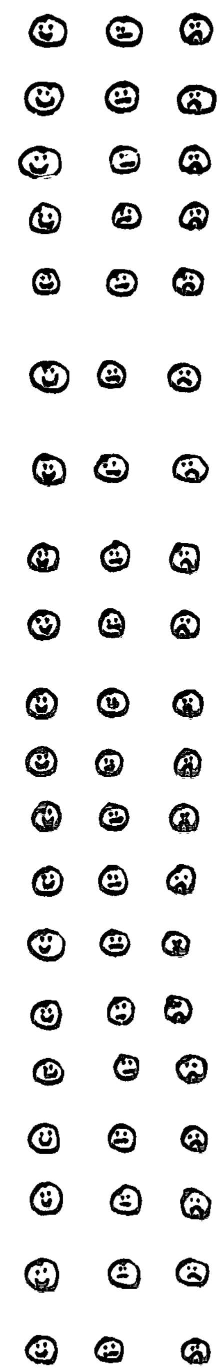
Questionnaire

APPENDIX E

Date ______________________________

Name ______________________________

Yes No



1. I like working in math groups.
2. I help other people in my group.
3. Other people have helped me to learn.
4. I can learn more in a group than by myself.
5. I would like to learn all of my math from a textbook.
6. I would like to do all of my math in groups in the math lab.
7. It helps me when I can use things like cubes to learn new ideas.
8. I need to use the things to help me learn.
9. I could learn more from a math textbook than I do in the math lab.
10. My teacher cares about me.
11. My teacher cares if I learn my math.
12. I have learned a lot of math this year.
13. I like the people in my math group.
14. I am proud of my work in math.
15. I am good at doing math.
16. I like to write in my math journal.
17. My journal helps me to learn.
18. When I have problems, I ask someone in my group to help me.
19. I like to help other people in my group.
20. I learn better when I talk about math.

APPENDIX F

Transcripts of Individual Interviews

APPENDIX F

Individual Interview with Alex

AHW - I don't know anything about multiplication. Ms. C tells me that you are all experts and really smart.

Alex - You don't know about multiplying?

AHW - No, I have no idea. What I need you to do is to use any of this stuff, the pencil, the paper, anything ... and I need you to teach me how to multiply. Everything you know so I'm an expert on multiplying.

Alex - Okay. I'll use these (took pile of cubes). If they give you a question like 4 x 5? You get, you like, add four five times like (made five four-rods) you try and do this in your mind. You count them all. Like you count the cubes and you can put it the other way too. (Took spare cubes and put on top of four of the four-rods; took away one four-rod). You can have four groups of five in there. The way that I figure these out to go faster if they ask me a question, I just count the five, the fives, five, ten, fifteen, twenty.

AHW - What about the five groups of four? How would you do that?

Alex - How would I figure it out? Well I would just (recreated the five four-rods) I would just count four five times. But I get really mixed up in my head when I do it that way.

AHW - Is it harder to count by fours?

Alex - For me it is.

AHW - It is for a lot of people, counting by fours and sixes is hard for a lot of people. Okay, so is five multiplied by four the same as four multiplied by five?

Alex - Yeah but the numbers just look different but you get the same answer.

AHW - How do the numbers look different?

Alex - Can I use the paper? (Wrote 4 x 5 = 20). Four times five. Four groups with five in them, and (wrote 5 x 4 = 20) that is five groups with four in them. And they have the same numbers and the same answer but the numbers are just mixed up.

AHW - Do the numbers mean the same thing?

Alex - No.

AHW - Can you tell me what the difference is then?

Alex - This one, the four in here (in 4 x 5 = 20) that tells how much groups there is. (Wrote 'groups' above the four). And this is how much in each group. And this is how much you get.

AHW - So in this one, the four means how many groups. What does the four mean down there (in 5 x 4 = 20)?

Alex - How many in each. And the five down here (in 5 x 4 = 20) means how many groups too. The number here means how much things (second factor) but you still get the same answer.

AHW - What else do I need to know about multiplying? (Twelve second pause). I know about four multiplied by five and five multiplied by four. I know that equals twenty. What else do I need to know? (Eleven second pause). Could I get through life knowing those two multiplication equations and that's that?

Alex - No.

AHW - Okay, then you need to teach me more.

Alex - Okay. Um ... (seven second pause).

AHW - Is that the only multiplying equation you can think of?

Alex - No, I can think of lots more.

AHW - Okay, why don't you show me another one and see if that'll make me smarter too.

Alex - (Put down pencil; made one four-rod into an eight-rod; counted it and then added one more cube; pushed rods aside and went back to the paper). Oh, just a minute. Nine times five. (Wrote 9 x 5 =). Oh, that's the same thing.

AHW - The same thing as what?

Alex - As this one (5 x 4 = 20) but you just count by fives nine times.

AHW - And what does it equal?

Alex - Forty-five.

AHW - And if you made it with the cubes, what would it look like? (Three second pause). Can you describe it to me?

Alex - What it would look like?

AHW - Yeah.

Alex - (Began to build rods).

AHW - Can you just tell me what it would look like?

Alex - It would look ... (seven second pause).

AHW - It would look big, right?

Alex - Big, yeah.

AHW - What do the numbers....

Alex - It would look like it would be a hard problem.

AHW - Is it?

Alex - No.

AHW - Why not? Those are big numbers.

Alex - Not a hard problem, though.

AHW - Why not?

Alex - 'Cause you just count by fives.

AHW - Oh. Can you tell me what the numbers mean in this sentence? (9 x 5 =).

Alex - This nine in this one means groups and this one....

AHW - The five.

Alex - The five means how many in each group.

AHW - Does the first number always tell us how many groups?

Alex - (Nodded).

AHW - And does the second number always tell us how many in each group?

Alex - Yeah.

AHW - Does this number always tell us how many we have? (Indicated product).

Alex - The answer means ... the answer means how many there are in each group. Like there's nine times five, you have forty-five with all the cubes that you have. (Indicated five groups of cubes lying beside paper; had made only one nine-rod but seemed to indicate that had five nine-rods). So, that's the answer.

AHW - Okay, so the answer means how many you have in every one of the groups or in all of the groups together?

Alex - In all the groups together.

AHW - So is it like adding two plus three and getting five and the five tells you how many you have altogether.

Alex - Let me explain this another way. The answer always means how much ... (began to make rods; stopped). I'll just do an easier one. It always means. Like this one (pointed to one of his equations). This one here (indicated first factor) means how much groups there are and this one (indicated second factor) means how much in each group. But the answer always means how much cubes there is together. (Counted five four-rods by ones).

AHW - So the answer always means all of the groups all put together?

Alex - Yeah.

AHW - All right, fine. I'll give you one and you tell me what the multiplying sentence would be, okay?

Alex - Okay.

AHW - (Made two three-rods and three five-rods). Can you tell me the multiplying sentence for that?

Alex - (Examined rods for seven seconds; looked up; re-examined for another twelve seconds). I could do something but I'd have to take one away.

AHW - Leaving them exactly the way they are. Can you tell me what the multiplying sentence would be? (Seventeen second pause). Can you describe to me what you have in front of you, Alex?

Alex - Well, the problem is, why I can't figure it out is because there are two groups of three and other ones with five in them.

AHW - Is that a problem?

Alex - Because I can't make it go five with five in each. I can't make it five times five.

AHW - Why not?

Alex - Because the only way I could do that is if I took one away.

AHW - How many groups do you have? How many are in each group?

Alex - Five and three.

AHW - It makes a problem, right? You can't do that. Can you tell me why you can't do that?

Alex - Because. I can't put them. Like multiplying always has to be. The things in each group always has to be the same amount of things. So you can multiply.

AHW - Because you have five in some groups and three in some groups, is that what the problem is?

Alex - Yeah.

AHW - So even though you have five groups altogether, you can't multiply it?

Alex - No.

AHW - Can you tell me what you could do to change it around so you could multiply it? Or show me how you could change it?

Alex - (Looked at rods). To make it a multiplying sentence?

AHW - To make it a good multiplying sentence that you could do.

Alex - Without adding or taking anything away.

AHW - Without adding or taking anything away. Just change around what you have.

Alex - (Put together two three-rods). That would be six, though.

AHW - Does that work?

Alex - No.

AHW - Well, keep going.

Alex - (Twenty-two second pause; no more changes attempted).

AHW - It's okay. It's a tough thing to ask you to do. I think it's a fair thing to ask but it's kind of tough. Let's leave this (the five-rods) and use these to change. (The three-rods). So we're not going to make these (five-rods) any smaller. Can you take these (three rods) and make these groups (fives) change?

Alex - Can I take these ones (three-rods) and make these (fives) change?

AHW - Yeah. Not taking anything away from here (fives). So are these groups going to stay the same or become bigger?

Alex - Stay the same.

AHW - No.

Alex - Become bigger?

AHW - They have to become bigger (six seconds). Now can we use those (three rods) to make these groups bigger so you could use them for a multiplying sentence?

Alex - And I can't add anything to these? (Fives).

AHW - You can. You can't take anything away.

Alex - But I can add to these.

AHW - You can add....

Alex - I got it.

AHW - You can add however many you want from those six you have in your hand.

Alex - I got it. (Broke apart one three-rod and added one to each five-rod).

AHW - You had five in each group. Now you have...?

Alex - Six. (Broke apart other three-rod and added to six-rods; now had three seven-rods). There.

AHW - Now what do you have? Instead of two groups of three and three groups of five, what do you have?

Alex - Three groups of seven.

AHW - And how would you write the multiplying sentence for that?

Alex - (Wrote 3 x 7 =; counted first rod, ran pencil down length of middle rod; counted third rod by ones; wrote down 21).

AHW - What did you just do to figure that out?

Alex - Well, I know the ones like seven plus seven is fourteen, so I have seven groups of three here. So I just take these two sevens and fourteen instead of wasting my time going one, two, three ... fourteen.

AHW - So you know that these two together make fourteen and then what did you do with this one?

Alex - Counted it.

AHW - So you counted it by ones and got the answer twenty-one. Good. Is there anything else I need to know about multiplying? (Six seconds). So you've told me all the important ideas.

Alex - That I can think of.

AHW - Without using the paper and pencil and without the cubes, if I said to you, "Tell me what multiplying is", can you do that?

Alex - Maybe.

AHW - See what you can come up with. (Eight second pause). Multiplying is when ...? (Twelve second pause). When you do what?

Alex - When you ... when you just ... when you do multiplying, it's adding. It's just adding because you're adding the numbers in each group together.

AHW - So if I had that (indicated the unequal rods) that's adding and I could multiply that?

Alex - No.

AHW - Why not? Did I just do what you told me to do? Why can't that be multiplying?

Alex - Because. It can't be multiplying. Some questions you got to add how much there is in each group.

AHW - So I could have four plus four plus three which would give me eleven. Is that the same as multiplying them?

Alex - No, not really.

AHW - I don't think I can multiply these three groups and get an answer.

Alex - No.

AHW - Can you tell me why not?

Alex - Because the three needs one more.

AHW - Why?

Alex - So you can multiply.

AHW - Why?

Alex - So it can be even numbers.

AHW - Ah. That's what I wondered about. When we were talking before

... it sounded like.... Do all the groups have to be the same size?

Alex - Yeah.

AHW - Do they absolutely have to? So when I'm multiplying, the groups have to be the same size and it's like adding?

Alex - Yeah.

AHW - That's it? Is that all I need to know about multiplying?

Alex - (Nine second pause). Well ... (ten second pause) that's all. When I came back from vacation, that's all I needed to know.

AHW - Have you learned anything more since you came back?

Alex - Learned how to multiply.

AHW - Was the multiplying you did with Ms. C different from what you had learned before?

Alex - (Nodded).

AHW - How?

Alex - Well, it wasn't different because. Like I sort of knew multiplying before but it took me a long time to figure it out.

AHW - Can you tell me the difference between how you were doing it and how Ms. C does it? Is that what you're saying? You learned a different way from Ms. C?

Alex - (Nodded).

AHW - Can you tell me what the difference is between the two? How did you do it before you came back?

Alex - Well it wasn't ... it wasn't very different. My friends just told me that, not in Ms. C's class but in the other one, just said that the first number is always how many groups you have and the other one is how much in each group and you got to count how much there is in each group. You got to count each one of the things that you've got and then you get the answer. Like you count (made three four-rods) and then you count one, two, three, four ... twelve. (Pointed as he counted). And then that's the answer. Ms. C told me pretty much the same thing.

AHW - I'm satisfied. I think I know how to do multiplying now. Anything else you'd like to tell me about this?

Alex - (Six second pause; laughed).

AHW - Can you answer one last question for me? I promise it's the very last one. Can you tell me the difference between multiplying and dividing?

Alex - Okay. Dividing is when you get a number and it's like separated and you've got to share it into groups. (Took three four-rods and separated into single cubes; added two extras). You got to put it into groups. Like twelve divided by two, you can put it in two groups with six in each or you can just put it like this (made seven groups of two). There. And that's how you do dividing.

AHW - (Indicated seven groups of two). But this isn't two groups with six in each group. What's this?

Alex - This is six groups with two in it.

AHW - So when you're dividing, you take something that's big and you do what?

Alex - You share it.

AHW - So what's multiplying? How is it different?

Alex - You got to count how much ... you got ... like in multiplying? You got groups with things in them and you've got to count. Okay. Like dividing? It's like one, two, three, four, five. (Realized he had seven groups).

AHW - You have seven groups. So how many did we start out with? (Pause). Fourteen, not twelve. All right.

Alex - And so like seven groups. That gives you the answer seven. And then you get ... (put cubes into two groups of seven). The other way of dividing, is just putting two groups with seven in them and then you get the answer of how much is in here (indicated one group). But multiplying, when they ask you the question, they already have groups.

AHW - And what do you do with the groups?

Alex - You count how much there is in each group.

AHW - And is that all you do? You just count, "This group has four, this group has four and this group has four" and then you're finished?

Alex - No.

AHW - So what do you have to do after that?

Alex - You got to (put together three four-rods). You got to count them and not go "This group has four, this group has four and this group has four." You got to count each one of them. (Counted three four-rods by ones). And they, when you multiply ... and in division these same cubes, the way that you get the answer is by how much groups there is or how much there is in each group.

AHW - So dividing, you're starting with a big group and making it into little groups and multiplying you're...?

Alex - Starting with little groups and you just, you make, when they ask you the question they already have groups like three times seven, they already have three groups with seven in them.

AHW - So in multiplying you don't make the groups? The groups are already made for you?

Alex - Yeah. And you just got to count how much there is in each group.

AHW - To find out?

Alex - The answer.

AHW - And the answer tells you how much all together?

Alex - Yeah.

AHW - If when you came in, I had that sitting there (indicated the seven groups of two) just exactly that way, would you think that's multiplying or dividing?

Alex - Dividing.

AHW - If it was just sitting there like that?

Alex - Oh.

AHW - If I had put it out before you came in and I said, "Okay, Alex. Look at this. Tell me whether it's multiplying or dividing?"

Alex - But they were groups, right?

AHW - It's exactly that way.

Alex - It would be ... (ten second pause) multiplying.

AHW - Good, because they're already in little groups, right? But if you came in and I said divide it into two (pushed cubes into single pile) then you'd actually have to separate it into the groups, right?

Alex - Yeah.

AHW - Okay, that's it.

Strengths	Weaknesses
a) equal groups b) difference between x and ÷ c) commutativity d) skip counting strategy (by 5's) e) could visualize completed rods even without making them (p 229) f) demonstrated groups (concrete) g) wrote sentence (symbolic)	a) meaning of factors tied to specific locations because his friend said: 1st # = groups 2nd # = quantity b) some instrumental ("rules") understanding? or just recitation of convenient way to remember?

Individual Interview with Jenny

AHW - What you are going to have to do this morning is to teach me to multiply. Ms. C's been trying really hard to get you guys to multiply, but I haven't learned. So, there are chips in here, chips here, cubes and paper and pencil; you can use anything you want. Teach me to multiply.

Jenny - If you have two groups of five, (made with cubes) and you add them together, that's ten. That's part of multiplying. You add two equal groups together. And you can turn them around and five, you can turn them around and two multiplied by five. (Made five-rods into five two-rods). Two multipled by five. And the two stands for, sometimes it stands for how many in a group and sometimes it stands for how many groups.

AHW - When you have something like this (pointed to five two-rods) what does the two mean?

Jenny - It means how in a group. Um....

AHW - So that's it? Just twos and fives? I can't do anything else for multiplying?

Jenny - Yeah, you could.

AHW - Okay, well can you show me something else?

Jenny - (Eleven second pause; put out six individual cubes). If you have six and one in a group, that would be the number six 'cause you add them all up (put all six together in one group) and then six in each ... one in each group and if you add them all up, there would be six. 'Cause there's only six groups and one in each group. And if you have one group and you have six in the group, you would, then it would just be six 'cause there's only one group. And if you have zero and you have three ... (put out three cubes) say you have zero groups and there's three in each group. You can't have that. It would always be zero.

AHW - Why?

Jenny - 'Cause there's no group and there has to be something to hold the threes. There has to be a group. And if you have one group and nothing in it, (put aside cubes) then that wouldn't add up to anything. It would still be zero because there's nothing in the group.

AHW - So you'd have a space for the group but nothing in it?

Jenny - No.

AHW - Sort of like an empty plate.

Jenny - Uh huh.

AHW - Can you write a multiplying sentence for me?

Jenny - Okay. (Took paper and wrote 5 x 9 = 45). Five times nine equals forty-five. It equals forty-five because if you have five groups and nine in each of them, (made groups of nine; started by putting out five cubes and then built the groups around these). That's forty-five. You can count by fives. If you have five groups and nine in each of them, you can count by nines. Or if you have nine groups with five in them, (rearranged cubes to show nine groups of five) if you have five ... if you have nine groups and five in them, you can count by fives. Five, ten, fifteen, twenty, twenty-five, thirty, thirty-five, forty, forty-five. And that's nine multiplied by five.

AHW - Why is that nine multiplied by five instead of five multiplied by nine (pointed to 5 x 9 = 45 Jenny had written earlier). What's the difference?

Jenny - Five multiplied by nine means that there's five groups and nine in each group. That's one way. Or five ... five in each group and nine groups but right now we're learning about five groups and nine in each group. So the second number is how many in each group and the first one is how many groups. And if you turn those two numbers around (wrote 9 x 5 = 45) nine multiplied by five is nine groups with five in each group. But you still get the same answer.

AHW - Why do you get the same answer?

Jenny - Because you use the same numbers and nine groups with five in each group is forty-five still and the same as five multiplied by nine.

AHW - So if you show them the two different ways, do you still have the same number of things?

Jenny - Um, yeah.

AHW - So this is the same as the number that you had when you did groups of nine?

Jenny - Oh, no. Not the same but the numbers are the same.

AHW - How is it different?

Jenny - It's different because there's five multiplied by nine, there's more in a group.

AHW - Okay. So this is nine multiplied by five? (Indicated nine groups of five).

Jenny - Yeah.

AHW - Can you show me five multiplied by nine again?

Jenny - (Made groups of nine; confirmed by counting each group; counted groups twice; pushed one cube back into 'extra' pile).

AHW - What's the matter?

Jenny - I just took one from the pile.

AHW - It sort of slipped in by accident? What's this? (Indicated arrangement of cubes).

Jenny - This is five multiplied by nine. There's five groups with nine in each group.

AHW - Do you have the same number of cubes when you showed me nine groups of five?

Jenny - (Nodded).

AHW - Is that why the answer's the same?

Jenny - Yeah.

AHW - Is there anything else I need to know?

Jenny - I don't really think so.

AHW - Can you tell me just in a sentence or two what multiplying is?

Jenny - It's when you have how many groups and how many in them and you add them together.

AHW - All right. If I gave you ... (set out unequal groups) can you tell me what the multiplying sentence would be?

Jenny - (Ten second pause). Is it these two groups? (Separated rods so she had the like groups together).

AHW - No, you have two groups with four and then these three groups each have two. Can you tell me what the multiplying sentence is?

Jenny - (Nineteen second pause). I don't know.

AHW - What's the problem?

Jenny - 'Cause they're not equal numbers in a group.

AHW - So if you have two different sizes of groups, can you do the multiplying?

Jenny - No.

AHW - Is that an important idea to know about multiplying?

Jenny - Yeah.

AHW - So the groups have to be....

Jenny - Equal.

AHW - And they have to be fair groups, too. Alex kept calling them "fair" groups. So I know that multiplying is groups. I know that they have to be equal groups. This number means ... (pointed to first number in equation).

Jenny - How many in a group. I mean, how many groups and how many in a group?

AHW - What does this mean ... after the equals sign?

Jenny - That means how many altogether.

AHW - Okay. That's an important thing to know, too, isn't it?

Jenny - Yeah.

AHW - You said that multiplying is kind of like adding. Can you tell me how they're the same?

Jenny - It's kind of like adding because you have to add two numbers together.

AHW - Does that mean you add five and nine together here? (In 5 x 9 = 45).

Jenny - Oh no, it means that in the groups you have to add all the ... or you have to add numbers together. You have to kind of use adding.

AHW - Can you show me how you'd use adding to figure out nine multiplied by five?

Jenny - Um. (Looked at four-rods, picked up one three-rod and began to pull it apart).

AHW - Is that what you did when you had all the groups here?

Jenny - Nine multiplied by five. I had nine groups with five in them and I added all of them together and got forty-five.

AHW - How did you add them together?

Jenny - I added them together by when I ... sometimes I break them up into ... and sometimes I have to add a couple of another group into the pile of the other group into fives and then I count them by fives.

AHW - Is counting by fives the same as the adding?

Jenny - That's a hard question.

AHW - It is kind of. Can you answer it?

Jenny - No.

AHW - No? Okay, that's fine. Can you tell me how multiplying is different from dividing?

Jenny - Because you have to ... in multiplying you have to add them together and in dividing you have to make them into groups. It's one whole big group and you have to make them into equal groups.

AHW - And in multiplying...?

Jenny - You have to put the groups together.

AHW - So you have a whole bunch of groups and you're making them like this? (Pushed four-rods together). Making them into a bigger group?

Jenny - Yeah. And when you're dividing, you have to make them into equal groups.

AHW - That's a very simple way to tell the difference.

Jenny - If you know multiplying, all you do for dividing is to do it backwards.

AHW - Oh. Is there anything else that's important that I need to know about multiplying?

Jenny - That there's ... that you have to have equal groups or else you won't be dividing ... I mean multiplying. And that the answer's going to be the same if you change the numbers around.

AHW - Do these numbers have to be the same, though? (Indicated numbers in her equations).

Jenny - These numbers? If you change them around?

AHW - If I had five and nine here and down here I had six and five, will it give me the same answer still?

Jenny - No.

AHW - Why not?

Jenny - Because the six....

[Interrupted by school intercom; I evidently lost my train of thought]

AHW - Can you show me a picture of a multiplying sentence ... say, three groups of two?

Jenny - (Drew three trees with two apples on each). Three groups with two in each group.

AHW - And what would the multiplying sentence look like?

Jenny - (Wrote 3 x 2 = 6).

AHW - You've done very well. Thank you.

Strengths	Weaknesses
a) zero b) uses and explains commutativity c) groups d) concrete/symbolic/1 picture (prompted) e) skip counting f) knowledge of ÷ and x as inverse operations	

Individual Interview with Fred

AHW - What you need to do is to teach me to multiply. You may use any of the things which are in front of you ... the paper, the pencil, anything at all, but I need you to teach me everything you know about multiplying.

Fred - Do I write everything down?

AHW - That's completely up to you. You can tell me, show me, write it ... doesn't matter.

Fred - Well ... are you going to ... are you ... I don't know how to.... How do you spell multiplying?

AHW - (Spelled for him).

Fred - (Wrote "Multiplying is where you have a number of groups and a number in each group.").

AHW - Could you read your sentence to me?

Fred - (Read above sentence).

AHW - So does that mean I can have ... one, two, four. (Set out cubes as named amounts). Would that be multiplying?

Fred - (Laughed). No. You have to have the same number of things in each group. So ... (made three groups of two) like that.

AHW - Can you tell me why?

Fred - Here? (Pointed to paper).

AHW - If you want to write it that's fine or if you just want to show me and explain that's fine, too. Whatever you think will teach me so that I know how to multiply.

Fred - (Wrote "You have to have the same number in each group."). There, I wrote, "You have to have the same number in each group." Then if someone says, three times two, you can count by twos or you can count on your fingers by ones.

AHW - Would you get the same number?

Fred - Yes.

AHW - Could you show me the two different ways that you'd do it?

Fred - Two, four, six, (counted his groups of two) or one, two, three, four, five, six.

AHW - How would you show this in your writing? (Indicated three groups of two-rods).

Fred - (Wrote "3 groups of two").

AHW - Is there any other way you could write it?

Fred - I could draw it. (Drew three circles with two smaller circles inside).

AHW - Is there another way you could do it?

Fred - (Wrote "Jane had three bag and two candys [sic] in each bag.") This is just like my writing down here.

AHW - That's okay.

Fred - (Wrote 3 x 2 = 6).

AHW - You said this sentence about Jane had three bags was the same as what other part?

Fred - Three groups of two.

AHW - How are they the same?

Fred - Because if she had three bags, that would mean three groups and two candies. That would mean two in each group.

AHW - Can you explain the three? The last part that you wrote? This multiplying sentence. Can you explain what it means?

Fred - Three?

AHW - This one (pointed to 3 x 2 = 6). What does that mean?

Fred - Three groups of two equals six.

AHW - So what does the three stand for?

Fred - Groups.

AHW - And what does the two stand for?

Fred - Number in each group.

AHW - Does the three always have to mean the number of groups that you have?

Fred - Yes.

AHW - Is that what the three always means or is that something about the way that you write the multiplying sentence?

Fred - What do you mean? The number of groups is always at the start or it can be here (indicated second factor).

AHW - How do you know where it is?

Fred - My teacher taught me.

AHW - Can you teach me some more about multiplying?

Fred - No, I don't know it.

AHW - You don't know anything more about multiplying?

Fred - I know answers.

AHW - Well, why don't you do some of that and show me.

Fred - Okay. (Wrote 11 x 2 = 21).

AHW - Can you tell me what you just did?

Fred - Eleven times ... eleven groups with two in each group ... (stopped and looked puzzled). Oops.

AHW - Oops, what?

Fred - I made two groups of eleven, so I have to switch it around. Or it will be eleven groups of two. So it doesn't matter which way it goes.

AHW - What do you mean, it doesn't matter which way it goes?

Fred - The number here doesn't always have to be here. It could be the second number or it could be ... the number that's in the front doesn't have to the front. It could be here or it could be down here (indicated that could be first or second factor).

AHW - And it doesn't make any difference at all?

Fred - No, because two times eleven is twenty-one and eleven times two is twenty-one.

AHW - So does that mean you get the same answer?

Fred - Uh huh.

AHW - How do you know you get the same answer?

Fred - Because ... I do it with my fingers.

AHW - Can you show me?

Fred - Two, four, six, eight, ten.... How many is that? Five?

Twelve, fourteen, sixteen, eighteen ... (counted by putting up two fingers at a time; after eighteen, stopped and looked puzzled) ... twenty, twenty-one? (Looked again at his 11 x 2 = 21).

AHW - Is there a problem? You look puzzled.

Fred - (Set out eleven individual cubes; counted by twos). Am I doing...?

AHW - Tell me what you have here (indicated rods).

Fred - I have eleven.

AHW - Eleven cubes? And what are you doing with them?

Fred - Making eleven groups ... (pointed to 11 x 2 = 21 equation) ... of two. (Made groups of two by adding additional cubes; counted). Two, four, ... eighteen ... twenty, (hesitated) ... twenty-one, twenty-two. (Looked puzzled). That's wrong, right? (Indicated 11 x 2 = 21 on paper).

AHW - What's wrong?

Fred - This. (Changed to 11 x 2 = 22).

AHW - You had twenty-one, now you have twenty-two, Fred. Why did you change it?

Fred - Because eleven plus eleven is twenty-two. And two makes two groups of eleven, so you just add eleven and eleven.

AHW - Does this show two groups of eleven? (Pointed to eleven groups of two).

Fred - No.

AHW - What's this?

Fred - Eleven groups of two. That's what this is (indicated 11 x 2 = 22).

AHW - Oh, so this is eleven groups of two (equation). Is it different if you have two groups of eleven?

Fred - Yes.

AhW - How?

Fred - (Made two groups with cubes).

AHW - So what do you have in each group?

Fred - (Counted). Eleven.

AHW - So this isn't eleven groups of two (indicated cubes), now this is. . .?

Fred - Two groups of eleven.

AHW - How is that different from what you did before?

Fred - They were in smaller groups before.

AHW - How is it the same as what you did before?

Fred - The answer.

AHW - Why is the answer the same? They're completely different groups, aren't they?

Fred - Uh huh.

AHW - So why is the answer the same?

Fred - 'Cause all you're doing is switching them around and ... I don't know how to explain it.

AHW - Think about the cubes that you put out there. Does that help you, by looking at that, can you tell me why the answer is the same?

Fred - No.

AHW - All right. You had two groups of eleven here. Can you show me eleven groups of two?

Fred - (Arranged cubes).

AHW - Tell me what you just did.

Fred - Just switched them around. Made it smaller.

AHW - By doing what?

Fred - By putting them into eleven groups with two in each group.

AHW - Does that help you to figure out why the answer's the same?

Fred - Uh huh.

AHW - Can you tell me?

Fred - (Shrugged). I don't know how.

AHW - Give it a try.

Fred - Why it's? Why the answer's the same?

AHW - Uh huh.

Fred - (Pause) Because all you're doing is switching it around and it's just like plus. You're making it into groups.

AHW - That's a good explanation. Can you tell me a bit more about multiplying?

Fred - (Eight second pause). No?

AHW - What are the important ideas that you've told me so far?

Fred - (Read the two sentences he wrote earlier). It's about how you can ... I just told you about it. I told you about multiplying. It's ... how it's ... what multiplying is. Like you have the same number in each group ... and three groups of two and how to do it. And why the answer is the same.

AHW - That's a lot of important ideas. (Gave Fred three four-rods). What would that multiplying sentence be?

Fred - Three groups of four equals twelve.

AHW - (Added one two-rod). What about that?

Fred - Seven groups of two.

AHW - How did you get seven groups of two?

Fred - If you have the same number in each group. (Began to break four-rods into two-rods).

AHW - All right. But if we left them this way ... (put back into four-rods) can you explain how to multiply?

Fred - You could take away something.

AHW - Leaving it exactly the way it is. Can you multiply it?

Fred - No.

AHW - Why not?

Fred - Because there's not the same number in each group.

AHW - Which is why you took this apart into groups of two?

Fred - Yes.

AHW - That's a very smart answer. And that a trick question just to see if you're awake yet this early in the morning. Tell you what. Just one last thing. Can you tell me how multiplying and dividing are different?

Fred - In dividing you're taking ... (nine second pause; began to write).

AHW - You can use these too if you want (indicated cubes).

Fred - (Took some and moved them around but without any final product). In multiplying you're taking away. You're making groups. You're seeing how many groups there is but in multiplying you're seeing how many in each group or you're seeing how many there is.

AHW - In multiplying, we're making groups and seeing how many in each group?

Fred - Yeah.

AHW - And in dividing?

Fred - We're taking a number apart to see how many groups there is.

AHW - Okay. When we start multiplying, would we see this (three two-rods) or would we see this? (Pile of cubes).

Fred - This. (Pointed to rods).

AHW - What about this? (Pile). Is this multiplying?

Fred - No, that's dividing.

AHW - So what wo"ld you do with this? (Pile).

Fred - If it said ... (counted cubes) eleven divided by two, you just make groups. (Separated all the cubes which had been joined) ... Multiply, I think. I forget how to do dividing.... Groups. (Wrote 11 divided by 2 =) Oh, a remainder. (Set out groups of two cubes; counted aloud). Two ... two ... and one left over.

AHW - Elevan divided by two ...?

Fred - (Counted groups of twos). One, two, three, four, five ... (wrote answer in equation) five ... (wrote rm 1) remainder one.

AHW - What did you do with the three divided by two? (Reference to first division equation he had written).

Fred - I was trying to think how to do it again because I forgot that ... I was thinking ... I remembered that this means two in each group or it can mean two groups. (Referred to 2 in equation).

And I took it and I found that there was one left. And if you did two groups, (took three cubes; showed them shared into two groups) one in each group and one remainder.

AHW - When you're multiplying, can you have a remainder?

Fred - No.

AHW - Why not?

Fred - 'Cause it has to be the same number in each group.

AHW - So they have to be exactly equal? And you're never, ever allowed to have any kind of a remainder with multiplying?

Fred - No.

AHW - That's exactly right. You can have a remainder in dividing, but you can't with multiplying. If you have a remainder with multiplying, it should tell you that there's something wrong. So you figure I'm a multiplying expert now?

Fred - Uh huh.

AHW - You've told me all the most important ideas?

Fred - Uh huh.

AHW - I think you have too. Good, thank you Fred.

Strengths	Weaknesses
a) = groups b) used rods to check 11 x 2 when fingers didn't work (had 11 x 2 = 21) c) self-correcting (p 245) d) knew commutativity would produce same answer (p 246) e) able to visualize rods in different sized groups (p 249) f) multiple representations of same multiplication situation (p 246-47) g) uses one cube to represent each group (p 247) h) used symbolic, pictorial and concrete levels i) wrote definitions as first means of explaining	a) is not sure why anwers = (i.e., 11 x 2; 2 x 11)

Individual Interview with Peter

AHW - What I need you to do is to teach me to multiply. There are chips, cubes, paper, pencil. You can use anything you want.

Peter - Make a multiplying sentence or something?

AHW - Teach me to multiply. What do I need to know?

Peter - Okay. What you need to know ... what you have to do is start of with any number, a number, ... oh man. You start off, you see the number then you look at the second number like uh, (made six-rod) five. You gotta put five. I mean times. (Made five-rod; counted first rod by ones; removed one cube; put two five-rods next to each other). Okay. Five times five. Let's see ... there's five, right? (Put up hands to show five fingers on each). There's five times another five. And you add like five to another five, right? (Picked up five-rod). And you put five in one group. And another five and another five and another five. (Indicated each five by pointing to single cube in five-rod). Then you'll get the answer.

AHW - Can you tell me more? Explain a little more about these fives (indicated two five-rods) and how you get the answer from those?

Peter - (Eleven second pause). Hm ... I don't know.

AHW - Okay, just take your time. there's no time limit on doing this, so just stop and start when you need to. Then you can keep going with your explanation.

Peter - Five is the group and the second five is how much in each group. (Twelve second pause). I think that's all.

AHW - Okay, so this (two five-rods) means five times five? Is that what these groups mean?

Peter - Yeah.

AHW - If you write down five times five, what do the numbers tell you?

Peter - How much groups and how much in each group.

AHW - How do the numbers tell you that? How do I know when I look at a multiplying sentence what the numbers mean?

Peter - The first number ... (six second pause).

AHW - You can use this (paper) to help you explain it too.

Peter - (Twenty-one second pause). I don't know.

AHW - It might be easier if you start with something where you have two different numbers, instead of five and five. That might be part of the problem.

Peter - Let's see. (Made one two-rod and left one five-rod). Two. (Seven second pause). What was the question again?

AHW - What I did was to suggest that you should use two different numbers instead of five and five, use two completely different numbers like three and two so you can tell the difference between them.

Peter - Okay. (Twelve second pause). I don't know.

AHW - So are you having trouble explaining what multiplying is?

Peter - Yeah.

AHW - Can you tell me the difference between multiplying and dividing?

Peter - Dividing is putting groups and dividing is when you put one in the ... like you give the children and you one. Something like that.

AHW - So when we're dividing, what do we start with?

Peter - The number. Of how much things you got.

AHW - And then...?

Peter - Divide by two or something to give the children two or three.

AHW - So you'd take a big group and give you some and give me some....

Peter - But the same number.

AHW - Why is it important to have the same number?

Peter - So it would be fair.

AHW - Okay, what do we do if we don't have the same number?

Peter - Then....

AHW - Does it work?

Peter - No.

AHW - What about if I had eleven things. And I want to divide them between you and me....

Peter - There'd be one left over.

AHW - Why?

Peter - 'Cause that's an odd number.

AHW - And what do we do with the one left over?

Peter - Cut in in half, whatever it is.

AHW - That's okay if it's a chocolate chip cookie. If it was a lemon or something, I don't know if we'd want to cut it in half and share it. So dividing is taking a group and sharing it between people?

Peter - Yeah.

AHW - Can dividing mean anything else?

Peter - No.

AHW - How is that different from multiplying?

Peter - In multiplying, you don't give children stuff.

AHW - What do you do in multiplying?

Peter - Put them in groups.

AHW - So can you show me with the cubes or with the counters, can you show me some multiplying groups?

Peter - (Twenty-one seconds as Peter moved around some cubes). I don't know how to make them.

AHW - Well, groups of what? How many in each group?

Peter - Five. (Put out one five-rod).

AHW - What multiplying sentence would you write for this one? (Indicated five-rod).

Peter - Five times two or something. Five times five.

AHW - Can you explain to me how this shows five times two?

Peter - Hmm?

AHW - Well, you said you could write five times two for this one. Or would it have to be five times five?

Peter - Five times five.

AHW - So can you explain to me, looking at this one, how you'd figure out five times five?

Peter - (Made five-rod; placed at right angle to original five-rod). Five here, (indicated five-rod just made) another five here, here, here, here. (Indicated where rows of five-rods would go).

AHW - Why don't you do that and show me how it would look.

Peter - (Made four more five-rods and placed in array: had five five-rods and one original one at right angles).

AHW - Car you explain to me what you did?

Peter - An array.

AHW - What does the array show us?

Peter - There's five here (bottom row) and five here (left column). Five times five.

AHW - Is this part of the array? (Five-rod along edge).

Peter - No.

AHW - So what's it doing here?

Peter - I don't know. (Removed extra five-rod).

AHW - Oh, it's just extra? So we have five across here and five down each of these?

Peter - Yeah.

AHW - How many do we have altogether?

Peter - Twenty-five.

AHW - How do you know that?

Peter - Five times five is twenty-five.

AHW - Can you show me an array that has six times five?

Peter - (Added one more cube to each five-rod; included the extra five-rod from earlier 5 x 5 array; had six by six array; counted number of rods; removed the extra).

AHW - How many down this way? (Column).

Peter - Five.

AHW - How many this way? (Row).

Peter - Six.

AHW - Does that show us six times five?

Peter - Backwards.

AHW - Why is it backwards?

Peter - I don't know.

AHW - How could you have it forwards?

Peter - (Shifted array so was oriented in opposite direction).

AHW - All right. So it's six this way (row) and five this way (column) so it's six times five?

Peter - Yeah.

AHW - When you had it this way before, you had this one in with it (extra six-rod). Why did you take that one away?

Peter - 'Cause I put groups? And I put one group of five (put rod along to show its original position in five by five array) well, it used to be five. So I had one group here and another group here, another group, another group. (Removed one cube to make five-rod again).

AHW - So what did you use this rod for?

Peter - To put the groups. Not the ... how much in each group.

AHW - So were you using it just to keep track? Of how many groups you had?

Peter - Yeah.

AHW - Why did you take it away at the end?

Peter - 'Cause I didn't need it.

AHW - So you just used it to keep track and then it was extra?

Peter - Yeah.

AHW - (Pointed to array). So this is called an array. Where have you seen arrays before?

Peter - In the math lab. With Mr. G.

AHW - How does an array help you with multiplying?

Peter - Just count one, two, three, four, five ... (indicated cubes in one column) and you keep on counting down, down (indicated columns) until you get the answer.

Peter - Okay. Can this array show you more than one multiplying sentence?

Peter - Two. (Counted down one column). Five times six (indicated one six row) or six times five (indicated appropriate rows and columns).

AHW - How can it show you two different things?

Peter - I don't know. 'Cause they're a family?

AHW - Is the answer going to be the same?

Peter - Yes.

AHW - How do you know?

Peter - The teachers told me.

AHW - And you believe everything you're told?

Peter - Yup.

AHW - So if I told you the sky was green would you believe me?

Peter - No.

AHW - Can you show me another multiplying....

Peter - Array?

AHW - All right, an array. Whatever. You can arrange it however you want.

Peter - (Made two by four array). Two times four.

AHW - So you have...? Can you explain why it's two times four?

Peter - There's two down and four. (Indicated columns of two and rows of four).

AHW - This is two times four and it's also....

Peter - Two times four or four times two.

AHW - Is the answer the same?

Peter - Yes.

AHW - (Separated the array into four two-rods; just created spaces between Peter's groups; did not regroup or change anything but the spacing). What does that show us?

Peter - Apart. It's apart.

AHW - What multiplying sentence could you use for that?

Peter - (Seven second pause). Four times four.

AHW - Can you explain to me why it's four times four?

Peter - (Indicated two of the two-rods). One, two. There's two groups and then there's four. (Indicated other two two-rods).

AHW - I'm not sure that I understand how you got the fours. Can you just keep explaining?

Peter - One, two, one, two. (Counted the first two two-rods by ones). There's four. One, two, one, two. (Counted other two two-rods by ones).

AHW - How many do we have here? (In first two-rod).

Peter - Two.

AHW - And how many here? (In second two-rod).

Peter - Two.

AHW - So you put these together to make four? (Indicated first two two-rods).

Peter - Yeah.

AHW - And then...? (Indicated other two two-rods).

Peter - Another two, another two.

AHW - And we put them together.

Peter - Four.

AHW - So this shows us four times four?

Peter - What does four times four equal?

Peter - Eighteen?

AHW - Is that what you have here? (Indicated four two-rods).

Peter - Yeah.

AHW - Suppose I give you something that looks like that. (Two three-rods and three two-rods). Can you explain to me how to multiply that?

Peter - Just put them together. (Pushed together one three-rod and one two-rod).

AHW - You have to leave them the way they are. You can't put anything together or take anything apart. How would you multiply that?

Peter - (Twenty-seven second pause). I don't know.

AHW - What's the problem with it?

Peter - It's not equal.

AHW - What difference does that make?

Peter - (Six second pause). I don't know.

AHW - Is there something about multiplying that has to be equal?

Peter - Yeah.

AHW - Can you tell me?

Peter - No.

AHW - Can you show me how you could make that so you could multiply it? Change them around somehow?

Peter - (Counted the two-rods first then the three-rods; made three four-rods; then changed to two six-rods). Six times six.

AHW - Which equals?

Peter - (Used fingers to count; counted six points on each of six fingers to keep track of groups). Thirty-four?

AHW - Do you have thirty-four cubes there?

Peter - (Counted one six-rod). Twelve. Twelve cubes. (Looked again at the two six-rods). The answer is twelve. No, it's ... (drew six circles with six smaller circles in each one). Thirty-six.

AHW - You made how many groups?

Peter - Six.

AHE - And how many did you put in each one?

Peter - Six.

AHW - Is this six multiplied by six?

Peter - Yeah.

AHW - And that equals thirty-six?

Peter - Yeah.

AHW - Does that match with what you have here? (Indicated the two six-rods).

Peter - Yeah.

AHW - So is there anything else I need to know about multiplying?

Peter - No.

AHW - So that's it. It has to be in groups and the groups have to be...?

Peter - Equal or something.

AHW - What do I do once I have the groups? How do I figure out the answer?

Peter - Hm. (Fifteen second pause). Just add six plus six plus six or something like that.

AHW - Multiplying is like adding?

Peter - Just add six times, plus six times ... oops. (Wrote 6*6+6); began again and wrote 6*6*6*6+6+6).

AHW - Now can you explain to me what you just wrote?

Peter - Six plus six plus six plus six plus six plus six.

AHW - That's hard to say isn't it? So what does this go with? (Indicated the equation).

Peter - This. (Indicated the six large circles with the smaller circles inside).

AHW - With this? Okay, so up here you have a group with six in it so that's this one? (First adend in equation).

Peter - Yeah.

AHW - And that's that one? (Showed group and six from equation).

Peter - Yeah.

AHW - This is like adding all of these sixes together? (In equation).

Peter - Yeah.

AHW - How does that fit with the multiplying

Peter - It's just a faster way of doing like six plus six plus six.

AHW - And an easier way to say it, too, right? Could I have your pencil, please. (Wrote 4 x 3 =). If I asked you to show me what four multiplied by three loooked like, what would you do?

Peter - (Drew four circles; put three smaller circles into each).

AHW - Can you tell me what you just did?

Peter - A picture.

AHW - Of?

Peter - Of a circle with a three in each group.

AHW - How many circles did you start by doing?

Peter - Four.

AHW - Four circles and three little circles inside each one?

Peter - Yeah.

AHW - Can you tell me how that goes with these numbers, with the four and the three?

Peter - Four groups, three in each group.

AHW - Does the four have to mean how many groups?

Peter - Yeah.

AHW - Why?

Peter - (Thirteen second pause). 'Cause if you do three. Wait. No, it doesn't. You could just do a three. (Drew three circles). Three groups of four. (Put four small circles inside each larger one).

AHW - And does this mean the same as this? (Indicated sets of three circles and sets of four circles).

Peter - Yeah.

AHW - So the four can mean how many groups there are or how many things there are in each group?

Peter - Yeah.

AHW - Does it make any difference?

Peter - No.

AHW - And what does the three mean?

Peter - How much in all. Make it three circles and four in each.

AHW - Or down here (in set of four circles) the three means?

Peter - How much in all.

AHW - What's the answer to this?

Peter - (Counted the smaller circles inside the four). Twelve. (Put answer into equation).

AHW - Can you show me four multiplied by three using the cubes?

Peter - (Made one three-rod and one four-rod; put them side by side). Just put a times there. (Wrote on table with finger in between the rods).

AHW - Four cubes multiplied by three cubes equals....

Peter - Is twelve.

AHW - Is there anything else I need to know?

Peter - No.

AHW - So that's all multiplying is?

Peter - Yeah.

AHW - Okay. Thank-you.

Strengths	Weaknesses
a) "shorthand" - uses one three-rod and one four-rod to represent 3 x 4 (p 262)	a) appears to understand multiplication mainly in arrays (p 258) I separated his array into groups; he couldn't explain)
b) invented finger counting method	b) pp. 3-4, 6 x 5 = 5 x 6 "because the teachers told me" (p 257)
c) concrete/pictorial/symbolic levels used	c) unclear about meanings of numbers in equations (p 262); confused the meanings of products and factors
d) could use same array for 6 x 5 and 5 x 6 (p 257)	

Individual Interview with Theresa

AHW - What I need you to do is to teach me to multiply. There are cubes, there are counters, you have paper.

Theresa - (Looked at counters). I call them cookies.

AHW - You call them cookies. That's great. You can use any of those things you want or just tell me but I want you to teach me how to multiply.

Theresa - (Thirty second pause). You mean tell you how to do it?

AHW - Tell me. Show me. Explain it. I need to know what you know about multiplying.

Theresa - (Twelve second pause). Too bad this wasn't dividing.

AHW - We may get to dividing later. Right now, we'll just worry about multiplying. Would it be easier to start with dividing?

Theresa - Uh huh.

AHW - Okay. Well, why don't you do that. But keep in mind I need to know about multiplying later. You can start with dividing.

Theresa - (Began counting out cubes).

AHW - Can you tell me what you're doing?

Theresa - I'm taking out twenty-two cubes and making them into groups.

AHW - Okay.

Theresa - (Continued counting).

AHW - You said twenty-two cubes?

Theresa - (Nodded; put cubes into two groups).

AHW - So what did you do?

Theresa - I took the twenty-two cubes and put them into two groups and after I got done there was eleven in each group.

AHW - Why were there eleven in each group?

Theresa - (Thirty-six second pause). 'Cause it's like if you take twenty-two and you take eleven away and you put them in the one group and you take the other group and you'll have eleven and that'll be the answer.

AHW - Suppose we started with this (shoved all the cubes into one group). And I made them into two groups. And I started like this (made two unequal groups). Is that twenty-two divided by two? ... How many are in this group? (smaller).

Theresa - Eight.

AHW - How many are in this group?

Theresa - Fourteen.

AHW - So I took the twenty-two cubes and I made them into two groups. Eight in one group and fourteen in the other. That's division, isn't it?

Theresa - No. They have to be equal groups.

AHW - Why?

Theresa - 'Cause if they aren't equal groups, you'll get a different answer and then if you were to do it again, someone would say that you did that wrong. And then you'd get another different answer and you'd keep on getting different answers all the time.

AHW - Is it important to have the same answer?

Theresa - No. Well, if you're doing it over and over again it is. But if there were (??).

AHW - When would it be important to have the same answer?

Theresa - (Twenty-five second pause). I don't know.

AHW - All right. So explain to me what dividing is. You take a group and you do what?

Theresa - You take the group and you put it in the ... you can either ... the dividing sentence is two things. You either put it into two groups or you put two in each group. If you have three, you make it into three groups or you put three in each group. It's kind of like what you have after the dividing.

AHW - So when we're dividing, are we taking little groups or are we taking a big group? To start with?

Theresa - What we're doing right now?

AHW - Yeah. How do you start when you're dividing? ... Did we start with small groups or did we start with a big group?

Theresa - We started with a big group.

AHW - You start with a big group and you do what?

Theresa - Either ... if it's written down on paper and it says make two groups of whatever, then you just take all the cubes and you make them into two equal groups or if it says something else and it doesn't say make it into two equal groups, then you make it into groups of two in each group.

AHW - So we're taking a big group and we're making it into little groups when we're dividing. Is anything about the groups important?

Theresa - That they have the same amount of cubes in each one so that you don't get a different answer.

AHW - We started with twenty-two. If we had started with twenty-one, what would have happened?

Theresa - We would have had one remainder and ten in each group.

AHW - Is the one remainder a problem?

Theresa - (Shook head).

AHW - What do you do with the remainder? Do you put it in with one of the groups?

Theresa - No. You just put it away somewhere and you just keep on going.

AHW - Is it important or can you forget it?

Theresa - You can forget it, but you have to write it down. So it would be important.

AHW - That sounds like you're telling me two different things, that I can forget it but I have to remember it at the same time.

Theresa - You have to remember it.

AHW - 'Cause otherwise, will the answer be right?

Theresa - No.

AHW - Now can you teach me to multiply, now that you've taught me to divide?

Theresa - When you're multiplying, you can take twenty-two but you can put ten multiplied by two. Then you can just add ten on instead of taking ten away and putting it into groups. It's like doubles. You could just add it on. And then you say ... then you could write down the answer and then ... (thirteen second pause). It's hard to explain. It's easier to do it than explain it.

AHW - Go ahead and do it. You can do it here (paper) or here (cubes) or do it with the cookies. Or with these (counters).

Theresa - (Wrote 9 x 5 =; drew nine circles and put five dots inside each). Sometimes you can just get ... if it's nine multiplied by five or it's eight multiplied by five, if it's anything multiplied by five, you can just count by fives. It's the same as counting by twos, or threes, or fours or whatever it is. And, but sometimes it's hard so you can draw the first number of circles and you can put ... if it's seven, you could put it ... seven dots or something in each circle or you could just write the seven. And you could count it all up and see how much it made, then you could write the answer down. Or sometimes you could just put seven plus seven plus seven and just keep on going until you get it the right number of times.

AHW - Can you tell me how this math sentence that you wrote and these pictures go together?

Theresa - Um....

AHW - Or do they go together?

Theresa - Well, there's nine multiplied by five so I can just take nine circles and if I put five in each one, it helps me sometimes if I think I know what it is I do this just to make sure. I guess first but then I would ... I'd think of a guess and then I would write it down on one part of the paper and then I would do something just to make sure if it was done properly.

AHW - So are the pictures your way of checking to see whether your answer up here is correct? (Referred to 9 x 5 =).

Theresa - Yes.

AHW - What does the nine being multiplied by five mean?

Theresa - The nine means that you ... like ... I don't (??).

AHW - Start with whatever's more comfortable.

Theresa - The nine means ... if the second number's five, the nine means to add the second number on that much times if that's what the first number means. The second number, you just keep adding it on until you do it nine or eight or however much many times you're supposed to do it.

AHW - So the nine tells you how many times you're supposed to add it?

Theresa - Yeah.

AHW - What does the five mean?

Theresa - It means what you have to add.

AHW - If you had three multiplied by two, what does the three mean?

Theresa - You add two on three times.

AHW - Can you say it any other way?

Theresa - Two multiplied by three.

AHW - And then what does the two mean?

Theresa - Add it on three times.

AHW - In two multiplied by three, what are you adding?

Theresa - The three.

AHW - What is three multiplied by two?

Theresa - You're taking the two and you're adding it three times instead of two times and it's the other way around.

AHW - What does it equal? What's the answer?

Theresa - Six.

AHW - And what does the six mean?

Theresa - It means how much both ... it means that it added up to altogether. Sometimes they teach you you can turn them around.

AHW - Does it make any difference if you turn them around?

Theresa - I don't know. I only did it a couple of times but I don't think so.

AHW - We were just talking about two multiplied by three and three multiplied by two. Does it make any difference the order the numbers come in?

Theresa - (Shrugged).

AHW - Does it change the answer?

Theresa - No, because in three multiplied by two, you're just taking two and adding it three times and you can just count by twos and it goes two, four, six and you added it three times. And the other way around, you would just ... it's like three multiplied by two but it's not. You just have to add the three two times and then it's two multiplied by three equals six.

AHW - Can you use some of these and show me three multiplied by two? (Cubes).

Theresa - (Made three groups of two).

AHW - So we have three...?

Theresa - Groups. And two in each group.

AHW - Now can you show me two multiplied by three?

Theresa - (Rearranged cubes into two groups of three).

AHW - That was easy. What does it equal?

Theresa - Six.

AHW - Do you know why the answer to both questions is six?

Theresa - It's like adding three plus three equals six and there's two threes and that's all I do for the groups. I just add. If it's twenty-four plus twenty-four, I just ... if it's twenty-four times twenty-four, I just add another twenty-four to one twenty-four and it would be forty-eight.

[Recess break]

AHW - We were doing multiplying before recess and you were explaining what the nine and the five meant in this equation, in this multiplying sentence. If I showed you ... (began to put out cubes; put out one two-rod).

Theresa - That's twenty-two.

AHW - That's twenty-two?

Theresa - No, that's two.

AHW - (Put out unequal groups). Can you tell me how to multiply that, please?

Theresa - (Twenty second pause).

AHW - Can you tell me what you're thinking about? ... Can you talk to me and tell me as you're thinking? I want to know how you figure it out.

Theresa - (Six second pause).

AHW - What do you have in front of you?

Theresa - Twelve cubes.

AHW - Okay, but it isn't just a pile of twelve cubes. How are they arranged?

Theresa - Three of them have two in each and then two of them have three in each group.

AHW - So?

Theresa - Three multiplied by two.

AHW - Leaving it exactly the way it is, you can multiply it by two?

Theresa - Yeah.

AHW - Can you show me how to do that?

Theresa - (Looked at cubes).

AHW - How would you multiply this by two?

Theresa - You can take those three and ... (two-rods; forty-five second pause).

AHW - Is there a problem?

Theresa - I don't know how you'd multiply this. Two groups of three and three groups of two.

AHW - Why not?

Theresa - 'Cause I always thought in every group it'd have to be equal.

AHW - Is it possible to multiply when you have groups that look like this?

Theresa - (Shrugged). I never tried it.

AHW - If you have two unequal size groups, do you think you can multiply it?

Theresa - (Shrugged). I don't know what the question would be. It would be....

AHW - Think back --

Theresa - I know what the answer would be, though.

AHW - What's the answer?

Theresa - If you would take these away. (Indicated one cube at the end of each three-rod).

AHW - If we took one cube away from each of these? (Removed the cubes). Can I just take them away or could I do something else with them? ... I'd like to leave them as part of the set that we have here. So what could we do with them?

Theresa - I don't know. I never tried it this way.

AHW - (Put the two extra cubes together). Could we do this? Could we put them together and include them?

Theresa - Yeah.

AHW - You could. So what would this multiplying sentence be? (For six two-rods).

Theresa - Six multiplied by two?

AHW - Equals?

Theresa - Twelve.

AHW - And we started out with twelve, didn't we, because you told me that to begin with. You gave me an answer a minute ago that told me why I can't multiply this. You said 'cause I think the groups have to be....?

Theresa - Equal. I always thought in every group it'd have to be equal.

AHW - And that's a very important part of multiplying.

Theresa - I got it from what I said in the math lab.

AHW - How did you get it from the math lab?

Theresa - I don't know. We were just playing a thing. I think it was Giants and I just said something about the giant's strides and I don't know how I said it.

AHW - Was it the same idea? Was it about equal groups?

Theresa - (Nodded).

AHW - Why did you remember that you said that?

Theresa - 'Cause you said it out loud.

AHW - Okay. What else did I say about it? Do you remember?

Theresa - No.

AHW - Just that it's a very, very important idea for multiplying. So, multiplying is about equal groups and what else? If you didn't have any of this kind of stuff in front of you, and I said to you, "Tell me about multiplying", no cubes, no cookies. Tell me what multiplying is.

Theresa - When you multiply the groups have to be in ... all the groups equal (ten second pause).

AHW - Is that all there is to multiplying? Just equal groups?

Theresa - No. (Seven second pause). I don't know how to explain it.

AHW - Okay then. Can you tell me the difference between multiplying and dividing?

Theresa - When you're multiplying, you can't have one remainder. (Ten second pause). Multiplying means one thing and dividing means two things.

AHW - That's interesting. Can you tell me more about that?

Theresa - About what?

AHW - What do you mean, one thing and two things?

Theresa - The dividing sign, it tells you you can have two groups that have equal amounts in each group or else you can make a littler group with less in each group and then you would write the answer of how much groups you had and in the two big groups you would anwer how much was in each group. And when you're multiplying, you can only do one thing.

AHW - And what's the one thing do you do when you multiply?

Theresa - You take nine and you can ... oh, boy ... you can take the number and you can just, the way you have to do it is take it and put it in the groups. And you can get the answer.

AHW - (Put out three two-rods and a pile of individual cubes). If you came into the room and I had this, tell me which one is multiplying and which one is dividing.

Theresa - (Examined). That would be multiplying. (Pointed to assembled rods).

AHW - Because?

Theresa - One multiplied by three, there's only one group. Or it could be two multiplied by three.

AHW - (Indicated the pile). Is this multiplying as well?

Theresa - Could be.

AHW - Do you think it probably is? Multiplying or dividing?

Theresa - Dividing.

AHW - Why?

Theresa - Actually, multiplying because there's one group with eleven in each group and in dividing it'd have to have two groups. If you just set it down like this, it'd have to be multiplying.

AHW - What would dividing look like? If you came in and saw something, what would make you say, okay that's going to be dividing?

Theresa - Either if you could have lots of little groups like this (two rods) or you could have two big groups like this. (Pile of cubes). And you can ... that's about it.

AHW - If you came in and we had ... (made groups with and without remainders) ... is it easier to tell which is multiplying and which is dividing?

Theresa - No.

AHW - Why not?

Theresa - 'Cause there is both two groups.

AHW - Is there any difference between these, these two sets of things?

Theresa - No.

AHW - Be really careful. Is there any difference at all?

Theresa - This one has one remainder and this one doesn't.

AHW - What did you tell me earlier about a remainder?

Theresa - I said you couldn't have a remainder.

AHW - So if you saw the groups and the remainder....

Theresa - That's dividing (with remainder) and that's multiplying (without).

AHW - Can you tell if you just see the groups like this when you come in? (One pile of cubes and several rods). Can you tell right away whether it's multiplying or dividing?

Theresa - No.

AHW - If you opened a book and you saw pictures, when you start multiplying what do they usually give you for a picture?

Theresa - Um....

AHW - Would they give you a big group like this or little groups?

Theresa - Probably little groups.

AHW - Right. Is there anything else I need to know about multiplying?

Theresa - I don't know.

AHW - Do you think you've done a good job of teaching me?

Theresa - You're the one that's writing the book.

AHW - It's up to you. Do you think there's anything else that you could teach me that I need to know about multiplying?

Theresa - No.

AHW - Okay then. Thank-you.

Strengths	Weaknesses
a) good grasp of × b) uses concrete, pictorial, symbolic levels c) can verbalize differences in situations d) aware that cannot have multiplication remainder e) much information given orally; seems to be able to discuss multiplication without having to demonstrate with manipulatives; seems to represent more advanced ability f) knows that groups must be = g) (p 265) seems to know that × can be done two ways h) compares to repeated addition; has strategy for finding answers i) can describe commutativity; knows answer stays same	a) (p 273) responses contradictory; first says that there is no difference between two sets of cubes; when challenged can immediately identify difference and what it signifies b) (p 265) confused about whether equations need to have same answers if done over c) little motivation to explore to find unknown answers