Distributionally Robust Chance-Constrained Energy Management for Islanded Microgrids

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***Abstract*—With the development of smart grid, energy management becomes critical for reliable and efficient operation of power systems. In this paper, we develop a chance-constrained energy management model for an islanded microgrid, which includes distributed generators, energy storage system (ESS), and renewable generation, such as wind power. The objective function of this model consists of generation cost, emission cost, and ESS degradation cost. To capture the uncertainty of renewable generation, a novel ambiguity set is introduced without knowing its probability distribution or exact moment information. Based on the ambiguity set, the chance constraint can be processed with distributionally robust optimization method and the energy management problem is reformulated as a tractable second-order conic programming problem. The proposed approach is tested with a case study and simulation results indicate that it is effective and reliable. Moreover, the comparison with the method based on known moment information and some other methods is also conducted to show the performance of the proposed method.**

***Index Terms*—Ambiguity set, chance-constrained energy management, distributionally robust optimization (DRO), microgrid, renewable energy.**

Nomenclature

The main notations used in this paper are listed below for quick reference.

Indices and Numbers

i, N_{dg}	Index and number of conventional generators.
j, N_{ess}	Index and number of energy storage system (ESS).
k, N_{def}	Index and number of deferrable loads.
t	Index of time slot.

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Decision Variables

$P_{i,t}$	The output power of generator i at time slot t [kW].
$P_{j,t}^{ch}$	Charging power of ESS j at time slot t [kW].
$P_{j,t}^{dch}$	Discharging power of ESS j at time slot t [kW].
$P_{k,t}^{def}$	Demand of deferrable load k at time slot t [kW].
x_t	General representation of decision variables.
$\mathbf{x}$	Set of decision variables.

Parameters and Auxiliary Variables

P_i^{min}	Minimum output power of generator i [kW].
P_i^{max}	Maximum output power of generator i [kW].
R_i^{up}, R_i^{dn}	Ramp-up and ramp-down limits of generator i [kW].
a_i	Quadratic coefficient of cost function of generator i [\$/kWh^2].
b_i	Linear coefficient of cost function of generator i [\$/kWh].
c_i	Constant term of cost function of generator i [\$/h].
$CG_{i,t}$	Generation cost of generator i at time slot t [\$].
d_i	Quadratic coefficient of emission function of generator i [kg/kWh^2].
e_i	Linear coefficient of emission function of generator i [kg/kWh].
f_i	Constant term of emission function of generator i [kg/h].
$Em_{i,t}$	Emission of generator i at time slot t [kg].
$CE_{i,t}$	Emission cost of generator i at time slot t [\$].
c^{emis}	Emission cost coefficient [\$/kg].
$E_{j,t}$	Energy storage of ESS j at time slot t [kWh].
$P_j^{ch,max}$	Maximum charging power of ESS j [kW].
$P_j^{dch,max}$	Maximum discharging power of ESS j [kW].
E_j^{min}	Minimum energy storage of ESS j [kWh].
E_j^{max}	Maximum energy storage of ESS j [kWh].
η_j^{ch}, η_j^{dch}	Charging and discharging efficiency of ESS j.
Δt	Absolute time between two adjacent time steps [h].
$CS_{j,t}$	Degradation cost of ESS j at time slot t [\$].
c^{ess}	Degradation cost efficient of ESS [\$/kWh].
L_t	Critical load at time slot t [kW].
P_k^{def}	Total demand of deferrable load k over the specified time interval [kW].
T_k^a, T_k^b	Start time and end time to serve deferrable load k.

$P_k^{def,min}$	Minimum serving rate for deferrable load k [kW].
$P_k^{def,max}$	Maximum serving rate for deferrable load k [kW].
w_t	Uncertain wind power at time slot t [kW].
ϵ	Violation probability of power balance constraint.
$\mathcal{P}_t^0$	The set of all the probability distributions of random variables.
$\mathcal{P}_t^1, \mathcal{P}_t^2$	The ambiguity set of the probability distribution of wind power.
$\mathbb{P}_t$	Distribution of random variable w_t.
$\mathbb{E}_{\mathbb{P}_t}$	Expectation under distribution $\mathbb{P}_t$.
μ_t	Mean value of random variable w_t.
σ_t^2	Variance of random variable w_t.
$\underline{\mu_t}, \overline{\mu_t}$	Lower and upper bounds of μ_t.
$\underline{\sigma_t^2}, \overline{\sigma_t^2}$	Lower and upper bounds of σ_t^2.
$h^0(x_t)$	Auxiliary functions of x_t.
$h(x_t)$	Auxiliary coefficient functions of x_t.
$\beta, v, \tau, z, s, \eta_1, \eta_2, r, \xi$	Auxiliary variables in the second-order conic constraint formulation.

Abbreviations

CC	Chance-constrained.
CCP	Chance-constrained programming.
CVaR	Conditional value-at-risk.
DG	Distributed generator.
DRO	Distributionally robust optimization.
ESS	Energy storage system.
SDP	Semidefinite programming.
SOCP	Second-order conic programming.

I. Introduction

As IMPORTANT parts of the future smart grid, microgrids have attracted much attention and have been widely studied over the last decade. A microgrid is usually integrated with distributed energy resources (DERs) such as distributed generators (DGs), energy storage systems (ESS) and controllable loads [1]. It can either operate in grid-connected mode or islanded mode and the energy schedule can be handled by an energy management system (EMS). With advanced technologies, renewable energy generation such as wind power and solar power increasingly penetrating into the microgrid, the microgrid energy management problem has become complex and uncertain.

To deal with the uncertainty caused by renewable energy generation in microgrid energy management, various methods have been investigated including robust optimization, stochastic programming and chance constrained programming. Among these methods, the application of robust optimization [2] in microgrid energy management has been extensively explored recently. For example, the energy management for a grid-connected microgrid with high penetration of wind power and demand-side management is studied in [3]. The formulated robust optimization problem is solved by a distributed algorithm to minimize the system net cost. Similarly, a scenario-based robust energy management method for a grid-connected microgrid is proposed to deal with the uncertain renewable generation and load in [4]. By exploring the worst-case scenario, a robust energy management solution for the proposed model could be obtained. In [5], a two-stage adaptive robust optimization method is studied for microgrid energy management which considers the uncertainties of renewable generation and grid-connection condition. The uncertainties are controlled by the "budget of uncertainty" parameters in this work.

Stochastic programming is another common approach to handle the uncertainty in microgrid operation and energy scheduling. With this method, the uncertainties associated with renewable generation and load are usually represented by certain distributions such as normal distribution [6], [7]. Although not so many as the research on robust and stochastic optimization model, chance constrained programming (CCP) method is also investigated in microgrid energy management [8]. For instance, in [9], two CCP problems are formulated for grid-connected microgrids energy management and the problems are solved by a linear programming transformation. Also, a grid-connected microgrid based on combined heat and power (CHP) system is studied in [10]. The CCP method is employed to describe the uncertainty of renewable generation and load and the optimal schedule problem is solved by a particle swarm optimization (PSO) based algorithm. In [11], chance constrained optimization is employed to solve the demand response problem for a home energy management system and the chance constraint is used to describe the variable power interaction between the household and the utility grid due to uncertain load demand. In addition, the CCP method has also been studied for other power system problems such as transmission expansion planning problem with load and wind uncertainties [12].

Although the microgrid energy management has been widely studied, most of the research focuses on the grid-connected microgrids with uncertainties compared with few research works on islanded microgrids [13], [14]. In addition, the constraints with uncertainties have not been sufficiently studied which mostly concentrate on the balance of generation and load demand [9]. However, for real islanded microgrids integrated with renewable energy, the power balance may not always be satisfied due to the uncertainty of renewable generation which will lead to a reliability problem. To ensure the system reliability, in this paper, we propose a chance-constrained (CC) problem for islanded microgrids energy management involving the uncertain renewable generation. The uncertainty of renewable generation is represented by a new box-type ambiguity set, based on which the CC problem can be solved by distributionally robust optimization (DRO) method.

The DRO method is an intermediate approach between stochastic programming and robust optimization which was first introduced by Scarf in 1958 [15]. Instead of making an assumption of a certain distribution, the DRO method optimizes the expected objective over a set of unknown distributions sharing certain statistical characteristics such as moment information. Recently, this idea has been applied to power system optimization such as unit commitment problem [15], [16], energy reserve and scheduling [17], [18] and optimal power flow [19]. In addition, DRO method has also been applied to power system planning problem [20], distributed generation capacity assessment of active distribution networks [21], optimal bidding problem of electricity markets [22], transmission line congestion management problem [23] and so on. A general framework of these works is that an ambiguity set is proposed first based on the assumption of the research problem and a tractable reformulation is then derived based on dual and approximation techniques.

Although DRO method has become a hot topic recently, its application to CC problems is still not so sufficient. For distributionally robust CC model, it has been mostly studied in optimal power flow problems so far [19], [24], [25]. However, research on CC microgrid energy management with DRO approach has not been reported, to the best of our knowledge. In this study, we try to achieve optimal robust energy management for islanded microgrids by utilizing the DRO method, i.e., we formulate a distributionally robust CC model that accounts for the uncertain renewable generation with a new ambiguity set.

The main contributions of this work are summarized as follows:

1) A CC energy management problem is formulated for an islanded microgrid considering DGs, ESS, renewable generation and various load demand. Unlike the previous chance constraints in the literature, the common power balance is presented in a probabilistic version for islanded microgrids in this work. The objective of the proposed model is to minimize the total system cost including the generation cost and emission cost of DGs and degradation cost of ESS where the consideration of emission cost is necessary in practice with increasing attention of environment problem;
2) A novel ambiguity set is proposed to describe the uncertain probability distribution of renewable generation. In particular, we use the box-type ambiguity set to capture the uncertain moment information (e.g., mean and variance) of renewable generation. This ambiguity set has not been studied previously for microgrid energy management problem with uncertainty. Based on the ambiguity set, the DRO method is utilized to solve the microgrid energy management problem by transforming the CC problem into a tractable second-order conic programming (SOCP) problem which can be solved by off-the-shelf solvers efficiently;
3) A case study with real datasets to verify the effectiveness of the proposed method is presented. The comparison with the DRO method with known moment information is also carried out to show the robustness of the approach. In addition, sample average approximation (SAA) and stochastic optimization with normal distribution method, which are two common methods to deal with chance constraint, are also applied to solve the problem for comparison purpose.

The remainder of this paper is organized as follows. The problem formulation of islanded microgrid energy management is presented in Section II. Section III proposes the solution method to reformulate the distributionally robust chance constraint to be second-order conic constraints so that the complete problem can be solved as an SOCP problem. In Section IV, a case study with real datasets is carried out and the conclusions are finally drawn in Section V.

II. Problem Formulation

A typical islanded microgrid is composed of conventional generators, ESS, renewable generation and different kinds of load. It is usually assumed that the microgrid energy management is controlled by an EMS in a centralized mode. In this section, we introduce different components of the studied microgrid system and formulate the corresponding energy management problem.

A. Distributed Generation

Distributed generation in a microgrid usually includes micro-turbines, diesel generators, fuel cells and renewable generation such as uncertain wind power which is introduced in the next section. The conventional generation units are important components of a microgrid and they are dispatchable to meet the load demand. In this study, we mainly focus on diesel generators which burn fossil fuel to generate electricity. The output power of generators is restricted by the maximum and minimum limits as follows:

$$P_i^{min} \leq P_{i,t} \leq P_i^{max}, \forall i, t. \tag{1}$$

In addition, the DG units should satisfy the ramping up/down constraints:

$$P_{i,t+1} - P_{i,t} \leq R_i^{up}, \forall i, t \tag{2}$$

$$P_{i,t} - P_{i,t+1} \leq R_i^{dn}, \forall i, t. \tag{3}$$

Note that the generators are assumed to have on status over a finite time horizon in the energy management [3]. If the unit commitment problem is considered in an extended model, then the start-up and shut-down constraints should also be included.

Generally, the operation cost of generators is mainly the fuel consumption cost or generation cost which can be expressed as a quadratic model [13] as follows:

$$CG_{i,t} = \left(a_i P_{i,t}^2 \Delta t + b_i P_{i,t} + c_i\right)\Delta t. \tag{4}$$

The linear cost model [5] is included in this quadratic model. In order to achieve environmentally friendly microgrid energy management, the emission effect should also be considered. In this study, we assume that only diesel generators produce emission and the emission model is also represented as a quadratic function [26], [27]. Then the emission cost model can be expressed as follows:

$$Em_{i,t} = \left(d_i P_{i,t}^2 \Delta t + e_i P_{i,t} + f_i\right)\Delta t \tag{5}$$

$$CE_{i,t} = c^{emis} * Em_{i,t} \tag{6}$$

where c^{emis} is the emission cost coefficient.

B. Energy Storage System

In a microgrid integrated with renewable generation, the ESS plays a critical role in mitigating the system uncertainties and maintaining the power balance. Considering a battery ESS, we have the following dynamic model and constraints:

$$E_{j,t+1} = E_{j,t} + \eta_j^{ch} P_{j,t}^{ch} \Delta t - P_{j,t}^{dch} \Delta t / \eta_j^{dch}, \forall j, t \quad (7)$$

$$0 \leq P_{j,t}^{ch} \leq P_j^{ch,max}, \forall j, t \quad (8)$$

$$0 \leq P_{j,t}^{dch} \leq P_j^{dch,max}, \forall j, t \quad (9)$$

$$E_j^{min} \leq E_{j,t} \leq E_j^{max}, E_{j,T} = E_{j,0}, \forall j, t \quad (10)$$

where constraint (7) represents the dynamics of the stored energy; constraints (8) and (9) are used to limit the charging and discharging power. In constraint (10), the ESS capacity is restrained by a lower and upper bound to avoid overcharging and deep discharging. In addition, the final stored energy is assumed to be equal to its initial energy level. Note that the complementary constraint $P_{j,t}^{ch} P_{j,t}^{dch} = 0$ is usually used to avoid simultaneous charging and discharging which results in a mixed integer linear programming (MILP) model for ESS in some references [7], [13]. Actually, this constraint is redundant when charging and discharging efficiency are considered as in this work and the MILP model can be exactly relaxed to a linear model to reduce the computational burden [28], [29].

The ESS will degrade with frequent charging and discharging process. Therefore, the ESS degradation cost should be considered in energy management. In this work, we adopt a linear model to calculate the ESS degradation cost [5] for simplicity as follows:

$$CS_{j,t} = c_j^{ess} \left(\eta_j^{ch} P_{j,t}^{ch} \Delta t + P_{j,t}^{dch} \Delta t / \eta_j^{dch} \right) \quad (11)$$

where c^{ess} is the degradation cost coefficient.

C. Load Demand

The load demand in a microgrid can be classified into two groups: critical loads and deferrable loads. Critical loads are non-dispatchable and must be satisfied in highest priority such as the hospital load demand. In this study we use L_t to represent the critical load at each time slot.

Unlike critical loads, deferrable loads are dispatchable and can be scheduled according to the real-time power supply and demand. For these kind of loads, using the electrical vehicle as an example, their load only needs to be satisfied over a specified time horizon. Hence, the deferrable load demand model can be expressed as follows [3]:

$$\sum_{t=T_k^a}^{T_k^b} P_{k,t}^{def} = P_k^{def}, \forall k, t \in \left[T_k^a, T_k^b\right] \quad (12)$$

$$P_k^{def,min} \leq P_{k,t}^{def} \leq P_k^{def,max}, \forall k, t \in \left[T_k^a, T_k^b\right] \quad (13)$$

$$P_{k,t}^{def} = 0, \forall k, t \notin \left[T_k^a, T_k^b\right]. \quad (14)$$

D. Chance Constraint for Power Balance

For power balance constraint, most of the existing research focuses on the strict balance of power generation and load demand. However, in an islanded microgrid, the power demand may not always be satisfied due to the uncertain renewable generation, or the strict power balance will result in high cost. To deal with this problem, CCP can be used to allow the solutions to violate the constraint with no more than a small specified probability, i.e., the constraint should be met with a certain confidence level [30]. Consequently, we propose a chance constraint for power supply and demand in this study which can be represented as follows:

$$Pr\left\{ \sum_{i=1}^{N_{dg}} P_{i,t} + \sum_{j=1}^{N_{ess}} \left(P_{j,t}^{dch} - P_{j,t}^{ch} \right) + w_t \geq L_t + \sum_{k=1}^{N_{def}} P_{k,t}^{def} \right\} \geq 1 - \epsilon \quad (15)$$

where w_t is the aggregated random renewable power output and the uncertainty set of its probability distribution is introduced in the next section, ϵ is a predefined small probability index.

Based on the above notations, we can get the objective function of the microgrid energy management, i.e., the total operation cost of the microgrid, which includes the generation cost and emission cost of diesel generators and the ESS degradation cost as shown below:

$$C_{tot} = \sum_{t=1}^{T} \left[\sum_{i=1}^{N_{dg}} \left(CG_{i,t} + CE_{i,t} \right) + \sum_{j}^{N_{ess}} CS_{j,t} \right]. \quad (16)$$

Thus, the complete chance constrained microgrid energy management problem is formulated as follows:

$$\min_{\mathbf{x}} \{(16) : (1) - (3), (7) - (10), (12) - (15)\}$$

where $\mathbf{x}$ denotes the set of decision variables.

The objective of the proposed chance constraint in the system model is to maximize the system reliability, i.e., the load demand should be satisfied with a high probability. Therefore, the dumping load can be added to absorb the excess power supply in practical system operation to keep the power balance [31]. In addition, the proposed energy management model is a single-stage one as the unit commitment (UC) problem is not considered here. We assume that the UC decision has been done. The integration of UC problem which elicits a two-stage energy management model is left for future research. In practice, the optimal solution can provide the decision maker a preliminary and robust dispatch plan when the uncertainties are unknown and the proposed microgrid model may be applied in some remote islands where real-time dispatch is not so convenient.

III. Solution Methodology

The microgrid energy management problem formulated above is difficult to solve directly due to the chance constraint and the uncertain renewable generation. To handle this problem, in this section, we first introduce a novel ambiguity set to describe the uncertain probability distribution of renewable power output. Then, based on this ambiguity set, we apply the DRO method to process the chance constraint and the problem is reformulated as a tractable SOCP problem.

A. An Ambiguity Set for Wind Power Output

In the power balance chance constraint, the renewable generation w_t is a random variable. In this work, without loss of generality, we consider wind power as the renewable generation. To describe the uncertainty of wind power output, different methods have been studied in the literature, e.g., the polyhedral and ellipsoid uncertain set in robust optimization method [3], [32], a particular probability distribution in stochastic optimization [7]. Unlike the robust optimization and stochastic optimization method, the DRO method handles the uncertain wind power with an ambiguity set. The ambiguity set is described as the family of all distributions that have the same moment information such as mean, variance and covariance and structural properties [33]. Different kinds of ambiguity sets have been researched on DRO in the literature including Markov ambiguity set [34], Chebyshev ambiguity set [35] and so on.

Among various ambiguity sets, the ambiguity sets with known moment information are widely studied on uncertainty quantification [35], [36] and they have been adopted to handle the random wind power [17]. The typical moment ambiguity set with known mean and variance can be expressed as follows:

$$\mathcal{P}_t^1 = \left\{ \mathbb{P}_t \in \mathcal{P}_t^0(W_t) \left| \begin{array}{c} \mathbb{P}\{w_t \in W_t\} = 1 \\ \mathbb{E}_{\mathbb{P}_t}\{w_t\} = \mu_t \\ \mathbb{E}_{\mathbb{P}_t}\left\{(w_t - \mu_t)^2\right\} = \sigma_t^2 \end{array} \right. \right\} \quad (17)$$

where μ_t and σ_t^2 represent the mean and variance which can be obtained from historical data. $\mathcal{P}_t^0(W_t)$ denotes the family of all the probability distributions on the support of W_t and $\mathbb{P}_t$ is the probability distribution of w_t.

Although the mean and variance of wind power can be estimated from abundant historical data, their actual values are hard to know in reality and the estimation may not be accurate. In other words, it is difficult to determine the exact moment values. To tackle the uncertain moment information, the ambiguity set with bounded moment such as ellipsoid and conic bound [19] is studied in uncertainty description. Inspired by the polyhedral uncertain set in robust optimization, we design a box-type ambiguity set to capture the uncertain moment information (mean and variance) in this work [37]. In such a set as shown below, the moments are assumed to lie in a box region specified by upper and lower bounds [38]:

$$\mathcal{P}_t^2 = \left\{ \mathbb{P}_t \in \mathcal{P}_t^0(W_t) \left| \begin{array}{c} \mathbb{P}_t\{w_t \in W_t\} = 1, \mathbb{E}_{\mathbb{P}_t}\{w_t\} = \mu_t \\ \mathbb{E}_{\mathbb{P}_t}\left\{(w_t - \mu_t)^2\right\} = \sigma_t^2 \\ \underline{\mu_t} \le \mu_t \le \overline{\mu_t}, \underline{\sigma_t^2} \le \sigma_t^2 \le \overline{\sigma_t^2} \end{array} \right. \right\} \quad (18)$$

where the first and second row in (18) have the same definition with those in (17), and the third row is used to describe the estimated intervals of unknown mean and variance. Note that μ_t and σ_t^2 in (18) are just mathematical symbols and they are unknown compared with those in (17).

Based on this ambiguity set, we can obtain the distributionally robust (DR) variant of chance constraint (15):

$$\mathbb{P}_t \left\{ \sum_{i=1}^{N_{dg}} P_i^t + \sum_{j=1}^{N_{ess}} \left(P_{j,t}^{dch} - P_{j,t}^{ch} \right) + w_t \ge L_t + \sum_{k=1}^{N_{def}} P_{k,t}^{def} \right\} \ge 1 - \epsilon, \forall \mathbb{P}_t \in \mathcal{P}_t^2. \quad (19)$$

Then we have the following DR chance constrained (CC) problem:

$$\min_{\mathbf{x}} \{(16) : (1) - (3), (7) - (10), (12) - (14), (19)\}.$$

B. Problem Reformulation

Despite the fact that chance constrained problems with moment ambiguity sets have been investigated by a few works, they mainly concentrate on theoretical derivation and the study on DR-CC problem is still limited. To the best of our knowledge, this is the first application of this method to islanded microgrid energy management with uncertain wind power output. In this section, we handle the chance constraint with a conservative approximation, i.e., we first derive the sufficient condition of the constraint, which is, then, analyzed and processed based on our ambiguity set.

To solve the DR-CC problem introduced above, we first cope with the DR chance constraint with worst-case Conditional Value-at-Risk (CVaR) approximation and then it can be transformed into a tractable SOCP constraint. For convenience, we consider the DR chance constraint (19) in a general form

$$\mathbb{P}_t\left\{h^0(x_t) + h(x_t)w_t \le 0\right\} \ge 1 - \epsilon, \forall \mathbb{P}_t \in \mathcal{P}_t^2, x_t \in \mathbf{x}. \quad (20)$$

Note that we drop the index t of decision variables x_t and other auxiliary variables in the following for simplicity. It has been demonstrated that [35], [39]:

$$\sup_{\mathbb{P}_t \in \mathcal{P}_t^2} \mathbb{P}_t\text{-CVaR}_\epsilon\left(h^0(x) + h(x)w_t\right) \le 0$$
$$\Rightarrow \inf_{\mathbb{P}_t \in \mathcal{P}_t^2} \mathbb{P}_t\left\{h^0(x) + h(x)w_t \le 0\right\} \ge 1 - \epsilon \quad (21)$$

where the CVaR at level ϵ with respect to probability distribution $\mathbb{P}_t$ is defined as follows [40]:

$$\mathbb{P}_t\text{-CVaR}_\epsilon\left(h^0(x) + h(x)w_t\right) = \inf_{\beta \in \mathbb{R}} \left\{ \beta + \frac{1}{\epsilon} \mathbb{E}_{\mathbb{P}_t}\left[\left(h^0(x) + h(x)w_t - \beta \right)^+ \right] \right\} \quad (22)$$

where $(\theta)^+ = max\{\theta, 0\}$. Therefore, according to (21), we can consider the following conservative approximation which is a sufficient condition to derive (20):

$$\sup_{\mathbb{P}_t \in \mathcal{P}_t^2} \mathbb{P}_t\text{-CVaR}_\epsilon\left(h^0(x) + h(x)w_t\right) \le 0. \quad (23)$$

Then we can investigate the above worst-case CVaR approximation and the problem solution based on this constraint is also feasible for the original problem. First, the left side of constraint (23) can be processed equivalently as follows:

$$\sup_{\mathbb{P}_t \in \mathcal{P}_t^2} \inf_{\beta \in \mathbb{R}} \left\{ \beta + \frac{1}{\epsilon} \mathbb{E}_{\mathbb{P}_t}\left[\left(h^0(x) + h(x)w_t - \beta \right)^+ \right] \right\}$$
$$= \inf_{\beta \in \mathbb{R}} \left\{ \beta + \frac{1}{\epsilon} \sup_{\mathbb{P}_t \in \mathcal{P}_t^2} \mathbb{E}_{\mathbb{P}_t}\left[\left(h^0(x) + h(x)w_t - \beta \right)^+ \right] \right\} \quad (24)$$

where the interchange of sup and inf is based on the stochastic saddle point theorem [41]. By introducing another uncertainty set $\mathcal{Q} = \{(\mu_t, \sigma_t^2) : \underline{\mu_t} \le \mu_t \le \overline{\mu_t}, \underline{\sigma_t^2} \le \sigma_t^2 \le \overline{\sigma_t^2}\}$, we can reformulate the inner maximization problem in (24) as follows:

$$\sup_{\mathcal{Q}} \sup_{\mathbb{P}_t \in \mathcal{P}_t^1} \mathbb{E}_{\mathbb{P}_t}\left[\left(h^0(x) + h(x)w_t - \beta\right)^+\right]. \tag{25}$$

Further, let $r = h(x)w_t$, then the mean and variance of r are $h(x)\mu_t$ and $(h(x))^2\sigma_t^2$, respectively. Thus the inner maximization problem of (25) can be represented equivalently in integral form:

$$\sup_{\xi \in \mathcal{M}} \int_{\mathbb{R}} \left(\left(h^0(x) + h(x)w_t - \beta\right)^+\right)\xi(dr) \tag{26}$$

$$s.t. \int_{\mathbb{R}} \xi(dr) = 1, \int_{\mathbb{R}} r\xi(dr) = h(x)\mu_t \tag{27}$$

$$\int_{\mathbb{R}} r^2\xi(dr) = h^2(x)\sigma_t^2 + (h(x)\mu_t)^2 \tag{28}$$

where $\mathcal{M}$ is the cone of nonnegative Borel measures on $\mathbb{R}$ including the decision variable ξ. Based on duality theory and using change of variables, the problem above is transformed into the following equivalent problem [37] (see Appendix):

$$\inf_{v,\tau,z,s} \ v + s \tag{29a}$$

$$s.t. \ 4zs \ge \tau^2 + (h(x))^2\sigma_t^2 \tag{29b}$$

$$v - h^0(x) + \beta + \tau - h(x)\mu_t - z > 0 \tag{29c}$$

$$z > 0, \ v \ge 0. \tag{29d}$$

Considering the outer uncertainty set $\mathcal{Q}$, the constraint (25) is thus equivalent to

$$\inf_{v,\tau,z,s} \ v + s \tag{30a}$$

$$s.t. \ 4zs \ge \tau^2 + \max_{\sigma_t^2} (h(x))^2\sigma_t^2 \tag{30b}$$

$$v - h^0(x) + \beta + \tau - \max_{\mu_t} h(x)\mu_t - z > 0 \tag{30c}$$

$$z > 0, \ v \ge 0 \tag{30d}$$

$$\underline{\mu_t} \le \mu_t \le \overline{\mu_t}, \underline{\sigma_t^2} \le \sigma_t^2 \le \overline{\sigma_t^2}. \tag{30e}$$

Since $\sigma_t^2 \ge 0$ in (30b), we can easily get that $\max(h(x))^2\sigma_t^2 = (h(x))^2\overline{\sigma_t^2}$. For the maximization problem in (30c), it has the following dual form:

$$\min \ \eta_1\overline{\mu_t} - \eta_2\underline{\mu_t} : \text{s.t. } h(x) = \eta_1 - \eta_2, \ \eta_1, \eta_2 \ge 0. \tag{31}$$

Therefore, combining all the equations above, we can get the equivalent version of constraint (23) as follows:

$$\inf_{\beta,v,\tau,z,s,\eta_1,\eta_2} \ \beta + \frac{1}{\epsilon}(v + s) \le 0 \tag{32a}$$

$$s.t. \ 4zs \ge \tau^2 + (h(x))^2\overline{\sigma_t^2}, \ h(x) = \eta_1 - \eta_2 \tag{32b}$$

$$v - h^0(x) + \beta + \tau - \left(\eta_1\overline{\mu_t} - \eta_2\underline{\mu_t}\right) - z > 0 \tag{32c}$$

$$z > 0, \ v \ge 0, \ \eta_1 \ge 0, \ \eta_2 \ge 0. \tag{32d}$$

Note that the constraint (32b) is a rotated SOCP constraint which can be transformed into a tractable standard SOCP constraint [38]. In summary, the chance constraint (19) in the original energy management model is transformed into the SOCP constraint (32) for which we only need to determine the auxiliary functions $h^0(x)$ and $h(x)$ with respect to decision variables from (19).

Thus, based on the CVaR approximation of the DR chance constraint, the islanded microgrid energy management problem is reformulated as follows:

$$\min_{\mathbf{x},\Theta} \{(16) : (1) - (3), (7) - (10), (12) - (14), (32)\}.$$

where Θ is the set of auxiliary variables including $\beta, v, \tau, z, s, \eta_1$ and η_2 which are introduced in the above reformulation process. Despite the conservatism of CVaR constraint, this approximation is advantageous since the original problem can be reformulated as a tractable SOCP problem. Note that the CVaR approximation in (21) is actually equivalent when the chance constraint function is concave in w_t [35], [37].

C. Including Unimodality Information

DRO method intends to find the optimal solution of the problem considering the worst-case distribution in the ambiguity set. However, the worst-case distribution which usually consists of some discrete points [42] is rarely encountered in practice. Thus, only considering the moment information in the ambiguity set will lead to a very conservative solution. In this regard, the unimodality information or strengthened supports can be investigated to reduce the conservatism [43]. In this work, the inclusion of α-unimodality is further studied [44], i.e., the unimodality information of random variable is assumed to be known except for the moment information in the ambiguity set.

The α-unimodality is defined as follows [45]: for any fixed positive α, a random variable ω is said to have an α-unimodal distribution with mode 0 if $q^\alpha\mathbb{E}[g(q\omega)]$ is nondecreasing in $q > 0$ for every bounded, nonnegative, Borel measurable function g on $\mathbb{R}^n$. Based on the moment and unimodality information, let $\widetilde{\mu}_t = \frac{\alpha+1}{\alpha}\mu_t$, $\widetilde{S}_t = \frac{\alpha+2}{\alpha}\sigma_t^2$, we have the following inequation according to [44]:

$$\sup_{\mathbb{P}_t \in \mathcal{P}_t^1} \mathbb{E}_{\mathbb{P}_t}\left[\left(h^0(x) + h(x)w_t - \beta\right)^+\right] \ge \sup_{\mathbb{P}_t \in \mathcal{P}_t^1(\widetilde{\mu}_t, \widetilde{S}_t)} \mathbb{E}_{\mathbb{P}_t}\left[(L(w_t) - \beta)^+\right] \tag{33}$$

where $\mathcal{P}_t^1(\widetilde{\mu}_t, \widetilde{S}_t)$ is defined similarly as $\mathcal{P}_t^1$ in (17) with the mean $\widetilde{\mu}_t$ and variance $\widetilde{S}_t$, and $L(w_t) = h^0(x) + (\frac{\alpha}{\alpha+1})h(x)w_t$. Combining (33) with (24) and (25), we can derive the following constraint from (23):

$$\inf_{\beta \in \mathbb{R}} \left\{\beta + \frac{1}{\epsilon}\sup_{\mathcal{Q}} \sup_{\mathbb{P}_t \in \mathcal{P}_t^1(\widetilde{\mu}_t, \widetilde{S}_t)} \mathbb{E}_{\mathbb{P}_t}\left[(L(w_t) - \beta)^+\right]\right\} \le 0. \tag{34}$$

With the same reformulation method introduced in III.B, the equivalent version of (34) can be attained similarly as (32), given by:

$$\inf_{\beta,v,\tau,z,s,\eta_1,\eta_2} \ \beta + \frac{1}{\epsilon}(v + s) \le 0 \tag{35a}$$

TABLE I
PARAMETERS OF CONVENTIONAL GENERATORS

Unit	P_i^{min} (kW)	P_i^{max} (kW)	R_i^{up}/R_i^{down} (kW)	a_i (\$/kWh^2)	b_i (\$/kWh)	c_i (\$/h)	$d_i(\times 10^{-4})$ (kg/kWh^2)	$e_i(\times 10^{-4})$ (kg/kWh)	$f_i(\times 10^{-4})$ (kg/h)
G1	10	150	30	0.02	0.05	0.1	6.49	-5.55	4.09
G2	8	135	25	0.1	0.04	0.14	5.64	-6.05	2.54
G3	15	280	40	0.01	0.02	0.02	3.38	-3.55	5.33

$$s.t. \quad 4zs \geq \tau^2 + \frac{\alpha+2}{\alpha}(h(x))^2\overline{\sigma_t^2}, \;\; h(x) = \eta_1 - \eta_2 \tag{35b}$$

$$v - h^0(x) + \beta + \frac{\alpha}{\alpha+1}\tau - \left(\eta_1\overline{\mu_t} - \eta_2\underline{\mu_t}\right) - \left(\frac{\alpha}{\alpha+1}\right)^2 z > 0 \tag{35c}$$

$$z > 0, \;\; v \geq 0, \;\; \eta_1 \geq 0, \;\; \eta_2 \geq 0. \tag{35d}$$

Note that α is set to 1 in this study considering the wind power characteristic and the nesting property of α-unimodality [46]. Thus, the DR-CC microgrid energy management problem based on the moment and unimodality assumption is reformulated as follows:

$$\min_{\mathbf{x},\Theta} \{(16) : (1) - (3), (7) - (10), (12) - (14), (35)\}.$$

The inclusion of unimodality information is expected to induce a less conservative solution theoretically.

IV. CASE STUDY

In this section, a case study is conducted to evaluate the proposed DRO method in solving the CC islanded microgrid energy management. We first describe the studied microgrid configuration and relevant datasets. Then the simulation results and discussion are presented. The microgrid energy management is implemented over a finite time horizon (e.g., T = 24 hours) in this study and the time step is set to be 1 hour. All the experiments are performed in MATLAB with the modeling tool YALMIP [47] and CPLEX 12.71 solver on a desktop with an Intel Core i7-6700 CPU 3.40 GHz and 8 GB of RAM.

A. Description of Microgrid

In this work, we consider a microgrid composed of three conventional generators, an ESS, a wind turbine, a critical load and a deferrable load [5]. For the three generators, their parameters are given in Table I which are collected and modified from [3] and [26]. The same emission coefficients are used here and the emission cost coefficient is 1 \$/kg. In addition, an ESS with storage capacity of 200 kWh is deployed in the microgrid whose parameters are summarized in Table II. The initial and final energy levels of ESS are set to be half of its capacity and the degradation cost coefficient is 0.0035 \$/kWh [5].

As for wind power, we only need the upper and lower bounds of the mean and variance. Based on the estimated mean and variance taken from [36], the upper and lower bounds can be obtained by deviating 10% from the rated values. The mean values of wind power are illustrated in Fig. 1 as an example.

TABLE II
PARAMETERS OF ESS

E^{min} (kWh)	E^{max} (kWh)	$P^{ch,max}$ (kW)	$P^{dch,max}$ (kW)	η^{ch}	η^{dch}
40	180	100	100	0.95	0.95

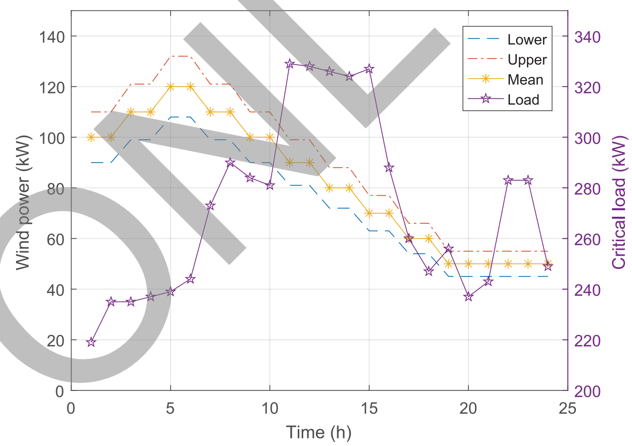

Fig. 1. Wind power and critical load.

Moreover, the critical load demand from [48] is approximately scaled and used as shown in Fig. 1. Note that only the real power is considered here. The deferrable load is assumed to have a total demand of 100 kWh which needs to be satisfied between 13th and 18th time slot. The minimum and maximum serving rates for deferrable load are set to be 10 kW and 50 kW, respectively. The chance constraint confidence level is set as $1 - \epsilon = 95\%$.

B. Simulation Results

With the parameters and datasets introduced above, we solve the DR-CC microgrid energy management problem in this section. We first analyze the output power of conventional generators. The optimal energy schedule of the three generators is shown in Fig. 2. As can be seen from this figure, the output of three generators has great difference: the unit G3 has the largest output, followed by the unit G1, while the unit G2 produces the least output power. This phenomenon is consistent with the generation cost as we can find that unit G3 has the smallest generation cost coefficients. However, the generation cost coefficients of unit G2 are quite large. Additionally, the emission cost only takes a small proportion compared with the generation cost which will be introduced later.

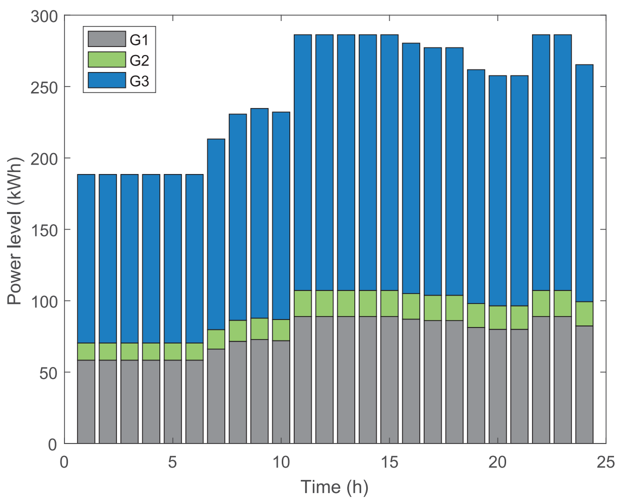

Fig. 2. Optimal power schedule of conventional generators.

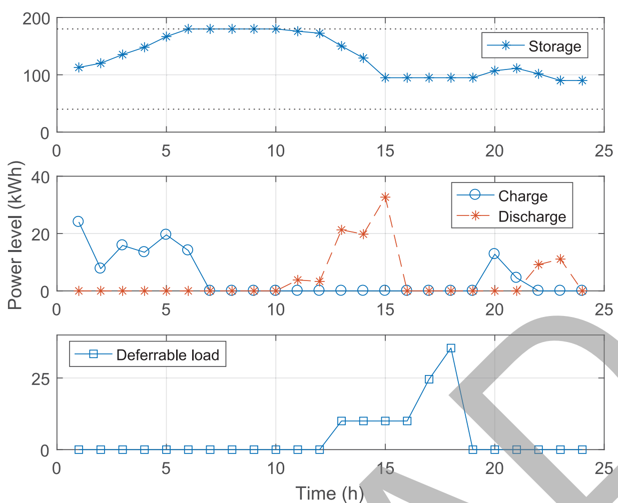

Fig. 3. Optimal energy schedule of ESS and deferrable load.

ESS plays a significant role in microgrid energy management which helps achieve peak load shifting. The energy schedule of ESS including the stored energy level, the charging and discharging power is shown in Fig. 3. As we can see from this figure, the ESS charges in the first few hours when the load demand is low, it stays unchanged for several hours when the maximum capacity is reached. As the load increases later, the generators produce more power and the ESS starts to discharge. It can also be found that the change of ESS state is influenced by the load change most of the time. In addition, the energy schedule of deferrable load (e.g., EV) is also presented in Fig. 3. The deferrable load is served almost at a uniform rate over the specified time interval except the sudden increase in the 17th and 18th period. The total microgrid energy management cost in this case is \$9847.5.

For the method considering the unimodality information, the energy management has a similar dispatch solution which has been omitted here. However, the total system cost with this method is \$8381.5 which substantiates the improved conservatism by incorporating the unimodality information.

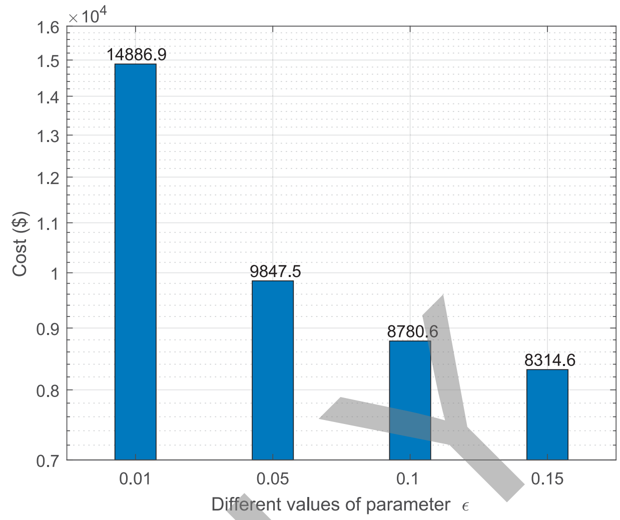

Fig. 4. System cost with different ϵ.

C. Discussion

1) Reliability and Robustness of Our Method: The DRO method solves the CC islanded microgrid energy management with a worst-case CVaR approximation. To validate the effectiveness of our solutions, we use Monte Carlo simulation method to test the reliability and robustness of our approach. Based on the estimated mean and variance values of wind power, we randomly generate a million scenarios by assuming a normal distribution. With these wind power samples, our energy management solutions are checked. The percentage of the scenarios that satisfy the power balance constraint is up to 99.99934%, which is higher than the setting confidence level 95%. In addition, this probability value ensures that the chance constraints violation probability in the total scheduling horizon is very low. Similarly, the method with unimodality information can also be checked and the percentage is about 99.9919%. Therefore, this result further verifies that our method is reliable and robust against the unknown probability distributions sharing the same moment information.

2) Influence of Different Parameter ϵ and Interval Size: Various confidence levels will result in various solutions in microgrid energy management. Moreover, the uncertainty of microgrid system can be controlled by adjusting the confidence level or the parameter ϵ. Although the confidence level $1 - \epsilon = 95\%$ is mostly studied, we investigate the influence of some other parameter settings in this study. The total system cost with different parameter ϵ, including 0.01, 0.05, 0.1 and 0.15, are summarized in Fig. 4 and the corresponding energy management solutions are omitted for simplicity.

As can be seen from this figure, the smaller the ϵ value, the higher the total system cost. A small ϵ value represents a high confidence level. In other words, we have to pay more to achieve a more reliable system. In addition, comparing the difference between the neighbouring system cost, we can find that a dramatic decrease occurs when the parameter ϵ increases from 0.01 to 0.05 which means that the marginal cost becomes larger as the confidence level increases. Thus, we have to select

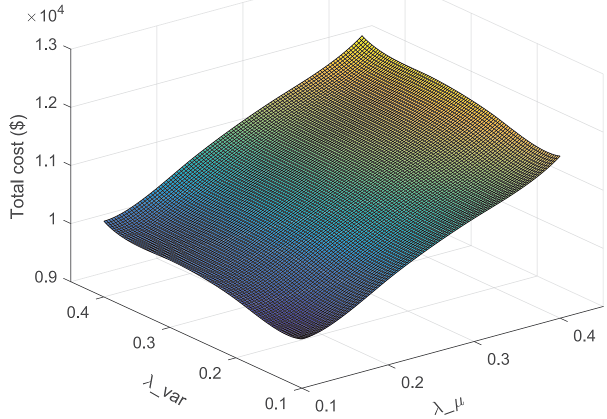

Fig. 5. System cost with different interval size.

a proper confidence level in practice to avoid the extremely high cost caused by a strict reliability constraint.

The critical feature of the proposed ambiguity set is the inclusion of the interval for uncertain mean and variance. To investigate the impacts of the interval on the simulation results, the interval size is enlarged gradually. In this case study, the intervals for mean and variance are attained as follows: $(1 \pm \lambda\_\mu)\mu_t$ and $(1 \pm \lambda\_var)\sigma_t^2$, where the deviation coefficients λ_μ and λ_var are increased from 0.1 to 0.4 with a step size of 0.1. The corresponding results of total cost are plotted in Fig. 5 with interpolation, from which we can see that the system total cost increases as the intervals expand. Also, the intervals for mean values have a larger impact on the cost in this case caused by the larger nominal values. Therefore, to reduce the total cost, the interval in the ambiguity set should be shortened in practice. In this regard, different data-driven techniques can be used to estimate the interval based on historical dataset, such as the popular direct interval forecast methods [49].

3) Comparison With Other Methods: In this part, we compare our method denoted as M_1 with the DRO method based on known moment information (e.g., mean and variance), sample average approximation (SAA) method and stochastic optimization with normal distribution (SND) in solving the CC microgrid energy management problem. The method with known moment information is denoted as M_2, where the ambiguity set $\mathcal{P}_t^1$ is used instead of $\mathcal{P}_t^2$ and the other constraints remain unchanged. In addition, the inclusion of unimodality information in M_1 and M_2 is also studied, denoted as M_1^{uni} and M_2^{uni}, respectively. In these methods, we set the confidence level as 95%. Note that the CC problem is usually transformed into a semidefinite problem (SDP) with the method M_2 [35]. The SDP problem can be solved directly by off-the-shelf solvers or we can use the same method in Section III to further reformulate it as a tractable SOCP problem.

SAA is an effective method to cope with CC problems and a number of theoretical research can be found in previous literature [50], [51]. The basic idea is to approximate the true distribution in chance constraint with an empirical distribution obtained from Monte Carlo sampling technique. For the chance constraint $Pr\{h^0(x) + h(x)w_t > 0\} \leq \epsilon$ in this work, it can be approximated with SAA method as follows:

$$N_s^{-1} \sum_{l=1}^{N_s} \mathbb{I}_{(0,\infty)}\left(h^0(x) + h(x)w_t^l\right) \leq \gamma \tag{36}$$

where $\mathbb{I}_{(0,\infty)}$ is the indicator function of $(0, \infty)$, i.e., $\mathbb{I}(y) = 1$ if $y > 0$, otherwise, $\mathbb{I}(y) = 0$; N_s is the number of samples and γ is the risk level of the SAA chance constraint; w_t^l represents the sample of the random variable. According to [50], the SAA chance constraint can be replaced with mixed-integer constraints, thus the SAA problem is a mixed-integer quadratic programming (MIQP) problem. In this case study, N_s and γ are set to 500 and 0.05, respectively, and the samples are generated from a normal distribution with nominal mean and variance. More details about the SAA method can be found in relevant references.

For the SND method, we assume that the uncertain wind power follows a normal distribution with deterministic mean and variance. With this assumption, the original chance constraint can be transformed into a deterministic constraint as follows:

$$\begin{aligned} &\left\{Pr\left\{h^0(x) + h(x)w_t \leq 0\right\} \geq 1 - \epsilon\right\} \\ &\quad = \left\{h(x)\mu_t + h^0(x) + |h(x)|\sigma_t z_\epsilon \leq 0\right\} \end{aligned} \tag{37}$$

where $z_\epsilon = \Phi^{-1}(1 - \epsilon)$ is the $(1 - \epsilon)$ quantile of standard normal distribution. In this case, the mean and variance are also set to the nominal values introduced before.

The system cost results of these different methods are summarized in Table III. It can be seen that the cost of our method (M_1) is higher than that of M_2 which implies that our method with the ambiguity set $\mathcal{P}_t^2$ is more conservative. However, our method M_1 is more reliable and robust than M_2 whose CC satisfaction percentage is 99.909% calculated from the method discussed above. This comparison gives us an intuitive effect of the method M_1 and the result is rational since M_1 assumes that less information is known about the uncertain wind power compared with M_2. Also, the inclusion of unimodality information produces a less conservative solution for both methods as expected. Additionally, we can also find that the generation cost accounts for a considerable proportion of the total system cost compared with the emission cost and ESS degradation cost. This large difference is mainly caused by the setting of different cost coefficients. This verifies the analysis of different output power of conventional generators in the simulation results.

As the SAA method is based on Monte Carlo sampling, the system cost may vary for each run. Hence, we repeat the simulation for ten times and calculate the average cost as given in Table III. As can be seen, the result of SAA is more conservative and its performance is not as good as M_1 despite the fact that the samples are generated from a normal distribution. For SND method, we can see that the system cost is lower due to the more certain information about the uncertain wind power in the assumption.

TABLE III
COMPARISON OF SYSTEM COST WITH DIFFERENT METHODS

Method	Generation ($)	Emission ($)	ESS ($)	Total ($)
M_1	9553.9	292.9	0.7	9847.5
M_1^{uni}	9101.8	278.9	0.8	9381.5
M_2	8816.8	270.0	0.7	9087.5
M_2^{uni}	8403.0	257.1	0.7	8660.8
SAA	22542.6	697.9	0.8	23241.3
SND	6978.3	212.9	0.8	7192.0

V. CONCLUSION

Microgrid energy management is of great significance in the future smart grid environment. In this study, we design a CC energy management model for an islanded microgrid which consists of conventional generators, ESS, wind turbines and various load demand. In addition to the common generation cost in the objective function, the emission cost and ESS degradation cost are also considered in our model. The uncertainty of wind power is captured by a novel ambiguity set in this work, based on which the individual chance constraint can be tackled with the DRO method and the microgrid energy management problem is reformulated as a tractable SOCP problem. The proposed method has been analyzed through a case study and the simulation results show its effectiveness and reliability. Moreover, the comparison with the approach with known moment information validates the robustness of the proposed method, which is more applicable in practice. The comparison with SAA and stochastic optimization method also reveals the advantage of the proposed method.

APPENDIX

From (26)-(28), we can get the dual problem by introducing corresponding dual variables θ_0, θ_1 and θ_2 as follows:

$$\inf_{\theta_0,\theta_1,\theta_2} \quad \theta_0 + \theta_1 h(x)\mu_t + \theta_2\Big[h^2(x)\sigma_t^2 + (h(x)\mu_t)^2\Big] \tag{38a}$$

$$s.t. \quad \theta_0 + \theta_1 r + \theta_2 r^2 \geq 0 \tag{38b}$$

$$\theta_0 + \beta - h^0(x) + (\theta_1 - 1)r + \theta_2 r^2 \geq 0. \tag{38c}$$

It can be verified that strong duality holds as $h^2(x)\sigma_t^2$ is positive [35] and the dual problem has feasible solutions when $\theta_2 > 0$. The constraints (38b) and (38c) can be transformed into the following equivalent constraints by considering the minimum value of the left hand side:

$$\inf_{\theta_0,\theta_1,\theta_2} \quad \theta_0 + \theta_1 h(x)\mu_t + \theta_2\Big[h^2(x)\sigma_t^2 + (h(x)\mu_t)^2\Big] \tag{39a}$$

$$s.t. \quad \theta_0 - \frac{\theta_1^2}{4\theta_2} \geq 0, \;\; \theta_0 + \beta - h^0(x) - \frac{(\theta_1 - 1)^2}{4\theta_2} \geq 0. \tag{39b}$$

Then using the following variables change with auxiliary variables v, τ and $z > 0$, we can obtain the problem (29).

$$\theta_0 = v + \frac{(\tau - h(x)\mu_t)^2}{4z}, \;\; \theta_1 = \frac{\tau - h(x)\mu_t}{2z}, \;\; \theta_2 = \frac{1}{4z}.$$

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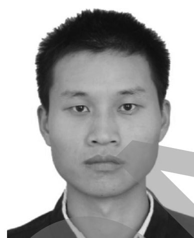

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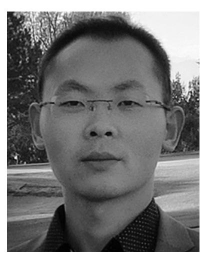

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