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TRANSFER EFFECTS OF MEASURE CONCEPTS ON THE LEARNING OF FRACTIONAL NUMBERS

by

© ABDUSSALAMI A. SAMBO

A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled TRANSFER EFFECTS OF MEASURE CONCEPTS ON THE LEARNING OF FRACTIONAL NUMBERS submitted by ABDUSSALAMI ABUBAKAR SAMBO in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Secondary Education.

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Date 12th Sept. 1980

DEDICATION

To my wife Dorcas
whose moral support was
very crucial to the completion
of this project

ABSTRACT

The purpose of this study was to investigate transfer effects of measure concepts on the learning of fractional numbers. Three instructional units were designed to investigate this effect. Instructional unit one called the transfer unit consisted of measure and fractional number concepts. It was designed according to Osborne's (1976) theory of transfer. The second unit, called the review units, consisted of a review of the topics in unit one as they were treated in a standard primary school test. The third unit called the measure unit, was identical to the measure section of instructional unit one. The measure ideas used in all units were those of linear and area measurement. Two parallel tests of achievement in fractional numbers were also constructed.

The study employed eleven grade seven classes and twelve teachers in three different secondary schools in Kaduna State of Nigeria. The teachers were instructed concerning the strategy to be employed in each unit. Before instruction started, the students were pre-tested. After instruction which lasted a minimum of four weeks, the students were tested with the second test. The first test (pre-test) was used as a retention test six weeks after instruction. The three sets of test scores were analysed in a three-way analysis of variance using the testing occasion as a repeated measure.

The result of data analysis indicated that school main effect was not significant. The treatment effect and test occasion effects were both significant ($p \leq .001$). The school-treatment interaction effect was not significant. But the school-testing occasion and treatment-testing interaction were significant ($p \leq .001$). The transfer group performed significantly better ($p \leq .001$) than the review group, the measure group and the control group on both the post and retention tests of fractional number achievement. This result was duplicated in all three schools. The measure group did not perform significantly better than the review group on the post test. But the measure group's performance on the retention test was significantly better than that of the review group ($p \leq .002$).

It was concluded that instructional designs for transfer should make conscious effort to emphasize similarities and note difference between two sets of concepts before transfer could take place. A simple review of related material was not sufficient for transfer to take place.

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Principal, Government Secondary School, Giwa

Principal, Government Girls Secondary School, Soba;

Finally, to the teachers who worked so hard in teaching the units of this experiment. Their names are listed in the appendix against the units they taught.

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THE PROBLEM

Introduction

The rational number system is of great importance to the mathematical education of every student. The system provides mathematical models for a large number of the practical problems that students are likely to encounter. But research studies since the beginning of this century have shown that children have great difficulties in learning the concept of rational numbers. A simple survey of textbooks used before 1960 will show that the problem is not because of lack of emphasis on instruction in rational numbers (or fractions).

Early research into the problem of rational number learning centred on manipulation and operations with fraction symbols. Suydam (1968) found that the questions researchers asked were:

1. How can operations with fractions be taught effectively?

2. What is the best method for finding the common denominator for addition with fractions?

3. What is the best method for teaching division with fractions?

4. What is the best sequence for teaching division ideas?

5. Should addition or multiplication be taught first?

Suydam (1968) further reported that Bruechner (1928), Shane (1938) and Scott (1962) found that errors with fractions could be attributed to:

(1) Lack of comprehension of the process involved.

(2) Computation

(3) Inability to reduce fractions to the lowest terms.

(4) Difficulty in changing improper fractions to whole or mixed numbers.

(5) Difficulty in "reduction" in addition with fractions

(6) Difficulty in "borrowing" in subtraction with fractions.

(7) Faulty computation in multiplication with fractions, and

(8) The use of wrong process in division with fractions.

The results were not surprising considering the types of questions asked.

With the advent of modern mathematics, curriculum designers looked for more "meaningful" approaches to instruction in rational numbers. Several questions were asked. What is the best physical world representation of fractions? How should the various meanings that can be associated with fraction be taught? As a result of these questions many curriculum developers used one or more of these models for fractions; linear measurement and sets. Even with these efforts the problem of fractional number learning persisted.

Romberg (1968) reported that pupils using modern textbooks failed to cancel when multiplying with fractions more often than those using conventional texts. In summarising the result of research findings on fractions, Anderson (1969) found that there were three principal causes of unsatisfactory work in fractions:

(a) Inadequate conception of the principles involved.

(b) Confusion of operations.

(c) Lack of adequate degree of skills in the fundamental operations with integers.

The results of the National Assessment of Education Progress (Carpenter et al. 1975) supports some of the above findings. Commenting on the results of the survey, Carpenter et al (1975) suggested that instead of increasing the amount of time spent on fractions, teachers and curriculum developers should examine first what aspects of fractions are being emphasized in the curriculum.

The difficulty in learning rational number concepts may have persisted because of two reasons. Kieren (1975) has postulated that the concept of rational numbers has four subconstructs. Only one of these is used by most textbook writers as a model for teaching rational number concept; namely the measure subconstruct. Secondly, the use of the measurement model is based on some tacit assumptions: that

(a) Measure systems possess properties or structures which are similar to the properties or structures of rational numbers.

(b) Learners possess the knowledge of these properties and structures by the time they begin learning about rational numbers, and

(c) That learners can transfer this knowledge to the new learning situation.

Even for this model, Osborne (1975) stated that

> ".... if the rationale is based on measure concepts and understanding, insufficient attention has been given to the learning context for measure concepts and understanding and to the common attributes of the two learning sets in terms of how transfer to numbers occurs...."

More recently researchers are turning their attentions to these basic questions.

Statement of the Problem

This study was planned to investigate the transfer effects of measurement learning on the learning of fractional numbers. In attempting to measure this effect the following research questions will be answered:

1. What is the effect of stressing the similarities between linear and area measure concepts and those of fractional number on the learning of fractional number concept?

2. What is the effect of learning measure concepts on the learning of fractional numbers.

Definitions

These terms used in this report will be defined as follows:

Linear measure - The assignment of number to lengths of physical objects.

Area measure - The assignment of number to the surfaces of physical objects.

Fraction - The symbol $\frac{a}{b}$ where a and b are natural numbers and $b \neq 0$.

Fractional number - The equivalence class of fractions.

Rational number - The equivalence class of fractions $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

Basic Assumptions

This study is based on the following assumptions:

1. That the most commonly used models for teaching the concept of fractional numbers are linear and measurement models.

2. That differences in mean score on an achievement test between treatment groups will be a reflection of a measure of transfer effects of measure concepts on fractional number learning.

3. That because the way students were assigned to treatment groups was not in any particular order is assumed to be equivalent to random assignment.

4. That the students used in the study possess all the psychological pre-requisites for measurement learning.

Delimitations

1. There are many interpretations of rational numbers. Each of these interpretations could be used as a model for developing a

subconstruct of rationals. This study used only the linear and area measure models to develop the concept of fractional numbers.

2. The models are used to develop the following ideas only:

   (a) meaning of fraction

   (b) equivalence of fraction

   (c) operations of addition and subtraction.

3. The study is delimited to six weeks of instruction in grade seven, or first year of secondary school.

Limitations

The following are some major limitations in this study:

1. The amount of time spent on instruction in each school varied from four to six periods per week. As a result, different schools completed the units at different times.

2. There was a variation from school to school in the use of teaching aids due to the variation of facilities available to the schools.

Importance of the Study

In an effort to make mathematical instruction and learning more meaningful, mathematics educators are always on the look out for best ways of presenting mathematical notions to learners. One of these ways is the use of models; the more physical and less abstract, the better.

Theoretically, one could see that the linear and area measure systems have similar structure to that of fractional number systems. And because measurement activity is amendable to physical and pictorial demonstration, it seems ideal for use as a model for teaching the more abstract and complex fractional number concept. Therefore many curriculum designers and textbook writers have used these ideas in teaching fractions. In spite of these efforts the learning of fractional numbers remains problematic.

One is therefore led to believe that similarities of structure between a model and the process that it simulates may not be enough to cause subsequent learning. It may be that deliberate effort must be made to point out and emphasize these similarities in order that transfer of learning from one learning situation to another can be affected. The purpose of this study is to see whether such an emphasis may not cause the desired transfer of learning. If such a result is obtained, then it will have an effect on the design of curriculum in the use of models in instruction in mathematics.

Outline of the Report

In this chapter, the research problem and questions are stated. Chapter II deals with review of related literature establishing the theoretical framework for the study. In chapter III, the research questions are translated into hypotheses. The accompanying statistical design was discussed. The results of data analysis are reported in Chapter IV. Chapter V summarises the research findings and discusses their implications.

Chapter II

REVIEW OF LITERATURE

Introduction

Many curriculum and instructional materials have used measurement models, especially linear and area models in developing the concept of fractional numbers. The practice has been occasioned by the similarities of structure between measurement and fractional number concepts. The hope is entertained that learning of measurement concept will transfer or aid the learning fractional number concepts. The purpose of this study is to show that despite the similarities between the two learning situations, transfer of learning will take place best if the instructional sequence continuously emphasize the similarities. The purpose of this literature review is to establish the following:

(a) The theoretical bases for the use of measurement concepts in teaching fractional numbers.

(b) Research in measurement that deals with transfer to fractional number concepts; and

(c) Research in fractional number concept based on measure concepts.

Theoretical Framework

This study is based on certain principles of transfer postulated by Osborne (1975). These principles are based on the assumptions that:

1. Two sets (P and S) of concepts, principles, or generalizations are to be learned. P is the initial or prior learning set and S is the subsequent learning set.

2. That the set of common attributes for the prior learning, P, and the subsequent learning, S, have been identified. Then

Principles

1. Instructional materials and design for P must be explicit in terms of the attributes of P that are common to P and S.

2. The more complete and thorough the learning of P, the greater the probability of transfer.

3. The design and instruction for S must be in terms of the attributes and conditions of learning that characterise P.

4. More powerful and inclusive concepts, principles, and generalizations have greater potential for facilitating transfer than the less powerful or inclusive.

5. Instruction for S must focus on the differences as well as the commonalities of the attributes of P and S in order to protect the learner from over generalization and misapplication of the concepts, principles and generalization of P to S.

In this study the set P is the set of measure concepts, linear and area. The set S is the set of fractional number concepts. For the principles of transfer to apply to this study, it is shown that linear and area measure have the same or similar structure as the fractional numbers. This is done in the next two subsections.

Linear and Area Measures

A measure system consists of a measure space, a measure function and a numerical range space. The description of linear and area measure here will not be as detailed mathematically as the treatment of the topics in Blakers (1969). Linear measure is described in some detail. The similarities and differences between linear and area measure are then described.

Let L be a set of objects that possess a property called length. Then by comparing elements of L it can be determined which of the following relations exists between them:

If l_1 and $l_2 \in$ L then

1. $l_1 = l_2$ that is l_1 has the same length as l_2
2. $l_1 < l_2$ that is l_1 is shorter than l_2
3. $l_1 > l_2$ that is l_1 is longer than l_2

The relation 'has the same length as' has the following properties:

If l_1, l_2 and $l_3 \in$ L then

1. $l_1 = l_1$
2. If $l_1 = l_2$ then $l_2 = l_1$
3. If $l_1 = l_2$ and $l_2 = l_3$ then $l_1 = l_3$

These properties make the relation "has the same length as" an equivalence relation which partitions the set L into equivalence classes. Let M be the set of all the equivalence classes m of L. Then either the relation "shorter" or "longer" can be used to order the elements of M as follows:

If m_1, $m_2 \in M$ then $m_1 < m_2$ iff $l_1 \in m_1$, $l_2 \in m_2$ and $l_1 < l_2$. This provides an order relation in the set M with the following properties:

1. If $m_1 < m_2$ and $m_2 < m_3$ then $m_1 < m_3$
2. Given any two m_1 and m_2 there exists at least one m, and so infinitely many, such that $m_1 < m < m_2$

These properties say that M is a dense ordered set.

An operation called joining or combining lengths can be introduced into the set M. This is accomplished as follows:

Let m_1 and $m_2 \in M$, then $m_1 * m_2 \in M$ iff there exist an $m \in M$ such that $l_1 \in m_1$, $l_2 \in m_2$ and $l_1 * l_2 = l \in m$.

The operation has these properties:

1. $m_1 * m_2 = m_2 * m_1$
2. $(m_1 * m_2) * m_3 = m_1 * (m_2 * m_3)$
3. Given any m_1 and $m_2 \in M$ there exist a natural number n such that $m_1 \leq nm$, where $\leq$ means "is shorter or equal in length to".

In summary, the set L has been shown to possess the following properties:

A. The set L can be partitioned into mutally disjoint subsets by an equivalence relation "has the same length as".

B. The set of all equivalence classes M is dense under an order relation "shorter than" or "longer than"

C. $(M, <)$ is closed under a commutative, associative operation * of "joining length".

D. $(M, <, \star)$ is Archimedian

By the same procedure as above it can be shown that the set of regions A has a similar structure as the set L of rods or sticks. The differences between the two systems of measure arise from two sources:

1. The comparison procedure which establishes the equivalence relation
2. The nature of their measure function

Where it is easy to decide when two rods or sticks have the same length, it is not easy to make such a decision in the case of regions having the same area. To determine equivalence of area both size and shape must be taken into account. In length measure, once a unit is decided upon, it is easy to determine the length of other rods by using the unit iteratively. This is not always the case in area measure. For if the unit has a circular shape it is impossible to use it iteratively to determine the area of other regions by covering.

These problems are resolved in beginning measure learning by considering only the subset of the set of all regions A which is made up of those regions which have either rectangular shape or can be reduced to that shape through a determinate number of cuts and rearrangements. For these regions the following relations impose a similar structure in the set A of regions:

By the relation "has the same area as or $\sim$ " set A has these properties:

1. $\forall a \in A \quad a \sim a$

2. If $a_1 \sim a_2$ then $a_2 \sim a_1$

3. If $a_1 \sim a_2$ and $a_2 \sim a_3$ then $a_1 \sim a_3$

This partitions the set A into equivalence classes. If A*is the set of these equivalence classes, then the relation " has less area than" is an order relation. The set A* is dense because given any a_1* and a_2* there exist at least one a* such that

$a_1* < a* < a_2*$

There is also an operation o between any two contiguous non-overlapping regions producing a third region such that:

1. $a_1* \text{ o } a_2* = a_2* \text{ o } a_1*$
2. $(a_1* \text{ o } a_2*) \text{o} a_3* = a_1* \text{o} (a_3 \text{ o } a_3*)$
3. Given any a_1* and a_2* in A* there exist a natural number n such that $a_1* \leq na$.

In elementary measure the range space normally used is the set of natural numbers. The set N of natural numbers is an ordered commutative semigroup. But unlike the sets M and A*, N is not dense. Hence there are an infinite number of rods and regions which cannot be assigned numerical values in N. The problem is solved for all practical purposes, by the set F of fractional numbers. It will soon be seen that F has the similarity of structure needed to serve as a range space of a homomorphic function from either (M, < *) or (A*, <, 0).

Fractional Number

The set S of all symbols of the type $\frac{a}{b}$ where a and b are natural numbers is called the set of fractions. Two fractions $\frac{a}{b}$ and $\frac{g}{h}$ are equivalent written as

$$\frac{a}{b} \sim \frac{g}{h} \quad \text{iff } ah=bg$$

This relation partitions S into a set of equivalence classes F called the set of fractional numbers. The relation has the following properties:

Let $f, f_1, f_2, f_3 \in F$ then

1. $f \sim f$
2. If $f_2 \sim f_2$ then $f_2 \sim f_1$
3. If $f_1 \sim f_2$ and $f_2 \sim f_3$ then $f_1 \sim f_3$

The order relation in F is defined by:

Let $f_1 = \frac{a_1}{b_1}$, $f_2 = \frac{a_2}{b_2}$ then

$f_1 < f_2$ iff $a_1 b_2 < a_2 b_1$

From which the following properties follow

1 If $f_1 < f_2$ and $f_2 < f_3$ then $f_1 < f_3$

2. Given any f_1 and f_2 in F there exist an f in F such that: $f_1 < f < f_3$

In F there is an operation "+" called addition defined as

$$\frac{a_1}{b_1} + \frac{a_2}{b_2} = \frac{a_1b_2 + a_2b_1}{b_1b_2}$$

with the following properties

1. $f_1 + f_2 = f_2 + f_1$

2. $(f_1 + f_2) + f_3 = f_1 + (f_2 + f_3)$

Therefore F has all the properties and structures of a densely ordered commutative semi-group. It can easily be shown that it is Archimedian where nf means adding f, n times. Table I summarises the structures of linear and area measure and fractional numbers.

Research on Measurement Concept

The last section dealt with the mathematical similarities of linear measure, area measure and fractional numbers. In this section research literature dealing with measurement learning is reviewed. The purpose is to find any literature that dealt with measurement concept in relation to its transfer effects on learning number.

Until recently research in measurement did not consider transfer effects on the basis of the model outlined at the beginning of this chapter. Carpenter (1975) Osborne (1975) and Carpenter and Osborne (1975) stated that most research in measurement dealt with:

(a) Pre-measurement learning in the paradigm of Piaget's theory of cognitive development, and

(b) The concepts of conservation and transitivity.

Researchers wanted to

1. Validate the existence of individual cognitive operations and describe their development,
2. Validate or establish the relationship between different cognitive skills,

TABLE I

SIMILARITIES OF STRUCTURE BETWEEN LINEAR MEASURE AREA MEASURE AND FRACTIONAL NUMBERS

	LINEAR MEASURE	AREA MEASURE	FRACTIONAL NUMBERS
set elements and property	Rods or stick with length	Rectangular or polygonal regions with area	Fraction symbols for number
Equivalence relation and properties	'Has the same length as' reflexive, commutative and transitive	'Has the same area as' reflexive commutative and transitive	$\frac{a}{b} \sim \frac{c}{d}$ iff ad = bd reflexive, commutative and transitive
Order relation	'Is shorter than' transitive and dense	'has less area than', transitive and dense	$\frac{a}{b} < \frac{c}{d}$ iff $ad < bc$ transitive and dense
Operation	joining rods Commutative, associative and archimedian	Joining polygonal regions, commutative, associative and archimedian	addition commutative, associative and archimedian

3. Identify the nature of transitions between stages of development described by Piaget,

4. Or experiment with all three above using information processing techniques to describe and simulate Piagetian structures.

Piaget and his associates (Piaget et. 1960) have postulated four stages of cognitive development of measurement concepts. The last stage is achieved at age 12 and older. At this stage measurement development is complete characterised by the ability to calculate area on the basis of linear dimensions.

Osborne (1975), Steffe and Hirstein (1975) have summarised the findings of research on the psychological pre-requisites of measurement learning. These pre-requisites are:

1. Transitive property. If region A has the same area as region B and region B has the same area as region C, then region A has the same area as region C. (Similarly, for "less area than" and "greater area than").

2. Subsitutive property. If region A has the same area as region B and region B has more (less) area than region C, then region A has more (less) area than region A.

3. Assymetric property. If region A has more (less) area than region B, then region B does not have more (less) area than region A.

4. Symmetric property. If region A has the same area as region B, then region B has the same area as region A.

5. Reciprocal property. If region A has less area than region B, then region B has more area than region A.

In addition to the above, the following concepts are also important:

1. Additivity. The join of two non-overlapping segments (regions) has the same measure as the sum of the measures of the individual segments.

2. Unit. There exists a segment (region) that maps to one in the range space.

3. Congruence. Congruent segment (regions) map to the same real number.

The students that took part in this study were from form one (grade seven). Their mean age was over twelve years. It is one of the assumptions of this study that the children possess all the psychological pre-requisites for measurement learning.

Research on Curriculum and Instruction on Fractional Number

The purpose of this study was to measure the transfer effect of measurement concepts on the learning of fractional numbers. Osborne's (1976) theory suggests that this transfer is possible if similarities exist between the concepts of measure and those of fractional numbers and also if instructional designs in both measure and fractional number concepts emphasize these similarities. It has so far been shown that the similarities between the concepts exist mathematically. In this section research reports which establish this relationship either as psychological constructs or in instructional designs are reviewed.

In introducing the problem of this study, it has been shown that research in fractional numbers has been going on for decades. Early researchers concentrated their attention on how children operate (add, subtract, multiply and divide) with fractional number symbols. The modern mathematics approach to the problem was to find meaningful settings within which to develop the concepts of rational numbers and operations with them. These approaches used different models (ratio, regions, number line, and sets), different modes of presentation (diagrams, pictures, and manipulating materials), and different sequences of presentations in investigating the learning of fractional numbers. That these efforts have not been very successful has been documented e.g. The National Assessment of Educational Progress, (Carpenter et al (1975)).

This lack of success may be due to several factors In analysing the mathematical, cognitive and instructional foundations of rational numbers, Kieren (1976) showed that rational numbers have seven interpretations viz: rationals as fractions, decimals, ordered pairs, measures, quotients, operators, and ratios. He argued that to understand rational numbers is to have adequate experience with their many interpretations. But previous research and instruction on rational number learning has been limited to only a few of these interpretation while others were totally neglected. He advocated a conglomerate view of rational numbers in curriculum design and instruction.

Another factor for the lack of success of previous research may be due to the inadequacy or lack of a sound theoretical formulation of the psychological construct of rational numbers. Kieren (1980) presented a theory of the rational number construct which involves five sub-constructs, namely: part-whole, ratios, quotients measures, and operators. These subconstructs could be considered as different. To acquire the 'megaconcept' of rational numbers is to acquire these five sub-constructs. Kieren (1980, 1980a, 1980b) stated further that knowing rational numbers has three levels involving two kinds of mental mechanisms. The first level of knowing or developing the rational number construct is the development or acquisition of the five subconstructs. The second level of knowing is the perceiving of the similarities between the subconstructs and generalising them. The final level of knowing is to consider the rational numbers as an axiomatic system.

The mental mechanisms necessary for acquisition of these levels are developmental and constructive. The developmental mechanism follows stages of mental development, while the constructive mechanisms involve the abilities to manipulate and use symbols partitioning and equivalence.

Some studies have been conducted within the context of this paradigm of rational number learning, while the results of others provide supportive evidence of some of its formulation. For example, Noelting (1978) obtained results that supports the independence of the ratio and quotient subconstructs of rational numbers. An exploratory study conducted by Kieren and Nelson (1978) found that the operator subconstruct of rational numbers depends on stages of mental development. They hypothesized three stages of this development ranging between the ages of nine and fourteen years. A later study by Kieren and Southwell (1979) supported the three developmental stages but between the ages of eight and fifteen. In another study Ganson and Kieren (1980c) compared the developmental stages of the operator subconstruct with that of the ratio subconstruct of rational numbers. They found evidence to suggest that the level of cognitive thinking necessary for a child to perform general operator tasks is relatively the same as the level needed to perform the multiplicative equivalence comparisons in the ratio tasks.

There are a number of studies which relate the measure subconstruct to rational number achievement. Babcock (1978) studied the nature of the relationship between measurement concepts and rational number concepts at the three grade levels. She correlated scores on a Test of Basal Measurement Concepts with scores on a Fraction Achivement Test.at the three grade levels.

She found that:

> "....a relationship exists between measure and fractional number behaviours as assessed by a paper and pencil test. Specifically initial fraction concepts and the operations of addition and subtraction are related to area measure and linear/area subdivision concepts at all grade levels. Linear measure and number line concepts are strongly related at the grade eight level only."

The Babcock study is related to this study in some essential ways. First in both studies the measure concepts studied are those of linear and area measure. Both studies investigated the effects or relationship of the measure concepts on fractional number learning. A major difference between the two studies is that the Babcock study was correlational while this study was experimental. Also this study was interested in measuring the relationship of the two sets of concepts through Osborne's theory of transfer.

The focus of this study is on one subconstruct of the Kieren model-measure. Babcock's work indicated that without deliberate emphasis, the relationship between the structures of measure and fractional number achievement in chidren was lacking until grade 8. Such a relationship existed at that level. In this study the effect of deliberately teaching fractional numbers as measures will be tested compared with the teaching of measure alone and more traditional fractional number instructional/curriculum pattern.

This study is an instructional extension of

Babcock's as well as a test of part of the Kieren model-that is, that rational numbers as measures provide a useful curriculum approach.

Another study that investigated the rationship between the measure construct and rational number achievement was conducted by Owens (1977). He investigated children's ability to:

(a) Perform area measurement by counting units;

(b) Succeed in inclusion and multiple classification tasks; and

(c) Determine the effect of success on these tasks to learn meaning of a fraction and equivalence concepts.

The study was conducted in grades three and four. The results showed that area was a significant factor in determining fraction test score, but grade level was not significant. He concluded that when area models are used for instruction the area concept is related to fractional number learning.

A number of studies were conducted by Doctoral students at the University of Michigan between 1968 and 1975 on the learning of fractional numbers. The early studies investigated the uses of different approaches and materials in the teaching and learning of operations with fractional numbers. The later studies shifted their emphasis from operations to what happened when learning took place. They all used measurement ideas as the bases of instruction. Green (1970) compared two approaches, Area and finding a part of, and two instructional materials, diagrams and cardboard strips to teach the multiplication of fractional numbers. She found that the area-cardboard strip method was superior to all others in teaching multiplication. But no significant results were obtained on transfer tests. All four groups had difficulty in renaming numbers expressed in mixed form as fractions. This caused difficulty in multiplying fractional numbers expressed in mixed form.

Bohan (1971) compared three learning sequences for equivalent fractions. One sequence was a modern mathematics approach that uses diagrams and multiplying the numerator and denominator of a fraction by the same natural number. Another sequence was the same as the first but including the use of paper folding. The third sequence uses the property of multiplication with 1 to produce other equivalent fractions. He found no significant difference between adjusted means either on immediate post tests or retention tests on addition, subtraction or multiplication of fractional numbers with arithmetic achievement held constant. But he found that the paper folding treatment was better in attainment of objectives related to equivalent fractions and better attitudes toward fractions than were the other two treatments, although this did not extend to the addition of fractional numbers or in renaming fractions in the lowest terms.

Coburn (1974) compared a region approach with a ratio approach in teaching equivalence of fractions. He concluded that although the region model provided for a smoother connection with addition and subtraction of unlike fractions and renaming in higher terms, the level of mastery for the generalization was not sufficient in either treatment to build on for subsequent computational topics with fractional numbers where renaming was required. He recommended the removal of this topic from grade four and giving emphasis to the development of equivalent fractions.

Muangnapoe (1975) investigated the learning of initial concept and oral/written symbols for fractional numbers in grades three and four. He found that though the initial fraction sequence does not make a significant difference in immediate achievement, it was more effective on retention and transfer when taught using small groups. He also found that some major difficulties encountered by pupils were:

(a) Identifying a unit when many units were present.

(b) Realizing the need for equal-size parts of a unit.

(c) Comparing fractions.

(d) Dealing with fractions greater than one.

(e) Applying fractions to the number line.

In summary, Payne (1975) indicated that the major features of the initial fraction sequence of the Michigan Studies were:

1. The measurement idea was the overall theme for fractions.

2. Concrete objects were used for a longer period of time before diagrams were used.

3. Word names were taught before the fraction symbols to help with the "reversal problem".

Novillis (1975) placed the concept of fraction in a hierachy and tested the hierachy in grades four, five and six. She found that:

"1. Associating a fraction with part-whole (the unit was a geometric region) and part-group (the unit was a set) models were pre-requisite to associating a fraction with a point on a number line.

2. Associating a fraction with part-whole model or with part group model was pre-requisite to using a fraction in comparison situation involving the respective models.

3. Associating a fraction with part-whole model or part-group model was pre-requisite to associating a fraction with the model where the number of parts was a multiple of the denominator and the parts were arranged in an array that suggested the denominators.

4. Associating a fraction with part-whole model or with part-group model with congruent parts was pre-requsite to associating a fraction with the respective model with non-congruent parts where, in the case of part-whole models, the parts were equal in area".

Summarising the result of Piagetian research on fractional number concept, Copeland (1974) stated that:

1. There can be no thought of fraction unless there is a divisible whole.

2. A fraction implies a determinate number of parts.

3. The subdivision of the whole is exhaustive; there is no remainder.

4. There is a fixed relationship between the number of parts into which the whole is divided and the number of intersections (cuts). One cut produces two parts, two cuts produce three parts, etc.

5. The concept of an airthmetical fraction implies that all the parts are equal.

6. When the concept of subdivision is operational, children realize that fractions have a dual character. They are part of the original whole (as a nesting series), and they are also wholes in their own right which can be subdivided still further.

7. Since fractions relate to the whole from which they come, the whole remains invariant. Conservation of the whole is an essential condition for operational subdivision.

Summary

The major purpose of this research was to test the effect of measure concept on the learning of rational numbers. This is done on the basis of principles of transfer that put emphasis on the nature of concepts to be learnt and the instructional decisions, The review of literature has shown that there is a similarity of structure between linear measure, area measure and fractional numbers. It has also shown the preconditions for learning measure and rational number concepts. These preconditions are attained, according to Piagetian research, by age twelve and over.

A theoretical and psychological formulation of the concept of rational number was reviewed. Some researches based on this paradigm were also reviewed. The present study is conceived within the measure subconstruct of the above theory. By using children of age twelve and over the study assumed that the preconditions for learning measure and fractional numbers exist in the learners. This study investigated how learning of measure concepts affect the learning of fractional numbers.

Chapter III

DESIGN FOR THE STUDY

Introduction

The purpose of this study was to measure the transfer effect of measurement concepts on the learning of fractional numbers. The review of literature has shown that measure concepts and fractional number concepts have many similarities. According to Osborne (1975) maximal transfer will take place if instructional units in measurement and fractional numbers exphasised the similarities and differences between the two concepts. In this chapter the research procedures employed for the study of the problem are discussed. The design implementation includes a description of the sampling procedure, the construction of the teaching units, the construction of the tests and testing procedure, and the statistical procedures employed in testing research hypothesis.

Research Design

The problem for the study was translated into the following two questions:

1. What is the effect of stressing the similarities between linear and area measure concepts and those of fractional number on the learning of fractional number concept?

2. What is the effect of learning measure concepts on the learning of fractional number concept?

In seeking answers to these questions, three instructional units were designed (Appendix A). Instructional unit I consisted of linear measure concepts, area measure concepts and fractional number concepts. The development of the unit emphasised the similarities of these concepts whenever possible and pointed out their differences. Instructional Unit II consisted of a review of the same concepts in I as these concepts were treated in a primary school textbook series "The Oxford Primary School Mathematics". Instructional Unit III consisted of the linear and area measure sections of Instructional Unit I. These units were taught in experimental groups G_1, G_2 and G_3 respectively. A fourth group G_4 did not receive any instruction. Two tests T_1 and T_2 were constructed. Each group was tested three times: before instruction started with T_1; immediately after instruction with T_2; and six weeks after instruction with T_1. The design was as follows:

G_1	T_1	Unit I	T_2	6 weeks T_1
G_2	T_1	Unit II	T_2	6 weeks T_1
G_3	T_1	Unit III	T_2	6 weeks T_1
G_4	T_1	No Instruction	T_2	6 weeks T_1

To answer the research questions the following hypotheses were tested:

Hypothesis 1 Group G_1 will perform significantly better than G_2, G_3 and G_4 on an achievement test on fractional numbers;

Hypothesis 2. Group G_2 will perform significantly better than G_3 and G_4 on an achievement test on fractional numbers;

Hypothesis 3. Group G_3 will perform significantly better than G_4 on an achievement test on fractional numbers;

Hypothesis 4. Group G_1 will perform significantly better than G_2, G_3 and G_4 on a retention test on fractional numbers;

Hypothesis 5. Group G_2 will perform significantly better than G_3 and G_4 on a retention test on fractional numbers;

Hypothesis 6. Group G_3 will perform significantly better than G_4 on a retention test on fractional numbers.

Sample

The experiment was conducted in Kaduna State of Nigeria during the summer and first term of the 1979/80 School session. Permission was obtained from Education Authorities to use the following secondary schools:

(a) A Boys Boarding Secondary School - 4 classes

(b) A Girls Boarding Secondary School - 4 classes

(c) A mixed Day Secondary School - 3 classes

(d) A Teacher Training College - 3 classes

All the classes of Forms I (Grade seven) were used in the study. The average age of the students was 12.5 years. The students were assigned to these schools from primary schools from all over the state. In each class there was a mixture of students from rural primary schools and urban primary schools. The students were not assigned to the classes in the school according to ability, nor was the assignment done on the basis of any criterion. All students were new to their schools when the experiment was conducted. They, therefore, had no time to acquire characteristics peculiar to the schools. All students had gone through instruction on fractional numbers in primary school. Four hundred and eighty (480) students started the experiment. Three hundred and thirty (330) completed the experiment. The attrition was due to the closing of one of the schools by education authorities. The final sample was assigned to treatment as in the following table. Actual school classes were assigned to treatment groups.

TABLE II

STUDENT SAMPLE BY SCHOOL AND TREATMENT GROUPS

SCHOOL	I				II				III		
TREATMENT	G_1	G_2	G_3	G_4	G_1	G_2	G_3	G_4	G_1	G_2	G_3

Instructionsl Unit I and II*

In developing these units fractional number concepts that are developed through the use of measurement models

*The reader is encouraged to look at Appendix A for the Instructional Units I, II and III.

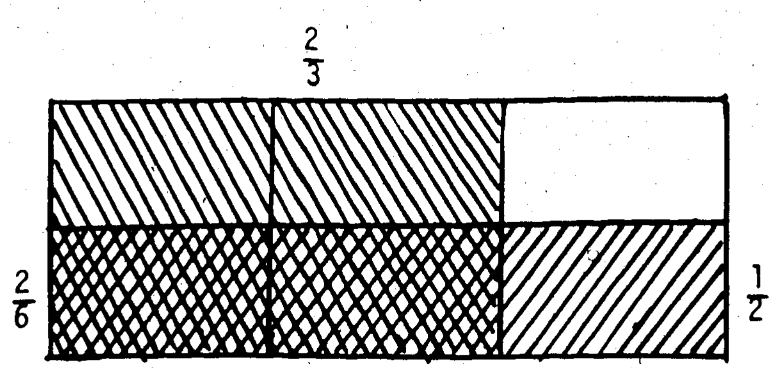
were first identified. These were found to fall under three main headings: Concept, Equivalence and Operations. Under these major headings the following topics were listed:

(a) Concepts: Subdivision of the whole, subdivisions must be equal. Relating the subdivisions to the whole. The meaning of the symbol $\frac{a}{b}$.

(b) Equivalence: After subdividing a whole into equal parts, the parts themselves can be subdivided. These subdivisions must be related to the fractional part and to the original whole. Ordering of fractions.

(c) Operations: Adding two like fractions. Adding two unlike fractions. Subtracting two like fractions. Subtracting two unlike fractions. Adding and subtracting mixed numbers.

The operations of multiplication and division were not included in the units because they have no corresponding operations in measure systems. Although multiplication is sometimes depicted thus



this illustrates the operator or "of" interpretation of rational numbers rather than the measure interpretation. Multiplying a fraction by a whole number can be illustrated by the Archimedian property of a measure system. But this too was not included in the teaching unit for the sake of simplicity.

For every topic in fractions the corresponding measure topic on which it was based was identified. The following measurement topics and activities were arrived at:

Linear measure

(a) Unit of measurement (whole), subdivision of the unit. The use of a measuring unit shorter than the object being measured. The use of a measuring unit longer than the thing being measured. Relating the measure in the last case to the whole unit.

(b) Using "has the same length as" to divide a set of rods or sticks into equivalence classes (sets of rods with the same length). Changing units of measurement (i.e. finding two sets of rods whose combined length equals the length of a given rod). Use of "longer than" and "shorter than" to order a set of rods.

(c) Operations: Joining two sticks of fractional length to determine the length of the combination when each of the sticks is measured in different units. Reverse the last process - that is determine the difference in their length. Repeat the whole operation with sticks whose lengths are longer than the unit.

The topics for area were the same except for the following subsitutions:

1. Only rectangular regions or regions that can easily be reduced to that shape through a determinate number of cuts and rearrangements were used.

(2) "Area" is substituted for "length"

(3) "More area", "less area" or has the "same area as" substituted for "longer", "shorter" or has the "same length as".

(4) "Regions" substitutes for rods or sticks. Only square or rectangular regions were used as units except when the regions can be easily joined together or be used to cover another region completely.

These topics were used to write out objectives for instructional units I. Activities and materials were suggested to teachers so that they could point out the similarities and differences between measurement and fractional number concepts. The set of objectives, activities and materials for Instructional Unit I are in Appendix A.

For Instructional Unit II the set of topics were used as guidelines in selecting review material from the Oxford Primary School Mathematics series. Sample lessons of Instructional Unit II are given in Appendix A.

The units were prepared by the researcher in conjunction with the people who finally taught them. A different teacher was assigned to teach each treatment in each school. The teachers were final year degree students in mathematics education at Ahmadu Bello

University, Zaria, Nigeria. Before entry into the degree programme each of the students had a Nigerian Certificate of Education - a teaching qualification. They all have taught in Secondary Schools. Some have taught also in Primary Schools.

Before the teaching started several meetings were held by the researcher with the teachers. During the meetings objectives of the research, the teaching units and the lessons were discussed. Each teacher was told the kind of emphasis he should place according to the treatment he was teaching. Instructional materials were suggested and developed. Teachers were told to keep a daily record of observations and comments. Testing materials and procedures were also discussed with the teachers. During the course of the experiment, the researcher visited the schools, observed some lessons and held discussions with the teachers and school authorities. Weekly meetings were held with all the teachers where they reported on any problems and solutions were discussed. The tests were administered by the teachers, corrected by them using the same format and the results were checked and collated by the researcher.

Tests I and II

The tests were developed using a set of objectives derived from the list of topics for Instructional Units I and II. The list of objectives are in Appendix B. For each objective a pool of items was drawn up. Two were taken for each, one for test I and the other for test II. The tests were kept as simple as possible because of the variation in student background and ability to use English. The questions therefore used as little language as possible. To keep the two tests parallel, even the stems of the questions were kept the same. The numbers and figures only were varied. The items were pooled under the following subheadings: Concept, Equivalence and Operations.

The tests were administered by the teachers using the same set of instructions. Each test was given during two fourty minute periods. The questions were read to the students. Explanations were given wherever they were needed. This took ten minutes. Five minutes were left at the end for collecting the papers.

When the tests were constructed they were administered twice on children from primary schools around the University. Means and standard deviations were calculated for each test. Split-half, test-retest and Cronbach's Alpha were calculated for each test. Test I was also correlated with test II. The results of these are presented in Table 3. Table 4 shows a set of intercorrelations between Tests I and II and their component parts of Concept, Equivalence and Operation.

TABLE III

RELIABILITY COEFFICENTS AND OTHER

STATISTICS FOR TESTS I & II

STATISTIC	TEST I	TEST II
MEAN	15.10	16.00
S.D.	5.60	6.00
SPLIT-HALF	.71, $p \leq .001$	.73, $p \leq .001$
TEST-RETEST	.49, $p \leq .001$	.49, $p \leq .001$
CRONBACH'S ALPHA	.80	.83
CORRELATION I vs II	.48 $p \leq .001$	.48

TABLE IV

INTER CORRELATIONS BETWEEN TEST I, TEST II

AND THEIR CONCEPT, EQUIVALENCE

AND OPERATION COMPONENTS

	TEST1	CON1	EQ1	OP1	TEST2	CON2	EQ2	OP2
TEST1	1.00	.86	.77	.81	.48	.40	.42	.32
CON1		1.00	.49	.57	.52	.43	.36	.23
EQ1			1.00	.50	.45	.17	.32	.20
OP1				1.00	.53	.37	.39	.41
TEST2					1.00	.88	.76	.71
CON2						1.00	.52	.39
EQ2							1.00	.45
OP2								1.00

Probability for all is $p \leq .001$

Statistical Procedures

In discussing the sample it was seen that two of the three schools had a fourth class which was used as a control group. The data from the two schools were first analysed. Then the data for the two control groups were dropped. The data from the third school were combined with that of groups G_1 G_2 and G_3 from the first two schools. The same analyses were performed. The results are reported in two separate sections in Chapter IV. Data analysis for the two school group is described here. The procedure, the questions and the hypothesis are exactly the same.

Each group in each school was tested three times using tests I and II. Since tests I and II are considered parallel forms of each other, the test results were taken as repeated measures of the same quality. The design model was a three factorial design with school, treatment and testing time as the factors. A three way analysis of variance with one factor repeated was used to test the design model. In order to find answers to the research questions the following hypothesis were tested:

Hypothesis 1. There is no significant difference between the performance of groups G_1, G_2, G_3 and G_4 on tests in fractional numbers.

When the hypothesis was rejected the treatment's main effect was significant. Post-hoc comparisons of treatment means were made on pre-test scores of the groups to determine any original differences between the groups. The comparisons provided evidence for the hypotheses.

Hypothesis I. There is no significant difference between mean scores groups G_1, G_2, G_3 and G_4 on a pre-test of fractional numbers.

Hypothesis II. There is no significant difference between mean scores of groups G_1, G_2, G_3 and G_4 on a post-test of fractional numbers.

Hypothesis III. There is no significant difference between mean scores of groups G_1, G_2, G_3 and G_4 on a retention test of fractional numbers.

A second main hypothesis was the following:

Hypothesis 2. There is no significant difference between the achievement of treatment groups at the three testing periods; pre-test, post-test and retention test.

When the hypothesis was rejected, the testing main effect was significant. Post-hoc comparisons provided evidence for these hypotheses.

Hypothesis I. There is no significant difference between the mean score on pre-test and post-test for any of the treatment groups.

Hypothesis II. There is no significant difference between the mean score on post-test and retention test for any of the treatment groups.

A third main hypothesis was that:

Hypothesis III. There is no significant difference between the performance of students from different schools.

Rejecting the hypothesis would indicate a school main effect. Post-hoc contrasts would indicate where these differences occur.

All analysis were carried out at the University of Alberta Division of Educational Research Services using ANOVA 30 and TEST 02, Winer, pp. 563 - 567.

Summary

This summary indicates the chronological sequence of activities associated with this research.

(a) Early September, 1979. The development of Test I, Test II and Teaching Units.

(b) Late September, 1979. Visit to the Ministry of Education, Kaduna State. Visit to Schools.

(c) First week October, 1979. First testing.

(d) October - November, 1979. Teaching Units. Second testing.

(e) First week of December, 1979. Third testing.

(f) January - February, 1980. Analysis of Data.

Chapter IV

RESULTS

Introduction

The study was originally planned with four schools in the sample. Four schools started the experiment, but later one of the schools was closed down due to student unrest. Of the three schools that completed the experiment, two had four classes. One of the four classes was used as a control group. The results of the data analysis are presented in two sections. Section one deals with the two schools, control group data analysis. Section two deals with the three schools data analysis.

Results of Data Analysis: Two Schools

The purpose of the study was to measure the extent to which transfer of learning takes place between measure concepts and fractional number concepts when instruction in both is designed for maximal transfer. The research questions were stated in chapter I and restated in Chapter III. The accompanying statistical hypotheses were also stated. In this chapter the research questions are restated again and the results of data analysis presented:

Question 1. What is the effect of stressing the similarities between linear and area measure concepts and those of fractional numbers on the learning of fractional numbers?

The experimental groups G_1, G_2, G_3, have already been described in chapter III. Group G_4 is simply a control group where no instruction was given. With this notation the research question is translated into the following research hypothesis:

Hypothesis 1. There is no significant difference between the performance of groups G_1, G_2, G_3 and G_4 on fractional number tests.

The groups were first pre-tested. After each treatment a post-test was administered. Then a retention test was given three weeks after the end of instruction. Table V shows the mean score of each treatment group in each school on the pretest, post-test and retention tests. The pre-test results show a slight difference between all the groups. The post-test scores, however, show a marked difference between group G_1 and all the other groups. Except for group G_1 and group G_2 in school II, all the groups actually did better in the retention test than on the post-test.

A three way analysis of variance was performed on the test scores. The three factors used were school, treatment and testing effect, that is, pre, post and retention. The test scores were considered as repeated measures. The ANOVA 30 programme of the Division of Educational Research Services of the Faculty of Education was used.

TABLE V

CELL MEANS SCHOOL BY TREATMENT, BY PRE, POST AND RETENTION TEST: TWO SCHOOLS TOTAL SCORE

SCHOOL	TREATMENT	PRE	SD	POST	SD	RETENTION	SD
I	G_1	5.73	3.07	14.73	3.39	13.26	5.27
	G_2	6.32	4.58	7.51	4.59	9.19	5.53
	G_3	7.30	4.96	7.67	4.92	11.06	6.26
	G_4	7.07	4.97	7.19	4.26	10.00	6.39
II	G_1	6.92	4.51	15.98	3.25	13.27	4.40
	G_2	5.77	4.38	8.77	4.68	8.23	5.09
	G_3	7.53	5.54	9.77	6.33	10.65	6.28
	G_4	7.23	5.81	8.53	6.04	8.77	7.02

The programme required an equal number of individuals per cell. The numbers in the treatment groups ranged from 26 to 32. The numbers in the cells were reduced to 26 in a random fashion. Table VI gives a summary of the analysis of variance.

Table VI shows that school effect was not significant; $F = 0.18$, $p \leqslant 0.05$. But treatment effect was significant $F = 6.95$, $p \leqslant 0.01$. Test time was also significant $F = 134.53$, $p \leqslant 0.01$. The following effects were also significant; the interaction of school and

TABLE VI

ANOVA OF FRACTION ACHIEVEMENT SCORES ON PRE, POST AND RETENTION TESTS: TWO SCHOOLS TOTAL SCORE

SOURCE		SS	df	MS	F	P ≤
SCHOOL	(A)	12.71	1	12.71	0.18	.673
TREATMENT	(B)	1479.02	3	493.01	6.95	.000
PRE, POST, RET.	(C)	1837.34	2	918.67	134.53	.000
A X B		17.26	3	5.75	0.08	.970
A X C		144.38	2	72.19	10.57	.000
B X C		1215.71	6	202.62	29.67	.000
A X B X C		26.45	6	4.41	0.65	.694
ERROR I (AB)		14181.07	200			
II ABC		2731.50	400			

test-time, and the interaction of treatment with test-time. Since there was no school effect, no school by treatment effect, the school by testing effect was not considered further.

The analysis of variance provided enough evidence to reject the research hypothesis stated above. There was a significant difference between the treatment means. These means are summarised in table VI and plotted in Figure 2. Group G_1 had the highest mean score of the four treatment groups on the post-test and retention test. Group G_2 did poorer than the control Group G_4 on both pre and retention tests. Group G_3 did better than both G_2 and G_4.

Since subjects were not assigned to treatment groups in any particular order, there should not be any difference between the performance of the groups on the

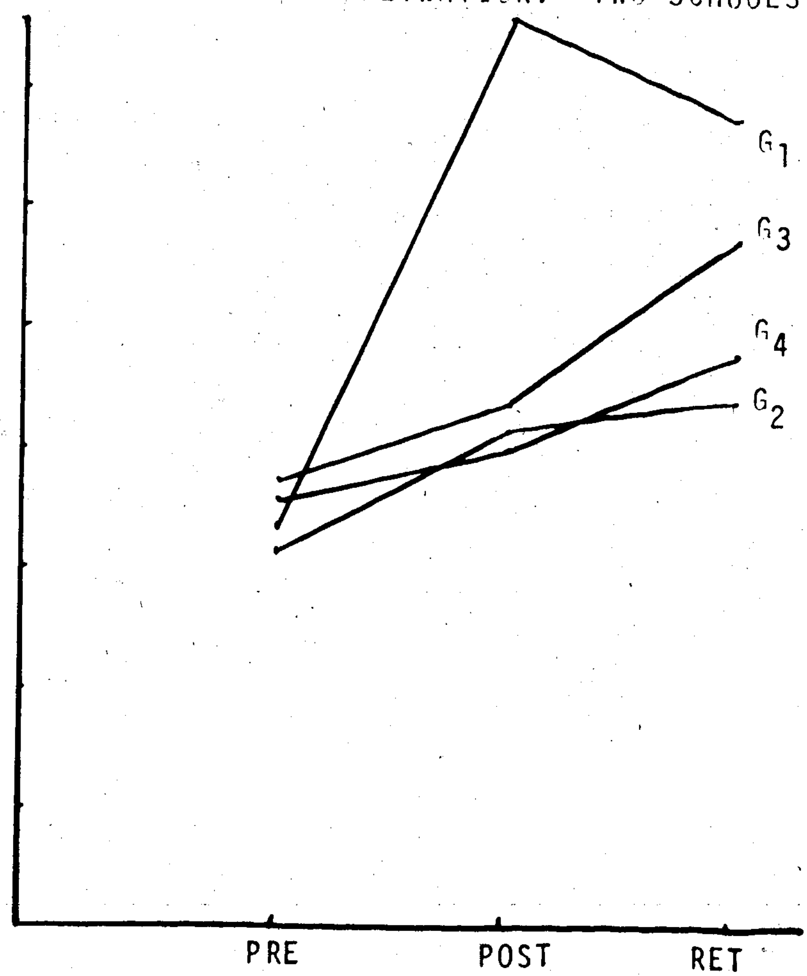
TABLE VII

CELL MEANS TREATMENT BY PRE, POST AND RETENTION: TWO SCHOOLS TOTAL SCORE

TREATMENT	PRE	SD	POST	SD	RETENTION	SD
G_1	6.33	3.90	15.31	3.37	13.27	4.86
G_2	6.06	4.67	8.15	4.89	8.71	5.47
G_3	7.42	5.26	8.67	5.77	11.30	6.30
G_4	7.15	4.81	7.87	5.73	9.39	6.55

FIGURE 2

CELL MEANS, TREATMENT BY PRE, POST AND RETENTION: TWO SCHOOLS



pre-test. Hence the following hypothesis was tested:

> Hypothesis I. There is no significant difference between the mean scores of any two treatment groups on the pre-test.

An orthogonal contrast of the treatment means showed that none of the differences between any two groups was significant. The hypothesis was, therefore, not rejected. The result of the contrasts is summarised in table VIII.

TABLE VIII

CONTRAST BETWEEN TREATMENTS ON PRE-TEST SCORES: TWO SCHOOLS TOTAL SCORE

CONTRAST TREATMENT	MEAN DIFF	MEAN SQ	df_1	df_2	F	P $\leq$
G_1 vs G_2	0.27	28.19	1	600	0.10	.752
G_1 vs G_3	-1.10	28.19	1	600	1.66	.198
G_1 vs G_4	-0.83	28.19	1	600	0.95	.331
G_2 vs G_3	-1.37	28.19	1	600	2.58	.109
G_2 vs G_4	-1.10	28.19	1	600	1.66	.198
G_3 vs G_4	0.27	28.19	1	600	0.10	.752

In order to test the following hypothesis pairwise, comparisons of the treatment means on the post-test were made.

> Hypothesis II. There is no significant difference between the mean scores of any two treatment groups on the post-test.

The comparisons showed that the mean difference of groups

G_1 and G_2, G_1 and G_3, and G_1 and G_4 were significant at $p < 0.01$ level. No other mean difference was significant. Hypothesis II was therefore rejected. The result of these comparisons is summarised in table IX. The order of means was $\bar{X}_1 > \bar{X}_3 > \bar{X}_2 > \bar{X}_4$ where $\bar{X}_1$ is the mean of G_1, $\bar{X}_2$ of G_2, $\bar{X}_3$ of G_3 and $\bar{X}_4$ of G_4.

TABLE IX

CONTRASTS BETWEEN TREATMENTS ON POST TEST SCORES: TWO SCHOOLS TOTAL SCORE

TREATMENT CONTRAST	MEAN DIFF	MEAN SQ	df_1	df_2	F	P $\leq$
G_1 vs G_2	7.15	28.19	1	600	70.81	.001
G_1 vs G_3	6.63	28.19	1	600	60.90	.001
G_1 vs G_4	7.44	28.19	1	600	76.63	.001
G_2 vs G_3	-0.52	28.19	1	600	0.37	.542
G_2 vs G_4	0.29	28.19	1	600	0.12	.735
G_3 vs G_4	0.81	28.19	1	600	0.90	.342

Similar comparisons were made between treatment means on the retention test. The hypothesis tested was:

Hypothesis III. There is no significant difference between the mean scores of any two of the treatment groups on the retention test.

Pairwise comprisons showed that the mean differences of the following groups were significant. G_1 and G_2, $p < 0.01$; G_1 and G_3, $p < 0.05$; G_1 and G_4, $p < 0.01$; G_2 and G_3, $p < 0.01$; G_3 and G_4, $p < 0.05$. Hypothesis III was therefore rejected. Table X gives a summary of the comparisons. The order of means in this case was:

$\bar{X}_1 > \bar{X}_3 > \bar{X}_4 > \bar{X}_2$

TABLE X

CONTRAST BETWEEN TREATMENTS ON RETENTION

TEST SCORES: TWO SCHOOLS TOTAL SCORE

TREATMENT CONTRAST	MEAN DIFF	MEAN SQ	df_1	df_2	F	P $\leq$
G_1 vs G_2	4.56	28.19	1	600	28.74	.001
G_1 vs G_3	1.96	28.19	1	600	5.32	.021
G_1 vs G_4	3.88	28.19	1	600	20.88	.001
G_2 vs G_3	2.60	28.19	1	600	9.33	.002
G_2 vs G_4	-0.67	28.19	1	600	0.63	.429
G_3 vs G_4	1.92	28.19	1	600	5.12	.024

Table VI shows that the testing main effect was significant. Therefore the second main hypothesis was rejected.

Hypothesis 2. There is no significant difference between the achievements of the treatment groups at the three testing periods; pre-test, post-test and retention test.

Let $\bar{X}_{11}$ be the mean of G_1 on pre-test. $\bar{X}12$ the mean of G_1 on post-test. $\bar{X}_{13}$ the mean of G_1 on retention test. And similarly, for G_2, G_3, G_4 to be $\bar{X}_{21}$, $\bar{X}_{22}$, $\bar{X}_{23}$; $\bar{X}_{31}$, $\bar{X}_{32}$; $\bar{X}_{41}$, $\bar{X}_{42}$, $\bar{X}_{43}$. If $X_{11} < X_{12} > X_{13}$ and Hypothesis 2 is rejected then

$\bar{X}_{11} - \bar{X}_{12} < 0$ is the pretest, post-test difference

$\bar{X}_{12} - \bar{X}_{13} > 0$ is the post-test, retention test difference.

Post-hoc pairwise comparisons were made of the test mean scores for each treatment groups. The result of these comparisons are summarised in table XI. The pre, post tests' difference for groups G_1, G_2 and G_3 were all significant at the level of $p < 0.01$. The results also show that only group G_1 had a post-test to retention test drop. The other groups actually improved their mean scores on the retention test!

TABLE XI

DIFFERENCES BETWEEN PRE, POST AND RETENTION TESTS MEAN SCORES FOR DIFFERENT TREATMENTS TOTAL SCORE

TREATMENT CONTRAST	MEAN DIFF	MEAN SQ	df_1	df_2	F	P $\leqslant$
$\bar{X}_{11}-\bar{X}_{12}$	-8.98	6.83	1	200	460.63	.001
$\bar{X}_{12}-\bar{X}_{13}$	2.04	6.63	1	200	23.73	.001
$\bar{X}_{21}-\bar{X}_{22}$	-2.10	6.83	1	200	25.09	.001
$\bar{X}_{22}-\bar{X}_{23}$	-0.56	6.83	1	200	1.78	.184
$\bar{X}_{31}-\bar{X}_{32}$	-1.25	6.83	1	200	8.92	.003
$\bar{X}_{32}-\bar{X}_{33}$	-2.63	6.83	1	200	39.64	.001
$\bar{X}_{41}-\bar{X}_{42}$	-0.71	6.83	1	200	2.89	.091
$\bar{X}_{42}-\bar{X}_{43}$	-1.52	6.83	1	200	13.18	.001

The evidence from these comparisons was enough to reject the following sub-hypotheses:

Hypothesis I. There is no significant difference between the mean score on the pre-test and the post-test for any of the treatment groups.

Hypothesis II. There is no significant difference between the mean score on the post-test and the retention test for any of the treatment groups.

Question 2. What is the effect of learning measurement concepts on the learning of fractional number concepts?

This second question can also be answered by the evidence obtained from the main hypotheses 1 and 2 and their sub-hypotheses.

Results of Data Analysis: Three Schools.

The sequence of this section is exactly the same as the previous section, only in this section the treatment groups were three as in the original design of the experiment. There were also three schools in the sample. Two of the three schools were the same as the two schools in section 1. The differences between the two sections are:

(a) The two school sections had a control group, while

(b) The three school sections had more individuals per treatment.

In this section sub-scores on concept, equivalence and operation are also discussed.

Question 1. What is the effect of stressing the similarities between linear and area measure concepts and those of fractional numbers on the learning of fractional numbers?

The accompanying research hypothesis is:

Hypothesis 1. There is no significant

difference between the performance of groups G_1, G_2 and G_3 on fractional number tests.

Table XII shows the cell means of each treatment in each school on pre, post and retention tests. As in the previous section some groups (G_2 and G_3) actually did better in the retention test than on the post-test.

TABLE XII

CELL MEANS, SCHOOL BY TREATMENT, BY PRE, POST AND RETENTION: THREE SCHOOLS TOTAL SCORE

SCHOOL	TREATMENT	PRE	SD	POST	SD	RETENTION	SD
	G_1	5.73	3.07	14.73	3.39	13.27	5.27
I	G_2	6.35	4.58	7.54	4.59	9.19	5.52
	G_3	7.30	4.97	7.58	4.91	11.96	6.26
	G_1	6.92	4.51	15.88	3.25	13.37	4.40
II	G_2	5.76	4.38	8.77	4.68	8.23	5.08
	G_3	7.54	5.54	9.77	6.33	10.65	6.28
	G_1	4.65	4.74	14.96	4.33	14.50	6.80
III	G_2	4.81	2.82	9.08	3.05	9.85	4.32
	G_3	4.69	4.00	12.31	5.48	11.69	5.64

A three-way analysis of variance was performed on the test scores. The factors were: School, treatment and tests. Table XIII shows the summary of this analysis.

The result is similar to the two schools case. table shows that there is no significant school rence. But treatment and testing effects were significant. This was enough evidence to reject hypothesis 1. The table also shows that the school-testing, and treatment-testing interactions were significant. Since schools main effect and school by treatment effect were not significant, the school-testing effect was not deemed important enough for further consideration.

TABLE XIII

ANOVA OF FRACTION ACHIEVEMENT SCORES ON PRE, POST AND RETENTION TESTS: THREE SCHOOLS TOTAL SCORE

SOURCE		SS	df	MS	F	P ≤
SCHOOL	(A)	17.66	2	8.83	0.16	.856
TREATMENT	(B)	1724.30	2	862.15	15.16	.001
PRE, POST, RET.	(C)	4414.70	2	2207.35	244.42	.001
A X B		28.95	4	7.24	0.13	.972
A X C		429.40	4	107.35	11.89	.001
B X C		1094.95	4	273.74	30.31	.001
A X B X C		183.10	8	22.89	2.53	.010
ERROR I (AB)		12798.36	225	56.88		
II (ABC)		4063.89	450	9.03		

Table XIV gives a summary of cell means for treatments on pre, post and retention tests. The means were plotted on a graph and shown in figure 3.

A pairwise comparison of group means on the pre-test was made to test the hypothesis that:

Hypothesis I. There is no significant difference between the mean scores of any two treatment groups on the pre-test.

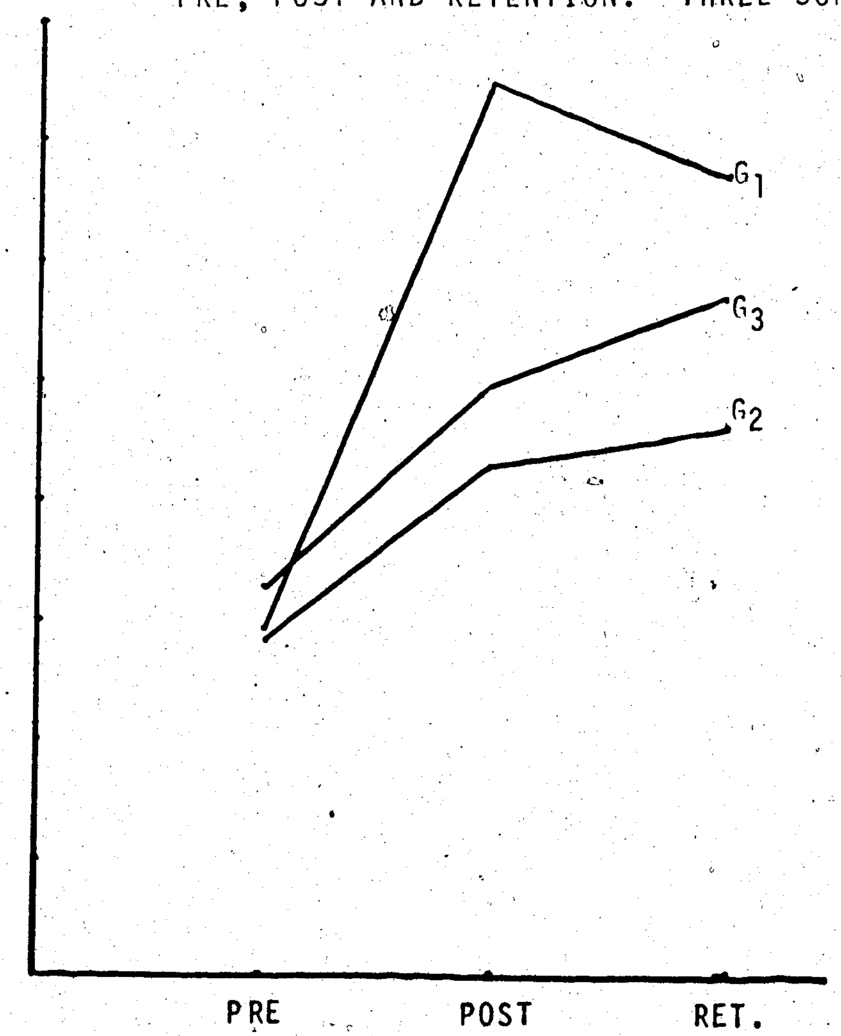
TABLE XIV

CELL MEANS, TREATMENT BY PRE, POST AND RETENTION: THREE SCHOOLS TOTAL SCORE

TREATMENT	PRE	SD	POST	SD	RETENTION	SD
G_1	5.77	4.28	15.19	3.72	13.67	5.61
G_2	5.64	4.19	8.46	4.39	9.09	5.14
G_3	6.51	5.05	9.88	5.93	11.44	6.09

FIGURE 3

PLOT OF CELL MEANS, TREATMENT BY PRE, POST AND RETENTION: THREE SCHOOLS



The result of the comparison is shown in table XV. None of the comparisons was significant. The hypothesis was, therefore, not rejected.

TABLE XV

CONTRASTS BETWEEN TREATMENTS ON PRE-TEST SCORES: THREE SCHOOLS TOTAL SCORE

TREATMENT CONTRASTS	MEAN DIFF	MEAN SQ	df_1	df_2	F	P $\leq$
G_1 vs G_2	0.13	24.98	1	675	0.03	.873
G_1 vs G_3	-0.74	24.98	1	675	0.86	.353
G_2 vs G_3	-0.87	24.98	1	675	1.19	.276

Pairwise comparisons of treatment means on the post-test were made to provide evidence for the following:

Hypothesis II. There is no significant difference between the mean scores of any two treatment groups on the post-test.

The comparisons showed that the mean difference between groups G_1 and G_2 and G_1 and G_3 are significant. In both cases $p < 0.01$. But the mean difference between G_2 and G_3 was not significant, $p > 0.05$. The order of means was $\bar{X}_1 > \bar{X}_3 > \bar{X}_2$. Table XVI gives a summary of the comparisons. Hypothesis II was rejected.

The same order of means was observed in the concept and equivalence subscore means. The order of means in the operation sub-scores was $\bar{X}_1 > \bar{X}_2 > \bar{X}_3$, although

$\bar{X}_1 - \bar{X}_2$ was not significant. Table XVII is summarv of these results.

TABLE XVI

CONTRASTS BETWEEN TREATMENTS ON POST-TEST SCORES: THREE SCHOOLS TOTAL SCORE

TREATMENT CONTRASTS	MEAN DIFF	MEAN SQ	df_1	df_2	F	P $\leq$
G_1 vs G_2	6.73	24.98	1	675	70.73	.001
G_1 vs G_3	5.31	24.98	1	675	43.98	.001
G_2 vs G_3	-1.42	24.98	1	675	3.16	.076

TABLE XVII

CONTRAST BETWEEN TREATMENT GROUPS ON POST-TEST CONCEPT, EQUIVALENCE AND OPERATION SUB-SCORES

TREATMENT CONTRASTS	MEAN DIFF	MEAN SQ	df_1	df_2	F	P $\leq$
CONCEPT						
G_1 vs G_2	4.69	7.15	1	675	120.17	.001
G_1 vs G_3	3.28	7.15	1	675	58.79	.001
G_2 vs G_3	-1.41	7.15	1	675	10.85	.001
EQUIVALENCE						
G_1 vs G_2	2.03	2.88	1	675	56.65	.001
G_1 vs G_3	1.23	2.88	1	675	20.54	.001
G_2 vs G_3	-0.79	2.88	1	675	8.57	.004
OPERATION						
G_1 vs G_2	0.17	3.03	1	675	0.36	.550
G_1 vs G_3	0.96	3.03	1	675	11.91	.001
G_2 vs G_3	0.79	3.03	1	675	8.14	.005

Pairwise comparisons of treatment means on the retention test scores were made to provide evidence for the following:

Hypothesis III. There is no significant difference between the mean scores of any two treatment groups on the retention test.

The comparison showed that all pairs of mean difference were significant, Table XVIII. The order of means was $\bar{X}_1 > \bar{X}_3 > \bar{X}_2$.

TABLE XVIII

CONTRASTS BETWEEN TREATMENTS ON RETENTION TEST SCORES: THREE SCHOOLS TOTAL SCORE

TREATMENT CONTRASTS	MEAN DIFF	MEAN SQ	df_1	df_2	F	P ≤
G_1 vs G_2	4.59	24.98	1	675	32.89	.001
G_1 vs G_3	2.24	24.98	1	675	7.86	.001
G_2 vs G_3	-2.35	24.98	1	675	8.59	.001

As before, the concept and equivalence sub-score means reflected the same order except the operation sub-score where the order was $\bar{X}_1 > \bar{X}_2 > \bar{X}_3$. But $\bar{X}_1-\bar{X}_2$ and $\bar{X}_2-\bar{X}_3$ were not significant.

The second main hypothesis was:

Hypothesis 2. There is no significant difference between the achievement of the treatment groups at the three testing periods: pre-test, post-test and retention test.

TABLE XIX

CONTRAST BETWEEN TREATMENT GROUPS

ON RETENTION TEST, CONCEPT, EQUIVALENCE

AND OPERATION SUB-SCORES

TREATMENT CONTRASTS	MEAN DIFF	MEAN SQ	df_1	df_2	F	P $\leqslant$
CONCEPT						
G_1 vs G_2	2.96	7.15	1	675	47.87	.001
G_1 vs G_3	1.12	7.15	1	675	6.79	.009
G_2 vs G_3	-1.85	7.15	1	675	18.60	.001
EQUIVALENCE						
G_1 vs G_2	1.41	2.88	1	675	26.97	.001
G_1 vs G_3	0.58	2.88	1	675	4.51	.034
G_2 vs G_3	-0.83	2.88	1	675	9.42	.002
OPERATION						
G_1 vs G_2	0.18	3.03	1	675	0.42	.520
G_1 vs G_3	0.60	3.03	1	675	4.68	.031
G_2 vs G_3	0.42	3.03	1	675	2.31	.129

Table XIII shows that the testing main effect was significant. Therefore the hypothesis was rejected. With the notation of the previous section pairwise comparisons were made between means of different treatment groups on the pre-test, post-test and retention test. The following mean differences were significant at $p < 0.01$ level. $\bar{X}_{11}-\bar{X}_{12}$, $\bar{X}_{21}-\bar{X}_{22}$ and $\bar{X}_{31}-\bar{X}_{32}$.

The mean difference of G_1 $\bar{X}_{11}-\bar{X}_{12}$ had the highest absolute value. The next highest was the mean difference of G_3 then G_2. As in the two school control group case the mean score of G_1 dropped from post test to retention, $\bar{X}_{12}-\bar{X}_{13} > 0$. But G_2 and G_3 improved their mean scores on the retention test, $\bar{X}_{22}-\bar{X}_{23} < 0$ and $\bar{X}_{32}-\bar{X}_{33} < 0$. These results are summarised in table XX.

TABLE XX

CONTRAST BETWEEN PRE, POST AND RETENTION TEST MEAN SCORES OF DIFFERENT TREATMENTS: THREE SCHOOLS TOTAL SCORE

TREATMENT CONTRASTS	MEAN DIFF	MEAN SQ	df_1	df_2	F	P ≤
$\bar{X}_{11}-\bar{X}_{12}$	-9.42	9.03	1	225	383.46	.001
$\bar{X}_{12}-\bar{X}_{13}$	1.51	9.03	1	225	9.88	.002
$\bar{X}_{21}-\bar{X}_{22}$	-2.82	9.03	1	255	34.36	.001
$\bar{X}_{22}-\bar{X}_{23}$	-0.63	9.03	1	225	1.70	.193
$\bar{X}_{31}-\bar{X}_{32}$	-3-37	9.03	1	225	49.10	.001
$\bar{X}_{32}-\bar{X}_{33}$	-1.55	9.03	1	255	10.39	.001

The evidence from these comparisons was sufficient to reject the following sub-hypotheses

Hypothesis I. There is no significant difference between the mean score on the pre-test and the post-test for any of the treatment groups.

Hypothesis II. There is no significant difference between the mean score on the post-test and the retention test for any of the treatment groups.

Question 2. What is the effect of learning measurement concepts on the learning of fractional number concept?

This second question can also be answered by the evidence obtained from the main hypothesis 1 and 2 and their sub-hypotheses.

In the next chapter, the results of the two sections are summarised. Their implication are discussed and conclusions arrived at. Suggestions for further research are also made.

Chapter V

SUMMARY, CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

Introduction

The purpose of this study was to measure the transfer effects of measure concepts on the learning of fractional numbers. Instructional units were designed and taught to a sample of Grade Seven students. Instructional units consisted of a sequence of linear measure, area measure and fractional number topics. The sequence design emphasized the similarities between these topics where any existed. Instructional unit two consisted of a review of the topics in unit one. The review was based on a standard elementary school textbook, The Oxford Mathematics Series. Instructional unit three consisted of the topics of linear and area measure treated in unit one. A fourth group received no instruction. Before instruction started, all the groups were pre-tested with a fractional number achievement test. A parallel form of the pre-test was used at the end of instruction as a post-test. The pre-test was then used as a retention test six weeks after instruction.

The results of the tests were analysed using analysis of variance methods. Post hoc comparisons of group means were performed to test the significance of differences between the groups. The results of the analysis are summarised below.

Summary of Results: Two Schools

Group G_1 which was taught instructional unit one is here referrred to as transfer group. Group G_2 is here referred to as the review group. Group G_3 which received instruction in measure concepts only is here referred to as the measure group. Group G_4 which received no instruction is here referred to as the control group. The result of data analyses showed that:

1. Treatment and testing effects were significant, $p \leq .001$. School effect was not significant p .673.

2. The school and testing interaction was significant, $p \leq .001$. The treatment and testing interaction was also significant, $p \leq .001$.

3. The mean difference between all the groups on the pre-test was not significant.

4. The transfer group performed significantly better than the review group, the measure group and the control group on a post-test of fractional number achievement, $p \leq .001$.

5. The performances of the review and measure groups were not significantly different from that of the control group on a post-test of fractional number achievement, $p \leq .735$ and $p \leq .342$ respectively.

6. There was no significant difference between the performance of the review and measure groups on a post-test of fractional number achievement, $p \leq .542$.

7. There was a significant mean difference between the performance of the transfer group and the review group on a retention test, $p \leq .001$.

8. There was a significant mean difference

between the performance of the transfer group and the measure group on a retention test, $p \leq .021$.

9. There was a significant mean difference between the performance of the transfer group and the control group on a retention test, $p \leq .001$.

10. The difference between the means of the review and measure groups on the retention test was significant, $p \leq 0.002$.

11. The difference between the means of the review and control groups on the retention test was not significant, $p \leq .429$.

12. The difference between the means of the measure group and the control group was significant, $p \leq 0.24$.

In cases 4 to 12 above, the mean of the transfer group was higher than the means of all the groups on the post-test. The measure group mean was higher than the mean of the review and control group on the post test. The order of means from the highest to the lowest was transfer group, measure group, review group and control group. The order of means on the retention test was transfer group, measure group, control group, then review group.

13. The differences between the mean pre-test and the mean post-test scores were significant for transfer group, p .001; review group, $p \leq .001$, measure group, $p \leq .003$ but not for the control group, $p \leq .091$.

14. The differences between the mean post-test and the mean retention test scores were significant for the transfer group, $p \leq .001$, the measure group, $p \leq .001$, but not for the review group, $p \leq .184$, nor the control group, $p \leq .091$.

Summary of Results: Three Schools

In this section the control group G_4 was dropped and the review group G_2 was considered as a conceptual control. The results of data analyses were as follows:

1. School effect was not significant, $p \leq .856$, but treatment and testing main effects were significant, $p \leq .001$ for both.

2. There was no significant school treatment interaction, $p \leq .972$ but school testing and treatment testing interactions were significant, $p \leq .001$ for both.

3. The mean difference between all groups on the pre-test was not significant, $p \leq .873$, .353 and .276.

4. The mean differences between the transfer group and the review and measure groups on a post-test of fractional number achievement were significant, $p \leq .001$.

5. The mean difference between the review and measure group on a post-test of fractional number achievement was not significant, $p \leq .076$.

6. The performance of the transfer group on concept and equivalence subtest of the post-test was significantly better than that of the review and measure groups. The transfer group did not do better than the review group on the operation sub-test. The measure group did better than the review group on all sub-tests.

7. The transfer group performed significantly better than the review and measure groups on a retention test of fractional numbers. The performance of the measure groups was better than that of the review group on the same test.

8. The performance of the transfer group on the concept and equivalence sub-tests of the retention test was significantly better than

the review and measure groups. The performance of the groups on the operation sub-test of the retention test was not significantly different. The measure group did better than the review group on both the concept and equivalence sub-tests.

9. The differences between the mean pre-test and the mean post-test scores were significant for all groups. The mean differences between post-test and retention test scores were significant for the transfer and measure groups but not for the review groups.

Discussion

The first things one notices about the results of the experiment are the following. There was no significant difference between the means of the groups in the pre-test. All groups improved their total scores from the pretest to post-test. The measure, review and control groups had higher scores in the retention test than on the post-test. The transfer group had the lowest mean in the pre-test and the highest means in the post and retention test. But there was a drop in the mean score of the transfer group from the post-test to the retention test. The results were the same in both the two-school-control group, and the three school group. See Tables V and VII, Tables XII and XIV.

These results were duplicated in all the three schools. Initially more than one school was chosen for the experiment for two main reasons. First the location of the school was thought to be a factor because of the expected differences between rural and urban schools. Secondly the number of schools increased the size of the sample in the study. But it has already been pointed out that the students used in the study were new to the schools and they also came from all over the State. For this reason rural-urban classification was not considered as a factor in the experiment. This has been supported by the results since neither school main effect nor school treatment interaction were significant. Since the setting of the experiment was the same in each school this fact could be considered as three replications of the experiment.

The experiment was conducted at the beginning of the school year with students who had come from all over the State. The environment was necessarily new and unfamiliar. Since all students have had instruction in fractional numbers in primary school, the increase in subsequent performance may be a reflection of some memory effects. Besides the retention test is exactly the same as the pre-test which is a parallel form of the post-test. Also two of the three schools used were boarding schools. Discussion of what went on in class was possible during preparation time.

The increase in scores could not be due to instruction in measure or fractional number concepts between the post-test and the retention test. The teachers were told not to teach anything relative to these topics. Their reports and observation showed that this was the case for the control group during the experiment, the measure group after completion of the measurement topics and all groups after the post-test. The topics covered during these times were review of topics in algebra and operation with integers. In any case, the performance of the transfer group was significantly better than all the groups on both the post and retention tests and in both the two and three schools settings. The performance of the measure group was slightly better than those of the review and control group. The control groups performance was also slightly better than that of the review group. But these differences were not significant.

The general performance of the students in all groups was low. This could be due to the unsettled nature of the period when the experiment was conducted. When primary six students were used to pilot the tests in their normal schools their performance was higher, see tables III and IV. The teachers also noted the following physical problems in all the schools:

1. There were not enough chairs to seat all the students at the beginning of the experiment.

2. There was always a loss of teaching time before students settled down after break.

3. In the day school students were tired after a long walk to school.

4. Some classes were scheduled after physical education classes or late in the day.

Except for the walk to school the problems were common to all the schools.

The observed difference between the treatment groups cannot be explained by the students' previous knowledge in fractional numbers for a number of reasons. First the assignment of students to classes was not done according to ability. Secondly, the group that reviewed measurement and fractional number concepts should have done better than all the other groups, especially the control group. Not only did the transfer group perform significantly better than all the groups, the measure group did slightly better than the review group. One would have expected the review group to do better than the measure group simply because the latter was not taught fractional number concepts.

A simple perusal of instructional units I and II would indicate that there was nothing about the transfer unit which was novel and unusual to sensitise the group about the experiment. In both cases materials and diagrams were used in instruction. It was not unusual for teachers from outside the schools to come and teach in these schools, because the universities and the Advanced Teachers Colleges are always placing their students in these and other primary and secondary schools for teaching practice and research. Neither the schools nor the students volunteered for the experiments, only the principals and the mathematics coordinators and teachers were aware that an experiment was taking place. Also students were taught in groups which would remain their classes for the rest of the time they were in these schools. Therefore, novelty of experience could not be used as explanation of the observed gains of the groups or the differences between them.

The numbers of students per treatment cells were different. This was not due to students dropping out of the classes. These numbers were of those students who reported to school at the time the expertiment started. Those who reported later were assigned to classes on the basis of which class had available seating facilities. Those who were not there at the beginning of the experiment were not included in the results.

It will also be noted that in both the two school and the three school group, there was no school treatment interaction. But in both groups there were school-testing and treatment-testing interactions. The school and test interaction would imply that there were characteristics of the schools that caused the tests to be more effective there than they would be in a larger population of schools. But the students that participated in the experiment were at these schools for the first time. They were drawn from primary schools all over Kaduna State. They had not been at these schools long enough to have acquired any characteristics peculiar to the schools. Two of the schools are in villages away from Zaria City while only one is in Zaria City. The posting of students to secondary schools away from their homes is a common practice all over Nigeria. This is because there are not enough secondary schools to enable a student to attend one in his or her home environment. Therefore a school-test interaction effect would not limit the applicatibility of these results to similar form one classes in other secondary schools.

A test-treatment interaction would indicate a sensitization of pre-test to the different treatments given. The tests used (Appendix B) are similar in form and content to the ones used by the West African Examination Council which conducts a common entrance examination from primary schools to all secondary schols in Nigeria. Students were selected for secondary education on the basis of the examination in English and Arithmetic. There was nothing unusual about the tests. Therefore, any test-treatment or test-school interaction effect would indicate that similar thing would happen with students from other schools in the state or the country. Campbell and STanley (1966) stated that:

> "But in research in teaching, one is interested in generalizing to a setting in which testing is a regular phenomenon. Especially if the experiment can use

regular classroom examinations as Os, (Observations) but probably also if the experimental Os are similar to those usually used, no undesirable interaction of test and X (treatment) would be presented," p 18.

On the basis of the results of the experiment, and the above discussion the following conclusions can be drawn.

Conclusions

1. That the transfer group performed significantly better than the review group and the measure group on a post-test of fractional number learning.

2. The transfer group performed significantly better than the review and measure group on a retention test of fractional number learning.

3. The performance of the transfer group was better than that of the review and measure groups on fractional number concept and equivalence sub-scores of both the post-test and retention tests. But the performance of all groups was not different on the operations sub-score.

4. The performance of the measure and review groups were not significantly different on a post-test of fractional number learning. But the measure group did significantly better than the review group on a retention test of fractional number learning.

5. The measure groups performance on the concept, equivalence and operation sub-scores of the post-test of fraction number achievement was significantly better than that of the review group. This difference was the same on the retention test except for the operation subscore.

6. The trend of the results were the same in all the three schools involved in

the experiment. This could be considered as three replications of the experiment

The following observations were recorded by the teachers that taught the transfer and review groups:

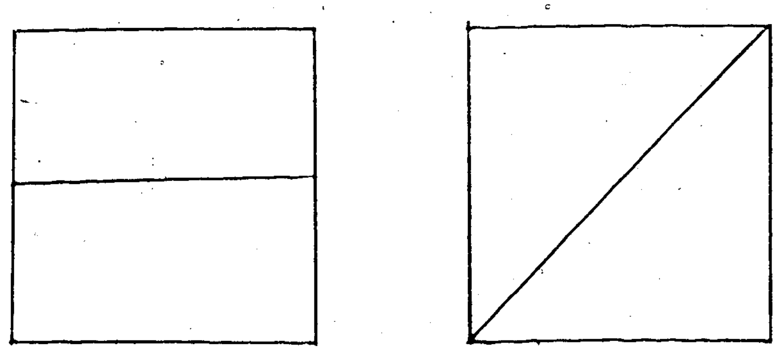
1. Students realised the need for standard unit of measurement after using paces, hand-span and foot-steps as units of measure.

2. The need for smaller units of measure was realised when things that were measured were less than an unmarked metre rule.

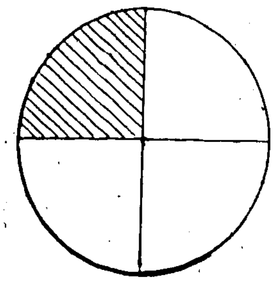
3. Students were able to find areas of rectangular shapes by covering and using the area rule. But problems occured when the shapes of the figures were non-rectangular.

4. Students were unable to relate the measurement of smaller distances in class to longer distances outside of class.

5. Some students could not see the equivalence of the following subdivision of area:



6. Some students responded to shaded region of the following figure as $\frac{1}{3}$ instead of $\frac{1}{4}$:



7. Some students solved addition and subtraction problems with simple unit fractions by finding the least common multiple of denominators instead of by finding equivalent fractions, for example, $\frac{1}{2} + \frac{1}{4}$ would be done as follows:

$$\frac{1}{2} + \frac{1}{4}$$

$$\frac{2 + 1}{4} = \frac{3}{4}$$

The above conclusions suggest that the research questions stated in Chapter I could be answered as follows:

Question I. What is the effect of stressing the similarities between linear and area measure concepts and those of fractional number on the learning of fractional number concepts?

From the results of this study it would seem that the effect of such an emphasis is significant for transfer of measurement ideas in the learning of fractional numbers. Emphasising these similarities in instructional design does a better job than a simple review of the conepts. The learning that takes place in such a setting is also retained much better. The instructional design is more effective in developing fractional number concepts and equivalence though not more effective in developing operation with fractional numbers than other designs.

Questions 2. What is the effect of learning measure concepts on the learning of fractional numbers?

The results of this experiment do not allow us to suggest that this method does a better job of teaching fractional number concepts than a review of measurement and fractional number concepts. But the results do indicate that a design which emphasises the similarities of the measure concept and fractional number concept is likely to do a better job of transfer to fractional number concepts, even if the fractional number concepts are not taught, than one which reviews both measure and fractional concepts. The design also helps in developing the basic concepts of fractional numbers and equivalence.

The results of a number of pilot studies Payne (1976) show that measurement concepts are a useful basis for instruction in fractional numbers. Owens (1976) has shown that the learning of fractional number concepts is enhanced when area models are used as a basis of instruction. Babcock (1978) has shown that there is a relationship between basal measurement concepts and fractional number learning in three grade levels. Her results show that without delibrate emphasis, the relationship between the structures of measure and fractional number achievement in children was lacking until grade 8. Such a relationship existed at that level. In this study the effect of delibrately teaching fractional numbers as measures was tested compared with the teaching of measure alone and more traditional fractional number instructional/ curriculum pattern. The results supports Babcock's findings. The results also suggest that these similarities and relations can be utilised in developing curriculum materials for instruction in fractional numbers and how this could be done. A simple review of measurement concepts does not seem to suffice. A concious effort should be made to emphasize similarities and point out differences between the two sets of conepts.

Suggestions for Further Studies

Many studies are suggested as a result of this investigation. In this investigation both linear and area measure concepts were used to study transfer to fractional number learning. One study could investigate a linear measure, fractional number sequence against an area, fractional number sequence to see which is better in developing initial fractional number concepts. The study could be conducted in three grade levels (four, six and eight for example) to see if there are any developmental factors involved.

In this study achievement in measurement was not measured. A similar study could be conducted where tests of measurement concepts similar to Babcock's (1978) Test of Basal Measurement Concepts could be given after instruction in measurement. A test of fractional number achievement would be given at the end of instruction. The study would try to find out how achievement in measure concepts relate to achievement in fractional number concepts in an instructional setting which emphasizes the similarities of the two concepts.

The results of this study indicated that there is a treatment-test interaction. The tests presented fractional numbers in linear and area modles. To study the effect of this interaction, a Solomon four group design could be used to study the effects of pre-testing on fractional number achievement.

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APPENDIX A

INSTRUCTIONAL UNIT I

EXPERIMENTAL UNIT

A LINEAR MEASURE

B AREA MEASURE

C FRACTIONS

TEACHERS: E.I. WAHALA

B.M. GETSO

A.Y. DARE

J.A. OYEDOYIN

TOPIC OUTLINE, OBJECTIVES AND SUGGESTED ACTIVITIES FOR LINEAR MEASURE

Topics

1. Equivalence

   (a) Equivalence relation is "has the same length as".

   (b) Properties of equivalence relation.

      (i) A rod has the same length with itself.

      (ii) If a rod A has the same length with rod B then rod B has the same length with rod A.

      (iii) If rod A has the same length as rod B and rod B has the same length as rod C then rod A has the same length as rod C.

2. Order

   (a) Order relation is "shorter than" or "longer than".

   (b) Properties of the order relation:

      (i) If rod A is shorter than rod B then rod B is longer than rod A.

      (ii) If rod A is shorter than rod B and rod B is shorter than rod C then rod A is shorter than rod C.

      (ii) Given any two rods A and B such that A is shorter than B then there exist rod C which is longer than rod A but shorter then rod B.

3. Operation

   (a) Join two rods A and B and find a third rod C with the same length as the combined lengths of A and B.

   (b) Properties:

      (i) The combined lengths of rod A joined with rod B is the same as the combined lengths of rod B joined with rod A.

      (ii) The result of joining the lengths of rod A to rod B then to rod C is the same as joining the lengths of rod A to the combined lengths of rods B and C.

(iii) Given a rod A and a set of rods all equal in length to rod B then a certain number of B-rods joined end-to-end will be longer than rod A.

4. Function; Unit

(a) Determine a unit either standard or non-standard.

(b) Use the unit iteratively to assign a natural number to a given rod by counting.

(c) Properties:

(i) Congruent rods have the same length

(ii) The combined length l of rod A and B is the same as the length r of rod A plus the length t of rod B. That is $l = r + t$.

(iii) For a given unit not every rod has a whole number measure of length. That is there are some rods whose length cannot be measured by a particular unit a whole number of times.

(iv) Different units give different measures for the same rod.

<u>Objectives</u>

At the end of this unit the students should be able to:

1. Determine the equivalence of a given set of rods (or sticks) using the equivalence relation "has the same length as".

2. Decide which rod (stick) is longer or shorter than the other among a given pair. Make similar comparison given two or more sticks using longest and shortest where appropriate. And make these comparisons physically and or pictorially or diagramatically.

3. Join two or more rods and determine another rod whose length is equivalent to the length of the combined rods. Do this using sticks and or line diagrams.

4. Assign a number to a given rod using a given unit iteratively.

5. Use a metre stick in measuring the lengths of objects in metres and centimetres.

6. Order lengths of objects measured in the same units and in different units.

7. Add and subtract two measurements when both are in the same unit and when they are in different units.

Suggested Activities

I. Give students sets of sticks. Make sure that there is a large number of sticks of different lengths and large number of sticks of the same length in each length set. For example, make a collection of sets of sticks made up as follows:.

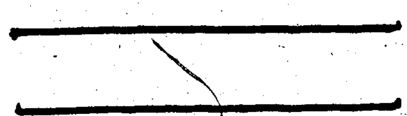
100, 5cm sticks
50, 10cm sticks
40, 15cm sticks
20, 20cm sticks
10, 25cm sticks
10, 30cm sticks

Each student should have a set similar to the one above. Compare sticks by putting them side by side to determine those of the same length.

Make the following comparisons:

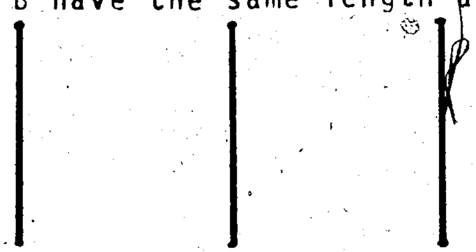
Stick A

Stick B



A has the same length as B

Does B have the same length as A?



A B C

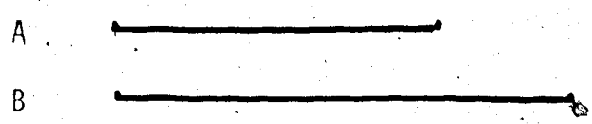
A has the same length as B

B has the same length as C

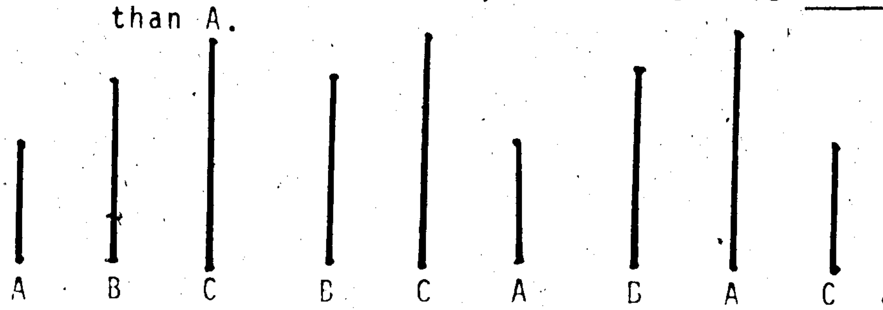
Does A have the same length as C?

Make these comparisons using actual sticks and in diagram form on the blackboard and on paper. Emphasize that two sticks are "equivalent" because 'they have the same length'. Make sure some of the sticks are differently coloured and of different thickness.

II Give students a set of rods of different length. Compare physically and by using diagrams on the board to decide which stick is longer or shorter. Put the sticks in order according to length, from the shortest to the longest.



A is shorter than B, therefore B is ________ than A.



A is shorter than B, B is shorter than C, therefore, A is ____________ than C.

III Using the same set of sticks above, the following demonstrations can be made:

1. Join two 5cm sticks to compare with a 10cm stick.

2. Two 10's to compare with a 20

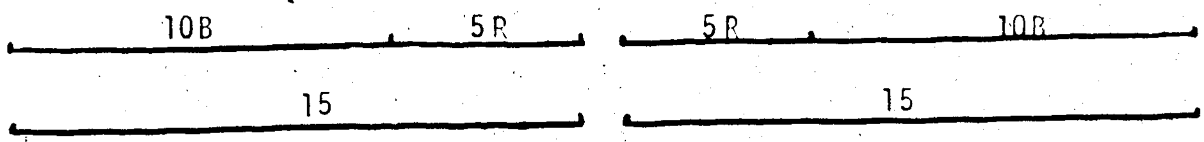
3. Two 15's to compare with a 30

4. Three 5's to compare with a 15

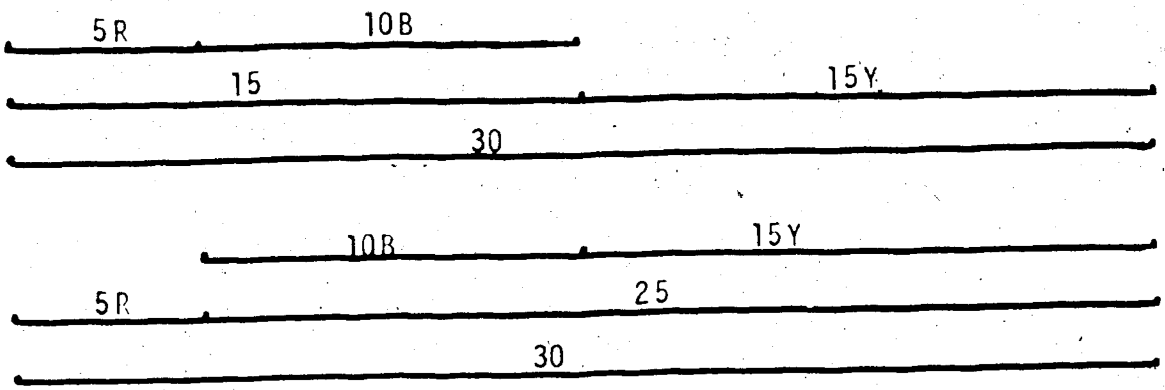
5. Four 5's to compare with a 20

Emphasize the following:

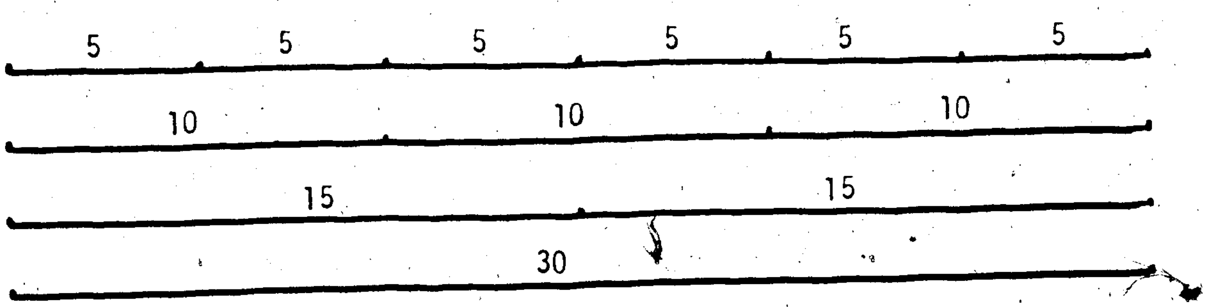
(a) Take a red, R, 5 and a blue, B, 10, show that R joined with B is the same as B with R both being equivalent to a 15.



(b) Take a red, R, 5; a blue, B, 10 and a a yellow Y, 15. Show that joining R and B then Y is equivalent to joining B and Y first and then R, both being equivalent to a 30



Take a 30, two 15s, three 10s and six 5s. Arrange them as follows



This shows that a 30 stick can be made up in many ways each of which is equivalent because they have the same length as the 30 stick. Point out and emphasize the following:

(i) A 30 stick can be made up of two 15

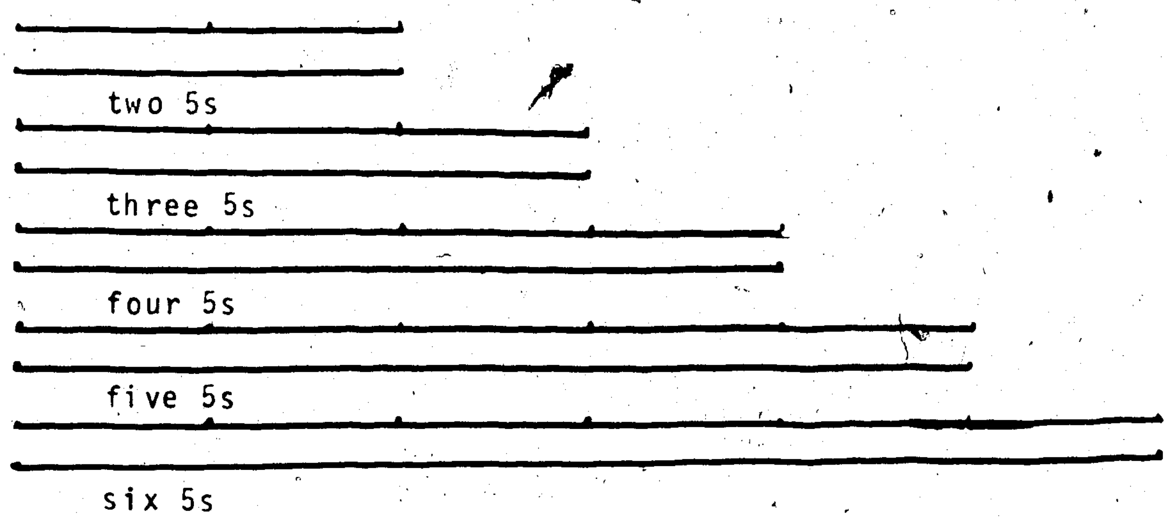
sticks. Both sticks are of equal length.

(ii) A 30 stick can be made up of three ten sticks. Each of the smaller sticks is of equal length.

(iii) A 30 stick can be made up of six 5-stick Each of the smaller sticks are of equal length.

These facts are important in both measuring and later in fractional numbers.

IV Take a 10, a 15, a 20, a 25, and a 30. Ask students to arrange them in order of length from shortest to longest. Compare any two. Which is longer? By how much? Students should join 5-sticks to make equivalent lengths for the other as follows:



Therefore 2 can be assigned to rod A

3 can be assigned to rod B

4 can be assigned to rod C

5 can be assigned to rod D

6 can be assigned to rod E

The 5-rod is the unit. It is used repeatedly to measure the other rods.

Ask students to use either their steps, feet, strides, arm span or hand span to measure lengths of classroom walls, tables, desks, etc. Point out the following:

1. Different units measuring the same length give different measures.

2. Some units are more convenient for measuring than others. For instance it is better to use arm span to measure classroom wall than foot.

The first point is similar to the fact in fractional numbers that two fractions are the same because they represent the same number (or length of a line). The first and second points suggest the use of a standard unit like the metre.

V Use an unmarked metre stick. Ask students to use it to measure the lengths of certain things. Make sure you have things whose lengths are a whole number multiple of the metre. Then measure the length of things whose length is more than one metre but less than two metres. Discuss what to do with the remainder. Bring a marked metre ruler and discuss the centimetre solution. Point out the following facts:

1. The metre is the unit.

2. In order to use it to measure smaller things the unit is subdivided.

3. The subdivisions are all equal. There are one hundred of them. Each one is called a centimetre or one hundredth of a metre.

Point out also that the above facts are exactly the way fractions are derived. A whole unit subdivided into equal number of parts, and some parts taken. Make a summary of the following:

1 metre = 100 centimetres

half a metre = 50 centimetres

Decimetre = 10 centimetres

A half metre is a metre divided into two. A decimetre is a metre divided into ten equal parts. Therefore 1 metre = 2 half metres = 10 decimetres = 100 centimetres. All these are equal because they represent the same length.

VI Use a metre rule to measure two walls of the classrom. Report the measurements in metres like:

Wall A is 4 metres

Wall B is 5 metres

Therefore wall B is longer than wall A. Repeat the activity with other measure. Now measure the length of an object in metres and of another object in centimetres. Assume the following results:

Object A 3 metres

Object B 250 centimetres.

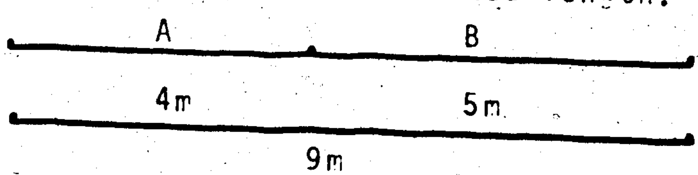
Which object is longer?

No comparison can be made here unless the units of measurement are the same. We can either convert length of A to centimetres or length of B to metres.

Object A is 3 metres. Each metre is 100 centimetre. Therefore object A is 300 centimetre long.

300 > 250. Therefore object A is longer than object B. What is done here is similar to comparing fractions. If two fractions have the same denominator, then they can be compared. But if the denominator is different then the comparison is not possible unless they are made the same.

VII 1. Measure the two objects A and B in metres. Say A is 4m long. B is 9m long. Join A and B. Find their combined length.

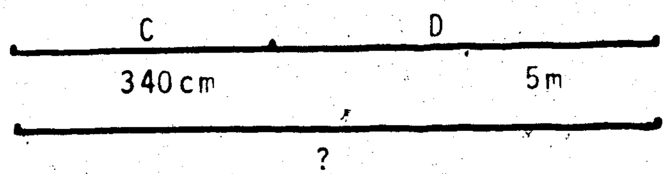


length A + length B = 4m + 5m = 9m

2. Measure two objects C and D

length C = 340cm

length D = 5m

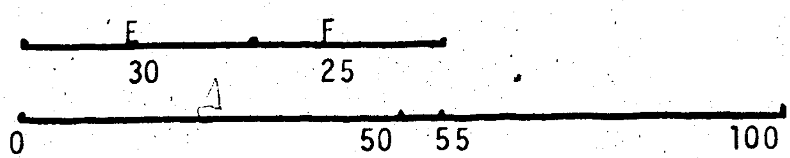


Convert 5m to cm. $\quad$ 5m = 5 x 100cm
= 500cm

Therefore length C + length D = 340cm + 500cm
= 340 + 500cm = 840cm

3. Measure objects E and F

length E = 30cm
length F = 25cm
length E + length F = 30cm + 25cm
= 30 + 25cm = 55cm



4. Measure objects G and H

length G = 40cm
length H = 3 decimetres

therefore length G + length H = 40cm + dcm

convert 3dm to cm $\quad$ 3dm = 3 x 10cm = 30cm

therefore length G + length H = 40cm + 30cm
= 40 + 30 cm
= 70cm

The following points should be noted in these four activities:

(a) Adding length is putting the objects end-to-end and determining the combined length.

(b) If the lengths of the pieces are in the same units, simply add the numbers.

(c) If the units are not the same, they have to be made the same by conversion before addition can take place.

5. Measure objects A and B
length A is 5 metres
length B is 3 metres
By how much is A longer than B?

length A - length B = 5m - 3m
= (5-3)m = 2m

6. Measure objects C and D
length C is 370cm
length D is 2m
Which is taller and by how much?

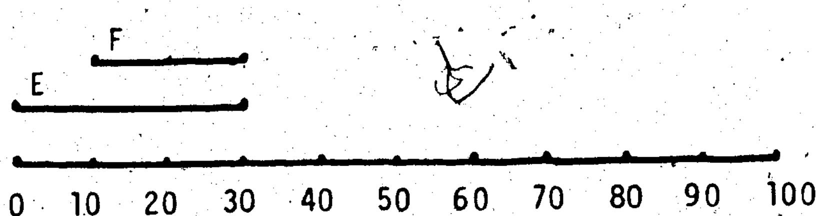
convert 2 metres to centimetres

2m = 200cm

length C - length D = 370cm - 200cm
= (370 - 200)cm = 170cm

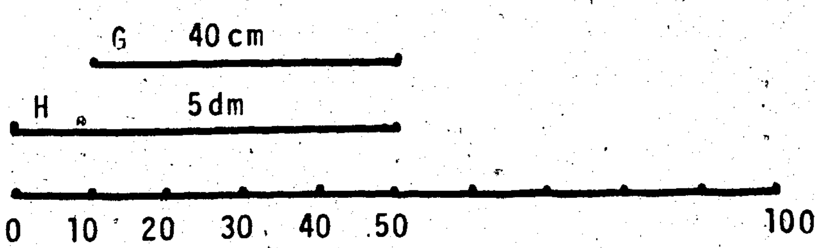
7. Measure objects E and F
length E = 30cm
length F = 20cm

How much longer is E than F?



length E - length F = 30cm - 20cm
= (30-20)cm = 10cm

8. Measure objects G and H
length G is 40cm
length H is 5dm
which is longer and by how much?



convert 5dm to cm. 5dm = 5 x 10 cm
= 50cm

length H - length G = 50cm - 40cm
= (50-40)cm = 10cm

The following points should be noted:

(a) Subtracting length is putting objects end-to-beginning and determining the difference.

(b) If the lengths of the pieces are in the same unit, simply subtract the numbers

(c) If the units are not the same, they have to be made the same by conversion before subtraction can be done.

These facts together with the ones noted earlier are similar to those in adding and subtracting fractions. For fractions the facts are:

(a) To add two fractions if the denominators (like units) are the same add or subtract the numerators (like the numbers in measurement).

(b) If the denominators are not the same make them same (conversion) and then add or subtract the numerators.

TOPIC OUTLINE, OBJECTIVES AND SUGGESTED ACTIVITIES FOR AREA MEASURE

Topics

Note: Regions referred to in this section are either rectangular or can be made into rectangular shape by a finite number of cuts and rearrangements.

1. Equivalence

   (a) Equivalence relation is "has the same area as"

   (b) Properties of equivalence relation:

      (i) A region has the same area as itself

      (ii) If region A has the same area as region B then region B has the same area as region A.

      (iii) If region A has the same area as region B and region B has the same area as region C then region A has the same area as region C.

2. Order

   (a) Order relation is "has less area than".

   (b) Properties of order relation:

      (i) If region A has more area than region B then region B has less area than region A.

      (ii) If region A has less area than region B and region B has less area than region C then region A has less area than region C.

      (iii) Given any two regions A and B such that A has less area than B, then there exists a region C which has more area than region A but less area than region B.

3. Operation

   (a) Join two contiguous regions A and B and find a third region C which has the same area as the combined area of A and B.

   (b) Properties:

      (i) The combined area of regions A and B is the same as the combined area of regions B and A

(ii) The combined area of region A and B joined with region C is the same as the combined area of region A joined with regions B and C.

(iii) Given a region A and a set of regions all of equal area to a given region B then a certain number of B regions joined end-to-end will be larger in area than region A.

4. Function

(a) Determine a square unit

(b) Use the unit iteratively to cover regions, count the number of repetitions and assign that number to the area of the region.

(c) Properties:

(i) Congruent regions have the same area

(ii) The combined area m of region A and B is the same as the area n of A plus the area s of B. That is $m = n + s$.

(iii) For a given unit not every region has a whole number measure of area. That is there are some regions whose area cannot be measured by a particular unit a whole number of times.

(iv) Different units give different areas for the same region.

<u>Objectives</u>

At the end of this unit the student whould be able to:

1. Sort out a given set of regions according to equality of area.

2. Order similar regions according to which region has more (less) area than the other region.

3. Join two regions (rectangular) together and determine another region with equal area.

4. Assign a number to a given region using a given unit (square) iteratively.

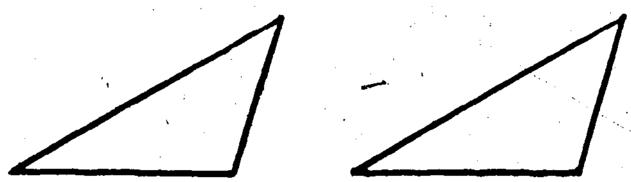
5. Order regions whose area has been measured in the same unit and in different units.

6. Add and subtract two area measurement when both measurements are in the same unit or different units.

Suggested Activities

1. Make a collection of cardboard cut-outs of triangles, circles, squares, rectangles, pentagons and hexagons. Have a large number of each shape of the same size and also of different sizes. Colour as many of them different as possible. For triangles a set of similar equilateral, isoceles, rightangled and sealene triangles could be made. To make comparisons of larger, smaller area than, similar figures must be used so that children can see that one region is completely contained in the other.

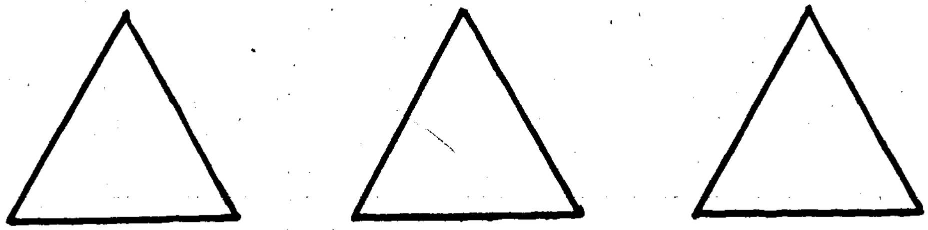
Make the following comparisons



A B

A has the same area as B

Does B have the same area as A?



A B C

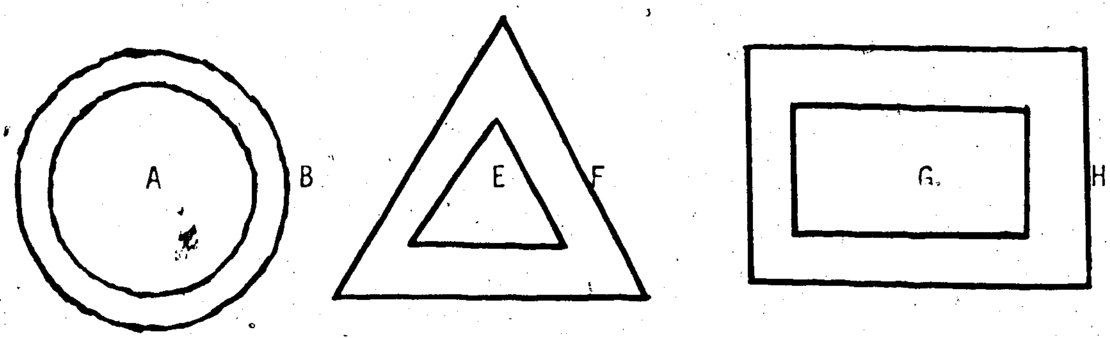
A has the same area as B

B has the same area as C

Does C have the same area as A?

Make comparisons using different shapes.

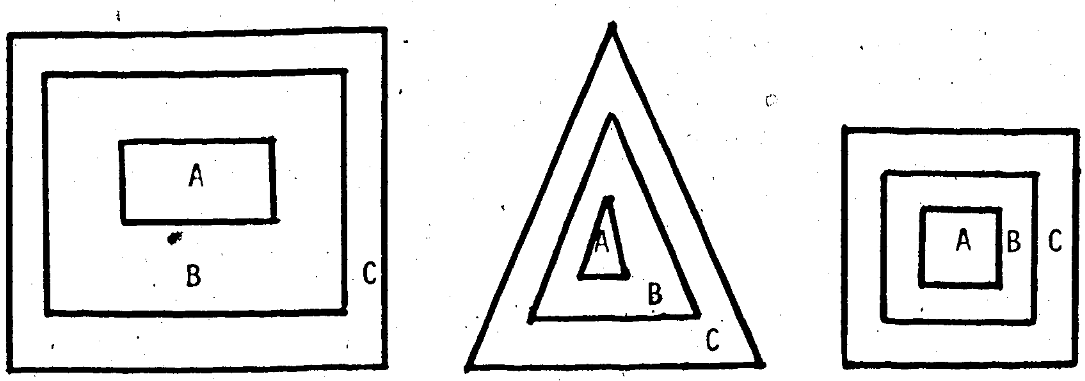
II Take a set of paper cut-outs of a particular shape (say circles). Compare these with each other. If a circle is completely embedded within another circle, the one embedded is smaller and has less area than the one embedding it.



1. Circle A has less area than circle B
2. Triangle F has more area than triangle E
3. Rectangle G has less area than rectangle H

Comparing two or more shapes and using largest and smallest the students should put the cut-outs in order of size of area.

Make the following comparisons:



Region A has less area than region B

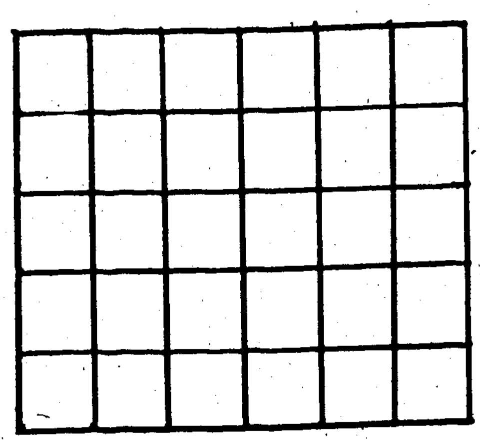
Region B has less area than region C

Region A has ____________ than region C

III Make a large number of large rectangular strips of paper with two or more smaller rectangular strips that can cover the large one. Students should practice using small strips of paper to cover larger one. The total area of the small strips should then be equal to the area of the large one. A unit as an element of a set of congruent regions which can be used to cover any given region.

IV Take a rectangular region of dimensions (say 50cm by 30cm). Cut out strips of one square centimetre ($1cm^2$). Use the squares to cover the rectangular region completely without leaving any space. Count the number of squares. Repeat with rectangular spaces of dimensions 10cm by 5cm; 20cm by 10cm. Draw diagrams illustrating the covering and the number of squares in the covering.

Example: illustrate with paper-folding



30, 1 cm squares written

as 30 cm^2 for short

Make a table as follows

Diamensions of rectangle		Area in square cm by counting.	l x w
Length	Width		
50	30	1500	1500
10	5	50	50
20	10	200	200

Repeat as many times as possible before making the generalization that area of a rectangle is given by the product of the length measurement with the width measurement.

Emphasize the following points:

(a) To find area the region is divided into equal subregions.

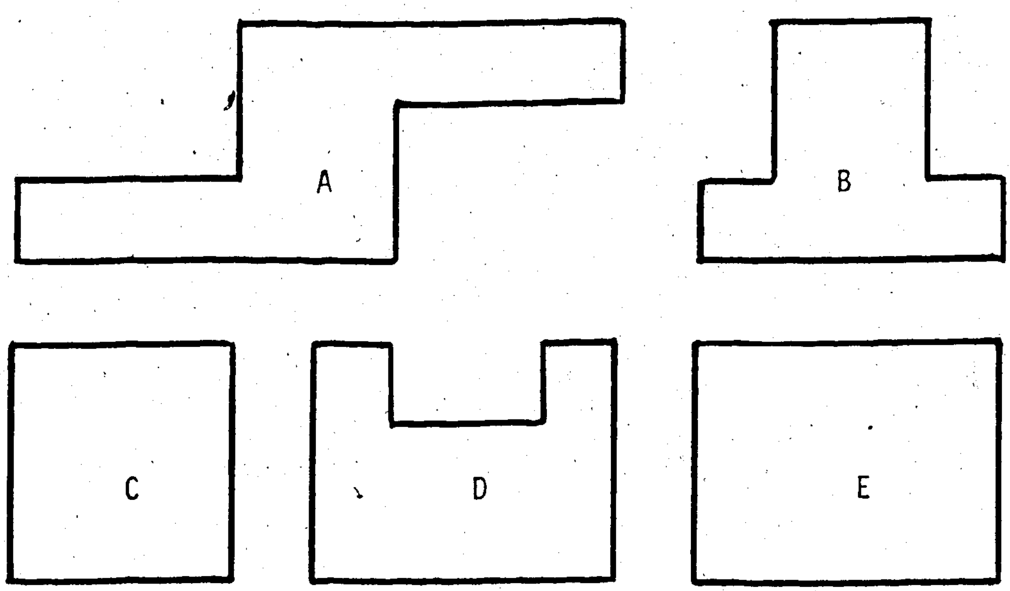
(b) Each subregion is a square

(c) The division or covering of a region divides the edges of the region into congruent segments.

(d) The subdivisions cover the region completely.

These points are used in introducing the concept of fractions using area of regions.

V With the use of a unit square regions can be ordered.



Which has most area? Which has least?

The area of desk top, book covers etc. can be measured with square centimetres. But a football field is too large to measure by covering with square centimetre. So a square whose sides are one metre long or a square metre (or m^2 for short) is used to measure such areas.

$10,000cm^2 = 1m^2$

To order regions according to area the units of measurement must be the same.

Examples.

Region A has area of $35700cm^2$
Region B has area of $27000cm^2$

Then region A has more area than region B But if region C has $35700cm^2$ and region D has $4m^2$ a comparison cannot be made unless the m^2 is converted to cm^2.

$4m^2 = (4 \times 10,000)cm^2$

$= 40,000cm^2$

Therefore region D has more area than region E.

VI (a) Measure two regions A and B

Area A = $30cm^2$

Area B = $40cm^2$

Area A + area B = $30cm^2 + 40cm^2$

$= (30 + 40)cm^2 = 70cm^2$

Area B - area A = $40cm^2 - 30cm^2$

$(40 - 30)cm^2 = 10cm^2$

(b) Measure two regions C and D

region C has area $5529cm^2$

region D has area $2m^2$

convert m^2 to cm^2

$2m^2 = (2 \times 10,000)cm^2 = 20,000cm^2$

Therefore area C + area D $= 5520cm^2 + 20,000cm^2$

$= (5520 + 20,000)cm^2$

$= 25,520cm^2$

Area D - area C $= 20,000cm^2 - 5520cm^2$

$= (20,000 - 5520)cm^2$

$= 14480cm^2$

Adding and subtracting area measure is similar to adding and subtracting length measure. The similarities between length measure and fractions has already been shown. It applies equally as well here. That is:

(1) Adding or subtracting area measure can be done if the measures are in the same unit.

(2) Adding and subtracting area measure in different units can be done only if the units are converted into the same one.

TOPIC OUTLINE, OBJECTIVES

AND SUGGESTED ACTIVITIES

FOR FRACTIONAL NUMBERS

Topics

(a) Unit subdivision

(b) Fraction symbol and meaning

(c) Other ways of making subdivisions

(d) Equivalence of different subdivisions

(e) Fractional numbers

(f) Addition of fractions

  (i) Fractions with the same denominator

  (ii) Fractions with different denominators

(g) Subtraction of fractions

  (i) Fractions with the same denominator

  (ii) Fractions with different denominators.

Objectives

At the end of this unit the student should be able to:

1. Divide a given unit (linear or area) into a given number of equal parts.

2. Show that each fractional part is also a whole in itself.

3. Indicate that the number below the line in this symbol $\frac{a}{b}$ tells the number of congruent parts a unit has been divided into and the number above the line tells how many of these equal parts are shaded or taken. The number above the line is called a numerator. The number below the line is called a denominator.

4. Given a subdivision of a unit the student should be able to produce another subdivision of the unit which is equivalent to the first. Write fraction symbols for the two subdivisions and show that these fraction symbols must be equivalent.

5. Find a short way of producing fraction equivalent to a given fraction.

6. Given two fractional numbers, tell which one is larger and which one is smaller.

7. Find the sum of two fractional numbers with the same denominator.

8. Find the sum of two fractional numbers with different denominators.

9. Find the difference of two fractional numbers with same denominator.

10. Find the difference of two fractional numbers with different denominators.

Suggested Activities

I

Above are two units of linear measure A and area measure B. Just like in measurement when a unit is too long to measure an object (like metre in linear measure, or square metre in area measure) the unit is broken down. Also as in measurement the subdivision will be made equal. The unit can be broken into two, four, six, eight or any number of parts until the smallest of the parts is big enough to measure a given length or area or a whole number of parts.

In the above cases the units are broken into four equal parts and three of these parts are required to complete a measurement.

Emphasize the following:

(a) Just like in measurement the subdivisions must be equal.

(b) Only parts of the subdivision of the whole is needed.

(c) Also the parts themselves are wholes which can be further subdivided.

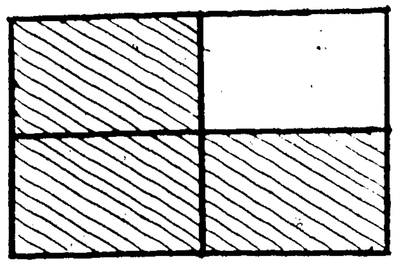
Summarise the activity as follows:

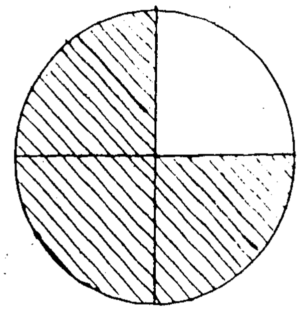
Into how many parts is the whole unit divided? ________4. How many of the parts were needed to complete measurement?__________3. A short hand way of writing this is $\frac{3}{4}$.

The number at the bottom of the line is called the denominator. It indicates into how many congruent parts the unit whole is divided. The number at the top of the line called the numerator tells how many of the equal parts were required to complete the measurement. The complete symbol is called a fraction.

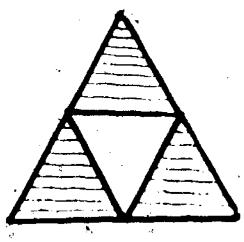
There are many more ways of showing a fraction in area measure as long as the whole is divided into

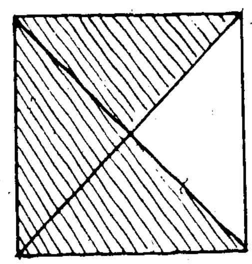
Congruent parts. Examples





$\frac{3}{4}$

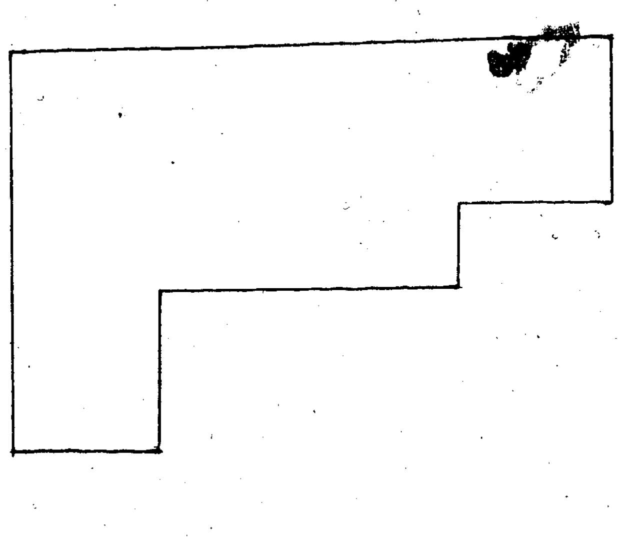




All of the above figures show the same fractional part, $\frac{3}{4}$.

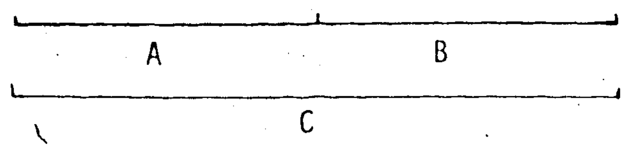
This activity should be done practically as in measuring length with sticks or area with paper strips. The teacher should prepare the material before hand. Then diagrams should be drawn on paper and the board illustrating the practical work.

Use an unmarked metre stick to measure the length of a stick $1\frac{3}{4}$ metres long. A strip of paper 10cm by 20cm to cover the area of the figure below:



II The following set of activities will be restricted to linear measure only. But it can also be demonstrated with area measure quite easily through paper cutting. Use sticks to demonstrate the following:

In measurement, given two sticks, A and B, we can find another stick C whose length is the combined length of A and B, thus



The process can be reversed. That is start with stick C find sticks A and B. The process can also be continued. Also the sticks A and B can be found to be of equal length (from the same equivalence class). Find other sticks A_1, A_2, B_1, B_2 of equal length such that their combined length is equal to C. This can be done by finding A_1 A_2 combined length equal to A. B_1 B_2 combined length equal to B. Similarly, A_{11}, A_{12}, A_{21}, A_{22}, B_{11}, B_{12} B_{21}, B_{22} are found with combined length equal to C.

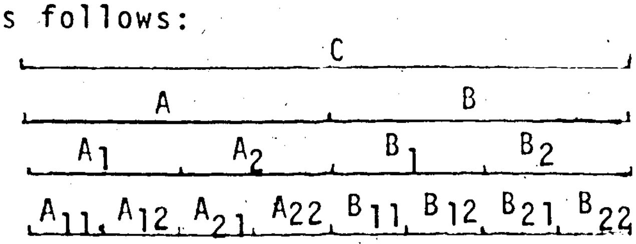
Combined length of A_{11} A_{12} equals A_1

A_{21} A_{22} equals A_2

B_{11} B_{12} equals B_1

B_{21} B_{22} equals B_2

Repeat the process until students realise that it can be continued indefinitely if one wishes. Then summarise as follows:



Now looking at stick C and line 1, we see that line 1 divides the stick into 2 equal parts and A is one of those parts. So the length of A is half of C or $\frac{1}{2}$.

Looking at line 2 and C we find that A_1 A_2 B_1 B_2 divide C into four equal parts. So the length of A_1 and A_2 is two quarters of that of C or $\frac{2}{4}$.

Similarly length of A_{11}, A_{12}, A_{21}, A_{22} is $\frac{4}{8}$ of C. Since the length of A is equivalent to the combined length of A_1 and A_2 which is in turn equivalent to the combined length of A_{11}, A_{12}, A_{21} A_{22} then we can write

$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$ and so on.

Emphasize the following:

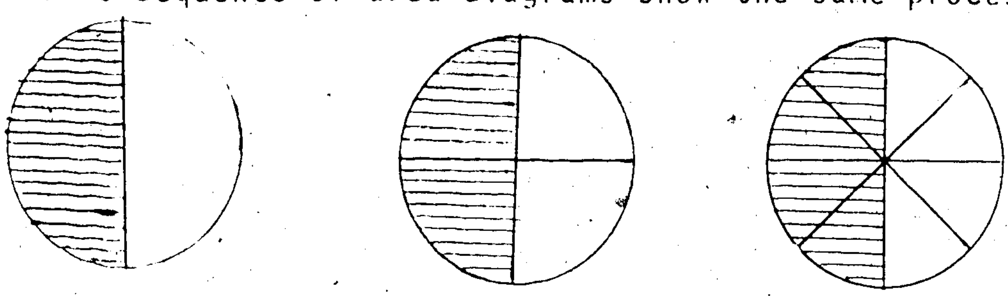
(a) The unit is broken into congruent parts

(b) Each part is a whole in itself which is broken down further

(c) Each fractional part is related to the one before it and the original whole

(d) The fractions are equal only because they represent the same length measure

This sequence of area diagrams show the same process



$\frac{1}{2}$ $\frac{2}{4}$ $\frac{4}{8}$

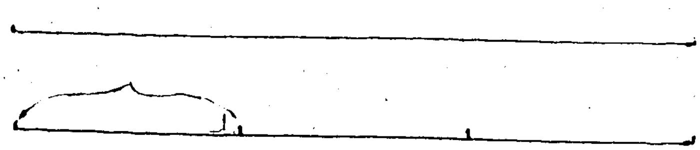
III Use a set of sticks to illustrate the following activities:

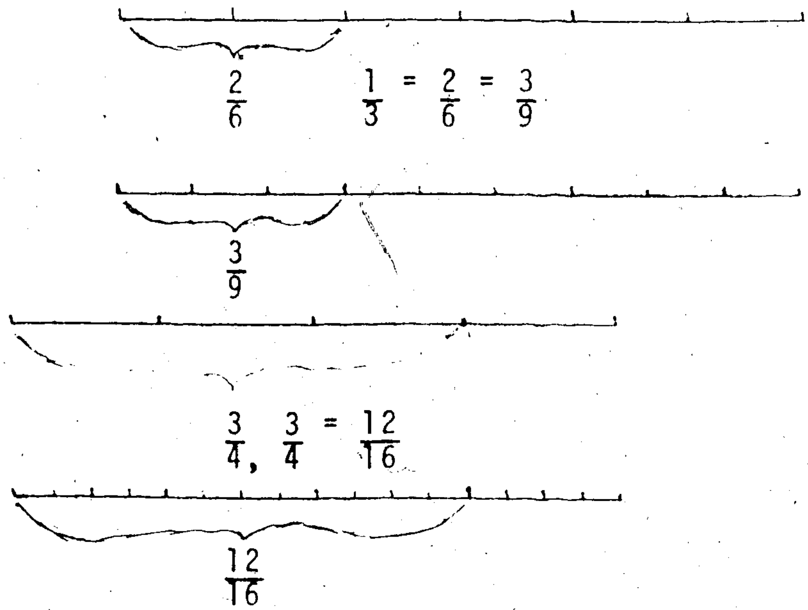
1. It has been shown that

$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$

With a further subdivision what will be the next fraction? $\frac{8}{16}$ etc.

2. $\frac{1}{3}$





Point out that in 1, numerator and denominator are multiplied by 2 and 4, in 2 by 2 and 3, and in 3, by 4. That is

$$\frac{1}{2} = \frac{2}{4} = \frac{1 \times 2}{2 \times 2}$$

$$\frac{1}{2} = \frac{4}{8} = \frac{1 \times 4}{2 \quad 4}$$

$$\frac{1}{3} = \frac{2}{6} = \frac{1 \times 2}{3 \times 2}$$

$$\frac{1}{3} = \frac{3}{9} = \frac{1 \times 3}{3 \times 3}$$

$$\frac{1}{4} = \frac{12}{16} = \frac{3 \times 4}{4 \times 4}$$

In all cases the numerator and denominator is multiplied by the same number or the number of parts the original fractional part is dubdivided.

Apply the rule to $\frac{2}{5}$

$$\frac{2}{5} = \frac{2 \times 3}{5 \times 3} = \frac{6}{15}$$

Illustrate using sticks.

IV In measurement in order for us to tell which stick is longer or shorter than which or which region is larger, smaller than which, we make physical comparison. Alternatively, we assign numbers to the measures and then compare the numbers. But it was also noted that the comparison is possible only if the units of measurement is the same. If the units are different then one had to be converted to the other or both to the same one. The same considerations occur in comparing fractions.

(1) Same denominator

Compare $\frac{2}{16}$ and $\frac{5}{16}$

certainly 5 sixteenth is more than 2 sixteenth.

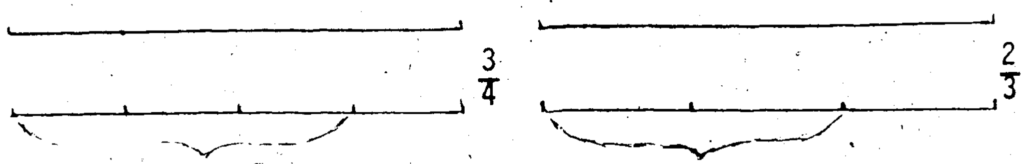
so $\frac{2}{16} < \frac{5}{16}$

In both cases the unit was divided into 16 equal parts.

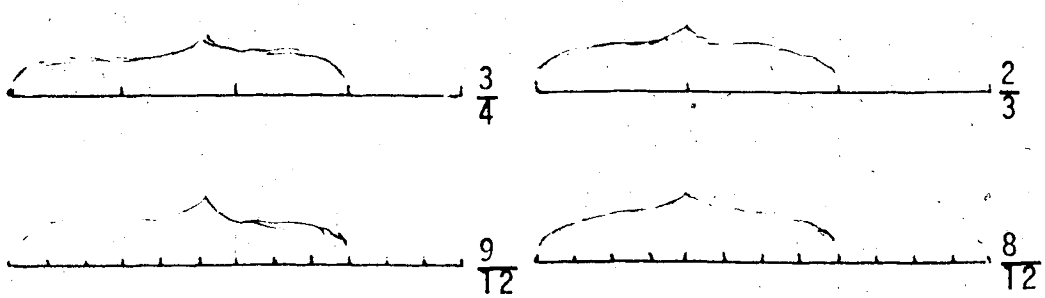
(2) Different denominators

Compare $\frac{3}{4}$, and $\frac{2}{3}$

We make a physical comparison to decide this case. But this will not be possible always. So it is better to find equivalent fractions for both $\frac{2}{3}$ and $\frac{3}{4}$ that have the same denominator.



The original fractional part in $\frac{3}{4}$ is one quarter. subdividing it by 3 into 3 equal parts gives a multiplier 3 for the fraction $\frac{3}{4}$. The original unit is then subdivided into 12 equal parts. Similarly for $\frac{2}{3}$, the original fractional part is one third. Subdividing into 4 equal parts breaks down the original unit into 12 equal parts.



Therefore $\frac{3}{4} = \frac{9}{12}$ and

$$\frac{2}{3} = \frac{8}{12}$$

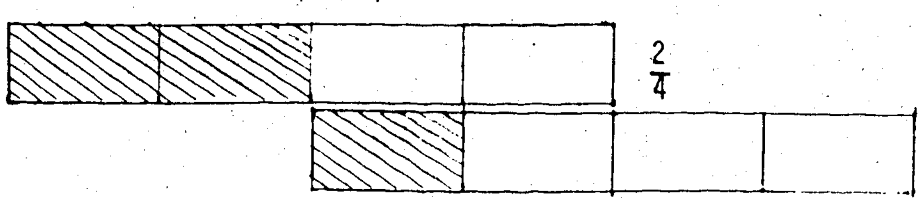
Since $\frac{8}{12} < \frac{9}{12}$ then $\frac{2}{3} < \frac{3}{4}$

In summary given any two fractions $\frac{a}{b}$ $\frac{c}{d}$ we can always subdivide the fractional part $\frac{1}{b}$ into d equal parts and $\frac{1}{d}$ into b equal parts. Then the fractions become $\frac{ad}{bd}$, $\frac{bc}{bd}$ with the same denominator.

V Find the sum of two fractional numbers.

Case 1 same denominator.

(a) Remember in adding length or area once the measurement is done in the same units we simply add the numbers and put the same unit. Adding fractions is very similar. Once the denominators are the same we simply add the numerators. This is because the denominator tells us that the original unit was divided into the same number of equal parts. Lets illustrate $\frac{2}{4} + \frac{1}{4}$



$\frac{2}{4}$

$\frac{1}{4}$

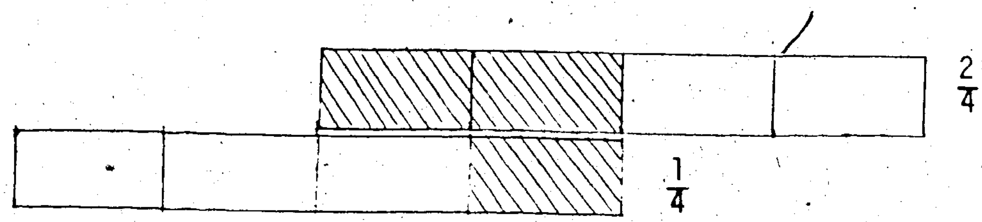
Therefore $\frac{2}{4} + \frac{1}{4} = \frac{2 + 1}{4} = \frac{3}{4}$

Another way of looking at it is this:

The original unit was broken down into quarters. One case has two quarters, the other has 1. So there are 3 quarters all together.

(b) To subtract two fractions we reverse the process of joining, and putting together. From two quarters take away one quarter. One quarter is left. That is

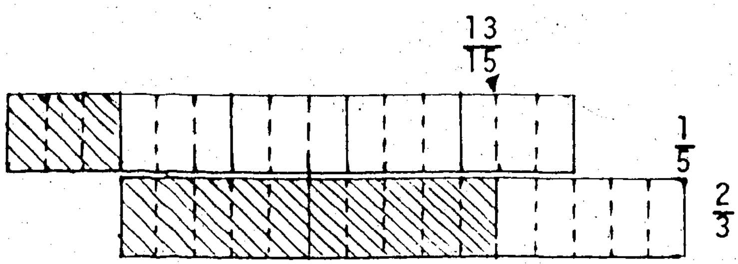
$$\frac{2}{4} - \frac{1}{4} = \frac{2 - 1}{4} = \frac{1}{4}$$



Case 2 different denominators

(a) Remember in adding two measures which are in different units one of the units was changed. Both units could have been changed to some other common unit. In adding fractions with different denominators we do a similar thing by finding equivalent fractions with the same denominator. To illustrate

$\frac{1}{5} + \frac{2}{3}$



To find equivalent fraction for $\frac{1}{5}$ break the original fifth into 3 equal parts.

then $\frac{1}{5}$ becomes $\frac{3}{15}$

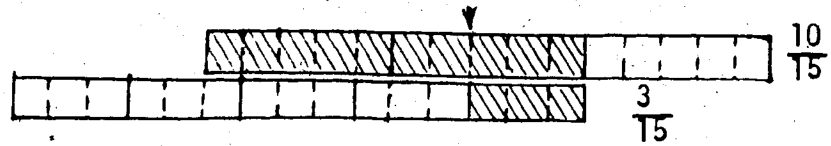
Break the third into 5 equal parts. Then $\frac{2}{3}$ become $\frac{10}{15}$

Now we can add $\frac{1}{5} + \frac{2}{3} = \frac{3}{15} + \frac{10}{15}$

$= \frac{3 + 10}{15} = \frac{13}{15}$

(b) To take away a fifth from two thirds, we do the reverse of addition.

$\frac{2}{3} - \frac{1}{5} = \frac{10}{15} - \frac{3}{15} = \frac{10 - 3}{15} = \frac{7}{15}$



INSTRUCTIONAL UNIT II

REVIEW UNIT

A LINEAR MEASURE

B AREA MEASURE

C FRACTIONS

TEACHERS:
1. B.J. MEDUGU
2. M.H. ALKALI
3. S.A. AHMED
4. I.K. OYEBAMIJI

LINEAR MEASUREMENT

LESSON I

Topic: Introduction to the Idea of Length.

Aim: To talk about length and introduce the idea of measurement to the students.

Materials: Wooden-rulers and paper-rulers.

Introduction: Choose suitable objects in the class and let each student come to the front to measure them with their rulers.

Presentation: Ask the students to mention those objects in the class which can be measured. The names of these objects shall include the desks, benches, blackboard, length and breadth of the room, their writing materials, etc.

Step II: With paper-rulers the students measure the edges of their desks.

Ask: 'How many paper-rulers did you combine to cover the length of the desks? - Yes, six paper rulers and a bit more.' 'Now each of these paper-rulers is divided into how many centimetres?' Yes, there are 10 centimetres in each.' 'How many centimetres do we find in a length of 6 paper-rulers? - Yes, there are (6 x 10) = 60 centimetres'. Measure the length of the desk again with the students. Explain that there are six paper-rulers and four centimetres left. Since each paper-ruler has 10 centimetres, we now have 60 centimetres plus the four centimetres remaining. The total length is therefore 64 centimetres.

Step III: Divide the class into groups. Let each group be involved in the measuring activities like finding the height of the table, length of blackboard, etc. They record each of these lengths.

Conclusion: The groups compare their recordings and correct any error they might have made. The teacher should go round to help them.

LESSON II

Topic: 'Changing' centimetres to Metres and centimetres.

Aim: To give practice in 'Changing' centimetres to metres and centimetres.

Materials: Wooden or paper-rulers.

Introduction: Draw a line 140cm. long on the blackboard. Get a student to measure it by using a metre-ruler.

Ask: 'How long isit? - Yes, it is one metre and 40 centimetres.' 'How many centimetres is that? - Yes, one hundred and forty centimetres.' Draw other lines on the blackboard, up to two metres. Let the students measure these lines and give answer in metres and centimetres and also in centimetres only.

Presentation: Discuss various lengths being measured by the students like say, the first line is 1 metre and 40cm. long. This is also 140cm. Therefore, 1 metre 40cm = 140cm. The second line is 1m. 90cm. long. This is also 190cm. long i.e. 1m. 90cm = 190cm.

Step II: Ask the students to change the following to centimetres:

(a) 1m. 65cm (b) 2m. 5cm.

(c) 3m. 42cm (d) 3m. 7cm.

(e) 10m. 5cm

Step III: Discuss the answers to the above 5 exercises together and let them correct their mistakes.

Step IV: Give the students more practice with measurement of lengths. Let them convert whatever they have in centimetres to metres. e.g. The length of the bench is 362cm. This is 3m. 65cm.

Conclusion: Appraisal of work: Give the students more practice with smaller objects like the length of their pens, the width of their exercise books etc.

LESSON III

Topic: Addition and subtraction of metres and centimetres.

Aim: To introduce the students to the addition and subtraction of metres and centimetres.

Materials: Strips of wood (or Cardboard, or string) with different lengths.

Introduction: Hold two strips (of different lengths) together. Ask each of the students to measure how long it is all together. For example, the first strip is 2m 50cm long while the second is 1m 17cm. By holding the two together, we have a total length of 3m 67cm.

Practise with other pairs of lengths like:

2m 40cm and 3m 35cm

8m 25cm and 5m 20cm

3m 65cm and 4m 28cm

4m 15cm and 4m 28cm

Find the difference in heights of the students. For example: Audu is 1m 35cm tall while Gani is 1m 48cm tall. Gani is taller than Audu by 13cm.

Presentation: Let the students practise in pairs adding two different lengths together and also [illegible]ing the difference between the pairs of [illegible]ths.

S[illegible] Give the following exercise to the students to [illegible]tise:

[illegible])	m	cm	(2)	m	cm	(3)	m	cm
	3	29		4	15		5	72
	+4	38		+2	17		+6	18
(4)	m	cm	(5)	m	cm			
	7	50		6	65			
	-5	45		-2	20			

Conclusion: Appraisal of work. Correction of common mistakes.

LESSON IV

Topic: Addition and subtraction of metres and centimetres with 'Changing'.

Aim: To introduce 'Changing' in the addition and subtraction of metres and centimetres.

Materials: Rulers, tape measure (or string, rope, etc. marked in metres and centimetres).

Introduction: Draw a line with red colour on the board with length 3m 64cm. Join this red line by another blue line with length 2m 86cm. Ask a student to measure each line and then measure the two lines together. We now have

m	cm
3	64
+2	86
6	50

Put this sum on the blackboard.

Presentation: The child measures both the red and blue lines together. He measures 650cm.

Ask: "What is another way of expressing this? - Yes, 6 metres 50 centimetres. Discuss the writing of the '50' in the 'cm' column of the answer. Then explain the 'carrying' of the "1" (metre) underneath the line in the "m" column. Since 64cm and 86cm make 150cm in the 'cm' column. This 150cm is 1 metre 50 centimetres. Ask the students to complete the addition as follows:

m	cm
3	64
2	86
6	50

Step II: Discuss with the students that, A carpenter brings a plank 7m 15cm long to his workshop. He then saws off 2m 45cm. Ask: "How can we find out how much is left? - Yes, by taking 2m 45cm from 7m 15cm. Ask a student to work this aloud. Make sure he deals with the centimetres and then with the metres.

Step III: Write on the blackboard:

m	cm
7	15
-2	45

Remind the students of the method 'Decomposition'. Go through the working with the students. The completed example should look like this:

If 'Decomposition' is used:

m	cm
6	100
	15
2	45
4	70

If 'Equal Addition' is used

m	cm
	100
7	15
3	45
4	70

Let the students go through the following on their own.

(1)

m	cm
3	73
+5	86

(2)

m	cm
6	75
3	85

(3)

m	cm
9	15
3	68

(4)

m	cm
7	35
4	72

Conclusion: Appraisal of work. Correction of any common mistakes.

LESSON V

Topic: Introducing Bigger Units.

Aim: To consider in detail the measure of lengths in metre and kilometres.

Materials: Rulers, tape-measure marked in metres and centimetres, metre-sticks.

Introduction: Show the students a metre-stick. Let them compare this with the centimetre. Discuss with the students that when measuring longer distances like the length and breadth of the football field, length of a block of the four classrooms, etc. we consider longer units like metres and decimetres.

Presentation: Give each child a unit measure of decimetre and metre. Let them find out how many decimetres in one metre. Remind the students of the following abbreviations: m = metre, dm = decimetre, cm = centimetre, mm = millimetre. Ask the students to convert the following to metres, decimetres and centimetres:

(1) 85cm (2) 125mm (3) 15dm

Step II: Ask students to describe how to measure the distance between Zaria City and Bassawa village. Explain that since it will take a very long time measuring such a long distance, we will use larger units of measurements such as Decametres, Hectometre and Kilometres. Divide the students into groups. Mark a kilometre distance from the classroom and let the students use their metre-stick to measure the distance. Ask: "How many metre-sticks do we find in a kilometre distance? - Yes, 1000 metre sticks. So there are 1000 metre-sticks."

Step III: Ask the students to do the following exercise:

(1) Change the following to metres:

(a) 3 kilometres

(b) 500 centimetres

(c) 60 decimetres

(d) 7km 6dm

(2) (a)

	m	cm
	15	92
+	2	15

(b)

	m	cm	mm
	3	44	3
-	1	29	60

(c)

	m	dm	cm
	3	8	5
-	1	5	6

(d)

km	m
7	18
3	88

Conclusion: Appraisal of work. Correction of any common mistake.

AREA MEASUREMENT

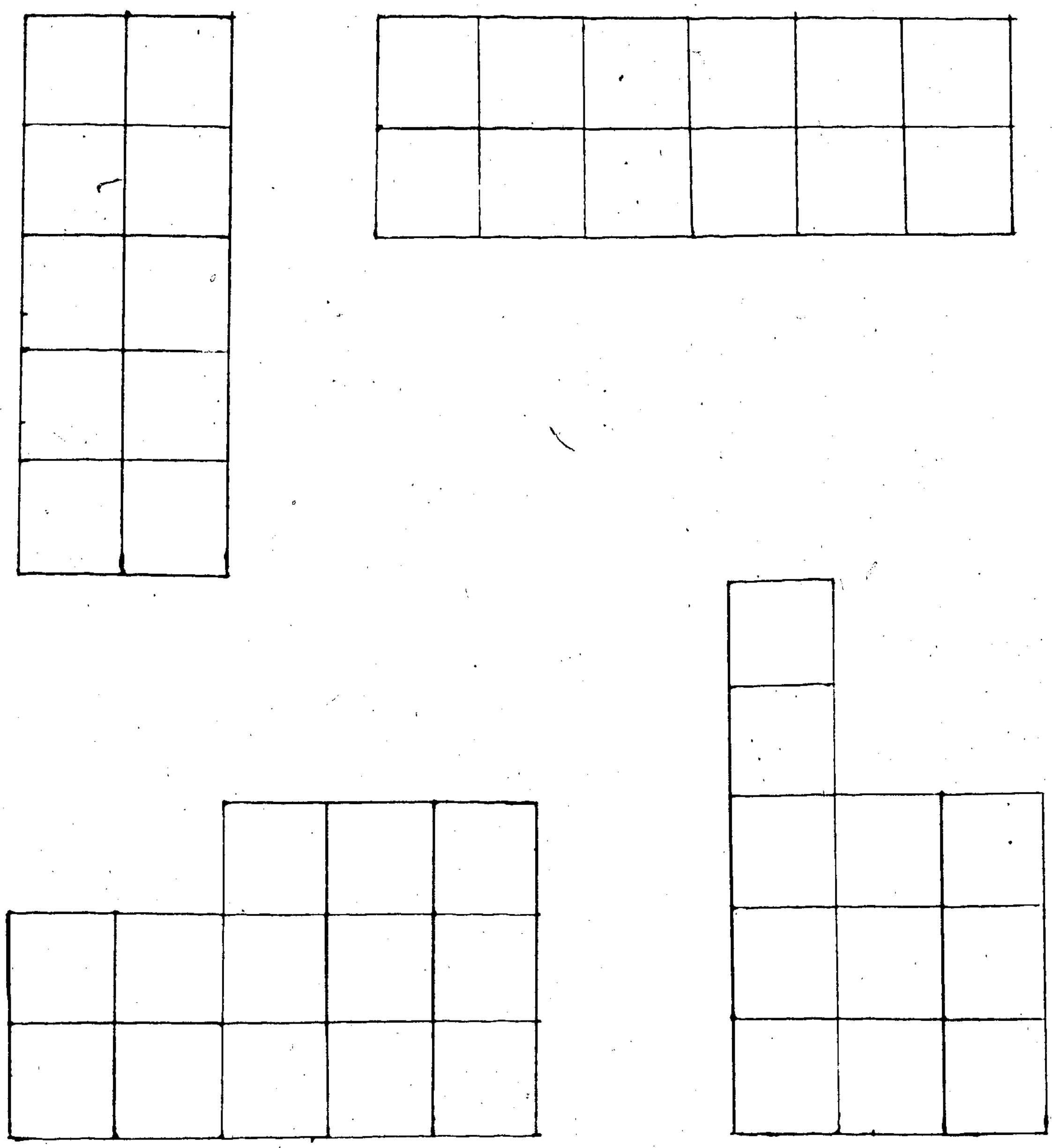
LESSON I

Topic: Area

Aim: To introduce the idea of area

Previous knowledge: The students already know about smallest, largest, wall sheet of stamps and shape. They also know about row.

Introduction: Four different sheets of stamps are drawn on the board as below.



Then the students are asked to look at the pictures, and tell what they see. One of the students answers by saying some small sheets of stamps. Again another student is asked to point out at the largest sheet, and through counting of the stamps in each, this is found.

Step I: Now students in the Post Office, the man at the counter uses large sheets of stamps from where he could cut smaller sheets of stamps of various sizes as required by the customer. By drawing a large sheet of stamps on the board say 245 stamps, the students are shown how 12 stamps could be taken using various ways e.g.

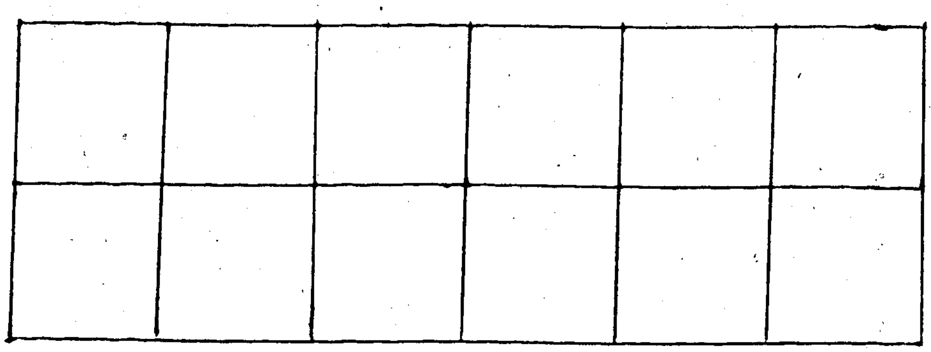
(a) a row of twelve stamps

(b) two rows of 6 stamps each

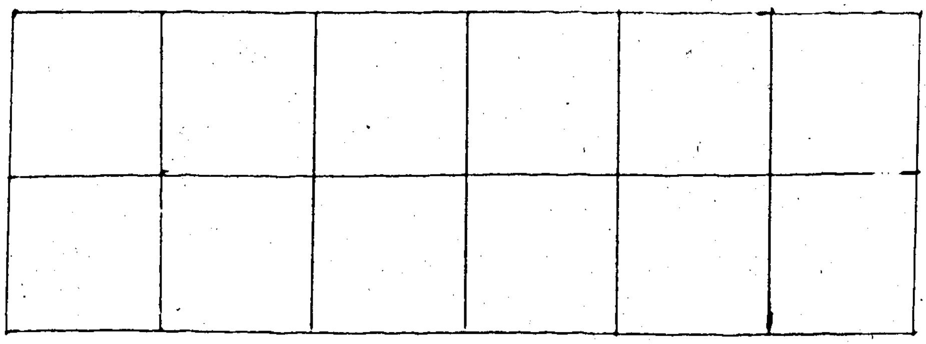
(c) four rows of 3 stamps each

Step II: Using suitable drawings on the black board, the students are asked to show the ways in which these twelve stamps could be counted.

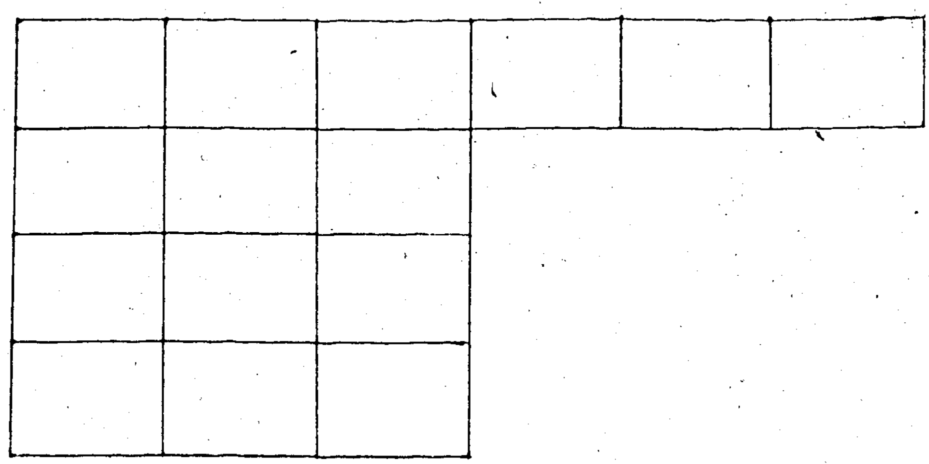
(a) By counting them one by one



(b) By counting them through two rows of six stamps each



Step III: Through another suitable drawing on the board, one of the students is asked to tell the class how we can count the stamps from the drawing on the board.



Here there is one row of six stamps and two rows of three stamps.

Summary: A large sheet of stamps of say 36 stamps is drawn on the blackboard and the teacher shows various ways of taking twelve stamps from the figure. Some of the ways may be:

(1) Using a single row

(2) Using two rows of six stamps each

(3) Three rows of four stamps each.

Assignment: Students are asked to draw a large sheet of say 20 stamps in their exercise books of 4 rows containing 5 stamps each. And show four ways of getting 12 stamps from the drawing.

LESSON II

Topic: Area

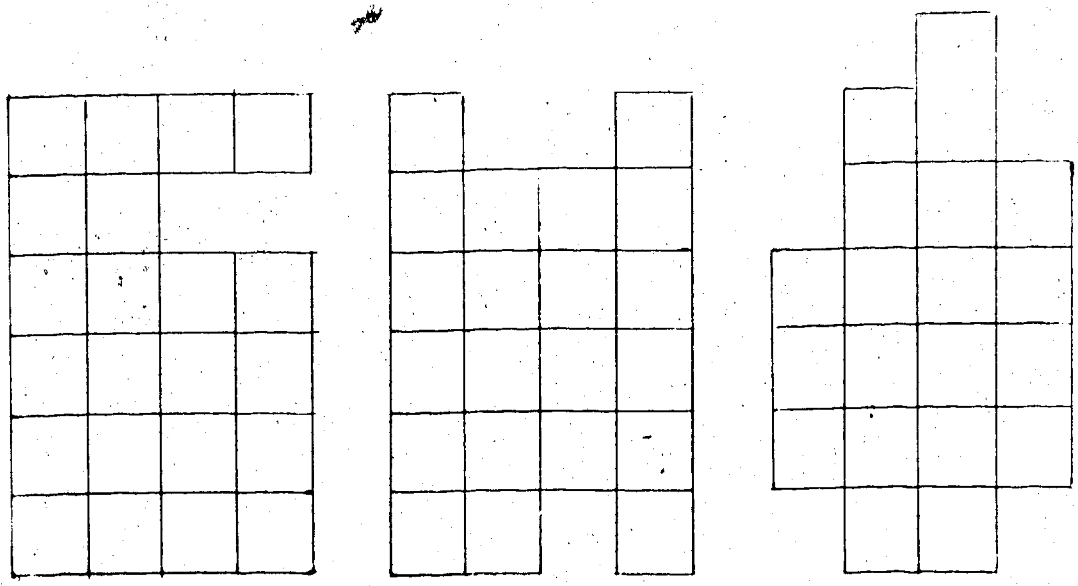
Aim: To lead the students to understand how to count squares of half squares.

Previous knowledge: The students know how to draw a large sheet of stamps and from it know various ways of getting say 12 stamps from the sheet. And also know how to count the number of stamps in a sheet i.e. they can count unit shapes and they know what is a unit square.

Introduction: The teacher draws three different sheets of stamps on the board, and sheet I contains 10 stamps, sheet II contains 15 stamps, sheet III contains 7 stamps. One of the students is asked to show the largest sheet. And by counting he finds out the correct one. Then the teacher says from the stamps we shall go to squares, and half squares.

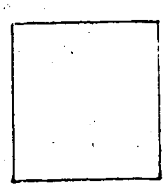
Presentation:

Step I: The teacher draws three shapes on the board, the first consisting of 21 squares, the second 22 squares and the third 20 squares as below:

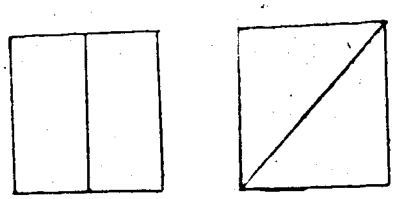


he then counts on the board the number of each squares in each shape. The teacher then shows them why shape (2) is the largest and shape (3) the smallest.

<u>Step II</u>: The teacher makes several large paper or card-board squares (with sides about 6cm), and the teacher discusses with the students how they can be cut into two equal parts as below:



a whole square

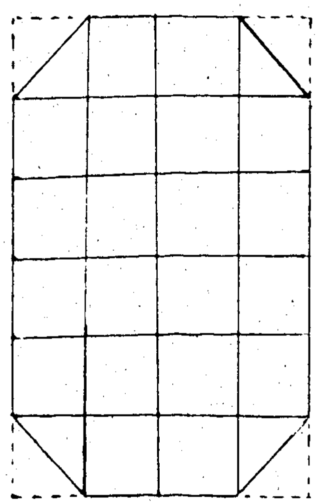


half squares

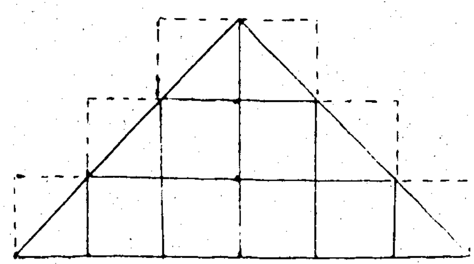
two half squares
make a whole square.

Here children participate in the cutting. And the teacher also places two half squares on each other, for the children to see that they are equal. The teacher ensures that two half squares make one whole squares etc.

Step III: The teacher then makes the following two drawings on the board:



(1)



(2)

In both cases the students are asked to count:

(1) The squares

(2) The half squares

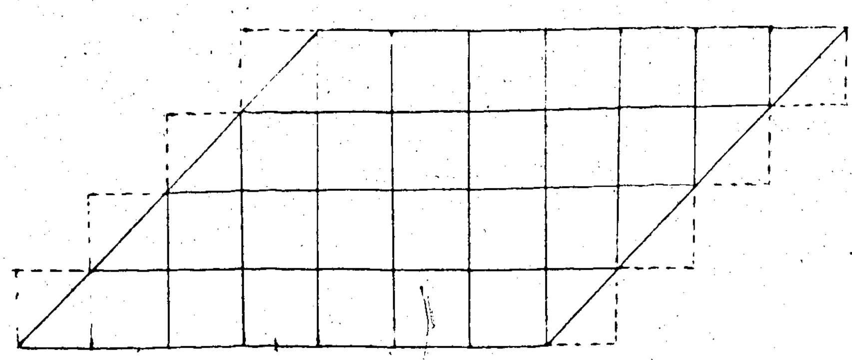
(3) How many squares are needed to make the shape in both cases.

The teacher ensures that they understand the questions. In shape (1) there are 20 squares and 4 half squares and since 4 half squares make 2 squares in the shape we have a total of 22 squares.

In shape (2) there are 6 squares, 6 half squares, all together giving 9 squares.

In addition the teacher asks them which is the largest shape: of course, shape (1)

<u>Summary</u>: The teacher draws the following diagram on the board for each student to draw in his exercise book and answer the questions following the diagram.



(1) Count the squares (24)

(2) Count the half squares (8)

(3) How many squares are needed to make the shape (28).

LESSON III

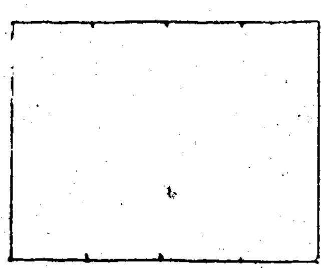
Topic: Area

Aim: To introduce to the students the use of centimetre square and covering shapes with centimetre squares.

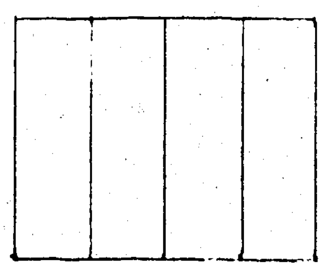
Previous Knowledge: The students already know how to count the number of squares and half squares in a shape, and then find the total number of squares there are in shape, and how to compare various shapes.

Introduction: The teacher tells the students that, in the previous lesson, they have learnt how to count the number of squares, half squares and the total number of squares in a shape then the teacher discusses with the students the need for a standard measure (mention the use of metre for length, grams for weight and litres for volum). And then tell them that they will soon see the use of centimetre square.

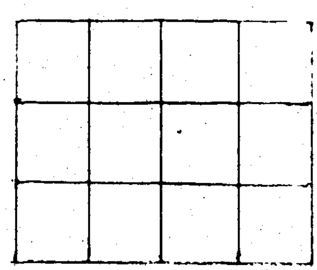
Presentation: Get the students to make some centimetre square from a rectangular sheet of paper each (coloured if possible) as below:



(1)



(2)



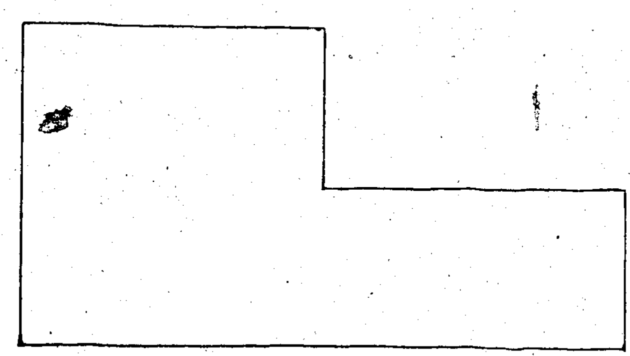
(3)

Start at figure (1) and mark as many centimetre as possible along AB, starts and D and mark centimetres along DC. In figure (2) join the marks on AB to the corresponding marks on DC. In figure (3) mark centimetres along AD and BC and again join the corresponding marks. Here a set of centimetre squares is formed ready to be cut. The teacher should make sure that the squares are properly drawn on the paper before they are cut out.

<u>Step II</u>: The teacher asks each student to draw a rectangle of length 3cm and breadth 2cm in his exercise book. Then each student is asked to cover the shape (rectangle) with centimetre squares and find how many of such squares there are. Answer is 6 square centimetre. This should be done carefully and neatly.

<u>Summary</u>: The teacher reviews step II above.

Assignment: The students are asked to draw the following figure in their exercise books:



Then they are asked to use their cut centimetre [illegible] to find the number of centimetre square in [illegible]. Answer is 24 cm. sq.

LESSON IV

[illegible]ic: [illegible]ea

A[illegible]: [illegible] lead the students to know covering of shapes [illegible]h centimetre squares and half-squares.

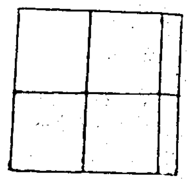
Pre[illegible]s Knowledge: The students know how to cut centi-[illegible]etre squares, and cover a shape with such squares.

Intr[illegible]ction: The teacher asks each student to draw a [illegible]ctangle 5cm long and 2cm wide, then cover it with t[illegible]eir cut centimetre squares and find the number of them. Answer is 10 sq. cm. The teacher then me[illegible]ions that the students will see how certain s[illegible] can only be covered with both centimetre and squares and half squares.

Presentation:

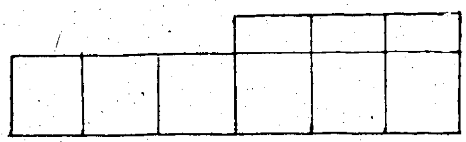
Step I: The teacher draws a shape of 2cm long and $1\frac{1}{2}$cm wide on a sheet of paper pinned on the wall. Then the teacher cuts one of his centimetre squares into two equal parts. He shows the students how to cover the shape by using 2cm squares and 2 half squares. Then he tells the students that the whole shape can be covered by 3cm squares.

Step II: The students are asked to draw the following figure in their exercise books and then cover the shape with their centimetre squares and half-squares.



4½cm squares

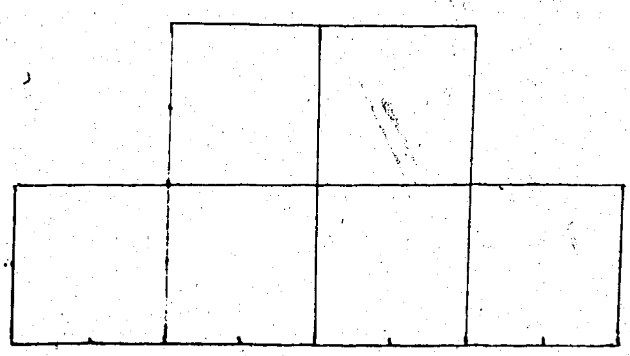
Step III: The students are asked to draw the following figure in their exercise books and then cover the shape with their centimetre squares and half squares.



answer 7½cm. squares.

Summary: The teacher reviews step II

Assignment: The students are asked to draw the following shape in their exercise books.



The students are then asked to find the number of centimetre squares covering the shape by using centimetre squares and half squares.

LESSON V

Topic: Area

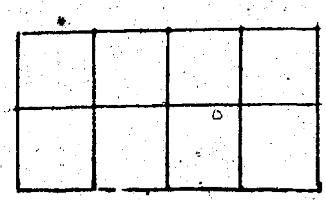
Aim: Introducing the students to the formala for finding the area of a rectangle.

Previous Knowledge: The students know how to cover shapes with centimetre squares and half squares.

Introduction: Students, in our previous lesson, we saw how to cover a shape with centimetre squares and half squares. This leads us to what is called area. The teacher tells them, that "the area of a shape is measured by the total number of squares which cover the shape". This definition is repeated with the students. The teacher explains the use of the word area, because we can have larger units like metre-squares, kilometres squares. Concequently the need to get the areas of our room, hous, town, country, or top of a table. Note the chart form, centimetre square or square centimetre is explained.

Presentation:

Step I: The teacher asks each student to draw a rectangle of length 4cm and with 3cm of length 4cm and 2cm.



No formula is introduced at this stage.

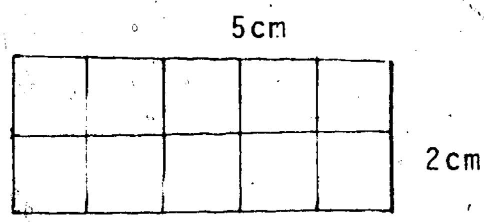
Then they are asked the following questions:

(1) How many squares are there in each row - 4

(2) How many rows are there - 2

(3) How many squares are ther altogether - 2 x 4 = 8. The area of the rectangle is 8 sq. cm.

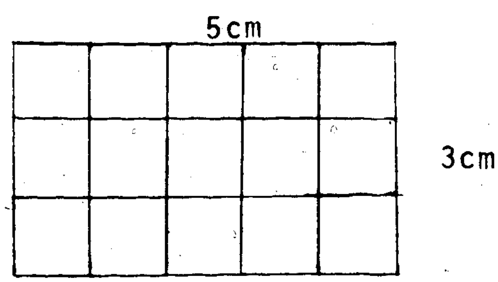
Step II: The teacher asks each student to draw a rectangle of length 5cm and width 2cm.



The previous questions are repeated.

Answer, area = 2 x 5 = 10sq. cm.

Step III: The teacher repeats the above questions and directives with the diagram below:



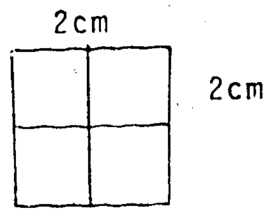
Area = 3 x 5 = 15 sq. cm

All the above examples are in turn done by the teacher and every step to ensure the students know what they are doing and how it is found.

Summary: The teacher reviews.

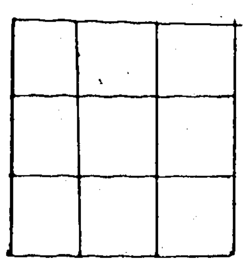
Assignment: The students are asked to draw the following figures in their exercise books and find their areas:

(1)



Area = 2 x 2 = 4 sq. cm.

(2)



Area = 3 x 3 = sq. cm.

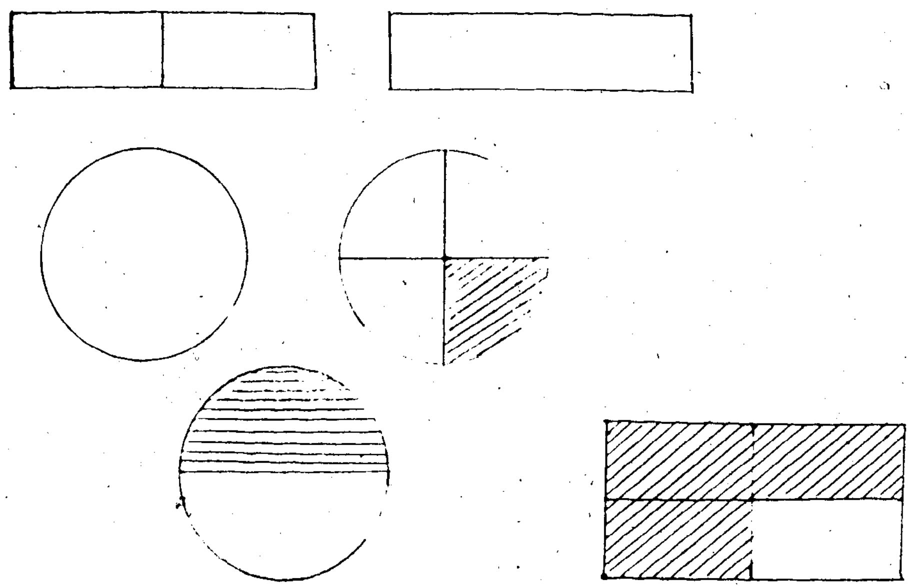
FRACTIONS

LESSON I

Topic: Fractions

Objective: Students should be able to identify pictorial presentation of fractions.

Diagrams

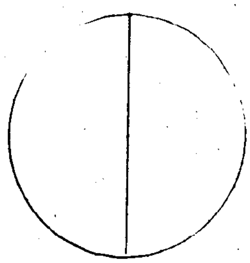


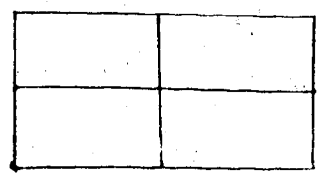
Method: We present geometrical shapes like circles, rectangles and squares fractions by first drawing the shapes to represent one whole, then dividing the shape into two and four equal parts, shade one of such divisions in each figure to represent $\frac{1}{2}$ and $\frac{3}{4}$ respectively. Shape drawn should be symmetrical for easy division into equal parts.

Conclusion: We should ask the students to come to the board and shade the area representing $\frac{3}{4}$ on a circle and $\frac{1}{2}$ on a square.

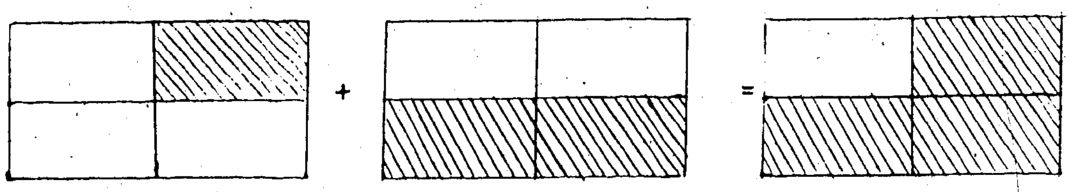
LESSON II

Objective: Students should be able to add simple fraction with the same denominator.



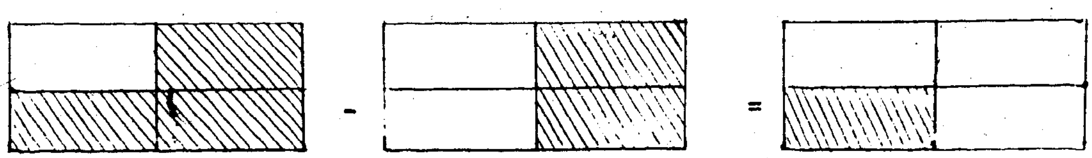


Method: Recall that in our last lesson we presented fractions pictorially, today we will add fractions of the same denominator pictorially. Example, we want to add $\frac{1}{4} + \frac{2}{4} =$



$\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$

The process is similar for subtraction



$\frac{3}{4}$ $\frac{2}{4}$ $\frac{1}{4}$

Conclusion: We should give fractions whose sum is a fraction (proper) and whose difference is also a proper fraction.

LESSON III

Objective: Students should be able to calculate what a given fraction of a certain quantity is say $\frac{1}{4}$ of 20, $\frac{3}{4}$ of 20 and so on.

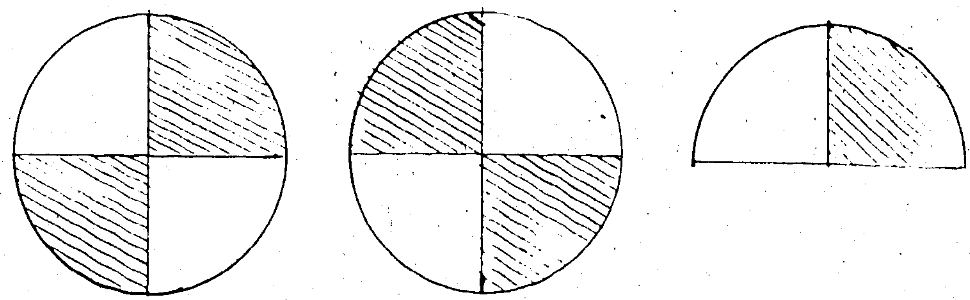
Activities: We shall cut a given length of a stick into twenty equal parts. Each of these twenty pieces represent a certain fraction of the original stick. We divide the pieces into two equal parts and count each group. Each group should contain ten pieces, implying that $\frac{1}{2}$ of 20 = 10. Similarly we can divide the whole bunch of pieces into four equal parts and count each group which should be five, $\frac{1}{4}$ of 20 = 5, and then ask the student to calculate $\frac{3}{4}$ of 20 using the method taught.

Conclusion: There is nothing specific about the number twenty except that it is convenient to get it divided into half and quarter, and any number that is convenient can be used.

LESSON IV

Objective: Students should be able to tell how many fixed fractional parts are there in a given number. e.g. how many quarters are there in $2\frac{1}{2}$.

Method: We shall take two whole and one half and break it into quarters, and count how many quarters we shall get.

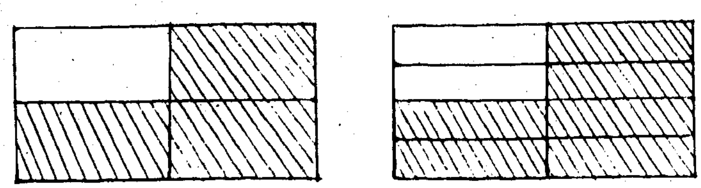


Shaded quarters and five unshaded quarters, in all we have ten quarters in $2\frac{1}{2}$. Further we shall consider another number and another fraction, but we should be sure that any number and fraction considered gives a whole fraction. That is we should not have $\frac{1}{6}$ while the fraction into which we shall divide the number is $\frac{1}{3}$.

LESSON V

Objective: Students should be able to break a given fraction into smaller fraction.

Method: Just as we have broken a given number into fraction in the last lesson, we shall today break a given fraction into smaller fractions.

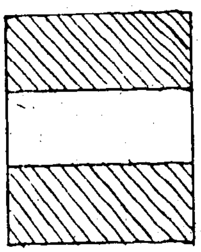


Suppose we want to break $\frac{3}{4}$ in eight, we take that unit, $\frac{3}{4}$, break it into quarters, take another unit and break it into eight, then we count the number eight we have in the exact space of three of the quarters. See diagram above, the shaded areas are equal. In one we have the area $\frac{3}{4}$ in another six of of the eights, therefore, $\frac{3}{4} = \frac{6}{8}$. We shall repeat this process using other fractions.

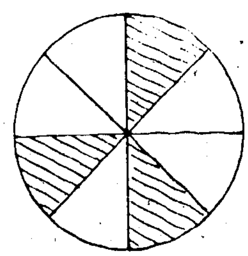
LESSON VI

Objective: Students should be able to say what fraction of a group or picture has certain properties, given the properties of that fraction.

Method: We draw a simple diagram on the board and divide it into several equal parts, shade some of these parts, then ask the students what fraction of the diagram is shaded, just like the diagram below:



$\frac{2}{3}$ is shaded



$\frac{3}{8}$ is shaded

It should be emphasized that the sum of the fraction representing the shaded and the unshaded area is equal to one whole.

Conclusion: Ask the students to draw a diagram and shade $\frac{1}{6}$ of the diagram.

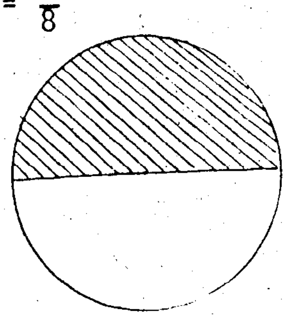
LESSON VII

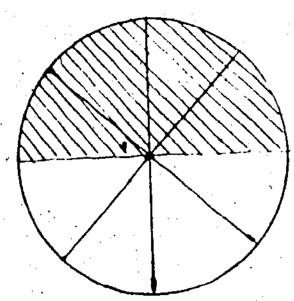
Objective: Addition of fraction with different denominators.

Method: Remember we have broken fractions into smaller fractions before, say $\frac{1}{2} = \frac{2}{4}$. We shall use this knowledge to add fractions of different sizes. We shall break the fractions to be added together into a smaller size common to both fractions so that we shall be able to add them together.

Consider adding $\frac{1}{2} + \frac{3}{8}$, half is of bigger size than eighths, therefore, we shall break half into eighths so that we can add them to the other eighth:

$\frac{1}{2} = \frac{4}{8}$





$\frac{1}{2} + \frac{3}{8} = \frac{4}{8} + \frac{3}{8} = \frac{7}{8}$

Sometimes it is necessary to break both fractions into smaller units in order to obtain a common size. e.g. $\frac{1}{2} + \frac{1}{3}$ we cannot break half in thirds, so we break both $\frac{1}{2}$ and $\frac{1}{3}$ into sixth.

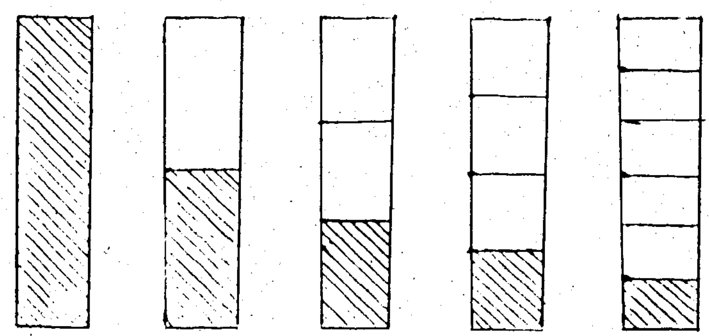
$\frac{1}{2} = \frac{3}{6}$ $\qquad$ $\frac{1}{3} = \frac{2}{6}$

$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

We should have many variations in the examples we consider.

LESSON VIII

Objective: Looking at a given diagram students should be able to say how many of a given fraction does a whole number contain. Which is greater $\frac{1}{2}$ or $\frac{1}{3}$.



Method: We should remember that the sum of the fraction represented by a shaded area and unshaded is equal to 1. So how many $\frac{1}{3}$s are there in 1 unit? How many $\frac{1}{6}$ths are there in one whole? Students are expected to answer these questions and explain how they arrive at this conclusion with the help of the teacher. We should point out that each of the columns represent one unit and are equal. We take the column of the given fraction and count all the divisions in it and the number of fractions in 1 unit. Similarly, ask them to tell which of two given fractions is greater. From the given diagram, the fraction whose shaded area is bigger is the greater of the two fractions. That is the shaded area representing the fraction $\frac{1}{2}$ is bigger than the shaded area representing $\frac{1}{3}$. Therefore $\frac{1}{2}$ is greater than $\frac{1}{3}$. We should repeat the process for other fraction.

LESSON IX

Objective: Students should be able to change a given fraction into smaller fractions without necessarily going through the process of pictoral subdivision.

Method: In our last lesson we have changed a given fraction into smaller fraction. Example:

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$$

Has anyone seen any sequence in the above change? Yes, for the top parts (numerator) we multiply by 2 in order to obtain the next numerator in the sequence. How about the denominator? Yes, the same

$$\frac{1}{2} = \frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$$

$$\frac{2}{4} = \frac{2}{4} \times \frac{2}{2} = \frac{4}{8}$$

$$\frac{4}{8} = \frac{4}{4} \times \frac{2}{2} = \frac{8}{16}$$

Therefore if we multiply the top and bottom of a given fraction by the same number, the fraction is changed into smaller fractions. Change $\frac{1}{3}$ into twelve, we multiply the denominator by another number in order to obtain a new denominator which is 12, by what number do we multiply 3 to get 12? by 4. Since we multiply the bottom by 4, we must also multiply the top by 4.

$\frac{1}{3} = \frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$

LESSON X

Objective: Students should be able to add or subtract fraction.

Method: We shall today add fraction of different size by changing the size of one of them, also subtract smaller fraction from a given bigger fraction. Suppose we want to add $\frac{2}{3} + \frac{1}{6}$, we have to change $\frac{2}{3}$ into sixth so that we shall add them together just like we have been adding fraction of the same denominator. Thus

$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$ so $\frac{2}{3} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6} = \frac{5}{6}$

Similarly $\frac{2}{3} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6} = \frac{3}{6} = \frac{3 \times 1}{3 \times 2}$

$= \frac{1}{2} \times \frac{3}{3} = \frac{1}{2}$

Since we know that $\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$ $\frac{3}{6} = \frac{1}{2}$

so $\frac{2}{3} - \frac{1}{6} = \frac{1}{2}$

Suppose we want to add $\frac{1}{3} + \frac{1}{4}$ since we cannot easily [illegible]rd into quarters. Let us see what we can break $\frac{1}{3}$ and $\frac{1}{4}$ into so that we can add them, can any one suggest something? $\frac{1}{3} + \frac{1}{4}$.

We shall see that third and fourth can only have twelve as their largest common size. That is the largest size into which we can break thirds and quarters.

$$\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$$

$$\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$$

$$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

Note: the L.C.M. of 3 and 4 is twelve, and is the denominator for the largest size of fraction common to both thirds and quarters.

Conclusion: Every one teaching these units should bear in mind that he should include as much variations in the examples as time permits and wherever possible the addition and fraction should be taught together just as we have done in the last lesson.

INSTRUCTIONAL UNIT III

A LINEAR MEASURE

B AREA MEASURE

TEACHERS: M.I. GURE

B. CALEB

E.E. EMAH

G.F. JAM

This unit consisted of
the linear and Area measure
sections of Unit I

APPENDIX B

TESTS

Concept

I Given a fraction the student should be able to name the numerator and the denominator.

II Given a linear representation of a fraction the student should be able to identify the fraction.

III Given an area representation of a fraction the student should be able to identify the fraction.

IV Given an unequally divided and shaded figure, the student should recognise that it does not form a fraction.

V Given a fraction the student should be able to sketch a simple area representation of it.

Equivalence

I Given a fraction, the student should be able to identify its equivalent fraction.

II Given a fraction, the student should be able to find another equivalent fraction by:

(a) multiplication

(b) by division

III The student should be able to order two fractions.

Operations

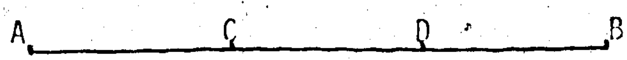
The student should be able to add and subtract two fractions.

FRACTIONS ACHIEVEMENT

TEST I

PRETEST AND RETENTION TEST

1. In the fraction $\frac{2}{3}$, 2 is known as:
   a. denominator
   b. L.C.M.
   c. numeral
   d. numerator
2. In the fraction $\frac{3}{5}$, 5 is known as the
   a. denominator
   b. numerator
   c. L.C.M.
   d. numeral



3. What fraction of the segment AB is the segment AC?
   a. $\frac{3}{6}$
   b. $\frac{7}{9}$
   c. $\frac{2}{7}$
   d. $\frac{1}{3}$
4. What fraction of the segment AB is the segment CD?
   a. $\frac{7}{9}$
   b. $\frac{4}{9}$
   c. $\frac{2}{7}$
   d. $\frac{3}{5}$

5. What fraction of a metre is one centimetre 1 cm? ______

6. Express a decimetre as a fraction of a metre ________

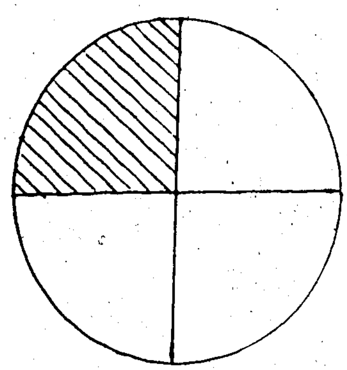
7. What fraction of the whole figure is the shaded region?

a. $\frac{1}{3}$

b. $\frac{3}{4}$

c. $\frac{1}{4}$

d. $\frac{4}{3}$



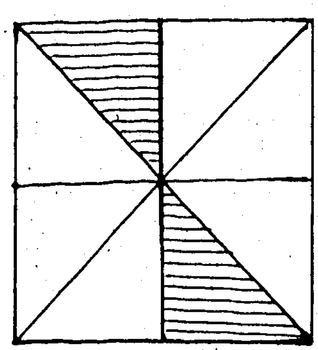
8. What fraction of the whole figure is shaded?

a. $\frac{2}{6}$

b. $\frac{6}{8}$

c. $\frac{2}{8}$

d. $\frac{3}{8}$



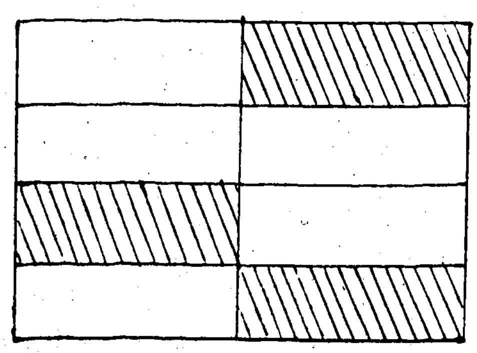
9. What fraction of the whole figure is the shaded region?

a. $\frac{3}{8}$

b. $\frac{3}{5}$

c. $\frac{4}{8}$

d. $\frac{3}{6}$



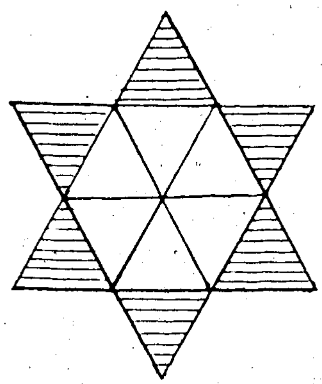
10. What fraction of the figure is shaded?

a. $\frac{6}{12}$

b. $\frac{6}{18}$

c. $\frac{6}{6}$

d. $\frac{3}{9}$



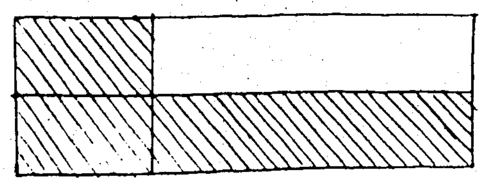
11. What fractions do these figures represent?

a. $\frac{3}{4}$

b. $\frac{1}{4}$

c. $\frac{1}{3}$

d. None of the above



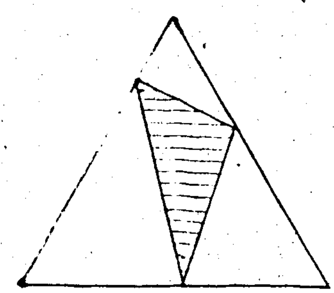
12.

a. $\frac{1}{4}$

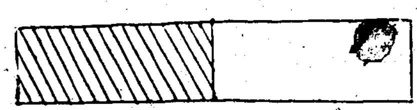
b. $\frac{1}{3}$

c. $\frac{3}{4}$

d. None of the above



13. Here is $\frac{1}{2}$



Now sketch the following fractions

$\frac{2}{4}$

14. $\frac{3}{8}$

15. The fraction $\frac{3}{4}$ is equivalent to

a. $\frac{9}{12}$

b. $\frac{6}{8}$

c. $\frac{9}{12}$

d. $\frac{4}{20}$

16. The fraction $\frac{3}{8}$ is equivalent to

a. $\frac{6}{8}$

b. $\frac{3}{16}$

c. $\frac{6}{16}$

d. $\frac{3}{4}$

17. Complete the following

$\frac{1}{3} = \frac{\ }{9} = \frac{6}{\ }$

18. Complete the following

$\frac{18}{24} = \frac{9}{\ } = \frac{\ }{4}$

The fraction $\frac{1}{2}$ is less than $\frac{2}{3}$ or $\frac{1}{2}$ $\quad$ $\frac{2}{3}$ and $\frac{3}{4}$ is greater than $\frac{1}{4}$ or $\frac{3}{4}$ $\quad$ $\frac{1}{4}$. Order the following fractions.

19. $\frac{3}{4}$ $\quad$ $\frac{2}{3}$

20. $\frac{5}{8}$ $\quad$ $\frac{1}{8}$

21 $\frac{2}{5}$ $\quad$ $\frac{4}{5}$

22. $\frac{1}{8}$ $\quad$ $\frac{1}{7}$

Add

23. $\frac{1}{4} + \frac{3}{4}$

a. $\frac{4}{8}$

b. $\frac{3}{16}$

c. $\frac{3}{4}$

d. $\frac{4}{4}$

24. $\frac{2}{5} + \frac{3}{5}$

a. $\frac{5}{10}$

b. $\frac{5}{5}$

c. $\frac{2}{10}$

d. $\frac{3}{10}$

25. $\frac{1}{2} + \frac{1}{4}$

a. $\frac{2}{6}$

b. $\frac{1}{6}$

c. $\frac{3}{4}$

d. $\frac{1}{3}$

Subtract

26. $\frac{13}{15} - \frac{5}{15}$

a. $\frac{8}{0}$

b. $\frac{7}{15}$

c. $\frac{18}{15}$

d. $\frac{8}{15}$

27. $\frac{3}{4} - \frac{5}{10}$

a. $\frac{8}{14}$

b. $\frac{2}{20}$

c. $\frac{5}{20}$

d. $\frac{8}{20}$

28. $\frac{1}{2} - \frac{1}{5}$

a. $\frac{3}{5}$

b. $\frac{3}{2}$

c. $\frac{3}{10}$

d. $\frac{2}{10}$

FRACTION ACHIEVEMENT

TEST II

POST TEST

1. In the fraction $\frac{5}{6}$, 5 is known as

   a. denominator

   b. L.C.M.

   c. numerator

   d. numeral

2. In the fraction $\frac{4}{9}$, 9 is known as

   a. numerator

   b. denominator

   c. L.C.M.

   d. numeral

3. What fraction of the segment AB is the segment AD?

   a. $\frac{5}{9}$

   b. $\frac{5}{4}$

   c. $\frac{2}{7}$

   d. $\frac{4}{5}$

4. What fraction of the segment AB is the segment CE?

   a. $\frac{5}{9}$

   b. $\frac{5}{4}$

   c. $\frac{2}{7}$

   d. $\frac{4}{5}$

5. What fraction of a metre is 5cm? __________

6. Express 5 decimetres as fraction of a metre________

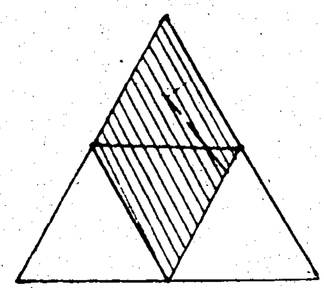
7. What fraction of the whole figure is the shaded region?

a. $\frac{1}{3}$

b. $\frac{2}{4}$

c. $\frac{2}{3}$

d. $\frac{1}{4}$



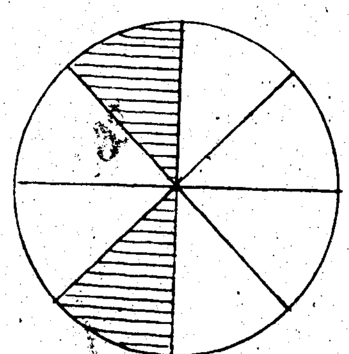
8. What fraction of the whole figure is shaded?

a. $\frac{2}{6}$

b. $\frac{2}{8}$

c. $\frac{2}{4}$

d. $\frac{6}{8}$



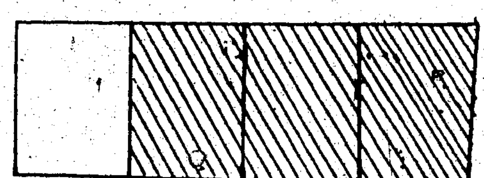
9. What fraction of the whole figure is the shaded region?

a. $\frac{3}{4}$

b. $\frac{1}{3}$

c. $\frac{1}{4}$

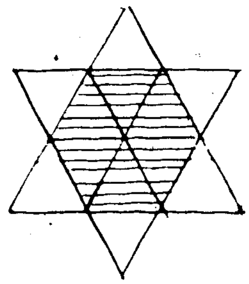
d. $\frac{3}{1}$



10. What fraction of the figure is shaded?

a. $\frac{6}{18}$

b. $\frac{6}{12}$

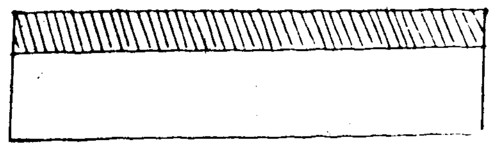


c. $\frac{3}{9}$

d. $\frac{6}{6}$

11. What fractions do these figures represent?

a. $\frac{1}{2}$

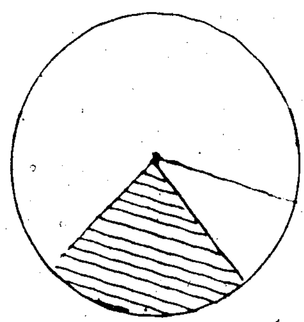


b. $\frac{1}{3}$

c. $\frac{1}{4}$

d. None of the above

12. a. $\frac{2}{6}$

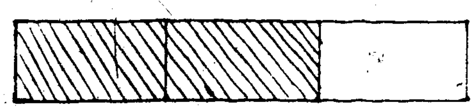


b. $\frac{4}{6}$

c. $\frac{2}{4}$

d. None of the above

13. Here is $\frac{2}{3}$



Now sketch the following fractions

$\frac{3}{4}$

14. $\frac{3}{6}$

15. The fraction $\frac{5}{8}$ is equivalent to

a. $\frac{25}{48}$

b. $\frac{10}{24}$

c. $\frac{15}{30}$

d. $\frac{25}{30}$

16. The fraction of $\frac{5}{6}$ is equivalent to

a. $\frac{10}{18}$

b. $\frac{25}{30}$

c. $\frac{15}{24}$

d. $\frac{30}{42}$

17. Complete the following

$\frac{1}{4} = \frac{\quad}{12} = \frac{6}{\quad}$

18. Complete the following

$\frac{36}{48} = \frac{18}{\quad} = \frac{\quad}{16}$

19. The fraction $\frac{1}{2}$ is less than $\frac{2}{3}$ $\frac{1}{2}$ $\frac{1}{3}$ and $\frac{3}{4}$ is greater than $\frac{1}{4}$ or $\frac{3}{4}$ $\frac{1}{4}$. Order the following fractions:

20. $\frac{1}{4}$ $\frac{1}{5}$

21. $\frac{1}{6}$ $\frac{1}{3}$

22. $\frac{3}{4}$ $\frac{2}{4}$

23. $\frac{3}{5}$ $\frac{3}{4}$

Add

24. $\frac{3}{8} + \frac{5}{8}$

a. $\frac{8}{8}$

b. $\frac{8}{16}$

c. $\frac{3}{32}$

d. $\frac{3}{8}$

25. $\frac{2}{5} + \frac{3}{5}$

a. $\frac{5}{10}$

b. $\frac{5}{5}$

c. $\frac{3}{10}$

d. $\frac{2}{10}$

26. $\frac{1}{4} + \frac{3}{8}$

a. $\frac{4}{12}$

b. $\frac{3}{12}$

c. $\frac{5}{8}$

d. $\frac{3}{32}$

Subtract

26. $\frac{9}{16} - \frac{3}{16}$

a. $\frac{9}{16}$

b. $\frac{6}{0}$

c. $\frac{6}{16}$

d. $\frac{7}{16}$

27. $\frac{3}{8} - \frac{1}{5}$

a. $\frac{2}{3}$

b. $\frac{3}{40}$

c. $\frac{4}{40}$

d. $\frac{7}{40}$

28. $\frac{3}{4} - \frac{1}{5}$

a. $\frac{11}{20}$

b. $\frac{2}{20}$

c. $\frac{4}{9}$

d. $\frac{2}{1}$