UNIVERSITY OF ALBERTA

INITIAL VELOCITY OF BUOYANT SPILLS

BY

DOUGLAS G. M. TOVELL

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF

MASTER OF SCIENCE

IN

WATER RESOURCES ENGINEERING

DEPARTMENT OF CIVIL ENGINEERING

EDMONTON, ALBERTA

FALL, 1991

ISBN 0-315-70204-4

UNIVERSITY OF ALBERTA

RELEASE FORM

NAME OF AUTHOR: Douglas G. M. Tovell

TITLE OF THESIS: Initial Velocity of Buoyant Spills

DEGREE: Master of Science

YEAR THIS DEGREE GRANTED: 1991

Douglas G. M. Tovell

11303 103 Ave., #104
Edmonton, Alberta
T5K 0S3

Date: August, 1991

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled "Initial Velocity of Buoyant Spills" submitted by Douglas G. M. Tovell in partial fulfilment of the requirements for the degree of Master of Science in Water Resources Engineering.

N. Rajaratnam
Supervisor

P. Steffler

K. C. Cheng

Date: 1 August 1991

ABSTRACT

As the public and governments become more concerned about the environment, it is becoming ever more important that the engineering community be able to accurately predict the consequences of a spill. Upon review of the problem, it was felt that one of the most fundamental characteristics of a spill that an engineer would be interested in finding is the initial velocity of the spill slick front. To do this, modifications were made to Benjamin's (1968) model of a surge front and O'Brien and Cherno's (1934) spill slumping model, which were then combined to produce a spill initiation model capable of predicting both the slick velocity and slick thickness.

To test the accuracy of this model, comparisons were made with data obtained from the University of Alberta, O'Brien and Cherno (1934), Middleton (1966) and Keulegan (1958). Based upon this analysis, it was found that the model has an average absolute error of approximately nine percent. In an attempt to further reduce this error, the effects of induced flows in the ambient ahead of the slick front were also considered. The results from the analysis suggest that the effects of these induced flows can be safely neglected under most conditions.

ACKNOWLEDGEMENTS

I would like to take this opportunity to extend a special thank you to my supervisor, Dr. Rajaratnam. If it was not for his support, trust and guidance I am not sure that I would have undertaken this work.

The number of other people to whom I am indebted are too numerous to mention. Thus, I would like to extend a broad thank you to all the students in the lab and the remaining staff making up the hydraulics department. I could not have done it without your support.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

Symbol	Description
a	- acceleration (ie. $F = ma$) - thickness of slick
b	- a generic model scale (O'Brien and Cherno) - an integration limit - virtual thickness of a slick
b_A	- virtual thickness of ambient intrusion
b_S	- virtual thickness of spill slick
c_1	- opposing ambient flow velocity
c_2	- velocity under spill
d	- depth of ambient
Δd	- surge height in front of head
g	- acceleration due to gravity
g'	- modified gravitational constant
h	- thickness of slick (Abbott) - depth under slick (Benjamin)
h_o	- thickness of ambient (Abbott)
h_f	- thickness of front of slick
Δh	- height of the surface of the spill above the ambient surface elevation
m	- mass (ie. $F = ma$)
m_H	- virtual mass associated with the passage of the head
r	- density ratio (ie. ρ_o/ρ) - radial coordinate
s	- salinity (ie. $G_1 - G_2$)
t	- time - elapsed time of spill event
t_o	- time at the start of experiment

Δt	- time step
u	- average fluid velocity
u_o	- instantaneous ambient velocity
u_f	- velocity of front of slick
A, B, C	- coefficients in quadratic equation (ie. $y = Ax^2+Bx+C$)
F_{net}	- net unbalanced force
Fr	- densimetric Froude number
G_1	- specific gravity of denser layer
G_2	- specific gravity of lighter layer
H	- fluid depth - depth of ambient (Barr) - slick thickness (Benjamin)
H_o	- original spill depth
J	- similarity criterion (Barr)
K	- similarity criterion (O'Brien and Cherno) - coefficient (Abbott)
K'	- modified similarity criterion
KE	- total kinetic energy
L	- length of lock (O'Brien and Cherno) - diameter or width of slick (Fay)
M	- total accelerated mass
N	- non-dimensional coefficient
P_i	- pressure at point 'i'
Q	- volume flow rate of spilled material
U	- average fluid velocity
V	- volume - velocity at which mass 'M' moves at
V_o	- initial velocity of saltwater front
V_Δ	- surge velocity (Keulegan)

V_r - radial velocity component of 'V'

V_θ - angular velocity component of 'V'

V_A - velocity of ambient intrusion front

V_H - velocity of spill head

V_S - velocity of slick head

X - distance of head from source
- rectangular coordinate along channel

X_A - penetration distance of ambient

X_S - penetration distance of slick

Y - rectangular coordinate perpendicular to bed

α - slick thickness as a fraction of the original spill depth

β - ambient intrusion thickness as a fraction of the original spill depth

γ - unit weight (ie. $\gamma = \rho g$)

θ - angular coordinate

λ - ratio of densities (Abbott)

μ - doublet source strength

ν - kinematic viscosity

ρ - density of heavier layer

ρ_o - density of spill

$\Delta\rho$ - difference in layer densities ($\rho-\rho_o$)

ρ_1 - density of lighter fluid (Keulegan)

ρ_2 - density of heavier fluid (Keulegan)

ρ_m - average of layer densities (Keulegan)

ρ_w - density of water

σ - net surface tension

Φ - velocity potential lines

Ψ	-	streamlines
ω	-	unit weight (O'Brien and Cherno)
r_H, θ_H	-	cylindrical coordinates of head doublet
r_V, θ_V	-	cylindrical coordinates of virtual doublet
X_H, Y_H	-	rectangular coordinates of head doublet
X_V, Y_V	-	rectangular coordinates of virtual doublet

CHAPTER 1

INTRODUCTION

In the process of developing scientific theories, perhaps the most difficult task that a researcher must first accomplish is to determine the fundamental assumptions upon which that theory or model will be based. As the model is developed, many of these constraints will become obvious since they are the result of explicit simplifications necessary to arrive at a workable solution. However, there are also implicit assumptions that exist due to an unintentionally biased view of reality. One such assumption that is commonly made in the field of engineering fluid mechanics, is the assumption of single layer fluid systems. In actual practice, though, truly single layer systems rarely exist.

On the surface, such a statement may appear unwarranted; so let me illustrate with an example. Let us consider the classical hydraulic jump. As most hydraulic engineers realize, the sequent depths for such a situation can be easily determined through the application of continuity and momentum equations to the control volume containing the hydraulic jump. The question to be asked, though, is upon what basis can we neglect the air overlying the jump? In most cases, a formal acknowledgment of this assumption is never made and to answer the question, a stratified flow analysis must be performed in which both the air and water layers are considered. It can then be shown that for large density differences between the layers and with the surface layer at rest, the equations do reduce to those of a classical hydraulic jump.

There are, indeed, many other examples of multi-layer or stratified flow systems. Before we look at other examples, however, it may be useful to first establish a working definition of what a stratified system is. For the purpose of this work, it will be assumed that a fluid may be considered stratified if it can be divided into distinct layers on the basis of differences in the parameters describing the flows. The significance of this definition is that it does not restrict our view of stratified flows to just those that possess density stratification.

To illustrate the significance of the above definition, let us consider an example. In the analysis of jets and plumes, the common approach is to start with the Reynolds equations and to simplify them to the point where they can be solved for the parameters of interest. An unfortunate side effect of this form of analysis, is that it tends to produce a narrower view of such flow systems and extracts them from any continuum to which they might belong. If one views such flows as being examples of momentum stratified flows, however, it becomes clear that any general analysis for jets and plumes should also be applicable to any flow expansion including hydraulic jumps.

In addition to momentum dominated flows, there are many other areas of practical significance where the concept of flow stratification is important. For example, River and

Coastal engineers must commonly deal with the problem of sediment transport, an example of flow stratification produced by non-uniform sediment distributions. Meteorologists must commonly face the problem of density stratification in the atmosphere since this is responsible for frontal systems and temperature inversions. Environmental and Water Resources engineers also must deal with the problems of density stratification since this strongly affects the ability of fluids to mix together. Examples where this occurs are in the discharge of cooling water from generating stations, industrial discharges, and spills.

Of the above examples, it is in the analysis of spills that this work will concentrate and it will be assumed that any release of a finite quantity of one fluid into another will constitute a spill. Using this definition, it becomes clear that the analysis of spills may be applied to a broad range of flow systems including chemical spills (both liquid and gaseous), turbidity currents, and dam break problems which include lock exchange problems.

From a review of the literature, it appears that there have been few comprehensive attempts to understand the mechanics of spills and as a result, there are few tools available to engineers for the modelling of spills. Of the tools available, most seem to be rather complicated and in a developmental stage making them unsuitable as practical engineering tools. Thus, it is the purpose of this work to provide a basis upon which a complete spill model, suitable for practical use, could be built. Specifically, we will be concerned with the mechanics of flow initiation, and will attempt to provide a model framework which will meet the following objectives:

- an attempt will be made to keep the physics of the model sufficiently simple that most practicing engineers will be able to easily understand the concepts and equations.

- the development of the model will be modular in design to simplify the updating or extension of the model.

As was stated previously, one of the most important and often most difficult activities of a researcher is to present all the assumptions upon which their model is based. Therefore, before proceeding further, the following are the global assumptions upon which this model will be based.

Stratified Flow: It is assumed that the spilled fluid produces a stratification within the ambient fluid and that the ambient is not stratified in any other way.

No Mixing: It is assumed that at no point along the interface between the two liquids will any flow entrainment or mixing between the fluids take place.

No Surcharge: It is assumed that both fluids are in hydrostatic equilibrium at the base of the spill.

Wide Rectangular Channel: It is assumed that the spill occurs in a rectangular channel of sufficient width that any possible wall effects may be neglected.

Occupies Full Channel Width: It is assumed that the spill occupies the full channel width and is essentially one-dimensional in nature.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

After reading the general introduction to this work, many individuals may be asking themselves the question

> "If this is a work dealing with the initiation of flow in a spill, why worry about stratified flows?"

The simple answer to this question is, of course, that spills are an example of stratified flows. If this is the only reason that comes to the readers mind, however, then careful consideration should be given to this review and at least some time invested in reviewing the more important articles mentioned. This is necessary because to appreciate the mechanics involved in spills, be it the slumping of the initial spill volume or the development of the head, it is essential to have a general understanding of multi-layered or stratified flows.

2.2 General

The area of research which attempts to understand stratified flows is very broad. To achieve a comprehensive literature review, it would be necessary to scour the literature in the fields of Classical Physics, Mathematics, Meteorology, Geology, Mechanical Engineering, and Civil Engineering as a minimum. Although it is beyond the scope of this work to provide such a review, it is possible to develop an acceptable 'feel' for this body of knowledge by referring to a few of the classic works in this area.

In any general discussion of stratified flows, there are at least four works that the reader must review if a broad understanding of the field is sought. These are the works by Harleman (1960), Turner (1979), Yih (1980), and Bo Pedersen (1986). Although it is certainly beyond the scope of this work to provide a review as extensive as that provided by these authors, following the format of Harleman's (1960) work, an attempt will be made to provide a brief overview of some of the more fundamental areas in the subject.

Harleman's (1960) work begins with a discussion of a simple two-layer stratified flow to show that in some situations, stratified flow equations are very similar to open-channel flow equations; the only difference being that the densimetric Froude number 'Fr', defined as

$$Fr = \frac{U}{\sqrt{g' H}} \tag{2.1}$$

where

$$g' = \frac{\Delta\rho}{\rho} g \tag{2.2}$$

must be used instead of the ordinary Froude number. In these equations, 'U' is the average velocity in the layer, 'ρ' is density of the denser layer, 'Δρ' is the difference in layer densities, 'g' is the acceleration due to gravity and 'H' is the thickness of the layer. Rajaratnam and Powley's (1990) work expands on this notion to extend previous work performed by Rajaratnam on hydraulic jumps to hydraulic jumps in a stratified environment with the surface layer at rest.

Pursuing the work of Harleman (1960), an introduction to uniform stratified flow is presented. To illustrate, he uses as an example a gravity current flowing over the bed beneath a still ambient. Neglecting the possibility of bed entrainment and taking into account the effects of both bed and interfacial shear stress, he develops a generalized form of the Chezy equation used in open-channel flow. In developing this expression, an analogy is made to flow between parallel plates in order to simulate the interfacial shear stress. For a more complete discussion of the modelling of interfacial shear stress, the reader is referred to the work of Bo Pedersen (1980).

Of special interest to Coastal Engineers is the gravity current that transports sediment. These currents are commonly referred to as turbidity currents and may be made unsteady due to the suspension of bed sediment into the current or the deposition of sediment being carried by the current; a phenomenon studied by Middleton (1966, 1967), Bo Pedersen (1980) and Pallesen (1983). In both Bo Pedersen's and Pallesen's work, several field case studies are reviewed in an attempt to further understand the mechanics of turbidity currents.

Another broad class of stratified flows that Harleman (1960) addresses in his work, are the nonuniform flows. A classic work in this area is that done by Schijf and Schönfeld (1953). In this paper, a closed system of differential equations is developed to describe a two-layer stratified flow system that is nonuniform in nature. By investigating the characteristics of the system of equations, it was found that four kinds of waves were possible. One set of waves are identical to those predicted for an ordinary one-layer system (referred to as 'external' waves), and the other set is unique to a two-layer system (referred to as 'internal' waves). To illustrate how these equations might be applied to engineering problems, the authors use as examples: exchange flows, internal hydraulic jumps, and salt wedges in rivers without tides.

An alternate method of analysis that may be used to solve problems involving nonuniform stratified flow was presented by Yih and Guha (1955). Following Hayakawa's (1970) procedure for simplifying the integral equations developed by Yih and Guha when applied to an internal hydraulic jump, Rajaratnam, Tovell, and Loewen (1991) were able to solve for the sequent depths of the stratified system even when both layers are in the motion and the density difference between the layers is not small. When one of the layers is at rest, the sequent depth equations reduce to a form very similar to Belanger's equation with the exception that the densimetric Froude number must be used instead of the ordinary Froude number. In the case of very small density differences between the layers, the equation becomes identical to Belanger's equation.

If the reader is interested in the behavior of salt wedges and related phenomena, in addition to the very brief treatment given by Schijf and Schönfeld (1953), an excellent introductory reference is that by Keulegan (1966). In this work, Keulegan summarizes his findings from a series of experimental studies designed to understand the nature of salt wedges and lock exchange problems. Another interesting work related to this same area of study is that done by O'Brien and Cherno (1934). In this work, a very simple spill slumping model is used to estimate the initiation velocity of the spill and some information is presented to indicate the reduction of velocity at the spill head with distance.

Returning once again to the work by Harleman (1960), a good introduction is presented to another major area of work in stratified flow; the area of internal wave motion. Internal waves are those waves created at the interface between two stratified liquids. In Harleman's article, specific attention is paid solitary internal waves as well as the relationship between internal waves and surface waves, illustrating the significance of the relationship with practical examples. For the reader not familiar with waves in general, Keulegan (1949) has written an excellent article describing the fundamental characteristics of both free-surface and internal waves.

In situations where the waves produced at the interface are severe, wave breaking may occur which leads to interfacial mixing. In the article by Harleman (1960), the definitions for critical flow are presented beyond which mixing is likely to occur. In general, if viscosity is neglected, the interface stability is described by a modified densimetric Froude number, but when viscosity is not neglected, a form of Reynolds number also becomes important. If the reader is specifically interested in the mechanics of interfacial mixing, reference should be made to the landmark paper by Ellison and Turner (1959) and an excellent summary of pertinent literature is provided in the work by Turner (1979) and Bo Pedersen (1980).

Related to the topic of interfacial mixing is the topic of diffusion in stratified flow. Following the summary presented by Harleman (1960), it appears that the difference between the two is in the nature of turbulence associated with the phenomena. In the case of interfacial mixing, the turbulence of interest is the macro scale turbulence created by the interfacial shear stresses which produce breaking waves along the interface. If this process continues, a two-layer system may eventually be converted into a three-layer system with the third layer being produced by the wave mixing at the interface.

For diffusion, however, the turbulence of interest is the small scale turbulence that may exist in the body of the flow. This turbulence acts upon the interface in a much more localized fashion, creating small cusps along the interface through which streamers of the other layer fluid enters and is mixed with the body of the turbulent layer. In the works by

Harleman (1966) and Pallesen (1983), the reader will find a much more detailed discussion of this process as well as a discussion of experiments conducted to simulate it.

When attempting to numerically analyze diffusion in stratified flows, the convection-diffusion equation is used. This equation is not applied to the entire multilayer system, but is rather applied to the layer into which fluid is diffusing. In nature such a system most commonly occurs in estuaries where, due to the turbulence in the fresh water flow, the concentration of salt is homogeneous at any section. This allows for the one-dimensional convection-diffusion equation to be used in the analysis. Whether an unsteady or steady analysis is performed will depend entirely on the length of the mixing zone compared to the intrusion distance of saltwater resulting from tidal effects. Once again, the reader is referred to Harleman's (1966) work for a more complete discussion.

A final topic that seems to have received a great deal of attention in the general stratified flow literature is that of selective withdrawal of fluid from stratified systems. In the article presented by Harleman (1960), there is a brief discussion of the literature considering specific points of withdrawal within a stratified fluid, such as when attempting to simulate the selective withdrawal in reservoirs. If the reader is interested in a more comprehensive review of the subject, reference should be made to the work of Yih (1980).

2.3 Models

By now one should have at least a general appreciation of the depth which exists in the area of stratified flows. It is important to realize that in the development of models to simulate stratified flow events, many of the preceding general areas of study may need to be considered. For example, in the development of a complete spill model, an unsteady analysis would need to be performed since the slick becomes thinner as it spreads, interfacial mixing may be significant if the spill is spreading rapidly, and knowledge of fluid extraction in stratified flows will be necessary if the spill is to be cleaned up.

Over the years, many different models have been created, of which, only a few will be reviewed here. With the models selected for review, it is hoped that the reader will gain an appreciation for the different techniques that have been used to simulate spills. In reviewing these techniques, it was found that basically three different modelling philosophies exist: physical models, order of magnitude models and analytical models.

2.3.1 Physical Models

Some of the earliest models created to understand spill and lock exchange phenomena were implemented as physical models. O'Brien and Cherno (1934) created such a model in an attempt to understand the nature of the exchange flow created when locks are operated in coastal areas. Following the definition sketch in Figure 2.1, we may develop the following expression for the initial velocity 'V_o' of the saltwater front

$$V_o = \pm\sqrt{\frac{gds}{2\,(G_1+G_2)}} \qquad (2.3)$$

if we adopt a simplified slumping model in which it is assumed that the inflection point on the interface (point O) always occurs at mid-depth. It should also be noted that in equation 2.3, 's' is the salinity, defined as

$$s = G_1-G_2 \qquad (2.4)$$

and 'G_1' and 'G_2' are the specific gravities of the heavier and lighter fluids respectively.

In order to compare the model to the prototype, the authors first developed a simplified model law based upon a similarity condition 'K'. In order to develop an expression for 'K', the authors considered the ratios between the model and the prototype of the following quantities: horizontal dimensions, vertical dimensions, velocities, time, and salinity. By using equation 2.3 as well as the one-dimensional Navier-Stokes equation, it was possible to show that the scales should be related by the following expression

$$\frac{b_v}{b_t} = \frac{b_d b_s}{b_L} = \frac{b_v}{b_d^2} \qquad (2.5)$$

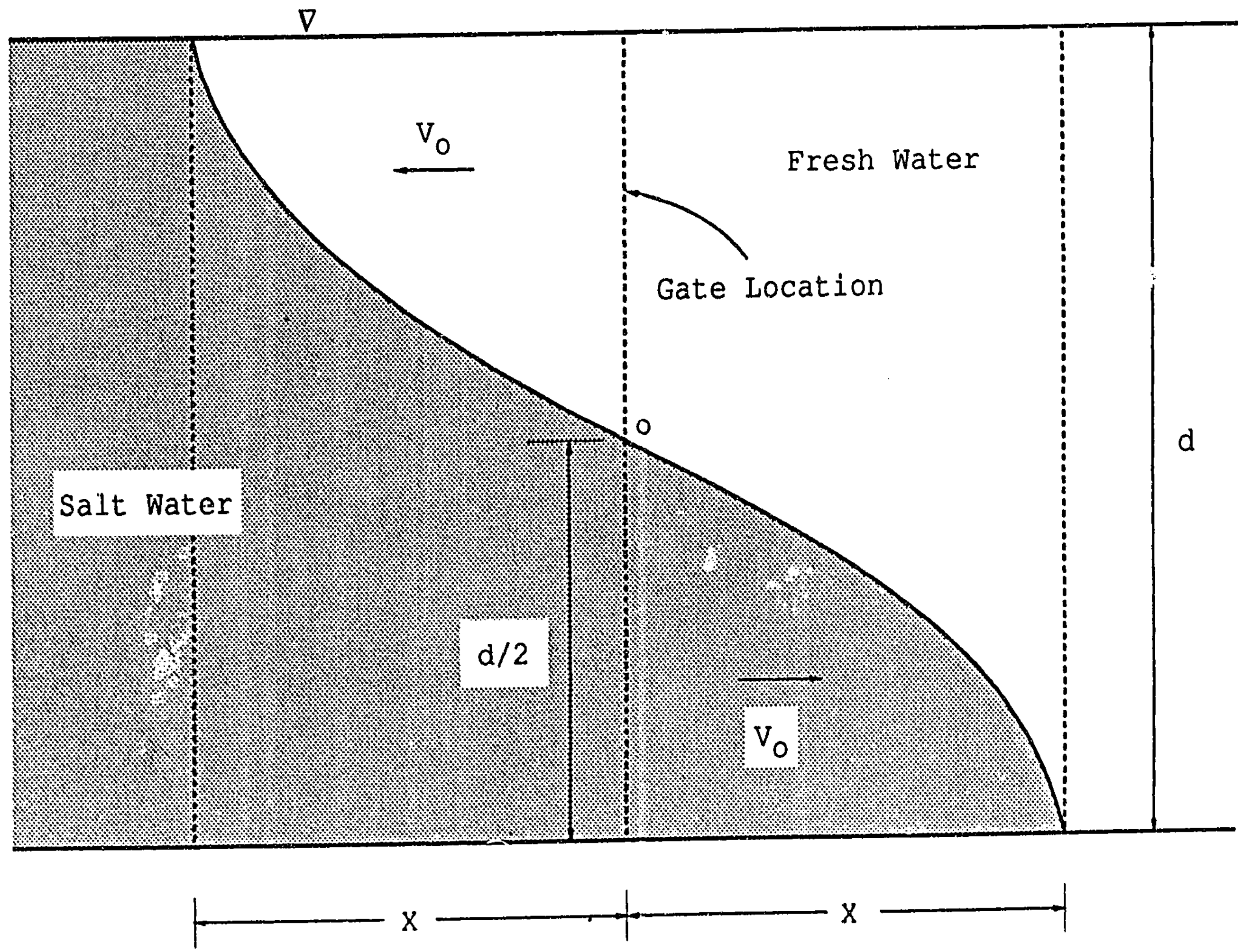


Figure 2.1 O'Brien and Cherno's Model

in which 'b' indicates a scale defined by the following subscripts:

v = velocity scale
t = time scale
d = depth scale
s = salinity scale
L = longitudinal length scale

With these scales it is possible to show that

$$K = \frac{L}{\sqrt{d^5 s}} \tag{2.6}$$

in which 'L' is the length of the lock. O'Brien and Cherno described equation 2.6 as the simplified model law since it is assumed that the viscosities are approximately the same in both the model and prototype (ie. b_v=1).

Another interesting set of papers which attempt to solve the exchange flow problem using physical models are those authored by Barr (1963, 1967) and Barr and Hassan (1963). In the first paper of this set (Barr, 1963), the author reviews the literature on exchange flows and establishes the definitions which will be used in future papers. Of special note in this paper, is the attempt to distinguish between exchange flows which initially occupy the full depth of flow (referred to as 'exchange flow') and exchange flows which initially occupy only a fraction of the initial flow depth (referred to as 'dam-burst analogy exchange flow'). Although the differences in physics between the two were not addressed, this does appear to be one of the first papers to consider exchange flows which do not occupy the full flow depth.

Another interesting aspect of Barr's (1963) paper is that he suggests the criterion for similarity between the model and prototype (for convenience referred to as 'J') is

$$\frac{\sqrt{g'H^3}}{\nu} = J \qquad (2.7)$$

in which 'H' is the original spill depth, 'v' is the kinematic viscosity of the spilled material and g' is as defined in equation 2.2. Thus, we may write

$$g' \approx gs \qquad (2.8)$$

as an approximation to equation 2.2. If 's' is the salinity as defined by equation 2.4, we may rewrite equation 2.8 as

$$\frac{\sqrt{gsH^3}}{\nu} = J \qquad (2.9)$$

Retaining both 'g' and 'v' as variables in the development of O'Brien and Cherno's criterion for similarity, equation 2.6 becomes

$$\frac{L\nu}{\sqrt{gsd^5}} = K' \qquad (2.10)$$

in which K' is the modified dimensionless similarity condition. If 'H' is equal to 'd', equations 2.9 and 2.10 may be combined to form the following relationship

$$J\frac{d}{L} = \frac{1}{K'} \qquad (2.11)$$

where, as before, 'L' is the lock length.

It is interesting to note that the two criteria for similarity are not the same. The reason for this is that in the work by Barr, long flumes are used with a lock length equal to one half the flume length. Thus, when his experiments were conducted, he could safely neglect the reflection of the surge of the back wall of the lock. In O'Brien and Cherno's work, however, locks of only a fraction of the length of the test flume were used making it necessary to include the additional scaling factor 'L/d'.

Thus, it can be said that this set of papers is an extension to the previous work by O'Brien and Cherno (1934). In the first paper, the emphasis is on reviewing past experimental results, paying particular attention to the work of Keulegan. In the second paper, the authors explain the nature of experiments that they conducted and comment on the structure of the flow with emphasis on the entrainment phenomenon associated with the head. In the third and final paper, the author considers lock exchange experiments in which short locks (ie. experiments similar to those of O'Brien and Cherno) have been used and also explores other aspects of exchange flows such as the effects of surface tension and cross sectional geometry.

2.3.2 Order of Magnitude Models

In order of magnitude models, the researcher typically relies upon his intuition and general understanding of the physics of the problem to develop approximate relationships describing the features of interest. Perhaps the best example of this approach in the area of spill modelling is that provided in the paper by Fay (1969).

Briefly stated, Fay approximates the spreading of a spill by assuming that only four major forces are involved in the phenomenon. He assumes that two of the forces act to spread the spill and two act to resist the motion of the spill. By setting each driving force against each resisting force, he is able to develop flow regimes in the spill event, each governed by a simple equation.

For driving forces, Fay reasons that only forces resulting from slumping of the spilled fluid (gravity force) and surface tension (surface tension force) are of importance. Opposing these forces, he feels that only fluid inertia (inertial force) and fluid viscosity (viscous force) are significant implying that this model only applies to buoyant spills. For dense spills, an additional term accounting for bed friction would need to be added. If it is assumed that in the surface tension regime, viscous forces are much larger than inertial forces, only three distinct flow regimes are possible: Gravity-Inertial, Gravity-Viscous, and Surface Tension-Viscous.

For an instantaneous spill of finite size, where 'L' is the diameter of the slick; g' is as described in equation 2.2; 'V','ρ'and 'σ' are, respectively, the volume, density and net surface tension of the spilled fluid; 't' is the elapsed time of the spill event; and 'ν' is the kinematic viscosity of the ambient; it is possible to describe the three regimes as follows:

Gravity-Inertia

$$L = \sqrt[4]{g'Vt^2} \tag{2.12}$$

Gravity-Viscous

$$L = \sqrt[6]{\frac{g'V^2 t^{1.5}}{\nu^{0.5}}} \tag{2.13}$$

Surface Tension-Viscous

$$L = \sqrt[4]{\frac{\sigma^2 t^3}{\rho^2 \nu}} \tag{2.14}$$

If the initial spilling of fluid occurs over a long period of time, a model assuming a continuous discharge should be used. For this situation, assuming the spill point is stationary, if 'L' is now the width of the slick; 'Q' the volume flow rate of the spilled fluid; 'u' the velocity of the ambient; 'x' the distance from the head to the source; and all other variables are as described previously, Fay has developed the following equations to describe the spreading regimes:

Gravity-Inertia

$$L = \sqrt[3]{g'Qx^2} \tag{2.15}$$

Gravity-Viscous

$$L = \sqrt[4]{\frac{g'Q^2x^{1.5}}{\nu^{0.5}u^{3.5}}} \qquad (2.16)$$

Surface Tension-Viscous

$$L = \sqrt[4]{\frac{\sigma^2x^3}{\rho^2\nu u^3}} \qquad (2.17)$$

If the reader is interested in a much more theoretical development of equations 2.12, 2.13 and 2.14, reference should be made to the paper by Hoult (1972). Starting from the generalized Navier-Stokes equations, the author shows that the equations developed by Fay (1969) are similarity solutions and compares the theoretical results to both laboratory and field observations.

2.3.3 Analytical Models

Since the subject of stratified flows is typically only discussed by academics and researchers, most of the models that have been created to simulate the growth of slicks resulting from spill events are analytical in design and rely heavily on the methods of calculus to solve the problem. As an example of how these techniques are applied to spills, let us refer to the set of papers authored by Abbott (1961).

In the first of these papers, Abbott considers a spill which is spreading in the Gravity-Inertial regime. It is assumed that the spill was instantaneous in nature, occurred in a long, straight, uniform channel and occupied the entire channel width. By also assuming that the pressure distributions in both layers are hydrostatic, and using the variables as defined in Figure 2.2, the continuity and momentum equations for the slick are:

$$\frac{\partial h}{\partial t} + h\frac{\partial u}{\partial x} + u\frac{\partial h}{\partial x} = 0 \tag{2.18}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\left(\frac{\partial h}{\partial x} + \frac{\partial h_o}{\partial x}\right) = 0 \tag{2.19}$$

and for the denser ambient are:

$$\frac{\partial h_o}{\partial t} + h_o\frac{\partial u_o}{\partial x} + u_o\frac{\partial h_o}{\partial x} = 0 \tag{2.20}$$

$$\frac{\partial u_o}{\partial t} + u_o\frac{\partial u_o}{\partial x} + g\left(\frac{\partial h_o}{\partial x} + \lambda\frac{\partial h}{\partial x}\right) = 0 \tag{2.21}$$

By writing equations 2.18 to 2.21 in matrix form and solving for the eigenvalues, Abbott is able to show that the system has four characteristics; two associated with the upper layer and two associated with the lower layer. He then goes on to show that if the lower layer is very deep, the equations decouple and, from the equations for the lower layer, proves that

$$\frac{\partial h_o}{\partial x} = -\lambda\frac{\partial h}{\partial x} \tag{2.22}$$

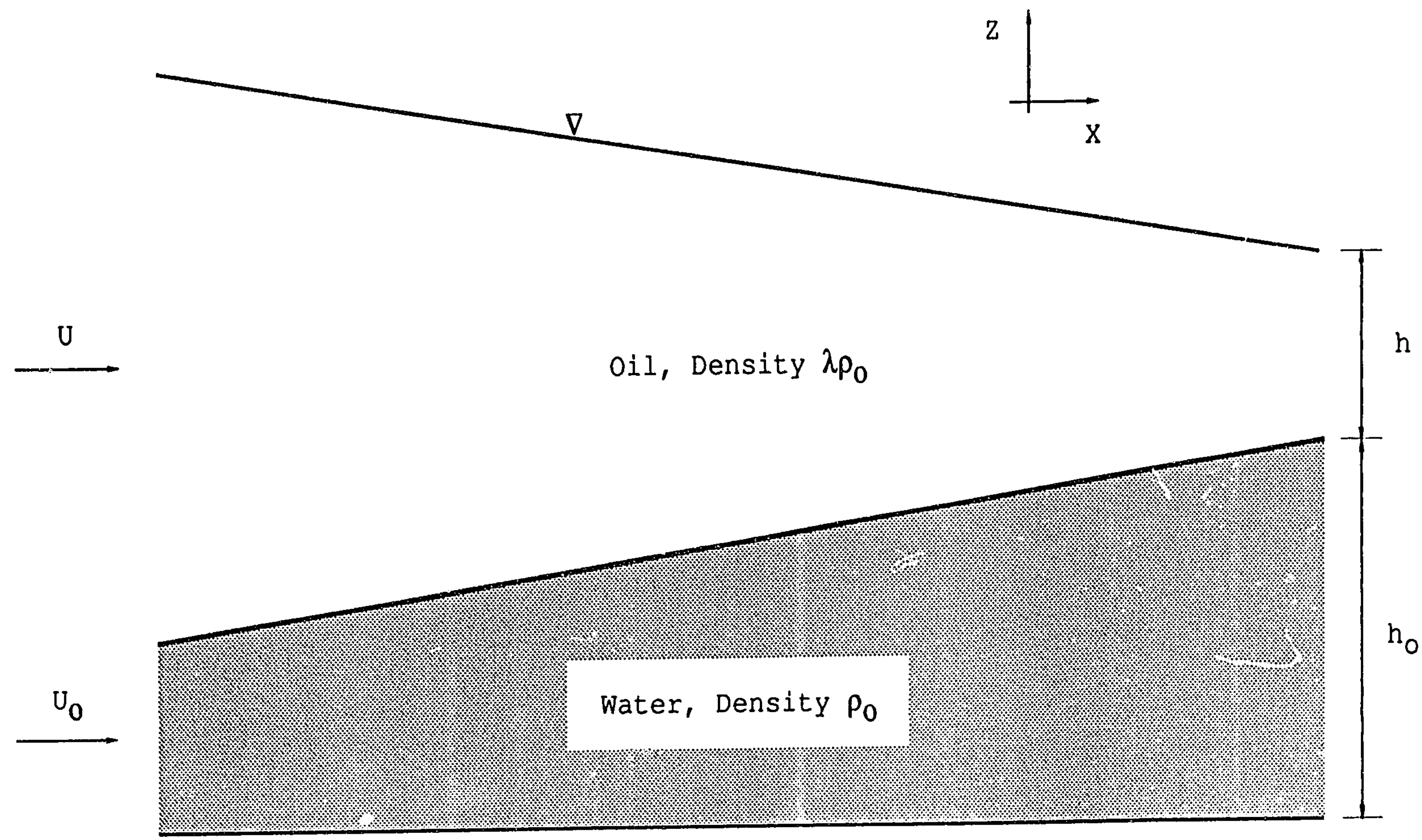


Figure 2.2: Abbott's Model

If equation 2.22 is substituted in the momentum equation for the slick, the following equation results

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g(1-\lambda)\frac{\partial h}{\partial x} = 0 \qquad (2.23)$$

It is interesting to note at this point that equations 2.19 and 2.23 are identical to the equations presented by Hoult (1972) for flow in the Gravity-Inertial regime. This is an important observation since it implies that Hoult's work only applies to spills spreading over a deep ambient; a fact not obvious from Hoult's discussion.

Proceeding now to Abbott's second paper in this series, it is here that he attempts to solve the reduced system of equations from the preceding paper by the method of characteristics. In his first attempt, he assumes a boundary condition at the head identical to that used in classical dam-break theory. This predicts the thickness of the slick reasonably well except in the vicinity of the leading edge. Since the velocity of the front is strongly dependent on the thickness of the front, this produces a frontal speed that is greatly in error.

In his next solution attempt, Abbott assumes that the following equation, in which 'K' is a constant, may be used to describe the relationship between the frontal height 'h_f' and velocity 'u_f'

$$u_f = K\sqrt{gh_f} \qquad (2.24)$$

Once again using the method of characteristics, he arrives at a solution which implies that the front of the slick is similar to a square wave and that the body of the slick is of uniform thickness. This solution does not simulate the gradually varied nature of the body of the slick, and although his analysis indicates that 'K' should be one, experiments support a value of two.

In his final solution attempt, Abbott acknowledges the existence of a head at the front of a slick and uses the following equation, in which 'K' is an empirical constant, to relate head speed 'u_f' to head thickness 'h_f'

$$u_f = K\sqrt{g(1-\lambda)h_f} \tag{2.25}$$

To determine an appropriate value for 'K', Abbott performed a series of experiments from which a value of unity was chosen. It should be noted that the apparatus used in these experiments produces a steady, uniform slick. For this reason, great care should be exercised if this value of 'K' is compared to that found by other researchers. Most researchers use lock exchange experiments which produce unsteady, non-uniform slicks.

Before discussing the final paper in this series, it is interesting to consider the question of why this technique is not capable of predicting the head velocity without using some form of empirical calibration. Although this point was not discussed by Abbott, it would seem that the most likely cause is that there is a discontinuity in the slick thickness in the vicinity of the head. Behind the head, the slick has a finite, reasonably uniform thickness and preceding the head, the slick has a thickness of zero. Thus, because of this discontinuity, calculus does not apply in this region.

In the final paper of this series, Abbott essentially repeats the analysis performed in the first two papers but this time considers a spill spreading in an axially symmetric fashion. If the reader is interested in these forms of spills, reference should be made to Hoult's (1972) paper as well as a paper by Lister and Kerr (1989). In the latter paper, the authors consider the motion of axially symmetric viscous gravity currents at the interface between two fluids.

Before moving on, attention should be drawn to three other papers of note. The first is by Fannelop and Waldman (1972); a classic paper in this area and one of the major references used in Hoult's (1972) work. The second is by Puskas, McBean and Kouwen (1987); one of the few papers to consider a stratified flow between two fixed boundaries. The third is by Reed, French, Feng and Knauss (1990); an excellent example of the type of complexity that can be built into spill analysis models.

At this point it may seem that there have already been numerous attempts at developing spill simulation models and that yet another attempt is unwarranted. As was mentioned earlier, however, most of the present models are really only suited for academic use. Since it is the practicing engineer who must solve most spill problems, there is a real need for a model designed to be used by this audience. Thus, this work attempts to provide one of the fundamental modules that should be incorporated into a finished model. This module deals specifically with initial spill slumping, induced velocities in the ambient fluid, and the simulation of the spill head.

2.4 Initial Spill Slumping

It was found that there is very little literature devoted exclusively to the mechanics of fluid slumping. Of course every spill model does simulate this process, but in most cases the phenomenon is handled implicitly since the governing equations are differential in nature. To stay within the mandate of this work, however, it was felt that a model based upon integral procedures would be more appropriate.

One of the few models found that met this criteria is the model used by O'Brien and Cherno (1934). Since the dynamics of this model are presented in a later chapter, only a brief description will be supplied here. Basically, the model simplifies the geometry of the event shown in Figure 2.1 to that shown in Figure 2.3. By conservation of mass, it can be said that fluid volumes 'A' and 'B' must be equal, and since Newton's second law (ie. F = ma) must also apply, we may say that the change in net force between any two times must be equal to the change in momentum of the fluid system. These two conditions are sufficient to describe the slumping event.

Another form of slumping model, common in the development of heavy gas spill models, is the 'box' model, an example of

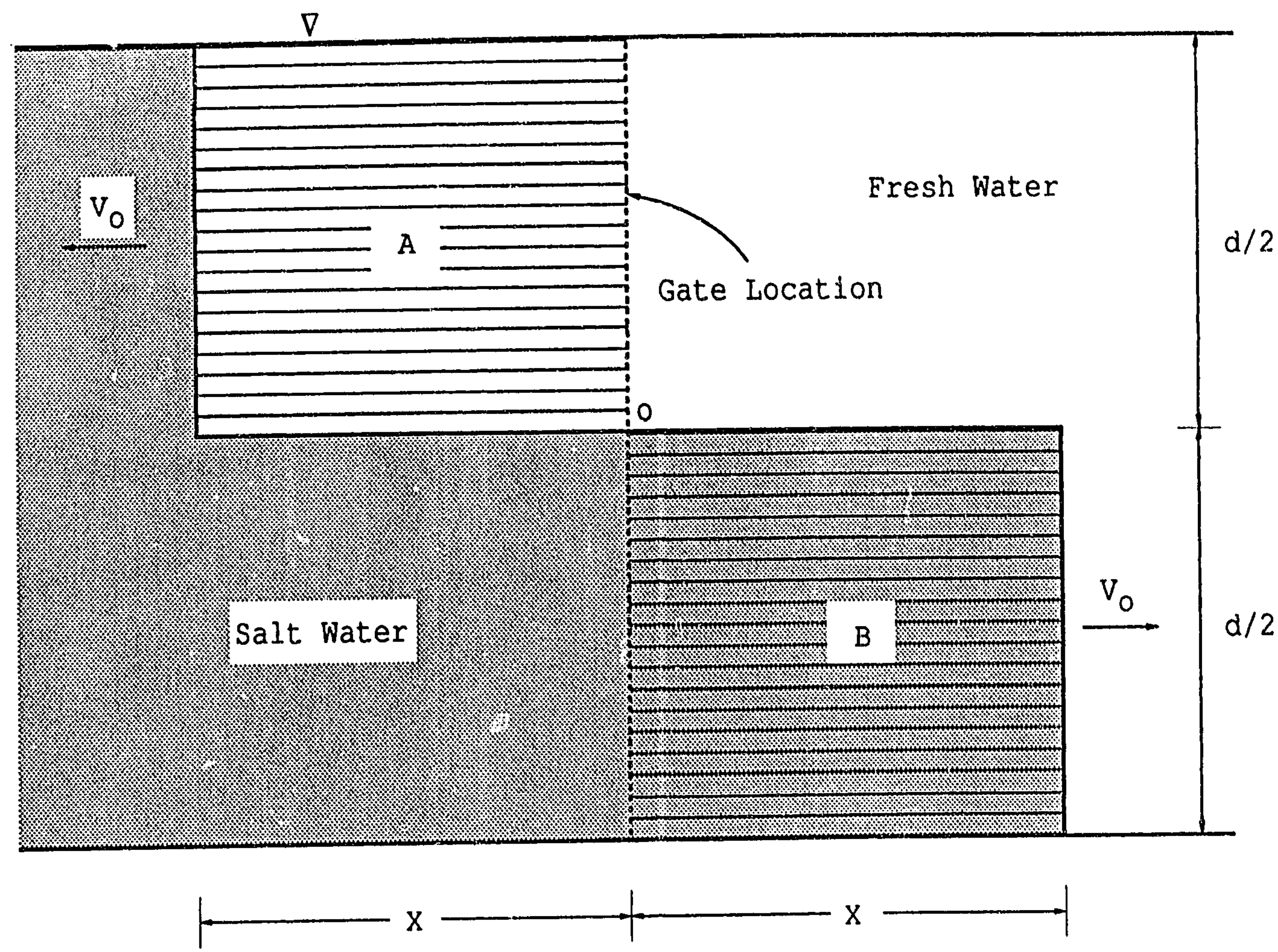


Figure 2.3: O'Brien and Cherno's Modified Model

which is presented in the paper by Bradley, Carpenter, Waite, Ramsay and English (1983). Considering Figure 2.4, it can be seen that the main difference between this type of model and that used by O'Brien and Cherno is that the spill is always assumed to have a box shape. In fact, it can be said that this is a simplification of the previous model since when volume 'A' in Figure 2.3 occupies the upper portion of the original spill volume, O'Brien and Cherno's model will predict that the spill has a 'box' shaped cross section.

2.5 Induced Ambient Velocities

It should be obvious to all readers that as the spilled fluid slumps and spreads, the ambient fluid will be pushed out of the region in front of the slick, and will fill the space vacated in the slumping process. What may not be obvious, however, is how to quantify the effects of these motions on the system.

On the surface, it may appear that the previously proposed slumping model takes this into account since in Figure 2.3, continuity dictates that volume 'A' must be equal to volume 'B'. This is, of course, a true statement, but it does not account for all the mass induced into motion. Referring to Figure 2.5, it should be clear that there is also mass induced into motion through regions 'C' and 'D'.

The analysis of the flow in regions such as 'C' and 'D' is made difficult due to the two-dimensional nature of the flow. Although in many cases a researcher may be justified in

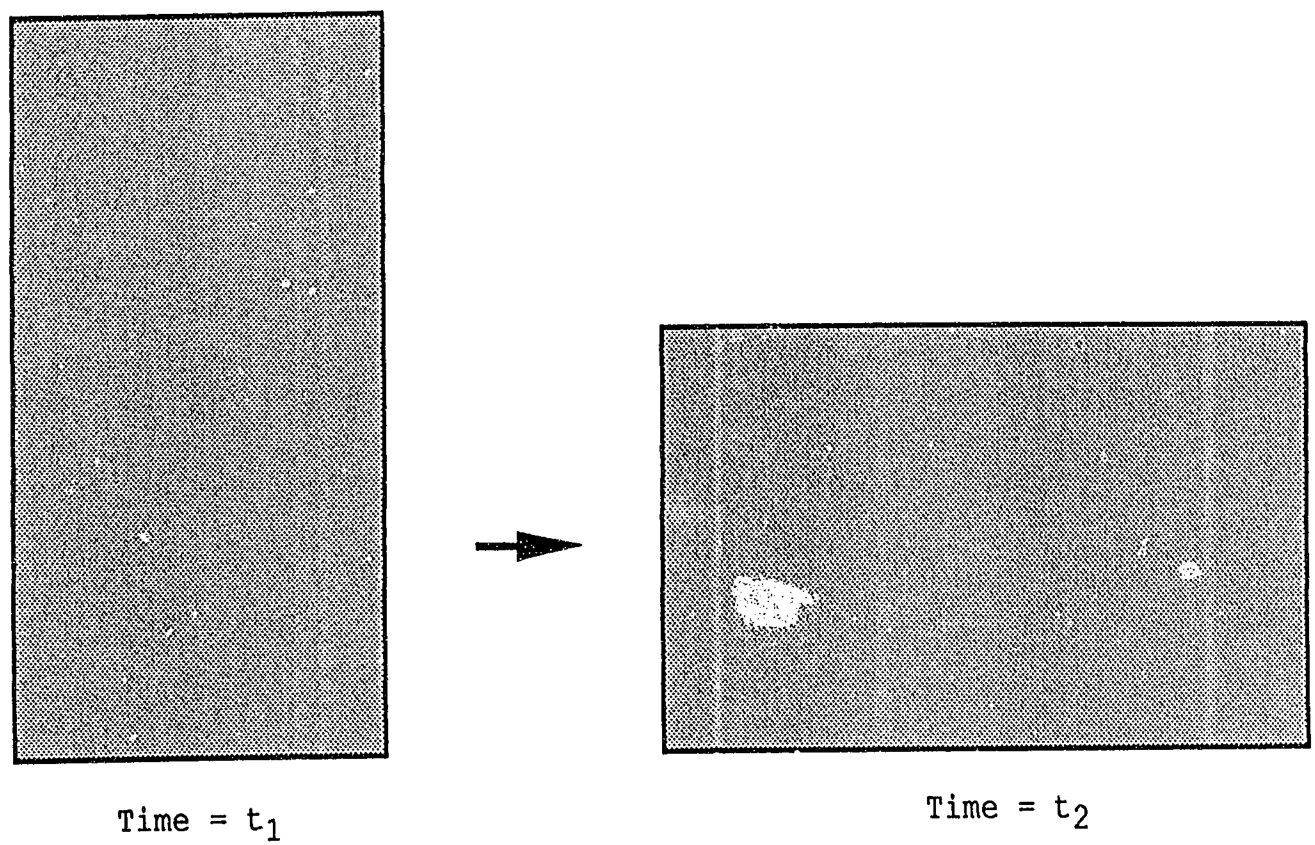


Figure 2.4: Evolution of Box Model

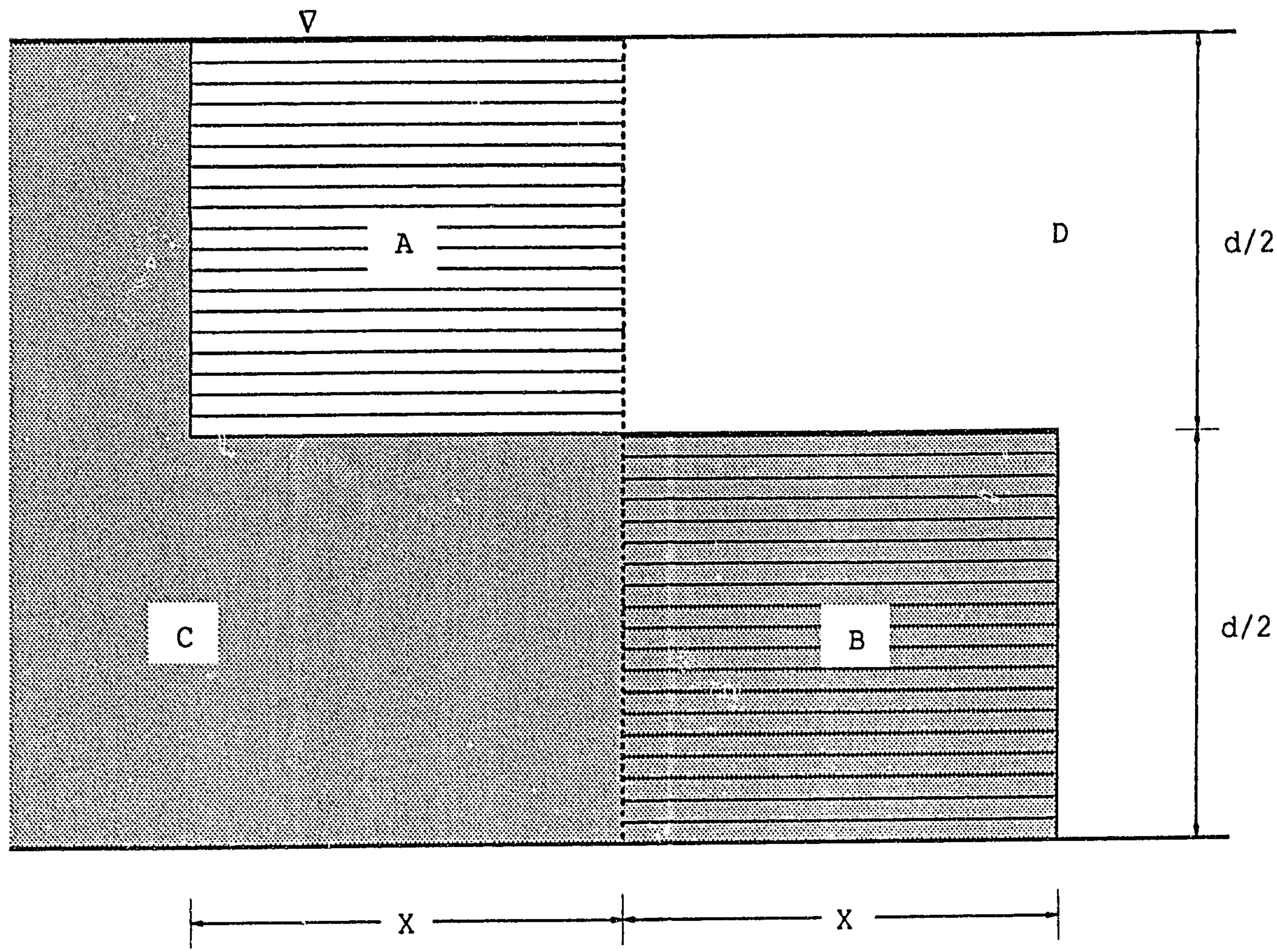


Figure 2.5: Definition of Flow Areas

neglecting these flows, care must be taken since even if a fluid has a small velocity, if the mass is large, the momentum may be considerable.

It is precisely this problem that Vallentine (1965) addresses, compensating for the induced mass with 'virtual' mass. Although the method of analysis will be discussed more fully in a later chapter, briefly stated, the method involves determining the total kinetic energy of the fluid displaced by a body due to the body's motion. From this, it is possible to discover the 'virtual' mass that must be added to the body such that, if induced flows were neglected, the new system would have the same total kinetic energy as the original system.

If the reader is interested in pursuing the subject of virtual mass, there is a discussion of this concept in the work by Bai (1977). Since this paper deals mostly with the virtual mass associated with wave motion, it was felt that the treatment presented by Vallentine was more appropriate.

2.6 Head Simulation

Although a great deal of research seems to have been done in this area, in general, most researchers seem to have been interested in either the shape of the head, the velocity of the head, or the stability of the interface behind the head. Let us first consider the shape of the head.

Often, the head is very distinct and considerably thicker than the body of the slick, and depending on the head velocity and density difference between the two fluids, considerable mixing may occur along the trailing interface. Of the people to consider this problem, one of the first was von Kármán (1940) who showed that the leading edge of the slick should form an angle of 120 degrees with the interface over which it is progressing. This analysis was pursued by Benjamin (1968), who developed an expression for the shape of the leading edge of the head using the method of conformal transformations. A good review of the experimental literature concerning head shape is presented by Middleton (1966) who shows, using his own experiments, that the leading edge is not smooth but has a series of small waves propagating along it produced by the high stress area near the tip. For comparison, the reader may be interested in the results of Keulegan (1958) who also investigated head shape.

As the slick advances, it must expel ambient fluid from its path. In Middleton's (1966) paper, a good description of the fluid motion in and around the head is presented with some discussion of the apparent mixing in the wake of the head. It is explained in Middleton's paper that the mixing in the head wake was so severe that it was impossible to determine the thickness of the current behind the head.

The phenomena of head wake mixing has attracted considerable research attention in an effort to understand its governing mechanisms. In addition to the work by Ellison and Turner (1959) considerable work has also been done in this area by Britter and Simpson (1978, 1979) and most recently by

Bühler, Wright and Kim (1991). In these papers, the authors use a unique experimental apparatus to simulate the head of a gravity current. With this system they were able to produce a stationary head from which they could observe more closely the structure of the head. Based upon theoretical and dimensional arguments, plots were then made of the experimental results in an attempt to develop correlations between the apparent governing parameters. In general, it was found that the height of the head and degree of mixing behind the head was strongly dependent on some form of densimetric Froude number and to a lesser degree, some form of Reynolds number. For a more complete discussion of the effects of Reynolds number, the reader is referred to the work of Keulegan (1958).

When considering the velocity of the head, perhaps the most extensive work is that done by Keulegan. In addition to this work, Middleton (1966) has also done a series of experiments which investigate the velocity of the head produced by density and turbidity currents. A good summary of the findings of the various researchers is supplied in the papers by Benjamin (1968) and Huppert and Simpson (1980).

In general, the velocity of the front of a spill will follow a relationship of the following form

$$V = C\sqrt{g'h} \qquad (2.26)$$

where 'C' is a "constant" of proportionality, g' is as defined in equation 2.2 and 'h' is a representative depth. Although many investigations have shown that the velocity should follow an expression of the above form, a particularly interesting treatment is presented in Benjamin's (1968) paper.

There are at least three major problems associated with expressing the head velocity relationship in the form defined in equation 2.26. Firstly, as is stated in Benjamin's discussion, there is little agreement as to what should be used as the representative depth 'h'. Although it is common to use either the total depth of the ambient, the depth of the slick, or the depth of the head, few researchers report all three values making it difficult to compare results.

The second major problem is that many researchers do not stress that the constant 'C' is not a constant but depends on the regime (eg. Gravity-Inertia) that the slick is in. This point is addressed by Keulegan (1958) when he establishes his "specific law for saline fronts". In his treatment, he shows that the constant has a Reynolds number dependency since his work follows the fronts into the Gravity-Viscous regime. Most researchers, however, are only investigating the Gravity-Inertial regime and hence they find 'C' to be a constant. In general, a value of 1.07 is typically given to this "constant" in accordance with the work by Keulegan.

The final major problem with equation 2.26 is that it assumes that bed slope has no effect on the frontal velocity. This factor was investigated by Middleton (1966) who found that although in some cases, as the slope increased, so did the constant 'C', in other cases, the slope seemed to have little effect. For a more thorough discussion of the effect of slope on the motion of a gravity current head, the reader is referred to the work by Britter and Linden (1980).

2.7 Summary

Before proceeding further, it should be stressed that the purpose of this review has not been to provide a comprehensive treatment involving all the literature that might be relevant to this work. The body of literature is simply too broad to make this possible. Instead, I have attempted to cover all the major areas that might be related to a spill spreading model, placing emphasis on a few representative papers so that the reader may expand on this review when felt necessary.

CHAPTER 3

SPILL HEAD SIMULATION

3.1 Introduction

After reviewing the literature, it became apparent that one of the major features associated with a spill is the development of a head at the beginning of the spill. This phenomena has received a great deal of research attention since, in some situations, it can be significantly thicker than the rest of the slick. Because of its potential size, many investigators have raised questions as to whether the head is the controlling feature of the spill. These same individuals point out that for spills with large heads, the wake area behind the head may exert a considerable drag force on the spill. Furthermore, some researchers have shown evidence that there may be a significant degree of entrainment associated with the head; another factor that may exert a considerable resistive force on the spill.

Although it is beyond the scope of this work to try and provide a detailed, comprehensive model of the head region, it would seem that any model would be negligent if it ignored this feature altogether. For this reason, we have decided to use a rather simplified model for the head based on work done by Benjamin (1968).

Benjamin's model was developed to analyze the shape and manner of progression of an air pocket formed when the end of a completely filled, horizontal pipe is removed. He shows that the leading edge of the pocket must develop at the angle calculated by von Kármán, and using conformal transformations, establishes a relationship for the profile of the head. Although it is not necessary to go into the detail that Benjamin did, an attempt will be made to follow his method of analysis to develop the final form of the head synthesis equations used in this work.

3.2 Development of Head Model

The model used in this work is a simplified model in which it is assumed there are no losses associated with the head and that there is no difference between the head thickness and the mean spill thickness. With this in mind, let us now consider Figure 3.1.

This figure presents a view of the head region of a spill of mean thickness 'H', density 'ρ_o', and mean spill velocity 'V_H', advancing over a still ambient of density 'ρ' and mean depth 'd'. The depth under the spill is 'h' and the height of the "bow wave" created in front of the spill is 'Δd'. To make this system easier to analyze, we will now bring the spill to rest by imposing an ambient flow velocity 'c_1' equal in magnitude to the spill velocity but opposite in direction as shown in Figure 3.2. If the velocity created under the spill is 'c_2', an expression for ambient continuity between sections A and B is shown below

$$c_1 d = c_2 h \qquad (3.1)$$

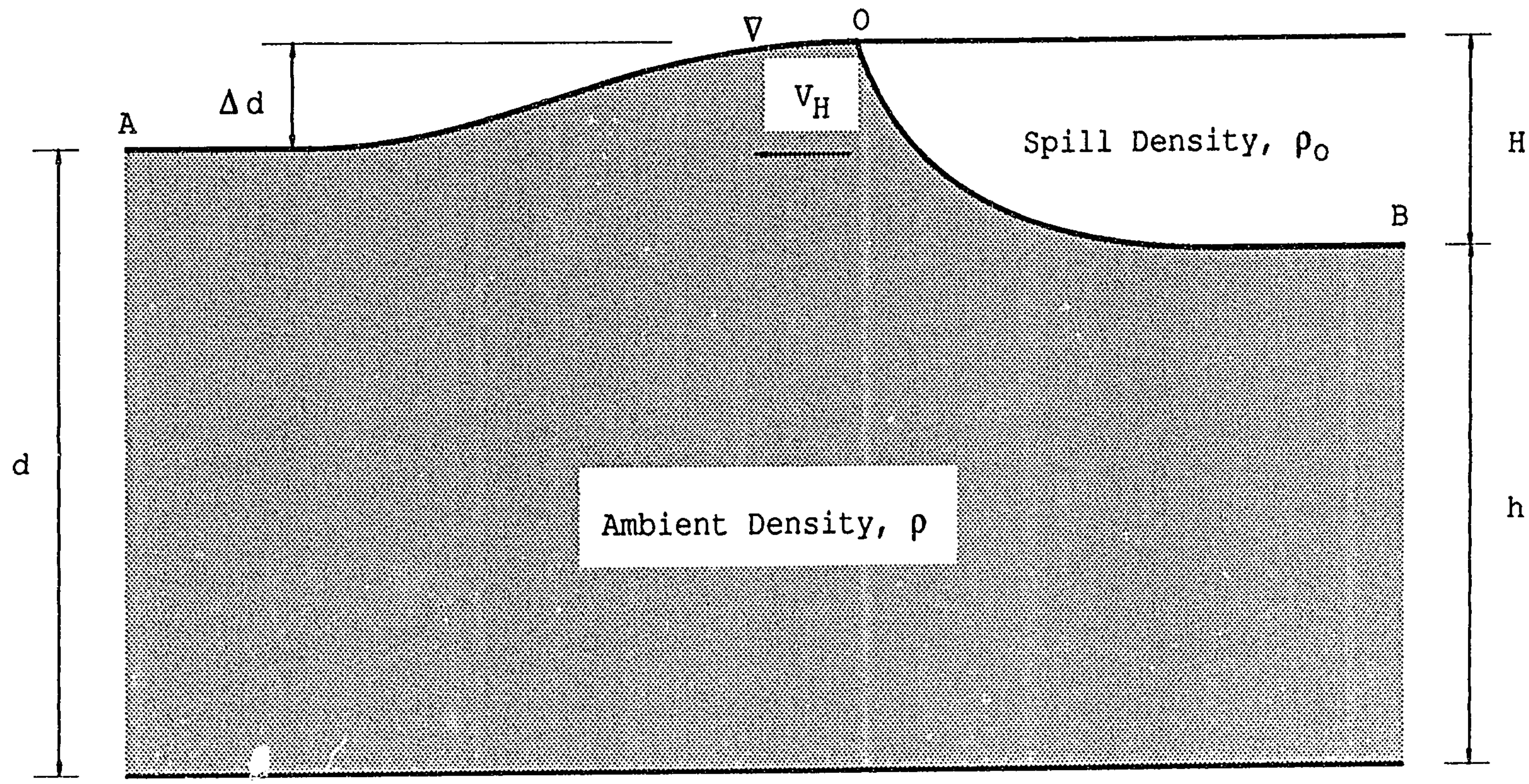


Figure 3.1: Model of Moving Spill Front

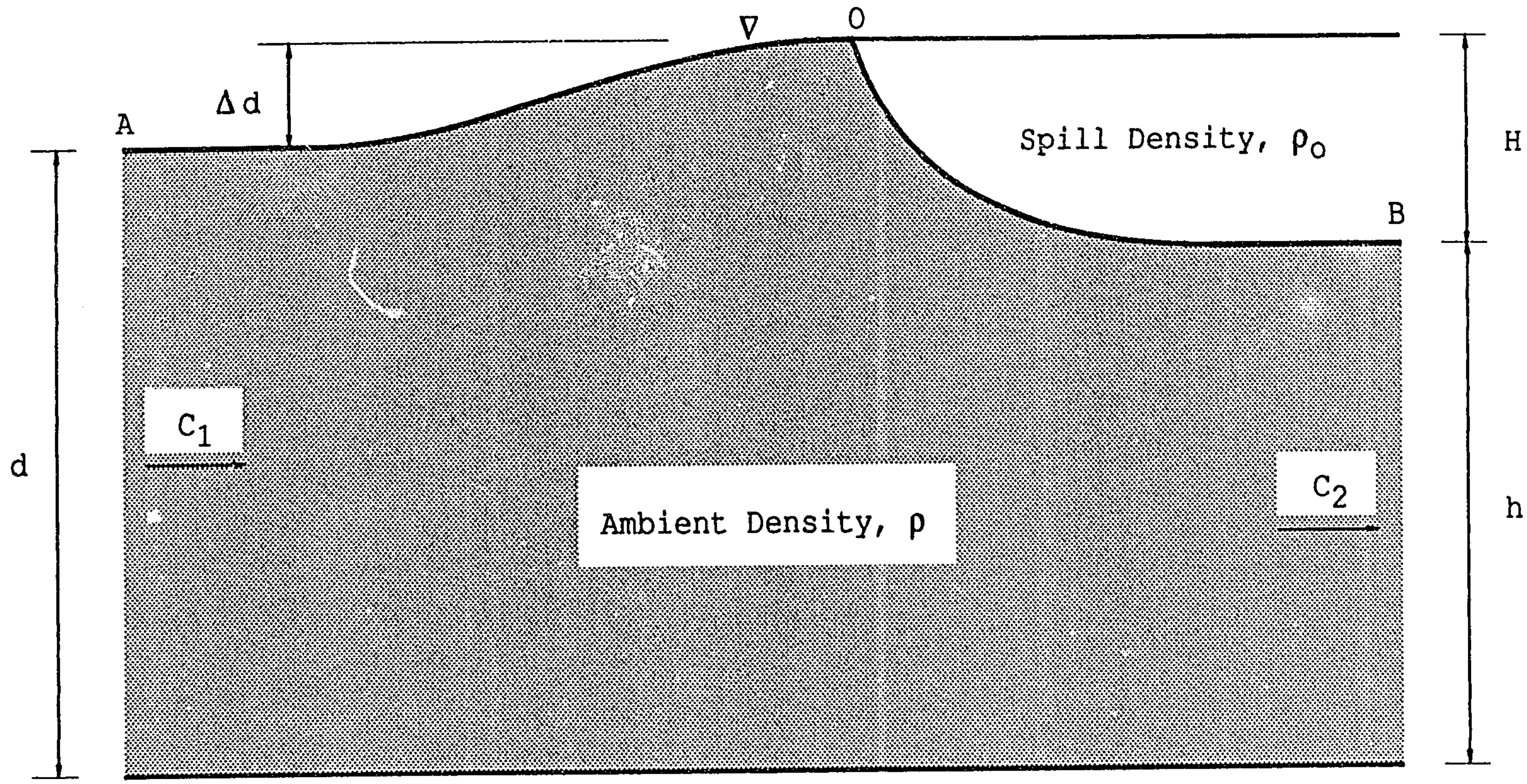


Figure 3.2: Model Spill Front Brought to Rest

If we now apply the Bernoulli equation between points A and O, assuming no losses, we find

$$\frac{P_A}{\gamma} + d + \frac{c_1^2}{2g} = \frac{P_O}{\gamma} + (d+\Delta d) + \frac{V_O^2}{2g} \tag{3.2}$$

Since the pressure at both points A and O is atmospheric and point O may be considered a stagnation point, equation 3.2 may be simplified to the following with the aid of the continuity equation

$$\Delta d = \frac{c_2^2 h^2}{2gd^2} \tag{3.3}$$

If we now apply the Bernoulli equation between points O and B, once again assuming no losses, we find

$$\frac{P_O}{\gamma} + (d+\Delta d) + \frac{V_O^2}{2g} = \frac{P_B}{\gamma} + h + \frac{c_2^2}{2g} \tag{3.4}$$

Using the continuity equation and assuming

$$P_B = \rho_o gH \tag{3.5}$$

equation 3.4 will simplify to

$$c_2^2 = \frac{2g\Delta\rho H}{\rho} \tag{3.6}$$

Substituting equation 3.6 into equation 3.3, we discover

$$\frac{\Delta d}{d} = \frac{\Delta\rho}{\rho}\frac{h^2 H}{d^3} \tag{3.7}$$

In equation 3.7, the density difference '$\Delta\rho$' is defined as

$$\Delta\rho = \rho - \rho_o \tag{3.8}$$

Returning now to the continuity equation, we may develop an expression for the spill velocity (i.e. c_1) if we also make use of equation 3.6. The resulting equation is

$$c_1 = \frac{h}{d}\sqrt{\frac{2g\Delta\rho H}{\rho}} \tag{3.9}$$

Before this expression can be evaluated, we must first develop an expression for h/d. To do this, let us write the momentum equation for the flow between sections A and B as shown below:

$$\rho g\frac{d^2}{2} + \rho c_1^2 d = \rho_o gHh + \rho g\frac{h^2}{2} + \rho_o g\frac{H^2}{2} + \rho c_2^2 h \tag{3.10}$$

If we use the continuity equation to eliminate the spill velocity, we arrive at the expression

$$c_2^2 = \frac{gd\left((d^2-h^2-H^2) - 2\frac{\rho_o}{\rho}Hh\right)}{2(d-h)h} \tag{3.11}$$

We now have two independently developed expressions for the velocity under the spill; equation 3.6 and equation 3.11. If we equate these two expressions, we will arrive at the following relationship after some manipulation

$$\left(\frac{1}{d^2} - \frac{4\Delta\rho H}{\rho d^3}\right)h^2 + \left(\frac{4\Delta\rho H}{\rho d^2} + \frac{2\rho_o H}{\rho d^2}\right)h + \frac{H^2}{d^2} - 1 = 0 \tag{3.12}$$

Remembering the relationship developed for Δd, equation 3.12 can be further simplified into the following form

$$\left(\frac{h}{d}\right)^2 + 2\frac{\rho_o}{\rho}\frac{H}{d}\left(\frac{h}{d}\right) + 4\left(\frac{1}{h} - \frac{1}{d}\right)\Delta d + \frac{H^2}{d^2} - 1 = 0 \qquad (3.13)$$

Equation 3.13 is almost in a form suitable for solution by the quadratic equation. To complete the conversion, we must make use of the relation

$$\frac{\Delta d}{h} = (1-r)\frac{H}{d}\left(\frac{h}{d}\right) \qquad (3.14)$$

where

$$r = \frac{\rho_o}{\rho} \qquad (3.15)$$

If we substitute equations 3.14 and 3.15 into equation 3.13, we will develop the following expression

$$\left(\frac{h}{d}\right)^2 + 2(2-r)\frac{H}{d}\left(\frac{h}{d}\right) + \frac{H^2}{d^2} - 4\frac{\Delta d}{d} - 1 = 0 \qquad (3.16)$$

Equation 3.16 is in the proper form for solution using the quadratic equation. It is interesting to note that if H/d is in the order of zero, the solution to this equation becomes

$$\frac{h}{d} = 1 \qquad (3.17)$$

which allows equation 3.9 to reduce to

$$c_1 = \sqrt{2g(1-r)H} \qquad (3.18)$$

Equation 3.18 is interesting in that it is precisely the equation recommended by von Kármán for the velocity of the head of a spill.

3.3 Summary

In this chapter we have attempted to develop a simple relationship to describe the velocity of the head of a spill. Using energy principles, we arrived at equation 3.9 which, in conjunction with the solution of equation 3.16, determines the head velocity since

$$V_H = c_1 \qquad (3.19)$$

CHAPTER 4

FLOW INITIATION WITHOUT VIRTUAL MASS

4.1 Introduction

Up to this point, most researchers have studied the behavior of slick dynamics after the initial spill slumping is complete and the maximum velocity has been reached. Very few, however, have attempted to describe the dynamics of the spill during the initial moments when the spill volume is slumping and the slick is experiencing an acceleration up to its maximum velocity. It is this region of flow initiation that we will attempt to describe in this section. In developing this aspect of the model, we will make use of an earlier model presented by O'Brien and Cherno (1934) which appears to have produced reasonable results. We will also assume in this development that the spill is buoyant, that there is no surcharge associated with the original spill volume, that the slick advances down a long, wide, rectangular channel, that no reflection of the front occurs, and that the effects of induced velocities in the ambient may be neglected.

4.2 O'Brien and Cherno Model

As was stated in the literature review, the model presented by O'Brien and Cherno was originally developed to investigate the behavior of saltwater entering fresh water as the result of locking operations. The model produced was one of the first models developed to explain slick growth and resulted in the following equation to predict the initial head velocity 'V_o'

$$V_o = \pm\sqrt{\frac{gds}{2(G_1+G_2)}} \tag{4.1}$$

In this equation, 's' is the salinity defined as

$$s = G_1 - G_2 \tag{4.2}$$

where 'G_1' and 'G_2' are the specific gravities of the heavier and lighter fluids respectively.

Let us now explore the formulation of equation 4.1. At its most fundamental level, this equation is developed from the expression

$$F_{net} = ma \tag{4.3}$$

which may be rewritten as

$$F_{net} = \frac{d}{dt}(mV_o) \tag{4.4}$$

Continuing with the analysis, if 'ω' is the unit weight of water defined as

$$\omega = \rho_w g \tag{4.5}$$

with reference to Figure 4.1, we may write that, at time t_o

$$F_{net} = \omega G_1 \frac{d^2}{2} - \omega G_2 \frac{d^2}{2} \tag{4.6}$$

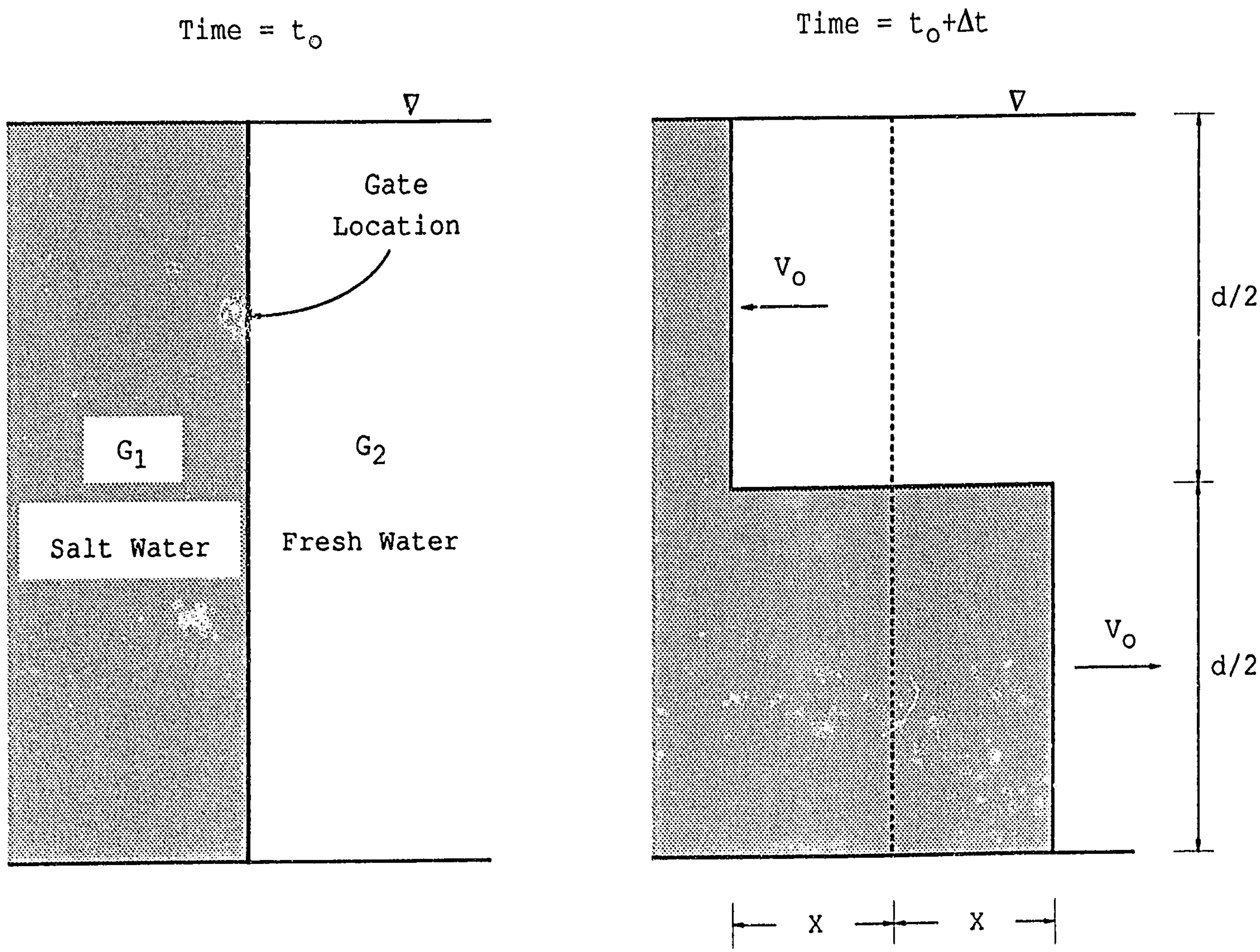


Figure 4.1: Time Evolution of O'Brien and Cherno's Modified Model

which, remembering the definition of salinity, may be rewritten as

$$F_{net} = \omega s \frac{d^2}{2} \tag{4.7}$$

Considering now the mass which is accelerated by the unbalanced force expressed in equation 4.7, if we first convert Figure 2.1 to the more idealized form expressed in Figure 4.1, at time $t_o+\Delta t$, it is easily seen that one possible description of this mass is as expressed in the following equation

$$m = 2x\frac{d}{2}G_1\frac{\omega}{g} + 2x\frac{d}{2}G_2\frac{\omega}{g} \tag{4.8}$$

If we now define 'V_o' as

$$V_o = \frac{dx}{dt} \tag{4.9}$$

after substitution of this equation as well as equations 4.7 and 4.8 into equation 4.4, we arrive at the expression

$$\omega s\frac{d^2}{2} = \frac{\omega}{g}d(G_1+G_2)\frac{d}{dt}\left(x\frac{dx}{dt}\right) \tag{4.10}$$

We would now like to integrate equation 4.10. After the first integration, we find

$$s\frac{d}{2}t = \frac{G_1+G_2}{g}x\frac{dx}{dt} + C_1 \tag{4.11}$$

in which, C_1 must equal zero, since at $t=t_o=0$, $x=0$. If we now perform the second integration, we find after simplification that

$$x = \pm t\sqrt{\frac{gds}{2\,(G_1 + G_2)}} \qquad (4.12)$$

After applying equation 4.9 to equation 4.12, we find that equation 4.1 is produced.

4.3 Modified Model

Since in the previous chapter we were able to develop an expression describing the head velocity, if we were now to add one more degree of complexity to O'Brien and Cherno's model, we would still be able to maintain a closed system of equations. From the preceding review of their model, it should be apparent that one of the more restrictive assumptions they use is to assume that the inflection point on the interface always occurs at mid-depth. Although in some situations this may true, it would seem that, in general, the depth of the inflection point should be one of the properties predicted by the model.

To do this, let us first modify Figure 4.1 to the form illustrated in Figure 4.2. From this figure, it can be seen that we have added two important improvements to the original model. Firstly, we have assumed that the original spill depth might not be equal to the depth of the ambient. This allows the model to be amendable to more than just lock exchange type flows. Secondly, we have assumed that the slick thickness

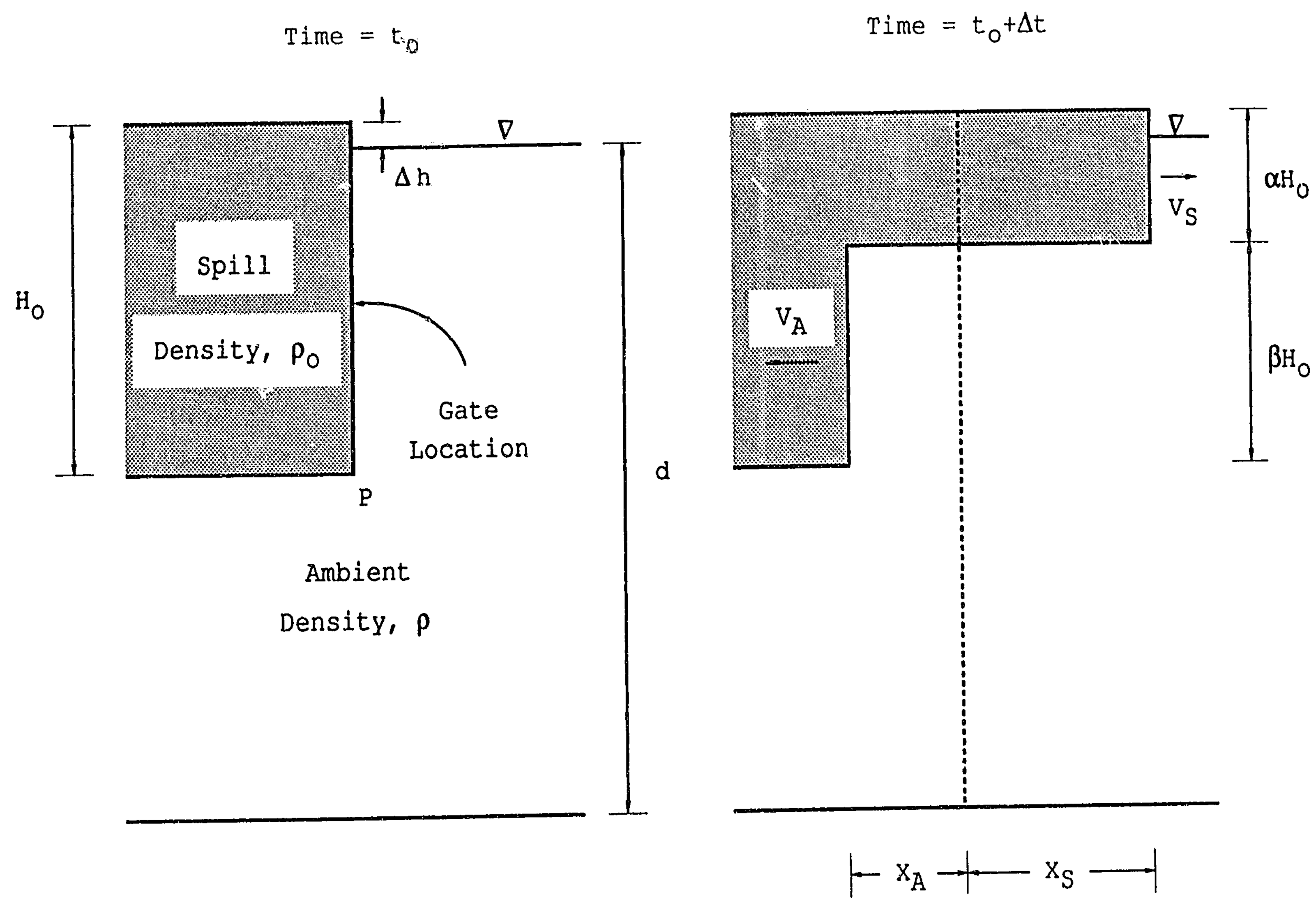


Figure 4.2: Time Evolution of Proposed Model

may not be equal to one half the original spill depth. This adds generality to the model and increases it's predictive characteristics.

Following O'Brien and Cherno's analysis procedure, we may write that at time $t_\circ$

$$F_{net} = \rho_\circ g\frac{H_\circ^2}{2} - \rho g\frac{(H_\circ-\Delta h)^2}{2} \tag{4.13}$$

Since at point 'P' the two fluids must be in hydrostatic equilibrium, it is easy to show that

$$rH_\circ = H_\circ - \Delta h \tag{4.14}$$

if we remember that 'r' is the density ratio as defined by equation 3.15. We find, after substituting this expression into equation 4.13 and simplifying, that

$$F_{net} = \frac{\rho_\circ(1-r)gH_\circ^2}{2} \tag{4.15}$$

Let us now turn our attention to describing the change in momentum which is produced by this unbalanced force. From Figure 4.2, it can be seen that at time $t_\circ+\Delta t$, the accelerated mass 'M' is

$$M = \rho_\circ(X_A+X_S)\alpha H_\circ + \rho(X_A+X_S)\beta H_\circ \tag{4.16}$$

The momentum 'MV' associated with the mass 'M' is

$$MV = \rho_\circ(X_A+X_S)\alpha H_\circ V_S + \rho(X_A+X_S)\beta H_\circ V_A \tag{4.17}$$

Before solving equation 4.17, it is useful to note that

$$V_A = \frac{dX_A}{dt} \tag{4.18}$$

$$V_S = \frac{dX_S}{dt} \tag{4.19}$$

Since during the slumping process, the volume of spill that slumps must be replaced by ambient, we may say that

$$X_A \beta H_o = X_S \alpha H_o \tag{4.20}$$

which may be rewritten as

$$X_A = \frac{\alpha}{\beta} X_S \tag{4.21}$$

Taking the time derivative of this equation, we arrive at the following expression which relates the two head velocities 'V_A' and 'V_s'

$$V_A = \frac{\alpha}{\beta} V_S \tag{4.22}$$

If we now substitute equations 4.18, 4.19, 4.21 and 4.22 into equation 4.17, we will find that

$$MV = \rho_o (\alpha + \beta) \frac{\alpha H_o}{\beta} X_S \frac{dX_S}{dt} + \rho (\alpha + \beta) X_S H_o \frac{\alpha}{\beta} \frac{dX_S}{dt} \tag{4.23}$$

which may be rewritten as

$$MV = \rho (\alpha + \beta) (1 + r) \frac{\alpha H_o}{\beta} X_S \frac{dX_S}{dt} \tag{4.24}$$

To determine the rate of change of the momentum 'MV', we must take the derivative of equation 4.24 with respect to time. After performing this operation, equation 4.24 becomes

$$\frac{d}{dt}(MV) = \rho\frac{\alpha H_o}{\beta}(\alpha+\beta)(1+r)\frac{d}{dt}\left(X_S\frac{dX_S}{dt}\right) \tag{4.25}$$

Continuing with the analysis, if we substitute equations 4.15 and 4.25 into equation 4.4, we find that

$$\rho_o g(1-r)\frac{H_o^2}{2} = \rho\frac{\alpha H_o}{\beta}(\alpha+\beta)(1+r)\frac{d}{dt}\left(X_S\frac{dX_S}{dt}\right) \tag{4.26}$$

After integration, this equation simplifies to

$$X_S = t\sqrt{\frac{rgH_o(1-r)\beta}{2(1+r)(\alpha+\beta)\alpha}} \tag{4.27}$$

Since the velocity of the slick head 'V_S' is described by equation 4.19, if we substitute equation 4.27 into this equation and perform the derivative, we find that

$$V_S = \sqrt{\frac{rgH_o(1-r)\beta}{2\alpha(\alpha+\beta)(1+r)}} \tag{4.28}$$

Realizing that

$$\beta = 1-\alpha \tag{4.29}$$

equation 4.28 may be simplified to

$$V_S = \sqrt{\frac{rgH_o(1-r)(1-\alpha)}{2\alpha(1+r)}} \tag{4.30}$$

4.4 System of Equations

With equation 4.28 it is possible to create a closed system of equations from which we may determine the initial velocity of the slick head 'V_S' if we already know the density ratio 'r' of the fluids, the original spill depth 'H_o' and the depth of the ambient 'd'. In addition to equation 4.28, we will also use from the previous chapter equations 3.7, 3.9 and 3.16 which may be rewritten as

$$V_S = \frac{h}{d}\sqrt{2g(1-r)\alpha H_o} \qquad (4.31)$$

$$\frac{h}{d} = \frac{-B + \sqrt{B^2-4AC}}{2A} \qquad (4.32)$$

in which

$$A = 1 - 4(1-r)\,\frac{\alpha H_o}{d} \qquad (4.33)$$

$$B = 2(2-r)\,\frac{\alpha H_o}{d} \qquad (4.34)$$

$$C = \left(\frac{\alpha H_o}{d}\right)^2 - 1 \qquad (4.35)$$

4.5 Data Sets

Now that we have a closed set of equations to predict the velocity of the slick head, it remains for us to test the predictive capabilities of the equations. To do this, data sets from the following sources were selected: University of

Alberta, O'Brien and Cherno (1934), Middleton (1966) and Keulegan (1958). Since each of these data sets has some unique feature associated with it, a brief discussion of each data set will be presented before proceeding with the analysis.

4.5.1 University of Alberta Data

This data set is the only set that seems to truly fit the model being developed. These experiments were performed in a rectangular flume 32.2 cm wide with a lock chamber at one end of the flume 30.1 cm in length. The experiments involved placing hot water in the lock and after removal of the gate, the slick spread over a cold water ambient. The lock gate was held in place with two small side boards and the length of the flume was sufficient that after release of the spilled material, the slick did not reach the end of the flume.

The results from this set of experiments are summarized in Table 4.1 and from these results, two important observations can be made. Firstly, since the spilled material is hot water which spreads over a cold ambient, this data set is comprised of buoyant spills. Of the four sets of information considered, this is the only one which considers buoyant spills. Secondly, it is important to note that the lock depth in these experiments is much less than the ambient depth. Once again, of the four sets of data considered, this is a feature unique to this data set.

Table 4.1: University of Alberta Data						
Run	Ambient		Spill		Density Ratio 'r'	Spill Velocity (m/s)
	Sp. Gr.	Depth (m)	Sp. Gr.	Depth (m)		
116	.992	.065	.999	.237	.993	.044
117	.992	.070	.998	.237	.994	.044
118	.991	.115	.998	.237	.993	.061
119	.991	.110	.999	.237	.993	.056
120	.993	.080	.999	.356	.994	.044
121	.993	.075	.999	.356	.995	.040
122	.989	.110	.999	.356	.990	.069
123	.985	.110	.999	.356	.987	.080

4.5.2 O'Brien and Cherno's Data

This data set was created from the information supplied in Table 1 on page 589 of O'Brien and Cherno's paper and is one of the two data sets that O'Brien and Cherno used to validate their model. The experiments were of a full depth, lock exchange type in which saltwater was released from the lock to spread under a fresh water ambient and were performed in a flume 24.4 feet long, 0.5 feet wide, and 1.25 feet deep.

The values obtained from O'Brien and Cherno's work is summarized in Table 4.2. It should be noted, however, that the parameters listed in this table are slightly different than those shown in O'Brien and Cherno's original table. Since their model used salinity as a parameter, they did not include the specific gravities (or densities) of each layer;

important parameters in this model. Thus, it was necessary to use the following expressions to obtain these values

$$G_1 = \frac{s}{2}\left(1+\frac{gd}{2V_o^2}\right) \qquad (4.36)$$

$$G_2 = G_1 - s \qquad (4.37)$$

Once the specific gravities are known, the density ratio 'r' can be found by simply taking the ratio of specific gravities.

Table 4.2: O'Brien and Cherno's Data

Run	Ambient		Spill		Density Ratio 'r'	Spill Velocity (ft/s)
	Sp. Gr.	Depth (ft)	Sp. Gr.	Depth (ft)		
A-2	.990	1.20	1.019	1.20	.972	.595
A-5	.997	0.80	1.027	0.80	.971	.416
A-6	.997	0.40	1.029	0.40	.969	.298
A-25	.995	0.42	1.025	0.42	.971	.297
A-23	.992	0.41	1.024	0.41	.969	.298
A-14	.991	1.20	1.023	1.20	.969	.532
A-10	.994	1.20	1.017	1.20	.977	.436
A-4	.996	1.20	1.011	1.20	.985	.379
A-12	.994	1.19	1.057	1.19	.940	.912
A-13	.986	1.20	1.087	1.20	.907	.995

4.5.3 Middleton's Data

The data gathered by Middleton is summarized in Table 4.3 and was obtained from the data supplied in Table 1 on page 536 of Middleton's original work.

Table 4.3: Middleton's Data				
Run	Ambient Depth (m)	Spill Depth (m)	Density Ratio 'r'	Spill Velocity (m/s)
1	.200	.200	.894	.234
2	.200	.200	.891	.250
3	.200	.200	.821	.291
4	.200	.200	.812	.298
5	.300	.300	.900	.268
6	.300	.300	.898	.270
7	.300	.300	.812	.363
8	.300	.300	.816	.358
9	.200	.200	.892	.210
10	.200	.200	.894	.207
11	.300	.300	.894	.290
12	.300	.300	.892	.250
13	.200	.200	.810	.288
14	.200	.200	.820	.266
15	.302	.302	.818	.356
16	.302	.302	.822	.339
17	.203	.203	.887	.230
18	.203	.203	.805	.313
19	.199	.199	.898	.213
20	.199	.199	.818	.282
21	.302	.302	.911	.237
22	.301	.301	.812	.355

This data set is very unique in that Middleton designed the experiment to investigate the behavior of gravity currents. To do this, he used a basic full depth, lock exchange apparatus in which the flume was 5 metres long, 0.50

metres deep and 0.154 metres wide, with a lock created at one end by inserting a rectangular box fitted with a sliding gate. In these experiments, the ambient was salt water and the lock was filled with a slurry mixture of small plastic beads and water. Since the plastic beads had a specific gravity of 1.52, it was necessary to stir the mixture in the lock prior to lifting the gate to ensure that a suspension was created.

In creating Table 4.3, it was necessary to convert between parameters supplied by Middleton and the parameters of importance to the present model. To do this, two important observations had to be made. Firstly, in Middleton's Table 1, there are typographical mistakes in the column headings. Any time that '$\Delta\rho$' is written, '$\Delta\rho/\rho$' was meant. Secondly, in the parameter '$\Delta\rho/\rho$', 'ρ' is the density of the saltwater, which in this situation is the density of the lighter fluid. With these observations, it is very easy to show that the density ratio 'r' may be calculated using the expression

$$r = \frac{1}{1+\frac{\Delta\rho}{\rho}} \tag{4.38}$$

4.5.4 Keulegan's Data

The data provided in this data set is only a fraction of the total volume of data that Keulegan gathered in his experiments. Once again, these experiments were of the full depth, lock exchange variety in which saltwater, released from the lock, was allowed to advance under a fresh water ambient. Although the exact dimensions of the flumes and locks are not known, it is known that more than one flume was used. For the first six runs in Table 4.4, the flume had a width of 0.229 metres and for the remaining seven runs, a flume with a width of 0.113 metres was used.

Table 4.4: Keulegan's Data

Run	Ambient Depth (m)	Spill Depth (m)	Density Ratio 'r'	Spill Velocity (m/s)
1	.455	.455	.991	.104
2	.455	.455	.991	.103
3	.455	.455	.980	.161
4	.455	.455	.963	.221
5	.455	.455	.928	.335
6	.455	.455	.890	.429
7	.112	.112	.994	.044
8	.112	.112	.989	.062
9	.112	.112	.978	.087
10	.112	.112	.960	.126
11	.112	.112	.934	.156
12	.112	.112	.885	.220
13	.112	.112	.849	.250

As was stated, it was only possible to use a small fraction of the data gathered by Keulegan. Unfortunately, in the presentation of his results, there seemed to be a tendency to only illustrate the results in graphical form and only provide a limited quantity of information on each figure describing the basic characteristics of the fluids in each experimental run. As a result, it was necessary to combine measured data from some figures with listed information provided in others in order to compile a data set with enough information to be used in this model.

To obtain the information in Table 4.4, for the first six runs, fluid information was gathered from Table 1 on page 29 of Keulegan's work and for the remaining seven runs, fluid information was obtained from Figure 1. For all runs, initial velocity information was measured from the data presented in Figure 7. In manipulating this data into a form suitable for this model, it was important to realize that

$$V_{\Delta} = \sqrt{\frac{\Delta\rho}{\rho_M} gH} \tag{4.39}$$

in which

$$\frac{\Delta\rho}{\rho_M} = \frac{\rho_2 - \rho_1}{\frac{\rho_1 + \rho_2}{2}} \tag{4.40}$$

Since our model requires the density ratio 'r', it is a simple matter to transform equation 4.40 into the following form

$$r = \frac{\left(2 - \frac{\Delta\rho}{\rho_M}\right)}{\left(2 + \frac{\Delta\rho}{\rho_M}\right)} \tag{4.41}$$

4.6 Results of Analysis

The information in the preceding tables was used as input data to the system of equations presented earlier. This system of equations was entered into the simultaneous equation solving package TKSolverPlus with the results being listed in Table 4.5 and the agreement between predicted and measured slick head velocities illustrated in Figure 4.3. It should be noted that the percent error illustrated in Table 4.5 was calculated using equation 4.42. Using this definition, a positive error is associated with a predicted velocity greater than the measured velocity. It is felt that this method makes it easier to appreciate the significance of the error.

$$\%\ Error = \frac{Pred\ Vel - Meas\ Vel}{Meas\ Vel} * 100 \qquad (4.42)$$

Table 4.5: Summary of Model Results

Investigator	Run	Predicted Velocity	Measured Velocity	Percent Error
O'Brien and Cherno	A-2	0.528	0.595	-11.2
	A-5	0.437	0.416	5.1
	A-6	0.319	0.298	7.1
	A-25	0.317	0.297	6.7
	A-23	0.324	0.298	8.6
	A-14	0.554	0.532	4.2
	A-10	0.470	0.436	7.8
	A-4	0.380	0.379	0.3
	A-12	0.768	0.912	-15.8
	A-13	0.972	0.935	4.0

It is important to note that in Table 4.5, the velocities are reported in the same system of measure as was used by the original researcher (ie. 'ft/s' for O'Brien and Cherno's data and 'm/s' for all others). Although this may cause some confusion, this method was chosen since it is consistent with the input data reported earlier.

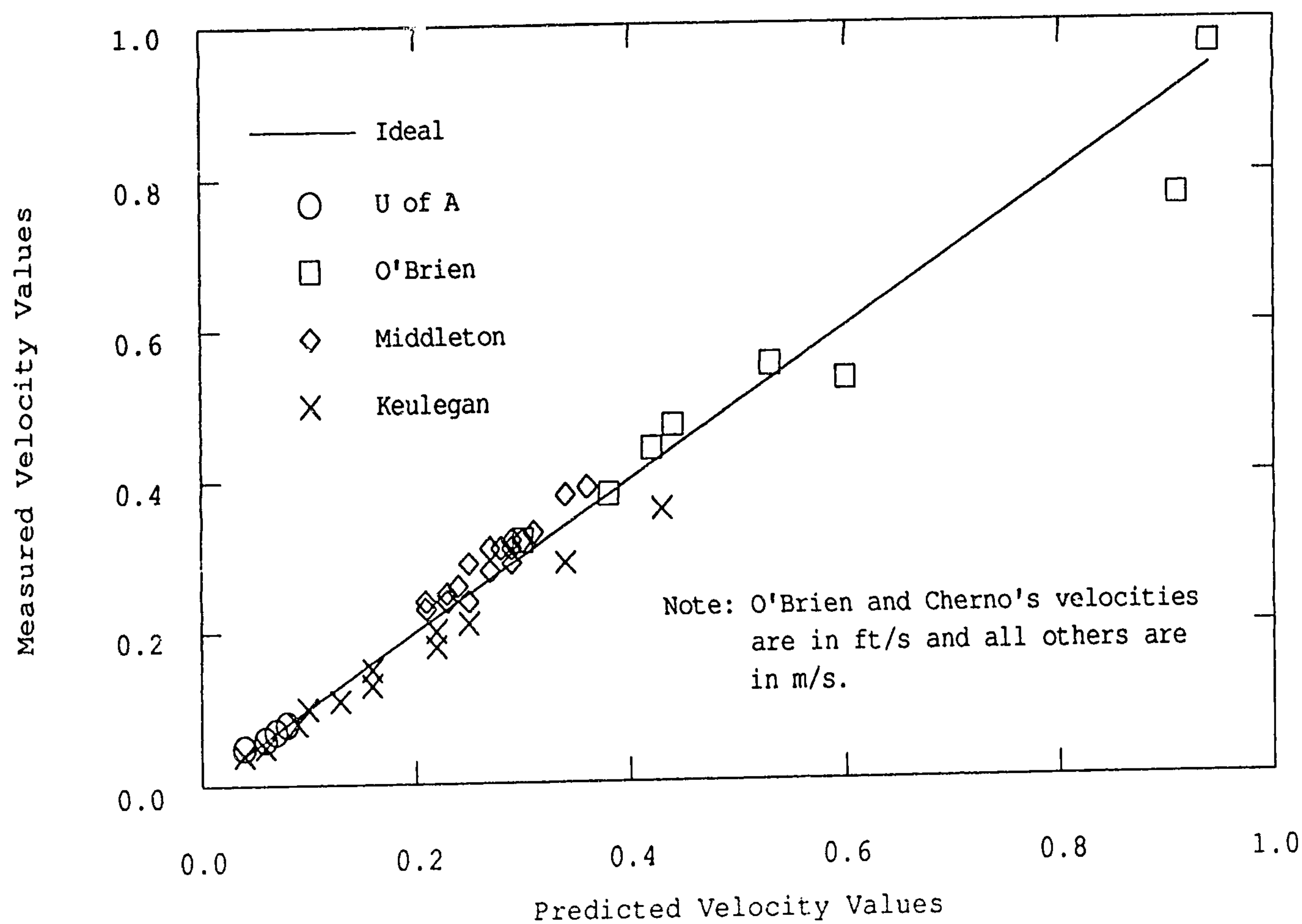
Table 4.5: Summary of Model Results (Continued)

Investigator	Run	Predicted Velocity	Measured Velocity	Percent Error
University of Alberta	116	0.048	0.044	9.0
	117	0.047	0.044	7.1
	118	0.063	0.061	3.3
	119	0.061	0.056	9.0
	120	0.049	0.044	11.2
	121	0.047	0.040	16.5
	122	0.074	0.069	7.6
	123	0.084	0.080	5.6
Keulegan	1	0.100	0.104	-3.7
	2	0.099	0.103	-3.8
	3	0.149	0.161	-7.7
	4	0.204	0.221	-7.7
	5	0.289	0.335	-13.6
	6	0.362	0.429	-15.6
	7	0.041	0.044	-7.8
	8	0.055	0.062	-11.4
	9	0.078	0.087	-10.7
	10	0.106	0.126	-15.8
	11	0.133	0.156	-14.9
	12	0.184	0.220	-16.6
	13	0.213	0.250	-15.0

Table 4.5: Summary of Model Results (Continued)				
Investigator	Run	Predicted Velocity	Measured Velocity	Percent Error
Middleton	1	0.235	0.234	0.5
	2	0.238	0.250	-4.8
	3	0.313	0.291	7.4
	4	0.322	0.298	7.9
	5	0.279	0.268	4.0
	6	0.281	0.270	4.1
	7	0.394	0.363	8.5
	8	0.389	0.358	8.7
	9	0.237	0.210	12.9
	10	0.235	0.207	13.7
	11	0.287	0.290	-1.0
	12	0.290	0.250	16.2
	13	0.323	0.288	12.1
	14	0.314	0.266	18.0
	15	0.387	0.356	8.8
	16	0.383	0.339	13.1
	17	0.245	0.230	6.7
	18	0.330	0.313	5.5
	19	0.230	0.213	7.9
	20	0.314	0.282	11.5
	21	0.263	0.237	11.2
	22	0.393	0.355	10.8

When reviewing Table 4.5, it is interesting to note that for Keulegan's data set, the theory consistently under estimates the head velocities. The reason for this becomes apparent when one reviews the technique by which Keulegan

Figure 4.3: Velocity Comrarisons Between Measured and Predicted Values



obtained his estimates of the initial velocity. Although he gathered his basic data using the same procedure as the other researchers (ie. recording the time required for the front to pass measured points on the side of the flume), his first data point was obtained at a distance of up to twenty lock lengths away from the lock gate. He would then take this data and plot it 'V_Δ/V' versus 'L/H'. By drawing a best fit line through this data and extrapolating back to the zero axis, he obtained the values plotted in his Figure 7. It can be seen in many of his figures, however, that this procedure under estimates the value of 'V_Δ/V_o', resulting in an over estimation of the initial head velocity 'V_o'.

In light of the observation regarding the manner in which Keulegan arrived at his estimate of the initial velocity, it was felt that summary statistics of the calculated error should consider the effect of Keulegan's data set. These statistics are summarized in Table 4.6, and from this table it can be seen that if one considers the absolute error, Keulegan's data set has only a small effect on the final statistics.

Table 4.6: Summary of Error Statistics

Statistic	Actual Error (%)	Absolute Error (%)	Comment
Average	2.37	9.05	With Keulegan's Data
Std. Dev.	9.80	4.46	
Average	6.74	8.38	Without Keulegan's Data
Std. Dev	6.57	4.27	

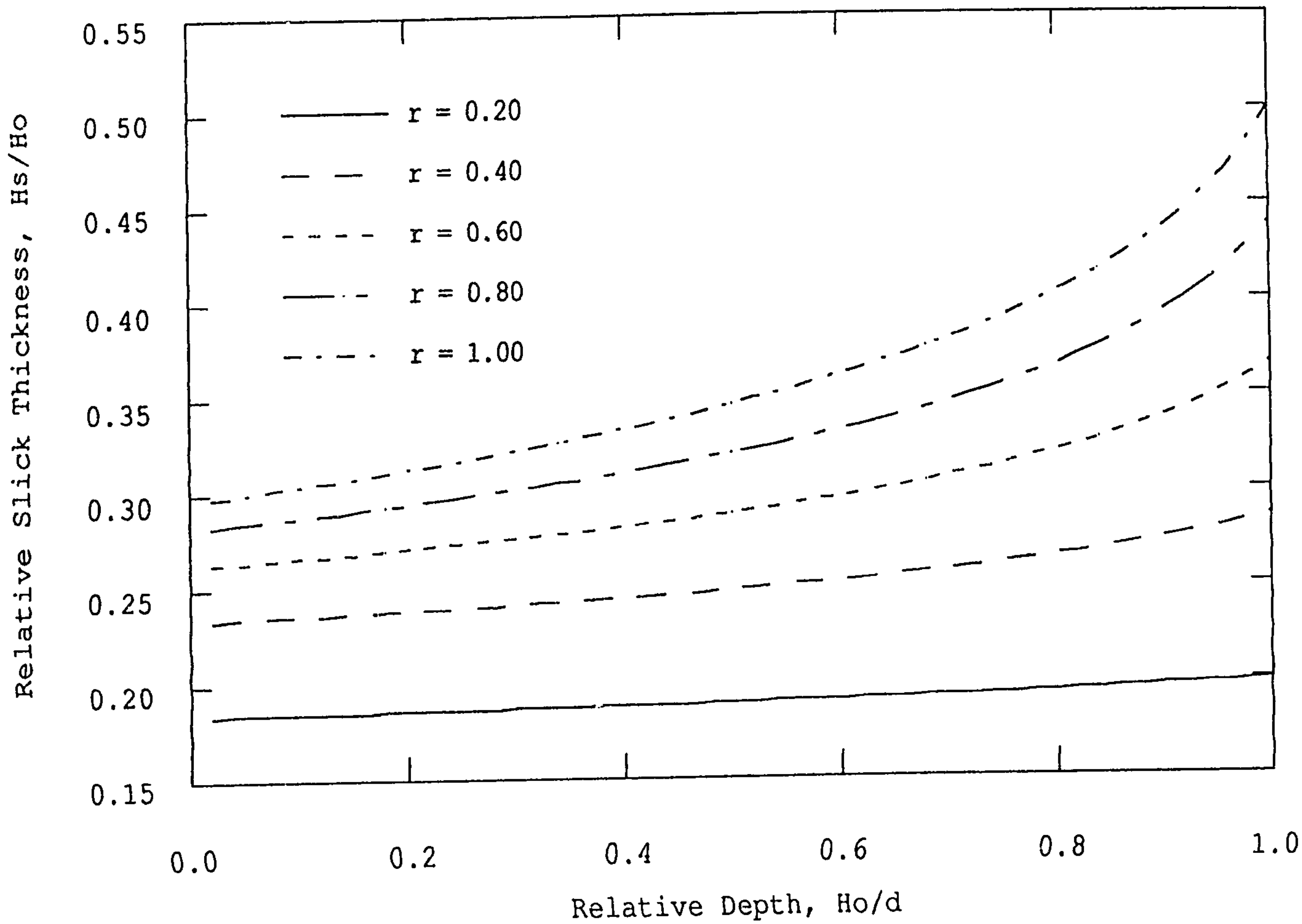
4.7 Design Figures

Since in practical design, engineers often require a mechanism for arriving at an approximate first guess of the final answer, an attempt has been made to summarize the results of this model in design figures. Although there are many parameters that could be summarized in such figures, it was felt that under most situations the parameters of greatest interest would be the slick thickness and the velocity of the slick head.

The first of these figures, Figure 4.4, attempts to summarize the change in slick thickness for different values of the density ratio 'r' and for different initial spill depths 'H_o'. One interesting feature of this plot is that, as the density ratio becomes smaller, the slick thickness also reduces for all values of the initial spill depth. Although on the surface this may appear false, since in a classic dam break problem the negative surge has a definite thickness, it is important to realize that a classic dam break problem is fundamentally different from this spill model.

Since this model assumes the fluids will experience the same hydrostatic pressure at the bottom of the gate, as the density ratio approaches zero, it becomes impossible for there to be any penetration of the spilled material into the ambient (Refer to equation 4.15). In a classic dam break problem, however, there exists a definite difference in hydrostatic pressure across the base of the gate and, thus, it is possible

Figure 4.4: Variation of Relative Slick Thickness with Relative Depth for Various Values of 'r'



to have a lock containing a fluid of considerably smaller density than the ambient.

The second design figure, Figure 4.5, attempts to illustrate the change in relative head velocity with the density ratio 'r' and the initial spill depth 'H_o'. If the relative head velocity is defined as

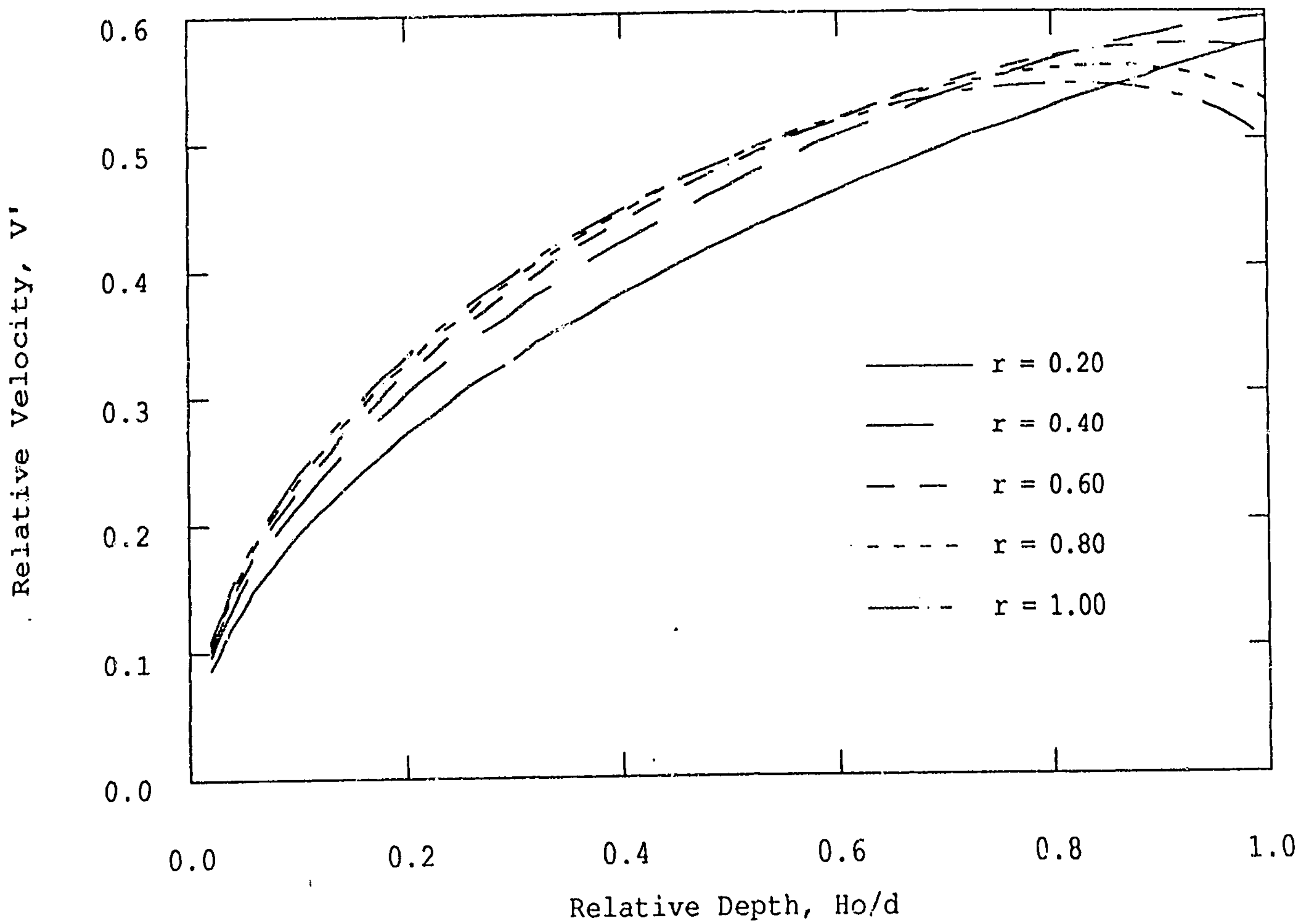
$$V' = \frac{V_s}{\sqrt{g(1-r)d}} \tag{4.43}$$

to be consistent with Keulegan's definition, it is interesting to note that for large density ratios, the maximum relative velocity does not occur when the initial spill depth is at its maximum.

4.8 Summary

Within this chapter, many things have been accomplished. We have reviewed the development of O'Brien and Cherno's spill initiation model and have shown how this model can be generalized to predict slick thicknesses. Using the head development model based upon Benjamin's work, we have managed to create a closed system of equations, and using four diverse data sets, we have shown that this model is capable of predicting slick head velocities within (on average) ten percent of the measured value. Having proven the validity of this model, we then created design figures which will allow an engineer to easily arrive at reasonable estimates of both the slick thickness and the slick head velocity.

Figure 4.5: Variation of Relative Slick Velocity with Relative Depth for Various Values of 'r'



Before continuing to the next chapter, there are a few additional points that should be discussed. Firstly, it is important to note that this model typically over estimates the velocity of the head. Since in most situations this will be considered a conservative estimate, it builds an implicit safety factor into an engineers calculations.

Secondly, since many of the data sets were derived from dense spill data, it could be argued that they should not be used to test the validity of the model since the model was developed explicitly for buoyant spills. If we were considering large travel times such that bed friction might be significant, this criticism would certainly be valid. However, for very short times, the friction must be small and, therefore, there is little reason to expect there to be a difference between dense and buoyant spills in the initial stages. This position is confirmed by the consistent distribution of errors in each data set.

Finally, it should be made explicit that the velocity predicted by this model will only be maintained until the intrusive front of ambient fluid penetrating under the original spill volume reaches the end of the lock. When the end of the lock is reached, there will be a sudden decrease in driving head and an accompanying dramatic decrease in slick head velocity.

CHAPTER 5

[illegible] MASS

5.1 Introduction

As was seen in the previous chapter, the present model seems to over predict the velocities of the head. Although on average the magnitude of the error is certainly within acceptable limits, the model would be more generally accepted by both the academic and practising engineering communities if the error were somewhat smaller. Thus, to decrease the error, we must attempt to broaden the model by including significant features that, up to this point, have been neglected. One such feature is associated with the modelling of the induced flows in the ambient ahead of the spill front.

5.2 Theory

As was briefly discussed in the literature review, Vallentine (1965) has described a method for determining the momentum associated with the induced velocities in an ambient produced by the passage of a body through the fluid. In the example that he presents, he simulates the velocity field produced in an ambient by the passage of a circular cylinder by modelling the cylinder as a doublet. By determining the total kinetic energy of the fluid displaced by the cylinder, it is possible to determine the 'virtual' mass that should be added to the original cylinder to compensate for the induced velocities in the ambient.

5.3 Exact Solution

Following the procedure used by Vallentine and referring to Figure 5.1, it may be possible to estimate the virtual mass associated with flows induced by the head of a spill if we assume that the profile of the head is approximately that of a quarter circle. Since we would also like to have this model apply in situations where head thickness is of the same order as the depth of the ambient, we must also develop a method for simulating the effects of the bed. Using the techniques of hydrodynamics, this can be accomplished by using a virtual doublet as illustrated in Figure 5.2.

Now that we have a conceptual model for describing the induced velocities in the ambient created by the passage of the spill head, let us now attempt to develop the equations which describe this model. For a doublet, the equations that describe the streamlines 'Ψ' and velocity potential lines 'Φ' are as follows:

$$\Psi = -\frac{\mu}{r}\sin\theta \qquad (5.1)$$

$$\Phi = \frac{\mu}{r}\cos\theta \qquad (5.2)$$

where 'μ' describes the source strength and 'r' and 'θ' are the cylindrical coordinates of a point of interest.

In order for a doublet to model the effects produced in the ambient by a moving head, we must first relate the velocity of the head 'V_H' to the doublet source strength 'μ'.

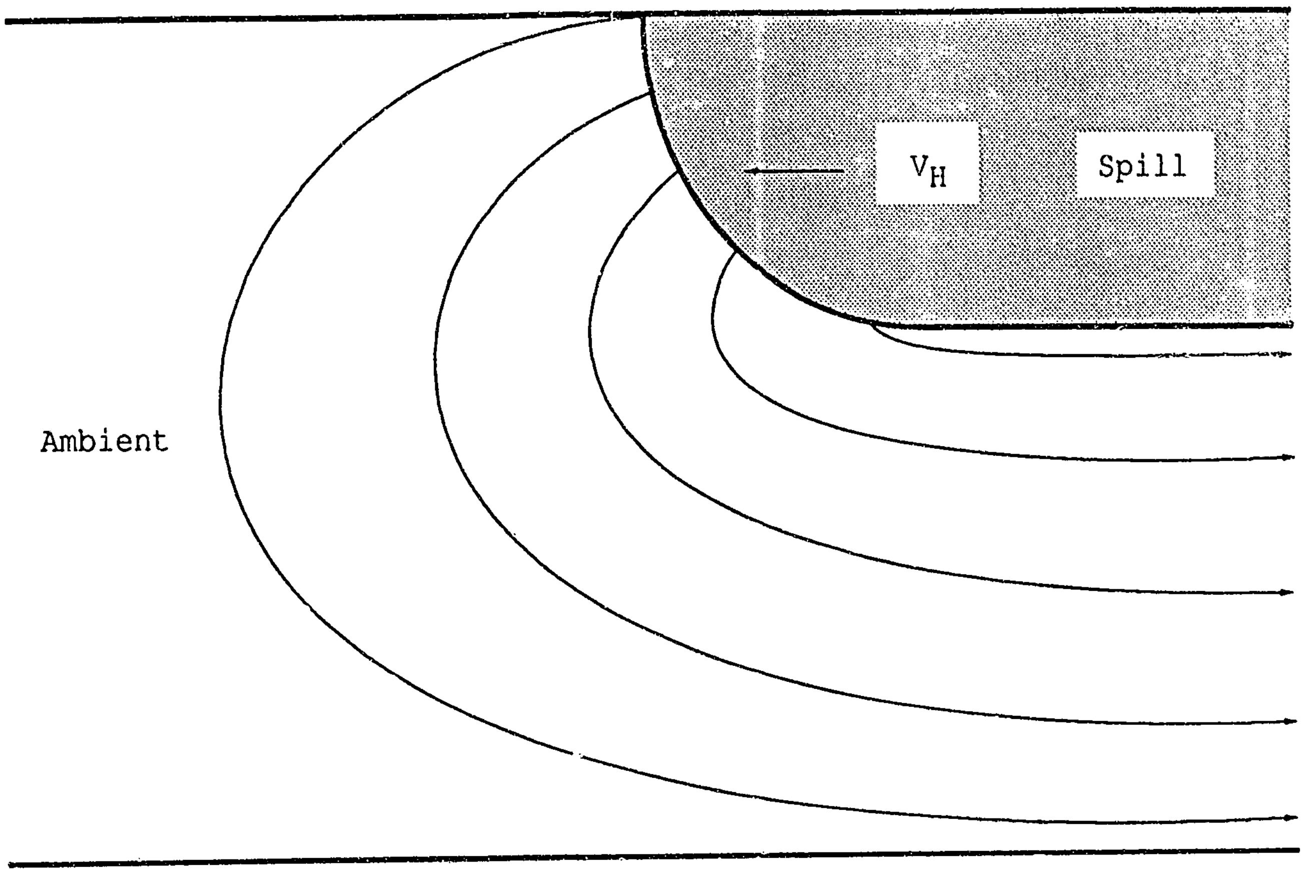


Figure 5.1: Induced Flow in a Still Ambient

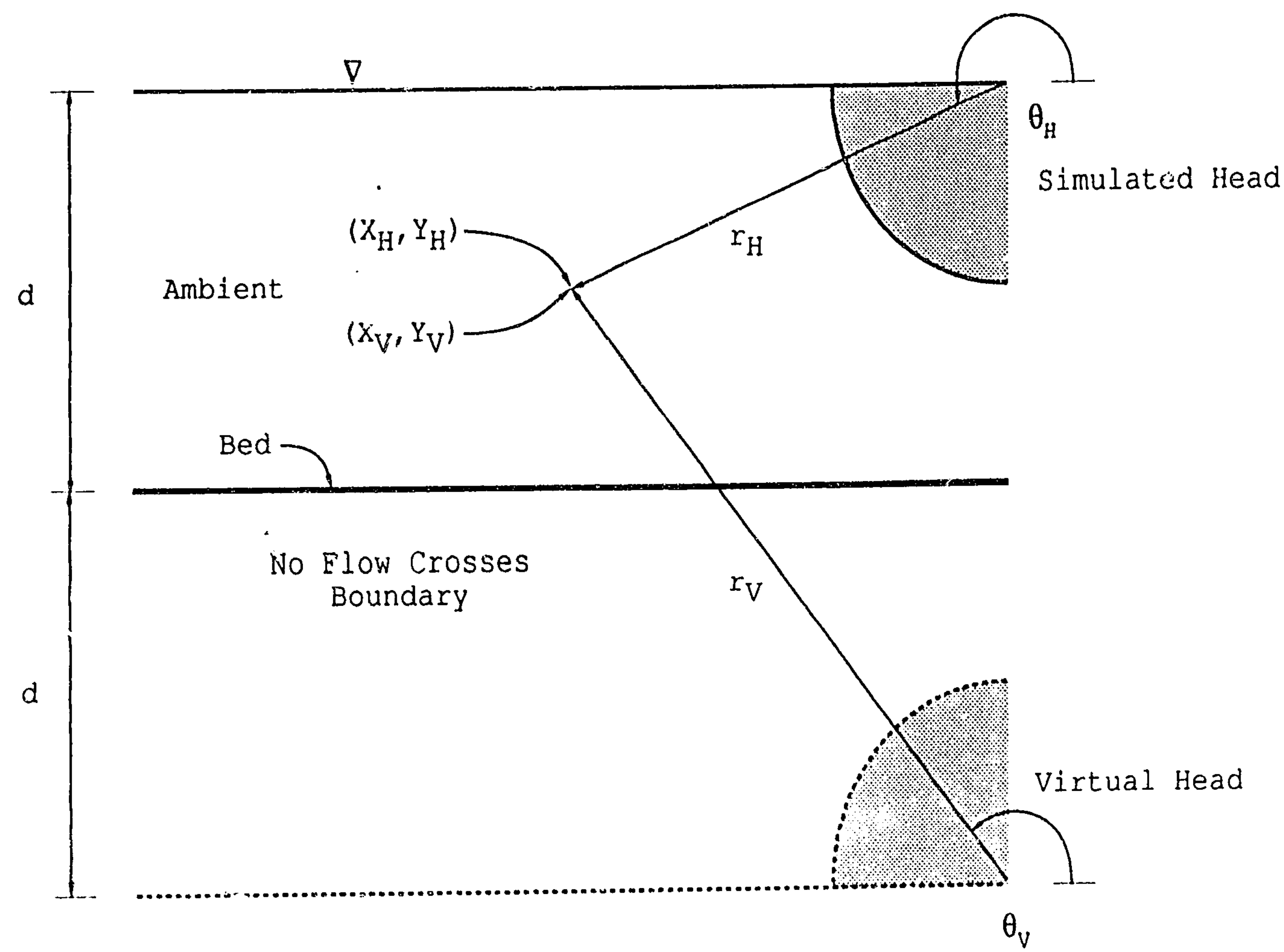


Figure 5.2: Arrangement for Simulating Induced Flow

To do this, let us consider a situation in which the head (modelled as a cylinder) is at rest and the ambient is flowing around it with a uniform approach velocity of 'V_H'. For this case, the following equations describe the streamlines and velocity potential lines

$$\Psi = V_H r \operatorname{Sin}\theta - \frac{V_H a^2}{r} \operatorname{Sin}\theta \tag{5.3}$$

$$\Phi = V_H r \operatorname{Cos}\theta + \frac{V_H a^2}{r} \operatorname{Cos}\theta \tag{5.4}$$

in which the first terms represent a uniformly flowing ambient and the second terms represent a stationary cylinder within that ambient.

If the ambient is now brought to rest, the first terms in equations 5.3 and 5.4 will disappear leaving us with the equations

$$\Psi = -\frac{V_H a^2}{r} \operatorname{Sin}\theta \tag{5.5}$$

$$\Phi = \frac{V_H a^2}{r} \operatorname{Cos}\theta \tag{5.6}$$

These equations describe a cylinder of radius 'a' moving with a velocity 'V_H' through a stationary ambient. By comparing these equations with equations 5.1 and 5.2, it is immediately apparent that

$$\mu = V_H a^2 \tag{5.7}$$

which is precisely the expression we need.

In our situation, however, we have two doublets; a real doublet representing the spill head and a virtual doublet simulating the bed. Thus, the following equations represent our system

$$\Psi = -V_H a^2 \left(\frac{1}{r_H} \mathrm{Sin}\theta_H + \frac{1}{r_V} \mathrm{Sin}\theta_V \right) \qquad (5.8)$$

$$\Phi = V_H a^2 \left(\frac{1}{r_H} \mathrm{Cos}\theta_H + \frac{1}{r_V} \mathrm{Cos}\theta_V \right) \qquad (5.9)$$

in which the variables are as defined in Figure 5.2.

In examining equations 5.8 and 5.9 it becomes immediately apparent that they are defining the flow field relative to two origins; one associated with the virtual doublet and one associated with the real doublet. To relate these two origins, we must use the expressions

$$X_H = X_V \qquad (5.10)$$

$$Y_H = Y_V \qquad (5.11)$$

By using equations 5.10 and 5.11 as well as the general expressions

$$X = r\,\mathrm{Cos}\theta \qquad (5.12)$$

$$Y = r\,\mathrm{Sin}\theta \qquad (5.13)$$

which transform polar coordinates into cartesian coordinates, it is possible to show that the general stream function and velocity potential function for the induced flow region may be rewritten as

$$\Psi = -V_H a^2\left(\frac{\text{Sin}\theta_H}{r_H}\right) - V_H a^2\left(\frac{r_H \text{Sin}\theta_H}{r_H^2 + 4dr_H \text{Sin}\theta_H + 4d^2}\right) - V_H a^2\left(\frac{2d}{r_H^2 + 4dr_H \text{Sin}\theta_H + 4d^2}\right) \tag{5.14}$$

$$\Phi = V_H a^2\left(\frac{\text{Cos}\theta_H}{r_H}\right) + V_H a^2\left(\frac{r_H \text{Cos}\theta_H}{r_H^2 + 4dr_H \text{Sin}\theta_H + 4d^2}\right) \tag{5.15}$$

With equations 5.14 and 5.15 we are able to describe the induced flows in the ambient in terms of streamlines and velocity potential lines in a cylindrical coordinate system relative to the head of the spill. However, since we would like to develop an expression for the total kinetic energy associated with the induced flow, we must first be able to describe the induced velocity field in the ambient. The theory of hydrodynamics allows us do this by using the following relationships to determine the radial 'V_r' and angular 'V_θ' component velocities

$$V_r = \frac{\partial \Phi}{\partial r} \tag{5.16}$$

$$V_\theta = -\frac{\partial \Psi}{\partial r} \tag{5.17}$$

By applying these relationships to equations 5.14 and 5.15, it is possible to develop the following expressions which describe the component velocities at any point in the induced flow

$$V_r = - V_H a^2 \left(\frac{\text{Cos}\theta_H}{r_H^2} \right) + V_H a^2 \left(\frac{(r_H^2 - 2r_H + 4d^2)\,\text{Cos}\theta_H}{(r_H^2 + 4d(r_H \text{Sin}\theta_H + d))^2} \right) \tag{5.18}$$

$$V_\theta = - V_H a^2 \left(\frac{\text{Sin}\theta_H}{r_H^2} \right) - V_H a^2 \left(\frac{r_H^2 \text{Sin}\theta_H + 4d(r_H + d\,\text{Sin}\theta_H)}{(r_H^2 + 4d(r_H \text{Sin}\theta_H + d))^2} \right) \tag{5.19}$$

The next step in determining the virtual mass of this system is to determine the total kinetic energy 'KE' of the induced ambient fluid. In general, the kinetic energy of a fluid element of mass 'm' and velocity 'V' is described as

$$KE = \frac{1}{2} m V^2 \tag{5.20}$$

in which 'V' is the resultant of the component velocities and is defined as

$$V = \sqrt{V_r^2 + V_\theta^2} \tag{5.21}$$

To determine the total kinetic energy, we must perform an integration over the entire induced flow region. This is possible if we rewrite equation 5.20 in the following form

$$KE = \frac{1}{2}\rho\int_{\pi}^{\frac{3\pi}{2}}\int_{a}^{b} V^2 r\, dr\, d\theta_H \tag{5.22}$$

in which 'b' is the boundary described by the equation

$$b = \frac{-d}{\mathrm{Sin}\theta_H} \tag{5.23}$$

At this point it is worth noting that the expression described by equation 5.22 is very difficult (if not impossible) to solve in an exact fashion. Since any solution will require numerical integration, it is felt that this violates the stated mandate upon which this model is being developed as it adds an order of complexity not appreciated by many practising engineers. For this reason, we will now attempt to develop an approximate model to describe the 'virtual' mass associated with the induced flow.

5.4 Approximate Solution

In the development of this approximate solution, we will still use the approach described by Vallentine and will follow the exact solution up to the development of equations 5.18 and 5.19. Before proceeding further, however, it is useful to observe that in equations 5.18 and 5.19, the first terms describe the velocities created if the spill occurred over an infinite depth, and the second terms modify the result to reflect the effects of the boundary. Furthermore, if it were not for the terms representing the effects of the boundary, an exact solution to equation 5.22 would be possible. Thus, in the development of this approximate solution, we will attempt to remove the second terms from each of these equations.

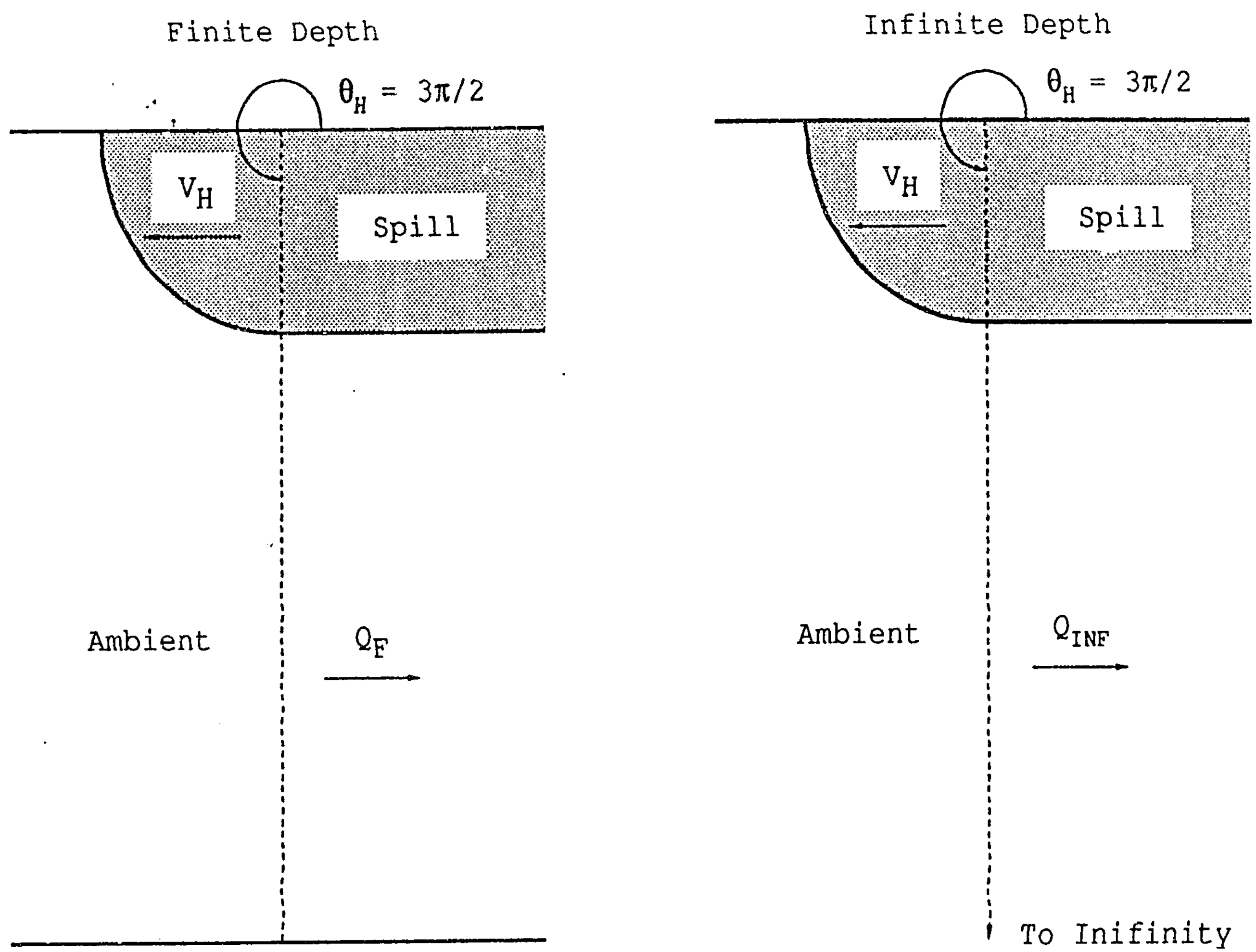
To do this, we will make the assumption that flow over a deep ambient and flow over a shallow ambient are dynamically similar. In addition, we will assume that the induced momentum in a shallow ambient will be equal to the induced momentum in an infinitely deep ambient if it can be shown that the kinetic energy of the fluid crossing a geometrically similar section in each system is equal. This situation is illustrated in Figure 5.3 and, for convenience, let us find the kinetic energy through the section at θ_H equal to $3\pi/2$.

Let us first consider the situation where a spill has occurred over an ambient of finite depth. Since there is no radial component to the velocity vector 'V' when θ_H is equal to $3\pi/2$, only equation 5.19 is required to completely describe the velocity profile. Substituting for θ_H in this equation, we get

$$V = V_\theta = V_H a^2 \left(\frac{1}{r_H^2} + \frac{1}{(r_H - 2d)^2} \right) \tag{5.24}$$

To determine the total kinetic energy of the fluid passing through this section, we will use the expression

$$KE = \frac{1}{2} \rho \int_a^d V^2 \, dr \tag{5.25}$$



Dynamically Similar if $KE_F = KE_{INF}$

Figure 5.3: Similarity Condition

If we substitute equation 5.24 into this equation and perform the integration, after some simplification, we will arrive at the result

$$KE = \frac{1}{2}\frac{\rho a^4 N}{d^3} V_H^2 \tag{5.26}$$

where

$$N = \frac{d^3-a^3}{3a^3} + \frac{1}{2}\left(\ln\left(\frac{2d-a}{a}\right) + \frac{d-a}{a} + \frac{a-d}{a-2d}\right) + \frac{(a-2d)^3+d^3}{3d^3(a-2d)^3} \tag{5.27}$$

Let us now consider the case of a spill with head thickness 'b' occurring over an ambient of infinite depth. As before, when θ_H is equal to $3\pi/2$, there is no radial component to the velocity and the velocity profile is described by the equation

$$V = V_\theta = \frac{b^2}{r^2} V_H \tag{5.28}$$

Substituting this expression into equation 5.25 and performing the integration with the upper limit now equal to infinity, we arrive at the following equation which describes the total kinetic energy of the fluid passing through the section per unit width of flow

$$KE = \frac{1}{2}\frac{\rho b}{3} V_H^2 \tag{5.29}$$

If we now equate equations 5.26 and 5.29, we find that

$$b = \frac{3a^4 N}{d^3} \tag{5.30}$$

Equation 5.30 is precisely the expression we were attempting to create and allows us to replace a slick of head thickness 'a' spreading over an ambient of finite depth with a slick of head thickness 'b' spreading over an ambient of infinite depth. Thus, if we use 'b' as the effective head thickness, we may use a single moving doublet to model the induced flow in the ambient due to the passage of the head.

For a single moving doublet, the stream and velocity potential functions are as defined by equations 5.5 and 5.6 if 'a' is replaced with 'b'. If we substitute these expressions into equations 5.16 and 5.17 and perform the differentiations, we will arrive at the following equations which describe the radial 'V_r' and angular 'V_θ' component velocities

$$V_r = -\frac{V_H b^2}{r^2}\mathrm{Sin}\theta_H \tag{5.31}$$

$$V_\theta = -\frac{V_H b^2}{r^2}\mathrm{Cos}\theta_H \tag{5.32}$$

Using equation 5.21, it is now possible to show that the magnitude of the resultant velocity 'V' is described by

$$V = \frac{V_H b^2}{r^2} \tag{5.33}$$

To determine the total kinetic energy for this system, let us first rewrite equation 5.22 as

$$KE = \frac{1}{2}\rho\int_b^\infty\int_\pi^{\frac{3\pi}{2}} V^2 r\, d\theta_H\, dr \tag{5.34}$$

Substituting equation 33 into this expression and performing the integrations leads us to

$$KE = \frac{1}{2}\frac{\rho\pi b^2}{4}V_H^2 \tag{5.35}$$

Comparing this expression to that of equation 5.20, it can be seen that the 'virtual' mass 'm_H' that must be added to the original spill mass to compensate for induced velocities in the ambient due the passage of the head is

$$m_H = \frac{\rho\pi b^2}{4} \tag{5.36}$$

5.5 Summary

With equation 5.36 we have an estimate of the 'virtual' mass which must be added to the original spill mass if the effects of induced velocities in the ambient are to be compensated for. In the development of this equation, we have departed from a rigorous mathematical treatment with only numerical solutions, to a simplified approximate model which has an analytical solution.

CHAPTER 6

SPILL INITIATION WITH VIRTUAL MASS

6.1 Introduction

In chapter four, we developed a version of the spill initiation model that seemed capable of predicting the initial velocity of the spill, on average, to within nine percent of the measured value. Although not discussed at the time, one possible cause of this error is the fact that induced velocities in the ambient, ahead of the slick, created by the passage of the head were neglected. Furthermore, since as the spill slumps there will be an intrusive surge of ambient material into the body of the original spill volume, there will also be induced velocities in the spill ahead of this ambient intrusion that were not accounted for. It is the goal of this chapter to incorporate the effects of these induced velocities into the earlier spill initiation model and to estimate the importance of this effect on the predicted initial slick velocity.

6.2 Model Development

To compensate for the effects of the induced velocities in the ambient, the typical procedure is to calculate a 'virtual' mass which must be added to the original mass of the moving body such that both the original and adjusted systems contain the same total kinetic energy. A more thorough discussion of this concept was presented in the previous chapter and it is from this chapter that we will extract the equations used to model the virtual mass.

In general, the virtual mass associated with the induced velocities ahead of a slick, created by the slick motion, is described by the equation

$$m_H = \frac{\rho \pi b^2}{4} \tag{6.1}$$

in which 'b' is the 'virtual' or equivalent head thickness required to convert a slick advancing over an ambient of finite depth to one advancing over an ambient of infinite depth. To determine the magnitude of 'b', we will use equation 5.30 in which 'N' is a scale factor described by equation 5.27. In both these equations 'a' is the original thickness of the slick and 'd' is the depth of the ambient.

As was mentioned earlier, when a spill volume slumps, there are two advancing 'slick' fronts. One is created by the advancement of the spilled material over the ambient, and the other is created by the advancement of ambient material under the original spill volume. This situation is illustrated in Figure 6.1 and associated with each front will be some quantity of virtual mass. If 'b_S' is the virtual thickness of the spill slick and 'b_A' is the virtual thickness of the ambient intrusion under the spill, Table 6.1 summarizes the values of 'a' and 'd' that must be substituted into their respective defining equations.

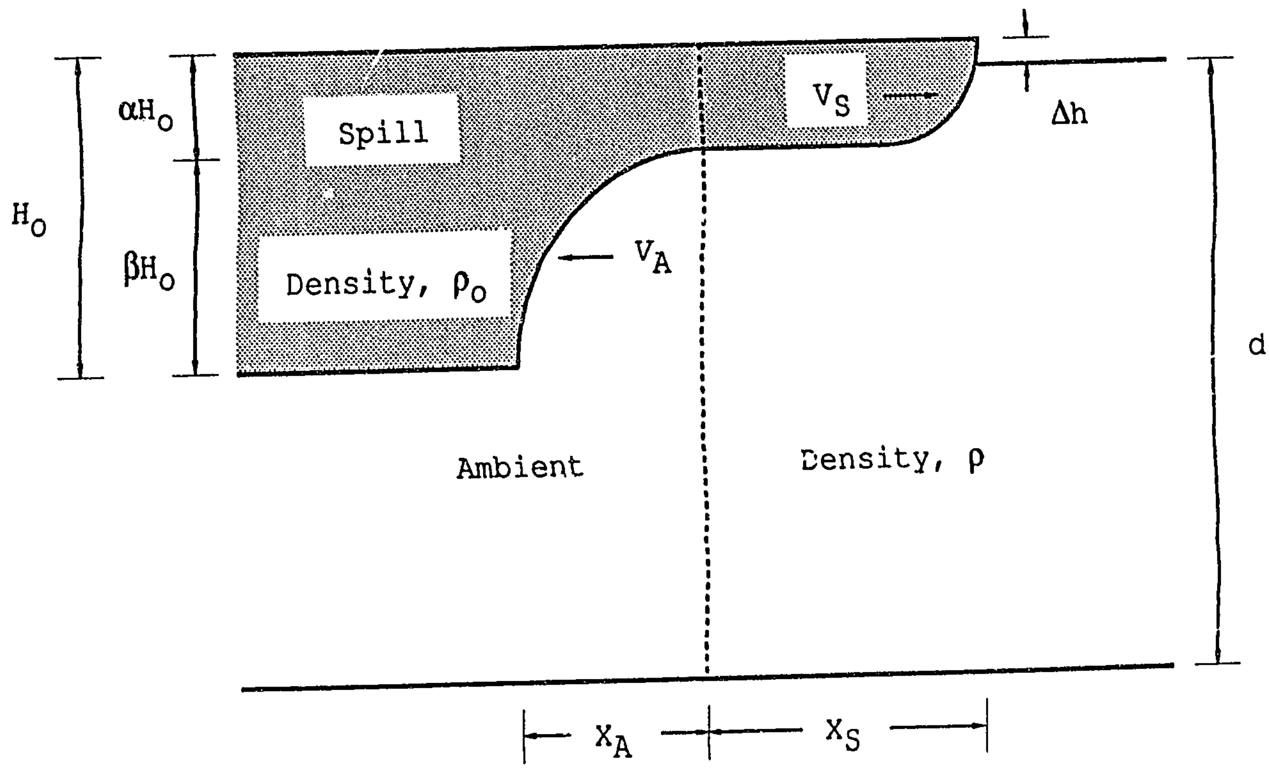


Figure 6.1: Proposed Model With Virtual Mass

Table 6.1: Summary of Virtual Slick Parameters		
Slick	a	d
Spill, b_S	$\alpha H_\circ$	d
Ambient, b_A	$\beta H_\circ$	$\alpha H_\circ$

As was done in the flow initiation model which neglected virtual mass, this model will be developed following the procedure of O'Brien and Cherno (1934). This model is based upon equation 4.4 and since Figure 6.1 is identical to Figure 4.2 at time $t_\circ$, equation 4.15 will still describe the net unbalanced force 'F_{net}' in equation 4.4.

It is in the description of the total mass 'M' induced into motion that this model will differ from the previous model. Referring to Figure 6.1, it can be seen that one possible description of the mass induced into motion is

$$M = \left(\rho_\circ (X_A+X_S)\,\alpha H_\circ + \frac{\rho\pi b_S^2}{4}\right) + \left(\rho\,(X_A+X_S)\,\beta H_\circ + \frac{\rho_\circ\pi b_A^2}{4}\right) \tag{6.2}$$

If the total momentum associated with this mass is 'MV', we may write an expression for the total momentum as follows

$$MV = \left(\rho_\circ (X_A+X_S)\,\alpha H_\circ + \frac{\rho\pi b_S^2}{4}\right) V_S + \left(\rho\,(X_A+X_S)\,\beta H_\circ + \frac{\rho_\circ\pi b_A^2}{4}\right) V_A \tag{6.3}$$

Since equation 4.4 requires an expression for the rate of change of momentum, it is necessary to take the derivative of equation 6.3 with respect to time. If we perform this operation and substitute equations 4.18, 4.19, 4.21 and 4.22 into the resulting expression in order reduce the number of parameters, after simplification we find that

$$\frac{d}{dt}(MV) = \frac{\rho(\alpha+\beta)(1+r)\alpha H_o}{\beta}\frac{d}{dt}\left(X_S\frac{dX_S}{dt}\right) + \frac{\rho\pi}{4\beta}\left(\beta b_S^2 + r\alpha b_A^2\right)\frac{d^2X_S}{dt^2} \tag{6.4}$$

After substituting this expression and equation 4.15 into equation 4.4, we find that

$$\frac{\rho_o g(1-r)H_o^2}{2} = \frac{\rho(\alpha+\beta)(1+r)\alpha H_o}{\beta}\frac{d}{dt}\left(X_S\frac{dX_S}{dt}\right) + \frac{\rho\pi}{4\beta}\left(\beta b_S^2 + r\alpha b_A^2\right)\frac{d^2X_S}{dt^2} \tag{6.5}$$

Continuing with the procedure described by O'Brien and Cherno, if we are to develop an expression for the initial velocity 'V_S' we must first find an expression for the slick penetration distance 'X_S'. This can be done by integrating equation 6.5. After the first integration, we find that

$$\frac{r g(1-r)H_o t}{2} = \frac{\alpha(\alpha+\beta)(1+r)}{\beta}X_S\frac{dX_S}{dt} + \frac{\pi}{4\beta H_o}\left(\beta b_S^2 + r\alpha b_A^2\right)\frac{dX_S}{dt} + Const \tag{6.6}$$

However, since at time t_o (ie. at t=0) the spill has not yet started to spread, the penetration distance 'X_s' must be zero and the constant in this equation must also be zero.

After the second integration, equation 6.6 becomes

$$\frac{rg(1-r)H_o t^2}{4} = \frac{\alpha(\alpha+\beta)(1+r)}{2\beta}X_S^2 + \frac{\pi}{4\beta H_o}\left(\beta b_S^2 + r\alpha b_A^2\right)X_S + Const \tag{6.7}$$

in which the constant must be zero for the same reasons as in the previous equation. This equation can be easily solved for X_s using the following form of the quadratic equation

$$X_S = \frac{-B+\sqrt{B^2-4AC}}{2A} \tag{6.8}$$

in which

$$A = \frac{\alpha(\alpha+\beta)(1+r)}{2\beta} \tag{6.9}$$

$$B = \frac{\pi}{4\beta H_o}\left(\beta b_S^2 + r\alpha b_A^2\right) \tag{6.10}$$

$$C = -\frac{rg(1-r)H_o t^2}{4} \tag{6.11}$$

Using the definition of the initiation velocity 'V_s' supplied by equation 4.19 and the simplification offered by equation 4.29, it is possible to show that

$$V_S = \frac{rg(1-r)H_o t}{2\sqrt{B^2-4AC}} \tag{6.12}$$

In examining equation 6.12, it is interesting to note that all of the effects of the virtual mass are contained in the coefficient 'B'. Thus, in situations under which the virtual mass is not important (ie. B=0), we would expect equation 6.12 to reduce to equation 4.30. After some work, it can be shown that the reduction does occur, implying that equation 4.30 is a special case of equation 6.12.

6.3 Model Confirmation

To test the predictive capabilities of equation 6.12, we used TKSolverPlus and the same data sets that were used to test equation 4.30. These data sets have been described previously and, as a group, span a very broad range of experimental conditions. Since time is also a parameter in this equation, it was necessary to test the model for different assumed slumping times. For convenience, times of 1 second, 5 seconds and 10 seconds were chosen. The results of this analysis are summarized in Table 6.2.

With reference to Table 6.2, several observations can be made. Firstly and perhaps most important, is that in terms of absolute error, the addition of virtual mass has changed the average error over all data sets by, at most, approximately one quarter of one percent. This would seem to indicate that in most situations the effect of virtual mass can be safely neglected.

The second interesting observation is that it appears that the errors are converging towards those for the case in which virtual mass was neglected. Although perhaps not obvious, this is expected since for very large spill durations the effects of the virtual mass must approach zero and the predictions of the two models must converge.

An observation that can be made that was not expected is the fact that the effect of the virtual mass is not a maximum at the shortest times. From Table 6.2, it appears that for the times used, the maximum effect occurred when the duration of spill slumping was 5 seconds. This suggests that it does take some time for the effects of the induced flows to become fully developed. Of coarse one must be careful in drawing such conclusions since the effect may not be common to all data sets.

Table 6.2: Summary of Error Statistics

Statistic	Actual Error (%)	Absolute Error (%)	Comment
Average	2.93	9.22	With Keulegan's Data, t=1 sec
Std. Dev.	9.71	4.21	
Average	7.46	8.64	Without Keulegan's Data, t=1 sec
Std. Dev.	5.88	3.94	
Average	2.85	9.29	With Keulegan's Data, t=5 sec
Std. Dev.	9.79	4.20	
Average	7.37	8.71	Without Keulegan's Data, t=5 sec
Std. Dev.	6.09	3.96	
Average	2.54	9.14	With Keulegan's Data, t=10 sec
Std. Dev.	9.78	4.31	
Average	6.96	8.51	Without Keulegan's Data, t=10 sec
Std. Dev	6.37	4.08	

Since of the three slumping times used, a time of 5 seconds seemed to produce the largest change in the average absolute error, a summary of the results from this run are listed in Table 6.3. As in Table 4.5, the units for each velocity are consistent with the original measurement system.

Table 6.3: Summary of Model Results, t=5 sec

Investigator	Run	Pred Vel	Meas Vel	Percent Error	Error Change
University of Alberta	116	0.048	0.044	9.0	0.0
	117	0.047	0.044	7.1	0.0
	118	0.063	0.061	3.3	0.0
	119	0.061	0.056	9.0	0.0
	120	0.049	0.044	11.2	0.0
	121	0.047	0.040	16.5	0.0
	122	0.074	0.069	7.6	0.0
	123	0.084	0.080	5.6	0.0
Keulegan	1	0.100	0.104	-3.6	0.0
	2	0.099	0.103	-3.8	0.0
	3	0.149	0.161	-7.7	0.0
	4	0.204	0.221	-7.7	0.0
	5	0.289	0.335	-13.6	0.0
	6	0.362	0.429	-15.6	0.0
	7	0.041	0.044	-7.8	0.0
	8	0.055	0.062	-11.4	0.0
	9	0.078	0.087	-10.7	0.0
	10	0.106	0.126	-15.8	0.0
	11	0.133	0.156	-14.9	0.0
	12	0.184	0.220	-16.6	0.0
	13	0.213	0.250	-15.0	0.0

Table 6.3: Summary of Model Results (Continued)					
Investigator	Run	Pred Vel	Meas Vel	Percent Error	Error Change
Middleton	1	0.235	0.234	0.5	0.0
	2	0.238	0.250	-4.8	0.0
	3	0.313	0.291	7.4	0.0
	4	0.322	0.298	7.9	0.0
	5	0.279	0.268	4.0	0.0
	6	0.281	0.270	4.1	0.0
	7	0.394	0.363	8.5	0.0
	8	0.389	0.358	8.7	0.0
	9	0.237	0.210	12.9	0.0
	10	0.235	0.207	13.7	0.0
	11	0.287	0.290	-1.0	0.0
	12	0.290	0.250	16.2	0.0
	13	0.323	0.288	12.1	0.0
	14	0.314	0.266	18.0	0.0
	15	0.387	0.356	8.8	0.0
	16	0.383	0.339	13.1	0.0
	17	0.245	0.230	6.7	0.0
	18	0.330	0.313	5.5	0.0
	19	0.230	0.213	7.9	0.0
	20	0.314	0.282	11.5	0.0
	21	0.263	0.237	11.2	0.0
	22	0.393	0.355	10.8	0.0

In calculating the error in this table, we once again used equation 4.42. This method has the advantage of using the sign of the error to indicate whether the model was over or under predicting when estimating the head velocity.

To illustrate the effect of virtual mass, the "Error Change" column was added. This illustrates the magnitude of the change in absolute error between the cases when virtual mass is included and when it is neglected. If the change is positive, this indicates that the addition of virtual mass caused the model to produce a larger absolute error than if the virtual mass were neglected.

Table 6.3: Summary of Model Results (Continued)

Investigator	Run	Pred Vel	Meas Vel	Percent Error	Error Change
O'Brien and Cherno	A-2	0.553	0.595	-7.1	-4.1
	A-5	0.438	0.416	5.2	0.2
	A-6	0.319	0.298	7.1	0.0
	A-25	0.317	0.297	6.7	0.0
	A-23	0.324	0.298	8.6	0.0
	A-14	0.578	0.532	8.7	4.5
	A-10	0.495	0.436	13.6	5.8
	A-4	0.406	0.379	7.1	6.8
	A-12	0.787	0.912	-13.8	-2.1
	A-13	0.989	0.935	5.7	1.8

One of the most interesting features of Table 6.3 is that only O'Brien and Cherno's set of experiments seem to show a significant change when the effects of virtual mass are added. At this point it is difficult to explain why this has occurred since the data set is not significantly different from that of Keulegan, and yet predictions based upon Keulegan's data are little affected by the virtual mass.

6.4 Summary

The purpose of this chapter has been to incorporate the effects of 'virtual' mass with the flow initiation model developed earlier. Although it appears that in most situations the virtual mass can be safely neglected, it was found that under some conditions the effect may be significant. This is one area that should be investigated further since it is not obvious which combination of physical parameters is responsible for this finding.

Also, before closing, it is interesting to note that one should really not have expected the effect of virtual mass to be large. Since the quantity of virtual mass must be of the same order of magnitude as the quantity of mass contained in the spill head, the effect must be very small since only a very small percentage of the total mass of the system is contained within the head. Although it can be argued that for very short spill slumping times a significant portion of the accelerated mass is in the head, it is doubtful that this model is applicable during these very early stages of slick development.

CHAPTER 7

SUMMARY

This chapter is the final chapter in this work and, as such, offers us an opportunity to review what we have learned from the previous sections. Starting with the introduction, an effort was made to emphasize the importance of careful scientific method and the application of appropriate levels of technology to the proposed solution of a problem. Throughout this work, considerable effort was made to illustrate all assumptions upon which the model development was based, and to restrain the development within guidelines that would allow an average practising engineer to follow the development.

Next came the literature review. Within this chapter, an effort was made to development in the reader an appreciation for the broadness of the field in which spill simulation techniques typically rest. From the material presented, it should be obvious that the model presented in this work is definitely one of the simplest in existence since it is based on large scale physical parameters and neglects many of the potentially complicating factors such as interface stability, mixing across the interface and the effects of the wake region behind the head. In spite of these simplifications, the model still presents excellent predictive capabilities suggesting that the factors mentioned may be of little importance during the period of initial slumping and flow development of the slick.

After reviewing the literature, we then presented the technique by which we modelled the head of the spill. This technique was based on the work of Benjamin (1968) and extended Benjamin's theory for a general two layer system with a free surface. In the process of extending the theory, a simple relationship was developed to predict the rise in fluid elevation ahead of the slick. This was an unexpected bonus and may have applications in other areas of engineering.

Once we had a method of modelling the head, we then went on to simulate the slumping process of the original spill volume. The technique used was based upon an approach developed by O'Brien and Cherno (1934) to estimate the behaviour of salt water released from locks in the form of lock exchange flows. In our development, we extended O'Brien and Cherno's theory to allow the thickness of the slick to be one of the parameters predicted by the model. Prior to this, the model assumed that the slick was equal in thickness to one half of the original spill depth. By using data sets obtained from the University of Alberta, O'Brien and Cherno (1934), Middleton (1966) and Keulegan (1958), it was possible to compare the predicted values of the initial head velocity with the measured values. From this analysis, it was found that on average, the absolute error of the model was in the order of nine percent.

In an attempt to explain the cause of this error, it was felt that the effects of the induced flows in the ambient ahead of the spill front may be significant. These effects were modelled using the 'virtual' mass technique described by Vallentine (1965). Once the equations describing the virtual mass were obtained, the virtual mass was added to the previous slumping model and the resulting model was again compared with the measured values in the data sets. Based on this analysis, it appears that the effect of virtual mass can be neglected in most cases. It is interesting to note, however, that in O'Brien and Cherno's data set, significant changes did occur. Since this data set is very similar to that of Keulegan, it is difficult to say at this time why this result was found.

Since it appears that virtual mass may have, under some situations, a significant effect, this is one area where it may be fruitful to perform additional work. In addition to this, other areas where additional research could extend this model are:

- in the spreading behaviour of immiscible fluids. Although this model will probably still show good predictive characteristics, it should be tested against data sets and expanded if necessary.

- the slumping behaviour of dense spills should be investigated further. Although the data sets used in this study suggest that the equations do give an acceptable description of dense spill flow initiation, it may be instructive to develop explicit equations for this situation.

- the theory could easily be expanded to include non-rectangular cross sectional geometries. This would be the first step in analyzing spills in irrigation canals, small streams and pipes.

- since many spills are not confined by channels, it would be very useful to expand this model to include axially symmetric spreading.

- since one of the primary assumptions in this analysis has been that the spill will not possess any driving head in excess of that based upon a hydrostatic equilibrium at the base of the spill, this is another area where research could be done. The results from this analysis would be particularly important in predicting the initial growth of tanker spills and plunging spills (ie. the spill initiated from some distance above or below the ambient's surface).
- work could be done to specialize the equations for the situation in which fluid elevations are equal on each side of the barrier separating the spill from the ambient.

The final suggested research topic may be particularly important when attempting to simulate the results from lock exchange experiments like the ones conducted by O'Brien and Cherno, Middleton and Keulegan. In the description of their experiments, each researcher stated that they started their experiment with equal depths of fluid on each side of the gate. This violates the assumption of hydrostatic equilibrium at the base of the gate and may be one of the major reasons why the models developed in this work over estimated the initial velocity of the slick.

As one final comment before closing this body of work, it should be stressed that this an <u>engineering</u> model. Many individuals may express concerns that interesting features have been left out of this model that, for correctness, should have been included. If this were physics, where the mandate is to attempt complete understanding of all processes, this argument would be valid. However, this is engineering. Our mandate is to develop understandable models that an average practising engineer can use safely. The models developed in this work are simple, understandable and conservative in their estimates; the key qualities that a good engineering model should have.

REFERENCES

Abbott, M. B., "On the Spreading of One Fluid Over Another. Part I: Discharge of Oil Into a Canal", La Houille Blanche, Vol. 16, 1961, pp. 622-628.

Abbott, M. B., "On the Spreading of One Fluid Over Another. Part II: The Wave Front", La Houille Blanche, Vol. 16, 1961, pp. 826-836.

Abbott, M. B., "On the Spreading of One Fluid Over Another. Part III: Discharge of Oil into Still Water Radial Flow", La Houille Blanche, Vol. 16, 1961, pp. 837-846.

Bai, K. J., "The Added-Mass of Two-Dimensional Cyllinders Heaving in Water of Finite Depth", Journal of Fluid Mechanics, Vol. 81, 1977, pp. 85-105.

Barr, D. I. H., "Densimetric Exchange Flow in Rectangular Channels. I: Definitions, Review and Relevance to Model Design", La Houille Blanche, Vol. 18, 1963, pp. 739-754.

Barr, D. I. H., "Densimetric Exchange Flow in Rectangular Channels. III: Large Scale Experiments", La Houille Blanche, Vol. 22, 1967, pp. 619-632.

Barr, D. I. H. and Hassan, A. M. M., "Densimetric Exchange Flow in Rectangular Channels. II: Some Observations of the Structure of Lock Exchange Flow", La Houille Blanche, Vol. 18, 1963, pp. 757-766.

Benjamin, T. B., "Gravity Currents and Related Phenomena", Journal of Fluid Mechanics, Vol. 31, 1968, pp. 209-248.

Bo Pedersen, F., "A Monograph on Turbulent Entrainment and Friction in Two-Layer Stratified Flow", Technical University of Denmark, Institute of Hydrodynamics and Hydraulic Engineering, Series Paper No. 25, 1980.

Bo Pedersen, F., "Environmental Hydraulics: Stratified Flows", Lecture Notes on Coastal and Estuarine Studies, Vol. 18, 1986.

Bradley, C. I., Carpenter, R. J., Waite, P. J., Ramsay, C. G. and English, M. A., "Recent Development of a Simple Box-Type Model for Dense Vapour Cloud Dispersion", in Heavy Gas and Risk Assessment - II, S. Hartwig, ed., D. Reidel Publishing Company, Dordrecht, Holland, 1983, pp. 77-89.

Britter, R. E. and Linden, P. F., "The Motion of the Front of a Gravity Current Travelling Down an Incline", Journal of Fluid Mechanics, Vol. 99, 1980, pp. 531-543.

Britter, R. E. and Simpson, J. E., "Experiments on the Dynamics of a Gravity Current Head", Journal of Fluid Mechanics, Vol. 88, 1978, pp. 223-240.

Buhler, J. Wright, S. J. and Kim, Y., "Gravity Currents Advancing into a Coflowing Fluid", Journal of Hydraulic Research, Vol. 29, 1991, pp. 243-257.

Ellison, T. H. and Turner, J. S., "Turbulent Entrainment in Stratified Flows", Journal of Fluid Mechanics, Vol 6, 1959, pp. 423-448.

Fannelop, T. K. and Waldman, G. D., "Dynamics of Oil Slicks", AIAA Journal, Vol 10, No. 4, 1972, pp. 506-510

Fay, J. A., "The Spread of Oil Slicks on a Calm Sea", in Oil on the Sea, D.P. Hoult, ed., Plenum Press, New York, 1969, pp. 53-63.

Harleman, D.R.F., "Stratified Flow", in Handbook of Fluid Dynamics, V.L. Streeter, ed., Section 26, McGraw-Hill Book Co., Inc., New York, N.Y., 1960.

Harleman, D.R.F., "Diffusion Processes in Stratified Flow", in Estuary and Coastline Hydrodynamics, A. T. Ippen, ed., Section 12, McGraw-Hill Book Co., Inc., New York, N.Y., 1966.

Hayakawa, N., "Internal Hydraulic Jump in Co-current Stratified Flow", Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers, Vol. 96, No. EM5, 1970, pp. 797-800.

Hoult, D. P., "Oil Spreading on the Sea", Annual Review of Fluid Mechanics, Vol. 4, 1972, pp. 341-368.

Huppert, H. E. and Simpson, J. E., "The Slumping of Gravity Currents", Journal of Fluid Mechanics, Vol. 99, 1980, pp. 785-799.

Keulegan, G. H., "Wave Motion", in Engineering Hydraulics, H. Rouse, ed., Chapter XI, John Willey and Sons, New York, 1949.

Keulegan, G. H., The Motion of Saline Fronts in Still Water, U.S. National Bureau of Standards, Report No. 5831, 1958.

Keulegan G. H., "The Mechanism of an Arrested Saline Wedge", in Estuary and Coastline Hydrodynamics, A. T. Ippen, ed., Section 11, McGraw-Hill Book Co., Inc., New York, N.Y., 1966.

Lister, J. R. and Kerr, R. C., "The Propagation of Two-Dimensional and Axisymmetric Viscous Gravity Currents at a Fluid Interface", Journal of Fluid Mechanics, Vol. 203, 1989, pp. 215-249.

Middleton, G. V., "Small Scale Models of Turbidity Currents and the Criterion for Auto-Suspension", Journal Sedimentary Petrology, Vol. 36, 1966, pp. 202-208.

Middleton, G. V., "Experiments on Density and Turbidity Currents I: Motion of the Head", Canadian Journal of Earth Sciences, Vol. 3, 1966, pp. 523-546.

Middleton, G. V., "Experiments on Density and Turbidity Currents III: Deposition of Sediment", Canadian Journal of Earth Sciences, Vol. 4, 1967, pp. 475-505.

O'Brien, M. P. and Cherno, J., "Model Law for Motion of Salt Water Through Fresh", Transactions of the American Society of Civil Engineers, Vol 99, 1934, pp. 576-609.

Pallesen, T. R., "Turbidity Currents", Technical University of Denmark, Institute of Hydrodynamics and Hydraulic Engineering, Series Paper No. 32, 1983.

Puskas, J., McBean, E. and Kouwen, N., "Behaviour and Transport of Oil Under Smooth Ice", Canadian Journal of Civil Engineering, Vol. 14, 1987, pp. 510-518.

Rajaratnam, N. and Powley, R., "Hydraulic Jumps in Two-Layer Flows", Proc. Instn. of Civil Engrs., London England, Vol. 89, 1990, pp. 127-142.

Rajaratnam, N., Tovell, D. and Loewen, M., "Internal Jumps in Two Moving Layers", Journal of Hydraulic Research, IAHR, Vol. 29, No. 1, 1991, pp.91-106.

Reed, M., French, D., Feng, S. S. and Knauss, W., "Three-Dimensional Oil and Chemical Spill Model and Coupled Geographical Information System", Proceedings of the Seventh Technical Seminar on Chemical Spills, 1990, pp. 1-16.

Schijf, J. B. and Schönfeld, J. C., "Theoretical Considerations on the Motion of Salt and Fresh Water", Proceedings of the Minnesota International Association for Hydraulic Research and American Society of Civil Engineers, 1953, pp. 321-333.

Simpson, J. E. and Britter, R. E., "The Dynamics of the Head of a Gravity Current Advancing Over a Horizontal Surface", Journal of Fluid Mechanics, Vol. 94, 1979, pp. 477-495.

Turner, J. S., "Bouyancy Effects in Fluids", Cambridge Monographs on Mechanics and Applied Mathematics, Cambridge University Press, 1979.

Vallentine, H. R., Applied Hydrodynamics, Butterworth & Co. Limited, London, England, 1965, pp. 120-122.

Von Kármán, T., "The Engineer Grapples with Nonlinear Problems", Bulletin of the American Mathematical Society, Vol. 46, 1940, pp. 615-683.

Yih, C. H. and Guha, C. R., "Hydraulic Jumps in a Fluid System of Two Layers", Tellus, Vol. 7, 1955, pp. 358-366.

Yih, C. S., Stratified Flows, Academic Press, 1980.