Electronic Appendix

This is the Electronic Appendix to the article

Modelling the mating system of polar bears –

a mechanistic approach to the Allee effect

by

Péter K. Molnár, Andrew E. Derocher, Mark A. Lewis and Mitchell K. Taylor

Sensitivity analyses of female mating success predictions to model parameters

The effects of varying the model parameters pair formation rate (σ), male mating ability (α), length of pair association ($1/\mu$), and mating season length (T) on female mating success ($1-F(T)/f_0$) are explored below. σ is varied by an order of magnitude from 0.2 km^2/h to 20 km^2/h to account for a possible underestimation of pair formation rate due to a lack of pairing data from March, $1/\mu$ is varied from 10.5 to 24.5 days, which likely encompasses the natural range of pair association lengths (Ramsay & Stirling 1986, Malyov 1988, Wiig *et al.* 1992), α is varied from 0 to 1, and T from 53 to 67 days.

Pair formation rate (σ)

Mating success is most sensitive to pair formation rate, particularly under low densities. For instance, applying σ=2.05 km^2/h to three hypothetical populations with densities ranging from 0.99 to 1.97 bears/1000 km^2 (densities one fourth to one half as high as in Lancaster Sound) and an operational sex ratio of m_0/f_0 =1.08, a component Allee effect due to difficulties in finding mates is predicted for all scenarios with female mating success ranging from 72% to 91%. In contrast, increasing pair formation rate to σ=20 km^2/h ensures 100% mating success in all three populations, reflecting that high pair formation rates ensure high mating success regardless of density for all but very female-biased operational sex ratios (figure S1a). In fact, as σ increases further, the threshold operational sex ratio, below which female mating success is expected to decline, approaches $m_0/f_0 \approx 0.29 = (1/\mu)/T$, showing that the maximum number of pairings a male can achieve is limited by pair association lengths and mating season length.

As expected, female mating success is greatly reduced if σ is decreased to σ=0.2 km^2/h. The three scenarios above would, for instance, only yield 13% to 24% female mating success (figure S1b).

The low sensitivity of mating success to pair formation rate at high densities is easily explained, because high densities ensure high encounter rates, and thus high pair formation rates, by themselves. In fact, increasing the density of available breeders while keeping pair formation rates constant is equivalent to increasing pair formation rates, while keeping density constant.

Male mating ability (α)

Male mating ability can also have strong effects on female mating success, however, under different conditions. While pair formation rate (and thus mate searching efficiency) is the most important factor determining mating success at low densities, and becomes less important as density increases (figure 4), male mating ability has little effect at low densities (figure S2a). For instance, increasing σ=2.05 km^2/h to σ=20 km^2/h raises predicted mating success from 59% to 100% at m_0/f_0=1.08 and a density of m_0+f_0=0.66 bears/1000 km^2 (one sixth the estimated density in Lancaster Sound), but decreasing α=1 to α=0 reduces mating success only from 59% to 51%. Furthermore, as expected, male mating ability has little effect on female mating success under male-biased operational sex ratios, regardless of population density (figure S2a).

In contrast to σ, the importance of α increases with increasing density, where pronounced effects on mating success can be found at balanced to female-biased operational sex ratios (figure S2a). At higher densities males and females can easily find each other, but at female-biased sex ratios the ability of males to mate with several females becomes crucial. If, for example, the male population in Lancaster Sound would be reduced to half its size (from 489 to 245 mature males), while the number of mature available females is kept at 451 (i.e. m_0/f_0=0.54), the model predicts 85% mating success using α=1, but only 52% mating success using α=0, a difference of 147 females (figure S2b).

Average length of pair association (1/μ)

The average length of pair association (and thus, whether or not females are polyandrous) has little influence on mating success, regardless of density (figure S3). However, the parameter estimate for $1/\mu$ provides some interesting evidence regarding the prevalence of polyandry in polar bears. Few observations exist fully documenting the length of pair associations in bears, one of 12.5 days in polar bears (I. Stirling, pers. comm.), and one of 14 days in brown bears (Herrero & Hammer 1977). A few opportunistic observations give pair associations in polar bears as ranging between at least 7 and at least 22 days (unpublished data, Ramsay & Stirling 1986, Wiig *et al.* 1992). If we assume that a single pairing lasts around 14 days on average, then the maximum likelihood estimate for the average length of pair association ($1/\mu$=17.5 days with a bootstrapped 95% confidence interval of 14.1-21.6 days) indicates the possibility that a significant proportion of females proceeds to mate with a second partner.

Mating season length (T)

Mating season length has little effect on mating success (figure S4). Thus, our choice of T=60 days to estimate female mating success does not affect our conclusions, as long as the mating season ends around the end of May, as previously suggested (Howell-Skalla *et al.*

2002, Rosing-Asvid *et al.* 2002), and supported by the fact that we observed no breeding pairs after May 28.

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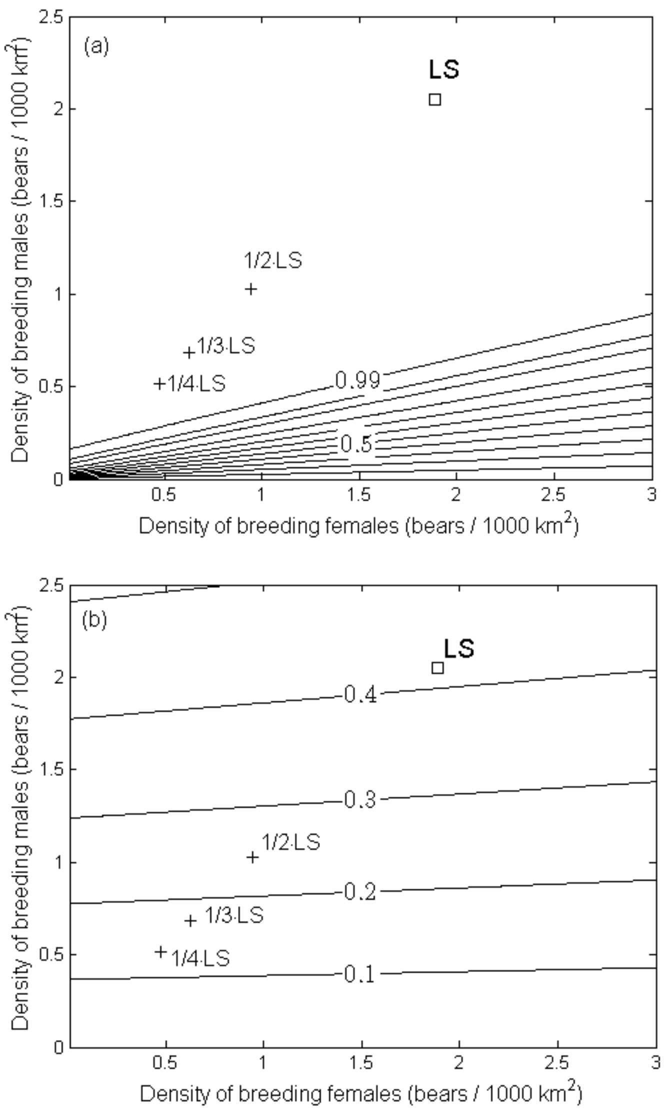

Figure S1. Contour lines giving female mating success as a function of male and female densities. The estimated male and female density of Lancaster Sound (LS) is indicated by a square. Three other hypothetical populations with one half, one third, and one fourth the density of Lancaster Sound, but the same operational sex ratio ($m_0/f_0 = 1.08$), are indicated by crosses. Panel (a) uses σ=20 km^2/h, panel (b) uses σ=0.2 km^2/h. Both panels use the maximum likelihood estimate from Lancaster Sound for pair dissolution rate (μ=(17.5 days)$^{-1}$), and assume maximal male mating ability (α=1), and T=60 days as mating season length.

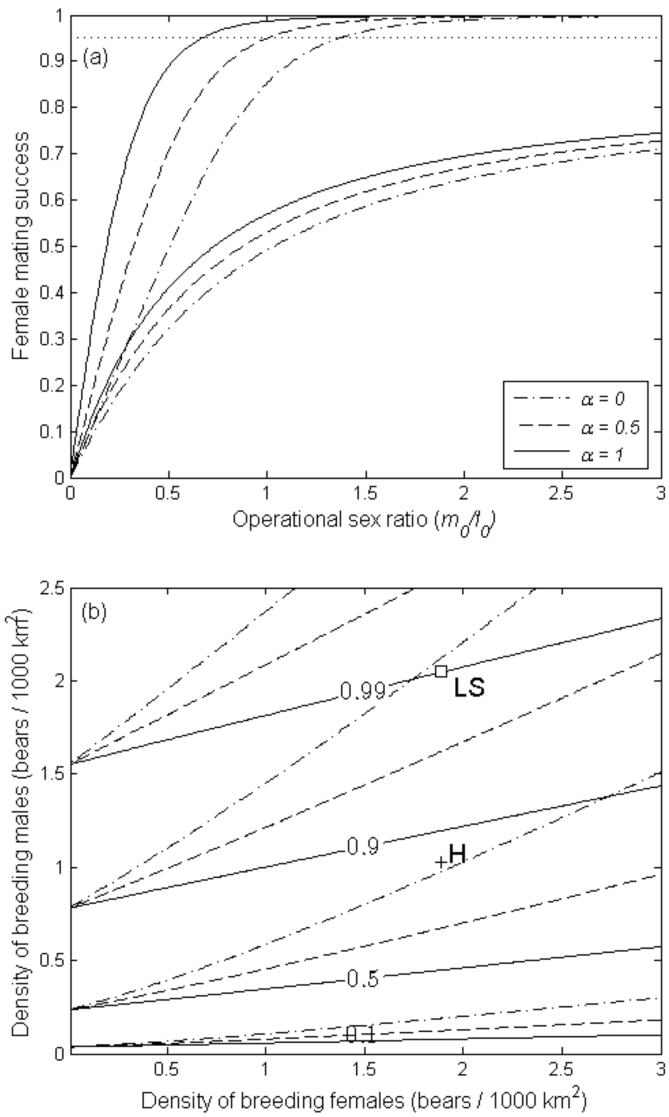

Figure S2. Sensitivity of female mating success to male mating ability, α. Panel (a): Female mating success as a function of the operational sex ratio m_0/f_0, with the overall density of breeding males and females, m_0+f_0, held constant. Two densities are shown for each parameter set: the estimated density in Lancaster Sound (3.94 bears/1000 km^2), and a density one sixth as high (0.66 bears/1000 km^2) with mating success decreasing as density decreases (graphs at the top left represent the high, the ones at the bottom right the low density scenario). The horizontal dotted line represents 95% mating success. Panel (b): Contour lines giving female mating success as a function of male and female densities. The estimated male and female density of Lancaster Sound (LS) is indicated by a square. H denotes a hypothetical population with the estimated female density of Lancaster Sound, but half the male density (see text for details). Both panels use the maximum likelihood parameter estimates from Lancaster Sound (σ=2.05 km^2/h, μ=(17.5 days)$^{-1}$), and assume T=60 days as mating season length. α is varied: α=1 (solid line), α=0.5 (dashed line), α=0 (dot-dashed line).

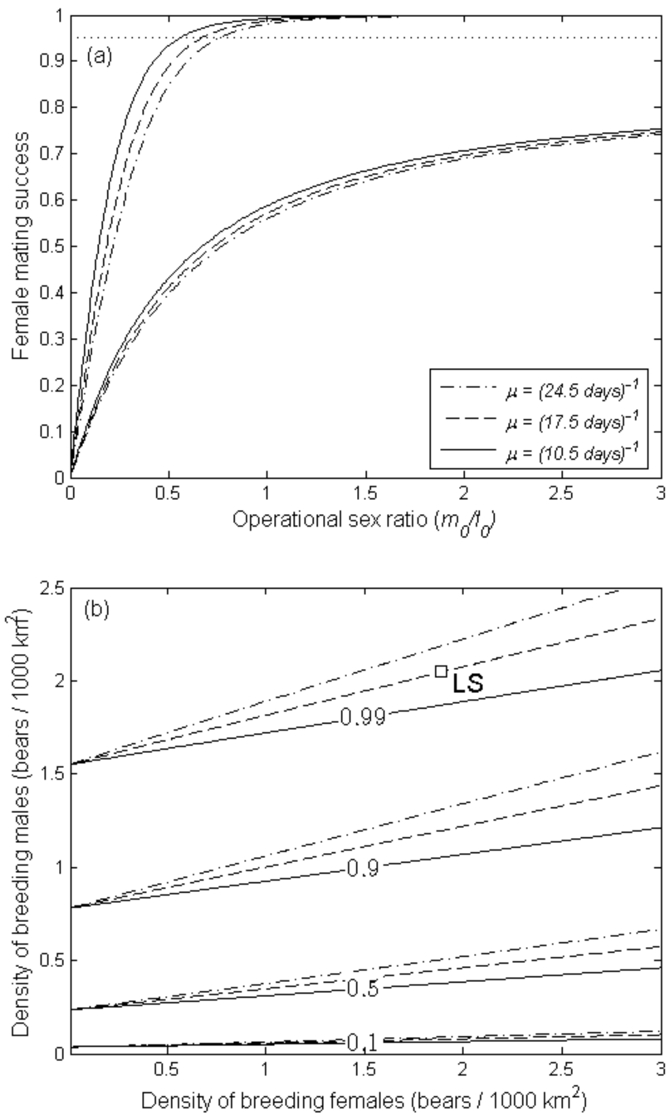

Figure S3. Sensitivity of female mating success to length of pairing association, $1/\mu$. Panel (a): Female mating success as a function of the operational sex ratio m_0/f_0, with the overall density of breeding males and females, m_0+f_0, held constant. Two densities are shown for each parameter set: the estimated density in Lancaster Sound (3.94 bears/1000 km^2), and a density one sixth as high (0.66 bears/1000 km^2) with mating success decreasing as density decreases (graphs at the top left represent the high, the ones at the bottom right the low density scenario). The horizontal dotted line represents 95% mating success. Panel (b):

Contour lines giving female mating success as a function of male and female densities. The estimated male and female density of Lancaster Sound (LS) is indicated by a square. Both panels use the maximum likelihood estimate for pair formation rate from Lancaster Sound (σ=2.05 km^2/h), and assume maximal mating ability (α=1) and T=60 days as mating season length. $1/\mu$ is varied: $1/\mu$=10.5 days (solid line), $1/\mu$=17.5 days (dashed line), $1/\mu$=24.5 days (dot-dashed line).

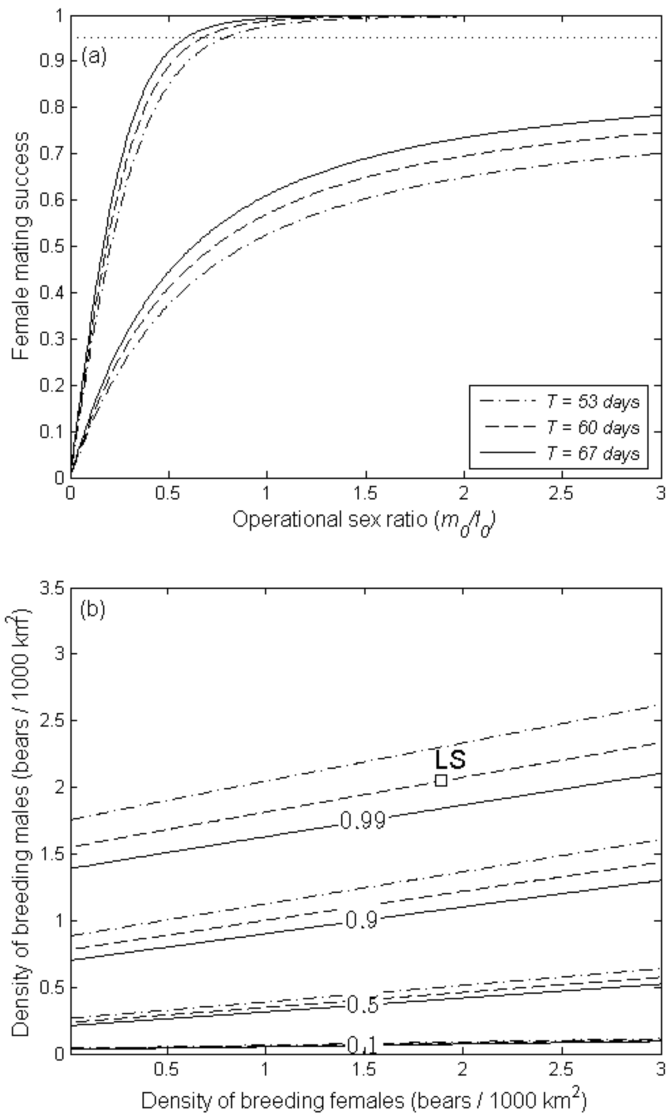

Figure S4. Sensitivity of female mating success to mating season length, T. Panel (a): Female mating success as a function of the operational sex ratio m_0/f_0, with the overall density of breeding males and females, m_0+f_0, held constant. Two densities are shown for each parameter set: the estimated density in Lancaster Sound (3.94 bears/1000 km^2), and a density one sixth as high (0.66 bears/1000 km^2) with mating success decreasing as density decreases (graphs at the top left represent the high, the ones at the bottom right the low density scenario). The horizontal dotted line represents 95% mating success. Panel (b):

Contour lines giving female mating success as a function of male and female densities. The estimated male and female density of Lancaster Sound (LS) is indicated by a square. Both panels use the maximum likelihood parameter estimates from Lancaster Sound (σ=2.05 km^2/h, μ=(17.5 days)$^{-1}$), and assume maximal mating ability (α=1). T is varied: T=67 days (solid line), T=60 days (dashed line), T=53 days (dot-dashed line).