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STABILITY COEFFICIENTS FOR EARTH SLOPES

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SYNOPSIS

The application of the effective stress analysis to earth slopes has suffered through lack of a general solution such as that presented by Taylor (1937) for the total stress analysis. Recent developments in computing technique have been applied to the slip circle method and have made it possible to present the results of the effective stress analysis in terms of *stability coefficients* from which the factor of safety can be rapidly obtained.

Illustrations are given of the use of these coefficients with the distributions of pore pressure encountered in typical earth dams and cuts.

L'application de principe des tensions efficace sur l'analyse des pentes de terre c'est empêcher de l'absence d'une solution générale, telle que celle du Taylor (1937) pour l'analyse des pentes en termes des tensions totales.

L'application des progrès récents dans la technique de computation, à la méthode du cercle de glissement, a rendu possible la présentation des résultats en termes de *coefficients de stabilité*, baser sur le principe des tensions efficace, dont on peut obtenir rapidement le facteur de sécurité.

Ici, on donne des illustrations graphiques sur l'emploi de ces coefficients avec la répartition des pressions interstitielles rencontrée dans des talus et des tranchées de terre typique.

INTRODUCTION

The practising engineer and the designer frequently require a rapid means of estimating the factor of safety of a cutting, an embankment, or a natural slope. A detailed analysis is often impracticable in the preliminary stages when a number of alternative schemes are under consideration. Taylor's stability charts (Taylor, 1937; 1948) are sometimes used in these circumstances; but as Taylor himself pointed out, they are strictly valid only for analyses based upon total stresses. Studies have indicated the advantages to be obtained by employing an effective stress failure criterion (Bishop, 1952; Henkel and Skempton, 1955; Bishop and Bjerrum, 1960) for the analysis and design of earth dams and slopes. Current methods of stability analysis used for the long-term stability of slopes and for most earth-dam problems include the observed or predicted pore-pressure distribution as a major factor in the calculation. These problems therefore call for a general solution of the type presented by Taylor but expressed in terms of effective stress rather than of total stress.

The number of variables to be considered in the effective stress solution is, of course, more formidable. Simple slopes without berms have been selected for analysis, and their geometry is specified by the parameters shown in Fig. 1. The height of the slope is H, and $D.H$ is the depth of the first hard stratum below the crest of the section; D being termed the depth factor. For example, for an earth dam founded directly on bedrock the depth factor would be 1. The slope angle with respect to the horizontal is β, and the inclination is expressed as the value of cot β, the slopes being referred to as 2 : 1, 3 : 1, . . . etc. The crest width is left unspecified since the most critical circle in an effective stress analysis begins close to the top of the slope in the cases considered in the Paper; the solution is thus applicable to earth dams as well as to cuts and natural slopes.

For purposes of analysis and tabulation, it is most convenient to express the pore-pressure u at any point in terms of the pore-pressure ratio r_u defined by:

$$r_u = \frac{u}{\gamma h} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where h is the depth of the point in the soil mass below the soil surface, and γ is the bulk density of the soil. The general solutions are based on the assumption that the pore-pressure ratio r_u is constant throughout the cross-section. This is called a homogeneous pore-pressure distribution. Approximations to allow the estimate of the factor of safety for non-homogeneous pore-pressure distributions in which the value of r_u varies within the cross-section (e.g. the steady seepage case) will be discussed in a subsequent paragraph.

The remaining variables are the cohesion intercept c', and the angle of shearing resistance ϕ', both in terms of effective stress, and the bulk density, γ.

The factor of safety, F, is therefore seen to depend on seven variables. An accurate and extensive general solution is made possible by three factors:

(1) For a given value of the dimensionless number $c'/\gamma H$ the factor of safety depends only on the geometry of the section, expressed by the values of cot β and D, on the pore-pressure ratio r_u, and on the angle of shearing resistance ϕ'. The use of a dimensionless number for expressing the influence of cohesion on stability has been suggested previously (Fellenius, 1927), and has been used by Taylor (1937, 1948) as an important simplification in the total stress analysis and further by Janbu (1954). Its application to the effective stress analysis is demonstrated in a later paragraph dealing with the development of the governing equation in a completely dimensionless form.

To reduce the amount of computation only three values of $c'/\gamma H$ have been used—0, 0·025, and 0·05. Considering that the cohesion intercept in terms of effective stress is generally somewhat lower than the cohesion intercept in terms of total stress, these values have been selected as representing the range commonly encountered in effective stress analysis and also a range within which a linear interpolation can be used without significant errors. Extrapolation beyond this range can be made and a measure of the inaccuracies involved is given in a further section. It should, however, be borne in mind that for cross-sections of natural slopes or wide embankments some errors may be incurred due to the neglect of tension cracks whose effect on stability becomes more pronounced at higher values of $c'/\gamma H$. For these problems, a modified analysis is generally required.

(2) For a simple soil profile and specified shear strength parameters it had been found that, to a close approximation, the factor of safety, F, varies linearly with the magnitude of the pore pressure expressed by the ratio r_u. It had been noted earlier (Bishop, 1952; 1955) that for a given slope the relationship between F and r_u, based upon a limited number of graphical solutions, did not differ appreciably from a straight line. For this reason the present calculations have been limited to only three values of r_u; i.e. 0, 0·3, and 0·7.

The more accurate solutions now available show that there is indeed no significant departure from linearity in the relationship between F and r_u for the range of r_u values, 0·0 to 0·7, usually encountered in practice. A typical example of this linear relationship is given in Fig. 2. It is therefore permissible, at least for the range of slopes, depth factors, and soil properties considered in this Paper, to use the expression:

$$F = m - n.r_u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where m and n are termed the *stability coefficients* for the particular slope and soil properties.

The presentation and tabulation of the results are greatly simplified by the use of these stability coefficients, which also have the advantage of giving an immediate picture of the influence of pore pressure on the factor of safety.

(3) The third factor making the present general solution possible has been the application of the electronic digital computer to the problem of slope stability analysis by Little and Price who have generously made available to the Authors the machine programme described recently (Little and Price, 1958). A general solution had been planned on a more modest scale before the electronic computer was programmed for this type of work. The scope of the present general solution, which has involved more than five thousand trial circles, could not have been contemplated using normal methods of computation.

This investigation has been concerned with values of slope inclination from 2 : 1 to 5 : 1,.

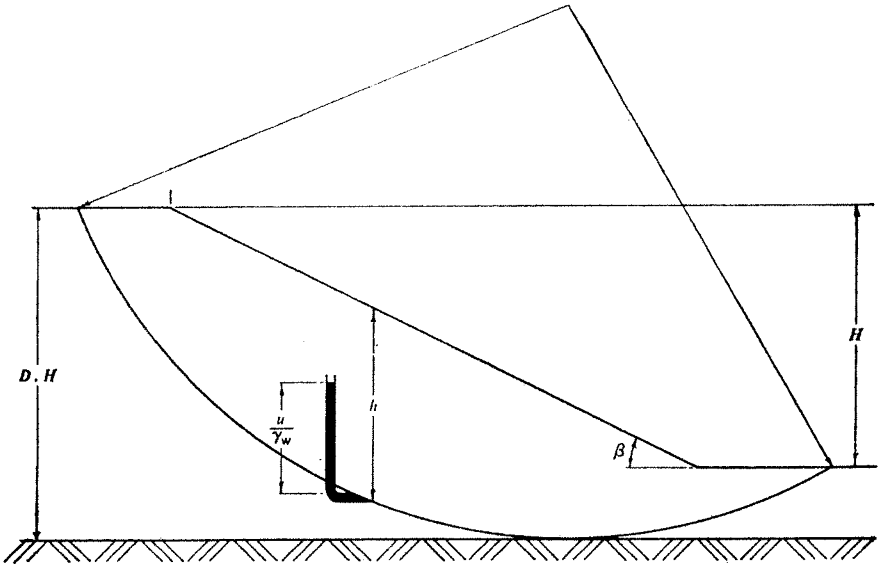

Fig. 1. Specification of parameters

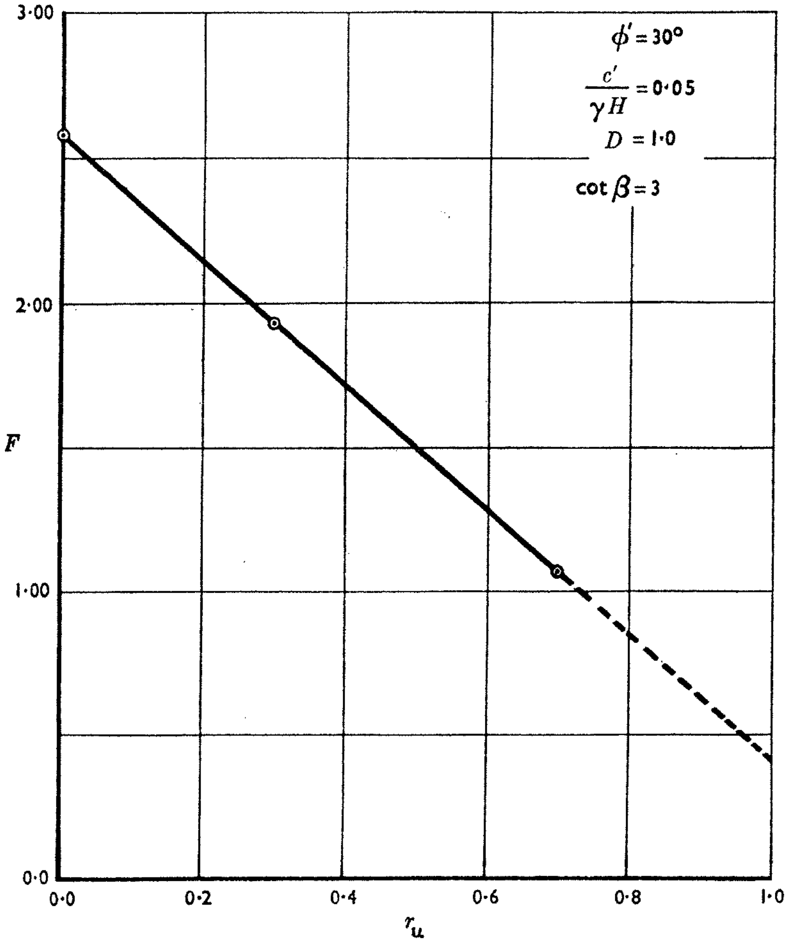

Fig. 2. Linear relationship between factor of safety, F, and pore-pressure ratio, r_u

and values of ϕ' from 10° to 40°. The range of $c'/\gamma H$ considered has already been given and depth factors, D, up to 1·5 have been treated where necessary.

METHOD OF ANALYSIS

For simple slopes having a homogeneous distribution of pore pressure (i.e. r_u being constant) it appears that the factor of safety is given by the circular arc method to a degree of accuracy adequate for all practical purposes. An investigation made to find a more critical surface for a typical earth dam section with a homogeneous pore-pressure distribution resulted in a reduction of only about 1% in the estimated value of the factor of safety (Kenney, 1956). This investigation was restricted to one section and the result should be regarded as only indicative. The circular arc method also has the great advantage of both mathematical simplicity and flexibility.

Two differences between the present solution and the approach adopted by Taylor may be noted. Taylor presented his results in terms of the value of stability number $c/F.\gamma.H$ required, for a given value of friction angle, ϕ, to maintain limiting equilibrium. This enabled the general solution to be presented in a very compact form but also meant that a step-by-step numerical method had then to be used to evaluate the factor of safety of any actual slope not in limiting equilibrium (Taylor, 1948). In the present solution the stability coefficients lead directly to the factor of safety. This presentation occupies rather more space, but permits the result in any particular case to be obtained by simple interpolations.

The factor of safety is defined as the factor by which the shear strength parameters in terms of effective stress, c' and $\tan \phi'$, can be reduced before the slope is brought into a state of limiting equilibrium. The shear strength, τ, mobilized under these conditions is given by the expression:

$$\tau = \frac{c'}{F} + (\sigma_n - u)\cdot\frac{\tan \phi'}{F} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where σ_n denotes the total stress normal to the potential failure surface and u denotes the pore-water pressure. This definition is the same as that adopted by Taylor as "the factor of safety with respect to shearing strength", and is in accord with that enunciated earlier by Fellenius (1927). It has the advantage of being applicable to circular and non-circular slip surfaces alike without modification and operates directly on the relevant strength parameters.

In his general solution Taylor used the friction circle method (Taylor, 1937; 1948), and from a comparison with results obtained by the method of slices he reached the conclusion that "these two general solutions show such close agreement that neither can be adjudged preferable". However, for routine work in which the shear parameters and the pore pressures vary throughout the cross-section, a numerical version of the slices method has considerable advantages (Bishop, 1955). For this reason the slices method had been used by Little and Price in programming the electronic computer and could therefore be readily applied to the present solution.

Taylor's solution can be considered a solution in terms of effective stress with no pore pressures. A direct comparison between the present solution and that of Taylor can be made for few cases, however, because Taylor calculated only one section with a specified depth factor, $D = 1$, for a frictional material, toe circles being used in most cases. The excellent agreement in this comparison is shown in Table 1. An example of the analysis of a circular sliding surface including pore pressures has also been calculated using the friction circle method by including a resultant pore-pressure vector in an analytical solution equivalent to the usual graphical process. The difference between this solution and that calculated by the method of slices is less than 2%, as shown in Table 1. This point is emphasized because results presented at the Conference on the Stability of Earth Slopes at Stockholm in 1954

Table 1

Comparison of factors of safety calculated by friction circle and slices methods

cot β	D	$\frac{c'}{\gamma H}$	ϕ'	r_u	F (Friction circle)	F (Slices)
4 : 1	1·00	0·05	30·4°	0	3·33*	3·30
3·4 : 1	1·00	0·025	10·0°	0	1·00*	1·00
3 : 1	1·00	0·05	30·0°	0·30	1·89†	1·92

* Obtained from Taylor's solution.
† Obtained analytically.

indicated wide differences between the results obtained by different methods of analysis. Provided comparisons are made on the basis of the same definition of the factor of safety, methods which represent adequately the statics of the problem will lead to the same numerical result. A similar conclusion based upon a comparative study of several methods of stability and using two different definitions of factor of safety has recently been reached by Borowicka (1959)*.

The derivation of the numerical slices method has already been given by Bishop (1955), but is repeated below for the sake of completeness.

Consider the system of forces on a slice as indicated in Fig. 3, where the potential sliding body is bounded by ABC, the free surface of the soil mass (of unit thickness), and the arc ADEC of the circular sliding surface under consideration.

In this diagram:

E_n and E_{n+1} denote the resultant horizontal forces on the sections n and $n+1$, respectively, and:
X_n and X_{n+1} denote the resultant vertical shear forces on the sections n and $n+1$, respectively.
W denotes the total weight of the slice above the sliding arc between n and $n+1$.
P denotes the total normal force acting on the base of the slice.
S ,, the shear force acting along the base of the slice.
h ,, the height of the slice.
l ,, the length of the arc DE.
b ,, the breadth of the slice.
α ,, the angle between DE and the horizontal.
x ,, the horizontal distance of the slice from the centre of rotation.

The total normal stress is σ_n where:

$$\sigma_n = \frac{P}{l} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

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* An alternative definition of factor of safety has been suggested on the basis of the ratio of the moment of the available shear strength about the circle centre to the moment of the forces about the circle centre tending to produce rotation (for example, Frohlich, 1954; 1955) such that:

$$F_f = \frac{Mr}{Ma} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

where Mr is the total resisting moment.
and Ma is the total activating moment.
The relation between this definition and the definitions of factor of safety with respect to shear strength has been considered by Odenstad (1955) and more recently by Borowicka (1959).

The equation for the shear strength mobilized under the condition of limiting equilibrium, equation (3), then becomes:

$$\tau = \frac{c'}{F} + \left(\frac{P}{l} - u\right)\frac{\tan \phi'}{F} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The condition of moment equilibrium about the centre of rotation 0 between the weight of the sliding body and the total shear force acting on the base of the sliding body leads to:

$$\Sigma Wx = \Sigma S.R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where S is the shear force mobilized along the base of the slice and is given by:

$$S = \tau l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

From equations (6), (7), and (8) we readily obtain:

$$F = \frac{R}{\Sigma W.x}\Sigma[c'l + (P - ul)\tan \phi'] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

From the condition of vertical equilibrium:

$$P \cos \alpha + \tau l \sin \alpha = W + X_n - X_{n+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and hence:

$$P' = \frac{W + X_n - X_{n+1} - ul \cos \alpha - \frac{c'}{F} l \sin \alpha}{\cos \alpha + \sin \alpha \frac{\tan \phi'}{F}} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where $P' = P - ul$.

Substituting for P' in equation (9), we obtain:

$$F = \frac{R}{\Sigma W.x}\Sigma\left[c'l + \tan \phi' \frac{\left\{W + X_n - X_{n+1} - ul \cos \alpha - \frac{c'}{F} l \sin \alpha\right\}}{\cos \alpha + \frac{\sin \alpha \tan \phi'}{F}}\right] \quad . \quad . \quad . \quad . \quad (12)$$

and introducing the following substitutions:

$$x = R \sin \alpha$$
$$b = l \cos \alpha$$

and

$$\frac{ub}{W} = \frac{u}{\gamma h} = r_u,$$

we obtain:

$$F = \frac{1}{\Sigma W \sin \alpha}\Sigma\left[\{c'b + [W(1 - r_u) + (X_n - X_{n+1})] \tan \phi'\} \frac{\sec \alpha}{1 + \frac{\tan \alpha \tan \phi'}{F}}\right] \quad . \quad (13)$$

For a further discussion of the conditions to be satisfied to determine the internal forces X and E, the reader is referred to the earlier work by Bishop (1955). The determination of these forces is necessary for a rigorous solution of equation (13), but it is sufficient for our purposes to note that the term $(X_n - X_{n+1})$ may be omitted from consideration with an estimated loss of accuracy of less than 1% for the range of circles considered in this Paper. The equation for the factor of safety, F, then becomes:

$$F = \frac{1}{\Sigma W \sin \alpha}\Sigma\left[\{c'b + W(1 - r_u) \tan \phi'\} \frac{\sec \alpha}{1 + \frac{\tan \alpha \tan \phi'}{F}}\right] \quad . \quad . \quad (14)$$

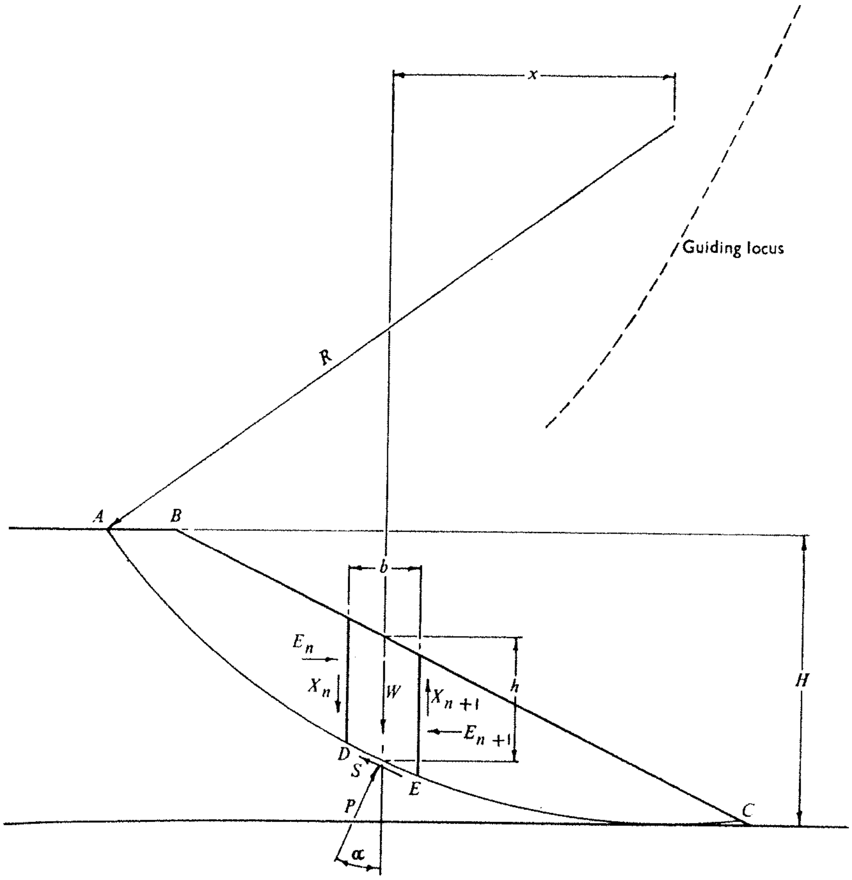


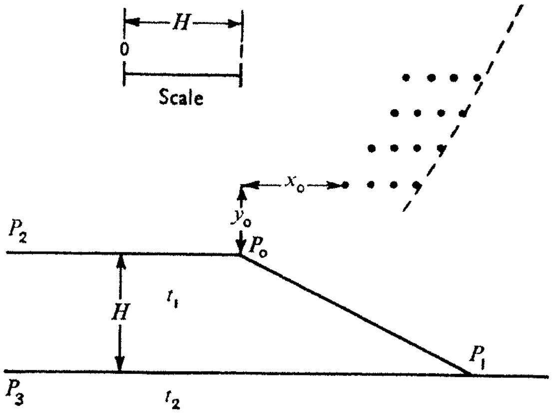
Fig. 3. Forces on a slice

Point data		
P	X	Y
P_0	0	0
P_1	2·00	1·00
P_2	−2·00	0
P_3	−2·00	1·00

Line data		
i	j	t
P_0	P_1	1
P_0	P_2	1
P_1	P_3	2

Soil data				
t	c'	γ	$\tan\phi'$	r_u
1	0·05	1	0·577	0·30
2	–	–	–	–

Circle data						
x_0	Δx	x_1	y_0	Δy	y_1	L
0·90	0·20	1·50	−0·60	0	−0·60	1·00
1·10	0·20	1·70	−0·90	0	−0·90	1·00
1·30	0·20	1·90	−1·20	0	−1·20	1·00
1·40	0·20	2·00	−1·50	0	−1·50	1·00



Section properties

$\cot\beta = 2:1$

$\phi' = 30°$

$\frac{c'}{\gamma H} = 0·05$

$r_u = 0·30$

Fig. 4. Typical data sheet for electronic computation

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To illustrate the simplification that can be achieved by the introduction of cohesion as a dimensionless number, W is expressed as $\gamma . b . h$, assuming γ to be uniform, and the linear dimensions are expressed as ratios of the height of the slope H (see Fig. 1). Equation (14) then becomes:

$$F = \frac{1}{\sum \frac{b}{H}\cdot\frac{h}{H}\cdot\sin\alpha} \sum \left[\left\{ \frac{c'}{\gamma H}\cdot\frac{b}{H} + \frac{b}{H}\cdot\frac{h}{H}\cdot(1 - r_u)\tan\phi' \right\} \frac{\sec\alpha}{1 + \frac{\tan\phi'\tan\alpha}{F}} \right] \quad . \quad . \quad (15)$$

For given values of $c'/\gamma H$, ϕ', and r_u, the value of the factor of safety, F, can then be seen to depend only on the geometry of the sliding body enclosed by the circular arc. The simplification resulting from the introduction of cohesion and pore pressure as dimensionless ratios thus greatly reduces the number of separate cases that have to be computed to provide a general solution.

The computer programme is based upon equation (14) (Little and Price, 1958); the necessary translation to the form of equation (15) is performed in the specification of the geometric and strength data to the digital computer rather than by modifying the existing programme. A typical set of machine instructions is shown in Fig. 4 to illustrate this point. The geometry of the section is specified in terms of the unit length, $H = 1$, such that the X and Y coordinates of the points required to specify the geometry and the centre of circles are determined by their position on the grid whose scale is defined by the unit height characteristic. This expresses all lengths as a function of the height of the section and the computer automatically reads the height and breadth of each slice as a ratio of the total height of the section. The bulk density is also taken as unity and then $c'/\gamma H$ takes the prescribed value for the ratio being investigated. After the data are specified, as shown in Fig. 4, they are then adjusted to conform to the number of significant digits demanded by the computer programme.

In the special case when $c' = 0$, the failure surface is a plane parallel to the free surface and α, therefore, is a constant and equal to β (Haefeli, 1948). Equation (14) then becomes:

$$F = \frac{(1 - r_u)\tan\phi' . \sec\beta}{\sin\beta + \frac{\tan\phi'}{F}\cdot\tan\beta\cdot\sin\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$\therefore \quad F = \frac{\tan\phi'}{\tan\beta}(1 - r_u\sec^2\beta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Equation (17) determines uniquely the variation of the factor of safety with geometry, friction angle, and pore-pressure distribution, and no machine computation is necessary. This expression tacitly assumes that the slope is semi-infinite and hence end effects are neglected. For design purposes, this is felt to be a reasonable approach to the analysis of cohesionless slopes. It is interesting to note from equation (17) that for F to be greater than zero r_u must be less than $\cos^2\beta$. No frictional resistance can be mobilized when the pore-pressure ratio is equal to or greater than $\cos^2\beta$. When r_u is equal to $\cos^2\beta$, the pore pressure at any point in the soil mass is equal to the normal stress on a plane parallel to the surface, and the effective normal stress is zero. It is obvious that in this case no shear strength can be mobilized.

CALCULATIONS AND DATA PROCESSING

Sets of instructions, as illustrated in Fig. 4, were fed into the computer for each combination of parameters being investigated. Instructions were also given to the computer to calculate a family of circles specified over an area where the centre of the critical circle was expected. Approximately twenty circles were needed to define a minimum. Rules for estimating the likely position of the critical circle (Fellenius, 1927) proved to be of little help. However, it has been found that the critical circle lies close to the locus of the centres of the circles passing through B (see Fig. 3) and the tangential to the level indicated by D. This is illustrated in Fig. 5. By specifying circles in the proximity of this locus the number of circles needed to determine the minimum factor of safety was reduced.

A set of contours of equal values of F based upon forty-nine circles has been obtained for a particular case, and is shown in Fig. 5. It is of interest to note the steep characteristic of the family of contours and the relative insensitivity to change in F with a change in the Y co-ordinate of the slip circle centre as compared to the change in F with a change in the X co-ordinate of the slip circle centre. An attempt to correlate the position of the critical slip circle centres with the shear strength and geometrical parameters has, so far, been unfruitful. The only definite correlation obtained is that the change in position of the centre of the critical slip surface is more sensitive to a change in slope angle than to any of the other variables involved. In addition, with a combination of low friction angle and high pore pressures the centre tends to drop. This, of course, would be expected because the cohesion is playing a greater rôle in maintaining the stability of the slope.

The number of slices that the computer uses to represent the sliding body is variable. The effect of this variation on the factor of safety has been investigated for typical cases of these simple slopes and has been found to be negligible. Over the range of fifteen to seventy slices, the maximum difference in factor of safety for a typical circle was of the order of one figure in the second decimal place for a problem whose minimum factor of safety was approximately 1·92. It would be expected that by increasing the number of slices considerably, the solution would converge on a unique value, but it is of interest to note that this convergence is not monotonic, probably due to the increasing effect of rounding errors in the machine computation.

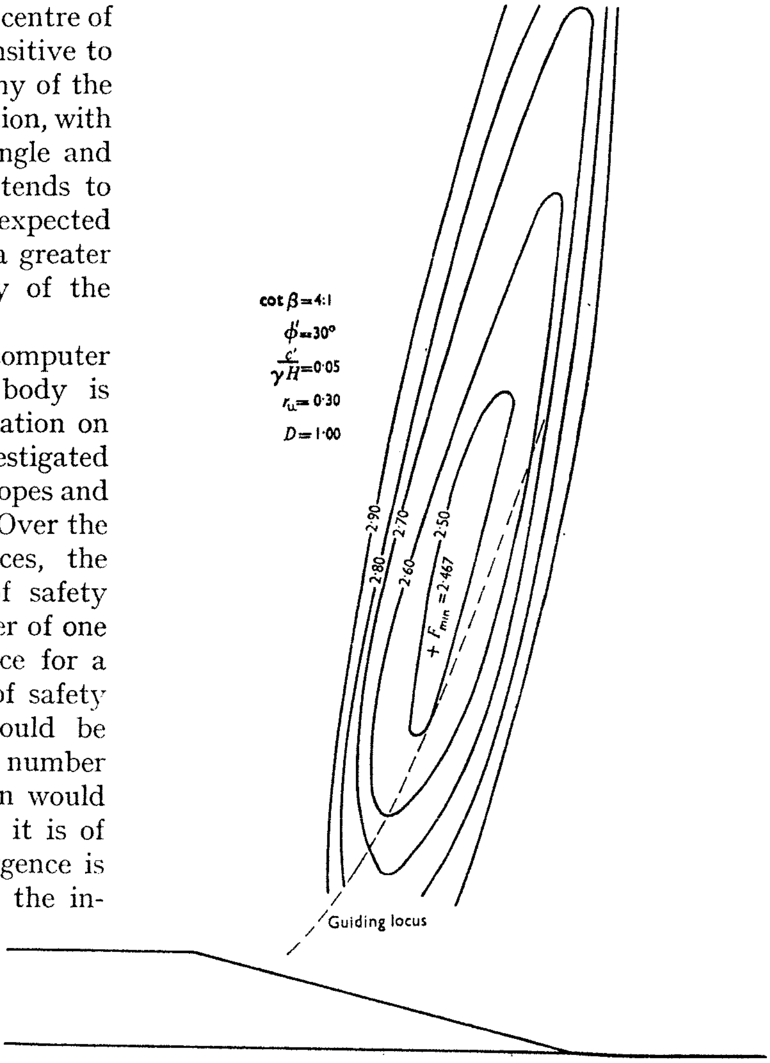

Fig. 5. Detailed contours for a typical case

As described previously, there is a linear relationship between F and r_u for given values of $c'/\gamma H$, ϕ', D, and β, as shown in Fig. 2. This linear relationship has been described in terms of two parameters m and n; where geometrically, m is the intersection with the F axis of the line describing the relationship between F and r_u and corresponds to the value of the factor of safety for the zero pore-pressure condition, and n is the slope of this line. Since the slope is always negative (i.e. the factor of safety decreases with increasing pore pressure, all other parameters being held constant) the factor of safety may be expressed in the form of equation (2):

$$F = m - n . r_u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For use in equation (2), m and n are always positive and will be indicated as such in both their graphical and tabular forms in a further section. Using the method of least squares (Worthing and Geffner, 1943) the two parameters, m and n, which define the linear relationship between F and r_u, may be calculated directly from the fitting process determining the best line passing through or near all three points. The three points are the values of F for r_u equal to 0, 0·3, and 0·7. Since there are three points the normal equations which allow the direct computation of the coefficients m and n are:

$$3m + n\sum r_{ui} = \sum F_i \quad \ldots \ldots \ldots \quad (18)$$

$$m\sum r_{ui} + n\sum r^2_{ui}F_i = \sum r_{ui}F_i \quad \ldots \ldots \ldots \quad (19)$$

where $i = 0, 0{\cdot}3, 0{\cdot}7$.

The machine computations and the least squares fitting process have provided values of m and n, the stability coefficients, at intervals in ϕ' of 10°. It was decided to obtain values of the stability coefficients at intermediate values of ϕ', by interpolation between the ten degree intervals initially determined. But as shown in Fig. 6, the variation of both m and n with ϕ', for a given $c'/\gamma H$, β, and D, is non-linear and any linear interpolation formula could only be used at the sacrifice of the computer accuracy. Because of this, the Lagrange interpolation formula for a function given at unequal intervals of the argument was adopted to determine the stability coefficients at intermediate values of ϕ' (Hartree, 1955). For the condition where four values of the function are known this formula is based upon the use of a third degree polynomial which takes the given values of n or m at four values of the argument, ϕ'. Intermediate values of the stability coefficients may then be calculated for intermediate values of ϕ'. Any further description of the numerical technique involved is considered to be beyond the scope of this Paper. Adopting an interval of $\phi' = 2{\cdot}5°$, the intermediate values of m and n have been calculated.

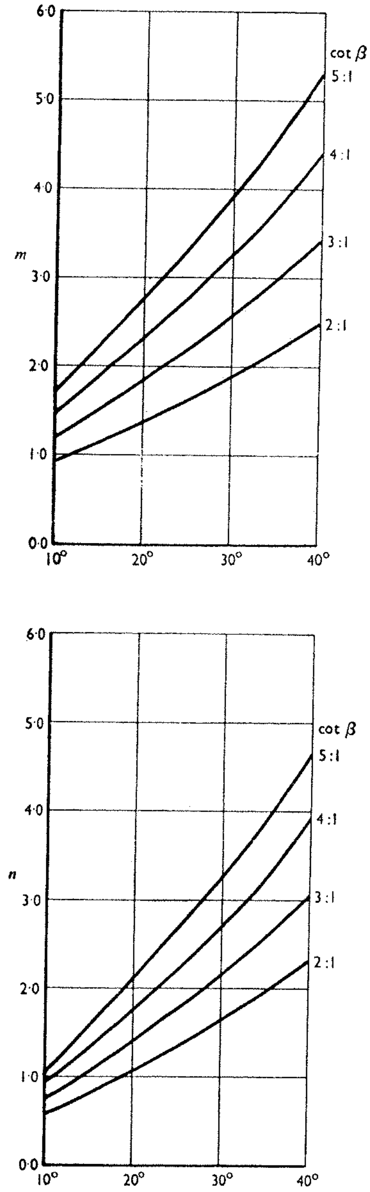

Fig. 6. Variation of stability coefficients with angle of shearing resistance for $\frac{c'}{\gamma H} = 0{\cdot}05$ and $D = 1{\cdot}00$

THE STABILITY COEFFICIENTS

The values of the stability coefficients have been plotted against β, the cotangent of the slope angle, for ϕ' varying between 10° and 40° in Figs 7–12, with values of $c'/\gamma H$ and D specified for each figure. Tabular values of the same results are given in Appendix A. The bold lines show values of m and n at intervals of 10°, whereas the lighter lines indicate the intermediate values that have been obtained by interpolation. The broken lines are those of equal r_u (denoted by r_{ue}) whose derivation and use will be described in a subsequent paragraph. To calculate the factor of safety of a section whose $c'/\gamma H$ lies within the range covered by these figures, it is necessary only to apply equation (2) to determine the factor of safety of the two nearest values of $c'/\gamma H$ and then perform a linear interpolation between these values, for the specified value of $c'/\gamma H$.

The variation of m and n with $c'/\gamma H$ is slightly non-linear as illustrated by Fig. 13. To obtain values of F for $c'/\gamma H$ greater than 0·05, with a minimum loss of accuracy, extrapolation should be carried out in terms of *F* against $c'/\gamma H$, rather than both *m* and *n* against $c'/\gamma H$. Fig. 14, based upon direct calculations, shows that the factor of safety is sensibly linear with increasing $c'/\gamma H$ for a typical case. Therefore, linear extrapolation based upon values obtained from the stability coefficients is permissible in the $c'/\gamma H$ range from 0·05 to 0·10.

For natural slopes or dams and embankments founded on alluvium whose properties do not differ significantly from the fill material, the critical slip circle may penetrate to below the level of the toe of the slope. Stability coefficients have been obtained for various depth factors to allow the ready computation of this condition. It is not immediately obvious which depth factor will provide the lowest factor of safety for a given set of parameters (β, ϕ', $c'/\gamma H$). The lines of equal pore-pressure ratio, r_{ue}, serve as a guide for the selection of the critical depth factor. These are shown as broken lines in Figs 7, 8, and 10.

For a given set of parameters (β, ϕ', $c'/\gamma H$), there is a value of the pore-pressure ratio for which the factor of safety, when *D* equals 1·00, is the same as the factor of safety when *D* equals 1·25. This value is denoted by r_{ue} and the equality can be expressed in the following manner:

$$r_{ue} = \frac{m_{1\cdot25} - m_{1\cdot00}}{n_{1\cdot25} - n_{1\cdot00}} \quad . \quad . \quad . \quad (20)$$

If the design value of the pore-pressure ratio is higher than r_{ue} for the given section and strength parameters, then the factor of safety with a depth factor $D = 1\cdot25$ has a lower value than with *D* equal to 1·00. This argument can be extended to discern whether the factor of safety with *D* equal to 1·50 is more critical

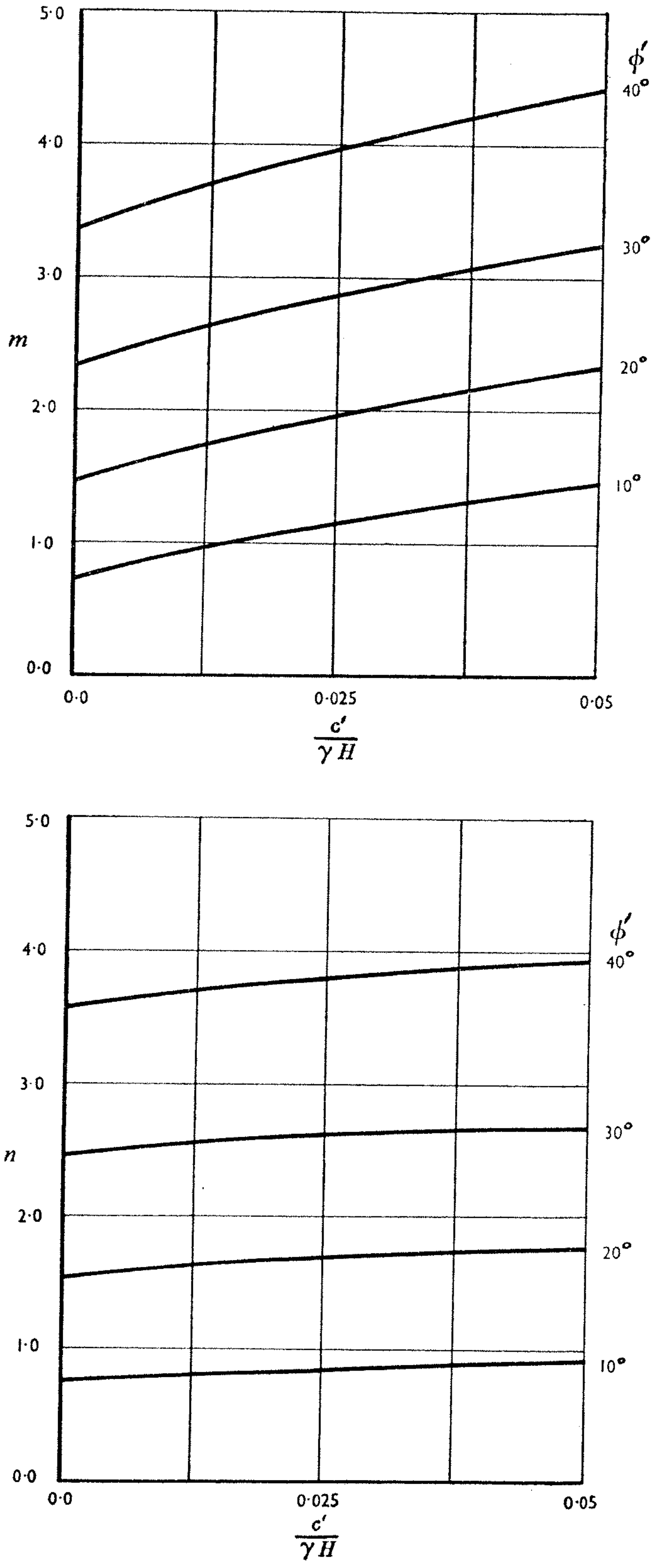

Fig. 13. Variation of m, n versus $\frac{c'}{\gamma H}$ for $\beta = 4{:}1$, $D = 1\cdot00$

than with D equal to 1·25. For this condition:

$$r_{ue} = \frac{m_{1\cdot 50} - m_{1\cdot 25}}{n_{1\cdot 50} - n_{1\cdot 25}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (21)$$

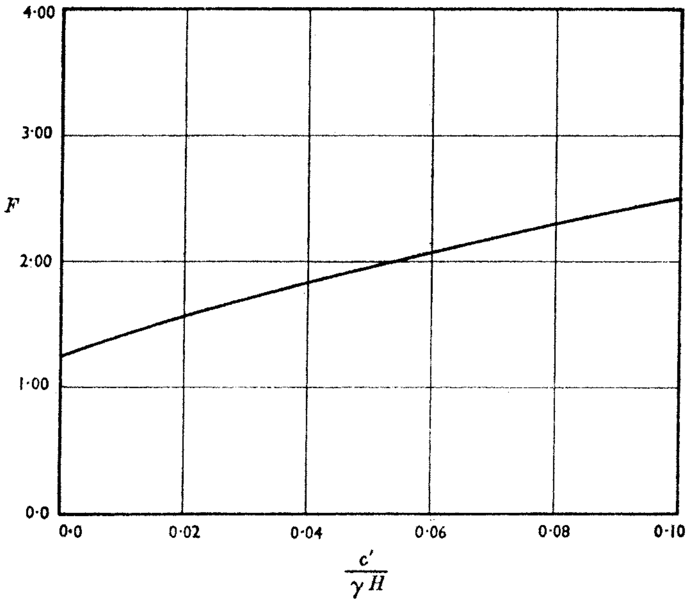

Fig. 14. Variations in F with $\frac{c'}{\gamma H}$ for a typical section

$\cot \beta = 3{:}1$
$D = 1{\cdot}00$
$\phi = 30°$
$r_u = 0{\cdot}30$

Calculating the values of r_{ue} using equations (20) and (21) for the full range of ϕ' and β that has been treated provides sufficient data to plot contours of r_{ue}. The contours of r_{ue} on Fig. 7 determine whether $D = 1{\cdot}25$ gives a more critical value of factor of safety than $D = 1{\cdot}00$ when $c'/\gamma H = 0{\cdot}05$. A family of lines of r_{ue} is plotted on Fig. 8 to indicate whether $D = 1{\cdot}50$ gives a factor of safety that is even lower for $c'/\gamma H = 0{\cdot}05$. Fig. 10 contains the family of lines of r_{ue} that determines whether $D = 1{\cdot}25$ is more critical than $D = 1{\cdot}00$ when $c'/\gamma H = 0{\cdot}025$.

If one wishes to determine the minimum factor of safety for sections not located directly

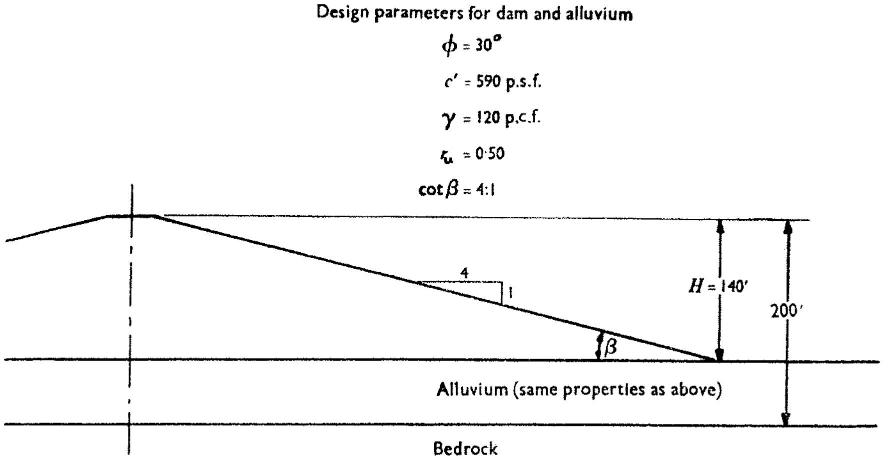

Fig. 15. Typical symmetrical earth dam section

on a hard stratum with specified values of $c'/\gamma H$, β, ϕ', and r_u, one enters the appropriate graph for the given $c'/\gamma H$ value and for $D = 1{\cdot}00$, initially (either Figs 7 or 10). The values of β and ϕ' then define a point on the curves of n with which is associated a value of r_{ue} given by the broken lines. If that value is less than the design value, the next depth factor, $D = 1{\cdot}25$, will give a more critical value of factor of safety. If one is operating with $c'/\gamma H = 0{\cdot}05$ a set of r_{ue} curves is available to determine in a similar manner whether the level given by $D = 1{\cdot}50$ is even more critical. $D = 1{\cdot}50$ has been inspected for $c'/\gamma H = 0{\cdot}025$ and has been found to be seldom more critical than $D = 1{\cdot}25$; hence it has not been presented. Depth factors play no part in the analysis of cohesionless slopes within the limits of the method already described.

AN EXAMPLE

The use of the stability coefficients is illustrated by the following example, for which the geometry and soil parameters are given in Fig. 15. From the given data we obtain:

$$\frac{c'}{\gamma H} = 0{\cdot}035$$

Proceeding to Fig. 10, for $c'/\gamma H = 0{\cdot}025$, and with $\phi' = 30°$ and $\beta = 4 : 1$ it is seen that:

$$r_{ue} < 0{\cdot}5$$

Therefore, $D = 1{\cdot}25$ is the more critical. Then, from Fig. 11, for $D = 1{\cdot}25$ and $\dfrac{c'}{\gamma H} = 0{\cdot}025$ it is found that:

$$\begin{aligned} m &= 2{\cdot}95, \\ n &= 2{\cdot}81. \end{aligned}$$

From equation (2), with $r_u = 0{\cdot}50$, it follows that:

$$F = 2{\cdot}95 - 1{\cdot}405 = 1{\cdot}545$$

For $c'/\gamma H = 0{\cdot}05$ it is evident in a similar manner from Fig. 7 that the factor of safety with $D = 1{\cdot}25$ is more critical than with $D = 1{\cdot}00$. Further, for $c'/\gamma H = 0{\cdot}05$, from Fig. 8:

$$r_{ue} \simeq 0{\cdot}72 > r_u$$

Therefore $D = 1{\cdot}25$ is the most critical level, and:

$$\begin{aligned} m &= 3{\cdot}23, \\ n &= 2{\cdot}83. \end{aligned}$$

Applying equation (2) with $r_u = 0{\cdot}50$:

$$F = 3{\cdot}23 - 1{\cdot}415 = 1{\cdot}815.$$

Interpolating linearly for the given value of $c'/\gamma H = 0{\cdot}035$ we obtain:

$$\begin{aligned} F &= 1{\cdot}545 + 0{\cdot}4 \times 0{\cdot}270 \\ &= 1{\cdot}65 \end{aligned}$$

THE PORE-PRESSURE RATIO, r_u

It has been shown in earlier paragraphs that the introduction of the pore pressure in terms of the ratio $r_u\left(= \dfrac{u}{\gamma h}\right)$ permits the results of stability analyses to be presented in a dimensionless form.

The use of the pore-pressure ratio r_u does not imply that the magnitude of the pore pressure is controlled by the product γh, though this may be approximately true of the pore pressures in embankment construction before consolidation occurs. It is equally applicable to pore pressures calculated for steady seepage by a flow net, and to observed pore pressures in either the initial or the steady state. Though the value of r_u is in general not constant over the whole cross section, in most slope stability problems an average value can readily be calculated and used in the stability analysis with little loss in accuracy.

In the prediction of pore pressure it is, however, necessary to distinguish between the two main classes of problem*:

(*a*) problems where pore pressure is an independent variable and is controlled either by ground-water level or by the flow pattern of impounded or underground water, and

(*b*) problems where the magnitude of the pore pressure depends on the magnitude of the stresses tending to lead to instability, as in rapid construction or excavation in soils of low permeability. In problems which fall initially into this class the pore-pressure distribution will change with time and at any point the pore pressure will either increase or decrease to adjust itself to the ultimate condition of equilibrium with the prevailing conditions of ground-water level or seepage. The rate at which this adjustment occurs depends on the permeability of the soil (as reflected in the coefficient of consolidation) and on the excess pore-pressure gradients.

The steady-seepage condition in an earth dam and the long-term stability of a natural slope are both example of class (*a*) problems for which the pore-pressure distribution maybe obtained either directly from field piezometer measurements (e.g. Sevaldson, 1956), or estimated from the construction of a flow net or by means of other analytical techniques for the solution of the differential equation governing the steady flow of water through soils (e.g. Casagrande, 1937).

In class (*b*) problems, the pore pressure u at any point in the soil mass is given in general by the following expression:

$$u = u_0 + \Delta u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where u_0 is the initial value of the pore pressure in the soil before any change in stress, and Δu is the change in pore pressure in the soil due to the change in stress.

The change in pore pressure, Δu, induced by a change in stress distribution has been expressed in the following manner (Skempton, 1954):

$$\Delta u = B\{\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

where $\Delta\sigma_1$ denotes the change in major principal stress,
$\Delta\sigma_3$ denotes the change in minor principal stress
and A and B are the pore-pressure parameters.

The application of the undrained triaxial test with pore-pressure measurement to the determination of these pore-pressure parameters has been fully described elsewhere (Bishop and Henkel, 1957).

The change in pore pressure with changes in both σ_1 and σ_3 is most conveniently expressed in terms of the relationship between pore pressure and major principal stress as given by the parameter $\bar{B}$. This parameter is obtained from an alternative form of equation (24):

$$\frac{\Delta u}{\Delta\sigma_1} = \bar{B} = B\left[\frac{\Delta\sigma_3}{\Delta\sigma_1} + A\left(1 - \frac{\Delta\sigma_3}{\Delta\sigma_1}\right)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

As illustrated by Fig. 16, the pore-pressure parameter $\bar{B}$ is a function of the principal stress ratio and also varies, in general, with the magnitude of the major principal stress. For a limited stress range, the variation with major principal stress is slight and $\bar{B}$ may be considered to be essentially a function of stress ratio alone.

It has been shown that the pore-pressure parameter $\bar{B}$ can be used to estimate the magnitude and distribution of pore pressure set up in a dam or embankment at the end of construc-

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* A discussion of the analysis of practical problems in terms of this classification is given by Bishop and Bjerrum (1960).

tion where no dissipation is assumed to occur* (Bishop, 1954), and the pore-pressure ratio as defined by equation (1) is related to $\bar{B}$:

$$r_u = \frac{u_0}{\gamma h} + \frac{\bar{B}}{\gamma h} \cdot \Delta\sigma_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

On the basis of the theory of elasticity, and averaging round a typical slip surface, it has also been shown that it is reasonable to assume the major principal stress at a point in earth fill embankments to be equal to the weight of soil above the point under consideration (Bishop, 1952). Then:

$$r_u = \frac{u_0}{\gamma h} + \bar{B} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

For earth fill placed at or below the optimum water content the initial pore pressure u_0 may be negative and in the low stress range the corresponding value of r_u will also be negative. Where negative values of r_u obtain they are usually neglected in design. For earth fill placed wet of the optimum water content, the initial pore pressure is often small enough to be considered negligible. Hence, stability analysis of the end of construction condition with predicted pore pressures has sometimes been based upon:

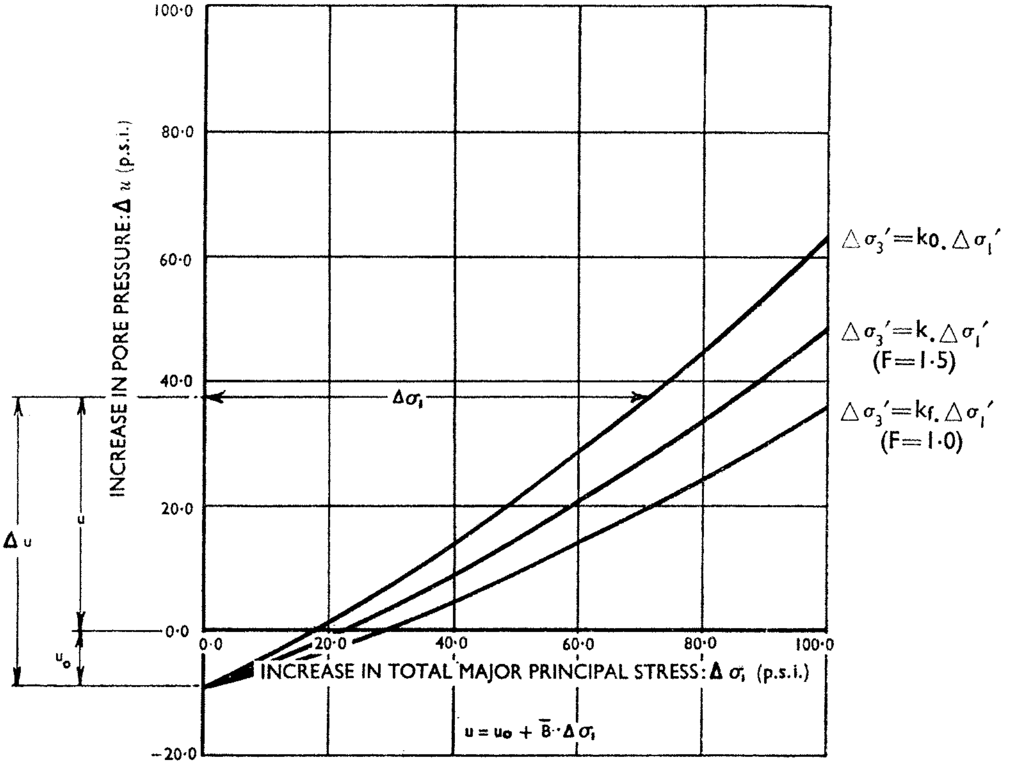

Fig. 16. Diagrammatic variation of pore-pressure parameter $\bar{B}$ with principal stress ratio and major principal stress

$$r_u = \bar{B} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

The early use of the pore-pressure ratio in stability analysis (Bishop, 1955) was based upon equation (27).

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* The effect of dissipation on the magnitude and distribution of pore pressures in a section has been treated by Gibson (1959).

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As mentioned previously, the pore-pressure ratio, r_u, in a section is not constant and an average value of r_u must be used to employ the stability coefficients in calculating the factor of safety. The averaging method that has been used has proved successful in giving values of the factor of safety that correspond closely to those obtained by direct calculation for cases in which the pore-pressure distribution in terms of the pore-pressure ratio r_u varied considerably throughout the section. To illustrate its application, take a cross-section as shown in Fig. 17 with three different pore-pressure zones given by r_{u1}, r_{u2}, r_{u3} and divide it into four areas, by dividing the base into four equal parts, starting from the middle of the crest and moving toward the toe. In the case of natural slopes begin the sectioning at a distance equal to $H/4$ from the crest. Then obtain the average pore-pressure ratio for each section area by averaging along the centre line of each section. For example for section area 1:

$$\text{Average } r_u = \frac{h_1 r_{u1} + h_2 r_{u2} + h_3 r_{u3}}{h_1 + h_2 + h_3}$$

or more generally:

$$\text{Section area average, } r_u = \frac{\sum_1^i h_i r_{ui}}{\sum_1^i h_i} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (28)$$

where i is the number of different pore-pressure zones intersected by the centre line of the section.

After finding the section area average of the pore-pressure ratios for the remaining three areas, the overall area average is found by summing the products of the area of each section times its own average pore-pressure ratio, given by equation (28) and dividing this sum by the sum of the areas of the four sections:

$$\text{Overall average, } r_u = \frac{\sum_1^n A_n r_{un}}{\sum_1^n A_n} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (29)$$

where n is the number of sections, and in this case equal to 4. Typical examples of overall average pore-pressure ratios, and the range of values from which they were derived are given in Table 2 for different classes of stability problem.

The four examples given in Table 2 have been calculated precisely, using the actual varying distribution of pore pressure. A comparison of the values of factor of safety obtained

Table 2

Pore-pressure ratios for four typical cases

Case	Range of r_u	Average r_u	Remarks
Vallecito Dam	0–0·55	0·23	Pore-pressure ratios calculated from measurements at end of construction (Niederhoff, 1951).
Steady seepage	0–0·40	0·22	Pore-pressure ratios calculated from construction of a flow net (Casagrande, 1937).
Lodalen	0–0·49	0·28	Pore-pressure ratios calculated from field measurements (Sevaldson, 1956).
Selset	0·48	0·48	Pore-pressure ratio based upon field observations (Imperial College).

by this more exact calculation with the values given by calculations using the stability coefficients and the average pore-pressure ratio shows that reasonable results are obtained by this approximate method when the pore-pressure ratio varies throughout the section.

The distribution of the pore-pressure ratio at the end of construction has been obtained by applying equation (1) to the measurements described by Niederhoff (1951) for the Vallecito Dam:

$$r_u = \frac{u}{\gamma h} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

The contours of r_u are shown in Fig. 18. Using typical shear strength values, a direct calculation of the factor of safety taking account of the varying pore-pressure ratios gave a value that was 6·8% *less* than that given by the average pore-pressure ratio (Table 2) and the stability coefficients. The section and strength parameters used in the calculations are also given in Fig. 18.

The steady seepage case for an earth dam (Fig. 19) is an artificially constructed problem which is, however, typical of this condition in a rolled fill earth dam. In this case, the calculation using the averaging technique and stability coefficients over-estimated the factor of safety given by the more precise calculation by only 2·3%.

The next two cases mentioned in Table 2 are natural slopes that have failed under long-term conditions, and for which the pore-pressure distribution at failure has been observed. For the slide at Lodalen (Sevaldson, 1956) the factor of safety calculated in terms of effective stress has been given as unity for the centre section. Recalculating the factor of safety using the stability coefficients and the average pore-pressure ratio, an over-estimate of 7% ($F = 1{\cdot}07$) has been obtained. For the slide at Selset, the ground-water level lies almost at the surface and the flow is very nearly horizontal. With the assumption of ground-water level at the surface and horizontal flow a homogeneous distribution of pore-pressure ratio is obtained and the averaging technique is not needed. As expected, the value given by the stability coefficients, namely, 0·99, coincides with that given by a direct analysis. It is expected that a full description of this slide will be given at a later date.

It is seen that for typical cases where the pore-pressure ratio varies throughout the section, the use of the averaging technique and stability coefficients for estimating the factor of safety gives sufficient accuracy for at least preliminary design purposes. Although exact for simple sections with a homogeneous distribution of pore-pressure ratio, this method tends to over-estimate the factor of safety in cases where the averaging technique must be applied. However, the error in the calculated factor of safety is only of the order of 7%, for an example representing an extreme case of non-uniformity of pore-pressure ratio. The practical value of this method in solving ordinary engineering problems with a minimum of labour is therefore apparent. It should be borne in mind that if there are isolated zones of high pore pressures, the failure surface may be controlled by their position and may deviate considerably from a circle; in this case a different type of analysis is required.

CONCLUSIONS

More general application of the effective stress analysis to routine work has been hindered by the lack of a general solution similar to that produced by Taylor for the total stress analysis. To obviate this difficulty, a set of coefficients that can be used to investigate the stability, in terms of effective stress, of most simple sections encountered in earth dam and embankment problems has been presented. The application of these coefficients gives results that are correct for simple sections composed of only one material and whose pore-pressure distribution can be described by the pore-pressure ratio, r_u, being constant throughout the given section.

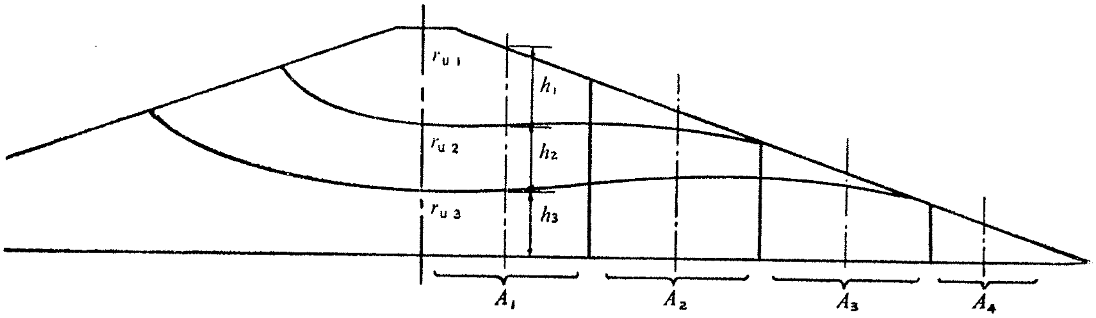

Fig. 17. Example of a non-homogeneous pore-pressure ratio distribution

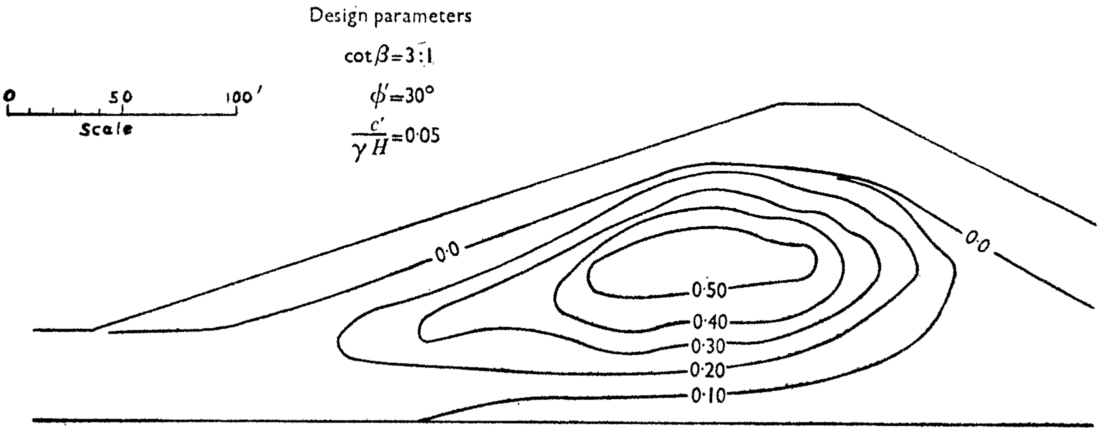

Fig. 18. Distribution of pore-pressure ratio r_u for end of construction case

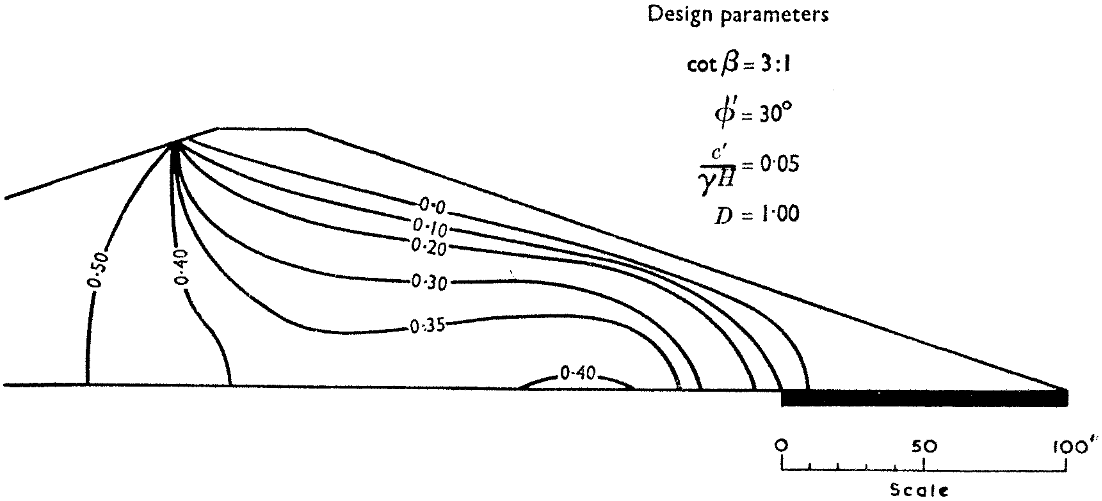

Fig. 19. Distribution of pore-pressure ratio r_u for a steady seepage case

A method has been described for determining the average pore-pressure ratio when the pore-pressure ratio distribution is variable within the cross-section. This allows the application of the stability coefficients to the more usual type of design or analysis that the engineer faces and the several cases of this type investigated reveal that only a comparatively small error is incurred in estimating the factor of safety. Examples of the end of construction and steady seepage stability in earth dams as well as natural slopes have been given to illustrate this application.

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APPENDIX A

The stability coefficients are presented in tabular form in this Appendix for use in the expression:

$$F = m - n . r_u \qquad \ldots\ldots\ldots\ldots \qquad (2)$$

Table A–1

Stability coefficients m and n for $\dfrac{c'}{\gamma H} = 0$

Stability coefficients for earth slopes								
ϕ'	Slope 2 : 1		Slope 3 : 1		Slope 4 : 1		Slope 5 : 1	
	m	n	m	n	m	n	m	n
10·0	0·353	0·441	0·529	0·588	0·705	0·749	0·882	0·917
12·5	0·443	0·554	0·665	0·739	0·887	0·943	1·109	1·153
15·0	0·536	0·670	0·804	0·893	1·072	1·139	1·340	1·393
17·5	0·631	0·789	0·946	1·051	1·261	1·340	1·577	1·639
20·0	0·728	0·910	1·092	1·213	1·456	1·547	1·820	1·892
22·5	0·828	1·035	1·243	1·381	1·657	1·761	2·071	2·153
25·0	0·933	1·166	1·399	1·554	1·865	1·982	2·332	2·424
27·5	1·041	1·301	1·562	1·736	2·082	2·213	2·603	2·706
30·0	1·155	1·444	1·732	1·924	2·309	2·454	2·887	3·001
32·5	1·274	1·593	1·911	2·123	2·548	2·708	3·185	3·311
35·0	1·400	1·750	2·101	2·334	2·801	2·977	3·501	3·639
37·5	1·535	1·919	2·302	2·558	3·069	3·261	3·837	3·989
40·0	1·678	2·098	2·517	2·797	3·356	3·566	4·196	4·362

Table A–2

Stability coefficients m and n for $\dfrac{c'}{\gamma H} = 0{\cdot}025$ and $D = 1{\cdot}00$

ϕ'	Slope 2 : 1		Slope 3 : 1		Slope 4 : 1		Slope 5 : 1	
	m	n	m	n	m	n	m	n
10·0	0·678	0·534	0·906	0·683	1·130	0·846	1·365	1·031
12·5	0·790	0·655	1·066	0·849	1·337	1·061	1·620	1·282
15·0	0·901	0·776	1·224	1·014	1·544	1·273	1·868	1·534
17·5	1·012	0·898	1·380	1·179	1·751	1·485	2·121	1·789
20·0	1·124	1·022	1·542	1·347	1·962	1·698	2·380	2·050
22·5	1·239	1·150	1·705	1·518	2·177	1·916	2·646	2·317
25·0	1·356	1·282	1·875	1·696	2·400	2·141	2·921	2·596
27·5	1·478	1·421	2·050	1·882	2·631	2·375	3·207	2·886
30·0	1·606	1·567	2·235	2·078	2·873	2·622	3·508	3·191
32·5	1·739	1·721	2·431	2·285	3·127	2·883	3·823	3·511
35·0	1·880	1·885	2·635	2·505	3·396	3·160	4·156	3·849
37·5	2·030	2·060	2·855	2·741	3·681	3·458	4·510	4·209
40·0	2·190	2·247	3·090	2·993	3·984	3·778	4·885	4·592

Table A–3

Stability coefficients m and n for $\frac{c'}{\gamma H}$ 0·025 and D = 1·25

ϕ'	Slope 2 : 1		Slope 3 : 1		Slope 4 : 1		Slope 5 : 1	
	m	n	m	n	m	n	m	n
10·0	0·737	0·614	0·901	0·726	1·085	0·867	1·285	1·014
12·5	0·878	0·759	1·076	0·908	1·299	1·089	1·543	1·278
15·0	1·019	0·907	1·253	1·093	1·515	1·312	1·803	1·545
17·5	1·162	1·059	1·433	1·282	1·736	1·541	2·065	1·814
20·0	1·309	1·216	1·618	1·478	1·961	1·775	2·334	2·090
22·5	1·461	1·379	1·808	1·680	2·194	2·017	2·610	2·373
25·0	1·619	1·547	2·007	1·891	2·437	2·269	2·897	2·669
27·5	1·783	1·728	2·213	2·111	2·689	2·531	3·196	2·976
30·0	1·956	1·915	2·431	2·342	2·953	2·806	3·511	3·299
32·5	2·139	2·112	2·659	2·585	3·231	3·095	3·841	3·638
35·0	2·331	2·321	2·901	2·841	3·524	3·400	4·191	3·998
37·5	2·536	2·541	3·158	3·112	3·835	3·723	4·563	4·379
40·0	2·753	2·775	3·431	3·399	4·164	4·064	4·958	4·784

Table A–4

Stability coefficients m and n for $\frac{c'}{\gamma H}$ = 0·05 and D = 1·00

ϕ'	Slope 2 : 1		Slope 3 : 1		Slope 4 : 1		Slope 5 : 1	
	m	n	m	n	m	n	m	n
10·0	0·913	0·563	1·181	0·717	1·469	0·910	1·733	1·069
12·5	1·030	0·690	1·343	0·878	1·688	1·136	1·995	1·316
15·0	1·145	0·816	1·506	1·043	1·904	1·353	2·256	1·567
17·5	1·262	0·942	1·671	1·212	2·117	1·565	2·517	1·825
20·0	1·380	1·071	1·840	1·387	2·333	1·776	2·783	2·091
22·5	1·500	1·202	2·014	1·568	2·551	1·989	3·055	2·365
25·0	1·624	1·338	2·193	1·757	2·778	2·211	3·336	2·651
27·5	1·753	1·480	2·380	1·952	3·013	2·444	3·628	2·948
30·0	1·888	1·630	2·574	2·157	3·261	2·693	3·934	3·259
32·5	2·029	1·789	2·777	2·370	3·523	2·961	4·256	3·585
35·0	2·178	1·958	2·990	2·592	3·803	3·253	4·597	3·927
37·5	2·336	2·138	3·215	2·826	4·103	3·574	4·959	4·288
40·0	2·505	2·332	3·451	3·071	4·425	3·926	5·344	4·668

Table A–5

Stability coefficients m and n for $\frac{c'}{\gamma H} = 0{\cdot}05$ and $D = 1{\cdot}25$

ϕ'	Slope 2 : 1		Slope 3 : 1		Slope 4 : 1		Slope 5 : 1	
	m	n	m	n	m	n	m	n
10·0	0·919	0·633	1·119	0·766	1·344	0·886	1·594	1·042
12·5	1·065	0·792	1·294	0·941	1·563	1·112	1·850	1·300
15·0	1·211	0·950	1·471	1·119	1·782	1·338	2·109	1·562
17·5	1·359	1·108	1·650	1·303	2·004	1·567	2·373	1·831
20·0	1·509	1·266	1·834	1·493	2·230	1·799	2·643	2·107
22·5	1·663	1·428	2·024	1·690	2·463	2·038	2·921	2·392
25·0	1·822	1·595	2·222	1·897	2·705	2·287	3·211	2·690
27·5	1·988	1·769	2·428	2·113	2·957	2·546	3·513	2·999
30·0	2·161	1·950	2·645	2·342	3·221	2·819	3·829	3·324
32·5	2·343	2·141	2·873	2·583	3·500	3·107	4·161	3·665
35·0	2·535	2·344	3·114	2·839	3·795	3·413	4·511	4·025
37·5	2·738	2·560	3·370	3·111	4·109	3·740	4·881	4·405
40·0	2·953	2·791	3·642	3·400	4·442	4·090	5·273	4·806

Table A–6

Stability coefficients m and n for $\frac{c'}{\gamma H} = 0{\cdot}05$ and $D = 1{\cdot}50$

ϕ'	Slope 2 : 1		Slope 3 : 1		Slope 4 : 1		Slope 5 : 1	
	m	n	m	n	m	n	m	n
10·0	1·022	0·751	1·170	0·828	1·343	0·974	1·547	1·108
12·5	1·202	0·936	1·376	1·043	1·589	1·227	1·829	1·399
15·0	1·383	1·122	1·583	1·260	1·835	1·480	2·112	1·690
17·5	1·565	1·309	1·795	1·480	2·084	1·734	2·398	1·983
20·0	1·752	1·501	2·011	1·705	2·337	1·993	2·690	2·280
22·5	1·943	1·698	2·234	1·937	2·597	2·258	2·990	2·585
25·0	2·143	1·903	2·467	2·179	2·867	2·534	3·302	2·902
27·5	2·350	2·117	2·709	2·431	3·148	2·820	3·626	3·231
30·0	2·568	2·342	2·964	2·696	3·443	3·120	3·967	3·577
32·5	2·798	2·580	3·232	2·975	3·753	3·436	4·326	3·940
35·0	3·041	2·832	3·515	3·269	4·082	3·771	4·707	4·325
37·5	3·299	3·102	3·817	3·583	4·431	4·128	5·112	4·735
40·0	3·574	3·389	4·136	3·915	4·803	4·507	5·543	5·171

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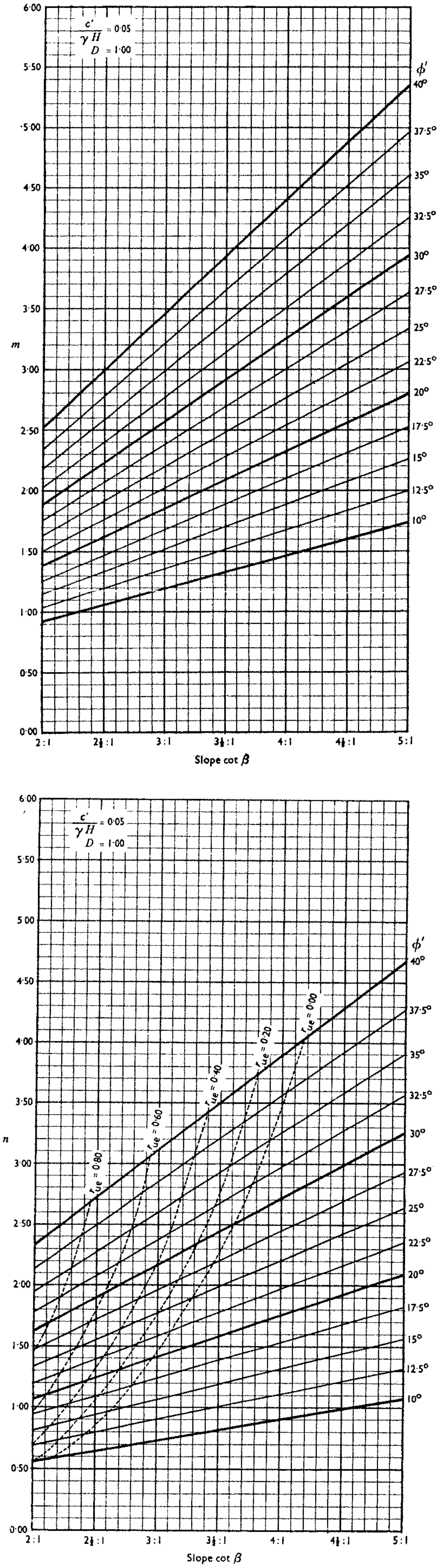

Fig. 7. Stability coefficients m and n for $\frac{c'}{\gamma H} = 0{\cdot}05$ and $D = 1{\cdot}00$

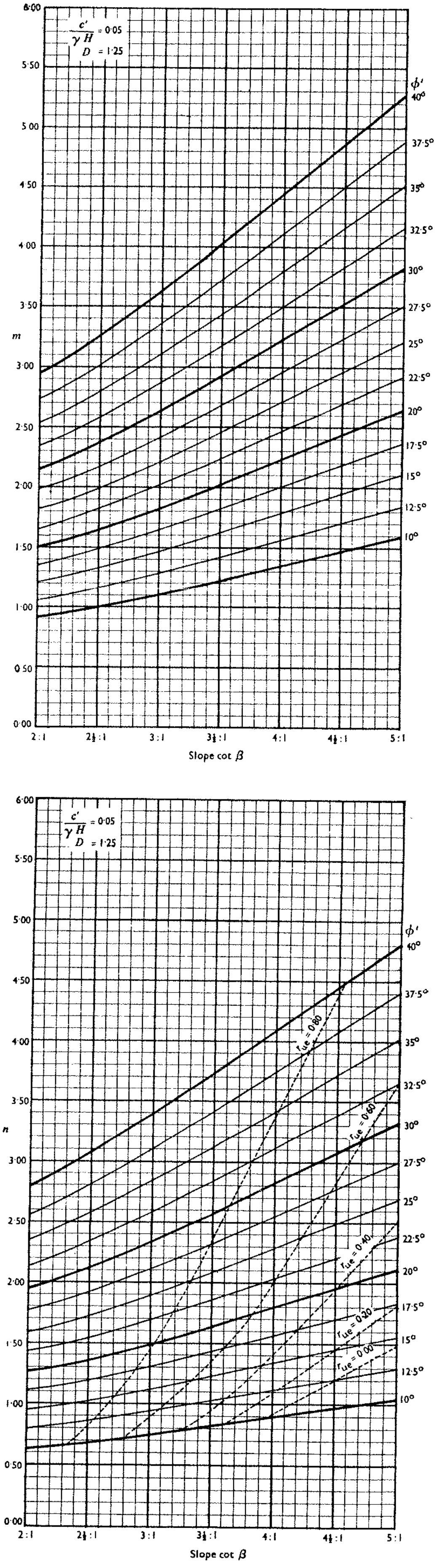

Fig. 8. Stability coefficients m and n for $\frac{c'}{\gamma H} = 0{\cdot}05$ and $D = 1{\cdot}25$

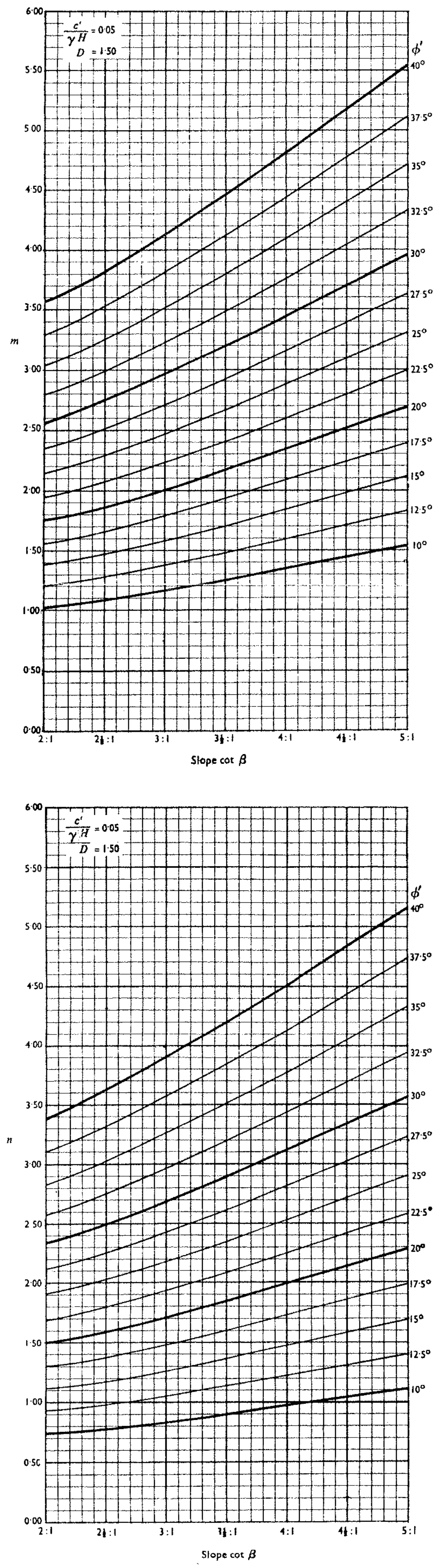

Fig. 9. Stability coefficients m and n for $\frac{c'}{\gamma H} = 0{\cdot}05$ and $D = 1{\cdot}50$

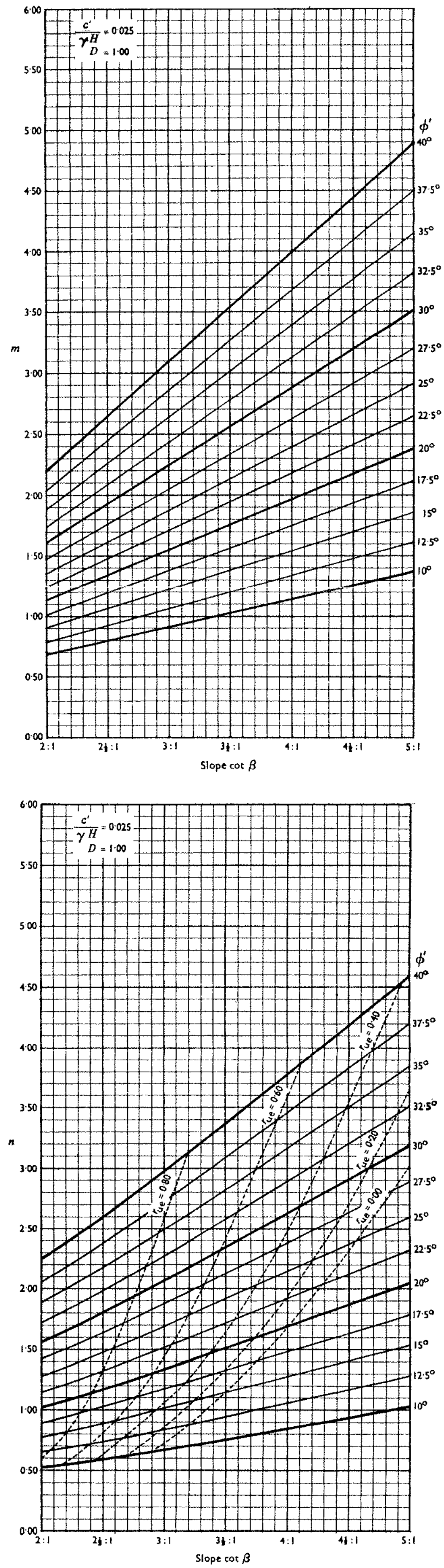

Fig. 10. Stability coefficients m and n for $\frac{c'}{\gamma H} = 0{\cdot}025$ and $D = 1{\cdot}00$

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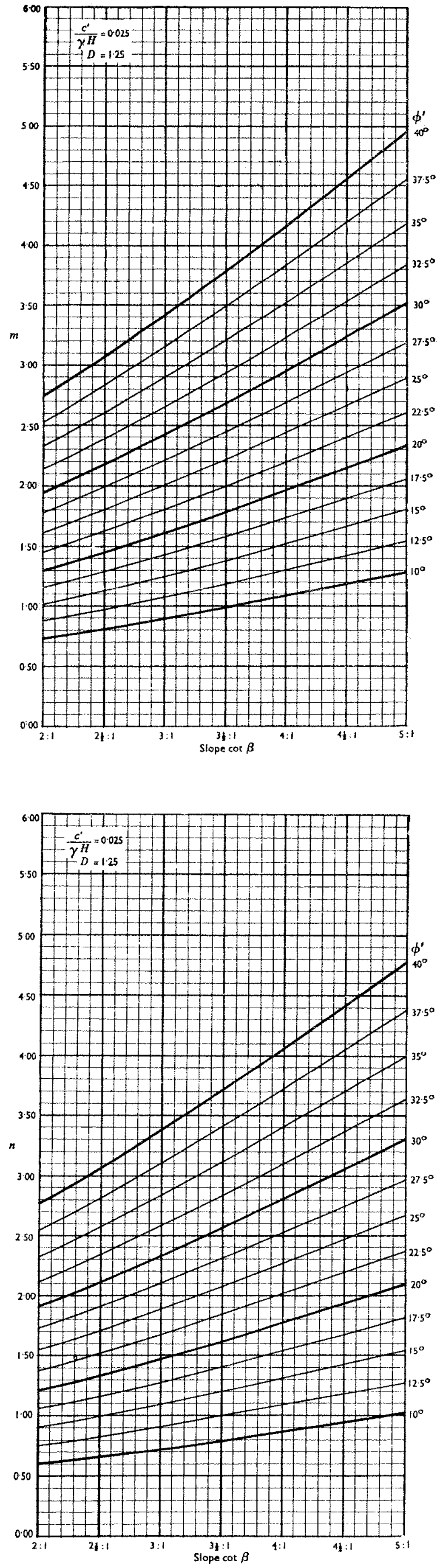

Fig. 11. Stability coefficients m and n for $\frac{c'}{\gamma H} = 0{\cdot}025$ and $D = 1{\cdot}25$

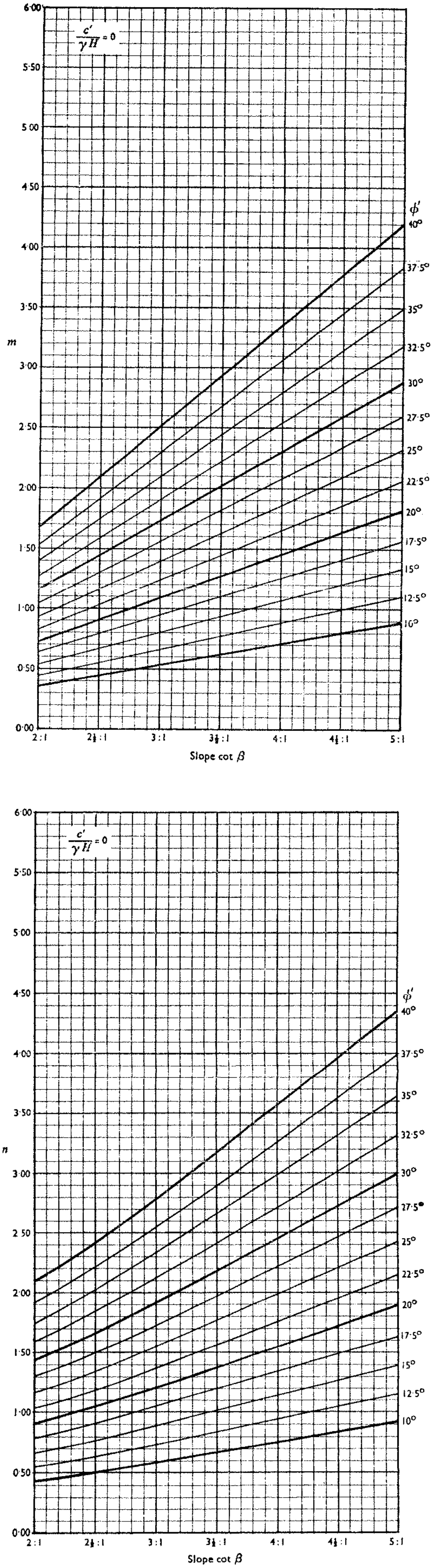

Fig. 12. Stability coefficients m and n for $\frac{c'}{\gamma H} = 0$

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