

# Mathematical Modeling and Machine Learning for Force Estimation on a Planar Catheter

H. Ali Zar<sup>1</sup>, M. Goharimanesh<sup>2</sup>, F. Janabi-Sharifi<sup>3\*</sup>

<sup>1</sup> (*Student*) Dept. of Electrical, Computer and Biomedical Engineering, Ryerson University, Toronto, Canada ([hzar@ryerson.ca](mailto:hzar@ryerson.ca))

<sup>2</sup>Mechanical Engineering Dept., University of Torbat Heydarieh, Iran ([m.goharimanesh@torbath.ac.ir](mailto:m.goharimanesh@torbath.ac.ir))

<sup>3</sup>Mechanical and Industrial Engineering Dept., Ryerson University, Toronto, Canada ([fsharifi@ryerson.ca](mailto:fsharifi@ryerson.ca))

**Abstract**—In this paper, estimation of the applied force on a planar catheter is considered. An image processing approach has been chosen as the most suitable tool. For this purpose, we create a comprehensive database of catheters with different shapes and operating forces. Using image processing algorithms, position data for the catheters' outer, middle, and inner layers in this database are obtained. Finally, using curve fitting, an exponential mathematical function is accepted for the middle layer of each catheter. Feeding this data into different machine learning algorithms, force estimation is obtained with a mean average error of 0.52 N. Later, the force applied to a tendon catheter is estimated using the SOFA framework and deep learning techniques, directing the research into a reliable applied force estimation.

**Keywords**—Force estimation; Machine learning; 2D catheter; Image processing; Mathematical modeling

## I. INTRODUCTION

Steerable catheters and endoscopes have been investigated for more operative minimally invasive treatment of cardiac problems [1-3]. Steerable catheter tubes in the ablation operation are inserted into a heart chamber through the vasculature and then ablate the target area in the heart using radiofrequency energy to scar or destroy the tissue responsible for passing on abnormal electrical signals [4, 5]. Due to the complex and delicate nature of the vasculature, the contact force applied at the tip of the catheter is vital information to improve safety and maintain a consistent force for an accurate operation [6, 7].

However, it is challenging to sense the contact force during the procedure. Cardiologists have minor haptic feedback during manual operations due to the partial transmission of the contact force that their fingers can sense through the long and flexible structure of the catheter [8]. Although studies are being conducted on direct sensors that can directly measure the applied force, recent results show that there is still a long way to build these measuring devices that operate with low cost and high accuracy [9]. Introduced dynamics through these devices could also be another issue in navigating the catheters. Fortunately, as the technologies of catheter intra-operative tracking advances, the 2D shape of catheters could be detected through the magnetic field, CT, MRI, and x-ray fluoroscopy [10]. Therefore, an alternative solution for this unadaptable problem is contact

forces estimation from the shape of the deflected flexible body of the catheter. This intrinsic force-sensing solution has already been implemented into conventional ablation catheters [11].

This research proposes a new contact force estimation method based on image processing and machine learning, following our previous work in this area [7, 11]. Creating a unified database to perform the image processing procedures is necessary. For this reason, using finite element methods, different samples of catheters with different diameters and shapes are subjected to a range of forces. In the image processing process, we obtain a grayscale outline of the catheter and use this outline to get position data for the samples' outer, middle, and inner edges. Using this data, we can obtain a mathematical model describing the curvature of the catheter.

Before starting the deep learning phase, using mathematical modeling techniques, suitable properties are obtained for the relationship between the mathematical model and the estimated force. Our goal is to use the images and videos of a catheter under force application and estimate the force applied through our proposed method.

In the second section of the paper, the two-dimensional (2D) catheter structure is explained. The third section presents the data set used in this research. The samples used in this section are created using SOLIDWORKS 2021 software, based on the Finite Element Method (FEM). The programming is done using Python and MATLAB languages. At the end of this section, the information obtained from the proposed exponential function is processed by a deep neural network, and finally, the applied force is estimated according to the accepted mathematical model. In the fifth section of the paper, the data of a tendon-driven catheter are analyzed in the SOFA framework [12] using image processing. The sixth part of the paper is dedicated to the conclusion and presenting future solutions.

## II. 2D CATHETER STRUCTURE

In this paper, a two-dimensional structure of a catheter is considered. The Global coordinate system is assumed to be located at the X-Y axis, and the external force is applied at the tip of the catheter. Using image processing techniques combined with a mathematical model, we detect the points during catheter deformation. In Fig. 1, a catheter's outer, middle, and inner layers with a diameter of cross-sectional profile ( $\phi$ ) is

considered. In the next section, we will prepare a database using five different diameter sizes with a range of forces applied.

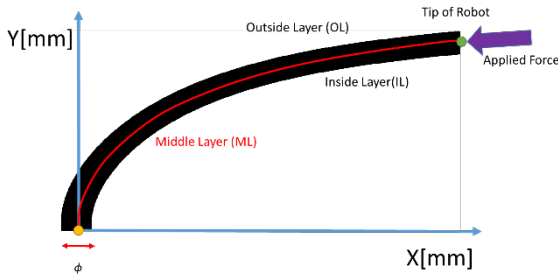


Figure 1: Schematic of a 2D catheter with applied force

### III. DATASET

Before using mathematical modeling or deep learning to estimate forces, we first need to establish a comprehensive database. Different catheter samples with diameters and shapes are designed using finite element methods (Fig. 2). We then subject these samples to a range of forces, and finally, the displacement of all parts of the catheter is captured through their images.

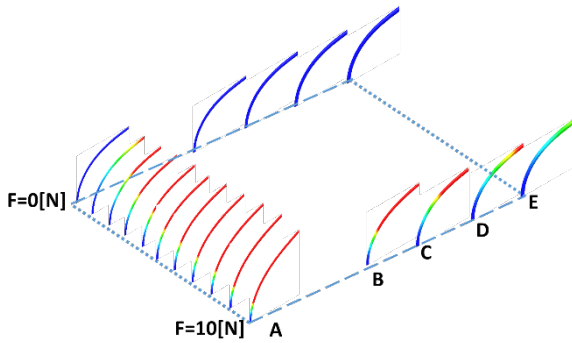


Figure 2: Database for catheters with different profiles and various applied forces

### IV. IMAGE PROCESSING AND MACHINE LEARNING

In this section, the following procedures are implemented in Python to extract the morphological features of catheters. Original images of the catheter dataset, as shown in Fig. 3, are considered and then processed by the following algorithms.

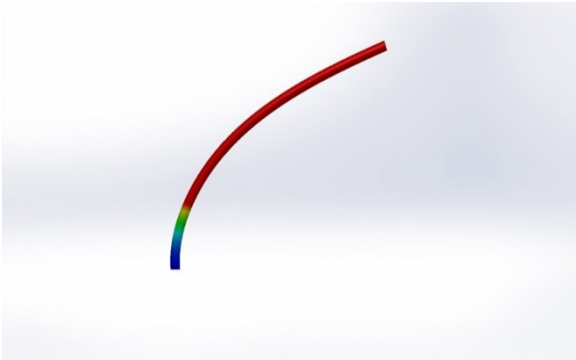


Figure 3: Original image of catheter sample,  $F=10[N]$ ,  $\phi=1[mm]$

#### A. Edge detection, Thresholding, and Cropping

Images from the database are loaded in grayscale Numpy arrays. The arrays can be thought of as a 2D grid, with each grid holding a specific pixel value in a range from 0 to 255. We first want to obtain an edge-detected view of these samples. This is done using the Canny edge detection algorithm. The library OpenCV provides a simple function to apply this algorithm. This function returns a grayscale image array with the edges highlighted in white (Fig. 4).

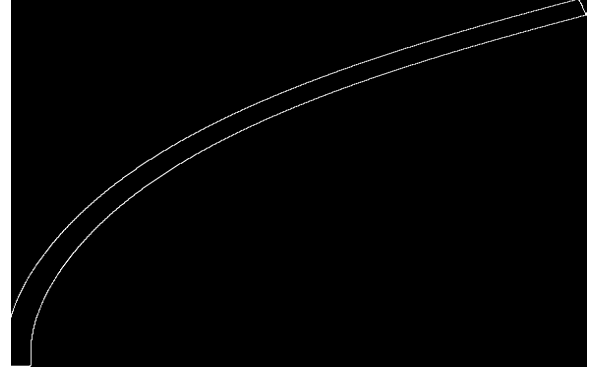


Figure 4: Cropped and edge detected image for a catheter,  $F=10\text{ N}$ ,  $\phi=1\text{ mm}$

After obtaining our edge detected image, we can crop this image ideally using Numpy. This is done by accessing every white pixel in our image array using a Numpy function name `argwhere()`. After obtaining these values, we crop the image according to the leftmost, rightmost, topmost, and bottommost white pixel, as shown in Fig. 5.

```
white_pt_coords=np.argwhere(img)
min_y = min(white_pt_coords[:,0])
min_x = min(white_pt_coords[:,1])
max_y = max(white_pt_coords[:,0])
max_x = max(white_pt_coords[:,1])
result = img[min_y:max_y,min_x:max_x]
```

Figure 5: Code for cropping catheter sample

#### B. Finding the outer, inner, and middle line

We use our processed image to extract data for the catheter's outer, inner, and middle curves. We loop through every row of pixels in the image from left to right for the external curve. As soon as we encounter a white pixel in a row, we record the position of this pixel in a and skip to the next row. Using this simple technique, we can record the parts of all the pixels on the outer layer of the catheter.



Figure 6: Plotting the data for the inner, outer, and middle lines

The process for obtaining the inner side is slightly different from the outer side. We loop through the rows of the image from right to left this time, again recording the first white pixel we encounter. The data for the middle line is simply by taking the average of the left and suitable arrays for each row. We now have three collections storing the outer, middle, and inner curves. These three arrays are plotted in Fig. 6.

### C. Converting to real-world coordinates

We can convert the pixel values to real-world distances using the diameter of each catheter, as shown below. First, we need to obtain the bottom left point and the bottom right point of the image. We can do this by using the last recorded value from the data for the outer side of the catheter. Since we looped from the top left to the top right, the final recorded value would be the bottommost left point of the catheter. For the bottommost right point, we can use the data obtained from the outer side of the catheter. The bottommost pixel value would be the first recorded data point.

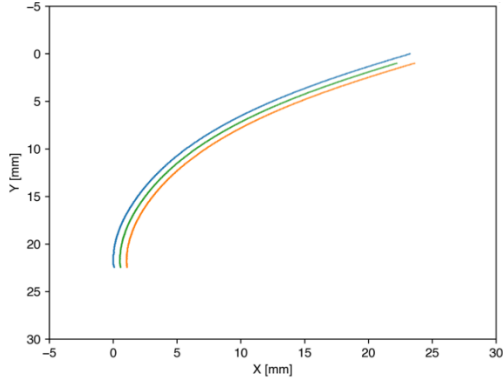


Figure 7: Actual coordinates of the detected catheter,  $F=10\text{N}$ ,  $\phi=1\text{mm}$

By calculating the difference between these two data points, we know now how many pixels make up the diameter of the catheter. We can then calculate the distance per pixel by dividing the number of pixels by the catheter's diameter. We must multiply all the pixel data we obtained for the outer, middle, and inner curves by the distance per pixel value. We can now graph the resulting data in terms of real-world coordinates (Fig. 7).

### D. Obtaining mathematical model

Before starting the deep learning phase, we obtain a mathematical model representing the catheter and the applied

force. This step is done using Matlab's curve fitting toolbox. We used an extended second-order exponential equation (1) to model our data. For all shapes of catheters, this curve fitting is done as shown in Fig. 8.

$$y_m = \left( \left( a e^{-bx_m} + c e^{-dx_m} \right) + e \right) x_m \quad (1)$$

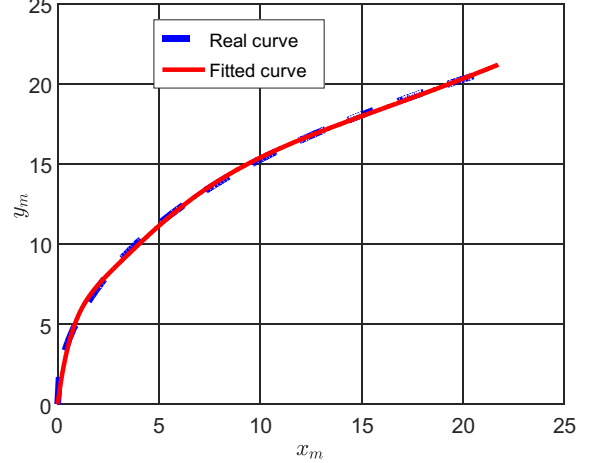


Figure 8: Actual coordinates of the detected catheter,  $F=10\text{N}$ ,  $\phi=1\text{mm}$  (axes in mm)

Our data now consists of 7 values,  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  (from the second-order exponential equation),  $\phi$  (diameter), and  $F$  (force). We obtained 87 different rows. A part of this result is shown in table 1.

TABLE I. MATHEMATICAL REPRESENTATIONS OF CATHETER SAMPLES

$a$	$b$	$c$	$d$	$e$	$\phi$	$F$ (N)
2.270	0.105	5.181	0.826	0.489	1	0
2.285	0.106	5.235	0.832	0.507	1	1
2.303	0.107	5.304	0.839	0.527	1	2
2.322	0.108	5.389	0.848	0.550	1	3
2.346	0.110	5.459	0.859	0.575	1	4

### E. Training using Random-Forest

This step predicts the force applied to the catheter using the mathematical models using the Random-Forest machine learning algorithm. This is a popular machine learning algorithm and will provide a good reference for future experiments. Implementation is given via the Scikit-learn Python library.

We split the data into training and validation sets with a split of 0.2. Scikit-learn provides dozens of different parameters for the Random-Forest algorithm. To find the best combination of parameters efficiently, we set up a function to iterate over 400 different varieties of parameters to find the ones that yield the best results. This is done using the RandomizedSearchCV function in SciKit-learn. The most optimal set of these 400 combinations gives us a mean average error of 1.36 N. Sample predictions are shown in Table 2.

TABLE II. SAMPLE PREDICTION OF RANDOM-FOREST ALGORITHM

True value N	Prediction N
3	3.25
0	0.84
4	5.03
1	0.92
5	5.63
6	5.65

F. Training using Deep Neural Network

Since the results using Random-Forest were promising, we decided to train the data on a deep neural net implemented with Tensor-Flow. This DNN has four hidden layers between the input and the output layers to allow accurate modeling of our non-linear relationship.

Data is loaded in a normalized, again with a 0.2 train/test split. The optimizer used is Adam, with a 0.001 learning rate. The loss function used is mean squared error (MSE), with 300 epochs and a batch size of 12. All parameters were chosen analytically with the help of the Taguchi statistical design method. The architecture summary is shown in Table 3.

TABLE III. ARCHITECTURAL SUMMARY FOR NEURAL NETWORK

Layer	Layer size	Param #
Normalization	6	13
Dense	6	42
Dense	256	1792
Dense	256	65792
Dense	256	65792
Dropout (0.2)	256	0
Dense	64	16448
Dense	1	65

We obtained a 0.52 N mean average error and an  $R^2$  of 96%. As shown in Figure 9, there are noticeable spikes of inaccuracy in our validation set. This is due to the relatively small dataset (87 rows) we are using. By developing a larger group of catheter samples, we can improve our mean average error and make more consistent predictions.

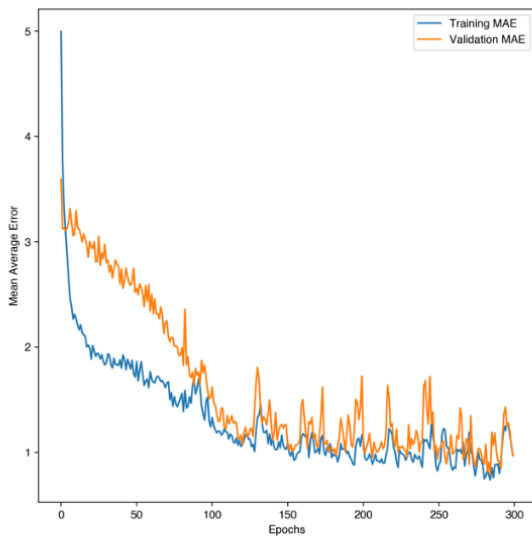


Figure 9: Training results using our Deep Neural Network

V. FORCE ESTIMATION VIA DEEP LEARNING IN SOFA FRAMEWORK

In this section, using the SOFA framework [12] and the Tendon Finger model, an extensive database of different shapes in terms of tendon force is examined. The most distinguishing factor of the SOFA framework with the previous FEM software is that image processing is very neat and presentable [13]. Part of this database is shown in Fig. 10. Edge detection is then performed using image processing algorithms via Python. Unlike the previous section, where we estimated the force on the catheter using a mathematical model and finally the neural network, in this section, we estimate force with a pure deep learning approach due to the high diversity of the database.

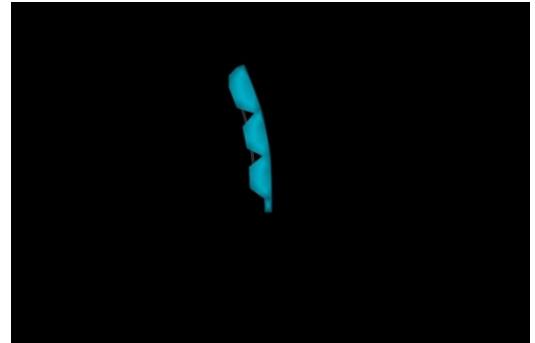


Figure 10: Tendon finger modeled with SOFA software

The final result in this study is that we will analyze the images and videos of a catheter and estimate the amount of force applied by the proposed method. We can combine temporal and spatial data with videos and pictures to achieve better results.

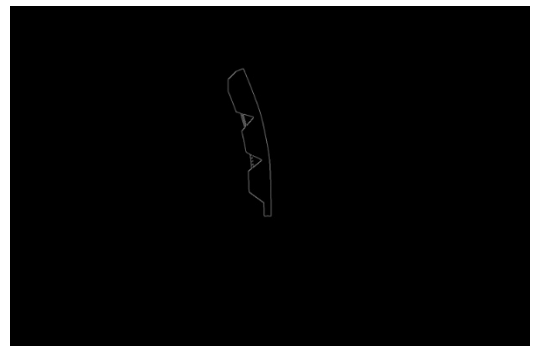


Figure 11: Edge detection of tendon finger modeled in SOFA framework

VI. CONCLUSION

Estimating forces acting on catheters is one of the most challenging topics in robot-assisted interventions. In the meantime, force estimation using image processing techniques can be a suitable solution for efficiently estimating external forces. This paper evaluated the various structures using a database based on SOLIDWORKS software. Finally, according to a mathematical model based on an exponential function and using neural networks, the logical relationship between the

parameters of the mathematical model and the applied force was set. In the following, we tested this force estimation separately in the SOFA software to estimate the force with deep learning techniques. In the future, we will extend our work to include a complete database in the SOFA environment for three-dimensional catheters and for different catheter shapes. We will also seek clinical examination of the proposed techniques.

#### REFERENCES

- [1] P. Breedveld, J. S. Sheltes, E. M. Blom, and J. E. I. Verheij, "A new, easily miniaturized steerable endoscope," *IEEE Engineering in Medicine and Biology Magazine*, vol. 24, no. 6, pp. 40–47, 2005, doi: 10.1109/EMEMB.2005.1549729.
- [2] P. E. Dupont *et al.*, "A decade retrospective of medical robotics research from 2010 to 2020," *Science Robotics*, vol. 6, no. 60, p. eabi8017, 2021, doi: doi:10.1126/scirobotics.abi8017.
- [3] S. Gunduz, H. Albadawi, and R. Oklu, "Robotic devices for minimally invasive endovascular interventions: A new dawn for interventional radiology," *Advanced Intelligent Systems*, vol. 3, no. 2, p. 2000181, 2021, doi: <https://doi.org/10.1002/aisy.202000181>.
- [4] C. Shi *et al.*, "Shape sensing techniques for continuum robots in minimally invasive surgery: A survey," *IEEE Transactions on Biomedical Engineering*, vol. 64, no. 8, pp. 1665–1678, 2017, doi: 10.1109/TBME.2016.2622361.
- [5] K. L. Hong, B. M. Glover, S. Y. Ho, D. Sanchez-Quintana, and P. Brugada, "Cardiac electrophysiology study, diagnostic maneuvers and ablation," in *Clinical Handbook of Cardiac Electrophysiology*, B. M. Glover and P. Brugada Eds. Cham: Springer International Publishing, 2021, pp. 31–80.
- [6] D. Kolpashchikov, O. Gerget, and R. Meshcheryakov, "Robotics in healthcare," in *Handbook of Artificial Intelligence in Healthcare: Vol 2: Practicalities and Prospects*, C.-P. Lim, Y.-W. Chen, A. Vaidya, C. Mahorkar, and L. C. Jain Eds. Cham: Springer International Publishing, 2022, pp. 281–306.
- [7] S. Hasanzadeh and F. Janabi-Sharifi, "Model-based force estimation for intracardiac catheters," *IEEE/ASME Transactions on Mechatronics*, vol. 21, no. 1, pp. 154–162, 2016, doi: 10.1109/TMECH.2015.2453122.
- [8] M. Li, A. Pal, A. Aghakhani, A. Pena-Francesch, and M. Sitti, "Soft actuators for real-world applications," *Nature Reviews Materials*, p. 1–15, 2021,
- [9] B. Behera, D. Padmanabhan, and H. J. Pandya, "A ring-shaped MEMS-based piezoresistive force sensor for cardiac ablation catheters," *IEEE Sensors Journal*, vol. 21, no. 22, pp. 26042–26049, 2021, doi: 10.1109/JSEN.2021.3118298.
- [10] M. E. M. K. Abdelaziz, L. Tian, M. Hamady, G.-Z. Yang, and B. Temelkuran, "X-ray to MR: the progress of flexible instruments for endovascular navigation," *Progress in Biomedical Engineering*, vol. 3, no. 3, p. 032004, 2021/07/01 2021, doi: 10.1088/2516-1091/ac12d6.
- [11] A. A. Nazari, F. Janabi-Sharifi, and K. Zareinia, "Image-based force estimation in medical applications: A review," *IEEE Sensors Journal*, vol. 21, no. 7, pp. 8805–8830, 2021, doi: 10.1109/JSEN.2021.3052755.
- [12] J. Allard *et al.*, "Sofa-an open source framework for medical simulation," in *MMVR 15-Medicine Meets Virtual Reality*, 2007, vol. 125: IOP Press, pp. 13–18.
- [13] O. Comas, Z. A. Taylor, J. Allard, S. Ourselin, S. Cotin, and J. Passenger, "Efficient nonlinear FEM for soft tissue modelling and its GPU implementation within the open source framework SOFA," in *International Symposium on Biomedical Simulation*, 2008: Springer, pp. 28–39.