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FROZEN STORAGE OF SUGARBEETS

by

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A THESIS

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IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

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ABSTRACT

Sugarbeet cylinders were subjected to a number of different freezing and thawing conditions. Each set of conditions resulted in a marked softening of the beet tissue. The thawed beets decayed rapidly. Consequently, if beets are stored frozen, they should not be allowed to thaw prior to processing.

In order to determine the effect of freezing rate on sugar loss in storage, samples were frozen at different rates before being stored at 263 K or 248 K. Experimental results indicated that all freezing methods produced a negligible change in sugar content after six months storage at either of the storage temperatures.

A numerical model was developed for the freezing or thawing of an infinite cylinder of biological material. This model predicted the variation of temperature with time and with radial position. The model predictions were in good agreement with experimental values.

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1. INTRODUCTION

Sugar is an important commercial product. It may reach the consumer in its pure form or as a component of various food products such as soft drinks, baked foods and ice cream. It is also used in the production of non-food items such as pharmaceuticals and explosives. Alberta's only sugar factory is located at Taber. In this factory sugar is extracted from sugarbeets which contain about 16% sugar. Approximately 90% of this sugar is recovered as bagged sugar. However, with poor beets the recovery may be as low as 70%. By-products of the factory are cattlefeed, molasses, and invert sugar. The factory operates continuously from the beginning of October to the end of January. About 450,000 tonnes of beets are processed per year to produce:

1. 60,000 tonnes of sugar,
2. 35,000 tonnes of beet pulp, which is a cattle feed, and
3. 20,000 tonnes of molasses, which is used in the production of cattle feed or yeast.

At harvest time the leaves and tops are removed from the beets, which are then placed in large storage piles to await processing. During storage, sugar is lost, mainly as a result of respiration. It has been estimated (229) that 10% of beet sugar in the U.S. is lost during storage. This loss may be reduced by lowering the storage temperature. Frozen storage is being considered in the U.S., and Russia, as a

means of reducing or eliminating sugar loss. This method of storage becomes economically attractive if cold outdoor air can be used to freeze the beets. However, rapid deterioration of the beets may occur if they are allowed to thaw in the storage piles. Deterioration results from the injury suffered during freezing and thawing. It may be possible to minimize this damage by a careful evaluation of freezing and thawing conditions. Texture measurements are useful in this evaluation. Freshly harvested beets have a firm texture whereas freeze-damaged beets are soft. A beet which has been softened by freezing and thawing is probably damaged and is likely to deteriorate rapidly.

Heat transfer models can also be used to estimate the severity of the freezing injury which is suffered by a biological sample when it is subjected to specific freezing and thawing conditions. Freezing injury may depend on the rate at which a sample is frozen and thawed. Different regions of large samples may experience different rates of temperature change and, consequently, sustain different degrees of injury. To obtain an overall estimate of damage, it is useful to predict how quickly the temperature changes throughout the sample during freezing and thawing.

1.0.1 Objectives of this Thesis

In this thesis it was desired to:

1. examine a number of different freezing and thawing conditions in order to produce a thawed beet with a firm texture,
2. determine if sucrose losses in frozen storage depend on the freezing conditions to which a beet is subjected prior to storage, and
3. develop a numerical model which would predict the transient temperature profile in an infinite cylinder of biological material during freezing or thawing.

2. PRODUCTION OF SUGAR FROM SUGARBEETS

The production of sugar from sugarbeets is briefly described in this chapter. This description is intended as an aid in understanding processing problems, which arise when sugarbeets are stored incorrectly.

Sugarbeets are removed from storage piles and placed in flumes outside the factory. Water then transports the beets into the factory. The beets are washed with water in an agitated vessel. After washing they are cut into thin slices ("cossettes"). Sugar is leached from the cossettes in a diffuser by hot water which flows countercurrently. The depleted cossettes ("pulp") are dried, combined with molasses, and pelletized to produce cattle feed. The extract solution ("raw juice") proceeds to the carbonation stage.

In carbonation, milk of lime (a mixture of calcium oxide and water) and carbon dioxide are added to the raw juice to precipitate impurities, such as organic acids and inorganic salts. The precipitate is removed by pumping the juice through filter presses. Sulfur dioxide is then added to prevent discolouration of the finished product. The resulting solution is "thin juice". This juice is concentrated in multiple effect evaporators to produce "thick juice", which is about 65% solids.

"Standard liquor" is obtained by adding recycled sugar to the thick juice. The liquor is pressure filtered (a filter aid is used) and boiled under vacuum in the "White Pan" until sugar crystals are formed. The sugar crystals

must now be separated from the syrup in which they are suspended. This is accomplished in the "spinners", which are large circular baskets rotated at high speed. The syrup is forced through the basket by centrifugal action. The sugar remains on the basket and is washed with Hot water. It is then dried and packaged. The syrup is processed to produce molasses and low grade sugars. The molasses is sold as a finished product or is added to pulp to form cattle feed. The low grade sugars are dissolved in thin juice and added to the thick juice to form standard liquor.

3. STORAGE OF UNFROZEN SUGARBEETS

When beets are stored, sucrose is lost as a result of aerobic and anaerobic respiration, biochemical transformations and attack by micro-organisms. Wyse (227) has defined "respiration in storage" to be "the process whereby the root converts sucrose into energy to maintain its physiological integrity". In aerobic respiration, sucrose is degraded according to the exothermic reaction:

$$C_{12}H_{22}O_{11} + 12O_2 \rightarrow 12CO_2 + 11H_2O$$

(sucrose)

Under ideal storage conditions aerobic respiration is responsible for 60 to 75% of the sucrose loss (213).

Anaerobic respiration may occur in regions of the storage pile where air ventilation is blocked by dirt, trash, or snow. It can be represented by:

$$C_{12}H_{22}O_{12} + H_2O \rightarrow 4CO_2 + 4C_2H_5OH$$

(ethanol)

This form of respiration can rapidly lead to cell death and invasion by micro-organisms.

Micro-organisms present in beet tissue use sucrose for their respiration and growth (88). They also deposit enzymes in the beet (138). These enzymes catalyze reactions which convert sucrose into glucose, fructose, and other

carbohydrates. Damaged beets are particularly susceptible to attack by micro-organisms.

Enzymes naturally present in the beet tissue are also responsible for some sucrose loss. One enzyme, invertase, catalyzes the hydrolysis of sucrose to glucose and fructose. This mixture of glucose and fructose is known as invert sugar because it rotates plane polarized light in the opposite direction to sucrose.

Raffinose, a trisaccharide, may also accumulate during storage, at the expense of sucrose. The mechanism by which raffinose is formed is still unknown (13) but it is thought that it results from the combination of sucrose and a compound (UDP galactose) formed from glucose.

Nonsucrose components ("impurities") of sugarbeets cause problems in the sugar factory. Invert sugars are degraded to organic acids, such as formic, acetic, lactic, and glucinic acid. These acids tend to increase colour formation (34) and reduce the pH of factory sugar solutions. This decrease in pH results in an increase in the amount of lime which must be added in carbonation. The concentration of lime in the thin juice is then increased. This may result in an increase in the quantity of lime which is deposited in the evaporators, causing scale formation. A high raffinose content will decrease crystallization rates and produce distorted and elongated sugar crystals. Impurities, in general, tend to increase the amount of sugar contained in molasses and decrease the quantity of bagged sugar obtained.

3.1 Factors Affecting Sugar Loss in Storage

3.1.1 Temperature

The quantity of sugar lost and impurities accumulated in storage will depend on a number of variables, of which temperature is probably the most important. The sucrose loss will approximately double for every 8 K rise in temperature (48) above 273 K. The invert sugar content of beets also increases as the temperature is increased. On the other hand, the raffinose content increases as the storage temperature is reduced below about 278 K.

If the temperature, anywhere in the storage pile, reaches 319 K then the beet structure will be destroyed within a week (156). Micro-organisms can then attack the dead cells. This results in a further rise in temperature and a rapid loss of sugar.

3.1.2 Time in Storage

As the storage time is increased so also will be the sugar loss. However, there is nonlinear relationship between sugar loss and storage time. A high respiration rate is obtained immediately after harvest. This may result from:

1. injuries sustained during the harvest,
2. metabolic changes resulting from the leaves being removed, or
3. the large exposed surface area, which is available for oxygen diffusion, before the surface becomes hardened by

dehydration.

Within a week of storage, the respiration rate drops to approximately half its initial value (227). After a few months, respiration may increase again as a result of mould growth (229).

3.1.3 Condition of Beet Piles

Since respiration is an exothermic reaction, whose rate increases with temperature, heat must be removed or high temperatures will be attained in the storage pile. The presence of dirt and trash in piles impedes air circulation and, consequently, the removal of heat. Local overheating in the pile may then occur. Beets may be killed by the high temperatures obtained or by a deficiency in oxygen supply. Microbial fermentation of the soil and trash may also cause overheating by as much as 30 K.

3.1.4 Condition of Beets Entering Storage

The condition of the beets entering the storage pile is also important. Beets suffering from diseases, such as Violet Root Rot, Clamp Rot, and Crown Rot, will decay rapidly during storage and will infect other beets. These diseases are caused by viruses, the most prevalent of which is Phoema betae. Wilted beets lose sugar quickly and are likely to undergo fermentation. Topped beets (crown removed) show greater spoilage and sucrose losses than untopped beets

(2). Despite this, beets are generally topped because the crown contains a high percentage of impurities. Zielke and Snyder (237) reported that the concentration of impurities was 70% higher in the crown than in the remainder of the root. If leaf buds are not properly removed from the beet, then shoots will grow at the expense of sugar during storage.

3.1.5 Use of Pile Covers

Numerous covers have been used for storage piles. These include storage sheds, covered trenches, straw, beet tops, earth, plastic canopies, and air inflated bubble structures. Covers have the advantage of minimizing dehydration and repeated freezing and thawing of the beets. While frozen storage reduces sugar losses, alternate freezing and thawing increase them (58). Covers have the disadvantage of making it more difficult to cool beet piles during warm weather. The relative merits of the different covers will depend on the weather conditions at the given location.

3.1.6 Controlled Atmosphere Storage

In controlled atmosphere storage, beets are stored in an atmosphere which contains about 5% of both carbon dioxide and oxygen. This type of storage reduces both the respiration rate and the rate of formation of impurities. The oxygen content should not be lower than 5% as anaerobic respiration will then occur (45,227). Despite its

advantages, controlled atmosphere storage of sugarbeets has been restricted to research studies (109,226,227).

3.1.7 Use of Ventilation

Pile ventilation with cold air has been used to lower the temperature in the pile and to prevent localized overheating. If ventilation is used, humidifiers may be necessary to prevent beet dehydration.

3.1.8 Growing Conditions

Growing conditions influence the storage behaviour of sugarbeets. For example, beets generally store better after a wet season of good growth than after a dry season (104). Also, beets store badly if large quantities of nitrogen fertilizer are added just before the harvest (104).

3.1.9 Beet Variety

Nelson and Wood (153) observed that the quantity of sugar lost, during storage, was dependent upon the variety of beet which was stored.

The storage and processing of frozen beets is considered in the next chapter.

4. STORAGE AND PROCESSING OF FROZEN SUGARBEETS

It is estimated that about 0.25 kg of sucrose/tonne of beets/day may be lost in storage piles in the Red River Valley of North Dakota and Minnesota (15). Based on a refined sugar price of $1/kg, the elimination of sugar loss in storage piles results in potentially increased revenues of $0.25/tonne of beets/day stored. If this figure is applied to the factory at Taber, then elimination of sugar loss will increase operating revenues by about $7 million. This is approximately 10% of their expected gross revenues for the 1980-81 season.

Frozen storage of sugarbeets has been observed to drastically reduce the amount of sugar lost during storage (15,58,77,201,213,230). It has also been concluded (201) that it is "easier" to process a frozen beet, in March, than one which has been stored unfrozen. The decision of whether or not to use frozen sugarbeet storage will, of course, depend on economic considerations. The cost of frozen storage must be balanced against the extra revenue obtained from a reduction in sucrose losses. Frozen storage is economically attractive in regions where cold outdoor air can be used to freeze the beets. Use of expensive freezing equipment is then avoided.

4.1 Frozen Storage in the U.S.S.R.

As the U.S.S.R. is the world's largest sugarbeet producer and has an ideal climate for freezing beets outdoors, it is not surprising that frozen storage of sugarbeets has been investigated there. In 1974, Moroghan (150) reported that about 200,000 tonnes of frozen beets were being stored in the U.S.S.R. Forced ventilation of cold outdoor air (at a temperature less than 261 K) is used to freeze the beets to a temperature below 268 K, which is considered to be a "thawing temperature". The air is saturated with moisture to prevent dehydration of the beets. The piles are covered with a layer of ice or snow which forms a heat transfer barrier to the surroundings. Ice covers are simply formed by spraying cold water on top of the pile. The ice layer is melted, when required, by spraying hot water on the pile.

4.2 Frozen Storage in the Red River Valley

In 1974-75, the American Crystal Sugar Company (201) examined two methods of freezing storage piles. One pile was frozen by blowing cold air through ducts (metal drums) underneath the pile. A second pile, the "split pile", was formed by removing beets from the centre of a regular pile. It was then allowed to freeze naturally. After 4 months of storage, each pile had lost an average of 0.1 kg of sugar/tonne of beets/day stored. This was an improvement on unfrozen piles where 0.22 kg of sucrose were lost/tonne of

beets/day stored.

A second study was undertaken in 1976-77. Two piles were frozen by blowing cold night air through A-shaped wooden ducts underneath the piles. It was reported that sucrose losses were insignificant after 150 days of storage. In contrast, beets in an unfrozen pile lost 16% of their initial sugar content.

Further experiments (217) in 1977-78 indicated a 20% sucrose loss in an unfrozen pile, whereas a frozen pile had only a 0.6% sucrose loss.

4.3 Problems with Frozen Storage

The main problem with frozen storage of sugarbeets is that thawing may occur. This is an important disadvantage as thawed beets become unprocessable within a week (157). Sugarbeet cells are very susceptible to freezing injury, a phenomenon discussed in Appendix B. Dexter et al. (59) stated that some thawed beets "disintegrate completely" during fluming and washing. Hull (104) observed that "when frozen beets begin to decay they heat very quickly, have a pungent, vinegary odour, and rapidly collapse into a putrid, slimy mess".

Some specific problems are encountered with thawed beets. Pectic substances are complex carbohydrates, which are present in cell walls. They occur in two forms: (1) water soluble pectins and (2) water insoluble protopectin. The soluble pectins form calcium pectate gels in the sugar

factory (232) and these gels reduce filtration rates. Enzymes degrade the pectic substances of thawed beets and increase the percentage of soluble pectins. Consequently, thawed beets have a high percentage of soluble pectins. They can, therefore, be expected to produce juices which are more difficult to filter than those produced by unfrozen beets.

Oldfield et al. (156) observed that thawed beets, stored at 293 K, had respiration rates (as measured by the amount of carbon dioxide evolved) more than ten times the rate for unfrozen beets. Most of the sucrose loss was due to respiration and very little was due to inversion.

Micro-organisms are responsible for much of the deterioration in thawed beets. Damaged cell membranes allow the exodus of the cell solution into the intercellular spaces where micro-organisms are present. It is not known whether these micro-organisms enter from soil after topping or are naturally present in the beet. Bacteria (Leuconostoc mesenteroides) act on the cellular solution to produce polysaccharide gums (dextran and levan), which cause filtration problems in the processing plant (13). The polysaccharide gums lower filtration rates because they (particularly dextran) reduce the size of precipitated particles and increase the viscosity of the juice, which is to be filtered. Measurable quantities of dextran or levan are not found in fresh, mould damaged, heat damaged or waterlogged beet (157). Consequently, the presence of these gums is taken to indicate that frost injury has occurred.

Yeasts also act on the cell solution to produce esters and the characteristic fruity odour of thawed beets (13).

After frozen beets have been thawed, their invert sugar content increases rapidly (157). The production of invert sugars starts soon after thawing, whereas there is some delay before gums are formed. The lower the temperature, reached during freezing, the faster will be the invert sugar formation on thawing (157). The increase in invert sugar content is important because:

1. It occurs at the expense of sucrose.
2. The invert sugar is degraded in the sugar factory, during carbonation and evaporation, to produce organic acids. Increased amounts of lime must then be added, during carbonation, to remove these acids.
3. The higher the invert sugar content of a beet the lower is the fraction of its sucrose which is recovered as bagged sugar in the sugar factory.
4. Degradation of invert sugars is accompanied by increased colour formation in factory sugar solutions.

Other disadvantages of thawed beets are:

1. They may lose as much as 6% of their sugar content into the factory transportation and cleaning water (196).
2. They may rot in the piles, causing the formation of "hot spots".

3. They have a soft texture and are, therefore, difficult to slice.

Because of the difficulties encountered with thawed beets, frozen beets are generally processed without being allowed to thaw.

4.4 Processing Frozen Sugarbeets

Frozen sugarbeets are more difficult to process than unfrozen beets. Additional power must be supplied to the slicers (201). Wider knives, producing thicker slices, are required. Poorly formed cossettes, which plug the diffuser, are obtained. Slicer knives must be replaced often, as they can be damaged by dirt and stones, which are frozen to the surface of the beets.

Extra heating is needed to bring the frozen cossettes to the diffuser operating temperature. This extra heating is not as much as would at first be expected as low diffuser temperatures can be used. This is because sucrose enters the diffuser water more easily from freeze-damaged than from healthy cells.

When frozen beets reach the hoppers, above the slicers, their surface is wet as a result of washing. If there is any hold-up in the sugar factory, this water will freeze, as a result of losing heat to the frozen beet. Freezing of the surface water causes the beets in the hopper to stick together. Consequently, when frozen beets are being

processed, fewer beets are allowed to build up in the hopper. This ensures faster movement of beets and facilitates the separation of the beets if they do stick together.

Overall, the processing of frozen beets is not considered to be a major problem. In Taber, the throughput of sugarbeets only falls about 5% when frozen beets are used. Swift (201) stated that the greatest problems are encountered when mixtures of frozen and unfrozen beets must be processed.

5. TEXTURE EXPERIMENTS

Following the frozen storage of sugarbeets, the beets may be processed in either the frozen or the thawed state. If the beets are thawed, prior to processing, then it is essential that freeze-thaw damage be kept to a minimum. Otherwise, rapid deterioration will occur after thawing. Beet texture is a good indicator of the condition of a thawed beet. A fresh undamaged sugarbeet will have a firm texture unless it has become wilted through loss of moisture. In contrast, a beet which has become damaged by freezing and thawing will have lost its rigidity and will have a soft texture.

The texture of a thawed beet is important in the sugar manufacturing process, where the beet must be cut into cossettes, before the sugar is extracted. If the beet is too soft, it will be difficult to cut cossettes into the required shape.

As discussed in more detail in Appendix B, the severity of freeze-damage depends on the following factors:

1. the rate at which the sample is frozen and thawed,
2. the lowest temperature reached by the sample during freezing, and
3. the temperature at which the sample is stored.

It was decided to investigate the effect, which a number of different freezing and thawing conditions, would have on the texture of sugarbeet samples. It was hoped to determine

freeze-thaw conditions, which would produce a thawed beet with a firm texture and little indication of freeze-damage. The freezing conditions, which were examined, ranged from rapid freezing in liquid nitrogen to very slow freezing (0.001 K/min). Thawing conditions varied from rapid thawing in a microwave oven to very slow thawing (0.001 K/min).

EXPERIMENTAL PROCEDURES

5.1 Preparation of Samples Prior to Freezing

The sugarbeets, which were used, were grown for 6 months in a growth chamber and then stored at 275 K until required. All the samples were cylinders, which were removed from along the length of a beet by means of a cork borer. The cylinders were then cut to a length of 5 cm. The cylinders were 1 cm in diameter unless they were frozen in liquid nitrogen. With one exception, the liquid nitrogen frozen samples had a diameter of 0.66 cm. This was because larger diameter cylinders tended to crack when placed in liquid nitrogen. Cylinders, which were to be subjected to microwave thawing after freezing in liquid nitrogen, were 0.33 cm in diameter. For diameters greater than 0.33 cm, the middle of a microwave thawed sample was burned before the ends were thawed.

Excluding the microwave-thawed samples, all beet cylinders were covered with two layers of aluminum foil prior to freezing. This was to minimize oxidation and

dehydration of the beet. The microwave-thawed samples were not covered during freezing and thawing. They were, however, covered immediately after thawing.

A 26 gauge copper-constantan thermocouple was used to measure the centre temperature, during freezing and thawing. The thermocouple was pushed through one end and along the axis of the cylinder until the thermocouple junction was at the cylinder centre. To facilitate the insertion of the thermocouple, a hole had been previously made along the axis of the cylinder with an 18 gauge syringe needle.

5.2 Freezing Procedures

5.2.1 Liquid Nitrogen Freezing

Preliminary experiments were performed in which samples were cut into pieces and visually examined immediately after they were removed from liquid nitrogen. These experiments indicated that 0.33 cm diameter cylinders were completely frozen after 15 s in liquid nitrogen. Consequently, samples, which were to be subsequently thawed in a microwave oven, were frozen for 15 s in liquid nitrogen. Samples, to be thawed by ambient air or in a water bath, were placed in a horizontal position on a sample holder (Figure 1) and then frozen in liquid nitrogen until the centre temperature reached 268 K. On removal from the liquid nitrogen, the centre temperature continued to fall to 263 K or below. Samples to be thawed at 0.001 K/min or at 0.008 K/min were frozen in a vertical position. They could then be

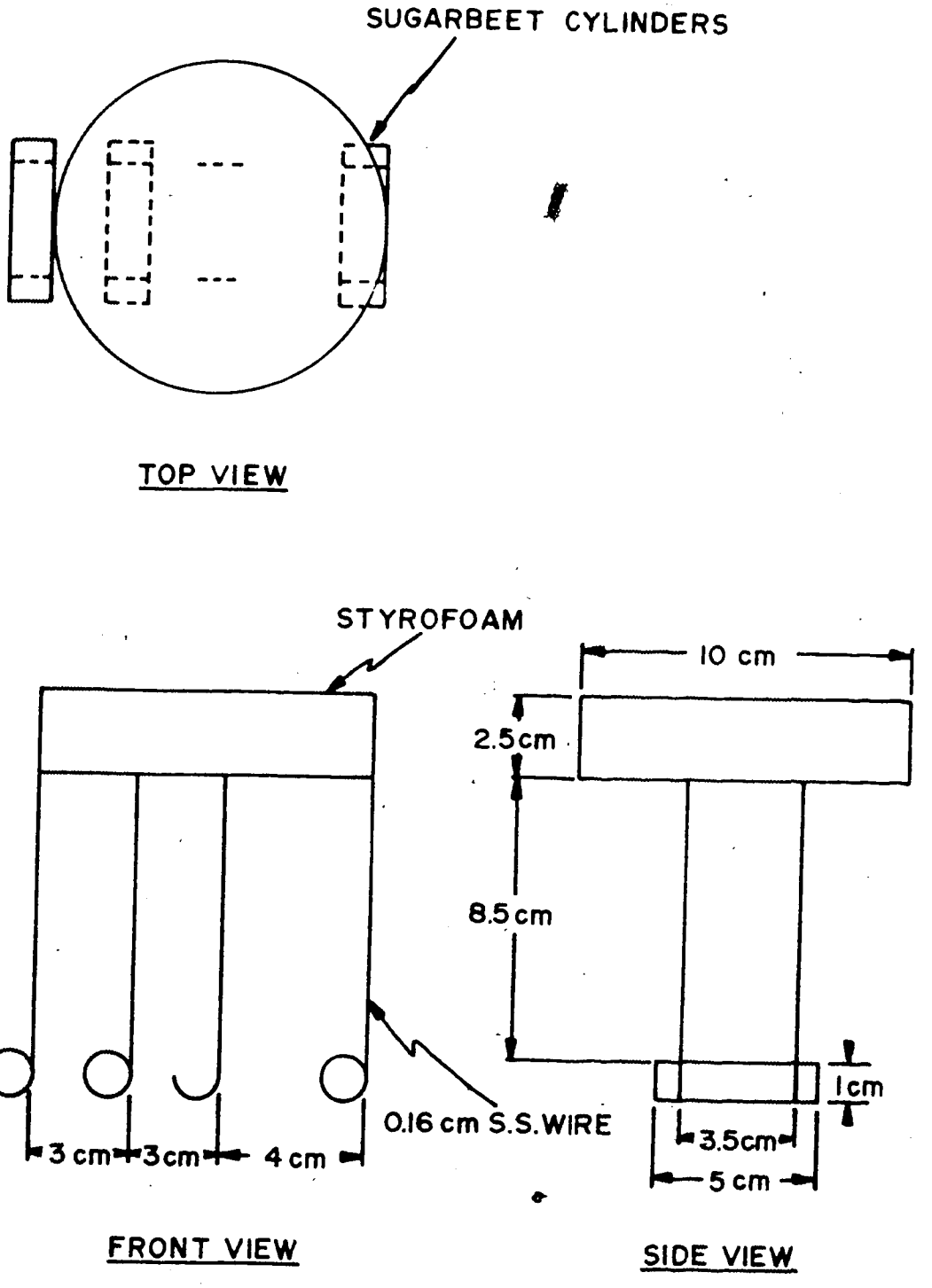

FIGURE 1: SAMPLE HOLDER FOR LIQUID NITROGEN FREEZING

transferred quickly to the centrifuge tubes (see Figure 2) in which they were thawed. They too were removed from the liquid nitrogen when the centre temperature was 268 K.

It was observed that liquid nitrogen freezing lowered the centre temperature, of 0.66 cm diameter cylinders, at a rate of approximately 50 K/min between 269 K and 267 K.

5.2.2 Freezing in 243 K Alcohol Bath

The cylinders were placed in a horizontal position on a sample holder, similar to that shown in Figure 3. The cylinders were then frozen in a NESLAB LT-50 bath, which contained 95% alcohol at 243 K. They were removed from the bath, for thawing, when their centres reached 263 K. During freezing in the 243 K bath, the centre temperature fell at the rate of 2.5 K/min between 269 K and 267 K.

5.2.3 Freezing at 0.008 K/min or 0.001 K/min

The samples were placed, in a vertical position (Figure 2), in closed 50 ml centrifuge tubes (to prevent attack by alcohol) which were kept in the NESLAB LT-50 bath at 273 K. When the samples attained a temperature of 273 K the bath temperature was programmed to fall at the desired rate (0.008 K/min or 0.001 K/min). When the bath temperature reached 263 K, the samples were removed and thawed.

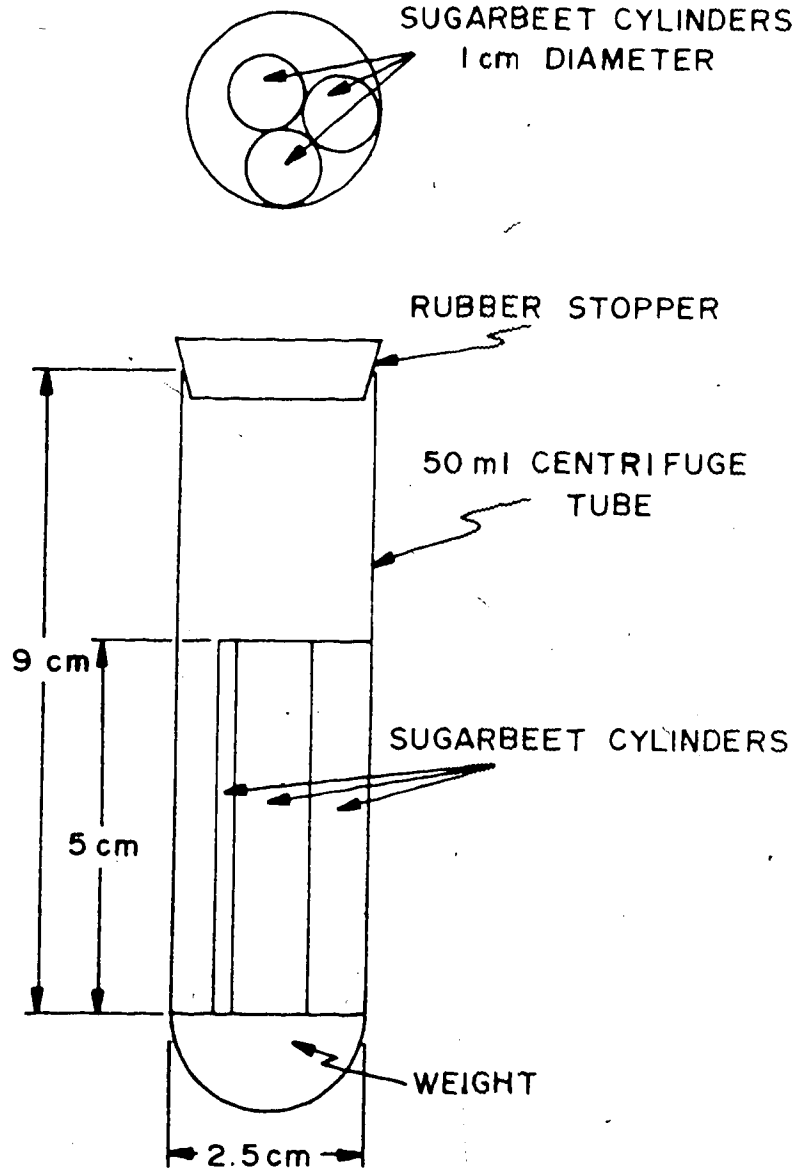

FIGURE 2: ARRANGEMENT OF BEET CYLINDERS FOR ULTRASLOW FREEZING AND THAWING

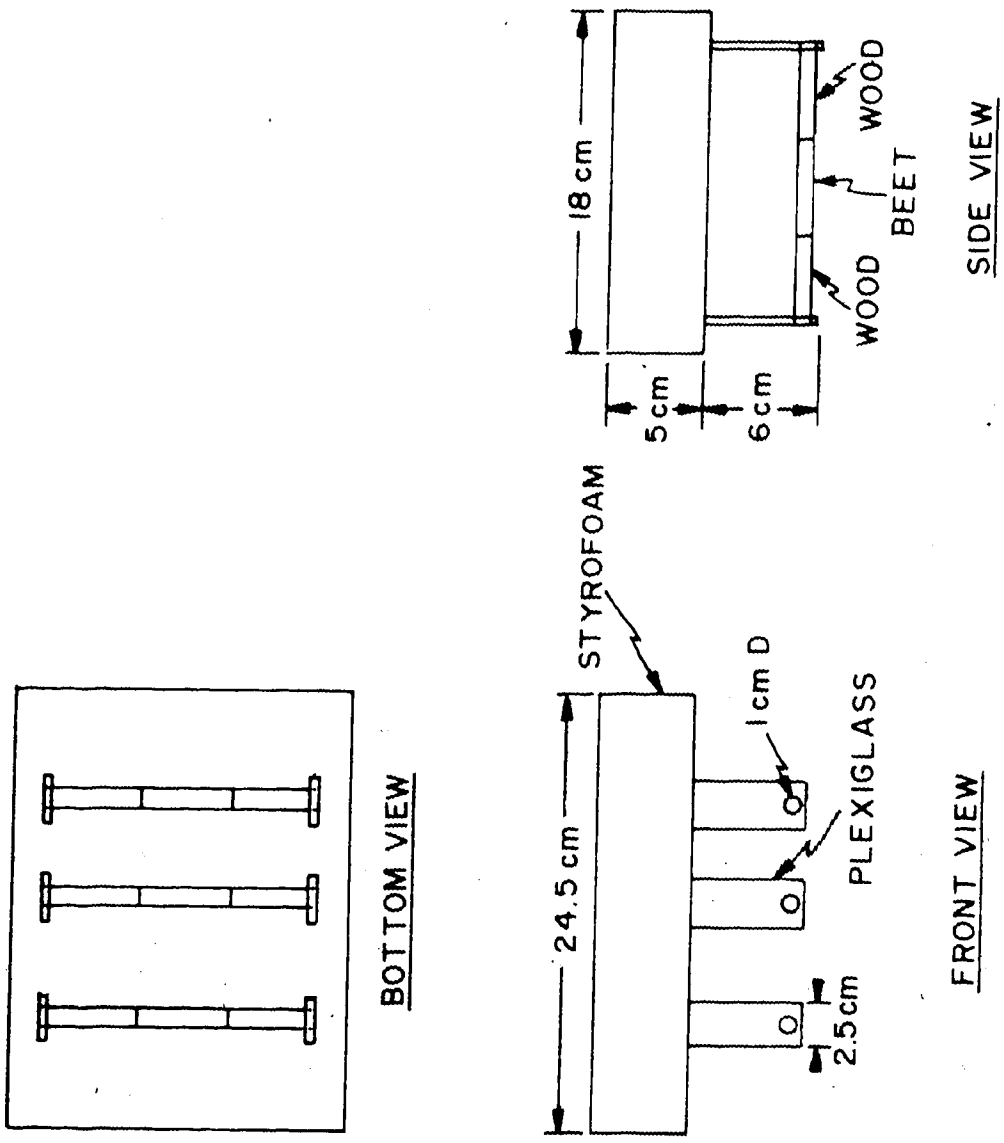

FIGURE 3: SAMPLE HOLDER FOR FREEZING IN ALCOHOL BATH

5.3 Thawing Procedures

5.3.1 Thawing in Microwave Oven

Microwave thawing was performed in a TOSHIBA DELUXE SERIES microwave oven. This oven has nine levels of power available for heating. The lowest power level was used in these experiments.

The 0.33 cm diameter samples were placed in liquid nitrogen for 15 s and then transferred immediately to the microwave oven for 75 s. Samples placed in the microwave oven for less than 75 s were still partially frozen on being removed. In contrast, samples left in the oven for periods longer than 75 s were too hot to touch.

5.3.2 Thawing in 303 K or 310 K Water Bath

The samples were thawed in water, maintained at either 303 K or 310 K, in a BRAUN THERMOMIX 1420 constant temperature bath.

Between 268 K and 270 K, the temperature at the centre of the sample rose at a rate of 2.5 K/min.

5.3.3 Slow Thawing

The samples were placed on a bench in the laboratory until they were thawed.

The centre temperature was observed to rise, at a rate of 0.5 K/min between 268 K and 270 K.

5.3.4 Thawing at 0.008 K/min or 0.001 K/min

Closed centrifuge tubes (capacity 50 ml) were submerged in 263 K alcohol, in the LT-50 bath. The tubes contained sufficient weights to allow them to float vertically in the bath. Samples, prefrozen to 263 K, in a prescribed manner, were placed in the tubes. The bath temperature was then programmed to rise at the desired rate (0.008 K/min or 0.001 K/min). When the bath temperature reached 273 K, the samples were removed and brought to room temperature in a 310 K water bath.

5.4 Texture Measurements

After thawing, the cylinders were cut into five pieces, each 1 cm long. The three interior pieces were then tested on an INSTRON UNIVERSAL TESTING MACHINE. This machine has been widely used (25,26,80,108,167) in Food Science to examine the effect of various treatments on food texture. The Instron was arranged as in Figure 4. A sugarbeet cylinder (1 cm diameter, 1 cm long) was placed on the compression table. The moving crosshead was then brought down slowly, using manual controls, until the anvil just touched the sample. The crosshead was then lowered at a speed of 0.5 cm/min until the cylinder was compressed to 0.24 cm.

When a sample is compressed it exerts a force, which is detected by the load cell. The load cell essentially consists of four strain gauges attached to a cantilever beam. The strain gauges are arranged as a Wheatstone Bridge. Any deflection in the cantilever beam, due to a force exerted on the load cell by a sample being compressed, will change the resistance of each strain gauge in proportion to the exerted force. The change in the strain gauges causes the bridge to become unbalanced and this results in a change in the output voltage to the recorder. The Instron is calibrated so the recorder output indicates force versus time. As the crosshead speed is known, this output can be easily converted to a stress-strain relationship.

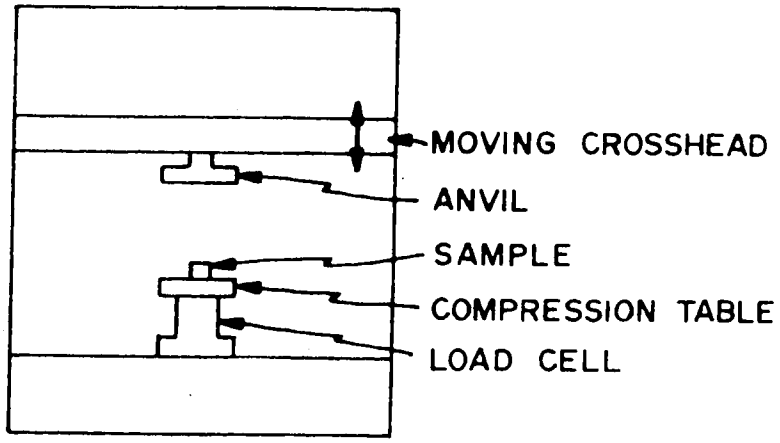

FIGURE 4: SAMPLE TESTING ON THE INSTRON

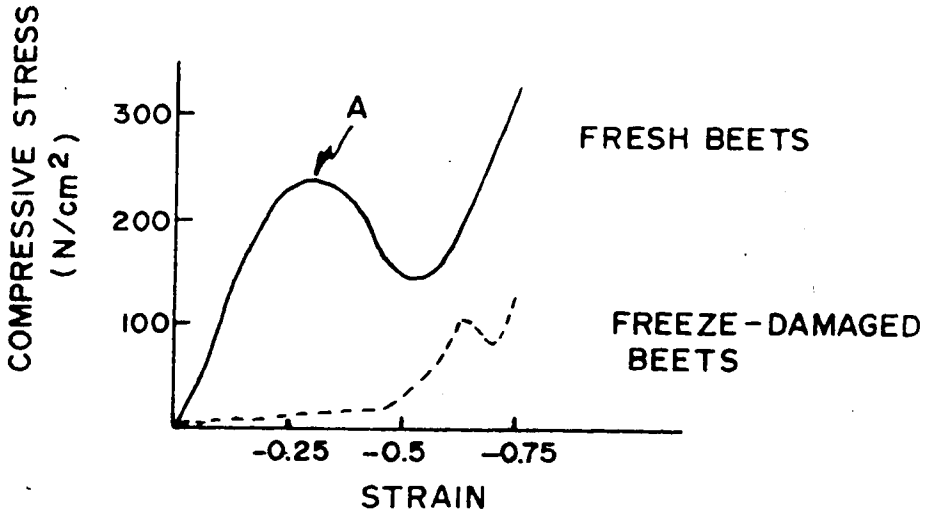

FIGURE 5: TEXTURE PROFILES

5.5 Experimental Results

Prior to performing the freeze-thaw experiments, preliminary stress measurements were taken on unfrozen samples. This was to test the variability of samples within a beet and between different beets. The results are listed in Table 1. This table shows the compressive stress of 1 cm long cylinders, after they have been compressed to a length of 0.9 cm. From this table, it can be seen that an unfrozen beet cylinder has a compressive stress of about 65 N/cm^2 at a strain of -10%. However, compressive stress measurements, for different beets, can vary between 54.4 and 72.8 N/cm^2. This is a variation of about 35%. Within a particular beet, variations of the order of 20% were obtained.

Typical results for fresh, and freeze-damaged beets are shown in Figure 5. Point A is called the "brittleness point". At this point the tissue of the beet collapses and the compressive stress drops sharply. The brittleness point of a freeze-damaged beet occurs at a strain of about -70%, whereas for a fresh beet it is observed at a strain of about -25%. Also the compressive stress at the brittleness point is smaller for a freeze-damaged beet. For low values of strain, the curve for an unfrozen beet is much steeper than that of a freeze-damaged beet. Consequently, it was decided to use the value of the compressive stress, at a strain of -10%, as an indicator of freezing injury. Low values of

TABLE 1: COMPRESSIVE STRESS OF UNFROZEN SAMPLES

Beet	No. of Samples[1]	Compressive Stress (N/cm^2) at a Strain of -10 %
1	14	68.8 ± 11.7
2	14	72.8 ± 11.5
3	14	65.4 ± 5.7
4	10	67.1 ± 3.1
5	15	54.4 ± 4.8

[1] No. of Samples = number of 1 cm long, 1 cm diameter cylinders which were examined.

this stress can be considered synonymous with softness.

Compressive stress measurements (at a -10% strain), for unfrozen and thawed beet cylinders, are tabulated in Table 2. The unfrozen and thawed cylinders, in each row of this table, were taken from the same beet. A wide range of freeze-thaw conditions was considered. They all produced soft cylinders, with a compressive stress which was an order of magnitude less than that of an unfrozen cylinder. Two trends are noticeable in Table 2. The first one is for cylinders frozen in liquid nitrogen. For these beets, it appears that, with the exception of microwave thawing, the faster the thawing rate the firmer will be the thawed beet. With microwave thawing, 0.33 cm diameter cylinders were used and this may have affected the stress measurements. The second trend was for very slowly frozen samples (0.008 K/min or 0.001 K/min). Firmness appeared to increase as the thawing rate was decreased. Thus, these texture experiments supported the general rule, explained in Appendix B, that fast freezing should be followed by fast thawing and slow freezing by slow thawing. However, the experiments were repeated an insufficient number of times to conclusively prove this rule for sugarbeets. It was not considered worthwhile to carry out more repeats to determine which freeze-thaw conditions produced the "least soft" beets.

TABLE 2: COMPRESSIVE STRESS OF FROZEN - THAWED SAMPLES

FREEZE - THAW CONDITIONS	COMPRESSIVE STRESS (N/cm^2) AT A STRAIN OF -10%	
	FROZEN	THAWED
Liquid Nitrogen Freezing with Thawing:		
-in microwave oven[2]	70.2±3.5 (2)[1]	2.3±1.2 (16)
-in 310 K water bath[3]	-	6.7±3.5 (12)
-at 0.5 K/min[3]	74.5±3.8 (4)	5.3±2.6 (8)
-at 0.008 K/min[3]	74.5±3.8 (4)	3.5±0.6 (9)
-at 0.001 K/min[3]	84.5±0 (2)	3.8±0.9 (7)
Freezing in 243 K Bath and thawing:		
-in 303 K water bath	67.1±4.7 (9)	4.1±0.4 (9)
-at 0.5 K/min	76.0±8.4 (8)	7.4±1.0 (9)
Freezing to 263 K at 0.008 K/min and thawing:		
-in 310 K water bath	80.7±2.8 (4)	5.1±0.1 (7)
-at 0.5 K/min	80.7±2.8 (4)	8.5±2.7 (8)
-at 0.008 K/min	80.7±2.8 (4)	10.6±4.2 (9)

Freezing to 263 K at 0.001 K/min and thawing	UNFROZEN	THAWED
-in 310 K water bath	82.1±3.9 (2)	3.7±0.8 (9)
-at 0.001 K/min	82.1±3.9 (2)	5.6±1.3 (9)

[1] The figure inside the brackets is the number of 1 cm long cylinders tested.

[2] indicates 0.33 cm diameter cylinders were used.

[3] indicates 0.66 cm diameter cylinders were used.

The above experiments indicated that thawed beets are soft, irrespective of the freezing and thawing method used. This softening of beet tissue is directly related to water loss from the beet cells. During freezing, water leaves the interior of the cell and freezes in the intercellular region or on the surface of the cylinder. On thawing, all of this water may not re-enter the cell. This may be due to cellular damage (discussed in Appendix B), or to nonequilibrium conditions, where the water has insufficient time to re-enter the cell during thawing. In either case, the water is lost as "drip". Loss of water reduces the firmness of the tissue. Some experiments were performed to test whether the softness of thawed beets was due to nonequilibrium thawing conditions or to cellular damage. Thawed cylinders were left uncovered in water, for periods of up to 16 hours, to determine if they would absorb water and increase their firmness. It was observed that soaking in water did not appreciably change the compressive stress of the samples. Consequently, it was concluded that loss of firmness in thawed cylinders is caused by cellular damage. It is, therefore, expected that all the freeze-thaw conditions, examined in this thesis, result in beets which deteriorate rapidly and become unprocessable. Therefore, frozen beets should not be allowed to thaw before processing.

6. STORAGE EXPERIMENTS

It is generally accepted (15,58,77,201,213,230) that if beets are stored frozen then the rate of sucrose degradation is decreased. As part of this thesis, experiments were performed to determine the effect of a number of variables on sucrose loss in frozen storage.

The amount of sucrose lost during frozen storage should depend primarily on the storage temperature and the storage time. To consider the effect of temperature some samples were stored at 263 K and others at 248 K. The effect of storage time was removed from the study by storing all the samples for six months. This time was selected because the use of frozen storage will probably increase the factory operating period from four to six months. Frozen storage will be limited to six months because outdoor air temperatures in April may be high enough to cause thawing of the beets. If shorter storage times were used in the experiments then smaller changes in sugar content would result. These small changes would be more difficult to measure experimentally.

In addition to storage time and temperature, the rate at which beets are frozen may affect the amount of sugar lost during storage. As explained in Appendix B, the rate at which a plant is frozen can determine the amount of injury it experiences during freezing. It would appear likely that beets experiencing the most damage during freezing would be the least stable during storage and would show the greatest

decline in sugar content.

Sucrose loss, during frozen storage, may also depend on the lowest temperature attained by the beets during freezing. For example, consider the effect of low temperature on invertase. Fennema (77) stated that the activity of invertase can be considerably reduced by exposing it to a temperature of 233 K. This decrease in activity may be important as invertase catalyzes the hydrolysis of sucrose to glucose and fructose. Consequently, if beets are frozen to 233 K prior to storage, sucrose losses may be decreased. However, exposing the beets to this lower temperature may increase the damage inflicted on the beet cells.

6.1 First Set of Storage Experiments

In these experiments, beets were subjected to a number of treatments. A "treatment" is considered to be defined by the freezing and storage conditions which a sample experiences. The eight treatments considered are now defined:

Treatment 1: Freeze and store samples in liquid nitrogen at 77 K.

Treatment 2:[1] Freeze samples to 233 K and store at 263 K (± 1.5 K).

---

[1] Samples subjected to treatments 2-5 were frozen in a 233 K alcohol bath.

Treatment 3:Freeze samples to 233 K and store at 248 K (± 1K).

Treatment 4:Freeze samples until the centre temperature reaches 263 K and store at 263 K (± 1.5 K).

Treatment 5:Freeze samples until the centre temperature reaches 248 K and store at 248 K (± 1 K).

Treatment 6:Freeze samples at a rate of 0.004 K/min to 262 K and store at 263 K (± 1.5 K).

Treatment 7:Freeze samples at a rate of 0.004 K/min to 247 K and store at 248 K (± 1 K)

Treatment 8:Freeze samples in liquid nitrogen for 30 s and store at 248 K (± 1 K).

Any sucrose loss resulting from treatments 2-8 was to be evaluated by comparing them to treatment 1. It was assumed that beet cylinders, subjected to treatment 1, would not experience a decrease in sucrose content since they were stored in liquid nitrogen. Treatments 2 and 3 were undertaken to test if exposing the samples to 233 K would reduce sucrose losses during storage. Treatments 4 and 5, 6 and 7, and 8 were investigated to determine the effect of fast (10 K/min), very slow (0.004 K/min), and very rapid (50 K/min) freezing on sucrose losses in storage.

6.1.1 Experimental Procedures

A Randomized Block Experiment was set up for the eight treatments. A replicate consisted of a set of three beet

cylinders, each 5 cm long and 1 cm in diameter. Every set contained one cylinder from each of three beets. The eight sets were then assigned at random to the eight different treatments. This procedure was followed twelve times to produce twelve replicates.

The beets, used in replicates 4 - 6, were taken from a storage pile, outside the sugar factory in Taber. They were stored at 293 K for three weeks, prior to freezing. Unless stated otherwise, all the other replicates were taken from beets grown for six months in a growth chamber and stored at 275 K until they were removed for freezing.

Detailed Description of Treatments

In all the following treatments, the beet cylinders were covered by two layers of aluminum foil in order to minimize dehydration and surface oxidation in storage. It also afforded some protection to the cylinders from the alcohol in which they were frozen.

For the four intermediate rate freezing treatments (2-5), the samples were put in the sample holder (Figure 3) which was then placed in a 233 K alcohol bath for various periods of time, depending on the treatment. The samples were quickly transferred, after being shaken to remove alcohol from their surface, to one of the storage freezers. Preliminary experiments indicated the centre temperature would be at 263 K, 248 K, and 233.5 K after 2 min ± 10 s, 2 min 30 s ± 15 s, and 4 min 40 sec ± 30 s, respectively, in

the 233 K bath. Based on these preliminary experiments, the intermediate rate freezing treatments in Table 3 were performed.

The ultraslow freezing treatments (6 and 7) were performed by placing the cylinders in closed 50 ml centrifuge tubes (Figure 2). The tubes were then put in a 273 K alcohol bath. The bath temperature was programmed to fall at a rate of 0.004 K/min. When the bath temperature reached 262 K the treatment 6 samples were transferred to the 263 K storage freezer. At a bath temperature of 247 K the treatment 7 samples were moved to the 248 K storage

TABLE 3: INTERMEDIATE RATE FREEZING TREATMENTS

	TIME IN 233 K BATH(min)	STORAGE TEMPERATURE (K)
Treatment 2	10	263
Treatment 3	10	248
Treatment 4	2	263
Treatment 5	2.67	248

freezer. The samples were frozen to 1 K below their storage temperature. This was because preliminary experiments had

shown that the temperature of the samples rose about 1 K, while being transferred from the alcohol bath to the storage freezer.

The rapidly frozen samples (treatment 8) were put on a holder (Figure 1) and then into liquid nitrogen for 30 s. The samples were next transferred to the 248 K storage freezer. Preliminary experiments had been performed in which samples were placed in liquid nitrogen for 30 s and then warmed in ambient air. It was observed that the centre temperature was between 279 K and 271 K, on removal from the nitrogen. It, subsequently, continued to fall, to a value between 249 K and 213 K.

6.1.2 Experimental Results

The sucrose contents, reproducible to ± 0.2% sucrose, obtained after 26 weeks (± 2 days) of storage are shown in Table 4. A detailed description of the procedure, used to estimate sugar contents, is listed in Appendix C. From the results in Table 4, an Analysis of Variance (AOV) table was constructed and it is shown as Table 5. From the F values in this table it can be seen that there is a significant difference between treatments at the 1% level i.e. there is a 99% probability that a difference between treatments exists.

In Table 6, the 8 treatments are listed in order of the mean sugar contents they produced.

TABLE

AFTER STORAGE (First Set of Experiments)

REPLICATE	TRT.1	TRT.2	TRT.3	TRT.4	TRT.5	TRT.6	TRT.7	TRT.8
1	2	18.3	18.2	18.4	17.4	19.2	19.1	18.6
2	18.6	16.2	17.2	17.1	17.6	18.4	18.6	18.1
3	17.6	16.6	16.6	17.0	17.7	17.4	17.4	17.8
4	15.7	14.5	14.3	15.6	14.6	15.8	#16.0	14.9
5	17.6	16.7	17.3	16.6	16.4	17.2	#18.3	17.3
6	16.9	14.9	16.2	15.5	16.7	17.0	#16.1	17.6
7	18.7	17.7	17.9	17.6	17.8	18.9	18.9	18.4
8	17.6	16.8	17.6	15.7	16.9	17.9	17.8	18.2
9	17.5	16.1	17.3	16.5	17.4	17.2	17.5	17.1
10	16.1	15.7	15.4	15.4	16.0	16.0	16.2	*16.3
11	17.9	16.8	17.5	17.3	17.1	17.7	18.3	18.0
12	17.2	16.1	15.8	16.3	17.3	16.8	17.5	17.4
AVG.	17.6	16.4	16.8	16.6	16.9	17.5	17.6	17.5

#The 9 cylinders were distributed incorrectly among reps. 4 - 6. The total sucrose content of reps. 4 - 6 is correct but the sucrose content of each rep. is incorrect.

*Rep. 10 of treatment 8 was not measured. The value in the table is estimated in Appendix D

TABLE 5: ANALYSIS OF VARIANCE TABLE FOR FIRST SET OF STORAGE EXPERIMENTS

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARES	F	F FOR SIGNIFICANCE 5%	F FOR SIGNIFICANCE 1%
REPLICATES	11	79.20	7.200	38.9	1.91	2.49
TREATMENTS	7	#20.51	2.930	15.8	2.12	2.88
ERROR	#76	14.05	0.1849			
TOTAL	#94	113.79				

includes correction because replicate 10 of treatment 8 was estimated.

TABLE 6: MEAN SUGAR CONTENTS OF TREATMENTS 1-8

TREATMENT	MEAN SUGAR CONTENT (%sucrose)
Trt.1	17.6
Trt.7	17.6
Trt.8	17.5
Trt.6	17.5
Trt.5	16.9
Trt.3	16.8
Trt.4	16.6
Trt.2	16.4

Duncan's Multiple Range Test (69,70) was used to determine which treatments were significantly different from treatment 1 (the "control"). The detailed analysis is given in Appendix D. The results may be summarized as follows:

5% level of significance

Trt.2 Trt.4 Trt.3 Trt.5 Trt.6 Trt.8 Trt.7 Trt.1

1% level of significance

Trt.2 Trt.4 Trt.3 Trt.5 Trt.6 Trt.8 Trt.7 Trt.1

Any two treatments which are underlined with the same line are not significantly different. If two treatments are not underlined with the same line then they are significantly different. With one exception, the use of either a 1 or 5% level of significance lead to the same conclusions. Treatment 2 was significantly different from treatments 3 and 5 at the 5% but not at the 1% level.

It can be reasonably concluded that ultraslow freezing (0.004 K/min) produced a negligible loss of sucrose for both storage temperatures (treatments 6 and 7). Liquid nitrogen freezing, followed by 248 K storage (treatment 8) also lead to an insignificant change in sucrose content. However, an intermediate freezing rate (treatments 2-5) caused a significant sucrose loss. This was true for both storage

temperatures. Freezing the samples to 233 K (treatments 2 and 4), instead of to the storage temperature, did not appear to reduce the amount of sucrose lost during storage. However, this sucrose loss could have been due either to the freezing rate or to attack by alcohol. In treatments 2-5 the beet cylinders, covered in aluminum foil, were placed in a 233 K, 95% ethanol, bath for 2-10 minutes depending on the treatment. The samples were then removed from the bath, shaken to remove alcohol, and placed in 50 ml containers in a 263 K or 248 K freezer. Despite this shaking, a few drops of alcohol did enter the container. As these containers were closed during storage, a few drops of alcohol would have been sufficient to saturate the air inside the containers. Also, a small amount of liquid alcohol did, despite the aluminum foil, reach the surface of the cylinders. Prior to an examination of the experimental results, it was assumed that the minute amounts of alcohol involved would not affect the beets, especially as they were being stored at low temperatures. However, as the four treatments which showed a loss in sucrose were the only treatments in which the cylinders were physically contacted by alcohol, this may not have been a valid assumption.

The low sugar contents may also have resulted from the experimental procedure which was used to measure them. As part of this procedure, the cylinders were cut into minute pieces in an Oster blender. Before blending, the aluminum foil cover was removed from the cylinders. The surfaces of

the intermediate rate frozen samples were moist. Consequently, a small amount of liquid ("drip"), containing sugar, may have been lost when the foil was removed. The ice layer on the surfaces of ultraslowly frozen samples may have prevented any significant decrease in measured sugar content due to drip loss.

As well as being part of the Randomized Block Design, treatments 2-7 also formed a Complete Factorial Design. This latter design consisted of two levels of storage temperature (263 K and 248 K) and three levels of freezing method (ultraslow freezing, intermediate rate freezing to the storage temperature, and intermediate rate freezing to 233 K). For these six treatments the experimental results are rewritten in Table 7 in the form of a Factorial Design. Table 8 is a summary of Table 7. It simply shows the column totals of Table 7 in a more succinct form. The AOV table for the Factorial Design is shown as Table 9. From the F values in this table it can be seen that there is a significant difference between different freezing methods and between different storage temperatures.

TABLE 7: SUCROSE CONTENT RESULTS DISPLAYED AS FACTORIAL DESIGN

	STORAGE TEMPERATURE = 248 K			STORAGE TEMPERATURE = 263 K			
REPLICATE	ULTRASLOW FREEZE	INTERMED. FREEZE TO STORAGE T	INTERMED. FREEZE TO 233 K	ULTRASLOW FREEZE	INTERMED. FREEZE TO STORAGE T	INTERMED. FREEZE TO 233 K	TOTAL
1	19.1	17.4	18.2	19.2	18.4	18.3	110.6
2	18.6	17.6	17.2	18.4	17.1	16.2	105.1
3	17.4	17.7	16.0	17.4	17.0	16.6	102.7
4	16.0	14.6	14.3	15.8	15.6	14.5	90.8
5	18.3	16.4	17.3	17.2	16.6	16.7	102.5
6	16.1	16.7	16.2	17.0	15.5	14.9	96.4
7	18.9	17.8	17.9	18.9	17.6	17.7	108.8
8	17.8	16.9	17.6	17.9	15.7	16.8	102.7
9	17.5	17.4	17.3	17.2	16.5	16.1	102.0
10	16.2	16.0	15.4	16.0	15.4	15.7	94.7
11	18.3	17.1	17.5	17.7	17.3	16.8	104.7
12	17.5	17.3	15.8	16.8	16.3	16.1	99.8
TOTAL	211.7	202.9	201.3	209.5	199.0	196.4	1220.8

****The above results are expressed as %sucrose by weight****

TABLE 8: EFFECTS OF DIFFERENT TREATMENT LEVELS

STORAGE CONDITIONS	ULTRASLOW FREEZING	INTERMEDIATE FREEZING TO STORAGE T	INTERMEDIATE FREEZING TO 233 K	TOTAL
248 K	211.7	202.9	201.3	615.9
263 K	209.5	199.0	196.4	604.9
TOTAL	421.2	401.9	397.7	1220.8

TABLE 9: ANALYSIS OF VARIANCE TABLE FOR THE FACTORIAL DESIGN

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARES	F	F FOR SIGNIFICANCE	
					5%	1%
REPLICATES	11	58.74	5.34	25.43	1.97	2.59
(TREATMENTS)	(5)	(14.93)	(2.99)	(14.22)	(2.38)	(3.37)
FREEZING	2	13.09	6.55	31.17	3.17	5.01
STORAGE	1	1.68	1.68	8.0	4.02	7.12
FREEZING X STORAGE	2	0.16	0.08	0.381	3.17	5.01
ERROR	55	11.53	0.21			
TOTAL	71	85.20				

Duncan's Multiple Range Test (Appendix D) produced the following results for 5 and 1% levels of significance:

5% Level of Significance

Trt.2	Trt.4	Trt.3	Trt.5	Trt.6	Trt.7

1% Level of Significance

Trt.2	Trt.4	Trt.3	Trt.5	Trt.6	Trt.7

From the results of Duncan's Test, it can be concluded that when using an intermediate rate of freezing it is immaterial whether the sample is frozen to 233 K (treatments 2 and 4) or merely to the storage temperature (treatments 3 and 5). Also, ultraslow freezing leads to a significantly higher sucrose content than intermediate rate freezing.

The low F value for the Freezing X Storage interaction indicates that the interaction between the freezing method and the storage temperature is not significant. Thus, the freezing conditions which produce the highest sugar content after storage at 263 K will also be the best conditions for storage at 248 K. Similarly, 248 K storage following any given freezing conditions will produce higher sugar content beets than those subjected to 263 K storage, following the same freezing conditions.

The conclusions from the Randomized Block and Factorial

Designs are now summarized:

1. It is significantly better to store beets at 248 K than at 263 K.
2. There is an insignificant interaction between freezing method and storage temperature.
3. Ultraslow freezing produces higher sucrose content beets than intermediate rate freezing.
4. Freezing to 233 K, instead of to the storage temperature, has an insignificant effect on the sucrose content.

However, as noted in the Randomized Block analysis, the lower sucrose contents resulting from intermediate rate freezing may have been due to alcoholic contamination or to drip loss during the sugar content determination.

6.1.3 Physical Condition of Beets After Storage

A number of beet cylinders were visually examined immediately after storage. It was apparent that the condition of these cylinders depended on the freezing method and storage temperature used. Liquid nitrogen frozen samples stored at 77 K or at 248 K were as white (immediately after thawing) as when they were placed in storage.

Ultraslowly frozen cylinders, stored at 248 K, had slightly off-white coloured surfaces but their interior was

white. Ultraslowly frozen cylinders, stored at 263 K, had some surface browning but their interiors were white or very light brown. These samples had a much thicker surface layer of ice than cylinders frozen more quickly.

Intermediate rate frozen samples stored at 248 K (treatments 3 and 5) were generally as white as the unfrozen samples. Occasionally some browning occurred. Intermediate rate freezing, to 263 K or to 233 K, usually resulted in a marked deterioration of at least part of the surface following storage at 263 K. The surfaces of these cylinders had either black, brown, or grey areas. This surface colouring was also present in the interior beneath it, although generally in a lighter form. Usually, deterioration started at one or both ends of the cylinder. This could have been due to the faster freezing rates and lower temperatures which were obtained at the ends during freezing. It could also have resulted from the fact that alcohol may have been trapped under the aluminum foil at the ends of the cylinder.

6.1.4 Additional Experiments

Some experimental difficulties were experienced with replicates 1-3 of treatment 6 and with replicates 1-6 of treatment 7. Consequently, it was decided to carry out an extra three replicates of treatment 6 and an extra six replicates of treatment 7. The results obtained are given in Table 10. These results support the earlier conclusion that ultraslow freezing, followed by six months storage, at

TABLE 10: SUCROSE CONTENTS AFTER 6 MONTHS STORAGE - ADDITIONAL RESULTS

REPLICATE	CONTROL (TRT.1)	ULTRASLOW FREEZE TO 262 K (TRT.6)	ULTRASLOW FREEZE TO 262 K (TRT.7)
13	19.2	19.0	18.9
14	15.0	14.6	14.7
15	17.5	17.4	17.4
16	17.7	-	17.7
17	20.1	-	20.2
18	18.7	-	18.2

	TRT.1	TRT.6	TRT.7
MEAN OF REPS. 13 - 15	17.2	17.0	17.0
MEAN OF REPS. 13 - 18	18.0	-	17.9

****The above results are expressed as %sucrose by weight****

either 263 K or 248 K, produces insignificant changes in sucrose content. The mean sucrose content of replicates 4-15 for treatments 1 and 6 were 17.2 and 17.1%, respectively. For replicates 7-18, both treatments 1 and 7 had a mean sucrose content of 17.8%.

The behaviour of refrozen samples during storage was tested for three replicates. It was reasoned that refreezing the samples would cause extensive tissue damage and would be a good test to determine if badly damaged beets would lose sugar when stored at 248 K. The samples were frozen slowly (0.004 K/min) to 249 K (± 2 K) and then immediately thawed, at a rate of 0.5 K/min, to 271.5 K. They were refrozen, without delay, at a rate of 0.25 K/min, to 247 K and stored at 248 K for six months. These freezing and storage conditions were considered to be treatment 9. The results are summarized in Table 11. From this table, it would appear that even refrozen samples, stored at 248 K, will have a negligible loss in sucrose. However, additional replicates would be required to validate this conclusion.

TABLE 11: SUCROSE CONTENTS OF REFROZEN SAMPLES[1]

REPLICATE	CONTROL (Trt.1)	REFROZEN (Trt.9)
	(%sucrose)	(%sucrose)
4	15.7	15.1
5	17.6	17.7
6	16.9	17.2
MEAN	16.7	16.7

[1] Taber beets were used in these experiments

6.2 Second Set of Storage Experiments

These experiments were designed to test if the sucrose lost in the intermediately fast frozen samples of the first set of experiments was actually due to the freezing rate. At the same time it was decided to determine if liquid nitrogen frozen samples would lose sucrose when stored at 263 K. The following four treatments were considered in this set of experiments:

Treatment 1: Freeze and store samples in liquid nitrogen at 77 K.

Treatment 10: Freeze cylinders in liquid nitrogen and store them at 264 K (± 1.5 K).

Treatment 11: Freeze cylinders in a 233 K alcohol bath until the centre reaches 248 K and then store at 250 K (±1 K).

Treatment 12: Freeze cylinders in a 233 K alcohol bath until the centre reaches 263 K and then store them at 264 K (±1.5 K).

Prior to freezing, the treatment 11 and 12 samples were wrapped in aluminum foil and placed in sealed polystyrene tubes to prevent contamination by alcohol. The polystyrene tubes reduced the freezing rate by a factor of three. The time for the geometric centre to go from 269 K to 267 K increased from 15 s to 45 s.

For each replicate, four groups of three cylinders were taken from two beets. Each group contained two cylinders from the first beet and one from the second. The four groups were then assigned at random to the four treatments.

6.2.1 Experimental Procedures

Treatment 10 consisted of freezing the samples (covered with aluminum foil) in liquid nitrogen for 30 s and then transferring them rapidly to the 264 K storage freezer.

In treatments 11 and 12, the samples were wrapped in aluminum foil and placed in polystyrene tubes (7.5 cm long, 1 cm I.D.) which were then sealed. An epoxy glue, which dried in 5 min, was used to prevent alcohol leaking into the tube (Figure 6). The tubes were then put in the holder (Figure 7) which kept them in a horizontal position in the 233 K alcohol bath. In treatment 11, the tubes were in the bath for 4.5 min before being moved to the 264 K storage freezer. In treatment 12, the tubes were in the bath for 5.5 min before being transferred to 248 K storage. It had earlier been observed that the centre of the sample would reach 258 ± 3 K and 244 ± 2 K after 4.5 and 5.5 min, respectively in the bath.

The procedure, used to determine sugar content, was modified for these experiments. The sugarbeet cylinders were placed in the blender jars without first having their aluminum foil cover removed. This modification prevented any drip loss of sugar.

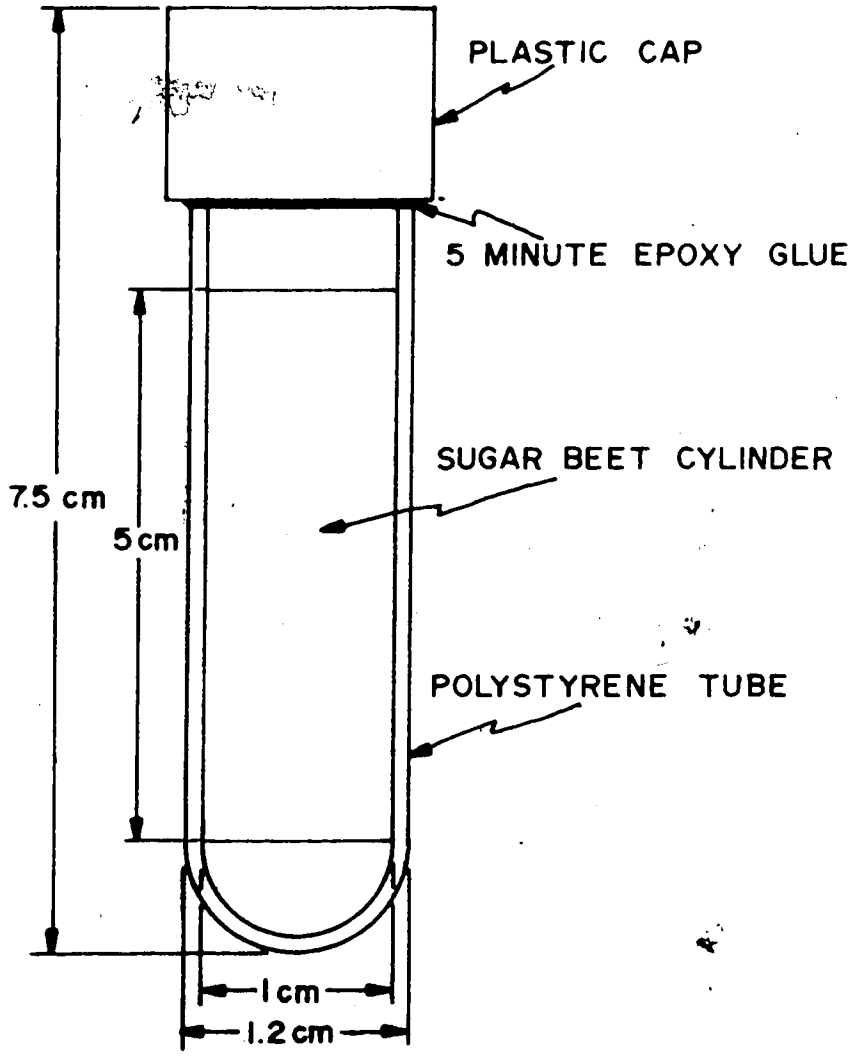

FIGURE 6: ARRANGEMENT TO PREVENT ALCOHOL CONTAMINATION

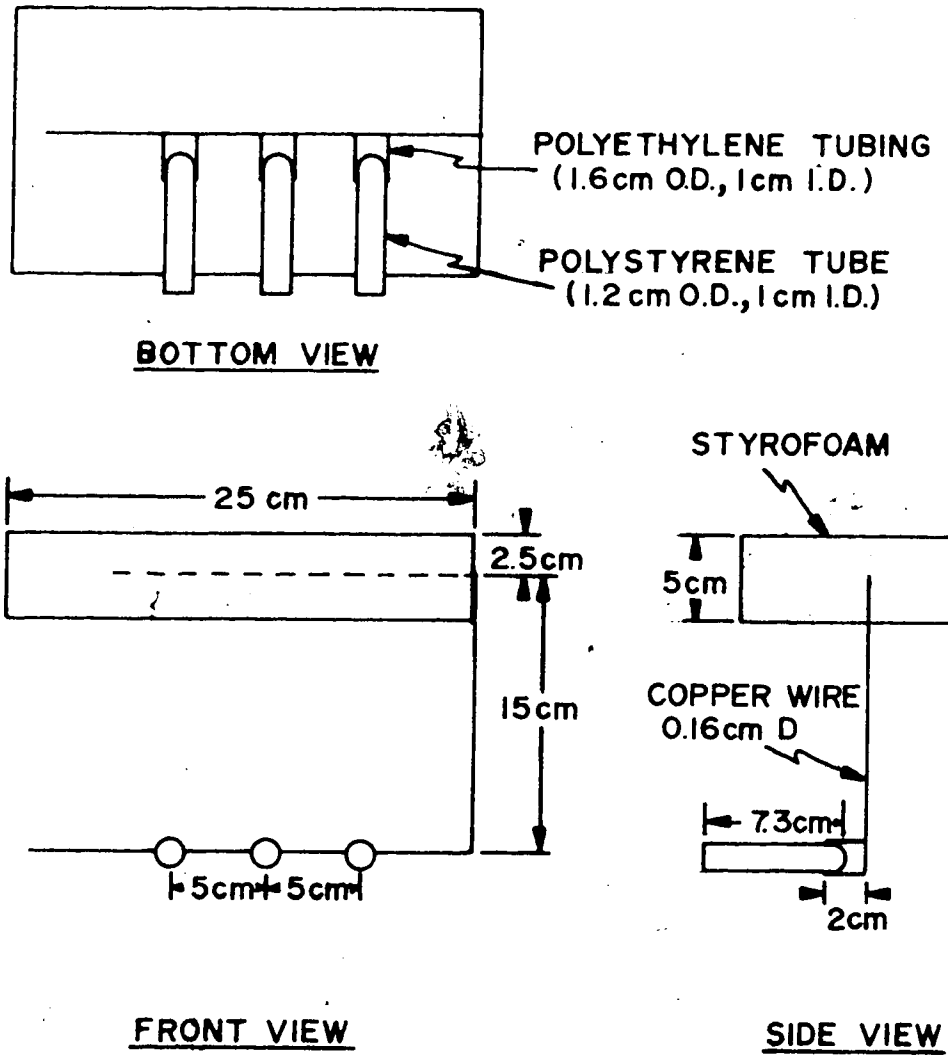

FIGURE 7: SECOND SAMPLE HOLDER FOR FREEZING IN ALCOHOL BATH

6.2.2 Experimental Results

Physical Condition of Beets After Storage

Cylinders frozen in liquid nitrogen and stored at 263 K (treatment 10) usually contained golden and white regions, both at the surface and in the interior.

Cylinders frozen at an intermediate rate and stored at 250 K (treatment 11) were all white.

The treatment 12 cylinders (frozen at an intermediate rate and stored at 264 K) had a similar appearance to those subjected to treatment 10. However, dark brown regions were sometimes obtained. Generally, the treatment 12 cylinders showed more sign of deterioration than treatment 10 cylinders.

Sucrose Content

The sucrose contents, which were obtained after 180 ± 4 days of storage, are summarized in Table 12. This data was used to construct Table 13, which is an Analysis of Variance table. From the F values in this table, it can be concluded that a significant difference between treatments existed at the 1% level.

TABLE 12: SUCROSE CONTENTS[1] AFTER 180 DAYS OF STORAGE - RANDOMIZED BLOCK DESIGN

Replicate	Trt.1	Trt.10	Trt.11	Trt.12
1	17.4	17.6	17.7	17.2
2	18.5	18.3	18.2	18.1
3	17.2	17.3	17.6	17.2
4	17.4	17.8	17.9	17.8
5	16.9	16.6	16.7	16.0
6	17.6	17.7	17.9	17.5
7	17.1	16.9	16.8	16.2
8	17.5	17.4	17.7	17.0

[1] Results expressed as %sucrose by weight.

TABLE 13: ANALYSIS OF VARIANCE TABLE FOR SECOND SET OF STORAGE EXPERIMENTS

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARES	F	F FOR SIGNIFICANCE	
					5%	1%
REPLICATES	7	8.44	1.206	14.4	2.49	3.64
TREATMENTS	3	1.56	0.520	6.2	3.07	4.87
ERROR	21	1.77	0.084			
TOTAL	31	11.77				

The four treatments are now listed in order of their mean sugar contents:

Trt.12	Trt.10	Trt.1	Trt.11
17.1	17.5	17.5	17.6

The results of a Duncan's Multiple Range Test, performed in Appendix D, are summarized below:

5% Level of Significance

Trt.12	Trt.10	Trt.1	Trt.11

1% Level of Significance

Trt.12	Trt.10	Trt.1	Trt.11

From the above results, it is concluded that treatments 10 and 11 caused a negligible loss in sugar. Treatment 12 is significantly different from treatment 1 at the 5% level but not at the 1% level of significance. However, it must be mentioned that replicates 5-8 of treatment 12 were stored at 264 K, whereas replicates 1-4 were stored at 263 K. This was because of the temperature variation within the storage freezer. For replicates 1-4, treatments 1 and 12 both had an average sucrose content of 17.6%. For replicates 5-8, treatment 1 had an average sucrose content of 17.3%, whereas

it was only 16.7% for treatment 12. It would, therefore, appear that a negligible loss in sucrose is obtained, for treatment 10, if the storage temperature is maintained at 263 K. However, a small rise in storage temperature may cause a significant loss in sucrose. This theory is supported by the experiments of Wyse (230). He observed that the respiration rate of sugarbeets decreased gradually with decreasing temperature until a temperature of 265 K was reached. At this temperature the respiration rate "declined sharply to near zero".

From the above experiments, it is concluded that an insignificant loss of sucrose results from:

1. six months storage at 250 K or at 263 K, following freezing in a 233 K alcohol bath, or
2. six months storage at 263 K, following freezing in liquid nitrogen.

6.2.3 Supplementary Experiments

The above conclusions are supported by the results of supplementary experiments. The results of additional replicates of treatments 10 and 11 are listed in Tables 14 and 15. The average of the sugar contents of fourteen replicates (eight from Table 12 and six from Table 14) is 17.8% for both treatments 10 and 11. Similarily, eleven replicates (eight from Table 12 and three from Table 15) of treatment 1 and of treatment 10 have the same average

TABLE 14: SUCROSE CONTENTS[1] OF EXTRA TREATMENT 10 SAMPLES

Replicate	Trt.1	Trt.10
1	18.5	19.0
2	18.5	19.3
3	18.8	18

[1] Results expressed as %sucrose by weight.

TABLE 15: SUCROSE CONTENTS[1] OF EXTRA TREATMENT 11 SAMPLES

Replicate	Trt.1	Trt.11
1	17.4	16.8
2	18.7	18.7
3	18.4	17.9
4	18.2	18.1
5	18.5	18.5
6	18.9	18.8

[1] Results expressed as %sucrose by weight.

sucrose content (17.8%). One extra replicate of treatment 12 (stored at 263 K) produced a sugar content of 18.3%, whereas the treatment 1 sample had a sugar content of 18.5%.

6.3 Conclusions from Storage Experiments

Based on all the storage experiments, it may be concluded that sucrose losses will be negligible in beets, stored for six months at 248 K or 263 K, irrespective of the freezing method used. However, ultraslow freezing has the advantage of providing a thick surface layer of ice. This is important as it may (at least until the ice melts) reduce the amount of sugar which is lost in the beet factory to the water, which is used for the transportation and washing of the frozen beets.

A storage temperature of 248 K is preferred to one of 263 K as it minimizes:

1. the probability of the beets becoming thawed prior to processing,
2. the quantity of sugar which is lost to the transportation and cleaning water, and
3. the beet deterioration which occurs during storage.

Of course if mechanical refrigeration must be used, storage at 248 K will be the more expensive.

7. SUGGESTED EXPERIMENTS ON STORAGE PILES

The laboratory experiments indicated that frozen storage can reduce the sugar loss in storage piles from a value of 0.25 kg/tonne/day (obtained in conventional outdoor piles) to a negligible amount. Using a refined sugar price of $1/kg, frozen storage could increase operating revenues by $0.25/tonne of beets/day stored. Frozen storage can be expected to increase the gross operating revenues of the Taber factory by about $7 million. It, therefore, appears worthwhile to extend the experiments to the frozen storage of sugarbeet piles at Taber. A pilot scale study on one storage pile should first be performed. If the results from this study are satisfactory then frozen storage of beets could be carried out on a large scale.

There are two alternative methods of storing frozen piles of beets. The first method entails the use of cold night air to freeze the beets, with subsequent outdoor storage. The main problem with this method is that it may be difficult to prevent the beets from thawing during periods of warm weather. This is a particularly valid concern at Taber because of the presence of warm Chinook winds. The second frozen storage method involves freezing and storing the beets inside a building, cooled by mechanical refrigeration. It should not be necessary to use a Quickfreeze unit to freeze the beets. This is because sucrose losses are negligible in frozen storage even if the beets are frozen slowly.

7.1 Outdoor Frozen Storage of Beet Piles

Research efforts should, at first, be concentrated on the outdoor storage of beets. This is because the cost of freezing and storage is minimal. Pile temperatures should be monitored to determine if the beets will thaw during storage. The use of a pile cover (ice, snow, straw, plastic etc.) in the prevention of thawing should be investigated.

7.2 Use of Mechanical Refrigeration

If it appears that thawing of beets cannot be avoided in outdoor piles then the use of mechanical refrigeration should be considered. This would, of course, significantly increase the cost of storage. However, frozen storage may still be economically attractive.

7.2.1 Pilot Study Using Mechanical Refrigeration

A pilot study could be performed by storing 2,500 tonnes of beets at 253 K in an insulated (10 cm thick styrofoam) archrib structure of dimensions:

20 m wide X 40 m long X 8m high.

The building would have a six inch concrete floor. The archrib structure was chosen because it is sturdier than an inflatable building. This is especially relevant because of the high wind velocities obtained in the Taber area.

7.2.2 Requirements of Refrigeration System

The refrigeration system will have its greatest load requirement in October when outdoor conditions are warmest. The following assumptions were made in estimating the requirements of the refrigeration system:

1. The average outdoor temperature in October is 288 K,
2. the average outdoor temperature, during the whole storage period, is 273 K,
3. the overall heat transfer coefficient is 1 J/(s . m^2 . K) for both heat transfer through the floor and through the walls,
4. the effect of the sun's radiation is accounted for by adding 3 K to the ΔT across the walls.

The capacity and average requirements of the refrigeration system were calculated to be 76.7 kW (22 tons of refrigeration) and 46.7 (13 tons), respectively.

7.2.3 Refrigeration Requirement to Cool Beets to 253 K

It was assumed that the 2,500 tonnes of beets were at 283 K when placed in storage in October. The heat, which must be removed to cool them to 253 K, was then calculated (using the enthalpy data in Table 18) to be 7.65 X 10^5 kJ. Assuming the beets are cooled in one week, the rate of heat removal required to freeze the beets to 253 K is 1.3 kW. This rate is negligible relative to the capacity of the

refrigeration system.

7.2.4 Removal of Heat of Respiration

The heat of respiration of sugarbeets at 283 K is 0.019 kW/tonne of beets (48). The respiration rate drops to near zero at about 265 K (230). It is, therefore, assumed that the heat of respiration is negligible below 265 K. The heat of respiration to be removed is greatest when the beets are placed in storage (their temperature is then 283 K). The maximum value of the heat of respiration, evolved by 2,500 tonnes of beets, was calculated to be 47.2 kW. This refrigeration load, although much greater than that required to cool the beets to 253 K, is less than the capacity of the refrigeration system. The heat of respiration does not alter the required capacity of the system as it is negligible when the refrigeration unit is operating at full capacity. This, of course, occurs when the storage temperature is 253 K.

7.2.5 Economics of Using Mechanical Refrigeration

This economic assessment applies to the pilot scale equipment already described. It basically estimates the net cost of performing the experiment. It, also, gives an indication of the economic feasability of storing frozen beets on a large scale, using mechanical refrigeration.

The potential increase in annual revenues obtained by storing the 2,500 tonnes of beets in the frozen state was calculated after making the following assumptions:

1. e beets are stored for 120 days, and
2. the price of refined sugar is $1/kg.

Annual operating costs were calculated after making the following assumptions:

1. no additional labour will be required to pile the beets in the archrib building over that required for conventional outdoor piles,
2. the electrical power required to run the refrigeration system is 0.4 kW per kW of refrigeration, and
3. the cost of electricity is 1.5 ¢/kW-hour.

The results of the economic calculations are summarized in Table 16. From this table it can be quickly determined that the increased revenues would pay for the frozen storage system in about 3 to 4 years.

A capital investment of $200,000 would be required to perform the pilot scale experiment. However, if it was discovered that this form of storage was not feasible then the investment would not be wasted. The archrib building could always be used as a general storage building. The refrigeration system could be sold for $50,000.

As shown above, frozen storage of beets may be economically feasible when applied to an exis g sugar factory. The possibilities for a new sugar factory are even better. Frozen storage could then be used to extend the

TABLE 16: SUMMARY OF ECONOMIC CALCULATIONS

CAPITAL COSTS	
Archrib building[1]	$100,000
Refrigeration system	$100,000
OPERATING COSTS	
Additional labour	0
Electricity	$1,000
Maintenance	$10,000[2]
OPERATING REVENUES	
Increase in refined sugar production	$75,000

[1] includes cost of installation and concrete floor.

[2] 5% of capital cost.

operating period of the factory from four to six, or more, months. This would result in a significant reduction in the required daily capacity of the factory. The savings in capital costs would then be substantial.

In conclusion, the frozen storage of sugarbeets in outdoor piles is economically attractive. A pilot study should be undertaken to determine if the beets can be maintained frozen during storage in outdoor piles in the Taber area. If the beets are observed to thaw during storage in these piles then the frozen storage of beets in mechanically cooled archrib structures should be investigated.

8. FREEZING MODELS - LITERATURE REVIEW

Heat transfer models are useful in estimating the freezing injury incurred by biological samples. The severity of this injury depends on the rate at which the biological material is frozen and thawed. Different regions of large samples may be injured to different extents since they may experience different rates of temperature change. Thus, in estimating the overall freeze-damage in an entire sample, it is advantageous to be able to predict how the temperature varies with time throughout the sample during freezing and thawing.

Freezing of biological materials is difficult to model because all the heat of fusion is not removed at one temperature i.e. they do not have a unique freezing point. Assuming there is no supercooling, the water in a biological material starts to freeze when the temperature is lowered to the initial freezing point. More water is frozen as the temperature is further reduced. Approximately 75% of a food's water content is frozen by 268 K.

Thermal properties change rapidly during freezing. Water is the largest component of most foods. For example, lean beef contains 74% water, potatoes 78%, and apples 84%. When this water freezes, the thermal conductivity of the food increases rapidly. In fact, the thermal conductivity may triple in the first 5 K below the initial freezing point. This dramatic increase results from the fact that the thermal conductivity of ice is four times that of water. In

contrast, the specific heat of a food falls during freezing. This also stems from the behaviour of water, which experiences a 50% reduction in its specific heat on freezing.

Another complication in food freezing is the fact that the heat of fusion is removed over a range of temperatures, beginning at, or just below (supercooling), the initial freezing point. This is sometimes allowed for by the use of an "effective specific heat", which includes a heat of fusion term. The effective specific heat is a complicated function of temperature just below the initial freezing point.

A literature review was undertaken to examine existing two phase heat transfer models. Some of these models will now be discussed.

8.1 Analytical Solutions

Analytical solutions are based on a number of simplifying assumptions. For example, Plank's formula (27), which is perhaps the most well-known food freezing equation, is based on the following assumptions:

1. the food freezes at a unique temperature, rather than over a range of temperatures,
2. physical properties are independent of temperature, although they do have different values in the frozen and unfrozen regions, and

3. the food is initially at its freezing point.

Plank's formula may be written as:

$$t_f = \frac{D\ L\ \rho}{T_f - T_a}\left[\frac{1}{h} + \frac{Gl^2}{k_f}\right] \qquad (1)$$

where:

t_f = time for centre of food sample to reach the freezing point,

l = thickness of infinite slab or diameter of sphere or cylinder,

T_f = freezing point of food,

T_a = temperature of cooling medium,

L = heat of fusion per unit mass,

ρ = density,

h = heat transfer coefficient at the surface,

D = 1/2 for infinite slab or infinite cylinder,

D = 1/3 for sphere,

G = 1/4 for infinite slab,

G = 1/2 for infinite cylinder or sphere,

k_f = thermal conductivity of frozen food.

Many researchers (27,40,42,49,68,152) have modified Plank's equation. Cleland and Earle (40,43,44,175) produced perhaps the best adaptation of Plank's formula. Based on a least squares fit of empirical data, the shape factors D and G (in Plank's equation) were modified to allow for the

situation where:

1. the food is initially at a temperature above its freezing point, and
2. the final temperature at the centre of the food is below the freezing point.

The heat of fusion, L, was replaced by the enthalpy change in going from the freezing point to the final temperature. Freezing times (time for the centre of the food to reach 263 K) were predicted for Karlsruhe Test Substance, meat and potatoes, which were within ± 10% of experimental values. They concluded (42) that their equations predicted food freezing times as accurately as Bonacina and Comini's numerical model (22-24,46). Their method was also shown (44) to be a significant improvement on the equations of Plank and Nagaoka (152).

Nagaoka et al. (152), also, modified Plank's equation to allow for cooling above and below the initial freezing point. However, their equation was based on experimental data for fish freezing (elliptical shape). Consequently, Nagaoka's equation is not as generally applicable as that of Cleland and Earle.

Mott (246) proposed an equation, without stating his assumptions, for calculating freezing times of irregularly shaped foods. However, this equation predicted freezing times which were about 80% too long.

Starting with the freezing point depression equation, Schwartzberg et al. (185) developed equations to predict the effective specific heat as a function of temperature during freezing or thawing. They then assumed that one phase solutions to Fourier's heat equation could be extended, for low Biot numbers, to the two phase problem by replacing the constant one phase specific heat with the effective specific heat. They also assumed that, during freezing, the thermal conductivity was the average value for the frozen and unfrozen food. An equation was then obtained which allowed the calculation of the time required for the average food temperature to reach a given value. This equation only applied to the freezing range. The time to go from the starting temperature to the initial freezing point, or vice versa during thawing, was calculated from an analytical solution for single phase heat transfer. Schwartzberg tested his equations against experimental thawing times for slabs of chopped beef. The agreement was generally within about 5% but a deviation as large as 22% was obtained. He concluded that his equations produced better freezing times than the methods of Plank, Tao, and Mott, all of which gave maximum deviations from the experimental values of the order of 40%. The maximum deviation obtained with Nagaoka's equation was 27%.

Bakal and Hayakawa (16,17,96) developed semi-empirical formulae which predicted temperature-time distributions in an infinite slab. Their method was more realistic than that

of Plank in that they allowed for the fact that foods freeze over a range of temperatures. However, their solutions were based on many other assumptions. For example, they assumed that:

1. Thermal properties are independent of temperature, although they do have different values in frozen and unfrozen regions. The value of a thermal property in a partially frozen region is the average of its values in the frozen and unfrozen states.
2. Before the temperature at the centre of the slab has reached the initial freezing point, the fraction of an elemental volume which is frozen is inversely proportional to the distance between the element and the surface. After the centre starts to freeze, this fraction becomes linearly related to the surface temperature.

Their model results were generally within 3 K of their experimental values but deviations as great as 11 K were observed.

8.2 Numerical Models

Semi-empirical equations are useful in estimating freezing times. However, in predicting the transient temperature distributions in a food during a phase-change, one should use a numerical approach. Increased flexibility

is also gained by using a numerical method. For example, a numerical model can allow for a heat transfer coefficient which varies with time. This is especially important for free convection situations, such as the still air thawing carried out in this thesis.

Charm (37,38) presented a numerical method which predicted transient temperature distributions in an infinite cylinder or an infinite slab. This method applied to both freezing and thawing. In developing his equations, he assumed foods had a unique freezing point. It was also assumed that specific heat and thermal conductivity, in both the frozen and unfrozen phases, were independent of temperature. However, the value of these thermal properties changed during the transition from the frozen to the unfrozen state. Charm tested his model against experimental centre temperature measurements, during freezing of cod and haddock slabs. For haddock, his model predicted a freezing plateau which lasted about 15 min. In contrast, no plateau was obtained in his experimental results. The long plateau in the model predictions resulted from the use of a unique freezing point approach. Differences as great as 10 K were obtained between model predictions and experimental measurements.

Fleming (83) developed a finite difference method for determining temperature distributions during freezing of bodies of arbitrary geometry. He allowed for thermal properties which change drastically in the freezing range.

The variation of these properties with temperature was approximated by a discontinuous function in order to reduce the computing time required. He tested his model against experimental data for foodstuffs and concluded that his model precision was limited by "an adequate knowledge of the thermal properties".

Cullwick and Earle (54) used a numerical approach to predict temperature as a function of time in an infinite slab. They allowed for the variation of specific heat and thermal conductivity with temperature. They compared their results to experimental data for minced beef. For slabs 7.5 cm and 10 cm thick, the experimental and model results were essentially the same. However, it should be noted that the heat transfer coefficient at the surface, which was used in the model equations, was not measured. Instead it was selected so the calculated freezing time ("measured from the beginning of the cooling and freezing cycle to the time at which the centre temperature falls from the latent heat plateau") was equal to the experimental value. For a 12.5 cm thick slab, model predictions deviated from experimental results by as much as 10 K.

Bonacina and Comini (22-24,46) numerically solved the one-dimensional heat equation:

$$\rho(T)c(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial y}\left[k(T)\frac{\partial T}{\partial y}\right] \qquad (2)$$

by replacing $\partial T/\partial t$ by a central difference value. The derivative $\partial T/\partial y$ was averaged over three time periods, j-1,

j, and j+1, where j.t is the time since the start of cooling. Their model is thus referred to as a "three tier time level implicit scheme" (22). By using this approach the thermal conductivity can be evaluated at the present time (time increment = j) and this results in a set of linear equations. Linearity can also be obtained by using an explicit model. However, convergence problems can then arise. In contrast to explicit models, Bonacina and Comini (22) proved that their system was unconditionally stable and convergent. Cleland and Earle (41) showed that Bonacina and Comini's model predicted freezing times which were accurate to ± 8%.

All the numerical models, which have been discussed, used specific heat in the fundamental energy balance equations. As stated by Fennema (77) "it is often more accurate to deal with enthalpy" rather than specific heat. Also, the effective specific heat is difficult to use as it increases rapidly when the temperature is lowered below the initial freezing point. It then falls quickly again as the temperature is further reduced. A two-phase numerical model is developed in the next chapter which is based on enthalpy rather than on specific heat. Shamsundar and Sparrow (186,187) did derive an implicit model which was based on enthalpy. However, their model was for a material with a unique freezing point.

8. DERIVATION OF MODEL EQUATIONS

In estimating the freeze-damage which will occur in a biological sample, it is useful to know the rate at which every region of the sample is frozen and thawed. A numerical heat transfer model which accomplishes this is now developed. The model predicts the transient temperature profile in an infinite cylinder during freezing and thawing. The model derivation is based on the following assumptions:

1. the cylinder is homogeneous,
2. the density remains unchanged, even during a phase change, and
3. the sample does not have a unique freezing point but rather freezes over a range of temperatures.

The cylinder is divided into N + 1 concentric elements as shown in Figure 8. The surface element has a thickness of $\Delta R/2$, where $\Delta R = R/N$. All the other elements are ΔR units thick.

Energy balance equations are constructed for the various elements. In the following equations $-\left(k \frac{\partial T}{\partial r}\right)_{i+\frac{1}{2}}$ is evaluated at the boundary between element i and element i + 1.

CENTRAL ELEMENT (i=1)

HEAT IN - HEAT OUT = ENERGY ACCUMULATED

$$0 + 2\pi \frac{\Delta R}{2} \left(k \frac{\partial T}{\partial r}\right)_{1\frac{1}{2}} = \pi \left(\frac{\Delta R}{2}\right)^2 \rho \frac{\partial H_1(T)}{\partial t} \qquad (3)$$

where:

k = thermal conductivity,

T = temperature,

r = radial position,

ρ = density,

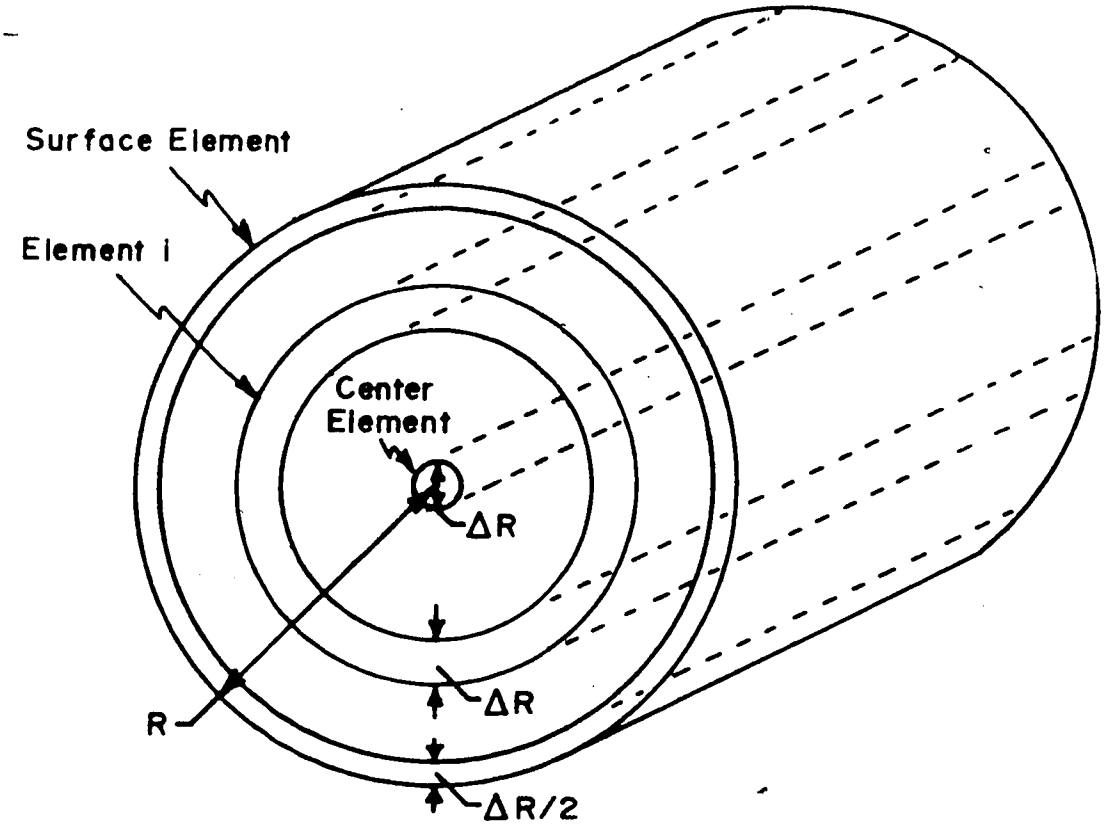

FIGURE 8: DIVISION OF CYLINDER INTO CONCENTRIC ELEMENTS

H_1 = enthalpy per unit mass of element 1, and

t = time.

Equation (3) is rearranged to yield:

$$\left(k\frac{\partial T}{\partial r}\right)_{i+\frac{1}{2}} = \frac{\Delta R}{4}\ \rho\ \frac{\partial H_1(T)}{\partial t} \qquad (4)$$

SURFACE ELEMENT, (i=N+1)

HEAT IN - HEAT OUT = ENERGY ACCUMULATED

$$-2\pi\left(R - \frac{\Delta R}{2}\right)\left(k\ \frac{\partial T}{\partial r}\right)_{N+\frac{1}{2}} - h\ 2\pi R\ (T_{N+1} - T_a) = \rho\pi\left[R^2 - \left(R - \frac{\Delta R}{2}\right)^2\right]\ \frac{\partial H_{N+1}(T)}{\partial t} \qquad (5)$$

where:

h = heat transfer coefficient at the surface,

T_a = environment temperature, and

T_{N+1} = surface temperature,

The next equation follows from Equation (5)

$$-\left(R - \frac{\Delta R}{2}\right)\left(k\ \frac{\partial T}{\partial r}\right)_{N+\frac{1}{2}} - hR\ (T_{N+1} - T_a) = \frac{\rho}{2}\ \left[R\ \Delta R - \frac{(\Delta R)^2}{4}\right]\ \frac{\partial H_{N+1}(T)}{\partial t} \qquad (6)$$

INTERIOR ELEMENTS (i: 2,N)

HEAT IN - HEAT OUT = ENERGY ACCUMULATED

$$-2\pi\ (i-1\tfrac{1}{2})\ \Delta R\ \left(k\ \frac{\partial T}{\partial r}\right)_{i-\frac{1}{2}} + 2\pi\ (i-\tfrac{1}{2})\ \Delta R\ \left(k\ \frac{\partial T}{\partial r}\right)_{i+\frac{1}{2}} = \rho\pi\ \left[\ \{(i-\tfrac{1}{2})\ \Delta R\}^2 - \{(i-1\tfrac{1}{2})\ \Delta R\ \}^2\right]\ \frac{\partial H_i(T)}{\partial t} \qquad (7)$$

Equation (7) is rearranged to produce:

$$-(i-1\tfrac{1}{2})\left(k\frac{\partial T}{\partial r}\right)_{i-\frac{1}{2}} + (i-\tfrac{1}{2})\left(k\frac{\partial T}{\partial r}\right)_{i+\frac{1}{2}} = \rho\,\Delta R(i-1)\,\frac{\partial H_i(T)}{\partial t}\qquad i=2,N \tag{8}$$

Equations (4), (6) and (8) are rewritten in terms of dimensionless temperatures, times and distances to yield:

$$\left(k\frac{\partial u}{\partial x}\right)_{1\frac{1}{2}} = \frac{\Delta x}{4}\,\frac{\rho\alpha}{T_o-T_a}\,\frac{\partial H_1(u)}{\partial \tau} \tag{9}$$

$$-\left(1-\frac{\Delta x}{2}\right)\left(k\frac{\partial u}{\partial x}\right)_{N+\frac{1}{2}} - hR\,u_{N+1} = \frac{\rho\alpha}{2(T_o-T_a)}\left[\Delta x - \frac{(\Delta x)^2}{4}\right]\frac{\partial H_{N+1}(u)}{\partial \tau} \tag{10}$$

and

$$-(i-1\tfrac{1}{2})\left(k\frac{\partial u}{\partial x}\right)_{i-\frac{1}{2}} + (i-\tfrac{1}{2})\left(k\frac{\partial u}{\partial x}\right)_{i+\frac{1}{2}} = \frac{\alpha\rho\Delta x}{(T_o-T_a)}\,(i-1)\,\frac{\partial H_i(u)}{\partial \tau}\qquad i=2,N \tag{11}$$

where:

$$u = \frac{T-T_a}{T_o-T_a} \tag{12}$$

and

T_o = initial uniform temperature of the cylinder,

$$x = r/R, \tag{13}$$

$$\Delta x = \Delta R/R, \text{ and} \tag{14}$$

$$\tau = \alpha t/R^2 \tag{15}$$

FIRST TIME INCREMENT

In the first time increment, $\left(\frac{\partial u}{\partial x}\right)_{i+\frac{1}{2}}$ is replaced in equations (9) - (11) by a two tier time level approximation, which is:

$$\left(\frac{\partial u}{\partial x}\right)_{i+\frac{1}{2}} = \frac{1}{2}\left[\frac{u_{i+1}^{1} - u_{i}^{1}}{\Delta x} + \frac{u_{i+1}^{0} - u_{i}^{0}}{\Delta x}\right] \qquad (16)$$

and where:

u_i^0 = initial dimensionless temperature at the centre of element i, and

u_i^1 = dimensioness temperature at the centre of element i at the end of the first time increment.

From the definition of u (Equation 12) it is apparent that:

$$u_{i+1}^{0} = u_{i}^{0} = 1 \qquad (17)$$

Equation (16) can then be simplified to yield:

$$\left(\frac{\partial u}{\partial x}\right)_{i+\frac{1}{2}} = \frac{u_{i+1}^{1} - u_{i}^{1}}{2\Delta x} \qquad (18)$$

The $\frac{\partial H(u)_i}{\partial \tau}$ terms are replaced by a forward difference approximation, which is:

$$\frac{\partial H(u)_i}{\partial \tau} = \frac{H(u)_i^1 - H(u)_i^0}{\Delta \tau} \qquad (19)$$

and where:

$\Delta\tau$ is the length of a dimensionless time increment.

When the finite difference approximations are substituted into Equations (9) -(11) the following equations are obtained.

$$u_2^1 - u_1^1 - \frac{(\Delta x)^2}{2\Delta\tau}\frac{\rho\alpha}{k_o\,(T_o - T_a)}\,(H(u)_1^1 - H(u)_1^0) = 0 \qquad (20)$$

$$\frac{2-\Delta x}{2\Delta x}\,k_o\,(u_{N+1}^1 - u_N^1) + hR\,(u_{N+1}^1 + u_{N+1}^0)$$

$$+ \frac{\rho\alpha}{\Delta\tau(T_o - T_a)}\left(\Delta x - \frac{(\Delta x)^2}{4}\right)(H(u)_{N+1}^1 - H(u)_{N+1}^0) = 0, \text{ and} \qquad (21)$$

$$(i-1\tfrac{1}{2})\ k_o\ (u_i^1 - u_{i-1}^1) - (i-\tfrac{1}{2})\ k_o\ (u_{i+1}^1 - u_i^1)$$

$$+ \frac{2\alpha\rho(\Delta x)^2\ (i-1)}{\Delta\tau\ (T_o-T_a)}\ (H(u)_i^1 - H(u)_i^0) = 0 \qquad 1=2,N \tag{22}$$

and where:

k_o = initial value of thermal conductivity.

SUBSEQUENT TIME INCREMENTS

For all times, after the first time increment, $(\frac{\partial u}{\partial x})_{i+\frac{1}{2}}$ is replaced in Equations (9) -(11) by a three tier time level approximation. For time increment j, the approximation is:

$$(\frac{\partial u}{\partial x})_{i+\frac{1}{2}}^j = \frac{1}{3}\left[\frac{u_{i+1}^{j+1} - u_i^{j+1}}{\Delta x} + \frac{u_{i+1}^j - u_i^j}{\Delta x} + \frac{u_{i+1}^{j-1} - u_i^{j-1}}{\Delta x}\right] \tag{23}$$

The $\frac{\partial H(u)_i}{\partial\tau}$ terms are replaced by a forward difference approximation:

$$\frac{\partial H(u)_i}{\partial\tau} = \frac{H_i^{j+1} - H_i^j}{\Delta\tau} \tag{24}$$

Equations (9) - (11) then become:

$$k_{1\frac{1}{2}}^j\left[(u_2^{j+1} - u_1^{j+1}) + (u_2^j - u_1^j) + (u_2^{j-1} - u_1^{j-1})\right]$$

$$- \frac{3(\Delta x)^2\ \rho\alpha}{4\Delta\tau\ (T_o-T_a)}\ (H(u)_1^{j+1} - H(u)_1^j) = 0 \tag{25}$$

$$\frac{k_{N+\frac{1}{2}}^j\ (2-\Delta x)}{6\Delta x}\left[(u_{N+1}^{j+1} - u_N^{j+1}) + (u_{N+1}^j - u_N^j) + (u_{N+1}^{j-1} - u_N^{j-1})\right]$$

$$+ \frac{hR}{3}\ (u_{N+1}^{j+1} + u_{N+1}^j + u_{N+1}^{j-1})$$

$$+ \frac{\rho\alpha}{8\ \Delta\tau\ (T_o-T_a)}\ (4\Delta x-(\Delta x)^2)\ (H(u)_{N+1}^{j+1}) - H(u)_{N+1}^j) = 0 \tag{26}$$

and

$$- (i-1\tfrac{1}{2})\, k^{j}_{i-\frac{1}{2}} \left[(u_i^{j+1} - u_{i-1}^{j+1}) + (u_i^{j} - u_{i-1}^{j}) + (u_i^{j-1} - u_{i-1}^{j-1}) \right]$$

$$+ (i - \tfrac{1}{2})\, k^{j}_{i+\frac{1}{2}} \left[(u_{i+1}^{j+1} - u_i^{j+1}) + (u_{i+1}^{j} - u_i^{j}) + (u_{i+1}^{j-1} - u_i^{j-1}) \right]$$

$$- \frac{3\alpha\rho(\Delta x)^2 (i-1)}{\Delta\tau (T_o - T_a)} (H(u)_i^{j+1} - H(u)_i^{j}) = 0 \qquad i=2,N \qquad (27)$$

where:

$k^{j}_{i-\frac{1}{2}}$ = k evaluated at $\dfrac{u_i^j + u_{i-1}^j}{2}$ (28)

$k^{j}_{i+\frac{1}{2}}$ = k evaluated at $\dfrac{u_i^j + u_{i+1}^j}{2}$ (29)

To solve the Equations (20) - (22) and (25) - (27) it is necessary to know how the enthalpy changes with temperature. As explained in the next chapter, H(T) can be adequately represented by:

$$H(T) = a_0 + a_1 T \qquad T > T_{c_1} \qquad (30)$$

$$H(T) = \exp [b_0 + b_1 T + b_2 T^2 + b_3 T^3] \qquad T_{c_2} \leq T \leq T_{c_1} \qquad (31)$$

$$H(T) = \exp [c_0 + c_1 T] \qquad T < T_{c_2} \qquad (32)$$

where:

a_0, a_1, b_0, b_1, b_2, b_3, c_0, and c_1 are constants, and

T_{c_1} = initial freezing point of the sample, and

T_{c_2} = temperature at which ln H vs T becomes linear (during freezing)

The dependence of thermal conductivity on temperature is also required. The k(T) profile is approximated by:

$$k(T) = d_0 + d_1 T \qquad T > T_{c_1} \qquad (33)$$

$$k(T) = e_0 + e_1 T \qquad T_{c_3} \leq T \leq T_{c_1} \qquad (34)$$

$$k(T) = f_0 + f_1 T + f_2 T^2 + f_3 T^3 \qquad T < T_{c_3} \qquad (35)$$

where:

d_0, d_1, e_0, e_1, f_0, f_1, f_2, and f_3 are constants, and

T_{c_3} = temperature at which k vs T is no longer linear (during freezing).

Prior to being used in the model equations, the H(T) and k(T) functions are first converted to H(u) and k(u) functions. This results in the following set of equations:

$$H(u) = a_2 + a_3 u \qquad u > u_{c_1} \tag{36}$$

$$H(u) = \exp (b_4 + b_5 u + b_6 u^2 + b_7 u^3) \qquad u_{c_2} \leq u \leq u_{c_1} \tag{37}$$

$$H(u) = \exp (c_2 + c_3 u) \qquad u < u_{c_2} \tag{38}$$

and

$$k(u) = d_2 + d_3 u \qquad u > u_{c_1} \tag{39}$$

$$k(u) = e_2 + e_3 u \qquad u_{c_3} \leq u \leq u_{c_1} \tag{40}$$

$$k(u) = f_4 + f_5 u + f_6 u^2 + f_7 u^3 \qquad u < u_{c_3} \tag{41}$$

where:

$$u_{c_1} = \frac{T_{c_1} - T_a}{T_o - T_a} \tag{42}$$

$$u_{c_2} = \frac{T_{c_2} - T_a}{T_o - T_a} \tag{43}$$

$$u_{c_3} = \frac{T_{c_3} - T_a}{T_o - T_a} \tag{44}$$

$$a_2 = a_0 + a_1 T_a \tag{45}$$

$$a_3 = (T_o - T_a) a_1 \tag{46}$$

$$b_4 = b_0 + b_1 T_a + b_2 T_a^2 + b_3 T_a^3 \tag{47}$$

$$b_5 = (T_o - T_a) (b_1 + 2b_2 T_a + 3b_3 T_a^2) \tag{48}$$

$$b_6 = (T_o - T_a)^2 b_2 + 3b_3 T_a) \quad (49)$$

$$b_7 = (T_o - T_a)^3 b_3 \quad (50)$$

$$c_2 = c_o + c_1 T_a \quad (51)$$

$$c_3 = (T_o - T_a) c_1 \quad (52)$$

$$d_2 = d_0 + d_1 T_a \quad (53)$$

$$d_3 = (T_o - T_a) d_1 \quad (54)$$

$$e_2 = e_0 + e_1 T_a \quad (55)$$

$$e_3 = (T_o - T_a) e_1 \quad (56)$$

$$f_4 = f_0 + f_1 T_a + f_2 T_a^2 + f_3 T_a^3 \quad (57)$$

$$f_5 = (T_o - T_a)(f_1 + 2f_2 T_a + 3f_3 T_a^2) \quad (58)$$

$$f_6 = (T_o - T_a)^2 (f_2 + 3f_3 T_a), \text{ and} \quad (59)$$

$$f_7 = (T_o - T_a)^3 f_3 \quad (60)$$

The Newton-Raphson procedure is used to solve the non-linear Equations (20) - (22) and (25) - (27).

The temperature at the midpoint of an element is used in determining its enthalpy. There are two exceptions to this rule. In the centre element, the temperature at the midpoint is greater than in the remainder of the element during freezing and is lower during thawing. Thus, the midpoint temperature cannot be considered to be a representative temperature for this element. Instead, it is assumed that the temperature varies linearly with radial position between the midpoints of the centre and adjacent elements. It then follows that the mass average temperature in the centre element is $\frac{u_2^j + 2u_1^j}{3}$. This temperature is used in the determination of the enthalpy of the centre element. In calculating

the enthalpy of the surface element it is assumed that the temperature at the inner boundary of the surface element is $(u_{N+1} + u_N)/2$. The average of the two boundary temperatures of the surface element is then $(3u_{N+1} + u_N)/4$. This temperature is used to calculate the enthalpy of the surface element.

A computer program was written for the above heat transfer model. It is listed in Appendix G. The program was run on a HEWLETT PACKARD computer (21 MXE-series). Twenty one space elements (N=20) were used in the model computer runs for both freezing and thawing of a 1 cm diameter sugarbeet cylinder. For freezing a time increment of 0.005 min was used. Under these conditions 9 hours of CPU time were required to execute the program. However, for N=5, the CPU time was only 1.5 hours. Reducing N from 20 to 5 changed temperatures at a given time by less than 0.5 K. For thawing, a time increment of 0.1 min was used.

10. ESTIMATION OF THERMAL PROPERTIES

10.1 Literature Review

In order to use the numerical model developed in the last chapter, it is necessary to know the dependence of both thermal conductivity and enthalpy on temperature.

Thermal properties of various foods have been experimentally determined by a number of authors (10,11,63,77,115,222,225).

Literature values for thermal conductivity should be carefully examined before being used. Fennema (77) pointed out a number of inconsistencies in the literature values which he examined. The literature on thermal conductivity and enthalpy will now be reviewed.

10.2 Thermal Conductivity

Cuevas and Cheryan (53) summarized a number of different methods of experimentally measuring or theoretically predicting the thermal conductivity of foods.

Experimental methods for the measurement of thermal conductivity are basically divided into two classes:

1. Steady state methods (e.g. parallel plate and concentric cylinders or spheres) in which the temperature within the food is kept constant with time (12,194).
2. Transient methods in which a heated probe is allowed to

change the temperature of the food for a short period of time (114,61).

The thermal conductivity may alternatively be determined indirectly from experimentally measured diffusivity (α) values using :

$$k = \rho c \alpha$$

A number of simple formulae can also be used to estimate the thermal conductivity. Comini (47) obtained k_f and k_ℓ from :

$$k_f = w_w \, k_i + (1 - w_w) \, k_d \qquad (61)$$

$$k_\ell = w_w \, k_w + (1 - w_w) \, k_d \qquad (62)$$

where :

w_w = mass fraction of water.

k_ℓ = thermal conductivity of the unfrozen food, J/(s . m . K)

k_f = thermal conductivity of the frozen food, J/(s . m . K)

k_i = thermal conductivity of ice, J/(s . m . K)

k_w = thermal conductivity of water, J/(s . m . K)

k_d = thermal conductivity of dry matter,
= 0.2602 J/(s . m . K).

In the freezing range k is assumed to vary linearly with temperature from k_ℓ to k_f.

Alternatively, Fikiin's (78) formula can be used for temperatures below the initial freezing point. It is:

$$k(T) = 1.745w_w\left[1 - \frac{T_{c_i} - 273.2}{T - 273.2}\right] + 0.233 \qquad (63)$$

where:

T_{c_i} = initial freezing point (K).

By assuming that the thermal conductivity was linearly dependent on the percent of water frozen, Schwartzberg (184) obtained the following equation for the freezing range:

$$k(T) = k_f + (k_\ell - k_f)\,(T_w - T_{c_i})\,/(T_w - T) \qquad (64)$$

where:

T_w = freezing point of water.

He concluded that this equation predicted thermal conductivities which were within about ± 10% of most experimental values he examined in the literature.

Riedel's empirically obtained formula (176) for thermal diffusity may also be applied, for an unfrozen food, to give:

$$k(T) = 6.978\rho(T)c(T)(0.053+(\alpha_w(T)-0.053)(w_w)) \qquad (65)$$

where:

$\alpha_w(T)$ = thermal diffusity of water at temperature T in cm^2/min.

Lentz and Van den Berg (115,116,211) developed a model for predicting the thermal conductivity of foodstuffs. Their model assumes that an unfrozen food consists of nonaqueous components dispersed, as spheres, in water. They assumed that the thermal conductivity of frozen foods was the average of two k values. The first k value was obtained by assuming water was the continuous phase in which nonaqueous components and ice were dispersed. The second k value was obtained by assuming ice was the continuous phase. They observed that their calculated thermal conductivities at 253 K were within ±15% of their experimental data.

10.3 Enthalpy

The enthalpy-temperature (H(T)) relationship can be experimentally evaluated (131,174) or theoretically predicted. Two theoretical methods will now be discussed. Starting with the freezing point depression equation, Schwartzberg (184) derived equations for the calculation of H(T) by making the following assumptions :

1. Enthalpies of food components are additive i.e. heat of solution = 0.
2. The solution contained in the food is ideal i.e. water

activity (a_w) = mole fraction of water.

3. $\ln(a_w) = a_w - 1$.

For lean beef, Schwartzberg's predicted enthalpy values which deviated by about 5% from the experimental results of Riedel.

10.3.1 Riedel's Method

Riedel (174) proposed a method for calculating H(T) for fruits and vegetables during freezing. Riedel's method was used in this thesis and thus is considered here in some detail.

Based on experimental observation Riedel (174) concluded that the specific heat of vegetable juices (c_J) could be calculated from :

$$c_J = 4.186 - 2.386w_o \quad (66)$$

where :

w_o = mass fraction of soluble solids in unfrozen juice.

From further experimental data it was concluded that the mass fraction of soluble solids in the residual solution (w_s) during freezing is related to the temperature by :

$$T = 273.2 - 10w_s - 50w_s^3 \quad (67)$$

Riedel used Randall's formula to calculate the heat of

fusion of ice (L) at the temperature T.

$$L = 334.12 + 2.118(T-273.2) - 0.00783(T-273.2)^2 \quad (68)$$

where:

L in J/g, and

T in K.

The mass fraction of the juice which has become ice (w_i) can be calculated from :

$$w_i = 1 - w_o/w_s \quad (69)$$

Riedel assumed that, in warming a sample from T to 293.2 K, all the ice is melted at T and the unfrozen solution is warmed to 293.2 K. He justified this assumption on the basis that, since enthalpy is a state function, the enthalpy change in going from one temperature to another is independent of the path. Thus, the enthalpy change ($\Delta H_J(T)$) in going from T to 293.2 K is given by :

$$\Delta H_J(T) = H_J(293.2) - H_J(T)$$
$$= w_i L + c_J(293.2 - T) \quad (70)$$

where:

$H_J(T)$ = enthalpy (J/g) of juice at temperature T.

Equations (66) and (69) are substituted into Equation (70) to yield:

$$\Delta H_J(T) = (L+4.186(293.2-T))-(L/w_s+2.386(293.2-T))w_o \quad (71)$$

To obtain the $H_J(T)$ profile select a w_s value. Equation (67) determines a T corresponding to this w_s. Equation (68) then calculates an L value. Using these w_s, T and L values Equation (71) predicts a $\Delta H_J(T)$ value. By selecting a number of w_s values the $\Delta H_J(T)$ profile is obtained. Assuming the enthalpy at 233.2 K is zero then the $H_J(T)$ profile is calculated from :

$$H_J(T) = \Delta H_J(233.2) - \Delta H_J(T) \quad (72)$$

The enthalpy versus temperature profile for the vegetable is then determined from:

$$H(T) = (1-w_u)H_J(T) + c_u w_u(T-233.2) \quad (73)$$

where :

$H(T)$ = enthalpy (J/g) of the vegetable at T.

w_u = mass fraction of insoluble solids.

c_u = specific heat of insoluble solids

= 1.21 J/(g . K) (value suggested by Riedel)

11. DETERMINATION OF THERMAL PROPERTIES OF SUGARBEETS

11.1 Estimation of k(T) of Sugarbeets

Karlsruhe Test Substance (175) is a gel containing 77% water, methylcellulose, and salt. Sugarbeets have a water content of 78% (133) which is very close to that of Karlsruhe Test Substance. In fact, a sugarbeet examined in this study had a 76% water content (determined using a vacuum oven at 343 K and a pressure of 6.8 kPa). The densities of Karlsruhe Test Substance and sugarbeets are very similar; 1.06 and 1.05 g/cm , respectively. Thus, these two materials should have almost equal water contents on a volume as well as on a weight basis. Van den Berg and Lentz (211) observed that the measured thermal conductivities of a number of foods, with the same volumetric water content, at 253 K were within a range of ± 15%. As the thermal conductivity of a food is primarily dependent on water content (211) it was considered reasonable to use the published k versus T graph (22) for Karlsruhe Test Substance to represent the k(T) profile for sugarbeets. This graph was approximated by the following set of equations:

$$k(T) = 0.5143 + 0.002199(T-273.2) \qquad T > 271.0\ K \qquad (74)$$

$$k(T) = -0.7620 - 0.57657(T-273.2)$$

$$269.7\ K \leq T \leq 271.0\ K \qquad (75)$$

$$k(T) = 0.9504 - 0.10697(T-273.2) - 0.005726(T-273.2)^2$$

$$-0.0000984(T-273.2)^3 \qquad T < 269.7\ K \qquad (76)$$

The k(T) equations used for sugarbeets were not quite identical to those used for Karlsruhe Test Substance (listed in Appendix F). They were altered slightly to allow for the fact that the initial freezing point of sugarbeets is 271 K, whereas for Karlsruhe Test Substance it is 272 K. Thus, the unfrozen region, with its lower k(T) values, extends for sugarbeets to 271 K instead of to 272 K. Below 270 K this modification only changes the k(T) values by about 2%. However, at 271 K it has a noticeable effect in that the k(T) value used for sugarbeets is 0.510 J/(s . m . K) whereas for Karlsruhe Test Substance it is equal to 0.935 J/(s . m . K). The published k(T) values for Karlsruhe Test Substance and the predicted k(T) values for sugarbeets are listed in Table 17. Assuming Bonacina and Comini's measured k(T) values were accurate to ± 15%, the k(T) values, used in this thesis for sugarbeets, were accurate to ± 30%

TABLE 17: k(T) VALUES FOR KARLSRUHE TEST SUBSTANCE AND SUGARBEETS

TEMPERATURE (K)	THERMAL CONDUCTIVITY (J/(s . m . K))	
	KARLSRUHE[1]	SUGARBEETS[2]
243.2	1.67	1.67
246.2	1.65	1.60
248.2	1.64	1.59
253.2	1.61	1.59
257.2	1.59	1.60
263.2	1.53	1.55
265.2	1.49	1.50
267.2	1.44	1.40
268.2	1.40	1.35
269.2	1.29	1.29
270.2	1.20	0.97
271.2	0.85	0.51
272.2	0.50	0.51
283.2	0.53	0.53
293.2	0.57	0.57

[1] Read from a published k versus T graph (22).

[2] Approximate values based on the published graph for Karlsruhe Test Substance.

11.2 Enthalpy of Sugarbeets

Riedel's method was used to predict the enthalpy-temperature relationship for sugarbeets. The H(T) values obtained were checked against experimental measurements made by personnel of the Food Science Department of the University of Alberta. The experimental enthalpy values were obtained by means of a bomb calorimeter (see Figure 9). A small sugarbeet piece , weighing about 50 mg, was placed in the sample holder. The calorimeter was put in an alcohol bath until it reached the bath temperature of 213 K. The bath was then warmed at a rate of 1 K/min.

The temperature of the block (T_b) and the temperature difference between the block and the sample (ΔT) were continually recorded by means of chromel-alumel thermocouples attached to a Cole Parmer recorder.

During warming the rate of heat flow ($\dot{Q}(t)$) from the block to the sample and sample holder is given by:

$$\dot{Q}(t) = B\,\Delta T(t) \qquad (77)$$

where:

B = a constant which is obtained from a calibration run with a material of known specific heat.

A "blank" run is performed to remove the effect of the sample holder. This run is undertaken without a sample in the holder and $\Delta T_b(t)$ is measured. The rate at which the sample obtains heat during a normal run is then given by:

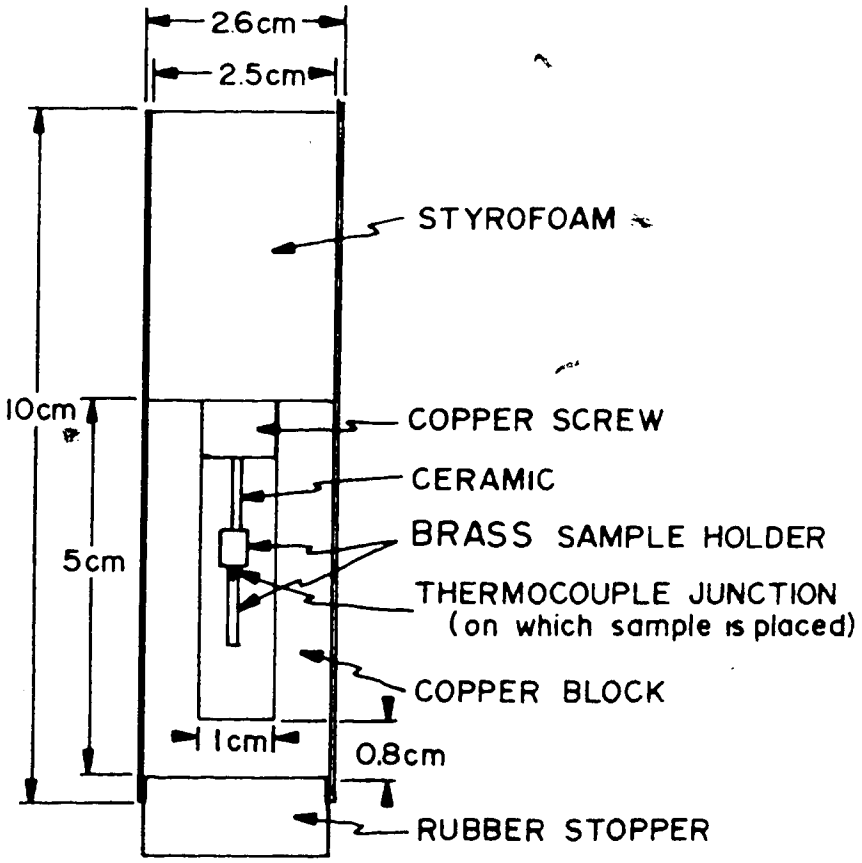

FIGURE 9: CALORIMETER FOR MEASURING H(T)

$$\dot{Q}(t) = B(\Delta T(t) - \Delta T_b(t)) \tag{78}$$

However ,

$$\dot{Q}(t) = m_s c_s(T) \frac{dT}{dt} \tag{79}$$

where:

m_s = mass of sample,

$c_s(T)$ = effective specific heat of sample (includes heat of fusion) at T, and

T = temperature of sample = $T_b - \Delta T$.

Equations (78) and (79) are combined to yield:

$$m_s c_s(T)dT = B(\Delta T(t) - \Delta T_b(t))dt \tag{80}$$

Integration of this equation leads to:

$$\int_{233.2}^{T} c_s(T)dT = \frac{B}{m_s} \int_0^t (\Delta T(t) - \Delta T_b(t))dt \tag{81}$$

or

$$H(T) = \frac{B}{m_s} \int_0^t (\Delta T(t) - \Delta T_b(t))dt \tag{82}$$

From experiments, using acetic acid as the sample, it was determined that:

$$B = 0.223 \text{ J/(min . K)} \tag{83}$$

A blank run resulted in a graph of ΔT_b versus time. Thus, the experimentally determined values of $\Delta T(t)$ and $\Delta T_b(t)$ produced (by means of numerical integration of Equation

(82)) the required H(T) data for the sample.

As can be seen from Table 18 and Figure 10, results obtained using Riedel's method agree well with the experimental data below the initial freezing point (271 K). Above this temperature, the experimental results were apparently in error as they produced specific heats greater than that of water (4.2 J/(g . K)).

To facilitate the use of the enthalpy data in the model, it was decided to approximate the H(T) function by mathematical expressions which could be easily differentiated. From Figure 10 it can be seen that the theoretical H(T) function is linear above the initial freezing point but has a non-linear dependence on T below it. In Figure 11, ln H(T) is plotted as a function of T. Based on this figure, ln H was represented by a cubic function of T between 271 K and 266 K and by a linear function of T below 266 K. From the results of a least squares curve fit, in the region 266 K to 271 K, it was determined that H(T) could be approximated by:

$$H(T) = 321.98+3.695(T-273.2) \qquad T > 271\ K \qquad (84)$$

$$H(T) = \exp[6.7584+0.6093(T-273.2) + 0.0775(T-273.2)^2+0.00399(T-273.2)^3]$$

$$266\ K \leq T \leq 271\ K \qquad (85)$$

$$H(T) = \exp[5.420+0.0722(T-273.2)] \qquad T < 266\ K \qquad (86)$$

TABLE 18: ENTHALPY - TEMPERATURE DATA

TEMPERATURE	ENTHALPY (J/g)				
(K)	RIEDEL[1]	EXPERIMENTAL			
		RUN 1	RUN 2	RUN 3	RUN 4
233.2	0.0	0.0	0.0	0.0	0.0
243.2	23.4	38.1	30.1	29.3	25.5
245.2	31.0	41.0	36.4	34.7	31.0
247.2	33.9	48.6	43.1	40.6	37.3
249.2	40.2	56.5	50.7	46.5	43.5
251.2	46.9	64.5	58.6	52.7	50.2
253.2	53.6	72.4	67.0	59.0	56.9
255.2	61.5	80.4	76.6	66.1	63.6
257.2	70.3	88.7	87.1	73.3	69.5
259.2	80.0	98.0	98.0	81.6	76.2
261.2	91.7	109.7	109.3	90.8	85.8
263.2	106.3	126.0	122.6	103.4	104.2
265.2	125.6	148.6	140.2	118.0	121.4
267.2	154.0	175.8	162.8	135.2	136.8
269.2	220.2	235.7	212.6	172.9	181.7
271.2	316.0	329.4	303.1	272.9	285.1
273.2	323.2	357.5	360.8	344.1	316.0
277.2	337.8	385.5	367.1	354.1	351.6
283.2	359.6	432.4	416.1	406.0	391.4

TEMPERATURE	RIEDEL	RUN 1	RUN 2	RUN 3	RUN 4
287.2	374.2	453.3	435.8	426.1	412.3
293.2	396.0	481.0	460.9	448.7	439.9

[1] Calculated using Riedel's method (174).

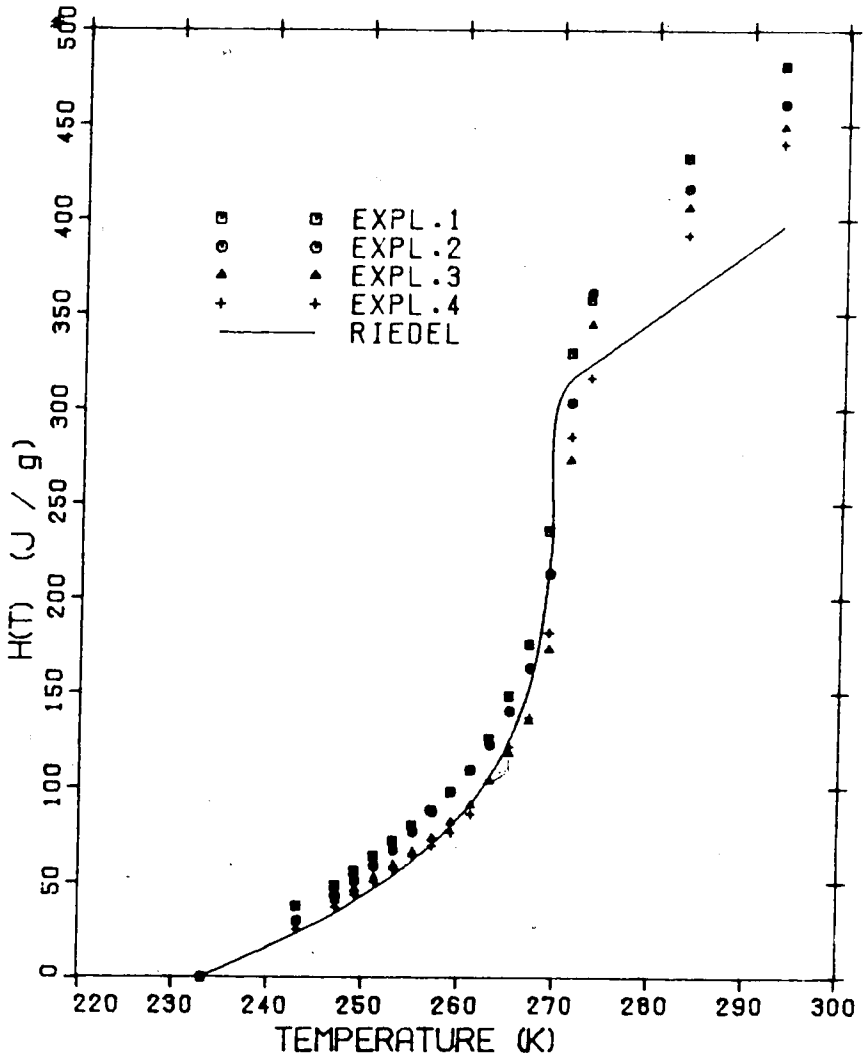

FIGURE 10: ENTHALPY VS. TEMPERATURE

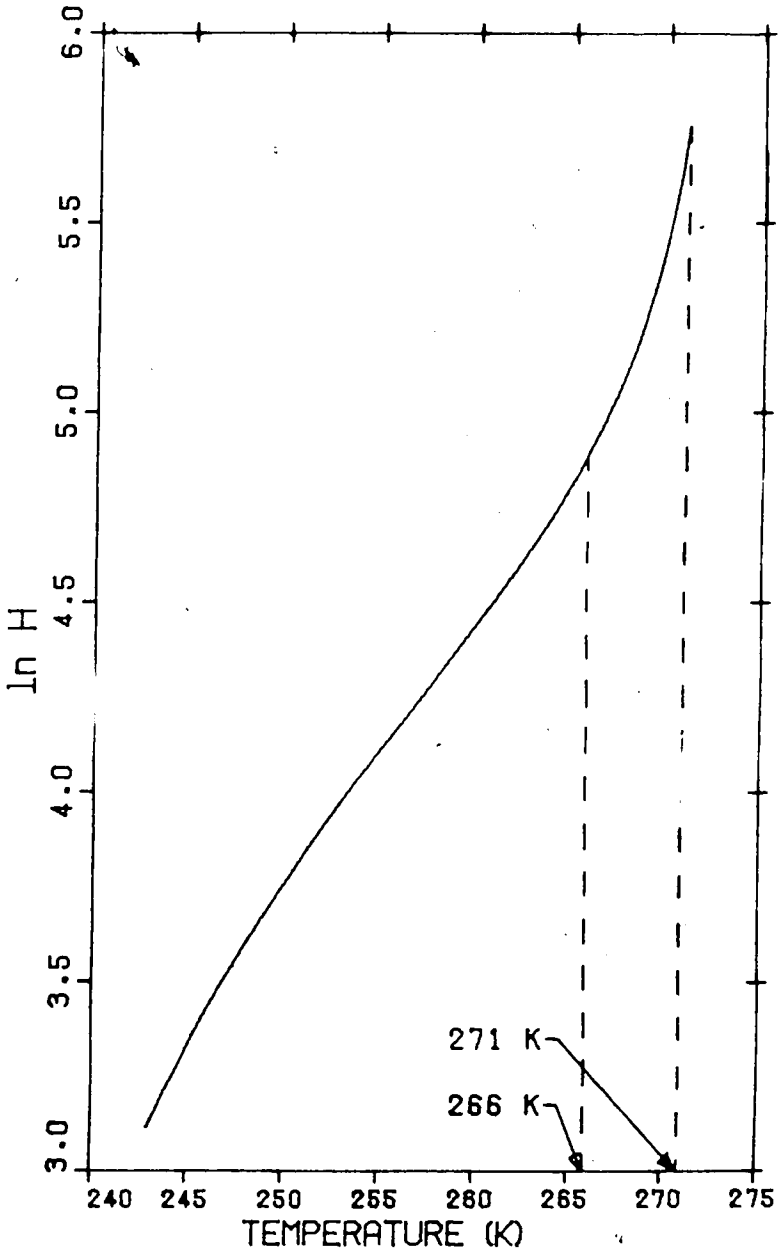

FIGURE 11: Ln H VS. TEMPERATURE

From Figure 12 it can be seen that the above equations adequately represent the experimental data below the initial freezing point.

11.3 Determination of Initial Freezing Point

At the initial freezing point the fraction of the beet juice which has become ice is zero i.e.:

$$w_i = 0 \tag{87}$$

Therefore from Equation (69):

$$w_s = w_o \tag{88}$$

and Equation (67) becomes:

$$T_{ci} = 273.2 - 10w_o - 50w_o^2 \tag{89}$$

where:

T_{ci} = initial freezing point of beet juice (K).

One g of beet is composed of 0.78 g water, 0.18 g soluble solids and 0.04 g insoluble solids. The beet juice consists of water and soluble solids.

Therefore,

$$w_o = 0.18/(0.78 + 0.18)$$
$$= 0.1875 \tag{90}$$

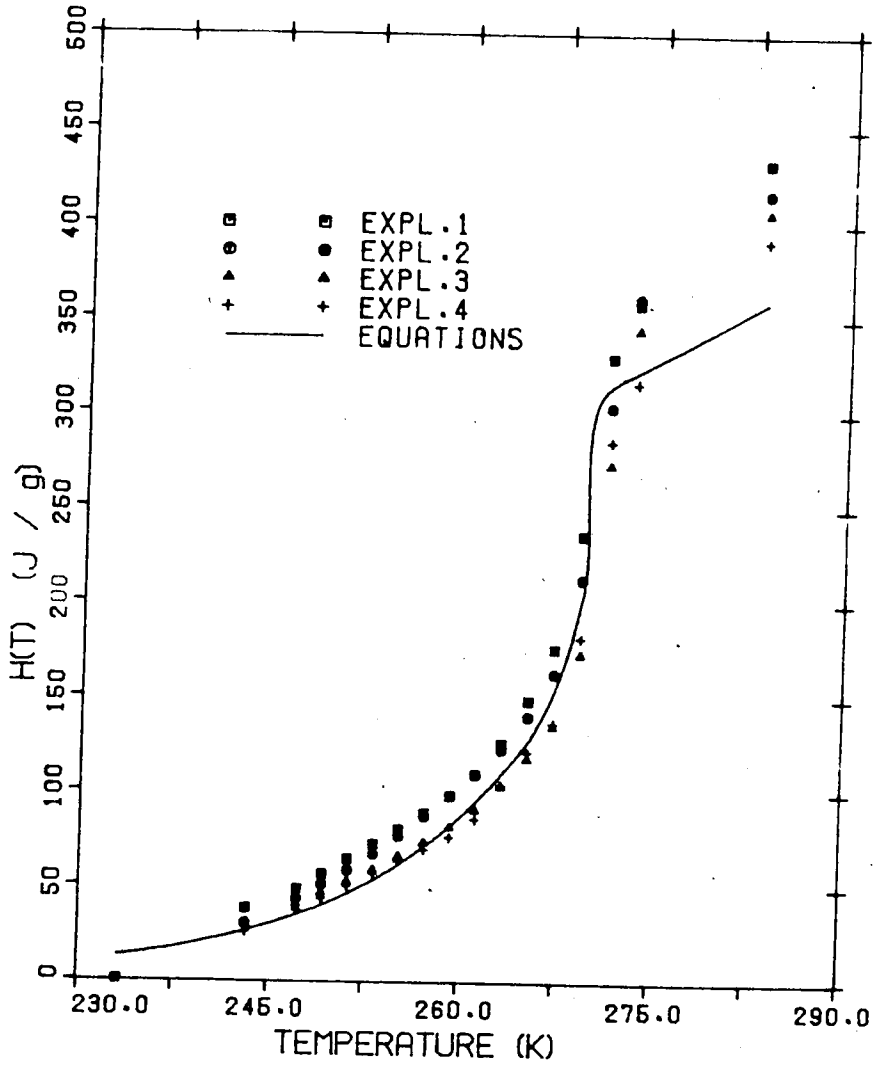

FIGURE 12: PLOT OF H(T) FROM EQUATIONS

Thus,

$$T_{c_1} = 273.2 - 10 \times 0.1875 - 50 \times 0.1875^2$$
$$= 271 \text{ K} \qquad (91)$$

The initial freezing point was also measured experimentally using a Differential Thermal Analyzer and was determined to be 271 K. From the recorded T(t) curves during air thawing of beet cylinders it was estimated to be between 270.5 K and 271 K.

11.4 Heat Transfer Coefficient at the Surface

The determination of the heat transfer coefficient involved subjecting a copper cylinder to the same experimental conditions as those experienced by beet cylinders during freezing and thawing. Analytical solutions for a single phase material were then used to calculate the heat transfer coefficient from the T(t) data which was recorded for the copper cylinder.

11.4.1 Experimental Procedures

The ends of a copper cylinder (15 cm long, 1 cm diameter) were insulated with 1 cm long, 1 cm diameter cork cylinders. A sheathed copper-constantan thermocouple (THERMOELECTRIC model no. T125G304-0-12-12-5H, wire diameter = 0.13 cm, sheath diameter = 0.102 cm) was placed at the geometric centre of the cylinder. For thermal insulation,

the part of the thermocouple sheath, which was outside the sample, was covered with 0.64 cm o.d. POLYFLOW tubing. The thermocouple lead wire was insulated with asbestos tape. The voltage produced by the thermocouple was recorded by a Hewlett Packard recorder (model 7100B), maintained at a full scale range of 5 mV. Published charts showing mV versus temperature for a copper-constantan thermocouple were used to prepare a table, showing temperature as a function of recorder reading. This table was then used to convert the recorded mV readings to temperatures. Measured temperatures were observed to be accurate to ± 0.5 K.

Imitating the situation for the beet cylinders (discussed in the next chapter) the middle of the copper cylinder was covered with aluminum foil. The ends were covered with Scotch plastic tape. A schematic diagram of the experimental set-up is shown in Figure 13.

Air Warming

The copper cylinder, at known initial temperatures of 243 K, 253 K, or 263 K, was placed on the sample holder (see Figure 14) and heated by ambient air. The temperature at the cylinder centre was measured as a function of time.

Cooling in Alcohol Bath

The copper cylinder, at a temperature of approximately 298 K, was placed in the LT-50 alcohol bath which was maintained at either 243 K, 253 K, or 263 K. The variation

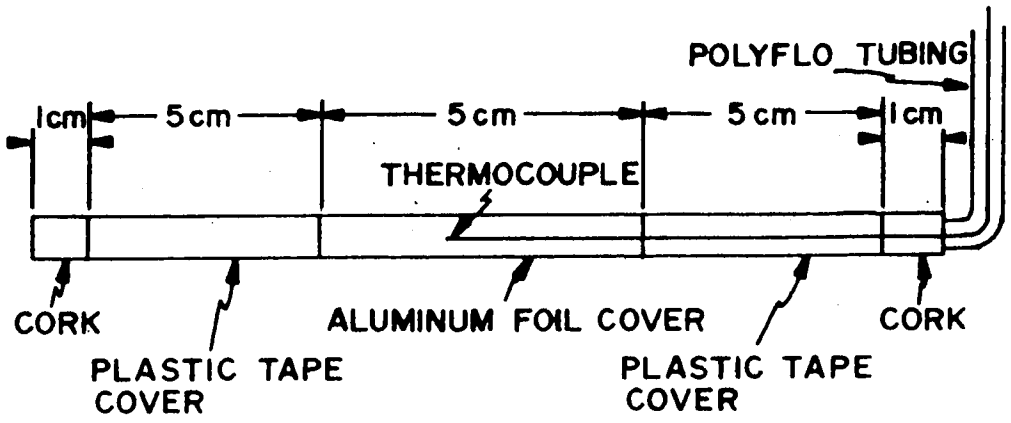

FIGURE 13: COPPER CYLINDER FOR DETERMINING h

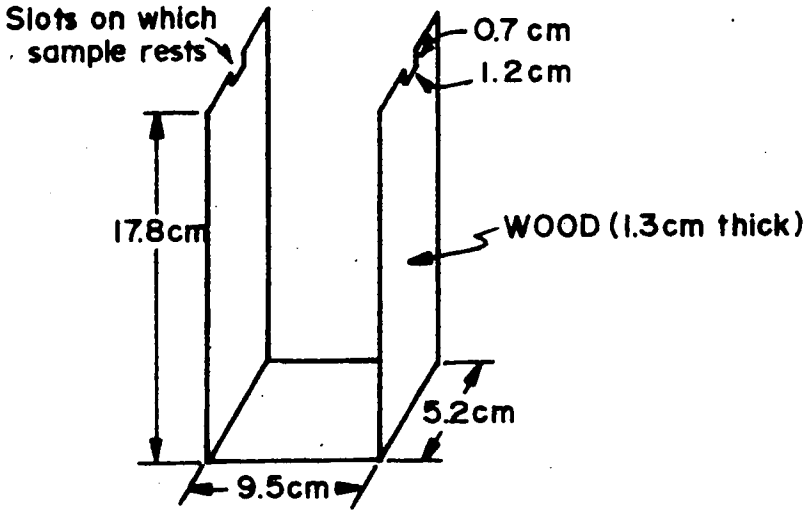

FIGURE 14: SAMPLE HOLDER FOR AIR HEATING EXPERIMENTS

of the cylinder centre temperature with time was then recorded.

11.4.2 Experimental Results

The T(t) data obtained for air warming are listed in Tables (E-1) to (E-3). The results for liquid cooling are summarized in Tables (E-4) to (E-6).

11.4.3 Calculation of h

Air Warming

As copper has a very high thermal conductivity, the internal resistance to heat transfer is negligible and a uniform temperature is maintained in the cylinder. For this situation, it can be shown that:

$$h = \frac{m\,c}{A\,t} \ln \frac{T(t-\Delta t)-T_a}{T(t) - T_a} \qquad (92)$$

where:

m = mass of cylinder = 101.4 g,

c = specific heat of copper = 0.383 J/(g . K),

A = surface area = πDL = 0.004668 m^2,

Δt = time difference between two temperature measurements (s),

T(t) = temperature of copper at time t (K),

T_a = temperature of cooling or heating medium (K).

During air warming of the copper cylinder, Biot numbers (hR/k) of the order of 0.0001 were obtained. Consequently, Equation (92) is applicable for air thawing. Using the data in Tables (E-1) to (E-3), the variation of h with time is calculated from Equation (92) and is shown in Tables (E-7) to (E-9). The value of the dimensionless temperature at the surface (u_{N+1}) where:

$$u_{N+1} = \frac{T(t)-T_a}{T_o - T_a} \qquad (93)$$

is calculated at the various times and is also listed in Tables (E-7) to (E-9). The data in Tables (E-7) to (E-9) are plotted in Figures 15 - 17 in the form of h versus u_{N+1}. Least-squares curve fitting of the $h(u_{N+1})$ data produced the following second degree approximations for the heat transfer coefficient:

$T_o = 243$ K

$$h = 0.8953 + 49.3939u_{N+1} - 30.5409(u_{N+1})^2 \qquad (94)$$

$T_o = 253$ K

$$h = 0.0363 + 39.0551u_{N+1} - 19.3128(u_{N+1})^2 \qquad (95)$$

$T_o = 263$ K

$$h = 1.9937 + 26.9908u_{N+1} - 9.1479(u_{N+1})^2 \qquad (96)$$

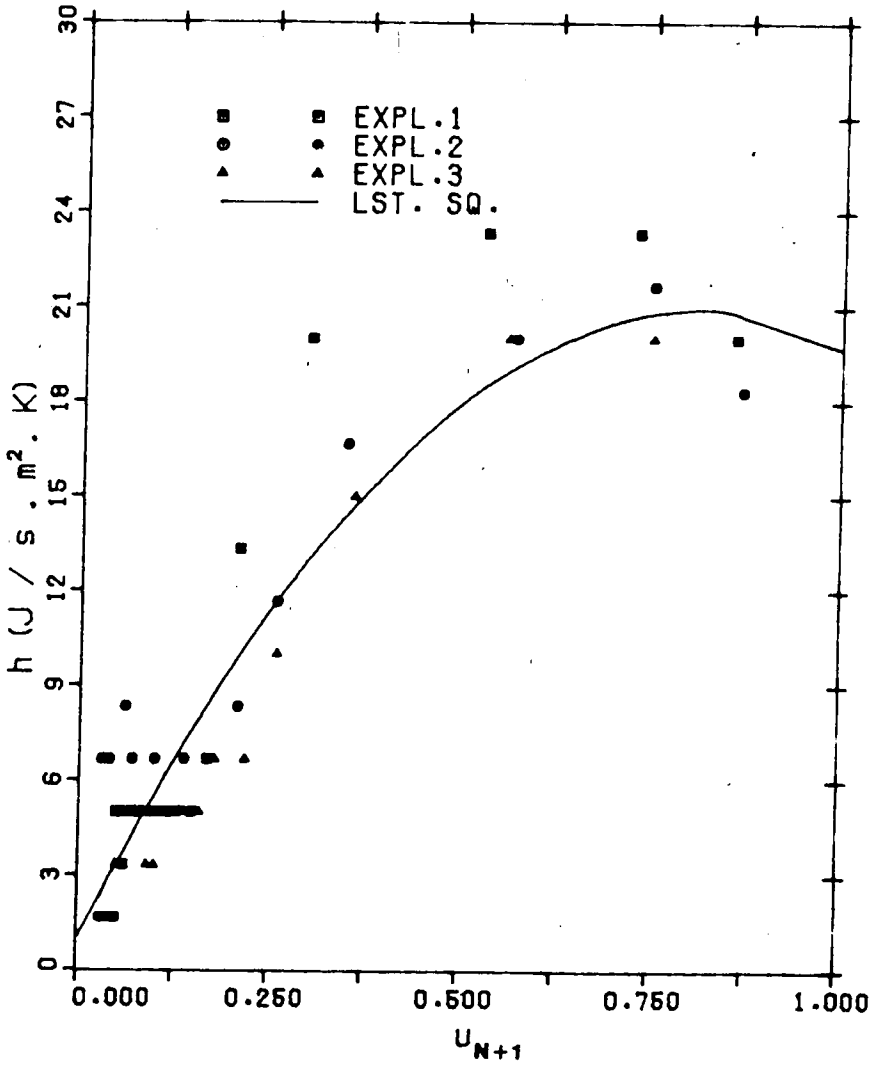

FIGURE 15 : h VERSUS U_{N+1} (T_0 =243 K)

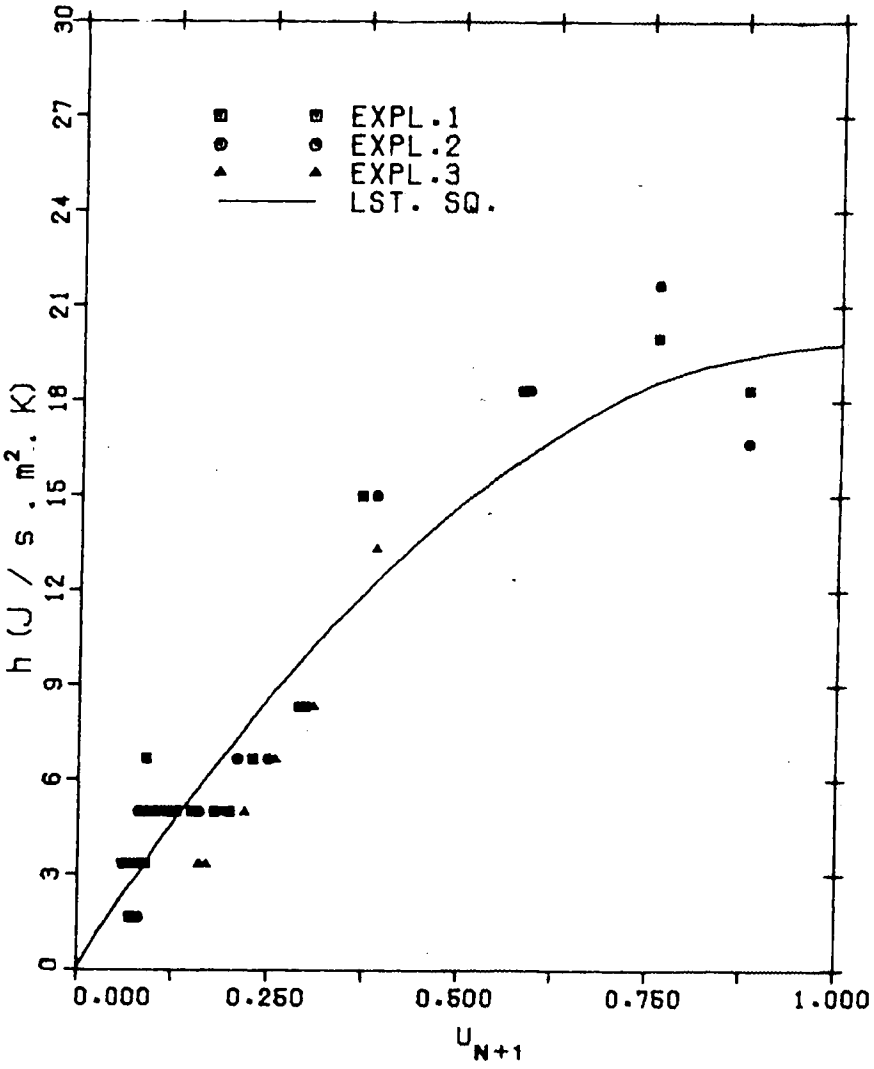

FIGURE 16 : h VERSUS u_{N+1} (T_0 =253 K)

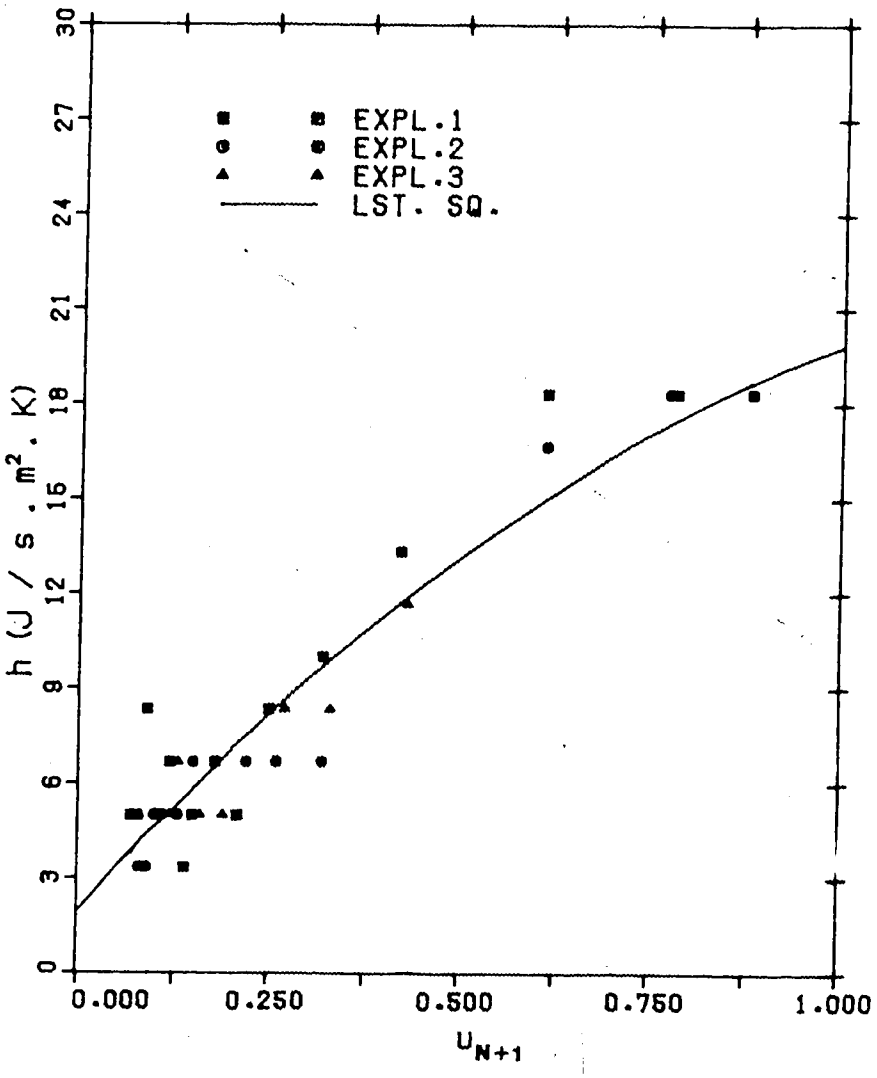

FIGURE 17 : h VERSUS u_{N+1} (T_0 =263 K)

Error Analysis of h Used in Air Thawing

The measured temperature of the copper cylinder is accurate to ± 0.5 K. For $0.25 \leq u_{N,1} \leq 1.0$, this leads to a maximum error of approximately 10% in the heat transfer coefficient, calculated from Equation (92). The maximum percentage error in h increases as $u_{N,1}$ is further reduced. This is because $T(t) - T_a$ and $T(t - \Delta t) - T_a$ in Equation (92) are decreasing with $u_{N,1}$ and are affected to a greater degree by an error in the temperature measurement. When $u_{N,1}$ reaches about 0.1 the maximum error in h will have risen to about 50%. However, for $u_{N,1}$ values less than 0.25 an accurate estimate of h is not very important for air thawing. This is because the sample temperature is above 283 K and has already been thawed.

Cooling in Alcohol Bath

The heat transfer coefficient is now evaluated for cooling in the LT-50 bath. The alcohol in this bath is continually circulated. Consequently, forced convection is the primary method of heat transfer between the cylinder and the alcohol. It is, therefore, assumed that the heat transfer coefficient remains constant during cooling. The h values are calculated from Carslaw and Jaeger's (35) analytical solution for one phase heat transfer in an infinite cylinder. This solution is:

$$\frac{T(r,t) - T_a}{T_o - T_a} = 2P\sum_{n=1}^{\infty} e^{-(\beta_n)^2 \alpha t/R^2} \frac{J_o(r\beta_n/R)}{((\beta_n)^2+P^2)J_o(\beta_n)} \qquad (97)$$

where:

$\pm\beta_n$, n = 1,2,-------- are the roots of

$\beta_n J_1(\beta_n) = PJ_o(\beta_n)$ (98)

$J_o(\beta_n)$ and $J_1(\beta_n)$ are zero and first order Bessel functions, and

$P = Rh/k$ (99)

A trial and error procedure is used to calculate h from Equation (97). Different values of h are substituted into this equation until the calculated and measured values of T(0.5) are equal. The temperature after 0.5 min was selected

because at least 50% of the temperature change inflicted on the copper cylinder occurs during the first 0.5 min. It can thus be considered to yield an "average" h value for the cooling process. For times greater than 0.5 min, the calculated heat transfer coefficient becomes more sensitive to errors in the measured temperature.

For a constant heat transfer coefficient, Equation (97) yields more accurate h values than Equation (92). This is because it allows for a nonuniform temperature pro le inside the copper cylinder. Equation (97) was not used for air thawing because the heat transfer coefficient changed dramatically with u_{N+1}.

The second computer program in Appendix H was used to determine h. The calculated heat transfer coefficients are listed in Table 19. The mean values in this table are used as the heat transfer coefficients for freezing of sugarbeet cylinders in the LT-50 bath. They are summarized in Table 20.

Error Analysis of h used for Alcohol Bath

The h values for cooling were also calculated, as a function of time, using Equation (92). These values are listed in Tables (E-10) to (E-12). From these values, it is apparent that errors in h are of the order of 50% and h generally decreases with u_{N+1}. Equation (92) was used to estimate the error involved in measuring h because the variation of h with time could be easily obtained from this

TABLE 19: CALCULATED h VALUES[1] FOR THE FIRST 0.5 min OF COOLING

T (K)	RUN 1	RUN 2	RUN 3	MEAN	S.D.[2]	S.D./MEAN
243	238	283	328	283	45	0.159
253	285	317	370	324	43	0.133
263	432	377	423	411	30	0.072

[1] The h values (J/s . m² . K) were calculated using Carslaw and Jaeger's analytical solution.

[2] S.D. = standard deviation.

TABLE 20: HEAT TRANSFER COEFFICIENTS FOR FREEZING IN THE LT-50 BATH

BATH TEMPERATURE	h (J/s . m² . K)
243 K	283
253 K	324
263 K	411

equation. This variation gives some indication of the accuracy involved in the h measurements. For cooling of the copper cylinder in the alcohol bath, Biot numbers of the order of 0.001 were obtained. Thus, for constant h and a given T_o, T_a and Δt, Equation (92) predicted a h value within 5% of that calculated from Equation (97).

12. EVALUATION OF FREEZING MODEL

12.1 ANALYTICAL SOLUTION

The numerical model was first tested for a one-dimensional heat transfer problem. The thermal conductivity and specific heat were assumed to be constant. An analytical solution (Equation (97)) is given in Carslaw and Jaeger (35) for this problem. The second computer program in Appendix H was used to calculate the T(t) profile from the analytical solution. As shown in Table 21, the model results are within 0.1 K of the analytical solution.

12.2 KARLSRUHE TEST SUBSTANCE

The numerical model was next tested against experimental data, obtained by Cleland and Earle (44), for cylinders of Karlsruhe Test Substance. This material is a methylcellulose gel which has been used (22,41-44) for testing food freezing models. Cleland and Earle considered the "freezing time" to be the time required for the centre of a food to reach 263 K. They (44) published experimental data for the freezing time of a 5.2 cm diameter cylinder of Karlsruhe Test Substance under various freezing conditions. This data consisted of freezing times which corresponded to specified h, T_o, and T_a values. The h values were reported to be correct to within ± 4%.

TABLE 21: RESULTS FOR ONE-PHASE HEAT TRANSFER

TIME	TEMPERATURE (K)					
	SURFACE		r/R = 0.5		CENTRE	
(min)	ANAL.	MODEL	ANAL.	MODEL	ANAL.	MODEL
0.375	274.1	274.1	287.7	287.6	291.5	291.5
0.75	265.8	265.8	279.6	279.5	284.4	284.4
1.125	261.3	261.3	272.7	272.6	276.9	276.9
1.5	257.8	257.8	267.0	267.0	270.5	270.5
1.875	255.0	255.0	262.5	262.4	265.3	265.3
2.25	252.8	252.7	258.8	258.7	261.1	261.0
2.625	250.9	250.9	255.8	255.8	257.6	257.6

CONDITIONS

$h = 293$ J/(s . m^2 . K)

$c_s = 8.37$ J/(g . K)

$k = 1.0$ J/(s . m . K)

$T_a = 243.2$ K

$T_o = 293.2$ K

$R = 0.005$ m

$\rho = 1.05 \times 10^6$ g/m^3

$\Delta t = 0.225$ s

$N = 20$

In this thesis published H(T) and k(T) data, for Karlsruhe Test Substance, were put in a form suitable for use in the numerical model. More details are given in Appendix F. Cleland and Earle's , T_∞, and $_a$ values were then entered and a freezing time calculated. The model and experimental results are both listed in Table 22. It can be seen that the model freezing times deviated from the average experimental value by between -2 and +11.6%. This is essentially the same deviation as reported by Cleland and Earle (44) for the best finite difference scheme which they tested.

The model predicts freezing times which are generally around the upper limit of experimental values. It thus predicts freezing times which err on the safe side i.e. it ensures nearly all the samples are frozen to 263 K, rather than just the average sample. However, in one case (T_∞=298.5 K, T_a=243 K, h=23.9 J/(s . m² . K)) the model actually predicted a freezing time 2.5 min shorter than the average experimental value. This indicates that there is not a fundamental error in the model which would always make it predict freezing times longer than those which actually occur.

TABLE 22: FREEZING TIMES FOR KARLSRUHE TEST SUBSTANCE

RUN CONDITIONS	FREEZING TIME (min)[1]		
	EXPERIMENTAL[2]	MODEL	% DIFF[3]
T_o=301.2 K ; T_a=253.3 K h=35.5 J/(s . m² . K)	12. o±10.	140.9	8.7
T_o=292.9 K ; T_a=253.2 K h=35.5	118.8±9.6	132.6	11.6
T_o=290.4 K ; T_a=240.0 K h=35.5	70.8±4.8	77.0	8.8
T_o=298.5 K ; T_a=243.0 K h=23.9	126.0±11.4	123.5	-2.0
T_o=288.6 K ; T_a=233.0 K h=23.9	80.4±7.8	85.2	6.0
T_o=277.5 K ; T_a=239.7 K h=35.5	63.6±4.2	65.6	3.1
T_o=277.2 K ; T_a=239.5 K h=35.5	62.4±4.2	65.0	4.2

[1] FREEZING TIME = time for centre of sample to reach 263 K

[2] Cleland and Earle's experimental results

[3] $$\% \text{ DIFF} = \frac{\text{MODEL RESULT-MEAN EXPERIMENTAL VALUE}}{\text{MEAN EXPERIMENTAL VALUE}} \times 100\%$$

12.3 FREEZING AND THAWING OF SUGARBEETS

The model was also tested against experimental results obtained by the author for 1 cm diameter, 15 cm long sugarbeet cylinders.

12.3.1 Experimental Procedures Used to Test Model

The temperature at the geometric centre of a sugarbeet cylinder (1 cm diameter, 15 cm long), insulated at both ends by cork (1 cm diameter, 1 cm long), was measured as a function of time during freezing and thawing. The 15 cm long cylinder was obtained by glueing together three 5 cm long cylinders. This was necessary because of the small size of the beets. The middle cylinder was covered with two layers of aluminum foil to prevent dehydration. For ease of handling, the two outside cylinders and cork insulation were covered with plastic tape instead of aluminum foil.

A sheated copper-constantan thermocouple (wire diameter 0.013 cm, sheath diameter 0.102 cm) was used to measure the temperature at the geometric centre (see Figure 18). Heat transfer from the thermocouple wire to the surrounding fluid was minimized by covering the sheath with 0.635 cm (1/4 inch) polyflo tubing. The thermocouple lead wire was insulated with asbestos tape.

In order to test the model results, the sugarbeet cylinder was frozen in the LT-50 bath, which was maintained at a constant temperature of either 243 K, 253 K, or 263 K,

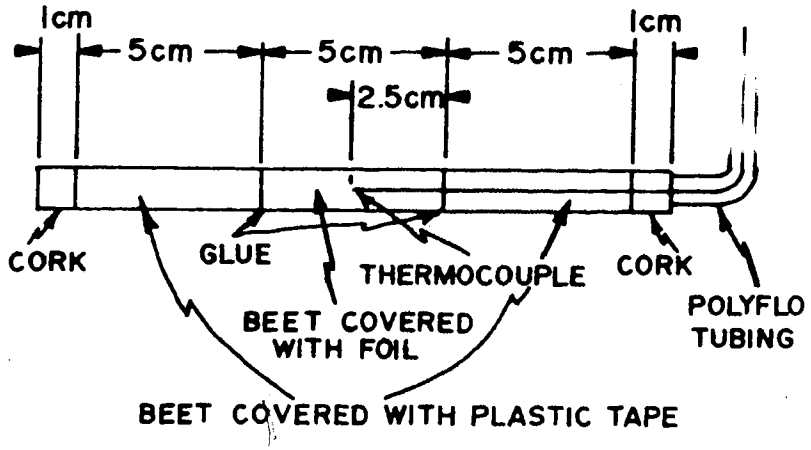

FIGURE 18: EXPERIMENTAL ARRANGEMENT FOR TESTING MODEL

and the centre temperature recorded as a function of time. The sugarbeet cylinder was later thawed in still air, by natural convection. For still air thawing, the cylinder was placed on a wooden holder (Figure 14). This holder was located beside the LT-50 bath so the frozen sample could be quickly placed on the holder. The temperature of the ambient air was measured with a thermometer, accurate to ± 0.5 K. During both freezing and thawing the cylinder was in a horizontal position.

The experimental results and model predictions are listed, in Appendix G, for various freezing and thawing conditions.

The freezing times (time for centre of sample to reach 263 K) in the 243 K and 253 K alcohol baths are summarized in Table 23. For the 243 K bath the average of the three experimental freezing times was 2.2 min, whereas the model predicted a freezing time of 2.5 min. In contrast, the predicted freezing time (3.5 min) in the 253 K bath was less than the average experimental value (4.1 min).

TABLE 23: FREEZING TIMES IN 243 K AND 253 K BATHS

T_a = 243 K

	FREEZING TIME (min)[1]
RUN 1	2.1
RUN 2	2.1
RUN 3	2.3
MODEL (h=283)[2]	2.5

T_a = 253 K

	FREEZING TIME (min)
RUN 1	4.1
RUN 2	4.0
RUN 3	4.0
RUN 4	4.2
MODEL (h=317)	3.5

[1] FREEZING TIME = time for centre of sample to reach 263 K.

[2] h is in units of J/(s . m^2 . K)

Another indication of how close the predicted temperatures are to experimental values is given by the "standard deviation from the predicted curve". It is defined by:

$$\text{standard deviation} = \sqrt{\frac{\sum[\bar{T}_{exp}(t) - T_{pre}(t)]^2}{(n-1)}} \qquad (98)$$

where:

n = number of time increments at which the temperatures are compared,

$\bar{T}_{exp}(t)$ = average experimental value at time t, and

$T_{pre}(t)$ = predicted value at time t.

The denominator is n - 1 because one degree of freedom is lost as a result of setting $T_{pre}(0)$ equal to $\bar{T}_{exp}(0)$. The standard deviations were calculated for the different freezing and thawing conditions from the data in Tables G-1 to G-6. They are summarized in Table 24 along with the maximum differences between experimental and predicted temperatures.

TABLE 24: COMPARISON OF EXPERIMENTAL AND PREDICTED RESULTS

FREEZING CONDITIONS	MAXIMUM[1] DIFFERENCE (K)	STANDARD[2] DEVIATION (K)
$T = 243$ K	9.2	4.1
$T = 253$ K	4.8	2.2
$T = 263$ K	3.2	1.9

THAWING CONDITIONS	MAXIMUM DIFFERENCE (K)	STANDARD DEVIATION (K)
$T_o = 243$ K	6.3	2.4
$T_o = 253$ K	4.2	1.3
$T_o = 263$ K	4.9	1.6

[1] MAXIMUM DIFFERENCE is the maximum value of $|\bar{T}_{exp}(t) - T_{pre}(t)|$.

[2] STANDARD DEVIATION $= \sqrt{\dfrac{\sum[\bar{T}_{exp}(t) - T_{pre}(t)]^2}{(n-1)}}$

The tables in Appendix G are summarized in Figures 19 - 26. For freezing in the 243 K bath, Figure 19 shows that the model predicts the correct general trend but, for $h = 283\ J/(s \cdot m^2 \cdot K)$, lags the experimental data by about 0.5 min, after the initial freezing point has been reached. It was thought that the difference between predicted and experimental curves might result from an error in the determination of h. Consequently, an h value of 417 was also used. From the dashed line in Figure 19 it is apparent that increasing the value of h actually caused the predicted results to precede the experimental values by about 0.5 min. Thus, an incorrect h value could be responsible for the differences between predicted and experimental results.

The results for the 253 K bath are somewhat improved. After the initial freezing point is reached, the theoretical curve, in Figure 20, initially lags the experimental curve but the situation is later reversed. This may be due to the fact that the actual h value is decreasing whereas a constant h is used in the model.

The experimental data in Figure 21 appears to suggest the existence of two plateaus, a short one at 270.5 K and a longer one at 269 K. Pereira (167) discussed this phenomenon in some detail. Numerous cells are damaged when a thermocouple is placed inside a solid food. Liquid then exudes from these cells and surrounds the thermocouple junction. According to Maximov's "wound sap theory", freezing of this liquid is responsible for the first

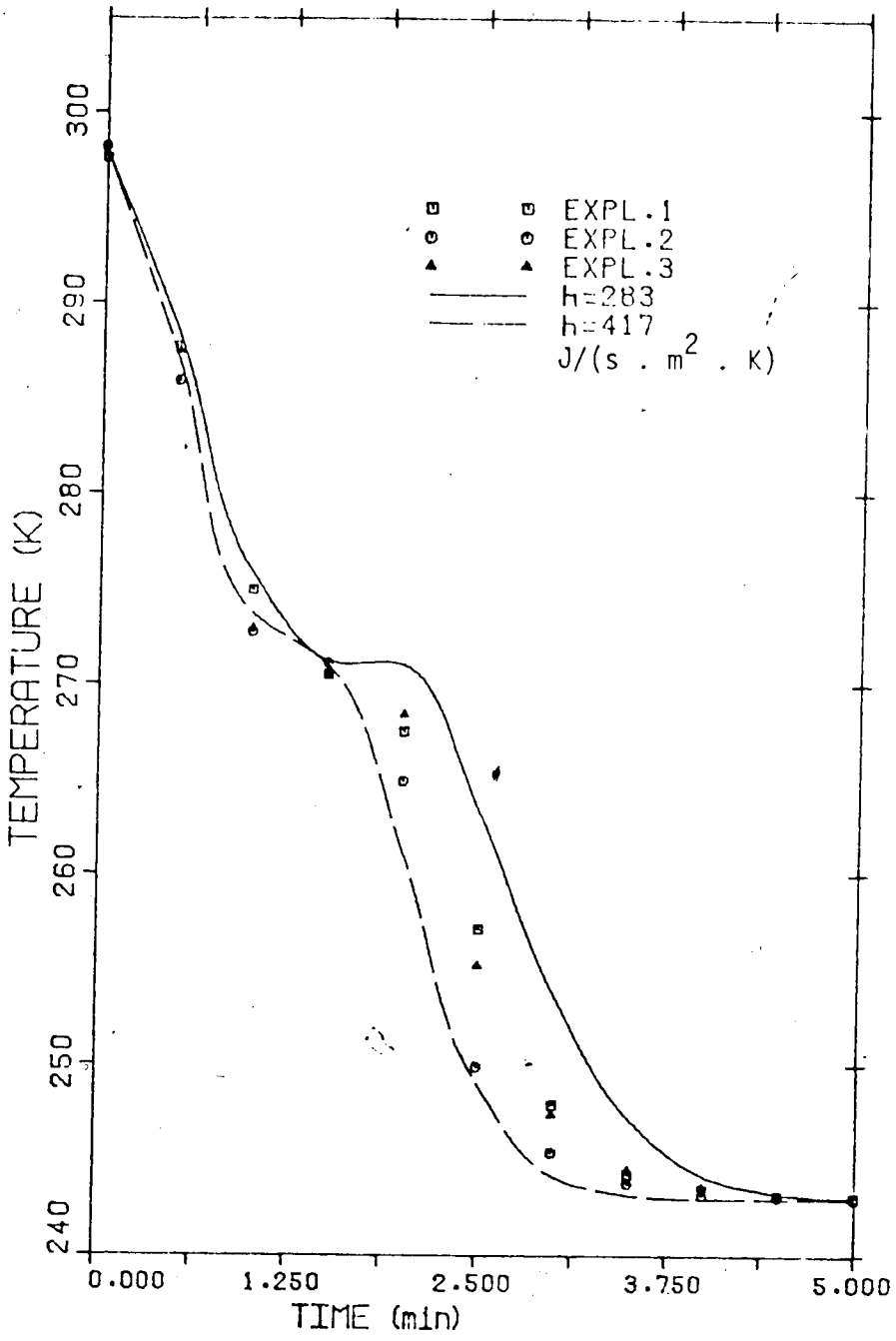

FIGURE 19: FREEZING IN 243 K BATH

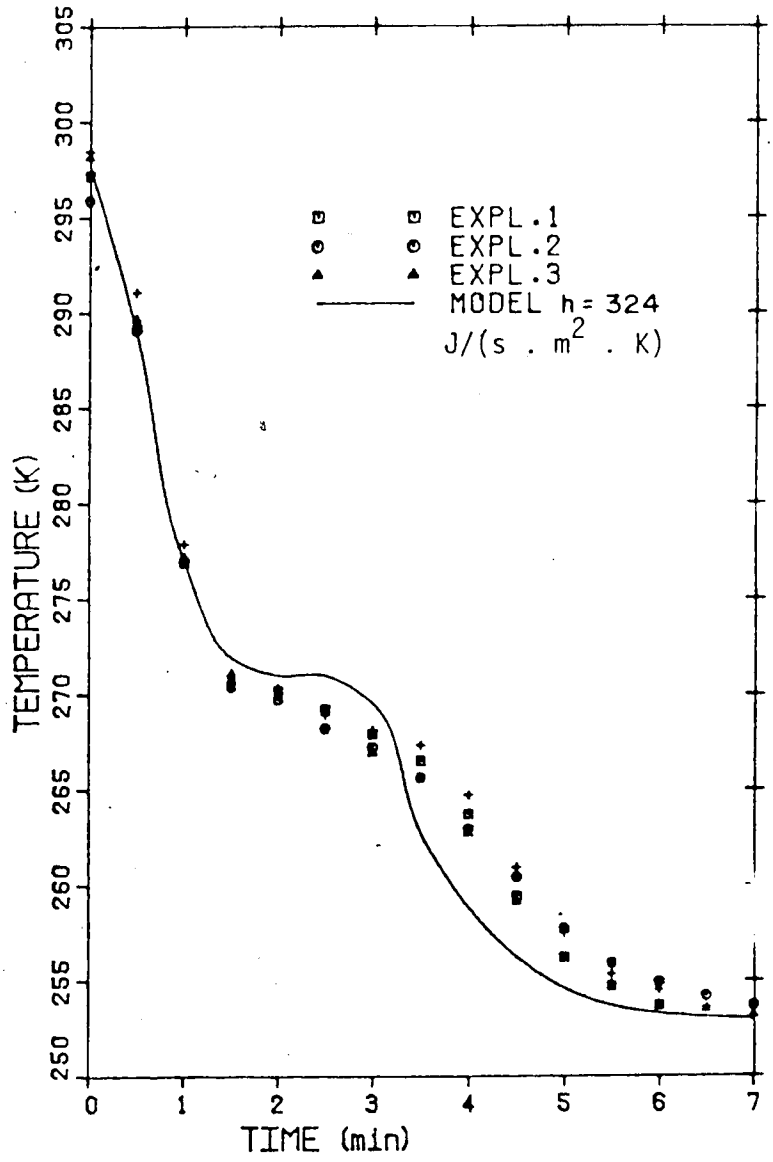

FIGURE 20: FREEZING IN 253 K BATH

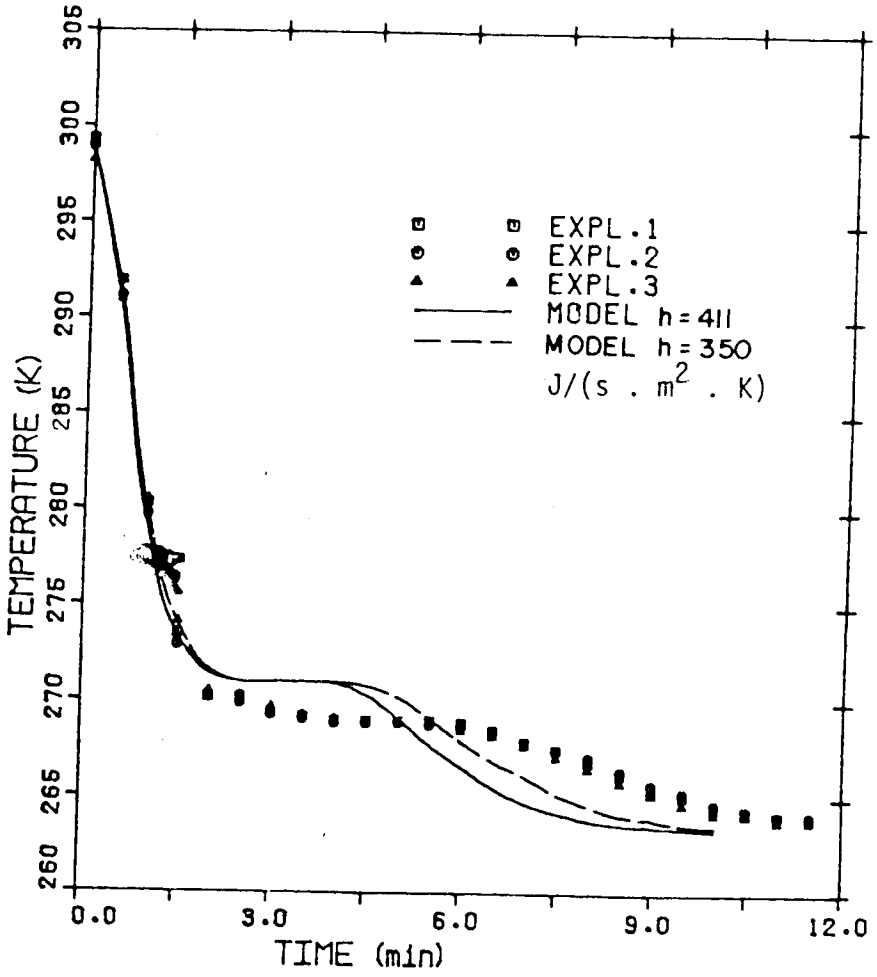

FIGURE 21: FREEZING IN 263 K BATH

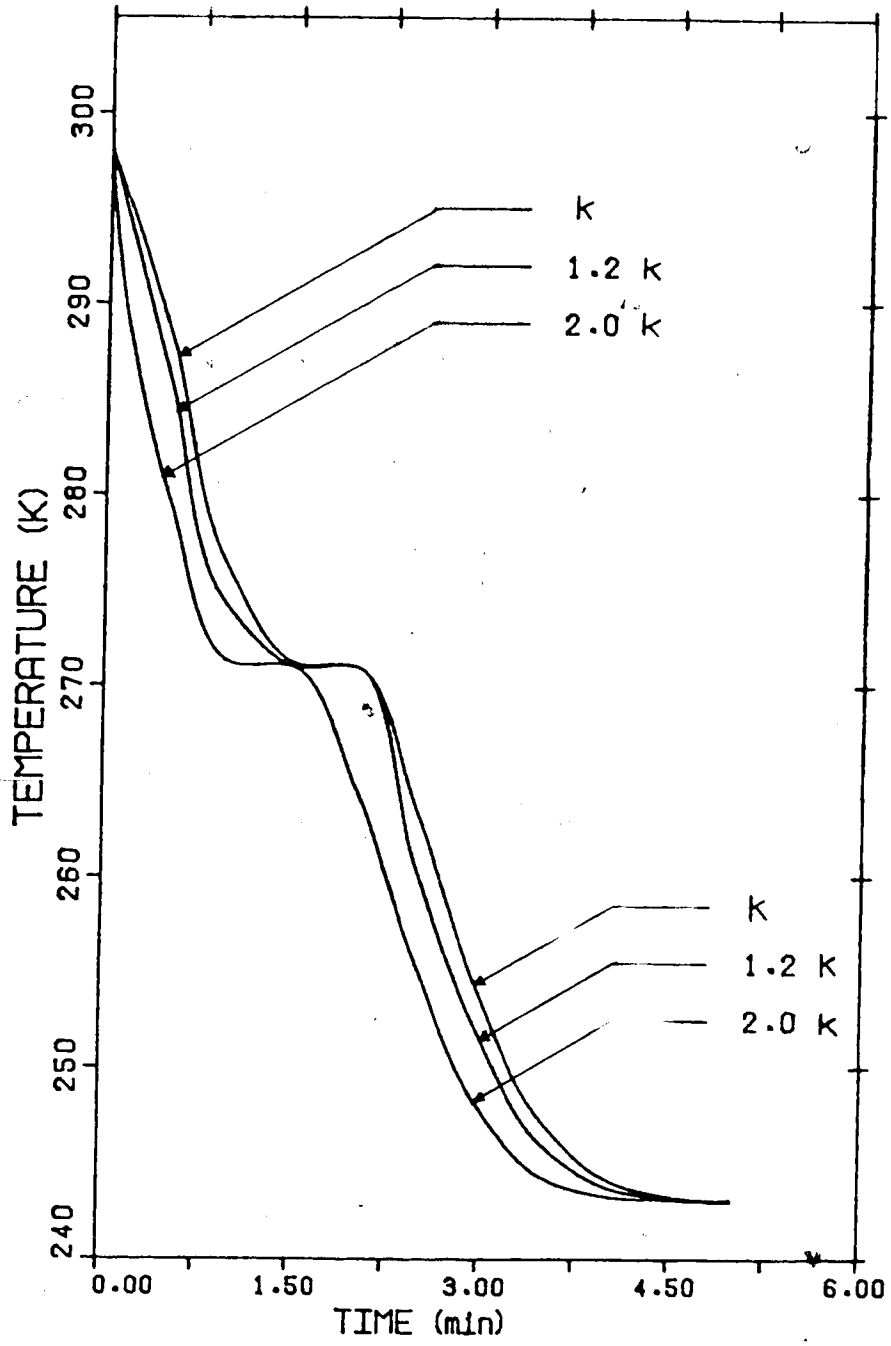

FIGURE 22: EFFECT OF CHANGING k

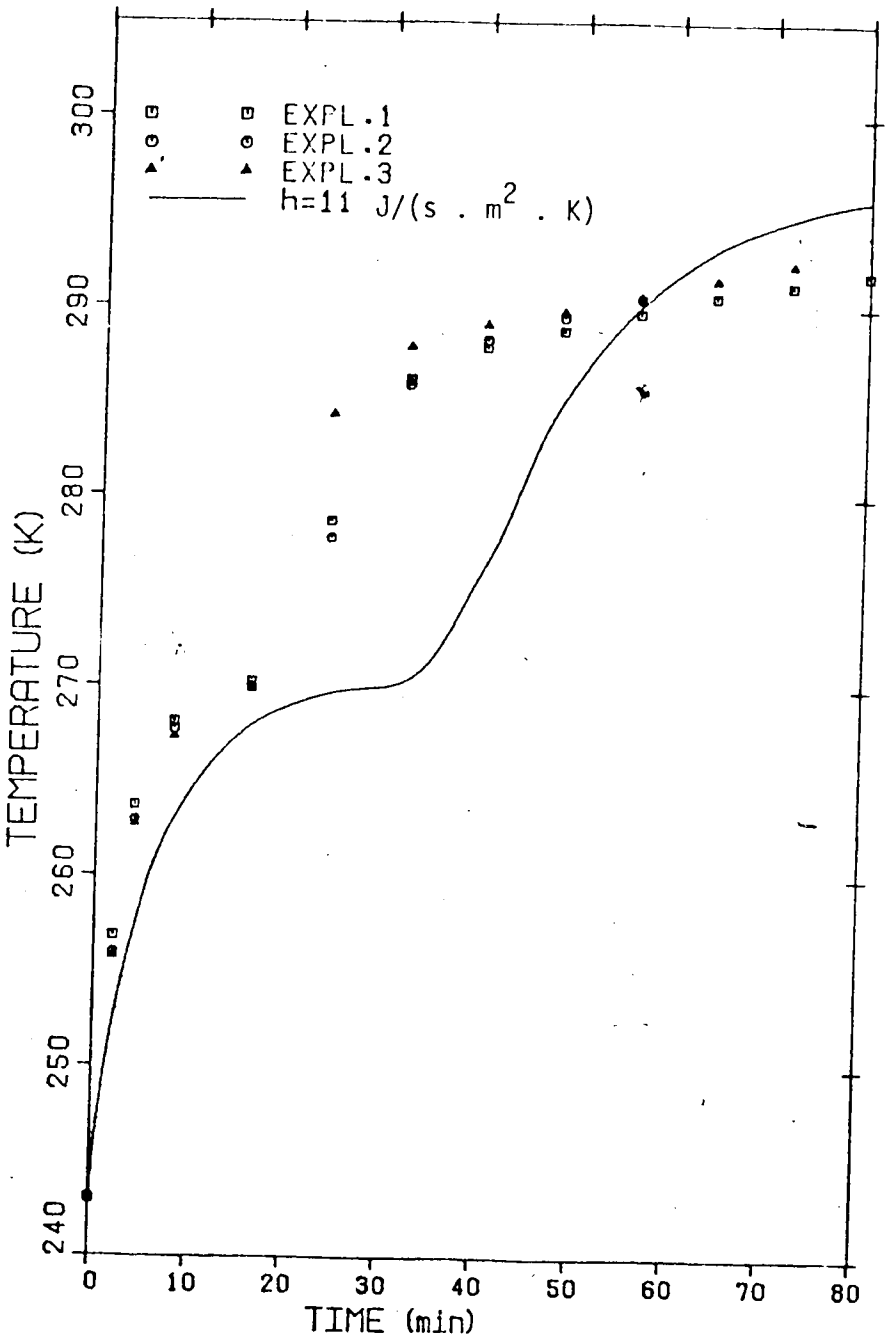

FIGURE 23: AIR THAWING - EFFECT OF CONSTANT h

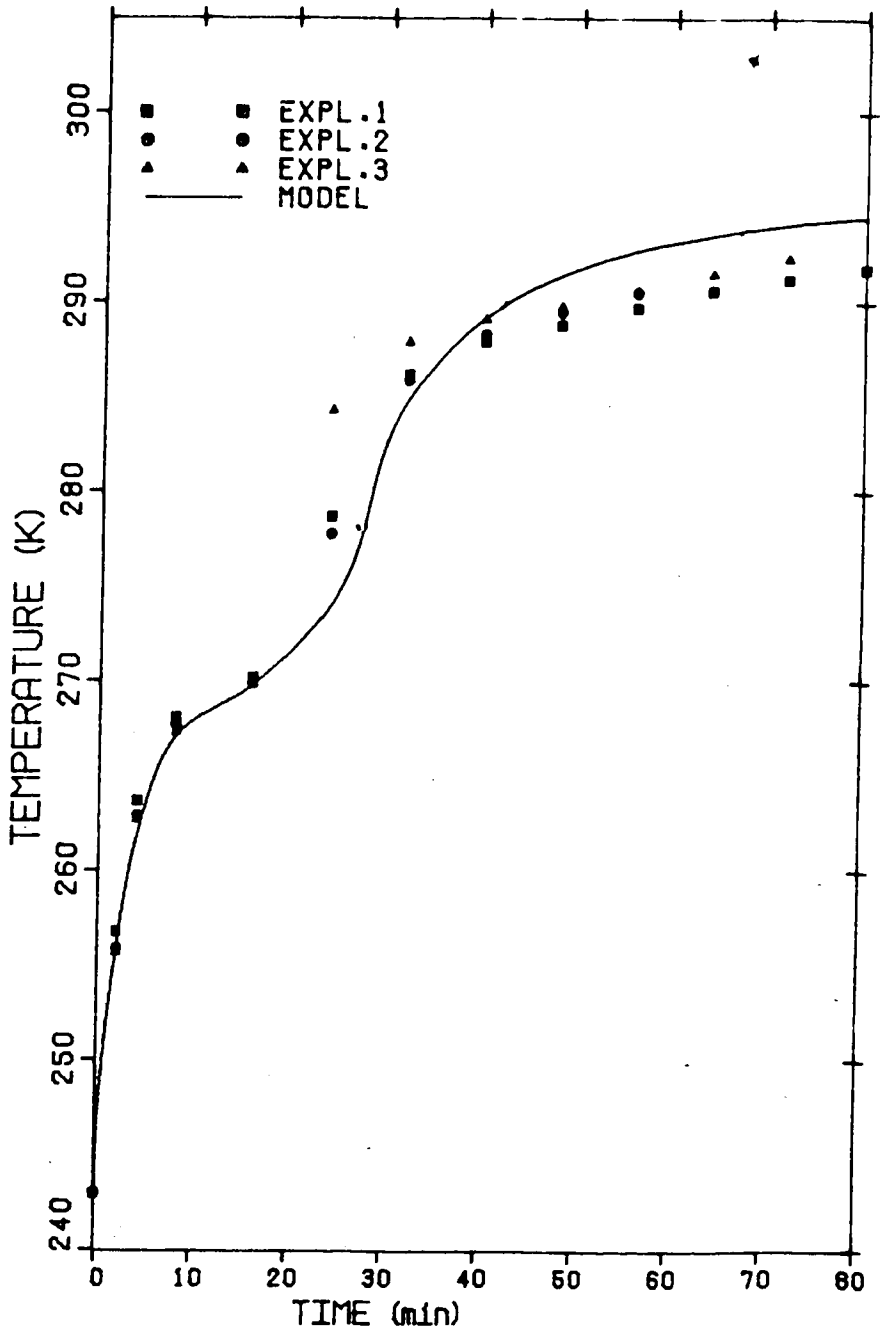

FIGURE 24: AIR THAWING OF 243 K BEET

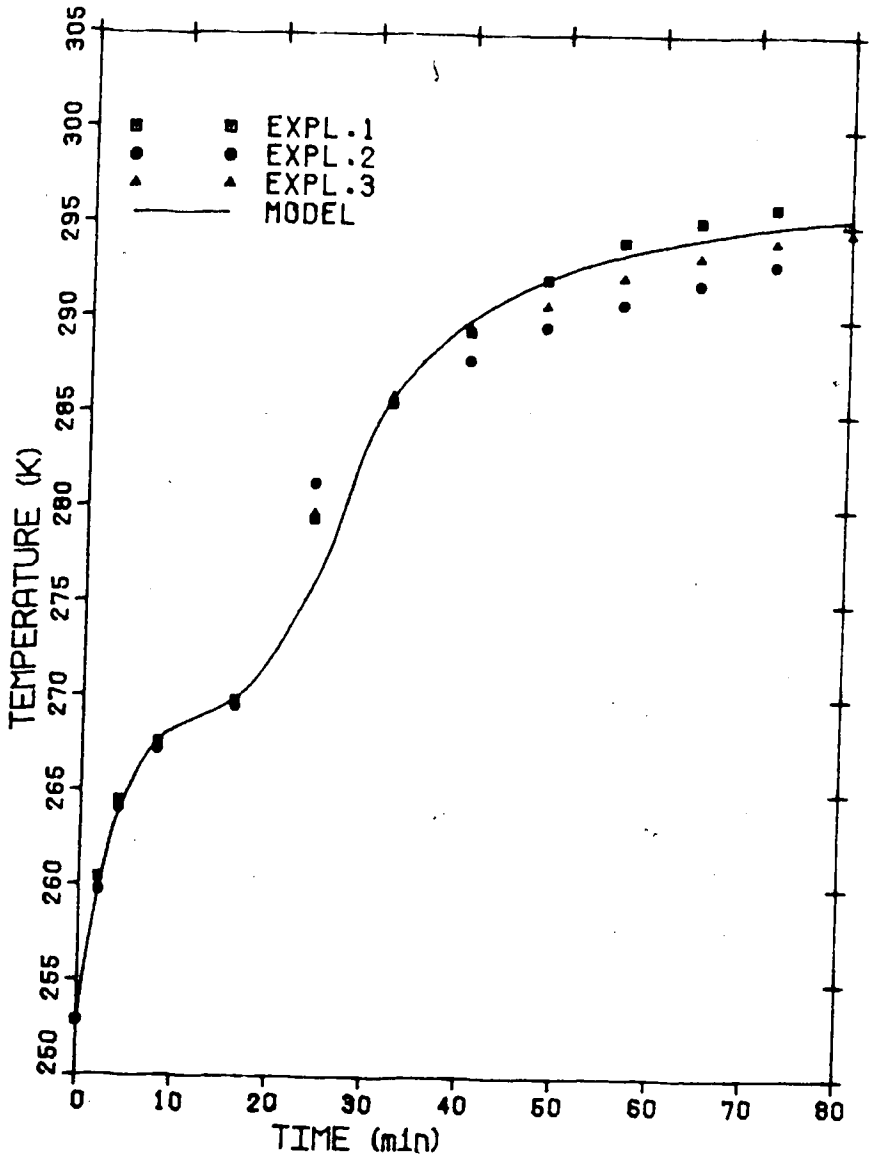

FIGURE 25: AIR THAWING OF 253 K BEET

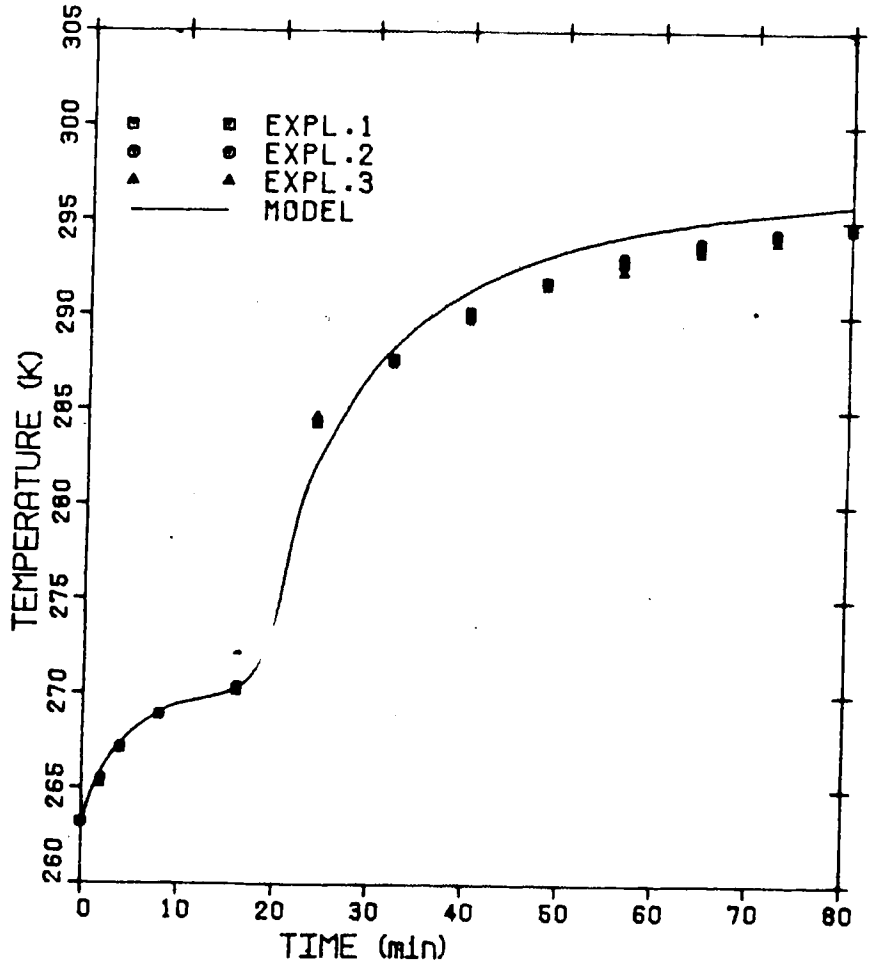

FIGURE 26: AIR THAWING OF 263 K BEET

plateau. Pereira supported Maximov's theory. His experimental results also indicated that the second plateau is caused by freezing within individual cells close to the thermocouple. The fact that the second plateau is about 2 K below the initial freezing point may be a result of supercooling.

To estimate how sensitive the model is to errors in k(T), it was decided to increase all the k(T) values by 20 or 100% and then examine the changes in the model results. From Figure 22 it can be seen that a 20% increase in k(T) values reduces the time to reach a given temperature by less than 0.2 min. Figure 22 shows that even a 100% increase in k(T) does not prevent a plateau occurring in the model T(t) profile. The existence of this plateau, in the model predictions, may result from the use of an incorrect heat transfer coefficient. Alternatively, the lack of a plateau in experimental results may be caused by supercooling.

The model is based on equilibrium enthalpy values and thus does not allow for supercooling. Initially, an attempt was made to consider supercooling but then it was realized that different parts of the cylinder would have different degrees of supercooling. To predict the amount of supercooling throughout the cylinder it would be necessary to allow for water diffusion. This would be a possible future extension of the model. The presence of supercooling might explain why the model results lag behind the experimental values just below the initial freezing point.

As heat of fusion need not be removed, supercooled foods will initially cool faster than predicted.

Originally, in testing the model for still air thawing, a constant h value of 11 J/(s . m² . K) was used. This estimate of h was suggested by Charm (37) for slow moving air. From Figure 23, it is apparent that use of this h value resulted in poor agreement being obtained between model predictions and experimental results. The experimental h values in Equations (33) - (35) were then employed. As can be seen from Figures 24 - 26, this led to excellent agreement between experimental and predicted temperature profiles.

The experimental $h(u_{N+1})$ data, for air thawing, was also approximated by cubic and discontinuous functions. Use of these functions resulted in T(t) curves which were as close to experimental values as the model results in Figures 24 - 26. It is, therefore, concluded that many different types of approximating functions can be used to represent $h(u_{N+1})$ and produce accurate T(t) profiles.

Model results (for T_o = 243 K, h = 283 J/(s . m² . K)) are used in Figure 27 to show how the T(t) profile changes with position, during freezing. This figure indicates that a very fast freezing rate is obtained at the centre relative to the outer regions. As $T_{N+1} - T_a$ at the surface is small when the centre is being frozen one might expect the latter to freeze slowly. However, the model predictions agree with the experimental results of Meryman (142) and Luyet (129).

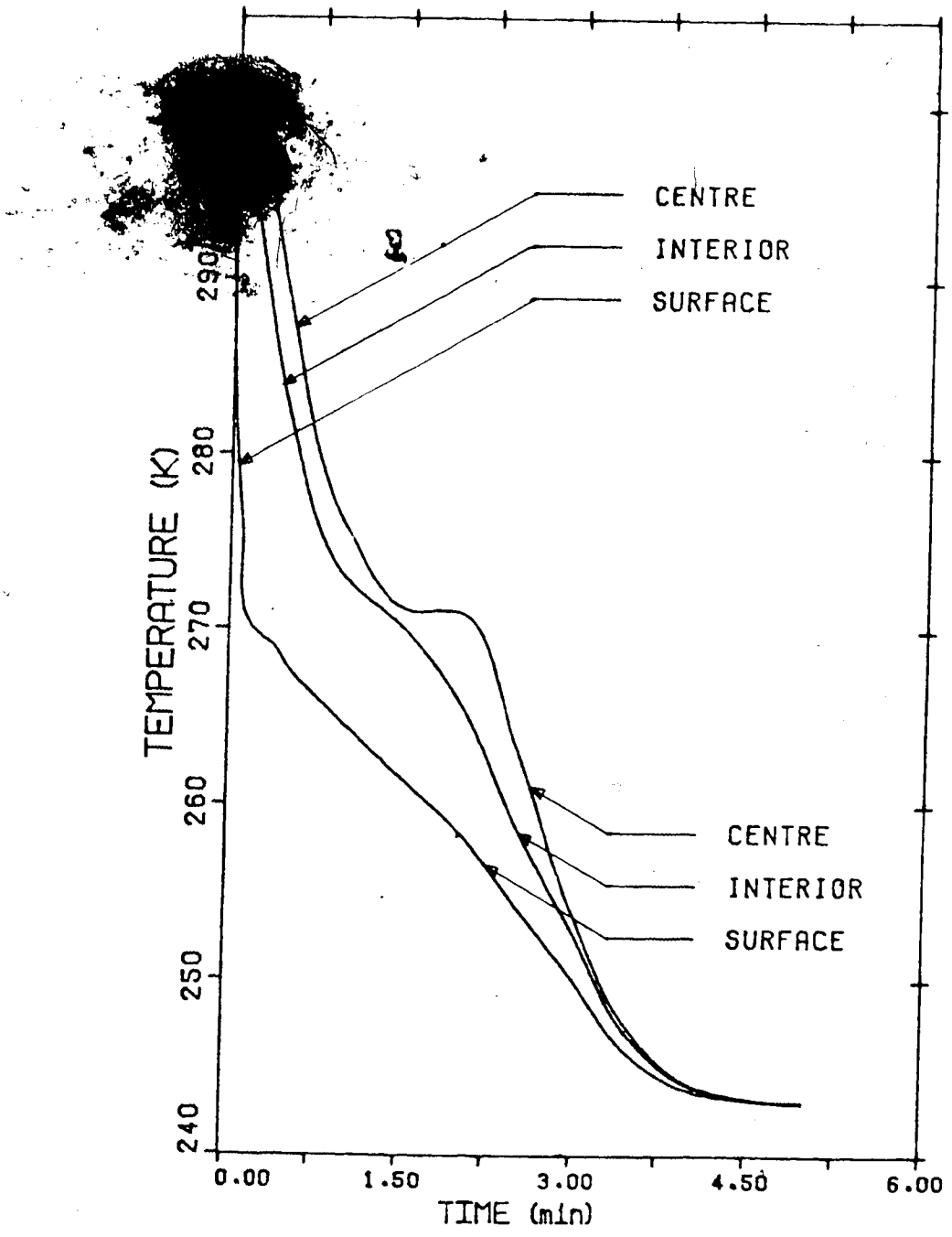

FIGURE 27: T(t) VARIATION WITH POSITION

Fast freezing rates are obtained at the centre because it is easier to remove the heat of fusion when the centre starts to freeze. At this stage most of the heat of fusion has already been removed from the exterior regions. Also, the thermal conductivity has been significantly increased as a result of freezing.

In conclusion, the numerical model predicted temperature-time relationships which were within 0.1 K of the analytical solution for one phase heat transfer. For Karlsruhe Test Substance, the model predicted freezing times which were within -2 to +11.6% of experimental values. For freezing of sugarbeet cylinders in the 243 K bath, the predicted freezing time was 13.6% greater than the average experimental value. The situation was reversed for the 253 K bath. In this bath, the predicted freezing time was 14.6% lower than the average experimental value. The maximum difference between experimental and predicted temperatures was 9.2 K for freezing and 6.3 K for thawing. The greatest value of the standard deviation from the predicted curve was 4.1 K for freezing and 2.4 K for thawing. The differences between model predictions and experimental values could have resulted from the inaccuracies involved in the determination of the thermal properties or from failure to allow for supercooling.

13. CONCLUSIONS

1. Frozen beets should not be allowed to thaw prior to processing. This is because thawed beets are soft and decay rapidly.
2. Insignificant sucrose losses occur in beets which are stored frozen, at 263 K or 248 K, for six months. This conclusion applies to a wide range of freezing rates.
3. Two frozen storage methods appear economically attractive and warrant further investigation. The first one is the storage of frozen beets in outdoor piles. The second method is the frozen storage of beets in mechanically cooled buildings.
4. Use of enthalpy, in the energy balance equations, simplifies the development of heat transfer models for freezing and thawing of biological materials. No loss in accuracy results from this simplification.

14. RECOMMENDATIONS

1. A numerical model should be developed for the freezing and thawing of storage piles of sugarbeets or other plant material. This model could then be used to predict the effect of unusual weather conditions, such as Chinook winds in Southern Alberta, on the temperature of stored beets.
2. Frozen storage experiments should be performed on beet piles in the Taber area. Cold night air should be blown through the pile to freeze the beets to 263 K or below. Temperatures in the pile should then be monitored to determine how long the beets remain frozen. The use of pile covers in the prevention of thawing should also be investigated.
3. Accurate experimental measurements of thermal conductivity and enthalpy, in the freezing range, should be obtained for numerous biological materials.
4. The enthalpy approach should be used to develop heat transfer models for other shapes, such as finite cylinders, finite slabs and cones.
5. The numerical model, developed in this thesis, should be extended to allow for the effects of both supercooling and water diffusion.

15. NOMENCLATURE

a_0, a_1, a_2, a_3	constants used in calculating enthalpy
a_w	water activity
A	surface area, cm^2
$b_0, b_1, .., b_7$	constants used in calculating enthalpy
B	constant used in the experimental determination of enthalpy, J/(min . K)
c	specific heat, J/(g . K)
c_J	specific heat of vegetable juice, J/(g . K)
c_s	specific heat of sample, J/(g . K)
c_u	specific heat of insoluble solids, J/(g . K)
c_0, c_1, c_2, c_3	constants used in calculating enthalpy
d_0, d_1, d_2, d_3	constants used in calculating thermal conductivity
D	shape factor used in Plank's equation
e_0, e_1, e_2, e_3	constants used in calculating thermal conductivity
EMS	error mean square
$f_0, f_1, .., f_7$	constants used in calculating thermal conductivity
G	shape factor used in Plank's equation
H	enthalpy per unit mass, J/g
H_J	enthalpy per unit mass of juice, J/g
h	heat transfer coefficient at the surface, J/(s . m^2 . K)
$J_0( )$	zero order Bessel function
$J_1( )$	first order Bessel function
k	thermal conductivity, J/(s . m . K)

k_d	thermal conductivity of dry matter, J/(s . m . K)
k_f	thermal conductivity of frozen food, J/(s . m . K)
k_i	thermal conductivity of ice, J/(s . m . K)
k_ℓ	thermal conductivity of unfrozen food, J/(s . m . K)
k_w	thermal conductivity of water, J/(s . m . K)
l	thickness of infinite slab, or diameter of sphere or cylinder, cm
LSR	least significant range
m	mass of copper cylinder, g
m_s	mass of sample, g
n	number of time increments at which experimental and predicted temperatures are compared
N	number of elements
P	Rh/k
$\dot{Q}$	rate of heat flow from block to sample, J/min
r	radial position, cm
rep	number of replicates
R	radius of cylinder, cm
s_m	standard error of the mean
Sall	sum of all the measured sucrose contents
Sr10	sum of the 7 treatments which had a replicate 10 measured
St8	sum of the 11 measured replicates of treatment 8
SSR	significant studentized range
t	time, min

t_f	time for centre of sample to reach the freezing point, min
trt	number of treatments
T	temperature, K
T_o	initial temperature, K
T_a	temperature of cooling or heating medium, K
T_b	temperature of block, K
T_{c1}	initial freezing point, K
T_{c2}	temperature at which ln H vs. T becomes linear (during freezing)
T_{c3}	temperature at which k vs. T is no longer linear (during freezing)
$T_{exp}(t)$	average experimental temperature at time t, K
T_f	unique freezing point, K
T_{N+1}	surface temperature, K
$T_{pre}(t)$	predicted temperature at time t, K
T_w	freezing point of water, K
u	dimensionless temperature, $\frac{T - T_a}{T_o - T_a}$
u_{c1}	$\frac{T_{c1} - T_a}{T_o - T_a}$
u_{c2}	$\frac{T_{c2} - T_a}{T_o - T_a}$
u_{c3}	$\frac{T_{c3} - T_a}{T_o - T_a}$
w_i	mass fraction of juice which has become ice
w_o	mass fraction of soluble solids in unfrozen juice

w_s	mass fraction of soluble solids in residual solution
w_u	mass fraction of insoluble solids
w_w	mass fraction of water
x	dimensionless position, r/R
y	distance in one-dimensional heat equation, cm
Y	estimated missing value in Table 4, % sucrose

Greek Symbols

α	thermal diffusity, cm^2/min
α_w	thermal diffusity of water, cm^2/min
β_n	roots of $\beta_n J_1(\beta_n) = PJ_0(\beta_n)$
$\Delta H_J(T)$	$H_J(293.2) - H_J(T)$, J/g
ΔR	element thickness, cm
Δt	time increment, min
ΔT	$T_b - T$
ΔT_b	ΔT without sample
Δx	dimensionless element thickness, $\Delta R/R$
ρ	density, g/m^3
τ	dimensionless time, $\alpha t/R^2$

Subscripts and Superscripts

d	dry matter

f	frozen
i	element number
j	time increment
J	juice
ℓ	unfrozen
s	sample

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17. APPENDIX A

17.1 The Sugarbeet

The sugarbeet is a biennial plant which produces a large tap root in the first year and seed in the following year. Sugar is produced by photosynthesis, in the leaves of the plant, and is used for the plant's metabolism and growth. Excess sugar is stored in the tap root. The top part of the root, from where the leaves grow, is called the "crown". The sugar content varies throughout the root. It's minimum value is attained at the crown where fewer storage cells exist. The maximum sugar content occurs about halfway down the root.

The sugarbeet is composed of white storage tissue which is penetrated by darker vascular rings. The vascular rings are composed of xylem and phloem vessels. The xylem conducts water and the phloem conducts nutrients. For example, sugar is transported from the leaves to the root by means of the phloem. The storage tissue consists of thin walled "parenchyma" cells. Small parenchyma cells surround the vessels whereas larger parenchyma cells are further away. The large cells are primarily responsible for water storage and have a much lower sugar content than the small cells. The "intercellular space" between cells occupies about 0.5 to 2% of the total volume (67).

It is important to remember that the sugarbeet root is composed of different types of cells of varying composition.

Each of these cells may freeze at a different temperature. Oldfield et al. (157) observed that sugarbeets start to freeze at 271 K. However, Wyse (230) stated that the intercellular region and some cells (epidermal cells) near the surface of the beets may freeze at 272 K. Different cells may also differ in their tolerance to freezing when it does occur. Phloem cells are more resistant to frost injury than parenchyma cells (214).

18. APPENDIX B

18.1 Freezing Injury in Plants

Plants are composed of small units called "cells", each of which is bounded by a "cell wall". Every cell is divided into a number of different compartments, each of which has its own specific functions. Separating the various compartments is the "ground cytoplasm" which is a complex colloidal solution. Changes in the concentration or pH of this solution can irreversibly damage the cell (106).

Biological materials are injured by freezing and thawing. This injury results from the conversion of water into ice. When the temperature falls below the initial freezing point, ice begins to form in the intercellular region. Water then leaves the inside of the cell in an attempt to equalize the chemical potentials inside and outside the cell. Thus, a growing ice layer is formed outside the cell. At the same time the solute concentration inside the cell is increased and this lowers the freezing point of the solution inside the cell ("intracellular solution"). If the temperature is decreased sufficiently slowly so water can leave the cell then intracellular ice formation will be avoided. However, if the sample temperature falls very rapidly then water cannot leave the cell quickly enough and ice is eventually formed inside the cell. This intracellular ice is considered to be lethal for cells. For frozen foods lethality may not be important as

cells need not survive freezing and thawing if the food is consumed within a short time of thawing.

The increased solute concentration, obtained during freezing, may result in precipitation of some solutes (65), attack on cell membranes (5,6,87,125,183,191), and acceleration of some chemical reactions (119). Damage to cell membranes can result in the movement of enzymes to regions of the tissue where they can act on reactants to which they do not normally have access. Loss of water leads to cell shrinkage, which, in turn, brings potential reactants closer together. This can result in protein denaturation (119,120) and degradation of lipid components (6,219,236) of cell membranes.

The rate constant for most chemical reactions decreases with decreasing temperature. However, one must consider that the cell solution is concentrated when water is transformed into ice. Thus, the rate of some chemical reactions is actually increased during freezing. These reactions are subsequently slowed down when the temperature falls below about 263 K. This is because: (1) the reaction rate constant continues to fall with temperature, and (2) the viscosity of the cell solution is increasing with decreasing temperature and will eventually become so great as to essentially stop the movement of potential reactants.

When water freezes it expands about 9%. This expansion may cause the growing ice layer to push adjoining cells apart or to even penetrate some of the cells

(57,77,119,145,173,198).

During thawing intercellular ice melts and flows back into the cell and this causes a stress on the cell wall and membranes (117). In addition, thawing of ice momentarily results in large concentration gradients inside the cell. Regions of almost pure water are adjacent to regions of concentrated solutes. The resulting high osmotic gradient can damage the cell.

The plasma membrane is situated just inside the cell wall and it maintains the integrity of the cell. It allows some molecules to enter the cell but stops others. If the plasma membrane is damaged then it may lose this semipermeability and solutes are lost from the interior of the cell. This results (because of a reduction in osmotic pressure) in a decrease in the amount of water which re-enters the cell during thawing. Loss of water may also be partly attributed to the reduction in water holding capacity of proteins denatured during freezing. A decrease in cell water content causes a reduction in the pressure on the cell wall and results in a softening of the tissue. Cell walls, because of their rigid nature, are partly responsible for the firmness of a plant tissue. Thus, damage to cell walls and membranes during freezing contributes to the softening of the tissue.

As stated above, cell membranes can be irreversibly damaged during freezing and lose their semipermeability. There are many possible causes of membrane damage.

Lovelock (125) stated that concentration of electrolytes, rather than the mechanical effect of ice crystals, was the main cause of damage in the freezing of red blood cells. When the cells were placed in NaCl solutions stronger than 0.8 M the plasma membrane became permeable to sodium ions.

Levitt (119) considered that when the cell contracts during slow freezing a "break" occurs in the lipid layer of its membranes. This break is then stabilized by the formation of disulfide linkages between membrane proteins and is not removed during thawing. For intracellular freezing, Levitt (122) speculated that submicroscopic crystals "expand into the plasma membrane, creating small holes in the now solid lipid layer". During thawing, these holes cause a loss of semipermeability in the plasma membrane.

Meryman et al. (143) postulated that irreversible damage to cell membranes occurs during slow freezing when the cell volume is reduced below a certain critical value (about 40 to 50% of the unfrozen volume"). This damage was attributed to loss of lipid or protein components from the cell membrane. The loss of the membrane material was considered to be caused by either a direct osmotic stress or physical forces resulting from the reduction in cell volume.

Araki's (6) experimental results, for mitochondrial membranes, indicated that phospholipids (particularly phosphatidylethanolamine) may be degraded during slow freezing by the action of a phospholipase. This enzyme is

apparently activated by changes inflicted on the membrane following exposure to high solute concentrations.

18.2 Effect of Freezing and Thawing Rates

Fast freezing generally produces food of a better quality than slow freezing (28,77,180,220). Fast freezing results in small distributed ice crystals. Slow freezing produces large ice masses which tend to physically disrupt the cell walls. Another disadvantage of slow freezing is that cells lose a large amount of water during freezing. The remaining solution then becomes highly concentrated. The probability of unfavourable reactions or solute precipitation occurring is therefore increased.

Chemical reactions can take place quite rapidly above 263 K. The faster the freezing rate the quicker the food temperature will go below 263 K and the less quality deterioration will occur. However, it should be noted that if large samples are frozen too quickly, for example in liquid nitrogen, the surface will be frozen solid before the interior starts to freeze. When the interior expands during freezing severe cracking occurs.

Slow thawing allows growth of ice crystals (recrystallization) and thus minimizes the advantages of fast freezing. It also permits more time for undesirable reactions to occur. However, it should be noted that fast thawing has the following disadvantages :

1. Ice is melted rapidly and neighbouring regions of almost pure water and concentrated solution will exist.
2. Water is returned very quickly through the cell wall and membrane, thus creating a shear stress on these structures.

The disadvantages of fast thawing are greater following slow freezing as more water must be returned to the cell. Thus, as a general rule, fast freezing should be followed by fast thawing and slow freezing by slow thawing.

Rates of thawing are limited by the fact that the surface temperature cannot be allowed to reach too high a value. If it is desired to keep a plant alive after thawing then the surface must be kept below 318 K. When dealing with a food, higher surface temperatures may be used but care must still be taken to avoid damaging the surface. Thawing rates are also limited by the fact that heat must be transferred through a growing layer of thawed food which has a thermal diffusity about nine times less than that of the frozen food.

18.3 Storage Temperature

Frozen vegetables are usually stored at 255 K as biochemical reactions take place very slowly at this temperature. Frozen storage above 263 K is not recommended as above this temperature :

1. Micro-organisms can grow.
2. Some biochemical reaction rates are high.

The actual storage temperature is decided by economics i.e. the potential improvement in vegetable quality or storage life must be balanced against the increased refrigeration cost of a lower storage temperature.

As a result of ice crystal growth the benefits of fast freezing may be lost during storage (180,224). The lower the storage temperature the slower will be the ice growth rate. It is generally accepted (103,180,208) that fluctating storage temperatures encourage the growth of ice crystals and thus the storage temperature should be kept as constant as possible. However, some experiments (92) have indicated that fluctuations in the storage temperature are not important for food storage.

Enzymatic reactions can continue at frozen storage temperatures. Thus, vegetables usually undergo a blanching treatment prior to freezing. It consists of placing the vegetable in steam or hot water (353 to 373 K) for approximately one minute. Blanching deactivates some enzymes and kills many micro-organisms (159). It also removes air from vegetable tissues and thus reduces the probability of oxidation occurring (91).

19. APPENDIX C

19.1 Estimation of Sucrose Content of Beet Cylinders

The following method was used to determine the sucrose content of beet cylinders. It was adapted from a procedure developed by Karr and Norman (109a).

1. Weigh a closed container to 4 decimal places (=A).
2. Add 3 beet cylinders and reweigh to 4 decimal places (=B).
3. Transfer the 3 cylinders to a 4 oz. oster blender jar.
4. Cut the cylinders into small pieces and add 100 ml of distilled water.
5. Add 0.4 g of dry basic lead subacetate (aids in subsequent filtration).
6. Blend for 6 min at the top speed of a CYCLOMATIC 14 OSTERIZER.
7. Filter the solution obtained into a preweighed (to 1 decimal place) plastic container (weight of container, including top, =C).
8. In this procedure it is assumed that no sucrose is lost during extraction. Consequently, after pouring the extract solution into the funnel the remaining extract, in the blender jar and on the cutter blades etc., is washed into the filtration funnel with distilled water. The precipitate left on the filter paper (WHATMAN no. 4, 24 cm diameter) after filtration is also washed with distilled water. The total amount of distilled water

used for washing is about 170 ml.

9. The top is then put on the container and the container and extract solution are weighed to 1 decimal place (=D).
10. Weigh a 5 ml volumetric flask (=E).
11. Add about 0.9 ml of the extract solution and reweigh (=F).
12. Add about 0.9 ml of a trehalose solution of known concentration (should be about 0.007 g trehalose/g of solution) and reweigh (=G).

    The above three weights are all recorded to 4 decimal places.
13. The solution in the volumetric flask is mixed well and then 0.8 ml of the solution is added to a labelled, empty, 6 ml hypovial.
14. The hypovial is placed in a vacuum oven (at 333 K and 10.2 kPa (3 in. Hg)) for 1 hour to remove all the water from the solution.
15. The hypovial is sealed and 0.5 ml of trimethylsilylímidazole and 0.5 ml of dimethylformamide are injected. The resulting solution is left overnight at room temperature or for 15 minutes at 333 K.
16. Then 9.8 microlitres of this solution are injected into a Gas Chromatograph (F and M model 810).

Using the resulting ratio of the sucrose and trehalose peak areas, the % sucrose can be calculated from the

formula:

$$\%\ \text{SUCROSE IN COSSETTES} = \text{AREA RATIO} \times \text{CONC.TREH. SOLUTION} \times K \times \frac{(G-F)(D-C)}{(F-E)(B-A)} \times 100\% \qquad \text{(C-1)}$$

The constant K is the calibration constant for the Gas Chromatograph and is defined by:

$$K = \frac{\text{trehalose peak area/trehalose mass injected}}{\text{sucrose peak area/sucrose mass injected}} \qquad \text{(C-2)}$$

It is obtained by carrying out calibration runs using standard sucrose and trehalose solutions of known concentration. These standard solutions were injected, every day samples were being analysed, to detect any variations in K.

Gas Chromatograph Details

A single column flame ionization detector was used to detect the sugar peaks. The response from the detector was transmitted to a HEWLETT PACKARD computer (21MX E-series), via an A/D converter, where the peak areas were determined and reported on a telytype.

G.C. Operating Conditions

Helium flow rate: 30 ml/min.

Hydrogen flow rate: 45 ml/min.

Air flow rate: 440 ml/min.

Helium inlet pressure: 380 kPa (40 psig).

Hydrogen inlet pressure: 250 kPa (22 psig).

Air inlet pressure: 380 kPa (40 psig).

Oven temperature: 533 K

Injection port temperature: 573 K

Flame detector temperature: 583 K

20. APPENDIX D

20.1 Estimation of Missing Value in Table 4

Due to experimental problems the percent sucrose for replicate 10 of treatment 8 in the first set of experiments was not measured. As a value was required for the Randomized Block Design it was decided to estimate one using the formula in Snedecor and Cochran (195):

$$Y = \frac{trt \times St8 + rep \times Sr10 - Sall}{(trt - 1)(rep - 1)} \qquad (D\text{-}1)$$

where:

Y = estimated %sucrose of treatment 8, replicate 10.

trt = number of treatments = 8

rep = number of replicates = 12

St8 = sum of the 11 measured replicates of treatment 8 = 193.4

Sr10 = sum of the 7 treatments which had a replicate 10 measured = 110.8

Sall = sum of all the measured sucrose contents = 1624.8

Thus,

$$Y = \frac{8 \times 193.4 + 12 \times 110.8 - 1624.8}{7 \times 11} \qquad (D\text{-}2)$$

$$= 16.3\%$$

20.2 Duncan's Multiple Range Test

20.2.1 First Set of Storage Experiments

Randomized Block Design

a) Rank Treatment (Trt.) Means

Trt.2	Trt.4	Trt.3	Trt.5	Trt.6	Trt.8	Trt.7	Trt.1
16.4	16.6	16.8	16.9	17.5	17.5	17.6	17.6

b) Calculate the Standard Error of the Mean (s_m)

From Table 4 the Error Mean Square (EMS) = 0.1849

Number of replicates (rep) = 12

$$s_m = \sqrt{EMS/rep} = \sqrt{0.1849/12} = 0.1241 \qquad \text{(D-3)}$$

c) Degrees of Freedom (DF) for Error

DF = (trt - 1)(rep-1)

= (8-1)(12-1) = 77 (D-4)

d) 5% Level of Significance

For DF=77 a linear interpolation of Harter's (94) Table 1 yields:

No. of means	2	3	4	5	6	7	8
SSR	2.82	2.97	3.06	3.13	3.19	3.23	3.27

where:

SSR = significant studentized range.

e) Calculate the Least Significant Ranges (LSR)

$$LSR = s_m \times SSR \qquad (D\text{-}5)$$

This yields the following LSR values:

No. of Means	2	3	4	5	6	7	8
LSR	.35	.37	.38	.39	.40	.40	.41

f) Determine the Treatments which are Significantly Different

Trt.1 - LSR_4 = 17.6 - 0.41 = 17.19

Trt.5 = 16.9

Therefore, Trt.1 is significantly greater than treatments 2-5.

Trt.1 - LSR_4 = 17.6 - 0.38 = 17.22

Trt.6 = 17.5

Therefore, Trt.1 is not significantly different from treatments 6-8.

Trt.8 - LSR_4 = 17.5 - 0.40 = 17.10

Trt.5 = 16.9

Therefore, Trt.8 is significantly different from treatments 2-5.

Trt.5 - LSR_4 = 16.9 - 0.38 = 16.52

Trt.2 = 16.4

Therefore, Trt.5 is significantly different from Trt.2.

Trt.5 - LSR_3 = 16.9 - 0.37 = 16.53

Trt.4 = 16.6

Therefore, Trt.5 is not significantly different from

treatments 4 and 3.

Trt.3 - LSR_3 = 16.8 - 0.37 = 16.43

Trt.2 = 16.4

Therefore, Trt.3 is significantly different from Trt.2.

Trt.4 - LSR_2 = 16.6 - 0.35 = 16.25

Therefore, Trt.4 is not significantly different from Trt.2.

The above results can be summarized as follows:

Trt.2 Trt.4 Trt.3 Trt.5 Trt.6 Trt.8 Trt.7 Trt.1

Two treatments which are underlined with the same line are not significantly different at the 5% level. If two treatments are not underlined with the same line then they are significantly different. The calculations were repeated at the 1% level and the following results were obtained:

Trt.2 Trt.4 Trt.3 Trt.5 Trt.6 Trt.8 Trt.7 Trt.1

Factorial Design

The six treatments in order of ranking are:

Trt.2	Trt.4	Trt.3	Trt.5	Trt.6	Trt.7
16.4	16.6	16.8	16.9	17.5	17.6

From Table 7:

EMS = 0.210

rep = 12

$$s_m = \sqrt{EMS/rep} = \sqrt{0.210/12} = 0.1323 \qquad \text{(D-6)}$$

DF for Error = (trt - 1)(r-1)

= (6-1)(12-1) = 55 (D-7)

For a 5% level of significance

use of Harter's Table 1 gives:

No. of Means	2	3	4	5	6
SSR	2.84	2.98	3.08	3.15	3.20
LSR	0.38	0.39	0.41	0.42	0.42

$Trt.7 - LSR_6 = 17.6 - 0.42 = 17.18$

Therefore, Trt.7 is significantly greater than treatments 2-5.

$Trt.7 - LSR_2 = 17.6 - 0.38 = 17.22$

Therefore, Trt.7 is not significantly different from Trt.6

$Trt.6 - LSR_5 = 17.5 - 0.42 = 17.08$

Therefore, Trt.6 is significantly greater than treatments 2-5.

$Trt.5 - LSR_4 = 16.9 - 0.41 = 16.49$

Therefore, Trt.5 is significantly different from Trt.2.

$Trt.5 - LSR_3 = 16.9 - 0.39 = 16.51$

Therefore, Trt.5 is not significantly different from treatments 4 and 3

$Trt.3 - LSR_3 = 16.8 - 0.39 = 16.41$

Therefore, Trt.3 is significantly different from Trt.2

$Trt.4 - LSR_2 = 16.6 - 0.38 = 16.22$

Therefore, Trt.4 is not significantly different from Trt.2.

The above results are now summarized:

Trt.2	Trt.4	Trt.3	Trt.5	Trt.6	Trt.7

The following result was obtained for a 1% level of significance:

Trt.2	Trt.4	Trt.3	Trt.5	Trt.6	Trt.7

20.2.2 Second Set of Storage Experiments

The four treatments in order of ranking are:

Trt.12	Trt.10	Trt.1	Trt.11
17.1	17.5	17.5	17.6

From Table 13:

EMS = 0.084

rep = 8

$s_m = \sqrt{EMS/rep} = \sqrt{0.084/8} = 0.1025$ (D-8)

DF for Error = 21 (D-9)

For a 5% level of significance, use of Harter's Table 1 yields:

No. of means:	2	3	4
SSR	2.94	3.09	3.18
LSR	0.30	0.32	0.33

Trt.11 - LSR_4 = 17.6 - 0.33 = 17.27

Therefore, Trt.11 is significantly greater than Trt.12.

Trt.11 - LSR_3 = 17.6 - 0.32 = 17.28

Therefore, Trt.11 is not significantly greater than treatments 1 and 10.

Trt.1 - LSR_3 = 17.5 - 0.32 = 17.18

Therefore, Trt.1 is significantly greater than T...

Trt.10 -LSR_2 = 17.5 - 0.30 = 17.20

Therefore, Trt.10 is significantly greater than Trt.12

The above results are summarized as follows:

Trt.12 Trt.10 Trt.1 Trt.11

The following result was obtained for a 1% level of significance:

Trt.12 Trt.10 Trt.1 Trt.11

21. APPENDIX E

21.1 Experimental Results Used in Determining h

TABLE E-1: AIR HEATING OF COPPER CYLINDER, T_0=243 K

TIME (min)	TEMPERATURE (K)		
	RUN 1 (T_a=297.7)	RUN 2 (T_a=297.5)	RUN 3 (T_a=297.5)
0	243.1	242.9	242.9
1	250.5	249.9	250.5
2	257.7	256.5	256.7
4	268.9	266.5	266.7
8	281.2	278.3	278.0
12	286.2	283.5	283.2
16	288.2	286.2	285.5
20	289.3	288.2	287.6
24	290.5	289.8	288.9
28	291.3	290.9	289.9
32	292.1	292.1	290.8
36	292.7	292.9	291.7
40	293.3	293.7	292.3
44	293.9	294.5	292.7
48	294.2	294.7	293.3
52	294.7	295.2	293.9

	RUN 1	RUN 2	RUN 3
56	295.1	295.6	294.3
60	295.2	315.9	294.6
64	295.3	316.0	294.8

TABLE E-2: AIR HEATING OF COPPER CYLINDER, T_o=253 K

TIME (min)	TEMPERATURE (K)		
	RUN 1 (T_a=297.8)	RUN 2 (T_a=297.9)	RUN 3 (T_a=297.9)
0	252.9	252.9	252.9
1	258.3	258.2	258.4
2	263.7	263.7	263.7
4	271.7	271.4	271.4
8	281.2	280.5	280.3
12	284.9	284.3	284.1
16	287.3	286.5	286.4
20	288.9	288.3	288.2
24	289.9	289.8	289.3
28	290.9	290.9	290.2
32	291.9	291.9	290.9
36	292.7	292.7	291.9
40	293.6	293.3	292.6
44	293.9	293.8	293.2
48	294.3	294.3	293.7
52	294.6	294.5	294.2
56	294.7	294.7	294.5
60	294.9	294.8	294.7
64	295.2	295.1	294.9
68	295.6	295.5	295.1
72	-	295.6	-

TABLE E-3: AIR HEATING OF COPPER CYLINDER, T_o=263 K

TIME (min)	TEMPERATURE (K)		
	RUN 1 (T_a=297.9)	RUN 2 (T_a=297.8)	RUN 3 (T_a=297.9)
0	263.2	263.2	263.2
1	267.3	267.3	267.2
2	270.9	271.2	271.1
4	276.8	276.8	276.7
8	283.4	283.3	282.9
12	286.9	286.7	286.3
16	289.3	288.8	288.6
20	290.5	290.3	290.2
24	291.7	291.6	291.4
28	292.6	292.6	292.3
32	293.1	293.3	293.3
36	293.9	293.9	293.8
40	294.7	294.5	294.4
44	295.1	294.8	294.7
48	295	295.1	295.2

TABLE E-4: COOLING OF COPPER CYLINDER, T_a = 243 K

TIME (min)	TEMPERATURE (K)		
	RUN 1	RUN 2	RUN 3
0	297.6	297.2	296.1
0.167	284.7	281.9	276.6
0.5	266.7	263.1	259.8
1.0	253.5	251.3	250.1
1.5	247.9	246.7	246.1
2.0	245.4	244.5	244.2
2.5	244.1	243.4	243.3
3.0	243.5	243.3	-
3.5	243.1	-	-

TABLE E-5: COOLING OF COPPER CYLINDER, T_a = 253 K

TIME (min)	TEMPERATURE (K)		
	RUN 1	RUN 2	RUN 3
0	297.3	296.8	296.6
0.167	286.2	281.3	278.8
0.5	269.3	267.5	264.9
1.0	259.1	258.4	257.2
1.5	255.5	255.2	254.7
2.0	254.1	253.8	253.7
2.5	253.2	253.2	-
3.0	253.1	-	-

TABLE E-6: COOLING OF COPPER CYLINDER, T_a=263 K

TIME (min)	TEMPERATURE (K)		
	RUN 1	RUN 2	RUN 3
0	297.9	298.6	298.5
0.167	282.3	284.3	282.2
0.5	270.9	272.7	271.3
1.0	265.4	266.2	265.6
1.5	264.0	264.1	264.1
2.0	263.5	263.7	263.5
2.5	-	263.3	-

TABLE E-7: AIR HEATING - h AS A FUNCTION OF u_{N+1}

(T_o = 243 K)

TIME (min)	RUN 1		RUN 2		RUN 3	
	u_{N+1}	h	u_{N+1}	h	u_{N+1}	h
0	1.0	-	1.0	-	1.0	-
1	0.864	20.2	0.872	18.8	0.861	20.2
2	0.733	23.0	0.751	20.9	0.747	19.5
4	0.527	23.0	0.568	19.5	0.564	19.5
8	0.302	19.5	0.352	16.7	0.357	15.3
12	0.211	12.6	0.256	11.2	0.262	10.5
16	0.174	6.3	0.207	7.7	0.220	6.3
20	0.154	4.2	0.170	7.0	0.181	7.0
24	0.132	5.6	0.141	6.3	0.158	4.9
28	0.117	4.2	0.121	5.6	0.139	4.2
32	0.103	4.9	0.099	7.0	0.123	4.2
36	0.092	4.2	0.084	5.6	0.106	4.9
40	0.081	4.2	0.070	6.3	0.095	3.5
44	0.070	4.9	0.055	8.4	0.088	2.8
48	0.064	2.8	0.051	2.1	0.077	4.9
52	0.055	5.6	0.042	7.0	0.066	5.6
56	0.048	4.9	0.035	6.3	0.059	4.2
60	0.046	1.4	0.029	6.3	0.053	3.5
64	0.044	1.4	0.027	2.1	0.049	2.8

h is expressed in units of J/(s . m^2 . K)

TABLE E-8: AIR HEATING - h AS A FUNCTION OF u_{N+1}

(T_0 = 253 K)

TIME (min)	RUN 1		RUN 2		RUN 3	
	u_{N+1}	h	u_{N+1}	h	u_{N+1}	h
0	1.0	-	1.0	-	1.0	-
1	0.880	17.4	0.882	16.7	0.878	18.1
2	0.759	20.2	0.760	20.9	0.760	20.2
4	0.581	18.8	0.589	17.4	0.589	17.4
8	0.369	15.3	0.387	14.7	0.391	14.0
12	0.287	9.1	0.302	8.4	0.307	8.4
16	0.234	7.0	0.253	6.3	0.256	6.3
20	0.198	5.6	0.213	6.3	0.216	5.6
24	0.176	4.2	0.180	5.6	0.191	4.2
28	0.154	4.9	0.156	4.9	0.171	3.5
32	0.131	5.6	0.133	5.6	0.156	3.5
36	0.114	4.9	0.116	4.9	0.133	5.6
40	0.094	7.0	0.102	4.2	0.118	4.2
44	0.087	2.8	0.091	4.2	0.104	4.2
48	0.078	3.5	0.080	4.2	0.093	4.2
52	0.071	2.8	0.076	2.1	0.082	4.2
56	0.069	1.4	0.071	2.1	0.076	2.8
60	0.065	2.1	0.069	1.4	0.071	2.1
64	0.058	3.5	0.062	3.5	0.067	2.1
68	0.049	5.6	0.053	5.6	0.062	2.1
72	-	-	0.051	1.4	-	-

TABLE E-9: AIR HEATING - h AS A FUNCTION OF u_{N+1}

(T_o = 263 K)

TIME (min)	RUN 1		RUN 2		RUN 3	
	u_{N+1}	h	u_{N+1}	h	u_{N+1}	h
0	1.0	-	1.0	-	1.0	-
1	0.882	17.4	0.882	17.4	0.882	17.4
2	0.778	17.4	0.769	18.8	0.770	18.8
4	0.608	17.4	0.607	16.0	0.609	16.0
8	0.418	13.3	0.419	12.6	0.431	11.9
12	0.317	9.8	0.321	7.0	0.333	9.1
16	0.248	8.4	0.260	7.0	0.267	7.7
20	0.213	5.6	0.217	6.3	0.221	6.3
24	0.179	6.3	0.179	6.3	0.187	5.6
28	0.150	5.6	0.150	6.3	0.161	4.9
32	0.138	3.5	0.130	4.9	0.132	7.0
36	0.115	6.3	0.113	4.9	0.118	4.2
40	0.092	7.7	0.095	5.6	0.101	5.6
44	0.081	4.9	0.087	3.5	0.092	2.8
48	0.069	5.6	0.078	3.5	0.078	5.6

h is expressed in units of $J/(s \cdot m^2 \cdot K)$

TABLE E-10: LIQUID COOLING h AS A FUNCTION OF u_{N+1}

(T_a = 243 K)

TIME (min)	RUN 1		RUN 2		RUN 3	
	u_{N+1}	h	u_{N+1}	h	u_{N+1}	h
0	1.0	-	1.0	-	1.0	-
0.167	0.764	223	0.718	275	0.633	380
0.5	0.435	235	0.372	273	0.318	288
1	0.194	225	0.155	243	0.135	237
1.5	0.091	208	0.070	155	0.060	225
2	0.046	193	0.029	105	0.024	250
2.5	0.022	203	0.009	322	0.008	327
3	0.011	193	0.007	62	-	-
3.5	0.004	305	-	-	-	-

h is expressed in units of J/(s . m^2 . K)

TABLE E-11: LIQUID COOLING - h AS A FUNCTION OF u_{N+1}

(T_a = 253 K)

TIME (min)	RUN 1		RUN 2		RUN 3	
	u_{N+1}	h	u_{N+1}	h	u_{N+1}	h
0	1.0	-	1.0	-	1.0	-
0.165	0.750	240	0.647	362	0.593	435
0.5	0.369	295	0.333	277	0.275	320
1	0.140	270	0.125	270	0.098	285
1.5	0.059	240	0.052	242	0.041	242
2	0.027	215	0.021	260	0.018	225
2.5	0.007	385	0.007	305	-	-
3	0.005	112	-	-	-	-

h is expressed in units of J/(s . m^2 . K)

TABLE E-12: LIQUID COOLING - h AS A FUNCTION OF u_{N+1}

(T_a = 263 K)

TIME (min)	RUN 1		RUN 2		RUN 3	
	u_{N+1}	h	u_{N+1}	h	u_{N+1}	h
0	1.0	-	1.0	-	1.0	-
0.167	0.550	497	0.596	430	0.538	515
0.5	0.222	378	0.268	332	0.229	355
1	0.063	348	0.085	320	0.068	337
1.5	0.023	358	0.025	335	0.025	272
2	0.009	272	0.014	163	0.009	305
2.5	-	-	0.003	447	-	-

h is expressed in units of J/(s . m^2 . K)

22. APPENDIX F

22.1 Karlsruhe Test Substance

Karlsruhe Test Substance is a methylcellulose gel which was first produced by Riedel (175) in Karlsruhe, Germany. This gel also contains 77% water, a preservative called parachlorometacresol, and enough common salt to lower the initial freezing point to 272.2 K. It is extremely useful in testing a freezing model because:

1. It's enthalpy and thermal conductivity have been measured as a function of temperature in the fre i. range.
2. It's thermal properties are less variable than those of a food.
3. It behaves very similarly to foods (especially lean beef) during freezing.

Riedel tabulated (175) enthalpy versus temperature data. For ease of use in the numerical model these values were approximated by the following sets of equa or s:

$$H(T) = 285.44 + 4.272(T-273.2) \qquad T \text{ - - } K$$

(F-1)

$$H(T) = \exp[6.0956 + 0.51751(T-273.2) + 0.06387(T-273.2)^2 + 0.003089(T-273.2)^3] \qquad 264.2\ K \leq T \leq 272.2\ K$$

(F-2)

$$H(T) = \exp[4.928 + 0.0632(T-273.2)] \qquad T < 264.2\ K$$

(F-3)

The reason for choosing these particular functions is explained in the chapter on "Determination of Thermal Properties of Sugarbeets". The above approximations lead to a maximum deviation from Riedel's tabulated values of 2.6%. Also, the enthalpy calculated from the equations agreed with the actual data at 293.2 K and 263.2 K. Thus, the approximations should have a negligible effect on predicted freezing times.

Bonacina and Comini (22) published a graph displaying the dependence of thermal conductivity on temperature. The graph was approximated in this thesis by the following equations:

$$k(T) = 0.495 \quad 0.0(\ 0(T-273.2) \qquad T > 272.2 \text{ K} \qquad \text{(F-4)}$$

$$k(T) = 0.1312 - 0.36456T \qquad 270.2 \text{ K} \leq T \leq 272.2 \text{ K} \qquad \text{(F-5)}$$

$$k(T) = 0.95271 - 0.10682(T-273.2) - 0.00572(T-273.2)^2 - 0.000098(T-273.2)^3 \qquad T < 270.2 \text{ K} \qquad \text{(F-6)}$$

These approximations lead to thermal conductivity values within the accuracy (5%) of reading data from the published graph. The constants in Equation (F-6) were obtained by means of a least squares fit through data read from Bonacina's graph. The constants in Equation (F-4) were

determined by making k(T) equal to the published value at 293.2 K and 272.2 K. Similarly, the constants for Equation (F-5) were obtained by making k(T) equal to the published value at 272.2 K and the value calculated from Equation (F-6) at 270.2 K. This procedure ensured that the k(T) function was continuous at both 272.2 K and 270.2 K.

23. APPENDIX G

23.1 Results for Freezing and Thawing of Sugarbeets

TABLE G-1: FREEZING OF SUGARBEET CYLINDER, T_a = 243 K

TIME (min)	CENTRE TEMPERATURE (K)			
	RUN 1	RUN 2	RUN 3	MODEL
0	297.6	298.2	298.2	298.0
0.5	287.7	285.9	287.5	288.3
1	274.9	272.7	272.9	275.8
1.5	270.5	271.1	270.5	271.2
2	267.5	264.9	268.4	271.0
2.5	257.1	249.9	255.2	263.3
3	247.9	245.4	247.4	253.7
3.5	244.3	243.8	244.5	247.2
4	243.5	243.2	243.6	244.2
4.5	243.2	243.1	243.2	243.3
5	243.1	243.0	-	243.1

A constant h value of 283 J/(s . m² . K) was used in the model run.

TABLE G-2: FREEZING OF SUGARBEET CYLINDER, T_a = 253 K

TIME (min)	CENTRE TEMPERATURE (K)				
	RUN 1	RUN 2	RUN 3	RUN 4	MODEL
0	297.2	295.9	298.2	298.7	297.5
0.5	289.2	289.1	289.7	291.3	288.8
1	276.9	276.9	277.2	278.1	276.9
1.5	270.5	270.4	271.1	271.1	271.9
2	270.2	269.7	270.3	270.2	271.0
2.5	269.2	268.2	269.1	269.1	271.0
3	267.9	267.2	266.9	268.3	269.5
3.5	266.5	265.6	265.6	267.5	262.7
4	263.7	262.9	262.7	264.9	258.8
4.5	259.4	260.4	259.1	261.1	256.2
5	256.2	257.7	256.2	257.7	254.6
5.5	254.7	255.9	254.7	255.5	253.7
6	253.7	254.9	253.7	254.7	253.3
6.5	-	254.2	253.5	253.8	253.1
7	-	253.7	253.2	253.3	253.0
7.5	-	253.5	-	-	252.9

A constant h value of 317 J/(s . m² . K) was used in the model run.

TABLE G-3: FREEZING OF SUGARBEET CYLINDER, T_a = 263 K

TIME (min)	CENTRE TEMPERATURE (K)			
	RUN 1	RUN 2	RUN 3	MODEL
0	299.3	298.9	298.2	298.8
0.5	291.9	291.1	290.9	290.6
1	280.3	279.7	280.5	279.0
1.5	273.5	272.9	274.1	273.4
2	270.2	270.2	270.5	271.4
2.5	270.2	270.5	270.3	271.0
3	269.4	269.3	269.7	271.0
3.5	269.2	269.1	269.3	271.0
4	269.0	268.9	269.1	270.9
4.5	269.0	268.9	268.9	270.3
5	269.0	268.9	268.9	269.1
5.5	269.0	268.8	268.9	267.7
6	268.9	268.7	268.5	266.6
6.5	268.5	268.4	268.2	265.5
7	267.9	267.9	267.7	264.7
7.5	267.5	267.5	267.1	264.2
8	266.9	267.1	266.5	263.8
8.5	266.2	266.4	265.8	263.6
9	265.3	265.7	265.2	263.5
9.5	265.1	265.3	264.7	263.4

	RUN 1	RUN 2	RUN 3	MODEL
10	264.5	264.7	264.2	263.3
10.5	264.3	264.4	264.1	-
11	264.1	264.1	263.8	-
11.5	263.9	264.1	-	-
12	-	263.7	-	-
12.5	-	263.6	-	-

A constant h value of 417 J/(s . m² . K) was used in the model run.

TABLE G-4: THAWING OF SUGARBEET CYLINDER, T_o=243 K

TIME (min)	CENTRE TEMPERATURE (K)			
	RUN 1 (T_a=297.1)	RUN 2 (T_a=297.1)	RUN 3 (T_a=296.9)	MODEL (T_a=297.0)
0	243.0	243.0	243.2	243.0
2	256.8	255.9	256.1	256.1
4	263.7	262.5	262.7	261.9
8	268.1	267.7	267.3	267.1
12	269.5	269.3	269.1	268.9
16	270.2	269.9	269.8	269.8
20	270.5	270.3	278.7	270.3
24	278.7	277.8	284.3	274.0
28	284.1	283.5	286.9	280.9
32	286.2	285.9	287.9	284.9
36	287.3	287.3	288.6	287.
40	287.9	288.3	289.1	289.2
44	288.5	288.8	289.3	290.5
48	288.8	289.5	289.8	291.4
52	289.2	289.9	290.1	292.1
56	289.7	290.5	290.6	292.7
60	290.1	-	291.1	293.1
64	290.6	-	291.5	293.5
68	290.9	-	291.9	293.8

	RUN 1	RUN 2	RUN 3	MODEL
72	291.2	-	292.3	294.1
76	291.5	-	-	-
80	291.8	-	-	294.5

Heat Transfer Coefficient Used in Model

$$h = 0.8953 + 49.3939u_{N+1} - 30.5409(u_{N+1})^2$$

$$J/(s \cdot m^2 \cdot K)$$

TABLE G-5: THAWING OF SUGARBEET CYLINDER, T_o=253 K

TIME (min)	CENTRE TEMPERATURE (K)			
	RUN 1 (T_a=299.0)	RUN 2 (T_a=297.9)	RUN 3 (T_a=297.9)	MODEL (T_a=298.4)
0	252.9	252.9	252.9	252.9
2	260.4	259.7	259.9	259.7
4	264.5	264.1	264.1	263.9
8	267.6	267.2	267.3	267.7
12	269.3	268.8	268.7	269.2
16	269.8	269.5	269.7	269.9
20	270.3	274.9	271.2	270.3
24	279.4	281.3	279.7	275.9
28	283.7	284.3	283.6	282.1
32	285.5	285.6	285.9	285.8
36	287.4	286.7	287.7	288.2
40	289.3	287.8	289.5	289.9
44	290.8	288.7	289.8	291.1
48	292.1	289.6	290.7	292.1
52	293.2	290.2	291.6	292.8
56	294.1	290.8	292.2	293.4
60	294.7	291.4	292.7	293.9
64	295.2	291.9	293.3	294.3
68	295.5	292.3	293.7	294.6

	RUN 1	RUN 2	RUN 3	MODEL
72	295.9	292.9	294.1	294.9
76	296.4	-	294.5	295.1
80	-	-	294.6	295.3

Heat Transfer Coefficient Used in Model

$$h = 0.0363 + 39.0551u_{N\cdot 1} - 19.3128(u_{N\cdot 1})^2$$

$$J/(s \cdot m^2 \cdot K)$$

TABLE G-6: THAWING OF SUGARBEET CYLINDER, T_o=263 K

TIME (min)	CENTRE TEMPERATURE (K)			
	RUN 1 (T_a=297.9)	RUN 2 (T_a=297.9)	RUN 3 (T_a=297.7)	MODEL (T_a=297.8)
0	263.2	263.2	263.2	263.2
2	265.3	265.5	265.5	265.9
4	267.1	267.2	267.2	267.4
8	268.9	268.9	268.9	269.0
12	269.7	269.7	269.7	269.9
16	270.2	270.4	272.2	270.3
20	280.3	280.5	281.5	275.9
24	284.3	284.5	284.7	282.1
28	285.9	286.5	286.2	285.8
32	287.7	287.5	287.5	288.3
36	289.1	288.7	289.5	290.0
40	290.2	289.8	290.1	291.3
44	291.1	290.9	290.7	292.3
48	291.8	291.7	291.5	293.1
52	292.3	292.3	291.9	293.7
56	292.8	293.1	292.3	294.2
60	293.3	293.8	292.7	294.6
64	293.6	293.9	293.3	294.9
68	293.9	294.2	293.6	295.2
72	294.3	294.4	293.9	295.4

	RUN 1	RUN 2	RUN 3	MODEL
76	294.5	294.5	294.1	296.1
80	294.7	-	294.5	296.3

Heat Transfer Coefficient Used in Model

$$h = 1.9937 + 26.9908u_{N+1} - 9.1479(u_{N+1})^2$$

$$J/(s \cdot m^2 \cdot K)$$

24. APPENDIX H

24.1 Computer Programs

24.1.1 Model for Freezing and Thawing of Sugarbeets

```
FTN4
      PROGRAM EMMA(,1000)
      IMPLICIT DOUBLEPRECISION(A - H,O - Z)
      DIMENSION U(21,22), T(21,22), XYZ(75), JANE(10), IEMM1(3),
     1          IEMM2(3), IEMMB(3)
      COMMON U, T, XYZ, JANE
      EQUIVALENCE (LU,JANE(3)), (JKAK,JANE(4))
C THIS PROGRAM PREDICTS THE TRANSIENT TEMP. DISTRIBUTION OF AN INFINITE
C SUGAR BEET CYLINDER
C THE PROGRAM ASSUMES 3 DIFFERENT REGIONS IN BEET FREEZING
C 1. UNFROZEN TISSUE( TEMP GREATER THAN TTC1)
C 2. TISSUE IN PROCESS OF BEING FROZEN ( TEMP BETWEEN TTC1 AND TTC2 OR
C  EQUAL TO ONE OF THEM)
C 3. FROZEN TISSUE ( TEMP. LESS THAN TTC2)
C APART FROM THE CENTRAL ELEMENT ( I=1) THE TEMP AT THE CENTER OF THE
C ELEMENT IS USED TO CALC. THE ENTHALPY OF AN ELEMENT. FOR THE CENTRAL
C ELEMENT THE TEMP. USED IS (2U(1,1)+U(2,1))/3
C THE PROGRAM ASSUMES THAT THERE IS A UNIFORM INITIAL BEET TEMP.
C THE PROGRAM ASSUMES THE ENVIRONMENT TEMP. CAN BE EXPRESSED AS A CUBIC
C FUNCTION OF TIME
C THE BEET IS ASSUMED TO BE ISOTROPIC
C THE NEWTON RAPHSON METHOD IS USED TO SOLVE EQUATIONS
C   NOMENCLATURE
C  **************
C      ALPHA=THERMAL DIFFUSITY OF UNFROZEN BEET
C      ATA(J)=AMBIENT TEMP AT TIME J
C      ATA(J)=TA+TA1*TIME+TA2*TIME**2+TA3*TIME**3
C       DTT=TIME INCREMENT IN MINUTES
C       DT=DIMENSIONLESS TIME INCREMENT
C      ENTHALPY=C1+C2*T+C3*T**2+C4*T**3
C      WHERE T IN DEGREES CENTIGRADE
C       C1,C2,C3,C4  ARE DETERMINED EXPERIMENTALLY
C       ENTHALPY ALSO = GO+G1*U+G2*U**2+G3*U**3
C      C1-C4,G1-G4ANDP1-P4 ARE THE CONSTANTS FOR UNFROZEN PHASE
C AC1 AC4,AG1-AG4 AND AP1-AP4 ARE THE CONSTANTS FOR 2-PHASE REGION
C BC1 BC4,BG1-BG4 AND BP1-BP4 ARE THE CONSTANTS FOR FROZEN REGION
C       H=SURFACE HEAT TRANSFER COEFFICIENT - DETERMINED
C EXPERIMENTALLY(CAL/SQ.CM.MIN.DEG CENT.)
C      R=RADIUS IN CM.
C      ROE=DENSITY OF BEET(FROZEN AND UNFROZEN) IN GRAMS/CUBIC CM.
C      TA=INITIAL AMBIENT TEMP
C      THERMAL CONDUCTIVITY=C5+C6*T+C7*T**2+C8*T**3
C       THERMAL CONDUCTIVITY=P+P1*U+P2*U**2+P3*U**3
C      TIME=(JA+1)*DTT=TIME IN MINUTES
```

```
C      TO=INITIAL UNIFORM TEMP OF BEET (DEG.CENT.)
C UC1  AND TTC1 = INITIAL FREEZING POINT
C UC2 AND TTC2 = TEMP AT WHICH FOOD CAN BE CONSIDERED FROZEN
C THAT IS CHANGE OF ENTHALPY AND THERMAL CONDUCTIVITY
C WITH TEMPERATURE IS SMALL
C  IF TO GREATER THAN TA THEN UC1 GREATER THAN UC2
C  IF TO LESS THAN TA THEN UCI LESS THAN UC2
C      W=RELAXATION FACTOR
       DATA IEMM1 /2HEM, 2HM1, 2H  /
       DATA IEMMB /2HEM, 2HMB, 2H  /
       DATA IEMM2 /2HEM, 2HM2, 2H  /
       WRITE (1,20)
       READ (1,*) LU
C      DO 1 JKAK=1,4
       JKAK = 1
       CALL LDSEG(IEMM1)
       CALL LDSEG(IEMMB)
       CALL LDSEG(IEMM2)
   10  CONTINUE
       STOP
   20  FORMAT (5X, 'ENTER LISTING LU')
       END
```

```
FTN4
      PROGRAM EMM1(5)
      IMPLICIT DOUBLEPRECISION(A - H,O - Z)
      DIMENSION XYZ(75), JANE(10), U(21,22), T(21,22)
      COMMON U, T, XYZ, JANE
      EQUIVALENCE (GO,XYZ(1)), (G1,XYZ(2)), (G2,XYZ(3)), (G3,XYZ(4)),
     1            (AGO,XYZ(5)), (AG1,XYZ(6)), (AG2,XYZ(7)),
     2            (AG3,XYZ(8)), (BGO,XYZ(9)), (BG1,XYZ(10)),
     3            (BG2,XYZ(11)), (BG3,XYZ(12)), (TO,XYZ(13)),
     4            (TA,XYZ(14)), (P,XYZ(15)), (P1,XYZ(16)), (P2,XYZ(17)),
     5            (P3,XYZ(18)), (AP,XYZ(19)), (AP1,XYZ(20)),
     6            (AP2,XYZ(21)), (AP3,XYZ(22)), (BP,XYZ(23)),
     7            (BP1,XYZ(24)), (BP2,XYZ(25)), (BP3,XYZ(26)),
     8            (UC2K,XYZ(27)), (UC1,XYZ(28)), (UC2,XYZ(29)),
     9            (TCO,XYZ(30)), (A1,XYZ(31)), (A,XYZ(32)),
     *            (GGO,XYZ(33)), (FAT,XYZ(34)), (GTOT,XYZ(35)),
     1            (DX2,XYZ(36)), (WEN3,XYZ(37)), (WEN4,XYZ(38)),
     2            (WEN5,XYZ(39)), (WEN6A,XYZ(40)), (WEN7,XYZ(41)),
     3            (H,XYZ(42)), (DT,XYZ(43)), (DX,XYZ(44)),
     4            (ALPHA,XYZ(45)), (ROE,XYZ(46)), (R,XYZ(47)),
     5            (DTT,XYZ(48)), (TTC1,XYZ(49)), (TTC2,XYZ(50)),
     6            (TTC2K,XYZ(51)), (NEXP,JANE(1)), (N,JANE(2)),
     7            (LU,JANE(3)), (W,XYZ(52)), (TA1,XYZ(53)),
     8            (TA2,XYZ(54)), (TA3,XYZ(55)), (TAD,XYZ(56))
     9            (JKAK,JANE(4)), (N2,JANE(5)), (N3,JANE(6))
     *            (N4,JANE(7)), (H2,XYZ(57)), (H3,XYZ(58)),
     1            (H4,XYZ(59)), (TO2,XYZ(60)), (TO3,XYZ(61)),
     2            (TO4,XYZ(62)), (TTA2,XYZ(63)), (TTA3,XYZ(64)),
     3            (TTA4,XYZ(65)), (DTT2,XYZ(66)), (DTT3,XYZ(67)),
     4            (DTT4,XYZ(68)), (AT,XYZ(69))
      GO TO (10, 20, 30, 40), JKAK
   10 WRITE (1,200)
      READ (1,*) H1, H2, H3, H4
      WRITE (1,210)
      READ (1,*) TO1, TO2, TO3, TO4
      WRITE (1,220)
      READ (1,*) TTA1, TTA2, TTA3, TTA4
      WRITE (1,230)
      READ (1,*) N1, N2, N3, N4
      WRITE (1,240)
      READ (1,*) DTT1, DTT2, DTT3, DTT4
      WRITE (1,250)
      WRITE (1,260)
      WRITE (1,270)
      READ (1,*) AT
      H = H1
```

```
      TO = TO1
      TA = TTA1
      DTT = DTT1
      N = N1
      GO TO 50
   20 H = H2
      TO = TO2
      TA = TTA2
      DTT = DTT2
      N = N2
      GO TO 50
   30 H = H3
      TO = TO3
      TA = TTA3
      DTT = DTT3
      N = N3
      GO TO 50
   40 H = H4
      TO = TO4
      TA = TTA4
      DTT = DTT4
      N = N4
   50 CONTINUE
      R = 0.4953DO
      ROE = 1.05DO
      ALPHA = 0.076DO
      NEXP = 1
      DX = 1.O / FLOAT(N)
      DT = ALPHA * DTT / R ** 2
      TA1 = O.ODO
      TA2 = O.ODO
      TA3 = O.ODO
      C1 = 76.91858DO
      C2 = 0.8827741DO
      C3 = O.ODO
      C4 = O.ODO
      GO = C1 + C2 * TA + C3 * TA ** 2 + C4 * TA ** 3
      G1 = (TO - TA) * (C2 + 2.O*C3*TA + 3.O*C4*TA**2)
      G2 = (C3 + 3.O*C4*TA) * (TO - TA) ** 2
      G3 = C4 * (TO - TA) ** 3
      AC1 = 5.3266701SDO
      AC2 = O.6092703SDO
      AC3 = O.0774607 2DO
      AC4 = O.0039860 1DO
      AGO = AC1 + AC2 * TA + AC3 * TA ** 2 + AC4 * TA ** 3
```

```
AG1 = (TO - TA) * (AC2 + 2.0*AC3*TA + 3.0*AC4*TA**2)
AG2 = (AC3 + 3.0*AC4*TA) * (TO - TA) ** 2
AG3 = AC4 * (TO - TA) ** 3
BC1 = 3.9876183D0
BC2 = 0.0722371D0
BC3 = 0.0D0
BC4 = 0.0D0
BG0 = BC1 + BC2 * TA + BC3 * TA ** 2 + BC4 * TA ** 3
BG1 = (TO - TA) * (BC2 + 2.0*BC3*TA + 3.0*BC4*TA**2)
BG2 = (BC3 + 3.0*BC4*TA) * (TO - TA) ** 2
BG3 = BC4 * (TO - TA) ** 3
C5 = 0.073696D0
C6 = 0.0003152D0
C7 = 0.0D0
C8 = 0.0D0
P = C5 + C6 * TA + C7 * TA ** 2 + C8 * TA ** 3
P1 = (TO - TA) * (C6 + 2.0*C7*TA + 3.0*C8*TA**2)
P2 = (C7 + 3.0*C8*TA) * (TO - TA) ** 2
P3 = C8 * (TO - TA) ** 3
AC5 = -0.1091889D0
AC6 = -0.0826254D0
AC7 = 0.0D0
AC8 = 0.0D0
AP = AC5 + AC6 * TA + AC7 * TA ** 2 + AC8 * TA ** 3
AP1 = (TO - TA) * (AC6 + 2.0*AC7*TA + 3.0*AC8*TA**2)
AP2 = (AC7 + 3.0*AC8*TA) * (TO - TA) ** 2
AP3 = AC8 * (TO - TA) ** 3
BC5 = 0.1362035D0
BC6 = -0.0153012D0
BC7 = -0.0008207D0
BC8 = -0.0000141D0
BP = BC5 + BC6 * TA + BC7 * TA ** 2 + BC8 * TA ** 3
BP1 = (TO - TA) * (BC6 + 2.0*BC7*TA + 3.0*BC8*TA**2)
BP2 = (BC7 + 3.0*BC8*TA) * (TO - TA) ** 2
BP3 = BC8 * (TO - TA) ** 3
TTC1 = -2.205D0
TTC2 = -7.205D0
TTC2K = -3.5D0
UC2K = (TTC2K - TA) / (TO - TA)
UC1 = (TTC1 - TA) / (TO - TA)
UC2 = (TTC2 - TA) / (TO - TA)
WRITE (LU,280) N, NEXP
WRITE (LU,290) R, ROE, ALPHA, DTT
WRITE (LU,300) TO, TA, TA1, TA2, TA3
WRITE (LU,310) TTC1, TTC2, TTC2K, H
```

```
      WRITE (LU,320) C1, C2, C5, C6
      WRITE (LU,330) AC1, AC2, AC3, AC4, AC5, AC6
      WRITE (LU,340) BC1, BC2, BC5, BC6, BC7, BC8
      WRITE (LU,350)
      IF (TO - TTC1) 60, 60, 70
   60 IF (TO - TTC2K) 90, 80, 80
   70 TCO = P + P1 + P2 + P3
      GO TO 100
   80 TCO = AP + AP1 + AP2 + AP3
      GO TO 100
   90 TCO = BP + BP1 + BP2 + BP3
  100 CONTINUE
      A1 = ALPHA * ROE * DX ** 2 / (2.0*DT*(TO - TA)*TCO)
      A = 3.0 * ALPHA * ROE * DX ** 2 / (4.0*(TO - TA)*DT)
      GGO = -3.0 * ROE * ALPHA * DX ** 2 / ((TO - TA)*DT)
      FAT = 2.0 * ALPHA * ROE * DX ** 2 / ((TO - TA)*TCO*DT)
      IF (TO - TTC1) 110, 110, 120
  110 IF (TO - TTC2) 160, 130, 130
  120 GTOT = GO + G1 + G2 + G3
      GO TO 190
  130 IF (NEXP - 2) 140, 140, 150
  140 GTOT = DEXP(AGO + AG1 + AG2 + AG3)
      GO TO 190
  150 GTOT = AGO + AG1 + AG2 + AG3
      GO TO 190
  160 IF (NEXP - 2) 170, 170, 180
  170 GTOT = DEXP(BGO + BG1 + BG2 + BG3)
      GO TO 190
  180 GTOT = BGO + BG1 + BG2 + BG3
  190 CONTINUE
      DX2 = DX ** 2 / 4.0D0
      WEN3 = ROE * ALPHA * (DX - DX2) / (2.0D0*(TO - TA)*DT)
      WEN4 = TCO * (1.0D0-DX/2.0D0) / (2.0D0*DX)
      WEN5 = R * H / 2.0D0
      WEN6A = (1.0D0-DX/2.0D0) / (3.0D0*DX)
      CALL RETRN
      CALL EMMA
  200 FORMAT (5X, 'ENTER H1,H2,H3,H4')
  210 FORMAT (5X, 'ENTER TO1,TO2,TO3,TO4')
  220 FORMAT (5X, 'ENTER TTA1,TTA2,TTA3,TTA4')
  230 FORMAT (5X, 'ENTER N1,N2,N3,N4')
  240 FORMAT (5X, 'ENTER DTT1,DTT2,DTT3,DTT4')
  250 FORMAT (5X, ' AT  VALUE SPECIFIED LESS THAN 0.0 FOR AIR THAW.')
  260 FORMAT (5X, 'AND GREATER THAN 0.0 FOR LT-50 BATH ')
  270 FORMAT (5X, 'ENTER  AT  VALUE')
```

```
280 FORMAT (1X, '     N=', I2, '   NEXP=', I2)
290 FORMAT (1X, '   R=', F4.1, '   ROE=', F6.3, '   ALPHA=', F6.3, '   DTT
   1=', F8.6)
300 FORMAT (1X, '   TO=', F6.1, '   TA=', F6.1, '   TA1=', F4.1, '   TA2=
   1', F4.1, '   TA3=', F4.1)
310 FORMAT (1X, '    TTC1=', F7.3, '    TTC2=', F7.3, '    TTC2K=', F7.3,
   1          '     H=', F7.4)
320 FORMAT (1X, '   C1=', F7.4, '    C2=', F8.5, '    C5=', F7.5, '    C6=
   1', F7.5)
330 FORMAT (1X, '   AC1=', F7.4, '    AC2=', F7.4, '    AC3=', F7.5, '
   1AC4=', F6.4, '   AC5=', F8.5, '   AC6=', F8.5)
340 FORMAT (1X, '   BC1=', F8.5, '    BC2=', F8.5, '    BC5=', F7.5, '
   1BC6=', F8.6, '    BC7=', F8.6, '    BC8=', F10.8)
350 FORMAT (1X, 'C3=C4=C7=C8=AC3=AC4=AC7=AC8=BC3=BC4=0.0')
    END
```

```
FTN4
      PROGRAM EMMB(5)
      IMPLICIT DOUBLEPRECISION(A - H,O - Z)
      DIMENSION U(21,22), T(21,22), XYZ(75), JANE(10)
      COMMON U, T, XYZ, JANE
      EQUIVALENCE (GO,XYZ(1)), (G1,XYZ(2)), (G2,XYZ(3)), (G3,XYZ(4)),
     1            (AGO,XYZ(5)), (AG1,XYZ(6)), (AG2,XYZ(7)),

     2            (AG3,XYZ(8)), (BGO,XYZ(9)), (BG1,XYZ(10)),
     3            (BG2,XYZ(11)), (BG3,XYZ(12)), (TO,XYZ(13)),
     4            (TA,XYZ(14)), (P,XYZ(15)), (P1,XYZ(16)), (P2,XYZ(17)),
     5            (P3,XYZ(18)), (AP,XYZ(19)), (AP1,XYZ(20)),
     6            (AP2,XYZ(21)), (AP3,XYZ(22)), (BP,XYZ(23)),
     7            (BP1,XYZ(24)), (BP2,XYZ(25)), (BP3,XYZ(26)),
     8            (UC2K,XYZ(27)), (UC1,XYZ(28)), (UC2,XYZ(29)),
     9            (TCO,XYZ(30)), (A1,XYZ(31)), (A,XYZ(32)),
     *            (GGO,XYZ(33)), (FAT,XYZ(34)), (GTOT,XYZ(35)),
     1            (DX2,XYZ(36)), (WEN3,XYZ(37)), (WEN4,XYZ(38)),
     2            (WEN5,XYZ(39)), (WEN6A,XYZ(40)), (WEN7,XYZ(41)),
     3            (H,XYZ(42)), (DT,XYZ(43)), (DX,XYZ(44)),
     4            (ALPHA,XYZ(45)), (ROE,XYZ(46)), (R,XYZ(47)),
     5            (DTT,XYZ(48)), (TTC1,XYZ(49)), (TTC2,XYZ(50)),
     6            (TTC2K,XYZ(51)), (NEXP,JANE(1)), (N,JANE(2)),
     7            (LU,JANE(3)), (W,XYZ(52)), (TA1,XYZ(53)),
     8            (TA2,XYZ(54)), (TA3,XYZ(55)), (TAD,XYZ(56))
      JA = 0
      J = 1
      IF (TO - TA) 50, 10, 10
   10 HOOKE = DABS(1.0D0-UC1)
   20 IF (HOOKE - 0.1) 40, 40, 30
   30 W = 0.5D0
      GO TO 60
   40 W = 0.1D0
      GO TO 60
   50 HOOKE = DABS(1.0D0-UC2)
      GO TO 20
   60 CONTINUE
      N1 = N + 1
C     INITIAL GUESSES FOR TEMPS AFTER FIRST TIME INCREMENT
C ***************************************************************
   70 CON1 = 0.5 / (1.0 - FLOAT(N1))
      CON2 = 1.0 - CON1
      DO 80 I = 1, N1
```

```
      U(I,1) = CON2 + CON1 * FLOAT(I)
   80 CONTINUE
      TAF = TA + TA1 * DTT + TA2 * DTT ** 2 + TA3 * DTT ** 3
      TAD = (TA - TAF) / (TO - TA)
      K = 0
   90 CONTINUE
      JAN = 1
      K = K + 1
C     CALC. OF BEET SURFACE TEMP AFTER FIRST TIME INCREMENT
C ***************************************************************
      WEND1 = (3.0D0*U(N + 1,1) + U(N,1)) / 4.0D0
  100 WEND2 = WEND1 ** 2
      WEND3 = WEND1 ** 3
      IF (TO - TA) 110, 110, 130
  110 IF (UC1 - WEND1) 120, 120, 150
  120 IF (UC2 - WEND1) 170, 160, 160
  130 IF (WEND1 - UC1) 140, 140, 150
  140 IF (WEND1 - UC2) 170, 160, 160
  150 RO = GO
      R1 = G1
      R2 = G2
      R3 = G3
      GO TO 180
  160 RO = AGO
      R1 = AG1
      R2 = AG2
      R3 = AG3
      GO TO 180
  170 RO = BGO
      R1 = BG1
      R2 = BG2
      R3 = BG3
  180 IF (TO - TA) 190, 190, 200
  190 IF (UC1 - WEND1) 220, 220, 210
  200 IF (WEND1 - UC1) 220, 220, 210
  210 WEST1 = RO + R1 * WEND1 + R2 * WEND2 + R3 * WEND3
      WEST2 = R1 + 2.0D0 * R2 * WEND1 + 3.0D0 * R3 * WEND2
      GO TO 240
  220 IF (NEXP - 2) 230, 230, 210
  230 WEST1 = DEXP(RO + R1*WEND1 + R2*WEND2 + R3*WEND3)
      WEST2 = (R1 + 2.0D0*R2*WEND1 + 3.0D0*R3*WEND2) * WEST1
  240 CONTINUE
      JAN = JAN + 1
      GO TO (250, 250, 260, 290), JAN
  250 WEST2 = 0.75D0 * WEST2
```

```
      WEST1 = WEST1 - GTOT
      U(N + 1,1) = U(N + 1,1) - W * (WEN4*(U(N + 1,1) - U(N,1)) + WEN5*(
     1U(N + 1,1) + 1.0D0+TAD) + WEN3*WEST1) / (WEN4 + WEN5 + WEN3*WEST2)
      SUM = 0.0D0
      SUM = SUM + DABS(WEN4*(U(N + 1,1) - U(N,1)) + WEN5*(U(N + 1,1) +
     11.0D0+TAD) + WEN3*WEST1)
C       CALCULATION OF INTERNAL TEMPS AFTER FIRST TIME INCREMENT
C ***************************************************************************
      I = N
      WEND1 = U(I,1)
      GO TO 100
  260 G = -FAT * FLOAT(I - 1)
      QQ = WEST1 - GTOT
      Q2 = WEST2
      U(I,1) = U(I,1) - W * ((FLOAT(I) - 0.5)*U(I + 1,1) - 2.0D0*FLOAT(
     1I - 1)*U(I,1) + (FLOAT(I) - 1.5)*U(I - 1,1) + G*QQ) / (-2.0*FLOAT(
     2I - 1) + G*Q2)
      SUM = SUM + DABS((FLOAT(I) - 0.5)*U(I + 1,1) - 2.0*FLOAT(I - 1)*U(
     1I,1) + (FLOAT(I) - 1.5)*U(I - 1,1) + G*QQ)
      IF (I - 2) 270, 270, 280
  270 WEND1 = (2.0D0*U(1,1) + U(2,1)) / 3.0D0
      GO TO 100
C     CALCULATION OF CENTER TEMP AFTER FIRST TIME INCREMENT
C ***************************************************************************
  280 JAN = JAN - 1
      I = I - 1
      WEND1 = U(I,1)
      GO TO 100
  290 ZZ = A1 * (WEST1 - GTOT)
      ZZZ = -1.0D0-(2.0D0/3.0D0) * A1 * WEST2
      U(1,1) = U(1,1) - W * (U(2,1) - U(1,1) - ZZ) / ZZZ
      SUM = SUM + DABS(U(2,1) - U(1,1) - ZZ)
      IF (K - 5) 90, 90, 300
  300 IF (SUM - 0.00001) 330, 330, 310
  310 IF (K - 3000) 90, 90, 320
  320 WRITE (LU,360)
      WRITE (LU,350) JA
  330 CONTINUE
      DO 340 I = 1, N1
        T(I,1) = U(I,1) * (TO - TA) + TA
  340 CONTINUE
      WRITE (LU,390)
      JAK = 1
      WRITE (LU,380) JAK
      WRITE (LU,400) (T(I,1),I=1,N1)
```

```
      WRITE (LU,370) TCO, WEST1
      CALL RETRN
      CALL EMMA
  350 FORMAT (1X, 'JA=', I4)
  360 FORMAT (5X, 'NO CONVERGENCE')
  370 FORMAT (2X, 'TCO=', F5.3, '    CENTER ENTHALPY=', F6.2)
  380 FORMAT (1X, 'J=', I4)
  390 FORMAT (1X, 'TEMPERATURES IN DEGREES CENTIGRADE')
  400 FORMAT (1X, 12F8.2)
      END
```

```
FTN4
      PROGRAM EMM2(5)
      IMPLICIT DOUBLEPRECISION(A - H,O - Z)
      DIMENSION U(21,22), UD(22), WD(22), V(21,22), UU(21,22), T(21,22),
     1          TWN(22), ATA(22), DATA(22), WN(22), WN2(22), TWN2(22),
     2          XYZ(75), JANE(10), TC1(21), TC2(21)
      COMMON U, T, XYZ, JANE
      EQUIVALENCE (GO,XYZ(1)), (G1,XYZ(2)), (G2,XYZ(3)), (G3,XYZ(4)),
     1            (AGO,XYZ(5)), (AG1,XYZ(6)), (AG2,XYZ(7)),
     2            (AG3,XYZ(8)), (BGO,XYZ(9)), (BG1,XYZ(10)),
     3            (BG2,XYZ(11)), (BG3,XYZ(12)), (TO,XYZ(13)),
     4            (TA,XYZ(14)), (P,XYZ(15)), (P1,XYZ(16)), (P2,XYZ(17)),
     5            (P3,XYZ(18)), (AP,XYZ(19)), (AP1,XYZ(20)),
     6            (AP2,XYZ(21)), (AP3,XYZ(22)), (BP,XYZ(23)),
     7            (BP1,XYZ(24)), (BP2,XYZ(25)), (BP3,XYZ(26)),
     8            (UC2K,XYZ(27)), (UC1,XYZ(28)), (UC2,XYZ(29)),
     9            (TCO,XYZ(30)), (A1,XYZ(31)), (A,XYZ(32)),
     *            (GGO,XYZ(33)), (FAT,XYZ(34)), (GTOT,XYZ(35)),
     1            (DX2,XYZ(36)), (WEN3,XYZ(37)), (WEN4,XYZ(38)),
     2            (WEN5,XYZ(39)), (WEN6A,XYZ(40)), (WEN7,XYZ(41)),
     3            (HO,XYZ(42)), (DT,XYZ(43)), (DX,XYZ(44)),
     4            (ALPHA,XYZ(45)), (ROE,XYZ(46)), (R,XYZ(47)),
     5            (DTT,XYZ(48)), (TTC1,XYZ(49)), (TTC2,XYZ(50)),
     6            (TTC2K,XYZ(51)), (NEXP,JANE(1)), (N,JANE(2)),
     7            (LU,JANE(3)), (W,XYZ(52)), (TA1,XYZ(53)),
     8            (TA2,XYZ(54)), (TA3,XYZ(55)), (TAD,XYZ(56)),
     9            (AT,XYZ(69))
C     RESULTS ARE PRINTED AFTER THE FIRST AND NPR'TH TIME INCREMENTS
C     AND THEN AFTER EVERY NPR*NPK'TH TIME INCREMENT
      NTIME = 6000
      NTIM = NTIME - 1
      NPK = 1
      NPK1 = NPK - 1
      NK = 1
      N1 = N + 1
C     NPR SHOULD NOT BE SPECIFIED GREATER THAN 20 . OTHERWISE T(I,J) ETC.
C     WILL NOT BE DEFINED AS THE DIMENSION STATEMENT IS LIMITED BY THE
C SMALL COMPUTER MEMORY
      NPR = 20
      NPR1 = NPR - 1
      NPR2 = NPR - 2
      NPR3 = NPR + 1
      NPNT = 6
C     IF NPNT IS SPECIFIED LESS THAN OR EQUAL TO 4 THE RESULTS
C     WILL ALSO BE PRINTED AFTER EVERY TIME INCREMENT.
```

```
C     OTHERWISE THEY WILL ONLY BE PRINTED AS SPECIFIED BY NPR.
C     IF WANT TO IGNORE NPR THEN JUST SET NPR GREATER THAN NTIME.
C     CANT HAVE NPR=1 OR 2 AS V(I,NPR-2) WILL BE UNDEFINED.
      J = 1
      DO 990 JA = 1, NTIM
        IF (AT) 10, 10, 60
   10   IF (U(N + 1,J) - 0.4) 30, 20, 20
   20   H = HO
        GO TO 70
   30   IF (U(N + 1,J) - 0.20) 50, 50, 40
   40   H = HO - 0.01
        GO TO 70
   50   H = 0.006
        GO TO 70
   60   H = HO
   70   WEN7 = R * H / 3.0D0
        TIME = FLOAT(JA + 1) * DTT
        IF (W - 0.1) 130, 130, 80
   80   IF (TO - TTC1) 120, 90, 90
   90   HOOK = DABS(U(N + 1,J) - UC1)
  100   IF (HOOK - 0.1) 110, 110, 130
  110   W = 0.1
        GO TO 130
  120   HOOK = DABS(U(N + 1,J) - UC2)
        GO TO 100
  130   CONTINUE
C     INITIAL GUESSES FOR TEMPS AFTER MORE THAN ONE TIME INC.
C ********************************************************************
        DO 140 I = 1, N1
          U(I,J + 1) = U(I,J) - 0.01
  140   CONTINUE
        JCK = 1
        UAV = (U(2,J) + U(1,J)) / 2.0
  150   IF (TO - TA) 160, 160, 180
  160   IF (UC1 - UAV) 170, 170, 200
  170   IF (UC2K - UAV) 220, 210, 210
  180   IF (UAV - UC1) 190, 190, 200
  190   IF (UAV - UC2K) 220, 210, 210
  200   R4 = P
        R5 = P1
        R6 = P2
        R7 = P3
        GO TO 230
  210   R4 = AP
        R5 = AP1
```

```
      R6 = AP2
      R7 = AP3
      GO TO 230
  220 R4 = BP
      R5 = BP1
      R6 = BP2
      R7 = BP3
  230 UAV2 = UAV ** 2
      UAV3 = UAV ** 3
      TKA = R4 + R5 * UAV + R6 * UAV2 + R7 * UAV3
      GO TO (240, 250, 260, 270), JCK
  240 TC = TKA
      JCK = 2
      UAV = (U(N + 1,J) + U(N,J)) / 2.0D0
      GO TO 150
  250 TCN = TKA
      I = N
      JCK = 3
      UAV = (U(I + 1,J) + U(I,J))./ 2.0D0
      GO TO 150
  260 TC1(I) = TKA
      JCK = 4
      UAV = (U(I,J) + U(I - 1,J)) / 2.0D0
      GO TO 150
  270 TC2(I) = TKA
      JCK = 3
      I = I - 1
      IF (I - 1) 290, 290, 280
  280 UAV = (U(I + 1,J) + U(I,J)) / 2.0D0
      GO TO 150
  290 CONTINUE
      WEN6 = TCN * WEN6A
      K = 0
      ATA(J + 1) = TA + TA1 * TIME + TA2 * TIME ** 2 + TA3 * TIME ** 3
      DATA(J + 1) = (TA - ATA(J + 1)) / (TO - TA)
  300 CONTINUE
      JAN = 1
C     BEET SURFACE TEMP AFTER MORE THAN ONE TIME INCREMENT
C ***************************************************************
  310 WN(J + 1) = U(N + 1,J + 1) - U(N,J + 1)
      IF (J - 2) 320, 350, 350
  320 IF (JA .LE. 2) GO TO 340
      IF (JA .LE. NPR3) GO TO 330
      TWN(J) = WN(NPR) + WN(NPR3) + WN(J + 1)
      UTOT = U(N + 1,NPR) + U(N + 1,NPR3) + U(N + 1,J + 1)
```

```
      UTOTA = DATA(NPR) + DATA(NPR3) + DATA(J + 1)
      WN(J) = WN(NPR3)
      DATA(J) = DATA(NPR3)
      GO TO 360
330   TWN(J) = WN(NPR1) + WN(NPR) + WN(J + 1)
      UTOT = U(N + 1,NPR1) + U(N + 1,NPR) + U(N + 1,J + 1)
      UTOTA = DATA(NPR1) + DATA(NPR) + DATA(J + 1)
      WN(J) = WN(NPR)
      DATA(J) = DATA(NPR)
      GO TO 360
340   WN(1) = U(N + 1,1) - U(N,1)
      DATA(1) = TAD
      TWN(1) = WN(2) + WN(1)
      UTOT = U(N + 1,2) + U(N + 1,1) + 1.0D0
      UTOTA = DATA(2) + TAD
      GO TO 360
350   TWN(J) = WN(J + 1) + WN(J) + WN(J - 1)
      UTOT = U(N + 1,J + 1) + U(N + 1,J) + U(N + 1,J - 1)
      UTOTA = DATA(J - 1) + DATA(J) + DATA(J + 1)
360   WND1 = (3.0D0*U(N + 1,J) + U(N,J)) / 4.0D0
370   WND2 = WND1 ** 2
      WND3 = WND1 ** 3
380   IF (TO - TA) 390, 390, 410
390   IF (UC1 - WND1) 400, 400, 430
400   IF (UC2 - WND1) 450, 440, 440
410   IF (WND1 - UC1) 420, 420, 430
420   IF (WND1 - UC2) 450, 440, 440
430   RO = GO
      R1 = G1
      R2 = G2
      R3 = G3
      GO TO 460
440   RO = AGO
      R1 = AG1
      R2 = AG2
      R3 = AG3
      GO TO 460
450   RO = BGO
      R1 = BG1
      R2 = BG2
      R3 = BG3
460   GO TO (470, 560, 470, 560, 470, 560), JAN
470   IF (TO - TA) 480, 480, 490
480   IF (UC1 - WND1) 510, 510, 500
490   IF (WND1 - UC1) 510, 510, 500
```

```
  500   QBBO = RO + R1 * WND1 + R2 * WND2 + R3 * WND3
        GO TO 530
  510   IF (NEXP - 2) 520, 520, 500
  520   QBBO = DEXP(RO + R1*WND1 + R2*WND2 + R3*WND3)
  530   CONTINUE
        JAN = JAN + 1
        IF (JAN .EQ. 6) GO TO 550
        IF (JAN .EQ. 4) GO TO 540
        WND1 = (3.0*U(N + 1,J + 1) + U(N,J + 1)) / 4.0DO
        GO TO 370
  540   WND1 = U(I,J + 1)
        GO TO 370
  550   WND1 = (2.0DO*U(1,J + 1) + U(2,J + 1)) / 3.0DO
        GO TO 370
  560   IF (TO - TA) 570, 570, 580
  570   IF (UC1 - WND1) 600, 600, 590
  580   IF (WND1 - UC1) 600, 60   590
  590   QBB1 = RO + R1 * WND1 +     * WND2 + R3 * WND3
        WNDY2 = R1 + 2.0DO * R2 * WND1 + 3.0DO * R3 * WND2
        GO TO 620
  600   IF (NEXP - 2) 610, 610, 590
  610   QBB1 = DEXP(RO + R1*WND1 + R2*WND2 + R3*WND3)
        WNDY2 = (R1 + 2.0DO*R2*WND1 + 3.0DO*R3*WND2) * QBB1
  620   CONTINUE
        IF (JAN .EQ. 6) GO TO 750
        IF (JAN .EQ. 4) GO TO 630
        JAN = JAN + 1
        WNDY1 = QBB1 - QBBO
        WNDY2 = 0.75DO * WNDY2
        U(N + 1,J + 1) = U(N + 1,J + 1) - W * (WEN6*TWN(J) + WEN7*(UTOT
     1  + UTOTA) + WEN3*WNDY1) / (WEN6 + WEN7 + WEN3*WNDY2)
        SUM = 0.0DO
        SUM = SUM + DABS(WEN6*TWN(J) + WEN7*(UTOT + UTOTA) + WEN3*WNDY1)
        I = N
        WND1 = U(I,J)
        GO TO 370
C       CALC. OF INTERNAL TEMPS AFTER MORE THAN ONE TIME INCREMENT
C ***************************************************************************
  630   C = (FLOAT(I) - 0.5) * TC1(I)
        D = -(FLOAT(I) - 0.5) * TC1(I) - (FLOAT(I) - 1.5) * TC2(I)
        E = (FLOAT(I) - 1.5) * TC2(I)
        GU = GGO * FLOAT(I - 1)
        QBB = QBB1 - QBBO
        QDD = WNDY2
        IF (J - 2) 640, 670, 670
```

```
  640   IF (JA .LE. 2) GO TO 650
        GO TO 660
  650   VIO = C + D + E
        V(I,1) = C * U(I + 1,1) + D * U(I,1) + E * U(I - 1,1)
        V(I,2) = C * U(I + 1,2) + D * U(I,2) + E * U(I - 1,2)
        VTOT = V(I,1) + V(I,2) + VIO
        GO TO 720
  660   CONTINUE
  670   V(I,J + 1) = C * U(I + 1,J + 1) + D * U(I,J + 1) + E * U(I - 1,
     1  J + 1)
        IF (J - 1) 680, 680, 700
  680   IF (JA .GE. NPR3) GO TO 690
        VTOT = V(I,J + 1) + V(I,NPR1) + V(I,NPR)
        V(I,J) = V(I,NPR)
        GO TO 710
  690   VTOT = V(I,J + 1) + V(I,NPR) + V(I,NPR3)
        V(I,J) = V(I,NPR3)
        GO TO 710
  700   VTOT = V(I,J + 1) + V(I,J) + V(I,J - 1)
  710   CONTINUE
  720   CONTINUE
        U(I,J + 1) = U(I,J + 1) - W * (VTOT + GU*QBB) / (D + GU*QDD)
        SUM = SUM + DABS(VTOT + GU*QBB)
        IF (I - 2) 730, 730, 740
  730   JAN = JAN + 1
        WND1 = (2.0D0*U(1,J) + U(2,J)) / 3.0D0
        GO TO 370
  740   JAN = JAN - 1
        I = I - 1
        WND1 = U(I,J)
        GO TO 370
C       CALC. OF CENTER TEMP AFTER MORE THAN ONE TIME INCREMENT
C ***************************************************************************
  750   UD(J + 1) = U(2,J + 1) - U(1,J + 1)
        IF (J - 2) 760, 790, 790
  760   IF (JA .LE. 2) GO TO 780
        IF (JA .LE. NPR3) GO TO 770
        WD(J) = UD(NPR) + UD(NPR3) + UD(J + 1)
        UD(J) = UD(NPR3)
        GO TO 800
  770   WD(J) = UD(NPR1) + UD(NPR) + UD(J + 1)
        UD(J) = UD(NPR)
        GO TO 800
  780   UD(1) = U(2,1) - U(1,1)
        WD(1) = UD(2) + UD(1)
```

```
      GO TO 800
  790 WD(J) = UD(J + 1) + UD(J) + UD(J - 1)
  800 QBC = QBB1 - QBB0
      QADC = (2.0D0*WNDY2*A) / (TC*3.0D0)
      U(1,J + 1) = U(1,J + 1) - W * (WD(J) - A*QBC/TC) / (-1.0 - QADC)
      SUM = SUM + DABS(WD(J) - A*QBC/TC)
      K = K + 1
      IF (U(N + 1,J + 1) - 0.1) 810, 810, 820
  810 SUMV = 0.00001
      GO TO 830
  820 SUMV = 0.0001
  830 CONTINUE
      IF (K - 5) 300, 300, 840
  840 IF (SUM - SUMV) 860, 860, 850
  850 IF (K - 1000) 300, 300, 1010
  860 T(1,J + 1) = U(1,J + 1) * (TO - TA) + TA
      IF (TO - TA) 900, 870, 870
  870 IF (T(1,J) + 10.0D0) 900, 890, 880
  880 IF (T(1,J + 1) + 10.0D0) 890, 890, 900
  890 WRITE (LU,1090)
      WRITE (LU,1100)
      WRITE (LU,1110) JA, T(1,J), T(1,J + 1)
  900 CONTINUE
C     CONVERT NORMALIZED TEMPS INTO CENTIGRADE
C ***************************************************************
      IF (JA - NPR1) 940, 940, 910
  910 IF (J .LE. NPR1) GO TO 980
      NK = NK + 1
      DO 920 I = 1, N1
        U(I,1) = U(I,NPR3)
  920 CONTINUE
      IF (NK .LE. NPK1) GO TO 970
      NK = 0
      DO 930 I = 1, N1
        T(I,J + 1) = U(I,J + 1) * (TO - TA) + TA
  930 CONTINUE
      T(1,1) = U(1,1) * (TO - TA) + TA
      GO TO 960
  940 IF (J .LE. NPR2) GO TO 980
      DO 950 I = 1, N1
        U(I,1) = U(I,NPR)
        T(I,NPR) = U(I,NPR) * (TO - TA) + TA
  950 CONTINUE
      T(1,1) = U(1,1) * (TO - TA) + TA
  960 CONTINUE
```

```
      JAA = JA + 1
      WRITE (LU,1250) JAA
      WRITE (LU,1240) (T(I,J + 1),I=1,N1)
      WRITE (LU,1120) TC, TCN, QBB1, H
      ECK = DABS(T(1,J + 1) - TA)
      IF (ECK - 0.5) 1080, 1080, 970
  970 CONTINUE
      J = 0
  980 CONTINUE
      J = J + 1
  990 CONTINUE
 1000 CONTINUE
      GO TO 1030
 1010 WRITE (LU,1230)
      WRITE (LU,1130) JA, SUM
      DO 1020 I = 1, N1
        T(I,J) = U(I,J + 1) * (TO - TA) + TA
 1020 CONTINUE
      WRITE (LU,1140) (T(I,J),I=1,N1)
 1030 CONTINUE
      IF (NPNT - 4) 1040, 1040, 1080
 1040 WRITE (LU,1160)
      DO 1060 J = 1, NTIME
        DO 1050 I = 1, N1
          T(I,J) = U(I,J) * (TO - TA) + TA
 1050   CONTINUE
 1060 CONTINUE
      DO 1070 J = 1, NTIME
        WRITE (LU,1170) J, (T(I,J),I=1,N1)
 1070 CONTINUE
 1080 CONTINUE
      CALL RETRN
      CALL EMMA
 1090 FORMAT (3X, 'TIME FOR CENTER TO REACH -10 DEG. C')
 1100 FORMAT (3X, '*******************************')
 1110 FORMAT (3X, 'J=', I4, '   T(1,J)=', F8.2, '   T(1,J+1)=', F8.2)
 1120 FORMAT (2X, 'K CENTER @ J=', F5.3, '   K SURF. @ J=', F5.3, '   CE
     1NTER ENTH @ J+1=', F6.2, '   H=', F5.3)
 1130 FORMAT (1X, 'JA=', I4, '   SUM=', F10.6)
 1140 FORMAT (1X, 12F8.2)
 1150 FORMAT (1X, 'TEMPERATURES IN DEGREES CENTIGRADE')
 1160 FORMAT (1X, 'TEMPERATURES AFTER EVERY TIME INCREMENT')
 1170 FORMAT (1X, 'J=', I4, 12F8.2)
 1180 FORMAT (5X, 'TEMPERATURES AFTER FIRST TIME INCREMENT')
 1190 FORMAT (1X, 12F9.4)
```

```
1200 FORMAT (5X, 'TEMPERATURES AFTER MORE THAN ONE TIME INCREMENT')
1210 FORMAT (5X, 'J=', I4)
1220 FORMAT (5X, 'K=', I4)
1230 FORMAT (5X, 'NO CONVERGENCE')
1240 FORMAT (1X, 12F8.2)
1250 FORMAT (1X, 'J=', I4)
     END
```

24.1.2 Analytical Solution for One-phase Heat Transfer

This program calculates the analytical solution for one-phase heat transfer in an infinite cylinder.

```
      EXTERNAL AJO, F; AJ1
      DIMENSION B(100), VAL(100), TEMP(50,100), TA(4)
C  THIS PROGRAM ONLY WORKS IF THERE IS NO PHASE CHANGE
C  ***************************************************
C  THIS PROGRAM GIVES THE ANALYTICAL SOLUTION FOR AN INFINITE
C  CYLINDER OR THE VALUE OF THE HEAT TRANSFER COEFFICIENT AT
C  DIFFERENT TIME INTERVALS. IF NP GREATER THAN OR EQUAL TO 2 THEN
C  THIS PROGRAM CALCULATES A TEMPERATURE PROFILE FOR GIVEN H.
C  IF NP LESS THAN 2 THEN THIS PROGRAM ESTIMATES H FOR EACH TIME
C  INTERVAL SO THAT THE CALCULATED TEMP. T(NN,J) IS WITHIN 0.001
C  OF THE ACTUAL TEMP TA(J). USUALLY NN=1 AND TA(J) IS THE MEAS-
C  -URED TEMP AT THE CENTER. HOWEVER NN CAN BE ANY GRID POINT.
C  AL=THERMAL DIFFUSITY  (SQ.CM./MIN)
C  DH=INCREMENT USED IN TRIAL AND ERROR DETERMINATION OF H
C   T=DIMENSIONLESS TIME STEP
C   DT=TIME STEP IN MINUTES
C  IFLAG=1 IMPLIES CONVERGENCE
C  IFLAG=2 IMPLIES FUNCTION VALUE EXACTLY ZERO
C  IFLAG=3 IMPLIES CONVERGENCE BUT F(B) IS GREATER THAN THE VALUES
C  F(B) AND F(C) WERE ON INPUT.
C  IFLAG=4 IMPLIES NO ROOT IN THE INITIAL INTERVAL
C  H=HEAT TRANSFER COEFF. AT THE SURFACE
C  M=NUMBER OF TERMS FROM THE INFINITE SERIES WHICH ARE USED
C  IN THE ANALYTICAL SOLUTION
C  NN=POSITION OF THERMOCOUPLE IN CYLINDER
C  N1=NUMBER OF SPACE INCREMENTS
C  NTIME=NO. OF TIME INCREMENTS
C  RAD=RADIUS OF CYLINDER IN CM.
C  TA(J)=EXPERIMENTALLY MEASURED TEMP. AT NN AND TIME INC-
C  REMENT J IN DEGREE CENTIGRADE
C  TC=THERMAL CONDUCTIVITY (CAL./CM.*MIN.*DEG.CENT.)
C  TINIT+DTT=INITIAL TIME(MIN.) FOR WHICH TEMPS. ARE REPORTED.
C  TINIT+DTT SHOULD NOT BE LESS THAN 2, IF THE PROGRAM IS BEING USED
C  TO ESTIMATE H.  VERY HIGH VALUES OF H CAN BE PREDICTED FOR SMALL
C  TIMES AND A LOT OF COMPUTER TIME IS WASTED IN CALCULATING THESE H
C  VALUES
C  TO=INITIAL UNIFORM TEMP. IN DEG. CENT.
C  TTA=ENVIRONMENT TEMP. IN DEG. CENT.
      KTIM = 0
      NN = 1
      NP = 1
C  NTIME SHOULD BE 4 OR INPUT & OUTPUT FORMAT STATEMENTS
C  WILL HAVE TO BE CHANGED IF ESTIMATING H
      NTIME = 4
```

```
C   IF NP LESS THAN 2 THEN DAMO USED TO ESTIMATE H VALUES FROM THE
C   ACTUAL TEMP. VERSUS TIME AT A GIVEN LOCATION.  THE CALCULATED
C   TEMPS. AT A GIVEN TIME ARE THE TEMPS. CALCULATED USING H VALUE
C   CALCULATED SO THAT THE CALCULATED TEMP. AT THE SPECIFIED LOCATION
C   IS EQUAL TO THE ACTUAL TEMP.    THE VARIATION OF H WITH TIME
C   IS PRINTED OUT.
C    IF NP GREATER THAN OR EQUAL TO 2 THEN H IS SPECIFIED AND THE
C   TEMP. PROFILE VERSUS TIME IS CALCULATED
      IF (NP - 2) 10, 20, 20
   10 READ (5,540) (TA(J),J=1,NTIME)
      WRITE (6,410)
      WRITE (6,420).
      WRITE (6,430) (TA(J),J=1,NTIME)
      WRITE (6,420)
      WRITE (6,440)
      WRITE (6,450)
   20 CONTINUE
      TC = 55.327
      N = 10
      N1 = N + 1
      TO = 22.9
      TTA = -30.3
      AL = 67.4
      RAD = 0.5
      TINIT = 0.0
      TIN = TINIT * AL / RAD ** 2
      DTT = 0.5
      DT = AL * DTT / RAD ** 2
      M = 10
      DR = RAD / FLOAT(N)
C     H=CONSTANT IF NP GREATER THAN OR EQUAL TO 2
C     H=INITIAL ESTIMATE IF NP LESS THAN 2
      TRI = 0.0
      DO 370 J = 1, NTIME
        H = 0.001
        DH = 100.0
        KVAL = 0
   30   CONTINUE
        T = FLOAT(J) * DT + TIN
   40   CONTINUE
        AA = RAD * H / TC
        IF (AA - 0.5) 50, 50, 60
   50   BNO = 0.0
        BN1 = 1.0
        GO TO 90
```

```
 60    IF (AA - 5.0) 70, 70, 80
 70    BNO = 0.9
       BN1 = 2.0
       GO TO 90
 80    BNO = 1.9
       BN1 = 3.0
 90    CONTINUE
       BNOI = BNO
       DO 150 K = 1, M
100      CALL ZEROIN(F, AA, BNO, BN1, 0.0, 1.E-5, IFLAG)
         IF (BNO - BNOI) 110, 120, 120
110      BNO = BNOI + 4.5
         BN1 = BNO + 3.0
120      IF (IFLAG - 4) 140, 130, 390
130      WRITE (6,460) H, BNO, BN1
         BNO = BNO - 2.5
         BN1 = BN1 + 2.5
         GO TO 100
140      CONTINUE
         RESIDL = F(BNO,AA)
         B(K) = BNO
         BNOI = BNO
         BNO = BNO + 2.5
         BN1 = BNO + 2.0
150   CONTINUE
      SUM = 0.0
      DO 160 K = 1, M
        Z = B(K)
        AJOZ = AJO(Z)
        VAL(K) = EXP(-(B(K)**2*T)) * 2.0 * AA / ((B(K)**2 + AA**2)*
     1  AJOZ)
        SUM = SUM + VAL(K)
160   CONTINUE
      TEMP(1,J) = TTA + (TO - TTA) * SUM
      DO 180 I = 2, N1
        SUM = 0.0
        DO 170 K = 1, M
          Z = B(K)
          R = FLOAT(I - 1) * DR
          Y = R * B(K) / RAD
          AJOY = AJO(Y)
          AJOZ = AJO(Z)
          VAL(K) = EXP(-(B(K)**2*T)) * 2.0 * AA * AJOY / ((B(K)**2 +
     1    AA**2)*AJOZ)
          SUM = SUM + VAL(K)
```

```
  170      CONTINUE
           TEMP(I,J) = TTA + (TO - TTA) * SUM
  180    CONTINUE
         IF (NP - 2) 190, 360, 360
  190    TRIO = TRI
         TRI = TEMP(NN,J) - TA(J)
         TRIX = ABS(TEMP(NN,J) - TA(J))
         TRIP = TRI * TRIO
         IF (TRIX - 0.01) 200, 200, 210
  200    WRITE (6,550) J, H
         GO TO 350
  210    KVAL = KVAL + 1
         IF (KVAL - 100) 230, 220, 220
  220    KVAL = KVAL - 100
         WRITE (6,470) KVAL, H
  230    IF (TO * TTA) 250, 250, 240
  240    IF (TEMP(NN,J) .LE. TA(J)) GO TO 300
         IF (TRIP) 260, 270, 270
  250    IF (TEMP(NN,J) .GE. TA(J)) GO TO 300
         IF (TRIP) 260, 270, 270
  260    DH = DH / 10.0
         H = H - 1.0 * DH
         GO TO 40
  270    IF (KTIM) 280, 280, 290
  280    H = H + 0.001
         KTIM = 1
         GO TO 40
  290    H = H + DH
         GO TO 40
  300    IF (TRIP) 310, 320, 320
  310    DH = DH / 10.0
         H = H - 1.0 * DH
         GO TO 40
  320    H = H - DH
         IF (H) 330, 330, 340
  330    H = 0.001
         DH = DH / 10.0
         GO TO 30
  340    CONTINUE
         GO TO 40
  350    TRI = 0.0
  360    CONTINUE
  370 CONTINUE
      WRITE (6,480) NN, NP, N, M, TINIT
      WRITE (6,490) AL, TC, RAD
```

```
      WRITE (6,500) TO, TTA, DTT, H
      WRITE (6,510)
      WRITE (6,520)
      DO 380 J = 1, NTIME
        WRITE (6,560) (TEMP(I,J),I=1,N1)
  380 CONTINUE
C     GO TO 400
  390 WRITE (6,530) IFLAG, BNO, J
  400 CONTINUE
      STOP
  410 FORMAT (2X, 'MEASURED TEMPERATURES AT CENTER AT EACH TIME INC.')
  420 FORMAT (1X, '*****************************************************')
  430 FORMAT (1X, 4F5.1)
  440 FORMAT (1X, 'H VALUE FOR EACH TIME INCREMENT')
  450 FORMAT (1X, '*******************************')
  460 FORMAT (1X, '  H=', F12.6, '  BNO=', F12.6, '  BN1=', F12.6)
  470 FORMAT (1X, 'KVAL=', I3, '  H=', F9.3)
  480 FORMAT (2X, 'NN=', I2, 2X, 'NP=', I2, 2X, 'N=', I2, 2X, 'M=', I3,
     1       2X, 'TINIT=', F6.2)
  490 FORMAT (2X, 'AL=', F7.3, 2X, 'TC=', F7.4, 2X, 'RAD=', F8.4)
  500 FORMAT (1X, 'TO=', F8.4, 2X, 'TTA=', F8.4, 2X, 'DTT=', F8.4, 2X, '
     1H=', F6.4)
  510 FORMAT (2X, 'TEMPERATURE PROFILE FROM ANALYTICAL SOLUTION')
  520 FORMAT (1X, '*********************************************')
  530 FORMAT (1X, 'IFLAG=', I3, '  B(K)=', E20.7, '  J=', I3)
  540 FORMAT (4F5.1)
  550 FORMAT (2X, 'J=', I2, 2X, 'H=', F5.3)
  560 FORMAT (1X, 11F8.4)
      END


      FUNCTION AJO(X)
      IF (X - 2.0) 10, 10, 20
   10 AJO = 1.0 - X ** 2 / 4.0 + X ** 4 / 2.0 ** 6 - X ** 6 / (2.0**8*9.
     1) + X ** 8 / (2.0**10*144.) - X ** 10 / (2.0**12*3600.0) + X **
     212 / (2.0**12*720.**2)
      RETURN
   20 QOX = -0.125 / X + 0.0732422 / X ** 3 - 0.2271080 / X ** 5 + 1.
     17277275 / X ** 7
      POX = 1.0 - 0.0703125 / X ** 2 + 0.1121521 / X ** 4 - 0.5725014 /
     1X ** 6 + 6.074042001 / X ** 8
      AJO = SQRT(0.636618/X) * (POX*COS(X - 0.7854) - QOX*SIN(X - 0.
     17854))
      RETURN
      END
```

```
      FUNCTION F(X, AA)
      IF (X - 2.0) 10, 10, 20
   10 AAJO = 1.0 - X ** 2 / 4.0 + X ** 4 / 2.0 ** 6 - X ** 6 / (2.0**8*
     19.) + X ** 8 / (2.0**10*144.) - X ** 10 / (2.0**12*3600.0) + X **
     212 / (2.0**12*720.**2)
      AAJ1 = 0.5 * X - X ** 3 / 16. + X ** 5 / 384. - X ** 7 / 18432. +
     1X ** 9 / 1474560.
      GO TO 30
   20 QOX = -0.125 / X + 0.0732422 / X ** 3 - 0.2271080 / X ** 5 + 1.
     17277275 / X ** 7
      POX = 1.0 - 0.0703125 / X ** 2 + 0.1121521 / X ** 4 - 0.5725014 /
     1X ** 6 + 6.074042001 / X ** 8
      AAJO = SQRT(0.636618/X) * (POX*COS(X - 0.7854) - QOX*SIN(X - 0.
     17854))
      P1X = 1. + 0.1171875 / X ** 2 - 0.144195557 / X ** 4 + 0.
     1676592588 / X ** 6 - 6.883914268 / X ** 8
      Q1X = .375 / X - .102539063 / X ** 3 + .277576447 / X ** 5 - 1.
     1993531734 / X ** 7
      AAJ1 = SQRT(0.636618/X) * (P1X*COS(X - 2.3562) - Q1X*SIN(X - 2.
     13562))
   30 F = X * AAJ1 - AA * AAJO
      RETURN
      END


      FUNCTION AJ1(X)
      IF (X - 2.0) 10, 10, 20
   10 AJ1 = 0.5 * X - X ** 3 / 16. + X ** 5 / 384. - X ** 7 / 18432. +
     1X ** 9 / 1474560.
      RETURN
   20 P1X = 1. + 0.1171875 / X ** 2 - 0.144195557 / X ** 4 + 0.
     1676592588 / X ** 6 - 6.883914268 / X ** 8
      Q1X = .375 / X - .102539063 / X ** 3 + .277576447 / X ** 5 - 1.
     1993531734 / X ** 7
      AJ1 = SQRT(0.636618/X) * (P1X*COS(X - 2.3562) - Q1X*SIN(X - 2.
     13562))
      RETURN
      END
```

```
      SUBROUTINE ZEROIN(F, AA, B, C, ABSERR, RELERR, IFLAG)
      U = 9.0E-7
      RE = AMAX1(RELERR,U)
      IC = 0
      ACBS = ABS(B - C)
      A = C
      FA = F(A,AA)
      FB = F(B,AA)
      FC = FA
      KOUNT = 2
      FX = AMAX1(ABS(FB),ABS(FC))
   10 IF (ABS(FC) .GE. ABS(FB)) GO TO 20
      A = B
      FA = FB
      B = C
      FB = FC
      C = A
      FC = FA
   20 CMB = 0.5 * (C - B)
      ACMB = ABS(CMB)
      TOL = RE * ABS(B) + ABSERR
      IF (ACMB .LE. TOL) GO TO 80
      IF (KOUNT .GE. 500) GO TO 120
      P = (B - A) * FB
      Q = FA - FB
      IF (P .GE. 0.0) GO TO 30
      P = -P
      Q = -Q
   30 A = B
      FA = FB
      IC = IC + 1
      IF (IC .LT. 4) GO TO 40
      IF (8.0*ACMB .GE. ACBS) GO TO 60
      IC = 0
      ACBS = ACMB
   40 IF (P .GT. ABS(Q)*TOL) GO TO 50
      B = B + SIGN(TOL,CMB)
      GO TO 70
   50 IF (P .GE. CMB*Q) GO TO 60
      B = B + P / Q
      GO TO 70
   60 B = 0.5 * (C + B)
   70 FB = F(B,AA)
      IF (FB .EQ. 0.0) GO TO 90
      KOUNT = KOUNT + 1
      IF (SIGN(1.0,FB) .NE. SIGN(1.0,FC)) GO TO 10
```

```
      C = A
      FC = FA
      GO TO 10
   80 IF (SIGN(1.0,FB) .EQ. SIGN(1.0,FC)) GO TO 110
      IF (ABS(FB) .GT. FX) GO TO 100
      IFLAG = 1
      RETURN
   90 IFLAG = 2
      RETURN
  100 IFLAG = 3
      RETURN
  110 IFLAG = 4
      RETURN
  120 IFLAG = 5
      RETURN
      END
```