Economic Modeling of Software Platform Design and Software Anti-Piracy

by

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ABSTRACT

This thesis is composed of three individual papers in information system area. In the first paper, we build an economic model to study the problem of offering a new, high-certainty channel on an existing business-to-consumer platform such as Taobao and eBay. In the second paper, we use game theoretical models to study how software firms should determine their anti-piracy efforts and product prices when the network effect exists. In the third paper, we investigate a setting where heterogeneous healthcare providers (HPs) can join one or two competing HIEs (monopoly and duopoly case). We use a game theoretical model to investigate how HIEs should price the basic and value-added services to maximize their profits.

PREFACE

Research projects that I report in this thesis are the result of research collaborations. Chapter 2 has been published as “Can Sun, Yonghua Ji, Bora Kolfal, and Ray Patterson. 2017. Business-to-consumer platform strategy: How vendor certification changes platform and seller incentives. ACM Trans. Manage. Inf. Syst. 8, 2–3, Article 6 (July 2017), 42 pages.” In this paper, I was responsible for model formulation and model solving. My supervisor Ji and other two coauthors give me directions and help me to overcome the difficulties. They also help me to editing the paper and response to the journal reviewers' comments. Chapter 3 has been submitted and is currently under review. I worked on the model formulation and model solving. My supervisor gave me the direction and suggestions when I met difficulties.

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Chapter 1

Introduction

This thesis is composed of three individual papers in information system area. I will briefly describe what I have done in the thesis in the introduction.

In the first paper, we build an economic model to study the problem of offering a new, high-certainty channel on an existing business-to-consumer platform such as Taobao and eBay. On this new channel, the platform owner exerts effort to reduce the uncertainty of service quality. Sellers can either sell through the existing low-certainty channel, or go through additional screening in order to sell on this new channel. We model the problem as a Bertrand competition game where sellers compete on price and exert effort to provide better service to consumers. In this game, we consider a reputation spillover effect which refers to the impact of the high-certainty channel on the perceived service quality in the low certainty-channel. Counter-intuitively, we find that low-certainty channel demand will decrease as the reputation spillover effect increases, in the case of low inter-channel competition. Also, low-certainty channel demand increases as the quality uncertainty increases, in the case of intense inter-channel competition. Furthermore, the platform owner should offer a new high-certainty channel when: (i) the perceived quality for this channel is sufficiently high, or (ii) sellers in this channel are able to efficiently provide quality service, or (iii) consumers in this channel are not so sensitive to the quality uncertainty, or (iv) the reputation spillover effect is high. In the one-channel case, the incentives of the platform owner and sellers are aligned for all model parameters. However, this is not the case for the two-channel solution, and our model reveals where tensions will arise between parties.

In the second paper, we use game theoretical models to study how software firms should determine their anti-piracy efforts and product prices when the network effect exists. A unique aspect of our model is that anti-piracy efforts have both a direct effect and a cross effect on software piracy. We explore the problem in a monopoly setting and then in a duopoly setting. We contribute to research on software anti-piracy in three ways. First, we analyze how the network effect and competition influence the firm's anti-piracy efforts and product prices. We find that an increase in the network effect does not necessarily mean lower anti-piracy efforts or higher product prices, as previous literature has suggested; instead, higher network effects may require higher anti-piracy efforts and lower product prices. Second, we study the impact of anti-piracy effort's cross effect which has not been studied in the previous literature. Since an increase in one firm's cross effect could cause its competitor's pirated product to be less attractive and could potentially benefit its competitor, we find a counter-intuitive result: a firm should exert more, instead of less, effort in anti-piracy to control software piracy when its cross effect increases. Third, we have obtained other interesting results through comparative statics. Those results could have important managerial implications for managing software piracy and pricing.

In the third paper, we investigate a setting where heterogeneous healthcare providers (HPs) can join one or two competing HIEs (monopoly and duopoly case). The utility for a healthcare provider is determined by the intrinsic value offered by an HIE and also the network effect, i.e., the number of healthcare providers adopting the same HIE. We use a game theoretical model to investigate how HIEs should price the basic and value-added services to maximize their profits. We investigate the government subsidy differences between monopoly and duopoly case. We also compare different settings in the monopoly and duopoly cases, and find out how parameters affect the basic service price and value-added service price.

Chapter 2

Business-to-Consumer Platform Strategy: How Vendor Certification Changes Platform and Seller Incentives

2.1. Introduction

Online retail sales worldwide reached 1.55 trillion US dollars in 2015 and are projected to grow to 3.4 trillion US dollars in 2019 (Statista.com, 2016). Much of online retailing is carried out through business-to-consumer (B2C) platforms. B2C platforms such as eBay.com and Taobao.com allow retailers to sell products directly to consumers. As more and more consumers purchase products conveniently online, such platforms play an increasingly important role in today's economy. However, a B2C platform's value diminishes when issues related to counterfeit products and perceived uncertainty of product quality arise.

One issue is the sale of counterfeit products (products that imitate more expensive, well-known brand-name products). This is a pervasive problem on e-Business platforms. Counterfeit products not only hurt consumers' utility, but also harm firms that sell legitimate products and the related e-Business platforms, as illustrated by a 4% decline in Alibaba's U.S. stock price when a counterfeit product issue became known (Wu, 2015). The Chinese e-Business company, Alibaba Group, which owns Taobao.com, was reported to have removed 114 million suspect listings in the first nine months of 2013 (Grant, 2014). In addition, it spends approximately $16 million U.S. dollars each year to combat this threat, including the establishment of a professional IP protection team of more than 5000 people (Simpson, 2014). eBay has also taken extensive measures to reduce counterfeits (eBay, 2015).

Another issue is the consumer's uncertainty of perceived product quality. Compared to brick-and-mortar stores, B2C firms are at a disadvantage as products are not immediately available for physical inspection by consumers before purchasing. This requires B2C platforms to provide services which reduce consumer uncertainty and garner consumer trust to compete with traditional brick-and-mortar stores. For instance, sellers can be encouraged to employ more customer service representatives to better handle consumers' questions and problems in a timely manner. Another mechanism is generous return policies and free shipping for returns which reduces the consumer's purchase risk by lowering the transaction costs associated with product returns (eBay Seller Center, 2016). Additionally, companies such as Best Buy offer low-price guarantees. All of these quality assurance actions on the part of sellers lead to higher consumer trust in the products sold through online platforms.

It may not be economically viable for an e-Business platform to solve these issues solely on their existing platform. An emerging solution is for an e-Business platform to create a second *vetted* channel where only certified sellers passing additional screening are allowed to sell. For instance, Alibaba, the owner of Taobao.com, established a second vetted channel called Tmall.com. Sellers wanting to sell in Tmall.com have to pay higher deposits, provide more certification material, and pay high fines if caught selling counterfeit products. At the same time, they pay higher transaction fees to Alibaba, the platform owner, to participate in this vetted channel (Tmall, 2016, Don, 2015). Product searches on Taobao.com yield both Tmall.com and Taobao.com sellers, with Tmall.com designations being prominently displayed. eBay also has created a similar secondary vetting mechanism. The Canadian version of eBay (eBay.ca) has a special website area called "Brand Vault", in which only vetted sellers can participate. Thus, we see examples of efforts by platform owners to establish second vetted channels, seemingly in response to problems with counterfeit products and consumer uncertainty.

In this paper we use a game theoretic model with price (Bertrand) competition. Consistent with Bertrand competition, we observe competition on quality (and therefore price) between channels such as Tmall.com and Taobao.com. We examine the conditions and effects of introducing a second, vetted channel. We will show the impact on platform owner and seller demand and profit when this vetted channel is introduced. Reputation spillover, quality, seller efficiency to provide high-quality service, and consumer sensitivity to quality in the vetted channel will be examined in terms of the impact of these model parameters on demand and profits. We will illustrate conditions when the one-channel or two-channel solution is best for the platform owner. Our model will reveal where tensions will arise between parties when a second, vetted channel is introduced.

2.2. Literature Review

There are four streams of literature related to our paper. The first stream is related to product competition. Vandenbosch and Weinberg (1995) investigates product differentiation on two dimensions. It finds that two firms tend to maximize differentiation on one dimension and minimize differentiation on the other dimensions, which is different from the one dimension case where firms tend to maximize differentiation. Dewan et al. (2003) incorporates the roles of the Internet and flexible manufacturing technologies (which can reduce design costs to produce tailored consumer goods) into the model of product customization and flexible pricing. It shows that when customization and information collection technologies improve, a monopoly seller may earn the highest profits by producing both standard and custom products, and can raise prices for both types of products. It also shows that in the duopoly case simultaneous adoption of customization reduces the differentiation between their standard products but does not intensify price competition. This illustrates that competition on quality and price can occur simultaneously. Mendelson and Parlaktürk (2008) investigates two firms competing on price and product variety. The first firm is a traditional firm choosing a limited set of product configurations, and the second firm is a customizing firm producing any configuration to order. It finds that the customizing firm's profit may decrease with the market size and its ease of customization and the traditional firm's profit may decrease with its holding cost. This illustrates that efforts to customize a product offering can be affected by stimuli in unique ways for different sellers. Casadesus-Masanell and Zhu (2010) analyzes the optimal strategy of a high-quality incumbent facing a low-price competitor with ads. It shows that the incumbent needs to reconfigure the business model when an ad-sponsored rival enters the market. This illustrates that high quality sellers will adjust strategies when competing with low quality sellers.

Caro and Martínez-de Albéniz (2012) investigates the satiation effect: when purchasing too much too quickly, consumers become satiated with a product. It finds that when a firm competes with a strategic competitor without managing its product's satiation effects, its profit may significantly reduce while the competitor will largely benefit. It also finds that when a firm manages the satiation effect more efficiently, the competitor may benefit if competition is on the product only, but not if the competition is on the price and product. The satiation effect illustrates that the actions of one competitor may benefit another competitor. Zeithammer and Thomadsen (2013) finds that when consumers seek varieties, then price competition will either soften or intensify, depending on the difference in firm qualities and the strength of consumer preference for variety.

While in our paper the difference in consumers' perceived quality between the two-channel is caused by platform owner's channel design, in the previous papers the sellers have different product qualities which enable competition. These papers often assume duopoly competition. In our model, since many sellers participate, the competition is more likely to be perfect competition instead of duopoly competition.

The second stream of related literature is on channel competition and coordination. Forman et al. (2009) empirically investigates the trade-off between the benefits of buying online versus buying in a local retail store. It finds that the disutility costs of purchasing online and the transportation costs of physical channels are important. Overby and Jap (2009) examines buyer and seller use of electronic and physical channels in the used vehicle market with uncertain quality. It finds that when quality uncertainty is high, products tend to be sold in a physical channel, and when quality uncertainty is low, products tends to be sold in an online channel. Tsay and Agrawal (2004) develops a model with key attributes of "channel conflict". It finds that when a manufacturer can adjust the price, a new direct channel will not necessarily hurt the reseller. Balakrishnan et al. (2014) studies how the browse-and-switch option effect affect physical retail and online pricing strategies and profits. It demonstrates that browse-and-switch behavior can indeed occur under equilibrium. The analysis further shows that the option for consumers to browse-and-switch intensifies competition, reducing the profits for both firms. In this stream of literature, the sellers determine whether to have a new channel (for instance, the e-business channel). In our paper, it is the platform owner who determines whether to open a new channel. Additionally, since there are many sellers in our problem, the sellers have less bargaining power compared with only a few sellers as is traditionally found in this stream.

The third stream of literature discusses the economics of B2C platforms. Liu et al. (2015) studies the problem of a website which maximizes its profit through optimally scheduling personalization services. Ryan et al. (2012) considers a single retailer, who currently sells its product only through its own website, but who may also choose to contract with Amazon to sell its product through the marketplace system. It finds conditions when the retailer should choose to sell through the marketplace system. Hagiu and Wright (2014) investigates whether an intermediary should choose to be a marketplace (in which sellers sell their products directly to buyers) or be a reseller (by purchasing products from sellers and reselling to buyers). However, none of these previous works take the perspective of the marketplace owner considering whether or not to establish a second vetted channel with high product quality and certainty. Bhargava and Choudhary (2004) also studies the decision of whether to establish a second channel. In contrast to Bhargava and Choudhary (2004), our paper considers both sellers' and the platform owner's effort. Their paper finds that the two-channel case is always better with respect to the channel owner's profit, while we find that the two-channel case is better than the one-channel case in certain parameter regions.

Finally, we present the fourth area of literature. Our paper introduces the concept of reputation spillover effect with respect to the platform itself. There are mainly three streams of literature related to reputation spillover effect. The first type of spillover effect, which is not considered in our paper, is where a firm's action affects the reputation of its competitors and collaborators (Yu and Lester, 2008, Lester and Sengul, 2002, Barnett and Hoffmanross, 2008, Kang, 2008, Lee and Rim, 2016); For example, Yu and Lester (2008) finds that that a reputational crisis may spillover from one organization to other organizations that are either geographically or structurally close to the focal organization. In the second type of spillover effect, one branded product's reputation affects another same-firm similarly branded product's reputation. Sullivan (1990) investigates the practice of umbrella branding (i.e., labelling more than one product with a single brand name). Umbrella branding is commonly used by multi-product companies. This paper finds that spillovers happen to identically branded products when information about one product affects the others. Voss and Gammoh (2004) examines the effect of an alliance with two, one, or zero well-known brand allies on evaluations of a previously unknown focal brand. It finds that the presence of a single brand ally significantly increases perceived quality and hedonic and utilitarian attitudes. In the third type of spillover effect, a product sold by different sellers is perceived differently. In this situation, the seller's reputation has an impact on the perceived product quality (e.g., an identical pair of pants sold by Walmart versus Neiman Marcus may be perceived as having different product quality due to the reputation spillover from the seller to the perceived product quality). Purohit and Srivastava (2001) assesses the effects of manufacturer reputation, retailer reputation, and product warranty on consumer perceptions of product quality. It highlights the important role that the retailer plays in assessments of product quality. Roggeveen et al. (2014) finds retailer's reputation can improve the effect of a guarantee policy. Wang et al. (2016) finds that the effect of product presentation on product evaluation is weakened by seller reputation under low-involvement situations.

Both the second and third streams of reputation spillover literature relate to our work. In our work, all authentic products are the same and have the same quality (similar to the third example). However, there is a chance that the product purchased will turn out to be non-authentic or a poor match and we consider consumer's perceived product quality at the time of purchase as an expected value that considers all these possible scenarios. Therefore, even though authentic products have the same innate quality, consumer's perceived product quality at the time of purchase might be different based on the channel (e.g., Tmall.com or Taobao.com). When products are sold in a vetted channel such as Tmall.com, consumers' perceived product quality is high, while in the unvetted channel such as Taobao.com, consumers' perceived product quality can be considerably lower. This relates to the third type of reputation spillover literature where the seller's reputation has an effect on consumers' perceived product quality. In our situation it is the reputation of the channel that is spilling over onto customer perceptions.

2.3. Exploratory Empirical Investigation

In this section, we present the results of an empirical analysis of Alibaba's unvetted seller platform Taobao.com and their vetted seller platform Tmall.com. The data was collected during March and April, 2016 from all 6 key product categories identified by the selling platform (baby, home life, electronics, women, men, and outdoor sports). Each category contains 100 best selling products as identified by the platform.

Table 2.1: Case Distribution for Each Category

	Baby	Homelife	Electronic	Men	Women	Sports
Case 1: Only Tmall	24	27	60	31	15	25
Case 2: Both Tmall and Taobao	15	23	40	36	12	18
Case 3: Only Taobao	61	50	0	33	73	57

For any given product, there are three possible channel distribution cases (see Table 2.1).

Table 2.2: Chi-squared Test Results (p-values)

	Homelife	Electronic	Men	Women	Sports
Baby	0.2286	0.0000	0.0001	0.1750	0.8070
Homelife	x	0.0000	0.0364	0.0037	0.5642
Electronic	x	x	0.0000	0.0000	0.0000
Men	x	x	x	0.0000	0.0014
Women	x	x	x	x	0.0587

Table 2.3: Product Classification by Transaction Volume in Case 2

	Baby	Homelife	Electronic	Men	Women	Sports
Tmall transaction ¿ Taobao transaction	13	22	40	35	12	18
Tmall transaction ¡ Taobao transaction	2	1	0	1	0	0

Table 2.4: Product Classification by Price in Case 2

	Baby	Homelife	Electronic	Men	Women	Sports
Tmall price ¿ Taobao price	8	12	25	24	9	14
Tmall price ¡ Taobao price	4	9	12	6	2	2
Tmall price = Taobao price	3	2	3	6	1	2

In Case 1, only Tmall sellers have 1 or more transactions, while no transactions exist for Taobao. For all six categories, there are 182 products for which only Tmall sellers have 1 or more transactions, while Taobao sellers have no transactions. It is possible that sellers with no sales exist in Taobao that offer the product. In Case 2, both Tmall and Taobao have sellers with 1 or more transactions. For all six categories, there are 144 products for which both Taobao and Tmall sellers have 1 or more transactions. In Case 3, only Taobao sellers have 1 or more transaction. For all six categories, there are 274 products for which only Taobao sellers have 1 or more sales transactions.

We use the Chi-squared test to determine whether categorical variables are independent. We test whether categories effect the products' distribution between the three cases. The null hypothesis is that categories do not effect the products' distribution among the three cases. We find that categories affect the distribution of products among the three cases. For instance, in the digital home products category, there are no instances in which products are sold by Taobao sellers only (Case 3). However, in other categories, there are products which are sold on Taobao only. Of the 15 pairwise comparisons between categories conducted to determine if the distribution between the cases are the same or different (see Table 2.2), 10 are significantly different at the p=0.05 level, and an 11th at the p=0.10 level. We therefore find strong evidence that product categories affect the products' distribution of sellers between the two channels.

When a product is simultaneously sold in both Taobao and Tmall, the Tmall transactions are typically greater than in Taobao (see Table 2.3), but the Tmall price is not necessarily higher than prices found in Taobao (see Table 2.4). A possible reason is that when consumers search for a product, they search by keywords, and different keywords may lead to different results. For example, in some searches, the Taobao product seller with the higher price may be shown, and the vetted Tmall seller with the lower price may not be shown. Thus, once in a while, consumers may only observe Taobao sellers, depending on the keywords selected. Our conclusion is that Taobao sellers with higher prices are possibly seeking extremely small volume sales at higher prices, and are in search of uninformed customers.

In the next section, we present a model to explore the phenomena where the B2C platform owner has an option to open a second vetted channel in direct competition with the first channel.

2.4. Model

Sellers and consumers sell and buy a single type of a product at an online platform. A game theoretic model is used to examine the impact of the platform owner introducing a second vetted channel. This vetted channel is a high-certainty channel with respect to service quality, which includes all experiences with the seller. The unvetted channel is considered to be a low-certainty channel. During the first stage of the game theoretic model, the platform owner decides whether to establish a new high-certainty channel. At the second stage, the platform owner decides how much effort to exert in order to reduce uncertainty in the high-certainty channel, and how much to charge sellers for selling on the platform. Then at the third stage, sellers decide whether to sell, and if they do sell, they must decide on the selling channel and how much effort they want to exert to increase certainty which will increase consumers' utility. At the final stage, the set of sellers engage in a Bertrand competition where they compete on price, given the quality, and consumers choose the channel and the seller (consumers may opt-out from the market entirely). Model notation is presented in Table 2.5.

For simplicity, we assume that each seller has one unit of the product for sale and one consumer buys one unit of the product. A seller with multiple units of product for sale can be considered as a segment of sellers, each with one unit of product. A consumer can not physically inspect a product before purchasing it online, but rather can only learn about the product and the seller through online descriptions and consumer reviews. Therefore, the consumers have uncertainty about the product quality and seller dependability, such as how well a product fits and whether a product sold by a particular seller is authentic or not.

The platform owner can offer two channels: high-certainty and low-certainty channels. We consider all purchase experience, after-sales experience from owning the product, and all experience with the seller as *service experience*, which we will refer to as service. When only the low-certainty channel exists, the perceived quality of the service has a distribution with mean 1 and variance σ_0^2. Before purchasing from a certain channel, the consumers' belief includes the following factors: the probability that the product is authentic, the perceived quality of the authentic product, and the probability of finding that the product does not match his or her needs. The product of these two factors determines the consumers' expected quality of product. In general, the consumers' expected quality of product is defined as:

Expected quality of product = Probability(authentic product) * Quality(authentic product)

+ Probability(non-authentic product) * Quality(non-authentic product)

- Probability(authentic but a poor match to consumer's needs) * Cost (poor match)

where Probability(authentic product) + Probability(non-authentic product) =1. For simplicity, we normalize the quality of the non-authentic product to be zero. Note that Qual-

Table 2.5: Model Notation

Parameters:	
α	Seller efficiency, uniformly distributed $U(0,1)$
ζ_h, ζ_l	Seller effort coefficient when they exert effort to increase consumer utility in the high- and low-certainty channels
k_p	Platform owner's effort coefficient when they exert effort to decrease quality variance in the high-certainty channel
$v_0(e)$, $v_h(e)$, $v_l(e)$	Consumer utility as a function of seller effort e in one-channel, high-certainty, and low-certainty channel cases
q	Perceived quality in the high-certainty channel
θ	Consumer quality preference parameter, uniformly distributed $U(0,1)$
r	Reputation spillover parameter
σ_0^2, σ_l^2, σ_h^2	Service quality variance in the one-channel, low- and high-certainty channels
s_h	Consumer sensitivity to quality variance in the high-certainty channel
Platform owner's decision variables:	
c_0, c_l	Seller unit transaction cost for using the low-certainty channel in the one- and two-channel cases
c_e	Additional unit transaction cost for using the high-certainty channel in the two-channel case
e_p	Platform owner's effort to decrease quality variance in the high-certainty channel
Seller decision variables:	
e_0, e_h, e_l	Seller effort in one-channel, high-certainty, and low-certainty channel cases
Intermediate variables:	
p_0, p_h, p_l	Product price in one-channel, high-certainty, and low-certainty channel cases
D_0, D_h, D_l, D_t	Demand in one-channel, high-certainty, low-certainty, and combined two-channel cases
S_0, S_h, S_l, S_t	Supply in one-channel, high-certainty, low-certainty, and combined two-channel cases
Outcome variables:	
Π_1, Π_2	Platform owner's profit in the one- and two-channel cases
π_0, π_h, π_l	Seller profit in one-channel, high-certainty, and low-certainty channel cases
SP_0, SP_h, SP_l, SP_2	Total seller profit in one-channel, high-certainty, low-certainty, and combined two-channel cases

ity(authentic product) can be improved with outstanding service such as a better return policy and rapid shipping. In the benchmark's case, let P_0 be the authenticity probability, and q_0 be the corresponding quality. Thus, we obtain:

Expected quality of product for benchmark case = $P_0 * q_0$

+ Probability(non-authentic product) * Quality(non-authentic product)

- Probability(authentic but a poor match to consumer's needs) * Cost(poor match)

= $P_0 * q_0$ - Probability(poor match to consumer's needs) * Cost(poor match),

and we normalize the expected quality of product for benchmark case to be 1 in our paper.

In the high-certainty channel, sellers offer generous return policies, pay higher deposit (which will be used to guarantee the authenticity of the product), or display the manufacturer's permit to sell. Then consumers' belief about the authenticity probability in the high-certainty channel (P_H) may rise (i.e., $P_H > P_0$). Also, it is in the sellers' interest to describe the products truthfully given the generous return policies and other restrictions of the high-certainty channel. Having more information on the true state of the product, a consumer will have a lower probability of finding the product unfit and incurring a cost to return it. Therefore, expected quality of product in the high-certainty channel (q_H) should be higher than that in the benchmarks' case (i.e., $q_H > q_0$).

Furthermore, a consumer, evaluating his or her utility of buying at the low-certainty channel, might also visit the high-certainty channel. Because the sellers in the high-certainty channel may be required to provide a generous return policy, for example, the high-certainty channel sellers are likely to provide a much clearer and more accurate description of the product. The description by sellers in the high-certainty channel could confirm this consumer's understanding of the product and reduce the chance of returning the product, thereby also leading to a higher expected quality in the low-certainty channel. That is the positive effect of reputation spillover between channels in our model. On the other hand, if a seller in the high-certainty channel displays the manufacturer's permit to sell while the counterpart in the low channel does not, then a consumer might trust such a seller in the low-certainty channel less and the authenticity probability in the low-certainty channel (P_L) becomes smaller than P_0. This is the negative reputation spillover effect, where $P_L < P_0$. As a result, the channel spillover effect (r) could be greater than, equal to, or less than 1. However, our analytical results are derived from comparative statics and hold regardless of whether r is larger than 1 or less than 1. We do not have any a priori presumption of the expected directionality of r.

As a platform owner exerts more effort on verifying and monitoring sellers' certification on the high-certainty channel, consumers' uncertainty of buying via the high-certainty channel is lower. Therefore, we model the variance σ_h^2 for the high-certainty channel as equation $\sigma_h^2 = (1 - e_p)\sigma_l^2$, where e_p is the platform owner's effort. The platform owner can require the sellers in the high-certainty channel to provide more certification materials or other activities which could increase the workload of the platform owner. The platform owner can also become involved in the transactions between sellers in the high-certainty channel and consumers by adjudicating disputes between sellers and consumers and distributing deposits held in escrow from sellers in the high-certainty channel to consumers in the event of seller misbehavior. These measures can make consumers more confident to purchase from the high-certainty channel, which will decrease the consumers' perceived quality variance.

Sellers need to determine whether to sell the product, and if they do, which channel to sell in for the two-channel case. Once they have chosen the channel, competition between these sellers and demand from consumers, together, will determine the price of the product. We denote the price as p_0 for the one-channel case, and p_h and p_l for high- and low-certainty channels in the two-channel case. At the same time, each seller can exert some effort e to serve customers and increase consumers' utility. We assume that the cost of effort is e_0/α for the one-channel case, and e_h/α and e_l/α for high- and low-certainty channels in the two-channel case. Here α denotes the seller's type and has a uniform distribution between 0 and 1, representing the fact that some sellers are very efficient at serving customers and some are not. The consumer utility gained by the seller's effort is $v_0(e_0)$ for the one-channel case, and $v_h(e_h)$ and $v_l(e_l)$ for high- and low-certainty channels in the two-channel case. For tractability of the model, we assume $v_0(e_0) = \zeta_l e_0^{1/2}$, $v_h(e_h) = \zeta_h e_h^{1/2}$ and $v_l(e_l) = \zeta_l e_l^{1/2}$, where the utility is an increasing function of effort with diminishing marginal returns. That is, when effort is high, the seller should exert more effort to gain the same extra unit of utility. Continuing with the concept of diminishing marginal returns, in this paper we are interested in the case when $\zeta_l > \zeta_h$, where consumers usually get more incremental satisfaction from the low-certainty channel given an identical amount of incremental service effort. We discuss the case when $\zeta_h > \zeta_l$ in Section 2.6.4.

In the one-channel case, consumer utility is determined by three factors: the channel chosen, the price of a particular seller p_0 and the additional utility $v_0(e_0)$ due to the effort exerted by this seller. We have:

$$u_0 = \theta - \sigma_0^2 - p_0 + v_0(e_0) \tag{2.1}$$

where θ denotes the consumer's type of quality preference, and it is uniformly distributed between 0 and 1. We have normalized consumer sensitivity to quality uncertainty in the low-certainty channel to 1. In the two-channel case, the consumer's utility of buying from the high-certainty channel is:

$$u_h = \theta q - s_h \sigma_h^2 - p_h + v_h(e_h) \tag{2.2}$$

where s_h is the consumer sensitivity to the quality uncertainty in the high-certainty channel. If a consumer is more sensitive to the quality uncertainty, then the consumer's utility would be lower given the same amount of uncertainty. To ensure consumers can get positive utility even when sellers don't exert any effort, we require $q > s_h \sigma_l^2$. In the two-channel case, the consumer's utility of buying from the low-certainty channel is:

$$u_l = \theta r - \sigma_l^2 - p_l + v_l(e_l) \tag{2.3}$$

where r is the perceived quality in the low-certainty channel including the impact of the reputation spillover effect.

In a particular channel, all sellers present the same utility value to a customer. At equilibrium, consumers who decided to buy the product are indifferent to buying from one seller or another. Otherwise, if this is not the case, a seller could adjust the price to attract consumers. Therefore, we can see from (2.1) that the difference between price and the extra utility generated by a seller's effort should be the same across sellers. As a result, we define the following:

$$p_0 - v_0(e_0) = \beta_0 \tag{2.4}$$

$$p_h - v_h(e_h) = \beta_h \tag{2.5}$$

$$p_l - v_l(e_l) = \beta_l \tag{2.6}$$

As explained above, β_0, β_h and β_l should be constants. Otherwise, suppose in the one-channel case, there are two sellers with different ${\beta_0}'$ and ${\beta_0}''$ values, where ${\beta_0}'$ is greater than β_0''. Then the seller with high β_0' could increase their price and obtain a higher profit. Thus, at equilibrium, we should have ${\beta_0}' = \beta_0''$ and β_0 is a constant. We can show that β_h and β_l are constants in the same way.

Seller profit is determined by three factors: the price of the product, the effort exerted, and fees charged by the platform owner. We denote the fees charged by the platform owner as c_0 in the one-channel case, and c_l in the two-channel case. In the two-channel case, a seller pays an extra transaction fee of c_e to be able to sell in the high-certainty channel. For example, sellers at Tmall (the high-certainty channel of Alibaba) have to pay higher transaction fees. Thus, in the one-channel case, the seller profit is:

$$\pi_0 = p_0 - (c_0 + \frac{e_0}{\alpha}) \tag{2.7}$$

In the two-channel case, the seller profit in the high-certainty channel is:

$$\pi_h = p_h - (c_l + \frac{e_h}{\alpha} + c_e) \tag{2.8}$$

while the seller profit in the low-certainty channel is:

$$\pi_l = p_l - (c_l + \frac{e_l}{\alpha}) \tag{2.9}$$

2.5. Analysis

In this section, we present the analysis of one- and two-channel cases. We begin with the one-channel case.

2.5.1 One-channel case

In the one-channel case, sellers who decide to sell the product choose the effort and price to maximize their profit (2.7) subject to the constraint (2.4). We can solve p_0 from (2.4) and substitute it into (2.7). Then the optimization problem is transformed to maximizing π_0 with respect to e_0, where:

$$\pi_0 = \beta_0 + \zeta_l e_0^{1/2} - (c_0 + \frac{e_0}{\alpha}) \tag{2.10}$$

From (2.10), optimal seller effort level is obtained as follows:

$$e_0 = (\frac{\alpha\zeta_l}{2})^2 \tag{2.11}$$

Only sellers with profit higher than 0 will participate. Substituting equation (2.11) into (2.10), we have:

$$\pi_0 = \beta_0 + \frac{\alpha\zeta_l^2}{2} - (c_0 + \frac{\alpha\zeta_l^2}{4}) > 0 \tag{2.12}$$

and therefore the market participation condition is:

$$\alpha > \alpha_0 \equiv \frac{4(c_0 - \beta_0)}{\zeta_l^2} \tag{2.13}$$

In other words, only sellers who serve customers more efficiently than the type α_0 seller will be in the market. Thus, the supply for the market is:

$$S_0 = 1 - \alpha_0 \tag{2.14}$$

For consumers, only those with utility u_0 (given by (2.1)) higher than 0 will be in the market. Substituting (2.4) into (2.1), we get:

$$u_0 = \theta - \sigma_0^2 - p_l + v_0(e_0) = \theta - \sigma_0^2 - \beta_0 > 0 \tag{2.15}$$

which implies:

$$\theta > \theta_0 \equiv \sigma_0^2 + \beta_0 \tag{2.16}$$

That is, only consumers who value the product sufficiently high ($\theta > \theta_0$) will buy the product. Thus, the demand of the product is given by:

$$D_0 = 1 - \theta_0 \tag{2.17}$$

In equilibrium, the supply of the product (2.14) equals the demand (2.17). That is:

$$\frac{4(c_0 - \beta_0)}{\zeta_l^2} = \sigma_0^2 + \beta_0 \tag{2.18}$$

which gives the expression of β_0:

$$\beta_0 = \frac{4c_0 - \sigma_0^2\zeta_l^2}{\zeta_l^2 + 4} \tag{2.19}$$

The platform owner's profit Π_1 in the one-channel case is given by the product demand D_0 multiplied by the unit cost charged by the platform owner (c_0). By using (2.16), (2.17), and (2.19):

$$\Pi_1 = D_0 c_0 = (1 - \frac{4c_0 - \sigma_0^2 \zeta_l^2}{\zeta_l^2 + 4} - \sigma_0^2) c_0 \tag{2.20}$$

Next, we find the optimal cost c_0 that maximizes the platform owner's profit (2.20) by using the first-order condition and get:

$$c_0 = ((\zeta_l^2 + 4) - 4\sigma_0^2)/8 \tag{2.21}$$

Then, by using (2.21), we obtain the optimal demand of the product from (2.17):

$$D_0 = \frac{1}{2} - \frac{2\sigma_0^2}{\zeta_l^2 + 4} \tag{2.22}$$

and the optimal platform owner's profit from (2.20):

$$\Pi_1 = \frac{(\zeta_l^2 - 4\sigma_0^2 + 4)^{\,2}}{16\,(\zeta_l^2 + 4r)} \tag{2.23}$$

From the expression of seller profit (2.12), we can get the profit for seller of type α:

$$\pi_0(\alpha) = \frac{1}{8}\zeta_l^2 \left(2\alpha - \frac{4\sigma_0^2}{\zeta_l^2 + 4r} - 1\right) \tag{2.24}$$

By using (2.13) and (2.23), we find the total seller profit in the one-channel case:

$$SP_0 = \int_{\alpha_0}^{1} \pi_0(\alpha) d\alpha = \frac{\zeta_l^2\,(\zeta_l^2 - 4\sigma_0^2 + 4)^{\,2}}{32\,(\zeta_l^2 + 4)^{\,2}} \tag{2.25}$$

2.5.2 Two-channel case

For the two-channel case, the platform owner's profit function is:

$$\Pi_2 = D_l c_l + D_h(c_l + c_e) - k_p {e_p}^2 \tag{2.26}$$

The first term $D_l c_l$, and the second term $D_h(c_l + c_e)$ are total transaction fees from the low- and high-certainty channels, respectively. The third term, $k_p {e_p}^2$, is the cost of effort for the platform owner which increases quadratically due to increasing marginal costs. The platform owner chooses e_p, c_l, and c_e to maximize their profit.

The sellers in the high-certainty channel choose the effort and price to maximize their profits (2.8) under constraint (2.5). Using (2.5), we transform the problem into profit maximization with respect to e_h:

$$\pi_h = \beta_h + \zeta_h e_h^{1/2} - (c_l + \frac{e_h}{\alpha} + c_e) \tag{2.27}$$

Therefore, by using the first-order condition for (2.27), we obtain the optimal seller effort e_h:

$$e_h = (\frac{\alpha\zeta_h}{2})^2 \tag{2.28}$$

The sellers in the low-certainty channel choose the effort and price to maximize their profit (2.9) under constraint (2.6). Using (2.6), we similarly transform the problem into profit maximization with respect to e_l:

$$\pi_l = \beta_l + \zeta_l {e_l}^{1/2} - (c_l + \frac{e_l}{\alpha}) \tag{2.29}$$

Similarly, we obtain the optimal effort e_l:

$$e_l = (\frac{\alpha\zeta_l}{2})^2 \tag{2.30}$$

A seller will only participate in a channel if the profit is greater than zero:

$$\pi_h = \beta_h + \alpha\zeta_h^2/4 - c_l - c_e > 0 \tag{2.31}$$

and

$$\pi_l = \beta_l + \alpha\zeta_l^2/4 - c_l > 0 \tag{2.32}$$

Sellers will only choose the high-certainty channel when π_h is greater than π_l and 0. Similarly, sellers will only choose the low-certainty channel when π_l is greater than π_h and 0. At $\pi_h = \pi_l$ we have the seller of type α_l who is indifferent between selling via the low- and high-certainty channels:

$$\alpha_l = \frac{-4c_e + 4\beta_h - 4\beta_l}{-\zeta_h^2 + \zeta_l^2} \tag{2.33}$$

Since $\zeta_h < \zeta_l$, we have $\pi_h < \pi_l$ when $\alpha > \alpha_l$. In other words, sellers who serve customers more efficiently than seller of type α_l will choose the low-certainty channel. The intuition is that those sellers can exert more effort with less cost, and therefore choose the low-certainty channel which values effort more. On the other hand, sellers who serve customers less efficiently than seller of type α_l will either choose the high-certainty channel, or will not sell if $\pi_h < 0$ in which case the seller is below α_h. By setting $\pi_h = 0$ in (2.31), we obtain:

$$\alpha_h = \frac{4c_e + 4c_l - 4\beta_h}{\zeta_h^2} \tag{2.34}$$

As a result, the supply of the high- and low-certainty channels are:

$$S_l = 1 - \alpha_l \tag{2.35}$$

$$S_h = \alpha_l - \alpha_h \tag{2.36}$$

Both channels exist with positive supply and demand when the following holds:

$$\alpha_l > \alpha_h \tag{2.37}$$

On the consumer side, a consumer will buy from the high-certainty channel when their utility u_h (given by (2.2)) is greater than u_l (given by (2.3)) and 0. Likewise, a consumer will buy from the low-certainty channel when u_l is greater than u_h and 0. At $u_h = u_l$, consumer of type θ_h is indifferent between buying from the low- and high-certainty channels. By using (2.2) and (2.3), we obtain:

$$\theta_h = \frac{\beta_h - \beta_l - \sigma_l^2 + s_h\sigma_l^2 - e_p s_h \sigma_l^2}{q - r} \tag{2.38}$$

Since the perceived quality in the high-certainty channel is higher ($q > r$), we can see from (2.2) and (2.3) that, $u_h > u_l$ when $\theta > \theta_h$. That is, a consumer who values service quality more than the consumer of type θ_h will buy from the high-certainty channel. It follows then that consumers in the range θ_l to θ_h will buy from the low-certainty channel, where customer of type θ_l is indifferent between buying the product or not. By setting $u_l = 0$ in (2.3), we obtain:

$$\theta_l = \frac{\beta_l + \sigma_l^2}{r} \tag{2.39}$$

As a result, the demand of the high- and low-certainty channels are:

$$D_h = 1 - \theta_h \tag{2.40}$$

$$D_l = \theta_h - \theta_l \tag{2.41}$$

Both channels exist with positive supply and demand when the following holds:

$$\theta_h > \theta_l \tag{2.42}$$

We define A, B, and C as follows, in order to simplify expressions in this section and in the appendix:

$$A \equiv \sigma_l^2 \left(\zeta_h^2 - s_h \left(\zeta_l^2 + 4r\right) + 4r\right) + (q - r) \left(\zeta_l^2 + 4r\right) \tag{2.43}$$

$$B \equiv k_p \left(-\zeta_h^4 + \zeta_h^2 \left(\zeta_l^2 - 4r\right) + 4q\zeta_l^2 + 16r(q - r)\right) - s_h^2 \sigma_l^4 \left(\zeta_l^2 + 4r\right) \tag{2.44}$$

$$C \equiv \frac{B\left(\zeta_l^2 - 4\sigma_l^2 + 4r\right)}{\zeta_l^2 + 4r} - \frac{A\left(4k_p\left(\zeta_h^2 + 4r\right)\right)}{\zeta_l^2 + 4r} \tag{2.45}$$

In equilibrium, the supply equals the demand: $S_l = D_l$ and $S_h = D_h$. From these two equations and by using (2.33) to (2.41), we obtain β_l and β_h. Substituting β_l and β_h into the platform owner's profit in (2.26), we find the optimal c_l and c_e that maximize the owner's profit as follows:

$$c_l = \frac{4r + \zeta_l^2 - 4\sigma_l^2}{8} \tag{2.46}$$

$$c_e = \frac{\zeta_h^2 - \zeta_l^2 + 4(q - r + (1 + (-1 + e_p)s_h)\sigma_l^2)}{8} \tag{2.47}$$

Substituting c_l and c_e back into (2.26), we get the optimal effort e_p that maximizes the owner's profit (through the first order condition):

$$e_p = \frac{s_h \sigma_l^2 A}{B} \tag{2.48}$$

Then, the optimal demands D_h and D_l can be derived from (2.40) and (2.41) as:

$$D_h = \frac{2k_p A}{B} \tag{2.49}$$

$$D_l = \frac{C}{2B} = \frac{1}{2}\left(\frac{\left(\zeta_l^2 - 4\sigma_l^2 + 4r\right)}{\zeta_l^2 + 4r} - \frac{D_h\left(2\left(\zeta_h^2 + 4r\right)\right)}{\zeta_l^2 + 4r}\right) \tag{2.50}$$

and the total demand in the two-channel case is given by

$$D_t = D_h + D_l = \frac{2D_h\left(\zeta_l^2 - \zeta_h^2\right) + \zeta_l^2 - 4\sigma_l^2 + 4r}{2\left(\zeta_l^2 + 4r\right)} \tag{2.51}$$

The platform owner's optimal profit in (2.26) can be written as:

$$\Pi_2 = \frac{8D_h A + (\zeta_l^2 - 4\sigma_l^2 + 4r)^2}{\zeta_l^2 + 4r} \tag{2.52}$$

Total seller profit in the high- and low-certainty channels are given by:

$$SP_h = \frac{k_p^2 \zeta_h^2 A^2}{2B^2} \tag{2.53}$$

$$SP_l = \frac{(\zeta_l^2 C + 8k_p \zeta_h^2 A)C}{32B^2} \tag{2.54}$$

2.6. Results and Insights

In this section, we summarize the results of our model. All proofs for theorems and table entries are presented in the Appendix. Our model assumptions and constraints are also listed in the Appendix.

The one-channel case will be used to compare the effects of introducing a second vetted channel. As discussed before, the one-channel case is assumed to be a low-certainty channel with respect to service quality. The impacts of parameters on demand and profit for the one-channel case are presented in Theorem 1 and Table 2.6.

Table 2.6: Parameter Impacts on Demand and Profit in One-Channel Case

	D_0	Π_1	SP_0
σ_0^2	$\frac{\partial D_0}{\partial \sigma_0^2} < 0$	$\frac{\partial \Pi_1}{\partial \sigma_0^2} < 0$	$\frac{\partial SP_0}{\partial \sigma_0^2} < 0$
ζ_l	$\frac{\partial D_0}{\partial \zeta_l} > 0$	$\frac{\partial \Pi_1}{\partial \zeta_l} > 0$	$\frac{\partial SP_0}{\partial \zeta_l} > 0$

Theorem 1. *For the one-channel case, the following hold:*

i. When σ_0^2 increases, the demand in the one-channel case D_0, the platform owner's profit Π_1, and seller profit SP_0 decreases.

ii. When ζ_l increases, the demand in the one-channel case D_0, the platform owner's profit Π_1, and seller profit SP_0 increases.

It is intuitive that when the quality variance σ_0^2 increases, consumers' utility of buying from the channel will decrease. In that case, more consumers may prefer not to buy and stay out of the market. As a result, the demand for the one-channel case will decrease. In turn, the platform owner will charge less for sellers to use the channel, so the platform owner's profit will decrease. When the seller effort coefficient ζ_l increases, sellers will be more efficient at increasing consumers' utility, everything else being the same. Therefore, sellers will be able to attract more consumers and the demand will increase. Consequently, the platform owner will be able to charge more from sellers to use the channel, so the platform owner's profit will increase.

We argue that, when the sign of the partial derivatives for a particular parameter is the same for two parties, then the incentives with respect to that parameter for those two parties are aligned. Note that as presented in Table 2.6, the incentives for the platform owner are perfectly aligned with the sellers in the one-channel case.

2.6.1 Parameter impacts on demand in two-channel case

In the two-channel case, the partial derivatives of demand with respect to model parameters are presented in Table 2.7. In terms of demand, the platform owner (represented by total demand D_t) is no longer perfectly aligned with either the low- or high-certainty channel sellers. We first analyze the comparative statics on the demand of the high-certainty channel D_h.

Theorem 2. *In the two-channel case, the following hold for the demand in the high-certainty channel D_h:*

1. *When q or ζ_h increases, demand in the high-certainty channel D_h increases.*

2. *When s_h, r, σ_l^2, or ζ_l increases, demand in the high-certainty channel D_h decreases.*

Table 2.7: Parameter Impacts on Demand in Two-Channel Case

	D_h	D_l	D_t
q	$\frac{\partial D_h}{\partial q} > 0$	$\frac{\partial D_l}{\partial q} < 0$	$\frac{\partial D_t}{\partial q} > 0$
s_h	$\frac{\partial D_h}{\partial s_h} < 0$	$\frac{\partial D_l}{\partial s_h} > 0$	$\frac{\partial D_t}{\partial s_h} < 0$
r	$\frac{\partial D_h}{\partial r} < 0$	$\frac{\partial D_l}{\partial r} \lesseqgtr 0^*$	$\frac{\partial D_t}{\partial r} \lesseqgtr 0^*$
σ_l^2	$\frac{\partial D_h}{\partial \sigma_l^2} < 0$	$\frac{\partial D_l}{\partial \sigma_l^2} \lesseqgtr 0^*$	$\frac{\partial D_t}{\partial \sigma_l^2} < 0$
ζ_l	$\frac{\partial D_h}{\partial \zeta_l} < 0$	$\frac{\partial D_l}{\partial \zeta_l} > 0$	$\frac{\partial D_t}{\partial \zeta_l} > 0$
ζ_h	$\frac{\partial D_h}{\partial \zeta_h} > 0$	$\frac{\partial D_l}{\partial \zeta_h} < 0$	$\frac{\partial D_t}{\partial \zeta_h} \lesseqgtr 0^*$

* Indicates sign changes depending on the cutoff point(s).

When the service quality of the high-certainty channel q increases, some consumers will move from the low- to high-certainty channel. When consumers become more sensitive to quality variance in the high-certainty channel (s_h increases), then the utility penalty for quality variance in the high-certainty channel ($s_h(1-e_p)\sigma_l^2$) will increase. This will decrease demand in the high-certainty channel D_h. Likewise, the same occurs when the quality variance in the low-certainty channel σ_l^2 increases, because this causes the high-certainty channel to become less attractive than before, due to the dependence of σ_h^2 on σ_l^2.

When seller effort becomes more effective in attracting consumers in the low-certainty channel (ζ_l increases), some consumers will move from the high- to low-certainty channel. Thus, the demand in the high-certainty channel D_h will decrease. When seller effort becomes more effective in attracting consumers in the high-certainty channel (ζ_h increases), some consumers will move from the low- to high-certainty channel. Thus, the demand in high-certainty channel D_h will increase.

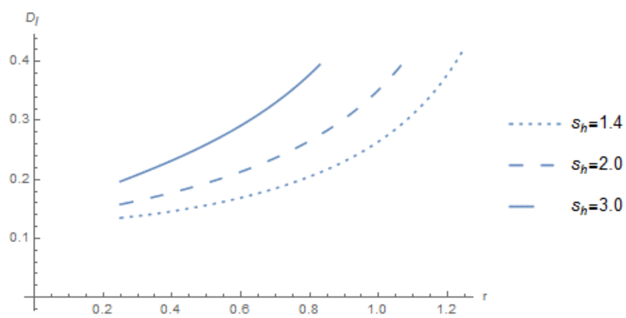
Theorem 3. *In the two-channel case, the following hold for the demand in the low-certainty channel D_l:*

1. *When q or ζ_h increases, demand in the low-certainty channel D_l decreases.*

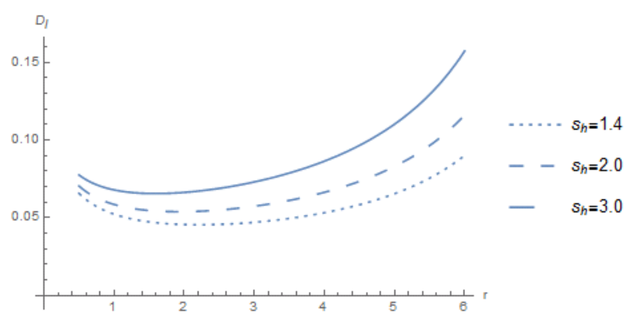
2. *When s_h or ζ_l increases, demand in the low-certainty channel D_l increases.*

3. *When $r < r_1'$ and r increases, then demand in the low-certainty channel D_l decreases. Conversely when $r > r_1'$ and r increases, then demand in the low-certainty channel D_l increases. See proof for the definition of r_1'.*

4. *When s_h is sufficiently small, i.e., $s_h < (\zeta_h^2 + 4q) / (\zeta_h^2 + 4r)$, demand in the low-certainty channel D_l decreases as σ_l^2 increases.*

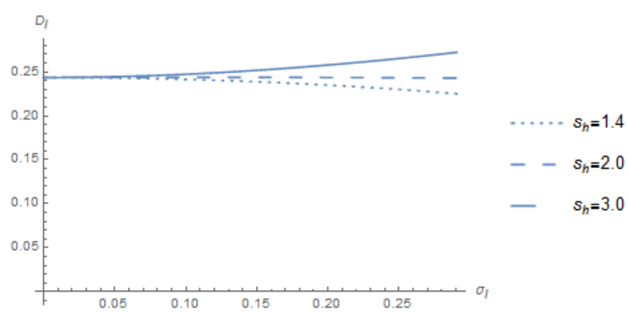


(a) Small q ($q = 1.4$)

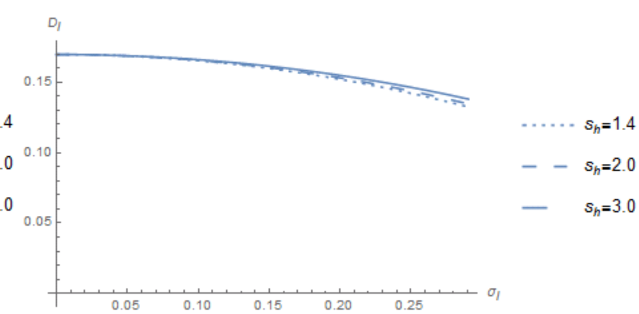


(b) Large q ($q = 8$)

Figure 2.1: D_l vs. r ($\zeta_h = 0.3, \zeta_l = 1.2, k_p = 1, \sigma_l = 0.5$)



(a) Small q ($q = 1.4$)



(b) Large q ($q = 8$)

Figure 2.2: D_l vs. σ_l ($\zeta_h = 0.3, \zeta_l = 1.2, k_p = 1, r = 0.7$)

For demand in the low-certainty channel D_l, parameters q, s_h, ζ_l and ζ_h have the opposite compared to their effect on the high-certainty channel demand D_h. The impact of reputation spillover r and quality variance σ_l^2 both become conditional in the low-certainty channel.

When the perceived quality in the high-certainty channel q increases, some consumers will move from the low- to high-certainty channel. When consumer's sensitivity to quality variance in the high-certainty channel s_h increases, consumer utility in the high-certainty channel will decrease, and some consumers will move from the high- to low-certainty channel.

When reputation spillover r is high enough, then as r increases, the low-certainty channel becomes more attractive to the consumers than the high-certainty channel. Thus, the low-certainty channel demand D_l will increase when r is high enough. Numerical analysis provides more insight on the impact of parameter q on the cutoff point r_1'. It should be noted that parameter r should fall within a certain range, specifically $\sigma_l^2 < r < \hat{r}$, where $\hat{r}$ is the upper bound. Consequently, a valid threshold r_1' should be within the same range as well. Otherwise, we set the threshold to the corresponding boundary value. For example, if $r_1' \leq \sigma_l^2$, then the threshold is set to $r_1' = \sigma_l^2$ and D_l will increase with all values of r (see proof for details). A similar situation holds for other threshold values mentioned in the paper, as well. The ranges for which thresholds are valid are presented in the corresponding proofs. Keeping this in mind, we see from Figure 2.1a that when q is small, r_1' equals its lower bound and therefore D_l increases for all possible values of r. Also, D_l is higher for any given value of r as s_h increases. As seen in Figure 2.1b, the situation changes when q is large. In this case, r_1' is valid (i.e., r_1' takes an interior value within its bounds) and therefore D_l decreases with r before r_1' and increases afterwards. Again, D_l is higher for any given value of r as s_h increases.

Intuitively, one may think that when reputation spillover effect r increases, the low-certainty channel demand D_l will always increase, since the consumers in the low-certainty channel will gain more utility. However, we find that demand in the low-certainty channel D_l decreases with r for a parameter region where r is quite low relative to q (see Figure 2.1b). This is primarily due to the fact that the inter-channel competition is not so intense in this region. For small values of r, as r increases, sellers in the low-certainty channel increase their price p while keeping their effort e_l the same (see (2.30)), while the channel owner increases transaction cost c_l (see (2.46)). As a result, the demand in the low-certainty channel D_l initially decreases. However, as r gets closer to q, the inter-channel competition intensifies and as r increases, the low-certainty channel start to take some demand away from the high-certainty channel, leading to an increase in D_l.

Similar numerical examination of D_l versus σ_l^2 also yields interesting results. As seen in Figure 2.2a, when q is small, D_l decreases with small values of s_h, but increases with large values of s_h. In contrast, when q is large, as seen in Figure 2.2b, D_l only decreases as σ_l^2 increases. The intuition behind this effect is that, when q is small, there is an intense inter-channel competition. In this case, for large values of s_h, as σ_l^2 increases, sellers in the high-certainty channel will be penalized more than sellers in the low-certainty channel. Then, the low-certainty channel gains market share at the expense of the high-certainty channel.

Theorem 4. *In the two-channel case, the following hold for the total demand D_t:*

i. When q or ζ_l increases, total demand in the two-channel case D_t increases.

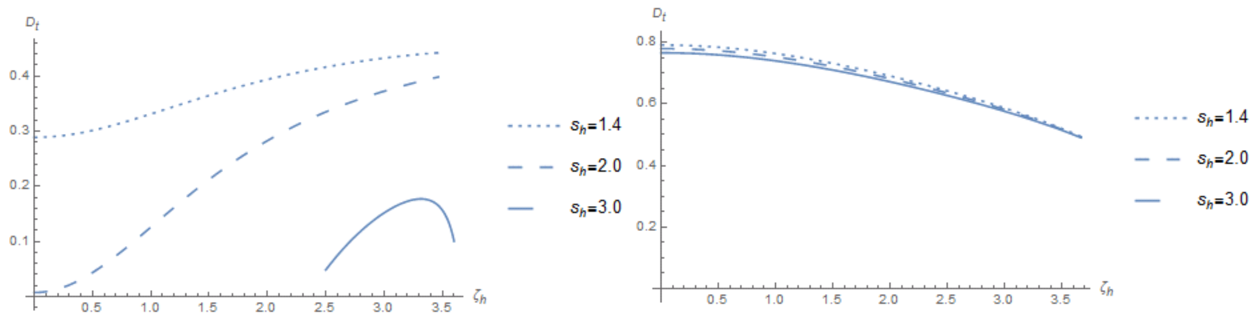
ii. When s_h or σ_l^2 increases, total demand in the two-channel case D_t decreases.

iii. When $r < r_2'$ and r increases, then total demand in the two-channel case D_t increases. Conversely, when $r > r_2'$ and r increases, then total demand in the two-channel case D_t decreases. See proof for definition of r_2'.

iv. When $\zeta_h < \zeta_h'$ and ζ_h increases, then total demand in the two-channel case D_t increases. Conversely, when $\zeta_h > \zeta_h'$ and ζ_h increases, then total demand in the two-channel case D_t decreases. See proof for definition of ζ_h'.

v. There exists a region with respect to the reputation spillover ($r_2'' < r < r_2'''$) for which the total demand in the two-channel case D_t will be greater than demand in the one-channel case D_0. See proof for definitions of r_2'' and r_2'''.

Total demand D_t for the platform owner is aligned with the high-certainty channel demand D_h for parameters q, s_h, and σ_l^2, while ζ_l is aligned with the low-certainty channel



(a) Small q ($q = 1.2$)　　(b) Large q ($q = 8$)

Figure 2.3: D_t vs. ζ_h $(r = 0.7, \zeta_l = 4, k_p = 1, \sigma_l = 0.8)$

demand D_l. When q increases, the high-certainty channel will become more attractive and get some consumers from the low-certainty channel. To compete with the high-certainty channel, sellers in the low-certainty channel will lower their price, attracting some consumers who previously did not buy. Then, the total demand D_t will increase.

When s_h increases, consumer utility in the high-certainty channel will decrease and some high-certainty channel consumers will switch to the low-certainty channel. As the channel becomes more desirable, the low-certainty channel sellers will increase their price. As a result, some marginal consumers will leave the market and choose not to buy the product, and total demand D_t will decrease.

When σ_l^2 increases, the low-certainty channel becomes less attractive to marginal consumers (near the indifferent customer with $\theta = \theta_l$), causing them to leave the market. Therefore, the total demand D_t decreases. The opposite is true for ζ_l, hence D_t increases with ζ_l.

As seen in Figure 2.3, D_t increases as s_h decreases for any given value of ζ_h. As illustrated in Figure 2.3a, when q and s_h are small, threshold ζ_h' is set to its upper bound and therefore, D_t is increasing with ζ_h. However, for a large s_h ($s_h = 3.0$), threshold ζ_h' is within bounds and therefore, D_t first increases until ζ_h' and then decreases as ζ_h continues to increase. For a large value of q, as seen in Figure 2.3b, we observe that threshold ζ_h' is set to its lower bound and therefore, D_t only decreases as ζ_h increases.

The last part of Theorem 4 defines the range for which the total demand for the two-channel case D_t will exceed the demand for the one-channel case D_0 with respect to r. It is interesting to see that the total demand in the two-channel case is higher only if the spillover effect is not too high or not too low. If the reputation spillover effect is low ($r < r_2''$), then a low-certainty channel is not so valuable to consumers and the total demand drops. On the other hand, if the spillover effect is too high ($r > r_2'''$), the low-channel channel sellers can charge high price and the marginal consumers near the indifferent consumer ($\theta = \theta_l$) drop out of the market and the total demand D_t becomes less than the demand in the one-channel case D_0. As a result, adding a second certified channel increases the total demand only if the reputation spillover effect is in a middle range.

We illustrate the effect of reputation spillover r on the platform owner's choice to open a second vetted channel in Figure 2.4. In this figure, the y-axis is labeled D for demand. Reputation spillover is key to understanding the addition of the second vetted channel. Both very low r and very high r values will cause the two-channel demand D_t to be lower than the alternative one-channel demand D_0.

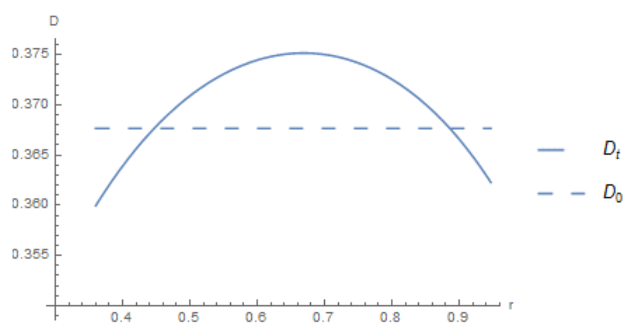


Figure 2.4: D_t and D_0 vs. r ($q = 1.4, \zeta_h = 0.3, \zeta_l = 1.2, k_p = 1, s_h = 2, \sigma_l = 0.6$)

2.6.2 Platform owner's effort in two-channel case

We now examine the impact of changes in various parameters on the platform owner's effort in the two-channel case. Results are presented in Table 2.8 and Theorem 5 below.

Theorem 5. *In the two-channel case, the following hold for the platform owner's effort e_p:*

Table 2.8: Parameter Impacts on Platform Owner's Effort in Two-Channel Case

	q	s_h	r	σ_l^2	ζ_l	ζ_h
e_p	$\frac{\partial e_p}{\partial q} > 0$	$\frac{\partial e_p}{\partial s_h} \lesseqgtr 0^*$	$\frac{\partial e_p}{\partial r} < 0$	$\frac{\partial e_p}{\partial \sigma_l^2} \lesseqgtr 0^*$	$\frac{\partial e_p}{\partial \zeta_l} < 0$	$\frac{\partial e_p}{\partial \zeta_h} > 0$

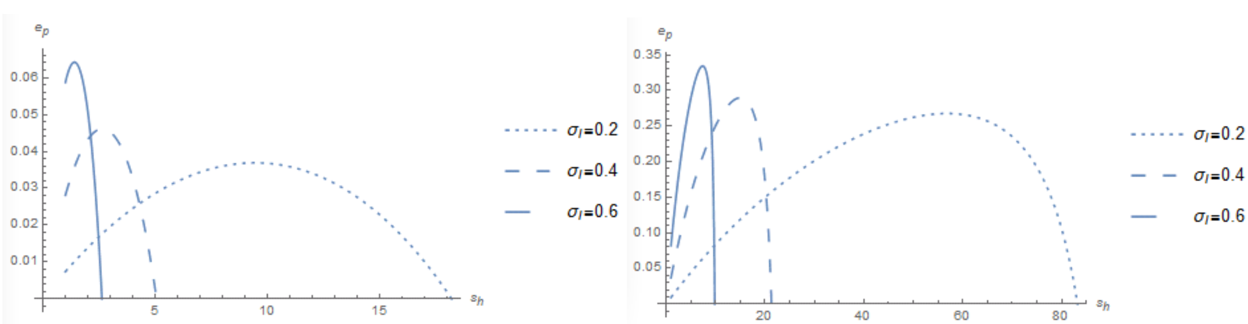
* Indicates sign changes depending on the cutoff point(s).

i. When q or ζ_h increases, the platform owner's effort e_p increases.

ii. When r or ζ_l increases, the platform owner's effort e_p decreases.

iii. When $s_h < s'_h$ and s_h increases, then the platform owner's effort e_p increases. Conversely, when $s_h > s'_h$ and s_h increases, then the platform owner's effort e_p decreases. See proof for definition of s'_h.

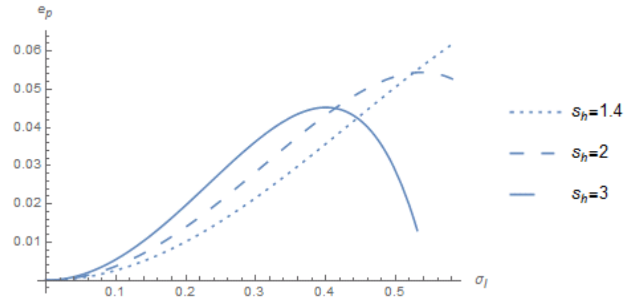
iv. When $\sigma_l^2 < \sigma_l''^2$ and σ_l^2 increases, then the platform owner's effort e_p increases. Conversely, when $\sigma_l^2 > \sigma_l''^2$ and σ_l^2 increases, then the platform owner's effort e_p decreases. See proof for definition of $\sigma_l''^2$.



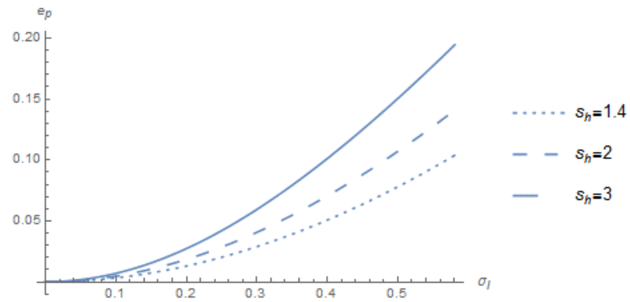
(a) Small q ($q = 1.4$) (b) Large q ($q = 4$)

Figure 2.5: e_p vs. s_h ($\zeta_h = 0.3, \zeta_l = 1.2, k_p = 1, r = 0.7$)

The incentives for the platform owner's effort e_p with respect to parameters q, r, ζ_l, and ζ_h are identical to the incentives for the high-certainty channel demand D_h. When q increases, the high-certainty channel becomes more attractive for consumers and the channel owner. When ζ_h increases, the high-certainty channel sellers exert more effort. In both of these cases for q and ζ_h, it is beneficial for the platform owner to increase their effort e_p to



(a) Small q ($q = 1.4$)



(b) Large q ($q = 4$)

Figure 2.6: e_p vs. σ_l ($\zeta_h = 0.3, \zeta_l = 1.2, k_p = 1, r = 0.7$)

attract more consumers and increase revenues. On the other hand, when r or ζ_l increases, the high-certainty channel becomes less attractive and platform owner will decrease their effort e_p.

From Figure 2.5, it is interesting to see that the platform owner's effort e_p first increases and then decreases as s_h increases. When s_h is small, as s_h increases, the platform owner reacts by increasing their effort in order to make the high-certainty channel more attractive. However, as s_h further increases to higher levels, the platform owner reacts by exerting less effort. This result occurs because of diminishing returns on effort for the platform owner.

In Figure 2.6, we observe that e_p increases as σ_l increases, especially when σ_l is small. When service quality uncertainty σ_l increases, both channels look less desirable to consumers. The platform owner reacts by reducing the uncertainty via e_p and make the high-certainty channel more attractive. In Figure 2.6b, the threshold $\sigma_l''^2$ is set to its upper bound, and therefore the platform owner's effort e_p increases for all possible values of σ_l. However, for a certain parameter region (for example, in Figure 2.6a when q is small and s_h is large) when σ_l further increases to higher levels, the platform owner reacts by reducing effort because of the diminishing returns.

2.6.3 Parameter impacts on profit

For the two-channel case, partial derivatives for total seller profit for the high-certainty channel SP_h, total seller profit in the low-certainty channel SP_l, and platform owner's profit Π_2 with respect to model parameters are presented in Table 2.9.

Table 2.9: Parameter Impacts on Profit in Two-Channel Case

	SP_h	SP_l	Π_2
q	$\frac{\partial SP_h}{\partial q} > 0$	$\frac{\partial SP_l}{\partial q} < 0$	$\frac{\partial \Pi_2}{\partial q} > 0$
s_h	$\frac{\partial SP_h}{\partial s_h} < 0$	$\frac{\partial SP_l}{\partial s_h} > 0$	$\frac{\partial \Pi_2}{\partial s_h} < 0$
r	$\frac{\partial SP_h}{\partial r} < 0$	$\frac{\partial SP_l}{\partial r} \lesseqgtr 0^*$	$\frac{\partial \Pi_2}{\partial r} \lesseqgtr 0^*$
σ_l^2	$\frac{\partial SP_h}{\partial \sigma_l^2} < 0$	$\frac{\partial SP_l}{\partial \sigma_l^2} \lesseqgtr 0^*$	$\frac{\partial \Pi_2}{\partial \sigma_l^2} < 0$
ζ_l	$\frac{\partial SP_h}{\partial \zeta_l} < 0$	$\frac{\partial SP_l}{\partial \zeta_l} > 0$	$\frac{\partial \Pi_2}{\partial \zeta_l} > 0$
ζ_h	$\frac{\partial SP_h}{\partial \zeta_h} > 0$	$\frac{\partial SP_l}{\partial \zeta_h} \lesseqgtr 0^*$	$\frac{\partial \Pi_2}{\partial \zeta_h} > 0$

* Indicates sign changes depending on the cutoff point(s).

Theorem 6. *In the two-channel case, the following hold for total seller profit in the high-certainty channel SP_h:*

i. When q or ζ_h increases, total seller profit in the high-certainty channel SP_h increases.

ii. When s_h, r, σ_l^2, or ζ_l increases, total seller profit in the high-certainty channel SP_h decreases.

The partial derivative directions for SP_h match the partial derivative directions for D_h for all parameters (see Tables 2.7 and 2.9). The intuition for the results of Theorem 6 is similar to the intuition for Theorem 2, which is presented in Section 2.6.1.

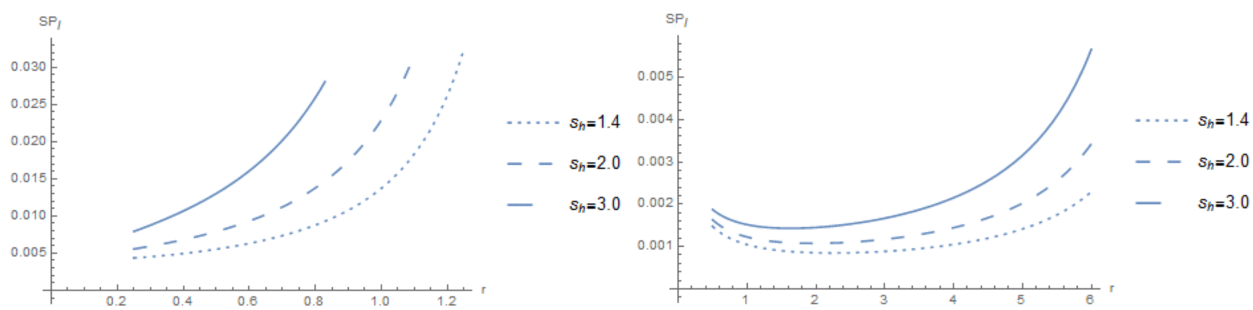
Theorem 7. *In the two-channel case, the following hold for total seller profit in the low-certainty channel SP_l:*

i. When q increases, total seller profit in the low-certainty channel SP_l decreases.

ii. When s_h or ζ_l increases, total seller profit in the low-certainty channel SP_l increases.

iii. When $r > r'_3$ and r increases, then total seller profit in the low-certainty channel SP_l increases. See proof for definition of r'_3.

iv. When $\zeta_h < \zeta''_h$ and ζ_h increases, then total seller profit in the low-certainty channel SP_l decreases. See proof for definition of ζ''_h.



(a) Small q ($q = 1.4$) (b) Large q ($q = 8$)

Figure 2.7: SP_l vs. r ($\zeta_h = 0.3, \zeta_l = 1.2, k_p = 1, \sigma_l = 0.5$)

The partial derivative directions for SP_l match the partial derivative directions for D_l for all parameters, except ζ_h (see Tables 2.7 and 2.9). Therefore, the intuition for the results of Theorem 7 is similar to the intuition for Theorem 3, except for the impact of ζ_h.

As the high-certainty channel sellers become more efficient in exerting effort (ζ_h increases), high-certainty channel sellers will increase their effort levels and sellers in both channels will raise prices. When ζ_h is small, the high-certainty channel sellers are very inefficient and therefore not so competitive, compared to the low-certainty channel sellers. In this case, when ζ_h increases, the effectiveness and competitiveness gain for the high-certainty channel relative to the low-certainty channel is high and therefore the decrease in D_l is large. The increase in price does not make up for the decrease in D_l initially and SP_l decreases.

Theorem 8. *In the two-channel case, the following hold for the platform owner's profit* Π_2*:*

i. When q*,* ζ_h*, or* ζ_l *increases, the platform owner's profit* Π_2 *increases.*

ii. When s_h *or* σ_l^2 *increases, the platform owner's profit* Π_2 *decreases.*

iii. When $r > r_4'$ *and* r *increases, the platform owner's profit* Π_2 *increases. See proof for definition of* r_4'*.*

From Table 2.9 and Theorem 8, we get the following results:

Corollary 8.1. *When* q*,* r*, or* ζ_h *is sufficiently high or when* s_h *is sufficiently low, the platform owner's profit increases if they offer a vetted channel.*

Theorem 8 and Corollary 8.1 help us gain more insights into the research question of when to offer a second, high-certainty channel. When the high-certainty channel consumers are not so sensitive to the quality uncertainty (s_h is sufficiently low) or when the perceived quality q in the new channel is sufficiently high, then this new channel is valuable to the channel owner and should be offered to the consumers. Another case in which the channel owner would benefit from offering the high-certainty channel is when the sellers in this channel are efficient in offering service (ζ_h is sufficiently high). We can draw some managerial insights from Corollary 8.1. In order to maximize the profit through offering a new channel, channel owner should focus on building brand recognition for the new channel, so that the perceived quality q of this new channel would be high. Also, the platform owner should focus on building system functionalities in the new channel so that it is easier for sellers in the new channel to provide service to consumers, which would allow high-certainty channel sellers to have a high ζ_h value.

2.6.4 Results when $\zeta_h > \zeta_l$

In the case where $\zeta_h > \zeta_l$, consumers in the high-certainty channel are more sensitive to sellers' extra effort than consumers in the low-certainty channel. An example of a plausible scenario where $\zeta_h > \zeta_l$ is as follows. If consumers return the product, they still need to pay the transportation fee. But if the sellers provide free return insurance that covers the transportation fee to consumers, consumers who pay more for the product will derive higher utility from this extra effort. Consumers in the high-certainty channel are more sensitive to the quality of the product because they pay more, thus consumers in the high-certainty channel will feel happier than consumers in the low-certainty channel when the free returns insurance is provided. This reversal of assumptions will cause the following changes to occur as shown in Table 2.10.

Table 2.10: Parameter Impacts on Demand, Effort and Profit in Two-Channel Case

	D_h	D_l	D_t	e_p	SP_h	SP_l	Π_2
q	•	•	$\frac{\partial D_t}{\partial q} = 0$	•	•	•	•
s_h	•	•	$\frac{\partial D_t}{\partial s_h} = 0$	$\frac{\partial e_p}{\partial s_h} \lesseqgtr 0^*$	•	•	•
r	•	$\frac{\partial D_l}{\partial r} > 0$	$\frac{\partial D_t}{\partial r} > 0$	•	•	$\frac{\partial SP_l}{\partial r} > 0$	$\frac{\partial \Pi_2}{\partial r} > 0$
σ_l^2	•	•	•	•	•	•	•
ζ_l	•	•	•	•	$\frac{\partial SP_l}{\partial \zeta_l} \lesseqgtr 0^*$	•	•
ζ_h	•	•	$\frac{\partial D_t}{\partial \zeta_h} = 0$	•	•	$\frac{\partial SP_l}{\partial \zeta_h} < 0$	•

* Indicates sign changes depending on the cutoff point(s).

• Indicates no change in the sign irrespective of whether $\zeta_h > \zeta_l$ or $\zeta_h < \zeta_l$.

Theorem 9. *When $\zeta_h > \zeta_l$, most results remains the same as the $\zeta_l > \zeta_h$ case, except the followings:*

i. When r increases, demand in the low-certainty channel D_l increases.

ii. When q, s_h and ζ_h changes, D_t remains the same. When r increases, D_t increases.

iii. When s_h increases, the platform owner's effort e_p will first increase and then decrease or always decrease.

iv. When ζ_l^2 increases, total seller profit in the high-certainty channel SP_h will always increase under certain conditions, and under other conditions there is a cutoff point. When ζ_l^2 is greater than this cutoff point, SP_h increases with ζ_l^2.

v. When r increases, total seller profit in the low-certainty channel SP_l increases. When ζ_h increases, total seller profit in the low-certainty channel SP_l decreases.

vi. When r increases, the platform owner's profit Π_2 increases.

2.7. Conclusion

Large B2C platforms have created channels for certified (e.g., vetted) sellers in response to customer concerns over seller service quality. We seek to explain key managerial issues related to the B2C platform owner's decision to open a second vetted B2C channel. This paper contributes to the literature through economic modeling analysis.

This paper provides an economic modeling analysis of an e-Business B2C platform owner's decision to expand from one-channel to add a second channel with vetted sellers. Insights are provided using partial derivatives for demand and profit with respect to a variety of model parameters. Parameters examined are the customers' sensitivity to service quality, the reputation spillover between channels, the perceived quality in the high-certainty channel, the channel quality uncertainty, and the seller efficiency coefficient for each channel. Insights regarding platform owner's effort are also presented. We examine when the second vetted channel will be added by the platform owner. Also, we examine the impact of reputation spillover on relative demand between the one- and two-channel cases.

The impact of parameter changes on demand and profits is generally in opposite directions for the low- and high-certainty channels. Thus, the low- and high-certainty channel sellers are at odds with each other, and they would like to see the parameters move in different directions. This natural tension causes incentive misalignment.

While the platform owner and sellers are aligned for all parameters in the one-channel case, there is misalignment for some parameters between the platform owner and sellers in both the low- and high-certainty channels in the two-channel case. Due to these misaligned incentives in the two-channel case, there will be conflict between the platform owner and sellers, and between the sellers in separate channels. The platform owner must therefore strike a between their incentives and the incentives of the low- and high-certainty channel sellers. With Corollary 8.1, we present some conditions under which it will be desirable for the platform owner to introduce the second high-certainty (or vetted) channel to increase their profits.

Our model has yielded several interesting results. We find that when reputation spillover r increases, consumers' perceived quality in the low-certainty channel increases. One might expect that the demand in the low-certainty channel D_l will increase. However, we show that D_l does not always increase. When r increases and the channel competition is not so intensive (r is small), sellers in the low-certainty channel can increase price, leading to a decrease in D_l. We also find that opposite to what one might expect, the total demand D_t could decrease in r in the region of high r since some customers are priced out of the market in this case. Another interesting and important result is that the platform owner's profit Π_2 decreases with r when the channel competition is not so intensive. Our managerial insight from this result is that the platform owner should only offer a second, vetted channel only if the reputation spillover effect is strong and the new channel can lead to an intensive competition. Our analytical model also yields other interesting results. For example, when σ_l increases, the channel owner's effort e_p first increases to reduce the uncertainty in the high-certainty channel and make it more attractive. However, as σ_l increases further, e_p starts to decrease due to the decreasing marginal return.

In this paper, a seller would only sell in one-channel. Future extensions to this work include the consideration of the case where a seller can sell in both channels simultaneously. Another possible extension could examine the impact of possible cooperation between sellers, such as defacto price-setting or quality-setting schemes between sellers. On the buyer side, the impact of consumer recommendations could be considered. Also, we consider an identical product sold in different channels by different sellers. Future work could consider the impact of non-identical rather than identical products being sold. How will different sellers with non-identical products choose the channel in which to sell? Additionally, we have considered whether the platform owner should establish a high-certainty channel. However, in many cases, the high-certainty channel is not established by the same platform owner, but is rather offered by another platform owner who does not own the low-certainty channel. A future research question is to consider the word-of-mouth (WOM) effect in a social network context (Bai et al., 2015). How will a channel owner's decisions be affected by the existence of such WOM? Another extension is to incorporate channel competition between different channel owners. If there is already another high-certainty channel in existence, how will this change the incentives of a low-certainty channel platform owner?

Chapter 3

Analyzing Software Anti-piracy Strategies in a Competitive Environment

3.1. Introduction

Software piracy has become a serious issue all over the world. In a report by the anti-piracy group The Software Alliance (BSA) (2016), out of 116 markets investigated around the world, more than half of PC software used in each of 72 markets is unlicensed. In 2013, the total commercial value of unlicensed installations was approximately \$62.7 billion (BSA, 2014); in 2015, that value was still around \$52.2 billion (BSA, 2016). Pirated software can be easily downloaded from many websites or peer-to-peer (P2P) file-sharing networks. According to International Data Corporation's Dangers of Counterfeit Software Survey (Gantz et al., 2013), 45% of pirated software was obtained from online websites and P2P networks.

Facing the threat of software piracy, software firms take various anti-piracy measures. For example, Microsoft has a worldwide anti-piracy team to track and trace criminal activity related to software piracy. This team includes former police officers, prosecutors, IP attorneys, and intelligence analysts. It works closely with law enforcement agencies to support criminal prosecutions (Microsoft, 2010). In 2012, it settled 3,265 counterfeiting suits worldwide (Kerr, 2013) and it continues to prevent software piracy by filing lawsuits (Keizer, 2016). Furthermore, software firms also form industrial alliances in order to jointly control software piracy activities, with anti-piracy watchdog BSA being a good example. Hereafter, this paper uses the terms "publisher" and "software firm" interchangeably to identify the software firm that is the legitimate owner of the software. Many governments have taken legal actions to reduce piracy. In the United States, people convicted of copyright infringements could be imprisoned for up to five years and fined up to $250,000. Repeat offenders could be imprisoned for up to 10 years and held responsible for damages or lost profits up to $150,000 per work (U.S. Copyright Office, 2011). Australia has similar legal measures (Australia Copyright Act 1968).

Previous literature has argued that piracy can sometimes be beneficial to publishers for several reasons including network effect (Conner and Rumelt, 1991) – a product or service becomes more valuable when more people use it. The network effect can be generated in several ways. For example, users can get help from other users around them more easily if more people use the same product. Also, a user can search on the Internet for help. If fewer people are using the product, it is less likely that a particular question has been answered on the Internet. Also, users often post their questions to online user forums to seek answers. There, a similar situation happens: if more people use this software, quick feedback is more likely. Another source of the network effect is that when more coworkers use the same software, it is more likely that a user can share files with them directly. In summary, the utility of a software package will be higher when more people use it.

In this paper, we consider both the *direct effect* and *cross effect* of anti-piracy efforts. On the one hand, a publisher's anti-piracy effort can directly increase the cost of pirating its software – a direct effect. On the other hand, the firm's anti-piracy effort can also increase the cost of pirating software from a similar firm – a cross effect which has received little attention in the existing literature. We consider two sources of the cross effect. The first source is that when a publisher solicits a government's help in anti-piracy, this effort will benefit other software firms as well. When the government makes stricter regulations and laws to combat anti-piracy and puts more effort into anti-piracy, all software piracy offenders will face higher fines or greater probability of being caught, leading to higher piracy costs. All software firms in the industry will benefit as a result. The second cross effect source is that when a publisher sues a piracy channel such as websites, P2P networks, or suppliers which pirate its products, this channel could be shut down. Then individual users would have greater trouble finding pirated software produced by a different firm so the time cost of pirating one firm's software could increase as a result of a competitor's anti-piracy effort.

In this paper, we study the case of two software firms which sell similar products. Each product also has a pirated version. One example pair of products is MATLAB and Mathematica, which are widely used in science, engineering, and business. The two products have many similar functions and are viewed as competitors. In this paper, we will build game theoretical models to study the competition between the two publishers where each firm determines its anti-piracy efforts and product prices. The following are the main research questions we study: First, how will the network effect and competition affect a publisher's decisions about anti-piracy effort and product price? Second, how will the cross effect influence a publisher's decisions? Third, how will anti-piracy coordination through an industrial alliance or a government affect a publisher's pricing decisions?

To answer these questions, we first investigate the case of a monopoly. Previous literature (Conner and Rumelt, 1991, Shy and Thisse, 1999) has shown that if the network effect increases, publishers tend to use weaker copyright protection. In contrast, we find that a firm's anti-piracy efforts should actually increase with the network effect when both the quality of the pirated product and the anti-piracy effort costs are high, but the network effect is low. In the duopoly case, we study the impact of the network effect and other parameters in both symmetric and asymmetric scenarios. In both scenarios, when the network effect increases, we find that a software firm's anti-piracy efforts should also increase; when its network effect increases, its optimal price reduces when the anti-piracy efforts are not costly, contrary to the result in the monopoly case. The additional competition between two pirated products drives this counter-intuitive result. If a pirated product, say Product

1, gains significant competitive power from the other pirated product, the software firm of Product 1 then needs to reduce its price to compete with the pirated Product 1, even if the network effect increases. We also study the cases of industrial alliance and government planning. In the industrial alliance case, an industrial alliance (e.g., BSA) can represent the industry to determine the anti-piracy efforts necessary to control the piracy and maximize all software firms' profits. In the government planning case, a government determines the anti-piracy efforts and maximizes the social welfare that includes both publishers' profit and legitimate consumers' surplus. Interestingly, compared with a government agency to manage anti-piracy, an alliance can under- or over-invest in anti-piracy. Our findings also have implications for research in gray market (Zhang and Feng, 2017) where a consumer can get a product at a lower price through an unauthorized channel. There is a great similarity between gray market and software piracy.

The rest of the paper is organized as follows. Section 3.2 reviews the related literature and Section 3.3 builds an analytical model for a duopoly setting. In Section 3.4, we analyze the model and obtain results through comparative statics. We consider model extensions in Section 3.5 and conclude the paper in Section 3.6.

3.2. Literature Review

Our paper is related to a stream of literature on the network effect. Katz and Shapiro (1985) propose three sources for the network effect: the direct network effect, indirect network effect, and post-purchase service network effect. They investigate the impact of the network effect on competition and on the firms' compatibility decisions. Brynjolfsson and Kemerer (1996) build a hedonic model to determine the impact of the network effect on the price of microcomputer spreadsheet software. They find that the network effect significantly increases the publisher's optimal price of spreadsheet products. Cheng and Liu (2012) develop a unified framework to investigate which free trial strategy was preferred in the presence of the network effect. In our model, users of legitimate or pirated software both enjoy the network effect.

We show that the network effect can increase or decrease the optimal level of anti-piracy efforts in a nonlinear way.

Our paper is also related to a second stream of literature on software piracy. Chen and Png (2003) explore how a monopolistic publisher should set both prices and spending levels on detection when a government sets the cost of piracy. Sundararajan (2004) investigates how a publisher should choose the optimal pricing schedules and technological deterrence level when digital piracy exists in the market and the degree of piracy can be influenced by implementing digital rights management (DRM) systems. Gu and Mahajan (2005) study the effects of piracy on the profits of software firms when competition exists. They show that piracy can reduce price competition and can be beneficial to firms when their markets have high wealth gaps. Wu and Chen (2008) find that when there is no piracy, a single version is the optimal strategy for an information goods provider (i.e., a publisher). However, when piracy exists, such firms tend to offer more than one version; this versioning strategy is an effective and profitable instrument to fight piracy under some conditions. August and Tunca (2008) consider whether a software firm should allow pirating users to update security patches. They find that if the piracy tendency is low, then the publisher's software security patch restriction is optimal only when the piracy enforcement level is high. When patching costs are sufficiently low, an unrestricted patch release policy by the publisher can maximize its profit. Johar et al. (2012) investigate a publisher who gains profit through advertisement when providing content to consumers who have heterogeneous valuations. The publisher needs to determine two dimensions, the content quality and content distribution delay, in its content provision strategy. They find that when piracy exists, the publisher should improve on at least one dimension of content provision. Zhang et al. (2012) investigate strategies to fight counterfeits when there are two competing brand name products and a counterfeit product. Tunca and Wu (2013) explore the effects of suing file-sharing P2P networks or consumers who share copyrighted material on the P2P network; such action can turn out to hurt legitimate publishers of information goods. Lahiri and Dey (2013) find that when piracy enforcement is low, the monopolist publisher has more incentive to invest in quality. Our paper considers the network effect and cross effect. The focus of our paper is on the network effect's impact on software anti-piracy and software pricing.

Here we highlight some papers investigating the positive impact of piracy. This stream of literature closely relates to our paper. Conner and Rumelt (1991) incorporate the network effect into the model and examine piracy's effect on a software firm's profit. When more people use the software, either legitimate or pirated version, consumers can gain higher utility and are willing to pay more for the product. They find that if the network effects are large, then publishers can benefit from piracy. Shy and Thisse (1999) extend the monopoly results of Conner and Rumelt (1991) to a duopoly framework. They show that software firms will allow piracy (i.e., not combat it intensely) in order to increase the market size. If the network effects are strong, then firms can benefit from not exerting effort on anti-piracy. Jain (2008) finds that strong network effects may lead to higher levels of copyright protection in some cases. When the network effect is strong, stronger copyright enforcement can reduce price competition. Dey et al. (2016) investigate how piracy affects the supply chain of information goods. They find that piracy can increase the profits of the publisher and the retailer on the supply chain as well as increase consumer welfare. Herings et al. (2017) employ a dynamic stochastic model to determine the optimal pricing policy of music recordings when P2P file-sharing (piracy) exists. They find that if a music publisher exerts large effort to fully enforce the intellectual property rights, then consumer surplus and total welfare decrease. Different from the works by (Conner and Rumelt, 1991, Shy and Thisse, 1999), our results show that a stronger network effect can encourage publishers to invest more in controlling software piracy.

Our work is most relevant to Jain (2008)'s work. However, our scenario is different in three important ways. First, Jain (2008)'s paper assumes that the number of potential pirating users is proportional to that of the potential legitimate users. This proportion is exogenously given. Also Jain (2008)'s paper does not consider anti-piracy efforts. In our paper, we assume each potential consumer will choose to buy or pirate, and that the competition between legitimate and pirated products determines the demand for the pirated product. Second, in Jain (2008)'s work, consumers of legitimate products are located on a Hotelling line. Then demands for legitimate products are derived based only on direct competition between legitimate products. Demand for a pirated product is (exogenously) proportional to demand for the legitimate product. In our paper, consumers for both pirated and legitimate software are located on a single Hotelling line. Product demands are derived from more complex competition which includes competition involving pirated products. Third, we introduce the cross effect, which was not considered in the previous paper. We show that the cross effect influences anti-piracy efforts in an unexpected way – a firm's optimal anti-piracy effort increases with the cross effect although such an increase in effort could benefit its competitors.

3.3. Model

We consider a one-period model where two software firms sell substitutable software, labeled as Product 1 and Product 2. These two firms are located at the endpoints of a unit Hotelling line. Following the literature on information goods (Essegaier et al., 2002, Fishburn and Odlyzko, 1999), we assume the marginal cost of producing an extra copy of the software is 0. Each product also has a pirated version. We assume that the consumer demand is normalized to 1 and individual demands are uniformly distributed on the Hotelling line. Also, the quality of both products is assumed to be large enough so that the market is fully covered (otherwise, each firm will act as a local monopolist and there is no competition between two firms). Table 4.1 contains the notation used in this paper.

3.3.1 Consumers' Decision Making

Under the assumption of full market coverage, consumer will take one of the following four actions: buying Product 1 or 2, or pirating Product 1 or 2.

Table 3.1: Summary of notation

Notation	Description
Parameters	
q_i	quality of product i ($i = 1, 2$ in this table)
θ_i	discount factor relative to a legitimate version
k_i	coefficient of a product's network effect
r_i	coefficient of anti-piracy effort cost
x	a consumer's location on the Hotelling line
t	coefficient of unfitness cost of using a legitimate version
Intermediate Variables	
π_i	profit of software firm i
U_i	Consumers' utility of using ith legitimate software product
U_{2+i}	Consumers' utility of using ith pirated software product
D_i	demand of legitimate product i
D_{2+i}	demand of pirated product i
Decision Variables	
p_i	price of product i
e_i	anti-piracy effort exerted by software firm i
p_m	price of Product 1 in monopoly case, a special case of p_1
e_m	anti-piracy effort of software firm 1 in monopoly case
p_d	price of Product 1 and 2 in duopoly case
e_d	anti-piracy effort of software firm 1 and 2 in duopoly case

For a consumer located at x, the utility of buying Product 1 is given by

$$U_1 = q_1 - tx + k_1(D_1 + D_3) - p_1. \tag{3.1}$$

where t is the unfitness cost coefficient of Product 1 and k_1 is the network effect coefficient of Product 1. Also D_1 is the demand of legitimate Product 1 and D_3 is the demand of the pirated version.

Similarly, the utility for a consumer located at x of buying Product 2 is given by

$$U_2 = q_2 - t(1 - x) + k_2(D_2 + D_4) - p_2. \tag{3.2}$$

However, if consumers choose to use a pirated version, they might not get the full functionality offered by the legitimate version. For example, software firms usually provide customer service and technical support to legitimate users only (Microsoft, 2016). Also, software firms constantly create new features and add-ons for their products as upgrades.

These upgrades might be available for legitimate versions only (Omron, 2016). As a result, compared with a user of the legitimate software, a user of a pirated version can only use part of the product's features, and therefore both network externality and unfitness cost t are discounted correspondingly. Similar to Jain (2008), our model uses a single discount factor θ_1 (θ_2) to represent the percentage of quality, unfitness cost, and the network effect associated with the pirated Product 1 (pirated Product 2), relative to the legitimate version (Our main results and insights would still apply with three different discount factors). For simplicity, we will call θ_1 (θ_2) the discount factor of the pirated software from now on. Accordingly, the utility of a consumer located at x who uses the pirated version of Product 1 is

$$U_3 = \theta_1[q_1 - tx + k_1(D_1 + D_3)] - c_1(e_1, e_2). \tag{3.3}$$

In Equation (3.3), the term $c_1(e_1, e_2)$ represents the cost of using the pirated Product 1, detailed as follows. First, as software firms may not provide patches for pirated versions (August and Tunca, 2008), users of pirated software could face security risk and therefore incur costs. Second, those people face the possibility of being caught using pirated software and then paying a costly fine (Copyright Law of the United States 2011). Also, using pirated software could damage those users' reputation among their peers. This cost $c_1(e_1, e_2)$ is not only a function of Firm 1's anti-piracy effort e_1 but also Firm 2's effort e_2 due to the cross effect of anti-piracy efforts: when Firm 2 exerts effort, the penalty cost of using Firm 1's pirated product will also increase. In particular, we assume $c_1(e_1, e_2) = a_1e_1 + b_1e_2$ to simplify the model. Since the direct effect of the anti-piracy effort is likely to be larger than the cross effect, we assume $a_1 > b_1$.

Similarly, the utility of a consumer located at x who uses the pirated version of product 2 is given by

$$U_4 = \theta_2[q_2 - t(1 - x) + k_2(D_2 + D_4)] - c_2(e_1, e_2). \tag{3.4}$$

where $c_2(e_1, e_2) = a_2e_1 + b_2e_2$. Also, we assume $b_2 > a_2$.

We can prove the following lemma by using the above utility functions (3.1) to (3.4) (see the appendices for all the proofs of theorems and lemmas). Lemma 1 describes the consumers' choice of a particular product according to their unfitness level. Furthermore, consumers will choose a product that yields a higher utility in the duopoly case, as depicted in Figure 3.1.

Lemma 1. *For a particular product, user who have a lower software unfitness level will favor the option of buying; otherwise they favor the option of pirating.*

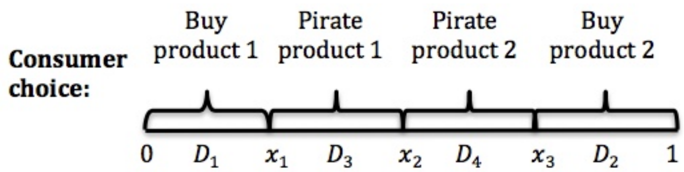


Figure 3.1: Consumer demands in the duopoly case

3.3.2 Firms' Decision Making

For the two firms, their profits are given by

$$\pi_i(p_i, e_i) = D_i p_i - r_i e_i^2, i = 1, 2, \tag{3.5}$$

where p_i, D_i, e_i are respectively price, demand, and anti-piracy effort for product i, $i = 1, 2$. Each publisher chooses a price and effort to maximize its profit. We assume that firm i's effort cost is a quadratic function of effort e_i, denoted as $r_i e_i^2$. This formulation captures the property that when the effort e_i increases, the cost will increase; as it is more costly to exert an anti-piracy effort, the marginal cost of e_i will also increase.

The decision time sequence of players in this model is the following. In the first stage, both firms simultaneously decide their anti-piracy effort e_1 and e_2. In the second stage, both firms simultaneously decide the prices p_1 and p_2. In the third stage, consumers choose which product to obtain and whether to buy or pirate it.

3.4. Analysis of the Model

We first consider the case of a monopolist selling a software product to differentiated consumers. This analysis serves as a benchmark for the analysis of the duopoly case.

3.4.1 Benchmark: Monopoly Case

We assume that the monopolist (Product 1) is at one end of the Hotelling line, Point 0, without loss of generality. Similar to the duopoly case, the firm decides on anti-piracy effort e_m in the first stage and price p_m in the second stage. In the third stage, consumers make their decision of purchasing, pirating, or not using it. Since there is only one product, no cross effect of anti-piracy effort exists. Then $c_i(e_1, e_2)$ becomes $c(e_m)$. In particular, $c(e_m) = a_1 e_m$.

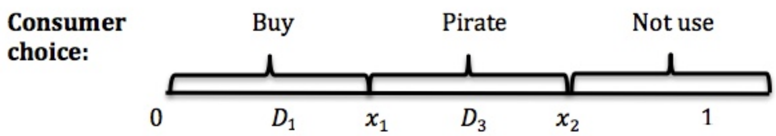


Figure 3.2: Consumer demands in the monopoly case

In the monopoly case, consumers can purchase legitimate Product 1, gaining utility U_1 given by (3.1), or use a pirated version, gaining utility U_3 given by (3.3). Similar to Figure 3.1, Figure 3.2 depicts the consumers' choices in the monopoly case. Consumers located at x_1 are indifferent between buying Product 1 or pirating it; and those at x_2 are indifferent between pirating Product 1 and not using it. Variables x_1 and x_2 satisfy the following two equations:

$$U_1|_{x=x_1} = U_3|_{x=x_1} \tag{3.6}$$

and

$$U_3|_{x=x_2} = 0. \tag{3.7}$$

We can get x_2 from (3.7):

$$x_2 = \frac{\theta_1 q_1 - a_1 e_m}{\theta_1(t - k_1)} \tag{3.8}$$

and then x_1 from (3.6):

$$x_1 = -\frac{-a_1 e_m k_1 + a_1 e_m \theta_1 t + \theta_1 k_1 p_m - \theta_1 p_m t + \theta_1^2 q_1(-t) + \theta_1 q_1 t}{(\theta_1 - 1)\,\theta_1 t(t - k_1)}. \tag{3.9}$$

The boundary case $x_2 = 1$ does not give interesting results and therefore is not considered in this analysis.

The firm's profit can be written as:

$$\pi_m(p_m, e_m) = D_m p_m - r_1 e_m^2, \tag{3.10}$$

where the demand for legitimate product D_m is x_1. In the monopoly case, maximizing π_m by e_m and p_m sequentially is equivalent to doing so simultaneously, so we can get the optimal anti-piracy effort e_m^* and optimal price p_m^* simultaneously by maximizing (3.13) :

$$e_m^* = \frac{a_1\,(\theta_1 - 1)\,\theta_1 q_1 t\,(\theta_1 t - k_1)}{a_1^2\,(k - \theta_1 t)^2 + 4\,(\theta_1 - 1)\,\theta_1^2 r_1 t(k_1 - t)^2} \tag{3.11}$$

and

$$p_m^* = \frac{2\,(\theta_1 - 1)^2\,\theta_1^2 q_1 r_1 t^2 (k_1 - t)}{a_1^2\,(k_1 - \theta_1 t)^2 + 4\,(\theta_1 - 1)\,\theta_1^2 r_1 t(k_1 - t)^2}. \tag{3.12}$$

where in Appendix 6.2.3, we have verified that the optimal solutions (3.11) and (3.12) satisfy the second-order conditions as well. Substituting (3.11) and (3.12) into (3.10), we can obtain the firm's profit π_m^*:

$$\pi_m^* = -\frac{(\theta_1 - 1)^2\,\theta_1^2 q_1^2 r_1 t^2}{a_1^2\,(k_1 - \theta_1 t)^2 + 4\,(\theta_1 - 1)\,\theta_1^2 r_1 t(k_1 - t)^2} = \frac{q_1 p_m^*}{2(t - k_1)}. \tag{3.13}$$

Comparative Statics of Network Effect k_1

From Equations (3.11) to (3.13), we can obtain further analytical results through comparative static analysis. In Theorem 10, we present the results related to the optimal effort e_m^* concerning the network effect. For clarity, the results are also summarized in Table 3.2.

Theorem 10. *In the monopoly case, when the network effect increases, there are two regions.*

1. *When the discount factor of the pirated software is small ($\theta < 1/2$), the publisher's anti-piracy effort decreases with the network effect, i.e., $\dfrac{\partial e_m^*}{\partial k_1} < 0$.*

2. *When the discount factor of the pirated software is large ($\theta > 1/2$), there are two sub-regions.*

   (a) *When anti-piracy effort cost is small ($r_1 < r_{m1}$), the publisher's optimal anti-piracy effort decreases with the network effect, i.e., $\dfrac{\partial e_m^*}{\partial k_1} < 0$.*

   (b) *When anti-piracy effort cost is large ($r_1 > r_{m1}$): (1) when the network effect is small ($0 < k_1 < \bar{k}_1$), the publisher's optimal anti-piracy effort increases with the network effect, i.e., $\dfrac{\partial e_m^*}{\partial k_1} > 0$; (2) when the network effect is large ($\bar{k}_1 < k_1 < \theta_1 t$), the publisher's optimal anti-piracy effort e_m^* decreases with the network effect, i.e., $\dfrac{\partial e_m^*}{\partial k_1} < 0$.*

The threshold values $\bar{k}_1$ and r_{m1} are defined in the proof.

Table 3.2: Impact of k_1 on e_m^*

	r small	r large	
		θ small	θ large
k_1 small	$\dfrac{\partial e_m^*}{\partial k_1} < 0$	$\dfrac{\partial e_m^*}{\partial k_1} < 0$	$\dfrac{\partial e_m^*}{\partial k_1} > 0$
k_1 large			$\dfrac{\partial e_m^*}{\partial k_1} < 0$

In Theorem 11, we provide the comparative static result related to price p_m^* with respect to the network effect.

Theorem 11. *In the monopoly case, the product optimal price p_m^* increases with the network effect, that is, $\dfrac{\partial p_m^*}{\partial k_1} > 0$.*

From Theorem 11, the product price increases monotonically with the network effect. The explanation is the following. When the network effect increases, the product becomes more attractive and the consumers' utility of using legitimate software increases. Then the monopolistic firm can charge a higher price and obtain a higher profit. Theorem 10 shows a very interesting result concerning the network effect. As previous literature (Shy and Thisse, 1999, Jain, 2008) has shown, if the network effect increases, firms tend to use weaker copyright protection. Our results contrast with the previous literature by showing that this result is only true in two cases: (i) when the quality of anti-piracy product (θ_1) is small so that using pirated software is not attractive, or (ii) when the anti-piracy effort cost (r_1) is small so that a publisher could exert a sufficient amount of anti-piracy effort. In either case, the firm can keep the piracy activity under control. Then when k_1 increases, it is more valuable to take advantage of the increased network effects and enlarge the whole user base by reducing its anti-piracy effort.

However, in the case of large θ_1 and r_1, when the network effect k_1 increases, a producer should increase its anti-piracy effort in the small k_1 range but reduce it in the large k_1 range. This non-intuitive result is due to the trade-off between two conflicting factors. On the one hand, to encourage the purchase of legitimate software, a firm needs to control the demand for the pirated product. On the other hand, it also wants to grow its network and make its product more attractive by taking advantage of the network effect. Then, it could charge a higher price for legitimate software. In the case of a strong network effect (large k_1), it is more beneficial for a publisher to increase its user base and charge more for its product. Therefore, it would reduce the anti-piracy effort to expand its user network by allowing the previous non-users to use the software through pirating. When k_1 is small, the result is more interesting. Given that the pirated product quality is large and it is costly to exert an anti-piracy effort, the demand for pirated software is high. When k_1 increases, the demand for pirated software becomes higher. Since the network effect by pirated software is not significant in this small k_1 region, the software firm increases its anti-piracy effort to bring the piracy under control. The insight, in this case, is that the firm can simply focus on discouraging piracy and increasing the demand for its legitimate software.

To summarize, when responding to a change in network effect, a manager responsible for a publisher's anti-piracy effort needs to jointly consider various factors such as the cost of anti-piracy efforts (r_1), discount factor of pirated software (θ_1), and network effect (k_1). Care must be taken in the case of large r_1 and θ_1: the change in anti-piracy effort could be opposite, depending on the value of the network effect.

Other Comparative Statics

For the purpose of comparison with the results in the duopoly case later, we summarize in Table 3.3 the results about the impacts of q_1, θ_1, a_1, and r_1 on e_m^* and p_m^*.

Table 3.3: Impacts of q_1, θ_1, a_1 and r_1 on e_m^* and p_m^* in the monopoly case

	q_1	a_1	r_1	θ_1
e_m^*	$\frac{\partial e_m^*}{\partial q_1} > 0$	$\frac{\partial e_m^*}{\partial a_1} > 0$	$\frac{\partial e_m^*}{\partial r_1} < 0$	$\frac{\partial e_m^*}{\partial \theta_1} > 0$
p_m^*	$\frac{\partial p_m^*}{\partial q_1} > 0$	$\frac{\partial p_m^*}{\partial a_1} > 0$	$\frac{\partial p_m^*}{\partial r_1} < 0$	$\frac{\partial p_m^*}{\partial \theta_1} < 0$

When the product quality q_1 increase, the consumer's utility increases, which implies that a software firm can charge a higher price to gain more profit. Also, as the value of adding an additional consumer increases due to a higher price, the firm will exert more anti-piracy effort to make customers purchase instead of pirating, further increasing profit. Similarly, when the anti-piracy effort (a_1) is more effective, a publisher can exert more effort and also charge more. On the other hand, when anti-piracy effort cost (r_1) increases, the results are opposite to those of a_1: the optimal effort and price decrease. Finally, when θ_1, the quality of the pirated software (relative to the legitimate version) increases, the competition between the legitimate and pirated products intensifies. Then a firm needs to increase the anti-piracy effort and decrease the price to make the legitimate version more attractive, causing the profit to decrease.

Will such comparative static results in the monopoly case still hold in the duopoly case? We will explore this question in Section 3.4.2.

3.4.2 Duopoly Case

We now consider the case that two competing software firms (publishers), Firm 1 and Firm 2, are in the market. They lie at the opposite ends of the Hotelling line. Without loss of generality, let Firm 1 be located at Point 0 and Firm 2 at Point 1.

We use backward induction to solve this case according to the decision sequence given in Section 3.3. At Stage 3, a consumer makes a decision to buy or pirate to maximize his or her utility, as shown in Figure 3.1 of Section 3.3.1. Variables x_1, x_2, and x_3 represent indifference points. For a consumer located at x_1, the utility U_1 of using the legitimate software (given by Equation (3.1)) equals the utility U_3 of using the pirated version (given by Equation (3.3)). That is,

$$q_1 - x_1 t + k_1(D_1 + D_3) - p_1 = \theta_1[q_1 - x_1 t + k_1(D_1 + D_3)] - c_1(e_1, e_2). \tag{3.14}$$

At $x = x_3$, utilities U_2 and U_4, given by Equations (3.2) and (3.4) respectively, should be the same:

$$q_2 - (1 - x_3)t + k_2(D_2 + D_4) - p_1 = \theta_2[q_1 - x_3 t + k_2(D_2 + D_4)] - c_2(e_1, e_2). \tag{3.15}$$

Furthermore, at $x = x_2$, $U_3 = U_4$:

$$\theta_1[q_1 - x_2 t + k_1(D_1 + D_3)] - c_1(e_1, e_2) = \theta_2[q_2 - (1 - x_2)t + k_2(D_2 + D_4)] - c_2(e_1, e_2), \tag{3.16}$$

where

$$D_1 + D_3 = x_2 \tag{3.17}$$

and

$$D_2 + D_4 = 1 - x_2. \tag{3.18}$$

From Equations (3.14) – (3.16), we can obtain x_1, x_2, and x_3.

The publishers' decisions about e_i and p_i are made in the first and second stages to maximize their own profits, as given by Equation (3.5). We can find the solutions of e_i^* and p_i^* through backward induction. They are complicated so that we do not present them here. To gain insights from the results, we again resort to comparative statics.

Comparative Statics in an Asymmetric Setting

In this *asymmetric* setting, when studying the impact of a particular parameter (for instance k_1), we first take the derivative of this parameter by holding the other parameters constant and then set the parameters to be symmetric: $k_1 = k_2 = k$ etc. For each parameter, k_i, a_i, b_i, q_i, and r_i, $i = 1, 2$, we study its impact on each software firm's anti-piracy effort and price by using comparative statics. We first show the impact of the network effect k_i on the anti-piracy effort in Theorem 12. For ease of understanding, we also summarize the results in Table 3.4.

Theorem 12. *In the duopoly asymmetric setting, there are two regions where the impacts of Firm 1's network effect are different.*

1. *In the region where the discount factor of the pirated software is low ($\theta \leq \theta''$), Firm 1's anti-piracy effort decreases with its network effect, i.e.,* $\frac{\partial e_1}{\partial k_1}|_{k_1=k_2=k} < 0$. *Also,*
   (a) *When the effort cost is small ($r < r_{dDC}$), Firm 2's anti-piracy effort decreases with Firm 1's network effect, i.e.,* $\frac{\partial e_2}{\partial k_1}|_{k_1=k_2=k} < 0$.
   (b) *When the effort cost is large ($r > r_{dDC}$), Firm 2's anti-piracy effort will first increase and then decrease with Firm 1's network effect, i.e., there is a threshold value $\bar{k}_{dDC}$ for a given r: when $k < \bar{k}_{dDC}$,* $\frac{\partial e_2}{\partial k_1}|_{k_1=k_2=k} > 0$; $\frac{\partial e_2}{\partial k_1}|_{k_1=k_2=k} < 0$ *otherwise.*

2. *In the region where the discount factor of the pirated software is high ($\theta > \theta''$), Firm 2's anti-piracy effort decreases with Firm 1's network effect* ($\frac{\partial e_2}{\partial k_1}|_{k_1=k_2=k} < 0$) *when r is small enough or large enough. Also,*

(a) When the effort cost is small ($r < r_{dDB}$), Firm 1's anti-piracy effort decreases with its network effect, i.e., $\frac{\partial e_1}{\partial k_1}|_{k_1=k_2=k} < 0$.

(b) When the effort cost is large ($r > r_{dDB}$), Firm 1's anti-piracy effort will first increase and then decrease with its network effect, i.e., there is exactly one threshold value $\bar{k}_{dDB}$: $\frac{\partial e_1}{\partial k_1}|_{k_1=k_2=k} > 0$ if $k < \bar{k}_{dDB}$ and $\frac{\partial e_1}{\partial k_1}|_{k_1=k_2=k} < 0$ otherwise.

The threshold values θ'', r_{dDB}, r_{dDC}, k_{dDB}, and k_{dDC} are defined in the proof.

Table 3.4: Impacts of k_1 on e_i^*, $i = 1, 2$

	r small		r large			
			θ small		θ large	
k_1 small	$\frac{\partial e_1^*}{\partial k_1} < 0$	$\frac{\partial e_2^*}{\partial k_1} < 0$	$\frac{\partial e_1^*}{\partial k_1} < 0$	$\frac{\partial e_2^*}{\partial k_1} > 0$	$\frac{\partial e_1^*}{\partial k_1} > 0$	$\frac{\partial e_2^*}{\partial k_1} < 0$
k_1 large				$\frac{\partial e_2^*}{\partial k_1} < 0$	$\frac{\partial e_1^*}{\partial k_1} < 0$	

When Firm 1's own network effect coefficient k_1 increases, its anti-piracy effort changes in a similar way as in the monopoly case (shown in Theorem 10). The explanation is similar to that in the monopoly case. We find that when the effort cost r is small, e_1^* and e_2^* change in the same direction. Similar to the monopoly case, software piracy is under control and the network effect is important to both firms. When the network effect coefficient k_1 increases, both firms choose to decrease the anti-piracy effort. When both θ and r are large, software piracy is severe. When k_1 increases, pirated Product 1 will attract some demand from consumers who are using pirated software 2, causing software piracy to be less attractive and therefore less problematic for Firm 2. Then Firm 2 can decrease its anti-piracy effort.

The impact of k_1 on Firm 2's anti-piracy effort is more interesting when r is large and θ is small. In this case, the pirated software is not so attractive and software piracy is also under control. Firm 1 values the growth of the network more than controlling piracy. Firm 1 will choose to decrease its anti-piracy effort as the network effect increases. Then as pirating Firm 1's software becomes easier, the demand for Firm 2's pirated product decreases, causing the overall network effect of Product 2 to decrease. When the network effect coefficient k_1

is small, Firm 2 will value anti-piracy more than network growth and will choose to exert more effort in anti-piracy. However, when k_1 is large enough, exerting effort in anti-piracy will be less attractive than increasing the network effect, so Firm 2 will choose to decrease its anti-piracy effort.

To sum up, we find that the cost of anti-piracy effort, the discount factor of pirated software, and the size of network effect influence the optimal reaction of anti-piracy effort when the network effect changes. The managerial implication is that a manager needs to consider each of these three factors when responding to a change in the network effect.

We have also studied the impact of the network coefficient k_1 on the publishers' optimal prices. The results are shown in Theorem 13.

Theorem 13. *In the duopoly asymmetric setting, there are three regions where the impacts of Firm 1's network effect are different. When Firm 1's network effect increases,*

1. *In the case that the effort cost is small ($r < r_{dDD}$), Firm 1's price decreases with its network effect, i.e.,* $\frac{\partial p_1}{\partial k_1}|_{k_1=k_2=k} < 0$. *When r is small enough, Firm 2's price decreases with Firm 1's network effect, i.e.,* $\frac{\partial p_2}{\partial k_1}|_{k_1=k_2=k} < 0$.
2. *In the case that the effort cost is medium ($r_{dDD} < r < r_{d4}$), Firm 1's price first increases and then decreases with its network effect, i.e., there is a threshold value $\bar{k}_6$: when $k < \bar{k}_6$,* $\frac{\partial p_1}{\partial k_1}|_{k_1=k_2=k} > 0$; $\frac{\partial p_1}{\partial k_1}|_{k_1=k_2=k} < 0$ *otherwise. Due to complexity, we are unable to determine the impact of Firm 1's network effect on Firm 2's price.*
3. *In the case that the effort cost is large ($r > r_{d4}$), Firm 1's price increases with its network effect, i.e.,* $\frac{\partial p_1}{\partial k_1}|_{k_1=k_2=k} > 0$. *When r is large enough, Firm 2's price increases with Firm 1's network effect, i.e.,* $\frac{\partial p_2}{\partial k_1}|_{k_1=k_2=k} > 0$.

The threshold values r_{dDD}, r_{d4}, and $\bar{k}_6$ are defined in the proof.

A counter-intuitive result arises in the case of small effort cost r. When the network effect k_1 increases, Firm 1's product becomes more attractive. Therefore, one would expect

Firm 1's price to increase. Surprisingly, we find that Firm 1 should actually reduce its price, contrary to what we have seen in the monopoly case. We can explain the results by considering the competition between legitimate and pirated products. As k_1 increases, Firm 1's pirated product becomes more attractive than Firm 2's pirated product. Then Firm 1 gains market share by attracting consumers who are using Firm 2's pirated product, and those consumers become more important to Firm 1. In this sense, we can view that Firm 1's pirated product has increased its bargaining power. Furthermore, as Theorem 12 shows, Firm 1 reduces its anti-piracy effort as k_1 increases, making software piracy more attractive. Therefore, Firm 1 needs to reduce its price to compete with software piracy, contrary to the monopoly case. Also from Theorem 13, we can see that the sign of $\frac{\partial p_1^*}{\partial k_1}$ is affected by the effort cost r. In other words, given a change in the network effect k_1, Firm 1 in the duopoly case needs to react differently regarding price in the different regions of effort cost. However, in the monopoly case, a firm's price always increases with the network effect k_1.

Table 3.5: Impact of q_1, a_1, r_1 and b_1 on e_1^*, e_2^*, p_1^*, and p_2^* in the asymmetric setting

	q_1	a_1	r_1	b_1
e_1^*	$\frac{\partial e_1^*}{\partial q_1} > 0$	$\frac{\partial e_1^*}{\partial a_1} > 0$	$\frac{\partial e_1^*}{\partial r_1} < 0$	$\frac{\partial e_1^*}{\partial b_1} > 0$
p_1^*	$\frac{\partial p_1^*}{\partial q_1} > 0$	$\frac{\partial p_1^*}{\partial a_1} > 0$	$\frac{\partial p_1^*}{\partial r_1} < 0$	$\frac{\partial p_1^*}{\partial b_1} > 0$
e_2^*	$\frac{\partial e_2^*}{\partial q_1} \lesseqgtr 0^\dagger$	$\frac{\partial e_2^*}{\partial a_1} > 0$	$\frac{\partial e_2^*}{\partial r_1} < 0$	$\frac{\partial e_2^*}{\partial b_1} > 0$
p_2^*	$\frac{\partial p_2^*}{\partial q_1} \lesseqgtr 0^\dagger$	$\frac{\partial p_2^*}{\partial a_1} > 0$	$\frac{\partial p_2^*}{\partial r_1} < 0$	$\frac{\partial p_2^*}{\partial b_1} > 0$

† indicates sign changes depending on the threshold value(s).

From Table 3.5, we see that when any of Product 1's quality q_1, its anti-piracy effort's direct effect a_1, or the cross effect b_1 increases (other factors staying the same), Firm 1's optimal effort and price increase. When Firm 1's effort cost r_1 increases, Firm 1's effort and price decrease. The intuition for the impacts of product quality, direct effect, and effort cost is the same as that in the monopoly case. Different from the monopoly case, the duopoly case includes the anti-piracy effort's cross effect b_1. When the cross effect increases, it becomes more costly to pirate Product 1 due to Firm 2's anti-piracy effort. Then the competition between pirated Product 1 and legitimate Product 1 will be less intensive and Firm 1 will have the motivation to increase the price to extract more profit. As the price increases, Firm 1 can gain more profit when converting a pirating user to a legitimate one. Therefore Firm 1 will be motivated to exert higher anti-piracy effort e_1^*. As e_1^* increases, it also becomes more costly to pirate Product 2. Therefore, similar to Firm 1, Firm 2 can increase both price and effort.

It is interesting to see that when Product 1's quality q_1 increases, Firm 2's effort and price will not change monotonically. In Appendix 6.2.9, we show that when the effort cost is large enough, Firm 2's effort and price decrease; When the effort cost is small enough, Firm 2's effort and price increase. The explanation is the following. When q_1 increases, Firm 1's anti-piracy effort increases. On the one hand, if the effort cost r_1 is large, the anti-piracy effort will not change by much. Then pirated Product 1 will be more attractive and draw additional users who were pirating Product 2. Consequently, Firm 2's concern about software piracy is somewhat alleviated, implying that Firm 2 should decrease its anti-piracy effort to save cost. Since the anti-piracy effort decreases, Firm 2 will also decrease its price to compete with its own pirated version. On the other hand, when r_1 is small, the anti-piracy effort increases greatly as q_1 increases. Then pirating Product 1 becomes less attractive and some users of pirated Product 1 will switch to pirated product 2. To keep the piracy activity under control, Firm 2 increases its anti-piracy effort. Then Firm 2 can increase its price since using pirated Product 2 becomes less attractive. Finally, the impact of its anti-piracy effort's direct effect, cross effect, or effort cost on Firm 2 is the same as for Firm 1.

Next, we study the setting where the two firms are symmetric.

Comparative Statics in a Symmetric Setting

In this subsection, we consider the setting where two firms are symmetric, i.e., $k_i = k$, $r_i = r$, $q_i = q$, and $\theta_i = \theta$, $i = 1, 2$; $a_1 = b_2$, and $a_2 = b_1$.

Since the optimal anti-piracy efforts for the two publishers Firm 1 and Firm 2 are symmetric, we can denote the optimal anti-piracy effort and optimal product price as e_d^* and p_d^*, which are given by

$$e_d^* = \frac{(\theta-1)(k+2q)\left(a_1(\theta k+k-2\theta t)+b_1(\theta-1)k\right)}{2\left((a_1+b_1)\left(a_1(\theta k+k-2\theta t)+b_1(\theta-1)k\right)+8(\theta-1)\theta rt(k-t)\right)} \tag{3.19}$$

and

$$p_d^* = \frac{-2(\theta-1)^2\theta rt(k+2q)(k-t)}{(a_1+b_1)\left(a_1(\theta k+k-2\theta t)+b_1(\theta-1)k\right)+8(\theta-1)\theta rt(k-t)}. \tag{3.20}$$

We can get the following comparative statics results in Theorem 14 (also shown in Table 3.6) about the network effect coefficient k:

Theorem 14. *In the duopoly symmetric setting, when the network effect increases, there are two regions.*

1. *In the region where the discount factor of the pirated software is low ($\theta < \theta''$), anti-piracy efforts decrease with the network effect, i.e., $\frac{\partial e_d^*}{\partial k} < 0$.*
2. *In the region where the discount factor of the pirated software is high ($\theta > \theta''$),*
   (a) *When the effort cost is small ($r < r_{dDF}$), the anti-piracy effort decreases with the network effect, i.e., $\frac{\partial e_d^*}{\partial k} < 0$.*
   (b) *When the effort cost is large ($r > r_{dDF}$), the anti-piracy effort will first increase and then decrease with the network effect, i.e., there is a threshold value $\bar{k}_{dDF}$ for any given r within the feasible region: when $k < \bar{k}_{dDF}$, $\frac{\partial e_d^*}{\partial k} > 0$, $\frac{\partial e_d^*}{\partial k} < 0$ otherwise.*

The threshold values θ'', r_{dDF}, and $\bar{k}_{dDF}$ are specified in the proof.

These results are similar to those in the asymmetric setting (see Table 3.4). We can understand Theorem 14 in the following way. In either case: (i) when θ or r is small, the piracy activity is under control, or (ii) when both θ and r are large, and k is also sufficiently

Table 3.6: Impact of k on e_d^*

	θ small	θ large	
		r small	r large
k_1 small	$\frac{\partial e_d^*}{\partial k_1} < 0$	$\frac{\partial e_d^*}{\partial k_1} < 0$	$\frac{\partial e_d^*}{\partial k_1} > 0$
k_1 large			$\frac{\partial e_d^*}{\partial k_1} < 0$

large, both firms will value the network effect more than controlling piracy activity. As the network effect increases, they will choose to decrease the anti-piracy effort. However, in the case when θ and r are large, and k is small, the network effect is small, and software piracy level is high. Then both firms will value controlling piracy activity more than the network effect. They will exert more anti-piracy effort as k increases.

We can also understand the results in Theorem 14 by using the results from the asymmetric setting. Mathematically, $\frac{\partial e_d^*}{\partial k} = (\frac{\partial e_1^*}{\partial k_1} + \frac{\partial e_1^*}{\partial k_2})|_{k_1=k_2=k} = (\frac{\partial e_1^*}{\partial k_1} + \frac{\partial e_2^*}{\partial k_1})|_{k_1=k_2=k}$. When r is small or when both r and k are large, we have $\frac{\partial e_1^*}{\partial k_1} < 0$ and $\frac{\partial e_2^*}{\partial k_1} < 0$ from Table 3.4. Then we can conclude $\frac{\partial e_d^*}{\partial k} < 0$. However, when r is large and k is small, $\frac{\partial e_1^*}{\partial k_1}$ and $\frac{\partial e_2^*}{\partial k_1}$ have different signs. Since the network effect k_1 affects Firm 1's anti-piracy effort directly and Firm 2's anti-piracy effort indirectly, we can expect k_1 has a larger effect on e_1^* than on e_2^*. Therefore, the sign of $(\frac{\partial e_1^*}{\partial k_1} + \frac{\partial e_2^*}{\partial k_1})|_{k_1=k_2=k}$ is determined by $\frac{\partial e_1^*}{\partial k_1}$.

We can also analyze the impact of k on price, and the results are summarized in Theorem 15.

Theorem 15. *In the duopoly symmetric setting, when the network effect increases, there are three regions.*

1. *When the effort cost is small ($r < r_{dDG}$), the price decreases with the network effect, i.e.,* $\frac{\partial p_d^*}{\partial k} < 0$.
2. *When the effort cost is medium ($r_{dDG} < r < r_{dF}$), the price first increases and then decreases with the network effect, i.e., there is a threshold value $\bar{k}_{dDG}$: when $k < \bar{k}_{dDG}$,* $\frac{\partial p_d^*}{\partial k} > 0$; $\frac{\partial p_d^*}{\partial k} < 0$ *otherwise.*

3. *When the effort cost is large ($r > r_{dF}$), the price increases with the network effect, i.e.,* $\dfrac{\partial p_d^*}{\partial k} > 0$.

The threshold values r_{dDG}, r_{dF}, and $\bar{k}_{dDG}$ are specified in the proof.

Table 3.7: Impacts of k on p_d^*

	k small	k large
r small	$\dfrac{\partial p_d^*}{\partial k} < 0$	
r medium	$\dfrac{\partial p_d^*}{\partial k} > 0$	$\dfrac{\partial p_d^*}{\partial k} < 0$
r large	$\dfrac{\partial p_d^*}{\partial k} > 0$	

When r is small, we know that the anti-piracy effort decreases in k (Theorem 14) and so the utility of pirating products increases. To compete with pirated software, firms decrease the product price. When r is large, the anti-piracy effort will not change much as k increases. Then both firms can increase their product price to exploit the increased network effect. Similarly, we can also explain the results of Theorem 15 from Theorem 13.

Table 3.8: Impact of q, a_1, r, and b_1 on e_d^* and p_d^* in the duopoly symmetric setting

	q	a_1	r	b_1
e_d^*	$\dfrac{\partial e_d^*}{\partial q} > 0$	$\dfrac{\partial e_d^*}{\partial a_1} > 0$	$\dfrac{\partial e_d^*}{\partial r} < 0$	$\dfrac{\partial e_d^*}{\partial b_1} > 0$
p_d^*	$\dfrac{\partial p_d^*}{\partial q} > 0$	$\dfrac{\partial p_d^*}{\partial a_1} > 0$	$\dfrac{\partial p_d^*}{\partial r} < 0$	$\dfrac{\partial p_d^*}{\partial b_1} > 0$

From Table 3.8 (proofs are in Appendix 6.2.13), we can see that each firm's effort and price increase with the product quality, anti-piracy effort's direct effect, and cross effect; and they decrease with anti-piracy effort cost. The results concerning product quality, direct effect, and cross effect can be generated from results shown in Table 3.5 in the asymmetric case. Here we find that when the cross effect increases, both the effort and price of both firms increase. We can explain the results as follows. As the cross effect b_1 increases, each firm has a greater impact on the other firm's overall anti-piracy effort. By increasing its anti-piracy effort, each firm can make the other firm's pirated software less attractive to use. As a result, its own pirated software is also less competitive. Then each firm can increase its price. Firms react in the same way when the direct effect a_1 increases but for a different reason. When it is more cost effective to control software anti-piracy (a_1 increases), each firm increases its anti-piracy effort to discourage software piracy. As a result, each firm can increase its price.

3.5. Extensions

In this section, we study how coordination via industrial alliance or government regulation can affect anti-piracy effort and product price.

3.5.1 Industrial Alliance

As is observed in reality, individual software firms can form industrial alliances such as BSA that exert an anti-piracy effort on individual firms' behalf. An industrial alliance's objective is to maximize the total benefit of the system which consists of the alliance and member firms.

In this case, each consumer's utility of buying the legitimate products is the same as before, given by Equations (3.1) and (3.2). However, the utility of pirating a product is different from the previous duopoly case. There is no cross effect any more since only the alliance will exert anti-piracy effort. We assume the alliance's anti-piracy effort will have the same effect on both pirated products, with the piracy cost being given by $a_0 e_a$ (subscript "a" stands for industrial alliance). Then the consumers' utility functions of pirating Product 1 and 2, U_{3a} and U_{4a}, are

$$U_{3a} = \theta_1 \left[q_1 - tx + k_1 (D_{1a} + D_{3a}) \right] - a_0 e_a \tag{3.21}$$

and

$$U_{4a} = \theta_2 \left[q_2 - t\left(1 - x\right) + k_2 \left(D_{2a} + D_{4a}\right)\right] - a_0 e_a. \tag{3.22}$$

By letting $U_1 = U_{3a}$, $U_2 = U_{4a}$, $U_{3a} = U_{4a}$, we can get legitimate Product 1 and 2's demand D_{1a} and D_{2a} as a function of e_a.

At the first stage, the industrial alliance chooses the optimal effort e_a^* to maximize the total system profit given by

$$\pi_a = D_{1a}p_{1a} + D_{2a}p_{2a} - re_a^2, \tag{3.23}$$

where r is the effort cost coefficient. The optimal effort e_a^* satisfies $\dfrac{\partial \pi_a}{\partial e_a} = 0$. Then in the second stage, the firms choose their price simultaneously to maximize their profit, which is given by:

$$\pi_{1a} = D_{1a}p_{1a} - re_a^2/2 \tag{3.24}$$

and

$$\pi_{2a} = D_{2a}p_{2a} - re_a^2/2 \tag{3.25}$$

where the cost of the anti-piracy effort is shared by the two firms equally. In Equations (3.24) and (3.25), we assume that each firm shares the anti-piracy cost equally. We can find p_{1a}^* and p_{2a}^* by solving $\dfrac{\partial \pi_{1a}}{\partial p_{1a}} = 0$ and $\dfrac{\partial \pi_{2a}}{\partial p_{2a}} = 0$.

To simplify the analysis, we assume the two firms are symmetric, i.e., $k_i = k$, $q_i = q$, $\theta_i = \theta$, $i = 1, 2$. Through backward induction, we can obtain

$$e_a^* = \frac{a_0\left(\theta - 1\right)\left(k + 2q\right)}{2\left(a_0^2 + 2\left(\theta - 1\right)rt\right)} \tag{3.26}$$

and

$$p_a^* = p_{1a}^* = p_{2a}^* = -\frac{(\theta - 1)^2 rt(k + 2q)}{2\left(a_0^2 + 2(\theta - 1)rt\right)} \tag{3.27}$$

from which we can find the impact of the network effect k on the anti-piracy effort and price:

Lemma 2. *In the industry alliance case, when the network effect increases, both the industrial alliance's anti-piracy effort e_a^* and each member firm's product price p_a^* increase.*

In the industry alliance case, the anti-piracy effort changes in a pattern different from those in the monopoly case and symmetric duopoly case. In the monopoly case and symmetric duopoly case, anti-piracy effort increases with the network effect only when (i) the quality of pirated software and the anti-piracy effort cost are large, and (ii) the network effect is low. However, in the industrial alliance case, the anti-piracy effort will always increase with the network effect. We can explain this result as follows. In the industry alliance case, the anti-piracy effort is not determined by the individual firms but by the alliance. When the alliance determines the anti-piracy effort, it considers the total system profit. The total demand for each firm's product (legitimate and pirated) is $1/2$ since these two firms are symmetric. Then the network effect of using a legitimate product is $k/2$. As the network effect coefficient k increases, the legitimate product becomes more valuable, and therefore the alliance should exert more anti-piracy effort to discourage software piracy, leading to higher demand for the legitimate software. Then each firm can charge a higher price to gain more profit.

3.5.2 Government Planning

In the government planning case, the government exerts anti-piracy effort in order to maximize the total social welfare by considering the software firms' profit, legitimate users' surplus, and the cost of anti-piracy effort.

In this case, the consumers' utility functions are similar to those in the industrial alliance case. From the utility functions, the demands of the legitimate products of both firms, D_{1g} and D_{2g} (subscript "g" stands for government planning), are similar to those in the industrial alliance's case.

The total consumer surplus for those who buy Product 1 is

$$CS_1 = \int_0^{D_{1g}} U_{1g} dx, \tag{3.28}$$

and that for buying Product 2 is

$$CS_2 = \int_{1-D_{2g}}^{1} U_{2g} dx. \tag{3.29}$$

In the first stage, the government needs to maximize the social welfare given by

$$SW = CS_1 + CS_2 + D_{1g}p_{1g} + D_{2g}p_{2g} - re_g^2, \tag{3.30}$$

by choosing an optimal e_g^*. In the second stage, each firm chooses its price to maximize its profit. When the firms are symmetric, we can find the optimal effort and price e_g^* and p_g^* through backward induction:

$$e_g^* = -\frac{a_1(\theta - 1)(k + 2q)}{2\left(a_1^2 + 4(\theta - 1)^2 rt\right)} \tag{3.31}$$

and

$$p_g^* = -\frac{(\theta - 1)(k + 2q)\left(a_1^2 + 2(\theta - 1)^2 rt\right)}{2\left(a_1^2 + 4(\theta - 1)^2 rt\right)}. \tag{3.32}$$

Then from (3.31) and (3.32), we can obtain the impact of the network effect k, as expressed in the following lemma:

Lemma 3. *In the government planning case, the optimal anti-piracy effort e_g^* and optimal product price p_g^* increase with the network effect.*

The results in Lemma 3 are similar to those in Lemma 2. In both cases, as the network effect increases, the central planner (the alliance or the government) increases anti-piracy effort to discourage piracy. This allows the firms to increase their price to take advantage of the increased network effect and higher piracy cost. In the government planning case, the government cares about consumer surplus. It is interesting to see how the consideration of consumer surplus affects the price and anti-piracy effort. We find the results as follows:

Theorem 16. *When the discount factor of pirated software θ is large ($\theta > 1/2$) and the piracy cost r is large ($r > r_p$), the anti-piracy effort, product price, and demand for legitimate software in the industrial alliance case are smaller than those in the government planning case. Otherwise, the anti-piracy effort, product price, and legitimate demand in the industrial alliance case are larger than those in the government planning case. The threshold r_p is specified in the Appendix 6.2.16.*

When software piracy is more tempting due to a high discount factor of pirated software θ ($\theta > 1/2$), intuitively we would expect an industrial alliance to invest more than a government does to prevent software piracy and maximize the total profit for the whole industry. However, Theorem 16 shows that this intuition is only true when the anti-piracy cost r is low. When the anti-piracy effort becomes expensive, the situations differ because unlike the government, which needs to prevent too many consumers from using pirated software as well as consider the industry's profit, the industrial alliance only needs to maximize the total profit of the two firms. In other words, without considering legitimate consumers' surplus, the industrial alliance invests less than the socially optimal level in the anti-piracy measure in order to save more on the anti-piracy effort. Then due to a lower anti-piracy measure, firms have to set their prices lower in the industrial alliance case than they do in the government planning case. As a result of under-investment in anti-piracy, the demand for legitimate software will also be lower in the industrial alliance case. In other scenarios when software piracy is not tempting, either because of a low discount factor of pirated software θ or high-level of anti-piracy measure due to a low r, an alliance over-invests in anti-piracy in pursuing the total profit of software firms, compared with the government planning case. Then firms can charge more, therefore hurting consumer surplus for buying and using legitimate software. According to the above discussion, an industrial alliance would over-react when left alone. If a government regulatory body wants to increase social welfare, it should help the industrial alliance in controlling software piracy when piracy issue is severe (high θ and r); on the other hand, if the piracy issue is not so severe, the government regulator should discourage the alliance from over-investing.

3.6. Conclusion

In this paper, we analyze software firms' optimal strategy to control software piracy through anti-piracy effort and product price. We find in both the monopoly and duopoly cases, a firm's anti-piracy efforts decrease with network effect if one of the following conditions holds: (i) the quality of the pirated software is small, (ii) the anti-piracy effort cost is small, or (iii) the quality of pirated product, the anti-piracy effort cost, and the network effect are large. Different from previous literature, a counter-intuitive result in our paper is that a firm's anti-piracy effort increases with the network effect under certain conditions. Although an increase in the network effect could make software piracy potentially more beneficial to the firm, the software firm does not always decrease its anti-piracy effort and tolerate piracy more. In other words, the software firm can exploit the network effect of pirated software to increase profit even while it also faces the problem of controlling the piracy activity. When determining the anti-piracy effort, the firm should balance the gain from the network effect and the loss from piracy.

However, the impact of the network effect on product price in the monopoly case is different from that in the duopoly case. In the monopoly case, the product price always increases with the network effect. The reason is the following. When the network effect increases, a consumer's utility of using legitimate software increases; then a firm can charge a higher price. However, in the duopoly case, we have obtained a counter-intuitive result: When the anti-piracy effort cost is small and a firm's software network effect increases, its optimal product price actually decreases, different from that in the monopoly case. The difference between the monopoly case and duopoly case arises from competition between legitimate products and their pirated counterparts. When a firm's network effect increases, its pirated product becomes more attractive. Given that the firm needs to lower its anti-piracy effort in the region of small effort cost, the firm has to lower its price for the legitimate software as well in order to compete with the pirated software. The implication is that in the duopoly case, the product price does not always increase with the network effect; a firm should also consider the competition effect when determining its price.

In both the asymmetric case and symmetric case in the duopoly scenario, we find that when one firm's cross effect increases, both firms' anti-piracy effort and price increase. An explanation is that an increase in cross effect can make pirated products less attractive and weaken the competition between legitimate products and their pirated counterparts. Then firms can increase their prices. As a result, firms increase anti-piracy efforts to reduce software piracy and increase sales.

We have also studied the industry alliance and the government planning cases. In both cases, when network effect increases, firms will exert more anti-piracy effort and charge a higher price. This result of the anti-piracy effort is different from the duopoly symmetric case. The reason is that in the industry alliance and government planning cases, the anti-piracy effort is not determined by the firms. An industry alliance or a government increases the anti-piracy effort as the network effect increases. As a result, pirated products become less attractive and thus the software firms can increase their product prices. Our result also shows that an industry alliance can over- or under-invest compared with a government agency. The policy implication is that when the piracy tendency is very strong, a government regulator should help the industry alliance in fighting software piracy in order to increase social welfare. In other situations, the government regulator should discourage over-investment in an anti-piracy effort by the industry alliance.

Here are some possible extensions to our paper. One could make product quality a decision variable and study vertical competition based on quality. New insights could be generated when adding this dimension of quality competition, such as considering whether a higher-quality firm has more incentive to control software piracy. It would also be interesting to see the interaction between vertical and horizontal competitions. One could also consider the existence of ethical consumers such as corporate users who will only choose between legitimate products. Such consideration could also introduce direct competition between legitimate products. Finally, one could extend the model in this paper to include the arrival of new consumers and study their impact on firms' anti-piracy investment.

Chapter 4

Analyzing Healthcare Information Exchanges' Strategies in a Competitive Environment

4.1. Introduction

Healthcare spending constitutes the largest share of public spending in most OECD countries, reaching 8.9% in 2015 (OECD, 2015). In US, it reached $3.0 trillion and accounted for 17.5% of the nation's GDP (Centers for Medicare and Medicaid Services, 2014). In Canada, the total health expenditure is expected to reach $219 billion and represent 10.9% of GDP (Canadian Institute for Health Information, 2016).

Despite of the enormous amount of expenditure, there are constant deep concerns of quality and efficiency of healthcare systems such as overuse of diagnostic testing services, avoidable hospitalization and readmission, preventable deaths and etc (Kohn et al., 2001, Weinberger, 2011, Mishra et al., 2012). One underlying reason of such high spending in healthcare is fragmented information infrastructure. For example, a patient may be required to have a CT test, even though he or she has already done some similar tests in a different hospital before. Another example is drug prescription. A physician without access to a patient's previous prescription history may not be able to prescribe effective medicine and as a result prolong treatment.

Healthcare information technology (HIT) has been identified as a potential solution to reducing cost and improving quality of service (Kohn et al., 2001, Garg et al., 2005, Hillestad et al., 2005, Chaudhry et al., 2006). Given the importance of health IT for transforming health care, US Congress passed Health Information Technology for Economic and Clinical Health Act (HITECH) in 2009. It provided more than $30 billion in stimulus fund to encourage healthcare providers to adopt HIT under the coordination of the Office of the National Coordination for Health Information Technology (ONC); it requires all medical records to be in standardized electronic forms by 2014 with establishing Health Information Exchange (HIE) programs as being one main objective (Blumenthal and Tavenner, 2010). HIE is a technological platform that allows health providers to 1). Securely access and share patients' vital medical history, no matter where patients are, whether in physicians' offices, labs, or emergency rooms and 2). provide safer, more effective care tailored to patients' unique medical needs. Therefore, adopting HIE could ultimately lead to better and more efficient healthcare.

In general, there are two types of services provided by an HIE. One is the basic service which provides the core functionalities of an HIE to support the sharing of healthcare information. For instance, Southeast Texas Health System (SETHS) is a collaborative HIE of rural hospitals in Texas. It provides basic services for its members such as patient management, record locator provision, and so on (Demirezen et al., 2016). In addition to providing base functionalities, an HIE can provide additional services, called value-added service, which could be valuable to healthcare providers. For example, an HIE could offer an integrated portal with data warehousing and data mining capability. A HP could use such portal to discover disease patterns and provide innovative care.

In our paper, we consider several types of network effect. The first type of network effect comes from using basic service. When more HPs use basic service, a provider can gain more utility for using the same basic service. The second type of network effect comes from value-added service. When there are more HPs using basic service, a provider with value-added service could utilize patients' records from those providers to discover patterns and design new treatment methods. In other words, a provider with value-added service could benefit from the installed base of basic service. Finally, among HPs adopting value-added service, they could benefit from each other by practices such as sharing solution development and deployment experiences. That is, there is a network effect among HPs using value-added service.

We study the effect of competition on the pricing of basic and value-added services. Our main research questions are:

1. How should an HIE determine the basic service price, value-added service price, and value-added service quality?

2. When will an HIE provide basis service only, and when will an HIE provide both basic and value-added services?

3. How will an HIE's decision changes when parameters change?

4. Should the government subsidize HIEs so that HIEs can gain higher profit and sustain?

We build game-theoretical models to address the above research questions. Our contributions will be mainly in three ways. First we study the competition among HIEs and how competition affects the pricing and service strategies. Second, our results show how HIE(s) determine the basic service price, the value-added service price, and the value-added service quality. Third, our results will show how a government should optimally subsidize the HIEs in different setting.

4.2. Literature

There are two streams of literature related to our paper. One stream of literature is network effect (network externality), which means that the value of a product or service will be affected by the total number of users. It is often assumed that consumers will have the same network effect. In particular, even though consumers valuate a product differently, they have the same network effect (Fudenberg and Tirole, 2000, Niculescu et al., 2012). In our paper, when other HPs use basic service (value-added service), a HP will gain network effect for using basic service (value-added service), which is the same as the network effect described by the literature. However, we also include a type of network effect that when other HPs use basic service, a HP can gain network effect for value-added service.

Another stream of literature is HIE policy. The policy could be from the government's perspective, and it could also be from HIEs' perspective. Since HIE can seamlessly transfer patients' information from one place to another, the U.S. government encourages HIEs to effectively connect HPs (Walker et al., 2005, Adler-Milstein et al., 2011). Khuntia et al. (2017) use surveys data of HIEs in the United States from 2008 to 2010 and find that HIEs need to provide value-added service besides basic service. They also find that the services should be bundled appropriately using transaction-, subscription- or mixed-fee models. Adjerid et al. (2018) use a national panel data set of the largest insurer in the United States to study whether HIEs can reduce spending for the insurer. They find that HIE can significantly reduce the spending in healthcare markets. Yeager et al. (2014) examines what affect use of the HIE in Louisiana. They find that "Meaningful Use" requirements play a critical role in participating in the HIE. Yaraghi et al. (2014) investigate actual adoption and use behaviors of 2,054 physicians. At the level of medical practices, they find what affect HIE adoption and use. Demirezen et al. (2016)'s work related to our paper the most. They use game game-theoretic to investigate sustainability of HIEs. They find the equilibrium behaviors of an HIE provider and the HPs. The similarity between Demirezen et al. (2016) and our paper comes in two ways. First both paper discuss the basic service price, the value-added service price, and the quality of the value-added service. Second, both paper has investigated the monopoly case. However, there are several difference between Demirezen et al. (2016) and our paper. First, we assume even though consumers valuate a product differently, they have the same network effect (Fudenberg and Tirole, 2000, Niculescu et al., 2012), while in Demirezen et al. (2016)'s paper, HIE valuations and the benefits HPs obtain from the network are correlated. Second, we investigate the duopoly case and solve the result. However, in Demirezen et al. (2016)'s paper, they only list the duopoly case model but cannot solve it.

4.3. Model Setup and Notations

We first consider a monopoly case and then a duopoly case. In each case, we have two sub-cases, basic service (hereby we use BS for abbreviation) sub-case and basic and value-added service (hereby we use B&VS for abbreviation) sub-case. In the BS case, HIE(s) will only provide basic service, while in the B&VS case, HIE(s) will provide both basic and value-added service.

We use Hotelling model instead of vertical competition model to investigate the duopoly case. In the vertical competition case, the functions of one HIE contains the functions of another HIE, while in the Hotelling model, both HIEs may have some unique functions. The difference between two cases implies that when the two HIEs' prices are the same, the HIE with higher quality is always preferred in the vertical competition case, while both HIEs will be chosen by some HPs in the Hotelling case, better matching the reality.

In the duopoly case, we assume two HIEs are located on two endpoints of a Hotelling line. Without loss of generality, let HIE 1 be at Point 0 and HIE 2 at Point 1. We also assume that HPs are located uniformly on the Hotelling line. In the comparative static analysis, we first investigate the *asymmetric* case, that is, only one parameter of an HIE changes. Then we study the *symmetric* case, that is, a common parameter of both HIEs changes.

4.3.1 Monopoly Case

In the monopoly case, we assume that only HIE 1 exists, and is located at the endpoint 0. For the monopoly case, we have two sub-cases, BS sub-case and B&VS sub-case.

Table 4.1: Summary of notation

Notation	Description
Parameters	
q_i	the value of HIE i's basic service for HPs
k_1 (k_{1i})	Network effect of using basic service in the monopoly case (with regard to HIE i in the duopoly case)
k_2 (k_{2i})	Network effect of using value-added service due to the users of basic service in the monopoly case (with regard to HIE i in the duopoly case)
k_3 (k_{3i})	Network effect of using value-added service due to the users of value-added service in the monopoly case (with regard to HIE i in the duopoly case)
M_h	Government subsidy to an HIE per adopted HP
x	A HP's location on the Hotelling line, representing its ideal choice of service
t	HP's unfitness cost of choosing an HIE
c_v (c_{vi})	value-added service cost coefficient in monopoly (duopoly) case
Decision Variables	
$p^{m\alpha}$	HIE's price for service type α in the monopoly case ($\alpha = b$ for basic service and $\alpha = v$ for value-added service in this paper)
$p_i^{d\alpha}$	HIE i's price for service type α in the duopoly case
Q_i	the value of HIE i's value-added service for HPs
Intermediate Variables	
$U^{m\alpha}$	A HP's net utility of using service type α in the monopoly case
$\pi^{m\alpha}$	the HIE's total profit for providing service type α in the monopoly case
$U_i^{d\alpha}$	a HP's net utility of using HIE i's service type α in the duopoly case
$\pi_i^{d\alpha}$	the HIE i's total profit of providing service type α in the duopoly case

BS Sub-case

We consider the case of HIE 1 offering only basic service to HPs. In this case, HIE 1 acts as a monopolist. When a HP joins an HIE and chooses the basic service, it will obtain utility q_1. At the same time, it will face the unfitness cost for the location difference from the endpoint 0. Assume the unfitness cost coefficient is t_1, and the location of HP is x, then the unfitness cost of the HP with respect to HIE 1 is $t_1 x$. Since an HIE has strong network effect, we need to incorporate the network effect into our model. Let x_1^{mb} be the indiffernce point at which HP will have the same utility between using the basic service and not joining HIE 1. Then, D_1^{mb}, the demand of HPs joining HIE 1 is given by $D_1^{mb} = x_1^{mb}$. Let k_1 be the network effect coefficient of using the basic service. Then we have a HP's reservation price of using HIE 1,

$$u^{mb} = q_1 - tx + k_1 D^{mb}. \tag{4.1}$$

Let HIE 1 charge HP price p^{mb} for basic service, then we conclude HPs' utility of using HIE 1's basic service is

$$U^{mb} = u^{mb} - p^{mb}. \tag{4.2}$$

Let HPs located at $x = x^{mb}$ are indifferent between joining and not. Then we have

$$U^{mb}|_{x=x^{mb}} = 0. \tag{4.3}$$

HIE 1's profit is given by

$$\pi^{mb} = D^{mb}\left(M_h + p^{mb}\right) \tag{4.4}$$

where M_h is the subsidy per HP from the government to the HIE for providing service. The HIE's objective is to maximize its profit given by Equation (4.4) by choosing the basic service price, i.e., $\partial \pi^{mb}/\partial p^{mb} = 0$. Then we can have

$$p^{mb*} = \frac{1}{2}\left(q_1 - M_h\right), \tag{4.5}$$

$$x^{mb*} = D^{mb*} = \frac{M_h + q_1}{2\left(t_1 - k_1\right)}, \tag{4.6}$$

and

$$\pi^{mb*} = \frac{\left(M_h + q_1\right)^2}{4\left(t_1 - k_1\right)}. \tag{4.7}$$

The B&VS Case

We consider the case of HIE 1 offering both basic and value-added service to HPs. Then some HPs will only choose the basic service, while others will choose both basic and value-added service. By following the literature Demirezen et al. (2016), we assume that HPs can not use value-added service without basic service.

For health providers that choose both basic service and value-added service, they have two reservation prices, the basic service reservation price and the value-added service reservation price. The basic service reservation price is the same as before, given by Equation (4.1). For a HP joining HIE 1 and choose value-added service, it will obtain utility Q_1 from the value-added service. Assume the unfitness cost coefficient is t_2, then the unfitness cost of the HP's value-added service with respect to HIE 1 is $t_2 x$. Let x_1^{mv} be the indifference point at which HPs will have the same utility when they choose both services or only the basic service. Then the demand of HPs using value-added service is $D_1^{mv} = x_1^{mv}$.

When a HP uses the value-added service, it can benefit from both the user base of basic service and that of the value-added service. Let k_2 be the network effect coefficient due to the adoption base of basic service, and k_3 be the network effect coefficient due to the base of value-added service. Then a HP's reservation price of using value-added service, u^{mv}, is

$$u^{mv} = Q_1 - tx + k_2 D^{mb} + k_3 D^{mv} \tag{4.8}$$

and its utility of using HIE's value-added service is

$$U^{mv} = u^{mv} - p^{mv}. \tag{4.9}$$

Let $x = x^{mv}$ be the location that an HP is indifferent between choosing only basic service and both basic and value-added service. Then we have

$$U^{mb}|_{x=x^{mv}} = (U^{mb} + U^{mv})|_{x=x^{mv}} \tag{4.10}$$

We can prove the following lemma by using (4.10) (see the appendices for all the proofs of theorems and lemmas). Lemma 1 describes HPs' choice of service according to their unfitness level.

Lemma 4. *For a particular HIE, HPs who have a lower unfitness level will favor the option of using both basic and value-added service; otherwise they favor the option of using basic service only.*

Then we can depict HPs' choice of services in the B&VS case in Figure 4.1.

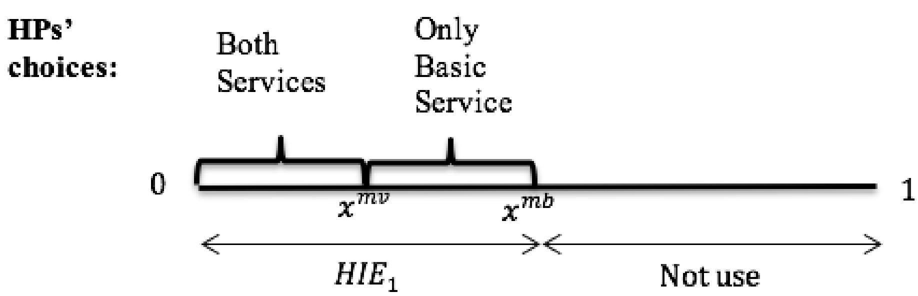


Figure 4.1: HPs' choice of service(s) in monopoly B&VS sub-case

The profit of HIE 1 will be

$$\pi^{mv} = D^{mb}\left(M_h + p^{mb}\right) + D^{mv}p^{mv} - Q_1^2 c_v D^{mv} \tag{4.11}$$

HIE 1 will first determine the quality of the value-added service Q_1 and then the prices of the basic service and the value-added service. Since there is only one decision maker (HIE 1), the sequential decision result is equivalent to the simultaneous decision result. To maximize Equation (4.11), HIE 1 needs to determine the value-added service quality, basic service price, and value-added service price. We have

$$p^{mb*} = \frac{1}{4}\left(\frac{k_2\left(2k_2c_v\left(M_h+q_1\right)-k_1+t_1\right)}{c_v\left(k_2^2-4\left(k_1-t_1\right)\left(k_3-t_2\right)\right)}+2\left(q_1-M_h\right)\right), \tag{4.12}$$

$$p^{mv*} = \frac{4k_2c_v\left(k_3-t_2\right)\left(M_h+q_1\right)-6\left(k_1-t_1\right)\left(k_3-t_2\right)+k_2^2}{4c_v\left(k_2^2-4\left(k_1-t_1\right)\left(k_3-t_2\right)\right)}, \tag{4.13}$$

$$Q_1^* = \frac{1}{2c_v}, \tag{4.14}$$

$$\pi^{mv*} = \frac{-4c_v\left(M_h+q_1\right)\left(4c_v\left(t_2-k_3\right)\left(M_h+q_1\right)+k_2\right)+k_1-t_1}{16c_v^2\left(k_2^2-4\left(k_1-t_1\right)\left(k_3-t_2\right)\right)}, \tag{4.15}$$

$$x^{mb*} = -\frac{k_2-8c_v\left(k_3-t_2\right)\left(M_h+q_1\right)}{4c_v\left(k_2^2-4\left(k_1-t_1\right)\left(k_3-t_2\right)\right)}, \tag{4.16}$$

and

$$x^{mv*} = \frac{-2k_2c_v\left(M_h+q_1\right)+k_1-t_1}{2c_v\left(k_2^2-4\left(k_1-t_1\right)\left(k_3-t_2\right)\right)}. \tag{4.17}$$

In the previous sub-section 4.3.1 we have studied the case where an HIE provider offers only basic service. Then one will ask: what is the optimal strategy for an HIE to provide services? In other words, is it always optimal to offer value-added service? By comparing the two profit functions (4.7) and (4.15), we find that

$$\pi^{mv*} - \pi^{mb*} = -\frac{\left(2k_2c_v\left(M_h+q_1\right)-k_1+t_1\right)^2}{16c_v^2\left(k_1-t_1\right)\left(4\left(k_1-t_1\right)\left(k_3-t_2\right)-k_2^2\right)} > 0. \tag{4.18}$$

which leads to the following lemma

Lemma 5. *In the monopoly case, the value-added service should always be provided.*

Will this result hold in the duopoly case? This is one of the questions we are going to investigate next.

4.3.2 Duopoly Case

In the duopoly case, we use Hotelling model to investigate the competition between two HIEs. The duoply case also has two sub-cases, the BS sub-case and B&VS sub-case. The offering of services is depicted in Figure 4.2.

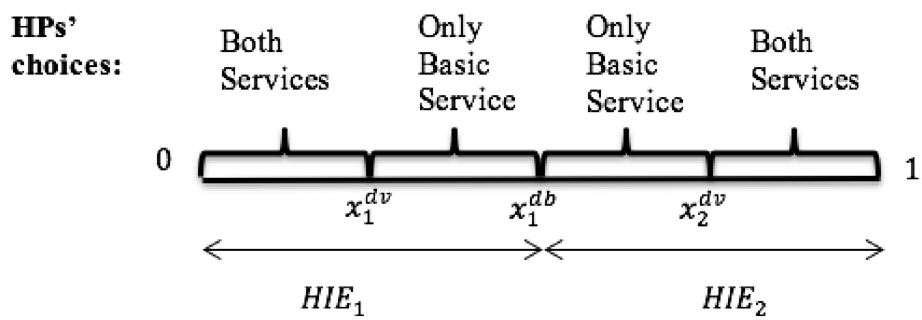


Figure 4.2: HPs' choice of service(s) in duopoly B&VS sub-case

BS Case

We consider the case of two HIEs offering only basic service to HPs. When a HP joining HIE i and chooses basic service, it will obtain utility q_i. To ensure all HPs are covered (otherwise, each HIE will act as a local monopolist and there is no competition between two HIEs), we assume q_i is large enough. At the same time, HPs will face the unfitness cost for the location difference from the endpoints. Assume the unfitness cost coefficient is t_1, and the location of HP is x, then the unfitness cost of the HP with respect to HIE 1 is $t_1 x$, and the unfitness cost of the HP with respect to HIE 2 is $t_1(1-x)$. Let x_{ih}^{db} be the indifference point at which HPs will have the same utility between using the basic service of HIE i and both services of HIE i. Let x_{1h}^{dv} be the indifference point at which HPs will have the same utility between using the basic service of HIE 1 and the basic service of HIE 2. Then $D_{1h}^{db} = x_{1h}^{db}$ will be the demand of HIE 1's basic service; $D_{2h}^{db} = 1 - x_{1h}^{db}$ will be the demand of HIE 2's basic service. Also, let k_{11} and k_{12} be network effect coefficient of using basic service. Then we can have

HPs' reservation price of choosing HIE 1 and HIE 2.

$$u_1^{db} = q_1 - t_1 x + k_{11} D_1^{db} \tag{4.19}$$

$$u_2^{db} = q_2 - t_1(1-x) + k_{12} D_2^{db} \tag{4.20}$$

then the utility function

$$U_1^{db} = u_1^{db} - p_1^{db} \tag{4.21}$$

$$U_2^{db} = u_2^{db} - p_2^{db} \tag{4.22}$$

The profit function will be

$$\pi_1^{db} = D_1^{dB} \left(M_h + p_1^{dB} \right) \tag{4.23}$$

and

$$\pi_2^{db} = D_2^{dB} \left(M_h + p_2^{dB} \right) \tag{4.24}$$

By maximizing π_1^{db} with respect to p_1^{db} and π_2^{db} with respect to p_2^{db} under the condition

$$U_1^{db}|_{x=x_1^{dB}} = U_2^{db}|_{x=x_1^{dB}}, \tag{4.25}$$

we have

$$p_1^{db*} = \frac{1}{3}\left(-k_{11} - 3M_h - 2k_{12} + q_1 - q_2\right) + t_1, \tag{4.26}$$

$$p_2^{db*} = \frac{1}{3}\left(-2k_{11} - 3M_h - k_{12} - q_1 + q_2\right) + t_1, \tag{4.27}$$

$$\pi_1^{db*} = -\frac{\left(k_{11} + 2k_{12} - q_1 + q_2 - 3t_1\right)^2}{9\left(k_{11} + k_{12} - 2t_1\right)}, \tag{4.28}$$

and

$$\pi_2^{db*} = -\frac{\left(2k_{11} + k_{12} + q_1 - q_2 - 3t_1\right)^2}{9\left(k_{11} + k_{12} - 2t_1\right)}. \tag{4.29}$$

When the two HIEs are symmetric, let $q_2 = q_1$, $k_{11} = k_{12} = k_1$, we have

$$p_1^{db*} = p_2^{db*} = -M_h - k_1 + t_1 \tag{4.30}$$

and

$$\pi_1^{db*} = \pi_2^{db*} = \frac{1}{2}\left(t_1 - k_1\right). \tag{4.31}$$

B&VS Case

We consider the case of two HIEs offering both basic and value-added service to HPs. HPs joining HIE 1 and 2 will gain utility U_1^{db} and U_2^{db} from basic service, given by Equation (4.21) and Equation (4.22). At the same time, when HPs use the value-added service, they can also gain extra utility

$$U_1^{dv} = u_1^{db} + u_1^{dv} - p_1^{db} - p_1^{dv} \tag{4.32}$$

and

$$U_2^{dv} = u_2^{db} + u_2^{dv} - p_2^{db} - p_2^{dv}, \tag{4.33}$$

where u_1^{dv} and u_2^{dv} are the reservation price for the value-added service, and

$$u_1^{dv} = Q_1 - t_2 x + k_{21} D_1^{db} + k_{31} D_1^{dv} \tag{4.34}$$

and

$$u_2^{dv} = Q_2 - t_2(1-x) + k_{22} D_2^{db} + k_{32} D_2^{dv}. \tag{4.35}$$

Let x_{ih}^{db} be the indifference point at which HPs will have the same utility between using the basic service of HIE i and both services of HIE i. Let x_{1h}^{dv} be the indifference point at which HPs will have the same utility between using the basic service of HIE 1 and the basic service of HIE 2. Then $D_{1h}^{dv} = x_{1h}^{dv}$ will be the demand of HIE 1's value-added service; $D_{1h}^{db} = x_{1h}^{db}$ will be the demand of HIE 1's basic service; $D_{2h}^{db} = 1 - x_{1h}^{db}$ will be the demand of HIE 2's basic service; $D_{2h}^{dv} = 1 - x_{2h}^{dv}$ will be the demand of HIE 2's value-added service.

Two HIEs' profit function will be

$$\pi_1^{dv} = D_1^{db}\left(M_1 + p_1^{db}\right) + D_1^{dv} p_1^{dv} - Q_1^2 c_{vf} D_1^{dv} \tag{4.36}$$

and

$$\pi_2^{dv} = D_2^{db}\left(M_1 + p_2^{db}\right) + D_2^{dv} p_2^{dv} - Q_2^2 c_{vs} D_2^{dv} \tag{4.37}$$

To maximize each HIE's profit, each HIE will first determine the quality of value-added service, and then the price of basic service and value-added service. From Equation (4.25),

$$D_1^{db} + D_1^{db} = 1, \tag{4.38}$$

$$U_1^{db}|_{x=x_1^{dv}} = U_1^{dv}|_{x=x_1^{dv}}, \tag{4.39}$$

and

$$U_2^{db}|_{x=x_2^{dv}} = U_2^{dv}|_{x=x_2^{dv}}, \tag{4.40}$$

we can express D_{ih}^{dv} and D_{ih}^{dv} as a function of p_{ih}^{dv} and p_{ih}^{dv}. Substituting the demand expression into the profit function, we can express the profit as a function of p_{ih}^{dv} and p_{ih}^{dv}. Then by solving

$$\frac{\partial \pi_i^{dv}}{\partial p_{ih}^{db}} = 0, \tag{4.41}$$

and

$$\frac{\partial \pi_i^{dv}}{\partial p_{ih}^{dv}} = 0, \tag{4.42}$$

we can express p_{ih}^{dv} and p_{ih}^{dv} as a function of Q_1 and Q_2. Substituting the price function into the profit function, by solving

$$\frac{\partial \pi_1^{dv}}{\partial Q_1} = 0 \tag{4.43}$$

and

$$\frac{\partial \pi_2^{dv}}{\partial Q_2} = 0, \tag{4.44}$$

we can solve the value-added service quality,

$$Q_1 = \frac{1}{2c_{vf}} \tag{4.45}$$

$$Q_2 = \frac{1}{2c_{vs}} \tag{4.46}$$

Substituting them into the price function, we have p_1^{db*}, p_2^{db*}, p_1^{dv*}, p_2^{dv*}, p_1^{dv*}, and π_1^{dv*}. However, there expressions are too complex, and we write them in the Mathematica file.

When the two HIEs are symmetric, let $q_2 = q_1$, $k_{11} = k_{12} = k_1$, $k_{21} = k_{22} = k_2$, and $k_{31} = k_{32} = k_3$, we will have

$$Q_1^* = Q_2^* = \frac{1}{2c_v} \tag{4.47}$$

$$p_1^{db*} = p_2^{db*} = \frac{2k_2^2 c_v + k_2}{8k_3 c_v - 8t_2 c_v} - M_h - k_1 + t_1 \tag{4.48}$$

and

$$p_1^{dv*} = p_2^{dv*} = \frac{1}{4}\left(\frac{3}{2c_v} + k_2\right). \tag{4.49}$$

Then, we have the profit

$$\pi_1^{dv*} = \pi_2^{dv} = \frac{4k_2^2 c_v^2 - 1}{64c_v^2\left(k_3 - t_2\right)} + \frac{1}{2}\left(t_1 - k_1\right) \tag{4.50}$$

By comparing the two profit functions (4.31) and (4.49), we find that

$$\pi_i^{dv*} - \pi_i^{db*} = \frac{4k_2^2 c_v^2 - 1}{64c_v^2\left(k_3 - t_2\right)}. \tag{4.51}$$

from which we can obtain the following lemma:

Lemma 6. *In the duopoly case, the value-added service should be provided if the network effect k_2 or the value-added service quality cost coefficient c_v is low enough; otherwise only basic service should be provided.*

Compared with Lemma 5 in the monopoly's case, we can see that an HIE provides the value-added service only if the network effect k_2 or the quality cost coefficient c_v is low enough. Where there is competition, offering value-added service does not always benefit an HIE. One needs to make a careful cost-benefit trade-off to determine the service offering.

4.4. Analysis

In this section, we do the comparative statics. We first list the parameters' impact on the basic service price and the value-added service price of both monopoly and duopoly case in Table 4.2. Then we will analyze these result in detail and list the results in theorem format.

Table 4.2: Results Related to price

	Monopoly			Duopoly		
	B	B+V		B	B+v	
	p^{mb*}	p^{mb*}	p^{mv*}	p_1^{db*}	p_1^{db*}	p_1^{dv*}
M_h	$\frac{\partial p^{mb*}}{\partial M_h} < 0$	$\frac{\partial p^{mb*}}{\partial M_h} < 0$	$\frac{\partial p^{mv*}}{\partial M_h} > 0$	$\frac{\partial p_1^{db*}}{\partial M_h} < 0$	$\frac{\partial p_1^{db*}}{\partial M_h} < 0$	$\frac{\partial p_1^{dv*}}{\partial M_h} = 0$
q_1	$\frac{\partial p^{mb*}}{\partial q_1} > 0$	$\frac{\partial p^{mb*}}{\partial q_1} \lesseqgtr 0^\dagger$	$\frac{\partial p^{mv*}}{\partial q_1} > 0$	$\frac{\partial p_1^{db*}}{\partial q_1} > 0$	$\frac{\partial p_1^{db*}}{\partial q_1} > 0$	$\frac{\partial p_1^{dv*}}{\partial q_1} > 0$
q_2	•	•	•	$\frac{\partial p_1^{db*}}{\partial q_2} < 0$	$\frac{\partial p_1^{db*}}{\partial q_2} < 0$	$\frac{\partial p_1^{dv*}}{\partial q_2} < 0$
k_1	$\frac{\partial p^{mb*}}{\partial k_1} = 0$	$\frac{\partial p^{mb*}}{\partial k_1} < 0$	$\frac{\partial p^{mv*}}{\partial k_1} > 0$	•	•	•
k_{11}	•	•	•	$\frac{\partial p_1^{db*}}{\partial k_{11}} < 0$	$\frac{\partial p_1^{db*}}{\partial k_{11}} < 0$	$\frac{\partial p_1^{dv*}}{\partial k_{11}} > 0$
k_{12}	•	•	•	$\frac{\partial p_1^{db*}}{\partial k_{12}} < 0$	$\frac{\partial p_1^{db*}}{\partial k_{12}} < 0$	$\frac{\partial p_1^{dv*}}{\partial k_{12}} < 0$
k_2	•	$\frac{\partial p^{mb*}}{\partial k_2} < 0$	$\frac{\partial p^{mv*}}{\partial k_2} > 0$	•	•	•
k_{21}	•	•	•	•	$\frac{\partial p_1^{db*}}{\partial k_{21}} < 0$	$\frac{\partial p_1^{dv*}}{\partial k_{21}} > 0$
k_{22}	•	•	•	•	$\frac{\partial p_1^{db*}}{\partial k_{22}} < 0$	$\frac{\partial p_1^{dv*}}{\partial k_{22}} < 0$
k_3	•	$\frac{\partial p^{mb*}}{\partial k_3} < 0$	$\frac{\partial p^{mv*}}{\partial k_3} > 0$	•	•	•
k_{31}	•	•	•	•	$\frac{\partial p_1^{db*}}{\partial k_{31}} < 0$	$\frac{\partial p_1^{dv*}}{\partial k_{31}} > 0$
k_{32}	•	•	•	•	$\frac{\partial p_1^{db*}}{\partial k_{32}} < 0$	$\frac{\partial p_1^{dv*}}{\partial k_{32}} < 0$
c_v	•	$\frac{\partial p^{mb*}}{\partial c_v} > 0$	$\frac{\partial p^{mv*}}{\partial c_v} < 0$	•	•	•
c_{v1}	•	•	•	•	$\frac{\partial p_1^{db*}}{\partial c_{v1}} > 0$	$\frac{\partial p_1^{dv*}}{\partial c_{v1}} < 0$
c_{v2}	•	•	•	•	$\frac{\partial p_1^{db*}}{\partial c_{v2}} > 0$	$\frac{\partial p_1^{dv*}}{\partial c_{v2}} > 0$

† indicates sign changes depending on the threshold value(s).

4.4.1 Government Subsidy

From our intuition, government's subsidy to HIEs can decrease the HIEs' price and increase HIEs' profit. This result is true in the monopoly BS sub-case. However, the results is different in other sub-cases.

In the monopoly B&VS sub-case, the subsidy can decrease the basic service price, and increase the HIE's profit. These results are consist with the monopoly BS sub-case. However, we find that government's subsidy will increase the value-added service price. The intuition is that when the government increases the subsidy, more HPs will use the basic service. Then HPs' network effect of using value-added service will also increase. An HIE can extract more profit from HPs by increase the value-added service price.

In the duopoly case, the basic service price will decrease, which is consistent with the monopoly case. However, the value-added service price will not change. The reason is that in the duopoly case, the competition exists. When the government increases the subsidy, both HIEs will have motivation to decrease the basic service price. In euqilibrium, their basic service demand will not change. HIEs' network effect of value-added service due to basic service demands will not change. Then the value-added service price will not change. We also find that the profit of both HIEs will not be affected by government subsidy. The explanation is that the competition between two HIEs leads two HIEs to decrease their price, which will transfer all the subsidy to the HPs. We can gain some insights from this result. When the competition between two HIEs is very intense and all potential HPs will join one HIE, then government's subsidy to sustain HIEs is not effective when their subsidy strategy is paying the HIEs per new HP joing them.

Summarizing our analysis, we have the following theorem.

Theorem 17. *When government increase the subsidy,*

1. *In all cases, the basic service price will decrease.*

2. *In the monopoly B&VS sub-case, the value-added service price will increase.*

3. In the duopoly B&VS sub-case, the value-added service price will not change.

4. In duopoly case, HIEs' profit is not a function of government's subsidy.

4.4.2 Basic Service Quality

When one HIE's basic service quality increases, one may conclude the HIE's basic service price and the value-added service price increase. The intuition is higher basic service quality can lead to higher HPs' utility. Then HIEs can charge higher price to maximize its profit. At the same time, more HPs will be willing to join the HIE. When there are more HPs using the basic service, HPs can gain more network effect from the value-added service. Then HIEs will charge higher price for the value-added service.

However, we find that in the monopoly B&VS sub-case, the basic service price does not increase monotonically. When the network effect of value-added service due to the basic service demand (k_2) is small, the basic service price increases with the basic service quality; else, the basic service price decreases with the basic service quality. When k_2 is large, the basic service demand has large effect on the network effect of value-added service. When the basic service quality increases, decreasing the basic service price will lead to higher basic service demand. Then value-added service's network effect will increase. HIEs can charge higher price for the value-added service. The intuition is that the value gained by the higher value-added service offsets the loss of lower basic service. when k_2 is small, the basic service demand does not play an important role in the network effect of value-added service. The explanation will be similar to the explanation in the last paragraph.

In the duopoly case, because of the competition, even the basic service price decrease, the basic service demand will not increase as much as in the monopoly case. Then the network effect of value-added service due to basic service demand will not change too much. HIEs have no motivation to decrease the basic service price to increase the network effect of value-added service due to basic service demand. Instead, HIEs have motivation to increase the basic service price.

In the monopoly case, HIE's profit will increase with network effect. Also in the duopoly case, when one HIE's basic service quality increases, its profit will increase, at the same time, the other HIE's profit will decrease. However, when two HIEs are symmetric, HIEs' basic service quality does not have impact on their profits. The reason is the competition between two HIEs. Two HIEs' quality effect are offset by each other. In summary we have the following theorem

Theorem 18. *When one HIE's basic service quality increase,*

1. *In the duopoly case and monopoly BS sub-case, the basic service price will increase.*

2. *In the monopoly B&VS sub-case, the basic service price increases with the basic service quality when the network effect k_2 is small, otherwise, the basic service price decreases with the basic service quality.*

3. *HIE's profit will increase.*

In the symmetric case, when HIEs' basic service quality increase, their profits will not change.

4.4.3 Network Effect k_1

When the network effect k_1 increases, the results in duopoly and monopoly case are quite different. In the monopoly BS sub-case, the basic service price is not a function of the network effect k_1. The intuition is like this. When the network effect k_1 increases, although HPs' utility will increase, the network effect will also increase. By balancing the gain for higher price and the gain from more HPs, the HIE will choose to increase the total demand. In the monopoly B&VS sub-case, the basic service price decreases with the network effect k_1. This result is not obvious. The reason is that sacrificing the benefit gaining from basic service, the demand in value-added service will increase, at the same time, the price of value-added service will increase. Then the HIE can gain more from the value-added service than the lose from the basic service.

In the duopoly BS sub-case, the HIE's basic service price will increase. Although the basic service price is decreasing in both B&VS cases and the duopoly BS sub-case, the reason behind them are different. In the duopoly BS sub-case, the reason is that the competition between two HIEs are intensified. In the B&VS cases, the basic service price decrease. There are two reasons, one is the intensified competiton between two HIEs and the other one is to increase the demand of basic service so that the value-added service's network effect increase, leading to high profit from the value-added service. For the other HIE, the basic service price will decrease in the duopoly case. In summary, we have the following theorem,

Theorem 19. *When one HIE's network effect k_1 increases,*

1. *In the monopoly BS sub-case, the basic service price does not change.*
2. *In the monopoly B&VS sub-case, the basic service price decreases.*
3. *In the duopoly case, the basic service price decreases.*
4. *In the duopoly case, the other HIEs' basic service price will decrease.*

4.4.4 Network Effect k_2 (k_{2i}) and k_3 (k_{2i})

The result and explanation related to network effect k_2 and k_3 are similar, so we discuss them together. When network effect k_2 and k_3 increase, HPs will have higher network effect from the value-added service. Then HIEs have higher motivation to increase the network of value-added service. By decreasing the basic service price, the value-added service network effect will directly and indirectly increase. HIEs can increase the value-added service price and gain more profit from the value-added service.

Here we can see that in the duopoly B&VS case, when one HIE's network effect k_{2i} and k_{3i} increase, the other HIE's basic service price and value-added service price will decrease. The reason is that facing the higher competition for the HIE, the other HIE should lower its price. In summary, we have the following theorem.

Theorem 20. *When one HIE's network effect k_2 (k_{2i}) or k_3 (k_{3i}) increases,*

1. *In all cases, the basic service price decreases.*

2. *In all cases, the value-added service price increases.*

3. *In the duopoly case, the other HIE's basic service price and value-added service price decrease.*

4.4.5 Value-added Service Quality Cost Coefficient

In the monopoly case, when the value-added service quality cost coefficient increases, HIE will incline to decrease the value-added service quality and decrease the value-added service price. Then some HPs will choose not to use the value-added service. Since the demand for value-added service decreases, the value-added service network effect will decrease. The HIE will find that keep the basic service price to a low level to increase the value-added service network effect is not worthy. Then the HIE will increase the basic service price.

However, in the duopoly case, when one HIE's quality cost coefficient c_{v1} increases, we find the other HIE's the basic service price will increase. We have explained that when one HIE's quality cost coefficient increases, its basic service price will increase, which means that the competition between two HIEs decrease. Then the other HIE can also increase its basic service price. In summary,

Theorem 21. *When one HIE's quality cost coefficient increase,*

1. *In both monopoly case and duopoly case, the value-added service price decreases and the basic service price increases.*

2. *In the duopoly case, the other HIE's basic service price increases.*

4.5. Conclusion

In this paper, we have investigated the problem of HIEs providing basic and value-added services. We analyzed the cases when HIE(s) choose to provide basic service only and both basic and value added services. We find that in the monopoly case, the value-added service should always be provided. However, in the duopoly case, the value-added service should be provided if the network effect k_2 or the value-added service quality cost coefficient c_v is low enough; otherwise only basic service should be provided.

We have investigated how government subsidy affect basic service price, value-added service price, and HIE(s)' profit. We find that in the duopoly case, HIEs' profits are not affected by the government subsidy. We also find that the value-added service price increases with the subsidy. The implication is that if the government intends to sustain HIEs' operation, it is not effective in the duopoly case than in the monopoly case.

With regard to basic service quality, we find that in the monopoly B&VS sub-case, the basic service price does not increase monotonically with basic service quality. When the network effect of value-added service due to the basic service demand (k_2) is small, the basic service price increases with the basic service quality; otherwise, the basic service price decreases with basic service quality. We have studied how network effect and value-added service cost impact the basic service price and value-added service price. The results show that competition plays an important role in the optimal pricing decisions.

One possible extension to our paper is that we can include different types of HPs to our model such as hospitals and clinics. When different types of HPs exist simultaneously, the model will be more complex but could yield more interesting results. For example, an HIE needs to consider the network effect in different types of HPs. Another extension could be considering the effect of multi-homing – an HP could subscribe to the services of multiple HIEs simultaneously. There, an interesting question is to investigate how multi-homing could change the intensity of competition. Finally, one could also consider an interesting scenario that by subscribing to an HIE, an HP could benefit from the network effect of other HIEs due to the information sharing among HIEs. Then, one can explore the impact of sharing and compatibility decisions on service offering and pricing.

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Chapter 6

Appendices

6.1. Proof of Chapter 2

6.1.1 Appendix 1: Proofs for $\zeta_l > \zeta_h$ Case

Assumptions and Constraints

In this paper, we make the following assumptions: (6.1)-(6.4), (6.8)-(6.12), (6.14), (6.17), and (6.19). The rest of the sub-section explains the reasons why we make these assumptions.

The utility functions are given by (2.1), (2.2), and (2.3). Since $0 < \theta < 1$, to ensure that the utility function (2.2) is positive even when a platform owner's effort is 0,

$$q > s_h \sigma_l^2 \tag{6.1}$$

Similarly, we make the following assumptions

$$r > \sigma_l^2 \tag{6.2}$$

to ensure (2.3) to be positive and

$$\sigma_0^2 < 1 \tag{6.3}$$

to ensure (2.1) to be positive. Since consumers in the high-certainty channel are more sensitive than consumers in the low-certainty channel, we shall have:

$$s_h > 1 \tag{6.4}$$

Let c_{p1} be the solution to $B = 0$. Then we have $e_p = 0$ when $k_p = \infty$, and $e_p = \infty$ when $k_p = c_{p1}$. Therefore the feasible range of k_p is between c_{p1} and ∞. Then from (2.44), we have $B > 0$ since the coefficient of k_p in B is positive. As a result, to ensure $D_h > 0$ and $D_l > 0$, we have

$$A > 0 \tag{6.5}$$

$$B > 0 \tag{6.6}$$

Then from Equation (2.45), we can see that

$$B > C \tag{6.7}$$

Define $\hat{s_h}$ as $A|_{s_h=\hat{s_h}} = 0$, and $\widehat{\sigma_l^2}$ as $A|_{\sigma_l^2=\widehat{\sigma_l^2}} = 0$. We have $A > 0$ when

$$s_h < \hat{s_h} = \frac{\zeta_h^2\sigma_l^2 + q\zeta_l^2 - r\zeta_l^2 + 4r\sigma_l^2 + 4qr - 4r^2}{\sigma_l^2\left(\zeta_l^2 + 4r\right)} \tag{6.8}$$

since A is a decreasing function of s_h. Or equivalently, $A > 0$ when

$$\sigma_l^2 < \widehat{\sigma_l^2} = \frac{(q-r)\left(\zeta_l^2 + 4r\right)}{s_h\left(\zeta_l^2 + 4r\right) - \left(\zeta_h^2 + 4r\right)} \tag{6.9}$$

Using the same way, we can find an upper bound of r, a lower bound of ζ_l, and an upper bound of ζ_h. That is

$$r < \hat{r} \tag{6.10}$$

$$\zeta_l > \hat{\zeta}_l \tag{6.11}$$

$$\zeta_h > \hat{\zeta}_h \tag{6.12}$$

To ensure $D_l > 0$, we have

$$C > 0 \tag{6.13}$$

Furthermore, to make the problem realistic, e_p should be finite. Therefore, we need an lower bound on k_p. We make the following assumption of the lower bound on k_p which will simplify our analysis as well:

$$k_p > s_h \sigma_l^2 / 4 \tag{6.14}$$

Then

$$C|_{k_p = s_h \sigma_l^2} = -k_p \left(\zeta_h^2 - \zeta_l^2 + 4\sigma_l^2 \right) \left(\zeta_h^2 - 4 s_h \sigma_l^2 + 4q \right) \tag{6.15}$$

which should be greater then 0, so

$$\zeta_h^2 - \zeta_l^2 + 4\sigma_l^2 < 0 \tag{6.16}$$

Since $\sigma_h^2 = \sigma_l^2 (1 - e_p)$, the platform owner's effort should be less than 1. Then from (2.48),

$$s_h \sigma_l^2 A - B < 0 \tag{6.17}$$

When $D_h > 0$, $D_l > 0$, and $D_h + D_l < 1$, we have

$$0 < \theta_l < \theta_h < 1 \tag{6.18}$$

To ensure $c_e > 0$ for all e_p, we shall have

$$\zeta_h^2 - 4 s_h \sigma_l^2 - \zeta_l^2 + 4\sigma_l^2 + 4q - 4r > 0 \tag{6.19}$$

Then we have

$$4q - 4r - 4 s_h \sigma_l^2 > \zeta_l^2 - \zeta_h^2 - 4\sigma_l^2 > 0. \tag{6.20}$$

That is

$$q - r - s_h \sigma_l^2 > 0 \tag{6.21}$$

Proof of Theorem 1

For D_0 in (2.22), since $\frac{2\sigma_0^2}{\zeta_l^2+4}$ increases with σ_0^2 and decreases with ζ_l^2, we can conclude D_0 is a increasing function of ζ_l, and a decreasing function of σ_0^2. From (2.23), we have

$$\frac{\partial \Pi_1}{\partial \zeta_l^2} = \frac{1}{16} - \frac{\sigma_0^4}{(\zeta_l^2 + 4)^2} < 0. \tag{6.22}$$

So, Π_1 decreases with ζ_l^2. Since $\zeta_l^2 - 4\sigma_0^2 + 4$ decreases with σ_0^2 and is greater than 0. We can conclude Π_1 decreases with σ_0^2. From (2.25), we get

$$\frac{\partial SP_0}{\partial \zeta_l^2} = \frac{-16\,(\zeta_l^2 - 4)\,\sigma_0^4 + (\zeta_l^2 + 4)^3 + 8\,(\zeta_l^2 + 4)\,\sigma_0^2}{32\,(\zeta_l^2 + 4)^3} > 0 \tag{6.23}$$

$$\frac{\partial SP_0}{\partial \sigma_0^2} = \frac{\zeta_l^2 \sigma_0^2}{(\zeta_l^2 + 4)^2} - \frac{\zeta_l^2}{4\,(\zeta_l^2 + 4)} < 0 \tag{6.24}$$

So SP_0 increases with ζ_l^2, and decreases with σ_0^2.

Proof of Theorem 2

Using the expression of D_h in (2.49), we can have

$$\frac{\partial D_h}{\partial x} = 2k_p \frac{\frac{\partial A}{\partial x} B - \frac{\partial B}{\partial x} A}{B^2} \tag{6.25}$$

where x is a parameter of interest.

Also, we can have

$$\frac{\partial}{\partial x}\left(\frac{\partial A}{\partial x} B - \frac{\partial B}{\partial x} A\right) = \frac{\partial^2 A}{\partial x^2} B - \frac{\partial^2 B}{\partial x^2} A \tag{6.26}$$

<u>(i) D_h **vs.** q</u>

We have:

$$\frac{\partial A}{\partial q} = \zeta_l^2 + 4r \tag{6.27}$$

$$\frac{\partial B}{\partial q} = 4k_p\left(\zeta_l^2 + 4r\right) \tag{6.28}$$

Then we have:

$$\frac{1}{(\zeta_l^2 + 4r)}\left(\frac{\partial A}{\partial q}B - \frac{\partial B}{\partial q}A\right) = k_p\left(\zeta_h^2 + 4r\right)\left(-\zeta_h^2 + \zeta_l^2 - 4\sigma_l^2\right) + s_h\sigma_l^2\left(\zeta_l^2 + 4r\right)\left(4k_p - s_h\sigma_l^2\right) > 0 \tag{6.29}$$

according to the conditions (6.14) and (6.16). So D_h increases with q.

<u>*(i)* D_h **vs.** ζ_h^2</u>

We have

$$\frac{\partial A}{\partial \zeta_h^2} = \sigma_l^2 > 0 \tag{6.30}$$

$$\frac{\partial B}{\partial \zeta_h^2} = -k_p\left(2\zeta_h^2 + 4r - \zeta_l^2\right) \tag{6.31}$$

Then we have

$$\left(\frac{\partial A}{\partial \zeta_h^2}B - \frac{\partial B}{\partial \zeta_h^2}A\right)|_{\zeta_h^2=\hat{\zeta}_h^2} = \frac{\partial A}{\partial \zeta_h^2}B|_{\zeta_h^2=\hat{\zeta}_h^2} > 0 \tag{6.32}$$

by using $A|_{\zeta_h^2=\hat{\zeta}_h^2} = 0$.

From (6.26),

$$\frac{\partial}{\partial \zeta_h^2}\left(\frac{\partial A}{\partial \zeta_h^2}B - \frac{\partial B}{\partial \zeta_h^2}A\right) = 2k_pA > 0 \tag{6.33}$$

by using (6.30) and (6.31). Therefore, for all $\hat{\zeta}_h \le \zeta_h < \zeta_l$, we get

$$\frac{\partial D_h}{\partial \zeta_h^2} > 0 \tag{6.34}$$

<u>*(ii)* D_h **vs.** s_h</u>

First we have

$$\frac{\partial A}{\partial s_h} = \sigma_l^2\left(-\left(\zeta_l^2 + 4r\right)\right) < 0 \tag{6.35}$$

$$\frac{\partial B}{\partial s_h} = -2s_h\sigma_l^4\left(\zeta_l^2 + 4r\right) < 0 \tag{6.36}$$

Then we can get

$$\frac{\partial A}{\partial s_h}B - \frac{\partial B}{\partial s_h}A = \sigma_l^2\left(\zeta_l^2 + 4r\right)\left(2s_h\sigma_l^2 A - B\right) \tag{6.37}$$

It is easy to see that $(2s_h\sigma_l^2 A - B)$ is concave since the coefficient of s_h^2 in $(2s_h\sigma_l^2 A - B)$ is negative. By using the expressions of A and B (given by (2.43) and (2.44)) and (6.26), we can get

$$\frac{\partial(2s_h\sigma_l^2 A - B)}{\partial s_h}|_{s_h=\hat{s_h}} = 2\sigma_l^2 A|_{s_h=\hat{s_h}} = 0 \tag{6.38}$$

Define

$$N = k_p\left(\zeta_l^2 + 4r\right)\left(\zeta_h^4 - \zeta_l^2\left(\zeta_h^2 + 4q\right) + 4r\zeta_h^2 - 16qr + 16r^2\right) + \left(\sigma_l^2\left(\zeta_h^2 + 4r\right) + (q-r)\left(\zeta_l^2 + 4r\right)\right)^2. \tag{6.39}$$

We can show that:

$$N|_{s_h=\hat{s_h}} < (-s_h^2\sigma_l^4\left(\zeta_l^2 + 4r\right)^2 + \left(\sigma_l^2\left(\zeta_h^2 + 4r\right) + (q-r)\left(\zeta_l^2 + 4r\right)\right)^2)|_{s_h=\hat{s_h}} = 0 \tag{6.40}$$

That is, at $s_h = \hat{s_h}$, $(2s_h\sigma_l^2 A - B)$ reaches the maximum value which is negative:

$$\left(2s_h\sigma_l^2 A - B\right)|_{s_h=\hat{s_h}} = \frac{N}{\zeta_l^2 + 4r}|_{s_h=\hat{s_h}} < 0 \tag{6.41}$$

according to (6.40). Then we can conclude

$$\frac{\partial D_h}{\partial s_h} < 0 \tag{6.42}$$

<u>*(iii)* D_h **vs.** r</u>

First we have

$$\frac{\partial A}{\partial r} = -4\left(s_h - 1\right)\sigma_l^2 - \zeta_l^2 + 4q - 8r \tag{6.43}$$

$$\frac{\partial B}{\partial r} = 4k_p\left(-\zeta_h^2 + 4q - 8r\right) - 4s_h^2\sigma_l^4 \tag{6.44}$$

Plugging the expression of A and B into $\frac{\partial A}{\partial r}B - \frac{\partial B}{\partial r}A$ and after some simplification, we have

$$\frac{\partial A}{\partial r}B - \frac{\partial B}{\partial r}A < k_p\left(\zeta_h^2\left(4s_h\sigma_l^2 + \zeta_l^2 - 4q + 8r\right) + 4\zeta_l^2\left(q - s_h\sigma_l^2\right) + 16r^2\right)\left(\zeta_h^2 - \zeta_l^2 + 4\sigma_l^2\right) \tag{6.45}$$

by using (6.14). From both equations (6.1) and (6.16), we can show that the second term of the right side of (6.45) is positive

$$\begin{aligned} &\zeta_h^2\left(4s_h\sigma_l^2 + \zeta_l^2 - 4q + 8r\right) + 4\zeta_l^2\left(q - s_h\sigma_l^2\right) + 16r^2 \\ &> \zeta_h^2\left(4s_h\sigma_l^2 + \zeta_l^2 - 4q + 8r\right) + 4\zeta_h^2\left(q - s_h\sigma_l^2\right) + 16r^2 \\ &= \zeta_h^2\left(\zeta_l^2 + 8r\right) + 16r^2 > 0 \end{aligned} \tag{6.46}$$

while the third term is negative according to (6.16). Therefore we have

$$\frac{\partial A}{\partial r}B - \frac{\partial B}{\partial r}A < 0 \tag{6.47}$$

Therefore $\frac{\partial D_h}{\partial r} < 0$.

(iv) D_h **vs.** σ_l^2

First we have

$$\frac{\partial A}{\partial \sigma_l^2} = \zeta_h^2 - s_h\left(\zeta_l^2 + 4r\right) + 4r < 0 \tag{6.48}$$

by $\zeta_h < \zeta_l$ and $s_h > 1$.

$$\frac{\partial B}{\partial \sigma_l^2} = -2s_h^2\sigma_l^2\left(\zeta_l^2 + 4r\right) < 0 \tag{6.49}$$

Also,

$$\frac{\partial A}{\partial \sigma_l^2}B - \frac{\partial B}{\partial \sigma_l^2}A = s_h^2\sigma_l^4\left(\zeta_l^2+4r\right)\left(\zeta_h^2 - s_h\left(\zeta_l^2+4r\right)+4r\right) + 2s_h^2\sigma_l^2(q-r)\left(\zeta_l^2+4r\right)^2 +$$
$$k_p\left(-\zeta_h^4+\zeta_h^2\left(\zeta_l^2-4r\right)+4q\zeta_l^2+16r(q-r)\right)\left(\zeta_h^2-s_h\left(\zeta_l^2+4r\right)+4r\right) \quad (6.50)$$

which is a quadratic and concave function of σ_l^2, and according to (6.26), we have

$$\frac{\partial}{\partial \sigma_l^2}\left(\frac{\partial A}{\partial \sigma_l^2}B - \frac{\partial B}{\partial \sigma_l^2}A\right) = 2s_h^2\left(\zeta_l^2+4r\right)A \geq 0 \quad (6.51)$$

Therefore, by setting (6.51) to be zero, the maximum point of $\frac{\partial A}{\partial \sigma_l^2}B - \frac{\partial B}{\partial \sigma_l^2}A$ is

$$\sigma_l^2 = \widehat{\sigma_l^2} \quad (6.52)$$

At this maximum point,

$$\left(\frac{\partial A}{\partial \sigma_l^2}B - \frac{\partial B}{\partial \sigma_l^2}A\right)|_{\sigma_l^2=\widehat{\sigma_l^2}} = \frac{\partial A}{\partial \sigma_l^2}B|_{\sigma_l^2=\widehat{\sigma_l^2}} < 0 \quad (6.53)$$

So, $\frac{\partial A}{\partial \sigma_l^2}B - \frac{\partial B}{\partial \sigma_l^2}A < 0$ for all σ_l^2. That is

$$\frac{\partial D_h}{\partial \sigma_l^2} < 0 \quad (6.54)$$

<u>*(v)* D_h **vs.** ζ_l^2</u>

We have

$$\frac{\partial A}{\partial \zeta_l^2} = (q-r) - s_h\sigma_l^2 \quad (6.55)$$

$$\frac{\partial B}{\partial \zeta_l^2} = k_p\left(\zeta_h^2+4q\right) - s_h^2\sigma_l^4 > 0 \quad (6.56)$$

Then

$$\frac{\partial A}{\partial \zeta_l^2}B - \frac{\partial B}{\partial \zeta_l^2}A = \left(\zeta_h^2 + 4r\right)\left[s_h^2\sigma_l^6 + k_p\left(\sigma_l^2\left(4rs_h + \zeta_h^2\left(s_h - 1\right) - 4q\right) - \left(q - r\right)\left(\zeta_h^2 + 4r\right)\right)\right] \tag{6.57}$$

which is a quadratic and convex function of s_h. Also,

$$\left(\frac{\partial A}{\partial \zeta_l^2}B - \frac{\partial B}{\partial \zeta_l^2}A\right)|_{s_h=0} = -k_p\left(\zeta_h^2 + 4r\right)\left[\sigma_l^2\left(\zeta_h^2 + 4q\right) + \left(q - r\right)\left(\zeta_h^2 + 4r\right)\right] < 0 \tag{6.58}$$

Since

$$\frac{\partial A}{\partial \zeta_l^2}|_{s_h=\hat{s_h}} = -\frac{\sigma_l^2\left(\zeta_h^2 + 4r\right)}{4r + \zeta_h^2} < 0 \tag{6.59}$$

we have

$$\left(\frac{\partial A}{\partial \zeta_l^2}B - \frac{\partial B}{\partial \zeta_l^2}A\right)|_{s_h=\hat{s_h}} = \frac{\partial A}{\partial \zeta_l^2}B|_{s_h=\hat{s_h}} < 0. \tag{6.60}$$

So, we can conclude for all $1 < s_h < \hat{s_h}$,

$$\frac{\partial D_h}{\partial \zeta_l^2} < 0. \tag{6.61}$$

Proof of Theorem 3

<u>*(i)* D_l **vs.** q</u>

From (2.50), we have

$$\frac{\partial D_l}{\partial q} = -\frac{2\left(\zeta_h^2 + 4r\right)}{\zeta_l^2 + 4r}\frac{\partial D_h}{\partial q} < 0 \tag{6.62}$$

according to (6.29).

<u>*(ii)* D_l **vs.** ζ_h^2</u>

Similarly, according to (6.34), D_h increases with ζ_h^2. From (6.64), we can conclude D_l decreases with ζ_h^2.

<u>*(iii)* D_l **vs.** s_h</u>

Similarly,

$$\frac{\partial D_l}{\partial s_h} = -\frac{2\left(\zeta_h^2 + 4r\right)}{\zeta_l^2 + 4r}\frac{\partial D_h}{\partial s_h} > 0. \tag{6.63}$$

according to (6.42).

<u>(*iv*) D_l **vs.** ζ_l^2</u>

D_l in (2.50) can be written as

$$D_l = \frac{1}{2}\left(1 - \frac{2D_h\left(\zeta_h^2 + 4r\right) + 4\sigma_l^2}{\zeta_l^2 + 4r}\right) \tag{6.64}$$

According to (6.61), D_h decreases with ζ_l^2. Then we can conclude D_l increases with ζ_l^2.

<u>(*v*) D_l **vs.** r</u>

From (6.2) and(6.10), we have $\sigma_l^2 < r < \hat{r}$ where $A_{r=\hat{r}} = 0$.

Then from the expression of D_l in (2.50), we have

$$\frac{\partial D_l}{\partial r}|_{r=\hat{r}} = \frac{1}{2}\left(\frac{16\sigma_l^2}{\zeta_l^2 + 4\hat{r}} - \frac{\zeta_h^2 + 4\hat{r}}{\zeta_l^2 + 4\hat{r}}\frac{\partial D_h}{\partial r}|_{r=\hat{r}}\right) > 0 \tag{6.65}$$

Also, we can show that $\frac{\partial C}{\partial r}B - \frac{\partial B}{\partial r}C = A_1 r^2 + A_2 r + A_3$ where $A_1 = 64k_p s_h \sigma_l^2\left(4k_p - s_h\sigma_l^2\right) > 0$ according to (6.14). Also, we can show the coefficient A_2 is positive:

$$A_2 = 8k_p s_h \sigma_l^2\left(\zeta_l^2 - 4\sigma_l^2\right)\left(4k_p - s_h\sigma_l^2\right) - 8k_p^2\left(\zeta_h^2 - \zeta_l^2 + 4\sigma_l^2\right)\left(\zeta_h^2 - 4s_h\sigma_l^2 + 4q\right) > 0.$$

Let r_1 and $r_1{}'$ be the two roots of $\partial D_l/\partial r = 0$ and $r_1 < r_1{}'$. If $A_3 \geq 0$, $r_1{}' \leq 0$ and D_l increases with r. On the other hand, if $A_3 < 0$, $r_1 < 0$ and $r_1{}' > 0$. We have $\frac{\partial D_l}{\partial r}$ positive when $r > r_1{}'$ and negative otherwise.

For r_1' to be valid, $\sigma_l^2 < r_1' < \hat{r}$ should hold, otherwise a valid threshold does not exist and we set the threshold to the corresponding boundary value. For example, if $r_1' \leq \sigma_l^2$, then the threshold is $r_1' = \sigma_l^2$ and D_l will increase with r, for all valid values of r. A similar situation holds for the upper bound as well.

<u>(*vi*) D_l **vs.** σ_l^2</u>

From (2.50), we have

$$\frac{\partial D_l}{\partial \sigma_l^2} = -\frac{2B^2 + (\zeta_h^2 + 4r)(2k_p(\frac{\partial A}{\partial x}B - \frac{\partial B}{\partial x}A))}{B^2(\zeta_l^2 + 4r)} \tag{6.66}$$

When $\sigma_l = 0$, the numerator of $\frac{\partial D_l}{\partial \sigma_l^2}$ is $-8k_p \left(\zeta_h^2 - s_h \left(\zeta_h^2 + 4r\right) + 4q\right) B|_{\sigma_l=0}$. When $s_h > \left(\zeta_h^2 + 4q\right) / \left(\zeta_h^2 + 4r\right)$, the numerator of $\frac{\partial D_l}{\partial \sigma_l^2}$ is positive and when $s_h < \left(\zeta_h^2 + 4q\right) / \left(\zeta_h^2 + 4r\right)$, the numerator of $\frac{\partial D_l}{\partial \sigma_l^2}$ is negative.

$$\begin{aligned}\frac{\partial^2 D_l}{\partial(\sigma_l^2)^2} = \frac{32k_p s_h^2 \left(\zeta_h^2 + 4r\right)}{-8B^3}&[s_h^2\sigma_l^4 \left(\zeta_l^2 + 4r\right)\left(\zeta_h^2\sigma_l^2 - s_h\sigma_l^2\left(\zeta_l^2 + 4r\right) + 3\zeta_l^2(q-r) + 4r\left(\sigma_l^2 + 3q - 3r\right)\right) \\ &+ B|_{\sigma_l=0}\left(3\zeta_h^2\sigma_l^2 - 3s_h\sigma_l^2\left(\zeta_l^2 + 4r\right) + \zeta_l^2(q-r) + 4r\left(3\sigma_l^2 + q - r\right)\right)]\end{aligned} \tag{6.67}$$

The expression in square brackets in (6.67) can be expressed as a cubic function of σ_l^2, $C_3\sigma_l^6 + C_2\sigma_l^4 + C_1\sigma_l^2 + C_0$, where $C_3 = s_h^2\left(\zeta_l^2 + 4r\right)\left(\zeta_h^2 - s_h\left(\zeta_l^2 + 4r\right) + 4r\right)$, $C_2 = 3s_h^2(q - r)\left(\zeta_l^2 + 4r\right)^2$,
$C_1 = 3B|_{\sigma_l=0}\left(\zeta_h^2 - s_h\left(\zeta_l^2 + 4r\right) + 4r\right)$ and $C_0 = (q-r)\left(\zeta_l^2 + 4r\right)B|_{\sigma_l=0}$.

We have C_2, and C_0 are positive. When $s_h < \left(\zeta_h^2 + 4q\right) / \left(\zeta_h^2 + 4r\right)$, C_3, and C_1 are positive. That is, the numerator is positive. Since the denominator is negative, we have $\frac{\partial^2 D_l}{\partial(\sigma_l^2)^2} < 0$.

Therefore, we can conclude that D_l decreases with σ_l^2 for all σ_l^2 for a sufficiently small s_h, i.e., $s_h < \left(\zeta_h^2 + 4q\right) / \left(\zeta_h^2 + 4r\right)$.

Proof of Theorem 4

Define $D_t = \frac{G}{B}$ where

$$\begin{aligned}G = {}& s_h^2\sigma_l^4\left(-\zeta_l^2 + 4\sigma_l^2 - 4r\right) + \\ & k_p\left(-\zeta_h^4 + 4\left(\zeta_l^2\left(-s_h\sigma_l^2 + 2q - r\right) + 4(q-r)\left(r - \sigma_l^2\right)\right) + \zeta_h^2\left(4s_h\sigma_l^2 + \zeta_l^2 - 4q\right)\right)\end{aligned} \tag{6.68}$$

Since $D_t < 1$, we have $B > G$.

<u>(i) D_t **vs.** q</u>

From (2.51), we can see that D_t increases with q since $\zeta_l^2 - \zeta_h^2 > 0$ and D_h increases with q according to (6.29).

<u>(ii) D_t **vs.** ζ_l^2</u>

$$\frac{\partial G}{\partial \zeta_l^2} = k_p \left(\zeta_h^2 - 4s_h\sigma_l^2 + 8q - 4r\right) - s_h^2\sigma_l^4 \tag{6.69}$$

$$\frac{\partial B}{\partial \zeta_l^2} = k_p \left(\zeta_h^2 + 4q\right) - s_h^2\sigma_l^4 \tag{6.70}$$

From(6.14) and (6.21), we can have $\frac{\partial G}{\partial \zeta_l^2} > 0$ and $\frac{\partial B}{\partial \zeta_l^2} > 0$.

$$\frac{\partial G}{\partial \zeta_l^2} - \frac{\partial B}{\partial \zeta_l^2} = k_p \left(4q - 4r - 4s_h\sigma_l^2\right) \tag{6.71}$$

From (6.21), $\frac{\partial G}{\partial \zeta_l^2} - \frac{\partial B}{\partial \zeta_l^2} > 0$. Since $B > G$,

$$\frac{\partial G}{\partial \zeta_l^2} B - \frac{\partial B}{\partial \zeta_l^2} G > 0 \tag{6.72}$$

So, $\frac{\partial D_t}{\partial \zeta_l^2} > 0$.

<u>(iii) D_t **vs.** s_h</u>

Similarly, D_t decreases with s_h since D_h decreases with s_h according to (6.42).

<u>(iv) D_t **vs.** σ_l^2</u>

From (2.51), we can see that D_t decreases with σ_l^2, since $\zeta_l^2 - \zeta_h^2 > 0$ and D_h decreases with σ_l^2 according to (6.54).

<u>(v) D_t **vs.** r</u>

First we have

$$\frac{\partial G}{\partial r} = -4\left(k_p \left(\zeta_l^2 - 4\left(\sigma_l^2 + q - 2r\right)\right) + s_h^2\sigma_l^4\right) \tag{6.73}$$

and

$$\frac{\partial B}{\partial r} = 4k_p \left(-\zeta_h^2 + 4q - 8r\right) - 4s_h^2\sigma_l^4 \tag{6.74}$$

Then

$$\frac{\partial}{\partial r}\left(\frac{\partial G}{\partial r}\right) = -32k_p \tag{6.75}$$

$$\frac{\partial}{\partial r}\left(\frac{\partial B}{\partial r}\right) = -32k_p \tag{6.76}$$

which implies $\frac{\partial}{\partial r}\left(\frac{\partial G}{\partial r}\right)B - \frac{\partial}{\partial r}\left(\frac{\partial B}{\partial r}\right)G = -32k_p(B-G) < 0$. That is $\frac{\partial G}{\partial r}B - \frac{\partial B}{\partial r}G$ is a decreasing function of r. Let $\frac{\partial G}{\partial r}B - \frac{\partial B}{\partial r}G|_{r=r_2'} = 0$.

Then if $r < r_2'$,

$$\frac{\partial G}{\partial r}B - \frac{\partial B}{\partial r}G > 0 \tag{6.77}$$

If $r > r_2'$,

$$\frac{\partial G}{\partial r}B - \frac{\partial B}{\partial r}G < 0 \tag{6.78}$$

For r_2' to be valid, $\sigma_l^2 < r_2' < \hat{r}$ should hold. Otherwise a valid threshold does not exist and we set the threshold to the corresponding boundary value.

<u>*(vi)* D_t **vs.** ζ_h^2</u>

Let

$$\left(\frac{\partial G}{\partial \zeta_h^2}B - \frac{\partial B}{\partial \zeta_h^2}G\right)|_{\zeta_h=\zeta_h'} = 0. \tag{6.79}$$

We have

$$\frac{\partial G}{\partial \zeta_h^2} = k_p\left(-2\zeta_h^2 + 4s_h\sigma_l^2 + \zeta_l^2 - 4q\right) \tag{6.80}$$

$$\frac{\partial B}{\partial \zeta_h^2} = k_p\left(-2\zeta_h^2 + \zeta_l^2 - 4r\right), \tag{6.81}$$

so

$$\frac{\partial}{\partial \zeta_h^2}\left(\frac{\partial G}{\partial \zeta_h^2}B - \frac{\partial B}{\partial \zeta_h^2}G\right) < 0 \tag{6.82}$$

That is $\left(\frac{\partial G}{\partial \zeta_h^2}B - \frac{\partial B}{\partial \zeta_h^2}G\right)$ is a decreasing function of ζ_h^2. So D_t will increase with ζ_h when $\zeta_h < \zeta_h'$, and decrease with ζ_h^2 when $\zeta_h > \zeta_h'$.

For ζ_h' to be valid, $\hat{\zeta}_h < \zeta_h' < (\zeta_l^2 - 4\sigma_l^2)^{1/2}$ should hold. Otherwise a valid threshold does not exist and we set the threshold to the corresponding boundary.

(v) D_t **vs.** D_0,

In order to have $D_h^* + D_l^* > D_0^*$, we need to ensure that:

$$2\left(\zeta_l^2+4\right)\left(k_p\left(-\zeta_h^4+4\left(\zeta_l^2\left(-s_h\sigma_l^2+2q-r\right)+4(q-r)\left(r-\sigma_l^2\right)\right)+\zeta_h^2\left(4s_h\sigma_l^2+\zeta_l^2-4q\right)\right)\right.$$
$$\left.+s_h^2\sigma_l^4\left(-\zeta_l^2+4\sigma_l^2-4r\right)\right)-\left(\zeta_l^2-4\sigma_0^2+4\right)$$
$$\times\left(2k_p\left(-\zeta_h^4+\zeta_h^2\left(\zeta_l^2-4r\right)+4q\zeta_l^2+16r(q-r)\right)-2s_h^2\sigma_l^4\left(\zeta_l^2+4r\right)\right)>0 \quad (6.83)$$

The left hand-side is a quadratic function of r, which can be written as $A_1r^2 + A_2r + A_3$, where:

$A_1 = -128k_p\sigma_0^2 < 0$.

$A_1r^2+A_2r+A_3 = 0$ has two roots, r_2'' and r_2'''. Without loss of generality, let $r_2'' < r_2'''$. When $r_2'' < r < r_2'''$, we have $A_1r^2 + A_2r + A_3 > 0$, and thus $D_t^* > D_0^*$.

Proof of Theorem 5

(i) e_p **vs.** q **or** ζ_h^2

e_p can be written as a function of D_h, and

$$e_p = \frac{s_h\sigma_l^2 D_h}{2k_p} \quad (6.84)$$

Since D_h increases with q (or ζ_h^2), e_p will increases with q (or ζ_h^2).

(ii) e_p **vs.** r **and** ζ_l^2

From (6.84), since D_h decreases with r (or ζ_l^2), e_p will decreases with r (or ζ_l^2).

(iii) e_p **vs.** s_h

From (2.48), We have e_p's first order derivative with respect to s_h

$$\frac{\partial e_p}{\partial s_h} = \frac{\sigma_l^2(s_h\frac{\partial A}{\partial s_h}B + AB - s_h\frac{\partial B}{\partial s_h}A)}{B^2} \quad (6.85)$$

We can find $s_h\frac{\partial A}{\partial s_h}B + AB - s_h\frac{\partial B}{\partial s_h}A$ is a quadratic function of s_h and the coefficient of the quadratic term is $\zeta_h^2\sigma_l^6\left(\zeta_l^2+4r\right)+\sigma_l^4(q-r)\left(\zeta_l^2+4r\right)^2+4r\sigma_l^6\left(\zeta_l^2+4r\right)$, which is positive.

Also,

$$\frac{\partial A}{\partial s_h}|_{s_h=\hat{s_h}}=\sigma_l^2\left(-\left(\zeta_l^2+4r\right)\right)<0 \tag{6.86}$$

$$A|_{s_h=\hat{s_h}}=0 \tag{6.87}$$

Therefore, we have

$$\frac{\partial e_p}{\partial s_h}|_{s_h=\hat{s_h}}=\frac{\sigma_l^2 s_h\frac{\partial A}{\partial s_h}B}{B^2}<0 \tag{6.88}$$

$$\begin{aligned}\frac{\partial e_p}{\partial s_h}|_{s_h=1}=\sigma_l^4\left(\zeta_l^2+4r\right)\left(\sigma_l^2\left(\zeta_h^2+4r\right)+(q-r)\left(\zeta_l^2+4r\right)\right)\\ -B|_{\sigma_l^2=0}k_p\left(\sigma_l^2\left(-\zeta_h^2+2\zeta_l^2+4r\right)+(r-q)\left(\zeta_l^2+4r\right)\right)\end{aligned} \tag{6.89}$$

By (6.20), we can prove $\frac{\partial e_p}{\partial s_h}|_{s_h=1}>0$.

Let s_h' be the cutoff point, where

$$\frac{\partial e_p}{\partial s_h}|_{s_h=s_h'}=0 \tag{6.90}$$

So, e_p will first increase and then decrease with s_h.

<u>*(iv)* e_p **vs.** σ_l^2</u>

We have

$$\frac{\partial e_p}{\partial \sigma_l^2}=\frac{s_h(\sigma_l^2\frac{\partial A}{\partial \sigma_l^2}B+AB-\sigma_l^2\frac{\partial B}{\partial \sigma_l^2}A)}{B^2} \tag{6.91}$$

Since

$$\frac{\partial A}{\partial \sigma_l^2}=\zeta_h^2-s_h\left(\zeta_l^2+4r\right)+4r<0 \tag{6.92}$$

$$\frac{\partial B}{\partial \sigma_l^2}=-2s_h^2\sigma_l^2\left(\zeta_l^2+4r\right)<0 \tag{6.93}$$

we find that $\sigma_l^2\frac{\partial A}{\partial \sigma_l^2}B+AB-\sigma_l^2\frac{\partial B}{\partial \sigma_l^2}A$ is a quadratic function of σ_l^2, and the quadratic term is $s_h^2(q-r)\left(\zeta_l^2+4r\right)^2$, which is positive.

$$\left(\sigma_l^2\frac{\partial A}{\partial \sigma_l^2}B + AB - \sigma_l^2\frac{\partial B}{\partial \sigma_l^2}A\right)|_{\sigma_l^2=0} = AB|_{\sigma_l^2=0} > 0 \tag{6.94}$$

We define $\sigma_l''^2$ as the cutoff point:

$$\frac{\partial e_p}{\partial \sigma_l^2}|_{\sigma_l^2=\sigma_l''^2} = 0 \tag{6.95}$$

So we can conclude when $\sigma_l^2 < \sigma_l''^2$, $\frac{\partial e_p}{\partial \sigma_l^2} > 0$ else if $\sigma_l^2 > \sigma_l''^2$, $\frac{\partial e_p}{\partial \sigma_l^2} < 0$.

For $\sigma_l''^2$ to be valid, $0 < \sigma_l''^2 < \hat{\sigma_l^2}$ should hold. Otherwise a valid threshold does not exist and we set the threshold to the corresponding boundary value

Proof of Theorem 6

From

$$SP_h = \frac{A^2k_p^2\zeta_h^2}{2B^2} = \frac{\zeta_h^2D_h^2}{8} \tag{6.96}$$

We can conclude that SP_h and D_h will change in the same direction as a parameter changes.

Proof of Theorem 7

From (2.45) and (2.54), we have

$$SP_l = \frac{\left(\zeta_l^2\left(\frac{B\left(\zeta_l^2-4\sigma_l^2+4r\right)}{\zeta_l^2+4r} - \frac{A\left(4k_p\left(\zeta_h^2+4r\right)\right)}{\zeta_l^2+4r}\right) + 8k_p\zeta_h^2A\right)\left(\frac{B\left(\zeta_l^2-4\sigma_l^2+4r\right)}{\zeta_l^2+4r} - \frac{A\left(4k_p\left(\zeta_h^2+4r\right)\right)}{\zeta_l^2+4r}\right)}{B^2} \tag{6.97}$$

Since

$$\frac{A}{B} = \frac{D_h}{2k_p} \tag{6.98}$$

$$SP_l = \zeta_l^2F^2 + 4\zeta_h^2D_hF \tag{6.99}$$

where $F = \left(\frac{\left(\zeta_l^2-4\sigma_l^2+4r\right)}{\zeta_l^2+4r} - \frac{D_h\left(2\left(\zeta_h^2+4r\right)\right)}{\zeta_l^2+4r}\right)$.

$$\frac{\partial SP_l}{\partial D_h} = -\frac{8\left(\zeta_h^2+4r\right)\left(\zeta_h^2\left(\zeta_l^2+8r\right)-4r\zeta_l^2\right)}{\left(\zeta_l^2+4r\right)^2} D_h + \frac{16r\left(\zeta_h^2-\zeta_l^2\right)\left(\zeta_l^2-4\sigma_l^2+4r\right)}{\left(\zeta_l^2+4r\right)^2} \tag{6.100}$$

which is a linear function of D_h. D_h approaches the limit of $1/2$ as q goes to infinity. Then

$$\frac{\partial SP_l}{\partial D_h}|_{D_h=0} = \frac{16r\left(\zeta_h^2-\zeta_l^2\right)\left(\zeta_l^2-4\sigma_l^2+4r\right)}{\left(\zeta_l^2+4r\right)^2} < 0 \tag{6.101}$$

$$\frac{\partial SP_l}{\partial D_h}|_{D_h=\frac{1}{2}} = -\frac{4\left(4r\zeta_h^2\left(4\left(\sigma_l^2+r\right)-\zeta_l^2\right)+\zeta_h^4\left(\zeta_l^2+8r\right)+4r\zeta_l^2\left(\zeta_l^2-4\sigma_l^2\right)\right)}{\left(\zeta_l^2+4r\right)^2} < 0 \tag{6.102}$$

we can conclude $\frac{\partial SP_l}{\partial D_h} < 0$.

(*i*) SP_l **vs.** q

$$\frac{\partial SP_l}{\partial q} = \frac{\partial SP_l}{\partial D_h}\frac{\partial D_h}{\partial q} \tag{6.103}$$

Since $\frac{\partial SP_l}{\partial D_h} < 0$ and $\frac{\partial D_h}{\partial q} > 0$, $\frac{\partial SP_l}{\partial q} < 0$.

(*ii*) SP_l **vs.** s_h

$$\frac{\partial SP_l}{\partial s_h} = \frac{\partial SP_l}{\partial D_h}\frac{\partial D_h}{\partial s_h} \tag{6.104}$$

Since $\frac{\partial SP_l}{\partial D_h} < 0$ and $\frac{\partial D_h}{\partial s_h} < 0$, $\frac{\partial SP_l}{\partial s_h} > 0$

(*iii*) SP_l **vs.** ζ_l^2

$$\frac{dSP_l}{d\zeta_l^2} = \frac{\partial SP_l}{\partial \zeta_l^2} + \frac{\partial SP_l}{\partial D_h}\frac{\partial D_h}{\partial \zeta_l^2} \tag{6.105}$$

We can easily see $\frac{\zeta_l^2-4\sigma_l^2+4r}{\zeta_l^2+4r} - \frac{D_h\left(2\left(\zeta_h^2+4r\right)\right)}{\zeta_l^2+4r}$ increases with ζ_l^2. Therefore, $\frac{\partial SP_l}{\partial \zeta_l^2} > 0$

Since $\frac{\partial SP_l}{\partial D_h} < 0$ and $\frac{\partial D_h}{\partial \zeta_l^2} < 0$, $\frac{\partial SP_l}{\partial D_h}\frac{\partial D_h}{\partial \zeta_l^2} > 0$

We can see $\frac{\partial SP_l}{\partial \sigma_l^2} + \frac{\partial SP_l}{\partial D_h}\frac{\partial D_h}{\partial \sigma_l^2} > 0$.

(*iv*) SP_l **vs.** r

$$\frac{dSP_l}{dr} = \frac{\partial SP_l}{\partial r} + \frac{\partial SP_l}{\partial D_h}\frac{\partial D_h}{\partial r} \tag{6.106}$$

Since $\frac{\partial SP_l}{\partial D_h} < 0$ and $\frac{\partial D_h}{\partial r} < 0$, $\frac{\partial SP_l}{\partial D_h}\frac{\partial D_h}{\partial r} > 0$. $\frac{\partial SP_l}{\partial r}$'s sign is the same as F where D_h is considered as a constant. When $D_h = 0$ $(r = \hat{r})$, $F = \frac{\zeta_l^2 - 4\sigma_l^2 + 4r}{\zeta_l^2 + 4r}$ increases with r. Then we can see $\frac{dSP_l}{dr} > 0$ at $r = \hat{r}$.

When $D_h = 1/2$, $F = \frac{\zeta_l^2 - 4\sigma_l^2 - \zeta_h^2}{\zeta_l^2 + 4r}$ decreases with r. There could be several roots to $\frac{\partial SP_l}{\partial r} + \frac{\partial SP_l}{\partial D_h}\frac{\partial D_h}{\partial r} = 0$. Let the largest cutoff point be r_3'. Therefore we shall have $\frac{dSP_l}{dr} > 0$ when $r_3' < r \leq \hat{r}$.

For r_3' to be valid, $\sigma_l^2 < r_3' < \hat{r}$ should hold. Otherwise a valid threshold does not exist and we set the threshold to the corresponding boundary value.

<u>(v) SP_l **vs.** ζ_h^2</u>

$$\frac{dSP_l}{d\zeta_h^2} = \frac{\partial SP_l}{\partial \zeta_h^2} + \frac{\partial SP_l}{\partial D_h}\frac{\partial D_h}{\partial \zeta_h^2} \tag{6.107}$$

Since $\frac{\partial SP_l}{\partial D_h} < 0$ and $\frac{\partial D_h}{\partial \zeta_h^2} > 0$, $\frac{\partial SP_l}{\partial D_h}\frac{\partial D_h}{\partial \zeta_h^2} < 0$. At $\zeta_h = \hat{\zeta}_h$ $(D_h = 0)$, we have the following from (6.99)

$$\frac{\partial SP_l}{\partial \zeta_h^2}|_{D_h=0} = 0 \tag{6.108}$$

Then we can see that $\frac{dSP_l}{d\zeta_h^2} < 0$ at $\zeta_h = \hat{\zeta}_h$.

$$\frac{\partial SP_l}{\partial \zeta_h^2}|_{D_h=\frac{1}{2}} = -\frac{2\left(8r\left(\zeta_h^2 + 2\sigma_l^2\right) + \zeta_l^2\left(\zeta_h^2 - 4r\right)\right)}{\left(\zeta_l^2 + 4r\right)^2} \tag{6.109}$$

which can be positive or negative. There could be several roots to $\frac{\partial SP_l}{\partial \zeta_h^2} + \frac{\partial SP_l}{\partial D_h}\frac{\partial D_h}{\partial \zeta_h^2} = 0$. Let ζ_h'' be the smallest cutoff point that is greater than $\hat{\zeta}_h$. Therefore we shall have $\frac{dSP_l}{d\zeta_h^2} < 0$ when $\hat{\zeta}_h < \zeta_h < \zeta_h''$. For ζ_h'' to be valid, $\hat{\zeta}_h < \zeta_h'' < (\zeta_l^2 - 4\sigma_l^2)^{1/2}$ should hold. Otherwise a valid threshold does not exist and we set the threshold to the corresponding boundary.

Proof of Theorem 8

<u>(i) Π_2 **vs.** q</u>

From (2.52), since D_h and A increase with q (Theorem 2), we can conclude Π_2 increases with q.

<u>(ii) Π_2 **vs.** ζ_l^2</u>

Define $\Pi_2 = \frac{H}{16B}$, then

$$\frac{\partial \Pi_2}{\partial \zeta_l^2} = \frac{1}{16B^2}\left(\frac{\partial H}{\partial \zeta_l^2}B - \frac{\partial B}{\partial \zeta_l^2}H\right) \tag{6.110}$$

From

$$\frac{\partial \Pi_2}{\partial \zeta_l^2} = \frac{-8D_h\sigma_l^2\left(\zeta_h^2+4r\right)+\left(\zeta_l^2+4r\right)^2-16\sigma_l^4}{\left(\zeta_l^2+4r\right)^2} + \frac{\sigma_l^2 A}{\zeta_l^2+4r}\frac{\partial D_h}{\partial \zeta_l^2} \tag{6.111}$$

we find that

$$\frac{\partial \Pi_2}{\partial \zeta_l^2}|_{\zeta_l=\hat{\zeta}_l} = \frac{\left(\zeta_l^2+4r\right)^2-16\sigma_l^4}{\left(\zeta_l^2+4r\right)^2} > 0 \tag{6.112}$$

So $\left(\frac{\partial H}{\partial \zeta_l^2}B - \frac{\partial B}{\partial \zeta_l^2}H\right)|_{\zeta_l=\hat{\zeta}_l} > 0$

Since we have

$$\frac{\partial}{\partial \zeta_l^2}\left(\frac{\partial H}{\partial \zeta_l^2}B - \frac{\partial B}{\partial \zeta_l^2}H\right) = \left(2k_p\left(\zeta_h^2+4q\right)-2s_h^2\sigma_l^4\right)B > 0, \tag{6.113}$$

then for all $\zeta_l > \hat{\zeta}_l$, $\left(\frac{\partial H}{\partial \zeta_l^2}B - \frac{\partial B}{\partial \zeta_l^2}H\right) > 0$. That is Π_2 increases with ζ_l.

(iii) Π_2 **vs.** ζ_h^2

From (2.52), since A and D_h increase with ζ_h (Theorem 2), we can conclude Π_2 increases with ζ_h.

(iv) Π_2 **vs.** s_h

From (2.52), since A and D_h decrease with s_h (Theorem 2), we can conclude Π_2 decreases with s_h.

(v) Π_2 **vs.** σ_l^2

First A and D_h decrease with σ_l^2 (Theorem 2). Also, $\left(\zeta_l^2-4\sigma_l^2+4r\right)^2$ decreases with σ_l^2. Then we can conclude Π_2 decreases with σ_l^2.

(iv) Π_2 **vs.** r

From (2.52), we can see

$$\frac{\partial \Pi_2}{\partial r} = \frac{D_h\left(32\sigma_l^2\left(\zeta_l^2-\zeta_h^2\right)-8\left(\zeta_l^2+4r\right)^2\right)+4\left(\zeta_l^2+4r\right)^2-64\sigma_l^4}{\left(\zeta_l^2+4r\right)^2} + \frac{\sigma_l^2 A}{\zeta_l^2+4r}\frac{\partial D_h}{\partial r} \tag{6.114}$$

From (6.2), we can see that the first term of (6.114) is a decreasing function of D_h. When $D_h = 1/2$, this term is positive. So we conclude that for all $0 < D_h < 1/2$, this term is positive. Since D_h decreases with r, we can conclude $\frac{\sigma_l^2 A}{\zeta_l^2+4r}\frac{\partial D_h}{\partial r}$ is negative.

When $r = \hat{r}$, we have $A = 0$ and then $\frac{\partial \Pi_2}{\partial r} > 0$.

We denote r_4' as the largest root of (6.114). Then we have $\frac{\partial \Pi_2}{\partial r} > 0$ when $r_4' < r < \hat{r}$.

For r_4' to be valid, $\sigma_l^2 < r_4' < \hat{r}$ should hold. Otherwise a valid threshold does not exist and we set the threshold to the corresponding boundary value.

6.1.2 Appendix 2: Proofs for $\zeta_h > \zeta_l$ Case

We will make the following additional assumptions: (6.122), (6.124), and (6.125). Reasons for the assumptions are explained below. The demand side is the same as the $\zeta_l > \zeta_h$ case, while the supply side is different. When $\zeta_h > \zeta_l$, we have $\pi_h > \pi_l$ when $\alpha > \alpha_h$. To ensure $\pi_l > 0$, we have $\alpha > \alpha_l$. By letting $\pi_h = \pi_l$ at α_h and $\pi_l = 0$ at α_l, we can get the expression

$$\alpha_h = \frac{-4c_e + 4\beta_h - 4\beta_l}{\zeta_l^2 - \zeta_h^2} \tag{6.115}$$

$$\alpha_l = \frac{4c_l - 4\beta_l}{\zeta_l^2} \tag{6.116}$$

The demand in the high-certainty channel will be $D_h = 1 - \alpha_h$, and the demand in the low-certainty channel will be $D_l = \alpha_h - \alpha_l$. By letting the demand and supply in each channel equal, we can get the following expressions:

$$D_h = \frac{k_p\left(\zeta_h^2 - 4\left(s_h - 1\right)\sigma_l^2 - \zeta_l^2 + 4q - 4r\right)}{2k_p\left(\zeta_h^2 - \zeta_l^2 + 4q - 4r\right) - 2s_h^2\sigma_l^4}. \tag{6.117}$$

$$D_l = \frac{\sigma_l^2\left(4k_p\left(\zeta_h^2 - s_h\left(\zeta_l^2 + 4r\right) + 4q\right) + s_h^2\sigma_l^2\left(\zeta_l^2 - 4\sigma_l^2 + 4r\right)\right)}{2\left(\zeta_l^2 + 4r\right)\left(k_p\left(-\zeta_h^2 + \zeta_l^2 - 4q + 4r\right) + s_h^2\sigma_l^4\right)}. \tag{6.118}$$

$$D_t = \frac{1}{2} - \frac{2\sigma_l^2}{\zeta_l^2 + 4r} \tag{6.119}$$

$$e_p = \frac{s_h \sigma_l^2}{2k_p} D_h \tag{6.120}$$

We can also find the solution for SP_h, SP_l, and Π_2 in a similar way. Since

$$\zeta_h^2 - 4\left(s_h - 1\right)\sigma_l^2 - \zeta_l^2 + 4q - 4r > 0, \tag{6.121}$$

to ensure $D_h > 0$,

$$k_p\left(\zeta_h^2 - \zeta_l^2 + 4q - 4r\right) - s_h^2 \sigma_l^4 > 0 \tag{6.122}$$

To make sure $D_h < D_t < 1/2$, we should have

$$k_p\left(\zeta_h^2 - \zeta_l^2 + 4q - 4r\right) - s_h^2 \sigma_l^4 > k_p\left(\zeta_h^2 - 4\left(s_h - 1\right)\sigma_l^2 - \zeta_l^2 + 4q - 4r\right) \tag{6.123}$$

That is

$$4k_p\left(s_h - 1\right)\sigma_l^2 - s_h^2 \sigma_l^4 > 0 \tag{6.124}$$

Since the denominator of D_l is negative, and $s_h^2 \sigma_l^2 \left(\zeta_l^2 - 4\sigma_l^2 + 4r\right) > 0$ from (6.2), to ensure $D_l > 0$, we need

$$\zeta_h^2 - s_h\left(\zeta_l^2 + 4r\right) + 4q < 0 \tag{6.125}$$

Proofs about D_h

<u>(*i*) D_h **vs.** q**,** r **,** ζ_l^2**, and** ζ_h^2</u>

$$\frac{\partial D_h}{\partial q} = \frac{2k_p \sigma_l^2 \left(4k_p\left(s_h - 1\right) - s_h^2 \sigma_l^2\right)}{\left(k_p\left(\zeta_h^2 - \zeta_l^2 + 4q - 4r\right) - s_h^2 \sigma_l^4\right)^2} > 0 \tag{6.126}$$

by (6.124). Then we have:

$$\frac{\partial D_h}{\partial r} = -\frac{\partial D_h}{\partial q} < 0 \tag{6.127}$$

$$\frac{\partial D_h}{\partial \zeta_l^2} = -\frac{1}{4}\frac{\partial D_h}{\partial q} < 0 \tag{6.128}$$

$$\frac{\partial D_h}{\partial \zeta_h^2} = \frac{1}{4}\frac{\partial D_h}{\partial q} > 0 \tag{6.129}$$

(ii) D_h **vs.** s_h

$$\frac{\partial D_h}{\partial s_h} = k_p\sigma_l^2 \frac{(2k_p\left(-\zeta_h^2+\zeta_l^2-4q+4r\right)+s_h\sigma_l^2\left(\zeta_h^2-2\left(s_h-2\right)\sigma_l^2-\zeta_l^2+4q-4r\right))}{\left(k_p\left(\zeta_h^2-\zeta_l^2+4q-4r\right)-s_h^2\sigma_l^4\right)^2} \tag{6.130}$$

When $s_h = \frac{\zeta_h^2-\zeta_l^2+4q-4r}{4\sigma_l^2}+1$, the numerator of $\frac{\partial D_h}{\partial s_h}$ will get the maximum value: $2k_p\left(-\zeta_h^2+\zeta_l^2-4q+4r\right)+2s_h\sigma_l^2$, which is negative by (6.122). We see that $\frac{\partial D_h}{\partial s_h}<0$.

(iii) D_h **vs.** σ_l^2

$$\frac{\partial D_h}{\partial \sigma_l^2} = \frac{k_p\left(s_h^2\sigma_l^2\left(\zeta_h^2-2\left(s_h-1\right)\sigma_l^2-\zeta_l^2+4q-4r\right)-2k_p\left(s_h-1\right)\left(\zeta_h^2-\zeta_l^2+4q-4r\right)\right)}{\left(k_p\left(\zeta_h^2-\zeta_l^2+4q-4r\right)-s_h^2\sigma_l^4\right)^2} \tag{6.131}$$

When $\sigma_l^2 = \frac{\zeta_h^2-\zeta_l^2+4q-4r}{4(s_h-1)}$, the numerator of $\frac{\partial D_h}{\partial \sigma_l^2}$ will get the maximum value: $k_p\left(2\left(s_h-1\right)s_h^2\sigma_l^2\sigma_l^2-2k_p\left(s_h-1\right)\left(\zeta_h^2-\zeta_l^2+4q-4r\right)\right)$, which is negative by (6.122). We have $\frac{\partial D_h}{\partial \sigma_l^2}<0$.

Proofs about D_t

$$D_t = \frac{1}{2}-\frac{2\sigma_l^2}{\zeta_l^2+4r} \tag{6.132}$$

We can find D_t increases with r and ζ_l; D_t decreases with σ_l^2; D_t remains the same when q, s_h, and ζ_h.

Proofs about D_l

(i) D_l **vs.** q **and** ζ_h^2

$$\frac{\partial D_l}{\partial q} = \frac{2k_p\sigma_l^2\left(s_h^2\sigma_l^2-4k_p\left(s_h-1\right)\right)}{\left(k_p\left(\zeta_h^2-\zeta_l^2+4q-4r\right)-s_h^2\sigma_l^4\right)^2}<0 \tag{6.133}$$

$$\frac{\partial D_l}{\partial \zeta_h^2} = \frac{k_p\sigma_l^2\left(s_h^2\sigma_l^2-4k_p\left(s_h-1\right)\right)}{2\left(k_p\left(-\zeta_h^2+\zeta_l^2-4q+4r\right)+s_h^2\sigma_l^4\right)^2}<0 \tag{6.134}$$

by (6.124).

(ii) D_l **vs.** s_h

Since $D_t = \frac{1}{2}-\frac{2\sigma_l^2}{\zeta_l^2+4r}$, which does not change with s_h, and D_h decreases with s_h, we can conclude D_l increases with s_h.

$\underline{(iii)\ D_l\ \textbf{vs.}\ r}$

Since D_t increases with r, and D_h decreases with r, we can conclude D_l increases with r.

$\underline{(iv)\ D_l\ \textbf{vs.}\ \sigma_l^2}$

$$\frac{\partial D_l}{\partial \sigma_l^2} = \frac{k_p s_h^2 \sigma_l^2 \left(\left(\zeta_l^2+4r\right)\left(-\zeta_h^2+\zeta_l^2-4q+4r\right)+2\sigma_l^2\left(\left(s_h-3\right)\zeta_l^2+2\left(\zeta_h^2+2rs_h+4q-6r\right)\right)\right)}{\left(\zeta_l^2+4r\right)\left(k_p\left(-\zeta_h^2+\zeta_l^2-4q+4r\right)+s_h^2\sigma_l^4\right)^2} + \frac{-2k_p^2\left(\zeta_h^2-\zeta_l^2+4q-4r\right)\left(\zeta_h^2-s_h\left(\zeta_l^2+4r\right)+4q\right)-2s_h^4\sigma_l^8}{\left(\zeta_l^2+4r\right)\left(k_p\left(-\zeta_h^2+\zeta_l^2-4q+4r\right)+s_h^2\sigma_l^4\right)^2} \tag{6.135}$$

When $\sigma_l^2 = 0$, we can see the numerator is positive. From (6.124), we can get the upper bound of σ_l^2 at which point the numerator will be $s_h^4\sigma_l^6\left(\zeta_l^2+4r\right)\left(\frac{4(s_h-1)\sigma_l^2}{\zeta_h^2-\zeta_l^2+4q-4r}-1\right)$, which is negative by (6.121). So, D_l will first increase and then decrease with σ_l^2

$\underline{(v)\ D_l\ \textbf{vs.}\ \zeta_l^2}$

Since D_t increases with ζ_l^2 and D_h decreases with ζ_l, we can conclude D_l increases with ζ_l^2.

Proofs about D_t

$$D_t = \frac{1}{2} - \frac{2\sigma_l^2}{\zeta_l^2+4r} \tag{6.136}$$

We can find D_t increases with r and ζ_l; D_t decreases with σ_l^2; D_t remains the same when q, s_h, and ζ_h.

Proofs about e_p

$\underline{(i)\ e_p\ \textbf{vs.}\ q\textbf{,}\ r\textbf{,}\ \zeta_l^2\textbf{, and}\ \zeta_h^2}$

$$\frac{\partial e_p}{\partial q} = \frac{4k_p\left(s_h-1\right)s_h\sigma_l^4 - s_h^3\sigma_l^6}{\left(k_p\left(\zeta_h^2-\zeta_l^2+4q-4r\right)-s_h^2\sigma_l^4\right)^2} > 0 \tag{6.137}$$

$$\frac{\partial e_p}{\partial r} = \frac{s_h\sigma_l^4\left(s_h^2\sigma_l^2-4k_p\left(s_h-1\right)\right)}{\left(k_p\left(\zeta_h^2-\zeta_l^2+4q-4r\right)-s_h^2\sigma_l^4\right)^2} < 0 \tag{6.138}$$

$$\frac{\partial e_p}{\partial \zeta_l^2} = \frac{s_h^3\sigma_l^6-4k_p\left(s_h-1\right)s_h\sigma_l^4}{4\left(k_p\left(-\zeta_h^2+\zeta_l^2-4q+4r\right)+s_h^2\sigma_l^4\right)^2} < 0 \tag{6.139}$$

$$\frac{\partial e_p}{\partial \zeta_h^2} = \frac{4k_p\left(s_h-1\right)s_h\sigma_l^4 - s_h^3\sigma_l^6}{4\left(k_p\left(-\zeta_h^2+\zeta_l^2-4q+4r\right)+s_h^2\sigma_l^4\right)^2} > 0 \tag{6.140}$$

by (6.124).

<u>*(ii)* e_p **vs.** s_h</u>

$$\frac{1}{\sigma_l^2}\frac{\partial e_p}{\partial s_h} = \frac{\left(k_p\left(\zeta_h^2-\zeta_l^2+4q-4r\right)\left(\zeta_h^2+4\left(1-2s_h\right)\sigma_l^2-\zeta_l^2+4q-4r\right)+s_h^2\sigma_l^4\left(\zeta_h^2-\zeta_l^2+4\left(\sigma_l^2+q-r\right)\right)\right)}{4\left(k_p\left(\zeta_h^2-\zeta_l^2+4q-4r\right)-s_h^2\sigma_l^4\right)^2} \tag{6.141}$$

When $s_h = 1$, the sign of $\zeta_h^2+4\left(1-2s_h\right)\sigma_l^2-\zeta_l^2+4q-4r$ is undetermined. When the sign of this term is negative and k_p is large enough, the numerator will be negative. When the sign of this term is positive, the numerator will always be positive.

When s_h reaches its upper bound, which can be obtained by (6.121), the numerator will be $4s_h^2s_h\sigma_l^4\sigma_l^2 - 4k_ps_h\sigma_l^2\left(\zeta_h^2-\zeta_l^2+4q-4r\right)$, which is negative, since the denominator of D_h is positive. Therefore, we can conclude that e_p will either always decrease with s_h, or first increase and then decrease with s_h.

<u>*(iii)* e_p **vs.** σ_l^2</u>

$$\frac{\partial e_p}{\partial \sigma_l^2} = \frac{s_h\left(\zeta_h^2-\zeta_l^2+4q-4r\right)\left(k_p\left(\zeta_h^2-8\left(s_h-1\right)\sigma_l^2-\zeta_l^2+4q-4r\right)+s_h^2\sigma_l^4\right)}{4\left(k_p\left(-\zeta_h^2+\zeta_l^2-4q+4r\right)+s_h^2\sigma_l^4\right)^2} \tag{6.142}$$

The numerator is a quadratic function of σ_l^2. When $\sigma_l^2 = 0$, the numerator will be $k_ps_h\left(\zeta_h^2-\zeta_l^2+4q-4r\right)^2$, which is positive. From (6.122), when σ_l^2 get to the upper bound, we have $k_p\left(\zeta_h^2-\zeta_l^2+4q-4r\right) - s_h^2\sigma_l^4 = 0$. The numerator will be $-s_h\left(\zeta_h^2-\zeta_l^2+4q-4r\right)\left(k_p8\left(s_h-1\right)\right)$, which is negative, so we can conclude $\frac{\partial e_p}{\partial \sigma_l^2}$ will first be positive, and then be negative.

Proofs about SP_h

$$\pi_h = \frac{1}{8}\left(2D_h\left(\zeta_h^2-\zeta_l^2\right)+2\alpha\zeta_h^2-2\zeta_h^2+\zeta_l^2-\frac{4\zeta_l^2\sigma_l^2}{\zeta_l^2+4r}\right) \tag{6.143}$$

$$SP_h = \int_{1-D_h}^{1}\pi_h d\alpha = \frac{D_h\left(\zeta_l^2+4r\right)\left(\zeta_l^2\left(1-2k_pD_h\right)+\left(2k_p-1\right)D_h\zeta_h^2\right)-4x\zeta_l^2\sigma_l^2}{8\left(\zeta_l^2+4r\right)} \tag{6.144}$$

<u>*(i)* SP_h **vs.** q**,** s_h**,** r**, and** σ_l^2</u>

D_h and π_h increase with q and decrease with s_h, r, σ_l^2 , and thus we can conclude SP_h increases with q and decreases with s_h, r, σ_l^2 .

<u>*(ii)* SP_h **vs.** ζ_l^2</u>

$$\frac{\partial SP_h}{\partial \zeta_l^2} = k_p\left(-\zeta_h^2 + 4\left(\left(s_h - 1\right)\sigma_l^2 - q + r\right) + \zeta_l^2\right)$$
$$\frac{\left(k_p\left(4\sigma_l^2\left(\zeta_h^2\left(\left(s_h+1\right)\zeta_l^2 + 4r\left(s_h-1\right)\right) - 2\zeta_l^2\left(s_h\left(\zeta_l^2+4r\right) - 4q\right)\right) - \zeta_h^2\left(\zeta_l^2+4r\right)\left(\zeta_h^2 - \zeta_l^2 + 4q - 4r\right)\right) + M\right)}{32\left(\zeta_l^2+4r\right)\left(k_p\left(-\zeta_h^2+\zeta_l^2-4q+4r\right) + s_h^2\sigma_l^4\right)^2} \tag{6.145}$$

where $M = 2s_h^2\zeta_l^2\sigma_l^4\left(\zeta_l^2 - 4\sigma_l^2 + 4r\right)$.

When ζ_l^2 reaches the lower bound, that is 0, the numerator of the fraction part will be $4rk_p\zeta_h^2\left(4\left(\left(s_h-1\right)\sigma_l^2 - q + r\right) - \zeta_h^2\right)$, which is negative by (6.121).

From (6.121) and (6.122), we can find the upper bound for ζ_l^2. From (6.123), we can see the upper bound of ζ_l^2 by (6.121) is smaller. When ζ_l^2 reaches the upper bound given by (6.121), the numerator of the fraction part will be $2\sigma_l^2\left(s_h^2\sigma_l^2 - 4k_p\left(s_h-1\right)\right)\left(\zeta_h^2 - 4s_h\sigma_l^2 + 4q\right)\left(\zeta_h^2 - 4\left(s_h - 1\right.\right.$ which is negative by (6.121) and (6.124).

Then we can conclude SP_h will always increase with ζ_l^2, or there is a cutoff point; when ζ_l^2 is greater than this cutoff point, SP_h increases with ζ_l^2.

<u>*(iii)* SP_h **vs.** ζ_h^2</u>

$$\frac{\partial \pi_h}{\partial \zeta_h^2} = \frac{1}{8}\left(2\alpha + 2\frac{\partial D_h}{\partial \zeta_h^2}\left(\zeta_h^2 - \zeta_l^2\right) + 2D_h - 2\right) \tag{6.146}$$

which is positive. Since D_h increases with ζ_h^2, we can conclude SP_h increases with ζ_h^2.

Proofs about SP_l

$$\pi_l = \frac{1}{8}\zeta_l^2\left(2\alpha - \frac{4\sigma_l^2}{\zeta_l^2 + 4r} - 1\right) \tag{6.147}$$

$$SP_l = \int_{1-D_t}^{1-D_h} \pi_l d\alpha \tag{6.148}$$

<u>*(i)* SP_l **vs.** q**,** s_h **and** ζ_h^2</u>

π_l and D_t is not a function of q, s_h and ζ_h, D_h increases with q and ζ_h, and decreases with s_h. We can conclude SP_l decreases with q and ζ_h, and increases with s_h.

<u>*(ii)* SP_l **vs.** r</u>

π_l increases with r, D_t increases with r, and D_h decreases with r. We can conclude SP_l increases with r.

<u>*(ii)* SP_l **vs.** σ_l^2</u>

π_l decreases with σ_l^2, and D_l first increases and then decreases with σ_l^2. When $\sigma_l^2 = 0$, the FOC of SP_l with respect to σ_l^2 is positive. We can conclude SP_l first increases and then decreases with σ_l^2.

<u>*(iv)* SP_l **vs.** ζ_l^2</u>

π_l increases with ζ_l^2, D_t increases with ζ_l^2, and D_h decreases with ζ_l^2. We can conclude SP_l increases with ζ_l^2.

Proofs about Π_2

<u>*(i)* Π_2 **vs.** q **and** ζ_h</u>

$$\frac{\partial \Pi_2}{\partial q} = \frac{k_p\left(\zeta_h^2 - 4\left(s_h-1\right)\sigma_l^2 - \zeta_l^2 + 4q - 4r\right)\left(k_p\left(\zeta_h^2 + 4\left(s_h-1\right)\sigma_l^2 - \zeta_l^2 + 4q - 4r\right) - 2s_h^2\sigma_l^4\right)}{4\left(k_p\left(-\zeta_h^2 + \zeta_l^2 - 4q + 4r\right) + s_h^2\sigma_l^4\right)^2} \tag{6.149}$$

The numerator is an increasing function of k_p. From (6.124), when k_p gets to the lower bound, $\frac{2s_h^2\sigma_l^4}{8(s_h-1)\sigma_l^2}$, the numerator is $\frac{s_h^4\sigma_l^4\left(-\zeta_h^2+4\left((s_h-1)\sigma_l^2-q+r\right)+\zeta_l^2\right)^2}{16(s_h-1)^2}$,which is positive. We can conclude $\frac{\partial \Pi_2}{\partial q} > 0$.

We can also have the following result with the same logic:

$$\frac{\partial \Pi_2}{\partial \zeta_h^2} = \frac{k_p\left(\zeta_h^2 - 4\left(s_h-1\right)\sigma_l^2 - \zeta_l^2 + 4q - 4r\right)\left(k_p\left(\zeta_h^2 + 4\left(s_h-1\right)\sigma_l^2 - \zeta_l^2 + 4q - 4r\right) - 2s_h^2\sigma_l^4\right)}{16\left(k_p\left(-\zeta_h^2 + \zeta_l^2 - 4q + 4r\right) + s_h^2\sigma_l^4\right)^2} > 0 \tag{6.150}$$

<u>*(ii)* Π_2 **vs.** s_h</u>

$$\frac{\partial \Pi_2}{\partial s_h} = \frac{k_p \sigma_l^2 \left(\zeta_h^2 - 4\left(s_h - 1\right)\sigma_l^2 - \zeta_l^2 + 4q - 4r\right)\left(4k_p\left(-\zeta_h^2 + \zeta_l^2 - 4q + 4r\right) + s_h \sigma_l^2\left(\zeta_h^2 - \zeta_l^2 + 4\left(\sigma_l^2 + q - r\right)\right)\right)}{8\left(k_p\left(-\zeta_h^2 + \zeta_l^2 - 4q + 4r\right) + s_h^2 \sigma_l^4\right)^2} \tag{6.151}$$

The numerator is a decreasing function of k_p. From (6.124), when k_p gets to the lower bound, $\frac{2s_h^2\sigma_l^4}{8(s_h-1)\sigma_l^2}$, the numerator is $-\frac{s_h^3\sigma_l^6\left(-\zeta_h^2+4\left((s_h-1)\sigma_l^2-q+r\right)+\zeta_l^2\right)^2}{4(s_h-1)^2}$, which is negative. We can conclude $\frac{\partial \Pi_2}{\partial s_h} < 0$.

<u>(iii) Π_2 **vs.** r **and** ζ_l</u>

$$\frac{\partial \Pi_2}{\partial r} = \sigma_l^2 D_l * \frac{\left(s_h^2 \sigma_l^2\left(\zeta_l^2 + 4\left(\sigma_l^2 + r\right)\right) - 4k_p\left(\zeta_h^2 + \left(s_h - 2\right)\zeta_l^2 + 4rs_h + 4q - 8r\right)\right)}{4\left(\zeta_l^2 + 4r\right)^2\left(k_p\left(-\zeta_h^2 + \zeta_l^2 - 4q + 4r\right) + s_h^2\sigma_l^4\right)} \tag{6.152}$$

The numerator is a decreasing function of k_p. From (6.124), when k_p gets to the lower bound, $\frac{2s_h^2\sigma_l^4}{8(s_h-1)\sigma_l^2}$, the numerator is $\frac{s_h^2\sigma_l^2\left(-\zeta_h^2+4\left((s_h-1)\sigma_l^2-q+r\right)+\zeta_l^2\right)}{s_h-1}$, which is negative. Also from (6.122), the denominator is negative. Thus, we can conclude $\frac{\partial \Pi_2}{\partial r} > 0$.

We can also have the following result with the same logic:

$$\frac{\partial \Pi_2}{\partial \zeta_l^2} = \left(4k_p\left(\zeta_h^2 - s_h\left(\zeta_l^2 + 4r\right) + 4q\right) + s_h^2\sigma_l^2\left(\zeta_l^2 - 4\sigma_l^2 + 4r\right)\right) * \\ \frac{\sigma_l^4\left(s_h^2\sigma_l^2\left(\zeta_l^2 + 4\left(\sigma_l^2 + r\right)\right) - 4k_p\left(\zeta_h^2 + \left(s_h - 2\right)\zeta_l^2 + 4rs_h + 4q - 8r\right)\right)}{16\left(\zeta_l^2 + 4r\right)^2\left(k_p\left(-\zeta_h^2 + \zeta_l^2 - 4q + 4r\right) + s_h^2\sigma_l^4\right)^2} > 0 \tag{6.153}$$

<u>(iv) Π_2 **vs.** σ_l^2</u>

$$\pi_2 = D_h c_e + D_t c_l - k_p e_p^2 \tag{6.154}$$

Since

$$D_h c_e - k_p e_p^2 = \frac{k_p\left(-\zeta_h^2 + 4\left(\left(s_h - 1\right)\sigma_l^2 - q + r\right) + \zeta_l^2\right)^2}{16k_p\left(\zeta_h^2 - \zeta_l^2 + 4q - 4r\right) - 16s_h^2\sigma_l^4} = \frac{1}{8}\left(\zeta_h^2 - 4\left(s_h - 1\right)\sigma_l^2 - \zeta_l^2 + 4q - 4r\right) D_h \tag{6.155}$$

we can conclude $D_h c_e - k_p e_p$ decreases with σ_l^2. Since D_t decreases with σ_l^2 and $c_l = \frac{1}{8}\left(\zeta_l^2 - 4\sigma_l^2 + 4r\right)$ also decreases with σ_l^2¿ Thus, we can conclude Π_2 decreases with σ_l^2.

6.2. Proof of Chapter 3

On-line Appendix to "Analyzing Software Anti-piracy Strategies in a Competitive Environment"

6.2.1 Constraints of Parameter Values

Monopoly Case

To ensure $\pi_m > 0$, from (3.13), the denominator should be negative:

$$DEN_m \equiv a_1^2 (k_1 - \theta_1 t)^2 + 4(\theta_1 - 1)\theta_1^2 r_1 t (k_1 - t)^2 < 0. \tag{6.2.1}$$

Define the coefficient of k_1^2 in (6.2.1) as R which will be used frequently later on:

$$R = a_1^2 + 4(\theta_1 - 1) r_1 t \theta_1^2 \tag{6.2.2}$$

To ensure $e_m > 0$ and $p_m > 0$, we have

$$\theta_1 t > k_1. \tag{6.2.3}$$

To ensure $D_3 = x_2 - x_1 > 0$, from (3.8) and (3.9), we have

$$x_2 - x_1 = \frac{a_1 e_m - \theta_1 p_m}{(\theta_1 - 1)\theta_1 t} > 0. \tag{6.2.4}$$

That is, $a_1 e_m < \theta_1 p_m$. Then we have,

$$R_0 = a_1^2 (k_1 - \theta_1 t) + 2(\theta_1 - 1)\theta_1^2 r_1 t (k_1 - t) > 0, \tag{6.2.5}$$

which means

$$r_1 > r_m \equiv \frac{a_1^2 (k_1 - \theta_1 t)}{2(\theta_1 - 1)\theta_1^2 t (t - k_1)}. \tag{6.2.6}$$

It is easy to verify when (6.2.5) is satisfied, (6.2.1) will be satisfied. To summarize, to

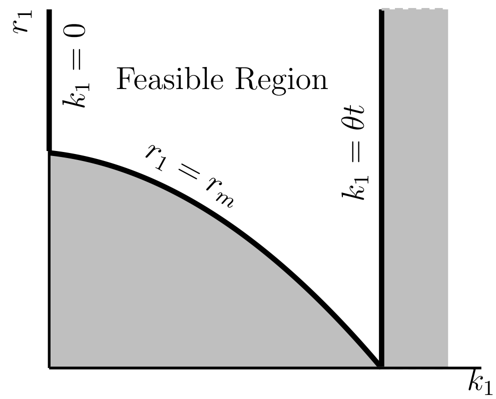

Figure 6.2.1: Feasible region in the (k_1,r_1) parameter space

ensure that solutions in the monopoly case are meaningful, the values of parameters should satisfy the constraints (6.2.3) and (6.2.6).

Using the constraints (6.2.3), (6.2.6) and $k_1 > 0$, we can visually display the feasible region in the (k_1, r_1) parameter space in Figure 6.2.1. Define the *left boundary* as B_L which is formed by two thick lines $k_1 = 0$ and $r_1 = r_m$ in Figure 6.2.1. Line $r_1 = r_m$ is a decreasing function of k_1 according to Equation (6.2.6).

Duopoly Case

First, the numerator of p_d in (3.20) is positive since $k < t$ from (6.2.3). Then to ensure $p_d > 0$, the denominator of p_d shall be positive:

$$DEN_d \equiv (a_1 + b_1)\,(a_1(\theta k + k - 2\theta t) + b_1(\theta - 1)k) + 8(\theta - 1)\theta rt(k - t) > 0. \tag{6.2.7}$$

which means $r > r'_d$, where

$$r'_d \equiv -\frac{(a_1 + b_1)\,R_1}{8(\theta - 1)\theta t(k - t)}. \tag{6.2.8}$$

The expression of R_1 is given by

$$R_1 \equiv DEN_d|_{r=0}/(a_1 + b_1) = a_1(\theta k + k - 2\theta t) + b_1(\theta - 1)k < 0 \tag{6.2.9}$$

to ensure e_d in (3.19) to be positive. This implies

$$k < k_{max} \equiv \frac{2a_1\theta t}{a_1(\theta+1)+b_1(\theta-1)} \tag{6.2.10}$$

We also define R_2 which will be used later:

$$R_2 \equiv \frac{\partial DEN_d}{\partial k} = (a_1+b_1)\left(a_1(\theta+1)+b_1(\theta-1)\right)+8(\theta-1)\theta rt. \tag{6.2.11}$$

From (3.14) to (3.16), we can find the demand of legitimate product D_d as a function of e_d and p_d. By using (3.20) and (3.19), we get D_d as the following:

$$D_d = \frac{2(\theta-1)\theta r(k+2q)(k-t)}{DEN_d} \tag{6.2.12}$$

To ensure both legitimate product demands are less than 1/2, we can have the following constraint:

$$R_3 \equiv 4(\theta-1)\theta r(k-t)(k+2q-2t)-(a_1+b_1)\,R_1 < 0 \tag{6.2.13}$$

which means

$$r > r_d \equiv \frac{(a_1+b_1)\,R_1}{4(\theta-1)\theta(k-t)(k+2q-2t)}. \tag{6.2.14}$$

It is easy to verify $DEN_d + R_3 > 0$. Then if $R_3 < 0$, we must have $DEN_d > 0$. That is

$$r'_d < r_d. \tag{6.2.15}$$

Also under the constraints that $R_1 < 0$ and $R_3 < 0$, we can derive the following property which will be used later:

$$k+2q-2t < 0. \tag{6.2.16}$$

Finally, when network effect $k = 0$, we assume that the software quality is sufficiently large so that the whole Hotelling line is still covered, i.e., a user located at $x = 1/2$ will still

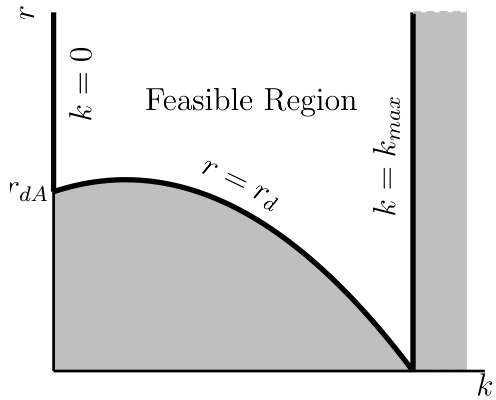

Figure 6.2.2: Feasible region in the (k,r) parameter space

pirate. Technically, from (3.3),

$$t < 2q \tag{6.2.17}$$

To summarize, to ensure that solutions in the duopoly case are meaningful, the values of parameters should satisfy the constraints (6.2.10), (6.2.14) and (6.2.17).

Properties of the Feasible Region

Using the constraints (6.2.10), (6.2.14) and $k > 0$, we can display the feasible region in the (k, r) parameter space in Figure 6.2.2. Similar to the monopoly case, B_L is the left boundary formed by two thick lines $k = 0$ and $r = r_d$ in Figure 6.2.2. Line $r = r_d$ can have at most one peak between $k = 0$ and $k = k_{max}$ according to Equation (6.2.14).

We also define the minimum-k curve B_{min} as the joint of line $k = 0$ and a segment of line $r = r_d$ with $r < r_{dA}$, i.e.,

$$k = \begin{cases} r_d^{-1}(r), & r < r_{dA} \\ 0, & r > r_{dA}. \end{cases} \tag{6.2.18}$$

That is, for a given r, this curve B_{min} yields the minimum k for a given r on B_L. In other words, curve B_L is B_{min} plus a segment of $r = r_d$ with $r > r_{dA}$.

We first prove the following lemma with regard to the roots of a general function $f(k, r)$

along the left boundary B_L.

Lemma 6.2.1. *Function $f(k,r)$ has a unique root along the left boundary B_L if the function $f(k,r)$ satisfies the following properties:*

1. *The sign of $\frac{\partial f(k,r)}{\partial r}|_{k=0}$ is different from that of $f(k,r)|_{k=k_{max},r=r_d}$,*

2. *Along the line $r = r_d$, $g(k) \equiv f(k,r)|_{r=r_d}$ can be written as $g(k) = g_1(k)g_2(k)$ where the sign of $g_1(k)$ does not change for $k \in (0, k_{max})$, and g_2 satisfies either of the following conditions:*

   Condition 1: *$g_2(k)$ is a monotonic function of k, or*

   Condition 2: *$g_2(k)$ is a quadratic function of k and the sign of $g_2(k)|_{k=k_{max}}$ is different from the sign of $\frac{\partial^2 g_2(k)}{\partial k^2}$.*

Proof of Lemma 6.2.1

We can separate the proofs into two cases according to the sign of $\frac{\partial f(k,r)}{\partial r}|_{k=0}$:

Case 1: $\frac{\partial f(k,r)}{\partial r}|_{k=0} > 0$

The sign of $f(k,r)$ at the right end point of the line $r = r_d$ is

$$f(k,r)|_{k=k_{max},r=r_d} < 0, \tag{6.2.19}$$

since $\frac{\partial f(k,r)}{\partial r}|_{k=0}$ and $f(k,r)|_{k=k_{max},r=r_d}$ have opposite signs according to the first property of the function f. We can show that there is only one root on B_L in the following two sub-cases according to the sign of $f(k,r)$ at the left end point of the line $r = r_d$:

Sub-case 1.1: $f(k,r)|_{k=0,r=r_d} > 0$

Since the signs of $g_1(k)$ does not change according to the second property of the function f, together with (6.2.19), we can conclude the sign of $g_2(k)|_{k=0}$ and $g_2(k)|_{k=k_{max}}$ are different. Then there is a root on the line $r = r_d$, whether $g_2(k)$ is a linear or quadratic function of k. Since $\frac{\partial f(k,r)}{\partial r}|_{k=0} > 0$ and $f(k,r)|_{k=0,r=r_d} > 0$, we can conclude there is no root on the line $k = 0$.

Sub-case 1.2: $f(k, r)|_{k=0, r=r_d} < 0$

In this case, the signs of $g_2(k)|_{k=0}$ and $g_2(k)|_{k=k_{max}}$ are the same since the sign of $g_1(k)$ does not change. We can show that the sign of $g_2(k)$ does not change along $r = r_d$ under either Condition 1 or Condition 2 identified in Lemma 6.2.1:

(i) If $g_2(k)$ satisfies Condition 1, i.e., it is monotonic, we can conclude the sign of $g_2(k)$ does not change along $r = r_d$.

(ii) Under Condition 2, if $g_2(k)$ is convex $\left(\frac{\partial^2 g_2(k)}{\partial k^2} > 0\right)$, then $g_2(k)|_{k=k_{max}} < 0$. Then $g_2(k) < 0$ along the line $r = r_d$, i.e., the sign of $g_2(k)$ does not change along the line $r = r_d$. Similarly, if $g_2(k)$ is concave, the sign of $g_2(k)$ also does not change along the line $r = r_d$.

Therefore, $f(k, r) < 0$ along the line $r = r_d$. Since $\frac{\partial f(k,r)}{\partial r}|_{k=0} > 0$ and $f(k, r)|_{k=0, r=r_d} < 0$, we can conclude there exists only one root on the line $k = 0$.

Case 2: $\frac{\partial f(k,r)}{\partial r}|_{k=0} < 0$

The proof process is similar to that in Case 1 and therefore omitted. ■

By using Lemma 6.2.1, we next prove the following proposition that will be used extensively in the future proofs.

Proposition 6.2.1. *Suppose a function $f(k, r)$ has a unique root on the left boundary B_L. On the minimum-k curve B_{min}, there is a threshold value of r_{dDA} (depending on function $f(k, r)$).*

1. *If $\frac{\partial f(k,r)}{\partial r}|_{k=0} > 0$, when $r > r_{dDA}$, $f(k, r) > 0$; $f(k, r) < 0$ otherwise.*
2. *If $\frac{\partial f(k,r)}{\partial r}|_{k=0} < 0$, when $r > r_{dDA}$, $f(k, r) < 0$; $f(k, r) > 0$ otherwise.*

Proof of Proposition 6.2.1

We denote the unique root along the left boundary B_L as (r_{dD}, k_{dD}). We first prove the first part of Proposition 6.2.1 when $\frac{\partial f(k,r)}{\partial r}|_{k=0} > 0$. We will discuss two cases: the root on the line $r = r_d$ and the root on the line $k = 0$.

Case 1: the root is on the line $r = r_d$

Then we have two sub-cases according to the sign of $r_{dD} - r_{dA}$.

<u>Sub-case 1.1:</u> $r_{dD} - r_{dA} > 0$

It is easy to see that when $r > r_{dA}$, $f(k,r) > 0$ on the curve B_{min}; when $r < r_{dA}$, $f(k,r) < 0$ on the curve B_{min}.

<u>Sub-case 1.2:</u> $r_{dD} - r_{dA} < 0$

We can see that, when $r > r_{dD}$, $f(k,r) > 0$ on the curve B_{min}; when $r < r_{dD}$, $f(k,r) < 0$ on the curve B_{min}.

<u>Case 2:</u> the root is on the line $k = 0$

Then in this case, when $r > r_{dD}$, $f(k,r) > 0$ on the curve B_{min}; when $r < r_{dD}$, $f(k,r) < 0$ on the curve B_{min}.

Combining these two cases, we can conclude there is a threshold value denoted as r_{dDA} (which is either r_{dA} in Sub-case 1.1 or r_{dD} in Subcase 1.2 and Case 2): when $r > r_{dDA}$, $f(k,r) > 0$ on the curve B_{min}; when $r < r_{dDA}$, $f(k,r) < 0$ on the curve B_{min}.

For the second part of Proposition 6.2.1, the proof process is similar to that of the first part and therefore omitted here for brevity. ■

6.2.2 Proof of Lemma 1

Let $U_1 - U_3|_{x=x_1} = 0$. Since $U_1 - U_3$ is a decreasing function of x, then when $x < x_1$, $U_1 - U_3 > 0$, that is, consumers will buy products; when $x > x_1$, $U_1 - U_3 < 0$, that is, consumers will choose pirated software. ■

6.2.3 Optimality of e_m and p_m

To show that e_m and p_m are optimal, we need to check the second-order conditions: $\frac{\partial^2 \pi_m}{\partial e_m^2} < 0$, and $Hessian_{\pi_m} = \frac{\partial^2 \pi_m}{\partial e_m^2}\frac{\partial^2 \pi_m}{\partial p_m^2} - (\frac{\partial^2 \pi_m}{\partial e_m \partial p_m})^2 > 0$.

By (3.9) and (3.10), π_m can be written as a function of e_m and p_m.

$$\pi_m = \frac{p_m \left(-\left(-a_1 e_m k_1 + a_1 e_m \theta_1 t + \theta_1 k_1 p_m - \theta_1 p_m t + \theta_1^2 q_1(-t) + \theta_1 q_1 t\right)\right)}{\left(\theta_1 - 1\right) \theta_1 t (t - k_1)} - r e_m^2 \quad (6.2.20)$$

we have

$$\frac{\partial^2 \pi_m}{\partial e_m^2} = -2r < 0 \quad (6.2.21)$$

$$\frac{\partial^2 \pi_m}{\partial p_m^2} = -\frac{2}{t - \theta_1 t} < 0 \quad (6.2.22)$$

$$\frac{\partial^2 \pi_m}{\partial e_m \partial p_m} = \frac{4r}{t - \theta_1 t} - \frac{a_1^2 \left(k_1 - \theta_1 t\right)^2}{\left(\theta_1 - 1\right)^2 \theta_1^2 t^2 (k_1 - t)^2} \quad (6.2.23)$$

The determinant of π_m's Hessian matrix is

$$Hessian_{\pi_m} = \frac{\partial^2 \pi_m}{\partial e_m^2} \frac{\partial^2 \pi_m}{\partial p_m^2} - \left(\frac{\partial^2 \pi_m}{\partial e_m \partial p_m}\right)^2 = -\frac{a_1^2 \left(k_1 - \theta_1 t\right)^2 + 4 \left(\theta_1 - 1\right) \theta_1^2 r t (k_1 - t)^2}{\left(\theta_1 - 1\right)^2 \theta_1^2 t^2 (k_1 - t)^2} \quad (6.2.24)$$

By (6.2.1), $Hessian_{\pi_m} > 0$. We can conclude e_m^* and p_m^* satisfy the second-order condition and therefore are optimal. ■

6.2.4 Proof of Theorem 10

From (3.11), we have

$$\frac{\partial e_m^*}{\partial k_1} = \frac{a_1 \left(\theta_1 - 1\right) \theta_1 q_1 t A}{DEN_m^2}, \quad (6.2.25)$$

where

$$A \equiv a_1^2 \left(k_1 - \theta_1 t\right)^2 + 4 \left(\theta_1 - 1\right) \theta_1^2 r_1 t \left(t - k_1\right) \left(\left(2\theta_1 - 1\right) t - k_1\right), \quad (6.2.26)$$

which is a quadratic function of k_1.

To study the sign of $\frac{\partial e_m^*}{\partial k_1}$ which is opposite to A, we first separate the entire parameter space according to the sign of $\frac{\partial A}{\partial r_1}|_{k_1=0}$. Since

$$\frac{\partial A}{\partial r_1}|_{k_1=0} = 4\left(\theta_1 - 1\right)\theta_1^2\left(2\theta_1 - 1\right)t^3, \tag{6.2.27}$$

we have two cases, $\theta_1 < 1/2$ and $1/2 < \theta_1 < 1$.

Case 1: $\theta_1 < 1/2$

Since

$$\frac{\partial A}{\partial r_1} = 4\left(\theta_1 - 1\right)\theta_1^2 t\left(t - k_1\right)\left(\left(2\theta_1 - 1\right)t - k_1\right) > 0, \tag{6.2.28}$$

together with

$$A|_{r_1=0} = a_1^2\left(k_1 - \theta_1 t\right)^2 > 0, \tag{6.2.29}$$

we can conclude $A > 0$ in the feasible region.

Case 2: $1/2 < \theta_1 < 1$

We have

$$A|_{r_1=r_m} = -a_1^2 A_m t\left(k_1 - \theta_1 t\right), \tag{6.2.30}$$

where

$$A_m = k_1 + \left(2 - 3\theta_1\right)t. \tag{6.2.31}$$

When k is close to θt, we can have $A|_{r_1=r_m} > 0$. Together with $\frac{\partial A}{\partial r_1}|_{k_1=0} < 0$ from (6.2.27), we can conclude there is a root on the left boundary according to Lemma 6.2.1. Then according to Proposition (6.2.1) and $\frac{\partial A}{\partial r_1}|_{k_1=0} < 0$ from (6.2.27), there is a threshold value r_{m1} on the left boundary (the left boundary and B_{min} are identical in this case): when $r_1 > r_{m1}$, $A < 0$; $A > 0$ otherwise.

On the right boundary $k_1 = \theta_1 t$,

$$A|_{k_1=\theta_1 t} = -4\left(\theta_1 - 1\right)^3\theta_1^2 r_1 t^3 > 0. \tag{6.2.32}$$

From (6.2.26),we can see A is a quadratic function of k. Then when $r_1 > r_{m1}$, it has a threshold value $\bar{k}_1$ for any given r_1: when $0 < k_1 < \bar{k}_1$, $A < 0$; when $\bar{k}_1 < k_1 < \theta_1 t$, $A > 0$ in the feasible region. When $r_1 < r_{m1}$, we have $A > 0$ in the feasible region since it is positive on both boundaries, is a quadratic function of k and reaches the extreme value on the right boundary $\left(\frac{\partial A}{\partial k_1}|_{k_1=\theta_1 t} = 0\right)$.

In conclusion,

1. When $\theta < 1/2$, $A > 0$, and then $\frac{\partial e_m^*}{\partial k_1} < 0$.

2. When $\theta > 1/2$,

   (a) when $r_1 > r_{m1}$, there is a threshold value $\bar{k}_1$. When $0 < k_1 < \bar{k}_1$, $A < 0$ and then $\frac{\partial e_m^*}{\partial k_1} > 0$; when $\bar{k}_1 < k_1 < \theta_1 t$, $A > 0$ and then $\frac{\partial e_m^*}{\partial k_1} < 0$ in the feasible region.

   (b) when $r_1 < r_{m1}$, $A > 0$, and then $\frac{\partial e_m^*}{\partial k_1} < 0$ in the feasible region. ■

6.2.5 Proof of Theorem 11

From (3.12), we have

$$\frac{\partial p_m^*}{\partial k_1} = -\frac{2\left(\theta_1 - 1\right)^2 \theta_1^2 q_1 r_1 t^2 C}{DEN_m^2}, \tag{6.2.33}$$

where

$$C \equiv a_1^2 \left(k_1 + (\theta_1 - 2)\, t\right)\left(k_1 - \theta_1 t\right) + 4\left(\theta_1 - 1\right)\theta_1^2 r_1 t \left(t - k_1\right)^2. \tag{6.2.34}$$

On the boundary $r_1 = r_m$,

$$C|_{r_1=r_m} = -a_1^2 \left(k_1 - \theta_1 t\right)^2 < 0. \tag{6.2.35}$$

Together with

$$\frac{\partial C}{\partial r_1} = 4\left(\theta_1 - 1\right)\theta_1^2 t \left(t - k_1\right)^2 < 0, \tag{6.2.36}$$

we can conclude $C < 0$ within the feasible region and $\frac{\partial p_m^*}{\partial k_1} > 0$. ■

6.2.6 Proof of Table 3.3

e_m^* **vs.** q_1

$$\frac{\partial e_m^*}{\partial q_1} = \frac{e_m^*}{q_1} > 0 \tag{6.2.37}$$

e_m^* **vs.** θ_1

$$\frac{\partial e_m^*}{\partial \theta_1} = \frac{a_1 q_1 t B}{DEN_m^2}, \tag{6.2.38}$$

where $B \equiv 4(\theta_1 - 1)^2 \theta_1^2 k_1 r_1 t(k-t)^2 - a_1^2 (k_1 - \theta_1 t)^2 (2\theta_1 k_1 - k_1 + \theta_1^2(-t))$.

Since

$$\frac{\partial B}{\partial r_1} = 4(\theta_1 - 1)^2 \theta_1^2 k_1 t (t - k_1)^2 > 0 \tag{6.2.39}$$

and

$$B|_{r_1=0} = -a_1^2 (k_1 - \theta_1 t)^2 \left((2\theta_1 - 1) k_1 - \theta_1^2 t\right) > 0, \tag{6.2.40}$$

we can conclude $B > 0$. That is $\dfrac{\partial e_m^*}{\partial \theta_1} > 0$.

e_m^* **vs.** a_1

$$\frac{\partial e_m^*}{\partial a_1} = \frac{(\theta_1 - 1)\theta_1 q_1 t (\theta_1 t - k_1)\left(4(\theta_1 - 1)\theta_1^2 r_1 t (k_1 - t)^2 - a_1^2 (k_1 - \theta_1 t)^2\right)}{DEN_m^2} > 0 \tag{6.2.41}$$

by (6.2.3).

e_m^* **vs.** r_1

$$\frac{\partial e_m^*}{\partial r_1} = \frac{4a_1 (\theta_1 - 1)^2 \theta_1^3 q_1 t^2 (k_1 - t)^2 (k_1 - \theta_1 t)}{DEN_m^2} < 0 \tag{6.2.42}$$

by (6.2.3).

p_m^* **vs.** q_1

$$\frac{\partial p_m^*}{\partial q_1} = \frac{p_m^*}{q_1} > 0 \tag{6.2.43}$$

p_m^* **vs.** θ_1

$$\frac{\partial p_m^*}{\partial \theta_1} = \frac{4\left(\theta_1 - 1\right)\theta_1 q_1 r_1 t^2 (k_1 - t) D}{DEN_m^2}, \tag{6.2.44}$$

where

$$D \equiv a_1^2 \left(k_1 - \theta_1 t\right)\left(2\theta_1 k_1 - k_1 - \theta_1^2 t\right) + 2\left(\theta_1 - 1\right)\theta_1^3 r_1 t (k_1 - t)^2. \tag{6.2.45}$$

Since

$$\frac{\partial D}{\partial r_1} = 2\left(\theta_1 - 1\right)\theta_1^3 t \left(t - k_1\right)^2 < 0 \tag{6.2.46}$$

and

$$D|_{r_1 = r_m} = a_1^2 \left(\theta_1 - 1\right)\left(k_1 - \theta_1 t\right)^2 < 0, \tag{6.2.47}$$

we can conclude $D < 0$. That is $\dfrac{\partial p_m^*}{\partial \theta_1} < 0$.

p_m^* **vs.** a_1

$$\frac{\partial p_m^*}{\partial a_1} = \frac{4a_1 \left(\theta_1 - 1\right)^2 \theta_1^2 q_1 r_1 t^2 \left(t - k_1\right)\left(k_1 - \theta_1 t\right)^2}{\left(a_1^2 \left(k_1 - \theta_1 t\right)^2 + 4\left(\theta_1 - 1\right)\theta_1^2 r_1 t \left(t - k_1\right)^2\right)^2} > 0 \tag{6.2.48}$$

according to (6.2.3).

p_m^* **vs.** r_1

$$\frac{\partial p_m^*}{\partial r_1} = -\frac{2a_1^2 \left(\theta_1 - 1\right)^2 \theta_1^2 q_1 t^2 \left(t - k_1\right)\left(k_1 - \theta_1 t\right)^2}{\left(a_1^2 \left(k_1 - \theta_1 t\right)^2 + 4\left(\theta_1 - 1\right)\theta_1^2 r_1 t \left(t - k_1\right)^2\right)^2} < 0 \tag{6.2.49}$$

according to (6.2.3). ■

6.2.7 Proof of Theorem 12

To find the expressions of e_i^* and p_i^*, we first find p_i^* from the first-order conditions $\dfrac{\partial \pi_i}{\partial p_i} = 0$, $i = 1, 2$ where π_i is given by Equation (3.5). Such p_i is a function of anti-piracy effort e_i, $i = 1, 2$. Then substituting p_i^* back into (3.5), we can find the optimal e_i^* by solving two first-order conditions simultaneously: $\dfrac{\partial \pi_i}{\partial e_i} = 0$, $i = 1, 2$ (A Mathematica file containing all the relevant derivations is available upon request). In this way, when $k_1 \neq k_2$, we have

$$e_1^* = -\frac{(\theta k_2 - k_1 + 2(\theta-1)q)\left(a_1\left(\theta\left(k_2-2t\right)+k_1\right)+b_1\left(\theta k_2-k_1\right)\right)}{2\left(-\theta k_2\left(\left(a_1+b_1\right)^2+8(\theta-1)rt\right)+\left(a_1+b_1\right)\left(a_1\left(2\theta t-k_1\right)+b_1k_1\right)+8(\theta-1)\theta rt^2\right)}. \tag{6.2.50}$$

and p_1 in (6.2.74).

e_1^* **vs.** k_1

Taking the derivative of (6.2.50) with respect to k_1 and then letting $k_1 = k_2 = k$, we can get:

$$\frac{\partial e_1}{\partial k_1}|_{k_1=k_2=k} = \frac{eNum_{k_1}}{4DEN_d^2}. \tag{6.2.51}$$

where

$$\begin{aligned} eNum_{k_1}/2 = {} & a_1(\theta-1)\left(b_1^2(-k)(3\theta k + k - 4\theta t) - 16\theta rt(k-t)(k-\theta(q+t)+q)\right) \\ & - a_1^2 b_1(\theta k + k - 2\theta t)((3\theta-1)k - 2\theta t) + a_1^3\left(-(\theta k + k - 2\theta t)^2\right) \\ & + b_1(\theta-1)^2\left(b_1^2\left(-k^2\right) - 16\theta rt(k+q)(k-t)\right). \end{aligned} \tag{6.2.52}$$

Then we have

$$\frac{\partial eNum_{k_1}}{\partial r} = 32(\theta-1)\theta t(k-t)\left(\left(a_1(q+t)-b_1(k+q)\right)\left(\theta-\theta''\right) - \frac{a_1 k\left(a_1-b_1\right)(q+t)}{a_1(q+t)-b_1 q}\right) \tag{6.2.53}$$

where

$$\theta'' \equiv \frac{q\left(a_1-b_1\right)}{a_1(q+t)-b_1 q} < 1. \tag{6.2.54}$$

To determine the sign of $eNum_{k_1}$, we separate the discussion into two cases:

Case 1: $\theta \leq \theta''$

It is easy to see that (6.2.53) is less than 0 when $\theta < \theta''$, i.e., $eNum_{k_1}$ is a decreasing function of r. Together with

$$eNum_{k_1}|_{r=0} = -2\left(a_1+b_1\right)\left(a_1(\theta k + k - 2\theta t) + b_1(\theta-1)k\right)^2 < 0, \tag{6.2.55}$$

we can conclude $eNum_{k_1} < 0$.

Case 2: $\theta > \theta''$

We first discuss the properties of $eNum_{k_1}$ on the r–k boundary shown in Figure 6.2.2.

a. On the boundary of $k = 0$

We have

$$\begin{aligned} eNum_{k_1}|_{k=0} &= -8\theta t^2 \left(a_1^2 b_1 \theta + 4(\theta-1) r \left(a_1((\theta-1)q+\theta t) - b_1(\theta-1)q\right) + a_1^3\theta\right) \\ &= -8\theta t^2 \left(4(\theta-1)\left(a_1(q+t) - b_1 q\right)\left(\theta-\theta''\right) r + a_1^2\theta\left(a_1+b_1\right)\right) \end{aligned} \tag{6.2.56}$$

which is an increasing function of r when $\theta > \theta''$.

b. On the boundary of $r = r_d$

We have

$$eNum_{k_1}|_{r=r_d} = -\frac{2\left(a_1+b_1\right)(k+2q) R_1 F_4}{k+2q-2t}, \tag{6.2.57}$$

where

$$F_4 = a_1(\theta k + k - 4\theta t + 2t) + b_1(\theta-1)(k+2t) = R_1 - 2(a_1-b_1)t(\theta-1). \tag{6.2.58}$$

c. On the boundary of $k = k_{max}$

We have

$$eNum_{k_1}|_{k=k_{max}} = \frac{32(\theta-1)^3\theta r t^2 \left(a_1-b_1\right)^2 \left(a_1(\theta(q+t)+q) + b_1(\theta-1)q\right)}{\left(a_1(\theta+1)+b_1(\theta-1)\right)^2} < 0, \tag{6.2.59}$$

and

$$\frac{\partial eNum_{k_1}}{\partial k}|_{k=k_{max}} = \frac{32(\theta-1)^2\theta r t\left(a_1-b_1\right) F_5}{a_1(\theta+1)+b_1(\theta-1)} \tag{6.2.60}$$

where

$$F_5 = \left(a_1(q+t) + b_1(q-t)\right)\left(\theta-\theta''\right) + \frac{a_1\left(a_1-b_1\right)(2q-t)(q+t)}{a_1(q+t) - b_1 q}. \tag{6.2.61}$$

which is positive given $\theta > \theta''$ and (6.2.17). Then when $\theta > \theta''$,

$$\frac{\partial eNum_{k_1}}{\partial k}|_{k=k_{max}} > 0. \tag{6.2.62}$$

Since $\theta > \theta''$, from (6.2.53), $\dfrac{\partial eNum_{k_1}}{\partial k}|_{k=0} > 0$. Together with (6.2.57), (6.2.58), and (6.2.59), we can conclude there is a root on the line $r = r_d$ according to Lemma 6.2.1. Then according to Proposition 6.2.1 and (6.2.53), we can conclude there is a threshold value r_{dDB}: when $r > r_{dDB}$, $eNum_{k_1} > 0$ on the curve B_{min}; when $r < r_{dDB}$, $eNum_{k_1} < 0$ on the curve B_{min}. We then have two sub-cases:

Sub-case 2.1: $r < r_{dDB}$

In this case, $eNum_{k_1} < 0$ is negative on the curve B_{min} and on the right boundary line $k = k_{max}$. Also we can see that $eNum_{k_1}$ in (6.2.52) is a quadratic function of k. If it is convex, then it must be negative in the feasible region. If it is concave, together with (6.2.62), we can also see that $eNum_{k_1} < 0$. So, $eNum_{k_1} < 0$ for $r < r_{dDB}$.

Sub-case 2.2: $r > r_{dDB}$

In this case $eNum_{k_1}$ is positive on the curve B_{min} and negative on the right boundary line $k = k_{max}$. Since it is a quadratic function of k, there is exactly one threshold value $\bar{k}_{dDB}$: $eNum_{k_1} > 0$ if $k < \bar{k}_{dDB}$ and $eNum_{k_1} < 0$ if $k > \bar{k}_{dDB}$.

e_2^* vs. k_1

We have:

$$\frac{\partial e_2}{\partial k_1}|_{k_1=k_2=k} = \frac{\partial e_1}{\partial k_2}|_{k_1=k_2=k} = \frac{eNum_{k_2}}{4DEN_d^2}. \tag{6.2.63}$$

where

$$\begin{aligned} eNum_{k_2}/(2\theta) &= a_1^2 b_1(\theta k + k - 2\theta t)((3\theta-1)k - 2\theta t) + a_1^3(\theta k + k - 2\theta t)^2 \\ &+ a_1(\theta-1)\left(b_1^2 k(3\theta k + k - 4\theta t) + 8rt\left(\theta^2\left(k^2 - 2kt + 2t(q+t)\right) - 2\theta(k(q+t) + qt) + k(k+2q)\right)\right) \\ &+ b_1(\theta-1)^2\left(b_1^2 k^2 + 8rt\left(\theta\left(k^2 - 2t(k+q)\right) + k(k+2q)\right)\right) \end{aligned} \tag{6.2.64}$$

From (6.2.55), we have

$$eNum_{k_2}|_{r=0} = -\theta eNum_{k_1}|_{r=0} > 0. \tag{6.2.65}$$

To determine the sign of $eNum_{k_2}$, we also consider two cases:

Case 1: $\theta < \theta''$

We first study the properties of $eNum_{k_2}$ on the r-k boundary shown in Figure 6.2.2. We have

$$eNum_{k_2}|_{k=0} = -\theta eNum_{k_1}|_{k=0} > 0 \tag{6.2.66}$$

according to (6.2.56). From (6.2.66) and (6.2.53), we have

$$\frac{\partial eNum_{k_2}}{\partial r}|_{k=0} > 0. \tag{6.2.67}$$

We also have

$$eNum_{k_2}|_{r=r_d} = \frac{2\left(a_1+b_1\right)(k+2q)R_1}{(k-t)(k+2q-2t)}F_6 \tag{6.2.68}$$

where

$$F_6 = a_1\left(\theta(\theta+1)k^2 + (2-3\theta(\theta+1))kt + 2\theta(2\theta-1)t^2\right) + b_1(\theta-1)(\theta(k+t)(k-2t)+2kt). \tag{6.2.69}$$

From (6.2.3), (6.2.9), (6.2.16), and (6.2.66), we have

$$F_6|_{k=0,r=r_d} < 0. \tag{6.2.70}$$

From

$$eNum_{k_2}|_{k=k_{max}} = \frac{32(\theta-1)^3\theta^2 rt^2\left(a_1^2-b_1^2\right)\left(a_1(\theta(q+t)+q)+b_1(\theta-1)q\right)}{\left(a_1(\theta+1)+b_1(\theta-1)\right)^2} < 0, \tag{6.2.71}$$

we can conclude

$$F_6|_{k=k_{max}} > 0. \tag{6.2.72}$$

Together with (6.2.70) and F_6 is a convex function of k, we can conclude there is a root to $F_6 = 0$ (and therefore to $eNum_{k_2} = 0$) on the line $r = r_d$. From (6.2.66), there is no root to $eNum_{k_2} = 0$ on the line $k = 0$. Then we can conclude there is one root on the boundary B_L. Together with (6.2.67) and Proposition 6.2.1, we can conclude there is a threshold value r_{dDC} on the curve B_{min}: when $r > r_{dDC}$, $eNum_{k_2} > 0$; otherwise, $eNum_{k_2} < 0$. Then we can discuss the sign of $eNum_{k_2}$ in two regions:

1. In the region where $r < r_{dDC}$, given that $eNum_{k_2} < 0$ on B_{min}, $eNum_{k_2}|_{r=0} > 0$ according to (6.2.65), and $eNum_{k_2}$ is a linear function of r, we have $eNum_{k_2} < 0$.

2. In the region where $r > r_{dDC}$, $eNum_{k_2} > 0$ on the curve B_{min}, and $eNum_{k_2}|_{k=k_{max}} < 0$ on the right boundary from (6.2.71). Since $eNum_{k_2}$ is a quadratic function of k, we can conclude there is a threshold value $\bar{k}_{dDC}$ for a given r: when $k < \bar{k}_{dDC}$, $eNum_{k_2} > 0$; $eNum_{k_2} < 0$ otherwise.

Case 2: $\theta > \theta''$

When $\theta > \theta''$, we have

$$\frac{\partial eNum_{k_2}}{\partial r}|_{k=0} = 32(\theta-1)\theta^2 t^2 \left(a_1((\theta-1)q+\theta t) - b_1(\theta-1)q\right)(\theta-\theta'') < 0. \qquad (6.2.73)$$

Then when k is small, $\dfrac{\partial eNum_{k_2}}{\partial r} < 0$ by continuity. Therefore, when r is large enough, $eNum_{k_1} < 0$. When k is large enough (close to k_{max}), $eNum_{k_1}|_{r=r_d} < 0$ by continuity according to (6.2.71). Since $eNum_{k_2}$ is a linear function of r, from (6.2.65), we can conclude when k is large enough, $eNum_{k_2} < 0$. ■

6.2.8 Proof of Theorem 13

We have

$$p_1^* = -\frac{2(\theta-1)\theta rt\,(t-k_2)\,(\theta k_2 - k_1 + 2(\theta-1)q)}{-\theta k_2\left((a_1+b_1)^2 + 8(\theta-1)rt\right) + (a_1+b_1)\left(a_1\left(2\theta t - k_1\right) + b_1 k_1\right) + 8(\theta-1)\theta rt^2}. \qquad (6.2.74)$$

$\underline{p_1^* \text{ vs. } k_1}$

We have

$$\frac{\partial p_1}{\partial k_1}|_{k_1=k_2=k} = \frac{4G_5(\theta-1)\theta rt(t-k)}{DEN_d^2}, \tag{6.2.75}$$

where

$$G_5 = -(a_1+b_1)(a_1(\theta(k+q-t)-q)-b_1(\theta-1)q) - 4(\theta-1)\theta rt(k-t). \tag{6.2.76}$$

Since $\frac{4(\theta-1)\theta rt(t-k)}{\mathrm{DEN}_d^2} < 0$, the sign of $\frac{\partial p_1}{\partial k_1}|_{k_1=k_2=k}$ is opposite to the sign of G_5. To determine the sign of G_5, we first have

$$G_5|_{r=0} = -(a_1+b_1)(a_1(\theta(k+q-t)-q)-b_1(\theta-1)q). \tag{6.2.77}$$

Since $G_5|_{r=0}$ is a linear function of k, together with

$$G_5|_{r=0,k=0} = -(a_1+b_1)(a_1((\theta-1)q-\theta t)-b_1(\theta-1)q) > 0 \tag{6.2.78}$$

and

$$G_5|_{r=0,k=k_{max}} = -\frac{(\theta-1)(a_1^2-b_1^2)(a_1(\theta(q+t)+q)+b_1(\theta-1)q)}{a_1(\theta+1)+b_1(\theta-1)} > 0, \tag{6.2.79}$$

we can conclude

$$G_5|_{r=0} > 0. \tag{6.2.80}$$

We also have

$$G_5|_{r=r_d} = -\frac{(a_1+b_1)(k+2q)H_1}{k+2q-2t}, \tag{6.2.81}$$

where $H_1 = a_1(\theta(k+q-2t)-q+t) - b_1(\theta-1)(q-t)$. Given that G_5 is a decreasing function of r from (6.2.76), $G_5|_{r=0,k=k_{max}} > 0$ from (6.2.79), and H_1 is a linear function of k, then according to Lemma 6.2.1, there is exactly one root to $G_5 = 0$ on the left boundary

B_L. Together with Proposition 6.2.1 and the fact that G_5 is a decreasing function of r, we can conclude there is a threshold value r_{dDD}: when $r > r_{dDD}$, $G_5 < 0$ on the curve B_{min}; otherwise $G_5 > 0$.

From (6.2.80) and

$$\frac{\partial G_5}{\partial r} = -4(\theta - 1)\theta t(k - t) < 0, \tag{6.2.82}$$

we can conclude there is a threshold value $r_{d4} = -\dfrac{(a_1 + b_1)\left(a_1(\theta(q + t) + q) + b_1(\theta - 1)q\right)}{4(\theta - 1)\theta t^2}$. When $r > r_{d4}$, $G_5|_{k=k_{max}} < 0$; otherwise, $G_5|_{k=k_{max}} > 0$.

Since G_5 is a linear function of k and

$$\frac{\partial G_5}{\partial k}|_{r=r_{d4}} = \frac{q\left(a_1 + b_1\right)\left(a_1(\theta + 1) + b_1(\theta - 1)\right)}{t} > 0, \tag{6.2.83}$$

we can conclude $\dfrac{\partial G_5}{\partial k}|_{r=r_{d4}} < 0$ on the curve B_{min}. Then $r_{d4} > r_{dDD}$.

To summarize,.

1. When $r < r_{dDD}$, G_5 is positive on the curve B_{min} and the right boundary. Also, it is a linear function of k. Then $G_5 > 0$ and therefore $\dfrac{\partial p_1}{\partial k_1}|_{k_1=k_2=k} < 0$.

2. When $r_{dDD} < r < r_{d4}$, G_5 is negative on the curve B_{min} and positive on the right boundary. Also, it is a linear function of k. Then there is a threshold value $\bar{k}_6$: when $k < \bar{k}_6$, $G < 0$ and $\dfrac{\partial p_1}{\partial k_1}|_{k_1=k_2=k} > 0$; when $k > \bar{k}_6$, $G > 0$ and $\dfrac{\partial p_1}{\partial k_1}|_{k_1=k_2=k} < 0$.

3. When $r > r_{d4}$, G_5 is negative on the curve B_{min} and the right boundary. Since it is a linear function of k, $G_5 < 0$ and $\dfrac{\partial p_1}{\partial k_1}|_{k_1=k_2=k} > 0$.

<u>p_2^* **vs.** k_1</u>

$\dfrac{\partial p_2}{\partial k_1}|_{k_1=k_2=k} = \dfrac{\partial p_1}{\partial k_2}|_{k_1=k_2=k}$. We have,

$$\frac{\partial p_2}{\partial k_1}|_{k_1=k_2=k} = \frac{\partial p_1}{\partial k_2}|_{k_1=k_2=k} = \frac{2(1 - \theta)\theta r t G_6}{\mathrm{DEN}_d^2}, \tag{6.2.84}$$

where

$$G_6 = 2a_1b_1\theta^2(k-t)^2 + a_1^2\left(\theta^2\left(k^2-4kt+2t(t-q)\right)+2\theta(k(k+q)+qt)-k(k+2q)\right) + (\theta-1)\left(b_1^2\left((\theta-1)k^2-2kq+2\theta qt\right)+8\theta^2rt(k-t)^2\right). \quad (6.2.85)$$

which is a linear decreasing function of r since $\dfrac{\partial G_6}{\partial r} = 8(\theta-1)\theta^2 t(k-t)^2 < 0$. Then we can conclude that when r is large enough, $G_6 < 0$.

We have $G_6|_{k=k_{max},r=0} = -\dfrac{2(\theta-1)^2\theta t\,(a_1-b_1)\,(a_1+b_1)^2\,(a_1(\theta(q+t)+q)+b_1(\theta-1)q)}{(a_1(\theta+1)+b_1(\theta-1))^2} < 0$. Then we can conclude that when k is large enough, or equivalently r is small enough, $G_6|_{r=r_d} < 0$ by continuity. Since G_6 is a decreasing function of r, we have $G_6 < 0$ in the feasible region when r is small enough.

In conclusion, when r is either small enough or large enough, we have $G_6 < 0$. ■

6.2.9 Proof of Table 3.5

Let x_i be the general parameter that represents q_i, a_i, b_i, or r_i, $i = 1,2$. Similar to the steps in Appendix 6.2.7, define

$$\frac{\partial e_1^*}{\partial x_i} \equiv \frac{eNum_{x_i}}{4DEN_d^2}, i = 1,2 \quad (6.2.86)$$

and

$$\frac{\partial p_1^*}{\partial x_i} \equiv \frac{pNum_{x_i}}{4DEN_d^2}, i = 1,2. \quad (6.2.87)$$

In the Mathematica file, we have derived $eNum_x$, and $pNum_x$ in each of the following cases.

e_1^* vs. q_1

$$eNum_{q_1} = 2(\theta-1)R_1M_1N_1 \quad (6.2.88)$$

where

$$M_1 = (a_1(\theta k+k-2\theta t)+b_1(\theta-1)k)\,(a_1(k-2\theta t)-b_1k)+8(\theta-1)\theta^2rt(k-t)(k-2t) \quad (6.2.89)$$

$$N_1 = (a_1(\theta k + k - 2\theta t) + b_1(\theta - 1)k)^2 \left(\left(a_1^2 - b_1^2\right)(k - \theta t) + 8(\theta - 1)\theta r t(k - t)\right) + 64(\theta - 1)^2\theta^3 r^2 t^2 (k - t)^3 \quad (6.2.90)$$

M_1 is a linear decreasing function of r, and

$$M_1|_{r=r_d} = \frac{R_1\left(a_1(k(k + 2q - 2t) - 4\theta q t) - b_1\left(k^2 + 2k(q - (\theta + 1)t) + 4\theta t^2\right)\right)}{k + 2q - 2t} < 0 \quad (6.2.91)$$

Then we have

$$M_1 < 0 \quad (6.2.92)$$

At $r = r'_d$ defined in (6.2.8),

$$N_1|_{r=r'_d} = 0 \quad (6.2.93)$$

and

$$\frac{\partial N_1}{\partial r}|_{r=r'_d} = -8(\theta - 1)\theta t(k - t)R_1\left(a_1(\theta - 1)k + b_1(\theta k + k - 2\theta t)\right) < 0. \quad (6.2.94)$$

Together with

$$\frac{\partial^2 N_1}{\partial r^2} = 128(\theta - 1)^2\theta^3 t^2 (k - t)^3 < 0, \quad (6.2.95)$$

we can conclude when $r > r'_d$, $N_1 < 0$. Then when $r > r_d > r'_d$,

$$N_1 < 0 \quad (6.2.96)$$

From (6.2.9), (6.2.88), (6.2.92), and (6.2.96), we have $eNum_{q_1} > 0$. Then we have $\frac{\partial e_1}{\partial q_1}|_{q_1=q_2=q} > 0$.

e_1^* vs. q_2

$$eNum_{q_2} = 2(\theta - 1)R_1 M_2 N_1 \quad (6.2.97)$$

where

$$M_2 = R_1\left(a_1 k - b_1(k-2\theta t)\right) + 8(\theta-1)\theta^2 krt(k-t). \tag{6.2.98}$$

from which we can conclude there is a threshold value $\hat{r}$. When $r < \hat{r}$, $eNum_{q_2} > 0$; $eNum_{q_2} < 0$ otherwise;

$\underline{p_1^* \textbf{ vs. } q_1}$

$$pNum_{q_1} = -2(\theta-1)^2\theta rt(k-t)M_1N_1 > 0 \tag{6.2.99}$$

from (6.2.92) and (6.2.96).

$\underline{p_1^* \textbf{ vs. } q_2}$

$$pNum_{q_2} = -2(\theta-1)^2\theta rt(k-t)M_2N_1 \tag{6.2.100}$$

From (6.2.98), we can conclude there is a threshold value $\hat{r}$. When $r < \hat{r}$, $pNum_{q_2} > 0$; $pNum_{q_2} < 0$ otherwise;

$\underline{e_1^* \textbf{ vs. } a_1}$

$$eNum_{a_1} = 2(\theta-1)(k+2q)M_3N_9 \tag{6.2.101}$$

where

$$M_3 = (a_1-b_1)(k-\theta t)\left(a_1(\theta k+k-2\theta t)+b_1(\theta-1)k\right) + 8(\theta-1)\theta^2 rt(k-t)^2 \tag{6.2.102}$$

and

$$\begin{aligned} N_9 &= 64(\theta-1)^2\theta^3 r^2 t^2 (k-t)^3(\theta k+k-2\theta t) \\ &\quad - a_1(k-\theta t)\left(a_1(\theta k+k-2\theta t)+b_1(\theta-1)k\right)^3 \end{aligned} \tag{6.2.103}$$

Since

$$\frac{\partial M_3}{\partial r} = 8(\theta-1)\theta^2 t(k-t)^2 < 0 \tag{6.2.104}$$

and

$$M_3|_{r=r_d'} = -R_1\left(a_1(\theta-1)k + b_1(\theta k+k-2\theta t)\right) < 0 \tag{6.2.105}$$

where r_d' is defined in Equation (6.2.8), together with (6.2.15), we can conclude

$$M_3 < 0. \tag{6.2.106}$$

Similarly, since

$$N_9|_{r=r_d'} = R_1^2 \left(a_1(\theta k + k - 2\theta t) + b_1\theta(k-t)\right)\left(a_1(\theta-1)k + b_1(\theta k + k - 2\theta t)\right) > 0 \tag{6.2.107}$$

and

$$\frac{\partial N_9}{\partial r} = 128(\theta-1)^2\theta^3 r t^2 (k-t)^3(\theta k + k - 2\theta t) > 0 \tag{6.2.108}$$

where r_d' is defined in equation (6.2.8), together with (6.2.15), we can conclude

$$N_9 > 0. \tag{6.2.109}$$

From (6.2.101), (6.2.106), and (6.2.109), we can conclude $eNum_{a_1} > 0$.

<u>e_1^* **vs.** b_2</u>

$$eNum_{b_2} = 2(\theta-1)(k+2q)R_1M_3N_{10} \tag{6.2.110}$$

where

$$\begin{aligned} N_{10} = &-2a_1(\theta-1)k(\theta k + k - 2\theta t)\left(b_1^2(\theta t - k) + 4(\theta-1)\theta r t(k-t)\right) \\ &+ a_1^2 b_1(k-\theta t)(\theta k + k - 2\theta t)^2 + b_1(\theta-1) \\ &\left(b_1^2(\theta-1)k^2(k-\theta t) + 8\theta r t(t-k)\left(\left(\theta^2+1\right)k^2 - 2\theta(\theta+1)kt + 2\theta^2t^2\right)\right) \end{aligned} \tag{6.2.111}$$

We have

$$\frac{\partial N_{10}}{\partial r} = 8(\theta-1)\theta t\left(b_1(t-k)\left(\left(\theta^2+1\right)k^2 - 2\theta(\theta+1)kt + 2\theta^2t^2\right) - a_1(\theta-1)k(k-t)(\theta k + k - 2\theta t)\right), \tag{6.2.112}$$

which is a linear function of b_1. Since

$$\frac{\partial N_{10}}{\partial r}|_{b_1=0} = -8a_1(\theta-1)^2\theta kt(k-t)(\theta k+k-2\theta t) < 0 \tag{6.2.113}$$

and

$$\frac{\partial N_{10}}{\partial r}|_{b_1=a_1} = -16a_1(\theta-1)\theta^3 t(k-t)^3 < 0, \tag{6.2.114}$$

we can conclude

$$\frac{\partial N_{10}}{\partial r} < 0. \tag{6.2.115}$$

Together with

$$N_{10}|_{r=0} = (k-\theta t)b_1 R_1^2 < 0, \tag{6.2.116}$$

we can conclude

$$N_{10} < 0. \tag{6.2.117}$$

From (6.2.106), (6.2.110), and (6.2.117), we can conclude $eNum_{b_2} > 0$.

<u>p_1^* **vs.** a_1</u>

$$pNum_{a_1} = 2(\theta-1)^2\theta rt(k+2q)(k-t)M_3N_{11} \tag{6.2.118}$$

where

$$\begin{aligned}N_{11} &= 4a_1(k-t)\left(2(\theta-1)\theta rt(\theta k+k-2\theta t)^2 - b_1^2\theta(k-\theta t)^2\right)\\ &\quad + 3a_1^2b_1(\theta-1)k(k-\theta t)(\theta k+k-2\theta t) + 2a_1^3(k-\theta t)(\theta k+k-2\theta t)^2\\ &\quad + b_1(\theta-1)k(\theta k+k-2\theta t)\left(b_1^2(\theta t-k) + 8(\theta-1)\theta rt(k-t)\right)\end{aligned} \tag{6.2.119}$$

Since

$$N_{11}|_{r=r'_d} = -R_1\left(a_1(\theta k+k-2\theta t) + b_1\theta(k-t)\right)\left(a_1(\theta-1)k + b_1(\theta k+k-2\theta t)\right) > 0 \tag{6.2.120}$$

where r_d' is defined in Equation (6.2.8), and

$$\frac{\partial N_{11}}{\partial r} = 8R_1(\theta-1)\theta t(k-t)(\theta k+k-2\theta t) > 0, \tag{6.2.121}$$

together with (6.2.15), we can conclude

$$N_{11} > 0 \tag{6.2.122}$$

From (6.2.106), (6.2.118), and (6.2.122), we can conclude $pNum_{a_1} > 0$.

<u>p_1^* **vs.** b_2</u>

$$pNum_{b_2} = 2(\theta-1)^2\theta rt(k+2q)(k-t)M_3N_{12} \tag{6.2.123}$$

where

$$\begin{aligned} N_{12} &= 2a_1(\theta-1)k(\theta k+k-2\theta t)\left(b_1^2(\theta t-k)+4(\theta-1)\theta rt(k-t)\right) \\ &\quad + a_1^2 b_1(k-\theta t)\left(-(\theta k+k-2\theta t)^2\right) + b_1(\theta-1) \\ &\quad \left(b_1^2(\theta-1)k^2(\theta t-k)+8\theta rt(k-t)\left(\left(\theta^2+1\right)k^2-2\theta(\theta+1)kt+2\theta^2t^2\right)\right) \end{aligned} \tag{6.2.124}$$

We have

$$\frac{\partial N_{12}}{\partial r} = 8(\theta-1)\theta t(k-t)\left(a_1(\theta-1)k(\theta k+k-2\theta t)+b_1\left(\left(\theta^2+1\right)k^2-2\theta(\theta+1)kt+2\theta^2t^2\right)\right), \tag{6.2.125}$$

which is a linear function of b_1. Since

$$\frac{\partial N_{12}}{\partial r}|_{b_1=0} = 8a_1(\theta-1)^2\theta kt(k-t)(\theta k+k-2\theta t) > 0 \tag{6.2.126}$$

and

$$\frac{\partial N_{12}}{\partial r}|_{b_1=a_1} = 16a_1(\theta-1)\theta^3t(k-t)^3 > 0, \tag{6.2.127}$$

we have

$$\frac{\partial N_{12}}{\partial r} > 0. \tag{6.2.128}$$

Together with

$$N_{12}|_{r=0} = b_1(-(k-\theta t))\left(a_1(\theta k + k - 2\theta t) + b_1(\theta-1)k\right)^2 > 0, \tag{6.2.129}$$

we have

$$N_{12} > 0. \tag{6.2.130}$$

From (6.2.106), (6.2.123), and (6.2.130), we have $pNum_{b_2} > 0$.

<u>e_1^* **vs.** a_2</u>

$$eNum_{a_2} = 2(\theta-1)(k+2q)R_1N_5 \tag{6.2.131}$$

where

$$\begin{aligned} N_5 = &- \left(\left(a_1 - b_1\right)\left(k - \theta t\right)\left(a_1(\theta k + k - 2\theta t) + b_1(\theta-1)k\right) + 8(\theta-1)\theta^2 rt(k-t)^2\right) \\ &[a_1(\theta-1)k(\theta k + k - 2\theta t)\left(8(\theta-1)\theta rt(k-t) - 3b_1^2(k-\theta t)\right) \\ &- 4a_1^2 b_1\theta(k-t)(k-\theta t)^2 + a_1^3(\theta-1)k(k-\theta t)(\theta k + k - 2\theta t) \\ &+ 2b_1(\theta-1)^2k^2\left(b_1^2(\theta t - k) + 4(\theta-1)\theta rt(k-t)\right)] \\ &- 2(k-\theta t)\left(a_1(k-\theta t) + b_1(\theta-1)k\right) \\ &[(a_1(\theta k + k - 2\theta t) + b_1(\theta-1)k)^2 \\ &\left(\left(a_1^2 - b_1^2\right)(k-\theta t) + 8(\theta-1)\theta rt(k-t)\right) + 64(\theta-1)^2\theta^3 r^2 t^2 (k-t)^3] \end{aligned} \tag{6.2.132}$$

We have

$$\frac{\partial^2 N_5}{\partial r^2} = 128(\theta-1)^2\theta^3 t^2(k-t)^3\left(a_1\left(-\left(\left(\theta^2+1\right)k^2 - 2\theta(\theta+1)kt + 2\theta^2 t^2\right)\right) - b_1(\theta-1)k(\theta k + k - 2\theta t)\right), \tag{6.2.133}$$

which is a linear function of b_1. Since

$$\frac{\partial^2 N_5}{\partial r^2}|_{b_1=0} = 128a_1(\theta-1)^2\theta^3 t^2(k-t)^3\left(-\left(\theta^2+1\right)k^2+2\theta(\theta+1)kt-2\theta^2t^2\right) > 0 \quad (6.2.134)$$

and

$$\frac{\partial^2 N_5}{\partial r^2}|_{b_1=a_1} = -256a_1(\theta-1)^2\theta^5t^2(k-t)^5 > 0, \quad (6.2.135)$$

we can conclude

$$\frac{\partial^2 N_5}{\partial r^2} > 0. \quad (6.2.136)$$

At $r = r'_d$ defined in Equation (6.2.8), we have

$$\frac{\partial N_5}{\partial r}|_{r=r'_d} = 8(\theta-1)\theta t(k-t)R_1N_{51}\left(a_1(\theta-1)k+b_1(\theta k+k-2\theta t)\right) \quad (6.2.137)$$

where

$$N_{51} = a_1\left(\theta^2\left(2k^2-4kt+3t^2\right)+k^2-2\theta kt\right)+b_1(\theta-1)k(2\theta k+k-3\theta t). \quad (6.2.138)$$

Since N_{51} is a linear function of b_1, together with

$$N_{51}|_{b_1=0} = a_1\left(\theta^2\left(2k^2-4kt+3t^2\right)+k^2-2\theta kt\right) > 0 \quad (6.2.139)$$

and

$$N_{51}|_{b_1=a_1} = a_1\theta(k-t)((4\theta-1)k-3\theta t) > 0, \quad (6.2.140)$$

we have $N_{51} > 0$. Then we can conclude

$$\frac{\partial N_5}{\partial r}|_{r=r'_d} > 0 \quad (6.2.141)$$

Together with (6.2.136) and

$$N_5|_{r=r'_d} = -\left(a_1\theta(k-t) + b_1(\theta-1)k\right) R_1^2 \left(a_1(\theta-1)k + b_1(\theta k + k - 2\theta t)\right)^2 > 0, \quad (6.2.142)$$

we can have when $r > r'_d$, $N_5 > 0$. From (6.2.15), we can conclude when $r > r_d$,

$$N_5 > 0. \quad (6.2.143)$$

From (6.2.131) and (6.2.143), we can conclude $eNum_{a_2} > 0$.

e_1^* **vs.** b_1

$$eNum_{b_1} = 2(\theta-1)(k+2q)M_3N_6 \quad (6.2.144)$$

where

$$N_6 = b_1(k-\theta t)\left(a_1(\theta k + k - 2\theta t) + b_1(\theta-1)k\right)^3 + 64(\theta-1)^3\theta^3 k r^2 t^2 (k-t)^3 \quad (6.2.145)$$

Since

$$\frac{\partial N_6}{\partial r} = 128(\theta-1)^3\theta^3 k r t^2 (k-t)^3 > 0 \quad (6.2.146)$$

and

$$N_6|_{r=0} = (k-\theta t)R_1^3 > 0, \quad (6.2.147)$$

we can conclude

$$N_6 > 0. \quad (6.2.148)$$

Together with (6.2.106) and (6.2.144), we can conclude $eNum_{b_1} > 0$.

p_1^* **vs.** a_2

$$pNum_{a_2} = -2(\theta-1)^2\theta r t(k+2q)(k-t)N_5 \quad (6.2.149)$$

From (6.2.143), we can conclude $pNum_{a_2} > 0$.

$\underline{p_1^* \textbf{ vs. } b_1}$

$$pNum_{b_1} = 2(\theta-1)^2\theta rt(k+2q)(k-t)M_3N_8 \tag{6.2.150}$$

where

$$N_8 = a_1(\theta-1)k(\theta k+k-2\theta t)\left(8(\theta-1)\theta rt(k-t)-3b_1^2(k-\theta t)\right)-4a_1^2b_1\theta(k-t)(k-\theta t)^2 + a_1^3(\theta-1)k(k-\theta t)(\theta k+k-2\theta t)+2b_1(\theta-1)^2k^2\left(b_1^2(\theta t-k)+4(\theta-1)\theta rt(k-t)\right) \tag{6.2.151}$$

Since

$$N_8|_{r=r'_d} = -\left(a_1\theta(k-t)+b_1(\theta-1)k\right)R_1\left(a_1(\theta-1)k+b_1(\theta k+k-2\theta t)\right) > 0 \tag{6.2.152}$$

and

$$\frac{\partial N_8}{\partial r} = 8(\theta-1)^2\theta kt(k-t)R_1 > 0, \tag{6.2.153}$$

we can conclude when $r > r'_d$, $N_8 > 0$. Then from (6.2.15), when $r > r_d$,

$$N_8 > 0. \tag{6.2.154}$$

From (6.2.106), (6.2.150), and (6.2.154), we have $pNum_{b_1} > 0$.

$\underline{e_1^* \textbf{ vs. } r_1}$

$$eNum_{r_1} = -8(\theta-1)^2\theta t(k+2q)(k-t)R_1M_3N_3 \tag{6.2.155}$$

where

$$N_3 = R_1^2 + 16(\theta-1)\theta^2rt(k-t)^2. \tag{6.2.156}$$

Since

$$N_3|_{r=r'_d} = -R_1\left(a_1(\theta-1)k+b_1(\theta k+k-2\theta t)\right) < 0 \tag{6.2.157}$$

and

$$\frac{\partial N_3}{\partial r} = 16(\theta-1)\theta^2 t(k-t)^2 < 0, \tag{6.2.158}$$

we can conclude when $> r'_d$, $N_3 < 0$. From (6.2.15), we can conclude when $> r_d$,

$$N_3 < 0. \tag{6.2.159}$$

Together with (6.2.106), we can conclude $eNum_{r_1} < 0$.

<u>e_1^* **vs.** r_2</u>

$$eNum_{r_2} = 8(\theta-1)^2\theta t(k+2q)(k-t)R_1^2\left(a_1(\theta-1)k + b_1(\theta k + k - 2\theta t)\right)M_3. \tag{6.2.160}$$

From (6.2.106), we can conclude $eNum_{r_2} < 0$.

<u>p_1^* **vs.** r_1</u>

$$pNum_{r_1} = -2(\theta-1)^2\theta t(k+2q)(k-t)\left(a_1(\theta k+k-2\theta t)+b_1(\theta-1)k\right)^2 M_3 N_4 \tag{6.2.161}$$

where

$$N_4 = \left(a_1^2 - b_1^2\right)(k-\theta t) + 4(\theta-1)\theta r t(k-t). \tag{6.2.162}$$

Since

$$N_4|_{r=r'_d} = -\frac{1}{2}\left(a_1+b_1\right)\left(a_1(\theta-1)k + b_1(\theta k+k-2\theta t)\right) > 0 \tag{6.2.163}$$

and

$$\frac{\partial N_4}{\partial r} = 4(\theta-1)\theta t(k-t) > 0, \tag{6.2.164}$$

we can conclude when $> r'_d$, $N_4 > 0$. From (6.2.15), we can conclude when $> r_d$,

$$N_4 > 0. \tag{6.2.165}$$

Together with (6.2.106), we can conclude $pNum_{r_1} < 0$.

p_1^* **vs.** r_2

$$pNum_{r_2} = -(\theta-1)^3\theta^2 rt^2(k+2q)(k-t)^2 R_1 \left(a_1(\theta-1)k + b_1(\theta k + k - 2\theta t)\right) M_3. \quad (6.2.166)$$

From (6.2.106), We can conclude $pNum_{r_2} < 0$. ■

6.2.10 Optimality of e_d and p_d

We have $\frac{\partial^2 \pi_1}{\partial p_1^2} = \frac{2}{(\theta-1)t} < 0$, which implies that p_d^* satisfies the second-order condition. Plugging the expressions of p_1, p_2 as a function of e_1 and e_2 into π_1 (Equation 3.5), we have

$$\frac{\partial^2 \pi_1}{\partial e_1^2} = -\frac{\left(a_1(\theta k + k - 2\theta t) + b_1(\theta-1)k\right)^2 + 16(\theta-1)\theta^2 rt(k-t)^2}{8(\theta-1)\theta^2 t(k-t)^2}. \quad (6.2.167)$$

which is a decreasing function of r and

$$\frac{\partial^2 \pi_1}{\partial e_1^2}|_{r=r'_d} = -\frac{\left(a_1(\theta k + k - 2\theta t) + b_1(\theta-1)k\right)\left(a_1(k-\theta k) - b_1(\theta k + k - 2\theta t)\right)}{8(\theta-1)\theta^2 t(k-t)^2} < 0. \quad (6.2.168)$$

Then, $\frac{\partial^2 \pi_1}{\partial e_1^2} < 0$ for $r > r_d > r'_d$. We can conclude π_1 can reach the maximum value at e_d^*. ■

6.2.11 Proof of Theorem 14

From Equation (3.19), we have

$$\frac{\partial e_d^*}{\partial k} = \frac{F}{2DEN_d^2}, \quad (6.2.169)$$

where

$$F = F_0 + F_1 k + F_2 k^2 \quad (6.2.170)$$

$$F_0 = 4(\theta-1)\theta t^2\left(a_1^2 b_1\theta + 4(\theta-1)r\left(a_1((\theta-1)q + \theta t) - b_1(\theta-1)q\right) + a_1^3\theta\right), \quad (6.2.171)$$

$$F_1 = -4(\theta-1)\theta t\left(a_1(\theta+1) + b_1(\theta-1)\right)\left(a_1\left(a_1+b_1\right) + 4(\theta-1)rt\right), \quad (6.2.172)$$

$$F_2 = (\theta-1)\left(a_1(\theta+1) + b_1(\theta-1)\right) R_2. \quad (6.2.173)$$

To determine the sign of F, which is the same as that of $\frac{\partial e_d^*}{\partial k}$, we first have

$$F|_{r=0} = (\theta - 1)(a_1 + b_1) R_1^2 < 0. \tag{6.2.174}$$

We also have

$$\frac{\partial F}{\partial r} = 8(\theta - 1)^2 \theta t \left(a_1 \left((\theta+1)k^2 - 2(\theta+1)kt + 2\theta t(q+t) - 2qt\right) + b_1(\theta - 1)\left(k^2 - 2t(k+q)\right)\right), \tag{6.2.175}$$

which is a convex function of k. Also,

$$\frac{\partial F}{\partial r}|_{k=0} = 16(\theta - 1)^2 \theta t^2 \left(a_1(q+t) - b_1 q\right)(\theta - \theta'') \tag{6.2.176}$$

and

$$\frac{\partial F}{\partial r}|_{k=k_{max}} = \frac{16(\theta - 1)^3 \theta t^2 (a_1 - b_1)\left(a_1(\theta(q+t) + q) + b_1(\theta - 1)q\right)}{a_1(\theta + 1) + b_1(\theta - 1)} < 0, \tag{6.2.177}$$

We need to consider two cases according to the sign of (6.2.176) in order to determine the sign of F:

<u>Case 1: $\theta < \theta''$</u>

According to (6.2.176) (which is negative in this case) and (6.2.177), together with the fact that $\frac{\partial F}{\partial r}$ is a convex function of k, we can conclude

$$\frac{\partial F}{\partial r} < 0, \quad 0 < k < k_{max}. \tag{6.2.178}$$

From (6.2.178) and (6.2.174), we can conclude $F < 0$, that is $\frac{\partial e_d^*}{\partial k} < 0$.

<u>Case 2: $\theta > \theta''$</u>

We have

$$F|_{k=k_{max}} = \frac{16(\theta-1)^3\theta r t^2\,(a_1-b_1)\,(a_1(\theta(q+t)+q)+b_1(\theta-1)q)}{a_1(\theta+1)+b_1(\theta-1)} < 0 \tag{6.2.179}$$

and

$$F|_{r=r_d} = \frac{(\theta-1)\,(a_1+b_1)\,(k+2q)R_1}{(k-t)(k+2q-2t)}K \tag{6.2.180}$$

where

$$K = a_1\left((\theta+1)k^2 - k(3\theta t+t) + 2(2\theta-1)t^2\right) + b_1(\theta-1)(k+t)(k-2t), \tag{6.2.181}$$

which is a convex function of k. From (6.2.179) and (6.2.180), we have $K|_{k=k_{max}} < 0$. Given (6.2.176) which is positive in this case, (6.2.179), $K|_{k=k_{max}} < 0$, and the fact that K is a a convex function of k, from Lemma 6.2.1, we can conclude there is a root on the left boundary B_L. Given (6.2.176) which is positive in this case, from Proposition 6.2.1, we can conclude there is a threshold value r_{dDF}: when $r > r_{dDF}$, $F > 0$ on the curve B_{min}; $F < 0$ otherwise.

On the line $r = r_d$, we have

$$\frac{\partial F}{\partial k}|_{r=r_d} = \frac{2(\theta-1)\,(a_1+b_1)\,(k+2q)\,(a_1(\theta+1)+b_1(\theta-1))\,R_1}{k+2q-2t} < 0. \tag{6.2.182}$$

In particular, $\frac{\partial F}{\partial k}|_{r=r_d,k=0} < 0$. Given that

$$\frac{\partial^2 F}{\partial k \partial r} = 6(\theta-1)^2\theta t(k-t)\,(a_1(\theta+1)+b_1(\theta-1)) < 0, \tag{6.2.183}$$

we can see that $\frac{\partial F}{\partial k} < 0$ along the line $k = 0$ on B_L. Together with (6.2.182), we can conclude on the curve B_{min}, $\frac{\partial F}{\partial k} < 0$. On the right boundary $k = k_{max}$,

$$\frac{\partial F}{\partial k}|_{k=k_{max}} = 16(\theta-1)^3\theta r t^2\,(a_1-b_1) < 0. \tag{6.2.184}$$

Then $\frac{\partial F}{\partial k} < 0$ in the feasible region since it is a quadratic function according to (6.2.170). By using (6.2.179), we can conclude: (i) when $r > r_{dDF}$, there is a threshold value $\bar{k}_{dDF}$ for any given r within the feasible region: when $k < \bar{k}_{dDF}$, $F > 0$; $F < 0$ otherwise. (ii) when $r < r_{dDF}$, $F < 0$. ■

6.2.12 Proof of Theorem 15

From (3.20), the FOC of p_d with respect to k is:

$$\frac{\partial p_d^*}{\partial k} = \frac{-2(\theta-1)^2\theta rtG}{DEN_d^2}, \tag{6.2.185}$$

where

$$G = 2a_1b_1\theta(k-t)^2 + a_1^2\left((\theta+1)k^2 - 4\theta kt + 2t(-\theta q + q + \theta t)\right) + (\theta-1)\left(b_1^2\left(k^2+2qt\right) + 8\theta rt(k-t)^2\right). \tag{6.2.186}$$

which is a quadratic function of k. The sign of $\frac{\partial p_1}{\partial k}$ is opposite to the sign of G.

To determine the sign of G, we study the properties of G on the boundary. On the line $r = r_d$,

$$G|_{r=r_d} = \frac{(k+2q)G_7}{k+2q-2t} \tag{6.2.187}$$

where

$$G_7 = \left(2a_1b_1\theta(k-t)^2 + a_1^2\left((\theta+1)k^2 - 4\theta kt + 2t(-\theta q + q + 2\theta t - t)\right) + b_1^2(\theta-1)\left(k^2 + 2t(q-t)\right)\right). \tag{6.2.188}$$

Since G_7 is a quadratic function of k, together with

$$\frac{\partial G_7}{\partial k}|_{k=0} = -4t\theta a_1\left(a_1+b_1\right) < 0 \tag{6.2.189}$$

and

$$\frac{\partial G_7}{\partial k}|_{k=k_{max}} = 0, \tag{6.2.190}$$

we can see that G_7 is a convex function and reaches minimum at $k = k_{max}$. Given that G_7 decreases with k within the feasible region,

$$G|_{k=k_{max},r=r_d} = -\frac{2(\theta-1)t\,(a_1^2-b_1^2)\,(a_1(\theta(q+t)+q)+b_1(\theta-1)q)}{a_1(\theta+1)+b_1(\theta-1)} > 0, \tag{6.2.191}$$

and

$$\frac{\partial G}{\partial r} = 8(\theta-1)\theta t(k-t)^2 < 0, \tag{6.2.192}$$

we can conclude there is exactly one root for $G = 0$ on the left boundary B_L from Lemma 6.2.1. Together with (6.2.192) and Proposition 6.2.1, there is a threshold value r_{dDG}: when $r > r_{dDG}$, $G < 0$ on the curve B_{min}; $G > 0$ otherwise.

From (6.2.191) and (6.2.192), we can conclude there is a threshold value r_{dF} on the right boundary $k = k_{max}$: when $r < r_{dF}$, $G|_{k=k_{max}} > 0$; $G|_{k=k_{max}} < 0$ otherwise.

Next we show that $r_{dF} > r_{dDG}$. From

$$\frac{\partial G}{\partial k}|_{r=r_d} = \frac{2\,(a_1+b_1)\,(k+2q)R_1}{k+2q-2t} > 0, \tag{6.2.193}$$

we have $\frac{\partial G}{\partial k}|_{k=0,r=r_d} > 0$. Together with

$$\frac{\partial^2 G}{\partial k \partial r} = 16(\theta-1)\theta t(k-t) > 0, \tag{6.2.194}$$

we have $\frac{\partial G}{\partial k} > 0$ along the boundary line $k = 0$. Together with (6.2.193), we can conclude $\frac{\partial G}{\partial k} > 0$ on the curve B_{min}. Together with

$$\frac{\partial G}{\partial k}|_{k=k_{max}} = \frac{16(\theta-1)^2\theta r t^2\,(a_1-b_1)}{a_1(\theta+1)+b_1(\theta-1)} > 0, \tag{6.2.195}$$

and the fact that G is a quadratic function of k, we can conclude $\frac{\partial G}{\partial k} > 0$ within the feasible region. Then $G|_{r=r_{dF}} < 0$ on the curve B_{min}. Therefore $r_{dF} > r_{dDG}$.

1. When $r < r_{dDG}$, G is positive on the curve B_{min} and the right boundary. Since $\frac{\partial G}{\partial k} > 0$, then $G > 0$. That is, $\frac{\partial p_d^*}{\partial k} < 0$.

2. When $r_{dDG} < r < r_{dF}$, G is negative on the curve B_{min} and positive on the right boundary. Given that $\frac{\partial G}{\partial k} > 0$, there is a threshold value $\bar{k}_{dDG}$: when $k < \bar{k}_{dDG}$, $G < 0$ and $\frac{\partial p_d^*}{\partial k} > 0$; $G > 0$ and $\frac{\partial p_d^*}{\partial k} < 0$ otherwise.

3. When $r > r_{dF}$, G is negative on the curve B_{min} and the right boundary. Since $\frac{\partial G}{\partial k} > 0$, we have $G < 0$ and $\frac{\partial p_d^*}{\partial k} > 0$. ■

6.2.13 Proof of Table 3.8

Since $\frac{\partial e_d^*}{\partial a_1} = \frac{\partial e_1^*}{\partial a_1} + \frac{\partial e_2^*}{\partial a_1}|_{a_1=b_2}$, $\frac{\partial e_d^*}{\partial b_1} = \frac{\partial e_1^*}{\partial b_1} + \frac{\partial e_2^*}{\partial b_1}|_{a_2=b_1}$, and $\frac{\partial e_d^*}{\partial r} = \frac{\partial e_1^*}{\partial r_1} + \frac{\partial e_2^*}{\partial r_1}|_{r_1=r_2=r}$, from Table 3.5, we can prove $\frac{\partial e_d^*}{\partial a_1} > 0$, $\frac{\partial e_d^*}{\partial b_1} > 0$, and $\frac{\partial e_d^*}{\partial r} < 0$. From Equation (3.19), $\frac{\partial e_d^*}{\partial q} = \frac{2e_d^*}{k+2q} > 0$.

Similarly, we can prove $\frac{\partial p_d^*}{\partial a_1} > 0$, $\frac{\partial p_d^*}{\partial r} < 0$, $\frac{\partial p_d^*}{\partial b_1} > 0$, and $\frac{\partial p_d^*}{\partial q} > 0$. ■

6.2.14 Proof of Lemma 2

$$\frac{\partial e_a^*}{\partial k} = \frac{e_a^*}{k+2q} > 0 \tag{6.2.196}$$

$$\frac{\partial p_a^*}{\partial k} = \frac{p_a^*}{k+2q} > 0 \tag{6.2.197}$$

6.2.15 Proof of Lemma 3

$$\frac{\partial e_g^*}{\partial k} = \frac{e_g^*}{k+2q} > 0 \tag{6.2.198}$$

$$\frac{\partial p_g^*}{\partial k} = \frac{p_g^*}{k+2q} > 0 \tag{6.2.199}$$

6.2.16 Proof of Theorem 16

Since

$$e_a^* - e_g^* = \frac{2P_1}{a_0^2 + 4(\theta-1)^2 rt} e_a^* \tag{6.2.200}$$

and

$$p_a^* - p_g^* = -\frac{a_0^2 P_1}{(\theta-1)rt\,(a_0^2 + 4(1-\theta)^2 rt)} p_a^*, \tag{6.2.201}$$

where $P_1 = a_0^2 + (\theta-1)(2\theta-1)rt$. To determine the sign of $e_a^* - e_g^*$ and $p_a^* - p_g^*$, we need to determine the sign of P_1.

Case 1: $\theta_1 < 1/2$

In this case, $P_1 > 0$. Then, $e_a^* > e_g^*$ and $p_a^* > p_g^*$.

Case 2: $1/2 < \theta_1 < 1$

In order for anti-piracy effort in (3.26) to be positive, we shall have $r > r_1 \equiv \dfrac{a_0^2}{2(1-\theta)}$. Let $P_1 = 0$, we have $r = r_p \equiv \dfrac{a_0^2}{(1-\theta)(2\theta-1)} > r_1$. When $r > r_p$, $P_1 < 0$; $P_1 > 0$ otherwise.

■

6.3. Proof of Chapter 4

6.3.1 Constraints of Parameter Values

Monopoly Case with Only Basic Service

To ensure the demand for basic service is positive, from (4.6), we can conclude

$$t_1 - k_1 > 0. \tag{6.3.1}$$

To ensure the HIE's price is positive, from (6.3.11), we have

$$q_1 - M_h > 0 \tag{6.3.2}$$

Monopoly Case with Additional Value-Added Service

From (4.17) and (6.3.1), to ensure $x^{mv*} > 0$, we can conclude

$$k_2^2 - 4\left(k_1 - t_1\right)\left(k_3 - t_2\right) < 0. \tag{6.3.3}$$

Together with (4.16), to ensure $x^{mb*} > x^{mv*}$, we have

$$8c_v\left(k_3 - t_2\right)\left(M_h + q_1\right) + k_2\left(4c_v\left(M_h + q_1\right) - 1\right) - 2k_1 + 2t_1 < 0 \tag{6.3.4}$$

to ensure

Duopoly Case With Only Basic Service

To ensure the price is greater than 0, we can conclude we have

$$t_1 - M_h - k_1 > 0 \tag{6.3.5}$$

Duopoly Case With Additional Value-Added Service

From (4.47) to (4.49), we can solve D_1^{dv},

$$D_1^{dv} = \frac{2k_2c_v + 1}{8c_v\left(t_2 - k_3\right)}. \tag{6.3.6}$$

To ensure $D_1^{dv} > 0$, we have

$$t_2 - k_3 > 0. \tag{6.3.7}$$

TO ensure $D_1^{dv} < 1/2$, we have

$$2k_2c_v + 1 < 4c_v\left(t_2 - k_3\right) \tag{6.3.8}$$

6.3.2 Proof of Lemma 4

From (4.10),we can find that

$$U^{mb}|_{x<x^{mv}} < (U^{mb} + U^{mv})|_{x<x^{mv}} \tag{6.3.9}$$

and

$$U^{mb}|_{x>x^{mv}} > (U^{mb} + U^{mv})|_{x>x^{mv}}. \tag{6.3.10}$$

Then we can prove the result of Lemma 4.

6.3.3 Proof of Table 4.2

Basic service monopoly case

$$p^{mb*} = \frac{1}{2}\left(q_1 - M_h\right) \tag{6.3.11}$$

<u>p^{mb*} **vs.** M_h</u>

$$\frac{\partial p^{mb*}}{\partial M_h} = -1/2 < 0 \tag{6.3.12}$$

<u>p^{mb*} **vs.** q_1</u>

$$\frac{\partial p^{mb*}}{\partial q_1} = 1/2 > 0 \tag{6.3.13}$$

<u>p^{mb*} **vs.** k_1</u>

$$\frac{\partial p^{mb*}}{\partial k_1} = 0 \tag{6.3.14}$$

monopoly B&VS sub-case

$$p^{mb*} = \frac{1}{4}\left(\frac{k_2\left(2k_2c_v\left(M_h + q_1\right) - k_1 + t_1\right)}{c_v\left(k_2^2 - 4\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)} + 2\left(q_1 - M_h\right)\right) \tag{6.3.15}$$

$\underline{p^{mb*} \textbf{ vs. } M_h}$

$$\frac{\partial p^{mb*}}{\partial M_h} = \frac{1}{2}\left(\frac{k_2^2}{k_2^2 - 4\left(k_1 - t_1\right)\left(k_3 - t_2\right)} - 1\right) < 0 \tag{6.3.16}$$

$\underline{p^{mb*} \textbf{ vs. } q_1}$

$$\frac{\partial p^{mb*}}{\partial q_1} = \frac{1}{2}\left(\frac{k_2^2}{k_2^2 - 4\left(k_1 - t_1\right)\left(k_3 - t_2\right)} + 1\right) \tag{6.3.17}$$

When $k_2^2 < 2\left(k_1 - t_1\right)\left(k_3 - t_2\right)$, $\frac{\partial p^{mb*}}{\partial q_1} > 0$. else when $2\left(k_1 - t_1\right)\left(k_3 - t_2\right) < k_2^2 < 4\left(k_1 - t_1\right)\left(k_3 - t_2\right)$, $\frac{\partial p^{mb*}}{\partial q_1} < 0$.

$\underline{p^{mb*} \textbf{ vs. } k_1}$

$$\frac{\partial p^{mb*}}{\partial k_1} = -\frac{k_2^2\left(8c_v\left(t_2 - k_3\right)\left(M_h + q_1\right) + k_2\right)}{4c_v\left(k_2^2 - 4\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)^2} < 0 \tag{6.3.18}$$

$\underline{p^{mb*} \textbf{ vs. } k_2}$

$$\frac{\partial p^{mb*}}{\partial k_2} = \frac{\left(k_1 - t_1\right)\left(-16k_2c_v\left(k_3 - t_2\right)\left(M_h + q_1\right) + 4\left(k_1 - t_1\right)\left(k_3 - t_2\right) + k_2^2\right)}{4c_v\left(k_2^2 - 4\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)^2} < 0 \tag{6.3.19}$$

$\underline{p^{mb*} \textbf{ vs. } k_3}$

$$\frac{\partial p^{mb*}}{\partial k_3} = \frac{k_2\left(k_1 - t_1\right)\left(2k_2c_v\left(M_h + q_1\right) - k_1 + t_1\right)}{c_v\left(k_2^2 - 4\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)^2} < 0 \tag{6.3.20}$$

$$p^{mv*} = \frac{4k_2c_v\left(k_3 - t_2\right)\left(M_h + q_1\right) - 6\left(k_1 - t_1\right)\left(k_3 - t_2\right) + k_2^2}{4c_v\left(k_2^2 - 4\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)} \tag{6.3.21}$$

$\underline{p^{mv*} \textbf{ vs. } M_h}$

$$\frac{\partial p^{mv*}}{\partial M_h} = \frac{k_2\left(k_3 - t_2\right)}{k_2^2 - 4\left(k_1 - t_1\right)\left(k_3 - t_2\right)} > 0 \tag{6.3.22}$$

$\underline{p^{mv*} \textbf{ vs. } q_1}$

$$\frac{\partial p^{mv*}}{\partial q_1} = \frac{k_2 (k_3 - t_2)}{k_2^2 - 4 (k_1 - t_1) (k_3 - t_2)} > 0 \tag{6.3.23}$$

<u>p^{mv*} **vs.** k_1</u>

$$\frac{\partial p^{mv*}}{\partial k_1} = -\frac{k_2 (k_3 - t_2) (8c_v (t_2 - k_3) (M_h + q_1) + k_2)}{2c_v (k_2^2 - 4 (k_1 - t_1) (k_3 - t_2))^2} > 0 \tag{6.3.24}$$

<u>p^{mv*} **vs.** k_2</u>

$$\frac{\partial p^{mv*}}{\partial k_3} = \frac{(k_3 - t_2) (k_2 (k_1 - t_1) - c_v (4 (k_1 - t_1) (k_3 - t_2) + k_2^2) (M_h + q_1))}{c_v (k_2^2 - 4 (k_1 - t_1) (k_3 - t_2))^2} > 0 \tag{6.3.25}$$

<u>p^{mv*} **vs.** k_3</u>

$$\frac{\partial p^{mv*}}{\partial k_3} = \frac{k_2^2 (2k_2 c_v (M_h + q_1) - k_1 + t_1)}{2c_v (k_2^2 - 4 (k_1 - t_1) (k_3 - t_2))^2} > 0 \tag{6.3.26}$$

Basic service duopoly case Define

$$N \equiv \{q_2 = q_1, k_{11} = k_{12} = k_1, k_{21} = k_{22} = k_2, k_{31} = k_{32} = k_3, c_{v1} = c_{v_2} = c_v\} \tag{6.3.27}$$

$$p_1^{db*} = \frac{1}{3} (-k_{11} - 3M_h - 2k_{12} + q_1 - q_2) + t_1 \tag{6.3.28}$$

<u>p_1^{db*} **vs.** M_h</u>

$$\frac{\partial p_1^{db*}}{\partial M_h}|_N = -1 < 0 \tag{6.3.29}$$

<u>p_1^{db*} **vs.** q_1</u>

$$\frac{\partial p_1^{db*}}{\partial q_1}|_N = 1/3 > 0 \tag{6.3.30}$$

<u>p_1^{db*} **vs.** q_2</u>

$$\frac{\partial p_1^{db*}}{\partial q_2}|_N = -1/3 < 0 \tag{6.3.31}$$

$\underline{p_1^{db*}\ \textbf{vs.}\ k_{11}}$

$$\frac{\partial p_1^{db*}}{\partial k_{11}}|_N = -1/3 < 0 \tag{6.3.32}$$

$\underline{p_1^{db*}\ \textbf{vs.}\ k_{12}}$

$$\frac{\partial p_1^{db*}}{\partial k_{12}}|_N = -2/3 < 0 \tag{6.3.33}$$

Basic and value-added service duopoly case

$\underline{p_1^{db*}\ \textbf{vs.}\ M_h}$

$$\frac{\partial p_1^{db*}}{\partial M_h}|_N = -1 < 0 \tag{6.3.34}$$

$\underline{p_1^{db*}\ \textbf{vs.}\ q_1}$

$$\frac{\partial p_1^{db*}}{\partial q_1}|_N = \frac{1}{6}\left(\frac{k_2^2}{k_2^2 - 6\left(k_1 - t_1\right)\left(k_3 - t_2\right)} + 2\right) > 0 \tag{6.3.35}$$

$\underline{p_1^{db*}\ \textbf{vs.}\ q_2}$

$$\frac{\partial p_1^{db*}}{\partial q_2}|_N = -\frac{k_2^2}{6\left(k_2^2 - 6\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)} - \frac{1}{3} < 0 \tag{6.3.36}$$

$\underline{p_1^{db*}\ \textbf{vs.}\ k_{11}}$

$$\frac{\partial p_1^{db*}}{\partial k_{11}}|_N = \frac{1}{12}\left(\frac{k_2^2}{k_2^2 - 6\left(k_1 - t_1\right)\left(k_3 - t_2\right)} - 4\right) < 0 \tag{6.3.37}$$

$\underline{p_1^{db*}\ \textbf{vs.}\ k_{12}}$

$$\frac{\partial p_1^{db*}}{\partial k_{12}}|_N = -\frac{k_2^2}{12\left(k_2^2 - 6\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)} - \frac{2}{3} < 0 \tag{6.3.38}$$

$\underline{p_1^{db*}\ \textbf{vs.}\ k_{21}}$

$$\frac{\partial p_1^{db*}}{\partial k_{21}}|_N = \frac{\left(k_2^2 - 8\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)\left(4k_2c_v + 1\right)}{16c_v\left(k_2^2 - 6\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)\left(k_3 - t_2\right)} < 0 \tag{6.3.39}$$

<u>p_1^{db*} **vs.** k_{22}</u>

$$\frac{\partial p_1^{db*}}{\partial k_{22}}|_N = \frac{\left(k_2^2 - 4\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)\left(4k_2c_v + 1\right)}{16c_v\left(k_2^2 - 6\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)\left(k_3 - t_2\right)} < 0 \tag{6.3.40}$$

<u>p_1^{db*} **vs.** k_{31}</u>

$$\frac{\partial p_1^{db*}}{\partial k_{31}}|_N = -\frac{k_2\left(k_2^2 - 8\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)\left(2k_2c_v + 1\right)}{16c_v\left(k_2^2 - 6\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)\left(k_3 - t_2\right)^2} < 0 \tag{6.3.41}$$

<u>p_1^{db*} **vs.** k_{32}</u>

$$\frac{\partial p_1^{db*}}{\partial k_{32}}|_N = -\frac{k_2\left(k_2^2 - 4\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)\left(2k_2c_v + 1\right)}{16c_v\left(k_2^2 - 6\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)\left(k_3 - t_2\right)^2} < 0 \tag{6.3.42}$$

<u>p_1^{dv*} **vs.** M_h</u>

$$\frac{\partial p_1^{dv*}}{\partial M_h}|_N = 0 \tag{6.3.43}$$

<u>p_1^{dv*} **vs.** q_1</u>

$$\frac{\partial p_1^{dv*}}{\partial q_1}|_N = \frac{k_2\left(k_3 - t_2\right)}{2\left(k_2^2 - 6\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)} > 0 \tag{6.3.44}$$

<u>p_1^{dv*} **vs.** q_2</u>

$$\frac{\partial p_1^{dv*}}{\partial q_2}|_N = \frac{k_2\left(t_2 - k_3\right)}{2\left(k_2^2 - 6\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)} < 0 \tag{6.3.45}$$

<u>p_1^{dv*} **vs.** k_{11}</u>

$$\frac{\partial p_1^{dv*}}{\partial k_{11}}|_N = \frac{k_2\left(k_3 - t_2\right)}{4\left(k_2^2 - 6\left(k_1 - t_1\right)\left(k_3 - t_2\right)\right)} > 0 \tag{6.3.46}$$

<u>p_1^{dv*} **vs.** k_{12}</u>

$$\frac{\partial p_1^{dv*}}{\partial k_{12}}|_N = \frac{k_2\left(t_2-k_3\right)}{4\left(k_2^2-6\left(k_1-t_1\right)\left(k_3-t_2\right)\right)} < 0 \qquad (6.3.47)$$

<u>p_1^{dv*} **vs.** k_{21}</u>

$$\frac{\partial p_1^{dv*}}{\partial k_{21}}|_N = -\frac{24c_v\left(k_1-t_1\right)\left(k_3-t_2\right)+k_2}{16c_v\left(k_2^2-6\left(k_1-t_1\right)\left(k_3-t_2\right)\right)} > 0 \qquad (6.3.48)$$

<u>p_1^{dv*} **vs.** k_{22}</u>

$$\frac{\partial p_1^{dv*}}{\partial k_{22}}|_N = \frac{k_2\left(4k_2c_v+1\right)}{16c_v\left(k_2^2-6\left(k_1-t_1\right)\left(k_3-t_2\right)\right)} < 0 \qquad (6.3.49)$$

<u>p_1^{dv*} **vs.** k_{31}</u>

$$\frac{\partial p_1^{dv*}}{\partial k_{31}}|_N = \frac{k_2^2\left(2k_2c_v+1\right)}{16c_v\left(k_2^2-6\left(k_1-t_1\right)\left(k_3-t_2\right)\right)\left(k_3-t_2\right)} > 0 \qquad (6.3.50)$$

<u>p_1^{dv*} **vs.** k_{32}</u>

$$\frac{\partial p_1^{dv*}}{\partial k_{32}}|_N = -\frac{k_2^2\left(2k_2c_v+1\right)}{16c_v\left(k_2^2-6\left(k_1-t_1\right)\left(k_3-t_2\right)\right)\left(k_3-t_2\right)} < 0 \qquad (6.3.51)$$