University of Alberta

Issues in Symmetry Breaking and Superunification

By

Alick Lachlan Macpherson ©

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Alick Macpherson

Alick Lachlan Macpherson

Department of Physics

University of Alberta

Edmonton, Alberta

T6G 2J1

Date: 26/9/96

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled "Issues in Symmetry Breaking and Superunification" submitted by Alick Lachlan Macpherson in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Theoretical Physics.

B.A. Campbell
Prof. B.A. Campbell Advisor

Prof. F. Khanna

James Pinfold
Prof. J.L. Pinfold

Doug Gingrich
Prof. D. Gingrich

Prof. N. Rodning

G. Ludwig
Prof. G. Ludwig

P J O'Donnell
Prof. P. O'Donnell

Date: 24/9/96

Abstract

Internal symmetries play a vital role in the development of particle physics theory, and the implications of these symmetries and their subsequent breakdown can produce a variety of physical implications for the low energy effective theory. Three such case studies are presented, which examine some of the more unusual aspects of internal symmetries and their breakdown. The issues considered are the low energy effective theory signatures of strong coupling induced Higgs bag formation, biased spontaneous symmetry breaking of a discrete symmetry, and the effect on nucleon decay from a supersymmetric $SO(10)$ grand unified theory with a non-minimal Higgs sector. The conclusions resulting from these studies range from the complete rejection of any observable Higgs bag signatures associated with toponium bound states through the production of novel Fermi Ball dark matter candidates, to prediction of a branching fraction spectrum for nucleon decay. The latter, if observed at experiments like Super-KAMIOKANDE, would elucidate much on the structure of the supersymmetric grand unified theory extension to the Standard Model of particle physics.

Preface

The results in this thesis were obtained over the course of the author's Ph.D. programme at the University of Alberta between 1992 and 1995. This presentation of this work is in accordance with the "Paper Format" regulations of the Faculty of Graduate Studies of the University of Alberta, and is based on the following published papers:

- Alick L. Macpherson and Bruce A. Campbell, *Toponium Tests of Top Quark Higgs Bags*, Phys. Lett **B306**, 379 (1993).
- Alick L. Macpherson and Bruce A. Campbell, *Biased Discrete Symmetry Breaking and Fermi Balls*, Phys. Lett **B347**, 205 (1995).
- Alick L. Macpherson, *Nucleon Decay in Non-Minimal Supersymmetric SO(10)*, Nucl. Phys. **B472**, 79 (1996).

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CHAPTER 1

Introduction

Simplicity and elegance are often considered desirable qualities. Unfortunately, the complicated jumble of the observed low energy particle physics that we call the subatomic zoo appears to mirror neither of these qualities. Because of this, it should be of no surprise that one of the most compelling and aesthetic notions put forward in present day particle physics is that the underlying theory should move toward simplicity and elegance as the energy scale is increased. Adhering to this notion of reformation then requires two primary tenets on the underlying physical theory:

1 The underlying physical theory is constructed from a set of symmetry principles and the invariances under these symmetries.

2 The complicated nature of the physics of the low energy limit is a result of the breaking of these internal symmetries.

Although these tenets suggest that first we build up the theory, and then we tear it down, the construction of the theory is done along very specific guidelines, with the tearing down being well regulated, in as much as the symmetry breaking follows specific patterns.

The standard example of this approach in particle physics is the issue of the origin of mass. Classically, all particles were thought to be massive, and it is only through the development of electrodynamics, quantum mechanics, and quantum field theory that a limited set of massless particles have been revealed. These observations raise questions as to the origin of mass, as explicit vector boson mass terms in the formulation of the underlying physical theory are not permitted in a renormalisable field theory[1]. Thus, it is simply not enough, as a physicist, to play God, and decree the masses of the particles in the subatomic zoo. Clearly there must be a mechanism by which mass is generated, but it is also clear that this mechanism must be selective, so that particles like the photon are not assigned a mass. As will be seen, there is indeed such a mechanism, which manages to selectively generate masses for all of the observed massive particles, as well as maintaining the masslessness of the others. This mechanism requires the development of an underlying structure specified by the invariance under internal symmetries, followed by a carefully constructed breakdown of an internal symmetry that causes the lowest energy state of a particular field component to disrespect the internal symmetry.

The mechanism in question is of course the Higgs mechanism[2], which evolved from the development of gauge theories by means of a deft use of spontaneous symmetry breaking in the local gauge symmetry environment. As will be seen, the Higgs mechanism generates a non-zero vacuum expectation value for a particular field component, and through the couplings of other physical fields to this vacuum field configuration, effective mass terms result. It is by means of the gauge theory construction, and the subsequent breaking of the internal gauge symmetry, that masses for all the presently observed massive particles in the subatomic zoo are generated in a self-consistent way.

Such a process can occur as the physics of the early Universe is taken to be at a very high energy scale, implying that its energy density is sufficient to force the physical theory into some simple highly symmetric and unified form. One can imagine that this high energy limit of the physical theory is explicitly invariant under all the internal symmetries of the theory, and so all particle states are massless. However, as the Universe expands, the subsequent cooling induces the conditions necessary for the Higgs mechanism to kick in, thereby breaking some of the internal symmetries and hence generating mass for some subset of the originally massless particle states. Whatever the specifics of the particular symmetry breaking pattern the Higgs mechanism induces, the phenomenological upshot of the whole procedure is that at least one massive scalar particle is necessarily produced.

Indeed, all known models of mass generation for standard model fermions and gauge bosons involve Higgs bosons, or some composite states masquerading as Higgs bosons. Clearly, the Higgs mechanism is the most feasible answer to the question of the origin of mass, and it relies strongly on the two tenets described above; the development of gauge theory is a response to the desire for a simple self-consistent high energy theory, while the symmetry breaking Higgs mechanism offers the most acceptable means of reproducing the spectrum of particle masses.

Yet gauge theories and the Higgs mechanism are only one aspect of the issues associated with internal symmetries in a physical theory. Due to the fundamental importance placed on the role of symmetries in physical theory, there are in fact many far reaching consequences of the aforementioned tenets, that provide a full range of far-reaching predictions for those willing to look. These predictions take the form of unexpected phenomena, whether they be unusual experimental signatures, relic particles, or predictions from the low energy limit resulting from the unification generated by internal symmetries.

This thesis presents three separate studies dealing with the issues of internal symmetries in particle theory, and the associated physical phenomena induced by symmetry breaking. Each focuses on a particular feature or characteristic of symmetry breakdown, and attempts to resolve the predictions resulting from these issues in terms of experimental signatures. Further, the work is designed to show that internal symmetries and the breaking of such symmetries are responsible for much more than just the gauge theories, the Higgs mechanism, and mass generation (as if that weren't already enough!).

1.1 The Gauge Age

Before launching into some of the issues surrounding internal symmetries and symmetry breaking, a review of gauge theories and their implications is necessary to set the stage. Thus, the remainder of this introduction is devoted to the achievements of the era in modern particle physics colloquially known as "the age of the gauge". First however, a little pre-history is in order.

1.1.1 Pre-History

In comparison to our everyday world, the length scales associated with the subatomic particle zoo are very very very small. Such small scales imply the subatomic world is quantum mechanical by nature, and not the classical billiard-ball physics of our macro-scale world. Upon descent into this quantum regime, the fundamental laws of classical physics are found to be replaced by the quantum mechanical action. Then, in this quantum mechanical formulation, the classical motion of a particle is replaced by the transition amplitude, which is the weighted sum of all possible trajectories in the quantum mechanical configuration space. The weighting factor for each trajectory is $e^{\frac{iS}{\hbar}}$, where S is the action associated with the particular trajectory or path. The constant $\hbar$ is defined as Planck's constant, and is generally minute in comparison to the value of the action. The only exception to this arises when the action is extremal, and as the path length of a trajectory is unbounded above, this extremal corresponds to the minimum of the action. This then gives the principle of least action[3], which states that in the limit $\hbar \to 0$, the correct quantum mechanical field equations correspond to the trajectory of least action (the configuration for which $\delta S = 0$.).

Therefore, in order to formulate a quantum mechanical description of subatomic physics, a formulation of the quantum mechanical action is necessary. Following Feynman[4], for a set of local fields[‡] Φ, the action takes the form

$$S = \frac{1}{4}\int_A^B d^4x\sqrt{g}\mathcal{L}(\Phi, \partial_\mu\Phi) \tag{1.1}$$

where A and B are the initial and final points of the trajecory in a general 4-dimensional spacetime, g is the determinant of the spacetime metric $g_{\mu\nu}$, and $\mathcal{L}(\Phi, \partial_\mu\Phi)$ is the Lagrangian density associated with the set of local fields Φ. On restriction to the normal default of Minkowskian flat spacetime, which is appropriate for the particle physics regime, the action reduces to

$$S = \int_B^A d^4x L(\Phi(x), \partial_\mu\Phi(x)) \tag{1.2}$$

with $L(\Phi(x), \partial_\mu\Phi(x))$ being the Lagrangian of the system. In the classical limit the Lagrangian defaults to the kinetic minus the potential energy of the system, but in the quantum regime, it is a real function of the fields of the system and their partial derivatives[§]. Typically, the terms of the Lagrangian are constrained

[‡]Here local fields means that the fields have no spatial size, and are considered at a single spacetime point.

[§]The terms in L are required to contain at most two ∂_μ operators, as otherwise the associated classical equations of motion will be higher than second order in derivatives, which is unacceptable as it implies the equations of motion develop non-causal solutions.

by the imposition of Poincaré invariance (a.k.a invariance under translations and Lorentz transformations) which preserves the postulates of Special Relativity, thereby permitting theories compatible with the observed physical world[3]. However, it is the imposition of additional internal symmetries (symmetries pertaining to the set of local fields Φ) within the Poincaré invariant structure, that are used to specify the form of the terms that comprise the Lagrangian. In attempts to describe the subatomic world, it has been this freedom to choose the internal symmetries of the Lagrangian that has led to the development of gauge theories, and the arrival of the "Gauge Age".

1.1.2 The Gauge Age

Gauge theories[5] are theories in which the physics predicted by the theory remains unchanged after a set of transformations on a set of local fields. Such an invariance of the physics is intimately related to the presence of an exact symmetry - either manifest or hidden - in the underlying physical theory. This connection between physical invariance and internal symmetries is best seen by considering the case of phase rotations of a set of complex scalar fields ϕ_i. Being complex, these fields and their hermitian conjugates are distinct, allowing for the identification of ϕ_i and $\phi_i^\dagger$ with charged particle and antiparticle states. A phase rotation on these fields of

$$\phi_i(x) \rightarrow e^{-igQ_i\lambda}\phi_i(x) \tag{1.3}$$

can be seen to leave a simple "kinetic − potential" Lagrangian of the form

$$L(\Phi, \Phi^\dagger, \partial_\mu\Phi, \partial_\mu\Phi^\dagger) = \partial_\mu\phi_i^\dagger\partial^\mu\phi_i - V(\phi_i^\dagger\phi_i) \tag{1.4}$$

unaltered. As phase space rotations are identified with the Abelian $U(1)$ group, this invariance of the Lagrangian is equivalent to an invariance of the theory under the continuous $U(1)$ internal symmetry. From the form of the $\phi_i(x)$ transformation, gQ_i corresponds to the eigenvalues of the $U(1)$ group, and in this simple charged particle model, g is a coupling constant corresponding to a fundamental unit of charge[‡]. Therefore, this invariance under the $U(1)$ group implies the physical property of charge conservation in this particular model. For more sophisticated theories, where the underlying internal symmetry group is more complicated, $gQ_i\lambda$ is replaced by $gQ_i^a\lambda^a$, where the Q^a are the hermitian generators of the internal symmetry group, and λ^a a set of parameters. Invariance under such larger symmetries implies conservation of more generalised charges.

Thus, this simple model, with its invariance under the $U(1)$ group, constitutes an example of a gauge theory, albeit a trivial one. This triviality is due mainly to the global nature of the phase rotation parameter in the $\phi_i(x)$ transformation - λ does not depend on spacetime position at all. A much more instructive example of gauge theory is one where the phase invariance is made local, so that the phase rotation itself is spacetime dependent ($\lambda \to \lambda(x^\mu)$). With this relaxing of the constraints on the phase rotation, the transformation of $\phi_i(x^\mu)$ retains the same form as before, but the kinetic term of the Lagrangian loses its invariance due to the appearance of "extra" terms in the associated transformation of the derivative:

$$\partial_\mu\phi_i(x) \to [\partial_\mu\phi_i(x) - igQ_i\phi_i(x)\partial_\mu\lambda(x)]e^{-igQ_i\lambda(x)} \tag{1.5}$$

In order to restore invariance to the Lagrangian under this local phase rotation of $\phi_i(x)$, necessity implies that terms must be added to the Lagrangian, but in such a way that the transformation of these terms under the symmetry group cancel out the unwanted "extra" terms already present. As the transformation of the derivative

‡In $U(1)$ theory, only one generator Q_i exists, and so the fields are often rescaled to absorb the coupling constant present in the phase rotation. This coupling constant then only appears in the normalisation of the kinetic term of the vector gauge field. This has not been done here, in order to emphasize the generality of the procedure to any gauge group.

generates "extra" terms that carry a vector index, a vector field must be introduced into the theory in order to cancel, or "gauge away" the unwanted terms in the Lagrangian. Restoration of the invariance of the Lagrangian under the symmetry group then requires that the vector gauge field A_μ (or set of gauge fields, if the symmetry group is larger than $U(1)$) must transform as

$$A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \lambda(x) \tag{1.6}$$

To guarantee invariance of the kinetic term under this local gauge transformation, it is also necessary to redefine the derivative to allow for the presence of the gauge field. The "new" covariant derivative is

$$D_\mu \phi_i(x) = [\partial_\mu + igQ_i A_\mu(x)]\phi_i(x) \tag{1.7}$$

Thus, once the symmetry (gauge) group is specified by means of the transformation on $\phi_i(x)$, the introduction of the gauge field and a redefinition of the derivative combine to give the necessary cancellation of the troublesome "extra" terms, thereby returning the invariance under the internal symmetry group to the kinetic term. However, consistency implies that one must also add kinetic terms for the gauge field to the Lagrangian, which is done by adding the term

$$L_{Kin}^{A_\mu} = -\frac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x) \tag{1.8}$$

where the field strength $F_{\mu\nu}$ is given by

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) \tag{1.9}$$

This then results in a consistent self-contained theory, as the field strength is itself invariant under the symmetry group due to the transformation given by equation (1.6), and so the entire Lagrangian is invariant[‡].

[‡]It should be noted that the discussion presented has only been for scalar fields, but that an equivalent argument applies for fermionic fields[5].

In summary, we see that the underlying internal symmetry by means of local gauge invariance, has resulted in the introduction of a massless vector boson (gauge boson), and subsequently, introduced new terms in the Lagrangian. For the case at hand, equation (1.7) shows that this vector field couples to charge, so the natural identification for the massless $A_\mu(x)$ would be that of the photon. The effect of demanding a local gauge symmetry has been that it fixes the dynamical behaviour of the theory; this is the magic of gauge theories!

1.1.3 If It Ain't Fixed, Bust It!

With gauge theories under our belt, we feel ready as physicists to take on the particle physical world. This feeling soon subsides however, as it becomes obvious that gauge theories themselves are not sufficient - the reason being that the gauge symmetries are exact and the associated gauge bosons are all massless, in contrast to the observed particle spectrum. As far as is known, the subatomic world is governed by the strong, the weak, and the electromagnetic force‡, and so three different types of force mediators (gauge bosons) are expected[5]. If the gauge symmetries were all exact, then all of the gauge bosons would be massless. Such a conclusion is completely unsatisfactory, as the weak interaction requires rather massive gauge bosons to mediate the weak force in order to explain the short range of the weak force. (Note the gauge boson mediating the electromagnetic force is of course the photon, which is massless, thereby explaining the infinite range of the electromagnetic interaction). To add further insult to injury, explicit addition of mass terms in the Lagrangian for the observed chiral fermions destroys the invariance under the gauge group[5], so suggesting that an exact gauge symmetry is synonymous with massless fermions - this

‡Gravity is only significant at length scales of order the Planck scale, and so it is unnecessary to try an attempt to include it within the particle physics model - at least at this stage.

is a rather unpalatable thought for any particle phenomenologist. What is needed is a mass generating mechanism within the context of a gauge theory.

Such a mass generating mechanism is indeed possible within the confines of a gauge theory, if the theory is constructed so that the gauge symmetry is an exact symmetry of the theory, but not a symmetry of the vacuum. Clearly, this symmetry "breaking" cannot be done by a simple explicit symmetry breaking. In order to have the symmetry "hidden" in the vacuum while maintained in the full theory, a particular type of symmetry breaking must be invoked. The required breaking scheme necessary for a mass generating mechanism compatible with the gauge theory movement is that of spontaneous symmetry breaking. Yet as will be seen, the spontaneous symmetry breaking of a continuous global symmetry leads to the generation of massless Nambu-Goldstone bosons[6], which are by themselves, insufficient for mass generation. It was only with the application of spontaneous symmetry breaking to local gauge theories[2], by Anderson, Higgs, Brout and Englert, Guralnick, Hagen, and Kibble, that the massless Nambu-Goldstone bosons produced by the spontaneous symmetry breaking of the gauge symmetry were shown to be absorbed in the theory by means of supplying the longitudinal components to the gauge fields. The generation of longitudinal components for vector gauge fields imply that these fields are massive, and so the spontaneous symmetry breaking of a local gauge symmetry results in a mass generation mechanism. As will be seen, this mechanism, known as the Higgs mechanism, has dramatic consequences.

Spontaneous symmetry breaking occurs when the Lagrangian retains its invariance under the symmetry group, but the vacuum state (ground state of the system) does not exhibit the same invariance - the symmetry is hidden, as the vacuum state breaks the symmetry. To understand how this is achieved, we return to the case of a single complex scalar field ($\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$), and the $U(1)$ phase rotation invariant

Lagrangian given by equation (1.4), with $V(\phi, \phi^\dagger) = \mu^2|\phi|^2 + \Lambda|\phi|^4$. The vacuum state of the system, ϕ_0 then corresponds to the solution of the field equation for ϕ, and for this system the vacuum state must satisfy

$$\phi_0(\mu^2 + 2\Lambda|\phi_0|^2) = 0 \tag{1.10}$$

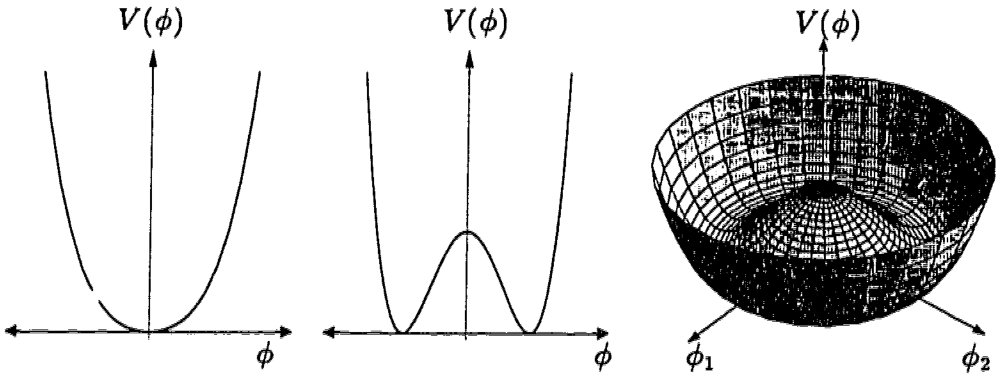
This condition offers two possibilities; either both μ^2 and Λ are positive, or μ^2 is negative while Λ remains positive (Λ cannot be negative as the potential must be bounded below.). The former case implies that there is only one solution, that of conventional vacuum with $\phi_0 = 0$, while the latter case permits a second solution;

$$|\phi_0|^2 = -\frac{\mu^2}{2\Lambda} \equiv \frac{v^2}{2} \qquad \phi_0 \equiv <\phi> = \frac{1}{\sqrt{2}} v e^{i\zeta} \tag{1.11}$$

This latter solution implies that the true vacuum corresponds to a non-zero vacuum expectation value (vev) for ϕ. In the two dimensional space of real components of the complex scalar field, the spontaneous symmetry breaking potential is the famous "Mexican Hat" potential, and the vev ($<\phi>$) is a circle of minima of radius $\frac{v}{\sqrt{2}}$ centred on the origin. Figure 1.1 shows the potential for both solutions to the spontaneous symmetry breaking condition, as well as a sketch of the 3-dimensional "Mexican Hat" potential.

Initially, one may find this second solution a little disturbing, as the scalar field appears to have an unphysical negative mass. Actually, the physical masses are positive semi-definite, and the apparent negative mass is an artifact of the expansion of ϕ around something other than its vev. However, this situation can be remedied by means of a field redefinition of ϕ, where ϕ is now expanded around its vev. The form of this expansion is

$$\phi = \frac{1}{\sqrt{2}}(\rho(x) + v)e^{i[\alpha + \frac{\xi(x)}{v}]} \tag{1.12}$$



(a) (b) (c)

Figure 1.1: Various variations of the $U(1)$ invariant Higgs potentials: (a) The conventional vacuum Higgs potential with $<\phi>=0$, (b) the spontaneous symmetry breaking Higgs potential with $<\phi>\neq 0$, and (c) the Mexican Hat Higgs potential.

with $\rho(x)$ and $\xi(x)$ being real fields. The effect of this field redefinition on the Lagrangian is to cause it to dramatically alter its form;

$$L=\frac{1}{2}\partial_\mu\rho\partial^\mu\rho+\frac{1}{2}\frac{(\rho+v)^2}{v^2}\partial_\mu\xi\partial^\mu\xi+\mu^2\rho^2-\Lambda v\rho^3-\frac{\Lambda}{4}\rho^4-\frac{\mu^2v^2}{4} \tag{1.13}$$

The justification for the field redefinition becomes apparent. Equation (1.13) is now the Lagrangian for two real scalar fields, one of which $(\rho(x))$ has a positive mass while the other $(\xi(x))$ is massless. Both are physically acceptable fields. Further, this Lagrangian does not explicitly exhibit the original invariance under phase rotations (due to the expansion of ϕ around its vev), yet by its very construction the Lagrangian still possesses the underlying internal symmetry of the gauge group. Because of this, the symmetry has been spontaneously broken, and resulting in the appearance of a massless field (Nambu-Goldstone boson[6]), which is free to move along the minima of the "Mexican Hat" potential.

In general, for a gauge group G, of dimension N, spontaneous symmetry breakdown of G to the subgroup H, of dimension M, results in $N - M$ massless Nambu-Goldstone bosons. These $N - M$ massless bosons correspond to the $N - M$ broken generators of the coset space $\frac{G}{H}$.

Now we are in a position to understand the Higgs mechanism - namely the application of spontaneous symmetry breaking to a local gauge group. Again consider the complex scalar field, but this time require it to be invariant under local gauge transformation, as well as having the "Mexican Hat" potential. From the above discussion, the Lagrangian takes the form

$$L = \frac{-1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger D^\mu \phi - \mu^2 |\phi|^2 - \Lambda |\phi|^4 \tag{1.14}$$

with the covariant derivative and field strength defined as in equations (1.7) and (1.9). If μ^2 is negative, then spontaneous symmetry breaking is induced, and a non-zero vev for ϕ results. This in turn leads to an expansion of ϕ around the vev in order to recover physical scalar fields, however the expansion is a little more complicated due to the presence of the covariant derivative. Specifically, the scalar field kinetic term is reparameterised as

$$\begin{aligned} (D_\mu \phi)^\dagger D^\mu \phi &= \frac{1}{2} \partial_\mu \rho \partial^\mu \rho + \frac{1}{2} \frac{(\rho + v)^2}{v^2} (\partial_\mu \xi)^\dagger \partial^\mu \xi \\ &\quad - g \frac{(\rho + v)^2}{v} A^\mu \partial_\mu \xi + \frac{1}{2} g^2 (\rho + v)^2 A^\mu A_\mu \end{aligned} \tag{1.15}$$

Again the field redefinition has generated mass, but this time for the vector gauge field $A_\mu(x)$! An additional term proportional to $A^\mu \partial_\mu$ has also been generated, which does not correspond to a standard interaction. However, by means of a field redefinition of the gauge field, one can introduce a massive field $B_\mu(x)$ such that

$$B_\mu(x) = A_\mu(x) + \frac{1}{gv} \partial_\mu \xi(x) \tag{1.16}$$

which restores the Lagrangian to a more conventional form, namely

$$\begin{aligned} L &= \frac{-1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{2}m_B^2 B^\mu B_\mu + \frac{g^2}{2}(\rho^2 + 2\rho v)B^\mu B_\mu \\ &+\frac{1}{2}\partial_\mu\rho\partial^\mu\rho - \frac{1}{2}m_\rho^2\rho^2 - \frac{\Lambda}{4}\rho^4 - \Lambda v\rho^3 \end{aligned} \tag{1.17}$$

All trace of the Nambu Goldstone boson field $\xi(x)$ has been removed, and instead of a massless gauge field $A_\mu(x)$ the theory contains a massive vector gauge field $B_\mu(x)$, as well as the massive scalar $\rho(x)$. The masses are $m_B = gv$ and $m_\rho = \sqrt{-2\mu^2}$ respectively.

What has happened is that the spontaneous symmetry breaking has caused the original massless gauge field $A_\mu(x)$ to absorb the Nambu-Goldstone boson $\xi(x)$, thereby gaining the additional longitudinal degree of freedom that is necessary for a massive vector gauge field. Colloquially, the massless $A_\mu(x)$ has "eaten" the scalar field $\xi(x)$, and in doing so has become the massive vector gauge field $B_\mu(x)$.

Thus, by means of spontaneous breaking of a local gauge symmetry (aka the Higgs mechanism), gauge bosons can be given mass[‡]. This procedure also solves the problem of fermion mass generation, as the general gauge invariant coupling of fermions to a Higgs field is in the form of a Yukawa coupling ($\lambda_Y f\phi f$ where f corresponds to a fermion field, λ_Y is a coupling constant, and the group structure of the interaction has been ignored for the moment). When ϕ takes its vev, $\phi \rightarrow <\phi>$, and the Yukawa interaction reduces to an effective mass term. Further, as a by-product of this symmetry breaking procedure, the scalar field $\rho(x)$, itself attains a mass. (Corresponding to excitations about the potential well centred on the vev.) Within the context of the Standard Model[7], this massive scalar is of course the infamous Higgs particle - the one remaining Standard Model particle still to be discovered, and the one particle that can offer confirmation of this mass generation mechanism.

[‡]For a renormalisable field theory with massive gauge bosons, 't Hooft showed that these theories must be spontaneously broken Yang-Mills gauge theories[1]

1.1.4 The Best Gauge Theory We Have

Having established the power of the gauge principle, and that the Higgs mechanism is a consistent mass generation mechanism, a return to the problem of describing the subatomic zoo is in order. Such a discussion constitutes the "coming of age of the gauge", and of course deals with the Yang-Mills gauge theories[8] of QCD[9] and Glashow, Salam, and Weinberg's unified electroweak model[10]. These combine to give a model known as the Standard Model of particle physics, and is at present, the most accurate model of the particle physics world that we have.

The Standard Model is built up from three gauge symmetries, corresponding to the three fundamental forces relevant to particle physics, namely the strong, the weak, and the electromagnetic force. The Standard Model gauge group is the product group $SU(3) \times SU(2) \times U(1)$, with the $SU(3)$, $SU(2)$, and $U(1)$ component groups being associated with the colour, weak, and hypercharge symmetries[11]. The corresponding gauge bosons are the massless gluons of QCD, the massive W^+, W^- and Z of the weak interaction, and the massless photon of electromagnetism. As the weak gauge bosons are massive, a Higgs field is necessary to induce spontaneous symmetry breaking, and as the gauge group for the weak interaction is $SU(2)$, the simplest scenario for the Higgs field is that it be an $SU(2)$ doublet. Lastly, there are the fermion representations - these depend on the gauge group to which particular fermions couple. Starting with the quarks, their interactions are mediated by the gluons of QCD, implying that they are labelled by the colour quantum number of QCD. This in turn implies that quarks must transform as a fundamental representation of $SU(3)$ (a colour triplet). As $SU(3)$ is a non-Abelian gauge theory, the coupling will only become small at large momentum scales (this is the property of asymptotic freedom[12]), so that the only physical quark states are colour singlet baryon (qqq) and meson ($q\bar{q}$) states (ie colour confinement[13]). For the electroweak ($SU(2)\times U(1)$) section of the theory, the fermion assignment is based on the observation that the charged $SU(2)$ gauge bosons mediate the interaction between charged leptons and their associated (left-handed) neutrinos[7]. Thus, the left-handed states must transform non-trivially under $SU(2)$, while the right handed states are $SU(2)$ singlets. The simplest scenario that works for the electroweak assignments is that of left-handed doublets and right handed singlets. Also, as the quarks carry charge, they also must fit into the $SU(2)\times U(1)$ electroweak framework, and as the observed hadronic charged weak currents are left-handed, the representation structure follows that of the leptonic sector. Thus, the Standard Model fermions have electroweak representations given by

$$\begin{array}{cccccc} \begin{bmatrix} \nu_e \\ e^- \end{bmatrix}_L & \begin{bmatrix} \nu_\mu \\ \mu^- \end{bmatrix}_L & \begin{bmatrix} \nu_\tau \\ \tau^- \end{bmatrix}_L & e_R & \mu_R & \tau_R \\ \begin{bmatrix} u^i \\ d^i \end{bmatrix}_L & \begin{bmatrix} c^i \\ s^i \end{bmatrix}_L & \begin{bmatrix} t^i \\ b^i \end{bmatrix}_L & u^i_R, d^i_R & c^i_R, s^i_R & t^i_R, b^i_R \end{array} \tag{1.18}$$

where $i = 1, 2, 3$ is an $SU(3)$ colour index. Note, in the Standard Model there is no provision for right handed neutrinos, and so the left handed neutrinos are taken to be massless. Unfortunately, the Standard Model makes no attempt to explain the observed three-family structure of quarks and leptons.

This chiral structure of the fermion sector reinforces the idea that fermion masses are generated by Yukawa couplings to the Higgs field, as any explicit fermion mass term would mix left and right handed components, thereby destroying gauge invariance. Further, due to the doublet structure of the left-handed fermions, the Higgs field must be an $SU(2)$ doublet (as suggested by the $W^\pm$ and Z being massive), in order to produce a Yukawa interaction term that is a singlet under the full gauge group ($L_{Yuk} \sim \lambda^{ab}_Y \bar{F}_a H f_b$, where F and f are a fermion doublet and singlet respectively, and a and b are family indices.). A full description of the Standard

Model Lagrangian is then obtained by application of the gauge principle to the gauge group $SU(3) \times SU(2) \times U(1)$ with the particle content as outlined above; a detailed description of the resulting Lagrangian is inappropriate for this introduction, but the interested reader is referred to some excellent texts [5, 7]. As a final comment on the Standard Model, it should be noted that while it still leaves some unanswered questions, it is the most well-tested theory that physics has produced, and the degree with which predictions match with experimental verification is nothing short of remarkable.

1.2 Bigger Things

Unfortunately, the Standard Model does have its limitations, which suggest that the Standard Model may not be the whole picture. Some of these short-falls are: The model is inflicted with too many unspecified parameters (19 at minimalist counting), the $U(1)$ charge quantization is left unexplained, there is no explanation as to the general features of the fermion mass spectrum nor the overall 3-family structure of the model, and there is no real justification as to why neutrinos should be massless. What these unexplained problems are telling us is that the Standard Model has to be viewed as a low energy effective theory, and that at some higher energy scale, a deeper underlying theory should emerge that will resolve at least some of these issues. Of course there may be more than one layer of effective theory, and we may have to bootstrap ourselves up one underlying theory at a time[‡]. Nevertheless, the desirable course of action is to construct a higher level theory, valid up to some higher energy scale, that defaults to the Standard Model in the low energy limit. To do so, one

[‡]The boostrap process may go something like Standard Model → super GUTs → superstrings → ...

is faced with two possible paths. Either the Standard Model fields are composite or they are not. If one takes the composite field approach[14], then the particles of both the scalar and fermion sectors can easily be thought of as composite particles, and the preons (constituents of quarks and leptons) offer the possibility of solving the family structure of the Standard Model, just as quarks provided a way to understand the multitude of hadronic states observed. Yet observation requires that compositeness occur at length scales above 1 TeV, while quark and lepton masses are well below this scale. This implies that the theory must be such that composite quarks and leptons stay light, yet this is rather difficult to arrange, given that the low energy limit has to match with the Standard Model. Thus, this route will not be considered, as a full discussion of such compositeness is outside the scope of this thesis. Instead, if Standard Model fields are taken to be fundamental, the challenge is to produce a suitable extension to the Standard Model, all the while staying within the comfortable confines of gauge theory. This approach results in the development of Grand Unified Theories[15].

1.2.1 Grand Unified Theories

A Grand Unified Theory (GUT) is one in which the gauge group has been enlarged to a single simple group G (or the product of two identical groups, so that only one coupling constant is necessary), so that at some high energy scale M_G the Standard Model forces are unified under this gauge symmetry G. The Standard Model is then expected to appear through a spontaneous symmetry breaking pattern of G down to $SU(3) \times SU(2) \times U(1)$. Further, if the GUT is such that all the fundamental fields of a given spin fit into irreducible representations of G, then their interactions are also governed by the gauge group symmetry. With these expectations of a GUT, it is then a matter for the physicist to construct an appropriate GUT model, and thanks to the machinery of gauge theories, such constructions follow a straight-forward recipe.

This GUT recipe is as follows:

- Pick a gauge group G, such that $SU(3) \times SU(2) \times U(1)$ is contained within G. Local gauge invariance then specifies the spin 1 gauge bosons of the theory.

- Prescribe the fermions of the theory to representations of G. Again, local gauge invariance acts to specify the nature of the coupling of these fermions to the gauge bosons. The only constraint on the assignment of the fermion content to representations of G is that in the low energy limit, the $SU(3) \times SU(2) \times U(1)$ fermion structure should emerge. This implies that the fermion representations must be complex in order to accommodate the chiral structure of the Standard Model‡.

- Arrange the scalar sector so that the pattern of spontaneous symmetry breaking takes the gauge group G down to $SU(3) \times SU(2) \times U(1)$. Again, the scalar content must fit into representations of the gauge group G in order to maintain local gauge invariance.

- Finally, specify the Yukawa couplings, so that after spontaneous symmetry breaking from G down to $SU(3) \times SU(2) \times U(1)$, the fermion mass spectrum matches the observed Standard Model spectrum.

‡Real representations are possible for the fermion assignment, but only at the cost of introducing "mirror" fermions[15] - which then requires that these mirror fermions be consistent within the context of the low energy theory. This turns out to be very difficult to arrange, and so this possibility is not considered here.

Application of this procedure results in a phenomenological model that is a candidate for a GUT based Standard Model extension; whether the model is a valid/reasonable description of the high energy theory depends on the degree of accuracy of its low energy predictions.

1.2.2 Leading By Example

In order to understand the implications of GUTs, it is perhaps best to consider the prototypical GUT; $SU(5)$[16]. $SU(5)$ is the natural candidate for an extension of the gauge group, as its rank (number of diagonal generators and hence the number of conserved quantum numbers the symmetry permits) is the same as $SU(3) \times SU(2) \times U(1)$[‡], it contains $SU(3) \times SU(2) \times U(1)$, and it has complex representations for the fermion assignments. There are 24 vector gauge bosons corresponding to the $N^2 - 1|_{N=5} = 24$ generators of the adjoint representation of $SU(5)$, and in unbroken $SU(5)$ these gauge bosons are massless. As to the fermion representations, the GUT structure requires that all fermions within a representation be of a single helicity, which suggests that it is best to deal with left-handed fermions and write the right-handed fermions as left-handed charge conjugate spinors. The small dimensional representations of $SU(5)$ that are available for assignment of the fermions are the singlet, $\underline{1}$, the fundamental representation, $\underline{5}$ and the antisymmetric product of two fundamental representations, $\underline{10}$. Given these representations, a single family of the Standard Model fermion content is found to fit into $SU(5)$ representations in the

[‡] $SU(3) \times SU(2) \times U(1)$ has 4 diagonal generators, but the $U(1)$ generator is trivially diagonal, and so is usually overlooked. This is especially true when one uses Coxeter-Dynkin diagram notation to aid in gauge group selection. For example, in Dynkin diagram notation, $SU(5)$ is ○–○–○–○, which contains the ○ – ○   ○ of $SU(3) \times SU(2) \times U(1)$.

combination $\underline{\bar{5}} \oplus \underline{10}$, with the specific assignment of a single family being

$$\underline{\bar{5}} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ l \\ -\nu \end{bmatrix}_L \qquad \underline{10} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & U_3 & -U_2 & -u^1 & -d^1 \\ -U_3 & 0 & U_1 & -u^2 & -d^2 \\ U_2 & -U_1 & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -L \\ d^1 & d^2 & d^3 & L & 0 \end{bmatrix}_L \tag{1.19}$$

where U_i, D_i and L_i are the charge conjugations of the right handed SU(2) singlet up, down, and charged lepton fields.

This fermion assignment immediately starts to reveal some of the appealing aspects of GUT models. In particular, as the Standard Model is embedded in $SU(5)$, the photon is a gauge boson of $SU(5)$, and so the charge operator Q, must be associated with one of the 4 traceless generators of $SU(5)$. This implies a traceless condition on the fermion multiplets, and for the $\underline{5}$ this gives the condition

$$0 = 3Q_q + Q_{l^c} + Q_{\nu^c} \tag{1.20}$$

Not only does this imply that the $U(1)$ charge is now quantised, but also that the down quarks carry one third of the electron charge (similarly for the other families). In addition, the left-handed $SU(2)$ lepton doublet must be combined with the left-handed charge conjugates of the right-handed $SU(2)$ quark singlets in order to maintain this trace condition, which means that $SU(5)$ predicts the right-handed quarks to be $SU(2)$ singlets. Similar constraints also exist for the $\underline{10}$, thereby determining the above assignment. Furthermore, the $\underline{\bar{5}} \oplus \underline{10}$ assignment results in an anomaly free theory[‡].

[‡]An anomaly in a theory occurs when an apparent symmetry at the classical (or tree) level is not respected by radiative corrections, and as such anomaly cancellation is critical for the renomalisability of a theory. The simplest example of an anomaly is that of the triangle anomaly in QED, which is a three fermion loop with two vector and one axial coupling to three external gauge boson legs.

The next step is to fix the scalar sector, and here the requirement is that it breaks $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$, followed by the electroweak breaking $SU(2) \times U(1) \rightarrow U(1)_{EM}$. In the first symmetry breaking from $SU(5)$ directly to $SU(3) \times SU(2) \times U(1)$, 12 of the 24 gauge bosons acquire masses that are of order the GUT breaking scale M_G, leaving the 12 remaining gauge bosons associated with the Standard Model massless. The second spontaneous symmetry breaking is that of the electroweak theory, which generates masses for the W^+, W^-, and Z. The simplest Higgs representation is of course, a Higgs $\underline{5}$, and due to the fermion assignments, this 5-plet is composed of an $SU(3)$ triplet and an $SU(2)$ doublet. The $SU(2)$ doublet component of this Higgs field is exactly what is required to facilitate the spontaneous electroweak symmetry breaking, and so can be identified with the standard Standard Model Higgs doublet. Alas, this 5-plet is not suitable for inducing the first symmetry breaking stage, as this would imply the unfavourable situation that $M_W \sim M_G$. Instead, it is necessary to extend the Higgs sector by introducing an adjoint representation (the $\underline{24}$ representation). Inclusion of this $\underline{24}$ not only breaks $SU(5)$ and thereby generates M_G scale masses for both the gauge bosons and the Higgs scalars of the $\underline{24}$ itself, but the cross coupling between the $\underline{24}$ and $\underline{5}$ of Higgs scalars result in contributions to the masses of both the colour triplet and weak doublet Higgs fields of the $\underline{5}$ that are of order M_G. Unfortunately, this is not desirable, as the mass of the doublet in the 5-plet must be of order to M_W if electroweak breaking is to proceed as desired. For a typical GUT M_W is 12 orders of magnitude smaller than M_G. While it is possible to arrange a cancellation in Higgs doublet mass term contributions so that its mass is of the M_W scale, such a cancellation within the context of GUTs is rather extreme and unjustified, and at best, must be considered fine tuning. This necessity of fine tuning is known as the Hierarchy problem[17, 15].

Despite this problem, the minimal $SU(5)$ GUT offers some significant predictions. Perhaps the two most pertinent predictions are that of coupling unification and nucleon decay!

As the Standard Model coupling constants depend on the energy scale at which they are measured (the dependence is introduced via radiative corrections), the action of the renormalisation group equations is to cause these coupling constants to flow toward a unification point as the energy scale is increased. This is exactly what is expected within the GUT formalism, and from initial measurements of the coupling constants made at the 1 GeV scale, the couplings appeared to unify, with the unification occurring at an energy scale of the order of $M_G \simeq 10^{15}$ GeV[18]. Subsequent precision measurements have shown that coupling unification under the GUT formalism doesn't actually work, but this is remedied by the inclusion of supersymmetry. Supersymmetry is to be discussed in the following section.

With the fermion assignments in a GUT placing quarks and leptons in the same multiplet, it is automatic that the gauge bosons mediate interactions between quarks and leptons. Further, as $SU(5)$ has both the $\underline{\bar{5}}$ and $\underline{10}$ fermion representations, this 4-fermion interaction mediated by the heavy gauge boson exchange can result in effective interactions that violate both the baryon quantum number B, and the lepton quantum number L (but not the combination $B-L$). This implies that within the context of a GUT, nucleon decay is permitted, which is a radical departure from the Standard Model, where the proton is considered a stable baryon. However, in the low energy limit of the GUT, these nucleon decay inducing quark-quark-quark-lepton effective vertices are weighted by two inverse powers of the GUT breaking scale M_G (this weighting comes from the low energy limit of the gauge boson propagator), and so are heavily suppressed. With $M_G \simeq 10^{15}$ GeV, the proton lifetime predictions obtained from these standard GUT nucleon decay channels, are too small compared with the experimental lower bound of $\tau_p > 5.5 \times 10^{32}$ years for the partial lifetime of the dominant $p \rightarrow \pi^0 + e^+$ mode[19].

1.2.3 Variations

As has been mentioned, the non-supersymmetric $SU(5)$ GUT is problematic. It appears to answer a number of questions, but either fails or partially answers others, as well as raising a few new questions. Yet the $SU(5)$ GUT is only a prototype, and a variety of GUT models can be constructed that attempt to improve on the minimal $SU(5)$ GUT. Typical variations involve either (or both) an extension of the Higgs sector (in an attempt to give accurate predictions of the Standard Model fermion mass spectrum generated from the Yukawa interaction), or an expansion of the gauge group to a higher rank. The work to be presented in chapter 4 is in fact based on a supersymmetric version of one such variation - a non minimal $SO(10)$ GUT model. The $SO(10)$ gauge group is perhaps an ideal GUT candidate, as $SU(5)$ and hence $SU(3) \times SU(2) \times U(1)$ are contained within it, it is rank 5 and so the quantum number $B - L$ can be associated with a generator of the group rather than a residual symmetry (as in $SU(5)$), it is automatically anomaly free, and the spinorial representation is $2^4 = 16$ dimensional so that an entire family of Standard Model fermions plus a right handed neutrino can be assigned to the 16-plet. (The $SU(5)$ decomposition of the $SO(10)$ $\underline{16}$ is $\underline{16} \rightarrow \underline{10} \oplus \underline{\bar{5}} \oplus \underline{1}$. The singlet in this decomposition is ideal for identification with the right-handed neutrino, and can be taken as being very massive‡.) However, these variations do not address issues such as the Hierarchy problem or the failure of the coupling constant unification in the GUT

‡The inclusion of the right-handed neutrino gives the possibility that the left-handed neutrinos are indeed massive, but very light as a result of the neutrino see-saw mass mechanism[7, 15].

scheme. These issues require either that there is an increase in the influence of non-perturbative physics at high energy scales, or that a further symmetry is imposed. In this thesis the second option is adopted.

1.3 The Politically Correct Symmetry

In order not to extend the gauge group, imposition of a new symmetry requires that the symmetry itself not be of the local gauge variety, but rather some other internal symmetry residing within the theory. As GUTs compartmentalise the particle content in terms of particle spin, the obvious choice for such a symmetry is one that relates particles of different spin. In particular, this spin symmetry would relate fermions to bosons, and vice versa. Such a symmetry does indeed exist[20], and it is called supersymmetry. As the algebra of supersymmetry can be rather laborious, we restrict ourselves to a colloquial discussion of this symmetry, which is adequate for discussing the relevance of supersymmetry to GUTs, and suitable preparation for chapter 4. (A full description of the formalities of supersymmetry can be found in a number of texts and papers [21].)

Supersymmetry, simply put, follows the maxim

For every HE there is a SHE, where SHE is a super-HE‡.

As this maxim implies, for every particle in the theory, simple (N=1) supersymmetry requires there be a partner particle (sparticle) that is identical in mass, charge, and all relevant quantum numbers, except that it differs in spin by half a unit of spin. This results in a doubling of the particle content of the theory, so that in addition to the

‡Attributed to Bruce Campbell

vector gauge bosons, fermions and Higgs particles, one has the superpartner gauginos, sfermions, and Higgsinos. The gauginos are taken to be spin $\frac{1}{2}$ (spin $\frac{3}{2}$ sparticles are partnered with the spin 2 graviton of supergravity), but alas, the fermions of the model cannot acts as gauginos. The reason for this is that as the gauginos must fit into the adjoint representation in the same way the gauge bosons do, while this is not possible for the chiral representations of the fermions. Likewise the sfermions (squarks and sleptons) cannot be spin 1, as all the spin 1 particles must be gauge bosons if the theory is to be renormalisable, and so having spin 1 sfermions would entail an enlargement of the gauge group. Instead, the sfermions are taken to be spin 0 and placed in a chiral supermultiplet with their partner fermions. The only choice for the Higgsinos is that they be spin $\frac{1}{2}$, and one may be tempted to equate the Higgsinos with some of the quarks and leptons of the theory. Unfortunately this fails to work, as it does not generate acceptable masses for the Standard Model fermions, implying that Higgs particles have "new" and distinct Higgsino fermion partners.

Given the supermultipet structure of a supersymmetric particle model (vector supermultiplets composed of gauge bosons and their partner gauginos, and chiral supermultiplets composed of scalars and their partner fermions), the potential and Yukawa interaction terms of the Lagrangian can be easily generated by means of a superpotential[‡]. The definition of these interaction terms in terms of the superpotential, W, is

$$L_{\text{int}} = \frac{1}{2}\sum_{i,j}\left(\frac{\partial^2 W}{\partial\Phi_i\partial\Phi_j}|_{\Phi=\phi}\psi_i\psi_j + h.c.\right) - \sum_i \left|\frac{\partial W}{\partial\Phi_i}\right|^2_{\Phi=\phi} \tag{1.21}$$

with Φ represents a chiral superfield, and ϕ and ψ the scalar and fermionic components of the Φ. However, the key feature to this superpotential is that due to the requirement of invariance under supersymmetry, the superpotential W can only be composed from chiral superfields with the same chirality - that is, the superpotential

[‡]This algebraic detour is necessary as a particular superpotential is considered in chapter 4.

W must have the functional form $W(\Phi)$ but not $W(\Phi, \Phi^\dagger)$. Also, renormalisability constrains the general form of the superpotential to being at most cubic in chiral superfields[15]. Thus, the most general form for $W(\Phi)$ is

$$W(\Phi) = \lambda_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k \tag{1.22}$$

From this, one deduces that the superpotential, being rather simple in form, can play a key role in the evaluation of the physical properties of a supersymmetric model - especially one where Yukawa couplings and fermion masses are a central issue - as will be seen in chapter 4.

Another interesting feature of a theory with exact supersymmetry is that the masses of scalars are not renormalised when one ventures beyond the tree level, as the associated radiative corrections are cancelled to all orders of perturbation theory. Such cancellation is due to a non-renormalisation theorem[22], which states that in perturbation theory, no radiative corrections can be generated that would result in corrections to the superpotential. Technically, no F-terms are generated at any order of perturbation theory above the tree level: F-terms are terms that are constructed from superfields of a single chirality. An understanding of how this theorem works is perhaps best obtained by considering the cancellation of the quadratically divergent 1-loop corrections to a Higgs field propagator in the presence of exact supersymmetry. As shown in Figure 1.2, the 1-loop corrections involve an internal fermion and sfermion loop, with the fermion and its fermionic partner being degenerate in mass (supersymmetry unbroken), and so the amplitude of these two 1-loop corrections are found to be identical in magnitude. However, from field theory it is well known that the presence of a closed fermion loop results in an additional factor of -1, and it is this factor that causes the exact cancellation of these two 1-loop radiative corrections. This reasoning applies to all orders of perturbation theory, and so implies that no radiative corrections, and hence no corrections to the superpotential, are induced.

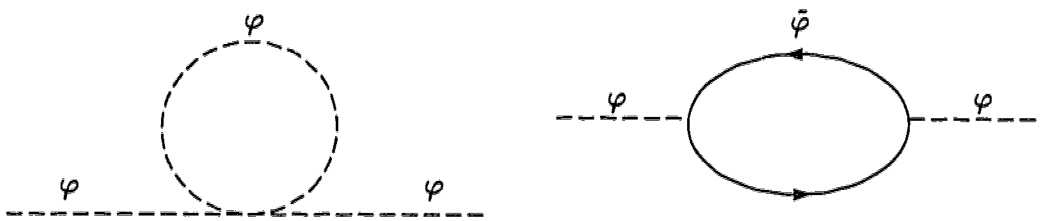


Figure 1.2: The cancelling 1-loop corrections to a Higgs propagator in exact supersymmetry.

Finally, when considering supersymmetric particles, the question that naturally arises is "Where are they?". If supersymmetry were an exact symmetry, the superpartner particles would be degenerate with their particle partners, which is in clear contradiction to the observed particle zoo. This suggests that at some scale, supersymmetry must be broken, either explicitly or spontaneously, thereby removing the mass degeneracy. There are numerous ways in which to break supersymmetry[15], but as will be seen in the next subsection, certain model independent consequences can be ascertained without going into the detailed mechanics of the supersymmetry breaking mechanism. With supersymmetry broken, it is then possible that the supersymmetric particles may themselves decay into lighter non-supersymmetric particles, resulting in no supersymmetric particles in the low energy limit. Also, nucleon decay can, in general, occur via renormalisable interactions generated by the gauge invariant superpotential. These somewhat unpalatable consequences are easily rectified by the imposition of an additional discrete symmetry called R-parity[23], which assigns a multiplicative quantum number to each particle/sparticle state. In fact, while the imposition of R-parity is not essential in a supersymmetric theory, it was precisely the prohibiting of the dimension 4 proton decay operators which are unsuppressed by any inverse powers of M_G, and hence so deadly for proton lifetime predictions that motivated its introduction. Further, by definition, every non-supersymmetric particle is assigned an R-parity value of $+1$, and each supersymmetric particle a value of -1, so that if R-parity is an exact symmetry, the R-parity quantum number is conserved. R-parity conservation implies that a supersymmetric particle cannot decay into purely conventional non-supersymmetric particle states, and conversely conventional non-supersymmetric states cannot produce an odd number of supersymmetric states. Thus, demanding R-parity conservation guarantees that a lightest supersymmetric particle (LSP) must exist in the low energy limit of the theory.

1.3.1 What SUSY Can Do For You

With the properties of supersymmetry established, attention must now turn to what the inclusion of supersymmetry can do for GUT models.

One of the primary features of a supersymmetric GUT scheme is that it offers a solution to the Hierarchy problem. From earlier discussion, it was seen that the structure of the GUT scheme implied that there was nothing, apart from "unnatural" fine tuning of the theory, to prevent the light Standard Model Higgs doublet from becoming extremely heavy (masses $\sim M_G$) as a result of radiative corrections. The only "natural" way of preventing the Standard Model Higgs from becoming excessively heavy is to impose a symmetry that prohibits contributions to the Higgs mass from radiative corrections. Due to the existence of the non-renormalisation theorems, supersymmetry is exactly such a symmetry, and as far as is known, is the only such symmetry that provides such cancellation. Thus, supersymmetry provides a resolution to the Hierarchy problem. Yet as supersymmetry is not an exact symmetry in the low energy limit, the cancellation of radiative corrections to the Standard Model

Higgs doublet is not exact below the scale at which supersymmetry is broken. Further, the requirement that the Higgs doublet stay relatively light ($m_H < 1$TeV in order to maintain perturbative unitarity up to energy scales approaching the Planck scale[24].) implies that the mass splitting induced by supersymmetry breaking, for both the vector and chiral supermultiplets, also be of order 1 TeV. Such a result is of considerable interest, as it suggests that particles from the supersymmetric spectrum are on the verge of accessibility for current particle physics experiments.

Thus, the requirement of a light Higgs, which defines the electroweak breaking scale, is directly related to supersymmetry breaking and the supersymmetry breaking scale M_S. If supersymmetry is explicitly broken, then M_S typically is of order the electroweak breaking scale M_W, and the mass splitting of the chiral supermultiplets containing the Standard Model fermions is of order M_W. Unfortunately, explicit symmetry breaking is rather arbitrary, and so spontaneous symmetry breaking is often preferred. In contrast, spontaneous breaking of supersymmetry, such that the electroweak Higgs doublet and mass splitting of the supermultiplets are acceptably small, requires only a weak constraint on M_S. In fact, phenomenologically, a large value (of order 10^{11} GeV) is preferred for M_S[‡]. At such a breaking scale, gravity can not be neglected, and the influence of gravity on spontaneous supersymmetry breaking, the resulting supermultiplet mass splittings, and the Higgs doublet mass requires an extension to local supersymmetry (supergravity); unfortunately this topic is beyond the scope of this introduction.

Perhaps the most obvious feature that supersymmetry brings to the GUT scheme is that the "doubling of the particle content" alters the running of the Standard Model gauge coupling constants. As a supersymmetric GUT requires a light

[‡]Due to gravitational effects induced by the supergravity extension of the model[25].

Standard Model Higgs, and thus a relatively small mass splitting of the supermultiplets, the theory is populated with a plethora of light supersymmetric particles. This "doubling" of the possible radiative corrections causes a slowing in the evolution of the Standard Model coupling constants as they are renormalised up to higher energy scales. Fortunately, this slowing in the coupling constant flow results in an improvement in the projected unification of the gauge couplings - with the present precision measurements of coupling constants at 100 GeV, and their subsequent extrapolation via the Renormalisation Group equations, a gauge coupling unification is found to be consistent with current measurements. However, the price of this doubling of the particle content, and the subsequent slowing of the coupling constant flow, is that the unification scale is pushed to higher energies. For typical supersymmetric GUTs, this new unification scale is $M_G \simeq 3 \times 10^{16}$ GeV[26, 27].

A consequence of this increase in the unification scale is that the nucleon decay operators of standard GUT models, that are generated by means of gauge boson exchange, are suppressed due to this higher unification scale. (The SUSY GUT decay rate for these operators is typically suppressed by eight orders of magnitude over the non-SUSY GUT equivalent.) As a result, proton decay via these operators is pushed well beyond the present experimentally determined lower bound, making these decay channels unmeasurable by present standards. Yet one can still confront nucleon decay with experimental evidence, as the imposition of supersymmetry opens up new decay channels that were not possible in a conventional GUT scheme. In particular, fermion-fermion-sfermion-sfermion effective vertices that mediate nucleon decay are generated by means of Higgsino exchange, and as the Higgsino is fermionic, the low energy limit of its propagator is proportional to inverse mass, implying that these effective vertices are suppressed by $\frac{1}{M_G}$ and not $\frac{1}{M_G^2}$. Therefore, within the SUSY GUT structure, it is possible to have nucleon decay such that the lifetimes are comparable with the current experimental lower bounds. In chapter 4, such effective vertices, and their resulting phenomenological implications, are considered for a "realistic" SUSY GUT model.

One final point regarding SUSY GUTs is necessary, and this concerns the Higgs superfield sector. In order to build a "realistic" model, it is often the case that the Higgs sector requires extension. Such a practice is not new, as even in the case of the supersymmetric version of the Standard Model, it is necessary to introduce a second Higgs doublet superfield in order to generate both the up and the down quark masses.[‡]

1.4 Onwards

In this thesis, the reader will find three relatively self contained investigations dealing with aspects of symmetry breaking, and in particular, the implications one can draw from the structure and observed signatures of our particle world. Two of these studies look closely at physical effects generated by the actual symmetry breaking process, while the other focuses on the combined result of demanding an elaborate set of symmetries necessary for superunification, along with a symmetry breaking pattern sufficient to generate a realistic low energy effective theory. Specifically, the topics addressed are non-perturbative strong coupling within the Standard Model[28], biased discrete Higgs symmetries[29], and a realistic supersymmetric Grand Unified Theory[30]. As a means of preparation, this chapter was designed to provide the necessary background required for these topics, and as such I hope that the reader

[‡]Yukawa terms are generated by the superpotential, which is composed of superfields of the same chirality, and so one cannot use the charge conjugate of the original Higgs doublet to generate the up quark mass, as in the non-supersymmetric case. Instead one must introduce a second Higgs superfield[15].

will now enjoy the pages that follow.

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CHAPTER 2

Higgs Bags

2.1 Preliminaries

Particle physicists, in order to have a calculational scheme, often stay well within the realm of perturbative field theory, and pay little or no attention to the non-perturbative strong coupling regime. Yet strong coupling may indeed play a role in our physical world, and so the physical signatures generated by such strong coupling must be understood. However, exact derivation of such signatures is typically very difficult, due to the inherent non-linearities associated with a strongly coupled system. Thus, in order to deal with these non-linear characteristics of the model, reasonable quantitative approximations are necessary. The importance of non-perturbative approximations can only be borne out through examination of the experimental signatures that they imply.

The issue of strong coupling is raised within the Standard Model due to the unspecifed nature of the coupling constants associated with the Yukawa interaction. These Yukawa terms imply a set of non-linear field equations for the fermion and Higgs fields, with the severity of the non-linearity depending of the strength of the particular Yukawa coupling involved. Therefore, any fermion representations that we allow within the context of the Standard Model, or any of its extensions, may contain a sufficiently strong Yukawa coupling so that a non-linear feedback mechanisms is established between the Higgs and fermion field. Fortunately, only one Standard Model fermion is sufficiently massive to warrant such attention. Such reasoning has led to the some recent suggestions (to be discussed in the next section) that top quarks, or very massive fourth generation quarks, might surround themselves with a Higgs "bag" of deformation of the Higgs expectation value from its vacuum magnitude. In this chapter, the issue addressed is not the nature or various aspects of the interaction itself, but rather, whether such non-linear Higgs-top interaction effects are subject to experimental verification. Specifically, the experimental signatures resulting from both the Yukawa interaction itself, as well as the bound state system formed from a fermion-antifermion pair will be discussed.

It should be noted that this work assumed that no massive fourth generation quarks exist, and that the top quark - although undiscovered at the time of publication of this work - was the primary fermion candidate for a strong or non-perturbative coupling to the Higgs field, due to its apparently heavy mass. Since then, strong experimental evidence for the top quark's existence has been collected[1], and a preliminary measurement of the top quark mass has been made. This measurement of the top mass does not alter the results of this work, but rather, acts to constrain the experimental signature beyond what was originally presented. Because of this, the reader will find an updated conclusion appended to the end of this chapter.

2.2 Toponium Tests Of Top-Quark Higgs Bags

2.2.1 Introduction

In the standard model the Higgs field acts, through its vacuum expectation value (vev), as the source of mass for all particles, with the mass obtained depending on the strength of the particle's coupling to the Higgs. Of the particles in the standard model, the only one with potentially very large mass, and hence large coupling to the Higgs, is the top quark. This opens the possibility that there are nonperturbative, strong-coupling effects, with Higgs particles, that will occur uniquely in interaction with the top quark. The idea that a fermion which is strongly coupled to an order parameter may locally deform that order parameter, and surround itself with a "bag" of field deformation, dates at least as far back as Feynman's treatment of polarons [2], and more recently has been generally explored in relativistic field theories of scalars and spinors [3, 4, 5, 6, 7, 8, 9]. Recently it has been suggested [10] that for large values of the top quark mass just such nonlinear effects occur, with the top quark digging a hole in the Higgs vev, and surrounding itself with a "bag" or "dimple" of deformation of the Higgs field (a posteriori such a possibility would also appear for very massive quarks, or leptons, of a hypothetical fourth generation). More detailed quantitative examinations of this proposal have come to the conclusions that: semiclassical "bag" formation implies couplings sufficiently strong to jeapordize vacuum stability, or imply a breakdown of perturbation theory at energies not too far above the top quark mass range [11]; perturbative couplings result in "dimples", that as quantum superpositions involve on average a fraction of a quantum [12]; strong non-perturbative couplings result in quantum fluctuations that tend, at least in a large N expansion, to "deflate" the "bag" [13]. In this work we adopt a slightly different approach to the problem; we ask what would be the observable signatures of formation of Higgs "bags", both for individual top quarks, and also for toponium bound states. We then evaluate the magnitude of these effects for top quarks of moderate mass, where we may treat the Higgs-top coupling in perturbation theory, and examine where the nonlinear higher-order effects should begin to dominate, giving observable signatures of "bag" formation. In agreement with the previous analyses [11, 12, 13] we find for standard model Higgs masses in the range allowed by vacuum stability, and perturbative non-triviality, that the effects of Higgs "bag" formation are not strong enough to be significant. We then extend our analysis to the case of two Higgs doublets, where one of the Higgs may have enhanced coupling to the top, to examine whether in this case observable effects of Higgs "bag" formation may occur.

The possibility of the formation of Higgs "bags" around heavy quarks is suggested by simple energetic considerations. A heavy quark obtains its large mass by virtue of a large Yukawa coupling to the Higgs field vev. If the value of that vev could be locally diminished in the vicinity of the top quark, then the mass of the top quark could be lowered. Provided that the gain in energy from decreasing the mass of the top quark can more than compensate for the kinetic and potential energy invested in deforming the Higgs field around the top quark, and the kinetic energy localizing the top quark, then the top quark will dig a hole for itself in the Higgs vev, and inhabit the region of diminished vev. For this to be energetically favourable, we need the energy saved from lowering the quark mass to dominate, which means the possibility depends on a large Yukawa coupling, and so it may occur only for (very) heavy quarks. If this scenario is correct, then a heavy quark such as the top should be thought of not as an isolated fermion, but rather as a structured object consisting of a fermion surrounded by a coherent superposition of Higgs bosons representing the deformation of the Higgs vev.

2.2.2 The Higgs-Top Yukawa Coupling and Higgs Bag Explosions

Since this coherent superposition of Higgs quanta is supported by the energy saved in reducing the mass of the heavy quark source, the disappearance of that quark would, result in the dispersal of the Higgs quanta. In the case of top quarks, this means that their normal charged current weak decay, via $t \rightarrow bW^+$ would remove the source of the Higgs "bag" (the b being too weakly coupled to the Higgs), and hence lead to the sudden disruption of the "bag". This would in turn mean that the dominant decay modes of such a top quark (with "bag") would involve a copious shower of Higgs bosons from the disruption of the "bag", as well as the b and W. Decay to the bW mode (without Higgs) would be strongly suppressed by the small wave function overlap of the "bag" state with the final state absence of Higgs. This means that the observation of the standard decay mode of the top would provide prima facie evidence against the formation of Higgs "bags". Conversely, a fermion strongly enough coupled to the Higgs field to engender "bag" formation, may be expected to have complex decay modes, that display the complexity of the coherent Higgs superposition in which it reposes.

2.2.3 The Toponium Bound State Signature

A second way that one might imagine obtaining experimental evidence concerning the possibility of Higgs "bag" formation, is by examining toponium bound states. A priori, these seem like ideal systems to probe the possibility of "bags": first they represent already localized top quark sources for the Higgs; second the bound state spectrum provides a sensitive test of the structure of the potential well in which the $\bar{t}t$ find themselves, and should surely be sensitive to as qualitatively distinct a feature as Higgs "bag" formation. To reduce the problem to its essential form, let us consider a $\bar{t}t$ bound state, held together by the QCD potential, which for the heavy toponium we may consider to be approximately Coulombic, and which interacts with the Higgs field via a Lagrangian of the form: (we ignore everything else in the standard model, as we expect it to be quantitatively insignificant in our considerations)

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m_H^2}{2}\phi^2 + \bar{\Psi}(i\not{\partial} - g\phi)\Psi - m_t\bar{\Psi}\Psi - \frac{\lambda}{4!}\phi^4 \tag{2.1}$$

The interaction term $\mathcal{L} = -g\bar{\Psi}\phi\Psi$ will cause a minor deformation of the Higgs field in the presence of the top quarks. Moreover, due to the assumed heaviness of the top, one can apply non-relativistic quasi-classical methods to bound state systems (toponium) composed of (anti)top quarks and a Higgs field. For quasi-static (anti)top quarks in a toponium bound state, which act as a source of deformation of the Higgs field from its vev, we have classically for the Higgs deformation:

$$(\nabla^2 - m_H^2)\phi = g\psi^\dagger\psi \tag{2.2}$$

where in the preceding equation (and hereafter) the ψ represents the "large" components of the non-relativistic spinor Ψ. Here the time dependence has been disregarded as the lowest energy state of the toponium is stationary. Further, the scalar coupling of the Higgs to the top quark, and the non-relativistic treatment of the toponium, implies that the spin degrees of freedom of the top can be neglected, and the $\psi^\dagger\psi$ can be treated as a scalar source for the Higgs vev deformation.

We consider the S-wave fermion wave functions of our toponium bound states as Higgs sources. In view of the spherical symmetry of the S-wave states, the source term composed of the $\bar{t}t$ can be written in terms of the top wave function, expressed in polar coordinates, centred on the toponium. Assuming that the QCD binding potential is approximately Coulombic, then over the distance scale probed by the toponium wave function, these wave functions are exponential in nature; they act as an exponentially falling (radially) Higgs source term; and the 1S and the 2S wave functions represent strong, localized Higgs sources. For Coulombic toponium, the

Higgs field source terms are:

$$
\begin{array}{lrcl}
1S: & g\psi^\dagger\psi & = & \frac{g}{\pi a_0^3} e^{-\frac{2r}{a_0}} \\
2S: & g\psi^\dagger\psi & = & \frac{g}{32\pi a_0^3}(2 - \frac{r}{a_0}) e^{-\frac{r}{a_0}}
\end{array} \tag{2.3}
$$

with $a_0 = \frac{1}{2\pi \alpha_s m_t}$ as the Bohr radius of the unperturbed toponium bound state. α_s is the effective strong coupling constant on scales corresponding to the size of the toponium bound state, which we take to have a value of $\alpha_s \simeq 0.32$, in approximate agreement with the values used by Athanasiu et al. [14] in their study of the $\bar{t}t$ system. The deformation of the Higgs field in the neighborhood of the toponium source causes a decrease in the observed top (and hence toponium) mass.

To solve for the deformation of the Higgs vev, one uses the three dimensional Green's function associated with the equation of motion of the Higgs field.

$$
G(\bar{r}_1, \bar{r}_2) = \frac{-1}{4\pi} \frac{e^{m_H|\bar{r}_1 - \bar{r}_2|}}{|\bar{r}_1 - \bar{r}_2|} \tag{2.4}
$$

Utilising this, one can then analytically obtain the first order position dependent deviation of the Higgs vev from its asymptotic value of $v = 246$ GeV. For the 1S and 2S toponium wave function sources the form of the Higgs vev deviation is

$$
\phi_{1S}^{1st}(r) = \frac{\alpha_s^3\, m_t^3\, (2\,\alpha_s\, e^{m_H\, r}\, m_t - 2\,\alpha_s\, e^{\alpha_s\, m_t\, r}\, m_t - e^{m_H\, r}\, m_H^2\, r + \alpha_s^2\, e^{m_H\, r}\, m_t^2\, r)}{8\, e^{(m_H + \alpha_s\, m_t)\, r}\, (m_H - \alpha_s\, m_t)^2\, (m_H + \alpha_s\, m_t)^2\, \pi\, r} \tag{2.5}
$$

$$
\begin{aligned}
\phi_{2S}^{1st}(r) \quad = \quad & \frac{\alpha_s\, e^{-(m_H\, r) - \frac{\alpha_s\, m_t\, r}{2}}\, m_t}{512\,(-2\, m_H + \alpha_s\, m_t)^4\, \pi\, r} \Big(-128\, e^{m_H\, r}\, m_H^3 + 128\, e^{\frac{\alpha_s\, m_t\, r}{2}}\, m_H^3 \\
& +256\, \alpha_s\, e^{m_H\, r}\, m_H^2\, m_t - 256\, \alpha_s\, e^{\frac{\alpha_s\, m_t\, r}{2}}\, m_H^2\, m_t - 128\, \alpha_s^2\, e^{m_H\, r}\, m_H\, m_t^2 \\
& +128\, \alpha_s^2\, e^{\frac{\alpha_s\, m_t\, r}{2}}\, m_H\, m_t^2 + 64\, \alpha_s^3\, e^{m_H\, r}\, m_t^3 - 64\, \alpha_s^3\, e^{\frac{\alpha_s\, m_t\, r}{2}}\, m_t^3 \\
& -64\, \alpha_s\, e^{m_H\, r}\, m_H^3\, m_t\, r + 64\, \alpha_s^2\, e^{m_H\, r}\, m_H^2\, m_t^2\, r - 64\, \alpha_s^3\, e^{m_H\, r}\, m_H\, m_t^3\, r \\
& +24\, \alpha_s^4\, e^{m_H\, r}\, m_t^4\, r + 16\, \alpha_s^2\, e^{m_H\, r}\, m_H^3\, m_t^2\, r^2 - 12\, \alpha_s^4\, e^{m_H\, r}\, m_H\, m_t^4\, r^2 \\
& +4\, \alpha_s^5\, e^{m_H\, r}\, m_t^5\, r^2 - 8\, \alpha_s^3\, e^{m_H\, r}\, m_H^3\, m_t^3\, r^3 + 12\, \alpha_s^4\, e^{m_H\, r}\, m_H^2\, m_t^4\, r^3 \\
& -6\, \alpha_s^5\, e^{m_H\, r}\, m_H\, m_t^5\, r^3 + \alpha_s^6\, e^{m_H\, r}\, m_t^6\, r^3 \Big)
\end{aligned} \tag{2.6}
$$

To see the effect of the coupling on the mass of the toponium bound state, we examine the change in the splitting between the 2S and 1S energy levels; we focus on the energy level splitting as it is a physical observable, and may reasonably be expected to be sensitive to Higgs "bag" formation, in as much as the 1S and 2S states represent Higgs sources with a different degree of localization, so they should be deformed differently by the formation of a Higgs "bag". Our strategy is to determine the ratio of the leading perturbative correction to the 2S-1S splitting, to corrections that appear at second order, after the feedback of the Higgs field on the toponium source wave function has recorrected the energies of the toponium states. We would interpret second order corrections to the splitting that were a significant fraction of the first order correction, as evidence of a nonlinear feedback in the Higgs-toponium system, representing the onset of "bag" formation.

To examine the effect of the interaction term, first consider as the zeroth order approximation, a QCD toponium bound state. The energy level for the nS state of such a system is given approximately by the Coulombic QCD binding potential for heavy quarkonium

$$E^0_{nS} \simeq -\frac{4}{3}\frac{(4\pi\alpha_s)^2 m_t}{4n^2} \tag{2.7}$$

Here $\frac{4}{3}$ is the colour factor. For the 2S-1S splitting, ΔE^0 this gives $\Delta E^0 \simeq -1.7$ GeV with our assumed value of the effective QCD coupling. The modification of the splitting due to the presence of the Higgs-top interaction is given by the change in the energy level splitting, ΔE, for which time independent non-degenerate perturbation theory is used. The perturbing Hamiltonian is given by

$$\mathcal{H}_1 = -g\phi \tag{2.8}$$

The first order correction to the energy levels due to the presence of the condensate

ϕ is then:

$$E_{nS}^{1st} = < \psi_{nS}^{0} | - g\phi | \psi_{nS}^{0} > \qquad (2.9)$$

Applying equation (2.9) one can obtain numerical values for the first order correction to the 2S-1S splitting for various values of m_H and m_t. Figure 1(a) shows the ratio of the first order correction to the 2S-1S splitting to the zeroth order splitting, as a function of the top quark mass and the mass of the Higgs. In Figure 1(b) contour lines are shown corresponding to first order fractional shifts in the splitting of 5% and of 1%; also shown on the figure is the top and Higgs mass parameter range allowed in the standard model by the constraints of vacuum stability, and perturbative non-triviality up to the Planck scale [15]. Clearly, a measurable shift in the splitting from first order corrections is restricted to a small region of the allowed m_H and m_t parameter space. To test for evidence of "bag" formation, one has to consider the higher order corrections to the energy perturbation. In particular, Higgs "bag" effects would be observable if the non-linear feedback in the Higgs-toponium system, represented by the second order correction, was large in comparison with the first order correction (say of the same order or more). A large second order correction implies that the fermion wave function is pulled in tighter, giving stronger binding to the toponium, and thereby indicating strong binding in a Higgs "bag" potential well. This in turn would increase the influence of the source term in equation (2.2), and so result in a significant increase in the deviation of the Higgs field around the toponium which would then cause a further correction to the splitting. This nonlinear feedback would proceed to dig a hole in the Higgs field, and produce observable Higgs "bag" effects.

Using the first order perturbations, and maintaining the top normalisation,

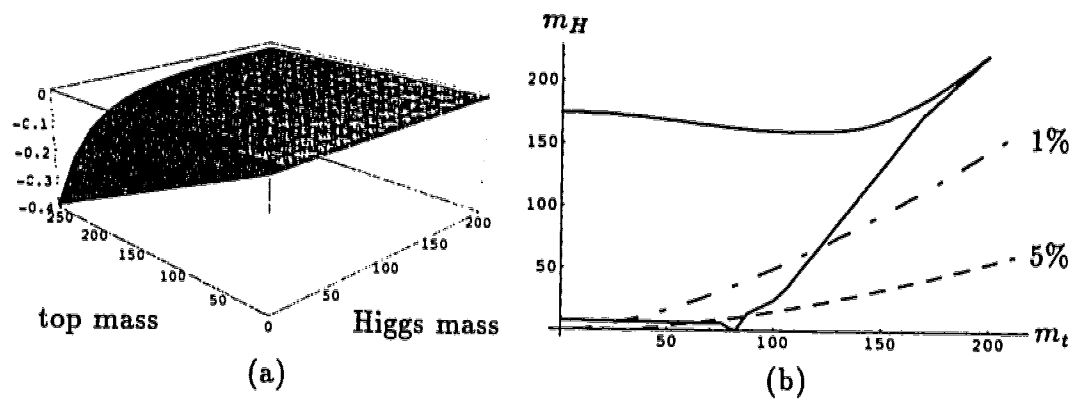

Figure 2.1: (a) The ratio of the first order correction to the 2S-1S splitting to the zeroth order splitting, as a function of the top quark mass and the mass of the Higgs; (b) Contour lines corresponding to first order fractional shifts in the splitting of 5% (dashed line) and of 1% (dot dashed line).

one has

$$\begin{aligned} \psi_{1S} &= \psi_{1S}^0 + \psi_{1S}^1 \\ \psi_{2S} &= \psi_{2S}^0 + \psi_{2S}^1 \end{aligned} \tag{2.10}$$

for the first order corrected top wave functions. It should be noted that for m_t in the range 0 to 250 GeV the adjustment is slight. Given these corrected wave functions, the correction to the Higgs field can be computed. The correction to ϕ_0 is given by

$$(\nabla^2 - m_H^2)\phi_1 = g\psi_{nS}^{0\dagger}\psi_{nS}^1 \tag{2.11}$$

As this equation only differs from equation (2.2) in the inhomogenous term, the Green's function is unaltered, and the ϕ_1 can be found. This then allows one to evaluate the second order correction to the toponium energy levels. For the nS top wave functions, the second order energy correction is

$$E_{nS}^{2nd} =< \psi_{nS}^0| - g\phi_0| \sum_{m\neq n} b_m\psi_{mS}^0 > + < \psi_{nS}^0| - g\phi_1|\psi_{nS}^0 > \tag{2.12}$$

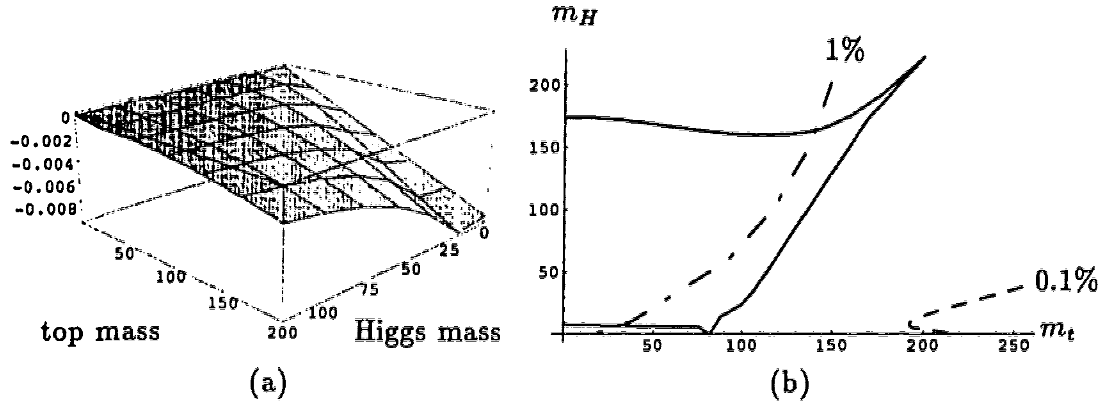

Figure 2.2: (a) The ratio R of the second order correction to the first order correction for the 2S-1S energy splitting as a function of the top quark mass and the mass of the Higgs; (b) Contour lines corresponding to .1% (dot dashed line) and 1% (dashed line) in R.

where the b_m are the coefficients of the first order correction to the wave function. The ratio of concern is

$$R = \frac{\Delta E^{2nd}}{\Delta E^{1st}} = \frac{E_{2S}^{2nd} - E_{1S}^{2nd}}{E_{2S}^{1st} - E_{1S}^{1st}} \tag{2.13}$$

If R (plotted in Figure 2(a))is large then the feedback will have a significant effect on the toponium bound state. On the other hand if R is negligible, then then feedback is insignificant. Figure 2(b) displays the values of the mass parameters required to give a 0.1% and 1% value for R. Clearly, the m_H and m_t for even such slight feedback are not physically acceptable as they lie outside the range of the allowed mass parameters. Also, for any larger value of R the predicted values of m_H and m_t fall further away from the acceptable region. The smallness of R in the mass parameter range allowed by the standard model tells one that the feedback corrections to the energy splitting are negligible, and thus Higgs "bag" are experimentally unobservable. The only circumstance with marginally significant feedback is the case where the top mass is

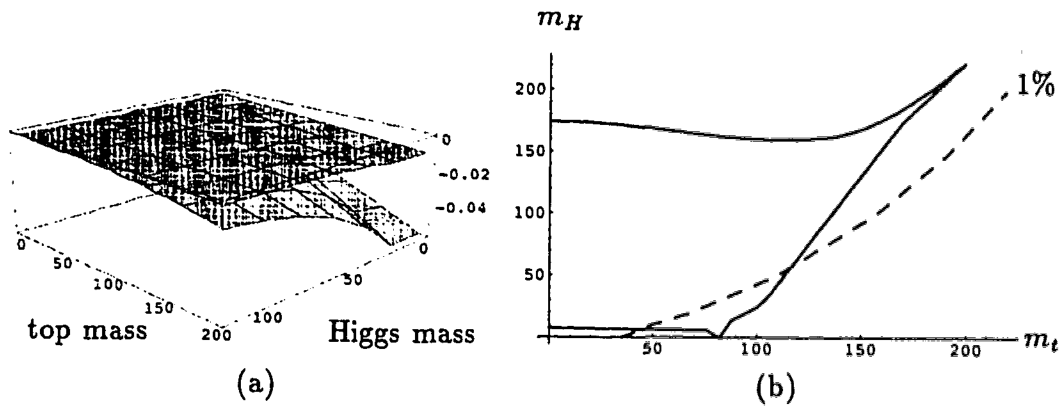

Figure 2.3: (a) The R ratio for normal Higgs coupling (upper surface), and for a coupling enhanced by a factor of 5 (lower surface) as a function of the top quark mass and the mass of the Higgs; (b) Contour line corresponding to 1% (dashed line) in R, with the enhanced coupling.

large ($m_t \approx 150$ GeV) and the Higgs mass is of the order of a few GeV: this situation is already ruled out by LEP limits on the Higgs mass [16].

2.2.4 Non-minimal Higgs Sector Models

If the Higgs sector is extended to a non-minimal content consisting of two Higgs doublets, then there will be extra physical scalar Higgs fields, each of which must be considered. Consider the possibility that one or more of the physical scalars in this non-minimal scenario has enhanced coupling to the top quark. Such an enhancement will in general result in an increase in the corrections to the energy level splittings, thus reopening the possibility for detectable Higgs "bag" effects. In Figure 3(a) the feedback ratio has been plotted for a coupling that has been enhanced by a factor of 5 over the standard model Higgs coupling, while Figure 3(b) indicates the mass parameters required for R to reach the 1% level. Clearly, while enhancement of the coupling increases the second order correction, even a factor of five increase in the coupling has not resulted in significant nonlinear feedback. As such, we do not find evidence for Higgs "bag" formation around toponium, even with substantially enhanced couplings that could appear in models with non-minimal Higgs content.

2.3 Conclusion

2.3.1 The 1993 Conclusion

In conclusion, we have considered the possible observable effects of formation of a Higgs "bag" around toponium, as has been recently suggested. For values of the Higgs and top mass expected in the standard model, the potentially observable effects that could occur in toponium bound states are sufficiently small, that no indication of non-linear feedback characteristic of "bag" formation has appeared. This conclusion remains essentially unaltered, even with the ad hoc enhancement of the top-Higgs coupling by a factor of five, as might occur in a model with a non-minimal Higgs sector.

2.3.2 The 1996 Conclusion

Since the work presented here was originally published, the 'Collider Detector at FermiLab' (CDF) collaboration have found evidence for top quark production in $\bar{p}p$ collisions at $\sqrt{s} = 1.8$ TeV [1]. The original announcement, released in April of 1994, found a small number of events (12) that consituted evidence of top quark production, and from this, it was inferred that the top quark mass is $m_t = 174 \pm 10^{+13}_{-12}$ GeV. Given that the top quark mass, as determined from the Yukawa interaction, is $m_t = \frac{gv}{\sqrt{2}}$ and the value of the Higgs vev is 246 GeV, this top mass measurement implys that the Higgs-top Yukawa coupling g is very close to unity, thereby confirming the non-perturbative strong coupling nature of the Higgs-top Yukawa interaction. Yet the observation of top decay by its standard signature revealed no anomalous signal that could be associated with a Higgs bag explosion. Further, this value of the top mass eliminates completely any chance of observable feedback corrections to the toponium energy level splittings, and the CDF measurement acts only to reaffirm the original conclusion that no experimental evidence for Higgs "bag" production via a non-linear feedback mechanism is expected.

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CHAPTER 3

Biased Discrete Symmetry Breaking and Fermi Balls

3.1 Preliminaries

Enforcement of symmetries and their subsequent breaking has to be considered one of the most crucial cornerstones of modern physics. As has been shown, this is particularly true in the case of the gauge theory development of particle physics. The onset of symmetry breaking, whether spontaneous, explicit, or dynamical, reduces the the degree of symmetry in the model, leaving a gauge group of smaller rank, as well as the possibility of additional accidental symmetries that can be either continuous or discrete. Thus, symmetry breaking can introduce a variety of physical effects that are symptomatic of the underlying symmetries, but length scale dependent. This in turn suggests a wide array of possible physical phenomena, depending on the symmetries involved, and the pattern of symmetry breaking chains employed. The standard example is of course the Standard Model, with its $SU(3) \times SU(2) \times U(1)$ group representation content, and the Higgs Mechanism (aka the spontaneous symmetry breaking of a local gauge symmetry). As discussed in chapter 1, it is this spontaneous symmetry breaking by the Higgs field that offers what appears to be the only consistent mechanism by which mass is generated. Inclusion of one or more Higgs fields in such a way that the resulting Higgs potential can evolve to a configuration that has non-trivial minimum (global or local) in the Higgs field configuration space, and is bounded below, implies that the Higgs potential can induce one or more symmetry breaking phases. When a Higgs symmetry is broken, a non-zero vacuum expectation value (vev) for the associated Higgs field results, implying that couplings of the Higgs field to other particle representations in the theory induce "new" effective interaction terms (aka mass terms) when the Higgs field takes its vev.

This brief outline of the Higgs mechanism has been to emphasise what the Higgs mechanism actually does for us; it produces non-zero vacuum expectation values of Higgs fields. Unlike the previous chapter, where the symmetry breaking was standard and the local variations of the single Higgs vev was the issue, the work presented in this chapter considers the astrophysical implications of a non-traditional symmetry breaking. It shows that residual symmetries from some higher gauge symmetry breaking can produce a scenario that has physical implications that extend well beyond the simple mass generation associated with the vevs of scalar fields. Specifically, this chapter deals with a discrete symmetry rather than the usual continuous $SU(2) \times U(1)$ symmetry responsible for the Standard Model's "Mexican Hat" Higgs potential, and the astrophysical predictions resulting from its breaking.

Discrete symmetries obviously come in many varieties, with the number of minima specifying the labelling scheme - Z_2 is equivalent to the parity symmetry, while Z_n corresponds to a symmetric potential with nth level degeneracy in the minima. To visualise examples of such potentials, one needs only to think of a vertical slice of the "Mexican Hat" potential, or alternatively, the bottom of a 1-litre pop bottle (see figure 3.1). The key characteristic of a discrete symmetry over its continuous cousin is that with a discrete symmetry, one can not continuously deform from one minima (vacuum expectation value) to another without the field being forced out of the minimal (lowest energy) state. If necessary, the n-level degeneracy of a Z_n symmetry

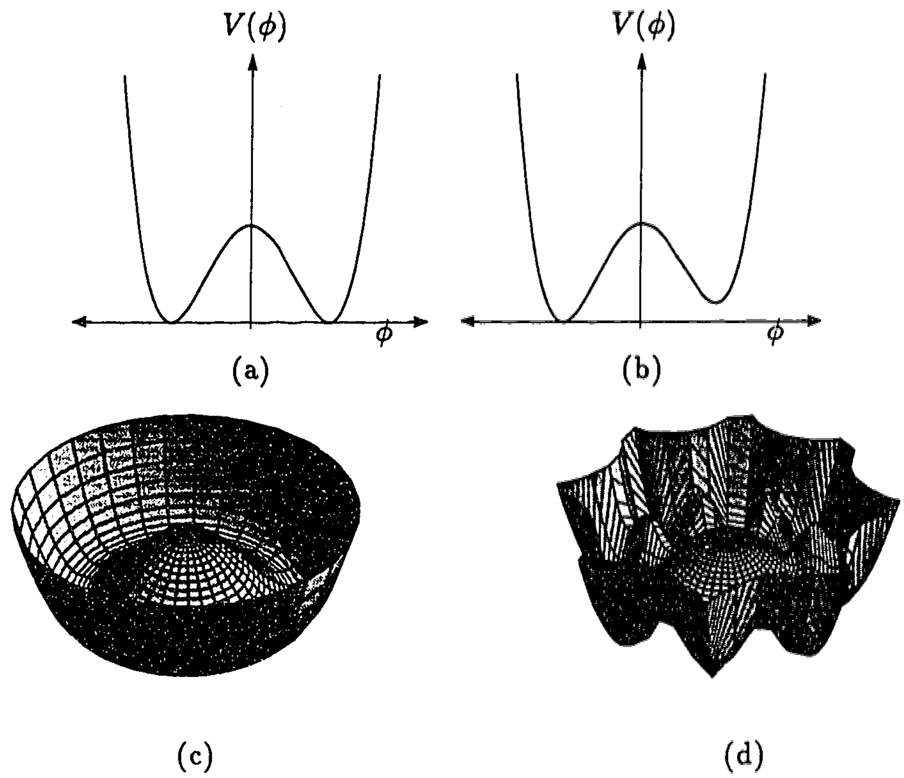


Figure 3.1: Various spontaneous symmetry breaking potentials: (a) A Z_2 discrete symmetry potential, (b) a biased Z_2 discrete symmetry potential, (c) the "Mexican Hat" potential, and (d) The discrete "pop-bottle" potential.

can be removed by biasing the discrete symmetry. A biased discrete symmetry is one where degeneracy of the minima of the potential is destroyed due to the presence of a term in the Lagrangian that does not respect the symmetry of the potential. Biased discrete symmetries therefore require explicit symmetry breaking, but with the strength of the explicit symmetry breaking being sufficiently small so that the symmetry of the potential is approximately maintained for the length scale under consideration. It is such a biased discrete symmetry that is to be considered in the following sections.

3.2 Biased Discrete Symmetry Breaking and Fermi Balls

3.2.1 Discrete Symmetry Breaking and the Astrophysical Implications

It is well known that spontaneous breaking of a discrete symmetry can produce topological structures composed of different domains separated by topological defects [1, 2, 3]. In the simplest such physical scenario, the topological defects produced are domain walls [1] (transition regions between spatial domains that possess topologically different vacuum orientations), which within the context of cosmological models, have been applied to phenomena ranging from energetically soft topological defects [4] and structurons [5, 6] for the formation of large scale structure [7, 8], to significant deviations from thermal equilibrium at the QCD scale [9], neutrino balls [10, 11], and an origin for cosmological Gamma Ray Bursts [12]. In this chapter, the interaction of domain walls with a fermion sector is considered, which suggests the possible production of composite microscopic cosmological relics referred to henceforth as Fermi balls. These Fermi balls, under certain conditions, provide an unusual source for cold dark matter, and may be relics of the seeds for possible structure formation in the cold dark matter scenario.

The simplest model exhibiting topological structure is that of a real scalar field φ with a Lagrange density of the form

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\lambda^2}{8}(\varphi^2 - \varphi_0^2)^2 \tag{3.1}$$

Clearly, equation (3.1) possess a Z_2 symmetry (invariance under $\varphi \to -\varphi$), which if spontaneously broken results in a vacuum expectation value (vev) for φ that has two possible values; $<\varphi>=\pm\varphi_0$. These two vev's correspond to topologically distinct vacuum orientations (distinct values of the order parameter); here the notion of topologically distinct vacua implies that one vacuum orientation cannot be continuously deformed into the other. Yet due to the Z_2 symmetry being exact, neither vev is preferred, so the determination of the vev in a particular spatial region is set by random fluctuations in φ. Thus the spontaneous symmetry breaking results in a randomly generated network of spatial domains of both vacuum orientations that are separated by transition regions called domain walls (topological defects). The form of the domain wall solution is a topological soliton of class π_0 [13], and is easily obtained from the equation of motion for φ. The simplest such solution is that of a planar domain wall in the xy plane at $z=0$ with the boundary conditions $\varphi(z\rightarrow\pm\infty)=\pm\varphi_0$, and has the form $<\varphi>=\varphi_0\tanh(\frac{z}{\delta})$. Here $\delta=\frac{2}{\lambda\varphi_0}$ is the wall thickness. Typically, δ is assumed to be small compared to the average radius of curvature of the walls (the thin wall approximation), so that the domain walls can be treated as two dimensional surfaces. For the planar domain wall, the associated stress-energy tensor is $T^{\mu}_{\nu}=\frac{\lambda^2\varphi_0{}^4}{4}\cosh^{-4}(\frac{z}{\delta})$ diag$(1,1,1,0)$, indicating that the only non zero pressure components are within the plane of the wall, and both are equal to minus the energy density. Due to the form of the stress energy tensor, the surface tension ($\int T^i_i dz$) is exactly equal to the surface energy density of the wall ($\int T^0_0 dz$), which has the form

$$\sigma=\frac{2\lambda\varphi_0^3}{3} \tag{3.2}$$

The cosmological implication of spontaneous breaking of an exact discrete symmetry, as first analysed by Zel'dovich et al. [1], is the formation of stable domain walls separating protodomains (spatial regions with distinct vacuum orientations) of topologically distinct energetically degenerate ground states. These walls evolve to planar structures that dominate the energy density of the Universe. Clearly, this is in contradiction with our present observations. To avoid this prediction, the self coupling of φ could be fine tuned so to sufficiently delay the wall dominance of the energy density. A more creditable alternative, suggested in [1] is to remove the wall stability by requiring the discrete symmetry to be only approximate. Then, spontaneous symmetry breaking results in topologically distinct ground states that are non degenerate, as the symmetry breaking is biased. This non degeneracy manifests itself in the form of protodomains of true and false vacuum, that are separated by domain walls.

Upon formation, the domain walls evolve in accordance with the protodomain ensemble minimising its energy, so that the wall motion can be described in terms of the pressure imbalance across the domain wall [14]. In a φ self coupling model [14], only the false vacuum volume pressure and the normal component of the wall surface tension contribute to the pressure imbalance. The false vacuum volume pressure is typically constant, and pulls the wall towards the false vacuum protodomain, whilst the normal component of the surface tension acts to straighten the wall, and decreases with decreasing wall curvature. Thus, finite sized false vacuum protodomains (vacuum bags) collapse on themselves, whilst infinite domain walls are pulled toward the false vacuum region [14]. It is this biased discrete symmetry breaking, with its inevitable conversion of false to true vacuum that cause domain walls to disappear, and by which a wall dominated energy density disaster is avoided [2, 15, 16]. Obviously, the degree of biasing between the vacua dictates the average domain wall lifetime, and if their longevity is sufficient for them to dominate the energy density of the Universe, then power law inflation can be induced [2, 1, 15].

3.2.2 The Effects of Bias

As no φ self coupling model of biased discrete symmetry breaking produces stabilised finite size vacuum bags from topological defects, other more novel couplings have been investigated [17, 10, 18], each with there own cosmological implications. The coupling advocated in this work is one which relies on the presence of fermions strongly coupled to the scalar, with φ symmetrically coupled to a fermion via standard Yukawa couplings:

$$\mathcal{L} = \frac{1}{2}\bar{\psi}(i\partial - G\varphi)\psi + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\lambda^2}{8}(\varphi^2 - \varphi_0^2)^2 + A(\varphi) \tag{3.3}$$

The Lagrangian now contains both a Yukawa coupling of fermions to the scalar field φ, and a term $A(\varphi)$ that explicitly breaks the discrete symmetry to an approximate one. The actual form of $A(\varphi)$ is specified only to the extent that the energy difference between the two vev orientations is Λ. (For specific examples of $A(\varphi)$ consult [17] .) The Yukawa coupling implies that after spontaneous breaking, fermions acquire a mass proportional to $< \varphi >$, and so it is energetically favourable for the fermions to inhabit the domain wall as they become effectively massless there. (In the infinite planar wall there exists an analytic solution for the zero mode of the fermion bound to the domain wall [19].) Thus, any off wall fermions (that are strongly coupled) are swept up by, and reside in the domain wall. Since immediately after the phase transition each fermion will be, on average, within a correlation length of the percolating wall structure, we expect the fermions to be efficiently stuck to the walls. Domain walls quickly become populated with fermions, so that the walls (in the thin wall approximation) are essentially two dimensional surfaces inhabited by a Fermi gas of massless fermions. The associated Fermi gas pressure contributes to the pressure imbalance and acts to modify the wall dynamics. In order to halt the collapse of a finite sized false vacuum protodomain, and give stable false vacuum bubbles, the

Fermi gas pressure must cancel the surface tension and false vacuum volume pressure components. This will occur if the energy of a false vacuum protodomain that has accumulated a wall gas of N fermions can be minimised for some finite radius. For a vacuum bag of arbitrary shape, the energy of the bag is

$$E = V\Lambda + S\sigma + E_F \tag{3.4}$$

(V = the volume of the vacuum bag, S = its surface area, and E_F = the energy of the Fermi gas composed of N wall fermions.) Assuming the wall gas is composed of massless degenerate fermions with $g = 2$ internal degrees of freedom, the Fermi energy of the fermi gas in the zero temperature limit, with a wall number density n, is

$$\varepsilon_F = \tfrac{E_F}{N} = \frac{4\sqrt{\pi}}{3\sqrt{g}}\sqrt{\frac{N}{S}} = \frac{4\sqrt{\pi}}{3\sqrt{g}}\sqrt{n} \tag{3.5}$$

and for a spherical vacuum bag, minimising with respect to radius results in a stabilised bag being found, with a radius given by

$$N^{\frac{3}{2}} = 6\pi\sqrt{g}\left(R^4\Lambda + 2R^3\sigma\right) \tag{3.6}$$

This halting of the collapse process is due entirely to the presence of the fermions on the domain wall, and so for spherical false vacuum protodomains, one might expect stabilised false vacuum bags with a bounding outer skin of massless fermions.

But assuming the collapse of false vacuum bags to be completely described by the process of spherical shrinking until the pressure imbalance is nullified is incorrect. The collapse process is driven by a minimisation of the bag energy, to which there are three competing elements: volume energy density splitting, surface tension energy, and surface Fermi gas energy. As the surface tension energy and the energy of the two dimensional Fermi gas, E_F, are dependent on the surface area of the bag and not its volume (equation (3.5)), the vacuum bag energy can be reduced by a decrease in the bag volume, with the surface area held constant. Thus, bags are unstable with respect to "pancake" deformations, implying the bag flattens into a sheet-like structure. In conjunction with this flattening, the vacuum bag lowers its energy by fragmenting into smaller vacuum bags. To see that fragmenting is favoured, consider the energy for an arbitrary vacuum bag, but first neglect the volume contribution. By minimising this energy with respect to the surface area S, the energy of the stabilised bag is found to be

$$E\,|_{V\Lambda=0} = 3\left(\frac{4\sigma\pi}{9g}\right)^{\frac{1}{3}} N \tag{3.7}$$

which is proportional to N. This implies that one vacuum bag with a domain wall Fermi gas composed of N fermions is energetically equivalent to two vacuum bags each with $\frac{N}{2}$ fermions on their domain wall, and so vacuum bags may fragment but are not compelled to do so. However, on inclusion of the false vacuum volume energy, minimisation of energy favours bag fragmentation. These facets of the collapse process for a finite sized false vacuum protodomain result in a more involved vacuum bag evolution than the simple shrinkage to a minimal surface area stabilised by N wall inhabiting fermions, as all three act concurrently. The physical collapse process of a false vacuum bag is one of repeated shrinking, flattening, and fragmenting, that results in numerous smaller vacuum bags.

3.2.3 The "Creation" of a Particle

However, for a sufficiently strong coupling of the fermions to the scalar order parameter the collapse process does not continue ad infinitum, as the soliton origin of the bag structure will eventually arrest the collapse. This onset of the quantum regime is signified by the breakdown of the thin wall approximation, and implies that the domain wall radius of curvature is comparable to the size of the vacuum bag. When this occurs, the vacuum bag is no longer a bubble of false vacuum with a domain wall skin containing a two dimensional Fermi gas, but rather a ball composed almost exclusively of the domain wall, with almost all the interior false vacuum having been destroyed. Such a ball of domain wall still carries the Fermi gas, but now the massless fermions of the Fermi gas constitute a three dimensional Fermi gas inhabiting the interior of the domain wall ball. It is these balls of fermion populated domain wall that we refer to as Fermi balls, and they represent true non topological defects. If the fermions are strongly coupled to the scalar, then the Fermi balls will be stable if the energy invested in the scalar field configuration is less than the total mass the trapped fermions would have to obtain if the wall disappeared. To get a crude estimate of the size of the stabilised Fermi balls, we note that our Lagrangian contains only one dimensional parameter which, in the wall solution, determines its intrinsic thickness. By equating the minimum size of the stabilised Fermi balls R_{min} to the wall thickness δ, and assuming these stabilised Fermi balls adopt a minimum surface area configuration, the typical stabilised radius (radius at which the collapse process stops) is estimated by

$$R_{min} \sim \frac{2}{\lambda\varphi_0} \tag{3.8}$$

The radius R_{min} is small, indicating the collapse of false vacuum protodomains produces in a mist of tiny Fermi balls distributed throughout the 3-space. This mist of Fermi balls should be considered as possible cosmological relics, since their stability against further collapse may be assured by energetic considerations, and Fermi ball annihilation is ruled out if the fermions are Dirac particles with conserved fermion number.

Yet biased spontaneous symmetry breaking doesn't necessarily result in the formation of finite sized false vacuum protodomains. The nature of the protodomain structure at formation depends on the degree of anomalous breaking Λ; for Λ small compared to the potential barrier, a percolating domain wall structure [20, 3, 8] is expected, whilst a Λ comparable to the barrier height implies the formation of finite sized false vacuum bags. For the dynamical evolution of percolating domain walls, the analysis and conclusions differ little from that of Gelmini et. al. [14], who show that although there are several different cosmological scenarios, in which the domain walls straighten out on various scales, the false vacuum volume pressure eventually dominates the pressure imbalance. This causes the domain walls to be driven inward on the false vacuum protodomain structure, inducing a "melting" of the false vacuum. Once the false vacuum volume pressure becomes dominant, the conversion of false to true vacuum is relentless, and eventually leads to a fragmentation of the percolating domain wall structure into finite sized false vacuum bags. This fragmentation is essentially the conversion of topological defects to nontopological ones, and is a result of the system's desire to minimise its energy. Inclusion of a strong coupling to a fermion sector causes a modification to the constraints on Λ that define the different dynamical regimes (The surface tension σ is replaced by $\sigma - \mathcal{P}$ to account for the two dimensional Fermi gas pressure $\mathcal{P}$.), but the conclusions of [14] remain unaltered. This implies that irrespective of the protodomain structure formed at symmetry breaking, finite sized false vacuum bags are eventually produced, which in turn evolve into the Fermi ball structures discussed above.

Thus, biased discrete symmetry breaking with strongly coupled Dirac fermions may result in a mist of nontopological objects (Fermi balls) comprised of a superposition of the massless fermions and a local deformation of the order parameter $< \varphi >$. These Fermi balls are expected to be approximately spherical, with a radius R_{min} (equation (3.8)). Their stability against further collapse is assured by sufficiently strong spinor scalar coupling, but stability under fermion anti-fermion annihilation has not been addressed. Such annihilations could significantly affect the Fermi ball lifetime.

A strong Yukawa coupling implies that after the symmetry breaking, the fermions collect on the domain walls, thereby enhancing the fermion anti-fermion annihilation rate. Fermion anti-fermion annihilations reduce the Fermi gas pressure, so destabilising the false vacuum bag so that collapse continues until the pressure balance is restored. Thus, confinement of the fermions to the domain wall prohibits freeze out of the number density of fermions, and so Fermi balls can exist only if there is a net fermion anti-fermion asymmetry.

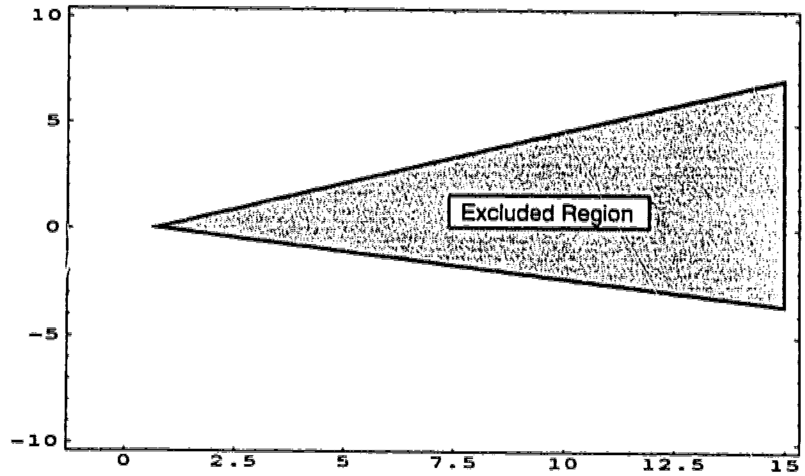
3.2.4 Where Are They?

Given that Fermi balls are produced, equations (3.6), (3.7), and (3.8) imply that they would be composed of approximately 50 fermions, independent of the symmetry breaking scale, and possess a mass of the order of $100\varphi_0$ GeV. This suggests that a Fermi ball would appear as a very heavy slow moving particle, which if the individual wall fermion had electric charge, would carry a charge in the order of $10-50$ times the electron charge. Such objects therefore have characteristics similar to either heavy ions or nuclearites [21], and so analysis of the Fermi ball stopping power [22] and the negative results of nuclearite searches by collaborations such as MACRO [23] can place a constraint on the relation between the Fermi ball mass and number density.

Alternatively, Fermi balls could be composed of a new Dirac fermion that possesses no standard model gauge charges. Fermi balls would then be neutral, heavy, and non relativistic, and due to their absence of gauge charges, would interact extremely weakly with standard model matter; barring new couplings, the only interaction (apart from the gravitational one) would be via couplings of the real scalar field φ to the standard Higgs fields. Thus, the heavy non relativistic neutral Fermi balls would constitute an ideal candidate for cold dark matter. This suggests a possible constraint on these neutral Fermi balls, as gravity results in an accumulation of Fermi balls around massive objects such as the sun. Gravitationally bound Fermi balls may orbit through or within the solar interior, thereby transporting energy away from the solar core by their weak scattering from solar core baryons (protons). If such heat diffusion is sufficiently efficient, the gravitationally bound Fermi balls become incompatible with the standard solar model.

Assuming the Fermi balls are the sole source of dark matter and that their contribution is such that the Universe attains closure density ($\Omega = 1$), the magnitude of the luminosity diffusion, as a function of the Fermi ball mass, can be evaluated. The analysis is based on the work of Press and Spregel [24, 25], which deals with the solar capture and the subsequent luminosity transport of cosmions [26]. For this closure density scenario, with gravitationally bound Fermi balls in approximate thermal equilibrium with the solar core, Figure 3.2 shows a contour plot of the luminosity transported by them relative to the solar luminosity, as a function of the Fermi ball mass relative to the proton mass, and the Fermi ball-baryon cross section relative to a fiducial cross section of reference [25]. A relative luminosity contour of unity is used to restrict the relative cross section and relative mass of the Fermi balls (which in turn can be related to φ_0), as a relative luminosity of unity or greater implies that for fixed total energy transport, the Fermi ball transport would more than halve the core temperature gradient, in contradiction with the solar model [25]. The restriction on parameter space isn't particularly severe, considering that neutral Fermi balls are expected to have extremely weak non-gravitational interactions, and so would free stream through the sun.

Finally, if Fermi ball closure density is assumed, the fermion anti-fermion asymmetry required just prior to the biased spontaneous symmetry breaking in order to



$\log_{10}\beta$

$\log_{10}\alpha$

Figure 3.2: A contour plot of the relative solar luminosity carried by the Fermi balls, as a function of $\alpha = \frac{m_{FB}}{m_p}$, the Fermi ball mass relative to the proton mass, and $\beta = \frac{\sigma_{FB}}{\sigma_c}$, the Fermi ball-baryon cross section, relative to the fiducial cross section $\sigma_c \equiv \frac{m_p}{M_\odot} R_\odot^2 = 4.0 \times 10^{-36}\mathrm{cm}^2$. The contour shown is that of relative luminosity of unity, and the excluded region is where the relative luminosity is greater than 1.

produce Fermi balls can be estimated. The constraint of closure density sets a restriction on the present day Fermi ball number density, which is in turn related to the relative fermion anti-fermion asymmetry just prior to symmetry breaking, defined by

$$B = \frac{n - \bar{n}}{n} \tag{3.9}$$

(here n and $\bar{n}$ represent the number density of fermions and anti-fermions). The present day Fermi ball number density is obtained from the number density of excess fermions at the symmetry breaking by evolving this number density forward to the present day, and then dividing this number density by the number of fermions in a typical Fermi ball. From this, the constraint on the relative fermion asymmetry is found to be of order

$$B \sim \frac{10^{-7}\text{GeV}}{\varphi_0} \tag{3.10}$$

which for a breaking scale of $\varphi_0 = 1\text{GeV}$ implies an asymmetry of 10^{-7}, which is of similar magnitude to the baryon asymmetry at the 1GeV scale.

3.3 Conclusion

As this work shows, biased discrete Higgs symmetries can result in "creation" of composite particles. These composite particles, or Fermi Balls as they were dubbed, have been shown to form in a rather generic and model-independent scenario, which only depends on the presence of a biased discrete Higgs symmetry, and generic Yukawa couplings to some fermion content. This study emphasises the phenomenological and astrophysical implications to the low energy limit of the theory if discrete symmetries, whether they be accidental or residual, are present.

The actual characteristics of Fermi Balls are divided into two categories; general and specific. The general properties are that they are very massive, extremely stable in the low energy limit, and of very small size (with size decreasing with an increasing discrete symmetry breaking scale). The specific properties of Fermi Balls are much more subtle, and depend on the Standard Model gauge charges carried by the Fermi Ball's fermion constituent. An analysis of these specific properties would require a model-specific study, which was not the intent of the work in this chapter. Despite the lack of specific characteristics, the general characteristics, combined with the fact that Fermi Balls have not been observed (so telling us that they must at least be WIMPs - weakly interacting massive particles) suggest that Fermi Balls are ideal dark matter candidates. Also, their massive nature suggests that detection of these objects is most likely to come from nuclearite-type searches, or in the dark matter scenario, from astrophysical constraints such as the solar luminosity study presented in the chapter.

Recently, Morris and Bazeia[27] added to the credibility of the Fermi Ball scenario, by showing that Fermi Ball production arises rather naturally from a supersymmetric theory that admits a domain wall solution. Specifically, they show that given a domain wall within the context of a supersymmetric theory, and a fermion zero mode on the domain wall, soft supersymmetry breaking results in the addition of terms to the Lagrangian that shift the spontaneously broken discrete symmetry from exact to biased, thereby initiating the production of Fermi Balls.

Whatever the nature of these Fermi Balls, it is hoped that this chapter has shown the reader that some unexpected physics can be generated from the breaking of more exotic symmetries than the now familiar gauge symmetries. Also, such physical/astrophysical phenomena can be compatible with the more familar physics, as the required discrete symmetry can be in addition to other symmetries present in the theory.

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CHAPTER 4

Nucleon Decay in Non-Minimal Supersymmetric SO(10)

4.1 Preliminaries

The realisation that the Standard Model is at best only an effective theory, is one of the most exciting observations of particle physics. It immediately implies that some deeper underlying theory is lurking at shorter length scales‡, and has been reduced to the Standard Model as the Universe cooled (Presumably through a pattern of symmetry breakings.). Naturally, this has led to much discussion as to the form of this underlying theory, and its associated phenomenological consequences. Both the discussions in chapter 2 of the phenomenology of strong coupling, and the astrophysical implications of discrete symmetries considered in chapter 3 constitute examples of such theorising, as they attempt to elucidate possible Standard Model extensions by means of phenomonology. However, the most feasible, physically reasonable, predictive, and downright attractive proposal to date, is the one that employs to the fullest, the concepts of symmetries, and is to be the subject of this chapter. The theory in question is that of a supersymmetric Grand Unified Theory (SUSY GUT).

As discussed in chapter 1, a GUT is a Standard Model extension in which the underlying gauge group symmetry is expanded to a larger simple group, G, with the

‡Here the reader is advised to see the film "Creature from the Black Lagoon".

particle content of the theory fitting into representations (reps) of this group. Not only does this provide for a simple high energy structure, but also, when supersymmetrised, implies a unification of the Standard Model coupling constants at some high energy scale. While such a structure may or may not be a faithful representation of the actual high energy physics, the benefit of a GUT is that it provides a robust and consistent theoretical framework in which to model the physics at the high energy scale, allows definite phenomenological predictions, and appears to give the most consistent description of the high energy physics as seen from our low energy world. (It may well be that the actual fundamental physical theory is some version of the present-day string theory, which breaks directly to the Standard Model, thereby passing the GUT stage altogether, but such a theory would be difficult to test and indeed, at the low energy scale, difficult to distinguish from a GUT approximation.) Once a GUT structure is assumed as the form of the Standard Model extension, the challenge is then to choose an appropriate gauge group, and construct the particle representations in such a way that the low energy effective theory that results from symmetry breaking of the gauge structure does actually mirror the Standard Model. Again, I refer the reader back to chapter 1 for the sort of constraints the model must respect. In line with this, the GUT model adopted in this chapter is not the usual minimal GUT model, but rather one that was chosen on the basis that its prediction of the low energy fermionic mass spectrum is the most "realistic".

In addition to imposition of the GUT scheme, a further symmetry is adopted: Supersymmetry. Supersymmetry, the symmetry that relates fermions to bosons and vice versa, is primarily justified in the GUT scheme by the fact that the "doubling of the particle content of the theory" slows the Renormalisation Group running of the Standard Model coupling constants so that they actually converge (within error) to a single unification point. (Again, the reader is refered back to chapter 1.) Yet this doubling of the particle content automatically introduces interactions between the particles and their superpartner particles, and it is these new interactions that open up the door to a variety of new physical processes, which can greatly influence the low energy signatures of the resulting effective theory. This is what the inclusion of supersymmetry does for the phenomenologist. Further, the influence of SUSY on the low energy signatures is more pronounced the lower the SUSY breaking scale, and if we require SUSY model to solve the hierarchy problem (return again to chapter 1), then the superpartners to the light Standard Model particles must be relativly low in mass (ie a low SUSY breaking scale). Thus, one should expect the influence of SUSY on the low energy signatures of the effective theory to be significant.

Obviously, the low energy signatures have to be identified, and it is the appeal to the role of symmetries in the "gauge age" (aka the gauge principle) that provide these signatures of the high energy theory. Specifically, it is the belief that all continuous symmetries are gauge symmetries, and that only conserved quantum numbers correspond directly to these gauge symmetries and their associated massless gauge fields. To date, we are only aware of the photon, gluon, and the graviton as massless fields, impling that quantum numbers such as the baryon number B or the lepton number L, cannot be absolutely conserved quantities. Violation of such conservation laws is then due to the breaking of the gauge symmetry, with the rate depending on some inverse power of the gauge boson mass generated in the symmetry breaking. (Once more the reader is refered back to the gauge theory discussion in chapter 1 for a discussion on the relation between the rank of the gauge group and conserved quantum numbers.) Thus, any GUT or SUSY GUT extension of the Standard Model must have both B and L violation as a low energy signature, although considerably suppressed due to the large value of the conjectured GUT breaking scale ($M_X \sim 10^{14} - 10^{16}$ GeV). From this baryon number non-conservation it can be inferred that the lightest baryons, which the Standard Model considers stable, can decay via $\Delta B \neq 0$ interactions. The classic low energy signature for GUT or SUSY GUT theories is therefore evidence of nucleon decay. Prediction of the relative decay rates then implies a method of distinguishing between different GUT and SUSY GUT candidates, once nucleon decay is observed. The work that follows is a specific inquiry into the set of phenomenological predictions associated with nucleon decay, with the model used being the SUSY extension of what the author considers to be the most "realistic" GUT in the literature.

4.2 Nucleon Decay in Non-Minimal Supersymmetric SO(10)

4.2.1 Introduction

Nucleon decay is by definition a baryon number violating process, and within the context of the standard model (SM) of particle physics is forbidden ‡. Yet there is a strong motivation for assuming that baryon number violation occurs; particularly the fact that there is no baryonic analog of the electromagnetic gauge invariance [2](which guarantees the conservation of electric charge), the presence of a baryonic asymmetry in the Universe [3], and the violation of baryon number conservation by black holes [4]. Allowing baryon number violation then suggests that the SM is only a low energy effective theory, and as such the stability of the nucleon is brought into question. This view was further reinforced when the adoption of Grand Unified Theories (GUTs) as a SM extension appeared to explain a large number of questions left unanswered by

‡Baryon number is not conserved in the SM, as violation occurs in weak interactions via instanton effects and the triangle anomaly, but the rate is suppressed and also involves violation of 3 units of baryon number for 3 standard model generations, thus making it irrelevant to nucleon decay [1].

the SM [5]. The GUT scheme was introduced to attempt to unify the SM interactions under a single simple gauge group. Imposition of such an underlying GUT structure then provided a new mechanism by which baryon number violation could occur, and nucleon decay induced [6, 7].

This nucleon decay mechanism is due to the fact that for conventional GUTs, quarks and leptons are placed in the same multiplets of the GUT gauge group. The coupling of these multiplets to either gauge or Higgs boson representations then gives interactions that couple quarks to leptons, and below the GUT scale, produce effective operators that induce nucleon decay. These tree level operators are four fermion dimension 6 operators [8, 9] built from two fermion-fermion-boson vertices by means of a gauge or Higgs boson exchange. As the low energy limit of the internal boson propagator is $\frac{1}{M_G^2}$ (M_G is the mass scale at which the GUT is spontaneously broken), the four fermion interaction reduces at low energy to an effective four fermion vertex scaled by two inverse powers of M_G. It is this class of effective vertex that would mediate nucleon decay in non-supersymmetric GUT models.

In the archetypal GUT - minimal non-supersymmetric SU(5) - first proposed by Georgi and Glashow [6], the unification scale is $M_G \sim 5 \times 10^{14}$ GeV [10, 11], and predicts the most dominant decay mode to be $p \rightarrow \pi^0 + e^+$ with a partial lifetime of $\tau_p \sim 4.5 \times 10^{29\pm1.7}$ yrs [11]. This is to be contrasted with the experimental lower bound obtained from IMB-3 Collaboration, of $\tau_p > 5.5 \times 10^{32}$ yrs [12, 13]. Clearly the minimal SU(5) model predicts proton decay at too rapid a rate, thereby ruling it out as a realistic GUT candidate. Nucleon decay channels and partial lifetime predictions have been calculated for a variety of GUT models [14], including non-minimal SU(5) (which includes a $\underline{45}$ Higgs rep in an attempt to predict the fermion masses), minimal and non-minimal SO(10), and an E_6 GUT model. Unfortunately, all these models tend to fail on the basis of a unification scale $M_G \sim (2-7) \times 10^{14}$ GeV, which implies an overly rapid nucleon decay rate as well as a prediction for $\sin^2\theta_W$ that is inconsistent with the high precision LEP measurements [15].

As conventional GUTs are essentially condemned by these failings, attention has turned to the supersymmetric GUT models (SUSY GUTs). Imposing supersymmetry - a symmetry that relates bosons and fermions - has the effect of doubling the particle content below the GUT scale, which results in the slowing of the SM gauge coupling running, and consequentially predicts a consistent gauge coupling unification at a higher scale. Thus, a SUSY GUT model not only addresses the matter of the consistency of the $\sin^2\theta_W$ prediction, but it also predicts a unification scale that is typically two orders of magnitude larger than that of conventional GUTs. This increase in the unification scale induces a suppression factor of order 10^{-8} in the decay rates of four fermion dimension 6 operators generated by boson exchange, placing the dimension 6 mediated nucleon partial lifetime predictions well beyond the experimental lower bound. However, with the advent of Super-KAMIOKANDE, even the decay mediated by the dimension 6 operators may be observable.

Yet the extension to a SUSY GUT model permits a new operator, capable of being the dominant contribution to nucleon decay. This operator is a dimension 5 fermion-fermion-sfermion-sfermion effective operator [16, 17] constructed from either two fermion-sfermion-Higgsino vertices or a fermion-fermion-Higgs and a sfermion-sfermion-Higgs vertex by means of a heavy colour triplet Higgino or Higgs exchange below the GUT scale. Such an operator then evolves down to the SUSY breaking scale, at which point the sfermions are 'dressed' by gaugino exchange to give an effective four fermion vertex that mediates nucleon decay. As the low energy limit of the dimension 5 operator is scaled by $\frac{1}{M_G}$, nucleon decay via this operator generally dominates over those mediated by the conventional dimension 6 operators[‡].

[‡]It is assumed that [illegible] invoked so to rule out dangerous dimension 4 operators.

Investigations of nucleon decay in a number of SUSY GUT models have been carried out, beginning with the supersymmetrised version of minimal SU(5) [18, 19]. Unlike its non-SUSY cousin, this model predicts the dominant nucleon decay modes to be $p \rightarrow K^+ + \bar{\nu}_\mu$ and $n \rightarrow K^0 + \bar{\nu}_\mu$, and as the unification scale is $M_G \sim 2.5 \times 10^{16}$ GeV the partial lifetime prediction is $\tau_{p \rightarrow K^+ + \bar{\nu}_\mu} \sim 10^{29\pm4}$ yrs [20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38] (with M_{H_3} set to M_G). This prediction does not disagree with the experimental bound of $\tau_{p \rightarrow K^+ + \bar{\nu}} > 10^{32}$ yrs (obtained from the water Čerenkov detector of the KAMIOKANDE Collaboration [39, 13]) due mainly to the large uncertainty resulting from the value of the Higgs/Higgsino colour triplet mass. Likewise, predictions for non-minimal SUSY SU(5), minimal SUSY SO(10) result in a marginal degree of compatibility with the experimental lower bounds on the partial lifetimes of the various nucleon decay channels [14]. This marginal consistency suggests that an improvement on the experimental lower bounds could lead to either a rejection of nucleon decay via SUSY GUT generated dimension 5 operators, or an observation of nucleon decay. Yet the uncertainties in nucleon partial lifetime predictions preclude model discrimination by rate. In order to distinguish the underlying SUSY GUT structure, the relative decay rate predictions within a model should be determined, and then used to identify the SUSY GUT candidate, once nucleon decay has been observed.

With this strategy in mind, this work presents the branching ratios for nucleon decay in a particular 'realistic' SUSY GUT model. The model chosen is a supersymmetrised version [40] of the non-minimal SO(10) GUT proposed by Harvey, Reiss, and Ramond [41], which was constructed primarily to reproduce a consistent phenomenological fit to the observed SM fermion masses and mixing angles. This realistic non-minimal SUSY SO(10) model, like its non-SUSY counterpart, can be viewed as a sophisticated phenomenological one, as it supports a rather expansive

Higgs sector that is responsible for the required Yukawa coupling texture. It will be shown that analysis of the various nucleon decay channels mediated by the dimension 5 operators of this model results in branching ratio predictions depending on a single parameter, with the branching ratios for some observable modes enhanced by factors of order 100 over the minimal SUSY SU(5) predictions. This in turn suggests that if nucleon decay is observed at Super-KAMIOKANDE, the $p \rightarrow K^0 + \mu^+$, $p \rightarrow \pi^0 + \mu^+$, and $n \rightarrow \pi^- + \mu^+$ decay channel may play a significant role in identifying the structure of the underlying SUSY GUT.

In this chapter, section 2 presents the non-minimal SUSY SO(10) model to be used, section 3 examines in detail the low energy quark-level effective Lagrangian, while section 4 discusses the effective Lagrangian at the hadronic level and presents the branching ratio predictions. Finally, in section 5 a discussion of these predictions and the conclusions that can be drawn from them is given.

4.2.2 The Non-Minimal SUSY SO(10) Model

As mentioned, this analysis is based on the non-minimal SO(10) GUT model of Harvey, Reiss, and Ramond [41], which has been explicitly constructed to generate a mass spectrum (including mixing angles) of the SM fermions from the GUT. An advantage of the choice of SO(10) as the gauge group is that the lowest dimensional chiral representation that accommodates the observed SM fermions is the $\underline{16}$, which allows for the assignment of one family of SM fermions plus a right handed neutrino, and does not include any mirror fermions. This in turn places constraints on the possible Higgs sector representations, since the fermion masses transform under SO(10) as $\underline{16} \times \underline{16} = (\underline{10} + \underline{126})_S + \underline{120}_A$, (where S and A refer to the symmetric and antisymmetric parts respectively), implying that the allowed Higgs reps that couple to fermions to form SO(10) invariant Yukawa terms are the $\underline{10}, \underline{120}$, and the $\overline{\underline{126}}$. The 'realistic' model of Harvey et al. [41] is then constructed from the representations in such a way that SO(10) GUT is broken directly to the SM gauge structure of $SU(3)_C \times SU(2) \times U(1)$, and the GUT scale texture of Yukawa couplings incorporates the up quark mass matrix ansatz of Fritzsch [42] and the down quark and charged lepton mass matrix ansatz of Georgi and Jarlskog [43] in such a way that the Oakes relation [44] results. The cost of such a model is the expansion of the Higgs sector well beyond that of most minimal models. The particle content of this model is given in terms of a $\underline{45}$ that is the adjoint of vector bosons, three families of fermions ($\underline{16}_1, \underline{16}_2, \underline{16}_3$), and a scalar sector composed of a $\underline{54}$, a complex $\underline{10}$, and three families of $\underline{126}$ ($\underline{126}_1, \underline{126}_2, \underline{126}_3$) - all of which are required for a viable spectrum of fermion masses. Note that the phenomenologically observed mass spectrum can be produced without requiring the presence of the $\underline{120}$ rep, which has a Yukawa coupling to the fermions that is antisymmetric in generation indices (as the SM fermions are expressed in terms of a single chirality, and the spin 0 fields occur in a product that is symmetric in Lorentz indices).

The extension [40] of this model to that of a SUSY SO(10) model is straight forward, as the gauge, fermion and conjugate Higgs fields are converted into vector and chiral superfields, giving a superfield content of:

$$\begin{aligned} \text{Vector superfields} &: \underline{45} \\ \text{Chiral superfields} &: \underline{10}, \underline{16}_1, \underline{16}_2, \underline{16}_3, \underline{54}, \overline{\underline{126}}_1, \overline{\underline{126}}_2, \overline{\underline{126}}_3 \end{aligned}$$

This SUSY SO(10) model, like its non-SUSY counterpart, is distinguished by its sophisticated Yukawa texture, composed of the $\underline{10}, \underline{16}$, and $\overline{\underline{126}}$ chiral superfield reps. Specifically, the model is defined in terms of its superpotential, and for the purposes of nucleon decay, the relevant terms of the superpotential for this SO(10) model are

$$
\begin{aligned}
W &= (A\underline{16}_1 \times \underline{16}_2 + B\underline{16}_3 \times \underline{16}_3) \times \overline{\underline{126}}_1 + (a\underline{16}_1 \times \underline{16}_2 + b\underline{16}_3 \times \underline{16}_3) \times \underline{10} \\
&\quad + c(\underline{16}_2 \times \underline{16}_2) \times \overline{\underline{126}}_2 + d(\underline{16}_2 \times \underline{16}_3) \times \overline{\underline{126}}_3 + M_G\underline{10} \times \underline{10} \qquad (4.1)
\end{aligned}
$$

with the superpotential expressed in terms of the SO(10) representations, and A, B, a, b, c, d as the undetermined GUT scale Yukawa couplings.

The beauty of this globally supersymmetric model is that as $\underline{10} \times \underline{10} \supset \underline{1}$ and $\overline{\underline{126}} \times \overline{\underline{126}} \not\supset \underline{1}$, the only SO(10) invariant F-term that contributes to nucleon decay below the spontaneously broken SO(10) GUT scale is given by Figure 4.1.

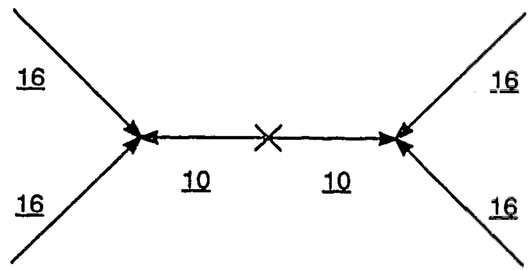

Figure 4.1: The only F-term supergraph that contributes to nucleon decay.

The key point here is that this superfield diagram has a Higgs/Higgsino mass insertion that involves only the $\underline{10}$ (the $\underline{120}$ reps are absent!), which implies that only the GUT scale Yukawa couplings of the $\underline{16}$'s to the $\underline{10}$ are of relevance to the predictions of nucleon decay (i.e. a and b in equation (4.1) are the only relevant couplings). In terms of the particle diagrams, the only tree level diagrams of concern are given in Figure 4.2.

Here the first dimension 5 diagram exhibits the exchange of a Higgsino of GUT scale mass (M_G), and so for momentum below the GUT scale the Higgsino propagator reduces to a factor of $\frac{1}{M_G}$. The second diagram in Figure 4.2 involves the exchange of a GUT scale Higgs scalar whose propagator reduces to $\frac{1}{M_G^2}$, but due to the weighting of the sfermion-sfermion-Higgs trilinear coupling by one power of M_G, the resulting diagram also contributes to the dimension 5 operator with weight $\frac{1}{M_G}$. Thus the effective dimension 5 operator, valid between the SUSY GUT scale and the SUSY breaking scale (assumed to be of order the electroweak scale) is a combination of both diagrams, and is represented by the effective vertex in Figure 4.2.

4.2.3 The Dimension 5 Operators

In order to evaluate these dimension 5 operator contributions to nucleon decay, the superpotential must be re-expressed in terms of the superfields corresponding to the SM content. This may be done in a two step process, which first involves the re-expression of the superpotential in a compact SU(5) notation, followed by a decomposition of the SU(5) superfields into their SM components. Such a decomposition can be used as the F-term of Figure 4.1 is the only dimension 5 contribution to nucleon decay, and it relies only on the Higgs $\underline{10}$ of SO(10) which has the SU(5) decomposition $\underline{10} \rightarrow \underline{5} + \underline{\bar{5}}$. (Note that the SU(5) decomposition of the $\underline{16}$ is $\underline{16} \rightarrow \underline{10} + \underline{\bar{5}} + \underline{1}$.) Thus the superpotential terms that contribute to nucleon decay can be written as

$$W_{SU(5)} = \sqrt{2}\chi_a^{\alpha\beta} M_{ab}^D \psi_{b\alpha} H_{2\beta} - \frac{1}{4}\epsilon_{\alpha\beta\gamma\delta\epsilon}\chi_c^{\alpha\beta} M_{cd}^U \chi_d^{\gamma\delta} H_1^\epsilon + M_G H_1^\alpha H_{2\alpha} \tag{4.2}$$

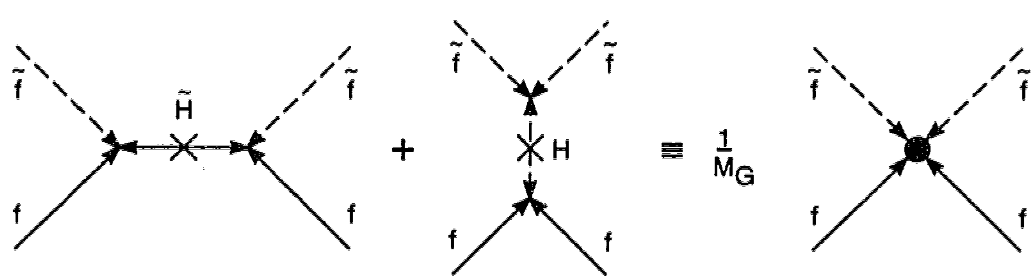

Figure 4.2: The two particle diagrams generated by the F-term supergraph of Figure 4.1.

with $W_{SU(5)}$ being valid at the GUT scale. Here M^U and M^D are 3×3 matrices in generation space that express in a compact form the Yukawa coupling texture expressed in equation (4.1). Below the SO(10) scale, the heavy Higgs superfield can be integrated out to give an effective superpotential (that is appropriate below the GUT scale but above the SUSY breaking scale). This effective superpotential is

$$W^{\text{eff}}_{SU(5)} = \frac{\sqrt{2}}{4M_G}\epsilon_{\alpha\beta\gamma\delta i}\chi^{\alpha\beta}_a M^U_{ab}\chi^{\gamma\delta}_b \chi^{i\epsilon}_c M^D_{cd}\psi_{d\epsilon} \tag{4.3}$$

Here the Greek indices $\alpha, \beta, \ldots$ are SU(5) indices, the family indices are (a, b, c, d), and the index i runs from 1 to 3. Also, the Lorentz structure is suppressed, so as to focus on the generation and SU(5) structure. Restriction to the tree level diagrams relevant to nucleon decay (Figure 4.1) then implies the Yukawa texture matrices for this effective superpotential are of the form

$$M^U = \begin{bmatrix} 0 & a & 0 \\ a & 0 & 0 \\ 0 & 0 & b \end{bmatrix} = M^D \tag{4.4}$$

As it is assumed that this SUSY SO(10) breaks straight to the minimally supersymmetric standard model (MSSM), the superpotential can then be further decomposed into the SM quark and lepton superfields. Typically the decomposition for one family of left-handed SU(5) matter superfields are

$$\psi_\alpha = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ l \\ -\nu \end{bmatrix}_L \qquad \chi^{\alpha\beta} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & U_3 & -U_2 & -u^1 & -d^1 \\ -U_3 & 0 & U_1 & -u^2 & -d^2 \\ U_2 & -U_1 & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -L \\ d^1 & d^2 & d^3 & L & 0 \end{bmatrix}_L \tag{4.5}$$

where U_i, D_i and L_i are the charge conjugations of the right handed SU(2) singlet up, down, and charged lepton fields. Substitution of this decomposition into the superpotential (equation (4.3)), results in an effective superpotential relevant to nucleon decay that is expressed in chiral superfields associated with the SM. The form of the effective superpotential in question is

$$\begin{aligned} W_{\text{eff}}^{\text{SM}} &= \frac{-1}{2M_G}[\frac{\epsilon^{ijk}}{4}L_a M_{ab}^U U_{bi} U_{ck} M_{cd}^D D_{dj} + \frac{\epsilon^{ijk}}{4} U_{ai} M_{ab}^U L_b U_{ck} M_{cd}^D D_{dj} \\ &\quad -\epsilon_{ijk}(u_a^i M_{ab}^U d_b^j - d_a^i M_{ab}^U u_b^j)(u_c^k M_{cd}^D l_d - d_c^k M_{cd}^D \mu_d)] \end{aligned} \tag{4.6}$$

From this superpotential it is clear that as a result of the SU(2) content, there are two classes of F-terms; the $(\mathrm{LLLL})_F$ and the $(\mathrm{RRRR})_F$ terms (here the notation of reference [17] is used to emphasise the SU(2) weak content of the operators). However as the $(\mathrm{RRRR})_F$ terms are antisymmetric in generation indices (a, b, c) - due to the Bose statistics of superfields in a superpotential - their composition is such that they must contain either a charm or a top SU(2) singlet superfield. This superfield generation remains, to a first approximation, unchanged on the dressing of the operator by gluino or bino exchange at the SUSY breaking scale (SU(2) gaugino exchange is forbidden for these singlet superfields), and so the low energy four fermion operator contains either a charm or a top quark. This implies that the $(\mathrm{RRRR})_F$ term contribution to nucleon decay is suppressed, leaving only the $(\mathrm{LLLL})_F$ terms. The effective Lagrangian relevant to nucleon decay is then obtained from the $(\mathrm{LLLL})_F$ term of the superpotential by the usual method ($\mathcal{L}_{\text{Int}} = \frac{1}{2}\sum_{i,j}(\frac{\partial W}{\partial\Phi_i\partial\Phi_j}|_{\Phi=\phi}\psi_i\psi_j + h.c.) - \sum_i |\frac{\partial W}{\partial \Phi_i}|^2_{\Phi=\phi}$ with Φ representing a chiral superfield, and ϕ and ψ the scalar and fermionic parts), which results in vertices composed of two particles and two sparticles. These vertices are then evolved down to the SUSY breaking scale ($\sim O(M_W)$) using the renormalisation group equations, at which point the dimension 5 operator is converted to a dimension 6 operator via gaugino and Higgsino exchanges. This dressing is schematically shown in Figure 4.3.

Of all the gaugino and higgsino exchanges associated with the dressing of the $(\mathrm{LLLL})_F$ dimension 5 operators, the dominant contribution comes from the charged

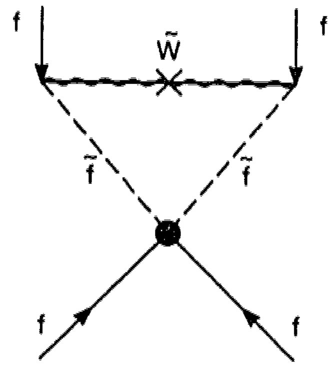

Figure 4.3: The particle diagram for the nucleon decay operators after being dressed by the wino or charged higgsino exchange.

wino. The gluino, neutral gaugino, and neutral Higgsino exchange contributions to the dressed operator are suppressed as their exchange is approximately generation diagonal and their contribution is thereby suppressed due to small Yukawa couplings of the first and second generation fields present in the dimension 5 operators [20, 21]. The charged-Higgsino exchanges are also suppressed, due to their Higgs strength Yukawa couplings to the first and and second generation fermions, thereby leaving the charged wino as the dominant contribution to the loop integral. The calculation of the loop dressing by chargino eigenstates has been performed by Sakai [19], and their implications for nucleon decay rates have been explored for minimal and non-minimal versions of SU(5) [29]. Here however, the simplifying assumption of wino dressing dominance in the decay amplitude is invoked. Performing the loop integration results in a triangle diagram factor, that although it depends on mass eigen-values and mixing angles of the sparticles in the loop, can be approximated (in the pure charged wino exchange limit) by [19]

$$\begin{aligned}\frac{\alpha_2}{2\pi} f(\tilde{u}, \tilde{d}, \tilde{W}) &= g_2^2 \int \frac{d^4k}{i(2\pi)^4} \frac{1}{m_{\tilde{u}}^2 - k^2} \frac{1}{m_{\tilde{d}}^2 - k^2} \frac{1}{m_{\tilde{W}} - \not{k}} \\ &\simeq \frac{\alpha_2}{2\pi} \frac{m_{\tilde{W}}}{m_{\tilde{u}}^2 - m_{\tilde{d}}^2} \left(\frac{m_{\tilde{u}}^2}{m_{\tilde{u}}^2 - m_{\tilde{W}}^2} \ln \frac{m_{\tilde{u}}^2}{m_{\tilde{W}}^2} - \frac{m_{\tilde{d}}^2}{m_{\tilde{d}}^2 - m_{\tilde{W}}^2} \ln \frac{m_{\tilde{d}}^2}{m_{\tilde{W}}^2} \right)\end{aligned} \tag{4.7}$$

and so becomes a multiplive factor of the dressed four fermion operator.

From the superpotential of equation (4.6), the effective Lagrangian for the dressed quark level operators of Figure (4.3) can be obtained, and with the use of equation (4.7) it has the form

$$\begin{aligned}\mathcal{L} &= \frac{\alpha_2}{2\pi M_G} R_S R_L M^U_{ab} M^D_{cd} \epsilon_{ijk} \Big[(u^i_a d^j_b)(d^k_c \nu_d)\{f(u_c, l_d, m_{\tilde{W}}) + f(u_a, d_b, m_{\tilde{W}})\} \\ &+ (d^i_a u^j_b)(u^k_c l_d)\{f(d_c, \nu_d, m_{\tilde{W}}) + f(d_a, u_b, m_{\tilde{W}})\} \\ &+ (u^i_b d^j_c)(u^k_a l_d)\{f(u_c, d_b, m_{\tilde{W}}) + f(d_a, \nu_d, m_{\tilde{W}})\} \\ &+ (d^i_a \nu_d)(d^j_b u^k_c)\{f(u_a, l_d, m_{\tilde{W}}) + f(u_b, d_c, m_{\tilde{W}})\} \Big] + h.c.\end{aligned} \tag{4.8}$$

Here the R_S and R_L are the short and long range renormalisation factors. The short range renormalisation accounts for the renormalisation effects from the SO(10) to the SUSY breaking scale, while the long range factor is from the SUSY breaking scale to a low energy scale (assumed here to be 1 GeV). R_S can be shown to be generation independent, and can be taken to be [20, 21]

$$R_S = \left[\frac{\alpha_3(m_S)}{\alpha_G}\right]^{\frac{-4}{9}} \left[\frac{\alpha_2(m_S)}{\alpha_G}\right]^{\frac{-3}{2}} \left[\frac{\alpha_1(m_S)}{\alpha_G}\right]^{\frac{5}{396}} \simeq 0.91 \tag{4.9}$$

where m_S is the SUSY breaking scale (which here, has been set to the electroweak scale m_W). The long range renormalisation is predominantly a result of QCD interactions between the SUSY scale and 1 GeV, and encompasses the renormalisation of the Yukawa couplings and anomalous dimension corrections to the four fermion operators. Again, following reference [21],

$$R_L \simeq \left[\frac{\alpha_3(1GeV)}{\alpha_3(m_c)}\right]^{\frac{-2}{3}} \left[\frac{\alpha_3(m_c)}{\alpha_3(m_b)}\right]^{\frac{-18}{25}} \left[\frac{\alpha_3(m_b)}{\alpha_3(m_Z)}\right]^{\frac{-18}{23}} \simeq 0.22 \tag{4.10}$$

This effective Lagrangian, as written, is for four fermion operators with the quarks and leptons expressed in their gauge interaction eigenstates; this however, is easily remedied by rotating from a gauge interaction to a mass eigenstate basis. Due to the mismatch in the rotations of the charge $\frac{2}{3}, -\frac{1}{3}$, and -1 fields, the operators incur additional generation mixing. The rotation matrices appropriate to this model are obtained from the diagonalisation of the mass matrices which are defined in terms of the Yukawa coupling texture (as specified in equation (4.1)) and the vevs of the various Higgs reps. For these $\Delta I_W = \frac{1}{2}$ Dirac masses (I_W denotes weak isospin), it is convenient to consider the SO(10) vev contributions in terms of their SU(5) content. The contribution of SU(5) vevs to the quark and charged lepton masses is as follows [41]:

$$
\begin{array}{lcl}
< \ldots > & \sim & \underline{5} \text{ gives a contribution to the charge } \frac{2}{3} \text{ mass} \\
< \ldots > & \sim & \overline{\underline{5}} \text{ gives an equal weight contribution to the charge } -\frac{1}{3} \text{ and } -1 \text{ masses} \\
< \ldots > & \sim & \overline{\underline{45}} \text{ relative weight contribution of } 1:-3 \text{ for charge } -\frac{1}{3} \text{ and } -1 \text{ masses}
\end{array}
$$

The SO(10) Higgs vevs structure is then decomposed as

$$
\begin{array}{rcl}
< \underline{10} > & = & r(\text{along } \overline{\underline{5}}) + p(\text{along } \underline{5}) \\
< \overline{\underline{126}}_1 > & = & t(\text{along } \underline{5}) \\
< \overline{\underline{126}}_2 > & = & s(\text{along } \overline{\underline{45}}) \\
< \overline{\underline{126}}_3 > & = & q(\text{along } \underline{5})
\end{array}
\tag{4.11}
$$

where p, q, r, s, t are taken as complex vevs. The assumption of complex vevs allows for the generation of soft CP violation through the process of symmetry breaking. Yet it is assumed that soft CP violation is not the sole source of CP violation in the model. Hard CP violation is also permitted due to the fact that, unlike the non-SUSY model of reference [41], the Yukawa couplings of equation (4.1) are taken to be complex [40].

As the masses in the low energy effective SUSY theory arise from the Yukawa couplings of the quarks and charged leptons to a single light Higgs doublet of $\Delta I_W = \frac{1}{2}$, the mass matrices can be formulated in terms of the vev of this light Higgs. From the SU(5) decomposition of the SO(10) Higgs vevs, this light Higgs vev is a linear combination of the doublets in the $\underline{10}, \overline{\underline{126}}_1, \overline{\underline{126}}_2$, and $\overline{\underline{126}}_3$ (in the ratio $|r+p| : t : s : q$), and so the GUT scale couplings appearing in the mass matrices can be read off. Yet as it is the mass texture at the SUSY breaking scale that must be diagonalised, the entries in these Yukawa coupling texture matrices at the GUT scale must be evolved down to the SUSY scale via the renormalisation group equations, as was done by Dimopoulos, Hall, and Raby [45] for 'realistic' Yukawa matrices of this form. The quark and charged lepton Yukawa matrices specified at the SUSY breaking scale are then the mass matrices that are diagonalised. From equations (4.1) and (4.11) the GUT and SUSY scale mass matrix textures of the quarks and charged leptons are:

$$\begin{array}{ccc} \text{GUT scale texture} & & \text{SUSY scale texture} \\ U = \begin{bmatrix} 0 & P_G & 0 \\ P_G & 0 & Q_G \\ 0 & Q_G & V_G \end{bmatrix} & \longrightarrow & U = \begin{bmatrix} 0 & P & 0 \\ P & \delta_u & Q \\ 0 & Q & V \end{bmatrix} \\ D = \begin{bmatrix} 0 & R_G e^{i\varphi_G} & 0 \\ R_G e^{-i\varphi_G} & S_G & 0 \\ 0 & 0 & T_G \end{bmatrix} & \longrightarrow & D = \begin{bmatrix} 0 & Re^{i\varphi} & 0 \\ Re^{-i\varphi} & S & \delta_d \\ 0 & 0 & T \end{bmatrix} \\ L = \begin{bmatrix} 0 & R_G & 0 \\ R_G & -3S_G & 0 \\ 0 & 0 & T_G \end{bmatrix} & \longrightarrow & L = \begin{bmatrix} 0 & R & 0 \\ R & -3S & 0 \\ 0 & 0 & T \end{bmatrix} \end{array} \tag{4.12}$$

with the assignments

$$P = ap + At \quad V = bp + Bt$$
$$R = ar \quad T = br \quad S = cs \quad Q = dq \tag{4.13}$$

and the subscript G indicating entries defined at the GUT scale. Here, the zero entries in the mass textures are the result of accidental discrete symmetries, which if broken, allow the generation of non-zero entries by means of the renormalisation group equations as the mass matrices are renormalised down to lower energies. This is indeed the case for the entries δ_u and δ_d which occur due to the violation of a discrete symmetry at the GUT scale.

Although both the Yukawa couplings and the SO(10) Higgs vevs are complex, thereby permitting both hard and soft CP violation, the entries in the mass matrix textures, as given in (4.12), have been rendered explicitly real by means of quark and charged lepton field redefinitions. It is then these (SUSY scale) matrices, with 8 real parameters and one phase, that are diagonalised and the mass eigenvalues fitted to the low energy data, following Dimopoulos, Hall, and Raby [45]. The diagonalisation proceeds by means of unitary and biunitary transformations of the form $U^{\rm diag} = V_u U V_u^\dagger$, $D^{\rm diag} = V_d^L D V_d^{R\dagger}$, and $L^{\rm diag} = V_l L V_l^\dagger$, and in following the assumptions of reference [45], that $V >> Q \sim \delta_u >> P$ and $T >> S \sim \delta_d >> R$, the approximate mixing matrices are of the form

$$\begin{aligned}
V_u &= \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \end{bmatrix} \\
V_d^L &= \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ c_4 & s_4 & 0 \\ -s_4 & c_4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\varphi} & 0 \\ 0 & 0 & e^{i\varphi} \end{bmatrix} \\
V_l &= \begin{bmatrix} c_5 & s_5 & 0 \\ -s_5 & c_5 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{4.14}$$

with $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$. The angles defined in these rotation matrices can then be determined by fitting the mass eigenvalues to the low energy data. Using the low energy input data of [45], the resulting phenomenological fit specifies the angles as

$$s_1 \simeq 0.196 \quad s_2 \simeq 0.05 \quad s_3 \simeq 0.046 \tag{4.15}$$

$$s_4 \simeq 0.0066 \quad s_5 \simeq 0.070 \quad \cos\varphi \simeq 0.41^{+0.22}_{-0.15}$$

This phenomenological fit may need some revision in view of the subsequent and more precise low energy data (especially in light of the recent improvement to the bounds on the CKM matrix entry V_{cb} [46]) but it is expected that any revisions will have small effects on our results, and so we continue to use the original fit.

4.2.4 The Hadronic Lagrangian and Branching Ratios

With the mass eigenstate rotations defined by equations (4.14) and (4.16), the low energy effective Lagrangian of equation (4.8) can be explicitly evaluated in terms of the dimension 6 four fermion quark level operators. In focusing on nucleon decay, these quark level ($qqql$) operators can be restricted by energy conservation, to have a quark composition of only the u, d, and s quarks. This in turn results in only five distinct operators, namely

$$\begin{aligned} &O^q(dud\nu_a) = \epsilon_{ijk}(d^i u^j)(d^k \nu_a) \quad O^q(sud\nu_a) = \epsilon_{ijk}(s^i u^j)(d^k \nu_a) \\ &O^q(uds\nu_a) = \epsilon_{ijk}(u^i d^j)(s^k \nu_a) \\ &O^q(duul_a) = \epsilon_{ijk}(d^i u^j)(u^k l_a) \quad O^q(suul_a) = \epsilon_{ijk}(s^i u^j)(u^k l_a) \end{aligned} \tag{4.16}$$

Thus, the effective Lagrangian, expressed at the quark level, can then be written as $\mathcal{L}_{\text{nucleon}} = \sum C(qqql)O^q(qqql)$. Here the $C(qqql)$'s are the coefficients of the distinct quark level operators $O^q(qqql)$, and are determined from equation (4.8) by summing the coefficients of the equivalent four fermion effective operators, modulo Fierz transformations. By classifying nucleon decay in terms of its various allowed channels, the effective Lagrangian for nucleon decay can be written as

$$\begin{aligned} \mathcal{L}(n,p \to \pi + \bar{\nu}_i) &= C(dudv_i)O^q(dud\nu_i) \\ \mathcal{L}(n,p \to \pi + l_i^+) &= C(duul_i)O^q(duul_i) \end{aligned} \tag{4.17}$$

$$
\begin{aligned}
\mathcal{L}(n,p \rightarrow K + l_i^+) &= C(suul_i)O^q(suul_i) \\
\mathcal{L}(n,p \rightarrow K + \bar{\nu}_i) &= C(sud\nu_i)O^q(sud\nu_i) + C(dus\nu_i)O^q(dus\nu_i)
\end{aligned}
$$

Yet these effective Lagrangian contributions are in terms of quark level operators, and so inappropriate for hadronic decay rate calculations. Instead, they must be converted to effective Lagrangian contributions at the hadronic level, thereby permitting evaluation of the nucleon decay rates, which although calculated at the hadronic level, are expressed in terms of the coefficients of the quark level four fermion effective operators specified by equations (4.8) and (4.17). This conversion may be performed using the chiral Lagrangian techniques developed in references [47], [24], and [25], which express general hadronic level decay widths in terms of coefficients of generic four fermion quark level operators. The results of these decay width calculations, in the notation of [34], are as follows:

$$
\begin{aligned}
\Gamma(p \rightarrow K^+ + \bar{\nu}_i) &= \frac{(m_p^2 - m_K^2)^2}{32\pi m_p^3 f_\pi^2} \left| \frac{2m_p}{3m_B}\mathcal{D}C(sud\nu_i) + \left[1 + \frac{m_p}{3m_B}(\mathcal{D} + 3\mathcal{F})\right] C(dus\nu_i) \right|^2 \\
\Gamma(p \rightarrow \pi^+ + \bar{\nu}_i) &= \frac{m_p}{32\pi f_\pi^2} \left|[1 + \mathcal{D} + \mathcal{F}]C(dud\nu_i)\right|^2 \\
\Gamma(p \rightarrow K^0 + l_i^+) &= \frac{(m_p^2 - m_K^2)^2}{32\pi m_p^3 f_\pi^2} \left| \left[1 - \frac{m_p}{m_B}(\mathcal{D} - \mathcal{F})\right] C(suul_i) \right|^2 \\
\Gamma(p \rightarrow \pi^0 + l_i^+) &= \frac{m_p}{64\pi f_\pi^2} \left|[1 + \mathcal{D} + \mathcal{F}]C(duul_i)\right|^2 \qquad (4.18) \\
\Gamma(p \rightarrow \eta + l_i^+) &= \frac{3(m_p^2 - m_\eta^2)^2}{64\pi m_p^3 f_\pi^2} \left| \left[1 - \frac{1}{3}(\mathcal{D} - 3\mathcal{F})\right] C(duul_i) \right|^2 \\
\Gamma(n \rightarrow K^0 + \bar{\nu}_i) &= \frac{(m_n^2 - m_K^2)^2}{32\pi m_n^3 f_\pi^2} \left| \left[1 - \frac{m_n}{3m_B}(\mathcal{D} - 3\mathcal{F})\right] C(sud\nu_i) \right. \\
&\qquad \left. + \left[1 + \frac{m_n}{3m_B}(\mathcal{D} + 3\mathcal{F})\right] C(dus\nu_i) \right|^2 \\
\Gamma(n \rightarrow \pi^0 + \bar{\nu}_i) &= \frac{m_n}{64\pi f_\pi^2} \left|[1 + \mathcal{D} + \mathcal{F}]C(dud\nu_i)\right|^2 \\
\Gamma(n \rightarrow \pi^- + l_i^+) &= \frac{m_n}{32\pi f_\pi^2} \left|[1 + \mathcal{D} + \mathcal{F}]C(duul_i)\right|^2 \\
\Gamma(n \rightarrow \eta + \bar{\nu}_i) &= \frac{3(m_n^2 - m_\eta^2)^2}{64\pi m_n^3 f_\pi^2} \left| \left[1 - \frac{1}{3}(\mathcal{D} - 3\mathcal{F})\right] C(dud\nu_i) \right|^2
\end{aligned}
$$

Here $m_B \equiv m_\Sigma = m_\Lambda = 1150$ MeV is the mass to be associated with the virtual baryon exchange, $m_n = m_p$ is the nucleon mass, and $\mathcal{D} = 0.81$ and $\mathcal{F} = 0.44$ are numerical factors.

From these decay rates, it is then very simple to construct branching ratios, which have the advantage over decay rates in that most of the as yet unspecified factors hidden in the quark level operators $C(qqql)$ divide out, leaving the branching ratios parameterised by the ratio of the GUT scale Yukawa couplings of the complex $\underline{10}$. The numerical predictions for the branching ratios of the most dominant proton and neutron decay channels, for a large range of this parameter, $\frac{a}{b}$, are presented in Figures 4.4 and 4.5 respectively. For this numerical evaluation, the branching ratios are defined as

$$Br(N \to x + y) = \frac{\Gamma(N \to x + y)}{\Gamma(N \to \text{anything})} \tag{4.19}$$

where N represents either the nucleon, and the decay rate for $N \to$ anything has been taken as the sum of all the relevant decay rates listed in (4.18).

4.3 Conclusions

With the results of the analysis of nucleon decay in this non-minimal SUSY SO(10) model presented in Figures 4.4 and 4.5, a number of important conclusions can be drawn. The first and most significant point is that this model gives one-parameter predictions for all the relevant nucleon decay branching ratios. Once nucleon decay is observed through any two channels, the ratio $\frac{a}{b}$ is determined, and all the remaining partial lifetimes of the proton and the neutron then have a definite prediction. As with the SUSY SU(5) models, this model predicts that for a large region of $\frac{a}{b}$ parameter space, $p \to K^+ + \bar{\nu}_\mu$ and $n \to K^0 + \bar{\nu}_\mu$ are the most dominant proton and neutron

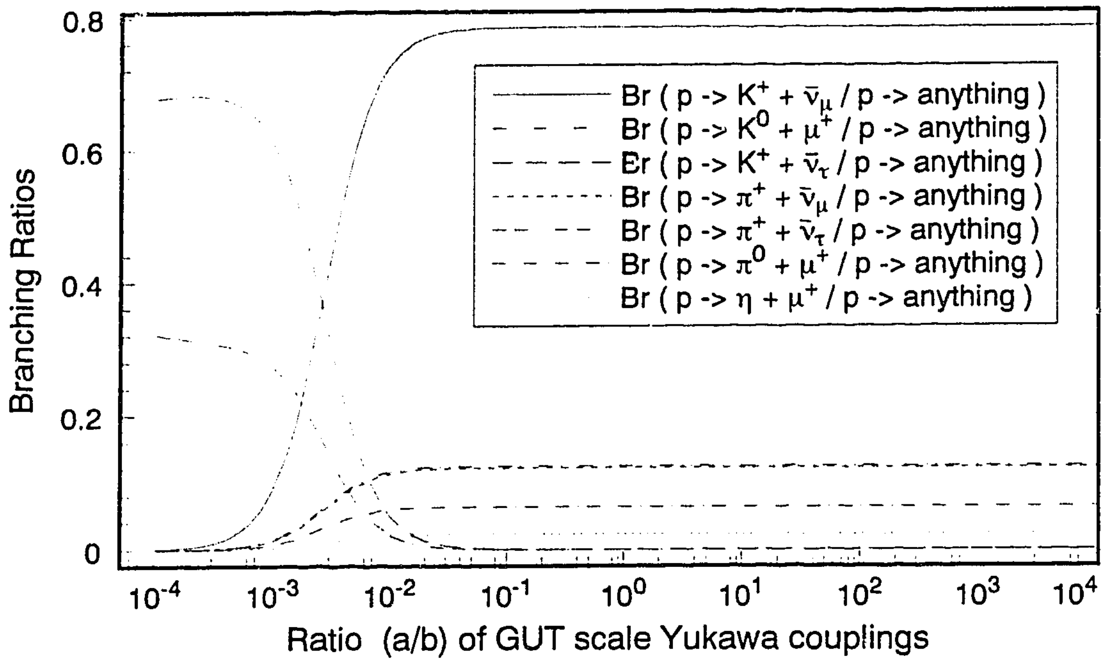

Figure 4.4: The branching ratios of the most dominant proton decay channels.

decay modes. This prediction could only be altered by a strong suppression of the GUT scale Yukawa coupling a relative to the third family self-coupling b, as shown by the prominence of the $p \rightarrow K^+ + \bar{\nu}_\tau$, $p \rightarrow \pi^+ + \bar{\nu}_\tau$, $n \rightarrow K^0 + \bar{\nu}_\tau$, and $n \rightarrow \pi^0 + \bar{\nu}_\tau$ decay modes for $\frac{a}{b} < 3 \times 10^{-2}$. Another striking feature is that for $\frac{a}{b} > 10^{-2}$ the branching ratio predictions are insensitive to the actual value of the parameter, thereby implying a degree of robustness to the predictions, regardless of the relative importance of the $\underline{10}$ of SO(10) in the assumed form of the GUT scale texture.

However, it is the relative strengths of some of the individual branching ratios that serve to identify this model, and in particular, it is the nucleon decay channels involving the μ^+ and the $\bar{\nu}_\mu$ that are the distinctive fingerprints of this model. For both the proton and the neutron, the branching ratio predictions for channels involving the charged muon show a marked enhancement over corresponding predictions of minimal SUSY SU(5). Specifically, the branching ratio predictions for the $p \rightarrow K^0 + \mu^+$,

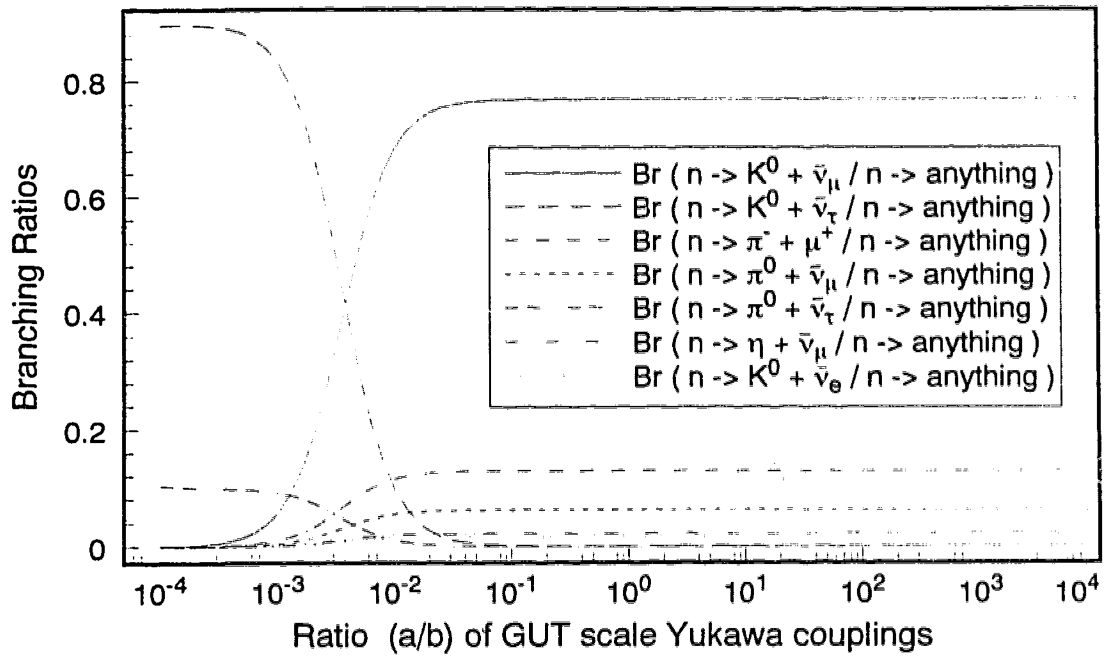

Figure 4.5: The branching ratios of the most dominant neutron decay channels.

$p \to \pi^0 + \mu^+$, and $n \to \pi^- + \mu^+$ relative to the dominant proton and neutron decay channels are enhanced over the minimal SUSY SU(5) predictions by factors of 50-500, 10-100, and 20-200 respectively (the ranges given in these enhancement factors are due to the uncertainty of the minimal SUSY SU(5) predictions as quoted by [29, 25, 34]). To a lesser extent, the $p \to \pi^+ + \bar{\nu}_\mu$ and $n \to \pi^0 + \bar{\nu}_\mu$ decay channels show a similar enhancement, but only by factors of 3.6 and 2.6 respectively. Thus, these enhancements in the decay rate predictions result in branching ratios for this non-minimal SUSY SO(10) model that are both qualitative and quantitatively different from that of the SUSY SU(5) nucleon decay spectrum, thereby making this 'realistic' non-minimal model a testable candidate for a SUSY GUT extension to the standard model. The issue of testing the predictions of this model could be addressed at Super-KAMIOKANDE, provided that Super-KAMIOKANDE in fact observes nucleon decay.

In sum, the distinctive tests of this realistic supersymmetric SO(10) GUT which arise from the consideration of nucleon decay come not from the actual decay rates or partial lifetimes of the nucleon, as the nature of the Higgs sector and the uncertainty of the Higgs and Higgsino colour triplet masses make the SUSY dimension 5 operator decay rate predictions uncertain. Rather, they come from the calculation of nucleon decay branching ratios. The fact that this realistic model predicts ratios of branching ratios $\frac{Br(p\to K^0+\mu^+)}{Br(p\to K^++\bar{\nu}_\mu)}$, $\frac{Br(p\to\pi^0+\mu^+)}{Br(p\to K^++\bar{\nu}_\mu)}$, and $\frac{Br(n\to\pi^-+\mu^+)}{Br(n\to K^0+\bar{\nu}_\mu)}$ of order 20%, shows the relevance of 'observable' channels such as $p \to K^0 + \mu^+$, $p \to \pi^0 + \mu^+$, and $n \to \pi^- + \mu^+$ to the testing of models of GUT unification. (For related considerations involving mass textures induced by higher dimensional operators see [48].) These enhanced branching ratio predictions are instead simply a result of the composition of the Higgs superfield sector, which is such that the GUT scale Yukawa couplings relevant to nucleon decay are not the full set of couplings that contribute to SM fermion mass generation.

The results presented here may be seen as some of the possible implications of a viable SUSY GUT model, and any observation of $p \to K^0 + \mu^+$, $p \to \pi^0 + \mu^+$ or $n \to \pi^- + \mu^+$ at a level significantly enhanced above the expected SUSY SU(5) predictions is an indication that the underlying structure of a realistic extension to the standard model is best described in terms of a SUSY GUT model with a non-minimal Higgs sector. Unfortunately, because only a partial set of the GUT scale Yukawa couplings is directly involved in the analysis of nucleon decay, whereas the light Higgs is a linear combination of contributions from the various SO(10) Higgs reps, the actual values of the GUT scale couplings remain undetermined and the texture unexplained, at least in this model.

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CHAPTER 5

Conclusion

With the three case studies on the implications of symmetry breaking for low energy experimental signatures completed, it is time to take stock. Clearly, from the results presented, some definite conclusions can be reached, both on the individual chapter results, and on the overall study.

First the individual chapters. The work in chapter 2 results in a very definite statement - no experimental signature of Higgs bag formation around a toponium bound state is expected. This essentially kills any discussion on the use of non-linear feedback produced by the strong Higgs-top coupling, as any effect that is there is at the present time, unobservable. Further, this work shows that the degree of relevance of the debate on the classical and/or semiclassical nature of the Higgs bag formation is rather small, as any hypothesis is virtually unaccessible to experimental verification. This conclusion is further supported by the fact that observation of the $t \rightarrow b + W^+$ decay is without anomalous signals that could be associated with a Higgs bag explosion.

Chapter 3 provides a conclusion that is a little more upbeat, in that the outcome of the investigation is a novel class particles. The results of considering a biased discrete symmetry (either accidental or residual) on an unspecified extension of the Higgs sector has shown that the production of composite particles is possible. The composite particles, or Fermi Balls as they were dubbed, form a rather odd low energy state, that if observed would be definite evidence of physics beyond the Standard

Model. While many physical properties of these novel astrophysical objects are model dependent, the Fermi Ball construction was specifically done in a model independent way. Nevertheless, such a construction does yield some discernible Fermi Ball characteristics, namely that they would be very massive, small in size, slow moving (unless accelerated by some cosmic dynamo), and almost certainly quite weakly interacting. These properties would suggest Fermi Balls are very difficult to detect. Evidence for their existence is most likely to come from dedicated experiments that focus on the very massive nature of these nuclearite-type particles, rather than from searches based on their model dependent characteristics. In lieu of direct evidence, one is restricted to astrophysical constraints such as the one derived from the solar luminosity argument, but these are typically not overly constraining. Due to the difficulty of defining the specifics of model independent Fermi Ball constructions, one soon realises that these objects make ideal, if somewhat novel, WIMP dark matter candidates.

The results of the third study are the most definitive of the three, and they should be of considerable interest to those pushing back the lower bound on nucleon lifetimes. Unlike the Higgs sector extension associated with the production of Fermi Balls, the Higgs sector prescribed in the evaluation of the non-minimal supersymmetric $SO(10)$ GUT of chapter 4 is very specific. It is in fact the non-standard Higgs sector that is responsible for the phenomenological predictions that come from this work. Firstly, the expansion of the Higgs sector has been large, with the three copies of the $SO(10)$ $\overline{126}$ representations implying a dramatic increase in the number of Higgs fields. Although some may question the validity of such a non-minimal Higgs sector, it does achieve an acceptable low energy fermion mass spectrum for the Standard Model fermions, which is essential if the Standard Model extension is to be realistic. Also the multitude of Higgs states present are conveniently hidden by the high energy scale GUT breaking - in a sense, all the uncertainty in the Standard

Model extension has been put into the Higgs sector and pushed to a high scale by the GUT scheme. Secondly, the imposition of supersymmetry does not just "double" the Higgs sector, but rather, shows that the Higgs sector is not necessarily composed of only scalar Higgs fields, and that thanks to Higgsino (as well as squark and slepton) states, the door is opened for "new" nucleon decay channels. The results of chapter 4 indicate is that when nucleon decay is observed, the relevant branching fractions of the dominant channels can be used to determine the validity of this realistic non-minimal supersymmetric $SO(10)$ GUT. As has been seen, the branching fraction predictions of this model make definite model-specific predictions, and are a classic example of employing phenomenological signatures in the low energy limit to unravel the underlying high energy theory.

From these case studies, it is hoped that the reader has realised the usefulness of internal symmetries in particle physics. Certainly, it is seen that the deduced low energy predictions can cover the whole spectrum of relevance (negative conclusions, novel proposals, and definitive predictions), but this is just a characteristic of the variety of ways that symmetry breaking can manifest itself. Further, it should also be noted that the implications of the internal symmetries and their breakings became more far reaching with each new case study. In a sense, this reflects the increase in exoticness of the internal symmetry structure. Obviously, the supersymmetric $SO(10)$ GUT case study was the most exotic, and as such has the most far reaching and exciting predictions.

Nevertheless, it is hoped that this thesis has shown that the consideration of the issues of symmetry breaking and superunification is well worth the while, and that symmetry breaking does much more than simply generate mass.