A Comparison of Rheological Drag Reduction in Wall Turbulence Using Different Additives

by

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Abstract

It is well known that long-chain polymers and surfactants can significantly reduce the skin-friction drag of turbulent liquid flows; a phenomenon often referred to as rheological drag reduction. However, it is unclear if the mechanism for drag reduction is common among different types of polymers and surfactants. In the present dissertation the rheology and drag-reducing capabilities of three different additives are compared, including a flexible polymer, a rigid polymer and a cationic surfactant. Educated predictions regarding each additives mechanism for drag reduction are made.

Aqueous solutions of flexible polymers exhibit viscoelastic non-Newtonian rheology, and a good ability to reduce drag in turbulent channel and boundary layer flows. Measurements of steady shear rheology indicate that drag-reducing flexible polymer solutions are only marginally shear thinning. That being said, the same solutions have an appreciable extensional relaxation time, as demonstrated by extensional rheology measurements using a capillary break up extensional rheometer (CaBER) and dripping onto substrate (DoS) rheometer. In a turbulent channel flow with a Reynolds number $Re$ of approximately 30 000, flexible polymer solutions achieve drag reduction (DR) percentages as large as 70% and a mean velocity profile that straddles the maximum drag reduction (MDR) limit. A turbulent boundary layer comprised of flexible polymers with low amounts of DR, indicate that skin-friction drag is reduced from a near-wall attenuation of vorticity and extensional flow motions – particular biaxial extension. As a result, the polymer-laden boundary layer exhibits more two-dimensional and shear-dominate flow within the conventional limits of the buffer layer, indicative of an expansion in the viscous sublayer and flow parabolization.

Similar to flexible polymers, solutions of rigid polymers can exhibit large amounts of DR in a turbulent channel flow; however, the mechanism for reducing drag in rigid polymers is seemingly different. A rigid polymer solution that is capable of imparting the same amount of DR as a flexible polymer solution in a turbulent channel flow tends to have a larger overall shear viscosity, more shear thinning, but no measurable extensional relaxation time using CaBER and DoS. Therefore, drag reduction using rigid polymers is largely
driven by the shear thinning rheology of the solution. Gradients in the mean velocity coupled with the solutions shear thinning rheology generate an effective slip within the buffer layer and a reduction in skin friction drag.

Unlike the polymeric solutions, drag-reducing solutions of cationic surfactants do not have a shear thinning viscosity, nor do they have a measurable extensional relaxation time from CaBER and DoS rheometry. Instead, surfactant solutions have a shear and extensional rheology similar to water, despite their ability to achieve a large DR of 70% in a turbulent channel flow. To discern the non-Newtonian qualities of the surfactant solution, the laminar flow of the drag-reducing fluids were compared in a periodically constricted tube (PCT), where the tube walls vary sinusoidally with respect to the streamwise direction. Although the PCT flow is not rheometric, it is also not as complex as wall turbulence and provides a comparison among the polymeric and surfactant fluids in a nontrivial flow with mixed kinematics. Above an $Re$ of 100 within the PCT, certain surfactant solutions exhibit a similar inertioelastic flow pattern as flexible polymers. Due to the sudden onset of inertioelastic flow with increasing $Re$, evidence is provided that flow-induced structures develop within the surfactant solutions. These flow-induced structures produce similar rheological features as flexible polymers, and most likely, a common means for reducing drag.
Preface

The content in Chapters 5 and 6 of the current thesis were published in the *Journal of Fluid Mechanics*:


Chapter 7 contains content published in a separate article, but also in the *Journal of Fluid Mechanics*:


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The content in Chapter 9 has been submitted for publication in the *Journal of Fluid Mechanics*.

I am responsible for generating all of the content contained within this thesis. In creating this work I performed an extensive review of the relevant literature, formulated the project objectives and methodology, designed the necessary facilities and experiments, collected and processed the experimental data, and assembled the content into a manuscript for journal publication. My PhD supervisor, Dr. Sina Ghaemmi, provided guidance and was co-author on all of the previously listed publications.
Dedicated to Dr. Joseph Warwaruk P.Eng, PhD, Professor Emeritus
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To the many colleagues I have made at University of Alberta, much of the work in this thesis was made easier from our conversations, your guidance and our time-spent helping one another in the lab. In particular, Bayode Owolabi helped teach and train me on rheology. Bradley Gibeau was very instrumental in helping me prepare for my candidacy and final examinations (including providing me with a formatting template for this thesis). Sen Wang and Prashant Das were both helpful when it came to setting up my measurement equipment. Wagih Abu Rowin provided me with the initial code, which I then adapted, for processing the Lagrangian velocity measurements of my first work.

During my time as a graduate student, the help and encouragement from my parents, Dave and Corinne Warwaruk, has been consistent. I am truly privileged to have grown up in such a loving home with two incredibly talented and caring people.
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⟨⋯⟩ Ensemble average of a quantity
· · · Average along streamwise x-direction in the PCT
∥⋯∥ L2-norm of a vector
\( \mathcal{R}(\cdots) \) standard deviation of a quantity

\( A \) Fitting coefficient for EC thinning regime
\( a \) Fitting coefficient for CY viscosity model
\( B \) Logarithmic law of the wall intercept
\( C_f \) Skin-friction coefficient
\( C_{f,N} \) Newtonian skin-friction coefficient
\( C_{f,NN} \) non-Newtonian skin-friction coefficient
\( c \) Additive concentration
\( D_0 \) Nozzle diameter of the DoS rheometer
\( D_h \) Hydraulic diameter
\( D_{mid} \) Mid-point diameter measured using CaBER
\( D_{min} \) Minimum diameters measured using DoS
\( D \) Rate of deformation tensor
\( d_p \) Seeding particle diameter
\( d \) Fluctuating rate of deformation tensor
\( De \) Deborah number
\( DR \) Drag reduction
\( DR_1 \) Drag reduction based on pressure drop
\( DR_2 \) Drag reduction based on fit of linear viscous sublayer
\( F_γ \) Strain coefficient
\( F_τ \) Stress coefficient
\( f \) Focal length
\( f \) Body force vector
\( Fr \) Froude number
\( G \) Elastic modulus
\( G' \) Gain modulus
\( G'' \) Loss modulus
\( G^* \) Complex modulus
\( H \) Full channel height
\(h\) Half channel height
\(h_{DG}\) Average gap width of double gap geometry
\(h_{PP}\) Gap height between parallel plates
\(I\) Single gap cylinder moment of inertia
\(I_{max}\) Maximum image intensity
\(\mathcal{K}\) Kinematical vorticity number
\(k\) Shear thinning flow index
\(L_{SG}\) Immersion height of single gap geometry
\(L_{DG}\) Immersion height of double gap geometry
\(L\) Velocity gradient tensor
\(M\) Shear thinning consistency
\(\dot{m}\) Mass flow rate
\(N\) Total number of samples
\(Oh\) Ohnesorge number
\(P\) Static pressure
\(P_D\) First invariant of the rate of deformation tensor
\(P_{DG}\) First invariant of the velocity gradient tensor
\(Q\) Volumetric flow rate
\(Q_{DG}\) Second invariant of the rate of deformation tensor
\(Q_{L}\) Second invariant of the velocity gradient tensor
\(Q_W\) Second invariant of the rate of rotation tensor
\(R\) Average radius of the PCT wall
\(R_1\) Inside radius of fixed double gap cylinder
\(R_2\) Outside radius of fixed double gap cylinder
\(R_3\) Inside radius of rotating double gap cylinder
\(R_4\) Outside radius of fixed double gap cylinder
\(R_D\) Third invariant of the rate of deformation tensor
\(R_L\) Third invariant of the velocity gradient tensor
\(R_i\) Minimum radius of the PCT geometry
\(R_{max}\) Maximum radius of the single gap geometry
\(R_{min}\) Minimum radius of the single gap geometry
\(R_o\) Maximum radius of the PCT geometry
\(R_{PP}\) Radius of the parallel plates
\(R_{uu}\) Two-point correlation of velocity fluctuations
\(R_w\) Wall radius of the PCT
\(R_{\mu'\mu'}\) Two-point correlation of viscosity fluctuations
\(Re\) Reynolds number of the PCT
\(Re_H\) Reynolds number based on full channel height
Re\textsubscript{b} Reynolds number based on hydraulic diameter
Re\textsubscript{d} Reynolds number of entrance region to PCT
Re\textsubscript{pp} Reynolds number of the parallel plate geometry
Re\textsubscript{\phi} Reynolds number based on momentum thickness
Re\textsubscript{\tau} Friction Reynolds number
r Radial direction
SF Velocity profile shape factor
St Stokes number
T Rheometer torque
Ta Taylor number
Tr Trouton ratio
T Large eddy turnover time
t Time
t\textsubscript{b} Filament break up time
t\textsubscript{e} Elastic relaxation time
t\textsubscript{f} Flow time scale
t\textsubscript{p} Particle response time
t\textsubscript{v} Viscous time scale
t\textsubscript{R} Rayleigh time
U Instantaneous streamwise velocity
U\textsubscript{0} Centreline velocity
U\textsubscript{\infty} Free-stream velocity
U\textsubscript{H} Velocity of upper moving plate
U\textsubscript{b} Bulk velocity
U\textsubscript{r} Radial velocity component
U\textsubscript{x} Streamwise velocity component
U\textsubscript{\theta} Angular velocity component
U Instantaneous velocity vector
u Fluctuating streamwise velocity
u\textsubscript{f} Representative flow velocity
u\textsubscript{p} Particle settling velocity
u\textsubscript{\tau} Friction velocity
u Fluctuating velocity vector
V Instantaneous wall-normal velocity
V\textsubscript{A} Vorticity advection
V\textsubscript{SD} Vorticity solvent diffusion
v Fluctuating wall-normal velocity
W Instantaneous spanwise velocity or channel width
w Fluctuating spanwise velocity
\( \textbf{W} \) Rate of rotation tensor
\( Wi \) Weissenberg number
\( x \) Streamwise coordinate
\( x \) Position vector
\( y \) Wall-normal coordinate
\( y_v \) Thickness of the linear viscous sublayer
\( z \) Spanwise coordinate
\( \alpha \) Multiplicative pre-factor in equation for IC thinning
\( \Gamma \) Eigenvalues of the rate of deformation tensor
\( \gamma \) Shear strain
\( \gamma_0 \) Shear strain amplitude
\( \dot{\gamma} \) Shear strain rate
\( \dot{\gamma}_w \) Wall shear rate
\( \dot{\gamma} \) Strain rate based on invariants in the rate of deformation tensor
\( \Delta \) Discriminant of the velocity gradient tensor
\( \Delta_D \) Discriminant of the rate of deformation tensor
\( \delta \) Boundary layer thickness
\( \delta^* \) Displacement thickness
\( \delta_v \) Viscous length scale of turbulent wall flow
\( \epsilon \) Amplitude of the PCT walls
\( \dot{\epsilon} \) Extensional strain rate
\( \zeta \) Indicator function
\( \theta \) Angular position
\( \kappa \) Von Kármán constant for the logarithmic law of the wall
\( \Lambda \) Eigenvalues of the velocity gradient tensor
\( \lambda \) Wavelength of the PCT wall
\( \mu \) Dynamic viscosity
\( \mu' \) Dynamic viscosity fluctuations
\( \tilde{\mu} \) Pseudo-viscosity
\( \mu_0 \) Zero-shear-rate viscosity
\( \mu_{\infty} \) Infinite-shear-rate viscosity
\( \mu_E \) Extensional viscosity
\( \mu_s \) Solvent viscosity
\( \mu_w \) Dynamic viscosity at the wall
\( \nu \) Kinematic viscosity
\( \nu_w \) Kinematic viscosity at the wall
\( \xi \) Divergence error
\( \rho \) Density
\( \rho_p \) Density of tracer particles
\( \sigma \) Surface tension
\( \tau \)  Shear stress
\( \tau_0 \)  Shear stress amplitude
\( \tau'_0 \)  In-phase shear stress amplitude
\( \tau''_0 \)  Out-of-phase shear stress amplitude
\( \tau_R \)  Reynolds shear stress
\( \tau_v \)  Viscous shear stress
\( \tau'_v \)  Fluctuating viscous shear stress
\( \tau_w \)  Shear stress at the wall
\( \tau_{w,1} \)  Shear stress at the wall based on pressure drop
\( \tau_{w,2} \)  Shear stress at the wall based on fit of linear viscous sublayer
\( \tau \)  Stress tensor
\( \tau_{nn} \)  Non-Newtonian stress tensor
\( \tau_s \)  Solvent stress tensor
\( \tau_w \)  Shear stress at the wall
\( \phi \)  Momentum thickness
\( \psi \)  Phase offset between \( G' \) and \( G'' \)
\( \Omega \)  Angular velocity of steady shear viscosity measurements
\( \omega \)  Angular frequency of dynamic shear viscosity measurements
\( \omega \)  Vorticity vector
## List of Abbreviations

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<th>Description</th>
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<tr>
<td>2D</td>
<td>Two-dimensional</td>
</tr>
<tr>
<td>3D</td>
<td>Three-dimensional</td>
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<tr>
<td>CaBER</td>
<td>Capillary break-up extensional rheometer</td>
</tr>
<tr>
<td>C14</td>
<td>Trimethyl Tetradecyl Ammonium Chloride</td>
</tr>
<tr>
<td>CY</td>
<td>Carreau-Yasuda model</td>
</tr>
<tr>
<td>DG</td>
<td>Double gap geometry</td>
</tr>
<tr>
<td>DNS</td>
<td>Direct numerical simulation</td>
</tr>
<tr>
<td>DOF</td>
<td>Depth of focus</td>
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<tr>
<td>DoS</td>
<td>Dripping onto substrate</td>
</tr>
<tr>
<td>DRA</td>
<td>Drag-reducing additive</td>
</tr>
<tr>
<td>EC</td>
<td>Elastocapillary</td>
</tr>
<tr>
<td>EIT</td>
<td>Elastoinertial turbulence</td>
</tr>
<tr>
<td>FOV</td>
<td>Field of view</td>
</tr>
<tr>
<td>GN</td>
<td>Generalized Newtonian</td>
</tr>
<tr>
<td>HDR</td>
<td>High drag reduction</td>
</tr>
<tr>
<td>IC</td>
<td>Inertiocapillary</td>
</tr>
<tr>
<td>IW</td>
<td>Interrogation window</td>
</tr>
<tr>
<td>JPDF</td>
<td>Joint probability density function</td>
</tr>
<tr>
<td>LDR</td>
<td>Low drag reduction</td>
</tr>
<tr>
<td>LVE</td>
<td>Linear viscoelastic</td>
</tr>
<tr>
<td>MDR</td>
<td>Maximum drag reduction</td>
</tr>
<tr>
<td>PAM</td>
<td>Polyacrylamide</td>
</tr>
<tr>
<td>PCT</td>
<td>Periodically constricted tube</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability density function</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle image velocimetry</td>
</tr>
<tr>
<td>PP</td>
<td>Parallel plate geometry</td>
</tr>
<tr>
<td>PSV</td>
<td>Particle shadow velocimetry</td>
</tr>
<tr>
<td>PTU</td>
<td>Programmable timing unit</td>
</tr>
<tr>
<td>PTV</td>
<td>Particle tracking velocimetry</td>
</tr>
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</table>
SI  Sisko model
SIS  Shear-induced structure
STB  Shake the box
TBL  Turbulent boundary layer
VGT  Velocity gradient tensor
VOI  Volume of interest
XG  Xanthan gum
Part I

Background
Chapter 1

Introduction

1.1 Motivation

Turbulence is a flow state characterized by a chaotic velocity field that varies in both time and space (Davidson, 2015). The chaos and disorder that is associated with turbulence promotes enhanced energy dissipation, mixing, heat transfer, and drag. Turbulent flows are ubiquitous in engineering applications, many of which involve flows that are bounded by one or more solid surfaces. In some instances, turbulence can be undesirable. For example, the efficiency of fluid transport in thermofluid systems is encumbered by the additional drag needed to sustain turbulence in pipes and ducts. In other circumstances, turbulence can be advantageous; turbulence has been known to prevent stall on aircraft wings and enhance heat transfer in heat exchangers. Turbulence control techniques seek to improve the desired outcomes of these engineering applications by amplifying or attenuating the turbulent motions within the flow.

Turbulence control techniques can be categorized as active or passive (Ghaemi, 2020). Active flow control methods include sensors and actuators that detect and respond to disturbances within the flow (Cattafesta III & Sheplak, 2011). On the other hand, passive techniques are more simple. Many passive techniques involve modifications to the geometry of the solid boundary. For example, the dimples on a golf ball promote the formation of a turbulent boundary layer that delays flow separation, shrinks the size of the wake, and decreases pressure drag (Quintavalla et al., 2013). Other passive control techniques involve modifications to the fluid. Dissolving trace amounts of polymers or surfactants into liquid wall-bounded flows has been a common method for attenuating turbulence and reducing skin friction drag (Qi & Zakin, 2002; White & Mungal, 2008). Dissolving these additives into a compatible liquid solvent produces a complex fluid with non-Newtonian rheology. Therefore, this passive flow control method is often referred to as rheological drag reduction (Graham, 2014).

Discovered in the late 1940s, rheological drag reduction is arguably one of the most successful, yet poorly understood passive flow control strategies (Toms, 1948; Mysels, 1949; Graham, 2014). A variety of industries have largely benefited from the use of polymers and surfactants for drag reduction or enhanced mass flow. For example, the Trans Alaska Pipeline System used polymers to increase their daily oil delivery by 300,000 barrels per day – a 20% improvement to their daily throughput (Burger et al., 1980). Japan has adopted the use of surfactants in over 180 district heating and cooling facilities, some of which have demonstrated a 60% reduction in their total energy demands (Saeki, 2014). Despite its utilization in several
industry applications, key aspects of rheological drag reduction remain unknown. Generally, researchers have yet to determine the mechanism by which polymers and surfactants attenuate turbulence and reduce skin friction drag.

There are incentives to better understanding the mechanism of rheological drag reduction. From an engineering perspective, this would help develop design tools and methods for better predicting the performance of drag-reducing additives (DRAs) in various industry applications. From a scientific perspective, understanding how non-Newtonian rheology interacts with turbulence can provide insights into the physics of fluid turbulence, of which much is still unknown (White & Mungal, 2008). Motivated by the possibility of improving the efficiency of thermo-fluid applications and better understanding the physics of rheology, fluid turbulence and turbulence control, the present research attempts to determine the traits and mechanism of rheological drag reduction for different DRAs. The remainder of this chapter summarizes existing knowledge of polymer and surfactant DRA rheology and theories regarding their rheological mechanism for drag reduction. Lastly, the objective for the current thesis is presented, followed by an overview of the thesis layout.

1.2 Rheology of polymer drag-reducers

Polymer drag-reducers are classified as having either a flexible or a rigid molecular structure (Virk & Wagger, 1990). When dissolved in a Newtonian solvent, both flexible and rigid polymers form a solution that is generally shear thinning (Escudier et al., 1999; Pereira et al., 2013). However, the rheological traits that are typically attributed to drag reduction are extensional viscosity and viscoelasticity (Lumley, 1973; de Gennes, 1990). For solutions of flexible polyacrylamide polymers, Owolabi et al. (2017) demonstrated a correlation between the amount of drag reduction in various turbulent duct flows and a characteristic relaxation time. The latter feature was obtained based on measurements of extensional stress growth in a small liquid filament contained between two rapidly displaced parallel plates using a device known as a capillary breakup extensional rheometer (CaBER). However, such a relaxation time has not been reported for samples of rigid polymer solutions. The filament of the rigid polymer solutions tends to break up rapidly upon extension using the standard CaBER systems, owing to its significantly lower extensional viscosity (Pereira et al., 2013; Mohammadtabar et al., 2020). With regards to viscoelasticity, a correlation between the elastic moduli of polymer solutions and drag reduction has yet to be confirmed experimentally (Pereira et al., 2013; Mohammadtabar et al., 2020). Therefore, a common rheological property among flexible and rigid polymer solutions that correlates with their ability to reduce drag has not been determined.

1.3 Rheology of surfactant drag-reducers

Drag-reducing solutions of surfactants can exhibit various rheological characteristics depending on the type of surfactant and the canonical flow. Qi & Zakin (2002) summarized the three qualities of dilute surfactant solutions that are of significance to drag reduction: shear-induced structures (SISs), viscoelasticity and a large extensional viscosity. The latter two properties share similarities with polymeric solutions, while SISs allude to a structural transformation of the surfactant molecules caused by deformation of the fluid. Shear-induced
structures are best demonstrated in steady shear viscosity measurements. At sufficiently low shear rates the shear viscosity is Newtonian, but above a critical shear rate the viscosity increases (i.e., shear thickening). After increasing the shear rate further, the viscosity begins to decrease, becoming shear thinning. While certain surfactant solutions exhibit SISs, viscoelasticity and a large extensional viscosity, some surfactant solutions only show one, or occasionally none of these rheological traits. Lin (2000) observed that several dilute surfactant solutions had a Newtonian shear viscosity (i.e., no SISs or shear thinning), no elasticity and a Newtonian resistance to uniaxial extension. Yet, the same dilute solutions could produce large amounts of drag reduction in a turbulent pipe flow – around 70%. Therefore, a rheological property of surfactant solutions that correlates with their ability to reduce drag remains unknown.

1.4 Non-Newtonian rheology and turbulence

Understanding how non-Newtonian rheology interacts with turbulence is critical to unravelling the mechanism of rheological drag reduction. Although experimental investigations have correlated drag reduction with certain rheological features of the solution (Owolabi et al., 2017), a particular interaction between the non-Newtonian rheology and the coherent patterns within the turbulent flow has yet to be determined. For example, drag-reducing flexible polymer solutions were shown to have an appreciable extensional viscosity, but it is unclear how this large extensional viscosity can reduce drag in a turbulent wall flow. It is counter-intuitive that a solution with a larger viscosity than its solvent would have less skin friction drag. Two classical theories attempt to reconcile how flexible polymers interact with turbulence and reduce drag (Lumley, 1973; de Gennes, 1990). The viscous theory of Lumley (1973) asserts that the large extensional viscosity of polymer solutions strongly inhibits turbulent fluctuations just outside the viscous sublayer, causing the buffer layer to expand and wall friction to reduce (Lumley, 1973; White & Mungal, 2008). In a channel flow simulation that utilized a simplified constitutive model of polymer stresses – the retarded-motion expansion (Bird et al., 1987) – Roy et al. (2006) demonstrated that the non-Newtonian extensional viscosity opposed flow in both uniaxial and biaxial flow regions, which mitigated the strength and formation of quasi-streamwise vortices and reduced drag. On the other hand, the elastic theory of de Gennes (1990) speculates that polymers are not sufficiently stretched within a turbulent flow to produce a large local enhancement in extensional viscosity. Rather, drag reduction occurs when turbulent kinetic energy becomes comparable to the elastic energy of the flexible polymers. Each theory has their own merit (Roy et al., 2006; Min et al., 2003a,b); however, evidence that one is more valid than the other has not been established (White & Mungal, 2008; Xi, 2019). Furthermore, the viscous and elastic theories are only considered applicable for flexible polymers and not rigid polymers or surfactants; although, de Gennes (1990) conjectured that the viscous theory could apply to rigid polymers. It is therefore unclear whether the mechanism for drag reduction is similar among these different additives.

1.5 Objective and methods

The overarching objective of the present dissertation is to compare the rheology and drag-reducing capabilities of different DRAs – namely a flexible polymer, rigid polymer and surfactant. Prior evidence suggests that
their respective mechanisms for drag reduction are unique, given their different chemical composition and rheological features. Therefore, it is expected that the fluids will respond differently within a turbulent environment. The DRA solutions are evaluated using the following methods.

1. The rheology of the three DRA solutions are compared at similar concentrations.

2. One-point and two-point ensemble statistics are measured and contrasted for the three DRA solutions in a turbulent channel flow with similar amounts of drag reduction.

3. Coherent flow patterns are identified within a polymer drag-reduced turbulent boundary layer. The influence of non-Newtonian rheology on the topology of the turbulent flow is determined.

Surprisingly, it is found that different DRA solutions, with unique rheology, can produce similar velocity statistics within a turbulent wall flow. Assertions are made regarding each DRAs respective mechanism for drag reduction.

1.6 Thesis overview

This thesis is separated into five parts: Background, Rheology, Turbulent channel flow, Turbulent boundary layer, and Closing. A description of the chapters contained with each part are detailed below.

Part I: Background

- Chapter 1 Introduction
  This chapter describes the motivation for investigating DRAs, the existing understanding of the mechanism for drag reduction, and the objectives and overview of the current thesis.

- Chapter 2 Complex fluids and rheology
  Rheological traits and principles, common in drag-reducing solutions, are derived and discussed. These traits include shear viscosity, linear viscoelasticity and extensional viscosity.

- Chapter 3 Wall-bounded turbulence
  An overview of wall-bounded turbulence is presented. Details regarding the canonical Newtonian turbulent channel and boundary layer flows are provided, followed by brief discussions of drag-reduced wall flows. Background information is also provided about identifying flow topology and turbulent coherent patterns utilizing a method known as the $\Delta$-criterion.

- Chapter 4 Experimental methods
  A description of the experimental facilities and measurements are provided. These include rheological measurements (steady shear, dynamics shear and extensional), wall-bounded turbulent flow facilities (channel and boundary layer flows), and the flow measurements used to analyze the wall-bounded turbulent flows.
Part II: Rheology

- Chapter 5 Shear and extensional rheology
  Steady shear, dynamic shear and extensional rheology of DRA solutions, including a flexible polymer, rigid polymer and a cationic surfactant solution, are documented and compared.

- Chapter 6 Nontrivial rheology
  Conventional rheology done in Chapter 5 does not yield a non-Newtonian response for certain DRA solutions. To discern the non-Newtonian features of these fluids, the flow through nontrivial apparatus, a periodically constricted tube, is considered.

Part III: Turbulent channel flows

- Chapter 7 Comparing drag-reduced channel flows of polymer and surfactants
  One-point and two-point ensemble velocity statistics are compared among the drag-reduced flows of flexible polymers, rigid polymer and suractants. Flows are compared at a common drag reduction percentage.

- Chapter 8 Lubricating layer in drag-reduced flow of rigid polymers
  Assuming the rigid polymer solution is inelastic, a generalized Newtonian constitutive model is used to comment on the rheological features of the rigid polymer solution within the turbulent flow. A near-wall lubricating layer, with a lower viscosity than the channel core is found.

Part IV: Turbulent boundary layer

- Chapter 9 Local flow topology of a polymer-laden boundary layer
  High spatial resolution velocity measurements in a thick boundary layer are used to derive velocity gradients within the turbulent and drag-reduced wall flows. Velocity gradients and the $\Delta$-criterion are used to establish the distribution of fine scale motions within a turbulent drag-reduced flow. The effect of non-Newtonian rheology on the flow topology is presented.

Part V: Closing

- Chapter 10 Conclusions
  The conclusions of the thesis are presented and potential future works are suggested.

- Appendix A Uncertainty analysis
  Supplemental discussions on measurement uncertainty for the respective investigations in chapters 6, 7, 8 and 9 are presented.
Chapter 2

Complex fluids and rheology

With the advancement of the chemical industry at the turn of the 20th century, the production of synthetic plastics produced a number of unique materials with unconventional flow behaviours (Macosko 1994). These complex materials sparked a wave of investigations and research that was eventually organized into a new field of study, rheology. The concept of rheology was developed by Dr. Eugene C. Bingham in 1920 at Lehigh University; it refers to the study of the deformation and flow of matter (Barnes et al. 1989). Rheology encompasses several topics including those related to fluid dynamics (also, aeronautics, hydrodynamics, hydraulics), and solid mechanics. More specifically, rheology involves determining the constitutive equations that describe the relationship between force and deformation in materials. A rheologist (i.e., someone who studies rheology) attempts to derive constitutive relations by investigating material behaviours in very simple deformation (Macosko 1994). On the other hand, a mechanist applies the constitutive relations developed by the rheologist to study the forces of materials in complex deformations. This philosophy is central to the present dissertation, particularly when investigating unique materials in highly complex turbulent flows.

Under simple shear deformation, the most basic relation between force and deformation in solid mechanics is Hooke’s law,

\[ \tau_{xy} = G \gamma_{xy} \quad (2.1) \]

where \( \tau_{xy} \) is the shear stress (force per unit area), \( \gamma_{xy} \) is the change in length relative to the initial configuration, and \( G \) is an intrinsic property of the material known as the elastic modulus. At the other end of the spectrum, the simplest constitutive relation for fluids is Newton’s law of viscosity,

\[ \tau_{xy} = \mu \dot{\gamma}_{xy} \quad (2.2) \]

where \( \dot{\gamma}_{xy} = d\gamma_{xy}/dt \) is the rate of shear straining and \( \mu \) is the shear viscosity of the material. There are a number of ideal elastic solids or Hookean solids, such as metals and ceramics, that obey equation (2.1) (Macosko 1994). Similarly, gases and most small molecule liquids, such as water and oils, are Newtonian fluids and obey equation (2.2). However, many materials – including blood, polymers and foods – have properties that fall somewhere between an ideal Hookean solid and Newtonian fluid. The focus of the present dissertation is on liquid-like non-Newtonian materials with a propensity to reduce drag in a turbulent flow regime. These include dilute aqueous solutions of polymers and surfactants. The following section will
review the common rheological traits (shear viscosity, linear viscoelasticity, and extensional viscosity) of non-Newtonian drag-reducing liquids. Three different types of simplified flow deformations or rheometric flows (steady shear, dynamic shear, and uniaxial extension) that help discern these rheological traits will be considered. Before discussing these traits, the conservation equations of a moving fluid are defined.

Conservation of mass and momentum are represented by the following equations,

\[
\nabla \cdot \mathbf{U} = -\frac{1}{\rho} \frac{D\rho}{Dt},
\]

\[
\rho \frac{D\mathbf{U}}{Dt} = -\nabla P + \nabla \cdot \mathbf{\tau} + \mathbf{f},
\]

where \( \rho \) is the fluid density, \( \mathbf{U} \) is the velocity vector, \( P \) is the static pressure, \( \mathbf{\tau} \) is the symmetric deviatoric stress tensor, and \( \mathbf{f} \) is the body force vector. Here, the total or material derivative of \( \mathbf{U} \) is represented as \( D\mathbf{U}/Dt = \partial\mathbf{U}/\partial t + \mathbf{U} \cdot \nabla \mathbf{U} \). Liquids are generally incompressible with \( \rho \) that is constant; therefore, \( D\rho/Dt = 0 \) and equation (2.3a) simplifies to \( \nabla \cdot \mathbf{U} = 0 \). The velocity gradient tensor (VGT) \( \mathbf{L} = \nabla \mathbf{U} \) describes the relative rate of separation between neighbouring points within a material. The VGT can be decomposed into a symmetric rate of deformation tensor \( \mathbf{D} = (\nabla \mathbf{U} + \nabla \mathbf{U}^\top)/2 \) and antisymmetric rate of rotation tensor \( \mathbf{W} = (\nabla \mathbf{U} - \nabla \mathbf{U}^\top)/2 \), where \( \mathbf{L} = \mathbf{D} + \mathbf{W} \) and \( \mathbf{U}^\top \) is the transpose operation. The constitutive equation is the relationship between the deviatoric stress tensor \( \mathbf{\tau} \) and the rate of deformation tensor \( \mathbf{D} \), which describes the rate of stretching and straining. For a Newtonian fluid, the constitutive equation in full tensor notation is \( \mathbf{\tau} = 2\mu\mathbf{D} \), and equation (2.3b) reduces to the well-known Navier-Stokes equation.

### 2.1 Shear viscosity

A schematic of Couette flow, the most simple steady shear flow, is shown in figure 2.1. Here, fluid is contained between two parallel surfaces separated by a distance \( H \) along \( y \), where \( x \), \( y \) and \( z \) are the streamwise, wall-normal and spanwise directions respectively. The upper surface moves tangentially relative to the lower surface at a constant speed of \( U_H \) along \( x \). The fluid velocity is homogeneous (no velocity gradients) along the \( x \) and \( z \) directions. Fluid velocity along the streamwise, wall-normal and spanwise directions are \( U \), \( V \) and \( W \) respectively. Due to no-slip boundary conditions, the fluid velocity at the lower surface is zero, while the velocity of the fluid at the upper surface is \( U_H \).

Fluid velocity for Couette flow is listed as follows,

\[
U = \dot{\gamma} y, \quad V = 0, \quad W = 0.
\]

where \( \dot{\gamma} = \partial U/\partial y = U_H/H \) is the shear rate. Therefore, the VGT for steady shear flow is,

\[
\mathbf{L} = \begin{bmatrix} 0 & \dot{\gamma} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

the rate of deformation tensor is,
For a Newtonian fluids, the deviatoric stress is proportional to the rate of deformation $\tau = 2\mu D$; therefore, the only nonzero components of $\tau$ is $\tau_{xy} = \tau_{yx} = \mu \dot{\gamma}$. Note that $\tau$ is usually presented as the shear stress, and hence going forward, it should be assumed that $\tau = \tau_{xy}$. Therefore, the viscosity can be derived according to $\mu = \tau/\dot{\gamma}$. For simplicity, the shear strain and shear rate are also represented as $\gamma_{xy} = \gamma$ and $\dot{\gamma}_{xy}$, respectively.

In various non-Newtonian fluids $\mu$ has a strong dependence on the shear rate, $\dot{\gamma}$. Generalized Newtonian fluids have a shear-rate dependent stress and viscosity. The most simple example of a shear-rate dependent fluid is a power-law fluid with a shear stress and viscosity of the form

$$\tau = M \dot{\gamma}^k, \quad \mu = M \dot{\gamma}^{k-1}. \quad (2.7)$$

where $M$ and $k$ are constants called the consistency and flow index. Non-Newtonian materials with $k < 1$ exhibit shear thinning where $\mu$ decreases with increasing $\dot{\gamma}$. This is arguably the most commonly observed non-Newtonian feature of drag-reducing fluids comprising polymers (Escudier et al. 1999, 2009; Mitishita et al. 2023); however, it is unclear if shear thinning is necessary for drag reduction. Boger fluids are an example of a fluid that exhibits no shear thinning, but is viscoelastic and proven to reduce drag (James 2009; Min et al. 2003a, b).
2.2 Linear viscoelasticity

The term “viscoelastic” implies the simultaneous existence of viscous and elastic properties within a material \cite{Barnes1989}. Linear viscoelasticity relies on the notion that the differential equation that relates stress and strain are linear. Mechanical models or spring and dashpot diagrams provide analogs or visual depictions of linear viscoelastic materials. Hookean deformation is represented by a spring with an elastic modulus of $G$, similar to (2.1). While Newtonian flow is represented using a dashpot with a viscosity of $\mu$, similar to (2.2). A Maxwell material consists of a spring and dashpot in series, as shown in figure 2.2. When the spring and dashpot are in series, the strain or strain-rates are additive. Therefore, the linear ordinary differential equation that describes the deformation within the Maxwell element is,

$$\frac{1}{G} \dot{\tau} + \frac{1}{\mu} \tau = \dot{\gamma},$$

where $\dot{\tau} = d\tau/dt$. A relaxation time can be derived according to $t_e = \mu/G$, and the previous equation can be rearranged to,

$$t_e \dot{\tau} + \tau = \mu \dot{\gamma}. \hspace{1cm} (2.8)$$

This is one example of a linear viscoelastic model. Other examples include the Kelvin model (a spring and dashpot in parallel) and the Jeffrey model (a dashpot in series with a spring-dashpot in parallel) \cite{Barnes1989}. Two different types of methods are used to measure linear viscoelastic behaviour, including static and dynamic methods. Static methods include creep and relaxation tests where a step change in the shear stress or strain is applied to a material and temporal development in the strain or stress is observed. Dynamic tests involve the application of a harmonically varying shear stress or strain, and are often called small amplitude oscillatory shear measurements. In more liquid-like materials, static tests do not produce a very obvious viscoelastic response; therefore, dynamic viscoelastic tests are prioritized.

![Figure 2.2: Mechanical model or spring and dashpot diagram of a Maxwell material.](image)

Small amplitude oscillatory shear measurements involve deforming a sample with a sinusoidally varying shear strain $\gamma$ in time $t$. A viscoelastic material will respond with a shear stress $\tau$ that oscillates with the same frequency as $\gamma$, but with a phase offset $\psi$ between $0^\circ$ and $90^\circ$. Here, the strain and stress are represented as,

$$\gamma = \gamma_0 \sin(\omega t), \hspace{1cm} (2.9a)$$

$$\tau = \tau_0 \sin(\omega t + \psi), \hspace{1cm} (2.9b)$$
where \( \omega \) is the oscillation frequency in rad s\(^{-1} \), \( \gamma_0 \) is the amplitude of the strain wave, and \( \tau_0 \) is the amplitude of the stress response. In this type of deformation \( \gamma \) can be applied in the same configuration as figure 2.1 where a fluid sample is contained between two parallel surfaces and the upper surfaces oscillates such that the strain rate within the sample is,

\[
\dot{\gamma} = \frac{d\gamma}{dt} = \omega \gamma_0 \cos(\omega t).
\]

Recall that the velocity of the upper surface can be similarly represented as \( U_H = H \dot{\gamma} \).

From trigonometric identities, it can be shown that the shear stress \( \tau \) can be decomposed into a component that is in-phase with \( \gamma \), i.e., \( \tau_0' = \tau_0 \cos(\psi) \), and another that is \( 90^\circ \) out-of-phase with \( \gamma \), i.e., \( \tau_0'' = \tau_0 \sin(\psi) \). In other words, \( \tau \) can be represented according to,

\[
\tau = \tau_0' \sin(\omega t) + \tau_0'' \cos(\omega t).
\]

Using each of these amplitudes, an in-phase and out-of-phase elastic moduli can be derived,

\[
G' = \frac{\tau_0'}{\gamma_0}, \quad (2.10a)
\]

\[
G'' = \frac{\tau_0''}{\gamma_0}, \quad (2.10b)
\]

where \( G' \) is also referred to as a gain modulus and \( G'' \) is a loss modulus. The phase offset \( \psi \), can be determined according to,

\[
\tan(\psi) = \frac{G''}{G'}.
\]

For a Hookean solid \( G' \) is finite, \( G'' = 0 \) and \( \psi = 0^\circ \). While for a Newtonian fluid \( G' = 0 \), \( G'' \) is finite and \( \psi = 90^\circ \). For a Maxwell material of figure 2.2 and (2.8), with mixed contributions of elastic and viscous features, the gain and loss moduli are,

\[
G' = \frac{\mu t_e \omega^2}{1 + \omega^2 t_e^2}, \quad (2.12a)
\]

\[
G'' = \frac{\mu \omega}{1 + \omega^2 t_e^2}, \quad (2.12b)
\]

and the phase offset between the strain and stress signals is,

\[
\tan(\psi) = \frac{1}{t_e \omega}.
\]

Small amplitude shear viscosity or dynamic shear viscosity measurements are a useful method for discerning the non-Newtonian properties, namely the linear viscoelasticity, of a complex material. Although better than static viscoelastic measurements, dynamic shear viscosity measurements have their limitations, particularly when evaluating dilute aqueous solutions with low shear viscosities. Therefore, this analysis is used less
often in the present dissertation than steady shear and extensional rheology. However, elasticity is believed by many to be a key rheological feature of drag reduction (Tabor & de Gennes, 1986; de Gennes, 1990). As such, dynamic shear viscosity measurements are performed when possible.

### 2.3 Extensional viscosity

Non-Newtonian elastic liquids generally exhibit significantly different extensional or elongational flow features than Newtonian fluids. Lumley (1973) and Roy et al. (2006) argued that the nonmonotonic trend in the so-called extensional viscosity with strain rate in dilute liquid-like polymer solutions is the rheological property responsible for turbulent drag reduction. Therefore, quantifying the extensional properties of the drag-reducing liquids is important. Three of the most simple types of steady extensional flows are uniaxial, biaxial and planar extension (Barnes et al., 1989). For brevity, only uniaxial extensional flow is reviewed.

Steady uniaxial extension consists of a velocity of the form,

\[ U = \dot{\varepsilon} x, \quad V = -\dot{\varepsilon} y/2, \quad W = -\dot{\varepsilon} z/2. \]

where \( \dot{\varepsilon} \) is the extension rate. Therefore, the VGT and rate of deformation tensor for the steady uniaxial extensional flow is,

\[
L = D = \begin{bmatrix}
\dot{\varepsilon} & 0 & 0 \\
0 & -\dot{\varepsilon}/2 & 0 \\
0 & 0 & -\dot{\varepsilon}/2
\end{bmatrix},
\]

(2.13)

while the rate of rotation tensor is \( W = 0 \). The uniaxial extensional viscosity \( \mu_E \) can be represented as,

\[ \tau_{xx} - \tau_{yy} = \dot{\varepsilon} \mu_E. \]

For a Newtonian fluid,

\[ \tau_{xx} - \tau_{yy} = 3\mu \dot{\varepsilon}. \]

The Trouton ratio \( Tr = \mu_E/\mu \) defines the ratio between the extensional and shear viscosity. For a Newtonian fluid under uniaxial deformation, the Trouton ratio is identically 3. Elastic polymer solutions with a shear thinning viscosity are known to exhibit large \( Tr \) greater than 3 (Barnes et al., 1989). Contrary to shear thinning, the extensional viscosity of viscoelastic polymer solutions \( \mu_E \) tends to increase dramatically with increasing \( \dot{\varepsilon} \), a trend known as tension thickening. Measuring \( \mu_E \) is difficult, considering imposing uniaxial deformation (or biaxial and planar extension) of the form of (2.13) at a constant strain rate \( \dot{\varepsilon} \) and without any shear deformation is challenging. That being said, some recent techniques have been developed (Rodd et al., 2005; Dinic et al., 2015), some of which are utilized in the current dissertation and detailed in §4.1.
Chapter 3

Wall-bounded turbulence

Many turbulent flows encountered in engineering systems are bounded by one or more solid surfaces. When fluid flows over a solid wall a boundary layer profile develops due to viscous shear stresses and the no-slip boundary condition. For internal duct flows, e.g., pipe flows, the boundary layers develop along all walls of the conduit and their thickness increases along the flow direction. With increasing streamwise distance, the flow eventually becomes fully developed and the profiles merge, occupying the complete duct cross-section. External flows, e.g, the boundary layer along the surface of a vehicle, develop freely and continuously along the flow direction. They also consists of a turbulent/non-turbulent interface, where the turbulent boundary layer meets the uniform free stream. An internal channel flow and external boundary layer are considered in the present dissertation. The current chapter provides details regarding the canonical turbulent channel and boundary layer flows, respectively. Background information about polymer and surfactant drag-reduced channel and boundary layer flows are also provided. The final section of this chapter discusses the concept of flow topology and the utilization of the $\Delta$-criterion for identifying coherent patterns within turbulent flows.

3.1 General equations and definitions

The conservation of mass and momentum was defined previously in §2. Equations (2.3) are valid for both laminar and turbulent flows. In a turbulent regime, a Reynolds decomposition can be used to segregate the velocity and pressure into a time average and fluctuating component. The Reynolds decomposed velocity components are as follows:

\[
\begin{align*}
U &= \langle U \rangle + u, \\
V &= \langle V \rangle + v, \\
W &= \langle W \rangle + w
\end{align*}
\]

(3.1a) (3.1b) (3.1c)

where $u$ is the fluctuating velocity vector with components $u$, $v$, $w$. The angle brackets $\langle \cdots \rangle$ represent the Reynolds time average. For the streamwise velocity, the Reynolds time average is defined according to,

\[
\langle U \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{0}^{T} U(x, y, z, t) dt.
\]
For a Newtonian fluid, the deviatoric stress tensor is represented as, \( \tau = 2 \mu D \). When the terms in equation (3.1) are substituted into (2.3) and the resulting equation is time averaged, the mean mass and momentum equations for an incompressible Newtonian fluid are,

\[
\nabla \cdot \langle U \rangle = 0, \quad (3.2a)
\]

\[
\rho \langle U \rangle \cdot \nabla \langle U \rangle = -\nabla \langle P \rangle + \mu \nabla^2 \langle U \rangle - \rho \nabla \cdot \langle uu \rangle, \quad (3.2b)
\]

where \( \langle uu \rangle \) is the Reynolds stress tensor and is representative of turbulent velocity fluctuations. Equation (3.2) describes the mean balance of momentum within a Newtonian, turbulent flow and is more commonly known as the Reynolds averaged Navier-Stokes equations.

### 3.2 Channel flows

A depiction of a fully-developed two-dimensional (2D) channel flow is shown in figure 3.1. Fluid is contained between two parallel walls separated by height \( H \) and propelled along the positive \( x \)-direction by a favourable pressure gradient \( \partial \langle P \rangle / \partial x < 0 \). At the walls (\( y = 0 \), and \( y = H \)) there is a no-slip boundary condition, \( U = W = 0 \), and a non-permeable boundary condition, \( V = 0 \). Along the centreline of the channel, where \( y = h = H/2 \), mean streamwise velocity \( \langle U \rangle \) is at a maximum. Fluid is also bound along the \( z \)-direction between two side walls that are into and out of the plane of the page with reference to figure 3.1. Flow within the channel is considered two-dimensional if the width is significantly larger than \( H \). Dean (1978) prescribed that an aspect ratio larger than 7 produces approximately two-dimensional flow. Similarly, the flow is fully-developed if the length of the channel is significantly longer than \( H \). Fully-developed, two-dimensional flow implies that the ensemble velocity statistics, such as \( \langle U \rangle \), do not vary along \( x \) or \( z \) and the mean spanwise velocity \( \langle W \rangle \) equals zero. From mass conservation (3.2a) it can also be shown that \( \langle V \rangle = 0 \).

The Reynolds number defines the ratio of inertial to viscous forces within the flow. A bulk Reynolds number \( Re_H \) is defined according to,

\[
Re_H = \frac{\rho U_b H}{\mu} \quad (3.3)
\]

where \( U_b \) is the bulk or average velocity across \( y \),

\[
U_b = \frac{1}{h} \int_0^h \langle U \rangle dy.
\]

Above an \( Re_H \) of 5772, flow within the channel is linearly unstable and turbulent (Orszag, 1971). Hence, velocity fluctuations or Reynolds stresses are present.

Simplifying (3.2b), the mean streamwise momentum balance of the turbulent channel flow is,

\[
0 = -\frac{\partial \langle P \rangle}{\partial x} + \mu \frac{\partial^2 \langle U \rangle}{\partial y^2} - \rho \frac{\partial \langle uu \rangle}{\partial y}.
\]

Integrating the above equation produces the mean streamwise stress balance,
Figure 3.1: Schematic of a two-dimensional channel flow. Representative mean velocity profiles $\langle U \rangle$ are drawn for Newtonian turbulent, Newtonian laminar and non-Newtonian drag-reduced turbulence at a similar $U_b$. Grey shaded regions are the channel walls.

\[ \tau(y) = \tau_v + \tau_R = \mu \frac{d\langle U \rangle}{dy} - \rho \langle uv \rangle, \]  

(3.4)

where $\tau$ is the total mean stress, $\tau_v$ is the viscous stress, and $\tau_R$ is the Reynolds shear stress (Pope, 2000). In the case of a channel flow, the streamwise pressure gradient and wall-normal gradient of shear stress are constant and equal. Therefore, the wall shear stress can be defined according to,

\[ \tau_w = -h \frac{\Delta \langle P \rangle}{\Delta x}, \]  

(3.5)

and,

\[ \tau(y) = \tau_w \left(1 - \frac{y}{h}\right), \]  

(3.6)

In equation (3.4), $\tau_R$ becomes zero close to the wall due to the no-slip boundary condition. Therefore, $\tau_w$ can be similarly represented as,

\[ \tau_w = \mu \frac{\partial\langle U \rangle}{\partial y} \bigg|_{y=0}, \]  

(3.7)

A non-dimensional wall shear stress or skin friction coefficient is defined according to,

\[ C_f = \frac{\tau_w}{\frac{1}{2} \rho U_b^2}. \]  

(3.8)

Dean (1978) found that $C_f$ varied with respect to $Re_H$ according to,
\[ C_f = 0.073Re^{-0.25} \]  

(3.9)

based on an empirical fit of various experimental measurements of \( C_f \) and \( Re \) in Newtonian channel flows.

Near the wall, Reynolds stresses approach zero, due to the no-slip boundary condition and viscous stresses are more significant. The so-called viscous scales are used to define the appropriate velocity and length scales for which these viscous effects are important. These include the friction velocity,

\[ u_\tau = \sqrt{\frac{\tau_w}{\rho}}, \]  

(3.10)

and viscous length scale,

\[ \delta_v = \frac{\nu}{u_\tau}, \]  

(3.11)

where \( \nu = \mu/\rho \) is the kinematic viscosity. The friction Reynolds number \( Re_\tau \), is used to define the separation between the viscous and geometric length scales according to,

\[ Re_\tau = \frac{u_\tau h}{\nu} = \frac{h}{\delta_v}. \]  

(3.12)

As \( Re_\tau \) increases, \( \delta_v \) decreases, and so does the thickness of the viscous wall region. The friction and bulk Reynolds numbers can be approximately related with one another according to, \( Re_\tau \approx 0.09Re_H^{0.88} \) (Pope, 2000).

The velocity and wall-normal distance normalized by the viscous scales are defined as \( \langle U \rangle^+ = \langle U \rangle/u_\tau \) and \( y^+ = y/\delta_v \). Derived from various scaling arguments, i.e., the law of the wall, the velocity profile within a Newtonian turbulent channel flow takes on a piece-wise function of the form,

\[ \langle U \rangle^+ = \begin{cases} y^+, & y^+ \leq 5, \\ \frac{1}{\kappa} \ln(y^+) + B, & y^+ > 30, \end{cases} \quad y^+/h < 0.3. \]  

(3.13)

where \( \kappa \) is the Von Kármán coefficient, and \( B \) is a constant. The flow region defined by \( y^+ < 5 \) is referred to as the viscous sublayer, while the region with \( y^+ > 30 \) is denoted the logarithmic layer. Between the viscous sublayer and log layer \( 5 < y^+ < 30 \), is referred to as the buffer layer. Another profile of \( \langle U \rangle^+ \) exists for \( y/h \) greater than \( 0.3 \), which is derived from the velocity defect law (Pope, 2000). However, this profile does not differ significantly from the log layer and is omitted in (3.13) for brevity. The approximate values for the Von Kármán coefficient and log layer intercept are, \( \kappa = 0.41 \) and \( B = 5.2 \) (Pope, 2000). Figure 3.2(a) demonstrates that the distributions of (3.13) show good agreements and overlap with the profiles of \( \langle U \rangle^+ \) derived from direct numerical simulation (DNS) of a Newtonian turbulent channel flow by Lee & Moser (2015) at an \( Re_\tau \) of 550. When not normalized by the viscous scales, the velocity profile \( \langle U \rangle \) takes on a distribution similar to that shown schematically in figure 3.1 where velocity gradients are much larger near the wall than closer to the channel centreline. At a similar bulk or average velocity \( U_b \), the turbulent profile of \( \langle U \rangle \) is more “flat” or blunted near the channel core compared to the parabolic laminar velocity profile shown in red.
Drag-reduced flows of polymers and surfactants have a lower skin friction coefficient $C_f$ relative to Newtonian turbulent channel flows at a similar $Re_H$. A drag reduction percentage is defined according to,

$$ DR = \left(1 - \frac{C_{f,NN}}{C_{f,N}}\right) \times 100\% $$

(3.14)

where $C_{f,NN}$ is the skin friction coefficient of the non-Newtonian drag-reduced flow, and $C_{f,N}$ is the skin friction coefficient of the turbulent Newtonian flow at a similar $Re_H$. For generally all types of drag-reducing additives, polymers or surfactants, $DR$ increases monotonically with the additive concentration $c$. Eventually, the $DR$ saturates and no longer increases with further enhancements of $c$ (Virk et al., 1970). The limit at which $DR$ saturates is the maximum drag reduction (MDR) asymptote, represented by the distribution

$$ \frac{1}{\sqrt{C_f}} = 19.0 \log_{10}(Re_H \sqrt{C_f}) - 32.4. $$

(3.15)

The $DR$ that corresponds to the MDR asymptote is generally around 60-80% depending on $Re_H$.

One of the most noticeable effects of polymer and surfactant drag-reducers is the modification to $\langle U \rangle$ relative to the Newtonian profile. The elastic sublayer model of Virk (1971) described drag-reduced flows with intermediate $DR$ as having three layers: a viscous sublayer, a buffer layer — that was re-termed the elastic sublayer — and a logarithmic layer that was referred to as the Newtonian plug layer. Relative to Newtonian flows, the viscous and elastic sublayers of a polymer drag-reduced flow are thicker. The Newtonian plug layer possesses a similar slope $1/k$ as the logarithmic layer of a Newtonian flow, but a larger intercept $B$ due to the thickened buffer or elastic sublayer. When the flow attains MDR, the Newtonian plug layer is eradicated and the elastic sublayer demonstrates an ultimate profile (Virk et al., 1970), represented as
\[ \langle U \rangle^+ = 11.7 \ln(y^+) - 17.0, \]  
\hspace{1cm} (3.16)

and shown by the black dash-dotted line in figure 3.2(a). Drag-reduced flows not at MDR will have a \( \langle U \rangle^+ \) profile that falls between the Newtonian log layer of (3.13) and the ultimate profile of (3.16). The elastic sublayer model and ultimate MDR velocity profile have been observed in a number of experimental and numerical investigations \cite{Warholic1999, Ptasinski2001, Ptasinski2003, Min2003}. When not normalized by the viscous scales, the velocity profile \( \langle U \rangle \) for a polymer or surfactant drag-reduced channel flow takes on a distribution similar to that shown schematically in figure 3.1. At a similar \( \langle U \rangle \), the drag-reduced flow lies somewhere between the laminar and turbulent distributions. Note, that if the viscosity \( \mu \) of the drag-reduced flow is similar to the Newtonian fluid, flows with an identical \( \langle U \rangle \) also constitutes a matching \( \text{Re}_H \) as per (3.3). Even at MDR, drag-reduced flows are not laminar. Instead, a drag-reduced flow has velocity fluctuations that are significantly attenuated relative to Newtonian turbulence.

Reynolds stress profiles demonstrate the significance of velocity fluctuations within the turbulent flow. Plots of the non-zero components of the Reynolds stress tensor \( \langle uu \rangle \) are shown in figure 3.2(b) for the Newtonian turbulent channel flow DNS of \cite{LeeMoser2015} at a \( \text{Re}_\tau \) of 550. Here, the Reynolds stresses are normalized by \( u^2_\tau \), e.g., \( \langle u^2 \rangle^+ = \langle u^2 \rangle / u^2_\tau \). Based on figure 3.2(b), the largest Reynolds stress component is \( \langle u^2 \rangle^+ \), followed by \( \langle w^2 \rangle^+ \), and then \( \langle v^2 \rangle^+ \). The Reynolds shear stress \( \tau_R = -\langle uv \rangle^+ \), is the negative distribution shown in figure 3.2(b). The most energetic turbulence activity occurs within the buffer layer of the flow, where the large peak in \( \langle u^2 \rangle^+ \) is situated.

Several experimental investigations have documented the modification to the Reynolds stresses caused by polymers and surfactants \cite{Ptasinski2001, Escudier2009, Mohammadtabar2017}. \cite{Warholic1999} showed that polymer drag-reduced flows have different inner-normalized Reynolds stress profiles depending on whether \( DR \) was “low” or “high”. The transition between these two states, coined low drag reduction (LDR) and high drag reduction (HDR), occurred at a \( DR \) of approximately 40% \cite{Warholic1999}. The main distinction in the Reynolds stresses of LDR and HDR flows was the change in the peak value of the Reynolds stresses \cite{Warholic1999, Escudier2009}. For polymer drag-reduced flows at LDR, an increase in \( DR \) was accompanied by an increase in the peak streamwise Reynolds stress, \( \langle u^2 \rangle^+ \), and an attenuation in the wall-normal, \( \langle v^2 \rangle^+ \), and spanwise Reynolds stresses, \( \langle w^2 \rangle^+ \). In contrast, HDR flows showed a decrease in all Reynolds stresses with increasing \( DR \). The Reynolds shear stress, \( \langle uv \rangle^+ \), of a polymer drag-reduced flow decreased monotonically with increasing \( DR \) in both LDR and HDR regimes. \cite{Warholic1999} found that the profile of \( \langle uv \rangle^+ \) for drag-reduced flows close to MDR was approximately zero for all \( y^+ \). Contrary to the findings of \cite{Warholic1999}, other experiments and simulations have suggested a \( \langle uv \rangle^+ \) profile equal to zero is not a necessary condition for MDR \cite{Ptasinski2003}. The discrepancy still remains unexplained, but it is generally accepted that flows near MDR have a significantly attenuated Reynolds shear stress profile \cite{WhiteMungal2008}.
3.3 Boundary layers

A schematic of a boundary layer flow over a smooth flat surface is shown in figure 3.3. The boundary layer is formed when a uniform laminar free-stream, with a streamwise velocity $U_{\infty}$, flows over a flat plate. Compared to the channel flow of figure 3.1, the boundary layer flow has a thickness along $y$ that is unconfined. Instead, the boundary layer thickness $\delta$ increases with $x$, shown in figure 3.3. The statistics are therefore, dependent both on $y$ and $x$. That being said, statistics are still independent of the spanwise direction, assuming the boundary layer is not bounded along $z$. Fluid at the wall $y = 0$ satisfies the no-slip ($U = W = 0$) and non-permeable boundary conditions ($V = 0$). At the outer region of the flow there is a turbulent/non-turbulent interface where the boundary layer meets the uniform free-stream or $y = \delta$.

![Figure 3.3: Schematic of a boundary layer flow. Representative mean velocity profiles $\langle U \rangle$ are drawn for Newtonian turbulent, Newtonian laminar and non-Newtonian drag-reduced turbulence at a similar $U_{\infty}$. The grey shaded region is the wall.](image)

The boundary layer thickness $\delta$ is defined as the value of $y$ where $\langle U \rangle$ is equal to 99% of $U_{\infty}$. Other length scales that define the boundary layer flow include the displacement thickness,

$$\delta^* = \int_0^\infty \left( 1 - \frac{\langle U \rangle}{U_{\infty}} \right) dy,$$

and the momentum thickness,

$$\phi = \int_0^\infty \frac{\langle U \rangle}{U_{\infty}} \left( 1 - \frac{\langle U \rangle}{U_{\infty}} \right) dy.$$

In the present dissertation, two Reynolds numbers are used to uniquely define the boundary layer flows. They include the friction Reynolds number,

$$Re_t = \frac{u_t \delta}{v},$$

and momentum thickness Reynolds number,
\( Re_\phi = \frac{U_\infty \phi}{\nu} \) \hspace{1cm} (3.20)

Utilizing Bernoulli’s equation in the free-stream of the flow, it can be demonstrated that the streamwise pressure gradient can be represented as,

\[
\frac{dP_\infty}{dx} = \rho U_\infty \frac{dU_\infty}{dx} \hspace{1cm} (3.21)
\]

where \( P_\infty \) is the pressure within the free-stream. Applying dimensionless scaling arguments to simplify the mean mass and momentum equation (3.2b), yields the Newtonian boundary layer equations,

\[
\frac{\partial \langle U \rangle}{\partial x} + \frac{\partial \langle V \rangle}{\partial y} = 0
\]

\[
\langle U \rangle \frac{\partial \langle U \rangle}{\partial x} + \langle V \rangle \frac{\partial \langle U \rangle}{\partial y} = -\frac{\partial \langle P \rangle}{\rho \partial y} + \nu \frac{\partial^2 \langle U \rangle}{\partial y^2} - \frac{\partial \langle uv \rangle}{\partial y}, \hspace{1cm} (3.22b)
\]

\[
0 = -\frac{\partial \langle P \rangle}{\rho \partial y} - \frac{\partial \langle v^2 \rangle}{\partial y} \hspace{1cm} (3.22c)
\]

From (3.22c) it follows that the \( \langle P \rangle \) is constant with respect to \( y \), considering \( \langle v^2 \rangle = 0 \) at \( y = 0 \) and \( y = \delta \) due to the no-slip boundary condition and the laminar flow conditions within the free-stream. Therefore, the following equality holds, \( \frac{\partial \langle P \rangle}{\partial x} = \frac{dP_\infty}{dx} \). Utilizing (3.21), equation (3.22b) can be re-written as,

\[
\langle U \rangle \frac{\partial \langle U \rangle}{\partial x} + \langle V \rangle \frac{\partial \langle U \rangle}{\partial y} = \frac{1}{\rho} \frac{\partial \tau}{\partial y} + U_\infty \frac{dU_\infty}{dx}, \hspace{1cm} (3.23)
\]

where the total shear stress \( \tau \) is defined as,

\[
\tau(x, y) = \tau_v + \tau_R = \mu \frac{\partial \langle U \rangle}{\partial y} - \rho \langle uv \rangle, \hspace{1cm} (3.24)
\]

Unlike the channel flow, the total stress \( \tau \) is not a linear function of \( y \) and the streamwise pressure gradient (3.6) due to the convective terms on the left hand side of (3.23). That being said, (3.24) demonstrates that the flow still exhibits a balance between viscous and Reynolds stresses. Therefore, many of the properties defined for the channel flow are still “locally” applicable; locally meaning at a particular location along \( x \). For example, definitions of \( \tau_v, u_\tau \) and \( \delta_v \) from (3.7), (3.10), and (3.11) are still valid; however, they change with respect to \( x \). A new skin friction coefficient is defined utilizing the free-stream velocity,

\[
C_f = \frac{\tau_v}{\frac{1}{2} \rho U_\infty^2} \hspace{1cm} (3.25)
\]

Compared to the streamwise velocity profile of a Newtonian laminar boundary, the mean velocity profile of the turbulent boundary layer is more flat and blunted, depicted in figure 3.3 and similar to that of the channel flows detailed previously in §3.2. Velocity profiles, normalized by the viscous scales also follow the law of the wall (3.13) for the turbulent boundary layer flow. Boundary layer DNS by Jiménez et al. (2010) at \( Re_\phi \) of 1551 and \( Re_\tau \) of 578 is shown in figure 3.4(a) alongside the profiles of (3.13). The DNS profile shows good agreements with the linear viscous sublayer and log layer within their respective \( y^* \)
ranges. Reynolds stress profiles for the Newtonian DNS of Jiménez et al. (2010) are also visually similar to those of the channel flow. The most energetic turbulence activity within the boundary layer flow is contained within the buffer layer, where the large peak in $\langle u^2 \rangle^+$ is situated.

![Figure 3.4: Profiles of (a) mean velocity and (b) Reynolds stresses for the Newtonian boundary layer flow DNS by Jiménez et al. (2010) at $Re_\phi = 1551$ and $Re_\tau = 578$. In (a) the solid red line is the $\langle U \rangle^+$ versus $y^+$ profiles from Jiménez et al. (2010), the solid black line is the linear viscous sublayer and the dashed black line is the log layer profile of (3.12). The dash-dotted black line in (a) is the MDR ultimate profile of (3.16).](image)

Polymer and surfactant drag-reduced boundary layers can be produced using two approaches. The first involves injecting a concentrated polymer or surfactant solution into the turbulent boundary layer from a slot cut into the wall. The second involves mixing a large homogeneous solution (often called a polymer ocean) and pumping the fluid over the flat plate similar to the Newtonian solvent. Both methods have been shown to produce large quantities of $DR$, where $DR$ is defined similarly to the channel flow (3.14); however, $C_{f,N}$ is defined as a Newtonian flow with a similar $Re_\phi$ (White et al. 2004; Tamano et al. 2011; Elbing et al. 2013; Tamano et al. 2018; Farsiani et al. 2020). Modifications to the mean velocity profile are also similar to the channel flow. With increasing $DR$, the buffer layer expands and the intercept of the log layer $B$ increases (Tamano et al. 2018; Farsiani et al. 2020). At a large enough concentration, the distribution $\langle U \rangle^+$ overlaps with the MDR ultimate profile of (3.16). Assuming the change in $\delta$ is the same between the drag-reduced and Newtonian flows, and both flows have the same $U_\infty$, profiles of $\langle U \rangle$ appear as shown in figure 3.3. For the drag-reduced boundary layer wall shear stress is lower than the Newtonian turbulent flow. Therefore, shear near the wall is greatly diminished and the profile of $\langle U \rangle$ for the drag-reduced flow falls somewhere between the Newtonian laminar and turbulent distributions, depending on $DR$. Reynolds stresses within the boundary layer flow are also modified similar to that of the channel flow. White et al. (2004) and Tamano et al. (2011) demonstrated that profiles of $\langle u^2 \rangle^+$ are enhanced, while distributions of $\langle v^2 \rangle^+$, $\langle w^2 \rangle^+$ and $\langle uv \rangle^+$ are attenuated with increasing $DR$. 

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3.4 Flow topology

Turbulent flows consist of various coherent patterns that persist in both time and space (Graham & Floryan, 2021). There are a number of different types of coherent flow motions; in wall-bounded turbulence, examples of coherent flow patterns include quasi-streamwise and hairpin vortices. Together, these elemental vortical structures are believed to account for the measured ensemble statistics (e.g., mean velocity and Reynolds stresses) within the different canonical turbulent flows. Therefore, understanding the distribution and dynamics of these coherent flow patterns is crucial to comprehending the nature of turbulence and unravelling methods on how it can be controlled or manipulated. There are several methods for identifying coherent flow patterns. The present work utilizes that of Chong et al. (1990), where the eigenvalues and invariants of the velocity gradient tensor (VGT) are used to identify the local topology and streamline patterns about critical points within the flow.

The method established by Chong et al. (1990), herein referred to as the Δ-criterion, groups the flow into regions that are focal (vortical) and dissipative (saddle points), depending on the sign convention of the invariants and the real/complex nature of the eigenvalues. Here, Δ is defined as the discriminant of the characteristic equation of the VGT. Although the Δ-criterion can be applied to both compressible and incompressible fluid flows, the focus of the present work is only on incompressible flows, which narrows down the number of flow classifications. Using the Δ-criterion, various works have demonstrated that quasi-streamwise and hairpin vortices can be visualized in numerical simulations and flow measurements of Newtonian wall-bounded turbulence. Furthermore, the joint probability density function (JPDF) of the invariants in the VGT ear-drop or pear-shaped distribution. This tear-drop pattern in the JPDF of the VGT invariants is not only found in wall-bounded turbulence, but also turbulent mixing layers, jets and isotropic turbulence, implying a universal distribution of topologies exists among different types of Newtonian turbulence (Soria et al., 1994; Chong et al., 1998; Ooi et al., 1999; da Silva & Pereira, 2008).

The following section will serve to summarize the Δ-criterion of Chong et al. (1990). Recall from §2 that the VGT is \( \mathbf{L} = \nabla \mathbf{U} \), and \( \mathbf{U} \) is the velocity vector. The characteristic equation for the tensor \( \mathbf{L} \) is,

\[
\Lambda^3 + P_L \Lambda^2 + Q_L \Lambda + R_L = 0,
\]

where \( P_L, Q_L, \) and \( R_L \) are the invariants of \( \mathbf{L} \). The eigenvalues are the roots to (3.26), and are defined as \( \Lambda_1, \Lambda_2 \) and \( \Lambda_3 \) in descending order of magnitude. In an incompressible flow, the first invariant \( P_L = -\text{tr}(\mathbf{L}) \) is equal to zero, while \( Q_L \) and \( R_L \) are the only non-zero invariants of \( \mathbf{L} \) and can be expressed as,

\[
Q_L = -\frac{1}{2} \text{tr}(\mathbf{L}^2),
\]

\[
R_L = -\text{det}(\mathbf{L}).
\]

Here, \( \text{tr}(\ldots) \) represents the trace operator on a square matrix, and \( \text{det}(\ldots) \) the determinant. The nature of the eigenvalues of \( \mathbf{L} \) are dictated by the sign convention of the discriminant \( \Delta \) of (2.1),

\[
\Delta = \frac{27}{4} R_L^2 + Q_L^3.
\]
Figure 3.5: Local topologies for different $R_L$ and $Q_L$ in an incompressible flow with $P_L = 0$.

where $\Delta > 0$ produces one real and two complex eigenvalues, and $\Delta \leq 0$ produces three real eigenvalues. Figure 3.5 describes the different possible local flow topologies that depend on the sign convention of $\Delta$ and $R_L$. The lines corresponding to $\Delta = 0$, and shown in figure 3.5, are referred to as the Vieillefosse tail’s. Here, ($\Delta = 0, R_L < 0$) is the left-Vieillefosse tail and ($\Delta = 0, R_L > 0$) is the right-Vieillefosse tail. Flow conditions above the Vieillefosse tail’s with $\Delta > 0$, consist of motions that are focal and primarily vortical. Regions of the flow with $\Delta \leq 0$ take on a node/saddle/saddle streamline pattern. Flow topology is also divided about the $R_L = 0$ axis, where flows with $R_L < 0$ are stable (stretching) and $R_L > 0$ are unstable (compressing).

JPDFs of $Q_L$ and $R_L$ in various Newtonian turbulent flows take on a tear-drop pattern [Soria et al., 1994; Blackburn et al., 1996; Chong et al., 1998; Ooi et al., 1999; da Silva & Pereira, 2008]. The point or tip of the tear-drop falls on the right-Vieillefosse tail ($\Delta = 0, R_L > 0$), while the bulb of the tear-drop is situated in the quadrant of stable focus-stretching ($\Delta > 0, R_L < 0$).

Similar to $L$, the tensors $D$ and $W$ have their own characteristic equation. For the tensor $D$, the characteristic equation is

$$
\Gamma^3 + P_D \Gamma^2 + Q_D \Gamma + R_D = 0,
$$

(3.29)

where $P_D = -\text{tr}(D) = 0$, and the non-zero invariants are defined according to,

$$
Q_D = -\frac{1}{2} \text{tr}(D^2),
$$

(3.30a)

$$
R_D = -\text{det}(D) = -\frac{1}{3} \text{tr}(D^3).
$$

(3.30b)

The roots of (3.29) are the eigenvalues of $D$ and are defined as $\Gamma_1$, $\Gamma_2$ and $\Gamma_3$ in descending order of magnitude. Similar to (3.28) for $L$, the discriminant of (3.30) for $D$ is,
Figure 3.6: Ratios of eigenvalues for different $R_D$ and $Q_D$ for an incompressible flow with $P_D = 0$. The eigenvalues are listed in the descending order of $\Gamma_1:\Gamma_2:\Gamma_3$.

$$\Delta_D = \frac{27}{4} R_D^2 + Q_D^3.$$  \hfill (3.31)

Because $D$ is a real and symmetric tensor, its eigenvalues will always be real and $\Delta_D \leq 0$. A plot of $Q_D$, $R_D$ space is shown in figure [3.6] where black solid lines represent curves with the same ratio of principal strain rates or eigenvalue of $D$, defined as $\Gamma_1:\Gamma_2:\Gamma_3$ [Blackburn et al., 1996]. The different eigenvalue ratios are commonly associated with unique straining motions.

Unlike tensors $L$ and $D$, the rate of rotation tensor has only one non-zero invariant for an incompressible flow, that being

$$Q_W = -\frac{1}{2} \text{tr}(W^2).$$  \hfill (3.32)

Note that the second invariant of $L$ can be equally represented as $Q = Q_D + Q_W$. Values of $Q_D$ are always negative, while values of $Q_D$ are always positive. [Truesdell] [1954] established a kinematical vorticity number

$$\mathcal{K} = \left(\frac{Q_W}{-Q_D}\right)^{1/2},$$  \hfill (3.33)

which defines the local strength of rotation relative to stretching [Ooi et al., 1999]. The change in $\mathcal{K}$ is shown schematically in figure [3.7] for different $Q_D$ and $Q_W$, similar to the diagram provided in [Soria et al., 1994]. Regions of the flow with small $Q_W$ and $\mathcal{K} \approx 0$ are more irrotational and dominated by dissipative motions, while flow regions with negligible $Q_D$ and large values of $\mathcal{K}$ that approach $\infty$ experience solid body rotation. Regions with large enstrophy density and dissipation fall on the line with $\mathcal{K} = 1$, where
\[ Q_W = -Q_D. \] From simulations of an incompressible mixing layer, [Soria et al. (1994)] described how flow motions with \( K = 1 \) consist of vortex sheets. From a rheological perspective, the values of \( K \) also translate to the different elementary rheometric flows, i.e., extension, shear and rotation.

The rheology of viscoelastic fluids depends heavily on whether the flow is dominated by extension or shear. Therefore, concerted efforts have been made within the rheology community to establish methods for distinguishing extensional flow motions from shear [Astarita (1967, 1979)]. Many of these methods utilize the invariants in the VGT (more so \( D \) and \( W \)), and were established long before the \( \Delta \)-criterion was documented by [Chong et al. (1990)].

Consider the elementary rheometric flows, i.e., steady shear, extension and rigid body rotation. Note that steady extensional flows can be further divided into uniaxial, biaxial and planar extension. A review of these basic rheometric flows is not presented, however deriving their respective invariants in \( D \) and \( W \) is trivial – see e.g., page 73–75 of [Macosko (1994)]. In the \( R_D, Q_D \) space, the ratio of principal strains also correspond to the limits of possible rheometric flows. The different rheological flows are labelled on the schematic of \( R_D, Q_D \) space of figure 3.6. Uniaxial and biaxial extension correspond to \( \Delta_D = 0 \) and the limits of \( R_D, Q_D \) space. Uniaxial extension flows have an eigenvalue ratio of \( \Gamma_1: \Gamma_2: \Gamma_3 = 2:-1:-1 \) (i.e., negative \( R_D \)), while biaxial extensional flows have an eigenvalue ratio of \( \Gamma_1: \Gamma_2: \Gamma_3 = 1:1:-2 \) (positive \( R_D \)). Shear and planar extension both exist on the \( R_D = 0 \) axis and are two-dimensional flows, with an eigenvalue ratio of \( \Gamma_1: \Gamma_2: \Gamma_3 = 1:0:-1 \). Comparing the invariants \( Q_W \) and \( Q_D \) is also commonly used to distinguish rheometric flows. [Astarita (1979)] derived a criteria that was adapted from the work of [Astarita (1967)], for distinguishing steady shear, extension and solid body rotation in non-Newtonian flows and served functionally the same
as the kinematical vorticity number $\mathcal{K}$. Flow regions that are extension dominant ($\mathcal{K} = 0$), shear dominant ($\mathcal{K} = 1$) or in rigid body rotation ($\mathcal{K} = \infty$) are annotated on the schematic of $Q_D$, $Q_W$ space shown in figure 3.7. Dimensionless indicators similar to $\mathcal{K}$ are generally referred to as a “flow-type” and can be commonly found in a variety of works involving non-Newtonian flows (Haward et al., 2016, 2018b; Walkama et al., 2020; Ekanem et al., 2020; Kumar et al., 2022).
Chapter 4

Experimental methods

Experimental measurements were performed using a consistent approach across all projects in this dissertation. Experiments of the turbulent flow of non-Newtonian fluids were performed in a large scale flow facility (e.g., channel flow or water flume). Samples of the fluid were extracted from the flow facility and their shear and extensional rheology is measured. The following chapter reviews the rheological methods in §4.1, followed by the large scale turbulent flow facilities in §§4.2 and 4.3, as well as the utilized flow measurements in §4.4.

4.1 Rheometric measurements

4.1.1 Steady shear rheology

The steady and dynamic shear viscosity of the Newtonian and non-Newtonian fluids were measured using a single-head stress-controlled torsional rheometer (HR-2, TA Instruments). The spindle head of the torsional rheometer is shown schematically in figure 4.1. The rheometer consists of a shaft that is driven by a drag cup alternating current (AC) motor. A torque $T$ is applied to the shaft via the drag cup motor and the resulting angular velocity $\Omega$ or displacement is measured using an optical encoder. An air bearing helps support the spindle shaft and mitigate friction between the shaft and spindle head assembly. A geometry is fastened to the end of the spindle head; in figure 4.1 a single gap concentric cylinder geometry is depicted. Fluid is loaded between the upper geometry and a compatible and immovable lower fixture. The applied torque $T$ of the drag cup motor can be converted into a stress $\tau$ imposed on the fluid sample, depending on the geometry. The shear strain $\gamma$ and shear rate $\dot{\gamma}$ within the fluid sample can be established based on the measurements of the angular displacement and velocity $\Omega$, again, depending on the geometry. The HR-2 single-head rheometer from TA Instruments has a minimum and maximum torque $T$ of 10 nN m and 200 mN m. The maximum measurable angular velocity $\Omega$ is 300 rad s$^{-1}$. The biggest limitation of the single head rheometer, particularly for dynamic shear viscosity measurements, is the need to overcome the inertia of the spindle shaft. Three different types of geometries were used for measurements of the steady and dynamic shear viscosity. Limitations from low torque and inertia are discussed for the different geometries and the types of measurements.

Three different types of geometries were used for measurements of steady shear viscosity. An illustration
of the geometries is shown in figure 4.2. A cylindrical coordinate system is shown alongside each geometry, where $r$ is the radial direction, $\theta$ is the angular component and $x$ is the axial component. The single gap concentric cylinder geometry is depicted in figure 4.2(a). A fluid sample is loaded between an inner cylinder of radius $R_{\text{min}} = 14$ mm and an outer fixed cylinder of radius $R_{\text{max}} = 15.2$ mm, with an immersion height of $L_{\text{sg}}$ of 42.04 mm. The inner cylinder is fastened to the end of spindle shaft depicted in figure 4.1 and rotates due to the balance between the torque $T$ applied from the drag cup and a counter-torque caused by the viscous skin friction of the sample. The flow between the cylinders is homogeneous Taylor-Couette flow with a constant shear rate $\dot{\gamma}$ between the gaps of the cylinders. This configuration emulates the simple shear Couette flow depicted in §2.1, but in cylindrical coordinates.

Figure 4.2(b) illustrates a double gap concentric cylinder geometry. The fixed cylinder has an inner cylinder of $R_1 = 15.1$ mm and outer radius of $R_2 = 18.5$ mm. The rotating cylinder fastened to the spindle head consists of an inner radius of $R_3 = 16.0$ mm and outer radius of $R_4 = 17.5$. Fluid is contained between two gaps from $R_1$ to $R_3$, and $R_2$ to $R_4$, at an immersion height of $L_{\text{dg}} = 53.0$ mm. Similar to the single gap concentric cylinder of figure 4.2(a), the double gap concentric cylinder imposes a homogeneous Taylor-Couette flow. The advantage of the double gap configuration over the single gap setup is the larger contact area between the fluid and the geometry. A larger contact area permits for lower measurements of $\Omega$, and hence measurements of $\mu$ at much lower shear rates $\dot{\gamma}$.

Lastly, a parallel plate geometry is depicted in figure 4.2(c). Fluid is contained between a rotating circular plate or disk with a radius $R_{\text{pp}} = 30$ mm and another parallel fixed plate of larger radius. The gap $h_{\text{pp}}$ between the fixed and rotating parallel plates is 0.2 mm. Unlike the Taylor-Couette flows of the single gap and double gap concentric cylinders of figure 4.2(a, b), the parallel plate geometry is not homogeneous and does not impose a constant shear rate $\dot{\gamma}$ within the sample. Instead, the shear rate is taken to be largest shear
rate within the sample, which is shear rate at the edge or rim of the parallel plate. The advantage of the parallel plate geometry is that it allows for measurements of \( \mu \) at very large \( \Omega \) and \( \dot{\gamma} \), when \( h_{pp} \) is small. However, this can be accompanied with some additional challenges and measurement errors. These errors are discussed in §8.3 when the measurements using the parallel plate are presented.

The rotational velocity \( \Omega \), in rad s\(^{-1}\), can be converted to \( \dot{\gamma} \), using \( \dot{\gamma} = F_\gamma \Omega \), where \( F_\gamma \) is the strain coefficient for the particular geometry. Similarly, the torque, \( T \), can be converted to stress, \( \tau \), using \( \tau = F_\tau T \), where \( F_\tau \) is the stress coefficient of the geometry. After which, the shear viscosity can be derived based on, \( \mu = \tau/\dot{\gamma} = (F_\tau/F_\gamma)(T/\Omega) \) (Barnes et al., 1989; Ewoldt et al., 2015). Each geometry has their own strain and stress coefficients, the values of which are listed in table 4.1 (Barnes et al., 1989; Macosko, 1994).

<table>
<thead>
<tr>
<th>Geometry</th>
<th>( F_\gamma )</th>
<th>( F_\tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single gap concentric cylinder</td>
<td>( \frac{R_{\text{max}}}{R_{\text{max}}-R_{\text{min}}} )</td>
<td>( \frac{1}{2\pi R_{\text{max}} L_{\text{sg}}} )</td>
</tr>
<tr>
<td>Double gap concentric cylinder</td>
<td>( \frac{R_1^2}{R_1^2-R_2^2} ) + ( \frac{R_3^2}{R_3^2-R_4^2} )</td>
<td>( \frac{1}{2\pi(R_3^2+R_4^2) L_{\text{dg}}} )</td>
</tr>
<tr>
<td>Parallel plates</td>
<td>( \frac{R_{pp}}{h_{pp}} )</td>
<td>( \frac{2}{\pi R_{pp}^2} )</td>
</tr>
</tbody>
</table>

Table 4.1: Strain and stress coefficients for geometries used in the torsional rheometer.
The maximum shear rate limit of the steady shear viscosity measurements corresponds to the onset of inertial instabilities and turbulence. For the Taylor-Couette viscometers depicted in figure 4.2 (a, b), this generally occurs when the Taylor number $Ta$, exceeds 1700 (Ewoldt et al., 2015). For the single gap configuration $Ta = \rho^2 \Omega^2 (R_{\text{max}} - R_{\text{min}})^3 R_{\text{min}} / \mu^2$. While for the double gap geometry, $Ta = \rho^2 (R_3 + R_4) h_{dg}^3 \Omega^2 / 2 \mu^2$, where $h_{dg} = [(R_3 - R_1) + (R_4 - R_2)] / 2$ (Pereira et al., 2013). For the parallel plate geometry, secondary instabilities corrupt the viscosity measurements when $Re_{pp} = \rho \Omega R_{pp} h_{pp} / \mu$ is greater than 100 (Davies & Stokes 2008).

4.1.2 Dynamic shear rheology

Two types of dynamic shear viscosity or small amplitude oscillatory shear measurements were performed, in accordance with §2.2. The first was a sweep of stress amplitude $\tau_0$ with a constant oscillation frequency $\omega$. These experiments were used to establish the limit of linear viscoelasticity (LVE); for large $\tau_0$, the relationship between stress and strain no longer follows a linear differential equation similar to (2.8) (Mezger, 2020). The second set of dynamic shear viscosity measurements was a sweep of $\omega$ using a sufficiently small value of $\tau_0$ that is within the LVE regime. Trios software (TA Instruments) was used to determine the phase offset, $\psi$, using a cross-correlation of the sinusoidal stress, $\tau(t)$, and the measured strain, $\gamma(t)$ signals. The complex stress modulus was derived from the quotient of the stress and strain amplitudes, $G^* = \tau_0 / \gamma_0$, where $\gamma_0$ is the measured strain amplitude. After determining the complex stress modulus and the phase offset, the gain modulus, $G'$, and loss modulus, $G''$, could be determined using, $G^* = G' + iG''$, and $\tan(\psi) = G'' / G'$, (2.11). Distributions of $G'$ and $G''$ for the non-Newtonian solutions were then used to comment on the linear viscoelasticity of the complex fluids.

Unlike steady shear viscosity measurements, dynamic shear viscosity measurements are much more constrained by the torque and inertia limitations of the device – especially when using a single head torsional rheometer (Läuger & Stettin, 2016). Correcting the torque measurements to compensate for the inertia of the spindle head and geometry can be effective, but not always perfect (Ewoldt et al., 2015; Läuger & Stettin, 2016). Ewoldt et al. (2015) recommended ensuring that the torque imposed by the material exceed the torque required to overcome the inertia of the geometry. They derived a limitation on the shear moduli,

$$G > \frac{1F_\tau}{F_\gamma \omega^2},$$

(4.1)

where $G$ can be either $G'$ or $G''$, and $I$ is the moment of inertia of the geometry. Of the three geometries depicted in figure 4.2, the single gap concentric cylinder has the lowest geometry inertia, $I$, which was approximately equal to $4.3 \times 10^{-6}$ kg m$^2$. Therefore, only the single gap concentric cylinder was used for the dynamic shear viscosity measurements. Measurements of $G'$ and $G''$ that fall below the inertia limitation were disregarded.

4.1.3 Extensional rheology

The extensional rheology of the non-Newtonian fluids were evaluated using two types of devices. The first was a bespoke dripping onto substrate (DoS) apparatus, depicted in figure 4.3 (a). In this measurement
technique, a small droplet was discharged from a blunt-end nozzle with a diameter $D_0$ of 1.27 mm. A syringe pump (Legacy 200, KD Scientific Inc.) was used to expel the droplet from the nozzle at a rate of 0.02 ml min$^{-1}$. Pumping was terminated once the droplet made contact with a glass substrate that was situated 3$D_0$ or 3.81 mm below the blunt-end of the nozzle outlet. An apparatus with similar features was used in [Dimic et al. (2015), Dimic et al. (2017) and Zhang & Calabrese (2022)]. After the droplet made contact with the substrate, a liquid bridge was formed between the nozzle outlet and the substrate. The diameter of the liquid bridge $D_{min}$ decayed rapidly due to capillary forces. Images of the liquid bridge were collected using a high-speed camera (v611, Vision Research) and back-lit illumination from a light emitting diode. Figure 4.3(b) shows a sample image of the liquid bridge for an aqueous solution of polyacrylamide with a concentration of 500 ppm. The camera had a $1280 \times 800$ pixel complementary metal oxide semiconductor sensor with pixels that were $20 \times 20 \mu m^2$ in size and had a bit-depth of 12 bit. A zoom lens was used to achieve a magnification of 3.8 and a scale of 5.16 $\mu m$ pixel$^{-1}$. Images were collected at an acquisition rate of 2 kHz. The minimum diameter $D_{min}$ of the liquid bridge was determined using a script developed in MATLAB (Mathworks Inc.).

![Diagram](image)

Figure 4.3: (a) Isometric view of a 3D model depicting the DoS setup. (b) A sample image taken for an aqueous solution of polyacrylamide with a concentration of 500 ppm in elastocapillary thinning.

The pinch-off dynamics of the liquid bridge in the DoS apparatus depends on forces attributed to inertia, surface tension, viscosity and elasticity [Dimic et al. (2017)]. The Ohnesorge number $Oh = t_v/t_R$ relates the time scale associated with viscous forces to the Rayleigh time $t_R$, which pertains to surface tension and inertial forces. Here $t_v = \mu D_0/2\sigma$ is the characteristic timescale of visco-cappillary thinning, $t_R = (\rho D_0^3/8\sigma)^{1/2}$, and $\sigma$ is the surface tension. Low viscosity fluids typically have $Oh < 1$ and a necking process dominated by inertial and capillary forces. In this regime, inertio-capillary (IC) thinning is described by a $2/3$ power law,

$$\frac{D_{min}(t)}{D_0} = a\left(\frac{t_b - t}{t_R}\right)^{2/3}, \quad (4.2)$$

where $t_b$ is the filament break-up time, and $a$ is a multiplicative pre-factor between 0.4 and 1 [Zhang & Calabrese (2022)]. If $Oh > 1$, viscous forces are significant, and the evolution of $D_{min}$ is described by
viscocapillary thinning, $D_{\text{min}}(t)/D_0 = 0.0709(t_b - t)/t_v$ (McKinley & Tripathi, 2000). For elastic fluids, the Deborah number $De = t_e/t_R$ describes the ratio of the extensional relaxation time $t_e$ and the Rayleigh time (Tirtaatmadja et al., 2006). If $De > 1$, the necking process is dominated by elastic and capillary forces. This elastocapillary (EC) regime is described by,

$$\frac{D_{\text{min}}(t)}{D_0} = A \exp \left(-\frac{t}{3t_e}\right), \quad (4.3)$$

where $A = (GD_0/2\sigma)^{1/3}$. Generally, the fluids measured using the DoS apparatus exhibited thinning in an IC ($Oh < 1$) or EC regime ($De > 1$). Nonlinear least square regression was used to establish $t_e$ for fluids that exhibited EC thinning using measurements of $D_{\text{min}}$ and equation 4.3. Values of $t_e$ are listed in table 5.1.

The second extensional rheometer was a commercial piece of equipment called a capillary break-up extensional rheometer (CaBER) from Thermo Scientific. Similar to the DoS apparatus, the CaBER device relies on capillary forces to induce elastocapillary thinning in a fluid sample. Unlike the DoS apparatus, however, the CaBER device applies an initial step strain to the fluid that then triggers the break-up of the fluid filament.

Within the CaBER apparatus, a small sample is loaded between two 6 mm diameter circular plates that are parallel and separated 3mm apart from one another. After loading the sample, the top plate was then rapidly displaced causing the solution to stretch in uniaxial extension, similar to §2.3. The final gap between the plates was 9 mm and the strike time to attain the final position was 50 ms. A laser micrometer was used to measure the midpoint diameter $D_{\text{mid}}$ as a function of time $t$. The extensional relaxation time $t_e$ was then derived utilizing (4.3), but substituting $D_{\text{mid}}$ for $D_{\text{min}}$.

### 4.2 Turbulent channel flow

A recirculating flow loop with an in-line channel section, as shown in figure 4.4, was used for measurements of a turbulent channel flow. The channel section had a rectangular cross-section with a height, $H$, of 15 mm and width, $W$, of 120 mm. It also consisted of four sub-sections connected with flanges as seen in figure 4.4. The third section from the channel inlet was made with glass walls for optical measurements. The measurements were carried out at the middle of this third section which was situated $107H$ downstream from the inlet of the channel section. This ensured a fully developed turbulent channel flow. The walls of the channel sections immediately upstream and downstream of the measurement section were cast acrylic. Transition fittings, 30 cm in length, were used to convert the cross-section from circular to rectangular, and vice versa. The complete length of the channel section was $168H$. Figure 4.5 demonstrates the cross-section of the measurement section and the coordinate system used here. Position along the streamwise direction is denoted as $x$, while $y$ is the wall-normal direction and $z$ is the spanwise direction. The coordinate system is centred at the mid-span of the lower channel wall.

Fluid was driven using a centrifugal pump (LCC-M 50-230, GIW Industries Inc.) controlled by a variable frequency drive. A thermocouple (Type K) and a double pipe heat exchanger were used to measure and maintain a constant temperature. The mass flow rate, $\dot{m}$, was measured using a Coriolis flow meter (Micro Motion F-series, Emerson Process Management) with an accuracy of $\pm 0.2\%$. A proportional integral
A derivative controller was used to maintain a constant $\dot{m}$ by controlling the input frequency to the pump. Static pressure loss along the channel was measured using a differential pressure transducer (DP-15, Validyne). Ports for the pressure transducer were separated $109H$, with the upstream port being $34H$ from the channel inlet.

![Diagram of flow facility](image)

Figure 4.4: Annotated top view of experimental flow facility showing the pipe loop connected to the channel section.

4.3 Turbulent boundary layer

Turbulent boundary layers were formed along the floor of a closed-loop water flume illustrated in figure 4.6(a). The flume consists of a 5 m long channel that bridges two cubic reservoirs. The channel was 0.68 m in width $W$. The free surface was situated at a height $H$ that was 0.2 m above the bottom floor of the channel. The channel cross-section with respect to the Cartesian coordinate system is shown in figure 4.6(b). The total volume of liquid within the flume was 3500 l. The walls of the channel consist of 12.7 mm thick glass.

![Diagram of channel cross-section](image)

Figure 4.5: Cross-section of the channel flow test section.
panels. Two centrifugal pumps (Deming 4011 4S, Crane Pumps and Systems) in a parallel configuration were used to circulate the fluid within the flume. Variable frequency drives enabled control of the rotational speed of each pump. In all boundary layer experiments within the flume both pumps were operated at the same rotational speed. Measurements of the turbulent boundary layer of water were collected for pump speeds between 300 rpm and 1000 rpm, which corresponds to free-stream velocities $U_\infty$ between 0.124 and 0.430 m s$^{-1}$. A series of mesh screens within the upstream reservoir of the water flume ensured that the turbulence intensity of the free-stream was less than 2%. Fluid temperature was monitored using a K-type thermocouple and a data logger (HH506, Omega Engineering).

Figure 4.6: (a) Isometric view of a model that depicts the water flume facility, and (b) a cross-section of the open channel section.

### 4.4 Flow measurements

Particle image velocimetry (PIV) is a non-intrusive technique used to measure the local displacement ($\Delta x$) of fluid elements over a short time interval ($\Delta t$) [Adrian, 1984]. The technique infers the velocity field based on the broad movement of tracer particles that are evenly dispersed within the moving fluid. Tracer particle are illuminated using a light source – which in the case of planar PIV is a thin laser sheet, that is typically 1-2 mm in thickness. Light scatted by the particles are then recorded by a camera that is synchronized with the light source. Illumination is provided as a short pulse that “freezes” the motion of the tracers in one image. More advanced three-dimensional (3D) methods, such as tomographic PIV, utilize multiple cameras at different perspectives to measure the velocity of the fluid in all three spatial directions [Elsinga et al., 2006]. Particles are typically illuminated using a laser volume. Concurrent images are captured with a short intermittent time delay $\Delta t$, such that the motion of the tracer particles between subsequent images can be recorded. Images are then divided into interrogation windows. In planar PIV, the displacement of the tracers
has two components, $\Delta x = (\Delta x, \Delta y)$. The two-dimensional velocity vector is then inferred according to,

$$U = \frac{\Delta x}{\Delta t}$$  \hspace{1cm} (4.4)

The displacement $\Delta x$ within each interrogation window is determined using a cross-correlation technique. The cross-correlation operator is applied to the 2D light intensity signal recorded within each interrogation window.

Particle tracking velocimetry (PTV) is a Lagrangian approach that tracks individual particles as they travel through the measurement domain. The PTV setup consists of the same equipment used for PIV. A laser is used to illuminate tracers in the flow, and cameras are used to record the light scattered from the particles over several instances of time. Traditionally, PTV required lower seeding densities than PIV, in order to locate individual particles and track their position across different time instances without interference from neighbouring particles. An algorithm called Shake-The-Box (STB) was established for 3D-PTV measurements that permits higher seeding densities and a larger number of measured particle trajectories within the measurement domain \cite{Schanz2016}.

The PIV and PTV measurement techniques rely on the assumption that tracer particle faithfully follow the fluid flow. Two criteria are used to convey the validity of this assumption \cite{Bewley2008}. The first criteria utilizes the Stokes number,

$$St = t_p/t_f,$$  \hspace{1cm} (4.5)

where $t_p = \rho_p d_p^2 / 18 \mu$ is the particle response, $t_f$ is a representative time scale of the flow, $\rho_p$ is the density of the particles and $d_p$ the diameter. If $St$ is less than 0.1, tracer particles follow the flow well \cite{Bewley2008}. The second criteria utilizes the Froude number,

$$Fr = u_p/u_f,$$  \hspace{1cm} (4.6)

where the particle settling velocity is $u_p = (\rho_p - \rho)d_p^2 g / 18 \mu$, $g$ is the gravitational acceleration, and $u_f$ is a representative velocity of the flow. If $Fr$ is less than 1, the influence of particle settling caused by gravity is negligible. Each investigation that utilizes PIV and PTV, including §§6, 7, 8 and 9, provides a depiction of the measurement setup, a description of the equipment and a discussion of measurement uncertainty. In the discussion of measurement uncertainty, the values of $St$ and $Fr$ are provided.
Part II

Rheology
Chapter 5

Shear and extensional rheology

Before investigating the drag-reducing capabilities of different non-Newtonian solutions, a review of their rheological features is first presented. The present chapter compares measurements of steady shear viscosity, dynamic shear viscosity and extensional rheology for aqueous solutions comprised of three different types of drag-reducing additives and using the methods detailed in §4.1. All of the additives are known to induce drag reduction in turbulent wall flows (Escudier et al. 1999, Qi & Zakin 2002).

5.1 Non-Newtonian fluids

The three additives included a flexible polymer, a rigid biopolymer and a cationic surfactant. Additives in their solid powder form were weighed using a digital scale (Explorer Analytical, OHAUS Corporation) with a 1 mg resolution. Solid powders were then gradually added to 15 l of distilled water and agitated for 8 h using a stand mixer equipped with a 100 mm diameter impeller (Model 1750, Arrow Engineering Mixing Products). After mixing, the aqueous non-Newtonian solutions were left to rest for 16 h. Fluid samples were then collected for rheology measurements.

The flexible polymer was polyacrylamide (PAM) from a sample batch contributed by SNF Floerger (6030S, molecular weight of 30-35 Mg mol$^{-1}$). The rigid biopolymer was xanthan gum (XG) (43708, MilliporeSigma). Both polymers, PAM and XG, have been readily used in various experimental investigations involving rheology and turbulent drag reduction (Escudier et al. 1999, Mohammadtabar et al. 2020, Warwaruk & Ghaemi 2021). Cationic surfactants are quaternary ammonium salts of the form $C_nH_{2n+1}N^+(CH_3)_3Cl$, where $n$ is an integer, generally between 12 and 18 (Qi & Zakin 2002). When paired with a counterion, such as sodium salicylate (NaSal), the molecules combine to form complex molecular agglomerates known as micelles (Bewersdorff & Ohlendorf 1988, Zhang et al. 2005). For the present measurements, Trimethyl Tetradecyl Ammonium Chloride ($n = 14$) (T0926, Tokyo Chemical Industry Co., Ltd.) combined with NaSal (71945, MilliporeSigma) at a molar ratio of 1:2 was used, as this was combination was shown to produce considerable amounts of drag reduction in other studies (Bewersdorff & Ohlendorf 1988, Warwaruk & Ghaemi 2021). Going forward, the surfactant solution is referred to as C14. A parametric sweep of five concentrations were considered for each additive (i.e. PAM, XG, and C14). The concentrations, $c$, were the same for all additives: 100ppm, 200ppm, 300ppm, 400ppm, and 500ppm.
5.2 Steady shear viscosity

Shear rheology measurements were performed using the controlled-stress single-head torsional rheometer detailed in §4.1.1 and shown schematically in figure 4.1. The single gap concentric cylinder detailed in figure 4.2 (a) was used for all viscosity measurements in the current chapter. Steady shear viscosity measurements involved a logarithmic sweep in the shear rate \( \dot{\gamma} \) from 0.1 s\(^{-1}\) to 1000 s\(^{-1}\) with 10 data points per decade, and the corresponding stress \( \tau \) was monitored. Recall from §4.1 that the rheometer has a lower torque limit of 10 nN m, or \( \tau = 0.2 \) mPa, according to the manufacturer, TA Instruments. In practice, it was found that the lower limit for steady shear viscosity measurements was higher, \( \tau = 100 \) nN m, or \( \tau = 2 \) mPa. A power-law model was fit to shear rheograms for fluids that exhibited shear thinning tendencies. The power law was defined according to \( \mu = M \dot{\gamma}^{k-1} \) or equation (2.7), where \( M \) is the consistency and \( k \) is the flow index. Fits were performed on profiles of \( \mu(\dot{\gamma}) \) with \( \tau > 2 \) mPa and \( Ta < 1700 \), using nonlinear least square regression.

Figure 5.1 displays measurements of \( \mu \) as a function of \( \dot{\gamma} \) for the Newtonian and non-Newtonian fluids. Shear viscosity distributions of distilled water are shown in figure 5.1 (a). Measurements of \( \mu \) for water are constant with respect to \( \dot{\gamma} \) provided \( \tau > 2 \) mPa and \( Ta < 1700 \). For \( \tau < 2 \) mPa, measurements of \( \mu \) for water are noisy and scattered. When \( Ta > 1700 \), measurements of \( \mu \) for water increase abruptly and are no longer constant with respect to \( \dot{\gamma} \); Taylor vortices have corrupted the measurements of \( \mu \). The average viscosity of water for \( \tau > 2 \) mPa and \( Ta < 1700 \) is 0.97 mPa s. This is 3\% lower than the theoretical shear viscosity of water at 20.1\(^\circ\)C, 1.00 mPa s.

Figure 5.1 (b) shows profiles of \( \mu \) for the five PAM solutions. All five concentrations of PAM exhibit larger values of \( \mu \) than water. They also exhibit shear thinning, where \( \mu \) decreases monotonically with increasing \( \dot{\gamma} \). At the higher values of \( \dot{\gamma} \), \( \mu \) appears to increase sharply for \( \dot{\gamma} \) with a \( Ta \) less than 1700. Nonetheless, the trend by which \( \mu \) reduces with respect to \( \dot{\gamma} \) is well represented by the power law model (equation 2.7) for measurements with \( \dot{\gamma} > 0.1 \) s\(^{-1}\), and \( \dot{\gamma} < 100 \) s\(^{-1}\) – sufficiently below the shear rate that \( \mu \) increases abruptly. Values of the consistency, \( M \), and flow index, \( k \), for PAM are provided in table 5.1

\[
\begin{array}{cccc}
 c (\text{ppm}) & M (\text{Pa s}^{n-1}) & k & t_e (\text{ms}) \\
 100 & 3.0 \times 10^{-3} & 0.92 & 2.2 \\
 200 & 6.0 \times 10^{-3} & 0.86 & 6.3 \\
 300 & 9.9 \times 10^{-3} & 0.82 & 16.3 \\
 400 & 32.8 \times 10^{-3} & 0.62 & 32.2 \\
 500 & 40.5 \times 10^{-3} & 0.62 & 48.6 \\
\end{array}
\]

Table 5.1: Rheological parameters of PAM from steady shear rheology and DoS.

Figure 5.1 (c) demonstrates profiles of \( \mu \) for the five XG solutions. Similar to PAM all concentrations of XG exhibit larger values of \( \mu \) than water and prevalent shear thinning. Unlike PAM, \( \mu \) appears to increase sharply for \( \dot{\gamma} \) with a \( Ta \) equal to 1700. The shear thinning trend is well represented by the power law model
Figure 5.1: Steady shear viscosity distributions for (a) the baseline Newtonian fluids, (b) flexible polymer solution PAM, (c) rigid biopolymer solution XG, and (d) cationic surfactant solution C14. The horizontal black solid line is $\mu$ for water determined from the empirical correlation of Cheng (2008). Dashed black lines indicate the lower torque limit ($\tau < 2$ mPa) and the onset of Taylor vortices ($\mathcal{T}a > 1700$). In (b) and (c) solid coloured lines represent the power-law fits for shear thinning fluids given by equation (2.7) and with values provided in tables 5.1 and 5.2.

(eqation 2.7) for measurements with $\dot{\gamma} > 0.1$ s$^{-1}$, and $\mathcal{T}a < 1700$. Values of the consistency, $M$, and flow index, $k$, for XG are provided in table 5.2.

Lastly, figure 5.1(d) demonstrates shear rheograms for the five C14 solutions. Interestingly, all five C14 solutions have values of $\mu$ similar to water (around 1.00 mPa s) and independent of $\dot{\gamma}$. Unlike PAM and XG, C14 does not augment the viscosity of the solvent.

5.3 Dynamic shear viscosity

Measurements of linear viscoelasticity are shown in figure 5.2 for high concentration solutions of PAM and XG. Details regarding the dynamic shear viscosity measurements are provided in §4.1.2. Figure 5.2(a) demonstrates sweeps of stress amplitudes $\tau_0$ for PAM solutions, and figure 5.2(b) demonstrates the same stress amplitude sweep for the XG solutions. Both amplitude sweeps are conducted at a constant $\omega$ of 0.625 rad s$^{-1}$. Stress amplitude sweeps for PAM with $c = 400$ ppm and 500 ppm, shown in figure 5.2(a), have $G'$ and $G''$ values greater than the inertia limit of (4.1). Lower concentration solutions, with $c \leq 300$ ppm, are
not shown as their $G'$ and $G''$ measurements fall below the inertia limit of (4.1). Both PAM solutions with $c = 400$ppm and 500ppm, have $G'' > G'$ for all values of $\tau_0$, implying the solutions are viscous dominant when $\omega = 0.628$ rad s$^{-1}$. The difference between $G''$ and $G'$ diminishes as $c$ increases, i.e. the solution becomes more elastic as $c$ grows. Values of $G'$ and $G''$ are constant for $\tau_0 < 10^{-2}$ Pa. Therefore, the LVE regime is confined to stress amplitudes less than 10 mPa for PAM. Stress amplitude sweeps for XG solutions, shown in figure 5.2(b), are similar to PAM. All XG solutions are viscous dominant for $\omega = 0.628$ rad s$^{-1}$. The disparity between $G''$ and $G'$ decreases as the concentration of XG grows. Values of $G'$ and $G''$ are constant with respect to $\tau_0$ for $\tau_0 < 4$ mPa.

Sweeps of $\omega$ are shown in figure 5.2(c) for PAM and figure 5.2(d) for XG at a constant $\tau_0$ of 3.3 mPa, which is within the LVE regime. PAM solutions have finite values of $G'$ and $G''$ for $\omega$ between 0.1 and 10 rad s$^{-1}$. For both $c = 400$ppm and 500ppm, the solutions are viscous dominant, $G'' > G'$. As $\omega$ increases the elastic and viscous moduli become more similar in magnitude, implying the cross-over frequency where $G' = G''$ is slightly greater than 10 rad s$^{-1}$. Similar to PAM, XG also demonstrates finite $G'$ that are lower in magnitude than $G''$, i.e., viscous dominant. As $c$ increase, $G'$ becomes more similar in magnitude to $G''$. Unlike PAM, XG with $c = 300$ppm and 400ppm have profiles of $G'$ and $G''$ that are parallel. In other words, the difference between $G'$ and $G''$ is not changing with respect to $\omega$. When the concentration is increased to 500ppm, profiles of $G'$ and $G''$ appear to begin converging towards one another, implying that the cross-over frequency becomes lower as $c$ increases. Nonetheless, it is likely that the cross-over frequency is well above 10 rad s$^{-1}$ for the XG solutions. Overall, both PAM and XG demonstrate characteristics of uncrosslinked polymer solutions with predominantly viscous behaviour. Solutions of C14 had no measurable $G'$ or $G''$ values for the same reason the viscous moduli of water could not be measured; dynamic oscillation tests were overcome by the inertia of the geometry for $0.1 \text{ s}^{-1} < \omega < 10 \text{ s}^{-1}$.

### 5.4 Extensional rheology

Measurements of $D_{\text{min}}/D_0$ using the dripping onto substrate (DoS) apparatus are shown in figure 5.3 for the PAM solutions – the only solutions that demonstrated EC thinning. Details regarding the DoS setup are provided in §4.1.3. For $t$ less than the inertial break up time $t_b$, the evolution of $D_{\text{min}}$ is in an IC regime

<table>
<thead>
<tr>
<th>$c$ (ppm)</th>
<th>$M$ (Pa s$^{-n}$)</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>$1.7 \times 10^{-3}$</td>
<td>0.94</td>
</tr>
<tr>
<td>200</td>
<td>$11.2 \times 10^{-3}$</td>
<td>0.73</td>
</tr>
<tr>
<td>300</td>
<td>$19.5 \times 10^{-3}$</td>
<td>0.68</td>
</tr>
<tr>
<td>400</td>
<td>$32.8 \times 10^{-3}$</td>
<td>0.62</td>
</tr>
<tr>
<td>500</td>
<td>$53.3 \times 10^{-3}$</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Table 5.2: Power law model parameters according to equation (2.7) for XG.
and well described by equation (4.2). The inertial break-up time $t_b$ was not significantly different among the PAM solutions of different $c$ and was approximately 7.2 ms ± 0.6 ms. Measurements of $D_{\text{min}}/D_0$ for PAM with $c = 100$ppm, are shown in the inset axes of figure 5.3. The IC thinning represented by equation (4.2), and shown by the black solid line in the inset axes of figure 5.3 has $\alpha = 0.5$ and $t_R = 1.9$ ms. This value of $\alpha$ is between 0.4 and 1, which is within the margin of experimental expectations (Zhang & Calabrese, 2022). The theoretical Rayleigh time $t_R = (\rho D_0^2/8\sigma)^{1/2}$ should be 1.89 ms (assuming $\sigma \approx 72$ mN m$^{-1}$) – not significantly different than $t_R$ derived from fitting equation (4.2) onto measurements of $D_{\text{min}}/D_0$ in the IC regime. Recall that the Ohnesorge number is defined as $Oh = t_v/t_R$, where $t_v = \mu D_0/2\sigma$. If $\mu$ in the equation for $t_v$ is taken to be the largest measured viscosity in figure 5.3 (about 0.1 mPa s for PAM with $c = 500$ppm), then $t_v \approx 0.9$ ms, and the largest $Oh$ is about 0.5.

For $t > t_b$, all PAM solutions demonstrate EC thinning, well represented by equation (4.3) and the coloured lines shown in figure 5.3. As the concentration grows, the extensional relaxation time $t_e$ increases. Values of $t_e$ are provided in table 5.1. If it is assumed that $t_R = 1.89$ ms for all PAM solutions, $De$ was
between 1.2 and 25.7 depending on $c$. For the high concentration PAM solutions, the 2 kHz image acquisition rate coupled with the spatial resolution of the camera results in repetitive measurements of $D_{min}/D_0$ over several time instances (i.e., the small horizontal lines).

Figure 5.3: Normalized minimum filament diameter with respect to time for the PAM solutions, as determined from the DoS system. The inset figure demonstrates a zoomed in distribution along time for PAM with $c = 100$ ppm. Coloured solid lines indicate the fits of the EC regime using equation (4.3). The solid black line in the inset denotes the fit of the IC regime using equation (4.2).

Solutions of C14 and XG do not demonstrate EC thinning, and therefore, $De < 1$. A lack of EC thinning is either a result of a low $t_e$ or a large $t_R$, by definition of the Deborah number $De = t_e/t_R$. It is well known that surfactant solutions have a much lower $\sigma$ than the solvent and hence, a large $t_R$ (Zhang et al. 2005). It is possible that the lack of EC thinning in C14 could be attributed to low surface tension. Surface tension $\sigma$ is generally 40% lower for large concentration solutions of cationic surfactants relative to water (i.e., 35-45 mN m$^{-1}$). This would mean that $t_R$ could be approximately 30% larger for surfactants. However, it is suspected that a 30% increase in $t_R$ is not sufficient enough to explain the lack of EC thinning for C14. The present investigation does not measure $\sigma$, hence no definitive conclusion can be made in this regard. That being said, it is expected that the lack of EC thinning is attributed to low $t_e$ at the conditions imposed from the DoS rheometer. This is another example of how difficult it is to measure $t_e$ using capillary-driven extensional rheometers for drag-reducing surfactant solutions, as previously seen in Warwaruk & Ghaemi (2021) and Fukushima et al. (2022). It also highlights a need to develop other techniques for measuring extensional features of non-Newtonian solutions, as done in Wunderlich & James (1987). Ultimately, PAM solutions have relatively large $t_e$ that could be derived from the DoS rheometer, while C14 and XG solutions have $t_e$ that could not measured using the DoS apparatus.
Chapter 6

Nontrivial rheology

To better understand the features of dilute non-Newtonian solutions, there is merit in considering flows of moderate complexity – those that are not trivial enough to be considered viscometric, but not overly complex such as turbulence. Bird & Wiest (1995) referred to these flows as “nontrivial flows,” as they involved the laminar flow of non-Newtonian fluids through complex geometries. Some of these geometries include an abrupt contraction, periodically constricted tube, porous media, and undulating surfaces (Deiber & Schowalter, 1979; Pilitsis et al., 1991; Poole et al., 2005; Page & Zaki, 2016). Bird & Wiest (1995) referred to a few of these nontrivial flows as benchmark experiments, that could aid in the development of numerical methods for modelling the flow of non-Newtonian fluids. The features and phenomena observed from these nontrivial flows, particularly those involving dilute polymer solutions, are also believed by some to be of significance to polymer drag reduction or related to the onset of the self-sustaining chaotic state known as elasto-inertial-turbulence (EIT) (Joseph, 1990; Haward et al., 2018a). Experiments of dilute polymer solutions, at relatively low Reynolds numbers, in pressure-driven contraction and periodic contraction-expansion channels demonstrated an increased streamwise pressure gradient, near-wall velocity overshoots, and an augmented vorticity, not observed for Newtonian fluids (Poole et al., 2005; Ober et al., 2013; Haward et al., 2018a). Few experiments have considered dilute surfactant solutions in these nontrivial flows geometries. Based on the viscometric flows detailed in §5, dilute surfactant solutions had no apparent non-Newtonian features. The investigation in the present chapter seeks to unravel the non-Newtonian traits of the dilute surfactant solution by considering its flow in a nontrivial geometry, that being a periodically constricted tube (PCT). The same three types of non-Newtonian additives used in §5 are investigated within the PCT flow. Five concentrations are considered for each non-Newtonian fluid (15 solutions in total). A flow measurement technique known as particle shadow velocimetry (Santiago et al., 1998; Estevadeordal & Goss, 2006; Khodaparast et al., 2013) is used to directly measure the velocity of each fluid in the PCT at five different flow rates.

6.1 Periodically constricted tube

Figure 6.1(a) demonstrates a 2D cross-section of the flow setup used for the experiments. The flow consists of several stages. Each stage is detailed starting from the farthest upstream location on the left hand side of figure 6.1(a) and moving downstream or to the right. The entrance region was of radius, $R_o = 1.07$ mm,
and was $68R_o$ in length – a sufficient length to ensure fully developed Poiseuille flow entered the sections to follow. Farther downstream of the entrance, the flow entered the PCT, where the radius of the tube wall, $R_w$, varied sinusoidally along the streamwise direction, $x$, according to,

$$R_w = R_o + \epsilon \left( \cos \left( \frac{2\pi x}{\lambda} \right) - 1 \right),$$

(6.1)

where the sinusoidal amplitude of wall radius was, $\epsilon = 0.14$ mm, and the wavelength, $\lambda$, was 4.7 mm. The maximum radius of the PCT was $R_o$, the minimum radius $R_i$ was 0.79 mm and the average radius $R$ was 0.93 mm. The length of the PCT section was 7.1. Figure 6.1(b) demonstrates a magnified depiction of the PCT portion of the test section. The cylindrical coordinate system is shown for reference on figure 6.1(b). The streamwise, radial and azimuthal directions are denoted as $x$, $r$, and $\theta$, respectively. The radius of the tube downstream of the PCT returned to $R_o$ for a length of $28R_o$. The radius then gradually increased to 5.5 mm via a 3-degree axisymmetric conical expansion farther downstream from the PCT.

![Figure 6.1: Two-dimensional schematic of the (a) complete acrylic test section, and (b) the periodically constricted tube.](image)

The 3D axisymmetric tube was built from two halves of 12.7 mm thick acrylic. The radial profile shown in figure 6.1 was cut into the two acrylic halves using a computer numerical control router with a precision ball nose end mill. The scallop height – the height of the surface imperfections caused by the curvature and step length of the ball nose tool – was less than 1 $\mu$m or 0.1% of $R_i$. The two halves were pressed together to form the 3D axisymmetric tube without using any adhesive. Custom milled steel flanges with lag bolts and nuts were used to apply sufficient compression to the two halves, such that fluid did not expel out the sides of the test section.

Fluid entered the test section from a straight, 1.2 m long stainless-steel tube with an inner radius of $R_o$, that was face-sealed to the left hand side of the test section shown in figure 6.1(a). Fluid that exited the test section entered a 0.3 m long stainless-steel tube with an inner radius of 5.5 mm that was joined to the downstream portion of the test section. Fluid temperature was monitored using a K-type thermocouple and a data logger (HH506, Omega Engineering). The average fluid temperature of all experiments was
20.1°C ± 0.2°C. A syringe pump (Legacy 200, KD Scientific Inc.) with an accuracy of ± 1% was used to propel fluid through the flow facility. Glass syringes (Micro-Mate, Popper & Sons Inc.) with 10 ml and 30 ml volumes were equipped in the syringe pump; the choice in the syringe volume depended on the required volumetric flow rate \( Q \). Flexible PVC tube with an inner radius of 3.18 mm connected the syringe to the 1.2 m long stainless steel tube. Five flow rates were considered for each Newtonian and non-Newtonian fluid: 1, 3, 6, 9 and 12 ml min\(^{-1}\).

The Reynolds number was defined based on, \( Re = \frac{2\bar{U}_0 R}{\nu_w} \), where, \( \nu_w = \mu_w / \rho \), is the kinematic wall viscosity, \( \mu_w \) is the dynamic wall viscosity and \( \rho \) is the density. This definition of \( Re \) is similar to that used in \( [\text{Ahrens et al.,} 1987] \), where the flow of viscoelastic fluids was simulated through a wavy-walled tube. Within the PCT, the centreline velocity \( U_0 \) oscillates with respect to \( x \). As such, the average centreline velocity \( \bar{U}_0 \) along \( x \) was determined from flow measurements in the PCT. Here, the overbar is used to denote spatial averaging along the \( x \) direction. Within the PCT, the fluid is subjected to a combination of shear and extensional deformation. A characteristic near-wall shear rate within the PCT was defined similar to the straight-walled section as \( \dot{\gamma}_w = \frac{2\bar{U}_0}{R} \). A characteristic extensional strain rate \( \dot{\varepsilon} \) was defined as the range in \( U_0 \) (maximum subtracted by minimum) divided by \( \lambda/2 \). In the present investigation, \( \dot{\gamma}_w \) was between 13 and 300 s\(^{-1}\) and \( \dot{\varepsilon} \) was between 2 and 58 s\(^{-1}\) depending on the fluid and \( Re \). The dynamic wall viscosity \( \mu_w \) was derived from shear rheograms shown in figure 5.1 for non-Newtonian fluids and using \( \dot{\gamma} = \dot{\gamma}_w \) in equation (2.7), as discussed in §5.2. For water, \( \mu_w = \mu_s \), where \( \mu_s \) is the viscosity of the solvent and was considered to be 1.00 mPa s according to \( [\text{Cheng,} 2008] \).

### 6.2 Particle shadow velocimetry

Particle image velocimetry (PIV) with backlight illumination, denoted as particle shadow velocimetry (PSV), was used to measure the velocity of the fluid within the test section. In PSV, the thickness of the measurement domain is driven largely by the depth of focus (DOF) of the imaging system. Provided a sufficient magnification and lens aperture, images can be acquired with a thin focal plan that enables 2D planar flow measurements along a select and narrow region of interest (Santiago et al., 1998; Estevadeordal & Goss, 2006; Khodaparast et al., 2013).

The PSV system consisted of a digital camera (Imager Pro X, LaVision GmbH) with a 2048 × 2048 pixel charged-coupled device sensor. Each pixel was 7.4 × 7.4 μm\(^2\) in size and had a 14-bit digital resolution. A Nikon lens with a focal length of \( f = 105 \) mm was equipped to the camera with an aperture diameter of \( f/2.8 \). The camera focus was adjusted such that images were focused on the radial mid-span of the test section. Two fields of view (FOVs) were considered, as shown in figure 5.2(a). The first FOV, i.e. FOV1, considered the entrance or development region immediately upstream of the PCT, as demonstrated in the left hand side of figure 6.2(a). The FOV1 captured the complete tube radius, \( R_o \), and approximately 3.\( \lambda \) along the \( x \) direction and immediately upstream of the first oscillation in the PCT. Only the Newtonian flow of water was considered in FOV1. The objective was to determine if the flow entering the PCT was fully-developed laminar Poiseuille flow. Experimental results for FOV1 were presented separately in Appendix A.1. The second field of view, FOV2, measured the velocity between the second to fifth oscillation of the PCT, that is from \( x \approx 2.\lambda \) to 5.\( \lambda \). For FOV2, flows of the three non-Newtonian fluids through the PCT were measured.
Both FOVs were approximately the same size, $(\Delta x, \Delta r) = 3.24 \times 14.1 \text{ mm}^2$, with a scale of 6.88 $\mu$m pixel$^{-1}$ after the sensor was cropped to remove unnecessary data for $r/R_o > 1$. The magnification was 1.07 and the DOF was 87 $\mu$m, which was approximately 10% the minimum radius in the PCT, $R_i$.

Backlight illumination of the PIV recordings was achieved using a 15 mJ pulse$^{-1}$ Nd:YAG laser (Solo I-15, New Wave Research Inc.) equipped with a diffuser. A diffuser expanded the laser beam, made the incident light incoherent and changed the wavelength to 610 nm using fluorescent disks. A programmable timing unit (PTU-9, LaVision GmbH) and DaVis 8.4 software (LaVision GmbH) were used to synchronize the camera and laser. Silver coated hollow glass spheres with diameter, $d_p = 10 \mu$m, were used as tracer particles in the flow (S-HGS-10, Dantec Dynamics). These particles were opaque, which was ideal for projecting a shadow on the camera in backlight illumination. The density of the particles, $\rho_p$, was 1400 kg m$^{-3}$. As a result the particle response time, $t_p = \rho_p d_p^2/18 \mu s$, and particle settling velocity, $u_p = (\rho_p - \rho)d_p g/18 \mu s$, could be established. Here, $g$ is the gravitational acceleration. The particle response time, $t_p$, was 7.8 $\mu$s and the particle settling velocity, $u_p$, was 21.8 $\mu$m s$^{-1}$. The Stokes number is estimated to be, $St = t_p \dot{\gamma}_w$, and the Froude number to be, $Fr = 2u_p/U_0$. The largest $St$ was 0.003 and the largest $Fr$ was 0.005, depending on $Q$. Both the Stokes and Froude number are small (less than 0.1) and errors attributed to particle inertia and particle settling are negligible.

For FOV1, five sets of measurements were performed for water, each for the different values of $Q$ that were previously listed in §6.1. The results for FOV1 are presented in Appendix A.1. The measurements of velocity within the entrance region show good agreement with the theoretical expectations for all values of
\( Q \), providing good confidence in PSV to produce reasonable measurements. For FOV2, measurements were performed for three different non-Newtonian fluids, each having five different concentrations, and five flow rates \( Q \) (75 data sets in total). As well, five measurements were performed for distilled water at FOV2 for each value of \( Q \). Each data set consisted of 600 pairs of double-frame images recorded at an acquisition frequency of 7.3 Hz. A sample image of the first frame for C14 at a mass concentration of 400 ppm is shown in figure 6.2(b). The time delay, \( \Delta t \), between image frames was between 500 and 7000 \( \mu s \) depending on the value of \( Q \), such that the maximum particle displacement between the image frames was no greater than 15 pixel.

Image processing was performed using DaVis 8.4 software (LaVision Gmbh). First, the images were inverted; the intensity signal at each pixel was subtracted from a constant intensity value. Next, the minimum intensity within each pixel and along the complete image ensemble was determined and subtracted from all images in each data set. Third, the intensity signals at each pixel were normalized by the average intensity of the ensemble. A sample image (C14 at a concentration of 400 ppm) after performing the previously detailed processing steps can be seen in figure 6.2(c). Compared to the native image, seen in figure 6.2(b), the processed image has more clearly defined bright particles for all values of \( r \).

Vector fields were established using the ensemble-of-correlation method with an initial interrogation window (IW) size of 64 \( \times \) 64 pixel (0.44 \( \times \) 0.44 mm\(^2\) or 0.41\( R \) \( \times \) 0.41\( R \)) and a final IW size of 16 \( \times \) 16 pixel (0.11 \( \times \) 0.11 mm\(^2\) or 0.10\( R \) \( \times \) 0.10\( R \)) with 75\% overlap between neighboring IWs (Meinhart et al., 2000). The velocity vector was denoted as \( U \), with components in cylindrical coordinates being, \( U_r, U_\theta, U_x \), and corresponding to the velocity along the \( r, \theta, x \) directions respectively. The flow is laminar and steady, with presumably no swirl, i.e. \( U_\theta = 0 \), given the geometric dimensions of the PCT and the Reynolds numbers of the flows in the present investigation (Deiber & Schowalter, 1979). Evidence of secondary flow re-circulations, turbulence or swirl is not observed. An evaluation of the uncertainty in measurements of \( U \) is provided in Appendix A.1. Throughout this chapter, error bars are used to convey the uncertainty in the results.

### 6.3 Flow field analysis

Simplifying equation (2.3), the steady flow of complex and Newtonian fluids in the PCT are governed by the following equations for mass and momentum conservation,

\[
\begin{align*}
\nabla \cdot U &= 0, \\
\rho U \cdot \nabla U &= -\nabla P + \nabla \cdot \tau,
\end{align*}
\]

(6.2)

where \( P \) is the indeterminate component of the Cauchy stress tensor, and \( \tau \) is the deviatoric stress tensor. The nonzero components of the rate of deformation tensor \( D \), and rate of rotation tensor \( W \) are listed,

\[
\begin{align*}
D_{rr} &= \frac{\partial U_r}{\partial r}, & D_{\theta\theta} &= \frac{U_r}{r}, & D_{xx} &= \frac{\partial U_x}{\partial x}, \\
D_{rx} &= D_{xr} &= \frac{1}{2} \left( \frac{\partial U_r}{\partial x} + \frac{\partial U_x}{\partial r} \right),
\end{align*}
\]

(6.3)
\[ \omega_\theta = -2W_{xy} = \frac{\partial U_r}{\partial x} - \frac{\partial U_x}{\partial r}, \quad (6.4) \]

where \( \omega \) is the vorticity vector, whose only non-zero component is \( \omega_\theta \). Equations (6.1) reduces to the Navier-Stokes equation for Newtonian fluids when the deviatoric stress tensor is represented by the constitutive equation, \( \tau = 2\mu_s \mathbf{D} \). For non-Newtonian fluids, the constitutive relation is much more complex and can be a partial differential equation with nonlinear terms (e.g. Phan-Thien-Tanner and Giesekus models). For most non-Newtonian constitutive models, it is common to segregate the deviatoric stress tensor into a solvent and non-Newtonian stress, i.e. \( \tau = \tau_s + \tau_{nn} \) \cite{Alves2020}. Here, \( \tau_s = 2\mu_s \mathbf{D} \), is the solvent stress, and \( \tau_{nn} \) is the non-Newtonian stress introduced from the polymers or micelles. Note that if \( \tau_{nn} = 0 \), then \( \tau = \tau_s \) and the constitutive equation is Newtonian. When substituted into equation (6.2), the divergence of the non-Newtonian stress, \( \nabla \cdot \tau_{nn} \), acts as an additional forcing term and for polymeric flows is often referred to as a “polymer force” \cite{Kim2007}.

Equations (6.3) and (6.4) can be explicitly evaluated using the measured \( U_x \) and \( U_r \). To circumvent the need for pressure, \( P \), the vorticity transport equation is considered, from taking the curl of the momentum transport equation shown in (6.2). The only non-zero component of the vorticity in the PCT flow is \( \omega_\theta \); therefore, the vorticity transport equation is only considered along the azimuthal direction,

\[
\begin{align*}
U_r \frac{\partial \omega_\theta}{\partial r} + U_x \frac{\partial \omega_\theta}{\partial x} - \frac{U_r \omega_\theta}{r} &= \nu_s \left(\frac{\partial^2 \omega_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial \omega_\theta}{\partial r} + \frac{\partial^2 \omega_\theta}{\partial x^2} - \frac{\omega_\theta}{r^2}\right) + T_\theta.
\end{align*}
\quad (6.5)
\]

The additional term on the right hand side of equation (6.5) is the azimuthal component of the non-Newtonian torque, \( T = (\nabla \times \nabla \cdot \tau_{nn})/\rho \). The non-Newtonian torque is a vector, whose only non-zero component in the PCT is \( T_\theta \). Previous numerical investigations have denoted \( T \) the “polymer torque” as it can be represented as the curl of the polymer force \cite{Kim2007, Kim2008, Kim2013, Page2015, Page2016, Biancofiore2017, Lee2017}. Its simplified units are \( s^{-2} \) – when multiplied by moment of inertia, the units are force times unit distance, consistent with the true torque definition. The under-braces shown in equation (6.5) isolate the different combinations of terms within the vorticity transport equation. On the left hand side of equation (6.5), \( VA \) denotes the azimuthal vorticity advection. The first term on the right hand side of equation (6.5), \( VSD \), represents vorticity solvent diffusion, where \( \nu_s = \mu_s/\rho \). For each flow, the azimuthal non-Newtonian torque was calculated based on the deficit between \( VA \) and \( VSD \), i.e. \( T_\theta = VA - VSD \).

To establish the first-order spatial gradients of velocity, a moving second-order polynomial surface was fit on profiles of \( U_x \) and \( U_r \). The size of the second-order polynomial filter was \( 20 \times 20 \) pixels, \( 138 \times 138 \mu m^2 \), or \( 0.15R \times 0.15R \). Coefficients of the polynomial surface were used to establish first-order spatial derivatives of \( U_x \) and \( U_r \). Azimuthal vorticity, \( \omega_\theta \) was then established using equation (6.4). To determine the higher order spatial gradients in the flow, a moving third-order polynomial surface was fit on profiles of \( U_x \) and \( U_r \). The size of the cubic polynomial filter was \( 76 \times 76 \) pixels, \( 522 \times 522 \mu m^2 \), or \( 0.56R \times 0.56R \). Coefficients of the third-order polynomial were used to the determine the second- and third-order spatial derivatives of \( U_x \) and \( U_r \). Three orders of differentiation in \( U \) are required due to the \( VSD \) term in equation (6.5). These higher-order derivatives were then used to calculate the azimuthal non-Newtonian torque \( T_\theta \).
using equation (6.5). Polynomial filters that overlapped with the PCT wall were neglected, and results of \( \omega_\theta \) and \( T_\theta \) were not considered close to the wall.

All parameters including \( U \), \( \omega_\theta \), and \( T_\theta \) exhibited symmetry about \( r = 0 \). Therefore, \( U \), \( \omega_\theta \), and \( T_\theta \) on the lower half of the domain \( (r < 0) \) were averaged with the upper half \( (r > 0) \). When comparing \( U \), \( \omega_\theta \), and \( T_\theta \) in one oscillation to prior or subsequent oscillations, the parameters are not dramatically different for all flow conditions and fluids. Therefore, \( U \), \( \omega_\theta \), and \( T_\theta \) were periodically averaged over three oscillations, i.e. for \( x \)-coordinates that share the same wall radius, \( R_w \).

The volumetric flow rate, \( Q \) can be determined from flow measurements based on a volume integration of \( U_x \), i.e. \( Q = 2\pi \int_0^{R_w} U_x r \, dr \). The bulk velocity can be defined according to, \( U = Q/(\pi R_w^2) \). Because of mass conservation and the variation of \( R_w \) along \( x \), the bulk velocity \( U_b \) changes along the streamwise \( x \) direction. Therefore an average value of \( U_b \) along \( x \) was determined, and an overbar was used to denote the spatial averaging along the \( x \) direction, i.e., \( \bar{U}_b \). Recall from §6.1 that the same overbar was used to define the average centreline velocity along \( x \), \( \bar{U}_0 \). An average shape factor can be determined from the ratio of centreline to bulk velocity, \( \bar{U}_0/\bar{U}_b \). For Poiseuille flow in a straight-walled tube, \( SF = 2 \). Lastly, distributions of \( U \), \( \omega_\theta \) and \( T_\theta \) for flows within the PCT (i.e., FOV2) were normalized by \( \bar{U}_0 \), \( \bar{\gamma}_w \) and \( \bar{\gamma}_w \), respectively. Spatial variables \( x \) and \( r \) were normalized by \( \lambda \) and \( R \), respectively.

### 6.4 Flows in the periodically constricted tube

#### 6.4.1 Water

Figure 6.3 demonstrates contours of the velocity magnitude \( ||U|| = (U_x^2 + U_r^2)^{0.5} \) along with streamlines, for the flow of water at five different \( Re \) within the PCT. All flows with unique \( Re \) have a centreline velocity \( U_0 \) that attains a maximum value around \( x/\lambda = 0.5 \). When \( x/\lambda = 0.5 \) the wall radius of the PCT, \( R_w \), is at its smallest value, \( R_w = R_t \). For the lowest \( Re \) flow (i.e., \( Re = 15.7 \)), the centreline velocity at \( x/\lambda = 0.5 \) attains 1.25\( \bar{U}_0 \). The lowest magnitude in \( U_0 \) occurs when \( x/\lambda = 0 \) and 1, and is approximately equal to 0.7\( \bar{U}_0 \) for \( Re = 15.7 \). At larger \( Re \), the centreline velocity at \( x/\lambda = 0.5 \), is smaller in magnitude – around 1.1\( \bar{U}_0 \) Values of \( U_0 \) are also slightly larger for the high \( Re \) cases when \( x/\lambda = 0 \) and 1 compared to the case with \( Re = 15.7 \) – approximately equal to 0.8\( \bar{U}_0 \). Therefore, when \( Re \) increases, the normalized centreline velocity decreases. In all flow conditions, streamlines at large \( r/R \) tend to follow the sinusoidal profile of the wall. Near the core, streamlines are more parallel with respect to the streamwise \( x \) direction.

Profiles of \( U_x/\bar{U}_0 \) with respect to \( r/R \) at different points of \( x/\lambda \) are shown in figure 6.4(a) for water at \( Re = 15.7 \), 106 and 203. Sample error bars are shown for the flow condition with \( Re \) of 15.7 and at \( x/\lambda = 0.5 \). Relative errors were conservatively estimated to be 0.042\( \bar{U}_0 \) near the centreline of PCT and 0.108\( \bar{U}_0 \) near the wall at \( x/\lambda = 0.5 \), as discussed in Appendix A. As noted in the discussion pertaining to figure 6.3, the low \( Re \) flow of 15.7 has a large variation in \( U_0 \). When \( x/\lambda = 0 \), \( U_0 \) becomes 0.75\( \bar{U}_0 \) and when \( x/\lambda = 0.5 \), \( U_0 \) equals 1.25\( \bar{U}_0 \). Newtonian flows with larger \( Re \) of 106 and 203 have a centreline velocity of approximately 0.81\( \bar{U}_0 \) when \( x/\lambda = 0 \) and 1.1\( \bar{U}_0 \) when \( x/\lambda = 0.5 \). Within the PCT contractions and expansions (i.e., \( x/\lambda = 0.25 \) and 0.75 respectively), radial profiles of \( U_x/\bar{U}_0 \) are approximately the same. In other words, the velocity is symmetric about \( x/\lambda = 0.5 \). Figure 6.4(b) demonstrates that the streamlines
Figure 6.3: Velocity magnitude normalized by the average centreline velocity $\tilde{U}_0$ for different $Re$ of water. Solid black lines overlaid on filled contours are streamlines. The solid black line at the limit of the filled contour is the sinusoidal wall profile.

Figure 6.4: (a) Velocity profiles of water at $Re = 15.7, 106$ and $203$ at different $x$ locations along the PCT. Down sampled error bars are shown for the flow of water at $Re = 15.7$ and $x/\lambda = 0.5$. (b) Overlaid streamlines of the water flows at different $Re$. The black line in (b) indicates the wall profile $R_w$. Symbol colours in (a) correspond to the different $Re$ as indicated in (b).

of the Newtonian flows also depend on $Re$. When $Re$ is low, streamlines are more curved and their radial position is closer to the PCT centreline at $x/\lambda = 0.5$.

As noted in regards to figure 6.4(a), the velocity in the PCT for water demonstrates a dependence on $Re$ most notable by the differences in the amplitude of the centreline velocity $U_0$. The standard deviation in the centreline velocity $\mathcal{R}(U_0)$ was computed for each $Re$ and normalized by their respective average centreline velocities $\tilde{U}_0$. Values of $\tilde{U}_0$ and $\mathcal{R}(U_0)/\tilde{U}_0$ are listed in table 6.1 for the different water flow. The inverse proportionality between the amplitude of $U_0$ and $Re$ is clearly demonstrated by the decreasing trend in $\mathcal{R}(U_0)/\tilde{U}_0$ as $Re$ grows. In addition to centreline velocity, table 6.1 also lists the average bulk velocity $\tilde{U}_b$ and shape factor $SF = \tilde{U}_0/\tilde{U}_b$ for the flows of water in the PCT. For Newtonian Poiseuille flow in a straight-walled pipe, $SF$ equals 2. Although the PCT is not straight-walled, values of $SF$ for all water flows are around 2.1 and not too different from the theoretical $SF$ for straight-walled Poiseuille pipe flow.
Table 6.1: Bulk and centreline velocity statistics for the flow of water within the PCT at different Re.

<table>
<thead>
<tr>
<th>Re</th>
<th>$\bar{U}_0$ mm s$^{-1}$</th>
<th>$R(U_0)/\bar{U}_0$</th>
<th>$\bar{U}_b$ mm s$^{-1}$</th>
<th>SF</th>
</tr>
</thead>
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Contours of azimuthal vorticity, $\omega_\theta/\dot{\gamma}_w$ are shown in figure 6.5 for water within the PCT. Near the centreline, $\omega_\theta/\dot{\gamma}_w$ is approximately equal to zero. For all radial and streamwise coordinates, $\omega_\theta$ is positive. The maximum $\omega_\theta/\dot{\gamma}_w$ is situated near the wall and at $x/\lambda = 0.5$ for all Re. Recall from §6.3 that measurements of $\omega_\theta/\dot{\gamma}_w$ within close proximity (15% of $R_w$) of the wall were not calculated. This is because the differentiation filter conflicted with the wall.

Figure 6.5: Vorticity normalized by average wall shear rate $\dot{\gamma}_w$ for different Re of water. The solid black line is the sinusoidal wall profile.

6.4.2 Xanthan gum solutions

Velocity contours and streamlines are shown in figure 6.6 for XG solutions at different Re. For brevity, only the results of two concentrations, $c = 200$ppm and 500ppm are shown. Similar to water, both of the XG solutions with $c = 200$ppm and 500ppm have magnitudes of $||U||/\bar{U}_0$ that are lowest when $x/\lambda = 0$ and 1, and largest when $x/\lambda = 0.5$. Compared to Newtonian water flows seen in figure 6.3 the zone with larger values of $||U||/\bar{U}_0$ is extended farther towards the tube wall. As $c$ increases from 200ppm to 500ppm, $||U||/\bar{U}_0$ also increases at $r/R > 0$ locations. Streamlines at large $r/R$ take on a similar sinusoidal profile as the wall pattern. Similar to the water flows, the streamlines for XG at $c = 200$ppm and 500ppm are approximately symmetric with respect to $x/\lambda = 0.5$.

Streamwise velocity profiles $U_x/\bar{U}_0$ at different $x/\lambda$ coordinates are shown in figure 6.7(a) for XG with $c = 500$ppm and at $Re = 10.2$. For comparison, the profiles of water at a similar $Re$ are presented alongside XG. Relative to water at the same $x/\lambda$ coordinates, XG has larger $U_x/\bar{U}_0$ values. The distributions of $U_x/\bar{U}_0$
Figure 6.6: Velocity magnitude normalized by the average centreline velocity $\bar{U}_0$ for different $c$ and $Re$ of XG. Solid black lines overlaid on filled contours are streamlines. The solid black line at the limit of the filled contour is the sinusoidal wall profile are more flat in the PCT centre; a blunted profile that is common in shear thinning fluids (Bird et al., 2007). Despite the different shaped velocity profile, the range of $U_0$ appears to be similar among water and XG. For the XG flow, $U_0 = 0.75\bar{U}_0$ at $x/\lambda = 0$, and $U_0 = 1.25\bar{U}_0$ at $x/\lambda = 0.5$ – the same as water. Lastly, figure 6.7(b) compares the streamlines of the same flows of XG and water seen in figure 6.7(a). Despite having different velocity profiles with respect to $r/R$, the streamlines for water and XG are approximately the same.

Figure 6.7: (a) Velocity profiles along different $x$ locations for XG with $c = 500$ppm at $Re = 10.2$, and water at $Re = 15.7$. Down sampled error bars are shown for the flow of water at $Re = 15.7$ and $x/\lambda = 0.5$. (b) Overlaid streamlines of XG and water. The black line in (b) indicates the wall profile $R_w$. Red symbols in (a) correspond to the water flow with $Re = 15.7$, while blue symbols represent the XG flow with $c = 500$ppm and $Re = 10.2$.

Based on figure 6.7(a), it was shown that the shape of $U_x/\bar{U}_0$ profiles with respect to $r/R$ were different,
but the relative variations in $U_0$ were the same among the flows of water and XG at similar Re. Figure 6.8(a) demonstrates the standard deviation in $U_0$ normalized by the average centreline velocity, $\mathcal{R}(U_0)/\bar{U}_0$, for XG and water. Low concentration solutions of XG ($c < 300$ ppm) appear to have $\mathcal{R}(U_0)/\bar{U}_0$ values that overlap with water at high $Re$. However, for $Re < 20$ and high XG concentrations, the values of $\mathcal{R}(U_0)/\bar{U}_0$ appear to be independent of the Reynolds number and relatively constant. The larger concentration XG solutions of 400 ppm and 500 ppm appear to have subtly larger values of $\mathcal{R}(U_0)/\bar{U}_0$ than water and the other XG solutions; however, the difference is not substantial. Generally, $\mathcal{R}(U_0)/\bar{U}_0$ for XG appears to be independent of concentration, and similar to water at higher $Re$ values. Figure 6.8(b) presents the average shape factor $SF = \bar{U}_0/\bar{U}_b$ for water and XG at different $Re$ and $c$ within the PCT. Relative to water, XG flows at all $c$ and $Re$ have lower $SF$ values. As the $c$ of XG increases, $SF$ decreases. The reducing trend in $SF$, aptly summarizes how shear thinning makes the profile more blunt as the concentration of XG increases.

Figure 6.8: (a) The standard deviation in the centreline velocity divided by the mean centreline velocity, and (b) the shape factor with respect to different $Re$ for XG solutions and water.

Vorticity contours for the XG flows with $c = 200$ ppm and 500 ppm are shown in figure 6.9. Similar to the flows of water in the PCT, $\omega_\theta/\dot{\gamma}_w$ attains a maximum value near the wall and at $x/\lambda = 0.5$. Both the XG flows with $c = 200$ ppm and 500 ppm have a noticeably attenuated $\omega_\theta/\dot{\gamma}_w$ in regions farther from the tube centreline. In other words, the thickness (along $r/R$) of the region near the pipe centreline with $\omega_\theta/\dot{\gamma}_w = 0$ is larger for XG relative to water. The thickness also grows with increasing $c$. The attenuated $\omega_\theta/\dot{\gamma}_w$ is attributed to the more uniform profiles of $U_x/\bar{U}_0$ caused by shear thinning.

6.4.3 Polyacrylamide solutions

Relative to water and XG, different patterns in the velocity are encountered for the flow of PAM within the PCT. Figure 6.10 demonstrates contours of $||U||/\bar{U}_0$ for PAM at different $c$ and $Re$. In this figure, $c$ increases from bottom to top, and $Re$ increases from left to right. Despite the low $Re$ flows showing some visual resemblance to the results for water, flows at high $c$ and large $Re$ are asymmetric about $x/\lambda = 0.5$. For these cases, the large velocity contours takes on a triangular or half chevron appearance leaning towards the upstream direction. Therefore, within the contraction regions (i.e. from $x/\lambda = 0$ to 0.5) the maximum
velocity is not necessarily situated at the centreline of the PCT. Within the tail of the chevron, streamlines appear to be tilted farther towards the centreline and non-conforming to the sinusoidal profile of the walls. Despite the PAM solutions having seemingly comparable steady shear rheology as the XG solutions (figure 5.1), the flow of PAM within the PCT produces an entirely different velocity distribution. It is clear that another rheological property, not present in XG, is causing the chevron pattern in the flows of PAM through the PCT.

Streamwise velocity profiles $U_x/\bar{U}_0$ along different $x/\lambda$ values are shown in figure 6.11(a) for PAM with $c = 300$ppm. Two different $Re$ are compared to contrast the change in the velocity from when the contours transition to the half chevron seen in figure 6.10. For the low Reynolds number case of $Re = 3.02$ (red symbols), profiles of $U_x/\bar{U}_0$ are similar to water or XG. The shape of the $U_x/\bar{U}_0$ profiles appear to be subtly more blunted than the parabolic Poisueille profile, and the variations in $U_0$ are slightly larger than the values encountered for water at $Re = 15.7$ seen in figure 6.5(a). Similar to XG, the more blunted velocity profile for PAM at low $Re$ can likely be explained by shear thinning. Recall that PAM with $c = 300$ppm has a lower power-law index than XG with $c = 500$ppm, as seen in tables 5.1 and 5.2. Therefore, it is expected that $U_x/\bar{U}_0$ profiles are not to be parabolic, but also not as blunted as the higher concentration XG solutions. Although the low $Re$ flow reflects some similarities to previous findings for XG, the higher $Re$ flow of PAM (blue symbols) exhibits entirely unique distributions in $U_x/\bar{U}_0$. At $x/\lambda = 0$ the $U_x/\bar{U}_0$ profile has two local maxima – one at the centreline, the other at $r/R = 0.9$. Within the contraction, where $x/\lambda = 0.25$, the maximum value of $U_x/\bar{U}_0$ is no longer situated at the centreline, but at $r/R = 0.6$. Prior works have observed large velocity overshoot near the wall in gradual planar contraction flows of PAM solutions (Poole et al., 2005) and numerical investigations that utilized various viscoelastic constitutive models (Afonso & Pinho, 2006; Poole et al., 2007; Alves & Poole, 2007; Poole & Alves, 2009). Poole et al. (2005) referred to these velocity overshoots as “cat’s ears” given their appearance.
Coupled with the near-wall velocity overshoots are highly curved streamlines, as shown in figure 6.11(b). At sufficiently large Re, PAM with $c = 300$ ppm has streamlines that are directed away from the PCT core and more towards the tube wall for $x/\lambda = 0$ to 0.5. The works by Cable & Boger (1978a,b, 1979) referred to the state of these curved streamlines as “divergent flow.” In general, solutions of PAM with $c \geq 200$ ppm and sufficiently large Re are subjected to near-wall velocity overshoots and divergent flow within the contracting portions of the PCT (i.e. $0 < x/\lambda < 0.5$), as seen in figure 6.10.

The pattern of $U_z/\bar{U}_0$ for PAM is clearly dependent on Re and $c$, as observed in figure 6.10. Compared to water, it can be observed that variations in $\bar{U}_0$ are larger for PAM at large $c$ and Re based on figure 6.11(a). Figure 6.12(a) demonstrates $\mathcal{R}(\bar{U}_0)/\bar{U}_0$ as a function of Re for different $c$ of PAM. When the concentration
Figure 6.11: (a) Velocity profiles along different $x$ locations for PAM with $c = 300$ppm at $Re = 3.02$, and $Re = 60.5$. Red symbols show $Re = 3.02$ and blue symbols show $Re = 60.5$. Down sampled error bars are shown for the flow of PAM with $c = 300$ppm and $Re = 3.02$ at $x/\lambda = 0.5$. (b) Overlaid streamlines of PAM at different $Re$. The black line in (b) indicates the wall profile $R_w$.

of PAM is low ($c = 100$ppm), values of $R(U_0)/\bar{U}_0$ are similar to water. This is expected; contours of velocity for PAM with $c = 100$ppm do not exhibit a prevalent asymmetric half chevron pattern in figure 6.10. At large concentrations, PAM enhances the variations in $U_0$. For more moderate PAM concentrations of $c = 200$ppm and 300ppm, $R(U_0)/\bar{U}_0$ increases up until an $Re$ of about 35, before decreasing with further growth in $Re$. At large concentrations of $c = 400$ppm and 500ppm, values of $R(U_0)/\bar{U}_0$ are similar.

Rheological measurements of PAM solutions demonstrated that the solutions are viscoelastic – see figures 5.2 and 5.3. Therefore, values of $R(U_0)/\bar{U}_0$ are also contrasted with elastic properties of the flow. A Deborah number within the PCT was defined as $De = te/tf$, where $tf = \lambda/\bar{U}_0$ is the timescale of the flow along the PCT centreline. Figure 6.14(b) shows values of $R(U_0)/\bar{U}_0$ with respect to $De$ for different concentrations of PAM. For all flows of PAM with $c = 100$ppm, the values of $De$ are less than 0.1, and corresponding values of $R(U_0)/\bar{U}_0$ are less than 0.2. Larger concentration PAM solutions with $De > 0.1$ have large values of $R(U_0)/\bar{U}_0$ that are greater than 0.2 and tend to increase with growing $De$ – that is, up until the point where $Re$ has attained 35, with reference to figure 6.12(a). Based on the trend in $R(U_0)/\bar{U}_0$ versus $Re$ for water, a decreasing $R(U_0)/\bar{U}_0$ is likely attributed to inertial effects – perhaps producing more stagnant flow or small, unseen recirculations in the expansion regions with adverse pressure gradients and where $R_w = R_o$ (Deiber & Schowalter 1981). On the other hand, elasticity acts to augment $R(U_0)/\bar{U}_0$. The increasing-decreasing trend in $R(U_0)/\bar{U}_0$ is most likely a result of the competing effects of elasticity and inertia. When $c$ is sufficiently large, elasticity dominates and the trend in $R(U_0)/\bar{U}_0$ as function of $De$ show better overlap for different $c$. Cases with $De > 0.1$ also tend to have a pronounced half chevron velocity pattern in figure 6.10.

Lastly, contours of vorticity $\omega_\theta/\dot{\gamma}_w$ are shown for the flows of PAM in the PCT in figure 6.13. At low $c$ and $Re$, $\omega_\theta/\dot{\gamma}_w$ is everywhere positive, similar to water and XG. However, PAM with sufficiently large $c$ and $Re$ exhibits negative values of $\omega_\theta/\dot{\gamma}_w$ within the PCT contractions. In certain cases, e.g. $c = 300$ppm
and $Re = 60.5$, there is a strong contrast between the tilted negative contour of $\omega_\theta/\dot{\gamma}_w$ and the surrounding positive $\omega_\theta/\dot{\gamma}_w$ values.

### 6.4.4 Surfactant solutions

Velocity contours are shown for the C14 solutions in figure 6.14. At the lowest concentration of $c = 100$ ppm and 200 ppm, the contours are similar to water. Half chevron patterns that are similar to those of PAM appear for all $c$ greater than 300 ppm and $Re$ that exceed 115. The chevrons result in curved streamlines that are asymmetric with respect to $x/\lambda = 0.5$. Despite a water-like shear rheogram, shown in figure 5.1, C14 demonstrates a complex, non-Newtonian response within the PCT that is similar to flexible polymers and unlike rigid polymers. Therefore, the current measurements show that the rheological trait responsible for the asymmetric chevron pattern in PAM is clearly also inherent in C14.

Profiles of $U_x/\bar{U}_0$ with respect to $r/R$ and at different $x/\lambda$ are shown in figure 6.15(a) for C14 with $c = 500$ ppm and $Re = 119$ and compared with PAM at $c = 200$ ppm and $Re = 83.1$ – the closest possible $Re$. The near-wall velocity overshoots encountered for the PAM flows, are also present in the C14 solution. Within the contraction, from $x/\lambda = 0$ to 0.25, distributions of $U_x/\bar{U}_0$ can be described by a higher-order polynomial with two local peaks along $r/R$. Streamlines are also compared for C14 and PAM in figure 6.15(b). Both solutions demonstrate divergent flow patterns within the PCT contraction (Cable & Boger [1978a,b, 1979]). Streamlines for PAM are projected farther towards the wall relative to C14. This is despite PAM having slightly lower near-wall velocity overshoots. In general, there is good qualitative agreement between the velocity field of PAM and C14 – cat’s ears and divergent flow.

Distributions of $R(U_0)/\bar{U}_0$ as a function of $Re$ are shown in figure 6.16 for C14 solutions of different $c$. Solutions that do not exhibit asymmetric velocity patterns, namely C14 with $c = 100$ ppm and 200 ppm, have values of $R(U_0)/\bar{U}_0$ that overlap with water and demonstrate the same decreasing trend in $R(U_0)/\bar{U}_0$ with increasing $Re$. For more concentrated solutions of C14, such as $c = 300$ ppm and 400 ppm, values of $R(U_0)/\bar{U}_0$ overlap with measurements for water at low $Re$. As $Re$ is increased further, values of $R(U_0)/\bar{U}_0$...
Figure 6.13: Vorticity normalized by $\dot{\gamma}_w$ for different $c$ and $Re$ of PAM. The solid black line is the sinusoidal wall profile.

abruptly increase. This is different than the monotonic increase in $\mathcal{R}(U_0)/\bar{U}_0$ with growing $Re$ observed for PAM in figure 6.12(a). The $Re$ at which $\mathcal{R}(U_0)/\bar{U}_0$ abruptly increases appears to be sensitive to small discrepancies in $Re$. It appears as though transition to large $\mathcal{R}(U_0)/\bar{U}_0$ occurs earlier for the $c = 400$ ppm C14 solution compared to the $c = 300$ ppm solution. However, the $Re$ at which $\mathcal{R}(U_0)/\bar{U}_0$ increases for $c = 300$ ppm is subtly larger than the $Re$ of the $c = 400$ ppm solution at a comparable flow rate. Evidently, the resolution of $Re$ is too sparse to capture the sudden augmentation in $\mathcal{R}(U_0)/\bar{U}_0$. Ultimately, the trend in the velocity pattern, namely $\mathcal{R}(U_0)/\bar{U}_0$ as a function of $Re$, is different for C14 compared to PAM. Beyond a critical $Re$, the asymmetric velocity patterns that are formed by the C14 solution exhibit qualitatively the same pattern as PAM, with values of $\mathcal{R}(U_0)/\bar{U}_0$ that are also larger than the Newtonian and XG flows.

Figure 6.17 presents contours of $\omega_\theta/\dot{\gamma}_w$ for the C14 solutions at different $c$ and $Re$. Similar to the PAM
solutions, the flows of C14 with sufficiently large $c$ and $Re$ demonstrate negative values of $\omega_\theta/\dot{\gamma}_w$ within the contractions of the PCT ($0 < x/\lambda < 0.5$). As expected, the conditions where half chevrons appear in velocity contours also reflect negative values in $\omega_\theta/\dot{\gamma}_w$. The $c = 100$ ppm and $200$ ppm C14 solution have $\omega_\theta/\dot{\gamma}_w$ distributions that are seemingly identical to water, seen in figure 6.5. At large $c$ and $Re$, negative contours of $\omega_\theta/\dot{\gamma}_w$ begin to appear. It is notable that the C14 solutions exhibit water-like rheology, yet they respond in a manner similar to PAM within the PCT at larger $c$ and $Re$ conditions.

Figure 6.14: Velocity magnitude normalized by the average centreline velocity $\bar{U}_0$ for different $c$ and $Re$ of C14. Solid black lines overlaid on filled contours are streamlines. The solid black line at the limit of the filled contour is the sinusoidal wall profile.
Figure 6.15: (a) Velocity profiles along different $x$ locations for C14 with $c = 500$ ppm at $Re = 119$, shown by the red symbols, and PAM with $c = 200$ ppm at $Re = 83.7$, shown with blue symbols. Down sampled error bars are shown for the flow of PAM with $c = 200$ ppm and $Re = 83.7$ at $x/\lambda = 0.5$. (b) Overlaid streamlines of C14 and PAM.

Figure 6.16: The standard deviation in the centreline velocity divided by the mean centreline velocity as a function of $Re$, for the various C14 solutions.

### 6.4.5 Non-Newtonian torque

The non-Newtonian torque was established based on the deficit between the advection of vorticity ($VA$) and the vorticity solvent diffusion ($VSD$) – see equation (6.5). The normalized azimuthal component of the non-Newtonian torque is $T_\theta/\gamma_w^2$. The distributions of $T_\theta/\gamma_w^2$ are presented in figure 6.18, 6.19 and 6.20 for water, XG, PAM and C14. In these figures, the open contours show $T_\theta/\gamma_w^2$ and are overlaid on filled contours of $\omega_\theta/\gamma_w$. Contour levels for $T_\theta/\gamma_w^2$ are from -0.4 to +0.4 in steps of 0.2. Contours greater than or equal to zero are solid lines and negative contours are dotted lines. Values of $T_\theta/\gamma_w^2$ were not computed or shown near the wall (within 42% of $R_w$) due to difficulties in computing spatial gradients within this region, as discussed in §6.3.
Figure 6.17: Vorticity normalized by $\dot{\gamma}_w$ for different $c$ and $Re$ of C14. The solid black line is the sinusoidal wall profile.

Based on equation (6.5), $T_\theta/\dot{\gamma}_w^2$ should be equal to zero in the flow of water. In other word, the dynamics of vorticity should be entirely described by vorticity advection ($VA$) and diffusion ($VSD$). Figure 6.18(a) presents contours of $\omega_\theta/\dot{\gamma}_w$ and $T_\theta/\dot{\gamma}_w^2$ for water at $Re = 170$. Contours of $T_\theta/\dot{\gamma}_w^2$ are relatively low in magnitude and noisy. Although XG is a non-Newtonian flow, with evidently large amounts of shear thinning and linear viscoelasticity (see figure 5.2), it too does not have contours of $T_\theta/\dot{\gamma}_w^2$ with large magnitude, as seen in figure 6.18(b). Generally, the plug-like flow of XG within the PCT has a larger region where $\omega_\theta/\dot{\gamma}_w$ and $T_\theta/\dot{\gamma}_w^2$ are equal to 0.

Contours of $\omega_\theta/\dot{\gamma}_w$ and $T_\theta/\dot{\gamma}_w^2$ for PAM with $c = 300$ppm and $Re = 60.5$ are shown in figure 6.19(a). A zone of large $T_\theta/\dot{\gamma}_w^2$ values is interspersed between regions of negative and positive $\omega_\theta/\dot{\gamma}_w$ within the PCT contraction. This $T_\theta/\dot{\gamma}_w^2$ zone appears in areas where values of $\omega_\theta/\dot{\gamma}_w$ significantly vary in space. The
Figure 6.18: Contours of vorticity and the non-Newtonian torque for the flows of (a) of water with Re = 106, and (b) XG at c = 200ppm, Re = 71.7. Positive and zero contours are solid lines, while dashed lines are negative contours.

The opposite can be observed within the PCT expansion (0.5 < x/λ < 1); the vorticity reduces with increasing x/λ, and hence Tθ/γ_w^2 is at its most negative. Similar observations can be made for the flow of PAM with c = 500ppm and Re = 35.5 in figure 6.19(b). Large values of Tθ/γ_w^2 are interspersed between positive and negative contours of ωθ/γ_w. In both cases, the Newtonian diffusion term (VSD) cannot account for the large spatial variations in ωθ/γ_w, implying that the non-Newtonian torque is needed to balance the vorticity equation.

Figure 6.19: Contours of vorticity and the non-Newtonian torque for the flow of PAM solutions with (a) c = 300ppm, Re = 60.5, and (b) c = 500ppm, Re = 35.5. Positive and zero contours are solid lines, while dashed lines are negative contours.

Figure 6.20(a) demonstrates contours of ωθ/γ_w and Tθ/γ_w^2 for C14 with c = 500ppm at Re = 119. Similar to the flows of PAM, C14 exhibits large values of Tθ/γ_w^2 intermittent between the regions of positive and negative ωθ/γ_w. The largest positive value of Tθ/γ_w^2 occurs within the contraction, where ωθ/γ_w...
changes abruptly from negative to positive with increasing $x/\lambda$. The same can be observed for larger $Re$ flows, such as C14 with $c = 500$ppm and $Re = 254$, seen in figure 6.20(b). As $Re$ increases – comparing figure 6.20(a) to (b) – the large positive contour of $T_{\theta}/\dot{\gamma}_w^2 = 0.2$ within the PCT contraction moves closer towards the centreline. Overall, the large values of $T_{\theta}/\dot{\gamma}_w^2$ are coupled with the strong spatial variations in $\omega_{\theta}/\dot{\gamma}_w$. Distributions in $T_{\theta}/\dot{\gamma}_w^2$ are relatively consistent among solutions of flexible polymers and surfactants as the two solutions apply the same mechanism via non-Newtonian torque for disrupting $\omega_{\theta}/\dot{\gamma}_w$. This mechanism is potentially associated with a common rheological feature that produces the non-Newtonian torque.

Figure 6.20: Contours of vorticity and the non-Newtonian torque for the flow of C14 solutions with (a) $c = 500$ppm, $Re = 119$, and (b) $c = 500$ppm, $Re = 254$. Positive and zero contours are solid lines, while dashed lines are negative contours.

6.5 Discussion

Rheometric measurements showed that PAM and XG have prevalent shear thinning and linear viscoelasticity, while C14 has a Newtonian and water-like shear viscosity, as shown in figures 5.1 and 5.2. On the other hand, high $c$ and $Re$ flows of PAM and C14 within the PCT demonstrate noticeably similar features. These features include the asymmetric half chevron velocity pattern, negative vorticity contours and non-Newtonian torque – all of which are not encountered in the flows of water or XG. This peculiar observation in the velocity and vorticity profiles of PAM and C14 can be explained by non-Newtonian qualities that do not exist for XG. Indeed, XG solutions do not exhibit elastocapillary thinning in DoS rheometry, unlike the PAM solutions shown in figure 5.3. It is plausible that the chevron-shaped velocity pattern for PAM in the PCT can be explained by a resistance to extensional flow. However, this does not explain the existence of the same chevron-shaped pattern observed for C14 flows within the PCT since the C14 solutions do not exhibit elastocapillary thinning. It is hypothesized that structures induced by shear, elongational or mixed kinematics are formed within the PCT flow of the surfactant solution when $Re$ is sufficiently large. These structures behave similarly as flexible polymers. The remaining discussion interprets the results for PAM and C14 further in an attempt to reconcile the cause for their non-Newtonian velocity and vorticity patterns within
In viscoelastic flows through gradual planar contractions, large near-wall velocity overshoots have been observed experimentally. Poole et al. (2005) were among the first to observe near-wall velocity overshoots in the flow of a PAM solution through a duct that gradually contracted along one Cartesian direction. Poole et al. (2005) coined the near-wall velocity overshoot as “cat’s ears” due to their appearance. The canonical flow of Poole et al. (2005) was not axisymmetric, and later numerical investigations by Afonso & Pinho (2006) and Poole et al. (2007) demonstrated that the magnitude of the velocity overshoot was different depending on the Cartesian plane of interest. Subsequent investigations by Alves & Poole (2007) and Poole & Alves (2009) of viscoelastic flows through planar contractions concluded that the cat’s ears and divergent streamlines were inherently elastic, and attributed to a large extensional viscosity and first normal stress differences along the centreline of the duct. Velocity statistics for PAM, seen in figure 6.10 and 6.11, reflect both cat’s ears and divergent flow, implying the PAM solutions impose a large resistance to extensional flow along the centreline of the PCT – as per the conclusion of Alves & Poole (2007) and Poole & Alves (2009).

Moreover, a complex interplay between elasticity and inertia within the PCT was alluded to, based on the trend in $R(U_0)/\bar{U}_0$ with respect to $Re$ and $De$ for PAM. From figure 6.12 it was observed that $R(U_0)/\bar{U}_0$ increased provided $De > 0.1$ and $Re < 35$. Using these threshold values to delineate the different flow regimes, a qualitative phase diagram shown in figure 6.21 was constructed. In figure 6.21 the $De$ and $Re$ of each PAM flow is shown with a colour that corresponds to their respective value of $R(U_0)/\bar{U}_0$. Inset axes in figure 6.21 show samples of the vorticity field (from figure 6.13) within each flow regime. The different flow regimes are summarized as follows.

1. Inelastic: $De < 0.1$ and $Re < 35$. Velocity and vorticity are symmetric with respect to $x/\lambda = 0.5$, as shown in figures 6.10 and 6.13. Velocity contours are similar to water or shear thinning XG solutions. Vorticity is everywhere positive. As $De$ approaches 0.1, $R(U_0)/\bar{U}_0$ is marginally enhanced relative to water flows.

2. Inertial: $De < 0.1$ and $Re > 35$. Mainly distinguished by the decreasing trend in $R(U_0)/\bar{U}_0$ with increasing $Re$ that was similarly observed for water flows in the PCT – seen in table 6.1 and figures 6.8(a) and 6.12(a). Possibly a result of small recirculations or more stagnant flow within the PCT expansion (Deiber & Schowalter, 1981; Pilitsis et al., 1991).

3. Elastic: $De > 0.1$ and $Re < 35$. Near-wall velocity overshoots are apparent, as shown by figure 6.11(a). The negative vorticity contours occupy a large region of the PCT contraction. Values of $R(U_0)/\bar{U}_0$ are significantly augmented relative to water and increase further with growing $De$ and $Re$.

4. Inertioelastic: $De > 0.1$ and $Re > 35$. Values of $R(U_0)/\bar{U}_0$ decrease with increasing $Re$; however, near-wall velocity overshoots are present – see figure 6.12(a). The negative vorticity contour occupies a smaller region of the PCT contraction compared to the elastic flows.

Far more fundamentally interesting is the observation that C14 solutions also demonstrate cat’s ears and divergent flow, as shown in figures 6.14 and 6.15, which hints at their elastic features. Evidently, the PCT stimulates the viscoelastic properties of C14 through the formation of structures induced from
shear, elongation or mixed deformation. The shape of the flow-induced structures are unknown, but they are conjectured to be groupings of micelles that can be conceived as polymer-like aggregates (Rothstein & Mohammadigoushki, 2020). The sudden jump in $R(U_0)/\bar{U}_0$ with increasing $Re$, shown in figure 6.16, demonstrates that these flow-induced structures are formed when $Re$ is greater than 100 within the PCT flows of C14. This corresponds to a value of $\dot{\gamma}_w$ of approximately 90 s$^{-1}$. From figure 5.1(d) no shear-induced structures (SISs) were observed in the shear rheograms of C14 near 90 s$^{-1}$; however, the PCT undergoes mixed deformations, both shear and extension. An explanation is that extension, or the combination of shear and extension, within the PCT is needed for the formation of these structures – similar to the so-called “elongation-induced structures” alluded to by Sachsenheimer et al. (2014); Omidvar et al. (2018); Recktenwald et al. (2019). It was also observed that extensional DoS rheometry does not demonstrate EC thinning for C14, implying these elongation-induced structures are not formed within the filament necking process of the DoS rheometer. The reason extensional DoS rheometry does not reveal these elongation-induced structures for C14 is either a result of insufficient extensional deformation, or perhaps the lower surface tension of the surfactant solution, which in turn reduces the Rayleigh time $t_R$ and Deborah number $De$ of the necking process. If these structures are shear-induced, perhaps pre-shearing the samples before DoS could enable measurements of $t_e$, similar to prior works such as Wunderlich & James (1987); Vissmann & Bewersdorf (1990); Bhardwaj et al. (2007); Fukushima et al. (2022). Regardless of how the structures are formed within the PCT (shear, elongation, or mixed kinematics), they produce the same qualitative net-effect as PAM, revealed by the velocity contours of figure 6.14 and the vorticity and non-Newtonian torque contours of figure 6.17.

Figure 6.21: Phase diagram of the different PAM flows in $De$, $Re$ space. The solid black lines separate the different flow regimes, which are labelled in each quadrant. The four inset axes show sample vorticity contours of flows within each regime. Data point colours correspond to the values of $R(U_0)/\bar{U}_0$ identified from the colourbar.
With reference to figure 6.21, the C14 flows that exhibit cat’s ears fall within the inertioelastic regime, considering their $Re$ is larger than 35. The similarity between the vorticity patterns for C14 and PAM, or more precisely the $R(U_0)/\bar{U}_0$ value, can be used to estimate the relaxation time $t_e$ of the C14 solutions. For example, the C14 solution shown in figure 6.17 with $c = 500$ ppm and $Re = 254$ has a $R(U_0)/\bar{U}_0$ value of 0.26, which is equal to the $R(U_0)/\bar{U}_0$ value of the PAM flow with the label A shown in figure 6.21. Estimating the flow time scale $t_f$ of the C14 flow based on $t_f = \lambda/\langle U_0 \rangle$ and extracting the $De = 0.12$ from figure 6.21, the relaxation time of this C14 solution is estimated to be approximately 4.1 ms. This example shows that measurements of $R(U_0)/\bar{U}_0$ using the PCT along with a phase diagram similar to figure 6.21 can be used for estimating the relaxation time of the C14 solutions. However, figure 6.21 is currently too sparse to provide an accurate map of $R(U_0)/\bar{U}_0$ values. It is envisaged that a larger and more dense matrix of $R(U_0)/\bar{U}_0$ can be used to obtain an accurate phase diagram for extracting the $De$, and therefore the relaxation time of the C14 solutions. Ultimately, the PCT is able to uncover the non-Newtonian features of the dilute C14 solutions.

6.6 Summary

In §6, three non-Newtonian solutions, comprised of XG, PAM and C14 were experimentally investigated in a steady, laminar flow through a periodically constricted tube (PCT). The tube with undulating walls imposed a mixture of shear and extensional deformation, where shear rates were as large as 300 s$^{-1}$ and extensional strain rates as large as 58 s$^{-1}$. The experimental campaign compared several concentrations of each non-Newtonian solution at five unique Reynolds numbers (Re) within the PCT. 

Particle shadow velocimetry (PSV) was used to determine the streamwise and radial velocity within the PCT. The vorticity transport equation was used to derive the non-Newtonian contribution to the vorticity field, referred to as the “non-Newtonian torque.” Our experimental investigation is the first to produce measurements of the non-Newtonian torque – providing another means for comparison with numerical investigations that can derive the non-Newtonian torque explicitly from constitutive models. 

Shown previously in §5, the steady shear rheology of XG and PAM was shear thinning. PAM solutions were the only non-Newtonian fluids to exhibit elastocapillary thinning from extensional rheology. Within the PCT, solutions of XG demonstrated evidence of a plug-like flow, consistent with expectations for pipe flow of inelastic shear thinning solutions. PCT flows of PAM solutions exhibited different dynamics depending on the Deborah number ($De$) and $Re$. A phase diagram that delineated the different flow regimes in $De – Re$ space was constructed for the PAM solutions, based on the change in the amplitude of the centreline velocity along the streamwise direction of the PCT. Above a $De$ of 0.1, PAM flows within the PCT exhibited “chevron” velocity contours, near-wall velocity overshoots and divergent streamlines with shape and curvature that departed dramatically from the sinusoidal wall profile. Within the contractions of the PCT were regions of negative vorticity and non-Newtonian torque. Despite having a shear viscosity that was identical to water and no elastocapillary extensional rheology, C14 exhibited similar non-Newtonian features as PAM within the PCT when $Re$ exceeded 100. The C14 solutions that demonstrated a non-Newtonian response within the PCT, reflected qualitative similarities with inertioelastic PAM flows with $De > 0.1$ and $Re > 35$. 

66
Part III

Turbulent channel flows
Chapter 7

Comparing drag-reduced channel flows of polymers and surfactants

The current investigation compares three drag-reducing additives in the channel flow facility detailed in §4.2. The different additives include the same additives from §§5 and 6, those being PAM, XG and C14. The additive solutions are prepared such that the solutions impose the same level of wall shear stress at the same mass flow rate, i.e. same drag reduction $DR$. Two scenarios of $DR$ are considered: a $DR$ of approximately 58% referred to as high drag reduction (HDR), and a MDR case with $DR$ of approximately 70%. To measure all three components of the velocity field with a high spatial resolution, the novel technique of three-dimensional particle tracking velocimetry (3D-PTV) based on the “Shake-The-Box” (STB) algorithm is employed (Schanz et al. 2013). In addition, the rheology of the drag-reduced solutions is evaluated using the torsional rheometer discussed in §4.1.1 and a CaBER, detailed in §4.1.3.

7.1 Assessment of drag reduction

Two methods are used to determine the wall shear stress, $\tau_w$, and drag reduction percentage $DR$ of the non-Newtonian fluids within the turbulent channel flow depicted in §4.2. The first method used measurements of the pressure drop, $\Delta P$, where $\tau_{w,1} = h\Delta P/\Delta x$ similar to (3.5), and $h$ is the half-channel height ($H/2$). Subscript, 1, is used to distinguish this first method and, going forward, will denote variables calculated based on $\Delta P$. In the second method, $\tau_{w,2}$ characterized by the subscript 2, was determined using a wall-normal gradient of the mean velocity obtained from 3D-PTV measurements, which is equivalent to equation (3.7) and will be elaborated on further in §7.3. The drag-reduction percentage $DR$ was assessed similarly to (3.14), although based on a comparison of $\tau_w$ of a drag-reduced flow and that of water at the same mass flow rate $\dot{m}$, according to,

$$DR = \left(1 - \frac{\tau_{w,A}}{\tau_{w,N}}\right) \times 100\%$$

(7.1)

where $\tau_{w,A}$ is the wall shear stress of the additive solution and $\tau_{w,N}$ is the wall shear stress of the Newtonian flow of water at the same $\dot{m}$. Additionally, $DR$ can be derived from $\Delta P$ (and $\tau_{w,1}$) as $DR_1$, which is equivalent to $DR_1 = (1 - \Delta P_A/\Delta P_N) \times 100\%$. In this equation, $\Delta P_A$ is the streamwise pressure drop for an additive.
solution and $\Delta P_N$ is the streamwise pressure drop for the flow of water at the same $\dot{m}$. All experiments with drag-reducing additives were performed at a $\dot{m}$ of 3.294 kg s$^{-1}$, which corresponds to a bulk velocity, $U_b$, of 1.839 m s$^{-1}$. For the flow of water, this flow rate equates to a bulk Reynolds number $Re_H$, from (3.3) of 31 900 and friction Reynolds number $Re_\tau$, from (3.12), of 793. Certain drag-reducing solutions have a viscosity that is larger than that of water (Escudier et al., 2009). Such an increase in kinematic viscosity of the flow will result in a decrease in $Re_H$ although $\dot{m}$ and $\Delta P$ are kept constant. It is challenging to maintain a constant $Re_H$ for the drag-reduced flows, since $Re_H$ is calculated using the viscosity of the fluid at the wall-shear-rate, which is unknown a priori. In addition, changing $\dot{m}$ will vary $\Delta P$ and therefore $DR$.

Additional measurements were also performed for water at lower $\dot{m}$ to match the $Re_\tau$ of the drag-reduced flows. Table 7.1 lists $U_b$, $Re_H$, $\Delta P$ and $\tau_{w,1}$ for each flow case of water. Table 7.1 also provides $\tau_{w,2}$, the friction velocity $u_\tau = (\tau_{w,2}/\rho)^{1/2}$, viscous lengthscale $\delta_v = \nu/u_\tau$, and $Re_\tau$ of each water flow experiment. Here $\rho$ is the density of the fluid. The variables in the last four columns of table 7.1 are derived based on the estimated $\tau_{w,2}$ from 3D-PTV measurements. The method will be discussed and evaluated in §7.3.

<table>
<thead>
<tr>
<th>$U_b$ (m s$^{-1}$)</th>
<th>$Re_H$</th>
<th>$\Delta P$ (Pa)</th>
<th>$\tau_{w,1}$ (Pa)</th>
<th>$\tau_{w,2}$ (Pa)</th>
<th>$u_\tau$ (mm s$^{-1}$)</th>
<th>$\delta_v$ (μm)</th>
<th>$Re_\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.613</td>
<td>10 630</td>
<td>290</td>
<td>1.330</td>
<td>1.248</td>
<td>35.42</td>
<td>24.42</td>
<td>307</td>
</tr>
<tr>
<td>0.736</td>
<td>12 770</td>
<td>385</td>
<td>1.766</td>
<td>1.739</td>
<td>41.81</td>
<td>20.69</td>
<td>363</td>
</tr>
<tr>
<td>0.859</td>
<td>14 890</td>
<td>496</td>
<td>2.275</td>
<td>2.394</td>
<td>49.05</td>
<td>17.63</td>
<td>425</td>
</tr>
<tr>
<td>0.981</td>
<td>17 020</td>
<td>695</td>
<td>2.821</td>
<td>2.749</td>
<td>52.57</td>
<td>16.45</td>
<td>456</td>
</tr>
<tr>
<td>1.103</td>
<td>19 140</td>
<td>748</td>
<td>3.431</td>
<td>3.458</td>
<td>58.95</td>
<td>14.67</td>
<td>511</td>
</tr>
<tr>
<td>1.839</td>
<td>31 900</td>
<td>1790</td>
<td>8.211</td>
<td>8.317</td>
<td>91.43</td>
<td>9.46</td>
<td>793</td>
</tr>
</tbody>
</table>

Table 7.1: Flow properties for channel flow experiments using water as the working fluid.

### 7.2 Drag-reducing additives

To prepare the additive solutions, drag-reducing powders were weighed using a digital scale (AB104-S, Mettler Toldeo) with a 0.1 mg resolution, and added to 15 l of tap water. The combination was then agitated for approximately 2 h using a stand mixer equipped with a three-blade impeller set to 100 revolutions per minute (Model 1750, Arrow Engineering Mixing Products) and left to rest for approximately 16 h (Abur-Rowin et al., 2018). The master solution was then added to the reservoir labelled in figure 4.4. The pump effectively mixed and diluted the 15 l concentrated master solution with 120 l of tap water, to bring the fluid to the desired concentration, $c$.

Two different cases of $DR$ were considered for the present experiments. The first was a comparison of additive solutions at a high level of drag reduction (HDR). This case evaluated three drag-reduced solutions at a similar $DR_1$, approximately equal to 57.7% ± 1.2%. Seeing as the $DR_1$ is greater than 40%, this comparison is in the “HDR” regime according to Warholic et al. (1999b). The HDR amount of 57.7% was
selected based on the largest $DR$ that could be obtained using the rigid polymer. The second scenario was a comparison of the flexible polymer and surfactant solutions at MDR, which occurs at $DR_1$ of approximately $70.3\% \pm 1.8\%$ for the $Re_H$ considered here.

When the concentration of PAM increased beyond 50 ppm, it was observed that $DR_1$ plateaued at approximately 68.5%, as demonstrated by figure 7.1(a). This suggested that 50 ppm of PAM could generate the required MDR state. To achieve the HDR case, with smaller $DR_1$, the rotational speed of the centrifugal pump was increased to reduce $DR_1$ to the desired value by using mechanical degradation. Figure 7.1(b) demonstrates how this procedure was executed on a 50 ppm PAM solution. Upon initially adding the master solution to the reservoir and letting the loop mix the solution for about 2 minutes at a low pump speed, $DR_1$ was 68.5% for a pump speed of 600 revolutions per minute (desired $\dot{m}$ of 3.294 kg s$^{-1}$). At this pump speed mechanical degradation is negligible and $DR_1$ remains constant. At $t = 360$ s, the pump speed was increased significantly to promote mechanical degradation. After approximately 720 s at a high pump speed, the pump speed was then returned to 600 revolutions per minute and the $DR_1$ became approximately equal to 58.0%. While lower levels of $c$ for PAM could produce the same effect, mechanical degradation at lower values of $c$ would have been greater, making flow measurements challenging (Virk & Wagger, 1990; Pereira et al., 2013). Therefore, a degraded 50 ppm PAM solution was used instead of a lower concentration solution of PAM, for the case of HDR.

![Figure 7.1: (a) Value of $DR_1$, as a function of $c$ for PAM, (b) $DR_1$ of $c = 50$ ppm solution of PAM as a function of time, $t$, (c) $DR_1$ of XG as a function of $c$, (d) $DR_1$ of C14/NaSal (1:2 mM) as a function of $c$.](image)

Figure 7.1(c) demonstrates that the largest $DR_1$ achieved was 58.5%, exhibited by 300 ppm of XG. The
XG solution showed negligible amounts of degradation, similar to the findings of Pereira et al. (2013). The largest $DR_1$ achieved using XG was chosen as the common HDR value. Due to the limited drag-reduction capability of XG, no MDR case was achieved.

Figure 7.1(d) shows that a 200 ppm (0.685 mM) solution of C14 produced $DR_1$ of 72.0%. No increase in $DR_1$ was observed if the $c$ of C14 was increased further. Therefore, 200 ppm of C14 was perceived to produce MDR. Choosing a $c$ equal to 150 ppm of C14 (0.521 mM), with the same 1:2 molar ratio of C14 to NaSal, produced $DR_1$ of 56.5% for HDR tests. The measurements of $\Delta P$ and $DR_1$ are listed in table 7.2 for each drag-reduced flow.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>$c$ (ppm)</th>
<th>$U_b$ (m s$^{-1}$)</th>
<th>$Re_H$</th>
<th>$\tau_{w,1}$ (Pa)</th>
<th>$DR_1$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>–</td>
<td>0.613-1.839</td>
<td>10 6300-31 900</td>
<td>1.330-8.211</td>
<td>–</td>
</tr>
<tr>
<td>PAM solution</td>
<td>50*</td>
<td>1.839</td>
<td>25 550</td>
<td>3.445</td>
<td>58.0 (HDR)</td>
</tr>
<tr>
<td>PAM solution</td>
<td>50</td>
<td>1.839</td>
<td>25 260</td>
<td>2.578</td>
<td>68.5 (MDR)</td>
</tr>
<tr>
<td>XG solution</td>
<td>300</td>
<td>1.839</td>
<td>17 060</td>
<td>3.399</td>
<td>58.5 (HDR)</td>
</tr>
<tr>
<td>C14 solution</td>
<td>150</td>
<td>1.839</td>
<td>30 130</td>
<td>3.564</td>
<td>56.5 (HDR)</td>
</tr>
<tr>
<td>C14 solution</td>
<td>200</td>
<td>1.839</td>
<td>30 120</td>
<td>2.294</td>
<td>72.0 (MDR)</td>
</tr>
</tbody>
</table>

Table 7.2: Bulk flow measurements from Coriolis flow meter and pressure transducer. To reiterate, $DR_1$ is calculated based on $\Delta P$. *Solution was subject to mechanical degradation.

The skin friction coefficient $C_f$ derived from equation (3.8), as a function of $Re_H$, is demonstrated in figure 7.2 for flows of drag-reducing solutions and water. For drag-reduced flows, the kinematic viscosity, $\nu$, that is used to calculate $Re_H$, corresponds to the measured shear viscosity at the wall shear rate. The procedure will be discussed in §7.4 and §7.5. The error bars shown in figure 7.2 propagate from random and systematic uncertainties in measurements of the flow rate, viscosity and streamwise pressure gradient. Figure 7.2 also presents two empirical correlations. The upper line in figure 7.2 corresponds to the equation (3.9) from Dean (1978) for a Newtonian turbulent channel flow that has a cross-section with $W/H$ greater than 7. The measured $C_f$ for the experimental data of water, shown by the blue markers in figure 7.2, are marginally lower than the Dean (1978) correlation equation. However, the results are in agreement with other turbulent channel flow experiments, several of which were used by Dean (1978) to obtain the correlation. The lower line in figure 7.2 corresponds to the MDR asymptote proposed by Virk et al. (1970) or (3.15). The original correlation was intended to be used for pipe flows. To adapt the equation to a channel flow, similar to Owolabi et al. (2017), the MDR asymptote is plotted using a Reynolds number that is calculated based on the hydraulic diameter, $Re_b = U_b D_h / \nu$, where $D_h = 2HW/(H+W)$. The $C_f$ of drag-reduced flows at MDR are about 15% greater than the $C_f$ of the correlation. It should be noted that there is considerable ambiguity in the equation describing the MDR asymptote in channel flows. Escudier et al. (2009) applied a correction factor to the Reynolds number to account for potential secondary flows, while Ptasinski et al. (2003) simply used $Re_H$. The choice of the length scale in defining the Reynolds number will raise or lower
the MDR asymptote along the vertical axis of the plot of $C_f$. Also, Virk et al. (1970) remarked that the $C_f$ relationship was derived from an integration of the asymptotic mean velocity profile. White et al. (2012), among others, had cast doubt on the exactness of the mean velocity profile of drag-reduced flows at MDR. Therefore, the $C_f$ distribution at MDR may also be erroneous and conditional on the canonical flow type, Reynolds number and additive type (White et al. 2012).

Figure 7.2: Skin friction coefficient as a function of bulk Reynolds number for drag-reduced flows and water. The upper equation shows the Dean (1978) correlation for Newtonian channel flows and the lower equation shows the MDR asymptote adapted for channel flows (Virk et al. 1970).

Shear and extensional viscosity measurements were performed on samples of each drag-reducing solution. The samples were collected from the flow loop using an outlet valve at the corresponding $DR$ and the rheology measurements were performed immediately afterwards. The shear viscosity $\mu$ as a function of shear rate $\dot{\gamma}$ for each additive solution and water, was determined using the torsional rheometer depicted in figure 4.1 equipped with the double gap cylinder geometry shown in figure 4.2(b). Shear viscosity measurements were performed three times for each sample listed in table 7.2 (including water) to establish the uncertainty of the measurements. The extensional relaxation time $t_e$ was established using the CaBER apparatus detailed in §4.1.3.

### 7.3 Lagrangian 3D-PTV measurements

Flow measurements were carried out using 3D-PTV based on the state-of-the-art STB algorithm devised by Schanz et al. (2016). The STB algorithm predicts the three-dimensional particle position based on the established trajectories of previous time steps. The prediction is then corrected using an iterative particle reconstruction (Wieneke 2012), where the particles are shifted (“shaked”) in the volume (“box”) until residual errors are minimized and a trajectory is established. The algorithm can analyse images with high seeding densities, allowing measurement of spatially resolved turbulent statistics and instantaneous flow
structures. The efficacy of STB was exemplified by Schröder et al. (2015), where the turbulent Reynolds stresses were accurately measured for $y^+$ as low as 1.5.

The 3D-PTV system consisted of four high-speed cameras (v611, Phantom) and a high-repetition Nd:YLF laser (DM20-527 Photonoics Industries). Figure 7.3 provides a visual representation of the cameras and laser configuration. The laser emitted light with a wavelength of 532 nm and a maximum pulse energy of 20 mJ pulse$^{-1}$. As seen in figure 7.3, the circular laser beam was directed in the spanwise direction of the channel (negative $z$). A lens combination shaped and collimated the beam into an oval profile. The resulting oval profile was then cropped to form a rectangular cross-section with 5 mm thickness in the wall-normal direction, covering from $y = 0$ to 5 mm. The laser sheet was 16 mm in the streamwise direction, $x$. To increase the light intensity for the backward scattered camera, the laser sheet was also reflected back onto itself using a large mirror situated on the opposite side of the test section (Ghaemi & Scarano 2010).

![Figure 7.3: Three-dimensional rendering of high-speed laser and camera array for 3D-PTV.](image)

The four Phantom v611 cameras had a 1280 $\times$ 800 pixel complementary metal oxide semiconductor sensor with pixel size of $20 \times 20$ $\mu$m$^2$ and 12 bit resolution. Scheimpflug adapters and Nikon lenses with a focal length of $f = 105$ mm were connected to the cameras. A reduced sensor resolution of $900 \times 800$ pixel was used to enable higher recording rates. The forward/backward scattering cameras (cameras 2 and 3 in figure 7.3) were placed along the $z$-direction and set to a lens aperture of $f/16$. The side scattering cameras (cameras 1 and 4) were placed along the streamwise $x$-direction with a lens aperture setting of $f/11$. The line of sight of cameras 2 and 3 had an angle of 60° with respect to each other, while the side scattering cameras were placed at 30° with respect to each other. The distance of the cameras to the measurement location was approximately 290 mm. This imaging configuration resulted in a magnification of approximately 0.56 and a resolution of 27.9 $\mu$m pixel$^{-1}$. The cameras and laser were synchronized using a programmable timing unit (PTU X, LaVision GmbH). Fluids were seeded with 10 $\mu$m silver coated hollow glass spheres (S-HGS-10, 73...
Dantec Dynamics). The density of the tracers in the images was approximately 0.05 particles per pixel. The fidelity for which the tracer particles can follow the fluid flow can be defined by two parameters, the Stokes number, $St$, and Froude number, $Fr$ (Bewley et al., 2008). The local values of $St$ and $Fr$ of the particles can be approximated as $St = \frac{t_p}{t_f}$ and $Fr = \frac{u_p}{u_r}$, and describe the significance of particle inertia and particle settling. The particle response time is $t_p = \frac{\rho_p d_p^2}{18 \mu}$, and the settling velocity is $u_p = \frac{(\rho_p - \rho) d_p^2 g}{18 \mu}$. Here $\rho_p$ is the density of the particles and $d_p$ the diameter. The characteristic fluid response time, $t_f$, was approximated as $\frac{\delta}{\nu}$, and the settling velocity is $u_p = \frac{1}{2} \rho \frac{d^2}{18 \mu}$. Here $\rho$ is the density of the particles and $d$ the diameter. While the $Fr$ for all flows was of the order of magnitude, $10^{-3}$ to $10^{-4}$. Therefore, particle inertia and particle settling was considered inconsequential.

One time-resolved data set, for each drag-reduced and Newtonian flow, consisted of 6800 single-frame images captured at a frequency between 2.5 and 4.5 kHz. Therefore, one data set was between 1.5 and 2.7 s in duration. Depending on $U_b$ of the flow being measured, the image capture rate was determined such that a maximum particle displacement of approximately 10 pixels across successive frames was maintained. After recording the images, the minimum intensity of each data set was computed and subtracted to remove any glare points caused by surface scratches and tracer particles stuck to the bottom wall. Images were further enhanced by applying a sliding minimum subtraction with kernel of 7 pixels and local intensity normalization over a kernel of 50 pixels.

Calibration of the imaging system was carried out by fitting a third-order polynomial mapping function onto images recorded from a dual-plane calibration target (058-5, LaVision GmbH). To improve the accuracy of the mapping function, volume self-calibration was employed (Wieneke, 2008), which brought the average disparity down to 0.02 pixels. An optical transfer function was generated for iterative particle reconstruction in STB (Schanz et al., 2013). The measurement volume was in the mid-span of the test section and had dimensions of $(\Delta x, \Delta y, \Delta z) = 670 \times 180 \times 670$ voxel $= 24 \times 5 \times 24$ mm$^3$. Additional image and volume cropping mitigated noise common along the borders of the volume. Lastly, the STB algorithm was performed in DaVis 8.4 (LaVision GmbH). The maximum triangulation error was constrained to 1 voxel. Particle displacement was limited to a maximum value of 15 voxels. In addition, particles with a change in velocity exceeding 2 pixels or 20% in successive image frames were discarded.

A moving second-order polynomial was fit on the particle trajectories in MATLAB. The length of the polynomial (kernel) was five time steps (1.11–2 ms) for obtaining first-order turbulence statistics. To mitigate noise in Reynolds stresses, a kernel with a length of 11 time steps (2.4–4.4 ms) was used. Trajectories less than the respective kernel length were removed from consideration. To obtain the velocity statistics, particle tracks were binned into slabs parallel with the wall, covering the entire measurement domain in the $x$ and $z$ directions. Each slab was 10 $\mu$m thick in the $y$ direction for evaluating the mean velocity profiles ($\Delta y^+ = 0.4 – 0.7$) and 100 $\mu$m in the $y$ direction for the Reynolds stresses ($\Delta y^+ = 4.0 – 6.7$). Both procedures incorporated a 75% overlap between neighbouring slabs in the $y$-direction. The statistics were obtained by averaging in time and the homogenous directions ($x$ and $z$), and are indicated by angle brackets, $\langle \cdots \rangle$. To obtain instantaneous velocity fields in a Eulerian frame of reference, the particle tracks were binned into $24 \times 24 \times 24$ voxel cubes with 75% overlap in all three directions. The instantaneous velocities in $x$, $y$ and $z$ directions were denoted by $U$, $V$ and $W$, respectively. The corresponding velocity fluctuations were
As previously established, a superscript of + is indicative of inner normalization by friction velocity \( u_\tau \) defined according to (3.10), and viscous length scale \( \delta_v \) from (3.11). For the inner normalization, the wall shear stress is calculated as \( \tau_{w,2} = \mu_w \partial \langle U \rangle / \partial y \rvert_w \) according to (3.7), where \( \partial \langle U \rangle / \partial y \rvert_w \) is the mean velocity gradient at the wall. Drag-reducing solutions can exhibit shear thinning characteristics, where \( \mu \) decreases with respect to \( \dot{\gamma} \) (Warholic et al. 1999b; Ptasinski et al. 2001; Escudier et al. 2009). Therefore, the shear viscosity measurements, discussed in §7.2, were used to estimate \( \mu_w \) at the wall shear rate, i.e. \( \dot{\gamma} = \partial \langle U \rangle / \partial y \rvert_w \). To determine \( \partial \langle U \rangle / \partial y \rvert_w \), a linear fit was applied on the mean velocity profile within \( 2 - 4 < y^+ < 5 \) in the linear viscous sublayer. The lower bound varied depending on the flow \( Re_\tau \) but it corresponded to \( y \approx 60 \mu m \). The efficacy of this procedure is discussed in §7.5 by comparing the normalized mean velocity profile and Reynolds stresses for turbulent channel flow of water with results from direct numerical simulation (DNS) at a similar \( Re_\tau \). Such an estimate of \( \tau_w \) using the near-wall gradient of the mean velocity profile is an approximation for the drag-reduced flows. Solutions that are shear thinning can exhibit instantaneous variations in \( \partial \langle U \rangle / \partial y \rvert_w \), and therefore variations in \( \mu_w \) with time. To ensure \( \tau_{w,2} \) of the drag-reduced flows was reasonable, results were validated by comparing the estimated \( DR_2 \) with the \( DR_1 \) that was obtained using measurements of \( \Delta P \).

Uncertainty in the normalized velocity and Reynolds stresses are quantified based on two sources of error. The first source propagates from the uncertainty in measurements of \( \mu \). This was estimated by repeating the measurements of \( \mu \), which will be shown in §7.4. The uncertainty in \( \mu \) affects variables used for inner scaling, that is \( u_\tau \) and \( \delta_v \), following a root-sum-of-squares propagation of uncertainty (Wheeler & Ganji, 2010). The second source of uncertainty is a random noise in the measured flow velocity associated with particle positioning in 3D-PTV. Using a spectral analysis of the particle tracks, Abu-Rowin & Ghaemi (2019) and Ebrahimian et al. (2019) showed that an error of 0.1, 0.2 and 0.1 pixel was present in particle displacements along the \( x \), \( y \) and \( z \) directions, respectively. Combined, these two sources of uncertainty contribute to the total uncertainty in normalized mean velocity, Reynolds stresses and wall-normal location. The estimated uncertainty is shown as error bars in the figures demonstrated in §7.5 to §7.7.

### 7.4 Fluid rheology

The results of the shear viscosity measurements using the torsional rheometer are shown in figure 7.4(a). The demonstrated shear viscosities are the average of the thrice repeated measurements for each sample. Error bars are the range in the measurements at each \( \dot{\gamma} \). Within the presented values of \( \dot{\gamma} \), the measurements of \( \mu \) show good repeatability and low random error; the range in the measurements are less than 5.7 %, Based on figure 7.4(a), the measured \( \mu \) of domestic tap water at 25°C is 0.861 ± 0.049 mPa s. The results for water can be contrasted with shear viscosity measurements of Nagashima (1977) and Collings & Bajenov (1983). They measured the viscosity of distilled water at 25°C; finding it to be 0.891 mPa s. The discrepancy between the results of figure 7.4(a) for water and the measurements of Nagashima (1977) and Collings & Bajenov (1983) is within the estimated uncertainty based on the three repeated measurements, and is attributed to systematic uncertainties inherent with the torsional rheometer.

From visual inspection of figure 7.4(a), it is apparent that the XG solution is shear thinning. The
viscosity of the XG solution reduces by 80.4% between $\dot{\gamma}$ of 5 and 400 s$^{-1}$. For $\dot{\gamma} > 400$ s$^{-1}$, Taylor instabilities produce a sudden increase in $\mu$ and the results were discarded. The values of $\partial \langle U \rangle / \partial y |_w$ for the drag-reduced, turbulent flows being investigated are beyond 2000 s$^{-1}$, much greater than the maximum achievable $\dot{\gamma}$ of 400 s$^{-1}$ using this rheometer. Therefore, a predictive model is used to extrapolate the data and estimate $\mu_w$ of the drag-reduced turbulent flows. For the XG solution, the Carreau–Yasuda (CY) model fit the measurements appropriately and is shown by the solid line in figure 7.4(a). The CY model is represented by the following equation,

$$\frac{\mu - \mu_{\infty}}{\mu_0 - \mu_{\infty}} = \frac{1}{(1 + (M \dot{\gamma})^a)^{1/a}}$$  \hspace{1cm} (7.2)

where $\mu_0$ is the zero-shear-rate viscosity, $\mu_{\infty}$ is the infinite-shear-rate viscosity, $M$ is the consistency, $k$ is the flow index and $a$ is an additional fitting parameter introduced by Yasuda et al. (1981). For XG, $\mu_0$ is 0.019 Pa s, $\mu_{\infty}$ is 0.937 mPa s, $M$ is 0.517 s, $k$ is 0.466 and $a$ is 1.935. The uncertainty in the extrapolated shear viscosity for XG is taken to be the maximum range in the thrice-repeated measurements of $\mu$. Using the above (7.2), the $\mu_w$ of XG at HDR, which corresponds to the value of $\dot{\gamma}$ that was equal to $\partial \langle U \rangle / \partial y |_w$, is 1.576 mPa s. Extrapolating the CY model may be subject to errors that can influence the variables derived for inner scaling, including $\tau_{w,2}$, $u_\tau$ and $\delta_v$ (Singh et al., 2016). It will be demonstrated that the $DR_2$ derived using these rheology measurements is within 5% of the $DR_1$ determined from measurements of the streamwise pressure gradient. Propagation of uncertainty accounts for additional errors in the inner-scaling variables that can be seen by error bars in the above plots of the mean velocity profile and Reynolds stresses.

Solutions of PAM also demonstrate shear thinning qualities, but to a much lesser extent than XG. The viscosity of PAM at MDR reduced by 7.4% between $\dot{\gamma}$ of 10 and 180 s$^{-1}$. The viscosity of PAM at HDR

Figure 7.4: Rheology of aqueous solutions of drag-reduced additives including (a) shear viscosity as a function of shear rate, and (b) mid-point filament diameter with respect to time from uniaxial filament extension.
reduces by 6.1% across the same range in $\dot{\gamma}$. Below $\dot{\gamma}$ of 10 s$^{-1}$, measurements of $\mu$ are noisy and ambiguous. In either scenario, measurements of $\mu$ are approximately constant for $\dot{\gamma} > 180$ s$^{-1}$, which is the maximum measurable $\dot{\gamma}$ of both PAM solutions (HDR and MDR) before Taylor instabilities impair the measurements. The Sisko (SI) model (Sisko, 1958) was used to represent $\mu$ of the PAM solutions at moderate and large values of $\dot{\gamma}$. This model is typically used when measurements close to the zero-shear-rate viscosity are lacking (Barnes et al., 1989). The fitted SI model is shown in figure 7.4 using a dashed line and is represented by the following equation,

$$
\mu = \mu_{\infty} + M \dot{\gamma}^{k-1}
$$

(7.3)

where $M$ and $k$ are constants used to describe the power law decay in $\mu$. The infinite-shear-rate viscosity, $\mu_{\infty}$, for PAM at HDR and MDR are estimated to be 1.072 and 1.087 mPa s, respectively. The fitting parameter $k$ and $M$ are 0.349 and 0.455 mPa s$^k$ for PAM at HDR and 0.101 and 0.985 mPa s$^k$ for PAM at MDR. Using the above (7.3), the $\mu_w$ of PAM at HDR and MDR is 1.074 and 1.088 mPa s respectively, not much greater than the corresponding values of $\mu_{\infty}$.

There is a negligible difference in measured values of $\mu$ for the 150 ppm C14 solution at HDR and the 200 ppm C14 solution at MDR. Unlike PAM and XG, solutions of C14 exhibit a Newtonian trend with constant $\mu$ for $10$s$^{-1} < \dot{\gamma} < 100$s$^{-1}$. Therefore, their viscosities were assumed constant for $\dot{\gamma} > 100$ s$^{-1}$. The estimated $\mu_w$ of C14 at HDR is 0.911 ± 0.036 mPa s and C14 at MDR is 0.912 ± 0.024 mPa s. No SISs are observed for C14; however, that does not rule out the possibility of their presence at higher values of $\dot{\gamma}$.

Using the CaBER system, it was not feasible to measure $t_e$ of XG and C14 solutions, since the filament immediately ruptured upon moving the end plates. Similar findings for rigid polymer and surfactant solutions have been reported by previous investigations (Lin, 2000; Escudier et al., 2009; Mohammadtabar et al., 2020). The two PAM solutions were the only fluids that showed a measurable $t_e$ using the CaBER apparatus. Figure 7.4(b) demonstrates the filament mid-point diameter, $D_{mid}$, as a function of time, $t$. Here $t = 0$ indicates the end of the top plate displacement. Similar to the shear viscosity measurements, the thrice-repeated measurements of $D_{mid}(t)$ were averaged for each sample and the error bars show the range of the measurements. The solid black line represents the exponential fit of $D_{mid}(t)$ using (4.3). The resulting $t_e$ for PAM at HDR and MDR were 4.3 and 11.0 ms, respectively. For the purposes of the current analysis, a comprehension that solutions of PAM have significantly larger extensional characteristics than those of XG and C14, will suffice.

Despite producing similar $DR$ at HDR or MDR (see table 7.2), each drag-reducing solution exhibits a different shear viscosity and extensional characteristics. Of the additive solutions, XG has the largest overall $\mu$ and a strong shear thinning behaviour. PAM has the next largest distribution in $\mu$; however, only approximately 20% larger than the average $\mu$ of water. C14, on the other hand, has a water-like distribution in $\mu$. Although $t_e$ could not be measured for C14 and XG using the CaBER system, the fact that $t_e$ for PAM solutions could be measured implies that PAM has a larger $t_e$ than C14 and XG. Rodd et al. (2005) specified that the operable range of the CaBER is constrained to fluids with $t_e$ larger than approximately 1 ms when $\mu$ is smaller than 70 mPa s. Given the measured shear viscosities of XG and C14 are less than 70 mPa s, it is possible that their $t_e$ are less than 1 ms. However, further measurements of the extensional rheology are
needed to confirm this hypothesis, one possible method being the dripping-onto-substrate technique detailed in [Dinic et al.] (2017). Such a method was capable of measuring the pinch-off dynamics of fluids with \( \mu \) less than 20 mPa s and \( t_e \) less than 1 ms, according to [Dinic et al.] (2017). Nonetheless, a correlation relating \( DR \) to \( t_e \), similar to that proposed by [Owolabi et al.] (2017) for flexible polymers, may not apply to solutions of XG or C14. The above analysis using conventional torsional and extensional rheometers highlights that the drag-reduced solutions demonstrate different rheological characteristics.

Other authors have demonstrated that flows obtained from DNS and using the FENE-P (finitely extensible non-linear elastic spring, with a Peterlin approximation) model with large Weissenberg number, \( Wi = t_e \partial \langle U \rangle / \partial y|_{w} \), have an effective viscosity that increases with distance from the wall ([Procaccia et al.] 2008). A viscosity that increases monotonically with distance from the wall is achieved inherently by shear thinning fluids. It is intriguing that \( DR \) exists for both XG with relatively small \( t_e \) and large shear thinning behaviour, and PAM with large \( t_e \) and minimal shear thinning characteristics. This could suggest that polymers achieve \( DR \) using a viscosity that increases monotonically with \( y \). Flexible polymers achieve this viscosity gradient using polymer elasticity (i.e. \( Wi \)), while rigid polymers are naturally shear thinning. Such a hypothesis is only speculative. Measurements connecting the role of shear thinning characteristics to \( DR \) are warranted.

### 7.5 Newtonian turbulent channel flow

The following section seeks to evaluate the 3D-PTV measurements for water by comparing them with DNS of [Iwamoto et al.] (2002) at \( Re_\tau = 300 \), [Moser et al.] (1999) at \( Re_\tau = 395 \), and [Lee & Moser] (2015) at \( Re_\tau = 550 \). The previously listed DNS data, in that order, are compared with the experimental water data at \( Re_\tau = 307, 425 \) and 511, respectively, in figures 7.5 and 7.6. The comparison involves an evaluation of \( \langle U \rangle^+ \) in figure 7.5 and the Reynolds stress distributions in figure 7.6. The error bars in figures 7.5 and 7.6 originate from a propagation of uncertainty stemming from errors in velocity and shear viscosity measurements. For clarity of the figures, the error bars are down sampled in figures 7.5 and 7.6.

As demonstrated in figure 7.5, the 3D-PTV measurements of mean velocity at the three \( Re_\tau \) agree with the distributions established using DNS and the law of the wall. Rather remarkable is the spatial resolution at which these measurements can be attained. For the lowest velocity case of \( Re_\tau = 307 \), the spacing of data points along \( y^+ \) is 0.4\( \delta_v \) and the velocity measurements are obtained for \( y^+ \) as low as 2 (60 \( \mu m \) from the wall). The spatial resolution of the velocity measurements with respect to inner scaling decreases with increasing \( Re_\tau \). For \( Re_\tau = 511 \), the spatial resolution is 0.7\( \delta_v \) and a minimum \( y^+ \) of 4 (60 \( \mu m \) from the wall). The closest data point to the wall is limited by the size of the tracer particles and glare spots that formed due to a reflection of the laser sheet from imperfections on the surface (small scratches and particles stuck to the wall). As shown in figure 7.5, there is no observable noise in the velocity distributions obtained from 3D-PTV based on STB.

The 3D-PTV measurements of the Reynolds stress profiles are compared with those of DNS in figure 7.6. The results from 3D-PTV and DNS agree well with one another, although there are some minor deviations. The maximum discrepancy in the peak streamwise Reynolds stress, \( \langle u^2 \rangle^+ \), shown in figure 7.6(a), is approximately 0.4\( u_\tau^2 \). The maximum deviation in the \( y^+ \) location of the peak in \( \langle u^2 \rangle^+ \) is 2.6\( \delta_v \). The wall-normal Reynolds stress profile, \( \langle v^2 \rangle^+ \), overlaps well with DNS for \( Re_\tau \) of 425 and 511, as shown
Figure 7.5: Inner-normalized mean streamwise velocity from 3D-PTV measurement for water in comparison with DNS and the law of the wall. The three profiles are shifted upward along the vertical axis by 10. 3D-PTV measurements at $Re_\tau = [307, 425, 511]$ are compared with DNS from Iwamoto et al. (2002) with $Re_\tau = 300$; MMoser et al. (1999) with $Re_\tau = 395$; and Lee & Moser (2015) with $Re_\tau = 550$.

The profiles of $\langle v^2 \rangle^+$ and $\langle uv \rangle^+$, shown in figure 7.6(b), both have visible low-amplitude noise. This is associated with the larger particle positioning error of 3D-PTV in the out-of-plane direction and the smaller flow motions in this direction ($v$ component). The largest peak-to-peak noise oscillation in figure 7.6(b) is approximately $0.03u_\tau^2$, occurring between $y^+ = 230$ and 250 for the case of $Re_\tau = 425$. This peak-to-peak noise corresponds roughly to a pixel disparity of 0.1 pixel, given the digital resolution of 27.9 μm pixel$^{-1}$ and the image acquisition rate of 2.9 kHz. Since 0.1 pixel is less than the assumed error of 0.2 pixel for $v$, the visible low-amplitude noise in figure 7.6(b) is within the assumed margin of uncertainty discussed in §7.3 and is captured by the error bars.

### 7.6 Mean velocity profile

The mean velocity profiles normalized using outer scaling are compared for drag-reduced flows at HDR and MDR in figures 7.7(a) and 7.7(b), respectively. Error bars are excluded from this figure, as the estimated 3D-PTV uncertainty is equivalent to the line thickness used here. In these figures, the mean velocity profile
Figure 7.6: Reynolds stresses from 3D-PTV of water compared with DNS. (a) $\langle u^2 \rangle^+$, where each data set is shifted upward along the vertical axis by 5, (b) $\langle v^2 \rangle^+$ where each data set is shifted by 1, (c) $\langle w^2 \rangle^+$ where each data set is shifted by 1 and lastly (d) $\langle uv \rangle^+$ where each data set is shifted by -1. The legends are similar to figure 7.5. The 3D-PTV results with $Re_\tau = [307, 425, 511]$ are compared with DNS from Iwamoto et al. (2002) with $Re_\tau = 300$; MMoser et al. (1999) with $Re_\tau = 395$; and Lee & Moser (2015) with $Re_\tau = 550$.

Figure 7.6: Reynolds stresses from 3D-PTV of water compared with DNS. (a) $\langle u^2 \rangle^+$, where each data set is shifted upward along the vertical axis by 5, (b) $\langle v^2 \rangle^+$ where each data set is shifted by 1, (c) $\langle w^2 \rangle^+$ where each data set is shifted by 1 and lastly (d) $\langle uv \rangle^+$ where each data set is shifted by -1. The legends are similar to figure 7.5. The 3D-PTV results with $Re_\tau = [307, 425, 511]$ are compared with DNS from Iwamoto et al. (2002) with $Re_\tau = 300$; MMoser et al. (1999) with $Re_\tau = 395$; and Lee & Moser (2015) with $Re_\tau = 550$.

for water at the same $U_b$ as the drag-reduced flows is also presented. For water, this flow rate results in $Re_\tau$ of 793, which is larger than $Re_\tau$ of the drag-reduced flows. The magnitudes of mean velocity in the near-wall region for the drag-reduced solutions is smaller than mean velocity of water. Although not fully captured within the wall-normal extent of the 3D-PTV domain, farther away from the wall, mean velocity of the drag-reduced flows is expected to become larger than that of water to maintain a similar $U_b$.

Based on the shape of velocity profiles in figure 7.7(a), it can also be seen that the wall-normal gradient of mean velocity at the wall, $\partial \langle U \rangle / \partial y|_w$, for all three drag-reduced cases is smaller than $\partial \langle U \rangle / \partial y|_w$ of water. The profiles of C14 and PAM at HDR appear to approximately overlap in figure 7.7(a). The XG solution, on the other hand, starts with a lower $\partial \langle U \rangle / \partial y|_w$, and its $\langle U \rangle / U_b$ profile is smaller up until $y/h$ of 0.42. The greater $\mu_w$ of XG compensates for its smaller $\partial \langle U \rangle / \partial y|_w$, resulting in a similar wall shear stress as PAM and C14. Within the region of $y/h < 0.4$ shown in figure 7.7(b), mean velocity for the two MDR cases of PAM and C14 are significantly lower than water. The profiles also demonstrate that $\partial \langle U \rangle / \partial y|_w$ of PAM and C14 are smaller than $\partial \langle U \rangle / \partial y|_w$ of water. PAM at MDR has a marginally lower velocity for $y/h < 0.5$ when compared to C14.

Figure 7.7 confirms that a similar $DR$ does not ensure overlap of the mean velocity profile for different drag-reducing additives when the profiles are normalized using outer scaling. This was observed clearly for the XG solution in figure 7.7(a). The results also show that the difference in the mean velocity profiles of
different drag-reducing additives at a similar $DR$ is not associated with the difference in their $Re_H$. In both figures 7.7(a) and 7.7(b), the mean velocity profiles of PAM and C14 solutions are similar while their $Re_H$ is different (see table 7.2). The properties of the solutions suggest that their shear viscosity plays an important role in setting the outer-normalized mean velocity profiles. At a similar $DR$, drag-reduced solution with larger $\mu_w$ have a lower $\frac{\partial \langle U \rangle}{\partial y}|_w$ and $\frac{\langle U \rangle}{U_b}$ in the near-wall region. While solutions with a similar $\mu_w$ result in a similar $\frac{\partial \langle U \rangle}{\partial y}|_w$ and $\frac{\langle U \rangle}{U_b}$ in the near-wall region.

The inner-normalized mean velocity profile, $\langle U \rangle^+$, in the immediate wall vicinity at $y^+ < 15$ is demonstrated for all additives and for water in figure 7.8. The inner scales of the turbulent flows are estimated here by calculating $\frac{\partial \langle U \rangle}{\partial y}|_w$ using a linear fit of the data at $2 - 4 < y^+ < 5$. The lower wall-normal limit corresponds to the first valid data point from the 3D-PTV system which is determined to be at $y \approx 60 \mu m$. For consistency, the upper bound is chosen to be the maximum limit of the linear viscous sublayer for a Newtonian flow. Figure 7.8 shows the linear fit used to calculate $\frac{\partial \langle U \rangle}{\partial y}|_w$, and confirms the presence of a linear region for all the flows. The estimated $\frac{\partial \langle U \rangle}{\partial y}|_w$ values are presented in table 7.3 and are used to calculate the corresponding $\mu_w$ based on the shear viscosity models described in §7.3. This results in $\mu_w$ and the other inner-scaling variables for the drag-reduced flows that are presented in table 7.3. The comparison of the estimated $DR_2$ (based on $\frac{\partial \langle U \rangle}{\partial y}|_w$) in table 7.3 with the $DR_1$ (based on $\Delta P$) in table 7.2 shows a reasonable agreement of the two methods. The difference between $DR_1$ and $DR_2$ is small and varies between 1.6% to 4.8%. The discrepancy is associated with several factors including the finite aspect ratio of the channel, deviation from the fully developed turbulence at the upstream pressure port, and the uncertainty in determining $\frac{\partial \langle U \rangle}{\partial y}|_w$.

The relatively good agreement amongst the wall statistics and $DR$ using measurements of $\Delta P$ and 3D-PTV for XG, suggests the extrapolation of the CY model from §7.3 can reasonably estimate $\mu_w$. A further
means of communicating the agreement of these measurements is by determining \( \mu_w \) using \( \frac{\partial \langle U \rangle}{\partial y} \big|_w \) and \( \tau_{w,1} \). Here, \( \frac{\partial \langle U \rangle}{\partial y} \big|_w \) is obtained from 3D-PTV measurements, and \( \tau_{w,1} \) is derived from measurements of \( \Delta P \). Such a validation has been done in experiments by [Warholic et al. (1999)] and [Ptasinski et al. (2001)]. If the same analysis is performed, the viscosity of the XG solution at a shear rate of 2364 s\(^{-1}\) (\( \frac{\partial \langle U \rangle}{\partial y} \big|_w \) from table 7.3) is 1.44 mPas (using \( \tau_{w,1} \) in table 7.1). This viscosity is approximately 0.14 mPas lower than the \( \mu_w \) listed in table 7.3, which is roughly 8%. The majority of this uncertainty is reflected in the error bars that propagate from a random error in repeated viscosity measurements and are shown in figures of mean velocity profile and Reynolds stresses to follow.

As alluded to earlier in §7.3, the method of multiplying \( \frac{\partial \langle U \rangle}{\partial y} \big|_w \) and \( \mu_w \) to establish \( \tau_{w,2} \) for the non-Newtonian fluids is an approximation. Fluctuations in \( \frac{\partial \langle U \rangle}{\partial y} \big|_w \) with respect to time can be significant and the instantaneous distribution of \( \mu_w \) may not be simply determined by the mean shear rate. This is most significant for the XG solution, whose shear viscosity is described by the CY model. [Gubian et al. (2019)] demonstrated that \( \tau_w \) can fluctuate by as much as 35% of the nominal value of \( \tau_w \) for a Newtonian turbulent channel flow with a \( Re_\tau \) of approximately 300. Assuming such a variance in \( \tau_w \) is applicable to XG, an uncertainty in \( \mu_w \) of approximately 0.06 mPas is expected. Such a fluctuation in \( \mu_w \) is captured by the error bars in the mean flow statistics demonstrated in the figures to follow.

In addition to demonstrating the fit of the linear viscous sublayer, figure 7.8 presents some insight into the
Table 7.3: The estimated inner scaling based on the wall-normal gradient of mean velocity at the wall for the drag-reduced flows.

<table>
<thead>
<tr>
<th>Solution</th>
<th>$\frac{\partial \langle U \rangle}{\partial y}$ (s$^{-1}$)</th>
<th>$\mu_w$ (mPa s)</th>
<th>$u_\tau$ (mm s$^{-1}$)</th>
<th>$\delta_v$ (μm)</th>
<th>$Re_\tau$</th>
<th>$DR_2$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PAM, HDR</td>
<td>3458</td>
<td>1.074</td>
<td>61.10</td>
<td>17.67</td>
<td>424</td>
<td>55.3</td>
</tr>
<tr>
<td>PAM, MDR</td>
<td>2042</td>
<td>1.088</td>
<td>47.24</td>
<td>23.14</td>
<td>324</td>
<td>73.3</td>
</tr>
<tr>
<td>XG, HDR</td>
<td>2364</td>
<td>1.576</td>
<td>61.89</td>
<td>26.16</td>
<td>287</td>
<td>55.2</td>
</tr>
<tr>
<td>C14, HDR</td>
<td>4113</td>
<td>0.911</td>
<td>61.38</td>
<td>14.92</td>
<td>503</td>
<td>54.9</td>
</tr>
<tr>
<td>C14, MDR</td>
<td>2145</td>
<td>0.912</td>
<td>44.33</td>
<td>20.66</td>
<td>363</td>
<td>76.5</td>
</tr>
</tbody>
</table>

thickness of the viscous sublayer for drag-reduced flows. The elastic sublayer model of Virk (1971) proposed that all drag-reduced flows have a viscous sublayer thickness of $y^+ = 11.6$ (corresponding to the tri-section point of the MDR asymptote, $y^+ = \langle U \rangle^+$, and the log law). However, figure 7.8 demonstrates that none of the drag-reduced flows have a viscous sublayer thickness of $y^+ = 11.6$ (represented by the maximum extent of the black line). Nonetheless, there is still a considerable thickening of the linear viscous subregion relative to water for the drag-reduced flows. At $y^+ = 11.6$, HDR flows of XG, C14 and PAM solutions deviate from the linear fit by $1.98u_\tau$, $1.44u_\tau$ and $1.12u_\tau$, respectively. The largest deviation corresponds to the XG solution, which has the largest shear viscosity. Water has a deviation from the linear fit at $y^+ = 11.6$ of $1.97u_\tau$, which is equivalent to the deviation of XG. For MDR flows of C14 and PAM, the relative deviation from the linear profile at $y^+ = 11.6$ is smaller and equal to $0.6u_\tau$ and $1.0u_\tau$, respectively.

The results in figure 7.8 show that the thickness of the viscous sublayer is smaller for drag-reduced flows at HDR than MDR, suggesting that viscous sublayer thickens with increasing $DR$. It can also be seen that the thickness of the viscous sublayer depends on the additive type, i.e., the thickness varies for different solutions at a similar $DR$. The results also suggest that in general the thickness of the viscous sublayer in inner scaling reduces with increasing shear viscosity. The XG solution has the highest shear viscosity and has an almost identical viscous sublayer thickness as water, while other HDR flows with lower shear viscosity have a thicker viscous sublayer.

The velocity profiles normalized by inner scaling and presented in a log–linear format are shown in figure 7.9. The inner-normalized mean velocity profiles are compared with both the Newtonian law of the wall and the ultimate profile for drag-reduced flows at MDR, or equation (3.16) (Virk et al., 1970). The results for flows at HDR in figure 7.9(a) are discussed first, followed by the results for MDR in figure 7.9(b).

The mean velocity profiles of the HDR flows in figure 7.9(a) are close to each other in the near-wall region. It can also observed that with increasing $y^+$, the HDR profiles of the three drag-reduced cases start to diverge and appear to have different slopes. Subject to the Virk (1971) elastic sublayer model for polymer flows at an intermediate $DR$, the $\langle U \rangle^+$ profile in the elastic sublayer (or buffer layer) is supposed to overlap with the ultimate profile, and for larger $y^+$ a Newtonian plug layer with a logarithmic profile with a similar slope as the Newtonian log layer should propagate. As shown in figure 7.9(a), none of the HDR profiles...
overlap with the ultimate asymptote. Therefore, $DR$ does not uniquely define the inner-normalized mean velocity profile since the type of additive plays a role in shaping the profile. In comparing the mean velocity profiles of different experiments, White et al. (2012) similarly observed variability in the outer layer of the mean velocity profile for polymer solutions with the same $DR$; albeit for cases of low $DR$, smaller than 40%. Due to the differences amongst the data sets, White et al. (2012) postulated that the velocity distribution in the outer layer depends on the Reynolds number, properties of the additive, and the canonical flow type. It is important to note that the results in figure 7.9(a) do not exclude the effect of Reynolds number. In other words, the variations can be partly attributed to differences in the Reynolds number of the drag-reduced flows.

The mean velocity profile of the two drag-reduced flows at MDR are shown in figure 7.9(b). The profile of C14 has a higher $\langle U \rangle^+$ than PAM outside the viscous sublayer, which is consistent with its slightly higher $DR_2$; 76.5% for C14 versus 73.3% for PAM solution. The C14 profile is also marginally greater than the MDR asymptote for $y^+ > 60$. Both previous experimental and numerical simulations have observed a small overshoot of the MDR asymptote for velocity profiles of polymer solutions (Escudier et al. 2009; White et al. 2012; Graham, 2014). Both profiles do not adhere to the MDR asymptote of (Virk et al. 1970) and intersect with it at different $y^+$. In addition, the profile of C14 does not agree with the asymptote for drag-reducing surfactant solutions proposed by Zakin et al. (1996); $\langle U \rangle^+ = 23.4 \ln y^+ - 65$. This asymptote is not shown in figure 7.9 for brevity. Considering the error bars and the slight difference in $DR$ of C14 and PAM, the MDR asymptote seems to be unique and independent of the additive type and the Reynolds number. However, the drag-reduced flows of PAM and C14 at MDR do not follow the logarithmic trend proposed by
they share a similar S-shaped profile that straddles or at least intersects the asymptote of Virk et al. (1970). To further evaluate the logarithmic behaviour, the indicator function, \( \zeta = y^+ \partial \langle U \rangle^+ / \partial y^+ \), is investigated next in figure 7.10. Using the indicator function to evaluate logarithmic dependency, White et al. (2012) found that the inner-normalized mean velocity of polymer drag-reduced flows at MDR were not truly logarithmic functions of \( y^+ \).

To establish \( \partial \langle U \rangle^+ / \partial y^+ \), and calculate \( \zeta \), a moving second-order polynomial filter, of length 10 – 15\( \delta_v \) (250 \( \mu \)m), was applied to the distribution of \( \langle U \rangle^+ \) as a function of \( y^+ \). The polynomials were then differentiated analytically. Figures 7.10(a) and 7.10(b) demonstrates \( \zeta \) as a function of \( y^+ \) for HDR and MDR flows, respectively. A region of \( y^+ \) where \( \zeta \) is constant is indicative of a layer where \( \langle U \rangle^+ \) varies logarithmically as a function of \( y^+ \). For example, the distribution of \( \zeta \) for water, shown in both figures 7.10(a) and 7.10(b), is approximately constant and equal to 2.5 for \( y^+ > 30 \), which is indicative of a logarithmic layer for the Newtonian turbulent channel flows. White et al. (2012), Elbing et al. (2013) and White et al. (2018) proposed that for a polymer drag-reduced flow, the shape of the mean velocity profile, and similarly \( \zeta \), depends on the Reynolds number, polymeric properties and the canonical flow type. Figure 7.10(a,b) addresses the second postulate by comparing flows comprised of different additives at HDR and MDR.

Figure 7.10(a) shows that the HDR flows of C14 and PAM have similar distributions of \( \zeta \). White et al. (2012) stated that HDR flows are distinct in their lack of a Newtonian plug. By observation of figure 7.10(a) none of the HDR flows have a \( y^+ \) range where \( \zeta \) appears constant and a Newtonian plug does not exist within the measurement domain. However, this does not rule out the possibility of a Newtonian plug existing at larger \( y^+ \). The profile of \( \zeta \) for XG show relative similarity with the other HDR flows for \( y^+ < 30 \); however,
the peak in its profile, though subject to experimental noise, appears to be marginally higher and located at larger \( y^+ \). The larger \( y^+ \) location of \( \zeta \) peak for XG solution indicates that the centre of the elastic sublayer (buffer layer) is farther away for the wall. Therefore, the indicator function also provides further evidence that the shape of the velocity profile and the thickness of the sublayers is not uniquely defined by \( DR \). Here, the thicker elastic sublayer of the XG solution is associated with its larger shear viscosity and lower Reynolds number. The \( y^+ \) location of the peak in the distribution of \( \zeta \), shows that the elastic sublayer is thinner for drag-reduced solution with higher Reynolds number (\( Re_H \) or \( Re_T \)).

Figure 7.10(b) compares the plots of \( \zeta \) for C14 and PAM at MDR. The two profiles appear similar for all \( y^+ \). The \( y^+ \) location and value in the peak of \( \zeta \) is approximately \((y^+, \zeta) = (70, 14)\) for both drag-reduced flows. The peak is larger and farther away from the wall relative to the HDR cases, indicating a thicker elastic sublayer. Due to the lack of a region with constant \( \zeta \), White et al. (2012) concluded that the exact shape of the MDR profile was not logarithmic. Instead, MDR was achieved when the peak in \( \zeta \) equals 11.7, corresponding to the slope in the MDR asymptote proposed by Virk et al. (1970). Figure 7.10(b) demonstrates that the peak exceeds this limit for both PAM and C14 solutions. In plotting \( \zeta \) for experimental data from Escudier et al. (2009) collected for a rigid polymer solution at MDR with \( DR \) of 67\%, White et al. (2012) demonstrated a similar overshoot of \( \zeta = 11.7 \). Elbing et al. (2013) also shows a peak in \( \zeta \) greater than 11.7 for a flexible polymer solution with \( DR = 65\% \). Therefore, further doubt is cast on the exactness of the slope of the MDR profile of Virk et al. (1970). Figure 7.10(b) also appends the conclusion of White et al. (2012) to state that surfactant drag-reduced flows at MDR, in addition to polymer flows, also do not possess a logarithmic layer. Furthermore, while the shape of the two mean velocity profiles at MDR are not exactly logarithmic, they are similar. This implies that a universal distribution of \( \langle U \rangle^+ \) and \( \zeta \), for drag-reduced flows at MDR, that is irrespective of the additive type and the Reynolds number, may exist.

### 7.7 Reynolds Stresses

The Reynolds stresses profiles for the HDR cases are compared in figure 7.11. In addition to the drag-reduced flows, the Reynolds stress profiles for water at four \( Re_T \) that are similar to \( Re_T \) of the drag-reduced cases are presented. For example, the Reynolds stress profiles of C14, PAM and XG, with \( Re_T \) of 503, 424 and 287, are shown alongside those for water with a \( Re_T \) of 511, 425 and 307. As expected, all of the Reynolds stress profiles of water show similar distributions, relative to one another, within the linear sublayer and buffer layer. Larger differences in the outer layer amplify with increasing \( y^+ \), as expected.

Figure 7.11(a) shows that all HDR flows possess a large peak value of \( \langle u^2 \rangle^+ \) that is also shifted away from the wall, relative to water at a similar \( Re_T \). The \( \langle u^2 \rangle^+ \) profiles of C14 and PAM appear similar for \( y^+ < 70 \) although the \( \langle u^2 \rangle^+ \) peak is smaller for PAM. The two profiles deviate with further increase of \( y^+ \). Compared to C14 and PAM, XG has a smaller peak value of \( \langle u^2 \rangle^+ \), which is displaced farther from the wall. Therefore, \( \langle u^2 \rangle^+ \) peak is smaller and farther away from the wall for solutions with higher shear viscosity. In addition, the notion that drag-reduced flows of different additives at the same \( DR \) have a similar \( \langle u^2 \rangle^+ \) peak appears to be invalid. The shift in the peak of \( \langle u^2 \rangle^+ \) away from the wall is an indication of a thicker buffer layer that is consistent with previous observations.

Figure 7.11(b,c) demonstrates significant attenuation in the profile of \( \langle v^2 \rangle^+ \) and \( \langle w^2 \rangle^+ \) of the drag-reduced
flows relative to water. For $\langle v^2 \rangle^+$, this agrees with the observations of Escudier et al. (2009) for polymers and also Warholic et al. (1999a) for surfactants. Attenuation in the profile of $\langle w^2 \rangle^+$ has been shown by White et al. (2004) for polymers. To the authors’ knowledge, $\langle w^2 \rangle^+$ has never been demonstrated for surfactant drag-reduced flows. Similar to their $\langle u^2 \rangle^+$ profiles, C14 and PAM display rather similar profiles for $\langle v^2 \rangle^+$ and $\langle w^2 \rangle^+$ with subtle discrepancies. The $\langle v^2 \rangle^+$ and $\langle w^2 \rangle^+$ profiles for XG, on the other hand, are noticeably more attenuated than the other HDR flows. The peak value in the $\langle v^2 \rangle^+$ and $\langle w^2 \rangle^+$ distributions of XG are approximately 50% those of C14. Figure 7.11(d) demonstrates similar profiles in $\langle uv \rangle^+$ for C14 and PAM, but again a more attenuated distribution for XG. The larger attenuation in $\langle uv \rangle^+$ is likely attributed to a larger imposition of viscous stresses due to the larger overall shear viscosity of the XG solution. Therefore, different drag-reduced solutions at an identical $DR$ do not exhibit identical distribution of Reynolds shear stresses, in particular when their shear viscosity is different. A lack of consistency in the shear viscosity of the drag-reduced solutions is also reflected by differences in the Reynolds number number of the solutions with similar $DR$ (i.e. similar $u_\tau$). Therefore, the discrepancy in the Reynolds stress distributions of the HDR flows can be similarly explained by differences in the Reynolds number of the drag-reduced solutions.

Figure 7.12 demonstrates the Reynolds stresses of C14 and PAM at MDR. Having observed that the Reynolds stresses of XG were much lower than the other HDR flows in figure 7.11, it was perceived to be prudent to include XG at HDR in the comparison with the MDR flows in figure 7.12. This was based on prior knowledge that the Reynolds stresses are more attenuated for flows with larger $DR$ (Warholic et al. 1999b; Ptasinski et al. 2001; Escudier et al. 2009). Similar to figure 7.11 figure 7.12 presents the Reynolds stresses
of the drag-reduced flows alongside the distributions of water that share a similar $Re_\tau$. C14 and PAM at MDR, alongside XG at HDR, with $Re_\tau$ of 363, 324 and 307, are presented together with the distributions of water with $Re_\tau$ of 363 and 307.

![Figure 7.12: Inner-normalized mean Reynolds stress profiles of drag-reduced flows at MDR and XG at HDR; (a) streamwise Reynolds stress, (b) wall-normal Reynolds stress, (c) spanwise Reynolds stress profiles and (d) Reynolds shear stress.](image)

In figure 7.12(a), there is relatively good overlap in the distributions of $\langle u^2 \rangle^+$ for the three solutions. Here the similarity in the XG profile with the other two profiles is striking, despite 18–21% difference in $DR_2$ of XG at HDR and the other two MDR flows. For polymer flows, Escudier et al. (2009) demonstrated that for $DR > 40\%$, $\langle u^2 \rangle^+$ decreases as a function of $DR$; albeit, results appeared mixed for other authors (Warholic et al., 1999b). In the current investigation, the $\langle u^2 \rangle^+$ peak of C14 and PAM at MDR decreased relative to their corresponding HDR cases. However, the peaks did not decrease to a point where they are lower than the peak measured for water. While Li et al. (2005) and Warholic et al. (1999a) demonstrate a lower peak in $\langle u^2 \rangle^+$ for surfactant drag-reduced flows with large $DR$ they have similarly shown that the peak in $\langle u^2 \rangle^+$ largely depends on the $Re_\tau$ of the flow. Warholic et al. (1999a) demonstrated this in their sweep of Reynolds number for different HDR flows, where the peak in $\langle u^2 \rangle^+$ was larger than water for surfactant drag-reduced flows with a large Reynolds number, but smaller than water for low Reynolds number. Thais et al. (2012) showed the peak in $\langle u^2 \rangle^+$ had a similar dependence on the Reynolds number based on DNS using the FENE-P model. Figure 7.12(b,c) demonstrates that the distributions of $\langle v^2 \rangle^+$ and $\langle w^2 \rangle^+$ for C14 and PAM at MDR, and XG at HDR, have nearly identical profiles that are also significantly suppressed relative to water. Li et al. (2005) and Warholic et al. (1999a) also observed significant attenuation in profiles of $\langle v^2 \rangle^+$ for surfactant drag-reduced flows near MDR. The overlap in $\langle u^2 \rangle^+$, $\langle v^2 \rangle^+$ and $\langle w^2 \rangle^+$ implies that the
mean turbulent kinetic energy is the same for the three drag-reduced flows.

Lastly, figure 7.12(d) demonstrates that \( \langle uv \rangle^+ \) profiles of C14 and XG are slightly larger than the \( \langle uv \rangle^+ \) profile of PAM at \( y^+ < 100 \). However, for all three flows, the \( \langle uv \rangle^+ \) magnitudes are small and have the same order of magnitude as the error bars. Therefore, the values should be considered negligible and differences are Tamano et al. (2018) presented a finite \( \langle uv \rangle^+ \) distribution, while Warholic et al. (1999a) demonstrated a \( \langle uv \rangle^+ \) profile approximately equal to zero for flows of surfactant drag-reducing additives at MDR. The discrepancies in the small residual values of \( \langle uv \rangle^+ \) is potentially associated with measurement uncertainties as they are also present in the current measurements.

Considering PAM and C14 at MDR, the measurements presented in figure 7.12 show that Reynolds stress profiles of drag-reduced flows at MDR overlap. A perfect overlap can be seen for all components except Reynolds shear stress. For the latter component, there are subtle differences with the same magnitude as the measurement uncertainties. Therefore, it can be concluded, that at MDR the Reynolds stress profiles are not a function of additive type and Reynolds number. At MDR, the Reynolds stress profiles converge to a common set of distributions for polymer and surfactant drag-reduced flows with different Reynolds number.

The \( C_f \) values presented based on \( \Delta P \) in figure 7.2 and mean velocity profiles of figure 7.9(a), suggest that XG is not at MDR. In contrast, the results of figure 7.12 demonstrate that Reynolds stress profiles of XG are similar to those of PAM and C14 at MDR. The measurements of \( DR_1 \) (based on \( \Delta P \)) for XG in figure 7.1(c) also show that a higher level of \( DR_1 \) was not achievable for XG with increasing its concentration; \( DR_1 \) plateaus to a constant 58.5% for \( c \) in excess of 300 ppm. Why XG has a lower asymptotic \( DR_1 \), relative to C14 and PAM at MDR, is likely attributed to the imposition of larger viscous stresses. To summarize, it is evident that the \( DR_1 \) of XG has attained an asymptotic state, according to figure 7.1(c). The Reynolds stresses also demonstrate that XG shares dynamical similarities with other MDR flows (see figure 7.12). Therefore, with respect to the turbulent flow and production of turbulent kinetic energy, XG is at an MDR state. The discrepancies in \( DR \) and mean velocity profile of XG with respect to the MDR state of the other drag-reduced flows is associated with larger inherent viscous stresses of this polymer solution.

### 7.8 Low- and high-speed streaks

The following analysis evaluates the length scale of the dominant flow structures at HDR and MDR using two-point correlation of streamwise velocity fluctuations. The spatial, two-point correlation is computed as

\[
R_{uu}(\Delta z) = \frac{\langle u_{x_0,y_0,z_0} u_{x_0,y_0,z_0+\Delta z} \rangle}{\sqrt{\langle u^2_{x_0,y_0,z_0} \rangle} \sqrt{\langle u^2_{x_0,y_0,z_0+\Delta z} \rangle}} \tag{7.4}
\]

Here, \((x_0, y_0, z_0)\) is the coordinate of the reference point selected at \((0, 0.4h, 0)\), which is positioned within the logarithmic layer for Newtonian flows. The dominant coherent structures at this location are low and high-speed streaks that have also been observed in drag-reduced flows (White et al. 2004; Mohammadtabar et al. 2017). At higher Reynolds numbers and in Newtonian flows, these streaks form the very large-scale motions (Hutchins & Marusic 2007). The incremental displacement along the spanwise direction is indicated as \( \Delta z \), relative to the \( z_0 \) reference point. As a result, \( R_{uu} \) characterizes the spanwise scale of the low and high-speed streaks in the drag-reduced flows.
Figure 7.13(a) presents $R_{uu}$ along $\Delta z/h$ for the HDR flows. The $R_{uu}$ functions for water are shown alongside the drag-reduced flows. The overlap in the $R_{uu}$ profiles indicate that the width of the streaks for the Newtonian cases are similar. The $R_{uu}$ profiles for C14 and PAM at HDR are also approximately similar, indicating a similar streak spacing. This suggests that the $R_{uu}$ distribution for drag-reduced flow may not be a strong function of the Reynolds number as PAM and C14 flows have different $Re_H$. The XG demonstrates a rather larger $R_{uu}$ relative to C14 and PAM, which indicates even wider streaks. Therefore, the turbulent streaks of drag-reduced flows of PAM and C14 with similar shear viscosities appear to be more alike, while XG – a solution with a much larger overall shear viscosity – is distinct.

Figure 7.13: Two-point correlation of streamwise velocity fluctuations in the spanwise direction for drag-reduced flows at (a) HDR and (b) MDR. The reference location for the two-point correlations is at $(x_0, y_0, z_0) = (0, 0.4h, 0)$.

Figure 7.13(b) presents $R_{uu}$ of drag-reduced flows of PAM and C14 at MDR, and XG at HDR. The profiles approximately overlap, and therefore streak spacing is expected to be similar for the three drag-reduced flows. Using a similar two-point correlation analysis, Li et al. (2006), White et al. (2004) and Tamano et al. (2018) demonstrated a monotonic increase in the spanwise width of the low- and high-speed streaks for polymer and surfactant drag-reduced flows with increasing $DR$. Comparing figure 7.13(a), with figure 7.13(b), both C14 and PAM exhibit growth in the average streak spacing with respect to $DR$. The XG profile appears to show more similarities in the width of its streaks with respect to solutions of C14 and PAM at MDR. This reinforces the notion that XG has attained a state of MDR regarding turbulent dynamics.

### 7.9 Summary

The main objective of this investigation was to compare the rheological features and turbulence statistics of three drag-reducing additives, PAM, XG and C14, in a turbulent channel flow. To ensure that the comparison of the additives is subject to similar conditions, the drag reducing solutions were prepared such that they all produced a similar level of drag reduction ($DR$) at a common mass flow rate. This is equivalent to
maintaining a similar wall shear stress and mass flow rate. Two DR values were considered; the first being a high drag reduction (HDR) case with DR of 57.7% ± 1.2%, and the second being a maximum drag reduction (MDR) case with DR of 70.3% ± 1.8%. Based on measurements of the streamwise pressure gradient along the channel, solutions of PAM, XG, and C14 achieved the HDR condition, while only PAM and C14 could attain the larger MDR limit. Although the mass flow rate and DR were constant, the flows had different Reynolds numbers ($Re_H$) due to the difference in their shear viscosity.

Samples of each drag-reduced flow at HDR and MDR were collected for shear viscosity measurements in a torsional rheometer and measurements of their extensional relaxation time using a capillary breakup extensional rheometer (CaBER). Despite having the capability of generating similar levels of DR, none of the different types of additive solutions exhibited overlap in their apparent shear viscosity curves or similarities in their extensional relaxation times. Solutions of C14 exhibited low, and relatively constant shear viscosities that were almost identical to the shear viscosity of water. PAM solutions demonstrated only marginal shear thinning trends. The overall shear viscosity of PAM was approximately 20% larger than the shear viscosity of water. In contrast, the shear viscosity of the XG solution at low strain rates, was an order of magnitude larger than the other solutions, and had a pronounced shear thinning trend. Regarding the extensional relaxation time, CaBER measurements could only be performed for solutions of PAM. Solutions of XG and C14 failed to show considerable uniaxial filament stretching; the samples disintegrated rapidly upon a marginal imposition of strain from the CaBER system. Therefore, only solutions of PAM demonstrated measurable extensibility characteristics using CaBER, with a relaxation time of 4 to 11 ms. Although the current measurements, alongside previous experimental measurements from the literature, have not identified a common rheological trait for different drag reducing additives, the possibility of such a common feature existing cannot be ruled out. However, these results pose the question of how different drag-reducing solutions manipulate the wall turbulence. This question is addressed using detailed measurements of the turbulence statistics.

The turbulent channel flow of the drag-reduced additives and several Newtonian flows were characterized using three-dimensional particle tracking velocimetry. The drag-reduced solutions of PAM, XG, and C14 at the HDR state demonstrated different mean velocity profiles when normalized using outer and inner scaling. The indicator function showed inconsistencies in the inner-normalized mean velocity distributions and were a result of variations in the wall-normal thickness of the constituent sublayers of the three drag-reduced solutions. Drag-reduced solutions with a larger overall shear viscosity, and therefore a smaller $Re_H$, had a thinner linear viscous sublayer and a thicker elastic sublayer. At HDR, the Reynolds stress profiles of the PAM, XG, and C14 solutions did not overlap. In particular, the XG solution, which had the highest shear viscosity, had more attenuated Reynolds stresses. Two-point correlation of streamwise velocity also demonstrated larger spanwise streak spacing for the XG solution relative to the other HDR flows. However, similar to previous observations, the drag-reduced additives resulted in the same qualitative net-effect: that is, relative to a Newtonian turbulent wall flow, the buffer layer of all drag-reduced flows were thicker, the streamwise Reynolds stress profile was significantly larger, and the other Reynolds stress components were much smaller. The observations demonstrated that turbulent flows of different drag reducing additives generated mean velocity and Reynolds stresses profiles that were qualitatively similar, but quantitatively different. The discrepancy in the magnitude of flow statistics appeared to be mainly due to the difference in
the flow $Re_H$.

In contrast to the HDR flows, the outer and inner-normalized mean velocity profiles of PAM and C14 at MDR approximately overlapped. The small deviation between the two profiles was associated with the marginal differences in their $DR$. The indicator function showed that the wall-normal spacing of the sublayers were similar for the two flows at MDR. Plots of the indicator function also demonstrated that a region where mean streamwise velocity varied logarithmically with distance from the wall, does not exist. That being said, the mean velocity profile at MDR was still asymptotic and independent of the type of additive and $Re_H$, despite not being precisely logarithmic in its distribution. The Reynolds stress profiles and two-point correlation of streamwise velocity fluctuations were also independent of additive type and $Re_H$ as they converged to a common profile for PAM and C14 at MDR.

Although XG had a much lower $DR$, its Reynolds stress profile overlapped with the Reynolds stress distributions of PAM and C14 at MDR. The overlap in the Reynolds stresses indicated that the XG solution achieved a maximum level of attenuation in its turbulence, similar to PAM and C14 at MDR. In contrast, the $DR$ and mean velocity profile of the XG solution at HDR was not consistent with those of PAM and C14 at MDR. The discrepancy was associated with the greater shear viscosity and therefore, lower $Re_H$ of the XG solution. The large shear viscosity and lower $Re_H$ of XG appeared to have hindered the solutions ability to produce a larger $DR$, and have its mean velocity profile intersect with the MDR asymptote. This observation refines the previous conclusions. It hints that the dependence of mean velocity profile and Reynolds stresses on the additive type was attributed to differences in the shear viscosity and $Re_H$, and not a rheological feature typically associated with drag reduction, such as the extensibility of the solution.
Chapter 8

Lubricating layer in drag-reduced channel flows of rigid polymers

The present investigation provides high-fidelity turbulence statistics of a drag-reduced channel flow of rigid polymers with varying $Re_H$. Few experiments of rigid polymers have explored the effect of $Re_H$ on flow statistics. The existing measurements of rigid polymers in a turbulent channel flow have low spatial resolutions (Escudier et al., 2009) or appear to be in an arguably transitional flow regime due to small $Re_H$ (Mohammadtabar et al., 2017). To alleviate this gap in the research, an experimental investigation is performed for a 170 ppm xanthan gum (XG) solution in a turbulent channel flow with a friction Reynolds number $Re_\tau$ between 170 and 700. The resulting levels of $DR$ are between 27% and 33%, demonstrating little dependence on $Re_H$. Planar particle image velocimetry (PIV) measurements are used to measure the instantaneous velocity of the drag-reduced flows. Shear rheology is characterized using a double gap and a parallel plate geometry to capture the viscosity of the XG solution over a large range of shear rates. A mechanism for rigid polymer drag reduction is asserted from the perspective of lubricated flows. A thin layer of low-viscosity fluid near the wall is observed for the rigid polymer solution at all flow conditions, which is proposed to be essential for $DR$ using rigid polymers.

8.1 Flow conditions

Experiments were performed in the same recirculating flow facility as shown in figure 4.4. For more information regarding the facility, refer to §4.2 or Warwaruk & Ghaemi (2021). The same right-hand orthonormal basis was used in the present experiments, with positions along the streamwise, wall-normal, and spanwise directions denoted by $x$, $y$, and $z$, respectively. The Cartesian coordinate system was placed at the midspan of the lower channel wall, as shown in figure 4.5. A shell and tube heat exchanger and a thermocouple were used to maintain a constant fluid temperature of $25^\circ C \pm 0.3^\circ C$. A differential pressure transducer (DP15, Validyne) with a 1 psi diaphragm was used to measure the streamwise gradient in the static pressure, i.e., $\Delta P/\Delta x$. Here the streamwise separation between the pressure ports was $\Delta x = 109H$, as detailed in §4.2.

Measurements were conducted for seven different conditions of bulk velocity $U_b = \dot{m}/\rho HW$, all of which are shown in Table 8.1 for water. In the case of water, the Reynolds number $Re_H$ was between 9100
and 37,000. Recall that the symbol $\mu_w$ represents the dynamic viscosity of the fluid corresponding to the shear rate at the wall. While this is a variable for the polymer solutions, for a Newtonian fluid such as water, the dynamic viscosity is consistently 0.89 mPa s at 25°C (Nagashima, 1977; Collings & Bajenov, 1983). Therefore, the $Re_H$ of the polymer solutions are calculated later in §8.3 when the wall shear rates and steady shear viscosity are obtained. The wall shear stress $\tau_w$ was established using measurements of the streamwise pressure gradient, i.e., $(3.5)$ – equivalent to $\tau_{w,1}$ from the previous experiments detailed in §7.1 and Warwaruk & Ghaemi (2021). The friction velocity $u_\tau$ from $(3.10)$, viscous lengthscale $(3.11)$, and friction Reynolds number $Re_\tau$ $(3.12)$ were then subsequently determined, the results for which are listed in Table 8.1 for the flow of water.

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<th>$\Delta P$ (Pa)</th>
<th>$\tau_w$ (Pa)</th>
<th>$u_\tau$ (mm s$^{-1}$)</th>
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Table 8.1: Flow properties for channel flow of water.

### 8.2 Rigid polymer solution

The same rigid polymer, XG, used in §§6 and 7 was utilized for the present experimental investigation. Solid XG, in powder form, was weighed using a digital scale (AB104-S, Mettler Toledo) with a 0.1 mg resolution. The powder was then gradually added to 15 l of tap water and agitated using a stand mixer (Model 1750, Arrow Engineering Mixing Products). The concentrated 15 l master solution was then left to rest overnight for approximately 12 h. The following day, the master solution was added to 100 l of moving tap water within the flow loop. This diluted the master solution to the desired concentration of 170 ppm. A 170 ppm solution of XG produced a solution of good transparency for PIV measurements. To ensure the solution was homogeneous, the pump was operated at 1400 rpm ($U_b = 4.380$ m s$^{-1}$) for 1 h. Near the end of the 1 h duration, $\Delta P$ was marginally growing at a rate of approximately 10 Pa min$^{-1}$, about a 0.1% increase in $\Delta P$ every minute. This was considered sufficiently steady state. After the 1 h time mark, the pump speed was reduced to 800 rpm, corresponding to $U_b = 2.197$ m s$^{-1}$, for the first PIV measurement at the highest $Re_H$. The pump speed was then reduced in increments such that PIV measurements for each flow condition listed in Table 8.2 were taken. At all of the measured flow rates listed in Table 8.2, no variation in $\Delta P$ was observed during the PIV acquisition time. Therefore, any mechanical degradation or polymer
deagglomeration was likely negligible after the 1 h mixing phase. Finally, fluid samples were collected for shear viscosity measurements using an access port along the flow loop.

\[
\begin{array}{cccccccc}
U_b (\text{m s}^{-1}) & Re_H & DR (%) & \mu_w (\text{mPa s}) & u_\tau (\text{mm s}^{-1}) & \delta_\eta (\mu\text{m}) & Re_\tau \\
0.542 & 6200 & 27 & 1.285 & 29.2 & 44.1 & 170 \\
0.819 & 10500 & 30 & 1.152 & 40.6 & 28.5 & 260 \\
1.094 & 14800 & 31 & 1.087 & 51.4 & 21.2 & 350 \\
1.371 & 19300 & 32 & 1.047 & 61.8 & 17.0 & 440 \\
1.647 & 23800 & 33 & 1.021 & 72.1 & 14.2 & 530 \\
1.924 & 28300 & 33 & 1.002 & 82.2 & 12.2 & 610 \\
2.197 & 32800 & 33 & 0.988 & 92.3 & 10.7 & 700
\end{array}
\]

Table 8.2: Flow properties for channel flow of 170 ppm XG solution.

### 8.3 Steady shear viscosity

Shear viscosity $\mu$ versus shear rate $\dot{\gamma}$ was measured for water and the 170 ppm XG solution using the torsional rheometer depicted in §4.1.1 and figure 4.1. Two geometries were used, the double-gap (DG) concentric cylinder shown in figure 4.2(b) for low to moderate $\dot{\gamma}$ and a parallel plate (PP) geometry shown in figure 4.2(c) for moderate to high $\dot{\gamma}$.

Figure 8.1(a) displays measurements of $\mu$ as a function of $\dot{\gamma}$ for the 170 ppm XG solution and water at 25 °C. The $\mu$ for water was measured between $\dot{\gamma}$ of 2 and 140 s$^{-1}$ using the DG geometry and $\dot{\gamma}$ between 60 and 2000 s$^{-1}$ using the PP geometry. For the rigid polymer solution, measurements of $\mu$ using the DG geometry are presented for 0.8 s$^{-1} < \dot{\gamma} < 180$ s$^{-1}$. Results using the PP geometry were performed between $\dot{\gamma}$ of 10 and 2500 s$^{-1}$ for the XG solution. The lower limit of $\dot{\gamma}$ for the viscosity measurements is a result of the low-torque limit of the rheometer (Ewoldt et al., 2015). The upper limit is a result of secondary or inertial flow instabilities that produce an increase in the measured torque and hence tamper with the measurements of $\mu$. In the DG geometry, the secondary instabilities are Taylor vortices, while for the PP geometry secondary instabilities are radial flows or turbulence (Ewoldt et al., 2015). Davies & Stokes (2008) demonstrated that secondary flows tampered with the PP measurements when the Reynolds number $Re_{pp} = \rho \Omega R_{pp} h_{pp}/\mu$ was greater than 100. Here $\Omega$ is the angular velocity of the upper plate in radians per second. Therefore, measurements of $\mu$ using the PP geometry with $Re_{pp} > 100$ were disregarded. Measurements using the PP geometry at low $h_{pp}$ can also be subjected to errors caused by gap offsets and surface tension (Davies & Stokes, 2008; Johnston & Ewoldt, 2013; Ewoldt et al., 2015). Appendix A.2 critically evaluates the consistency in measurements of $\mu$ for different gap heights and with alterations in the surface tension of the fluid by adding a small amount of TWEEN 20 to the XG solution. The measurements of $\mu$ were consistent for different $h_{pp}$, the $Re_{pp} = 100$ conservatively predicted the onset of inertial instabilities for different $h_{pp}$, and TWEEN 20 had little influence on the measurements of $\mu$. Based on the results presented in Appendix
it can be concluded that gap offset errors were minimal, the assumed inertial limitation from Davies & Stokes [2008] was valid, and surface tension did not corrupt the measurements of $\mu$.

Figure 8.1: (a) Steady shear viscosity measurements of 170 ppm XG solution and water. (b) Skin friction coefficient as a function of Reynolds number for water and the 170 ppm XG solution.

The average and standard deviations in measurements of $\mu$ for water were 0.86 mPa s ± 3.2%. The average value of $\mu$ for water was approximately 3.5% different from the theoretical viscosity of water at 25 °C, i.e., 0.89 mPa s (Nagashima, 1977; Collings & Bajenov, 1983). Therefore, a 3.5% relative systematic uncertainty was assumed for all measurements of $\mu$, including measurements of the XG solution. This uncertainty propagates to other variables, including those used for inner normalization of flow velocity. The trend in $\mu$ as a function of $\dot{\gamma}$ for the XG solution, shown in figure 8.1(a), was well approximated by the Carreau-Yasuda (CY) model, represented by equation (7.2) (Carreau, 1972; Yasuda et al., 1981). A Levenberg-Marquardt nonlinear least-squares method was used to fit equation (7.2) to the measurements of $\mu$ as a function of $\dot{\gamma}$ in MATLAB. The resulting CY fit for the XG solution had $\mu_0$ of 5.4 mPa s, $\mu_\infty$ of 0.89 mPa s, $M$ of 0.11 s, $k$ of 0.55 s, and $a$ of 0.67. Equation (7.2) with these values is shown for reference in figure 8.1(a) by the black solid line. The root mean square (rms) in the absolute deviation between the measurements and the CY model was 0.05 mPa s. The rms of the relative deviation was 2.1%. This was considered a relative random uncertainty in the measurements of $\mu$ for XG. Together with the 3.5% systematic uncertainty assumed from our viscosity measurements of water, the total relative uncertainty in our measurements of $\mu$ for XG was conservatively assumed to be 5.6%.

8.4 Skin friction coefficient and drag reduction

Plots of the skin friction coefficient $C_f$ as a function of $Re_H$ are shown for water and XG in figure 8.1(b). To determine $C_f$ for the XG flows, the wall shear stress had to first be established. The $\tau_w$ of each rigid polymer
flow condition was derived based on measurements of $\Delta P$, i.e., (3.5). The near-wall shear rate $\dot{\gamma}_w$ was determined invoking the CY model coupled with pressure drop measurements by substituting $\mu_w = \tau_w/\dot{\gamma}_w$ into the left-hand side of equation (7.2) and using $\tau_w$ from (3.5), after which the values of $\mu_w = \tau_w/\dot{\gamma}_w$ of each XG flow were determined. Subsequently, the variables $Re_H$, $u_\tau$, $\delta_v$, and $Re_\tau$ were obtained, all of which are listed in table 8.2 for the rigid polymer flows. The resulting values of $Re_H$ were then used in plots of $C_f$ shown in figure 8.1. Error bars in the data points of $C_f$ as a function of $Re_H$ propagate from random errors in measurements of $U_b$ and $\Delta P$, as well as the assumed uncertainty in $\mu_w$ determined in the preceding section.

Measurements of $C_f$ for water and XG show consistency with previous investigations. Equation (3.9), shown at the top of figure 8.1, is the empirical correlation relating $C_f$ and $Re_H$ for two-dimensional (2D) Newtonian turbulent channel flows prescribed by Dean (1978). The current measurements of $C_f$ for water agree well with the equation derived by Dean (1978) and are within 5% of the $C_f$ power-law equation. The lower equation shown in figure 8.1 is the Virk et al. (1970) MDR asymptote (3.15). The measurements of $C_f$ for the XG flows are between the $C_f$ correlations of Dean (1978) and Virk et al. (1970). Therefore, the XG flows do exhibit DR; however, none of the drag-reduced flows are at MDR. The $C_f$ measurements for XG also reasonably agree with the expected trend for flows of type B drag-reducing additives with increasing $Re_H$. Virk & Wagger (1990) detailed that type B additives exhibit a ladder effect, where the trend in $C_f$ as a function of $Re_H$ would be lower but parallel to the Newtonian $C_f$ correlation equation. In figure 8.1 a trend in $C_f$ for XG that is approximately parallel to the Dean (1978) correlation with increasing $Re_H$ can be observed. Drag-reduction was quantified by the attenuation in $\tau_w$ of the polymer solution relative to a turbulent Newtonian flow of a similar $Re_H$. The level of attenuation in $C_f$ was described by the percent drag reduction $DR$, represented by equation (3.14). Values of $DR$ were determined for each flow condition of XG, the values for which are listed in table 8.2.

### 8.5 Planar particle image velocimetry

Planar PIV was used to characterize the velocity of the Newtonian and non-Newtonian channel flows. Images were collected using a digital camera (Imager Intense, LaVision GmbH) with a 1376×1040 pixel$^2$ charged-coupled device sensor. Each pixel was $6.45 \times 6.45 \mu m^2$ in size with a digital resolution of 12 bits. A reduced sensor size of 1376×605 pixel$^2$ was used to enable a higher image acquisition rate and therefore a faster convergence in velocity statistics. A Sigma lens with a focal length of 105 mm and an aperture size of $f/8$ was used to focus on the full height of the channel at its midspan. The resulting magnification was 0.55, the depth of field was 1.30 mm, and the scaling factor was 11.81 $\mu m$ pixel$^{-1}$. Figure 8.2 illustrates the flow measurement setup relative to the test section. The camera was arranged in a portrait orientation such that the 1376 pixel dimension of the sensor was parallel to the height of the channel. Therefore, the field of view (FOV) of the images was $(\Delta x, \Delta y) = 12.28 \times 16.25 \ mm^2$. Along the $x$-direction, the center of the FOV was placed at the center of the glass test section, which is 107$H$ downstream of the channel inlet. The illumination source for the planar PIV measurements was a 90 mJ pulse$^{-1}$ Nd:YAG laser (Gemini PIV 30, New Wave Research Inc.). Two spherical lenses (one concave, the other convex) and one concave cylindrical lens expanded the 4.5 mm diam beam output from the laser head into a 20 mm wide (along the
x-direction) by 1-mm-thick (along the z-direction) laser sheet at the measurement location. Silver-coated hollow glass spheres, 2 µm in diameter, were used to seed the flows (SG02S40 Potters Industries). Das & Ghaemi (2021) demonstrated that these small silver-coated particles have strong side scattering and relatively consistent sizing. Synchronization between the camera and the laser was achieved using a programmable timing unit (PTU 9, LaVision GmbH) and DaVis 7.3 software (LaVision GmbH). One data set consisted of 9000 pairs of double-frame images, recorded at an acquisition rate of 7.4 Hz. The time delay \( \Delta t \) between image frames was 50–400 µs depending on the \( Re_H \) of the flow. The specific value of \( \Delta t \) was chosen such that the maximum particle displacement between image frames was approximately 12 pixels.

![Diagram of PIV setup](image)

Figure 8.2: Isometric three-dimensional model of the planar PIV setup relative to the glass test section and channel section.

All PIV processing was performed using DaVis 8.4 software (LaVision GmbH). First, the minimum intensity of all images was subtracted from each image. Next, each data set was normalized with their respective average ensemble intensity. The instantaneous velocity vector was defined as \( U \). Its components along the streamwise and wall-normal directions were defined as \( U \) and \( V \), respectively. Angular brackets were used to denote the ensemble average of the variables over time and the x-direction. The latter averaging is applied due to the homogeneity of the fully developed turbulent channel flows in the streamwise direction. Fluctuations in the streamwise and wall-normal velocities were denoted by \( u \) and \( v \), respectively. High spatial resolution profiles of mean streamwise velocity \( \langle U \rangle \) were established using the ensemble-of-correlation method with a final interrogation window (IW) size of 6 x 6 pixel\(^2\) (0.07 x 0.07 mm\(^2\)) and 83% overlap between neighboring IWs (Kähler et al., 2012). The resulting profiles of \( \langle U \rangle \) had a single pixel spatial resolution (0.3\( \delta_v \) – 1.5\( \delta_v \), depending on \( Re_H \)). The lower limit of the measurements in \( \langle U \rangle \) was \( y = 35 \mu m \), which corresponds to \( y^+ = 0.76 – 3.15 \), depending on \( Re_H \). The instantaneous velocities \( U \) and \( V \) were determined using a multipass cross-correlation algorithm with an initial IW size of 64 x 64 pixel\(^2\)
and a final IW size of $32 \times 32$ pixels ($0.38 \times 0.38$ mm$^2$), both with 75% overlap between adjacent IWs. The spatial resolution of instantaneous velocity measurements was 8 pixels or 0.09 mm ($2\delta_v - 12\delta_v$). Vector postprocessing using the universal outlier detection algorithm developed by Westerweel & Scarano (2005) was used to remove any spurious vectors in the measurements of $U$ and $V$, after which the Reynolds normal stresses ($u'^2$) and ($v'^2$) and the Reynolds shear stress ($uv$) were determined. All first- and second-order velocity statistics attained reasonable statistical convergence with minimal random errors, as demonstrated in Appendix A.3.

The wall location was determined based on the local intensity maximum $I_{max}$ that forms due to the glare line of the wall in the average intensity distribution of the PIV images. The uncertainty in the wall location was considered to be the extent of the high-intensity glare, which was assumed to be the $\Delta y$ separating $I_{max}$ and $I_{max}/e^2$ (Abu-Rowin et al., 2017). The corresponding uncertainty in the wall location was estimated to be approximately 3 pixels or 35.4 $\mu$m ($0.8\delta_v - 4.4\delta_v$). Errors in the wall location were treated as an uncertainty in $y$ and were a contributing factor to the error bars in wall-normal distributions of mean velocity and Reynolds stresses.

Variables scaled using inner normalization were identified with the superscript $\dagger$. Velocity statistics were normalized with the friction velocity $u_\tau$, positional coordinates were normalized with the wall units $\delta_v$, and viscosity variables were normalized by the wall viscosity $\mu_w$, as listed in tables 8.1 and 8.2. Error propagation was used to derive the uncertainties in $u_\tau$ and $\delta_v$ based on the assumed errors in $\mu$ (see §8.3) and random errors in $\Delta P$. A conservative 0.1 pixel uncertainty in the PIV measurements of $U$ and $V$ was also assumed (Raffel et al., 2018). Such uncertainties in the inner scaling variables and the velocity measurements were reflected by error bars in plots of $⟨U⟩^\dagger$, $⟨u'^2⟩^\dagger$, $⟨v'^2⟩^\dagger$, and $⟨uv⟩^\dagger$.

### 8.6 Flow field analysis

Assuming the present XG solution follows the shear thinning trend shown in figure 8.2(a), an approximation for the 2D instantaneous distribution of $\mu$ was obtained within the turbulent channel flow using the following procedure. First, a 2D version of the strain rate, $\dot{\gamma} = (2D : D)^{1/2}$, was determined. Here $D = (\nabla U + \nabla U^\dagger)/2$, is the rate of strain tensor, the dagger symbol, $\dagger$, denotes a matrix transpose, and the colon operator represents the double dot product of the rank two tensors. Considering the PIV vectors were 2D, $\dot{\gamma}$ was determined using $U$ and $V$ alone, i.e. $\dot{\gamma} = [2((\partial U/\partial x)^2 + 1/2(\partial U/\partial y + \partial V/\partial x)^2 + (\partial V/\partial y)^2)]^{1/2}$. Therefore, our version of $\dot{\gamma}$ was an approximation that does not take into account spanwise velocity, $W$, or spatial gradients along the spanwise direction. A moving second-order polynomial plane with a size of $40 \times 40$ pixels, or $0.45 \times 0.45$ mm$^2$, was fit along instantaneous distributions of $U$ and $V$. Each 2D polynomial function was differentiated to obtain the spatial gradients in the velocity, i.e. $\partial U/\partial x$, $\partial U/\partial y$, $\partial V/\partial x$, $\partial V/\partial y$. The 2D instantaneous distribution of $\mu$ was then established by substituting $\dot{\gamma}$ into the CY model or equation (7.2), that relates shear viscosity to shear rate for the XG solution. Time-averaging was performed on the instantaneous viscosity profile to obtain a mean viscosity, $⟨\mu⟩$, and fluctuating viscosity, $\mu' = \mu - ⟨\mu⟩$, similar to those derived in DNS using GN constitutive models (Singh et al., 2017, 2018, Arosemena et al., 2020, 2021). After which, plots of the inner-normalized mean viscosity, $⟨\mu⟩^\dagger = ⟨\mu⟩/\mu_w$, and the inner-normalized standard deviation of the viscosity, $R(\mu)^\dagger = \sqrt{(\mu'^2)/\mu_w}$ were determined. A two-point correlation of $\mu'$
was used to characterize the length scale of the viscosity fluctuations. The correlations coefficient, $R_{\mu'\mu'}$ can be represented by the following equation,

$$R_{\mu'\mu'}(\Delta x, \Delta y) = \frac{\langle \mu'_{x_0,y_0} \mu'_{x_0+\Delta x,y_0+\Delta y} \rangle}{\sqrt{\mu'^2_{x_0,y_0} \mu'^2_{x_0+\Delta x,y_0+\Delta y}}}, \tag{8.1}$$

where $(x_0, y_0)$ is the streamwise and wall-normal coordinate of the reference point and $\Delta x, \Delta y$ represent the spatial shift along the $x$- and $y$-directions. Two reference points were considered, the first being $(x_0, y_0) = (0.1h, 0.07h)$, and the second being $(x_0, y_0) = (0.1h, 0.42h)$.

Assuming the GN constitutive model holds for XG, and the 2D approximation of $\dot{\gamma}$ is appropriate, the inner-normalized mean stress, $\tau^+$, across the half channel can be determined based on,

$$\tau^+ = \tau^+_v + \tau^+_\gamma - \langle uv \rangle^+, \tag{8.2}$$

where $\tau^+_v = \langle \mu \rangle^+ \partial \langle U \rangle^+/\partial y^+$ is the mean viscous stress, and $\tau^+_\gamma = 2\langle \mu^+ d_{xy}^+ \rangle$ is the turbulent viscous stress, named by [Singh et al., 2017]. Note that $d = D - (D)$, is the fluctuating component of the rate of deformation tensor, and $d_{xy} = (\partial u/\partial y + \partial v/\partial x)/2$. When normalized, $d_{xy}^+ = d_{xy}/u_+ = d_{xy}/\dot{\gamma}_w$. Alternatively, the mean shear stress can be equally represented as $\tau^+ = 1 - y^+/Re_\tau$. Previous investigations have denoted $\tau^+_\gamma$ as a “polymer stress”, estimated from the deficit $\tau^+_\gamma = \tau^+ - \tau^+_v + \langle uv \rangle^+$ [Warholic et al., 1999b; Ptasinski et al., 2003]. Given that all components listed in equation (8.2) can be explicitly determined, the CY shear thinning GN constitutive equation can be used to establish $\tau^+_\gamma$ and comment on its contribution to $\tau^+$.

Another component of our analysis involved a spatial gradient in the mean velocity profile along $y$, i.e. $\partial \langle U \rangle/\partial y$. To remove high frequency experimental noise and to differentiate the profile, a moving second-order polynomial filter was applied to the distribution of $\langle U \rangle$ with respect to $y$. The length of the filter was 24 pixel or 283 µm ($6\delta_v - 35\delta_v$, depending on $Re$). Coefficients of the fitted second-order polynomial were used to calculate $\partial \langle U \rangle/\partial y$, and then established the indicator function, $\zeta = y^+ \partial \langle U \rangle/\partial y^+$. Calculating $\langle \mu \rangle$ near the wall is limited by the spatial resolution of measurements in $U$ and $V$. Better spatial resolutions were achieved in $\langle U \rangle$ due to the utilization of the ensemble-of-correlation method. To approximate $\langle \mu \rangle$ near the wall it was assumed that the dominant component of $\dot{\gamma}$ very close to the wall was $\partial \langle U \rangle/\partial y$. The wall-normal gradient in the mean viscosity was then substituted into the CY model to obtain an approximation of $\langle \mu \rangle$ near the wall. This is an assumption; one that is rather bold for a turbulent flow. As such, these profiles were denoted a “pseudo-mean viscosity,” and is indicated by $\bar{\mu}$.

### 8.7 Newtonian turbulent channel flow

The following section begins by comparing measurements of the mean velocity profiles for water with the Newtonian law of the wall in figure 8.3. For brevity, only experimental data for water with a $Re_\tau$ less than or equal to 620 are plotted. These conditions of $Re_\tau$ were chosen because they are similar in magnitude to the $Re_\tau$ conditions of the XG flows listed in table 8.2. Following the plots of $\langle U \rangle^+$, measurements of the Reynolds stresses for water are shown in figure 8.4. Three experimental Reynolds stress profiles with $Re_\tau$ of 270, 390 and 510 are presented on the same axes as the Reynolds stresses derived from Newtonian channel
flow DNS by Iwamoto et al. (2002) at $Re_\tau = 300$ and Lee & Moser (2015) at $Re_\tau = 550$. The error bars in figures 8.3 and 8.4 are a result of uncertainties propagating from $\mu$, $\Delta P$, $U$ and $y$. For clarity, only two error bars are shown for each profile, one approximately in the buffer layer, the other within the outer layer.

Figure 8.3(a) demonstrates that all experimental profiles of water show good agreement with the law of the wall. The profiles were limited to $y > 35 \mu m$, which corresponds to $y^+ = 1.29$ to 2.89, for $Re_\tau$ between 270 and 620. For $y^+ < 5$ and greater than their respective lower limit, experimental measurements overlap with the profile of the linear viscous sublayer, $\langle U \rangle^+ = y^+$. Farther from the wall, all of the experimental distributions in figure 8.3(a) overlap with the log law, $\langle U \rangle^+ = 1/\kappa \ln y^+ + B$. A Von Kármán constant, $\kappa$, of 0.41 and intercept, $B$, of 5.17, as prescribed by Dean (1978) for 2D Newtonian channel flows, is shown for comparison. Distributions of $\zeta$ shown in figure 8.3(b) accentuate the logarithmic dependence of $\langle U \rangle^+$ with respect to $y^+$. The profiles of $\zeta$ imply that $\kappa$ is larger than 0.41 in the logarithmic layer for all profiles of water. Comparing the experimental profiles of $\langle U \rangle^+$ for different $Re_\tau$, all distributions appear to overlap with one another within the boundaries of measurement uncertainties. The DNS of a Newtonian channel flow by Lee & Moser (2015) demonstrated that profiles of $\langle U \rangle^+$ over a wider $Re_\tau$ range of 180 to 5000 also overlapped. The current experimental results for water also reflect universality in their distributions of $\langle U \rangle^+$ and $\zeta$ among different $Re_\tau$.

Figure 8.4(a) presents experimental profiles of $\langle u^2 \rangle^+$ relative to Newtonian channel flow DNS. For water with a $Re_\tau = 510$, instantaneous PIV measurements with IWs of $32 \times 32$ pixels and 75% overlap, translates to a spatial resolution of $7.8\delta_v$. As a result, the linear viscous sublayer and a portion of the buffer layer is missed in these measurements. However, for lower $Re_\tau$ the spatial resolution of the measurements...
improve. The scenario with $Re_\tau = 270$ has a spatial resolution of $3.5\delta_v$ and has measurements that extend to wall-normal locations as small as $y^+ = 9$. Within the logarithmic and outer layers, the experimental results overlap with their DNS counterparts at similar $Re_\tau$. The moderate $Re_\tau = 390$ case demonstrates consistency, considering it lies between the two DNS and experimental profiles at lower and higher $Re_\tau$. Figure 8.4(b) shows experimental and DNS profiles of $\langle v^2 \rangle^+$ and $\langle uv \rangle^+$. Similar to the distributions in $\langle u^2 \rangle^+$, experimental profiles in $\langle v^2 \rangle^+$ and $\langle uv \rangle^+$ agree well with the DNS results at similar $Re_\tau$. However, there are some small discrepancies. For example, the experimental profile of $\langle v^2 \rangle^+$ at $Re_\tau = 510$ appears to be minutely larger than the DNS profile of $\langle v^2 \rangle^+$ at $Re_\tau = 550$ for $y^+ > 100$. Overall, the experimental mean velocity and Reynolds stress measurements show consistency and agreement with 2D Newtonian channel flow DNS. Therefore, subsequent results of the non-Newtonian solution can be presented with relatively good confidence in the validity of the measurements. It should also be noted that the spatial resolution of the measurements will improve with the addition of polymers, considering $DR$ is coupled with a reduction in $u_\tau$ and an increase in $\delta_v$. This can be observed by comparing the larger values of $\delta_v$ for XG flows with the $\delta_v$ values of water in tables 8.1 and 8.2.

### 8.8 Non-Newtonian turbulent channel flow

The current section investigates the turbulent flow of the XG solution with varying $Re_\tau$. The section is divided into three portions. The first subsection presents wall-normal distribution of the mean velocity profile, $\langle U \rangle^+$, indicator function, $\zeta$, and pseudo-viscosity profile, $\tilde{\mu}$, obtained from the vector fields with high-spatial-resolution. The second subsection investigates spatial distributions of the viscosity derived from the 2D shear rate. Lastly, the final subsection delves into the Reynolds stresses and viscous stresses of non-Newtonian flows at different $Re_\tau$.

![Figure 8.4: Inner normalized profiles of (a) streamwise Reynolds stress, (b) wall-normal and Reynolds shear stresses, for Newtonian flows.](image)
8.8.1 Mean velocity profile

Profiles of $\langle U \rangle^+$ for the XG scenarios are shown in figure 8.5(a). Near the wall, experimental distributions of $\langle U \rangle^+$ conform well with the linear viscous sublayer profile, $y^+ = \langle U \rangle^+$, for all $Re_\tau$ under consideration. The upper limit of the linear viscous sublayer appears to grow relative to Newtonian wall turbulence. For a Newtonian turbulent channel flow, the linear approximation of the viscous sublayer is valid to within 10% at $y^+=5$ (Pope, 2000). If a 10% confidence interval from $y^+=\langle U \rangle^+$ is used as a threshold, the size of the linear viscous sublayer for the non-Newtonian profiles shown in figure 8.5(a) can be approximated. The following table 8.5 lists the size of the linear viscous sublayer for the flows, both in inner- and outer-scaling. The size in inner-scaling is denoted, $y^+_v$, while the size in outer-normalization is $y_v/h$. All values of $y^+_v$ are between 8 and 12, demonstrating that the linear viscous sublayer is expanded relative to Newtonian wall turbulence, which has a $y^+_v$ between 3 and 5 (Pope, 2000). With increasing $Re_\tau$, the non-Newtonian values of $y^+_v$ increase subtly, implying that the very near wall profiles might be slightly different, and potentially depend on the small increase in $DR$ with increasing $Re_\tau$, as shown in table 8.2. However, with error bars, these differences could be a result of uncertainty in the measurements. At the larger $Re_\tau$, between 530 and 700, the linear sublayer appears to saturate and nearly approach the tri-section point, $(y^+,\langle U \rangle^+) = (11.6, 11.6)$, where the Virk MDR asymptote, equation (3.16), intersects with $y^+=\langle U \rangle^+$ and the Newtonian log law. Values of the outer-scaled thicknesses, $y_v/h$, decrease with increasing $Re_\tau$, mainly due to the large shrinkage in $y_v$ caused by increasing $Re_\tau$.

Farther from the wall at $y^+ > 30$, figure 8.5(a) demonstrates a larger $\langle U \rangle^+$ relative to the logarithmic

![Figure 8.5: Inner-normalized distributions of (a) mean streamwise velocity and (b) the indicator function, for flows with 170ppm XG solution.](image)
\[ Re_\tau \quad y^*_v \quad y_v/h \]

<table>
<thead>
<tr>
<th>( Re_\tau )</th>
<th>( y^*_v )</th>
<th>( y_v/h )</th>
</tr>
</thead>
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<tr>
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</tr>
<tr>
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</tr>
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<td>0.030</td>
</tr>
<tr>
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</tr>
<tr>
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<td>11.5</td>
<td>0.023</td>
</tr>
<tr>
<td>610</td>
<td>11.2</td>
<td>0.019</td>
</tr>
<tr>
<td>700</td>
<td>11.4</td>
<td>0.017</td>
</tr>
</tbody>
</table>

Table 8.3: Linear viscous sublayer sizes for non-Newtonian flows in inner- and outer-scaling.

law of the wall; an observation common for drag-reduced flows. Virk (1971), and later Warholic et al. (1999b), demonstrated that LDR flows form a Newtonian plug profile, which is observed as an increase in the log law intercept, \( B \), but a similar \( \kappa \), relative to the log law distribution of a Newtonian fluid. Virk (1971) detailed that the growth in \( B \) was proportional with \( DR \). A larger \( DR \) would result in an increased buffer layer thickness (deemed the elastic sublayer) and hence an enhancement in \( B \). Findings from Warholic et al. (1999b) showed that a Newtonian plug exists only for LDR flows with \( DR < 35\% \). Given \( DR \) of the present XG flows are between 27-33\% (see table 8.2), the current XG flows satisfy the criteria for LDR. Therefore, our measurements agree well with previous observations of \( \langle U \rangle^+ \) profiles for polymer drag-reduced LDR flows. Furthermore, figure 8.5(a) demonstrates that profiles of \( \langle U \rangle^+ \) for XG have little dependence on \( Re_\tau \). There is perhaps a subtle increase in \( B \) for \( 170 < Re_\tau < 440 \); however, this could be attributed to the small growth in \( DR \) with increasing \( Re_\tau \). The uncertainty in the flow measurements, shown by the error bars, also captures the small variations in \( B \).

White et al. (2012) re-evaluated the efficacy of the Virk (1971) elastic sublayer model using the indicator function, \( \zeta \), which highlights regions of strong logarithmic dependence. They compared mean velocity profiles from various experimental and numerical investigations of different \( DR \), canonical flows and \( Re_H \).

For LDR flows, White et al. (2012) observed constant \( \zeta \) (generally for \( y^+ > 50 \)), which is indicative of a Newtonian plug. Profiles of \( \zeta \) shown in figure 8.5(b) also demonstrate regions of constant \( \zeta \), providing further evidence of a Newtonian plug for rigid polymer solutions. For all \( Re_\tau \), these regions of constant \( \zeta \) are observed for \( y^+ > 60 \). This lower limit of \( y^+ = 60 \) is larger than the lower limit of \( y^+ = 30 \) for the Newtonian log layer (Pope 2000), demonstrating an expansion of the viscous sublayer. The peak values of \( \zeta \) for XG at \( y^+ = 15 \) is greater than the peak values of \( \zeta \) for water as seen in figure 8.3(b). The implication is that the slope of \( \langle U \rangle^+ \) within the buffer layer is larger for flows of XG relative to water. A larger slope in \( \langle U \rangle^+ \) is indicative of an “effective slip” in the buffer layer which, in turn, results in an increase in \( \langle U \rangle^+ \) within the logarithmic layer Lumley (1969); Virk (1971). Another observation is that the constant value of \( \zeta \) for the XG flows in the Newtonian plug layer, are marginally larger than the values of \( \zeta \) observed for water in the logarithmic layer shown in figure 8.3(b). White et al. (2012) similarly observed that \( \kappa \) was slightly larger than water in the
Newtonian plug for LDR flows. White et al. (2012, 2018) broadly suggested that the inner-normalized mean velocity profile of a polymer drag-reduced flow depends on the Reynolds number, polymeric properties and the canonical flow. Therefore, if $DR$ is constant, distributions of the inner-normalized mean velocity profiles of a rigid polymer solution are relatively independent of $Re\tau$ within the inner layer of the flow.

Figure 8.6 demonstrates distributions of the normalized pseudo-viscosity, $\tilde{\mu}^+$, with respect to $y^+$, for the XG flows of different $Re\tau$. The profiles of $\tilde{\mu}$ are an approximation of the mean viscosity in the near-wall region. Intuitively, the decreasing trend in $\tilde{\mu}^+$ with increasing $Re\tau$ at a given $y^+$ is plausible. Flows of higher $Re\tau$ have larger $\partial\langle U \rangle / \partial y$, hence $\tilde{\mu}$ should be correspondingly lower relative to a flow of smaller $Re\tau$. For $y^+ < 10$, all XG flows have distributions of $\tilde{\mu}^+$ that are approximately constant; only growing subtly by about 1% with increasing $y^+$. As $y^+$ increases beyond 10 all profiles experience a dramatic increase in the magnitude of $\tilde{\mu}^+$. The precise $y^+$ location where this inflection in $\tilde{\mu}^+/\mu_w$ occurs depends on the $Re\tau$ being considered. The thickness of the near-wall region of approximately constant $\tilde{\mu}$ appears to conform well with the peak in profiles of $\zeta$, shown in figure 8.5(b) and indicative of the central location of the buffer layer. The inner-normalized thickness of this region of constant $\tilde{\mu}$ grows with increasing $Re\tau$. However, the value of $\tilde{\mu}^+$ appears to monotonically decrease with increasing $Re\tau$ at any chosen value of $y^+$. Flows of large $Re\tau$ experience a less aggressive change in $\tilde{\mu}$ with respect to $y^+$, but the size of their near-wall region of low viscosity is larger. Generally, all flows experience a large and sudden change in $\tilde{\mu}$ for $y^+$ between of 10 and 30. For example, the XG flow with $Re\tau = 170$ has a $\tilde{\mu}$ that is 50% larger than $\mu_w$ at $y^+ = 30$. A near wall region of constant mean viscosity that suddenly and dramatically increases with respect to $y^+$ has also been observed from numerical simulations using generalized Newtonian (GN) models (Singh et al., 2017, 2018; Arosemena et al., 2020, 2021). Our results appear to qualitatively agree with the results of DNS using inelastic shear thinning GN models near the wall (Singh et al., 2017, 2018; Arosemena et al., 2020). This is despite the approximation used to derive the pseudo-viscosity profile, $\tilde{\mu}$, based on 2D velocity data.
8.8.2 Turbulent shear viscosity

Figure 8.7(a) shows an instantaneous contour of \( u \) for XG with \( Re_\tau \) of 170, while figure 8.7(b) shows a snapshot of \( \mu \). Contours (a) and (b) are extracted at the same time instance. In both figures 8.7(a), zones of low and high speed flow are observed. Figure 8.7(b) demonstrates that the viscosity near the wall is low and within 20% of \( \mu_w \) for \( y/h < 0.2 \). Away from the wall, \( y/h > 0.2 \), most of the fluid has a \( \mu \) between 1.5 to 3 times larger than \( \mu_w \). In general, the spatial distribution of \( \mu \) shows large streamwise-elongated zones of low and high viscosity that contain small-scale viscosity fluctuations. For example a large, low viscosity slug can be found at around \( y/h = 0.6 \) and extending from \( x/h = 0 \) to 0.6 in the snapshot shown in figure 8.7(b).

The location of this low viscosity slug appears to coincide roughly with the interface between the low and high speed zones, shown in figure 8.7(a). A second streamwise-elongated zone of low viscosity is observed extending from the wall at a shallow angle. Similarly, this low viscosity zone overlaps with the shear layer between low and high speed zones.

![Figure 8.7: Instantaneous contour of (a) streamwise velocity fluctuations and (b) viscosity for XG at \( Re_\tau = 170 \).](image)

Probability density functions (PDFs) of \( \mu^+ \) are shown in figure 8.8 for the XG flows at different \( Re_\tau \) within the inner and outer layers of the flow. Figure 8.8(a) demonstrates the PDFs of \( \mu^+ \) at \( y/h \) of 0.07. While figure 8.8(b) shows the PDFs at \( y/h \) of 0.42. Within both the inner and outer layers of the flow, PDFs of \( \mu^+ \) are positively skewed. Within the inner layer, flows with smaller \( Re_\tau \) (e.g. \( Re_\tau = 170 \)) tend to have a more narrow PDF than flows of larger \( Re_\tau \) and demonstrate a smaller PDF peak. In contrast, figure 8.8(b) demonstrates that at \( y/h = 0.42 \), the peak PDF in \( \mu^+ \) is larger for flows of high \( Re_\tau \). This implies that viscosity fluctuations are likely larger for low \( Re_\tau \) flows within the outer layer. Wall-normal profiles of \( \langle \mu \rangle^+ \)
and rms($\mu'$) better demonstrate these differences.

Figure 8.9 provides inner-normalized profiles of the mean viscosity, $\langle \mu \rangle^+$, and the rms of $\mu'$, for the non-Newtonian flows of different $Re_\tau$. Figure 8.9(a) demonstrates that distributions of $\langle \mu \rangle^+$ appear to be logarithmic, consistent with DNS using GN constitutive models (Singh et al., 2017, 2018; Arosemena et al., 2020, 2021). The profiles of $\langle \mu \rangle^+$ for different $Re_\tau$ do not overlap; flows with lower $Re_\tau$ have larger $\langle \mu \rangle^+$ in the outer-layer of the flow. Figure 8.9(b) shows the inner-normalized rms profiles of $\mu'$, which also reflect a similar dependency as $\langle \mu \rangle^+$ with respect to $Re_\tau$. Unlike the present findings of figure 8.9 Singh et al. (2018) observed that $\langle \mu \rangle^+$ and the rms($\mu'$) overlapped for pipe flow DNS with a power law GN model. The overlap is suspected to be contingent on the choice of the rheological model, i.e. the power law model. The nominal wall viscosities listed in table 8.2 encroach on the second Newtonian regime of the CY model and are likely not well described by a power law equation. Therefore, it appears that $\mu_w$ is an insufficient scaling parameter. Nonetheless, distributions of $\langle \mu \rangle^+$ demonstrate a lower average viscosity near the surface and a substantially larger viscosity closer to the core, much like the implication of the $\tilde{\mu}^+$ profiles shown in figure 8.6.

To characterize the length scale of the viscous fluctuations, a two-point correlation of $\mu'$ using equation (8.1) was performed for each of the XG channel flows with different $Re_\tau$. As mentioned in §8.6, two reference points were considered, the first being $(x_0, y_0) = (0.1h, 0.07h)$, and the second being $(x_0, y_0) = (0.1h, 0.42h)$. Therefore, the first point falls within the inner layer of the channel flow, while the second point is well into the outer layer of each flow ($y/h > 0.1$).

Figure 8.10(a) demonstrates distributions of the correlation coefficient, $R_{\mu'\mu'}$, along the streamwise direction, $\Delta x$, and at $y/h$ of 0.07. Although, figure 8.10 does not observe the value of $\Delta x$ at which $R_{\mu'\mu'}$ becomes zero, it can be reasonably inferred that the size of the viscosity fluctuations along the x-direction decrease in magnitude with increasing $Re_\tau$. The same observation can be made in the outer layer based on

![Probability density function of fluctuating viscosity taken at x of 0.1h and y of (a) 0.07h, and (b) 0.42h.](image-url)
Figure 8.9: Wall-normal profiles of (a) mean viscosity, and (b) the root mean square of the fluctuating viscosity. Error bars are shown at $y/h$ of 0.07 and 0.42.

plots of $R_{\mu'\mu'}$ as a function of $\Delta x$ and at a constant $y/h$ of 0.42, seen in figure 8.10(b). Figure 8.11 presents profiles of $R_{\mu'\mu'}$ at $x/h$ of 0.1, and along the wall-normal direction, $\Delta y$. For all $Re_\tau$, $R_{\mu'\mu'}$ decays to zero within $0.06h$ when $y_0$ is $0.07h$, as seen in figure 8.11(a). For lower $Re_\tau$ cases, (e.g. 170 and 260), there is a significant anti-correlation between $\Delta y/h$ of 0.06 and 0.2. The anti-correlation indicates a streaky pattern in the viscosity field, potentially generated by the shear layer structures between the streamwise elongated low and high-speed zones. It is suspected that this prevalent anti-correlation cannot be observed for large $Re_\tau$

Figure 8.10: Two-point correlation of viscosity fluctuations along the streamwise direction at wall-normal locations of (a) $y/h = 0.07$ and (b) $y/h = 0.42$. 

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due to the choice of \( y_0 \). For the case with the lowest \( Re_\tau \) of 170, \( y_0 = 0.07h \) is equivalent to a \( y^+ \) of 12, which lies near the centre of the buffer layer or the peak in \( \zeta \). For \( Re_\tau = 700 \), a \( y_0 \) of 0.07h corresponds to a \( y^+ \) of 49, which is close to the upper \( y^+ \) limit of the buffer layer. Therefore, \( \mu' \) within the viscous sublayer appears to be opposite in sign convention to \( \mu' \) within the log and outer regions of the flow. When \( y_0 \) is set to 0.42h, profiles of \( R_{\mu'\mu'} \) are generally the same for all \( Re_\tau \) cases, seen in figure 8.11(b). The correlation coefficient attains a value of zero, or very close to zero (< 0.01), within \( \Delta y \) of 0.3h. Based on figure 8.7(d), viscosity fluctuations are marginally more elongated along the \( x \)-direction relative to \( y \). The size of the structures become more isotropic with growing distance from the wall.

### 8.8.3 Reynolds stresses and mean shear stress budget

Figure 8.12(a) presents plots of \( \langle u^2 \rangle^+ \) for the XG flows alongside experimental data of water with \( Re_\tau = 510 \) and Newtonian channel flow DNS from Lee & Moser (2015) with \( Re_\tau = 550 \). Unlike the experimental results for water shown in figure 8.4(a), the peak in \( \langle u^2 \rangle^+ \) could be resolved for at least the two lowest \( Re_\tau \) scenarios, i.e. \( Re_\tau = 170 \) and 260. The use of XG makes resolving the peak in \( \langle u^2 \rangle^+ \) easier, since drag-reducing additives have been shown to shift the peak in \( \langle u^2 \rangle^+ \) farther from the wall relative to Newtonian fluids (Warholic et al., 1999b; Escudier et al., 2009). In general, the magnitude in \( \langle u^2 \rangle^+ \) for all \( Re_\tau \) scenarios is increased relative to the experimental profile for water shown in figure 8.12(a). The amount by which the XG profile of \( \langle u^2 \rangle^+ \) increases depends on the \( Re_\tau \) being considered. For example, comparing XG and water at similar \( Re_\tau \) of 510, the XG profile of \( \langle u^2 \rangle^+ \) is larger for nearly all \( y^+ \).

Profiles of \( \langle v^2 \rangle^+ \) are the positive distributions shown in figure 8.12(b). Relative to Newtonian profiles of similar \( Re_\tau \), distributions of \( \langle v^2 \rangle^+ \) for the XG solutions demonstrate significant attenuation along all values of \( y^+ \). This can easily be seen by comparing the plots of \( \langle v^2 \rangle^+ \) for XG at \( Re_\tau = 530 \) with the experimental
profile of water at $Re_\tau = 510$. Distributions of $\langle uv \rangle^+$ correspond to the negative profiles shown in figure 8.12(b). Unlike $\langle v^2 \rangle^+$, profiles of $\langle uv \rangle^+$ are only strongly attenuated near the wall, relative to Newtonian distributions of comparable $Re_\tau$. The values of $\langle uv \rangle^+$ are similar for $y^+ > 150$ when comparing XG and water at a $Re_\tau$ of 510. While for $y^+ < 150$, the XG solution shows a large reduction in the magnitude of $\langle uv \rangle^+$, when contrasted with the profile of water with a similar $Re_\tau$ of 510. Therefore, relative to Newtonian profiles of similar $Re_\tau$, solutions of XG at LDR exhibit strong attenuation in $\langle v^2 \rangle^+$ throughout the complete half-channel; however attenuation in $\langle uv \rangle^+$ is confined to a portion of the channel near the wall. Comparing the Reynolds stress profiles of XG with one another, all distributions for XG shown in figure 8.12 increase in magnitude monotonically with increasing $Re_\tau$ at a given $y^+$, similar to the trend in the Reynolds stresses for Newtonian fluids of increasing $Re_\tau$.

Different components of the mean stress balance are presented in figure 8.13. For brevity and to avoid clutter, the mean stress balance is shown for only three of the seven $Re_\tau$ cases (170, 440 and 700) of XG. The XG flows exhibit a trade-off in the budget or contribution of $\tau_v^+$ and $-\langle uv \rangle^+$ to the total mean stress, $\tau^+$, depending on the $y^+$ location. Specifically, near the wall $\tau_v^+$ contributes more to $\tau^+$ than $-\langle uv \rangle^+$, while closer to the core of the channel, the opposite can be observed, i.e. $-\langle uv \rangle^+$ is larger than $\tau_v^+$. For all XG flows, the turbulent viscous stress, $\tau_v^+$, contributes little to $\tau^+$, regardless of the $y^+$ location being considered. Distributions of $\tau^+$, represented by the solid lines in figure 8.13 and determined from the summation of $\tau_v^+$, $\tau_v^{++}$ and $-\langle uv \rangle^+$ i.e., \[8.2\], agree well with $1 - y^+ / Re_\tau$ within the margin of experimental uncertainty, represented by the down-sampled error bars. Therefore, it can be assumed that measurements of $\tau_v^+$, $\langle uv \rangle^+$, and $\tau_v^{++}$, are approximately valid. Arosemena et al. (2020, 2021) demonstrated that $\tau_v^+$ accounted for less than 5% of $\tau^+$ within the inner layer, based on DNS using a channel flow with a shear thinning GN constitutive model that had $DR \approx 10\%$. Although the present non-Newtonian flows have almost three times the $DR$ as Arosemena et al. Arosemena et al. (2020, 2021), $\tau_v^+$ also appears to be less than 5% for XG. Therefore,
a drag-reduced turbulent flow of XG can largely be explained by Reynolds and viscous stresses, with very little influence from stresses imposed by the fluctuating non-Newtonian viscosity.

In summary, the rigid polymer solution demonstrates larger profiles in $\langle U \rangle^+$ within the logarithmic layer relative to water, conducive of a Newtonian plug. Non-Newtonian flows of different $Re$ and similar $DR$ had overlapping profiles in $\langle U \rangle^+$, within the margin of measurement uncertainty. When compared to experiments of Newtonian turbulence at a similar $Re$, XG exhibits larger profiles in $\langle u^2 \rangle^+$, and smaller profiles in $\langle v^2 \rangle^+$, for all $y^+$. Attenuation in $\langle uv \rangle^+$ is observable, but only near the wall. These findings share similarities with numerical investigations using inelastic models, such as the GN power-law or Carreau constitutive equations. Singh et al. (2018) used a power-law model to simulate an inelastic non-Newtonian turbulent pipe flow of $Re$ between 323 and 750. Constant material properties were maintained across their different cases of $Re$ to evaluate the effect of Re on the flow statistics, much like what is demonstrated in the present experimental investigation. Singh et al. (2018) observed a Newtonian plug for all flow conditions, profiles of $\langle U \rangle^+$ that overlapped across different $Re$, an enhancement in $\langle u^2 \rangle^+$, attenuation in the radial and azimuthal Reynolds stresses, and a confined near wall attenuation in $\langle uv \rangle^+$, relative to a Newtonian flow of similar $Re$. Contrasting this with experiments using flexible polymers or DNS using elastic models, such as FENE-P, the same observations can be made for mean velocity statistics of generally any LDR flow, including the current findings. Consistency in the mean velocity statistics of elastic and inelastic DR suggests that the net effect of DR using elastic or inelastic additives is the same, at least for flows at LDR. This is despite their dramatically different rheology and potentially unique mechanisms for mitigating drag.

Figure 8.13: Inner-normalized mean stress balance of XG at three of the seven $Re$ conditions. The lines correspond to $\cdots 1 - y^+/Re$, $\cdots \tau^+$, $\cdots \langle uv \rangle^+$, $\cdots \tau_v^+$, $\cdots \tau_{v^*}$. Error bars are shown at $y/h$ of 0.07 and 0.42.
8.9 Discussion - lubricating layer

The classical theories of polymer DR have insinuated that polymers interact with turbulence in a manner that quells regions of high strain and vorticity through either an enhanced extensional viscosity or elasticity (Lumley, 1973; de Gennes, 1990). Indeed, experiments with flexible polymers in isotropic homogeneous grid turbulence demonstrate suppression of the small scale turbulent eddies that correspond to regions of the flow with high extensional strain, and thus large extensional viscosities (Van Doorn et al., 1999). However, shear thinning properties of rigid polymers work against these postulates, in that regions with large shear rates have lower viscosities, not enhanced. A comparison of isotropic turbulence using FENE-P versus inelastic shear thinning constitutive models could directly contrast the local instantaneous effect of flexible and rigid polymers on turbulence. Rather, it is argued that the phenomenon of DR for inelastic shear thinning fluids is primarily attributed to a wall-normal gradient in shear viscosity induced from the wall. Numerical investigations that employ inelastic shear thinning constitutive models seem to support this claim. Arosemena et al. (2021), performed channel flow DNS using an inelastic Carreau constitutive model and commented on the near wall turbulent structures within the flow. They demonstrated that forces arising from fluctuations in the viscosity do not necessarily act in opposition of turbulent structures, such as quasi-streamwise vortices and low/high-speed streaks. Instead, Arosemena et al. (2021) surmised that the local enhancement in the viscosity with increasing distance from the wall produces less energetic vortices and DR.

In the present experimental investigation, evidence of a striking demarcation in the viscosity, and the viscosity fluctuations, with growing distance from the wall are observed. For example, figure 8.6(b) and 8.9(a) imply that the viscosity within the outer layer of the channel can be 20% to 300% larger than the nominal wall viscosity. Figures 8.10 and 8.11 demonstrate that the size of correlated viscosity fluctuations are thin (Δy/h ≈ 0.06) and long (Δx/h > 0.4) within the buffer layer, but become more isotropic with increasing y. Moreover, spatial two-point correlations along the wall-normal direction show an anti-correlation between viscosity fluctuations within the near-wall region and the outer layer of the flow. It is apparent that the characteristics of the viscosity field are considerably different between the inner and outer layers of the flow. Furthermore, the mean stress balance, shown in figure 8.13 demonstrates that DR can largely be accounted for by a balance between viscous and Reynolds stresses alone, with little dependence on turbulent viscous stresses that arise from viscosity fluctuations. What is common among the present experimental investigation and DNS involving inelastic GN fluids (Singh et al., 2017, 2018; Arosemena et al., 2020, 2021), is a thin layer of nearly constant low viscosity fluid close to the wall followed by a sharp increase in the mean viscosity with increasing distance from the wall.

This thin near-wall layer of low viscosity is perhaps analogous to the low viscosity lubricating layer in the DNS of Roccon et al. (2019). In this numerical investigation, a thin layer of immiscible fluid with a different viscosity was introduced in the near wall region. When the near wall region had a viscosity comparable with that of the bulk fluid, Roccon et al. (2019) observed that the surface tension between the two fluids produced DR. However, for the cases where the near wall fluid had a lower viscosity, they commented that the near wall fluid acts as a lubricating layer that results in a smaller wall friction and consequently DR. In addition to this observation, there are some notable similarities with respect to the current investigation. In their DNS, Roccon et al. (2019) demonstrated that the average thickness of the lubricating layer was similar
to the thickness of the expanded linear viscous sublayer, $y_v/h$, in the present experimental findings for XG. The DNS by [Roccon et al.] (2019) attained $DR$ of 24% with a lubricating layer that was $0.038h$ in thickness; a value comparable to those of $y_v/h$ for XG, which are between $0.017h$ and $0.051h$, as listed in table 8.3. However, it should be noted that the $DR$ measured by [Roccon et al.] (2019) is based on an enhancement of volumetric flow rate considering they maintain a constant pressure gradient in their DNS – similar to most numerical investigations involving turbulent DR, including those of [Arosemena et al.] (2020, 2021) using GN constitutive models. In contrast, the present investigation considers a constant Re and evaluates the change in pressure gradient (a saving of “money” according to [Frohnapfel et al.] (2012)).

Turbulent DR using shear thinning liquids may also share commonalities with DR using superhydrophobic surfaces. Adding micro-scale roughness to a hydrophobic material produces a thin layer of air between the liquid and the solid boundary (Rothstein, 2010). The air layer causes the moving liquid to “slip”, generally resulting in large quantities of DR (Ling et al., 2016; Abu-Rowin & Ghaemi, 2019). This apparent slip of the liquid phase produces a mean velocity profile where values of $\langle U \rangle^+$ are larger for all $y^+$, but parallel to the Newtonian law of the wall – seemingly reminiscent of the Newtonian plug in polymer DR. Indeed, Lumley (1969) and Virk (1971) have regarded the Newtonian plug for polymer DR as being an “effective slip”. The Newtonian plug is realized in a polymer drag-reduced flow when the log layer is displaced upwards to larger $\langle U \rangle^+$ (Virk, 1971). The Newtonian plug and the “effective slip” were alluded to in the results pertaining to profiles of $\langle U \rangle^+$, and was realized by the large peak in $\zeta$. For rigid polymer solutions, slippage and the Newtonian plug are perhaps a manifestation of the fluids shear thinning rheology and the near-wall lubricating layer.

8.10 Summary

Solutions of xanthan gum (XG) polymer have historically demonstrated little viscoelastic and extensional properties; two rheological features often attributed to polymer drag-reduction (DR). Few existing experimental investigations have demonstrated the turbulence statistics of rigid polymers in a turbulent channel flow. The primary objective of the investigation in §8 was to scrutinize the effect of varying Reynolds number ($Re_H$) on the mean velocity and Reynolds stress profiles, independent of changes in $DR$. Our second objective was to evaluate the wall-normal gradient in the shear viscosity for drag-reduced flows of rigid polymers.

Inner-normalized mean velocity profiles for the XG flows of different $Re_H$ approximately overlapped. Relative to the Newtonian law of the wall, the intercept of the log layer was considerably larger, and the slope demonstrated marginal growth (i.e., a Newtonian plug flow). Compared to Newtonian Reynolds stress profiles of similar $Re_H$, distributions for XG exhibited enhancement in streamwise Reynolds stresses and attenuation in wall-normal Reynolds stresses for all inner normalized wall-normal coordinates. Attenuation in the Reynolds shear stress was only observed near the wall. The effect of increasing $Re$ in the non-Newtonian flows was the same as Newtonian, i.e., the Reynolds stresses increased in the logarithmic layer monotonically with increasing $Re_H$. The modification to the first- and second-order velocity statistics reflected consistency with results obtained from DNS using elastic and inelastic constitutive models and previous experiments with flexible polymers.
Instantaneous viscosity statistics were determined for each drag-reduced flow using the Carreau-Yasuda constitutive model and velocity gradients. The flows of XG possessed a thin near wall region with low mean viscosity. At wall-normal locations above the thin “lubricating layer”, the fluid had a much larger mean viscosity. Fluctuations in the viscosity reflected different size and characteristics with increasing distance from the wall. That being said, these viscosity fluctuations have a negligible contribution to the mean stress balance of the flow. The lubricating layer consisted of the expanded linear viscous sublayer and portions of the buffer layer within the XG flows. It is hypothesized that rigid polymer \( DR \) is largely attributed to gradients in the mean velocity coupled with the solutions shear thinning rheology. The lubricating layer is a product of this interaction and a mechanism for generating an effective slip within the buffer layer.
Part IV

Turbulent boundary layer
Chapter 9

Local flow topology of a polymer-laden boundary layer

The invariants in the VGT of a Newtonian and polymer-laden turbulent boundary layer were experimentally analyzed using velocity vectors measured from 3D particle tracking velocimetry (3D-PTV) based on the shake-the-box (STB) algorithm developed by Schanz et al. (2016). Polymer-laden and Newtonian flows were compared at a similar friction Reynolds number $Re_\tau$ and momentum thickness Reynolds number $Re_\phi$ in a boundary layer formed on the floor of the water flume depicted in §4.3. Based on the VGT, the local flow topology is analyzed using the $\Delta$-criterion, discussed in §3.3. Evidence is provided to test a hypothesis regarding the mechanism of polymer drag reduction. This hypothesis is inspired by the viscous theory of drag reduction mentioned in §1.3 – that being, the large extensional viscosity of polymer solutions strongly inhibits turbulent fluctuations just outside the viscous sublayer, causing the buffer layer to expand and wall friction to reduce (Lumley, 1973; White & Mungal, 2008). In a similar fashion, the work of Roy et al. (2006) proposed a mechanism by which polymers influence the nature of coherent structures that also pertains to the extensional viscosity of the polymer solution. In a channel flow simulation that utilized a simplified constitutive model of polymer stresses (the retarded-motion expansion), Roy et al. (2006) demonstrated that the non-Newtonian extensional viscosity opposed flow in both biaxial and uniaxial flow regions, which mitigated the strength and formation of quasi-streamwise vortices and reduced drag. Based on the findings of Roy et al. (2006), it is therefore, expected that changes in the topology will predominately occur in regions of strong uniaxial/biaxial extension. Uniaxial/biaxial flow regions are dissipative with $\Delta < 0$, and are strongly concentrated around the Vieillefosse tails in the JPDF of the VGT invariants discussed in §3.3 and shown in figure 3.5. They can also be identified from the invariants in the rate of deformation tensor or the symmetric component of the VGT, as demonstrated in §3.3 and figure 3.6. Details regarding the polymer solution and measurement apparatus are first presented, followed by an analysis of velocity statistics and flow topology based on the $\Delta$-criterion.

9.1 Polymer solution preparation and characterization

The flexible polymer polyacrylamide (PAM) (6030S, SNF Floerger) with a molecular weight of 30-35 MDa, was chosen for the polymer-laden boundary layer experiments. A 3500 l homogeneous PAM solution (a
polymer ocean) with a concentration $c$ of 140 ppm was utilized. To prepare the polymer ocean, an 1140 l concentrated master solution ($c = 430$ ppm) was first mixed and then diluted to achieve the desired 140 ppm concentration within the flume. The master solution was mixed in two 570 l cylindrical vessels. Solid polymer powder was weighed using a scale with a 0.1 g resolution, and gently added to each container (245 g to each vessel) along with tap water. A stand mixer equipped with a 150 mm diameter impeller and set to a rotational speed of 50 rpm was used to mix the master solution in each vessel for 2 hours. The master solution was then slowly added to 2360 l of tap water that was contained within the flume. An air operated diaphragm pump was used to transfer the master solution from the mixing containers to the flume at a flow rate of 1 l s$^{-1}$. Upon adding the master solution to the flume, the 3500 l solution was then circulated for 30 min, where the rotational speed of the centrifugal pumps was set to 300 rpm. The 140 ppm solution was then left to rest for 12 h. The resulting fluid was visibly transparent and had no heterogeneous clumps of polymers.

Flow measurements were performed immediately after the PAM solution was left to rest for 12 h. The rotational speed of the pumps were set to 1000 rpm, which produced a $U_{\infty}$ of 0.432 m s$^{-1}$. To avoid degradation of the PAM solution, the pumps were turned off intermittently between instances of image acquisition for 3D-PTV. For a single set of flow measurements, the pumps were turned on for 2 min. After which, the pumps were turned off for approximately 10 min to allow time for the 3D-PTV images to be saved. Eight sets of images were collected for 3D-PTV, therefore, this procedure of turning the pumps on for 2 min and off for 10 min was repeated eight times. Fluid samples were collected for rheology measurements immediately after each instance of image acquisition (eight fluid samples in total) while the pumps were turned off. Rheology measurements were necessary for characterizing the material properties of the fluid (i.e., shear viscosity and extensional relaxation time) and were also useful for diagnosing the effects of degradation.

Steady shear rheology was used to evaluate the viscous features of water and the 140 ppm PAM solution. Shear rheology measurements were performed using the torsional rheometer in §4.1.1 and figure 4.1. The double-gap concentric cylinder geometry of figure 4.2(b) was utilized for the measurements. Measurements of $\mu$ were performed over a logarithmic sweep of shear rate $\dot{\gamma}$ from 0.1 to 1000 s$^{-1}$, as shown in figure 9.1(a) for water and the 140 ppm PAM solution. Measurements of $\mu$ are limited by a minimum measurable torque $T$ and the inception of Taylor vortices. The lower torque limit provided by TA instruments was 10 nN m §4.1.1; in practice, the lower limit was larger and equal to $T = 600$ nN m. Taylor instabilities occur at larger $\dot{\gamma}$ when the Taylor number $Ta$ exceeds 1700 (Ewoldt et al. 2015). The dashed lines labelled $T = 600$ nN m and $Ta = 1700$ in figure 9.1(a) represent the lower and upper limits of $\dot{\gamma}$, between which $\mu$ can be measured accurately.

Figure 9.1(a) demonstrates the average measurements of $\mu$ for water and PAM with $c = 140$ ppm. For water, $\mu$ was measured for three samples. The three measurements were then averaged at their respective values of $\dot{\gamma}$. The down-sampled error bars in figure 9.1(a) convey the range in the measurements of $\mu$ at each $\dot{\gamma}$. As expected for a Newtonian fluid, the values of $\mu$ are relatively constant with respect to $\dot{\gamma}$ for water. The average $\mu$ of water across all values of $\dot{\gamma}$ (with $T > 600$ nN m and $Ta < 1700$) was 0.98 cP – approximately 2.0% lower than the expected value according to Cheng (2008). This 2% deviation between the expected and
Figure 9.1: Rheology measurements of tap water and the 140 ppm PAM solution. Here (a) corresponds to measurements of steady shear viscosity as a function of shear rate and (b) demonstrates the diameter versus time of thinning droplet expelled from a needle. Black dashed lines in (a) represent the lower and upper shear rate limits of the torsional rheometer. The solid red line in (a) is the fitted line of (9.1) representative of the Carreau shear-thinning trend. The red solid line in (b) is the fitted line of (4.3) which describes elastocapillary thinning.

The measured value of $\mu$ for water is assumed to be a systematic error in the shear viscosity. The measurements of $\mu$ for the 140 ppm PAM solution were taken for eight samples corresponding to different sets of 3D-PTV flow measurements. The data points in figure 9.1(a) are the average measurements of $\mu$ at each corresponding $\dot{\gamma}$ for the eight samples. Similar to water, the down-sampled error bars represent the range in the measurements of $\mu$ at each $\dot{\gamma}$. The error bars in $\mu$ are slightly larger for the PAM solution than water (for $T > 600$ nN m and $Ta < 1700$) and can be attributed to some degradation in the samples as the fluids are pumped within the flume. The largest relative error in $\mu$ is 4.9%; therefore, despite degradation being present, its influence on the measurements of $\mu$ are minimal. For a conservative estimate, it is assumed that the total uncertainty in measurements of $\mu$ is the root sum of the squared systematic uncertainty, determined from the measurements of $\mu$ for water, and the squared relative uncertainty of 4.9% caused by degradation. In other words, the total relative uncertainty in $\mu$ was assumed to be 5.3%.

The Carreau model was fit on the shear rheogram of the 140 ppm PAM solution to approximate the trend in $\mu$ as function of $\dot{\gamma}$ (Carreau, 1972). The model was of the form,

$$\frac{\mu - \mu_\infty}{\mu_0 - \mu_\infty} = \frac{1}{[1 + (M\dot{\gamma})^2]^{(1-k)/2}},$$

(9.1)

where $\mu_0$ is the viscosity at $\dot{\gamma} = 0$, $\mu_\infty$ is the viscosity at $\dot{\gamma} = \infty$, $M$ is the consistency, and $k$ is flow index. Nonlinear least square regression was used to fit (9.1) onto the average values of $\mu$ for the PAM solution between $\dot{\gamma}$ of 1 and 200 s$^{-1}$. The red solid line in figure 9.1 shows the Carreau model; the resulting fit of (9.1) agrees well with the experimental measurements for PAM. The values of $\mu_0$, $\mu_\infty$, $M$, and $k$ were 3.4 cP, 1.0 cP, 0.29 s and 0.76 respectively.
To evaluate the extensional rheology of water and the PAM solution, the deformation of a small droplet of fluid undergoing capillarly-driven thinning was measured, using the DoS apparatus depicted in §4.1.3. Three repeated measurements in the extensional rheometer were performed for water. Recall that eight samples of the PAM solution were collected immediately following 3D-PTV data collection. Three repeated measurements of the extensional rheology were performed for each sample of the PAM solution, resulting in 24 measurements in total. The minimum diameter $D_{\text{min}}$ of the liquid bridge was established using MATLAB software (Mathworks Inc.).

Figure 9.1(b) demonstrates the evolution of $D_{\text{min}}/D_0$ with respect to time $t$. The markers in figure 9.1(b) represent the average values of the repeated measurements of $D_{\text{min}}$ for each instance of $t$. The down sampled error bars indicate the range in the repeated measurements of $D_{\text{min}}$ for each instance of $t$. For water, the liquid bridge ruptures quickly in $t_b$ of 25 ms due to inertial and capillary forces and according to (4.2). The Ohnesorge number $Oh = t_v/t_R$ relates the time scale associated with viscous forces $t_v = \mu_0 D_0/2\sigma$ to that of surface tension and inertial forces, i.e., the Rayleigh time $t_R = (\rho D_0^3/8\sigma)^{1/2}$. Here $\sigma$ is the surface tension, which for water and low concentration solutions of PAM is generally 72 mN m$^{-1}$ (Miller et al., 2009). For both water and PAM, $Oh$ is less than 1, and the thinning process is dominated by inertial and capillary forces (Dinic et al., 2017). However, for the PAM solution, elastic forces also contribute to the pinch-off dynamics. The Deborah number $De = t_e/t_R$ represents the ratio of elastic forces to inertio-capillary forces, where $t_e$ is the elastic relaxation time of the fluid. When $De$ is greater than 1, the droplet exhibits elastocapillary thinning described by (4.3). Nonlinear least square regression is used to fit (4.3) on the average measurements of $D_{\text{min}}/D_0$ for $t > t_b$ of the PAM solution. The solid red line in figure 9.1(b) demonstrates the fitted (4.3) with respect to the measurements of $D_{\text{min}}/D_0$. Using (4.3), $t_e$ of the PAM solutions was determined to be 9.90 ms.

9.2 Flow measurements

Two types of flow measurements were used to characterize the Newtonian and non-Newtonian turbulent boundary layers. The first was 3D particle tracking velocimetry (3D-PTV) based on the shake-the-box (STB) algorithm (Schanz et al., 2016), which was used primarily to measure the VGT. The second consisted of a two-camera planar particle image velocimetry (PIV) setup, that was used to obtain bulk properties of the flow, including $U_\infty$, the momentum thickness $\phi$ and the boundary layer thickness $\delta$. These measurements were done concurrently, after the pumps for the flume were turned on. Both systems are described in the following sections.

9.2.1 3D particle tracking velocimetry

To obtain 3D measurements of the velocity vector $U$ within the Newtonian and non-Newtonian turbulent boundary layers, 3D-PTV using the STB algorithm was used (Schanz et al., 2016). The 3D-PTV measurements produce Lagrangian trajectories representative of particles that travel through the discrete measurement domain. The STB algorithm enhances the 3D-PTV technique by allowing for large particle seeding densities and a significantly greater number of trajectories within the measurement volume (Wieneke, 2012).
et al., 2016). The velocities of the Lagrangian trajectories are then projected onto an Eulerian grid at each instance of time \( t \). This effectively produces 3D time resolved measurements of \( U \).

An isometric view that illustrates the 3D-PTV measurement apparatus, with reference to a section of the water channel, is shown in figure 9.2(a). The 3D-PTV measurement apparatus consisted of four high-speed cameras (Phantom v611, Vision Research Inc.), each of which is labelled from 1 to 4 in figure 9.2. A high-repetition Nd:YLF laser (DM20-527, Photonics Industries), was used to illuminate the volume of interest (VOI). A zoomed in depiction of the VOI is shown in figure 9.2(b) with reference to the Cartesian coordinate system, where \( x \), \( y \) and \( z \) are the streamwise, wall-normal and spanwise directions respectively. The beam that exited the head of the laser had a diameter of 5.8 mm, a wavelength of 532 nm and a maximum pulse energy of 20 mJ pulse\(^{-1}\). The beam exited the laser head along the positive \( z \)--direction and underneath the water channel (i.e., negative \( y \)). One cylindrical lens was used to expand the beam along the \( x \)--direction. A mirror was then used to re-direct the resulting ovular beam along the positive \( y \)--direction. The beam then penetrated the channel from beneath the glass floor. Four knife edges secured to the underside of the glass floor of the water channel were used to crop the ovular laser beam, such that it captured the desired dimensions of the VOI along \( x \) and \( z \) shown on figure 9.2(b). The centre of the laser volume was positioned such that the VOI was at the channel mid-span (\( W/2 \)) along \( z \) and 4.5 m downstream of the inlet to the water channel along \( x \). The cropped laser volume was 3.5 mm thick along \( z \) and approximately 15 mm in width along \( x \), and had a rather uniform intensity profile along those respective directions.

Each of the four high-speed cameras had a 1280 × 800 pixel complementary metal oxide semiconductor sensor. The pixels that comprised each sensor were 20 × 20 \( \mu \)m\(^2\) in size and had a 12 bit digital resolution. To achieve larger acquisition times, the sensors on all cameras were cropped to 1280 × 304 pixel. The four cameras were arranged in a cross-like configuration, as depicted in figure 9.2(a). All cameras were placed in a portrait orientation such that the 1280 pixel dimension of each sensor was parallel to the \( y \)--direction. The three side-scattering cameras, i.e., cameras 1, 2 and 3, were placed along the same horizontal plane, which was parallel to the bottom wall of the channel (or the \( xz \)--plane). Cameras 1 and 3 had a viewing angle of ±30° rotated about the positive \( y \)--axis and depicted in figure 9.2(c). Camera 2 directly imaged the \( xy \)--plane with no viewing angle. The forward-scattering camera, i.e., camera 4, was positioned directly above camera 2 and on the same plane parallel to \( yz \) -- as shown in figure 9.2(d). Camera 4 had a viewing angle that was 20° rotated clockwise about the positive \( x \)--axis. Water-filled prisms helped mitigate image distortion caused by refraction for cameras 1, 3 and 4, which had large viewing angles. Each prism consisted of a 3D printed nylon frame with a glass viewing pane that was bonded to the exterior of the channel side wall and filled with distilled water. Sigma lenses with a focal length \( f \) of 105 mm and 2× teleconverters (Teleplus pro300, Kenko) were used to achieve a magnification of approximately 0.72 for all four of the cameras. All cameras had a lens aperture of \( f/16 \), with an approximated depth-of-focus of 7 mm. Schiempflug adapters were also used for cameras 1, 3 and 4 to ensure images of the VOI were in focus. The cameras and laser were synchronized using a programmable timing unit (PTU X, LaVision GmbH) and image acquisition was performed using DaVIS 8.4 software (LaVision GmbH). The fluids within the flume were seeded with 2 \( \mu \)m silver coated hollow glass spheres (SG02S40, Potters Industries). The density of tracers within the images was approximately 0.05 particles per pixels.
One time-resolved data set, for both measurements of the Newtonian and non-Newtonian turbulent boundary layers, consisted of 14354 single-frame images captured at a frequency between 0.52 kHz and 1.82 kHz. Therefore, one data set took between 7.9 s and 27.6 s or $36.2\delta/U_\infty$ and $62.7\delta/U_\infty$, where $\delta/U_\infty$ is a representative advection time or large eddy turnover time and $\delta$ is the boundary layer thickness. The frequency was selected depending on $U_\infty$, and such that a maximum particle displacement of 5 pixels across subsequent images was achieved. Image processing consisted of first determining the minimum intensity at each pixel and over the complete image ensemble, and then subtracting the minimum from all images in a data set. Second, the intensity signal at each pixel was normalized by the average intensity of the ensemble. Lastly, a sliding minimum subtraction with a kernel size of 5 pixels and local intensity normalization with a kernel size of 500 pixels were applied to every image. For the different Newtonian and non-Newtonian flows, eight data sets, equivalent to 114832 images, were collected to ensure sufficient convergence of the different ensemble statistics in the analysis. Therefore, the total duration of the eight data sets used for computing
ensemble statistics was between $260^T$ to $500^T$ depending on the flow condition. The statistical convergence of the 3D-PTV measurements were evaluated in Appendix A.4. It is shown that all velocity statistics attain sufficient statistical convergence, with low random errors, within the last 5700 realizations.

Calibration of the imaging setup was achieved by fitting a third-order polynomial mapping function onto images of a dual-plane 3D calibration target (025-3.3, LaVision GmbH). Volume self-calibration was used to significantly improve the accuracy of the mapping function (Wieneke, 2008). Self-calibration reduced the average and maximum disparity vector magnitude, or error in the mapping function, to 0.02 and 0.06 pixels respectively. After self-calibration, an optical transfer function was generated to account for changes in the imaged particle patterns across the 3D volume (Schanz et al., 2013). The resulting measurement volume or VOI had dimensions $(\Delta x, \Delta y, \Delta z) = 272, 1220, 102$ voxel $= 8.0, 35.8, 3.0$ mm$^3$, as shown in figure 9.2(b). Finally, the STB algorithm was performed using DaVIS 10.2 software (LaVision GmbH). The maximum triangulation error was set to 1 voxel, and particle displacements were limited to a maximum of 8 voxel. Particles with an acceleration that was larger than 2 pixels or 20% between subsequent image frames were discarded. The STB algorithm yielded approximately 6200 Lagrangian trajectories per time step within the VOI.

A moving first-order polynomial with a length of nine time steps was fit on the particle trajectories. Two types of binning were used to convert the Lagrangian trajectories into Eulerian vector components. The first involved averaging the trajectories into slabs that were parallel with the wall and covered the entire measurement domain along $x$ and $z$. Each slab was 6 voxels or 0.18 mm thick in the $y$–direction. Neighbouring slabs along $y$, overlapped by 75%. This binning procedure was used exclusively for establishing the mean streamwise velocity $\langle U \rangle$ with high spatial resolution. Here, the angle brackets $\langle \cdots \rangle$ denote averaging in time and along the spatially homogeneous direction $z$. It was also assessed that $\langle U \rangle$ did not vary significantly along $\Delta x$ within the VOI; hence, the statistics were also averaged along the $x$–direction within the VOI. The second binning procedure involved averaging particle tracks for each time step in $32 \times 32 \times 32$ voxel or $0.94 \times 0.94 \times 0.94$ mm$^3$ cubes to obtain the instantaneous velocity vector $U$ within the domain. Neighbouring cubes had 75% overlap with one another along the three Cartesian directions. Therefore, adjacent vectors were separated by 8 voxels or 0.235 mm. In terms of viscous wall units $\delta_v$, the bins were between $6.9\delta_v \times 6.9\delta_v \times 6.9\delta_v$ and $9.3\delta_v \times 9.3\delta_v \times 9.3\delta_v$ depending on the flow considered. The streamwise, wall-normal and spanwise components of the instantaneous velocity $U$ are denoted as $U, V$ and $W$, respectively. Velocity fluctuations were represented using lower case symbols, i.e., $u, v$ and $w$.

A moving first-order polynomial surface was fitted to the velocity components at each instance of time and then differentiated to obtain spatial gradients in velocity. The size of polynomial surface was three velocity vector components along each Cartesian direction, which equates to $24 \times 24 \times 24$ voxels or $0.704 \times 0.704 \times 0.704$ mm$^3$. Spatial velocity gradients were then used to analyze the topology of the Newtonian and non-Newtonian turbulent boundary layers according to §3.3.

The uncertainty in the 3D-PTV measurements is scrutinized in Appendix A.5. Uncertainty is primarily assessed based on how well the velocity vectors satisfy the divergence free condition, where $\nabla \cdot U = 0$. Appendix A.5 demonstrates that the present measurements adequately satisfy the divergence-free condition compared to other investigations that have utilized experimental flow measurements to measure the VGT.
The variables for inner scaling were established by fitting a linear function to the mean velocity profile $\langle U \rangle$ of each flow near the wall. The linear function was then differentiated in order to determine the near-wall shear rate $\dot{\gamma}_w$ of each flow. Here, $\dot{\gamma}_w$ is established by differentiating the mean velocity, i.e., $\partial \langle U \rangle / \partial y$, for $y > 0.2$ mm and $y^+ < 3$. The lower bound of the fit was the smallest measurable value of $y$ with a slab that did not overlap with the wall. While the upper bound of the fit is within the theoretical limit of the linear viscous sublayer. The wall shear stress $\tau_w$ was then established according to $\tau = \mu(\dot{\gamma}) \dot{\gamma}_w$ (similar to [3.7]), where $\mu(\dot{\gamma})$ is the viscosity of the fluid evaluated at the near-wall shear rate $\dot{\gamma}_w$ using the Carreau model that was fitted to measured values of $\mu$ for PAM detailed §9.1. For the water flows, $\mu$ does not vary with shear rate, and was equal to 1.00 cP according to Cheng (2008) and measurements of $\mu$ for water in §9.1. After establishing $\tau_w$, the friction velocity $u_\tau$ from (3.10) and viscous lengthscale $\delta_v$ from (3.11) were determined. Several other variables were also used to characterize the flows. For example the skin friction coefficient $C_f = 2\tau_w/\rho U_\infty$ was used to defined the local friction of the boundary layer. The friction Reynolds number $Re_\tau$ was determined according to (3.19). Lastly, the Weissenberg number $Wi = t_e \dot{\gamma}_w$ was used to define the ratio between the elastic and viscous forces of the flow. The different variables of the flow are listed in table 9.1.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>$U_\infty$ (m s$^{-1}$)</th>
<th>$\phi$ (mm)</th>
<th>$\delta$ (mm)</th>
<th>$u_\tau$ (mm s$^{-1}$)</th>
<th>$\delta_v$ (mm)</th>
<th>$Re_\phi$</th>
<th>$Re_\tau$</th>
<th>$C_f \times 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
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<td>9.77</td>
<td>81.87</td>
<td>7.60</td>
<td>0.132</td>
<td>1814</td>
<td>612</td>
<td>3.35</td>
</tr>
<tr>
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<td>77.94</td>
<td>9.90</td>
<td>0.101</td>
<td>2257</td>
<td>765</td>
<td>3.22</td>
</tr>
<tr>
<td>PAM</td>
<td>0.432</td>
<td>10.33</td>
<td>94.31</td>
<td>14.20</td>
<td>0.137</td>
<td>2290</td>
<td>687</td>
<td>2.16</td>
</tr>
</tbody>
</table>

Table 9.1: Inner and outer scaling variables of the Newtonian and non-Newtonian turbulent boundary layers.

### 9.2.2 Planar particle image velocimetry

For all of the flows considered, the VOI measured using 3D-PTV did not capture the complete boundary layer thickness along $y$. Therefore, a planar PIV setup was used to obtain measurements of $\langle U \rangle$ over a larger field of view ($0 < y < \delta$), in order to determine the bulk flow properties of the Newtonian and non-Newtonian turbulent boundary layers. These bulk properties include the momentum thickness $\phi$ from (3.18), boundary layer thickness $\delta$ and free-stream velocity $U_\infty$, all of which are listed in table 9.1. The boundary layer thickness is assessed as the $y$ location where $\langle U \rangle = 0.99 U_\infty$. The measurement location of the planar PIV apparatus was situated at the centre of the water channel along $z$, and 200 mm upstream of the VOI along $x$.

The planar PIV setup consisted of two double-frame digital cameras (Imager Intense, LaVision GmbH), each of which had a $1376 \times 1040$ pixels charged-coupled device sensor. Each pixel in the sensor was $6.45 \times 6.45$ $\mu$m$^2$ in size and had a 12 bit digital resolution. The sensors were cropped to $1376 \times 128$ pixels to enable higher acquisition rates, where the 1376 pixel dimension was parallel to the $y$–direction. Double-
frame images were acquired at a frequency of 14.3 Hz. The fields of view (FOVs) of both cameras were stacked along the wall-normal direction $y$, and covered a region with a size of $\Delta x = 7.0$ mm and $\Delta y = 143.1$ mm. The FOVs were placed at the centre of the channel along $z$ and 200 mm upstream of the VOI for 3D-PTV, along $x$. Illumination was provided from a 15 mJ pulse$^{-1}$ Nd:YAG laser (Solo I-15, New Wave Research Inc.), that was synchronized with the cameras using a programmable timing unit (PTU 9, LaVision GmbH) and DaVIS 7.3 software (LaVision GmbH). Two spherical lenses (one concave, the other convex) and one concave cylindrical lens expanded the laser beam into a 20-mm-wide (along $x$) and a 1-mm-thick (along $z$) laser sheet. One data set consisted of 800 pairs of double-frame images, which took 56 s to collect. The time delay $\Delta t$ between subsequent frames was between 1.43 and 5.00 ms depending on $U_\infty$. The value of $\Delta t$ was chosen such that the maximum particle displacement between image frames was approximately 15 pixels. Recall from §9.1 that eight data sets were collected for the two cases of water (corresponding to different $Re$) and the one condition of PAM. Therefore, each flow scenario consisted of 6400 double frame images.

Image processing was performed using DaVIS 8.4 software (LaVision GmbH). First the minimum intensity in each pixel was determined in each data set and subtracted from every image in the ensemble. Second, the intensity signals in each pixel were normalized by the average intensity of the ensemble. Vector fields were then established using cross-correlation with an initial interrogation window (IW) size of 64 x 64 pixels and a final IW size of 24 x 24 pixels with 75% overlap between neighbouring IWs. The mean streamwise velocity $\langle U \rangle$ with respect to $y$ was determined by averaging $U$ over all instances of time $t$ and along the $x$-direction. Profiles of $\langle U \rangle$ with respect to $y$ were then used to establish the free-stream velocity $U_\infty$, the boundary layer thickness $\delta$ and momentum thickness $\phi$.

### 9.3 Velocity statistics

Figure 9.3(a) demonstrates inner-normalized mean streamwise velocity $\langle U \rangle^+$ with respect to $y^+$ for the experimentally measured turbulent boundary layers of water with different $Re_\phi$ and the 140ppm PAM solution. Experimental $\langle U \rangle^+$ profiles are shown alongside the mean velocity profile derived from Newtonian turbulent boundary layer DNS in Jiménez et al. (2010) at an $Re_\phi$ of 1968, and also the law of the wall. All flows, both water and PAM, closely follow the linear viscous sublayer $\langle U \rangle^+ = y^+$ for $y^+ < 3$. For $y^+ > 30$ the boundary layers of water with different $Re_\phi$ both overlap with a logarithmic profile $\langle U \rangle^+ = 1/\kappa \ln(y^+) + B$ that has a Von Kármán coefficient $\kappa$ of 0.384 and an intercept $B$ of 4.5 – similar to the values prescribed by Nagib & Chauhan (2008) for Newtonian turbulent boundary layers. The polymer-laden flow exhibits enhanced values of $\langle U \rangle^+$ relative to the Newtonian boundary layers for $y^+ > 30$, a feature common in drag-reduced flows of polymer solutions. The slope in the log layer $B$ of the polymer-laden boundary layer is larger than $B$ for water, and visually $\kappa$ is approximately the same. Although $\langle U \rangle^+$ is enhanced within the outer layer of the polymer-laden flow, it does not overlap with the maximum drag reduction asymptote $\langle U \rangle^+ = 11.7 \ln(y^+) - 17.0$ of Virk et al. (1970).

Figure 9.3(b) demonstrates inner-normalized plots of the four non-zero components of the Reynolds stress tensor with respect to $y^+$. Listed in descending order of magnitude, $\langle u^2 \rangle^+$, $\langle w^2 \rangle^+$ and $\langle v^2 \rangle^+$ are the streamwise, spanwise and wall-normal Reynolds stresses respectively, and $\langle uv \rangle^+$ is the Reynolds shear stress.
The experimentally measured profiles of $\langle u^2 \rangle^+$, $\langle v^2 \rangle^+$ and $\langle uv \rangle^+$ for water overlap well with the DNS of Jiménez et al. (2010) at a comparable $Re_\phi$. That being said, the measured Reynolds stress profiles of $\langle w^2 \rangle^+$ for water are marginally less than that of the Newtonian DNS. That being said, profiles of $\langle w^2 \rangle^+$ for water with slightly different $Re_\phi$ show consistency with one another. Therefore, it is speculated that the lower than normal profile of $\langle w^2 \rangle^+$ is a unique condition of the present flow. Relative to the boundary layers of water, the polymer-laden boundary layer has augmented values of $\langle u^2 \rangle^+$ for $y^+ < 150$ and attenuated values of $\langle w^2 \rangle^+$, $\langle v^2 \rangle^+$ and $-(uv)^+$ for $y^+ < 100$. The peak in $\langle v^2 \rangle^+$ is also shifted away from the wall for the PAM flow relative to water; for PAM, the peak in $\langle u^2 \rangle^+$ is at a $y^+$ of 21, while for both of the water flows, the peak in $\langle u^2 \rangle^+$ is at a $y^+$ of approximately 13.

Experimentally measured mean velocity and Reynolds stress profiles of PAM, shown in figure 9.3, reflect consistency with prior measurements of polymer drag-reduced flows with low drag reduction percentages (LDR) that are less than 38% (Warholic et al., 1999b). LDR flows typically have an expanded buffer layer and a log layer with a larger $B$ – often referred to as a Newtonian plug (Virk et al., 1970; Warholic et al., 1999b). The larger $B$ is visually apparent in figure 9.3(a), and the expanded buffer layer is evident based on the shift in the peak of $\langle u^2 \rangle^+$ to larger $y^+$, seen in figure 9.3(b). Warholic et al. (1999b) similarly demonstrated that polymer drag-reduced channel flows at LDR consist of augmented $\langle u^2 \rangle^+$ values and attenuated $\langle w^2 \rangle^+$, $\langle v^2 \rangle^+$ and $-(uv)^+$ values relative to water at a comparable $Re_\tau$. Generally, the ensemble velocity statistics of PAM are in good agreement with the LDR flows depicted in Warholic et al. (1999b) and other investigations (Escudier et al., 2009; Warwaruk & Ghaemi, 2022; Mitishita et al., 2023).

Two-point correlation of the streamwise velocity fluctuations $u$ is used to obtain a depiction of the integral length scale within the buffer layer of each flow. Moreover, it is the most common metric used in prior investigations of polymer drag-reduced flows for quantifying the size of large scale coherent motions. Therefore, it provides another good baseline comparison between the current and prior investigations of LDR flows. The spatial two-point correlation is calculated according to,
\[
R_{uu}(\Delta x^+, \Delta y^+, \Delta z^+) = \frac{\langle u(x_0^+, y_0^+, z_0^+), u(x_0^+ + \Delta x^+, y_0^+ + \Delta y^+, z_0^+ + \Delta z^+) \rangle}{\sqrt{\langle u^2(x_0^+, y_0^+, z_0^+) \rangle} \sqrt{\langle u^2(x_0^+ + \Delta x^+, y_0^+ + \Delta y^+, z_0^+ + \Delta z^+) \rangle}}, \tag{9.2}
\]

where \((x_0^+, y_0^+, z_0^+)\) is the coordinate of a reference point, and \((\Delta x^+, \Delta y^+, \Delta z^+)\) are small displacements relative to the point of reference. Here, the point of reference is taken to be \((x_0^+, y_0^+, z_0^+) = 0, 20, 0\), which is at the border of the domain along \(x\) and \(z\), and in a buffer layer of the flow along \(y\).

Open contours of \(R_{uu}\) along the \(xy\)-plane and at \(z^+ = 0\) are shown in figure 9.4(a). Contours are coloured according to the different flows and similar to that of figure 9.3. For the water flows at different \(Re_\phi\), contours of \(R_{uu}\) overlap, implying that the length of the large scale motions in viscous wall units are the same. Evidently, the VOI is not wide enough along \(x\) to capture the complete integral length scale of each flow, or where \(R_{uu}\) becomes zero. That being said, it is clear that the PAM flow has a different distribution of \(R_{uu}\) than water. Compare, for example, \(R_{uu}\) with a value 0.95 or 0.85 in figure 9.4(b) for PAM to water. Values of \(R_{uu}\) = 0.95 for water extend to \((\Delta x^+, \Delta y^+) = 23.0, 4.8\) while for PAM, values of \(R_{uu}\) = 0.95 stretch to \((\Delta x^+, \Delta y^+) = 42.3, 2.8\). This demonstrates that the large scale motions within the buffer layer of the PAM boundary layer are double the length along \(x\) compared to those of water, and less angled upwards along \(y\).

![Figure 9.4: Two-point correlation \(R_{uu}\) of streamwise velocity fluctuations along the (a) \(xy\)-plane at \(z^+ = 0\), and (b) along the \(xz\)-plane at \(y^+ = 20\). The reference point for the correlation is \((x_0^+, y_0^+, z_0^+) = 0, 20, 0\). The colours of the contours correspond to the same line colours and conditions of figure 9.3. Grey is water with \(Re_\phi = 1814\), black is water with \(Re_\phi = 2257\), and red is the 140 ppm PAM flow with \(Re_\phi = 2290\).](image)

Contours of \(R_{uu}\) along the \(xz\)-plane and at \(y^+ = 20\) can be seen in figure 9.4(b). Similar to the \(xy\)-plane, contours of \(R_{uu}\) along the \(xz\)-plane overlap for the water flows with different \(Re_\phi\), implying that the width of the large scale motions in viscous wall units are the same. Based on figure 9.4(b), it is also apparent that the VOI is not wide enough along \(z\) to capture the complete width of the large scale motions where \(R_{uu} = 0\). However, much like figure 9.4(a), there is an unambiguous difference in the contours of \(R_{uu}\) along the \(xz\)-plane among PAM and water. Contours of \(R_{uu}\) that are similar in value extend to larger \(\Delta z^+\) for PAM compared to water. For example, when \(\Delta z^+ = 0\), values of \(R_{uu}\) equal to 0.85 extend to \(\Delta z^+\) of 10.6 for water. For PAM, the contour of \(R_{uu} = 0.85\) extends farther, to \(\Delta z^+\) of 17.6, implying that the large scale motions in
the flow of PAM are wider compared to water. An elongation and widening of high- and low-speed velocity streaks is a common feature of polymer drag-reduced flows (Warholic et al., 1999b; White et al., 2004; Farsiani et al., 2020; Warwaruk & Ghaemi, 2021). The difference in $R_{uu}$ among PAM and water observed in figure 9.4 implies the same augmentation to size of the large scale flow motions within the buffer layer.

Overall, the results of the current section demonstrates that the PAM boundary layer has one-point and two-point velocity statistics common for an LDR flow. It does not, however, provide a complete depiction of how, and why, the velocity statistics within the polymer-laden flow are different than a Newtonian turbulent boundary layer. For this, the distribution of fine scale motions and streamline patterns within the Newtonian and non-Newtonian boundary layers are scrutinized using the $\Delta$-criterion.

### 9.4 Flow topology

The topology of the Newtonian and non-Newtonian boundary layer is evaluated using the $\Delta$-criterion detailed in §3.3. Previous investigations of wall-bounded turbulence generally separate the topology of the flows into different regions of $y^+$, e.g., viscous sublayer, buffer layer, log layer, and wake region. Before separating the flow into these different wall-normal regions, the invariants in $L$, $D$, and $W$ are evaluated for the complete spatial domain. Note, that all gradients are made dimensionless by multiplying the components of $L$ by the large eddy turnover time $T = \delta / U_\infty$ of each flow. Probability density functions (PDFs) are used to establish a histogram of the invariants $P_L$, $Q_L$, $R_L$, $Q_D$, $Q_W$ and $R_D$, as well as the discriminant in (3.28) $\Delta$ and (3.31) $\Delta_D$. Certain PDFs, such as $P_L$ and $\Delta_D$, also demonstrate the accuracy of the 3D-PTV flow measurements.

PDFs of $P_L$, $Q_L$ and $R_L$ are shown in figure 9.5(a) for the boundary layers of water at $Re_\phi = 1814$ and 2257, and PAM. For an incompressible fluid flow the first invariant in $L$, $P_L$ is equal to zero; however, the present experiments are subject to a divergence error ($P_L \neq 0$) caused by experimental noise and the limited spatial resolution involved with binning the Lagrangian trajectories produced from 3D-PTV. This error has been shown to have a significant impact on the measured topology of each flow using the $\Delta$-criterion (Ganapathisubramani et al., 2007; Buxton et al., 2011). Therefore, a stringent evaluation of the divergence error is made in Appendix A.5. It is shown in Appendix A.5 that the divergence errors are comparable or better than those of prior experimental investigations that have utilized multi-probe hot wire techniques, holographic PIV, dual-plane stereoscopic PIV, and stereoscopic PIV utilizing Taylor’s hypothesis to measure the components of the VGT (Tsinober et al., 1992; Zhang et al., 1997; Ganapathisubramani et al., 2007; Buxton et al., 2011; Gomes-Fernandes et al., 2014). Based on figure 9.5(a), it is also apparent that values of $P_L$ are significantly smaller than other invariants, such as $Q_L$ and $R_L$ for all flow conditions. For the flows of water at different $Re_\phi$, PDFs of $Q_L$ and $R_L$ overlap. Values of $Q_L$ tend to be more positively skewed, while values of $R_L$ are more negatively skewed. The PDF in $R_L$ also cover a larger range of values than $Q_L$. Relative to water, the PAM flow has much fewer instances of non-zero values in both $Q_L$ and $R_L$.

Interestingly, PDFs of $Q_L$ and $R_L$ for PAM do not have a noticeable skewness and reflect a similar range of values, unlike the PDFs for water.

Figure 9.5(b) demonstrates PDFs of $Q_W$, $Q_D$ and $R_D$ for the flows of water and PAM. As expected, all values of $Q_D$ are negative, while all values of $Q_W$ are positive. Similar to figure 9.5(a) PDFs of $Q_W$, $Q_D$, and $R_D$...
Figure 9.5: Probability density functions of (a) $P_L$, $Q_L$ and $R_L$, (b) $Q_D$, $Q_W$ and $R_D$, (c) $\Delta$, and (d) $\Delta_D$, for all measured $y^+$. In (a) PDFs of $P_L$ are the solid lines, PDFs of $Q_L$ are the dashed lines, and PDFs of $R_L$ are the dotted lines. In (b) PDFs of $Q_D$ are the solid lines, PDFs of $Q_W$ are the dashed lines, and PDFs of $R_D$ are the dotted lines. All flow gradients are made dimensionless by multiplying by the large eddy turnover time $T$.

and $R_D$ overlap for the flows of water at different $Re_\phi$. The flow of PAM, on the other hand, has a higher likelihood of non-zero values of $Q_W$ and $Q_D$ compared to water – an opposite trend than the PDFs shown in figure 9.5(a). The PDF of $R_D$ is similar for PAM and water for negative values of $R_D$; however, the probability of positive $R_D$ values is lower for PAM compared to water.

Figure 9.5(c) shows PDFs of the discriminant $\Delta$ established using (3.28). PDFs of $\Delta$ are positively skewed and overlap for the two boundary layer flows of water. The boundary layer flow of PAM, on the other hand, has fewer instances of non-zero $\Delta$. PDFs of $\Delta_D$ determined from (3.31) are provided in figure 9.5(d). Recall from §3.3 that the discriminant $\Delta_D$ is always less than 0. This is predicated on the assumption that $P_L = P_D = 0$ and (3.29) only consists of the invariants $Q_D$ and $R_D$. Therefore, the positive values of $\Delta_D$ seen in the PDFs of figure 9.5(d) are a result of the divergence error or non-zero values of $P_L$. Despite the
Joint probability density functions (JPDFs) of the different invariants in $L$, $D$, and $W$ are used to determine the distribution of fine scale motions within certain wall-normal bounds of each flow ($\text{§}$3.3). The wall-normal bounds include the buffer layer ($5 < y^+ < 30$), the log layer ($y^+ > 30$, $y/\delta < 0.3$), and the wake region ($y/\delta > 0.3$) (Pope 2000). The JPDFs of $Q_L$ and $R_L$ (similar to figure 3.5), $Q_D$ and $R_D$ (figure 3.6), $-Q_D$ and $Q_W$ (figure 3.7) are presented for each wall-normal region of the flow. The results for the boundary layers of water are first shown, followed by PAM. Figure 9.6 presents JPDFs of the different tensor invariants for water with an $Re_\phi$ of 1814 alongside the other flow of water with an $Re_\phi$ of 2257. Filled contours correspond to the lower $Re_\phi$ case, while the open contours with black dashed lines are the higher $Re_\phi$ scenario.

The JPDF of $Q_L$ and $R_L$ within the buffer layer, log layer and wake region are presented in figures 9.6(a, b, c) respectively, for the different boundary layers of water. Within the buffer layer, i.e., figure 9.6(a), the $Q_L - R_L$ JPDF is skewed towards positive $Q_L$, but rather evenly distributed among positive and negative values of $R_L$. Overall, there is preference towards focal topologies with $\Delta > 0$. Moving farther away from the wall and into the log layer, the $Q_L - R_L$ JPDF in figure 9.6(b) continues to reflect a preference for topologies with $\Delta > 0$. As expected, the strength of the velocity gradients diminishes with increasing distance from the wall and the range of possible $Q_L$ and $R_L$ values decreases. Within the log layer, the shape of the $Q_L - R_L$ JPDF takes on a more well-defined tear-drop pattern with a clear point at the right-Vieillefosse tail ($\Delta = 0$, $R_L > 0$) compared to the JPDF of the buffer layer in figure 9.6(a). Moving into the wake region, figure 9.6(c) demonstrates that the range in possible values of $Q_L$ and $R_L$ continues to decrease with increasing $y$. That being said, the general shape of the $Q_L - R_L$ JPDF is similar to that of the log-layer in figure 9.6(b). A similar enhancement in the shape of the tea-drop pattern with increasing $y$ was also observed in Newtonian DNS of channel flows by both Blackburn et al. (1996) ($Re_\tau = 395$) and Mortimer & Fairweather (2022) ($Re_\tau = 180$), and boundary layers by Chong et al. (1998), who used the boundary layer DNS of Spalart (1988) with an $Re_\phi$ of 670. When comparing the JPDFs of $Q_L$ and $R_L$ for water at different $Re_\phi$, similar contour levels overlap within their respective wall-normal region of the flow, implying the flows at different $Re_\phi$ posses a similar distribution of fine scale motions.

The most notable difference between the $Q_L - R_L$ JPDFs of figure 9.6(a, b, c) and that of Newtonian wall-bounded DNS (Blackburn et al. 1996; Chong et al. 1998; Mortimer & Fairweather 2022), is that DNS produces a more "pointed" ridge at the right Vieillefosse tail. Buxton et al. (2011) demonstrated that divergence errors, inherent in most experimentally derived velocity vectors, do not alter the general shape and limits of the $Q_L - R_L$ JPDF, with the exception that it erodes the tip of the $Q_L - R_L$ JPDF along the right-Vieillefosse tail, making it more rounded. Although the tip of the tear-drop pattern becomes more rounded from divergence errors, Buxton et al. (2011) demonstrated that it continues to remain centred on the right-Vieillefosse tail, i.e., $\Delta = 0$. Considering the present measurements have a comparable divergence error to that of prior experimental investigations of the VGT, as demonstrated in Appendix A.5, the JPDFs shown
Figure 9.6: Joint probability density functions of the invariants in the VGT, rate of deformation tensor and rate of rotation tensor for boundary layers of water. Rows of figure correspond to different wall-normal locations: (a, b, c) buffer layer, (d, e, f) log layer, (g, h, i) wake region. Columns of figure correspond to JPDFs of different invariants: (a, d, g) $Q_L$ and $R_L$, (b, e, h) $Q_D$ and $R_D$, (c, f, i) $-Q_D$ and $Q_W$. Filled contours are the JPDFs of water with $Re_\phi = 1814$, open contours with black dashed lines are the JDFs of water with $Re_\phi = 2257$ at $10^{-5}$ and $10^{-4}$.

Figure 9.6(a, b, c) should provide a reasonable depiction of the distribution of fine scale motions within the Newtonian boundary layer. Moreover, the $Q_L - R_L$ JPDFs overlap for the different flows of water with unique $Re_\phi$ and spatial resolutions. Also, the JPDFs of $Q_L$ and $R_L$ take on a similar shape as those derived experimentally in a Newtonian turbulent boundary layer with approximately 5 times the spatial resolution of the present measurements and a presumably higher divergence error (Elsinga & Marusic, 2010).

Figure 9.6(d, e, f) demonstrates the JPDFs of the invariants in $\mathbf{D}$ for the water boundary layers with different $Re_\phi$. JPDFs of $Q_D$ and $R_D$ are presented alongside lines of different eigenvalue ratios, namely $\Gamma_2/\Gamma_1$, similar to that shown in figure 3.6. Recall from §3.3 that $\Gamma_2/\Gamma_1 = 1$ corresponds to biaxial extension.
\( \Gamma_2/\Gamma_1 = 0 \) represents steady shear or planar extension, and \( \Gamma_2/\Gamma_1 = -1/2 \) is uniaxial extension. Ashurst et al. (1987) demonstrated that the most probable eigenvalue ratio was \( \Gamma_2/\Gamma_1 = 1/3 \) using DNS of Newtonian isotropic turbulence, hence \( \Gamma_2/\Gamma_1 = 1/3 \) is also shown on figure 9.6(d, e, f). Within the buffer layer, shown in figure 9.6(d), there is a higher preference towards unstable node-saddle-saddle flow events with \( R_D > 0 \) and \( \Gamma_2/\Gamma_1 \) between 0 and 1. Interestingly, a large ridge in the \( Q_D - R_D \) JPDF within the buffer layer appears to align with the preferential eigenvalue ratio of \( \Gamma_2/\Gamma_1 = 1/3 \) for the Newtonian isotropic turbulence found by Ashurst et al. (1987). Moving away from the wall to the log layer and wake region shown in figure 9.6(e, f), the flow becomes increasingly skewed toward biaxial extensional flow events with \( R_D > 0 \) and \( \Gamma_2/\Gamma_1 > 0 \). Compared to the \( Q_D - R_D \) JPDF of the buffer layer, shown in figure 9.6(d), the log layer and wake regions have more events with \( \Delta_D > 0 \), indicative of divergence errors. Based on Appendix A.5 and also shown in Ganapathisubramani et al. (2007) and Gomes-Fernandes et al. (2014), regions of the flow with lower velocity gradients are generally coupled with larger divergence errors. Therefore, it is expected that the log and wake layers, with overall smaller velocity gradients than the buffer layer, may exhibit higher divergence errors – an effect of this being a positive \( \Delta_D \). That being said, the JPDFs of \( Q_D \) and \( R_D \) are generally similar to those derived from Blackburn et al. (1996) and Chong et al. (1998) using DNS of a Newtonian channel flows (\( Re_\tau = 395 \)) and boundary layers (\( Re_\phi = 670 \)), where preference to \( R_D > 0 \) grows as \( y \) increases.

Similar to the VGT invariants, \( Q_D - R_D \) JPDFs of water at different \( Re_\phi \) overlap, implying the straining motions within Newtonian flow are also similar among turbulent boundary layers at different \( Re_\phi \).

JPDFs of the invariants \( -Q_D \) and \( Q_W \) are presented for the water boundary layers at different \( Re_\phi \) in figure 9.6(g, h, i), similar to that of figure 3.7. Much like the previously detailed JPDFs of \( Q_L - R_L \) and \( Q_D - R_D \), the JPDFs of \( -Q_D \) and \( Q_W \) overlap for the water flows at different \( Re_\phi \). Within the buffer layer of the flow, shown in figure 9.6(g), there is a preference towards flow motions exhibiting conditions consistent with steady shear, with \( K = 1 \). Soria et al. (1994) detailed that turbulent mixing layers with flow regions having \( K = 1 \) consisted almost entirely of vortex sheets. Chong et al. (1998) demonstrated a similar preference to \( K = 1 \) and vortex sheet topologies within the buffer layer of a turbulent boundary flow with an \( Re_\phi \) of 670, that was derived from Newtonian DNS (Spalart 1988). Although the JPDF of figure 9.6(g) is concentrated around \( K = 1 \), there are deviations, particularly at smaller values of \( -Q_D \) and \( Q_W \). Chong et al. (1998) similarly observed subtle deviations from \( K = 1 \) within the buffer layer near the origin of \( -Q_D \) and \( Q_W \) in their analysis of Newtonian boundary flow DNS by Spalart (1988). Within the log and wake layers of the flow, shown in figure 9.6(h, i), a large spread between \( K \) of 0 and \( \infty \) emerges. Therefore, fine scale motions within the log and wake layers take on a variety of patterns, ranging from extensional to rotational, and the topology is similar to isotropic turbulence seen in Ooi et al. (1999).

JPDFs of the invariants \( L \), \( D \), and \( W \) are shown in figure 9.7 for the polymer-laden boundary layer at different wall-normal regions of the flow. The limits of the wall-normal regions are the same as those from figure 9.6. Open contours with black dashed lines in figure 9.7 are the JPDFs of water with an \( Re_\phi \) of 2257. Figure 9.7(a) provides the JPDF of \( Q_L \) and \( R_L \) for \( 5 < y^+ < 30 \). Compared to the flow of water at a similar \( Re_\phi \), the PAM boundary layer has attenuated values of \( Q_L \) and \( R_L \). The range in possible \( R_L \) values narrows considerably compared to water – almost a two fold reduction in the largest magnitude of \( R_L \). A narrower range in \( R_L \) was similarly observed by Mortimer & Fairweather (2022) for drag-reduced viscoelastic channel
flows at an $Re_\tau$ of 180 and derived from DNS. This is a general indication that stretching and extensional motions within the flow are diminished. Moving away from the wall, figure 9.7(b) demonstrates the $Q_L - R_L$ JPDF for $y^+ > 30$ and $y/\delta < 0.3$. Evidently, a reduction in the magnitude of $Q_L$ and $R_L$ relative to water at a similar $Re_\phi$ is still present farther from the wall. The tear-drop pattern no longer exists in the $Q_L - R_L$ JPDF of PAM, and a well-defined tip does not appear along the right-Vieillefosse tail. The trend continues into the wake region of the flow; figure 9.7(c) demonstrates again how the range of possible $Q_L$ and $R_L$ values is diminished for PAM relative to water. This is despite the fact that the boundary layers of PAM and water have comparable velocity fluctuations within the outer layer of the flow, as seen in figure 9.3(b).

Figure 9.7: Joint probability density functions of the invariants in the VGT, rate of deformation tensor and rate of rotation tensor for boundary layers of water. Rows of figure correspond to different wall-normal locations: (a, b, c) buffer layer, (d, e, f) log layer, (g, h, i) wake region. Columns of figure correspond to JDFs of different invariants: (a, d, g) $Q_L$ and $R_L$, (b, e, h) $Q_D$ and $R_D$, (c, f, i) $-Q_D$ and $Q_W$. Filled contours are the JPDFs of the PAM boundary layer, open contours with black dashed lines are the JDFs of water with $Re_\phi = 2257$ at $10^{-5}$ and $10^{-4}$.
Perhaps the most obvious difference between the topology of PAM and water are revealed in the JPDFs of \( Q_D \) and \( R_D \). Figure 9.7(d) demonstrates the \( Q_D - R_D \) JPDF for the polymer-laden boundary layer relative to water for \( 5 < y^+ < 30 \). Although there is still a bias towards an unstable node-saddle-saddle flow type, the preference to \( R_D > 0 \) is greatly diminished relative to water at a similar \( Re_\phi \). Rather, the flow tends towards an eigenvalue ratio \( \Gamma_2/\Gamma_1 \) of 0, where the flow is more two-dimensional with conditions comparable to steady shear or planar extension. Farther from the wall for \( y^+ > 30 \), straining motions within the flow of PAM shown in figure 9.7(e, f) become more biased towards biaxial stretching, but do not show as strong of a preference to \( \Gamma_2/\Gamma_1 = 1 \) as water.

JPDFs of \(-Q_D\) and \( Q_W\) also demonstrate an unambiguous difference between fine scale motions within the polymer-laden and Newtonian boundary layers. Compared to water, the flow of PAM near the wall shown in figure 9.7(g) consists of \(-Q_D\) and \( Q_W\) values that are almost always equivalent and concentrated on the line \( K = 1 \). Together, with figure 9.7(d), this implies that the near-wall flow of PAM is primarily two-dimensional and shear-dominate, with sheet-like motions. Moving farther from the wall and into the range of \( y^+ > 30 \) and \( y/\delta < 0.3 \), the higher tendency for the PAM flow to exhibit features with \( K = 1 \) continues. Compared to the flow of water at a similar \( Re_\phi \), the JPDF of \(-Q_D\) and \( Q_W\) shown in figure 9.7(h) shows more of a preference towards shear-dominate flow with \( K = 1 \), albeit less so than the PAM flow near the wall for \( 5 < y^+ < 30 \). Within the wake region, figure 9.7(i) demonstrates a more scattered JPDF of \(-Q_D\) and \( Q_W\) for the flow of PAM with no clear preference to a particular value of \( K \), but also no overlap with the JPDF of water at a similar \( Re_\phi \).

Based on the JPDFs of \( Q_D \) and \( R_D \) shown in figure 9.6(d, e, f) most straining motions within the Newtonian boundary layers were unstable node-saddle-saddle, with \( \Gamma_2/\Gamma_1 \) greater than 0. However, for the polymer-laden boundary layer, shown in figure 9.7(d, e, f), straining motions near the wall were more two-dimensional where \( \Gamma_2/\Gamma_1 = 0 \), and the local fine scale motions are akin to steady shear or planar extension. PDFs of \( \Gamma_2/\Gamma_1 \) are provided for the Newtonian and polymer-laden boundary layers within the buffer, log and wake regions in figure 9.8(a, b, c) respectively. The eigenvalues \( \Gamma_1 \) and \( \Gamma_2 \) are determined from locally solving (3.29) at every spatial coordinate and time instance. For the boundary layers of water, PDFs of \( \Gamma_2/\Gamma_1 \) overlap within the buffer layer, log layer and wake region. The probability of \( \Gamma_2/\Gamma_1 > 0 \) for the boundary layers of water are 58%, 70% and 70% for the buffer layer, log layer and wake region respectively, demonstrating the overall preference to an unstable node-saddle-saddle topology. Based on the near-wall PDF of \( \Gamma_2/\Gamma_1 \) for the PAM boundary layer, shown in figure 9.8(a), there is much higher probability of \( \Gamma_2/\Gamma_1 \) being zero compared to the flows of water. Moreover, the \( \Gamma_2/\Gamma_1 \) PDF of the near wall boundary layer of PAM, depicted in figure 9.8(a), has a probability of \( \Gamma_2/\Gamma_1 > 0 \) of 49%, which is 9% lower than water. Therefore, biaxial stretching events with \( \Gamma_2/\Gamma_1 > 0 \) are less abundant and two-dimensional shear or planar extensional flow features with \( \Gamma_2/\Gamma_1 = 0 \) are more common in the polymer-laden boundary layers for \( y^+ < 30 \). Despite the appearance of subtle difference in the JPDFs of \( Q_D \) and \( R_D \) in the log and wake regions among PAM and water in figure 9.7(e, f), the PDFs of \( \Gamma_2/\Gamma_1 \) shown in figure 9.8(b, c) demonstrate that the distribution of \( \Gamma_2/\Gamma_1 \) values is similar for the Newtonian and polymer-laden flows.

JPDFs of \(-Q_D\) and \( Q_W\) for the Newtonian boundary layers, shown in figure 9.6(g), demonstrated that the flow near the wall consisted mostly of two-dimensional vortex sheets where \( K = 1 \). However, in the log
and wake layers, shown in figure [9.6](h, i), the flow had a variety of dissipative and vortical motions with $\mathcal{K}$ between 0 and $\infty$, similar to isotropic turbulence [Ooi et al., 1999]. For the polymer-laden boundary layer, the flow was even more concentrated around $\mathcal{K} = 1$ within the buffer and log layers in figures [9.7](g, h) compared to water, while the topology within the wake region was scattered, with $\mathcal{K}$ between 0 and $\infty$. PDFs of $\mathcal{K}$ for the Newtonian and polymer-laden boundary layers are shown for the buffer, log and wake regions of the flows in figure [9.8](d, e, f). PDFs of $\mathcal{K}$ overlap for the flows of water at different $Re_\phi$ at all wall-normal regions. Within the buffer layer, figure [9.8](e) demonstrates visibly Gaussian PDFs of $\mathcal{K}$ where the average of $\mathcal{K}$ for water and the flow of PAM are both 1. However, the standard deviation in $\mathcal{K}$ for the polymer-laden flow is smaller and approximately equal to 0.15, compared to water where the standard deviation in $\mathcal{K}$ is 0.45. Within the log layer, water is slightly more biased towards $\mathcal{K} < 1$; the mode and median in the PDF of $\mathcal{K}$ for water shown in figure [9.8](e) is 0.685 and 0.910, respectively. For the polymer-laden boundary layer, the mode and median in the PDF of $\mathcal{K}$ within the log layer, shown in figure [9.8](e), is larger compared to water and equal to 0.945 and 0.975, respectively. Therefore, the polymer laden flow has less likelihood to exhibit dissipative topologies compared to water. In the wake region, the PDFs of water boundary layers shown in figure [9.8](f) are not significantly different than those of water in the log layer seen in figure [9.8](e). Also similar to the log layer, the wake region of the polymer-laden flow has a lower probability of exhibiting dissipative flow topologies and a higher preference towards $\mathcal{K}$ of 1.
9.5 Summary

The topology of a polymer-laden boundary layer was compared with two Newtonian turbulent boundary layers, one at a similar friction Reynolds number $Re_{\tau}$ of 687, and the other at a similar momentum thickness based Reynolds number $Re_\phi$ of 2290. Relative to the Newtonian boundary layer with a similar $Re_\phi$, the polymeric flow had a 33% lower skin friction coefficient. Joint probability density functions (JPDFs) of the invariants in the velocity gradient tensor, the rate of deformation tensor and the rate of rotation tensor were used to establish a distribution of the different fine scale motions within the polymer-laden and Newtonian boundary layers, some of which include extensional- and vortical-type flow motions.

Unambiguous difference in the JPDFs of the invariants in the velocity gradient tensor, $Q_L$ and $R_L$, were observed between the polymer-laden and Newtonian boundary layers. The JPDFs of $Q_L$ and $R_L$ for the Newtonian boundary layers overlapped with one another and exhibited the well-known tear-drop shaped pattern with a clear ridge at the right-Vieillefosse tail. Relative to the Newtonian flows, the polymer-laden boundary layer had attenuated values of $Q_L$ and $R_L$; although values of $R_L$ were diminished much more than $Q_L$. A narrowing of $R_L$ provides evidence that uniaxial and biaxial stretching is less abundant within the polymer-laden flows.

Alterations to the invariants in the rate of deformation and rate of rotation tensor are more telling of the attenuation in the uniaxial and biaxial extension within the polymer-laden flow – particularly within the inner layer or $y/\delta < 0.3$. Here, the invariants of the rate of deformation tensor, $Q_D$ and $R_D$, imply that straining motions of the polymeric flow are more two-dimensional and there is a higher preference for the second eigenvalue in the rate of deformation tensor to be zero compared to water. Moreover, JPDFs of $Q_D$ and the invariant in the rate of rotation tensor $Q_W$, suggest that extensional flow motions (particularly biaxial extension) within the polymer-laden flow are less abundant and there is a larger bias towards shear-dominate flow and sheet-like motions. These sheet-like structures are similar to those seen in the viscous sublayer of Newtonian turbulence. However, in the polymer-laden flow these sheet-like motions are found at $y^+$ larger than the conventional limit of the viscous sublayer, implying that the viscous sublayer of the polymer-laden flow is thicker compared to water.
Part V

Closing
Chapter 10

Conclusions

The present thesis compared the rheology and turbulence of three different non-Newtonian solutions comprising drag-reducing additives (DRAs). The different drag-reducing solutions consisted of a flexible polymer polacrylamide (PAM), rigid polymer xanthan gum (XG) and a cationic surfactant referred to as C14. Conventional rheological measurements included steady shear, dynamic shear and extensional viscosity measurements. Nontrivial rheometry was evaluated by experimentally investigating the fluids in a steady, laminar flow through a periodically constricted tube (PCT). Lastly, the turbulence of the drag-reducing solutions were measured in a high Reynolds number channel flow and boundary layer. Particle image velocimetry and particle tracking velocimetry were used to measure the velocity statistics and coherent flow patterns within the wall-bounded turbulent flows of drag-reducing fluids.

10.1 Rheology of drag-reducing solutions

Chapter 5 documented the results of the conventional shear and extensional rheology measurements of the DRA solutions at concentrations between 100 ppm and 500 ppm. Steady shear rheology of XG and PAM demonstrated that both fluids exhibit shear thinning, while the C14 solutions had a viscosity similar to water for all concentrations. Dynamic shear rheology showed that large concentration solutions of PAM and XG were viscoelastic, but mostly viscous dominant with a loss modulus greater than gain modulus. Linear viscoelasticity measurements could not be performed for the C14 solutions; however, it is expected that the gain modulus for the C14 solutions is negligible considering their shear viscosity is comparable to water. PAM solutions were the only non-Newtonian fluids to exhibit elastocapillary thinning from extensional rheology. XG and C14 had no measurable extensional relaxation time, implying the extensional viscosity of XG and C14 is significantly smaller than PAM. Although shear and extensional rheological measurements were exclusively compared in §5, other investigations throughout this dissertation (including §§7, 8 and 9) reciprocated these findings.

In chapter 6 the velocity of the DRA solutions in a PCT were experimentally measured using particle shadow velocimetry. The PCT revealed that C14 solutions have similar non-Newtonian features as PAM solutions when the Reynolds number \( Re \) exceeded 100 in the PCT. Unlike PAM solutions, the non-Newtonian features of the C14 solutions were not detectable from conventional shear and extensional rheometric measurement techniques, as shown in §5. Therefore, the measurements using the PCT proved
to be a novel technique for uncovering the elastic features of dilute surfactant solutions. Extension or mixed kinematics within the PCT flows of C14 promoted the formation of flow-induced structures. It is hypothesized that these structures are long wormlike aggregates of micelles that are analogous to flexible polymers. These wormlike aggregates do not form in the conventional shear and extensional rheometric flows. C14 solutions that exhibited non-Newtonian features within the PCT, reflected qualitative similarities with inertioelastic PAM flows with Deborah number ($De$) greater than 0.1 and $Re > 35$. A preliminary estimate of the elastic relaxation time of the flow-induced structures was established based on comparisons with PAM flows. However, fine tuning this estimate of the relaxation time requires a denser sweep of $Re$ and $De$ for the PAM flows within the PCT.

10.2 Velocity statistics of drag-reduced channel flows

Chapter 7 measured the one-point and two-point velocity statistics of the different DRA solutions at a common drag reduction percentage ($DR$). The experimental investigation demonstrated that different DRAs generate drag-reduced channel flows with similar turbulence statistics, provided $DR$ and $Re$ are similar. Although the drag-reduced flows had similar velocity statistics, a common rheological feature that can be associated with drag reduction could not be identified. The extensional relaxation time, that has been shown to correlate with drag reduction for flexible polymers, does not seem to be pertinent for drag-reducing solutions of rigid polymers and surfactants. This ambiguity in our understanding can be explained two fold.

1. DRAs have a common rheological property that has yet to be identified from rheological measurements. This implies that the different additives reduce the turbulent drag via a common mechanism. This appears to be plausible for flexible polymers and surfactants, based on the observations of §6 and the similar response between PAM and C14 in the PCT.

2. The rheological feature responsible for drag reduction is different among the DRAs. This suggests that wall turbulence responds similarly to the different drag reduction mechanisms induced by fluids with unique rheology. Of the three DRAs, rigid polymers are the outlier. At a similar $DR$, solutions of XG are more shear thinning than the PAM solutions, but have no measurable extensional features. They also do not exhibit a chevron-pattern response in the PCT, like PAM and C14. Therefore, it is asserted that the XG solution reduces drag differently than the other DRAs, and primarily due to shear thinning. This notion was explored further in chapter 8.

In chapter 8, the Carreau-Yasuda model and the spatial gradient in the velocity were used to approximate the instantaneous viscosity of different drag-reduced channel flows of XG. All XG flows possessed a near wall region that was thin and had a low mean viscosity. Fluid at wall-normal locations immediately above this region demonstrated dramatic growth in the mean viscosity. Viscosity fluctuations similarly reflected different size and characteristics with increasing distance from the wall. However, these viscosity fluctuations were shown to have a negligible contribution to the mean stress balance of the flow. Instead, drag reduction was primarily driven by a trade-off between viscous and turbulent Reynolds stresses in the budget of mean stress. The thin low viscosity layer is denoted as a “lubricating layer,” analogous to the wall-normal
viscosity stratification observed in lubricated wall-bounded flows of immiscible fluids. This lubricating layer encapsulated the expanded linear viscous sublayer and portions of the buffer layer for flows of the XG solution. Its extent corresponded roughly to the peak in the indicator function, \( \zeta \). Unlike the classical theories of polymer drag reduction, it is hypothesized that rigid polymer drag reduction is largely attributed to gradients in the mean velocity coupled with the solutions shear thinning rheology. The lubricating layer is a product of this interaction and a mechanism for generating an effective slip within the buffer layer.

10.3 Local flow topology of polymer-laden boundary layer

Based on the viscous theory of drag reduction, the large extensional viscosity of flexible polymer solutions is believed to oppose regions of the flow exhibiting uniaxial and biaxial extension, and mitigate the strength and formation of counterrotating streamwise vortices (Lumley, 1973; Roy et al., 2006). Chapter 9 sought to observe this effect by measuring the distribution of extensional and vortical motions within a polymer-laden boundary layer using three-dimensional particle tracking velocimetry and the \( \Delta \)-criterion of Chong et al. (1990). The assertion that extensional flow motions are opposed within the polymer-laden boundary layer was shown to be plausible. It was demonstrated that extensional straining motions, predominately biaxial extension, are less pervasive within the inner layer of the polymeric flow compared to water at similar \( Re \). Furthermore, strong vortical motions are also less abundant. Instead, the flow exhibits sheet-like structures similar to those found in the viscous sublayer of Newtonian turbulence, but at \( y^+ > 5 \), implying an expansion of the viscous sublayer. Each of these observations supports the assertions of Lumley (1973), and the simulations of Roy et al. (2006), that an attenuation of biaxial extensional flow motions inhibits vortical motions near the wall, expands the buffer layer and reduces skin friction.

10.4 Suggested future works

There are many research opportunities that can be explored to expand upon the findings of the current thesis. Three suggestions are provided.

Flow-induced structures of surfactants

At certain temperatures and concentrations, surfactants are known to form micelles of different shape (e.g., spherical, rodlike and wormlike micelles). Upon exposure to flow, it is hypothesized that these micelles group together to form higher-order structures or bundles of micelles called flow-induced structures. In §6 it was assumed that flow-induced structures were formed within the PCT; however, the shape of the micelles, let alone the flow-induced structures, were not determined. Measurements of the shape and conformation of these micelles and the resulting flow-induced structures (using for example, transmission electron microscopy, small angle neutron scattering and flow-induced birefringence) could provide better evidence of how these additives compare with polymers.
Simulations of generalized Newtonian models

Numerical simulations have commonly used viscoelastic constitutive equations, such as FENE-P, to model the flow of drag-reducing flexible polymer solutions. Although such simulations have observed drag reduction, it is unclear how the inputs to these simulations map to realistic flows and fluids. On the other hand, generalized Newtonian fluids use rheologically measured trends and constants to construct the constitutive model. The work in §8 along with the simulations of Owolabi et al. (2023), demonstrated that these models could be viable, although this requires further exploration. Direct comparisons between experiments and simulations using rheological measurements are needed.

Measuring in-situ elastic properties

The debate between the viability of the viscous versus elastic theory of drag reduction is ongoing. The present work of §9 drew inspiration from the viscous theory and provided some evidence in support of its applicability; however, this does not discredit the elastic theory. The elastic theory of de Gennes (1990), states that drag reduction occurs when the elastic energy becomes comparable to turbulent kinetic energy. While turbulent kinetic energy can be measured using flow measurements, elastic energy is more difficult to discern within a turbulent flow. Elasticity is a Lagrangian quality that depends on the initial configuration of the material. Kumar et al. (2022) provided a means for discerning elastic stresses using Lagrangian coherent structures, which can be measured. However, this has yet to be explored experimentally.
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Appendices

A Uncertainty analysis

A.1 Errors in periodically constricted tube measurements

Sources of uncertainty in the PSV measurements were assumed to include (1) errors due to subpixel interpolation of the correlation function, (2) the finite DOF, and (3) optical distortion near the walls of the tube from radial curvature and differences in the refractive index. Each source of uncertainty was conservatively estimated, the details for which are listed below.

1. Errors from subpixel interpolation are conservatively estimated to be 0.1 pixels according to Raffel et al. (2018). A 0.1 pixel error in displacement translates to an error in velocity of 0.1 to 1.4 mm s\(^{-1}\) depending on \(\Delta t\). If this error is normalized by the average centreline velocity, \(\bar{U}_0\), the largest velocity error among all flow conditions was 0.012\(\bar{U}_0\).

2. Quantifying the uncertainties attributed to radial distortion and differences in the refractive index was challenging and would require ray tracing analysis (Minor et al., 2007). Instead, errors from radial distortion were conservatively estimated based on how well the velocity within FOV1 could match the theoretical Poiseuille profile, as shown in figure A.1. The largest deviation from the parabolic velocity profile was 0.04\(\bar{U}_0\).

3. Slower moving particles within the DOF but outside the centre plane of the tube, will bias velocity vectors to lower values. If a parabolic velocity profile is assumed when the wall radius \(R_w\) is equal to \(R_i\), a DOF that is 0.1\(R_i\) in thickness would produce a relative error in \(U_x\) of about 0.003\(\bar{U}_0\) near the centreline of the PCT and 0.1\(\bar{U}_0\) near the wall of the PCT. These errors reduce when considering regions of the PCT with a larger wall radius.

The total uncertainty in measurements of \(U\) from PSV was estimated to be the root sum squared value of the three previously listed sources of uncertainty. This was about 0.042\(\bar{U}_0\) near the PCT centreline and 0.108\(\bar{U}_0\) near the PCT walls, when considering regions of the PCT where \(R_w = R_i\). In subsequent plots of velocity within the PCT, error bars are used to display the uncertainty in the velocity measurements from PSV.

Profiles of streamwise velocity \(U_x\) normalized by the centreline velocity \(U_0\) for the flows of water at different Re\(_d\) within the entrance region (FOV1) are shown in figure A.1(a). Recall, that the tube walls have constant radius \(R_o\) in the entrance region, and \(U_0\) does not vary with respect to \(x\). Therefore, the Reynolds number is defined as Re\(_d\) = \(2U_0R_o/\mu_s\). Shown alongside the measurements of \(U_x/U_0\) is the
Figure A.1: Radial profiles of (a) streamwise velocity, and (b) the shear rate, at FOV1 for the flow of water at various $Re_d$. Error bars are shown for $Re = 13.2$ and correspond to the $0.042\langle U_0 \rangle$ uncertainty assumed from §6.2.

Theoretical Poiseuille velocity profile for laminar pipe flow of a Newtonian fluid within a straight-walled pipe, $U_x/U_0 = 1 - r^2/R_o^2$. All measurements of $U_x/U_0$ are within 4% of the theoretical Poiseuille profile for different coordinates of $r/R_o$ and agree well with theoretical expectations. Profiles of the shear component of the rate of deformation tensor $D_{rx}$ are shown in figure A.1(b). When the Poiseuille profile for Newtonian pipe flow is differentiated, the relationship $\partial U_x/\partial r = 2D_{rx} = -2U_0 r/R_o^2$ is obtained. When simplified, it can be shown that $D_{rx} R_o/U_0 = -r/R_o$. Similar to the streamwise velocity profiles, measurements of $D_{rx} R_o/U_0$ agree well with the theoretical profile for all $Re_d$. In general, figure A.1 demonstrates that measurements within the entrance region reasonably satisfy the expectations for laminar fully-developed Newtonian pipe flow. We can proceed to measurements of the PCT knowing that the flow entering the PCT section is fully developed and the measurement technique is valid.

A.2 Influence of gap height and surface tension on parallel plate shear rheology

Steady shear viscosity measurements using a parallel plate (PP) geometry can be subjected to several sources of error, especially when dealing with small gap heights, $h_{PP}$ (Ewaldt et al. 2015; Davies & Stokes 2008). Inertial flow instabilities, viscous heating, gap offsets and surface tension are some of the many factors that can corrupt the viscosity measurements. Techniques have been introduced to correct or account for these errors. For example, gap offset errors can be corrected by measuring $\mu$, for different $h_{PP}$ (Davies & Stokes 2008). Measurements of $\mu$ at different $h_{PP}$ are shown for the 170 ppm XG solution in figure A.2. In this figure, the upper shear rate threshold depended on the gap height and measurements were often terminated due to radial ejection of the fluid from the sides of the plates. Secondary flow instabilities are well demonstrated by the steep increase in $\mu$ at high $\dot{\gamma}$. The $Re_{PP} = 100$ threshold (Davies & Stokes 2008), demonstrated by the colour coordinated dashed lines in figure A.2, conservatively estimated the critical $\dot{\gamma}$ at which the inertial instabilities corrupted the measurements of $\mu$. At $\dot{\gamma}$ between 10 s$^{-1}$ and 2500 s$^{-1}$, and ignoring viscosity measurement with $Re_{PP} > 100$, the measurements of $\mu$ for different $h_{PP}$ are in good agreement; therefore, gap offset errors were considered negligible when $Re_{PP} < 100$. 

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Surface tension can corrupt measurements of $\mu$ using the PP geometry when rotational symmetry is not maintained. The most likely scenario where this may occur is when the fluid sample is improperly added between the plates (sample underfilling or overfilling) (Johnston & Ewoldt, 2013). To identify if interfacial tension influenced the measurements of $\mu$, we performed additional viscosity measurements that compared the 170 ppm XG solution with and without a small amount of TWEEN 20 (CAS 9005-64-5, Sigma Aldrich). Bąk & Podgórska (2016) performed interfacial tension measurements of various aqueous Tween 20 solutions. They observed that a TWEEN 20 concentration of 0.2 mM reduced the interfacial tension of water by about 30% and a concentration of 0.6 mM reduced the surface tension of water by 40%. The XG solution was given enough TWEEN 20 to achieve a concentration of 0.5 mM. Based on Bąk & Podgórska (2016), a TWEEN 20 concentration of 0.5 mM would have a significant influence on the surface tension of the solution. Figure A.3 demonstrates the measurements of the XG solution with and without TWEEN 20 at $h_{PP}$ of 0.2 mm. There is good agreement among the measurements of $\mu$ using the DG geometry and the PP geometry with and without TWEEN 20. Therefore, we can confidently assume that the solution was loaded properly into the PP and surface tension has little influence on the shear viscosity measurements.

Although using the PP allowed us to obtain measurements of $\mu$ for much higher $\dot{\gamma}$ than we would have otherwise been able to achieve using just the DG geometry, there are more ideal measurement techniques for obtaining high shear rate viscometry. For example microfluidic channels or dedicated high shear rate rheometers can obtain viscosity measurements for $\dot{\gamma}$ on the order of $10^5$ s$^{-1}$, with high accuracy and a low probability for human error. Pipe et al. (2008) were able to measure $\mu$ for $\dot{\gamma}$ up to 80000 s$^{-1}$ using a microfabricated channel. Similarly, Sepulveda et al. (2021) measured the viscosity of various XG solutions using a microfluidic rheometer for $\dot{\gamma}$ up to $2 \times 10^5$ s$^{-1}$. Utilizing such measurement techniques could yield better quality of the CY fit and more certainty in the near wall scaling.
A.3 Statistical convergence of planar PIV measurements of 170 ppm XG channel flows

The following figure A.3 demonstrates the convergence distributions of the first- and second-order statistics of velocity, as well as $\gamma$ and $\mu$, for XG with $Re_\tau$ of 170 and 700 and at a $y^+$ location of 100. The variable $n$ denotes an instantaneous data point, while $N$ is the total number of data points. Variables with a subscript of $n$, i.e. $\langle ... \rangle_n$, represent the average from the first data point to the $n$’th data point. Each convergence plot is normalized with their respective average over the complete ensemble of data points, $\langle ... \rangle_N$. All distributions converge to the ensemble average approximately within the last 20% of the data (from $n/N = 0.8$ to 1).

A random error is calculated by determining the range (maximum subtracted from the minimum) in the convergence from $n/N$ of 0.8 to 1, the results of which are shown in table A.1. The random errors for $y^+$ of 50 and 200 are also provided. Generally all random errors listed in table A.1 are less than 5%, implying good statistical convergence for all variables.

A.4 Statistical convergence of 3D-PTV measurements of 140 ppm PAM boundary layer

Figure A.4 demonstrate plots of the statistical convergence of the first- and second-order velocity statistics for the flows of water at different $Re_\phi$ and PAM at different wall-normal locations, namely $y^+$ of 20, 100, and $y/\delta$ of 0.4. Here $N$ equals 114832, which is equivalent to eight datasets of 14354 vectors in time. All statistics in figure A.4 converge by $n/N$ of 0.95, or within the last 5700 realizations. A random error is calculated from the range (maximum less minimum) of each convergence diagram from $n/N$ of 0.95 to 1. The random errors of the velocity statistics are listed for each flow scenario and at $y^+$ of 20 in table A.2. Generally all random error listed in table A.2 are less than 5%, and the variables are considered adequately converged.
Figure A.4: Statistical convergence of (a) $\langle U \rangle$, (b) $\langle u^2 \rangle$, (c) $\langle v^2 \rangle$, (d) $\langle uv \rangle$, (e) $\langle \gamma \rangle$, and (f) $\langle \mu \rangle$, for the flow of XG at its smallest and largest $Re_T$ cases of 170 and 700, and at a $y^+$ of 100.

A.5 Divergence errors in 3D-PTV measurements of 140 ppm PAM boundary layer

The accuracy of the velocity gradients, computed from the 3D-PTV measurements, is assessed by evaluating the divergence of the velocity, $\nabla \cdot U = \text{tr}(L) = -P_L$. For an incompressible flow $\nabla \cdot U = 0$. A similar assessment of the divergence-free condition is performed in other experimental investigations that utilize the $\Delta$-criterion (Gomes-Fernandes et al., 2014). Figure A.6(a) shows the JPDF of $\partial U/\partial x$ and $-(\partial V/\partial y + \partial W/\partial z)$ for the flows of water at different $Re_\phi$ and PAM. All velocity gradients are made dimensionless by multiplying them by the large eddy turnover time $T$. Deviations from the diagonal dotted line in figure A.6(a), where $\partial U/\partial x = -(\partial V/\partial y + \partial W/\partial z)$, are indicative of divergence errors. JPDFs of $\partial U/\partial x$ and $-(\partial V/\partial y + \partial W/\partial z)$ agree reasonably well with the divergence-free line for all flows, compared to prior works that similarly
utilize the $\xi$-criterion (Gomes-Fernandes et al. 2014). The correlation coefficient between $\partial U/\partial x$ and $-(\partial V/\partial y + \partial W/\partial z)$ is 0.91, 0.94 and 0.84 for the flows of water at $Re_\phi = 1814$, water at $Re_\phi = 2257$ and PAM at $Re_\phi = 2290$, respectively. These are comparable or better than the correlation coefficients derived from Tsinober et al. (1992) (0.70) who used a multi-hot-wire probe technique, and Ganapathisubramani et al. (2007) (0.82) and Gomes-Fernandes et al. (2014) (0.5-0.6) who both used stereoscopic PIV with Taylor’s hypothesis, to derive the VGT.

Another estimate for the divergence error is the ratio,

$$\xi = \frac{(\partial U/\partial x + \partial V/\partial y + \partial W/\partial z)^2}{(\partial U/\partial x)^2 + (\partial V/\partial y)^2 + (\partial W/\partial z)^2},$$

developed by Zhang et al. (1997), who used holographic PIV to measure the turbulent flow of water in a square duct. The closer $\xi$ is to 0, the better the divergence-free condition is satisfied. PDFs of $\xi$ are shown in figure A.6(b) for the flows of water at different $Re_\phi$ and PAM. The mean value of $\xi$ for water at an $Re_\phi = 1814$, water at $Re_\phi = 2257$, and PAM at an $Re_\phi = 2290$ is 0.16, 0.12 and 0.25 respectively. These are comparable to the mean values of $\xi$ from holographic PIV performed by Zhang et al. (1997) (0.74-0.12), as well as stereoscopic PIV performed by Ganapathisubramani et al. (2007) (0.18).

Mullin & Dahm (2006) assessed the divergence error of their dual-plane stereoscopic PIV measure-

<table>
<thead>
<tr>
<th>Fluid</th>
<th>$Re_\phi$</th>
<th>$\langle U \rangle$</th>
<th>$\langle u^2 \rangle$</th>
<th>$\langle v^2 \rangle$</th>
<th>$\langle w^2 \rangle$</th>
<th>$\langle uv \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1814</td>
<td>0.30%</td>
<td>1.39%</td>
<td>1.03%</td>
<td>1.33%</td>
<td>1.84%</td>
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<tr>
<td>Water</td>
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<td>1.87%</td>
<td>1.21%</td>
<td>1.00%</td>
<td>1.13%</td>
</tr>
<tr>
<td>PAM</td>
<td>2290</td>
<td>0.70%</td>
<td>2.14%</td>
<td>1.03%</td>
<td>1.12%</td>
<td>3.27%</td>
</tr>
</tbody>
</table>

Table A.2: Random errors estimated from the range in the convergence from $n/N = 0.95$ to 1, for the different flow conditions of water and PAM at $y^+$ of 20.

$\sum_{i=1}^{n}$
Figure A.5: Statistical convergence of (a) $\langle U \rangle$, (b) $\langle u^2 \rangle$ (c) $\langle v^2 \rangle$, (d) $\langle w^2 \rangle$, and (e) $\langle uv \rangle$. The solid line is the convergence at $y^+$ of 20, the dashed line is the convergence at $y^+$ of 100 and the dotted line is the convergence at $y/\delta = 0.4$.

Assessments by calculating the divergence of the velocity vectors relative to the norm of the VGT. Figure A.7 demonstrates the PDFs of the divergence of velocity divided by the norm of the VGT. Here, the trace of the VGT or divergence in the velocity is written in index notation, i.e., $L_{ii} = \text{tr}(L) = \nabla \cdot U = 0$, and the norm in the VGT is $(L_{jk}L_{jk})^{1/2}$. PDFs in $L_{ii}/(L_{jk}L_{jk})^{1/2}$ shown in figure A.7 are visibly Gaussian, with a mean that is approximately equal to 0 for all flow conditions considered. Mullin & Dahm (2006) assumed a divergence error equal to the root mean square (rms) in $L_{ii}/(L_{jk}L_{jk})^{1/2}$. The rms value of $L_{ii}/(L_{jk}L_{jk})^{1/2}$ for water at an $Re_\phi = 1814$, water at an $Re_\phi = 2257$, and PAM at an $Re_\phi = 2290$ is 0.119, 0.095 and 0.170 respectively. These divergence errors are better than or comparable to the divergence error of the dual-plane stereoscopic PIV measurements of Mullin & Dahm (2006) (0.35), and both the stereoscopic PIV measurements of Ganapathisubramani et al. (2007) (0.25) and Gomes-Fernandes et al. (2014) (0.33–0.41). Ganapathisubramani et al. (2007) demonstrated that divergence errors are strong functions of the magni-
Figure A.6: (a) Joint probability density function of $\partial U / \partial x$ and $-(\partial V / \partial y + \partial W / \partial z)$. The open red contour in (a) is PAM with $Re_\phi = 2290$ and a JPDF value of $10^{-4}$. The open black contour in (a) is water with $Re_\phi = 2257$ and a JPDF value of $10^{-4}$. (b) Probability density function of local divergence error ratio $\xi$ from Zhang et al. (1997).

Figure A.7: (a) Probability density function of the velocity divergence $L_{ii}$ normalized by the norm in the VGT $(L_{jk}L_{jk})^{1/2}$. (b) Joint probability density function of the norm in the VGT and velocity divergence normalized by the norm in the VGT. The open red contour in (b) is PAM with $Re_\phi = 2290$ and a JPDF value of $10^{-3}$. The open black contour in (b) is water with $Re_\phi = 2257$ and a JPDF value of $10^{-3}$.

The divergence error, characterized by the horizontal spread in the JPDF of the VGT. JPDFs of $(L_{jk}L_{jk})^{1/2}$ and $L_{ii}/(L_{jk}L_{jk})^{1/2}$ are shown in figure A.7(b) for the boundary layers of water at different $Re_\phi$ and PAM, similar to those seen in Ganapathisubramani et al. (2007) and Gomes-Fernandes et al. (2014).
figure A.7(b) along $L_{ii}/(L_{jk}L_{jk})^{1/2}$, is larger when $(L_{jk}L_{jk})^{1/2}T$ is lower for all flow conditions. Therefore, it is expected that velocity gradients that are lower in magnitude are more corrupted by divergence error than those with a higher magnitude, similar to the conclusion of Ganapathisubramani et al. (2007) and Gomes-Fernandes et al. (2014).