University of Alberta

GAME THEORETICAL POWER ALLOCATION IN MULTI-USER WIRELESS COOPERATIVE SYSTEMS

by

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Abstract

Cooperative system is a promising concept to improve the performance of the communication in wireless networks. This new paradigm of wireless communication imposes new challenges to traditional problems such as resource allocation. To model the behaviors of selfish and autonomous nodes in a cooperative system, game theory is an appropriate tool. This thesis focuses on power allocation in wireless cooperative systems based on game theory, with three research components.

First, we study the power allocation in multi-user relay networks with altruistic relays. We propose an asymmetric Nash bargaining solution-based relay power allocation scheme, which can achieve a balance between global network performance and user fairness. We also give a distributed implementation of the proposed scheme. Second, we consider the power allocation and relay cooperation stimulation problem in multi-user relay networks. We use Stackelberg game to analyze the interaction between the relays and the users. Based on the proposed fair relay power allocation rule, the optimal relay power price is derived analytically. Third, we study the power allocation and user cooperation stimulation problem in multi-user cooperative networks. We propose an iterative double auction-based power allocation algorithm. We show that this algorithm achieves global optimality in the sense of weighted sum-signal-to-noise ratio.

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Chapter 1

Introduction

It is not rare that in certain period of history, a technology changed humans' life in such a fundamental way that it became indispensable in almost every walk of people's life. One of such technologies that dominate the last decades is *Wireless Communication*. We do not need to trace back far in order to sense the fundamental changes brought by wireless communication to our daily life. Everything nowadays is *Wireless*. We can watch Netflix live-streaming directly from our mobile devices; we can use Google Maps to navigate anywhere we go; we can even Skype with friends while traveling on a high-speed bullet train. All those marvelous convenience are enabled by the tremendous advancement of wireless communication.

Although wireless communication has become increasingly indispensable in our daily life, its further advancement is impeded by the inherent fading effect of wireless channels. In wireless networks, channel fading may be due to shadowing when there are physical obstacles between the transceiver, due to path loss that is the signal strength loss from a line-of-sight (LOS) path through the air, or due to multipath fading where the receiver sees the superposition of multiple copies of the transmitted signal resulting from numerous reflectors in the environment.

Many efforts have been invested to tackle this prominent problem. One of such is the multi-input-multi-output, or MIMO system, which utilizes multiple antennas at both the transmitter and the receiver. In MIMO systems, multiple copies of the signal arrive at the receiver using different paths, and each path may experience a different and independent interference environment. Collectively such a system can provide a reliable communication link.

Although the instalment of multiple antennas is clearly advantageous, it may not be practical for some scenarios. To get independent transmission channels, the distance between antennas on a device should be in the order of the carrier signal's wavelength. For example, the antenna distance should be larger than 15 cm when the carrier frequency is 1 GHz. Thus, due to size and hardware limitations, a wireless agent, e.g., a smart handset, may not be able to support multiple transmit antennas.

To help single-antenna mobile users reap the benefits of MIMO systems, the concept of cooperative communication is proposed. The basic idea of cooperative communication is to have multiple nodes in the network help each other's transmission. Specifically, the signals are transmitted along different paths composed by the multiple nodes to generate spatial diversity and to effectively combat the deleterious effects of fading. Cooperative communication can provide substantial benefits to wireless networks, e.g., enhancing system capacity, increasing network coverage, and improving power efficiency. These are of great value in many applications, including ad-hoc networks, mesh networks, and next generation wireless local area networks.

There are two options to deploy cooperative communication in wireless networks, supportive relaying and user cooperation. For supportive relaying, dedicated relays are installed to assist the communication between users and their destinations. This type of networks is usually referred to as *relay networks*. The second way to use cooperative communication in wireless networks is through user cooperation. Here, a user can transmit its own information while also acting as a relay for other users. In this thesis, we call a user "relay" when it is relaying other users' information. Networks employing user cooperation are usually called *cooperative networks*. In this thesis, we collectively call relay networks and cooperative networks as *cooperative systems*. In the remaining of this chapter, we first survey the basics of wireless relay networks, and then those of wireless cooperative networks. After that, we study the power allocation problem in the two networks. At last, we give the contributions and outline of this thesis.

1.1 Wireless Relay Networks

An example of relay networks is shown in Figure 1.1, where a mobile user is com-

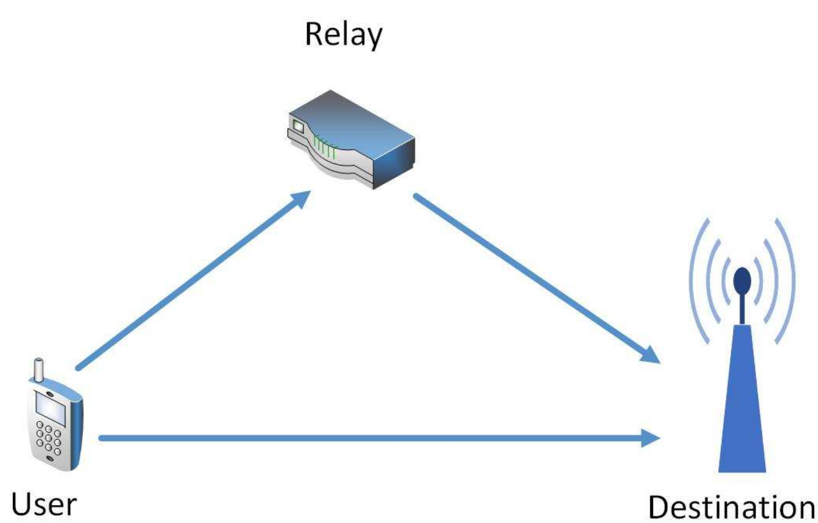

Figure 1.1: A single-user single-relay network.

municating with its destination with the assistance of a relay. In this network, when the user transmits its signal, due to the broadcast nature of wireless media, both the relay and its destination can hear this signal. The relay can then resend a processed version of the signal to the destination following some cooperation protocol. The destination can combine the signals received from the user and the relay. When the fading paths from the user and the relay are statistically independent, spatial diversity is generated during this process.

1.1.1 Single-User Relay Networks

The first study of relay channels dates back to the 1970s in the information theory community. Early works in this area are on performance analysis of single-user single-relay networks. In [3], Meulen studies three-terminal networks with one user, one relay, and one destination. The upper and lower bounds of their channel capacity are given in this paper. [4] analyzes the capacity of single-user single-relay networks. It is shown that the network can be decomposed into a broadcast channel from the user and a multiple access channel at the destination. [3, 4] have set the theoretical basis for subsequent research work in cooperative systems.

Since the early 2000s, cooperative communication has experienced rapid development to meet the high data-rate demands of next-generation wireless communication. Various cooperative protocols have been designed for wireless relay networks [5–14, 16]. In 2004, Laneman et al. provide the outage performance of single-user single-relay networks in [6]. Two basic cooperation protocols amplify-and-forward (AF) and decode-and-forward (DF) are proposed. In the AF strategy, the relays simply forward an amplified version of the received signal to the destinations. For DF, the relays decode the received signal from the users and then retransmit the decoded signal to their destinations. More generalized single-user multi-relay networks have been studied in [7–12, 14, 16, 19–21]. In [7–11], relay selection schemes are developed to realize cooperative diversity in multi-relay networks. [12] proposes the distributed space-time coding scheme then analyzes its performance in multi-relay networks. In [14], distributed relay beamforming is proposed and analytical solution is found for the relay beamformer design.

1.1.2 Multi-User Relay Networks

The number of mobile devices that are accessing wireless networks worldwide is dramatically increasing. In one research conducted by Cisco [15], it is forecasted that by 2017, there will be 8.6 billion handsets and 1.7 billion machine-to-machine connections. To meet the demands of networks with such a large number of users, research on relay networks has shifted its focus towards the understanding of *multi-user relay networks*, in which transmissions of multiple users are supported by relays. Potential applications of the results are in future communication systems such as next generation relay-assisted cellular networks, and wireless ad-hoc, sensor, and mesh networks where users are well supported by relay stations.

For multi-user relay networks, one model is the *multi-user single-relay networks* (also referred as multiple-access relay networks in some papers), where one relay assists communications between multiple users and their destinations. An application of such networks is the cooperative high-speed internet access and media sharing in a vehicle-to-vehicle scenario, where a road access point (AP) acts as a relay and helps users forward packets in vehicular networks. When there are multiple relays available, the networks are called *multi-user multi-relay networks*. Examples are wireless sensor and ad-hoc networks in which multiple intermediate relays are added to assist the communication from wireless users to their destinations. In this thesis, we consider both single-relay and multi-relay networks.

In the literature, there are many streams of research on multi-user relay networks. One stream studies cooperative schemes in networks where each user is helped by a predetermined relay, thus relay selection is not considered. Examples of such works are [28, 29], where different relaying strategies are proposed to maximize the sum-rate [28] and weighted sum-rate [29] in frequency/time-division multiple access (F/TDMA) relay networks. It should be noted that, as in [28, 29], FDMA and TDMA have been widely adopted in multi-user relay network designs to avoid user interference. Another stream of study on multi-user relay networks exploit cooperative diversity in combination with multiuser diversity, where the user with the best channel is chosen to transmit at each instant [34–39]. In this thesis, we focus on a more general case where each relay can help the concurrent transmissions of multiple users, and relay resources are allocated among these users. The resource allocation problem, especially the power allocation problem in multi-user relay networks will be discussed in Section 1.3.

1.2 Wireless Cooperative Networks

In Section 1.1, we have introduced the first way to deploy cooperative communication in wireless networks: supportive relaying. In this section, we will introduce the second way: user cooperation. User cooperation can been seen as an extension from supportive relaying, where at least two users are each other's respective relays to boost the other's communication links.

Figure 1.2 shows a cooperative network where two wireless users help each other by propagating each other's information to their destination. In Figure 1.2, the two users first exchange their information, and then jointly relay the information following some cooperation protocols. With the data exchange among themselves, the cooperative users form a distributed antenna array, which can be viewed as a virtual MIMO system.

A seminal work in cooperative networks is proposed by Sendonaris et al. in

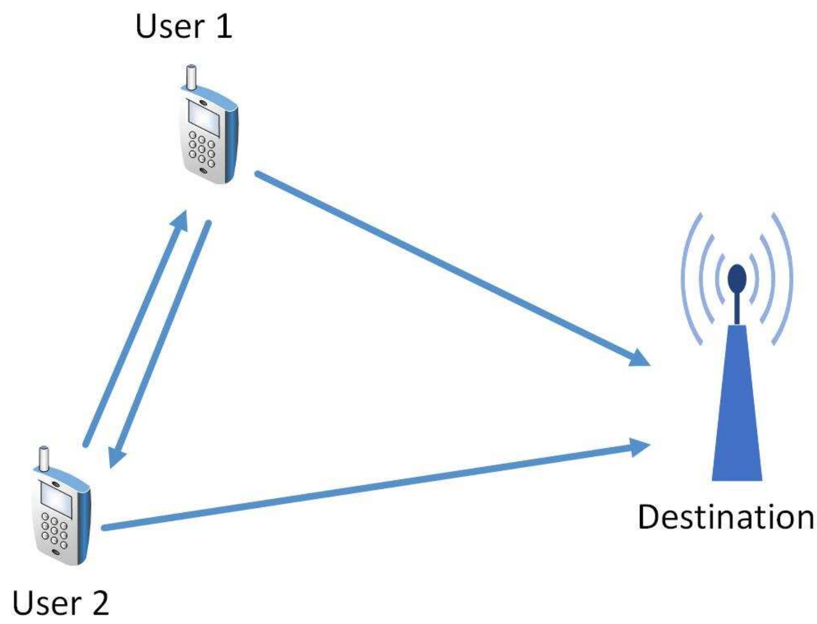

Figure 1.2: A two-user cooperative network.

1998 [18]. In this work, the authors propose a user cooperation protocol in order to increase network capacity in mobile uplink networks. Then in 2003, the same authors extend the cooperative protocol to more sophisticated schemes [5, 17]. Distributed space-time coding in wireless cooperative networks has been analyzed by Laneman in [13]. They show that spatially adjacent users can form distributed antenna arrays to yield full diversity. Note that [5, 13, 17, 18] focus on cooperative scheme design with equal resource allocation. In recent years, there have been more and more research efforts on resource allocation among users to further improve the performance of cooperative networks. In the next section, we will discuss the resource allocation problem, especially the power allocation problem in cooperative networks and give a literature review in this area.

1.3 Power Allocation in Wireless Cooperative Systems

In wireless cooperative systems, resources include the power of the relaying nodes (supportive relays or cooperative users), and the frequency spectrum of the network. Frequency spectrum has been considered as a scarce resource because of the dramatically increasing number of users and their demands for high data-rates. The bandwidth scarcity problem has been studied in [50–52].

Power is also a scarce resource. This is because unlike base stations, users and relays in cooperative systems usually are less expensive mobile devises and have limited power (One example is wireless sensor networks where most sensor devices have limited battery supply). Thus, it is important to design schemes for the allocation of the limited power resource among the competitive users in the network. On the other hand, optimal power allocation has been proved to be an effective method to cancel the interference, improve the quality of the signal transmission, thus increasing the coverage and capacity of the overall network [46–49]. Therefore, power allocation is an important issue that needs to be addressed in cooperative system design.

In this thesis, we focus on power allocation problem in cooperative systems. In the following, we first give a literature review of research works on power allocation in multi-user relay networks, and then those on cooperative networks.

1.3.1 Power Allocation in Multi-User Relay Networks

In the literature, the major objectives of power allocation in multi-user relay networks fall into two categories: achieving optimal network performance and achieving user fairness. We will first give a literature review on papers focusing on network performance and then those on user fairness.

In [53], the joint power and subchannel allocation problem is studied to maximize the sum-rate in AF multi-user multi-relay networks. Based on the dual composition method, a distributed power allocation algorithm is proposed. In [54], the optimal relay power allocation problem is considered to maximize the network throughput in AF multi-user two-way single-relay networks. The problem is solved based on Lagrange dual decomposition approach. In [55], the power allocation problem is investigated to maximize the received signal-to-noise ratio (SNR) at the destination in multi-user multi-relay networks. Considering a total power constraint at the relay and users, the sum-rate maximization problem is addressed in [58] for DF multi-user single-relay networks. In [59], the power allocation problem is studied in wireless sensor cooperative networks. Optimal power allocation is derived to get the best outage performance.

Power allocation problem in relay networks with fairness concerns are investigated in [52, 60–63]. [61] studies the power allocation problem in DF multi-user multi-relay networks. A fair power allocation scheme is proposed such that the quality-of-service requirement of each user is guaranteed. In [52], the joint subcarrier pairing and power allocation in downlink multi-destination single-relay networks is investigated with proportional fairness constraint. In [62], the joint relay power and subcarrier allocation problem is considered for FDMA multi-user multi-relay networks. A fair power allocation algorithm is proposed such that all relays have the same probability to be used. [63] studies the relay power allocation and admission control problem in multi-user multi-relay networks. Heuristic algorithms are developed to get the optimal admission control and power allocation to maximize the network throughput and to achieve two goals with fairness concerns: to maximize the minimum SNR among all users and to minimize the maximum transmit power of all users.

1.3.2 Power Allocation in Cooperative Networks

In the literature, most works on power allocation problem in wireless cooperative networks aim to achieve optimal network performance [64–68]. In [64], a power allocation strategy that minimize total power consumption is proposed in multi-user cooperative networks. [65] studies the power allocation problem in two-user cooperative networks. An algorithm is proposed to minimize bit error rate (BER) performance of the network. In [66], an optimal power allocation algorithm is proposed to maximize the sum-rate over all relayed links in downlink cooperative cellular networks. Iterative implementation of the proposed algorithm is also given. [67] investigates the power allocation problem in downlink cooperative code-division multiple access networks. The authors propose a scheme to minimize the power consumption in the network. In [68], the power allocation algorithm that minimizes BER is studied in DF cooperative networks.

1.4 Game Theoretical Solutions for Power Allocation in Cooperative Systems

For research works discussed in Section 1.3, users and relays are assumed to be altruistic and willing to cooperate to optimize the overall network performance. This is true for applications where users and relays belong to a single authority and voluntarily cooperate to achieve a common goal. In many commercial applications, however, users and relays may belong to different agents and aim to optimize their own benefits. In specific, relaying nodes use their power to help only when it is beneficial and users compete for the limited power to optimize their own data-rates or quality-of-service. Therefore, game theory, which analyzes the conflict and cooperation among independent decision makers, is an excellent tool to cope with this problem and has been widely used in power allocation in cooperative systems.

Game theory is a branch of applied mathematics which analyzes the process of decision making of a group of individuals, where one individual's benefit might depend on other individuals' actions. Recently, there has been growing interest in adopting game-theoretic methods to solve today's communication and networking problems, especially to solve the power allocation problem in a competitive environment. The reasons for the general application of game theory in power allocation problem of cooperative systems are two-fold.

1. Wireless users are generally selfish. These users are autonomous agents, making their own decisions about transmit power, signal forwarding, and so on. In such scenarios, they compete for limited resources in the network to optimize their own performance, resulting in competing scenarios. Game theory provides us sufficient theoretical tools to analyze such behaviors and actions.

2. Relaying nodes in the networks are usually autonomous agents and they relay users' information only when it is beneficial. Game theory provides us with cooperation stimulation schemes (e.g. the pricing scheme where relaying nodes can sell their power to users) to analyze such behaviors and stimulate them for cooperation, which potentially improves network performance.

Research in cooperative network designs using game theory become increasingly popular in recent years. Many papers have appeared in literature, e.g., [73–75, 77–93]. Research on game theoretical modeling of the power allocation problem in multi-user cooperative systems can be generally divided into two categories. The first category focuses on modeling and solving the user competition and cooperation problem. The second category is on providing cooperation stimulation for relaying nodes to share their power.

1.4.1 User Competition and Cooperation

For user competition and cooperation, two game theoretical models are usually used. In the first one, users are modeled as independent players and aim to optimize their own utilities. Since each user has no knowledge of the information of all other users, iterative algorithms are often needed to achieve Nash equilibrium (NE). Examples of this modeling method are [73–75]. In [73], the relay power allocation problem in the downlink of multi-user multi-relay cellular networks is studied. Non-cooperative game theory is used to model the competition for relay power among users. An iterative scheme is proposed to ensure all users reach NE. [74] investigates the distributed power allocation problem in relay-assisted cellular networks. Non-cooperative game theory is used to achieve spectral efficiency with spatial reuse of the relaying slot. In [75], for DF single-relay networks with two users, each user's achievable rate is optimized with an iterative power allocation algorithm. The algorithm is proved to converges to NE.

Another popular model to resolve the conflicts among users assumes that users cooperate to improve their performance, e.g., [81–84]. [81] studies the power allocation and admission control problem in multi-user multi-relay networks. Two kinds of users are considered: variable-rate users for which minimum rates are required and constant-rate users who need constant transmission rates. A distributed method is proposed to implement fair power allocation based on Nash bargaining solution (NBS). The work in [82, 83] are on two-user cooperative networks. They consider the scenario where the users are willing to cooperate as long as cooperation is beneficial, and they use cooperative game theory to model how the sources negotiate to address their conflicting objectives. By employing a two-source bargaining game, fair bandwidth allocation [82] and power allocation [83] are found using NBS. In [84], power allocation problem is studied in multi-user cooperative networks, in which users can form coalitions and share their power resource to form virtual multi-antenna systems. A merge-and-split algorithm is proposed to construct coalitions among users to maximize their transmission rates.

1.4.2 Cooperation Stimulation

For cooperation stimulation, the primary mechanism designed to provide incentives for relaying nodes are the payment-based mechanism. In this mechanism, the relaying nodes get paid if they forward users' messages and users pay for the relaying service. In the literature, the payment-based mechanism can be formulated as either a pricing game or an auction game. Examples of research work on cooperation stimulation in cooperative systems are [85–90].

In the pricing game formulation, each relaying node allocates power according to network environment and power prices; while the users demand power resource based on their network conditions and power prices. Examples of this kind of game formulation are [85–90]. In [85], for single-user multi-relay networks, the relay selection and relay power control problems are investigated using a two-level Stackelberg game. In this game, the relays compete to provide service to the user to gain revenue. [86] studies the user power allocation and relay pricing problems in multi-user single-relay networks. In the game theoretic model, the relay sets the price to maximize its revenue, while a non-cooperative game is used to model the user behavior, in which each user adjusts its transmit power to selfishly maximize its own utility. For two-hop multi-user ad hoc networks, compensation frameworks are proposed in [87] and [88] for power allocation, in which each user sets price and receives revenue if it cooperatively helps others' transmissions. For two-hop multi-user relay networks, based on the simplification that transmission of a frame is either successful or unsuccessful, [90] uses a pricing mechanism and Stackelberg game to encourage relay for sharing their power. The authors assume that the users set the payment rates and the payment is shared among relays who help the users.

In the auction game formulation, the relaying nodes are modeled as auctioneers and users are modeled as bidders. In an auction process, each user submits its bids to all the relays and the relays independently announce whether to sell, to which bidders to sell, and how much to sell. Examples of cooperation stimulation based on auction frameworks are [92] and [93]. In [92], the power allocation problem in single-user multi-relay networks is considered. The authors propose two auction mechanisms, SNR auction and power auction. Sufficient conditions are given for the existence and uniqueness of NE. In [93], the authors consider the power allocation problem in multi-user relay networks. An double-sided auction mechanism is designed to achieve maximum network throughput.

1.5 Thesis Contributions and Outline

In this thesis, we focus on the power allocation problem in multi-user cooperative systems based on game theory. While numerous research works have been done in this area, there are many issues that remain to be addressed. First, in practical networks, different applications may have different goals, e.g., some applications prefer fairness among the users, some applications desire better global network performance, while others require a balance between the two. Thus, it is important to study the impact of user interaction on system performance, analyze the interplay between user fairness and global network performance, and provide guidelines on cooperative system design for different applications with different goals. Second, as the scale of cooperative systems expands and the number of users increases, the centralized control mechanism almost becomes intractable. In order to effectively utilize limited power in cooperative systems to serve more customers, it is thus important to invent new techniques to exploit the distributed nature inherent in such networks. Finally, nodes in cooperative systems are usually simple devises and lack powerful computational capability. Thus it is important to design power allocation solutions with low implementation complexity. Closed-form solutions are suitable to meet this demand since they do not require iterations and are easy to obtain. However, in cooperative systems with many terminals participating in each transmission, the power allocation issue becomes very complicated, and closed-form solutions are impossible for most of the time. In such cases, numerical power allocation solutions with low implementation complexity are desirable.

In this thesis, we focus on these challenging issues in designing power allocation schemes for cooperative systems. The contributions of this thesis are summarized as follows.

- The first work focuses on power allocation strategy among the users in multi-user relay networks. We propose an NBS-based relay power allocation scheme, which can achieve a balance between global network performance and user fairness. To improve the scalability of the proposed scheme, we also propose a distributed implementation of our solution, which only requires local CSI at the users.

- In the second work, we study the power allocation problem in multi-user relay networks with cooperation stimulation. We propose a relay cooperation stimulation and power allocation scheme based on bargaining and Stackelberg game models. Based on the proposed fair relay power allocation rule, we derive the closed-form solution for the optimal relay power pricing problem.

- The last work focuses on power allocation in multi-user cooperative networks. We use iterative double auction (IDA) game to model the interaction among the users and the destination. We propose a distributed algorithm for the implementation of the IDA-based power allocation. We also show that the proposed algorithm achieves weighted sum-SNR optimal solution.

The thesis is organized as follows. Chapter 2 presents the background knowledge related to this thesis, including wireless channel models and game theory. In Chapter 3, we investigate the power allocation strategy in multi-user relay networks through bargaining. The power allocation and pricing problem in multi-user relay networks is studied in Chapter 4 using bargaining and Stackelberg games. In Chapter 5, we study the power allocation problem in cooperative networks based on double auction theory. Chapter 6 presents the conclusion and future research.

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Chapter 2

Background Knowledge

This thesis focuses on the power allocation in multi-user cooperative systems with game theoretical modeling. The basis background knowledge is reviewed in this chapter including two parts: wireless channel models and game theory.

2.1 Wireless Channel Models

The wireless channel models are fundamental for the analysis of cooperative systems. As introduced in Chapter 1, there are three phenomena that affect wireless transmission: pathloss, shadowing, and multi-path fading.

2.1.1 Pathloss

Pathloss measures the attenuation of wireless signals over a certain transmission distance from the LOS path. The free space model is formally expressed as

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi d} \right)^2, \tag{2.1}$$

where P_t is the transmit power, P_r is the received power of the free-space model at the distance d, G_t is the transmitter antenna gain, G_r is the receiver antenna gain, λ

is the wavelength. (2.1) is only valid in the far field, a more practical model can be defined as

$$P_r = P_t P_0 \left(\frac{d_0}{d} \right)^{\alpha}, \tag{2.2}$$

where d_0 is the reference distance, P_0 is the pathloss at the reference distance d_0, and α is the pathloss exponent in the range of 2 to 6.

2.1.2 Shadowing

Shadowing occurs when objects block the LOS between transmitter and receiver. The received power change caused by shadowing is often modeled using a log-normal distribution with a standard deviation according to the log-distance path loss model. The log-distance path loss model generalizes path loss to predict the mean signal strength for an arbitrary transmitter-receiver separation distance, which is given by

$$P_r(dB) = P_0(dB) + P_t(dB) + 10 \log_{10} \left(\frac{d_0}{d} \right)^{\alpha} + X_0, \tag{2.3}$$

where X_0 is a zero-mean Gaussian random variable with variance typically ranging from 3 to 12.

2.1.3 Multi-Path Fading

In the wireless transmission environment, there are numerous reflectors surrounding the transmitter and the receiver, which create multiple paths that a transmitted signal can traverse. For each path, the signal will experience different attenuation, delay and phase shift. This phenomenon is called multi-path fading. In the literature, there are many fading models that describe the distribution of signal attenuation. We give two as examples.

Rayleigh Fading

In Rayleigh fading channel, it is assumed that the LOS is obstructed and the receiver obtains only scattered waves from the surrounding objects in the environment. When there is a sufficiently large number of scatters, according to central limit theory, two quadrature components of the received signal are Gaussian random process. If there is no dominant component to the scatter, the envelope of the received signal will be Rayleigh distributed with probability density function

$$f_R(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}, \quad x \geq 0, \tag{2.4}$$

and the phase of the received signal will be evenly distributed between 0 and 2π radians. Often, Rayleigh fading is represented by a complex number, which is circularly-symmetric complex Gaussian with distribution $\mathcal{CN}(0, \sigma^2)$.

Rician Fading

Rician fading occurs when there is a dominant path, typically LOS, and other scattered waves from the surrounding objects. In Rician fading, the amplitude gain is characterized by a Rician distribution, which is expressed as

$$f_R(x) = \frac{x}{\sigma^2} e^{-\frac{x^2+A^2}{2\sigma^2}} \mathbf{I}_0 \left(\frac{Ax}{\sigma^2} \right), \quad x \geq 0, \tag{2.5}$$

where A is the peak amplitude of the dominant signal, and $\mathbf{I}_0()$ is the zeroth-order modified Bessel function of the first kind [134]. Rayleigh fading is a special case of Rician fading when $A = 0$.

2.2 Game Theory

In this section, we give some background knowledge of game theory. The three major components in a game model are a set of players, a set of actions of the players, and a set of utilities that represent the players' relative satisfaction of the outcome of the game. Formally, an N-player game can be modeled as

$$\mathscr{G} = \{\Omega, \{S_i | i \in \Omega\}, \{u_i | i \in \Omega\}\}, \tag{2.6}$$

where $\Omega = \{1, 2, \cdots, N\}$ is the player set and S_i is the strategy set of User i, including all strategies that it can use in the game. u_i is the utility of Player i.

In a noncooperative game with selfish players, each player aims to maximize its own utility by choosing an optimal strategy. Equilibrium is the strategy outcome of a game that is the best response of each player given the decisions of others. The most famous equilibrium is NE. An NE is defined as a strategy profile where Player i's strategy is the best response to all other players' strategies. Let $\mathscr{S}^* = (s_1^*, s_2^*, \cdots, s_N^*)$, where $s_1^* \in S_i$ is the strategy of Player i at NE. Let $\mathscr{S}_{-i}^* = (s_1^*, \cdots, s_{i-1}^*, s_{i+1}^*, \cdots, s_N^*)$, which is composed of all players' strategy at NE except Player i. NE satisfies the following condition

$$u_i(s_i^*, \mathscr{S}_{-i}^*) \geq u_i(s_i', \mathscr{S}_{-i}^*), \text{ for any } s_i' \in S_i. \tag{2.7}$$

(2.7) says that at NE, Player i cannot increase its utility by unilaterally changing its own strategy. Thus no player has the incentive to deviate from its current strategy at an NE.

A famous example in game theory is the prisoner's dilemma [135]. In this example, a policeman caught two suspects (A and B), but do not have sufficient evidence to accuse them. So the police separate them in different places, and offer both of them the following options:

If one of them confesses their sins, while the other is silent, the one who confesses obtains release, while the other will be in prison for 10 years;

If both of them are silent, both of them will be in prison for half a year;

If both of them confess their sins, both of them will be in prison for 2 years.

Table 2.1: Payoff matrix of prisoner's dilemma

	A silent	A confesses
B silent	(0.5 0.5)	(0 10)
B confesses	(10 0)	(2 2)

This game has two players A and B. Each of them has two actions, silent or confesses. The preference of the two players can be illustrated in Table 2.1. (m n) means A will be prison for m years and B will be in prison for n years.

The NE of this game is the (2 2) point, that is, both of them confess. Suppose that they are at this point, and A changes its status from confess to silent, then the payoff will be (0 10), so he has to stay 8 more years in prison, which is much worse than 2 years. It is the same story for B. Thus (2 2) is the strategy that both of them are unlikely to change.

From the prisoner's dilemma, we can see that game theory provides us tools to analyze the behaviors of selfish players where each of them makes their own decisions independently. And we can also see from this example that left to their own decisions, the selfish players usually behave inefficiently: the optimal solution of Table 2.1 should be (0.5 0.5) instead of (2 2). In later chapters, we will show that game theory provides other ways for us to obtain better outcomes in cooperative systems. For instance, it is possible for users in the network make agreements among each other to increase their utilities.

In the following, we give four game theoretic models and solutions which are especially useful for our research work.

2.2.1 Stackelberg Game

In the prisoner's dilemma, both players take actions independent of each other simultaneously. They do not have any knowledge of the decisions taken by the other player. Such games are called *strategic-form games*. In practice, players do not necessarily take actions at the same time and *dynamic game* can be used to model such behavior. In dynamic games, players take actions in orders and make their decisions sequentially. Thus, players have some information about each other's strategy and they can take actions more than once.

An example of a dynamic game is the Stackelberg game. In a Stackelberg game, one player acts as a leader who takes action first, and the other players are followers who observe the leader's action and act accordingly. The subgame perfect Nash equilibrium of a Stackelberg game can be found using the backward induction method. It first studies the followers' game: for each possible action of the leader, find the optimal followers' response that maximizes the followers' payoff. Then given the optimal followers' response strategy, it studies the leader's action and chooses the one that maximizes the leader's utility. The chosen strategy set is the subgame perfect Nash equilibrium [71].

Figure 2.1 shows an example of a Stackelberg game with one leader and one follower. The player set is $\{A, B\}$, where A is the leader and B is the follower. A takes action first and its strategy set is $S_A = \{x, y, z\}$. After observing A's action, B selects its strategy from its strategy set $S_B = \{m, n\}$. Their payoffs are listed at the bottom of this figure.

To find the subgame perfect Nash equilibrium, we use the backward induction method. We first find B's optimal strategy for each possible action of A. In Figure 2.1, if A selects x, B's optimal strategy is n and its payoff is 4. Similarly, if A chooses y, B's optimal strategy is m; and if A chooses z, B's optimal strategy is

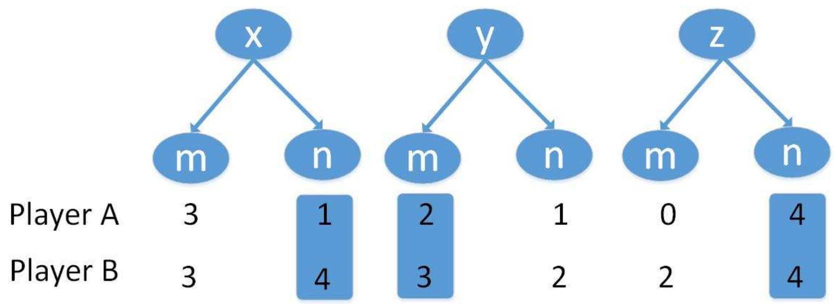

Figure 2.1: An example of Stackelberg game with two players.

n. After studying B' optimal response strategy, we study A's action and chooses the one that maximizes its utility. From the above analysis, if A chooses x, B will choose n, which gives A a utility of 1. Similarly, if A chooses y, its utility will be 2; and if A chooses z, its utility will be 4. Thus, A's optimal strategy is z and the subgame perfect Nash equilibrium of this game is $\{z, n\}$.

2.2.2 Auction Game

Auction theory is the branch of game theory dealing with how people behave in auction markets, and aims to find out their game theoretical properties. In traditional auctions, there is one seller and many buyers willing to buy the auctioned item. This is a one-to-many market structure and is called one-sided auction. Four popular one-sided auction models are: ascending-bid auction, descending-bid auction, first-price sealed-bid auction, and second-price sealed-bid (Vickrey) auction. In the ascending auction, the price is successively raised until the highest bidder wins the object. In the descending auction the auctioneer starts at a very high price, and then lowers the price continuously until one bidder claims the object. In the first-price sealed-bid auction, each bidder independently submits a single bid without seeing others' bids, and the object is sold to the bidder who makes the highest bid. Different from the first-price, in the second-price sealed-bid auction, the winning bidder pays the second highest bidders' bid [70].

In contrast with one-sided auctions, several buyers and sellers submit bids and offers simultaneously in double auctions, so the market structure is many-to-many. In wireless network designs, the following two double auction models have been adopted.

1. Preston-McAfee Double Auction (PMDA). This double auction model was developed by Preston and McAfee [120]. In this type of auction, each seller independently announces its trading price and how much to sell and each buyer decides its bidding price and how much to buy. An external auctioneer collects all asks and bids. The supply function is defined as the relationship between the ask prices of the sellers and the quality of their supply; the demand function is defined as the relationship between bid price of the buyers and the quantity they need. The supply and demand functions are described in Figure 2.2. The clearing price p is reached at the competitive equilibrium where quantity buyers willing to buy is equal to quantity sellers willing to sell. At competitive equilibrium, all the sellers who asked less than p sell and all buyers who bid more than p buy at price p.

2. IDA. Compared with PMDA which is a static model, IDA considers repeated double auction. Figure 2.3 illustrates an IDA game. In this figure, buyers submit bids for buying from others, and sellers submit asks for selling to others. The auctioneer determines the resource allocation based on these bids and asks. The auctioneers and the players interact in an iterative way until the market reaches the efficient market clearing point.

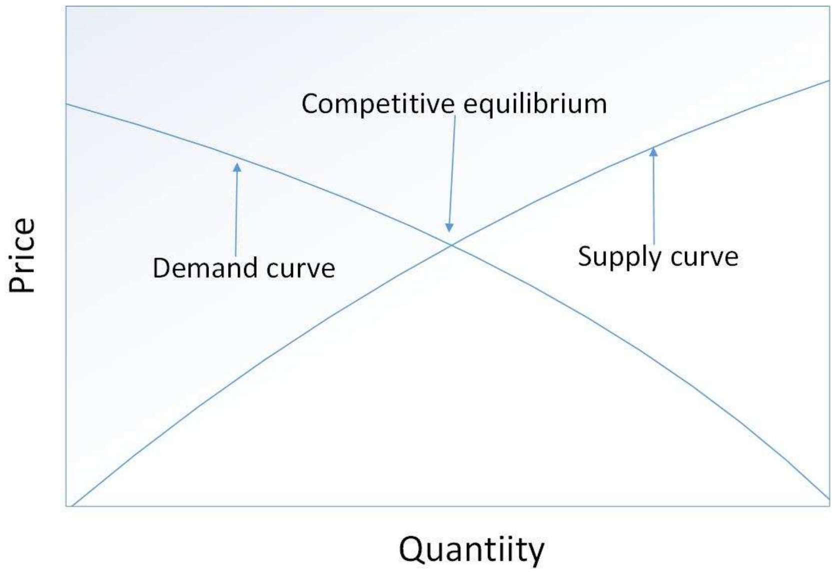

Figure 2.2: Illustration of supply and demand functions.

2.2.3 Asymmetric Nash Bargaining Solution

Now we discuss the bargaining problem. Its basic setting is as follows: there are N players with utility functions $u_1, u_2, \cdots, u_N$ and bargaining powers $\beta_1, \beta_2, \cdots, \beta_N$, where

$$\sum_{i=1}^{N} \beta_i = 1. \tag{2.8}$$

An utility vector $\mathbf{u} = (u_1 \ u_2 \ \cdots \ u_N)$ is called feasible if it is possible to find a strategy set that gives the ith player utility u_i for all $i = 1, \cdots, N$. Let $\mathcal{S}$ denote the set including all feasible utility vectors, which is assumed to be convex and compact [71]. The disagreement point, denoted as $\mathbf{u}_0 = (u_{1,0} \ u_{2,0} \ \ldots u_{N,0})$, is the vector of the minimal utility that each player expects if they do not reach an agreement and play non-cooperatively. It is the guaranteed utility for the players in the bargaining game.

Definition 2.1 Asymmetric NBS is a bargaining solution $\Phi(\mathcal{S}, u_0) = (\bar{u}_1, \bar{u}_2, \cdots, \bar{u}_N)$ which satisfies the following axioms.

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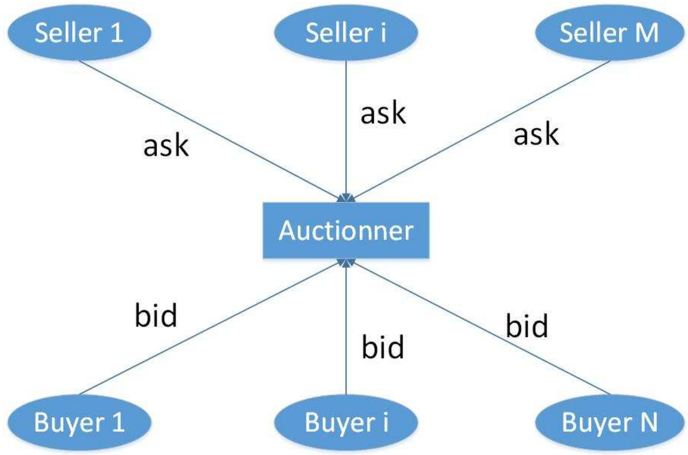

Figure 2.3: Illustration of an IDA game.

Axiom 1 *Invariance to Equivalent Utility Functions. Define a new bargaining problem, where* $f(u_i) = \alpha_i u_i + \lambda_i$ *and* $f(u_{i_0}) = \alpha_i u_{i_0} + \lambda_i$*, then* $\Phi(f(\mathcal{S}), f(u_0)) = f(\bar{u}_1, \bar{u}_2, \cdots, \bar{u}_N)$.

Axiom 2 *Independence of irrelevant alternatives. Let* $\mathcal{T} \in \mathcal{S}$*, and* $\mathcal{T}$ *be a feasible set. If* $\Phi(\mathcal{S}, u_0) \in \mathcal{T}$*, then* $\Phi(\mathcal{S}, u_0) = \Phi(\mathcal{T}, u_0)$.

Axiom 3 *Pareto optimality. If* $(u_1, u_2, \cdots, u_N) \in \mathcal{S}$*, and* $(u_1, u_2, \cdots, u_N) \geq (\bar{u}_1, \bar{u}_2, \cdots, \bar{u}_N)$*, then* $(u_1, u_2, \cdots, u_N) = (\bar{u}_1, \bar{u}_2, \cdots, \bar{u}_N)$.

Remark: Axiom 1 and 2 above guarantee the fairness of the solution. Pareto optimum in Axiom 3 means that there is no point in the feasible set that is superior to the bargaining solution.

Given the previous definition of NBS, the following theorem is proved [115], which provides a method to find NBS through optimization.

Theorem 2.1 If there is any point $\mathbf{u}$ such that $\mathbf{u} > \mathbf{u}_0$, then asymmetric NBS maximizes $\prod_{i=1}^{N}(u_i - u_{i_0})^{\beta_i}$.

2.2.4 Kailai-Smorodinsky Bargaining Solution

Compared with NBS, Kailai-Smorodinsky bargaining solution (KSBS) is also a popular solution to the bargaining game but does not require the feasible set $\mathcal{S}$ to be convex. To get KSBS, we need to define the ideal point. The ideal point $\mathbf{u}^I = (u_1^I \; u_2^I \; \ldots u_N^I)$ ('I' stands for ideal) is the vector of the maximum achievable utilities of the players in $\mathcal{S}$. We thus have $u_i^I \geq u_{i,0}$. Note that for players with $u_i^I = u_{i,0}$, cooperation does not increase their utilities and they will not enter the game. For the rest of the players, $u_i^I > u_{i,0}$, and they will participate in the bargaining process.

Given $\mathcal{S}$, the disagreement point $\mathbf{u}_0$, and the ideal point $\mathbf{u}^I$, KSBS is the solution to the optimization problem [72]

$$\max k \quad \text{s.t.} \quad \frac{u_i - u_{i,0}}{u_i^I - u_{i,0}} = k \tag{2.9}$$

for all players with $u_i^I > u_{i,0}$.

KSBS is an equilibrium point that guarantees fairness in the sense of equal penalty, which can be derived from the constraint in (2.9). Notice that $(u_i^I - u_{i,0})$ and $(u_i - u_{i,0})$ are Player i's maximum and actual net utility gains, respectively. Taking logarithm on both sides of the constraint in (2.9), we have

$$\log (u_i^I - u_{i,0}) - \log (u_i - u_{i,0}) = - \log k. \tag{2.10}$$

As $\log k$ is a constant independent of the players, the constraint in (2.9) forces all participating players to suffer the same utility penalty in the logarithmic scale, and thus ensures fairness in this sense. It is worth mentioning that KSBS is neither individual utility optimal nor global optimal in general. It is an equilibrium point that balances the proposed utility measure and fairness among users.

$\sim$

Chapter 3

Power Allocation in Multi-User Relay Networks through Bargaining

In this chapter, we study the power allocation strategy among users in multi-user relay networks. We use bargaining theory to model the negotiation among users on relay power allocation. By assigning a bargaining power to each user to indicate its priority, we propose an asymmetric NBS-based relay power allocation scheme. We analytically investigate the impact of the bargaining powers on the relay power allocation and show that via proper selection of the bargaining powers, the proposed power allocation can achieve a balance between the network sum-rate and the user fairness. Since centralized control is impossible in some networks as mentioned in Section 1.5, we propose a distributed implementation of the NBS-based power allocation, where each user only requires its local CSI. Simulation results are shown to compare the proposed NBS-based power allocation with sum-rate-optimal power allocation and rate-fair power allocation. The impact of the bargaining power selection on relay power allocation is also demonstrated via simulations.[1]

[1] *A version of this chapter has been published in IEEE Transactions on Wireless Communications, 12: 2870 - 2882 (2013).*

3.1 Introduction

As introduced in Section 1.1.2, power allocation problem is important in multi-user relay networks to harvest the potential benefit of cooperative communication. From the literature review in Section 1.3, we can see that all prior papers in this area focused exclusively on either global performance optimality, e.g., [54, 57, 58] or user fairness, e.g., [52, 60, 82, 83]. However, in practical networks, different applications may require different balances between fairness and global performance, e.g., some applications prefer fairness among the users while others desire better global network performance. Even for the same network application, the desired balance between global performance and fairness may change from time to time. Motivated by this, in this chapter we use bargaining game and propose an asymmetric NBS-based power allocation solution, which can jointly address these two issues. In addition, most previous works assume that there exists a trusted central controller who collects all the required CSI and who has sufficient computation capability to derive the proposed solutions. This is impractical in distributed systems such as ad hoc networks and sensor networks, where centralized controllers do not exist. Such systems therefore require a distributed cooperative protocol. To use our scheme for such scenarios, we provide a distributed implementation of the NBS-based power allocation scheme in which users with local information only are able to independently decide how to cooperate with other users and relays.

In this chapter, we consider a multi-user single-relay AF network, and use game theory to analyze the relay power allocation among the users. We model the interaction among the users as a bargaining problem, where they negotiate with each other on relay power allocation. We propose a new *asymmetric* NBS-based relay power allocation scheme, which can achieve a balance between global network performance and user fairness. We provide centralized implementation of the proposed power allocation. More importantly, to improve the scalability of the proposed scheme, we propose a distributed implementation of our solution, which only requires local CSI at the users. Convergence conditions are provided for this distributed algorithm. We then investigate the effect of bargaining power selection on network performance. We show analytically that via appropriate bargaining power selection, the proposed scheme can achieve the sum-rate-optimal solution for best global performance and even power allocation for best fairness. We also generalize the proposed NBS-based power allocation scheme and its distributed implementation to multi-user multi-relay networks. Simulations are conducted to compare the proposed NBS-based power allocation with the sum-rate-optimal power allocation, the even power allocation, and the rate-fair power allocation, to show that the proposed scheme can balance network sum-rate and user fairness. Via simulation, we also demonstrate the impact of the bargaining powers on the proposed relay power allocation solution. The results show the potential of using the proposed relay power allocation to address different network requirements in different applications, through proper selection of the bargaining powers.

The rest of this chapter is organized as follows. Section 3.2 elaborates the network model and the relay power allocation problem. The NBS-based relay power allocation scheme is proposed and studied in Section 3.3. In Section 3.4, we propose a centralized and a distributed schemes to implement the proposed relay power allocation. Discussions on bargaining power selection and how it can balance different network requirements are given in Section 3.5. In Section 3.6, we show the simulation results. The proposed NBS-based power allocation scheme is extended to multi-user multi-relay networks in Section 3.7. In Section 3.8, we give the conclusion of this chapter.

3.2 System Model

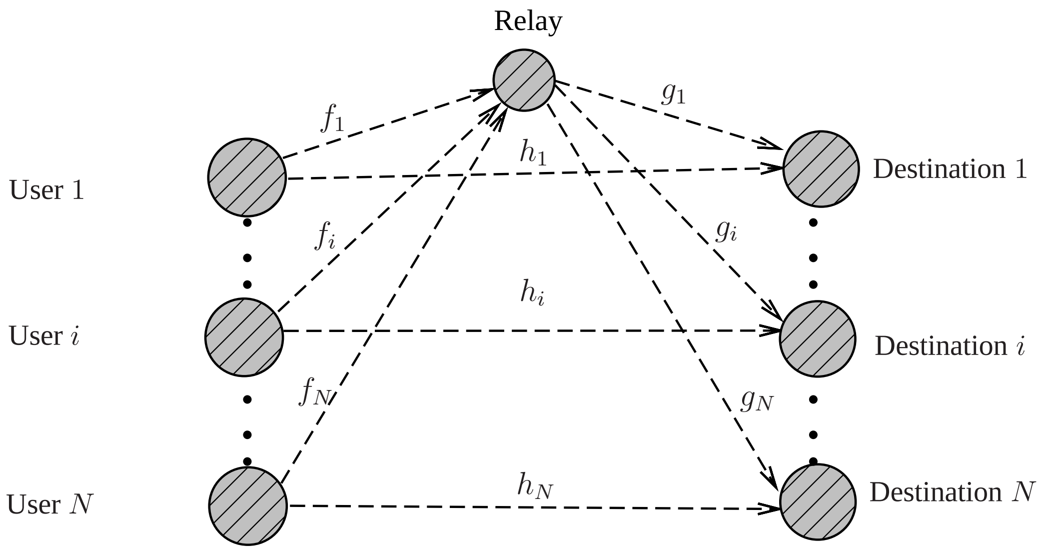

Figure 3.1: A multi-user single-relay network.

Consider a wireless network with N users communicating with their destinations with the help of one relay as shown in Figure 3.1. Denote the channel from User i to the relay as f_i, the channel from User i to Destination i (the direct link) as h_i, and the channel from the relay to Destination i as g_i. We consider two channel models in this work, Rayleigh flat-fading channel and path-loss channel. We denote the transmit power of User i as Q_i and the maximum transmit power of the relay as P. We also denote the power the relay uses in helping User i as P_i.

We assume a block-fading (or quasi-static) model: the channels remain invariant over a time interval, called the coherence time of the channels, but vary across successive coherence intervals according to a stationary and ergodic random process. The block-fading model is well justified for vehicular communication for rush-hour traffic scenarios (e.g., cooperative high-speed internet access and media sharing in a vehicle-to-vehicle scenario, where a road access point acts as a relay and helps users forward packets).

FDMA is used, so transmissions of different users are orthogonal and interference-free. Without loss of generality, we consider the transmission of User i's message on Channel i. To send one symbol from User i to Destination i, we use the popular half-duplex two-step AF protocol. In the first step, User i transmits $\sqrt{Q_i}s_i$, where s_i is the information symbol normalized as $\mathbf{E}(|s_i|^2) = 1$. The signals received by the relay and Destination i are

$$y_{iR} = \sqrt{Q_i} s_i f_i + n_{iR}, \tag{3.1}$$

and

$$y_{iD} = \sqrt{Q_i} s_i h_i + n_{iD}, \tag{3.2}$$

respectively, where n_{iR} and n_{iD} are additive noises at the relay and Destination i in the first step, respectively. They are assumed to be independent Gaussian following the distribution $\mathcal{CN}(0,1)$. In the second step, the relay amplifies y_{iR} and forwards it with power P_i on Channel i. The signal received at Destination i in the second step can be shown to be

$$y_{Ri} = \sqrt{\frac{Q_i P_i}{Q_i |f_i|^2 + 1}} s_i f_i g_i + \sqrt{\frac{P_i}{Q_i |f_i|^2 + 1}} g_i n_{iR} + n_{iRD}, \tag{3.3}$$

where n_{iRD} is the additive noise at Destination i in Step 2, which is assumed to be independent to other noises with the same distribution, $\mathcal{CN}(0,1)$.

To simplify the presentation, we introduce two variables, namely the *noise forwarding rate* and the *signal forwarding rate*. We define

$$\xi_i \triangleq \frac{|g_i|^2}{Q_i |f_i|^2 + 1}, \tag{3.4}$$

which is the power of the second term at the right hand side of (3.3) when $P_i = 1$. We call ξ_i the noise forwarding rate corresponding to User i since its physical meaning is the noise power that the relay forwards to Destination i if unit relay power is used. Intuitively, a large noise forwarding rate means low quality in the user's relay-path. Similarly, we define the signal forwarding rate of User i as

$$\rho_i = \frac{Q_i|f_i g_i|^2}{Q_i|f_i|^2+1}. \tag{3.5}$$

It is the power of the first term at the right hand side of (3.3) when $P_i = 1$. Its physical meaning is the signal power that the relay forwards to Destination i if unit relay power is used. A large signal forwarding rate intuitively means high quality in the user's relay-path.

After maximum ratio-combining of both the direct and relay paths, the effective received SNR of User i's transmission can be shown straightforwardly to be

$$\mathrm{SNR}_{iRD} = \frac{\rho_i P_i}{\xi_i P_i + 1} + Q_i|h_i|^2. \tag{3.6}$$

If User i's transmission is not helped by the relay and only the direct transmission is active, its received SNR becomes

$$\mathrm{SNR}_{iD} = Q_i|h_i|^2. \tag{3.7}$$

3.3 NBS-Based Power Allocation

We can see from (3.6) that all users desire the relay to allocate as much power as possible to help their own transmissions so they can achieve the highest SNRs. But the relay power is limited, so allocating more relay power to one user means less power available for the rest. To address this conflict among users, we model the interaction among the users as a bargaining game, and derive a fair relay power allocation scheme based on the NBS of the game.

3.3.1 Bargaining Game Model

In this section, we use bargaining game model to analyze the conflict and interaction among independent users. As defined in [122], bargaining theory studies the situation "in which two (or more) players can mutually benefit from reaching a certain agreement but have conflicting interests on the terms of the agreement". This fits our problem where the users have conflicting interests on the allocation of the relay power: each user tries to maximize its allocated relay power, and they have an interest in agreeing on the share, so they can all benefit and improve their SNR (achievable rate). The first step to formulate the power allocation problem as a bargaining game is to design the utility function. We define User i's utility function to be the effective received SNR of User i given in (3.6), that is,

$$u_i(P_i) \triangleq \mathrm{SNR}_{iRD} = \frac{\rho_i P_i}{\xi_i P_i + 1} + Q_i |h_i|^2. \tag{3.8}$$

It represents the received quality-of-service, and is directly related to the performance of the communication. It can be seen that $u_i(P_i)$ is an increasing function of P_i. Given the N users in our relay network, we define the utility vector as $\mathbf{u} = (u_1 \; u_2 \; \cdots \; u_N)$. We denote the disagreement point as $\mathbf{u}_0 = (u_{1,0} \; u_{2,0} \; \ldots \; u_{N,0})$, which is the vector of the minimal utility that each user expects if they do not reach an agreement and play non-cooperatively. Thus,

$$u_{i,0} \triangleq \mathrm{SNR}_{iD} = Q_i |h_i|^2, \tag{3.9}$$

which is the utility of User i when it does not get any power from the relay and uses the direct transmission only, i.e. $P_i = 0$. Note that this is a natural choice since if the users do not agree on the relay power allocation, the relay will not allocate any power to any user. Similar disagreement point setting is adopted in [83, 94–98].

Given the above definitions of the utility function and the disagreement point, a utility vector $\mathbf{u} = (u_1 \; u_2 \; \cdots \; u_N)$ is called feasible if there exists a power allocation strategy $(P_1\ P_2 \cdots\ P_N)$ where $P_i \geq 0$ and $\sum_{i=1}^{N} P_i \leq P$ that gives User i utility u_i for all $i = 1, \cdots, N$. Let $\mathcal{S}$ be the set of all feasible utility vectors. Thus

$$\mathcal{S} \triangleq \left\{ (u_1 \cdots u_N) \middle| \sum_{i=1}^{N} P_i \leq P, P_i \geq 0 \right\}. \tag{3.10}$$

The first inequality in (3.10), $\sum_{i=1}^{N} P_i \leq P$, is from the relay power constraint. Power allocations that do not satisfy this constraint are infeasible. The second inequality, $P_i \geq 0$, says that each user has to be allocated non-negative relay power, a natural condition from practical point of view. This inequality also guarantees that when cooperates, each user gets no less utility compared to the case that it does not cooperate and only the direct link is used for communication. This is a necessary condition for the game theory formulation of feasible set.

In our relay power bargaining game among the users, we consider the scenario where different users may have different priorities in obtaining the relay power. To model this, users are assigned bargaining powers, denoted as $\beta_1, \cdots, \beta_N$, that they agree upon before transmission [115]. The bargaining powers are normalized as $\sum_{i=1}^{N} \beta_i = 1$, which is defined in (2.8). In Section 3.5, we will investigate the effect of bargaining power selection on the proposed NBS-based power allocation and provide bargaining power allocation schemes that can bridge between global network performance and user fairness.

3.3.2 Nash Bargaining Solution

In our bargaining game model for the relay power allocation, given the feasible set $\mathcal{S}$ and the disagreement point $\mathbf{u}_0$, the users negotiate and select one feasible utility vector in $\mathcal{S}$ and the corresponding power allocation strategy. Depending on how they define "fairness", the users may choose different solutions in $\mathcal{S}$. In this work, we choose the asymmetric NBS [115] as the bargaining game solution for the following reasons. First, it has been proved in [122] that NBS is Pareto optimal, where no user can further improve its utility without decreasing others'. Thus NBS ensures that all relay power is efficiently utilized by the users, which is preferred on system design perspective. Second, NBS achieves proportional fairness by dividing the additional utility among users in a ratio that is equal to the rate at which this utility can be transferred [115]. Third, as will be discussed in Section 3.5, NBS has flexibility in bargaining power selection, which provides us a way to balance between global network performance and user fairness. In this work, we look for the NBS-based relay power allocation. For this purpose, we first prove the following two lemmas.

Lemma 3.1 Given the utility function $u_i(P_i)$ in (3.8), the feasible set $\mathcal{S}$ defined in (3.10) is convex.

Proof: From (3.8) and (3.9),

$$u_i(P_i) = \frac{\rho_i P_i}{\xi_i P_i + 1} + u_{i,0}. \tag{3.11}$$

It is a strictly increasing function of P_i and

$$\lim_{P_i \to \infty} u_i = \frac{\rho_i}{\xi_i} + u_{i,0} = Q_i |f_i|^2 + u_{i,0}. \tag{3.12}$$

Also, we can show that

$$P_i = \frac{u_i - u_{i,0}}{\rho_i - (u_i - u_{i,0})\xi_i}. \tag{3.13}$$

So $\mathcal{S}$ can be rewritten as

$$\mathcal{S} = \left\{ \mathbf{u} \middle| \phi(\mathbf{u}) \triangleq \sum_{i=1}^{N} \frac{u_i - u_{i,0}}{\rho_i - (u_i - u_{i,0})\xi_i} \leq P, \right. \\ \left. u_{i,0} \leq u_i < Q_i |f_i|^2 + u_{i,0}, i = 1, \cdots, N \right\}, \tag{3.14}$$

where the last constraint ensures that $0 \le P_i < \infty$ for all i's.

Define

$$\mathcal{S}_1 \triangleq \{\mathbf{u} | u_i \ge u_{i,0}, i = 1, \cdots, N\} \tag{3.15}$$

and

$$\mathcal{S}_2 \triangleq \{\mathbf{u} | \phi(\mathbf{u}) \le P, u_i < Q_i |f_i|^2 + u_{i,0}, i = 1, \cdots, N\}. \tag{3.16}$$

We thus have $\mathcal{S} = \mathcal{S}_1 \cap \mathcal{S}_2$. $\mathcal{S}_1$ is a convex set by definition. To prove that $\mathcal{S}$ is convex, we only need to show that $\mathcal{S}_2$ is also convex.

We first prove that $\phi(\mathbf{u})$ is a convex function. From the definition of ϕ in (3.14), the Hessian or the second-order derivative of $\phi(\mathbf{u})$ is

$$\nabla^2 \phi(\mathbf{u}) = \begin{bmatrix} \frac{\partial^2 \phi(\mathbf{u})}{\partial u_1^2} & 0 & \cdots & 0 \\ 0 & \frac{\partial^2 \phi(\mathbf{u})}{\partial u_2^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial^2 \phi(\mathbf{u})}{\partial u_N^2} \end{bmatrix}, \tag{3.17}$$

which is a diagonal matrix whose ith diagonal element is

$$\frac{\partial^2 \phi}{\partial u_i^2} = \frac{2 Q_i |f_i|^2}{\xi_i \left[Q_i |f_i|^2 - (u_i - u_{i,0})\right]^3}. \tag{3.18}$$

For any finite P_i , we have $Q_i|f_i|^2 - (u_i - u_{i,0}) > 0$, so $\frac{\partial^2 \phi}{\partial u_i^2} > 0$ for all $i = 1, \cdots, N$. Thus, $\nabla^2 \phi(\mathbf{u})$ is positive definite, which shows that $\phi(\mathbf{u})$ is a convex function. Consequently, from the definition of $\phi(\mathbf{u})$, $\mathcal{S}_2$ is convex [116], and this completes the proof. ■

Lemma 3.2 There is at least one point in $\mathcal{S}$ with $u_i > u_{i,0}$ for all $i = 1, \cdots, N$.

Proof: We show this lemma by construction. Consider the even power allocation where $P_i = P/N$ for all $i = 1, \cdots, N$. Since u_i is an increasing function of P_i, we have $u_i > u_{i,0}$ for all $i = 1, \cdots, N$. ■

With the results in Lemma 3.1 and Lemma 3.2, the asymmetric NBS is the solution to the following optimization problem [115]

$$\arg\max_{P_1,\cdots,P_N} \prod_{i=1}^{N} (u_i - u_{i,0})^{\beta_i}, \text{ s.t. } P_i \geq 0, \sum_{i=1}^{N} P_i \leq P, \tag{3.19}$$

where β_i is User i's bargaining power. This problem can be simplified by the following lemma.

Lemma 3.3 The optimization problem in (3.19) is equivalent to the following problem:

$$\begin{aligned} &\arg\max_{P_1,\cdots,P_N} \sum_{i=1}^{N} \beta_i \log\left(\frac{\rho_i P_i}{\xi_i P_i + 1}\right) \\ &\text{s.t.} \quad P_i > 0, \quad \sum_{i=1}^{N} P_i = P. \end{aligned} \tag{3.20}$$

Proof: As the logarithm function is monotonically increasing, we can take the logarithm of the objective function in (3.19) without changing its solution. Thus the objective function in (3.20) is obtained using the definitions in (3.8) and (3.9).

Furthermore, notice that when $P_i = 0$ for some i, the objective function of (3.20) becomes $-\infty$. This is obviously non-optimal since any feasible power allocation with non-zero P_i for all i's (e.g., $P_i = P/N$) will result in a higher objective function. Thus, we can replace $P_i \geq 0$ by $P_i > 0$. This ensures that all users will enter the bargaining game.

Next, we show by contradiction that the optimal solution, denoted as $\mathbf{P}^* = (P_1^* \cdots P_N^*)$ satisfies $\sum_{i=1}^{N} P_i^* = P$. Assume that the optimal solution $\mathbf{P}^*$ gives the utility vector $\mathbf{u}^* = (u_1^*, u_2^*, \cdots, u_N^*)$ and satisfies $\sum_{i=1}^{N} P_i^* < P$. Let

$$\Delta_P = P - \sum_{i=1}^{N} P_i^*. \tag{3.21}$$

We consider another power allocation strategy

$$\mathbf{P}' \triangleq (P_1^* + \Delta_P, P_2^*, \cdots, P_N^*), \tag{3.22}$$

which gives the utility vector $\mathbf{u}' = (u_1' \; u_2' \; \cdots \; u_N')$. It is straightforward to show that $\mathbf{u}'$ is in the feasible set $\mathcal{S}$, $u_1' > u_1^*$, and $u_i' = u_i^*$ for $i = 2, \cdots, N$. Thus this new solution results in a higher objective function than $\mathbf{P}^*$, which contradicts the assumption that $\mathbf{P}^*$ is optimal. This completes the proof. ■

Thus, to find the NBS-based relay power allocation, we should solve (3.20). Define

$$\mathbf{P} \triangleq \left[\; P_1 \quad \cdots \quad P_N \; \right]. \tag{3.23}$$

We write the Lagrangian function for Problem (3.20) as

$$\begin{aligned} \mathcal{L}(\mathbf{P}, \alpha) \triangleq & \sum_{i=1}^{N} \beta_i \log \frac{\rho_i P_i}{\xi_i P_i + 1} \\ & - \sum_{i=1}^{N} \lambda_i P_i + \alpha \left(P - \sum_{i=1}^{N} P_i \right). \end{aligned} \tag{3.24}$$

Here λ_i and α are Lagrangian multipliers associated with the inequality and equality constraints. In (3.20), the objective function can be shown straightforwardly to be concave, its inequality constraint functions are convex, and its equality constraint function is affine. Thus (3.20) is a convex optimization problem. Its first-order Karush-Kuhn-Tucker (KKT) conditions, which are necessary and sufficient for the solution of (3.20) (see (5.49) on Page 243 in [116]) are

$$\frac{\partial \mathcal{L}(\mathbf{P}, \alpha)}{\partial P_i} = \frac{\beta_i}{(\xi_i P_i + 1) P_i} - \lambda_i - \alpha = 0, \tag{3.25}$$

$$-P_i < 0, \quad \sum_{i=1}^{N} P_i = P, \quad \lambda_i \geq 0, \quad \lambda_i P_i = 0, \tag{3.26}$$

for $i = 1, \cdots, N$. As $P_i > 0$, we have $\lambda_i = 0$ and thus

$$P_i = \frac{2\beta_i}{\alpha} \left(\sqrt{1 + \frac{4\xi_i \beta_i}{\alpha}} + 1 \right)^{-1} \quad \text{and} \quad \sum_{i=1}^{N} P_i = P. \tag{3.27}$$

Using (3.27), we have

$$\frac{2}{\alpha}\sum_{i=1}^{N}\beta_i\left(\sqrt{1+\frac{4\xi_i\beta_i}{\alpha}}+1\right)^{-1}=P. \tag{3.28}$$

It can be shown that when α changes from 0 to ∞, the left-hand-side of (3.28) monotonically decreases from ∞ to 0. Thus, (3.28) has a unique positive solution and the solution can be found using bisection method[2]. Once the optimal α satisfying (3.28) is found, the NBS-based relay power allocation solution can be found using (3.27).

3.4 Implementation of NBS-Based Relay Power Allocation

In this section, we give possible implementations of the proposed NBS-based relay power allocation. First, we propose a centralized implementation, which requires no iterations and no computation at the users. But it requires global and perfect CSI at the relay. Also, the centralized implementation is based on the assumption that all computation is placed at the relay and the relay is trustworthy. We then propose a distributed implementation, which requires only local CSI at each user and no computation is required at the relay.

3.4.1 Centralized Implementation

For the centralized implementation of the proposed relay power allocation, the relay, assumed to have global and perfect CSI, computes the NBS-based power allo-

[2] The range of α can be set as $(0, \frac{1}{P})$. The upper bound of α can be derived as follows. Since $\xi_i = \frac{|g_i|^2}{Q_i|f_i|^2+1} > 0$ and $\beta_i > 0$, from (19), we have $P = \frac{2}{\alpha}\sum_{i=1}^{N}\beta_i\left(\sqrt{1+\frac{4\xi_i\beta_i}{\alpha}}+1\right)^{-1} < \frac{2}{\alpha}\sum_{i=1}^{N}\frac{\beta_i}{2} = \frac{1}{\alpha}$, which gives $\alpha < \frac{1}{P}$. For the lower bound of α, we can set it to 0 since α is nonnegative.

cation solution proposed in Section 3.3 and uses the corresponding power values to help the users. To get the NBS-based relay power allocation solution, the relay first finds the α that satisfies (3.28) using bisection method, then finds the NBS-based relay power allocation solution using (3.27). For the relay to know the channel gains from the users to itself, $f_1, \cdots, f_N$, training and channel estimations can be performed. For the relay to know the channel gain g_i from itself to Destination i, Destination i first estimates g_i, then feeds the coefficient back to the relay.

With this implementation, we actually assume that the relay is trustworthy. All users believe that 1) the relay will not change the parameter values (e.g., the bargaining powers and the CSI) to favor any user, and 2) the relay follows the NBS-based power allocation results to help all users in their transmissions.

3.4.2 Distributed Implementation

In practical wireless networks, especially for networks with a large number of users, it may be impractical to implement the aforementioned NBS-based power allocation in a centralized way at the relay. The reasons are threefold. First, the centralized scheme assumes accurate and complete CSI at the relay, which brings overhead for training, channel estimation, and CSI feedback from the destinations to the relay. Second, in the centralized scheme, all computational load is at the relay, which may not have high computational capability for many real network applications or may not be willing to conduct such computations. Third, in some applications, the users may distrust the relay and are unwilling to have the relay being the controller in power allocation.

To overcome these problems, we propose a distributed algorithm to solve (3.28) at the users, each having local CSI only, i.e., User i knows f_i and g_i. Similarly, the CSI can be obtained via training and feedback channel. Similar to [99, 100], we implement the distributed algorithm based on the gradient projection of the dual problem associated with the original problem (3.20).

The dual problem of (3.20) is:

$$\min_{\alpha \geq 0} D(\alpha), \tag{3.29}$$

where $D(\alpha)$ is the dual function defined as follows:

$$\begin{aligned} D(\alpha) &\triangleq \max_{\mathbf{P}} \mathcal{L}(\mathbf{P}, \alpha) \\ &= \max_{\mathbf{P}} \left\{ \sum_{i=1}^{N} \left(\beta_i \log \frac{\rho_i P_i}{\xi_i P_i + 1} - \alpha P_i \right) + \alpha P \right\}. \end{aligned} \tag{3.30}$$

$\mathcal{L}(\mathbf{P}, \alpha)$ is the Lagrangian function defined in (3.24). We have shown that $\lambda_i = 0$, so the term with λ_i is omitted.

Note that the summation term in $\mathcal{L}(\mathbf{P}, \alpha)$ is separable in P_i. Define

$$F_i(P_i) \triangleq \max_{P_i} \left(\beta_i \log \frac{\rho_i P_i}{\xi_i P_i + 1} - \alpha P_i \right).$$

Hence, we have from (3.30)

$$\begin{aligned} D(\alpha) &= \sum_{i=1}^{N} \left[\max_{P_i} \left(\beta_i \log \frac{\rho_i P_i}{\xi_i P_i + 1} - \alpha P_i \right) + \alpha P \right] \\ &= \sum_{i=1}^{N} \left[F_i(P_i) + \alpha P \right]. \end{aligned} \tag{3.31}$$

Since Problem (3.20) is a convex optimization problem, by duality theory, if α^* is the optimal solution of the dual problem in (3.29), $(P_1(\alpha^*), \cdots, P_N(\alpha^*))$ calculated from (3.27) is the optimal solution of (3.20). Therefore, we can focus on the dual problem (3.29).

The gradient of $D(\alpha)$ can be calculated to be:

$$\frac{\partial D(\alpha)}{\partial \alpha} = P - \sum_{i=1}^{N} P_i(\alpha). \tag{3.32}$$

We can now solve the dual problem with the gradient projection method [117] where α is adjusted in the opposite direction to $\frac{\partial D(\alpha)}{\partial \alpha}$ as:

$$\begin{aligned}\alpha(t+1) &= \max\left\{0, \alpha(t) - \gamma\frac{\partial D}{\partial \alpha}(\alpha(t))\right\}\\ &= \max\left\{0, \alpha(t) - \gamma\left[P - \sum_{i=1}^{N} P_i(\alpha(t))\right]\right\},\end{aligned} \tag{3.33}$$

where $\gamma > 0$ is the step-size.

The gradient projection method generates a sequence of α values: $\alpha(0), \cdots, \alpha(t)$, $\alpha(t+1), \cdots$ that approaches the optimal solution α^*. With a constraint on the step size γ, the convergence of the gradient projection method can be guaranteed, which is stated in the following theorem.

Theorem 3.1 Let $\beta_{\min} \triangleq \min\{\beta_1, \cdots \beta_N\}$ and $|g_{\max}| \triangleq \max\{|g_1|, \cdots, |g_N|\}$. If the step-size satisfies $0 < \gamma < \frac{2\beta_{\min}}{NP^2(|g_{\max}|^2 P+1)}$, for any initial $\alpha(0) \geq 0$, the gradient projection method will converge to the primal and dual optimal point, i.e.,

$$\lim_{t\to\infty} \alpha(t) = \alpha^*, \quad \lim_{t\to\infty} P_i(\alpha(t)) = P_i^*. \tag{3.34}$$

Proof: To prove Theorem 3.1, we first prove the following lemma:

Lemma 3.4 Functions $\Theta_i(P_i) = \beta_i \log \frac{\rho_i P_i}{\xi_i P_i + 1}, i = 1, \cdots, N$ are increasing, strictly concave and twice continuously differentiable. The curvatures of $\Theta_i(P_i)$ are bounded away from zero on feasible set $\mathcal{S}$.

Proof: To prove Lemma 3.4, we first show that

$$\begin{aligned}\Theta_i'(P_i) &= \frac{\beta_i}{P_i(\xi_i P_i + 1)} > 0,\\ \Theta_i''(P_i) &= \frac{-\beta_i(2\xi_i P_i + 1)}{P_i^2(\xi_i P_i + 1)^2} < 0, \text{and continuous.}\end{aligned}$$

Then since $\frac{\xi_i P_i}{\xi_i P_i+1} > 0$, we have

$$\begin{aligned} -\Theta_i''(P_i) &= \frac{\beta_i(2\xi_i P_i + 1)}{P_i^2(\xi_i P_i + 1)^2} = \frac{\beta_i(1 + \frac{\xi_i P_i}{\xi_i P_i+1})}{P_i^2(\xi_i P_i + 1)} \\ &\geq \frac{\beta_i}{P_i^2(\xi_i P_i + 1)}. \end{aligned}$$

This completes the proof. ■

From Lemma 3.4, we get that the dual objective function is convex, lower bounded, and continuously differentiable. To optimize $\Theta_i(P_i)$, the equation $\Theta_i'(P_i) = \alpha$ must be satisfied. Thus $P_i = \max\{0, \Theta_i'^{-1}(\alpha)\}$, where $\Theta_i'^{-1}$ is the inverse function of Θ_i'. Then we get $\frac{\partial P_i(\alpha)}{\partial \alpha} = \max\left\{0, \frac{1}{\Theta_i''(P_i(\alpha))}\right\}$. From (3.32), we get $\frac{\partial D(\alpha)}{\partial \alpha} = P - \sum_{i=1}^N P_i(\alpha)$, and hence

$$\frac{\partial^2 D(\alpha)}{\partial^2 \alpha} = -\sum_{i=1}^N \frac{1}{\Theta_i''(P_i(\alpha))}.$$

By using Taylor theorem, there exists a $t \in [0,1]$, such that

$$\frac{\partial D(\alpha)}{\partial \alpha} - \frac{\partial D(\beta)}{\partial \beta} = \frac{\partial^2 D(\mu)}{\partial^2 \mu}(\alpha - \beta),$$

where $\mu = t\alpha + (1-t)\beta$. Thus,

$$\left|\frac{\partial D(\alpha)}{\partial \alpha} - \frac{\partial D(\beta)}{\partial \beta}\right| \leq \left|\frac{\partial^2 D(\mu)}{\partial^2 \mu}\right| \left|(\alpha - \beta)\right|.$$

Now, from Lemma 3.4,

$$\left|\frac{\partial^2 D(\mu)}{\partial^2 \mu}\right| = \sum_{i=1}^N \frac{1}{|\Theta_i''(P_i(\alpha))|} \leq \sum_{i=1}^N \frac{P_i^2(\xi_i P_i + 1)}{\beta_i}.$$

As $Q_i|f_i|^2 + 1 > 1$, $\xi_i < |g_i|^2$. We have

$$\sum_{i=1}^N \frac{P_i^2(\xi_i P_i + 1)}{\beta_i} \leq \frac{NP^2(|g_{max}|^2 P + 1)}{\beta_{min}}.$$

From the analysis above, we conclude that $\frac{\partial D(\alpha)}{\partial \alpha}$ is Lipschitz [101] and the Lipschitz constant is $\kappa = NP^2(|g_{max}|^2 P + 1)/\beta_{min}$. Let γ be the step-size. If

$\gamma \in (0, \frac{2}{\kappa})$, then any accumulation point α^* generated by sequence $\alpha(t)$ is dual optimal. We can then follow the same proof statements in [100] to show that $P_i(\alpha(t))$ will converge to the unique primal optimal point P_i^*. ■

We now comment on the convergence speed of the distributed scheme. Using Tylor's theorem to $\frac{\partial D(\alpha(t))}{\partial \alpha(t)}$ at the optimal α^*, it can be readily shown that

$$\alpha(t) - \alpha^* \cong \frac{\partial D(\alpha(t))}{\partial \alpha(t)} \left(\frac{\partial^2 D(\alpha(t))}{\partial^2 \alpha(t)} \right)^{-1} + o(\alpha^* - \alpha(t)). \tag{3.35}$$

Combining the tth and $(t+1)$th iterations, we get that around α^*,

$$S = \frac{\alpha(t+1) - \alpha^*}{\alpha(t) - \alpha^*} \cong 1 + \gamma \sum_{i=1}^{N} \frac{1}{\Theta_i''(P_i(\alpha))} \tag{3.36}$$

where $\sum_{i=1}^{N} \frac{1}{\Theta_i''(P_i(\alpha))}$ is non-positive (as can be seen from the proof of Lemma 3.4 in Appendix A). Note that S determines the convergence speed [101] and a larger S means a higher convergence speed. So when $1 + \gamma \sum_{i=1}^{N} \frac{1}{\Theta_i''(P_i(\alpha))}$ is positive, a larger step size gives a higher convergence speed. When $1 + \gamma \sum_{i=1}^{N} \frac{1}{\Theta_i''(P_i(\alpha))}$ is negative, however, oscillation of the gradient projection method might occur, which impedes the convergence speed of our distributed algorithm.

We have shown how to get the NBS-based power allocation based on gradient projection method of the dual problem. Now, we discuss the distributed implementation of the proposed NBS-based power allocation scheme based on the above results.

Assume that each user has local CSI only. In each iteration of the distributed scheme, User i individually calculates $P_i(\alpha)$ according to (3.27) and broadcasts this information to all other users. Then each user updates α according to (3.33). We assume that user updates are synchronized. This cycle repeats until convergence. The distributed implementation is written as Algorithm 3.1.

To guarantee convergence, as specified in Theorem 3.1, the step size in updating α needs to satisfy the condition $0 < \gamma < \frac{2\beta_{\min}}{NP^2(|g_{\max}|^2 P + 1)}$. Thus the users need to

Algorithm 3.1 Distributed NBS-Based Relay Power Allocation

1: Initialize α and γ, e.g., $\alpha = \frac{1}{P}$ and $\gamma = \frac{\beta_{\min}}{NP^2(|g_{\max}|^2P+1)}$.

2: Each user calculates $P_i(\alpha)$ according to (3.27) and broadcasts it to all other users.

3: Each user updates α according to (3.33). Go to Step 2 until convergence.

know $\beta_{\min}$ and $|g_{\max}|$ to agree on a step size. $\beta_{\min}$ is the smallest bargaining power, which is pre-determined and known to all users. For the users to know $|g_{\max}|$, a distributed scheme based on timer [102] can be used: each user starts a timer whose value is an increasing function of $1/|g_i|$. The timer of the user with the smallest $1/|g_i|$ stops first, then it broadcasts its $|g_i|$, which is also $|g_{\max}|$. Other users will hear this signalling and get $|g_{\max}|$. Then the users decide on a step size inside the interval for convergence, e.g., $\gamma = \frac{\beta_{\min}}{NP^2(|g_{\max}|^2P+1)}$.

This distributed scheme based on updating $\alpha(t)$ can be seen as "price-based" power allocation. The parameter α can be interpreted as the price per unit power charged by the relay depending on the requested power from the users, and $F_i(P_i)$ defined in (3.31) represents the maximum benefit that User i can receive at price α. Equation (3.33) says that at time t, if the total demand $\sum_{i=1}^{N} P_i(\alpha(t))$ is larger than the available relay power P, the price should be raised; otherwise it should be reduced.

For the broadcasting of $P_i(\alpha)$, we can adopt a scheme similar to that in [95]: For each channel assigned to the users, a portion of the frequency band is used as the guard channel. Since the guard channels are orthogonal, users can broadcast their power demands simultaneously on these channels. To manage the error accumulation problem, we can use error-correcting codes [103] when broadcasting $P_i(\alpha)$.

3.5 Investigation on Bargaining Power Selection

In this section, we discuss the impact of the bargaining powers on the relay power allocation and show that by proper selection of the bargaining powers, the proposed NBS-based power allocation can bridge the even power allocation, which has the best fairness, and the sum-rate-optimal power allocation, which has the best global performance.

3.5.1 Impact of Bargaining Power Selection on Power Allocation

First, we investigate the effect of bargaining power selection on the proposed NBS-based power allocation. In the following proposition, we show that a user's bargaining power determines its priority and thus its allocated relay power.

Proposition 3.1 If User k's bargaining power β_k is increased while other users' bargaining powers are either decreased or remain unchanged, more power will be allocated to User k. •

Proof: We use contradiction to prove this proposition. For a given set of bargaining powers $\beta_1, \cdots, \beta_N$, let $(P_1 \ \cdots \ P_N)$ be the solution to (3.20), which satisfies (3.25)-(3.27). From (3.25) and the fact that $\lambda_i = 0$, we have

$$\psi(P_i) \triangleq (\xi_i P_i + 1)\, P_i = \beta_i \alpha^{-1}, \text{ for all } i. \tag{3.37}$$

Therefore,

$$\frac{\psi(P_k)}{\psi(P_j)} = \frac{\beta_k}{\beta_j}. \tag{3.38}$$

Now consider another set of bargaining powers $\beta_1', \cdots, \beta_N'$ with $(P_1' \ \cdots \ P_N')$

being the solution to (3.20). For the same reason, we have

$$\frac{\psi(P_k')}{\psi(P_j')} = \frac{\beta_k'}{\beta_j'}. \tag{3.39}$$

Assume that $\beta_k' > \beta_k$ and $\beta_j' \leq \beta_j$ for all $j \neq k$ but $P_k' \leq P_k$. We have

$$\frac{\beta_k'}{\beta_j'} > \frac{\beta_k}{\beta_j} \tag{3.40}$$

and thus

$$\frac{\psi(P_k')}{\psi(P_j')} > \frac{\psi(P_k)}{\psi(P_j)} \text{ for all } j \neq k. \tag{3.41}$$

Note that $\psi(P_i)$ is a strictly increasing function of P_i. So $\psi(P_k') \leq \psi(P_k)$ due to the assumption that $P_k' \leq P_k$. Consequently, from (3.41), we have $\psi(P_j') < \psi(P_j)$, and thus $P_j' < P_j$ for all $j \neq k$, since $\psi(\cdot)$ is monotonically increasing. Thus, $\sum_{i=1}^{N} P_i' < \sum_{i=1}^{N} P_i = P$ and $(P_1' \cdots P_N')$ cannot be a solution to (3.20). This completes the proof. ■

In this work, we assume that the bargaining powers of users are determined by the service provider and they are initiated before the bargaining process. Proposition 3.1 implies that we can adjust the NBS-based relay power allocation solution via adjusting the user bargaining powers. Priorities of users can be materialized with this adjustment. For example, in scenarios where the service provider aims to receive the most monetary revenue, larger bargaining powers can be assigned to users who pay higher price for higher priority. In this way, according to Proposition 3.1, these users will receive more relay power.

3.5.2 Bridging between Global Sum-Rate Optimum and Fairness

In this subsection, we connect the proposed NBS-based relay power allocation with even power allocation, which has the best fairness, and the global sum-rate-optimal power allocation, which has the best global performance. We show that via appropriate bargaining power selection, the proposed NBS-based solution provides a balance between fairness and global performance.

In the even power allocation, the amount of power the relay allocates to each user is P/N. The following proposition is proved.

Proposition 3.2 If

$$\beta_i = \frac{N + P\xi_i}{N^2 + P\sum_{j=1}^{N} \xi_j}, \tag{3.42}$$

the proposed NBS-based power allocation is the same as even power allocation. •

Proof: It is shown in the proof of Proposition 3.1 that with given bargaining powers $\beta_1, \cdots, \beta_N$, the NBS-based power allocation satisfies (3.37). With the value of β_i in (3.42), we have

$$\frac{(\xi_i P_i + 1)P_i}{(\xi_j P_j + 1)P_j} = \frac{\beta_i}{\beta_j} = \frac{N + P\xi_i}{N + P\xi_j}. \tag{3.43}$$

By observation, we can see that this is true if and only if $P_i = P_j = P/N$ for any i, j, which shows that the NSB-based power allocation coincides with the even power allocation when β_i is selected as in (3.42). ■

Recall that ξ_i defined in (3.4) is the noise forwarding rate of User i. From (3.42) we can see that to achieve even power allocation, a user with a larger noise forwarding rate (whose relay-path has a lower quality) should be assigned a larger bargaining power.

The sum-rate-optimal power allocation is the power allocation that maximizes the sum-rate of all users in the network. The sum-rate optimization problem of the network is as follows

$$\begin{aligned}
&\arg\max_{\mathbf{P}}(C_{1RD} + \cdots + C_{NRD}) \\
&= \arg\max_{\mathbf{P}} \sum_{i=1}^{N} \log_2 \left(\frac{\rho_i P_i}{\xi_i P_i + 1} + Q_i|h_i|^2 + 1 \right), \\
&\text{s.t.} \quad \sum_{i=1}^{N} P_i \le P. \qquad (3.44)
\end{aligned}$$

Using the same techniques as in (3.25)-(3.28), we can show that the solution of (3.44) satisfies,

$$\begin{aligned}
&P_i = \frac{-\left(\frac{Q_i|f_i|^2}{Q_i|h_i|^2+1} + 2\right) + \sqrt{\left(\frac{Q_i|f_i|^2}{Q_i|h_i|^2+1} + 2\right)^2 + 4\left(\frac{Q_i|f_i|^2}{Q_i|h_i|^2+1} + 1\right)\left(\frac{\rho_i}{\alpha_1(Q_i|h_i|^2+1)} - 1\right)}}{2\xi_i\left(\frac{Q_i|f_i|^2}{Q_i|h_i|^2+1} + 1\right)} \\
&\text{and} \quad \sum_{i=1}^{N} P_i = P, \qquad (3.45)
\end{aligned}$$

where α_1 is the Lagrangian multiplier associated with the equality constraint. The solution to (3.45), denoted as $\mathbf{P}^o$ (the superscript ‘o’ stands for sum-rate-optimal), can be found by first using bisection method to solve the optimal α_1 using the second equation in (3.45), then using the value of α_1 in the first equation in (3.45) to obtain the P_i’s.

Once $\mathbf{P}^o$ is found, we can find the bargaining powers that equate the NBS-based power allocation with the sum-rate-optimal solution as

$$\beta_i = \frac{\psi(P_i^o)}{\sum_{i=1}^{N} \psi(P_i^o)}, \qquad (3.46)$$

where ψ is defined in (3.37). The proof of this result is similar to the proof of Proposition 3.2, thus is omitted.

We would like to note that the representation of the bargaining power in (3.46) is not in a closed-form but in an implicit form. To find the values, a numerical bisection method as explain above is required. The purpose of the discussion is to show that through proper selection of the bargaining powers, the proposed NBS-based power allocation can achieve the global sum-rate-optimal.

In order to better understand how to select the bargaining powers for global performance, in the following, we use a high SNR approximation for further investigations. One of the widely-used high SNR approximations is to neglect the noise term that is forwarded by the relay, i.e., $\sqrt{\frac{P_i}{Q_i|f_i|^2+1}} g n_{iR}$. This approximation has shown to be sufficiently tight [73], especially in medium to high SNR regions, e.g., when the users are transmitting with a high power, or the relay is close to users. In the following proposition, we give the bargaining powers that equate the NBS-based power allocation with the sum-rate-optimal power allocation.

Proposition 3.3 Let

$$\beta_i = \frac{1}{N} + \sum_{j=1}^{N} \frac{Q_j|h_j|^2+1}{\rho_j NP} - \frac{Q_i|h_i|^2+1}{\rho_i P}. \tag{3.47}$$

For high SNR, if the relay noise is neglected, the proposed NBS-based power allocation maximizes the network sum-rate. •

Proof: When the noise at the relay is neglected, the utility of User i is approximated as

$$\mathrm{SNR}'_{iRD} = \rho_i P_i + Q_i|h_i|^2. \tag{3.48}$$

The disagreement point of User i is the same as in (3.9). So NBS is the solution to the following optimization problem:

$$\begin{aligned} &\arg \max_{P_1,\cdots,P_N} \sum_{i=1}^{N} \beta_i \log(\rho_i P_i) \\ &\text{s.t.} \quad P_i > 0, \quad \sum_{i=1}^{N} P_i = P. \end{aligned} \tag{3.49}$$

Using the same optimization techniques in (3.25)-(3.28), we can show straightforwardly that the solution to (3.49) is

$$P_i^{NBS} = \beta_i P. \tag{3.50}$$

For sum-rate-optimal solution, with the high-SNR approximation, (3.44) is equivalent to the following problem:

$$\begin{aligned} &\arg \max_{P_1,\cdots,P_N} \sum_{i=1}^{N} \log\left(\rho_i P_i + Q_i|h_i|^2 + 1\right) \\ &\text{s.t.} \quad P_i > 0, \quad \sum_{i=1}^{N} P_i = P. \end{aligned} \tag{3.51}$$

Again by using the KKT conditions, the solution is

$$P_i^o = \frac{P}{N} + \sum_{j=1}^{N} \frac{Q_j|h_j|^2 + 1}{\rho_j N} - \frac{Q_i|h_i|^2 + 1}{\rho_i}. \tag{3.52}$$

When β_i is defined as in (3.47), we have

$$P_i^{NBS} = P_i^o. \tag{3.53}$$

■

We can see that the first two terms in (3.52) are the same for all users. So the last term is the dominant factor in the bargaining power selection in achieving global sum-rate-optimal. Recall that ρ_i defined in (3.5) is the signal forwarding rate of User i. (3.47) shows that for global optimum, a user with a larger signal forwarding rate (whose relay-path has a higher quality) should be assigned a larger bargaining power. This has the opposite trend as the even power allocation case. The other coefficient $(Q_i|h_i|^2 + 1)$ in the last terms relates to the direct link and is independent of the relay link.

Based on the above discussions, for networks with different requirements, we can adjust the NBS-based relay power allocation toward the requirements by adjusting the bargaining powers. For example, in a network design, if the global sum-rate-optimal power allocation is desired, users whose relay-paths have higher quality should be allocated more relay power. With the proposed NBS-based power allocation, we can obtain good network sum-rate by assigning larger bargaining powers to such users. On the other hand, if fairness is the major concern, we can assign larger bargaining powers to users whose relay-paths have lower quality. Those users can thus obtain more relay powers to ensure a certain level of quality, which helps the fairness consideration of the network. But this improved fairness is at the cost of lower network sum-rate.

3.6 Simulation Results

In this section, we show the performance of our NBS-based power allocation solution and compare it with the sum-rate-optimal solution, the even power solution, and the rate-fair solution. The sum-rate-optimal solution is the relay power allocation that maximizes the network sum-rate while fairness is not considered. With the even power solution, the relay power assigned to each user is P/N. It has the best fairness in the sense of power. The rate-fair solution is the relay power allocation that makes all users in the network have the same achievable rate. It has the best fairness in the sense of achievable rate. It is not always possible, depending on the values of the channel coefficients. We compare four parameters: network sum-rate, individual achievable rate γ_i, the normalized-rate-difference, which is defined as $\mathbf{E}\{[max_i(\gamma_i)-min_i(\gamma_i)]/max_i(\gamma_i)\}$, and the normalized-power-difference $\mathbf{E}\{[max_i(P_i) - min_i(P_i)]/max_i(P_i)\}$. A smaller normalized-rate-difference (or normalized-power-difference) indicates a fairer solution. Other fairness metrics, e.g. Jain's fairness index [104], show the same performance trend. Two channel models are considered: Rayleigh flat-fading channels and static channels with path-loss only.

3.6.1 Rayleigh Flat-Fading Channels

For the Rayleigh flat-fading model, the channel gains, f_i, h_i, and g, are modeled as independent and identically distributed (i.i.d.) random variables following the distribution $\mathcal{CN}(0,1)$. We consider a three-user network and all users have the same bargaining power: $\beta_1 = \beta_2 = \beta_3 = 1/3$. The transmit power of each user is

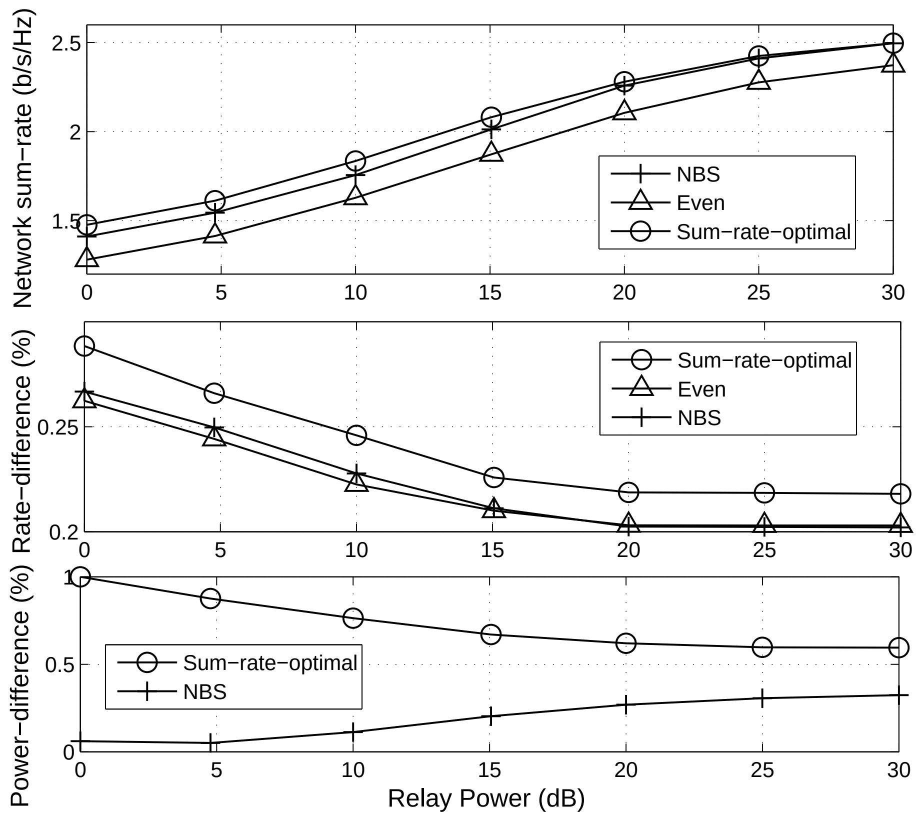

Figure 3.2: Sum-rate, normalized-rate-difference, and normalized-power-difference of a three-user relay network with Rayleigh fading channels.

set to be 10 dB. The relay power constraint P is in the range of 0 to 30 dB. Since for this channel mode, rate-fair solution is not always possible, the proposed solution is only compared with the sum-rate-optimal and the even power solutions.

Figure 3.2 compares the average sum-rate, normalized-rate-difference, and normalized-power-difference of the sum-rate-optimal solution, even power allocation, and the NBS-based power allocation. For even power allocation, as the relay allocates the same power to all three users, the normalized-power-difference is 0, thus is not shown in Figure 3.2. It can be seen that in the simulated power range, the sum-rate difference between the proposed NBS-based and the sum-rate-optimal solutions is within 4%, while it is within 14% between the sum-rate-optimal and the even power solutions. The proposed solution is about 4 dB superior to the even power solution in global sum-rate performance. From the normalized-rate-difference, we find that our NBS-based solution has similar rate-fairness to the even power solution and is fairer than the sum-rate-optimal solution. From the normalized-power-difference, we find that our NBS-based solution is fairer in the sense of power than the sum-rate-optimal solution.

3.6.2 Static Channels With Path-Loss Only

In this section, we consider a static network whose channels are only related to the path-loss, which is inverse proportional to the distance squared. The network has two users, one relay, and two destinations. The relative positions of the nodes are shown in Figure 3.3, where the coordinates of User 1, User 2, the relay, Destination 1, and Destination 2 are (-9, 0), (-3, 0), (0, 0), (7, 12), and (13, 0), respectively. Thus, User 2 has a better relay channel. The transmission power of both users are 20 dB, and the relay power constraint P ranges from 20 dB to 30 dB.

To investigate the global network sum-rate, the fairness, and the effect of the bargaining powers on network performance, we show the individual achievable rates of the users (in Figure 3.4), network sum-rate, and the normalized-rate-difference (in Figure 3.5) under the proposed solutions with two different sets of bargaining powers: $\beta_1 = 0.3, \beta_2 = 0.7$ and $\beta_1 = 0.7, \beta_2 = 0.3$. For comparison, the individual achievable rates under the sum-rate-optimal solution and the rate-fair solution are also shown. As the achievable rates of the two user are the same for the rate-fair solution, the normalized-rate-difference is 0 for this scheme and is not shown in Figure 3.5.

Comparing the two NBS-based power allocation schemes with different bargaining powers, we can see from the two figures that a user achieves a higher rate with a larger bargaining power, and the bargaining power can be tuned to gain the desired balance between the global network sum-rate and individual rate-fairness.

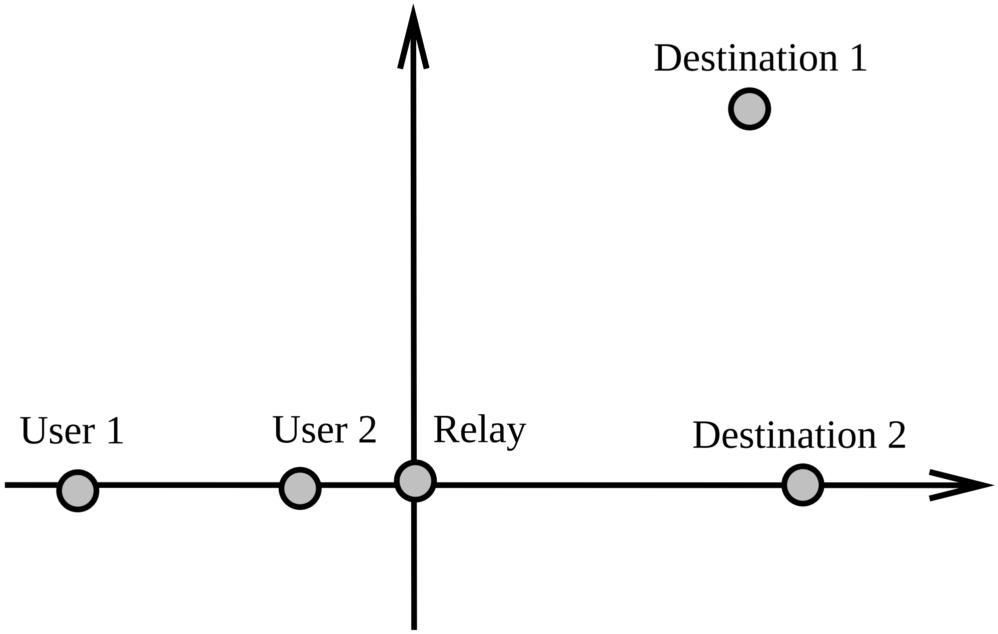

Figure 3.3: A two-user relay network with static channels.

When User 2, who has a better channel, is assigned a higher bargaining power, the NBS-based solution emphasizes more on the network sum-rate and allocates more relay power to User 2. In Figure 3.4 and 3.5, the sum-rate performance of the NBS-based solution with $\beta_1 = 0.3, \beta_2 = 0.7$ is very close to that of the sum-rate-optimal solution. In this case, User 1, with a worse channel, experiences low achievable rate, which is 37% to 50% of the achievable rate of User 2. On the contrary, when a larger bargaining power 0.7 is assigned to User 1, the NBS-base solution allocates more power to User 1, and the performance is closer to the rate-fair solution. In this case, the network sum-rate is reduced to 90% of that of the sum-rate-optimal solution when P is small and 93% when P is large. The normalized-rate-difference justifies the above-mentioned analysis, which shows that NBS-based solution with $\beta_1 = 0.7, \beta_2 = 0.3$ is fairer in the sense of rate than the other two schemes.

To further illustrate the effect of the bargaining powers on network performance, we show the network sum-rate, normalized-rate-difference, and normalized-power-

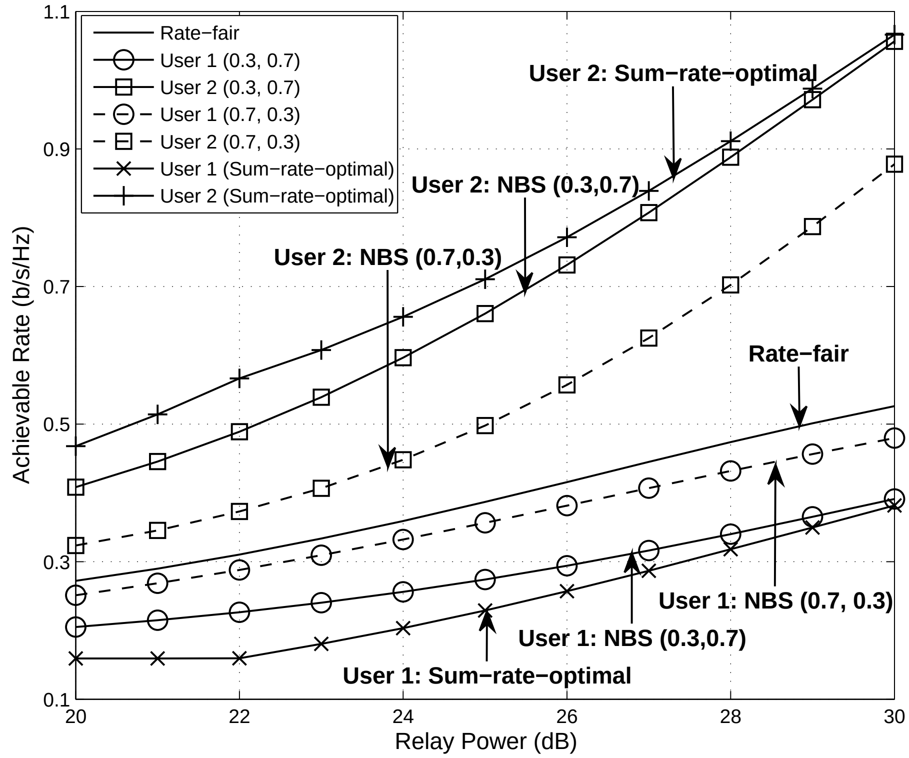

Figure 3.4: Achievable rates of a two-user relay network with static channels.

difference in Figure 3.6 under the proposed solution with the bargaining power of User 1 changing from 0 to 1. We consider three relay powers: 25 dB, 30 dB, and 35 dB. Other network conditions are the same as the static network shown in Figure 3.3. When $\beta_1 = 0$ or $\beta_1 = 1$, all relay power is allocated to User 2 or User 1, so the normalized-rate-difference is 1. For the three different relay powers, network sum-rate is maximized at approximately $\beta_1 = 0.25$. After that, we can see a reduction in the network sum-rate as β_1 increases, which verifies the conclusion in Section 3.5.2: by assigning a larger bargaining power to User 2 which has a higher signal forwarding power, the solution approaches the sum-rate-optimal solution. For fairness in the sense of both rate and power, the normalized-rate-difference and the normalized-power-difference decrease as β_1 increases until rate-fair or power-

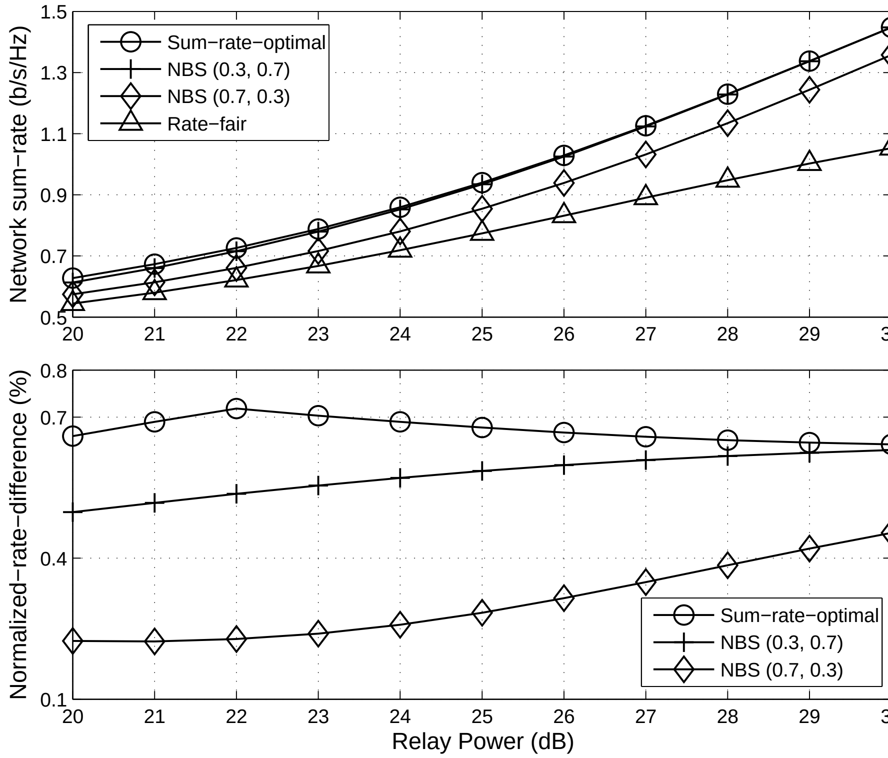

Figure 3.5: Sum-rate and normalized-rate-difference of a two-user relay network with static channels.

fair is achieved. For $P = 25, 30$, and 35 dB, when $\beta_1 = 0.6, 0.64$, and 0.675, the proposed NBS-based power allocation becomes even power allocation. These values of β_1 are the same as been calculated with Proposition 3.2. This verifies our claim in Section 3.5.2 that on the contrary to sum-rate optimum, power fairness can be approached by assigning higher bargaining power to User 1 which has a larger noise forwarding rate, and thus lower quality in the relay path. Similar to power-fairness, for rate-fairness, the user with a higher noise forwarding rate should be assigned a higher bargaining power. For $P = 25, 30$, and 35 dB, rate-fair power allocation can be achieved using the proposed NBS-based power allocation when $\beta_1 = 0.9, 0.95$, and 0.97, respectively.

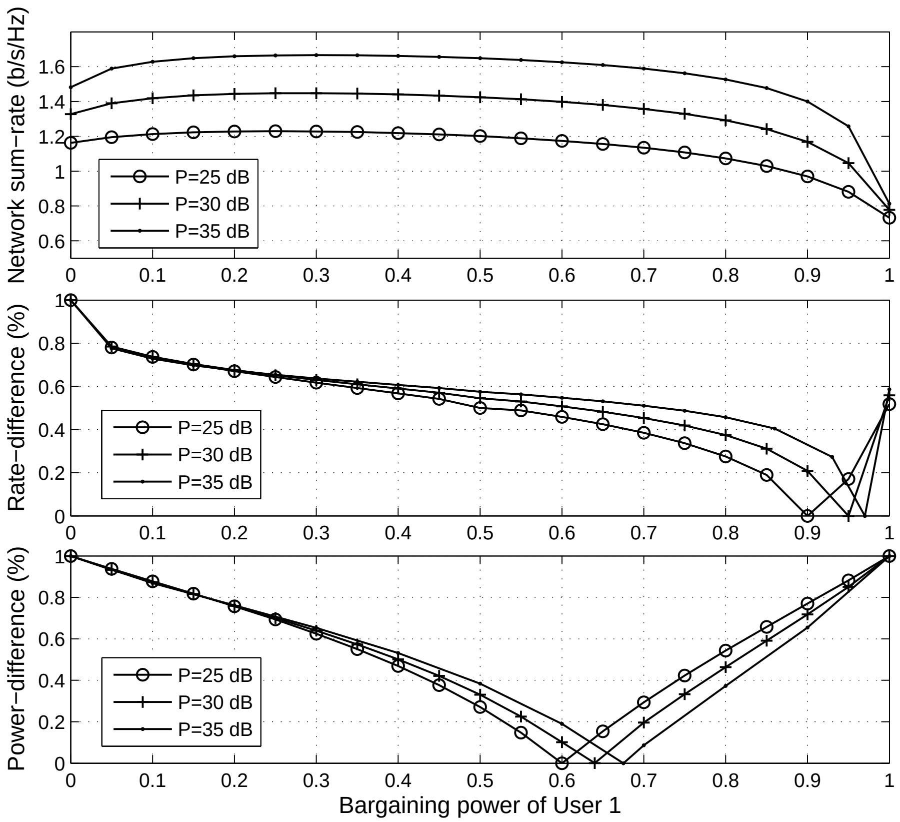

Figure 3.6: Sum-rate, normalized-rate-difference, and normalized-power-difference of a two-user relay network with different relay powers and varying bargaining powers.

Figure 3.7 illustrates the convergence of the distributed relay power allocation. In this simulation, the relay power is set to be 30 dB, the bargaining powers of the two users are $\beta_1 = 0.7, \beta_2 = 0.3$, and all other network settings are the same as the network in Figure 3.3. α is initialized as 0.1. We can see from Figure 3.7 that the proposed distributed scheme converges after 2 iterations and similar performance is verified with different initial values of α.

To further illustrate the convergence performance of the proposed distributed relay power allocation, we show the network sum-rate of a fifty-user single-relay network in Figure 3.8. In this simulation, the relay power is set to be 30 dB, the bargaining powers of all users are the same $1/50$. The transmit power of all users

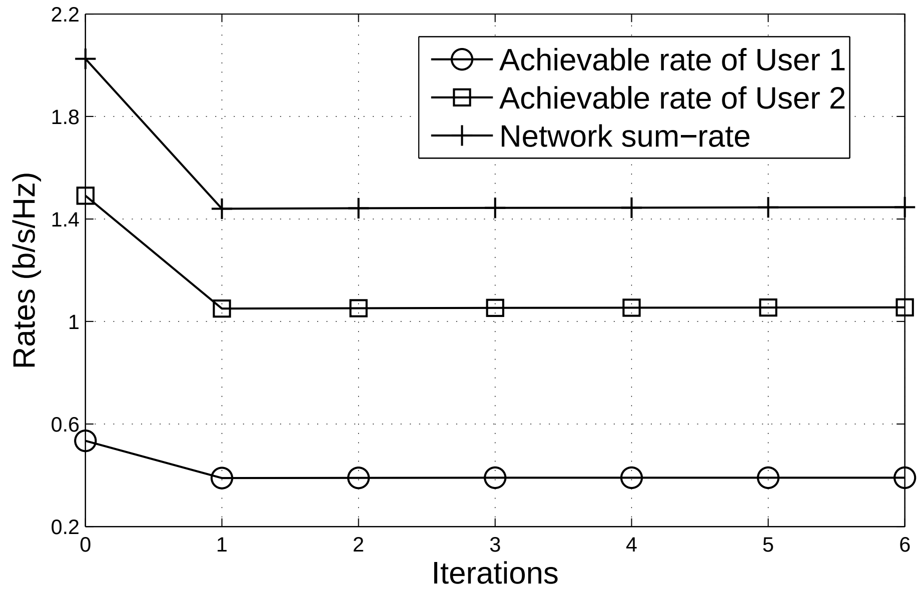

Figure 3.7: Convergence of the distributed NBS-based power allocation algorithm in a two-user relay network.

are 10 dB. We generate one realization of the Rayleigh flat-fading channels. α is initialized as 0.01. We can see from Figure 3.8 that the proposed distributed scheme converges after 10 iterations.

3.7 Extension to Multi-User Multi-Relay Networks

In this section, we discuss the extension of our work to multi-user multi-relay networks where users can receive help from multiple relays. Assume that there are N users and R relays as shown in Figure 3.9.

Assume that the relays also use orthogonal channels. Denote the channel gain from User i to Relay r as f_{ir}, and the channel gain from Relay r to Destination i as g_{ir}. Denote the power constraint of Relay r as $P^{(r)}$ and Relay r uses power P_{ir} to help User i. So the power allocation for all users from all relays can be denoted as a matrix $\{P_{ir}\}$, where the row index is the user index and the column index is the relay index. Denote the vector that contains the power allocation from all relays for

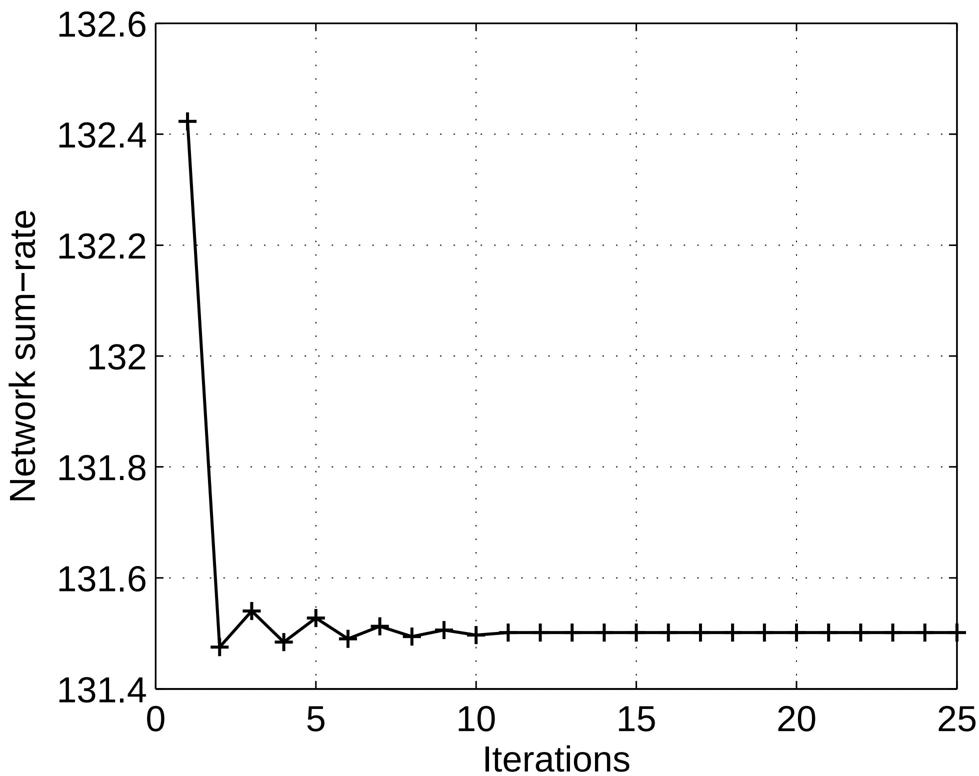

Figure 3.8: Convergence of the distributed NBS-based power allocation algorithm in a fifty-user relay network.

User i as

$$\mathbf{P}_i = [P_{i1}, P_{i2}, \cdots, P_{iR}]^T, \tag{3.54}$$

and the vector that contains power allocation of Relay r for all users as

$$\mathbf{P}^{(r)} = [P_{1r}, P_{2r}, \cdots, P_{Nr}]^T. \tag{3.55}$$

Define the noise forwarding rate of User i at Relay r as

$$\xi_{ir} \triangleq \frac{|g_{ir}|^2}{Q_i|f_{ir}|^2 + 1}, \tag{3.56}$$

and the signal forwarding rate of User i at Relay r as

$$\rho_{ir} \triangleq \frac{Q_i|f_{ir}g_{ir}|^2}{Q_i|f_{ir}|^2 + 1}. \tag{3.57}$$

Other assumptions and notation are the same as the single-relay case.

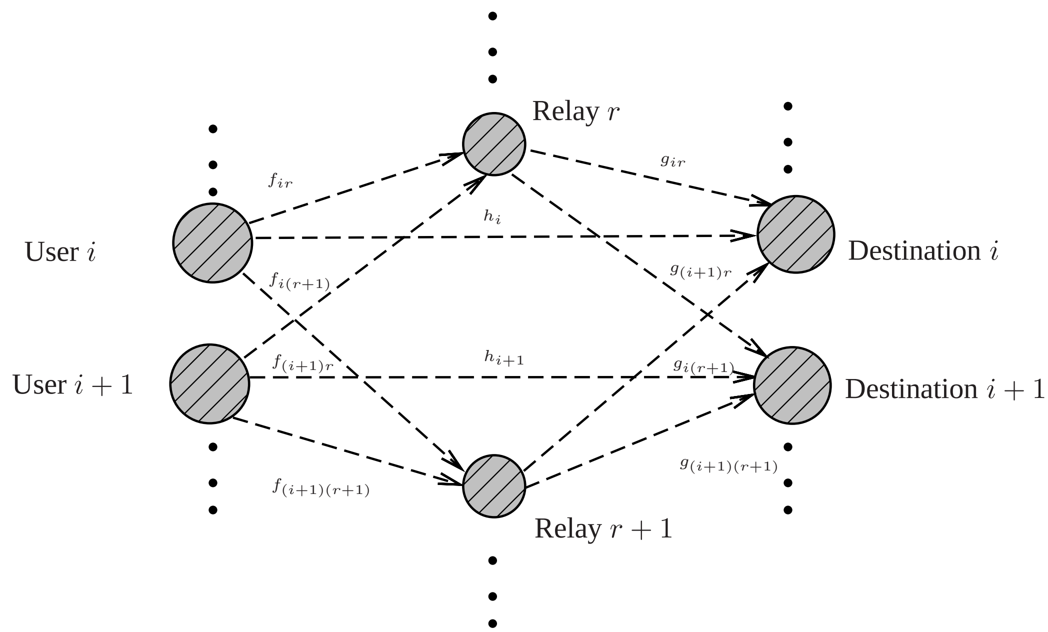

Figure 3.9: A multi-user multi-relay network.

With maximum ratio-combining, the received SNR of User i's transmission is

$$\mathrm{SNR}_{iRD} = \sum_{r=1}^{R} \frac{\rho_{ir} P_{ir}}{\xi_{ir} P_{ir} + 1} + Q_i |h_i|^2. \tag{3.58}$$

Similarly, define the utility of User i as:

$$u_i(\mathbf{P}_i) \triangleq \mathrm{SNR}_{iRD}. \tag{3.59}$$

Denote the minimum utility that User i expects as

$$u_{i,0} = Q_i |h_i|^2.$$

Similar to the single-relay case, to use the NBS-based power allocation, we first need to prove that the feasible set

$$\mathcal{S}^M \triangleq \left\{ (u_1 \cdots u_N) \middle| P_{ir} \geq 0, \quad \sum_{i=1}^{N} P_{ir} \leq P^{(r)}, \quad r = 1 \cdots R \right\} \tag{3.60}$$

is convex.

Lemma 3.5 Given the utility function $u_i(\mathbf{P}_i)$ in (3.59), the feasible set $\mathcal{S}^M$ in (3.60) is convex.

Proof: Given $\{x_{ir}\}$ as a feasible power allocation matrix where x_{ir} is the power allocation from Relay r to User i, denote the power allocation vector at Relay r for all users as

$$\mathbf{x}^{(r)} = [x_{1r} \cdots x_{Nr}]^T,$$

and the power allocation vector for User i from all relays as

$$\mathbf{x}_i = [x_{i1} \cdots x_{iR}]^T.$$

Similarly, we define another power allocation matrix y_{ir}. The corresponding power allocation vector at Relay r for all users will be $\mathbf{y}^{(r)}$ and the corresponding power allocation vector for User i from all relays will be $\mathbf{y}_i$. To prove that $\mathcal{S}^M$ is convex, we need to show that given two arbitrary power allocation matrices $\{x_{ir}\}$ and $\{y_{ir}\}$ and the corresponding utility vectors $\mathbf{u} = [u_1(\mathbf{x}_1), u_2(\mathbf{x}_2), \cdots, u_N(\mathbf{x}_N)]^T$ and $\mathbf{v} = [u_1(\mathbf{y}_1), u_2(\mathbf{y}_2), \cdots, u_N(\mathbf{y}_N)]^T$ in the feasible set $\mathcal{S}^M$, we have $\theta\mathbf{u} + (1-\theta)\mathbf{v} \in \mathcal{S}^M$ for any $0 \leq \theta \leq 1$.

Note that

$$\mathbf{u}=\begin{bmatrix} u_1(\mathbf{x}_1) \\ u_2(\mathbf{x}_2) \\ \vdots \\ u_N(\mathbf{x}_N) \end{bmatrix} = \begin{bmatrix} \frac{\rho_{11}x_{11}}{\xi_{11}x_{11}+1} + \cdots + \frac{\rho_{1R}x_{1R}}{\xi_{1R}x_{1R}+1} + u_{1,0} \\ \frac{\rho_{21}x_{21}}{\xi_{21}x_{21}+1} + \cdots + \frac{\rho_{2R}x_{2R}}{\xi_{2R}x_{2R}+1} + u_{2,0} \\ \vdots \\ \frac{\rho_{N1}x_{N1}}{\xi_{N1}x_{N1}+1} + \cdots + \frac{\rho_{NR}x_{NR}}{\xi_{NR}x_{NR}+1} + u_{N,0} \end{bmatrix} \tag{3.61}$$

$$= \begin{bmatrix} \frac{\rho_{11}x_{11}}{\xi_{11}x_{11}+1} \\ \frac{\rho_{21}x_{21}}{\xi_{21}x_{21}+1} \\ \vdots \\ \frac{\rho_{N1}x_{N1}}{\xi_{N1}x_{N1}+1} \end{bmatrix} + \cdots + \begin{bmatrix} \frac{\rho_{1R}P_{1R}}{\xi_{1R}P_{1R}+1} \\ \frac{\rho_{2R}P_{2R}}{\xi_{21}P_{2R}+1} \\ \vdots \\ \frac{\rho_{NR}P_{NR}}{\xi_{N1}P_{NR}+1} \end{bmatrix} + \begin{bmatrix} u_{1,0} \\ u_{2,0} \\ \vdots \\ u_{N,0} \end{bmatrix} \tag{3.62}$$

$$=\mathbf{f}^1\left(\mathbf{x}^{(1)}\right) + \mathbf{f}^2\left(\mathbf{x}^{(2)}\right) + \cdots + \mathbf{f}^R\left(\mathbf{x}^{(R)}\right) + \mathbf{u}_0, \tag{3.63}$$

where

$$\mathbf{f}^r\left(\mathbf{P}^{(r)}\right) \triangleq \left[\frac{\rho_{1r}x_{1r}}{\xi_{1r}x_{1r}+1} \cdots \frac{\rho_{Nr}x_{Nr}}{\xi_{Nr}x_{Nr}+1}\right]^T, \tag{3.64}$$

for $r = 1, \cdots, R$. Similarly, given the power allocation matrix $\{y_{ir}\}$, the corresponding utility vector is

$$\mathbf{v} = \mathbf{f}^1\left(\mathbf{y}^{(1)}\right) + \mathbf{f}^2\left(\mathbf{y}^{(2)}\right) + \cdots + \mathbf{f}^R\left(\mathbf{y}^{(R)}\right) + \mathbf{u}_0. \tag{3.65}$$

Therefore, we have

$$\begin{aligned}\theta\mathbf{u} + (1-\theta)\mathbf{v} =& \left[\theta\mathbf{f}^1\left(\mathbf{x}^1\right) + (1-\theta)\mathbf{f}^1\left(\mathbf{y}^1\right)\right] \\ &+ \cdots + \left[\theta\mathbf{f}^R\left(\mathbf{x}^R\right) + (1-\theta)\mathbf{f}^R\left(\mathbf{y}^R\right)\right] + \mathbf{u}_0. \end{aligned} \tag{3.66}$$

Note that $\left\{\mathbf{f}^r\left(\mathbf{P}^{(r)}\right) + \mathbf{u}_0 \middle| P_{ir} \geq 0, \sum_{i=1}^N P_{ir} \leq P^{(r)}\right\}$ is the feasible set of a network with a single relay, Relay r. It is proved to be convex from Lemma 3.1. Therefore, for any Relay r and any $0 \leq \theta \leq 1$, we can find another power allocation vector $\mathbf{z}^{(r)}$ with $z_{ir} \geq 0$ and $\sum_{i=1}^N z_{ir} \leq P^{(r)}$ such that

$$\mathbf{f}^r\left(\mathbf{z}^{(r)}\right) = \theta\mathbf{f}^r\left(\mathbf{x}^{(r)}\right) + (1-\theta)\mathbf{f}^r\left(\mathbf{y}^{(r)}\right). \tag{3.67}$$

Combining the power allocation vectors $\{\mathbf{z}^{(r)}\}$ for all relays, we can find the feasible power allocation matrix $\{z_{ir}\}$ such that

$$\theta\mathbf{u} + (1-\theta)\mathbf{v} = \mathbf{f}^1\left(\mathbf{z}^{(1)}\right) + \mathbf{f}^2\left(\mathbf{z}^{(2)}\right) + \cdots + \mathbf{f}^R\left(\mathbf{z}^{(R)}\right) + \mathbf{u}_0 \in \mathcal{S}^M. \tag{3.68}$$

This completes the proof. ■

In addition, Lemma 3.2 is also valid for the multi-relay case, that is, there is at least one point in $\mathcal{S}^M$ with $u_i > u_{i,0}$ for all $i = 1, \cdots, N$. This can also be shown by construction. Consider the even power allocation $P_{ir} = P^{(r)}/N$. Since u_i is an increasing function of P_{ir}, we have $u_i > u_{i,0}$ under even power allocation.

Therefore, the asymmetric NBS for the multi-relay network is the solution of the following optimization problem:

$$\begin{aligned} &\arg\max_{\mathbf{P}_1, \cdots, \mathbf{P}_N} \sum_{i=1}^N \beta_i \log\left(\sum_{r=1}^R \frac{\rho_{ir}P_{ir}}{\xi_{ir}P_{ir}+1}\right) \\ &\text{s.t.} \quad P_{ir} > 0, \quad \sum_{i=1}^N P_{ir} = P^{(r)}. \end{aligned} \tag{3.69}$$

This is a convex optimization problem and can be solved efficiently using standard convex optimization techniques [116] when centralized implementation is possible.

To implement the distributed NBS-based power allocation, we can follow the same technique in Section 3.4.2. First, we write the Lagrangian function for (3.69) as

$$\begin{aligned}\mathcal{L}(\{P_{ir}\},\vec{\alpha}) =&\sum_{i=1}^{N}\beta_i \log\left(\sum_{r=1}^{R}\frac{\rho_{ir}P_{ir}}{\xi_{ir}P_{ir}+1}\right)\\ &-\sum_{i=1}^{NR}\lambda_{ir}P_{ir}-\sum_{r=1}^{R}\alpha_r\left(\sum_{i=1}^{N}P_{ir}-P^{(r)}\right).\end{aligned} \tag{3.70}$$

Here λ_{ir} and $\vec{\alpha}\triangleq[\alpha_1\cdots\alpha_R]$ are Lagrangian multipliers associated with the inequality and equality constraints. Same as the analysis of the single-relay networks in Section 3.3.2, we have $\lambda_{ir}=0$ for all $i=1,\cdots,N$ and $r=1,\cdots R$ as $P_{ir}>0$.

Then, similar to the analysis of the single relay network in Section 3.4.2, the dual problem of (3.69) is: $\min_{\vec{\alpha}\geq 0} D^M(\vec{\alpha})$, where $D^M(\vec{\alpha})$ is the dual function defined as

$$\begin{aligned}&D^M(\vec{\alpha})\triangleq\max_{\{P_{ir}\}}\mathcal{L}(\{P_{ir}\},\vec{\alpha})\\ &=\max_{\{P_{ir}\}}\left\{\sum_{i=1}^{N}\beta_i\log\left(\sum_{r=1}^{R}\frac{\rho_{ir}P_{ir}}{\xi_{ir}P_{ir}+1}\right)-\sum_{r=1}^{R}\alpha_r\left(\sum_{i=1}^{N}P_{ir}-P^{(r)}\right)\right\}\\ &=\sum_{i=1}^{N}\left\{\max_{\mathbf{P}_i}\underbrace{\left[\beta_i\log\left(\sum_{r=1}^{R}\frac{\rho_{ir}P_{ir}}{\xi_{ir}P_{ir}+1}\right)-\sum_{r=1}^{R}\alpha_rP_{ir}\right]}_{\triangleq F_i(\mathbf{P}_i)}+\sum_{r=1}^{R}\alpha_rP^{(r)}\right\}.\end{aligned} \tag{3.71}$$

As explained in the single-relay case, the equality in (3.71) holds since the summation term in $\mathcal{L}(\{P_{ir}\},\vec{\alpha})$ is separable in $\mathbf{P}_i$.

The gradient of $D^M(\vec{\alpha})$ is

$$\frac{\partial D^M(\vec{\alpha})}{\partial\alpha_r}=P^{(r)}-\sum_{i=1}^{N}P_{ir}(\vec{\alpha}),\quad r=1,\cdots,R, \tag{3.72}$$

where $\{P_{ir}(\vec{\alpha})\}_{i=1}^{N}$ is the maximizer of $F_i(\mathbf{P}_i)$ in (3.71) for a given $\vec{\alpha}$. Since $F_i(\mathbf{P}_i)$ is a convex function, $\{P_{ir}(\vec{\alpha})\}_{i=1}^{N}$ can be calculated with standard convex optimization techniques.

The dual problem can be solved with the gradient project method where α_r can be adjusted in the opposite direction to $\frac{\partial D^M(\vec{\alpha})}{\partial \alpha_r}$ as:

$$\begin{aligned}\alpha_r(t+1) &= \max\left\{0, \alpha_r(t) - \gamma_r \frac{\partial D}{\partial \alpha_r}(\vec{\alpha}(t))\right\} \\ &= \max\left\{0, \alpha_r(t) - \gamma_r \left[P^{(r)} - \sum_{i=1}^{N} P_{ir}(\overrightarrow{\alpha})\right]\right\}. \end{aligned} \tag{3.73}$$

Similar to Theorem 1, we can show that the gradient projection method converges to the primal and dual optimal point for all relays if the step-size satisfies

$$0 < \gamma_r < \frac{2\beta_{\min}}{N{P^{(r)}}^2(|g_{\max}^{(r)}|^2 P^{(r)} + 1)}. \tag{3.74}$$

Here,

$$|g_{\max}^{(r)}| \triangleq \max\{|g_{1r}|, \cdots, |g_{Nr}|\}. \tag{3.75}$$

Assume that each user has local CSI only. In each iteration of the distributed scheme, User i individually calculates $P_{ir}(\overrightarrow{\alpha})$ (for $r = 1, \cdots, R$) and broadcasts this information to all other users. Then each user updates $\overrightarrow{\alpha}$ according to (3.73). This cycle repeats until convergence. The distributed implementation of the NBS-based power power allocation for multi-relay networks can be summarized as in Algorithm 3.2.

Figure 3.10 and Figure 3.11 show the performance of the proposed solutions in two-user three-relay networks with Rayleigh flat-fading channels and static channels, respectively. We also compare the proposed solutions with the sum-rate-optimal solution and the even power solution. We assume that the two users have the same bargaining power: $\beta_1 = \beta_2 = 1/2$.

For the Rayleigh flat-fading model, the channel gains, f_{ir}, h_i, and g_{ir}, are modeled as i.i.d. random variables following the distribution $\mathcal{CN}(0, 1)$. The transmit

Algorithm 3.2 Distributed NBS-Based Relay Power Allocation for Multi-Relay Networks

1: Initialize α_r and γ_r, e.g., $\alpha_r = \frac{1}{P^{(r)}}$ and $\gamma_r = \frac{\beta_{\min}}{NP^{(r)^2}(|g_{\max}^{(r)}|^2 P^{(r)}+1)}$, for $r = 1, \cdots, R$.

2: Each user calculates $P_{ir}(\overrightarrow{\alpha})$ (for $r = 1, \cdots, R$) that maximizes $F_i(\mathbf{P}_i)$ in (3.71) and broadcasts this information to all other users.

3: Each user updates $\overrightarrow{\alpha}$ according to (3.73). Go to Step 2 until convergence.

power of each user is set to be 10 dB. The power constraints at all relays are the same and are in the range of 0 to 30 dB. Figure 3.10 compares the average sum-rate and normalized-rate-difference of the sum-rate-optimal solution, even power allocation, and the NBS-based power allocation. It can be seen that the proposed solution is about 2 dB superior to the even power solution in global sum-rate performance. From the normalized-rate-difference, we find that our NBS-based solution has similar rate-fairness to the even power solution and is fairer than the sum-rate-optimal solution. This verifies our conclusion for the single-relay case in Figure 3.2.

For static channels with path-loss only, we add two more relays to the system setup in Figure 3.3 at (0, 1) and (0, -1), respectively. The transmit power of both users is set to be 10 dB. The power constraints at all relays are the same and in the range of 0 to 30 dB. Figure 3.11 compares the network sum-rate and normalized-rate-difference of the sum-rate-optimal solution, even power allocation, and the NBS-based power allocation. From Figure 3.11, we can see that the sum-rate performance of the NBS-based solution is very close to that of the sum-rate-optimal solution. The normalized-rate-difference of NBS-based solution is fairer in the sense of rate than sum-rate-optimal solution and has similar performance as even power allocation.

3.8 Conclusion

In this chapter, we consider a multi-user single-relay wireless network, and conduct the game-theoretic analysis of relay power allocation among the users. We propose an asymmetric NBS-based power allocation solution, where each user is assigned a bargaining power indicating its transmission priority. We first proposed a centralized algorithm to implement the NBS-based power allocation at the relay. Then, to improve the scalability of the proposed scheme, we provide a distributed algorithm for the NBS-based power allocation and its convergence conditions are provided. We show that bargaining powers can be adjusted to accommodate different requirements in different applications. After that, we generalize our NBS-based power allocation solution and its distributed implementation to multi-user multi-relay networks. Simulations are conducted to compare the proposed NBS-based power allocation with the sum-rate-optimal power allocation, the even power allocation, and the rate-fair power allocation. We find that the proposed NBS-based scheme has better sum-rate than even and rate-fair power allocation and is fairer than the sum-rate-optimal solution. Via simulation, we also demonstrate the impact of the bargaining powers on the proposed relay power allocation solution. We show that the proposed scheme can bridge the sum-rate-optimal power allocation, which has the best global performance and the even power allocation, which has the best fairness, by proper selection of bargaining powers.

$\sim$

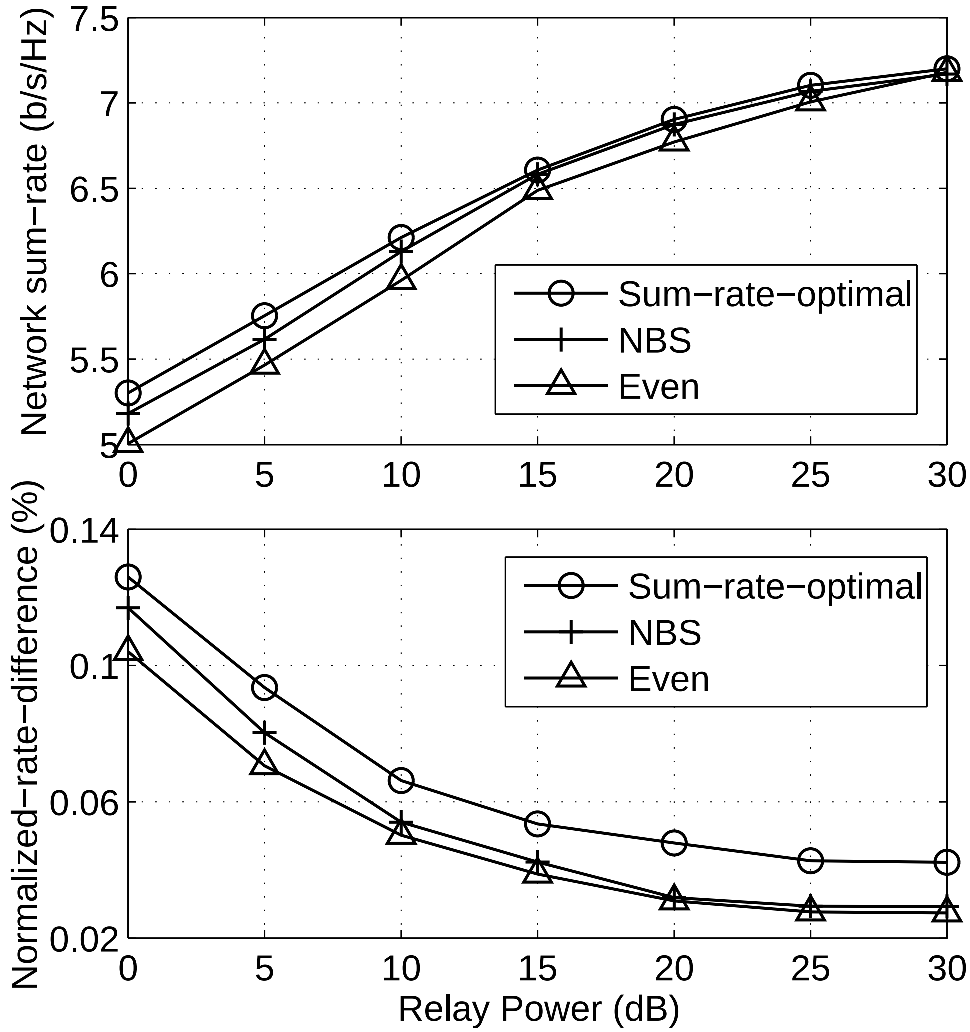

Figure 3.10: Sum-rate and normalized-rate-difference of a two-user three-relay network with Rayleigh fading channels.

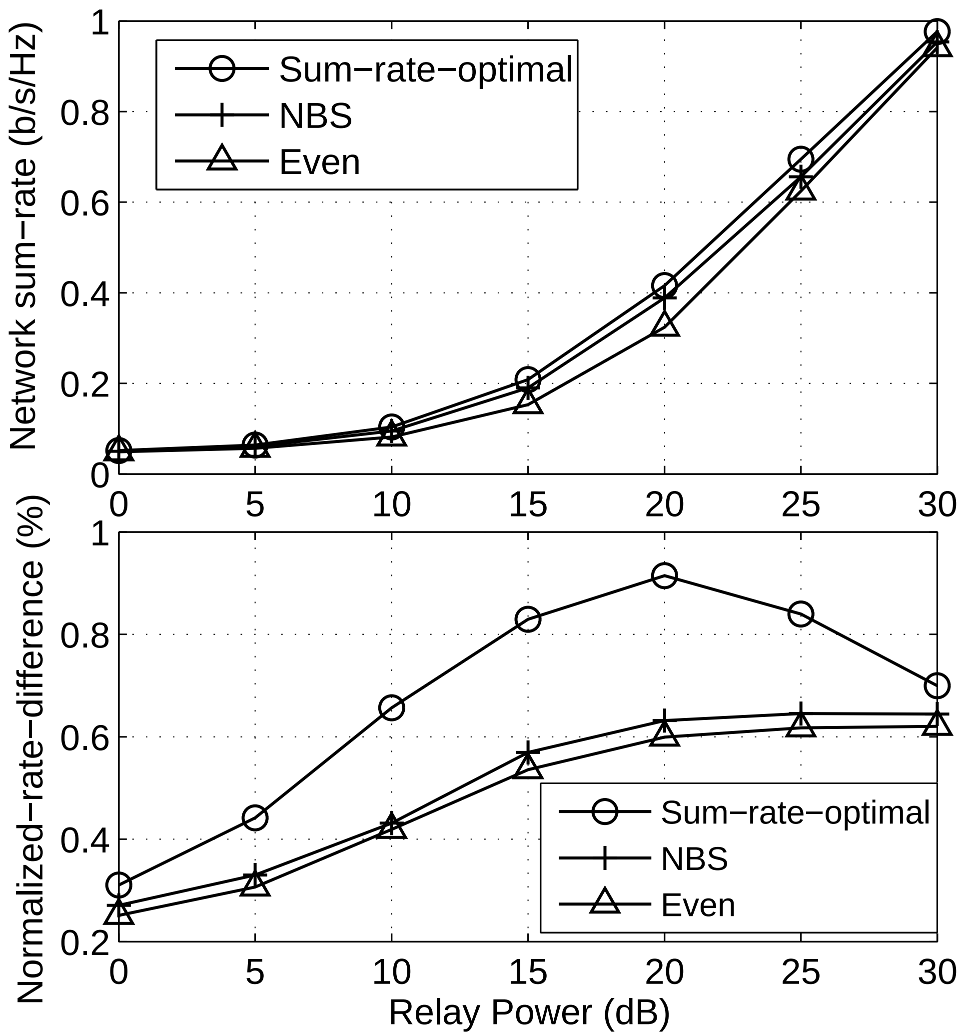

Figure 3.11: Sum-rate and normalized-rate-difference of a two-user three-relay network with static channels.

Chapter 4

Power Allocation and Pricing in Multi-User Relay Networks Using Bargaining and Stackelberg Games

In this chapter, we study the power allocation problem in multi-user relay networks with relay cooperation stimulation. We aim at finding the optimal relay pricing strategy and a fair power allocation corresponding to it. We use the Stackelberg game to model the interaction among the users and the relay. In Stackelberg game formulation, followers are normally modeled as non-cooperative players [122]. In this chapter, to get a fair relay power allocation among the users, we use bargaining theory to model the negotiation among them and use KSBS for relay power allocation. Based on the proposed fair relay power allocation rule, the optimal relay power price that maximizes the relay revenue is derived analytically. Simulation shows that the proposed power allocation scheme achieves higher network sum-rate and relay revenue than the even power allocation. Furthermore, compared with the sum-rate-optimal solution, simulation shows that the proposed scheme achieves better fairness with comparable network sum-rate for a wide range of network scenarios. The proposed pricing and power allocation solutions are also shown to be consistent with the laws of supply and demand.[1]

4.1 Introduction

As introduced in Section 1.4.2, the payment-based scheme is the primary mechanism for cooperation stimulation in multi-user relay networks. In this chapter, we consider an AF multi-user single-relay network. We use the pricing mechanism where the relay gets paid for signal forwarding and the users pay for the relay service. We model the interaction between the relay and the users as a two-level Stackelberg game, in which the relay is the leader and sets the unit power price for the relay service, and users are the followers where each user decides how much power to purchase from the relay. This work is different from [85, 87–91] in the network and channel models. Compared with [86], this work considers the relay power allocation among users, instead of the user power control; and also the relay power competition among users is modeled as a cooperative bargaining game. For the relay power allocation, KSBS is used for fairness. The power allocation problem is transformed into a convex optimization problem. With the KSBS-based relay power allocation, We analytically find the optimal relay price that maximizes the relay revenue. From our simulations, compared with the sum-rate-optimal power allocation, the proposed KSBS-based power allocation is fairer and achieves close-to-optimal sum-rate for a wide range of network scenarios. Compared with the even power allocation, the proposed KSBS-based power allocation achieves higher relay revenue and network sum-rate. It is also shown via simulations that the proposed relay pricing and power allocation solutions are consistent with the laws of supply and demand.

The rest of this chapter is organized as follows. Section 4.2 describes the Stack-

[1] *A version of this chapter has been published in IEEE Transactions on Vehicular Technology, 61: 3177 - 3190 (2012).*

elberg game and bargaining game models for the relay pricing and relay power allocation problems. In Section 4.3, we analyze the relay power pricing and power allocation problems. The optimal relay price is solved analytically, while the relay power allocation is transformed into a convex optimization problem. Section 4.4 discusses the properties of the proposed solutions and their possible implementation. We discuss the applications of the proposed solution and its extensions to multi-user multi-relay networks in Section 4.5. Simulation results are shown in Section 4.6. In Section 4.7, we give the conclusion of this chapter.

4.2 Game Models for Relay Pricing and Relay Power Allocation

We consider the same system model as described in Section 3.2 where N users communicate with their destinations with the help of one relay. We use the same notations and transmission protocols as in Section 3.2. The effective received SNR of User i's transmission with and without the help of the relay are given in (3.6) and (3.7) respectively.

In the remaining of this section, we elaborate the relay power pricing and relay power allocation problems, and propose the game theoretical models for the problems using a Stackelberg game and a bargaining game.

For the game theocratical modeling of the selfish behavior of the users and the relay, our goal is to find a fair power allocation among the users and the optimal relay pricing strategy. We use the Stackelberg game to model the interaction between the users and the relay, and the bargaining game to model the relay power allocation among the users, which, as explained in Section 2.2, is a natural fit.

4.2.1 Stackelberg Game Model for Relay Pricing

We consider the relay as the leader of the Stackelberg game who sets the price of its power in helping the users. The key point of the relay pricing game is for the relay to set the price to gain the maximum revenue. The relay revenue, denoted as u_R, is the total payment from the users. We use a simple pricing model by assuming that the relay revenue is linear in the amount of power it sells, i.e.,

$$u_R = \sum_{i=1}^{N} \lambda P_i, \tag{4.1}$$

where λ is the normalized unit price of the relay power and P_i is the power the relay uses to help User i. We consider the users as followers of the Stackelberg game that react in a rational way given the unit price of the relay power.

4.2.2 Bargaining Game Model for Relay Power Allocation among Users

We use the bargaining game to model the cooperative interaction among users. That is, we assume that users make agreements to cooperatively share the relay power. A key point of formulating the users as selfish players in a bargaining game is to design the utility function, which should reflect both the quality-of-service and the payment-for-service of users. Its physical meaning can be the benefits received by the users. In this work, we seek to design an appropriate utility function that is not only physically meaningful, but also mathematically attractive to ensure tractability and convergence.

We define the utility of User i, for $i = 1 \cdots N$, as

$$u_i \triangleq \frac{Q_i P_i |f_i g_i|^2}{P_i |g_i|^2 + Q_i |f_i|^2 + 1} + Q_i |h_i|^2 - \lambda P_i, \tag{4.2}$$

which, for a given network scenario, is a function of P_i, the power the relay uses to help User i. The first two terms of (4.2) correspond to the effective received SNR of User i given in (3.6). The last term λP_i represents the user's normalized cost in purchasing the relay service. If User i does not buy any power from the relay and uses the direct transmission only, i.e., $P_i = 0$, its utility is the minimum utility that User i expects. Thus

$$u_{i,0} = Q_i |h_i|^2. \tag{4.3}$$

4.3 Relay Power Allocation and Pricing Solutions

In this section, we analyze the above Stackelberg game and bargaining game models to find the optimal relay power pricing and a fair power allocation among the users. We solve the power allocation and pricing problems jointly using the backward induction method [71]. That is, we first solve the user game, i.e., the relay power allocation among the users for a given price of the relay power, then solve the relay game, i.e., the optimal price of the relay power, based on the derived user bargaining strategy. The user game and the relay game are formulated and analyzed in the following two subsections, respectively.

4.3.1 Relay Power Allocation Based on KSBS

The user game is to find the relay power allocation among the users for a given unit power price λ. We use the bargaining game as described in Section 4.2.2 for a fair power allocation. Specifically, we look for the KSBS of the bargaining game, the background of which is provided in Section 2.2.4.

We first calculate User i's ideal utility u_i^I of a given λ. To maximize its utility, User i's goal is

$$\max_{P_i} u_i \quad \text{s.t. } u_i \geq u_{i,0},\ 0 \leq P_i \leq P. \tag{4.4}$$

The first constraint in (4.4) ensures that User i gets no less utility than $u_{i,0}$, which is its utility when it receives no help from the relay, i.e., $P_i = 0$. The second constraint ensures that the power demand of User i does not exceed the total power budget P

of the relay. Given a relay power price, this optimization problem can be solved analytically and the result is given in Lemma 4.1.

Lemma 4.1 Define

$$b_i \triangleq \frac{Q_i|f_i g_i|^2}{Q_i|f_i|^2+1}. \tag{4.5}$$

Given the unit relay power price λ, the ideal power demand of User i that maximizes its utility u_i in (4.2) is:

$$P_i^I(\lambda) = \begin{cases} 0 & \text{if } \lambda \geq b_i \\ \frac{Q_i|f_i|^2}{\sqrt{b_i}}\left(\frac{1}{\sqrt{\lambda}} - \frac{1}{\sqrt{b_i}}\right) & \text{if } b_i > \lambda > b_i\left(\frac{b_i P}{Q_i|f_i|^2}+1\right)^{-2} \\ P & \text{if } \lambda \leq b_i\left(\frac{b_i P}{Q_i|f_i|^2}+1\right)^{-2} \end{cases} \tag{4.6}$$

The ideal utility of User i is

$$u_i^I(\lambda) = \begin{cases} u_{i,0} & \text{if } \lambda \geq b_i \\ Q_i|f_i|^2(1-\sqrt{\lambda/b_i})^2 + u_{i,0} & \text{if } b_i > \lambda > b_i\left(\frac{b_i P}{Q_i|f_i|^2}+1\right)^{-2} \\ \frac{b_i P}{(Q_i|f_i|^2)^{-1}b_i P+1} - \lambda P + u_{i,0} & \text{if } \lambda \leq b_i\left(\frac{b_i P}{Q_i|f_i|^2}+1\right)^{-2}. \end{cases} \tag{4.7}$$

Proof: From (4.2), we have

$$\frac{\partial u_i}{\partial P_i} = b_i\left(\frac{b_i P_i}{Q_i|f_i|^2}+1\right)^{-2} - \lambda \tag{4.8}$$

$$\text{and} \quad \frac{\partial^2 u_i}{\partial^2 P_i} = \frac{-2(Q_i|f_i|^2)^{-1}b_i^2}{[(Q_i|f_i|^2)^{-1}b_i P_i+1]^3}. \tag{4.9}$$

Thus

$$\frac{\partial^2 u_i}{\partial^2 P_i} < 0, \tag{4.10}$$

which means that u_i is a concave function of P_i.

When $\lambda \geq b_i$, $\frac{\partial u_i}{\partial P_i} \leq 0$ for all $P_i \geq 0$ as $[(Q_i|f_i|^2)^{-1}b_i P_i + 1]^2 > 1$. So u_i is a non-increasing function of P_i and its maximum is reached at $P_i^I(\lambda) = 0$. When $\lambda \leq b_i\left(\frac{b_i P}{Q_i|f_i|^2}+1\right)^{-2}$, $\frac{\partial u_i}{\partial P_i} \geq 0$ for all $P_i \leq P$. So u_i is a non-decreasing function of P_i, and $P_i^I(\lambda) = P$ in this case. When $b_i > \lambda > b_i \left(\frac{b_i P}{Q_i|f_i|^2} + 1\right)^{-2}$, u_i reaches its maximum when $\frac{\partial u_i}{\partial P_i} = 0$, i.e.,

$$P_i = \frac{Q_i|f_i|^2}{\sqrt{b_i}}\left(\frac{1}{\sqrt{\lambda}} - \frac{1}{\sqrt{b_i}}\right).$$

This proves the ideal power solution in (4.6). The results in (4.6) shows that b_i is the price above which User i will not purchase any relay power. Using this solution and the equalities (4.2) and (4.3), we can obtain the ideal utility for User i in (4.7). ■

From Lemma 4.1, we see that $P_i^I(\lambda)$ is independent of User i's direct link h_i. Intuitively, this is because the contribution of the direct link to User i's receive SNR and utility is fixed and keeps unchanged for any amount of relay power that User i obtains.

Lemma 4.1 also shows that when the price is too high, Case 1 in (4.6), User i will not buy any relay service. When the price is too low, Case 3 in (4.6), User i wants to purchase all relay power to maximize its utility. For the price range shown in Case 2 in (4.6), User i asks for part of the relay power that gives the ideal balance between its SNR and its payment to maximize its utility. The ideal power demand of User i depends not only on the relay power price, but also on its power constraint Q_i and the quality of its local channels f_i and g_i. The b_i defined in (4.5), whose value depends on User i's condition only, is an important parameter. As shown in (4.6), it is the price above which User i will not purchase any relay power. In addition, it also affects how much power a user asks for ideally. We can see b_i as a quality measure for User i to some extent. For any two users, User i and User j, assume that $b_i > b_j$. We can see that if User i is not allocated any relay power, which happens when $b_i \leq \lambda$, User j will not be allocated any relay power either because its b_j is smaller. Also, for a given price λ, increasing the Q_i and $|f_i|^2$ of User i will increase b_i, which then results in higher or the same relay power demand from User i. This is shown in the following lemma.

Lemma 4.2 Given a relay power price λ, $P_i^I(\lambda)$ is a non-decreasing function of Q_i and $|f_i|^2$.

Proof: From (4.6), we get, when $b_i > \lambda > b_i \left(\frac{b_i P}{Q_i|f_i|^2} + 1\right)^{-2}$,

$$\begin{aligned} P_i^I(\lambda) &= \frac{Q_i|f_i|^2}{b_i}\left(\sqrt{\frac{b_i}{\lambda}} - 1\right) \\ &= \frac{Q_i|f_i|^2 + 1}{|g_i|^2}\left\{\sqrt{\frac{|g_i|^2}{\lambda[1 + 1/(Q_i|f_i|^2)]}} - 1\right\}, \end{aligned}$$

which is a non-decreasing function of Q_i and $|f_i|^2$ for a given λ. For the other two price ranges, when $\lambda \geq b_i$ $P_i^I(\lambda) = 0$; and when $\lambda \leq b_i \left(\frac{b_i P}{Q_i|f_i|^2} + 1\right)^{-2}$, $P_i^I(\lambda) = P$. So, in all price ranges,

$$P_i^I(\lambda) = \max\left[0, \min\left(\frac{Q_i|f_i|^2 + 1}{|g_i|^2}\left\{\sqrt{\frac{|g_i|^2}{\lambda[1 + 1/(Q_i|f_i|^2)]}} - 1\right\}, P\right)\right].$$

$\max$ and $\min$ are also non-decreasing functions. So we conclude that $P_i^I(\lambda)$ is a non-decreasing function of Q_i and $|f_i|^2$. ∎

To find the KSBS of the user bargaining game, without loss of generality, we assume that the users are sorted in descending order of their b_i values, that is

$$b_1 \geq b_2 \geq \cdots \geq b_N. \tag{4.11}$$

With the given price λ, for users satisfying $b_i \leq \lambda$, as shown in Lemma 4.1, their ideal power demand is 0, thus do not enter the game.

Let $L(\lambda)$ be the number of users satisfying $b_i > \lambda$. That is, with the ordering in (4.11), assume that

$$b_{L(\lambda)} > \lambda > b_{L(\lambda)+1}. \tag{4.12}$$

The first $L(\lambda)$ users will participate in the bargaining game and purchase the relay service. Given λ, to find the KSBS-based power allocation of the $L(\lambda)$ users is equivalent to solving the following optimization problem [72]:

$$\begin{aligned}
&\max_{P_i} k \\
&\text{s.t.} \quad \frac{\frac{b_i P_i}{(Q_i|f_i|^2)^{-1} b_i P_i + 1} - \lambda P_i}{u_i^I - u_{i,0}} = k, \\
&\text{and} \quad \sum_{i=1}^{L(\lambda)} P_i \leq P, \quad 0 < P_i < Q_i|f_i|^2 \left(\frac{1}{\lambda} - \frac{1}{b_i}\right),
\end{aligned} \tag{4.13}$$

where u_i^I and $u_{i,0}$ are the ideal and minimal utilities of User i. Their values are given in (4.7) and (4.3) respectively. The second constraint in (4.13) is due to the total power constraint of the relay, and the last constraint is to ensure the feasibility of the solution and is derived from rewriting $u_i > u_{i,0}$.

In the proof of Lemma 4.1, we have shown that u_i is a concave function of P_i. Also, $u_i = u_{i,0}$ when $P_i = 0$ or $P_i = Q_i|f_i|^2 (1/\lambda - 1/b_i)$, and u_i reaches its maximum $u_i^I(\lambda)$ when $P_i = P_i^I(\lambda)$. An example of u_i as a function of P_i is given in Figure 4.1. It can be shown from the definition in (4.2) that for each $u \in (u_{i,0}, u_i^I(\lambda))$, there are two possible choices of P_i that satisfy $u_i(P_i) = u$ in the range

$$\left(0, Q_i|f_i|^2 \left(\frac{1}{\lambda} - \frac{1}{b_i}\right)\right).$$

One is in the range $\left(0, P_i^I(\lambda)\right]$, and the other is in the range

$$\left[P_i^I(\lambda), Q_i|f_i|^2 \left(\frac{1}{\lambda} - \frac{1}{b_i}\right)\right).$$

Thus we can shrink the feasible region of P_i from $(0, Q_i|f_i|^2 (1/\lambda - 1/b_i))$ to either one of the smaller regions. We choose the first region for two reasons. First, for the same u_i value, this choice results in a smaller P_i than choosing the second region, and the users prefer to buy less power to gain the same utility. Second, smaller power consumption for each user saves relay power, so more users can be helped.

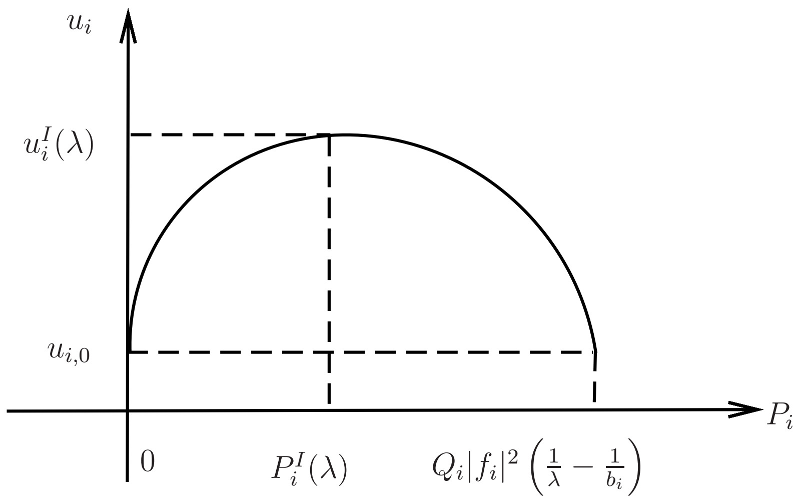

Figure 4.1: Concavity of the utility function.

With this choice, (4.13) becomes

$$\begin{aligned}
&\max_{P_i} k \\
&\text{s.t.} \quad \frac{\frac{b_i P_i}{(Q_i|f_i|^2)^{-1} b_i P_i + 1} - \lambda P_i}{u_i^I - u_{i,0}} = k, \\
&\text{and} \quad \sum_{i=1}^{L(\lambda)} P_i \leq P, \quad 0 < P_i \leq P_i^I(\lambda).
\end{aligned} \tag{4.14}$$

To solve this optimization problem, we prove the following lemma.

Lemma 4.3 The relay power allocation problem in (4.14) is equivalent to the following max-min problem:

$$\begin{aligned}
&\max_{P_i} \min_i \left\{ \frac{\frac{b_i P_i}{(Q_i|f_i|^2)^{-1} b_i P_i + 1} - \lambda P_i}{u_i^I - u_{i,0}} \right\}, \\
&\text{s.t.} \quad \sum_{i=1}^{L(\lambda)} P_i \leq P, \quad 0 < P_i \leq P_i^I(\lambda).
\end{aligned} \tag{4.15}$$

Proof: First we use the notation

$$\psi_i(P_i) \triangleq \frac{\frac{b_i P_i}{(Q_i|f_i|^2)^{-1} b_i P_i + 1} - \lambda P_i}{u_i^I - u_{i,0}}. \tag{4.16}$$

To prove this lemma, it is sufficient to show that the power allocation solution in (4.15), denoted as $(P_1^*, \cdots, P_{L(\lambda)}^*)$, satisfies

$$\psi_1(P_1^*) = \cdots = \psi_{L(\lambda)}(P_{L(\lambda)}^*). \tag{4.17}$$

We prove this by contradiction. Without loss of generality, assume that

$$\psi_1(P_1^*) < \psi_2(P_2^*) < \min\{\psi_3(P_3^*), \cdots, \psi_{L(\lambda)}(P_{L(\lambda)}^*)\}. \tag{4.18}$$

Thus,

$$\max_{P_i} \min_i \psi_i(P_i^*) = \psi_1(P_1^*). \tag{4.19}$$

Since $\psi_1(P_1), \psi_2(P_2)$ are increasing and continuous functions of P_1, P_2 in the feasible region given in (4.15), there exists a small enough positive ϵ such that $P_1^* + \epsilon, P_2^* - \epsilon$ are still in the feasible region and

$$\psi_1(P_1^*) < \psi_1(P_1^* + \epsilon) < \psi_2(P_2^* - \epsilon) < \psi_2(P_2^*).$$

The new power allocation $(P_1^* + \epsilon, P_2^* - \epsilon, P_3^*, \cdots, P_{L(\lambda)}^*)$ satisfies all power constraints in (4.15). Its max-min value is $\psi_1(P_1^* + \epsilon)$ which is larger than the max-min value of the solution $(P_1^*, \cdots, P_{L(\lambda)}^*)$. This contradicts the assumption that $(P_1^*, \cdots, P_{L(\lambda)}^*)$ is optimal, thus completes the proof. ■

(4.15) is a convex optimization problem and can be solved efficiently using standard convex optimization techniques [116]. We call the solution of (4.15) the KSBS-based power allocation. Recall that in (4.15), only the $L(\lambda)$ users whose b_i's are larger than the relay price λ participate in the game. The remaining $N - L(\lambda)$ users request no relay power.

In the game theoretical model in (4.15), the power constraint at the relay is taken into consideration. For any relay price λ, (4.15) will result in a feasible power allocation among users, i.e., the total power demanded by the users does not exceed the relay power constraint. Without the game theoretical model, if, for example, for a given price, the users request their ideal relay powers to maximize their individual utilities, it may happen that the total power demand of the users exceeds the relay power constraint, which is infeasible. With the proposed KSBS-based relay power allocation, when the sum of the ideal power demands of all users does not exceed the relay power constraint, the users will be allocated their ideal powers, in which case, k in (4.14) reaches its maximum 1; when the sum of the ideal power demands of all users exceeds the relay power constraint, the proposed KSBS-based power allocation will allocate all relay power to the users fairly. This is shown in the following lemma.

Lemma 4.4 For a fixed λ, let the ideal power allocation of User i be $P_i^I(\lambda)$, which is given in (4.6); and let the KSBS-based power allocation be $P_i^K(\lambda)$ (K stands for KSBS). When $\sum_{i=1}^{L(\lambda)} P_i^I(\lambda) \leq P$, we have

$$P_i^K(\lambda) = P_i^I(\lambda); \tag{4.20}$$

when $\sum_{i=1}^{L(\lambda)} P_i^I(\lambda) > P$, we have

$$\sum_{i=1}^{L(\lambda)} P_i^K(\lambda) = P. \tag{4.21}$$

Proof: Again, we use the notation $\psi_i(P_i)$ in (4.16). With the new feasible region of P_i in (4.15), $\psi_i(P_i)$'s are increasing functions and reach their maximum 1 when $P_i = P_i^I(\lambda)$. Thus $k \in [0, 1]$ and achieves the maximum $k = 1$ if and only if $\sum_{i=1}^{L(\lambda)} P_i^I(\lambda) \leq P$, that is, when all users can reach their ideal utilities with a feasible relay power allocation. In this case, $P_i^K(\lambda) = P_i^I(\lambda)$.

If $\sum_{i=1}^{L(\lambda)} P_i^I(\lambda) > P$, not all users can reach their ideal utilities and thus $k < 1$. From the equivalent form (4.14), actually no user can reach its ideal utility. That is, $P_i^K(\lambda) < P_i^I(\lambda)$. Suppose that $\sum_{i=1}^{L(\lambda)} P_i^K(\lambda) < P$. Define ϵ as

$$\epsilon \triangleq \min_i \left\{ \frac{P - \sum_{i=1}^{L(\lambda)} P_i^K(\lambda)}{L(\lambda)}, P_1^I(\lambda) - P_1^K(\lambda), \cdots, P_{L(\lambda)}^I(\lambda) - P_{L(\lambda)}^K(\lambda) \right\}.$$

ϵ is a positive number. Now consider the power allocation

$$\tilde{P}_i(\lambda) \triangleq P_i^K(\lambda) + \epsilon. \tag{4.22}$$

First, this new power allocation satisfies all power constraints due to its construction. Also, as ψ_i's are increasing functions, the new power allocation results in a higher minimum value, that is

$$\min_i \psi_i(\tilde{P}_i(\lambda)) > \min_i \psi_i(P_i^K(\lambda)), \tag{4.23}$$

which contradicts the assumption that $P_i^K(\lambda)$ is optimal. This completes the proof. ∎

4.3.2 Optimal Relay Power Price

Now we investigate the relay pricing problem. The price of the relay power is crucial to the relay revenue and the relay power allocation among the users. If the relay sets the price too high, no user will buy any power, and the relay revenue will be zero. If the relay sets the price too low, all users will ask for as much power as possible; and even though all relay power can be sold, the relay revenue will not be maximized.

With the unit price of the relay power λ, from Section 4.2, and by using the KSBS-based relay power allocation in Section 4.3.1, the revenue of the relay is $\sum_{i=1}^N \lambda P_i^K(\lambda)$, where $P_i^K(\lambda)$ is the relay power allocated to User i based on the KSBS for the given price λ. The relay pricing problem can be formulated as:

$$\max_\lambda \sum_{i=1}^N \lambda P_i^K(\lambda). \tag{4.24}$$

Note that the relay power constraint $\sum_{i=1}^N P_i^K(\lambda) \leq P$ is always guaranteed by the KSBS-based power allocation, thus needs not to appear explicitly in the relay revenue maximization.

To solve the relay pricing problem, we first prove the following lemma.

Lemma 4.5 The optimal price is inside the interval $[b_{lb}, b_1)$, where b_{lb} satisfies the following equation:

$$\phi(b_{lb}) \triangleq \sum_{i=1}^{N} \max\left\{0, \frac{Q_i|f_i|^2}{\sqrt{b_i}}\left(\frac{1}{\sqrt{b_{lb}}} - \frac{1}{\sqrt{b_i}}\right)\right\} = P \tag{4.25}$$

and

$$b_{lb} \geq \max_i \left\{ b_i \left(\frac{b_i P}{Q_i|f_i|^2} + 1\right)^{-2} \right\}. \tag{4.26}$$

Proof: First we can see that $\phi(b_{lb})$ monotonically decreases from ∞ to 0 as b_{lb} increases from 0 to b_1. Thus, Equation (4.25) has a unique positive solution inside $(0, b_1)$.

Then we prove (4.26) by contradiction. Assume that

$$b_{lb} < b_1 \left(\frac{b_1 P}{Q_1|f_1|^2} + 1\right)^{-2}. \tag{4.27}$$

Thus,

$$\phi(b_{lb}) \geq \max\left\{0, \frac{Q_1|f_1|^2}{\sqrt{b_1}}\left(\frac{1}{\sqrt{b_{lb}}} - \frac{1}{\sqrt{b_1}}\right)\right\} > P,$$

which conflicts (4.25). So

$$b_{lb} \geq b_1 \left(\frac{b_1 P}{Q_1|f_1|^2} + 1\right)^{-2}. \tag{4.28}$$

Similarly, we can show that

$$b_{lb} \geq b_i \left(\frac{b_i P}{Q_i|f_i|^2} + 1\right)^{-2} \quad \text{for} \quad i = 2, \cdots, N. \tag{4.29}$$

Thus (4.26) is proved.

Now we show that the optimal price is no less than b_{lb}. Using the result in (4.26) and from (4.6), when the relay power price is b_{lb}, i.e., $\lambda = b_{lb}$, we have

$$\sum_{i=1}^{N} P_i^I(b_{lb}) = \phi(b_{lb}) = P. \tag{4.30}$$

Also from (4.6), $P_i^I(\lambda)$ is a continuous and non-increasing function of λ. So $\sum_{i=1}^{N} P_i^I(\lambda)$ is a continuous and non-increasing function of λ. Inside the price range $[0, b_{lb}]$, i.e., $\lambda < b_{lb}$, we have $\sum_{i=1}^{L(\lambda)} P_i^I(\lambda) \geq P$ based on (4.30). With the KSBS-based power allocation, according to Lemma 4.4, all power of the relay will be allocated to the users , i.e.,

$$\sum_{i=1}^{L(\lambda)} P_i^K(\lambda) = P. \tag{4.31}$$

The relay revenue maximization when the price is within $[0, b_{lb}]$ becomes:

$$\max_{0 \leq \lambda \leq b_{lb}} \lambda \sum_{i=1}^{L(\lambda)} P_i^K(\lambda) = b_{lb} P, \tag{4.32}$$

which is reached at $\lambda = b_{lb}$. So the optimal price in the range $[0, b_{lb}]$ is b_{lb}.

To prove the upper bound on the relay price, note that when $\lambda \geq b_1$, from (4.6), $P_i^I(\lambda) = 0$ for all i, i.e., no user will buy any power from the relay and the relay revenue will be 0. So any price in the range $[b_1, +\infty)$ is not optimal, and the optimal price must be in the range $[b_{lb}, b_1)$. ■

The value of b_{lb} can be obtained by solving the equation in (4.25). This is a generalized waterfilling problem [105], where $1/\sqrt{\lambda}$ is the water-level, $1/\sqrt{b_i}$ is the ground level of User i, and $Q_i|f_i|^2/\sqrt{b_i}$ are the weights that can be visually interpreted as the width of each patch. In this work, we can find the value of b_{lb} analytically. Notice that $\phi(b_{lb})$ is a decreasing function of b_{lb} and b_i's are in non-increasing order. We can first find the M such that $\phi(b_M) < P$ and $\phi(b_{M+1}) > P$. Thus, $b_{lb} \in [b_M, b_{M+1}]$. Within this interval,

$$\phi(b_{lb}) = \sum_{i=1}^{M} \frac{Q_i|f_i|^2}{\sqrt{b_i}} \left(\frac{1}{\sqrt{b_{lb}}} - \frac{1}{\sqrt{b_i}} \right) = \left(\sum_{i=1}^{M} \frac{Q_i|f_i|^2}{\sqrt{b_i}} \right) \frac{1}{\sqrt{b_{lb}}} - \left(\sum_{i=1}^{M} \frac{Q_i|f_i|^2}{b_i} \right).$$

Thus, from $\phi(b_{lb}) = P$, we have

$$b_{lb} = \left(\sum_{i=1}^{M} \frac{Q_i|f_i|^2}{\sqrt{b_i}} \right)^2 \left(P + \sum_{i=1}^{M} \frac{Q_i|f_i|^2}{b_i} \right)^{-2}. \tag{4.33}$$

In what follows, we solve the optimal relay power price analytically. First, several notation are introduced. Recall the ordering of the users based on their b_i values in (4.11) and M is the index such that

$$b_M \geq b_{lb} \geq b_{M+1} \tag{4.34}$$

That is, the b_i's of the first M users are no less than b_{lb}, while the b_i's of the remaining users are no larger than b_{lb}. We have shown in Lemma 4.5 that only the price range $[b_{lb}, b_1)$ needs to be considered for the optimal price. For the simplicity of notation, we consider the range $[b_{lb}, b_1]$. Define

$$\gamma_i \triangleq b_i, \quad \text{for} \quad i = 1, \cdots, M, \tag{4.35}$$

and

$$\gamma_{M+1} \triangleq b_{lb}. \tag{4.36}$$

Further define the price range where i users purchase the relay service as

$$\Gamma_i \triangleq [\gamma_{i+1}, \gamma_i], \quad \text{for} \quad i = 1, \cdots, M. \tag{4.37}$$

We thus can divide the price range $[b_{lb}, b_1]$ into the following M intervals:

$$\begin{aligned}[b_{lb}, b_1] =& [b_{lb}, b_M] \cup [b_M, b_{M-1}] \cup \cdots \cup [b_3, b_2] \cup [b_2, b_1] \\ \triangleq& \Gamma_M \cup \Gamma_{M-1} \cdots \cup \Gamma_2 \cup \Gamma_1. \end{aligned} \tag{4.38}$$

Inside the price range $[b_{lb}, b_1]$, because $\sum_{i=1}^{N} P_i^I(\lambda)$ is a non-increasing function of λ and (4.25), we have

$$\sum_{i=1}^{N} P_i^I(\lambda) \leq P. \tag{4.39}$$

Thus, from Lemma 4.4,

$$P_i^K(\lambda) = P_i^I(\lambda). \tag{4.40}$$

We can thus rewrite the price optimization problem in (4.24) into

$$\max_{i=1,2,\cdots M} \max_{\lambda \in \Gamma_i} \sum_{j=1}^{i} \lambda P_j^I(\lambda). \tag{4.41}$$

In (4.41), we have decomposed the optimization problem into M subproblems, where the ith subproblem is to find the optimal price within the range Γ_i where User 1 to i purchase non-zero power from the relay:

$$\text{Sub-problem } i: \quad \max_{\lambda \in \Gamma_i} \sum_{j=1}^{i} \lambda P_j^I(\lambda). \tag{4.42}$$

The following proposition is proved to solve the sub-problem.

Proposition 4.1 For $i = 1, 2, \cdots, M$, define

$$c_i \triangleq \left(\frac{\sum_{j=1}^{i} Q_j |f_j|^2 / \sqrt{b_j}}{2 \sum_{j=1}^{i} Q_j |f_j|^2 / b_j} \right)^2. \tag{4.43}$$

The solution to (4.42) is

$$\lambda_i \triangleq \begin{cases} \gamma_{i+1} & \text{if } c_i < \gamma_{i+1}, \\ \gamma_i & \text{if } c_i > \gamma_i, \\ c_i & \text{if } \gamma_{i+1} \leq c_i \leq \gamma_i. \end{cases} \tag{4.44}$$

•

Proof: When $\lambda \in \Gamma_i$, for $1 \leq j \leq i$, from (4.6), the power that User j will ask for is

$$\frac{Q_i |f_i|^2}{\sqrt{b_i}} \left(\frac{1}{\sqrt{\lambda}} - \frac{1}{\sqrt{b_i}} \right),$$

and User $(i+1)$ to User M will ask for zero relay power. Subproblem (4.42) can be rewritten as

$$\max_{\lambda \in \Gamma_i} \left\{ \lambda \sum_{j=1}^{i} \frac{Q_i |f_i|^2}{\sqrt{b_i}} \left(\frac{1}{\sqrt{\lambda}} - \frac{1}{\sqrt{b_i}} \right) \right\} = \max_{\lambda \in \Gamma_i} \phi_{R,i}(\lambda), \tag{4.45}$$

where

$$\phi_{R,i}(\lambda) \triangleq \left(\sum_{j=1}^{i} \frac{Q_j |f_j|^2}{\sqrt{b_j}} \right) \sqrt{\lambda} - \left(\sum_{j=1}^{i} \frac{Q_j |f_j|^2}{b_j} \right) \lambda. \tag{4.46}$$

In (4.45), $\phi_{R,i}(\lambda)$ is the relay revenue given the price $\lambda \in \Gamma_i$. It can be shown through straightforward calculation that when $\lambda = c_i$, as defined in Proposition 4.1,

$$\frac{d\phi_{R,i}(\lambda)}{d\lambda} = 0, \tag{4.47}$$

and when $\lambda \in \Gamma_i$,

$$\frac{d^2\phi_{R,i}(\lambda)}{d\lambda^2} < 0. \tag{4.48}$$

Therefore, if $c_i > \gamma_i$, $\phi_{R,i}(\lambda)$ reaches its maximum at γ_i; if $c_i < \gamma_{i+1}$, it reaches its maximum at γ_{i+1}; and if $\gamma_{i+1} \leq c_i \leq \gamma_i$, it reaches its maximum at c_i. ■

With the subproblems solved, we are ready to find the optimal relay power price. The result is given in the following theorem.

Theorem 4.1 The optimal relay power price, denoted as λ^*, is

$$\lambda^* = \arg\max_{\lambda_i} \left\{ \left(\sum_{j=1}^{i} \frac{Q_j |f_j|^2}{\sqrt{b_j}} \right) \sqrt{\lambda_i} - \sum_{j=1}^{i} \frac{Q_j |f_j|^2}{b_j} \lambda_i \right\}, \tag{4.49}$$

where λ_i is defined in Proposition 4.1.

Proof: This is a natural result of Proposition 4.1 and (4.41). ■

With Theorem 4.1, we can find the optimal price for the relay power by solving the M subproblems in (4.41) analytically using Proposition 4.1, then find the optimal price among the M sub-problem solutions that results in the maximum relay revenue. This is written as Algorithm 4.1.

Algorithm 4.1 Optimal Relay Power Price for the Relay Pricing Problem with Stackelberg Game Formulation.

1: Calculate b_i's using (4.5). Order the N users such that $b_1 \geq b_2 \geq \cdots \geq b_N$.
2: Find M then b_{lb} using (4.33).
3: Initialize γ_i: $\gamma_i = b_i$ for $i = 1, \cdots, M$ and $\gamma_{M+1} = b_{lb}$.
4: Calculate c_i's for $i = 1, \cdots, M$ using (4.43).
5: For $i = 1, \cdots, M$, find λ_i using (4.44).
6: Find the optimal price λ^* using (4.49).

We also would like to clarify that in this work, an analytical result is found for the optimal relay power price, and our proposed Algorithm 4.1 does not require any iteration or numerical calculation. After ordering (whose average complexity is $N \log N$), its complexity is linear in the number of users in the network. Thus our proposed scheme has very low computational complexity, and is suitable for networks with a large number of users and large or moderate coherence intervals.

Previously, we have shown that b_i is an important factor for the ideal relay power. Here we can see that it is also important for the optimal relay price. We prove the following lemma, which further reflects the importance of b_i.

Lemma 4.6 If $b_1 < 4b_{lb}$, the optimal price for the relay is b_{lb}.

Proof: First recall that $b_1 \geq \cdots \geq b_{M-1} \geq b_M \geq b_{lb}$. When $b_1 < 4b_{lb}$, for $i = 1, \cdots, M$ and $j = 1, \cdots, i$, we have

$$b_j \leq b_1 < 4\gamma_{M+1} \leq 4\gamma_{i+1}. \tag{4.50}$$

Therefore,

$$\frac{Q_j|f_j|^2}{\sqrt{4\gamma_{i+1}}} < \frac{Q_j|f_j|^2}{\sqrt{b_j}} \Leftrightarrow \frac{Q_j|f_j|^2}{\sqrt{b_j}} < \frac{2Q_j|f_j|^2\sqrt{\gamma_{i+1}}}{b_j},$$

and

$$\sqrt{c_i} = \frac{\sum_{j=1}^{i} Q_j|f_j|^2/\sqrt{b_j}}{2\sum_{j=1}^{i} Q_j|f_j|^2/b_j} < \frac{2\sqrt{\gamma_{i+1}}\sum_{j=1}^{i} Q_j|f_j|^2/b_j}{2\sum_{j=1}^{i} Q_j|f_j|^2/b_j} = \sqrt{\gamma_{i+1}},$$

for $i = 1, \cdots, M$. From Proposition 1, within the range Γ_i, the optimal price is γ_{i+1}, the lower bound of Γ_i. So the optimal price in the range $[\gamma_{M+1}, \gamma_1]$ is γ_{M+1}, which is b_{lb}. ∎

Lemma 4.6 says that when the difference between b_1 and b_{lb} is small, that is, the conditions of the users are not too separate apart, the relay should set its price to be low so all users can gain some benefits. On the contrary, when some users have a much higher b_i than others, the price will be higher than b_{lb} and those users with lower b_i's may not purchase the relay service because the price is too high compared to the SNR gain they may receive.

4.4 Discussion on the Proposed Solutions

In this section, we discuss possible implementation of the proposed relay power allocation and pricing solutions and properties of the power allocation solution.

It is assumed in this work that the users employ orthogonal channels to avoid interference. In reality, there may be more users than channels and medium access control (MAC) is needed. We can use a straightforward TDMA-based channel assignment scheme as follows. Suppose that there are T channels available (for example, the IEEE 802.11G standard specifies 3 orthogonal channels, thus $T = 3$) and a total of $N > T$ users in the network. In the MAC layer, the N users are divided into $\lceil \frac{T}{N} \rceil$ groups. We use the round robin method with shared wireless channels, where each group of nodes transmit in consecutive rounds. In each round, users in the current group use the proposed power allocation and pricing strategy in Section 4.3 to decide the relay power allocation. More research on bandwidth allocation, user scheduling, and joint bandwidth and power allocation can be found in [106–108, 114].

Next we discuss the implementation of the proposed relay power allocation and power pricing solutions. As discussed in Section 3.2, we assume a block-fading channel model. Within each time slot, a training process is first conducted for the relay to obtain global CSI. Research on efficient channel training and estimation can be found in [110–112]. For the relay to know the channel gains from the users to itself, training and channel estimation should be performed at the relay. For the relay to know the channel gains from itself to the destinations, feedback from the destinations to the relay is required. Then, the relay power price and power allocation are updated using Algorithm 4.1. The proposed algorithm is a centralized one instead of distributed. With this optimal price, the relay finds the KSBS-based solution for the relay power allocation problem given in (4.15). With this implementation, we actually assume that the relay is trustworthy. All users believe that the relay will not change the parameter values (e.g., the CSI) but uses the aforementioned procedure to set the price and determine the KSBS-based power allocation, and follows the results to help all users in their transmissions.

Now we discuss robustness of the proposed KSBS-based power allocation to CSI error. When the relay sets its price to be the optimal, from the analysis in Section 4.3.2, all users will be allocated their ideal relay powers, $P_i^I(\lambda)$, and the individual utilities of the users are maximized. This is the ideal case and requires the relay to have perfect CSI. However, in reality, CSI at the relay is subject to error and delay, in which case, the relay may set a price different to the optimal one. Sometimes, the relay may want to set its price different to the optimal one due to other reasons such as marketing considerations. Our bargaining game model and KSBS-based power allocation is robust to the relay price fluctuation in the sense that a "fair" relay power allocation among the users can still be made. Specifically, if the relay power price is set to be higher than or equal to b_{lb}, defined in (4.25), with the KSBS-based power allocation, each user gets its ideal power demand (see Lemma 4.4); if the relay power price is set to be lower than b_{lb}, no user can get its ideal relay power but the relay power will be fairly allocated to the users based on Lemma 4.3, where the utility losses of the users are the same in the logarithmic scale; and all relay power will be allocated (see Lemma 4.4).

4.5 Applications and Extension to Multi-User Multi-Relay Networks

In this section, we discuss the applications of the proposed solution to two special network scenarios and its extensions to multi-user multi-relay networks.

One application of the proposed solution is the multi-user, single-relay, and single-destination networks, also addressed as multi-access relay networks (MARNs) [25, 27, 57, 123, 124, 129, 130]. The proposed scheme can be directly applied to

MARNs by setting $g_1 = \cdots = g_N$ in all network formulation. From Lemma 4.2, $P_i^I(\lambda)$ is a non-decreasing functions of Q_i and f_i. Thus, with the relay to destination channel the same for all users, users with better user-relay channels or higher transmit powers will be allocated more relay power.

Another popular network scenario is the multi-user single-relay networks with no direct links. Our solutions again can be applied straightforwardly as the solutions are independent of the direct link. And we can apply our results to such networks by setting $h_1 = \cdots = h_N = 0$.

Last, we discuss possible extensions of our work to multi-user multi-relay networks. A straightforward extension to multiple-relay network is to divide the network into several independent clusters, where each cluster contains one relay. Then, our result can be directly applied to each cluster. This is a simple but sub-optimal solution. There are of course other ways to generalize our results to multi-relay networks that allow a user to receive help from multiple relays and/or a relay to help multiple users. One possibility is as follows.

For multi-relay networks where all relays belong to the same agent and a total power constraint is assumed, the relays should have the same goal of maximizing the total revenue of all relays, and we assume a fixed price for all relays.

Assume that there are N users and R relays as shown in Figure 3.8, and the relays use orthogonal channels. Denote the channel gain from User i to Relay r as f_{ir}, and the channel gain from Relay r to Destination i as g_{ir}. Denote the total power constraint of all relays as P. Relay r uses power P_{ir} to help User i. Define

$$b_{ir} = \frac{Q_i|f_{ir}g_{ir}|^2}{Q_i|f_{ir}|^2 + 1}. \tag{4.51}$$

Other assumptions and notation are the same as the single-relay case.

For the relay power allocation problem, define the utility of User i as:

$$u_i \triangleq \sum_{r=1}^{R} \frac{Q_i P_{ir}|f_{ir}g_{ir}|^2}{P_{ir}|g_{ir}|^2 + Q_i|f_{ir}|^2 + 1} + Q_i|h_i|^2 - \sum_{r=1}^{R} \lambda P_{ir}. \tag{4.52}$$

The first two terms of (4.52) correspond to the effective received SNR of User i and the last term represents the user's total normalized cost in purchasing service from the relays. Also let

$$u_{i,0} = Q_i |h_i|^2, \tag{4.53}$$

which is the minimum utility that User i expects when it does not buy power from the relays.

Similar to the single-relay case, User i's goal is to maximize its utility. The problem can be formulated as follows.

$$\max_{P_{ir}} u_i \quad \text{s.t. } u_i \geq u_{i,0},\ 0 \leq \sum_{r=1}^{R} P_{ir} \leq P. \tag{4.54}$$

This is a convex optimization problem and can be solved efficiently using standard convex optimization techniques [116]. The ideal utility of User i can be calculated correspondingly.

For the relay power pricing problem, similar to the single-relay case, we can find a price b_{lb} such that the total ideal power demands of the users are P and any price below b_{lb} is not optimal. When the price is larger than b_{lb}, the KSBS-based power allocation for User i at Relay r is

$$P_{ir}(\lambda) = \frac{Q_i |f_{ir}|^2}{\sqrt{b_{ir}}} \left(\frac{1}{\sqrt{\lambda}} - \frac{1}{\sqrt{b_{ir}}} \right). \tag{4.55}$$

The optimal relay price problem is equivalent to that of a single-relay network with $N \times R$ users purchasing power from one relay with power constraint P and can be solved using Algorithm 4.1.

4.6 Simulation Results

In this section, we show the simulated performance of the proposed relay power allocation and pricing solutions, and compare them with the sum-rate-optimal power allocation and the even power allocation. Sum-rate-optimal power allocation solution is the relay power allocation among the users that maximizes the network sum-rate. For the even power allocation, the relay allocates $1/N$ of its total power to each of the N users, and each user decides how much power to buy from the relay to maximize its utility. That is, the relay power allocated to User i is $\min\{P_i^I(\lambda), P/N\}$. Two channel models are considered: the Rayleigh flat-fading channel and the static channel with path-loss only.

4.6.1 Rayleigh Flat-Fading Channels

In the first numerical experiment, the channels are modeled as i.i.d. Rayleigh flat-fading, i.e., f_i, h_i, and g_i are generated as i.i.d. random variables following the distribution $\mathcal{CN}(0,1)$. We consider a network with three users. The transmit powers of the users are set to be 10 dB. The simulation results follow the same trend for other values of user powers.

We first investigate the network performance when the relay power ranges from 10 dB to 40 dB. We set the relay power price to be the optimal according to Theorem 4.1. Figure 4.2 shows the optimal relay power price, the relay power actually sold, and the corresponding relay revenue. We can see that when the relay has more power to sell, the optimal relay power price is lower, more relay power is sold, and the relay receives more revenue. This complies with one of the laws of supply and demand [118], which says that if supply increases and demand remains unchanged, then it leads to lower equilibrium price and higher quantity.

Figure 4.3 compares the network sum-rate and fairness of the proposed KSBS-based power allocation with those of the sum-rate-optimal power allocation and the even power allocation. We set the relay power price to be the optimal according to Theorem 4.1. It can be seen that for the sum-rate, the difference between our algorithm and the sum-rate-optimal solutions is within 3.5%, while it is within 13% between the sum-rate-optimal and the even power solutions. The proposed solution

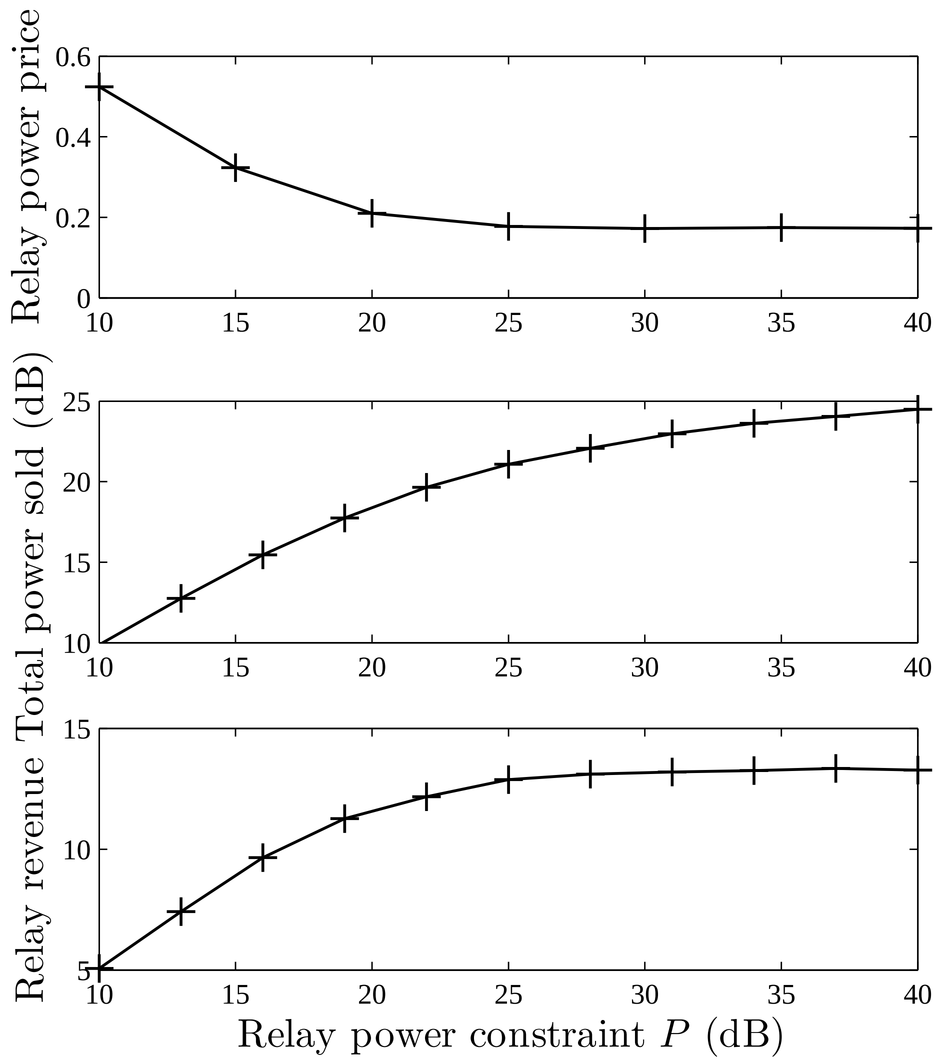

Figure 4.2: Optimal relay power price, total relay power sold, and relay revenue in a three-user relay network with Rayleigh fading channels and different relay power constraints.

is about 5 dB superior to the even power allocation. To quantify the fairness, we use the average value of the normalized difference: $[\max_i(r_i) - \min_i(r_i)]/\max_i(r_i)$, where r_i is the achievable rate of User i. A smaller difference indicates a fairer solution. We can see that our solution achieves similar fairness to the even power solution and is fairer than the sum-rate-optimal one.

Next, we examine the trend of the optimal relay price with an increasing demand. From Lemma 4.2, $P_i^I(\lambda)$ is a non-decreasing function of $|f_i|^2$. So, we can use an increasing $|f_i|^2$ to simulate increasing user demand. In this numerical experiment, we again consider a three-user network and model all channels as independent circularly symmetric complex Gaussian random variables with zero-mean. The variances of all g_i's and h_i's are 1, while the variance of all f_i's ranges from 1 to 20. A larger variance means a higher average value of $|f_i|^2$, which on average means a higher power demand from the users. The transmit power of the users is set to be 10 dB and relay power is set to be 20 dB. Figure 4.4 shows the optimal relay power price, the actual relay power sold, and the corresponding relay revenue with different variances of f_i. We can see that as the variance of f_i increases, the optimal relay price increases, more relay power is sold, and the relay revenue increases. This fits one of the laws of supply and demand, which says, if the supply is unchanged and demand increases, it leads to higher equilibrium price and quantity.

In the third numerical experiment, we examine the relationship between the optimal relay price and the number of users. The relay power is fixed to be 20 dB. The user power is fixed as 10 dB but the number of users vary from 5 to 15. All channels are generated following the distribution $\mathcal{CN}(0,1)$. Figure 4.5 shows the optimal relay power price, the total relay power sold, and the corresponding relay revenue with different numbers of users. We can see that as the number of users increases, the optimal relay power price increases, the relay power actually sold increases, and the relay revenue increases. Figure 4.5 verifies the same law as Figure 4.4, which says, if the supply is unchanged and demand increases, it leads

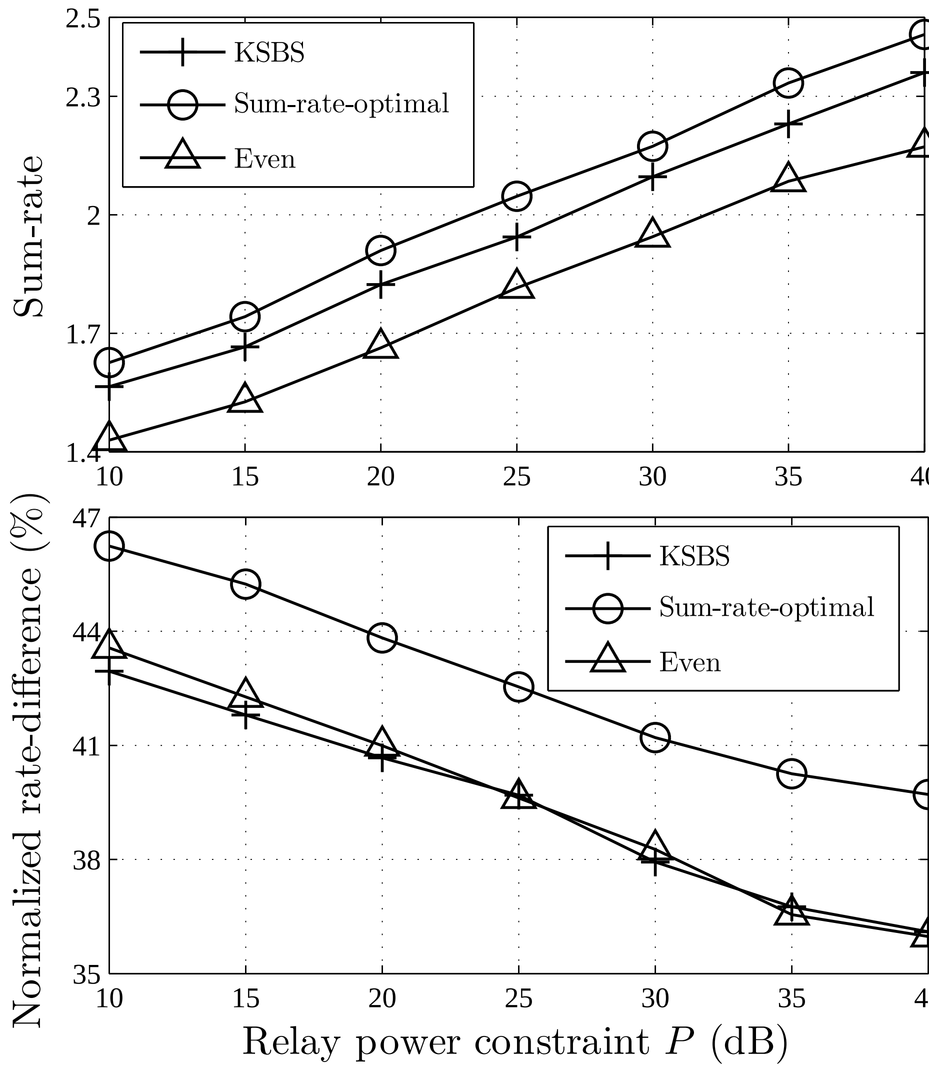

Figure 4.3: System sum-rate and fairness of a three-user relay network with Rayleigh fading channels.

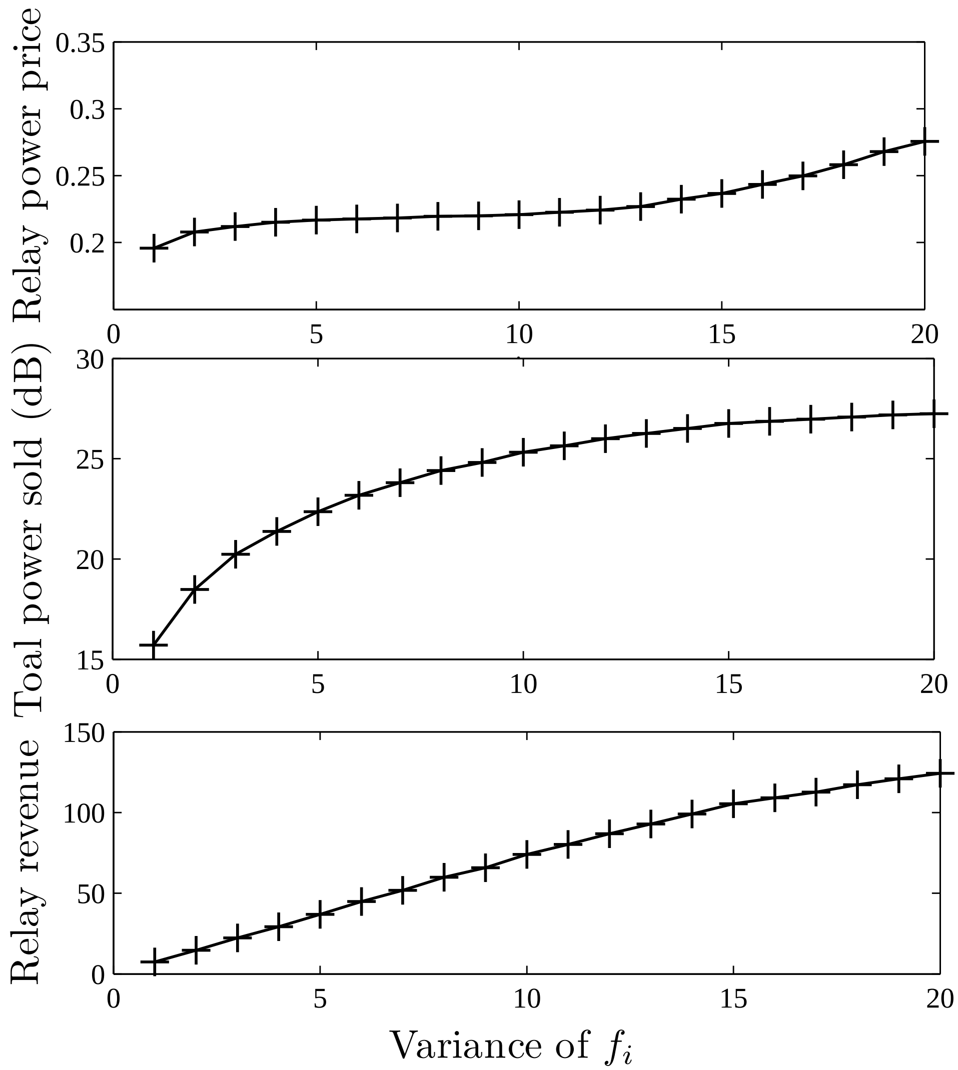

Figure 4.4: Optimal relay power price, total relay power sold, and relay revenue in a three-user relay network with Rayleigh fading channels and different variances of f_i.

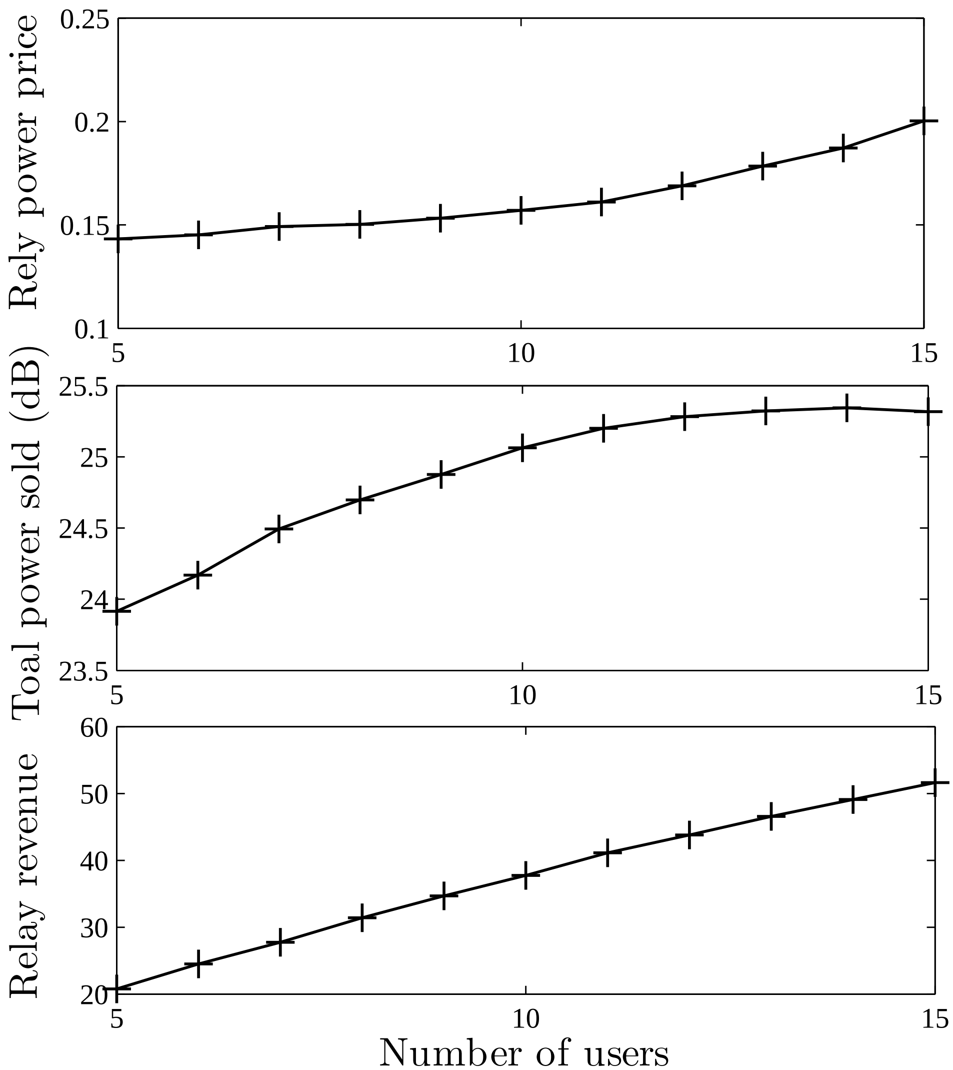

Figure 4.5: Optimal relay power price, total relay power sold, and relay revenue in a three-user relay network with Rayleigh fading channels and different numbers of users.

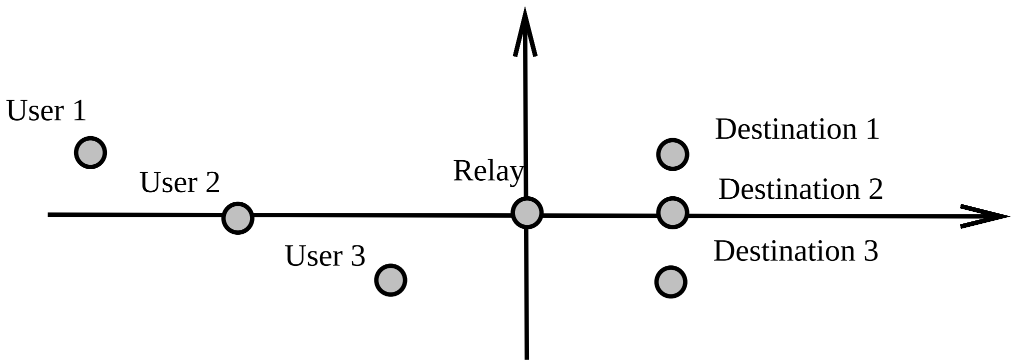

Figure 4.6: A three-user relay network with static channels.

to higher equilibrium price and quantity.

4.6.2 Static Channels with Path-Loss Only

In this subsection, we study a static network whose channels are deterministic instead of random. The network has three users, one relay, and three destinations. The relative positions of the nodes are shown in Figure 4.6, where the coordinates of Users $1-3$, the relay, and Destinations $1-3$ are $(-15, 3)$, $(-10, 0)$, $(-5, -3)$, $(0, 0)$, and $(5, 3)$, $(5, 0)$, $(5, -3)$, respectively. We consider the path-loss effect of wireless channels only by assuming that the channel gains are inversely proportional to the distance squared. In Figure 4.6, User 1 is the farthest from its destination thus has the worst channel; while User 3 is the closest to its destination and has the best channel. The power of the users is set to be 10 dB and the power of the relay is set to be 15 dB.

In Figure 4.7, the total power sold to the three users, the relay revenue, and the network sum-rate are shown as the relay power price varies. Three power allocation solutions are presented: the proposed KSBS-based power allocation, the sum-rate-optimal power allocation, and the even power allocation. Note that the sum-rate-optimal allocation solution aims to maximize the network sum-rate, is independent of the relay power price, and allocates all the relay power P to the three users. We

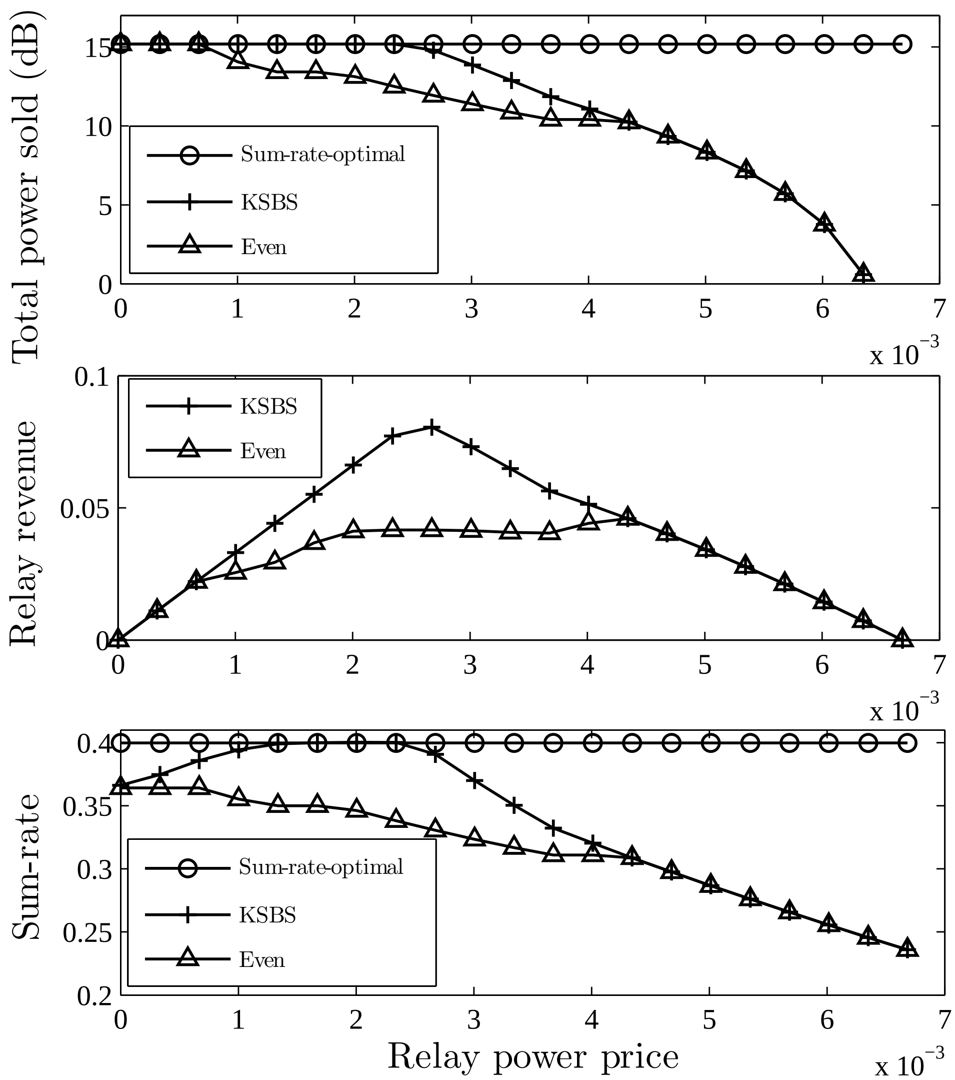

Figure 4.7: Power allocation, relay revenue, and sum-rate in a three-user relay network with static channels.

can observe from Figure 4.7 that, when the price is higher, with the KSBS-based and the even power allocation schemes, the users purchase less power from the relay and the total power demand is smaller. For example, using the KSBS-based allocation scheme, the total power demand is less than P when the price is higher than 0.0023. Now let us look at different price ranges separately. First, we can see that in the price range $[0, 0.0007]$, both KSBS-based and the even power allocation schemes sell all relay power to the users. This is because in this price range, $P_i^I(\lambda) \geq P/3$ for $i = 1, 2, 3$, thus with the even power allocation, each user will buy $P/3$, and all relay power will be sold; for the KSBS-based power allocation, $\sum_{i=1}^{3} P_i^I(\lambda) \geq P$, so all power of the relay will be purchased by the users based on Lemma 4.4. Second, when $\lambda \geq 0.0047$, the even power and the KSBS-based schemes give the same power allocation results. This is because in this price range, all three users' ideal power demands are no more than $P/3$, that is, $P_i^I(\lambda) \leq P/3$ for $i = 1, 2, 3$ and $\sum_{i=1}^{3} P_i^I(\lambda) \leq P$. In this scenario, from Lemma 4.4, both the even power allocation and the KSBS-based schemes assign the ideal power demand $P_i^I(\lambda)$ to User i, and the two schemes have the same performance. And when λ is in the range $[0.0007, 0.0047]$, the KSBS-based power allocation demands more relay power than the even power allocation, and thus the relay receives a higher revenue in this range. This is because with the even power allocation, a user cannot request more than $1/3$ of the total relay power, while the KSBS-based scheme does not have this constraint and thus enables users to request more power. Furthermore, when λ is 0.0027, the KSBS-based scheme demands 91% of the relay power to be sold to the users and the relay revenue is maximized. At this relay power price, the network sum-rate difference between the proposed KSBS-based solution and the sum-rate-optimal one is only about 2%. The relay revenue is maximized at $\lambda = 0.0047$ under the even power allocation. However, at this price, the sum-rate difference between the even power and the sum-rate-optimal schemes is 23%. For any relay price, the sum-rate difference between the even power and the sum-rate-optimal schemes is no less

Table 4.1: Achievable rates, normalized rate-difference, and the system sum-rate in a three-user relay network with static channels

		r_1	r_2	r_3	Rate-difference	Sum-rate
Sum-rate-optimal		0.0356	0.0838	0.2802	0.8729	0.3997
0	Even	0.0498	0.1017	0.2127	0.7658	0.3641
	KSBS	0.0499	0.0994	0.2169	0.7701	0.3662
0.0013	Even	0.0356	0.1017	0.2127	0.8325	0.3500
	KSBS	0.0356	0.0991	0.2643	0.8652	0.3989
0.0027	Even	0.0356	0.0823	0.2127	0.8325	0.3306
	KSBS	0.0356	0.0823	0.2727	0.8694	0.3907
0.0047	Even	0.0356	0.0627	0.1992	0.8211	0.2975
	KSBS	0.0356	0.0627	0.1992	0.8211	0.2975
0.0053	Even	0.0356	0.0627	0.1777	0.7995	0.2760
	KSBS	0.0356	0.0627	0.1777	0.7995	0.2760

than 9%.

To further compare the performance of the three schemes, Table 4.1 shows user's individual achievable rate, the normalized rate-difference, and the network sum-rate with the three power allocation schemes at the relay power prices 0, 0.0013, 0.0027, 0.0047, and 0.0053. As can be seen from Table 4.1, the proposed KSBS-based scheme achieves a smaller normalized rate difference than the sum-rate-optimal solution for all relay prices, while the sum-rate difference between these two is small. This shows that the proposed solution is fairer than the sum-rate-optimal one with comparable network sum-rate. In sum, Figure 4.7 and Table 4.1 show that for the simulated network, the proposed KSBS-based power allocation and relay pricing solutions achieve close-to-optimal sum-rate, at the same time maximize the relay revenue and achieve fairness among users.

To compare the sum-rates of the proposed solutions and the sum-rate-optimal solution, we show in Figure 4.8 the network sum-rate of the proposed relay pricing and power allocation solutions as the relay power constraint P varies. We can see that when P is small, indicating high demand and low supply, the sum-rate of the proposed solution is almost the same as the maximum sum-rate of the network. As P increases, indicating low demand and high supply, the sum-rate difference between the proposed solutions and the sum-rate-optimal solution increases. When the relay power is 25 dB, the difference is about 6%. The optimal relay price, on the other hand, decreases as P increases. These verifies the same law of supply and demand as Figure 4.2, which says, if supply increases and demand remains unchanged, then it leads to lower equilibrium price.

4.7 Conclusion

In this chapter, we study the relay power allocation problem in a multi-user single-relay network. By introducing a relay power price, we take into consideration the incentives for cooperation at the relay. Stackelberg game is used to model the interaction between the relay and the users, in which the relay acts as the leader who sets the price of its power to gain the maximum revenue and the users act as followers who pay for the relay service. To model the competition among users, a bargaining game and its KSBS are used for a fair power allocation. We analytically solve the optimal relay price, while the problem of relay power allocation among users is transformed into a convex optimization problem and can be solved with efficient numerical methods. Simulation results show that our solutions reflect the laws of supply and demand, give better user utilities and relay revenue than even power allocation, and approach the sum-rate-optimal power allocation in terms of network sum-rate for a wide range of network scenarios.

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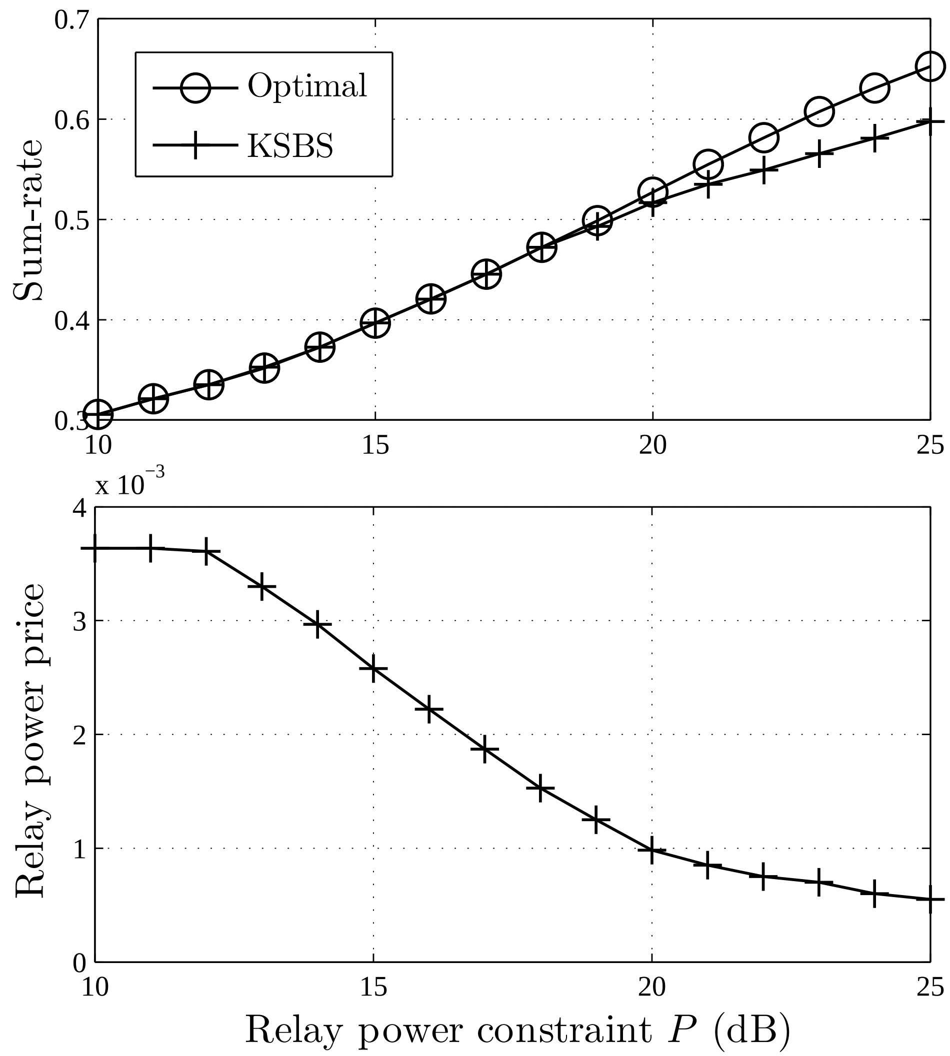

Figure 4.8: Sum-rate and relay power price in a three-user relay network with static channels.

Chapter 5

Power Allocation in Multi-User Cooperative Networks Using Double Auction Theory

In this chapter, we study the power allocation problem in a multi-user cooperative network. We use IDA to model the interaction among the users and the AP. In each iteration of this game, the users first submit bids for buying other users' power and asks for selling its own power, and then the AP determines the power allocation based on users' bids and asks. We propose a distributed algorithm for the implementation of the IDA-based power allocation. We also show that the proposed algorithm achieves weighted sum-SNR optimal solution. Simulation results are conducted to verify the performance of the proposed algorithm.[1]

5.1 Introduction

As introduced in Section 1.3, the efficient allocation of available power resource is a critical issue in cooperative networks. In Section 1.3.2, we give a literature

[1] *A version of this chapter has been submitted to IEEE Transactions on Vehicular Technology, (2013).*

review of works in this area. In these works, nodes in a network are assumed to be altruistic and willing to cooperate to optimize the overall network performance. However, as introduced in Section 1.4, nodes are selfish and aim to optimize their own benefits or quality-of-service in many practical applications. To model and analyze these behaviors, game theory is a appropriate tool. There are a handful of works that are on game theoretical solutions for power allocation in cooperative networks, e.g., [82–84]. In these networks, the authors focus on user behavior analysis, but optimal network performance cannot be achieved.

[125, 128] develop game theoretical frameworks that can provide system-level optimization. In [125, 128], the authors study the resource allocation problem for mobile data offloading and in autonomous networks. They use a double-sided auction market framework to model the interactions among the nodes.They show that their game theoretical solutions maximize sum-utilities of all nodes. In this chapter, we study power allocation problem in cooperative networks with the double-auction framework.

We study an AF cooperative network where multiple users help each other's transmission to an AP. We assume that each user has a fixed power constraint. As introduced in Section 1.5, two natural questions arise in the network: 1) How much power should a user reserve for itself and provide for other users? 2) How to provide user incentives for cooperation while maintaining good network performance? In this work, we use an IDA [125] game to model the selfish user behaviors and answer the aforementioned questions. We assume that each user plays two roles: a buyer and a seller. The user announces bids for buying other users' power and asks for selling its own power. The AP collects the bids and asks from the users, then determines the power allocation. The interaction between the users and the AP is in a iterative way until the network reaches global optimality. We also propose a distributed algorithm for the implementation of the IDA-based power allocation where each user only needs its local CSI for bid and ask updates.

5.2 System Model

Consider a wireless network with N users and one AP as shown in Figure 5.1. The AP is the destination of all user information. Each user can act as a source as well as a relay for other users. Denote the channel from User i to User j as f_{ij}, the channel from User i to the AP (the direct link) as h_i.We denote the maximum transmit power of User i as P_i. We also denote the power User i uses in helping User j as P_{ij}.

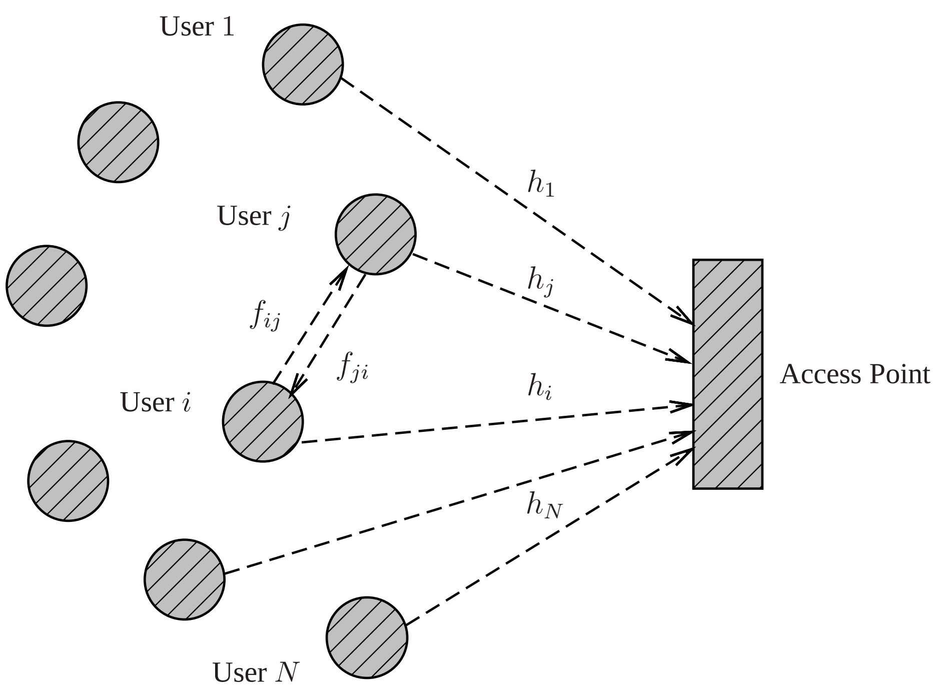

Figure 5.1: A multi-user cooperative network.

FDMA is used, so transmissions of different users are orthogonal and interference-free. Without loss of generality, we elaborate the transmission of User i's message on Channel i. We use the popular half-duplex two-step AF relaying protocol. Let s_i be the information symbol of User i. It is normalized as $\mathbf{E}(|s_i|^2) = 1$. In the first step, User i transmits $\sqrt{P_{ii}}s_i$. The signals received by User j and the AP are

$$y_{ij} = \sqrt{P_{ii}}s_i f_{ij} + n_{ij} \quad \text{and} \quad y_{iA} = \sqrt{P_{ii}}s_i h_i + n_{iA}, \tag{5.1}$$

respectively, where n_{ij} and n_{iA} are the additive noises at User j and the AP in the first step, respectively. They are assumed to be independent Gaussian following the distribution $\mathcal{CN}(0,1)$. In the second step, all users other than User i amplify their received signals and forward them to the AP on Channel i in turn [126]. For example, User j amplifies y_{ij} and forwards it with power P_{ji} on Channel i. The signal received at the AP in the second step can be shown to be

$$y_{ijA} = \sqrt{\frac{P_{ii}P_{ji}}{P_{ii}|f_{ij}|^2+1}} s_i f_{ij} h_j + \sqrt{\frac{P_{ji}}{P_{ii}|f_{ij}|^2+1}} h_j n_{ij} + n_{jA}, \tag{5.2}$$

where n_{jA} is the additive noise at the AP in the second step, which is assumed to be independent to other noises with the same distribution, $\mathcal{CN}(0,1)$.

The effective received SNR of User i's transmission with User j's help can be shown to be

$$\mathrm{SNR}_{ij} = \frac{P_{ii}P_{ji}|f_{ij}h_j|^2}{P_{ji}|h_j|^2 + P_{ii}|f_{ij}|^2 + 1}, \quad i \neq j. \tag{5.3}$$

To make the analysis tractable, we use a high SNR approximation of (5.3) as

$$\mathrm{SNR}_{ij} \approx \widetilde{\mathrm{SNR}}_{ij} = \frac{P_{ii}P_{ji}|f_{ij}h_j|^2}{P_{ji}|h_j|^2 + P_{ii}|f_{ij}|^2}. \tag{5.4}$$

This approximation is widely used in literature and has been shown to be sufficiently tight [127].

The received SNR of the AP from the direct link is

$$\mathrm{SNR}_{ii} = P_{ii}|h_i|^2. \tag{5.5}$$

After maximum-ratio combining of both the direct path and the relay path signals, the total effective SNR of User i's transmission can be calculated as

$$\mathrm{SNR}_i = \sum_{j=1}^{N} \mathrm{SNR}_{ij} \approx \sum_{j=1, j\neq i}^{N} \frac{P_{ii}P_{ji}|f_{ij}h_j|^2}{P_{ji}|h_j|^2 + P_{ii}|f_{ij}|^2} + P_{ii}|h_i|^2. \tag{5.6}$$

5.3 IDA-Based Power Allocation

We can see from (5.6) that each user desires all users in the network, including itself, to allocate as much power as possible to help its own transmission, so it can achieve the highest SNR. If the users independently decide their power allocation, each of them would use full power for its own transmission, which is not optimal from the global performance point of view and the potential benefit of cooperative communication is lost. Therefore, it is important to find a scheme that provides incentives for user cooperation as well as ensures good network performance. To achieve this goal, we design an IDA mechanism to model the interaction among the users and the AP, where users try to maximize its own utility with local information only. We show that, with the proposed IDA-based power allocation scheme, we can achieve the globally optimal power allocation that maximizes weighted sum-SNR of the network.

5.3.1 IDA Game Design

In the IDA game, each user submits its bids for buying power from other users, and asks for selling its own power. The AP is the auctioneer who determines the power allocation based on these bids and asks. The AP and the users interact iteratively until the market reaches the efficient market clearing point. In this work, we design an IDA game such that the market clearing point is the globally optimal power allocation which maximizes the weighted sum-SNR of the network.

Before introducing the IDA mechanism, we define the auction rules for the users as follows: User i submits bid b_{ji} to User j for each unit of power that User i is willing to buy from User j. With P_{ji} being the power that User j uses to help User i, User i's expected payment to User j is $b_{ji}P_{ji}$. User i submits ask a_{ij} to User j for each square unit of power that User i is willing to sell to User j, and its expected payoff from User j is $a_{ij}P_{ij}^2$. Note that we will design a game frame work to guarantee global optimality, which will be shown in Section 5.3.3. To achieve this, we assume that the AP uses different pricing rules for payment and payoff.

In each iteration of the IDA mechanism, there are two stages. In the first stage, the AP first determines the power allocation with collected bids and asks as

$$P_{ij} = \left[\frac{2(b_{ij} - \mu_i)}{a_{ij}}\right]^+, \tag{5.7}$$

where $[x]^+ = \max\{0, x\}$. In (5.7), $\mu_i > 0$ is the ith reserve bid, which is a design parameter. We will show shortly how the AP adjusts μ_i in each iteration to ensure that IDA-based power allocation achieves global optimality. With the design in (5.7), P_{ij} is increasing with bid b_{ij} and decreasing with respect to ask a_{ij}. This is intuitive: when User j places a higher bid for User i's power, or when User i announces a lower ask for User j, then User i should allocate more power to User j. Similar allocation rules have been adopted in [125, 128].

The AP then updates the reserve bid as follows

$$\mu_i(t+1) = \max\left\{0, \mu_i(t) - \gamma_i\left[P_i - \sum_{j=1, j\neq i}^{N} P_{ij}\right]\right\}, \tag{5.8}$$

where γ_i is a small constant step-size. (5.8) is designed to ensure the IDA-based power allocation is the same as the globally optimal solution. The intuition behind (5.8) is that, at time t, if the total power allocation $\sum_{j=1,j\neq i}^{N} P_{ij}$ is larger than the power constraint P_i, the reserve bid should be raised; otherwise it should be reduced.

Now we look at the biding process. We define User i's utility function as

$$u_i = w_i \mathrm{SNR}_i + \sum_{j=1, j\neq i}^{N} a_{ij} P_{ij}^2 - \sum_{j=1, j\neq i}^{N} b_{ji} P_{ji}, \tag{5.9}$$

where w_i is User i's weight and $\sum_{i=1}^{N} w_i = 1$. It is suitable for scenarios where users have different priorities and QoS differentiation has to be performed for them. In (5.9), SNR_i is the effective received SNR of User i given in (5.6) and represents the quality-of-service of the user. It is directly related to the performance of User i, e.g., its achievable rate. $\sum_{j=1,j\neq i}^{N} a_{ij} P_{ij}^2$ represents the expected payment received from all other users and $\sum_{j=1,j\neq i}^{N} b_{ji} P_{ji}$ represents the payment of User i to all other users. Note that, for User i, to maximize SNR_i, the transmit power of its own signal should be

$$P_{ii} = P_i - \sum_{j=1, j \neq i}^{N} P_{ij}. \tag{5.10}$$

User i finds its optimal bids and asks by solving the following user problem (**UP**):

$$\textbf{UP:} \quad \max_{a_{ij}, b_{ji}} u_i, \tag{5.11}$$

By evaluating the Hessian matrix of u_i with respect to a_{ij} and b_{ji}, it can be proved that u_i is jointly concave in a_{ij} and b_{ji}. Thus, the optimal asks and bids that maximize user i's utility satisfy the following equations

$$w_i \frac{\partial \mathrm{SNR}_i}{\partial P_{ij}} + a_{ij} P_{ij} = 0 \quad \text{and} \quad w_i \frac{\partial \mathrm{SNR}_i}{\partial P_{ji}} + u_j - 2b_{ji} = 0, \tag{5.12}$$

With straightforward calculations, (5.12) is equivalent to

$$\begin{aligned} a_{ij} &= -\frac{w_i}{P_{ij}} \frac{\partial \mathrm{SNR}_i}{\partial P_{ij}} = \frac{w_i}{P_{ij}} \left(\sum_{j=1, j \neq i}^{N} \frac{P_{ji}^2 |f_{ij} h_j^2|^2}{(P_{ji}|h_j|^2 + P_{ii}|f_{ij}|^2)^2} + |h_i|^2 \right), \\ \text{and} \quad b_{ji} &= \frac{w_i}{2} \frac{\partial \mathrm{SNR}_i}{\partial P_{ji}} + \frac{u_j}{2} = \frac{w_i}{2} \frac{P_{ii}^2 |f_{ij}^2 h_j|^2}{(P_{ji}|h_j|^2 + P_{ii}|f_{ij}|^2)^2} + \frac{u_j}{2}. \end{aligned} \tag{5.13}$$

5.3.2 Implementation of IDA-Based Power Allocation

In this subsection, we propose the distributed implementation of the IDA mechanism proposed in Section 5.3.1. With distributed implementation, we mean that each user has local CSI only and there is no central controller with full CSI of all users in the network. IDA is executed in successive rounds, as summarized in Algorithm 5.1.

5.3.3 Global Optimality

In this section, we show that the proposed IDA-based power allocation achieves optimal network performance in the sense of weighted sum-SNR in the network.

Algorithm 5.1 Distributed Implementation of IDA-Based Power Allocation

1: The AP initializes P_{ij} and μ_i and broadcasts this information to users.

2: Each user individually calculates its optimal bid and ask according to (5.13) and broadcasts this information to the AP.

3: The AP collects all bids and asks, determines power allocation based on (5.7), and updates the reserve bid based on (5.8). Then the AP broadcasts the updated power allocation and reserve bid to users. Go to Step 2 until convergence.

With w_i as the weight factor of User i, the weighted sum-SNR maximization problem (**GO**) can be posed as:

$$\textbf{GO:} \quad \max_{P_{ij,i\neq j}} \sum_{i=1}^{N} w_i \mathrm{SNR}_i$$

$$\text{s.t.} \quad P_{ij,i\neq j} \geq 0, \sum_{j=1,j\neq i}^{N} P_{ij} \leq P_i,$$

Before introducing the relationship between the IDA-based power allocation and the globally optimal solution, we first prove the following lemma.

Lemma 5.1 **GO** is a convex optimization problem.

Proof: The constraints of **GO** are convex by definition. Thus, to prove Lemma 5.1, we only need to show that the objective function of **GO** is concave. From the definition (5.6), we can see that the objective function is a weighted summation of SNR_{ii} and $\widetilde{\mathrm{SNR}}_{ij}$. Thus, we only need to show that SNR_{ii} is a concave function of P_{ik} and $\widetilde{\mathrm{SNR}}_{ij}$ is a concave function of P_{ik} and P_{ji}, where $k = 1, \cdots, N$ and $k \neq i$ [116].

From (5.10), we can see that SNR_{ii} is a linear combination of P_{ik}, thus it is concave in P_{ik}. For $\widetilde{\mathrm{SNR}}_{ij}$, its Hessian matrix with respect to P_{ii} and P_{ji} is

$$\nabla^2 \widetilde{\mathrm{SNR}}_{ij} = \frac{1}{(P_{ji}|h_j|^2 + P_{ii}|f_{ij}|^2)^3} \begin{bmatrix} -2P_{ji}^2|h_j|^4 & 2P_{ii}P_{ji}|f_{ij}h_j|^2 \\ 2P_{ii}P_{ji}|f_{ij}h_j|^2 & -2P_{ii}^2|f_{ij}|^4 \end{bmatrix}, \quad (5.14)$$

which is a negative semidefinite matrix. Thus, it is a concave function of P_{ii} and P_{ji}. As $\widetilde{\mathrm{SNR}}_{ij}$ is non-decreasing in P_{ii} and P_{ji} and P_{ii} is a linear combination of P_{ik}, we get that it is a concave function of P_{ik} and P_{ji}. This completes the proof. ■

Now we show the relationship between the IDA-based power allocation and the globally optimal solution in Theorem 5.1.

Theorem 5.1 The IDA-based power allocation achieves the weighted sum-SNR maximization.

Proof: We write the Lagrangian function of **GO** as

$$\mathcal{L}(P_{ij}) = \sum_{i=1}^{N} w_i \mathrm{SNR}_i - \sum_{i=1}^{N} \lambda_i \left(\sum_{j=1}^{N} P_{ij} - P_i \right). \tag{5.15}$$

Here λ_i are Lagrangian multipliers associated with the inequality constraints. The first-order KKT conditions of **GO**, which are necessary and sufficient for its solution are

$$\frac{\partial \mathcal{L}(P_{ij})}{\partial P_{ij}} = w_i \frac{\partial \mathrm{SNR}_i}{\partial P_{ij}} + w_j \frac{\partial \mathrm{SNR}_j}{\partial P_{ij}} - \lambda_i = 0, \tag{5.16}$$

$$P_{ij} \geq 0, \quad \lambda_i \geq 0, \quad \lambda_i \left(\sum_{j=1, j\neq i}^{N} P_{ij} - P_i \right) = 0. \tag{5.17}$$

The optimal solution of **GO** should satisfy equations (5.16-5.17).

If we check the IDA-based power allocation solution, and put the a_{ij} and b_{ij} solutions in (5.13) into (5.7), we find that the resulted (5.7) is equivalent as (5.16). Thus, Algorithm 5.1 is equivalent to solving **GO** by gradient projection method with constraint (5.17) . The convergence of this method has been proved in [125]. After convergence, the power constraints of **GO** are satisfied. Therefore, the IDA-based power allocation and the optimal solution of **GO** are the same. ■

5.3.4 Discussion

In this section, we compare the proposed IDA-based power allocation algorithm and the centralized implementation of the globally optimal solution in three perspectives: overhead, computational load at the AP, and user behavior modeling.

First, for centralized implementation of the globally optimal solution, a centralized controller, e.g., the AP, is required to have accurate and complete CSI, which brings significant overhead for training, channel estimation, and CSI feedback among users and the AP, especially for networks with a large number of users. However, for the proposed IDA-based power allocation, only local CSI is required at each user.

Second, for the centralized implementation of the globally optimal solution, all computational load is placed at the AP. However, APs may not have high computational capability for many practical network applications. For the proposed IDA-based power allocation algorithm, the AP only calculates the power allocation based on (5.7) and updates the reserve bids based on (5.8). Thus, the burden on the AP is reduced.

Third, the centralized implementation of the globally optimal solution assumes that the users are altruistic and willing to cooperate to optimize the overall network performance. In many practical applications, however, users are rational and selfish and they aim to maximize their own benefits. In the proposed IDA-based power allocation algorithm, we model the selfish behaviors of the users so that they can maximize their utilities. Our solution also guarantees global optimality in the sense of weighted sum-SNR.

5.4 Simulation Results

In this section, we show simulation results. We consider a static network whose channels are only related to the path-loss, which is inverse proportional to the dis-

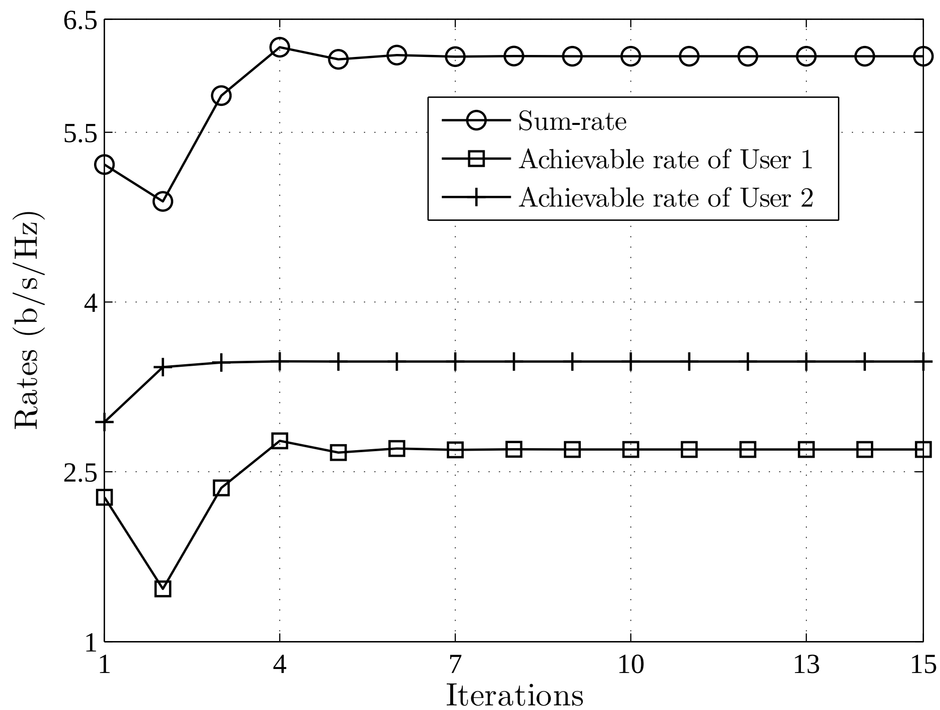

Figure 5.2: Convergence of the double auction-based power allocation algorithm in a two-user cooperative network.

tance squared. The network has two users and one AP. The coordinates of User 1, User 2, and the AP are (-2, 0), (-0.5, 0.5), and (0, 0), respectively. Thus, User 2 has a better channel to the AP. We assume that the two users have the same transmission power which ranges from 20 to 30 dB.

Figure 5.2 illustrates the convergence of the double auction-based power allocation algorithm with user weights $w_1 = w_2 = 0.5$ and transmit power 20 dB. P_{ij} is initialized as 50, μ_i is initialized as 0.1, and γ_i is set to be 0.01. We use the same initialization for all simulations. We can see from Figure 5.2 that the proposed distributed algorithm converges after 5 iterations. Similar performance is verified with different initial values selections. After convergence, the achievable rate of User 2, who has a better direct path, has a larger value.

In Figure 5.3, we show the network sum-rates with sum-rate-optimal solution and with the proposed double auction-based solutions under two different sets of user weights: $w_1 = 0.3, w_2 = 0.7$ and $w_1 = 0.7, w_2 = 0.3$. From this figure, we can see that when User 2, who has a better direct path, is assigned a higher weight, the sum-rate of the proposed solution is very close to that of the sum-rate-optimal solution. This is because with $w_1 = 0.3, w_2 = 0.7$, more emphasis is placed on User 2's achievable rate which increases network sum-rate. On the contrary, when a larger weight 0.7 is assigned to User 1, the network sum-rate is reduced to 78% of that of the sum-rate-optimal solution when P is small and 84% when P is large.

In Figure 5.4, we show the network sum-rate and User 1's achievable rate under the proposed solution with User 1's weight changing from 0.1 to 0.9. User 2's achievable rate is the difference between the sum-rate and User 1's achievable rate. We consider two user power constraints: 20 dB and 25 dB. For these two different transmission powers, network sum-rate is maximized when $w_1 = 0.2$. After that, we can see a reduction in the network sum-rate as w_1 increases, which verifies the conclusion in Figure 5.3: by assigning a larger weight to User 2, the solution approaches the sum-rate-optimal solution.

5.5 Conclusion

In this chapter, we consider a multi-user cooperative network and conduct the game-theoretic analysis of power allocation among the users. We propose a double auction-based power allocation algorithm, where users announce bids and asks to optimize their utility. Then the AP collects all the bids and asks and determines the power allocation. We show that the proposed algorithm achieves weighted sum-SNR optimal solution. Simulation results are conducted to verify the convergence performance of the proposed algorithm. The impact of user weights on network sum-rate is also demonstrated via simulation.

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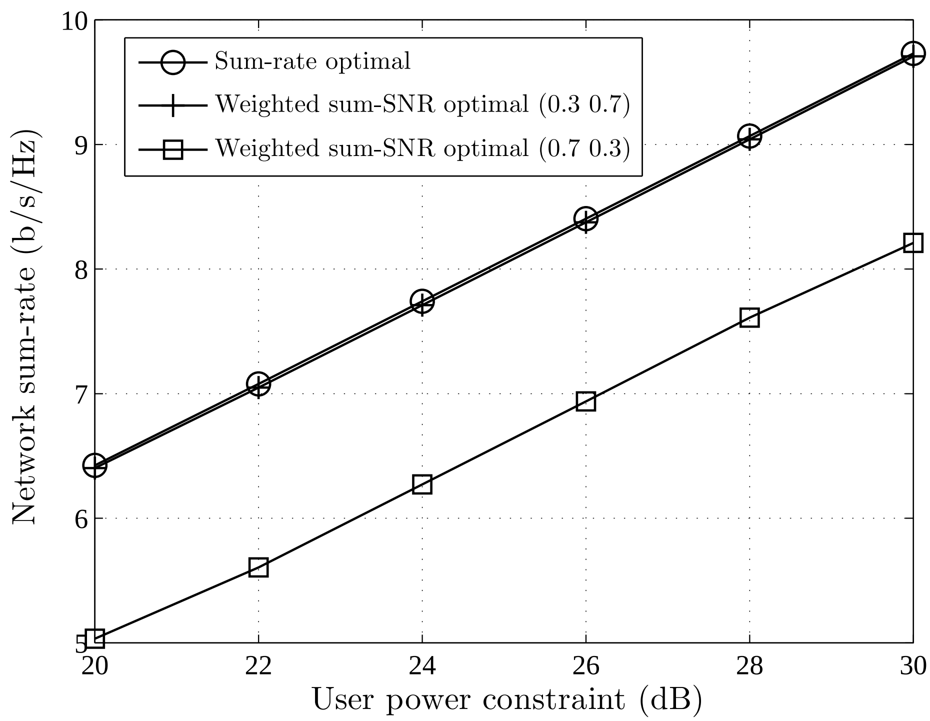

Figure 5.3: Sum-rate of a two-user cooperative network with different user power constraints.

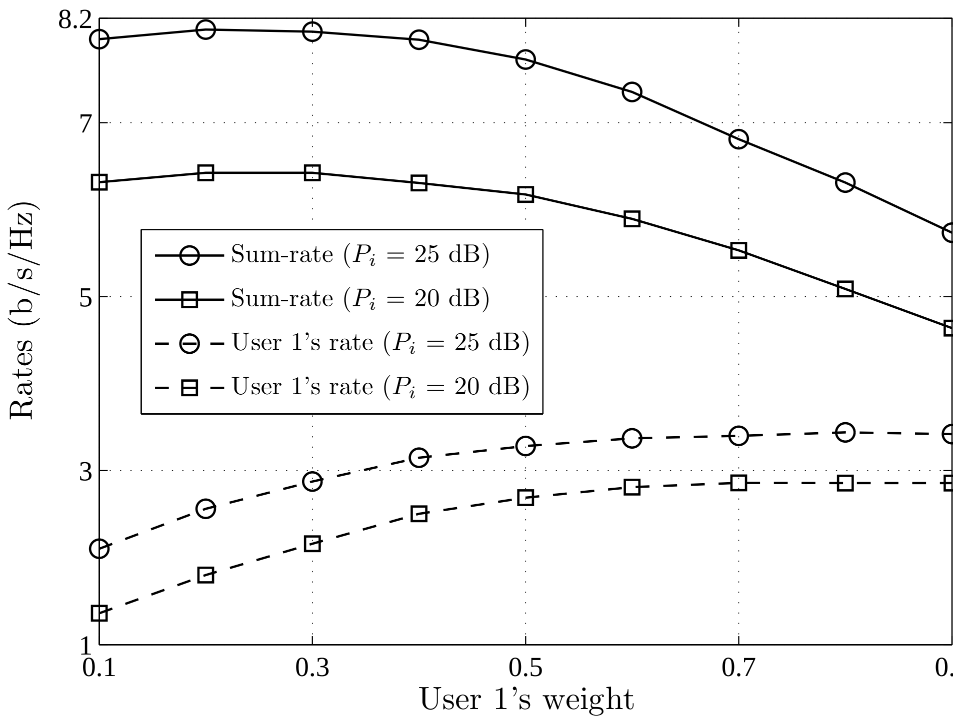

Figure 5.4: User achievable rate and sum-rate of a two-user cooperative network with different user weights.

Chapter 6

Conclusion and Future Works

This chapter summarizes the contributions of this thesis and discusses future works.

6.1 Conclusion

Future wireless applications demand high date rates and large coverage area, which can be achieved by cooperative systems. The limited power resource in cooperative systems can lead autonomous network nodes to be selfish and aim at optimizing their own benefits. Game theory has been proved to be an effective tool to model such behaviors of the autonomous nodes. In this thesis, we focus on power allocation in cooperative systems based on game theory. We propose game theoretical power allocation schemes that can address different requirements for different applications. Moreover, we provide distributed algorithms with low implementation complexity for the proposed schemes, which can be easily implemented in real cooperative systems. The proposed research fill the void of current studies. Moreover, the proposed models, methodologies, and results can be useful for other wireless research problems as well, for example, spectrum allocation in cognitive radio networks and resource allocation in wireless ad hoc networks.

The detailed contributions contributions of the thesis are summarized as follows.

- In Chapter 3, an NBS-based scheme is derived for relay power allocation among users in multi-user relay networks. It is shown that the bargaining powers of users can be adjusted to accommodate different requirements in different applications. Considering the scalability of the proposed scheme, a distributed algorithm for the NBS-based power allocation is proposed and its convergence conditions are provided.

- In Chapter 4, the power allocation and cooperation stimulation problem is studied in multi-user relay networks. Stackelberg game is used to model the interaction between the relays and the users and a bargaining game and its KSBS are used for a fair power allocation among the users. The optimal relay power price is derived analytically, while the problem of relay power allocation among users is transformed into a convex optimization problem which can be solved with efficient numerical methods.

- In Chapter 5, an IDA-based power allocation scheme is proposed in multi-user cooperative networks. It is also shown that the proposed scheme achieves weighted sum-SNR optimal solution. A distributed algorithm is proposed for the implementation of the IDA-based power allocation. The easy implementation of the distributed algorithm is of interest in potential applications.

6.2 Future Research Directions

In this thesis, we use FDMA to avoid user interference. For the future work, we may consider the transmission in an interference environment. In interference channel scenarios, the proposed solutions in this thesis may not perform well. One popular solution to this suboptimality in recent years is through the use of competitive strategies in repeated games [140]. We may also tackle this problem by changing the pricing rules [141].

For game theoretical formulation of the power allocation problem, we define the utility function of a user as the gain minus the cost. The gain is the received SNR and the cost is its payment to the relay[1]. There are two possible extensions to our utility function design. First, in the wireless scenario, a user might have other preferences for gains, e.g., achievable rate [73] or power efficiency [87]. In such applications, how to generalize the proposed methodologies and algorithms are of interest. Furthermore, nailing down a single utility function that represents the preferences of all users is not appropriate in some applications, since different users will have different preferences. In these applications, we can design game theoretical power allocation schemes with different utility functions for different users.

In this thesis, we limit our game theoretic models to ideal scenario that the relays or users have perfect knowledge of CSI through training and feedback. But in reality, CSI at the users or relays is generally imperfect due to channel fluctuations and channel estimation errors [137, 138], resulting in suboptimal performance of the proposed algorithm. Hence, future research could investigate more practical network setting with imperfect or limited channel information available. In this case, Bayesian game can be used to analyze the power allocation problem. In Bayesian game, players have beliefs about the types of other players, where a belief is the probability distribution of the possible types of other players. Each user tries to maximize its expected benefit based on his beliefs, and the corresponding equilibrium is the Bayesian Nash equilibrium, which is the strategy profile that maximizes each player's expected payoff given their beliefs and given the strategies played by the other players [71].

Finally, in our game theoretic models, it is assumed that all users are unmalicious. In practical applications, however, due to the broadcast nature of the wireless

[1] In the NBS-based power allocation scheme, we assume that the relay is unselfish and the payment is 0.

channel, cooperative systems face security threats such as eavesdropping (the malicious nodes listen to the signal between the legitimate transmitters and receivers) or jamming (the malicious nodes degrade the quality of the legitimate communication through broadcasting interference in the network). One important future direction is to study game theoretical power allocation schemes in the presence of malicious nodes. The presence of malicious nodes would strongly impact the user strategies, as they would be required to learn the trust value of each user prior to making a decision on the power allocation. In this regards, it would be of interest to combine the proposed power allocation schemes with a learning algorithm that can help in identifying malicious nodes. One possible design of such scheme is to use adaptive Q-learning RL algorithms that allow each node to interact optimally against a variety of known and unknown opponents and maximize their expected utilities [139].

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