UNIVERSITY OF ALBERTA

FINITE ELEMENT ANALYSIS OF FIBRE-REINFORCED COMPOSITE PIPELINE

By

DAVID J. THORNTON ©

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Name of Author: David J. Thornton

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Box 106
Oakville, Manitoba, Canada
R0H 0Y0

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University of Alberta

Faculty of Graduate Studies and Research

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled *Finite Element Analysis of Fibre-reinforced Composite Pipeline* submitted by David J. Thornton in partial fulfillment of the requirements for the degree of Master of Science.

Dr. F. Ellyin

Dr. Z. Xia

Dr. J. Szymanski

Dr. A. Glover

99-09-27

Abstract

The many desirable properties of fibre-reinforced composites, combined with the ability to custom tailor the final material properties to the given loading situation, make fibre-reinforced composites a viable pipeline material. The purpose of this thesis was to perform a parametric study on fibre-reinforced composite pipe under pipeline loading conditions by finite element analysis. The study was based on an elastic analysis of an E-glass/epoxy composite with a $[\pm\theta]_{2S}$ layup, for fibre angles, θ, varying from 0 to 90°. The strains were resolved into the material directions of the layers, and analysed with respect to the failure strains of the constituent materials according to the maximum strain failure theory. Three design studies based on the results of the finite element analysis are presented, indicating that an optimum range of fibre angles exist which will satisfy the pipeline loading conditions.

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Nomenclature

A	axial stress in hoop to axial stress loading
B_d	horizontal width of trench at top of pipe (m)
c_1	pipeline installation coefficient
c_2	pipeline installation coefficient, $c_2 = 6.26c_1$
Cd	load coefficient for trench installations
CLT	Classical Laminate Theory
CSA	Canadian Standards Association
E_a	Young's modulus in the axial direction of a pipe
E_x	Young's modulus in the fibre direction (GPa)
E_y	Young's modulus in the transverse to fibre direction (GPa)
F_b	buoyancy force per unit length (N/m)
FRP	fibre reinforced polymer
g	acceleration due to gravity, $9.81 m/s^2$
GFRP	glass fibre reinforced polymer
G_{xy}	Shear modulus (GPa)
H	height of backfill in trench loading (m),
	hoop stress in hoop to axial stress loading
I	second moment of area for a pipe (m^4)

ID	inside diameter (m)
K_u	trench loading coefficient
L	load
LSD	Limit States Design
OD	outside diameter (m)
M	bending moment (N·m)
P_i	internal pressure (MPa)
R	resistance
t	sidewall thickness (m)
t/D	sidewall thickness to outside diameter ratio
u	displacement
V	volume per unit length (m^3/m)
w	weight of pipe per unit length (N/m)
w_d	load on pipe, per unit length, from overburden force (N/m)
w_t	unit weight of filing material in trench loading (kN/m^3)
α	load factor
θ	angle
δ	displacement (m)
π	pi, 3.14159
ρ	density (kg/m^3)
φ	resistance factor
σ	stress (MPa)
σ_f	fibre stress (MPa)
σ_{Amax}	maximum axial stress (MPa)
σ_h	average hoop stress (MPa)
υ_{xy}	Poisson's ratio, fibre to matrix
υ_{yz}	Poisson's ratio, matrix to matrix
ε_i	strain, fibre matrix or shear
ε_{if}	failure strain, fibre matrix or shear

ε_1	fibre strain
ε_{1f}^{+}	fibre failure strain in tension
ε_{1f}^{-}	fibre failure strain in compression
ε_{12}	shear strain
ε_{12f}	shear failure strain
ε_2	matrix strain
ε_{2f}^{+}	matrix failure strain in tension
ε_{2f}^{-}	matrix failure strain in compression

Chapter 1

Advanced Pipeline Materials - State of the Art Survey

1.1 Introduction

The composite pipe industry grew out of a US government research grant issued during World War II to find a viable alternative to steel, stainless steel and other materials in short supply [1]. Since then, there have been over three hundred thousand metres of composite pipe put into service in USA alone, in applications such as: gasoline, jet fuel, chlorine, sodium hypochlorite, carbon dioxide, sulfuric acid, seawater, wastewater, deionized water, etc.[1] Composite pipes are also a feasible proposition for high performance offshore applications, as well as being capable of withstanding the severe loading conditions encountered in such an environment, their light-weight, excellent corrosion resistance and fatigue failure resistance can provide significant advantages over equivalent steel products[2]. As a result, composite pipe is becoming the system of choice in onshore as well as offshore oil and gas and petro-chemical industries.

In 1990, the US Department of Defence Critical Technology Plan classified composite materials as a critical technology[1]. This is not surprising, as new developments and further understanding of composite materials lead to new uses. Fibres in a resin, it appears to be such a simple concept yet can produce very strong and very lightweight products. The many desirable properties of fibre reinforced polymers are the reasons for its increasing popularity. Fibre-reinforced polymers (FRPs) have a high strength to density ratio, high corrosion resistance, and excellent hydraulic characteristics, to name a few. With filament wound products, such as pressure vessels and pipelines, the final properties can be altered by the choice of layup (fibre orientation), and the selection of fibre and resin types. The final cost of the pipe is dependent on the method of manufacture, fibre and resin types, and resin additives which may be required to further enhance the characteristics of the composite.

There are many field performance issues of FRPs that require attention. FRPs are anisotropic by nature, but the final properties are tailorable to the design loading conditions. However, due to this customization, considerations must be given to loading conditions that differ from the design conditions. A second issue is the environment, FRPs are fairly inert to environmental degradation, but this is dependant upon the particular environment. Other field performance issues include service life and safety considerations.

With regard to design and design issues, a summary of failure analysis techniques is presented in this chapter. A vast variety of available life prediction theories exist, but no theory can accurately predict failure in multi-directional composite layups. Numerical analysis techniques are often used, and a table comparing the analysis capabilities of commercial codes is included.

The economics of FRP use are not only dependant upon the raw-materials and manufacture, but also on the life cycle and maintenance requirements. For FRPs, the life cycle costs are dependent upon the specific situation, but in many cases, the costs are reduced compared to a steel equivalent due to reduced maintenance and increased safety.

1.2 General Overview

1.2.1 Material Type, Description and Method of Primary Manufacture

Continuous fibre-reinforced polymeric composite pipes are formed by wrapping continuous filaments and resin around a mandrel. There are a variety of both resins and fibres available, such as epoxy, Poly-Ethyl-Ethyl-Ketone [PEEK], vinyl-ester, and polyester for resin; and Carbon, E-glass, S-glass, and Kevlar for fibres [3]. Wrapping techniques include hand layup of a pre-preg tape, or machine layup of pre-preg tape or of resin-wet fibres (wet winding). Continuous filaments are the cheapest and strongest form of fibre reinforcement, but filament wound products are generally limited to geometrically convex outer contours.

Mandrel materials include water-soluble sand mandrels, plaster mandrels for low-volume products, collapsible mandrels, and load sharing or non-load sharing unremovable liners. Careful attention must be paid to mandrel design and material selection to minimize fibre damage during part removal, dimensional tolerances and residual stresses. The mandrel must be able to resist sagging under its own weight and the applied winding tension. It must also be able to retain sufficient strength during cure at elevated temperatures, and must be able to be easily removed following the cure[3].

The increasing popularity of FRPs are mainly due to their many desirable properties. These properties include: high strength to density ratio, high modulus to density ratio, high corrosion resistance, low thermal conductivity, high impact and shatter resistance, low notch sensitivity, excellent hydraulic characteristics, and lightweight[4]. A comparison of the strength to weight ratios (relative to a mild steel) for some materials is given in Table 1 [5].

Table1.1: Comparison of relative strength to density ratios (as compared to mild steel)[5]

Material	strength to weight ratio (as vs mild steel)*
High tensile steel NV36	1.53:1
Aluminum	1.8:1
Glass fibre reinforced polymer, typical	2.46:1
Advanced composites, typical	12.7:1

*The strength to weight ratio of mild steel is 0.030 Pa/kg/m^3

The fibre orientation in a FRP composite pipe has the notation of [$\pm\theta_1$, $\pm\theta_2$]. The notation symbolizes a [$+\theta_1$,$-\theta_1$,$+\theta_2$,$-\theta_2$] angle ply layer order, starting with the $+\theta_1$ as the innermost layer. If the layup is repeated, a numerical value is added, if the layup is symmetric with respect to the midplane, a subscript 's' is added. For example, for a notation of $[\pm\theta]_{2S}$, the plies are stacked through the thickness from the inside radius to the outside radius:

inside[$+\theta$,$-\theta$,$+\theta$,$-\theta$,$-\theta$,$+\theta$,$-\theta$,$+\theta$]outside

with the fibre angle orientation measured from the axis of the pipe, Fig 1.1.

With filament wound products, the final products properties are dictated by its layup, fibre, and resin specifications. Some examples of unique properties due to fibre/resin/layup are zero coefficient of expansion composite tubes for space structures[6], negative coefficient of expansion tubes[7], and creating other products which react quite "unnaturally". One such unnatural product is created by producing a pipe with an unsymmetric layup. As a result, under certain bending loadings, a positive twist loading causing negative twist deformation can occur or under certain twist loadings, a positive bending loading can produce a negative deflection[7].

Filament wound fibre-reinforced composite pipes and tubes are being developed into a variety of products other than high and low pressure pipes and pipelines. These other products include high pressure storage tanks, rocket motor cases such as the solid rocket boosters on the space shuttle, golf club shafts, fishing rods, automotive drive shafts and tension leg platform risers on oil platforms[8]. In the case of the space shuttle's solid rocket boosters, the weight savings from using composites in the manufacture of the solid rocket boosters added 2100kg to the total payload[3].

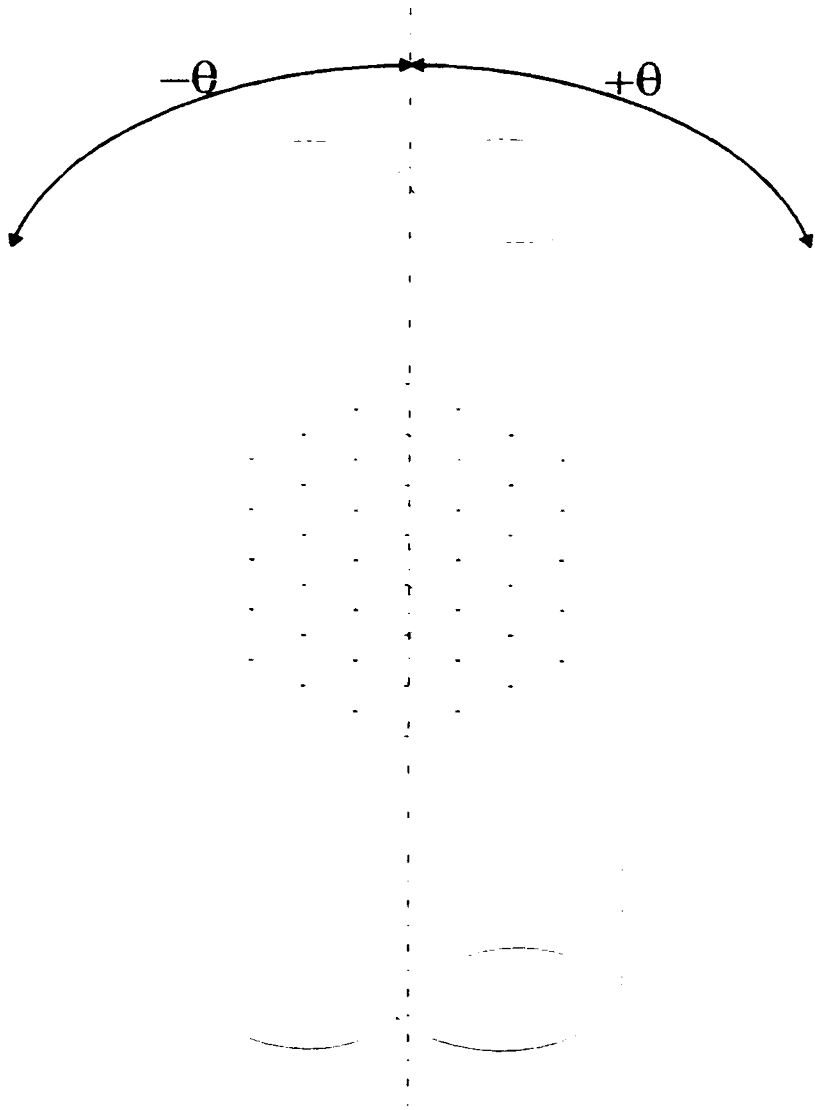

Figure 1.1: Fibre orientation in a FRP composite pipe

1.2.2 Primary Material Cost

The primary material cost is dependant on choice of fibre and resin materials. The performance level and cost are generally interrelated. In the case of fibres, for instance, glass fibres are quite inexpensive but yield a low level of performance. Carbon fibres on the other hand are quite expensive, but generally yield much higher performance, depending upon the application. The cost of the resin depends on the choice of resin material, as well as other considerations such as additives to combat fungal and microbial corrosion, or fire and smoke retarders.

1.2.3 Applicable Codes and Standards/Regulatory Requirements

A variety of codes and standards exist which are applicable to FRPs. These codes and standards cover aspects of test methods for raw material properties, to test methods for final product properties, to practices for classifying visual defects to health and safety aspects of the raw materials.

Associations and organizations developing and refining standards and guidelines include: American Petroleum Institute [API], American Society for Testing and Materials [ASTM], Compressed Gas Association Inc [CGA], Canadian Standards Association [CSA], European Committee for Standardization [CEN], Fibreglass Pipe Institute [FPI], International Maritime Organization [IMO], Norwegian Petroleum Directorate [NPD], Suppliers of Advanced Composites Materials Association [SACMA], Thermal Insulation Manufacturers Association [TIMA], United Kingdom Offshore Operators Association [UKOOA] and others.

1.3 Field Performance Issues

1.3.1 Mechanical Loading and Failure Considerations

A number of issues must be considered when replacing steel pipes with FRP pipes. FRP pipes are anisotropic and are usually tailored to fit a given loading situation. As a result, loading situations which differ from the design parameters may adversely affect the performance of the FRP pipe. It is to be noted that the adequate design of structures with reliability criteria require knowledge of pertinent probability distribution of loads and material properties, as well as knowledge of the effects of environment on material behaviour. All of these considerations would then affect the applicable safety factor.

Research is currently ongoing into all aspects of filament wound composite pipes, this includes investigation of mechanical loading and failure under various service conditions.

Environment and Failure Considerations

FRPs are fairly inert to environmental degradation [1], research is currently ongoing for specific conditions. Of special note is that the resins are flammable and thus require insulation and protection from direct fire[1]. Other considerations include: in acidic environments E-glass fibres can become susceptible to stress corrosion cracking [9], and composite materials may be susceptible to microbial attack by providing nutrients for growth from chemicals in the resins and on the fibres[10].

Durability and Service Life

Since FRPs are a relatively new material, little is known about their ultimate field life expectancy. However, there is documented experience of fibre-glass epoxy pipe being in continual oil-field service for over twenty-five years[11].

One of the problems with determining the ultimate service life of FRP's, is that once put into service, there is a reluctance to remove functioning equipment from service if no problems are encountered. Thus, there exists practical experience, but little published work.

1.3.2 Field Construction and Manufacturing Issues

Many practical joining techniques are proprietary and therefore manufacturer dependent. A variety of joining techniques are being developed/established. These include composite to metal, composite to composite, adhesive joints, lap joints, threaded joints, and others.

Composites are non-sparking, thus repair and replacement of composite components does not necessitate shutting down platform production in offshore applications [12].

1.3.3 Safety Considerations

Elements such as the low thermal conductivity, ease of handling, reduced maintenance (and hence reduced exposure), as well as reduction of "hot work" in construction improve safety and reduce personal risk in dealing with FRPs on site[13]. Many of the resins used to create FRPs are flammable, and therein lie safety concerns such as strength integrity in fire, spread of fire, smoke, and toxicity[13]. As with many other materials, there can be safety aspects regarding airborne dusts from mechanical cutting. In the case of some of the resins, there is emission of aromatic hydrocarbons from laser beam cutting. Much work has been done in the past with fibre-glass products. As a result, glass fibre specific safety documentation exists[14-15].

1.4 Design Issues

1.4.1 Available Stress and Failure Analysis Techniques

There are many variables to consider in the design of FRP's. These variables include fibre, resin, layup, thickness of layers, pretensioning of fibres, and quality of manufacture issues which include initial defects such as voids, cracks, fibre clustering, and fibre misalignments.

The behaviour of laminated composites under loading is rather complex, especially when possible degradation with pre-existing damage, crack initiation and propagation leading to structural fracture is to be considered.

Classical Laminate Theory, e.g. see [16], is typically used for initial design to limit design variables, but is not sufficient to accurately predict final product properties beyond the elastic range. Generally, prototype testing is done, since accurate analysis models do not exist. In most cases, there is limited experimental data to validate analytical or numerical models

The analytical and numerical methods usually employed cannot accurately predict the finer characteristics of the composite. Experimental work at the Advanced Composite Materials Engineering (ACME) group at the University of Alberta has shown that for composite coupon specimens consisting of only variations of [±45°,0°] layups, a significant sequence effect was observed due to the placement of the 0° layer. An increase of 11% in strength and 6% in stiffness between the weakest laminate $[0^{\circ}_{2},45^{\circ}_{2},-45^{\circ}_{2}]_S$ and the strongest laminate,$[45^{\circ}_{2},-45^{\circ}_{2},0^{\circ}_{2}]_S$, were noted[17]. Classical Laminate Theory, for example, cannot predict such an observation.

The design for optimization is a proper balance between material selection and cross section geometry[18]. The final product must be suited to both the loading conditions and the environmental conditions. Thus selection of fibre type and size, as well as matrix material and matrix additives are very important, as is the laminate layup of the final product.

1.4.2 Life Prediction Methodologies

Accurate prediction of failure in multi-directional composite layups is not yet available, particularly under fatigue loading. This is an area of current research.

A large number of failure theories exist for fibre reinforced composites. Of the vast collection of theories [over 30], most are simply special cases of others. The most common theories include Maximum Stress, Maximum Strain, Tsai-Wu and Azzi-Tsai[19]. A brief overview of the common theories is given in Table 1.2.

Table 1.2: Critique of failure theories [19]

Theory	Comments
Maximum Stress	-composite layer fails when any stress component in principal direction reaches strength value from a uni-directional stress experiment, -no interaction between stress components, -material properties based on uni-directional test results.
Maximum Strain	-composite layer fails when any principal strain value reaches that determined by a uni-directional test -no interaction between strain components, -interaction between stresses due to Poisson ratio effect, -material properties based on uni-directional test results.
Tsai-Wu	-a tensor-polynomial failure criterion, -often difficult to determine the tensor values (F_{ij}), -basis of many other theories, -material properties based on biaxial test results.
Azzi-Tsai	-generalization of von Mises-Hencky's Maximum Distortion Energy Theory to include anisotropic materials, -simple to use, -modified Hill Criterion, -often used for quick design checks.

For a comparison of some of the theories with experimental data, see Labossiere and Neale [20].

1.4.3 Environmental Concerns

Under hot-wet conditions, the degree of damage is dependent on the matrix material. Absorption of water by an epoxy matrix generally leads to plasticisation and/or hydrolysis of the matrix, thus weakening of the matrix [21]. In the case of FRP pipes however, absorption of water leads to the expansion of the matrix, and therefore the closure of cracks. Consequently, the effect of an aqueous environment on FRP pipes at low temperatures may not be significant.

1.4.4 Numerical Analysis

Many finite element codes are used to predict the characteristics of the final FRP product. Some finite element codes are developed in-house, the rest are commercial codes. Typical commercial codes include ABAQUS, ADINA, ANSYS, NASTRAN, and others.[22] Table 1.3 provides a brief overview of the analysis capabilities of several commercial finite element codes.

Table 1.3: Brief overview of commercial finite element codes [22]

		ABAQUS	ADINA	ANSYS	CASSE	EASOR	NASTRAN	NISA	PATCHES 3
Analysis	static	Y	Y	Y	Y	Y	Y	Y	Y
	vibration	Y	Y	Y	N	Y	Y	Y	N
	buckling	Y	Y	Y	N	Y	Y	Y	N
Material Properties	isotropic material	Y	Y	Y	Y	Y	Y	Y	Y
	orthotropic material	Y	Y	Y	Y	Y	Y	Y	Y
	anisotropic material	Y	Y	Y	N	Y	Y	Y	Y
Dependency	linear material	Y	Y	Y	Y	Y	Y	Y	Y
	incompressible material	Y	N	N	Y	Y	Y	Y	Y
	bimodular material	Y	U	Y	N	N	Y	N	Y
	non-linear behaviour	Y	Y	Y	N	N	Y	Y	Y
Lamination	uni-directional laminate	Y	N	Y	Y	Y	Y	Y	Y
	orthotropic material	Y	N	Y	Y	Y	Y	Y	Y
	anisotropic laminate	Y	N	N	N	Y	Y	Y	Y
	involute laminate	N	N	N	N	N	N	U	Y

Y = yes, N = no, U = unknown

1.5 Economic Evaluation

The economics associated with a FRP pipe are dependent upon the choice of fibre, resin, and method of manufacture. Fibre costs vary depending upon the fibre type, size and quality. Glass fibres can cost $2/lb, while carbon fibres can cost upwards of $30/lb. Resin costs vary depending on the characteristic properties needed in order to maintain the intended function. These characteristic properties include mechanical material properties which must match the fibre properties, and the environmental conditions which may require additives to combat fungal and microbial corrosion, fire and smoke retarders and considerations for robust curing cycle. The resin costs can vary from $1/lb for a basic resin, while a multi-additive epoxy can cost significantly more. Mechanical and adhesion properties may vary with the type of additive used, and each manufacturer has its own specification, and will provide data for their product.

The method of manufacture also plays a part in the final cost of the composite. Using a pre-impregnated (pre-preg) cloth or tape has added cost due to the pre-fabrication of the cloth or tape. Wet-winding on the other hand, a process which wraps resin-wet fibres around a mandrel, can reduce the cost significantly by eliminating the pre-manufacture of the pre-preg. For example, a carbon fibre FRP pipe, using a wet winding technique, a $30/lb fibre with a $2/lb resin are combined for a $19/lb composite. The same product made with a pre-preg tow, tape, or cloth would cost in excess of $38/lb. [23] Wet-winding also eliminates the special storage requirements of the pre-preg, as the pre-preg can deteriorate if improperly stored. The pre-fabrication of the pre-preg material usually makes for a higher quality product, however.

There is often significant savings in the usage of FRP products. A study conducted recently by the Composites Engineering and Applications Centre for Petroleum Exploration and Production at the University of Houston [12] concluded that the use of composite materials would provide economic payoff to offshore developments by capturing the integrated benefits of weight reduction, lower operating costs through reduced maintenance, and reduced production downtime. The study replaced topside steel equipment, steel production risers, and tension leg platform (TLP) tendons with suitable composite equivalents. The total weight savings were 3,150 tons, which is about 48% of the weight of the original steel equipment. A value of $8500 was credited for each ton of topside weight and riser/tendon pretension saved on the TLP. After calculating the composite cost premium, due to some composite systems being more costly then their steel counterparts, the total net cost savings for the study was $25,000,000[12].

Similarly, during the conceptual phase of a redevelopment project, Winkel of Phillips Petroleum Norway reported that when FRP pipe was used to replace corroded carbon steel sea water pipe, the installed cost of the FRP pipe was 90% that of carbon steel while life-cycle cost was only 60% [24].

1.6 Overview of the Current Study

The many desirable properties of FRPs, combined with the ability to custom tailor the final material properties to the given loading situation, as well as the reduced life-cycle costs, make FRPs a viable pipeline material. From an economic viewpoint, FRPs are an attractive alternative to steel pipes. Reduced maintenance of FRPs leads to lower operating costs and reduced downtime. The non-sparking characteristics lead to increased safety and reduced downtime in fire hazard environments such as oil-fields and oil-rigs. The high strength to density ratio of FRPs can lead to significant savings in weight sensitive situations. The combination of these and other attributes are continually leading to the increased usage of FRPs.

The future of FRPs and FRP pipes in particular is quite positive. Continued research will further the knowledge and understanding of the properties of FRPs, particularly with regards to response to loading conditions such as fatigue and specific environments. It is predicted that the economics and better understanding of FRPs will lead to increased usage, and increased usage will lead to increased research.

The purpose of the current study is to perform a parametric study of a FRP pipe under pipeline loading conditions by finite element analysis. The study was limited to an E-glass fibre and epoxy matrix, a rather inexpensive yet useful combination. The study was completed using the general finite element package ANSYS, Revision 5.4.

The following chapters detail the loading conditions investigated, the finite element model and its assumptions and limitations, the fibre angle dependent results of the applied loading conditions, and a design study.

The idealized loading conditions concerning a pipeline are detailed in Chapter 2. In Chapter 3, the finite element model is developed, detailing the assumptions and limitations of the element and its usage, along with the pipe model. The results from the finite element model are discussed in Chapter 4, with Chapter 5 detailing three design studies of a high pressure pipeline based on the numerical results of Chapter 4.

Chapter 2
Pipeline Loadings

2.1 Introduction

In order to properly design a pipeline, there must be an accounting for the loadings that it will incur during its life. These loadings consist of construction and operational loads. The loads arise as a result of the existence of a pipeline and the conditions of its use. These loads primarily are weights, internal pressures, and external forces.

2.2 Loadings

The loadings on a pipeline can be divided into three basic categories: weights, pressures and forces.

2.2.1 Weights

The weights involved in a pipeline, both during construction and operation include the pipe, the contents of the pipeline, and any attachments applied to the pipeline.

The weight of the pipeline is determined by the following equation:

$$w = g \cdot \rho \cdot \pi \cdot \left(\left(\frac{OD}{2} \right)^2 - \left(\frac{ID}{2} \right)^2 \right) = g \cdot \rho \cdot \pi \cdot \left(\frac{OD}{2} \right)^2 \cdot \left(1 - \left(1 - 2 \cdot \frac{t}{D} \right)^2 \right) \qquad (2.1)$$

where:

w	= weight of pipe per unit length (N/m)
g	= acceleration due to gravity
	= 9.81 m/s^2
ρ	= material density (kg/m^3)
OD	= outside diameter (m)
ID	= inside diameter (m)
t/D	= sidewall thickness to outside diameter ratio

Pipeline attachments are typically geotextiles or anchors attached to the pipe to maintain buoyancy control with respect to particular soil or water conditions. For this thesis, the pipeline attachments are assumed to be a line load force, distributed along the length of the pipeline. It is also assumed that the weight of the contents of the pipeline has a negligible affect on the state of stress and strain in the pipe, i.e. gas transmission.

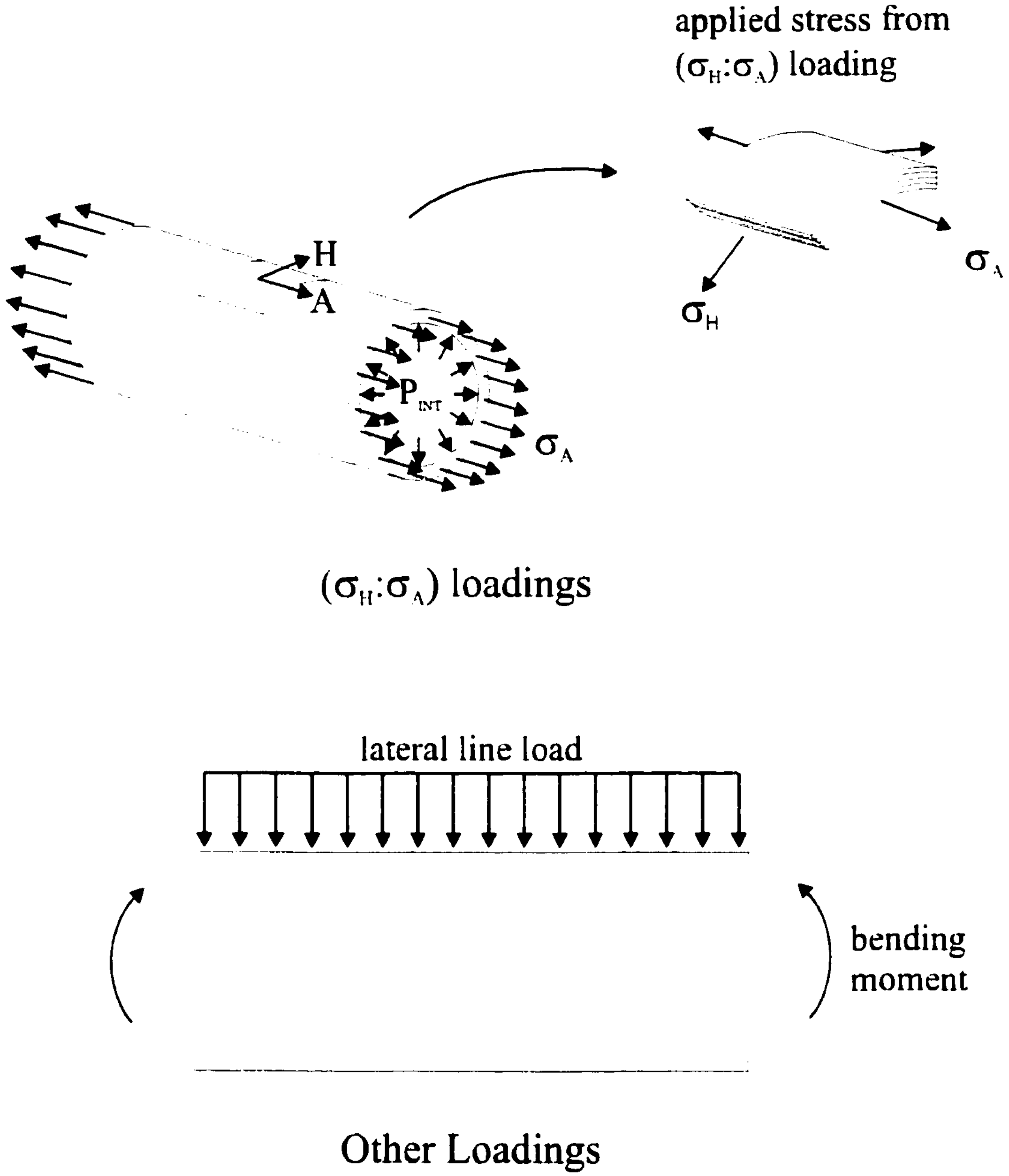

Figure 2.1: Loadings under consideration for the FEA of FRP pipe

2.2.2 Pressures

The purpose of a pipeline is to transport pressurized fluids, either pressurized liquids or gases. During the construction of the pipeline, the only pressurization of the pipeline will be pressure testing, usually one of the final phases of the construction process. While the pipeline is operational, it can be expected that the pipeline will always be under an internal pressure loading, though that pressure may not remain constant [25]. Since fatigue loading is beyond the scope of this thesis, the internal pressure was assumed constant.

The average hoop stress on the pipe wall as caused by internal pressure is determined by:

$$\sigma_h = \frac{P_i \cdot OD}{2 \cdot t} = \frac{P_i}{2 \cdot (t / D)} \tag{2.2}$$

where

σ_h	= hoop stress (MPa)
P_i	= internal pressure (MPa)
t	= sidewall thickness (m)
OD	= outside diameter (m)
t/D	= sidewall thickness to OD ratio

Pressure loadings are typically given as ratios of applied hoop stress to the applied axial stress. The window of possible stress ratios ranges from (1H:0A) for a pure pressure loading, to a (0H:1A) loading for pure axial loading. For this thesis, the variety of hoop to axial stress ratios considered are shown in Table 2.1.

Table 2.1 : Hoop stress to axial stress ratios considered

pure pressure (1H:0A)
(8H:1A)
(4H:1A)
pressure vessel (2H:1A)
(1H:1A)
(1H:2A)
(1H:4A)
(1H:8A)
pure axial (0H:1A)

Poisson's effects were not considered as the effects easily fall within the stress ratio loadings.

2.2.3 Forces

Force loadings include any forces applied to a pipeline during its construction and operation. These forces consist mainly of forces caused by the lowering a pipeline into the ground and the applied force of soil overburden in a buried pipeline.

Pipeline Installation

The loadings caused by the laying of a pipeline can be modelled by a bending moment, according to the equation:

$$M = c_1 \cdot (w \cdot E \cdot I)^{\frac{1}{2}} \quad (2.3)$$

where

M = maximum bending moment (Nm)

c_l = coefficient dependant upon the lift height and depth of ditch (see Table 2.1)

E_a = Young's modulus for the axial direction of the pipe (Pa)

I = second moment of area for the pipe (m^4)

Table 2.2 Pipeline installation loading coefficient, c_l

lift (m) \ depth, δ (m)	1	2	3	4	5	6
0	0.96	1.36	1.67	1.93	2.16	2.36
0.25	1.21	1.55	1.82	2.06	2.28	2.47
0.5	1.40	1.71	1.96	2.19	2.39	2.58
0.75	1.57	1.85	2.09	2.30	2.50	2.68
1	1.73	1.98	2.21	2.41	2.60	2.78

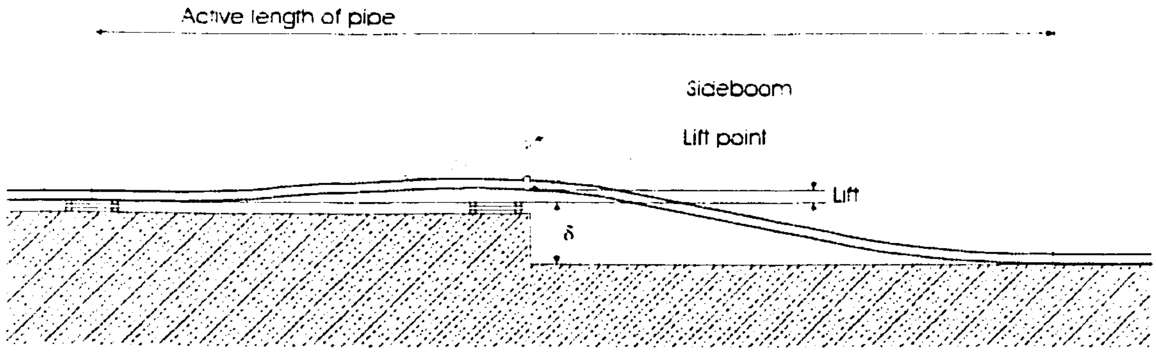

Figure 2.2 Schematic of pipeline installation

This loading is acting at the point of lift acting in the axial direction of the pipe.

The maximum stresses and strains due to the installation loading can be found directly using equations (2.4) and (2.5):

$$\sigma_{max} = c_2 \cdot (E \cdot \rho)^{\frac{1}{2}} \cdot \left(1 + \left(1 - 2\frac{t}{D}\right)^2\right)^{-\frac{1}{2}} \tag{2.4}$$

$$\varepsilon_{max} = c_2 \cdot \left(\frac{\rho}{E}\right)^{\frac{1}{2}} \cdot \left(1 + \left(1 - 2\frac{t}{D}\right)^2\right)^{-\frac{1}{2}} \tag{2.5}$$

with coefficient $c_2 = 6.26c_1$.

Further derivation and explanation of this equations can be found in Appendix A.

Overburden Force

The backfilled earth above and about a buried pipeline is known as soil overburden. Soil overburden from a trench type pipeline installation imposes vertical force loadings on the pipeline in accordance with the following equation[26]:

$$W_d = C_d \cdot w_f \cdot B_d^2 \tag{2.6}$$

where:

W_d = load on pipe (N/m)

C_d = load coefficient for trench installations

$$= \frac{1 - e^{-2\frac{K_u \cdot H}{B_d}}}{2 \cdot K_u}$$

K_u = combination of the conjugate ratio and the coefficient of sliding friction between the backfill material and the trench walls

= 0.165 for sand and gravels

H = height of backfill above the top of the pipe (m)

B_d = horizontal width of trench at top of pipe (m)

w_f = unit weight of filling material

= 15.7 kN/m^3 for sand and gravel

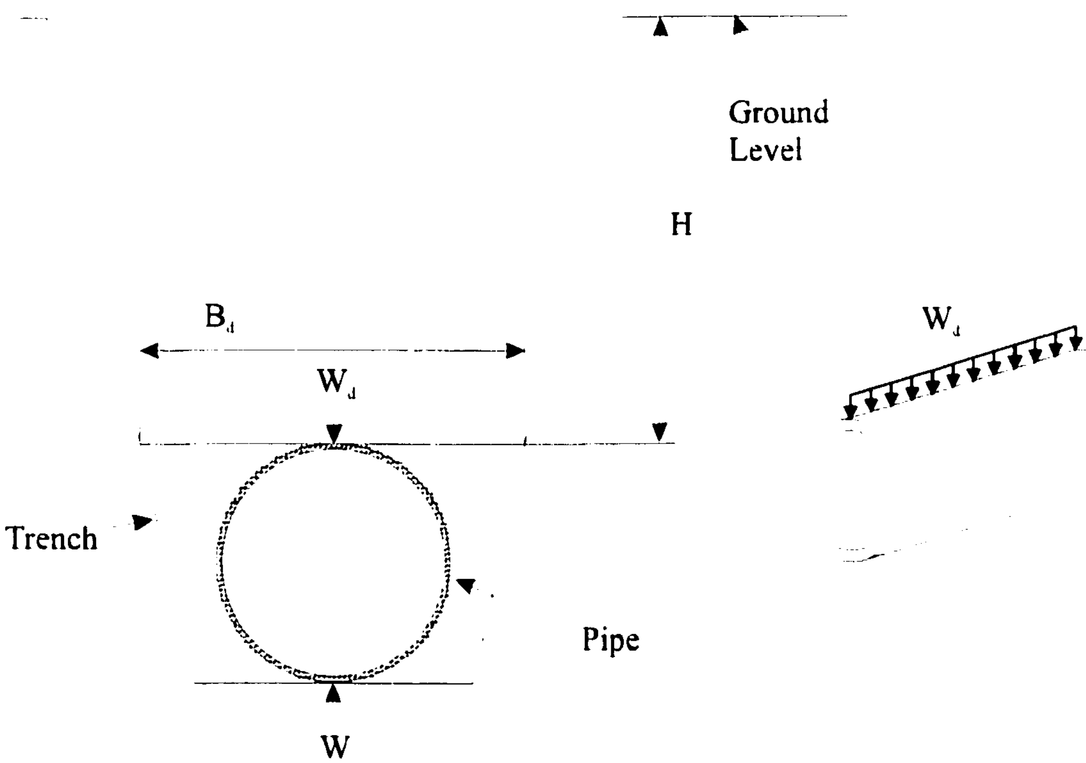

Figure 2.3 Schematic of soil overburden for a buried pipeline installation

Buoyancy Load

As the pipeline may cross rivers or pass through marshland, it is often necessary to determine the buoyancy of the pipeline caused by the displaced water. The buoyant force acting on a pipeline in water is determined by:

$$F_b = \rho \cdot g \cdot V \tag{2.7}$$

with:

F_b = buoyancy force per unit length (N/m)

ρ = density of water (kg/m^3)

= 1000 kg/m^3

g = acceleration due to gravity

= 9.81 m/s^2

V = volume of water displaced by the pipe, per unit length (m^3/m)

$$= \pi \cdot \left(\frac{OD}{2}\right)^2$$

2.2.4 Other Loadings

Pipelines may also be subjected to other loadings that can not always be accounted for during design. These loadings can include puncture or scarring caused by a backhoe, or a loading caused by ground instabilities such as frost heave or soil movements. For this thesis, the loadings caused by ground instabilities were interpreted as a bending loading.

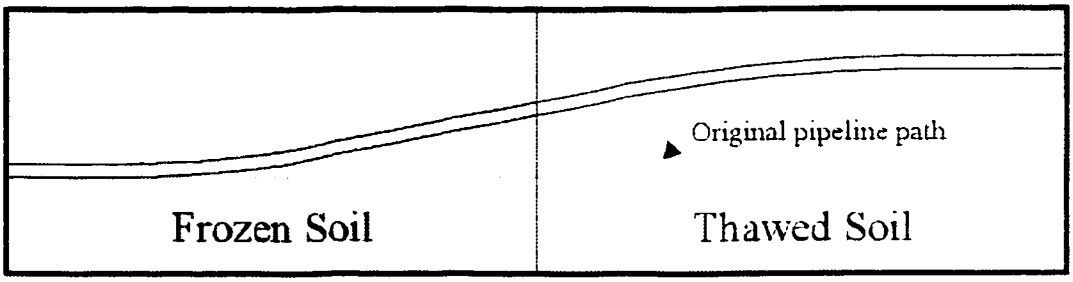

Figure 2.4 Soil instability caused by frost heave

2.3 Limit States Design

Limit states design (LSD)[27] is a non-mandatory appendix to the Canadian Standards Association (CSA) pipeline design code, CSA-Z662, Oil and Gas Pipeline Systems. Its purpose is to supplement existing requirements as part of an effort to make reliability-based design procedure available to pipeline designers. LSD is offered as a practical method of incorporating reliability concepts into the normal design process. It is suggested that LSD results in a more rational, logical design process and that it provides more consistent levels of safety.

Limit states design is a reliability based design methodology which utilizes factored loads and factored resistances to ensure a certain level of safety in the pipeline. The approach is based on designing a pipeline such that the factored resistance equals or exceeds the sum of the factored load effects for all relevant limit states. The probability of failure, p_f, depends upon the variability and resistance type associated with the resistance and load.

The basic design equation is given by:

$$\phi R \geq \sum_{i=1}^{n} \alpha_i L_i \tag{2.8}$$

with ϕR representing the factored resistance, and αL representing the factored load effects from the simultaneous loads.

2.3.1 Limit States Design Factors

For this thesis, the design factors used are given in Table 2.3, and are the same as those used for steel pipe, as given in the Limit States Design appendix of CSA-Z662. For the sake of simplicity, a class factor equal to 1 was used.

Table 2.3: Limit States Design factors from CSA-Z662

LSD Factor	value
Class Factor	1.00
Resistance Factor	0.80
Load Factor (pressure)	1.25
Load Factor (force)	1.25

The load factor of 1.40 was used for the bending moment loadings, to account for uncertainties in the application of moment loadings for installation and soil instability.

2.3.2 Provisions for Fibreglass or Composite Pipes in CSA-Z662

According to section 13.1.1.2 of CSA Z662 fibreglass pipe is currently only permitted for use in multi phase fluid, oilfield water and other low vapour pressure pipelines.

As the composite material considered in this thesis would qualify as fibreglass pipe, its capability to withstand high internal pressures must be proven before it is allowed to be used as a viable pipeline material. The purpose of this thesis is to explore a such a possibility through a finite element analysis as a first step to the qualification of FRP as a pipeline material.

Chapter 3
The Finite Element Model

3.1 Introduction

A finite element analysis allows for the exploration of variables and loadings when no analytical model is available. Modelling of the physical shape and the discretization of it into discrete elements allows for a numerical solution that should closely predict the physical behaviour of the actual material, provided that the material model is an appropriate one.

By using the numerical model, numerous variables can be examined and scenarios explored in order to limit the range of practical variables for physical testing. The numerical model can further be used to complement the physical testing by modelling the stresses, strains and the support reactions to aid in the understanding of the physical test.

In the case of the work presented in this thesis, finite element modelling allowed for exploration of the different parameters involved in a pipe under pipeline loading conditions. This was done in order to limit the range of feasible fibre wind angles and range of thickness to diameter ratios for future physical testing.

3.2 ANSYS Element Solid46

The finite element analysis was completed using the general purpose finite element code ANSYS Revision 5.4. The pipe was modelled using an eight node, three degrees of freedom per node, 3-D structural layered solid element named Solid46 [28]. The input of the element allows for specification of thickness, fibre orientation, and material properties for each layer.

The use of this element implies certain assumptions with regards to the laminate lay-up. For example, it implies that the layers of the element are ideal, i.e. there is no inclusion or provision to account for manufacturing defects, or defects such as layer overlap or bandwidth effects from filament winding. It is also assumed that there is perfect bonding between the layers, with no allowance for delamination or slippage between the layers. This analysis further assumes that each layer in the element is homogeneous and anisotropic, with a linear elastic material response.

The setup of the element allows for post-processing on each individual layer, at the users discretion.

3.3 Understanding the element

In the ANSYS manuals [28-29] not much information is given regarding the derivation of element Solid46. In order to further understand the layered element, a single cube Solid46 element, modelling a glass fibre reinforced polymer (GFRP) laminate, was monotonically loaded to determine its global material properties.

Table 3.1: Young's modulus in the axial direction of a GFRP as determined by material testing, the Classical Laminate Theory (CLT), and ANSYS element Solid46

Layup	Experimental	CLT	ANSYS
[0/90]s	92.46 GPa	91.44 GPa	91.44 GPa
[±45]s	16.4 GPa	16.45 GPa	16.44 GPa

In the above table, the experimental results were taken from tests of physical specimens described in reference [30].

Classical Laminate Theory (CLT) [15] is a homogenization technique which calculates the global properties of the laminate by translating the material properties of each layer into the global directions, then weighing the properties of each layer, and calculating the global stiffness of the laminate. Once the global properties are calculated, the global stresses and strains, based on the applied load, are calculated. These results can then be translated back into each layer's coordinates. This method is well described in all composite materials textbooks, and will not be discussed further, except by referring to the results obtained through its usage.

Table 3.2 Comparison of individual layer strains for a $[\pm 65]_{2S}$ composite loaded with an axial stress of 68.95 MPa, as calculated by Classical Laminate Theory and ANSYS

	ε_1		ε_{12}		ε_2	
Layer	CLT	ANSYS	CLT	ANSY	CLT	ANSYS
1	-1.5895e-5	-1.5895e-5	-3.5889e-3	-3.5886e-3	-1.5895e-5	-1.5895e-5
2	-1.5895e-5	-1.5895e-5	3.5889e-3	3.5886e-3	-1.5895e-5	-1.5895e-5
3	-1.5895e-5	-1.5895e-5	-3.5889e-3	-3.5886e-3	-1.5895e-5	-1.5895e-5
4	-1.5895e-5	-1.5895e-5	3.5889e-3	3.5886e-3	-1.5895e-5	-1.5895e-5
5	-1.5895e-5	-1.5895e-5	3.5889e-3	3.5886e-3	-1.5895e-5	-1.5895e-5
6	-1.5895e-5	-1.5895e-5	-3.5889e-3	-3.5886e-3	-1.5895e-5	-1.5895e-5
7	-1.5895e-5	-1.5895e-5	3.5889e-3	3.5886e-3	-1.5895e-5	-1.5895e-5
8	-1.5895e-5	-1.5895e-5	-3.5889e-3	-3.5886e-3	-1.5895e-5	-1.5895e-5

The comparison of the stresses and strains in each layer of the composite, as calculated by CLT and ANSYS revealed that the ANSYS 3-D structural layered element, Solid46, is clearly based upon the Classical Laminate Theory.

3.4 The Model

The laminate layup used for the finite element model was $[\pm\theta]_{2S}$, which is balanced and symmetric about the middle of the wall thickness, creating an eight layer solid. Since the Solid46 element is based on Classical Laminate Theory, as long as the layup is balanced and symmetric, the macroscopic homogenized properties of the laminate would be the same regardless of the number of layers. This means that based on a CLT approach, the macromechanical properties of a $[\pm\theta]_S$, $[\pm\theta]_{2S}$, or a $[\pm\theta]_{4S}$ layup would be the same when non-dimensionalized with respect to the thickness divided by the area. A commercial filament wound composite pipe would certainly consist of more than eight layers, and depending upon the exact location in the pipe, the layup may be $[\pm\theta]_{nS}$ or $[\mp\theta]_{nS}$. However, the homogenized macroscopic mechanical properties would be the same as for a $[\pm\theta]_{2S}$ pipe when non-dimensionalized with respect to the thickness divided by the area.

For the present parametric study, the simple $[\pm\theta]$ layup was selected to limit variables involved. A compound layup such as $[\pm\theta_1,\pm\theta_2,\pm\theta_3,\ldots\pm\theta_n]_s$ could have just as easily been studied, however the increase in parameters would have been substantial.

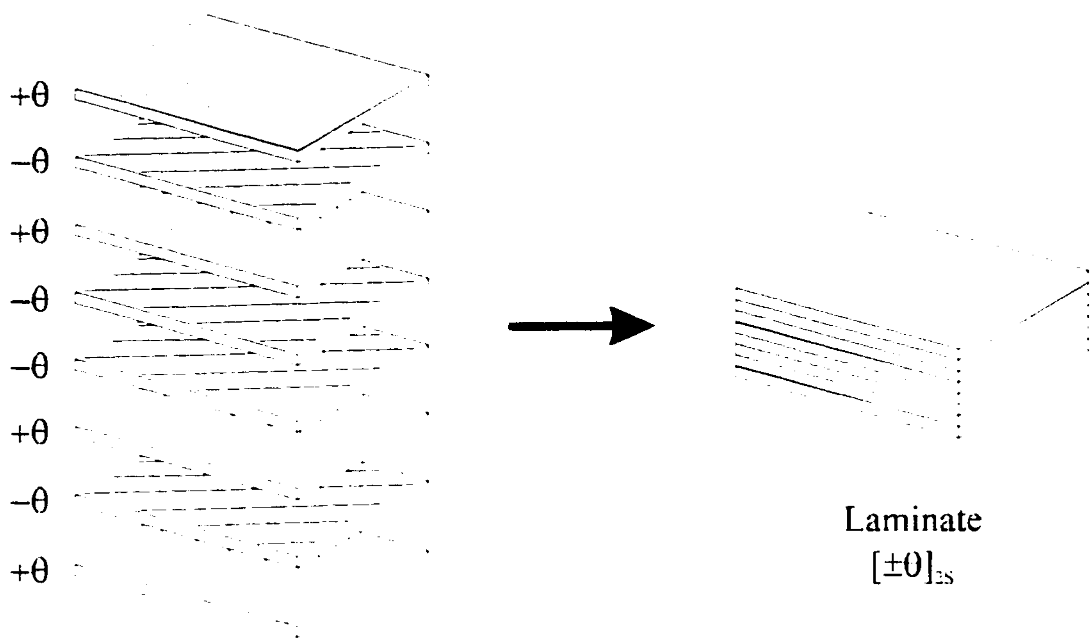

Figure 3.1 : Schematic showing the layup for a $[\pm\theta]_{2S}$ laminate (where θ is the fibre angle orientation with respect to the structural or loading axis)

3.5 Loading

As discussed in the previous chapter, the basic loadings involved in this analysis are internal pressure, axial load, bending moment, and external forces. The symmetry of the loading on the pipe allows for only one half of a cylindrical cross section to be modelled. The assumption of a long pipe, with the stresses and strains not varying along the length, allows for a short length of pipe of only one element along the axis of the cylinder to be modelled. The cylinder was cantilevered at one end but allowed to expand in the radial direction.

Since the model was of a section cut from a long pipe, the ends of the model must remain plane after deformation. The cantilevered end remained plane due to its cantilever boundary conditions, while for the axial loadings and bending moment loadings, the other end remained plane due to the applied loadings. For the force loadings, a constraint equation was applied to the non-cantileverd end of the model so that after deformation the end remained plane.

The constraint equation for the force loading is as follows:

$$\frac{u_{za} - u_{zi}}{u_{za} - u_{zb}} = \frac{y_a - y_i}{y_a - y_b} \tag{3.1}$$

with u_z representing the displacement in the Z direction, 'y' representing the location on the Y axis, 'a' representing the uppermost node (OD/2,π/2,length), and 'b' representing the lowermost node (OD/2,-π/2,length), and 'i' representing a node between 'a' and 'b'.

The average hoop stress was applied as an internal pressure, according to equation 3.2.

$$\sigma_h = \frac{P_i \cdot OD}{2 \cdot t} = \frac{P_i}{2 \cdot (t / D)} \tag{3.2}$$

with σ_h = hoop stress (MPa), P_i= internal pressure (MPa),t= sidewall thickness (m), OD= outside diameter (m), and t/D= sidewall thickness to OD ratio.

The axial stress was directly applied as a pressure in the axial direction.

The force due to backfill was applied as a line load, distributed along the length of the pipe at the top of the cylinder instead of a load distributed both around the circumference and along the length of the pipe. This is a worst case scenario, thus the stresses and strains are exaggerated, making for a conservative design.

The external force from buoyancy is also modelled as a line load at the top of the pipe, distributed along the length, but not distributed around the circumference. Other force loadings such as anchors or other pipeline attachments could be similarly applied.

Due to the input restrictions of ANSYS, the moment loading on the finite element model could not be applied directly as a moment. Instead, the moment load was applied as a ramped axial stress, as shown in Figure 3.1, with the maximum axial stress calculated according to equation 3.3.

$$\sigma_{A\max} = \frac{M \cdot OD}{2 \cdot I} \tag{3.3}$$

with $\sigma_{A\max}$= maximum axial stress from moment loading (Pa),M= applied moment (Nm), OD= outer diameter (m), and I= second moment of area for the pipe (m^4).

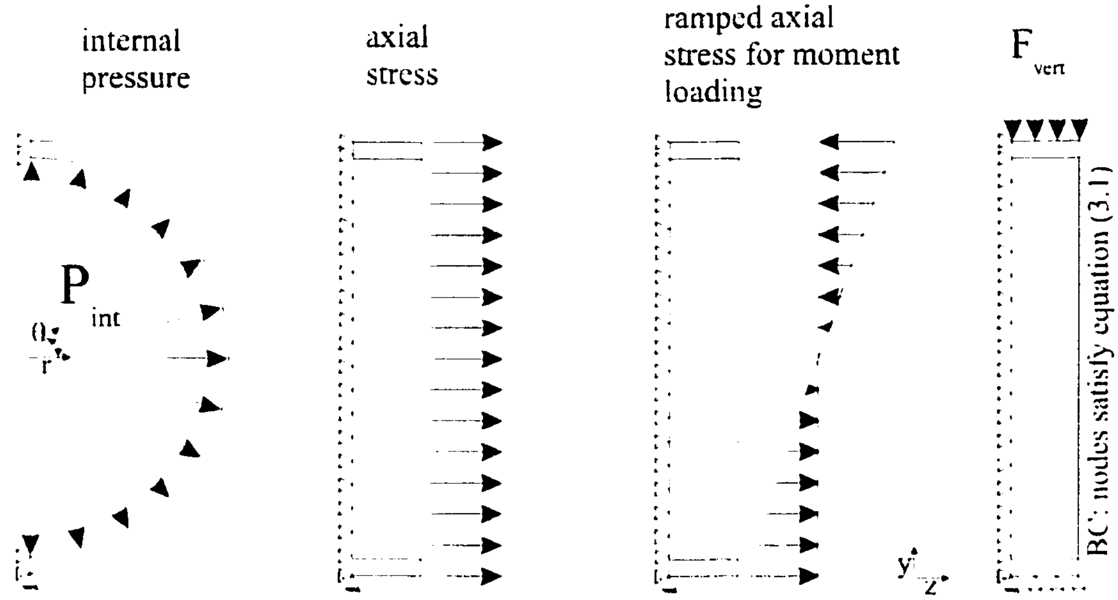

Figure 3.2 Loadings and Boundary Conditons Applied to the Finite Element Model

To account for a possible soil instability, the loading was applied as a pure bending moment. From the bending moment, the vertical displacement of the pipe can easily be calculated, based on equations and relations from beam theory, depending upon the assumed deformed shape.

Now that the finite element model has been developed and the assumptions explained, the model can be run under the various loadings, and the parametric study can be accomplished.

Chapter 4
Results

4.1 Introduction

The numerical model in and of itself is insufficient for full evaluation of the pipe. There are other items that require consideration: failure determination in the composite, the material properties used, and the effects of variations in the material properties of the laminate. Once these items have been taken into account, the results of the finite element model can then be suitably used.

4.2 Assumptions

In order to correctly interpret the results of the finite element model, certain assumptions must be either made or acknowledged. These assumptions range from the determination of failure, to the material properties used and the effects of variances in the material properties, to the finite element model itself.

4.2.1 Failure Determination

There are essentially two methods for defining failure in a composite: first failure* - the point at which an irreversible material damage occurs within the composite, or final failure - where damage has progressed into all plies of the layup. With reference to traditional isotropic materials such as steel, the point of irreversible material damage is determined when the stress or strain in any direction exceeds the yield stress or the yield strain. The yield point is the point at which the modulus deviates from linearity. This deviation from linearity and irreversible damage occurs at first failure within a composite, be that fibre failure or matrix damage. For the discussion and presentation of results that follow, the first failure method is used.

It has been shown by a number of authors [31-34] that leakage does not occur until after the limit of linear elastic response of the composite pipe. However, the exact relation between the departure from linearity and the leakage failure is not fully understood.

The first failure condition is failure initiation within the composite, and may be merely an isolated crack within one layer of a multi-layer composite and not functional or structural failure. By restricting failure to first failure, progressive damage modelling is eliminated.

The elimination of progressive damage is important, as the ANSYS manual [28-29] makes no reference to incorporating failure into the Solid46 element. Therefore it is inferred that the element is not directly capable of accounting for failure within the element. The use of a user programmed incremental solution could be used to incorporate failure within the element. This study is limited to the elastic range of the composite behaviour up until the point of first failure, thus the factoring of failure into the element is not required.

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* first failure referred to first ply failure in most references

Furthermore, since the modelling is concerned with a high pressure pipeline, it is assumed that there is a need for a high degree of reliability, hence almost zero tolerance for damage.

Failure in the composite was determined using the theory of maximum strain. [19] In the maximum strain theory, failure is assumed to occur when one of the following conditions is met:

for tensile fibre failure: $$\frac{\varepsilon_1^+}{\varepsilon_{1f}^+} = 1$$

for compressive fibre failure: $$\frac{\varepsilon_1^-}{\varepsilon_{1f}^-} = 1$$

for tensile matrix failure: $$\frac{\varepsilon_2^+}{\varepsilon_{2f}^+} = 1$$

for compressive matrix failure: $$\frac{\varepsilon_2^-}{\varepsilon_{2f}^-} = 1$$

for shear failure: $$\frac{\varepsilon_{12}}{\varepsilon_{12f}} = 1$$

where superscripts + and - denote tensile and compressive loading. The failure strains, ε_{if}, are determined by mechanical characterization tests.

Failure was assumed to have occurred when any of the above conditions is met in any of the layers within the composite. Due to the restrictions and assumptions made in the formulation of the element, only strains in the plane of the fibres were considered.

The maximum strain failure criteria requires that the strains be resolved into the material directions. Application of this theory accounts for the ultimate tensile and compressive limits of the individual constituents of the composite material.

The advantages of the theory of maximum strain are its simplicity and the ability to differentiate between failure modes. In addition, there is also a growing body of evidence that the matrix failure in a composite is goverened by strain [36].

4.2.2 Material Properties

The material properties used (Tables 4.1 and 4.2) are taken from a variety of sources [35-36]. The failure strains, shown in Table 4.2, are generally determined from characterization tests on unidirectional lamina specimens.. There is some question to the validity of the application of results from uniaxial specimens when applied to laminates. It has been shown in certain studies that the failure strains of a ply within a laminate are affected by the neighbouring plies. [36-38]. The failure strains have also been shown to be dependant upon the thickness of the layers. Classical laminate theory is unable to predict this, and as the element used is based on CLT, it must be said that the numerical results are representative of the physical material, but are a conservative estimate, as the layer effects may lead to a higher strength material.

Since the failure strains used in the post-processing of the finite element model are based on physical testing, the discrepancies in the physical testing can cause some variability in the strain ratio based failure in the numerical model. Variability in the values of the failure strains can shift the strain ratio curves of the constituents up or down, and thus change the failure mode of the pipe.

Table 4.1: Elastic material properties for a typical E-glass/epoxy FRP

Young's modulus in the fibre direction (E_x)	41.31GPa
Young's modulus in the transverse to fibre direction (E_y)	8.652GPa
Poisson's ratio, fibre to matrix (υ_{xy})	0.313
Poisson's ratio, matrix to matrix (υ_{yz})	0.0655
Shear modulus (G_{xy})	4.103GPa
Density (ρ)	1510 kg/m^3

reference: [35-36]

Table 4.2: Failure strains for use with maximum strain failure criteria

fibre failure in tension (ε_{1f}^{+})	0.025
fibre failure in compression (ε_{1f}^{-})	0.015
transverse to fibre failure in tension (ε_{2f}^{+})	0.007
transverse to fibre failure in compression (ε_{2f}^{-})	0.00135
shear failure (ε_{12f})	0.01

reference: [35]

4.3 Results

Having discussed the variations in the material properties and limitations of the element, the results of the finite element analysis are presented as follows:

4.3.1 Biaxial Loadings

There has been a fair amount of investigation done regarding the hoop to axial stress ratio loading of pipes. [36,39-43]. The finite element analysis was carried out for the range of hoop stress to axial stress ratio loadings given in Table 4.3. As this study is based on a first failure type criteria, only the elastic material range is considered. Therefore, for each hoop to axial stress ratio loading, an average hoop stress across the cross section of 1 MPa was applied. For other hoop values of stress, the strain ratio results obtained can simply be scaled.

The reasons for specifying loadings as a hoop stress averaged through the thickness rather than an internal pressure are twofold: the axial stress can be directly calculated, and the strain ratio results become independent of the thickness and diameter of the pipe. For a specific pipe diameter, thickness, and pressure, the average applied hoop stress can easily be calculated from equation (3.1) and the strain ratios can be scaled accordingly.

Table 4.3: Stress ratio, hoop stress to axial stress, loadings considered in the FEA

(1H:0A)
(8H:1A)
(4H:1A)
(2H:1A)
(1H:1A)
(1H:2A)
(1H:4A)
(1H:8A)
(0H:1A)

The response of the finite element model, resolved into the fibre, perpendicular to fibre, and shear directions of the layup, to the applied hoop stress to axial stress loadings can be seen in Figure 4.1 to Figure 4.17 (odd). In these figures, the ordinate is dimensionless strain, $\varepsilon_i/\varepsilon_{if}$, where ε_{if} is the failure strain (see §4.2.1). The abscissa is the fibre winding angle (see Figure 1.1).

Due to the different failure strains for tensile or compressive loadings, there is a change in slope of the strain ratio curves of the fibre and matrix as the strain ratio passes from tension to compression, or vise versa.

In order to determine the maximum allowable hoop stress, or the hoop stress which would bring the pipe to failure under the given loading condition, a scaling factor was determined which would bring the maximum strain ratio to unity.

The hoop stress to failure results thus determined are in presented in Figures 4.2-4.18 (even). In these figures, the ordinate is the applied hoop stress which causes the first failure and the abscissa is the fibre winding angle.

It can be seen from the results presented in Figures 4.1 to 4.17 (odd) that the strains, and therefore the stresses, within the composite are highly dependant upon the fibre wind angle. From these plots it can also be seen that for a given biaxial loading, there is an optimum fibre angle at which strains caused by the loading are at a minimum. This optimum occurs when the fibres are generally oriented in the principal direction of the loading. An approximate analytical method exists for determining this optimum fibre angle for biaxial loading called "netting analysis". This is discussed further in Appendix B.

The failure mode of the composite is also evident in the biaxial results (Figures 4.1 to 4.17 (odd)). In deciding upon an appropriate wind angle for the layup of the composite, the mode of failure requires particular consideration. From a safety perspective, a shear or transverse to fibre failure mode (matrix cracking) is preferred to a fibre failure mode. This preference is due to the tendency of a shear or transverse to fibre failure mode to initiate cracks in the matrix material, thus leading to leakage, whereas a fibre failure would be more of a structural failure, which may lead to a catastrophic burst. Netting analysis cannot predict the failure mode, as it assumes that the matrix material carries no load.

Figure 4.1 shows the strains occurring in the composite pipe due to a (1H:0A) loading. For fibre angles of 0 to 38 degrees, the maximum strain ratio occurs in the direction transverse to the fibre direction, or in the matrix material. For fibre angles of 38 to 75 degrees, the maximum strain ratio occurs in shear, and from 75 to 90 degrees, the maximum ratio will occur in the fibre directions. Thus, for fibre angles less than 75 degrees, the pipe would form cracks in the matrix initially, while the fibres remained intact. Thus the pipe would tend to leak, while maintaining structural integrity. For fibre angles greater than 75 degrees, failure would occur initially in the fibres. It is expected then that structural integrity would be lost, and the pipe would burst catastrophically with very little warning.

In Figure 4.2, it is shown that the maximum allowable hoop stress can vary quite significantly with the fibre angle. The maximum hoop stress is directly related to the utilization of the fibre strength. The effects of fibre utilization, or the lack thereof, are shown by the significant loss of material strength for fibre angles less than 75 degrees, compared to fibre angles greater than 75 degrees.

Table 4.4 summarizes the failure mode and fibre angle relations for the biaxial loadings listed in Table 4.3.

4.3.2 Bending Moment Loading

The finite element pipe model was also loaded with a pure bending moment. As discussed in Chapter 3, the bending moment was applied to the model as a ramped axial stress. By specifying the loading of the model as a maximum applied axial stress caused by bending, rather than a moment specification, the strain ratio results became independent of pipe's diameter and thickness.

For any given bending moment, the maximum axial stress can be easily calculated, and similar to the biaxial loadings, the strain ratios can be scaled to determine the ratios for that loading. This loading differs from the biaxial loadings covered in §4.3.1 as the loading produces both positive (tensile) and negative (compressive) strain ratios across the cross section in each of the material constituents. As a result, the strain ratio results produced by the loading of a maximum axial stress of 1MPa, as shown in Figure 4.19, includes both the maximum and minimum strain ratios occurring in the cross section, as determined by the FEA. The applied loading is anti-symmetric, with the maximum applied axial stress being equal in magnitude to the absolute value of the minimum applied axial stress, but due to the different failure strains of the composite constituents in tension or compression, the strain ratio plot in Figure 4.19 is not symmetric with respect to the ordinate axis.

As with the biaxial loadings, this bending moment loading was also scaled to produce failure in the composite, to make the maximum absolute value of the strain ratios equal to unity. The results of this are shown in Figure 4.20, with the abscissa being the maximum applied axial stress, and the ordinate the fibre wind angle. The failure modes for the bending moment loading are fibre compression (0-18 degrees), shear (22-52 degrees) and matrix tension (18-22 degrees, 52-90 degrees). As would be expected with the axial loading of the moment, maximum loading before failure is significantly higher for lower fibre angles, when the fibres are generally oriented in the principal direction of the loading.

4.3.3 Lateral Line Load

A further loading case for a pipeline is the line load acting along the length of the pipe from soil overburden or buoyancy. As with the previous loading cases, a unit load of 1 kN per unit length was applied and the strains were calculated by the finite element model. Unlike the other loadings, however, the strains proved to be dependant upon the thickness and diameter of the pipe. When normalizing the thickness and the applied force by the diameter of the pipe, the strains were shown to be dependant upon the thickness to diameter ratio. This normalization also revealed that the applied force required to attain a specific strain was also dependant upon the diameter of the pipe. For example, to achieve a fibre strain ratio of 0.1 in a pipe with a layup angle of 50 degrees and t/D of 0.01, the force required for a diameter of 0.1m is 486.1 kN/m, while for the force required to achieve the same fibre strain ratio in a pipe with a diameter of 1m is 4861 kN/m.

As with the other loadings, a scaling factor can easily be used to factor the strain ratios to any given force loading.

Figures 4.21-4.25 and 4.27-4.31 show the strain ratios from a lateral line load of 1 kN per meter length per meter diameter for t/D varying from 0.01 to 0.10. Figures 4.26 and 4.32 show the force per meter length per meter diameter at failure.

The strain ratio results for a line load of 1 kN per metre length per metre diameter and a t/D of 0.01 are shown in figure 4.21. The applied loading creates strains which are both tensile and compressive for each constituent of the composite, thus the maximum and minimum strain ratios occurring are included in the figure. The strain ratio curves for the other t/D values are all identical, with only the values in the abscissa changing from figure to figure.

Figure 4.26 shows the maximum allowable line load per metre per diameter for t/D values of 0.01 to 0.05. The effect of the thickness of the pipe and its ability to withstand the line load is quite evident in this figure, as the abscissa requires log scale to display the results.

The relation of the failure mode to the fibre angle is given in Table 4.4. The failure mode/fibre angle relations are the same for all t/D ratios considered.

4.4 Chapter Summary

It has been demonstrated in this chapter that the strains which occur in a fibre reinforced epoxy composite are highly dependant upon the wind angle of the fibre. This was demonstrated for the hoop to axial stress ratio loading, bending moment, and distributed line load loadings. It can be thus said that there must exist an optimum fibre wind angle, or range of allowable fibre wind angles for any given set of possible loading conditions. This is further discussed in the following chapter.

Table 4.4: Summary of failure modes, by fibre angle, for the loadings considered

Loading	Fibre Angles (degrees)	Failure Mode
(1H:0A)	0-37	matrix tension
	37-75	shear
	75-90	fibre tension
(8H:1A)	0-38	matrix tension
	38-70	shear
	70-90	fibre tension
(4H:1A)	0-40	matrix tension
	40-62	shear
	62-90	fibre tension
(2H:1A)	0-90	matrix tension
(1H:1A)	0-90	matrix tension
(1H:2A)	0-90	matrix tension
(1H:4A)	0-23	fibre tension
	23-50	shear
	50-90	matrix tension
(1H:8A)	0-18	fibre tension
	18-51	shear
	51-90	matrix tension
(0H:1A)	0-14	fibre tension
	14-52	shear
	52-90	matrix tension
bending moment	0-18	fibre compresson
	18-21	matrix tension
	21-52	shear
	52-90	matrix tension
force	0-40	matrix tension
	40-78	shear
	78-90	fibre compression

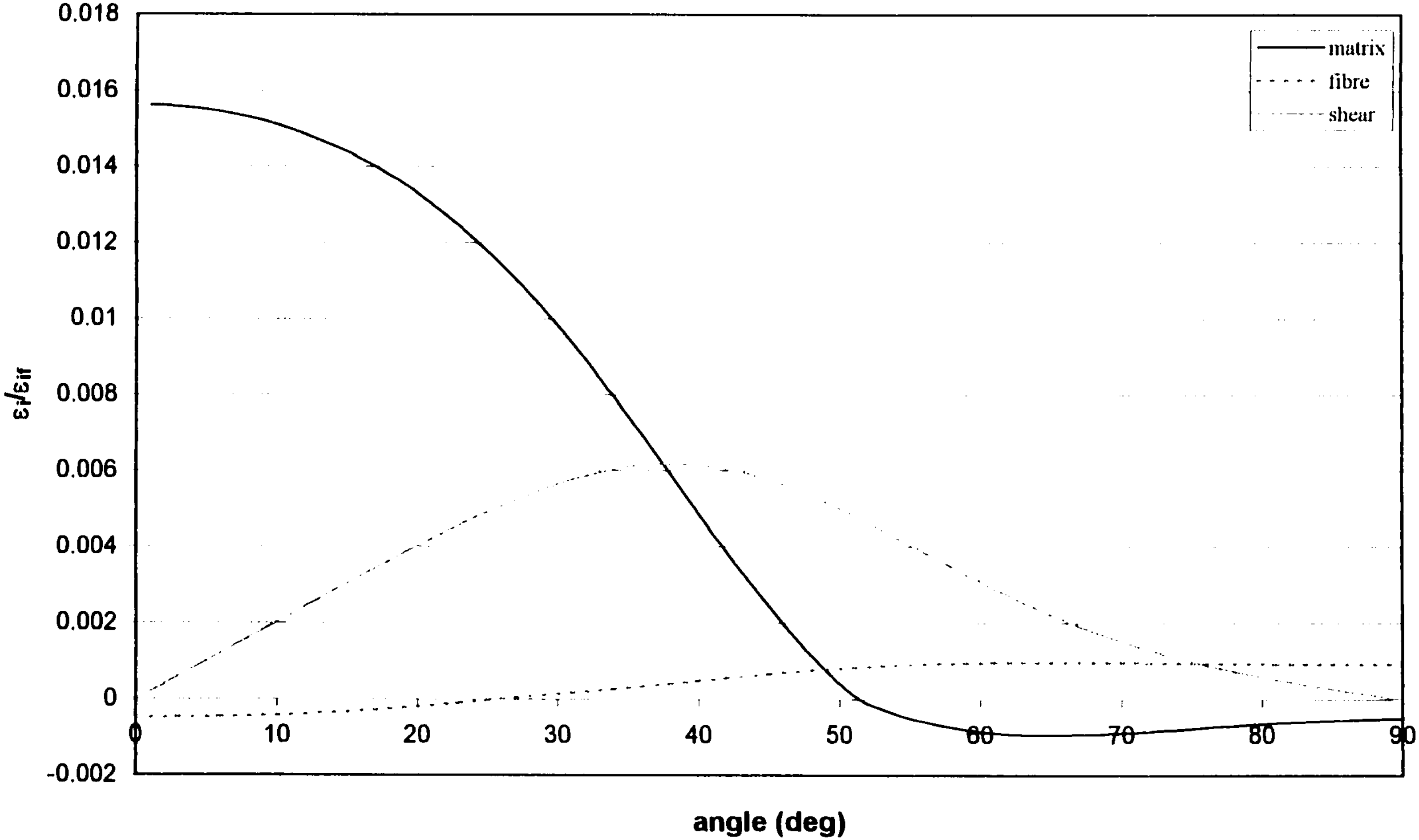

Figure 4.1: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for an applied hoop stress of 1 MPa in a stress ratio loading of (1H:0A)

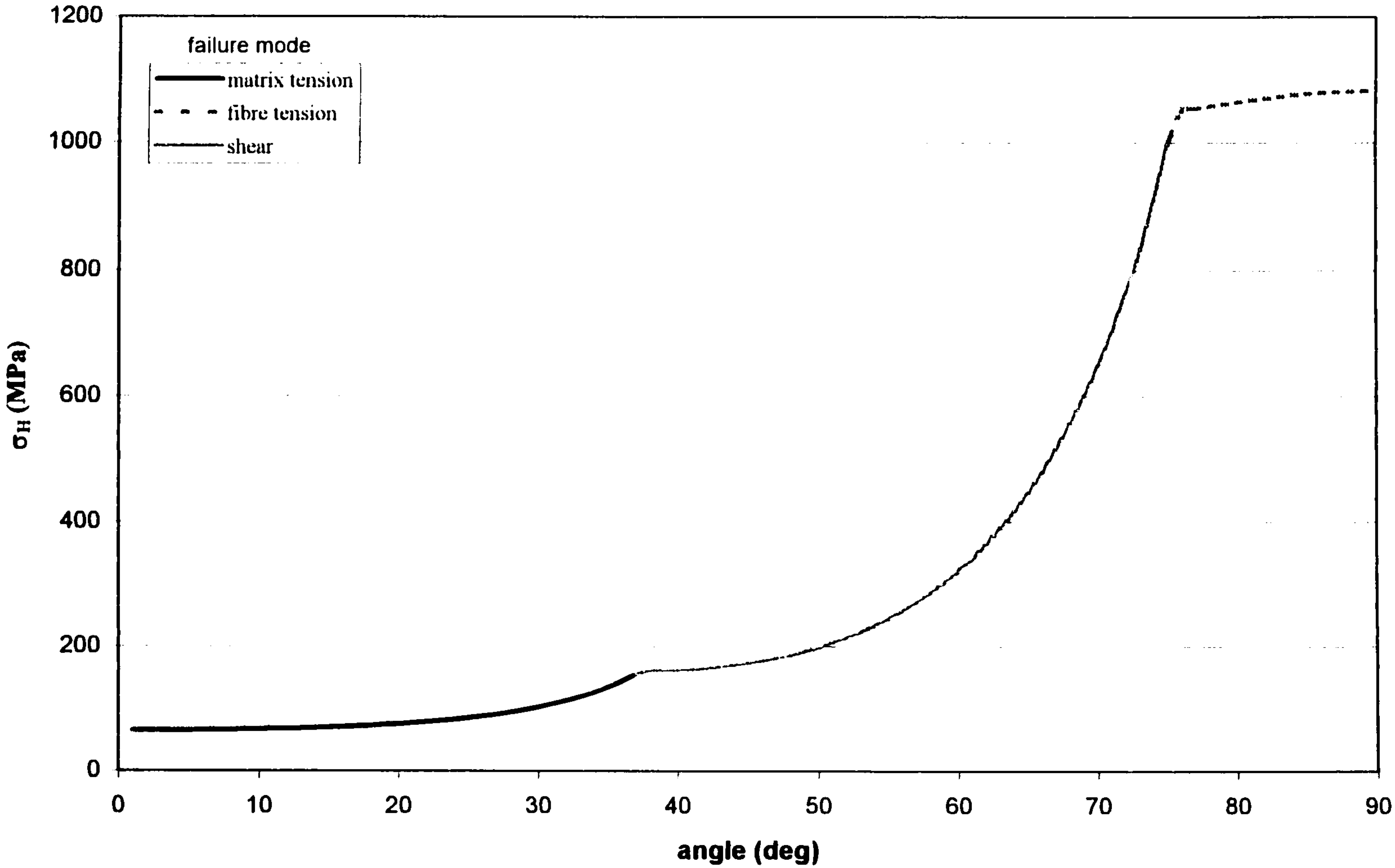

Figure 4.2: Hoop stress at failure ($\varepsilon_{imax}/\varepsilon_{if} = 1$) for fibre angles of 0 to 90 degrees, for a loading of (1H:0A)

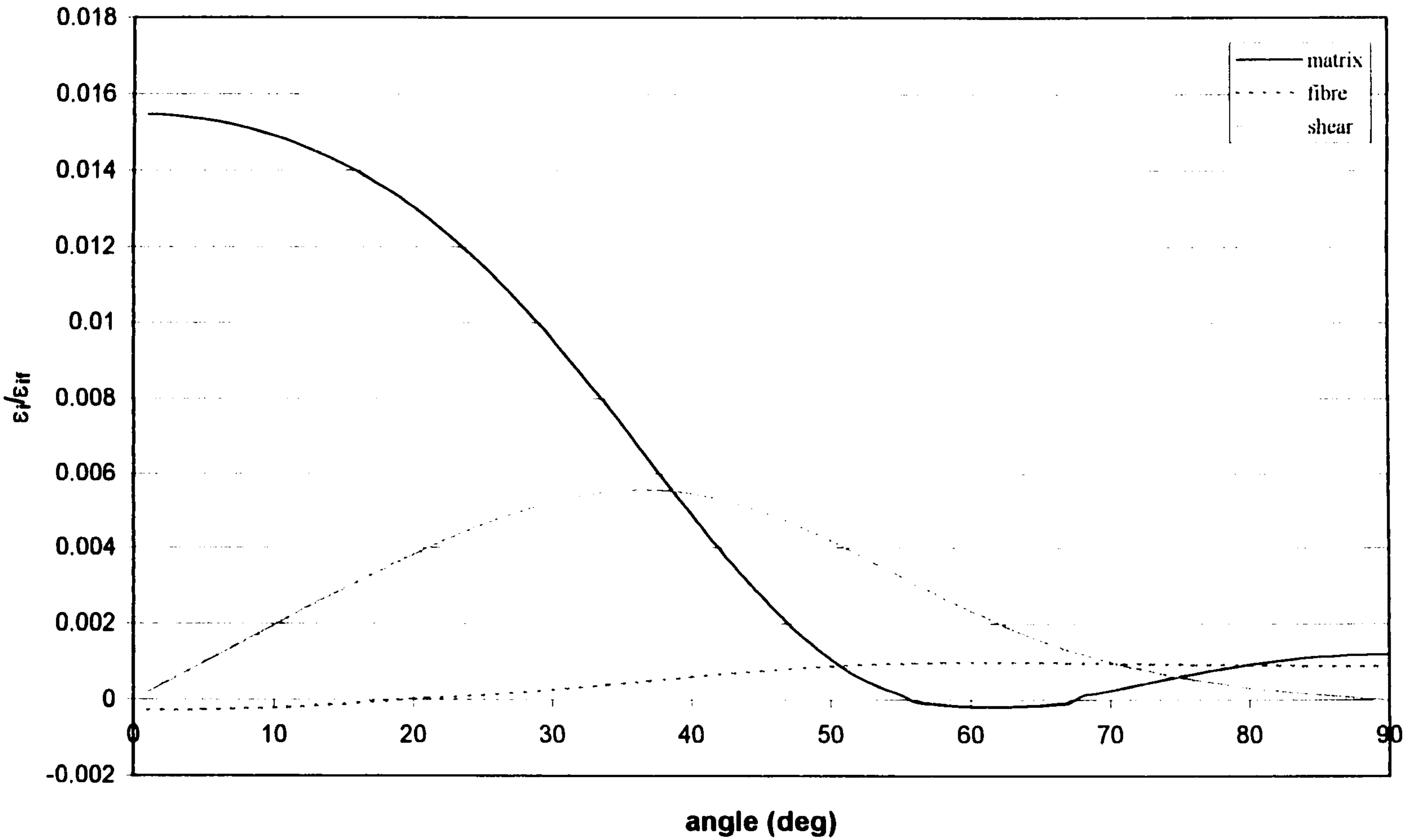

Figure 4.3: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for an applied hoop stress of 1 MPa in a stress ratio loading of (8H:1A)

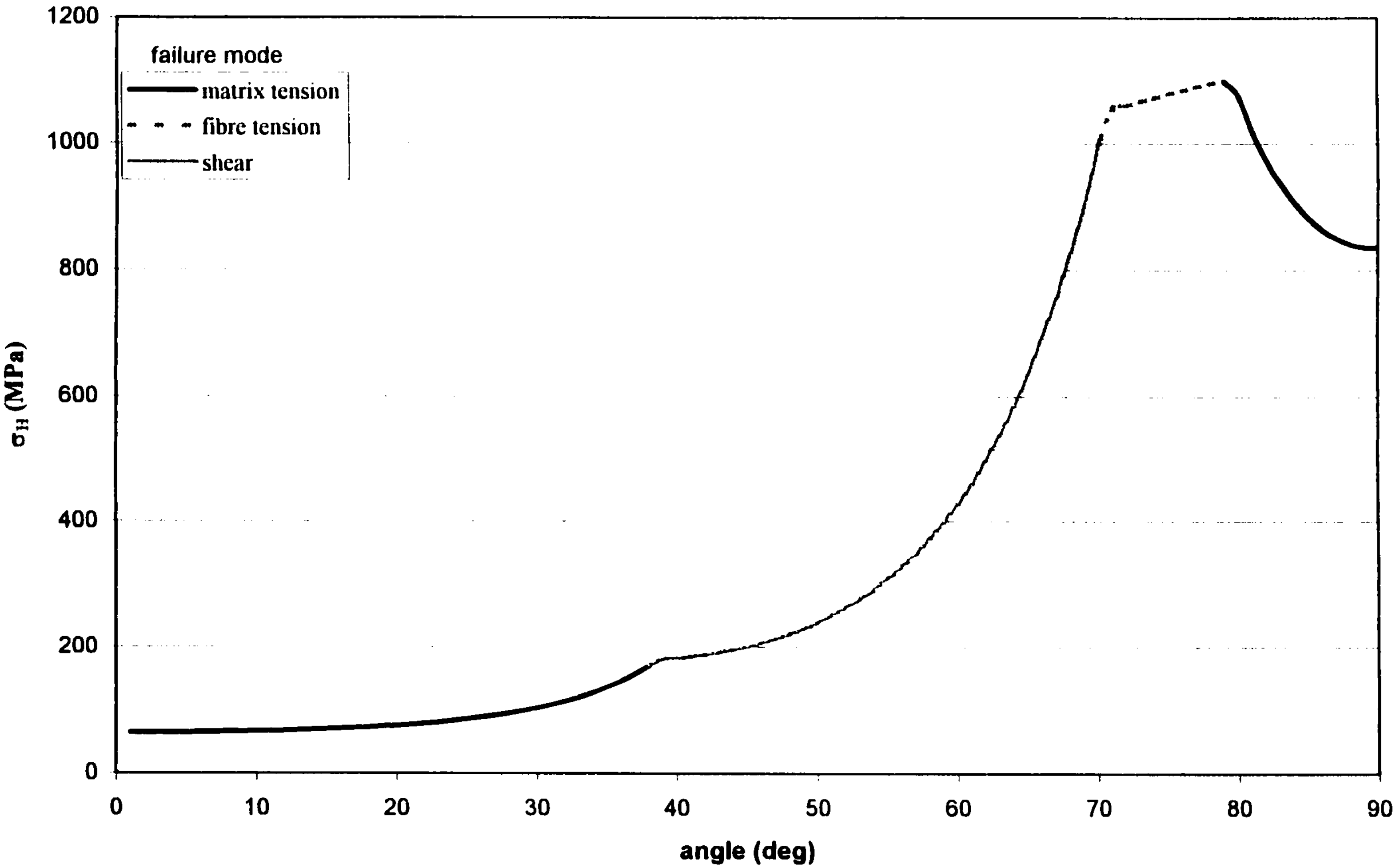

Figure 4.4: Hoop stress at failure ($\varepsilon_{imax}/\varepsilon_{if} = 1$) for fibre angles of 0 to 90 degrees, for a loading of (8H:1A)

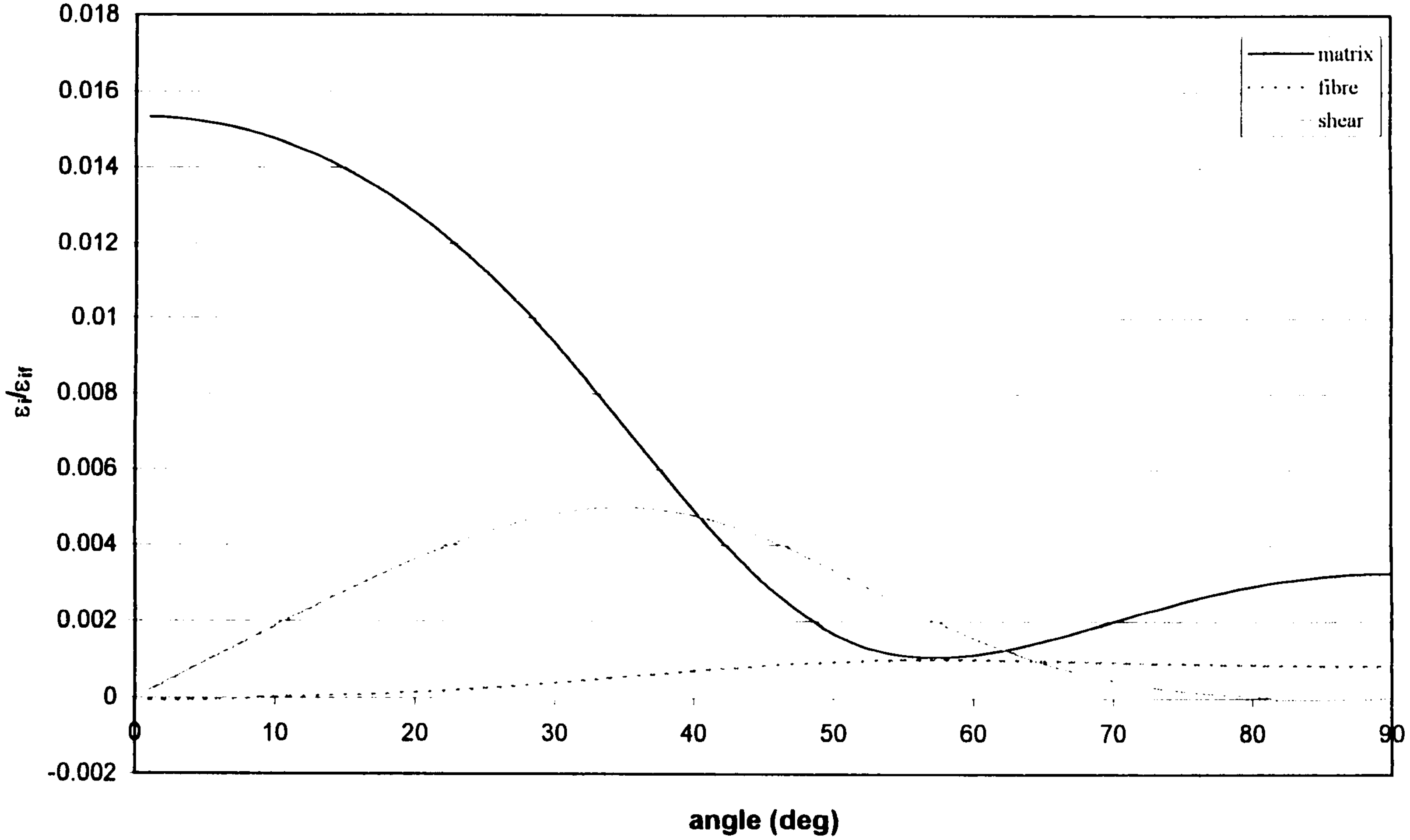

Figure 4.5: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for an applied hoop stress of 1 MPa in a stress ratio loading of (4H:1A)

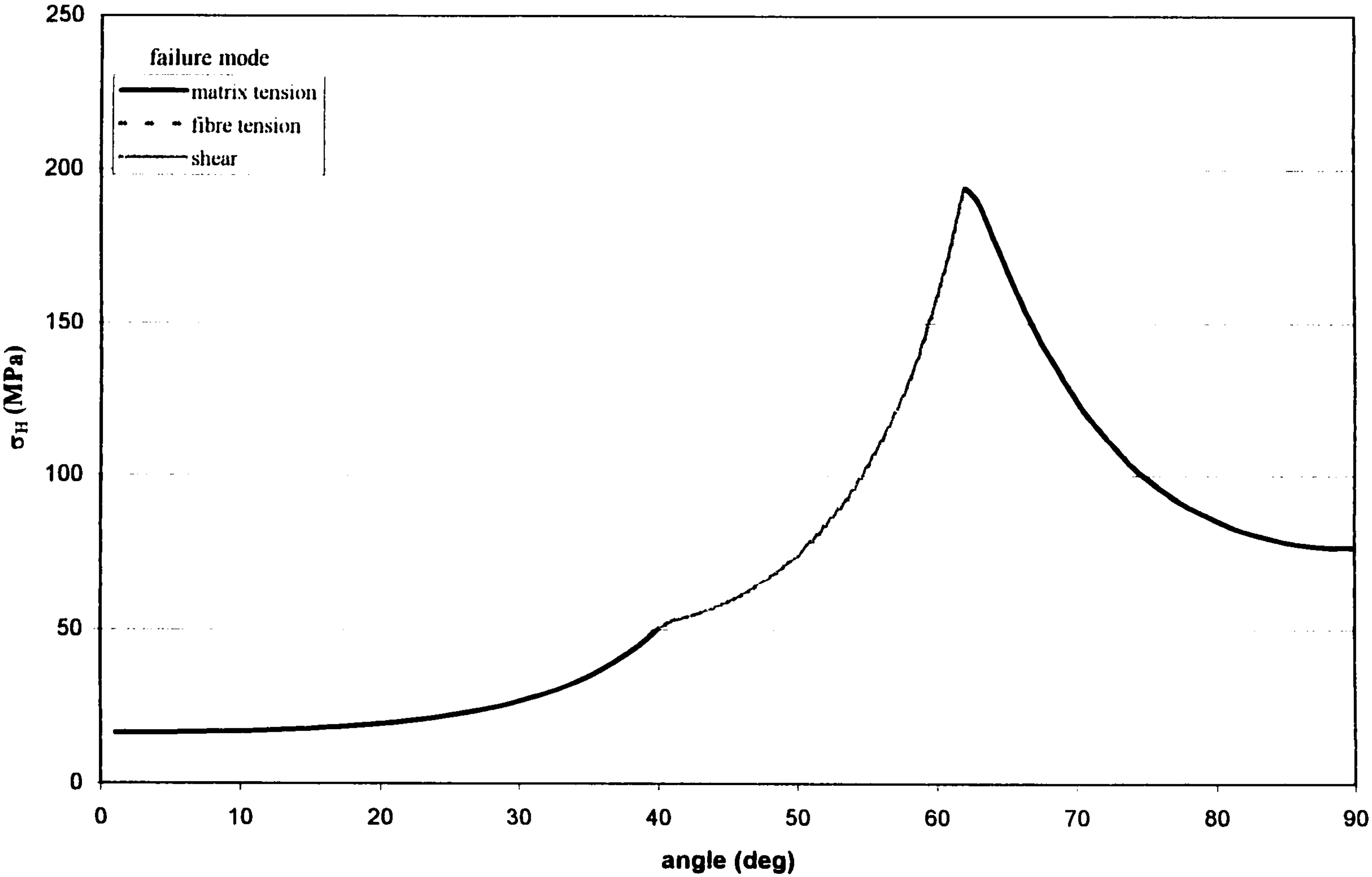

Figure 4.6: Hoop stress at failure ($\varepsilon_{imax}/\varepsilon_{if} = 1$) for fibre angles of 0 to 90 degrees, for a loading of (4H:1A)

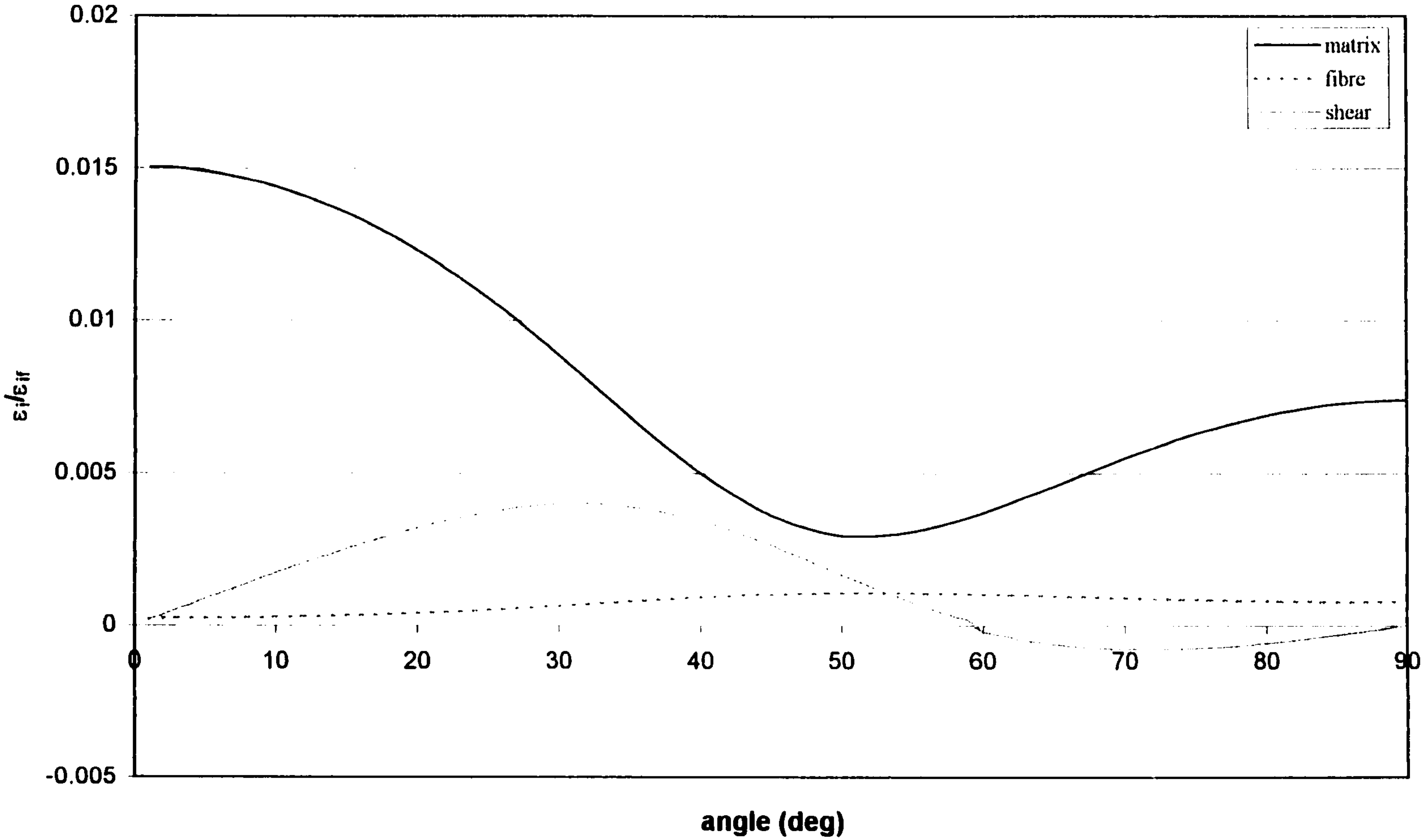

Figure 4.7: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for an applied hoop stress of 1 MPa in a stress ratio loading of (2H:1A)

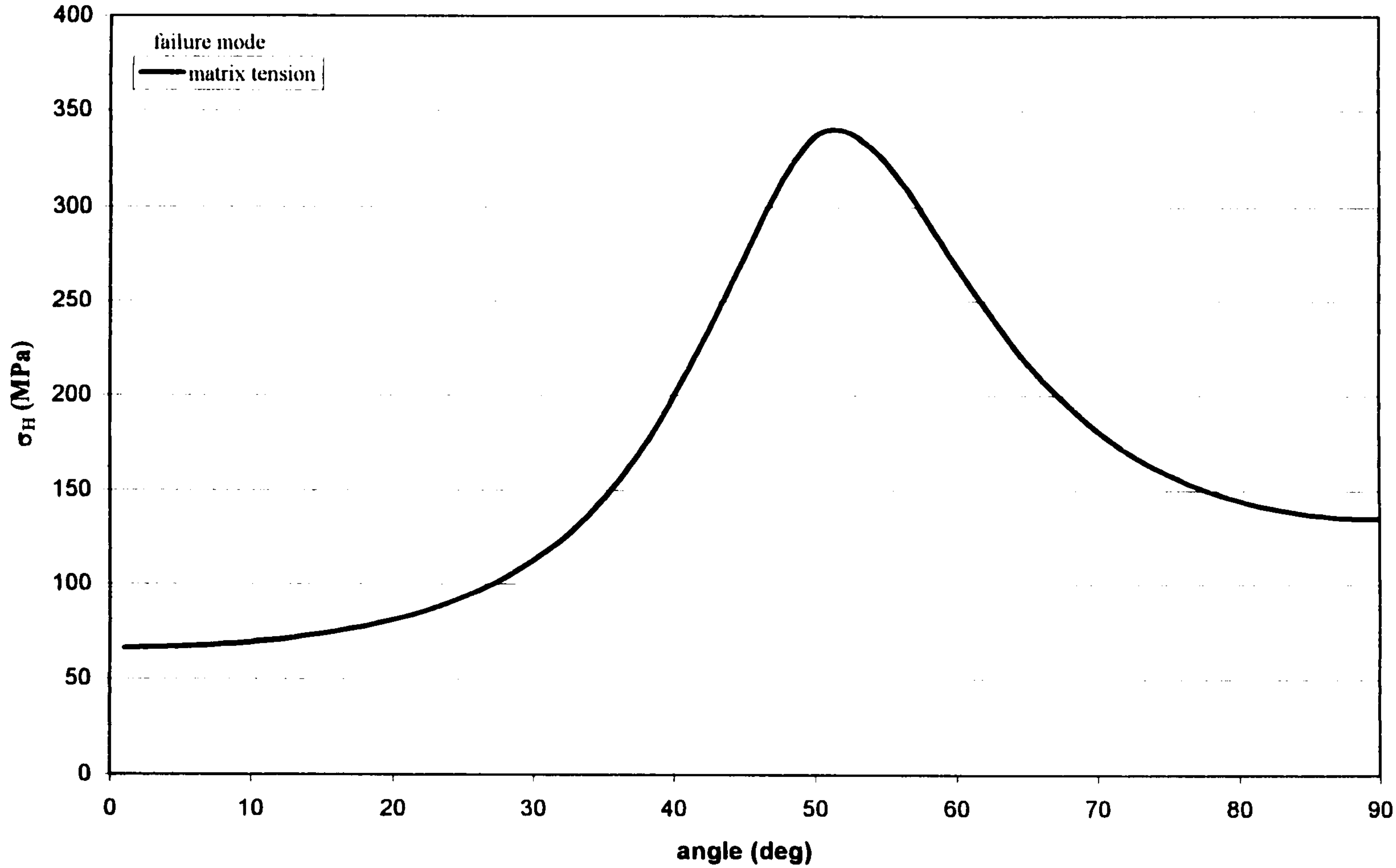

Figure 4.8: Hoop stress at failure ($\varepsilon_{imax}/\varepsilon_{if} = 1$) for fibre angles of 0 to 90 degrees, for a loading of (2H:1A)

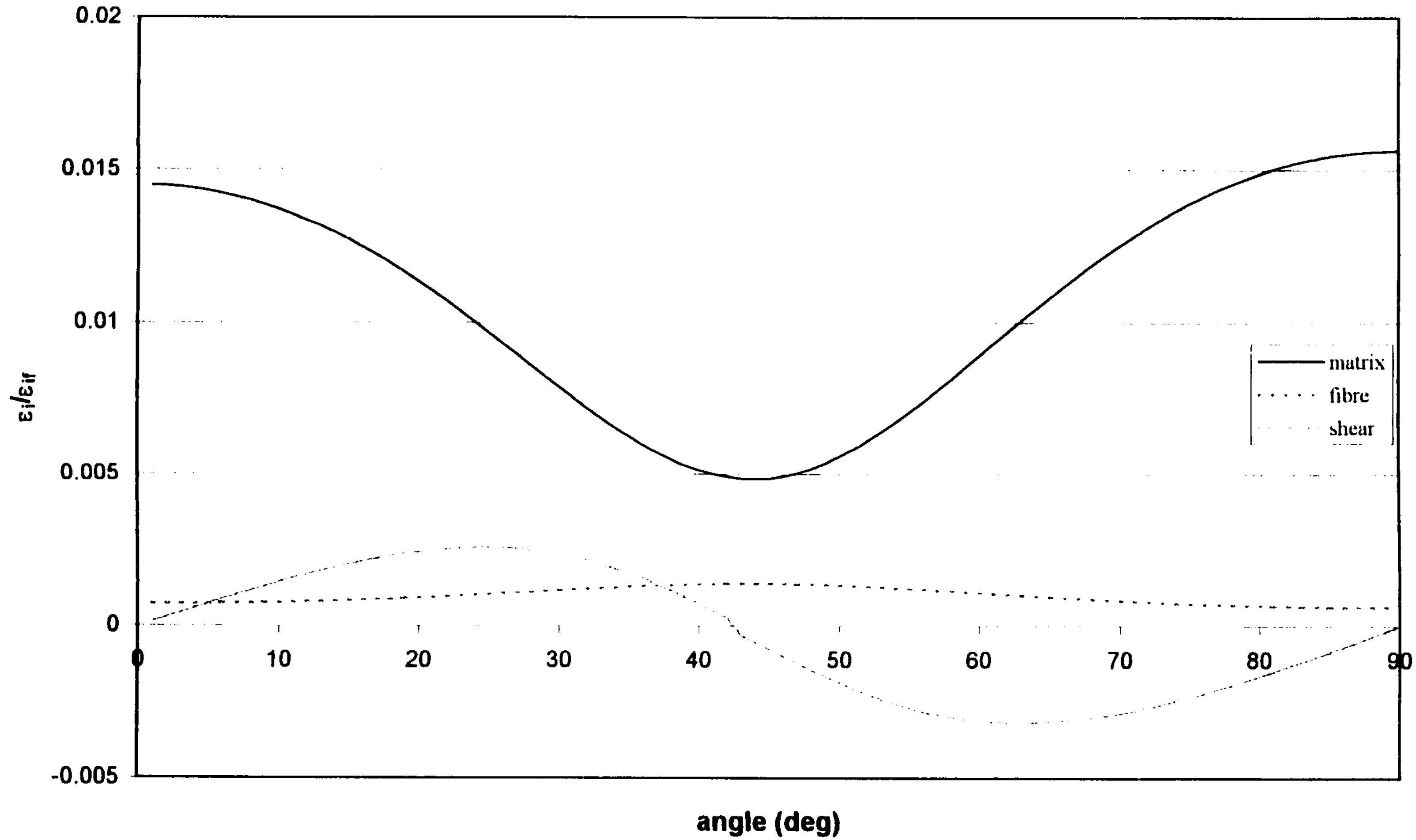

Figure 4.9: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for an applied hoop stress of 1 MPa in a stress ratio loading of (1H:1A)

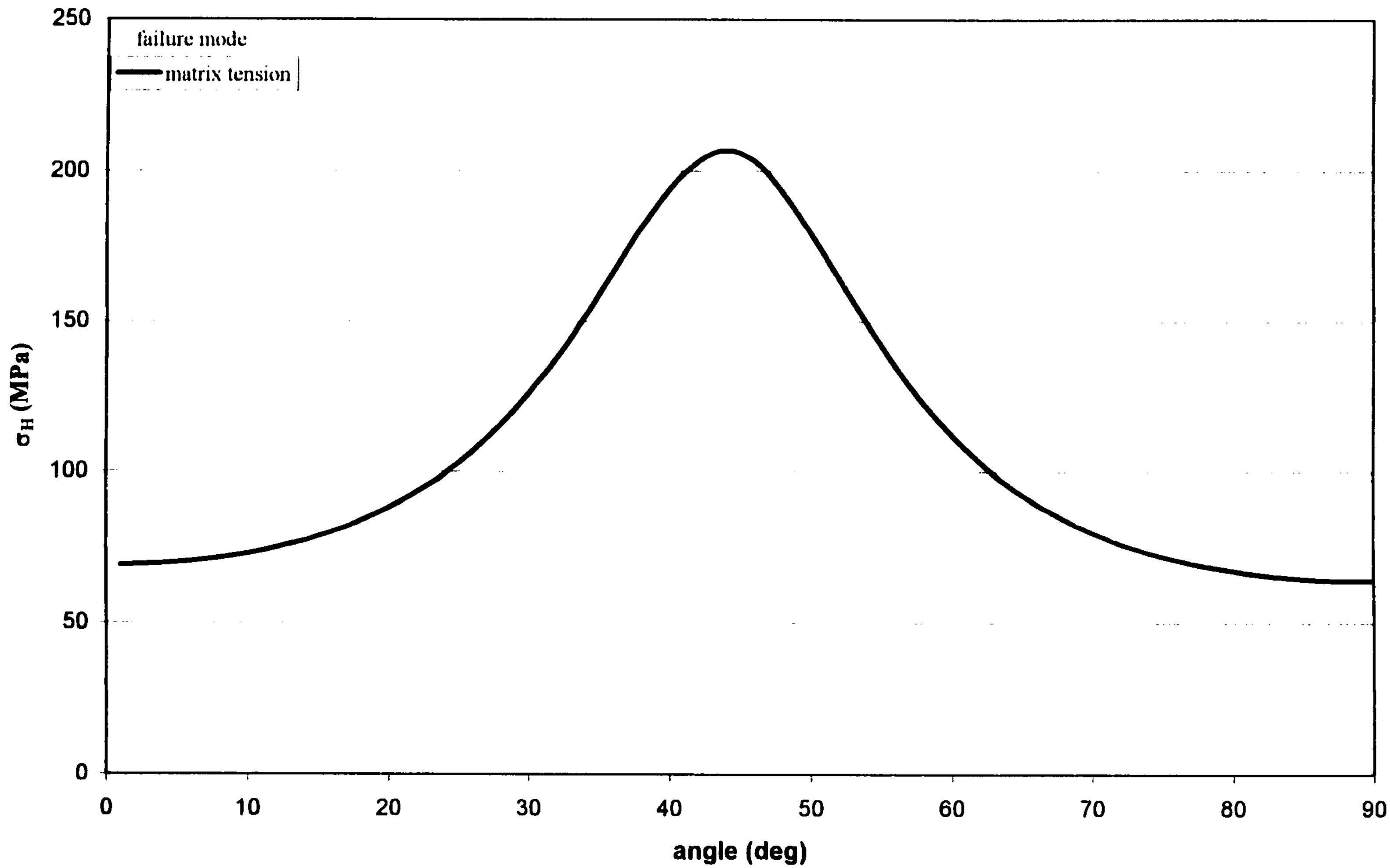

Figure 4.10: Hoop stress at failure ($\varepsilon_{imax}/\varepsilon_{if} = 1$) for fibre angles of 0 to 90 degrees, for a loading of (1H:1A)

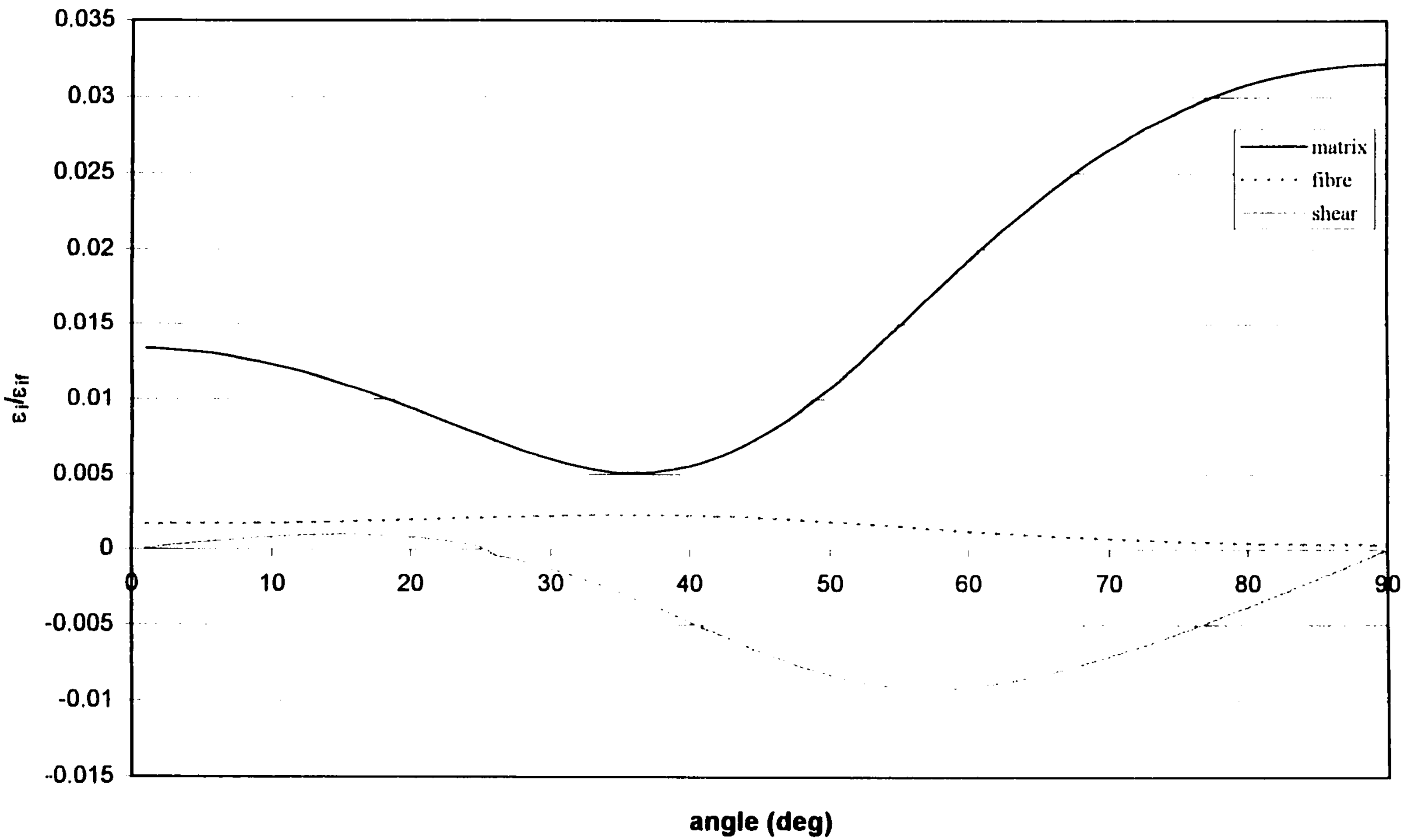

Figure 4.11: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for an applied hoop stress of 1 MPa in a stress ratio loading of (1H:2A)

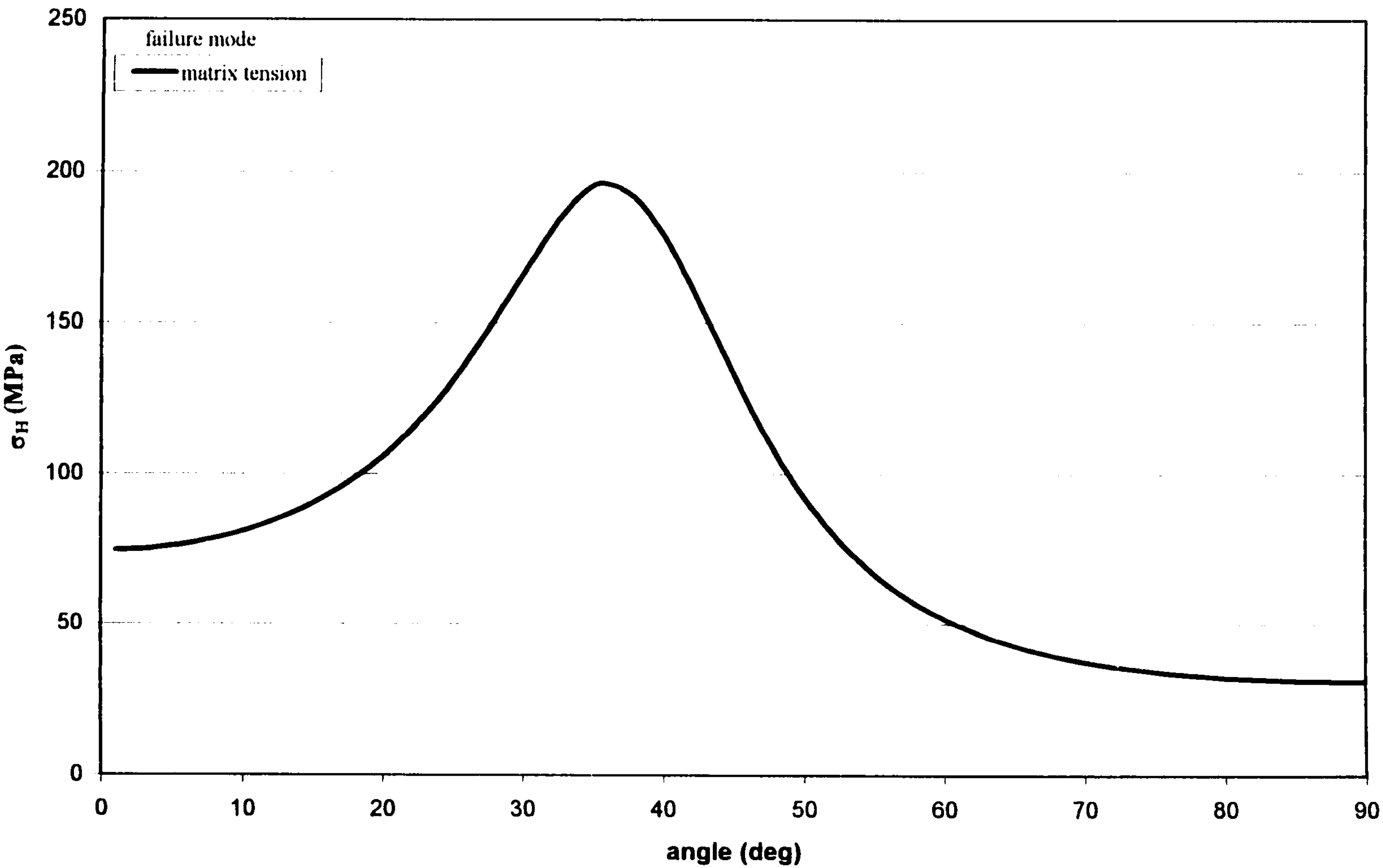

Figure 4.12: Hoop stress at failure ($\varepsilon_{imax}/\varepsilon_{if} = 1$) for fibre angles of 0 to 90 degrees, for a loading of (1H:2A)

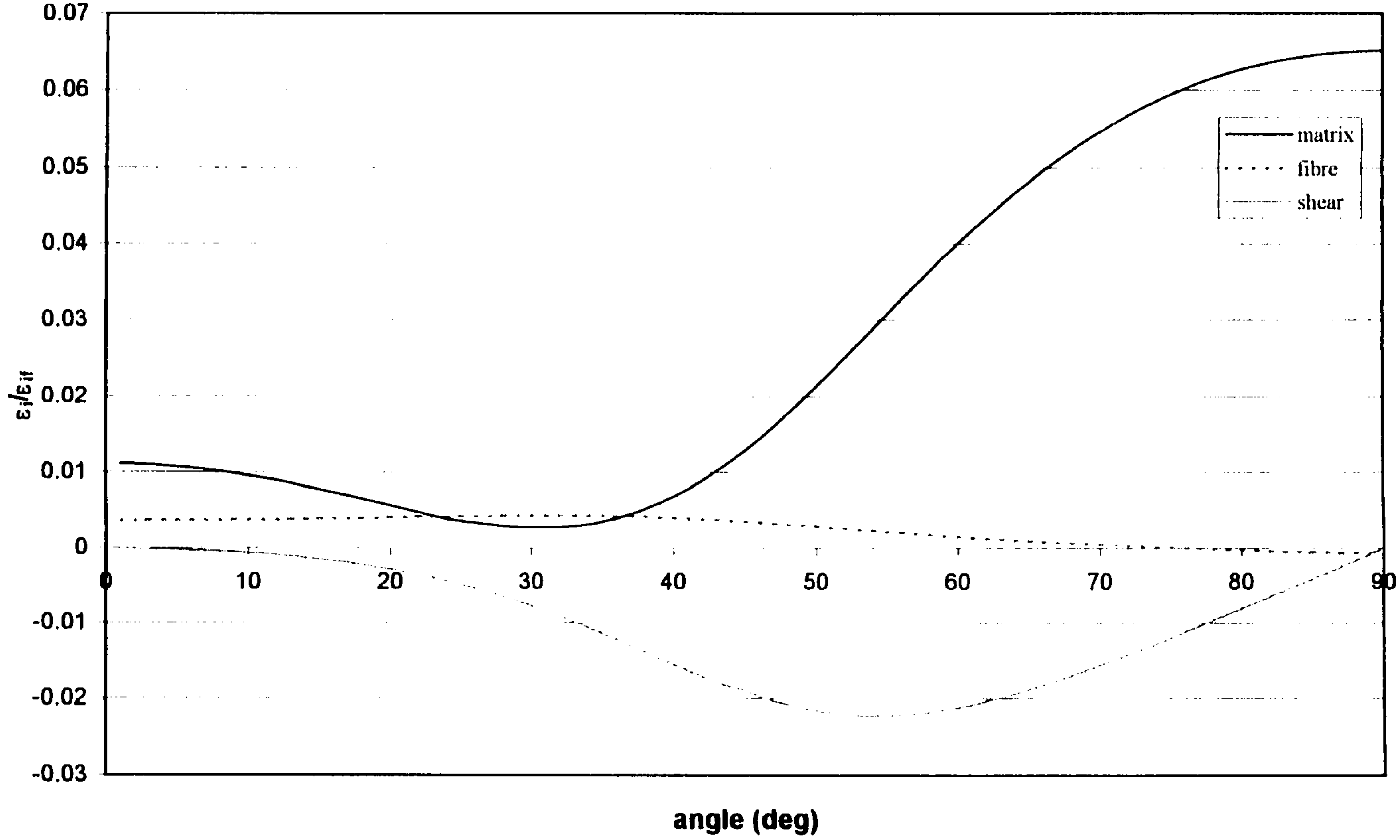

Figure 4.13: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for an applied hoop stress of 1 MPa in a stress ratio loading of (1H:4A)

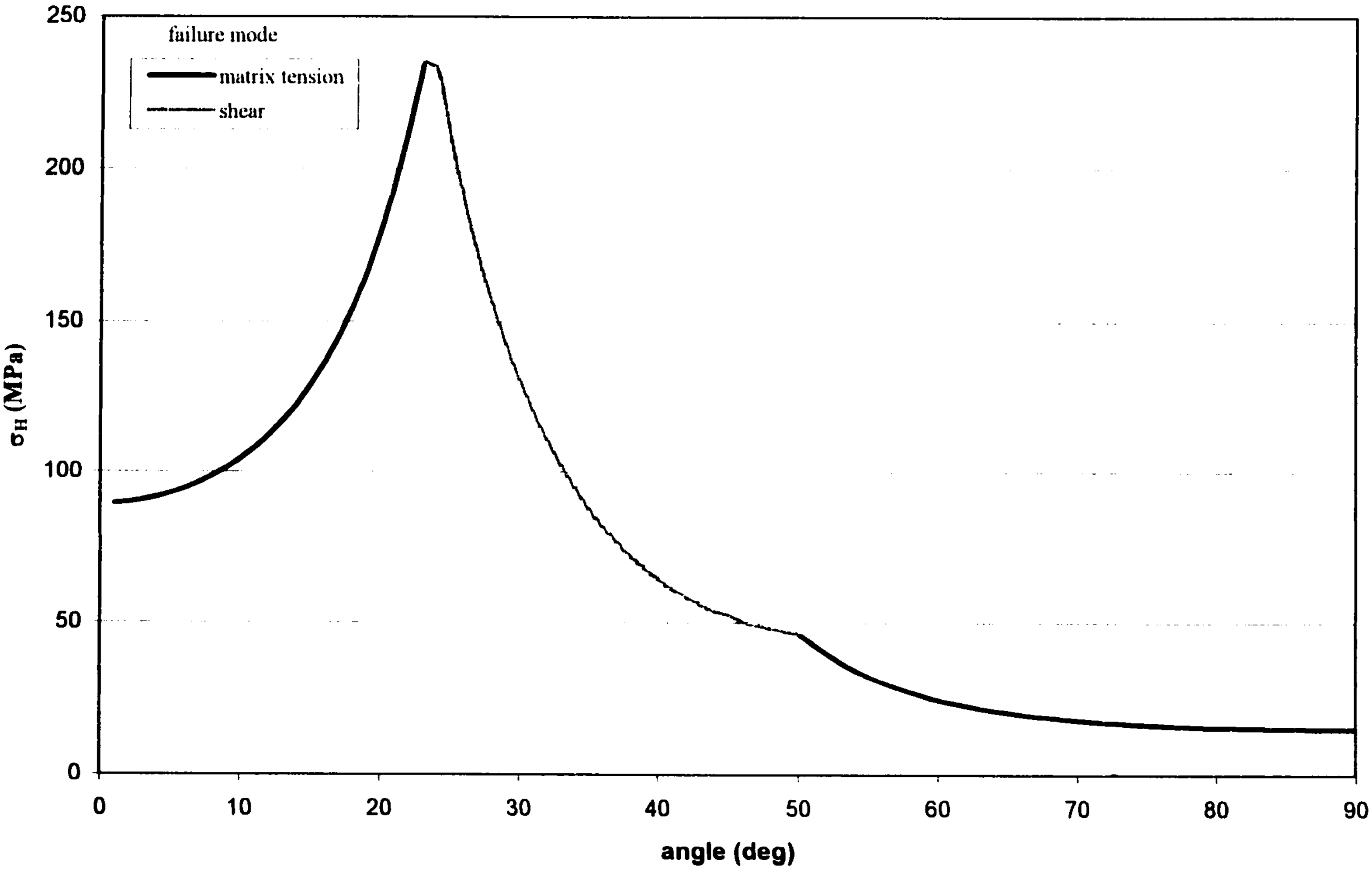

Figure 4.14: Hoop stress at failure ($\varepsilon_{imax}/\varepsilon_{if} = 1$) for fibre angles of 0 to 90 degrees, for a loading of (1H:4A)

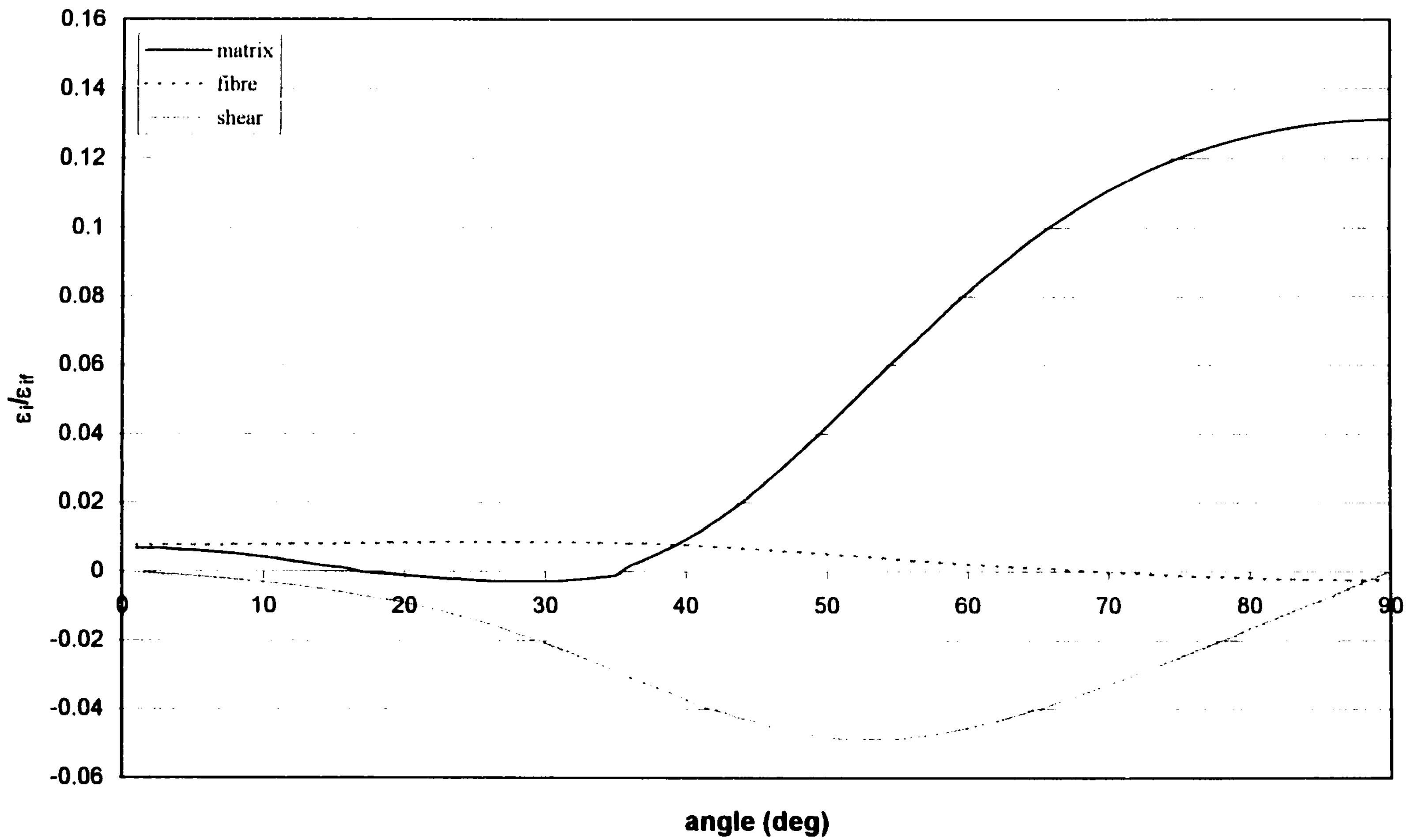

Figure 4.15: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for an applied hoop stress of 1 MPa in a stress ratio loading of (1H:8A)

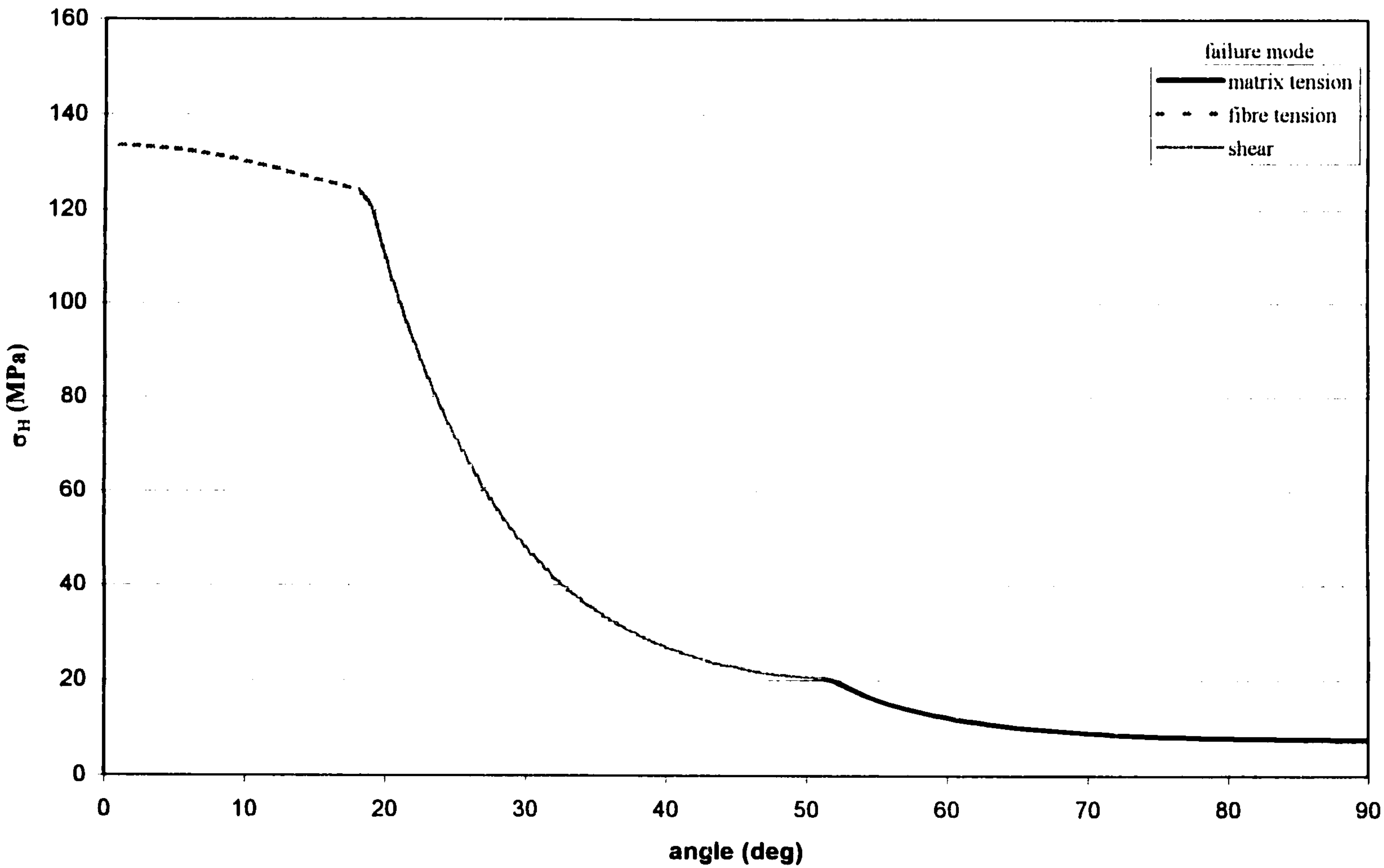

Figure 4.16: Hoop stress at failure ($\varepsilon_{imax}/\varepsilon_{if} = 1$) for fibre angles of 0 to 90 degrees, for a loading of (1H:8A)

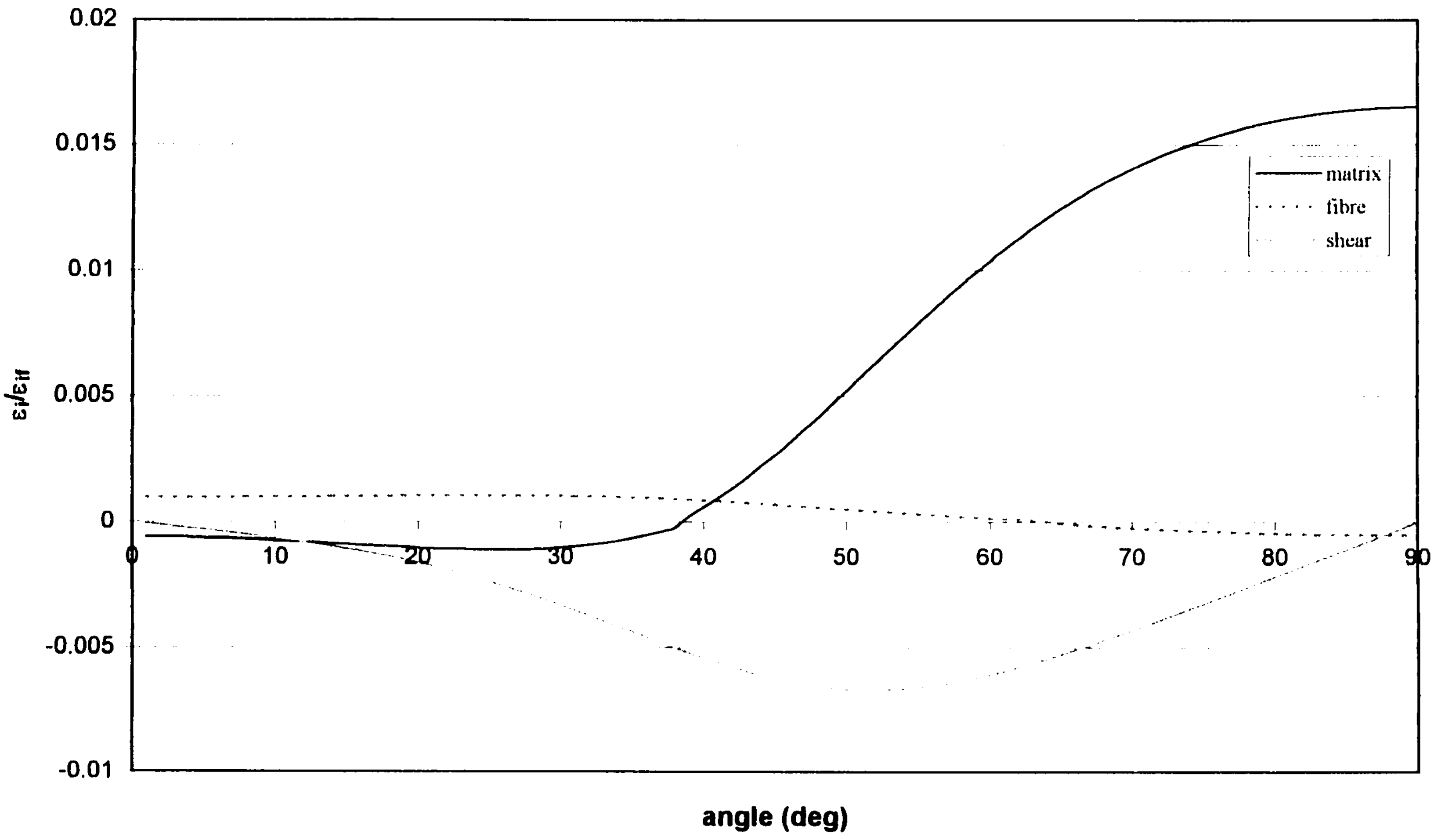

Figure 4.17: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for an applied axial stress of 1 MPa in a stress ratio loading of (0H:1A)

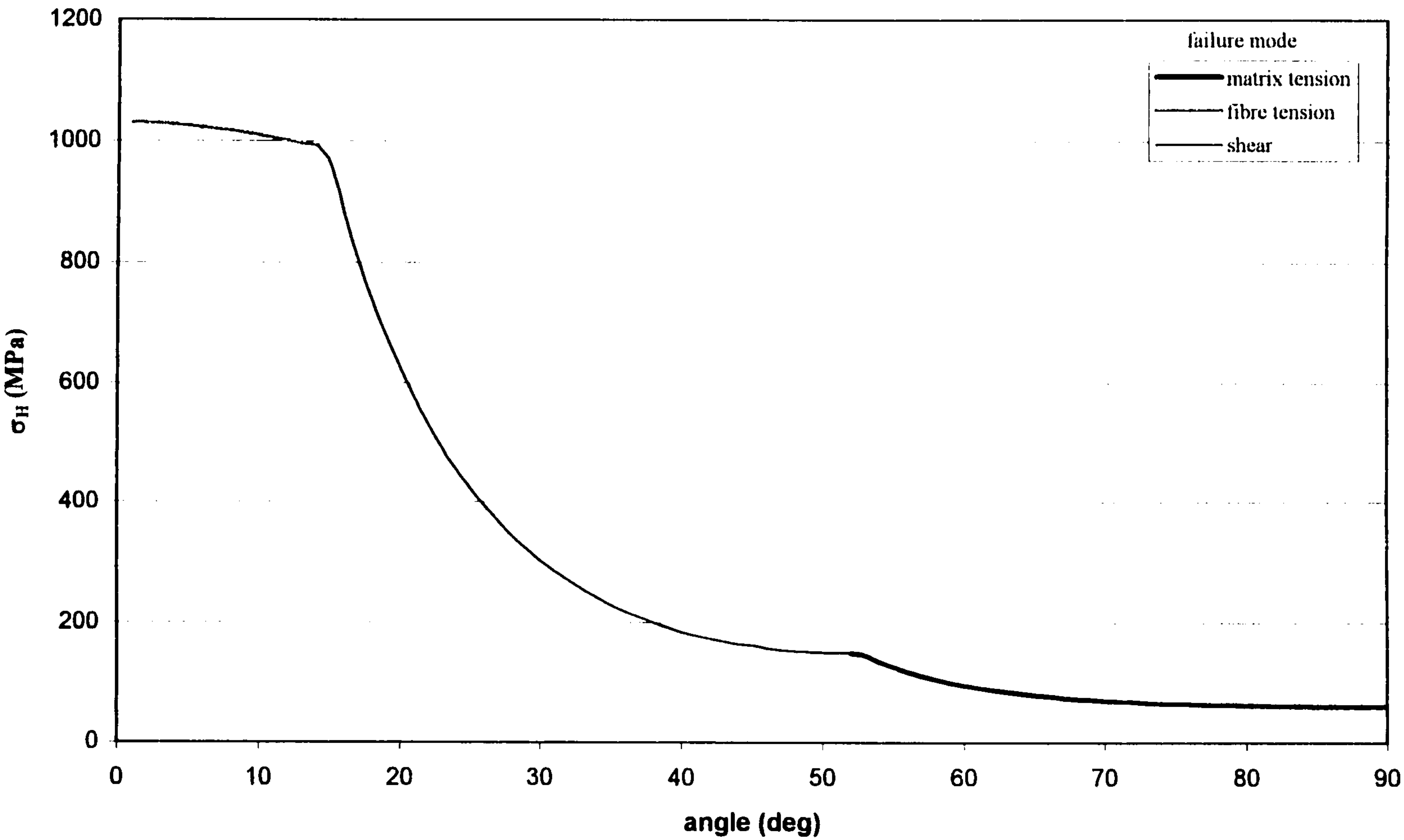

Figure 4.18: Axial stress at failure ($\varepsilon_{lmax}/\varepsilon_{lf} = 1$) for fibre angles of 0 to 90 degrees, for a loading of (0H:1A)

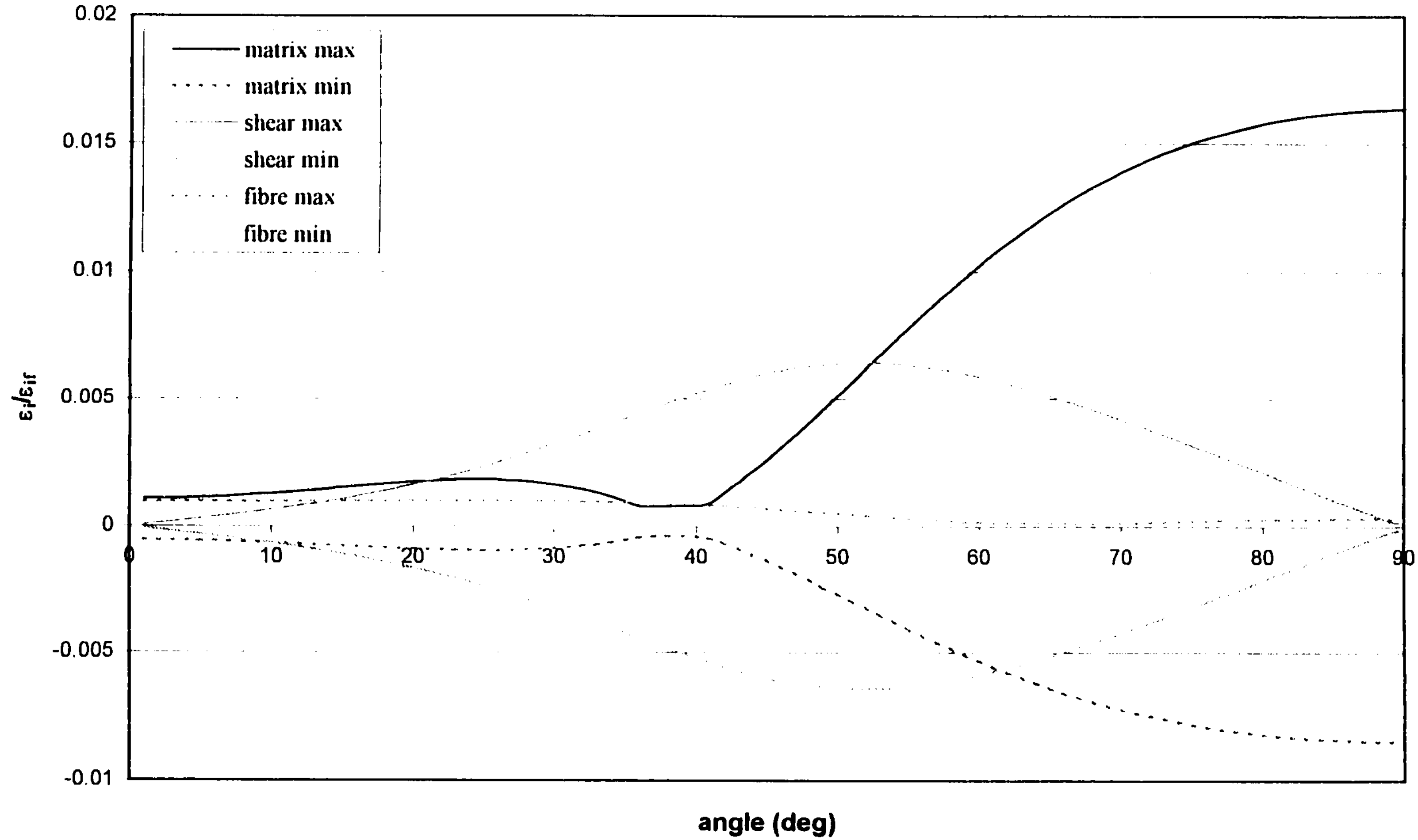

Figure 4.19: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for a moment loading with a maximum axial stress of σ_{Amax} = 1MPa

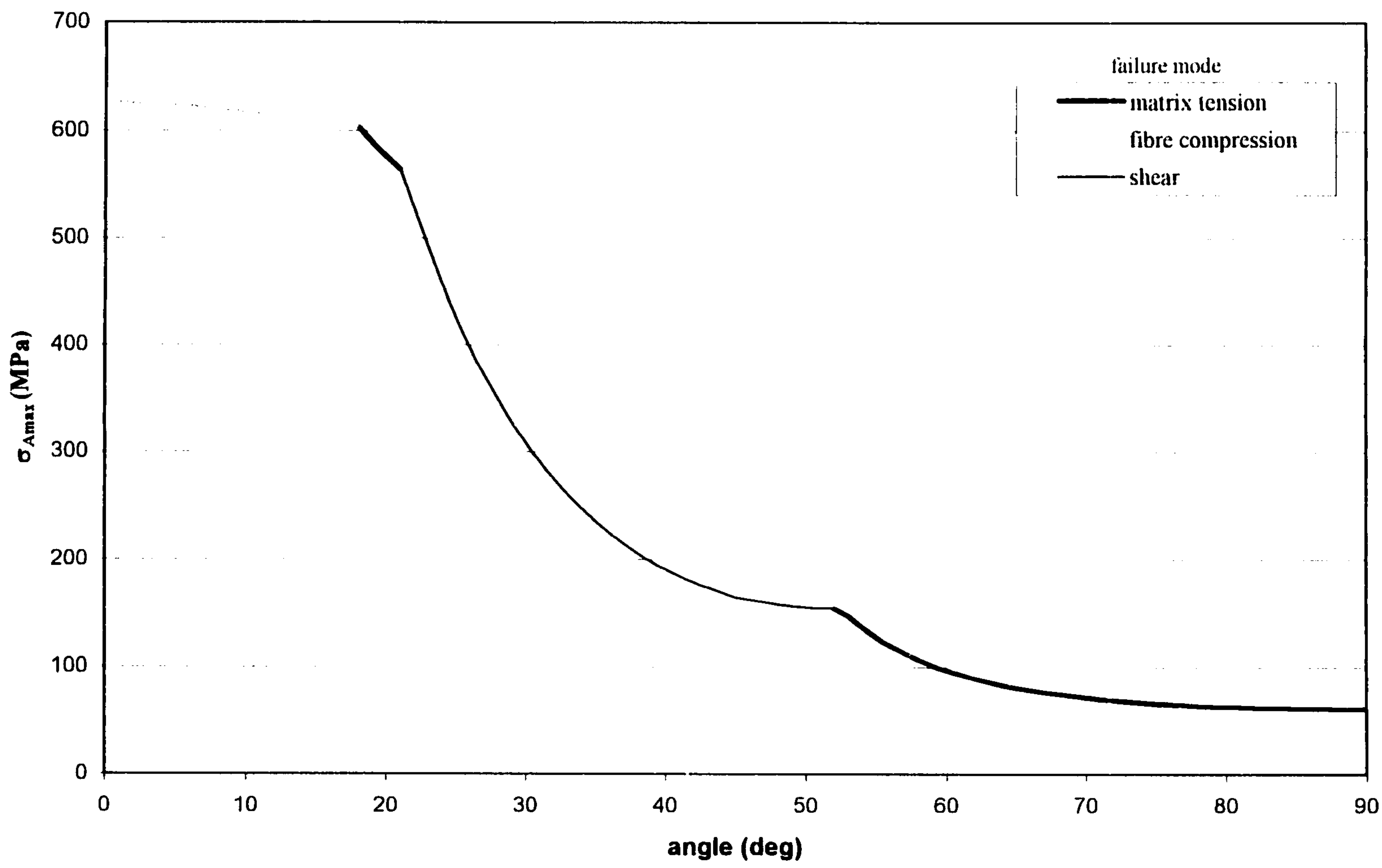

Figure 4.20: σ_{Amax} **at failure (**$\varepsilon_{lmax}/\varepsilon_{lf} = 1$**) for fibre angles of 0 to 90 degrees, for a moment loading**

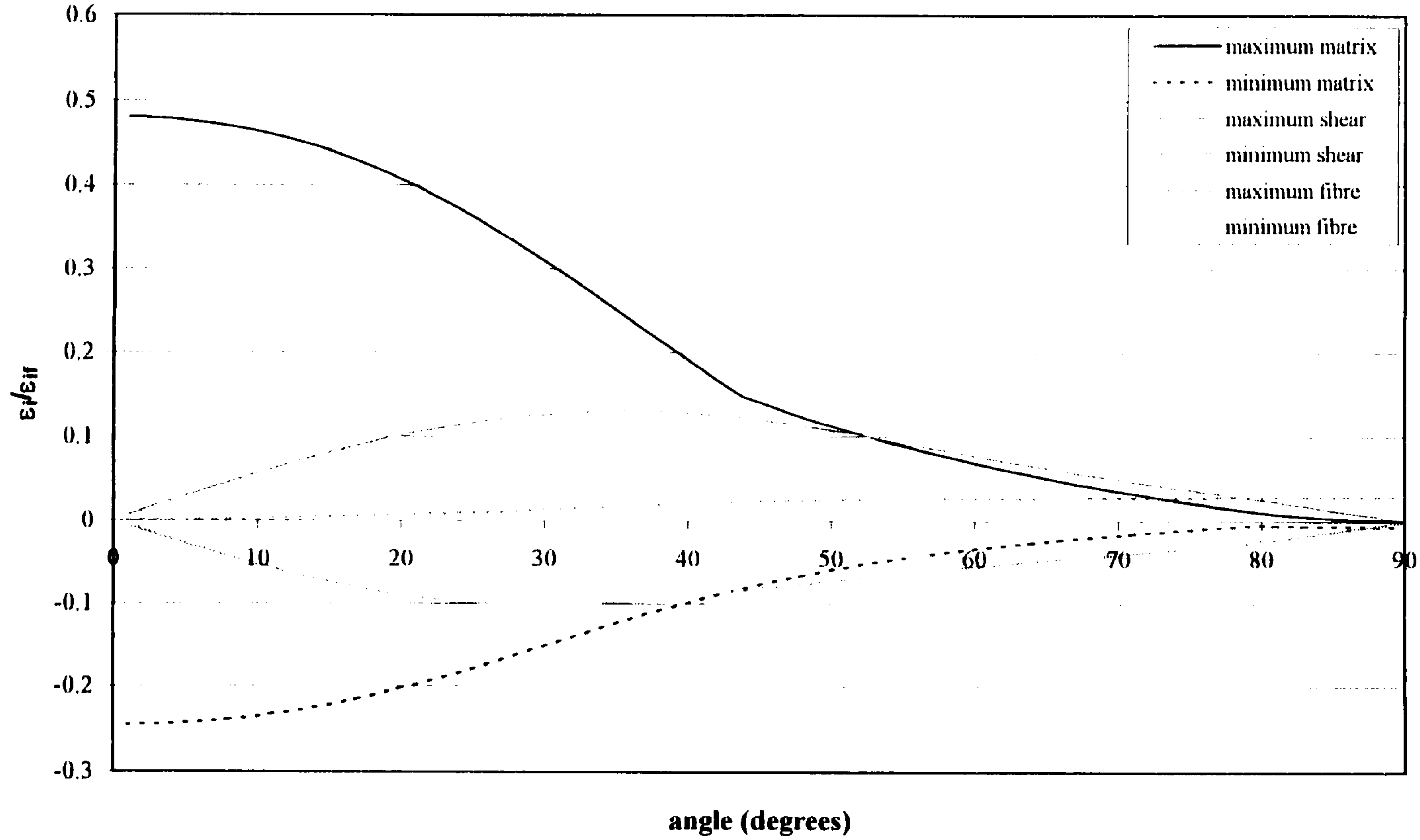

Figure 4.21: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for a distributed lateral line load normalized by the diameter (w_d/D) for a value of 1(kN/m)/m and a t/D of 0.01

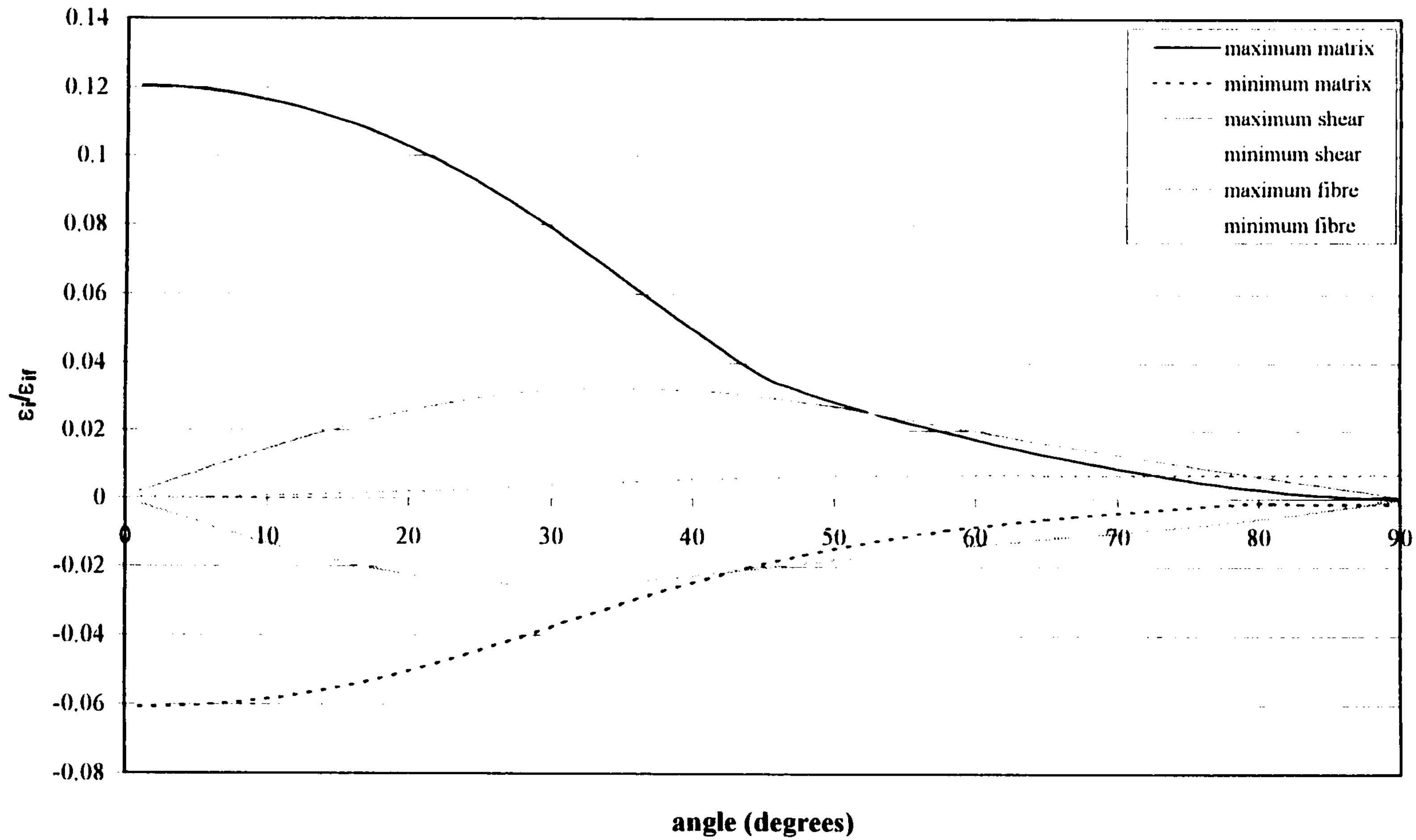

Figure 4.22: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for a distributed lateral line load normalized by the diameter (w_d/D) for a value of 1(kN/m)/m and a t/D of 0.02

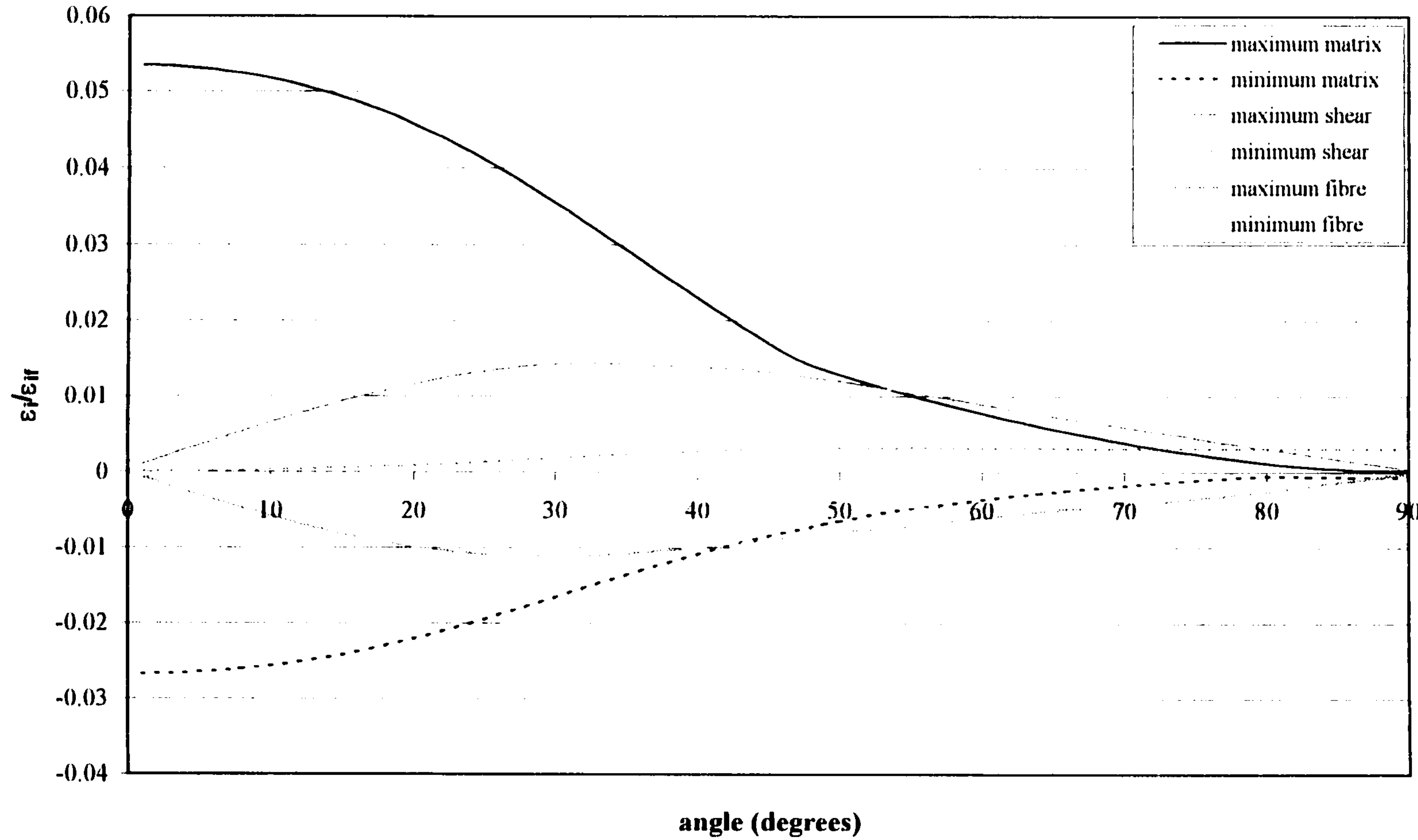

Figure 4.23: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for a distributed lateral line load normalized by the diameter (w_d/D) for a value of 1(kN/m)/m and a t/D of 0.03

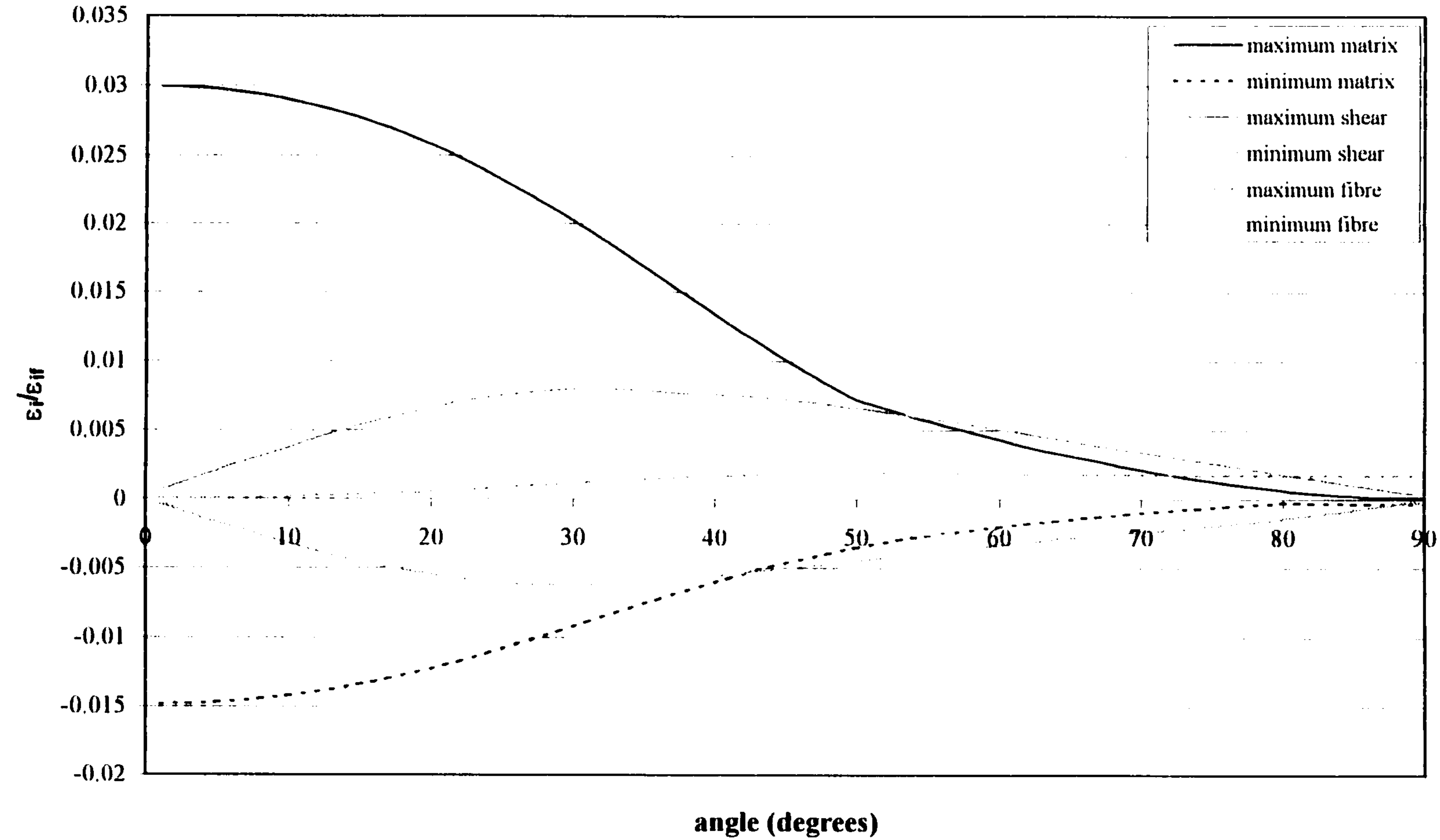

Figure 4.24: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for a distributed lateral line load normalized by the diameter (w_d/D) for a value of 1(kN/m)/m and a t/D of 0.04

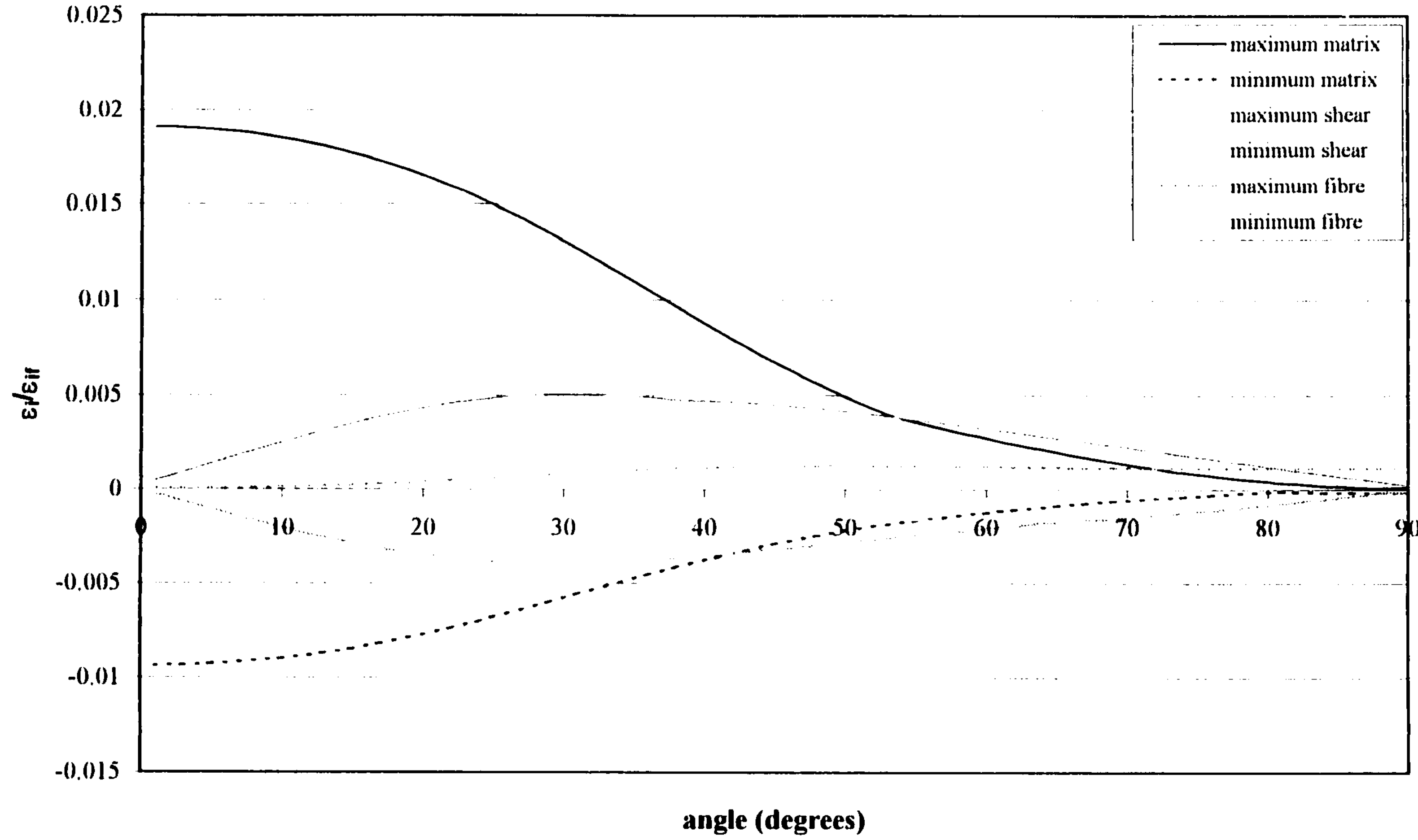

Figure 4.25: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for a distributed lateral line load normalized by the diameter (w_d/D) for a value of 1(kN/m)/m and a t/D of 0.05

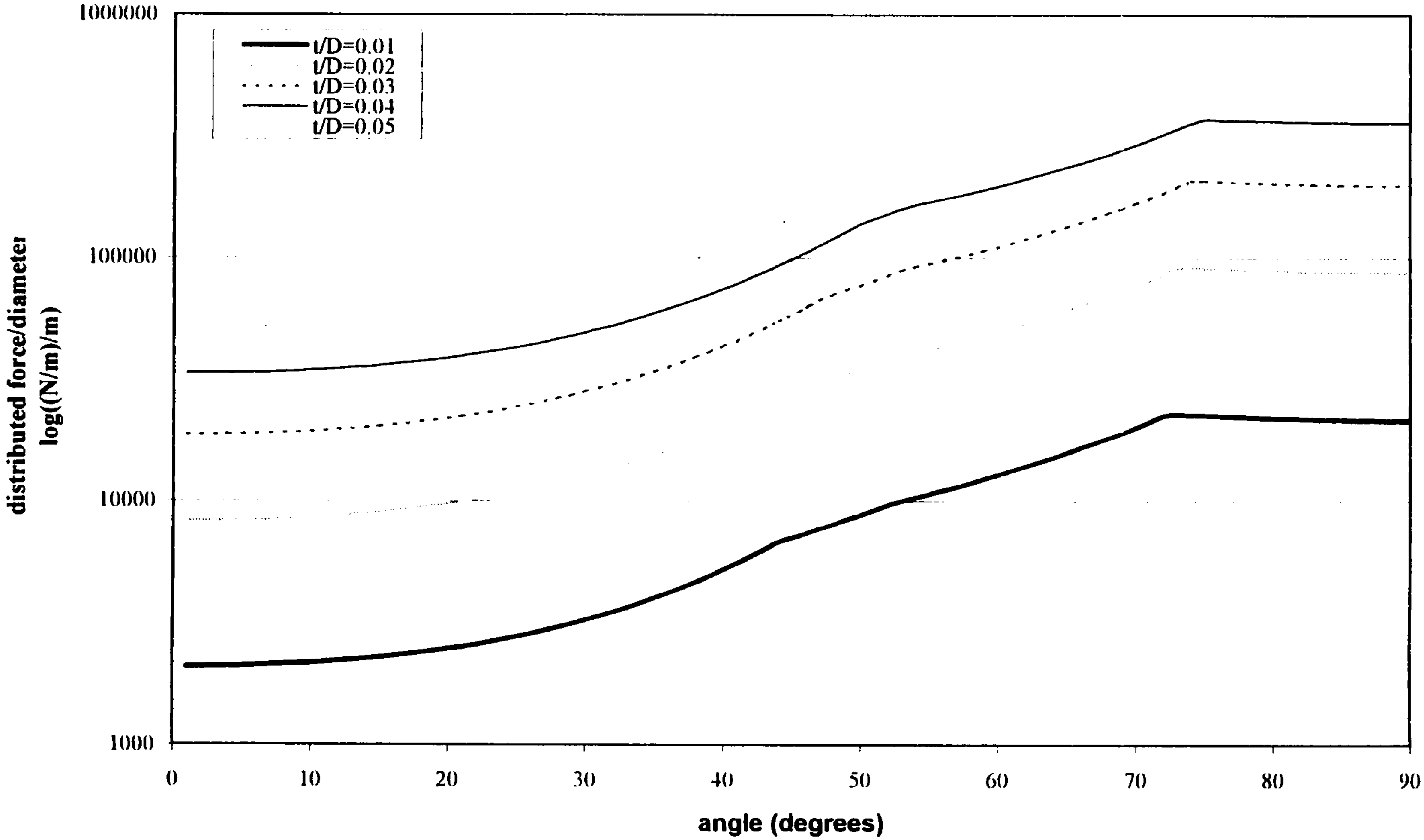

Figure 4.26: Distributed lateral line load at failure ($\varepsilon_{lmax}/\varepsilon_{lf} = 1$), normalized by the diameter, for fibre angles of to 90 degrees for $0.01 \leq t/D \leq 0.05$

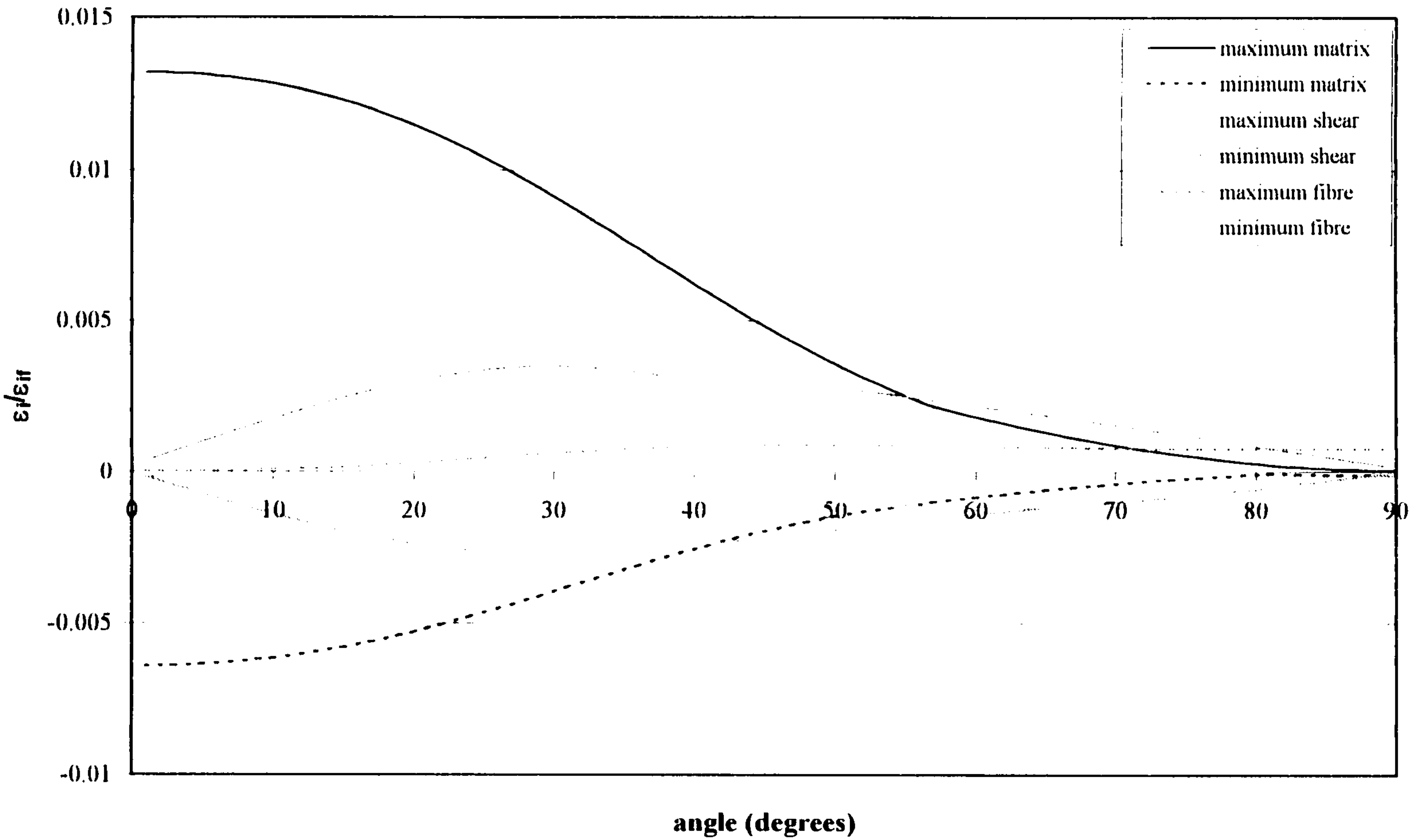

Figure 4.27: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for a distributed lateral line load normalized by the diameter (w_d/D) for a value of 1(kN/m)/m and a t/D of 0.06

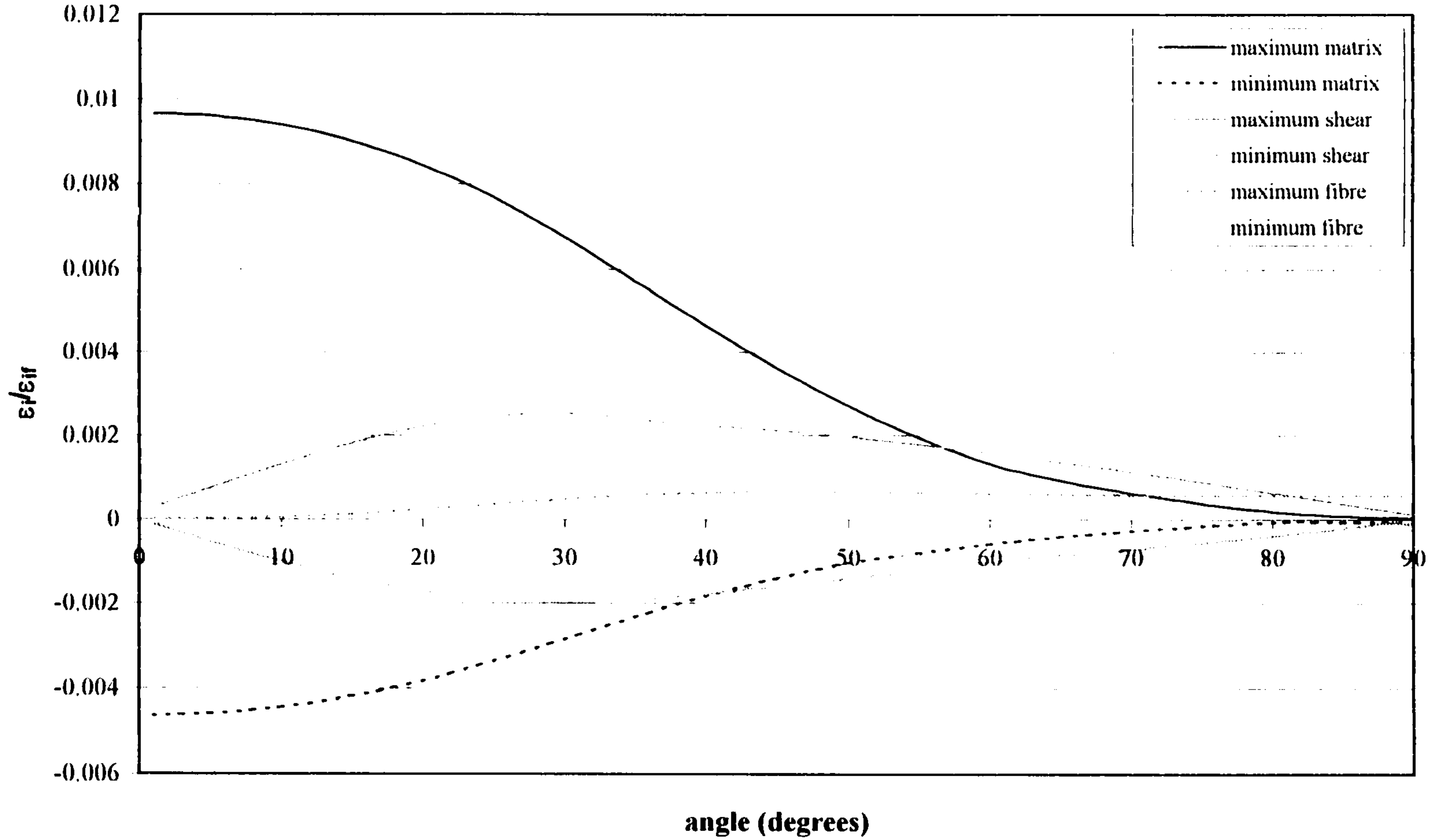

Figure 4.28: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for a distributed lateral line load normalized by the diameter (w_d/D) for a value of 1(kN/m)/m and a t/D of 0.07

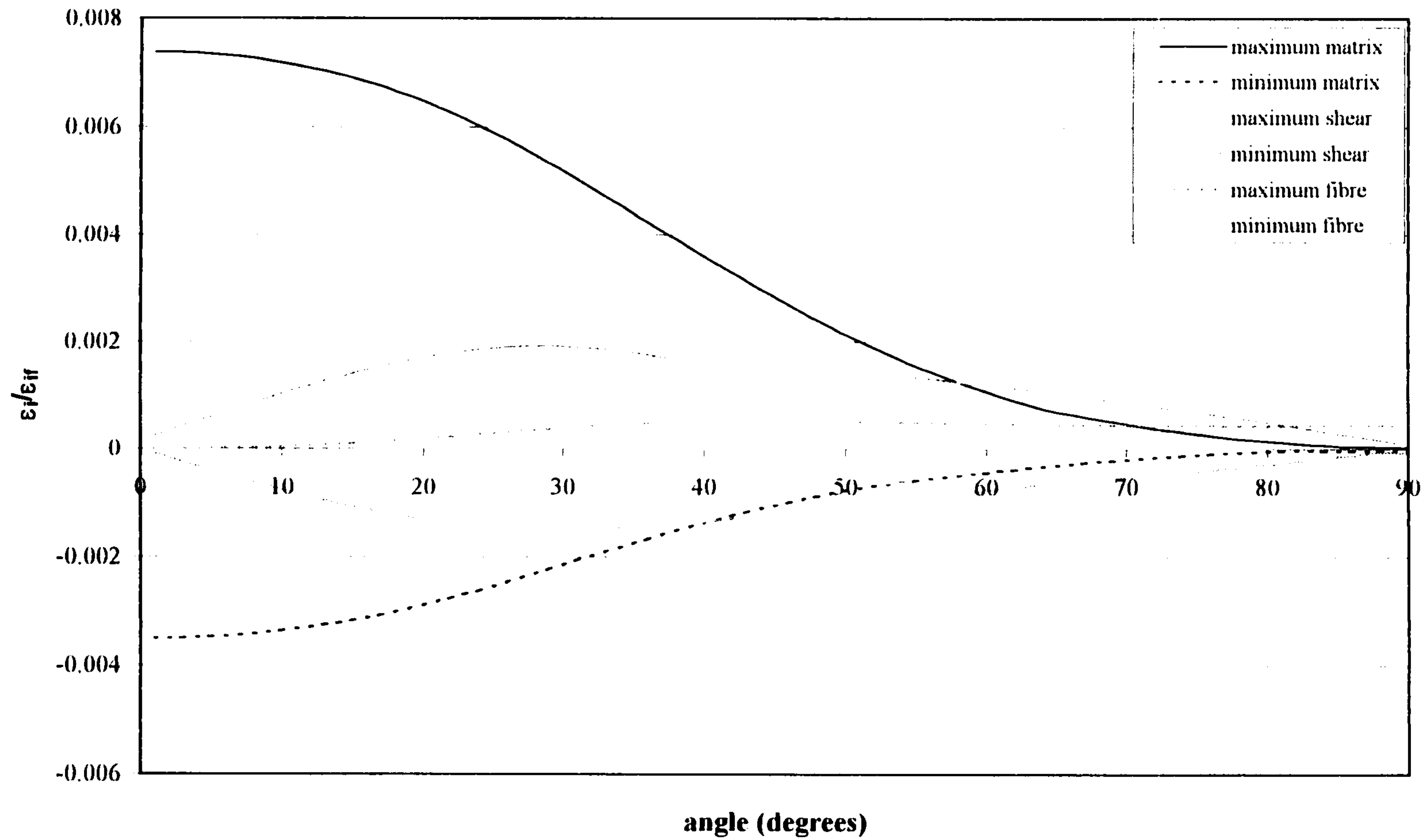

Figure 4.29: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for a distributed lateral line load normalized by the diameter (w_d/D) for a value of 1(kN/m)/m and a t/D of 0.08

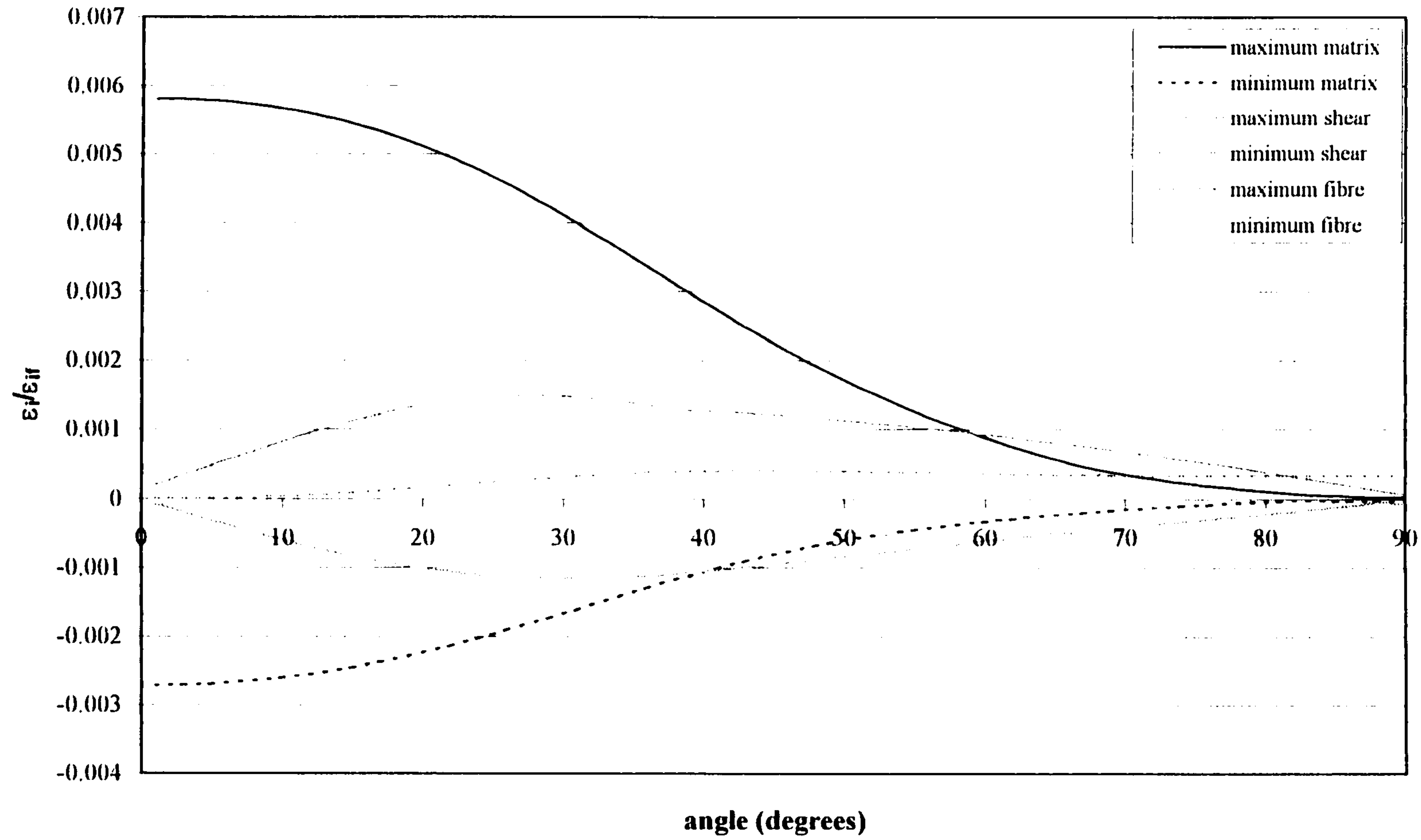

Figure 4.30: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for a distributed lateral line load normalized by the diameter (w_d/D) for a value of 1(kN/m)/m and a t/D of 0.09

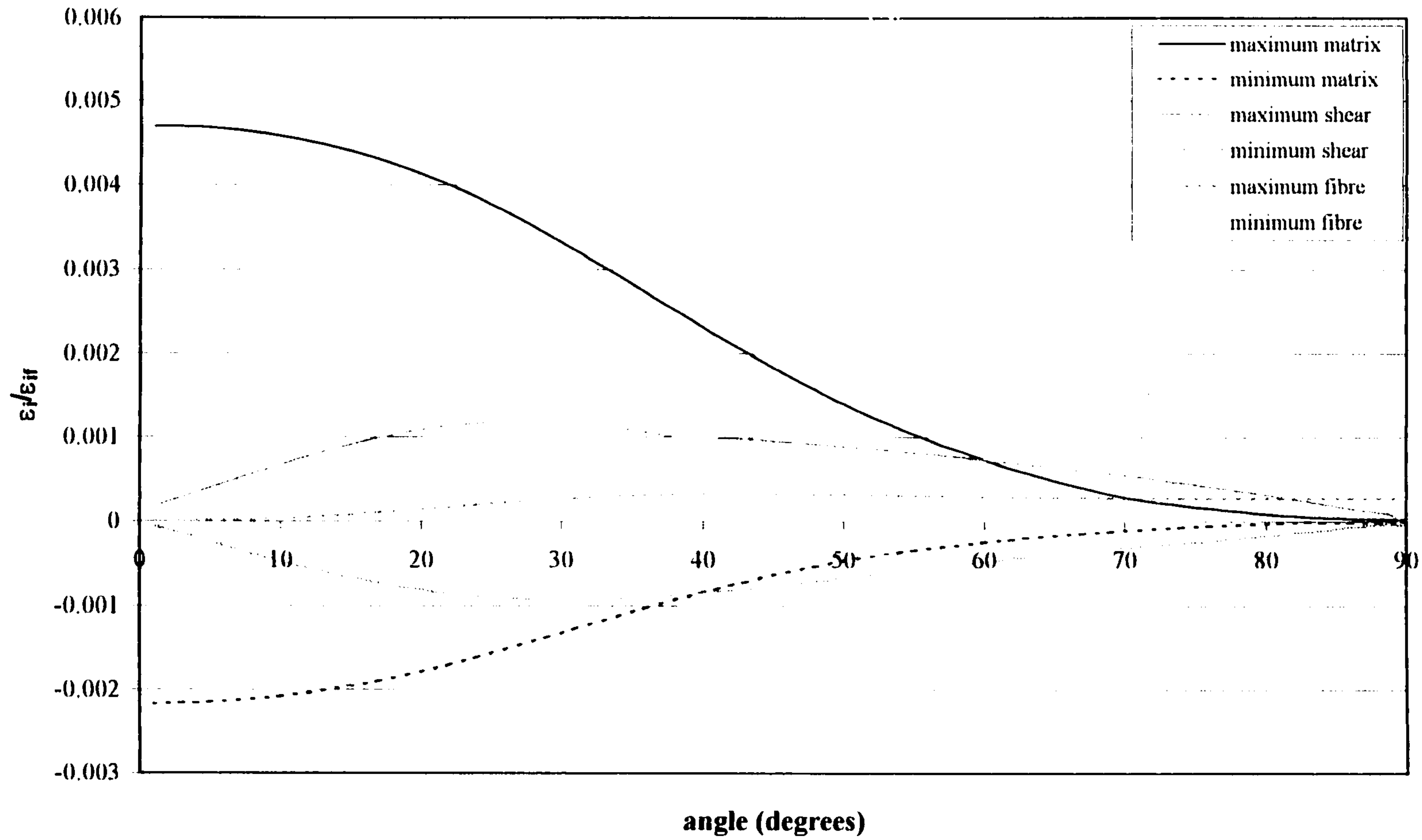

Figure 4.31: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for a distributed lateral line load normalized by the diameter (w_d/D) for a value of 1(kN/m)/m and a t/D of 0.10

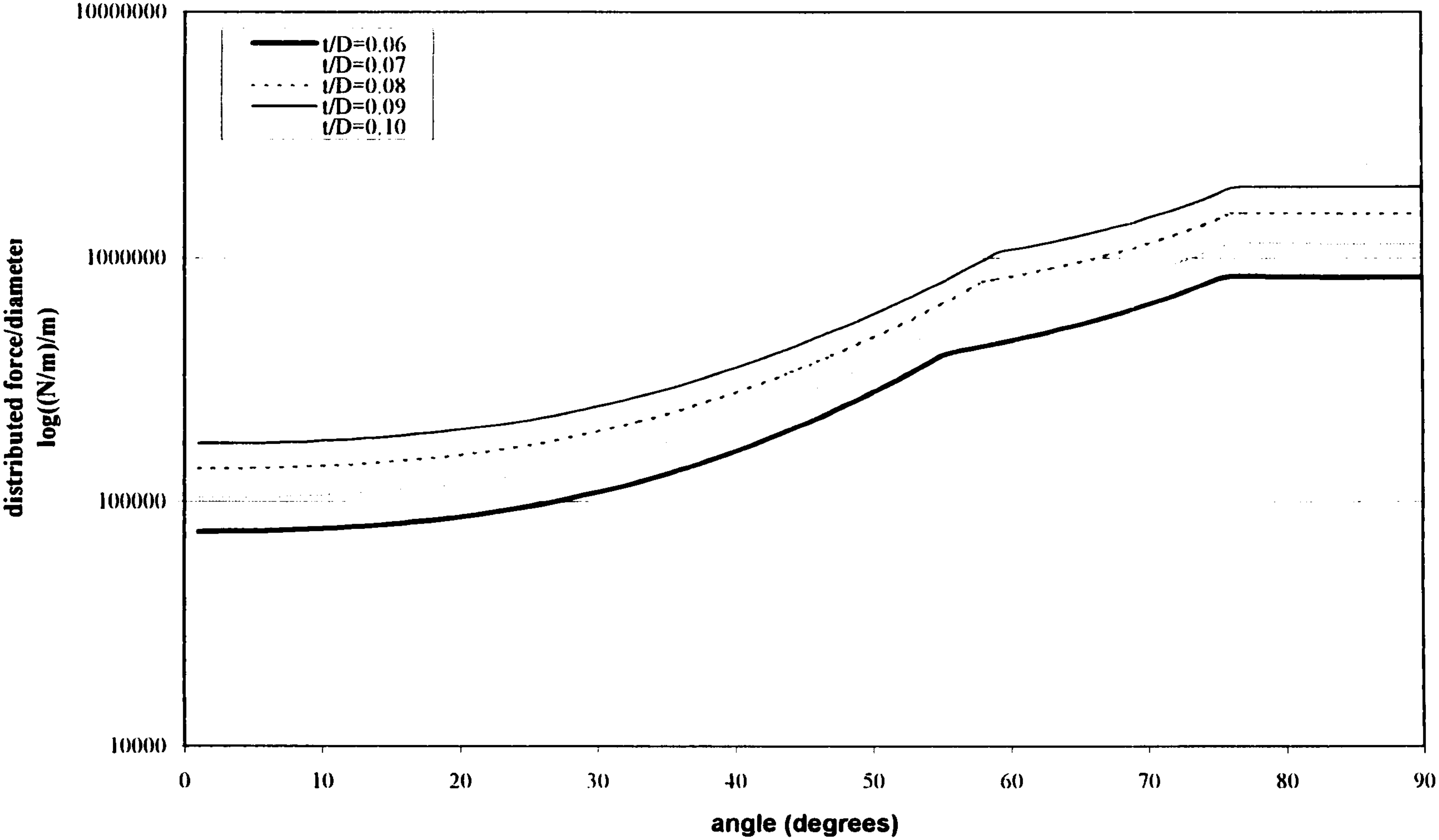

Figure 4.32: Distributed lateral line load at failure ($\varepsilon_{imax}/\varepsilon_{if} = 1$), normalized by the diameter, for fibre angles of to 90 degrees for $0.01 \leq t/D \leq 0.05$

Chapter 5
FRP Pipeline Design Study Based on FE Results

5.1 Overview

One of the challenges of composite pipeline design is to design a material which utilizes the strengths of the constituents and provides an acceptable failure mode, while maintaining structural integrity under prescribed loading conditions with appropriate safety margins.

The purpose of this chapter is to demonstrate how the strain ratios and numerical values from the parametric study conducted in this thesis can be used to determine the acceptable fibre angles for a high-pressure pipeline under the prescribed pipeline loading conditions. Three different scenarios and loading situations are explored.

Since this design study deals with the elastic region of the stress-strain curve, load combinations can simply be obtained by superposition, with the failure of the pipe still defined as a maximum strain ratio equal to one. A worst case scenario was assumed for all load combinations, assuming that all of the maximum strain ratios occurring were cumulative, or occurring at the same location. This addition of maxima to maxima and minima to minima causes some of the strains to be exaggerated, leading to a conservative design.

For the design studies reported herein, the range of fibre angles under consideration was restricted to 50 to 70 degrees. The lower limit angle of 50 degrees was selected since the loadings on the pipes in the three studies are of primarily hoop and not axially dominated. The high end angle of 70 degrees was selected since for fibre angles greater than 70° there is very little strength gain in the hoop direction. This is demonstrated in Figure 4.2 of Chapter 4.

The design studies are based upon the limit states design methodology as discussed previously in this thesis. The design studies include load factors for the applied pressures, bending moments, and forces. A resistance factor is also utilized, however, it is applied in an unconventional manner. With the use of the maximum strain failure criteria, failure is defined as a strain ratio equal to unity. Incorporation of the resistance factor is achieved by limiting the maximum allowable strain ratio to the resistance factor. The class factor was assumed to be 1.0, which would have no effect on any of the calculations, therefore it was omitted.

For the design studies that follow, the hoop stress in the pipe is used as a loading parameter, rather than the internal pressure. This allows for easier scaling of the strain ratio results from Chapter 4.

5.2 Summary of the Loading Conditions for the Design Studies

Three design studies are presented, demonstrating various aspects of pipeline design using FRP composite materials. The pressure loadings were primarily hoop stress dominated, as is expected for horizontal run pipelines. Design study No. 1 is based on an internal pressure of 15 MPa, an internal diameter of 0.5m and a ditch depth of 2.5m.

The second design study (No. 2) has an internal pressure of 30 MPa and a ditch depth of 3.5m, in addition it also incorporates a consideration for pipeline buoyancy. The third study (No. 3) is based on an internal pressure of 11 MPa, a ditch depth of 2m, and an internal diameter of 1.0m. Design example No. 3 also includes a consideration for the maximum allowable bending moment under pressure and backfill loading conditions. The third study also demonstrates the effects of changing only the thickness and the effect which it has on the range of allowable fibre wind angles. Table 5.1 gives a summary of the above three examples.

Table 5.1: Summary of the loading conditions for the design studies

	case 1	case 2	case 3
pressure (MPa)	15	30	11
ID (m)	0.5	0.5	1.0
installation parameters			
lift (m)	1	1	1
ditch depth (m)	2.5	3.5	2
comments		buoyancy condition	minimum allowable moment restriction
loadings (common to all)	(1H:0A) (8H:1A) (4H:1A) (2H:1A) installation backfill combinations of the biaxial loads and backfill calculation of maximum allowable bending moment above biaxial ratios and backfill loading		
factors: [27]			
resistance factor	0.8	0.8	0.8
load factor for pressure	1.25	1.25	1.25
load factor for backfill	1.25	1.25	1.25
load factor for bending moment	1.4	1.4	1.4

5.3 Background on Limit States Design

Limit States Design was incorporated into the three design studies. The Limit States Design methodology is based on incorporating reliability concepts into practical design using separate safety factors, determined by probabilistic methods, for the load and resistance parameters involved in design. The basic design equation is given by:

$$\phi R \geq \sum_{i=1}^{n} \alpha_i L_i \tag{5.1}$$

The factored loads were calculated by multiplying a specified load, L_i, by its load factor, α_i, with unique load factors as required by the probability of each load. For the design studies that follow, the averaged hoop stress through the cross-section is the loading condition considered, so the hoop stress is factored, instead of the internal pressure.

Traditionally, the factored resistance would be calculated by multiplying a specified resistance value, R, by a resistance factor, φ. In the case of this thesis, the resistance factor, 0.8, is multiplied by the strain ratio occurring at failure, unity. Thus when the factored loads are combined by summing the strain ratios from each load, the resultant strain ratios cannot exceed the factored resistance, 0.8, in order to satisfy the Limit States Design criteria.

5.4 Design Study 1

The purpose of this design study is to find the feasible fibre angles for a composite pipe, based on a set of given loading conditions summarized below.

Given:

- Internal pressure = 15 MPa (2.18 ksi)
- Possible hoop to axial stress ratio loadings:
  (1H:0A), (8H:1A), (4H:1A), (2H:1A)
- Factors:
  Resistance factor = 0.8
  Load factor for pressure = 1.25
  Load factor for backfill = 1.25
  Load factor for moment = 1.4
- ID = 0.5m
- Installation Parameters:
  lift = 1m
  ditch depth = 2.5m

Solution:

For a pressure of 15 MPa, assume t/D = 0.06

therefore:

$$OD = \frac{ID}{\left(1 - 2 \cdot \frac{t}{D}\right)} = 0.56\text{m}$$

and

$$I = \frac{\pi}{4} \cdot \left(\left(\frac{OD}{2} \right)^4 - \left(\frac{ID}{2} \right)^4 \right) = 1.759\text{x}10^{-3}\,\text{m}^4$$

to calculate the factored hoop stress:

$$\sigma_H = \frac{\text{load factor} \cdot \text{pressure}}{2 \cdot \frac{t}{D}} = \frac{1.25 \cdot 15}{2 \cdot 0.06} = 156.25\text{MPa}$$

5.4.1 Biaxial Pressure Loading of (1H:0A)

The strain results obtained from the finite element model were all for an applied hoop stress of 1MPa. Since this is an elastic analysis, a scaling factor can be used to determine the strain ratios for the given factored hoop stress of 156.25 MPa. Table C.1, in Appendix C, contains the numerical values of the strain ratios for fibre angles of 50° to 70° obtained from a (1H:0A) loading with a hoop stress of 1MPa. Figure 4.1 in Chapter 4 is the graphical representation of the values in Table C.1 in Appendix C.

From Table C.1, a scaling factor can be used to scale the strain ratios to the factored hoop stress of 156.25 MPa:

$$\text{scaling factor} = \frac{\text{factored hoop stress}}{\text{applied hoop stress}} = \frac{156.25\text{MPa}}{1\text{MPa}} = 156.25$$

So, for a fibre angle of 50 degrees the strain ratios are scaled as demonstrated in Table 5.2.

Table 5.2: Strain ratio scaling sample calculation for a hoop stress of 156.26 MPa, in a (1H:0A) loading for a fibre angle of 50 degrees

	hoop stress	
strain ratio	1MPa	156.25MPa
maximum matrix	427.3×10^{-6}	0.06680
minimum matrix	398.3×10^{-6}	0.06220
maximum shear	5.010×10^{-3}	0.7832
minimum shear	-4.110×10^{-3}	-0.6426
maximum fibre	814.6×10^{-6}	0.1273
minimum fibre	809.7×10^{-6}	0.1265

Applying the scaling factor to all of the strain ratios for fibre angles of 50° to 70° produces Table 5.3.

Table 5.3: Scaled strain ratios for (1H:0A) with factored hoop stress of 156.25 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.0668	0.0622	0.7832	-0.6426	0.1273	0.1265	0.7832	shear
51	0.0165	0.0118	0.7551	-0.6247	0.1310	0.1302	0.7551	shear
52	-0.0168	-0.0191	0.7259	-0.6060	0.1343	0.1335	0.7259	shear
53	-0.0368	-0.0395	0.6959	-0.5867	0.1374	0.1366	0.6959	shear
54	-0.0560	-0.0589	0.6654	-0.5669	0.1401	0.1393	0.6654	shear
55	-0.0731	-0.0760	0.6346	-0.5468	0.1425	0.1417	0.6346	shear
56	-0.0880	-0.0910	0.6037	-0.5265	0.1446	0.1439	0.6037	shear
57	-0.1009	-0.1040	0.5730	-0.5062	0.1465	0.1457	0.5730	shear
58	-0.1117	-0.1150	0.5426	-0.4858	0.1480	0.1473	0.5426	shear
59	-0.1207	-0.1241	0.5126	-0.4656	0.1493	0.1486	0.5126	shear
60	-0.1280	-0.1315	0.4832	-0.4456	0.1504	0.1497	0.0005	shear
61	-0.1337	-0.1373	0.4546	-0.4259	0.1512	0.1506	0.4546	shear
62	-0.1378	-0.1415	0.4268	-0.4066	0.1518	0.1512	0.4268	shear
63	-0.1406	-0.1444	0.3999	-0.3876	0.1523	0.1517	0.3999	shear
64	-0.1423	-0.1461	0.3739	-0.3690	0.1525	0.1520	0.3739	shear
65	-0.1428	-0.1467	0.3490	-0.3509	0.1527	0.1521	0.3490	shear
66	-0.1424	-0.1463	0.3251	-0.3332	0.1527	0.1521	0.3251	shear
67	-0.1412	-0.1451	0.3023	-0.3160	0.1525	0.1520	0.3023	shear
68	-0.1393	-0.1432	0.2806	-0.2993	0.1523	0.1518	0.2806	shear
69	-0.1369	-0.1407	0.2599	-0.2830	0.1520	0.1516	0.2599	shear
70	-0.1339	-0.1377	0.2403	-0.2671	0.1517	0.1512	0.2403	shear

From the pure pressure loading case (1H:0A), all of the maximum strain ratios are less than the resistance factor of 0.8. Thus any fibre angle from 50° to 70° is acceptable.

5.4.2 Biaxial Pressure Loading of (8H:1A)

As with the (1H:0A) loading case, the (8H:1A) loading case can also be scaled. Table C.2, from Appendix C, contains the strain ratios obtained from a (8H:1A) loading with a hoop stress of 1 MPa.

The strain ratios in Table C.2 are for an applied hoop stress of 1MPa, therefore:

$$\text{scaling factor} = \frac{156.25\text{MPa}}{1\text{MPa}} = 156.25$$

for a fibre angle of 50 degrees, scaling the strain ratios from Table C.2, Table 5.4 is obtained.

Table 5.4: Strain ratio scaling sample calculation for a hoop stress of 156.26 MPa, in a (8H:1A) loading for a fibre angle of 50 degrees

	hoop stress	
strain ratio	1MPa	156.25MPa
maximum matrix	1.060×10^{-3}	0.1660
minimum matrix	-1.030×10^{-3}	-0.1608
maximum shear	4.180×10^{-3}	0.6535
minimum shear	4.160×10^{-3}	0.6498
maximum fibre	879.3×10^{-6}	0.1374
minimum fibre	873.6×10^{-6}	0.1365

Table 5.5: Scaled Strain Ratios for (8H:1A) with Factored Hoop Stress of 156.25 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.1660	0.1608	0.6535	0.6498	0.1374	0.1365	0.6535	shear
51	0.1266	0.1212	0.6248	0.6211	0.1403	0.1394	0.6248	shear
52	0.0916	0.0860	0.5955	0.5918	0.1428	0.1419	0.5955	shear
53	0.0636	0.0584	0.5658	0.5621	0.1451	0.1442	0.5658	shear
54	0.0403	0.0350	0.5360	0.5322	0.1470	0.1461	0.5360	shear
55	0.0208	0.0154	0.5062	0.5025	0.1487	0.1478	0.5062	shear
56	-0.0032	-0.0064	0.4767	0.4730	0.1500	0.1492	0.4767	shear
57	-0.0108	-0.0140	0.4476	0.4439	0.1511	0.1503	0.4476	shear
58	-0.0165	-0.0198	0.4191	0.4155	0.1519	0.1511	0.4191	shear
59	-0.0205	-0.0238	0.3914	0.3879	0.1525	0.1518	0.3914	shear
60	-0.0230	-0.0264	0.3645	0.3611	0.1529	0.1522	0.3645	shear
61	-0.0240	-0.0275	0.3387	0.3353	0.1531	0.1524	0.3387	shear
62	-0.0237	-0.0273	0.3138	0.3106	0.1531	0.1524	0.3138	shear
63	-0.0223	-0.0260	0.2901	0.2869	0.1529	0.1523	0.2901	shear
64	-0.0199	-0.0236	0.2675	0.2645	0.1526	0.1520	0.2675	shear
65	-0.0166	-0.0204	0.2460	0.2432	0.1522	0.1516	0.2460	shear
66	-0.0126	-0.0164	0.2258	0.2230	0.1517	0.1512	0.2258	shear
67	-0.0080	-0.0117	0.2067	0.2040	0.1511	0.1506	0.2067	shear
68	0.0177	-0.0066	0.1888	0.1861	0.1504	0.1500	0.1888	shear
69	0.0283	0.0211	0.1720	0.1694	0.1497	0.1493	0.1720	shear
70	0.0393	0.0322	0.1564	0.1538	0.1490	0.1485	0.1564	shear

It is seen from Table 5.5 that all strain ratios are less than the resistance factor, 0.8, therefore, there are no wind angle restrictions from the (8H:1A) loading condition.

5.3.3 Biaxial Pressure Loading of (4H:1A)

As with the other load ratios, the (4H:1A) loading can also be scaled. Table C.3 is used as a basis for this operation, and Table 5.6 is obtained by the scaling process.

Table 5.6: Scaled Strain Ratios for (4H:1A) with Factored Hoop Stress of 156.25 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.2652	0.2594	0.5238	0.5203	0.1475	0.1465	0.5238	shear
51	0.2367	0.2307	0.4946	0.4911	0.1496	0.1486	0.4946	shear
52	0.2139	0.2083	0.4652	0.4616	0.1513	0.1503	0.4652	shear
53	0.1972	0.1915	0.4358	0.4322	0.1528	0.1518	0.4358	shear
54	0.1844	0.1786	0.4066	0.4030	0.1539	0.1530	0.4066	shear
55	0.1752	0.1693	0.3778	0.3743	0.1548	0.1538	0.3778	shear
56	0.1695	0.1635	0.3496	0.3461	0.1554	0.1545	0.3496	shear
57	0.1670	0.1609	0.3222	0.3188	0.1557	0.1548	0.3222	shear
58	0.1674	0.1614	0.2957	0.2923	0.1558	0.1550	0.2957	shear
59	0.1708	0.1646	0.2702	0.2669	0.1557	0.1549	0.2702	shear
60	0.1766	0.1702	0.2459	0.2426	0.1554	0.1546	0.2459	shear
61	0.1847	0.1781	0.2227	0.2196	0.1550	0.1542	0.2227	shear
62	0.1946	0.1879	0.2008	0.1978	0.1543	0.1536	0.2008	shear
63	0.2062	0.1994	0.1803	0.1773	0.1536	0.1529	0.2062	matrix tension
64	0.2191	0.2123	0.1610	0.1581	0.1527	0.1521	0.2191	matrix tension
65	0.2333	0.2263	0.1430	0.1401	0.1518	0.1511	0.2333	matrix tension
66	0.2483	0.2413	0.1264	0.1236	0.1508	0.1502	0.2483	matrix tension
67	0.2640	0.2570	0.1111	0.1083	0.1497	0.1491	0.2640	matrix tension
68	0.2801	0.2731	0.0970	0.0943	0.1486	0.1481	0.2801	matrix tension
69	0.2966	0.2896	0.0842	0.0815	0.1474	0.1470	0.2966	matrix tension
70	0.3131	0.3063	0.0725	0.0700	0.1463	0.1458	0.3131	matrix tension

It is observed from Table 5.6 that the (4H:1A) loading condition puts no restriction on the wind angle.

5.4.4 Biaxial Pressure Loading of (2H:1A)

For the biaxial pressure loading of (2H:1A), Table C.4 is used as a basis, and is then scaled to determine the strain ratios in Table 5.7, for an applied hoop stress of 156.25MPa.

Table 5.7: Scaled Strain Ratios for (2H:1A) with Factored Hoop Stress of 156.25 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.4636	0.4567	0.2643	0.2611	0.1677	0.1665	0.4636	matrix tension
51	0.4597	0.4529	0.2342	0.2309	0.1682	0.1670	0.4597	matrix tension
52	0.4602	0.4533	0.2045	0.2013	0.1684	0.1671	0.4602	matrix tension
53	0.4645	0.4576	0.1757	0.1725	0.1682	0.1670	0.4645	matrix tension
54	0.4726	0.4656	0.1478	0.1446	0.1678	0.1666	0.4726	matrix tension
55	0.4840	0.4771	0.1210	0.1179	0.1671	0.1659	0.4840	matrix tension
56	0.4985	0.4916	0.0955	0.0924	0.1662	0.1650	0.4985	matrix tension
57	0.5159	0.5090	0.0714	0.0684	0.1650	0.1639	0.5159	matrix tension
58	0.5357	0.5289	0.0488	0.0458	0.1637	0.1626	0.5357	matrix tension
59	0.5578	0.5510	0.0278	0.0248	0.1621	0.1611	0.5578	matrix tension
60	0.5816	0.5749	-0.0307	-0.0307	0.1605	0.1595	0.5816	matrix tension
61	0.6070	0.6004	-0.0467	-0.0467	0.1587	0.1578	0.6070	matrix tension
62	0.6336	0.6272	-0.0612	-0.0612	0.1569	0.1560	0.6336	matrix tension
63	0.6612	0.6548	-0.0740	-0.0740	0.1549	0.1541	0.6612	matrix tension
64	0.6896	0.6832	-0.0851	-0.0851	0.1529	0.1522	0.6896	matrix tension
65	0.7185	0.7120	-0.0947	-0.0947	0.1509	0.1502	0.7185	matrix tension
66	0.7475	0.7409	-0.1028	-0.1028	0.1489	0.1482	0.7475	matrix tension
67	0.7764	0.7698	-0.1094	-0.1094	0.1469	0.1462	0.7764	matrix tension
68	0.8050	0.7985	-0.1145	-0.1145	0.1449	0.1443	0.8050	matrix tension
69	0.8332	0.8267	-0.1183	-0.1183	0.1429	0.1424	0.8332	matrix tension
70	0.8608	0.8544	-0.1207	-0.1207	0.1410	0.1405	0.8608	matrix tension

It is seen from Table 5.7, that the strain ratio is greater than the resistance factor of 0.8 for fibre angles higher than 67°, in an applied stress loading of (2H:1A). Thus the (2H:1A) loading condition restrains the range of possible fibre angles from 50° to 67°.

5.4.5 Pipeline Installation

It is possible that the loads applied on the pipeline during installation may be the maximum stresses or strains ever to be sustained by the pipeline. In order to determine if this is the case, the strains from the installation loading must be calculated, based upon the installation parameters specified.

For a lift of 1m, and a ditch depth of 2.5m, assuming that the pipeline is supported 0.5m above ground level gives:

lift = 1m
effective depth (delta) = 0.5m + 2.5m = 3m

where delta is the distance from the top of the above ground pipe supports to the bottom of the ditch (Figure 2.2 of Chapter 2).

The load coefficient for a lift of 1 and delta of 3 can be found in Table A.2 of Appendix A.

load coef. (c_2)= 13.84

The installation loading is equivalent to an applied bending moment, with the maximum axial stress being dependant upon the Young's modulus in the axial direction of the pipe, and therefore, the loading is dependant upon the fibre wind angle.

The installation equation is given by equation 2.4:

$$\sigma_{max} = c_2 \cdot (E \cdot \rho)^{\frac{1}{2}} \cdot \left(1 + \left(1 - 2\frac{t}{D}\right)^2\right)^{-\frac{1}{2}}$$

For a fibre angle of 50°, with ρ = 1510 kg/m^3, t/D = 0.06 and

E_{axial} = 11.02GPa (from Table C.10)

$$\sigma_{max} = 13.83 \cdot \left(11.02x10^3 \cdot 1510\right)^{\frac{1}{2}} \cdot \left(1 + \left(1 - 2(.06)\right)^2\right)^{-\frac{1}{2}}$$

$$\sigma_{max} = 42.36\text{MPa}$$

Since the composite material is anisotropic, the Young's modulus for the hoop and axial directions of the pipe differ. The stress caused by the installation acts in the axial direction of the pipe, the Young's modulus in the axial direction of the pipe, as determined by the Classical Laminate Theory, was used. Table C.10, in Appendix C, includes the axial Young's modulus for fibre angles of 50 to 70°.

Young's modulus in the axial direction of the pipe is dependant upon the fibre wind angle. Since the installation loading is dependant upon the axial Young's modulus, the moment loading, and therefore the scaling factor, is also dependant upon the fibre wind angle.

Using a 50 degree fibre angle,

$$\text{scaling factor} = \frac{\text{moment load factor} \cdot \text{max axial stress}}{\text{applied max axial stress}} = \frac{1.4x42.36x10^6}{1x10^6} = 59.30$$

For a fibre angle of 50 degrees, scaling the strain ratios for a moment loading (Table C.5), one gets the values summarized in Table 5.8.

Table 5.8: Strain ratio scaling sample calculation for an installation load with a lift of 1m and delta of 3m for a fibre angle of 50 degrees

	maximum axial stress	
strain ratio	1MPa	59.30MPa
maximum matrix	0.0051512	0.3055
minimum matrix	-0.002666	-0.1581
maximum shear	0.006417	0.3805
minimum shear	$-6.38x10^{-3}$	-0.3786
maximum fibre	0.0005111	0.0303
minimum fibre	-0.0008422	-0.0499

Due to the uncertainty involved in the use of the equation for installation loading, and the use of the CLT to determine the E_{axial} of the pipe, a load factor of 1.4 was used.

Table 5.9 contains the calculated maximum axial stresses and the factored maximum axial stresses for fibre angles of 50 to 70 degrees. The strain ratios for the loadings of Table 5.9 are shown in Figure 5.10.

Table 5.9: Maximum axial stress from the installation loading for a lift of 1, and a delta of 3.

ang	maximum axial stress (MPa)	factored maximum axial stress (MPa)
50	42.4	59.3
51	41.9	58.6
52	41.4	58.0
53	41.0	57.4
54	40.6	56.8
55	40.2	56.3
56	39.9	55.9
57	39.6	55.5
58	39.3	55.1
59	39.1	54.7
60	38.9	54.4
61	38.7	54.2
62	38.5	53.9
63	38.3	53.7
64	38.2	53.5
65	38.1	53.3
66	38.0	53.2
67	37.9	53.0
68	37.8	52.9
69	37.7	52.8
70	37.6	52.7

Table 5.10: Scaled strain ratios for the installation load with t/D = .06, lift = 1, delta = 3

		strain ratios							
ang	scaling factor	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	59.30	0.3055	-0.1581	0.3805	-0.3786	0.0303	-0.0499	0.3805	shear
51	58.61	0.3334	-0.1726	0.3776	-0.3757	0.0277	-0.0456	0.3776	shear
52	57.97	0.3609	-0.1870	0.3738	-0.3720	0.0252	-0.0414	0.3738	shear
53	57.37	0.3881	-0.2011	0.3693	-0.3676	0.0227	-0.0373	0.3881	matrix tension
54	56.83	0.4147	-0.2149	0.3642	-0.3625	0.0203	-0.0333	0.4147	matrix tension
55	56.33	0.4407	-0.2284	0.3584	-0.3567	0.0180	-0.0295	0.4407	matrix tension
56	55.88	0.4660	-0.2416	0.3519	-0.3503	0.0158	-0.0258	0.4660	matrix tension
57	55.46	0.4907	-0.2544	0.3449	-0.3434	0.0137	-0.0222	0.4907	matrix tension
58	55.09	0.5145	-0.2668	0.3374	-0.3360	0.0117	-0.0188	0.5145	matrix tension
59	54.74	0.5375	-0.2787	0.3294	-0.3281	0.0097	-0.0156	0.5375	matrix tension
60	54.44	0.5597	-0.2902	0.3210	-0.3198	0.0090	-0.0127	0.5597	matrix tension
61	54.16	0.5810	-0.3013	0.3122	-0.3111	0.0086	-0.0123	0.5810	matrix tension
62	53.91	0.6014	-0.3119	0.3031	-0.3020	0.0081	-0.0122	0.6014	matrix tension
63	53.69	0.6209	-0.3220	0.2937	-0.2926	0.0076	-0.0120	0.6209	matrix tension
64	53.49	0.6395	-0.3317	0.2839	-0.2830	0.0072	-0.0118	0.6395	matrix tension
65	53.31	0.6573	-0.3409	0.2740	-0.2731	0.0067	-0.0116	0.6573	matrix tension
66	53.15	0.6741	-0.3496	0.2638	-0.2630	0.0063	-0.0114	0.6741	matrix tension
67	53.01	0.6900	-0.3579	0.2535	-0.2528	0.0058	-0.0111	0.6900	matrix tension
68	52.89	0.7051	-0.3657	0.2430	-0.2423	0.0053	-0.0108	0.7051	matrix tension
69	52.78	0.7193	-0.3730	0.2324	-0.2318	0.0055	-0.0105	0.7193	matrix tension
70	52.69	0.7326	-0.3800	0.2217	-0.2211	0.0065	-0.0105	0.7326	matrix tension

From Table 5.10, it can be concluded that even with a large load factor of 1.4, the entire range of possible fibre angles is capable of satisfactorily withstanding the installation loading of this design study.

5.4.6 Force Due to Backfill

Just as the pipe must be able to withstand the moment loading from installation, the pipe must be able to withstand the applied force due to backfill while the pipe is not internally pressurized.

Applying equation 2.6 and assuming a sand and gravel backfill, and a ditch width at the top of the pipe equal to 1.7 times the OD, the force due to backfill is given by:

$$W_d = C_d w_f B_d^2$$

with

$$C_d = \frac{1 - e^{-2\frac{K_u \cdot H}{B_d}}}{2 \cdot K_u}$$

where

W_d	= load on pipe (N/m)
Ku	= 0.165 for sand and gravel
H	= height of backfill above the top of the pipe (m)
B_d	= horizontal width of trench at top of pipe (m) = 1.7 x OD
w_f	= unit weight of filling material = 15.7 kN/m^3 for sand and gravel
C_d	= load coefficient for trench installations

The height of backfill above the pipe is given by:

$$H = (\text{depth of ditch}) - (\text{OD}) = 2.50\text{-}.56 = 1.94\text{m},$$

For an OD of 0.56m, B_d=1.7xOD, K_u = 0.165, and w_f = 15.7 kN/m^3, and H= 1.94m:

$$C_d = \frac{1 - e^{-2\frac{(0.165)(1.94)}{1.7\times 0.56}}}{2(0.165)} = 1.484$$

and

$$W_d = (1.484)(15.7\times10^3)(1.7\times0.56)^2 = 21.1\ \text{kN / m}$$

Therefore the vertical force per metre of pipe = 21.1 kN/m

and the factored force per metre of pipe = 1.25x21.1 = 26.39 kN/m

For a line loading, the strain ratio is dependant on the magnitude of the loading and the diameter of the pipe. From Table C.8, the applied force from Table C.8 must be multiplied by the diameter of the pipe. Thus the scaling factor is given by:

$$\text{scaling factor} = \frac{\text{factored force}}{\text{applied force} \cdot \text{OD}} = \frac{26.36\times10^3}{1\times10^3 \cdot 0.56} = 47.118$$

For a wind angle of 50 degrees, and scaling the strain ratios from Table C.8 produces Table 5.11:

Table 5.11: Strain ratio scaling sample calculation for a backfill loading, with a depth of backfill of 1.94m for a fibre angle of 50 degrees

strain ratio	force/length 1kN/m per diameter	 47.118 kN/m
maximum matrix	0.003589	0.1691
minimum matrix	-0.001461	-0.0688
maximum shear	0.002790	0.1315
minimum shear	-0.001934	-0.0911
maximum fibre	0.000895	0.0422
minimum fibre	-0.001120	-0.0528

The scaled strain ratios for the backfill loadings are shown in Table 5.12.

Table 5.12: Scaled strain ratios for a 1.94m backfill with t/D = 0.06

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.1691	-0.0688	0.1315	-0.0911	0.0422	-0.0528	0.1691	matrix tension
51	0.1585	-0.0649	0.1290	-0.0886	0.0421	-0.0530	0.1585	matrix tension
52	0.1482	-0.0617	0.1265	-0.0862	0.0419	-0.0533	0.1482	matrix tension
53	0.1383	-0.0585	0.1238	-0.0837	0.0417	-0.0535	0.1383	matrix tension
54	0.1288	-0.0554	0.1211	-0.0812	0.0415	-0.0537	0.1288	matrix tension
55	0.1197	-0.0524	0.1184	-0.0787	0.0413	-0.0538	0.1197	matrix tension
56	0.1110	-0.0496	0.1156	-0.0763	0.0410	-0.0540	0.1156	shear
57	0.1028	-0.0468	0.1128	-0.0738	0.0407	-0.0542	0.1128	shear
58	0.0972	-0.0442	0.1099	-0.0714	0.0404	-0.0543	0.1099	shear
59	0.0917	-0.0416	0.1070	-0.0691	0.0401	-0.0544	0.1070	shear
60	0.0864	-0.0391	0.1041	-0.0668	0.0398	-0.0546	0.1041	shear
61	0.0813	-0.0367	0.1011	-0.0645	0.0395	-0.0547	0.1011	shear
62	0.0763	-0.0344	0.0982	-0.0622	0.0393	-0.0548	0.0982	shear
63	0.0715	-0.0321	0.0952	-0.0600	0.0390	-0.0549	0.0952	shear
64	0.0669	-0.0300	0.0922	-0.0577	0.0387	-0.0550	0.0922	shear
65	0.0624	-0.0279	0.0891	-0.0555	0.0384	-0.0551	0.0891	shear
66	0.0581	-0.0259	0.0861	-0.0535	0.0382	-0.0552	0.0861	shear
67	0.0539	-0.0239	0.0830	-0.0519	0.0380	-0.0553	0.0830	shear
68	0.0498	-0.0220	0.0799	-0.0503	0.0378	-0.0554	0.0799	shear
69	0.0459	-0.0202	0.0768	-0.0486	0.0376	-0.0555	0.0768	shear
70	0.0422	-0.0185	0.0737	-0.0468	0.0374	-0.0555	0.0737	shear

From Table 5.12, it can be concluded that all fibre wind angles considered in this study will be able to support the backfill. It is assumed that instability, i.e. buckling, will not occur.

5.4.7 Load Combinations: (1H:0A) and Backfill

The load combinations are obtained by simply combining the strain ratios of the pressure loading and the backfill loading. This creates a "worst case" condition, assuming that the maximum strains from the backfill loading occurs at the same locations as the maximum strains from the pressure loading. The (1H:0A) strain ratios are from §5.4.1 while the backfill strain ratios are from §5.4.6.

Table 5.13 gives an example of the summed strain ratios for the combined loadings for a fibre angle of 50 degrees. The summed strain ratios for the entire range of fibre angles is shown in Table 5.14.

Table 5.13: Summed strain ratio sample calculation for a combined loading of (1H:0A) and backfill, with a hoop stress of 156.25 MPa and 1.94m of backfill, for a fibre angle of 50 degrees

strain ratio	(1H:0A)	backfill	sum
maximum matrix	0.0668	0.1691	0.2359
minimum matrix	0.0622	-0.0688	-0.0066
maximum shear	0.7832	0.1315	0.9147
minimum shear	-0.6426	-0.0911	-0.7337
maximum fibre	0.1273	0.0422	0.1695
minimum fibre	0.1265	-0.0528	0.0737

Table 5.14: Summed strain ratios for a 1.94m backfill and a hoop stress from a (1H:0A) loading of 156.25 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comments
50	0.2359	-0.0066	0.9147	-0.7337	0.1695	0.0737	0.9147	shear	too high
51	0.1750	-0.0532	0.8841	-0.7133	0.1730	0.0772	0.8841	shear	too high
52	0.1314	-0.0807	0.8523	-0.6921	0.1762	0.0803	0.8523	shear	too high
53	0.1016	-0.0979	0.8197	-0.6704	0.1791	0.0831	0.8197	shear	too high
54	0.0728	-0.1143	0.7865	-0.6481	0.1816	0.0857	0.7865	shear	
55	0.0466	-0.1285	0.7530	-0.6255	0.1838	0.0879	0.7530	shear	
56	0.0230	-0.1406	0.7194	-0.6028	0.1856	0.0899	0.7194	shear	
57	0.0019	-0.1508	0.6858	-0.5800	0.1872	0.0916	0.6858	shear	
58	-0.0146	-0.1592	0.6525	-0.5572	0.1885	0.0930	0.6525	shear	
59	-0.0290	-0.1657	0.6196	-0.5347	0.1895	0.0942	0.6196	shear	
60	-0.0416	-0.1706	0.5873	-0.5124	0.1902	0.0951	0.5873	shear	
61	-0.0523	-0.1739	0.5557	-0.4904	0.1907	0.0959	0.5557	shear	
62	-0.0615	-0.1759	0.5249	-0.4688	0.1911	0.0964	0.5249	shear	
63	-0.0691	-0.1765	0.4950	-0.4475	0.1912	0.0968	0.4950	shear	
64	-0.0754	-0.1761	0.4661	-0.4267	0.1912	0.0970	0.4661	shear	
65	-0.0804	-0.1746	0.4382	-0.4064	0.1911	0.0970	0.4382	shear	
66	-0.0844	-0.1722	0.4112	-0.3867	0.1909	0.0970	0.4112	shear	
67	-0.0874	-0.1690	0.3854	-0.3679	0.1905	0.0968	0.3854	shear	
68	-0.0895	-0.1652	0.3605	-0.3495	0.1901	0.0965	0.3605	shear	
69	-0.0909	-0.1609	0.3367	-0.3315	0.1896	0.0961	0.3367	shear	
70	-0.0917	-0.1562	0.3139	-0.3139	0.1890	0.0957	0.3139	shear	

This loading combination reveals that the maximum strain ratios for fibre wind angles below 53° is greater than the resistance factor of 0.8, and are thus not acceptable.

5.4.8 Load Combinations: (8H:1A) and Backfill

The other load combinations simply follow the same format as that used in §5.4.7. The summed strain ratios for this loading combination are in Table 5.15.

Table 5.15: Summed strain ratios for a 1.94m backfill and a hoop stress from a (8H:1A) loading of 156.25 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comments
50	0.3351	0.0920	0.7850	0.5587	0.1796	0.0837	0.7850	shear	
51	0.2851	0.0563	0.7538	0.5325	0.1823	0.0864	0.7538	shear	
52	0.2398	0.0244	0.7220	0.5056	0.1847	0.0887	0.7220	shear	
53	0.2019	0.0000	0.6897	0.4784	0.1868	0.0907	0.6897	shear	
54	0.1692	-0.0204	0.6571	0.4510	0.1885	0.0925	0.6571	shear	
55	0.1406	-0.0370	0.6246	0.4237	0.1899	0.0940	0.6246	shear	
56	0.1078	-0.0560	0.5923	0.3967	0.1910	0.0952	0.5923	shear	
57	0.0920	-0.0608	0.5604	0.3701	0.1918	0.0961	0.5604	shear	
58	0.0807	-0.0639	0.5290	0.3441	0.1924	0.0968	0.5290	shear	
59	0.0712	-0.0654	0.4984	0.3188	0.1927	0.0973	0.4984	shear	
60	0.0634	-0.0655	0.4686	0.2943	0.1927	0.0976	0.4686	shear	
61	0.0573	-0.0642	0.4398	0.2708	0.1926	0.0977	0.4398	shear	
62	0.0526	-0.0617	0.4120	0.2484	0.1923	0.0976	0.4120	shear	
63	0.0492	-0.0581	0.3852	0.2270	0.1919	0.0974	0.3852	shear	
64	0.0470	-0.0536	0.3596	0.2067	0.1913	0.0970	0.3596	shear	
65	0.0458	-0.0483	0.3352	0.1876	0.1907	0.0966	0.3352	shear	
66	0.0454	-0.0422	0.3119	0.1694	0.1899	0.0960	0.3119	shear	
67	0.0459	-0.0357	0.2897	0.1521	0.1891	0.0953	0.2897	shear	
68	0.0675	-0.0287	0.2687	0.1359	0.1882	0.0946	0.2687	shear	
69	0.0742	0.0008	0.2489	0.1209	0.1873	0.0938	0.2489	shear	
70	0.0815	0.0137	0.2301	0.1070	0.1864	0.0930	0.2301	shear	

This load combination indicates that for a (8H:1A) loading with an applied hoop stress of 156.25MPa and a backfill loading of 1.94m put no constraints on the fibre angles.

5.4.9 Load Combinations: (4H:1A) and Backfill

The resultant strain ratios from the load combination of (4H:1A) and backfill is shown in Table 5.16.

Table 5.16: Summed strain ratios for a 1.94m backfill and a hoop stress from a (4H:1A) loading of 156.25 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comment
50	0.4343	0.1906	0.6552	0.4291	0.1897	0.0937	0.6552	shear	
51	0.3952	0.1658	0.6236	0.4024	0.1916	0.0955	0.6236	shear	
52	0.3621	0.1466	0.5917	0.3755	0.1932	0.0971	0.5917	shear	
53	0.3356	0.1330	0.5596	0.3485	0.1945	0.0983	0.5596	shear	
54	0.3133	0.1232	0.5277	0.3218	0.1954	0.0993	0.5277	shear	
55	0.2950	0.1169	0.4962	0.2955	0.1961	0.1000	0.4962	shear	
56	0.2805	0.1139	0.4652	0.2698	0.1964	0.1005	0.4652	shear	
57	0.2698	0.1141	0.4350	0.2449	0.1964	0.1007	0.4350	shear	
58	0.2646	0.1172	0.4056	0.2209	0.1963	0.1007	0.4056	shear	
59	0.2625	0.1230	0.3772	0.1978	0.1959	0.1005	0.3772	shear	
60	0.2631	0.1311	0.3500	0.1759	0.1953	0.1001	0.3500	shear	
61	0.2660	0.1414	0.3239	0.1551	0.1945	0.0995	0.3239	shear	
62	0.2709	0.1535	0.2990	0.1356	0.1936	0.0988	0.2990	shear	
63	0.2777	0.1672	0.2754	0.1173	0.1926	0.0980	0.2777	matrix tension	
64	0.2860	0.1823	0.2532	0.1003	0.1914	0.0971	0.2860	matrix tension	
65	0.2957	0.1984	0.2322	0.0846	0.1902	0.0961	0.2957	matrix tension	
66	0.3063	0.2154	0.2125	0.0701	0.1890	0.0950	0.3063	matrix tension	
67	0.3178	0.2330	0.1941	0.0564	0.1877	0.0939	0.3178	matrix tension	
68	0.3300	0.2511	0.1769	0.0440	0.1863	0.0927	0.3300	matrix tension	
69	0.3425	0.2694	0.1610	0.0330	0.1850	0.0915	0.3425	matrix tension	
70	0.3553	0.2878	0.1462	0.0231	0.1837	0.0903	0.3553	matrix tension	

It is noted from Table 5.16 that all fibre angles between 50° and 70° are valid for this load combination.

5.4.10 Load Combinations: (2H:1A) and Backfill

Table 5.17 contains the summed strain ratios from the combined loading of (2H:1A) and backfill.

Table 5.17: Summed strain ratios for a 1.94m backfill and a hoop stress from a (2H:1A) loading of 156.25 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comment
50	0.6327	0.3878	0.3958	0.1699	0.2099	0.1137	0.6327	matrix tension	
51	0.6182	0.3880	0.3632	0.1423	0.2103	0.1139	0.6182	matrix tension	
52	0.6084	0.3917	0.3310	0.1152	0.2103	0.1139	0.6084	matrix tension	
53	0.6029	0.3992	0.2995	0.0888	0.2099	0.1136	0.6029	matrix tension	
54	0.6014	0.4102	0.2689	0.0634	0.2093	0.1130	0.6014	matrix tension	
55	0.6037	0.4246	0.2394	0.0391	0.2083	0.1121	0.6037	matrix tension	
56	0.6096	0.4421	0.2111	0.0161	0.2071	0.1110	0.6096	matrix tension	
57	0.6187	0.4622	0.1842	-0.0055	0.2057	0.1098	0.6187	matrix tension	
58	0.6329	0.4848	0.1587	-0.0256	0.2041	0.1083	0.6329	matrix tension	
59	0.6495	0.5094	0.1349	-0.0443	0.2023	0.1067	0.6495	matrix tension	
60	0.6680	0.5358	0.0734	-0.0975	0.2003	0.1050	0.6680	matrix tension	
61	0.6883	0.5637	0.0544	-0.1112	0.1983	0.1031	0.6883	matrix tension	
62	0.7099	0.5928	0.0370	-0.1234	0.1961	0.1012	0.7099	matrix tension	
63	0.7328	0.6227	0.0212	-0.1339	0.1939	0.0992	0.7328	matrix tension	
64	0.7565	0.6532	0.0070	-0.1429	0.1916	0.0972	0.7565	matrix tension	
65	0.7809	0.6841	-0.0056	-0.1503	0.1894	0.0951	0.7809	matrix tension	
66	0.8055	0.7150	-0.0167	-0.1563	0.1871	0.0930	0.8055	matrix tension	too high
67	0.8302	0.7459	-0.0264	-0.1613	0.1848	0.0910	0.8302	matrix tension	too high
68	0.8549	0.7764	-0.0346	-0.1648	0.1826	0.0889	0.8549	matrix tension	too high
69	0.8792	0.8065	-0.0415	-0.1668	0.1804	0.0869	0.8792	matrix tension	too high
70	0.9030	0.8359	-0.0471	-0.1676	0.1783	0.0849	0.9030	matrix tension	too high

The load combination of (2H:1A) and backfill eliminates fibre wind angles over 65 degrees, i.e. the acceptable fibre wind angles are $50 \leq \theta \leq 64$ degrees.

5.4.11 Load Combinations: (1H:0A), Backfill and Bending Moment

Utilizing the strain ratios from the load combinations from §5.4.7 through §5.4.10, it is possible then to calculate the allowable maximum bending moment which will make the maximum strain ratio equal to the resistance factor (0.8).

In the following tables, a maximum ratio not equal to 0.8 or of "#N/A" means that the strain ratio was greater than 0.8 before the application of the moment, and thus that fibre angle need not be considered.

For this loading combination, the scaling factor is determined by finding the minimum value which will make any strain ratio equal to 0.8

$$\text{scaling factor} = \min\left(\text{abs}\left(\frac{\text{resistance factor} - \text{strain ratio(i)}}{\text{moment strain ratio(i)}}\right)\right)$$

where:

resistance factor =	0.8
ratio(i) =	strain ratio (min or max of fibre, matrix or shear) from combined pressure and backfill loading
moment strain ratio(i) =	strain ratio (min or max of fibre, matrix or shear) from moment loading

The ratio(i) values are taken from sections 5.4.7-5.4.10 of this design study, while the moment strain ratio(i) values are taken from Table C.5.

For a fibre angle of 60 degrees, the scaling factor is determined by finding the minimum of possible scaling factors. Table 5.18 demonstrates the calculation of the determination of the minimum possible scaling factor for a fibre angle of 60 degrees.

Table 5.18: Determination of minimum scaling factor for determination of maximum allowable bending moment on a pipe loaded with (1H:0A) with a hoop stress of 156.25 MPa and a backfill of 1.94m for a fibre angle of 60 degrees

strain ratio	strain ratio from (1H:0A) and backfill	strain ratio from unit moment loading	$\text{abs}\left(\frac{\text{resistance factor} - \text{strain ratio}}{\text{moment strain ratio}}\right)$
maximum matrix	-0.0416	0.0103	81.85
minimum matrix	-0.1706	-0.0053	182.0
maximum shear	0.5873	0.0059	36.06
minimum shear	-0.5124	-0.0059	223.4
maximum fibre	0.1902	0.0002	3680
minimum fibre	0.0951	-0.0002	3028

Thus the minimum scaling ratio for a fibre angle of 60 degrees is 36.06.

Scaling the strain ratios from the moment unit loading, using the minimum scaling factor from Table 5.18, and summing with the strain ratios from the combined (1H:0A) and backfill loading, the maximum strain ratio should be equal to 0.8, the resistance factor. This calculation is shown in Table 5.19 for a fibre angle of 60 degrees.

Table 5.19: Sample calculation of summed and scaled strain ratios for (1H:0A) and backfill and maximum allowable bending moment for a fibre angle of 60 degrees.

strain ratio	strain ratio from (1H:0A) and backfill	strain ratio from unit moment loading	scaled moment strain ratio	summed strain ratio from (1H:0A) and backfill + scaled moment
maximum matrix	-0.0416	0.0103	0.3708	0.3292
minimum matrix	-0.1706	-0.0053	-0.1923	-0.3629
maximum shear	0.5873	0.0059	0.2127	0.8000
minimum shear	-0.5124	-0.0059	-0.2119	-0.7242
maximum fibre	0.1902	0.0002	0.0060	0.1962
minimum fibre	0.0951	-0.0002	-0.0084	0.0867

Thus, the maximum strain ratio for the combined loading in the above example is equal to 0.8, with the maximum ratio occurring in shear.

The maximum axial stress is found by multiplying the maximum axial stress in Table C.5 by the scaling factor.

Scaling the maximum axial stress from the moment loading is obtained by multiplying the scaling factor by the unit load from Table C.5, i.e.

Scaled maximum axial stress = 36.06 x1MPa = 36.06MPa

The actual moment load which causes the maximum axial stress can be calculated from:

$$\sigma_{max} = \frac{M \cdot OD}{2I} \therefore M = \frac{2\sigma_{max} I}{OD}$$

Therefore, for the fibre angle of 60 degrees:

$$M = \frac{2x36.06 \cdot 1.759x10^{-3}}{0.56} = 226.6 \text{ kN} \cdot \text{m}$$

The strain ratios occurring from the combined loadings of backfill, (1H:0A), and bending moment are shown in Table 5.20. Table 5.21 includes the scaled maximum axial stress and the bending moment as determined by the minimum scaling factor.

Table 5.20: Strain Ratios resulting from a backfill of 1.94m, hoop stress of 156.25 MPa in a (1H:0A) loading, and the maximum allowable moment

		strain ratios							
ang	scaling factor	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	17.88	0.3280	-0.0543	1.0294	-0.8479	0.1786	0.0587	1.0294	#N/A
51	13.05	0.2492	-0.0916	0.9681	-0.7970	0.1792	0.0670	0.9681	#N/A
52	8.111	0.1819	-0.1069	0.9046	-0.7442	0.1797	0.0745	0.9046	#N/A
53	3.064	0.1223	-0.1087	0.8394	-0.6900	0.1803	0.0811	0.8394	#N/A
54	2.100	0.0881	-0.1222	0.8000	-0.6615	0.1823	0.0844	0.8000	shear
55	7.386	0.1044	-0.1584	0.8000	-0.6723	0.1861	0.0840	0.8000	shear
56	12.80	0.1298	-0.1960	0.8000	-0.6831	0.1893	0.0840	0.8000	shear
57	18.37	0.1644	-0.2351	0.8000	-0.6937	0.1917	0.0842	0.8000	shear
58	24.08	0.2104	-0.2758	0.8000	-0.7041	0.1935	0.0848	0.8000	shear
59	29.98	0.2653	-0.3183	0.8000	-0.7144	0.1948	0.0856	0.8000	shear
60	36.06	0.3292	-0.3629	0.8000	-0.7242	0.1962	0.0867	0.8000	shear
61	42.37	0.4022	-0.4097	0.8000	-0.7338	0.1975	0.0863	0.8000	shear
62	48.92	0.4843	-0.4589	0.8000	-0.7428	0.1984	0.0854	0.8000	shear
63	55.75	0.5757	-0.5110	0.8000	-0.7514	0.1992	0.0843	0.8000	shear
64	62.90	0.6767	-0.5661	0.8000	-0.7595	0.1997	0.0830	0.8000	shear
65	70.40	0.7876	-0.6247	0.8000	-0.7671	0.2000	0.0817	0.8000	shear
66	69.74	0.8000	-0.6308	0.7574	-0.7318	0.1991	0.0820	0.8000	matrix tension
67	68.18	0.8000	-0.6293	0.7114	-0.6930	0.1980	0.0825	0.8000	matrix tension
68	66.73	0.8000	-0.6266	0.6671	-0.6553	0.1968	0.0828	0.8000	matrix tension
69	65.38	0.8000	-0.6230	0.6246	-0.6186	0.1964	0.0831	0.8000	matrix tension
70	64.13	0.8000	-0.6187	0.5838	-0.5831	0.1970	0.0829	0.8000	matrix tension

Table 5.21 Scaled maximum axial stresses and moments for backfill of 1.94m and a hoop stress of 156.25 MPa in a (1H:0A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	#N/A	#N/A
51	#N/A	#N/A
52	#N/A	#N/A
53	#N/A	#N/A
54	2.100	13.20
55	7.386	46.42
56	12.80	80.46
57	18.37	115.41
58	24.08	151.34
59	29.98	188.38
60	36.06	226.63
61	42.37	266.25
62	48.92	307.43
63	55.75	350.34
64	62.90	395.25
65	70.40	442.41
66	69.74	438.22
67	68.18	428.43
68	66.73	419.32
69	65.38	410.86
70	64.13	403.01

5.4.12 Load Combinations: (8H:1A), Backfill and Bending Moment

For the this load combination, the combined strain ratios are given in Table 5.22 and the scaled maximum axial stress and bending moment are given in Table 5.23.

5.4.13 Load Combinations: (4H:1A), Backfill and Bending Moment

For the this load combination, the combined strain ratios are given in Table 5.24 and the scaled maximum axial stress and bending moment are given in Table 5.25.

5.4.14 Load Combinations: (2H:1A), Backfill and Bending Moment

For the this load combination, the combined strain ratios are given in Table 5.26 and the scaled maximum axial stress and bending moment are given in Table 5.27.

Table 5.22: Strain Ratios resulting from a backfill of 1.94m, hoop stress of 156.25 MPa in a (8H:1A) loading, and the maximum allowable moment

		strain ratios							
ang	scaling factor	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	2.340	0.3471	0.0858	0.8000	0.5438	0.1808	0.0818	0.8000	shear
51	7.165	0.3258	0.0352	0.8000	0.4866	0.1857	0.0808	0.8000	shear
52	12.10	0.3151	-0.0146	0.8000	0.4280	0.1900	0.0800	0.8000	shear
53	17.14	0.3179	-0.0601	0.8000	0.3686	0.1936	0.0796	0.8000	shear
54	22.29	0.3319	-0.1047	0.8000	0.3088	0.1965	0.0794	0.8000	shear
55	27.57	0.3563	-0.1488	0.8000	0.2491	0.1987	0.0795	0.8000	shear
56	32.98	0.3829	-0.1986	0.8000	0.1899	0.2004	0.0799	0.8000	shear
57	38.53	0.4329	-0.2375	0.8000	0.1315	0.2013	0.0807	0.8000	shear
58	44.23	0.4938	-0.2782	0.8000	0.0743	0.2017	0.0817	0.8000	shear
59	50.11	0.5633	-0.3206	0.8000	0.0185	0.2015	0.0830	0.8000	shear
60	56.19	0.6412	-0.3651	0.8000	-0.0357	0.2021	0.0845	0.8000	shear
61	62.48	0.7276	-0.4118	0.8000	-0.0880	0.2025	0.0835	0.8000	shear
62	66.99	0.8000	-0.4493	0.7886	-0.1269	0.2024	0.0825	0.8000	matrix tension
63	64.91	0.8000	-0.4475	0.7403	-0.1269	0.2011	0.0828	0.8000	matrix tension
64	62.98	0.8000	-0.4441	0.6940	-0.1265	0.1998	0.0831	0.8000	matrix tension
65	61.18	0.8000	-0.4394	0.6496	-0.1258	0.1984	0.0832	0.8000	matrix tension
66	59.50	0.8000	-0.4336	0.6072	-0.1250	0.1969	0.0832	0.8000	matrix tension
67	57.94	0.8000	-0.4268	0.5668	-0.1242	0.1954	0.0832	0.8000	matrix tension
68	54.95	0.8000	-0.4086	0.5212	-0.1159	0.1937	0.0833	0.8000	matrix tension
69	53.27	0.8000	-0.3756	0.4834	-0.1130	0.1928	0.0832	0.8000	matrix tension
70	51.68	0.8000	-0.3589	0.4475	-0.1098	0.1928	0.0827	0.8000	matrix tension

Table 5.23 Scaled maximum axial stresses and moments for backfill of 1.94m and a hoop stress of 156.25 MPa in a (8H:1A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	2.340	14.70
51	7.165	45.03
52	12.10	76.02
53	17.14	107.7
54	22.29	140.1
55	27.57	173.3
56	32.98	207.2
57	38.53	242.1
58	44.23	278.0
59	50.11	314.9
60	56.19	353.1
61	62.48	392.6
62	66.99	421.0
63	64.91	407.9
64	62.98	395.8
65	61.18	384.4
66	59.50	373.9
67	57.94	364.1
68	54.95	345.3
69	53.27	334.7
70	51.68	324.7

Table 5.24: Strain Ratios resulting from a backfill of 1.94m, hoop stress of 156.25 MPa in a (4H:1A) loading, and the maximum allowable moment

ang	scaling factor	strain ratios maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	22.56	0.5505	0.1304	0.8000	0.2851	0.1898	0.0747	0.8000	shear
51	27.38	0.5509	0.0851	0.8000	0.2269	0.2046	0.0742	0.8000	shear
52	32.31	0.5633	0.0424	0.8000	0.1682	0.2073	0.0740	0.8000	shear
53	37.34	0.5881	0.0022	0.8000	0.1093	0.2093	0.0741	0.8000	shear
54	42.49	0.6233	-0.0375	0.8000	0.0508	0.2106	0.0744	0.8000	shear
55	47.76	0.6686	-0.0768	0.8000	-0.0069	0.2114	0.0750	0.8000	shear
56	53.15	0.7238	-0.1159	0.8000	-0.0634	0.2114	0.0759	0.8000	shear
57	58.69	0.7890	-0.1551	0.8000	-0.1185	0.2109	0.0771	0.8000	shear
58	57.32	0.8000	-0.1604	0.7567	-0.1288	0.2084	0.0810	0.8000	matrix tension
59	54.74	0.8000	-0.1557	0.7067	-0.1302	0.2056	0.0848	0.8000	matrix tension
60	52.22	0.8000	-0.1473	0.6579	-0.1309	0.2039	0.0879	0.8000	matrix tension
61	49.78	0.8000	-0.1355	0.6109	-0.1308	0.2024	0.0882	0.8000	matrix tension
62	47.42	0.8000	-0.1208	0.5656	-0.1301	0.2007	0.0881	0.8000	matrix tension
63	45.16	0.8000	-0.1036	0.5224	-0.1288	0.1990	0.0879	0.8000	matrix tension
64	42.98	0.8000	-0.0843	0.4813	-0.1271	0.1972	0.0876	0.8000	matrix tension
65	40.91	0.8000	-0.0632	0.4424	-0.1249	0.1954	0.0871	0.8000	matrix tension
66	38.93	0.8000	-0.0406	0.4057	-0.1226	0.1935	0.0866	0.8000	matrix tension
67	37.05	0.8000	-0.0170	0.3712	-0.1202	0.1917	0.0861	0.8000	matrix tension
68	35.26	0.8000	0.0073	0.3389	-0.1175	0.1899	0.0855	0.8000	matrix tension
69	33.57	0.8000	0.0321	0.3088	-0.1144	0.1885	0.0848	0.8000	matrix tension
70	31.98	0.8000	0.0571	0.2807	-0.1111	0.1876	0.0839	0.8000	matrix tension

Table 5.25 Scaled maximum axial stresses and moments for backfill of 1.94m and a hoop stress of 156.25 MPa in a (4H:1A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	22.56	141.8
51	27.38	172.1
52	32.31	203.0
53	37.34	234.7
54	42.49	267.0
55	47.76	300.1
56	53.15	334.0
57	58.69	368.8
58	57.32	360.2
59	54.74	344.0
60	52.22	328.2
61	49.78	312.8
62	47.42	298.0
63	45.16	283.8
64	42.98	270.1
65	40.91	257.1
66	38.93	244.6
67	37.05	232.8
68	35.26	221.6
69	33.57	211.0
70	31.98	201.0

Table 5.26: Strain Ratios resulting from a backfill of 1.94m, hoop stress of 156.25 MPa in a (2H:1A) loading, and the maximum allowable moment

		strain ratios							
ang	scaling factor	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	32.46	0.8000	0.3013	0.6041	-0.0373	0.2265	0.0864	0.8000	matrix tension
51	31.97	0.8000	0.2938	0.5691	-0.0626	0.2254	0.0891	0.8000	matrix tension
52	30.78	0.8000	0.2924	0.5295	-0.0823	0.2236	0.0919	0.8000	matrix tension
53	29.15	0.8000	0.2970	0.4872	-0.0979	0.2215	0.0946	0.8000	matrix tension
54	27.22	0.8000	0.3073	0.4433	-0.1102	0.2190	0.0970	0.8000	matrix tension
55	25.09	0.8000	0.3229	0.3990	-0.1197	0.2164	0.0990	0.8000	matrix tension
56	22.83	0.8000	0.3433	0.3549	-0.1270	0.2136	0.1005	0.8000	matrix tension
57	20.50	0.8000	0.3682	0.3116	-0.1324	0.2108	0.1015	0.8000	matrix tension
58	17.89	0.8000	0.3981	0.2683	-0.1347	0.2079	0.1022	0.8000	matrix tension
59	15.33	0.8000	0.4314	0.2271	-0.1361	0.2050	0.1023	0.8000	matrix tension
60	12.83	0.8000	0.4674	0.1491	-0.1729	0.2025	0.1020	0.8000	matrix tension
61	10.41	0.8000	0.5058	0.1144	-0.1710	0.1999	0.1008	0.8000	matrix tension
62	8.072	0.8000	0.5461	0.0824	-0.1686	0.1973	0.0994	0.8000	matrix tension
63	5.813	0.8000	0.5878	0.0530	-0.1656	0.1947	0.0979	0.8000	matrix tension
64	3.634	0.8000	0.6307	0.0263	-0.1621	0.1921	0.0964	0.8000	matrix tension
65	1.552	0.8000	0.6742	0.0024	-0.1582	0.1896	0.0948	0.8000	matrix tension
66	0.435	0.8110	0.7122	-0.0146	-0.1585	0.1871	0.0929	0.8110	#N/A
67	2.323	0.8605	0.7302	-0.0152	-0.1724	0.1851	0.0905	0.8605	#N/A
68	3.408	0.9003	0.7529	-0.0189	-0.1804	0.1830	0.0882	0.9003	#N/A
69	0.919	0.8917	0.8000	-0.0374	-0.1709	0.1805	0.0867	0.8917	#N/A
70	4.977	0.9722	0.8000	-0.0261	-0.1885	0.1790	0.0839	0.9722	#N/A

Table 5.27 Scaled maximum axial stresses and moments for backfill of 1.94m and a hoop stress of 156.25 MPa in a (2H:1A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	32.46	204.0
51	31.97	200.9
52	30.78	193.4
53	29.15	183.2
54	27.22	171.0
55	25.09	157.7
56	22.83	143.5
57	20.50	128.8
58	17.89	112.4
59	15.33	96.3
60	12.83	80.7
61	10.41	65.4
62	8.072	50.72
63	5.813	36.53
64	3.634	22.84
65	1.552	9.750
66	#N/A	#N/A
67	#N/A	#N/A
68	#N/A	#N/A
69	#N/A	#N/A
70	#N/A	#N/A

5.4.15 Summary of Design Study 1

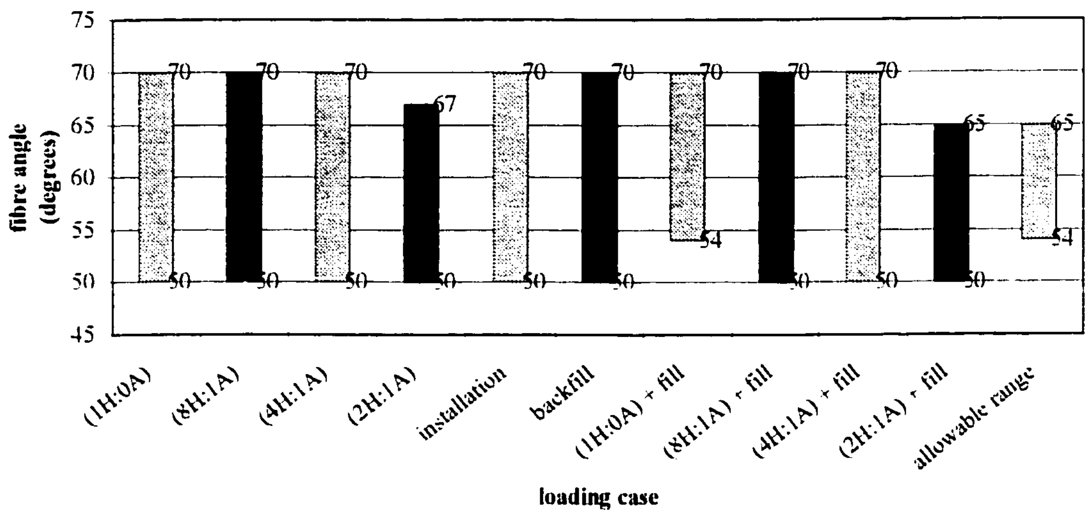

Figure 5.1: Fibre angle compatibilities for Design Study 1

Figure 5.1 is a summary of the allowable fibre angles as determined by the loadings considered in this design study. The limitations to the fibre angles tend to come from the most extreme loading cases. For instance, the combination of pure hoop stress and back fill restricts the lowest allowable fibre angle to 54°, while the combination of backfill and (2H:1A), in which the largest axial stresses of all of the loading cases occur, limits the largest allowable fibre angle to be less than 65°.

Thus, from the given loading conditions, the allowable range of fibre angles that will satisfy the prescribed design criteria is 54 - 65 degrees. From a design point of view, noting that the service load is generally the combination of (1H:0A) and backfill, one would tend to choose a fibre angle near the maximum of the fibre range, i.e. 65°.

5.5 Design Study 2

The purpose of this design study is to find the feasible fibre angles for a composite pipe, based on a set of given loading conditions summarized below. Design study 2 also considers possible buoyancy effects.

Given:

- Internal pressure = 30 MPa (4.4 ksi)

- Possible hoop to axial stress ratio loadings:
  (1H:0A), (8H:1A), (4H:1A), (2H:1A)

- Factors:
  Resistance factor = 0.8
  Load factor for pressure = 1.25
  Load factor for backfill = 1.25
  Load factor for moment = 1.4

- ID = 0.5m

- Installation Parameters:
  lift = 1m
  ditch depth = 3.5m

Solution:

For a pressure of 30 MPa, assume t/D = 0.10

therefore:

$$OD = \frac{ID}{\left(1 - 2 \cdot \frac{t}{D}\right)} = 0.60\text{m}$$

and

$$I = \pi \cdot \left(\left(\frac{OD}{2}\right)^4 - \left(\frac{ID}{2}\right)^4 \right) = 3.29\text{x}10^{-3}\,\text{m}^4$$

to calculate the factored hoop stress:

$$\sigma_H = \frac{\text{load factor} \cdot \text{pressure}}{2 \cdot \frac{t}{D}} = \frac{1.25 \cdot 30}{2 \cdot 0.10} = 187.5\text{MPa}$$

5.4.1 Biaxial Pressure Loading of (1H:0A)

The strain results obtained from the finite element model were all for an applied hoop stress of 1MPa. Since this is an elastic analysis, a scaling factor can be used to determine the strain ratios for the desired factored hoop stress of 187.5 MPa. Table C.1, in Appendix C, contains the numerical values of the strain ratios for fibre angles of 50° to 70° obtained from a (1H:0A) loading with a hoop stress of 1MPa.

From Table C.1, a scaling factor can be used to scale the strain ratios to the factored hoop stress of 187.5 MPa:

$$\text{scaling factor} = \frac{\text{factored hoop stress}}{\text{applied hoop stress}} = \frac{187.5\text{MPa}}{1\text{MPa}} = 187.5$$

So, for a fibre angle of 50 degrees the strain ratios are scaled as demonstrated in Table 5.28.

Table 5.28: Strain ratio scaling sample calculation for a hoop stress of 187.5 MPa, in a (1H:0A) loading for a fibre angle of 50 degrees

	hoop stress	
strain ratio	1MPa	187.5MPa
maximum matrix	427.3E-6	0.08012
minimum matrix	398.3E-6	0.07468
maximum shear	5.0E-3	0.93989
minimum shear	-4.1E-3	-0.77114
maximum fibre	814.6E-6	0.15274
minimum fibre	809.7E-6	0.15182

Applying the scaling factor to all of the strain ratios for fibre angles of 50° to 70° produces Table 5.29.

Table 5.29: Scaled strain ratios for (1H:0A) with factored hoop stress of 187.5 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.0801	0.0747	0.9399	-0.7711	0.1527	0.1518	0.9399	shear
51	0.0198	0.0141	0.9061	-0.7496	0.1571	0.1562	0.9061	shear
52	-0.0201	-0.0229	0.8710	-0.7272	0.1612	0.1602	0.8710	shear
53	-0.0441	-0.0474	0.8351	-0.7040	0.1648	0.1639	0.8351	shear
54	-0.0673	-0.0706	0.7985	-0.6803	0.1681	0.1672	0.7985	shear
55	-0.0877	-0.0912	0.7615	-0.6562	0.1710	0.1701	0.7615	shear
56	-0.1056	-0.1092	0.7245	-0.6318	0.1736	0.1727	0.7245	shear
57	-0.1211	-0.1248	0.6876	-0.6074	0.1758	0.1749	0.6876	shear
58	-0.1341	-0.1380	0.6511	-0.5830	0.1776	0.1768	0.6511	shear
59	-0.1449	-0.1490	0.6151	-0.5587	0.1792	0.1783	0.6151	shear
60	-0.1536	-0.1578	0.5799	-0.5348	0.1805	0.1796	0.5799	shear
61	-0.1604	-0.1647	0.5455	-0.5111	0.1814	0.1807	0.5455	shear
62	-0.1654	-0.1698	0.5121	-0.4879	0.1822	0.1814	0.5121	shear
63	-0.1688	-0.1733	0.4798	-0.4651	0.1827	0.1820	0.4798	shear
64	-0.1707	-0.1753	0.4487	-0.4428	0.1831	0.1824	0.4487	shear
65	-0.1714	-0.1760	0.4188	-0.4211	0.1832	0.1825	0.4188	shear
66	-0.1709	-0.1756	0.3902	-0.3999	0.1832	0.1826	0.3902	shear
67	-0.1695	-0.1741	0.3628	-0.3792	0.1831	0.1824	0.3628	shear
68	-0.1672	-0.1718	0.3367	-0.3591	0.1828	0.1822	0.3367	shear
69	-0.1642	-0.1688	0.3119	-0.3396	0.1824	0.1819	0.3119	shear
70	-0.1607	-0.1652	0.2883	-0.3205	0.1820	0.1815	0.2883	shear

From the pure pressure loading case (1H:0A), the maximum strain ratios for angles less than 54° are greater than the resistance factor of 0.8. Thus the (1H:0A) loading condition restrains the range of possible fibre angles from 54° to 70°.

5.5.2 Biaxial Pressure Loading of (8H:1A)

As with the (1H:0A) loading case, the (8H:1A) loading case can also be scaled. Table C.2, from Appendix C, contains the strain ratios obtained from a (8H:1A) loading with a hoop stress of 1 MPa. Therefore the scaling factor can be found from:

$$\text{scaling factor} = \frac{187.5\text{MPa}}{1\text{MPa}} = 187.5$$

For a fibre angle of 50 degrees, scaling the strain ratios from Table C.2, Table 5.30 is obtained.

Table 5.30: Strain ratio scaling sample calculation for a hoop stress of 187.5 MPa, in a (8H:1A) loading for a fibre angle of 50 degrees

	hoop stress	
strain ratio	1MPa	187.5MPa
maximum matrix	1.062E-3	0.1992
minimum matrix	1.029E-3	0.1930
maximum shear	4.182E-3	0.7842
minimum shear	4.159E-3	0.7798
maximum fibre	879.3E-6	0.1649
minimum fibre	873.6E-6	0.1638

Table 5.31 shows the scaled strain ratios for fibre angles of 50 to 70 degrees.

Table 5.31: Scaled Strain Ratios for (8H:1A) with Factored Hoop Stress of 187.5 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.1992	0.1930	0.7842	0.7798	0.1649	0.1638	0.7842	shear
51	0.1519	0.1455	0.7498	0.7453	0.1683	0.1673	0.7498	shear
52	0.1099	0.1032	0.7146	0.7101	0.1714	0.1703	0.7146	shear
53	0.0763	0.0701	0.6790	0.6745	0.1741	0.1730	0.6790	shear
54	0.0484	0.0420	0.6432	0.6387	0.1764	0.1754	0.6432	shear
55	0.0250	0.0185	0.6074	0.6030	0.1784	0.1774	0.6074	shear
56	-0.0038	-0.0077	0.5720	0.5676	0.1800	0.1790	0.5720	shear
57	-0.0129	-0.0168	0.5371	0.5327	0.1813	0.1803	0.5371	shear
58	-0.0198	-0.0237	0.5030	0.4986	0.1823	0.1814	0.5030	shear
59	-0.0246	-0.0286	0.4697	0.4655	0.1830	0.1821	0.4697	shear
60	-0.0276	-0.0317	0.4375	0.4333	0.1835	0.1826	0.4375	shear
61	-0.0288	-0.0330	0.4064	0.4024	0.1837	0.1828	0.4064	shear
62	-0.0285	-0.0328	0.3766	0.3727	0.1837	0.1829	0.3766	shear
63	-0.0268	-0.0312	0.3481	0.3443	0.1835	0.1827	0.3481	shear
64	-0.0239	-0.0283	0.3209	0.3173	0.1832	0.1824	0.3209	shear
65	-0.0200	-0.0244	0.2952	0.2918	0.1827	0.1820	0.2952	shear
66	-0.0152	-0.0196	0.2709	0.2676	0.1820	0.1814	0.2709	shear
67	-0.0096	-0.0141	0.2480	0.2447	0.1813	0.1807	0.2480	shear
68	0.0212	-0.0079	0.2266	0.2233	0.1805	0.1799	0.2266	shear
69	0.0339	0.0253	0.2065	0.2033	0.1797	0.1791	0.2065	shear
70	0.0472	0.0386	0.1877	0.1846	0.1788	0.1782	0.1877	shear

It is seen from Table 5.31 that all strain ratios are less than the resistance factor, therefore, there are no wind angle restrictions from the (8H:1A) loading condition.

5.5.3 Biaxial Pressure Loading of (4H:1A)

As with the other load ratios, the (4H:1A) loading can also be scaled. Table C.3 is used as a basis for this operation. The scaled strain ratios are given in Table 5.32.

Table 5.32: Scaled Strain Ratios for (4H:1A) with Factored Hoop Stress of 187.5 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.3182	0.3113	0.6285	0.6243	0.1770	0.1758	0.6285	shear
51	0.2840	0.2769	0.5935	0.5893	0.1795	0.1783	0.5935	shear
52	0.2567	0.2499	0.5582	0.5540	0.1816	0.1804	0.5582	shear
53	0.2367	0.2298	0.5229	0.5187	0.1833	0.1822	0.5229	shear
54	0.2213	0.2143	0.4879	0.4836	0.1847	0.1836	0.4879	shear
55	0.2103	0.2032	0.4534	0.4491	0.1858	0.1846	0.4534	shear
56	0.2034	0.1962	0.4195	0.4153	0.1865	0.1854	0.4195	shear
57	0.2004	0.1931	0.3866	0.3825	0.1869	0.1858	0.3866	shear
58	0.2009	0.1937	0.3548	0.3508	0.1870	0.1859	0.3548	shear
59	0.2050	0.1975	0.3243	0.3203	0.1869	0.1859	0.3243	shear
60	0.2120	0.2043	0.2950	0.2912	0.1865	0.1855	0.2950	shear
61	0.2216	0.2137	0.2673	0.2635	0.1860	0.1850	0.2673	shear
62	0.2335	0.2255	0.2410	0.2374	0.1852	0.1843	0.2410	shear
63	0.2474	0.2393	0.2163	0.2127	0.1843	0.1835	0.2474	matrix tension
64	0.2630	0.2547	0.1932	0.1897	0.1833	0.1825	0.2630	matrix tension
65	0.2799	0.2716	0.1716	0.1682	0.1821	0.1814	0.2799	matrix tension
66	0.2979	0.2895	0.1517	0.1483	0.1809	0.1802	0.2979	matrix tension
67	0.3167	0.3083	0.1333	0.1300	0.1796	0.1790	0.3167	matrix tension
68	0.3362	0.3278	0.1164	0.1132	0.1783	0.1777	0.3362	matrix tension
69	0.3559	0.3476	0.1010	0.0978	0.1769	0.1764	0.3559	matrix tension
70	0.3758	0.3675	0.0870	0.0840	0.1755	0.1750	0.3758	matrix tension

The (4H:1A) loading condition puts no restriction on the wind angle.

5.5.4 Biaxial Pressure Loading of (2H:1A)

For the biaxial pressure loading of (2H:1A), Table C.4 is used as a basis, and is then scaled to determine the strain ratios obtained by an applied hoop stress of 187.5MPa. The scaled ratios are given in Table 5.33

Table 5.33: Scaled Strain Ratios for (2H:1A) with Factored Hoop Stress of 187.5 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.5564	0.5480	0.3172	0.3133	0.2013	0.1998	0.5564	matrix tension
51	0.5516	0.5435	0.2810	0.2771	0.2018	0.2004	0.5516	matrix tension
52	0.5522	0.5440	0.2455	0.2416	0.2020	0.2006	0.5522	matrix tension
53	0.5574	0.5492	0.2108	0.2070	0.2019	0.2004	0.5574	matrix tension
54	0.5671	0.5588	0.1773	0.1735	0.2013	0.1999	0.5671	matrix tension
55	0.5808	0.5725	0.1452	0.1414	0.2005	0.1991	0.5808	matrix tension
56	0.5983	0.5900	0.1146	0.1109	0.1994	0.1980	0.5983	matrix tension
57	0.6191	0.6108	0.0857	0.0821	0.1980	0.1967	0.6191	matrix tension
58	0.6429	0.6347	0.0586	0.0550	0.1964	0.1951	0.6429	matrix tension
59	0.6693	0.6612	0.0334	0.0298	0.1946	0.1934	0.6693	matrix tension
60	0.6979	0.6899	-0.0368	-0.0368	0.1926	0.1914	0.6979	matrix tension
61	0.7284	0.7205	-0.0561	-0.0561	0.1905	0.1894	0.7284	matrix tension
62	0.7603	0.7526	-0.0734	-0.0734	0.1882	0.1872	0.7603	matrix tension
63	0.7935	0.7858	-0.0887	-0.0887	0.1859	0.1849	0.7935	matrix tension
64	0.8276	0.8198	-0.1022	-0.1022	0.1835	0.1826	0.8276	matrix tension
65	0.8622	0.8544	-0.1137	-0.1137	0.1811	0.1802	0.8622	matrix tension
66	0.8969	0.8891	-0.1234	-0.1234	0.1787	0.1778	0.8969	matrix tension
67	0.9316	0.9238	-0.1313	-0.1313	0.1762	0.1755	0.9316	matrix tension
68	0.9660	0.9582	-0.1374	-0.1374	0.1738	0.1731	0.9660	matrix tension
69	0.9999	0.9921	-0.1419	-0.1419	0.1715	0.1708	0.9999	matrix tension
70	1.0330	1.0253	-0.1449	-0.1449	0.1692	0.1686	1.0330	matrix tension

It is seen from Table 5.33, that the strain ratio is greater than the resistance factor of 0.8 for fibre angles greater 63°, in an applied stress loading of (2H:1A). Thus the (2H:1A) loading condition restrains the range of possible fibre angles to 50° to 63°.

5.5.5 Pipeline Installation

It is possible that the loads applied on the pipeline during installation may be the maximum loadings ever to be sustained by the pipeline. In order to determine if this is the case, the strains from the installation loading must be calculated, based upon the installation parameters specified.

For a lift of 1m, and a ditch depth of 3.5m, assuming that the pipeline is supported 0.5m above ground level gives:

lift = 1m
effective depth (delta) = 0.5m + 3.5m = 4m

where delta is the distance from the top of the above ground pipe supports to the bottom of the ditch (see Figure 2.2 of Chapter 2).

The load coefficient for a lift of 1 and delta of 4 can be found in Table A.2 of Appendix A.

load coef.(c_2) = 15.09

The installation loading is equivalent to an applied bending moment, with the maximum axial stress being dependant upon the Young's modulus in the axial direction of the pipe, and therefore, the loading is dependant upon the fibre wind angle.

The installation equation is given by equation 2.4:

$$\sigma_{max} = c_2 \cdot (E \cdot \rho)^{\frac{1}{2}} \cdot \left(1 + \left(1 - 2\frac{t}{D}\right)^2\right)^{-\frac{1}{2}}$$

For a fibre angle of 50°, with ρ = 1510 kg/m^3, t/D = 0.10 and

E_{axial} = 11.02GPa (from Table C.10)

$$\sigma_{max} = 15.09 \cdot \left(11.02 x 10^3 \cdot 1510\right)^{\frac{1}{2}} \cdot \left(1 + \left(1 - 2(.10)\right)^2\right)^{-\frac{1}{2}}$$

$$\sigma_{max} = 48.05 MPa$$

Since the installation loading is dependant upon the axial Young's modulus, the moment loading, and therefore the scaling factor, is also dependant upon the fibre wind angle. Table C.10, in Appendix C, includes the axial Young's modulus for fibre angles of 50 to 70°, as determined by the Classical Laminate Theory.

Using the same 50 degree fibre angle,

$$\text{scaling factor} = \frac{(\text{moment load factor}) \cdot (\text{max axial stress})}{\text{applied max axial stress}} = \frac{1.4 x 48.05 x 10^6}{1 x 10^6} = 67.27$$

For a fibre angle of 50 degrees, scaling the strain ratios for a moment loading (Table C.5), one gets the values summarized in Table 5.34.

Table 5.34: Strain ratio scaling sample calculation for the installation loading with a lift of 1m and a delta of 4m for a fibre angle of 50 degrees

	maximum axial stress	
strain ratio	1MPa	67.27MPa
maximum matrix	5.152E-3	0.3466
minimum matrix	-2.667E-3	-0.1794
maximum shear	6.417E-3	0.4317
minimum shear	-6.380E-3	-0.4294
maximum fibre	511.1E-6	0.0344
minimum fibre	-842.2E-6	-0.0567

Due to the uncertainty involved in the use of the equation for installation loading, and the use of the CLT to determine the E_{axial} of the pipe, a load factor of 1.4 was used.

The maximum axial stresses and the factored maximum axial stresses, for each fibre wind angle, due to the installation loading is given in Table 5.35.

Table 5.35: Maximum axial stress from the installation loading for a lift of 1m, and a delta of 4m

ang	maximum axial stress (MPa)	factored maximum axial stress (MPa)
50	48.0	67.3
51	47.5	66.5
52	47.0	65.8
53	46.5	65.1
54	46.0	64.5
55	45.6	63.9
56	45.3	63.4
57	44.9	62.9
58	44.6	62.5
59	44.4	62.1
60	44.1	61.7
61	43.9	61.4
62	43.7	61.1
63	43.5	60.9
64	43.3	60.7
65	43.2	60.5
66	43.1	60.3
67	43.0	60.1
68	42.9	60.0
69	42.8	59.9
70	42.7	59.8

The scaled strain ratios due to the installation loading, for fibre angles of 50 to 70 degrees are given in Table 5.36.

Table 5.36: Scaled strain ratios for the installation load with t/D = 0.10, lift = 1m, delta = 4m

		strain ratios							
ang	scaling factor	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	67.279	0.3465	-0.1794	0.4316	-0.4294	0.0344	-0.0567	0.4316	shear
51	66.48	0.3782	-0.1958	0.4283	-0.4261	0.0314	-0.0517	0.4283	shear
52	65.75	0.4094	-0.2121	0.4240	-0.4219	0.0286	-0.0469	0.4240	shear
53	65.08	0.4402	-0.2281	0.4189	-0.4169	0.0258	-0.0423	0.4402	matrix tension
54	64.46	0.4704	-0.2438	0.4131	-0.4111	0.0231	-0.0378	0.4704	matrix tension
55	63.90	0.4999	-0.2591	0.4065	-0.4046	0.0205	-0.0334	0.4999	matrix tension
56	63.38	0.5286	-0.2740	0.3992	-0.3974	0.0180	-0.0293	0.5286	matrix tension
57	62.91	0.5566	-0.2886	0.3913	-0.3896	0.0155	-0.0252	0.5566	matrix tension
58	62.48	0.5836	-0.3026	0.3827	-0.3811	0.0132	-0.0214	0.5836	matrix tension
59	62.10	0.6097	-0.3162	0.3737	-0.3722	0.0110	-0.0177	0.6097	matrix tension
60	61.75	0.6349	-0.3292	0.3641	-0.3627	0.0102	-0.0144	0.6349	matrix tension
61	61.43	0.6590	-0.3418	0.3542	-0.3528	0.0097	-0.0139	0.6590	matrix tension
62	61.15	0.6822	-0.3538	0.3438	-0.3426	0.0092	-0.0138	0.6822	matrix tension
63	60.90	0.7043	-0.3653	0.3331	-0.3319	0.0087	-0.0136	0.7043	matrix tension
64	60.67	0.7254	-0.3762	0.3221	-0.3210	0.0081	-0.0134	0.7254	matrix tension
65	60.47	0.7455	-0.3866	0.3108	-0.3098	0.0076	-0.0132	0.7455	matrix tension
66	60.29	0.7646	-0.3965	0.2993	-0.2984	0.0071	-0.0129	0.7646	matrix tension
67	60.13	0.7827	-0.4059	0.2876	-0.2867	0.0066	-0.0126	0.7827	matrix tension
68	60.00	0.7998	-0.4148	0.2757	-0.2749	0.0061	-0.0123	0.7998	matrix tension
69	59.87	0.8159	-0.4231	0.2636	-0.2629	0.0062	-0.0119	0.8159	matrix tension
70	59.77	0.8310	-0.4310	0.2515	-0.2508	0.0074	-0.0119	0.8310	matrix tension

From Table 5.36, as a resultant of the large load factor of 1.4, fibre wind angles greater than 68° are greater than the resistance factor of 0.8, and therefore must be eliminated from the design study.

5.5.6 Force Due to Backfill

Just as the pipe must be able to withstand the moment loading from installation, the pipe must be able to withstand the applied force due to backfill while the pipe is not internally pressurized.

Applying equation 2.6 and assuming a sand and gravel backfill, and a ditch width at the top of the pipe equal to 1.7 times the OD, for a backfill depth given by:

$$H = (\text{depth of ditch}) - (\text{OD}) = 3.50\text{-}.60 = 2.90\text{m}$$

For an OD of 0.56m, B_d=1.7xOD, K_u = 0.165, and w_f = 15.7 kN/m^3:

$$C_d = \frac{1 - e^{-2\frac{(0.165)(2.9)}{1.7\times 0.60}}}{2(.0165)} = 1.844$$

and

$$W_d = (1.844)(15.7\text{x}10^3)(1.7\text{x}0.60)^2 = 30.1 \text{ kN / m}$$

Therefore the force per metre of pipe = 30.1 kN/m
and the factored force per metre of pipe = 1.25x30.1 = 37.66 kN/m

For a line loading, the strain ratio is dependant on the magnitude of the loading and the diameter of the pipe. From Table C.8, the applied force from Table C.8 of Appendix C, must be multiplied by the diameter of the pipe. Thus the scaling factor is given by:

$$\text{scaling factor} = \frac{\text{factored force}}{(\text{applied force})\cdot \text{OD}} = \frac{37.66\text{x}10^3}{1\text{x}10^3 \cdot 0.60} = 62.767$$

For a wind angle of 50 degrees, and scaling the strain ratios from Table C.9 produces Table 5.37:

Table 5.37: Strain ratio scaling sample calculation for a backfill loading, with a depth of backfill of 2.90m for a fibre angle of 50 degrees.

	force/length	
strain ratio	1kN/m per diameter	47.118 kN/m
maximum matrix	0.001402	0.08761
minimum matrix	-0.0004658	-0.02924
maximum shear	0.0008927	0.05603
minimum shear	-0.0006712	-0.04213
maximum fibre	0.0003190	0.02003
minimum fibre	-0.0003956	-0.02483

The scaled strain ratios due to a backfill of 2.90m for a t/D of 0.10 are given in Table 5.38.

Table 5.38: Scaled strain ratios for a 2.90m backfill with t/D = 0.10

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.08802	-0.02924	0.05603	-0.04213	0.02003	-0.02483	0.08802	matrix tension
51	0.08314	-0.02738	0.05514	-0.04092	0.01988	-0.02494	0.08314	matrix tension
52	0.07841	-0.02572	0.05423	-0.03972	0.01972	-0.02503	0.07841	matrix tension
53	0.07384	-0.02427	0.05329	-0.03851	0.01955	-0.02511	0.07384	matrix tension
54	0.06943	-0.02287	0.05233	-0.03731	0.01938	-0.02518	0.06943	matrix tension
55	0.06517	-0.02153	0.05135	-0.03611	0.01921	-0.02525	0.06517	matrix tension
56	0.06107	-0.02025	0.05035	-0.03491	0.01903	-0.02530	0.06107	matrix tension
57	0.05711	-0.01901	0.04934	-0.03373	0.01886	-0.02535	0.05711	matrix tension
58	0.05331	-0.01783	0.04831	-0.03255	0.01869	-0.02540	0.05331	matrix tension
59	0.04964	-0.01670	0.04726	-0.03139	0.01853	-0.02544	0.04964	matrix tension
60	0.04611	-0.01561	0.04620	-0.03023	0.01837	-0.02547	0.04620	shear
61	0.04272	-0.01457	0.04512	-0.02909	0.01821	-0.02550	0.04512	shear
62	0.03947	-0.01357	0.04403	-0.02796	0.01807	-0.02553	0.04403	shear
63	0.03634	-0.01261	0.04292	-0.02684	0.01793	-0.02556	0.04292	shear
64	0.03334	-0.01170	0.04179	-0.02573	0.01780	-0.02558	0.04179	shear
65	0.03046	-0.01082	0.04064	-0.02464	0.01768	-0.02560	0.04064	shear
66	0.02771	-0.00997	0.03947	-0.02356	0.01757	-0.02562	0.03947	shear
67	0.02508	-0.00917	0.03827	-0.02249	0.01748	-0.02563	0.03827	shear
68	0.02256	-0.00839	0.03705	-0.02143	0.01739	-0.02564	0.03705	shear
69	0.02016	-0.00765	0.03581	-0.02039	0.01731	-0.02566	0.03581	shear
70	0.01788	-0.00694	0.03454	-0.01945	0.01724	-0.02567	0.03454	shear

From Table 5.38, it can be concluded that all fibre wind angles considered in this study will be able to support the backfill.

5.5.7 Load Combinations: (1H:0A) and Backfill

The load combinations are obtained by simply combining the strain ratios of the pressure loading and the backfill loading. This creates a "worst case" condition, assuming that the maximum strains from the backfill loading occurs at the same locations as the maximum strains from the pressure loading. The (1H:0A) strain ratios are from §5.5.1 while the backfill strain ratios are from §5.5.6.

Table 5.39 details the summed strain ratios for the combined loading for a fibre angle of 50 degrees.

Table 5.39: Summed strain ratio sample calculation for a combined loading of (1H:0A) and backfill, with a hoop stress of 187.5 MPa and 2.90m of backfill, for a fibre angle of 50 degrees

strain ratio	(1h:0a)	backfill	sum
maximum matrix	0.0801	0.0880	0.1681
minimum matrix	0.0747	-0.0292	0.0454
maximum shear	0.9399	0.0560	0.9959
minimum shear	-0.7711	-0.0421	-0.8133
maximum fibre	0.1527	0.0200	0.1728
minimum fibre	0.1518	-0.0248	0.1270

The summed strain ratios for the entire range of fibre angles is given in Table 5.40.

Table 5.40: Summed strain ratios for a 2.90m backfill and a hoop stress from a (1H:0A) loading of 187.5 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comments
50	0.1681	0.0454	0.9959	-0.8133	0.1728	0.1270	0.9959	shear	too high
51	0.1030	-0.0133	0.9612	-0.7905	0.1770	0.1313	0.9612	shear	too high
52	0.0583	-0.0486	0.9252	-0.7669	0.1809	0.1352	0.9252	shear	too high
53	0.0297	-0.0716	0.8883	-0.7425	0.1844	0.1388	0.8883	shear	too high
54	0.0022	-0.0935	0.8508	-0.7176	0.1875	0.1420	0.8508	shear	too high
55	-0.0226	-0.1127	0.8129	-0.6923	0.1902	0.1448	0.8129	shear	too high
56	-0.0446	-0.1295	0.7748	-0.6667	0.1926	0.1474	0.7748	shear	
57	-0.0639	-0.1438	0.7369	-0.6411	0.1946	0.1495	0.7369	shear	
58	-0.0808	-0.1558	0.6994	-0.6155	0.1963	0.1514	0.6994	shear	
59	-0.0953	-0.1657	0.6624	-0.5901	0.1977	0.1529	0.6624	shear	
60	-0.1075	-0.1734	0.6261	-0.5650	0.1988	0.1542	0.6261	shear	
61	-0.1177	-0.1793	0.5906	-0.5402	0.1997	0.1552	0.5906	shear	
62	-0.1259	-0.1834	0.5561	-0.5158	0.2003	0.1559	0.5561	shear	
63	-0.1324	-0.1859	0.5227	-0.4919	0.2007	0.1564	0.5227	shear	
64	-0.1374	-0.1870	0.4905	-0.4685	0.2009	0.1568	0.4905	shear	
65	-0.1409	-0.1868	0.4594	-0.4457	0.2009	0.1569	0.4594	shear	
66	-0.1432	-0.1855	0.4296	-0.4234	0.2008	0.1569	0.4296	shear	
67	-0.1444	-0.1833	0.4011	-0.4017	0.2005	0.1568	0.4011	shear	
68	-0.1446	-0.1802	0.3738	-0.3806	0.2002	0.1566	0.3738	shear	
69	-0.1441	-0.1765	0.3477	-0.3600	0.1997	0.1562	0.3477	shear	
70	-0.1428	-0.1722	0.3229	-0.3400	0.1992	0.1558	0.3229	shear	

This loading combination reveals that the maximum strain ratios for fibre wind angles less than 55° are greater than the resistance factor of 0.8, and should therefore be excluded from the study.

5.5.8 Load Combinations: (8H:1A) and Backfill

The other load combinations simply follow the same format as that used for §5.5.7. The summed strain ratios for this loading are given in Table 5.41.

Table 5.41: Summed strain ratios for a 2.90m backfill and a hoop stress from a (8H:1A) loading of 187.5 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comments
50	0.2872	0.1637	0.8402	0.7377	0.1849	0.1390	0.8402	shear	too high
51	0.2351	0.1181	0.8049	0.7044	0.1882	0.1423	0.8049	shear	too high
52	0.1883	0.0775	0.7689	0.6704	0.1911	0.1453	0.7689	shear	
53	0.1502	0.0459	0.7323	0.6360	0.1936	0.1479	0.7323	shear	
54	0.1178	0.0192	0.6955	0.6014	0.1958	0.1502	0.6955	shear	
55	0.0902	-0.0030	0.6588	0.5669	0.1976	0.1521	0.6588	shear	
56	0.0572	-0.0279	0.6224	0.5327	0.1990	0.1537	0.6224	shear	
57	0.0442	-0.0358	0.5865	0.4990	0.2002	0.1550	0.5865	shear	
58	0.0335	-0.0416	0.5513	0.4661	0.2010	0.1560	0.5513	shear	
59	0.0250	-0.0453	0.5169	0.4341	0.2016	0.1567	0.5169	shear	
60	0.0185	-0.0473	0.4837	0.4031	0.2019	0.1571	0.4837	shear	
61	0.0139	-0.0476	0.4515	0.3733	0.2019	0.1573	0.4515	shear	
62	0.0110	-0.0463	0.4206	0.3447	0.2018	0.1573	0.4206	shear	
63	0.0095	-0.0438	0.3910	0.3175	0.2014	0.1572	0.3910	shear	
64	0.0094	-0.0400	0.3627	0.2916	0.2010	0.1568	0.3627	shear	
65	0.0105	-0.0353	0.3359	0.2671	0.2003	0.1564	0.3359	shear	
66	0.0126	-0.0296	0.3104	0.2440	0.1996	0.1558	0.3104	shear	
67	0.0155	-0.0233	0.2863	0.2222	0.1988	0.1551	0.2863	shear	
68	0.0438	-0.0163	0.2636	0.2019	0.1979	0.1543	0.2636	shear	
69	0.0541	0.0176	0.2423	0.1829	0.1970	0.1535	0.2423	shear	
70	0.0650	0.0317	0.2222	0.1652	0.1960	0.1526	0.2222	shear	

This load combination reveals that for a (8H:1A) loading with an applied hoop stress of 187.5MPa and a backfill loading of 2.90m, that the strain ratios occurring in the composite pipe for fibre angles less than 52° are higher than the resistance factor of 0.8.

5.5.9 Load Combinations: (4H:1A) and Backfill

The resultant strain ratios from the combined loadings of (4H:1A) and a backfill of 2.90m are given in Table 5.42.

Table 5.42: Summed strain ratios for a 2.90m backfill and a hoop stress from a (4H:1A) loading of 187.5 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comments
50	0.4063	0.2820	0.6846	0.5822	0.1970	0.1510	0.6846	shear	
51	0.3672	0.2495	0.6487	0.5483	0.1994	0.1534	0.6487	shear	
52	0.3351	0.2242	0.6125	0.5142	0.2013	0.1554	0.6125	shear	
53	0.3105	0.2055	0.5762	0.4801	0.2029	0.1571	0.5762	shear	
54	0.2907	0.1914	0.5402	0.4463	0.2041	0.1584	0.5402	shear	
55	0.2754	0.1816	0.5047	0.4130	0.2050	0.1594	0.5047	shear	
56	0.2644	0.1759	0.4699	0.3804	0.2055	0.1600	0.4699	shear	
57	0.2575	0.1741	0.4360	0.3488	0.2057	0.1604	0.4360	shear	
58	0.2542	0.1758	0.4031	0.3182	0.2057	0.1606	0.4031	shear	
59	0.2546	0.1808	0.3715	0.2889	0.2054	0.1604	0.3715	shear	
60	0.2581	0.1887	0.3412	0.2609	0.2049	0.1601	0.3412	shear	
61	0.2643	0.1992	0.3124	0.2344	0.2042	0.1595	0.3124	shear	
62	0.2730	0.2119	0.2850	0.2094	0.2033	0.1588	0.2850	shear	
63	0.2838	0.2266	0.2592	0.1859	0.2022	0.1579	0.2838	matrix tension	
64	0.2963	0.2430	0.2350	0.1639	0.2011	0.1569	0.2963	matrix tension	
65	0.3104	0.2607	0.2123	0.1435	0.1998	0.1558	0.3104	matrix tension	
66	0.3256	0.2796	0.1912	0.1247	0.1985	0.1546	0.3256	matrix tension	
67	0.3418	0.2992	0.1716	0.1075	0.1971	0.1533	0.3418	matrix tension	
68	0.3587	0.3194	0.1535	0.0917	0.1957	0.1520	0.3587	matrix tension	
69	0.3761	0.3399	0.1368	0.0775	0.1942	0.1507	0.3761	matrix tension	
70	0.3937	0.3606	0.1216	0.0645	0.1928	0.1493	0.3937	matrix tension	

The strain ratio results in Table 5.42 reveals that all fibre angles between 50° and 70° are valid for this load combination.

5.5.10 Load Combinations: (2H:1A) and Backfill

For a load combination of (2H:1A) with a hoop stress of 187.5 MPa and backfill of 2.90m, the resultant strain ratios are given in Table 5.43.

Table 5.43: Summed strain ratios for a 2.90m backfill and a hoop stress from a (2H:1A) loading of 187.5 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comments
50	0.6444	0.5187	0.3732	0.2712	0.2213	0.1749	0.6444	matrix tension	
51	0.6348	0.5161	0.3361	0.2362	0.2217	0.1754	0.6348	matrix tension	
52	0.6306	0.5183	0.2997	0.2019	0.2218	0.1755	0.6306	matrix tension	
53	0.6313	0.5249	0.2641	0.1685	0.2214	0.1753	0.6313	matrix tension	
54	0.6365	0.5359	0.2296	0.1362	0.2207	0.1748	0.6365	matrix tension	
55	0.6460	0.5510	0.1965	0.1053	0.2197	0.1739	0.6460	matrix tension	
56	0.6593	0.5697	0.1649	0.0760	0.2184	0.1727	0.6593	matrix tension	
57	0.6762	0.5918	0.1350	0.0483	0.2169	0.1714	0.6762	matrix tension	
58	0.6962	0.6169	0.1069	0.0224	0.2151	0.1697	0.6962	matrix tension	
59	0.7189	0.6445	0.0807	-0.0016	0.2131	0.1679	0.7189	matrix tension	
60	0.7440	0.6743	0.0094	-0.0671	0.2110	0.1660	0.7440	matrix tension	
61	0.7711	0.7059	-0.0110	-0.0852	0.2087	0.1639	0.7711	matrix tension	
62	0.7998	0.7390	-0.0294	-0.1013	0.2063	0.1616	0.7998	matrix tension	
63	0.8298	0.7732	-0.0458	-0.1156	0.2038	0.1594	0.8298	matrix tension	too high
64	0.8609	0.8082	-0.0604	-0.1279	0.2013	0.1570	0.8609	matrix tension	too high
65	0.8926	0.8436	-0.0730	-0.1383	0.1988	0.1546	0.8926	matrix tension	too high
66	0.9247	0.8791	-0.0839	-0.1469	0.1962	0.1522	0.9247	matrix tension	too high
67	0.9567	0.9146	-0.0930	-0.1537	0.1937	0.1498	0.9567	matrix tension	too high
68	0.9886	0.9498	-0.1004	-0.1589	0.1912	0.1475	0.9886	matrix tension	too high
69	1.0200	0.9844	-0.1061	-0.1623	0.1888	0.1452	1.0200	matrix tension	too high
70	1.0509	1.0183	-0.1104	-0.1643	0.1864	0.1429	1.0509	matrix tension	too high

The load combination of (2H:1A) and backfill eliminates fibre wind angles greater than 62 degrees.

5.5.11 Load Combinations: (1H:0A), Backfill and Bending Moment

Utilizing the strain ratios from the load combinations from §5.5.7 through §5.5.10 it is possible then to calculate the maximum bending moment which will make the maximum strain ratio equal to the resistance factor (0.8).

In the following tables, a maximum ratio not equal to 0.8 or of "#N/A" means that the strain ratio was greater than 0.8 before the application of the moment, and thus that fibre angle need not be considered.

For this loading combination, the scaling factor is determined by finding the minimum value which will make any strain ratio equal to 0.8

$$\text{scaling factor} = \min\left(\text{abs}\left(\frac{\text{resistance factor} - \text{strain ratio(i)}}{\text{moment strain ratio(i)}}\right)\right)$$

where:

resistance factor =	0.8
ratio(i) =	strain ratio (min or max of fibre, matrix or shear) from combined pressure and Backfill loading
moment strain ratio(i) =	strain ratio (min or max of fibre, matrix or shear) from moment loading

The ratio(i) values are taken from sections §5.5.7-§5.5.10 of this design study, while the moment strain ratio(i) values are taken from Table C.5 of Appendix C.

For a fibre angle of 60 degrees, the scaling factor is determined by finding the minimum of possible scaling factors. The calculations are shown in Table 5.44.

Table 5.44: Determination of minimum scaling factor for determination of maximum allowable bending moment on a pipe loaded with (1H:0A) with a hoop stress of 187.5 MPa and a backfill of 2.90m for a fibre angle of 60 degrees

strain ratio	strain ratio from (1H:0A) and Backfill	strain ratio from unit moment loading	$\text{abs}\left(\frac{\text{resistance factor} - \text{strain ratio}}{\text{moment strain ratio}}\right)$
maximum matrix	-0.1075	0.0103	88.26
minimum matrix	-0.1734	-0.0053	182.6
maximum shear	0.6261	0.0059	29.49
minimum shear	-0.5650	-0.0059	232.4
maximum fibre	0.1988	0.0002	3628
minimum fibre	0.1542	-0.0002	2775

Thus the minimum scaling ratio is 29.49.

Now, using the minimum scaling factor, the maximum strain ratio should be equal to 0.8, the resistance factor. The calculations are detailed in Table 5.45.

Table 5.45: Sample calculation of summed and scaled strain ratios for (1H:0A) and pressure and bending moment for a fibre angle of 60 degrees

strain ratio	strain ratio from (1h:0a) + backfill	strain ratio from unit moment loading	scaled moment strain ratio	summed strain ratio from (1h:0a) + backfill + scaled moment
maximum matrix	-0.1075	0.0103	0.3033	0.1958
minimum matrix	-0.1734	-0.0053	-0.1573	-0.3307
maximum shear	0.6261	0.0059	0.1739	0.8000
minimum shear	-0.5650	-0.0059	-0.1733	-0.7382
maximum fibre	0.1988	0.0002	0.0049	0.2037
minimum fibre	0.1542	-0.0002	-0.0069	0.1473

Thus the maximum strain ratio is equal to 0.8, the resistance factor. The maximum strain ratio occurs in shear.

The maximum axial stress is found by multiplying the maximum axial stress used in Table C.5 (from Appendix C) by the scaling factor.

Scaling the maximum axial stress from the moment loading is obtained by multiplying the scaling factor by the unit load from Table C.5

Scaled maximum axial stress = 29.49 x1MPa = 29.49MPa

The actual moment load which causes the maximum axial stress can be calculated from:

$$\sigma_{max} = \frac{M \cdot OD}{2I} \therefore M = \frac{2\sigma_{max} I}{OD}$$

So, for the fibre angle of 60 degrees:

$$M = \frac{2x29.49x3.29x10^{-3}}{0.60} = 324 \text{ kN} \cdot \text{m}$$

The strain ratios for the load combination of backfill, pure pressure, and bending moment are given in Table 5.46.

Table 5.46: Strain Ratios resulting from a backfill of 2.90m, hoop stress of 187.5 MPa in a (1H:0A) loading, and the maximum allowable moment

		strain ratios							
ang	scaling factor	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	30.53	0.3254	-0.0360	1.1918	-1.0082	0.1884	0.1013	1.1918	#N/A
51	25.02	0.2453	-0.0870	1.1224	-0.9509	0.1889	0.1118	1.1224	#N/A
52	19.42	0.1792	-0.1112	1.0505	-0.8915	0.1893	0.1213	1.0505	#N/A
53	13.72	0.1225	-0.1197	0.9767	-0.8304	0.1898	0.1299	0.9767	#N/A
54	7.928	0.0600	-0.1235	0.9016	-0.7682	0.1903	0.1373	0.9016	#N/A
55	2.024	-0.0067	-0.1210	0.8258	-0.7051	0.1909	0.1438	0.8258	#N/A
56	3.994	-0.0113	-0.1467	0.8000	-0.6918	0.1937	0.1455	0.8000	shear
57	10.14	0.0258	-0.1903	0.8000	-0.7039	0.1971	0.1455	0.8000	shear
58	16.43	0.0726	-0.2354	0.8000	-0.7157	0.1998	0.1457	0.8000	shear
59	22.87	0.1293	-0.2821	0.8000	-0.7272	0.2018	0.1464	0.8000	shear
60	29.49	0.1958	-0.3307	0.8000	-0.7382	0.2037	0.1473	0.8000	shear
61	36.32	0.2720	-0.3813	0.8000	-0.7488	0.2054	0.1469	0.8000	shear
62	43.37	0.3580	-0.4343	0.8000	-0.7588	0.2068	0.1461	0.8000	shear
63	50.69	0.4538	-0.4899	0.8000	-0.7682	0.2079	0.1451	0.8000	shear
64	58.30	0.5597	-0.5485	0.8000	-0.7770	0.2087	0.1439	0.8000	shear
65	66.26	0.6760	-0.6105	0.8000	-0.7852	0.2092	0.1425	0.8000	shear
66	74.38	0.8000	-0.6747	0.7988	-0.7915	0.2095	0.1410	0.8000	matrix tension
67	72.56	0.8000	-0.6731	0.7480	-0.7477	0.2085	0.1416	0.8000	matrix tension
68	70.86	0.8000	-0.6702	0.6994	-0.7052	0.2073	0.1420	0.8000	matrix tension
69	69.28	0.8000	-0.6661	0.6527	-0.6642	0.2069	0.1424	0.8000	matrix tension
70	67.81	0.8000	-0.6612	0.6081	-0.6245	0.2077	0.1423	0.8000	matrix tension

The maximum axial stresses and associated bending moments that yield the strain ratios in Table 5.46 are given in Table 5.47. The maximum axial stress is scaled by the scaling factor for each angle. The bending moment is calculated using the physical dimensions of the pipe and the scaled maximum axial stress.

Table 5.47 Scaled maximum axial stresses and moments for backfill of 2.90m and a hoop stress of 187.5 MPa in a (1H:0A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	#N/A	#N/A
51	#N/A	#N/A
52	#N/A	#N/A
53	#N/A	#N/A
54	#N/A	#N/A
55	#N/A	#N/A
56	3.994	43.85
57	10.14	111.3
58	16.43	180.4
59	22.87	251.1
60	29.49	323.8
61	36.32	398.7
62	43.37	476.2
63	50.69	556.5
64	58.30	640.1
65	66.26	727.5
66	74.38	816.6
67	72.56	796.6
68	70.86	778.0
69	69.28	760.7
70	67.81	744.5

5.5.12 Load Combinations: (8H:1A), Backfill and Bending Moment

For the this load combination, the combined strain ratios are given in Table 5.48 and the scaled maximum axial stress and bending moment are given in Table 5.49.

5.5.13 Load Combinations: (8H:1A), Backfill and Bending Moment

For the this load combination, the combined strain ratios are given in Table 5.50 and the scaled maximum axial stress and bending moment are given in Table 5.51.

5.5.14 Load Combinations: (8H:1A), Backfill and Bending Moment

For the this load combination, the combined strain ratios are given in Table 5.52 and the scaled maximum axial stress and bending moment are given in Table 5.53.

Table 5.48: Strain Ratios resulting from a backfill of 2.90m, hoop stress of 187.5 MPa in a (8H:1A) loading, and the maximum allowable moment

ang	scaling factor	strain ratios maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	6.271	0.3195	0.1470	0.8805	0.6977	0.1881	0.1337	0.8805	#N/A
51	0.7662	0.2394	0.1159	0.8099	0.6995	0.1886	0.1417	0.8099	#N/A
52	4.8293	0.2184	0.0619	0.8000	0.6394	0.1932	0.1418	0.8000	shear
53	10.518	0.2213	0.0090	0.8000	0.5686	0.1978	0.1411	0.8000	shear
54	16.306	0.2368	-0.0425	0.8000	0.4974	0.2016	0.1406	0.8000	shear
55	22.198	0.2638	-0.0931	0.8000	0.4263	0.2047	0.1405	0.8000	shear
56	28.204	0.2925	-0.1499	0.8000	0.3558	0.2070	0.1407	0.8000	shear
57	34.336	0.3480	-0.1933	0.8000	0.2864	0.2087	0.1412	0.8000	shear
58	40.608	0.4128	-0.2382	0.8000	0.2184	0.2096	0.1421	0.8000	shear
59	47.037	0.4869	-0.2848	0.8000	0.1522	0.2099	0.1432	0.8000	shear
60	53.642	0.5701	-0.3333	0.8000	0.0880	0.2107	0.1446	0.8000	shear
61	60.448	0.6624	-0.3839	0.8000	0.0261	0.2115	0.1436	0.8000	shear
62	67.482	0.7638	-0.4368	0.8000	-0.0333	0.2119	0.1421	0.8000	shear
63	68.343	0.8000	-0.4537	0.7648	-0.0550	0.2112	0.1419	0.8000	matrix tension
64	66.117	0.8000	-0.4500	0.7137	-0.0582	0.2098	0.1422	0.8000	matrix tension
65	64.037	0.8000	-0.4447	0.6650	-0.0609	0.2084	0.1424	0.8000	matrix tension
66	62.092	0.8000	-0.4380	0.6186	-0.0633	0.2069	0.1425	0.8000	matrix tension
67	60.276	0.8000	-0.4301	0.5745	-0.0651	0.2054	0.1424	0.8000	matrix tension
68	56.730	0.8000	-0.4085	0.5243	-0.0580	0.2036	0.1427	0.8000	matrix tension
69	54.741	0.8000	-0.3692	0.4833	-0.0574	0.2027	0.1425	0.8000	matrix tension
70	52.858	0.8000	-0.3495	0.4446	-0.0566	0.2026	0.1421	0.8000	matrix tension

Table 5.49 Scaled maximum axial stresses and moments for backfill of 2.90m and a hoop stress of 187.5 MPa in a (8H:1A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	#N/A	#N/A
51	#N/A	#N/A
52	4.829	53.02
53	10.52	115.5
54	16.31	179.0
55	22.20	243.7
56	28.20	309.7
57	34.34	377.0
58	40.61	445.8
59	47.04	516.4
60	53.64	588.9
61	60.45	663.7
62	67.48	740.9
63	68.34	750.4
64	66.12	725.9
65	64.04	703.1
66	62.09	681.7
67	60.28	661.8
68	56.73	622.8
69	54.74	601.0
70	52.86	580.3

Table 5.50: Strain Ratios resulting from a backfill of 2.90m, hoop stress of 187.5 MPa in a (4H:1A) loading, and the maximum allowable moment

ang	scaling factor	strain ratios							
		maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	17.99	0.49896	0.23406	0.80000	0.46733	0.19715	0.13581	0.80000	shear
51	23.49	0.50079	0.18028	0.80000	0.39778	0.21048	0.13507	0.80000	shear
52	29.08	0.51615	0.13043	0.80000	0.32764	0.21395	0.13462	0.80000	shear
53	34.76	0.54564	0.08373	0.80000	0.25746	0.21666	0.13446	0.80000	shear
54	40.54	0.58651	0.03812	0.80000	0.18777	0.21861	0.13461	0.80000	shear
55	46.42	0.63858	-0.00660	0.80000	0.11907	0.21984	0.13507	0.80000	shear
56	52.41	0.70160	-0.05068	0.80000	0.05179	0.22035	0.13585	0.80000	shear
57	58.53	0.77530	-0.09437	0.80000	-0.01367	0.22020	0.13695	0.80000	shear
58	58.43	0.80000	-0.10716	0.76106	-0.03819	0.21807	0.14056	0.80000	matrix tension
59	55.55	0.80000	-0.10204	0.70579	-0.04401	0.21525	0.14458	0.80000	matrix tension
60	52.71	0.80000	-0.09236	0.65207	-0.04867	0.21362	0.14779	0.80000	matrix tension
61	49.93	0.80000	-0.07865	0.60027	-0.05238	0.21207	0.14820	0.80000	matrix tension
62	47.24	0.80000	-0.06140	0.55063	-0.05523	0.21038	0.14812	0.80000	matrix tension
63	44.63	0.80000	-0.04109	0.50336	-0.05739	0.20860	0.14790	0.80000	matrix tension
64	42.12	0.80000	-0.01820	0.45859	-0.05895	0.20674	0.14756	0.80000	matrix tension
65	39.71	0.80000	0.00681	0.41640	-0.05993	0.20483	0.14711	0.80000	matrix tension
66	37.41	0.80000	0.03354	0.37683	-0.06038	0.20290	0.14657	0.80000	matrix tension
67	35.20	0.80000	0.06156	0.33989	-0.06037	0.20096	0.14594	0.80000	matrix tension
68	33.10	0.80000	0.09051	0.30557	-0.05995	0.19903	0.14524	0.80000	matrix tension
69	31.11	0.80000	0.12004	0.27380	-0.05915	0.19747	0.14448	0.80000	matrix tension
70	29.22	0.80000	0.14982	0.24451	-0.05811	0.19643	0.14353	0.80000	matrix tension

Table 5.51 Scaled maximum axial stresses and moments for backfill of 2.90m and a hoop stress of 187.5 MPa in a (4H:1A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	17.99	197.5
51	23.49	257.9
52	29.08	319.3
53	34.76	381.6
54	40.54	445.1
55	46.42	509.7
56	52.41	575.5
57	58.53	642.6
58	58.43	641.5
59	55.55	609.9
60	52.71	578.7
61	49.93	548.2
62	47.24	518.7
63	44.63	490.0
64	42.12	462.5
65	39.71	436.0
66	37.41	410.7
67	35.20	386.5
68	33.10	363.5
69	31.11	341.6
70	29.22	320.9

Table 5.52: Strain Ratios resulting from a backfill of 2.90m, hoop stress of 187.5 MPa in a (2H:1A) loading, and the maximum allowable moment

		strain ratios							
ang	scaling factor	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	30.20	0.8000	0.4382	0.5670	0.0783	0.2367	0.1495	0.8000	matrix tension
51	29.05	0.8000	0.4306	0.5233	0.0500	0.2355	0.1528	0.8000	matrix tension
52	27.21	0.8000	0.4305	0.4751	0.0273	0.2336	0.1561	0.8000	matrix tension
53	24.95	0.8000	0.4375	0.4247	0.0087	0.2313	0.1591	0.8000	matrix tension
54	22.41	0.8000	0.4512	0.3732	-0.0067	0.2287	0.1616	0.8000	matrix tension
55	19.69	0.8000	0.4711	0.3218	-0.0194	0.2260	0.1636	0.8000	matrix tension
56	16.87	0.8000	0.4968	0.2712	-0.0298	0.2232	0.1650	0.8000	matrix tension
57	13.99	0.8000	0.5276	0.2220	-0.0383	0.2203	0.1657	0.8000	matrix tension
58	11.11	0.8000	0.5630	0.1750	-0.0454	0.2174	0.1659	0.8000	matrix tension
59	8.255	0.8000	0.6025	0.1304	-0.0511	0.2146	0.1656	0.8000	matrix tension
60	5.442	0.8000	0.6453	0.0414	-0.0990	0.2119	0.1647	0.8000	matrix tension
61	2.692	0.8000	0.6910	0.0045	-0.1006	0.2091	0.1632	0.8000	matrix tension
62	0.019	0.8000	0.7389	-0.0293	-0.1015	0.2063	0.1616	0.8000	matrix tension
63	2.577	0.8596	0.7577	-0.0317	-0.1296	0.2042	0.1588	0.8596	#N/A
64	1.314	0.8766	0.8000	-0.0534	-0.1348	0.2015	0.1567	0.8766	#N/A
65	6.812	0.9766	0.8000	-0.0380	-0.1732	0.1996	0.1531	0.9766	#N/A
66	9.829	1.0493	0.8145	-0.0351	-0.1956	0.1974	0.1501	1.0493	#N/A
67	12.04	1.1134	0.8333	-0.0354	-0.2111	0.1950	0.1473	1.1134	#N/A
68	14.15	1.1772	0.8520	-0.0354	-0.2237	0.1927	0.1446	1.1772	#N/A
69	16.15	1.2401	0.8703	-0.0350	-0.2332	0.1905	0.1419	1.2401	#N/A
70	18.04	1.3018	0.8882	-0.0344	-0.2401	0.1886	0.1393	1.3018	#N/A

Table 5.53 Scaled maximum axial stresses and moments for backfill of 2.90m and a hoop stress of 187.5 MPa in a (2H:1A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	30.20	331.6
51	29.05	318.9
52	27.21	298.7
53	24.95	273.9
54	22.41	246.0
55	19.69	216.2
56	16.87	185.2
57	13.99	153.7
58	11.11	122.0
59	8.255	90.64
60	5.442	59.75
61	2.692	29.56
62	0.019	0.203
63	#N/A	#N/A
64	#N/A	#N/A
65	#N/A	#N/A
66	#N/A	#N/A
67	#N/A	#N/A
68	#N/A	#N/A
69	#N/A	#N/A
70	#N/A	#N/A

5.5.15 Consideration for Buoyancy

A pipeline often has to cross rivers and wetlands. It is therefore often necessary to determine the buoyancy of the pipeline caused by the displaced water. According to equation (2.5), the buoyant force per unit length is determined by:

$$F_b = \rho \cdot g \cdot V = \rho \cdot g \cdot \pi \left(\frac{OD}{2} \right)^2$$

where:

F_b	= buoyancy force per unit length (N/m)
ρ	= density of water (kg/m^3)
	= 1000 kg/m^3
g	= acceleration due to gravity
	= 9.81 m/s^2
V	= volume of water displaced by the pipe, per unit length (m^3/m)

So for the pipeline from the current study, with OD = 0.60m, the buoyant force would be:

$$F_b = 1000 \text{x} 9.81 \cdot \pi \cdot \left(\frac{0.60}{2} \right)^2 = 2.77 \text{ kN / m}$$

For an empty pipeline, which is the worst case condition, the weight of the pipeline per metre length is given by equation (2.1):

$$w_p = \rho_p \cdot g \cdot \pi \cdot \left(\left(\frac{OD}{2} \right)^2 - \left(\frac{ID}{2} \right)^2 \right)$$

$$= 1510 \cdot g \cdot \pi \cdot \left(\left(\frac{0.60}{2} \right)^2 - \left(\frac{0.50}{2} \right)^2 \right) = 1.28 \text{ kN / m}$$

where the density of the pipeline is 1510 kg/m^3, from Table 4.1 of Chapter 4.

Thus there is a positive buoyant force of 1.49 kN per metre length of the pipeline. To stop the pipeline from floating to the surface, negative buoyancy is required. To accomplish this, the pipeline requires weights or anchors to provide a minimum force of 1.49 kN per metre of pipeline length. By comparison to the backfill loading calculations in section 5.5.6, the pipe proved to be capable of withstanding a load of 37.66 kN per metre length for every fibre angle under consideration in this study. Thus the pipe would easily be able to withstand the forces applied by the anchors or weights required to achieve negative buoyancy.

5.5.16 Summary of Design Study 2

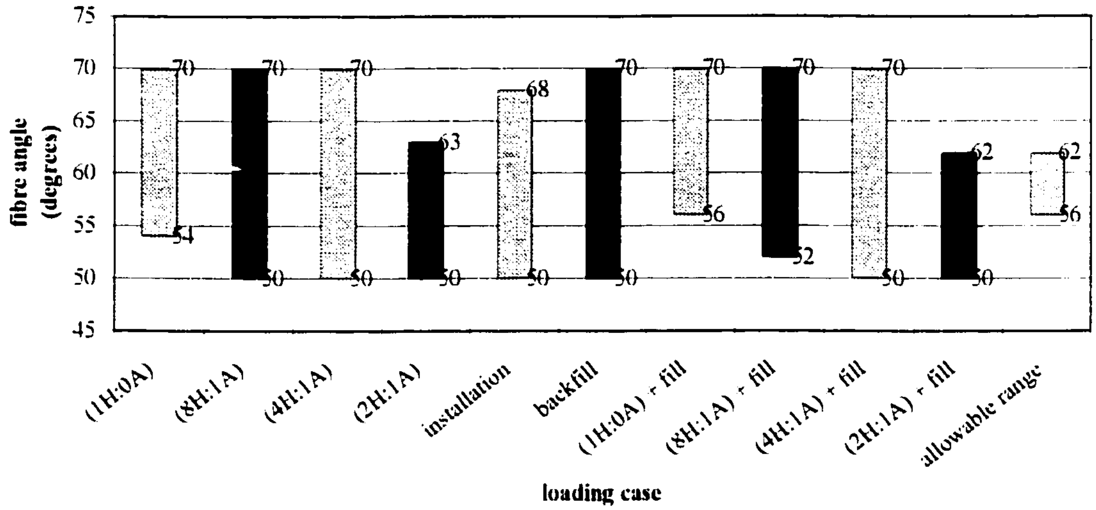

Figure 5.2: Fibre angle compatibilities for Design Study 2

Figure 5.2 is a summary of the allowable fibre angles as determined by the loadings considered in this design study. As with the previous design study, the limitations to the fibre angles tend to come from the most extreme loading cases. In this case, the fibre angles are most restricted by the (1H:0A) and backfill loading combination and the (2H:1A) and backfill loading combination.

The design criteria for this study is satisfied for fibre angles between 56 and 62 degrees. Assuming that the primary loading condition to be the combination of (1H:0A) and backfill, the fibre wind angle should be near the high end of the allowable range, i.e. 62°.

5.6 Design Study 3a

The purpose of this design study is to find the feasible fibre angles for a composite pipe, based on a set of given loading conditions. Design study 3 also considers a the ability of the pipe to withstand a moment loading. The first part of design study 3 is based on a thickness to diameter ratio of 0.04. The second part of design study 3 repeats the same calculations for a t/D ratio of 0.05 to show the effects of increasing the thickness of the pipe on the "reserve" of the pipe to withstand a bending moment while under the loadings of internal pressure and backfill. This design study also shows the effect of changing the thickness on the window of fibre angles which satisfy the design criteria.

Given:

- Internal pressure = 11 MPa (1.6 ksi)

- Possible pressure loadings:
  (1H:0A), (8H:1A), (4H:1A), (2H:1A)

- Factors:
  Resistance factor = 0.8
  Load factor for pressure = 1.25
  Load factor for backfill = 1.25
  Load factor for moment = 1.4

- ID = 1m

- Installation Parameters:
  lift = 1m
  ditch depth = 2m

Solution:

For a pressure of 11 MPa, assume t/D = 0.04

therefore:

$$OD = \frac{ID}{\left(1 - 2 \cdot \frac{t}{D}\right)} = 1.08m$$

and

$$I = \frac{\pi}{4} \cdot \left(\left(\frac{OD}{2} \right)^4 - \left(\frac{ID}{2} \right)^4 \right) = 17.69x10^{-3} m^4$$

To calculate the factored hoop stress:

$$\sigma_H = \frac{\text{load factor} \cdot \text{pressure}}{2 \cdot \frac{t}{D}} = \frac{1.25 \cdot 11}{2 \cdot 0.04} = 171.9 \text{ MPa}$$

5.5.1 Biaxial Pressure Loading of (1H:0A)

As with the previous design studies, the strain results obtained from the hoop stress of 1MPa finite element models are scaled to the applied hoop stress of 171.9 MPa, occurring in this loading. Table C.1, in Appendix C, contains the numerical values of the strain ratios for fibre angles of 50° to 70° obtained from a (1H:0A) loading with a hoop stress of 1MPa.

From Table C.1, a scaling factor can be used to scale the strain ratios to the factored hoop stress of 171.9 MPa:

$$\text{scaling factor} = \frac{\text{factored hoop stress}}{\text{applied hoop stress}} = \frac{171.9\text{MPa}}{1\text{MPa}} = 171.9$$

The factoring of strain ratios is demonstrated in Table 5.54 for a fibre angle of 50 degrees.

Table 5.54: Strain ratio scaling sample calculation for a hoop stress of 171.9 MPa, in a (1H:0A) loading for a fibre angle of 50 degrees

	hoop stress	
strain ratio	1MPa	171.9MPa
maximum matrix	427.3E-6	0.0734
minimum matrix	398.3E-6	0.0685
maximum shear	5.013E-3	0.8616
minimum shear	-4.113E-3	-0.7069
maximum fibre	814.6E-6	0.1400
minimum fibre	809.7E-6	0.1392

Applying the scaling factor to all of the strain ratios for fibre angles of 50° to 70° produces Table 5.55.

Table 5.55: Scaled strain ratios for (1H:0A) with factored hoop stress of 171.9 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.0734	0.0685	0.8616	-0.7069	0.1400	0.1392	0.8616	shear
51	0.0182	0.0129	0.8306	-0.6871	0.1440	0.1432	0.8306	shear
52	-0.0185	-0.0210	0.7984	-0.6666	0.1477	0.1469	0.7984	shear
53	-0.0404	-0.0434	0.7655	-0.6453	0.1511	0.1502	0.7655	shear
54	-0.0617	-0.0647	0.7319	-0.6236	0.1541	0.1532	0.7319	shear
55	-0.0804	-0.0836	0.6981	-0.6015	0.1568	0.1559	0.6981	shear
56	-0.0968	-0.1001	0.6641	-0.5792	0.1591	0.1583	0.6641	shear
57	-0.1110	-0.1144	0.6303	-0.5568	0.1611	0.1603	0.6303	shear
58	-0.1229	-0.1265	0.5968	-0.5344	0.1628	0.1620	0.5968	shear
59	-0.1328	-0.1365	0.5638	-0.5122	0.1643	0.1635	0.5638	shear
60	-0.1408	-0.1447	0.5315	-0.4902	0.1654	0.1647	0.0005	shear
61	-0.1470	-0.1510	0.5000	-0.4685	0.1663	0.1656	0.5000	shear
62	-0.1516	-0.1557	0.4694	-0.4472	0.1670	0.1663	0.4694	shear
63	-0.1547	-0.1588	0.4398	-0.4263	0.1675	0.1668	0.4398	shear
64	-0.1565	-0.1607	0.4113	-0.4059	0.1678	0.1672	0.4113	shear
65	-0.1571	-0.1613	0.3839	-0.3860	0.1679	0.1673	0.3839	shear
66	-0.1567	-0.1609	0.3577	-0.3665	0.1679	0.1673	0.3577	shear
67	-0.1554	-0.1596	0.3326	-0.3476	0.1678	0.1672	0.3326	shear
68	-0.1533	-0.1575	0.3087	-0.3292	0.1676	0.1670	0.3087	shear
69	-0.1505	-0.1548	0.2859	-0.3113	0.1672	0.1667	0.2859	shear
70	-0.1473	-0.1515	0.2643	-0.2938	0.1668	0.1663	0.2643	shear

As demonstrated in Table 5.55, the (1H:0A) loading condition limits the range of possible fibre angles from 52° to 70°.

5.6.2 Biaxial Pressure Loading of (8H:1A)

As with the (1H:0A) loading case, the (8H:1A) loading case can also be scaled. Table C.2, from Appendix C, contains the strain ratios obtained from a (8H:1A) loading with a hoop stress of 1 MPa. The scaled strain ratios for this loading condition are found in Table 5.56.

Table 5.56: Scaled Strain Ratios for (8H:1A) with Factored Hoop Stress of 171.9 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.1826	0.1769	0.7189	0.7148	0.1511	0.1502	0.7189	shear
51	0.1393	0.1334	0.6873	0.6832	0.1543	0.1533	0.6873	shear
52	0.1008	0.0946	0.6551	0.6510	0.1571	0.1561	0.6551	shear
53	0.0700	0.0643	0.6224	0.6183	0.1596	0.1586	0.6224	shear
54	0.0444	0.0385	0.5896	0.5855	0.1617	0.1608	0.5896	shear
55	0.0229	0.0169	0.5568	0.5527	0.1635	0.1626	0.5568	shear
56	-0.0035	-0.0070	0.5243	0.5203	0.1650	0.1641	0.5243	shear
57	-0.0118	-0.0154	0.4924	0.4883	0.1662	0.1653	0.4924	shear
58	-0.0182	-0.0218	0.4610	0.4571	0.1671	0.1662	0.4610	shear
59	-0.0226	-0.0262	0.4305	0.4267	0.1678	0.1669	0.4305	shear
60	-0.0253	-0.0290	0.4010	0.3972	0.1682	0.1674	0.4010	shear
61	-0.0264	-0.0303	0.3725	0.3688	0.1684	0.1676	0.3725	shear
62	-0.0261	-0.0300	0.3452	0.3416	0.1684	0.1676	0.3452	shear
63	-0.0246	-0.0286	0.3191	0.3156	0.1682	0.1675	0.3191	shear
64	-0.0219	-0.0260	0.2942	0.2909	0.1679	0.1672	0.2942	shear
65	-0.0183	-0.0224	0.2706	0.2675	0.1674	0.1668	0.2706	shear
66	-0.0139	-0.0180	0.2484	0.2453	0.1669	0.1663	0.2484	shear
67	-0.0088	-0.0129	0.2274	0.2243	0.1662	0.1656	0.2274	shear
68	0.0194	-0.0073	0.2077	0.2047	0.1655	0.1649	0.2077	shear
69	0.0311	0.0232	0.1893	0.1864	0.1647	0.1642	0.1893	shear
70	0.0432	0.0354	0.1720	0.1692	0.1639	0.1634	0.1720	shear

It is seen from Table 5.56 that all strain ratios are less than the resistance factor, therefore, there are no wind angle restrictions from the (8H:1A) loading condition.

5.6.3 Biaxial Pressure Loading of (4H:1A)

As with the other load ratios, the (4H:1A) loading can also be scaled. Table C.3, from Appendix C, is used as a basis for this operation. The scaled values are found in Table 5.57.

Table 5.57: Scaled Strain Ratios for (4H:1A) with Factored Hoop Stress of 171.9 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.2917	0.2853	0.5761	0.5723	0.1623	0.1611	0.5761	shear
51	0.2604	0.2538	0.5441	0.5402	0.1645	0.1634	0.5441	shear
52	0.2353	0.2291	0.5117	0.5078	0.1665	0.1654	0.5117	shear
53	0.2170	0.2107	0.4794	0.4754	0.1681	0.1670	0.4794	shear
54	0.2028	0.1964	0.4472	0.4433	0.1693	0.1683	0.4472	shear
55	0.1927	0.1862	0.4156	0.4117	0.1703	0.1692	0.4156	shear
56	0.1864	0.1798	0.3846	0.3807	0.1709	0.1699	0.3846	shear
57	0.1837	0.1770	0.3544	0.3506	0.1713	0.1703	0.3544	shear
58	0.1842	0.1775	0.3253	0.3215	0.1714	0.1705	0.3253	shear
59	0.1879	0.1810	0.2972	0.2936	0.1713	0.1704	0.2972	shear
60	0.1943	0.1873	0.2705	0.2669	0.1710	0.1701	0.2705	shear
61	0.2031	0.1959	0.2450	0.2415	0.1705	0.1696	0.2450	shear
62	0.2141	0.2067	0.2209	0.2176	0.1698	0.1690	0.2209	shear
63	0.2268	0.2193	0.1983	0.1950	0.1689	0.1682	0.2268	matrix tension
64	0.2411	0.2335	0.1771	0.1739	0.1680	0.1673	0.2411	matrix tension
65	0.2566	0.2489	0.1573	0.1542	0.1670	0.1663	0.2566	matrix tension
66	0.2731	0.2654	0.1390	0.1359	0.1658	0.1652	0.2731	matrix tension
67	0.2904	0.2827	0.1222	0.1191	0.1647	0.1640	0.2904	matrix tension
68	0.3081	0.3005	0.1067	0.1037	0.1634	0.1629	0.3081	matrix tension
69	0.3262	0.3186	0.0926	0.0897	0.1622	0.1617	0.3262	matrix tension
70	0.3445	0.3369	0.0798	0.0770	0.1609	0.1604	0.3445	matrix tension

From the scaled strain results in Table 5.57, the (4H:1A) loading condition puts no restriction on the wind angle.

5.6.4 Biaxial Pressure Loading of (2H:1A)

For the biaxial pressure loading of (2H:1A), Table C.4, in Appendix C, is used as a basis, and is then scaled to determine the strain ratios obtained by an applied hoop stress of 171.9MPa. The scaled strain ratio results are presented in Table 5.58.

Table 5.58: Scaled Strain Ratios for (2H:1A) with Factored Hoop Stress of 171.9 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.5100	0.5023	0.2907	0.2872	0.1845	0.1831	0.5100	matrix tension
51	0.5057	0.4982	0.2576	0.2540	0.1850	0.1837	0.5057	matrix tension
52	0.5062	0.4987	0.2250	0.2215	0.1852	0.1839	0.5062	matrix tension
53	0.5110	0.5034	0.1933	0.1897	0.1850	0.1837	0.5110	matrix tension
54	0.5198	0.5122	0.1625	0.1591	0.1846	0.1833	0.5198	matrix tension
55	0.5324	0.5248	0.1331	0.1296	0.1838	0.1825	0.5324	matrix tension
56	0.5484	0.5408	0.1050	0.1016	0.1828	0.1815	0.5484	matrix tension
57	0.5675	0.5599	0.0785	0.0752	0.1815	0.1803	0.5675	matrix tension
58	0.5893	0.5818	0.0537	0.0504	0.1800	0.1789	0.5893	matrix tension
59	0.6135	0.6061	0.0306	0.0273	0.1784	0.1773	0.6135	matrix tension
60	0.6398	0.6324	-0.0338	-0.0338	0.1765	0.1755	0.6398	matrix tension
61	0.6677	0.6605	-0.0514	-0.0514	0.1746	0.1736	0.6677	matrix tension
62	0.6970	0.6899	-0.0673	-0.0673	0.1725	0.1716	0.6970	matrix tension
63	0.7273	0.7203	-0.0813	-0.0813	0.1704	0.1695	0.7273	matrix tension
64	0.7586	0.7515	-0.0936	-0.0936	0.1682	0.1674	0.7586	matrix tension
65	0.7903	0.7832	-0.1042	-0.1042	0.1660	0.1652	0.7903	matrix tension
66	0.8222	0.8150	-0.1131	-0.1131	0.1638	0.1630	0.8222	matrix tension
67	0.8540	0.8468	-0.1203	-0.1203	0.1616	0.1609	0.8540	matrix tension
68	0.8855	0.8783	-0.1260	-0.1260	0.1593	0.1587	0.8855	matrix tension
69	0.9166	0.9094	-0.1301	-0.1301	0.1572	0.1566	0.9166	matrix tension
70	0.9469	0.9398	-0.1328	-0.1328	0.1551	0.1545	0.9469	matrix tension

It is seen from Table 5.58, that the strain ratio is greater than the resistance factor of 0.8 for fibre angles higher than 65°, in an applied stress loading of (2H:1A). Thus the (2H:1A) loading condition restrains the range of possible fibre angles to 50° to 65°.

5.6.5 Pipeline Installation

For design study 3, the lift is specified as 1m, with a ditch depth of 2m, assuming that the pipeline is supported 0.5m above ground level gives:

lift = 1m
effective depth (delta) = 0.5m + 2.0m = 2.5m

where delta is the distance from the top of the above ground pipe supports to the bottom of the ditch. (see Figure 2.2 in Chapter 2)

The load coefficient for a lift of 1 and delta of 2.5 is estimated by averaging the load coefficients for deltas of 2m and 3m, for a lift of 1m. The load coefficient can be found in Table A.2 of Appendix A.

$$\text{load coefficient } (c_2) = \frac{12.39 + 13.83}{2} = 13.11$$

The installation equation is given by equation 2.4:

$$\sigma_{max} = c_2 \cdot (E \cdot \rho)^{\frac{1}{2}} \cdot \left(1 + \left(1 - 2\frac{t}{D}\right)^2\right)^{-\frac{1}{2}}$$

For a fibre angle of 50°, with ρ = 1510 kg/m^3, t/D = 0.04 and

E_{axial} = 11.02GPa (from Table C.10)

then:

$$\sigma_{max} = 13.11 \cdot \left(11.02x10^3 \cdot 1510\right)^{\frac{1}{2}} \cdot \left(1 + \left(1 - 2(.04)\right)^2\right)^{-\frac{1}{2}}$$
$$\sigma_{max} = 39.36\text{MPa}$$

Young's modulus in the axial direction of the pipe is dependant upon the fibre wind angle. Since the installation loading is dependant upon the axial Young's modulus, the moment loading, and therefore the scaling factor, is also dependant upon the fibre wind angle.

Using the same 50 degree fibre angle,

$$\text{scaling factor} = \frac{(\text{moment load factor})\cdot(\text{max axial stress})}{\text{applied max axial stress}} = \frac{1.4x39.36x10^6}{1x10^6} = 55.10$$

The scaling of the strain ratios for a fibre angle of 50 degrees and a scaling factor of 55.10, is demonstrated in Table 5.59.

Table 5.59: Strain ratio scaling sample calculation for an installation load with a lift of 1m and delta of 2.5m for a fibre angle of 50 degrees

	maximum axial stress	
strain ratio	1MPa	55.10MPa
maximum matrix	0.005152	0.2839
minimum matrix	-0.002667	-0.1470
maximum shear	0.006417	0.3536
minimum shear	-0.006380	-0.3515
maximum fibre	0.000511	0.0282
minimum fibre	0.000842	0.0464

Due to the uncertainty involved in the use of the equation for installation loading, and the use of the CLT to determine the E_{axial} of the pipe, a load factor of 1.4 was used.

The calculated maximum axial stresses and factored maximum axial stresses for the range of investigated fibre angles (50 - 70 degrees) are shown in Table 5.60. The scaled strain ratios for this loading, including the scaling factor, for each fibre angle are in Table 5.61.

Table 5.60: Maximum axial stress from the installation loading for a lift of 1, and a delta of 2.5

ang	maximum axial stress (MPa)	factored maximum axial stress (MPa)
50	39.35	55.09
51	38.89	54.44
52	38.46	53.85
53	38.07	53.30
54	37.71	52.79
55	37.38	52.33
56	37.08	51.91
57	36.80	51.52
58	36.55	51.17
59	36.33	50.86
60	36.12	50.57
61	35.94	50.31
62	•35.77	50.08
63	35.62	49.87
64	35.49	49.69
65	35.37	49.52
66	35.27	49.38
67	35.18	49.25
68	35.10	49.13
69	35.03	49.04
70	34.96	48.95

Table 5.61: Scaled strain ratios for the installation load with t/D = .04, lift = 1, delta = 2.5

		strain ratios							
ang	scaling factor	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	55.09	0.2838	-0.1469	0.3535	-0.3517	0.0282	-0.0464	0.3535	shear
51	54.44	0.3097	-0.1604	0.3507	-0.3490	0.0257	-0.0424	0.3507	shear
52	53.85	0.3353	-0.1737	0.3473	-0.3455	0.0234	-0.0384	0.3473	shear
53	53.30	0.3605	-0.1868	0.3431	-0.3414	0.0211	-0.0346	0.3605	matrix tension
54	52.79	0.3852	-0.1996	0.3383	-0.3367	0.0189	-0.0310	0.3852	matrix tension
55	52.33	0.4094	-0.2122	0.3329	-0.3314	0.0168	-0.0274	0.4094	matrix tension
56	51.91	0.4329	-0.2244	0.3269	-0.3255	0.0147	-0.0240	0.4329	matrix tension
57	51.52	0.4558	-0.2363	0.3204	-0.3190	0.0127	-0.0207	0.4558	matrix tension
58	51.17	0.4780	-0.2478	0.3134	-0.3121	0.0108	-0.0175	0.4780	matrix tension
59	50.85	0.4993	-0.2589	0.3060	-0.3048	0.0090	-0.0145	0.4993	matrix tension
60	50.57	0.5199	-0.2696	0.2982	-0.2971	0.0084	-0.0118	0.5199	matrix tension
61	50.31	0.5397	-0.2799	0.2901	-0.2890	0.0080	-0.0114	0.5397	matrix tension
62	50.08	0.5587	-0.2897	0.2816	-0.2805	0.0075	-0.0113	0.5587	matrix tension
63	49.87	0.5768	-0.2991	0.2728	-0.2718	0.0071	-0.0112	0.5768	matrix tension
64	49.69	0.5941	-0.3081	0.2638	-0.2629	0.0067	-0.0110	0.5941	matrix tension
65	49.52	0.6106	-0.3167	0.2545	-0.2537	0.0062	-0.0108	0.6106	matrix tension
66	49.38	0.6262	-0.3248	0.2451	-0.2443	0.0058	-0.0106	0.6262	matrix tension
67	49.25	0.6410	-0.3324	0.2355	-0.2348	0.0054	-0.0103	0.6410	matrix tension
68	49.13	0.6550	-0.3397	0.2258	-0.2251	0.0050	-0.0101	0.6550	matrix tension
69	49.03	0.6682	-0.3465	0.2159	-0.2153	0.0051	-0.0098	0.6682	matrix tension
70	48.95	0.6806	-0.3530	0.2059	-0.2054	0.0061	-0.0097	0.6806	matrix tension

From Table 5.61, even with the large load factor of 1.4, no fibre angles are eliminated from consideration.

5.6.6 Force Due to Backfill

For design study 3, with a backfill depth given by:

$$H = (\text{depth of ditch}) - (\text{OD}) = 2 - 1.08 = 0.92\text{m}$$

then applying equation 2.6 and assuming a sand and gravel backfill, and a ditch width at the top of the pipe equal to 1.7 times the OD, the force due to backfill is calculated from:

$$C_d = \frac{1 - e^{-2\frac{(0.165)(0.92)}{1.7\text{x}1.08}}}{2(0.165)} = 0.4619$$

and the force due to backfill:

$$W_d = (0.4619)(15.7\text{x}10^3)(1.7\text{x}1.08)^2 = 24.44 \text{ kN / m}$$

Therefore the force per metre of pipe =24.44 kN/m
and the factored force per metre of pipe = 1.25x24.44 = 30.55 kN/m

For a line loading, the strain ratio is dependant on the magnitude of the loading and the diameter of the pipe. From Table C.8, the applied force from Table C.8 must be multiplied by the diameter of the pipe. Thus the scaling factor is given by:

$$\text{scaling factor} = \frac{\text{factored force}}{(\text{applied force}) \cdot \text{OD}} = \frac{30.55\text{x}10^3}{1\text{x}10^3 \cdot 1.08} = 28.29$$

For a wind angle of 50 degrees, and scaling the strain ratios from Table C.9 produces Table 5.62. The scaled strain ratios for all of the fibre angles are in Table 5.63.

Table 5.62: Strain ratio scaling sample calculation for a backfill loading, with a depth of backfill of 0.92m for a fibre angle of 50 degrees

	force/length	
strain ratio	1 kN/m per diameter	28.29 kN/m
maximum matrix	0.007224	0.2044
minimum matrix	-0.003505	-0.0992
maximum shear	0.006635	0.1877
minimum shear	-0.004439	-0.1256
maximum fibre	0.001890	0.0535
minimum fibre	-0.002579	-0.0730

Table 5.63: Scaled strain ratios for a 0.92m backfill with t/D = 0.04

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.2044	-0.0992	0.1877	-0.1256	0.0535	-0.0730	0.2044	matrix tension
51	0.1952	-0.0935	0.1837	-0.1223	0.0536	-0.0731	0.1952	matrix tension
52	0.1862	-0.0881	0.1795	-0.1191	0.0536	-0.0731	0.1862	matrix tension
53	0.1774	-0.0830	0.1753	-0.1159	0.0536	-0.0732	0.1774	matrix tension
54	0.1689	-0.0788	0.1709	-0.1127	0.0536	-0.0733	0.1709	shear
55	0.1606	-0.0748	0.1664	-0.1095	0.0535	-0.0733	0.1664	shear
56	0.1526	-0.0709	0.1619	-0.1063	0.0533	-0.0734	0.1619	shear
57	0.1447	-0.0671	0.1574	-0.1031	0.0532	-0.0734	0.1574	shear
58	0.1371	-0.0634	0.1528	-0.1000	0.0530	-0.0735	0.1528	shear
59	0.1296	-0.0599	0.1481	-0.0968	0.0528	-0.0735	0.1481	shear
60	0.1224	-0.0565	0.1435	-0.0937	0.0525	-0.0736	0.1435	shear
61	0.1154	-0.0531	0.1388	-0.0907	0.0523	-0.0737	0.1388	shear
62	0.1085	-0.0499	0.1342	-0.0876	0.0520	-0.0738	0.1342	shear
63	0.1019	-0.0468	0.1296	-0.0846	0.0518	-0.0740	0.1296	shear
64	0.0955	-0.0438	0.1249	-0.0815	0.0515	-0.0742	0.1249	shear
65	0.0892	-0.0409	0.1203	-0.0785	0.0513	-0.0744	0.1203	shear
66	0.0832	-0.0380	0.1157	-0.0764	0.0511	-0.0747	0.1157	shear
67	0.0773	-0.0353	0.1111	-0.0743	0.0509	-0.0749	0.1111	shear
68	0.0717	-0.0326	0.1065	-0.0721	0.0507	-0.0751	0.1065	shear
69	0.0662	-0.0300	0.1019	-0.0699	0.0505	-0.0753	0.1019	shear
70	0.0609	-0.0275	0.0974	-0.0676	0.0503	-0.0755	0.0974	shear

From Table 5.63, it can be concluded that all fibre wind angles considered in this study will be able to support the backfill.

5.6.7 Load Combinations: (1H:0A) and Backfill

The load combinations are obtained by simply combining the strain ratios of the pressure loading and the backfill loading. This creates a worst case condition, assuming that the maximum strains from the backfill loading occurs at the same locations as the maximum strains from the pressure loading. The (1H:0A) strain ratios are from §5.6.1 while the backfill strain ratios are from §5.6.6.

Table 5.64: Summed strain ratio sample calculation for a combined loading of (1H:0A) and Backfill, with a hoop stress of 171.9 MPa and 0.92m of backfill, for a fibre angle of 50 degrees

strain ratio	(1H:0A)	backfill	sum
maximum matrix	0.0734	0.2044	0.2778
minimum matrix	0.0685	-0.0992	-0.0307
maximum shear	0.8616	0.1877	1.0493
minimum shear	-0.7069	-0.1256	-0.8324
maximum fibre	0.1400	0.0535	0.1935
minimum fibre	0.1392	-0.0730	0.0662

The summing process demonstrated in Table 5.64 is completed for the remaining fibre angles, producing Table 5.65.

Table 5.65: Summed strain ratios for a 0.92m backfill and a hoop stress from a (1H:0A) loading of 171.9 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comments
50	0.2778	-0.0307	1.0493	-0.8324	0.1935	0.0662	1.0493	shear	too high
51	0.2134	-0.0806	1.0142	-0.8095	0.1976	0.0701	1.0142	shear	too high
52	0.1677	-0.1090	0.9780	-0.7857	0.2014	0.0737	0.9780	shear	too high
53	0.1370	-0.1264	0.9407	-0.7612	0.2047	0.0770	0.9407	shear	too high
54	0.1073	-0.1436	0.9028	-0.7363	0.2077	0.0800	0.9028	shear	too high
55	0.0802	-0.1584	0.8645	-0.7110	0.2102	0.0826	0.8645	shear	too high
56	0.0557	-0.1710	0.8261	-0.6855	0.2124	0.0849	0.8261	shear	too high
57	0.0337	-0.1815	0.7877	-0.6599	0.2143	0.0869	0.7877	shear	
58	0.0141	-0.1899	0.7496	-0.6344	0.2158	0.0885	0.7496	shear	
59	-0.0032	-0.1964	0.7120	-0.6090	0.2170	0.0899	0.7120	shear	
60	-0.0184	-0.2011	0.6750	-0.5839	0.2179	0.0910	0.6750	shear	
61	-0.0316	-0.2041	0.6389	-0.5592	0.2186	0.0919	0.6389	shear	
62	-0.0431	-0.2056	0.6036	-0.5348	0.2190	0.0925	0.6036	shear	
63	-0.0528	-0.2057	0.5694	-0.5109	0.2193	0.0928	0.5694	shear	
64	-0.0610	-0.2045	0.5362	-0.4875	0.2193	0.0929	0.5362	shear	
65	-0.0679	-0.2022	0.5042	-0.4645	0.2192	0.0929	0.5042	shear	
66	-0.0735	-0.1990	0.4733	-0.4430	0.2190	0.0927	0.4733	shear	
67	-0.0780	-0.1949	0.4437	-0.4219	0.2187	0.0924	0.4437	shear	
68	-0.0816	-0.1901	0.4152	-0.4013	0.2182	0.0919	0.4152	shear	
69	-0.0844	-0.1848	0.3878	-0.3812	0.2177	0.0914	0.3878	shear	
70	-0.0864	-0.1790	0.3617	-0.3614	0.2171	0.0908	0.3617	shear	

This loading combination reveals that the maximum strain ratios for fibre wind angles below 56° is greater than the resistance factor of 0.8. Thus, from this loading, the fibre range is restricted to 57-70 degrees.

5.6.8 Load Combinations: (8H:1A) and Backfill

The other load combinations simply follow the same format as that used for §5.6.7. The summed strain ratios for this loading combination are found in Table 5.66.

Table 5.66: Summed strain ratios for a 0.92m backfill and a hoop stress from a (8H:1A) loading of 171.9 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comments
50	0.3870	0.0777	0.9065	0.5893	0.2046	0.0772	0.9065	shear	too high
51	0.3344	0.0399	0.8710	0.5609	0.2079	0.0802	0.8710	shear	too high
52	0.2869	0.0066	0.8346	0.5318	0.2107	0.0830	0.8346	shear	too high
53	0.2474	-0.0187	0.7977	0.5024	0.2132	0.0854	0.7977	shear	
54	0.2133	-0.0403	0.7605	0.4728	0.2153	0.0875	0.7605	shear	
55	0.1835	-0.0578	0.7233	0.4432	0.2170	0.0893	0.7233	shear	
56	0.1490	-0.0779	0.6863	0.4140	0.2184	0.0907	0.6863	shear	
57	0.1328	-0.0825	0.6497	0.3852	0.2194	0.0919	0.6497	shear	
58	0.1189	-0.0852	0.6138	0.3571	0.2201	0.0928	0.6138	shear	
59	0.1070	-0.0861	0.5787	0.3298	0.2205	0.0934	0.5787	shear	
60	0.0971	-0.0855	0.5445	0.3035	0.2207	0.0937	0.5445	shear	
61	0.0890	-0.0834	0.5114	0.2782	0.2207	0.0939	0.5114	shear	
62	0.0824	-0.0800	0.4794	0.2540	0.2204	0.0938	0.4794	shear	
63	0.0774	-0.0754	0.4486	0.2311	0.2200	0.0935	0.4486	shear	
64	0.0736	-0.0698	0.4191	0.2094	0.2194	0.0930	0.4191	shear	
65	0.0709	-0.0633	0.3909	0.1889	0.2187	0.0923	0.3909	shear	
66	0.0693	-0.0560	0.3640	0.1688	0.2179	0.0916	0.3640	shear	
67	0.0685	-0.0482	0.3385	0.1500	0.2171	0.0908	0.3385	shear	
68	0.0911	-0.0399	0.3142	0.1326	0.2161	0.0899	0.3142	shear	
69	0.0972	-0.0068	0.2912	0.1165	0.2152	0.0889	0.2912	shear	
70	0.1041	0.0079	0.2694	0.1016	0.2142	0.0878	0.2694	shear	

This load combination reveals that for a (8H:1A) loading with an applied hoop stress of 171.9MPa and a backfill loading of 0.92m, that the strain ratios occurring in the composite pipe for fibre angles below 53° are higher than the resistance factor of 0.8.

5.6.9 Load Combinations: (4H:1A) and Backfill

The summed strain ratios for the load combination of (4H:1A) and backfill are found in Table 5.67.

Table 5.67: Summed strain ratios for a 0.92m backfill and a hoop stress from a (4H:1A) loading of 171.9 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comments
50	0.4961	0.1862	0.7638	0.4467	0.2157	0.0882	0.7638	shear	
51	0.4555	0.1603	0.7277	0.4178	0.2181	0.0904	0.7277	shear	
52	0.4215	0.1410	0.6913	0.3887	0.2201	0.0922	0.6913	shear	
53	0.3944	0.1277	0.6546	0.3595	0.2217	0.0938	0.6546	shear	
54	0.3718	0.1176	0.6181	0.3307	0.2229	0.0950	0.6181	shear	
55	0.3534	0.1114	0.5820	0.3022	0.2238	0.0959	0.5820	shear	
56	0.3390	0.1090	0.5465	0.2744	0.2243	0.0965	0.5465	shear	
57	0.3284	0.1099	0.5118	0.2475	0.2245	0.0969	0.5118	shear	
58	0.3212	0.1141	0.4780	0.2216	0.2244	0.0970	0.4780	shear	
59	0.3175	0.1211	0.4454	0.1968	0.2241	0.0968	0.4454	shear	
60	0.3167	0.1308	0.4140	0.1732	0.2235	0.0965	0.4140	shear	
61	0.3185	0.1428	0.3838	0.1509	0.2227	0.0959	0.3838	shear	
62	0.3226	0.1568	0.3551	0.1300	0.2218	0.0951	0.3551	shear	
63	0.3287	0.1725	0.3278	0.1105	0.2207	0.0941	0.3287	matrix tension	
64	0.3366	0.1897	0.3020	0.0923	0.2195	0.0930	0.3366	matrix tension	
65	0.3458	0.2081	0.2776	0.0756	0.2183	0.0918	0.3458	matrix tension	
66	0.3563	0.2274	0.2547	0.0595	0.2169	0.0905	0.3563	matrix tension	
67	0.3677	0.2474	0.2333	0.0448	0.2155	0.0892	0.3677	matrix tension	
68	0.3798	0.2678	0.2132	0.0316	0.2141	0.0878	0.3798	matrix tension	
69	0.3924	0.2886	0.1945	0.0198	0.2127	0.0863	0.3924	matrix tension	
70	0.4053	0.3094	0.1771	0.0094	0.2112	0.0849	0.4053	matrix tension	

Table 5.67 reveals that there are no fibre angles restrictions from this loading condition.

5.6.10 Load Combinations: (2H:1A) and Backfill

For the loading combination of (2H:1A) and backfill, the strain ratios are summed according to the format in §5.6.7. The results are in Table 5.68.

Table 5.68: Summed strain ratios for a 0.92m backfill and a hoop stress from a (2H:1A) loading of 171.9 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comments
50	0.7144	0.4032	0.4784	0.1616	0.2380	0.1102	0.7144	matrix tension	
51	0.7008	0.4047	0.4413	0.1317	0.2386	0.1106	0.7008	matrix tension	
52	0.6924	0.4106	0.4045	0.1023	0.2388	0.1107	0.6924	matrix tension	
53	0.6884	0.4204	0.3685	0.0738	0.2387	0.1105	0.6884	matrix tension	
54	0.6887	0.4334	0.3334	0.0464	0.2381	0.1100	0.6887	matrix tension	
55	0.6930	0.4500	0.2995	0.0202	0.2373	0.1092	0.6930	matrix tension	
56	0.7010	0.4699	0.2670	-0.0046	0.2361	0.1082	0.7010	matrix tension	
57	0.7122	0.4928	0.2359	-0.0279	0.2347	0.1069	0.7122	matrix tension	
58	0.7264	0.5184	0.2065	-0.0496	0.2330	0.1054	0.7264	matrix tension	
59	0.7431	0.5462	0.1788	-0.0695	0.2311	0.1037	0.7431	matrix tension	
60	0.7622	0.5760	0.1097	-0.1275	0.2291	0.1019	0.7622	matrix tension	
61	0.7831	0.6073	0.0874	-0.1421	0.2269	0.0999	0.7831	matrix tension	
62	0.8055	0.6399	0.0669	-0.1549	0.2246	0.0978	0.8055	matrix tension	too high
63	0.8293	0.6735	0.0482	-0.1659	0.2222	0.0955	0.8293	matrix tension	too high
64	0.8541	0.7077	0.0313	-0.1752	0.2198	0.0931	0.8541	matrix tension	too high
65	0.8796	0.7423	0.0161	-0.1828	0.2173	0.0908	0.8796	matrix tension	too high
66	0.9054	0.7770	0.0026	-0.1895	0.2148	0.0884	0.9054	matrix tension	too high
67	0.9313	0.8115	-0.0092	-0.1946	0.2124	0.0860	0.9313	matrix tension	too high
68	0.9572	0.8457	-0.0195	-0.1981	0.2100	0.0836	0.9572	matrix tension	too high
69	0.9827	0.8794	-0.0282	-0.2000	0.2077	0.0813	0.9827	matrix tension	too high
70	1.0078	0.9123	-0.0355	-0.2004	0.2054	0.0790	1.0078	matrix tension	too high

The load combination of (2H:1A) and backfill eliminates fibre wind angles above 61 degrees. This is shown in Table 5.68.

5.6.11 Load Combinations: (1H:0A), Backfill and Bending Moment

Utilizing the strain ratios from the load combinations from §5.6.7 through §5.6.10 it is possible then to calculate the maximum bending moment which will make the maximum strain ratio equal to the resistance factor (0.8).

In the following tables, a maximum ratio not equal to 0.8 or of "#N/A" means that the strain ratio was greater than 0.8 before the application of the moment, and thus that fibre angle need not be considered.

For this loading combination, the scaling factor is determined by finding the minimum value which will make any strain ratio equal to 0.8

$$\text{scaling factor} = \min\left(\text{abs}\left(\frac{\text{resistance factor} - \text{strain ratio}(i)}{\text{moment strain ratio}(i)}\right)\right)$$

where:

resistance factor =	0.8
ratio(i) =	strain ratio (min or max of fibre, matrix or shear) from combined pressure and backfill loading
moment strain ratio(i) =	strain ratio (min or max of fibre, matrix or shear) from moment loading

The ratio(i) values are taken from sections §5.6.7-5.6.10 of this design study, while the moment strain ratio(i) values are taken from Table C.5 of Appendix C.

For a fibre angle of 60 degrees, the scaling factor is determined by finding the minimum of possible scaling factors. This is demonstrated in Table 5.69.

Table 5.69: Determination of minimum scaling factor for determination of maximum allowable bending moment on a pipe loaded with (1H:0A) with a hoop stress of 171.9 MPa and a backfill of 0.92m for a fibre angle of 60 degrees

strain ratio	strain ratio from (1H:0A) and backfill	strain ratio from unit moment loading	$abs\left(\frac{\text{resistance factor} - \text{strain ratio}}{\text{moment strain ratio}}\right)$
maximum matrix	0.1224	0.0103	79.60
minimum matrix	-0.0565	-0.0053	187.8
maximum shear	0.1435	0.0059	21.19
minimum shear	-0.0937	-0.0059	3512
maximum fibre	0.0525	0.0002	3046
minimum fibre	-0.0736	-0.0002	235.6

Thus the minimum ratio is 21.19

Now, using the minimum scaling factor, the maximum strain ratio should be equal to 0.8, the resistance factor. A check for this is in Table 5.70.

Table 5.70: Sample calculation of summed and scaled strain ratios for (1H:0A) and pressure and bending moment for a fibre angle of 60 degrees

strain ratio	strain ratio from (1H:0A) + backfill	strain ratio from unit moment loading	scaled moment strain ratio	summed strain ratio from (1H:0A) + backfill + scaled moment
maximum matrix	-0.0184	0.0103	0.218257	0.1999
minimum matrix	-0.2011	-0.0053	-0.11231	-0.3134
maximum shear	0.675	0.0059	0.125021	0.8000
minimum shear	-0.5839	-0.0059	-0.12502	-0.7089
maximum fibre	0.2179	0.0002	0.004238	0.2221
minimum fibre	0.091	-0.0002	-0.00424	0.0868

Thus the maximum strain ratio is equal to 0.8, the resistance factor, occurring in shear.

The maximum axial stress is found by multiplying the maximum axial stress used in Table C.5 by the scaling factor. Scaling the maximum axial stress from the moment loading is obtained by multiplying the scaling factor by the unit load from Table C.5.

Scaled maximum axial stress = 21.19 x1MPa = 21.19MPa

The actual moment load which causes the maximum axial stress can be calculated from:

$$\sigma_{max} = \frac{M \cdot OD}{2I} \therefore M = \frac{2\sigma_{max} I}{OD}$$

So, for the fibre angle of 60 degrees:

$$M = \frac{2x21.19x1.769x10^{-2}}{1.08} = 694.4 \text{ kN} \cdot \text{m}$$

The strain ratios from this loading combination of (1H:0A), backfill and bending moment are found in Table 5.71.

Table 5.71: Strain Ratios resulting from a backfill of 0.92m, hoop stress of 171.9 MPa in a (1H:0A) loading, and the maximum allowable moment

		strain ratios							
ang	scaling factor	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	38.84	0.4779	-0.1343	1.2985	-1.0804	0.2133	0.0335	1.2985	#N/A
51	33.26	0.4025	-0.1785	1.2285	-1.0226	0.2134	0.0443	1.2285	#N/A
52	27.60	0.3396	-0.1980	1.1559	-0.9628	0.2134	0.0540	1.1559	#N/A
53	21.86	0.2849	-0.2030	1.0815	-0.9013	0.2134	0.0628	1.0815	#N/A
54	16.05	0.2244	-0.2042	1.0057	-0.8386	0.2134	0.0706	1.0057	#N/A
55	10.14	0.1595	-0.1995	0.9290	-0.7752	0.2135	0.0773	0.9290	#N/A
56	4.14	0.0902	-0.1889	0.8521	-0.7114	0.2136	0.0830	0.8521	#N/A
57	1.98	0.0513	-0.1906	0.8000	-0.6722	0.2148	0.0861	0.8000	shear
58	8.23	0.0910	-0.2298	0.8000	-0.6846	0.2175	0.0857	0.8000	shear
59	14.63	0.1404	-0.2709	0.8000	-0.6967	0.2196	0.0858	0.8000	shear
60	21.19	0.1995	-0.3141	0.8000	-0.7084	0.2214	0.0861	0.8000	shear
61	27.95	0.2682	-0.3596	0.8000	-0.7197	0.2230	0.0856	0.8000	shear
62	34.92	0.3466	-0.4076	0.8000	-0.7305	0.2243	0.0846	0.8000	shear
63	42.16	0.4348	-0.4585	0.8000	-0.7407	0.2253	0.0834	0.8000	shear
64	49.69	0.5331	-0.5126	0.8000	-0.7503	0.2260	0.0819	0.8000	shear
65	57.55	0.6417	-0.5702	0.8000	-0.7594	0.2265	0.0803	0.8000	shear
66	65.81	0.7611	-0.6318	0.8000	-0.7686	0.2267	0.0786	0.8000	shear
67	67.46	0.8000	-0.6503	0.7662	-0.7436	0.2260	0.0782	0.8000	matrix tension
68	66.14	0.8000	-0.6474	0.7190	-0.7044	0.2249	0.0784	0.8000	matrix tension
69	64.90	0.8000	-0.6435	0.6736	-0.6662	0.2245	0.0784	0.8000	matrix tension
70	63.75	0.8000	-0.6388	0.6299	-0.6289	0.2251	0.0781	0.8000	matrix tension

The calculated bending moments and maximum axial stress for the current loading combination are found in Table 5.72.

Table 5.72: Maximum axial stresses and moments for a backfill of 0.92m and a hoop stress of 171.9 MPa in a (1H:0A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	#N/A	#N/A
51	#N/A	#N/A
52	#N/A	#N/A
53	#N/A	#N/A
54	#N/A	#N/A
55	#N/A	#N/A
56	#N/A	#N/A
57	1.983	64.97
58	8.231	269.7
59	14.63	479.3
60	21.19	694.4
61	27.95	915.8
62	34.92	1144
63	42.16	1382
64	49.69	1628
65	57.55	1886
66	65.81	2157
67	67.46	2211
68	66.14	2167
69	64.90	2127
70	63.75	2089

5.6.12 Load Combinations: (8H:1A), Backfill and Bending Moment

For the this load combination, the combined strain ratios are given in Table 5.73 and the scaled maximum axial stress and bending moment are given in Table 5.74.

5.6.13 Load Combinations: (4H:1A), Backfill and Bending Moment

For the this load combination, the combined strain ratios are given in Table 5.75 and the scaled maximum axial stress and bending moment are given in Table 5.76.

5.6.14 Load Combinations: (2H:1A), Backfill and Bending Moment

For the this load combination, the combined strain ratios are given in Table 5.77 and the scaled maximum axial stress and bending moment are given in Table 5.78.

Table 5.73: Strain Ratios resulting from a backfill of 0.92m, hoop stress of 171.9 MPa in a (8H:1A) loading, and the maximum allowable moment

ang	scaling factor	strain ratios maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	16.60	0.4725	0.0335	1.0131	0.4833	0.2131	0.0632	1.0131	#N/A
51	11.02	0.3971	0.0074	0.9420	0.4903	0.2131	0.0717	0.9420	#N/A
52	5.37	0.3204	-0.0107	0.8692	0.4974	0.2131	0.0792	0.8692	#N/A
53	0.36	0.2498	-0.0200	0.8000	0.5001	0.2134	0.0852	0.8000	shear
54	6.17	0.2583	-0.0636	0.8000	0.4335	0.2175	0.0839	0.8000	shear
55	12.06	0.2779	-0.1067	0.8000	0.3669	0.2209	0.0829	0.8000	shear
56	18.06	0.2996	-0.1560	0.8000	0.3008	0.2235	0.0824	0.8000	shear
57	24.16	0.3466	-0.1933	0.8000	0.2356	0.2253	0.0822	0.8000	shear
58	30.40	0.4028	-0.2324	0.8000	0.1717	0.2265	0.0824	0.8000	shear
59	36.78	0.4682	-0.2734	0.8000	0.1094	0.2271	0.0829	0.8000	shear
60	43.33	0.5426	-0.3165	0.8000	0.0490	0.2279	0.0837	0.8000	shear
61	50.06	0.6261	-0.3619	0.8000	-0.0094	0.2286	0.0825	0.8000	shear
62	57.03	0.7186	-0.4099	0.8000	-0.0654	0.2290	0.0809	0.8000	shear
63	62.48	0.8000	-0.4502	0.7904	-0.1095	0.2289	0.0795	0.8000	matrix tension
64	60.75	0.8000	-0.4465	0.7416	-0.1121	0.2276	0.0795	0.8000	matrix tension
65	59.13	0.8000	-0.4414	0.6948	-0.1140	0.2262	0.0794	0.8000	matrix tension
66	57.62	0.8000	-0.4350	0.6500	-0.1163	0.2247	0.0793	0.8000	matrix tension
67	56.20	0.8000	-0.4276	0.6072	-0.1179	0.2232	0.0790	0.8000	matrix tension
68	53.18	0.8000	-0.4076	0.5585	-0.1111	0.2215	0.0790	0.8000	matrix tension
69	51.57	0.8000	-0.3713	0.5182	-0.1100	0.2205	0.0786	0.8000	matrix tension
70	50.05	0.8000	-0.3531	0.4800	-0.1084	0.2204	0.0779	0.8000	matrix tension

Table 5.74: Maximum axial stresses and moments for backfill of 0.92m and a hoop stress of 171.9 MPa in a (8H:1A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	#N/A	#N/A
51	#N/A	#N/A
52	#N/A	#N/A
53	0.3602	11.81
54	6.167	202.1
55	12.06	395.3
56	18.06	591.7
57	24.16	791.8
58	30.40	996.1
59	36.78	1205
60	43.33	1420
61	50.07	1641
62	57.03	1869
63	62.48	2047
64	60.75	1991
65	59.13	1938
66	57.62	1888
67	56.20	1842
68	53.18	1743
69	51.573	1690
70	50.050	1640

Table 5.75: Strain Ratios resulting from a backfill of 0.92m, hoop stress of 171.9 MPa in a (4H:1A) loading and the maximum allowable moment

		strain ratios							
ang	scaling factor	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	5.636	0.5251	0.1711	0.8000	0.4107	0.2158	0.0834	0.8000	shear
51	11.22	0.5193	0.1272	0.8000	0.3459	0.2234	0.0816	0.8000	shear
52	16.86	0.5265	0.0867	0.8000	0.2805	0.2274	0.0802	0.8000	shear
53	22.58	0.5471	0.0485	0.8000	0.2149	0.2306	0.0791	0.8000	shear
54	28.38	0.5788	0.0103	0.8000	0.1496	0.2331	0.0783	0.8000	shear
55	34.27	0.6214	-0.0275	0.8000	0.0852	0.2347	0.0780	0.8000	shear
56	40.25	0.6747	-0.0650	0.8000	0.0221	0.2357	0.0780	0.8000	shear
57	46.34	0.7384	-0.1026	0.8000	-0.0394	0.2359	0.0783	0.8000	shear
58	51.26	0.8000	-0.1342	0.7920	-0.0911	0.2352	0.0794	0.8000	matrix tension
59	49.14	0.8000	-0.1291	0.7411	-0.0978	0.2328	0.0828	0.8000	matrix tension
60	47.01	0.8000	-0.1198	0.6912	-0.1030	0.2313	0.0855	0.8000	matrix tension
61	44.88	0.8000	-0.1069	0.6426	-0.1069	0.2298	0.0857	0.8000	matrix tension
62	42.79	0.8000	-0.0908	0.5957	-0.1097	0.2282	0.0855	0.8000	matrix tension
63	40.75	0.8000	-0.0719	0.5507	-0.1116	0.2265	0.0850	0.8000	matrix tension
64	38.76	0.8000	-0.0507	0.5078	-0.1128	0.2247	0.0844	0.8000	matrix tension
65	36.84	0.8000	-0.0275	0.4670	-0.1131	0.2229	0.0838	0.8000	matrix tension
66	34.99	0.8000	-0.0028	0.4284	-0.1136	0.2210	0.0830	0.8000	matrix tension
67	33.22	0.8000	0.0232	0.3921	-0.1136	0.2191	0.0822	0.8000	matrix tension
68	31.52	0.8000	0.0499	0.3580	-0.1128	0.2173	0.0813	0.8000	matrix tension
69	29.91	0.8000	0.0772	0.3262	-0.1116	0.2158	0.0804	0.8000	matrix tension
70	28.39	0.8000	0.1046	0.2966	-0.1097	0.2148	0.0792	0.8000	matrix tension

Table 5.76: Maximum axial stresses and moments for backfill of 0.92m and a hoop stress of 171.9 MPa in a (4H:1A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	5.636	184.7
51	11.22	367.5
52	16.86	552.6
53	22.58	740.0
54	28.38	930.0
55	34.27	1123
56	40.25	1319
57	46.34	1519
58	51.26	1680
59	49.14	1610
60	47.01	1540
61	44.88	1471
62	42.79	1402
63	40.75	1335
64	38.76	1270
65	36.84	1207
66	34.99	1147
67	33.22	1088
68	31.52	1033
69	29.91	980.2
70	28.39	930.2

Table 5.77: Strain Ratios resulting from a backfill of 0.92m, hoop stress of 171.9 MPa in a (2H:1A) loading, and the maximum allowable moment

		strain ratios							
ang	scaling factor	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	maximum ratio
50	16.62	0.8000	0.3588	0.5851	0.0555	0.2465	0.0962	0.8000	matrix tension
51	17.43	0.8000	0.3534	0.5536	0.0199	0.2469	0.0970	0.8000	matrix tension
52	17.29	0.8000	0.3548	0.5160	-0.0086	0.2463	0.0984	0.8000	matrix tension
53	16.50	0.8000	0.3626	0.4747	-0.0319	0.2452	0.0998	0.8000	matrix tension
54	15.25	0.8000	0.3757	0.4312	-0.0509	0.2436	0.1011	0.8000	matrix tension
55	13.67	0.8000	0.3946	0.3865	-0.0664	0.2417	0.1021	0.8000	matrix tension
56	11.88	0.8000	0.4186	0.3418	-0.0791	0.2395	0.1027	0.8000	matrix tension
57	9.926	0.8000	0.4473	0.2976	-0.0894	0.2371	0.1029	0.8000	matrix tension
58	7.883	0.8000	0.4802	0.2548	-0.0977	0.2347	0.1027	0.8000	matrix tension
59	5.790	0.8000	0.5167	0.2136	-0.1042	0.2321	0.1021	0.8000	matrix tension
60	3.680	0.8000	0.5563	0.1314	-0.1491	0.2297	0.1010	0.8000	matrix tension
61	1.579	0.8000	0.5985	0.0965	-0.1511	0.2271	0.0995	0.8000	matrix tension
62	0.494	0.8110	0.6371	0.0697	-0.1576	0.2246	0.0976	0.8110	matrix tension
63	2.530	0.8585	0.6583	0.0620	-0.1797	0.2226	0.0949	0.8585	matrix tension
64	4.525	0.9082	0.6797	0.0553	-0.1991	0.2204	0.0921	0.9082	#N/A
65	6.453	0.9591	0.7010	0.0493	-0.2158	0.2181	0.0893	0.9591	#N/A
66	3.500	0.9498	0.7540	0.0200	-0.2068	0.2153	0.0876	0.9498	#N/A
67	1.708	0.9536	0.8000	-0.0011	-0.2028	0.2126	0.0856	0.9536	#N/A
68	6.613	1.0453	0.8000	0.0109	-0.2284	0.2107	0.0823	1.0453	#N/A
69	11.23	1.1357	0.8000	0.0213	-0.2493	0.2088	0.0790	1.1357	#N/A
70	14.94	1.2156	0.8045	0.0274	-0.2631	0.2072	0.0760	1.2156	#N/A

Table 5.78: Maximum axial stresses and moments for backfill of 0.92m and a hoop stress of 171.9 MPa in a (2H:1A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	16.62	544.6
51	17.43	571.3
52	17.29	566.5
53	16.50	540.6
54	15.25	499.7
55	13.67	448.1
56	11.88	389.2
57	9.926	325.3
58	7.883	258.3
59	5.790	189.7
60	3.680	120.6
61	1.579	51.73
62	#N/A	#N/A
63	#N/A	#N/A
64	#N/A	#N/A
65	#N/A	#N/A
66	#N/A	#N/A
67	#N/A	#N/A
68	#N/A	#N/A
69	#N/A	#N/A
70	#N/A	#N/A

5.6.15 Maximum Bending Moment Considerations

An installed pipeline may run through areas where the soil may be unstable. For this thesis, this instability was considered as a bending moment loading. Thus, for a pipeline which may run through unstable soil regions, there may be the further loading specification of an applied bending moment.

For the pipe considered in this study, the maximum allowable moments from sections §5.6.11 through §5.6.14 are included in Table 5.79.

Table 5.79: Summary of maximum allowable bending moments from sections §5.6.11 through §5.6.14

	maximum moment (kN·m) for combined stress ratio pressure loading and backfill loading			
ang	(1H:0A)	(8H:1A)	(4H:1A)	(2H:1A)
50	#N/A	#N/A	184.7	544.6
51	#N/A	#N/A	367.5	571.3
52	#N/A	#N/A	552.6	566.5
53	#N/A	11.81	740.0	540.6
54	#N/A	202.1	930.0	499.7
55	#N/A	395.3	1123	448.1
56	#N/A	591.7	1319	389.2
57	64.97	791.8	1519	325.3
58	269.7	996.1	1680	258.3
59	479.3	1205	1610	189.7
60	694.4	1420	1540	120.6
61	915.8	1641	1471	51.73
62	1144	1869	1402	#N/A
63	1382	2047	1335	#N/A
64	1628	1991	1270	#N/A
65	1886	1938	1207	#N/A
66	2157	1888	1147	#N/A
67	2211	1842	1088	#N/A
68	2167	1743	1033	#N/A
69	2127	1690	980.2	#N/A
70	2089	1640	930.2	#N/A

For the limited range of allowable fibre angles, 57-61 degrees, the maximum allowable moment loads range from 51.73 kN·m to 1641 kN·m. Should this range of "reserved" maximum bending moment be insufficient for the design purposes, then the designer should proceed to increase the pipe thickness, as documented in §5.6 Design Study 3b.

5.6.16 Summary of Design Study 3a

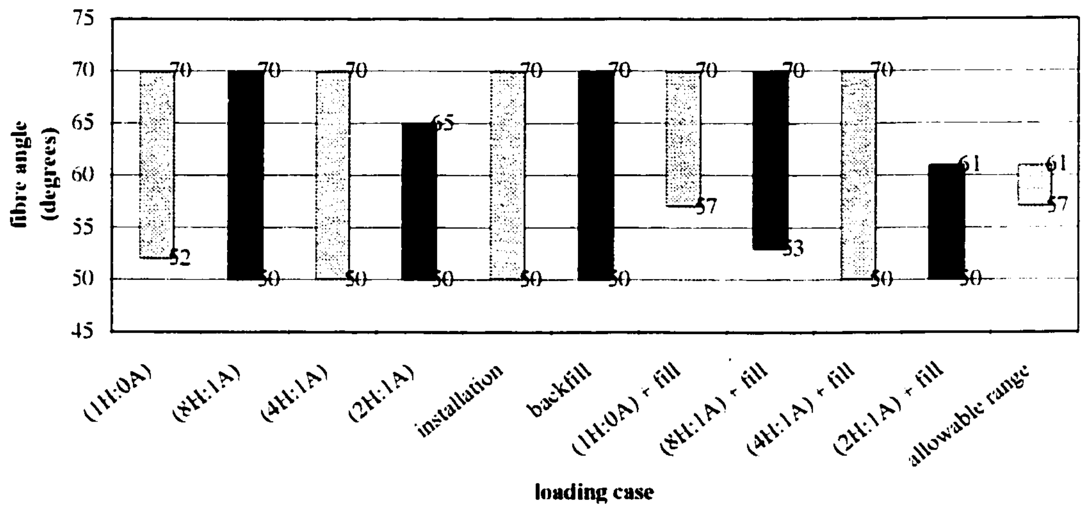

Figure 5.3: Fibre angle compatibilities for Design Study 3b

From Figure 5.3, the allowable range of fibre angles which satisfy the design criteria is 57-61 degrees. This 5 degree range is too small to be practical. It is obvious that the assumed t/D of 0.04 is too small for the applied loadings.

Design study 3b repeats the calculations from this study, but for a t/D of 0.05.

5.7 Design Study 3b

Design study 3b is repetition of design study 3a, with the exception of using a larger t/D ratio in an attempt to increase the load bearing capacity of the composite pipe and to open up, or make larger, the window of fibre angles which satisfy the design criteria.

Given:

- Internal pressure = 11 MPa (1.6 ksi)
- Possible pressure loadings:
  (1H:0A), (8H:1A), (4H:1A), (2H:1A)
- Factors:
  Resistance factor = 0.8
  Load factor for pressure = 1.25
  Load factor for backfill = 1.25
  Load factor for moment = 1.4
- ID = 1m
- Installation Parameters:
  lift = 1m
  ditch depth = 2m

Solution:

For a pressure of 11 MPa, a t/D ratio of 0.04 was used in design study 3a.

For design study 3b, a t/D ratio of 0.05 is used.

therefore:

$$OD = \frac{ID}{\left(1 - 2 \cdot \frac{t}{D}\right)} = 1.10\text{m}$$

and

$$I = \frac{\pi}{4}\cdot\left(\left(\frac{OD}{2}\right)^4 - \left(\frac{ID}{2}\right)^4\right) = 22.78x10^{-3}\,m^4$$

To calculate the factored hoop stress:

$$\sigma_H = \frac{\text{load factor} \cdot \text{pressure}}{2 \cdot \frac{t}{D}} = \frac{1.25 \cdot 11}{2 \cdot 0.05} = 137.5\ \text{MPa}$$

5.6.1 Biaxial Pressure Loading of (1H:0A)

As with the previous design studies, the strain results obtained from the hoop stress of 1MPa finite element models are scaled to the applied hoop stress of 137.5 MPa, occurring in this loading. Table C.1, in Appendix C, contains the numerical values of the strain ratios for fibre angles of 50° to 70° obtained from a (1H:0A) loading with a hoop stress of 1MPa.

From Table C.1, a scaling factor can be used to scale the strain ratios to the factored hoop stress of 137.5 MPa:

$$\text{scaling factor} = \frac{\text{factored hoop stress}}{\text{applied hoop stress}} = \frac{137.5\text{MPa}}{1\text{MPa}} = 137.5$$

The factoring of strain ratios is demonstrated in Table 5.80 for a fibre angle of 50 degrees.

Table 5.80: Strain ratio scaling sample calculation for a hoop stress of 137.5 MPa, in a (1H:0A) loading for a fibre angle of 50 degrees

	hoop stress	
strain ratio	1MPa	137.5MPa
maximum matrix	427.3E-6	0.0588
minimum matrix	398.3E-6	0.0548
maximum shear	5.013E-3	0.6892
minimum shear	-4.113E-3	-0.5655
maximum fibre	814.6E-6	0.1120
minimum fibre	809.7E-6	0.1113

Applying the scaling factor to all of the strain ratios for fibre angles of 50° to 70° produces Table 5.81.

Table 5.81: Scaled strain ratios for (1H:0A) with factored hoop stress of 137.5 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.0588	0.0548	0.6892	-0.5655	0.1120	0.1113	0.6892	shear
51	0.0146	0.0104	0.6644	-0.5497	0.1152	0.1146	0.6644	shear
52	-0.0148	-0.0168	0.6387	-0.5333	0.1182	0.1175	0.6387	shear
53	-0.0324	-0.0347	0.6124	-0.5163	0.1209	0.1202	0.6124	shear
54	-0.0493	-0.0518	0.5855	-0.4989	0.1233	0.1226	0.5855	shear
55	-0.0643	-0.0669	0.5585	-0.4812	0.1254	0.1247	0.5585	shear
56	-0.0775	-0.0801	0.5313	-0.4633	0.1273	0.1266	0.5313	shear
57	-0.0888	-0.0915	0.5042	-0.4454	0.1289	0.1282	0.5042	shear
58	-0.0983	-0.1012	0.4775	-0.4275	0.1303	0.1296	0.4775	shear
59	-0.1063	-0.1092	0.4511	-0.4097	0.1314	0.1308	0.4511	shear
60	-0.1126	-0.1157	0.4252	-0.3922	0.1323	0.1317	0.0004	shear
61	-0.1176	-0.1208	0.4000	-0.3748	0.1331	0.1325	0.4000	shear
62	-0.1213	-0.1245	0.3756	-0.3578	0.1336	0.1331	0.3756	shear
63	-0.1238	-0.1271	0.3519	-0.3411	0.1340	0.1335	0.3519	shear
64	-0.1252	-0.1286	0.3291	-0.3247	0.1342	0.1337	0.3291	shear
65	-0.1257	-0.1291	0.3071	-0.3088	0.1343	0.1339	0.3071	shear
66	-0.1253	-0.1287	0.2861	-0.2932	0.1343	0.1339	0.2861	shear
67	-0.1243	-0.1277	0.2661	-0.2781	0.1342	0.1338	0.2661	shear
68	-0.1226	-0.1260	0.2469	-0.2634	0.1340	0.1336	0.2469	shear
69	-0.1204	-0.1238	0.2287	-0.2490	0.1338	0.1334	0.2287	shear
70	-0.1178	-0.1212	0.2114	-0.2351	0.1335	0.1331	0.2114	shear

The (1H:0A) loading condition does not put any restraint on the range of possible fibre angles.

5.7.2 Biaxial Pressure Loading of (8H:1A)

As with the (1H:0A) loading case, the (8H:1A) loading case can also be scaled. Table C.2, from Appendix C, contains the strain ratios obtained from a (8H:1A) loading with a hoop stress of 1 MPa. The scaled strain results for this loading case are shown in Table 5.82.

Table 5.82: Scaled Strain Ratios for (8H:1A) with Factored Hoop Stress of 137.5 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.1461	0.1415	0.5751	0.5719	0.1209	0.1201	0.5751	shear
51	0.1114	0.1067	0.5498	0.5466	0.1234	0.1227	0.5498	shear
52	0.0806	0.0757	0.5241	0.5208	0.1257	0.1249	0.5241	shear
53	0.0560	0.0514	0.4979	0.4946	0.1277	0.1269	0.4979	shear
54	0.0355	0.0308	0.4717	0.4684	0.1294	0.1286	0.4717	shear
55	0.0183	0.0136	0.4455	0.4422	0.1308	0.1301	0.4455	shear
56	-0.0028	-0.0056	0.4195	0.4162	0.1320	0.1313	0.4195	shear
57	-0.0095	-0.0123	0.3939	0.3907	0.1330	0.1322	0.3939	shear
58	-0.0145	-0.0174	0.3688	0.3657	0.1337	0.1330	0.3688	shear
59	-0.0181	-0.0210	0.3444	0.3413	0.1342	0.1335	0.3444	shear
60	-0.0202	-0.0232	0.3208	0.3178	0.1346	0.1339	0.3208	shear
61	-0.0211	-0.0242	0.2980	0.2951	0.1347	0.1341	0.2980	shear
62	-0.0209	-0.0240	0.2761	0.2733	0.1347	0.1341	0.2761	shear
63	-0.0197	-0.0229	0.2552	0.2525	0.1346	0.1340	0.2552	shear
64	-0.0175	-0.0208	0.2354	0.2327	0.1343	0.1338	0.2354	shear
65	-0.0146	-0.0179	0.2165	0.2140	0.1340	0.1334	0.2165	shear
66	-0.0111	-0.0144	0.1987	0.1962	0.1335	0.1330	0.1987	shear
67	-0.0070	-0.0103	0.1819	0.1795	0.1330	0.1325	0.1819	shear
68	0.0156	-0.0058	0.1661	0.1638	0.1324	0.1320	0.1661	shear
69	0.0249	0.0186	0.1514	0.1491	0.1318	0.1313	0.1514	shear
70	0.0346	0.0283	0.1376	0.1354	0.1311	0.1307	0.1376	shear

It is demonstrated in Table 5.83 that all strain ratios are less than the resistance factor, therefore, there are no wind angle restrictions from the (8H:1A) loading condition.

5.7.3 Biaxial Pressure Loading of (4H:1A)

As with the other load ratios, the (4H:1A) loading can also be scaled. Table C.3 from Appendix C, is used as a basis for this operation. The scaled ratios are in Table 5.84.

Table 5.84: Scaled Strain Ratios for (4H:1A) with Factored Hoop Stress of 137.5 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.2334	0.2283	0.4609	0.4578	0.1298	0.1289	0.4609	shear
51	0.2083	0.2030	0.4353	0.4321	0.1316	0.1307	0.4353	shear
52	0.1882	0.1833	0.4094	0.4062	0.1332	0.1323	0.4094	shear
53	0.1736	0.1685	0.3835	0.3803	0.1345	0.1336	0.3835	shear
54	0.1623	0.1571	0.3578	0.3547	0.1355	0.1346	0.3578	shear
55	0.1542	0.1490	0.3325	0.3294	0.1362	0.1354	0.3325	shear
56	0.1491	0.1439	0.3077	0.3046	0.1367	0.1359	0.3077	shear
57	0.1469	0.1416	0.2835	0.2805	0.1370	0.1362	0.2835	shear
58	0.1474	0.1420	0.2602	0.2572	0.1371	0.1364	0.2602	shear
59	0.1503	0.1448	0.2378	0.2349	0.1370	0.1363	0.2378	shear
60	0.1554	0.1498	0.2164	0.2135	0.1368	0.1361	0.2164	shear
61	0.1625	0.1567	0.1960	0.1932	0.1364	0.1357	0.1960	shear
62	0.1712	0.1654	0.1767	0.1741	0.1358	0.1352	0.1767	shear
63	0.1814	0.1755	0.1586	0.1560	0.1352	0.1345	0.1814	matrix tension
64	0.1928	0.1868	0.1417	0.1391	0.1344	0.1338	0.1928	matrix tension
65	0.2053	0.1991	0.1259	0.1233	0.1336	0.1330	0.2053	matrix tension
66	0.2185	0.2123	0.1112	0.1087	0.1327	0.1321	0.2185	matrix tension
67	0.2323	0.2261	0.0977	0.0953	0.1317	0.1312	0.2323	matrix tension
68	0.2465	0.2404	0.0854	0.0830	0.1307	0.1303	0.2465	matrix tension
69	0.2610	0.2549	0.0741	0.0718	0.1297	0.1293	0.2610	matrix tension
70	0.2756	0.2695	0.0638	0.0616	0.1287	0.1283	0.2756	matrix tension

The (4H:1A) loading condition puts no restriction on the wind angle.

5.7.4 Biaxial Pressure Loading of (2H:1A)

For the biaxial pressure loading of (2H:1A), Table C.4 is used as a basis, and is then scaled to determine the strain ratios obtained by an applied hoop stress of 137.5MPa. Table 5.85 contains the scaled strain ratios for this loading condition.

Table 5.85: Scaled Strain Ratios for (2H:1A) with Factored Hoop Stress of 137.5 MPa

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.4080	0.4019	0.2326	0.2297	0.1476	0.1465	0.4080	matrix tension
51	0.4045	0.3986	0.2061	0.2032	0.1480	0.1469	0.4045	matrix tension
52	0.4049	0.3989	0.1800	0.1772	0.1482	0.1471	0.4049	matrix tension
53	0.4088	0.4027	0.1546	0.1518	0.1480	0.1470	0.4088	matrix tension
54	0.4159	0.4098	0.1300	0.1272	0.1476	0.1466	0.4159	matrix tension
55	0.4259	0.4198	0.1065	0.1037	0.1470	0.1460	0.4259	matrix tension
56	0.4387	0.4326	0.0840	0.0813	0.1462	0.1452	0.4387	matrix tension
57	0.4540	0.4479	0.0628	0.0602	0.1452	0.1443	0.4540	matrix tension
58	0.4715	0.4654	0.0430	0.0403	0.1440	0.1431	0.4715	matrix tension
59	0.4908	0.4849	0.0245	0.0218	0.1427	0.1418	0.4908	matrix tension
60	0.5118	0.5059	-0.0270	-0.0270	0.1412	0.1404	0.5118	matrix tension
61	0.5342	0.5284	-0.0411	-0.0411	0.1397	0.1389	0.5342	matrix tension
62	0.5576	0.5519	-0.0538	-0.0538	0.1380	0.1373	0.5576	matrix tension
63	0.5819	0.5763	-0.0651	-0.0651	0.1363	0.1356	0.5819	matrix tension
64	0.6069	0.6012	-0.0749	-0.0749	0.1346	0.1339	0.6069	matrix tension
65	0.6323	0.6265	-0.0834	-0.0834	0.1328	0.1322	0.6323	matrix tension
66	0.6578	0.6520	-0.0905	-0.0905	0.1310	0.1304	0.6578	matrix tension
67	0.6832	0.6774	-0.0963	-0.0963	0.1292	0.1287	0.6832	matrix tension
68	0.7084	0.7027	-0.1008	-0.1008	0.1275	0.1270	0.7084	matrix tension
69	0.7333	0.7275	-0.1041	-0.1041	0.1257	0.1253	0.7333	matrix tension
70	0.7575	0.7519	-0.1063	-0.1063	0.1241	0.1236	0.7575	matrix tension

It is demonstrated in Table 5.85, that the (2H:1A) loading condition puts no restriction on the fibre wind angle.

5.7.5 Pipeline Installation

For design study 3, the lift is specified as 1m, with a ditch depth of 2m, assuming that the pipeline is supported 0.5m above ground level gives:

lift = 1m
effective depth (delta) = 0.5m + 2.0m = 2.5m

where delta is the distance from the top of the above ground pipe supports to the bottom of the ditch. (see Figure 2.2 in Chapter 2)

The load coefficient for a lift of 1 and delta of 2.5 is estimated by averaging the load coefficients for deltas of 2m and 3m, for a lift of 1m. The load coefficient can be found in Table A.2 of Appendix A.

$$\text{load coefficient } (c_2) = \frac{12.39 + 13.83}{2} = 13.11$$

The installation equation is given by equation 2.4:

$$\sigma_{max} = c_2 \cdot (E \cdot \rho)^{\frac{1}{2}} \cdot \left(1 + \left(1 - 2\frac{t}{D}\right)^2\right)^{-\frac{1}{2}}$$

For a fibre angle of 50°, with ρ = 1510 kg/m^3, t/D = 0.04 and

E_{axial} = 11.02GPa (from Table C.10)

then:

$$\sigma_{max} = 13.11 \cdot \left(11.02x10^3 \cdot 1510\right)^{\frac{1}{2}} \cdot \left(1 + \left(1 - 2(.05)\right)^2\right)^{-\frac{1}{2}}$$
$$\sigma_{max} = 39.75\text{MPa}$$

Young's modulus in the axial direction of the pipe is dependant upon the fibre wind angle. Since the installation loading is dependant upon the axial Young's modulus, the moment loading, and therefore the scaling factor, is also dependant upon the fibre wind angle.

For a fibre angle of 50 degrees,

$$\text{scaling factor} = \frac{(\text{moment load factor})\cdot(\text{max axial stress})}{\text{applied max axial stress}} = \frac{1.4\text{x}39.75\text{x}10^6}{1\text{x}10^6} = 55.65$$

The scaling of the strain ratios for a fibre angle of 50 degrees and a scaling factor of 55.65, is demonstrated in Table 5.86.

Table 5.86: Strain ratio scaling sample calculation for an installation load with a lift of 1m and delta of 2.5m for a fibre angle of 50 degrees

	maximum axial stress	
strain ratio	1MPa	55.65MPa
maximum matrix	0.0052	0.2867
minimum matrix	-0.0027	-0.1484
maximum shear	0.0064	0.3571
minimum shear	-0.0064	-0.3553
maximum fibre	0.0005	0.0284
minimum fibre	-0.0008	-0.0469

Due to the uncertainty involved in the use of the equation for installation loading, and the use of the CLT to determine the E_{axial} of the pipe, a load factor of 1.4 was used.

The calculated maximum axial stresses and factored maximum axial stresses for the range of investigated fibre angles (50 - 70 degrees) are shown in Table 5.87. The scaled strain ratios for this loading, including the scaling factor, for each fibre angle are in Table 5.88.

Table 5.87: Maximum axial stress from the installation loading for a lift of 1, and a delta of 2.5

ang	maximum axial stress (MPa)	factored maximum axial stress (MPa)
50	39.74	55.64
51	39.28	54.99
52	38.85	54.39
53	38.45	53.83
54	38.09	53.32
55	37.75	52.85
56	37.45	52.43
57	37.17	52.04
58	36.92	51.68
59	36.69	51.36
60	36.48	51.07
61	36.30	50.81
62	36.13	50.58
63	35.98	50.37
64	35.85	50.18
65	35.73	50.02
66	35.62	49.87
67	35.53	49.74
68	35.45	49.63
69	35.38	49.53
70	35.31	49.44

Table 5.88: Scaled strain ratios for the installation load with t/D = 0.05, lift = 1, delta = 2.5

		strain ratios							
ang	scaling factor	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	55.64	0.2867	-0.1484	0.3570	-0.3552	0.0284	-0.0469	0.3570	shear
51	54.99	0.3128	-0.1620	0.3542	-0.3525	0.0260	-0.0428	0.3542	shear
52	54.39	0.3386	-0.1754	0.3507	-0.3490	0.0236	-0.0388	0.3507	shear
53	53.83	0.3641	-0.1886	0.3465	-0.3449	0.0213	-0.0350	0.3641	matrix tension
54	53.32	0.3891	-0.2016	0.3417	-0.3401	0.0191	-0.0313	0.3891	matrix tension
55	52.85	0.4135	-0.2143	0.3362	-0.3347	0.0169	-0.0277	0.4135	matrix tension
56	52.43	0.4373	-0.2267	0.3302	-0.3287	0.0149	-0.0242	0.4373	matrix tension
57	52.04	0.4604	-0.2387	0.3236	-0.3222	0.0129	-0.0209	0.4604	matrix tension
58	51.68	0.4827	-0.2503	0.3166	-0.3153	0.0109	-0.0177	0.4827	matrix tension
59	51.36	0.5043	-0.2615	0.3091	-0.3078	0.0091	-0.0146	0.5043	matrix tension
60	51.07	0.5251	-0.2723	0.3012	-0.3000	0.0085	-0.0119	0.5251	matrix tension
61	50.81	0.5451	-0.2827	0.2930	-0.2919	0.0080	-0.0115	0.5451	matrix tension
62	50.58	0.5643	-0.2926	0.2844	-0.2834	0.0076	-0.0114	0.5643	matrix tension
63	50.37	0.5826	-0.3021	0.2755	-0.2746	0.0072	-0.0113	0.5826	matrix tension
64	50.18	0.6001	-0.3112	0.2664	-0.2655	0.0067	-0.0111	0.6001	matrix tension
65	50.02	0.6167	-0.3198	0.2571	-0.2563	0.0063	-0.0109	0.6167	matrix tension
66	49.87	0.6325	-0.3280	0.2475	-0.2468	0.0059	-0.0107	0.6325	matrix tension
67	49.74	0.6474	-0.3358	0.2379	-0.2372	0.0054	-0.0104	0.6474	matrix tension
68	49.63	0.6615	-0.3431	0.2280	-0.2274	0.0050	-0.0102	0.6615	matrix tension
69	49.53	0.6749	-0.3500	0.2181	-0.2175	0.0052	-0.0099	0.6749	matrix tension
70	49.44	0.6874	-0.3565	0.2080	-0.2075	0.0061	-0.0098	0.6874	matrix tension

From Table 5.88, even with the large load factor of 1.4, no fibre angles are eliminated from consideration by this loading condition.

5.7.6 Force Due to Backfill

For design study 3, with a backfill depth given by:

$$H = (\text{depth of ditch}) - (\text{OD}) = 2 - 1.10 = 0.90\text{m}$$

then applying equation 2.6 and assuming a sand and gravel backfill, and a ditch width at the top of the pipe equal to 1.7 times the OD, the force due to backfill is calculated from:

$$C_d = \frac{1 - e^{-2\frac{(0.165)(0.90)}{1.7x1.10}}}{2(0.165)} = 0.4450$$

and

$$W_d = (0.4450)(15.7x10^3)(1.7x1.10)^2 = 24.43 \text{ kN / m}$$

Therefore the force per metre of pipe =24.43 kN/m
and the factored force per metre of pipe = 1.25x24.43 = 30.54 kN/m

For a line loading, the strain ratio is dependant on the magnitude of the loading and the diameter of the pipe. From Table C.8, the applied force from Table C.8 must be multiplied by the diameter of the pipe. Thus the scaling factor is given by:

$$\text{scaling factor} = \frac{\text{factored force}}{(\text{applied force}) \cdot \text{OD}} = \frac{30.54x10^3}{1x10^3 \cdot 1.10} = 27.76$$

For a wind angle of 50 degrees, and scaling the strain ratios from Table C.9 produces Table 5.89. The scaled strain ratios for the rest of the range of fibre angles is in Table 5.90.

Table 5.89: Strain ratio scaling sample calculation for a backfill loading, with a depth of backfill of 0.90m for a fibre angle of 50 degrees

	force/length	
strain ratio	1kN/m per diameter	27.76kN/m
maximum matrix	0.0049	0.1367
minimum matrix	-0.0022	-0.0603
maximum shear	0.0041	0.1149
minimum shear	-0.0028	-0.0781
maximum fibre	0.0013	0.0349
minimum fibre	-0.0016	-0.0452

Table 5.90: Scaled strain ratios for a 0.90m backfill with t/D = 0.05

	strain ratios							
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.1367	-0.0604	0.1149	-0.0781	0.0349	-0.0452	0.1367	matrix tension
51	0.1276	-0.0568	0.1126	-0.0760	0.0349	-0.0453	0.1276	matrix tension
52	0.1189	-0.0537	0.1102	-0.0740	0.0349	-0.0454	0.1189	matrix tension
53	0.1105	-0.0510	0.1078	-0.0719	0.0348	-0.0455	0.1105	matrix tension
54	0.1046	-0.0484	0.1053	-0.0698	0.0347	-0.0455	0.1053	shear
55	0.0994	-0.0459	0.1028	-0.0678	0.0345	-0.0456	0.1028	shear
56	0.0943	-0.0434	0.1002	-0.0657	0.0344	-0.0457	0.1002	shear
57	0.0893	-0.0410	0.0975	-0.0637	0.0342	-0.0459	0.0975	shear
58	0.0845	-0.0388	0.0949	-0.0617	0.0340	-0.0460	0.0949	shear
59	0.0799	-0.0366	0.0922	-0.0597	0.0338	-0.0461	0.0922	shear
60	0.0753	-0.0344	0.0895	-0.0577	0.0336	-0.0462	0.0895	shear
61	0.0709	-0.0323	0.0868	-0.0557	0.0334	-0.0464	0.0868	shear
62	0.0667	-0.0304	0.0841	-0.0538	0.0332	-0.0465	0.0841	shear
63	0.0625	-0.0284	0.0814	-0.0518	0.0330	-0.0466	0.0814	shear
64	0.0585	-0.0265	0.0786	-0.0499	0.0327	-0.0467	0.0786	shear
65	0.0546	-0.0247	0.0759	-0.0481	0.0326	-0.0468	0.0759	shear
66	0.0509	-0.0230	0.0731	-0.0468	0.0324	-0.0469	0.0731	shear
67	0.0472	-0.0213	0.0704	-0.0454	0.0322	-0.0470	0.0704	shear
68	0.0437	-0.0196	0.0676	-0.0441	0.0320	-0.0471	0.0676	shear
69	0.0403	-0.0181	0.0649	-0.0426	0.0319	-0.0472	0.0649	shear
70	0.0371	-0.0165	0.0621	-0.0412	0.0318	-0.0473	0.0621	shear

From Table 5.90, it can be concluded that all fibre wind angles considered in this study will be able to support the backfill.

5.7.7 Load Combinations: (1H:0A) and Backfill

The Load Combinations are obtained by simply combining the strain ratios of the pressure loading and the backfill loading. This creates a worst case condition, assuming that the maximum strains from the backfill loading occurs at the same locations as the maximum strains from the pressure loading. The (1H:0A) strain ratios are from §5.7.1 while the backfill strain ratios are from §5.7.6.

Table 5.91: Summed strain ratio sample calculation, for a fibre angle of 50 degrees

strain ratio	(1H:0A)	backfill	sum
maximum matrix	0.0588	0.1367	0.1955
minimum matrix	0.0548	-0.0604	-0.0056
maximum shear	0.6892	0.1149	0.8042
minimum shear	-0.5655	-0.0781	-0.6436
maximum fibre	0.1120	0.0349	0.1470
minimum fibre	0.1113	-0.0452	0.0661

The summed strain ratios for fibre angles of 50 to 70 degrees for this loading condition are found in Table 5.92.

Table 5.92: Summed strain ratios for a 0.90m backfill and a hoop stress from a (1H:0A) loading of 137.5 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comments
50	0.1955	-0.0056	0.8042	-0.6436	0.1470	0.0661	0.8042	shear	too high
51	0.1422	-0.0465	0.7771	-0.6258	0.1502	0.0692	0.7771	shear	
52	0.1042	-0.0705	0.7490	-0.6072	0.1531	0.0721	0.7490	shear	
53	0.0782	-0.0857	0.7202	-0.5882	0.1557	0.0747	0.7202	shear	
54	0.0553	-0.1002	0.6909	-0.5687	0.1580	0.0771	0.6909	shear	
55	0.0350	-0.1128	0.6612	-0.5490	0.1599	0.0791	0.6612	shear	
56	0.0168	-0.1235	0.6315	-0.5291	0.1616	0.0809	0.6315	shear	
57	0.0006	-0.1326	0.6018	-0.5091	0.1631	0.0824	0.6018	shear	
58	-0.0138	-0.1400	0.5723	-0.4892	0.1643	0.0836	0.5723	shear	
59	-0.0264	-0.1458	0.5433	-0.4694	0.1652	0.0847	0.5433	shear	
60	-0.0373	-0.1501	0.5148	-0.4498	0.1659	0.0855	0.5148	shear	
61	-0.0467	-0.1531	0.4868	-0.4305	0.1664	0.0861	0.4868	shear	
62	-0.0546	-0.1549	0.4596	-0.4115	0.1668	0.0866	0.4596	shear	
63	-0.0612	-0.1555	0.4332	-0.3929	0.1669	0.0869	0.4332	shear	
64	-0.0667	-0.1551	0.4077	-0.3747	0.1670	0.0870	0.4077	shear	
65	-0.0710	-0.1538	0.3830	-0.3569	0.1669	0.0871	0.3830	shear	
66	-0.0745	-0.1517	0.3593	-0.3400	0.1667	0.0870	0.3593	shear	
67	-0.0770	-0.1490	0.3364	-0.3235	0.1664	0.0868	0.3364	shear	
68	-0.0789	-0.1457	0.3146	-0.3074	0.1661	0.0865	0.3146	shear	
69	-0.0801	-0.1419	0.2936	-0.2917	0.1657	0.0862	0.2936	shear	
70	-0.0808	-0.1377	0.2735	-0.2762	0.1652	0.0857	0.2735	shear	

This loading combination reveals that the maximum strain ratios for fibre wind angles below 52° are greater than the resistance factor of 0.8.

5.7.8 Load Combinations: (8H:1A) and Backfill

The other load combinations simply follow the same format as that used for §5.7.7. The summed strain ratios for the loading combination of (8H:1A) and backfill are found in Table 5.93.

Table 5.93: Summed strain ratios for a 0.90m backfill and a hoop stress from a (8H:1A) loading of 137.5 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comments
50	0.2828	0.0812	0.6900	0.4938	0.1559	0.0749	0.6900	shear	
51	0.2391	0.0499	0.6625	0.4705	0.1584	0.0773	0.6625	shear	
52	0.1995	0.0220	0.6343	0.4468	0.1606	0.0795	0.6343	shear	
53	0.1665	0.0004	0.6057	0.4227	0.1625	0.0814	0.6057	shear	
54	0.1401	-0.0176	0.5770	0.3985	0.1640	0.0831	0.5770	shear	
55	0.1177	-0.0323	0.5482	0.3744	0.1653	0.0845	0.5482	shear	
56	0.0915	-0.0490	0.5196	0.3505	0.1664	0.0856	0.5196	shear	
57	0.0799	-0.0534	0.4914	0.3270	0.1672	0.0864	0.4914	shear	
58	0.0700	-0.0562	0.4637	0.3040	0.1677	0.0870	0.4637	shear	
59	0.0618	-0.0575	0.4366	0.2817	0.1680	0.0874	0.4366	shear	
60	0.0551	-0.0576	0.4103	0.2601	0.1681	0.0877	0.4103	shear	
61	0.0498	-0.0566	0.3848	0.2394	0.1681	0.0877	0.3848	shear	
62	0.0458	-0.0544	0.3602	0.2195	0.1679	0.0876	0.3602	shear	
63	0.0429	-0.0513	0.3366	0.2007	0.1675	0.0874	0.3366	shear	
64	0.0410	-0.0473	0.3140	0.1828	0.1671	0.0871	0.3140	shear	
65	0.0400	-0.0427	0.2924	0.1659	0.1665	0.0866	0.2924	shear	
66	0.0398	-0.0374	0.2718	0.1494	0.1659	0.0861	0.2718	shear	
67	0.0402	-0.0316	0.2523	0.1340	0.1652	0.0855	0.2523	shear	
68	0.0593	-0.0255	0.2338	0.1197	0.1644	0.0848	0.2338	shear	
69	0.0652	0.0005	0.2163	0.1065	0.1636	0.0841	0.2163	shear	
70	0.0716	0.0118	0.1997	0.0942	0.1628	0.0834	0.1997	shear	

This load combination reveals that there are no fibre angle restrictions for an (8H:1A) loading with an applied hoop stress of 137.5MPa and a backfill loading of 0.90m.

5.7.9 Load Combinations: (4H:1A) and Backfill

The summed strain ratios for this load combination are found in Table 5.94.

Table 5.94: Summed strain ratios for a 0.90m backfill and a hoop stress from a (4H:1A) loading of 137.5 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comments
50	0.3701	0.1679	0.5758	0.3797	0.1648	0.0837	0.5758	shear	
51	0.3359	0.1462	0.5479	0.3561	0.1666	0.0854	0.5479	shear	
52	0.3071	0.1296	0.5196	0.3323	0.1681	0.0869	0.5196	shear	
53	0.2841	0.1175	0.4913	0.3084	0.1692	0.0881	0.4913	shear	
54	0.2669	0.1088	0.4631	0.2848	0.1701	0.0891	0.4631	shear	
55	0.2536	0.1031	0.4352	0.2616	0.1708	0.0898	0.4352	shear	
56	0.2434	0.1005	0.4078	0.2389	0.1711	0.0902	0.4078	shear	
57	0.2363	0.1006	0.3811	0.2168	0.1712	0.0904	0.3811	shear	
58	0.2319	0.1032	0.3551	0.1956	0.1711	0.0904	0.3551	shear	
59	0.2302	0.1083	0.3300	0.1752	0.1708	0.0902	0.3300	shear	
60	0.2308	0.1154	0.3059	0.1558	0.1704	0.0898	0.3059	shear	
61	0.2334	0.1244	0.2828	0.1375	0.1697	0.0893	0.2828	shear	
62	0.2379	0.1350	0.2608	0.1203	0.1690	0.0887	0.2608	shear	
63	0.2440	0.1470	0.2400	0.1042	0.1681	0.0880	0.2440	matrix tension	
64	0.2514	0.1602	0.2203	0.0891	0.1672	0.0871	0.2514	matrix tension	
65	0.2599	0.1744	0.2018	0.0752	0.1661	0.0862	0.2599	matrix tension	
66	0.2693	0.1893	0.1844	0.0620	0.1650	0.0852	0.2693	matrix tension	
67	0.2795	0.2048	0.1681	0.0499	0.1639	0.0842	0.2795	matrix tension	
68	0.2902	0.2207	0.1530	0.0389	0.1628	0.0832	0.2902	matrix tension	
69	0.3013	0.2368	0.1389	0.0291	0.1616	0.0821	0.3013	matrix tension	
70	0.3126	0.2530	0.1259	0.0204	0.1605	0.0810	0.3126	matrix tension	

Table 5.94 reveals that there are no fibre angles restrictions resulting from this loading.

5.7.10 Load Combinations: (2H:1A) and Backfill

Table 5.95 contains the summed strain ratios for this loading condition.

Table 5.95: Summed strain ratios for a 0.90m backfill and a hoop stress from a (2H:1A) loading of 137.5 MPa

	strain ratios								
ang	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio	comments
50	0.5447	0.3415	0.3475	0.1516	0.1826	0.1013	0.5447	matrix tension	
51	0.5322	0.3418	0.3187	0.1272	0.1830	0.1016	0.5322	matrix tension	
52	0.5239	0.3452	0.2902	0.1032	0.1830	0.1017	0.5239	matrix tension	
53	0.5193	0.3517	0.2624	0.0799	0.1828	0.1015	0.5193	matrix tension	
54	0.5205	0.3614	0.2353	0.0574	0.1823	0.1011	0.5205	matrix tension	
55	0.5253	0.3740	0.2092	0.0359	0.1816	0.1004	0.5253	matrix tension	
56	0.5330	0.3892	0.1842	0.0156	0.1806	0.0995	0.5330	matrix tension	
57	0.5433	0.4069	0.1604	-0.0035	0.1794	0.0984	0.5433	matrix tension	
58	0.5560	0.4267	0.1379	-0.0214	0.1780	0.0971	0.5560	matrix tension	
59	0.5707	0.4483	0.1167	-0.0378	0.1765	0.0957	0.5707	matrix tension	
60	0.5871	0.4715	0.0625	-0.0847	0.1748	0.0941	0.5871	matrix tension	
61	0.6051	0.4960	0.0457	-0.0968	0.1730	0.0925	0.6051	matrix tension	
62	0.6242	0.5216	0.0303	-0.1076	0.1712	0.0908	0.6242	matrix tension	
63	0.6444	0.5478	0.0163	-0.1169	0.1693	0.0890	0.6444	matrix tension	
64	0.6654	0.5747	0.0037	-0.1249	0.1673	0.0872	0.6654	matrix tension	
65	0.6869	0.6018	-0.0075	-0.1315	0.1654	0.0854	0.6869	matrix tension	
66	0.7086	0.6290	-0.0173	-0.1373	0.1634	0.0835	0.7086	matrix tension	
67	0.7304	0.6562	-0.0259	-0.1417	0.1614	0.0817	0.7304	matrix tension	
68	0.7522	0.6830	-0.0331	-0.1448	0.1595	0.0799	0.7522	matrix tension	
69	0.7736	0.7095	-0.0392	-0.1467	0.1576	0.0781	0.7736	matrix tension	
70	0.7946	0.7353	-0.0442	-0.1474	0.1558	0.0763	0.7946	matrix tension	

The load combination of (2H:1A) and backfill eliminates no fibre wind angles.

5.7.11 Load Combinations: (1H:0A), Backfill and Bending Moment

Utilizing the strain ratios from the load combinations from 5.7.7 through 5.7.10 it is possible then to calculate the maximum bending moment which will make the maximum strain ratio equal to the resistance factor (0.8).

In the following tables, a maximum ratio not equal to 0.8 or of "#N/A" means that the strain ratio was greater than 0.8 before the application of the moment, and thus that fibre angle need not be considered.

For this loading combination, the scaling factor is determined by finding the minimum value which will make any strain ratio equal to 0.8

$$\text{scaling factor} = \min\left(\text{abs}\left(\frac{\text{resistance factor} - \text{strain ratio(i)}}{\text{moment strain ratio(i)}}\right)\right)$$

where:

resistance factor =	0.8
ratio(i) =	strain ratio (min or max of fibre, matrix or shear) from combined pressure and backfill loading
moment strain ratio(i) =	strain ratio (min or max of fibre, matrix or shear) from moment loading

the ratio(i) values are taken from sections §5.7.7-§5.7.10 of this design study, while the moment strain ratio(i) values are taken from Table C.5 of Appendix C.

For a fibre angle of 60 degrees, the scaling factor is determined by finding the minimum of possible scaling factors. This is demonstrated in Table 5.96.

Table 5.96: Determination of minimum scaling factor for determination of maximum allowable bending moment on a pipe loaded with (1H:0A) with a hoop stress of 137.5 MPa and a backfill of 0.90m for a fibre angle of 60 degrees

strain ratio	strain ratio from (1H:0A) and backfill	strain ratio from unit moment loading	$\text{abs}\left(\frac{\text{resistance factor} - \text{strain ratio}}{\text{moment strain ratio}}\right)$
maximum matrix	-0.0373	0.0103	81.43
minimum matrix	-0.1501	-0.0053	178.2
maximum shear	0.5148	0.0059	48.37
minimum shear	-0.4498	-0.0059	212.8
maximum fibre	0.1659	0.0002	3826
minimum fibre	0.0855	-0.0002	3070

Thus the minimum ratio is 48.37.

Now, using the minimum scaling factor, the maximum strain ratio should be equal to 0.8, the resistance factor. This is demonstrated in Table 5.97.

Table 5.97: Sample calculation of summed and scaled strain ratios for (1H:0A) and pressure and bending moment for a fibre angle of 60 degrees

strain ratio	strain ratio from (1H:0A) + backfill	strain ratio from unit moment loading	scaled moment strain ratio	summed strain ratio from (1H:0A) + backfill + scaled moment
maximum matrix	-0.0373	0.0103	0.4973	0.4600
minimum matrix	-0.1501	-0.0053	-0.2579	-0.4080
maximum shear	0.5148	0.0059	0.2852	0.8000
minimum shear	-0.4498	-0.0059	-0.2841	-0.7340
maximum fibre	0.1659	0.0002	0.0080	0.1739
minimum fibre	0.0855	-0.0002	-0.0113	0.0742

Thus the maximum strain ratio is equal to 0.8, the resistance factor, for the shear strain.

The maximum axial stress is found by multiplying the maximum axial stress used in Table C.5 by the scaling factor.

Scaling the maximum axial stress from the moment loading is obtained by multiplying the scaling factor by the unit load from Table C.5

Scaled maximum axial stress = 43.18 x1MPa = 43.18MPa

The actual moment load which causes the maximum axial stress can be calculated from:

$$\sigma_{max} = \frac{M \cdot OD}{2I} \therefore M = \frac{2\sigma_{max} I}{OD}$$

So, for the fibre angle of 60 degrees:

$$M = \frac{2x48.37x22.78x10^{-3}}{1.10} = 200.\ 3kN \cdot m$$

The strain ratios for fibre angles of 50 to 70 degrees for this loading combination are presented in Table 5.98. The bending moment and maximum axial stress values are in Table 5.99.

Table 5.98: Strain Ratios resulting from a backfill of 0.90m, hoop stress of 137.5 MPa in a (1H:0A) loading, and the maximum allowable moment

		strain ratios							
ang	scaling factor	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	0.6481	0.1988	-0.0073	0.8083	-0.6477	0.1473	0.0656	0.8083	#N/A
51	3.561	0.1625	-0.0570	0.8000	-0.6486	0.1519	0.0665	0.8000	shear
52	7.910	0.1534	-0.0960	0.8000	-0.6580	0.1565	0.0665	0.8000	shear
53	12.40	0.1620	-0.1292	0.8000	-0.6676	0.1606	0.0667	0.8000	shear
54	17.03	0.1796	-0.1646	0.8000	-0.6773	0.1641	0.0671	0.8000	shear
55	21.82	0.2057	-0.2012	0.8000	-0.6871	0.1669	0.0677	0.8000	shear
56	26.76	0.2400	-0.2392	0.8000	-0.6969	0.1692	0.0685	0.8000	shear
57	31.87	0.2825	-0.2788	0.8000	-0.7065	0.1710	0.0696	0.8000	shear
58	37.17	0.3333	-0.3200	0.8000	-0.7159	0.1721	0.0709	0.8000	shear
59	42.66	0.3925	-0.3630	0.8000	-0.7251	0.1728	0.0725	0.8000	shear
60	48.37	0.4600	-0.4080	0.8000	-0.7340	0.1739	0.0742	0.8000	shear
61	54.32	0.5360	-0.4553	0.8000	-0.7425	0.1750	0.0738	0.8000	shear
62	60.53	0.6207	-0.5051	0.8000	-0.7506	0.1759	0.0729	0.8000	shear
63	67.05	0.7143	-0.5577	0.8000	-0.7584	0.1765	0.0719	0.8000	shear
64	72.48	0.8000	-0.6046	0.7925	-0.7582	0.1767	0.0710	0.8000	matrix tension
65	70.65	0.8000	-0.6056	0.7461	-0.7188	0.1758	0.0716	0.8000	matrix tension
66	68.95	0.8000	-0.6053	0.7015	-0.6813	0.1748	0.0722	0.8000	matrix tension
67	67.38	0.8000	-0.6038	0.6587	-0.6448	0.1738	0.0726	0.8000	matrix tension
68	65.93	0.8000	-0.6015	0.6175	-0.6095	0.1727	0.0730	0.8000	matrix tension
69	64.59	0.8000	-0.5983	0.5780	-0.5753	0.1724	0.0733	0.8000	matrix tension
70	63.34	0.8000	-0.5945	0.5400	-0.5420	0.1731	0.0731	0.8000	matrix tension

Table 5.99: Maximum axial stresses and moments for backfill of 0.90m and a hoop stress of 137.5 MPa in a (1H:0A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	3.561	147.5
51	7.910	327.6
52	12.40	513.6
53	17.03	705.5
54	21.82	903.7
55	26.76	1108
56	31.87	1320
57	37.17	1539
58	42.66	1767
59	48.37	2003
60	54.32	2250
61	60.53	2507
62	67.05	2777
63	72.48	3002
64	70.65	2926
65	68.95	2856
66	67.38	2791
67	65.93	2731
68	64.59	2675
69	63.34	2624
70	62.92	2606

5.7.12 Load Combinations: (8H:1A), Backfill and Bending Moment

For the this load combination, the combined strain ratios are given in Table 5.100 and the scaled maximum axial stress and bending moment are given in Table 5.101.

5.7.13 Load Combinations: (4H:1A), Backfill and Bending Moment

For the this load combination, the combined strain ratios are given in Table 5.102 and the scaled maximum axial stress and bending moment are given in Table 5.103.

5.7.14 Load Combinations: (2H:1A), Backfill and Bending Moment

For the this load combination, the combined strain ratios are given in Table 5.104 and the scaled maximum axial stress and bending moment are given in Table 5.105.

Table 5.100: Strain Ratios resulting from a backfill of 0.90m, hoop stress of 137.5 MPa in a (8H:1A) loading, and the maximum allowable moment

ang	scaling factor	strain ratios maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	17.14	0.3711	0.0354	0.8000	0.3843	0.1646	0.0605	0.8000	shear
51	21.35	0.3605	-0.0130	0.8000	0.3337	0.1685	0.0607	0.8000	shear
52	25.69	0.3595	-0.0609	0.8000	0.2819	0.1717	0.0612	0.8000	shear
53	30.18	0.3706	-0.1053	0.8000	0.2294	0.1744	0.0618	0.8000	shear
54	34.80	0.3941	-0.1492	0.8000	0.1766	0.1765	0.0627	0.8000	shear
55	39.58	0.4273	-0.1928	0.8000	0.1238	0.1780	0.0638	0.8000	shear
56	44.51	0.4627	-0.2415	0.8000	0.0714	0.1790	0.0650	0.8000	shear
57	49.62	0.5188	-0.2809	0.8000	0.0197	0.1794	0.0665	0.8000	shear
58	54.90	0.5828	-0.3220	0.8000	-0.0309	0.1793	0.0682	0.8000	shear
59	60.38	0.6547	-0.3650	0.8000	-0.0802	0.1787	0.0702	0.8000	shear
60	66.08	0.7345	-0.4100	0.8000	-0.1281	0.1791	0.0723	0.8000	shear
61	69.93	0.8000	-0.4456	0.7880	-0.1623	0.1791	0.0719	0.8000	matrix tension
62	67.61	0.8000	-0.4455	0.7403	-0.1592	0.1780	0.0724	0.8000	matrix tension
63	65.46	0.8000	-0.4439	0.6947	-0.1561	0.1769	0.0728	0.8000	matrix tension
64	63.48	0.8000	-0.4410	0.6510	-0.1531	0.1756	0.0730	0.8000	matrix tension
65	61.64	0.8000	-0.4368	0.6092	-0.1499	0.1743	0.0732	0.8000	matrix tension
66	59.95	0.8000	-0.4317	0.5694	-0.1472	0.1729	0.0733	0.8000	matrix tension
67	58.38	0.8000	-0.4257	0.5314	-0.1443	0.1716	0.0733	0.8000	matrix tension
68	55.57	0.8000	-0.4096	0.4891	-0.1349	0.1700	0.0735	0.8000	matrix tension
69	53.92	0.8000	-0.3806	0.4537	-0.1303	0.1693	0.0734	0.8000	matrix tension
70	52.38	0.8000	-0.3659	0.4201	-0.1256	0.1694	0.0730	0.8000	matrix tension

Table 5.101: Maximum axial stresses and moments for backfill of 0.90m and a hoop stress of 137.5 MPa in a (8H:1A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	17.14	710.1
51	21.35	884.3
52	25.69	1064
53	30.18	1250
54	34.80	1442
55	39.58	1639
56	44.51	1844
57	49.62	2055
58	54.90	2274
59	60.38	2501
60	66.08	2737
61	69.93	2896
62	67.61	2800
63	65.46	2711
64	63.48	2629
65	61.64	2553
66	59.95	2483
67	58.38	2418
68	55.57	2302
69	53.92	2234
70	52.38	2170

Table 5.102: Strain Ratios resulting from a backfill of 0.90m. hoop stress of 137.5 MPa in a (4H:1A) loading and the maximum allowable moment

ang	scaling factor	strain ratios maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	34.94	0.5501	0.0748	0.8000	0.1567	0.1648	0.0543	0.8000	shear
51	39.14	0.5586	0.0309	0.8000	0.1052	0.1851	0.0550	0.8000	shear
52	43.48	0.5779	-0.0107	0.8000	0.0533	0.1869	0.0559	0.8000	shear
53	47.95	0.6085	-0.0505	0.8000	0.0012	0.1882	0.0570	0.8000	shear
54	52.58	0.6505	-0.0901	0.8000	-0.0505	0.1890	0.0583	0.8000	shear
55	57.34	0.7022	-0.1294	0.8000	-0.1015	0.1891	0.0598	0.8000	shear
56	62.27	0.7628	-0.1688	0.8000	-0.1516	0.1888	0.0615	0.8000	shear
57	63.72	0.8000	-0.1917	0.7774	-0.1778	0.1870	0.0648	0.8000	matrix tension
58	60.82	0.8000	-0.1913	0.7277	-0.1755	0.1840	0.0696	0.8000	matrix tension
59	58.03	0.8000	-0.1872	0.6792	-0.1726	0.1811	0.0736	0.8000	matrix tension
60	55.36	0.8000	-0.1798	0.6324	-0.1694	0.1795	0.0769	0.8000	matrix tension
61	52.81	0.8000	-0.1694	0.5873	-0.1658	0.1781	0.0774	0.8000	matrix tension
62	50.38	0.8000	-0.1565	0.5441	-0.1619	0.1766	0.0773	0.8000	matrix tension
63	48.07	0.8000	-0.1413	0.5030	-0.1578	0.1750	0.0772	0.8000	matrix tension
64	45.88	0.8000	-0.1243	0.4639	-0.1536	0.1733	0.0770	0.8000	matrix tension
65	43.81	0.8000	-0.1057	0.4269	-0.1492	0.1716	0.0767	0.8000	matrix tension
66	41.84	0.8000	-0.0859	0.3921	-0.1451	0.1700	0.0763	0.8000	matrix tension
67	39.99	0.8000	-0.0651	0.3594	-0.1408	0.1683	0.0758	0.8000	matrix tension
68	38.24	0.8000	-0.0437	0.3287	-0.1363	0.1667	0.0753	0.8000	matrix tension
69	36.60	0.8000	-0.0218	0.3001	-0.1316	0.1654	0.0748	0.8000	matrix tension
70	35.05	0.8000	0.0002	0.2734	-0.1267	0.1648	0.0740	0.8000	matrix tension

Table 5.103: Maximum axial stresses and moments for backfill of 0.90m and a hoop stress of 137.5 MPa in a (4H:1A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	34.94	1447
51	39.14	1621
52	43.48	1801
53	47.95	1986
54	52.58	2178
55	57.34	2375
56	62.27	2579
57	63.72	2639
58	60.82	2519
59	58.03	2404
60	55.36	2293
61	52.81	2188
62	50.38	2087
63	48.07	1991
64	45.88	1901
65	43.81	1815
66	41.84	1733
67	39.99	1656
68	38.24	1584
69	36.60	1516
70	35.05	1452

Table 5.104: Strain Ratios resulting from a backfill of 0.90m, hoop stress of 137.5 MPa in a (2H:1A) loading, and the maximum allowable moment

		strain ratio							
ang	scaling factor	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	49.55	0.8000	0.2094	0.6654	-0.1647	0.2079	0.0595	0.8000	matrix tension
51	47.08	0.8000	0.2031	0.6220	-0.1746	0.2052	0.0650	0.8000	matrix tension
52	44.35	0.8000	0.2022	0.5763	-0.1814	0.2023	0.0700	0.8000	matrix tension
53	41.50	0.8000	0.2063	0.5296	-0.1860	0.1993	0.0745	0.8000	matrix tension
54	38.31	0.8000	0.2165	0.4808	-0.1869	0.1960	0.0786	0.8000	matrix tension
55	35.11	0.8000	0.2316	0.4326	-0.1864	0.1928	0.0821	0.8000	matrix tension
56	32.01	0.8000	0.2508	0.3858	-0.1851	0.1897	0.0847	0.8000	matrix tension
57	29.01	0.8000	0.2738	0.3408	-0.1832	0.1866	0.0868	0.8000	matrix tension
58	26.12	0.8000	0.3002	0.2979	-0.1807	0.1835	0.0882	0.8000	matrix tension
59	23.35	0.8000	0.3294	0.2573	-0.1778	0.1806	0.0890	0.8000	matrix tension
60	20.70	0.8000	0.3612	0.1846	-0.2063	0.1782	0.0893	0.8000	matrix tension
61	18.17	0.8000	0.3950	0.1504	-0.2012	0.1759	0.0884	0.8000	matrix tension
62	15.75	0.8000	0.4304	0.1189	-0.1958	0.1736	0.0872	0.8000	matrix tension
63	13.45	0.8000	0.4672	0.0899	-0.1902	0.1712	0.0860	0.8000	matrix tension
64	11.26	0.8000	0.5049	0.0635	-0.1844	0.1688	0.0847	0.8000	matrix tension
65	9.175	0.8000	0.5431	0.0397	-0.1785	0.1665	0.0834	0.8000	matrix tension
66	7.205	0.8000	0.5816	0.0184	-0.1729	0.1642	0.0820	0.8000	matrix tension
67	5.344	0.8000	0.6201	-0.0003	-0.1672	0.1620	0.0806	0.8000	matrix tension
68	3.590	0.8000	0.6582	-0.0166	-0.1613	0.1599	0.0791	0.8000	matrix tension
69	1.938	0.8000	0.6958	-0.0307	-0.1552	0.1578	0.0777	0.8000	matrix tension
70	0.3874	0.8000	0.7325	-0.0425	-0.1491	0.1559	0.0762	0.8000	matrix tension

Table 5.105: Maximum axial stresses and moments for backfill of 0.90m and a hoop stress of 137.5 MPa in a (2H:1A) loading which make the maximum strain ratio 0.8.

ang	maximum axial stress (MPa)	moment (kN·m)
50	49.55	2052
51	47.08	1950
52	44.35	1837
53	41.50	1719
54	38.31	1587
55	35.11	1454
56	32.01	1326
57	29.01	1202
58	26.12	1082
59	23.35	967.3
60	20.70	857.5
61	18.17	752.6
62	15.75	652.6
63	13.45	557.2
64	11.26	466.2
65	9.175	380.0
66	7.205	298.4
67	5.344	221.4
68	3.590	148.7
69	1.938	80.29
70	0.3874	16.05

5.7.15 Maximum Bending Moment Considerations

The maximum allowable bending moment values for the combined loadings of sections §5.7.11 through §5.7.14 are summarized in Table 5.106.

Table 5.106: Summary of maximum allowable bending moments from sections §5.7.11 through §5.7.14

	maximum moment (kN·m) for combined stress ratio pressure loading and backfill loading			
fibre angle	(1H:0A)	(8H:1A)	(4H:1A)	(2H:1A)
50	147.5	710.1	1447	2052
51	327.6	884.3	1621	1950
52	513.6	1064	1801	1837
53	705.5	1250	1986	1719
54	903.7	1442	2178	1587
55	1108	1639	2375	1454
56	1320	1844	2579	1326
57	1539	2055	2639	1202
58	1767	2274	2519	1082
59	2003	2501	2404	967.3
60	2250	2737	2293	857.5
61	2507	2896	2188	752.6
62	2777	2800	2087	652.6
63	3002	2711	1991	557.2
64	2926	2629	1901	466.2
65	2856	2553	1815	380.0
66	2791	2483	1733	298.4
67	2731	2418	1656	221.4
68	2675	2302	1584	148.7
69	2624	2234	1516	80.29
70	2606	2170	1452	16.05

Comparing the maximum allowable bending moments from Design Study 3b(Table 5.106), to the bending moments from Design Study 3a (Table 5.79), for fibre angles of 57-61 degrees, for the range of allowable fibre angles from design study 3a, it is seen that the range of allowable bending moments changes from 51.73 kN·m to 1641 kN·m to 752.6 kN·m - 2896 kN·m.

As with design study 3b, if the moment "reserve" is insufficient, the designer would simply increase the thickness of the pipe and repeat the calculations.

5.7.16 Summary of Design Study 3b

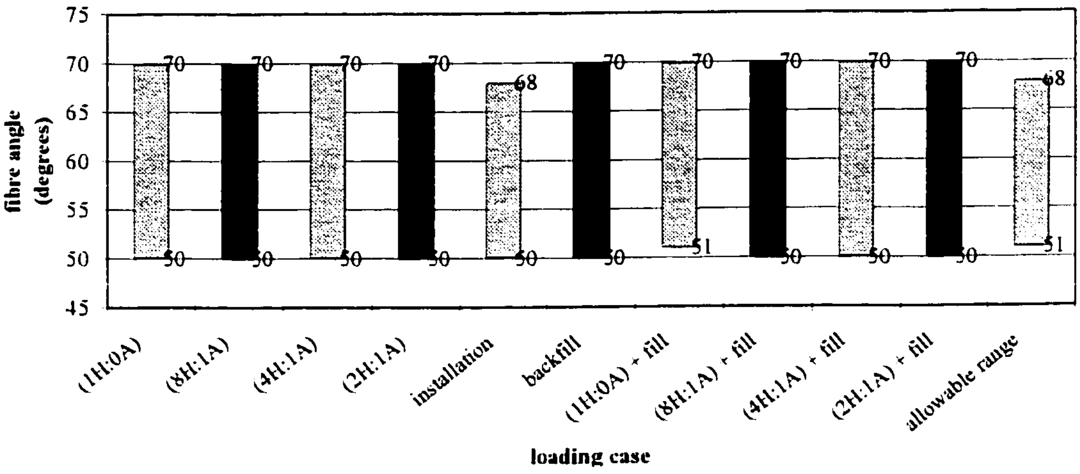

Figure 5.4: Fibre angle compatibilities for Design Study 3b

The change in t/D from 0.04 in design study 3a to 0.05 in design study 3b has changed the range of allowable fibre angles from 57-61 degrees to 51-68 degrees. The 1cm change in pipe thickness has made a dramatic change in the range of fibre angles. This is mainly due to the large change in the factored average hoop stress from 179.1 MPa to 137.5 MPa.

From section §5.7.15, it is also demonstrated that the simple change in thickness has a dramatic effect on the maximum allowable bending moment.

From Figure 5.4, fibre angles between 51 and 68 degrees satisfy the design criteria for this design study. As with the previous design studies, the service load would generally be the combination of (1H:0A) and backfill, so the fibre wind angle chosen for the pipe would tend to be towards the upper limit of the allowable range, i.e. 68 degrees.

5.8 Summary of the Design Studies

From the design studies in §5.4-5.7, the limitations to the fibre angles come from the most extreme of the loading cases. For instance, the upper end of the possible fibre angle range is limited by the loading case with the greatest axial load, while the lower end of the fibre range is limited by the loading with the greatest hoop loading. For the design studies completed here, the upper end of the fibre range was limited by the (2H:1A) and backfill loading, while the lower end was limited by the pure hoop (1H:0A) and backfill loading combination.

Design Study 1 had a design pressure of 15MPa, an ID of 0.5m, and a backfill of 1.94m. For a t/D of 0.06, the range of fibre angles was 54-65 degrees.

For Design Study 2, the design pressure was 30MPa, with an ID of 0.5m, and a backfill of 2.90m. For a t/D of 0.10, the range of fibre angles was found to be 56-62 degrees. Buoyant forces were also considered for this study, and it was demonstrated that the forces caused by the anchoring required to maintain negative buoyancy were significantly less than the forces due to backfill, thus the strains due to anchoring would also be much less than the strains caused by the backfill loading.

The third design study was for an internal pressure of 11MPa, and an ID of 1.0m. The calculations were completed twice, once for a t/D of 0.04 in §5.5, and for a t/D of 0.05 in §5.6. The backfill loadings are dependant upon the outside diameter of the pipe, and were 0.92m for Design Study 3a, and 0.90m for Design Study 3b.

For Design Study 3a, the range of fibre angles was found to be 57-61 degrees. Design Study 3b only differed from Design Study 3a by the t/D of the pipe. This increase in pipe thickness changed the range of fibre angles to 51-68 degrees.

Design Studies 3a and 3b also included considerations for the maximum allowable bending moments. The increase in t/D from 0.04 to 0.05 proved to make a significant change in the ability of the pipe to withstand the moment loading while under the combined biaxial loadings and backfill.

Table 5.107: Summary of the design studies

	case 1	case 2	case 3
pressure (MPa)	15	30	11
ID (m)	0.5	0.5	1.0
installation parameters			
lift (m)	1	1	1
ditch depth (m)	2.5	3.5	2
loadings (common to all)	(1H:0A) (8H:1A) (4H:1A) (2H:1A) installation backfill combinations of the biaxial loads and backfill calculation of maximum allowable bending moment above biaxial ratios and backfill loading		
t/D	0.06	0.10	0.04,0.05
depth of backfill (m)	1.94	2.90	0.92,0.90
range of fibre angles which satisfy the loading conditions (degrees)	54-65	56-62	57-61,51-68

Chapter 6

Summary and Conclusions

6.1 Summary

The objective of this study was to perform a design study of a FRP pipe under pipeline loading conditions. To achieve this a general finite element analysis code ANSYS was used. The composite pipe was modelled using ANSYS element Solid46, which is an eight node, three degree of freedom per node, 3-D structural layered solid. The assumptions and limitations inherent in the element limited the study to the elastic range of the material. Since the modelling is concerned with a high pressure pipeline, there is a need for a high degree of reliability and hence almost no tolerance for damage, therefore the elastic range limit is appropriate.

The pipeline loading conditions essentially reduce to three basic loading cases: applied biaxial stress ratios, bending moments, and line load forces. For all of these loadings, it was shown that in the range of elastic material properties, the strains can be scaled to a given loading from those obtained for a unit loading. For the case of applied biaxial stresses, by specifying the loading as an averaged hoop stress through the cross section, it was possible to back-calculate the applied pressure based on the physical dimensions of the pipe. Similarly for a moment loading, by specifying the loading as the maximum axial stress, the applied moment and the resultant strains can be calculated based on the physical dimensions of the pipe.

The only loading which did not entirely adhere to these principals was the longitudinal force loading, which resulted in strains being dependant upon the thickness to diameter ratio of the pipe, and the pipe's diameter. Similar to the other two loadings, however, the strain results can easily be scaled according to the specified loading, dependant upon the physical dimensions of the pipe.

Failure in the composite material was based on the theory of maximum strain, where failure is assumed to take place when the strain in either the fibre or the matrix equals its failure strain. The failure strain is determined by mechanical characterization tests, for both tension and compression. This failure criteria proved to be quite useful, as each constituent of the material was separately compared to its failure strain. From this, it was possible to determine the failure mode of the composite.

6.2 Conclusions

The work in this thesis provided valuable insight into the effects of the fibre wind angle on the strains in the composite and the failure mode of the material. It was demonstrated for a $[\pm\theta]_{2s}$ layup, that altering the fibre wind angle, θ, changed the strains in the laminate caused by the applied loadings. From this, it was demonstrated that it is possible to orient the fibre angle in the composite such that the strains were minimized. This was demonstrated for all loading cases considered. The classic example of this is for a pressure vessel loading of (2H:1A), the minimum strains occur for a fibre angle of ~53 degrees.

It was also demonstrated that for any given loading, the fibre wind angle dictates the mode at which failure will initiate within the pipe. This gives the designer not only the ability to custom tailor the material to the specific loading conditions, but also the freedom to design a material system based on a specific failure mode. For example it is feasible for a pipeline to be designed such that it will leak before it fails in a catastrophic burst mode.

The fibre angle orientation which gives the minimum strain for a given loading case will not always produce a "leak before burst" failure mode. In fact, the opposite is most often true. The minimum strains generally occur when the fibres carry most of the load. For example, for a (1H:0A) loading, the minimum strains will occur for a fibre orientation of 90 degrees from the axis of the pipe. The failure mode for this loading and fibre angle is failure of the fibre in tension, indicating a burst mode failure. For the same loading, but a fibre angle of 70 degrees, the strains will be higher, but the failure mode will be that of shear, indicating that the pipe will leak before ultimate failure.

For biaxial loadings where the ratios are more balanced, (2H:1A), (1H:1A), and (1H:2A), for instance, the failure mode for all fibre angles are the same, being failure of the matrix in tension. For all fibre angles, for these loading cases, the failure will be a "leak before burst". Thus, for these loadings, it is possible to both minimize the strains and have an acceptable failure mode.

Three design studies were completed in which the strain ratio results obtained from the numerical model were applied to actual pipe design criteria. The design studies demonstrated that a properly designed FRP pipe is capable of operating under pipeline loadings. Since the strains in the composite are dependant upon both the fibre angle and the loading conditions, the selection of the correct fibre angle for the composite pipeline is essential. The design studies indicated that for a range of biaxial loadings, there exists a range of fibre wind angles which satisfy the design criteria, without resorting to a pipe that is over designed, i.e. an "optimum" condition exists.

The design proceedure presented in the design studies determines a range of fibre angle placements for a pipeline with a variety of possible loadings in conjunction with a controlled failure mode. Simple analytical methods, such as netting analysis, may predict a fibre angle for optimal use of the fibre strength, but only for a single specific hoop to axial stress loading, and are based entirely on fibre failure.

The finite element modelling reported here demonstrated that a glass fibre reinforced epoxy matrix composite pipe could be safely used in a high pressure pipeline material. For the specific loading cases expected for a pipeline, an "optimum" fibre angle exists which will minimize the strains occuring within the composite material.

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Appendix A
Modelling of Pipeline Installation Load

A.1 Introduction

To ensure that a pipeline withstands load combinations that it may be subjected to, both operating and installation loads must be considered. Of particular concern was the proper modelling of the installation loading. Considerable time was spent investigating analytical modelling of the deflected shape of a pipeline during installation, and finally a numerical model was created which accurately models the installation of the pipeline.

A.2 Pipeline Installation Modelling Currently in Practice

NOVA Gas Transmission Limited (NGTL), a subsidiary of Trans Canada Pipeline (TCPL) uses an equation of the form[26]:

$$M = .9413 \cdot (E \cdot I \cdot w \cdot \delta)^{\frac{1}{2}} \tag{A.1}$$

where

E	= Young's modulus of the material (Pa)
I	= second moment of area for the pipe (m^4)
w	= weight of pipe per unit length (N/m)
δ	= vertical deflection of pipe

The equation is based on an idealized natural deflected shape and single sideboom support to calculate the installation stresses and strains on steel pipe due to the installation loading. This is a worst case scenario as a typical pipeline installation utilizes more than one sideboom support to distribute the loading along the pipe. Thus the stresses due to installation could be considerably less. Figure A.1 shows the deflected shape and single support of a pipe during installation. The derivation and ultimate source of equation (A.1) could not be traced at the time of writing.

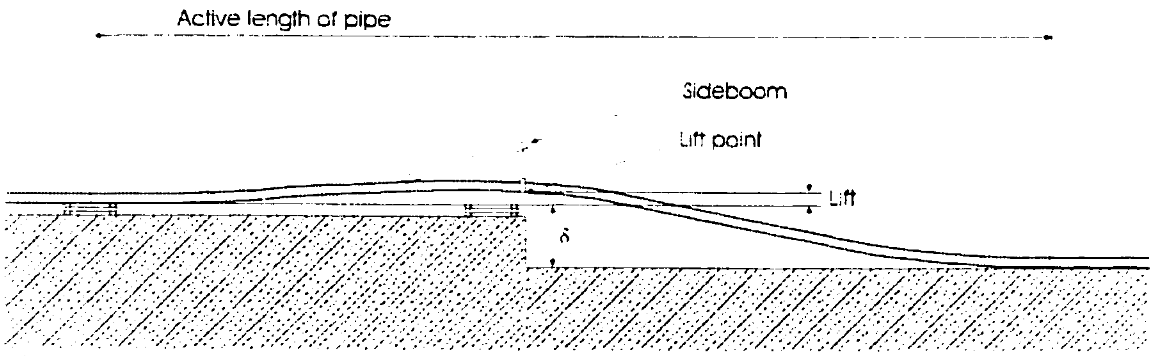

Figure A.1 Deflected Shape of a Pipe During Installation

A.3 Analytical Modelling

The loading equation used by NGTL is insufficient as it does not account for the possibility of a displacement or lift at the point of sideboom support which may be required to lift the pipe so that the sideboom's support sling will clear the pipeline supports. Analytical modelling was attempted to model the deformed shape of the pipeline in order to create either an equation of the form of Equation A.1 which included the lift condition or at least to confirm the equation in use by NOVA. Many difficulties were encountered with the analytical modelling. The length of the pipeline active with regards to the installation process, a dependant value for the analytical beam theory equations, is unknown. Various different analytical models were attempted, but all proved either insufficient at modelling the deformed shape or introduced unknown variables. The analytical model was thus abandoned in favour of a numerical model.

A.4 The Pipe Element and Contact Element Model

A numerical model was created which utilized ANSYS element Pipe16 [28], an elastic uniaxial straight pipe element and ANSYS element Contac26 [28], a point-to-line 2D contact element. This model allowed for modelling of a long pipeline with a point of lift and a "ditch" into which the pipe may settle, with the only loading being at the point of lift and the pipe's self weight. (See Figure A.2) The long length of the model and the contact elements allowed the pipeline to take its natural deformed shape with no length restrictions. The numerical model also allowed for rapid parametric evaluation of variables such as diameter, sidewall thickness ratio and Young's modulus.

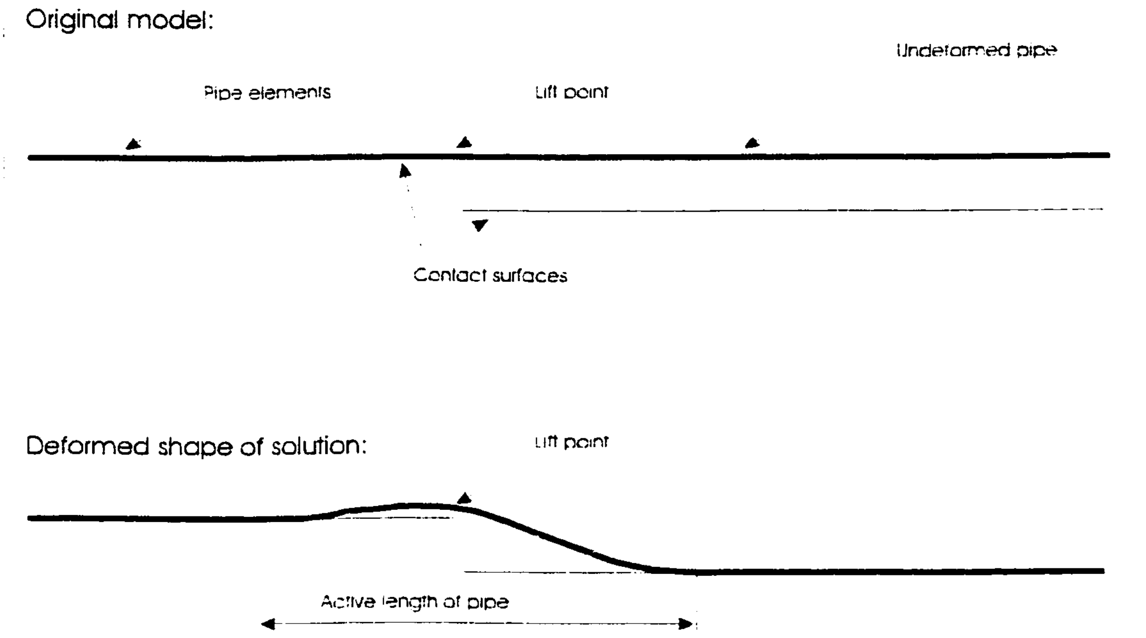

Figure A.2 The pipe element and contact element model (before solution and after solution)

Evaluation of the different parameters in the finite element model revealed that a pure bending moment equation of the form:

$$M = c_1 (E \cdot I \cdot w)^{\frac{1}{2}} \tag{A.2}$$

with c_1 given in Table A.1 was sufficient to model the stresses and strains in a pipeline due to the installation loading. It is seen that $0.96 \le c_1 \le 2.78$ depending on the pipe lift and depth of the trench.

Table A.1 Coefficient c_1 for determination of moment from installation loading

lift (m) \ depth(m)	1	2	3	4	5	6
0	0.96	1.36	1.67	1.93	2.16	2.36
0.25	1.21	1.55	1.82	2.06	2.28	2.47
0.5	1.40	1.71	1.96	2.19	2.39	2.58
0.75	1.57	1.85	2.09	2.30	2.50	2.68
1	1.73	1.98	2.21	2.41	2.60	2.78

The stresses and strains can be found directly by making a few substitutions into equation (A.2)

Knowing that:

$$\sigma_{Amax} = \frac{M \cdot OD}{2 \cdot I} \tag{A.3}$$

and

$$\varepsilon = \frac{\sigma}{E} \tag{A.4}$$

With a weight per unit length of :

$$w = \rho \cdot g \cdot \pi \cdot \left(\frac{OD}{2}\right)^2 \cdot \left(1 - \left(1 - 2\frac{t}{D}\right)^2\right) \tag{A.5}$$

and the second moment of area given by:

$$I = \frac{\pi}{4} \cdot \left(\frac{OD}{2}\right)^4 \cdot \left(1 - \left(1 - 2\frac{t}{D}\right)^4\right) \tag{A.6}$$

and substituting (A.2), (A.5), and (A.6) into (A.3)

$$\sigma_{max} = c_2 \cdot (E \cdot \rho)^{\frac{1}{2}} \cdot \left(1 + \left(1 - 2\frac{t}{D}\right)^2\right)^{-\frac{1}{2}} \tag{A.7}$$

and substituting (A.7) into (A.4)

$$\varepsilon_{max} = c_2 \cdot \left(\frac{\rho}{E}\right)^{\frac{1}{2}} \cdot \left(1 + \left(1 - 2\frac{t}{D}\right)^2\right)^{-\frac{1}{2}} \tag{A.8}$$

with:

$$c_2 = 2 \cdot g^{\frac{1}{2}} \cdot c_1 = 2 \cdot (9.81)^{\frac{1}{2}} \cdot c_1 = 6.26 \cdot c_1$$

Table A.2: Coefficient c_2 for determination of maximum axial stress and strain from installation loading

lift (m) \ depth (m)	1	2	3	4	5	6
0	6.01	8.51	10.45	12.08	13.52	14.77
0.25	7.57	9.70	11.39	12.90	14.27	15.46
0.5	8.76	10.70	12.27	13.71	14.96	16.15
0.75	9.83	11.58	13.08	14.40	15.65	16.78
1	10.83	12.39	13.83	15.09	16.28	17.40

Figures A.3 and A.4 graphically demonstrate coefficients c_1 and c_2.

It is worthy noting that the maximum axial stresses and strains (Equations A.7 and A.8) sustained by a pipeline during installation are dependant upon depth of ditch, lift, density, Young's modulus and the t/D ratio.

In a steel pipeline, the specified minimum yield stress (SMYS) is 483 MPa.. For example: for a pipe with t/D = 0.01, ρ=7860kg/m^3, and E=207 GPa, being installed into a ditch of 2m in depth, with a lift of 1m:

$$\sigma_{max} = c_2 \cdot (E \cdot \rho)^{\frac{1}{2}} \cdot \left(1 + \left(1 - 2\frac{t}{D}\right)^2\right)^{-\frac{1}{2}}$$

$$= 12.39 \cdot (207x10^9 \cdot 7860)^{\frac{1}{2}} \cdot \left(1 + (1 - 2(0.01))^2\right)^{-\frac{1}{2}}$$

$$= 357.1MPa$$

Or, as a function of SMYS,

$$\%SMYS = \frac{357.1MPa}{483MPa} x100 = 73.9\%$$

The stresses occurring in the pipeline for a range of lift of 0-1m and a ditch depth of 1-5m, for a t/D of 0.01 are given in Figure A.5 and Table A.3. The stresses are presented as a percentage of SMYS in Figure A.6 and Table A.4.

Table A.3 Pipeline lowering stresses for a steel pipe with t/D = 0.01 (MPa)

lift (m) \ depth (m)	1	2	3	4	5
0	173.1	245.3	301.2	348.1	389.5
0.25	218.2	279.5	328.2	371.5	411.2
0.5	252.5	308.4	353.5	395.0	431.0
0.75	283.1	333.6	376.9	414.8	450.9
1	312.0	357.1	398.6	434.6	468.9

Table A.4 Pipeline lowering stresses for a steel pipe with t/D = 0.01, presented as a percentage of the SMYS (%)

lift(m) \ depth(m)	1	2	3	4	5
0	35.8	50.8	62.4	72.1	80.7
0.25	45.2	57.9	68.0	76.9	85.1
0.5	52.3	63.8	73.2	81.8	89.2
0.75	58.6	69.1	78.0	85.9	93.3
1	64.6	73.9	82.5	90.0	97.1

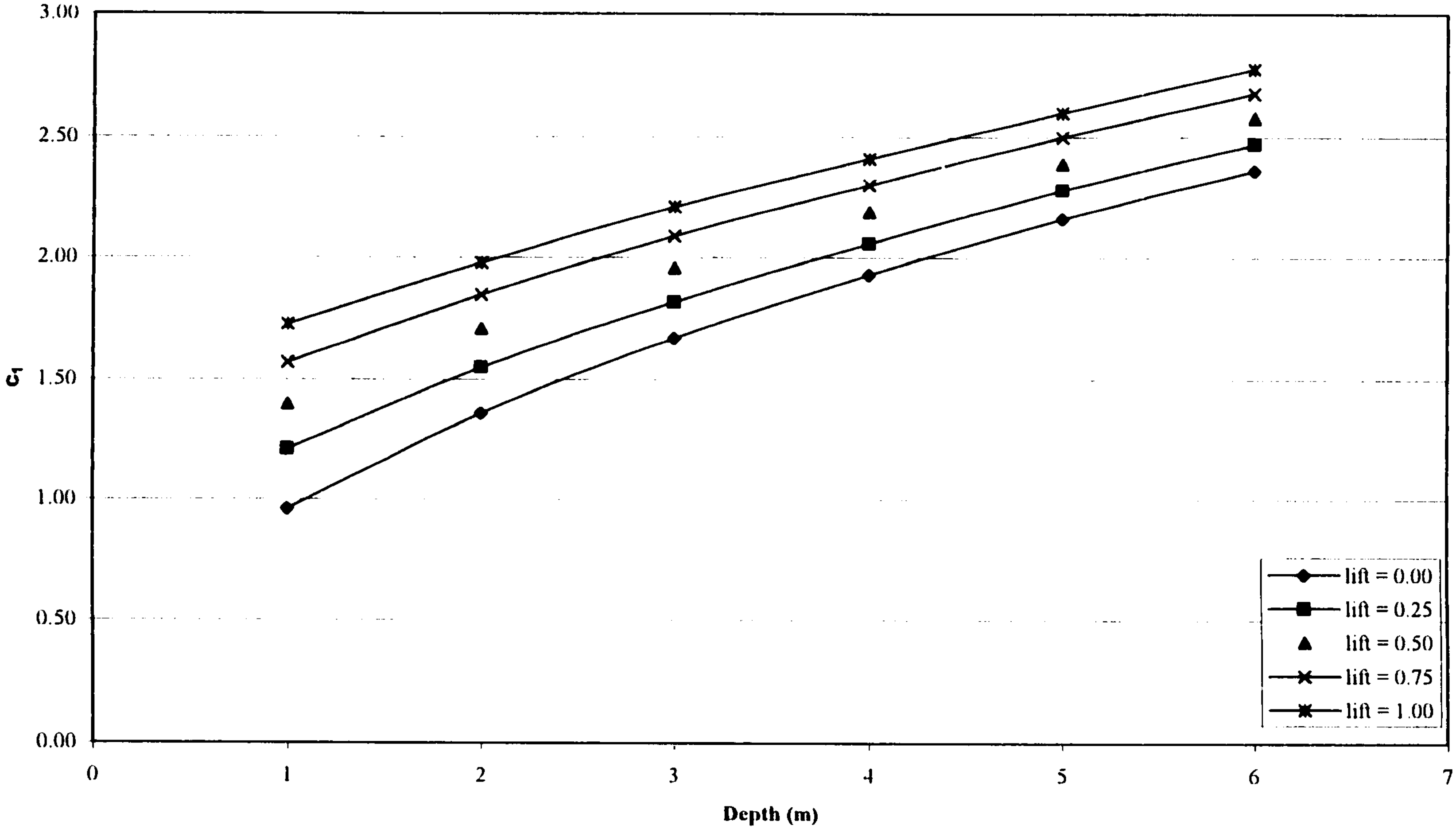

Figure A.3: Coefficient c_1 for determination of equivalent moment for installation loading from depth and lift values

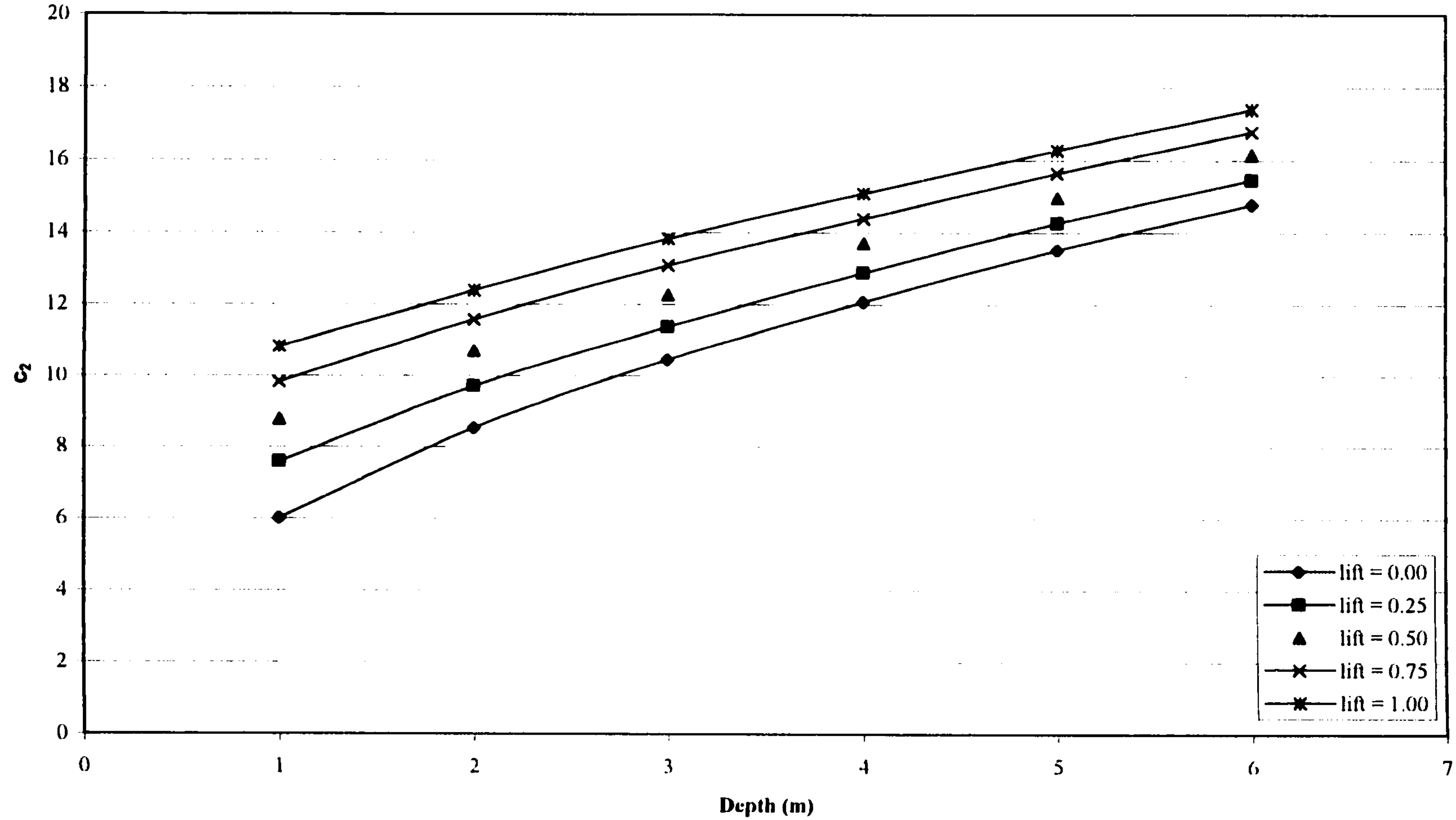

Figure A.4: Coefficient c_2 for determination of maximum axial stress and strain for installation loading from depth and lift values

200

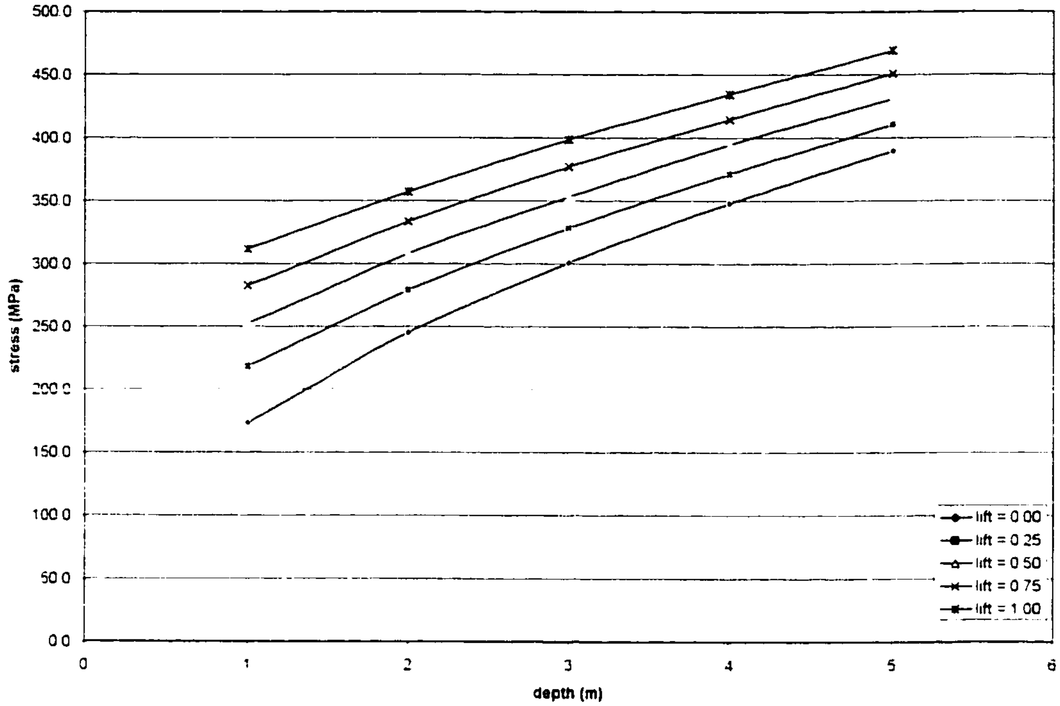

Figure A.5: Pipeline lowering stresses for a steel pipe with t/D = 0.01

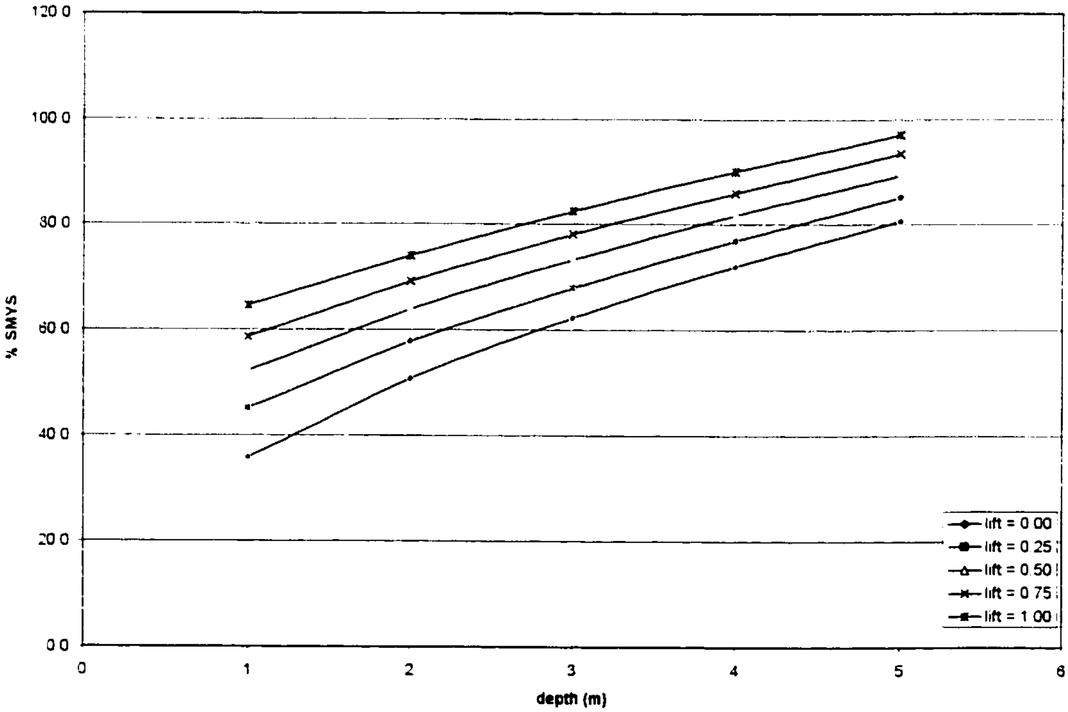

Figure A.6: Pipeline lowering stresses for a steel pipe with t/D = 0.01, presented as a percentage of the SMYS

Appendix B

Netting Analysis

Netting analysis is a purely geometric design methodology that has been used extensively in the design of pressure vessels since the early sixties. It is based on the assumption that all of the load is carried by the fibres/filaments with no assistance from the matrix/resin or interaction between the fibre layers. It also assumes that the tube wall acts as a membrane and carries no out of plane bending or shear loads.

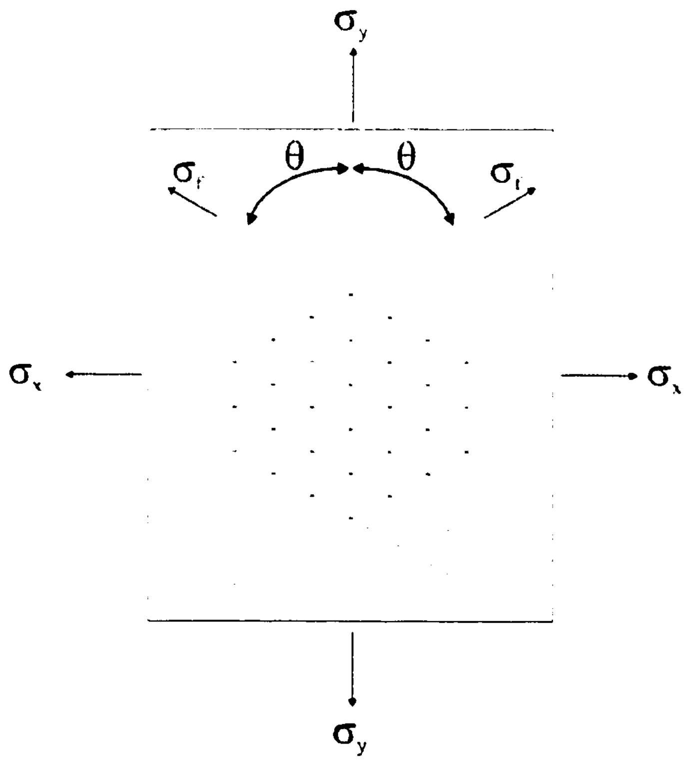

Figure B.1 Definitions for the netting analysis of composites

From Figure B.1, the fibre stresses can be determined from:

$$\sigma_f = \frac{\sigma_x}{\cos^2\theta}$$
$$\sigma_f = \frac{\sigma_y}{\sin^2\theta} \qquad \text{(B.1)}$$

Equating σ_f, yields:

$$\frac{\sigma_x}{\sigma_y} = \tan^2\theta \qquad \text{(B.2)}$$

For example, a (2H:1A) hoop to axial stress ratio, as is the loading case for an internal pressure loaded short pipe with closed ends, yields an "optimum" angle of ~±55°. Figure B.2 shows the strain ratios for a (2H:1A) stress ratio loading, as predicted by the finite element model. From this model, it can be seen that the minimum matrix strain occurs at ~53°, which is the optimum fibre wind angle predicted by the FEA. It is interesting to note that for the (2H:1A) loading, the maximum strain ratio always occurs in the matrix, and at the optimum fibre angle, where netting analysis predicts that all of the load is taken by the fibres, the load on the matrix is at a minimum.

Netting analysis is valuable for predicting the "optimum" fibre angles for a given hoop to axial stress loading, or for calculating the loading that a given fibre angle would be optimum for. However, in situations where the fibre angle is not optimized with respect to the loading case, equations B-1 are no longer valid, as the loads are no longer entirely carried by the fibres.

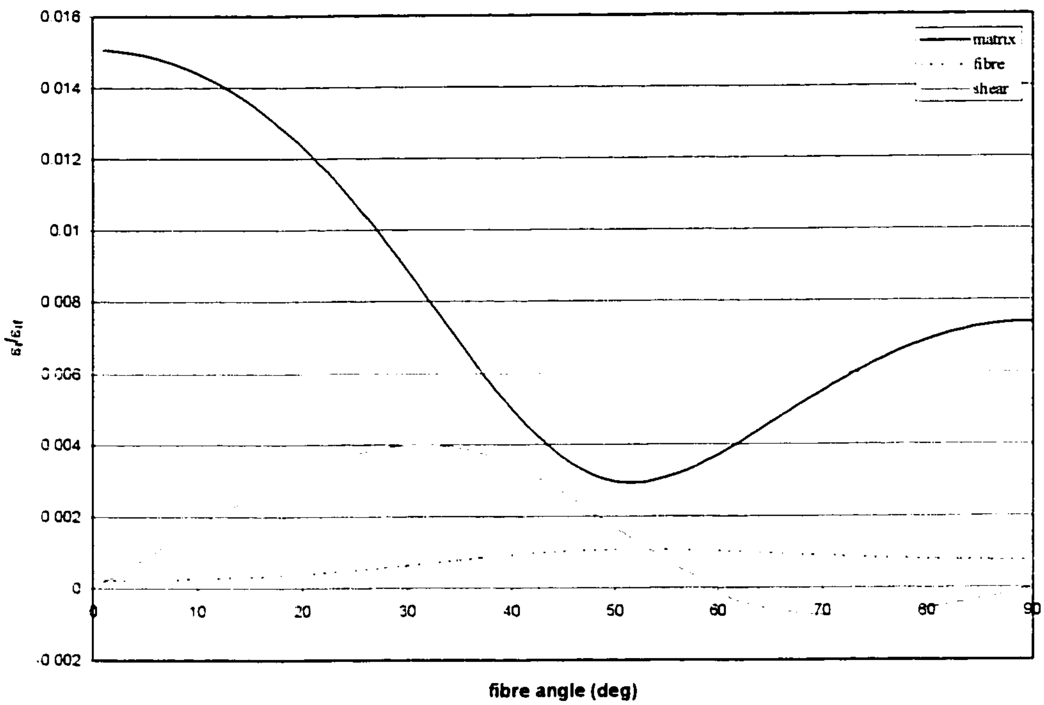

Figure B.2: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 0 to 90 degrees, for an applied hoop stress of 1 MPa in a stress ratio loading of (2H:1A)

Consider the maximum allowable hoop stress for a (4H:1A) loading case as shown in Figure B.3. Netting analysis yields an optimum angle of 63.5°, which is clearly indicated in the figure by the large spike in the hoop stress to failure curve. At fibre angles above and below the predicted netting analysis optimum, the maximum allowable hoop stress drops dramatically.

A pipe with a fibre angle of 63.5°, designed for a (4H:1A) loading, operating at an average hoop stress of 150 MPa in a (4H:1A) loading, would have a factor of safety of 5.3. However, were the loading situation of that pipe changed to a (2H:1A) loading, maintaining the 150 MPa hoop stress, the factor of safety in the pipe would be reduced to 1.5.

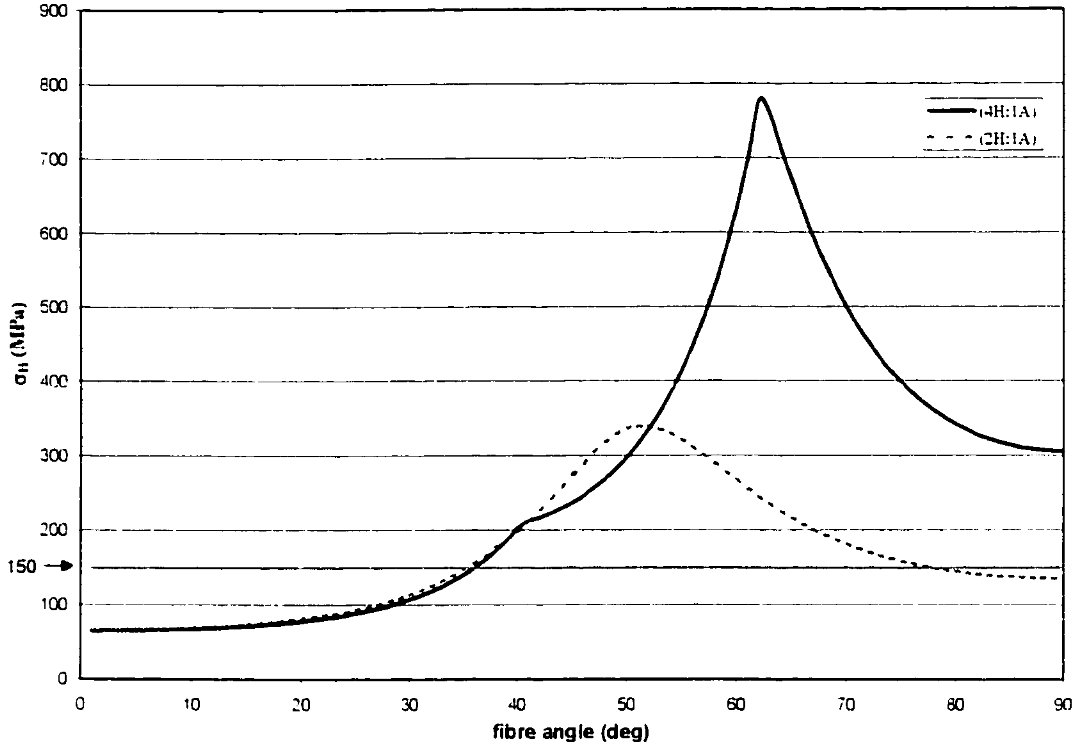

Figure B.3: Hoop stress at failure ($\varepsilon_{imax}/\varepsilon_{if} = 1$) for fibre angles of 0 to 90 degrees, for loadings of (2H:1A) and (4H:1A)

Table B.1 compares the optimum fibre angles as predicted by the FEA and netting analysis. The optimum fibre angle as determined by the FEA was determined from peaks of the maximum allowable hoop stress figures from Chapter 4. It is interesting to note that there is significant correlation between the values predicted by the FEA and netting analysis. The variations between the numbers could easily be accounted for by the different methods used to determine the optimum fibre angles. For instance, "tweaking" or using different values for the failure strains of the constituents of the composite could easily shift the optimum angles predicted by the FEA.

However, in the case of netting analysis, one could not predict the failure mode. Although by discounting the matrix, it is implied that it may have failed. This is not true for loading cases where the axial load is dominant, e.g. (1H:4A) (see Figure 4.14 in Chapter 4)

Table B.1: Optimum fibre angles as predicted by FEA and netting analysis

biaxial loading	FEA prediction	netting analysis
(1H:0A)	90	90
(8H:1A)	79	70.5
(4H:1A)	62	63
(2H:1A)	53	55
(1H:1A)	45	45
(1H:2A)	36	35
(1H:4A)	23	26.5
(1H:8A)	7	19.5
(0H:1A)	0	0

Appendix C

Numerical Tables of Strain Ratios

Table C.1: Maximum and minimum strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 50 to 70 degrees, for an applied hoop stress of 1 MPa in a stress ratio loading of (1H:0A)

	strain ratio							
fibre angle	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	maximum ratio
50	427.3E-6	398.3E-6	5.013E-3	4.988E-3	814.6E-6	809.7E-6	5.013E-3	shear
51	105.8E-6	75.3E-6	4.832E-3	4.808E-3	838.1E-6	833.1E-6	4.832E-3	shear
52	-107.4E-6	-122.0E-6	4.645E-3	4.620E-3	859.6E-6	854.6E-6	4.645E-3	shear
53	-235.3E-6	-252.6E-6	4.454E-3	4.428E-3	879.1E-6	874.1E-6	4.454E-3	shear
54	-358.7E-6	-376.7E-6	4.259E-3	4.233E-3	896.6E-6	891.6E-6	4.259E-3	shear
55	-468.0E-6	-486.5E-6	4.062E-3	4.036E-3	912.1E-6	907.2E-6	4.062E-3	shear
56	-563.4E-6	-582.5E-6	3.864E-3	3.839E-3	925.6E-6	920.8E-6	3.864E-3	shear
57	-645.6E-6	-665.6E-6	3.667E-3	3.642E-3	937.4E-6	932.7E-6	3.667E-3	shear
58	-715.1E-6	-736.0E-6	3.472E-3	3.448E-3	947.3E-6	942.7E-6	3.472E-3	shear
59	-772.8E-6	-794.4E-6	3.281E-3	3.257E-3	955.7E-6	951.2E-6	3.281E-3	shear
60	-819.2E-6	-841.6E-6	3.093E-3	3.069E-3	962.4E-6	958.1E-6	3.093E-3	shear
61	-855.4E-6	-878.4E-6	2.909E-3	2.887E-3	967.7E-6	963.5E-6	2.909E-3	shear
62	-882.0E-6	-905.6E-6	2.731E-3	2.709E-3	971.7E-6	967.7E-6	2.731E-3	shear
63	-900.1E-6	-924.2E-6	2.559E-3	2.538E-3	974.5E-6	970.7E-6	2.559E-3	shear
64	-910.5E-6	-934.9E-6	2.393E-3	2.373E-3	976.3E-6	972.6E-6	2.393E-3	shear
65	-914.1E-6	-938.7E-6	2.234E-3	2.214E-3	977.1E-6	973.5E-6	2.234E-3	shear
66	-911.6E-6	-936.3E-6	2.081E-3	2.062E-3	977.1E-6	973.6E-6	2.081E-3	shear
67	-903.9E-6	-928.7E-6	1.935E-3	1.917E-3	976.3E-6	973.0E-6	1.935E-3	shear
68	-891.7E-6	-916.5E-6	1.796E-3	1.778E-3	974.9E-6	971.8E-6	1.796E-3	shear
69	-875.9E-6	-900.4E-6	1.663E-3	1.646E-3	973.0E-6	970.0E-6	1.663E-3	shear
70	-857.0E-6	-881.3E-6	1.538E-3	1.521E-3	970.6E-6	967.8E-6	1.538E-3	shear

Table C.2: Maximum and minimum strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 50 to 70 degrees, for an applied hoop stress of 1 MPa in a stress ratio loading of (8H:1A)

	strain ratio							
fibre angle	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	1.1E-3	1.0E-3	4.182E-3	4.159E-3	879.3E-6	873.6E-6	4.182E-3	shear
51	810.3E-6	775.9E-6	3.999E-3	3.975E-3	897.7E-6	892.0E-6	3.999E-3	shear
52	586.2E-6	550.6E-6	3.811E-3	3.787E-3	914.1E-6	908.4E-6	3.811E-3	shear
53	407.0E-6	374.1E-6	3.621E-3	3.597E-3	928.5E-6	922.8E-6	3.621E-3	shear
54	258.1E-6	224.2E-6	3.430E-3	3.406E-3	940.9E-6	935.3E-6	3.430E-3	shear
55	133.3E-6	98.6E-6	3.240E-3	3.216E-3	951.4E-6	945.9E-6	3.240E-3	shear
56	-20.5E-6	-40.9E-6	3.051E-3	3.027E-3	960.1E-6	954.7E-6	3.051E-3	shear
57	-68.9E-6	-89.6E-6	2.865E-3	2.841E-3	967.0E-6	961.8E-6	2.865E-3	shear
58	-105.6E-6	-126.5E-6	2.682E-3	2.659E-3	972.4E-6	967.2E-6	2.682E-3	shear
59	-131.4E-6	-152.5E-6	2.505E-3	2.482E-3	976.2E-6	971.2E-6	2.505E-3	shear
60	-147.1E-6	-168.9E-6	2.333E-3	2.311E-3	978.6E-6	973.8E-6	2.333E-3	shear
61	-153.6E-6	-176.0E-6	2.167E-3	2.146E-3	979.7E-6	975.1E-6	2.167E-3	shear
62	-151.9E-6	-174.8E-6	2.008E-3	1.988E-3	979.7E-6	975.3E-6	2.008E-3	shear
63	-142.9E-6	-166.2E-6	1.856E-3	1.836E-3	978.7E-6	974.5E-6	1.856E-3	shear
64	-127.5E-6	-151.1E-6	1.712E-3	1.693E-3	976.8E-6	972.9E-6	1.712E-3	shear
65	-106.5E-6	-130.3E-6	1.575E-3	1.556E-3	974.2E-6	970.4E-6	1.575E-3	shear
66	-80.8E-6	-104.8E-6	1.445E-3	1.427E-3	970.9E-6	967.4E-6	1.445E-3	shear
67	-51.2E-6	-75.2E-6	1.323E-3	1.305E-3	967.1E-6	963.7E-6	1.323E-3	shear
68	113.1E-6	-42.3E-6	1.208E-3	1.191E-3	962.8E-6	959.7E-6	1.208E-3	shear
69	180.8E-6	134.9E-6	1.101E-3	1.084E-3	958.2E-6	955.3E-6	1.101E-3	shear
70	251.5E-6	206.1E-6	1.001E-3	984.6E-6	953.4E-6	950.6E-6	1.001E-3	shear

Table C.3: Maximum and minimum strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 50 to 70 degrees, for an applied hoop stress of 1 MPa in a stress ratio loading of (4H:1A)

	strain ratio							
fibre angle	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	1.697E-3	1.660E-3	3.352E-3	3.330E-3	944.0E-6	937.6E-6	3.352E-3	shear
51	1.515E-3	1.477E-3	3.165E-3	3.143E-3	957.3E-6	950.9E-6	3.165E-3	shear
52	1.369E-3	1.333E-3	2.977E-3	2.954E-3	968.6E-6	962.2E-6	2.977E-3	shear
53	1.262E-3	1.226E-3	2.789E-3	2.766E-3	977.8E-6	971.5E-6	2.789E-3	shear
54	1.180E-3	1.143E-3	2.602E-3	2.579E-3	985.2E-6	979.0E-6	2.602E-3	shear
55	1.121E-3	1.083E-3	2.418E-3	2.395E-3	990.7E-6	984.6E-6	2.418E-3	shear
56	1.085E-3	1.046E-3	2.238E-3	2.215E-3	994.5E-6	988.5E-6	2.238E-3	shear
57	1.069E-3	1.030E-3	2.062E-3	2.040E-3	996.7E-6	990.9E-6	2.062E-3	shear
58	1.072E-3	1.033E-3	1.892E-3	1.871E-3	997.4E-6	991.7E-6	1.892E-3	shear
59	1.093E-3	1.053E-3	1.729E-3	1.708E-3	996.7E-6	991.2E-6	1.729E-3	shear
60	1.130E-3	1.089E-3	1.574E-3	1.553E-3	994.8E-6	989.5E-6	1.574E-3	shear
61	1.182E-3	1.140E-3	1.425E-3	1.405E-3	991.7E-6	986.7E-6	1.425E-3	shear
62	1.245E-3	1.203E-3	1.285E-3	1.266E-3	987.8E-6	983.0E-6	1.285E-3	shear
63	1.320E-3	1.276E-3	1.154E-3	1.135E-3	983.0E-6	978.4E-6	1.320E-3	matrix tension
64	1.403E-3	1.358E-3	1.030E-3	1.012E-3	977.5E-6	973.2E-6	1.403E-3	matrix tension
65	1.493E-3	1.448E-3	915.4E-6	896.9E-6	971.4E-6	967.4E-6	1.493E-3	matrix tension
66	1.589E-3	1.544E-3	809.0E-6	790.8E-6	964.9E-6	961.1E-6	1.589E-3	matrix tension
67	1.689E-3	1.645E-3	710.9E-6	693.1E-6	958.0E-6	954.5E-6	1.689E-3	matrix tension
68	1.793E-3	1.748E-3	620.9E-6	603.5E-6	950.9E-6	947.6E-6	1.793E-3	matrix tension
69	1.898E-3	1.854E-3	538.7E-6	521.9E-6	943.6E-6	940.5E-6	1.898E-3	matrix tension
70	2.004E-3	1.960E-3	464.1E-6	447.8E-6	936.3E-6	933.4E-6	2.004E-3	matrix tension

Table C.4: Maximum and minimum strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 50 to 70 degrees, for an applied hoop stress of 1 MPa in a stress ratio loading of (2H:1A)

	strain ratio							
fibre angle	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	2.967E-3	2.923E-3	1.691E-3	1.671E-3	1.073E-3	1.065E-3	2.967E-3	matrix tension
51	2.942E-3	2.899E-3	1.499E-3	1.478E-3	1.077E-3	1.069E-3	2.942E-3	matrix tension
52	2.945E-3	2.901E-3	1.309E-3	1.288E-3	1.078E-3	1.070E-3	2.945E-3	matrix tension
53	2.973E-3	2.929E-3	1.124E-3	1.104E-3	1.077E-3	1.069E-3	2.973E-3	matrix tension
54	3.024E-3	2.980E-3	945.715E-6	925.432E-6	1.074E-3	1.066E-3	3.024E-3	matrix tension
55	3.098E-3	3.053E-3	774.296E-6	754.277E-6	1.069E-3	1.062E-3	3.098E-3	matrix tension
56	3.191E-3	3.146E-3	611.096E-6	591.401E-6	1.063E-3	1.056E-3	3.191E-3	matrix tension
57	3.302E-3	3.258E-3	456.940E-6	437.619E-6	1.056E-3	1.049E-3	3.302E-3	matrix tension
58	3.429E-3	3.385E-3	312.484E-6	293.219E-6	1.047E-3	1.041E-3	3.429E-3	matrix tension
59	3.570E-3	3.526E-3	178.229E-6	158.807E-6	1.038E-3	1.031E-3	3.570E-3	matrix tension
60	3.722E-3	3.680E-3	-178.213E-6	-196.522E-6	1.027E-3	1.021E-3	3.722E-3	matrix tension
61	3.885E-3	3.843E-3	-281.040E-6	-299.179E-6	1.016E-3	1.010E-3	3.885E-3	matrix tension
62	4.055E-3	4.014E-3	-373.461E-6	-391.428E-6	1.004E-3	998.3E-6	4.055E-3	matrix tension
63	4.232E-3	4.191E-3	-455.547E-6	-473.282E-6	991.5E-6	986.2E-6	4.232E-3	matrix tension
64	4.414E-3	4.373E-3	-527.381E-6	-544.847E-6	978.8E-6	973.8E-6	4.414E-3	matrix tension
65	4.598E-3	4.557E-3	-589.178E-6	-606.310E-6	965.9E-6	961.2E-6	4.598E-3	matrix tension
66	4.784E-3	4.742E-3	-641.192E-6	-657.924E-6	952.9E-6	948.5E-6	4.784E-3	matrix tension
67	4.969E-3	4.927E-3	-683.696E-6	-700.002E-6	939.9E-6	935.9E-6	4.969E-3	matrix tension
68	5.152E-3	5.110E-3	-717.055E-6	-732.901E-6	927.1E-6	923.4E-6	5.152E-3	matrix tension
69	5.333E-3	5.291E-3	-741.686E-6	-757.011E-6	914.5E-6	911.1E-6	5.333E-3	matrix tension
70	5.509E-3	5.468E-3	-758.004E-6	-772.752E-6	902.2E-6	899.0E-6	5.509E-3	matrix tension

Table C.5: Maximum and minimum strains in material directions, ε_i, normalized with respect to their failure strains, e_{if}, for fibre angles of 50 to 70 degrees, for a moment loading with a maximum axial stress of σ_{Amax} =1 MPa

	strain ratio							
fibre angle	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	5.152E-3	-2.666E-3	6.417E-3	-6.384E-3	511.1E-6	-842.2E-6	6.417E-3	shear
51	5.688E-3	-2.945E-3	6.442E-3	-6.410E-3	472.8E-6	-778.2E-6	6.442E-3	shear
52	6.227E-3	-3.225E-3	6.449E-3	-6.417E-3	434.4E-6	-714.0E-6	6.449E-3	shear
53	6.764E-3	-3.504E-3	6.437E-3	-6.406E-3	396.1E-6	-649.9E-6	6.764E-3	matrix tension
54	7.296E-3	-3.781E-3	6.408E-3	-6.378E-3	358.0E-6	-586.3E-6	7.296E-3	matrix tension
55	7.823E-3	-4.055E-3	6.361E-3	-6.332E-3	320.3E-6	-523.5E-6	7.823E-3	matrix tension
56	8.340E-3	-4.324E-3	6.298E-3	-6.270E-3	283.3E-6	-461.7E-6	8.340E-3	matrix tension
57	8.847E-3	-4.587E-3	6.219E-3	-6.192E-3	247.1E-6	-401.2E-6	8.847E-3	matrix tension
58	9.340E-3	-4.843E-3	6.125E-3	-6.100E-3	211.7E-6	-342.2E-6	9.340E-3	matrix tension
59	9.819E-3	-5.092E-3	6.018E-3	-5.993E-3	177.3E-6	-285.2E-6	9.819E-3	matrix tension
60	10.282E-3	-5.332E-3	5.897E-3	-5.874E-3	165.7E-6	-232.8E-6	10.28E-3	matrix tension
61	10.728E-3	-5.563E-3	5.765E-3	-5.744E-3	158.2E-6	-226.5E-6	10.73E-3	matrix tension
62	11.156E-3	-5.786E-3	5.622E-3	-5.602E-3	150.4E-6	-225.7E-6	11.16E-3	matrix tension
63	11.566E-3	-5.998E-3	5.470E-3	-5.451E-3	142.5E-6	-224.0E-6	11.57E-3	matrix tension
64	11.957E-3	-6.201E-3	5.309E-3	-5.291E-3	134.3E-6	-221.5E-6	11.96E-3	matrix tension
65	12.329E-3	-6.394E-3	5.140E-3	-5.123E-3	126.1E-6	-218.3E-6	12.33E-3	matrix tension
66	12.682E-3	-6.577E-3	4.964E-3	-4.949E-3	117.7E-6	-214.4E-6	12.68E-3	matrix tension
67	13.015E-3	-6.750E-3	4.782E-3	-4.768E-3	109.4E-6	-209.9E-6	13.02E-3	matrix tension
68	13.330E-3	-6.914E-3	4.595E-3	-4.582E-3	101.1E-6	-204.9E-6	13.33E-3	matrix tension
69	13.626E-3	-7.067E-3	4.403E-3	-4.391E-3	104.0E-6	-199.6E-6	13.63E-3	matrix tension
70	13.904E-3	-7.211E-3	4.207E-3	-4.196E-3	124.3E-6	-198.8E-6	13.90E-3	matrix tension

Table C.6: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 50 to 70 degrees, for a distributed lateral line load, normalized by the diameter (w_d/D) for a value of (1kN/m)/m and a t/D of 0.04

	strain ratio							
fibre angle	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	7.224E-3	-3.505E-3	6.635E-3	-4.439E-3	1.890E-3	-2.579E-3	7.224E-3	matrix tension
51	6.898E-3	-3.305E-3	6.493E-3	-4.325E-3	1.894E-3	-2.583E-3	6.898E-3	matrix tension
52	6.581E-3	-3.113E-3	6.346E-3	-4.211E-3	1.896E-3	-2.586E-3	6.581E-3	matrix tension
53	6.272E-3	-2.934E-3	6.195E-3	-4.097E-3	1.896E-3	-2.588E-3	6.272E-3	matrix tension
54	5.971E-3	-2.786E-3	6.041E-3	-3.983E-3	1.894E-3	-2.590E-3	6.041E-3	shear
55	5.678E-3	-2.643E-3	5.884E-3	-3.870E-3	1.890E-3	-2.592E-3	5.884E-3	shear
56	5.392E-3	-2.505E-3	5.724E-3	-3.757E-3	1.886E-3	-2.594E-3	5.724E-3	shear
57	5.115E-3	-2.371E-3	5.563E-3	-3.645E-3	1.879E-3	-2.595E-3	5.563E-3	shear
58	4.845E-3	-2.242E-3	5.400E-3	-3.534E-3	1.872E-3	-2.598E-3	5.400E-3	shear
59	4.582E-3	-2.117E-3	5.236E-3	-3.423E-3	1.865E-3	-2.600E-3	5.236E-3	shear
60	4.326E-3	-1.996E-3	5.072E-3	-3.313E-3	1.856E-3	-2.602E-3	5.072E-3	shear
61	4.078E-3	-1.879E-3	4.908E-3	-3.205E-3	1.848E-3	-2.606E-3	4.908E-3	shear
62	3.837E-3	-1.765E-3	4.744E-3	-3.096E-3	1.839E-3	-2.609E-3	4.744E-3	shear
63	3.603E-3	-1.655E-3	4.579E-3	-2.989E-3	1.830E-3	-2.617E-3	4.579E-3	shear
64	3.375E-3	-1.548E-3	4.416E-3	-2.882E-3	1.822E-3	-2.624E-3	4.416E-3	shear
65	3.155E-3	-1.445E-3	4.252E-3	-2.777E-3	1.813E-3	-2.632E-3	4.252E-3	shear
66	2.941E-3	-1.344E-3	4.089E-3	-2.701E-3	1.805E-3	-2.639E-3	4.089E-3	shear
67	2.734E-3	-1.247E-3	3.926E-3	-2.627E-3	1.798E-3	-2.647E-3	3.926E-3	shear
68	2.533E-3	-1.153E-3	3.764E-3	-2.550E-3	1.791E-3	-2.655E-3	3.764E-3	shear
69	2.339E-3	-1.062E-3	3.603E-3	-2.471E-3	1.784E-3	-2.662E-3	3.603E-3	shear
70	2.151E-3	-973.4E-6	3.442E-3	-2.389E-3	1.778E-3	-2.670E-3	3.442E-3	shear

Table C.7: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 50 to 70 degrees, for a distributed lateral line load, normalized by the diameter (w_d/D) for a value of (1kN/m)/m and a t/D of 0.05

	strain ratio							
fibre angle	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	4.925E-3	-2.174E-3	4.139E-3	-2.813E-3	1.259E-3	-1.629E-3	4.925E-3	matrix tension
51	4.598E-3	-2.047E-3	4.056E-3	-2.739E-3	1.258E-3	-1.632E-3	4.598E-3	matrix tension
52	4.283E-3	-1.935E-3	3.971E-3	-2.664E-3	1.256E-3	-1.635E-3	4.283E-3	matrix tension
53	3.981E-3	-1.837E-3	3.883E-3	-2.590E-3	1.253E-3	-1.638E-3	3.981E-3	matrix tension
54	3.768E-3	-1.743E-3	3.793E-3	-2.516E-3	1.249E-3	-1.640E-3	3.793E-3	shear
55	3.580E-3	-1.652E-3	3.701E-3	-2.441E-3	1.244E-3	-1.642E-3	3.701E-3	shear
56	3.396E-3	-1.564E-3	3.608E-3	-2.368E-3	1.238E-3	-1.647E-3	3.608E-3	shear
57	3.218E-3	-1.478E-3	3.513E-3	-2.294E-3	1.231E-3	-1.652E-3	3.513E-3	shear
58	3.045E-3	-1.396E-3	3.418E-3	-2.222E-3	1.224E-3	-1.657E-3	3.418E-3	shear
59	2.877E-3	-1.317E-3	3.321E-3	-2.149E-3	1.217E-3	-1.661E-3	3.321E-3	shear
60	2.713E-3	-1.240E-3	3.224E-3	-2.078E-3	1.210E-3	-1.666E-3	3.224E-3	shear
61	2.555E-3	-1.165E-3	3.127E-3	-2.007E-3	1.202E-3	-1.670E-3	3.127E-3	shear
62	2.401E-3	-1.093E-3	3.029E-3	-1.936E-3	1.194E-3	-1.674E-3	3.029E-3	shear
63	2.252E-3	-1.024E-3	2.931E-3	-1.866E-3	1.187E-3	-1.678E-3	2.931E-3	shear
64	2.108E-3	-956.2E-6	2.832E-3	-1.799E-3	1.180E-3	-1.682E-3	2.832E-3	shear
65	1.968E-3	-891.0E-6	2.734E-3	-1.733E-3	1.173E-3	-1.685E-3	2.734E-3	shear
66	1.832E-3	-827.9E-6	2.635E-3	-1.685E-3	1.166E-3	-1.689E-3	2.635E-3	shear
67	1.702E-3	-766.8E-6	2.535E-3	-1.637E-3	1.160E-3	-1.693E-3	2.535E-3	shear
68	1.575E-3	-707.7E-6	2.436E-3	-1.587E-3	1.154E-3	-1.697E-3	2.436E-3	shear
69	1.453E-3	-650.6E-6	2.336E-3	-1.536E-3	1.149E-3	-1.701E-3	2.336E-3	shear
70	1.335E-3	-595.4E-6	2.236E-3	-1.483E-3	1.144E-3	-1.705E-3	2.236E-3	shear

Table C.8: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 50 to 70 degrees, for a distributed lateral line load, normalized by the diameter (w_d/D) for a value of (1kN/m)/m and a t/D of 0.06

	strain ratio							
fibre angle	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	3.589E-3	-1.461E-3	2.790E-3	-1.934E-3	895.2E-6	-1.120E-3	3.589E-3	matrix tension
51	3.363E-3	-1.378E-3	2.738E-3	-1.881E-3	892.8E-6	-1.125E-3	3.363E-3	matrix tension
52	3.145E-3	-1.309E-3	2.684E-3	-1.829E-3	889.5E-6	-1.130E-3	3.145E-3	matrix tension
53	2.936E-3	-1.241E-3	2.628E-3	-1.776E-3	885.5E-6	-1.135E-3	2.936E-3	matrix tension
54	2.735E-3	-1.176E-3	2.571E-3	-1.723E-3	880.8E-6	-1.139E-3	2.735E-3	matrix tension
55	2.541E-3	-1.113E-3	2.513E-3	-1.671E-3	875.7E-6	-1.143E-3	2.541E-3	matrix tension
56	2.356E-3	-1.052E-3	2.454E-3	-1.619E-3	870.1E-6	-1.146E-3	2.454E-3	shear
57	2.181E-3	-993.6E-6	2.394E-3	-1.567E-3	864.1E-6	-1.149E-3	2.394E-3	shear
58	2.062E-3	-937.1E-6	2.333E-3	-1.516E-3	858.0E-6	-1.152E-3	2.333E-3	shear
59	1.946E-3	-882.5E-6	2.271E-3	-1.466E-3	851.8E-6	-1.155E-3	2.271E-3	shear
60	1.834E-3	-829.7E-6	2.209E-3	-1.417E-3	845.5E-6	-1.158E-3	2.209E-3	shear
61	1.726E-3	-778.8E-6	2.147E-3	-1.368E-3	839.2E-6	-1.160E-3	2.147E-3	shear
62	1.620E-3	-729.7E-6	2.084E-3	-1.320E-3	833.0E-6	-1.163E-3	2.084E-3	shear
63	1.518E-3	-682.2E-6	2.020E-3	-1.273E-3	827.0E-6	-1.165E-3	2.020E-3	shear
64	1.420E-3	-636.3E-6	1.956E-3	-1.225E-3	821.3E-6	-1.167E-3	1.956E-3	shear
65	1.324E-3	-592.0E-6	1.892E-3	-1.178E-3	815.8E-6	-1.169E-3	1.892E-3	shear
66	1.232E-3	-549.2E-6	1.827E-3	-1.136E-3	810.5E-6	-1.171E-3	1.827E-3	shear
67	1.143E-3	-507.9E-6	1.762E-3	-1.101E-3	805.7E-6	-1.173E-3	1.762E-3	shear
68	1.057E-3	-468.0E-6	1.696E-3	-1.067E-3	801.2E-6	-1.175E-3	1.696E-3	shear
69	974.7E-6	-429.5E-6	1.630E-3	-1.031E-3	797.1E-6	-1.177E-3	1.630E-3	shear
70	895.1E-6	-392.3E-6	1.563E-3	-994.0E-6	793.4E-6	-1.179E-3	1.563E-3	shear

Table C.9: Strains in material directions, ε_i, normalized with respect to their failure strains, ε_{if}, for fibre angles of 50 to 70 degrees, for a distributed lateral line load, normalized by the diameter (w_d/D) for a value of (1kN/m)/m and a t/D of 0.10

	strain ratio							
fibre angle	maximum matrix	minimum matrix	maximum shear	minimum shear	maximum fibre	minimum fibre	maximum ratio	mode of maximum ratio
50	1.402E-3	-465.8E-6	892.7E-6	-671.2E-6	319.0E-6	-395.6E-6	1.402E-3	matrix tension
51	1.325E-3	-436.2E-6	878.5E-6	-652.0E-6	316.7E-6	-397.3E-6	1.325E-3	matrix tension
52	1.249E-3	-409.8E-6	863.9E-6	-632.8E-6	314.1E-6	-398.7E-6	1.249E-3	matrix tension
53	1.176E-3	-386.6E-6	849.0E-6	-613.6E-6	311.5E-6	-400.0E-6	1.176E-3	matrix tension
54	1.106E-3	-364.4E-6	833.7E-6	-594.4E-6	308.8E-6	-401.2E-6	1.106E-3	matrix tension
55	1.038E-3	-343.0E-6	818.1E-6	-575.3E-6	306.0E-6	-402.2E-6	1.038E-3	matrix tension
56	972.9E-6	-322.5E-6	802.2E-6	-556.2E-6	303.2E-6	-403.1E-6	972.9E-6	matrix tension
57	909.9E-6	-302.9E-6	786.0E-6	-537.3E-6	300.5E-6	-403.9E-6	909.9E-6	matrix tension
58	849.2E-6	-284.1E-6	769.6E-6	-518.6E-6	297.8E-6	-404.7E-6	849.2E-6	matrix tension
59	790.8E-6	-266.0E-6	753.0E-6	-500.0E-6	295.2E-6	-405.3E-6	790.8E-6	matrix tension
60	734.7E-6	-248.7E-6	736.1E-6	-481.6E-6	292.6E-6	-405.8E-6	736.1E-6	shear
61	680.7E-6	-232.1E-6	718.9E-6	-463.4E-6	290.2E-6	-406.3E-6	718.9E-6	shear
62	628.8E-6	-216.2E-6	701.5E-6	-445.4E-6	287.9E-6	-406.8E-6	701.5E-6	shear
63	579.0E-6	-201.0E-6	683.7E-6	-427.6E-6	285.7E-6	-407.2E-6	683.7E-6	shear
64	531.2E-6	-186.3E-6	665.7E-6	-410.0E-6	283.6E-6	-407.5E-6	665.7E-6	shear
65	485.4E-6	-172.3E-6	647.4E-6	-392.6E-6	281.7E-6	-407.8E-6	647.4E-6	shear
66	441.5E-6	-158.9E-6	628.8E-6	-375.3E-6	280.0E-6	-408.1E-6	628.8E-6	shear
67	399.5E-6	-146.0E-6	609.7E-6	-358.3E-6	278.4E-6	-408.3E-6	609.7E-6	shear
68	359.4E-6	-133.7E-6	590.3E-6	-341.5E-6	277.0E-6	-408.6E-6	590.3E-6	shear
69	321.2E-6	-121.9E-6	570.5E-6	-324.8E-6	275.8E-6	-408.8E-6	570.5E-6	shear
70	284.9E-6	-110.6E-6	550.2E-6	-309.8E-6	274.7E-6	-408.9E-6	550.2E-6	shear

Table C.10: Young's modulus in the axial direction of a pipe, E_{axial}, as determined by Classical Laminate Theory, for fibre angles of 50 to 70 degrees.

fibre angle	Young's modulus (GPa)
50	11.02
51	10.76
52	10.53
53	10.31
54	10.12
55	9.940
56	9.780
57	9.636
58	9.505
59	9.388
60	9.282
61	9.188
62	9.103
63	9.028
64	8.961
65	8.902
66	8.850
67	8.804
68	8.763
69	8.728
70	8.697

Appendix D
ANSYS Batch Files

D.1 Composite pipe model

D.2 Post processor

D.3 Optimization file

D.4 Pipeline installation contact element model

D.1 ANSYS Batch File - composite pipe model   page 1 of 6

```
/filname,root
!
! comments:
! the following text is a sample batchfile used to model the GRP.
! This sample is for a 1 element thru the thickness, 8 layer solid.
!
! The model is created node by node, then element by element.
! This is done to insure that the fibre angle set in the element coordinate system is
defined
! with theta measured from the axis of the pipe, and layer 1 being on the inside of the
pipe,
! and layer 8 on the outside.
!

!---------------------------------------------------------------------
!       Given numbers:
!---------------------------------------------------------------------
        diameter=1                              !* exterior diameter
        thickrat=0.05                           !* thinwall  < 1/10
        ang=1                                   !*starting wind angle (degrees)


! fiailure strains
! 1=fibre       C=compression
! 2=matrix      T=tension
E1T=0.025
E1C=0.015
E2T=0.0070
E2C=0.0135
E12=0.01



thk=thickrat*diameter                           !* cylinder wall thickness
R1=diameter/2                                   !*outer radius
R0=R1-THK                                       !*inner radius
len=thk                                         !* model length

PI=3.14159                                      !*PI (radians)

volume=PI*(r1*r1-r0*r0)*len/2

NUM=8                                           !*number of layers
LTHK=THK/NUM                                    !* layer thickness
ethick=1                                        !* elements thru thickness
elen=1                                          !* elements along length
ecirc=25                                        !* elements around circumference
nthick=ethick+1                                       !* nodes thru thickness
nlen=elen+1                                     !* nodes along length
ncirc=ecirc+1                                   !* nodes around circumference


/prep7                                          !* enter pre-processor

*afun,deg                               !so sin, cos are in degrees not radians

!***    Material properties - glass/epoxy
!***    metric
density = 1510                                  ! density = 1510 kg/m3 = 1.51 g/cm3
```

```
 MP,EX,1,41.31e9
 MP,EY,1,8.652e9
 MP,EZ,1,8.652e9
 MP,PRXY,1,.313
 MP,PRYZ,1,.0655
 MP,PRXZ,1,.0655
 MP,GXY,1,4.103E9
 MP,GXZ,1,4.103E9
 MP,DENS,1,1510

finish

!*
!* set values to zero.

xmax=0                      !* maximum strain - global x direction (r)
xmin=0                      !* minimum strain - global x direction (r)
ymax=0                      !* maximum strain - global y direction (theta)
ymin=0                      !* minimum strain - global y direction (theta)
zmax=0                      !* maximum strain - global z direction (axial)
zmin=0                      !* minimum strain - global z direction (axial)
xymax=0                     !* maximum strain - global xy direction
xymin=0                     !* minimum strain - global xy direction
xzmax=0                     !* maximum strain - global xz direction
xzmin=0                     !* minimum strain - global xz direction
yzmax=0                     !* maximum strain - global yz direction
yzmin=0                     !* maximum strain - global yz direction
pri1max0                    !* maximum strain - principal 1
pri1min=0                   !* minimum strain - principal 1
pri2max=0                   !* maximum strain - principal 2
pri2min=0                   !* minimum strain - principal 2
pri3max=0                   !* maximum strain - principal 3
pri3min=0                   !* minimum strain - principal 3
e11max=0                    !* maximum strain - material direction 11
e11min=0                    !* minimum strain - material direction 11
e12max=0                    !* maximum strain - material direction 12
e12min=0                    !* minimum strain - material direction 12
e13max=0                    !* maximum strain - material direction 13
e13min=0                    !* minimum strain - material direction 13
e22max=0                    !* maximum strain - material direction 22
e22min=0                    !* minimum strain - material direction 22
e23max=0                    !* maximum strain - material direction 23
e23min=0                    !* minimum strain - material direction 23
e33max=0                    !* maximum strain - material direction 33
e33min=0                    !* minimum strain - material direction 33

/PREP7                          !* open preprocessor
```

```
!*
!* et,element-type-reference-number,elementname,keyopts
!*
!*  R=real constants
!* rmodif, nset,start location,v1,v2,v3,v4,v5,v6
!*                            (v* = new values)
!*  if keyopt(2)=0
!*     Number of layers(location 1), LSYM(2), LP1(3), LP2(4), blank(5), blank(6)
!*     Kref(location 7),blank,blank,blank,blank,blank(12)
!*     material(location 13),theta,thickness, material,theta,thick(18)
!*     material(location 19),theta,thickness, material,theta,thick(24)


ET,1,SOLID46,                              !* ET,material type,element type
keyopt,1,8,1                               !* keyopt 8: store data for all layers

R,1
RMODIF,1,1,8,0,0,0,                        !* rmodif, set 1, start location 1, 8layers,
                                           !*             no symm
RMODIF,1,7,0,                              !* rmodif, set 1, start location 7, 0
!*
RMODIF,1,13,1,ANG,LTHK,1,-ANG,LTHK,         !* element layer setup
RMODIF,1,19,1,ANG,LTHK,1,-ANG,LTHK,
RMODIF,1,25,1,-ANG,LTHK,1,ANG,LTHK,
RMODIF,1,31,1,-ANG,LTHK,1,ANG,LTHK,


!***********
csys,1                                     !* cylindrical coordinate system


/eshape,1                                  !* display layers in element

thick=thk/ethick                           !* element thickness

n,1,r0,-90,0                               !* create node 1 at (r0,-90,0)
n,2,r0+thick,-90,0                         !* create node 2 at (r0+element thickness,-!*
                                                       90,0)

!* generate nodes along length
!ngen, times, increment,    node1,    node2,ninc,dr,dthet,    dz
ngen,     nlen,    nthick,          1,         2,   1, 0,     0, len/elen

!* generate nodes around circumference
!1st layer of nodes
ngen,ncirc,  nthick*nlen,  1,    elen*nthick+1,nthick, 0,180/ecirc,0
!second layer of nodes
ngen,ncirc,  nthick*nlen,  2,    elen*nthick+2,nthick, 0,180/ecirc,0

nrotat,all                                 !* rotate nodes into global coord system
```

```
!* create elements


*do,i,1,elen*nthick,nthick
        type,1                                        !* element type 1
        e,i,nthick+i,nthick*nlen+nthick+i,nthick*nlen+i,i+1,nthick+i+1,nthick*nlen+nthick+
                i+1,nthick*nlen+i+1
        !* e (create element), bottom layer node numbers, top layer node numbers
*enddo

finish

/solu

!----------------------------------------------------------!
! begin basic loadings
!----------------------------------------------------------!

!* symmetry loading
nsel,s,node,,1,nthick*nlen,1                          !* select nodes
nsel,a,node,,ecirc*nlen*nthick+1,nthick*nlen*ncirc,1
dsym,symm,x,0                                         !* set symmetry

nsel,all                                              !* reselect all nodes


!* fixed end
        FLST,2,ncirc*nthick,1,orde,ncirc*nthick                 !* select nodes with another
                                                                !* method of selection
        *do,i,1,nlen*nthick*ecirc+1,nlen*nthick
           *do,j,0,ethick,ethick
             fitem,2,i+j
           *enddo
        *enddo
        D,P51X, , , , , ,UZ                           !* set constraint in Z direction


!* fixed points on fixed end to limit other movements

flst,2,nthick,1,orde,2                                !* constrain some nodes in Y direction
fitem,2,(ecirc*nthick*nlen)/2+1
fitem,2,((ecirc*nthick*nlen)/2+nthick)
D,P51X, , , , , ,uy

finish
!----------------------------------------------------------!
! end basic loadings
!----------------------------------------------------------!
```

```
/post1                                    !* enter post processor
!-----------------------------------------------------------!
! Loadings
!-----------------------------------------------------------!

hoopstrs=1e6                              !* set the hoop stress
pload=(thickrat*2)*hoopstrs               !* calculate the equivalent pressure

PRESS2=0                                  !* endcap pressure

!-----------------------------------------------------------!
! begin pressure loading
!-----------------------------------------------------------!

/solu                                     !* to ensure we're in the solution processor

!* internal pressure
!* pressure on every nthick'th node

FLST,2,ncirc*nlen,1,ORDE,ncirc*nlen       !* select nodes on inside surface to apply
                                          !*  pressure to
*do,i,1,nlen*nthick*ncirc-(nthick-1),nthick
       FITEM,2,i
*enddo
SF,P51X,PRES,pload,                       !* apply pressure


!*end pressure loading
       flst,2,ncirc*nthick,1,orde,2*ncirc    !* select nodes on end to apply load to
       *do,i,elen*nthick,nthick*nlen*ncirc,nthick*nlen
          fitem,2,i+1
          fitem,2,-(i+nthick)
       *enddo
       SF,P51X,PRES,-PRESS2,              !* apply end cap loading

!-----------------------------------------------------------!
! end pressure loading
!-----------------------------------------------------------!

       solve
       finish

!* load postprocessor to resolve strains into fibre angles
/INPUT,post1,lgw,,1,0                     !* for layer 1
/INPUT,post8,lgw,,1,0                     !* for layer 8
```

```
!--------------------------------------------------------------------
! calculate strain ratios
!--------------------------------------------------------------------

*if,e33max,gt,0.0,then                          !* 33 dir from post processor is fibre angle
       ratft=e33max/e1t
*else
       ratft=e33max/e1c
*endif
*if,e33min,lt,0.0,then
       ratfc=e33min/e1c
*else
       ratfc=e33min/e1t
*endif
*if,e22max,gt,0.0,then
       ratmt=e22max/e2t
*else
       ratmt=e22max/e2c
*endif
*if,e22min,lt,0.0,then
       ratmc=e22min/e2c
*else
       ratmc=e22min/e2t
*endif
ratst=e23max/e12
ratsc=e23min/e12

!* calculate absolute values of the shear strain ratios for comparison purposes
ratstp=sign(ratst,0)                            !* ratstp = RATio Shear Tension (max) !*
                                                Positive
ratscp=sign(ratsc,0)                            !* ratstp = RATio Shear Compression (min) !*
                                                Positive

ratmax=ratft>ratmt>ratstp>ratscp                !* determine maximum ratio
ratmin=ratfc<ratmc                              !* determine minimum ratio

ratfp=ratft>sign(ratfc,0)
ratmp=ratmt>sign(ratmc,0)
ratsp=ratstp>ratscp
```

```
!* what follows is the post processor used to resolve the strains, as calculated by ANSYS
!* into the fibre directions of the layer.
!* The 1st layer of the element is a +ve angle, thus the strains are rotated by + theta.
!* The format for the -ve rotation is also included
!* The postprocessing for layer 8 is the same, with the exception of the layer number
!* processed
!*
!* e3 - fibre
!* e2 - matrix
!* e1 - thickness

!* if z is the axis of the pipe, and y is the hoop direction, x thru thickness
!* x' is still thru thickness
!* wind angle measured from z
!* z' = wind angle
!* therefore y' = 90 deg from z'

!---------------------------------------------------------------------------
!axx                                          I+ve angleI -ve angle
!a11    ! cosine of angle between x' and x,I    0      I       0
!a12    ! cosine of angle between x' and y,I   90      I      90
!a13    ! cosine of angle between x' and z,I   90      I      90
!a21    ! cosine of angle between y' and x,I   90      I      90
!a22    ! cosine of angle between y' and y,I  ang      I     -ang
!a23    ! cosine of angle between y' and z,I  90+ang I   90-ang
!a31    ! cosine of angle between z' and x,I  90       I      90
!a32    ! cosine of angle between z' and y,I (ang-90)I -(90+ang)
!a33    ! cosine of angle between z' and z,I    ang  I     -ang
!---------------------------------------------------------------------------

/post1

rsys,1                        ! results in cylindrical coords
*afun,deg                     !so sin, cos are in degrees not radians


layer,1                                       !* results from layer 1


etable,x-strn,epel,x                          !etable column1 - x strain
etable,y-strn,epel,y                          !etable column2 - y strain
etable,z-strn,epel,z                          !etable column3 - z strain
etable,xy-strn,epel,xy                        !etable column4 - xy
etable,yz-strn,epel,yz                        !etable column5 - yz
etable,xz-strn,epel,xz                        !etable column6 - xz
etable,prin1,epel,1                           !etable column#7 - principal 1
etable,prin2,epel,2                           !etable column#8 - principal 2
etable,prin3,epel,3                           !etable column#9 - principal 3


!-----------------------------------------------------------------
!-----------------------------------------------------------------
!strain rotation for +ve layup
!-----------------------------------------------------------------
!-----------------------------------------------------------------
!sadd, new, old1, old2,mult1, mult2
!
!      e'11 =  1 x-strn 1

sadd,e11,x-strn,,1,                !etable column10
```

```
!
!       e'12 =  1 xy-strn cos(ang)
!               1 xz-strn cos(90+ang)

sadd,e12,xy-strn,xz-strn,cos(ang),cos(90+ang)                          !etable column11

!       e'13 =  1 xy-strn cos((ang-90))
!               1 xz-strn cos(ang)

sadd,e13,xy-strn,xz-strn,cos((ang-90)),cos(ang)                        !etable column12

!       !----------------------
!
!       e'21 =  cos(ang) xy-strn 1
!               cos(90+ang) xz-strn 1

sadd,e21,xy-strn,xz-strn,cos(ang),cos(90+ang)                          !etable column13


!       e'22 =  cos(ang) y-strn cos(ang)
!               cos(90+ang) yz-strn cos(ang)
!               cos(ang) yz-strn cos(90+ang)
!               cos(90+ang) z-strn cos(90+ang)

sadd,e22a,y-strn,z-strn,cos(ang)*cos(ang),cos(90+ang)*cos(90+ang)              !etable
column14
sadd,e22,e22a,yz-strn,1,2*cos(90+ang)*cos(ang)                         !etable column15

!       e'23 =  cos(ang) y-strn cos((ang-90))
!               cos(90+ang) yz-strn cos((ang-90))
!               cos(ang) yz-strn cos(ang)
!               cos(90+ang) z-strn cos(ang)

sadd,e23a,y-strn,z-strn,cos(ang)*cos((ang-90)),cos(ang)*cos(90+ang)
        !etable column16
sadd,e23,e23a,yz-strn,1,cos(ang)*cos(ang)+cos((ang-90))*cos(90+ang)
        !etable column17


!       !--------------------------
!
!       e'31 =  cos((ang-90)) xy-strn 1
!               cos(ang) xz-strn 1

sadd,e31,xy-strn,xz-strn,cos((ang-90)),cos(ang)                        !etable column18


!       e'32 =  cos((ang-90)) y-strn cos(ang)
!               cos(ang) yz-strn cos(ang)
!               cos((ang-90)) yz-strn cos(90+ang)
!               cos(ang) z-strn cos(90+ang)

sadd,e32a,y-strn,z-strn,cos(ang)*cos((ang-90)),cos(ang)*cos(90+ang)
        !etable column19
sadd,e32,e32a,yz-strn,1,cos(ang)*cos(ang)+cos(ang)*cos(90+ang)                 !etable
column20
```

```
!       e'33 =  cos((ang-90)) y-strn cos((ang-90))
!               cos(ang) yz-strn cos((ang-90))
!               cos((ang-90)) yz-strn cos(ang)
!               cos(ang) z-strn cos(ang)

sadd,e33a,y-strn,z-strn,cos((ang-90))*cos((ang-90)),cos(ang)*cos(ang)
        !etable column21
sadd,e33,e33a,yz-strn,1,2*cos(ang)*cos((ang-90))                  !etable column22


!-------------------------------------------------------------
!-------------------------------------------------------------
! strain rotation for the -ve layup
!-------------------------------------------------------------
!-------------------------------------------------------------
! select the other elements
!       esel,inve,elem
!       !-------------------------------------------------------------
!       !  strain rotaion for -ve layup
!       !-------------------------------------------------------------
!
!       !sadd, new, old1, old2,mult1, mult2
!       !
!       !       e'11 =  1 x-strn 1
!
!       sadd,e11,x-strn,,1,
!
!       !
!       !       e'12 =  1 xy-strn cos(-ang)
!       !               1 xz-strn cos(90-ang)
!
!       sadd,e12,xy-strn,xz-strn,cos(-ang),cos(90-ang)
!
!       !       e'13 =  1 xy-strn cos(-(ang+90))
!       !               1 xz-strn cos(-ang)
!
!       sadd,e13,xy-strn,xz-strn,cos(-(ang+90)),cos(-ang)
!
!       !       !---------------------
!       !
!       !       e'21 =  cos(-ang) xy-strn 1
!       !               cos(90-ang) xz-strn 1
!
!       sadd,e21,xy-strn,xz-strn,cos(-ang),cos(90-ang)
!
!
!       !       e'22 =  cos(-ang) y-strn cos(-ang)
!       !               cos(90-ang) yz-strn cos(-ang)
!       !               cos(-ang) yz-strn cos(90-ang)
!       !               cos(90-ang) z-strn cos(90-ang)
!
!       sadd,e22a,y-strn,z-strn,cos(-ang)*cos(-ang),cos(90-ang)*cos(90-ang)
!       sadd,e22,e22a,yz-strn,1,2*cos(90-ang)*cos(-ang)
!
!       !       e'23 =  cos(-ang) y-strn cos(-(ang+90))
!       !               cos(90-ang) yz-strn cos(-(ang+90))
!       !               cos(-ang) yz-strn cos(-ang)
!       !               cos(90-ang) z-strn cos(-ang)
!
!       sadd,e23a,y-strn,z-strn,cos(-ang)*cos(-(ang+90)),cos(-ang)*cos(90-ang)
!       sadd,e23,e23a,yz-strn,1,cos(-ang)*cos(-ang)+cos(-(ang+90))*cos(90-ang)
!
!
```

```
!       !       !----------------------------
!       !
!       !       e'31 =  cos(-(ang+90)) xy-strn 1
!       !               cos(-ang) xz-strn 1
!
!       sadd,xy-strn,xz-strn,cos(-(ang+90)),cos(-ang)
!
!       !       e'32 =  cos(-(ang+90)) y-strn cos(-ang)
!       !               cos(-ang) yz-strn cos(-ang)
!       !               cos(-(ang+90)) yz-strn cos(90-ang)
!       !               cos(-ang) z-strn cos(90-ang)
!
!       sadd,e32a,y-strn,z-strn,cos(-ang)*cos(-(ang+90)),cos(-ang)*cos(90-ang)
!       sadd,e32,e32a,yz-strn,1,cos(-ang)*cos(-ang)+cos(-ang)*cos(90-ang)
!
!       !       e'33 =  cos(-(ang+90)) y-strn cos(-(ang+90))
!       !               cos(-ang) yz-strn cos(-(ang+90))
!       !               cos(-(ang+90)) yz-strn cos(-ang)
!       !               cos(-ang) z-strn cos(-ang)
!
!       sadd,e33a,y-strn,z-strn,cos(-(ang+90))*cos(-(ang+90)),cos(-ang)*cos(-ang)
!       sadd,e33,e33a,yz-strn,1,2*cos(-ang)*cos(-(ang+90))
!

!-------------------------------------------------------------------
! the rest of the post processing
!-------------------------------------------------------------------


!* sort and get maximums/mins

esort,etab,x-strn,0,
*get,xmax,sort,,max
*get,xmin,sort,,min

esort,etab,y-strn,0,
*get,ymax,sort,,max
*get,ymin,sort,,min

esort,etab,z-strn,0,
*get,zmax,sort,,max
*get,zmin,sort,,min

esort,etab,xy-strn,0,
*get,xymax,sort,,max
*get,xymin,sort,,min

esort,etab,xz-strn,0,
*get,xzmax,sort,,max
*get,xzmin,sort,,min

esort,etab,yz-strn,0,
*get,yzmax,sort,,max
*get,yzmin,sort,,min

esort,etab,prin1,0,
*get,pri1max,sort,,max
*get,pri1min,sort,,min

esort,etab,prin2,0,
*get,pri2max,sort,,max
*get,pri2min,sort,,min
```

```
esort,etab,prin3,0,
*get,pri3max,sort,,max
*get,pri3min,sort,,min

esort,etab,e11,,0
*get,e11max,sort,,max
*get,e11min,sort,,min

esort,etab,e12,,0
*get,e12max,sort,,max
*get,e12min,sort,,min

esort,etab,e13,,0
*get,e13max,sort,,max
*get,e13min,sort,,min

esort,etab,e22,,0
*get,e22max,sort,,max
*get,e22min,sort,,min

esort,etab,e23,,0
*get,e23max,sort,,max
*get,e23min,sort,,min

esort,etab,e33,,0
*get,e33max,sort,,max
*get,e33min,sort,,min
```

```
!* What follows is the optimization file for calculating the
!* strains in the FRP pipe model
!* Since the model is small and the calculations are completed quickly
!* the optimization module of ANSYS is set to "run" type, and is
!* used to repeat the calculations
!* over and over for the different fibre angles.
!* This is no object variable optimized.


/opt                                         !* begin optimization

! set file to be optimized

opanl,root,lgw,


!* OPVAR - DEFINES PARAMETERS TO BE ALTERED/CHANGED
!* OPVAR,VARIABLE NAME,DESIGN VARIABLE,MIN-VALUE,MAX-VALUE,TOLERANCE-BTW-LOOPS

OPVAR,ratmaxab,OBJ, , , 1E-7

opvar,ang,dv,0.0,90,1                        !* so the fibre angle can be set by the opt
                                             !*             file

opvar,hoopstrs,dv,1e6,1e10
opvar,pload,dv,1,50E9                        !* so pressure can be changed in the opt file
opvar,press2,dv,1,50e9                       !* so endcap force can be changed in the opt
                                             !*             file


optype,run                                   !* optimization type


!-----------------------------
!-----------------------------
opdel,all                                    !* remove any previous optimization runs from
                                             !*             memory


!1H:0A
*do,oploopi,1,90                             !* loop to run through the fibre angles
       ang=oploopi
       hoopstrs=1e6                          !* set hoop stress
       pload=(thickrat*2)*hoopstrs           !* calculate pressure
       PRESS2=0                              !* set axial load to zero
       opexe                                 !* run optimization
*enddo

opsave,purehoop,opt,                         !* save results of the optimization variables
```

```
!* The sample ANSYS input code which follows is the code which was used to determine
!* the installation bending moment coefficients included in Appendix A.

/prep7
/title,install


neqit,1000                                 ! number of non-linear iterations

diameter=.8                                !* set a pipe diameter
thickrat=.02                               !* set a thickness
nlen=721                                   !* nodes along length
len=360                                    !* length
deep=5                                     !* ditch depth
lift=1                                     !* specify lift condition

pi=3.14159

density = 7860                             ! density = 7860 kg/m3 = 7.86 g/cm3 (steel)

volume=pi*(((.5+thickrat)*diameter)**2-(diameter/2)**2)
weight=density*volume*9.81
Icirc=3.14159/4*(((.5+thickrat)*diameter)**4-(diameter/2)**4)


et,1,pipe16                                !* element type
r,1,diameter+diameter*thickrat,diameter*thickrat
                                           !* set values for the element
                                           !* r,1st set,OD=1,thk=.02


! material properties for steel
 MP,EX,1,200e9
 MP,PRXY,1,.25
 MP,GXY,1,77E9
 MP,DENS,1,7860

ez=200e9

*do,j,1,nlen,1
   n,j,(j-1)*len/nlen                      !* create nodes along the length
*enddo

*do,j,1,nlen-1,1
   e,j,j+1                                 !* create elements
*enddo


d,1,all                                    !* XY and Zconstraints on node 1
d,(nlen*2)/5,uy,lift                       !* set lift point constraint at a node

acel,,9.81                                 !* load the elements with self weight
                                           !*             (gravity)

!contact surface stuff

ET,2,contac26                              !* element type
r,2,3e15                                   !* surface stiffness

!nodes

n,2001,0                                   !* specify nodes for contact surface
n,2002,2*len/5-1,                          !* nodes 2001,2002 at ground level
```

```
n,2003,2*len/5-1,-deep                   !* nodes 2003, 2004 at bottom of ditch
n,2004,len,-deep

type,2                                   !* specify element type
real,2

*do,j,2,(2*nlen)/5,1
  e,j,2001,2002                          !* create contact elements
*enddo                                   !* e,pipe node, surface nodes

*do,j,(2*nlen)/5+1,nlen,1
  e,j,2003,2004
*enddo


cnvtol,u,.00001                          !* set the convergance tolerance or it may
                                         !* not come to solution

/solu                                    !* enter solution module
solve                                    !* solve the model

/post1                                   !* enter the post processor

esel,s,type,,1                           !* select only the pipe elements
etable,disp,u,y                          !* etable 1 - displacements of each node
etable,axial0,lepel,1                    !* etable 2 - strains at 0 deg around
                                         !*            circumference
etable,axial45,lepel,5                   !* etable 3 - 45 deg
etable,axial90,lepel,9                   !* etable 4 - 90 deg
etable,axial135,lepel,13                 !* etable 5 - 135 deg
etable,axial180,lepel,17                 !* etable 6 - 180 deg
etable,prinmax,nmisc,86                  !* etable 7 - maximum principal strain


esort,etab,disp,0,                       !* sort displacements
*get,dspmax,sort,,max                    !* find maximum and minimum
*get,dspmin,sort,,min

esort,etab,axial0,0,                     !* sort strains at 0 deg, find max and min
*get,max0,sort,,max
*get,min0,sort,,max

esort,etab,axial45,0,                    !* sort strains at 45 deg, find max and min
*get,max45,sort,,max
*get,min45,sort,,max

esort,etab,axial90,0,                    !* sort strains at 90 deg, find max and min
*get,max90,sort,,max
*get,min90,sort,,max

esort,etab,axial135,0,                   !* sort strains at 135 deg, find max and min
*get,max135,sort,,max
*get,min135,sort,,max

esort,etab,axial180,0,                   !* sort strains at 180 deg, find max and min
*get,max180,sort,,max
*get,min180,sort,,max

esort,etab,prinmax,0                     !* sort principal strains, find max and min
*get,prinmax,sort,,max
*get,prinmin,sort,,min

eusort                                   !* unsort etable
```

```
*do,i,1,nlen-1                                  !* loop to find location of maximum strain
  *get,prin,etab,7,elem,i
        *if,prin,eq,prinmax,then
                prinmaxn=i
        *elseif,prin,eq,prinmin
                prinminn=i
        *endif
*enddo

  *get,prinmaxx,node,prinmaxn,loc,x

axialmax=max0>max45>max90>max135>max180
axialmin=min0<min45<min90<min135<min180


!----------------------------------------------------
! determine geometries of the pipe during installation
!----------------------------------------------------
!* to find point of lift off
!* define liftoff as Y>.0001
!* define settle as Y=deep

liftoff=.0001
settle=deep

liftptn=(nlen*2)/5
*get,liftpt,node,(nlen*2)/5,loc,x

*do,i,1,liftptn+2,                              !* determine x location of liftoff
   *get,disp,etab,1,elem,i

   liftnode=i
   *get,liftoffx,node,i,loc,x
   *if,disp,gt,liftoff,exit
*enddo

*do,i,1,nlen-1,                                 !* determine x location of set down
    *get,sdisp,etab,1,elem,i

   setnode=i
   *get,setdownx,node,i,loc,x
   *if,sdisp,eq,-settle,exit
*enddo


lenact=setdownx-liftoffx                        !* determine active length of pipe
lenup=liftpt-liftoffx                           !* determine length from liftoff to liftpt
lendown=setdownx-liftpt                         !* determine length from liftoff to set down

esel,s,elem,,liftptn,setnode-5
esort,etab,prinmax,0
*get,prinmin1,sort,,min
*do,i,liftptn,setnode
  *get,prin,etab,7,elem,i
        *if,prin,eq,prinmin1,then
                prinmi1n=i
        *endif
*enddo


c=(1/2+thickrat)*diameter
moment=prinmax*Icirc/c                          !* calculate bending moment which would cause
                                                !* the maximum strain occuring here
```

```
coef=moment/((Ez*icirc*weight)**.5)          !* calculate the coefficient for the equation
                                             !* moment = coef*(E*I*w)**.5

!* An optimization file was used here to run through various
!* diameters, Young's modulii, and densities to ensure that the coefficients
!* calculated were appropriate.
```