Alarm System Design Using Rank Order Filters

by

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Abstract

In the process industry, process variables are continuously monitored to ensure safety, reliability and efficiency of plant operations. Due to the advancement of the modern communication and computer technology, it is now possible to incorporate alarms to every process variable at little or no cost. As a result, operators are flooded with too many alarms beyond their capacity to respond accordingly. Many of these alarms are false or nuisance alarms. Therefore an efficient and dependable alarm system is needed for greater safety and productivity. Motivated by this, this thesis focuses on the application of a class of nonlinear filters, namely rank order filters, on process data and develops quantitative relationship among filter parameters and alarm performance indices.

In industries, filtering is a widely used alarm design technique. Moving average filters are most commonly applied filters in the industry because of their simplicity and ease of implementation. However, nonlinear filters have not been able to draw industrial attention due to nonlinearity and unknown relationship with alarm performance indices. Hence, we investigated the applicability of rank order filters and compared the performance with moving average filters under different input distributions. We established analytical relationships between filter order with different ranks and detection delay. Then we proposed a method to design filter order meeting

the performance requirements.

We obtained significant improvements in terms of reducing false and missed alarm rates and detection delay by applying rank order filters. The performance curve of a rank order filter lies between the performance curves of the moving average filter and the general optimal filter with the corresponding order. In the end, all the theoretical development and design techniques have been validated through numerical simulation and some industrial case study.

To my loving parents

and my wife

Shahela Akhand Laboni

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Chapter 1

Introduction

1.1 Motivation and Background

In any industrial setup, smooth and uninterrupted operation is desired to gain maximum productivity and profitability. To ensure this, modern industries are monitored by hundreds or thousands of sensors. These sensors are installed in different locations throughout the plant to monitor the actual condition. There are communications and interactions going on among these sensors as well. When the process operation is under normal operating conditions, process operators take routine actions. The emergence of alarm systems in process industry comes from the occurrence of faults or abnormality. The message that abnormality has occurred in some part of the plant is conveyed by alarms on the operator panel. Operators then take necessary control actions promptly to ensure cost efficiency, product quality, and safety of the workers and plants. Failure to take timely actions may result in serious consequences, even human injuries and casualties. So, alarm systems play a critical role in process industry.

The *International Society of Automation (ISA)* provides standards for process industry. In 2008, an online survey conducted by ISA indicated that "what automation industry observers and practitioners felt that near-term trends were going to be showed that alarm management and security scored at 14%, one of the top 5 technologies that the facility would rely on" [64]. In process industry, alarms are defined in [13]: "an alarm is some signal designed to alert, inform, guide or confirm, and an alarm system is a system for generating and processing alarms and presenting them to users". Detailed description of an alarm system is provided in ISA 18.2 Standards and EEMUA guidelines. According to [38], basic parts of an alarm system may include an alarm generating part in the basic process control system (BPCS) and safety instrumented system (SIS), the alarm log, and a human machine interface (HMI) for communicating with operator. Alarm historian is also important to store the alarm log files. In Figure 1.1, basic parts of an alarm system is shown.

Due to modern day Distributed Control Systems (DCS) and communication technology, it has become very easy to access thousands of process variables and support interaction among them. It has been found that, faults generated at one process variable propagate through the plant and affects other process variables. As a result, cascaded faults occur and may end up with plant upset. According to the Abnormal Situation Monitoring (ASM) Consortium [35], petrochemical plants on average suffer a major accident once every three years. Moreover, the US petrochemical industry alone loses 10-20 billion dollars annually because of abnormalities related to equipment failure, environmental damage and human casualties [40].

DCS systems have made it even easier to configure alarms than previous hard wired systems at little or no cost. As a result, alarms are configured at almost every point in a plant without proper analysis and rationalization. Too many alarms make

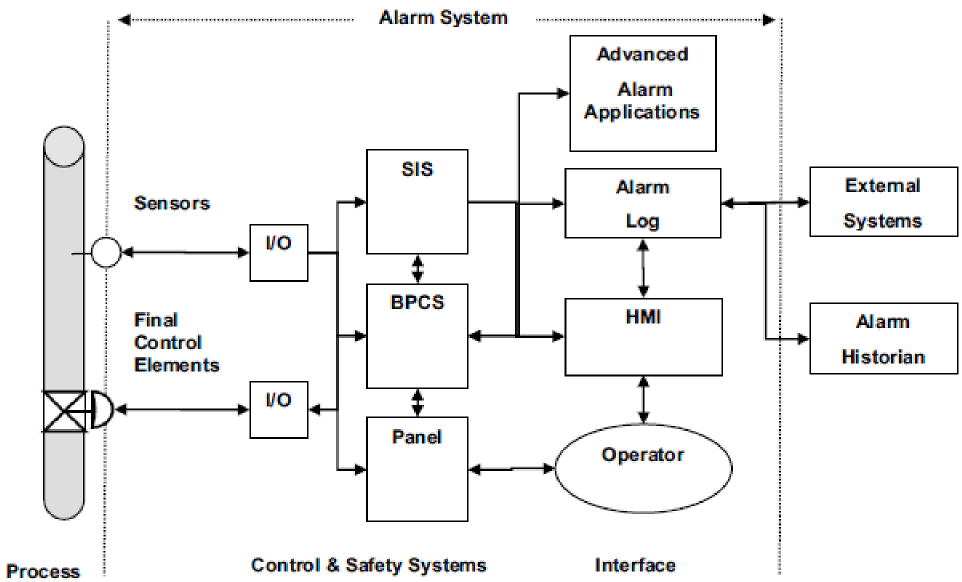


Figure 1.1: Alarm system dataflow [38]

the alarm system complex and operators experience difficulties during abnormal situations. Most of these alarms are false or nuisance alarms, generated due to interaction and fault propagation. Operators face floods of alarms and cannot respond promptly and effectively. This situation is defined as an alarm flood. According to EEMUA 191 [26], under normal conditions, an operator needs about 10 minutes to process and respond to an alarm, so an operator should not receive more than 6 alarms per hour. Alarm performance metrics suggested by EEMUA are given in Table 1.1. From the table, it is visible that there is a large gap between standard and observed industrial counts. There are a lot of room for improvement of alarm performance to comply with the standards.

Fault detection and identification (FDI) is closely related to alarm annunciation. In alarm systems, a fault is detected at first then operator is informed about the oc-

	EEMUA	Oil and Gas	Petrochemical	Power
average alarms/hour	≤ 6	36	54	48
average standing alarms	9	50	100	65
peak alarms/hour	60	1320	1080	2100
distribution % (low/med/high)	80/15/5	25/40/35	25/40/35	24/40/35

Table 1.1: EEMUA benchmark and average values received in industries

currence of fault by raising an alarm. There are two approaches in FDI: *model based* and *signal based* [1]. Due to mathematical complexity and implementation issues in DCS, the signal based approach is widely accepted in industry. In signal based methods, alarm system design can be classified in two ways: *univariate* and *multivariate*. In univariate design, alarm threshold and processing techniques (deadband, delay timer, filter etc.) are configured for individual process variable [24, 39]. In case of multivariate design, alarms are designed for a latent variable which is usually a linear combination of several process variables. Several multivariate design of alarm rationalization have been discussed in [7, 44, 54, 68]. In this thesis, we focus on the univariate alarm design approach for selecting filters that balance false alarm rate, missed alarm rate and detection delay.

In process industry, filtering is an accepted method for suppressing noise from the process data [17]. In signal based alarm design methods, each process variable is compared with an alarm threshold which depends on process specification, productivity and operating conditions [41]. In process plants, noises are present in process variables during regular operation. Such noise propagates and affects other variables as well. The process variables operating close to the alarm threshold may cross and come below the threshold within a very short time and keep doing that throughout the operation of the plant. This results in unwanted alarm annunciation to the operators repeatedly. If hundreds of process variables are configured this way, thousands of alarms will be raised and cleared within a short time. Due to this continuous on and off mode, these alarms are referred to as chattering alarms. It has been observed that chattering alarms are one of the main contributors for alarm floods. By applying filters we can reduce noise which in turn reduce chattering alarms and alarm floods.

In industry, the performance measurement of different alarm processing techniques (deadband, delay timer, filter) is an important concern. Due to poor design of alarm threshold and improper filtering of process data, false and missed alarms may be annunciated. On the other hand, applying filter may result in delay in the activation of alarms. The false alarm rate (FAR) and the missed alarm rate (MAR) quantify alarm accuracy; and the expected detection delay (EDD), which is the average time required to activate an alarm quantifies alarm system latency [40]. The exact quantitative relationship between performance specifications and design methods is a comparatively new area of research. This thesis focuses on obtaining quantitative expressions that relate filter parameters to the performance indices. Using these expressions, a practical alarm system design method is carried on in later chapters.

1.2 Literature Survey

Alarm management has drawn recent attraction in academia as a potential sector for research due to industrial demand. The complexity in process industries has increased many folds with the advancement of communication and computer technologies. Industries now try to monitor thousands of process variables to maximize their plant reliability, efficiency, quality of products and profitability. The negative side of all these improvements is that, there are hundreds or thousands of configured alarms and the operators are lost in a sea of information. Several techniques (filters, deadbands and delay timers) have been applied to manage these numerous alarms. In this section, we review existing and current trends in literature regarding alarm systems design and performance improvement.

Rank Order Filters

The main topic of this thesis is the application of nonlinear filters in the process industry. Due to complexity and nonlinearity, these filters were not applied in industry. Among all the nonlienar filters, a rank order filter is chosen due to two inherent properties: edge preservation and effective noise attenuation with robustness to impulsive noise [73]. Preserving edge of signals are very important in image processing and biomedical signal processing as well. In addition, rank order filters can effectively suppress impulsive noise in communication systems [43, 57]. Median filters, a special case of rank order filters, have been proven to be optimal filters for biexponential noise like moving average filters for Gaussian noise [17]. Rank order filters are the simplest form of nonlinear filters and easily implementable in DCS systems without any computational burden. For a selected window, a rank order filter organizes the data according to their rank and produces, e.g., the i^{th} maximum, as output specified by design. These filters only use rank information of the input data which makes it implementable. Two widely applied rank order filters are maximum and minimum filters which output the maximum and minimum values in the window respectively. In image processing, they are also known as erosion and dilation as well [28, 47, 60]. Since frequency and impulse responses are not relevant in rank order filters, the properties of such nonlinear filters are characterized statistically [73]. Different types of rank order filters have been developed depending on applications. Some of them are FIR-median hybrid (FMH) filters [32, 37, 53], log-likelihood (Ll) filters [46, 56], weighted median (WM) and weighted order statistics (WOS) filters [11, 15, 33, 69, 71] and stack filters [21, 66, 72, 74]. In this work, we mainly focus on rank order filters with different maximum as output and their application to alarm systems [73].

Alarm Design and Filtering

The most popular method in alarm system design is the univariate design method. Generally, a process variable is continuously monitored and compared with a threshold in a signal-processing based method. However, an incorrect setting of the alarm threshold may result in false, missed and redundant alarms and may cause delay in alarm activation [5,6,8,40,41]. A more conventional approach in alarm design is listing down top ten alarms and setting a 3σ limit on each process variable [40]. In [41,42], a general framework for univariate alarm design is introduced. The performance of the alarm system is evaluated in terms of three metrics, namely, false alarm rate (FAR), missed alarm rate (MAR) and expected detection delay (EDD). Receiver Operating Characteristics (ROC) curve is introduced to visualize the trade-off on FAR and MAR with respect to the alarm threshold. In [41], a threshold design process is presented for different signal processing techniques (filter, delay timer and deadband) based on ROC curves by balancing FAR and MAR. Detection delay for a moving average filter has been formulated in [4]. A 4-step alarm design method is presented using moving average filters and meeting performance requirements on FAR, MAR and EDD. In [17], authors designed optimal filters and proposed differential evolution (DE) based algorithm to estimate the PDF of the filtered output. They have also derived sufficient condition for a moving average filter to be the optimal linear filter.

Delay in Alarm Activation

The success of a fault detection method depends on how fast it can detect the abrupt change in the process [48]. Estimating the detection delay is very important for alarm system design. A designer is supposed to have a clear idea about detection delay for activating an alarm depending on a particular design scheme. This idea allows the designer to take preventive measures for compensating activation delay. The CUSUM type algorithm has been discussed as a mean of quantifying detection delay in [12, 75]. In [75], the probability distribution for detection delay and time between false alarms have been analytically and numerically proven. Analytical probability distribution for sequential fault detection schemes (CUSUM and GLR based methods) have been discussed in [70]. In [5,6] detection delays caused by simple limit checking, delay timers, deadbands and filters have been formulated. By using an estimate of the detection delay, a step-by-step alarm design method is also presented here by balancing false and missed alarm rates. The ISA 18.2 Standards [38] and EEMUA 191 publication [26] described alarm limits, delay timers and deadbands for alarm design for process industry for the first time. Wald's test [12, 61] discusses sequential hypothesis testing for detection of change. Above all, with expert process knowledge, fault occurrence and point of change can easily be identified using historical process data [2, 3].

Chattering and Nuisance Alarms

Chattering and repeating alarms are the most common form of nuisance alarms in industry. When several alarms are raised in a short period of time for a single process variable, then it is termed as chattering alarms [36]. In [51], a chattering index is calculated on the basis of statistical properties of the process variable. A chattering index based on run length distribution has been proposed in [45]. A modified chattering index has also been introduced in [62, 63] where instead of run length, total data length is considered. Filters, delay timers and deadbands [26, 34, 38, 41] are effective methods to reduce chattering and repeating alarms but there are some trade-offs in each method [40, 41]. Application of deadband in reducing chattering alarms is discussed in [52] where the authors mentioned that for a fixed alarm threshold, increasing deadband is less effective in reducing alarm chatter. In [63], an online method for removing chattering alarms has been proposed based on alarm duration and alarm interval. Application of filters in reducing chattering alarms has been proven effective in [4]. Filters reduce noise which in turn reduce chattering alarms for a single process variable. At the same time it introduces delay in alarm activation. In [4,67] detection delay for a moving average filter has been formulated; and the relation of detection delay with filter order and alarm threshold has also been established in [4].

Alarm Flood and Causality Analysis

An alarm flood is a serious issue in today's process industry. It is defined as a situation where more than 10 alarms per 10 minutes arrive at the operators panel [26]. Indecision and delay in identifying actions to be taken often lead to emergency plant shutdown or major plant upset. Recently, similarity investigation and pattern analysis of alarm sequences have drawn attention to academia with the application of machine learning and artificial intelligence techniques. In [7], a nonlinear time alignment method named dynamic time warping (DTW) has been discussed to group time stamped alarm floods. According to [7], root cause analysis of historical alarm floods can be used to take necessary actions in advance to prevent plant upset. Application of the Smith Waterman algorithm in alarm pattern sequencing and similarity measurement has been discussed in [18]. The algorithm is modified for application to alarm systems by including time-stamp information in calculating the similarity index. [27] proposed an automatic alarm data analyzer (AADA) algorithm considering causal dependencies in alarm data. A high density alarm plot is a handy tool to visualize top bad actors on the same plot and identify alarm floods from the color density discussed in [9]. Another important visualization tool is the alarm similarity color map mentioned in [9] where similar and redundant alarms are highlighted. Several industrial applications have been presented in [9] to show the usefulness of the visualization tool. Data driven Granger causality [29] and transfer entropy [25, 29] have been proposed in [9] to capture process connectivity. Though the results are not that reliable yet, they can be verified using the P&ID of the plant.

After reviewing literature on alarm system design and existing signal processing techniques, we find that there is plenty of room for univariate alarm system performance improvement. The scale of improvement varies depending on the specific industry. Moreover, filtering techniques have not been explored widely since most optimal filters are nonlinear. This is why, a simplest nonlinear filter, the rank order filter, has been introduced in this thesis for alarm system design. To the knowledge of the author, there has been no application of the filter in alarm systems. In this thesis an attempt has been made to formulate performance metrics for rank order filters and apply on industrial data to show the effectiveness of the proposed filter.

1.3 Thesis Contributions

The research on performance indices of alarm systems and filter design is relatively a new area. As a result, the relation among performance indices and filter parameters is not well known. The main goal of this thesis is to develop quantitative relationship between filter parameters and performance indices for alarm systems. So, the major contributions in this thesis are enlisted here:

1. Visualize the performance of rank order filters in terms of ROC curves in comparison with moving average filters and LLR-based general optimal filters considering both Gaussian and non-Gaussian distributions.

2. Propose a method to compute the expected detection delay for rank order filters, and compare with moving average filters with the same window size.

3. Utilize the expected detection delay information in alarm systems design. The design provided in this thesis balanced the false alarm rate and missed alarm rate while meeting detection delay requirements.

4. Finally, both simulation and industrial case studies have been demonstrated to show the application of developed design methods and derived equations.

1.4 Thesis Outline

This thesis has been prepared according to the guidelines from the Faculty of Graduate Studies and Research (FGSR) at the University of Alberta. The rest of the thesis is organized as follows.

In Chapter 2, basics of rank order filters have been discussed. A rank order filter is a non-linear filter which gives i^{th} maximum value as output from a specific window. The PDFs of rank order filters with 1^{st} maximum, 1^{st} minimum and median as outputs have been formulated and verified by Monte-Carlo simulations. Some data driven tests have also been run on filtered data in order to verify that rank order filtered data are not *i.i.d.* (independently and identically distributed).

Chapter 3 is concerned with the accuracy of the univariate alarm design based on rank order filters. Accuracy of alarms is measured in terms of the false alarm rate (FAR) and missed alarm rate (MAR). The ROC curve is a visualization plot to show the trade-off between FAR and MAR. The design of an optimal alarm filter by minimizing a weighted sum of false and missed alarm rates (probabilities) have been presented. In this chapter, the performance of a rank order filter with i^{th} maximum as output has been compared with the moving average filter of the same order in terms of ROC curves for Gaussian distributed input data. The performance of the rank order filter is also compared with the optimal alarm filter, the LLR filter, considering both Gaussian and non-Gaussian distributions. Performance comparison in terms of ROC curves allow us to visualize the effectiveness of the rank order filter in reducing false and missed alarm rates.

Chapter 4 focuses on swiftness or latency of alarm activation, namely, detection delay calculation for rank order filtering. Filtering has been applied widely in the industry to suppress noise from process data. The only downside of applying filter is the delay introduced by filters due to processing signals. An attempt has been made to formulate the detection delay for the rank order filter with i^{th} maximum as output. Numerical examples have been presented to validate the analytical result.

Chapter 5 includes application of rank order filter design to both simulated and industrial data. At first, a step-by-step univariate alarm design method has been presented to show the application of the proposed method based on simulated data. Since filtering is only applicable to process data, historical process data from oil-sand plants have been analysed and the proposed filtering technique has been applied to show the applicability of the filtering method in designing alarm systems.

Finally, Chapter 6 discusses on conclusion and future work. A summary of work presented in the thesis is given, and possible scopes for future extension are also discussed.

Chapter 2

Rank Order Filters

2.1 Introduction to Rank Order Filters

Filtering is a widely accepted method for reducing noise, smoothing the curves and changing distribution; it is applied in signal and image processing, time series analysis and process industries [34]. In process industry, with the advancement in distributed control systems (DCS), it has become much easier to monitor as many process variables as needed. To maintain normal operations of plants, process variables are continuously compared with some thresholds also known as alarm limits. When a variable crosses a limit, an alarm is raised announcing the occurrence of an abnormality. But, the alarms that are generated are not always correct ones, there are several false or nuisance alarms annunciated, which is due to poor configuration of the trip point, noise or oscillation in the process variables. For the process variables operating close to their trip points, a small noise can introduce chattering effect. To get rid of these situations, filtering has already been employed in process industry [1].

Moving average filters are the most common filters deployed in the industry due to its easy implementation and relatively better performance. Many variations of the median filter have also been proven applicable in achieving better performance; these include recursive medians, weighted medians and per-mutated medians. The most important characteristics of these filters is the robustness to outliers. Unlike moving average filters, the outlier would be removed and the rest of the output signals would be relatively smooth [59].

Median filters are a subclass of rank order filters [22, 23] where the median value in the window is selected as output. Rank order filters are obtained by putting the i^{th} largest sample as the output in a window length N. Two specially applied rank order filters are the minimum and maximum filters where minimum and maximum values in the window, respectively, are chosen as output [10]. In image processing [58], these are also mentioned as erosion and dilation filters. Frequency analysis and impulse response don't have any meaning in rank order filtering due to non-linearity. Output distribution properties are the basic descriptor of rank order filters [73]. In the next sections, mathematical formulation has been developed and verified for the rank order filters along with statistical property analysis.

2.2 Mathematical Formulation

Let $X_1, X_2, ..., X_N$ represent a set of *i.i.d.* random samples. If these samples are sorted in ascending order such that $X_{(1)} \leq X_{(2)} \leq ... \leq X_{(N)}$, then $X_{(i)}$ represents the i^{th} order statistic. Rank order (causal) filter of rank i at point n in the output sequence with window size N [23, 50, 73] :

$$Y(n) = i^{th} \quad largest \quad value \quad in \quad \{X(n-N+1), ..., X(n-1), X(n)\} \tag{2.1}$$

So, the i^{th} rank selection probability can be denoted by $\mathbb{P}(Y = X_{(i)}), 1 \leq i \leq N$ and is the probability that the output $Y = X_{(i)}$. Assume that the input variable $X(n)$ has PDF $f_X(t)$ and CDF $F_X(t)$, and the output variable $Y(n)$ has PDF $f_{Y_i}(t)$ and CDF $F_{Y_i}(t)$, $(i = 1, ..., N)$, then the density and distribution functions of the i^{th} ranked sample respectively are given as follows:

$$\begin{aligned} f_{Y_i}(t) &= i\binom{N}{i} F_X(t)^{i-1}(1-F_X(t))^{N-i} f_X(t) \\ F_{Y_i}(t) &= \sum_{k=i}^{N} \binom{N}{k} F_X(t)^k (1-F_X(t))^{N-k} \end{aligned} \tag{2.2}$$

Let us assume that Y_i are statistics (they are functions of random sample $(X_1, X_2, ..., X_N)$ and $Y_1 \leq Y_2 \leq ... \leq Y_N$. Unlike the random sample itself, the order statistics are clearly not independent, for if $Y_j \geq y$, then $Y_{j+1} \geq y$. Several data driven tests are run on the filtered data to verify dependency later in this chapter.

Let $Y_1 \leq Y_2 \leq ... \leq Y_N$ represent the order statistics from a cumulative distribution function $F(\cdot)$. The marginal cumulative distribution function of Y_α, $\alpha = 1, 2, ..., N$, can be obtained from [50]:

$$F_{Y_\alpha} = \sum_{k=\alpha}^{N} \binom{N}{k} [F(y)]^k [1 - F(y)]^{N-k} \tag{2.3}$$

In the next section, we validate Eqn.2.2 considering different distributions for input data. We try to select different samples from the window and show that the simulated result is following the analytical one.

2.3 Verification of PDFs

A rank order filter has two design parameters: filter order, N, and ordered position in the window, i [73]. Though any i^{th} ranked maximum value in the window can be selected, for simplicity, only the 1^{st} maximum, 1^{st} minimum and median values in the selected window are considered here. At first, we have tried to show that the estimate of the probability density function [31, 55] of the rank order filtered data matches with that of the corresponding analytic pdf equation given in Eqn.(2.2).

2.3.1 Gaussian input data

For the numeric simulation, normally distributed random data with mean, μ_n= 0 and variance, σ_n= 1 has been generated as normal data. Abnormality occurred in the sense of change in mean and variance. As a result, normally distributed data with mean, μ_{ab}=1 and variance, σ_{ab}= 2 is generated as abnormal segment of the data. Each normal and abnormal segment has 2000 samples totalling to 4000 samples shown in Figure 2.1.

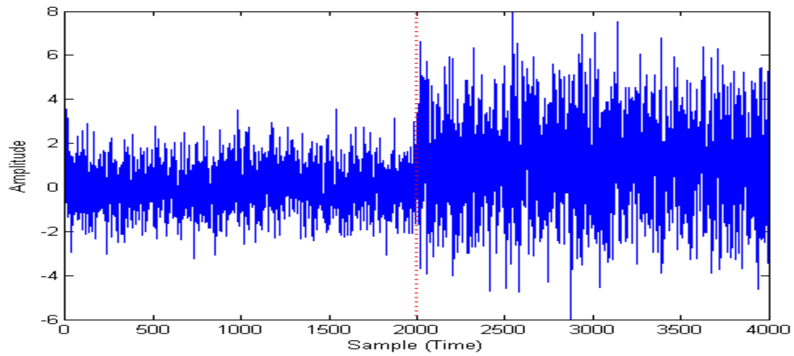


Figure 2.1: Normal and abnormal segment of data

To estimate the distribution of the input raw data and the filtered data, kernel density based estimation has been applied. For each density estimation 1000 Monte-Carlo simulations are run. The density estimate of the input signal with the trip point, y_{tp} is shown in Figure 2.2:

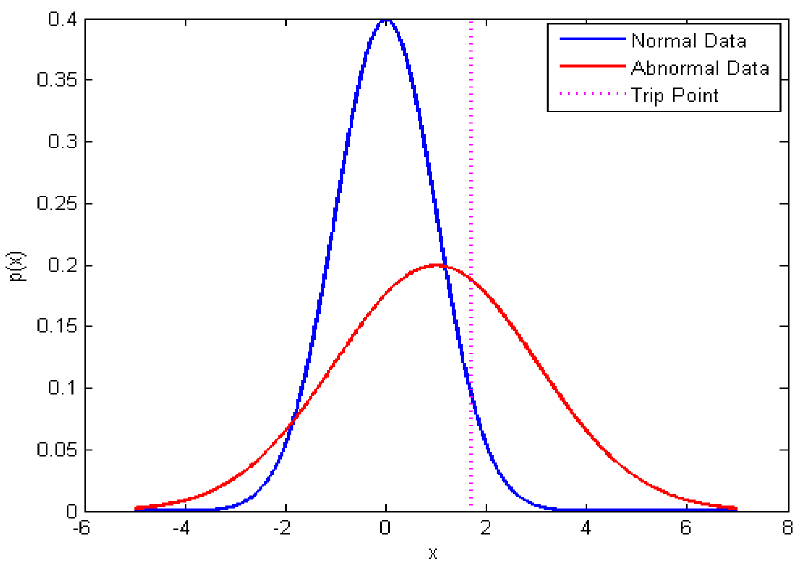
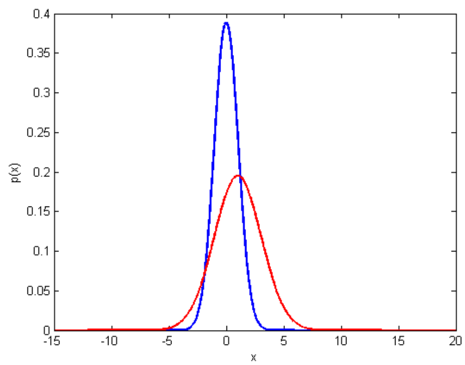
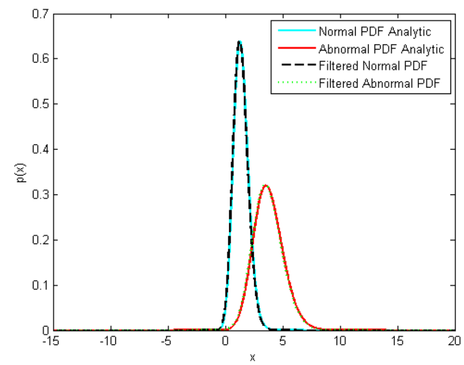


Figure 2.2: PDF of normal and abnormal data

Through numerical simulation, it has been shown below that for a fixed order, N with 1^{st} maximum, 1^{st} minimum and median value as output, the estimated densities of the filtered data match exactly with the corresponding analytical densities in Figure 2.3 (b)-(d).



(a)



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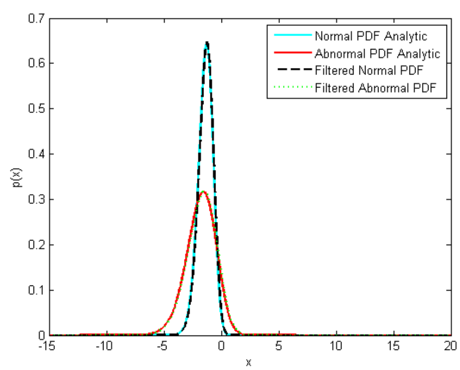

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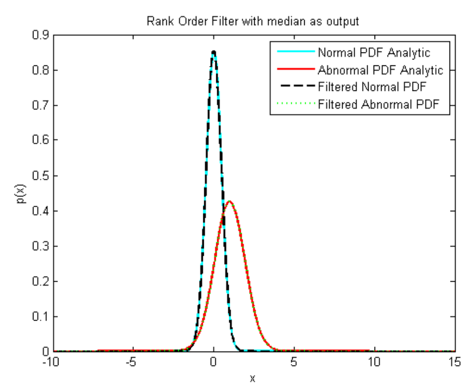
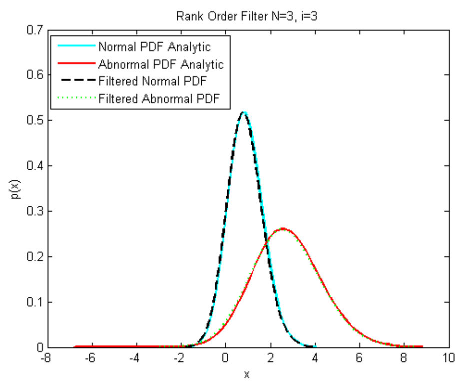


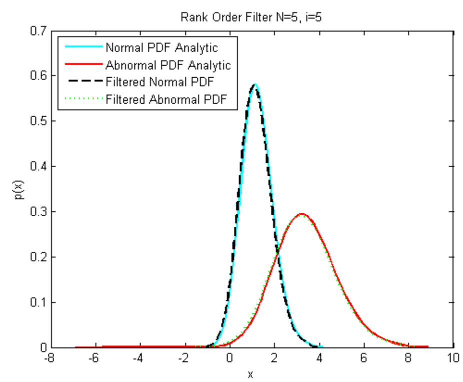
(d)

Figure 2.3: (a) PDF estimate of the normal and abnormal data, (b) verification of PDF with N=7, i=7 (1^{st} maximum), (c) verification of PDF with N=7, i=1 (1^{st} minimum), (d) verification of PDF with N=7, i=4 (median output)

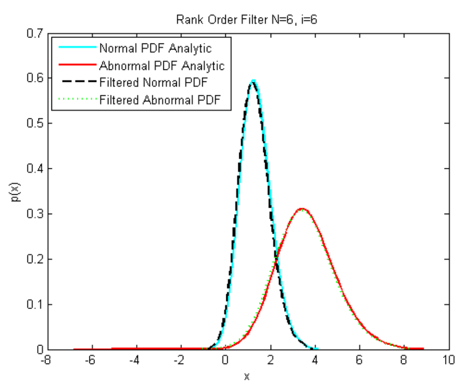
We can also visualize the effect of filtering data with different order of the rank order filter. The main goal of filtering data for alarm design is to reduce noise and also reduce the overlapping region of the distribution of normal and abnormal data. In this part, we change filter order with 1^{st} maximum as output and estimate the pdf of the filtered data. The estimated PDF's are shown is Figure 2.4.



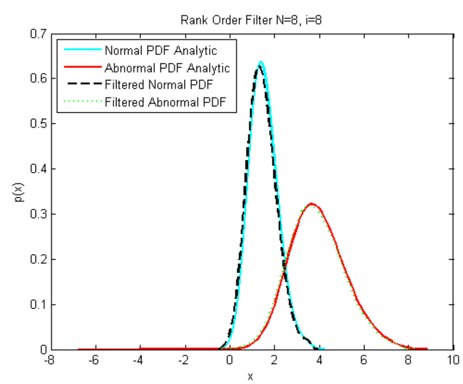
(a)



(b)



(c)



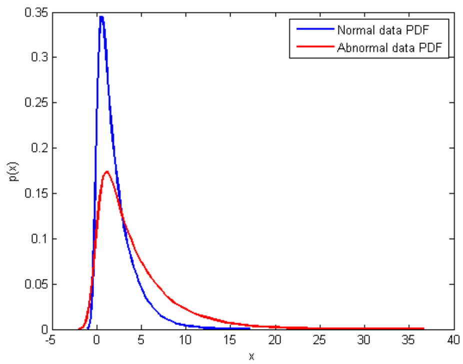
(d)

Figure 2.4: (a) Verification of PDF with N=3, i=3, (b) verification of PDF with N=5, i=5, (c) verification of PDF with N=6, i=6, (d) verification of PDF with N=8, i=8

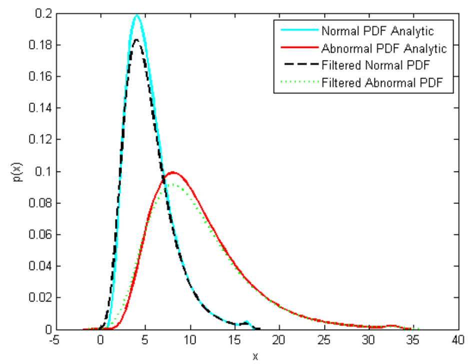
From the figures above, it is visible that with the increment of filter order, the mean of density plot is shifting and the overlapping region under the normal and abnormal density curves is getting smaller. Hence, the purpose of applying filter is served. An optimal trip point can be set for each filter order that would give minimum false and missed alarm rates for the given distribution.

2.3.2 Exponential input data

We now consider exponentially distributed [55] data as an input to the rank order filter to verify the PDF Eqn.(2.2). Normal data is assumed to be distributed with mean, $\beta_n = 2$ and abnormal data is distributed with mean, $\beta_{ab} = 4$. We filter the data with a fixed order filter but with i^{th} maximum as output. The filtered PDF is shown in Figure 2.5 which matches exactly with the analytical PDF.



(a)



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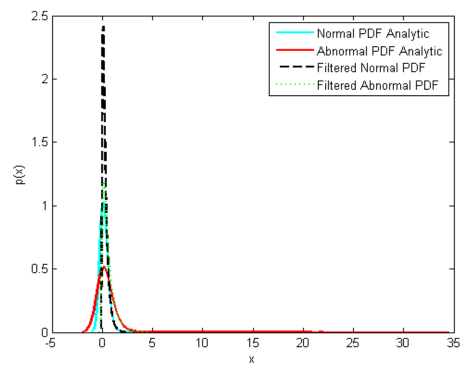

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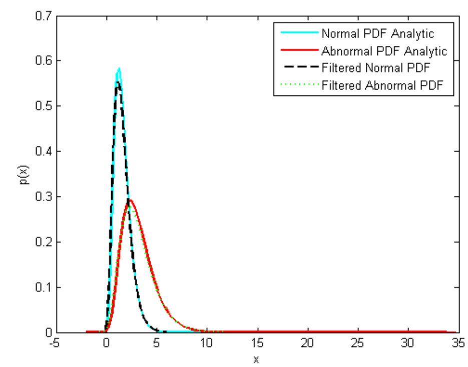


(d)

Figure 2.5: (a) PDF estimate of the normal and abnormal data, (b) verification of PDF with N=7, i=7 (1^{st} maximum), (c) verification of PDF with N=7, i=1 (1^{st} minimum), (d) verification of PDF with N=7, i=4 (median output)

2.3.3 Logistic input data

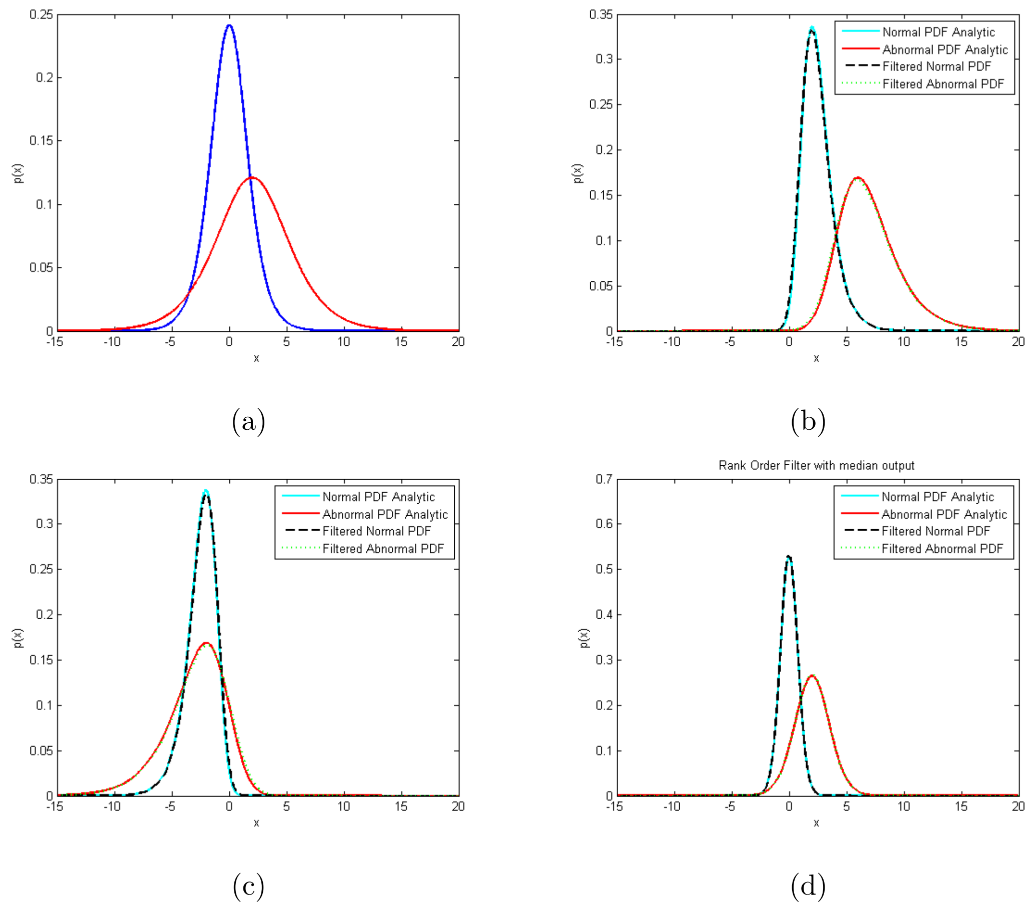


Figure 2.6: (a) PDF estimate of the normal and abnormal data, (b) verification of PDF with N=7, i=7 (1^{st} maximum), (c) verification of PDF with N=7, i=1 (1^{st} minimum), (d) verification of PDF with N=7, i=4 (median output)

For this section we use logistic distributed [55] data to validate the PDF Eqn.(2.2). The parameters for logistic distribution are the mean, μ, and the scaling parameter, s. So, for normal distribution we assume $\mu_n = 0$ and $s_n = 1$. For the abnormal segment of the data, we consider $\mu_{ab} = 2$ and $s_{ab} = 2$. The input data distribution is shown in Figure 2.6(a), whereas the estimated PDFs are verified for a fixed order filter with different outputs in Figure 2.6(b)-(d).

2.4 I.I.D. Hypothesis

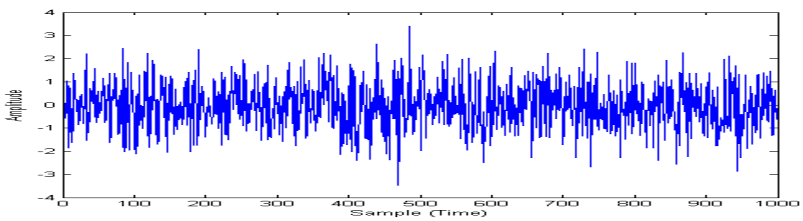
In this section, several data based methods [19,30] have been applied to check whether the rank order filtered data is independent and identically distributed (*i.i.d.*) or not. The advantage of proving *i.i.d.* data is that it will allow us to use the pre-derived analytic relationships for calculating false and missed alarm rates. To do so, it is assumed that we have an *i.i.d.* time series $\{x_1, x_2, ..., x_T\}$ as input to the filter.

2.4.1 Correlogram

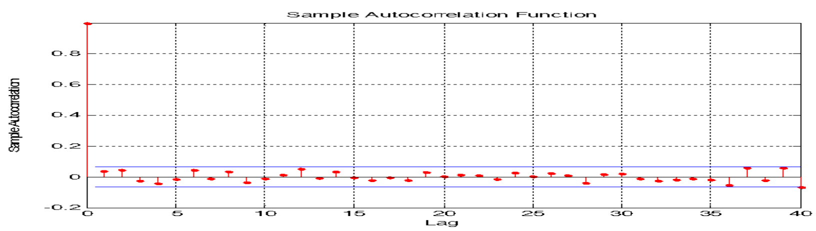
This is a simple method where autocorrelation is checked for the data under test. The null hypothesis is assumed that autocorrelation would be zero [30].

$$H_0 : \rho_k = 0$$

If we consider the following time series $x_t \sim N(0, 1)$, from the autocorrelation plots, it is visible that, for the raw data, the correlation is within the interval shown in Figure 2.7 (b). So, the null hypothesis is valid for unfiltered data, i.e., the input raw data is *i.i.d.*, which is obvious. However, for the filtered data, sample autocorrelation in Figure 2.8 (b) crosses the interval at lag 1-lag 7, a relevant time lag. Thus, we reject the *i.i.d.* hypothesis for the filtered data. From this test result, it can be concluded that the filtered data is not independently distributed.

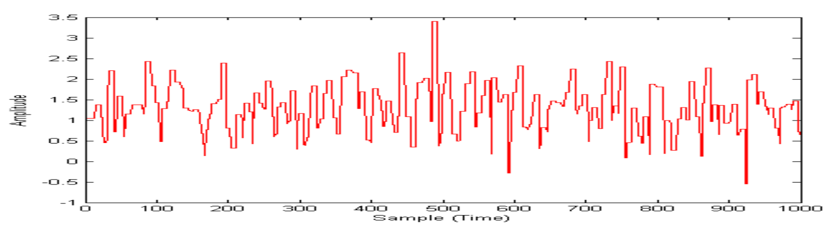


(a)

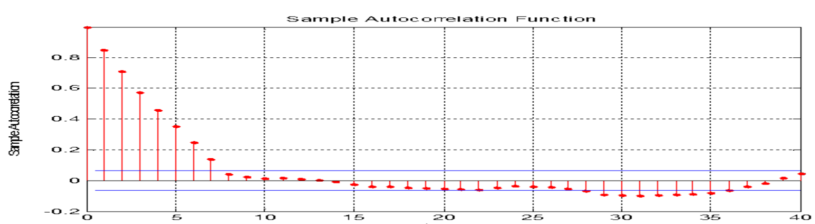


(b)

Figure 2.7: (a) Raw data, (b) correlogram of raw data



(a)



(b)

Figure 2.8: (a) Rank order filtered data, (b) correlogram of filtered data

2.4.2 QQ plot

In statistics, the QQ plot is a graphical method to plot quantiles of two probability distributions against each other for comparing their distributions. A point (X, Y) on the plot displays a quantile-quantile plot [20, 49] of two samples coming from two distributions. The plot will be a linear one, if the samples come from the same distribution. The sample data is displayed with the plot symbol '+' in a QQ plot. A line that joins the first and third quartiles of each distribution is superimposed on the plot. This line is extrapolated out to the ends of the sample to help evaluate the linearity of the data.

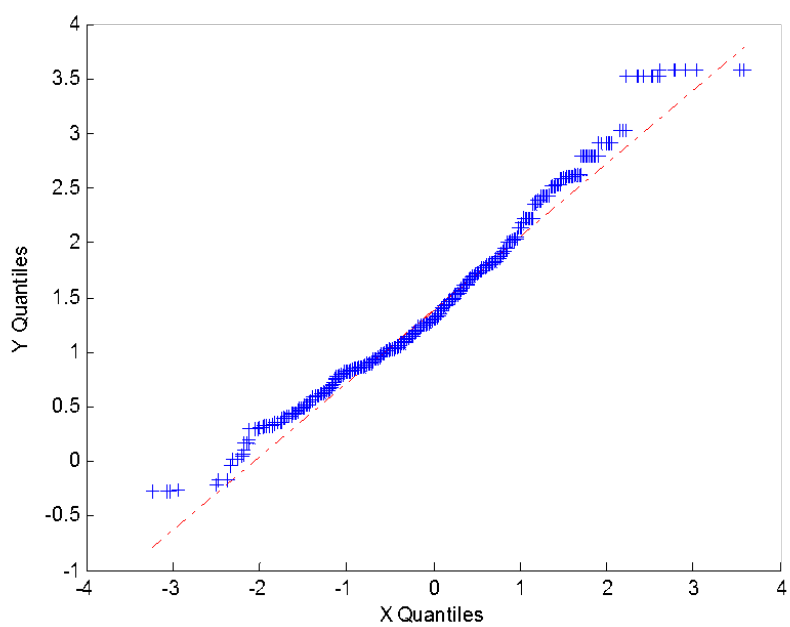


Figure 2.9: QQ plot of filtered data

The plot in Figure 2.9 shows a linear relationship between two samples which implies that they come from the same distribution. So, from this plot, it can be concluded that the filtered data is identically distributed.

2.4.3 Difference-sign test

The difference sign test is a test for randomness check. It is a non-parametric test [14, 65] which is used to test the null hypothesis that the inputs are values of independent and identically distributed random variables. We count the number of values of i such that $y(i) > y(i-1)$ or $y(i-1) > y(i)$.

The test statistic can be given as [14]:

$$\begin{aligned} S &= |\{i : Y_i > Y_{i-1}| = |\{i : (\nabla Y)_i > 0\}| \qquad (2.4) \\ ES &= \frac{n-1}{2} \end{aligned}$$

It can be shown that $S \sim AN(n/2, n/12)$, where n is the length of the data set. Reject (at 5% level) the hypothesis that the series is $i.i.d.$ if

$$\begin{aligned} \left|S - \frac{n}{2}\right| &> 1.96\sqrt{\frac{n}{12}} \qquad (2.5) \\ or \frac{\left|S - \frac{n}{2}\right|}{\sqrt{\frac{n}{12}}} &> 1.96 \end{aligned}$$

For the input data, the test statistic was found 1.4781 which is less than 1.96. We fail to reject the null hypothesis. For the filtered data, the test statistic was found 42.3178 which is greater than 1.96. We are convinced to reject the null hypothesis, i.e., the filtered data is not $i.i.d.$

From different data based test results, we come to a conclusion that, the rank order filtered data is not $i.i.d.$ As a result, we can not apply pre-derived equations for false and missed alarm rate calculation.

2.5 Summary

In this chapter, basics of rank order filters have been discussed. The density functions for 1^{st} maximum, 1^{st} minimum and median of the window as outputs have been simplified and verified for different orders of the filter using numerical simulation and analytical expressions. Different input distributions for the input data have also been considered and verified. In the end, different data based methods have been applied to test whether the rank order filtered data is independently and identically distributed $(i.i.d.)$ or not. A conclusion is drawn based on these tests that rank order filtered data is not $i.i.d.$, which matches with statistical inference.

Chapter 3

Performance Evaluation of Rank Order Filters

3.1 Alarm Performance: FAR and MAR

In industry, the overall performance of alarm systems is evaluated by the average annunciated alarm rate per operator, peak alarm rates, alarm floods, chattering or fleeting alarms, and priority distributions [34]. However, in design level, individual alarm tags play significant roles as a few poorly configured alarm tags may degrade the overall performance. For this reason, a systematic approach to address individual tags is more important. If alarms are well-configured at the individual level, the overall performance of the alarm system improves significantly. In this chapter a bottom-up approach is followed to improve accuracy of alarm systems [1, 16].

In designing univariate alarm systems, the accuracy of the alarm system depends on two performance indices, namely, the False Alarm Rate (FAR) and the Missed Alarm Rate (MAR) [41]. These two quantities can be discussed from the perspective of a two-class classification problem. If P and N are number of positive and negative instances with outcomes p (positive) and n (negative) respectively, then there are four possible outcomes namely, true positive, false positive, false negative, and true negative. The outcomes are summarized by a 2×2 confusion matrix (also known as contingency table) shown in Figure 3.1 [1].

		Actual Value		Total
		p	*n*	
Predicted Outcome	*p′*	**True Positive**	**False Positive**	**P′**
	n′	**False Negative**	**True Negative**	**N′**
	Total	**P**	**N**	

Figure 3.1: Confusion matrix for a two-class classification problem

If both the predicted and the actual outcomes are true (p), then it is a true positive instance. On the other hand, if the actual one is false (n) and the predicted one is true, then it is a false positive instance. In alarm systems, the false positive is defined as the false alarm. So, a false alarm is an alarm that is raised when the process variable is behaving normally or under normal operating region. The false alarms may lead to losing the trustworthy of alarm systems due to the so-called "cry wolf" effect.

On the other hand, if the predicted outcome is n and the actual outcome is p then it is a false negative instance. In alarm systems, it is defined as the missed alarm. So, missed alarms occur when the process variable is behaving abnormally but no alarm is raised. The designed functionality of alarm systems is severely degraded due to missed alarms. As shown in the Figure 3.3(a), for a configured high alarm, in the normal region, any signal lying above the trip point will generate false alarms; and in the abnormal region, samples lying below the trip point will generate missed alarms.

One of the main goals of a well-designed alarm system is to reduce the *false alarm rate (FAR)* and the *missed alarm rate (MAR)*. The higher the accuracy, the lower the FAR and MAR are. The accuracy of alarm can be well-represented by the receiver operating characteristics (ROC) curve. This concept of ROC curve is adapted from signal detection theory and the axes are interchanged for better visualization in alarm systems [41]. An ROC curve is a plot of the false alarm rate vs the missed alarm rate as presented in Figure 3.2. If minimizing both false alarm rate and missed alarm rate is the only objective, then the point closest to the origin on the ROC curve will give the optimum trip point for which FAR and MAR will be the minimum. The shape of the ROC curve and optimum point varies depending on the alarm design method used and assigned weights to both FAR and MAR, respectively, in the objective function mentioned in Chapter 2.

In the subsequent sections, false and missed alarms are discussed in detail. Performance of rank order filters is compared with widely applied moving average filters to better understand the applicability of rank order filters. Moreover, several case studies have been presented considering different distributions of the data.

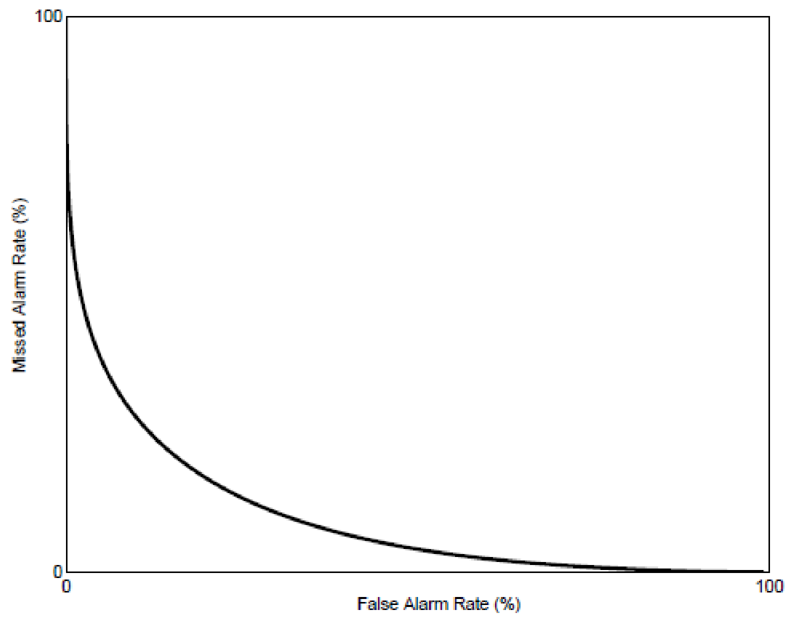
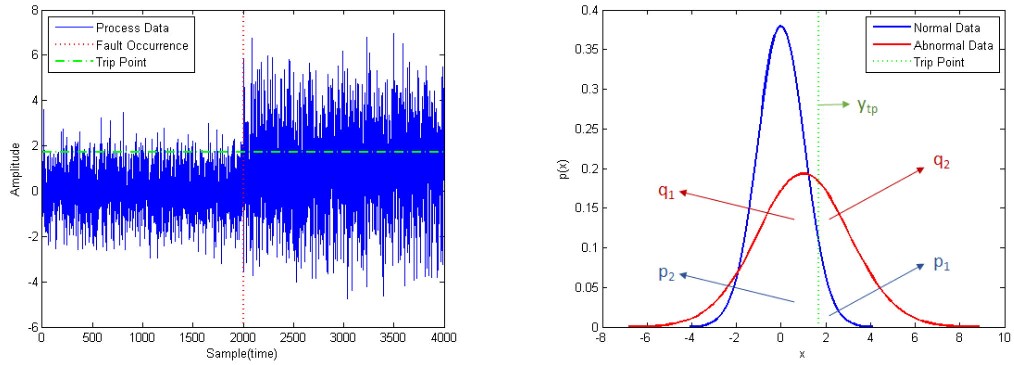


Figure 3.2: Receiver operating characteristic (ROC) curve

3.2 Problem Formulation

Let us assume that the PDFs of the process variable x in both normal and abnormal conditions are known a-priori with probability distribution function f_{X_n} and $f_{X_{ab}}$ respectively. For example, we consider that both the normal and abnormal segment of the process data are Gaussian distributed. Normal data has distribution $N(0,1)$ whereas abnormal data has distribution $N(1,2)$. Abnormality occurs in the sense of mean and variance change as depicted from the distribution properties. Figure 3.3 shows the distributions of the normal and abnormal regions along with the trip point y_{tp} and fault occurrence.

The FAR as the probability of false alarms can be computed [40] as the area under the normal PDF for the values of x greater than the trip point, y_{tp}. It is denoted by



(a) (b)

Figure 3.3: (a) Process data with normal and abnormal region, (b) PDF of the normal and abnormal region

p_1 on the Figure 3.3. Mathematically,

$$FAR = \int_{y_{tp}}^{+\infty} f_{X_n}(x)dx \tag{3.1}$$

Likewise, the MAR as the probability of missed alarms can be computed [40] as the area under abnormal PDF from $-\infty$ to y_{tp}. This is denoted as q_1 in Figure 3.3. Mathematically,

$$MAR = \int_{-\infty}^{y_{tp}} f_{X_{ab}}(x)dx \tag{3.2}$$

Rank order filter are non-linear filters which have already been implemented widely for image processing [58]. If the PDFs of the filtered data for normal and abnormal data are known, then the equations above can be utilized to find the FAR and MAR directly. Let the PDF of the rank order filtered data be denoted as f_{Y_n} and $f_{Y_{ab}}$ under normal and abnormal conditions respectively. Then from Eqn.(3.1) and Eqn.(3.2) it can be extended as:

$$\begin{aligned} FAR &= \int_{y_{tp}}^{+\infty} f_{Y_n}(x)dx \\ &= \int_{y_{tp}}^{+\infty} \frac{N!}{(i-1)!(N-i)!} F_{X_n}^{(i-1)}(1-F_{X_n})^{(N-i)} f_{X_n} dx \end{aligned} \tag{3.3}$$

$$\begin{aligned} MAR &= \int_{-\infty}^{y_{tp}} f_{Y_{ab}}(x)dx \\ &= \int_{-\infty}^{y_{tp}} \frac{N!}{(i-1)!(N-i)!} F_{X_{ab}}^{(i-1)}(1-F_{X_{ab}})^{(N-i)} f_{X_{ab}} dx \end{aligned} \tag{3.4}$$

From the definition of rank order filters, we found that there are two parameters N and i. If we take the 1^{st} maximum value as output, we can simplify Eqn.(3.3) and Eqn.(3.4) by putting $i = N$. Then the simplified form of FAR and MAR can be presented as:

$$\begin{aligned} FAR &= \int_{y_{tp}}^{+\infty} N F_{X_n}^{(N-1)} f_{X_n} dx \\ &= N \int_{y_{tp}}^{+\infty} F_{X_n}^{(N-1)} f_{X_n} dx \end{aligned} \tag{3.5}$$

$$\begin{aligned} MAR &= \int_{-\infty}^{y_{tp}} N F_{X_{ab}}^{(N-1)} f_{X_{ab}} dx \\ &= N \int_{-\infty}^{y_{tp}} F_{X_{ab}}^{(N-1)} f_{X_{ab}} dx \end{aligned} \tag{3.6}$$

where f_{X_n} and F_{X_n} denote the PDF and CDF of the normal data respectively, and $f_{X_{ab}}$ and $F_{X_{ab}}$ represent the PDF and CDF of the abnormal data respectively. There are no closed form solutions for Eqn.(3.5) and Eqn.(3.6). Rather these equations are solved numerically.

3.3 Optimal Trip Point Design

Assume PDF (CDF) of normal data is $f_{Xn}(x)(F_{Xn}(x))$, PDF (CDF) of abnormal data is $f_{Xab}(x)(F_{Xab}(x))$; and PDF (CDF) of the filtered normal data is $f_{Yn}(x)(F_{Yn}(x))$, and PDF (CDF) of the filtered abnormal data is $f_{Yab}(x)(F_{Yab}(x))$. Then the objective function for quantifying alarm accuracy can be chosen as a weighted sum of FAR and MAR:

$$\begin{aligned} \min J(x) &= c_1 \int_x^{+\infty} f_{Yn}(x)dx + c_2 \int_{-\infty}^{x} f_{Yab}(x)dx \\ &= c_1(1 - F_{Yn}(x)) + c_2 F_{Yab}(x) \end{aligned} \tag{3.7}$$

where c_1 and c_2 are two positive weights. To find the optimal trip point, let

$$\frac{\partial J}{\partial x} = -c_1 f_{Yn}(x) + c_2 f_{Yab}(x) = 0 \tag{3.8}$$

So, the optimal trip point is such that

$$\frac{f_{Yab}(x)}{f_{Yn}(x)} = \frac{c_1}{c_2} \tag{3.9}$$

For rank order with window size N and rank i, PDF equations for both normal and abnormal data are given below:

$$\begin{aligned} f_{Yn}(x) &= \frac{N!}{(i-1)!(N-i)!} F_{Xn}(x)^{i-1}(1 - F_{Xn}(x))^{N-i} f_{Xn}(x) \\ f_{Yab}(x) &= \frac{N!}{(i-1)!(N-i)!} F_{Xab}(x)^{i-1}(1 - F_{Xab}(x))^{N-i} f_{Xab}(x) \end{aligned} \tag{3.10}$$

If equal weights are assigned to both FAR and MAR, then $c_1 = c_2$. The optimal trip point that would give minimum J can be found by solving following equation:

$$\frac{f_{Yab}(x)}{f_{Yn}(x)} = 1 \tag{3.11}$$

Assigning equal weights means choosing the closest point to the origin on the ROC curve [17]. Eqn.(3.11) is then solved numerically. The results obtained from numerical solutions for different filter orders with maximum output is given in Table 3.1.

Filter Order	Trip Point
3	1.679
4	1.950
5	2.038
6	2.109
7	2.167
8	2.217

Table 3.1: Trip point for different filter lengths

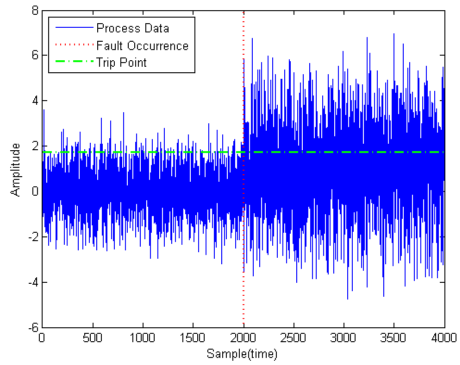
3.4 Performance Comparison of Rank Order Filters

Rank order filters are the simplest non-linear filters. Though industries demand simple and easily implementable filters like moving average filters, non-linear filters like rank order filters in the sense of performance are also worth studying. In this section the accuracy performance of the rank order filter with i^{th} maximum as output is compared with the moving average filter and general optimal filter of the same order.

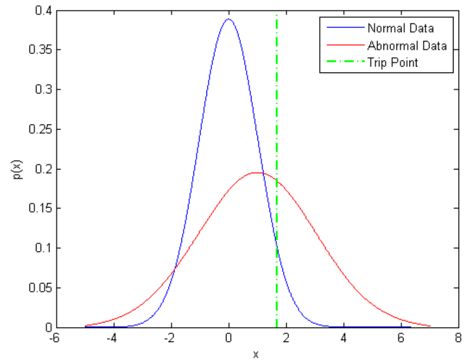
ROC Curves

As mentioned earlier, the concept of ROC curve is taken from signal detection theory to visualize the hit and miss rates. The same thing is modified in alarm systems to evaluate the performance of alarm systems [40]. The ROC curve, which is plot of false alarm rate vs missed alarm rate, is a performance evaluation method when filtering process data. The closer the curve is to the origin, the better the performance is. False alarms are generated in the normal segment of the data, whereas missed alarms are generated in the abnormal region of the data. Since there is no analytical forms of FAR and MAR in terms of filter parameters, false and missed alarm rates are calculated by varying the trip point and counting the numbers of points above and below that trip point, then dividing by the total lengths of the normal and abnormal data respectively. Then for each trip point FAR and MAR are plotted which forms the ROC curve after connecting those points.

In this section, rank order filters are applied on both the normal and abnormal segments of the process data shown in Figure 3.3(a). Then numerical simulation is run to obtain ROC curves for different orders, N, and the 1^{st} maximum value in the window. One of the output distributions for the filter with order $N = 8, i = 8$ is shown in Figure 3.4(d) which clearly indicates the reduction in MAR by comparing it with unfiltered PDF in Figure 3.4(b).



(a)



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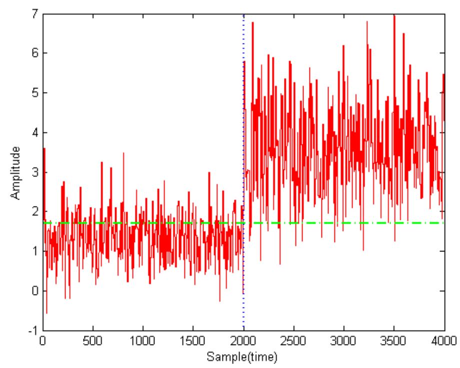
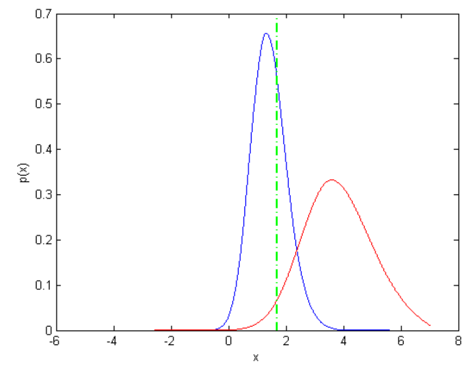


(c)



(d)

Figure 3.4: (a) Unfiltered data, (b) unfiltered data PDF, (c) rank order filtered data with N=8, i=8, (d) rank order filtered PDF with N=8, i=8

By changing the filter order, we change the output distribution. As a result, for the same trip point we may end up with different FAR and MAR from the unfiltered case. From the Figure 3.5 it is visible that with the increment of the filter order, the ROC curves are shifting toward the origin, i.e., less FAR and MAR are achieved. On the other hand, for a fixed filter order, e.g., $N = 7$, with different maximums as output, e.g., $i = 1, 2, ..., 7$, ROC curves have been plotted in Figure 3.6. The figure shows that with $i = 7$ (1^{st} maximum), the ROC curve is closer to origin compared to $i = 1$ (1^{st} minimum). The downside of increasing filter order is the increment of detection delay which is discussed in Chapter 4. As a result, we can not keep increasing filter order, N to achieve lower FAR and MAR.

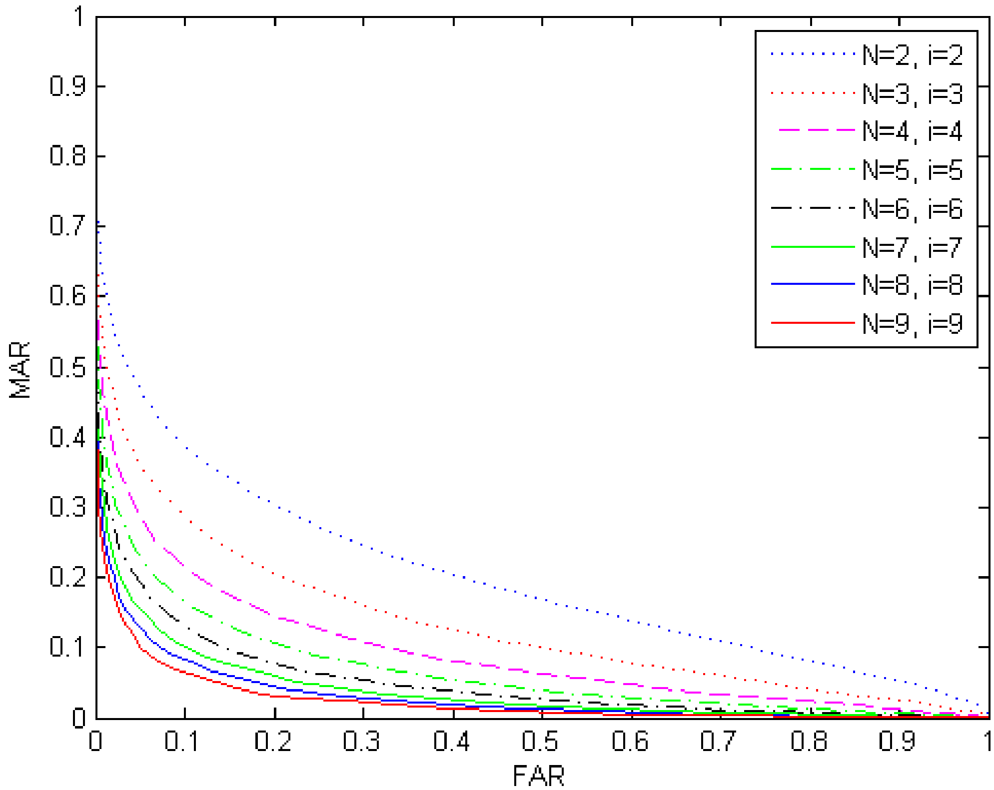


Figure 3.5: ROC curves for different orders of the rank order filter

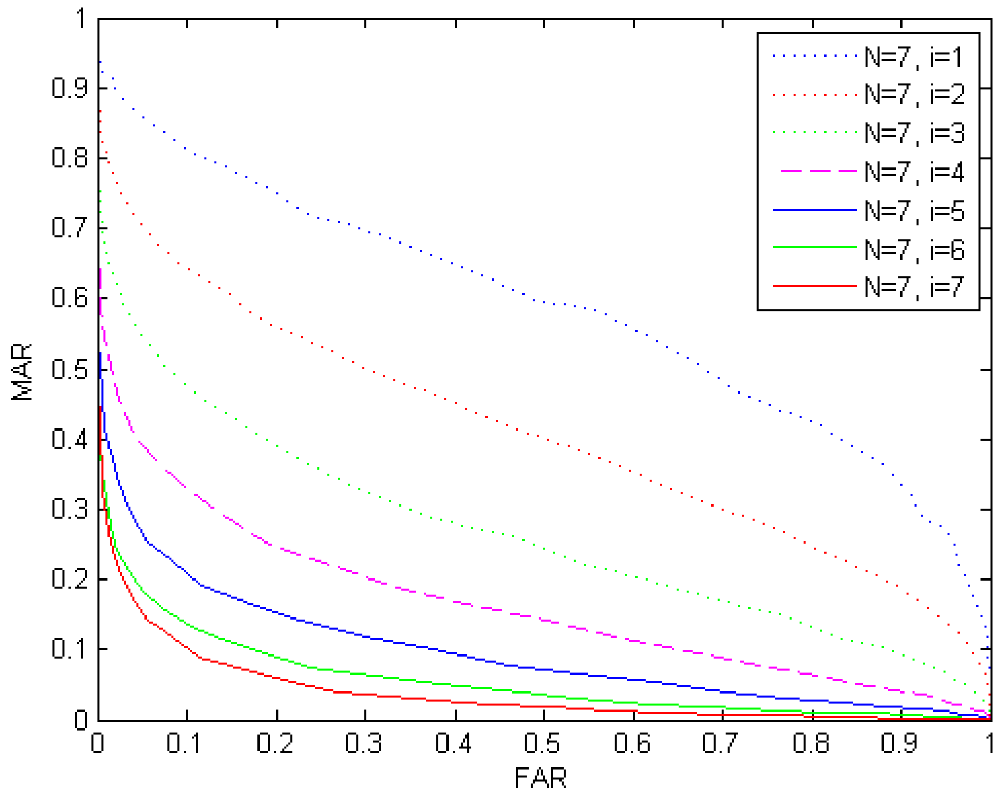


Figure 3.6: ROC curves for the rank order filter with N=7 and different outputs

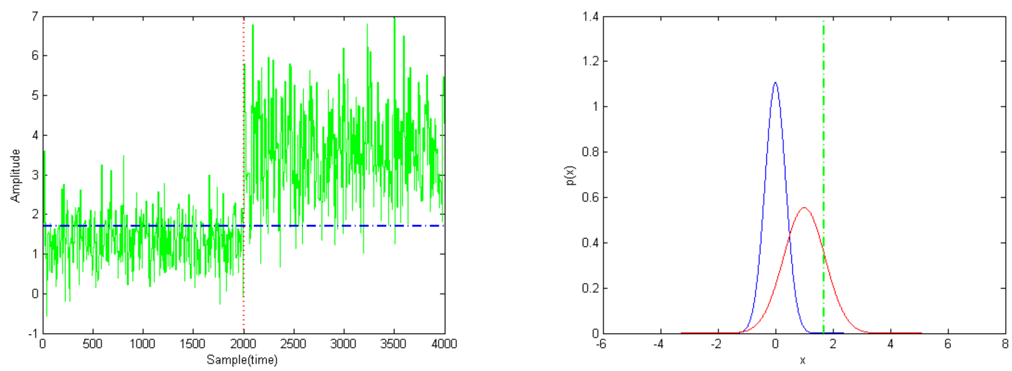
3.4.1 Comparison with moving average filters

A moving average filter is a causal linear filter which takes an average of the previous m samples defined by the filter order m. It has only one design parameter, namely, the filter order m. As mentioned in [17], under some conditions, moving average filters are the optimal filters among all FIR filters. Moreover, it has got wide industry application due to its simplicity and ease of application. This is why it is important to compare the performance of rank order filters with moving average filters.

A moving average filter can be represented mathematically as [4]:

$$Y_i = \frac{1}{m}(X_{i-m+1} + \cdots + X_{i-1} + X_i) \tag{3.12}$$

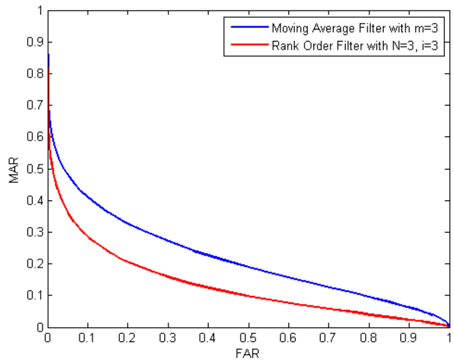
where m is the order of the filter and X is the unfiltered process data.



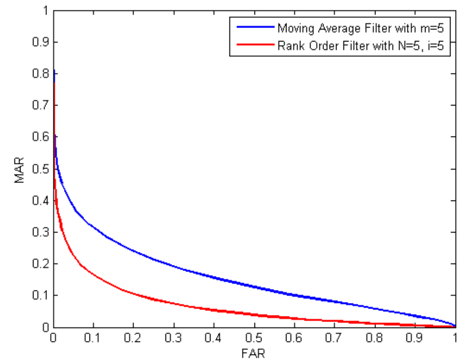
(a) (b)

Figure 3.7: (a) Moving average filtered data with m=8, (b) moving average filtered PDF with m=8

To understand the performance of rank order filters better, a comparison in terms of ROC curves has been presented in this section. If the ROC curve of the rank order filter of the same order as the moving average filter lies below the ROC curve of the moving average filter, it can be said that the rank order filter is performing better. The normal and abnormal segments of the input data are considered the same as before in Figure 3.3. Moving average filtered data for $m = 8$ and the distribution of the filtered data are shown in Figure 3.7(a) and Figure 3.7(b), respectively. By observation, we can directly say that for the same trip point as unfiltered case in Figure 3.4(b), FAR has improved significantly but MAR is still high.



(a)



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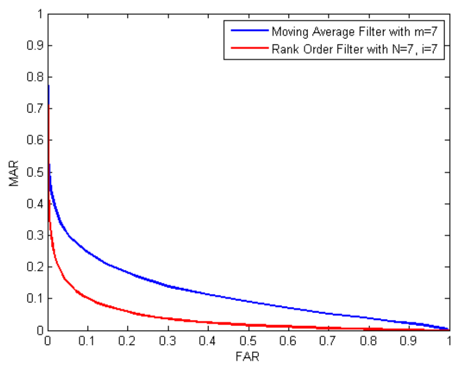

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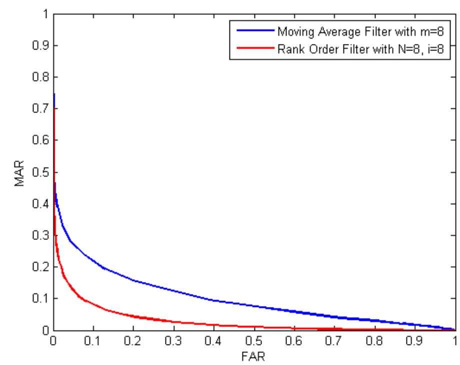


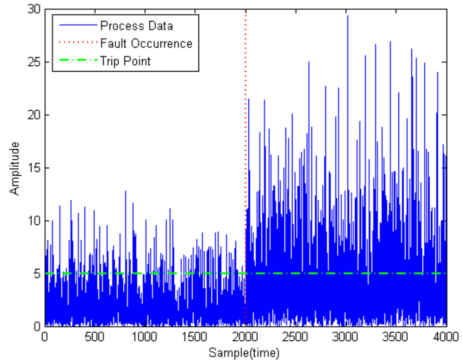
(d)

Figure 3.8: (a) ROC curves for rank order filter (N=3 and i=3) and moving average filter(m=3), (b) ROC curves for rank order filter (N=5 and i=5) and moving average filter(m=5), (c) ROC curves for rank order filter (N=7 and i=7) and moving average filter(m=7), (d) ROC curves for rank order filter (N=8 and i=8) and moving average filter(m=8)

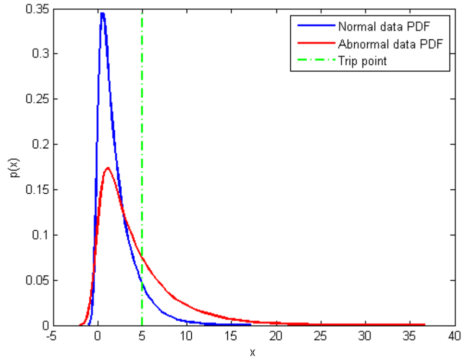
By varying the trip point, y_{tp}, from the minimum to the maximum of the process data for both the rank order and moving average filtered data and running 1000 simulations for each trip point, we came across these ROC curves. In Figure 3.8, we compared ROC curves for rank order filters with $N = 3, 5, 7, 8$ and $i = N$ and moving average filters with $m = 3, 5, 7, 8$, where we can see that the ROC curve for the rank order filter is lying below than the ROC curve of the moving average filter. This is indicating better alarm accuracy, i.e., lower FAR and MAR for rank order filters. We can come to a conclusion that, for Gaussian distributed data where abnormality occurs in the sense of mean and variance changes, rank order filters with 1^{st} maximum as output have shown better performance compared to moving average filters of the same order.

3.4.2 Performance comparison with exponential distribution

The input data may not always follow the Gaussian distribution. If the distribution of the input data is changed, we may derive some important conditions about the applicability of rank order filters. For example, if the normal and abnormal parts are assumed to be exponentially distributed with mean 2 and 4 respectively, the PDF of the input data, rank order filtered data and moving average filtered data can be easily estimated by the kernel method. In Figure 3.9(b), the PDF of the exponentially distributed data is shown. The change in output PDF is achieved by applying the rank order filter of order $N = 7, i = 7$ and moving average filter of order $m = 7$ in Figures 3.9(d) and 3.9(f), respectively. But the performance of the rank order filter is more visible when ROC curves are plotted and compared with the moving average filter.



(a)



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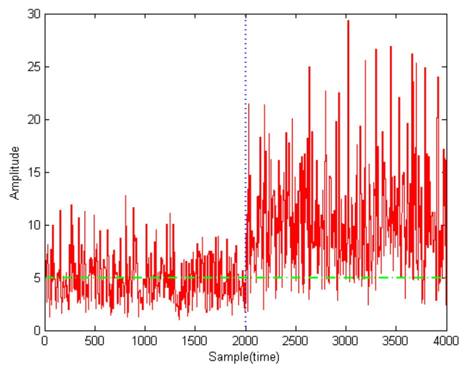

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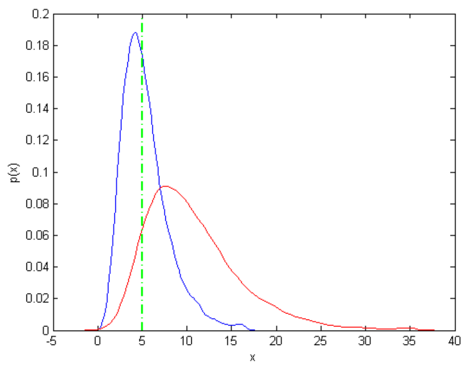

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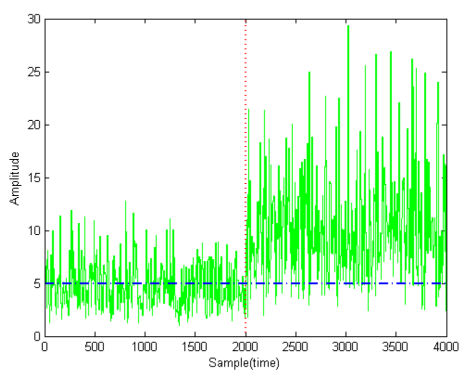
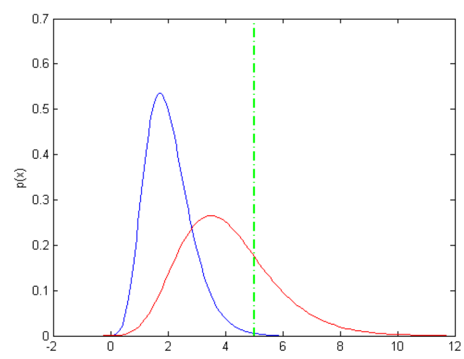
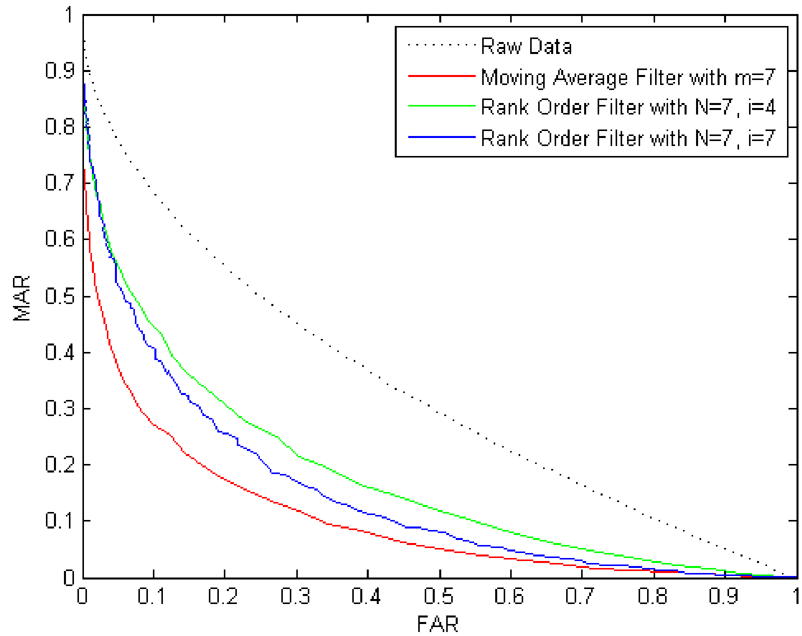


(e)



(f)

Figure 3.9: (a) Unfiltered data, (b) unfiltered data PDF, (c) rank order filtered data with N=7, i=7, (d) rank order filtered PDF with N=7, i=7, (e) moving average filtered data with m=7, (f) moving average filtered PDF with m=7



(a)

Figure 3.10: ROC curve comparison among different filters of same order on exponential distributed data

In Figure 3.10 ROC curves for rank order and moving average filters for exponentially distributed input data are compared. It is visible that the ROC curve for moving average filter is more closer to the origin compared to the ROC curves of rank order filters of the same order with different outputs.

3.4.3 Comparison with general optimal filters

The optimization problem of reducing false and missed alarm rates while varying the trip point is a classic classification problem. This optimal classification can be obtained by log-likelihood ratio (LLR) filters as discussed in [17]. The general optimal

LLR filter is as follows:

$$y[k] = \sum_{i=k-N+1}^{k} \ln\frac{p_{ab}(x[i])}{p_n(x[i])} \quad (3.13)$$
$$y_{tp} = \ln(\frac{c_1}{c_2})$$

where p_n and p_{ab} are the normal and abnormal PDFs of the input data.

Exapmple#1: If normal and abnormal data follow Gaussian distribution with $N(0,1)$ and $N(\mu,\sigma^2)$ respectively, then according to Eqn.(3.14), the optimal filter is obtained as follows [17]:

$$y[k] = \sum_{i=k-N+1}^{k} \left(\frac{\sigma^2 - 1}{2\sigma^2} x[i]^2 + \frac{\mu}{\sigma^2} x[i] - \left(\frac{\mu^2}{2\sigma^2} + \ln\sigma \right) \right) \quad (3.14)$$
$$y_{tp} = \ln(\frac{c_1}{c_2})$$

Now, for the the same parameters $\mu = 1$ and $\sigma = 2$, the application of general optimal filters reduces the overlapping of the normal and abnormal region of the PDFs. The performance is compared with the same order rank order filter and moving average filter in Figures 3.11.

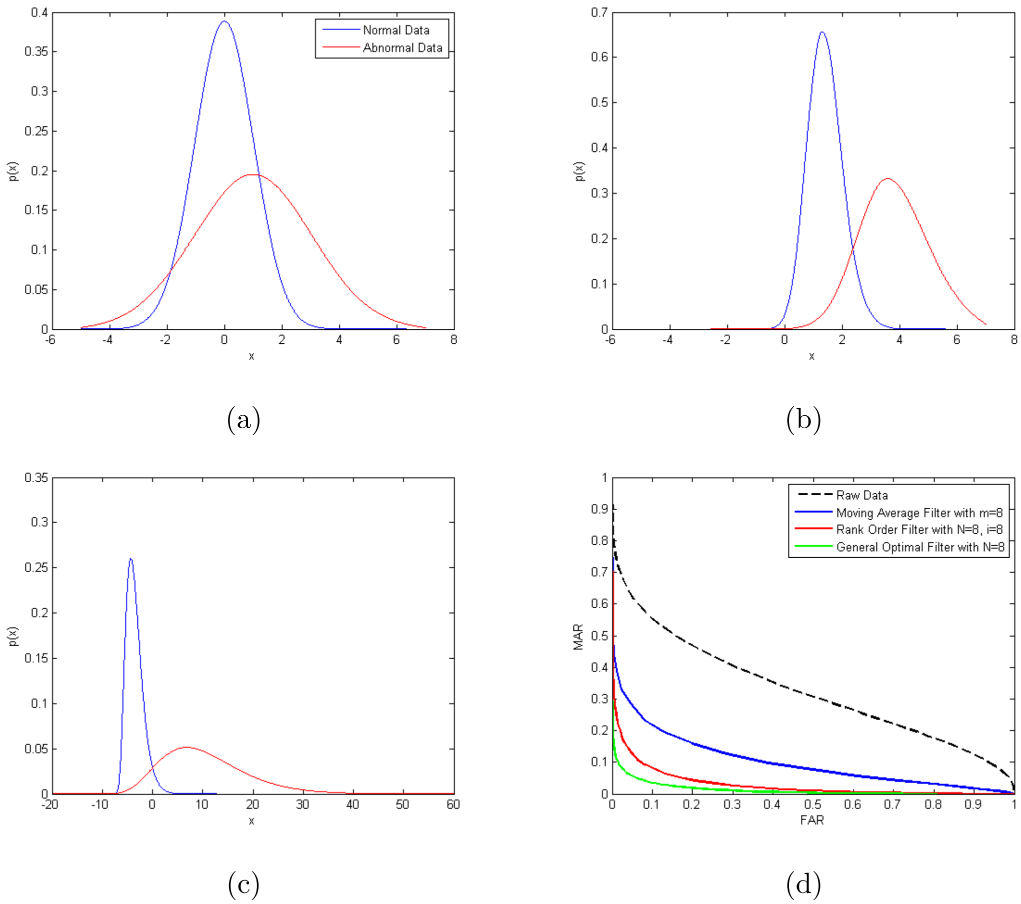


Figure 3.11: (a) Unfiltered data PDF, (b) rank order filtered data PDF with N=8, i=8, (c) general optimal filtered data PDF with N=8, (d) ROC curve comparison with general optimal filter

From the ROC curves, the performance of filters can be easily visualized. Application of filters bring the ROC curves close to the ideal point (0, 0). The rank order filter performs in between general optimal filter and moving average filter for the same distribution. One advantage of using rank order filters over general optimal filters is that the rank order filtered data variance does not change significantly, whereas general optimal filtered data has wide variance. In cases where wide variance of the data can cause other problems, rank order filters can be easily used for achieving better performance.

Example#2: Let, the normal and abnormal data follow logistic distributions respectively as follows [17]:

$$\begin{aligned} p_n(x) &= \frac{e^{-x}}{(1+e^{-x})^2} \\ p_{ab}(x) &= \frac{e^{-(x-2)/2}}{2(1+e^{-(x-2)/2})^2} \end{aligned} \tag{3.15}$$

By putting these equations in Eqn.(3.14), we get the general optimal filter for the distribution:

$$y[k] = \sum_{i=k-N+1}^{k} \left(0.5x[i] + 2\ln\frac{1+e^{-x[i]}}{1+e^{-(x[i]-2)/2}} + \ln 2 + 1\right) \tag{3.16}$$

with the trip point $y_{tp} = \ln(\frac{c_1}{c_2})$

In Figure 3.12(a)-(c), the PDFs of the unfiltered data, general optimal data and rank order filtered data are shown. The performance of the rank order filter with $N = 8, i = 8$ is compared with the general optimal filter and moving average filter of the same order in terms of ROC curves as visible from Figure 3.12 (d). In this case, the performance of the rank order filter lies in between the moving average filter and general optimal filter as well.

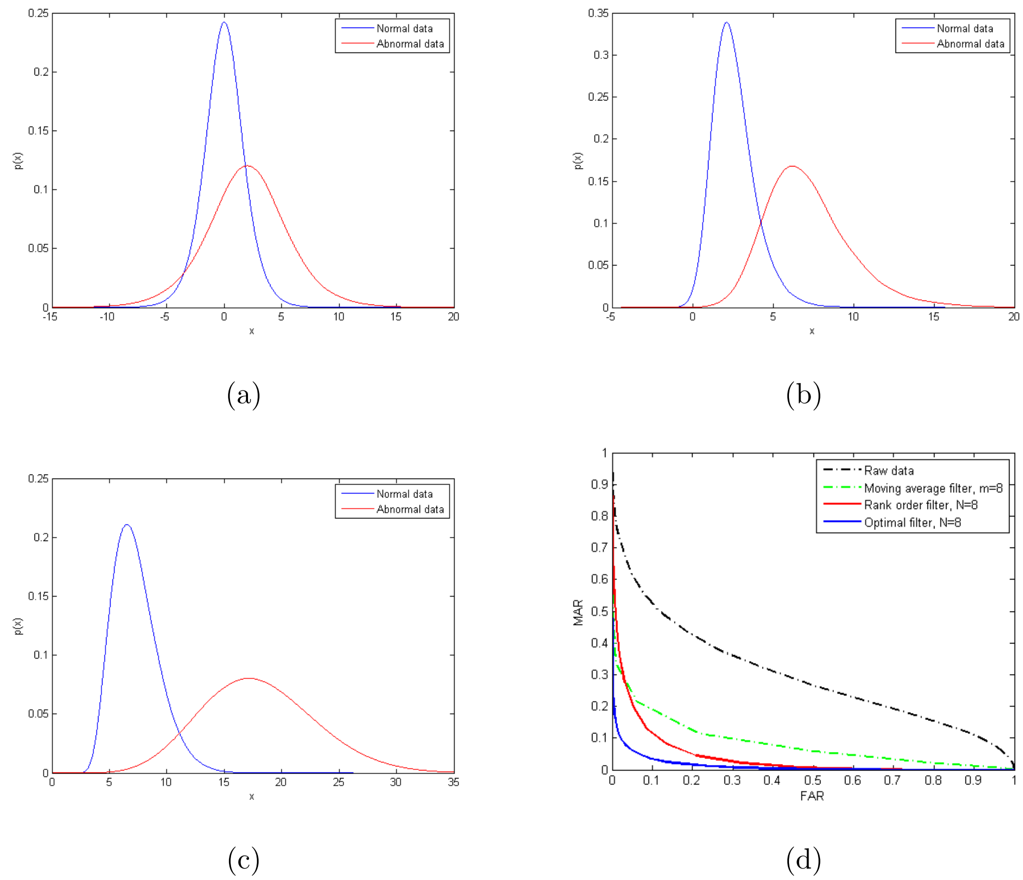
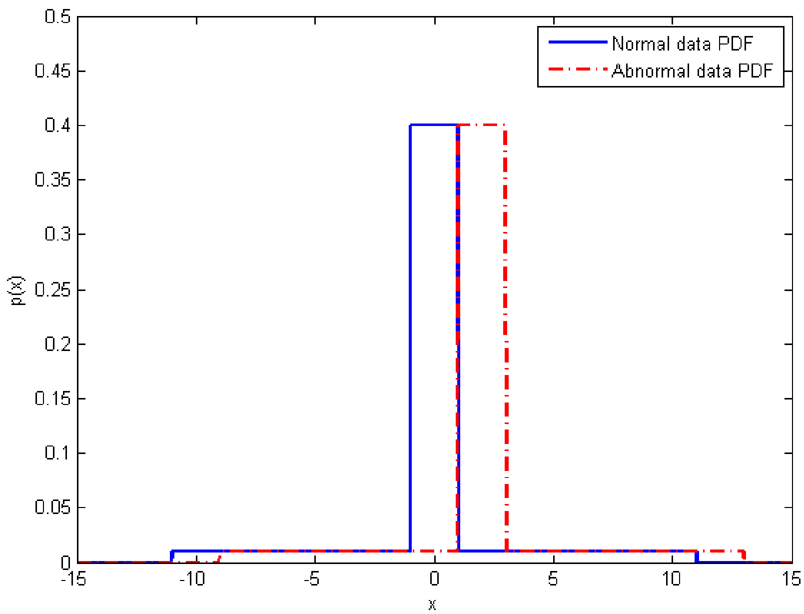


Figure 3.12: (a) Unfiltered data PDF, (b) rank order filtered data PDF with N=8, i=8 (c) general optimal filtered data PDF with N=8, (d) ROC curve comparison with general optimal filter

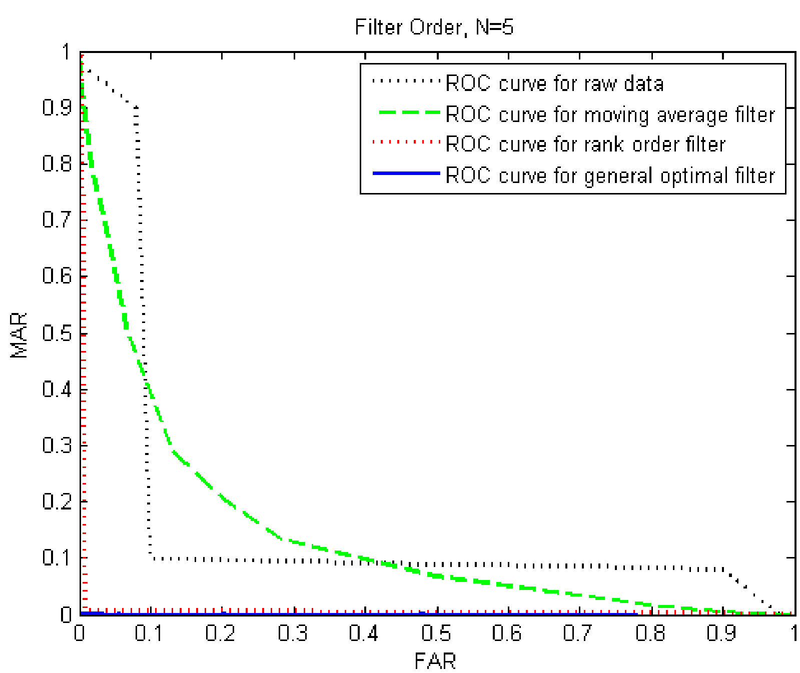
Example#3: If we assume discrete distributions for the normal and abnormal data like:

$$
\begin{aligned}
p_n(x) &= \begin{cases} 0.01 & x \in (-11,-1) \cup (1,11) \\ 0.40 & x \in [-1,1] \\ 0 \ \text{elsewhere} \end{cases} \\
p_{ab}(x) &= \begin{cases} 0.01 & x \in (-9,1) \cup (3,13) \\ 0.40 & x \in [1,3] \\ 0 \ \text{elsewhere} \end{cases}
\end{aligned} \tag{3.17}
$$

After applying the optimal filter from Eqn.(3.14), the ROC curves are compared in Figure 3.13(b). Here we considered the filter order $N = 5$. In Figure 3.13(b), it is visible that the ROC curve obtained using raw data is performing better than the moving average filter for some portions, but worse in other portions. On the other hand, applying a rank order filter allowed us to achieve performance curve closer to the optimal ROC curve. The ROC curve using the general optimal filter can almost reach the origin (0,0) while the rank order filter is performing in between the general optimal and moving average filter.



(a)



(b)

Figure 3.13: (a) Unfiltered data PDF, (b) ROC curve comparison with general optimal filter

Conditions for better performance

Through simulations, rank order filters perform better than moving average filters under these conditions:

- if both normal and abnormal data are Gaussian distributed,
- if the means of the normal data and abnormal data are close.

3.5 Summary

False and missed alarm rates are two important parameters for accurate alarm design. Even though we could not achieve any analytical solution for FAR and MAR for rank order filters, we have made justification by running numerical simulations. The optimal filter design method is presented as an optimization problem. In this chapter, the performance of rank order filters is visualized by comparing the ROC curve with moving average filters considering both Gaussian and non-Gaussian distributions. Moreover, examples have been shown to prove that the performance of the rank order filter lies between the general optimal filter and moving average filter (optimal linear filter for Gaussian distributions). The limitation of applying rank order filters are, they change the mean of the distribution. In cases where the mean of the distribution can not be changed, rank order filters can not be applied. In the end, based on our observation of ROC curves and data distributions, we have come up with some conditions under which rank order filters with the maximum value as output perform better. Based on these conditions we can apply rank order filters specially in those process tags where these conditions are visible.

Chapter 4

Expected Detection Delay

4.1 Detection Delay

One of the main goals of a reliable alarm system is that it should raise the alarm instantly at the moment a fault occurs and notify the operators. In reality, this is not the case. The activation of the alarm may be delayed due to network delays, bad implementation, hardware malfunction, sensor failure, data loss, etc. [1]. In addition to those, processing process and alarm data (filtering, delay timer, or deadband) may cause delay in alarm activation [5,6]. Detection delay is the difference in time/sample between the instant an actual fault occurs and the instant an alarm is raised. If a process variable moves from normal operating region to faulty region of operation at the instant t_f and alarm is raised at the instant t_a, then the detection delay (DD) is given by

$$DD = t_a - t_f$$

Due to different delays it is hardly seen that a fault is detected instantly at the time it occurs. Practically, the problem is to detect the occurrence of the change as soon as possible [12]. To improve the effectiveness of the limit checking/comparison method, several techniques like delay-timers, deadbands, and filters are widely used in alarm systems; even though these increase detection delay. However, there has been very limited study on the performance evaluation of different filtering methods. The analytic relationships between filter parameters and performance indices are not well established [67].

An interesting observation is that, even if no delay-timer, deadband or filter is used, the position of alarm trip point or threshold limit may cause some detection delays. In the next section, an example is presented to show the dependency of detection delay on the alarm threshold configuration.

In process industry, discrete time random variables are monitored to raise or clear an alarm. For any process tag, suppose an independent random variable $\{X_i\}_{i\geq 1}$ is collected. We assume that random variables $X_1, X_2, X_3, \ldots, X_{tf-1}$ are distributed according to probability distribution function (PDF) p_n known a-priori. Also, random variables $X_{tf}, X_{tf+1}, X_{tf+2}, \ldots$ belong to a PDF, p_{ab}. Here, p_n and p_{ab} denotes the PDF of X_i under normal and abnormal operating conditions respectively. Depending on alarm generation techniques, process variables X_i's are processed and compared with pre-set alarm trip point.

Under the normal operating region, let the probability of one sample exceeding the trip point (y_{tp}) be p_1 and the probability of one sample falling within the trip point is p_2. Similarly under the abnormal operating region, the probability of one sample falling within trip point is q_1 and, q_2 is the probability of one sample lying above the trip point. Therefore, $p_2 = 1 - p_1$ and $q_2 = 1 - q_1$ as visible from the Figure 4.1 (b).

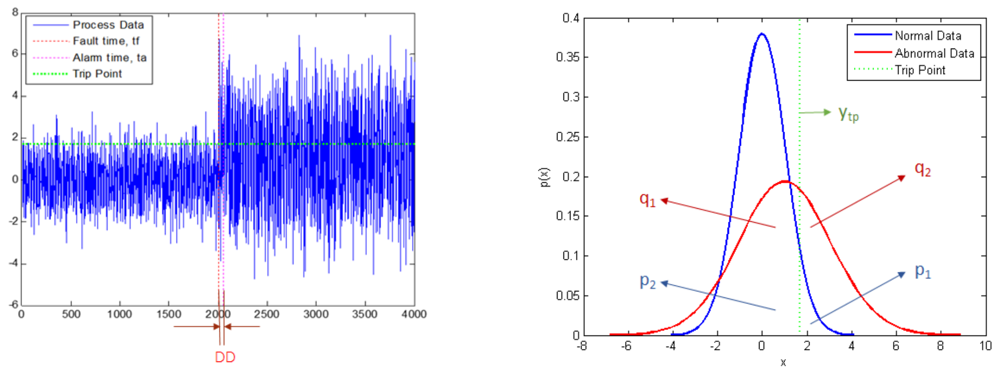


Figure 4.1: Process data with trip point and occurrence instance (left); corresponding distributions of the normal and abnormal data with the same trip point(right)

Detection delay being zero indicates that the alarm is raised instantly at the time fault occurs. Detection delay of one sample means the alarm is raised after one sample of fault occurrence. Likewise, n sample delay means the activation of the alarms is delayed by n samples from the fault instance. If we assume, fault arrived at time $t = t_f$, the probability of zero detection delay is the probability of raising an alarm at the ${t_f}^{th}$ instant.

$$\begin{aligned}
\mathbb{P}(\text{DD}=0) &= \mathbb{P}(Alarm \quad at \quad time \quad t=t_f)=q_2 \\
\mathbb{P}(\text{DD}=1) &= \mathbb{P}(Alarm \quad at \quad time \quad t=t_f+1 \quad \& \quad no \quad alarm \quad at \\
&\quad t=t_f)=q_2q_1 \\
&\vdots \\
\mathbb{P}(\text{DD}=n) &= \mathbb{P}(Alarm \quad at \quad time \quad t=t_f+n \quad \& \quad no \quad alarm \quad at \\
&\quad t=t_f+n-1\ldots\& \quad no \quad alarm \quad at \quad t=t_f+1 \\
&\quad \& \quad no \quad alarm \quad at \quad t=t_f) \\
&= q_2{q_1}^n \qquad (4.1)
\end{aligned}$$

Though detection delay has been represented in terms of samples, it can easily be converted to time since sampling time is constant. From the above equations it is also visible that the probability of detection delay is solely dependent on the distribution of abnormal data. Normal data distribution has no effect in activating/ deactivating alarms. As it is a discrete event, the expected value of detection delay is expressed in terms of summation as

$$\text{EDD}=\mathbb{E}(\text{DD})=\sum_{n=0}^{\infty}(n\mathbb{P}(\text{DD}))=\sum_{n=0}^{\infty}nq_2{q_1}^n=q_1/q_2 \qquad (4.2)$$

Given the distribution of abnormal data, the probabilities q_1 and q_2 are the areas under the abnormal distribution depending on the position of the trip point (y_{tp}). So without pre-processing the data, the expected detection delay is dependent on the position of the trip point. Changing the trip point will result in a different detection delays. For example, if the normal data is Gaussian distributed with mean 0 and variance 1 and abnormal data is Gaussian distributed with mean 1 and variance 2, then setting different trip points (y_{tp}) result in different detection delays as shown in Figure 4.2.

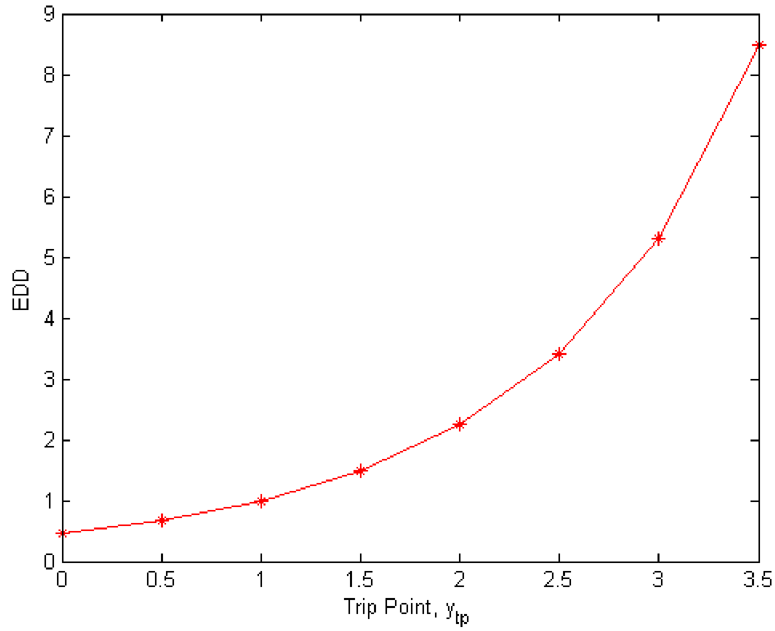


Figure 4.2: Effect of trip point on expected detection delay

Swiftness of alarm activation is an important performance evaluation parameter measured by expected detection delay as it indicates the average time it takes to raise an alarm when fault occurs. To design an effective alarm system, it is always desirable to reduce expected detection delay and keep it within a minimum value. Step by step procedure for alarm system design with the performance requirements will be presented in Chapter 5.

4.2 Detection Delay for Rank Order Filters

In this section, detection delay for rank order filters with i^{th} maximum as output has been formulated. Considering same probabilities for normal and abnormal distributions mentioned in Section 4.1, we will try to calculate the detection delay for the rank order filter of order 2 , 1^{st} maximum ($i = N$) and then discuss the general case for filter length N, rank i. Assume that the fault occurs at time t_f. For the rank order filter of order 2, the probability of alarm being raised at time $(t_f + 1)$ (i.e., detection delay (DD) equals one) is given by:

$$
\begin{aligned}
\mathbb{P}(\mathrm{DD}=1) &= \mathbb{P}\{\max[X_{tf-1}, X_{tf}] \leq y_{tp} \quad \& \quad \max[X_{tf}, X_{tf+1}] > y_{tp}\} \\
&= \mathbb{P}\{X_{tf-1} \leq y_{tp}\} \quad \& \quad \mathbb{P}\{X_{tf} \leq y_{tp}\} \quad \& \quad \mathbb{P}\{X_{tf+1} \geq y_{tp}\} \\
&= p_2 q_1 q_2 \qquad (4.3) \\
\mathbb{P}(\mathrm{DD}=2) &= \mathbb{P}\{\max[X_{tf-1}, X_{tf}] \leq y_{tp} \quad \& \quad \max[X_{tf}, X_{tf+1}] \leq y_{tp} \quad \& \\
&\quad \max[X_{tf+1}, X_{tf+2}] > y_{tp}\} \\
&= \mathbb{P}\{X_{tf-1} \leq y_{tp}\} \quad \& \quad \mathbb{P}\{X_{tf} \leq y_{tp}\} \quad \& \quad \mathbb{P}\{X_{tf+1} \leq y_{tp}\} \quad \& \\
&\quad \mathbb{P}\{X_{tf+2} > y_{tp}\} \\
&= p_2 . q_1 . q_1 . q_2 \\
&= p_2 {q_1}^2 q_2 \qquad (4.4) \\
&\vdots \\
\mathbb{P}(\mathrm{DD}=k) &= p_2 {q_1}^k q_2 \qquad (4.5)
\end{aligned}
$$

Likewise, for the rank order filter of order N, an alarm being raised at time (t_f+k),

k-sample detection delay can be formulated as:

$$\mathbb{P}(\mathrm{DD} = k) = {p_2}^{N-1}{q_1}^k q_2 \tag{4.6}$$

The expected detection delay can be calculated as:

$$\begin{aligned}
\mathrm{EDD} &= \sum_{k=0}^{\infty}(k\mathbb{P}(\mathrm{DD})) \\
&= \sum_{k=1}^{\infty}(k{p_2}^{N-1}{q_1}^k q_2) \\
&= {p_2}^{N-1}q_2\sum_{k=1}^{\infty}(k{q_1}^k) \\
&= {p_2}^{N-1}q_2\frac{q_1}{(1-q_1)^2} \\
&= {p_2}^{N-1}\frac{q_1}{q_2}
\end{aligned} \tag{4.7}$$

General case

The analytical relation developed earlier is extended here for other ranks. Due to insufficient conditions, we try to calculate the probability of k sample detection delay in an iterative way. Let us consider the general case with window size N and rank i.

Let $Q(k)$ denote the probability that there is no alarm (NA) up to time $(t_f + k)$ (i.e., sample k). Suppose we know the probability $Q(k-1)$ at $k-1^{th}$ instant, then at sample k, an iteration can be found for $Q(k)$ as

$$Q(k) = \alpha_1 Q(k-1) + \alpha_2 Q(k-2) + \cdots + \alpha_N Q(k-N) \tag{4.8}$$

where α_i's are to be determined below.

Let

$$x(k) = \begin{bmatrix} Q(k) \\ Q(k-1) \\ \vdots \\ Q(k-N+1) \end{bmatrix} \tag{4.9}$$

then

$$x(k) = Ax(k-1), (k \geq N) \tag{4.10}$$

where

$$A = \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_{N-1} & \alpha_N \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \tag{4.11}$$

with initial condition

$$x(N) = \begin{bmatrix} Q(N) \\ Q(N-1) \\ \vdots \\ Q(1) \end{bmatrix}$$

Let $P(k)$ denote the probability that the delay is exactly of sample k for the rank order filter, i.e.,

$$P(k) = \mathbb{P}(\mathrm{DD} = k)$$

then it is easy to show that

$$\begin{aligned} P(k) &= Q(k-1) - Q(k) \\ &= Cx(k) \\ &= CA^{k-N}x(N) \end{aligned} \tag{4.12}$$

where

$$C = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \end{bmatrix}$$

thus the expected delay (EDD) will be

$$\begin{aligned} \text{EDD} &= \sum_{k=0}^{\infty} k\mathbb{P}(\text{DD} = k) \\ &= \sum_{k=0}^{N-1} k(Q(k-1) - Q(k)) + \sum_{k=N}^{\infty} kCA^{k-N}x(N) \\ &= \sum_{k=0}^{N-1} Q(k) + CA(I-A)^{-2}x(N) \end{aligned} \tag{4.13}$$

The problem to compute EDD for rank order filters thus reduces to finding the probability ($Q(k)$) of NA at k by iteration, and the initial probabilities at the first $N-1$ samples.

4.2.1 Case $i = N$

With $i = N$, the largest elements of the past N data will be selected. In this case, the probability that there is NA at k will be

$$Q(k) = p_2^{N-1} q_1^{k+1}$$

or in iterative form

$$Q(k) = q_1 Q(k-1)$$

The probability that the delay is exactly k will be

$$\begin{aligned}P(k) &= Q(k-1) - Q(k) \\ &= Q(k-1) - q_1 Q(k-1) \\ &= (1-q_1)Q(k-1) = q_2 Q(k-1) \\ &= p_2^{N-1} q_1^k q_2 \qquad (4.14)\end{aligned}$$

$$\begin{aligned}\text{EDD} &= \sum_{k=0}^{\infty} k\mathbb{P}(\text{DD}=k) \\ &= \sum_{k=0}^{\infty} k p_2^{N-1} q_1^k q_2 \\ &= p_2^{N-1} q_1/q_2 \qquad (4.15)\end{aligned}$$

So, Eqn.(4.15) matches exactly with Eqn.(4.7), the one developed earlier for the maximum case. Now we can proceed with other cases in iterative way.

4.2.2 Case $i = 1$

With $i = 1$, the smallest elements of the past N data will be selected. In this case, the probability that there is NA at k will be

$$Q(k) = q_1 Q(k-1) + q_1 q_2 Q(k-2) + q_1 q_2^2 Q(k-3) + \cdots + q_1 q_2^{N-1} Q(k-N) \quad (4.16)$$

or

$$\alpha_j = q_1 q_2^{j-1}, (j = 1, \cdots, N)$$

thus

$$\begin{aligned} \text{EDD} &= \sum_{k=0}^{\infty} k\mathbb{P}(\text{DD} = k) \\ &= \sum_{k=0}^{N-1} Q(k) + CA(I-A)^{-2}x(N) \end{aligned} \tag{4.17}$$

with initial conditions

$$\begin{aligned} Q(0) &= 1 - p_1^{N-1}q_2 \\ Q(k) &= 1 - p_1^{N-1}q_2 - p_2p_1^{N-2}q_2^2 - \cdots - p_2p_1^{N-k-1}q_2^{k+1} \\ & \quad (k = 1, \cdots, N-1) \\ Q(N) &= Q(N-1) - q_1q_2^N \end{aligned} \tag{4.18}$$

4.2.3 Case $i = N - 1$

With $i = N - 1$, the second largest elements of the past N data will be selected. In this case, the probability that there is NA at k will be

$$Q(k) = q_1Q(k-1) + q_2q_1^{N-1}Q(k-N) \tag{4.19}$$

or

$$\alpha_1 = q_1; \alpha_l = 0 (l = 2, \cdots, N-1); \alpha_N = q_2q_1^{N-1}$$

thus

$$\begin{aligned} \text{EDD} &= \sum_{k=0}^{\infty} k\mathbb{P}(\text{DD} = k) \\ &= \sum_{k=0}^{N-1} Q(k) + CA(I-A)^{-2}x(N) \end{aligned} \tag{4.20}$$

with initial conditions

$$Q(0) = p_2^{N-1} + (N-1)p_2^{N-2}p_1q_1 \tag{4.21}$$

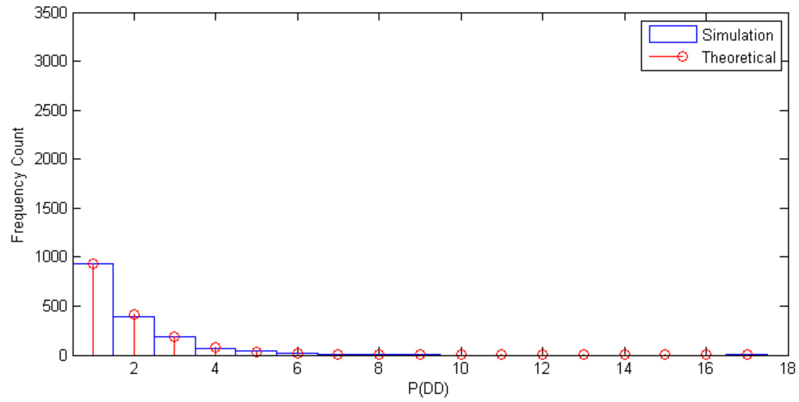
$$Q(k) = q_1Q(k-1) + q_2q_1^kp_2^{N-k-1}(p_2^{k-1} + (k-1)p_1p_2^{k-1}) \tag{4.22}$$

$$(k = 1, \cdots, N-1)$$

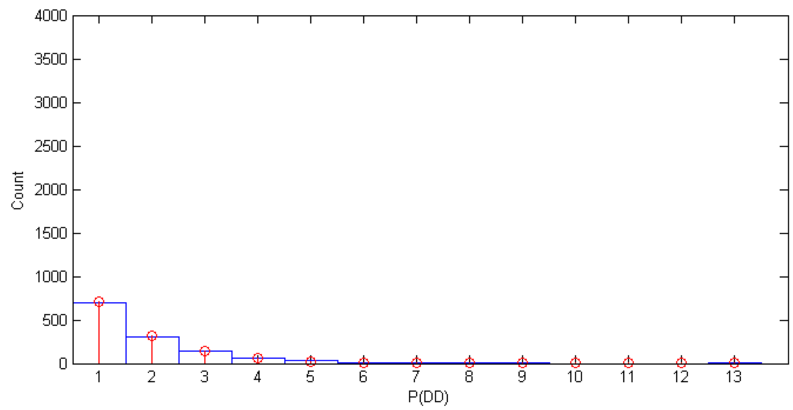
$$Q(N) = q_1Q(N-1) + q_2q_1^{N-1}Q(0) \tag{4.23}$$

4.3 Validation and Comparison

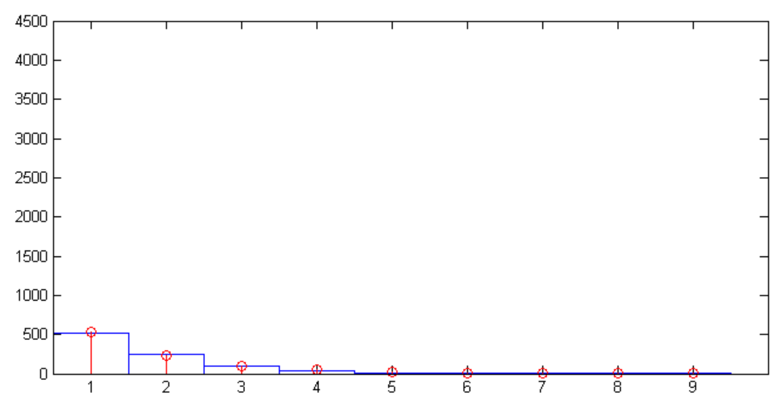
To validate the general equation of probability of detection delay given in Eqn.(4.6), we are considering Gaussian distributed normal and abnormal data. Let normal data follow Gaussian distribution with $N(0,1)$ and abnormal data follow Gaussian distribution with $N(1,2)$. The sampling period is kept at 1 sec. We ran 5000 Monte Carlo simulations for different filter orders from N = 2 to 4. The trip point was fixed at $y_{tp} = 0.7$ throughout the simulation. We count the number of occurrence of each delay and take average to get a single estimate for $\mathbb{P}$(DD). In Figure 4.3, simulation outcome is presented for different delays along with theoretical value obtained from Eqn.(4.6). Since, Eqn.(4.6) gives a single estimate, for consistency we multiplied this value with the number of Monte Carlo simulations. From Figure 4.3 it is visible that for different orders of the filter, the analytical solution exactly matches with the numerical results, which validates the Eqn.(4.6).



(a) Filter Order, N=2



(b) Filter Order, N=3



(c) Filter Order, N=4

Figure 4.3: Validation of analytical solution by Monte-Carlo simulation for different filter orders

For the same input distribution, the effect of changing filter order with 1^{st} maximum ($i = N$) on EDD is visualized from Figure 4.4. In each case, the simulation result is validated by the theoretical result obtained from Eqn.(4.7).

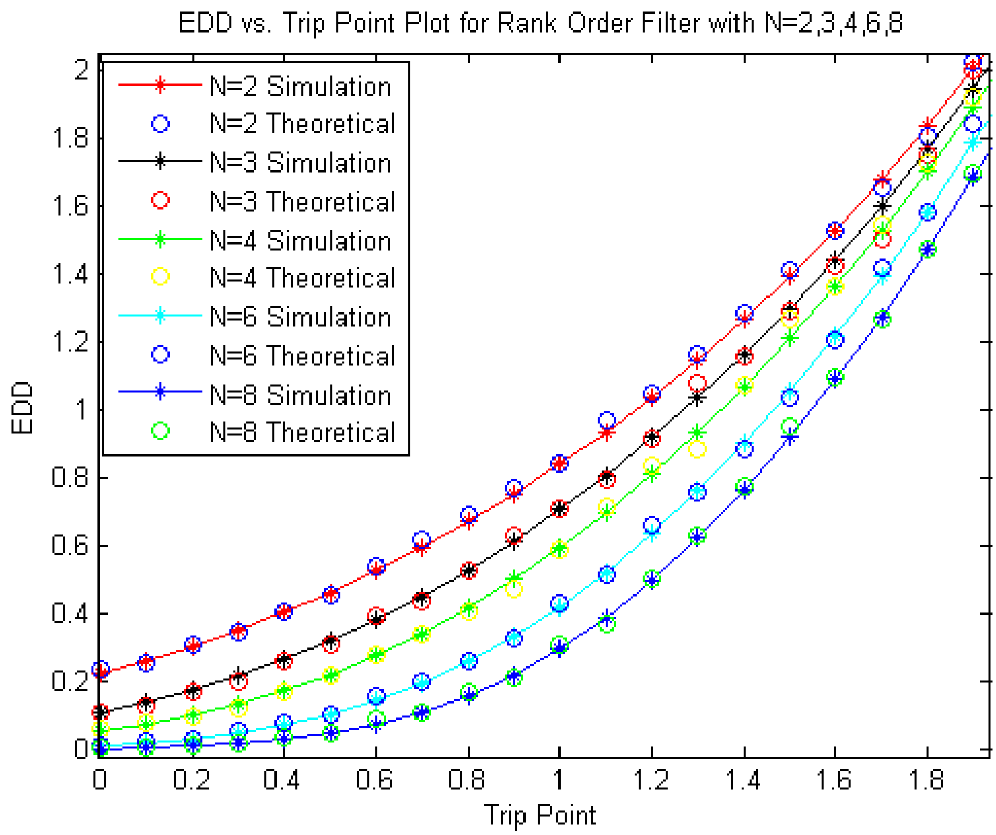
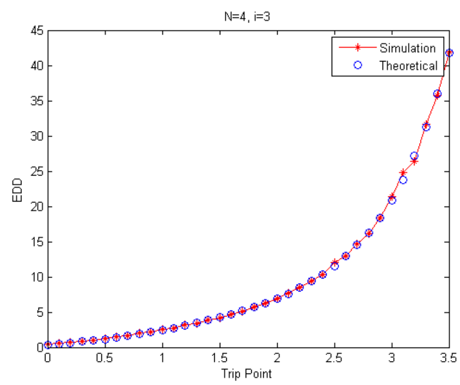
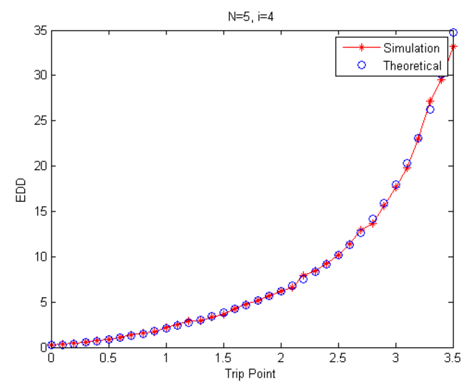


Figure 4.4: Expected detection delay for different filter order and $i = N$ validated by Monte-Carlo simulation

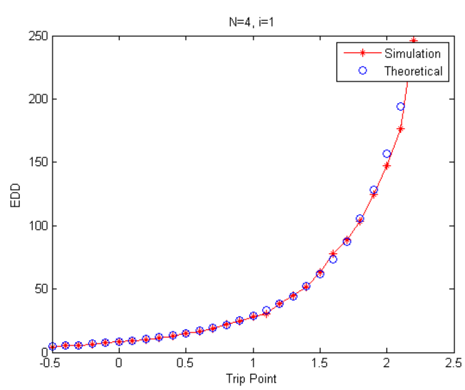
Expected detection delays for rank order filters with $i = N - 1$ (2^{nd} maximum) and $i = 1$ (1^{st} minimum) are simulated again for validation purpose. Eqn.(4.20) and Eqn.(4.17) are used to estimate the expected detection delays analytically. These results are validated by Monte Carlo simulations. In Figure 4.5 (a)-(b), both theoretical and simulation results are shown for filter orders 4 and 5 with rank $i = N - 1$ (2^{nd} maximum). Again in Figure 4.5 (c)-(d), EDD has been plotted against trip point for rank order filters with orders 4 and 5 with $i = 1$ (1^{st} minimum).



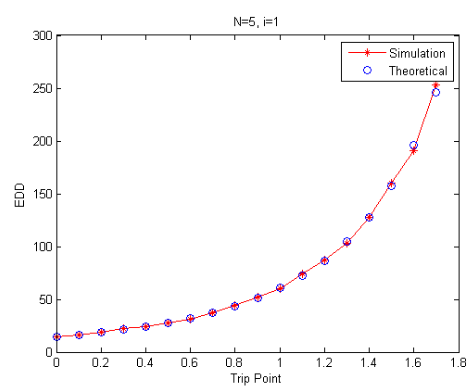
(a)



(b)



(c)



(d)

Figure 4.5: Expected detection delays for (a) $N = 4, i = 3$, (b) $N = 5, i = 4$, (c) $N = 4, i = 1$, (d) $N = 5, i = 1$

Comparison of Expected Detection Delay

The closed form solution for expected detection delays is obtained in Eqn.(4.7). In Figure 4.6, expected detection delay are plotted against the trip point for different filter orders. Each plot is validated by Monte Carlo simulation where for each trip point 5000 simulations are run. From Figure 4.6, we conclude that the theoretical and simulation results are consistent. For rank order filters with window size N and order i, if i is fixed, the expected detection delay is decreased with the increment of window size (N). With the same distributions of normal and abnormal data, expected detection delays for moving average filters are plotted in Figure 4.7 to compare the performance with rank order filters. We observe that, for the same trip point with higher filter order, detection delay for the rank order filter is smaller than that of the moving average filter.

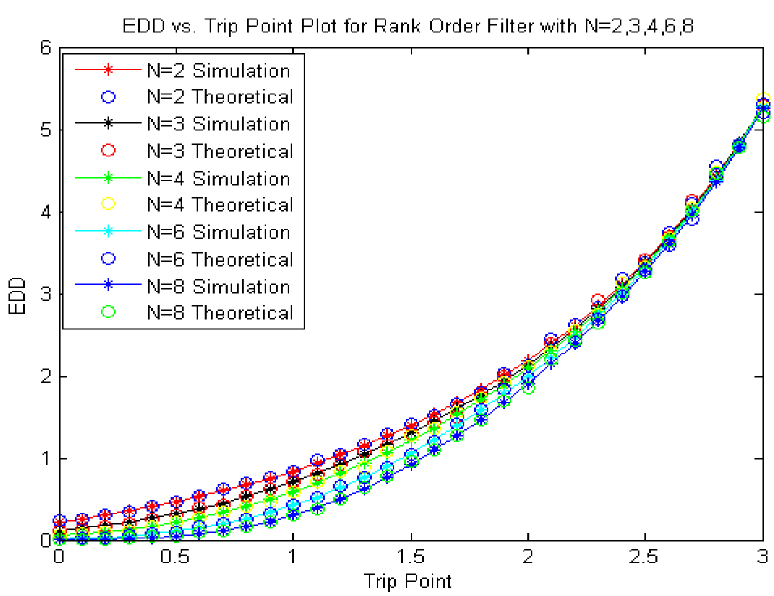


Figure 4.6: Comparison of expected detection delay for different filter orders

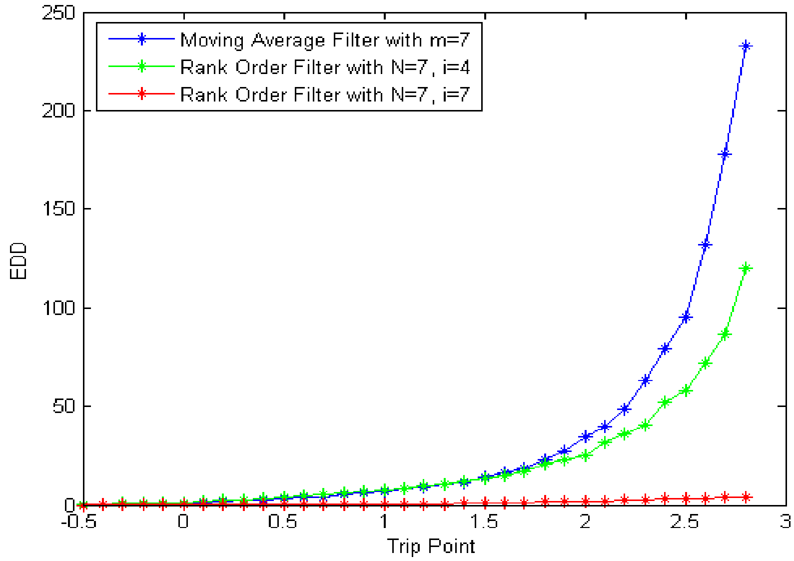


Figure 4.7: Expected detection delay for different filters compared with the moving average filter

4.4 Summary

Expected detection delay is an important parameter for designing alarm systems. Longer delay in alarm activation may lead to serious consequences. In this chapter, detection delay for rank order filters with 1^{st} maximum, 2^{nd} maximum and 1^{st} minimum as outputs have been formulated and validated through numerical simulations for Gaussian distributions of the data. A comparative performance overview for rank order filters is presented with respect to moving average filters (widely applied method in the process industry). We have derived some important conclusion based on observation of the performance comparison.

Chapter 5

Case Study

5.1 Overview

In this chapter, the proposed rank order filters have been applied to an actual industrial alarm data set. The alarm data has been taken from an oil-sand extraction plant. This is part of the ongoing alarm rationalization project where univariate alarm tags have been identified and rationalized. In the project the overall work for alarm rationalization has been done in several stages such as: selecting alarm to rationalize, justification and prioritization, operator decision support, classification, setting alarm limits, alarm tuning and advanced alarming, and safety analysis [16]. This is a step by step process of evaluating all alarm settings to check for legitimacy, accuracy and rationale. The goal of alarm rationalization is to meet the requirements of ISA 18.2 [38] and reduce burden on the operators. To efficiently rationalize alarms, knowledge and collaboration from control engineering team, operations, maintenance and other disciplines are required.

However, this chapter focuses on possible applications of rank order filters on identified top bad actors from the industrial data. Since filter design and alarm limit tuning are dependent on process data, the availability of normal and abnormal historical data is of concern. Even if the abnormal data is not present for some of the tags, we can apply our filter design approach based on normal data distribution and a required false alarm rate. Generally flow and level tags are noisier compared to pressure and temperature. As a result, a flow tag is analyzed in the following section. The original tag names of the industrial data are masked due to confidentiality.

5.2 Alarm System Design

The accurate and efficient design of an alarm system depends on three performance indices, namely, the false alarm rate, the missed alarm rate, and the expected detection delay [41]. For optimal configuration, we need to balance among these indices. In this section, an attempt has been made to demonstrate the step-by-step procedure for designing optimum alarm setting satisfying requirements of FAR, MAR and EDD. This configuration would be optimal in the sense that it would reduce FAR and MAR i.e., increase the accuracy, and also reduce EDD, i.e., increase the efficiency of alarm activation. The objective function proposed in Chapter 2 with these constraints can be described by minimizing

$$\begin{aligned} J(x) &= c_1 \int_x^{+\infty} f_{Y n}(x)dx + c_2 \int_{-\infty}^{x} f_{Y ab}(x)dx \\ s.t. \quad & EDD \leq \kappa \end{aligned} \tag{5.1}$$

Where $f_{Y n}$ and $f_{Y ab}$ are the probability density functions of normal and abnormal data respectively; κ is the maximum delay we can consider. The two integrals represent FAR and MAR, and c_1 and c_2 are the weights assigned to FAR and MAR respectively.

The objective function is plotted in terms of ROC curves in Chapter 3. The ROC curve is the plot of FAR vs MAR while the trip point y_{tp} is varied from minimum to the maximum of the process data. For a high alarm configuration, lower trip point results in higher FAR whereas changing trip point to higher points results in lower FAR but higher MAR. The weights (c_1, c_2) assigned to FAR and MAR change the optimal region for selecting trip points. In this case, we assigned equal weights to both FAR and MAR. As a result, the optimal point would be the one closest to the origin for a specific filter order [17, 41]. The other constraint in the objective function is expected detection delay. So, the objective function should be computed considering maximum allowable delay [4].

Simulated Data Case Study

In this work, a four step alarm design method is demonstrated for rank order filters with 1^{st} maximum as output similar to the method presented in [4]. The design parameters for rank order filters are the filter order, N, and the i^{th} maximum value. We fix the value of i to be N, i.e., taking the 1^{st} maximum. So, for designing the alarm system, the design parameters are the filter order, N, and the trip point, y_{tp}. We consider $N(0, 1)$ and $N(1, 2)$ for the normal and abnormal segment of the data respectively. Process data and corresponding PDF's are shown in Section 4.1.

For this example, we are considering to design an alarm system with FAR $\leq 15\%$, MAR $\leq 15\%$ and EDD ≤ 2 seconds. Rank order filter orders are changed from N = 2 to 8 and corresponding ROC curves are plotted in Figure 5.1. It is visible that with the increment of the filter order, the ROC curves move closer to the origin. Expected detection delay for different filter orders are also plotted in Figure 5.3. The trip point for equal FAR and MAR can be estimated from Figure 5.2.

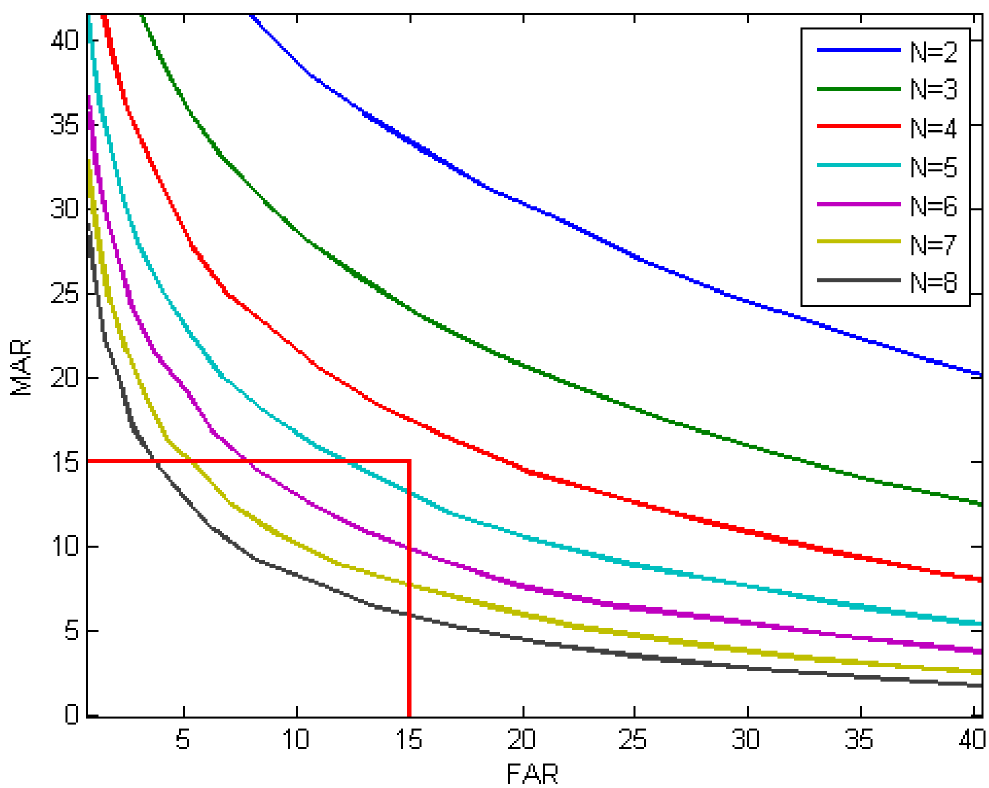


Figure 5.1: ROC curves when the filter order is changed

Step 1

At first, we address the accuracy of the alarm system. The filter order is selected from the ROC curves that lies below our specification i.e., FAR $\leq$ 15% and MAR $\leq$ 15%. The smallest filter order N_1 is selected that satisfies the above criteria from Figure 5.1. Any filter order $N \geq N_1$ would meet the selection criteria. For this case, rank order filter of order 5 or more meet the design requirements of FAR and MAR.

Step 2

The second step is to find the optimal operating limit or trip point (y_{tp}) for the selected orders of rank order filters. Since, FAR and MAR are calculated by varying the trip point, plotting both FAR and MAR against the trip point on the same plot would allow us to estimate the trip point for equal FAR and MAR. In Figure 5.2, FAR and MAR are plotted against the trip point where plots from the upper left are FAR curves and from upper rights are MAR curves. By observing the plots, trip points with equal FAR and MAR are estimated from the intersection of these two curves. The result is summarized in Table 5.1.

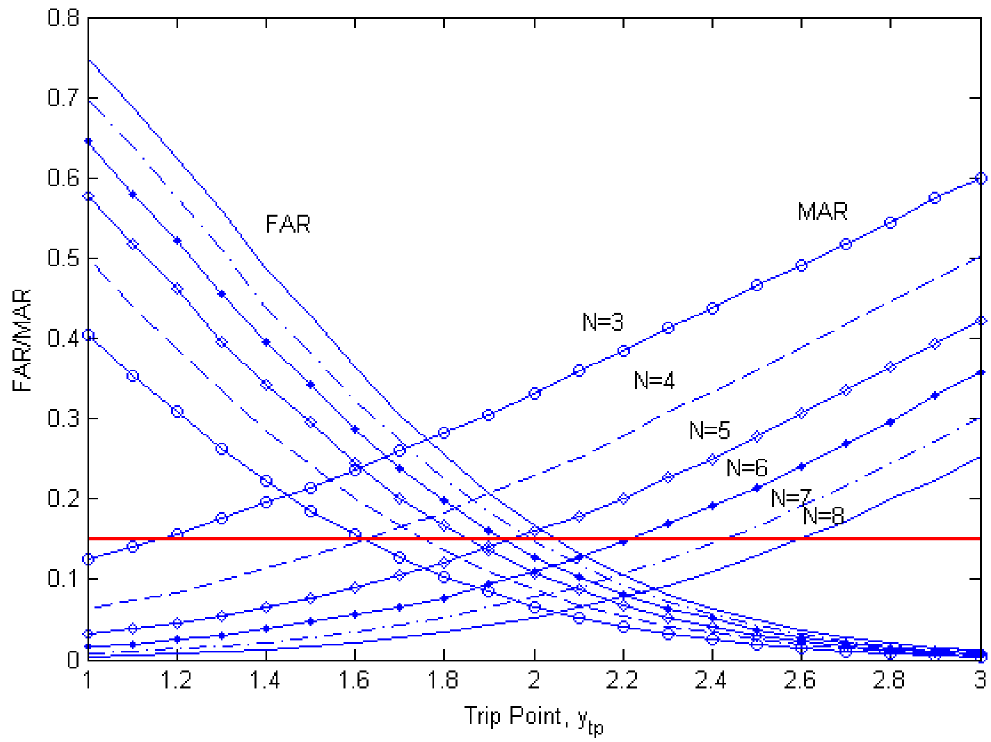


Figure 5.2: Estimation of trip points when FAR = MAR

Step 3

In this step, the expected detection delay is estimated for the trip points obtained in step 2. These trip points correspond to equal FAR and MAR for different filter orders. The largest value of the filter order N_2 for EDD ≤ 2 seconds is selected from Figure 5.3. The summary of the results is presented in Table 5.1. For this design, we found $N_2 = 5$.

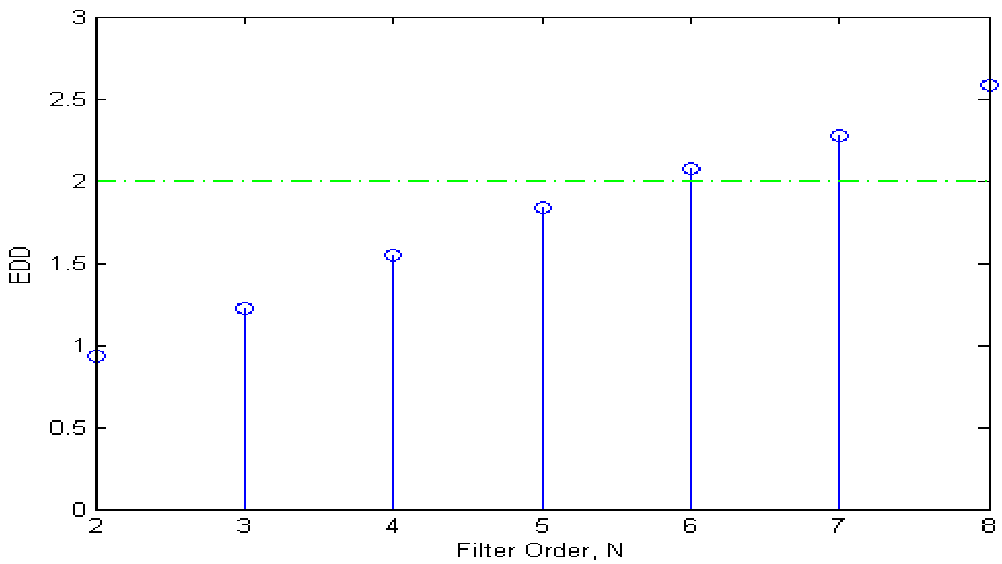


Figure 5.3: Expected detection delays for different filter orders with optimal trip points obtained from Step 2

Step 4

Once N_1 and N_2 are obtained, the last step is to select the rank order filter order. From step 1, we found that filter order 5 or more satisfy our accuracy design criteria, whereas from step 3, we found that filter order 5 or less satisfy out efficiency criteria.

If $N_2 \geq N_1$, then any order between $N_1 \leq N \leq N_2$ would satisfy the design criteria. For this example, we select filter order 5 with FAR and MAR of 13.85% and EDD 1.84 seconds respectively.

On the other hand, if no filter order N is found in the range $N_1 \leq N \leq N_2$ then we need to change the design requirements. For this case, the requirements may be too strict for achieving our goal. In a nutshell, we can say that this method not only provides us with steps for selecting filter order but also allows us to check whether we can achieve performance requirements or not.

Filter order (N)	Trip Point (y_{tp})	FAR = MAR (%)	EDD
2	1.07	26.38	0.93
3	1.448	20.46	1.221
4	1.703	16.56	1.549
5	1.892	13.85	1.842
6	2.043	11.74	2.075
7	2.163	10.07	2.28
8	2.271	8.86	2.583

Table 5.1: Design summary for rank order filters

5.3 Industrial Case Study

In this section, an industrial case study has been presented where univariate alarm tags have been configured based on three performance requirements on FAR, MAR and EDD. Since filtering is applied on process data, process tags have been selected from corresponding alarm tags that are the top bad actors.

Case Study

For this case study, we have considered an actual flow variable from a Hydrogen generation plant in an oil-sand processing facility for high alarm configuration. The sampling time is 15 *sec*, so the data represents 24 hours data starting from 14 August 2014 in Fugure 5.4. As visible from the figure, fault occurrence is identified from the abrupt change in variance. In Figure 5.6, histograms of the corresponding normal and abnormal segment of the data is presented. If the rank order filter of order 8 and 1^{st} maximum is applied on the data, the plotted ROC curve then allows us to visualize the trade-off between FAR and MAR. Figure 5.5 compares ROC curves filtered using rank order filter and moving average filter of order 8.

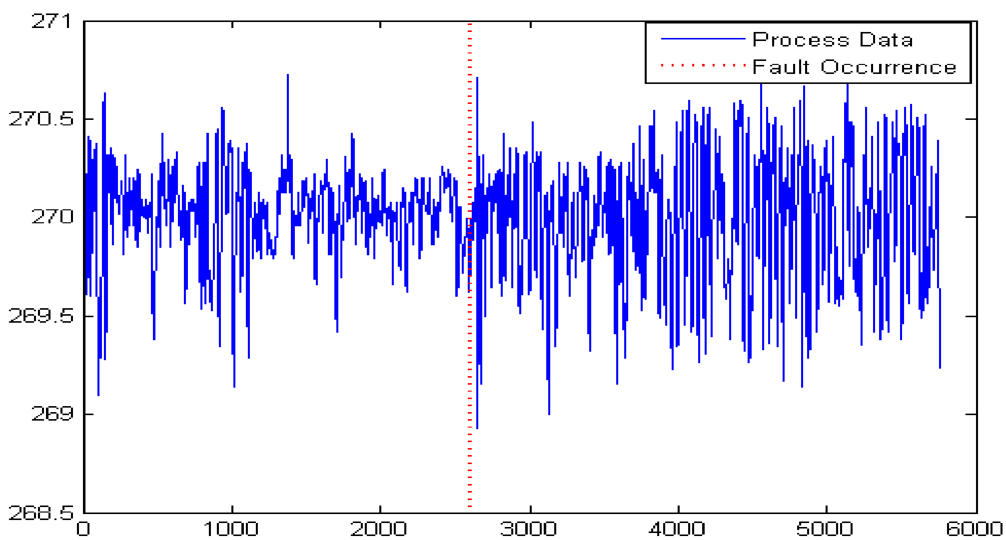


Figure 5.4: Flow variable from an oil-sand industry

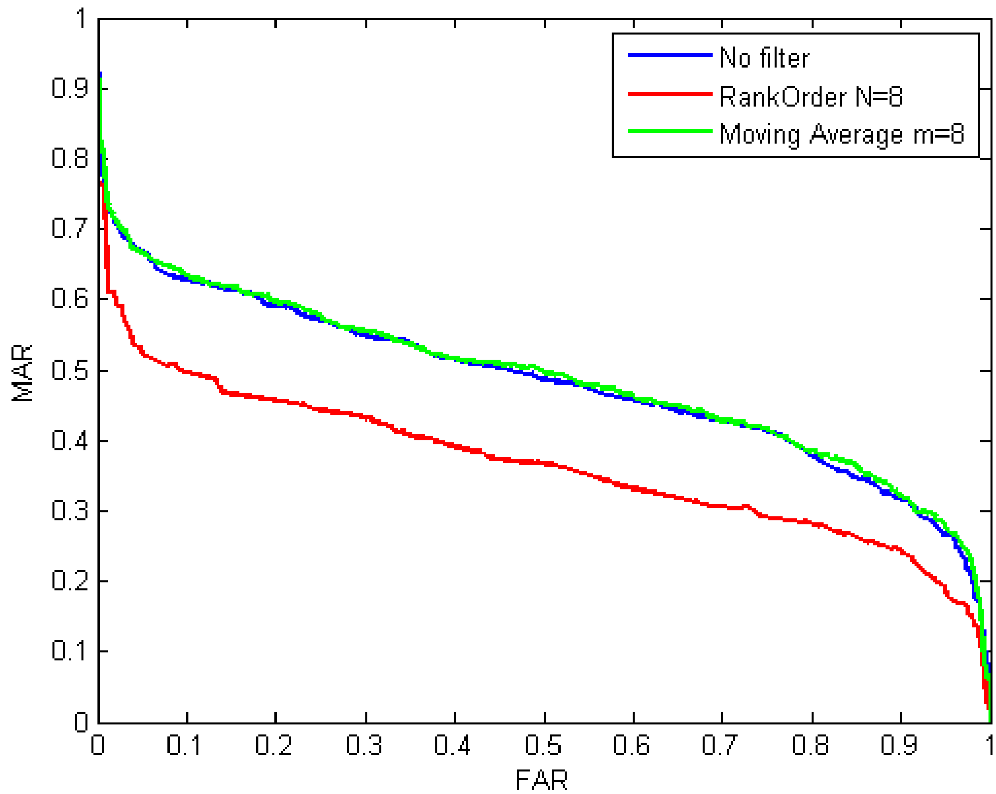
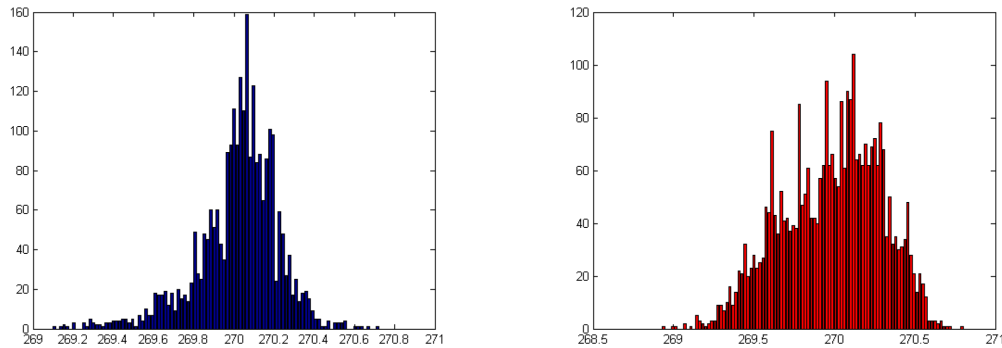


Figure 5.5: ROC curve comparison of rank order filter (N=8, i=8) with moving average filter (m=8)

In the next segment, we plan to choose the filter order for alarm system design based on empirical assumption on FAR and MAR. In this case, we follow the same four-step procedure mentioned earlier. The design requirements for the case study here are: FAR $\leq$ 50%, MAR $\leq$ 50% and EDD $\leq$ 30 samples. The first step is to identify the filter order from Figure 5.7. Rank order filters with order 4 and above satisfy the design criteria. We have selected the smallest order, N_1 of the identified filter orders that satisfies FAR=MAR=50%. Then, thresholds that satisfy FAR=MAR are obtained from Figure 5.7 for different filter orders and the results are



(a) (b)

Figure 5.6: Histograms of the normal and abnormal segments of the data

listed in Table 5.2. In the third step, we narrow down the filter orders from Figure 5.8 that satisfy EDD $\leq$ 30 samples and enlist them in Table 5.2.

Filter order (N)	Trip Point (y_{tp})	FAR = MAR	EDD
2	270.053	0.516	23.8
3	270.063	0.5032	24.44
4	270.074	0.4924	25.11
5	270.09	0.4849	34.3
6	270.093	0.4729	39.2
7	270.109	0.4640	40
8	270.127	0.4575	40.5

Table 5.2: Design summary for rank order filters

From the list, we select the largest order N_2. The last step is selecting the order of the filter, N. From the earlier example, we have decided that any N satisfying $N_1 \leq N \leq N_2$ can be used for alarm design. For our case $N = 4$ (shaded in the Table

5.2). The threshold for this filter is 270.0736.

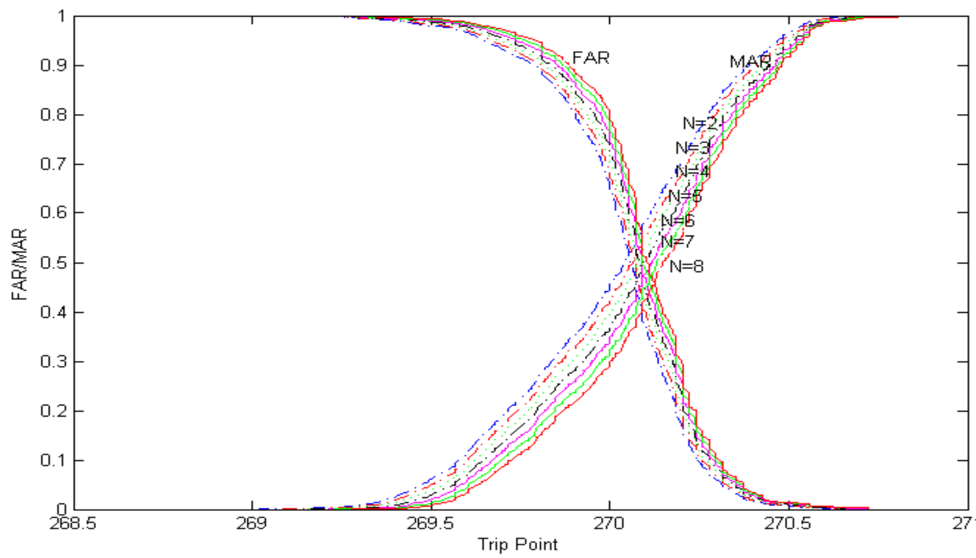


Figure 5.7: Estimation of thresholds when FAR=MAR

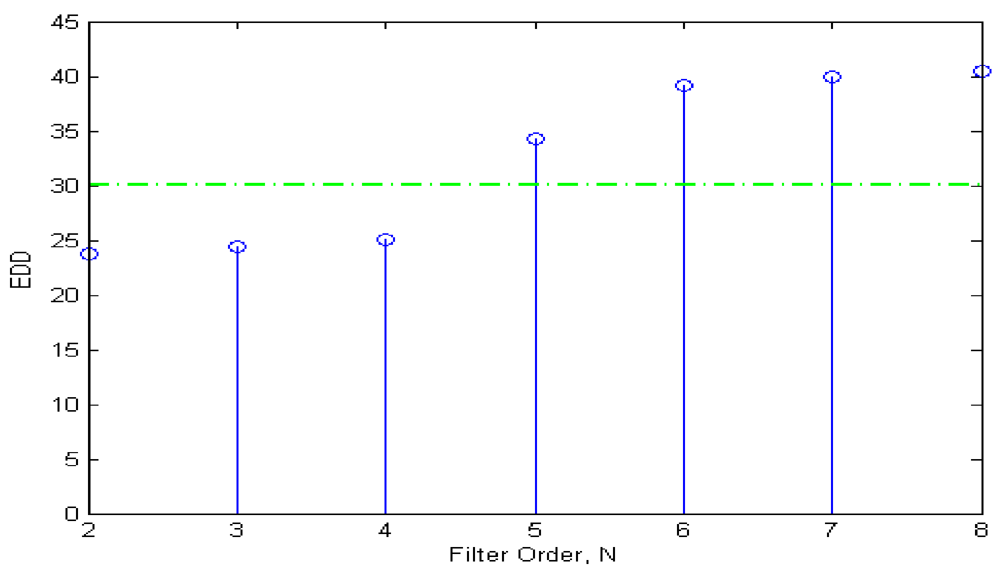


Figure 5.8: Effect of EDD on different filter orders

The result obtained here can be compared with the Shewhart chart where upper limit is set on 3σ interval. The mean of the normal data is 270.0209 and standard deviation is 0.2030. Setting 3-sigma limit sets the upper threshold to 270.6299. For this setting, the alarm generates 3151 missed alarms. By using the rank order filter and proposed threshold, the number of missed alarms reduces 50% to 1556 alarms.

5.4 Summary

The alarm design and rationalization procedure followed here is solely dependent on historical process data. Process behaves differently due to different operating conditions. Case studies presented here represent only a segment of the big picture. In practice, changing alarm configuration requires thorough process knowledge and needs proper safety analysis before any change is made. On the other hand, there is no general filter to be applied on all the process variables. Depending on the distribution of the data and process behaviour, a specific filter may perform better than others. However, the main goal of the study was to show the effectiveness of the nonlinear rank order filters over linear moving average filters. With simulated examples and industrial case studies, we can come to a conclusion that rank order filters can effectively reduce false and missed alarms and meet the detection delay limitation. In the future, further studies can be made by selecting different windows and comparing them to validate the effectiveness.

Chapter 6

Concluding Remarks

6.1 Contribution of the Thesis

Chattering and nuisance alarms are serious concerns in process industry where thousands of alarms are annunciated due to easily configurable modern DCS systems. For some cases, chattering alarms flood the operator panel, making it difficult for the operators to take necessary action in a timely manner. Many major accidents occurred in the past only because operators were misled by the overwhelming information. For this reason, chattering and nuisance alarms causing alarm floods are a major safety concern in process industry. The ISA 18.2 [38] and EEMUA 191 [26] guidelines for alarm management mention some standards for operators to handle efficiently. Though industries are still far away from the standard recommended numbers, they are trying to comply with these standards by initiating plant wide systematic alarm system design and rationalization. Several studies have been made to reduce the effect of chattering alarms; among them filtering and delay timers are most effective ones. As a result, properly designed filters and delay timers contribute significantly in alarm management and rationalization.

Filters are mainly applied in process industry for suppressing noises. Process variables operating close to the trip point are more vulnerable to noise as they may cross the trip point and activate or deactivate nuisance/chattering alarms. To alleviate this problem filters of different orders are applied directly on process variables before comparing with thresholds. However, like delay timers, filters introduce delay in the activation of the alarms when a fault occurs. In this thesis, an attempt has been made to build quantitative relationship among the rank order filter order and performance indices. In Chapter 2, the basics of rank order filter has been discussed and the PDF equation of the rank order filters have been verified by Monte Carlo simulation considering different input distributions. Several statistical tests have been conducted on filtered data to verify that the filtered data is not *i.i.d.*.

In Chapter 3, accuracy of alarm annunciation, i.e., FAR and MAR have been calculated numerically for rank order filters with the i^{th} maximum value as output. An optimal solution to design rank order filters has been formulated mathematically. Then the performance of the rank order filter has been compared with moving average filters for different input data distributions. Moreover, it has been shown that the performance curve of the rank order filter lies between the corresponding moving average filter and general log likelihood based optimal filter considering different input distributions. In the end, some conditions have been proposed based on the observations where rank order filters are suitable.

The major contribution of the thesis is presented in Chapter 4 where detection delay for rank order filters with 1^{st} maximum, 1^{st} minimum and $N-1^{th}$ maximum values as outputs have been analytically developed and verified by Monte Carlo simulations. Expected detection delays (EDD) of the rank order filters are compared with EDD of the moving average filters of the same order. For rank order filters with window size N and order i, if i is fixed, the expected detection delay is decreased with the increment of window size (N).

The developed method is applied on simulated and industrial data to show the effectiveness of the developed method in Chapter 5. A step-by-step univariate alarm design method has been presented to visualize the overall method based on simulated data. An industrial case study has been presented and the step-by-step alarm design method has been implemented. It has been shown that properly selected filter order can increase the accuracy and effectiveness of the alarm system.

6.2 Scope for the Future Work

In this work, analytical relations of expected detection delays have been established for rank order filters with 1^{st} maximum, 1^{st} minimum and $N-1^{th}$ maximum values as outputs. Analytical relations for other maximums in the window may be established as well. These theoretical analysis on rank order filters are more difficult due to non-linearity. The effectiveness of rank order filtering in reducing chattering/nuisance alarms can be compared to that of moving average filters. This in turn may give an idea on what kind of filters need to be used to prevent alarm flood situation since it is related to chattering alarms. Window based data selection and filtering are solely dependent on historical process data. So, it is a great source for effective alarm design. The data distribution is assumed to be known a-priori and data is assumed to be independent and identically distributed (*i.i.d.*). The *i.i.d.* assumption in detection delay calculation makes the application of the proposed method somewhat restrictive. The issue with *i.i.d.* distribution can be addressed in the future work. In addition to that, rank order filters can be incorporated in the Alarm Management Toolbox as an introduction of non-linear filtering in the toolbox.

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