# A Robust Two-Layer Network Equivalent for Transient Studies

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***Abstract*— This paper proposed a new systematic approach for constructing a Two-Layer Network Equivalent (TLNE) for external systems suitable for electromagnetic transient simulation. With full-order Vector Fitting (VF), Genetic Algorithms (GAs), Constrained Linearized Least-Square (CLLSQ) Optimization, and accurate low-order line parameter fitting routine, the generated low-order model is of high accuracy compared to its full model in frequency domain. The merits of this method are its robustness in terms of stability and positive-realness, its accuracy in not only transient frequencies but also at dc and power frequency, and its order determination feature. To validate the new method, a detailed three-phase multi-port case study has been elaborated. Comparison of results among the proposed method, the existing method and FDNE are made on the basis of RMS-error of external system input admittance. Time-domain simulation results with respect to the original system in ATP and PSCAD/EMTDC illustrate the accuracy and computational efficiency of the proposed approach.**

***Index Terms*— electromagnetic transient analysis, transmission lines, frequency domain analysis, network equivalent, least square methods, genetic algorithms.**

## I. Introduction

In electromagnetic transient studies of large systems, it is common practice to divide a system into a study zone where transient phenomena occur and an external system encompassing the rest of the system (Fig. 1), in order to reduce computational burden. The external system is commonly represented by a linear equivalent or Frequency-Dependent Network Equivalent (FDNE) [1], [2], which is obtained by the well-known routine Vector Fitting (VF) [3], [4] in $s$-domain.

In large systems, the complexities of external system result in high-order rational function (matrix), which requires excessive computations in transient simulations. This is not only an obstacle in off-line simulation, but also the main bottleneck in achieving real-time digital simulation of realistic size power systems. The Two-Layer Network Equivalent (TLNE) (Fig. 1), first proposed by Abdel-Rahman, et al. [5], in which the external system was further partitioned into a surface-layer comprising of low-order frequency-dependent transmission lines and a deep-region of low-order FDNE model, overcomes this obstacle. The contribution of surface layer and deep region on the external system input admittance varies with frequency. In particular, both surface layer and deep region have effects on the admittance at low frequency. However, since transients in study zone do not travel very far in external systems, the deep region mainly contributes to the lower frequency range.

Existing methods in [5] for obtaining TLNE rely on low-order VF for both surface layer and deep region parameters, and Sequential Quadratic Programming (SQP) with passivity constraints to improve model accuracy. Nonetheless, our experience with the methods revealed the following concerns:

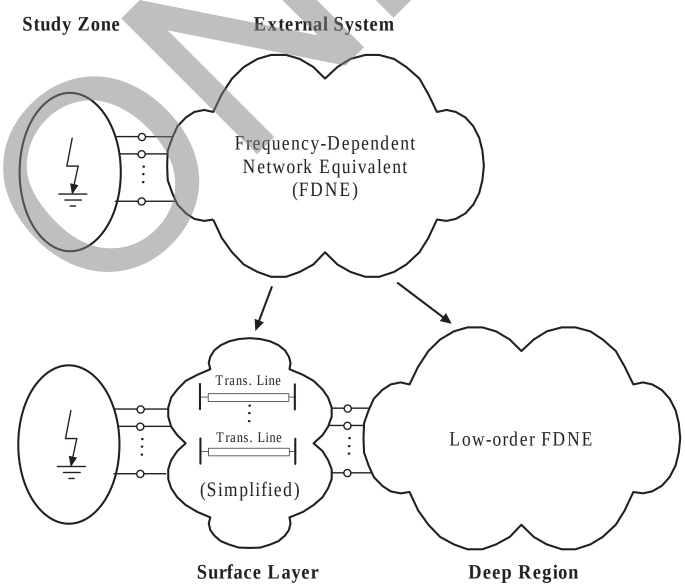


Fig. 1. Two-Layer Network Equivalent for an external system

1) With low-order VF in obtaining the deep region, it is difficult to control deviations with respect to the original deep region, which SQP may not be able to compensate.
2) Frequency response at dc is not specifically accentuated although it affects the dc offset in the transient.
3) SQP is prone to divergence. If better first approximations of input admittance of external systems can be found, SQP can be replaced by Constrained Linearized Least-Square (CLLSQ) optimization, in which the convergence is guaranteed.
4) Optimization of surface layer is especially helpful in increasing accuracy of frequency response in the low frequency range, e.g., dc and power frequency, with little cost of computational time. Thus, partial parameters of surface layer are optimized by default. Fig. 2, which shows the real part of input admittance of a sample system, illustrates the advantages accrued by optimizing

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the surface layer. The sample system diagram is shown in Fig. 14 in Appendix A.

5) In multi-port external systems with complex frequency response, the passivity constraint is very strong, so the freedom for changing the parameters is small. Therefore, the first approximation in a multi-port case, is required to be closer to the original than that for single-port case. Thus, transmission line parameters in surface layer require higher accuracy but low-order realization.

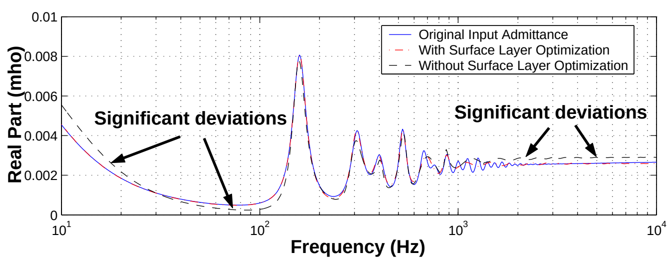


Fig. 2. Deviations due to not optimizing surface layer parameters

This paper introduces a robust approach on Two-Layer Network Equivalent (TLNE) based on full-order Vector Fitting (VF), Genetic Algorithms (GAs) [11], Constrained Linearized Least-Square (CLLSQ) Optimization, and accurate low-order line parameter fitting routine. The new approach not only overcomes the problems mentioned above, but also includes optimal determination of deep region order. A detailed case study of a three-phase multi-port system, as well as time-domain results from ATP and PSCAD/EMTDC simulations are presented to show the effectiveness of the proposed approach.

## II. The Robust Two-Layer Network Equivalent

### A. Introduction

The robustness of an equivalent is determined not only by its stability but also by its passivity or positive-realness. The passivity criterion is important due to its strong effect on stability of time-domain simulations; the electric network with passivity violations will more likely result in unstable and erroneous simulations. In a single-port network, the passivity criterion requires the the real part of the input admittance be positive at all frequencies. In a multi-port network, all eigenvalues of the real part of the admittance matrix must be positive in the entire frequency range.

For a passive external system, the first approximation of the input admittance (matrix) of the external system $\tilde{\mathbf{Y}}^0_{input}(\omega)$ is the initial frequency-domain mathematical combination of admittance matrix of the surface layer $\tilde{\mathbf{Y}}^0_{surface}(\omega)$ constituting low-order frequency-dependent transmission lines, and deep region admittance (matrix) $\tilde{\mathbf{Y}}^0_{deep}(\omega)$ comprising low-order FDNE. In the existing method, both the parameters of surface layer and deep region are first obtained from low-order VF.

Due to their globalism in searching and merit in multi-objective optimization, GAs are used to find the appropriate deep region and build the first approximation of input admittance (matrix) in the robust TLNE model. Indeed, the application of GAs is the main contribution in constructing the robust TLNE. Further improvements include CLLSQ optimization with inclusion of frequency response at dc and the optimal deep region order determination feature.

### B. Surface Layer

The surface layer is comprised of transmission lines of low-order frequency-dependent model. In our transient studies, Marti's frequency-dependent line model [6] is chosen, since it is widely used and its low-order realization [7], [8] has been shown to be suitable for real-time implementation [9], [10]. Furthermore, for the rational approximation of line parameters (e.g., $Z_c(\omega)$ or $Y_c(\omega)$, and $A(\omega)$ or $P(\omega)$) we used Bode's asymptotic technique for a simpler system (the sample system in Appendix A), and nonlinear Levenberg-Marquardt fitting method due to Fernandes and Neves [8] for higher accuracy in multi-port systems (the Case Study in Section III).

### C. Deep Region

Instead of low-order VF and SQP, the deep region is found out by full-order VF, GAs and CLLSQ. Commonly, transients in study zone do not travel very far into the external system, and in the TLNE, the deep region is further "insulated" from the study zone by the surface layer. Thus, the order of deep region may be significantly reduced, due to the relative insensitivity of input admittance (matrix) to that of the deep region. By removing those insensitive resonant peaks in the deep region, it is possible to reduce the order of deep region without significantly affecting the accuracy of the input admittance. Although those peak are likely to distribute in the lower frequency range, we are still uncertain about their locations. Therefore, a global search algorithm is required which captures the peaks that have most significant effects on input admittance. GAs are well-suited for this situation. The partial fractions representing rational functions generated by full-order VF are indexed and encoded as chromosomes so that GAs are able to find out best suitable partial fractions for the deep region. Further compensation technique is applied to eliminate the deviation effects at lower frequencies in this procedure. For a $m$-port external system, the objective function [11] is defined as

$$f_{obj} = \left\| \mathbf{Y}_{input}(\omega) - \tilde{\mathbf{Y}}^0_{input}(\omega) \right\|_F^2 + \mu = \sum_{i,j=1}^{m} \left| \mathbf{Y}_{input,ij}(\omega) - \tilde{\mathbf{Y}}^0_{input,ij}(\omega) \right|^2 + \mu \quad (1)$$

where $\mathbf{Y}_{ij}(\omega)$ is the $ij$-th element of the matrix $\mathbf{Y}(\omega)$; $\mu$ denotes a penalty term when positive-real criterion violation occurs in deep region. If the criterion is violated, $\mu$ will be a big enough positive number, otherwise $\mu = 0$. This ensures that the outputs from GAs are the best fitted deep regions, which are both stable and positive-real. Therefore, the first approximation of the input admittance (matrix) is built by combining the surface layer and deep region.

### D. Constrained Linearized Least-Square Optimization

Theoretically, the input admittance obtained from GAs is very close to the original. Nonetheless, the surface layer and deep region are subject to further fine-tuning to minimize deviations in the input admittance (matrix). In Marti's line model (characteristic impedance $Z_{eq}(s)$ and weighting function $P_a(s)$ and propagation constant $\tau$ are assumed), for faster algorithm convergence and accuracy, the parameters considered for further optimization are

- In the surface layer, constant terms, all poles and residues of $Z_{eq}(s)$.
- In the deep region, all constant terms and residues.

In our experience, optimizing surface layer parameters particularly improves the frequency response at dc and power frequency. By building linearized Jacobian matrix $\mathbf{J}(\omega)$, an iterative process is initiated by recursive evaluation of

$$\mathbf{x} = \mathbf{x}_0 + \Delta\mathbf{x} \tag{2}$$

where $\Delta\mathbf{x}$ is obtained by the overdetermined equation

$$\Delta\mathbf{Y}_{input}(\omega) = \mathbf{J}(\omega)\Delta\mathbf{x} \tag{3}$$

$\mathbf{x}$ is the model parameter column vector considered for optimization and $\Delta\mathbf{x}$ is the model parameter change column vector.

In order to obtain the best suitable order of deep region, a series of deep region orders are applied to the problem. Thus, we obtain a collection of discrete values representing the order of deep region v.s. percentage RMS-error. The optimal order is the one where in the orders lower than it, the percentage RMS-error increases dramatically, whereas in the orders higher than it, the percentage RMS-error does not decrease significantly. The later Case Study in Section III illustrates this idea.

### E. External Systems of Active Networks

The contributions of sources, in an active external system which is linear and time-invariant, are limited only to power frequency. This gives the idea that we only need to consider the external system in power frequency and construct Norton equivalent sources for sources at the input ports of external system. The Norton equivalent sources are found either by using an analytical method [5], or by measuring short-circuit current in EMTP.

The passive part of the external system is obtained by traditional method of eliminating voltage and current sources. Then the robust TLNE model discussed in Section II is applied to the passive part. Thus we have a robust TLNE model for generic external systems, whose flowchart is shown in Fig. 3.

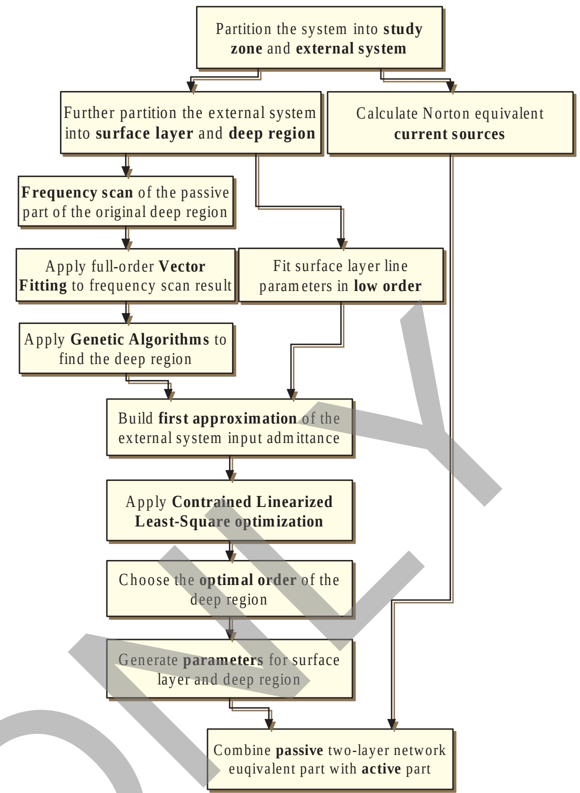


Fig. 3. Flowchart for obtaining robust TLNE model for generic external systems

## III. Case Study

The 220kV system used here is a modification of a system used for transient stability studies in [12]. Fig. 4 shows the system single-line diagram and partitioning. The transient phenomena to be analyzed are balanced capacitor switching events at Bus 15 and and three-phase to ground fault at Bus 16. In both cases, we measure the phase A voltage and current at Bus 15. The system is first partitioned into a study zone and an external system, as shown in Fig. 4. The decision to split the external system network into surface layer and deep region relies on engineering judgement. By applying the TLNE model to the passive part of the system, TL1, TL2 and TL3 are considered for surface layer and the remaining system comprises the deep region.

Due to the multi-port nature and passivity constraints of the system, in fitting the surface layer parameters $Z_c(\omega)$ and $P(\omega)$, non-linear fitting technique [8] is used. Following the flow chart in Fig. 3, the frequency scan of passive deep region produces phase-domain admittance matrix and short-circuit currents provide Norton equivalent current sources. Clarke's transformation further decouples the matrix to ground mode and aerial mode in modal domain. Since the transients are balanced, only aerial mode, which has complex frequency response, is considered for both surface layer transmission lines and deep region FDNE.

The computation time of finding deep region by GAs is approximately 15 hours on a Pentium IV 1.6GHz computer. However, since this is running off-line, it is of no concern as far as further real-time implementation is considered. The order versus percentage RMS-error is shown in Fig. 5, from which, order 21 is found to be the optimal order for the deep region with low RMS-error of 5.533%. Figures 6 through 8 show the deep region frequency response generated by GA and after CLLSQ optimization. Figures 9 through 11 show the input admittances of external system. It can be observed that the first approximations of input admittances are very close to original. Frequency responses of both the external

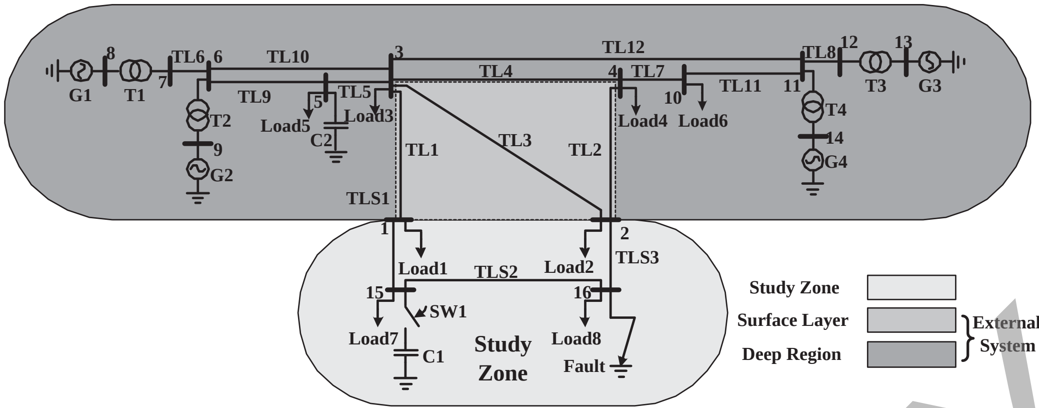


Fig. 4. System single-line diagram and partitioning for the Case Study

system and deep region after CLLSQ optimization overlap those generated from GAs. It is shown that due to their relative insensitivity to the input admittance, some pronounced resonant peaks in the deep region are not chosen by GAs. Further CLLSQ optimization mostly enhances the accuracy in the low frequency range, especially at dc, where the maximum RMS-error is only 2.4%.

Fig. 12 shows phase A voltage and current transients at Bus 15 where capacitor C1 is switched at 0.05s. Fig. 13 shows phase A fault current and voltage transients also at Bus 15, when the balanced fault is induced at Bus 16 with a 2Ω fault resistance per phase. The fault occurs at 0.05s and is cleared at 0.15s. All transients are verified via ATP and PSCAD/EMTDC with a time step of $\Delta T = 20\mu s$. Detailed agreement between the full model of the system and the the robust TLNE model is observed. The computational time is a major saving in the robust TLNE. Table I shows the CPU time differences among full model, FDNE model and the robust TLNE model on the same computer. The FDNE model with 160th order has 6.859% RMS-error (higher than the TLNE model) and does not demonstrate great savings on CPU time due to its high order. As seen from Table I, the robust TLNE model is 9 to 12 times faster than the full model, which makes it suitable for real-time digital simulation. Application of the existing method [5] to obtain TLNE of the same order produces 10.53% RMS-error, which is higher than the new approach.

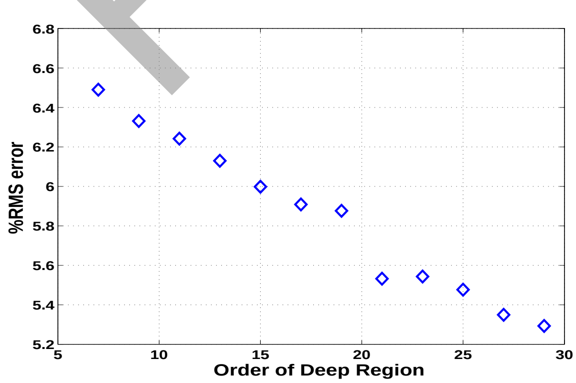


Fig. 5. Percentage RMS-error v.s. order of deep region in the Case Study

TABLE I

CPU TIME COMPARISON ($\Delta T = 20\mu s$) FOR THE CASE STUDY

| | Tmax | Full model | FDNE(160th) | Robust TLNE |
|---|---|---|---|---|
| C1 switching | 0.15s | 1.034s | 1.024s | 0.082s |
| Fault | 0.20s | 1.072s | 1.064s | 0.117s |

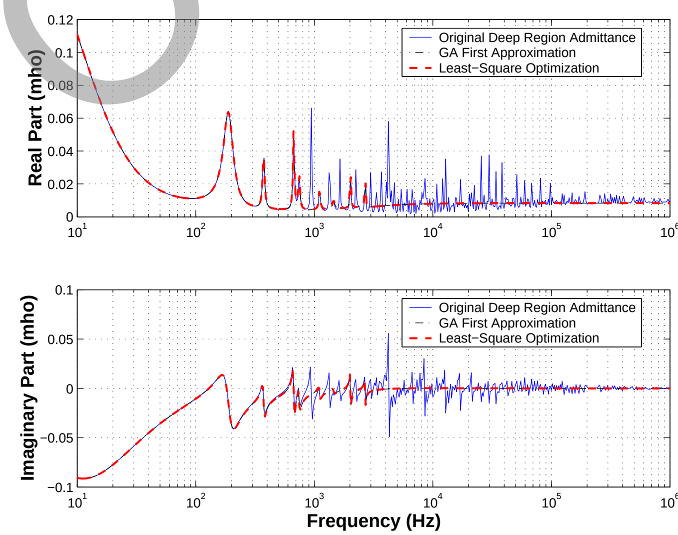


Fig. 6. Deep region admittance $\mathbf{Y}_{deep,11}$ in the Case Study

## IV. CONCLUSIONS

This paper presented a robust Two-Layer Network Equivalent (TLNE) for external system suitable for transient studies. Compared to the existing approach, the new method has the following merits:

1) Application of Genetic Algorithms in finding the appropriate deep region while its stability and passivity are ensured.
2) Guaranteed algorithm convergence in Constrained Linearized Least-Square Optimization.
3) Inclusion of frequency response at dc in the optimization for accurate transient dc offset.

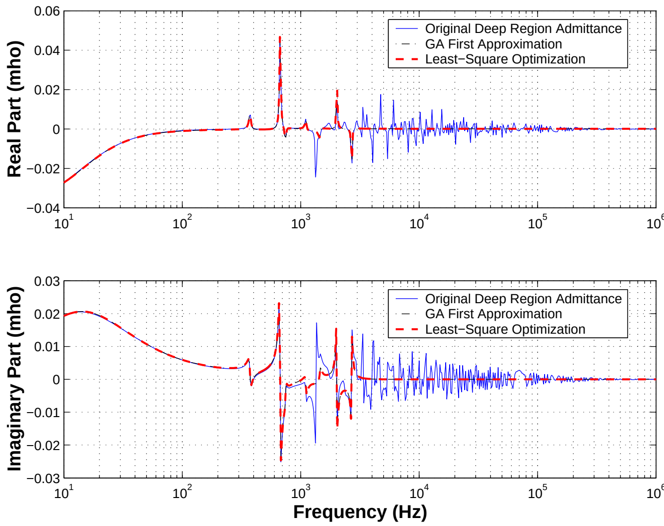


Fig. 7. Deep region admittance $\mathbf{Y}_{deep,12}$ in the Case Study

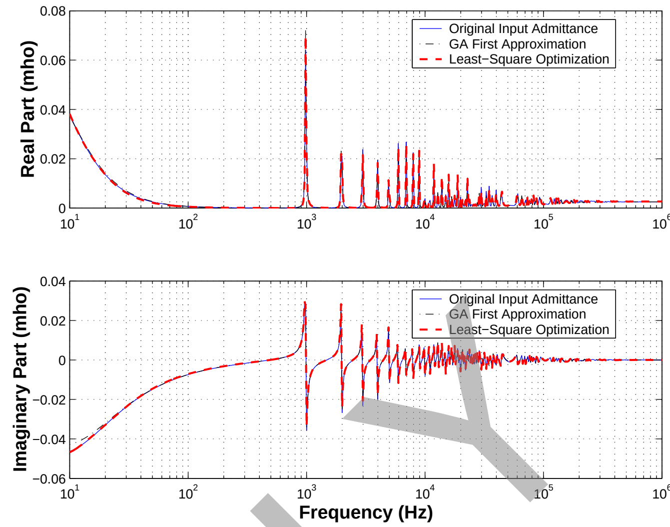


Fig. 9. Input admittance $\mathbf{Y}_{input,11}$ in the Case Study

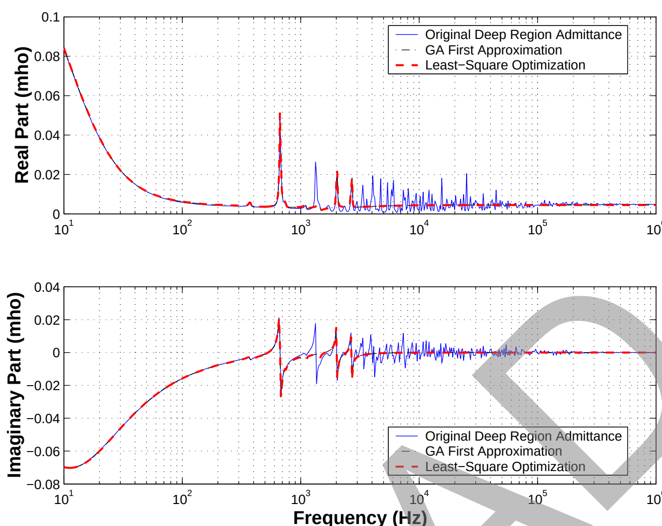


Fig. 8. Deep region admittance $\mathbf{Y}_{deep,22}$ in the Case Study

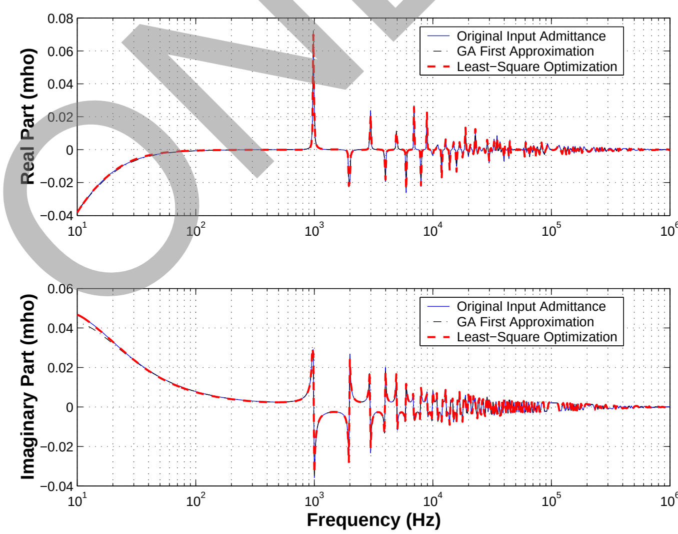


Fig. 10. Input admittance $\mathbf{Y}_{input,12}$ in the Case Study

4) Enhancement of model accuracy through optimizing surface layer parameters.
5) Deep region order determination feature.

A case study based on a three-phase multi-port system verified the effectiveness (accuracy and computational efficiency) of the robust TLNE model vis-á-vis the full model, FDNE model and the existing TLNE model. Time-domain simulation in ATP and PSCAD/EMTDC validated the model performance. For future research, a realistic test case involving the Alberta Interconnected Electric System (AIES) has been modeled by the robust TLNE approach, and its real-time implementation is currently being carried out.

## APPENDIX

### A. Sample System Diagram

The sample system diagram for generating frequency response of input admittance (Fig. 2) is shown in Fig. 14.



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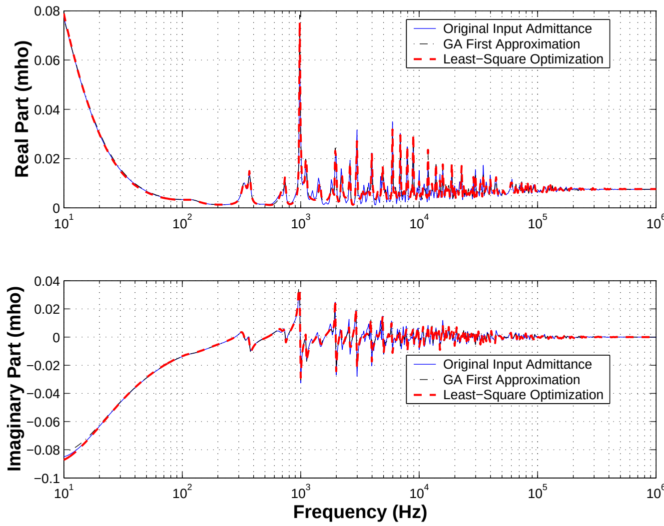


Fig. 11. Input admittance $\mathbf{Y}_{input,22}$ in the Case Study

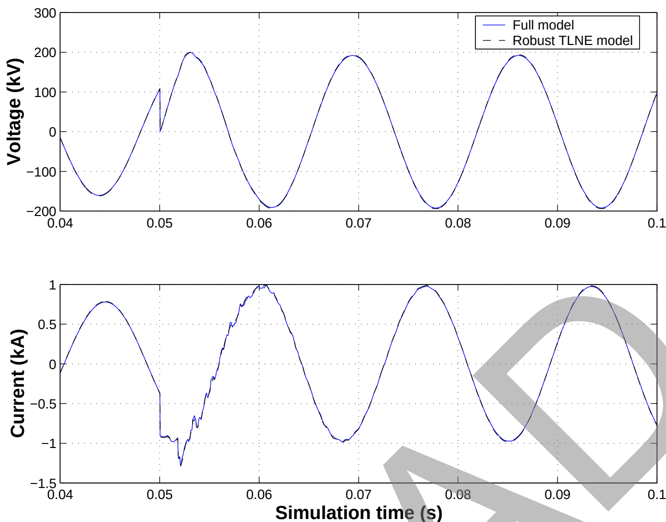


Fig. 12. $C1$ switching transient comparison in the Case Study


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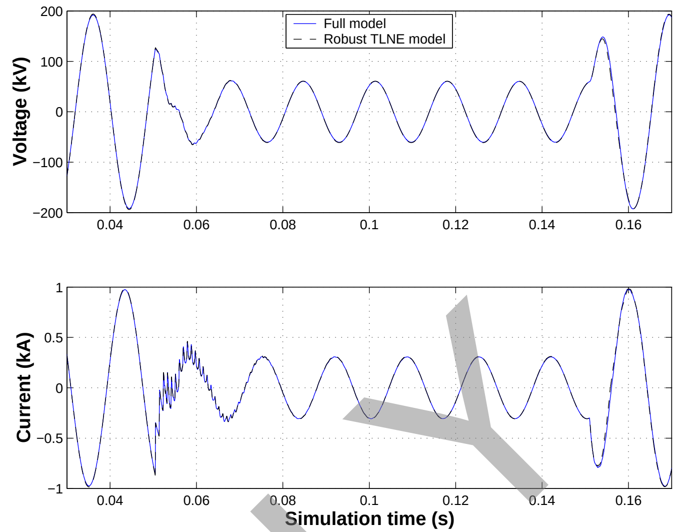


Fig. 13. Three-phase fault transient comparison in the Case Study

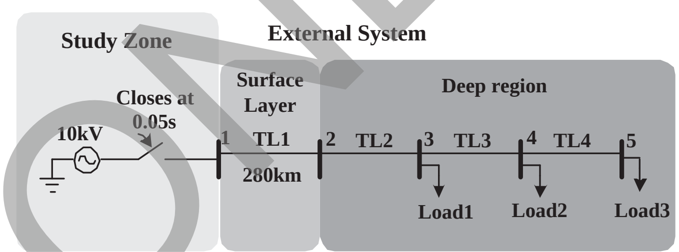


Fig. 14. Sample system diagram and partitioning for obtaining frequency response in Fig. 2

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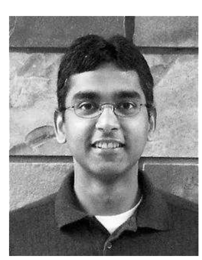

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