*The illiterate of the 21st century will not be those who cannot read and write, but those who cannot learn, unlearn, and re-learn.*

– Alvin Toffler

**University of Alberta**

# The Baseline Approach to Agent Evaluation

by

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*To Kim*
*My co-author in life*

# Abstract

Efficient, unbiased estimation of agent performance is essential for drawing statistically significant conclusions in multi-agent domains with high outcome variance. Naïve Monte Carlo estimation is often insufficient, as it can require a prohibitive number of samples, especially when evaluating slow-acting agents. Classical variance reduction techniques typically require careful encoding of domain knowledge or are intrinsically complex. In this work, we introduce the *baseline* method of creating unbiased estimators for zero-sum, multi-agent high-variance domains. We provide two examples of estimators created using this approach, one that leverages computer agents in self-play, and another that utilizes existing player data. We show empirically that these baseline estimators are competitive with state-of-the-art techniques for efficient evaluation in variants of computer poker, a zero-sum domain with notably high outcome variance. Additionally, we demonstrate how simple, yet effective, baseline estimators can be created and deployed in domains where efficient evaluation techniques are currently non-existent.

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# Chapter 1

# Introduction

Efficiently evaluating agent performance is an important problem for many highly stochastic artificial intelligence domains. An agent's performance in this context is often measured as the agent's expected value given a particular environmental configuration. Measuring this performance is typically achieved by averaging the values of the observed outcomes from random samples of the agent and the environment. This process, referred to as Monte Carlo estimation, provides an unbiased estimate of an agent's true performance and is probably the most common approach to evaluation given its simplicity.

Since the Monte Carlo approach only provides an estimate for the expected value of an agent, the statistical confidence of this estimate must also be taken into consideration when making claims regarding the agent's true performance. In order to increase the level of statistical significance when using Monte Carlo estimation, one can simply increase the number of random samples being used in the evaluation. While not problematic when the stochasticity of the agents or the environment is low, as the entropy in the system increases, so does the number of samples required by Monte Carlo estimation in order to produce statistically significant estimates. This problem is further compounded when the costs associated with generating samples is high, such as when evaluating human agents or slow-acting computer agents. In order to make such costs manageable, more sophisticated evaluation techniques must be put to use in any high variance domain.

The main focus of this work explores a new approach to evaluating agent performance in multi-agent, zero-sum domains. Entitled the *baseline* approach, this method is designed to be an effective framework for creating low-variance, unbiased estimators that is as simple as Monte Carlo estimation in agent evaluation settings. This work also provides two practical implementations of the baseline approach designed for use in two different agent evaluation scenarios. The first application of the baseline approach is designed for situations where access to competent computer agents for the evaluation domain exist and are easily accessible. We call estimators created in this manner *agent baseline* estimators and show results of their effectiveness in the domains of Texas hold 'em poker and Ad Auctions [22]. The second application of our approach is targeted at a more general case of the agent evaluation problem, one in which competent computer agents who can act in the evaluation domain may not exist. In this case, creation of the baseline estimator relies on the existence of player data for the domain of interest, which is often much more plentiful than competent computer agents. We refer to estimators created using player data as *data baseline* estimators, and we analyze their performance by experimenting in the domain of Texas hold 'em poker on a mixture of human and computer agent data.

The overall goal for the analysis of both applications of the baseline approach is to show that estimators of this nature are simple to implement, effective in their use, and should be considered for many agent evaluation scenarios.

# Chapter 2

# Background and Related Work

This chapter provides an overview of the core concepts regarding the problem of efficiently evaluating agents in zero-sum domains. We begin with the definition of extensive form games with imperfect information as a way of introducing some of the notation and terminology used throughout the body of this work. Following this, we give the definition of Monte Carlo estimation, since it is the method we will use as a base comparison for all of the evaluation techniques explored in this work. The remainder of the chapter is then dedicated to introducing various techniques for creating low-variance, unbiased estimators, which are often referred to as *variance reduction* techniques. For each variance reduction technique, we will describe its general formulation and when applicable, give examples of the technique applied to the problem of agent evaluation. This chapter concludes with a brief description of the various domains used in the empirical analysis of our approach.

## 2.1 Extensive Games With Imperfect Information

Extensive games provide a framework for describing multi-agent interaction in sequential decision making problems. Agents in an extensive game are able to choose policies that can reason about their plan of action whenever they are required to make a decision. Typically, the agents and chance alternate acting within the game until some terminal history is reached, upon which the players each receive some reward. In cases where an agent is unable to observe other agents' actions or the actions of chance, indistinguishable sequences of events can be created from the point of view of the observing agent. Games with this property are referred to as games with *imperfect information* and can be defined as follows:

**Definition 1 (Extensive Game)** *[[16], p. 200] A finite extensive game with imperfect information has the following properties and components:*

- *A finite set of $N$ **players**.*
- *A finite set $H$ of sequences, the possible **histories** of actions such that the empty sequence is in $H$ and every prefix of a sequence in $H$ is also in $H$. $Z \subseteq H$ is the set of **terminal histories**. $A(h) = \{a : (h, a) \in H\}$ are the actions available after a nonterminal history $h \in H \backslash Z$.*
- *A **player function** $P$ that assigns each nonterminal history to a member of $N \cup \{c\}$ where $c$ represents chance. $P(h)$ is the player who takes an action after history $h$. If $P(h) = c$, then chance determines the action after history $h$. Let $H_i = \{h \in H : P(h) = i\}$ be the set of histories where player $i$ is next to act.*
- *A function $\sigma_c$ that associates with every history $h$ for which $P(h) = c$ a probability measure $\sigma_c(\cdot|h)$ on $A(h)$ where each such probability measure is independent from one another. Let $\sigma_c(a|h)$ be the probability that action $a$ occurs after history $h$.*
- *For each player $i \in N$, a partition $\mathcal{I}_i$ of $H_i$ is the **information partition** for player $i$ with the property that $A(h) = A(h')$ whenever $h$ and $h'$ are in the same member of the partition. $I_i \in \mathcal{I}_i$ is an **information set** for player $i$.*
- *For each player $i \in N$, a utility function $u_i : Z \to \mathbb{R}$ that maps each of the the terminal histories in $Z$ to some real value, i.e.: $u_i(z)$ is the reward that player $i$ receives when reaching terminal history $z$. If $\sum\limits_{i \in N} u_i(z) = 0$ for all $z \in Z$ then the game is said to be **zero-sum**.*

Empirical analysis of the contributions presented in this work is performed in domains that can all be represented by extensive form games of imperfect information that exhibit the zero-sum constraint, although they need not necessarily be two-player .

## 2.2 Monte Carlo Estimation

In the context of agent evaluation, Monte Carlo estimation refers to the method of generating random repeated samples of agents interacting within a specific environment in order to compute their relative performance to one another. By simply sampling outcomes from the agents and the environment, the Monte Carlo method provides a naïve mechanism for computing the performance of the agents when the samples are drawn in an identically and independently distributed *(i.i.d.)* fashion [12]. The volume of samples needed to make statistically significant claims regarding an agent's performance depends highly on the stochasticity of the agent's policies and of the environment itself.

Formally, let $X = (X_1, \dots, X_n)$ be the random variable corresponding to the observed outcomes of an agent whose expected value we are trying to estimate. The Monte Carlo approach then averages together all of the samples of $X$ in order to create an unbiased estimator of $\mathbb{E}[X]$. This average, denoted as $\bar{X}$, is computed as

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{2.1}$$

where $X_i$ is the outcome of a single sample of the agent's behaviour and $n$ is the number of observed samples. It is clear from this that as we provide more samples, the variance of $\bar{X}$ decreases by the following:

$$\mathbb{V}\mathrm{ar}[\bar{X}] = \mathbb{V}\mathrm{ar}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n^2}\sum_{i=1}^{n}\mathbb{V}\mathrm{ar}[X_i] = \frac{\mathbb{V}\mathrm{ar}[X]}{n} \tag{2.2}$$

Often the biggest problem with using the Monte Carlo method to estimate $\mathbb{E}[X]$ is that the number of samples required to achieve some fixed level of accuracy may be arbitrarily large for a given domain, as it is directly related to the variance of $X$ [12].

## 2.3 Efficient Estimation

The goal of efficient estimation techniques is to increase the efficiency of the Monte Carlo method when estimating $\mathbb{E}[X]$ by reducing the variance of the estimate. This can be done by either reducing the number of samples required to compute an accurate estimate or by increasing each sample's quality without adding bias.

### 2.3.1 Antithetic Variates

Antithetic variates are one common approach to reducing the variance of an observed random variable by generating antitheses to the statistic being estimated [12]. These antitheses are produced by introducing negative dependancies between the observed variable, $X$, and the corresponding antithetic variable, $Y$. The antithetic unbiased estimator, $Z$, is then constructed by averaging the values of $X$ and $Y$

$$Z_i = \frac{X_i + Y_i}{2} \tag{2.3}$$

Given that $X$ and $Y$ have the same expected value, the expected value of $Z$ is

$$\begin{aligned}\mathbb{E}[Z] &= \frac{1}{2}(\mathbb{E}[X] + \mathbb{E}[Y]) \\ \mathbb{E}[Z] &= \mathbb{E}[X] \qquad (2.4)\end{aligned}$$

which shows that the antithetic estimator $Z$ is an unbiased estimator of $X$. Seeing that $X$ and $Y$ have the same distribution and therefore the same variance, the variance of $Z$ is computed as

$$\begin{aligned}\mathbb{V}\text{ar}[Z] &= \frac{\mathbb{V}\text{ar}[X] + \mathbb{V}\text{ar}[Y] + 2\mathbb{C}\text{ov}[X,Y]}{4} \\ \mathbb{V}\text{ar}[Z] &= \frac{\mathbb{V}\text{ar}[X] + \mathbb{C}\text{ov}[X,Y]}{2} \qquad (2.5)\end{aligned}$$

From the reduced variance equation, it is clear that antithetic variate estimators rely on $\mathbb{C}\text{ov}[X,Y] < 0$ in order to be more efficient than simple Monte Carlo estimation [12]. This of course assumes that the effort required to create instances of $Y$ is roughly equal to that of generating samples of $X$.

The biggest drawback of using antithetic variates comes from the complications associated with guaranteeing the negative covariance when generating the antithetic pairs. In the context of agent evaluation, some solutions to this problem can have undesirable side effects, such as introducing the inability to evaluate an individual agent's skill [11].

### Duplicate

*Duplicate* is a technique for producing antithetic pairs in multi-agent games. The duplicate approach has been used successfully in bridge [26] and computer poker [1, 13, 19] and is a potentially effective approach to variance reduction in scrabble-like games [21]. Duplicate methods create antithetic pairs by replicating the starting and future chance events of a game for each configuration of the players. For scrabble-like games, this is done by setting the order in which the tiles will be drawn and in card games, it is achieved by fixing the order of the cards in the deck. The score for each player is their average score across all the permutations that were sampled. The intuition behind the duplicate approach is that the variance associated with a particularly good or bad starting position will be reduced by scoring each instance in this way.

Using the duplicate approach for agent evaluation can create logistical problems, particularly when evaluating human agents. Since duplicate requires each player to play the same sequence of random events multiple times, the players are required to forget all past knowledge from previous replications. While not particularly problematic for many computer-agent scenarios, this restriction essentially makes it impossible to evaluate human agents. For instance, duplicate bridge is typically scored in a very different manner than the duplicate approach would suggest due to this problem [26] and using duplicate in computer poker requires the use of teams, forgoing any attempt at evaluating the performance of individual players [19]. Combined with the potentially laborious and error-prone task of creating each duplicate replication, the usefulness of using the duplicate approach to create efficient, unbiased estimators is questionable for many agent evaluation scenarios.

### 2.3.2 Control Variates

Control variates provide another way of creating unbiased estimators with reduced sample variance. The control variate approach is similar to that of antithetic variates in that it attempts to exploit certain properties of the observed random variable $X$ [12]. In particular given the variable whose quantity is being estimated, $X = (X_1, \ldots, X_i)$, the control variate approach is to generate a correlated *control* variable, $Y = (Y_1, \ldots, Y_i)$, whose expected value, $\mathbb{E}[Y]$, is a known quantity. Given that every $(X_i, Y_i)$ pair is i.i.d, the control variate estimator, $Z$, is defined such that each $Z_i$ is computed as

$$Z_i = X_i - c(Y_i - \mathbb{E}[Y]) \tag{2.6}$$

The sample mean of the estimator, denoted as $\bar{Z}$, is then equal to

$$\bar{Z} = \frac{1}{n}\sum_{i=1}^{n}(X_i - c(Y_i - \mathbb{E}[Y])) \tag{2.7}$$

For any $c \in \mathbb{R}$, $\bar{Z}$ is an unbiased estimator of $\mathbb{E}[X]$ since

$$\begin{aligned}\mathbb{E}[\bar{Z}] &= \mathbb{E}[\bar{X} - c(\bar{Y} - \mathbb{E}[Y])] \\ &= \mathbb{E}[\bar{X}] - \mathbb{E}[c(\bar{Y} - \mathbb{E}[Y])] \\ &= \mathbb{E}[\bar{X}] - c(\mathbb{E}[\bar{Y}] - \mathbb{E}[Y]) \\ &= \mathbb{E}[X] \end{aligned} \tag{2.8}$$

and the variance of each $Z_i$ is equal to

$$\begin{aligned}\mathbb{V}\text{ar}[Z_i] &= \mathbb{V}\text{ar}[X_i - c(Y_i - \mathbb{E}[Y])] \\ &= \mathbb{V}\text{ar}[X] + c^2\mathbb{V}\text{ar}[Y] - 2c\mathbb{C}\text{ov}[X, Y]\end{aligned} \tag{2.9}$$

From these equations, it follows that $\bar{Z}$ has lower variance than the Monte Carlo estimate when

$$c^2\mathbb{V}\text{ar}[Y] < 2c\mathbb{C}\text{ov}[X, Y] \tag{2.10}$$

This shows that the optimal choice of coefficient, $c^*$, is the one that minimizes the variance of the estimator and is equal to

$$c^* = -\frac{\mathbb{C}\text{ov}[X, Y]}{\mathbb{V}\text{ar}[Y]} \tag{2.11}$$

Thus, when using the optimal coefficient $c^*$, the resulting variance of $Z$ is

$$\begin{aligned}\mathbb{V}\text{ar}[Z] &= \mathbb{V}\text{ar}[X - c^*(Y - \mathbb{E}[Y])]\mathbb{V}\text{ar}[Y] \\ &= 1 - \rho^2_{X,Y} \qquad (2.12)\end{aligned}$$

where $\rho_{X,Y}$ is the correlation between $X$ and $Y$ and $1-\rho^2_{X,Y}$ corresponds to the magnitude of the resulting variance reduction. Essentially, the more the control variable and the observed random variable co-vary, the less variance there will be in the resulting estimator.

The difficulty in practice when trying to use $c^*$ comes from the observation that if $\mathbb{E}[X]$ is unknown, $\mathbb{V}\text{ar}[X]$ and $\rho_{X,Y}$ are likely to be unknown as well. In this situation, we can estimate the optimal coefficient, $\hat{c}$, by replacing $\mathbb{V}\text{ar}[X]$ and $\rho_{X,Y}$ with their sample counterparts. Let $\hat{Z}$ be the estimator computed using the estimated optimal coefficient $\hat{c}$. Note that by applying the strong law of large numbers, as the sample size $n \to \infty$, $\hat{c}$ converges to $c^*$ with probability 1 [12]. This estimation introduces some bias, precisely, the bias of $\hat{Z}$ is equal to

$$\begin{aligned}Bias(\hat{Z}) &= \mathbb{E}[\hat{Z}] - \mathbb{E}[X] \\ &= -\mathbb{E}[\hat{c}(Y - \mathbb{E}[Y])] \qquad (2.13)\end{aligned}$$

This bias is not necessarily zero since $\hat{c}$ and $\bar{Y}$ are not independent in this setting. However the bias is typically on the order of $\mathcal{O}(1/n)$ and the standard error being of $\mathcal{O}(1/\sqrt(n))$ [12].

Some of the same problems that arise with antithetic variates are also true of control variates. While the samples generated for use in the control do not have to be strictly negatively correlated, they do require a high degree of correlation in order to make the resulting estimator effective. Additionally, the fact that $\mathbb{E}[Y]$ needs to be a known value is often problematic. Control variates have, however, been used with success in agent evaluation [11] and as a method of reducing variance in Monte Carlo Tree Search techniques [15, 23]. Since the majority of this work is based off of this approach, we will describe an application of control variates in Chapter 3.

### 2.3.3 Advantage Sum Estimators

Zinkevich et al. provided a general framework for creating unbiased estimators for agent evaluation in stochastic domains, which are known as *advantage sum estimators* [27]. Advantage sum estimators are created by separating out the effects of *skill*, *luck* and *positional advantage* for each decision an agent makes. Let $H$ be the set of all reachable histories, $Z \subseteq H$ be the subset of all terminal histories and the utility function $u : Z \to \mathbb{R}$ be the function that maps terminal histories to real values. The goal of the advantage sum estimator is then to provide a low-variance, unbiased estimate for $\mathbb{E}[u]$ given the dynamics of the system [27].

Given a value function $V : H \to \mathbb{R}$ that maps histories to real values, let $S_V$, $L_V$ and $P_V$ be the components of $V$ that correspond to the skill, luck and positional values of $V$. These components can be described as follows [24]:

$$\begin{aligned}
S_{V_j}(z) &= \sum_{\substack{ha \sqsubseteq z \\ P(h) \neq c}} V_j(ha) - V_j(h) \\
L_{V_j}(z) &= \sum_{\substack{ha \sqsubseteq z \\ P(h) = c}} V_j(ha) - V_j(h) \\
P_{V_j} &= V_j(\emptyset)
\end{aligned}$$

$V_j$ refers to the value function for player $j$, $ha$ refers to the history $h$ followed by action $a$ and the function $P(h)$ determines which player acts after history $h$, either one of the agents or chance, $c$. Given that

$$u_j(z) = S_{V_j}(z) + L_{V_j}(z) + P_{V_j} \tag{2.14}$$

the advantage sum estimator is defined as

$$\hat{u}_{V_j}(z) = S_{V_j}(z) + P_{V_j}(z) \tag{2.15}$$

It follows that advantage sum estimator provides an unbiased estimate of $u_j(z)$ when $\mathbb{E}[L_{V_j}(z)]$ is zero from the observation that $\mathbb{E}[\hat{u}_{V_j}] = \mathbb{E}[u_j]$ and $\mathbb{V}\mathrm{ar}[\hat{u}_{V_j}] \leq \mathbb{V}\mathrm{ar}[u_j]$. Thus, the key to creating an unbiased advantage sum estimator is being able to craft a value function to satisfy this constraint, known as the *zero-luck* constraint.

### DIVAT and MIVAT

DIVAT [6] and MIVAT [24] are both implementations of advantage sum estimators in the domain of Texas hold 'em poker. The DIVAT estimator was created using a handcrafted value function based on an expert-knowledge, hand-crafted policy specifically designed for the two-player limit variant of Texas hold 'em. DIVAT evaluates the outcome for each action in a hand by assuming that players play according to the DIVAT policy from that action forward, much in the same way David Wolfe evaluated actions in the game of blackjack [25]. Each of the players is awarded a score based on how their play impacted the outcome of the game by comparing the value realized by the player versus the value that would have been achieved had the DIVAT policy been used in the same situation.

This type of evaluation was proven to be unbiased by Zinkevich et al. [27] and the empirical results showed that the DIVAT estimator was able to achieve a 75% to 85% reduction in the variance over that of Monte Carlo estimation when evaluating agents in two-player limit Texas hold 'em. Since the DIVAT estimator's value function is based on a hand-coded policy, its use is very limited outside the domain of two-player limit Texas hold 'em. Other simpler, more generic policies have been experimented with using this framework, such as *always-call* and *always-raise*, but these policies tend to underestimate or overestimate the values of hands and produce much weaker low-variance estimators.

MIVAT [24] is an extension of the DIVAT framework that removes the need to hand-code a value function. The MIVAT value function is instead learned from samples of player data based on an optimization, the objective of which is to find a value function that minimizes the variance of the estimator. By using a reformulation of the advantage sum estimator approach, the resulting optimization is defined as

$$\min_{V_j} \left( \hat{u}_{V_j}(z_t) - \frac{1}{T} \sum_{t'=1}^{T} \hat{u}_{V_j}(z_{t'}) \right)^2 \tag{2.16}$$

where $z_1, \ldots, z_T$ are sample outcomes from the data. For tractability, the MIVAT estimator uses a linear value function such that

$$V_j(h) = \phi(h)^T \theta_j \tag{2.17}$$

where $\phi : H \to \mathbb{R}^d$ is a mapping of histories to a vector of $d$ features. It turns out that the optimal $\theta_j^*$ has a closed-form solution [24], which is then used in the MIVAT value function.

MIVAT has the advantage over DIVAT in that it can be used in more domains than just two-player limit Texas hold 'em. For instance, MIVAT has been used in multi-player and no-limit variants of poker and can be used in any finite-horizon MDP or POMDP or in any extensive form game. Using a base set of features, MIVAT performs equally as well as DIVAT in two-player limit Texas hold 'em and outperforms DIVAT when DIVAT scores are included as a feature. MIVAT, although much more flexible than DIVAT, requires designing and selecting meaningful features in order to learn a value function capable of strong variance reduction, which can be a non-trivial task in many domains.

### 2.3.4 Importance Sampling

Importance sampling is a variance reduction technique that attempts to lower the variance of an observed random variable $X = (X_i, \ldots, X_n)$ by changing the underlying distribution from which samples of $X$ are generated. This is done by giving more weight to the *important* samples of $X$ in order to increase sampling efficiency of Monte Carlo estimation [12]. More specifically, assume that the random variable $X$ has probability density $f$, and that each $X_i \in X$ is drawn according to $f$. Importance sampling states that instead of drawing the samples $X_i, \ldots, X_n$ from the distribution $f$, they are drawn according to some other probability density, $g$, that satisfies the equation

$$f(x) > 0 \Rightarrow g(x) > 0 \tag{2.18}$$

This introduces bias into the sampling of $X$, however the bias can be accounted for by weighting the terms properly when estimating $\mathbb{E}[X]$. The weighting factor, $w$, is then done according the *likelihood ratio* between $f$ and $g$ where

$$w(X_i) = \frac{f(X_i)}{g(X_i)} \tag{2.19}$$

Let $Z$ be the estimator such that

$$Z_i = w(X_i)X_i \tag{2.20}$$

The sample mean of the estimator, $\bar{Z}$, is then

$$\bar{Z} = \frac{1}{n}\sum_{i=1}^{n} Z_i \tag{2.21}$$

By ensuring that the weighting is done according to Equation 2.19, $\bar{Z}$ is a provably unbiased estimator of $\bar{X}$ [12].

Comparing the variance when estimating $\mathbb{E}[X]$ with and without importance sampling depends entirely on the choice of $g$. Given that the variance of $Z$ is

$$\mathbb{V}\text{ar}[Z] = \mathbb{E}[Z^2] - (\mathbb{E}[Z])^2 \tag{2.22}$$

we only need to consider the $\mathbb{E}[Z^2]$ term when comparing the $\mathbb{V}\text{ar}[X]$ to the $\mathbb{V}\text{ar}[Z]$. If we expand this term out, we get

$$\mathbb{E}[Z^2] = w(X)X^2 \tag{2.23}$$

Comparing $\mathbb{E}[Z^2]$ and $\mathbb{E}[X^2]$ shows that an arbitrary choice of $g$ could lead to an infinitely larger or smaller variance in the importance sampling estimator. Unfortunately there is no general rule for choosing $g$, thus choosing an effective importance sampling density is often referred to as an "art" [12].

**Importance Sampling and Poker**

Importance sampling techniques have been used with success in the domain of computer poker for both agent evaluation and policy selection [4, 7]. Bowling et al. [7] applied the importance sampling technique in the domain of Texas hold 'em poker as a way of creating low-variance, unbiased estimators for post-game analysis. Although their work describes how to apply importance sampling for different *on-policy* and *off-policy* cases, we will summarize the *off-policy full-information* usage, as it is the most applicable to agent evaluation. Agent evaluation in this setting refers to the situation where, given a function that maps terminal histories to real values, $V : Z \to \mathbb{R}$, we would like to estimate $\mathbb{E}_{z|\sigma}[V(z)]$ where $\sigma$ is the policy we are creating the estimator for. The terms off-policy and full-information are used to describe how the outcomes are generated and evaluated. In the off-policy scenario, the outcomes are generated according to some distribution $\pi^{\hat{\sigma}}$, where $\hat{\sigma}$ differs only for player $i$ from $\sigma$ and the goal of the evaluation is to assess how well a different strategy would perform in place of player $i$. Full-information in this context merely states that all of the information for every outcome is observable, including all of the private information for each player.

The off-policy importance sampling estimator is constructed by examining the subset of terminal histories $U(z') \subseteq Z$ such that $z' \in Z$ and $z' \in U(z')$, where $U(z' \in Z) \subseteq Z$ is a mapping of terminal histories to a set of terminal histories. In this mapping, $z' \in U(z')$ and an unbiased estimator is constructed by considering the history $z'$ whenever a history from the set $U(z')$ is observed. Alternatively, the set of synthetic histories considered when observing z can be denoted as the set $U^{-1}(z)$. The importance sampling estimator, $V_U(z)$, is then

$$V_U(z) = \sum_{z' \in U^{-1}(z)} V(z') \frac{\pi^{\sigma}(z')}{\pi^{\hat{\sigma}}(U(z'))} \tag{2.24}$$

The estimator weights the value of every $z'$ by the probability of reaching $z'$ according to both $\pi^{\sigma}(z')$ and $\pi^{\hat{\sigma}}(U(z'))$. Note that $V_U(z)$ is not an estimate of $V(z)$, rather they have the same expected value and good choices of $U$ can be constructed to only consider known strategies $\sigma_i$ and $\hat{\sigma}_i$. Also, as long as $\pi_i^{\hat{\sigma}}(z)$ is non-zero for all $z \in Z$ then $V_U$ is unbiased [7]

$$\mathbb{E}_{z|\hat{\sigma}}[V_U(z)] = \mathbb{E}_{z|\sigma}[V(z)] \tag{2.25}$$

The simplest choice for $U$ is simply $U(z) = \{z\}$, which is simply known as *basic* importance sampling, where if $\sigma_i = \hat{\sigma}_i$, the weighting is 1 and is equal to simple Monte Carlo estimation. Two other choices for $U$ that only depend on $\sigma_i$ and $\hat{\sigma}_i$ are *game ending actions* and *private information* importance sampling [7]. Importance sampling with game ending actions only considers prefixes of $z$ where the remaining decisions are for player $i$ or chance. This allows for the importance sampling estimator to evaluate actions that could have been made by player $i$ earlier in the sequence that would have ended the game early, such as folding in Texas hold' em poker. Importance sampling with private information considers the histories where player $i$ had differing private information as these histories are indistinguishable from the perspective of the other players. An example of this in Texas hold 'em would be the situations where player $i$ held different private cards. These two choices can also be combined together and the value function $V$ can also be replaced with any unbiased estimate of $V$, such as values from a duplicate estimator. Importance sampling in this manner only works when the full strategy is known, which means it is limited to evaluating the performance of accessible computer agents.

### 2.3.5 Additional Applied Statistical Estimators

In addition to the applications of efficient estimators previously described, this section summarizes a collection of other work that does not specifically fall into any of the aforementioned general categories.

#### Monte Carlo Estimation in Blackjack

In the game of blackjack, David Wolfe explored the problem of evaluating how well an agent performs with respect to an optimal policy [3, 25]. Since a good blackjack player may have to play upwards of 1.2 million hands in order to get an accurate estimate of their own skill [25], Wolfe used a form of Monte Carlo estimation based on the control variate approach to reduce the variance for any given hand.

Wolfe's method first involved creating a base oracle or near-oracle policy, denoted as $\sigma$. This policy was then used to evaluate the performance of an agent, $P$, by comparing the winnings of the agent, $w(P)$, to the winnings of base policy, $w(\sigma)$, for a given state of the deck. The estimated expected winnings of the agent then becomes

$$\mathbb{E}[w(P)] = \mathbb{E}[w(\sigma)] + \mathbb{E}[w(P) - w(\sigma)] \tag{2.26}$$

Seeing that the strategy of the agent being evaluated may not be known for every possible deck configuration, Wolfe creates an estimator by assuming that after the agent acts, all following actions will be according to that of $\sigma$. Scores are then assigned to the agent based on the difference in the expected value it would have received for the actions in the agent's strategy that differ from $\sigma$. Formally, let $P_{i,j}$ denote using the agent's strategy for the sequence of actions from $i \to j$ and $\sigma_{i,j}$ be the sequence of actions from $i \to j$ according to the policy $\sigma$. The expected value for the player then is the sum of these differences for every action made in a given hand,

$$\mathbb{E}[w(P)] = \mathbb{E}[w(\sigma)] + \sum_{i=1}^{n} \mathbb{E}[w(P_{0,i}, \sigma_{i+1,n}) - w(P_{0,i-1}, \sigma_{i,n})] \tag{2.27}$$

Wolfe evaluated agents using a *high-low* counting strategy and observed that his evaluation technique converged anywhere from 100 to 2500 times faster than that of pure Monte Carlo estimation depending on the number of decks and the shuffling policy of the shoe. This approach proved to be a powerful technique for agent evaluation and the basis for techniques such as DIVAT. Unfortunately, Wolfe's approach may not be tractable for many domains as it requires creating a near oracle policy and computing the expected value of that policy for all possible instances of the domain.

### Monte Carlo Rollouts in Backgammon

In the game of backgammon, Gerald Tesauro considered the the task of computing the near-oracle policy necessary to evaluate agent decisions using Wolfe's approach highly impractical for evaluating the play of backgammon agents [17]. When building his TD-Gammon agent, he instead tackled this problem by using a form of Monte Carlo estimation, called *rollouts*, to evaluate each action in the game. The Monte Carlo rollout approach estimates the score of each action by randomly assigning future chance events to the game and playing the rest of the moves out according to some fixed *rollout policy*, $\sigma$. This is then repeated multiple times and the best play is recorded as the rollout that achieved the highest equity. The difference between each rollout and the best one observed provides a score for the action being evaluated and the average score for all of the rollouts provides a low-variance, unbiased estimate for the value of that action. While the rollout policy may not be optimal, Tesauro observed that the same policy tends to lose roughly equal amounts on either side of the match when used to decide actions for both players during the rollouts. This means that in backgammon, even intermediate policies tend to be sufficient in estimating the equity of many of the actions.

Tesauro and Galperin also surmised that if the rollouts could be performed in real-time, they could be used to provide low-variance estimates for action selection during play [18]. The main disadvantage to this approach comes from the speed at which the rollouts are computed, which can be very slow for deep searches and uninformative if the depth is too shallow.

### Variance Reduction in Monte Carlo Tree Search

Recent work has been done using variance reduction methods in Monte Carlo Tree Search [23]. Monte Carlo Tree Search (MCTS) algorithms are methods of finding optimal decisions by constructing search trees based on random samples for a given domain [8], a popular variant being the Upper Confidence Bound for Trees (UCT) algorithm [15]. The UCT algorithm builds a partial search tree by using UCB1 [2] as the tree policy, treating the value of each child node as the expected value approximated by the Monte Carlo rollouts that occurred from the child node. Since these values are being treated as random variables with unknown distributions and are being sampled according to some stochastic process, they can be subject to both error and bias.

While much work has been done on reducing the bias of these estimates, Veness et al. approached the problem of reducing their variance by leveraging the variance reduction techniques of control variates and antithetic variates [23]. Although general in approach, their work specifically focused on applying these techniques as modifications to the UCT algorithm.

The control variate modification to the UCT algorithm replaces the value of state-acton pairs by a control variate counterpart. That is, for every given state-action pair $(s, a) \in SxA$, the value, $X_{s,a}$, is replaced by

$$Z_{s,a} = X_{s,a} - c_{s,a}(Y_{s,a} - \mathbb{E}[Ys, a]) \tag{2.28}$$

Here $Y_{s,a}$ acts as the control variable and has a known expected value for all state-action pairs. The controls were generated such that for a random trajectory, $S_t = s_t, A_t = a_t \ldots S_n = s_n, A_n = a_n$,

$$Y_{s_t,a_t} = \sum_{i=t}^{n-1} \mathbb{E}[\mathbb{I}[b(S_{i+1}]|S_i = s_i, A_i = a_i] - \mathbb{P}[b(S_{i+1}|S_i = s_i, A_i = a_i]) \tag{2.29}$$

In this formulation, the function $b : S \to \{true, false\}$ acts as a boolean function applied to the state $S$ and $\mathbb{I}$ is the boolean indicator of that function. This simplifies the control variate for any state-action pair since the expected value, $\mathbb{E}[Y_{s_t,a_t}]$, is zero [23].

The antithetic variates approach was applied to UCT by storing the realized trajectory starting with $s_{i+1}, a_{i+1} \ldots s_n, a_n$ for each state-action pair, $s_i, a_i$, at each node in the tree. This stored trajectory can then be used to create antithetic paths through the space on future iterations and the values of the two outcomes are averaged together as per the standard antithetic variate approach. Veness et al. showed that using these variance reduction techniques in UCT provided a significant reduction in the number of simulations required to reach a desirable level of play in three different test domains [23].

## 2.4 Evaluation Domains

In this section we introduce the domains for which the agent evaluation approaches explored in this work were evaluated with. For each domain, a brief overview of the rules and any modifications that were made in the evaluation section are provided.

### 2.4.1 Texas Hold 'em Poker

Texas hold 'em is one of the most popular games in the family of poker card games. It has appeared on television, in movies [10] and as a part of the World Series of Poker, a venue where thousands of professionals compete every year for millions of dollars [20]. Texas hold 'em was also the variant of poker used for the Man versus Machine poker matches [19] and is currently the variant of poker used by the Annual Computer Poker Competition (ACPC) [1]. These reasons, coupled with the intrinsically high outcome variance associated with poker games, have made Texas hold 'em an enticing choice for benchmarking the performance of agent evaluation techniques. In this section, we will outline the general rules common to all forms of Texas Hold 'em poker, and describe the specific variants and rules used by the 2013 Annual Computer Poker Competition, with is the primary domain used in the evaluation of our approach.

#### General Rules

Texas hold 'em poker is a card game played with anywhere from two to ten players using the standard 52 card deck. Each *hand* of Texas hold 'em consists of four rounds, commonly known as the *pre-flop*, *flop*, *turn* and *river*. Each round begins with a dealing phase, the structure of which differs slightly between rounds. The dealing phase in the pre-flop has the dealer deal every player two face-down *hole* cards, which is their private information for the hand. In the remaining rounds, the dealer deals face-up cards which are shared between all players, which are referred to as the *board* cards. Three board cards are dealt at the start of the flop round and one more board card is dealt in each of the turn and river rounds. This gives each player a maximum of seven cards from which to assemble their poker hand.

After the dealing phase in each round, a betting phase begins where the players are able to take turns making one of three different actions. The players may place wagers, known as *bets*, *call* a previously made bet, or *fold* their hand, forfeiting any wagers they have made or called. The winner of a hand in Texas hold 'em is either the last player remaining if everyone else folds, or the player with the best five card poker combination of their hole cards and the board cards. The winner receives the other players' wagers and the goal of Texas hold 'em is to maximize one's profit over many hands.

#### Betting Structures

The betting structure in Texas hold 'em defines what legal betting actions are available to players for a given situation. The most common betting structures, and those of interest in this work, are that of *no-limit* and *fixed-limit*. The no-limit betting structure allows players to make any size of bet greater than or equal to some minimum value and less than or equal to the total amount of chips they currently have available to them. With fixed-limit, the betting actions are split into two different fixed bet sizes, known as the *small bet* and *big bet*. The small bet is the size of bet a player may make in the pre-flop and flop rounds and the big bet, which is typically twice the size of the small bet, is used in the turn and river rounds. The fixed-limit betting structure also specifies a maximum number of bets in each round. This maximum is commonly equal to three bets in the pre-flop round and four in every other round.

In both betting structures, the players may also *check* the bet, which is to make a bet of zero chips. Players may also call a bet larger than their remaining chips which allows them to continue in the hand, however they can only win or lose an amount equal to their own wager. Any player who has wagered all of their remaining chips is referred to as being *all-in* and may no longer perform any actions for the remainder of the hand.

#### ACPC Rule Modifications

The ACPC uses what is known as *Doyle's game* in order to circumvent issues that can arise regarding the amount of chips available to the players throughout the course of a match. In Doyle's game, all of the players have their chips reset to some pre-defined number before each hand begins. In this way, each hand is played for exactly the same value and can be considered independent of every other hand.

### 2.4.2 Trading Agent Competition

The Trading Agent Competition (TAC) is an annual computer agent competition where autonomous agents interact in various trading agent type problems. Typically, agents in this domain act as the broker or seller of goods and services in a simulated consumer market. The particular domain explored in this work is that of the TAC Ad Auctions (TAC-AA) game [22].

**TAC Ad Auctions**

TAC-AA is a competition where the customers are simulated users of an internet search engine and the agents must place bids for the placement of their advertisements in a home entertainment market. The agents bids are in the form of the cost-per-click fee they are willing to pay for a given advertisement with the highest bids receiving the best ad placement. Simulated users in the system are then shown ads, which they may click on. Should the user purchase the goods featured in the ad, revenue is generated for the agent who placed the ad. The goal of the agents in this competition is to maximize their profit over the course of a fixed number of days.

**Customer Model**

The customers in this environment are simulated users who act by querying a search engine, clicking on advertisements and purchasing goods. Each user's behaviour is decided by the *customer simulation model*. The customer simulation model generates product and brand dispositions for each user, which determine the queries that particular user will submit to the search engine. Each query produces a ranked list of advertisements, called *impressions*, from which the customer model provides transition probabilities for the user. These transition probabilities determine whether or not the user will click on an advertisement in the impression, and whether or not the user will purchase a good once they have clicked on an advertisement.

**Agent Interaction**

During each day of the simulation, the advertising agents simultaneously and secretively place cost-per-click bids for the ads they wish to show given a particular user query. The types of advertisements an agent may show are either *generic* or *targeted*, which effect a user's likelihood of viewing a particular ad. Each agent also has a product specialty which impacts the likelihood of turning a product view into a sale. Agents may impose a spending budget for the day, which forces the search engine to stop showing advertisements for the agent once its budget is exhausted. At the end of each day, all of the agents receive a general report which contains the average ranking of the agents and the type of ads each agent chose to display for a random sample of user queries. Each agent also receives a report containing information specific to their own advertisements, such as the average per-click cost, the number of impressions that contained their advertisements, and the total number of advertisement clicks they received.

**TAC-AA Rule Modifications**

The rules regarding the scoring of the TAC-AA competition were modified in order to create a zero-sum environment. This was done by averaging the scores of all of the agents after each match and subtracting the average from each individual agent's score.

# Chapter 3

# The Baseline Approach

This chapter outlines a general approach for creating low-variance unbiased estimators based on the control variate methods. We call this the *baseline* approach to variance reduction, which provides a simple framework for creating control variable pairings in agent evaluation scenarios. Estimators created using this framework are referred to as *baseline estimators*, two different applications of which are detailed in Chapters 4 and 5.

## 3.1 Creating Baseline Estimators

The general baseline approach is mainly a reformulation of the control variate technique described in Section 2.3.2. This reformulation facilitates easy control variable generation when the environmental randomness in a domain is independent of the agents' actions. In this setting, we assume that the randomness of a domain can be represented as a single random variable, $Q$, with $q \in Q$ denoting the set of possible values for $Q$, and $q_i$ denoting the $i$-th such instance drawn from $Q$. An example of this could be the seed of the random number generator used to generate all of the stochastic events in the environment.

Let $g : q \to \mathbb{R}$ be the function that specifies the performance of an agent for a particular instance of the domain and $X = g(Q)$ be the random variable whose expectation we are estimating. In order to generate the control pairs for each instance in $X$, the baseline approach utilizes a function $\beta : q \to \mathbb{R}$ which we call the *baseline scoring function.* Applying the baseline scoring function to an instance $q \in Q$ produces a *baseline score* for the instance, which can be seen as the expected level of performance for an agent given the random instance. Creating the control variate for the observed instances is then achieved by creating a baseline score for each $q_i$ instance.

It is possible that some of the instances in the observed data are incomplete, meaning that the chance events may be unknown. The baseline scoring function must be able to handle this case in an unbiased fashion in order to produce an unbiased estimator. One example of this would be to randomly assign the future chance events during the computation of the baseline score for the instance.

The control variable, $Y^{\beta}$, used in the baseline estimator is computed simply as

$$Y^{\beta} = \beta(Q) \tag{3.1}$$

and the equation for the baseline estimator, $Z^{\beta}$, is written as

$$Z_i^{\beta} = X_i - c(Y_i^{\beta} - \mathbb{E}[Y^{\beta}]) \tag{3.2}$$

The sample mean of this estimator, $\bar{Z^{\beta}}$, is then an unbiased estimator for $\bar{X}$ by Equation 2.8 and has lower variance than $\bar{X}$ when $X$ and $Y^{\beta}$ are highly correlated.

## 3.2 The Expected Value of $Y^{\beta}$ in Zero-Sum Domains

So far, the baseline approach differs little from the underlying control variate approach, and still suffers from the problem of using baseline scoring functions that have a known $\mathbb{E}[Y^{\beta}]$. One easy way to handle this problem is to produce a baseline scoring function whose expected value is zero, the approach suggested by Veness et al. in their work on using control variates in Monte Carlo Tree Search [23]. Imposing certain conditions on the domain and the baseline scoring function ensures that $\mathbb{E}[Y^{\beta}]$ will be zero, resulting in the baseline estimator being unbiased. The conditions are as follows:

1. The utility outcomes in the domain must be zero-sum.
2. The domain must be factorable into the environment structure itself and the assignment of agents to possible positions within the environment. An example of environment positions would be the order that an agent must act in a game such as poker.
3. The agents are equally as often observed in each of the possible positions.
4. The $\beta$ function must only depend on the environment structure, and not on any actions of the participating agents, or on the configurations of specific agents to environment positions.
5. The $\beta$ function must return the same baseline scores for a given instance of the environment.

Given these restrictions, we can examine how they impact the expected value of the baseline scoring function, $\beta(Q)$, and show that it is indeed zero.

The second and third points consider the situation where one is sampling the environment and randomly assigning agents to positions within the environment. The fourth and fifth points state that since the returned baseline utilities only depend on the environment structure, each positional assignment must map to the same vector of utilities. Combining the second, third, fourth and fifth points show that because each agent has a probability of $\frac{1}{n}$ of being placed in each of the $n$ environment positions, the expected baseline score is $\frac{1}{n}$ times the sum of the baseline utilities over all of the positions. This sum must be zero from condition one, and thus, the expected baseline score must be zero.

By substituting $\mathbb{E}[Y^\beta] = 0$ in Equation 3.2, we get a simplified equation for the baseline estimator

$$Z_i^\beta = X_i - c(Y_i^\beta) \tag{3.3}$$

and the expected value of the resulting baseline estimator, $\bar{Z^\beta}$, is equal to

$$\begin{aligned}
\mathbb{E}[\bar{Z^\beta}] &= \mathbb{E}[\bar{X} - c\bar{Y^\beta}] \\
&= \mathbb{E}[\bar{X}] - \mathbb{E}[c\bar{Y^\beta}] \\
&= \mathbb{E}[\bar{X}] - c(0) \\
&= \mathbb{E}[X]
\end{aligned} \tag{3.4}$$

This shows that imposing these restrictions will guarantee that the baseline estimator will be unbiased. While these restrictions are not required to produce baseline estimators, they do provide a simple way of ensuring that a chosen baseline scoring function will produce unbiased baseline estimators.

# Chapter 4

# Agent Baseline

The *agent baseline*[1] approach is the first major contribution of this work to the field of agent evaluation. Agent baseline is an application of the baseline approach from Chapter 3 designed to create high-quality unbiased estimators for use in zero-sum stochastic domains. This is achieved by leveraging the self-play of agents that exists in the evaluation domain. We begin with the motivation that led to this application of the baseline approach as well as the general details on how to create the baseline scoring function using existing agents. Additionally, we will walk through the implementation details and provide empirical results showing the performance of agent baseline estimators when evaluating computer poker and ad-auction agents.

## 4.1 Motivation

The agent baseline approach is specifically tailored for creating estimators in stochastic domains where obtaining competent computer agents is simple. Since baseline estimators rely on a baseline scoring function that provides a benchmark level of expected performance of agents within a domain, a natural way to produce this would be to use the actual values that agents obtain in the game itself. Although any agent could be used to provide a score for each observed environmental instance, slow-acting agents or those whose play is not indicative of the agents being evaluated will likely decrease the potential variance reduction of the resulting baseline estimator. This suggests that using competent and fast-acting computer agents are an ideal choice for creating the baseline scoring function when

[1]Portions of this chapter appeared in the Proceedings of the 11th International Conference on Autonomous Agents and Multi-agent Systems [11].

evaluating decently skilled agents. Luckily fast, skilled computer agents exist for many of the domains that require efficient agent evaluation.

The domain of Texas hold 'em poker provides an excellent example of an evaluation scenario that could benefit from the use of an agent baseline estimator. Evident from the volume of matches needed to create significant results in the Annual Computer Poker Competition [1], there exists a need for efficient agent evaluation due to the high outcome variance inherent in the game. Texas hold 'em poker is also a domain where there are many fast-acting, highly-skilled computer agents, some of which can outperform the best human players in the world [19]. The hypothesis, then, is that by creating baseline estimators by using the values of competitive computer agents as the baseline scores for the observed instances, the resulting estimators will yield highly-efficient, unbiased estimates of agent performance.

## 4.2 Creating the Baseline Estimator

Creating agent baseline estimators involves using computer agents to generate the required baseline scoring function, $\beta(Q)$, for each stochastic instance present in the evaluation. Specifically, the agent baseline approach is to choose a single agent, referred to as the *baseline agent*, and create this function by using that agent's self-play on the observed instances. This requires that the baseline agent be able to generate outcomes in self-play easily for every instance of the domain, which we will represent with the function $s : q \mapsto \mathbb{R}$. For each $q_i$ in $Q$, the values of the control variable are computed such that $Y_i^\beta = s(q_i)$. The equation representing the resulting agent baseline estimator, denoted as $A^\beta$, is then

$$A_i^\beta = X_i - c(Y_i^\beta - \mathbb{E}[Y^\beta]) \qquad (4.1)$$

which is equal to the underlying baseline estimator function defined in Equation 3.2.

### 4.2.1 Comparisons to DIVAT

One might observe that the agent baseline approach closely resembles the DIVAT approach [6], since both techniques require an agent, or policy, in order to preform evaluation. In fact, these two approaches can be thought of as nearly equivalent if we reformulate the way that chance acts for the domain. By moving all of the chance events to the start of the game, the DIVAT approach will then simply calculate one DIVAT score for the sequence of player actions that occurred, much in the same way that the agent baseline approach creates one baseline score for any given observation. In this setting, a simple implementation of both techniques would be in fact equivalent if they shared the policy used for creating the scores for each observation.

Although the agent baseline approach is similar to that of DIVAT, there exists a fundamental difference in the necessary behaviour of the policy needed by each technique. While creating an agent baseline estimator only requires that there exist some agent that can act within a domain given some predefined set of chance outcomes, the types of policies that can be used with the DIVAT approach are much more restrictive. DIVAT estimators require a strategy that must be able to make reasonable decisions at all possible points in the game, even those in which the strategy may never play itself. The baseline approach does not impose this requirement and allows the strategy to play freely when evaluating each observation. For instance, in the game of poker, the policy used for a DIVAT estimator must be able to act for all possible sequences of actions for all possible card combinations, whereas an agent baseline policy is allowed to never take certain actions with certain sets cards. Allowing for the use of less restrictive policies can make the agent baseline approach both simpler to implement as well as more applicable to a broader range of domains.

## 4.3 Proof of Unbiasedness for Zero-Sum Domains

Agent baseline estimators created using the aforementioned approach are provably unbiased for zero-sum domains since the $Y^{\beta}$ term from Equation 4.1 satisfies the conditions provided in Section 3.2. The first condition holds trivially by the zero-sum domain restriction. Further, since the second and third conditions only depend upon the domain, we assume that these properties hold for any application of the agent baseline approach. This is certainly true for the domains used for the evaluation of the baseline approach in this work. The fourth condition holds as the agent baseline scoring function, $s$, only considers the chance events from the observed instances in $Q$. The fifth condition holds true since there is a one-to-one mapping from baseline scores to observed instances under the agent baseline approach.

## 4.4 Implementation Details

In this section, we introduce some of the implementation details that were used in the empirical analysis portion of this work. The implementation choices were all made with the goal of reducing the potential variance introduced into the estimator while creating the baseline scores for each instance of the domain.

### 4.4.1 Strategy Sampling

While a single sample of the baseline agent in self-play can be used to create each $Y_i^\beta$, any stochasticity inherent in the baseline agent can add variance to the resulting agent baseline estimator, lowering its effectiveness. One way to overcome this variance would be to compute the expected value of the baseline agent in self-play for each $q_i$ observed instance, effectively reducing the variance of $Y^\beta$ to zero. This expected value computation may not be practical in many domains, as it can be computationally expensive to perform an enumeration of all possible self-play sequences for each $q_i$. Instead of doing the full expected value computation, multiple samples of the baseline agent's self-play can be used to compute the values of $Y^\beta$. That is, for each observed instance, $q_i$, we can compute $Y_i^\beta$ as the average of $m$ independent samples of the baseline agent self-play. Each $j^{th}$ sample is computed by drawing different self-play action sequences from the baseline agent, denoted as $s_j(q_i)$. The new equation used to create the control variable for the agent baseline estimator becomes

$$Y_i^\beta = \frac{1}{m}\sum_{j=1}^{m} s_j(q_i) \tag{4.2}$$

We refer to this technique as *strategy sampling*, which reduces the variance of the basic agent baseline estimator and results in more accurate estimates.

The magnitude of this reduction depends on both $m$, the number of samples being averaged, the amount of variance in the baseline agent's policy used in creating the baseline scoring function. Unfortunately this means that in order to set the value of $m$ in a principled manner for a particular domain, one must consider the costs of generating samples of the baseline agent's self-play scores and the variance inherent in the baseline agent's policy.

### 4.4.2 Random Completion

Analogous to strategy sampling, one can also sample certain aspects of the environment when creating the baseline scores. Similar to the private information importance sampling approach [7], future chance events that were unobserved in each $q_i$ instance, can be randomly sampled when computing the baseline score for $q_i$, as long as they would have had no impact on the values of $X_i$. For instance, in the game of poker, if one of the player folds before the final round, the unobserved cards have no impact on the score of the hand, and thus could have been any of valid remaining cards. Where the basic approach to creating agent baseline estimators would assign the unobserved future chance events to a single instance, the *random completion* method instead samples these unobserved future chance events, using the average self-play score of these samples as the baseline score for the particular instance. Let $c : q \rightarrow q'$ denote the function that generates an instance of the domain, $q'$, which shares a common chance prefix with $q$, but has randomly generated future chance events. For example, if $q$ represents a hand of Texas hold 'em poker that ended before all five of the public cards were dealt, $q'$ represents the same hand of poker where any missing board cards are randomly dealt from the deck.

Then, for each $q_i$, the agent baseline estimator's control is computed as the average over $m$ samples of $c(q_i)$. By combining this with the aforementioned strategy sampling approach, the resulting equation for the control is then

$$Y_i^{\beta} = \frac{1}{m} \sum_{j=1}^{m} s_j(c_j(q_i)) \tag{4.3}$$

The effectiveness of the random completion approach depends both on how much of the variance in the agent baseline estimator was due to randomness in the environment and how that randomness impacted the baseline score. A similar analysis to that of the strategy sampling approach must be done in order to set the value of $m$ effectively, taking into consideration the costs associated with sampling the environment and the agents. This method can also be combined with strategy sampling, and assigning the number of samples to each technique depends on the amount of variance that is inherent in the environment and the variance of the baseline agent's policy.

### 4.4.3 Use With Importance Sampling

The importance sampling techniques introduced by Bowling et al. [7] can also be used in conjunction with the agent baseline approach. By applying an importance sampling function to the outcome of the agent baseline scoring function for each observed instance, we can compute a lower variance control for use in the estimator. Given an importance sampling function, $\mathbb{I}$, the function that computes $Y_\beta$ becomes

$$Y_i^{\beta} = \mathbb{I}(s(q_i)) \tag{4.4}$$

Importance sampling can also be combined with the either the strategy sampling or random completion agent baseline implementations.

## 4.5 Evaluation

The agent baseline approach was evaluated using the variants of Texas hold 'em poker from the Annual Computer Poker Competition and the modified version of the Trading Agent

Ad-Auction Competition described in Section 2.4.2. For each of the evaluation domains, the details of the evaluation methodology are provided first, and followed by the results of the experimentation. In the poker domains, the agent baseline estimator was compared to the duplicate estimator, denoted as *DUP*, when applicable. The duplicate approach was chosen as the performance benchmark due to it's comparable simplicity to the agent baseline approach and the fact that it was the agent evaluation technique used to evaluate the matches in the 2013 ACPC [1].

For all of the no-limit evaluation, the data was pre-processed using *all-in equity*, a technique that adjusts the value of hands that went all-in to reflect the expected value given the cards known to the players at the time of an all-in. The all-in equity technique tends to help reduce variance since the range of payouts being evaluated becomes smaller after it is applied.

The performance of each estimator is measured as one minus the ratio between the standard error of the mean (SE) when using the estimator versus the standard error of the mean when using a pure Monte Carlo (MC) approach. This ratio is presented as a percentage value and referred to as the reduction percentage for a given estimator. Higher percentages correspond to better reductions in variance, with values of 100% translating to zero-variance estimators. Conversely, a value of 0% means that the estimator did not reduce the variance of the Monte Carlo estimation. Negative numbers are also possible, which would imply that the estimator actually increased the variance of the estimate. The intuition behind using this measure is that an estimator with lower standard error will have more accurate estimated confidence intervals, which translates to a better estimate of the true performance of an agent.

The question of cost is also an important part of comparing evaluation techniques, and will be discussed for the different estimators employed in each domain.

### 4.5.1 Texas Hold 'em Poker

#### Experimental Design

The agent baseline approach in the domain of Texas Hold 'em poker was predominately evaluated using data from the 2013 Annual Computer Poker Competition (ACPC). In addition to the analysis performed on the ACPC dataset, the agent baseline approach was evaluated using two 1000-hand human versus computer-agent matches and one 4500-hand human versus computer-agent match in the domain of two-player no-limit Texas hold 'em. The human competitors in these matches were all high-calibre poker players and the computer agents were of the same quality as the top entries in the 2013 ACPC two-player no-limit competition.

Evaluation of the ACPC matches was performed on two different datasets based on the competition data, referred to as the *top-competitors* and *all-competitors* datasets. The top-competitors dataset was a subset of the competition data that consisted of only the matches between the best competitors for the given competition. The all-competitors dataset simply contained all of the matches for the competition. Since there are two different ways winners are decided in the ACPC competitions, the top-competitors dataset for each of the two-player variants contained the five best agents as determined by each of the different winner selection methods. This resulted in the dataset containing a total of eight different agents for the two-player limit variant while there were only five in the two-player no-limit variant. Due to the lack of entrants in the three-player limit competition, only the matches containing two of the top three agents were included in the three-player top competitor dataset.

In order to fairly compare agent baseline estimators against duplicate and Monte Carlo estimators, we treated the duplicate ACPC matches as independent samples in both the Monte Carlo and agent baseline evaluations. As such, matches that did not contain a corresponding duplicate entry were not included in the evaluation data.

The agent baseline estimators used in these experiments were implemented using the random completion technique and setting $m = 50$. The choice of $m$ was determined to be an acceptable cost-performance trade-off for the evaluation of the agent baseline estimators. The baseline agent for each estimator was chosen to be a fast-acting competition-grade agent that did not appear in any of the evaluation data. The value of $c$ from Equation 4.1 was chosen to be an estimate of $c^*$ by using Equation 2.11 on a 10% hold-out dataset for each of the different agent match-ups.

### Cost Analysis

The cost of estimation when using agent baseline estimators depends predominantly on the time it takes to draw samples from the baseline agent in self-play. That is, if the cost of drawing one self-play sample of a baseline agent is $\alpha$, and the cost of drawing one sample of two observed agents playing is $\gamma$, then the cost of the estimation is simply $\alpha + \gamma$. Comparing the cost of creating a random completion estimator with the cost of creating a duplicate estimator means simply measuring the difference in $m\alpha$ to $2\gamma$. That is, when $m\alpha < 2\gamma$, agent baseline estimation is more efficient than the duplicate equivalent. This is almost always true when we are evaluating slow-acting computer agents or human agents, since we typically have control over both $\alpha$ and $m$ and not $\gamma$. Also, it may not be possible to draw further samples of the agents interacting, for which $\gamma$ is effectively infinite.

Furthermore, the cost of creating agent baseline estimators can also be reduced by reusing the baseline score calculations when creating estimators for multiple matches that share the same random seeds. For large competitions with many different competitor pairings that share random seeds, such as the ACPC competitions, this amortization effectively makes the cost of creating agent baseline estimators zero. All of this analysis assumes that the resulting performance of each estimator is equal.

### Two-Player Limit ACPC Results

A bar plot displaying the performance of the duplicate and agent baseline estimators, evaluating the agents from the 2013 ACPC two-player limit competition, is presented in Figure 4.1. This bar plot displays the average performance for each of the estimators when applied to each of the matches in the top-competitors and all-competitors datasets. Also displayed in this figure is an estimated 95% confidence interval for each estimator's average performance computed as $1.96 * SE$, where $SE$ is the standard error of the mean for the estimators performance on the dataset. A line denoting the value of the agent baseline's lower confidence interval in included to help visualize any claims of statistical significance.

Both of the bar plots show that the agent baseline estimators have a higher average performance than the duplicate estimators, although not by a statistically significant margin. Also clear is the higher average performance of both estimators when evaluating the top echelon of competitors. The average performance of the agent baseline estimators increases from 49.8% to 64.5% between these two datasets and the duplicate estimators have a nearly identical increase.

A more in-depth look at the results of the estimators used to evaluate the agents in the top-competitors dataset is shown in Table 4.1. Here we can see that when evaluating matches containing the best agents in the competition, the agent baseline estimators are strictly better than their duplicate counterparts. Appendix A, A.2, shows the full evaluation for each of the estimators performed on the all-competitors dataset. This table shows that agent baseline estimators outperform the duplicate estimators in 73.3% of the matches. Considering that the cost of the agent baseline evaluation is amortized by the shared seeds between each pairing of competitors in ACPC competitions, evaluation using agent baseline estimators should be considered as a replacement for duplicate estimation in future two-player limit ACPC tournaments.

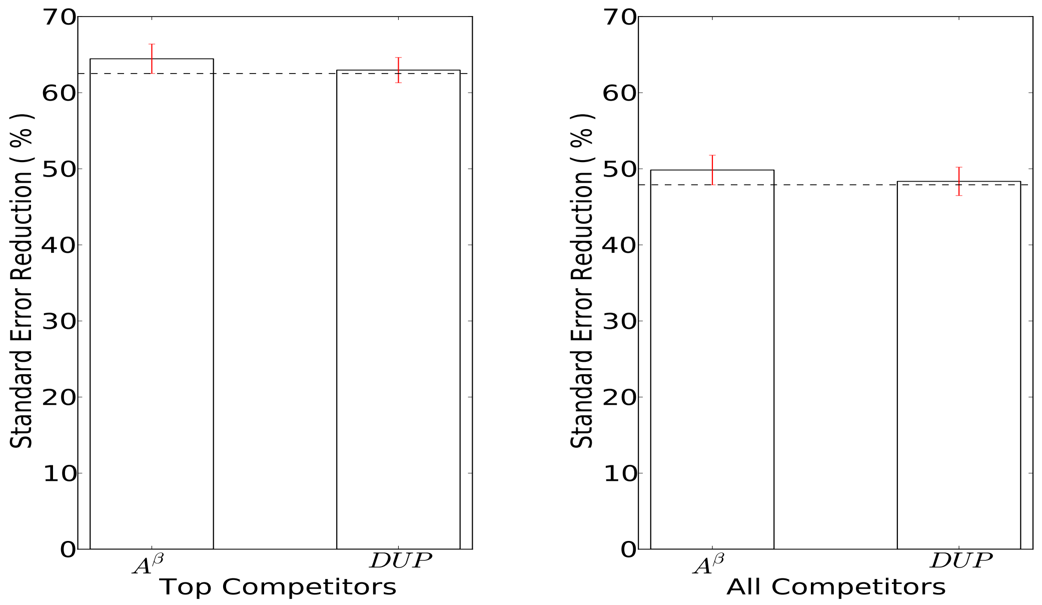


Figure 4.1: 2013 ACPC Two-Player Limit Average Estimator Performance

| Match Name | MC SE | $DUP$ SE | $A^\beta$ SE | $DUP$ % | $A^\beta$ % |
|---|---|---|---|---|---|
| hyperborean_iro/littlerock | 9.42 | 3.56 | **3.26** | 62.25 | **65.33** |
| hyperborean_iro/marv | 9.41 | 3.06 | **2.74** | 67.48 | **70.9** |
| hyperborean_iro/neopokerlab | 10.22 | 3.5 | **3.28** | 65.76 | **67.94** |
| hyperborean_iro/zbot | 6.49 | 2.33 | **2.15** | 64.03 | **66.78** |
| hyperborean_tbr/littlerock | 9.8 | 4.17 | **4.07** | 57.47 | **58.5** |
| hyperborean_tbr/marv | 10.51 | 4.05 | **3.96** | 61.43 | **62.29** |
| hyperborean_tbr/neopokerlab | 10.64 | 4.35 | **4.28** | 59.15 | **59.82** |
| hyperborean_tbr/zbot | 10.64 | 4.37 | **4.33** | 58.93 | **59.34** |
| littlerock/marv | 10.11 | 3.5 | **3.42** | 65.4 | **66.23** |
| littlerock/neopokerlab | 10.23 | 3.89 | **3.74** | 62.0 | **63.43** |
| littlerock/zbot | 10.26 | 4.01 | **3.87** | 60.95 | **62.29** |
| marv/neopokerlab | 10.14 | 3.35 | **3.22** | 66.96 | **68.2** |
| marv/zbot | 10.15 | 3.45 | **3.34** | 66.03 | **67.11** |
| neopokerlab/zbot | 10.27 | 3.73 | **3.67** | 63.7 | **64.22** |

Table 4.1: 2013 ACPC Two-Player Limit Top-Competitors Results

### Three-Player Limit ACPC Results

The average performance of the duplicate and agent baseline estimators for the 2013 ACPC three-player limit competition is displayed in Figure 4.2. Both estimators are not nearly as strong in this poker variant as they are for two-player limit, with the agent baseline estimators only achieving an average performance of 28.6% on the all-competitors dataset. Once again, both duplicate and agent baseline estimators see a boost in their average performance when evaluating agents in the top-competitors dataset. This gain is roughly 5% for the agent baseline estimators and only 2% for the duplicate estimators. Similar to the two-player limit analysis, the agent baseline estimators have a higher average performance for both datasets, although the difference is still without statistical significance.

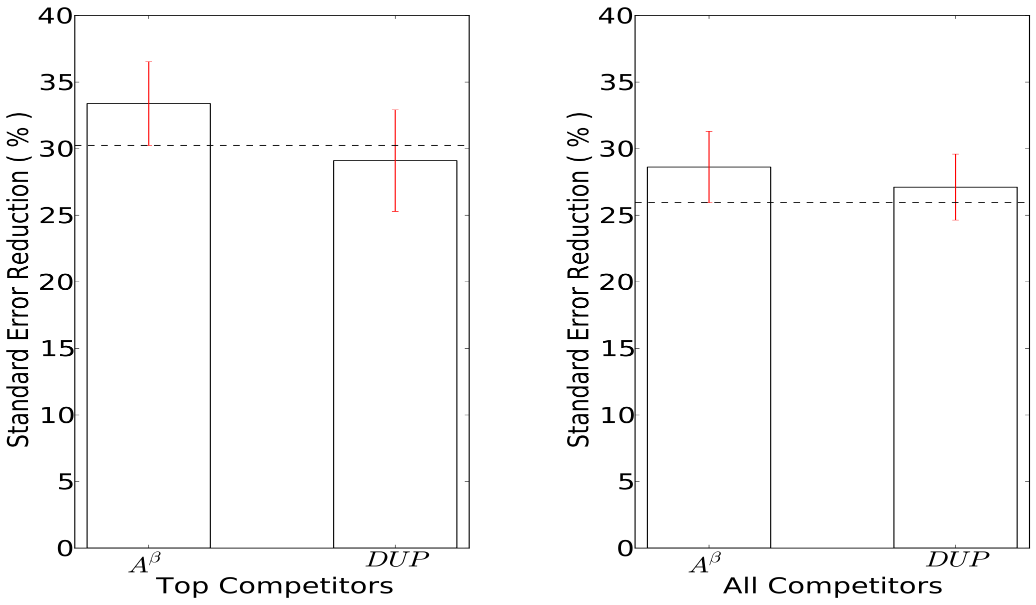


Figure 4.2: 2013 ACPC Three-Player Limit Average Estimator Performance

Table 4.2 provides a breakdown of the performance of both agent baseline and duplicate estimators on the top-competitors dataset for this domain. This table shows that the agent baseline estimators have better overall performance than the corresponding duplicate estimators in all but two of the matches. Table A.2, presented in Appendix A, contains the performance of the estimators on the all-competitors ACPC dataset, the results of which show the agent baseline estimators outperforming the duplicate estimators in 63.3% of the matches. The performance of agent baseline estimators, combined with the costs associated with the six-way duplicate evaluation performed by the ACPC suggest that using duplicate estimation in the three-player limit competition should be replaced by the agent baseline approach for future competitions.

### Two-Player No-Limit ACPC Results

Figure 4.3 shows the average performance of the agent baseline and duplicate estimators for the 2013 ACPC two-player no-limit competition matches. The graphs in this figure

| Match Name | MC SE | *DUP* SE | $A^{\beta}$ SE | *DUP* % | $A^{\beta}$ % |
|---|---|---|---|---|---|
| HITSZ/littlerock/hyperborean_iro | 22.58 | 16.33 | **15.88** | 28.31 | **29.77** |
| HITSZ/littlerock/hyperborean_tbr | 24.51 | **17.37** | 17.5 | **29.47** | 28.68 |
| liacc/littlerock/hyperborean_iro | 26.41 | 21.36 | **18.01** | 20.3 | **31.93** |
| liacc/littlerock/hyperborean_tbr | 27.75 | 21.41 | **19.0** | 23.59 | **31.62** |
| littlerock/hyperborean_iro/kempfer | 17.85 | 11.41 | **10.93** | 35.84 | **38.64** |
| littlerock/hyperborean_tbr/kempfer | 18.8 | 12.18 | **11.87** | 34.92 | **36.78** |
| neopokerlab/HITSZ/hyperborean_iro | 22.51 | 16.42 | **16.03** | 27.65 | **28.88** |
| neopokerlab/HITSZ/hyperborean_tbr | 24.61 | **17.53** | 17.77 | **29.06** | 27.84 |
| neopokerlab/HITSZ/littlerock | 22.16 | 16.32 | **15.95** | 26.95 | **28.09** |
| neopokerlab/hyperborean_iro/kempfer | 17.38 | 11.7 | **10.98** | 32.55 | **36.75** |
| neopokerlab/hyperborean_tbr/kempfer | 18.54 | 12.69 | **12.22** | 31.54 | **34.04** |
| neopokerlab/liacc/hyperborean_iro | 26.43 | 22.07 | **18.98** | 17.52 | **28.26** |
| neopokerlab/liacc/hyperborean_tbr | 27.74 | 22.29 | **19.99** | 20.3 | **27.93** |
| neopokerlab/liacc/littlerock | 26.3 | 22.25 | **19.18** | 16.51 | **27.12** |
| neopokerlab/littlerock/hyperborean_iro | 13.04 | 7.16 | **6.56** | 45.05 | **49.64** |
| neopokerlab/littlerock/hyperborean_tbr | 19.23 | 11.04 | **10.32** | 42.59 | **46.35** |
| neopokerlab/littlerock/kempfer | 17.36 | 11.7 | **11.23** | 32.54 | **35.25** |

Table 4.2: 2013 ACPC Three-Player Limit Top-Competitors Results

show that the duplicate and agent baseline estimators achieve a nearly identical average performance of approximately 20% when evaluating the top-competitors dataset. When comparing the estimators on the entire population of agents for this competition, the duplicate estimators perform only slightly better on average, with a mean performance of around 15% compared to the 14% realized by the agent baseline estimators. Once again however, these differences are not of statistical significance.

Analyzing the full top-competitor results from Table 4.3 further reinforces this equality. On this dataset, the agent baseline estimators perform only slightly better that the duplicate estimators on just over half of the matches. Table A.3 in Appendix A shows the results of these estimators for the all-competitors dataset, from which the duplicate estimators achieve a better average performance than the agent baseline estimators on just 55% of the matches. These results show that both duplicate and agent baseline estimators perform equally well in this domain.

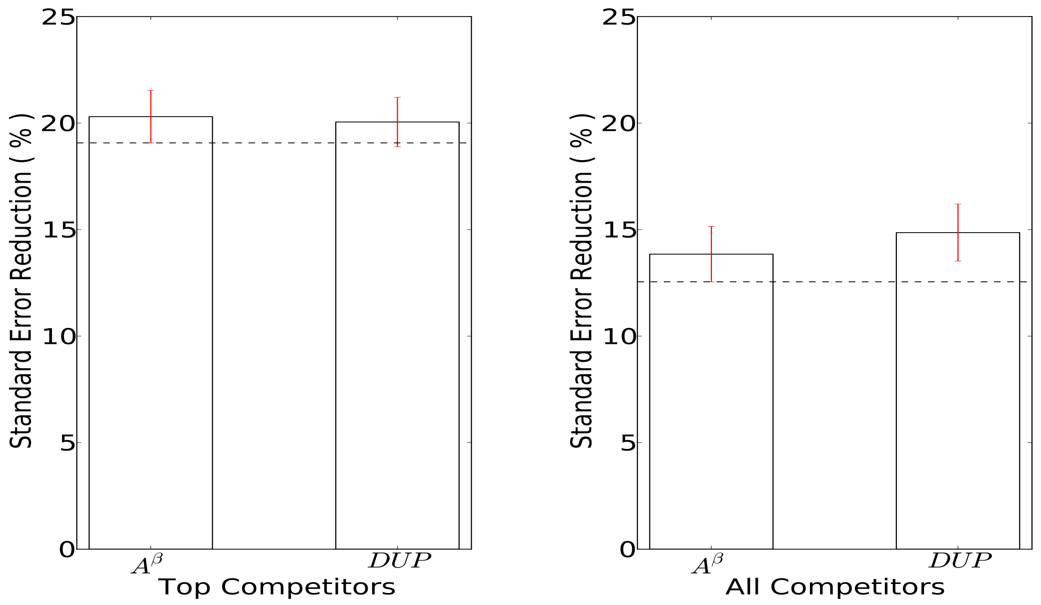


Figure 4.3: 2013 ACPC Two-Player No-Limit Average Estimator Performance

| Match Name | MC SE | $DUP$ SE | $A^\beta$ SE | $DUP$ % | $A^\beta$ % |
|---|---|---|---|---|---|
| hyperborean_iro/koypetition | 50.26 | 41.27 | **40.19** | 17.88 | **20.04** |
| hyperborean_iro/nyx | 27.63 | **22.13** | 22.14 | **19.91** | 19.86 |
| hyperborean_iro/slumbot | 11.44 | 8.94 | **8.85** | 21.87 | **22.68** |
| hyperborean_iro/tartanian6 | 46.58 | **35.95** | 36.96 | **22.82** | 20.66 |
| hyperborean_tbr/koypetition | 51.61 | 43.73 | **42.8** | 15.27 | **17.08** |
| hyperborean_tbr/nyx | 37.81 | **31.03** | 31.49 | **17.93** | 16.73 |
| hyperborean_tbr/slumbot | 34.55 | 28.24 | **28.02** | 18.27 | **18.89** |
| hyperborean_tbr/tartanian6 | 50.22 | **41.08** | 42.22 | **18.21** | 15.93 |
| koypetition/nyx | 45.17 | 35.7 | **35.21** | 20.97 | **22.06** |
| koypetition/slumbot | 46.59 | 37.02 | **35.9** | 20.54 | **22.95** |
| koypetition/tartanian6 | 48.18 | 38.14 | **37.5** | 20.85 | **22.16** |
| nyx/slumbot | 22.14 | 17.16 | **17.13** | 22.52 | **22.63** |
| nyx/tartanian6 | 39.56 | **30.93** | 31.35 | **21.8** | 20.75 |
| slumbot/tartanian6 | 37.73 | **29.46** | 29.49 | **21.91** | 21.83 |

Table 4.3: 2013 ACPC Two-Player No-Limit Top-Competitors Results

### Two-Player No-Limit Human vs Computer-Agent Results

Table 4.4 provides the results for the two 1000-hand human versus computer-agent matches that were run using the duplicate-like approach previously executed in the two-player limit man versus machine competitions [19]. The duplicate estimator in this approach is created by having both human players play against the computer agent with their cards swapped. Since this type of evaluation does not allow for individual skill to be assessed, the duplicate estimator is left blank for these entries in the table.

| Match | $\overline{MC}$ | MC SE | $A^\beta$ SE | $DUP$ SE | $A^\beta$ % | $DUP$ % |
|---|---:|---:|---:|---:|---:|---:|
| Match A Human 1 | 39.55 | 491.12 | **403.51** | — | **17.84** | — |
| Match A Human 2 | -1597.75 | 683.90 | 557.11 | — | **18.54** | — |
| Match A Team | -779.1 | 421.28 | 346.88 | **315.54** | 17.66 | **25.1** |
| Match B Human 1 | 78.78 | 465.62 | **379.94** | — | **18.40** | — |
| Match B Human 2 | -146.63 | 442.31 | **287.99** | — | **34.89** | — |
| Match B Team | -33.93 | 321.04 | **239.49** | 239.52 | **25.40** | 24.39 |

Table 4.4: Two-Player No-Limit Human vs Computer-Agent Results (1000-Hand Matches)

In match A, the duplicate estimator achieves the best performance when evaluating the team play, and both the agent baseline estimator and duplicate estimator are nearly equal for match B's team evaluation. The $\overline{MC}$ column shows the win-rate for each of the players or for the team, the units of which are in mili-big-blinds per hand (mbb/h), a standard unit of measure for this domain. This is provided in order to demonstrate how using duplicate estimators in the human versus computer-agent setting can be extremely misleading. When combining the two human players' scores and computing a 95% confidence interval, the team loses with confidence in match A. This is solely due to the horrible performance of Human 2 in this match. Given the choice, it is obvious that Human 1 would prefer to be evaluated alone since the results show a statistical tie for this player. Being able to individually evaluate players is one large advantage that agent baseline estimators have over duplicate estimators when evaluating human players.

### 4.5.2 Trading Agent Competition: Ad Auctions

#### Experimental Design

In order to test the performance of agent baseline estimators in domains where no other methods of efficient evaluation are typically performed, we evaluated agents in a mock Trading Agent Ad Auction tournament [22]. This experiment involved using six open-source agents and two random agents in order to create an eight-player tournament. The two random agents were used due to a lack of available open source agents and the software not supporting fewer than eight agent tournaments. The tournament consisted of 100 ten-day matches, where each of the agents was assigned a random specialization before the beginning of each match. This provided a mechanism for capturing the environmental stochasticity needed to create the agent baseline estimators. Three different agent baseline estimators were created using three of the six open-source agents from the tournament. For practicality reasons surrounding the computational time required to run a tournament, only one sample of each baseline agent's self-play was used to create the corresponding baseline estimator.

### Results

Table 4.5 shows the resulting standard error reduction for each of the competitors when being evaluated using the three agent baseline estimators based on the epfl, crocodile and metroclick bots, respectively labeled $A_1^\beta$,$A_2^\beta$ and $A_3^\beta$. The effects vary, ranging from a 0.1% to a 53% reduction in the standard error, with the agent baseline estimator $A_3^\beta$ having the best overall performance. As only one self-play sample was used in the calculation due to the unavailability of a fast-acting baseline agent, additional performance gains are likely to be realized by using the strategy sampling techniques. This experiment does however show that agent baseline estimators should be considered for competitions of this nature, especially in the absence of other efficient evaluation techniques.

| Player | MC SE | $A_1^\beta$ SE | $A_2^\beta$ SE | $A_3^\beta$ SE | $A_1^\beta$ % | $A_2^\beta$ % | $A_3^\beta$ % |
|---|---|---|---|---|---|---|---|
| tau | 0.689 | 0.635 | 0.636 | **0.551** | 7.78 | 7.64 | **19.98** |
| crocodile11 | 0.862 | 0.851 | 0.851 | **0.812** | 1.36 | 1.28 | **5.82** |
| crocodile | 1.01 | 0.825 | 0.953 | **0.805** | 18.30 | 5.65 | **20.31** |
| epfl | 1.245 | 1.217 | 1.210 | **1.178** | 2.26 | 2.80 | **5.34** |
| metroclick | 1.449 | 1.308 | 1.267 | **0.677** | 9.72 | 12.59 | **53.25** |
| Schlemazl | 0.923 | 0.922 | 0.923 | **0.922** | 0.12 | 0.01 | **0.17** |

Table 4.5: TAC-AA Tournament (100 Ten-Day Matches)

# Chapter 5

# Data Baseline

The *data baseline* approach is another application of the basic baseline approach from Chapter 3 and marks the second major contribution of this work to the field of agent evaluation techniques. Data baseline leverages the availability of large volumes of player data to create the baseline scoring function for use in agent evaluation. In this chapter, we will describe the motivation behind the data baseline approach, as well as the general approach to implementing estimators of this nature. We will also provide the details of the empirical analysis used to evaluate the performance of data baseline estimators in the domain of Texas hold 'em in addition to the specifics involved with the implementation of data baseline estimators for this domain.

## 5.1 Motivation

The data baseline approach is designed to address the problems with the agent baseline approach in situations where there are no readily available competent computer agents for the evaluation domain. One possible solution to this problem would be to try and use simple rule-based agents to create the necessary agent baseline scores. This solution is unlikely to be effective since the power of agent baseline estimators comes from the correlation between the baseline agent's self-play and that of the agents being evaluated. Another solution would be to simply create a competent agent to use as the baseline agent. This solution is also problematic due to the large amount of effort involved with engineering an agent of the required skill.

The data baseline approach solves this issue for domains where there are volumes of data consisting of players competing with one another. Data baseline uses this existing player data directly in the computation of the baseline scores, thus removing the need for an underlying baseline agent. Fortunately, player data is often a much more abundant resource than competitive computer agents in many domains, especially those for which there are large communities of human players. Also potentially beneficial to using this type of data is the likelihood of higher correlation when evaluating similar types of players. An example of this would be to use human data when evaluating human players. In fact, we hypothesize that using data baseline created with tailored data would produce highly efficient estimators capable of better performance than agent baseline estimators in domains where both can be created.

## 5.2 Creating the Baseline Estimator

In order to create a data baseline estimator, one must provide the necessary baseline scoring function, $\beta(Q)$, in order to create the control variable $Y^{\beta}$. While there are conceptually many different ways to create this mapping using existing player data, we will outline several possible implementations, starting with the obvious mapping that assumes there is a *perfect match* in the player data for each instance being evaluated. In what follows, it is assumed that a dataset $D = \{(q_1, u1), (q_2, u_2), \ldots, (q_n, u_n)\}$ exists, and contains examples of agents interacting on instances of the domain in question, $q_i$, along with the utilities, $u_i$, the agents received on that domain instance.

### 5.2.1 Perfect Matching

One option for creating $\beta(Q)$ is to simply find an exact match in the dataset for the domain instance being evaluated, which we call $q \in Val(Q)$. In other words, if the dataset $D$ contains instances where the exact environmental realization from the evaluation domain instance occurred ($q \in D$), then the mapping simply returns the value obtained for each agent on that instance from the dataset. If $q \notin D$, we return zero for the instance. If more than one instance in $D$ corresponds to any given $q$, then the average utility values are returned. Let $D(q) \subseteq D$ be the set of instances $(q_i, u_i)$ in the dataset $D$ such that $q = q_i$.We can now define the *Perfect Match* function $PM_D : Val(Q) \rightarrow \mathbb{R}$ as

$$PM_D(q) = \begin{cases} \frac{1}{|D(q)|} \sum_{(q_i,u_i) \in D(q)} u_i & \text{if } |D(q)| > 0 \\ 0 & \text{otherwise} \end{cases} \tag{5.1}$$

A perfect matching implementation of data baseline is then created using $\beta = PM_D$ to generate the values for $Y^{\beta}$. The agent baseline approach is actually a special case of the perfect matching implementation where the perfect matches are generated post-hoc using a computer agent in self-play, either by sampling the agent once for each instance or multiple times in the case of the strategy sampling technique.

### 5.2.2 Nearest Neighbour

Since in most domains it is unlikely that there exists an exact match in our dataset, $D$, for every possible domain instance, $q \in Q$, another option is to instead return the value of the *nearest* match. The hope here is that by choosing nearby matches, the values will be very close to the values of an exact match if it were to exist. For some domains, the notion of distance over domain instances may be ill-defined, leaving us with no explicit way to return the nearest match. In these situations, the intuitive approach is to first map domain instances to some $d$ dimensional feature space, $\mathbb{R}^d$, both when constructing $D$ and querying $D$ for a match. By doing this, we can use a well-defined distance measure such as Euclidean distance when searching $D$. Let $\phi : Val(Q) \to \mathbb{R}^d$ be the function that maps domain instances to a $d$ dimensional feature vector. We define the function $NE^L$, which returns the entry $(q_i, u_i)$ in $D$, such that $q_i$ minimizes the $L$-distance in the feature space to the input domain instance $q$.

$$NE_D^L(q) = \operatorname*{arg\,min}_{(q_i,u_i)\in D} L(\phi(q), \phi(q_i))$$

The *Nearest Neighbour* function $NN_D^L : Val(Q) \to \mathbb{R}$ then simply returns the utility value, $u$, of nearest entry,

$$NN_D^L(q) = NE_D^L(q)[2]$$

where the $[i]$ operator returns the $i$-th element of a dataset entry, which in this case is the utility value of the nearest database entry. The nearest neighbour data baseline approach then proceeds by setting $\beta = NN_D$.

### 5.2.3 k-Nearest Neighbour

One possible shortcoming of the previous idea is that depending on a single returned entry could lead to high variance in the control variate, which decreases the amount of variance reduction that can be achieved by the resulting baseline estimator. Instead of returning a single nearest entry, we can instead have the mapping function return the $k$ nearest entries in the dataset and average their values. We refer to this as the *k-Nearest Neighbour* mapping function. More precisely, if we let $k$-$NE_D^L(q)$ return the $k$ nearest entries from dataset $D$, according to distance metric $L$ in the feature space, then the k-Nearest Neighbour function $k\text{-}NN_D^L : Val(Q) \to \mathbb{R}$ can be defined as

$$k\text{-}NN_D^L(q) = \frac{1}{k}\left(\sum_{(q_i,u_i)\in k\text{-}NE_D^L(q)} u_i\right) \tag{5.2}$$

The k-Nearest Neighbour data baseline estimator is then created by setting $\beta = k\text{-}NN_D^L$.

In all cases, the resulting data baseline estimator, $D\beta$, is defined as

$$D_i^\beta = X_i - c(Y_i^\beta - \mathbb{E}[Y^\beta]) \tag{5.3}$$

### 5.2.4 Comparisons to MIVAT

Much in the same ways that agent baseline approach is similar to DIVAT, the data baseline approach also shares some properties with the MIVAT approach to agent evaluation. Both the data baseline and MIVAT techniques require a pre-existing volume of player data in order from which to create an estimator as well as a method of mapping player data to features space. The main difference between these two techniques, aside from the way in which the estimators are applied, revolves around the types of features required by each technique and how the features are used. Since the MIVAT approach makes use of the player data as a part of a closed-form optimization, in order for the optimization to be tractable, the value function needs to be linear in the features imposed on the data [24]. The data baseline approach however imposes no such restriction when using the proposed nearest-neighbour method of searching for baseline scores. Although the use of non-linear features within the data baseline approach is not explored in this work, removing such a restriction makes data baseline a more flexible approach than MIVAT for creating estimators from existing player data.

## 5.3 Proof of Unbiasedness for Zero-Sum Domains

Regardless of the chosen mapping from dataset instances to baseline scores, in order for the resulting data baseline estimator to be unbiased, we either need to know the value of $\mathbb{E}[Y^\beta]$ or show that it is zero. By assuming that the data baseline estimators are operating in zero-sum domains, we can use the conditions outlined in Section 3.2 to prove that these mapping functions produce an expected value of zero for the control variable $Y^\beta$. The first of these conditions is held trivially via our domain assumption. The second and third conditions only depend on the environment, and as such the choice of the $\beta$ baseline scoring function does not impact this. The fourth and fifth conditions are satisfied directly for each of the mapping functions we provided since none of them depend on the actions of the agents for a given domain instance nor will they return a different set of values anytime they are called for a given domain instance. Thus, any choice of mapping function that satisfies these conditions in a zero-sum domain will produce an unbiased data baseline estimator, the three aforementioned mapping functions being examples of this.

## 5.4 Implementation Details

In this section we introduce the data baseline implementation details that were used in the evaluation. Specifically, we provide the details regarding the data structure used to store the dataset and the choices involved when implementing the baseline scoring function. The data baseline estimators used in the empirical analysis were all created using the k-Nearest Neighbour ($k$-$NN$) baseline scoring function using Euclidean distance. Since all of the evaluation was done using Texas hold 'em domains, many of the implementation choices are specific to player data within this domain.

### 5.4.1 Storage

Recall that the k-Nearest Neighbour mapping function requires a search over the baseline dataset, $D$, where each instance is mapped to a feature vector in $\mathbb{R}^d$. A naïve search for a nearest entry in our dataset could take $\mathcal{O}(n)$ time, which for large datasets would be very costly. We instead use a kd-tree [5] data structure as a way to store the dataset while allowing for efficient searches over the data. A kd-tree is essentially a multi-dimensional binary search tree [9], where $k$ refers to the dimensionality of the search space. Nearest neighbour queries for a given point when using kd-trees are $O(log\ n)$ as long as the tree remains balanced, a condition that is fairly easy to enforce when no deletions or insertions are performed after initial construction of the tree.

### 5.4.2 Feature Representation

The features chosen to represent the data are based on the notion of *expected hand strength*, a function that maps poker hands to real numbers according to their likelihood of winning [14]. Each hand in the dataset is represented by an eight dimensional feature vector whose entries correspond to the expected hand strength for each player, given their private cards and the public cards available during each round of the game. For example, the hand Q♦J♠ vs Q♥K♥ with the board cards J♦K♠8♥6♥2♣, has a corresponding feature vector of [0.581, 0.634, 0.784, 0.866, 0.776, 0.899, 0.794, 0.891]. If a hand is incomplete, meaning that it is missing board cards due to players folding before the river round, random board cards are used for these rounds when generating the feature vector. This was done to ensure that information corresponding to the players' actions is not unintentionally encoded into the feature space representation for the hand, potentially biasing the estimation.

### 5.4.3 Sensitivity of $k$

The value of $k$ used in the k-Nearest Neighbour data mapping for each of the different types of Texas hold 'em was determined through a parameter study using past ACPC data. Data baseline estimators were created using the past ACPC data for each of the ACPC poker domains and evaluated using a small subset of holdout data. The average performance of the resulting estimators was recorded for varying values of $k$, the results of which can be seen in Figure 5.1. This graph shows that the best choice of $k$ within each of the poker variants was fairly robust to change with little variance in performance across $k$ values. The peak performance was around $k = 16$ for both two-player and three-player limit Texas hold 'em and around $k = 128$ for two-player no-limit Texas hold 'em. We speculate that the discrepancy between the optimal $k$ values for no-limit and limit games centre around the variance in the payouts between the two games coupled with the variance intrinsic in the policies of the various agents. Given that the range of payouts for no-limit games is much greater than their limit counterparts, it is very possible that any given hand may realize a wide variety of payouts in both the evaluation and query datasets. While averaging out the values of many *near* hands when creating the estimator helps to achieve a better baseline score for each no-limit hand, a better solution would be to have many samples of the stochastic agents playing the same hand and average the values of those samples. Based on these results, we set $k = 16$ for the limit variants of poker and $k = 128$ for the no-limit variants in our experimentation.

### 5.4.4 Dataset Size

In order to grasp the impact that the size of the data baseline dataset has on the performance of the resulting baseline estimator, we experimented with datasets of different sizes in the ACPC poker domains using random subsets drawn from the 2013 ACPC data. The graph in Figure 5.2 shows the effect that varying the amount of available data has on the performance

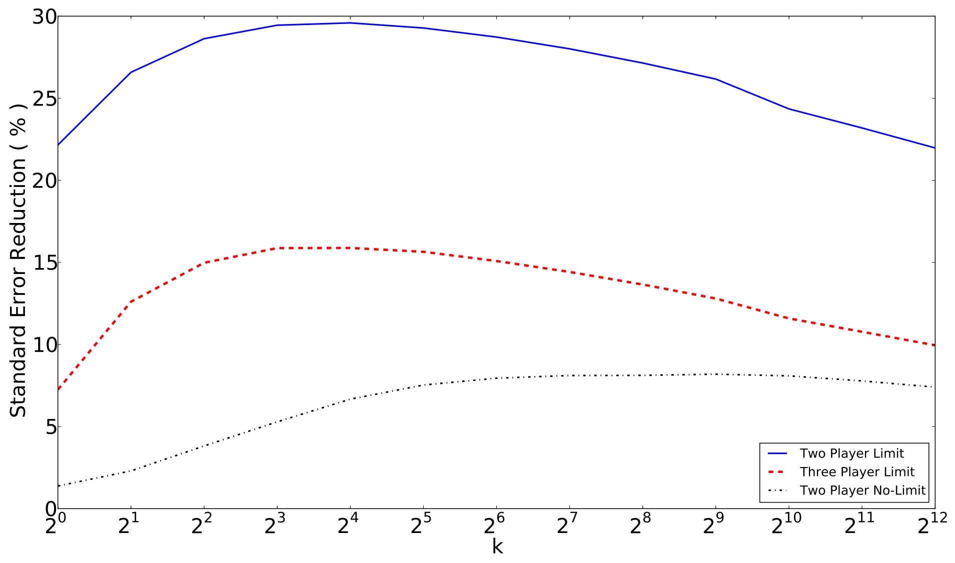


Figure 5.1: Sensitivity of k for $k$-$NN$ Search

of the data baseline estimator for each of the domains. As expected, the performance of the estimator increases as the number of hands contained in the dataset increases, with somewhat diminishing returns. As suggested by these results, in order achieve the best possible performance during the evaluation, we decided to include as much data as was available when implementing each data baseline estimator.

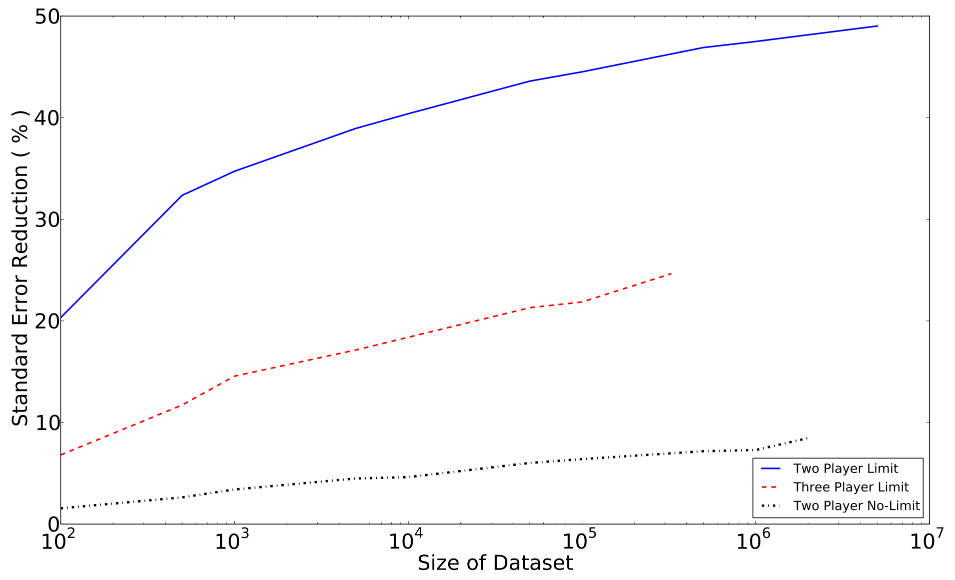


Figure 5.2: Performance for Varying Sizes of the Baseline Dataset

## 5.5 Evaluation

The data baseline approach was evaluated using the same three Texas hold 'em domains that were used in the agent baseline evaluation. This was done in order to compare the performance of data baseline estimators to agent baseline estimators in domains where both can be used. However, in many applications, a competent agent may be unavailable, making data baseline the only choice. In this evaluation, we explore how different datasets impact the performance of the data baseline estimators when evaluating both human and computer agents in order to test the hypothesis that using datasets representative of the types of players being evaluated will increase the performance of the data baseline estimator. Additionally, we also simulate the scenario of evaluating agents in domains where there are no competitive agents by comparing the performance of data baseline estimators to agent baseline estimators created using simple rule-based agents. As with the evaluation done with the agent baseline estimators, the results are mainly reported as the reduction in standard error between the various estimators and the Monte Carlo approach and presented as percentage values. Once again, all of the no-limit evaluation, the data was pre-processed using the *all-in equity* variance reduction technique.

### 5.5.1 ACPC Texas Hold 'em Poker

#### Experimental Design

Evaluation of the data baseline approach was performed using the ACPC match data from each of the three 2013 competitions. For each of the different competitions, we compared the performance of data baseline estimators to the performance of various agent baseline estimators. Replicating the agent baseline experimentation, the estimators are evaluated using the same top-competitor and all-competitor datasets for each of the competitions.

For all of the evaluation performed on the top-competitors and all-competitors ACPC dataset, past ACPC data was used as the internal dataset for the baseline estimators. This past data amounted to roughly two million unique hands for each of the the two-player games and around three hundred thousand unique hands for three-player limit. In the two-player limit analysis, data baseline estimators were also created using a dataset that consisted of nearly five million hands of human players of varying skill playing against competition grade computer agents.

As a benchmark, agent baseline estimators were created for each domain by using a competition level agent that did not appear in the 2013 competitions. In order to simulate a likely approach in a domain where this level of agent does not exist, we created three more agent baseline estimators using simple rule based approaches: *always-call*, *call-or-raise* and *random*. The always-call (AC) agent only performs the call action, which was shown by Kan et al. to be an effective choice of policy for creating DIVAT estimators in two-player limit games [6]. The call-or-raise (CR) agent chooses actions with a 50% chance of calling and a 50% chance of making any raise and the random (RAND) agent acts with equal probability of performing any valid action.

### Cost Analysis

The costs of creating data baseline estimators for use in this type of evaluation consist of the cost required to populate the data structure used to store the estimator's dataset and the cost of generating the various baseline scores for each observed hand. This assumes that there already exists a corpus of player data that can be used as the internal dataset. Storing the data in a kd-tree, as per our implementation, requires $\mathcal{O}(rn\ log\ n)$ time to build the tree, where $r$ is the dimensionality of the feature representation of the data and $n$ is the number of data points being added to the tree. The dominating cost then comes from creating the baseline scores for each observed hand. Unlike the agent baseline approach, where generating a baseline score for an observed hand was $\mathcal{O}(1)$, the cost of generating the baseline scores is on the order of $\mathcal{O}(k\ log\ n)$ (or in the worst case $\mathcal{O}(kn^2)$) for a fixed choice of $k$ when using the chosen k-Nearest Neighbours search to build the data baseline estimator. Fortunately, the cost of using data baseline for the evaluation is still likely to be lower than the cost of gathering additional samples from computer agents who are slow to act. Once again these costs can also be amortized by reusing data baseline estimators when evaluating competition data where matches share the same random seeds.

### Two-Player Limit ACPC Results

Figure 5.3 shows the results of the different baseline estimators when evaluating the 2013 ACPC two-player limit competition data. The data baseline estimators built using the past ACPC data and the human versus computer-agent data, $D^{\beta}_{(ACPC)}$ and $D^{\beta}_{(HUMAN)}$, achieve nearly equal performance in this analysis. Both estimators average a reduction in the standard error of approximately 44% when evaluating the all-competitors dataset, and this performance increases nearly 5% when evaluating the top-competitors dataset. The data baseline estimators also outperform the simple rule-based agent baseline estimators, $A^{\beta}_{(AC)}$, $A^{\beta}_{(CR)}$, and $A^{\beta}_{(RAND)}$, with statistical significance, albeit by a small margin. Their average performance is comparable to the competition level agent baseline estimators, $A^{\beta}_{(COMP)}$, with only a 5% difference separating the two techniques. The fact that the data baseline estimators built using human or ACPC data have almost identical performance in this domain suggest that there is an advantage to using a tailored dataset when evaluating agents, since the human dataset contained nearly 2.5 times as many entries as the ACPC dataset. This analysis also suggests that the data baseline estimators are less sensitive to the types of players being evaluated than agent baseline estimators, due to the significantly smaller drop in performance between the top-competitor and all-competitor evaluations.

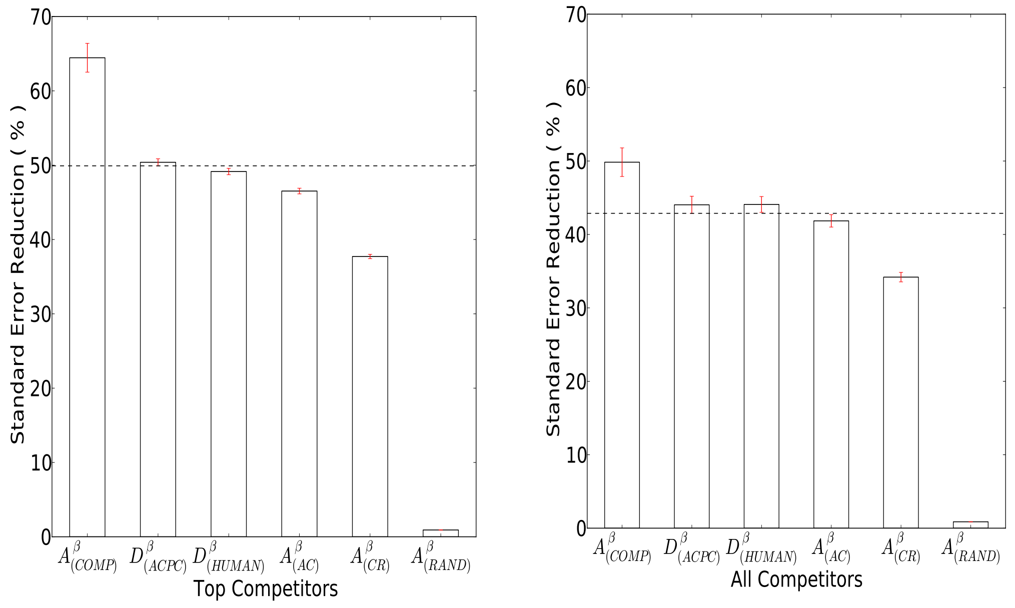


Figure 5.3: 2013 ACPC Two-Player Limit Average Estimator Performance

### Three-Player Limit ACPC Results

Consistent with the two-player limit ACPC evaluation, Figure 5.4 shows that the estimators built using the data baseline approach perform better than all of the simple rule based agent baseline estimators with 95% confidence. Also consistent with the two-player limit ACPC evaluation is the 5% gap between the average performance of the competitive agent baseline and data baseline estimators when evaluating the all-competitors dataset for this competition. The average performance of the data baseline estimators on the top-competitors dataset is nearly identical to their performance on the all-competitors dataset, whereas the estimators created with the agent baseline approach suffer nearly a 5% drop in average performance between the two evaluation datasets. This further suggest that the performance of data baseline estimators is not sensitive to the varying skill of the agents being evaluated.

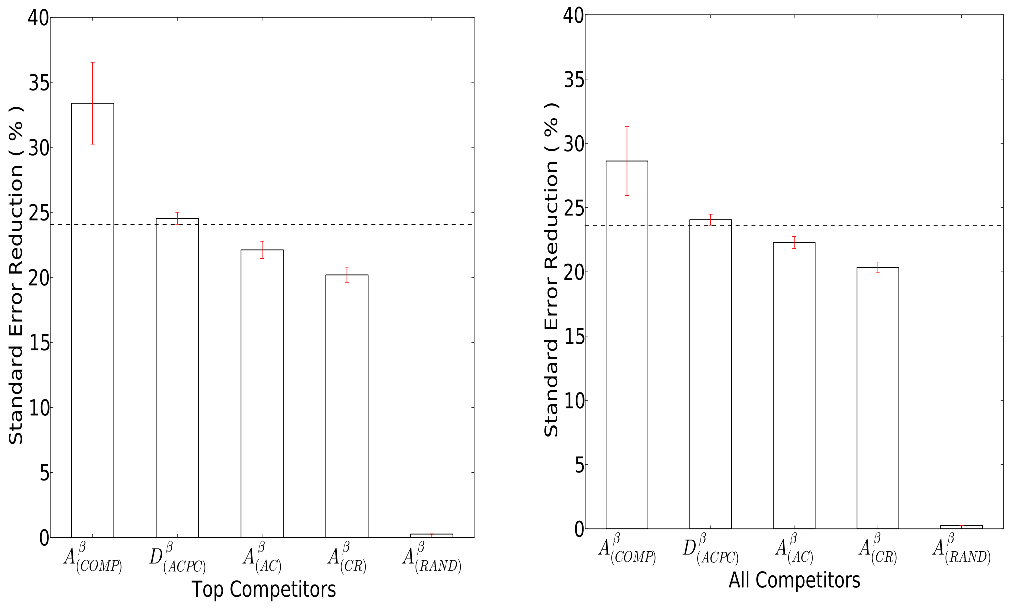
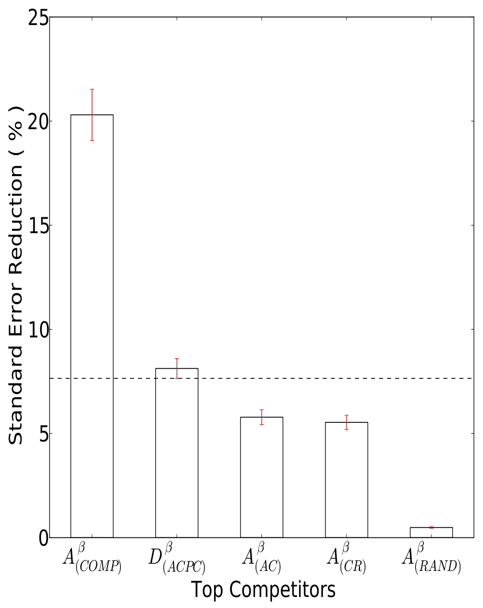


Figure 5.4: 2013 ACPC Three-Player Limit Average Estimator Performance

**Two-Player No-Limit ACPC Results**

The results for the evaluation of the 2013 ACPC two-player no-limit matches are presented in Figure 5.5. The average performance of the data baseline estimators are once again slightly better than all three of the rule-based agent baseline estimators with 95% confidence. When evaluating the all-competitors dataset, the estimators created using the data baseline approach are able to achieve an average reduction in the standard error of approximately 9%, which is still only 5% lower than that of the competition grade agent baseline estimators on the same data. It is worth noting that the lower overall performance of all of the estimators for this domain is consistent with the results seen in the agent baseline analysis from Chapter 4. Not obvious from the presentation of these results was the presence of two abysmal agents in the competition, who each lost almost the maximum possible amount in every match. The inclusion of these agents in the analysis helps to explain some of the drop in average performance between the top-competitor and all-competitors datasets. Once again however, the data baseline estimator suffers much less of a performance drop between the two evaluation scenarios, reinforcing their robustness when evaluating agents with varying levels of skill.



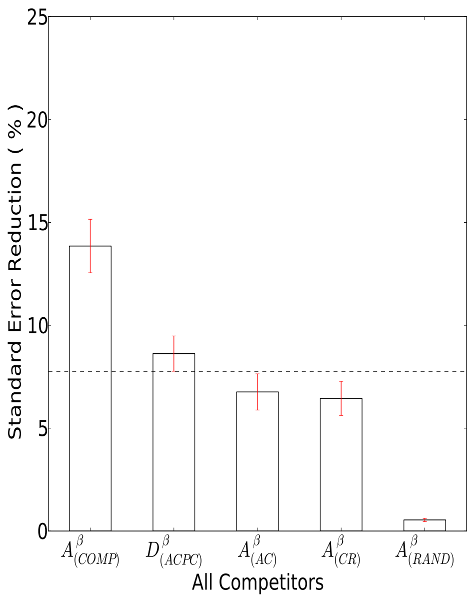


Figure 5.5: 2013 ACPC Two-Player No-Limit Average Estimator Performance

### 5.5.2 Human Texas Hold 'em Poker

#### Experimental Design

For the task of evaluating human play, the data baseline approach was evaluated using data gathered from four human versus computer-agent poker matches and one human versus human poker tournament. In the human versus computer-agent setting, data baseline estimators were used to evaluate the human players in four different matches, each match consisting of a different human player competing against a top-level computer agent in the one thousand hands of two-player no-limit Texas hold 'em. Two different data baseline estimators for this evaluation were created, one using a dataset of thirty thousand hands of human versus computer-agent play and the other using the past ACPC data. Both of these estimators were then compared to agent baseline estimators built using the same agent from the two-player no-limit ACPC analysis.

The human versus human analysis was executed using data from a competition held by the International Federation of Poker in 2011 [13]. This competition was a six-player no-limit duplicate poker tournament similar in format to the competitions in the ACPC. Due to the complexity of running their duplicate tournament, only 144 total unique hands were played by the competitors. Unfortunately, since data in this domain is highly guarded by both poker players and poker companies due to privacy concerns, we were forced to use this small amount of data for both evaluation and building the estimator. This was done using leave-one-out cross-validation on each of the six matches on the existing data. Two different data baseline estimators were created using the remaining data by either including or excluding the duplicate entries from the other matches. Including the duplicate entries in the validation helps to give an idea of what an estimator in this domain would be capable of, if it was built with a more sufficiently sized dataset. We compared the performance of the two data baseline estimators to an agent baseline estimator created using a simple always-call rule-based agent.

#### Cost Analysis

The experimentation involving human play used the exact same implementation choices that were made in the ACPC experiments and thus the costs associated with the evaluation are exactly the same as those in Section 5.5.1. The fact that gathering human samples can be orders of magnitudes slower than sampling computer agents, the overall costs associated with creating data baseline estimators have even less of an impact on the total evaluation cost for this type of domain.

#### Two-Player No-Limit Human versus Computer-Agent Results

Table 5.1 shows the performance of the data baseline and agent baseline estimators for the four different human versus computer-agent matches. In this domain, the data baseline estimators outperform the agent baseline estimators in three of the four matches as well as on average. Also suggested from these results is the presence of the same relationship between the size of the baseline estimator dataset and the quality of the data being used that was observed in the two-player limit ACPC analysis. Although the ACPC dataset is almost 100 times larger than the human versus computer-agent dataset, the data baseline estimators created with either dataset achieves similar performance in this domain. This result, combined with the performance versus dataset size analysis, reinforces the observation that using tailored datasets can increase overall performance in data baseline estimators.

#### Six-Player No-Limit Human versus Human Results

The cross-validation results for the six-player no-limit human tournament are shown in Table 5.2. Since a competition level agent from which to create an agent baseline estimator does not exist in this domain, we compared the data baseline estimator to an agent baseline estimator created using an always-call rule-based agent. Although the sample size is extremely

| | Match Reduction ( % ) | | | | |
|---|---|---|---|---|---|
| Estimator | 1 | 2 | 3 | 4 | Avg |
| $A^{\beta}_{(COMP)}$ | 1.37 | **10.91** | 8.62 | 0.79 | 5.42 |
| $D^{\beta}_{(Human)}$ | 4.31 | 2.30 | **16.43** | **15.25** | 9.57 |
| $D^{\beta}_{(ACPC)}$ | **5.53** | 10.66 | 15.71 | 10.61 | **10.63** |

Table 5.1: Two-Player No-Limit Human vs Computer-Agent Results (1000-Hand Matches)

small, the data baseline estimator created when leaving duplicate hands out of the dataset is able to achieve some reduction of the standard error. This performance is also better than that of the always-call agent baseline estimator in five of the six matches. The results are much better in the idealistic case where we include the duplicate hands in the data baseline estimator, gaining a substantial performance increase in five of the six matches. Although further experimentation with larger sample sizes is needed, the results indicate that data baseline estimators could be useful in these types of real world competitive human scenarios.

| | Match Avg Reduction ( % ) | | | | | | |
|---|---|---|---|---|---|---|---|
| Estimator | 1 | 2 | 3 | 4 | 5 | 6 | Avg |
| $A^{\beta}_{(AC)}$ | **2.60** | 0.54 | 0.77 | 0.40 | 0.97 | 0.37 | 0.94 |
| $D^{\beta}_{(No-Dup)}$ | 1.17 | **0.76** | **2.16** | **0.92** | **1.33** | **0.61** | **1.15** |
| $D^{\beta}_{(Dup)}$ | 7.40 | 5.18 | 7.11 | 10.24 | 1.60 | 6.45 | 6.33 |

Table 5.2: Six-Player No-Limit Human Tournament (144-Hand Matches)

# Chapter 6

# Conclusions

This work has demonstrated that the baseline approach to agent evaluation provides a simple method for creating efficient, unbiased estimators in highly stochastic multi-agent domains. Applications of this approach are easily implemented by merely choosing unbiased baseline scoring functions, two examples of which we have provided an in-depth analysis.

## 6.1 Agent Baseline

The agent baseline application of baseline estimators was shown to be a superior alternative to the duplicate approach currently used in the domain of Texas hold 'em computer poker. Not only is the agent baseline approach able to create unbiased estimators for computer poker domains, but for any zero-sum domain where there is a similarly skilled computer agent that can be used to create the necessary baseline scoring function, such as the Trading Agent Ad Auction Competition. The agent baseline approach is also particularly useful for evaluating computer-agent versus human scenarios where, unlike the duplicate approach, individual skill assessments can be made. While it is conceivable that there are situations where creating a duplicate estimator would be advantageous over agent baseline estimators, such as situations where a the agents being evaluated behave drastically different than the baseline agent, this scenario did not occur in any of the evaluation performed in this work. Agent evaluation using agent baseline estimators should be a serious consideration for any computer agent competition, human versus computer-agent competition or even human versus human matches in domains where strong computer agents exist.

## 6.2 Data Baseline

Creating baseline estimators using the data baseline approach addresses one major shortcoming of the agent baseline approach: the situation where there are simply no available competitive agents for the domain agent evaluation is being performed within. Data baseline estimators leverage existing player data to overcome this limitation with the intuition that for many domains, this data is much easier to obtain than competitive computer agents. By using player data to generate the baseline estimators, we have shown that creating efficient, unbiased estimators is possible for a much broader category of domains, and that the penalty in performance when compared to agent baseline estimators is minor. Data baseline estimators also outperform agent baseline estimators created with weak underlying baseline agents, and are well suited for creating tailored estimation based on both agent skill and style of play. This allows for efficient evaluation of a broader class of players. For domains where the agent baseline approach cannot be deployed, the data baseline approach should be strongly considered, especially if there is an abundance of player data for the domain.

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# Appendix A

# 2013 ACPC Baseline Full Results

## A.1 Two Player Limit

| Match Name | MC SE | DUP SE | $A^\beta$ SE | $D^\beta$ SE | DUP % | $A^\beta$ % | $D^\beta$ % |
|---|---|---|---|---|---|---|---|
| HITSZ/chump1 | 13.26 | **7.96** | 8.81 | 8.81 | **40.0** | 33.61 | 33.58 |
| HITSZ/chump12 | 9.73 | **5.86** | 6.21 | 6.61 | **39.76** | 36.15 | 32.07 |
| HITSZ/hyperborean_iro | 9.92 | 5.04 | **4.89** | 5.09 | 49.16 | **50.7** | 48.71 |
| HITSZ/hyperborean_tbr | 10.42 | **5.34** | 5.51 | 5.6 | **48.76** | 47.16 | 46.22 |
| HITSZ/liacc | 15.88 | **7.56** | 9.23 | 8.93 | **52.39** | 41.86 | 43.75 |
| HITSZ/littlerock | 9.85 | 5.01 | **4.97** | 5.15 | 49.09 | **49.56** | 47.7 |
| HITSZ/marv | 9.57 | 4.72 | **4.52** | 4.81 | 50.68 | **52.75** | 49.75 |
| HITSZ/neopokerlab | 9.91 | 5.1 | **4.97** | 5.12 | 48.58 | **49.83** | 48.36 |
| HITSZ/propokertools | 9.92 | **5.0** | **5.03** | 5.19 | **49.64** | 49.25 | 47.7 |
| HITSZ/slugathorus | 13.6 | **9.02** | 9.33 | 9.13 | **33.71** | 31.42 | 32.85 |
| HITSZ/unamur_iro | 9.63 | **5.26** | 5.4 | 5.38 | **45.33** | 43.93 | 44.1 |
| HITSZ/unamur_tbr | 10.28 | **5.44** | 5.87 | 5.74 | **47.05** | 42.88 | 44.13 |
| HITSZ/zbot | 10.02 | **5.09** | **5.0** | 5.16 | 49.16 | **50.13** | 48.46 |
| chump1/chump12 | 11.32 | 7.32 | **6.88** | 7.22 | 35.34 | **39.22** | 36.28 |
| feste_iro/HITSZ | 10.02 | **4.98** | 5.43 | 5.51 | **50.26** | 45.79 | 44.99 |
| feste_iro/hyperborean_iro | 9.2 | 4.13 | **3.72** | 4.18 | 55.1 | **59.5** | 54.57 |
| feste_iro/liacc | 14.9 | 8.42 | 8.51 | **8.24** | 43.53 | 42.9 | **44.74** |
| feste_iro/littlerock | 9.51 | 4.21 | **4.1** | 4.42 | 55.72 | **56.94** | 53.58 |
| feste_iro/marv | 10.19 | 4.31 | **4.01** | 4.46 | 57.71 | **60.67** | 56.25 |
| feste_iro/neopokerlab | 10.28 | 4.66 | **4.32** | 4.74 | 54.64 | **57.96** | 53.93 |
| feste_iro/propokertools | 10.32 | 4.52 | **4.45** | 4.77 | 56.2 | **56.85** | 53.74 |
| feste_iro/slugathorus | 12.57 | 8.04 | 8.04 | **7.99** | 36.03 | 36.03 | **36.42** |

. . . continued on next page

| Match Name | MC SE | DUP SE | $A^\beta$ SE | $D^\beta$ SE | DUP % | $A^\beta$ % | $D^\beta$ % |
|---|---:|---:|---:|---:|---:|---:|---:|
| feste_iro/unamur_iro | 9.26 | **4.52** | 4.62 | 4.68 | **51.2** | 50.18 | 49.46 |
| feste_iro/zbot | 10.32 | 4.68 | **4.38** | 4.77 | 54.7 | **57.54** | 53.84 |
| feste_tbr/HITSZ | 10.22 | **4.97** | 5.68 | 5.76 | **51.32** | 44.44 | 43.62 |
| feste_tbr/chump1 | 12.29 | **6.98** | 7.58 | 7.52 | **43.23** | 38.32 | 38.82 |
| feste_tbr/chump12 | 9.36 | 5.72 | **4.71** | 5.59 | 38.88 | **49.68** | 40.28 |
| feste_tbr/hyperborean_tbr | 9.73 | **4.41** | 4.5 | 4.56 | **54.69** | 53.73 | 53.15 |
| feste_tbr/liacc | 15.32 | **7.74** | 8.45 | 7.91 | **49.44** | 44.82 | 48.35 |
| feste_tbr/littlerock | 9.36 | 4.21 | **4.11** | 4.35 | 55.03 | **56.07** | 53.51 |
| feste_tbr/marv | 10.31 | 4.44 | **4.19** | 4.55 | 56.93 | **59.39** | 55.86 |
| feste_tbr/neopokerlab | 10.43 | 4.82 | **4.52** | 4.85 | 53.82 | **56.66** | 53.52 |
| feste_tbr/propokertools | 10.46 | **4.66** | **4.6** | 4.87 | 55.44 | **56.01** | 53.43 |
| feste_tbr/slugathorus | 14.41 | **9.04** | 9.28 | 9.06 | **37.25** | 35.63 | 37.12 |
| feste_tbr/unamur_tbr | 9.87 | **4.8** | 5.16 | 5.05 | **51.39** | 47.71 | 48.77 |
| feste_tbr/zbot | 10.48 | 4.8 | **4.55** | 4.86 | 54.18 | **56.57** | 53.64 |
| hyperborean_iro/liacc | 14.04 | 8.28 | 7.94 | **7.91** | 41.03 | 43.44 | **43.64** |
| hyperborean_iro/littlerock | 9.42 | 3.56 | **3.26** | 4.16 | 62.25 | **65.33** | 55.77 |
| hyperborean_iro/marv | 9.41 | 3.06 | **2.74** | 3.9 | 67.48 | **70.9** | 58.55 |
| hyperborean_iro/neopokerlab | 10.22 | 3.5 | **3.28** | 4.43 | 65.76 | **67.94** | 56.68 |
| hyperborean_iro/propokertools | 10.22 | 3.79 | **3.55** | 4.5 | 62.88 | **65.29** | 55.94 |
| hyperborean_iro/slugathorus | 12.35 | 7.91 | **7.66** | 7.83 | 35.97 | **37.98** | 36.63 |
| hyperborean_iro/unamur_iro | 9.16 | 4.39 | **4.09** | 4.47 | 52.09 | **55.39** | 51.2 |
| hyperborean_iro/zbot | 6.49 | 2.33 | **2.15** | 2.84 | 64.03 | **66.78** | 56.2 |
| hyperborean_tbr/chump1 | 12.31 | 7.45 | **7.41** | 7.46 | 39.48 | **39.81** | 39.39 |
| hyperborean_tbr/chump12 | 9.3 | 5.81 | **4.36** | 5.51 | 37.54 | **53.16** | 40.78 |
| hyperborean_tbr/liacc | 14.58 | 8.26 | 8.45 | **8.09** | 43.32 | 42.06 | **44.49** |
| hyperborean_tbr/littlerock | 9.8 | 4.17 | **4.07** | 4.46 | 57.47 | **58.5** | 54.46 |
| hyperborean_tbr/marv | 10.51 | 4.05 | **3.96** | 4.49 | 61.43 | **62.29** | 57.3 |
| hyperborean_tbr/neopokerlab | 10.64 | 4.35 | **4.28** | 4.78 | 59.15 | **59.82** | 55.12 |
| hyperborean_tbr/propokertools | 10.63 | 4.49 | **4.42** | 4.81 | 57.79 | **58.45** | 54.73 |
| hyperborean_tbr/slugathorus | 12.78 | 8.18 | 8.11 | **8.09** | 36.01 | 36.58 | **36.69** |
| hyperborean_tbr/unamur_tbr | 9.89 | **4.66** | 4.97 | 4.93 | **52.92** | 49.77 | 50.2 |
| hyperborean_tbr/zbot | 10.64 | 4.37 | **4.33** | 4.8 | 58.93 | **59.34** | 54.91 |
| liacc/chump1 | 17.6 | **9.46** | 11.13 | 11.0 | **46.23** | 36.74 | 37.47 |
| liacc/chump12 | 13.02 | 8.74 | **7.13** | 8.2 | 32.89 | **45.24** | 37.03 |
| liacc/littlerock | 14.17 | 8.42 | 8.14 | **8.06** | 40.58 | 42.6 | **43.17** |
| liacc/marv | 13.14 | 6.54 | 6.4 | **6.25** | 50.18 | 51.25 | **52.43** |
| liacc/neopokerlab | 14.02 | 8.47 | 8.23 | **8.15** | 39.58 | 41.3 | **41.86** |
| liacc/propokertools | 14.3 | 8.59 | 8.31 | **8.24** | 39.91 | 41.86 | **42.35** |
| liacc/slugathorus | 19.28 | **11.28** | 12.31 | 11.89 | **41.51** | 36.14 | 38.31 |
| liacc/unamur_iro | 13.92 | **8.06** | 8.34 | 8.13 | **42.05** | 40.06 | 41.57 |
| liacc/unamur_tbr | 14.31 | **7.63** | 8.3 | 7.93 | **46.67** | 42.02 | 44.58 |
| liacc/zbot | 14.21 | 8.44 | 8.16 | **8.08** | 40.6 | 42.59 | **43.14** |

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| Match Name | MC SE | DUP SE | $A^\beta$ SE | $D^\beta$ SE | DUP % | $A^\beta$ % | $D^\beta$ % |
|---|---|---|---|---|---|---|---|
| littlerock/chump1 | 11.52 | 7.0 | **6.87** | 6.95 | 39.27 | **40.32** | 39.67 |
| littlerock/chump12 | 8.61 | 5.3 | **3.82** | 5.1 | 38.43 | **55.66** | 40.81 |
| littlerock/marv | 10.11 | 3.5 | **3.42** | 4.3 | 65.4 | **66.23** | 57.47 |
| littlerock/neopokerlab | 10.23 | 3.89 | **3.74** | 4.55 | 62.0 | **63.43** | 55.48 |
| littlerock/propokertools | 10.25 | 3.95 | **3.94** | 4.63 | 61.48 | **61.57** | 54.77 |
| littlerock/slugathorus | 12.37 | 7.89 | **7.72** | 7.83 | 36.2 | **37.58** | 36.68 |
| littlerock/unamur_iro | 9.17 | 4.44 | **4.25** | 4.52 | 51.6 | **53.63** | 50.71 |
| littlerock/unamur_tbr | 9.86 | 4.79 | **4.73** | 4.88 | 51.43 | **52.06** | 50.5 |
| littlerock/zbot | 10.26 | 4.01 | **3.87** | 4.62 | 60.95 | **62.29** | 54.97 |
| marv/chump1 | 11.08 | 6.38 | **6.25** | 6.31 | 42.44 | **43.58** | 43.01 |
| marv/chump12 | 8.57 | 5.21 | **3.54** | 4.97 | 39.27 | **58.65** | 42.03 |
| marv/neopokerlab | 10.14 | 3.35 | **3.22** | 4.25 | 66.96 | **68.2** | 58.09 |
| marv/propokertools | 10.13 | 3.51 | **3.45** | 4.3 | 65.37 | **65.94** | 57.52 |
| marv/slugathorus | 12.03 | 7.13 | **6.91** | 7.05 | 40.71 | **42.55** | 41.38 |
| marv/unamur_iro | 10.15 | 4.6 | **4.32** | 4.73 | 54.7 | **57.41** | 53.42 |
| marv/unamur_tbr | 10.47 | 4.72 | **4.6** | 4.86 | 54.87 | **56.07** | 53.55 |
| marv/zbot | 10.15 | 3.45 | **3.34** | 4.3 | 66.03 | **67.11** | 57.68 |
| neopokerlab/chump1 | 15.85 | 9.66 | **9.43** | 9.51 | 39.05 | **40.52** | 39.97 |
| neopokerlab/chump12 | 11.89 | 7.37 | **5.21** | 7.02 | 38.02 | **56.2** | 40.92 |
| neopokerlab/propokertools | 10.25 | 3.92 | **3.8** | 4.59 | 61.77 | **62.96** | 55.18 |
| neopokerlab/slugathorus | 12.32 | 7.96 | **7.76** | 7.88 | 35.37 | **36.99** | 36.07 |
| neopokerlab/unamur_iro | 10.21 | 4.89 | **4.64** | 5.0 | 52.14 | **54.55** | 51.07 |
| neopokerlab/unamur_tbr | 10.52 | 5.04 | **4.99** | 5.2 | 52.05 | **52.58** | 50.53 |
| neopokerlab/zbot | 10.27 | 3.73 | **3.67** | 4.57 | 63.7 | **64.22** | 55.53 |
| propokertools/chump1 | 11.52 | **6.9**5 | **6.9** | **6.9**8 | 39.65 | **40.13** | 39.43 |
| propokertools/chump12 | 8.64 | 5.34 | **3.82** | 5.11 | 38.15 | **55.81** | 40.78 |
| propokertools/slugathorus | 12.42 | 7.96 | **7.79** | 7.9 | 35.94 | **37.25** | 36.44 |
| propokertools/unamur_iro | 10.25 | 4.91 | **4.74** | 5.04 | 52.1 | **53.77** | 50.89 |
| propokertools/unamur_tbr | 10.59 | 5.13 | **5.09** | 5.27 | 51.51 | **51.89** | 50.24 |
| propokertools/zbot | 10.27 | 4.01 | **3.88** | 4.61 | 60.98 | **62.21** | 55.1 |
| slugathorus/chump1 | 15.45 | **10.44** | 11.08 | 11.15 | **32.38** | 28.23 | 27.8 |
| slugathorus/chump12 | 11.34 | 8.04 | **7.27** | 7.79 | 29.05 | **35.89** | 31.26 |
| slugathorus/unamur_iro | 10.85 | **6.92** | 7.07 | 7.05 | **36.23** | 34.82 | 35.01 |
| slugathorus/unamur_tbr | 12.91 | **8.2** | 8.47 | 8.36 | **36.49** | 34.36 | 35.23 |
| slugathorus/zbot | 12.47 | 7.96 | **7.77** | 7.88 | 36.15 | **37.69** | 36.8 |
| unamur_iro/zbot | 10.28 | 4.94 | **4.73** | 5.07 | 51.96 | **53.99** | 50.65 |
| unamur_tbr/chump1 | 11.46 | **6.98** | 7.31 | 7.33 | **39.12** | 36.22 | 36.01 |
| unamur_tbr/chump12 | 8.99 | 5.71 | **4.78** | 5.46 | 36.51 | **46.85** | 39.3 |
| unamur_tbr/zbot | 10.62 | 5.08 | **5.07** | 5.27 | 52.16 | **52.29** | 50.43 |
| zbot/chump1 | 11.64 | 7.06 | **6.87** | 6.95 | 39.4 | **41.03** | 40.28 |
| zbot/chump12 | 8.66 | 5.37 | **3.82** | 5.15 | 37.99 | **55.9** | 40.57 |

Table A.1: 2013 Two-Player Limit ACPC Full Results

## A.2 Three Player Limit

| Match Name | MC SE | DUP SE | $A^\beta$ SE | $D^\beta$ SE | DUP % | $A^\beta$ % | $D^\beta$ % |
|---|---|---|---|---|---|---|---|
| HITSZ/hyper_iro/kempfer | 21.72 | **16.44** | 16.6 | 16.53 | **24.5** | 23.52 | 23.82 |
| HITSZ/hyper_tbr/kempfer | 23.55 | **16.98** | 18.14 | 17.85 | **27.79** | 22.86 | 24.05 |
| HITSZ/liacc/hyper_iro | 35.7 | **25.8** | 27.91 | 27.14 | **26.65** | 21.84 | 23.8 |
| HITSZ/liacc/hyper_tbr | 38.07 | **26.69** | 29.43 | 28.47 | **29.42** | 22.71 | 25.14 |
| HITSZ/liacc/kempfer | 35.16 | **24.08** | 28.5 | 27.41 | **30.45** | 18.69 | 21.67 |
| HITSZ/liacc/littlerock | 35.34 | **25.63** | 27.86 | 27.12 | **26.2** | 21.12 | 23.01 |
| HITSZ/littlerock/hyper_iro | 22.58 | 16.33 | **15.88** | 16.19 | 28.31 | **29.77** | 28.33 |
| HITSZ/littlerock/hyper_tbr | 24.51 | **17.37** | 17.5 | 17.6 | **29.47** | 28.68 | 28.19 |
| HITSZ/littlerock/kempfer | 21.54 | **16.2** | 16.65 | 16.5 | **25.02** | 22.7 | 23.37 |
| liacc/hyper_iro/kempfer | 25.72 | 20.92 | **19.44** | 19.46 | 18.96 | **24.29** | 24.23 |
| liacc/hyper_tbr/kempfer | 27.25 | 21.13 | 20.47 | **20.33** | 22.43 | 24.71 | **25.22** |
| liacc/littlerock/hyper_iro | 26.41 | 21.36 | **18.01** | 18.46 | 20.3 | **31.93** | 30.13 |
| liacc/littlerock/hyper_tbr | 27.75 | 21.41 | **19.0** | 19.2 | 23.59 | **31.62** | 30.8 |
| liacc/littlerock/kempfer | 25.45 | 20.85 | **19.34** | **19.34** | 18.49 | **23.9** | 23.89 |
| littlerock/hyper_iro/kempfer | 17.85 | 11.41 | **10.93** | 11.98 | 35.84 | **38.64** | 32.79 |
| littlerock/hyper_tbr/kempfer | 18.8 | 12.18 | **11.87** | 12.72 | 34.92 | **36.78** | 32.27 |
| neopoker/HITSZ/hyper_iro | 22.51 | 16.42 | **16.03** | 16.24 | 27.65 | **28.88** | 27.89 |
| neopoker/HITSZ/hyper_tbr | 24.61 | **17.53** | 17.77 | 17.75 | **29.06** | 27.84 | 27.87 |
| neopoker/HITSZ/kempfer | 21.17 | **15.87** | 16.38 | 16.24 | **25.27** | 22.64 | 23.27 |
| neopoker/HITSZ/liacc | 35.7 | **25.89** | 28.43 | 27.48 | **26.24** | 20.28 | 22.8 |
| neopoker/HITSZ/littlerock | 22.16 | 16.32 | **15.95** | 16.12 | 26.95 | **28.09** | 27.23 |
| neopoker/hyper_iro/kempfer | 17.38 | 11.7 | **10.98** | 11.91 | 32.55 | **36.75** | 31.42 |
| neopoker/hyper_tbr/kempfer | 18.54 | 12.69 | **12.22** | 12.89 | 31.54 | **34.04** | 30.45 |
| neopoker/liacc/hyper_iro | 26.43 | 22.07 | **18.98** | 19.15 | 17.52 | **28.26** | 27.55 |
| neopoker/liacc/hyper_tbr | 27.74 | 22.29 | 19.99 | **19.96** | 20.3 | 27.93 | **27.98** |
| neopoker/liacc/kempfer | 25.95 | 21.59 | 20.25 | **20.19** | 17.27 | 21.9 | **22.11** |
| neopoker/liacc/littlerock | 26.3 | 22.25 | **19.18** | 19.31 | 16.51 | **27.12** | 26.56 |
| neopoker/littlerock/hyper_iro | 13.04 | 7.16 | **6.56** | 7.99 | 45.05 | **49.64** | 38.68 |
| neopoker/littlerock/hyper_tbr | 19.23 | 11.04 | **10.32** | 11.84 | 42.59 | **46.35** | 38.4 |
| neopoker/littlerock/kempfer | 17.36 | 11.7 | **11.23** | 12.07 | 32.54 | **35.25** | 30.47 |

Table A.2: 2013 Three-Player Limit ACPC Full Results

## A.3 Two Player No-Limit

| Match Name | MC SE | DUP SE | $A^\beta$ SE | $D^\beta$ SE | DUP % | $A^\beta$ % | $D^\beta$ % |
|---|---|---|---|---|---|---|---|
| HITSZ/Sartre | 31.9 | **30.97** | 31.32 | 31.06 | **2.92** | 1.8 | 2.63 |
| HITSZ/entropy | 41.92 | **36.73** | 38.39 | 36.8 | **12.37** | 8.42 | 12.21 |
| HITSZ/hugh | 22.06 | **15.7** | 19.38 | 17.99 | **28.82** | 12.14 | 18.47 |
| HITSZ/hyperborean_iro | 33.4 | **30.61** | 31.3 | 30.74 | **8.35** | 6.28 | 7.96 |
| HITSZ/hyperborean_tbr | 86.78 | **77.5** | 82.97 | 80.96 | **10.7** | 4.4 | 6.7 |
| HITSZ/kempfer | 35.41 | **24.86** | 30.72 | 27.9 | **29.78** | 13.22 | 21.2 |
| HITSZ/koypetition | 20.06 | **17.01** | 17.95 | 17.29 | **15.2** | 10.53 | 13.81 |
| HITSZ/liacc | 228.45 | **139.88** | 212.17 | 199.17 | **38.77** | 7.13 | 12.82 |
| HITSZ/littlerock | 27.42 | **25.71** | 26.28 | 25.97 | **6.24** | 4.16 | 5.31 |
| HITSZ/neopokerlab | 35.26 | **25.31** | 30.17 | 28.03 | **28.22** | 14.45 | 20.5 |
| HITSZ/nyx | 22.16 | **19.66** | 20.55 | 20.07 | **11.27** | 7.26 | 9.43 |
| HITSZ/slumbot | 26.69 | **24.14** | 24.96 | 24.49 | **9.57** | 6.49 | 8.25 |
| HITSZ/tartanian6 | 47.85 | **44.0** | 45.13 | 44.2 | **8.05** | 5.69 | 7.63 |
| Sartre/entropy | 37.92 | **31.35** | 31.93 | 32.65 | **17.33** | 15.79 | 13.91 |
| Sartre/hugh | 34.81 | 30.07 | **29.47** | 30.39 | 13.62 | **15.35** | 12.7 |
| Sartre/hyperborean_iro | 29.79 | **24.18** | 24.36 | 25.3 | **18.84** | 18.24 | 15.07 |
| Sartre/hyperborean_tbr | 53.17 | **48.18** | 48.64 | 48.51 | **9.4** | 8.53 | 8.76 |
| Sartre/kempfer | 46.42 | 40.61 | **40.57** | 40.66 | 12.52 | **12.6** | 12.4 |
| Sartre/koypetition | 44.44 | 35.91 | **35.42** | 37.52 | 19.2 | **20.3** | 15.57 |
| Sartre/liacc | 84.07 | **78.78** | 81.58 | 81.37 | **6.29** | 2.96 | 3.22 |
| Sartre/littlerock | 26.26 | **22.33** | 22.34 | 23.2 | **14.94** | 14.92 | 11.64 |
| Sartre/neopokerlab | 42.4 | **33.76** | 34.69 | 35.95 | **20.38** | 18.19 | 15.21 |
| Sartre/nyx | 26.82 | **21.18** | 21.51 | 22.71 | **21.02** | 19.79 | 15.32 |
| Sartre/slumbot | 23.53 | **19.3** | **19.37** | 20.2 | **17.96** | 17.68 | 14.14 |
| Sartre/tartanian6 | 43.34 | **35.39** | 36.25 | 37.21 | **18.33** | 16.34 | 14.14 |
| entropy/hugh | 30.9 | **27.2** | 27.33 | 28.36 | **11.97** | 11.53 | 8.22 |
| entropy/hyperborean_iro | 34.36 | 28.81 | **28.51** | 30.45 | 16.14 | **17.01** | 11.38 |
| entropy/hyperborean_tbr | 50.5 | **43.96** | 46.08 | 46.25 | **12.94** | 8.76 | 8.41 |
| entropy/kempfer | 56.08 | 49.31 | **48.66** | 50.67 | 12.08 | **13.24** | 9.65 |
| entropy/koypetition | 51.1 | **42.36** | 42.64 | 45.22 | **17.1** | 16.56 | 11.52 |
| entropy/liacc | 71.08 | **68.47** | 69.66 | 69.6 | **3.67** | 1.99 | 2.08 |
| entropy/littlerock | 32.75 | 28.2 | **27.99** | 29.19 | 13.9 | **14.54** | 10.88 |
| entropy/neopokerlab | 41.58 | 36.7 | **36.38** | 37.92 | 11.73 | **12.52** | 8.81 |
| entropy/nyx | 33.2 | 27.89 | **27.79** | 29.12 | 15.99 | **16.28** | 12.26 |
| entropy/slumbot | 29.68 | 25.03 | **24.46** | 26.16 | 15.67 | **17.61** | 11.86 |
| entropy/tartanian6 | 50.67 | **42.72** | 42.87 | 44.56 | **15.68** | 15.38 | 12.06 |
| hugh/hyperborean_iro | 34.32 | 29.26 | **28.11** | 29.96 | 14.74 | **18.1** | 12.71 |
| hugh/hyperborean_tbr | 38.84 | 34.5 | **33.23** | 34.66 | 11.17 | **14.43** | 10.75 |
| hugh/kempfer | 56.77 | **47.96** | 48.98 | 51.03 | **15.53** | 13.72 | 10.11 |
| hugh/koypetition | 61.01 | 51.32 | **50.73** | 53.1 | 15.88 | **16.85** | 12.97 |
| hugh/liacc | 118.07 | **110.7** | 115.76 | 115.13 | **6.24** | 1.96 | 2.49 |
| hugh/littlerock | 36.37 | 31.58 | **30.99** | 32.35 | 13.19 | **14.81** | 11.07 |

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| Match Name | MC SE | DUP SE | $A^\beta$ SE | $D^\beta$ SE | DUP % | $A^\beta$ % | $D^\beta$ % |
|---|---|---|---|---|---|---|---|
| hugh/neopokerlab | 51.43 | **44.45** | 44.61 | 46.76 | **13.57** | 13.25 | 9.08 |
| hugh/nyx | 31.95 | 27.73 | **26.55** | 28.31 | 13.21 | **16.92** | 11.4 |
| hugh/slumbot | 27.99 | 24.59 | **23.19** | 24.94 | 12.13 | **17.15** | 10.91 |
| hugh/tartanian6 | 54.0 | 46.23 | **45.14** | 46.74 | 14.38 | **16.41** | 13.45 |
| hyperborean_iro/kempfer | 41.44 | 35.4 | **34.23** | 36.1 | 14.56 | **17.39** | 12.87 |
| hyperborean_iro/koypetition | 50.26 | 41.27 | **40.19** | 43.37 | 17.88 | **20.04** | 13.71 |
| hyperborean_iro/liacc | 56.28 | **54.4** | 54.98 | 55.16 | **3.34** | 2.31 | 1.98 |
| hyperborean_iro/littlerock | 26.77 | 22.03 | **21.87** | 23.5 | 17.7 | **18.29** | 12.21 |
| hyperborean_iro/neopokerlab | 44.44 | 37.15 | **36.0** | 38.52 | 16.41 | **18.98** | 13.31 |
| hyperborean_iro/nyx | 27.63 | **22.13** | 22.14 | 24.01 | **19.91** | 19.86 | 13.08 |
| hyperborean_iro/slumbot | 11.44 | 8.94 | **8.85** | 9.75 | 21.87 | **22.68** | 14.8 |
| hyperborean_iro/tartanian6 | 46.58 | **35.95** | 36.96 | 39.06 | **22.82** | 20.66 | 16.14 |
| hyperborean_tbr/kempfer | 86.9 | **75.61** | 76.39 | 75.97 | **12.99** | 12.09 | 12.58 |
| hyperborean_tbr/koypetition | 51.61 | 43.73 | **42.8** | 45.09 | 15.27 | **17.08** | 12.64 |
| hyperborean_tbr/liacc | 69.8 | **65.86** | 68.17 | 68.16 | **5.65** | 2.33 | 2.35 |
| hyperborean_tbr/littlerock | 34.82 | 30.14 | **29.81** | 31.07 | 13.42 | **14.39** | 10.75 |
| hyperborean_tbr/neopokerlab | 54.62 | **45.54** | 46.44 | 47.89 | **16.63** | 14.98 | 12.32 |
| hyperborean_tbr/nyx | 37.81 | **31.03** | 31.49 | 33.24 | **17.93** | 16.73 | 12.1 |
| hyperborean_tbr/slumbot | 34.55 | 28.24 | **28.02** | 30.33 | 18.27 | **18.89** | 12.21 |
| hyperborean_tbr/tartanian6 | 50.22 | **41.08** | 42.22 | 43.58 | **18.21** | 15.93 | 13.22 |
| kempfer/koypetition | 74.23 | **61.89** | 62.16 | 64.0 | **16.63** | 16.27 | 13.79 |
| kempfer/liacc | 132.17 | **120.57** | 127.32 | 126.35 | **8.78** | 3.67 | 4.4 |
| kempfer/littlerock | 31.68 | 28.33 | **26.41** | 28.29 | 10.58 | **16.61** | 10.69 |
| kempfer/neopokerlab | 49.96 | 43.93 | **41.0** | 44.26 | 12.08 | **17.93** | 11.4 |
| kempfer/nyx | 37.8 | 32.23 | **31.16** | 32.84 | 14.74 | **17.57** | 13.12 |
| kempfer/slumbot | 33.9 | 29.44 | **28.06** | 29.83 | 13.15 | **17.21** | 11.98 |
| kempfer/tartanian6 | 64.82 | 54.73 | **54.31** | 55.47 | 15.57 | **16.22** | 14.43 |
| koypetition/liacc | 116.74 | **113.05** | 113.83 | 113.9 | **3.16** | 2.49 | 2.44 |
| koypetition/littlerock | 47.57 | 38.14 | **37.97** | 40.97 | 19.83 | **20.19** | 13.87 |
| koypetition/neopokerlab | 50.12 | 41.1 | **40.1** | 42.96 | 18.0 | **19.99** | 14.28 |
| koypetition/nyx | 45.17 | 35.7 | **35.21** | 38.3 | 20.97 | **22.06** | 15.21 |
| koypetition/slumbot | 46.59 | 37.02 | **35.9** | 39.53 | 20.54 | **22.95** | 15.16 |
| koypetition/tartanian6 | 48.18 | 38.14 | **37.5** | 40.26 | 20.85 | **22.16** | 16.45 |
| liacc/littlerock | 58.82 | 57.46 | **57.45** | 57.66 | 2.31 | **2.33** | 1.97 |
| liacc/neopokerlab | 89.71 | **84.31** | 87.85 | 87.72 | **6.03** | 2.07 | 2.22 |
| liacc/nyx | 55.92 | 54.46 | **54.34** | 54.63 | 2.62 | **2.83** | 2.31 |
| liacc/slumbot | 49.18 | **48.03** | 48.04 | 48.24 | **2.34** | 2.32 | 1.91 |
| liacc/tartanian6 | 71.77 | **69.31** | 70.34 | 70.5 | **3.44** | 2.0 | 1.77 |
| littlerock/neopokerlab | 38.71 | 31.84 | **31.68** | 33.97 | 17.75 | **18.16** | 12.24 |
| littlerock/nyx | 25.87 | **20.91** | 21.41 | 22.8 | **19.17** | 17.23 | 11.86 |
| littlerock/slumbot | 22.52 | 18.42 | **18.2** | 19.73 | **18.24** | 19.2 | 12.41 |
| littlerock/tartanian6 | 38.7 | **30.58** | 31.39 | 33.13 | **20.99** | 18.9 | 14.39 |

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| Match Name | MC SE | DUP SE | $A^{\beta}$ SE | $D^{\beta}$ SE | DUP % | $A^{\beta}$ % | $D^{\beta}$ % |
|---|---|---|---|---|---|---|---|
| neopokerlab/nyx | 41.42 | 33.99 | **33.5** | 35.84 | 17.92 | **19.12** | 13.47 |
| neopokerlab/slumbot | 36.98 | 30.7 | **29.67** | 32.44 | 16.99 | **19.78** | 12.29 |
| neopokerlab/tartanian6 | 47.78 | **38.83** | 39.29 | 41.16 | **18.74** | 17.77 | 13.87 |
| nyx/slumbot | 22.14 | 17.16 | **17.13** | 18.9 | 22.52 | **22.63** | 14.67 |
| nyx/tartanian6 | 39.56 | **30.93** | 31.35 | 33.38 | **21.8** | 20.75 | 15.62 |
| slumbot/tartanian6 | 37.73 | **29.46** | 29.49 | 31.42 | **21.91** | 21.83 | 16.74 |

Table A.3: 2013 Two-Player No-Limit ACPC Full Results