

Nonlinear Axisymmetric Magnetostatic Analysis for Electromagnetic Device Using TLM-Based Finite-Element Method

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Finite-element method (FEM) has been widely used in industrial applications for many years. With the increase of finite-element model complexity and the number of elements, computation burden has become a serious issue and requires more attention. In this paper, the transmission line method (TLM) is adopted to the solution of FEM-based nonlinear axisymmetric magnetostatic problem and an equivalent finite-element network's circuit model is given. Several methods were investigated to speed up the computation, such as Newton preconditioners, simplification of nonlinear equation solution and parallel computation. Compared with traditional Newton FEM iteration, our results show that the proposed method has more obvious advantages on computational efficiency at the same calculation accuracy. The method is applicable and valuable in the characteristic analysis and design of the magnetic device with a cylindrical structure.

Index Terms—Axisymmetric magnetostatic analysis, electromagnetic device, finite-element method, multi-thread, nonlinear magnetostatic, parallel processing, transmission line method (TLM).

I. INTRODUCTION

CALCULATION of magnetostatics and dynamics of electromagnetic device with nonlinear magnet is a problem of permanent interest in various applications. Many papers have been published on its improvement. From the traditional magnetic circuit method to the recent numerical computation method, the simulation method is gradually transferring from the single domain to the multi-physical domain. Nowadays, the main calculation method is the finite-element method (FEM) and it has been adopted by main commercial simulation software, such as ANSYS, ABAQUS, PATRAN, FLUX, and so on. With the increase of the complexity and size of finite-element model, as well as the requirement of the optimum design of the device, even if the accuracy of the FEM model can meet the industrial requirement, the calculation efficiency is still a bottleneck. The reduction of computation time has been a vital issue of virtual prototype technique in the field of electrical apparatus.

Aiming at the improvement of the computational efficiency, many studies have been reported and researchers have focused on the domain decomposition method and the solution of the sparse linear equations. In terms of domain decomposition, [1]–[3] propose a parallel 3-D mesh refinement based on dynamic load balancing algorithm. The method allows for a relatively detailed description of the system and revealed key performance characteristics. References [4] and [5] adopt the finite-element tearing and interconnecting (FETI) method to divide the computational domain into non-overlapping

subdomains of smaller size, which allowed the possibility of parallel computing. Significant reduction in computation time was achieved using the highly parallelizable nature of FETI through parallel computing with multiple processors and the method was effectively used to solve electrothermal cosimulation problem. The dual-primal FETI (FETI-DP) method was combined with the Newton–Raphson (N–R) method to expand the capability and improve the efficiency of 3-D nonlinear Finite Element Analysis for electromechanical problems [6], [7]. Reference [8] proposes an approach to avoid the computationally expensive domain decomposition at each step by properly choosing an initial decomposition, which was applied to all steps throughout the simulation.

Another important aspect of FE computation is the solution of sparse nonlinear equations. For improving the calculation efficiency, the related research on serial algorithms, including the direct iteration method, N–R, and so on, was implemented and the relaxation method was used to reduce the iteration times [9], [10]. References [7] and [8] use a multigrid preconditioned conjugate gradients method to solve the discretized Partial Differential Equation (PDE) systems. The method properly solves the problem discretized on a hierarchy of grids while eliminating different scales of eigenfrequencies of the PDE operator separately. This makes the rate of convergence to be optimal with respect to the number of degrees of freedom. With the spread of multi-core computational platform, more studies focused on the parallelism of the solution method. Reference [12] studies the parallelizing technique of iterative process when FEM was used to calculate electromagnetic problems and pointed out the pre- and postprocessing of FEM should also be considered by parallelizing. Reference [13] combined the parallel boundary element method and serial FEM and used the approach in the Generalized Minimal Residual method iterations. The method not only speeds up

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the computation, but also overcomes the limited storage of a single computer. Reference [14] gave an evaluation of the numerical simulation of electrical machines by means of a hybrid parallelization of the FEM-package iMOOSE using the standardized paradigms OpenMP and Message Passing Interface (MPI). The evaluation showed that the solution process of the system of equations limited the possible speed up owing to the increasing number of required iterations with increasing number of used cores. Reference [15] tested significant improvements of parallel sparse direct solver MUMPS in 3-D numerical simulation of induction heating industrial applications. Results showed a remarkable reduction of both memory usage and number of performed operations. Furthermore, both tree and node parallelisms are exploited to reduce the solution time. For improving the FE computation, the transmission line method (TLM) is employed as a novel method of solution; [16]–[18] adopted this method to fulfill the time-domain induction motor analysis. Based on the above-mentioned research, [19] further compared the TLM-DD and N–R approaches and described the necessary steps to implement the TLM-DD approach for time-stepped eddy-current analysis of induction machines and [20] presented an approach to parallelize 2-D eddy-current finite-element analysis of skewed induction machines. A novel TLM method has also been proposed [21] for an accurate nonlinear permeance network simulation of induction machines.

However, little attention has been focused on the application of the TLM method of axisymmetric magnetostatic problem. And previous studies failed to point out how to obtain a good guess of the TL's admittance. In this paper, we focus on the TLM-based FEM and apply the method to solve a typical nonlinear axisymmetric magnetostatic problem of electromagnetic device. The method proposed can be extended into a more common form. Furthermore, we propose a new way of the TLM convergence and provide several methods to speed up the computation. Meanwhile, we have compared the method with traditional N–R method. Results show that the method in this paper can significantly speed up the computation and has good prospects in the future.

This paper is organized as follows. Section II briefly describes the TLM modeling strategy and the TLM-FEM formulation of axisymmetric magnetostatic problem. Section III provides simulation results for a dc electrical contactor, followed by the conclusions in Section IV.

II. TLM-BASED FEM METHOD

A. Principle of Approach

The TLM iteration for solution of electric circuits is based on the TL theory [22]. By adding a virtual TL between the linear network and the unknown electric component, as shown in Fig. 1(a), nonlinear iteration can be implemented by exploiting the travel time delay on the TL and it includes two procedures: incidence [Fig. 1(b)] and reflection [Fig. 1(c)]. In Fig. 1, U is the voltage source, and Z is its inner resistor. Y_0 is the TL's admittance. The nonlinear resistor R is unknown. And the solving process is shown in Fig. 2.

Compared with the traditional Newton iteration, the TLM has some exciting features on the iterative computation that

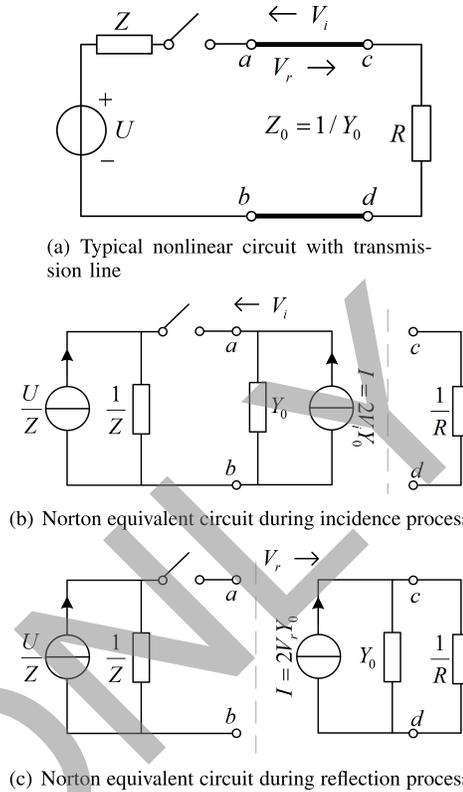


Fig. 1. Nonlinear circuit and its TLM solution. (a) Typical nonlinear circuit with TL. (b) Norton equivalent circuit during incidence process. (c) Norton equivalent circuit during reflection process.

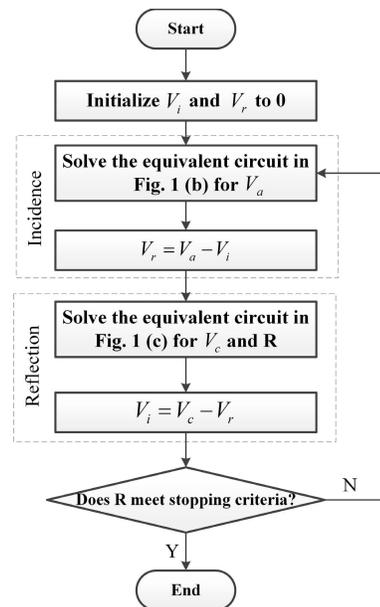


Fig. 2. Flowchart for solving nonlinear circuit using TLM.

can be used to the calculation of nonlinear network. The TL transforms the linear and the nonlinear hybrid network solution into a fully linear network solution and N separate small nonlinear problem solutions; thus, there is no need to assemble the global matrix and perform the LU factorization

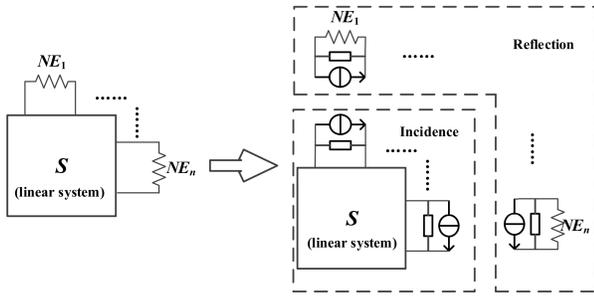


Fig. 3. Principle of TLM iteration.

repeatedly on each iteration which can save about 95% time in an iteration cycle [23]; at the same time, every nonlinear part solution can be put into an individual processing element to accelerate the solution process using parallel programming. Fig. 3 shows the principle of the TLM iteration method for a nonlinear circuit system. In Fig. 3, S represents the linear part of the system, and NE_i ($i = 1, 2, 3, \dots, N$) represents the nonlinear part of the system.

Based on the above-mentioned characteristics, the Newton iteration was replaced with the TLM method in the FE analysis as an alternative nonlinear solution method [17]. There is a strong similarity between the FEM method in static magnetic fields and nodal analysis in circuit solution for the reason that they eventually need to solve (1) and (2) separately. In (1), C is the global coefficient matrix of the finite-element network, A is the unknown nodal magnetic potential matrix, and J represents the excitation source matrix. Similarly, in (2), Y is the admittance matrix of the unresolved circuit, V is the unknown electric potential matrix, and I is the current source matrix. Obviously, the reason that C and Y are both positive-definite sparse matrices and can be assembled quickly makes it possible to transfer the finite-element mesh network into a nonlinear circuit network and combine the TLM method with the FEM method to solve the static magnetic field.

$$[C][A] = [J] \quad (1)$$

$$[Y][V] = [I]. \quad (2)$$

B. Typical Nonlinear Axisymmetric Magnetostatic Problem and Its TLM-FEM Solution

In static magnetic fields, to obtain magnetic flux density \vec{B} and permeability μ , we generally start with the magnetic potential \vec{A} , that is governed by Poisson's equation

$$\nabla \times v \nabla \times \vec{A} = \vec{J} \quad (3)$$

where the reluctivity $v = 1/\mu$.

For a 2-D (xy plane) nonlinear magnetostatic problem, the direction of current density \vec{J} and magnetic potential vector \vec{A} are both parallel to the z -axis, that is, $A_z = A$, $A_x = A_y = 0$, $J_z = J$, and $J_x = J_y = 0$. The corresponding partial differential equation can be described as

$$\frac{\partial}{\partial x} \left(v \frac{\partial A_z}{\partial x} \right) + \frac{\partial}{\partial y} \left(v \frac{\partial A_z}{\partial y} \right) = -J_z. \quad (4)$$

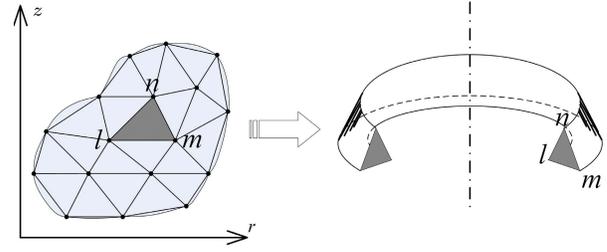


Fig. 4. Axisymmetric FE mesh.

The boundary condition is

$$\begin{aligned} A_z &= 0 \text{ on } \Gamma_1 \\ \frac{\partial A_z}{\partial n} &= 0 \text{ on } \Gamma_2. \end{aligned} \quad (5)$$

It can be noticed that when there is nonlinear magnetic material in the solution region, μ will be dependent on the variation of the magnetic flux density. References [16] and [17] have obtained a reduction in computational efforts using the TLM iterative method for this kind of problem. However, in the industry, most electromagnetic equipments are designed with axisymmetric feature. The 2-D computation model cannot be directly used in the axisymmetric example because of the axisymmetric structure. Some improvements are required.

In general, the cylindrical coordinate system (r, ϕ, z) is used in axisymmetric problems [24], and the direction of current density \vec{J} and magnetic potential vector \vec{A} are both perpendicular to the rz plane, that is, $A_\phi = A$, $A_r = A_z = 0$, $J_\phi = J$, and $J_r = J_z = 0$. The corresponding partial differential equation can be described as

$$\frac{\partial}{\partial z} \left(v \frac{\partial A_\phi}{\partial z} \right) + \frac{\partial}{\partial r} \left(v \frac{\partial (r A_\phi)}{\partial r} \right) = -J_\phi. \quad (6)$$

Let the boundary condition be

$$\begin{aligned} r A_\phi &= 0 \text{ on } \Gamma_1 \\ \frac{1}{r} \frac{\partial (r A_\phi)}{\partial n} &= 0 \text{ on } \Gamma_2. \end{aligned} \quad (7)$$

Using the FE method, a triangular mesh is generated in the solution region, as shown in Fig. 4. For an individual element (l, m, n) , the linear interpolation function is used to represent the unknown quantity in the interior of the element, namely

$$\begin{aligned} A^e(r, z) &= \sum_i a_i A_i \\ &= \sum_i \frac{1}{2\Delta} (a_i + b_i r + c_i z) A_i \quad (i = l, m, n) \end{aligned} \quad (8)$$

where

$$\begin{cases} a_l = r_m z_n - z_m r_n & a_m = r_n z_l - z_n r_l & a_n = r_l z_m - z_l r_m \\ b_l = z_m - z_n & b_m = z_n - z_l & b_n = z_l - z_m \\ c_l = r_n - z_m & c_m = z_l - z_n & c_n = z_m - z_l \end{cases}$$

and Δ is the area of the triangular element.

The axisymmetric magnetic potential field variation problem is

$$\begin{aligned} F(A) &= 2\pi \oint_S \frac{1}{2\mu} B^2 r dr dz - 2\pi \oint_S J A r dr dz \\ &= \min \end{aligned} \quad (9)$$

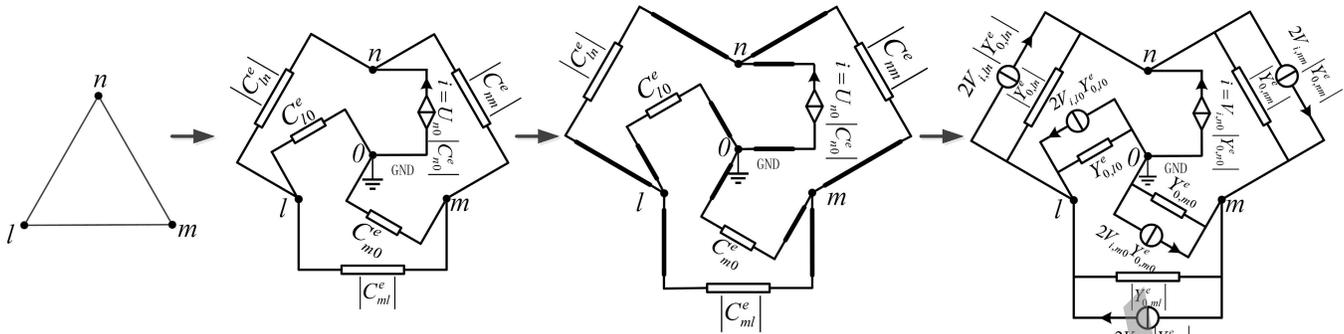


Fig. 5. Axisymmetric FEM triangular element's equivalent circuit.

where in each element

$$\begin{aligned}
 B^2 &= B_r^2 + B_z^2 \\
 B_r &= -\frac{\partial A}{\partial z} \approx -\sum_i \frac{c_i A_i}{2\Delta} \quad (i = l, m, n) \\
 B_z &= \frac{A}{r} + \frac{\partial A}{\partial r} \approx \frac{\sum_i A_i}{\sum_i r_i} + \sum_i \frac{b_i A_i}{2\Delta} \quad (i = l, m, n). \quad (10)
 \end{aligned}$$

By substituting (8) and (10) into (9), we can obtain the unit coefficient matrix

$$[C^e] = \begin{bmatrix} C_{ll}^e & C_{lm}^e & C_{ln}^e \\ C_{ml}^e & C_{mm}^e & C_{mn}^e \\ C_{nl}^e & C_{nm}^e & C_{nn}^e \end{bmatrix} \quad (11)$$

where

$$\begin{aligned}
 C_{rs}^e &= C_{sr}^e \\
 &= \frac{2\pi}{\mu} \left[\frac{r_c}{4\Delta} (b_r b_s + c_r c_s) + \frac{1}{6} (b_r + b_s) + \frac{\Delta}{9r_c} \right] \\
 &\quad (r, s = l, m, n) \\
 r_c &= (r_l + r_m + r_n)/3.
 \end{aligned}$$

Assuming that element's current density J_e is constant over the triangle, and then, the unit excitation source matrix is given as

$$[J^e] = \frac{2\pi \Delta}{3} J_e r_c [1 \quad 1 \quad 1]^T. \quad (12)$$

The matrix C^e 's diagonal items C_{ll}^e , C_{mm}^e , and C_{nn}^e can be seen as the self-admittances at the nodes, and the other admittances are the mutual admittances of the nodes. Thus, this finite-element network is equivalent to a circuit system. However, in a single element, because

$$C_{i0}^e = \sum_j C_{ij}^e = \frac{\sum_j b_j + 2b_i + \Delta}{3} \neq 0 \quad (i, j = l, m, n)$$

we know that the sum of the admittances in each row of C^e is not equal to zero in axisymmetric case which is the first part different from 2-D case. This means that there exists another component between the node and the ground whose admittance equals the sum. If the sum is positive, it should be a resistor, however, if the sum is negative, it should be a

controlled current source. Fig. 5 shows how we transform a single triangular element from the rough triangle to its TLM model. In the last sub-figure of Fig. 5, the circuit will be used when the incident voltages arrive at the linear network. In an individual mesh element, the circuit's admittance matrix is given as

$$[Y^e] = \begin{bmatrix} Y_{ll}^e & Y_{lm}^e & Y_{ln}^e \\ Y_{ml}^e & Y_{mm}^e & Y_{mn}^e \\ Y_{nl}^e & Y_{nm}^e & Y_{nn}^e \end{bmatrix} \quad (13)$$

where

$$\begin{aligned}
 Y_{ll}^e &= |Y_{0,ln}^e| + |Y_{0,ml}^e| + Y_{0,l}^e \\
 Y_{mm}^e &= |Y_{0,nm}^e| + |Y_{0,ml}^e| + Y_{0,m0}^e \\
 Y_{nn}^e &= |Y_{0,ln}^e| + |Y_{0,nm}^e| \\
 Y_{lm}^e &= Y_{ml}^e = -|Y_{0,ml}^e| \\
 Y_{ln}^e &= Y_{nl}^e = -|Y_{0,ln}^e| \\
 Y_{mn}^e &= Y_{nm}^e = -|Y_{0,nm}^e|. \quad (14)
 \end{aligned}$$

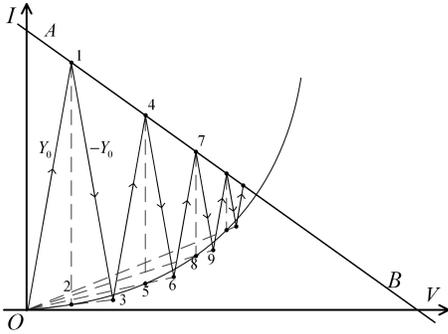
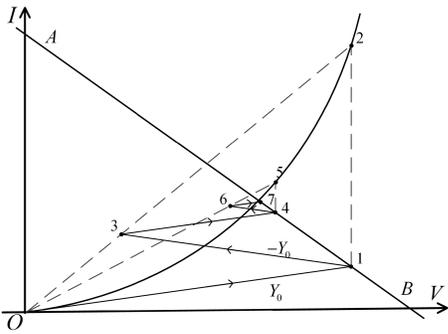
The right side current source matrix is

$$\begin{aligned}
 [I^e] &= \begin{bmatrix} 2V_{i,ml}|Y_{0,ml}^e| + 2V_{i,l0}Y_{0,l}^e - 2V_{i,ln}|Y_{0,ln}^e| \\ 2V_{i,nm}|Y_{0,nm}^e| + 2V_{i,m0}Y_{0,m0}^e - 2V_{i,ml}|Y_{0,ml}^e| \\ 2V_{i,ln}|Y_{0,ln}^e| + V_{i,n0}Y_{0,n0}^e - 2V_{i,nm}|Y_{0,nm}^e| \end{bmatrix} \\
 &\quad + \frac{2\pi \Delta}{3} J_e r_c \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\
 &= [I_{TLM}^e] + [J^e]. \quad (15)
 \end{aligned}$$

Once we assemble all the element matrix Y^e and I^e , a linear network can be obtained and its nodal admittance equation is

$$[Y][V] = [I_{TLM}] + [J]. \quad (16)$$

When the reflective voltages arrive at the nonlinear components, unlike the 2-D case, the nonlinear problem becomes more complex. We find it difficult to establish a relationship between the nonlinear resistor (or controlled current source)


 Fig. 6. Illustration of the convergence process when $Y_0 > Y_{\text{true}}$.

 Fig. 7. Illustration of the convergence process when $Y_0 < Y_{\text{true}}$.

and the potential differences across it because

$$\begin{aligned} \bar{B}^2 &= (\nabla \times \bar{A})^2, \\ &= \left(\frac{A}{r}\right)^2 + 2\frac{A}{r} \frac{\partial A}{\partial r} + |\nabla A|^2, \\ &= \left(\frac{\sum_i A_i}{\sum_i r_i}\right)^2 + \frac{\sum_i A_i}{\sum_i r_i} \sum_j \frac{b_j A_j}{2\Delta} - \sum_{i,j} \nabla a_i \nabla a_j (A_i - A_j)^2 \\ &\quad (i, j = l, m, n) \end{aligned} \quad (17)$$

which is the second part different from 2-D case. Here, we just utilize the vertex magnetic potential to obtain the flux density B by (10) and it directly determines the values of the nonlinear resistors or controlled current sources. Thus, we can solve the small circuit in Fig. 1(c) where the value of R is known and it is also a linear solution process. Figs. 6 and 7 show the convergence process of the TLM iteration of a single TL. Y_{true} is the true value of the nonlinear components. The straight line AB represents the linear network. At the beginning, the incident voltage and the reflective voltage are both equal to zero. When the incident voltage signals come into the linear network, we solve (16) for the voltage at point 1. When the reflective voltage signals come into the separate nonlinear parts, in axisymmetric case, we directly use magnetic potential value at point 1 to obtain flux density B by (10), i.e., we start from point 1 and draw a line up, ending at the point 2. In this way, there is no need to solve the nonlinear coupling which is a set of six-dependent nonlinear equations. In every small element, because we have obtained the nonlinear value, the solution is at point 3. After that, the process starts with a

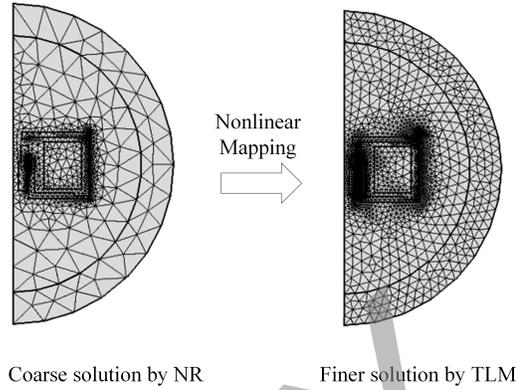


Fig. 8. Preconditioning before applying TLM-FEM.

new incident pulse and reflection process until the error of convergence reaches a small value which is a sequence of points 1, 4, 7, ... From the view of circuit, the TLM method will always converge unless the nonlinear problem is on the verge of instability and the rate of convergence is linear, slower than that of the Newton method which is quadratic [17].

C. Speed Up Strategies

1) *Choice of Y_0* : It is known that the initial guess of the reluctivity should be as close as possible to the true value if we want to get a shorter computation time. The Newton method is very efficient in nonlinear problem solution which means if we use the Newton method to solve the magnetostatic problem with a coarser mesh (Fig. 8) before applying the TLM method, we can obtain an ideal guess of Y_0 but with less time consumption. Assuming μ_{guess} is the result from the Newton method, we substitute it into (11) and get all items of matrix C^e which can be used to set the value of corresponding TL's admittance.

2) *Using Parallel Processing*: Multi-threaded parallel processing is employed to take advantage of multiple CPU cores. Two parts in the TLM method are considered. The first is the sparse matrix solution which is the most burdensome. Running on shared-memory and distributed-memory multiprocessors, there are several robust, efficient, and high-performance packages available, such as SuperLU_MT, MUMPS, PARDISO, and so on. The second part is the nonlinear solution in every element when the voltage pulse reflects into the nonlinear section. The OpenMP and MPI parallel programming Application Programming Interface are both taken into account for implementing this process. The solving process is shown in Fig. 9.

III. CASE STUDY

In this section, the TLM-based FEM is applied to solve axisymmetric magnetostatic problem of a dc power electrical contactor.

A. Structure of Typical Electrical Contactor

The dc power contactor usually adopts the solenoid structure in order to fulfill the fast making and breaking operations.

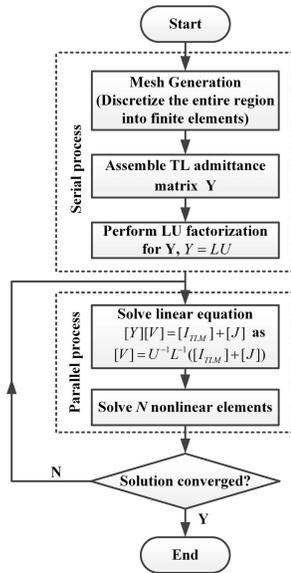


Fig. 9. TLM-FEM process flowchart.

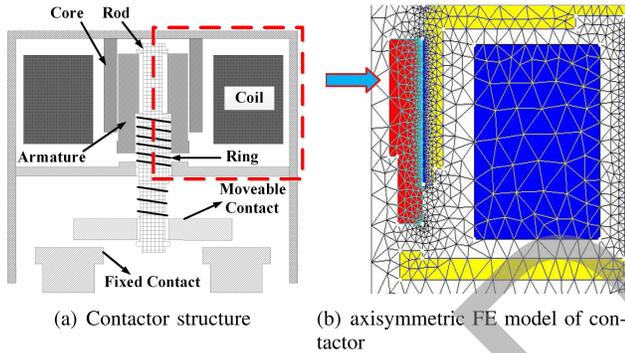


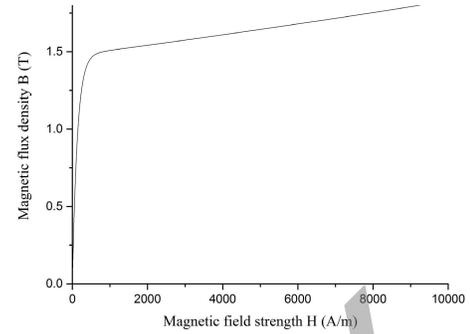
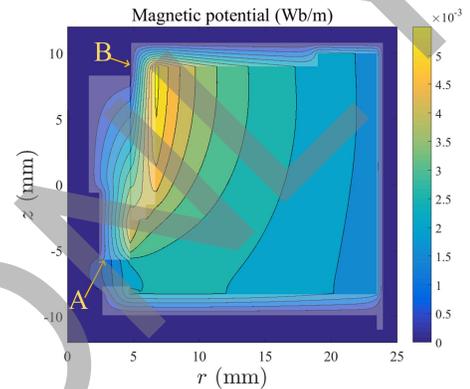
Fig. 10. Contactor structure and its 2-D FE model. (a) Contactor structure. (b) Axisymmetric FE model of contactor.

Contactors are constitutive of the electromagnetic system and the mechanical system. The typical structure of the contactor is shown in Fig. 10(a). Considering the solenoid structure is axisymmetric, an axisymmetric FE model of the contactor was built to calculate the magnetic field, as shown in Fig. 10(b). The voltage rating of the contactor is 12 V, the number of coil turns is 518 and the coil resistance is 3.2 Ω . The magnetic material used in the electromagnetic system is typical electrical pure iron (Type : DT4E) and we have measured magnetic induction B under different magnetic field intensity H using a dc hysteresis loop tracer whose model number is MATS-2010SD made by NextWare, Inc., Curve fitting is utilized to obtain the following analytical equation for the B - H curve (Fig. 11):

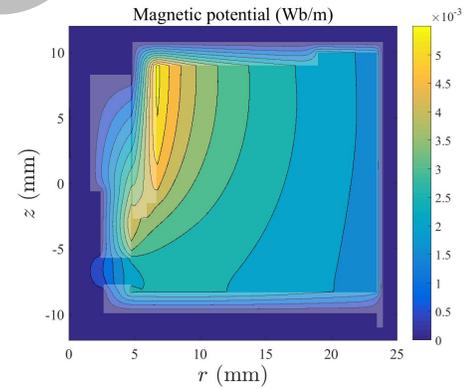
$$B(H) = 1.477e^{2.144 \times 10^{-5} H} - 1.473e^{-0.007577 H}. \quad (18)$$

B. Process of Solution and Comparison With Newton-Raphson Method

In the FE model, to achieve higher accuracy, the self-adaptive mesh technique is adopted, especially in the critical areas (e.g., the gap between the iron core and yoke), the

Fig. 11. B - H curve of DT4E material.

(a) Result of N-R solution

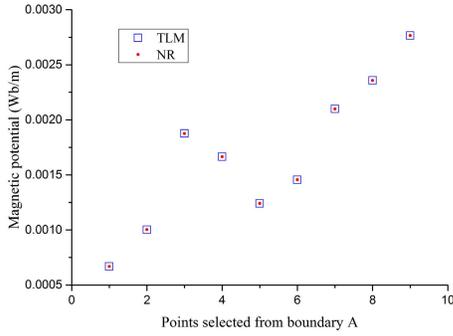


(b) Result of TLM-FEM solution

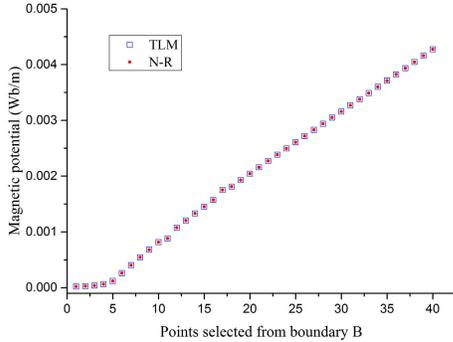
Fig. 12. Comparison of magnetic potential (Wb/m) distribution from two methods, ϕ component. (a) Result of N-R solution. (b) Result of TLM-FEM solution.

mesh is more dense. There are in total 60271 mesh nodes and 119677 surface elements in the model. The mesh shape is triangular and the minimum size of a mesh element is less than 0.1 mm.

The nonlinear algebraic equations derived from the FE model were solved by the TLM-FEM and N-R method, respectively. The magnetic potential distribution was calculated and comparison of the results is shown in Fig. 12. In Fig. 13, we also compare the results at points selected from the boundary A and B in Fig. 12 and it shows good agreement which has an absolute error of $1e-9$. Here, the sparse solver used is SuperLU (multi-thread version) and



(a) Points from boundary A



(b) Points from boundary B

Fig. 13. Comparison of magnetic potential at the boundary points. (a) Points from boundary A. (b) Points from boundary B.

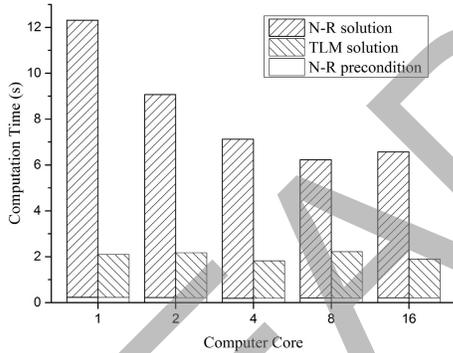


Fig. 14. Comparison of total computation time with respect to CPU cores.

OpenMP acceleration technique is chosen. Before we use the TLM and N-R iteration, we calculated a coarse FE mesh model (472 mesh elements and 1942 mesh nodes) by N-R iteration in advance.

To demonstrate the computation efficiency of our method, we compare some results on a multi-core workstation which has two Intel Xeon Processor E5-2670 v3(30M Cache, 2.30 GHz). The results in Fig. 14 show the comparison of the total computation time between N-R and TLM iteration both with an N-R preconditioner on different cores. The results in Fig. 15 show the comparison of single step's computation time of N-R and TLM method using 16 CPU cores. Fig. 16 shows the comparison of the computation time on different mesh sizes of the two methods. Table I shows the comparison of the iteration steps between TLM and N-R under different mesh size. The convergence error is $1e-6$.

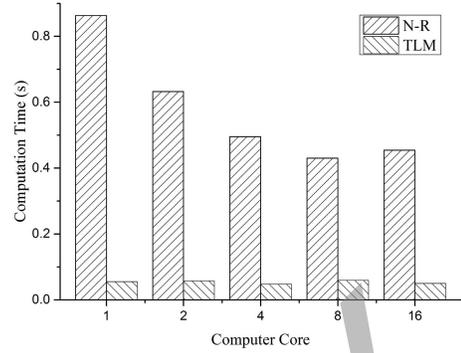


Fig. 15. Comparison of single step computation time with respect to CPU cores.

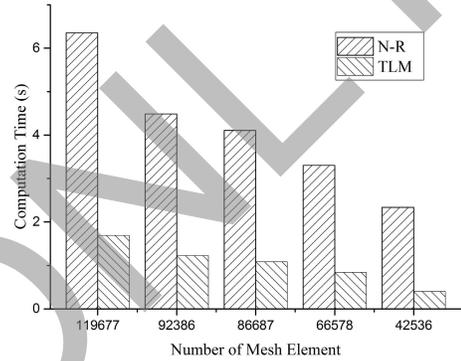


Fig. 16. Comparison of computation time with respect to number of finite elements.

TABLE I
COMPARISON OF ITERATION STEPS BETWEEN TLM AND N-R

Mesh size	Number of TLM Iterations	Number of N-R Iterations
472	24	14
42536	25	14
66578	28	14
86687	30	14
92386	32	14
119677	34	14

From Fig. 14, we notice that there is a large difference between N-R and TLM computation time and the N-R's time is almost six times of TLM's on one core. Although the parallel acceleration has been used, the TLM method still has a big advantage. In contrast with the parallel results, with the increment of the cores, it shows that the N-R iteration has a big promotion, while TLM's parallel effect is only marginal. In Fig. 15, because of no LU factorization, the comparison of the computation time in one step shows that the benefit from the TLM method is especially clear. The difference is more obvious when we increase the mesh size.

C. Results and Discussion

Overall, this paper has described the principle of the TLM method, the formulation and the implementation of the TLM method for the solution of axisymmetric magnetostatic problem. Compared with the 2-D TLM method, several features

in axisymmetric case have been discussed, in particular, the import of a negative admittance component—voltage controlled current source and the new iteration convergence process. The main contribution in [16] and [17] is introducing the circuit's TLM method into the 2-D FEM, and most of his conclusion is based on the fact that the nodal admittance matrix of a resistive network and the global matrix of 2-D FEM is equivalent. However, by the study of axisymmetric case, we can make it more general and practical. The FEM network can be transformed as a circuit network consisting of resistors and voltage controlled current sources.

Although there are important results revealed by this paper, there are also some limitations. The TLM method is easy to program, however, it requires a lot of memory storage which depends on the complexity of the structure and the number of mesh element. Each element should allocate memory for the incident and reflected voltage of the element nodes. In addition, it seems that the mesh element had better be triangle element. If not, the computation in every loop will increase with the number of element's nodes.

Furthermore, our results suggest that the TLM method in this paper is suitable to problems in other areas which can be solved by FEM, such as thermal, fluid, and mechanical analysis.

IV. CONCLUSION

In this paper, we focus on the TLM-based FEM and apply the method to solve a typical nonlinear axisymmetric magnetostatic problem of electromagnetic device. Meanwhile, we used Newton preconditioning, multi-thread sparse matrix solver SuperLU_MT and OpenMP to accelerate the problem solution. The axisymmetric results show that the proposed method in this paper has absolute advantage compared with the traditional Newton method in speeding up the computation and maybe have potential for further computation in 3-D field.

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