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University of Alberta

Estimating Labour Productivity Using Fuzzy Set Theory

by

Hongwei Mao



A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of Master of Science

in

Construction Engineering and Management

Department of Civil and Environmental Engineering

Edmonton, Alberta

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Sept 13/99

Abstract

This research involves the development of a fuzzy logic system to aid in the estimation of labour productivity. The objective of the research is to explore a method of using fuzzy set theory in the estimation of labour productivity. Concrete wall formwork is selected as an example application. The research includes identifying the factors that affect concrete formwork labour productivity, developing a fuzzy logic estimation model, and implementing it in a computer application.

The final fuzzy logic estimation model includes a fuzzy rule base, a fuzzy inference engine, a fuzzification module, and a defuzzification module. The productivity is predicted as a linguistic assertion describing the productivity level or as a single value. A construction company's Vancouver projects are used to validate the proposed fuzzy logic estimation model. A sensitivity analysis is conducted to assess the model's level of accuracy, flexibility, stability, and consistency.

The main conclusion of this thesis is that fuzzy logic provides a realistic and reasonable method of modeling the labour estimation problem. The contributions of this research include developing a fuzzy reasoning method that mirrors the decision-making process used in estimating labour productivity; illustrating a reasoning framework that can be modified to apply to different scenarios; and, understanding construction activities and the factors that affect labour productivity.

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Chapter 1

1. INTRODUCTION

1.1 Overview of Problem

Concrete formwork development has paralleled the growth of the construction industry throughout the centuries. Economy is always a major concern since formwork costs around 35 to 60 percent of the total cost of the concrete structure. Previous research (Touran 1988) demonstrates that the most important cost item in formwork is labor, which may account for more than 30 percent of the total concrete cost. The estimation accuracy of labor productivity in formwork activities is therefore of critical importance for a successful construction project.

Traditionally, estimators use companies' historical data and personal judgement to predict the labor productivity for formwork activity. Portas (1996) found that a contractor's estimate versus actual labor productivity has an accuracy of plus or minus 15% approximately 40% of the time for concrete wall formwork, and inaccuracies of 50% or 100% are possible. This result is not surprising because of the limitations of historical records and inconsistencies of judgement. Currently, statistical analysis, expert systems, and neural networks are applied in estimating formwork labor productivity, which help to improve the estimation accuracy.

Traditional methods and neural network methods all rely on estimators having available historical information or being able to quantify the factors affecting the formwork activity under consideration. However, estimating embraces numerous linguistic assertions of the relationship between the productivity and its influence factors, especially in the preliminary estimate phase. There is usually insufficient objective data to calculate the probability of construction events because of their unique nature, as when, for example, the project involves a new technology or new location. Both traditional methods and

neural network methods can not overcome this problem properly. Fuzzy set theory provides a suitable approach for solving this problem since it was developed specifically to deal with uncertainties that are not statistical in nature (Zadeh 1965). It is much closer in spirit to human thinking and natural language than the traditional logic systems. It has shown potential for quantitative evaluation of the effects of multiple inputs on output, especially when the relations between inputs and output can not be expressed in a mathematical equation and the problem involves linguistic variables.

This research examines the application of fuzzy set theory in predicting construction concrete formwork labor productivity.

1.2 Objectives

The primary objective of this research is to explore a method of using fuzzy set theory in the estimation of labor productivity when estimators do not have much exact information on the project being estimated. Concrete formwork activity is selected as a sample application. In addition, this research has the following sub-objectives:

- To develop a model to guide estimators in assessing where productivity lies.
- To train inexperienced estimators in the impact of factors affecting productivity.
- To explore a method of predicting productivity for new projects without similar historical records.
- To explore a method of defining membership functions based on objective data and the sensitivity of the results based on the shape and range of membership functions.
- To develop a fuzzy reasoning method that mirrors the decision-making process of estimators.
- To develop a model that accounts for the effect of numerous imprecise and subjective factors on outputs.

1.3 Methodology

In order to achieve the objectives, a fuzzy logic model was set up. This study focuses on concrete wall formwork of a general contractor in the building construction industry. Figure 1.1 shows the methodology used in this research.

The procedure used in conducting this research involves the following:

First, based on previous research and literature review, the factors affecting formwork labor productivity are identified and classified. Since each company has its own features, the factors are further identified based on the context of the study being undertaken. Each factor and productivity (output) are expressed by the appropriate linguistic states.

Second, a fuzzy membership function is introduced for each factor and productivity (input and output) to express the associated measurement uncertainty. The purpose of the fuzzy membership function is to interpret measurements of linguistic variables by means of a fuzzification function.

Third, the knowledge pertaining to the given problem is formulated in terms of a set of fuzzy inference rules. The rules are elicited from experienced engineers, common sense, and historical data from a general contractor in the building construction industry.

Fourth, a fuzzy inference engine is built to combine the relevant fuzzy rules to infer labor productivity (output). This stage of the study involves approximate reasoning with several conditional fuzzy propositions.

Finally, defuzzification methods are discussed. The purpose of defuzzification is to obtain a crisp value for labor productivity, derived from the fuzzy output. The productivity can then be estimated, based on the recommendations given by the model.

An example is given to illustrate the mechanism of the fuzzy logic estimation model. A company's Vancouver project data are used as an application to validate the model and conduct a sensitivity analysis.

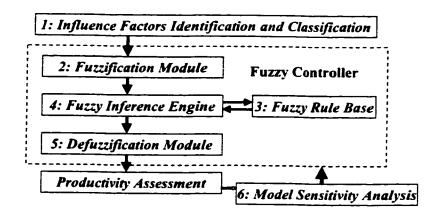


Figure 1-1: Components of the Fuzzy Logic Model

1.4 Expected Contribution

Fuzzy set theory is a branch of artificial intelligence that has been applied successfully in many fields. This thesis is expected to make the following contributions:

- 1. Developing a fuzzy reasoning method that mirrors the decision-making process used in estimating labor productivity that involves the consideration of numerous subjective factors affecting an output.
- 2. Providing a reasoning framework, based on sound techniques of fuzzy set theory, that can be modified to apply to different scenarios. Other factors can be substituted or added, with only the modification of the heuristic rules.
- 3. Using the framework with a sample application (formwork productivity) to illustrate how the decision-making process can be automated.

4. Defining data that needs to be collected in order to verify the framework, especially the heuristic rules. These data requirements provide the basis for a survey to collect such data.

1.5 Thesis Organization

Chapter 2 contains a literature review. It introduces fuzzy set theory and its applications in the construction industry. Labor productivity and the factors that affect it, and formwork productivity prediction models are described. Finally, fuzzy applications in formwork labor productivity estimation are discussed.

Chapter 3 identifies and classifies the factors affecting labor productivity of concrete wall formwork used in the fuzzy model. Fuzzy membership functions are developed for each factor based on the elements that define the factor.

Chapter 4 describes the method used in setting up a fuzzy logic model for estimating labor productivity for wall formwork. A fuzzy rule base and inference engine are presented. Several defuzzification methods are introduced. One defuzzification method developed by this study is recommended. An example is provided to illustrate the calculation process performed in the model.

Chapter 5 presents an application and a sensitivity analysis of the fuzzy logic model. The model is validated by a company's Vancouver project data. Different fuzzy inference methods, different shapes and ranges of fuzzy membership functions, and the influence of the different data sets are discussed

Chapter 6 presents the conclusions of this research and recommendations for future work.

5

Chapter 2

2. LITERATURE REVIEW

2.1 Introduction

This chapter reviews two areas of research related to this thesis:

- Applications of fuzzy set theory in construction.
- Formwork labor productivity estimation models.

First, a detailed introduction of fuzzy set theory and its applications in construction are presented. Then, labor productivity, the factors affecting it, and different concrete formwork labor productivity estimation models are described. This is followed by a discussion.

The objective of this literature review is:

- To illustrate the types of problems that are suited to fuzzy set theory modeling.
- To demonstrate how fuzzy set theory is applied in the construction industry.
- To understand why traditional formwork labor productivity estimation methods need to be improved.
- To explain why fuzzy set theory is suitable in formwork labor productivity estimation.

2.2 Fuzzy Set Applications in Construction

2.2.1 Fuzzy Set Theory

Fuzzy set theory was founded by L. A. Zadeh in 1965 (Zadeh 1965). Since then, virtually all disciplines have been affected to various degrees by this new methodology. The scope of fuzzy set applications ranges from theoretical to practical, and from the natural sciences and engineering to the humanities, medicine, and artificial intelligence.

Traditional mathematical methods usually require the transformation of problems from their intuitive basis into a mathematical format. They need suitable sets containing distinct objects, which are known as crisp sets. This method may require the transformation of notions which are, to some extent, only vaguely fixed, into clear, crisply determined ones, or the assumption that precise data or data with precise error bounds are available. In reality, this transformation or assumption may not be possible or correct. For example, we know that high temperatures (a vague concept) would affect construction labor productivity. We may have to specify that labor productivity will be influenced if temperature is above 40°C. But no one would doubt that 39.9 °C can also affect the productivity under the same circumstances.

In order to solve the problem above, Zadeh proposed a new, "rough" modeling method using fuzzy sets and fuzzy methods. He analyzed the problem and made two basic observations. First, humans have a capability to understand and analyze imprecise concepts, which is not easily incorporated into existing analytical methods. Second, current methodologies show a concern for precise representation of certain system aspects that are irrelevant to understanding the system's objectives (Kangari and Riggs 1989). In 1965, Zadeh advanced the concept of a fuzzy set. A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function that assigns to each object a grade of membership

ranging between zero and one. Fuzzy set theory was developed specifically to deal with uncertainties that are not statistical in nature (Zadeh 1965).

The difference between the fuzzy set concept and the conventional crisp set is mainly the degree to which an objective belongs to a set. In a crisp set, objects are either in or out of the set. A membership value of either 1 or 0 is assigned to each object in the universal set to discriminate between members and non-members of the crisp set under consideration. In a fuzzy set, however, a membership value between 1.0 and 0.0 can be assigned to each number in the universal set to indicate the degree to which the member belongs in the set under consideration, where zero means nonmembership and one signifies full membership.

For example: set A has elements $a_1, a_2, \dots a_{10}$.

If it is a crisp set, it can expressed as:

$$A = \{a_1, a_2, \dots a_{10}\}$$
 (2.1)

It also can be written as:

$$\forall_{x} \in X: \ m_{A}(x) = 1, \text{ if } x \in A;$$

$$m_{A}(x) = 0, \text{ otherwise.}$$
(2.2)

If it is a fuzzy set, it is defined as set of pairs $[\mu(a_i), a_i]$, where $\mu(a_i)$ is the membership value of element a_i . The set can be expressed as:

$$A = \{\mu(a_1)|a_1, \mu(a_2)|a_2, \mu(a_3)|a_3, ..., \mu(a_{10})|a_{10}\}$$
 (2.3)

Figure 2.1 clearly shows the difference in membership value between the crisp set and the fuzzy set.

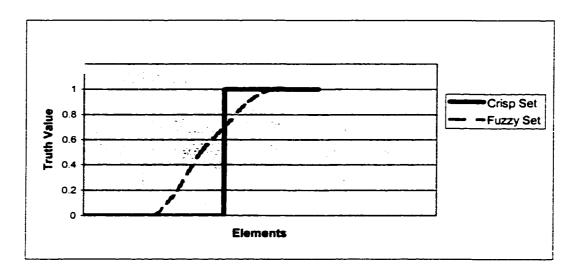


Figure 2-1: Fuzzy Set vs. Crisp Set

Since Zadeh initiated the concept of a fuzzy set, fuzzy set theory has been widely used in fields such as decision-making, information systems, uncertainty management, soft computing, image-processing, fuzzy hardware, robotics, etc. Table 2.1 shows the publications on fuzzy set theory and its applications during this decade.

Table 2.1: Number of Published Papers in Selected Databases

Engineering and Technology Database		
	Jan. 1997 - Oct. 1998	Jan. 1990 - Oct. 1998
Fuzzy Set or Fuzzy Logic	4064	11981
Fuzzy Set (or Logic) Applications	2056	6140
ASCE Civil Engineering Database	 	
Fuzzy Set	29	146

Development of fuzzy theory and its applications can be categorized into at least three phases. At first, fuzzy researchers did only "algorithmic" studies on the theory itself and tested it on some scenario cases or at the laboratory level. Soon real topics were successfully treated. Applications included classification, pattern recognition, database management, modeling of chemical processes, and operations research.

From the 1970s, Zadeh propagated new ideas of fuzzy set theory and applied them to knowledge representation and artificial intelligence. The main target of his research was towards natural language modeling. The topic is now referred to as "fuzzy logic" or "approximate reasoning".

Currently, much research has been done on fuzzy control and the use of fuzzy information in knowledge bases and expert systems. Fuzzy control is an advanced fuzzy application. It is used to convert a quality control strategy based on expert knowledge into an automatic control strategy. Detailed information on the development of fuzzy set theory can be found in many papers (Bandemer and Gottwald 1996; Lee 1990).

2.2.2 Fuzzy Set Applications

The construction industry is usually referred to as a knowledge-based industry. There is insufficient objective data to calculate the probability of construction events because of their unique nature. Many linguistic assertions (qualitative variables) are required. These characteristics make construction engineering a discipline in which traditional theories never fully fit the actual problem. As a result, the uncertainty in applying theoretical solutions to real projects is great. How to deal with this uncertainty becomes the major concern for construction engineers.

The uncertainty problems met in the construction industry include, for example, weather conditions, productivity levels, design quality, the evaluation of alternative technology, and the materials employed. Over the centuries, construction engineers have used their experience and "rules of thumb" derived from history to make decisions. Since the 1960s construction researchers have found that fuzzy logic is much closer in spirit to human thinking and natural language than the traditional logic systems. This seems to be the reason why research in the construction industry has so quickly found a strong affinity with fuzzy set theory.

Fuzzy applications in the construction engineering and management field (CEM) can be found in areas such as contractor evaluation and selection, cost estimating, bidding strategy, scheduling, reliability of construction operations (safety), and fuzzy expert systems. The number of applications has been growing so rapidly that now it is difficult to present a comprehensive survey of the wide variety of the applications that have been made. Here, four major categories of fuzzy application in CEM are presented: cost estimating, fuzzy expert systems, scheduling, and decision-making.

Cost Estimating

Tam et al. (1994) used fuzzy reasoning in tendering. They found that many risk factors which needed to be considered by estimators to decide mark-up were all qualitative and fuzzy in nature and therefore unable to be analyzed by the traditional quantitative mathematical models, such as market conditions, current workload of the contractor, labor supply, currency fluctuation, inflation, and project risks. They disclosed that the fuzzy reasoning approach accepted qualitative inputs and could handle the problem ideally.

Tam et al. set up a membership function for each risk factor. The data was passed into the inference engine where a rule base was established. Altogether there were 30 fuzzy rules derived from 30 sample projects. The rules were the backbone of the fuzzy reasoning technique. A computer program in Pascal was prepared for implementing the inference engine of the fuzzy reasoning mechanism, which consisted of three modules: the fuzzification module, the fuzzy operation module and the main program. The final result was obtained by means of a center of gravity method. One real project was used to verify this fuzzy model and there was a 15% relative error. They concluded that this result produced a reasonable level of acceptance and could prove its validity, a conclusion which may not be accepted by readers.

Mason and Kahn (1997) tried to use a fuzzy expert system in construction cost estimating. They found that during estimating, estimators would first identify attributes

of a project called cost drivers. The cost drivers were related to costs by cost estimating relationships (CERs). Due to lack of data or insight, estimators could usually only make linguistic assertions as to the relationship between the costs and cost drivers, instead of mathematical equations. Moreover, if a project involved an unconventional technology or unfamiliar location, numerical information would be impossible for estimators to obtain.

Based on this analysis, Mason and Kahn stated that a fuzzy approach "may be used to build a cost model". In their paper, they analyzed two attributes, groundwater level and political stability, as examples. A simple fuzzy expert system was built in order to show its fuzzy mechanism. Min-Max fuzzy composition was used as the inference engine principle. The final estimate value was given by means of the centroid method as a defuzzification approach.

Finally, the authors pointed out that membership functions must be obtained from the expert on that domain to interpret linguistic assertions. They concluded that currently little software is available for cost estimating with fuzzy set theory.

Fayek (1998) presented a competitive bidding strategy model for setting markup for civil engineering and building construction projects using fuzzy set theory. She stated that though many competitive bidding strategy models have been available since the 1950s, few of these are used in practice. The main reason is that they do not suit the actual practices of construction contractors.

Fayek pointed out that a good competitive bidding strategy model should have the capability of using qualitative and subjective contractor judgement and heuristic logic, rather than extensive mathematical or statistical techniques, and should display less reliance on historical project and competitor data.

She applied fuzzy set theory as the main mathematical inference tool. The reasons for choosing fuzzy set theory to develop the bidding model were:

- Fuzzy set theory could represent the linguistic approximations in numerical form so that a computer could manipulate them.
- Fuzzy set theory could generate solutions to problems affected by human subjectivity.
- A fuzzy bidding model would be suitable in practice.

In her paper, two fuzzy composition operations, max-min and cum-min, were tested. The center of area method was applied as the defuzzification tool. A prototype computer-aided estimating and bidding system called PRESTTO was developed to implement her competitive bidding strategy model.

Fayek's research demonstrated that fuzzy set theory is a suitable and realistic tool for setting markup for the construction industry. The most significant feature of her work is that her fuzzy bidding model was validated with data from actual project bids, collected from a survey of the Australian construction industry. This is the only fuzzy application model in construction that we know of which has been validated using real project data.

Fuzzy Expert Systems

Leung and Lam (1988) pointed out that the reason we use fuzzy concepts in expert systems is that traditional expert systems could not cope with inexact information. (Instead, they use certainty or confidence factors to handle uncertainties in knowledge.) They indicated that much human knowledge is vague and imprecise. Human thinking and reasoning involve inexact information. A practical expert system should be able to manipulate this situation.

Based on this analysis, they presented a comprehensive expert system-building tool, which could deal with exact, fuzzy, and combined reasoning, allowing fuzzy and normal terms to be freely mixed in the rules and facts of an expert system. In their paper, they defined uncertainty as meaning uncertainty about a piece of information, and fuzziness as meaning that the boundary of a piece of information was not clear-cut. They employed fuzzy logic to handle fuzzy reasoning and fuzzy numbers to handle uncertainty. Their

system consisted of three components: the knowledge-acquisition module, the consultation driver, and the fuzzy knowledge base.

Their system has been employed to build several expert systems. The obvious advantage is that the system could allow any mix of fuzzy and normal terms, numeric-comparison logic controls, and uncertainties. Their work shows the feasibility and effectiveness of applying fuzzy concepts into expert systems.

Russell and Fayek (1994) developed a fuzzy expert system for diagnosing problems on site and suggesting corrective actions. The fuzzy expert system consists of a set of user-assigned activity attributes, a set of problem sources, a set of corrective actions, and expert rules. The idea was that based on the daily site report, the system could "automatically identify activities experiencing difficulties, the sources of these difficulties, and the type of problems resulting, find corroborating information, validate the causes of these problems, and suggest likely corrective actions" on an activity-by-activity basis.

Fuzzy set concepts were used to manipulate the uncertainty and imprecision involved in assessing a problem. Fuzzy logic was applied to simulate the human reasoning process. Two fuzzy composition operations, max-min composition and cum-min composition, were examined. When a project met with trouble, the system could automatically give several corrective actions with different rankings. Theoretically, the action with the highest rank should be the recommended one. The system allowed the user to select the most suitable corrective actions according to his/her own individual situation.

Wirha et al. (1995) analyzed the reasons why traditional knowledge-based systems fail to satisfy construction managers in project control:

• Most present KBSs concentrate on identifying the presence of deviations on project plans and not on explaining detailed planning and control operations.

- Most construction project monitoring and control KBSs are rule-based, which severely limits knowledge representation that is non-modular in nature.
- Most KBSs in construction use probability theory to represent uncertainty, but subjective knowledge, as in construction project control, does not fit in properly with the traditional representational methods of Bayesian probability and certainty factors.

They presented a new approach that employed object-oriented paradigms to design and implement the knowledge system. Fuzzy logic was used to handle the uncertainty and vagueness in construction knowledge; projects are generally affected by many risk factors, most often described by construction engineers using linguistic variables such as high, very low, etc to describe the likelihood and the influence level. The fuzzy computations involved three major steps:

- The fuzzification: converting linguistic variables into a fuzzy set;
- The calculation of a fuzzy weighted mean;
- The defuzzification: returning a linguistic concept, using the Euclidean distance method.

The authors outlined a framework of this new approach. More detailed research and validation work needs to be done.

Scheduling

Many papers have described the use of fuzzy set concepts in construction scheduling. Among them, Ayyub and Haldar's paper (1984) is one of the most significant. They first applied the fuzzy set concept in construction project scheduling. Though the paper looks like educational material today, it does provide many ideas for current fuzzy research.

The authors stated that many different probabilistic methods with various degrees of complexity were being used in scheduling. However, when a parameter was expressed in linguistic rather than mathematical terms, classical probability theory failed to incorporate the information. The authors then tried to solve this problem as a fuzzy

application. An example was given, considering two parameters which were best described in linguistic terms, weather conditions and labor skills. The authors disclosed that their fuzzy approach could be combined with the conventional scheduling methods like PERT to produce a more accurate answer.

The significance of this paper is:

- Frequency of Occurrence is introduced as a parameter so that traditional statistical methods can be used in the fuzzy set model.
- The sensitivity test disclosed that the proposed technique is not sensitive to small variations in the membership values, but is sensitive to the choice of the fuzzy relation between the attributes and output. This point indicates the direction for future fuzzy research.

Motivated by Ayyub and Haldar's work, Abourizk and Sawhney (1993) developed an automated duration estimation system based on Aggregated Input-Process method (AIM). They indicated that scheduling is actually an estimate based on estimators' experience with past projects and knowledge of new conditions that have an impact on the work process. The influence of the new conditions, such as weather and level of labor skill, was difficult to quantify and incorporate in the traditional duration estimate.

They assumed that the probability density function of activity durations was well represented by a beta density function, and the user knew the activity's minimum and maximum time (as the low and upper end point). Fuzzy set theory was applied to manipulate a set of linguistic descriptions of the external factors that affected the duration to obtain two moments of that beta distribution. Traditional statistical methods could then be used to calculate the estimated duration. An example application was presented in the paper to demonstrate the use of their system.

Wu and Hadiprono (1994) recommended angular fuzzy set theory to quantify linguistic values which would affect the duration into numerical values to solve the scheduling problem. Figure 2.2 shows the typical linguistic truth values represented by this theory.

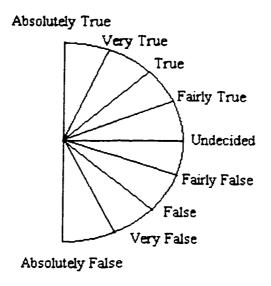


Figure 2-2: Angular Fuzzy Set Models for Truth Values

A new model, the Activity Duration Decision Support System (ADDSS) was presented, which employed fuzzy modus ponens deduction (FMPD) techniques to assess the impacts of duration factors on activity duration. ADDSS can give the adjustment factors, which are used to modify the most likely duration obtained from CA-SuperProject to get optimistic duration and pessimistic duration.

Lorterapong and Moselhi (1996) stated that previous research demonstrated the use of fuzzy set theory on project scheduling, but did not provide a means to generate a complete network-based schedule.

They presented a network scheduling method based on fuzzy set theory (FNET). Uncertainty degrees associated with activity duration were described as the trapezoidal distribution. The results were compared with those obtained using Monte Carlo simulation, and proved the model's correctness. A computerized FNET was needed.

Decision-Making

Decision-making seems to be a fairly mature field for fuzzy application. There are many papers on this topic. The following are the representative works.

Paek and Lee (1992) tried to develop a multicriterion decision-making methodology for selecting the best design/build proposal under uncertainty. Historically, owners usually chose the winner who had the lowest cost estimate. Nowadays, selection of the best proposal becomes complicated by a trade-off between cost and technical factors.

Paek and Lee thought that fuzzy set theory provided a good tool for this kind of decision-making problem, where there were conflicting objectives, the objectives had varying degrees of importance, and values of input variables were uncertain. They first defined 25 basic factors as critical and sensitive criteria for the evaluation, such as site utilization and development, site integration, grading, heating, material quality, cost, etc. A fuzzy-composition programming method was applied to formulate a methodology for the relative weighting mechanism of the technique factors, and the logical combining tool between cost factor and technique factors. Using this model, the proposal with the highest ordering value would be the best one. Since this result may vary with the weights and balancing factors assigned to each criterion and group by users (changing the membership functions), the authors recommended that a sensitivity analysis was needed to investigate the effect of the weights and balancing factors.

Elton et al. (1994) applied fuzzy set theory to improve contractor prequalification techniques. They found that current contractor prequalification methods could not satisfy users because they failed to address uncertainty in the evaluation process.

Nine categories of decision factors were examined: financial and experience, failed performance, performance, capacity for assuming new projects, management, bonding, location, resources, and safety performance. The factors were described as quality

variables. The bounded bell-shaped function was developed for characterizing fuzzy numbers used in linguistic assessments. The utility model was used as a defuzzification approach.

The authors provided a case study to show how fuzzy set theory could simplify and conform to the analysis of contractor prequalification decisions. The purpose of their paper was to demonstrate the potential use of fuzzy set theory in the decision-making field.

Chao and Skibniewski (1998) developed a fuzzy logic decision system for evaluating a new construction technology. Their point of view was that fuzzy logic systems could simulate human linguistic inference and achieve consistent judgement based on accountable rules.

The steps of their fuzzy-logic-based approach to decision-making were:

- 1. Producing probabilistic cost estimates for alternative technologies.
- 2. Consolidating probability distributions of cost into probability-profit-loss vectors.
- 3. Setting up fuzzy rules showing decision makers' preferences.
- 4. Formulating membership functions for instances of fuzzy variables.
- 5. Evaluating alternative technologies by fuzzy set operations on fuzzy rules by performing a sensitivity analysis.

Their paper provided a general method for decision-making. It could be applied to other similar aspects, such as evaluation of alternative procurement methods, evaluation of alternative layouts and project design. The paper presented an example, where triangular distribution was defined as the fuzzy membership function. The authors did not mention the validation of their model.

In conclusion, we find that fuzzy set theory, as one branch of artificial intelligence, has distinct advantages that can not be substituted by any other technique. The construction

industry, because of its unique, uncertain, knowledge-based nature, has seen applications of fuzzy set theory in many fields.

Through the literature review, we conclude that fuzzy set theory can be applied in the construction industry in the following situations:

- A problem that involves qualitative inputs rather than quantitative inputs.
- A problem that meets with uncertainties and needs much subjective judgement.
- A problem that computerizes many linguistic approximations.
- A system that demands its inference process be more like the human way of thinking instead of simple mathematical calculations.
- No clear mathematical equation can be written between inputs and outputs.
- No historical data is available.

2.3 Formwork Labor Productivity Estimation Models

Construction labor productivity is a popular topic in the CEM field. Much research has been done on identifying factors affecting it and developing methods to improve it. Labor productivity of concrete formwork, one activity of construction work, is an area that has not received much attention.

This section first reviews current research in the area of construction labor productivity and the factors affecting it. Formwork activity, as the subject of this research, is then examined and its labor productivity estimation models are introduced.

2.3.1 Labor Productivity and Its Influencing Factors

The term productivity has different meanings for different people. Traditionally, labor productivity has been defined as the number of units of work produced by a person in a specific unit of time. The U.S. Department of Commerce regards it as dollars of output per person-hour for labor input. In this research, construction labor productivity is

defined as man-hours per unit of work. A large productivity value represents a low-level productivity, while a small value means high efficiency.

A great deal of research has been done in identifying factors which cause low productivity.

Adrian and Boyer (1976) developed a Method Productivity Delay Model (MPDM). They modified the traditional "time" and "motion" studies, incorporated elements of work sampling, statistics, production function analysis, time study, and balancing models in their study. They categorized five types of productivity delay:

- Environment: change in soil condition, change in roadway alignment.
- Equipment: temporary breakdown, unscheduled maintenance.
- Labor: personal breaks, awaiting instructions.
- Material: not available on demand, defective.
- Management: poor planning, interfering with operations, unavailable for instructions.

The Construction Industry Development Council (1984) presented a report. They performed a survey of factors impairing construction productivity in Canada. Seven categories were identified:

- Project Conditions sixteen factors were listed, including remote location, scale of the project, difficult working conditions, the project's technological complexity, etc.
- 2 Market Conditions seven factors were listed, including overtime required, lack of experienced personnel, etc.
- Design and Procurement sixteen factors were listed, including poor quality of drawings and specifications, insufficient attention to constructability of design, change orders or rework, poor work packaging, etc.
- 4 Management of the Construction Phase twenty-six factors were listed, including inadequate use of planning and scheduling techniques, lack of speedy feedback and initiation of corrective action, poor coordination among contractors, etc.

- Labor fourteen factors were listed, including union rules being too restrictive, inadequate instruction regarding tasks and project goals, absenteeism, etc.
- Government Policy and Regulations nine factors were listed, including environmental regulations, mobility restrictions, etc.
- Education and Training seven factors were listed, including lack of appropriate training programs for skilled tradesmen, supervisory, and project management personnel, lack of effective safety programs, etc.

The survey requested respondents to rank the most important factors. The following were recognized as the factors most seriously impairing construction productivity for general contractors:

- Union rules too restrictive
- Lack of experienced tradesmen
- Labor opposed to productivity improvement efforts
- High seasonal variability in weather
- Training of tradesmen inadequate
- Jurisdictional disputes
- Lack of motivation
- Supervisory personnel lack sufficient management training
- Indecisive owners
- Inadequate use of planning and scheduling techniques.

Adrian (1987) divided the factors affecting labor productivity into industry factors, labor factors, and management factors.

Industry-Related factors included:

- Uniqueness of construction project
- Varied location
- Adverse, uncertain weather and seasonality
- Dependence on the economy
- Small size of firms

- Lack of research and development
- Building codes
- Regulations and laws.

Labor-Related causes included:

- High percentage of labor cost
- Supply-Demand characteristic
- Little potential for learning
- Risk of worker accident
- Work rules
- Lack of worker motivation.

Management-Related factors included:

- Scheduling methods
- Training
- Personnel management skills
- Accounting and control procedures to measure and monitor labor productivity.

Dozzi and Abourizk (1993) divided productivity issues into macro- and micro-level. Macro-level deals with contracting methods, labor legislation, and labor organization. Micro-level deals with the management and operation of a project, mainly at the job site.

Russell and Fayek (1994) grouped problem sources into ten categories, which might affect one or more project performance measures, through an extensive literature search, field experience, and numerous brainstorming and discussion sessions with construction personnel. The categories were:

- Environment: temperature, wind, precipitation, freeze-thaw cycles.
- Site Conditions: storage space, access, congestion, ground conditions, workspace.
- Owner and Consultants: decisions required, changes requested, interference with work orders, awaiting inspections/tests, excessive quality demanded.

- Design/Drawings: drawing errors, changes, drawings insufficient/incomplete, conflicting information, poor design coordination.
- Schedule: delay of activity predecessors, work done out of sequence, improper sequencing of activities, delay of off-site procurement.
- Workforce: undermanning, overmanning, skill level, turnover, motivation, inadequate instructions, accidents, fatigue, trade stacking, poor trade coordination.
- Work: estimating error, error in construction or layout, poor workmanship, rework.
- Supplies and Equipment: insufficient materials/equipment, tools breakdown, damaged deliveries, fabrication errors, inefficient materials handling.
- Utilities/City: awaiting permits/connection/inspections/tests, interference of existing utilities, damage to existing utilities.
- Miscellaneous: theft, strikes, vandalism, Worker's Compensation Board shutdown, delay/change in award of contract, noise levels too high, natural disasters.

2.3.2 Formwork and Its Labor Productivity Estimation Models

The term "formwork" has been employed to include the total system of support for the freshly placed concrete – sheathing plus all supporting members, hardware, and necessary bracing. Hurd (1995) defined formwork as "a temporary structure that supports its own weight and that of the freshly placed concrete as well as construction live loads including materials, equipment, and workmen".

Formwork was traditionally built in place, used once, and destroyed. The trend today is toward more ready-made or contractor-built prefabricated panels because of high labor costs. Such panels are simple and durable for many reuses and reduce the labor required at the job site. This research focuses on wall formwork activity, both loose and repetitive.

Economy is a major concern since formwork costs range anywhere from 35 to 60 percent of the cost of the concrete structure (Hurd 1995). The costs include the cost of material, such as lumber, nail and ties, the cost of using power equipment, such as saws and drills,

and the cost of labor making, erecting, and removing the forms. Labor costs may account for more than 30 percent of the total concrete cost, i.e. 50-90 percent of the formwork cost, especially when the forms are custom-built (Touran 1988).

Formwork labor productivity is affected by a variety of factors. The previous section provides ideas about these. The impact of factors on productivity has been quantified by several different formwork productivity models, which play an important role in construction estimating, scheduling, and planning decisions.

Generally speaking, the most reliable source of labor productivity is the accurate, up-to-date, well-kept records of the construction company. There is no better estimation of formwork productivity than the actual formwork productivity of the contractor from another recent job, modified to meet the requirements of the project being estimated. This, however, is an ideal scenario.

Traditionally, estimators apply considerable personal experience in selecting a productivity value. They consider the productivity obtained from projects constructed under conditions similar to the current one. Necessary modifications are made. If there are no company records for formwork productivity, estimators will refer to cost data books, such as Means Building Construction Cost Data or any cost engineering magazine, or textbooks. The following factors are the major reasons for modifying formwork labor productivity (Peurifoy and Oberlender 1989):

- 1. Size of the forms
- 2. Kind of material used
- 3. Shape of the structure
- 4. Location of the forms
- 5. The extent to which prefabricated form panels or sections may be used
- 6. Rigidity of dimension requirements
- 7. The extent to which power equipment is used to fabricate the forms.

Most construction companies employ the above method. Some advanced firms may use statistical methods to help their estimators work. Portas (1996) disclosed that the traditional method has an accuracy of plus or minus 15% approximately 40% of the time for concrete wall formwork, and inaccuracies of 50% or 100% are possible.

Touran (1988) noted that concrete formwork was one of the most difficult items to estimate. He thought that the factors that affect formwork labor productivity could be divided into two major groups:

- The first group consists of factors that do not depend on the type and shape of the structure. Weather, project location, type of labor (union vs. open shop), management and contractor's experience are among the more important factors in this group.
- The second group consists of factors that depend on the formwork requirements and the geometrical shape of the structural members. These factors are at least as important as the factors mentioned in the previous group.

Touran first proposed "Difficulty Factors". Difficulty factors or complicating factors are factors that quantify the effect of irregularities in formwork productivity. A methodology was proposed for quantifying these "Difficulty Factors". The following general equation was developed to calculate total man-hours spent:

$$Y_{j} = \sum_{i=1}^{n} A_{ij} X_{i}$$
 (2.4)

Where:

n = total number of unknown productivity rates,

j = floor number

For example, for Floor 1 there is:

$$A_{11}X_1 + A_{21}X_2 + A_{31}X_3 + A_{41}X_4 + A_{51}X_5 = Y_1$$
 (2.5)

Where:

```
A_{11} = area of beam side (ft.<sup>2</sup>) floor 1

A_{21} = area of beam soffit (ft.<sup>2</sup>) floor 1

.....

X_1 = productivity rate for beam side (man-hours/ft.<sup>2</sup>)

X_2 = productivity rate for beam soffit (man-hours/ft.<sup>2</sup>)

.....

Y_1 = total man-hours spent in Floor 1 on spandrel beams
```

Smith and Hanna (1991) discussed the factors affecting formwork productivity. They classified the factors into design, site conditions, and formwork system selection categories.

Design factors are fixed factors and can not be reconciled in the field. Table 2.2 lists some design factors.

Table 2-2: Design Factors Affecting Formwork Productivity

Height of the wall or column
Number of vertical intersections
Surface finish
Inconsistent column sizes
Shape irregularities

Examples of site factors include the area available for material storage, access to the formwork area, the maintenance of traffic through the area, and site management. Examples of formwork system would be the number of ties required, modularity of the system, wale design, hardware, and the number of loose parts.

Smith and Hanna admitted that there was a lack of research data to support the impact of these factors.

Kamarthi, et al. (1992) further discussed vertical formwork selection. They thought the following were key factors that affected the selection of vertical formwork systems:

- Building height and structural system
- Concrete finish
- Site characteristics
- Hoisting equipment
- Building shape.

Sonmez and Rowings (1997) pointed that previous labor productivity models developed by regression analysis for qualitative evaluation of the impact of factors on productivity usually addressed the effect of a single factor. They developed a methodology to model productivity for concrete pouring, formwork, and concrete finishing tasks using regression analysis and neural networks.

The initial regression model was used to identify the factors that might have an effect on production rate. The significant factors were then used as the input variables, and the production rate was used as the output variable in the neural network models. The data for their research were compiled from eight building projects of a contract. Nine factors were examined: quantities completed, job type, crew size, percent overtime, percent laborer, temperature, humidity, precipitation, and concrete pump.

Their research results indicated that productivity models including fewer significant factors predict better than models based on many factors without considering significance. They admitted that the limitation of their model was a lack of quantitative information for the other factors.

Portas (1996) researched the use of neural networks in estimating formwork labor productivity. Two formwork activities, loose walls and loose slabs, were chosen for his study. A great deal of work was done in data collection of factors affecting formwork productivity. Fifty-three factors were analyzed and their significance was ranked by his model (see Table 2.3, the factors in white cells are considered in this study).

Portas' model was tested using a general contractor's historical data, and an accuracy of within 15% of the actual, 80% of the time was achieved, a significant improvement over traditional accuracy rates.

Table 2-3: Ranking of Factors from Neural Network Analysis (Portas 1996)

Rank	Field Name	Rank	Field Name
1	Activity performance	28	Log_quantity
2	Crewsize input I	29	Project superintendent score
3	Activity – superintendent score	30	# floors_above_low
4	Number of reuse input 4	31	Wall_thick
5	Tie type_wall_snap tie	32	Formwork duty_loose
6	# floors_above_high	33	Project_site_factor
7	Tie spacing_vertical	34	Material handling and crant time problems
8	Number of reuse input 3	35	District_11_input
9	Number of reuse input 1	36	Shift duration
10	District_6_input	37	Formwork duty_repetitive
11	Panel area input 2	38	Tie spacing_horizontal
12	# floors_below_high	39	Project complexity
13	Crew skill rating	40	Union
14	Panel area input 1	41	Costcode 1
15	# floors_below_low	42	# floors_above_medium
16	Panel area input 3	43	Continuity of cycle
17	Crewsize input 2	44	Tie type_wall_waler
18	Crewsize input 4	45	Log_company_contract
19	Season mean temperature	46	Costcode 2
20	Tie type_wall_taper type&burke_	47	District_4_input
21	Log_total_contract	48	District_5_input
22	Height_wall_1	49	Activity - district performance score
23	Crewsize input 3	50	Project district performance score
24	Tie type_wall_anchor&camlock	51	Log_gross building area
25	Design rating	52	Costcode 3
26	Degree of repetition rating	53	Crewsize input 5
27	Number of reuse input 2		

Based on Portas' work, Knowles (1997) further studied the issues of stability and accuracy enhancement in the development of the neural network model needed for estimation.

Methods of enhancing stability included:

- An evaluation of the input factors was undertaken in order to identify additional input factors. New factors were identified, including location of work, formwork design drawings prepared, average crew experience, level of owner inspection, safety, and quality requirements.
- The research extended the collection of training records so that lack of input stability due to insufficient data could be avoided.
- The activity performance factor was analyzed in detail in order to obtain stability in
 its influence. The activity performance factor was replaced with five difficulty
 factors, which were complexity of geometry, formwork irregularities, required
 finishes, working conditions, overall difficulty.

Methods of enhancing accuracy included:

- Classifying an activity with a group of similar activities: loose walls formwork, loose slabs formwork.
- Predicting the activity from a neural network only trained on records from the similar group.
- The use of Kohonen classification neural networks in combination with prediction neural networks, which has the potential to be almost 100% accurate with accurate record classification.

Knowles' adjustment of the stability and accuracy properties of the formwork neural network models has essentially produced a more stable and accurate application.

2.4 Discussion

Much research has been done on identifying the factors that affect construction labor productivity. How these factors specifically affect productivity is still unknown.

Many methods have been presented on estimating formwork labor productivity. They are all based on estimators having available historical information or being able to quantify the factors affecting the activity under consideration. In reality, estimators never have sufficient objective data for their work because of a construction project's unique nature. Estimating embraces numerous linguistic assertions of the relationship between the productivity and the influence factors. Traditional quantitative mathematical methods can never fully fit the actual problem.

Fuzzy set theory was developed specifically to deal with uncertainties that are not statistical in nature. It has shown potential for quantitative evaluation of the effects of multiple attributes on output, especially when the relations between the attributes and the output can not be expressed using equations, and the problems involve linguistic judgement. This is meaningful when estimators do not have historical records, such as, when the project involves a new technology or a new place, or the project does not have much information.

This thesis focuses on using fuzzy set theory in estimating labor productivity of formwork. The decision to pursue a fuzzy set solution was based on its suitability to the problem at hand and a desire to develop a realistic and innovative method of determining formwork labor productivity.

Chapter 3

3. MODEL FOR FUZZY MEMBERSHIP FUNCTIONS OF FACTORS INFLUENCING FORMWORK LABOR PRODUCTIVITY

3.1 Introduction

The model for fuzzy membership functions of labor productivity influence factors, their consequences, and the resultant labor productivity consists of determining what factors to select and how to set up fuzzy membership functions for the variables involved.

Previous research delineated numerous factors affecting formwork labor productivity. Since each company has its own features, determining which factors affecting productivity will be based on the context of the study being undertaken. In this research, influence factors are classified into three categories: design factors, project factors, and activity factors.

The basic idea of the study is to set up a fuzzy logic estimation system as shown in Figure 3-1. This chapter discusses identification, classification, and fuzzification of the variables involved in the study. Setting up fuzzy membership functions for the factors provides the foundation for a fuzzy logic model.

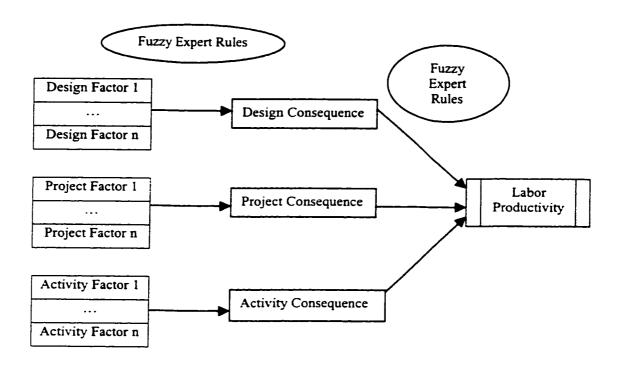


Figure 3-1: Basic Structure of the Fuzzy Logic Estimation System

3.2 Factor Identification

Average productivity rates are in some cases inaccurate when applied to specific jobs due to the numerous factors that affect formwork labor productivity. The following are the important factors that affect formwork labor productivity identified through an extensive literature search previously presented, analysis, and discussion sessions with construction personnel. These factors are classified into three categories: design factors, project factors, and activity factors.

Design Factors:

Formwork productivity can be constrained by the structural design. Design factors represent this impact. They are fixed factors and can not be reconciled in the field. Design factors include:

Formwork configuration (geometrical characteristics)

Irregularities

Surface finish

Rigidity of dimension requirements

Dimension (thickness, height)

Joint pattern

Accuracy of design

Project Factors:

Project factors do not depend on the type and shape of the structure but the whole concept of the project itself. Project factors include:

Weather

Project location

Labour availability

Type of labor (union vs. open shop)

Project site (site congestion, site access, and site conditions)

Project size

Contractor's experience

Activity Factors:

Activity factors represent activity-level aspects that influence the productivity of formwork construction. They are:

Formwork system

Size of the forms

Type of materials used

Activity superintendent skill

Activity repetition (degree of repetition, number of reuses, panel area)

Capacity of mixing and placing equipment, crane, hoist

Type of surface on which formwork is supported (concrete, sand, clay, wet, frozen, etc.)

Reshoring requirements

Crew efficiency (crew experience, crew skill, and crew size)

Location of the forms

The extent to which power equipment is used

Tie type

Tie spacing

Shift

Continuity of cycle

For example, formwork system selection is a very important factor with regard to labour productivity. Formwork systems include conventional forms, ganged forms, jump forms, slipforms, and self-raising forms. Kamarthi and Sanvido (1992) identified the following key factors that affect the selection of formwork systems: building height and structural system, concrete finish, site characteristics, hoisting equipment, and building shape. The aspects of this factor that influence labor productivity are the number of ties required, modularity of the system, wale design, hardware, and the number of loose parts.

Because of the individual nature of construction practice, which factors affect the labor productivity depends on the individual company and the project itself. It is meaningless to develop universal factors to be used by all contractors because of the various elements affecting productivity rates in different companies.

Recently, a great deal of research has been conducted on recognizing the factors that influence productivity and developing different productivity calculation models. The common situation is that there is a lack of research data, which support the impact of the factors. Good quality data are hard to find. Not only is it difficult to accumulate such data, but the competitive nature of the business discourages its dissemination.

In this research, one conventional formwork activity, wall formwork, was chosen as a test for the applicability of fuzzy set theory to aid in the prediction of labor productivity. The

data for this study are obtained from a company's historical database, a general contractor in the building construction industry in western Canada.

The following factors are considered in the fuzzy logic model used to estimate formwork labor productivity in this research (refer to Table 3-1). The reason for selecting these factors is that they were shown to be significant in the previous research (Portas 1996 and Knowles 1997) and the data of these factors are available in the company's database.

Design factors:

- Degree of difficulty: this factor represents the geometry, irregularities, and required surface finish for the formwork
- Accuracy of design.
- Dimension

Project factors:

- Project size: total contract amount, # of floors above grade, # of floors below grade
- Temperature
- Location: Vancouver projects are used for setting up the fuzzy logic model, Edmonton and Calgary projects are used for the sensitivity analysis
- Project management: site congestion, site access, site conditions.

Formwork activity factors:

- Skill: Activity superintendent skill, Crew skill
- Complexity: Tie type group, Tie spacing group, height, thickness
- Formwork quantity
- Activity repetition: degree of repetition, number of reuses, panel area
- Activity working conditions: crane time, continuity of cycle, shift duration.

Table 3-1: Influence Factors

Factor Category	Factors	Elements	
Design Factors	tors Degree of difficulty Geometry, irregularities, and required surf		
	Accuracy of design	Accuracy of design	
	Dimension	Height, thickness	
Project Factors	Project size	Original total contract, # of floors above grade, # of	
		floors below grade	
	Temperature	Temperature	
	Location	Location	
	Project site management	Site congestion, site access, site conditions	
Activity Factors	Skill	Activity superintendent skill, crew skill	
	Complexity	Tie type group, tie spacing group	
	Formwork quantity	Formwork quantity	
	Repetition	Degree of repetition, number of reuse, panel area	
	Activity working conditions	Crane time, continuity of cycle, shift duration	

3.3 Fuzzy Membership Functions Representing Factors

During the estimating phase, especially in the preliminary estimating phase, information regarding the factors affecting labor productivity is often inadequate. These variables are better described by linguistic assertions rather than by numbers.

The linguistic concepts can be represented using fuzzy set theory by fuzzy membership functions. Each fuzzy set, A, is defined in terms of a relevant universal set, X, by a function, which is called a fuzzy membership function. This function assigns to each element x of X a number, A(x), in the closed unit interval [0,1], that characterizes the degree of membership of x in A. Because of our cognitive limitations, we can only obtain an approximate membership function of a fuzzy set with limited data.

One commonly used membership function for characterizing fuzzy numbers used in linguistic assessments is a triangular function. The choice of function is discussed in Juang et al (1992). In this study, the triangular function is adopted because of its simple format. The parabolic function is used later for the sensitivity analysis.

Figure 3-2 shows the factors which need to be modeled using fuzzy membership functions. They are degree of difficulty, accuracy of design, dimension, project size, temperature, project site management, labor skill, activity work conditions, formwork quantity, repetition, complexity, labor consequence, system consequence, design consequence, project consequence, activity consequence, and resultant labor productivity.

There are three types of factors that are modeled using fuzzy sets:

- 1. Qualitative factors: linguistic information can be obtained from the company's historical database, such as Accuracy of Design (good, medium, bad).
- 2. Quantitative factors: such as temperature (e.g. 15 °C).
- 3. Fuzzification factors: these are factors that need to be subjectively evaluated, such as quality of project site management.

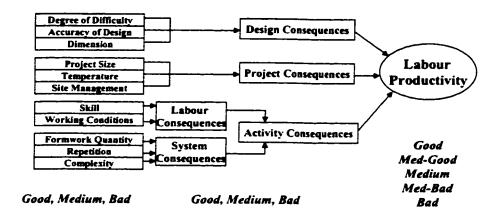


Figure 3-2: Factors Modeled Using Fuzzy Membership Function

The following sections describe how each factor was modeled using fuzzy membership functions.

3.3.1 Design Factors

This section describes how fuzzy membership functions for the design factors affecting formwork labor productivity were developed.

1: Degree of Difficulty

Scale the concept of "Degree of Difficulty" into 10 intervals. Point 0 represents very difficult and point 10 represents very easy. Consider a set of linguistic expressions that classify "degree of difficulty" into "Hard", "Medium", and "Easy". Assume that Figure 3-3 represents the membership functions for the concept of degree of difficulty. The fuzzy sets of these expressions are presented as follows:

Hard = [1.0|0, 0.8|1, 0.6|2, 0.4|3, 0.2|4, 0.0|5]

Medium = [0.0|2, 0.33|3, 0.67|4, 1.0|5, 0.67|6, 0.33|7, 0.0|8]

Easy = [0.0|5, 0.2|6, 0.4|7, 0.6|8, 0.8|9, 1|10]

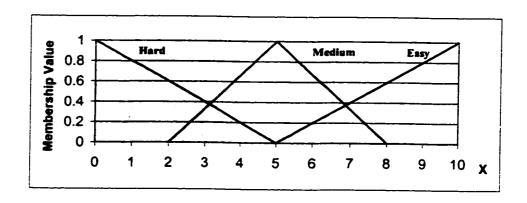


Figure 3-3: The Fuzzy Membership Functions of the Factor "Degree of Difficulty"

2: Accuracy of Design

Segment the concept of "Accuracy of Design" into 10 degrees (see Figure 3-4). Point 0 means not at all accurate and point 10 is very accurate. Three meaningful linguistic states for this factor are "Bad", "Medium", and "Good". Let the triangular functions represent the membership functions for the concept of accuracy of design (see Figure 3-2). The fuzzy sets of these expressions are presented as follows:

Bad = [1.0|0, 0.8|1, 0.6|2, 0.4|3, 0.2|4, 0.0|5]Medium = [0.0|2, 0.33|3, 0.67|4, 1.0|5, 0.67|6, 0.33|7, 0.0|8]Good = [0.0|5, 0.2|6, 0.4|7, 0.6|8, 0.8|9, 1|10]

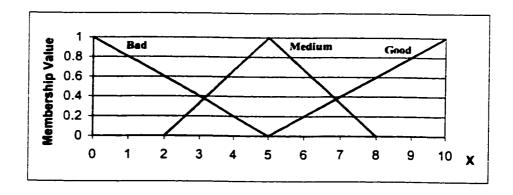


Figure 3-4: The Fuzzy Membership Functions of the Factor "Accuracy of Design"

3: Dimension

The dimension factor has two elements, height and thickness. A large dimension involves a more strictly supporting system, which leads to inefficient productivity.

According to Knowles' work (1997), concrete wall formwork can be rated as:

Height: Rating code = 1, if \leq 12 ft

Rating code = 0, if > 12 ft

Thickness: Rating code = 1, if \leq 12 inch

Rating code = 0, if > 12 inch

Let: Dimension rating code = height rating code + thickness rating code

Dimension rating code = 2: small dimension

Dimension rating code = 1: medium dimension

Dimension rating code = 0: large dimension

Scale "Dimension" into 10 intervals. 0 represents very small dimension and 10 means large dimension. Figure 3-5 shows the membership function for the concept of dimension. The fuzzy sets of "Large", "Medium", and "Small" dimension are presented as follows:

Large = [1.0|0, 0.8|1, 0.6|2, 0.4|3, 0.2|4, 0.0|5]

Medium = [0.0|2, 0.33|3, 0.67|4, 1.0|5, 0.67|6, 0.33|7, 0.0|8]

Small = [0.0|5, 0.2|6, 0.4|7, 0.6|8, 0.8|9, 1|10]

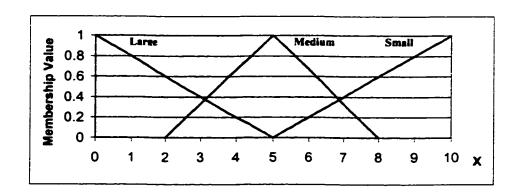


Figure 3-5: The Fuzzy Membership Functions of the Factor "Dimension"

3.3.2 Project Factors

1: Project Size

The factor "Project Size" includes three elements:

- Original total contract: assume that the greater the value of the total contract, the better the productivity.
- Number of floors above grade: the greater the value, the worse the productivity.
- Number of floors below grade: the greater the value, the lower the productivity.

Three categories are used to categorize the factor of the number of floors above grade.

They are low floor, medium floor and high floor, as follows (Portas 1996):

Low floor: ≤ 3 floors above grade

Medium floor: 4-10 floors above grade

High floor: >10 floors above grade

Two categories are used to categorize the factor of the number of floors below grade.

They are shallow basements and deep basements, as follows (Portas 1996):

Shallow basements: ≤ 2 floors below grade

Deep basements: >2 floors below grade

Scale the "Original Total Contract" into 5 levels, based on the company's historical wall formwork labor costs on Vancouver projects. Assume the following codes:

Table 3-2: "Original Total Contract" Rating Codes

Code	0	1	2	3	4
Project	Small project				Large project
Productivity	Low level				High level

Rate the "Number of Floors above Grade" and "Number of Floors below Grade" as:

Table 3-3: "# of Floors above Grade" Rating Codes

Code	0	2	4
Floor	High floor		Low floor
Productivity	Low level		High level

Table 3-4: "# of Floors below Grade" Rating Codes:

Code	0	2
Basement	Deep Basement	Shallow Basement
Productivity	Low level	High level

Rate the "Project Size" into 10 levels. Let:

Project Size Code = Original Total Contract Code + Number of Floors above Grade Code + Number of Floors below Grade Code

A code value of 0 represents the situation which is the most unfavorable to labor productivity, while 10 represents the condition that is most beneficial in terms of productivity. Three linguistic terms are used to describe the Project Size, which are "Bad", "Medium", and "Good". Figure 3-6 shows the membership function for the project size factor based on its rating code. The fuzzy sets of these expressions are presented as follows:

Bad = [1.0|0, 0.8|1, 0.6|2, 0.4|3, 0.2|4, 0.0|5]Medium = [0.0|2, 0.33|3, 0.67|4, 1.0|5, 0.67|6, 0.33|7, 0.0|8]

Good = [0.0|5, 0.2|6, 0.4|7, 0.6|8, 0.8|9, 1|10]

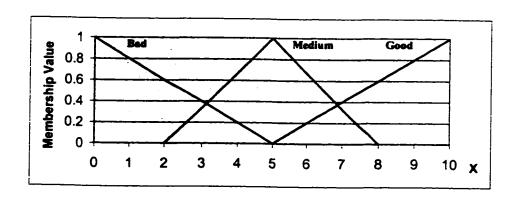


Figure 3-6: The Fuzzy Membership Functions of the Factor "Project Size"

2: Temperature

Three linguistic terms are used to describe the influence of the temperature factor on labor productivity. They are "Bad", "Medium", and "Good". Fuzzy membership functions of these qualitative expressions are assumed as follows:

Bad:
$$\mu = 1, \ x \le 0 \, ^{\circ}C$$

$$\mu = -0.25^{*}x + 1, \ 0 \, ^{\circ}C \le x \le 4 \, ^{\circ}C$$

$$\mu = 0, \ x > 4 \, ^{\circ}C$$
 Medium:
$$\mu = 0, \ x \le 0 \, ^{\circ}C$$

$$\mu = 0.25^{*}x, \ 0 \, ^{\circ}C \le x \le 4 \, ^{\circ}C$$

$$\mu = 1, \ 4 \, ^{\circ}C \le x \le 12 \, ^{\circ}C$$

$$\mu = 0.25^{*}(12 - x) + 1, \ 12 \, ^{\circ}C \le x \le 16 \, ^{\circ}C$$

$$\mu = 0, \ x > 16 \, ^{\circ}C$$
 Good:
$$\mu = 0, \ x \le 12 \, ^{\circ}C$$

$$\mu = 0.25^{*}(x - 12), \ 12 \, ^{\circ}C \le x \le 16 \, ^{\circ}C$$

$$\mu = 1, \ 16 \, ^{\circ}C \le x \le 28 \, ^{\circ}C \ (the maximum temperature value \le 28 \, ^{\circ}C)$$

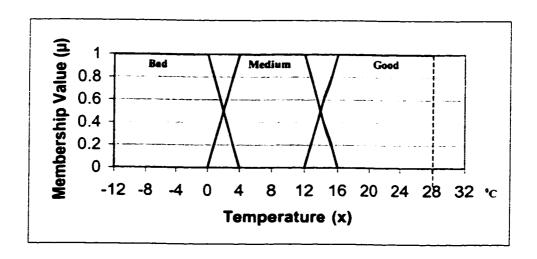


Figure 3-7: The Fuzzy Membership Functions of the Factor "Temperature"

3: Location

Vancouver is the area selected for this study because it has the largest number of data records.

4: Project Site Management

The project site management factor includes three elements: site access, site conditions, and site congestion. According to the company's database, each element is given the same rating code, as follows:

Table 3-5: "Site Access", "Site Conditions", and "Site Congestion" Rating Codes

Rating Code	1	2	3	4	5
Meaning	Significantly reduces prod.				Significantly improves prod.

Scale the concept of project site management into 10 levels. Use "Bad", "Medium", and "Good" to define this factor. Assume there is a relationship between project site management and its three elements as follows:

Project Site Management Code = Site Access Code + Site Condition Code + Site

Congestion Code

After normalizing the Project Site Management Code and making its code start from zero, we have the following formula:

Project Site Management Code = 10/12*(Site Access Code + Site Condition Code + Site Congestion Code -3)

The fuzzy membership functions for project site management are established using its rating code, as follows (see Figure 3-8):

Bad = [1.0|0, 0.8|1, 0.6|2, 0.4|3, 0.2|4, 0.0|5]

Medium = [0.0|2, 0.33|3, 0.67|4, 1.0|5, 0.67|6, 0.33|7, 0.0|8]

Good = [0.0|5, 0.2|6, 0.4|7, 0.6|8, 0.8|9, 1|10]

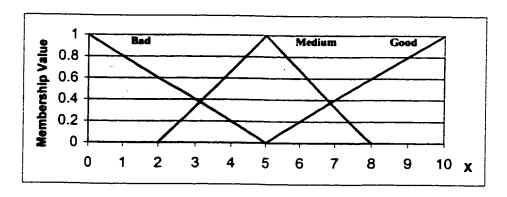


Figure 3-8: The Fuzzy Membership Functions of the Factor "Project Site Management"

3.3.3 Activity Factors

1: Skill

The skill factor includes activity superintendent skill and crew skill. According to the company's historical database, activity superintendent skill element is divided into 5 degrees, as follows:

Table 3-6: "Activity Superintendent Skill" Rating Codes

Rating Code	0	1	2	3	4
Meaning	Poor Skill				Good Skill

A previous study (Portas 1996) rates the crew skill in 5 degrees, as follows:

Table 3-7: "Crew Skill" Rating Codes

Rating Code	1	2	3	4	5
Meaning	Poor Skill				Good Skill

Scale the skill factor into 10 levels, as follows:

Skill code = 10 / 8 * (Activity Superintendent Skill Code + Crew Skill Code - 1)

Three linguistic terms are used to describe this factor, which are "Poor", "Medium", and "Good". The fuzzy membership functions for the linguistic terms are established as follows:

Poor = [1.0|0, 0.8|1, 0.6|2, 0.4|3, 0.2|4, 0.0|5]Medium = [0.0|2, 0.33|3, 0.67|4, 1.0|5, 0.67|6, 0.33|7, 0.0|8]Good = [0.0|5, 0.2|6, 0.4|7, 0.6|8, 0.8|9, 1|10]

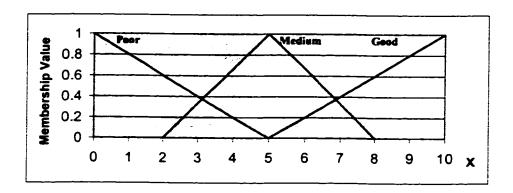


Figure 3-9: The Fuzzy Membership Functions of the Factor "Skill"

2: Complexity

Three elements are contained in the complexity factor, which are tie type group, tie spacing group, and formwork duty.

According to the degree of difficulty, five rating codes are given to define the difficulty of the tie type group, tie spacing group, and formwork duty: 1 means the most difficult, and 5 represents the least difficult.

Interviews were conducted with experienced engineers, and four rating codes are attached to four different tie types, as follows:

Table 3-8: "Tie Type" Rating Codes

Tie Type Id	Tie Type	Rating Code	
12	Snap tie and wedge	3	
13	Camlock	5	
14	Taper tie	i	
15	Single waler bracket	4	

The tie spacing group includes vertical tie spacing and horizontal spacing. The following is the rating codes for both (Portas, 1996):

Table 3-9: "Tie Spacing (Vertical and Horizontal)" Rating Codes

Rating Code	Spacing		
1	Spacing > 54 inch		
3	35 inch < spacing < 54 inch		
5	Spacing < 35 inch		

Assume the following is the rating code for formwork duty:

Table 3-10: "Formwork Duty" Rating Codes

ID	Formwork Duty	Rating Code
12	loose	3
13	Semi-panel	5

The complexity factor is scaled into 10 levels, as follows:

Complexity Code = 10 / 13 * (Tie Type Code + Vertical Tie Spacing Group Code + Horizontal Tie Spacing Group Code + Formwork Duty Code - 6)

Three linguistic terms are used for describing this factor, which are "Hard", "Medium", and "Easy". The fuzzy membership functions for the linguistic terms are set up based on the complexity code, as follows (refer to Figure 3-10):

Hard = [1.0|0, 0.8|1, 0.6|2, 0.4|3, 0.2|4, 0.0|5]Medium = [0.0|2, 0.33|3, 0.67|4, 1.0|5, 0.67|6, 0.33|7, 0.0|8]

Easy = [0.0|5, 0.2|6, 0.4|7, 0.6|8, 0.8|9, 1|10]

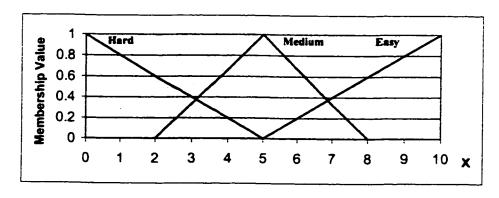


Figure 3-10: The Fuzzy Membership Functions of the Factor "Complexity"

3: Formwork Quantity

Based on the company's historical database, equally divide the recorded projects' formwork quantities into 10 intervals.

Interval = (Maximum Quantity - Minimum Quantity) / 10

Let Point 0 represent the smallest project and Point 10 represent the largest project. Three linguistic terms are used to define the quantity, which are "Small", "Medium", and "Large". The following membership functions are set up for the concept of formwork quantity (refer to Figure 3-11):

Small = [1.0|0, 0.8|1, 0.6|2, 0.4|3, 0.2|4, 0.0|5]

Medium = [0.0|2, 0.33|3, 0.67|4, 1.0|5, 0.67|6, 0.33|7, 0.0|8]

Large = [0.0|5, 0.2|6, 0.4|7, 0.6|8, 0.8|9, 1|10]

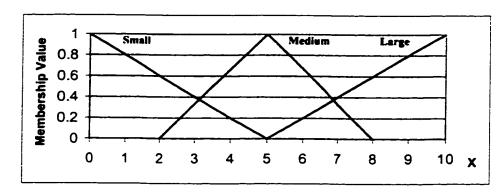


Figure 3-11: The Fuzzy Membership Functions of the Factor "Formwork Quantity"

4: Repetition

The repetition factor has three elements. They are degree of repetition, number of reuses, and panel area. The following tables show the rating codes for each element (Knowles, 1997):

Table 3-11 "Degree of Repetition" Rating Codes:

Rating Code	Repetition	
1	0% with panels	
2	25% with panels	
3	50% with panels	
4	75% with panels	
5	100% with panels	

Table 3-12 "Number of Reuses" Rating Codes:

Rating Code	Number of Reuses	
1	0 reuses	
2	x ≤ 8	
3	8 < x ≤ 15	
4	15 < x ≤ 25	
5	x > 25	

Table 3-13 "Panel Area" Rating Codes:

Rating Code	Panel Area	
1	0 sf	
2	0 – 175 sf	
3	175 – 275 sf	
4	> 275 sf	

Set:

Repetition code = 10/11 * (Degree of Repetition Code + Number of Reuse Code + Panel Area Code - 3)

The repetition code is used to set up the fuzzy membership functions for the repetition factor. Three linguistic terms are employed for this concept, which are "Bad", "Medium", and "Good". The following are the fuzzy membership functions of the linguistic expressions (refer to Figure 3-12):

Bad = [1.0|0, 0.8|1, 0.6|2, 0.4|3, 0.2|4, 0.0|5]

Medium = [0.0|2, 0.33|3, 0.67|4, 1.0|5, 0.67|6, 0.33|7, 0.0|8]

Good = [0.0|5, 0.2|6, 0.4|7, 0.6|8, 0.8|9, 1|10]

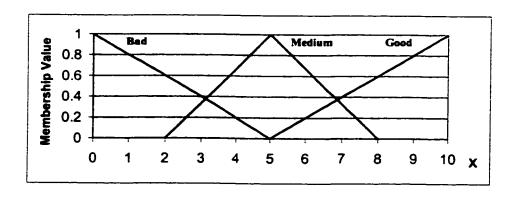


Figure 3-12: The Fuzzy Membership Functions of the Factor "Repetition"

5: Activity Working Conditions

The activity working conditions factor has three elements. They are crane time, continuity of cycle, and shift duration. The following tables show the rating codes for each element (Knowles 1997):

Table 3-14: "Crane Time" Rating Codes:

Rating Code	Crane	
1	Bad (limited resources)	
2	Medium-Bad	
3	Medium	
4	Medium-Good	
5	Good (no problem)	

Table 3-15: "Continuity of Cycle" Rating Codes:

Rating Code	Continuity	
1	Bad (numerous disruptions)	
2	Medium-Bad	
3_	Medium	
4	Medium-Good	
5	Good (uninterrupted manner)	

Table 3-16: "Shift Duration" Rating Codes:

Rating Code	Shift > 70 total hours / week	
1		
2		
3	50 total hours / week	
4		
5	No overtime	

Let:

Activity Working Conditions Code = 10/12* (Crane Time Code + Continuity of Cycle Code

+ Shift Duration Code - 3)

The activity working conditions code is used to set up fuzzy membership functions for the activity working conditions factor. Three linguistic terms are employed for this concept, which are "Bad", "Medium", and "Good". The following are the membership functions of these linguistic expressions (refer to Figure 3-13):

Bad = [1.0|0, 0.8|1, 0.6|2, 0.4|3, 0.2|4, 0.0|5]

Medium = [0.0|2, 0.33|3, 0.67|4, 1.0|5, 0.67|6, 0.33|7, 0.0|8]Good = [0.0|5, 0.2|6, 0.4|7, 0.6|8, 0.8|9, 1|10]

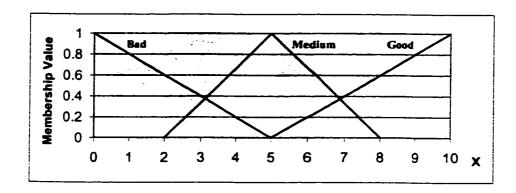


Figure 3-13: The Fuzzy Membership Functions of the "Activity Working Conditions"

3.3.4 Consequence Factors

Referring to Figure 3-2, the combined effect of a set of factors can be described by a single factor. Design consequences summarize the effect of the degree of difficulty, the accuracy of the design, and the dimensions of the wall formwork. Project consequences summarize the effect of the project size, temperature, and project site management. Labor consequences combine the effect of labor skill and working conditions, and system consequences combine the effect of formwork quantity, repetition, and complexity. Labor and system consequences can be further summarized into activity consequences.

Fuzzy membership functions are developed for design consequences, project consequences, labor consequences, system consequences, and activity consequences based on initial analysis. Three linguistic terms are selected to represent their meanings. They are "Bad", "Medium", and "Good". The following are the fuzzy membership functions of these terms for each consequence factor (refer to Figure 3-14):

Bad = [1.0|0, 0.8|1, 0.6|2, 0.4|3, 0.2|4, 0.0|5]

Medium = [0.0|2, 0.33|3, 0.67|4, 1.0|5, 0.67|6, 0.33|7, 0.0|8]

Good = [0.0|5, 0.2|6, 0.4|7, 0.6|8, 0.8|9, 1|10]

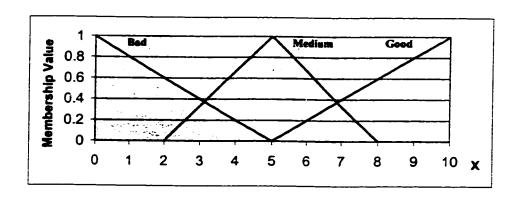


Figure 3-14: The Fuzzy Membership Functions of the Consequence Factors

3.3.3 Labor Productivity

The resultant labor productivity is also represented by a set of fuzzy membership functions. In order to improve the estimating model's accuracy, five linguistic expressions are used to represent the formwork labor productivity, rather than three. These are "Good", "Medium-Good", "Medium", "Medium-Bad", and "Bad". The company's historical productivity data of Vancouver projects are first normalized and the following fuzzy membership functions are built to represent the labor productivity (refer to Figure 3-15):

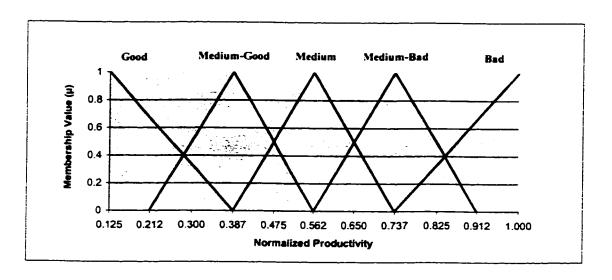


Figure 3-15: The Fuzzy Membership Functions of Labor Productivity

Good:	$\mu = x / (-0.262) + 1.477$	$0.125 \le x \le 0.387$
	$\mu = 0$	x > 0.387
Medium-Good:	$\mu = x / (0.175) - 1.211$	$0.212 \le x \le 0.387$
	$\mu = x / (-0.175) + 3.211$	$0.387 \le x \le 0.562$
	$\mu = 0$	otherwise
Medium:	$\mu = x / (0.175) - 2.211$	$0.387 \le x \le 0.562$
	$\mu = x / (-0.175) + 4.211$	$0.562 \le x \le 0.737$
	$\mu = 0$	otherwise
Medium-Bad:	$\mu = x / (0.175) - 3.211$	$0.562 \le x \le 0.737$
	$\mu = x / -0.175) +4.211$	$0.737 \le x \le 0.912$
	$\mu = 0$	otherwise
Bad:	$\mu = x / (0.263) - 2.80$	$0.737 \le x \le 1.000$
	$\mu = 0$	x < 0.737

3.4 Summary

There are numerous factors that influence formwork labour productivity. Through an extensive literature search, analysis, and discussion sessions with construction personnel, the important influence factors have been identified, and are classified into three categories: design factors, project factors, and activity factors.

Recently, a great deal of research has been conducted on recognizing the factors influencing labor productivity and on developing productivity calculation models. In most situations, however, there is a lack of research data to support these models. Not only is such data difficult to accumulate, but the competitive nature of the business discourages its dissemination.

The research data for this study are taken from previous research (Portas 1996; and Knowles 1997) and a commercial building contractor's historical records. There are some limitations to this data set. The limitations include not only the weakness of the data collection investigation in previous research and historical records, but also the suitability of available data for this study. For example, the crew efficiency is a factor affecting productivity, however, in the database, only the crew size information is available, which was used to represent crew efficiency in previous research. For this study, the crew size can not be used to represent problems with overstaffing, proper staffing or understaffing of an activity, since it has not been correlated with efficiency. This data collection limitation restricts the fuzzy model's accuracy to some extent.

Twelve factors are identified for the company based on available information. During the estimating phase, especially in the preliminary estimating phase, the information on the factors affecting productivity is inadequate. The information is better described in linguistic terms. In this study, the factors are all described in linguistic terms. Fuzzy membership functions are set up for each factor, each consequence, and labor productivity, which provide the foundation for the fuzzy productivity estimating model described in the next chapter.

It should be noted that methods of generating membership functions for construction is a new research area. Currently, little research has been done in this area. This study is not intended to solve this problem, but rather presents a method and ideas on how to set up membership functions for construction activities. Much work remains to be done in establishing methods of eliciting data and developing membership functions in a systematic way so that they can be calibrated to suit different contexts.

Chapter 4

4. A FUZZY LOGIC MODEL TO PREDICT LABOR PRODUCTIVITY

4.1 Introduction

The goal of this study is to aid in the estimation of labor productivity values for future projects. This goal will be realized by the completion of the following sub-objectives:

- To determine the correct fuzzy expert rules based on the information available.
- To determine appropriate defuzzification methods for explaining the fuzzy outputs and obtaining a crisp value for productivity.
- To construct a fuzzy logic estimation model through experimentation and development of an experimentation procedure.

Fuzzy logic is much closer in spirit to human thinking and natural language than traditional logic systems. It provides an effective means of capturing the approximate, inexact nature of construction activities and reasoning strategies. The essential part of the fuzzy logic control system is a set of linguistic control rules related by fuzzy implication and the compositional rule of inference. This system can then convert the linguistic control strategy based on expert knowledge into an automatic control strategy.

A fuzzy logic estimation system is developed in this research. It consists of four modules: a fuzzy rule base, a fuzzy inference engine, a fuzzification module, and a defuzzification module. The fuzzification module, which establishes the fuzzy membership functions for each factor, was described in chapter 3. This section discusses the other three modules.

The approach used for constructing the fuzzy logic estimation model is a trial and error process. Determining the proper expert rules, fuzzy inference mechanism, and defuzzification method is an iterative process, as shown in Figure 4-1.

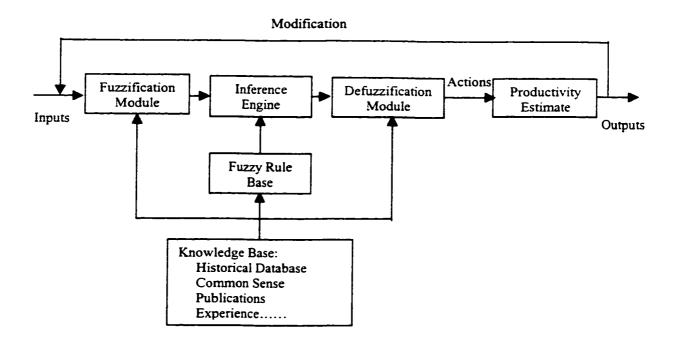


Figure 4-1: Structure of a Fuzzy Logic Estimation System

The results from the fuzzy estimation model are linguistic assertions that are translated into estimated labor productivity values. Analysis of the accuracy of the predictions is based on comparison to the actual productivity values. An example is given to demonstrate the operation of the fuzzy logic model.

Recommendations for future experimentation or other areas to investigate are an integral part of experimentation. The evolution of a fuzzy logic model for estimating productivity is slow and dependent on detailed investigation of the problem throughout. Limitations of the model are presented in the conclusion of this chapter.

4.2 Fuzzy Rule Base

In a fuzzy logic system, the knowledge pertaining to the given control problem is characterized by a set of linguistic rules based on expert knowledge. The expert knowledge is usually of the form:

If (a set of conditions are satisfied) (i.e. the antecedent),

Then (a set of consequences can be inferred) (i.e. the consequent).

For example, a simple fuzzy rule can be:

If A is good, and B is good, then C is good.

Where: A, B are linguistic variables representing factors affecting productivity.

C is the final result (productivity).

Three principal methods are employed to determine the relevant inference rules. One is to elicit them from experienced engineers. Another is to obtain them by common sense. The third method is to elicit them from the company's historical database. Figure 4-2 shows where the fuzzy rules are implemented in the fuzzy rule base.

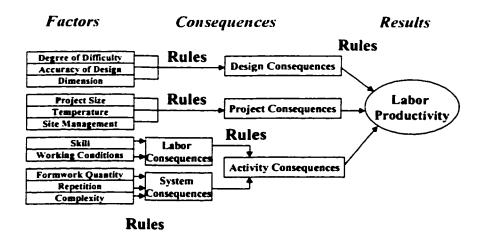


Figure 4-2: Fuzzy Rule Base

The experimentation for fine-tuning of the fuzzy rules is a trial and error process. Determining the proper relationship between the antecedents and their respective consequent is an iterative process. The procedure followed for fine-tuning of the rules is illustrated in Figure 4-3.

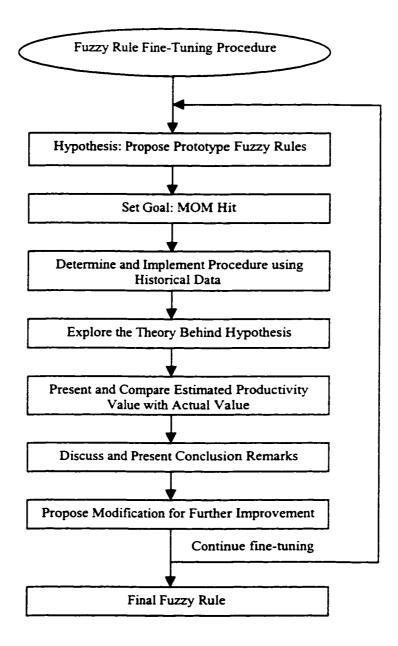


Figure 4-3: Fuzzy Rule Fine-Tuning Procedure

The objective for the fine-tuning is to increase the accuracy of the prediction of labor productivity by the fuzzy logic system. The results from this process are fuzzy membership values of predicted labor productivity. Analysis of the accuracy of the estimated productivity value is based on comparison to the actual value.

A MOM (Mean of Maximum) hit is used for measuring the estimated productivity versus actual productivity. If the linguistic term by which the actual productivity is represented is the same as the assertion the fuzzy estimation model predicted with the maximum membership value, we say MOM hits; otherwise, MOM does not hit.

The reason for adding the consequence variables (e.g. design consequences) is to make the fuzzy rule fine-tuning experimentation controllable. With them the user can easily understand which factors affect the result most strongly and which rule is unreasonable or reasonable.

4.3 Fuzzy Inference Engine

The purpose of the fuzzy inference engine is to combine properly the productivity influence factors with the relevant fuzzy rules to make inferences regarding the output variables.

Two fuzzy inference methods are explored, max-min composition and algebraic sumproduct composition methods. The former is used to build the fuzzy estimation model and the latter is used to test the model's sensitivity.

The composition of two binary fuzzy relations,
$$P(X,Y)$$
 and $Q(X,Y)$, is denoted by:
 $R(X,Z) = P(X,Y) \circ Q(Y,Z)$ (4.1)

A binary relation relates elements of two subsets, X and Z, through their respective relationship to a third and common subset of elements, Y. The most common fuzzy

composition operation is the maximum-minimum (max-min) composition. It is defined by:

$$\mu(x,z) = \max \min \left[\mu(x,y), \mu(y,z) \right] \quad \text{for all y}$$
 (4.2)

The max-min composition operation implies the following:

- The strength of each chain between elements x and z equals the strength of its weakest link (minimum) to a common element y.
- The strength of the relation between elements x and z is the strength of the strongest chain between them (maximum).

The max-min composition indicates the strength of a relation based on the strongest indicator or piece of evidence (i.e. based on the strongest chain between two elements).

The algebraic sum-product composition is usually used in fuzzy decision-making. It consists of algebraic product operation and algebraic sum operation. The algebraic product composition has the same principles as the max-min composition, except that in the algebraic product composition, algebraic product is used instead of minimum value and algebraic sum is used instead of maximum value. The following example demonstrates both composition operations.

Example

A fuzzy rule base is composed of three factors and a single consequence, as shown in Table 4-1.

Table 4-1: Example of Fuzzy Composition Operations

Factor 1		Fac	tor 2	Factor 3 Conse		Consequence
L.Term	M.Value	L.Term	M.Value	L.Term	M.Value	L.Term
Good	0.2	Medium	0.3	Good	0.5	Good
Good	0.2	Medium	0.3	Medium	0.4	Medium
Medium	0.4	Good	0.6	Good	0.5	Good
Medium	0.4	Medium	0.3	Medium	0.4	Medium

• L.Term: Linguistic term

M. Value: Fuzzy membership value

The rules can be explained as (each linguistic term has a different membership value):

Rule 1:

If Factor 1 is good, Factor 2 is medium, and Factor 3 is good, then Consequence is good.

Rule 2:

If Factor 1 is good, Factor 2 is medium, and Factor 3 is medium, then Consequence is medium.

Rule 3:

If Factor 1 is medium, Factor 2 is good, and Factor 3 is good, then Consequence is good.

Rule4:

If Factor 1 is medium, Factor 2 is medium, and Factor 3 is medium, then Consequence is medium.

Max-min composition:

Rule 1 \Rightarrow membership value of "Good" = min{0.2, 0.3, 0.5} = 0.2

Rule 2 \Rightarrow membership value of "Medium" = min{0.2, 0.3, 0.4} = 0.2

Rule 3 \Rightarrow membership value of "Good" = min{0.4, 0.6, 0.5} = 0.4

Rule 4 \Rightarrow membership value of "Medium" = min{0.4, 0.3, 0.4} = 0.3

Conclusion:

membership value of "Good" = $max\{0.2,0.4\} = 0.4$

membership value of "Medium" = $max\{0.2,0.3\} = 0.3$

Algebraic Product Composition:

Rule 1 \Rightarrow membership value of "Good" = 0.2 * 0.3 * 0.5 = 0.03

Rule 2 \Rightarrow membership value of "Medium" = 0.2 * 0.3 * 0.4 = 0.024

Rule 3 \Rightarrow membership value of "Good" = 0.4 * 0.6 * 0.5 = 0.12

Rule 4 \Rightarrow membership value of "Medium" = 0.4 * 0.3 * 0.4 = 0.048

Conclusion:

membership value of "Good" = 1 - (1 - 0.03) * (1 - 0.12) = 0.146

membership value of "Medium" = 1-(1-0.024)*(1-0.048) = 0.071

As we see in the example, it is possible that more than one rule may be involved in inferring consequences or productivity value. When this happens, the overall conclusion is expressed as a combination of the results of the individual rules, both for max-min or algebraic product composition.

A computer program (FIFT.EXE) was set up to store the rule base, and to process the fuzzy inference operations and fuzzy rule fine-tuning. Figure 4-5 shows the main form of this program.

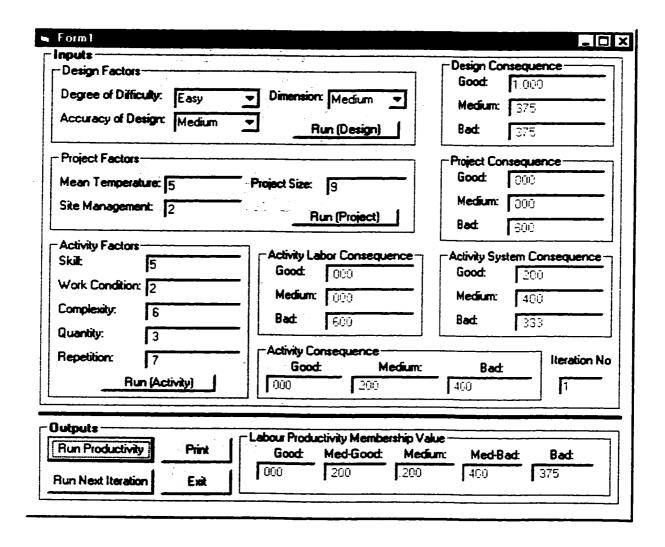


Figure 4-4: Fuzzy Inference & Rule Fine-Tuning Program Main Form (FIFT.EXE)

This program requires information from eleven influence factors (excluding the location factor). They are:

- Qualitative factors: Degree of Difficulty, Dimension, Accuracy of Design.
- Quantitative factors: Mean Temperature.
- Fuzzification variables: Project Size, Site Management, Skill, Work Conditions, Complexity, and Repetition.

The program calculates the fuzzy membership values of the consequence factors (Design Consequences, Project Consequences, Activity Labor Consequences, Activity System Consequences, and Activity Consequences). Then, the system determines the labor productivity membership values for each of the five linguistic terms describing labor productivity: good, medium-good, medium, medium-bad, and bad.

Finally, the system defuzzifies the results to provide a single recommended value or a linguistic term for labor productivity. Defuzzification is described in the next section.

4.4 Defuzzification

The computer program developed for the fuzzy estimation model presents the estimated labor productivity in terms of a fuzzy set. Defuzzification is used to explain the output, that is, to convert each fuzzy conclusion to a single real number or a linguistic term.

Many researchers (Lee 1990) give an overview of the defuzzification methods, concluding the lack of a systematic approach to the defuzzification problem. The following defuzzification methods are employed in this study:

1: Mean of Maximum Method (MOM)

In this method, the defuzzified value $x_d(C)$ is defined as a weighted average of the mean values of the intervals, in which the weights are interpreted as the relative lengths of the intervals. Formally:

$$x_d(C) = d_{MM}(C) = \sum x_k / |M|$$
(4.3)

Where: $M = \{x \in [-c,c] \mid C(x) = h(c)\},\$

C: fuzzy membership function

h(c): the maximum fuzzy membership truth value.

 $x_k \in M$

2: Centroid Method

In this method, which is sometimes called the center of gravity method, the defuzzified value, $x_d(C)$, is defined as the value within the range of productivity for which the area under the graph of membership function C is divided into two equal sub-areas. For this research, the range of productivity covers five linguistic assertions, ranging from good to bad. Formally:

$$x_d(C) = d_{CEN}(C) = [\sum C(x_k) * x_k] / [\sum C(x_k)]$$
 (4.4)

Where: $x_k \in [x_0, x_{10}]$

k = 1, 2, ..., n

3: Recommended Method

A new defuzzification method was developed in this research. The steps of this method are:

- Choose the linguistic term that has the maximum truth value.
- Compare the membership values of the two adjacent linguistic terms.
- There are two linear membership functions that make up each linguistic term. The
 defuzzified productivity is calculated by using the linear function of the linguistic
 term with the maximum truth value that is close to the linguistic term with the second
 largest truth value.

MOM method is better than the recommended method when the fuzzy model is used to predict the linguistic term of the productivity. However, if we want to obtain a single estimated productivity value using the fuzzy model, the recommended method can provide a more accurate data than other methods. The detailed comparison of these defuzzification methods will be described in the Chapter 5.

An application of these defuzzification methods to a particular fuzzy set (illustrated by the solid line) is shown in Figure 4-5.

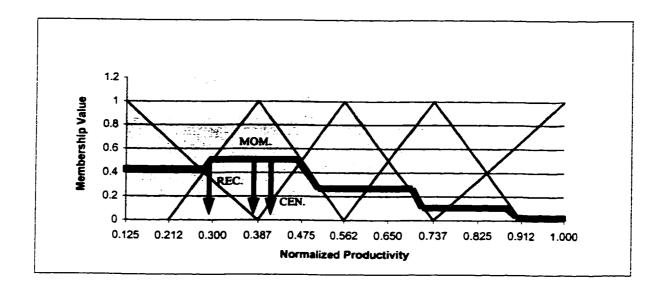


Figure 4-5: Illustration of the Described Defuzzification Methods

4.5 Example

A simple example is presented to illustrate the fuzzy logic approach to productivity prediction. Productivity influence factors are limited and simplified to make it easier to grasp the essential features of the fuzzy estimation system. Only two factors are considered: temperature and degree of difficulty. Max-min composition is used for fuzzy inference.

The factor "Degree of Difficulty" measures the difficulty level of constructing concrete formwork. We use a rating code (degree index) on a scale of 0 to 10 as the universe of discourse, with 10 being the easiest. Three notions associated with degree of difficulty are identified: "Easy", "Medium", and "Hard". The membership functions for these fuzzy sets are shown in Figure 3-3.

Figure 3-7 shows fuzzy sets describing the notions that the temperature is good, medium, and bad. Based on the company's historical records, the range of temperature is from -12 °C to 28 °C. For a temperature between -12 °C and 0 °C, the truth value of 1.0 is assigned to mean "Bad", indicating that we perceive temperature in this range to be definitely bad (it is cold enough to affect labor productivity). However, if the temperature is greater than 4 °C, the truth value of "Bad" is 0.0, indicating that it is definitely not bad. Between 0 °C and 4 °C, the belief that it is bad decreases linearly with the temperature values. Note that there are temperatures that we perceive as both bad and medium with different truth values, or good and medium with different truth values.

Five notions of productivity are identified: good, medium-good, medium, medium-bad, and bad. They are illustrated in Figure 3-15.

A simple fuzzy rule base is defined for this example. Table 4-2 shows the nine rules formulated. They are numbered 1 through 9, as follows:

Rule 1:

If Degree of Difficulty is easy and Temperature is good, then Productivity is good.

Rule 2:

If Degree of Difficulty is medium and Temperature is good, then Productivity is medium.

Rule 3:

If Degree of Difficulty is hard and Temperature is good, then Productivity is bad.

Rule 4:

If Degree of Difficulty is easy and Temperature is medium, then Productivity is good-medium.

Rule 5:

If Degree of Difficulty is medium and Temperature is medium, then Productivity is medium.

Rule 6:

If Degree of Difficulty is hard and Temperature is medium, then Productivity is medium-bad.

Rule 7:

If Degree of Difficulty is easy and Temperature is bad, then Productivity is good-medium.

Rule 8:

If Degree of Difficulty is medium and Temperature is bad, then Productivity is medium-bad. Rule 9:

If Degree of Difficulty is hard and Temperature is bad, then Productivity is bad.

Table 4-2: Simple Fuzzy Rule Base (Example)

		Degree of Difficulty			
Temperature	Easy	Medium	Hard		
Good	1. Good	2. Medium	3. Bad		
Medium	4. Good-Medium	5. Medium	6. Medium-Bad		
Bad	7. Good-Medium	8. Medium-Bad	9. Bad		

Scenario 1:

Assume the estimator is able to provide a fairly precise estimate of the temperature value and the rating code of degree of difficulty as a number on a scale of 1 to 10 (we assume that the degree of difficulty is a fuzzification variable).

Suppose that:

Degree of difficulty = 10 Temperature = 8 °C

Referring to Figure 3-3, this means that the degree of difficulty is easy with a truth value of 1.0, and the truth value of hard and medium are both 0.0.

Referring to Figure 3-7, it means that temperature is medium with a truth value of 1.0, and the truth value of temperature as good and bad are both 0.0.

Refer to the rule base in Table 4-2. For rule 4 (productivity is good-medium), the truth values for both premises (temperature is medium and degree of difficulty is easy) are 1.0. Therefore, the maximum truth value that can be assigned to the conclusion is 1.0. Moreover, for all the remaining rules, at least one of the truth values assigned to the premises is 0.0. Therefore, according to the max-min reasoning procedure, the truth value of the conclusion for any other rule is 0.0. So the only rule to succeed is rule 4.

The productivity is estimated as a fuzzy set (see Figure 4-6). Defuzzification is needed.

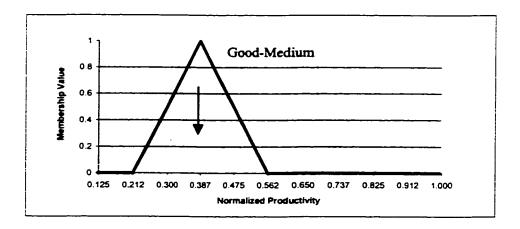


Figure 4-6: Productivity Chart (Example – Scenario 1)

Because of the symmetric shape of the fuzzy membership function, the MOM method and the centroid method provide the same defuzzified value. This value is also

equivalent to the productivity value obtained from the new proposed method, because the truth values of good, medium, medium-bad, and bad are zero.

Conclusion:

The estimated productivity is good-medium with a belief value of 1.

Productivity =
$$d_{MM}(C) = d_{CEN}(C) = d_{REC}(C) = 0.387$$

Scenario 2

Suppose that:

Degree of difficulty = 8

Temperature = 8 °C

Referring to Figure 3-3, this means that the degree of difficulty is easy with a truth value of 0.6, and the truth value of hard and medium are both 0.0.

Referring to Figure 3-7, it means that temperature is medium with a truth value of 1.0, and the truth value of temperature as good and bad are both 0.0.

Thus, the maximum truth value associated with the conclusion that productivity is good-medium is 0.6 (rule 4). As before, no other rules succeed and the conclusion is shown in Figure 4-7.

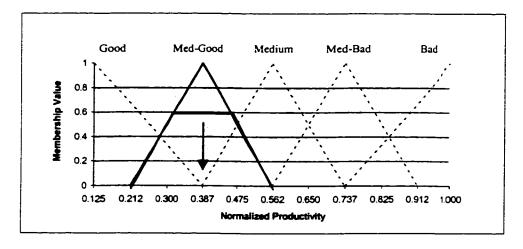


Figure 4-7: Productivity Chart (Example – Scenario 2)

Conclusion:

The estimated productivity is in the good-medium level with a belief value of 0.6.

Productivity =
$$d_{MM}(C) = d_{CEN}(C) = d_{REC}(C) = 0.387$$

Scenario 3

Suppose that:

Degree of difficulty = 5

Temperature = 15 °C

By referring to Figure 3-3, the membership truth values for degree of difficulty are:

$$\mu$$
 ? Easy = 0.0

$$\mu$$
 ? $_{Med} = 1.0$

$$\mu$$
 ? $_{Hard} = 0.0$

By referring to Figure 3-7, the membership truth values for temperature are:

$$\mu$$
 ? $Good = 0.25 * (15-12) = 0.75$

$$\mu$$
 ? $_{\text{Med}} = 0.25 * (12-15) + 1 = 0.25$

$$\mu$$
 ? $_{Bad} = 0.0$

Under this condition, two rules succeed (rule 2 and rule 5). Both rules imply that the productivity level is medium. However, since two rules succeed, the maximum of the truth values (0.25, 0.75) is taken in cases where there is a choice. The conclusion is shown in Figure 4-8.

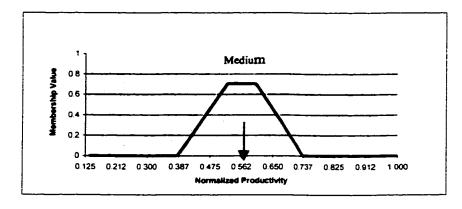


Figure 4-8: Productivity Chart (Example – Scenario 3)

Conclusion:

The estimated productivity is in the medium level with a belief value of 0.75.

Productivity = $d_{MM}(C) = d_{CEN}(C) = d_{REC}(C) = 0.562$

Scenario 4

One of the advantages of fuzzy set theory is that it is not necessary for the user to estimate precisely the values for the premises. In this case, the user states a premise using a linguistic expression (a qualitative variable).

Suppose that:

Degree of Difficulty = Hard

Temperature = $8 \, ^{\circ}$ C

As seen in Figure 3-3, the fuzzy set for "Medium" has the rating codes with nonzero truth values in common with the nonzero truth value for "Hard" in the range of 2 to 5. Accordingly, by asserting that the degree of difficulty is hard, the user is also implicitly asserting that it is medium but not with a truth value of 1.0. The truth value assigned to medium is where the two membership functions cross and is 0.375. It is the maximum value possibly assigned to medium. Referring to Figure 3-7, the temperature is medium with a truth value of 1.0, and the truth value of temperature as good and bad are both 0.0. This causes rule 5 and rule 6 to both succeed. The conclusion is shown in Figure 4-9.

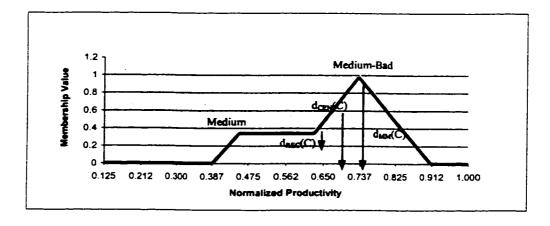


Figure 4-9: Productivity Chart (Example - Scenario 4)

Conclusion:

MOM defuzzification:

The estimated productivity is in the medium-bad level with a belief value of 1.0.

Productivity =
$$d_{MM}(C) = 0.737$$

• Centroid Defuzzification:

The defuzzified value is the gravity center of the area considered.

Productivity =
$$d_{CEN}(C) = 0.675$$

• Recommended Defuzzification Method:

Productivity =
$$d_{REC}(C) = 0.562 + 0.375 * (0.737 - 0.562) = 0.628$$

The following are trends of input and productivity changes of this example (see Table 4-3). We can find that with the premises (influence factors) turning worse, the productivity also becomes worse.

Table 4-3: Summary of the Example

Scenario #	Influence Factors	Trend	Productivity	Trend
1	Degree of Difficulty = 10 (very easy) Temperature = 8 °C (medium)		0.387 with a belief value of 1	
2	Degree of Difficulty = 8 (easy) Temperature = 8 °C (medium)		0.387 with a belief value of 0.6	
3	Degree of Difficulty = 5 (medium) Temperature = 15 °C (good and medium)	↓	0.562 with a belief value of 0.75	
4	Degree of Difficulty = Hard Temperature = 8 °C (medium)	Worse	$P_{MOM} = 0.733, P_{CEN} = 0.675, P_{REC} = 0.628$	Worse

This example shows that the results reflect changes in the inputs (factors) consistently. The fuzzy reasoning approach used in the model is suitable for productivity prediction.

4.6 Conclusion

A fuzzy logic productivity estimation model was set up, which contains a fuzzy rule base, a fuzzy inference engine, fuzzification and defuzzification modules. A fuzzy rule fine-tuning procedure was determined. A new defuzzification method was proposed. Figure 4-10 shows the flowchart of the fuzzy logic estimation model.

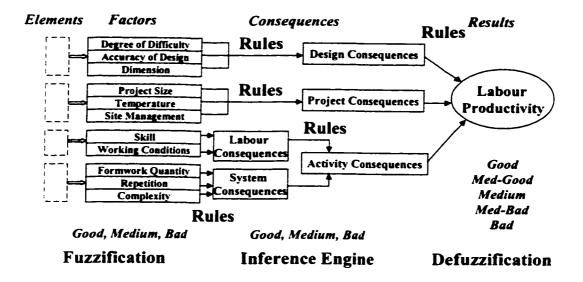


Figure 4-10: Fuzzy Logic Estimation Model Flowchart

There are some limitations while constructing this fuzzy logic estimation system, which should be noted and addressed:

A fuzzy logic system should be designed by domain experts or in close collaboration
with domain experts. Knowledge acquisition plays an important role in determining
the accuracy and reliability of a fuzzy logic system. It includes building membership
functions for the input factors and determining the fuzzy rules. Membership
functions are described numerically, which provides the foundation for the system to

understand the linguistic assertion in a mathematical form. Fuzzy rules summarize the domain experts' experience. Both require the participation of experienced estimators.

- 2. The historical data does not provide enough records. For the Vancouver area, there are altogether 26 projects recorded, of which 11 projects were used for fine-tuning the rules, an inadequate number for proper experimentation. The records also did not contain all possible combinations of factors. Further investigation for extensive data collection is essential.
- 3. The process of transferring expert knowledge into a usable knowledge base is time-consuming and tedious. Computer implementation is therefore necessary.

Despite these limitations, the fuzzy logic productivity estimation model illustrates the effectiveness of the fuzzy logic approach for linguistic knowledge representation.

Chapter 5

5. Application and Sensitivity Analysis of the Fuzzy Logic Model

5.1 Introduction

The objective of this study is to explore a method of using fuzzy set theory in the estimation of labor productivity. At present, it is difficult to develop a system that estimates productivity for all activities in each possible situation, though the approach would be similar for different activities. A practical method would be to choose a specific construction activity to explore the feasibility of applying fuzzy theory in estimating its productivity.

In this research, a company's database of productivity information was examined. Concrete wall formwork was selected as the activity for developing a fuzzy logic system to estimate its labor productivity. Vancouver projects were used to validate the fuzzy productivity model. Analysis of the estimation was based on comparison to the actual productivity value.

The sensitivity of the proposed fuzzy model to changes in the input is important in assessing the model's level of accuracy, flexibility, stability, and consistency. The sensitivity analysis can also help indicate directions for future research. In this research, the sensitivity analysis was conducted by changing the system's inference method and membership functions. Results are compared and conclusions are presented.

5.2 Application (Vancouver Projects)

The company's productivity database was carefully examined and different queries were set up in order to capture the necessary data. Information for 11 influencing factors (except the location factor) was collected for 26 Vancouver projects. Fuzzy expert rules were elicited from experience and common sense, but mainly from historical data records. Rule fine-tuning was based on 11 projects. Max-min composition was employed as the fuzzy inference method.

A computerized prototype was developed to implement the fuzzy estimation system. This prototype was programmed in ACCESS and Visual Basic. All data (inputs and outputs) were stored in an ACCESS database file. The user interface, the fuzzy rules, the fuzzy inference engine and the fuzzification module were programmed in Visual Basic.

There are 26 projects recorded in the company's Vancouver database. Eleven projects were used for fine-tuning (see Table 5-1). Among the 11 fine-tuning projects, 9 projects met the target (MOM Hit). The accuracy rate is 82%. Two projects failed to make a MOM Hit. Possible reasons for the failure is inconsistency of the data collection and inadequate representation of the reasoning process with the current influence factors. For the total 26 projects, the fuzzy rule base provides a 77% accuracy rate (MOM Hit). If we discard the 2 failed project data records, the fuzzy rule base provides an 83% accuracy rate (MOM Hit). These results are found in Appendix 2. The fuzzy rule base is shown in Appendix 1.

Table 5-1: Result of Fuzzy Rules Fine-Tuning

Project	Fine-	7s.,	Membership Value				Normalized	MOM Hit
# Tuning	Tuning	Good	Good-Medium	Medium	Medium-Bad	Bad	Productivity (Actual)	
1	yes	0.545	0.375	0.167	0.000	0.000	0.182	Yes
2	yes	0.375	0.545	0.000	0.000	0.000	0.262	Yes
3	yes	0.167	0.375	0.400	0.000	0.000	0.133	No
4	yes	0.375	0.500	0.000	0.375	0.000	0.216	Yes
5	yes	0.375	0.444	0.000	0.000	0.000	0.268	Yes
6	yes	0.400	0.375	0.000	0.000	0.000	0.268	Yes
7	yes	0.375	0.583	0.167	0.000	0.000	0.302	Yes
8	yes	0.375	0.400	0.000	0.375	0.000	0.349	Yes
9	yes	0.375	0.444	0.167	0.000	0.000	0.373	Yes
10	yes	0.000	0.583	0.000	0.000	0.000	0.383	Yes
11	yes	0.375	0.667	0.167	0.375	0.000	1.000	No

The results from the final prototype are: MOM (Mean of Maximum Method) Hit Rate is 77%, REC (Recommended Method) Hit Rate is 73%. After discarding bad data which were found in rule fine-tuning experimentation, MOM Hit Rate is 83%, REC Hit Rate is 79%.

The fuzzy model can provide an 83% accuracy rate for predicting the linguistic term representing the labor productivity. This means, for example, that if an actual productivity value falls in the linguistic expression "Good", there is an 83% chance that the fuzzy estimation model can predict the same answer "Good".

The premise of this study is that estimators do not have accurate information of a project at the estimating stage. Estimating embraces linguistic assertions of the relationship between productivity and its influencing factors. According to this premise, it is not realistic to demand that a system predict an exact productivity value. A feasible approach is to design a system to provide estimators with a data range to which a productivity value would belong. The fuzzy logic estimation model can predict a linguistic term of productivity level with high credibility (83% accuracy rate). The accuracy level can be

improved by defining more linguistic terms for variables and collecting more project records.

The REC Hit Rate is a parameter further measuring the accuracy of the estimated productivity versus the actual productivity. If the linguistic term that represents the actual productivity is the same as the linguistic term that describes the estimated value using the recommended method of defuzzification, we define it as a REC Hit; otherwise, REC does not hit. The REC Hit Rate has a lower accuracy rate (79%) than the MOM Hit Rate (83%), since the range of values covered by the recommended method is half of the range covered by the MOM method.

For the 26 Vancouver projects, six projects miss the MOM Hit, and seven miss the REC Hit. Table 5-2 shows the concept deviation* of these six projects.

Table 5-2: Concept Deviation of Vancouver Projects

МОМ Н	it (6 miss)	REC Hit (7 miss)		
Concept Deviation Number of Project		Concept Deviation	Number of Projects	
1	1 3		4	
2	2	2	2	
3	1	3	1	

^{*}If the predicted term is good but the actual concept is medium-good, this means that there is one concept deviation, and so on.

These results indicate that for these projects, the fuzzy logic estimation model does not predict productivities that vary extensively from the actual productivity. The majority of projects exhibit only one concept deviation, indicating reliability of the model's predictions.

The distribution of REC Hits is compared with the actual values for productivity. These results are shown in Figure 5-1.

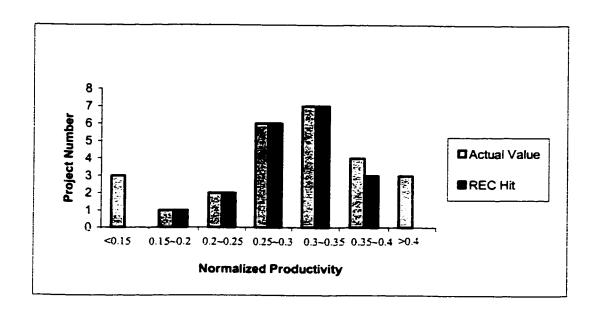


Figure 5-1: REC Hit Distribution Chart

Based on these results, the model does not predict well productivity values at the two extremes (i.e. high and low values). This may be due to lack of sufficient or consistent or lack of background information for these projects.

Appendix 3 shows the results of using a different defuzzification method to predict productivity value. Figure 5-2 illustrates that the recommended defuzzification method can provide the most accurate single productivity value compared with the others ($\pm 30\%$ accuracy, 70% of the time). The centroid method and the MOM method have almost the same accuracy level.

The model's accuracy is limited for a number of reasons. The list of factors influencing labor productivity is incomplete. Some factors that likely affect productivity, such as the appropriateness or efficiency of the crew size, are not contained in the company's database and could therefore not be taken into account. The data set contains some records that are inconsistent, and therefore the expert rules in the model have been trained and fine-tuned on inconsistent data. Furthermore, the data set contains an insufficient quantity of records for adequate development and training of the rules. The data set

consists of diverse records, which should be segregated into consistent clusters. Segregation of the data, however, would yield even smaller data sets.

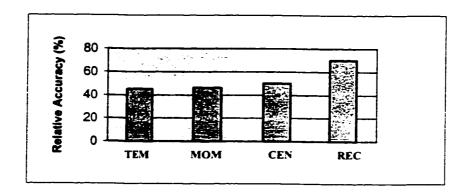


Figure 5-2: Defuzzification Method Comparison Chart (Vancouver Projects)

* TEM: Company's traditional estimating method (i.e. estimated values)

MOM: MOM defuzzification method CEN: Centroid defuzzification method REC: Recommended defuzzification method

The chart is based on $\pm 30\%$ relative error for concrete wall formwork.

Edmonton and Calgary project data are used to test the model. Detailed results are shown in Appendix 4.

The hit rates of the proposed fuzzy estimation model are:

Hit Rate of the recommended method = 39%

Hit Rate of the MOM method = 43%

The rule base of the model is built on the company's Vancouver project data. These results prove that the model is tailored for Vancouver only. The model will not work if the data set is out of the context undertaken.

Using the recommended method and the MOM method to predict a single productivity value, their accuracy levels are compared with the accuracy of the company's traditional estimating method in Figure 5-3.

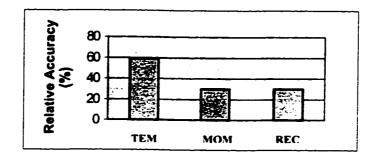


Figure 5-3: Defuzzification Method Comparison Chart (Edmonton and Calgary Projects)

* TEM: Company's traditional estimating method (i.e. estimated values)

MOM: MOM defuzzification method

REC: Recommended defuzzification method

The chart is based on $\pm 30\%$ relative error for concrete wall formwork.

To predict the exact productivity value, the MOM defuzzification method and the recommended defuzzification method both have a lower accuracy level. The conclusion can then be drawn that the proposed fuzzy estimation model is applicable only to the Vancouver projects for which it was designed.

5.3 Sensitivity Analysis

In order to evaluate the stability and consistency of the proposed fuzzy logic estimation model, a sensitivity analysis was conducted by:

- Using the sum-product composition instead of max-min composition operation.
- Changing the membership function shapes.
- Changing the input variables' ranges.

The sensitivity analysis was implemented by modifying the previous computer prototype (Vancouver Projects). The final program (FIN.EXE) was developed by adding more functional modules and linking it with the Calgary and Edmonton databases. The main form is shown in Figure 5-4.

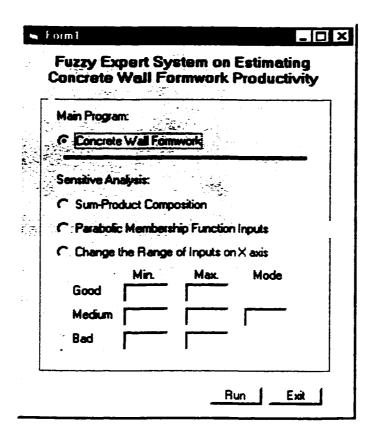


Figure 5-4: Main Form of the Fuzzy Estimation Prototype (FIN.EXE)

5.3.1 Sum-Product Composition

In order to evaluate the sensitivity of the proposed techniques to the fuzzy inference method, the Vancouver application was run again using the sum-product composition.

The results are found in Appendix 5. The conclusions from this sensitivity analysis are:

MOM Hit Rate is 79%, Recommended Method Hit Rate = 70%
 Using the sum-product composition, the fuzzy model provides a 79% accuracy rate for predicting the linguistic term representing the labor productivity by using the MOM method. The Recommended Method provides a 70% accuracy rate.

Based on these results, we find that the max-min composition operation provides slightly more accurate predictions than the sum-product composition. These results also indicate that the two inference methods are comparable inaccuracy.

• For the missed projects, Table 5-3 shows the concept deviations:

Table 5-3: Concept Deviation (Sum-Product Composition)

MOM Hit	(7 misses)	Rec. Hit (9 misses)		
Concept Deviation Project Numbers		Concept Deviation	on Project Numbers	
1	4	1	6	
2	2	2	2	
3	1	3	1	

These results indicate that for the missed projects, the majority has only one concept deviation, indicating reliability of the model's predictions.

• The relative accuracy rate from using different defuzzification methods to predict the productivity value is illustrated in Figure 5-5.

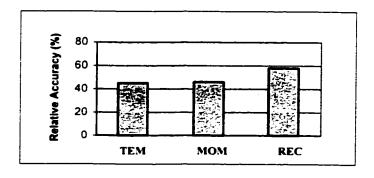


Figure 5-5: Defuzzification Method Comparison Chart (Sum-Product)

* TEM: Company's traditional estimating method (i.e. estimated values)

MOM: MOM defuzzification method

REC: Recommended defuzzification method

The chart is based on $\pm 30\%$ relative error for concrete wall formwork.

Figure 5-4 illustrates that the recommended defuzzification method still predicts the most accurate productivity value even though the fuzzy inference method is changed.

5.3.2 Parabolic Membership Function

The parabolic-shaped membership function is substituted for the triangular-shaped membership function to analyze the sensitivity of the proposed model to different fuzzy membership functions.

The parabolic-shaped fuzzy membership function, illustrated in Figure 5-6, is applied to all factors except the temperature factor.

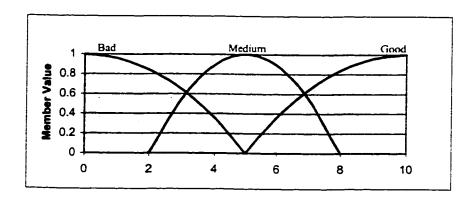


Figure 5-6: Parabolic-Shaped Fuzzy Membership Functions

Formally,

Bad:
$$y = -1/25 * x * x + 1$$
 $0 \le x \le 5$
 $y = 0$ $x > 5$
Medium: $y = 0$ $x < 2$
 $y = -1/9 * x * x + 10/9 * x - 16/9$ $2 \le x \le 8$
 $y = 0$ $x > 8$
Good: $y = 0$ $x < 5$
 $y = -1/25 * x * x + 4/5 * x - 3$ $5 \le x \le 10$
 $y = 1$ $x > 10$

Rerun the program and the estimation results are shown in Appendix 6. The following conclusions can be made:

- MOM Hit Rate = 83%, Recommended Method Hit Rate = 79%
 These results are exactly the same as those using the triangular function. Since the distribution of the triangular function is close to the parabolic function, it is obvious that the proposed fuzzy model is not sensitive to small variations in the membership values. This illustrates the stability and consistency of the fuzzy model.
- Table 5-4 shows the concept deviations of the missed projects.

Table 5-4: Concept Deviation (Parabolic-Shaped Membership Function)

MOM H	it (7 miss)	REC. Hit (9 miss)		
Concept Deviation Number of Projects		Concept Deviation	n Number of Projects	
l	3	1	4	
2	2	2	2	
3	1	3	1	

These results are the same as those using the triangular fuzzy membership function, again indicating little sensitivity to small changes in membership values.

• The recommended defuzzification method and the MOM defuzzification method are compared with the company's traditional estimating method. The results are shown in Figure 5-7.

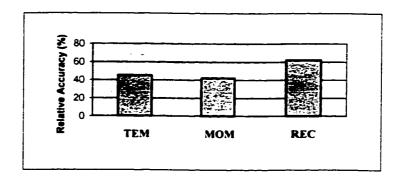


Figure 5-7: Defuzzification Method Comparison Chart (Parabolic-Shaped Function)

* TEM: Company's traditional estimating method (i.e. estimated values)

MOM: MOM defuzzification method

REC: Recommended defuzzification method

The chart is based on $\pm 30\%$ relative error for concrete wall formwork.

For predicting a single productivity value, the recommended defuzzification method provides the best answer. For this application, the use of triangular membership functions provides a better prediction than the parabolic-shaped functions.

5.3.3 Change Input Range

In order to evaluate the sensitivity of the fuzzy estimation model to the x-axis range of the linguistic input variables, the Vancouver application was solved using two different ranges in defining the input variables.

Case 1:

The fuzzy membership functions are changed as in Figure 5-8 (excluding temperature factor):

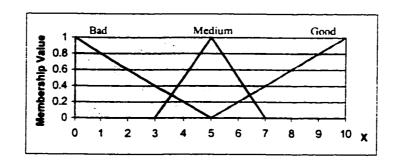


Figure 5-8: Fuzzy Membership Function (Small Change)

The membership function for "Medium" is changed slightly. The range of "Medium" is defined from 3 to 7, rather than $2 \sim 8$. The revised model provides the following results (refer to Appendix 7):

- MOM Hit Rate = 83%, Recommended Method Hit Rate = 79%
 The result does not change. It is clear that the proposed fuzzy model is not sensitive to small variations in the range of a single membership function on the x-axis. This feature represents the stability and consistency of the fuzzy model.
- Table 5-5 shows the concept deviations of the missed projects:

Table 5-5: Concept Deviation (Small Input Range Change)

MOM Hit	(7 misses)	REC. Hit (9 misses)		
Concept Deviation Number of Projects		Concept Deviation	Number of Projects	
1	1 3		4	
2	2	2	2	
3	1	3	1	

The concept deviations stay the same as in the original. This again proves that the model is not sensitive to small changes in defining the range of the linguistic variables on the x-axis.

 The recommended defuzzification method and the MOM defuzzification method are compared with the company's traditional estimating method. The results are shown in Figure 5-9.

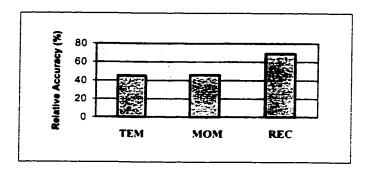


Figure 5-9: Defuzzification Method Comparison Chart (Small Change on Range)

TEM: Company's traditional estimating method (i.e. estimated values)

MOM: MOM defuzzification method

REC: Recommended defuzzification method

The chart is based on 630% relative error for concrete wall formwork.

The distribution in this chart is almost the same as in the original (see Figure 5-2). We can conclude that small changes in range will not affect the accuracy of the proposed fuzzy logic estimation model.

Case 2: The fuzzy membership functions are defined as in Figure 5-10 (excluding temperature factor):

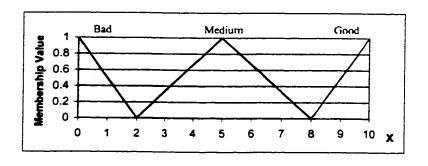


Figure 5-10: Fuzzy Membership Functions (Large Change)

The ranges of the membership functions are changed significantly. The range of "Bad" is defined from 0 to 2, rather than $0 \sim 5$. The range of "Good" is defined from 8 to 10, rather than 5 to 10. There is no overlap area between the concept "Bad" and "Medium", and the concept "Medium" and "Good". With this change the model provides the following results (refer to Appendix 8):

- The MOM Hit Rate is 67%. Since there is no overlap between linguistic terms, the estimation model predicts one linguistic concept whose truth value is not equal to 0; therefore the recommended method hit makes no sense. Comparing the MOM hit rate with the original model, there is a large change in the hit rate. This indicates that the proposed fuzzy model is sensitive to large variations in the range of membership function on the x-axis.
- Table 5-6 shows the concept deviations of the missed projects:

Table 5-6: Concept Deviation (Large Input Range Change)

MOM Hit (MOM Hit (10 misses)				
Concept Deviation	Number of Projects				
1	3				
2	0				
3	1				
Not Applicable	6				

The model can not predict answers for 6 projects. This again proves that the model is sensitive to large changes in defining the range of the linguistic variables on the x-axis.

• The recommended defuzzification method and the MOM defuzzification method are compared with the company's traditional estimating method. The results are shown in Figure 5-11.

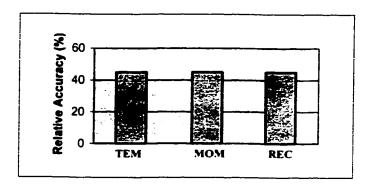


Figure 5-11: Defuzzification Method Comparison Chart (Large Change on Range)

* TEM: Company's traditional estimating method (i.e. estimated values)

MOM: MOM defuzzification method

REC: Recommended defuzzification method

The chart is based on 630% relative error for concrete wall formwork.

Figure 5-10 indicates that the accuracy levels of the MOM defuzzification method and the recommended defuzzification method for predicting a single productivity value are very poor. It is clear that the model is sensitive to significant changes in input ranges.

5.4 Conclusion

The company's Vancouver projects were used to validate the proposed fuzzy logic estimation model. The model can be used to predict a linguistic assertion describing the productivity level or a single productivity value.

For predicting a linguistic term, the accuracy rate is 83%. This result is sound and reasonable. The assumption of this study is that estimators do not have exact information on a project at the estimating stage. Linguistic assertions are involved in estimating. This model can provide a guideline for estimators to judge the productivity level for the situation considered.

The model's accuracy rate can be further improved by defining more linguistic terms both for factors and outputs, collecting more historical data, and involving experienced estimators in the development of the rule base.

The model was used to predict a single productivity value using different methods of defuzzification. It can achieve an accuracy of plus or minus 30%, approximately 70% of the time. The model is therefore useful, as a starting point for detailed estimating. It enables the estimator to account for the effect of numerous factors on labor productivity.

A sensitivity analysis of the model's performance was conducted in order to evaluate its flexibility, stability, and consistency. The sensitivity analysis was implemented by changing the fuzzy composition operation, membership function shape, and input range. A summary of results is shown in Table 5-7.

Table 5-7: Sensitivity Analysis Summary

Variation	Linguistic Te	rm Prediction	Relative Normalized Productivity Value Prediction		
	REC Hit	MOM Hit	REC. Def. Method	MOM Def. Method	Bud. Method
Original Application	79%	83%	100%	66%	64%
Sum-Product Composition	70%	79%	83%	66%	64%
Parabolic-Shaped Function	79%	83%	89%	66%	64%
Change Input Range (small)	79%	83%	99%	66%	64%
Change Input Range (large)	N/A	67%	N/A	50%	64%

Bud.Method: Budget method (method currently used by the company)

The results indicate that the proposed model is not sensitive to:

- A change in the fuzzy composition operation used in the inference process
- Small variations in the fuzzy membership values
- Small variations in the ranges of the membership function on the x-axis.

The model is sensitive to large variations in the ranges of the membership function on the x-axis

The sensitivity analysis shows that the fuzzy estimation model is stable. Small variations in inputs do not change the output much. The model is therefore consistent. It is trained for Vancouver projects, therefore it can not be used for other areas without further training. Large changes in the membership function could lead to the model's inconsistency. The model can employ different fuzzy composition operations. This again, proves the model's flexibility, stability, and consistency.

Chapter 6

6. Discussion and Recommendations

6.1 Discussion and Contributions

The objective of this research was to develop a model to aid in the estimation of labour productivity when estimators do not have much information on the project under consideration. Concrete wall formwork was selected as an example application. The objective was achieved by identifying the factors that affect concrete formwork labour productivity, establishing a fuzzy logic estimation model, and implementing it in a computer application for a company in the building construction industry.

The first stage of the study dealt with the factors that affect formwork labour productivity. Through an extensive literature search, analysis, and discussion sessions with construction personnel, these factors were identified and classified into 3 categories: design factors, project factors, and activity factors.

The second stage of the study dealt with the relevant factors for a specific company. Since each company has its own features, determining which factors influence productivity should be based on the context of the study being undertaken. The research data for this study are based on a company's historical records and on previous research (Portas 1996; Knowles 1997).

The third stage of the study dealt with building up fuzzy membership functions for all variables. We assumed that estimators do not have exact information on the project under consideration. All factors were described in linguistic terms. Three linguistic terms were used to describe the inputs. Five expressions were employed to describe the final productivity. The input factors were a mixture of qualitative variables, quantitative

variables, and fuzzification variables. Fuzzy membership functions were set up for factors, their consequences, and the resultant productivity.

The fourth stage of the study dealt with the development of the fuzzy logic model. The model consists of a fuzzification module, a fuzzy rule base, a fuzzy inference engine and a defuzzification module. The fuzzy membership functions provided the foundation for the fuzzification module. A fuzzy rule fine-tuning procedure was designed to develop the rule base. Vancouver project data (11 projects) were used as a sample to develop the fuzzy rules. Maximum-minimum composition was used as the fuzzy inference process. The mean of maximum and centroid methods were used in the defuzzification module. A new defuzzification method was also recommended.

The fifth stage of the study was an application to test the accuracy of the model's predictions. The company's Vancouver projects were used to test the model. Results were compared to the actual productivity data. The accuracy of the model is 83% for predicting linguistic terms. For estimating a single productivity value, the model can achieve plus or minus 30% accuracy, approximately 70% of the time. These results are sound and reasonable since the assumption of the study is that estimators do not have exact information on the project under consideration. Linguistic assertions are involved in estimating. It can be concluded that the fuzzy logic model's accuracy yields a reasonable level of acceptance, thus confirming the validity of the fuzzy logic approach developed.

The final stage of this research was a sensitivity analysis of the model's predictions. The results of the sensitivity analysis indicate that the model is sensitive to significant variations in the ranges of the membership functions, but is not sensitive to a change in the compositon operation used, small variations in the membership values, and small variations in the ranges of the membership functions. These results indicate a stability and consistency in the proposed model.

The study shows how the use of fuzzy logic can aid in the modeling of a process that contains linguistic and subjective evaluations. Fuzzy logic has the potential for the evaluation of the effects of multiple factors on an output, especially when interactions and ambiguous relations are present among the factors. Fuzzy logic can simulate the human decision-making process based on experienced judgement and heuristic rules.

The study has made both academic and industrial contributions. Academically, it has illustrated the usefulness of fuzzy logic in the development of a reasoning approach that mirrors the decision-making process involved when numerous subjective factors affect an output. This method is based on sound techniques of fuzzy set theory. Secondly, the model was used with a sample application (concrete wall formwork productivity) to illustrate how the proposed decision-making process can be implemented and automated. Thirdly, this study summarizes numerous factors affecting labour productivity and points out the weaknesses of the current research data; these findings provide a starting point for developing a survey to collect information for future development.

Industry contributions include providing a tool to guide inexperienced estimators in assessing labour productivity, providing a tool that predicts labour productivity as an initial input to the detailed estimating phase, exploring a method of predicting productivity without exact information (e.g. for the conceptual estimating phase), and illustrating a reasoning framework that can be modified to suit other activities.

6.2 Limitations and Recommendations for Future Research

There are limitations in the fuzzy logic estimation model developed in this research. The limitations should be known and addressed. These limitations are as follows:

Activity-Specific Fuzzy Membership Functions:
 Due to the unique nature of each activity, the membership function for each should be different. This study presents a general method and ideas for setting up membership

functions for activities. Further research is needed to develop activity-specific membership functions.

• Participation of Experienced Estimators:

Knowledge acquisition plays an important role in determining the accuracy and reliability of a fuzzy logic system. Setting up fuzzy membership functions and building a fuzzy rule base both require the participation of experienced estimators. Lack of participation of experienced estimators is the main limitation of this study.

• Research Data:

The fuzzy logic system was developed on the basis of historical records. Large amounts of data were needed in this study because the fuzzy expert rules were mainly elicited from historical records. The limitations of the research data include the inconsistency of the collected data, the number of data records available (especially at the two productivity extremes), and the suitability of the data sets. The company's database does not provide all the necessary information on factors affecting productivity, such as crew size efficiency. The available data restrict the model's accuracy to some extent.

Currently, the model is used as a prediction tool for labour productivity estimation. Future uses of the model include:

- A decision-making tool to help the estimator determine the changes required in the input factors to yield a desired productivity. In order to achieve this function, the relative importance of the input factors needs to be determined.
- An optimization tool, to determine the optimum set of input parameters (given that some are fixed, and some are variable) to achieve the optimum output (i.e. productivity).
- A tool to evaluate alternatives (of input factors) and assess their impact on productivity.

The following recommendations are made for future research:

- An exhaustive list of factors affecting labour productivity needs to be developed. Several factors and their elements are missed in this study, for example, crew efficiency and its elements, such as crew size, worker idle time, site layout, and activity size. This is because of the limitation of the data sets available. This weakness can be avoided in future study by defining all relevant factors and collecting data on these factors.
- A survey needs to be designed to collect information for further development of the fuzzy estimation model. The survey should be designed with experienced estimators in four areas: setting up activity-specific fuzzy membership functions, eliciting fuzzy expert rules, collecting project data, and validating the estimation model. The fuzzy estimation model is used when estimators do not have much exact information of the project being estimated. The survey should investigate, under this background, what information the estimators want to know and what answers they expect. The survey needs to be conducted to elicit expert knowledge for the formulation of the fuzzy membership functions and expert rules. In addition, the relative importance of the factors affecting labour productivity needs to be identified so that the rules can reflect their unequal weightings. The relative importance of the elements that make up each factor also needs to be identified in consultation with experts. Sufficient data needs to be collected to cover all possible combinations of inputs and outputs, for proper training for the rules. The rules need to be re-calibrated with the larger data set.
- The accuracy of the model's predictions can be increased by increasing the number of zones for the output (productivity). The expert rules need to be re-calibrated with a larger number of output zones.
- Different fuzzy composition methods and methods of defuzzification need to be explored.

- The domain-specific components of the model need to be isolated and modified in order to yield a generic approach that can then be tailored to suit different contexts. The generic aspects of the system consist of the membership functions, which can be used to represent different levels of any input or output factors; the rule structure, which can be applied to represent the relationship between any set of input and output factors; and the rule hierarchy, which can be expanded or collapsed to suit the structure of any problem. For each new application, one could use the framework developed with a set of context-specific rules to model the desired problem. Data required for the input and output components of the rules would have to be collected and used to train and test the new model.
- It would be significant to combine fuzzy set theory with statistics and neural networks in order to increase the estimating model's practicality and accuracy. Fuzzy set theory provides a good tool to manipulate qualitative information, statistics provides many methods of analysis, and neural networks can automatically learn experience from historical records. The combination of these techniques could provide a powerful tool for the design of estimating systems that emulate the human ability to learn, adapt to changes in the environment, and provide accurate answers. For example, the fuzzy estimation model does not work well when estimators want to perform an analysis with missing or incomplete input data. Missing or unknown input data is very likely if, for example, the estimator does not know yet the season of construction or the crew that will be used on the activity. However, with a fuzzy neural network model, even if some input data is missing, the model can still produce a reasonable answer.
- Commercial fuzzy logic software is needed. When the number of membership functions and factors increase, much time is spent on the fuzzy rule fine-tuning process. We need to develop a computer software which could automatically formulate fuzzy membership functions and elicit expert rules from historical data sets. This would reduce the guesswork in constructing a fuzzy system and increase the accuracy of the model.

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Appendix 1: Fuzzy Rule Base

1.A: Design Factor Rules

No.	Degree of Difficulty	Accuracy of Design	Dimension	Design Consequence
1	Easy	Good	Small	Good
2	Easy	Good	Medium	Good
3	Easy	Good	Large	Medium
4	Easy	Medium	Small	Good
5	Easy	Medium	Medium	Good
6	Easy	Medium	Large	Medium
7	Easy	Bad	Small	Medium
8	Easy	Bad	Medium	Medium
9	Easy	Bad	Large	Bad
10	Medium	Good	Small	Good
11	Medium	Good	Medium	Medium
12	Medium	Good	Large	Medium
13	Medium	Medium	Small	Medium
14	Medium	Medium	Medium	Medium
15	Medium	Medium	Large	Bad
16	Medium	Bad	Small	Medium
17	Medium	Bad	Medium	Bad
18	Medium	Bad	Large	Bad
19	Hard	Good	Small	Medium
20	Hard	Good	Medium	Medium
21	Hard	Good	Large	Bad
22	Hard	Medium	Small	Medium
23	Hard	Medium	Medium	Bad
24	Hard	Medium	Large	Bad
25	Hard	Bad	Small	Bad
26	Hard	Bad	Medium	Bad
27	Hard	Bad	Large	Bad

1.B: Project Factor Rules

No.	Project Size	Temperature	Project Site Management	Project Consequence
1	Good	Good	Good	Good
2	Good	Good	Medium	Good
3	Good	Good	Bad	Medium
4	Good	Medium	Good	Good
5	Good	Medium	Medium	Medium
6	Good	Medium	Bad	Bad
7	Good	Bad	Good	Medium
8	Good	Bad	Medium	Medium
9	Good	Bad	Bad	Bad
10	Medium	Good	Good	Good
11	Medium	Good	Medium	Medium
12	Medium	Good	Bad	Bad
13	Medium	Medium	Good	Medium
14	Medium	Medium	Medium	Medium
15	Medium	Medium	Bad	Bad
16	Medium	Bad	Good	Medium
17	Medium	Bad	Medium	Medium
18	Medium	Bad	Bad	Bad
19	Bad	Good	Good	Good
20	Bad	Good	Medium	Medium
21	Bad	Good	Bad	Bad
22	Bad	Medium	Good	Medium
23	Bad	Medium	Medium	Medium
24	Bad	Medium	Bad	Bad
25	Bad	Bad	Good	Medium
26	Bad	Bad	Medium	Medium
27	Bad	Bad	Bad	Bad

1.C: Activity System Rules

No.	Repetition	Complexity	Formwork Quantity	Activity System Consequence
<u> </u>	Good	Easy	Large	Good
2	Good	Easy	Medium	Good
3	Good	Easy	Small	Medium
4	Good	Medium	Large	Good
5	Good	Medium	Medium	Medium
6	Good	Medium	Small	Medium
7	Good	Hard	Large	Medium
8	Good	Hard	Medium	Medium
9	Good	Hard	Small	Bad
10	Medium	Easy	Large	Good
11	Medium	Easy	Medium	Medium
12	Medium	Easy	Small	Medium
13	Medium	Medium	Large	Medium
14	Medium	Medium	Medium	Medium
15	Medium	Medium	Small	Bad
16	Medium	Hard	Large	Medium
17	Medium	Hard	Medium	Medium
18	Medium	Hard	Small	Bad
19	Bad	Easy	Large	Medium
20	Bad	Easy	Medium	Medium
21	Bad	Easy	Small	Bad
22	Bad	Medium	Large	Medium
23	Bad	Medium	Medium	Medium
24	Bad	Medium	Small	Bad
25	Bad	Hard	Large	Medium
26	Bad	Hard	Medium	Medium
27	Bad	Hard	Small	Bad

1.D: Activity Labor Rules

No.	Skill	Work Condition	Labor Consequence
1	Good	Good	Good
2	Good	Medium	Good
3	Good	Bad	Medium
4	Medium	Good	Medium
5	Medium	Medium	Medium
6	Medium	Bad	Bad
7	Bad	Good	Medium
8	Bad	Medium	Bad
9	Bad	Bad	Bad

1.E: Activity Consequence Rules

No.	Labor Consequence	System Consequence	Activity Consequence
1	Good	Good	Good
2	Good	Medium	Good
3	Good	Bad	Medium
4	Medium	Good	Good
5	Medium	Medium	Good
6	Medium	Bad	Medium
7	Bad	Good	Medium
8	Bad	Medium	Medium
9	Bad	Bad	Bad

1.F: Productivity Rules:

No.	Design Consequence	Activity Consequence	Project Consequence	Productivity
1	Good	Good	Good	Good
_ 2	Good	Good	Medium	Good
3	Good	Good	Bad	Good-Medium
4	Good	Medium	Good	Good
_ 5	Good	Medium	Medium	Good
6	Good	Medium	Bad	Good-Medium
7	Good	Bad	Good	Medium
8	Good	Bad	Medium	Medium
9	Good	Bad	Bad	Medium-Bad
10	Medium	Good	Good	Good
11	Medium	Good	Medium	Good-Medium
12	Medium	Good	Bad	Medium
_13	Medium	Medium	Good	Good-Medium
14	Medium	Medium	Medium	Good-Medium
15	Medium	Medium	Bad	Medium
16	Medium	Bad	Good	Medium
17	Medium	Bad	Medium	Medium-Bad
18	Medium	Bad	Bad	Bad
19	Bad	Good	Good	Good-Medium
20	Bad	Good	Medium	Good-Medium
21	Bad	Good	Bad	Medium
22	Bad	Medium	Good	Medium
23	Bad	Medium	Medium	Medium-Bad
24	Bad	Medium	Bad	Bad
25	Bad	Bad	Good	Bad
26	Bad	Bad	Medium	Bad
27	Bad	Bad	Bad	Bad

APPENDIX 2: Fuzzy Rules Fine-Tuning (Vancouver Projects)

APPENDIX 2: Fuzzy Rules Fine-Tuning (Vancouver Projects)

										Γ							Τ		Γ				T			
MOM HIM	Ves	Yes	Yes	Yes	ž	2	2	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	S	% %	Yes	Yes	Yes	Yes	2
Normalised Productivity	0.182	0.262	0,263	0.294	0,389	0.133	0.137	0.216	0.268	0.268	0.279	0.302	0.317	0.337	0.339	0,349	0.373	0.375	0.383	0.448	0.125	0.231	0.325	0.347	0.553	1.000
Bad	0.000	0.000	0.000	0.000	0.000	0.000	0.375	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.200	0.000	0.000	0.375	0.000	0.000	0.000	0.000	0.000
MB	0.000	0.000	0.000	0.250	0.167	0.000	0.167	0.375	0.000	0.000	0.000	0.000	0.000	0.000	0.250	0.375	0.000	0.250	0.000	0.000	0.167	0.000	0.000	0.000	0.200	0.375
Medium	0.167	0.000	0.167	0.000	0.250	0.400	0.500	0.000	0.000	0.000	0.000	0.167	0.000	0.000	0.000	0.000	0.167	0.200	0.000	0.000	0.375	0.000	0.000	0.000	0000	0.167
СМ	0.375	0.545	0.385	0.400	0.375	0.375	0.375	0.500	0.444	0.375	0.385	0.583	0.583	0.545	0.444	0.400	0.444	0.444	0.583	0.375	0.385	0.500	0.545	0.444	0.444	0.667
Good	0.545	0.375	0.375	0.375	0.400	0.167	0.167	0.375	0.375	0.400	0.375	0.375	0.375	0.000	0.375	0.375	0.375	0.375	0.000	0.444	0.167	0.375	0.375	0.375	0.375	0.375
Fine-Tuning	yes	yes				yes		yes	yes	yes		yes				yes	yes		yes							yes
No.	-	2	3	4	S	9	7	∞	6	9	=	12	2	4	15	91	17	<u>«</u>	62	50	21	77	23	24	25	26

Appendix 3: Vancouver Projects Final Result

Appendix 3: Vancouver Projects Final Result

Hit (Rec)	700	2	S	S S	363	2 2		ves	yes	yes	yes	ves	ves	ves	yes	ves	ves	yes	yes	011	011	ves	ves	ves	92	ou
Hit (MOM) [Hit (Rec)		200	2 9	yes	2	2 2	2 2	yes	yes	yes	yes	ves	ves	yes	yes	yes	yes	yes	yes	2	92	yes	ves	ves	yes	2
RH/Rud 1%	68 513	A7A 0_	10.063	-17.813	287 101	40 878	87.818	25.712	4.513	30.065	17.100	-18.932	-58.596	-5.056		,	-30.267	-11.312	-65.730	-47.301	152.016		-8.699		-47.451	-57.659
RH(Rec)%	34.553	17 305	886 9	-4 221	9Pt 7C-	243 653	245,341	38.515	7.957	5.431	-0.048	4.058	-0.795	14.780	-14.521	-19.261	-22.385	-22.825	1.146	-39,356	296.825	29.648	-5.212	-16.530	-47.604	-67.145
RH(Cen)% RH(MOM)% RH(Rec)% RH(Rid)%	41.031	47 648	47 347	31 459	-34 213	322.524	308.932	78.997	44.320	-4.534	38.564	28.246	22.265	14.780	14.270	10.816	3.758	3.169	1.146	-42.811	210.295	67.539	19.214	11.584	-29.956	-61.300
RH(Cen)%	94.469	31.242	43.540	57.275	14.357	239.072	323.485	125.250	24.182	20.824	17.440	25.594	7.416	14.780	36.120	41.742	1.345	33.560	1.146	-28.737	366.645	45.460	4.120	-3.986	-19.639	-51.900
Rec. Prod.	0.244	0.308	0.279	0.282	0.283	0.457	0.475	0.299	0.289	0.283	0.279	0.314	0.314	0.387	0.289	0.282	0.289	0.289	0.387	0.271	0.495	0.299	0.308	0.289	0.289	0.329
MOM Prod.	0.256	0.387	0.387	0.387	0.256	0.562	0.562	0.387	0.387	0.256	0.387	0.387	0.387	0.387	0.387	0.387	0.387	0.387	0.387	0.256	0.387	0.387	0.387	0.387	0.387	0.387
Cen. Prod.	0.353	0.344	0.377	0.463	0.445	0.451	0.582	0.487	0.333	0.324	0.328	0.379	0.34	0.387	0.461	0.495	0.378	0.501	0.387	0.319	0.582	0.336	0.338	0.333	0.444	0.481
Bud.Norm.Prod.	0.390	0.332	0.271	0.308	1.000	0.239	0.329	0.346	0.357	0.444	0.417	0.312	0.167	0.408	N/A	V/A	0.331	0.424	0.167	0.300	0.400	N/A	0.377	N/A	0.370	0.539
Norm. Prod.	0.182	0.262	0.263	0.294	0.389	0.133	0.137	0.216	0.268	0.268	0.279	0.302	0.317	0.337	0.339	0.349	0.373	0.375	0.383	0.448	0.125	0.231	0.325	0.347	0.553	000.1
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Appendix 4: Estimation Result (Edmonton and Calgary Projects)

Appendix 4: Estimation Result (Edmonton and Calgary Projects)

ID	Good	GM	Medium	MB	Bad
1	0.167	0.400	0.375	0.000	0.000
2	0.200	0.375	0.500	0.000	0.000
3	0.385	0.375	0.167	0.000	0.000
4	0.200	0.500	0.375	0.000	0.000
5	0.385	0.375	0.167	0.375	0.167
6	0.375	0.385	0.375	0.167	0.000
7	0.375	0.500	0.000	0.000	0.000
8	0.375	0.385	0.167	0.375	0.167
9	0.000	0.000	0.385	0.167	0.375
10	0.375	0.385	0.375	0.167	0.000
11	0.167	0.375	0.455	0.000	0.000
12	0.375	0.500	0.167	0.000	0.000
13	0.375	0.400	0.333	0.000	0.000
14	0.375	0.400	0.375	0.200	0.000
15	0.167	0.375	0.500	0.000	0.000
16	0.444	0.375	0.250	0.167	0.000
17	0.500	0.375	0.200	0.200	0.000
18	0.385	0.375	0.200	0.200	0.167
19	0.200	0.375	0.455	0.000	0.000
20	0.375	0.385	0.200	0.000	0.000
21	0.375	0.500	0.167	0.000	0.000
22	0.400	0.375	0.000	0.000	0.000
23	0.167	0.375	0.455	0.000	0.000

Appendix 4: Estimation Result (Edmonton and Calgary Projects)

4.B: Calculation Results

Г	Т	Т	7	_	Τ-	_	_	_	_	-	т-	_		-		_	Т		-						_	_
DIVDd \0	0/ (Dad.) /0	99.14	-9.85	-6.28	-3.62	-3.84	-1.05	05 14-	40.78	49.65	40.05	19.67	20.02	37.07	10.82	29.72	095	-9.73	000	-6.50	01.0/	45.60	V/N	-0.35	715	-15.15
RH%/MOM	80.75	02.73	103.49	-27.21	104.94	-10.41	11.22	-30.71	01.9-	138 00	22.66	204 11	100 41	100.41	104.99	103.49	147.74	-10.94	29.10	01.72	06.70	75.53/	95.18	69.10	4.24	80.68
RH%(Rec.)	141 20	71.85	11.03	-18.4/	151.33	0.35	11.22	-46.38	-72.07	183.82	22.35	152.51	61 27	10.15	47.33	103.49	109.21	-5.69	-28 95	88.06	106.00	00.071	40.85	30.85	15.12	57.00
MOM Prod	0.387	695 0	730.0	0.230	0.387	0.256	0.387	0.387	0.387	0.562	0.387	0.562	0.387	0 397	0.00	0.38/	0.562	0.256	0.256	0.256	0.562	200.0	0.387	0.387	0.256	0.562
Rec. Prod.	0.492	0.475	0 287	0.207	0.475	0.287	0.387	0.299	0.279	0.670	0.387	0.467	0.299	0.787	0.307	0.387	0.475	0.271	0.257	0.287	0.467	0.10	0.279	0.299	0.283	0.467
Hit (MOM)	Miss	Miss	H		MISS	Hit	Hit	Hit	Miss	Miss	Hit	Miss	Miss	Miss	Miss	ecitat	Miss	Hit	Hit	Hit	Miss	Miss	MISS	Hit	Hit	Miss
Hit (Rec.)	Miss	Miss	Hit		MISS	Hit	Hit	Miss	Miss	Miss	Hit	Miss	Miss	Miss	Mice	141193	Miss	Hit	Hit	Hit	Miss	Mice	IMISS	111	Hit	Miss
Norm.Prod	0.204	0.276	0.352	0 100	0.107	0.286	0.348	0.559	1.000	0.236	0.316	0.185	0.186	0.189	0610	2000	0.22/	0.287	0.361	0.152	0.158	0 108	0.170	0.229	0.246	0.297
2	-	2	3	P	,	\ \ 	۰	7	∞	6	9	=	12	13	4	1	2	9]	17	18	61	2	3 5	17	77	23

Appendix 5: Sensitivity Analysis 1 (Sum-Product Composition)

Appendix 5: Sensitivity Analysis 1 (Sum-Product Composition)

ID	Good	GM	Medium	MB	Bad
1	0.181	0.189	0.037	0.000	0.000
1 0.181 0.189 0.0 2 0.177 0.243 0.0 3 0.083 0.132 0.0 4 0.102 0.140 0.0 5 0.124 0.062 0.0 6 0.026 0.113 0.1		0.000	0.000	0.000	
1 0.181 0.189 2 0.177 0.243 3 0.083 0.132 4 0.102 0.140 5 0.124 0.062		0.132	0.040	0.000	0.000
4	0.102	0.140	0.000	0.011	0.000
5	0.124	0.062	0.006	0.002	0.000
6	0.026	0.113	0.148	0.000	0.000
7	0.027	0.117	0.113	0.015	0.045
8	0.093	0.128	0.000	0.051	0.000
9	0.069	0.149	0.000	0.000	0.000
10	0.124	0.048	0.000	0.000	0.000
11	0.028	0.151	0.000	0.000	0.000
12	0.014	0.100	0.041	0.000	0.000
13	0.159	0.159	0.000	0.000	0.000
14	0.000	0.091	0.000	0.000	0.000
15	0.110	0.175	0.000	0.031	0.000
16	0.065	0.103	0.000	0.042	0.000
17	0.139	0.177	0.055	0.000	0.000
18	0.073	0.252	0.017	0.065	0.009
19	0.000	0.108	0.000	0.000	0.000
20	0.243	0.186	0.000	0.000	0.000
21	0.028	0.103	0.064	0.001	0.004
22	0.077	0.106	0.000	0.000	0.000
23	0.163	0.163	0.000	0.000	0.000
24	0.174	0.240	0.000	0.000	0.000
25	0.123	0.168	0.000	0.016	0.000
26	0.057	0.385	0.052	0.227	0.000

Appendix 5: Sensitivity Analysis 1 (Sum-Product Composition)

5.B: Calculation Results

0.254
0.235
0.236
0.355
0.413
0.542
0.234
0.238
0.355
0.238
0.545
0.293
0.387
0.243
0.230
0.243
0.256
0.387
0.324
0.544
0.230
0.293
0.254
0.241
0.279

Appendix 6: Sensitivity Analysis 2 (Parabolic-Shaped Membership Function)

Appendix 6: Sensitivity Analysis 2 (Parabolic-Shaped Membership Function)

ID	Good	GM	Medium	MB	Bad
1	0.793	0.462	0.306	0.000	0.000
2	0.462	0.793	0.000	0.000	0.000
3	0.462	0.621	0.306	0.000	0.000
4	0.462	0.640	0.000	0.438	0.000
5	0.640	0.462	0.438	0.306	0.000
6	0.306	0.462	0.640	0.000	0.000
7	0.306	0.462	0.750	0.306	0.462
8	0.462	0.750	0.000	0.462	0.000
9	0.462	0.691	0.000	0.000	0.000
10	0.640	0.462	0.000	0.000	0.000
11	0.462	0.621	0.000	0.000	0.000
12	0.462	0.826	0.306	0.000	0.000
13	0.462	0.826	0.000	0.000	0.000
14	0.000	0.793	0.000	0.000	0.000
15	0.462	0.691	0.000	0.438	0.000
16	0.462	0.640	0.000	0.462	0.000
17	0.462	0.691	0.306	0.000	0.000
18	0.462	0.691	0.360	0.438	0.360
19	0.000	0.826	0.000	0.000	0.000
20	0.691	0.462	0.000	0.000	0.000
21	0.306	0.621	0.462	0.306	0.462
22	0.462	0.750	0.000	0.000	0.000
23	0.462	0.793	0.000	0.000	0.000
24	0.462	0.691	0.000	0.000	0.000
25	0.462	0.691	0.000	0.360	0.000
26	0.462	0.889	0.306	0.462	0.000

Appendix 6: Sensitivity Analysis 2 (Parabolic-Shaped Membership Function)

6.B: Calculation Results

0.179 0.256 -1.58 40.66 0.321 0.387 33.91 47.71 0.324 0.387 21.95 47.15 0.324 0.387 10.20 31.63 0.324 0.387 10.20 31.63 0.219 0.256 -43.62 -34.19 0.0499 0.562 275.19 322.56 0.343 0.562 278.28 310.22 0.343 0.387 24.25 44.40 0.321 0.387 14.96 38.71 0.357 0.387 14.96 38.71 0.357 0.387 14.84 14.84 0.357 0.387 11.77 14.16 0.337 0.387 -1.77 14.16 0.338 0.387 -1.073 3.75 0.338 0.387 -1.073 3.75 0.453 0.387 -1.073 3.75 0.453 0.387 -1.04 1.04 0.343	Ð	Norm.Prod	Hit (Rec.)	Hit (MOM)	Rec. Prod.	MOM Prod	RH%(Rec.)	RHW/WOM	RH/Rud 1%
0.262 yes 0.351 0.387 33.91 47.71 0.263 yes 0.321 0.387 21.95 47.15 0.294 yes 0.324 0.387 10.20 31.63 0.289 no 0.029 0.256 -43.62 -34.19 0.389 no 0.049 0.562 275.19 322.56 0.137 no 0.518 0.562 275.19 322.56 0.216 yes 0.343 0.387 58.91 79.17 0.268 yes 0.343 0.387 24.25 44.40 0.268 yes 0.333 0.387 14.96 38.71 0.278 yes 0.329 0.387 14.96 38.71 0.279 yes 0.321 0.387 14.96 38.71 0.270 yes 0.327 0.387 14.96 38.71 0.371 yes yes 0.387 1.77 14.16 0.372 <td></td> <td>0.182</td> <td>yes</td> <td>yes</td> <td>0.179</td> <td>0.256</td> <td>-1.58</td> <td>40.66</td> <td>68.51</td>		0.182	yes	yes	0.179	0.256	-1.58	40.66	68.51
0.264 yes 0.321 0.387 21.95 47.15 0.284 yes 0.324 0.387 10.20 31.63 0.389 no 0.219 0.256 -43.62 -34.19 0.133 no 0.0499 0.562 275.19 332.56 0.216 yes 0.518 0.387 58.91 79.17 0.216 yes 0.333 0.387 24.25 44.40 0.208 yes 0.219 0.266 -18.16 -4.48 0.208 yes 0.219 0.287 24.25 44.40 0.208 yes 0.219 0.286 -18.16 -4.48 0.279 yes 0.219 0.256 -18.16 -4.48 0.279 yes 0.337 0.387 14.96 38.71 0.317 yes 0.337 0.387 -1.77 14.16 0.349 yes 0.337 0.387 -1.77 14.16 0.	\sim	0.262	yes	yes	0.351	0.387	33.91	47.71	-0.48
0.294 yes 0.324 0.387 10.20 31.63 0.389 no 0.219 0.256 -43.62 -34.19 0.133 no 0.499 0.562 275.19 32.56 0.137 no 0.649 0.562 275.19 32.56 0.216 yes 0.543 0.562 275.19 32.56 0.216 yes 0.343 0.387 28.91 79.17 0.268 yes 0.333 0.387 44.86 44.40 0.208 yes 0.219 0.256 -18.16 44.40 0.209 yes 0.219 0.387 14.96 38.71 0.302 yes 0.357 0.387 14.96 38.71 0.317 yes 0.387 0.387 14.84 14.84 0.317 yes 0.387 0.387 14.34 14.84 0.349 yes 0.387 0.387 -11.20 3.75 0.349	_	0.263	yes	yes	0.321	0.387	21.95	47.15	-19.05
0.389 no no 0.219 0.256 -43.62 -34.19 0.133 no no 0.499 0.562 275.19 32.56 0.216 yes 0.58 0.562 275.19 32.56 0.216 yes yes 0.343 0.387 24.25 44.40 0.268 yes yes 0.219 0.256 -18.16 44.40 0.208 yes yes 0.219 0.256 -18.16 44.40 0.208 yes 0.219 0.256 -18.16 44.40 0.209 yes 0.219 0.387 14.96 38.71 0.30 yes 0.357 0.387 14.84 14.84 0.31 yes 0.387 0.387 14.84 14.84 0.31 yes 0.387 0.387 -11.20 22.08 0.349 yes 0.387 0.387 -11.20 3.75 0.349 yes 0.333		0.294	yes	yes	0.324	0.387	10.20	31.63	-17.81
0.133 no 0.499 0.562 275.19 322.56 0.137 no 0.518 0.562 278.28 310.22 0.216 yes 0.343 0.387 58.91 79.17 0.268 yes 0.333 0.387 24.25 44.40 0.268 yes 0.219 0.256 -18.16 -4.48 0.279 yes 0.237 0.387 14.96 38.71 0.279 yes 0.357 0.387 18.09 28.15 0.317 yes 0.387 18.09 28.15 0.317 yes 0.387 1.250 22.08 0.339 yes 0.387 -1.17 14.16 0.349 yes 0.387 -1.12 1.089 0.343 yes yes 0.387 -1.12 1.04 0.348 yes 0.387 -1.12 1.04 1.04 0.488 yes yes 0.387 -1.12	\Box	0.389	ou	ou	0.219	0.256	-43.62	-34.19	101.79
0.137 no 0.518 0.562 278.28 310.22 0.216 yes 0.343 0.387 58.91 79.17 0.268 yes 0.333 0.387 24.25 44.40 0.268 yes 0.219 0.256 -18.16 -4.48 0.279 yes 0.21 0.387 14.96 38.71 0.279 yes 0.357 0.387 14.96 38.71 0.302 yes 0.357 0.387 14.84 14.84 0.317 yes 0.387 11.30 22.08 0.339 yes 0.387 -1.77 14.16 0.349 yes 0.333 0.387 -1.073 3.75 0.349 yes 0.333 0.387 -10.73 3.75 0.348 yes 0.334 0.387 -10.73 3.75 0.448 no no 0.0453 0.387 -4.04 11.53 0.231 yes		0.133	ou	no	0.499	0.562	275.19	322.56	40.83
0.216 yes 0.343 0.387 58.91 79.17 0.268 yes 0.333 0.387 24.25 44.40 0.268 yes 0.219 0.256 -18.16 -4.48 0.279 yes 0.219 0.256 -18.16 -4.48 0.279 yes 0.321 0.387 14.96 38.71 0.302 yes 0.357 0.387 12.50 22.08 0.317 yes 0.357 0.387 12.50 22.08 0.337 yes 0.387 0.387 14.84 14.84 0.339 yes 0.387 0.387 14.84 14.84 0.349 yes 0.387 0.387 -1.77 14.16 0.349 yes 0.337 0.387 -1.073 3.75 0.348 yes 0.338 0.387 -1.073 3.75 0.348 yes 0.387 0.387 -1.073 4.286 0.125		0.137	no	no	0.518	0.562	278.28	310.22	87.82
0.268 yes 0.333 0.387 24.25 44.40 0.268 yes 0.219 0.256 -18.16 -4.48 0.279 yes 0.321 0.387 14.96 38.71 0.302 yes 0.357 0.387 18.09 28.15 0.317 yes 0.357 0.387 12.50 22.08 0.337 yes 0.387 14.84 14.84 14.84 0.339 yes 0.387 1.77 14.16 10.89 0.349 yes 0.334 0.387 -1.77 14.16 10.89 0.349 yes yes 0.324 0.387 -1.120 3.75 10.89 0.357 yes yes 0.337 -1.120 3.75 10.44 1.04 1.04 0.357 yes yes 0.338 0.387 -10.73 3.75 10.44 1.04 1.04 1.04 1.04 1.04 1.04 1.04 1.04 <td></td> <td>0.216</td> <td>yes</td> <td>yes</td> <td>0.343</td> <td>0.387</td> <td>58.91</td> <td>79.17</td> <td>25.71</td>		0.216	yes	yes	0.343	0.387	58.91	79.17	25.71
0.268 yes 0.219 0.256 -18.16 -4.48 0.279 yes 0.321 0.387 14.96 38.71 0.302 yes 0.357 0.387 18.09 28.15 0.317 yes 0.357 0.387 12.50 22.08 0.337 yes 0.387 0.387 14.84 14.84 0.339 yes 0.387 0.387 14.84 14.84 0.349 yes 0.333 0.387 -1.77 14.16 0.349 yes 0.334 0.387 -7.16 10.89 0.375 yes 0.337 -1.77 14.16 10.89 0.375 yes 0.337 -1.120 3.75 10.4 0.48 yes 0.337 0.387 -1.073 3.75 0.48 yes 0.387 -1.04 1.04 1.04 0.12 yes 0.343 0.387 -24.05 19.08 0.34		0.268	yes	yes	0.333	0.387	24.25	44.40	4.51
0.279 yes 0.321 0.387 14.96 38.71 0.302 yes 0.357 0.387 18.09 28.15 0.317 yes 0.357 0.387 18.09 28.15 0.337 yes 0.387 0.387 14.84 14.84 0.339 yes 0.387 0.387 14.84 14.84 0.339 yes 0.333 0.387 -1.77 14.16 0.349 yes 0.324 0.387 -7.16 10.89 0.373 yes 0.333 0.387 -1.073 3.75 0.387 yes 0.387 -1.073 3.75 0.48 no 0.036 0.256 -54.05 -42.86 0.125 yes 0.387 -10.73 1.04 1.04 0.125 yes 0.343 0.387 -42.86 67.53 0.231 yes 0.343 0.387 -4.04 11.53 0.347 yes	_	0.268	yes	yes	0.219	0.256	-18.16	-4.48	30.06
0.302 yes 0.357 0.387 18.09 28.15 0.317 yes 0.357 0.387 12.50 22.08 0.339 yes 0.387 0.387 14.84 14.84 0.339 yes 0.387 0.387 -1.77 14.16 0.339 yes 0.333 0.387 -1.77 14.16 0.349 yes 0.333 0.387 -1.77 10.89 0.375 yes 0.333 0.387 -1.120 3.75 0.383 yes 0.387 -11.20 3.20 0.448 no no 0.387 -54.05 -42.86 0.125 no 0.387 0.387 -54.05 -42.86 0.125 yes 0.343 0.387 48.59 67.53 0.325 yes 0.343 0.387 -4.04 11.53 0.327 yes 0.333 0.387 -4.04 11.53 0.553 no	_	0.279	yes	yes	0.321	0.387	14.96	38.71	17.10
0.317 yes yes 0.357 0.387 12.50 22.08 0.339 yes 0.387 0.387 14.84 14.84 14.84 0.339 yes 0.333 0.387 -1.77 14.16 14.84 0.349 yes yes 0.333 0.387 -1.77 14.16 10.89 0.373 yes yes 0.333 0.387 -1.073 3.75 10.89 0.375 yes yes 0.387 -11.20 3.20 -42.86 0.448 no no 0.206 0.256 -54.05 -42.86 0.125 no no 0.453 0.387 262.62 209.60 0.231 yes yes 0.343 0.387 48.59 67.53 0.325 yes yes 0.343 0.387 -4.04 11.53 0.533 no yes 0.333 0.387 -4.04 11.53 0.553 no	\Box	0.302	yes	yes	0.357	0.387	18.09	28.15	-18.93
0.337 yes 0.387 0.387 14.84 14.84 14.84 0.339 yes 0.333 0.387 -1.77 14.16 10.89 0.349 yes 0.324 0.387 -7.16 10.89 10.89 0.373 yes 0.333 0.387 -7.16 10.89 10.89 0.375 yes 0.333 0.387 -11.20 3.20 10.4 0.448 no no 0.206 0.286 -54.05 -42.86 0.125 no 0.206 0.256 -54.05 -42.86 0.231 yes 0.343 0.387 48.59 67.53 0.347 yes 0.343 0.387 -4.04 11.53 0.553 no yes 0.333 0.387 -4.04 11.53 1.000 no 0.368 0.387 -4.04 -11.53		0.317	yes	yes	0.357	0.387	12.50	22.08	-58.60
0.339 yes yes 0.334 0.387 -1.77 14.16 16.89 0.349 yes 0.324 0.387 -7.16 10.89 10.89 0.373 yes 0.333 0.387 -10.73 3.75 10.89 0.375 yes 0.333 0.387 -11.20 3.20 3.20 0.383 yes 0.387 0.387 1.04 1.04 1.04 0.125 no no 0.0453 0.387 262.62 209.60 0.231 yes 0.347 48.59 67.53 19.08 0.347 yes 0.331 0.387 -4.04 11.53 0.553 no yes 0.333 0.387 -4.04 11.53 1.000 no no 0.368 0.387 -4.04 11.53		0.337	yes	yes	0.387	0.387	14.84	14.84	-5.06
0.349 yes 0.324 0.387 -7.16 10.89 10.89 0.373 yes 0.333 0.387 -10.73 3.75 7.15 0.375 yes 0.333 0.387 -11.20 3.20 3.20 0.383 yes 0.387 0.387 1.04 1.04 1.04 0.448 no no 0.206 0.256 -54.05 -42.86 7.286 0.125 no no 0.453 0.387 262.62 209.60 7.95 0.231 yes 0.343 0.387 48.59 67.53 7.95 0.347 yes 0.351 0.387 -4.04 11.53 7.05 0.553 no yes 0.333 0.387 -30.79 -30.02 1.000 no no 0.368 0.387 -30.79 -61.30		0.339	yes	yes	0.333	0.387	-1.77	14.16	V/N
0.373 yes 0.333 0.387 -10.73 3.75 0.384 yes 0.387 -11.20 3.20 0.383 yes 0.387 -11.20 3.20 0.448 no 0.206 0.256 -54.05 -42.86 0.125 no 0.453 0.387 48.59 67.53 0.231 yes 0.343 0.387 48.59 67.53 0.345 yes 0.351 0.387 -4.04 11.53 0.553 no yes 0.333 0.387 -4.04 11.53 0.553 no yes 0.333 0.387 -4.04 11.53 1.000 no no 0.368 0.387 -50.02 -30.02		0.349	yes	yes	0.324	0.387	-7.16	10.89	N/A
0.375 yes 0.333 0.387 -11.20 3.20 0.383 yes 0.387 0.387 1.04 1.04 1.04 0.448 no 0.206 0.256 -54.05 -42.86 -42.86 0.125 no 0.453 0.387 262.62 209.60 -67.53 0.251 yes 0.343 0.387 48.59 67.53 -67.53 0.347 yes 0.351 0.387 -4.04 11.53 -19.08 0.553 no yes 0.333 0.387 -4.04 11.53 1.000 no no 0.368 0.387 -59.79 -50.02		0.373	yes	yes	0.333	0.387	-10.73	3.75	-30.27
0.383 yes 0.387 0.387 1.04 1.04 1.04 0.448 no 0.206 0.256 -54.05 -42.86 -42.86 0.125 no 0.453 0.387 262.62 209.60 209.60 0.231 yes 0.343 0.387 48.59 67.53 8 0.347 yes 0.351 0.387 -4.04 11.53 19.08 0.553 no yes 0.333 0.387 -4.04 11.53 11.00 1.000 no no 0.368 0.387 -530.79 -61.30	_	0.375	yes	yes	0.333	0.387	-11.20	3.20	-11.31
0.448 no 0.206 0.256 -54.05 -42.86 0.125 no 0.453 0.387 262.62 209.60 0.231 yes 0.343 0.387 48.59 67.53 0.325 yes 0.351 0.387 7.95 19.08 0.347 yes 0.333 0.387 -4.04 11.53 0.553 no yes 0.333 0.387 -39.79 -30.02 1.000 no no 0.368 0.387 -63.24 -61.30	_	0.383	yes	yes	0.387	0.387	1.04	1.04	-65.73
0.125 no 0.453 0.387 262.62 209.60 0.231 yes 0.343 0.387 48.59 67.53 0.325 yes 0.351 0.387 7.95 19.08 0.347 yes 0.333 0.387 -4.04 11.53 0.553 no yes 0.333 0.387 -39.79 -30.02 1.000 no no 0.368 0.387 -63.24 -61.30	一	0.448	no	no	0.206	0.256	-54.05	-42.86	-47.30
0.231 yes 0.343 0.387 48.59 67.53 0.325 yes 0.351 0.387 7.95 19.08 0.347 yes 0.333 0.387 -4.04 11.53 0.553 no yes 0.333 0.387 -39.79 -30.02 1.000 no no 0.368 0.387 -63.24 -61.30		0.125	no	ou	0.453	0.387	262.62	209.60	152.02
0.325 yes 0.351 0.387 7.95 19.08 0.347 yes 0.333 0.387 -4.04 11.53 0.553 no yes 0.333 0.387 -39.79 -30.02 1.000 no no 0.368 0.387 -63.24 -61.30		0.231	yes	yes	0.343	0.387	48.59	67.53	N/A
0.347 yes 0.333 0.387 -4.04 11.53 0.553 no yes 0.333 0.387 -39.79 -30.02 1.000 no no 0.368 0.387 -63.24 -61.30	_	0.325	yes	yes	0.351	0.387	7.95	19.08	-8.70
0.553 no yes 0.333 0.387 39.79 30.02 1.000 no no 0.368 0.387 63.24 61.30		0.347	yes	yes	0.333	0.387	-4.04	11.53	N/A
1.000 no no 0.368 0.387 -63.24 -61.30		0.553	no	yes	0.333	0.387	-39.79	-30.02	-47.45
		1.000	no	no	0.368	0.387	-63.24	-61.30	-57.66

Appendix 7: Sensitivity Analysis 3 (Small Change in Range)

Appendix 7: Sensitivity Analysis 3 (Small Change in Range)

ID	Good	GM	Medium	MB	Bad
1	0.318	0.286	0.167	0.000	0.000
2	0.286	0.318	0.000	0.000	0.000
3	0.286	0.375	0.167	0.000	0.000
4	0.286	0.333	0.000	0.167	0.000
5	0.333	0.286	0.167	0.000	0.000
6	0.000	0.286	0.375	0.000	0.000
7	0.000	0.286	0.500	0.000	0.286
8	0.286	0.500	0.000	0.286	0.000
9	0.286	0.333	0.000	0.000	0.000
10	0.400	0.286	0.000	0.000	0.000
11	0.286	0.318	0.000	0.000	0.000
12	0.286	0.375	0.167	0.000	0.000
13	0.286	0.375	0.000	0.000	0.000
14	0.000	0.318	0.000	0.000	0.000
15	0.286	0.333	0.000	0.167	0.000
16	0.286	0.333	0.000	0.167	0.000
17	0.286	0.333	0.167	0.000	0.000
18	0.286	0.333	0.167	0.200	0.167
19	0.000	0.375	0.000	0.000	0.000
20	0.333	0.286	0.000	0.000	0.000
21	0.000	0.318	0.286	0.000	0.286
22	0.286	0.500	0.000	0.000	0.000
23	0.286	0.318	0.000	0.000	0.000
24	0.286	0.318	0.000	0.000	0.000
25	0.286	0.333	0.000	0.200	0.000
26	0.286	0.583	0.167	0.286	0.000

Appendix 7: Sensitivity Analysis 3 (Small Change in Range)

7.B: Calculation Results

0.182	**** (*****)	(NCC. LTOG.	MOM Prod	RH%(Rec.)	RH%(MOM)	RH(Bud.)%
2 8	yes	yes	0.304	0.256	67.55	41.03	68.51
8 2	yes	yes	0.268	0.387	2.11	47.65	-0.48
_	yes	yes	0.278	0.387	5.69	47.35	-19.05
	yes	yes	0.270	0.387	-8.19	31.46	-17.81
0.389	no	no	0.300	0.256	-22.86	-34.21	101.79
0.133	no	ou	0.453	0.562	240.36	322.52	40.83
0.137	no	ou	0.475	0.562	245.34	308.93	87.82
0.216	yes	yes	0.299	0.387	38.51	79.00	25.71
0.268	yes	yes	0.270	0.387	0.80	44.32	4.51
0.268	yes	yes	0.283	0.256	5.43	-4.53	30.06
0.279	yes	yes	0.268	0.387	-4.17	38.56	17.10
0.302	yes	yes	0.278	0.387	-8.01	28.25	-18.93
0.317	yes	yes	0.278	0.387	-12.30	22.26	-58.60
0.337	yes	yes	0.387	0.387	14.78	14.78	-5.06
0.339	yes	yes	0.270	0.387	-20.19	14.27	Y/X
0.349	yes	yes	0.270	0.387	-22.60	10.82	A/X
0.373	yes	yes	0.270	0.387	-27.53	3.76	-30.27
0.375	yes	yes	0.270	0.387	-27.95	3.17	-11.31
0.383	yes	yes	0.387	0.387	1.15	1.15	-65.73
0.448	no	ou	0.300	0.256	-32.94	-42.81	-47.30
0.125	no	ou	0.506	0.387	306.07	210.29	152.02
0.231	yes	yes	0.299	0.387	29.65	67.54	K/Z
0.325	yes	yes	0.268	0.387	-17.56	19.21	-8.70
0.347	yes	yes	0.268	0.387	-22.83	11.58	4 /Z
0.553	no	yes	0.270	0.387	-51.08	-29.96	-47.45
000.1	u0	no	0.314	0.387	-68.59	-61.30	-57.66

Appendix 8: Sensitivity Analysis 4 (Large Change in Range)

Appendix 8: Sensitivity Analysis 4 (Large Change in Range)

ID	Good	GM	Medium	MB	Bad
1	0.000	0.000	0.000	0.000	0.000
2	0.000	0.545	0.000	0.000	0.000
3	0.000	0.091	0.000	0.000	0.000
4	0.000	0.103	0.000	0.000	0.000
5	0.000	0.091	0.000	0.000	0.000
6	0.000	0.167	0.000	0.000	0.000
7	0.000	0.167	0.000	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000
9	0.000	0.091	0.000	0.000	0.000
10	0.000	0.103	0.000	0.000	0.000
11	0.000	0.167	0.000	0.000	0.000
12	0.000	0.091	0.000	0.000	0.000
13	0.000	0.000	0.000	0.000	0.000
14	0.000	0.167	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000
16	0.000	0.167	0.000	0.000	0.000
17	0.000	0.091	0.000	0.000	0.000
18	0.000	0.231	0.000	0.000	0.000
19	0.000	0.167	0.000	0.000	0.000
20	0.167	0.000	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.000
22	0.000	0.167	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.000
24	0.000	0.231	0.000	0.000	0.000
25	0.000	0.167	0.000	0.000	0.000
26	0.000	0.167	0.000	0.000	0.000

Appendix 8: Sensitivity Analysis 4 (Large Change in Range)

8.B: Calculation Results

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RH/Rnd 1%	68 513	61C:SC	-10.053	-17.813	101 788	40.828	87.818	25.712	4.513	30.065	17.100	-18.932	-58.596	-5.056	N/N	V/N	-30.267	-11.312	-65.730	-47.301	152.016	N/A	-8.699	N/A	-47.451	-57.659
RHWMOM)	V/V	47.65	47.35	3146	-0.55	190.96	181.60	N/A	44.32	44.32	38.56	28.25	N/A	14.78	V/N	10.82	3.76	3.17	1.15	-42.81	V/N	67.54	V/N	11.58	-29.96	-61.30
RH%(Rec.)	N/N	47.65	47.35	31.46	-0.55	190.96	181.60	N/A	44.32	44.32	38.56	28.25	N/A	14.78	N/A	10.82	3.76	3.17	1.15	-42.81	V/N	67.54	A/X	11.58	-29.96	-61.30
MOM Prod	٧×	0.387	0.387	0.387	0.387	0.387	0.387	N/A	0.387	0.387	0.387	0.387	N/A	0.387	K/N	0.387	0.387	0.387	0.387	0.256	Y/X	0.387	N/A	0.387	0.387	0.387
Rec. Prod.	N/A	0.387	0.387	0.387	0.387	0.387	0.387	N/A	0.387	0.387	0.387	0.387	N/A	0.387	N/A	0.387	0.387	0.387	0.387	0.256	N/A	0.387	N/A	0.387	0.387	0.387
Hit (MOM)	no	yes	yes	yes	yes	no	no	no	yes	yes	yes	yes	ou	yes	no	yes	yes	yes	yes	no	no	yes	no	yes	yes	no
Norm.Prod	0.182	0.262	0.263	0.294	0.389	0.133	0.137	0.216	0.268	0.268	0.279	0.302	0.317	0.337	0.339	0.349	0.373	0.375	0.383	0.448	0.125	0.231	0.325	0.347	0.553	1.000
e	_	2	3	4	5	9	7	∞	6	2	=	12	13	4-	2	91	17	8	6	20	21	22	23	24	25	56