Detecting Targets Hidden in Random Forests

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ABSTRACT

Military tanks, cargo or troop carriers, missile carriers or rocket launchers often hide themselves from detection in the forests. This plagues the detection problem of locating these hidden targets. An electro-optic camera mounted on a surveillance aircraft or unmanned aerial vehicle is used to capture the images of the forests with possible hidden targets, e.g., rocket launchers. We consider random forests of longitudinal and latitudinal correlations. Specifically, foliage coverage is encoded with a binary representation (i.e., foliage or no foliage), and is correlated in adjacent regions. We address the detection problem of camouflaged targets hidden in random forests by building memory into the observations. In particular, we propose an efficient algorithm to generate random forests, ground, and camouflage of hidden targets with two dimensional correlations. The observations are a sequence of snapshots consisting of foliage-obscured ground or target. Theoretically, detection is possible because there are subtle differences in the correlations of the ground and camouflage of the rocket launcher. However, these differences are well beyond human perception. To detect the presence of hidden targets automatically, we develop a Markov representation for these sequences and modify the classical filtering equations to allow the Markov chain observation. Particle filters are used to estimate the position of the targets in combination with a novel random weighting technique. Furthermore, we give positive proof-of-concept simulations.

Keywords: Target Detection, Correlation Structure, Random Weighting, Particle Filter

1. INTRODUCTION

1.1 Random Forests, Grounds, and Camouflages of Ground Objects

Military tanks, cargo or troop carriers, missile carriers and rocket launchers are the main modern military ground transportation vehicles or weapons. These objects usually obscure themselves from detection by moving or hiding under various forms of shelter . Kouritzin et al studied target tracking problems where targets obscure themselves by moving under random blockages, see.[8] In,[8] the blockages are random buildings or forests of many types classified by their shapes. However, the random forests described in that paper do not have a correlation structure, which we will formulate in this paper. Generally, forests contain many tree species within a small area and can be classified by the predominant composition of broadleaf trees, coniferous (needle-leaved) trees, or mixed. The forest type varies from region to region and the foliage shape and density vary from season to season. Moreover, the strong wind can change the features of the foliage. All the above factors may make the observation of the forest from the electro-optic equipment mounted on a drone random. Indeed, foliage changes as leaves move, grow, or fall but the overall density and correlations remain relatively constant from day to day. Herein, we focus on the characterization of the density of the foliage and the correlations among the foliage. We introduce the marginal probabilities to describe the density of the foliage and the covariances to characterize the correlations among them in the pixel by pixel scale. The ground in the forests is either covered with grass or just the soil. The sunshine, humidity and temperature may cause the appearance of the ground to change. Similarly as the forests, we characterize the correlation structure of the ground in terms of the marginal probabilities and

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the covariances. Henceforth, we consider the rocket launchers as an example of the latent ground object. Rocket launchers usually stand still in the forest, and use the turf as the camouflage to look like part of the surroundings. The best camouflaged strategy for rocket launchers is to imitate the correlation structure of ground colors, so the correlation structures of the rocket launcher and ground should be slightly different.

1.2 Detection Problem

The rocket launcher sits still in the forest and we use a camera mounted on a drone flying over the forest to capture the image of the forest with hidden target rocket launchers. The electro-optic equipment can not penetrate through the foliage, and the foliage blocks the latent ground objects from detection. However, we can observe partial images of the ground through the gaps among the leaves. The observed color of rocket launcher on the images is either green or brown, the inherent color on its surface and there is some type of correlation structure among pixels. Similarly, the ground is a mixture of the grass and soil, and its color is randomly either green or brown with some type of correlation structure. The information from within the forest called the observations, can be easily obtained from stored historical overhead pictures and analyzed pixel by pixel for each small area. These snapshots are characterized in terms of the correlation structure, i.e., the marginal probabilities and pairwise correlation matrices of the columns of observed colors of the foliage and ground or target below, both of which are correlated in the adjacent regions.

A method for generating Bernoulli variables with given marginal probabilities and pairwise covariances is discussed in.[7] In this paper, we propose an algorithm in Proposition 2.1 to generate random forests, and an algorithm in Proposition 2.5 to generate random ground or camouflage of rocket launchers, each of them having correlated random vectors with given marginal probabilities and pairwise correlation matrices or vectors, up to a lag l. These algorithms are very efficient, because the random forest, ground or rocket launcher are generated via component random variables. We also propose an algorithm in Proposition 2.7 to compute the conditional probabilities of random ground or rocket launcher vectors which will be employed to calculate the random weight functions for the SERP filter.

The observations are modelled as layered Markov chains; we only see part of the ground or rocket launcher since the foliage obscures the vision of hidden targets and creates random blockages. However, knowledge of the stochastic laws of the colors of rocket launcher and ground along with a sequence of observations over time enables detecting the presence of a target. In a very suspicious small area, the detection problem becomes determining whether there is a rocket launcher or not. When the plane flies over this area, a sequence of images are captured. Since the suspicious area is small, we can assume that the rocket launcher either occupies the entire image or not. For future work, we may consider the case that rocket launcher occupies part of the image, which is a complicated case of the detection problems.

1.3 SERP Filter

The classical particle filter method is a two-stage process. First, particles are evolved according to the law of the signal over time. Second, the weight for each particle is recalculated when the new observation is received. However, most of the particles tend to be unrepresentative of the real signal, so a resampling step has been added to the classical scheme by adapting the particles to observations. But the resampling can add unnecessary randomness to the filtering system, thereby degrading performance. A highly effective solution is the selectively resampling particle (SERP) filter which was first mentioned in Ballantyne, Kim and Kouritzin.[4] The basic idea is that particles are evolved as in the classical weighted method until such a time that the ratio of the largest particle weight to the smallest particle weight exceeds some factor ρ, which is often determined by solving a stochastic optimization problem. Then, particles are resampled pairwise until the ratio of the weights falls within the value ρ.

2. METHODS FOR GENERATING RANDOM FOREST, GROUND, AND ROCKET LAUNCHER

2.1 Formulation and Algorithm for Generating Random Forest

In this subsection, firstly we introduce notations to formulate the random forest, using marginal probabilities to express foliage densities and covariances to express the foliage correlation structure. Then, we state a proposition for generating the random forest. Lastly, we propose the corresponding algorithm for implementation.

We consider a raster of forest images containing $M \times N$ pixels, and each image is denoted by $\{\xi_1, \xi_2, ..., \xi_N\}$, where $\xi_j = (\xi_j^1, \xi_j^2, ..., \xi_j^M)^T$ $(1 \leq j \leq N)$. For $1 \leq i \leq M$, $1 \leq j \leq N$, define

$$\xi_j^i = \begin{cases} 2, & \text{if we observe tree leaf color at the } (i,j)^{th} \text{ pixel,} \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

In this subsection, we let $c_1 = 0$, $c_2 = 2$ and $C = \{c_1, c_2\}$ be the common state space of component random variables $\{\xi_j^i\}_{i=1,j=1}^{M,N}$. Let $q_F^{i,j,s} = P(\xi_j^i = c_s)$, for $s = 1$, 2, denote the density of leaves in a small area of the forest at the $(i,j)^{th}$ pixel. Let $l \in \mathbb{N}$ be the lag of the Markov chain structure which we will employ to generate the random forest. For the $(i,j)^{th}$ entry of the image, we denote the lags in the row and column directions by $n_i = l \wedge (i-1)$ and $n_j = l \wedge (j-1)$. For the $(i,j)^{th}$ node, we define the most recent n_i elements in the j^{th} column by $\xi_{i,j}^l = (\xi_j^{i-n_i}, \xi_j^{i-n_i+1}, ..., \xi_j^i)^T$. Define the pairwise correlations by $\mathbf{a}_F^{l,i,j,x} = cov(\xi_j^i, \xi_{i,j-x}^l)$, i.e., $\mathbf{a}_F^{l,i,j,x} = (a_{F,v}^{l,i,j,x})_{1\times(n_i+1)}$, where $a_{F,v}^{l,i,j,x} = cov(\xi_j^i, \xi_{j-x}^{i-n_i+v})$, $0 \leq v \leq n_i$; $\beta_F^{l,i,j,y} = cov(\xi_j^i, \xi_j^{i-y})$ for $1 \leq y \leq n_i$. Both $\{\mathbf{a}_F^{l,i,j,x}\}_{i=1,j=2,x=1}^{M,N,n_j}$ and $\{\beta_F^{l,i,j,y}\}_{i=2,j=1,y=1}^{M,N,n_i}$ partly characterize the correlation structure of tree leaf densities. For fixed $1 \leq i \leq M$, we introduce the (n_i+1)-dimensional vector $\mathbf{e}_i^l = (1, ..., 1)^T$. We assign conditional probabilities $P\left(\xi_j^i = c_s \middle| \begin{smallmatrix} \xi_j^{i-1}=m_j^{i-1},...,\xi_j^{i-n_i}=m_j^{i-n_i}; \\ \xi_{i,j-1}^l=\mathbf{m}_{i,j-1}^l,...,\xi_{i,j-n_j}^l=\mathbf{m}_{i,j-n_j}^l \end{smallmatrix}\right)$ for the component random variable ξ_j^i by Proposition 2.1, such that the desired covariances and marginal probabilities are maintained as i and j increase:

Proposition 2.1. *Suppose that $M, N, l \in \mathbb{N}$, $\left\{q_F^{i,j,s}\right\}_{i=1,j=1,s=1}^{M,N,2}$ are positive and $q_F^{i,j,1} + q_F^{i,j,2} = 1$; and $\mathbf{a}_F^{l,i,j,x}$, for $1 \leq x \leq n_j$, is a real (n_i+1)-vector and $\beta_F^{l,i,j,y}$, for $1 \leq y \leq n_i$, is a real number. Suppose also that $\left\{q_F^{i,j,s}\right\}_{i=1,j=1,s=1}^{M,N,2}$ and $\left\{\mathbf{a}_F^{l,i,j,x}\right\}_{i=1,j=2,x=1}^{M,N,n_j}$ and $\left\{\beta_F^{l,i,j,y}\right\}_{i=2,j=1,y=1}^{M,N,n_i}$ are such that the numbers on the RHS of (2) are between 0 and 1. Form the conditional probabilities recursively, starting with $i = 1, j = 1$, as*

$$\begin{aligned} & P\left(\xi_j^i = c_s \middle| \begin{smallmatrix} \xi_j^{i-1}=m_j^{i-1},...,\xi_j^{i-n_i}=m_j^{i-n_i}; \\ \xi_{i,j-1}^l=\mathbf{m}_{i,j-1}^l,...,\xi_{i,j-n_j}^l=\mathbf{m}_{i,j-n_j}^l \end{smallmatrix}\right) \\ = \quad & q_F^{i,j,s} + \frac{\sum_{x=1}^{n_j}(c_s-1)(\mathbf{a}_F^{l,i,j,x})^T(\mathbf{m}_{i,j-x}^l - \mathbf{e}_j^l) + \sum_{y=1}^{n_i}(c_s-1)\beta_F^{l,i,j,y}(m_j^{i-y}-1)}{2^{(n_i+1)(n_j+1)}P\left(\begin{smallmatrix} \xi_j^{i-1}=m_j^{i-1},...,\xi_j^{i-n_i}=m_j^{i-n_i}; \\ \xi_{i,j-1}^l=\mathbf{m}_{i,j-1}^l,...,\xi_{i,j-n_j}^l=\mathbf{m}_{i,j-n_j}^l \end{smallmatrix}\right)} \end{aligned} \tag{2}$$

for each $c_s \in \{0,2\}$, $m_j^{i-y} \in \{0,2\}$, $y = 1, ..., n_i$ and $\mathbf{m}_{i,j-x}^l = (m_{j-x}^{i-n_i}, m_{j-x}^{i-n_i+1}, ..., m_{j-x}^i)^T \in \{0,2\}^{n_i+1}$, $x = 1, ..., n_j$, where $n_i = l \wedge (i-1)$, $n_j = l \wedge (j-1)$. Then, $\{\xi_j = (\xi_j^1, \xi_j^2, ..., \xi_j^M)^T\}_{j=1}^N$ are correlated random vectors with component probabilities $q_F^{i,j,s} = P(\xi_j^i = c_s)$, $s = 1$, 2, and covariance vectors $\mathbf{a}_F^{l,i,j,x} = cov(\xi_j^i, \xi_{i,j-x}^l)$ and $x = 1, ..., n_j$ and $\beta_F^{l,i,j,y} = cov(\xi_j^i, \xi_j^{i-y})$, $y = 1, ..., n_i$, for all $1 < i \leq M$ and $1 \leq j \leq N$.

Remark 2.2. *l is the number of adjacent longitudinal or latitudinal correlations we are concerned with for each node. n_i and n_j allow us to start-up when we have fewer than l rows or columns; $\left\{q_F^{i,j,s}\right\}_{i=1,j=1,s=1}^{M,N,2}$, lag correlations $\left\{\mathbf{a}_F^{l,i,j,x}\right\}_{i=1,j=2,x=1}^{M,N,n_j}$ and $\left\{\beta_F^{l,i,j,y}\right\}_{i=2,j=1,y=1}^{M,N,n_i}$ are quantities that vary in time (e.g., $q_F^{i,j,2}$ is the probability of $(i,j)^{th}$ pixel being leaf color). For fixed $(i,j) \in \{1,2,...,M\} \times \{1,2,...,N\}$, we call $\mathbf{a}_F^{l,i,j,x}$ $(1 \leq x \leq n_j)$ x-step correlation and $\beta_F^{l,i,j,y}$ $(1 \leq y \leq n_i)$ y-step correlation (e.g. both $\mathbf{a}_F^{3,i,j,2}$ and $\beta_F^{3,i,j,2}$ are 2-step correlations, here lag $l = 3$).*

Remark 2.3. *If the right hand side of (2) is not between 0 and 1 for some $(i,j) \in \{1,2,...,M\} \times \{1,2,...,N\}$, some $m_j^{i-1}, ..., m_j^{i-n_i} \in \{0,2\}$, and some $\mathbf{m}_{i,j-1}^l, ..., \mathbf{m}_{i,j-n_j}^l \in \{0,2\}^{n_i+1}$, then we cannot use this proposition to* *create $\{\xi_1, \xi_2, ..., \xi_N\}$ with the desired marginal probabilities and covariances. However, we will see in the sequel that when adjacent correlations are not extremely strong, the right hand side of (2) is always between 0 and 1.*

Proposition 2.1 enables us to efficiently construct the random forest $\{\xi_1, \xi_2, \ldots, \xi_N\}$ to match the predescribed correlation structures $\mathbf{a}_F^{l,i,j,x} = cov(\xi_j^i, \xi_{i,j-x}^l)$ $(1 \le i \le M,\ 1 < j \le N$, and $x = 1, ..., l \wedge (j-1)$), and $\beta_F^{l,i,j,y} = cov(\xi_j^i, \xi_j^{i-y})$ $(1 < i \le M$ and $1 \le j \le N,\ y = 1, ..., l \wedge (i-1))$, and marginal probabilities $q_F^{i,j,s} = P(\xi_j^i = c_s)$ $(1 \le i \le M,\ 1 \le j \le N,\ c_s \in \{0, 2\})$ of component random variables $\{\xi_j^i\}_{i=1,j=1}^{M,N}$ up to a fixed lag l.

We use the following algorithm to simulate the correlated random forest $\{\xi_1, \xi_2, ..., \xi_N\}$. Repeat for the sequence $\xi_j, j = 1, 2, \ldots, N$:

1. Given the past $n_j = \min(j-1, l)$ random vectors, repeat for $\xi_j^i, i = 1, \ldots, M$.
2. Given the past $n_i = \min(i-1, l)$ random variables, compute $P\left(\xi_j^i = 0 \middle| \substack{\xi_j^{i-1} = m_j^{i-1}, \ldots, \xi_j^{i-n_i} = m_j^{i-n_i};\\ \xi_{i,j-1}^l = \mathbf{m}_{i,j-1}^l, \ldots, \xi_{i,j-n_j}^l = \mathbf{m}_{i,j-n_j}^l}\right)$, using Proposition 2.1.
3. Generate a $[0,1]$-uniform random variable U. Then,
$$\xi_j^i = \begin{cases} 0 & \text{If } U \le P\left(\xi_j^i = 0 \middle| \substack{\xi_j^{i-1} = m_j^{i-1}, \ldots, \xi_j^{i-n_i} = m_j^{i-n_i};\\ \xi_{i,j-1}^l = \mathbf{m}_{i,j-1}^l, \ldots, \xi_{i,j-n_j}^l = \mathbf{m}_{i,j-n_j}^l}\right), \\ 2 & \text{otherwise.} \end{cases}$$
When $i = M$, we obtain $\xi_j = (\xi_j^1, \cdots, \xi_j^M)^T$.

Remark 2.4. *The above algorithm based on Proposition 2.1 is optimal in the sense that the computation grows linearly with the number of component random variables MN. Simulations based on this algorithm proved the efficiency of this algorithm.*

2.2 Formulation and Algorithm for Generating Random Ground and Rocket Launcher

We describe the algorithm to generate the random ground and the camouflage of rocket launcher, which is very similar to the algorithm in the random forest subsection. We use notations with subscript or superscript G for ground and R for rocket launcher. Therefore, everything developed for the ground can be applied to the rocket launcher simply by replacing G with R.

We denote the ground image containing $M \times N$ pixels by $\{\mathbf{G}_1, \mathbf{G}_2, ..., \mathbf{G}_N\}$ where $\mathbf{G}_j = (G_j^1, G_j^2, ..., G_j^M)^T$ $(1 \le j \le N)$. For $1 \le i \le M$, $1 \le j \le N$, let

$$G_j^i = \begin{cases} 1, & \text{if we observe grass color at the } (i,j)^{th} \text{ pixel,} \\ -1, & \text{otherwise.} \end{cases} \tag{3}$$

In the current subsection, we let $C = \{c_1, c_2\}$ be the common state space of component random variables $\{G_j^i\}_{i=1,j=1}^{M,N}$, where $c_1 = -1$ and $c_2 = 1$. Denote by $q_G^{i,j,s} = P(G_j^i = c_s)$ $(s = 1, 2)$ the density of grass in a small area of the ground at the $(i,j)^{th}$ pixel. We use the same notation of lag $l \in \mathbb{N}$ for the Markov chain structure of the random ground, and keep the same implications for $n_i = l \wedge (i-1)$, $n_j = l \wedge (j-1)$ and $\mathbf{G}_{i,j}^l = (G_j^{i-n_i}, G_j^{i-n_i+1}, ..., G_j^i)^T$. Denote the covariances by $\mathbf{a}_G^{l,i,j,x} = cov(G_j^i, \mathbf{G}_{i,j-x}^l)$, i.e., $\mathbf{a}_G^{l,i,j,x} = (a_{G,v}^{l,i,j,x})_{1\times(n_i+1)}$, where $a_{G,v}^{l,i,j,x} = cov(G_j^i, G_{j-x}^{i-n_i+v})$, $0 \le v \le n_i$; and $\beta_G^{l,i,j,y} = cov(G_j^i, G_j^{i-y})$ for $1 \le y \le n_i$.

Proposition 2.5 is an analogue of Proposition 2.1. We maintain the desired covariances and marginal probabilities by assigning the conditional probabilities $P\left(G_j^i = c_s \middle| \substack{G_j^{i-1} = m_j^{i-1}, \ldots, G_j^{i-n_i} = m_j^{i-n_i};\\ \mathbf{G}_{i,j-1}^l = \mathbf{m}_{i,j-1}^l, \ldots, \mathbf{G}_{i,j-n_j}^l = \mathbf{m}_{i,j-n_j}^l}\right)$ for each component random variable G_j^i.

Proposition 2.5. *Suppose that $M, N, l \in \mathbb{N}$, $\left\{q_G^{i,j,s}\right\}_{i=1,j=1,s=1}^{M,N,2}$ are positive and $q_G^{i,j,1} + q_G^{i,j,2} = 1$; $\mathbf{a}_G^{l,i,j,x}$, for $1 \le x \le n_j$, is a real (n_i+1)-vector and $\beta_G^{l,i,j,y}$, for $1 \le y \le n_i$, is a real number. Suppose also that*

$\left\{q_G^{i,j,s}\right\}_{i=1,j=1,s=1}^{M,N,2}$ *and* $\left\{\mathbf{a}_G^{l,i,j,x}\right\}_{i=1,j=2,x=1}^{M,N,n_j}$ *and* $\left\{\beta_G^{l,i,j,y}\right\}_{i=2,j=1,y=1}^{M,N,n_i}$ *are such that the numbers on the RHS of (4) are between* 0 *and* 1*. Form the conditional probabilities recursively, starting with* $i = 1, j = 1$*, as*

$$\begin{aligned} & P\left(G_j^i = c_s \middle| \substack{G_j^{i-1}=m_j^{i-1},\ldots,G_j^{i-n_i}=m_j^{i-n_i};\\ G_{i,j-1}^l=\mathbf{m}_{i,j-1}^l,\ldots,G_{i,j-n_j}^l=\mathbf{m}_{i,j-n_j}^l}\right) \\ = \;& q_G^{i,j,s} + \frac{\sum_{x=1}^{n_j} c_s(\mathbf{a}_G^{l,i,j,x})^T\mathbf{m}_{i,j-x}^l + \sum_{y=1}^{n_i} c_s\beta_G^{l,i,j,y} m_j^{i-y}}{2^{(n_i+1)(n_j+1)}P\left(\substack{G_j^{i-1}=m_j^{i-1},\ldots,G_j^{i-n_i}=m_j^{i-n_i};\\ \mathbf{G}_{i,j-1}^l=\mathbf{m}_{i,j-1}^l,\ldots,\mathbf{G}_{i,j-n_j}^l=\mathbf{m}_{i,j-n_j}^l}\right)} \end{aligned} \tag{4}$$

for each $c_s \in \{-1,1\}$*,* $m_j^{i-y} \in \{-1,1\}$*,* $y = 1,...,n_i$ *and* $\mathbf{m}_{i,j-x}^l = (m_{j-x}^{i-n_i}, m_{j-x}^{i-n_i+1}, \cdots, m_{j-x}^i)^T \in \{-1,1\}^{n_i+1}$*,* $x = 1,...,n_j$*, where* $n_i = l \wedge (i-1)$*,* $n_j = l \wedge (j-1)$*. Then,* $\{\mathbf{G}_j = (G_j^1, G_j^2, ..., G_j^M)^T\}_{j=1}^N$ *are correlated random vectors with component probabilities* $q_G^{i,j,s} = P(G_j^i = c_s)$*,* $s = 1,\ 2$*, and covariance vectors* $\mathbf{a}_G^{l,i,j,x} = cov(G_j^i, \mathbf{G}_{i,j-x}^l)$*,* $x = 1,...,n_j$ *and* $\beta_G^{l,i,j,y} = cov(G_j^i, G_j^{i-y})$*,* $y = 1,...,n_i$*, for all* $1 < i \le M$ *and* $1 \le j \le N$*.*

Proposition 2.5 gives us a very efficient way of constructing the random ground $\{\mathbf{G}_1, \mathbf{G}_2, ..., \mathbf{G}_N\}$ to match the predescribed correlation structures $\mathbf{a}_G^{l,i,j,x} = cov(G_j^i, \mathbf{G}_{i,j-x}^l)$ ($1 \le i \le M$, $1 < j \le N$, and $x = 1,...,l\wedge(j-1)$) and $\beta_G^{l,i,j,y} = cov(G_j^i, G_j^{i-y})$ ($1 < i \le M$ and $1 \le j \le N$, $y = 1,...,l \wedge (i-1)$), and marginal probabilities $q_G^{i,j,s} = P(G_j^i = c_s)$ ($1 \le i \le M$, $1 \le j \le N$, $c_s \in \{-1,1\}$) of component random variables $\{G_j^i\}_{i=1,j=1}^{M,N}$, up to a fixed lag l.

We use the following algorithm to simulate the correlated random ground $\{\mathbf{G}_1, \mathbf{G}_2, ..., \mathbf{G}_N\}$. Repeat for the sequence $\mathbf{G}_j, j = 1, 2, \ldots, N$:

1. Given the past $n_j = \min(j-1, l)$ random vectors, repeat for $G_j^i, i = 1, \ldots, M$.
2. Given the past $n_i = \min(i-1, l)$ random variables, compute $P\left(G_j^i = -1 \middle| \substack{G_j^{i-1}=m_j^{i-1},\ldots,G_j^{i-n_i}=m_j^{i-n_i};\\ \mathbf{G}_{i,j-1}^l=\mathbf{m}_{i,j-1}^l,\ldots,\mathbf{G}_{i,j-n_j}^l=\mathbf{m}_{i,j-n_j}^l}\right)$, using Proposition 2.5.
3. Generate a $[0,1]$-uniform random variable U. Then,

$$G_j^i = \begin{cases} -1 & \text{If } U \le P\left(G_j^i = -1 \middle| \substack{G_j^{i-1}=m_j^{i-1},\ldots,G_j^{i-n_i}=m_j^{i-n_i};\\ \mathbf{G}_{i,j-1}^l=\mathbf{m}_{i,j-1}^l,\ldots,\mathbf{G}_{i,j-n_j}^l=\mathbf{m}_{i,j-n_j}^l}\right), \\ 1 & \text{otherwise.} \end{cases}$$

When $i = M$, we obtain $\mathbf{G}_j = (G_j^1, \cdots, G_j^M)^T$.

Remark 2.6. *The random forest and the random ground are assumed to be independent. The correlated structure of the random forest predetermined in Proposition 2.1 is about correlations of the foliage. Similarly, the correlated structure of random ground predescribed in Proposition 2.5 is about correlations of grass. It is assumed that there are no correlations between foliage and grass. This assumption only simplifies the presentation of the work. In fact, we have developed formula much stronger than (2) and (4) which allow for multiple colors instead of only two colors. We developed formula similar to (2) or (4), with the common state space* C *containing more than two values, in contrast to* $C = \{0,2\}$ *in subsection 2.1 or* $C = \{-1,1\}$ *in subsection 2.2. Therefore, we have a formula to support a more complicated structure of the problem, and the structure takes into account the correlations between the foliage and grass.*

2.3 Formula Used in Weighting functions

The following proposition enables us to compute the conditional probabilities of $N \in \mathbb{N}$ correlated random vectors with given marginal probabilities and pairwise correlation matrices up to a fixed lag l. For the first $l-1$ columns in the image, the lag could be at most the number of the column itself. In the following context, we still use n_j

to denote the lag, and $n_j = l \wedge (j-1)$ for $1 < j \leq N$. We define the marginal probabilities of each random vector of the random ground $\{\mathbf{G}_j = (G_j^1, G_j^2, ..., G_j^M)^T\}_{j=1}^N$: $q_{j\mathbf{s}} = P(\mathbf{G}_j = \mathbf{c_s})$ where $\mathbf{s} = (s_1, ..., s_M)^T \in \{1,2\}^M$, i.e., $\mathbf{c_s} = (c_{s_1}, ..., c_{s_M})^T \in \{-1,1\}^M$ $(1 \leq j \leq N)$. We define the pairwise correlation matrices of ground image $A_G^{l,j,k} = cov(\mathbf{G}_j, \mathbf{G}_{j-k})$, i.e., $A_G^{l,j,k} = (a_{G,u,v}^{l,j,k})_{M\times M}$, where $a_{G,u,v}^{l,j,k} = cov(G_j^u, G_{j-k}^v)$, for $1 < j \leq N$, $1 \leq k \leq n_j$ and $1 \leq u, v \leq M$. We compute conditional probabilities $P(\mathbf{G}_j = \mathbf{c_s} | \mathbf{G}_{j-1} = \mathbf{m}_{j-1}, \ldots, \mathbf{G}_{j-n_j} = \mathbf{m}_{j-n_j})$ in the following proposition if the covariance matrices and marginal probabilities satisfy aforementioned conditions.

PROPOSITION 2.7. *Suppose that $M, N, l \in \mathbb{N}$, $q_{j\mathbf{s}} = P(\mathbf{G}_j = \mathbf{c_s})$ where $\mathbf{c_s} = (c_{s_1}, ..., c_{s_M})^T \in \{-1,1\}^M$. Suppose that $A_G^{l,j,k} = cov(\mathbf{G}_j, \mathbf{G}_{j-k})$ for $1 < j \leq N$, $1 \leq k \leq n_j$. Then if $\{\mathbf{G}_j\}_{j=1}^N$ is constructed through Proposition 2.5, we have that*

$$\begin{aligned} & P(\mathbf{G}_j = \mathbf{c_s} | \mathbf{G}_{j-1} = \mathbf{m}_{j-1}, \ldots, \mathbf{G}_{j-n_j} = \mathbf{m}_{j-n_j}) \\ = \quad & q_{j\mathbf{s}} + \frac{\sum_{k=1}^{n_j} ((\mathbf{c_s})^T A_G^{l,j,k} \mathbf{m}_{j-k})}{2^{(n_j+1)M} P(\mathbf{G}_{j-1} = \mathbf{m}_{j-1}, \ldots, \mathbf{G}_{j-n_j} = \mathbf{m}_{j-n_j})} \end{aligned} \tag{5}$$

for each $\mathbf{c_s} = (c_{s_1}, ..., c_{s_M})^T \in \{-1,1\}^M$ and $\mathbf{m}_{j-k} \in \{-1,1\}^M$ ($j = 2, ..., N$, $k = 1, ..., n_j$), where $n_j = l \wedge (j-1)$.

3. SIGNAL AND OBSERVATION MODEL

We consider the signal $\mathbb{X}_k$ in our model to be the presence of rocket launcher in the suspicious area, which does not change with time. The signal dynamics are completely deterministic; it is just the initial condition that is random and unknown. In contrast to the signal model in,[8] we do not model the forest as part of the signal since including forest into the signal increases computation dimensionality significantly. Let $\mathbb{X}_k = \theta$, where

$$\theta = \begin{cases} 1, & \text{presence of rocket launcher,} \\ -1, & \text{otherwise.} \end{cases} \tag{6}$$

The particle filter is initialized with N_p particles $\mathbb{X}_0^j, j = 1, \cdots, N_p$. For the forest, although the leaves and grass colors are green, we assume that we can tell the difference of the color between foliage and grass. For simplicity, we say the leaves are black even though they are really just dark green. If there are leaves in the $(i,j)^{th}$ pixel, we observe the black color, otherwise, the color of the rocket launcher or ground. In a battle, the rocket launcher is always trying to hide itself and cover its surface with turf. So the surface of the rocket launcher has random colors, either green or brown. The correlation structures of the ground and rocket launcher are known and slightly different in our setting. The observations are made at a sequence of times $\{t_k, k \geq 0\}$, consisting of a discrete sequence $\mathbf{Y}_k$ of images, each of which is a two-dimensional raster of pixels. Although the camera can take photographs for a broad area, in this proof-of-concept work, we assume that each image is a two-dimensional raster of 60×60 pixels and each pixel on the image represents a ground area of 1.5×1.5 inches. The rocket launcher would be roughly 20×10 feet, and occupies 150×75 pixels. At time k, we observe $\mathbf{Y}_k = (\zeta_k, \zeta_{k-1}, \ldots, \zeta_{k-59})$, and the new observed column is $\zeta_k = (\zeta_k^1, \zeta_k^2, \ldots, \zeta_k^{60})^T$ when compared to $\mathbf{Y}_{k-1}$. The observation for each node ζ_k^i is defined by $Y_k^i = \xi_k^i + \frac{2-\xi_k^i}{2} R_k^i(\mathbb{X}_k)$. If we observe the leaf color black in $(i,k)^{th}$ bit, $\xi_k^i = 2$, otherwise 0, and R_k^i is the random color corresponding to the rocket launcher if the rocket launcher is present, otherwise the ground color. The value of R_k^i will be a random function of whether or not the pixel (i,k) is within the area of the observation domain that a rocket launcher occupies. We preprocess the observations and define the image as the most recent n_k columns with length 60, i.e., $\mathbb{Y}_k = (\zeta_k, \zeta_{k-1}, \ldots, \zeta_{k+1-n_k})$ where $n_k = l \wedge (k-1)$.

3.1 Measure Change

We treat the observations as a multi step Markov chain and assume we know the correlation structures for both ground and rocket launcher. Let the canonical process be $Y_j^i = \xi_j^i + \frac{2-\xi_j^i}{2} G_j^i$ under the fictitious measure Q, which amounts to changing $R_j^i(\mathbb{X}_j)$ into G_j^i, the random color corresponding to the ground. Then the weighting function η_k is,

$$\eta_k = \frac{dQ}{dP}\Big|_k = \prod_{j=0}^{k-1} \frac{P_{\mathbb{G}_{j-1}\to\mathbb{G}_j}}{P_{\mathbb{R}_{j-1}\to\mathbb{R}_j}(\mathbb{X}_j)}, \tag{7}$$

where $\mathbb{G}_j = (\mathbf{G}_j, \mathbf{G}_{j-1}, \ldots, \mathbf{G}_{j+1-n_j})$ and $\mathbb{R}_j = (\mathbf{R}_j, \mathbf{R}_{j-1}, \ldots, \mathbf{R}_{j+1-n_j})$ where $n_j = l \wedge (j-1)$, i.e., $\mathbb{G}_j$ is the most recent n_j columns of observations of the ground color at time j and $\mathbb{R}_j$ is the most recent n_j columns of observations of the rocket launcher color when $\mathbb{X}_j = 1$, otherwise of the ground color.

We need to approximate

$$\mu_k(dx) \doteq \frac{E^Q[1_{\mathbb{X}_k\in dx}\eta_k|\mathscr{F}_k^{\mathbb{Y}}]}{E^Q[\eta_k|\mathscr{F}_k^{\mathbb{Y}}]}, \tag{8}$$

where E^Q denotes the expectation with respect to the reference probability measure Q and the information, $\{\mathscr{F}_k^{\mathbb{Y}} = \sigma(\mathbb{Y}_0, \cdots, \mathbb{Y}_k)\}$ is generated by the observations up to time k. Sampling independent signal particles $\{\mathbb{X}_k^i, k = 1, 2, ...\}_{i=1}^{\infty}$ from the signal distribution. The weights are

$$\eta_k^i = \frac{dQ}{dP} = \prod_{j=0}^{k-1} \frac{P_{\mathbb{G}_{j-1}\to\mathbb{G}_j}}{P_{\mathbb{R}_{j-1}\to\mathbb{R}_j}(\mathbb{X}_j^i)}. \tag{9}$$

By deFinnetti's theorem and the law of large numbers,

$$\frac{1}{N_p}\sum_{i=1}^{N_p} \eta_k^i \delta_{\mathbb{X}_k^i}(dx) \Rightarrow \mu_k(dx), \tag{10}$$

then the approximated conditional distribution of the signal state is calculated via

$$\hat{\theta} = E^P(\mathbb{X}_k|\mathscr{F}_k^{\mathbb{Y}}) = \frac{\int x\mu_k(dx)}{\mu_k(1)} \approx \frac{\frac{1}{N_p}\sum_{i=1}^{N_p}\eta_k^i\mathbb{X}_k^i}{\frac{1}{N_p}\sum_{i=1}^{N_p}\eta_k^i}. \tag{11}$$

3.2 Approximation of Transition Probabilities

By looking at the expression of the weighting function in (7), we need the ratios of transition probabilities in the computation of the weighting function. When there is no target in the area, $\mathbb{G}_j = \mathbb{R}_j$ for all j, and the transition probabilities $P_{\mathbb{G}_{j-1}\to\mathbb{G}_j}$, and $P_{\mathbb{R}_{j-1}\to\mathbb{R}_j}$ are the same and hence the weighting function is just 1. When there is a target underneath the forest, we do not know what the actual colors of the rocket launcher and the ground are for those areas obscured by the foliage. To be able to compute the weighting function, we need an approximation to what is happening underneath the trees. As we know the correlation structures for the rocket launcher and the ground, we can make use of this information and fill in the color of those obscured portions using Proposition 2.5, and then evaluate the state-dependent transition probability $P_{\mathbb{R}_{j-1}\to\mathbb{R}_j}(\mathbb{X}_j)$ and $P_{\mathbb{G}_{j-1}\to\mathbb{G}_j}(\mathbb{X}_j)$ for the chain $\mathbb{R}_j$ and $\mathbb{G}_j$ separately through Proposition 2.7.

4. RESULTS

4.1 Simulation Description

In this simulation, we do not consider extremely dense forest, since the obscuration by foliage corrupts the measurement of the colors of rocket launcher or ground and makes the detection problem very hard. To apply Proposition 2.1 to generate forests, we specify the correlation structure characterized with time-homogeneous marginal probabilities and pairwise correlations between component random variables with a lag $l = 2$. Specifically, in experiment 1, for each pixel, the observation is foliage with a probability 0.5, and a gap with a probability

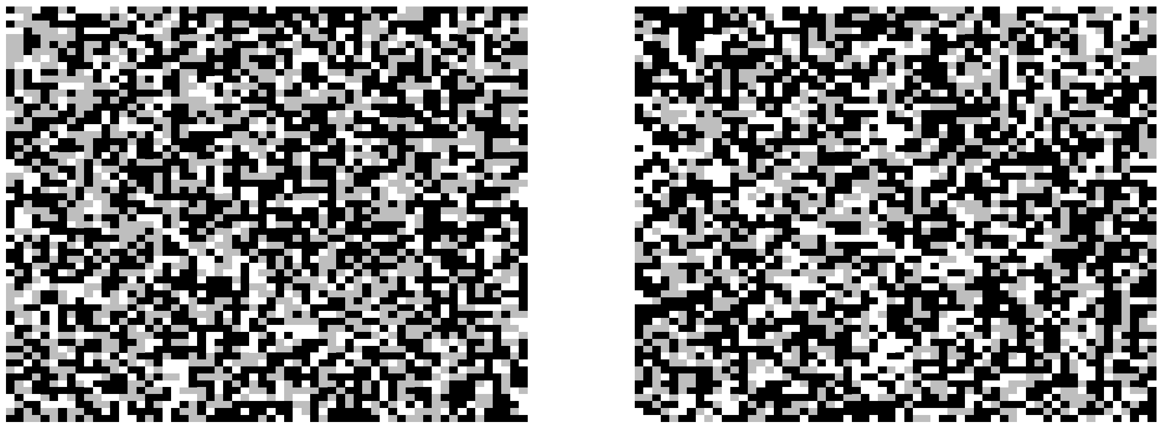

Figure 1. A camouflaged weapon compared with ground (white corresponding to brown color and grey corresponding to green color) under foliage (black); can you tell them apart?

0.5, and the covariances are 0.05 and 0.01 for 1-step and 2-step lags separately. We take the correlation structures of the rocket launcher and ground to be close to each other in terms of marginal probabilities, otherwise, the detection problem becomes trivial since we can easily distinguish the target and ground with our eyes. We choose equal marginal probabilities for the observation colors of the target and the covariances are 0.1 and 0.05 for 1-step and 2-step lags separately. For the ground, the observation is grass with a probability 0.6, and soil with a probability 0.4, and the covariances are 0.04 and 0.01 for 1-step and 2-step lags separately. We draw the pictures of the images with the rocket launcher present and just the ground in figure 1. We are also interested in the performance of our algorithm for the dense forest case. In experiment 2, we set the density of the forest to 0.8 and all other parameters are set as the same as experiment 1.

Next we make the detection problem more difficult by specifying much closer correlation structures between the rocket launcher and the ground. In experiment 3, we choose equal marginal probabilities and the covariance 0.02 and 0.01 for 1-step and 2-step lags separately for the observation colors of the ground. The marginal probabilities of the rocket launcher and ground are exactly the same, but covariances are a little bit more different when compared with experiment 1. Any other parameters are set exactly the same as experiment 1. Next, in experiment 4, we consider the sparse forest. Specifically, for each pixel, the observation is foliage with probability 0.2, and a gap with probability 0.8.

4.2 Filtering Results

We present the detection performance on small areas with either rocket launcher or ground present. The detection probability P is calculated as the number of correct detections divided by the number of trials $N_T = 100$; we use an equal number of areas of rocket launcher present or just ground $N_{RL} = N_G = 50$. We also present the performance in terms of the Type I and Type II errors, i.e., a miss and false alarm in radar terminology. The missing probability P_m is calculated as 1 - the correct detection percentage of rocket launcher. The false alarm P_f is calculated as 1 - the correct detection percentage of the ground. The number of particles initialized are 200.

Exp	$P_{RL=B}, Cov1, Cov2$	$P_{G=B}, Cov1, Cov2$	$P_{Tree=Black}, Cov1, Cov2$	P_m	P_f	P
1	0.5, 0.1, 0.05	0.4, 0.04, 0.01	0.5, 0.05, 0.01	0.04	0.0	0.98
2	0.5, 0.1, 0.05	0.4, 0.04, 0.01	0.8, 0.05, 0.01	0.5	0.0	0.75
3	0.5, 0.1, 0.05	0.5, 0.02, 0.01	0.5, 0.05, 0.01	0.0	1	0.5
4	0.5, 0.1, 0.05	0.5, 0.02, 0.01	0.2, 0.05, 0.01	0.0	0.28	0.86

4.3 Remark

The results coincide with what we expect. In experiment 1, the result is very good. In experiment 2, we consider the dense forests, but other parameters are set exactly the same as those in experiment 1. The performance is worse when compared with the previous experiment and it shows the density of the forest does matter and affect the performance greatly. The large miss, 0.5, can be partly explained by the random weighting functions. When there is a target underneath the forest, we approximate the weighting function by randomly filling in the color of those obscured portions. In the process of estimating the weighting function, we introduce more errors which lead to higher miss, especially when the marginal probabilities of the rocket launcher and the ground are much more different. In experiment 3, we set the marginal probabilities of the rocket launcher and the ground are exactly the same but the covariances are much more different. The outcome shows that the false alarm is 1, i.e., a rocket launcher is frequently detected when none is present. Since the marginal probabilities of the rocket launcher and the ground are the same, the 1-step and 2-step covariances are critical in the detection process. But the lager difference in covariances does not contribute too much to improve the performance in the relatively dense foliage case. In the last experiment, we consider the sparse forests and the performance is good. The false alarm rate 0.28 is relatively high compared to the miss 0. The density of the forest plays an important role, since the lower density allows us to take advantage of the 1-step and 2-step covariances information . Then the improvement of the correct detection probability from experiment 3 to experiment 4 can be explained. We only chose 200 particles in the simulation, the performance could be improved if more particles are used.

5. CONCLUSION

In this paper, we generalize the method of simulating correlated binary variables to correlated vectors. We apply filtering to solve the detection problem in the setting of correlation structures formulated by marginal probabilities and pairwise correlation matrices. We also propose a very novel random weighting technique.

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