Spectrum Sensing, Access, and Leasing in Cognitive Radio Networks

by

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Abstract

Cognitive radio has been considered as a promising way to deal with the overcrowded wireless spectrum. In cognitive radio, when licensed users (primary users) do not use their licensed spectrum, they may lease the spectrum to unlicensed users (secondary user). In a cognitive radio network, a secondary user may target at maximizing its utility, while a primary user may target at maximizing its revenue. In this thesis, the utility maximization of a secondary user and the revenue maximization of a primary user are both investigated.

For a secondary user's utility maximization, we investigate the spectrum sensing and access strategy of the secondary user. The secondary user pays rental fee to the primary user when accessing the licensed channel. In addition, a penalty fee is charge if the secondary user fails to detect primary activities and interferes with primary reception. The setting of the penalty price is discussed. The secondary utility maximization problem is formulated, which selects the optimal spectrum sensing duration and secondary transmission power. The problem is shown to be nonconvex. Some properties of the problem are derived, and accordingly, an iterative algorithm is provided to solve the problem.

For primary user's revenue maximization, long-term spectrum leasing with mul-

tiple rounds is considered, and the target is to find optimal price values over the rounds. Cases with discrete and continuous spectrum demand are investigated. For each case, revenue optimization problems are formulated, and solving methods are also provided. Some interesting properties of the optimal solutions are also presented as well.

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List of Symbols

f_s	sampling frequency
$f_m(y;x)$	probability mass function of accepted demand y with m available channels and given price x
$f_w(y;x)$	probability density function of accepted demand y with w available bandwidth and given price x
$g(y;x)$	probability mass function of demand value y given price x
H_0	hypothesis of absence of primary user
H_1	hypothesis of presence of primary user
I^{ps}	channel gain from primary transmitter to secondary receiver
I^{sp}	channel gain from secondary transmitter to primary receiver
I^{ss}	channel gain from secondary transmitter to secondary receiver
M	number of channels
N	number of stages
N_s	number of samples
P_d	detection probability
P_f	false alarm probability
P^p	primary transmission power
P^s	secondary transmission power
p_a	rental price
p_b	penalty price
$p_{n,m}$	price at Stage n when there are m available channels
$Q(\cdot)$	Q-function
$q_{n,w}$	price at Stage n when there is w available bandwidth

T	total duration of channel sensing and access (Chapter 3) or total leasing duration (Chapter 4)
ω	bandwidth
ξ	detection threshold
τ	sensing duration

List of Abbreviations

Acronyms	Definition
1G	first generation
2G	second generation
3G	third generation
4G	fourth generation
AWGN	additive white Gaussian noise
CDMA	code division multiple access
DTFT	discrete-time Fourier transform
FDMA	frequency division multiple access
i.i.d.	independent and identically distributed
MRC	maximal ratio combining
REC	receiver error count
ROC	receiver operating characteristic
SNR	signal-to-noise ratio
TDMA	time division multiple access
WSS	wide sense stationary

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Chapter 1

Introduction

1.1 Wireless Networks

Wireless communication networks have played more and more significant role and are already integrated in our daily life. Various wireless communication networks have been developed and deployed in the past two decades, including cellular networks [1]–[4], WiFi and ad hoc networks [5]–[8], and wireless sensor network [9]–[14].

Cellular networks have experienced rapid development, and can provide ubiquitous wireless coverage. The 1G (first generation) cellular network used analogue technique and frequency division multiple access (FDMA), to support voice services only [1]. The 2G (second generation) cellular network started to use digital technique and time division multiple access (TDMA). It mainly supported voice service, and very limited data service as well [2]. The 3G (third generation) cellular network adopted code diversion multiple access (CDMA) technique, supporting voice service as well as high-speed multimedia services (such as video service) to satisfy the increasing requirements of wireless customers [3]. The main purpose of the 4G (fourth generation) cellular network is to achieve much higher transmission rate than 3G networks [4].

Cellular networks are able to provide wireless service to a large service area. On the other hand, sometimes we need wireless service over a small area, and a fixed network infrastructure may not exist. For this purpose, WiFi and ad hoc networks have been developed. The advantages of WiFi and ad hoc networks can be summarized as follows: First, due to its flexibility, it can be deployed very quickly. Second, it is less costly than cellular network, since a network infrastructure is not necessary. Third, due to the small distance between transmitters and receivers, very high-speed transmission is possible. Thanks to those advantages, WiFi and ad hoc networks have been popularly deployed in hotspot areas (such as airport, cafe, and conferences) and in scenarios like earthquake rescue (in which cellular network infrastructure may have been destroyed).

Wireless sensor network has been popularly used in applications such as forest fire alarm, military sensing, and data collection in dangerous/hazardous environments. To set up a wireless sensor network, the sensor nodes can be placed manually or by the help of helicopters. Each sensor senses its local environment, and forwards its sensed data to a fusion center (also referred to as a base station). Since the sensors may not be evenly deployed, there may exist some areas that are not covered by sensors. Accordingly, one major research topic in wireless sensor networks is to improve the coverage. For coverage improvement [10], [11], one possible solution is to enhance some sensor nodes with moving capability, and move those mobile sensor nodes to uncovered areas. Sometimes, damages to the sensor nodes (for example, by enemy or animals) or node energy depletion may disconnect the sensor network. Therefore, bi-connectivity [12] has been introduced such that any two sensors in the network have at least two independent paths between them. Therefore, if any node in one path fails, the two sensors can still communicate by using the other path. In general, a bi-connected wireless sensor network can remain connected if any node failure happens. For maintaining the network connectivity, one important issue is to achieve energy efficiency so as to postpone the moment that the network does not function well due to energy depletion of sensor nodes. For this, efficient routing [13] method or effective packet scheduling [14] may be used, to minimize energy consumption of the sensor nodes.

1.2 Cognitive Radio: A Solution to Spectrum Scarcity

Although many research efforts have been conducted in the literature to improve the performance of wireless networks to meet the service requirement, the Global Mobile Data Traffic Forecast by Cisco expected that global mobile data traffic would increase 13-fold from the year of 2012 to the year of 2017. However, we do not have new available wireless spectrum to support the increase, and thus we will experience a spectrum scarcity problem in the near future [15]. Because of this, it has been a challenging research problem to efficiently utilize wireless spectrum. To solve this problem, the concept "Cognitive Radio" has been proposed [16]–[19]. According to the original idea, cognitive radio is a smart radio that can "probe" its environment and can adapt its communication parameters accordingly. In cognitive radio, users that have license to access a licensed spectrum band (referred to as *licensed channel*) are called *primary users*, while those that do not have the license are called *secondary users*. Secondary users are permitted to access the licensed channel if their transmissions do not interfere with primary activities, referred to as *opportunistic channel access*. Secondary users need to pay fees to primary users for their opportunistic channel access.

Generally, there exist two modes for secondary users in their accessing licensed channels: overlay mode and underlay mode [20]. In overlay mode, a secondary user could only use the licensed channels when there is no primary activity. Normally the secondary user will not know in advance whether or not primary activities exist. So before secondary users' channel access, they need to sense the channel, which is widely known as *spectrum sensing* or *channel sensing* [21]. In the overlay node, secondary users' transmission will not bring any interference to primary users; and further, secondary users do not receive interference from primary users. Therefore, the overlay mode is most popularly used in the research society. On the other hand, for underlay mode, a secondary user can access the channels when primary users are transmitting over the channels, but the secondary user needs to manage its transmission power such that its interference at the primary user's receiver side is minimized (or below a threshold level).

In the literature, many researchers have contributed a lot in the area of cognitive radio. Among all the research topics, two important topics are: channel sensing and access of secondary users [22]–[31], and spectrum leasing of primary users [32]–[41].

For channel sensing, the work in [23] analyzes the performance of energy detection based sensing under fading channels. A single-user case is first investigated, in which analytical expressions of average detection probability are given. Then, collaborative detection scenarios are studied, in which square-law selection (SLS) diversity is explored. It is shown that the detector performance is significantly affected by severe fading. When there are more collaborating users or branches, the detection performance can be enhanced. The work in [24] investigates spectrum sensing in a case that the primary signals are wide sense stationary. Spectral feature detection is adopted. Further, since it takes time to perform spectrum sensing, which reduces the channel access time, there are research efforts on sensing-throughput tradeoff. The work in [26] uses a Bayesian decision rule based algorithm to deal with sensing-throughput tradeoff, subject to a constraint on primary throughput. Then, a more general case is studied, in which the number of secondary users in cooperative sensing is limited. The work in [27] investigates sensing-throughput tradeoff when the wireless channels are Rayleigh faded. Since a decision statistic based on receiver error count (REC) does not work well, a new decision statistic with REC and combiner coefficient is provided, which works very well in maximal ratio combining (MRC) scenarios. The receiver operating characteristics are analyzed, which demonstrate the effectiveness of the new decision statistic. The work in [29] considers mobility of primary users. Optimal spectrum sensing efficiency is achieved.

For spectrum leasing, normally two typical modes are used: monopoly market and oligopoly market. In a monopoly market, there is only one single primary user that leases its spectrum. The work in [33] uses a model of a two dimensional power-time-price contract for a monopoly market. The necessary and sufficient condition of the formulated problem is investigated, as well as a feasible solution of the problem. The work in [35] uses Stackelberg game to model price-based power control. The formulated research problem is nonconvex, which is transformed into a convex optimization problem by some mathematical manipulations. Asymptotic analysis is obtained for the number of admitted secondary users with various interference-to-noise ratio levels. On the other hand, in an oligopoly market, there are multiple primary users that sell their licensed channels, called *sellers*. There are also multiple secondary uses that are willing to purchase channel access, called *buyers*. Each seller sets a price. The sellers may have competition or cooperation with each other, to attract more buyers and achieve more revenue. The work in [37] investigates spectrum leasing by using a three pricing model, namely Market-Equilibrium, Competitive, and Cooperative Pricing model. The work in [38] uses an evolutionary game to model buyers' behavior, while using a noncooperative game to model sellers' competition. Nash Equilibrium is analyzed and found. The work in [39] proposes an economic framework and uses it to study dynamic spectrum allocation and service pricing mechanism. Targeting at maximal revenue and spectrum utility, a knapsack based auction model is used. Optimal spectrum utility can be achieved by the proposed model, in which collisions among wireless service providers can be successfully avoided.

1.3 Thesis Motivation and Contributions

Although there are a lot of research efforts in spectrum sensing & access and spectrum leasing in cognitive radio networks, there are still some open research problems, as follows.

For spectrum sensing & access, in the literature, the optimal sensing duration setting has been investigated for single-channel and multiple channel cases. In those research efforts, a secondary user maximizes its average throughput, assuming that it can achieve a certain level of transmission rate even if it miss-detects the primary activities. When a missed detection happens, there is no penalty to the secondary user for the interference generated to primary users. In this thesis, penalty to secondary user is considered. In specific, in Chapter 3, the case of one primary user pair and one secondary user pair with one licensed channel is studied. The penalty price value is investigated, with lower and upper bounds discussed. Then, the channel sensing & access of the secondary user is formulated as an optimization problem. The problem is shown to be nonconvex. And an iterative algorithm is presented to find a solution to the problem.

For spectrum leasing, the works in the literature normally assume that all spectrum buyers have spectrum requests at the same time. However, in reality, since the buyers are independent from each other, it is more likely that they have spectrum requests at different time instants. This means the spectrum leasing of a primary user may take several rounds, and in each round, the spectrum price may have a unique value. In this thesis, the setting of spectrum prices in the multiple rounds is investigated, targeting at maximal revenue of the primary user. In specific, in Chapter 4, under the monopoly spectrum market with one seller and multiple buyers, we consider the dynamic spectrum pricing strategies over multiple rounds with both discrete and continuous spectrum demand cases. In each case, random and deterministic spectrum demand scenarios are investigated. Optimal pricing strategies in different scenarios are presented.

1.4 Thesis Outline

The remainder of this thesis is organized as follows. Background introduction of several spectrum sensing methods is given in Chapter 2. The channel sensing & access of a secondary user with penalty for missed detection is investigated in Chapter 3. In Chapter 4, dynamic pricing strategies are studied for long-term revenue maximization of a primary user. Chapter 5 concludes the thesis and indicates future work directions.

Chapter 2

Background: Spectrum Sensing Techniques

In cognitive radio, spectrum sensing is a core technology. Based on spectrum sensing, secondary users can know about current transmission status of primary users and then determine whether or not to access the channels. Three typical spectrum sensing techniques are energy detection, matched filter, and feature detection. Further, to improve spectrum sensing accuracy, cooperative sensing is a viable solution that needs participation of multiple secondary users [42].

2.1 Energy Detection

In energy detection, prior information about primary users' transmitted signals is not needed. Rather, a secondary user collects energy in the targeted spectrum band and compares with a pre-determined threshold, and considers the spectrum is busy (i.e., primary user is using the spectrum) if the collected energy is larger than the threshold, or considers the spectrum is idle otherwise.

The binary hypothesis problem for spectrum sensing is given as follows:

$$\begin{aligned} H_0: &\quad y(n) = v(n), & n = 1, 2, ..., N_s \\ H_1: &\quad y(n) = s(n) + v(n), & n = 1, 2, ..., N_s \end{aligned} \tag{2.1}$$

where H_0 and H_1 denote that the targeted spectrum band is idle and busy, respectively, n represents sample index, $s(n)$ is the n-th primary signal sample received at the secondary user, $v(n)$ is the background noise between primary and secondary user, assumed to be independent and identically distributed (i.i.d.) additive white Gaussian noise (AWGN) with zero mean and variance σ_v^2, $y(n)$ is summation of n-th signal sample and noise, and N_s is the number of samples. The detection statistic is given [43] as:

$$T(y) = \frac{1}{N_s} \sum_{n=1}^{N_s} |y(n)|^2. \tag{2.2}$$

Denote τ as sensing time that the secondary user spends in sensing the spectrum band and f_s as sampling frequency. Then from [43], the total number of samples can be expressed as $N_s = \tau f_s$. Under hypothesis H_0, the detection statistic follows a Chi-square distribution with N_s degree of freedom. Normally, the number of samples N_s is large enough. So by applying Central Limit Theorem [45], the distribution could be approximated as a Gaussian distribution with mean $\mu_0 = \sigma_v^2$ and variance $\sigma_0^2 = \frac{2}{N_s}\sigma_v^4$. Then the false alarm probability can be derived as:

$$P_f^E(\xi, \tau) = P_r(T(y) \geq \xi | H_0) = Q\left(\left(\frac{\xi}{\sigma_v^2} - 1\right)\sqrt{\frac{\tau f_s}{2}}\right) \tag{2.3}$$

where ξ represents detection threshold, $Pr(\cdot)$ means probability of an event, and $Q(\cdot)$ is Q-function, given as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{z^2}{2}\right) dz. \tag{2.4}$$

Similarly, under hypothesis H_1, denote γ as signal-to-noise ratio (SNR). Then the mean of approximated Gaussian distribution is $\mu_1 = (1+\gamma)\sigma_v^2$ while the variance is $\sigma_1^2 = \frac{2}{N_s}(2\gamma+1)\sigma_v^4$. Then detection probability is given as

$$P_d^E(\xi, \tau) = Pr(T(y) \geq \xi | H_1) = Q\left(\left(\frac{\xi}{\sigma_v^2} - \gamma - 1\right)\sqrt{\frac{\tau f_s}{2(2\gamma+1)}}\right). \tag{2.5}$$

For a target detection probability $\overline{P_d^E}$, based on formula (2.3) and (2.5), the receiver operating characteristic (ROC) expression of false alarm probability of energy detection is given as

$$P_f^E = Q\left(\sqrt{2\gamma+1}Q^{-1}\left(\overline{P_d^E}\right) + \sqrt{\frac{\tau f_s}{2}}\gamma\right). \tag{2.6}$$

2.2 Matched Filter Detection

In a communication system, matched filter can be used to improve SNR of output signals at the receiver side [46]. In spectrum sensing, assuming that secondary users know information of primary users' transmitted signals, matched filter can be employed to correlate the received signal with the known primary signal waveform, to make a detection decision. According to the binary hypothesis problem in (2.1), for a typical matched filter detection, its detection statistic is given as [42]:

$$Y(y) = \frac{1}{N_s} Re\left[\sum_{n=1}^{N_s} y(n)s^*(n)\right] \tag{2.7}$$

in which $s^*(n)$ is the conjugate of $s(n)$.

Similar to Section 2.1, the background noise is i.i.d. AWGN. Then the corresponding false alarm and detection probabilities are given as

$$P_f^M(\xi,\tau) = P_r(Y(y) \geq \xi | H_0) = Q\left(\frac{\xi}{\sigma_v^2}\sqrt{\frac{\tau f_s}{\gamma}}\right) \tag{2.8}$$

and

$$P_d^M(\xi,\tau) = P_r(Y(y) \geq \xi | H_1) = Q\left(\left(\frac{\xi}{\sigma_v^2} - \gamma\right)\sqrt{\frac{\tau f_s}{\gamma}}\right). \tag{2.9}$$

Consequently, for a target detection probability $\overline{P_d^M}$, we can obtain the ROC expression as

$$P_f^M(\tau) = Q\left(Q^{-1}\left(\overline{P_d^M}\right) + \sqrt{\tau f_s \gamma}\right). \tag{2.10}$$

Figure 2.1 shows the ROC curves of energy detection and matched filter de-

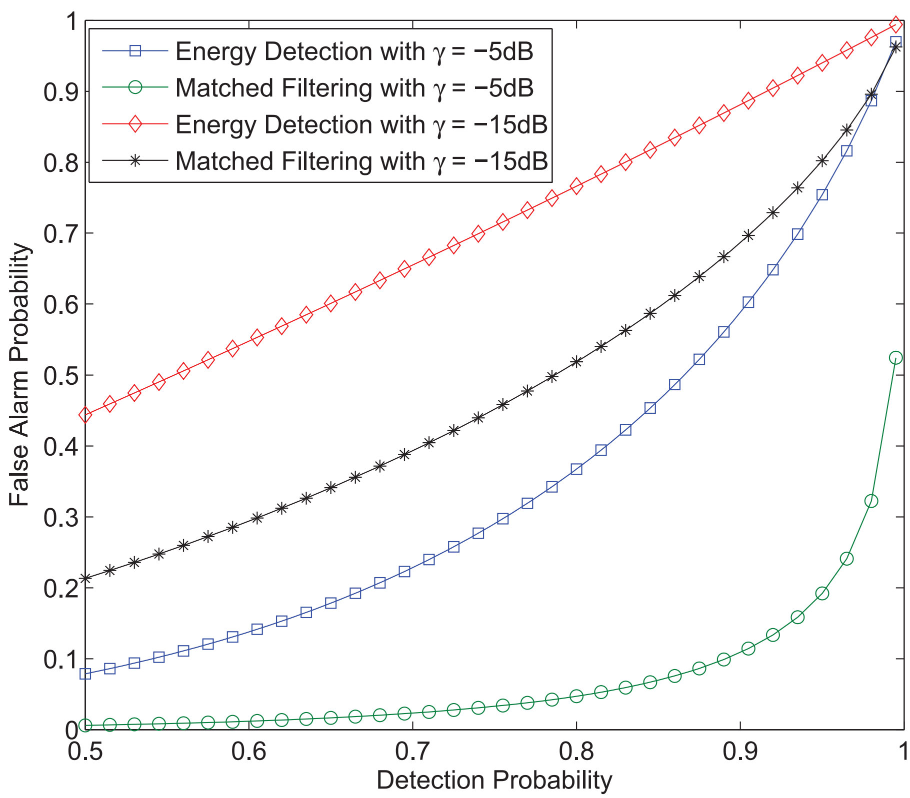

Fig. 2.1. Comparison of matched filter detection and energy detection.

tection. It can be seen that matched filter detection performs better than energy detection. The major drawbacks of matched filter detection are that it needs prior knowledge of transmitted signal waveforms and its complexity is higher.

2.3 Feature Detection

In some cases, the primary signals have some spectral features, for example, cyclic feature. Then feature detection can be an alternative spectrum sensing method. Its basic idea is to decide on whether or not the received signals have the features, to make a detection decision. Normally, this method is employed for detecting signals of wide sense stationary (WSS) such as video signals, television (TV) broadcast signals and so on. And the procedure of feature detection is as follows [24], for a binary hypothesis problem of received WSS signal similar to (2.1):

- Get autocorrelation function of the received signal as

$$r_{yy}(m) = E[y(t)y^*(t-m)].$$

- By using discrete-time Fourier transform (DTFT), calculate the power spectrum density (PSD) of received signal as

$$S_{YY}(\omega) = \sum_{m=-\infty}^{+\infty} r_{yy}(m) \exp(-jm\omega)$$

  where $0 \leq \omega \leq 2\pi$.

- Calculate $\int_0^{2\pi} S_{YY}(\omega)S_{XX}(\omega)d\omega$ (here $S_{XX}(\omega)$ is the PSD of the transmitted signal, the information of which is available at the detector), and decide on H_1 if the value is more than a pre-defined threshold, or decide on H_0 otherwise.

The feature detection needs prior knowledge of primary signals' spectrum features. It is more complex than energy detection. And it does not work well or even fails to work for non-WSS signals.

2.4 Cooperative Spectrum Sensing

To further improve the accuracy of spectrum sensing, cooperative sensing has been introduced in the literature, which makes a decision based on information from a number of secondary users. In specific, each secondary user first gets its own test statistic for primary signals. Then the secondary users send their data to a fusion center by using either of two methods: data fusion mode or decision fusion mode.

In data fusion mode, each secondary user sends the value of its test statistic to the fusion center. Then the fusion center takes a sum or a weighted sum of the collected test statistics and makes a decision on presence or absence of primary signals.

In decision fusion mode, each secondary user makes its own decision on presence or absence of primary signals, and sends the one-bit decision information to the fusion center. The fusion centre makes the final decision based on its fusion rule. Three typical fusion rules are as follows [43]:

- Logic OR Rule: among all the decisions received from secondary users, if one decision is presence of primary signals, then the fusion center decides on presence of primary signals. The fusion center decides on absence of primary signals only when all individual decisions from secondary users are absence of primary signals. Assuming that the individual decisions from the secondary users are independent, the detection and false alarm probabilities of the fusion center are given as

  $$P_{d,OR} = 1 - \prod_{l=1}^{M}(1 - P_d^{(l)})$$

  and

  $$P_{f,OR} = 1 - \prod_{l=1}^{M}(1 - P_f^{(l)})$$

  in which M is the number of individual decisions, $P_d^{(l)}$ and $P_f^{(l)}$ are detection and false alarm probability of the lth decision, respectively.

- Logic AND Rule: In this mode, the fusion center decides on presence of primary signals only when all the individual decisions are presence of primary signals. Then, the detection and false alarm probabilities of the fusion center are

$$P_{d,AND} = \prod_{l=1}^{M} P_d^{(l)}$$

and

$$P_{f,AND} = \prod_{l=1}^{M} P_f^{(l)}.$$

- Majority Rule: In this mode, the fusion center takes the majority decision among all individual decisions received from secondary users. In other words, the fusion center decides on presence of primary signals if $M/2$ or more individual decisions are on presence of primary signals; or decides on absence of primary signals otherwise. By assuming each individual decision has the same detection and false alarm probability denoted as P_d and P_f, respectively, the detection and false alarm probabilities of the fusion center are given as

$$P_{d,Majority} = \sum_{l=M/2}^{M} \binom{M}{l} (P_d)^l (1 - P_d)^{M-l}$$

and

$$P_{f,Majority} = \sum_{l=M/2}^{M} \binom{M}{l} (P_f)^l (1 - P_f)^{M-l}$$

respectively.

Chapter 3

Channel Sensing and Access of Secondary User

In this chapter, the channel sensing and access of a secondary user is investigated. If a secondary user accesses the licensed channel of a primary user, it pays rental fee to the primary user. However, if the secondary user accesses the channel when the primary user is using the channel, a penalty is charged based on interference generated to the primary activity. The bounds for the penalty price are investigated. Further, we investigate optimal sensing duration and transmission power of the secondary user. An alternative algorithm is provided to find a solution.

3.1 System Model

In this research, we consider a monopoly market, with one primary owner (the seller) and one secondary user. The primary spectrum owner is assumed to have a single licensed channel with bandwidth ω while the secondary user only needs one channel. The primary user is assumed to have a fixed network structure, while the secondary network is an ad hoc network. Figure 3.1 provides an example of them.

The secondary user shares the licensed channel in an overlay mode, i.e., it can access the channel as long as the primary user is idle. Therefore, the secondary user first senses the channel, and can transmit its information data if it senses the channel

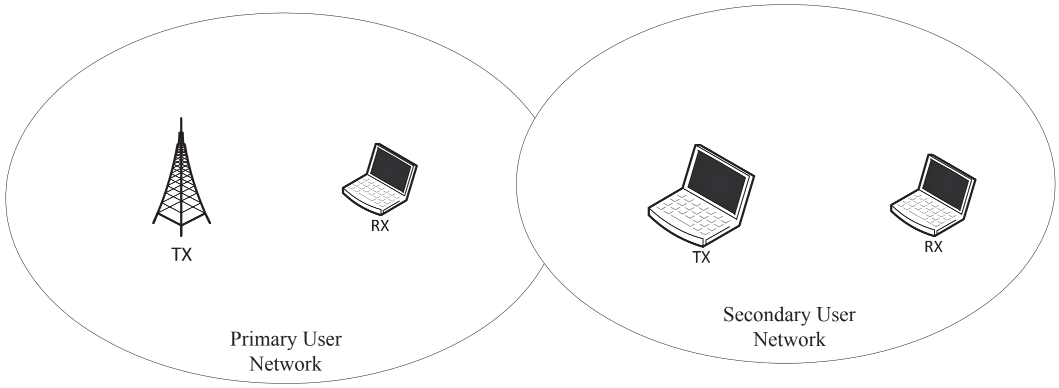

Fig. 3.1. Considered cognitive radio network.

to be free. The secondary user pays some rental fee for this opportunistic channel access. If a missed detection happens, the primary user is using the channel, but the secondary user mistakenly estimates the channel is idle and transmits. A penalty is charged to the secondary user based on the interference level generated to the primary receiver.

Assuming the background noise at the secondary receiver side is AWGN with distribution as $N(0, \sigma_v^2)$. Since it may be hard for the secondary user to know prior knowledge about primary transmitted signals, the secondary user adopts energy detection as its spectrum sensing method, with detection threshold being ξ and sampling frequency being f_s.

For the secondary user, assume it has a total duration of T, which is a constant. Within T, the secondary user needs to perform spectrum sensing for duration τ (a parameter to be determined). If channel is sensed idle, the secondary user transmits in the subsequent duration $T - \tau$. Denote the rental price as p_a per time unit per Hz, and penalty price as p_b per time unit per interference unit. Denote transmission power of the secondary user as P^s (a parameter to be determined). Denote the transmission power of the primary user as P^p. Denote channel gain from secondary transmitter to secondary receiver, from primary transmitter to secondary receiver, and from secondary transmitter to primary receiver as I^{ss}, I^{ps} and I^{sp}, respectively. Here only path loss attenuation is considered. Thus, I^{ss}, I^{ps} and I^{sp} depend only on distances of the corresponding paths, and are known in advance at the secondary user.

3.2 Setting of Price

Consider that the primary user is idle. The following is a model for the secondary user utility [47]. For a well-behaved secondary user (i.e., it successfully detects the availability of the channel), its utility can be given as

$$\begin{aligned} U_1 &= \omega(T-\tau)\left[\log_2\left(1+\tfrac{I^{ss}P^s}{\sigma_v^2}\right)-p_a\right] \\ &= \omega(T-\tau)V_1 \end{aligned} \tag{3.1}$$

in which $V_1 = \log_2\left(1+\frac{I^{ss}P^s}{\sigma_v^2}\right) - p_a$.

On the other hand, if the primary user is using the channel, and a missed detection happens, then the secondary user can still achieve a certain level of transmission rate. A penalty will also be charged. So the utility of the secondary user can be expressed as

$$\begin{aligned} U_2 &= \omega(T-\tau)\left[\log_2\left(1+\tfrac{I^{ss}P^s}{I^{ps}P^p+\sigma_v^2}\right)-p_bI^{sp}P^s\right] \\ &= \omega(T-\tau)V_2 \end{aligned} \tag{3.2}$$

in which $V_2 = \log_2\left(1+\frac{I^{ss}P^s}{I^{ps}P^p+\sigma_v^2}\right) - p_bI^{sp}P^s$.

For a proper rental price p_a, it should make U_1 in (3.1) no smaller than zero, because otherwise the secondary user will not rent the channel. Thus, we have

$$0 \le p_a \le \log_2\left(1+\frac{I^{ss}P^s}{\sigma_v^2}\right). \tag{3.3}$$

For a proper p_b, it should make U_2 in (3.2) a negative value, which means

$$p_b > \frac{\log_2\left(1+\frac{I^{ss}P^s}{I^{ps}P^p+\sigma_v^2}\right)}{I^{sp}P^s}. \tag{3.4}$$

Next we give further bounds of penalty price. Before that, we give the expression of the utility of the secondary user first, as follows.

Denote the idle and busy probabilities of the channel as $P(H_0)$ and $P(H_1)$, respectively. To obtain an expression for the secondary user's utility, similar to [43], [44], the following four scenarios should be taken into consideration.

- The channel is idle and is correctly estimated by the secondary user, the probability of which is given as

$$P(H_0)\left[1-Q\left(\left(\frac{\xi}{\sigma_v^2}-1\right)\sqrt{\frac{\tau f_s}{2}}\right)\right] \tag{3.5}$$

  with utility given in (3.1).

- The channel is idle but the secondary user estimates it as busy (i.e., a false alarm happens), the probability of which is given as

$$P(H_0)Q\left(\left(\frac{\xi}{\sigma_v^2}-1\right)\sqrt{\frac{\tau f_s}{2}}\right) \tag{3.6}$$

  with utility being zero.

- The channel is busy and is successfully detected by the secondary user, the probability of which is given as

$$P(H_1)Q\left(\left(\frac{\xi}{\sigma_v^2}-\gamma-1\right)\sqrt{\frac{\tau f_s}{2(2\gamma+1)}}\right) \tag{3.7}$$

  with utility being zero.

- The channel is busy but the secondary user estimates it as idle (i.e., a missed detection happens), the probability of which is given as

$$P(H_1)\left[1-Q\left(\left(\frac{\xi}{\sigma_v^2}-\gamma-1\right)\sqrt{\frac{\tau f_s}{2(2\gamma+1)}}\right)\right] \tag{3.8}$$

  with utility given in (3.2).

Overall, the utility of the secondary user can be expressed as:

$$\begin{aligned}\pi = \quad & P(H_0)\left[1 - Q\left(\left(\tfrac{\xi}{\sigma_v^2} - 1\right)\sqrt{\tfrac{\tau f_s}{2}}\right)\right] U_1 \\ & + P(H_1)\left[1 - Q\left(\left(\tfrac{\xi}{\sigma_v^2} - \gamma - 1\right)\sqrt{\tfrac{\tau f_s}{2(2\gamma+1)}}\right)\right] U_2.\end{aligned} \tag{3.9}$$

In general the detection threshold ξ should be set such that the detection probability is more than 0.5 and the false alarm probability is less than 0.5, given as

$$P_f(\xi, \tau) = Q\left(\left(\frac{\xi}{\sigma_v^2} - 1\right)\sqrt{\frac{\tau f_s}{2}}\right) < 0.5 \tag{3.10}$$

and

$$P_d(\xi, \tau) = Q\left(\left(\frac{\xi}{\sigma_v^2} - \gamma - 1\right)\sqrt{\frac{\tau f_s}{2(2\gamma + 1)}}\right) > 0.5. \tag{3.11}$$

To satisfy (3.10) and (3.11), ξ should satisfy

$$\sigma_v^2 < \xi < \sigma_v^2(1 + \gamma). \tag{3.12}$$

Lemma 3.1. proper penalty price should satisfy

$$\begin{aligned}&\frac{\log_2\left(1 + \frac{I^{ss}P^s}{I^{ps}P^p + \sigma_v^2}\right)}{I^{sp}P^s} + \frac{P(H_0)V_1}{P(H_1)I^{sp}P^s} < p_b \\ &\qquad < \frac{\log_2\left(1 + \frac{I^{ss}P^s}{I^{ps}P^p + \sigma_v^2}\right)}{I^{sp}P^s} + \frac{P(H_0)[1 - P_f(\xi, T)]V_1}{P(H_1)[1 - P_d(\xi, T)]I^{sp}P^s}.\end{aligned}$$

Proof. The secondary user's utility in (3.9) should be nonnegative, from which we have

$$\begin{aligned} p_b \quad & \leq \frac{\log_2\left(1+\frac{I^{ss}P^s}{I^{ps}P^p+\sigma_v^2}\right)}{I^{sp}P^s} + \frac{P(H_0)[1-P_f(\xi,\tau)]V_1}{P(H_1)[1-P_d(\xi,\tau)]I^{sp}P^s} \\ & < \frac{\log_2\left(1+\frac{I^{ss}P^s}{I^{ps}P^p+\sigma_v^2}\right)}{I^{sp}P^s} + \frac{P(H_0)[1-P_f(\xi,T)]V_1}{P(H_1)[1-P_d(\xi,T)]I^{sp}P^s}\end{aligned} \tag{3.13}$$

in which the second inequality is because when τ increases, $P_f(\xi, \tau)$ decreases and $P_d(\xi, \tau)$ increases.

On the other hand, assume there is a secondary user that does not perform spectrum sensing, but just transmits over the licensed channel. Its utility, demoted π', is given as

$$\pi' = P(H_0)\omega T V_1 + P(H_1)\omega T V_2.$$

Apparently, π' should be less than zero to avoid this kind of secondary user, from which we have

$$p_b > \frac{\log_2\left(1 + \frac{I^{ss}P^s}{I^{ps}P^p + \sigma_v^2}\right)}{I^{sp}P^s} + \frac{P(H_0)V_1}{P(H_1)I^{sp}P^s}.$$

This completes the proof. □

With properly set rental price p_a, penalty price p_b, and detection threshold ξ , we still need to set up sensing time τ and transmission power P^s to get maximal secondary user utility, to be investigated in the next section.

3.3 The Optimization Problem of Setting Sensing Duration and Transmission Power

The sensing period τ and transmission power P^s are continuous variables, to be optimized. The utility maximization problem of the secondary user can be formulated as follows:

Problem 3.1.

$$\begin{aligned} \max_{\tau, P^s} \quad & \pi = P(H_0)(1 - P_f(\tau))U_1 + P(H_1)(1 - P_d(\tau))U_2 \\ s.t. \quad & 0 < \tau \leq T, \\ & 0 < P^s \leq P^s_{\max}. \end{aligned}$$

Here $P^s_{\max}$ is the maximal allowable transmission power of the secondary user.

Problem 3.1 is nonconvex, and thus, cannot be solved by traditional convex optimization methods. Note that the constraints of Problem 3.1 are linear. Next we show that if one of the two variables, i.e., τ and P^s, is fixed, then the problem is convex. This equivalently means that the objective function of Problem 3.1 is concave with respect to one variable if the other variable is fixed, or in other words, the second-order partial derivative of the objective function with respect to each variable is negative.

We first consider $\frac{\partial^2 \pi}{\partial \tau^2}$. From the objective function of Problem 3.1, the expression of $\frac{\partial^2 \pi}{\partial \tau^2}$ should have two terms, one with $\frac{\partial^2\{[1-P_f(\tau)](T-\tau)\}}{\partial \tau^2}$ and the other with $\frac{\partial^2\{[1-P_d(\tau)](T-\tau)\}}{\partial \tau^2}$, given as follows:

$$\begin{aligned}
&\frac{\partial^2\{[1-P_f(\tau)](T-\tau)\}}{\partial \tau^2} \\
&= -P(H_0)\frac{1}{\sqrt{2\pi}}(\frac{f_s}{2})^{0.5}\left(\frac{\xi}{\sigma_v^2}-1\right)\tau^{-1.5}\exp\left(-\frac{1}{2}\left((\frac{\xi}{\sigma_v^2}-1)\sqrt{\frac{\tau f_s}{2}}\right)^2\right) \\
&\quad \times\left(\frac{1}{4}\left[1+\frac{f_s}{2}(\frac{\xi}{\sigma_v^2}-1)^2\tau\right](T-\tau)+\tau\right) \\
&< 0
\end{aligned} \tag{3.14}$$

$$\begin{aligned}
&\frac{\partial^2\{[1-P_d(\tau)](T-\tau)\}}{\partial \tau^2} \\
&= P(H_1)\frac{1}{\sqrt{2\pi}}(\frac{f_s}{2(2\gamma+1)})^{0.5}\left(1+\gamma-\frac{\xi}{\sigma_v^2}\right)\tau^{-1.5}\exp\left(-\frac{1}{2}\left((\frac{\xi}{\sigma_v^2}-\gamma-1)\sqrt{\frac{\tau f_s}{2(2\gamma+1)}}\right)^2\right) \\
&\quad \times\left(\frac{1}{4}\left[1+\frac{f_s}{2(2\gamma+1)}(\frac{\xi}{\sigma_v^2}-\gamma-1)^2\tau\right](T-\tau)+\tau\right) \\
&> 0.
\end{aligned} \tag{3.15}$$

Together with $V_1 > 0$ and $V_2 < 0$, we have $\frac{\partial^2 \pi}{\partial \tau^2} < 0$.

Next we consider $\frac{\partial^2 \pi}{\partial (P^s)^2}$. From the objective function of Problem 3.1, the expression of $\frac{\partial^2 \pi}{\partial (P^s)^2}$ should have two terms, one with $\frac{\partial^2 V_1}{\partial (P^s)^2}$ and the other with $\frac{\partial^2 V_2}{\partial (P^s)^2}$, given as follows:

$$\frac{\partial^2 V_1}{\partial (P^s)^2} = -\frac{|I^{ss}|^2}{\sigma_v^4}\frac{1}{\left(1+\frac{I^{ss}P^s}{\sigma_v^2}\right)^2\ln 2} < 0 \tag{3.16}$$

$$\frac{\partial^2 V_2}{\partial (P^s)^2} = -\frac{|I^{ss}|^2}{(I^{ps}P^p+\sigma_v^2)^2}\frac{1}{\left(1+\frac{I^{ss}P^s}{I^{ps}P^p+\sigma_v^2}\right)^2\ln 2} < 0. \tag{3.17}$$

Therefore, $\frac{\partial^2 \pi}{\partial (P^s)^2} < 0$.

Algorithm 3.1 Iterative Algorithm for Problem 3.1

1: Iteration index $i \leftarrow 1$, $P^{s*} \leftarrow$ an initial point of P^s, $\pi_0 \leftarrow 0$.

2: Fix P^s as P^{s*}, and get optimal value of τ by Lagrange decomposition as

$$\begin{cases} \frac{\partial L(\tau)}{\partial \tau} & = 0 \\ \lambda_L(\tau - T) & = 0. \end{cases}$$

3: Fix τ with the value determined in Step 2, find optimal value of P^s by Lagrange decomposition as

$$\begin{cases} \frac{\partial L(P^s)}{\partial P^s} & = 0. \\ \mu_L(P^s - P^s_{\max}) & = 0. \end{cases}$$

Denote π_i as the optimal objective function. $P^{s*} \leftarrow$ the optimal value of P^s determined above.

4: If $|\pi_i - \pi_{i-1}| \leq \delta$, terminate the algorithm and output the values of τ and P^s; otherwise, $i \leftarrow i + 1$, and go to Step 2.

From the analysis above, we can conclude that the objective function of Problem 3.1 is concave with respected to τ when P^s is fixed, and is also concave with respect to P^s when τ is fixed. In other words, for Problem 3.1, if one variable is fixed, it is a convex problem, and thus can be solved by traditional solving methods. Therefore, we present an iterative algorithm to solve Problem 3.1. We first fix P^s, and find the optimal value of τ that maximizes the objective function by using Lagrange decomposition. Then with the optimal τ, we find the optimal value of P^s. With this P^s, we find optimal τ again. This procedure is repeated until it converges.

When fixing P^s, assume $\lambda_L \geq 0$, the Lagrange function [48] with respected to τ can be derived as

$$L(\tau) = \pi(\tau) - \lambda_L(\tau - T).$$

When fixing τ, assume $\mu_L \geq 0$, the Lagrange function with respected to P^s can be derived as

$$L(P^s) = \pi(P^s) - \mu_L(P^s - P^s_{\max}).$$

Based on these notations, the procedure of the iterative algorithm is illustrated in Algorithm 3.1. In Step 4 of the algorithm, δ is a very small value to indicate the convergence of the procedure.

3.4 Simulation Results

Computer simulation is carried out to verify the theoretical analysis in this chapter. This section consists of two subsections. In the first subsection, the impact of penalty price value as well as the channel idle probability is demonstrated. In the second subsection, the features of the Problem 3.1 and the proposed iterative algorithm are evaluated.

Simulation setup is as follows. The noise at the secondary receiver side is AWGN, which power is 1mW. The channel gains I^{ss}, I^{ps} and I^{sp} are 0.81, 0.49 and 0.64, respectively. The transmission power of the primary transmitter is 20mW. The total duration for channel sensing and access is $T = 20$ms. The detection threshold ξ is set to be 5, which meets the requirements in (3.12). The rental price is $p_a = 2$. The bandwidth of the channel is 1 KHz. The sampling frequency is 3 KHz.

3.4.1 Impact of Penalty Value as well as the Channel Idle Probability

For different values of channel idle probability $P(H_0) = 0.2, 0.5,$ and 0.8 (which represent low channel idle probability, intermediate channel idle probability and high channel idle probability), we vary the penalty price value p_b, and calculate the utility of the secondary user given in (3.9). The results are shown in Figures 3.2–3.4 with different sensing time and transmission power of the secondary user.

From Figures 3.2–3.4, firstly, it could be seen that the secondary user utility decreases when the penalty increases. And the decreasing rate is larger with less sensing time or more secondary transmission power. This is because less sensing time leads to more missed detections, while more secondary transmission power means more interference to primary activities, and more penalty charge as well.

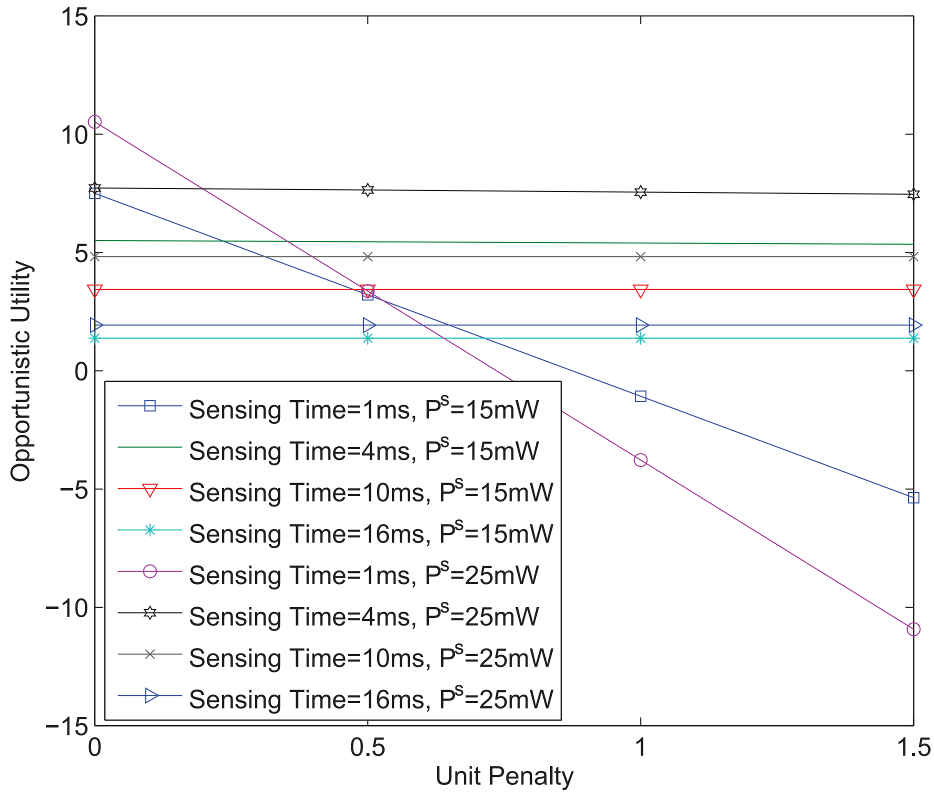

Fig. 3.2. Secondary user utility versus penalty price with $P(H_0) = 0.2$.

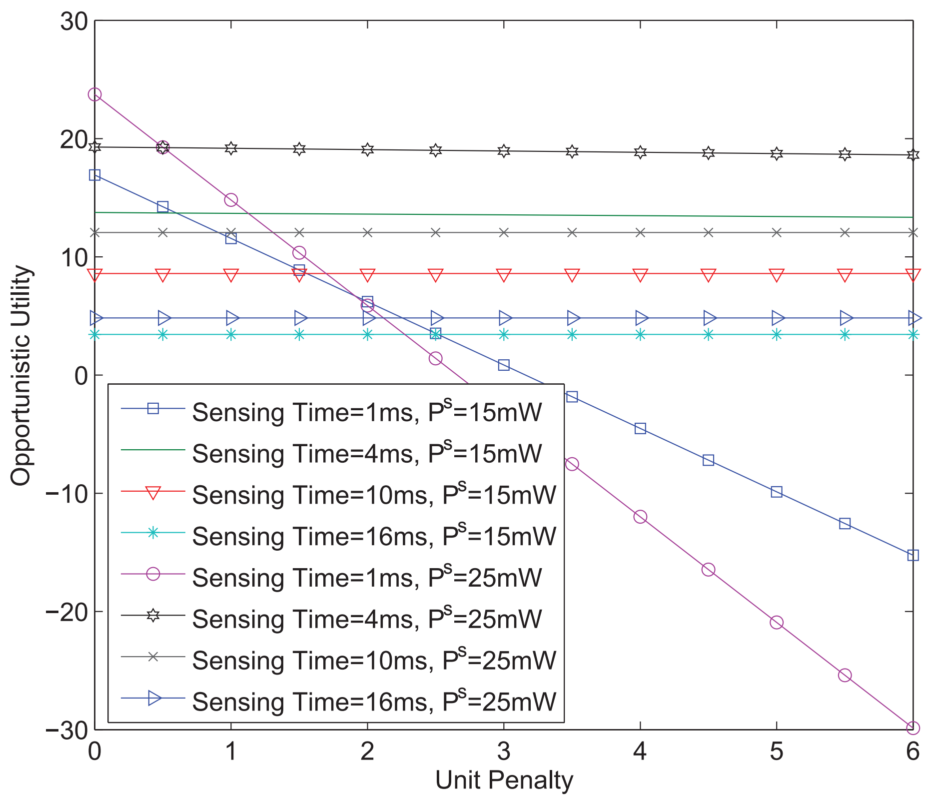

Fig. 3.3. Secondary user utility versus penalty price with $P(H_0) = 0.5$.

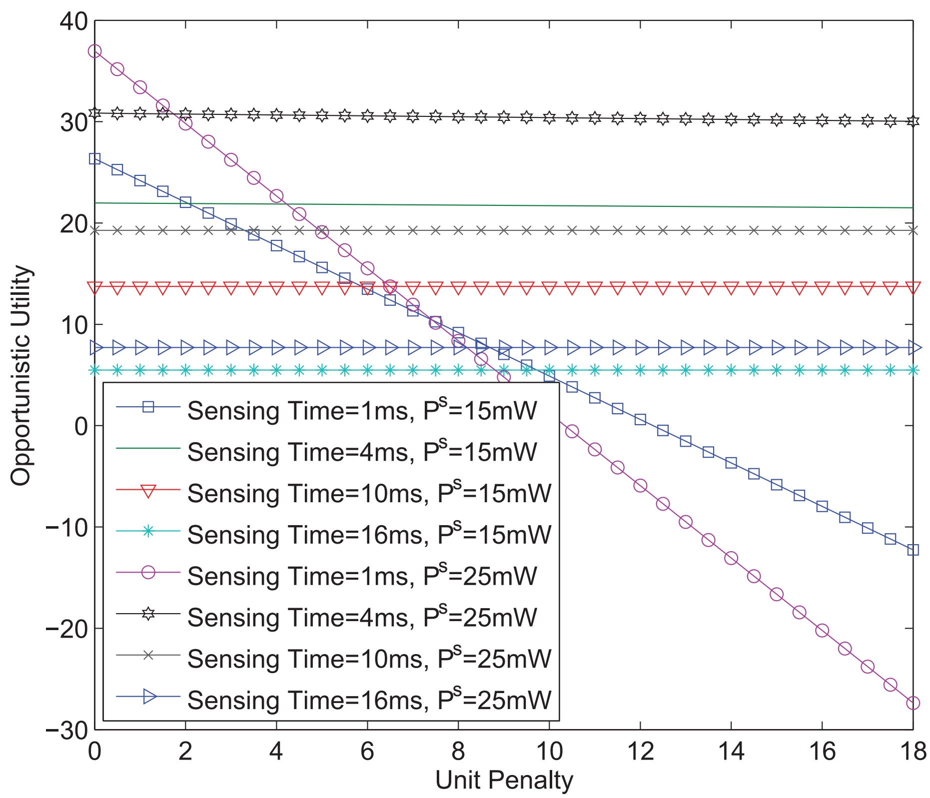

Fig. 3.4. Secondary user utility versus penalty price with $P(H_0) = 0.8$.

With a higher channel idle probability $P(H_0)$, it seems that the secondary user can tolerate more penalty price. This is because higher $P(H_0)$ means less chance to have missed detections.

3.4.2 Feature of Problem 3.1 and the Iterative Algorithm

As discussed before, for Problem 3.1, for the two variables τ and P^s, the objective function is concave with respect to one variable if the other variable is fixed. Here we use simulation to verify this feature. In the simulation setting, the maximal allowable transmission power of the secondary user is 35 mW; the channel idle and busy probabilities $P(H_0)$ and $P(H_1)$ are both 0.5; and the penalty price is 4.5. Among the two variables τ and P^s, we fix one variable and obtain the secondary user utility versus the other variable. The simulation results are shown in Figures 3.5 and 3.6.

From Figure 3.5 (or Figure 3.6), indeed the secondary user utility is a concave function with respected to sensing time (or secondary user transmission power) when secondary user transmission power (or sensing time) is fixed.

For the same simulation setup, we run the proposed iterative algorithm. Figure 3.7 shows how the objective function of Problem 3.1 changes with the iterations. It can be seen that after around 10 iterations, the proposed iterative algorithm converges.

3.5 Conclusion

In this chapter, we have investigated the channel sensing and access strategy of a secondary user to maximize its utility. The ranges of rental price and penalty price are analyzed. Some features of the formulated optimization problem, which optimizes the sensing duration and secondary transmission power, are explored, and accordingly an iterative algorithm is given to find a solution of the problem. This chapter should provide some insights for a secondary user to select its channel sensing and access strategy.

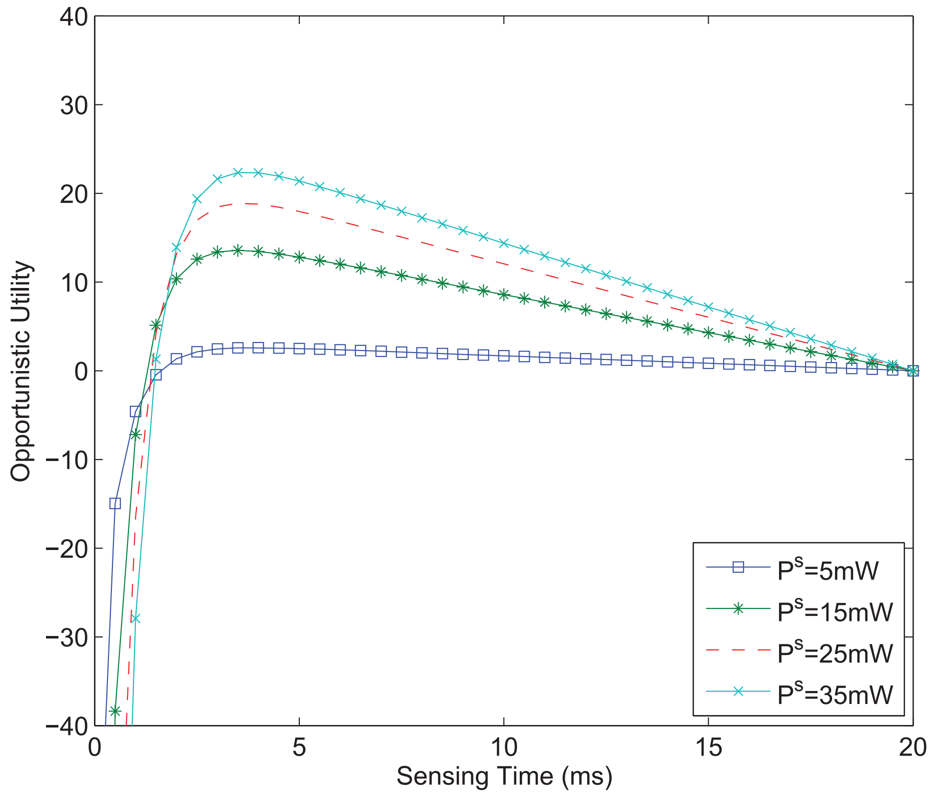

Fig. 3.5. Secondary user utility versus sensing time.

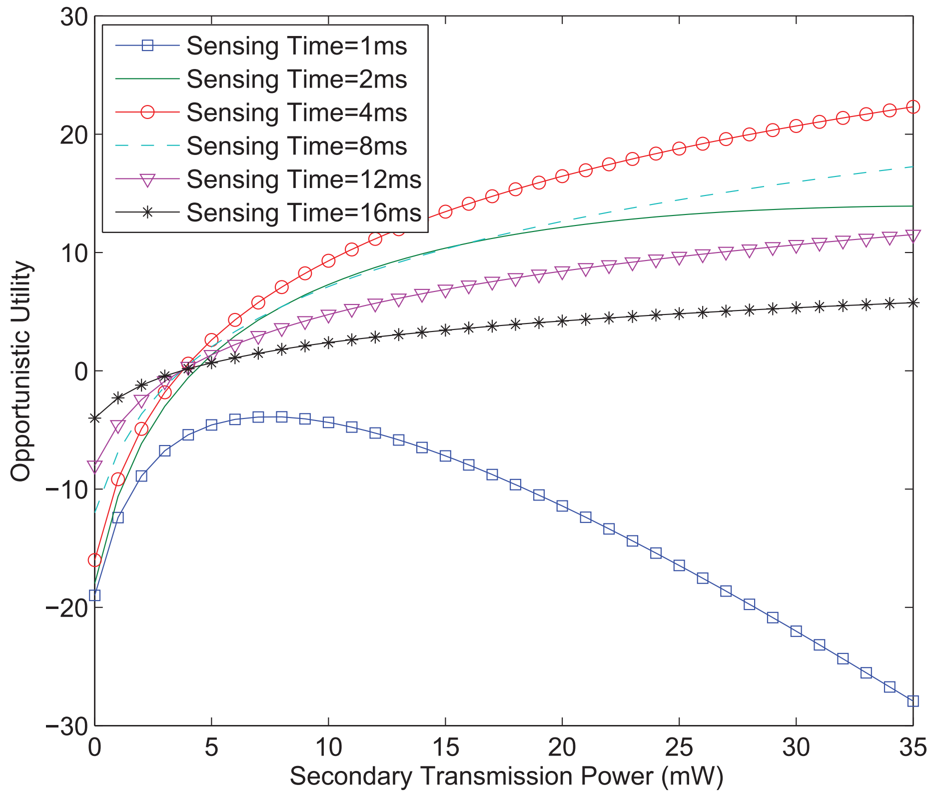

Fig. 3.6. Secondary user utility versus secondary transmission power.

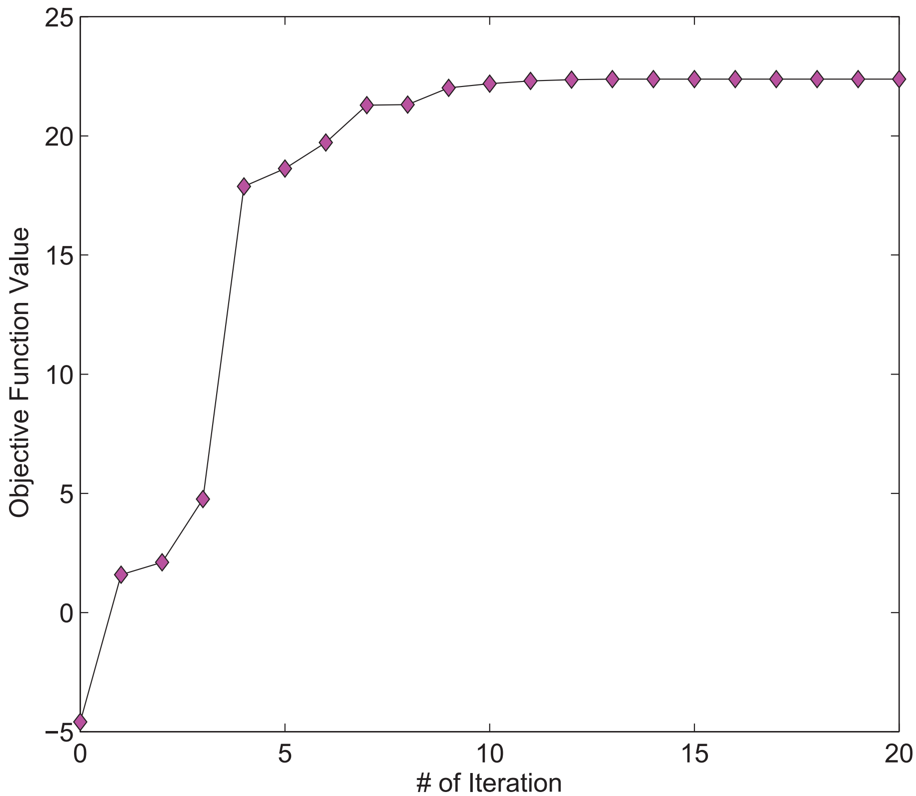

Fig. 3.7. Secondary user utility in the iterations.

Chapter 4

Dynamic Pricing over Multiple Rounds of Spectrum Leasing

In this chapter, the problem of dynamic pricing over multiple rounds of spectrum leasing is investigated. One primary user is considered. If the primary user does not use its licensed spectrum, it leases its licensed spectrum to secondary users. To accommodate different arrival instants of secondary users' spectrum requests, spectrum leasing is performed in multiple rounds, and in each round, a separate spectrum price is set. For cases with discrete or continuous spectrum demand, optimization problems are formulated to set up the spectrum prices in the multiple rounds, with the purpose of maximizing the total revenue of the primary user. The solving methods of the formulated optimization problems are presented. Additionally, some special properties of the optimal solutions are also presented, such as monotonicity and convexity of the maximal total revenue with respect to round index, lower/upper bounds of the maximal total revenue, and monotonicity of the optimal price with respect to round index. Numerical results are provided to verify the research findings.

4.1 Introduction

In spectrum leasing, the spectrum price is the most important design parameter [37], which directly affects the primary users' revenue as well as the willingness of secondary users to lease the spectrum. Spectrum leasing has been investigated in the literature, with two major settings: monopoly spectrum leasing and oligopoly spectrum leasing.

Monopoly spectrum leasing features one single primary user (or broker), targeting at revenue maximization of the primary user (or broker) [49]–[51]. A broker is considered in [49], which first decides on the spectrum amount that will be purchased from primary users, and then sets spectrum leasing price for secondary users to purchase. The research problem, i.e., to maximize the revenue of the broker, is formulated as a Stackelberg game. Authors of [50] take a similar model, but consider that secondary users' spectrum demand is random. In [51], a primary licence holder sets the spectrum price to achieve the optimal balance between the earned revenue and the cost due to extra interference (received from secondary transmissions) and reduced coverage area (by letting secondary users access the spectrum).

Oligopoly spectrum leasing features multiple primary users (or brokers). So the spectrum price is also affected by the competition among primary users (or brokers), and one major research focus in the literature is to achieve equilibrium among primary users (or brokers) [32], [38], [52]–[54]. The work in [52] considers two brokers, and uses a three-stage game. In Stage one, the two brokers purchase spectrum from primary users; In Stage two, the two brokers set and announce their spectrum prices; In Stage three, secondary users decide on their spectrum demand from one broker. The work in [32] also considers two brokers. Each broker has a common spectrum band to be shared by secondary users. So multiple secondary users that lease spectrum from the same broker will generate interference to each other. Potential interference is considered in secondary users' strategies. In [38], multiple primary and secondary users exist. When secondary users make purchase, they are unaware of the spectrum price or spectrum bandwidth that will be allocated.

The purchasing process of secondary users is formulated as an evolutionary game. In [53], there are multiple primary users, one broker and multiple secondary users. The utility function of a primary user reflects both the revenue earned and quality-of-service loss due to leasing some spectrum to secondary users. In [54], multiple primary users compete with each other by price setting, while each secondary user may have a unique criterion on whether or not to lease the spectrum. In all works listed above, Nash equilibrium among primary users (or brokers) is achieved.

All the research efforts mentioned above focus on the static decision making (i.e., the price of a primary user or broker is fixed, and secondary users have spectrum requests at the same time). However, for spectrum leasing in a long term, secondary users may have spectrum demand at different time moments, and thus, the stock of available spectrum should vary with time. In [55], a pricing strategy for dynamic cognitive networks in monopoly spectrum leasing is investigated, considering the arrivals and departures of secondary users. The primary user decides on spectrum price dynamically to maximize the average revenue over an infinite time duration.

In this chapter, we investigate dynamic pricing for monopoly spectrum leasing with a primary user. Since it is unlikely that the primary user always has spectrum to lease (i.e., the primary user may not have spectrum to lease to secondary users if it needs to use the spectrum), we consider spectrum leasing for a finite time duration. The time duration for spectrum leasing is equally divided into multiple stages, and in each stage a spectrum price is set. The spectrum demand in each stage depends on the spectrum price. Our target is to set up the spectrum price values in the stages such that the total revenue of the primary user over all stages is maximized.

The rest of this chapter is organized as follows. The considered system model is presented in Section 4.2. Dynamic pricing in the cases of discrete spectrum demand and continuous spectrum demand are investigated in Section 4.3 and 4.4, respectively. Numerical results are provided in Section 4.5. Finally concluding remarks are given in Section 4.6.

4.2 System Model

Consider one primary user with a licensed spectrum band and multiple secondary users. When the primary user does not need to use the spectrum for a period of time (say duration T), the primary user would like to lease its licensed spectrum to secondary users. As the secondary users may have spectrum access requests at different time instants, it is reasonable for the primary user to perform spectrum leasing once for a while.

For simplicity of presentation, the whole duration T is equally divided into N stages, indexed as Stages $N, N-1, ..., 1$, respectively (in other words, the first stage is called Stage N while the last stage is called Stage 1). At the beginning of each stage, the primary user first announces a price for leasing a spectrum unit with unit time. Then the secondary users who can accept the announced price make a contract with the primary user. Once a part of spectrum is leased to a secondary user, the lease will last until the end of the spectrum leasing duration T. This rule is easier for the primary user to manage its spectrum leasing. If a secondary user leases a portion of spectrum and finishes all its transmissions before the end of duration T, it can rent out its spectrum portion until the end of duration T in a secondary market.

In the following two sections, we consider two cases, when the spectrum demand of secondary users is discrete and continuous, respectively. With discrete spectrum demand, the total licensed spectrum band is equally partitioned into M sub-bands, called M channels. A secondary user can lease an integer number of channels. With continuous spectrum demand, a secondary user can lease a continuous value of spectrum bandwidth portion.

4.3 Dynamic Pricing with Discrete Spectrum Demand

With discrete spectrum demand, the price is the amount of money a secondary user needs to pay for using one channel for one stage duration.

4.3.1 Properties of dynamic pricing

Denote $p_{n,m}$ as the price at Stage n when there are m available channels (i.e., m channels remain un-leased). Here we assume that the price is taken from a finite set of discrete values. For a given price value x, the spectrum demand (i.e., the number of requested channels by secondary users) is a random variable (with integer values), and we denote $g(y;x)$ as the probability mass function of the demand value y (i.e., some secondary users agree on the price and request y channels). However, if the number of requested channels is more than the number of available channels (m), the primary user only accepts totally m channels' requests. Therefore, for a given price value x, the probability mass function of the accepted demand value y when there are m available channels is given by

$$f_m(y;x) = \begin{cases} g(y;x) & \text{if } y < m \\ \sum_{i=m}^{\infty} g(i;x) & \text{if } y = m \\ 0 & \text{if } y > m. \end{cases} \tag{4.1}$$

Denote $V(n,m)$ as the maximum attainable revenue of the primary user from Stage n with m available channels to the last stage (i.e., Stage 1). Then, we have

$$V(n,m) = \max_{p_{n,m}} \sum_{m'=0}^{m} f_m(m';p_{n,m}) \left[p_{n,m} \times n \times m' + V(n-1, m-m')\right] \tag{4.2}$$

in which the term $p_{n,m} \times n \times m'$ represents the revenue the primary user can collect from Stage n until Stage 1 by leasing m' channels to secondary users at Stage n at price $p_{n,m}$; $V(0,m) = 0, \forall m \in \mathcal{M} \triangleq \{1,2,...,M\}$; and $V(n,0) = 0, \forall n \in \mathcal{N} \triangleq \{1,2,...,N\}$.

Using the formula in Eqn. (4.2), $V(n,m)$ can be calculated iteratively from $V(n,0), \forall n \in \mathcal{N}$ and $V(0,m), \forall m \in \mathcal{M}$, by using dynamic programming, and the optimal price $p_{n,m}$ for $n \in \mathcal{N}, m \in \mathcal{M}$, denoted as $p^*_{n,m}$, can be determined accordingly.

Next we present some special properties of the dynamic pricing problem.

Lemma 4.1. The function $V(n,m)$ is an increasing function with respect to n, $\forall m \in \mathcal{M}$.

Proof. We use mathematical induction for proving. The proof consists of two steps.

In the first step, we should prove $V(1,m) - V(0,m) \geq 0, \forall m \in \mathcal{M}$. Since $V(1,m) \geq 0$ and $V(0,m) = 0$, apparently we have $V(1,m) - V(0,m) \geq 0$.

In the second step, we should prove that if $V(n,m) - V(n-1,m) \geq 0, \forall m \in \mathcal{M}$, then $V(n+1,m) - V(n,m) \geq 0, \forall m \in \mathcal{M}$. Suppose $V(n,m) - V(n-1,m) \geq 0, \forall m \in \mathcal{M}$ holds, then based on (4.2), we have

$$\begin{aligned}
& V(n+1,m) - V(n,m) \\
= & \sum_{m'=0}^{m} f_m(m'; p^*_{n+1,m}) \left[p^*_{n+1,m} \times (n+1) \times m' + V(n, m-m')\right] \\
& - \sum_{m'=0}^{m} f_m(m'; p^*_{n,m}) \left[p^*_{n,m} \times n \times m' + V(n-1, m-m')\right] \\
\geq & \sum_{m'=0}^{m} f_m(m'; p^*_{n,m}) \left[p^*_{n,m} \times (n+1) \times m' + V(n, m-m')\right] \\
& - \sum_{m'=0}^{m} f_m(m'; p^*_{n,m}) \left[p^*_{n,m} \times n \times m' + V(n-1, m-m')\right] \\
= & \sum_{m'=0}^{m} f_m(m'; p^*_{n,m}) \left[p^*_{n,m} \times m' + V(n, m-m') - V(n-1, m-m')\right] \\
\geq & \ 0
\end{aligned} \tag{4.3}$$

in which the first inequality is due to the fact that $p^*_{n+1,m}$ is the optimal price at Stage $n+1$ when there are m available channels, and the last inequality comes from $V(n,m) - V(n-1,m) \geq 0, \forall m \in \mathcal{M}$.

This completes the proof. □

Lemma 4.2. The function $V(n,m)$ is an increasing function with respect to m, $\forall n \in \mathcal{N}$.

Proof. We can use mathematical induction for proving, which includes two steps.

In the first step, it is apparent that we have $V(0,m+1) - V(0,m) \geq 0, \forall m \in \{0,1,2,...,M-1\}$.

In the second step, we should prove that if $V(n, m+1) - V(n, m) \geq 0, \forall m \in \{0, 1, 2, ..., M-1\}$, then we have $V(n+1, m+1) - V(n+1, m) \geq 0, \forall m \in \{0, 1, 2, ..., M-1\}$. We have

$$\begin{aligned}
& V(n+1, m+1) - V(n+1, m) \\
= & \sum_{m'=0}^{m+1} f_{m+1}(m'; p^*_{n+1,m+1}) \left[p^*_{n+1,m+1} \times (n+1) \times m' + V(n, m+1-m')\right] \\
& - \sum_{m'=0}^{m} f_m(m'; p^*_{n+1,m}) \left[p^*_{n+1,m} \times (n+1) \times m' + V(n, m-m')\right] \\
\geq & \sum_{m'=0}^{m+1} f_{m+1}(m'; p^*_{n+1,m}) \left[p^*_{n+1,m} \times (n+1) \times m' + V(n, m+1-m')\right] \\
& - \sum_{m'=0}^{m} f_m(m'; p^*_{n+1,m}) \left[p^*_{n+1,m} \times (n+1) \times m' + V(n, m-m')\right] \\
\overset{(a)}{=} & \sum_{m'=0}^{m} g(m'; p^*_{n+1,m}) \left[p^*_{n+1,m} \times (n+1) \times m' + V(n, m+1-m')\right] \\
& + \sum_{m'=m+1}^{\infty} g(m'; p^*_{n+1,m}) \left[p^*_{n+1,m} \times (n+1) \times (m+1) + V(n, 0)\right] \\
& - \sum_{m'=0}^{m} g(m'; p^*_{n+1,m}) \left[p^*_{n+1,m} \times (n+1) \times m' + V(n, m-m')\right] \\
& - \sum_{m'=m+1}^{\infty} g(m'; p^*_{n+1,m}) \left[p^*_{n+1,m} \times (n+1) \times m + V(n, 0)\right] \\
= & \sum_{m'=0}^{m} g(m'; p^*_{n+1,m}) \left[V(n, m+1-m') - V(n, m-m')\right] \\
& + \left[\sum_{m'=m+1}^{\infty} g(m'; p^*_{n+1,m})\right] \times p^*_{n+1,m} \times (n+1) \\
\overset{(b)}{\geq} & \; 0
\end{aligned} \tag{4.4}$$

in which (a) comes from (4.1), and (b) comes from the assumption that $V(n, m+1) - V(n, m) \geq 0, \forall m \in \{0, 1, 2, ..., M-1\}$.

This completes the proof. □

Remark: From Lemma 4.1 and Lemma 4.2, it can be seen that $V(n, m)$ grows with the increase of n and m. In other words, when there is more time or channels left for spectrum leasing, the maximum attainable revenue of the primary user is larger.

Lemma 4.3.

$$nV(1, m) \leq V(n, m) \leq \frac{n(n+1)}{2} V(1, m), \; \forall m \in \mathcal{M}, n \in \mathcal{N}.$$

Proof. First, we prove the left-handside inequality in Lemma 4.3. According to (4.2), we have

$$\begin{aligned}
& V(n,m) \\
= \;& \sum_{m'=0}^{m} f_m(m';p^*_{n,m}) \left[p^*_{n,m} \times n \times m' + V(n-1,m-m')\right] \\
\geq \;& \sum_{m'=0}^{m} f_m(m';p^*_{1,m}) \left[p^*_{1,m} \times n \times m' + V(n-1,m-m')\right] \\
\geq \;& \sum_{m'=0}^{m} f_m(m';p^*_{1,m}) \times p^*_{1,m} \times n \times m' \\
= \;& nV(1,m).
\end{aligned} \tag{4.5}$$

Next we prove the right-handside inequality in Lemma 4.3. Still according to (4.2), we have

$$\begin{aligned}
& V(n,m) \\
= \;& \sum_{m'=0}^{m} f_m(m';p^*_{n,m}) \left[p^*_{n,m} \times n \times m' + V(n-1,m-m')\right] \\
\overset{(c)}{\leq} \;& \sum_{m'=0}^{m} f_m(m';p^*_{n,m}) \left[p^*_{n,m} \times n \times m' + V(n-1,m)\right] \\
\overset{(d)}{=} \;& \left[\sum_{m'=0}^{m} f_m(m';p^*_{n,m}) \times p^*_{n,m} \times n \times m'\right] + V(n-1,m) \\
\overset{(e)}{\leq} \;& nV(1,m) + V(n-1,m)
\end{aligned} \tag{4.6}$$

in which (c) follows from Lemma 4.2, (d) comes from the fact $\sum_{m'=0}^{m} f_m(m';p^*_{n,m}) = 1$, and (e) is due to $V(1,m) = \max_{p_{1,m}} \sum_{m'=0}^{m} f_m(m';p_{1,m}) \times p_{1,m} \times 1 \times m'$. Accordingly,

$$\begin{aligned}
V(n,m) &= V(1,m) + \sum_{n'=2}^{n} \big(V(n',m) - V(n'-1,m)\big) \\
&\leq V(1,m) + \sum_{n'=2}^{n} n'V(1,m) \\
&= \tfrac{n(n+1)}{2} V(1,m)
\end{aligned} \tag{4.7}$$

where the inequality comes from (4.6).

This completes the proof. □

Remark: Lemma 4.3 shows that $V(n,m)$ grows with n superlinearly, but less than degree-2 polynomial. Specifically, when $n = N$ and $m = M$, the inequality in Lemma 4.3 becomes $NV(1,M) \leq V(N,M) \leq \frac{(N+1)}{2} \times NV(1,M)$. In this inequality, the term $NV(1,M)$ is the maximal attainable revenue over the whole spectrum leasing duration T by only performing spectrum leasing once (at the beginning of the spectrum leasing duration). The inequality $NV(1,M) \leq V(N,M)$ shows that the maximal attainable revenue by only performing spectrum leasing once for the whole spectrum leasing duration is less than the maximal attainable revenue by performing dynamic pricing over the N stages ($V(N,M)$). In other words, the way of dynamic pricing over multiple rounds of spectrum leasing can help to raise the revenue of the primary user. On the other hand, the inequality $V(N,M) \leq \frac{(N+1)}{2} \times NV(1,M)$ shows that the ratio of $V(N,M)$ to $NV(1,M)$, which represents the benefit of dynamic pricing, is upper bounded by $\frac{N+1}{2}$.

Lemma 4.4.

$$V(n,m) - V(n-1,m) \leq V(n+1,m) - V(n,m), \forall m \in \mathcal{M}, n \in \mathcal{N} \backslash \{N\}.$$

Proof. We use mathematical induction for proving. The proof consists of two steps. In the first step, it should be proved that the lemma holds for $n = 1$, i.e.,

$$V(1,m) - V(0,m) \leq V(2,m) - V(1,m), \forall m \in \mathcal{M}. \tag{4.8}$$

From Lemma 4.3, we have $2V(1,m) \leq V(2,m), \forall m \in \mathcal{M}$. Together with the fact that $V(0,m) = 0$, it can be seen that inequality (4.8) holds.

In the second step, we need to prove that

$$V(n,m) - V(n-1,m) \leq V(n+1,m) - V(n,m), \forall m \in \mathcal{M} \tag{4.9}$$

holds if

$$V(n-1,m) - V(n-2,m) \leq V(n,m) - V(n-1,m), \forall m \in \mathcal{M}. \tag{4.10}$$

Note that

$$
\begin{aligned}
& V(n-1,m) - V(n-2,m) \leq V(n,m) - V(n-1,m),\ \forall m \in \mathcal{M} && \text{(4.11a)}\\
\Leftrightarrow\quad & 2V(n-1,m) \leq V(n,m) + V(n-2,m),\ \forall m \in \mathcal{M} && \text{(4.11b)}\\
\Rightarrow\quad & \sum_{m'=0}^{m} f_m(m';p^*_{n,m})\,[2V(n-1,m-m')] \\
& \leq \sum_{m'=0}^{m} f_m(m';p^*_{n,m})\,[V(n,m-m') + V(n-2,m-m')] \\
\Rightarrow\quad & \sum_{m'=0}^{m} f_m(m';p^*_{n,m})\,\left[2p^*_{n,m} \times n \times m' + 2V(n-1,m-m')\right] \\
& \leq \sum_{m'=0}^{m} f_m(m';p^*_{n,m})\left[2p^*_{n,m} \times n \times m' + V(n,m-m') + V(n-2,m-m')\right] && \text{(4.11c)}\\
\Leftrightarrow\quad & \sum_{m'=0}^{m} f_m(m';p^*_{n,m})\,\left[2p^*_{n,m} \times n \times m' + 2V(n-1,m-m')\right] \\
& \leq \Big(\sum_{m'=0}^{m} f_m(m';p^*_{n,m})\,\left[p^*_{n,m} \times (n+1) \times m' + V(n,m-m')\right] \\
& + \sum_{m'=0}^{m} f_m(m';p^*_{n,m})\,\left[p^*_{n,m} \times (n-1) \times m' + V(n-2,m-m')\right] \Big) && \text{(4.11d)}\\
\Rightarrow\quad & 2\sum_{m'=0}^{m} f_m(m';p^*_{n,m})\,\left[p^*_{n,m} \times n \times m' + V(n-1,m-m')\right] \\
& \leq \Big(\sum_{m'=0}^{m} f_m(m';p^*_{n+1,m})\,\left[p^*_{n+1,m} \times (n+1) \times m' + V(n,m-m')\right] \\
& + \sum_{m'=0}^{m} f_m(m';p^*_{n-1,m})\,\left[p^*_{n-1,m} \times (n-1) \times m' + V(n-2,m-m')\right] \Big) && \text{(4.11e)}\\
\Leftrightarrow\quad & 2V(n,m) \leq V(n+1,m) + V(n-1,m) && \text{(4.11f)}\\
\Leftrightarrow\quad & V(n,m) - V(n-1,m) \leq V(n+1,m) - V(n,m) && \text{(4.11g)}
\end{aligned}
$$

in which the inequality in (4.11e) holds as

$$
p^*_{n+1,m} = \arg\max_{p} \left(\sum_{m'=0}^{m} f_m(m';p)\,[p \times (n+1) \times m' + V(n,m-m')] \right)
$$

and

$$
p^*_{n-1,m} = \arg\max_{p} \left(\sum_{m'=0}^{m} f_m(m';p)\,[p \times (n-1) \times m' + V(n-2,m-m')] \right)
$$

according to (4.2), and the inequality in (4.11f) holds by following the definition in (4.2).

This completes the proof. □

Remark: Lemma 4.1 and Lemma 4.4 show that $V(n,m)$ is an increasing convex-shaped function with respect to n.

4.3.2 Discussion when Spectrum Demand is Non-random

In this subsection, we investigate the special case when the spectrum demand is non-random and can be solely determined by the price.

For price p, the spectrum demand d (i.e., the number of requested channels) can be expressed as $d = D(p)$.[1] As $d \in \mathcal{I}$ where $\mathcal{I}$ means the set of non-negative integers, $D(p)$ is a piecewise function mapping intervals of price into non-negative integers. In Stage n, denote p_n as the price, and $d_n = D(p_n)$ as the demand for the number of channels. Without loss of generality, it is assumed that the demand d_n is not more than the number of remaining available channels at Stage n.[2] This means d_n is also the accepted demand at Stage n. The achieved revenue of the leased channels in Stage n can be expressed as $p_n d_n n$. So the total revenue over all the stages is $\sum_{n=1}^{N} p_n d_n n$, i.e., $\sum_{n=1}^{N} p_n D(p_n) n$. Then the revenue maximization problem can be formulated as

<u>Problem</u> 4.1.

$$\begin{aligned} \max_{p_1,p_2,\dots,p_N} \quad & \sum_{n=1}^{N} p_n D(p_n) n \\ s.t. \quad & \sum_{n=1}^{N} D(p_n) \le M, \\ & p_n \ge 0, n \in \mathcal{N}, \\ & D(p_n) \in \mathcal{I}, n \in \mathcal{N}. \end{aligned}$$

For the ease of analysing, by defining $P(d) = \max_{D(p)=d} p$,[3] Problem 4.1 can be reformulated as

[1] Here we assume that, for any demand value $d \in \{0, 1, 2, ..., M\}$, there exists at least a price p such that $D(p) = d$. It is also assume that $D(\cdot)$ is a decreasing function.

[2] This assumption is reasonable because, if d_n is more than the number of remaining available channels, the revenue of the primary user can always be increased by increasing the price p_n such that d_n is equal to the number of remaining available channels.

[3] The conceptional meaning of $P(\cdot)$ is the inverse function of $D(\cdot)$. Since $D(\cdot)$ is a piecewise function, its inverse function does not exist mathematically. Therefore, $P(d)$ is defined as the maximal price such that $D(P(d)) = d$, rather than $D^{-1}(d)$.

Problem 4.2.

$$\begin{aligned} \max_{d_1,d_2,\ldots,d_N} \quad & \sum_{n=1}^{N} d_n P(d_n) n \\ s.t. \quad & \sum_{n=1}^{N} d_n \le M, \\ & d_n \ge 0, d_n \in \mathcal{I}, n \in \mathcal{N}. \end{aligned}$$

For the price function with respect to demand, $P(d)$, three characteristics are assumed and justified in the following.

- $P(d)$ is a decreasing function with respect to demand d. This assumption is in concordance with the fact that when the announced price is higher, the spectrum for leasing is less attractive to secondary users, and there is less demand.

- $d \cdot P(d)$ is an increasing function with respect to demand d. This assumption is reasonable as the total revenue of the primary user should be more if more channels are leased.

- $d \cdot P(d)$ is "concave", which means

$$[(d+1)P(d+1) - dP(d)] \le [dP(d) - (d-1)P(d-1)], \forall d > 0, d \in \mathcal{I}.$$

  This assumption conforms to the *law of diminishing returns* [56] in economics: the increase of revenue slows down as the sale volume grows.

For Problem 4.2, the following lemma is in order.

Lemma 4.5. The maximal value of the objective function $\sum_{n=1}^{N} d_n P(d_n) n$ is achieved when $\sum_{n=1}^{N} d_n = M$.

Proof. We use proof by contradiction. According to the second assumption on $P(d)$, the objective function $\sum_{n=1}^{N} d_n P(d_n) n$ is an increasing function with respect to $d_n, n \in \mathcal{N}$. Define the optimal d_n as $d_n^*, n \in \mathcal{N}$. Suppose $\sum_{n=1}^{N} d_n^* = M' < M$, then the objective function in Problem 4.2 can be further increased by increasing d_1^*

to $d_1^* + M - M'$, which contradicts the assumption that $d_n^*, n \in \mathcal{N}$ is the optimal solution.

This completes the proof. $\square$

After substituting the constraint $\sum_{n=1}^{N} d_n \leq M$ with $\sum_{n=1}^{N} d_n = M$, Problem 4.2 has the following features: the objective function is separable and "concave", and the constraints are linear with variable coefficients all being 1's. Thus, an incremental algorithm [57] can be used to solve Problem 4.2.

The procedure of the incremental algorithm for Problem 4.2, referred to as Algorithm 4.1, is as follows.

Algorithm 4.1 Incremental Algorithm for Problem 4.2.

1: Set $d_n = 0, n \in \mathcal{N}$.
2: If $\sum_{n=1}^{N} d_n < M$, find $n^* = \arg\max_{n \in \mathcal{N}} \big((d_n + 1)P(d_n + 1)n - d_n P(d_n)n\big)$, and proceed to Step 3; Otherwise, proceed to Step 4.
3: $d_{n^*} = d_{n^*} + 1$, proceed to Step 2.
4: Output $\{d_n, n \in \mathcal{N}\}$.

Based on the procedure of Algorithm 4.1, the following lemma can be proved.

Lemma 4.6. d_n^* increases when n increases, and p_n^* decreases when n increases, where d_n^* and p_n^* are optimal d_n and p_n for Problem 4.2.[4]

Proof. To prove that d_n^* increases when n increases, we use proof by contradiction.

Suppose there exist $n_1, n_2 \in \mathcal{N}$ such that $n_1 > n_2$ and $d_{n_1}^* < d_{n_2}^*$. According to Algorithm 4.1, there are M rounds of search. In each round, the $n^* = \arg\max_{n \in \mathcal{N}} \big((d_n + 1)P(d_n + 1)n - d_n P(d_n)n\big)$ is found. Before the first round, $d_{n_1} = d_{n_2} = 0$, while after the last round, $d_{n_1}^* < d_{n_2}^*$. Then there should exist a round such that: before the round we have $d_{n_1} = d_{n_2}$, while after the round, d_{n_2}

[4] At Stage n, the optimal demand d_n^* is actually the demand corresponding to the optimal price p_n^*.

is increased by 1. In other words, before this round we have

$$[(d_{n_1}+1)P(d_{n_1}+1) - d_{n_1}P(d_{n_1})]\, n_1 \le [(d_{n_2}+1)P(d_{n_2}+1) - d_{n_2}P(d_{n_2})]\, n_2. \tag{4.12}$$

Since we have $d_{n_1} = d_{n_2}$ before this round, the inequality in (4.12) is equivalent to $n_1 \le n_2$, which contradicts the assumption that $n_1 > n_2$. So we have $d^*_{n_1} \ge d^*_{n_2}$, $\forall n_1 > n_2$, i.e., d^*_n increases with respect to n.

As $p^*_n = P(d^*_n)$, the function $P(\cdot)$ is a decreasing function, and d^*_n increases when n increases, it is easy to conclude that p^*_n decreases when n increases.

This completes the proof. □

Remark: According to Lemma 4.6, as the time approaches the end of the spectrum leasing duration T, the primary user should set the price higher, while in early stages, the primary user should set lower prices to attract more spectrum demand. This conclusion shares some similarity with pricing strategy in flight ticket booking: long before the flight departure date, the flight ticket price is low which can attract more bookings, while as the flight departure date is approaching, the flight ticket price goes higher.

4.4 Dynamic Pricing with Continuous Spectrum Demand

In this section, we assume that spectrum demand (i.e., the spectrum bandwidth requested) is a continuous variable.

4.4.1 Properties of Dynamic Pricing

Suppose W is the bandwidth of one channel as defined in Section 4.3. For a given price x, the spectrum demand is a random variable. When the available spectrum bandwidth amount is w at a stage, for spectrum price x, we denote the probability density function of the accepted spectrum demand y as $f_w(y; x)$, where $y \in [0, w]$.

Here the price means the fee to use one spectrum bandwidth unit for one stage duration.

Denote $q_{n,w}$ as the price at Stage n when the available spectrum bandwidth amount is w, and $Z(n, w)$ as the maximum attainable revenue of the primary user from Stage n to the last stage (Stage 1). Then the following formulation can be given:

$$Z(n, w) = \max_{q_{n,w}} \int_{w'=0}^{w} f_w(w'; q_{n,w}) \left[q_{n,w} \times n \times w' + Z(n-1, w-w')\right] dw' \tag{4.13}$$

in which $Z(n, w) \geq 0, \forall n \in \mathcal{N}, w \in [0, MW]$; $Z(0, w) = 0, \forall w \in [0, MW]$; and $Z(n, 0) = 0, \forall n \in \mathcal{N}$.

The optimal prices $q^*_{n,w}$ can be obtained by dynamic programming. Similar to Section 4.3.1, we have the following properties of the dynamic pricing:

- The function $Z(n, w)$ is an increasing function with respect to n, $\forall w \in [0, MW]$;
- The function $Z(n, w)$ is an increasing function with respect to w, $\forall n \in \mathcal{N}$;
- $nZ(1, w) \leq Z(n, w) \leq \frac{n(n+1)}{2} Z(1, w), \forall w \in [0, MW], n \in \mathcal{N}$;
- $Z(n, w) - Z(n-1, w) \leq Z(n+1, w) - Z(n, w)$, $\forall w \in [0, MW], n \in \mathcal{N} \backslash \{N\}$.

The proofs are similar to those in Section 4.3.1, and are omitted.

4.4.2 Discussion when Spectrum Demand is Non-random

In this subsection, we investigate the special case when the spectrum demand is non-random and can be solely determined by the spectrum price. Since the spectrum demand can take continuous values from $[0, MW]$, the spectrum price can also take continuous values.

Similar to Section 4.3.2, denote the price function for a continuous demand r as $q = P_c(r)$, and we have three similar assumptions on $P_c(\cdot)$:

- $P_c(r)$ is a decreasing function with respect to r;
- $r \cdot P_c(r)$ is an increasing function with respect to r;
- $r \cdot P_c(r)$ is a concave function with respect to r.

Denote q_n and r_n as the price and demand for spectrum in Stage n, respectively. The revenue maximization problem with non-random and continuous demand can be formulated as follow.

Problem 4.3.

$$\begin{aligned} \max_{r_1, r_2, \ldots, r_N} \quad & \sum_{n=1}^{N} r_n P_c(r_n) n \\ s.t. \quad & \sum_{n=1}^{N} r_n \leq MW, \\ & r_n \geq 0, n \in \mathcal{N}. \end{aligned}$$

Problem 4.3 is a convex problem and can be solved by traditional convex optimization techniques.

Similar to Lemma 4.6, the following lemma shows a property of the optimal solution of Problem 4.3.

Lemma 4.7. Demand r_n^* increases with respect to n, and price q_n^* decreases with respect to n, where r_n^* and q_n^* are optimal r_n and q_n for Problem 4.3.

4.5 Numerical Results

In this section, numerical results are given to verify the properties in Section 4.3.

4.5.1 Verification of Lemmas 4.1 – 4.4

In this subsection, Lemmas 4.1–4.4 are numerically verified. In the numerical example, N is set as 10 and M is set as 50. Define $g(y;x)$ as a discrete uniform distribution over $\{m_0, m_0+1, ..., m_0+4\}$ where $m_0 \triangleq \lfloor \frac{1}{x^2} \rfloor$ (here $\lfloor \cdot \rfloor$ is the floor function), i.e., $g(y;x) = \{0.2, 0.2, 0.2, 0.2, 0.2\}$ for $y = \{m_0, m_0+1, m_0+2, m_0+3, m_0+4\}$. The spectrum price in a stage is selected from a discrete set of 100 values that are evenly spaced between 0.1474 and 1.001, where the minimum selectable price 0.1474 makes $m_0 + 4 = 50$, and the maximum selectable price 1.001 makes $m_0 = 0$.

Figures 4.1–4.3 show $V(n, m)$ and its lower and upper bounds, given respectively as $nV(1, m)$ and $\frac{n(n+1)}{2}V(1, m)$, when n grows from 1 to N with $m = 10$, $m = 20$ and $m = 30$. It can be seen that $V(n, m)$ increases when n or m increases, as indicated in Lemma 4.1 and Lemma 4.2. In addition, $V(n, m)$ lies between the lower bound and upper bound, which is consistent with the conclusion in Lemma 4.3. And $V(n, m)$ is convex-shaped with n, as indicated by Lemma 4.4.

4.5.2 Verification of Lemma 4.6

For discrete and non-random spectrum demand, N is set as 10, the function $P(d)$ is set to be $\frac{1}{\sqrt{d}}$, which satisfies the three assumptions on $P(d)$ in Section 4.3.2. In Figures 4.4–4.5, the optimal price p_n^* and optimal demand d_n^* are plotted, when $M = 100$, $M = 200$ or $M = 400$. It can be seen that the optimal price p_n^* decreases with n, while the optimal demand d_n^* grows with n. This result matches Lemma 4.6.

4.6 Conclusions

In this chapter, we have investigated the problem of dynamic pricing over multiple rounds of spectrum leasing. With discrete and continuous spectrum demand, we have formulated optimization problems that find the optimal price in each round so as to maximize the total revenue of the primary user. We have presented the solving methods for the optimization problems, as well as properties of the optimal solutions, such as monotonicity and convexity of the maximal total revenue with respect to stage index n, lower/upper bounds of the maximal total revenue, and monotonicity of the optimal price with respect to stage index n.

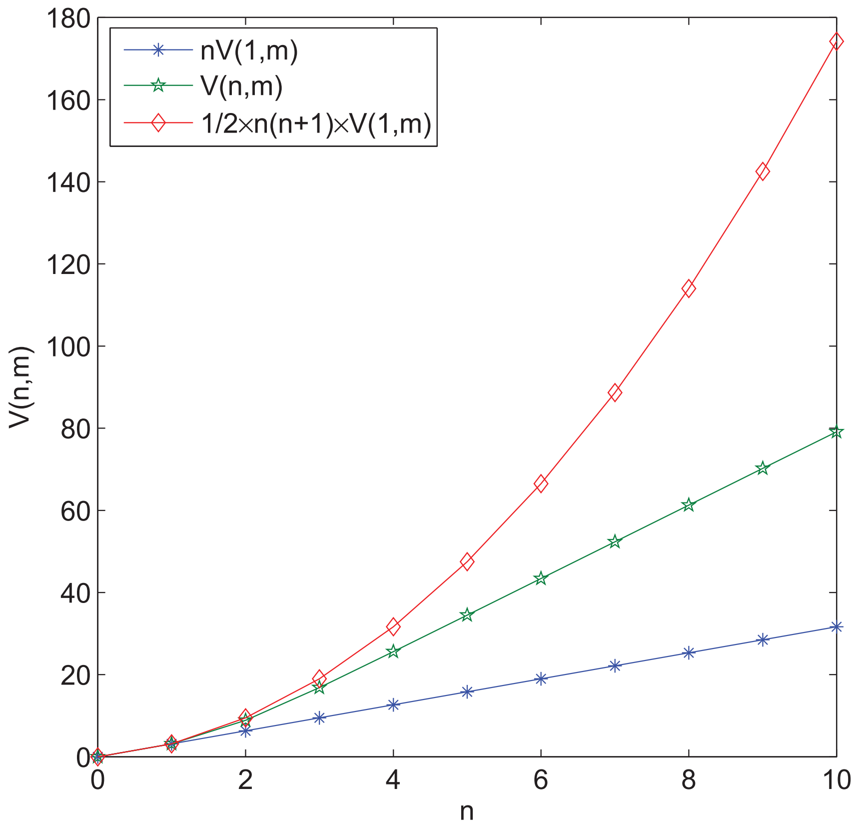

Fig. 4.1. $V(n,m)$ and its lower and upper bounds versus n when $m = 10$.

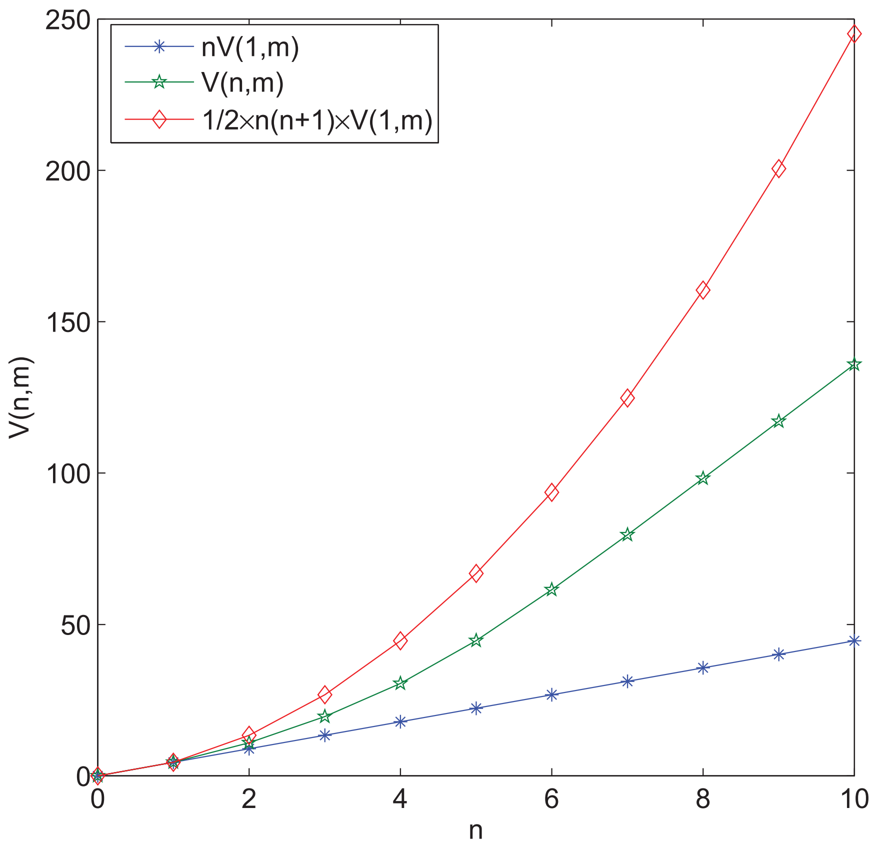

Fig. 4.2. $V(n,m)$ and its lower and upper bounds versus n when $m = 20$.

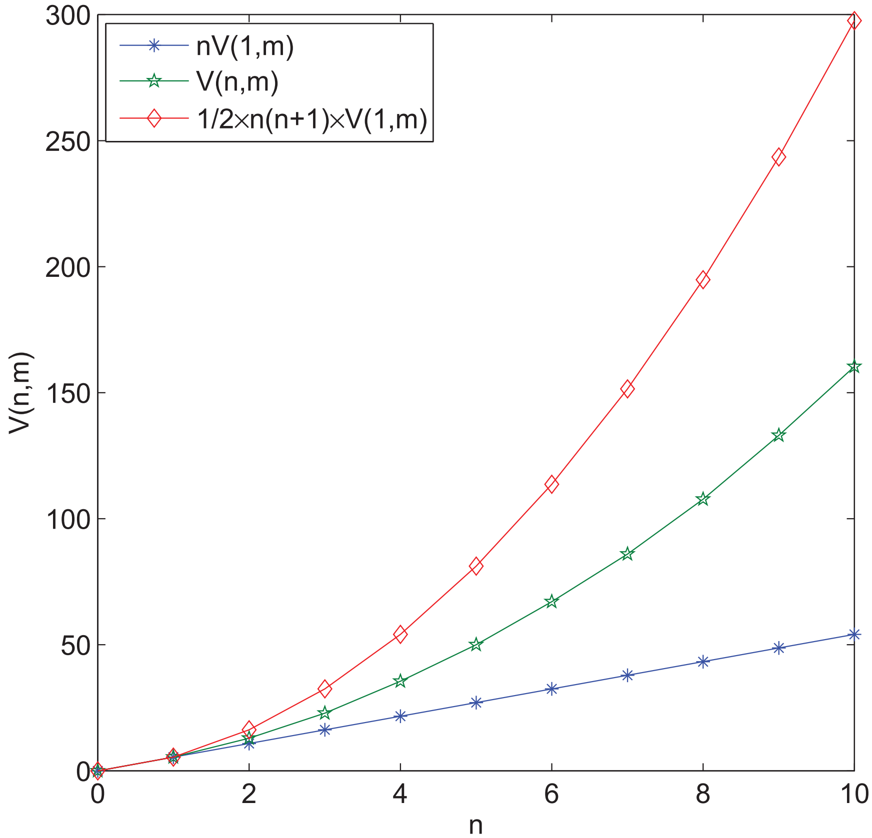

Fig. 4.3. $V(n,m)$ and its lower and upper bounds versus n when $m = 30$.

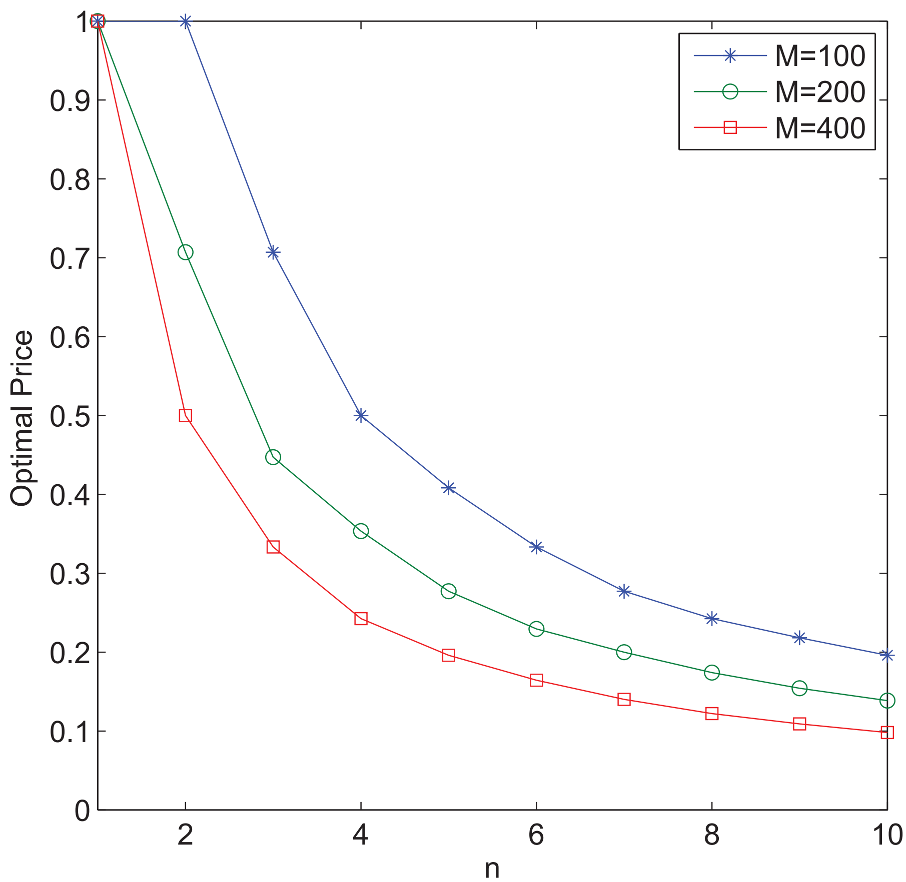

Fig. 4.4. Optimal price values.

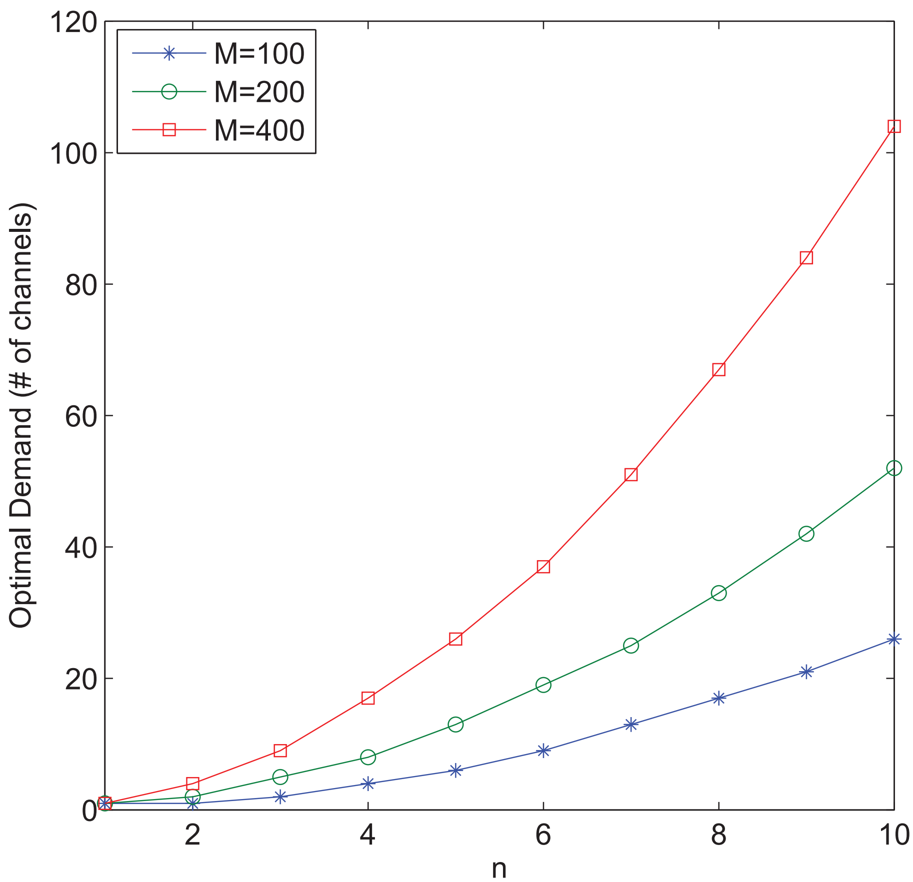

Fig. 4.5. Optimal demand.

Chapter 5

Conclusions and Future Research Works

This chapter summarizes the contributions of the thesis, and some future research directions are highlighted as well.

5.1 Conclusions

This thesis investigates the spectrum sharing, spectrum access, and spectrum leasing in cognitive radio, which has been demonstrated as a promising technique to improve spectrum efficiency in future wireless networks.

The spectrum sharing and access of secondary users are investigated in Chapter 3. Like existing research efforts, spectrum rental fee is charged to secondary users. Different from existing works, we also take into account penalty price for misbehaved secondary users. The setting of the penalty price is studied. A problem is formulated to maximize the secondary user's utility by setting the spectrum sensing time and the transmission power. Although the formulated problem is not a convex problem, we show that the problem can be convex if one of the two variables (spectrum sensing time and the transmission power) is fixed. Accordingly, we provide an iterative algorithm to find a solution of the problem. We also show that the iterative algorithm can converge quickly.

Chapter 4 investigates spectrum leasing from the primary user's perspective. A long-term leasing period is considered, and thus, the primary user takes several rounds in its spectrum leasing. Two cases, with discrete and continuous spectrum demand respectively, are studied. For each case, we formulate the revenue maximization problem, and provide methods to solve the problem. Some special features of the optimal pricing strategy are also derived. This research provides a solution for long-term spectrum leasing in varying scenarios.

5.2 Future Works

In the channel sensing and access in Chapter 3, only path loss is considered. In future work, we may take into account channel fading, which means the channel gains of the links vary with time. Therefore, if a missed detection happens, the penalty charge depends on the instantaneous channel gains of the links. An interesting research problem is to select sensing time and transmission power (which will be used for a long term) such that the average secondary user utility is maximized.

In the spectrum leasing in Chapter 4, only one seller (primary user) is considered. In future work, we may consider multiple sellers, and investigate the impact of competition among sellers on the price setting in the multiple rounds. Methods from game theoretical approaches might be helpful, and deserve further investigation.

References

[1] T. S. Rappaort, *Wireless Communications: Principles and Practice*, Prentice Hall, second edition, 2002.

[2] G. L. Stuber, *Principles of Mobile Communication*, Springer Press, 2011.

[3] International Telecommunication Union, *Cellular standards for the third generation*, Dec. 2005.

[4] J. Vilches, "Everything you need to know about 4G wireless technology," TechSpot, April 2010.

[5] R. Hekmat, *Ad Hoc Network: Fundamental Properties and Network Topologies*, Springer Press, 2006.

[6] R. Jurdak, C. V. Lopes, and P. Baldi, "A survey, classification and comparative analysis of media access control protocols for ad hoc networks," *IEEE Communications Surveys*, vol. 6, no. 1, pp. 2–16, 2004.

[7] I. Chlamtaca, M. Contib, and J. J.-N. Liuc, "Mobile ad hoc networking: Imperative and challenges," *Ad Hoc Networks*. vol. 1, no. 1, pp. 13–64, July 2003.

[8] L. A. DaSilva, A. B. MacKenzie, M. S. Thompson, and E. Q. Baumann, "The MANIAC challenge: Educational experiences in ad hoc networking," *IEEE Pervasive Computing*, vol. 8, no. 1, pp. 7–11, 2009.

[9] M.-H. Lu, P. Steenkiste, and T. Chen, "Time-aware opportunistic relay for video streaming over WLANs," in *Proceedings of IEEE International Conference on Multimedia and Expo*, pp. 1782–1785, 2007.

[10] H. Zhang and J. Hou, "Is deterministic deployment worse than random deployment for wireless sensor networks?" in *Proceedings of IEEE INFOCOM*, 2006.

[11] X. Bai, S. Kumar, D. Xuan, Z. Yun, and T. H. Lai, "Deploying wireless sensors to achieve both coverage and connectivity," in *Proceedings of the 7th ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*, pp. 131–142, 2006.

[12] Z. Yan, Y. Chang, H. Jiang, and Z. Shen, "Fault-tolerance in wireless ad hoc networks: Bi-connectivity through movement of removable nodes," *Wireless Communications and Mobile Computing*, vol. 13, no. 12, pp. 1095–1110, Aug. 2013.

[13] M. Patil and R.C. Biradar, "A survey on routing protocols in Wireless Sensor Networks," in *Proceedings of 2012 IEEE International Conference on Networks*, pp. 86–91, 2012.

[14] Y. Gu and T. He, "Dynamic switching-based data forwarding for low-duty-cycle wireless sensor networks," *IEEE Transactions on Mobile Compututing*, vol. 10, no. 12, pp. 1741–1754, Dec. 2011.

[15] D. Chandler and R. Munday, *A Dictionary of Media and Communication*, Oxford University Press, first edition, 2011.

[16] J. Mitola, "Cognitive radio for flexible mobile multimedia communications," in *Proceedings of IEEE International Workshop on Mobile Multimedia Communications*, 1999.

[17] A. F. Molisch, *Wireless Communications*, Wiley Press, second edition, 2011.

[18] Y.-C. Liang, K.-C. Chen, G. Y. Li, and P. Mahonen, "Cognitive radio networking and communications: An overview," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 7, pp. 3386–3407, Sept. 2011.

[19] J. Mitola and G. Q. Maguire, Jr, "Cognitive radio: Making software radios more personal," *IEEE Personal Communications*, vol. 6, no. 4, pp. 13–18, Aug. 1999.

[20] Y. Zhang, J. Zheng, and H.-H. Chen, *Cognitive Radio Networks: Architectures, Protocols and Standards*, CRC press, 2010.

[21] S. Haykin, D. J. Thomson, and J. H. Reed, "Spectrum sensing for cognitive radio," *Proceedings of the IEEE*, vol. 97, no. 5, pp. 849–877, May 2009.

[22] M. Lopez-Benitez and F. Casadevall, "Improved energy detection spectrum sensing for cognitive radio," *IET Communications*, vol. 6, no. 8, pp. 785–796, 2012.

[23] P. C. Sofotasios, E. Rebeiz, L. Zhang, T. A. Tsiftsis, D. Cabric, and S. Freear, "Energy detection based spectrum sensing over $k - \mu$ and $k - \mu$ extreme fading channels," *IEEE Transactions on Vehicular Technology*, vol. 62, no. 3, pp. 1031–1040, Mar. 2013.

[24] Z. Quan, W. Zhang, S. J. Shellhammer, and A. H. Sayed, "Optimal spectral feature detection for spectrum sensing at very low SNR," *IEEE Transactions on Communications*, vol. 59, no. 1, pp. 201–212, Jan. 2011.

[25] S. Stotas and A. Nallanathan, "On the throughput and spectrum sensing enhancement of opportunistic spectrum access cognitive radio network," *IEEE Transactions on Wireless Communications*, vol. 11, no. 1, pp. 97–107, Jan. 2012.

[26] S. Li, Z. Zheng, E. Ekici, and N. Shroff, "Maximizing system throughput by cooperative sensing in cognitive radio networks," *IEEE/ACM Transactions on Networking*, vol. 22, no. 4, pp. 1245–1256, Aug. 2014.

[27] E. Soltanmohammadi, M. Orooji, and M. Naraghi-Pour, “Improving sensing-throughput tradeoff for cognitive radio in Rayleigh fading channels,” *IEEE Transactions on Vehicular Technology*, vol. 62, no. 5, pp. 2118–2130, June 2013.

[28] Y. J. Choi, W. Pak, Y. Xin, and S. Rangarajan, “Throughput analysis of co-operative spectrum sensing in Rayleigh-faded cognitive radio systems,” *IET Communications*, vol. 6, no. 9, pp. 1104–1110, 2012.

[29] A. S. Cacciapuoti, I. F. Akyildiz, and L. Paura, “Optimal primary-user mobility aware spectrum sensing design for cognitive radio networks,” *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 11, pp. 2161–2172, Nov. 2013.

[30] A. A. El-Sherif and K. J. R. Liu, “Joint design of spectrum sensing and channel access in cognitive radio network,” *IEEE Transactions on Wireless Communications*, vol. 10, no. 6, pp. 1743–1753, June 2011.

[31] L. Tang, Y. Chen, E. L. Hines, and M.-S. Alouini, “Effect on primary user traffic on sensing-throughput tradeoff for cognitive radios,” *IEEE Transactions on Wireless Communications*, vol. 10, no. 4, pp. 1063–1068, Apr. 2011.

[32] A. W. Min, J. Choi, and K. G. Shin, “Exploiting spectrum heterogeneity in dynamic spectrum market,” *IEEE Transactions on Mobile Computing*, vol. 11, no. 12, pp. 2020–2032, Dec. 2012.

[33] Y. Cao, Q. Shi, X. Wang, X. Tian, and Y. Cheng, “Two-dimensional contract theory in Cognitive Radio networks,” *IEEE GLOBECOM*, pp. 1156–1161, 2012.

[34] H. Yu, L. Gao, Z. Li, X. Wang, and E. Hossain, “Pricing for uplink power control in cognitive radio networks,” *IEEE Transactions on Vehicular Technology*, vol. 59, no. 4, pp. 1769–1778, May 2010.

[35] Z. Wang, L. Jiang, and C. He, "Optimal price-based power control algorithm in cognitive radio networks," *IEEE Transactions on Wireless Communications*, accepted.

[36] N. H. Tran, C. S. Hong, Z. Han, and S. Lee, "Optimal pricing effect on equilibrium behaviors of delay-sensitive users in cognitive radio networks," *IEEE Journal of Selected Areas in Communications*, vol. 31, no. 11, pp. 2566–2579, Nov. 2013.

[37] D. Niyato and E. Hossain, "Market-equilibrium, competitive, and cooperative pricing for spectrum sharing in cognitive radio networks: Analysis and comparison," *IEEE Transactions on Wireless Communications*, vol. 7, no. 11, pp. 4273–4283, Nov. 2008.

[38] D. Niyato, E. Hossain, and Z. Han, "Dynamics of multiple-seller and multiple-buyer spectrum trading in cognitive radio networks: A game-theoretic modeling approach," *IEEE Transactions on Mobile Computing*, vol. 8, no. 8, pp. 1009–1022, Aug. 2009.

[39] S. Sengupta and M. Chatterjee, "An economic framework for dynamic spectrum access and service pricing," *IEEE/ACM Transactions on Networking*, vol. 17, no. 4, pp. 1200–1213, Aug. 2009.

[40] D. Niyato and E. Hossain, "Competitive pricing for spectrum sharing in cognitive radio networks: Dynamic game, inefficiency of Nash equilibrium, and collusion," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 1, pp. 192-202, Jan. 2008.

[41] D. Niyato and E. Hossain, "Competitive spectrum sharing in cognitive radio networks: A dynamic game approach," *IEEE Transactions on Wireless Communications*, vol. 7, no. 7, pp. 2651–2660, July 2008.

[42] T. Yucek and H. Arslan, “A survey of spectrum sensing algorithms for cognitive radio applications,” *IEEE Communications Surveys & Tutorials*, vol. 11, no. 1, pp. 116–130, 2009.

[43] Y.-C. Liang, Y. Zeng, E. C. Y. Peh, and A. T. Hoang, “Sensing-throughput tradeoff for cognitive radio,” *IEEE Transactions on Wireless Communications*, vol. 7, no. 4, pp. 1326–1337, Apr. 2008.

[44] R. Fan and H. Jiang, “Optimal multi-channel cooperative sensing in cognitive radio networks,” *IEEE Transactions on Wireless Communications*, vol. 9, no. 3, pp. 1128–1138, Mar. 2010.

[45] H. Kobayashi, B. L. Mark, and W. Turin, *Probability, Random Processes, and Statistical Analysis*, United Kingdom at the University Press, Cambridge, 2012.

[46] J. R. Barry, E. A. Lee, and D. G. Messerschmitt, *Digital Communications*, Kluwer Academic Publisher, third edition, 2004.

[47] L. Gao, X. Wang, Y. Xu, and Q. Zhang, “Spectrum trading in cognitive radio networks: A contract-theoretic modeling approach,” *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 4, pp. 843–855, Apr. 2011.

[48] S. P. Boyd and L. Vandenberghe, *Convex Optimization.* Cambridge University Press, 2004.

[49] L. Duan, J. Huang, and B. Shou, ”Investment and pricing with spectrum uncertainty: A cognitive operator’s perspective,” *IEEE Transactions on Mobile Computing*, vol. 10, no. 11, pp. 1590–1604, Nov. 2011.

[50] L. Qian, F. Ye, L. Gao, X. Gan, T. Chu, X. Tian, X. Wang, and M. Guizani, “Spectrum trading in cognitive radio networks: An agent-based model under demand uncertainty,” *IEEE Transactions on Communications*, vol. 59, no. 11, pp. 3192–3203, Nov. 2011.

[51] A. A. Daoud, M. Alanyali, and D. Starobinski, “Pricing strategies for spectrum lease in secondary markets,” *IEEE/ACM Transactions on Networking*, vol. 18, no. 2, pp. 462–475, April 2011.

[52] L. Duan, J. Huang, and B. Shou, “Duopoly competition in dynamic spectrum leasing and pricing,” *IEEE Transactions on Mobile Computing*, vol. 11, no. 11, pp. 1706–1719, Nov. 2012.

[53] P. Lin, J. Jia, Q. Zhang, and M. Hamdi, “Dynamic spectrum sharing with multiple primary and secondary users,” *IEEE Transactions on Vehicular Technology*, vol. 60, no. 4, pp. 1756–1765, May 2011.

[54] G. S. Kasbekar and S. Sarkar, “Spectrum pricing games with random valuations of secondary users,” *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 11, pp. 2262–2273, Dec. 2012.

[55] S. Li, J. Huang, and S-Y. R. Li, “Dynamic profit maximization of cognitive mobile virtual network operator,” *IEEE Transactions on Mobile Computing*, vol. 13, no. 3, pp. 526–540, Mar. 2014.

[56] R. Pindyck and D. Rubinfeld, *Microeconomics*. Prentice Hall, seventh edition, 2008.

[57] C. Floudas and P. M. Pardalos, *Encyclopedia of Optimization.* Springer Press, second edition, 2009.