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# Essays on Asset Pricing and Capital Markets

by

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## ABSTRACT

I. We propose the "artificial regression model" as an alternative conditional mean specification for asset returns. Our objectives are two-fold. First, we analytically demonstrate the benefit of this nonlinear specification in mitigating heteroskedasticity by incorporating return sign predictability. In the limit, perfect prediction of a return's sign is shown to completely eliminate heteroskedasticity. Second, we show how this model provides closed-form solutions for the traditional GARCH parameters and thus provides a simpler estimation approach for the popular class of GARCH models.

The major advantage of the artificial regression specification is that it generalizes to a multivariate approach in a natural way and enables large systems to be estimated with ease. Furthermore, this model is a flexible specification and can handle long decay structures that the traditional GARCH model cannot.

In the Monte Carlo analysis, we find that the artificial regression model is comparable to the popular approach in generating GARCH estimates. In the empirical analysis we test the impact of market direction in reducing heteroskedasticity. The results indicate that a richer information set and precision of the sign component help the mean and reduce the volatility. We also find that the proposed model is sensitive to misspecification of the underlying volatility process.

II. We examine the inter-day dynamic linkages among stock returns, return volatility and trading activity (hereafter volume) in the Canadian market in a multivariate framework and investigate the ability of volume to account for the persistence in stock-return volatility. We report a threshold-type nonlinearity in both returns and volume and document significant linear and nonlinear causality from stock returns and return volatility to volume. A surprising result is that volume is heteroskedastic. We also show that the U.S. stock returns serve as an effective public information measure for the Canadian market.

The multivariate analysis reveals that the economic significance of volume as an information flow measure is questionable. Volatility itself would be a more appropriate information flow measure. We find overwhelming evidence supporting the sequential information flow and the positive feedback trading hypotheses. We also document the inability of volume to explain the persistence in return volatility.

## DEDICATION

*To my parents*

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# CHAPTER 1

## INTRODUCTION

This thesis presents two papers representing research on asset pricing and capital markets. The unifying theme of the thesis is the nonlinearity of asset returns and its implications for theoretical and empirical asset pricing. From a specification point of view, the results of this research must be viewed as supportive of the premise that asset returns depend on the sample information as a nonlinear function of the variables in the information set.

In the second chapter, *The Estimation and Econometric Specifications of Asset Return Moments*, we propose the "artificial regression model" as an alternative nonlinear conditional mean specification, for asset returns. This chapter was motivated first, by the estimation problems encountered in the existing class of nonlinear models, specifically the Generalized Autoregressive Conditional Heteroskedastic (GARCH) class of models, in a multivariate setting and second, by the potential significance of return sign predictability on the heteroskedasticity of asset returns. The central idea underlying this model is the error decomposition into magnitude and sign components resulting in a threshold model in the mean. The uniqueness of this paper is that we then proceed to show the benefit of this nonlinear specification in mitigating heteroskedasticity in the residuals by incorporating return sign predictability. In the limit, perfect prediction of a return's sign is shown to completely eliminate heteroskedasticity. In addition, we show how this model provides closed-form solutions for the GARCH parameters and thus provides a simpler estimation approach for the popular class of GARCH models. The

Monte Carlo studies and empirical data analysis lend support to the proposed model. The major advantage of the artificial regression specification is that it generalizes easily to a multivariate approach and enables large systems to be estimated with ease. Furthermore, this model is a flexible specification and can handle long decay structures that the traditional GARCH model cannot.

In the third chapter, *The Informational Role of Volume in Financial Markets: An Empirical Study of the Canadian Stock Market*, we examine the inter-day dynamic linkages among stock returns, return volatility and trading activity in the Canadian market in a multivariate framework and investigate the ability of volume to account for the persistence in stock-return volatility. This study was motivated by the conflicting findings regarding the informational role of various trading variables. The primary contribution of our study is that we use a different estimation and inference approach, various trading activity measures, and a new data set, the Canadian market aggregates. We report a threshold-type nonlinearity in both returns and volume. We also find that volume is heteroskedastic. In addition, we also show that the U.S. stock returns serve as an effective public information measure for the Canadian market. The multivariate analysis reveals that the role of volume as an information flow measure is questionable. We find strong support for the sequential information flow and the positive feedback trading hypotheses. We also document the inability of volume to explain the persistence in return volatility. This finding suggests that the artificial regression model proposed in the preceding chapter could be a candidate model for investigating the inter-day dynamics for the Canadian markets. Chapter 4 concludes the thesis.

# CHAPTER 2

# THE ESTIMATION AND ECONOMETRIC SPECIFICATIONS OF ASSET RETURN MOMENTS

## 2.1. INTRODUCTION

Recent empirical research in financial markets has increasingly focused on the econometric modeling of temporal variations in higher moments, more specifically the conditional variance.[1] In particular, the Autoregressive Conditional Heteroskedastic (ARCH) family of models introduced by Engle (1982) is a popular specification for capturing volatility dependence. This approach has its merits since efficient econometric estimation of the conditional mean requires the correct specification of the conditional variance.[2] However in a multivariate setting, this class of models necessitates imposition of various constraints and is oftentimes computationally difficult to estimate. More specifically, the existing approaches for estimating Generalized ARCH (GARCH, Bollerslev 1986) systems include the maximum likelihood (ML) and the generalized method of moments (GMM), which use a numerical search procedure and often do not converge.

In this chapter, the "artificial regression model" is proposed as an alternative nonlinear conditional mean specification for asset returns. Our objectives are two-fold. First, we analytically demonstrate the benefit of this nonlinear specification in mitigating

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[1] See Bollerslev, Chou, and Kroner (1992) for an extensive survey of models on temporal variation in financial market volatility.

[2] Heteroskedasticity of security returns reduces the efficiency of empirical tests in which homoskedasticity is assumed. Pagan and Sabau (1987) show that an incorrect functional form of the ARCH process for errors of a regression model can result in inconsistent maximum likelihood estimators of the regression parameters. Stambaugh (1993) finds that conditional heteroskedasticity can produce large increases in the asymptotic variances of sample autocorrelations.

heteroskedasticity by incorporating return sign predictability. Second, we provide a simpler alternative estimation approach for the popular class of GARCH models. The intuition underlying the model is that increasing the precision of information should shrink volatility and lessen heteroskedasticity in the error structure that is caused by misspecifying the nonlinear conditional mean. Our formulation begins with an error decomposition into magnitude and sign components as in Granger and Ding (1996). The resultant model is a threshold model in the mean, which incorporates an indicator variable reflecting market direction. Conditional volatility is lessened on average through time as the precision of the prediction of the sign component increases. With perfect sign prediction, the error in the conditional mean is homoskedastic.

The major advantage of the "artificial regression" specification is that it simplifies the estimation procedure and enables large systems to be estimated with ease, circumventing the problem of non-convergence associated with numerical searches. Furthermore, this model is a flexible specification and can handle long decay structures that the traditional GARCH model cannot.[3] In addition, the artificial regression model has the advantage that any restriction including GARCH (1,1) can be imposed directly on the regression coefficients. Finally, estimation can be done by OLS, which should perform better in terms of convergence than ML and GMM in large systems.

The remainder of the paper is organized as follows. Section 2.2 describes the motivation and background material for the proposed model. Section 2.3 presents the

[3] Granger and Ding (1996) have found long memory temporal properties in absolute returns in many different speculative markets suggesting that shocks to the conditional variance die out at a slow hyperbolic rate of decay. In contrast, in traditional GARCH models, shocks die out at an exponential rate.

model development. Section 2.4 presents a Monte Carlo application. The practical importance of the model is explored in section 2.5, where we report the results of empirical applications to daily and monthly U.S. and Canadian stock index returns. Section 2.6 concludes.

## 2.2. Motivation and Background Material

### 2.2.1. Motivation for Model

Despite the fact that predictability of conditional variance is not exploitable for point prediction of returns, GARCH models have aroused considerable interest as they capture many empirical regularities of financial data returns such as nonlinear dependence, leptokurtosis, skewness, and volatility clustering.[4] The appeal of GARCH lies in the intuition that big surprises of either sign increase market uncertainty and therefore will be more likely to be followed by big surprises. These models permit nonlinear relationships in the second moments wherein forecasts of future volatility are made, but not the direction of price changes. The most general GARCH model assumes that the econometrician's information set consists of exogenous and lagged endogenous variables, being conditional economic variables and past innovations to the excess return. In these models, both conditional mean and variances jointly evolve over time. The brief

[4] Nonlinear models in the conditional mean include the threshold autoregressive model (TAR, Tong, 1990) and the bilinear process of Weiss (1986) and are forecastable. The TAR model has a functional form which may be regarded as a piecewise-linear approximation to a general nonlinear first order model. The bilinear terms include cross-product terms of the instrument variables and lags of error terms and thus affect the conditional mean. Bera and Higgins (1997) have shown the poor predictive ability of bilinear models.

review of the literature that follows is indicative of this area of research and is not an attempt at a complete review of all related research.[5]

Many variants of the GARCH model have been proposed. An important class of models is the ARCH-M specification as in Engle, Lilien and Robins (1987), which models a process with feedback from the conditional variance to the conditional mean. Risk premia are not time invariant but vary with the agent's perception of underlying uncertainty. Nelson's (1991) exponential GARCH (EGARCH) and Glosten, Jagannathan and Runkle's (1993) GARCH (GJR) improved on the GARCH formulation as their models captured the leverage effect in asset returns.[6] Hentschel (1995) develops a parametric family of GARCH models nesting the most popular symmetric and asymmetric GARCH models. He also provides evidence that the two types of asymmetry, shift and rotation of the news impact curve are distinct. According to Hentschel (1995), the shift is the dominant source of asymmetry for small shocks, while the rotation is more important for large shocks and when combined in one news impact curve, they can either reinforce or offset each other. Recent models include Bollerslev and Mikkelson's (1996) fractionally integrated GARCH model, which accommodates slow decay and Dueker's (1994) specification, where a compound GARCH/Markov switching process captures the mean reversion aspect of volatility.

The estimation of the various GARCH models can be problematic and some

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[5] Bollerslev, Chou, and Kroner (1992) provide an extensive but dated survey of the applications of this methodology.

[6] Leverage effect refers to the asymmetric response of stock return volatility to positive and negative return shocks i.e. stock return volatility tends to be negatively correlated with past returns, possibly due to the increased leverage following a fall in the stock price. The leverage effect in stock returns has been documented among others by Black (1976), Christie (1982), and Schwert (1989).

estimation issues relating to these models still remain unresolved. These models could be estimated by feasible generalized least squares (FGLS), one-step efficient estimation (Engle 1982), and maximum likelihood. The popular approach for estimating GARCH models is the latter approach i.e. by maximizing the conditional likelihood function. To obtain second order efficiency, an iterative procedure such as the Berndt, Hall, Hall, and Hausman (1974) (BHHH) algorithm can be used. Common distributional assumptions for the estimation of these models are the Gaussian or the t-distribution. However, French, Schwert and Stambaugh (1987) and Singleton and Wingender (1986) among others have documented violations of these distributional assumptions. Baillie and Bollerslev (1992) report that higher order deviations from conditional normality are important under high volatility scenarios. In order to overcome the distributional assumption problem, Bollerslev and Woolridge (1988) have shown that quasi-maximum likelihood estimation (QMLE) is consistent but not efficient. But, Engle and Gonzalez-Rivera (1991) show that in QMLE the loss of efficiency, due to misspecification of the density, could be as high as 84 per cent. They provide a semi-parametric approach that improves on QMLE, but the approach is still not efficient.

Going beyond estimation problems, Lamoureux and Lastrapes (1990) find that GARCH models, which do not allow for structural changes, will pick up high persistence due to misspecification. They have shown that occasional discrete shifts in the mean level of volatility cause substantial upward bias in the estimation of volatility persistence.

In terms of predictive power, ARCH models despite imputing high persistence to stock volatility, give poor forecasts. West and Cho (1995), report that for a one-week horizon, GARCH models tend to make slightly more accurate forecasts for weekly exchange rates, but for longer horizons, do no better than naive or ARMA models. Kim and Kon (1994) find that among the inter-temporal dependent models, the Glosten, Jagannathan and Runkle (1993) specification is the most descriptive for individual daily stock returns while Nelson's (1990) EGARCH is most appropriate for indexes. Pagan and Schwert (1990) looking at monthly stock returns find that non-parametric estimation procedures tend to give better in-sample explanations of the squared returns than any of the parametric models, but fared worse in out-of-sample predictions. They report the parametric methods use the persistent, smoother aspects of conditional volatility, while the nonparametric methods use a highly nonlinear response to large shocks.

Finally, in a univariate setting, we have a plethora of models but the computational complexities of GARCH have constrained their extension to a multivariate setting. However, asset-pricing models, asset-allocation models, and hedging strategies require multivariate generalizations of the GARCH model. While empirical work on asset pricing considers large numbers of assets and instruments, the existing GARCH models frequently use fewer asset-return series and simple specifications.[7] Furthermore, restrictions are imposed in order to reduce the number of estimated parameters. Often, such restricted multivariate GARCH models with the simplest specifications do not converge when estimated with ML or GMM.

Given the popularity of the GARCH functional form in modeling conditional volatility and keeping in mind its limitations, we ask two questions in the paper. First, is it possible to find a conditional mean specification that reduces heteroskedasticity in the

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[7] See Bollerslev, Engle and Woolridge (1988), Engle, Ng, and Rothschild (1990), and Turtle, Buse and Korkie (1994) for multivariate generalizations of the GARCH model in an asset-pricing context.

error term?[8] Second, is there a simpler estimation approach for the popular GARCH specification? Compared to the typical GARCH specification, our proposed "artificial regression" model is motivated by the following considerations: rationality in conditional moments, flexibility in the specification and ease of estimation. Our method is not subject to the lack of convergence problem because it provides an analytical solution for the GARCH estimates. Before moving on to the specifics of the GARCH (1,1) model, we provide an analytical proof for a proposition regarding the equivalence of various time series specifications of asset conditional moments.

### 2.2.2. The Equivalence of Econometric Specifications of Asset Conditional Moments

According to Engle (1982), serial dependence in higher moments may be the result of misspecification in the conditional mean caused by omitted variables. Consider this simple example from Mills (1993). Suppose that the true process is a bilinear model that is nonlinear in the conditional mean and is given by

$$r_t = \beta_1 r_{t-1} u_{t-1} + u_t, \qquad (2.2.1)$$

where,

$$E(u_t) = 0,\ E(u_t^2) = \sigma_u^2 \text{ and } cov(u_t u_{t+k}) = 0,\ k > 0. \qquad (2.2.2)$$

Suppose this model is mistaken for the linear model,

$$r_t = \tilde{u}_t \qquad (2.2.3)$$

[8] Previous work on nonlinear models in the conditional mean include the nonlinear moving average model of Robinson (1977), the bilinear model of Weiss (1986), the asymmetric moving average model of Wecker (1981) and the threshold autoregressive model of Tong (1990). Weiss (1986) notes that ignoring the bilinearity in the conditional mean can lead to residuals that appear to have ARCH errors even though they may not be autocorrelated; he proposes a specification which combines bilinearity and ARCH.

Substituting from equation (2.2.3) in (2.2.1), we get

$$\tilde{u}_t = \beta_1 \tilde{u}_{t-1} u_{t-1} + u_t \tag{2.2.4}$$

Then, squaring (2.2.4) and taking expectations, it follows that

$$E(\tilde{u}_t^2 \mid r_{t-1}, r_{t-2}, \ldots..) = \sigma_u^2 + \beta_1^2 \sigma_u^2 \tilde{u}_{t-1}^2 \tag{2.2.5}$$

This suggests that $\tilde{u}_t$ is ARCH (1) as the expectation implies that the squares of the residuals are autocorrelated. In general, equation (2.2.5) is similar to Engle's (1982) ARCH (1) test wherein residuals are regressed on the intercept and a lag of squared residuals with the appropriate test statistic $T.R^2$,being distributed as a $\chi^2(1)$. T represents the number of terms in the series being analyzed and $R^2$ is the squared multiple correlation coefficient between the squared residuals and the lagged squared residuals. Note that for a normally distributed error term, the squares of the residuals will not be autocorrelated. Thus, assuming a linear mean specification when the mean is nonlinear may indicate ARCH even though the true model is homoskedastic. We next present the specific propositions.

Our purpose in this section is to show that a specification that requires the first and second moments to be specified, can be closely approximated by one where only the first moment is specified. Such a specification would have appealing economic and statistical properties. Furthermore in a multivariate setting, this would provide a parsimonious framework, consistent with rational expectations and current financial theory.

Proposition 1a:

*A specification with a constant conditional mean and time varying conditional variance is approximately equivalent to one with a time varying conditional mean and a constant variance.*

Proof: See Appendix I

Proposition 1b

*A specification with a time varying conditional mean and a constant variance is approximately equivalent to one with a constant conditional mean and time varying conditional variance.*

Proof: See Appendix I

In Appendix I, we show analytically that, without distributional assumptions for disturbances, it is not possible to distinguish between models with time varying conditional means and time varying conditional variances and models with only time varying conditional means or only time varying conditional variances.

Our motivation is in the spirit of Granger's (1991) argument of treating nonlinearity in the conditional mean on an equal footing with nonlinearity in the conditional variance. We provide weak proofs for approximate equivalence of functional moments using second-order Taylor series approximations. This equivalence appears consistent with empirical evidence favoring the existence of either additive [nonlinear moving average model (Robinson, 1977), asymmetric moving average model (Wecker, 1981), and the threshold autoregressive model (Tong, 1990)] or multiplicative nonlinear models (ARCH family) or a combination of both (Weiss, 1986).

In the proofs in Appendix I, only the weakest rationality conditions are imposed on the conditional means and variances, so that forecasts of either variable contain no systematic errors.[9] Any further extensions, such as comparisons of the magnitude of the coefficients require specifying the error structure and the information instruments. Next, we focus on a popular specification in section 2.2.3, where we introduce the artificial regression model.

### 2.2.3. Introduction to Artificial Regression

We propose an artificial regression model, also called the Gauss-Newton regression (GNR), as an alternative to a GARCH (1,1) specification. In this model, the regressors and regressands are constructed in a linear regression to calculate one-step efficient estimates of the parameters in an asset's conditional first moment specification.

Our brief review of the artificial regression model is based on the discussions in Davidson and MacKinnon (1993). Consider the univariate nonlinear regression model

$$y_t = x_t(\beta) + u_t, \quad u_t \sim IID(0, \sigma^2 I) \qquad (2.2.6)$$

Taking a first-order Taylor series approximation of (2.2.6) around some parameter $\beta^*$ yields,

$$y_t = x_t(\beta^*) + X(\beta^*)(\beta - \beta^*) + \text{higher order terms} + u_t \qquad (2.2.7)$$

where $X(\beta)$ is a matrix with the $i^{th}$ element being the derivative of $x_t(\beta)$ with respect to $\beta_i$. Equation (2.2.7) can also be written as

$$y_t - x_t(\beta^*) = X(\beta^*)b + \text{residuals} \qquad (2.2.8)$$

[9] See Korkie and Turtle (1996) for the rationality conditions in a GMM framework.

where $(\beta - \beta^*)$ is replaced by a vector b. Equation (2.2.8) is the GNR in its popular form and if $\beta'$ is the initial estimate from (2.2.6) and $b'$ the estimate from the regression (2.2.8), then the one-step efficient estimator of $\beta$ is $\hat{\beta} = \beta' + b'$. Note that if $\beta' = \beta^*$, the GNR will not have any explanatory power. When $\beta'$ is close to $\beta^*$ and the sample is large enough, $\hat{\beta}$ should be close to the NLS estimator. Davidson and MacKinnon (1993) show that the one-step efficient estimator, $\hat{\beta}$ is asymptotically equivalent to the nonlinear least squares (NLS) estimator. However, the authors caution that the finite sample properties for the two approaches are similar only if the sample size is large and the initial estimator is good. Using this approach, we now present the formal development of our model.

## 2.3 The Proposed 'Artificial Regression' Model

Our goal is to first specify an alternative nonlinear functional form to capture an asset's return as a function of readily available information. Second, we demonstrate that incorporating return sign predictability in the conditional mean equation reduces the heteroskedasticity of the residuals.

We assume that return predictability manifests in asset returns in a nonlinear fashion. The endogenous variable in the model is the excess asset return and the information set consists of exogenous economic variables and lagged endogenous variables including past innovations to the excess return. Rationality conditions are imposed on the conditional means and variances, such that forecasts of either variable contain no systematic errors.

We choose the GARCH (1,1) specification as it is the most widely used model for a wide range of financial data. The univariate GARCH model consists of two equations. The mean equation describes the observed returns as a function of instrument variables and an error term. The variance equation specifies the conditional variance of the error term from the mean equation as a function of past conditional variances and lagged squared errors.

The discussion that follows is a departure from the traditional GARCH model, and focuses on our contribution through the respecification of the conditional moments.

### 2.3.1. The Granger-Ding Decomposition and the Impact of Forecasting Market Direction

Let $r_t$ represent the security return in period t. $\varphi_t$ is the econometrician's information set at time t, and consists of a vector of exogenous economic variables and lagged endogenous variables in the conditional mean equation. The return $r_t$ from an asset is given by

$$r_t = \alpha + \beta X_{t-1} + u_t, \qquad (2.3.1)$$

where,

$$\mu_t = E[r_t \mid \varphi_{t-1}] = \alpha + \beta X_{t-1} \qquad (2.3.2)$$

is the conditional mean, which is typically assumed to be linear. However, a nonlinear functional form could also be used. The error term, $u_t$, is heteroskedastic and has conditional variance, $\sigma_t^2$, which is known at time t, where

$$u_t^2 = \sigma_t^2 + \varepsilon_t; \quad \varepsilon_t \sim (0, \sigma_\varepsilon^2) \qquad (2.3.3)$$

As in Granger and Ding (1996), a decomposition of the error term into a magnitude and a sign component gives,[10]

$$u_t = D_t|u_t| \text{ where } D_t = 1 \quad \text{if } r_t > \mu_t \Rightarrow u_t > 0 \tag{2.3.4}$$

$$= -1 \quad \text{if } r_t \leq \mu_t \Rightarrow u_t \leq 0 \text{ and where}$$

$$|u_t| = \left|\sqrt{\sigma_t^2 + \varepsilon_t}\right| \text{ in a GARCH specification.} \tag{2.3.5}$$

After substitution, the return equation (2.3.1) becomes

$$r_t = \mu_t + D_t|u_t|, \tag{2.3.6}$$

which allows one to separate the sign, $D_t$, and magnitude, $|u_t|$, components of the return error, $u_t$.

The next step is to expand the square root function in equation (2.3.5).

### 2.3.2. Respecifying the GARCH (1,1) Model

The stochastic version of Bollerslev's (1986) GARCH (1,1) specification is

$$u_t^2 = \gamma_0 + \gamma_1 u_{t-1}^2 + \theta\sigma_{t-1}^2 + \varepsilon_t \tag{2.3.7}$$

where the conditional variance is

$$\sigma_t^2 = \gamma_0 + \gamma_1 u_{t-1}^2 + \theta\sigma_{t-1}^2 \text{ [11]} \tag{2.3.8}$$

Here we approximate this by a conditional variance that depends on n lags of $u_{t-1}^2$ as follows:

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[10] Granger and Ding (1996) propose the decomposition of the error term, but their main concern is with the magnitude component. They are not concerned with the sign component, which is an important part of this study.

[11] The nonnegativity requirement is satisfied if $\gamma_0 > 0$ and $\gamma_1$ and $\theta \geq 0$. The covariance stationarity requirement is satisfied if $\gamma_1 + \theta < 1$.

$$\sigma_t^2 \approx \gamma_0(1+\theta+\theta^2+\theta^3+...)+\gamma_1 u_{t-1}^2+\gamma_1\theta u_{t-2}^2+\gamma_1\theta^2 u_{t-3}^2+.. \quad (2.3.9)$$

Combining (2.3.5) and (2.3.9), we get,

$$|u_t| \approx \sqrt{\gamma_0(1+\theta+\theta^2+\theta^3+..)+\gamma_1 u_{t-1}^2+\gamma_1\theta u_{t-2}^2+\gamma_1\theta^2 u_{t-3}^2+..+\varepsilon_t} \quad (2.3.10)$$

and combining (2.3.6) and (2.3.10), the return equation may be expressed as

$$r_t = \mu_t + D_t\sqrt{\gamma_0(1+\theta+\theta^2+\theta^3+..)+\gamma_1 u_{t-1}^2+\gamma_1\theta u_{t-2}^2+...+\varepsilon_t} \quad (2.3.11)$$

At this point, we have consolidated the mean and variance into one return equation. In order to simplify estimation and ensure convergence, we use a Taylor series approximation of the square root in order to obtain $|u_t|$ as a linear function of squared lagged residuals. This results in a mean specification that is linear in the squared lag residuals and the estimation may proceed using OLS, as is shown next.

We consider $|u_t|$ as an n-dimensional, first order, Taylor series approximation in $u_{t-1}^2, u_{t-2}^2, ...., u_{t-n+1}^2$, and $\varepsilon_t$ expanded about $(\gamma_0, \gamma_0, \gamma_0, ....., 0)$, resulting in[12]

$$|u_t| = \frac{1}{2}\sqrt{\frac{\gamma_0(1+\theta+\theta^2+..+\theta^{n-2})(2+\gamma_1)^2}{(1+\gamma_1)}} + \sum_{j=1}^{n-1}\frac{\gamma_1\theta^{j-1}u_{t-j}^2}{2\sqrt{\gamma_0(1+\gamma_1)(1+\theta+..+\theta^{n-2})}} + \frac{\varepsilon_t}{2\sqrt{\gamma_0(1+\gamma_1)(1+\theta+..+\theta^{n-2})}} \quad (2.3.12)$$

[12] A second order Taylor series expansion of f(z) about $z_0$ is given by

$$f(z) = f(z_0) + \frac{f'(z)|z_0}{1!}(z-z_0) + \frac{f''(z)|z_0}{2!}(z-z_0)^2 + R_2$$

Substituting (2.3.12) in (2.3.6) and rearranging

$$\frac{r_t-\mu_t}{D_t}=\frac{1}{2}\sqrt{\frac{\gamma_0(1+\theta+\theta^2+..+\theta^{n-2})(2+\gamma_1)^2}{(1+\gamma_1)}}+$$
$$\sum_{j=1}^{n-1}\frac{\gamma_1\theta^{j-1}u_{t-j}^2}{2\sqrt{\gamma_0(1+\gamma_1)(1+\theta+\theta^2+..+\theta^{n-2})}}+\frac{\varepsilon_t}{2\sqrt{\gamma_0(1+\gamma_1)(1+\theta+..+\theta^{n-2})}} \tag{2.3.13}$$

Hence, (2.3.13) may be written as the artificial regression model,

$$\frac{r_t-\mu_t}{D_t}=b_0+\sum_{j=1}^{n-1}b_j u_{t-j}^2+\xi_t \tag{2.3.14}$$

where,

$$\xi_t=\frac{\varepsilon_t}{2\sqrt{\gamma_0(1+\gamma_1)(1+\theta+\theta^2+............+\theta^{n-2})}}$$

is the homoskedastic error. Alternatively, (2.3.14) may be written as

$$\frac{r_t}{D_t}=\frac{\mu_t}{D_t}+b_0+\sum_{j=1}^{n-1}b_j u_{t-j}^2+\xi_t \tag{2.3.15}$$

Notice from (2.3.15) that $D_t$ impacts both the slope and the intercept in the conditional mean specification, $\frac{\mu_t}{D_t}=\frac{\alpha}{D_t}+\frac{\beta X_{t-1}}{D_t}$. If one can predict $D_t$, the conditional mean expression will incorporate the predictable part of $D_t$ and the conditional volatility will be lessened through time, as is demonstrated in Section 2.3.3. Hence, a decomposition into sign and magnitude components would simplify the specification to one where only the first moment needs to be specified. Such a decomposition may prove helpful in the estimation of complex, heavily parametrized, multivariate systems.

### 2.3.3. The Impact of Forecasting Market Direction

If the direction of the market, given by the sign of $D_t$, is partially predictable then the conditional variance and the heteroskedasticity are reduced as is shown next. Perfect prediction of market direction completely eliminates the heteroskedasticity.

Let $F_{t-1}(D_t)$ be the time (t-1) forecast of $D_t$ (conditional on some information set) and $d_t$ is the forecast error. Consequently, $F_{t-1}(D_t) = \pm 1$ and $d_t = 0, \pm 2$, because of the identity

$$D_t = F_{t-1}(D_t) + d_t \tag{2.3.16}$$

Substituting for $D_t$ in (2.3.15) and rearranging gives the equation

$$r_t = \mu_t + F_{t-1}(D_t)(b_0 + \sum_{j=1}^{n-1} b_j u_{t-j}^2) + d_t(b_0 + \sum_{j=1}^{n-1} b_j u_{t-j}^2) + D_t \xi_t \tag{2.3.17}$$

From this, we can infer the new conditional moments.

The predicted asset return, given the forecastability of $D_t$, is

$$\mu_t^* = \mu_t + F_{t-1}(D_t)(b_0 + \sum_{j=1}^{n-1} b_j u_{t-j}^2) \tag{2.3.18}$$

and the new forecast error is

$$u_t^* = d_t(b_0 + \sum_{j=1}^{n-1} b_j u_{t-j}^2) + D_t \xi_t \tag{2.3.19}$$

and its square is

$$u_t^{*2} = d_t^2(b_0 + \sum_{j=1}^{n-1} b_j u_{t-j}^2)^2 + 2 d_t D_t (b_0 + \sum_{j=1}^{n-1} b_j u_{t-j}^2)\xi_t + \xi_t^2 \tag{2.3.20}$$

The conditional variance of the forecast error may be obtained by taking the expectation of $u_t^{*2}$ at time t-1; i.e.,

$$E\left(u_t^*\right)^2 = \sigma_\xi^2 + (b_0 + \sum_{j=1}^{n-1} b_j u_{t-j}^2)^2 E\left(d_t^2\right) \qquad (2.3.21)$$

where $E\left(d_t^2\right)$ is the expected squared error in the market direction, which depends upon the richness of the forecast $F_{t-1}(D_t)$.

Let q be the probability that $F_{t-1}(D_t)$ is a correct forecast, then the event tree relating to forecasts and outcomes appears in figure 2.3.1.[13]

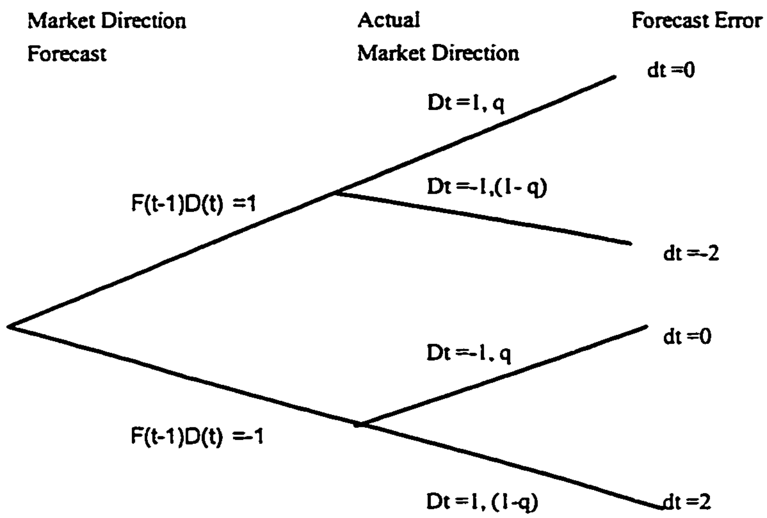


Figure 2.3.1

If the forecast is $F_{t-1}(D_t) = 1$,

$$E(d_t^2 | F_{t-1}(D_t) = 1) = 4(1-q) \qquad (2.3.22)$$

and if the forecast is $F_{t-1}(D_t) = -1$, then

$$E(d_t^2 | F_{t-1}(D_t) = -1) = 4(1-q) \text{, the identical result} \qquad (2.3.23)$$

So, the new conditional variance at time t is

[13]Note that our model assumes that the accuracy of the forecast is independent of the return on the asset.

$$\sigma_t^{*2} = \sigma_\xi^2 + 4(1-q)(b_0 + \sum_{j=1}^{n-1} b_j u_{t-j}^2)^2 \tag{2.3.24}$$

Notice that if the prediction is perfect, $q = 1$, and $\sigma_t^{*2} = \sigma_\xi^2$, which is homoskedastic. If the prediction is random, $q = 0.5$, then

$$\sigma_t^{*2} = \sigma_\xi^2 + 2(b_0 + \sum_{j=1}^{n-1} b_j u_{t-j}^2)^2 \tag{2.3.25}$$

Therefore, increasing precision of the forecast of market direction close to 1 decreases the conditional variance in any period t and reduces the heteroskedasticity because of the attenuation to zero of

$$4(1-q)(b_0 + \sum_{j=1}^{n-1} b_j u_{t-j}^2)^2$$

that occurs as the precision q, of the forecast increases. Next, we show how this model provides a simpler alternative estimation approach for the popular class of GARCH (1,1) models.

### 2.3.4. Alternative Approach to Estimating GARCH Parameters

The advantage of the specification in equation (2.3.15) is that the return is a linear function of endogenous and exogenous $X_{t-1}$ that can be estimated by OLS, rather than using maximum likelihood or GMM and a numerical algorithm. Contrary to section 2.3.3, we now assume that $D_t$ is not in the information set at t-1. However, the artificial regression model allows us to implicitly estimate both the conditional mean and GARCH parameters, in a traditional GARCH (1,1) model.

Running the regressions (2.3.14) or (2.3.15) and using the $b_j$ coefficient estimates, the implied values for $\gamma_0, \gamma_1$ and $\theta$, may be calculated resulting in an overidentified system of n equations with three unknowns. The values are given by

$$\theta = \frac{b_2}{b_1} = \frac{b_3}{b_2} = \frac{b_4}{b_3} = \ldots \quad (2.3.26)$$

$$\gamma_0 = \frac{4b_0^2(1+\gamma_1)}{(1+\theta+\theta^2+\theta^3+\ldots+\theta^{n-2})(2+\gamma_1)^2} \quad (2.3.27)$$

and,

$$\gamma_1 = -1 + 2b_0b_1 + \sqrt{1+4(b_0b_1)^2} \quad (2.3.28)$$

Equation (2.3.28) is a solution to a quadratic equation and we consider only the positive component of the square root.

The solutions for $\gamma_0, \gamma_1$ and $\theta$ do not depend upon the dimensions of the Taylor series, but the parameters, $b_j$ do. Our solution assumes the accuracy of the order (but not the dimension) of the Taylor series approximation.[14] It is possible that the expansion point of the Taylor series approximation could affect the artificial regression model. In order to test this, we have also derived the artificial regression model and the implied values of the GARCH parameters using a Taylor series approximation for the variance of the error term about the unconditional variance. These analytics are presented in Appendix II.

[14] In addition, we have also derived analytic solutions for the second order Taylor series approximation. For brevity, we report only the models using the first order Taylor series approximation in this study.

In addition we have also derived the artificial regression model when $|u_t|$ is first obtained as a first order Taylor series approximation consisting of lags of squared residuals and lagged conditional variance terms. The substitution for the conditional variance in terms of the lagged squared residuals is done next and the implied values of the GARCH parameters are obtained. These analytics are also presented in Appendix II.

### 2.3.5. A Summary of the Artificial Regression Approach

We have presented the fundamental ideas behind the artificial regression model. The intuition underlying this model is that misspecification of the mean manifests as heteroskedasticity in the residuals. Hence, an increase in the precision of information should lessen heteroskedasticity in the error structure. Our model is more in the tradition of Tong (1990) and Hamilton (1991). Tong (1990) proposes the threshold model in the mean and Hamilton (1991) develops the regime switching process, where it is shown that regime switching is the primary source for conditional heteroskedasticity. However, we adopt a different approach and focus on the decomposition of the residual into sign and magnitude components and then proceed to show that this results in the artificial regression model, a nonlinear specification. We then demonstrate that the benefit of this nonlinear specification is that it mitigates heteroskedasticity in the residuals by incorporating return sign predictability. In addition, we show how this model provides a simpler estimation approach for the popular class of GARCH models.

Notice that our approach considers assets with time varying first and second moments and simplifies the econometric specification to one where only the first moment needs to be specified. Hence, estimation can be done by OLS. Thus, multivariate extensions of the artificial regression model are straightforward in theory and application. Hence, compared to the typical GARCH specification, our estimation approach would model large systems and instrument variables and thus facilitate more complexity in modeling.

In the next two sections, we propose to test the effectiveness of the artificial regression model on two criteria.[15] First, we use a Monte Carlo study to test the effectiveness of the artificial regression model as a tool to obtain GARCH estimates. In effect, we test the tradeoff between easy econometrics and the costs of the Taylor series approximation. Second, using stock return data, we test the impact of market direction forecast on reducing heteroskedasticity in the artificial regression model.

## 2.4. MONTE CARLO SIMULATIONS

### 2.4.1. The Monte Carlo Design

In this section, using a simple Monte Carlo experiment, we examine the performance of the artificial regression model framework in estimating the GARCH (1,1) parameters. First, we generate a time series with a GARCH (1,1) data generating process. Next, we estimate the GARCH (1,1) parameters using the traditional numerical maximization approach. We then, estimate the artificial regression model using OLS.

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[15] In this study, we are not comparing the predictive power of the proposed artificial regression model with the traditional GARCH model. Future work would include out-of-sample tests of the models. As shown in West and Cho (1995), GARCH models have poor predictive power. However, the significant predictive power of simpler linear models as in Breen, Glosten, and Jagannathan (1994) and Beveridge and Bauer (1994) suggests that the proposed model should have better predictive powers.

Substituting the coefficient estimates from the preceding artificial regression model in equations (2.3.26) to (2.3.28), we obtain the corresponding implied values of the GARCH parameters. Finally, we compare the values of the GARCH parameters obtained using the two approaches with the true values. The mechanics of the Monte Carlo design are presented next.

In order to focus on the estimation properties for conditional variance parameters, we adopt a constant conditional mean. The Monte Carlo experiment is based on a GARCH (1,1) data generating process specified as,

$$r_t = \beta_0 + u_t; \quad u_t = \sqrt{\sigma_t^2}.v_t, \quad \text{where } v_t \sim N(0,1)$$
$$\sigma_t^2 = \gamma_0 + \gamma_1 u_{t-1}^2 + \theta\sigma_{t-1}^2 \qquad (2.4.1)$$

where $u_t$ is the mean equation innovation at time t, $\sigma_t^2$ is the conditional variance at time t, $v_t$ is the normalized residual at time t that is generated from a standard normal distribution, $r_t$ is the simulated return at time t, and $\beta_0$, $\gamma_0$, $\gamma_1$, and $\theta$ are constant parameters.

In order to determine how the specification responds to changes in persistence, we examine the artificial regression framework in two scenarios. Hence, we perform Monte Carlo simulations for high and medium persistence in volatility [c.f., Brock, Hsieh, and LeBaron (1991) and Engle and Ng (1993)]. We choose two sets of parameter values to reflect large variations in persistence: high persistence where $(\gamma_0, \gamma_1, \theta) = (0.05,0.05,0.90)$, and medium persistence where $(\gamma_0, \gamma_1, \theta) = (0.35,0.05,0.60)$. The parameter values chosen correspond to the empirical findings in the estimation of weekly financial stock returns (c.f., Brock, Hsieh, and LeBaron, 1992). Thus, in this experiment for each scenario, we run 100 simulations for each of the three samples of size 300, 1000 and 6000.[16]

Under the assumption of conditional normality, the log-likelihood function for the GARCH (1,1) model is given by

$$L(\vartheta) = -\frac{1}{2}\sum_{t=1}^{T}(\log(\sigma_t^2) + \frac{u_t^2}{\sigma_t^2}) \tag{2.4.2}$$

For each replication, a GARCH (1,1) model is estimated by numerical maximization of the conditional log-likelihood function in equation (2.4.2). Maximums are obtained using the BHHH algorithm, in accordance with the estimation methodology found in the literature [see for example Bollerslev (1986)]. For each draw, the artificial regression model in equation (2.3.15) is estimated using OLS as a two-step (T-S) procedure, as shown next.

Step 1: Since our purpose is to examine the performance of the conditional variance parameters, we use a constant mean, $\beta_0$.[17]

$$r_t = \beta_0 + u_t \tag{2.4.3}$$

The next step is to regress the return series $r_t$ on a constant. From the residual $\hat{u}_t$, define $D_t$ as,

$D_t = 1$, if $\hat{u}_t > 0$, else $D_t = -1$.

[16] We have chosen the number of runs for the Monte Carlo study arbitrarily. However, increasing the runs is likely to improve the estimates for the smaller sample sizes. We do not anticipate significant changes in the results for the larger samples in our study.

Step 2:

a. Form the artificial regression

For a given $D_t$ from step 1, we estimate the artificial regression model (2.4.4), using OLS,

$$\frac{r_t}{D_t} = b_0 + d_1 D_t + b_1 \hat{u}_{t-1}^2 + b_2 \hat{u}_{t-2}^2 + b_3 \hat{u}_{t-3}^2 + b_4 \hat{u}_{t-4}^2 + \ldots\ldots \\ + b_{15} \hat{u}_{t-k}^2 + \xi_t, \text{where} \xi_t \sim N(0, \sigma_\xi^2) \qquad (2.4.4)$$

b. Estimate the optimum number of lags, k, of $\hat{u}_t^2$ in equation (2.4.4) based on the length at which the Schwartz Information Criterion (SIC) is minimum.[18]

c. Using the new efficient estimate of $\beta_0, b_0$, the two-step procedure can be iterated for efficiency as in Ferson and Foerster (1994).

Notice that the coefficient of $D_t$ in equation (2.4.4) is equivalent to $\beta_0$ in equation (2.4.3). As an aside, note that the traditional GARCH estimation approach imposes positivity and stationarity constraints during estimation. However, the artificial regression approach tested in this study imposes no constraints on the system. Thus, our Monte Carlo study is by design biased in favor of the BHHH estimates because we estimate under a true null hypothesis.

We would like to point out that the two-step procedure simply replaces the unobserved component $D_t$ from the auxiliary model (2.4.3). It is well know that

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[17] In the Monte Carlo study we have not experimented any further with instruments in the mean equations. We focus on this aspect and its impact on the artificial regression model in the next section, when we examine stock returns data.
[18] The Akaike's (1974) information criterion which is defined as AIC = -2max L ($\psi$) + 2k, tends to overparametrize models (Hannan, 1980). We therefore choose the SIC where
SIC = -2max L ($\psi$) + k ln(T) where L is the log-likelihood function, k is the number of parameters in the estimated model, and T is the number of observations and $\psi$ is the parameter vector.

estimated standard errors and related test statistics based on these procedures are inconsistent. However, the iterated approach suggested by Ferson and Foerster (1994) used in step 2c. will yield consistent estimates.

### 2.4.2. The Monte Carlo Results

The results from the Monte Carlo simulation for both the high and the medium persistent univariate GARCH (1,1) models are tabulated in Tables 2.1 and 2.2 for three sample sizes, representative of weekly stock return data. Based on the SIC criterion, for high persistence, we find that 15 lags of $\hat{u}_t^2$ are optimal and for medium persistence, 5 lags are optimal. We report the GARCH parameters obtained using the maximum likelihood GARCH estimation technique (hereafter, BHHH) and the implied GARCH estimates (hereafter, AR), obtained analytically from the estimation of the artificial regression model using the Taylor expansion in equations 2.3.24 to 2.3.26[19]. We also present the corresponding t-statistic for each estimate and the results of testing the difference of means between the estimates and the true values. All the estimation results are based on 100 replications. For the artificial regression models, as the diagnostics in the simulations indicated that the residuals were homoskedastic, we performed only one

[19] Theoretically $\theta = \frac{b_2}{b_1} = \frac{b_3}{b_2} = \frac{b_4}{b_3} = \ldots\ldots$ However, empirically we have not imposed this constraint. $\theta$. Hence, the Monte Carlo simulation setup is biased towards GARCH. The results indicate that $\theta = \frac{b_2}{b_1}$ has the least variability. We report this value of $\theta$. As discussed in Section 2.4.3, it seems further improvements in the model could come from imposing the theoretical constraints on $\theta$. Moreover, in the proposed model, it is easy to impose such restrictions,

iteration.

**2.4.2.A. GARCH (1,1): High persistence ($\gamma_0$=0.05, $\gamma_1$=0.05, $\theta$=0.90)**

We now turn to the results for high persistence in Table 2.1. In Panel A, for a sample size of 6000, compared to the BHHH, the AR estimates are closer to the true values. However, both BHHH and AR estimates are significantly different from the true values. Notice, that with no constraints imposed on the AR system, the mean value of $\theta$ is greater than one and this is a violation of the positivity constraint. From Panel B, for a educed sample size of 1000, we find that the variability of the estimates increases. However, while the BHHH estimate for $\gamma_1$ is closer to the true parameter value compared to the AR, the $\theta$ estimate from AR is significantly closer to the true value. Finally, in Panel C we find that as the sample size is reduced to 300, the BHHH and the AR estimates for $\gamma_0$ and $\gamma_1$ are significantly different from the true values, but in contrast to BHHH, the AR estimate for $\theta$ is not significantly different from the true value

One can see that there are comparative advantages and disadvantages to both approaches. With large sample sizes, the AR seems to be performing better; but, with small sample sizes, both models seem to perform badly.

**2.4.2.B. GARCH (1,1): Medium persistence ($\gamma_0$=0.35, $\gamma_1$=0.05, $\theta$=0.60)**

We next report the results for moderate persistence in volatility in Table 2.2. As before, results are presented for the univariate GARCH (1,1) for three different sample sizes of 6,000, 1000 and 300 respectively, representative of weekly stock return data.

For a sample size of 6,000, in a departure from the earlier case of high persistence in volatility, the artificial regression model requires only 5 lags of squared residuals to minimize the SIC. Further, the BHHH estimates are closer to the true values than the AR estimates. With a sample size of 1,000 both the BHHH and AR estimates are significantly different from the true value for $\gamma_1$ but are not significantly different from the true values for $\gamma_0$ and $\theta$. As in the high persistence case, the variability of the estimates increases considerably as the sample size is reduced to 300. The AR estimate is significantly different from the true value for $\gamma_1$. However unlike the BHHH estimates, the AR estimates are not significantly different from the true values for $\gamma_0$ and $\theta$.

The simulation results suggest that the BHHH and AR estimates are comparable for high persistence in volatility in large samples. Both approaches perform poorly for reduced sample sizes. However, in the case of medium persistence, AR seems to outperform BHHH. While BHHH outperforms AR for large samples, for smaller sample sizes, AR performs better.

### 2.4.3. Summary of Monte Carlo analysis

In summary, based on the difference of means between the estimates and the true values of the GARCH parameters, the results illustrate that the artificial regression model seems to work as well as the popular approach in generating GARCH estimates. Although the model is a natural one and is mathematically straightforward, it is surprising that it does as well as it does, given that the simulation design is biased in favor of the BHHH, in as much as no restrictions are imposed on the coefficients of the artificial regression model. This is particularly promising as various asset return series exhibit a long memory property and the artificial regression model with no restrictions imposed should reveal this property.[20]

Some aspects about the artificial regression model remain troubling. For one thing, the estimated parameters, especially $\theta$ seems to be excessively sensitive to the sample size and tend to behave badly. Three approaches come to mind as possible solutions to this problem.

First, it is possible that the problem lies in the approximation used. We provide the analytics for the model using the Taylor series approximation about the unconditional variance in Appendix II. As the revised expansion point is now $\frac{\gamma_0}{1-\gamma_1-\theta}$, involving three parameters, the analytics are relatively more complex, compared to the Taylor expansion. However, we do not find significant improvements in AR estimates using this approach (results not reported here). On the other hand, the results indicate that the model would perform better if constraints were imposed on the AR estimates.

Second, we look at higher-order Taylor series expansions, specifically the second-order Taylor series expansion. The analytics get very unwieldy and are not reported here. In addition, the estimates are more sensitive than before to non-imposition of constraints in the AR system. Hence, increasing the complexity of the approximation takes away the simplicity of our approach, while reducing the tractability of the model.

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[20] In particular, it may be observed that for series exhibiting the long memory property, the GARCH restrictions are not true. Specifically, consider the implied parameter $\theta$ which is the ratio of adjacent coefficients. For a series with long memory property, this restriction would not be true and the artificial regression model would capture it.

Third, selecting the lags in the artificial regression model, using SIC on a draw specific basis could yield the "best" estimates for each run. We have not tested this approach, but we would like to point out that our lag selection was made based on 20 random runs. These first-pass tests do not indicate that the lag specification would yield significant improvements.

That said, it seems further improvements in the behavior of the estimates could come only from imposing the constraints in the AR model. While this does detract from the model, the advantage is that restrictions are easier to impose in our system. Furthermore, ease in imposing constraints gives additional flexibility as the artificial regression model could then accommodate various GARCH specifications.

Finally, it would be interesting to compare the properties of the estimators if errors were generated from a quasi-normal distribution. Since the QMLE is inefficient when the distribution is misspecified, we would expect the artificial regression model to outperform the BHHH estimator.

We are now ready to test the model on stock returns data. In the next section, we test the impact of market direction forecast on reducing heteroskedasticity in the artificial regression model.

## 2.5. Empirical Analysis: Canadian and U.S. stock index returns

### 2.5.1. Data and Estimation

Our objective in this section is to test the effectiveness of market direction forecasting on reducing heteroskedasticity in the artificial regression model. The data we analyze are the time series of the daily and monthly closing values of the TSE300 Total

Return Index (TRI) for the period January 1977 to December 1994 and the CRSP value weighted index for the period July 1962 to December 1994. Both indexes are value-weighted composite indexes with distributions. The first difference of the natural logarithm of the closing values of the index is used as the stock return measure.

In this part of the analysis, we adopt the following approach. For each of the four return series, we first estimate the benchmark model, which is the linear autoregressive model in the mean (2.5.1).

$$r_t = \delta_0 + \delta X_{t-1} + w_t ;\text{where} \quad w_t \sim N(0, \sigma^2) \qquad (2.5.1)$$

Next, we estimate the univariate GARCH (1,1) model. We then, estimate the corresponding artificial regression model. Next, we examine the diagnostics for each specification to check the reduction in heteroskedasticity. Finally, we compare the GARCH estimates obtained using the two approaches.

For the Canadian and U.S. indexes, we estimate univariate GARCH (1,1) models for daily and monthly returns as follows:

$$\begin{aligned} & r_t = \beta_0 + \beta X_{t-1} + u_t ;\ u_t = \sqrt{\sigma_t^2}.v_t, \text{where } v_t \sim N(0,1) \\ & \sigma_t^2 = \gamma_0 + \gamma_1 u_{t-1}^2 + \theta \sigma_{t-1}^2 \end{aligned} \qquad (2.5.2)$$

Our choice of a mean specification is guided by prior empirical work. For example, Lo and MacKinlay (1990) show that nonsynchronous trading in the stocks that constitute the index leads to serial dependence in the index returns. Seasonality effects including the day-of-the-week effect and January effect have been well documented. Hence, our benchmark information set ($\phi_S$), for the mean equation consists of the following instruments: lagged returns, a seasonal dummy and an October 1987 crash dummy. Our larger information set ($\phi_L$) includes the lag of the squared returns in addition to the preceding instruments. Our purpose in using the two information sets is to test the impact of the information set on the precision of the sign component and its implications for heteroskedasticity.

As in the Monte Carlo study, the GARCH (1,1) model is estimated by numerical maximization of the conditional log-likelihood function in equation (2.4.2) using the BHHH algorithm. This model is hereafter referred to as BHHH.

Using the same information set, the artificial regression model in equation (2.5.2) (hereafter, AR) is also estimated using OLS in the two-step approach similar to that used in the Monte Carlo study.

$$r_t = \alpha_0 + \beta X_{t-1} + u_t;\ u_t \sim N(0, \sigma_t^2)$$

$$D_t = 1 \text{ if } \hat{u}_t > 0, \text{ and } -1, \text{ otherwise.}$$

$$\frac{r_t}{D_t} = \frac{\hat{\alpha}_0}{D_t} + \frac{\hat{\beta} X_{t-1}}{D_t} + b_0 + \sum_{j=1}^{n-1} b_j \hat{u}_{t-j}^2 + \xi_t \tag{2.5.3}$$

Notice that $D_t$ here, does not indicate perfect foresight for $D_t$.[21] The precision of $D_t$ depends on the instruments in the information set.

Moving on to diagnostics, residual and squared residual correlations are commonly employed in the specification and diagnostic checking of GARCH-type models.[22] Ljung and Box (1978) portmanteau tests for serial correlation in the

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[21] $D_t$ could also be estimated using a procedure similar to Breen, Glosten, and Jagannathan (1989) or Beveridge and Bauer (1994). They use interest rates in the information set to forecast stock returns.
[22] Bollerslev and Mikkelsen (1996) find that AIC and SIC model selection criteria and the portmanteau tests for residual autocorrelation work effectively in specification tests in GARCH-type models.

standardized residuals $\frac{\hat{u}_t}{\sigma_t}$ and the squared standardized residuals are a popular diagnostic check.[23] If the AR specification is the correct specification, then the diagnostics should indicate that the residual, $\xi_t$, in equation (2.5.2) is homoskedastic. If the conditional variance is correctly specified in the GARCH (1,1) form, the standardized residuals, $\frac{\hat{u}_t}{\sigma_t}$ in equation (2.5.2) should behave as white noise.

We introduce the benchmark model (2.5.3) for two reasons. First, it gives a benchmark to interpret the improvement in the mean equation in the artificial regression model after the introduction of the return's sign component. Second, it serves as a baseline to gauge the reduction of heteroskedasticity in the AR and BHHH approaches.

### 2.5.2. Analysis and Results

Table 2.3 gives the summary statistics for the four return series. It is clear that the normality assumption is inappropriate for the returns data, as the series exhibit significant leptokurtosis and second-order dependence. The results suggest that the monthly and daily return series are not normally distributed.

Tables 2.4 and 2.5 provide residual-based diagnostic tests to evaluate the statistical adequacy of the models i.e. reduction of heteroskedasticity, for the daily and monthly return series. We first report the results for the benchmark, linear auto-regressive model (hereafter BM) estimated by OLS. This is the benchmark model for the

[23] Following Ljung and Box (1978), when testing the residuals from an ARCH-type model, the portmanteau test is asymptotically chi-square distributed with K-k degrees of freedom, where k denotes the estimated ARCH parameters and K, the number of observations in the series.

conditional mean equation. We then, provide comparative results for the AR and BHHH models. Specifically, we report the Ljung-Box (1978) (LB) portmanteau test statistics associated with the standardized residuals, Q(p) and the squared residuals, $Q^2(p)$ respectively[24]. The first one tests for linear dependence, while the second one is designed to pick up higher order dependence and the presence of conditional heteroskedasticity. Both these statistics are distributed as $\chi_p^2$. For the AR specification, we report the Ljung-Box statistics for 8 lags of the raw residuals $\xi_t$, and raw squared residuals, $\xi_t^2$. For the BHHH model, similar diagnostics are presented for the standardized residuals, $\frac{u_t}{\sigma_t}$ and the squared standardized residuals, $\frac{u_t^2}{\sigma_t^2}$. For the BM model, diagnostics are provided for the residual, $w_t$.

Table 2.4 presents the comparative diagnostics for the daily return models. Specifically, we examine Canadian daily returns in Panel A, for the small information set. The sample autocorrelations for the BM model indicate that the conditional mean is adequately specified. The Ljung-Box, Q(p) statistic indicates that the mean explains a significant part of the linear dependence. However the $Q^2(p)$ statistics suggests strong nonlinearity in the data. In the AR model, the R-squared is substantially higher compared to the BM model. Notice that, the basic difference between the BM and AR models is the introduction of the sign component. However, the Ljung-Box diagnostics suggest that

[24] $Q = T(T+2)\sum_{k=1}^{m}(T-k)^{-1}\hat{r}_k^2$ where T is the number of observations in the series being investigated, $\hat{r}_k$ is the sample autocorrelation of the residuals and m=8 for the present study.

the correlation for both residuals and squared residuals has increased. We next look at the BHHH model. Obviously the GARCH (1,1) is not an inadequate specification as the Ljung-Box statistic for the squared residual is significant for the BHHH model. Moving on to Panel B and the U.S.returns, we note that the BHHH model now seems well specified and performs better than the AR model. However, the AR model seems a big improvement over the corresponding BM model.

We now check if expanding the information set has any impact. The most notable feature for both daily U.S. and Canadian returns is the significant reduction in the Ljung-Box statistic for the AR model, when the larger information set is considered. This suggests that as the precision of $D_t$ increases, the errors will tend to be homoskedastic. This is consistent with Stambaugh's (1993) findings that, in the presence of heteroskedastic regression disturbances, it is possible that expanding the set of instruments used to compute an instrumental variables estimator can produce efficiency gains.

Overall the BHHH model outperforms the AR model for both U.S. and Canadian daily returns in both information sets in reducing the heteroskedasticity. Note that the artificial regression model does not provide a sufficient fit, as the squared standardized residuals are significantly correlated. Notice also that in the case of daily U.S. returns using a larger information set, $\phi_L$, the BHHH model does not converge.

In the case of monthly returns, the results in Table 2.5 indicate that the AR model's performance is comparable to BHHH for Canadian monthly returns in the larger information set. As before, the heteroskedasticity as indicated by the Box-Pierce statistics is reduced in the artificial regression model relative to the benchmark model. For Canadian returns, once again the AR model with the larger information set has a significant reduction in heteroskedasticity.

The empirical results suggest that heteroskedasticity is linked to the precision of the sign component and the data is illustrative of the convergence of the estimates in this specification. Using an expanded instrument set similar to Turtle, Buse and Korkie (1994) or Lo and MacKinlay (1995) or using the methodology of Breen, Glosten and Jagannathan (1989) in predicting the sign component should improve the specification. Further, the estimation problem of the BHHH model is evident, as the model does not converge for the large information set in daily U.S. returns.

Next we assume that the process is a GARCH (1,1) and we use real data to estimate the GARCH parameters with the AR and BHHH approaches. These results are presented in Table 2.6. In Panel A, we first compare the parameter estimates for the smaller information set. It is evident that the estimate of the parameter $\gamma_1$ using the AR approach is smaller than the BHHH estimate for all the series. The values of the constant in the mean equation and the parameter $\gamma_0$ are similar; however, the values of $\theta$ obtained from the two approaches are significantly different.

In the larger information set, the values of $\theta$ seem comparable; but, the $\gamma_1$ (AR) parameter is negative for daily returns.

Linking up the results from tables 2.4–2.6, it appears that when the diagnostics support the GARCH (1,1) specification for a return series, the parameters obtained using the AR and BHHH approach seem comparable. Notice for example that from tables 2.4

and 2.5, the GARCH (1,1) seems a better specification for the CRSP value-weighted returns compared to the TSE 300 TRI return. Interestingly, from table 2.6, the estimates from the two approaches are more divergent for the TSE 300 returns. Hence, the AR approach is sensitive to the correctness of the volatility specification.

### 2.5.3. Summary of the Empirical Analysis:

The results imply that a richer information set and precision of the sign component help the mean and reduce the volatility. Recent research (Bollerslev and Mikkelsen, 1996) has shown that an appropriate model for daily return series is the fractionally integrated GARCH model. For monthly series, the GARCH (1,1) might be an adequate specification. Hence, we would conjecture that since the AR model behaves better for the monthly series, the artificial regression model is more sensitive to misspecification of the underlying volatility process. This is supported by our findings in both the Monte Carlo study and the empirical analysis, where the AR model behaves well, when the underlying volatility process is correctly specified.

## 2.6 CONCLUSIONS

In this paper, we propose an "artificial regression model" as an alternative nonlinear specification for asset returns. The central idea underlying this model is the error decomposition into magnitude and sign components resulting in a threshold model in the mean. The uniqueness of this paper is that we then proceed to show the benefit of this nonlinear specification in mitigating heteroskedasticity in the residuals by incorporating return sign predictability. In the limit, perfect prediction of a return's sign is shown to completely eliminate heteroskedasticity. In addition, we show how this model provides closed-form solutions for the GARCH parameters and thus provides a simpler estimation approach for the popular class of GARCH models.

The economic intuition here is that increasing the precision of information should shrink volatility and lessen heteroskedasticity in the error structure that is caused by misspecifying the nonlinear conditional mean. From a specification point of view, the results of this research must be viewed as supportive of the premise that asset returns depend on the sample information as a nonlinear function of the variables in the information set. Incorporating these nonlinearities may increase the predictability of the dependent variable, whereas presence of GARCH does not necessarily provide additional forecastability of the dependent variable.

The artificial regression model simplifies the econometric specification from a two-moment (time-varying) specification to one where only the first moment needs to be specified. Hence, estimation can be done by OLS. Thus, an important practical importance of the model is that it generalizes to a multivariate approach in a natural way and there is a major reduction in computational complexity in estimation. Hence, compared to the typical GARCH specification, our estimation approach would model large systems and instrument variables and thus facilitate more complexity in modeling. Given that many issues in finance particularly asset pricing deal with multivariate systems, our proposed framework has important practical implications.

Further, the artificial regression model is a flexible specification as restrictions can be easily imposed. Hence it can handle long decay structures that the traditional GARCH model cannot.

We then proceed to test our model on two criteria. First, we use a Monte Carlo experiment to test the effectiveness of the artificial regression model in generating GARCH estimates. Our overall findings are that our model works as well as the popular approach for generating GARCH estimates. Additional comparisons could be made from out-of-sample tests of the differences between the artificial regression and GARCH (1,1) model in predicting conditional means and volatilities. The out-of-sample testing has been left for future research. That said, it seems further improvements in the behavior of the estimates could come only from imposing the constraints in the artificial regression model.

Second, we test the model on stock returns data to study the impact of market direction forecast on reducing heteroskedasticity in the artificial regression model. The results suggest that heteroskedasticity is linked to the precision of the sign component. A richer information set improves the precision of the sign component and helps the mean and reduces the volatility. Our analysis also indicates that the artificial regression model is more sensitive to misspecification of the underlying volatility process. The bottomline then, is that in the tradeoff between easy econometrics and the costs of the Taylor series approximation, we are better off using the artificial regression model.

Areas of future research that appear promising include the following: firstly testing the model in a multivariate setting; secondly, testing how a specification that incorporates the inherent nonlinearity in the data affects asset pricing tests; and finally, extending the artificial regression model to other popular GARCH models such as the exponential GARCH specification.[25]

[25] In terms of implementing the artificial regression model procedure with larger multivariate models, the major obstacle would be the assumption of the covariance matrix. Adopting the approach used in multivariate GARCH, first a constant correlation covariance matrix could be assumed, followed by more complicated covariance matrices.

**Table 2.1**

**Estimates of mean and variance equation parameters-Monte Carlo study: High Persistence**

This table presents the mean and variance equation parameter estimates from a Monte Carlo experiment (100 replications) based on the GARCH (1,1) data generating process. $u_t$ is the unpredictable return (the residual) at time t, $\sigma_t^2$ is the conditional variance at time t, $v_t$ is the normalized residual at time t which is generated from a standard normal random generator, $r_t$ is the simulated return at time t and $\beta_0, \gamma_0, \gamma_1$, and 0 are constant parameters. The set of parameter values considered is: high persistence where ($\gamma_0, \gamma_1$, 0) = (0.05,0.05,0.90).

**GARCH model: (Estimation by Maximum Likelihood -numerical algorithm) - BHHH**

$$r_t = \beta_0 + u_t ; u_t = \sqrt{\sigma_t^2}.v_t, \text{where} v_t \sim N(0,1); \quad \sigma_t^2 = \gamma_0 + \gamma_1 u_{t-1}^2 + 0\sigma_{t-1}^2$$

For each replication, a GARCH(1,1) model is estimated by numerically maximizing the conditional log-likelihood function using the BHHH algorithm. These estimates are termed BHHH in the panels below.

**ARTIFICIAL REGRESSION MODEL**: First Order 16-dimensional Taylor series approximation (Estimation by OLS) - **AR**

$$\frac{r_t}{D_t} = b_0 + d_1 D_t + b_1 u_{t-1}^2 + b_2 u_{t-2}^2 + b_3 u_{t-3}^2 + b_4 u_{t-4}^2 + \ldots\ldots\ldots\ldots b_{15} u_{t-15}^2 + \xi_t, \text{ where} \xi_t \sim N(0,\sigma_\xi^2).$$

For each replication, the artificial regression model is estimated by OLS and the implied GARCH estimates are calculated. These estimates are termed AR in the panels below.

| | $\beta_0$ BHHH | $\beta_0$ AR | $\gamma_0$ BHHH | $\gamma_0$ AR | $\gamma_1$ BHHH | $\gamma_1$ AR | 0 BHHH | 0 AR |
|---|---|---|---|---|---|---|---|---|
| PANEL A | | | | | | | | |
| **SAMPLE SIZE = 6000** | | | | | | | | |
| TRUE VALUE | 0.0025 | 0.0025 | 0.0500 | 0.0500 | 0.0500 | 0.0500 | 0.9000 | 0.9000 |
| MEAN | 0.0028 | 0.0027 | 0.2253 | 0.0716 | 0.0449 | 0.0231 | 0.7289 | 1.1170 |
| t-statistic | 12.1421 | 17.0446 | 4.0835 | 8.1981 | 28.4910 | 32.8765 | 13.4915 | 14.0826 |
| t-statistic: difference from true value = 0 | 1.1219 | 1.1075 | 3.1773 | 2.4749 | -3.2227 | -38.3316 | -3.1674 | 2.7360 |
| PANEL B | | | | | | | | |
| **SAMPLE SIZE = 1000** | | | | | | | | |
| TRUE VALUE | 0.0025 | 0.0025 | 0.0500 | 0.0500 | 0.0500 | 0.0500 | 0.9000 | 0.9000 |
| MEAN | 0.0023 | 0.0021 | 0.2080 | 0.1830 | 0.0467 | 0.0236 | 0.7455 | 1.2215 |
| t-statistic | 7.5427 | 6.9186 | 4.5679 | 8.2220 | 19.4884 | 11.9500 | 16.8535 | 2.4140 |
| t-statistic :difference from true value = 0 | -0.7072 | -1.4384 | 3.4697 | 5.9749 | -1.3628 | -13.3240 | -3.4935 | 0.6353 |
| PANEL C | | | | | | | | |
| **SAMPLE SIZE = 300** | | | | | | | | |
| TRUE VALUE | 0.0025 | 0.0025 | 0.0500 | 0.0500 | 0.0500 | 0.0500 | 0.9000 | 0.9000 |
| MEAN | 0.0020 | 0.0024 | 0.2859 | 0.2605 | 0.0373 | 0.0208 | 0.6729 | 0.7944 |
| t-statistic | 4.0095 | 4.1495 | 5.8499 | 8.1553 | 7.0212 | 5.0365 | 13.7395 | 1.5098 |
| t-statistic: difference from true value = 0 | -0.9707 | -0.2162 | 4.8267 | 6.5900 | -2.3807 | -7.0865 | -4.6370 | -0.2007 |

**Table 2.2**

**Estimates of mean and variance equation parameters-Monte Carlo study: Medium Persistence**

This table presents the variance equation parameter estimates from a Monte Carlo experiment (100 replications) based on the GARCH (1,1) data generating process. $u_t$ is the unpredictable return (the residual) at time t, $\sigma_t^2$ is the conditional variance at time t, $v_t$ is the normalized residual at time t which is generated from a standard normal random generator, $r_t$ is the simulated return at time t and $\beta_0, \gamma_0, \gamma_1$, and 0 are constant parameters. The set of parameter values considered is: high persistence where ($\gamma_0, \gamma_1$, 0) = (0.35,0.05,0.60).

**GARCH model: (Estimation by Maximum Likelihood –numerical algorithm) - BHHH**

$$r_t = \beta_0 + u_t; u_t = \sqrt{\sigma_t^2}.v_t, \text{where} v_t \sim N(0,1); \quad \sigma_t^2 = \gamma_0 + \gamma_1 u_{t-1}^2 + 0\sigma_{t-1}^2$$

For each replication, a GARCH (1,1) model is estimated by numerically maximizing the conditional log-likelihood function using the BHHH algorithm. These estimates are termed BHHH in the panels below.

**ARTIFICIAL REGRESSION MODEL:** First Order 6-dimensional Taylor series approximation (Estimation by OLS) - AR

$$\frac{r_t}{D_t} = b_0 + d_1 D_t + b_1 u_{t-1}^2 + b_2 u_{t-2}^2 + b_3 u_{t-3}^2 + b_4 u_{t-4}^2 + b_5 u_{t-5}^2 + \xi_t, \text{where } \xi_t \sim N(0, \sigma_\xi^2).$$

For each replication, the artificial regression model is estimated by OLS and the implied GARCH estimates are calculated. These estimates are termed AR in the panel below.

| | $\beta_0$ BHHH | $\beta_0$ AR | $\gamma_0$ BHHH | $\gamma_0$ AR | $\gamma_1$ BHHH | $\gamma_1$ AR | 0 BHHH | 0 AR |
|---|---|---|---|---|---|---|---|---|
| **PANEL A** | | | | | | | | |
| **SAMPLE SIZE = 6000** | | | | | | | | |
| TRUE | 0.0025 | 0.0025 | 0.3500 | 0.3500 | 0.0500 | 0.0500 | 0.6000 | 0.6000 |
| MEAN | 0.0028 | 0.0028 | 0.3648 | 0.2343 | 0.0486 | 0.0280 | 0.5854 | 0.7900 |
| t-statistic | 21.8280 | 19.1575 | 22.6350 | 15.0185 | 27.3055 | 27.1096 | 34.0768 | 11.8298 |
| t-statistic: difference from true value = 0 | 2.1063 | 2.1504 | 0.9166 | -7.4179 | -0.7877 | -21.2468 | -0.8526 | 2.8456 |
| **PANEL B** | | | | | | | | |
| **SAMPLE SIZE = 1000** | | | | | | | | |
| TRUE | 0.0025 | 0.0025 | 0.3500 | 0.3500 | 0.0500 | 0.0500 | 0.6000 | 0.6000 |
| MEAN | 0.0033 | 0.0033 | 0.3994 | 0.2989 | 0.0402 | 0.0284 | 0.5585 | 0.4878 |
| t-statistic | 10.0985 | 9.6643 | 8.9210 | 10.1585 | 10.5845 | 11.7161 | 12.0276 | 1.7597 |
| t-statistic: difference from true value = 0 | 2.3187 | 2.4171 | 1.1035 | -1.7379 | -2.5702 | -8.9412 | -0.8936 | -0.4046 |
| **PANEL C** | | | | | | | | |
| **SAMPLE SIZE =300** | | | | | | | | |
| TRUE | 0.0025 | 0.0025 | 0.3500 | 0.3500 | 0.0500 | 0.0500 | 0.6000 | 0.6000 |
| MEAN | 0.0028 | 0.0029 | 0.6446 | 0.3121 | 0.0545 | 0.0283 | 0.2929 | 3.7657 |
| t-statistic | 4.7778 | 4.8840 | 10.7222 | 10.5936 | 7.7249 | 7.1115 | 4.8082 | 1.0116 |
| t-statistic: difference from true value = 0 | 0.5169 | 0.5926 | 4.9003 | -1.2874 | 0.6342 | -5.4503 | -5.0397 | 0.8504 |

**Table 2.3: Summary Statistics for Daily and Monthly return series for the CRSP Value-Weighted (VWT) Index [July 1962-December 1994] and TSE 300 Total Return Index (TRI) [January 1977-December 1994]**

| Return – $r_t$ | N-no. of obs. | Mean | Standard Deviation | Skewness | Kurtosis | Maximum | Minimum | Residual Autocorrelation Ljung-Box stat Q(8) | Squared residual Autocorrelation Ljung-Box stat $Q^2(8)$ |
|---|---|---|---|---|---|---|---|---|---|
| CRSP VWT Daily Returns | 8179 | 0.0004 | 0.0080 | -1.2526** | 31.5055** | 0.0868 | -0.1718 | 316.2118** | 1107.3843** |
| TSE 300 TRI Daily Returns | 4536 | 0.0005 | 0.0077 | -1.2674** | 23.9702** | 0.0865 | -0.1186 | 293.3728** | 1866.3255** |
| CRSP VWT Monthly Returns | 828 | 0.0094 | 0.0557 | 0.2589** | 8.0520** | 0.3828 | -0.2900 | 29.1221** | 262.5615** |
| TSE 300 TRI Monthly returns | 467 | 0.0075 | 0.0445 | -0.7811** | 3.4144** | 0.1530 | -0.2552 | 17.1612** | 13.4022* |

**denotes significance at the 1% level.

**Table 2.4: Comparison of Model Diagnostics:**
**Daily return series for the CRSP Value-Weighted (VWT) Index [July 1962-December 1994]**
**and TSE 300 Total Return Index (TRI) [January 1977-December 1994]-**

This table presents specification diagnostics for the GARCH (1,1) model using the BHHH and the artificial regression (AR) approaches.

The sample period spans 4536 return observations from January 1977-December 1994 for the TSE 300 total return index and 8179 return observations from July 1962-December 1994 for the CRSP value-weighted index.

SMALL INFORMATION SET INSTRUMENTS, $\varphi_S$ : Lagged Returns (CRSP- 4, TSE-3), Seasonal dummy, and Crash Dummy

LARGE INFORMATION SET INSTRUMENTS, $\varphi_L$ : Lagged Returns, Lagged Squared Returns (CRSP- 4, TSE-3), Seasonal dummy, and Crash Dummy

Residual analysis refers to $w_t$ for BM, $\frac{u_t}{\sigma_t}$ for BHHH, and $\xi_t$ for OLS.

**BENCHMARK MODEL** (Estimation by OLS) – **BM**

$r_t = \delta_0 + \delta X_t + w_t$; where $w_t \sim N(0, \sigma^2)$

**TRADITIONAL GARCH (1,1) MODEL** – (Estimation by maximum likelihood – numerical algorithm) – **BHHH**

$r_t = \beta_0 + \beta X_t + u'_t$ ; $u'_t = \sqrt{\sigma_t^2}.v_t$, where $v_t \sim N(0,1)$; $\sigma_t^2 = \gamma_0 + \gamma_1 u'^2_{t-1} + \theta\sigma_{t-1}^2$

ML estimation by BHHH - $L(\vartheta) = -\frac{1}{2}\sum_{t=1}^{T}(\log(\sigma_t^2) + \frac{u_t^2}{\sigma_t^2})$

**ARTIFICIAL REGRESSION MODEL** – Estimation by OLS – **AR**

$r_t = \gamma_0 + \gamma X_t + u_t$; where $u_t \sim N(0, \sigma_t^2)$

$D_t = 1$, if $u_t > 0$, else $D_t = -1$. $\frac{r_t}{D_t} = b_0 + d_1 D_t + \frac{bX_t}{D_t} + \sum_{k=1}^{K} b_k u_{t-k}^2 + \xi_t$, where $\xi_t \sim N(0, \sigma_\xi^2)$

| | SMALL INFORMATION SET, $\varphi_S$ | | | LARGE INFORMATION SET, $\varphi_L$ | | |
|---|---|---|---|---|---|---|
| | BM | AR | BHHH | BM | AR | BHHH |
| PANEL A - TSE300 Returns | | | | | | |
| Adj. $R^2$ / Log-Likelihood | $R^2$=0.0894 | $R^2$=0.2665 | L=-99.4968 | $R^2$=0.1501 | $R^2$=0.3804 | L=-63.0378 |
| Residual Diagnostics: | | | | | | |
| Ljung-Box stats (8lags) Q(8) | 13.4195 | 147.1160 | 12.5721 | 15.6375 | 33.2609 | 12.3310 |
| p-value | 0.0506 | 0.0000 | 0.1274 | 0.0479 | 0.0001 | 0.1370 |
| Squared residual Diagnostics: | | | | | | |
| Ljung-Box stats (8lags) $Q^2(8)$ | 902.2313 | 1014.7726 | 51.4145 | 1443.6712 | 639.6085 | 16.1953 |
| p-value | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0397 |
| PANEL B – CRSP VWT returns | | | | | | |
| Adj. $R^2$ / Log-Likelihood | $R^2$=0.0560 | $R^2$=0.2449 | L=36578.7766 | $R^2$= 0.1129 | $R^2$= 0.3286 | No Convergence |
| Residuals Diagnostics: | | | | | | |
| Ljung-Box stats (8lags) Q(8) | 6.6732 | 402.6331 | 10.3238 | 12.1853 | 18.7491 | |
| p-value | 0.5722 | 0.0000 | 0.2430 | 0.1431 | 0.01626 | |
| Squared residuals Diagnostics: | | | | | | |
| Ljung-Box stats (8lags) $Q^2(8)$ | 3059.6553 | 1083.9341 | 9.2826 | 1786.3581 | 650.9206 | |
| p-value | 0.0000 | 0.0000 | 0.3190 | 0.0000 | 0.0000 | |

**Table 2.5: Comparison of Model Diagnostics:**
**Monthly return series for the CRSP Value-Weighted (VWT) Index [July 1962-December 1994] and TSE 300 Total Return Index (TRI) [January 1977-December 1994]**

This table presents specification diagnostics for the GARCH (1,1) model using the BHHH and the artificial regression (AR) approaches.

The sample period spans 467 return observations from January 1977-December 1994 for the TSE 300 total return index and 828 return observations from July 1962-December 1994 for the CRSP value-weighted index.

SMALL INFORMATION SET INSTRUMENTS , $\varphi_S$ : Lagged Returns (CRSP- 4, TSE-3) , Seasonal dummy, and Crash Dummy

LARGE INFORMATION SET INSTRUMENTS, $\varphi_L$ : Lagged Returns, Lagged Squared Returns (CRSP- 4, TSE-3), Seasonal dummy, and Crash Dummy

Residual analysis refers to $w_t$ for BM, $\frac{u_t}{\sigma_t}$ for BHHH, and $\xi_t$ for OLS.

**BENCHMARK MODEL** (Estimation by OLS) – **BM**

$r_t = \delta_0 + \delta X_t + w_t$; where $w_t \sim N(0, \sigma^2)$

**TRADITIONAL GARCH (1,1) MODEL** – (Estimation by maximum likelihood – numerical algorithm) – **BHHH**

$r_t = \beta_0 + \beta X_t + u'_t$ ; $u'_t = \sqrt{\sigma_t^2} . v_t$, where $v_t \sim N(0,1)$; $\sigma_t^2 = \gamma_0 + \gamma_1 u'^2_{t-1} + \theta \sigma_{t-1}^2$

ML estimation by BHHH - $L(\vartheta) = -\frac{1}{2} \sum_{t=1}^{T} (\log(\sigma_t^2) + \frac{u_t^2}{\sigma_t^2})$

**ARTIFICIAL REGRESSION MODEL** – Estimation by OLS – **AR**

$r_t = \gamma_0 + \gamma X_t + u_t$; where $u_t \sim N(0, \sigma_t^2)$

$D_t = 1$, if $u_t > 0$, else $D_t = -1$. $\frac{r_t}{D_t} = b_0 + d_1 D_t + \frac{bX_t}{D_t} + \sum_{k=1}^{K} b_k u_{t-k}^2 + \xi_t$, where $\xi_t \sim N(0, \sigma_\xi^2)$

| | SMALL INFORMATION SET, $\varphi_S$ | | | LARGE INFORMATION SET, $\varphi_L$ | | |
|---|---|---|---|---|---|---|
| | BM | AR | BHHH | BM | AR | BHHH |
| PANEL A - TSE300 Returns | | | | | | |
| Adj. $R^2$ / Log-Likelihood | $R^2$=0.0943 | $R^2$=0.2856 | L=1241.2947 | $R^2$=0. 1003 | $R^2$=0.2639 | L=1243.9318 |
| Residuals Diagnostics: | | | | | | |
| Ljung-Box stats (8lags) Q(8) | 10.4079 | 19.5861 | 11.3099 | 9.9011 | 18.5895 | 11.7698. |
| p-value | 0.2376 | 0.0120 | 0.1847 | 0.2720 | 0.0172 | 0.1618 |
| Squared residuals Diagnostics: | | | | | | |
| Ljung-Box stats (8lags) $Q^2(8)$ | 37.7776 | 27.2113 | 14.9509 | 28.8284 | 17.4399 | 15.4493. |
| p-value | 0.0000 | 0.0006 | 0.0601 | 0.0003 | 0.0258 | 0.0510 |
| PANEL B - CRSP VWT returns | | | | | | |
| Adj. $R^2$ / Log-Likelihood | $R^2$=0.0432 | $R^2$=0.2673 | L=2087.0209 | $R^2$= 0.0545 | $R^2$=0.3410 | 2093.3332 |
| Residuals Diagnostics: | | | | | | |
| Ljung-Box stats (8lags) Q(8) | 9.0880 | 15.3509 | 8.7007 | 10.9344 | 7.4069 | 10.6613 |
| p-value | 0.3349 | 0.0527 | 0.3682 | 0.2054 | 0.4934 | 0.2216 |
| Squared residuals Diagnostics: | | | | | | |
| Ljung-Box stats (8lags) $Q^2(8)$ | 271.1222 | 75.6155 | 10.0350 | 208.4354 | 84.8070 | 10.6730 |
| p-value | 0.0000 | 0.0000 | 0.2626 | 0.0000 | 0.0000 | 0.2209 |

**Table 2.6: Estimates of variance equation parameters:**
**Daily and Monthly returns for the CRSP Value-Weighted (VWT) Index [July 1962-December 1994] and TSE 300 Total Return Index (TRI) [January 1977-December 1994]**

**TRADITIONAL GARCH (1,1) MODEL**

$r_t = \beta_0 + \beta X_t + u_t;\ u_t = \sqrt{\sigma_t^2}.v_t$, where $v_t \sim N(0,1)$; $\sigma_t^2 = \gamma_0 + \gamma_1 u_{t-1}^2 + \theta\sigma_{t-1}^2$

| | N no. of obs. | $\beta_0$ BHHH | $\beta_0$ AR | $\gamma_0$ BHHH | $\gamma_0$ AR | $\gamma_1$ BHHH | $\gamma_1$ AR | $\theta$ BHHH | $\theta$ AR |
|---|---|---|---|---|---|---|---|---|---|
| PANEL A | SMALL INFORMATION SET, $\varphi_S$ : INSTRUMENTS: Lagged Returns, Seasonal dummy, Crash Dummy | | | | | | | | |
| CRSP VWT Daily Returns | 8179 | 0.0008 | 0.0007 | 0.0000 | 0.0000 | 0.1369 | 0.0063 | 0.8430 | 1.1824 |
| TSE 300 TRI Daily Returns | 4536 | 0.0637 | 0.0623 | 0.0128 | 0.0003 | 0.1011 | 0.0027 | 0.8743 | 0.5564 |
| CRSP VWT Monthly Returns | 828 | 0.0098 | 0.0068 | 0.0001 | 0.0001 | 0.2032 | 0.0129 | 0.7532 | 0.4757 |
| TSE 300 TRI Monthly returns | 467 | 0.0074 | 0.0039 | 0.0007 | 0.0000 | 0.2421 | 0.0065 | 0.3475 | 1.5998 |
| PANEL B | LARGE INFORMATION SET $\varphi_L$ : INSTRUMENTS : Lagged Returns, Seasonal dummy, Crash Dummy, and Lagged Squared Returns | | | | | | | | |
| CRSP VWT Daily Returns | 8179 | | -0.0002 | | 0.0000 | | -0.0016 | | 1.0759 |
| TSE 300 TRI Daily Returns | 4536 | 0.0334 | -0.0099 | 0.0104 | 0.0001 | 0.0845 | -0.0014 | 0.8934 | 0.8596 |
| CRSP VWT Monthly Returns | 828 | 0.0077 | 0.0034 | 0.0007 | 0.0000 | 0.2062 | 0.0064 | 0.3475 | 0.6273 |
| TSE 300 TRI Monthly returns | 467 | 0.0066 | 0.0023 | 0.0007 | 0.0000 | 0.2370 | 0.0032 | 0.3450 | 1.5129 |

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# Appendix I.

## Approximate Equivalence of Econometric Specifications of Conditional Asset Return Moments

Propositions and Proofs

We first set out the notation as follows.

$r_t$ is the security return in period t,

$X_t$ represents the information instruments in the conditional mean equation

$Y_t$ is a vector of information instruments in the conditional variance equation,

$\alpha$ is the parameter vector in the conditional mean equation ,

$\beta$ is the parameter vector in the conditional variance equation and

$\varphi_t$ is the information set at time t.

Consider the following three univariate models:

Model 1: Time-varying conditional mean and constant variance:

$$r_{at} = \alpha_{a0} + \alpha_{a1} X_{a1t} + \ldots + \alpha_{ak} X_{akt} + u_{at} \tag{A1.1a}$$

$$u_{at}^2 = \beta_{a0} + \nu_{at} \tag{A1.1b}$$

Model 2: Time-varying conditional mean and time-varying conditional variance:

$$r_{bt} = \alpha_{b0} + \alpha_{b1} X_{b1t} + \ldots + \alpha_{bL} X_{bLt} + u_{bt} \tag{A1.2a}$$

$$u_{bt}^2 = \beta_{b0} + \beta_{b1} Y_{b1t} + \ldots + \beta_{bm} Y_{bmt} + \nu_{bt} \tag{A1.2b}$$

Model 3: Constant conditional mean and time-varying conditional variance:

$$r_{ct} = \alpha_{c0} + u_{ct} \tag{A1.3a}$$

$$u_{ct}^2 = \beta_{c0} + \beta_{c1} Y_{c1t} + \dots\dots\dots + \beta_{cn} Y_{cnt} + \nu_{ct} \tag{A1.3b}$$

Only weak rationality requirements are imposed; that is the errors and information instruments are pairwise uncorrelated and that $u_{it}$ and $\nu_{it}$ in each model are uncorrelated. Hence, we do not make any distributional assumptions regarding the error terms other than the following:

$$E(u_{it}) = 0,\ E(\nu_{it}) = 0, \text{ and } E(\nu_{it}^2) = \sigma_i^2, \text{ where } i \in \{a, b, c\}$$

and

$$E(u_{at}^2) = \sigma_a^2,\ E(u_{bt}^2) = \beta_{b0} + \sum_{i=1}^{m} \beta_{bi} Y_{bit}, \text{ and } E(u_{ct}^2) = \beta_{c0} + \sum_{i=1}^{n} \beta_{ci} Y_{cit}$$

***Proposition 1: Without distributional assumptions regarding model disturbances, models with time-varying conditional means and constant variances and models with constant conditional means and time-varying conditional variances are empirically indistinguishable.***

Using second-order Taylor series approximations, we now furnish a weak proof of the approximate equivalence of Models 3 and 1.

*Proposition 1a: Model (3) ⇒ Model (1)*

*A specification with a constant conditional mean and time-varying conditional variance is approximately equivalent to one with a time varying-conditional mean and a constant variance.*

*Proof:*

The time-varying standard deviation for Model 3 can be written as $f(z) = z^{1/2}$ where,

$$z = u_{ct}^2 = \beta_{c0} + \beta_{c1} Y_{c1t} + \dots\dots + \beta_{cn} Y_{cnt} + v_{ct}$$

Consider a second order Taylor series expansion of f(z) about $z_0 = \beta_{c0}$,

$$f(z) = f(z_0) + \frac{f'(z) \mid z_0}{1!}(z - z_0) + \frac{f''(z) \mid z_0}{2!}(z - z_0)^2 + R_2$$

$$\begin{aligned} u_{ct} &= \beta_{c0}{}^{1/2} + \frac{1}{2}\beta_{c0}^{-1/2}\beta_{c1}Y_{c1t} + \dots\dots + \frac{1}{2}\beta_{c0}^{-1/2}\beta_{cn}Y_{cnt} - \frac{1}{8}\beta_{c0}^{-3/2}\beta_{c1}^2 Y_{c1t}^2 \\ &- \dots\dots - \frac{1}{8}\beta_{c0}^{-3/2}\beta_{cn}^2 Y_{cnt}^2 - \frac{1}{4}\beta_{c0}^{-3/2}\beta_{c1}Y_{c1t}v_{ct} - \dots\dots - \\ &\frac{1}{4}\beta_{c0}^{-3/2}\beta_{cn}Y_{cn1t}v_{ct} + [\frac{1}{2}\beta_{c0}^{-1/2}v_{ct} - \frac{1}{8}\beta_{c0}^{-3/2}v_{ct}^2 + R_2] \end{aligned} \tag{A1.4}$$

Assuming that $X_t$'s, the instruments in the conditional mean equation consist of Y's, $Y^2$'s and cross-products of Y's and $v$'s, from equation (A1.4) it follows that $u_{ct}$ is of the form,

$$u_{ct} = \gamma_{c0} + \gamma_{c1}X_{c1t} + \gamma_{c2}X_{c2t} + \dots\dots\dots + \gamma_{cp}X_{cpt} + \omega_{ct} \tag{A1.5}$$

$\omega_{ct}$ is the disturbance term $[\frac{1}{2}\beta_{c0}^{-1/2}v_{ct} - \frac{1}{8}\beta_{c0}^{-3/2}v_{ct}^2 + R_2]$ from equation (A1.4).

Substituting $u_{ct}$ from equation (A1.5) into equation (A1.3a) yields a linear model in the instruments,

$$r_t = \alpha_{c0} + u_{ct} = \gamma^*_{c0} + \gamma_{c1}X_{c1t} + \ldots\ldots\ldots\ldots\gamma_{cp}X_{cpt} + \varpi_{ct} \qquad \text{(A1.6)}$$

We add the expected portion of the disturbance $\omega_{ct}$, $(E(R_2) - \left(\frac{\beta_{c0}^{-3/2}}{8}\right)E(v_{ct}^2))$ to the constant $\gamma^*_{c0}$ in equation (A1.6). This implies that the disturbance in equation (A1.6) is mean zero ( $E(\varpi)=0$). Therefore, equation (A1.6) has a functional form approximately equivalent to equation (A1.1a). Q.E.D.

We have furnished this weak proof making minimal assumptions. Any further extensions, such as comparisons of magnitudes of coefficients require specifying the error structure and information instruments.

*Proposition 1b: Model (1) ⇒ Model (3)*

*A specification with a time-varying conditional mean and a constant variance is approximately equivalent to one with a constant conditional mean and time-varying conditional variance.*

*Proof:*

Equation (A1.1a) may be written as,

$$r_t = \alpha_{a0} + \alpha_{a1}X_{a1t} + \ldots\ldots. + \alpha_{ak}X_{akt} + u_{at} = \alpha_{a0} + \varepsilon_{at}$$

where $\varepsilon_{at}$ is the 'unconditional disturbance'.

$$\varepsilon_{at}^2 = (\alpha_{a1}X_{a1t} + \ldots.. + \alpha_{ak}X_{akt} + u_{at})^2$$

Assuming $E[u_{at} | \varphi_{t-1}] = 0$ and $E[u_{at}X_{ait} | \varphi_{t-1}] = 0$ ,

$$\varepsilon_{at}^2 = E[u_{at}^2 \mid \varphi_{t-1}] + \alpha_{a1}^2 X_{a1t}^2 + ..... + \alpha_{ak}^2 X_{akt}^2 + 2\alpha_{a1} X_{a1t} u_{at} + ..... + 2\alpha_{ak} X_{akt} u_{at} + (u_{at}^2 - E[u_{at}^2 \mid \varphi_{t-1}]) \quad \text{(A1.7)}$$

Equation (A1.7) is of the form,

$$\varepsilon_{at}^2 = \gamma_{a0} + \gamma_{a1} Y_{a1t} + \gamma_{a2} Y_{a2t} + .......... + \gamma_{ak} Y_{apt} + \nu_{at} \quad \text{(A1.8)}$$

Therefore, equation (A1.8) has a functional form approximately equivalent to equation (A1.3b). Q.E.D.

Propositions 1a and 1b together show that *Model (1)* ⇔ *Model (3)*.

***Proposition 2: Without distributional assumptions regarding model disturbances, models with time-varying conditional means and time-varying conditional variances and models with constant conditional means and time-varying conditional variances are empirically indistinguishable.***

We use a similar approach to that used in proving proposition 1.

*Proposition 2a:* *Model (3) → Model (2)*

*A specification with a constant conditional mean and time varying conditional variance is equivalent to one with a time-varying conditional mean and a time-varying conditional variance.*

*Proof:*

In eqn. (A1.3b) let

$$\varepsilon_{ct}^2 = u_{ct}^2 - \beta_{c2} Y_{c2t} - ....... - \beta_{cn} Y_{cnt} = \beta_{c0} + \beta_{c1} Y_{c1t} + \nu_{ct} \quad \text{(A1.9)}$$

$$r_t = \alpha_{c0} + \varepsilon_{ct} = \alpha_{c0} + [\beta_{c0} + \beta_{c1} Y_{c1t} + \nu_{ct}]^{1/2} \tag{A1.10}$$

Consider a second order Taylor series expansion of $f(z)=z^{1/2}=\varepsilon_{ct}$ about $z_0 = \beta_{c0}$.

$$f(z) = \beta_{c0}{}^{1/2} + \frac{1}{2}\beta_{c0}^{-1/2}\beta_{c1}Y_{c1t} - \frac{1}{8}\beta_{c0}^{-3/2}\beta_{c1}^2 Y_{c1t}^2 - \frac{1}{4}\beta_{c0}^{-3/2}\beta_{c1}Y_{c1t}\nu_{ct} + \frac{1}{2}\beta_{c0}^{-1/2}\nu_{ct} - \frac{1}{8}\beta_{c0}^{-3/2}\nu_{ct}^2 \tag{A1.11}$$

Substituting $\varepsilon_{ct}$ from (A1.11) into (A1.10) yields,

$$r_t = \alpha_{c0} + \beta_{c0}^{1/2} + \frac{1}{2}\beta_{c0}^{-1/2}\beta_{c1}Y_{c1t} - \frac{1}{8}\beta_{c0}^{-3/2}\beta_{c1}^2 Y_{c1t}^2 - \frac{1}{4}\beta_{c0}^{-3/2}\beta_{c1}Y_{c1t}\nu_{3t} + \xi_{ct} \tag{A1.12}$$

Equation (A1.12) can be written as,

$$r_t = \gamma_{c0} + \gamma_{c1}Y_{c1t} + \gamma_{c2}Y_{c2t}^2 + \gamma_{c3}Y_{c1t}\nu_{ct} + \xi_{ct} \tag{A1.13}$$

Without distributional assumptions on $u_{ct}$ and $\nu_{ct}$, equation (A1.13) is approximately equivalent to (A1.2a). Q.E.D.

*Proposition 2b:* *Model (2) → Model (3)*

*A specification with a time-varying conditional mean and time-varying conditional variance is equivalent to one with a constant conditional mean and a time-varying conditional variance.*

Proof:

As above, write equation (A1.2a) as $r_t = \alpha_{b0} + \varepsilon_{bt}$,

and,

$$\begin{aligned}\varepsilon_{bt}^2 &= (\alpha_{bl}X_{blt} + .... + \alpha_{bl}X_{blt} + u_{bt})^2 \\ &= \alpha_{bl}^2 X_{blt}^2 + (\beta_{b0} + \beta_{bl}Y_{blt} + .... + \beta_{bm}Y_{bmt} + \nu_{bt}) + \qquad \text{(A1.14)} \\ &\quad 2\alpha_{bl}X_{blt}(\beta_{b0} + \beta_{bl}Y_{blt} + .... + \beta_{bm}Y_{bmt} + \nu_{bt})^{1/2} + ...\end{aligned}$$

Without distributional assumptions on $u_{bt}$ and $v_{bt}$, (A1.14) is approximately equivalent to A1.3b. Q.E.D.

***Proposition 3: Without distributional assumptions regarding model disturbances, models with time-varying conditional means and time-varying conditional variances and models with time-varying conditional means and constant conditional variances are empirically indistinguishable.***

*Proposition 3a:* *Model (2) → Model (1)*

*A specification with time-varying conditional means and conditional variances is equivalent to one with time-varying conditional means and a constant variance.*

Proof:

The time-varying standard deviation for Model 2 can be written as

$$f(z) = z^{1/2},\ z = u_{bt}^2 = \beta_{b0} + \beta_{bl}Y_{blt} + ... + \beta_{bm}Y_{bmt} + \nu_{bt}.$$

Consider a second order Taylor series expansion about $z_0 = \beta_{b0}$,

$$f(z) = u_{bt} = \beta_{b0}{}^{1/2} + \frac{1}{2}\beta_{b0}^{-1/2}\beta_{b1}Y_{b1t} - \frac{1}{4}\beta_{b0}^{-3/2}\beta_{b1}Y_{b1t}v_{bt} - \ldots\ldots - \frac{1}{8}\beta_{b0}^{-3/2}\beta_{b1}^{2}Y_{b1t}^{2} + \ldots\ldots + \frac{1}{2}\beta_{b0}^{-1/2}v_{bt} - \frac{1}{8}\beta_{b0}^{-3/2}v_{bt}^{2} \qquad \text{(A1.15)}$$

Substitution $u_{bt}$ from (A1.15) into (A1.2a) yields,

$$r_t = \alpha_{b0} + \beta_{b0}^{1/2} + \alpha_{b1}X_{b1t} + [.]Y_{b1t} + [\cdot]Y_{b1t}v_{bt} + [\cdot]Y_{b1t}^{2} + (.)$$

which is of the form,

$$r_t = \gamma_{b0} + \gamma_{b1}Z_{b1t} + \gamma_{b2}Z_{b2t} + \gamma_{b3}Z_{b3t} + \gamma_{b4}Z_{b4t} + \omega_{bt} \qquad \text{(A1.16)}$$

Without distributional assumptions on $u_{bt}$ and $v_{bt}$, equation (A1.16) is approximately equivalent to equation (A1.1a).

Q.E.D.

Proposition 3b. *Model (1) → Model (2)*

*A specification with time-varying conditional means and constant conditional variance is equivalent to one with time-varying conditional means and a time-varying conditional variance.*

*Proof:*

As in proposition 2b above, arbitrarily move $K_1 < K$ mean regressors into the variance specification and the result follows. Q.E.D.

# Appendix II

Artificial Regression model: Taylor series first-order approximation of the variance of the error term about the unconditional variance, $\frac{\gamma_0}{1-\gamma_1-\theta}$

We expand $|u_t|$ as an n-dimensional, first order, Taylor series approximation in $u_{t-1}^2, u_{t-2}^2, \ldots, u_{t-n+1}^2$ and $\varepsilon_t$ about the unconditional variance

$(\frac{\gamma_0}{1-\gamma_1-\theta}, \frac{\gamma_0}{1-\gamma_1-\theta}, \frac{\gamma_0}{1-\gamma_1-\theta}, \ldots\ldots, 0)$ in an approach similar to that considered for the Taylor series approximation about $(\gamma_0, \gamma_0, \gamma_0 \ldots, 0)$.

This results in,

$$|u_t| = \frac{1}{2}\sqrt{\frac{(2\gamma_0+\gamma\gamma_1)^2(1+\theta+\theta^2+..+\theta^{n-2})}{(\gamma_0+\gamma\gamma_1)}} + \sum_{j=1}^{n-1}\frac{\gamma_1\theta^{j-1}u_{t-j}^2}{2\sqrt{(\gamma_0+\gamma\gamma_1)(1+\theta+\theta^2+..+\theta^{n-2})}} + \frac{\varepsilon_t}{2\sqrt{(\gamma_0+\gamma\gamma_1)(1+\theta+..+\theta^{n-2})}} \tag{A2.1}$$

where,

$$\gamma = \frac{\gamma_0}{1-\gamma_1-\theta}$$

The artificial regression model is now given as

$$\frac{r_t-\mu_t}{D_t} = \frac{1}{2}\sqrt{\frac{(2\gamma_0+\gamma\gamma_1)^2(1+\theta+\theta^2+\ldots+\theta^{n-2})}{(\gamma_0+\gamma\gamma_1)}} + \sum_{j=1}^{n-1}\frac{\gamma_1\theta^{j-1}u_{t-j}^2}{2\sqrt{(\gamma_0+\gamma\gamma_1)(1+\theta+\theta^2+..+\theta^{n-2})}} + \frac{\varepsilon_t}{2\sqrt{(\gamma_0+\gamma\gamma_1)(1+\theta+..+\theta^{n-2})}} \tag{A2.2}$$

Hence, (A2.2) may be written as the artificial regression model,

$$\frac{r_t - \mu_t}{D_t} = b_0 + \sum_{j=1}^{n-1} b_j u_{t-j}^2 + \xi_t \tag{A2.3}$$

where,

$$\xi_t = \frac{\varepsilon_t}{2\sqrt{(\gamma_0 + \gamma\gamma_1)(1+\theta+\theta^2+\ldots\ldots\ldots\ldots+\theta^{n-2})}}$$

is the homoskedastic error. Alternatively,

$$\frac{r_t}{D_t} = \frac{\mu_t}{D_t} + b_0 + \sum_{j=1}^{n-1} b_j u_{t-j}^2 + \xi_t \tag{A2.4}$$

Running the regression A2.4 and using the $b_j$ coefficient estimates, the implied values for $\gamma_0, \gamma_1$ and $\theta$ may be derived. The values are given by,

$$\theta = \frac{b_2}{b_1} = \frac{b_3}{b_2} = \frac{b_4}{b_3} = \ldots \tag{A2.5}$$

$$\gamma_1 = 1 - \theta + \sqrt{(1-\theta)^2 + 4b_0 b_1 (1-\theta)} \tag{A2.6}$$

and

$$\gamma_0 = \frac{\gamma_1^2 (1 - \gamma_1 - \theta)}{4b_1^2 (1-\theta)(1+\theta+\theta^2+\theta^3+\ldots+\theta^{n-2})} \tag{A2.7}$$

Notice that, while the value of $\theta$ remains unchanged from that obtained from the Taylor expansion, the values of $\gamma_0$ and $\gamma_1$ are different.

Artificial Regression model: Taylor series first-order approximation of $|u_t|$ followed by substitution of the conditional variance term:

We expand $|u_t|$ as a 3-dimensional, first order, Taylor series approximation in $u_{t-1}^2, \sigma_{t-1}^2$ and $\varepsilon_t$ about $(\gamma_0, \gamma_0, 0)$.

This results in,

$$|u_t| = \frac{2\gamma_0 + 2\gamma_0\gamma_1 + 2\gamma_0\theta - \gamma_0\gamma_1 - \gamma_0\theta}{2\sqrt{\gamma_0(1+\gamma_1+\theta)}} + \frac{\gamma_1 u_{t-1}^2}{2\sqrt{\gamma_0(1+\gamma_1+\theta)}} + \frac{\theta\sigma_{t-1}^2}{2\sqrt{\gamma_0(1+\gamma_1+\theta)}} + \frac{\varepsilon_t}{2\sqrt{\gamma_0(1+\gamma_1+\theta)}} \quad \text{(A2.8)}$$

Recursively substituting for lags of $\sigma_t^2$ in (A2.8), we now obtain the artificial regression model as,

$$\frac{r_t - \mu_t}{D_t} = \frac{2\gamma_0 + \gamma_0\gamma_1 + \gamma_0\theta(2 + \theta + \theta^2 + \ldots + \theta^{n-2})}{2\sqrt{\gamma_0(1+\gamma_1+\theta)}} + \sum_{j=1}^{n-1} \frac{\gamma_1\theta^{j-1}u_{t-j}^2}{2\sqrt{\gamma_0(1+\gamma_1+\theta)}} + \frac{\varepsilon_t}{2\sqrt{\gamma_0(1+\gamma_1+\theta)}} \quad \text{(A2.9)}$$

Hence, (A2.9) may be written as the artificial regression model,

$$\frac{r_t - \mu_t}{D_t} = b_0 + \sum_{j=1}^{n-1} b_j u_{t-j}^2 + \xi_t \quad \text{(A2.10)}$$

where,

$$\xi_t = \frac{\varepsilon_t}{2\sqrt{\gamma_0(1+\gamma_1+\theta)}}$$

# CHAPTER 3

# THE INFORMATIONAL ROLE OF VOLUME IN FINANCIAL MARKETS: AN EMPIRICAL STUDY OF THE CANADIAN STOCK MARKET

## 3.1. PURPOSE OF THE STUDY

The market meltdowns in the past decade highlight the strong inter-relationships among stock returns, return volatility and trading volume.[1] 1987, 1989, and 1997 witnessed sharp drops in prices, high trading volume and increased future volatility. Despite these strong linkages, while the informational role of stock prices has long been a subject of theoretical and empirical research, the informational role of volume in explaining asset prices continues to be peripheral in the financial economics literature. The exception has been Blume, Easley, and O'Hara (1994) who show that volume is informative by itself, as it serves as a signal of precision of beliefs. In particular, the authors show that volume provides information about the quality of traders' information that cannot be deduced from the price statistic.

In fact, Ross (1989) suggests that volume serves as a proxy for the rate of information flow. However, subsequent studies seem to suggest otherwise. For example, Mitchell and Mulherin (1994) in their rigorous empirical study do not find a significant relationship between dollar volume and public information. In addition, Jones, Kaul, and Lipson (1994) report that transactions per se and not their size generates volatility, a surprising result given that size of trades has been assumed in theoretical models to be

[1] Recent studies investigating the joint dynamics of stock returns and volume in U.S. are Gallant, Rossi, and Tauchen (1992) and Andersen (1996). See Karpoff (1987) for an excellent review of earlier research.

positively correlated with the precision of information of informed traders. Their study suggests that different trading variables, such as number of shares, dollar volume or number of transactions have different information implications. While these authors suggest that their findings support the mixture of distributions hypothesis (MDH), Lamoureux and Lastrapes (1994) and Hiemstra and Jones (1994) find contradictory evidence.[2] To confound the situation further, the linear and nonlinear causality results reported in Hiemstra and Jones (1994) and Gallant, Rossi, and Tauchen (1992) for U.S. markets are not in agreement.

In addition, earlier work has mainly focused on the U.S. markets. Evidence from a new sample such as the Canadian markets, which has similar market structures and security regulations as the U.S., reduces the data snooping bias connected to financial models. Furthermore, the U.S. market captures international influences and can serve as an information flow measure for the Canadian market, assuming that any event of international importance to Canada that does not originate in the U.S. will also be important to the U.S. and will therefore be reflected in the U.S. market.[3] In addition, while previous empirical work (see Gallant, Rossi, and Tauchen, 1992 ) did indicate that prices and volume should be investigated together, the focus of popular research has been on intra-day patterns rather than interday dynamics. Typically there are few explicit predictions regarding the relations between these variables at the daily frequency.

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[2] The MDH is primarily statistical in nature and asserts that volatility and volume are positively correlated because both are positively related to the number of information arrivals.

[3] Foerster and Karolyi (1993) have found that Canadian securities are priced to reflect their exposure to the US market. Karolyi (1995) showed that the magnitude and persistence of S&P500 shocks is greater for non-interlisted stocks, but overall the influence on the volatility of Canadian financial markets of US based stock price movements is weaker than previously understood and has declined over time.

In this paper, we combine these strands of literature and re-examine the informational role of volume in financial markets by empirically investigating the inter-day dynamic linkages between stock returns, return volatility and trading volume using a popular econometric model and a new data set, the Canadian market aggregates.[4] According to Gallant, Rossi, and Tauchen (1992) "theoretical models have not evolved sufficiently to guide the specification of an empirical model of daily stock market data." The task of current research is to develop a suitable parametric framework for using volume to understand market behavior.[5] As in Blume, Easley, and O'Hara (1994) volume is important in our model since it affects the behavior of the market rather than merely describing it. Our model assumes that stock returns and volume are not equal in their capacity to discover new information about asset prices. Traders use the information contained in volume. Traders learn from volume and use volume in their decision making. The volume statistic conveys information that is not contained in the price statistic.

This paper examines the short-run dynamic relationships among prices, volatility and trading volume in the Canadian stock market.[6] In particular, we investigate (i) the asymmetries and nonlinear dependencies in stock returns, return volatility and volume; (ii) the impact of U.S. returns on Canadian prices, volatility and volume; (iii) the

[4] We study market aggregates instead of individual stocks first, because our objective is to study the market-wide impact of information, in particular the effectiveness of volume proxying for rate of information flow. Furthermore we are interested in assessing the robustness of findings in the U.S. markets considering the similarities between U.S. and Canadian markets.
[5] Gallant, Rossi, and Tauchen (1992) in their study use nonparametric methods throughout.
[6] Trading volume refers to number of shares traded, dollar volume, number of transactions, size (share), and size (value).

importance of volume in price formation after using new proxies for information flows; (iv) the relationship between contemporaneous returns, volume and volatility in a parametric estimation context that accounts for conditional heteroskedasticity; (v) the interdependencies in return and volume volatility and (vi) the differences in information content in trading volume variables as reflected in prices. To our knowledge, this is the first comprehensive study investigating these issues. Furthermore, this is the first such study using Canadian data.

The remainder of the paper is organized as follows. In the next section we present the data description and the descriptive statistics. In section 3.3, we briefly review the stylized facts on the stock return-volatility-volume relationship in the U.S. markets and then present our results for the Canadian data. Here, we examine the correlation and asymmetric relationships among these variables. The results for the causality relationships are presented in section 3.4. Next, we present the univariate models in an estimation context that accounts for the heteroskedasticity in these variables in section 3.5. Sections 3.3-3.5 provide the motivation for section 3.6, where we present the empirical framework for analyzing the stock return-volume dynamics in a multivariate framework using the multivariate generalized autoregressive conditional heteroskedastic (MGARCH) family of statistical models. The advantage of this specification is that it permits the volatility of price changes and volume, as well as price changes and volume themselves to interact, while reflecting the asymmetric and causality relationships in returns and volume themselves. While our study is descriptive in nature, we believe that investigating the joint dynamics of stock prices and volume, will provide further insights into the role of volume. Section 3.7 concludes.

### 3.2. Data Description and Sample Characteristics

We investigate the Canadian aggregate stock market during the period 1980 to 1995.[7] Since the U.S. and Canadian markets are highly correlated and have similar market structures and security regulations, we can use the U.S. market to capture international influences and to serve as a proxy for the amount of daily public information that flows into the Canadian market.[8] We assume that any event of international importance to Canada that does not originate in the U.S. will also be important to the U.S. and will therefore be reflected in the U.S. market.[9] Moreover, our focus is on public information, since we are dealing with market aggregates. In addition, since the markets we examine are trading in the same time zone, we do not have the measurement problems associated with asynchronous trading.

All series analyzed in this study represent daily data. Canadian stock index returns and volume data have been obtained from the TSE-Monthly Review and the TSE 300 Index publications of the Toronto Stock Exchange. U.S. stock return data has been obtained from the tapes provided by the Center for Research on Security Prices (CRSP). We use the time series of closing values of daily TSE300 Total Return Index (TRI) and the CRSP value weighted index for the period January 2, 1980 to December 29, 1995. Both indexes are value-weighted composite indexes with distributions. The first difference of the natural logarithm of the closing values of each index series is used as a

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[7] While return and share volume data for the Canadian market is available from 1977, transaction data is available only from 1980. Hence, we choose the period 1980-1995 for the analysis.

[8] Cheung and Kwan (1992) find that absence of public information due to the closing of U.S. markets affects both volatility and trading volume in the Canadian markets.

[9] Our assumption is consistent with Hamao, Masulis, and Ng (1990) who investigate correlations between price changes and volatility in international markets and report that U.S. markets lead the Japanese markets.

measure of return. TSE returns will be hereafter referred to as returns and the CRSP value-weighted index returns will be hereafter referred to as U.S. returns.[10] We include daily returns for which the respective close-to-close trading periods in New York and Toronto are perfectly aligned. Although 30% of stocks listed in the TSE index are dually listed on either the AMEX or NYSE, Jorion and Schwartz (1986) could not reject the hypothesis that the behavior of the interlisted and purely domestic stocks is the same. However, Karolyi (1995) reports that the impact of U.S. innovations is different for interlisted and noninterlisted stocks. In as much as we are examining the joint dynamics of stock returns and volume in the Canadian market, we ignore the effect of any interdependence between the Canadian and U.S. markets due to interlisting of Canadian stocks in the U.S.

The raw trading volume represents the volume of all the stocks in the TSE300. We are interested in volume levels rather than changes in volume because, in the financial literature, the rate of information flow is revealed by the degree of price volatility (see Ross, 1989).[11] As volume is positively correlated with volatility, it in turn implies that trading volume is a proxy for the rate of information flow. For example, share volume proxies for information via its correlation with price volatility; value being a product of share volume and share price could reflect price or quantity volatility, both of which

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[10] The series represent own currency returns. Karolyi (1995) has shown that there is no significant difference in his results in his study of international transmission of return and volatility, whether he used U.S. $-denominated returns or own currency returns.

[11] It may be noted that our preliminary analysis with differenced trading variable series did not indicate siginficant differences from detrended levels of trading variables. In as much as we are interested in the informational role of volume in this study, we preferred to work with detrended levels in this study.

proxy for information. In addition, we are also interested in working with stationary series to avoid making spurious inferences. However, the volume series exhibits a trend and significant trend related variability. We therefore, take the natural log to stabilize the variance and detrend by subtracting a forty day backward moving average from the log volume series.[12] While trend is removed, considerable persistence remains.

We also consider value, transactions and size, as additional proxy variables for trading.[13] Earlier research as in Mitchell and Mulherin (1994) and Jones, Kaul, and Lipson (1994) suggests that various trading variables might be capturing different portions of the information flow with differing accuracies. Our study is also motivated by the premise that different trading variables might differ in their quality by capturing different portions of the information flow. For example, in contrast to volume, value and size (value) might better reflect the impact of institutional trading versus non-institutional trading. Transactions and volume might more effectively capture the impact of noise and feedback traders.

The trading variable, value is the difference from the forty-day backward moving average of the natural log of the aggregate trading value of the stocks in the TSE 300 composite index. Likewise, the variable, transaction is the difference from the forty-day backward moving average of the natural log of the aggregate number of transactions of all the stocks in the Toronto stock exchange. Similarly, the variable, size (shares) is the

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[12] Campbell et al. (1993) shows that detrended volume has superior explanatory power to total volume. LeBaron (1992) shows that the qualitative features of the time series of the trading volume measure are fairly robust to the method of detrending. Note that hereafter volume, value, transactions and size refer to the respective detrended variables.

[13] Mitchell and Mulherin (1994) while examining the impact of public information on the stock market consider value as a trading variable. Jones, Kaul, and Lipson (1994) show that the volatility of daily returns is more closely related to the number of transactions than to volume.

difference from the forty-day moving average of the natural log of the aggregate trading volume divided by the aggregate number of transactions of all the stocks in the Toronto stock exchange, while the variable, size (value) is the difference from the forty-day moving average of the natural log of the aggregate trading value divided by the aggregate number of transactions of all the stocks in the Toronto stock exchange. This procedure for detrending will not remove a unit root if one is present, but the Dickey-Fuller tests suggest that none of the series under analysis have a unit root.[14] Our model does not seek to explain the source of market growth of trading volume, value or transactions that could be due to market growth, institutional changes, increased adoption of risk management techniques, mutual funds, etc. Our model seeks to explain the fluctuations around the growth trend.

Returns observed after holidays in either markets and on weekends are denoted by separate dummy variables. As in Karolyi (1995), we eliminate the four influential daily observations from October 16 to October 21, 1987 around the market crash. Our complete data set consists of 3,921 observations.[15]

### 3.2.1. Summary Statistics and Autocorrelations of Returns and Trading Variables

In Table 3.1 we present the univariate descriptive statistics for the daily stock returns of the TSE300 TRI index, the CRSP value-weighted index, volume, value, transactions and size. All series are significantly negatively skewed and fat-tailed clearly

[14] We do not report these results here for brevity. For each series, the joint hypotheses test that each detrended data series has a unit root was rejected at a 1% significance level.

[15] As an aside, note that in our preliminary estimations in a multivariate GARCH framework in section 3.6, we used the entire series, without eliminating outliers. We used dummy variables for the crash days. However, in most cases, the models failed to converge

indicating that the unconditional distribution of these series is not the normal distribution. The significant excess kurtosis indicates that both returns and volume might have time varying conditional volatility. In this context, Pagan and Schwert (1990) emphasize that the underlying assumption, that the unconditional variance of the series is constant, should be tested before testing for conditional volatility. Following Pagan and Schwert (1990), we use the Cumulative Sum (CUSUM) tests and find that the stock return and the trading activity series are variance stationary.

Table 3.1 also reports the first-order sample autocorrelations and Ljung-Box (LB) portmanteau test statistics for up to 12 lags for all the series being investigated. For Canadian and U.S. returns series, the sample autocorrelations are positive and significant, possibly due to nonsynchronous trading. All the trading activity variables also exhibit strong positive first order autocorrelation. The Ljung-Box (LB) portmanteau statistics for the residuals at lags 6 and 12 indicate that all the trading activity series are highly persistent, consistent with Ajinkya and Jain's (1989) empirical findings for U.S. data. The LB statistics also show that the autocorrelation in the trading variables is much higher than that in the squared returns, implying a mixture model for returns and volume.[16] This autocorrelation in trading variables could also arise if traders receive information and rebalance their portfolios with a lag or if traders choose to trade periodically to minimize trading costs, as suggested by Admati and Pfleiderer (1988).

[16] The mixture of distribution hypothesis (MDH) is primarily statistical in nature and asserts that volatility and volume are positively correlated only because both are positively correlated to the number of daily information arrivals which is the mixing variable

The significant autocorrelations of the squared residuals from ARMA models suggest nonlinear dependence in the returns, volume, value, size and transaction series that arise possibly due to time-varying conditional variance. It may be noted that the size series exhibit the least heteroskedasticity. A class of models permitting dependence in second-order moments is the autoregressive conditional heteroscedasticity (ARCH) model introduced by Engle (1982). Univariate and multivariate ARCH tests indicate that all the series are conditionally heteroskedastic and imply that a GARCH specification might be appropriate to characterize their behavior over time. [17]

In general, the univariate analysis indicates that the Canadian series exhibit properties similar to that of U.S. However, we document the heteroskedasticity of the trading activity series. This has not been done for the U.S. markets. If volume is proxying for rate of information, the heteroskedasticity in the series perhaps suggests that information arrives in clusters. In summary, the preceding results indicate that the returns and volume series exhibit conditional heteroskedasticity and an ARCH type formulation might be appropriate to describe their behavior over time.

## 3.3. Stylized Facts - The Canadian Scenario-I

### 3.3.1. Contemporaneous Cross-Correlation Analysis- Canadian Markets

Previous empirical work on U. S. markets has documented a positive contemporaneous correlation between stock returns and trading volume and between absolute value of stock price changes and volume. This relationship has been

[17] We use the procedure suggested by Engle (1982) to test for ARCH in the univariate series. We follow Engle and Susmel's (1993) procedure to conduct multivariate ARCH tests. These results are not reported.

investigated at various frequencies.[18] Among others Karpoff (1987), Gerety and Mulherin (1992), and Gallant, Rossi, and Tauchen (1992) have documented the positive contemporaneous correlation between volume and price volatility in U.S. markets. It is hypothesized that volume is related to volatility because volume reflects the extent of disagreement about a security's value based on either differential information or differences in opinion.

Table 3.2. presents the contemporaneous correlations between the series. This table includes two additional series, the absolute TSE300 TRI index return series and the squared return series. Both, the absolute returns and squared returns series are commonly used as proxies for stock return volatility. The intuition underlying the analysis of squared returns correlations is similar to the intuition underlying ARCH, that news or information comes in clusters. While the innovations are serially uncorrelated, they are not likely to be independent since information tends to be positively autocorrelated. The clustering in information arrival sequences could likely arise due to a number of common macro factors impacting the overall economy. In fact, recent findings by Andersen and Bollerslev (1997) support the interpretation of volatility as a mixture of numerous heterogeneous short-run information arrivals.

The TSE returns and volatility are positively contemporaneously correlated with the trading variables, with magnitudes similar to that found in U.S. markets. This suggests that days with high volume are associated with high price volatility. In fact, the sizes of the correlations suggest that transactions, volume and value might be a better

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[18] Karpoff (1987) and Tauchen and Pitts (1983) provide surveys.

proxy for information flow when compared to size in capturing information flow with greater accuracy. Our subsequent analysis indicates that different trading variables appear to be capturing different portions of the information flow. However, considering the low sizes of the correlations, it is likely that volatility might be a better proxy for information flow as suggested by Ross (1989). Interestingly, contemporaneous returns and the proxy volatilities are negatively contemporaneously correlated. This is similar to the findings of Nelson (1991), who uses a well-specified econometric model to show that in the case of daily returns, the relation between conditional mean and conditional variance is significantly negative.

The preceding findings suggest that in addition to volume, transactions, value and size could also proxy for rate of information flows.

### 3.3.2. Contemporaneous Cross-Correlation Analysis- Canadian Markets with U.S. Stock Returns

The contemporaneous correlation between the U.S. and Canadian stock return series is positive and strongly significant, not surprisingly considering the significant integration of these two markets. Further, like TSE returns, U.S. returns are positively albeit, less strongly contemporaneously correlated with the trading variables. Interestingly, contemporaneous U.S. returns and Canadian return volatilities are negatively contemporaneously correlated, a relationship similar to that found between Canadian returns and volatility.

### 3.3.3. Non-contemporaneous Cross-Correlation Analysis- Canadian Markets

In contrast to contemporaneous relationships, lead-lag relationship between stock prices and volume has received much less attention. Black (1976) and Christie (1982) investigating stock returns and volatility have shown the well known leverage effect, where stock volatility tends to fall subsequent to an increase in stock prices and rise following a decline in stock prices. Investigating the lead-lag relationships between absolute stock returns and volume, Smirlock and Starks (1988) find a lead-lag relationship from volatility to volume.

In Table 3.3 we report the non-contemporaneous cross-correlations between returns, the squared return series and the trading variables. The leverage effect is clearly apparent as returns are significantly negatively correlated with future stock volatility. Panel A also indicates the asymmetry in the trade-return relationship with stronger positive feedback from returns to volume, value, transactions and size (value). The effect on size (shares) is symmetric and short-lived. Similarly, Panel B suggests the asymmetry in the trade-volatility relationship with stronger positive transmission of movements from return volatility to all trade variables contemporaneously and to volume, value and transactions at lag 1 in contrast to that found by Smirlock and Starks (1988). Panel C indicates the expected significant positive correlation between volume, value, size and transaction contemporaneously and at leads and lags. These correlations indicate the importance of considering both returns and return volatility in modeling the joint dynamics of returns and volume.

Overall, the correlation analysis indicates a difference with the U.S. market in that there is a unidirectional lead-lag effect from return volatility to volume. The analysis also suggests that in addition to volume, value, transactions, and size (value) could also proxy for information. Size (shares) seems to behave differently.

### 3.3.4. Non-contemporaneous Cross-Correlation Analysis- Canadian Markets with U.S. Stock Returns

Interestingly, panel C also shows the strong positive lead effect of U.S. stock returns on volume. Further, the effect of lagged U.S. returns on TSE returns and volume, imply that lagged U.S. return could serve as an information measure for the Canadian markets. The correlation analyses suggest possible interesting causality relationships that are discussed in the next section.

### 3.3.5. Asymmetries in Canadian Markets

Karpoff (1987) reports that previous researchers have found that volume is higher when prices increase than when prices decrease for U.S. markets, possibly due to short sale constraints. This effect could be shown via the following simple regressions:

$$V_t = \beta_{10} + \beta_{11} R_t + u_{1t} \tag{3.3.1}$$

$$V_t = \beta_{20} + \beta_{21} | R_t | + u_{2t} \tag{3.3.2}$$

where, $V_t$ represents the trading variable, $R_t$, stock returns and $|R_t|$, absolute value of stock returns. Estimates of $\beta_{11}$ and $\beta_{21}$ if positive, would support the preceding hypothesis. While previous research has focused on the above two relationships, Assogbavi et al. (1995) and Chamberlain et al. (1991) indicate that the impact of any asymmetry in the volume-price relationship can be tested directly using the following regression:

$$V_t = \beta_{30} + \beta_{31} | R_t | + \beta_{32} DR_t | R_t | + u_{3t} \tag{3.3.3}$$

In addition, we test the model in (3.3.4) to identify the precise nature of the nonlinearity.

$$V_t = \beta_{40} + \beta_{41} | R_t | + \beta_{42} DR_t | R_t | + \beta_{43} DR_t + u_{4t} \tag{3.3.4}$$

$DR_t$ is a dummy variable with $DR_t = 1$ when $R_t > 0$ and $= 0$ otherwise. Coefficients $\beta_{32}$, $\beta_{42}$, and $\beta_{43}$ if positive would support Karpoff's findings. We reproduce these specifications, with the addition of calendar dummies and lagged volume to account for the autocorrelation, to see if asymmetry holds for Canada. Model 1 of table 3.5 reveals that $\beta_{32}$ is significant at the 1% level suggesting an asymmetry in the relationship. This supports Assogbavi et al's (1995) conclusion that there could be an influence of short sales on volume in contrast to Chamberlain et al's (1991) findings. In addition, model 2 in table 3.5 reveals that $\beta_{43}$ is significant at the 10% level hinting at a stronger asymmetry in the relationship. Considering the significant asymmetry in the trading volume-return relationship, what may be also be occurring is an asymmetry in the pricing mechanism itself. We proceed to test it next.

Daily returns on the Canadian market are well specified by model 1 in Table 3.4 (p-values in parentheses), in as much as the errors terms are not serially correlated.

$$R_t = -0.069 - \underset{(0.0029)}{0.1443}\, M_t - \underset{(0.0000)}{0.0928}\, H_t + \underset{(0.0800)}{0.0693}\, R_{t-1} + \underset{(0.0171)}{0.1122}\, DR_{t-1} + \underset{(0.0004)}{}$$

$$\underset{(0.0000)}{0.2539}\, DR_{t-1} * R_{t-1} + \hat{u}_{5t}$$

(3.3.5)

where $M_t$ and $H_t$ are dummy variables for Mondays and holidays, respectively. This is a threshold autoregressive process (TAR) indicating that positive returns have a larger impact on next day's returns than do negative returns. A one percent increase today on average pushes up tomorrow's return by one third of a percent whereas the same magnitude fall in return causes tomorrow's return to diminish by under one-tenth of one percent. An old market adage is "buy on rumor, sell on news" and equation (3.3.5) may be the result as rumors are much more plentiful than news. Another possibility is that short sales restrictions have an impact on returns.

The U.S. experience is similar to the Canadian one. As shown in model 2 in Table 3.4, a regression similar in specification to (3.3.5) using daily returns on the CRSP value weighted index found the slope shifter dummy variable significant at the 5% level. The intercept dummy variable is not significant, neither is the lag of the return. Thus there appears to be a threshold type nonlinearity in both markets. In the U.S. it shows up most strongly in return's impact on trading activity whereas in the Canadian market it is also strong in returns themselves.

From section 2, it has been observed that returns lead volume. Hence, we test if the asymmetric effect is also noncontemporaneous. A regression of the form,

$$V_t = \beta_{50} + \beta_{51}|R_t| + \beta_{52}DR_t|R_t| + \beta_{53}DR_t + \beta_{54}|R_{t-1}| + \beta_{55}DR_{t-1}|R_{t-1}| + \beta_{56}DR_{t-1} + \dots\dots\dots + u_{6t} \tag{3.3.6}$$

where .... denotes lagged volume and calendar variables, would find $\beta_{55}$ and $\beta_{56}$ positive and significantly different from zero. Model 3 in Table 3.5 displays the result. Notice that $\beta_{55}$ is positive and significant, implying that returns lead volume in a nonlinear fashion. Thus the final connection between volume and return (without searching any further) is given by (estimates are in model 1 of table 3.6).

$$Vol_t = \alpha_{10} + \alpha_{11}|R_t| + \alpha_{12}DR_t*|R_t| + \alpha_{13}|R_{t-1}| + \alpha_{14}DR_{t-1}*|R_{t-1}| + \alpha_{15}|R_{t-2}| + \dots\dots\dots + v_{1t} \tag{3.3.7}$$

Notice that equation (3.3.7) confirms the correlations presented in section 3. The relationship between volume and returns is more than just contemporaneous. Repeating the analysis for the other trade variables yields for value (see model 2 in table 3.6),

$$Val_t = \alpha_{20} + \alpha_{21}|R_t| + \alpha_{22}DR_t*|R_t| + \alpha_{23}|R_{t-1}| + \alpha_{24}DR_{t-1}*|R_{t-1}| + \alpha_{25}|R_{t-2}| + \dots\dots + v_{2t} \tag{3.3.8}$$

for transactions (model 3 in table 3.6),

$$Trans_t = \alpha_{30} + \alpha_{31}|R_t| + \alpha_{32}DR_t*|R_t| + \alpha_{33}|R_{t-1}| + \alpha_{34}DR_{t-1}*|R_{t-1}| + \alpha_{35}|R_{t-2}| + \dots\dots + v_{3t} \tag{3.3.9}$$

and for size (shares) (model 4 in table 3.6),

$$Size(shares)_t = \alpha_{40} + \alpha_{41}|R_t| + \alpha_{42}DR_t*|R_t| + \alpha_{43}|R_{t-1}| + + \alpha_{44}|R_{t-2}| + \dots\dots + v_{4t} \tag{3.3.10}$$

and for size (value) (model 5 in table 3.6),

$$Size(value)_t = \alpha_{50} + \alpha_{51}|R_t| + \alpha_{52}DR_t*|R_t| + \alpha_{53}|R_{t-1}| + \alpha_{54}DR_{t-1}*|R_{t-1}| + \alpha_{55}|R_{t-2}| + \dots\dots + v_{4t} \tag{3.3.11}$$

Some points to note are that lagged absolute returns are important for all the trade variables and the impact of positive returns is similar across the different series. Comparisons of sums of the coefficients of $|R_{t-1}|$ and $DR_{t-1}|R_{t-1}|$ of the estimated versions of equations (3.3.10) and (3.3.11) show that the influence of positive returns today is positive on the size (value) of trading tomorrow but negative on the size (shares) of trading. This implies that market increases on average result in a fall of average order size (number of shares). The shift from larger to smaller orders may indicate the appeal of price increases to noise traders or informed traders disguising their trades by reducing trade size. The results also suggest that price increases tend to lead to increased trading in higher priced stocks the next day. Market lore has a "flight to quality" happening in bad times. At least in the very short run, the opposite seems to be true. The preceding suggest that positive returns should increase transactions contemporaneously and is consistent with our findings.

From this section, two important conclusions can be drawn. The dynamics between trade variables and returns must incorporate lagged effects and nonlinear influences. Of the five trade variables, size (shares) seems to be a noisy variable as it has the lowest correlation with volatility and is also unable to capture the asymmetry in the trading variable return relationship. To help sort out the paths of impact, we turn to linear and nonlinear causality analysis.

## 3.4. Stylized Facts - The Canadian Scenario - II

## (Linear and Nonlinear Causality Analysis)

There are several possible explanations for the presence of a causal relationship between stock returns and trading volume. The sequential information arrival models of Copeland (1976) and Jennings et al. (1981) imply a positive causal relation between stock prices and trading. Lakonishok and Smidt (1989) show that tax related motives imply a negative relationship from lagged returns to volume while non-tax related motives imply a positive relationship.[19] Thirdly, since noise traders do not trade on the basis of economic fundamentals, their trades impart a transitory mispricing in the short run [see DeLong et al. (1990)]. In the long run, the mispricing component disappears. A positive causal relation from volume to stock returns suggests that noise traders cause stock prices to move. Fourth, the positive feedback strategies of noise traders, who trade based on past prices, suggests a positive causal relation from stock returns to volume. According to De Long et al. (1990), feedback trading could arise due to extrapolative expectations about prices, or trend chasing. It could also be the result of stop-loss orders, liquidation of positions by investors unable to meet margin and short sale calls, and due to buyers of portfolio insurance.

In the case of U.S. markets, recent evidence in Hiemstra and Jones (1994) demonstrates unidirectional linear Granger causality from stock returns to trading volume and significant nonlinear bidirectional Granger causality between stock returns and trading volume. The authors find evidence of nonlinear causality from volume to returns

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[19] Tax-related reasons are associated with the optimal timing of capital gains and losses realized during the calendar year. Non-tax related motives include window dressing, portfolio rebalancing etc.

even after controlling for volatility persistence in returns. However, Gallant, Rossi, and Tauchen (1993) find no evidence of feedback from volume to returns or volatility of stock prices.

For Canadian markets, the preceding correlation analysis suggests significant nonlinear dependence in returns and trading activity variables. This implies possible nonlinear causality relationships due to simple volatility associated with information flows. Since the correlation analysis indicates important lead-lag relationships, we use linear and nonlinear Granger causality tests to investigate the dynamic relationships among stock returns, volume and volatility. To control for possible interaction among the variables, we also perform block exogeneity tests.

We use the traditional Granger causality tests to examine the linear predictive power among stock returns, volatility and volume. The nonlinear test is a simple modification of the linear Granger causality test using an additional asymmetric dummy indicator for the variable whose predictive power we are testing.[20]

### 3.4.1. Strict Linear Granger Causality Testing Procedure and Results

We test Granger causality by estimating a linear reduced-form vector autoregression(VAR):

$$\begin{aligned} X_t &= a_0 + \sum_{i=1}^{a} a_i X_{t-i} + \sum_{i=1}^{b} b_i Y_{t-i} + u_t \\ Y_t &= b_0 + \sum_{i=1}^{c} c_i X_{t-i} + \sum_{i=1}^{d} d_i Y_{t-i} + v_t, t = 1,2...... \end{aligned} \tag{3.4.1}$$

[20] Our nonlinear causality test is different from that of Hiemstra and Jones (1994) who use a nonparametric approach.

To test for strict Granger causality from Y to X, we use a joint F-test of exclusion restrictions to determine whether lagged Y significantly predict X. If the coefficients $b_i$ are jointly significantly different from zero, the null hypothesis that Y does not strictly Granger cause X is rejected. If the coefficients $b_i$ and $c_i$ are jointly significantly different from zero, the null hypothesis of no bidirectional causality is rejected.

We remove systematic day of the week effect by using dummy variables in equations (3.4.1). We estimate the parameters using OLS with heteroskedasticity consistent standard errors using White's (1980) procedure. We determine the lag lengths for the polynomials using the Schwartz's information criterion (SIC).[21] The residuals in the system of equations in (3.4.1) are each serially uncorrelated.

We report the linear Granger causality tests in Table 3.7. As in Hiemstra and Jones (1994), the tests show evidence of unidirectional strict Granger causality from stock returns to volume but not from volume to returns. In addition, we find unidirectional causality from returns to value, transactions and size (value). There is no strict Granger causal relationship between returns and size (shares). This probably arises due to a similar impact on volume and transactions washing out the effect on size. We find a similar unidirectional relationship between the trade variables and return volatility. We also find strict bidirectional Granger causality between returns and return volatility.[22] Overall, the causality results suggest the presence of traders, who use positive feedback

[21] The Akaike's (1974) information criterion which is defined as AIC = $-2*\max L(\psi) + 2*k$, tends to overparametrize models. We therefore choose the SIC where SIC = $-2*\max L(\psi) + k*\ln(T)$ where L is the log-likelihood function, k is the number of parameters in the estimated model, and T is the number of observations and $\psi$ is the parameter vector.

[22] We also find significant bidirectional contemporaneous causality between returns and return volatility, returns and the trade variables, return volatility and the trade variables and between the trade variables themselves. These findings suggest that common factors impact returns and the trade variables.

trading strategies based on past prices. It also implies that informed traders camouflage their order sizes, since size (share) is not impacted by either returns or return volatility. However, since size (value) is impacted by returns and return volatility, as before it suggests a movement toward higher priced stocks.

### 3.4.2. Nonlinear Granger Causality Testing Procedure and Results

We test nonlinear causality by estimating a modification of the linear reduced-form vector autoregression(VAR):[23]

$$X_t = a_0 + \sum_{i=1}^{a} a_i X_{t-i} + \sum_{i=1}^{b} b_i Y_{t-i} + eD_t + u_t$$
$$Y_t = b_0 + \sum_{i=1}^{c} c_i X_{t-i} + \sum_{i=1}^{d} d_i Y_{t-i} + fD_t^{'} + v_t, t = 1,2...... \quad (3.4.2)$$

where,

$D_t = 1$, if $Y_{t-1} > 0$, else $D_t = 0$

and,

$D_t^{'} = 1$, if $X_{t-1} > 0$, else $D_t^{'} = 0$

To test for nonlinear causality from Y to X, we use a joint F-test of exclusion restrictions to determine whether lagged Y and $D_t$ significantly predict X. If the coefficients $b_i$ and e and are jointly significantly different from zero, the null hypothesis that Y does not strictly nonlinearly Granger cause X is rejected.

[23] Hiemstra and Jones (1994) perform nonlinear causality tests using a nonparametric approach. They suggest that linear Granger causality tests have weak power when testing nonlinear relationships.

As in the case of linear causality testing, we remove systematic day of the week effect by using dummy variables in equations (3.4.2). We estimate the parameters using OLS with heteroskedasticity consistent standard errors using White's (1980) procedure. We determine the lag lengths for the polynomials using SIC. The residuals in the system of equations (3.4.2) are each serially uncorrelated.

We report the nonlinear Granger causality tests in Table 3.8. At a 5% significance level, the tests show evidence of unidirectional nonlinear Granger causality from TSE stock returns to volume/value/transactions/size (value) but not to size (share). In a departure from linear causality, we find that volume and the size variables do not nonlinearly cause stock return volatility, while value and transactions do. We also note that return volatility nonlinearly causes volume, value, transactions and size (value), but not size (share). As before, we also find bidirectional nonlinear Granger causality between returns and return volatility. Our results are consistent with Gallant, Rossi, and Tauchen (1992) and Tauchen, Zhang, and Liu (1996) who find evidence of strong nonlinear impacts from lagged stock returns to current and future trading volume but differ from Hiemstra and Jones (1994) who find bidirectional causality between volume and returns.

### 3.4.3. Summary of Causality Results

| | | |
|---|---|---|
| **A: STRICT LINEAR GRANGER CAUSALITY** | | |
| **Ret $\Rightarrow$Vol,** Vol$\nRightarrow$Ret, | **Val $\Leftrightarrow$Vol; Val $\Leftrightarrow$Trans** | **$\sigma\Leftrightarrow$Vol, $\sigma\Leftrightarrow$Val,** |
| **Ret $\Rightarrow$Trans,** Trans $\nRightarrow$Ret, | **Val $\Leftrightarrow$Size (Share/Val);** | **$\sigma\Leftrightarrow$Trans,** |
| **Ret $\Rightarrow$Val,** Val $\nRightarrow$Ret, | **Vol $\Leftrightarrow$Trans;** | **$\sigma\Rightarrow$Size (Val),** |
| **Ret $\Rightarrow$Size (Val),** Size (Val) $\nRightarrow$Ret | **Vol $\Leftrightarrow$Size (Share/Val)** | **Size (Val), $\nRightarrow\sigma$** |
| | **Trans $\Leftrightarrow$ Size (Share/Val)** | |
| Ret$\nRightarrow$Size (Share) | **Ret$\Leftrightarrow\sigma$** | $\sigma$ $\nRightarrow$Size (Share) |
| Size (share)$\nRightarrow$Ret | | Size (Share)$\nRightarrow\sigma$ |
| **B: NON-LINEAR GRANGER CAUSALITY[24]** | | |
| | | $\sigma\Rightarrow$**VOL**, VOL$\nRightarrow\sigma$ |

Overall, the Granger causality results indicate linear and nonlinear causality from returns to volume, value, size (value) and transactions and volatility to value, size (value) and transactions, consistent with noisy traders' feedback strategies.[25] There is surprisingly no causality from returns to size (shares) nor volatility to size (shares). This finding implies that an increase in price leads to an increase in volume and transactions leaving the size of the trades unchanged. It could also arise due to informed traders camouflaging their trades.

However, these results are different from that of U.S. where bidirectional nonlinear causality exists between returns and volume. Assuming that volatility is linked to the rate of information flow, as suggested by Ross (1989), the nonlinear causality from

[24] In panel B, only changes in causal relationships from those in panel A are shown.
[25] The causality tests from U.S. returns to the Canadian trade variables are similar but weaker than those with Canadian returns. These results are not reported here.

volatility to volume could be due to volatility effects associated with information flows.

The causality results are consistent with a number of explanations. Non-tax related motives, noise trading and feedback trading could be driving the causal relationships from stock returns to volume, value, size (value) and transactions. Further, causality from value and transactions to volatility tends to support a sequential information arrival. Interestingly, our results also provide evidence that value and transactions are the most effective proxy for information. Size being a ratio of volume/value to transactions is a weak proxy. Our results lend support to Gallant, Rossi and Tauchen's (1992) contention, that the studies of joint dynamics of stock returns and volume provide more information than a univariate analysis.[26] These findings provide the underlying structure for the empirical univariate and bivariate models of stock returns and volume that we test in the subsequent section.

## 3.5. Univariate Specifications for Stock Returns, Volume, Value, Transactions, and Size

It has now been widely documented that asset return series are conditionally heteroskedastic. Though, there has been extensive research on heteroskedasticity in the stock return series, none have investigated volume. Heteroskedasticity in volume suggests that information arrives in clusters. Indeed, in their theoretical model, Foster and Viswanathan (1993) show that if public information is substantially different from

[26] The block exogeneity tests (results not reported here) suggest that in a multivariate framework returns, return volatility and the trade variables have a contemporaneous causal relationship with respect to each other. The tests indicate the importance of studying the dynamics of returns, volume and volatility in a joint framework.

what investors expect, then an increase in price volatility and trading volume occurs. Their model separates the information and volume aspect and predicts conditional heteroskedasticity in trading volume and positive autocorrelation in volume.

Since the descriptive statistics in section 3.3 indicate the presence of heteroskedasticity in both the return series and the trade variable series, we propose appropriate specifications for the univariate dynamics of stock returns and volume. We consider the univariate GARCH model based on the methodology developed by Engle (1982) and Bollerslev (1986) to describe the dynamic behavior of volatility.[27] These models permit nonlinear relationships in the second moments wherein forecasts of future volatility are made based on past information. The univariate model is our benchmark model in that we permit the cross effects of returns and volume only in the mean equation but not in the conditional variance equations. We provide specifications for returns and each of the trade variables.

### 3.5.1. Univariate GARCH Model of Stock Returns

We present the specification for TSE returns in Table 3.9.[28] Our conditional mean specification models the mean return as a VAR process. We follow the approach of Gallant, Rossi, and Tauchen (1992) in using the dummy variable specification for weekend and holiday effects in the conditional mean. Since the correlation analysis in

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[27] Bollerslev, Chou, and Kroner (1992) provide an extensive survey of the applications of this methodology.

[28] We also estimated a GARCH (M) specification, with a function of the conditional variance in the mean equation. As in Baillie and DeGenarro (1990), we do not find the M-term significant and have therefore dropped it from our final specification.

section 3.3 confirms a significant lead from the U.S. returns to the TSE-300 return, we use lags of the U.S. return in our mean specification. Drawing on the results from section 3.3.5, we incorporate the asymmetry variable in our conditional mean specification. We have followed the approach of Baillie and Bollerslev (1987) in using the dummy variable specification for weekend and holiday effects in the conditional variance equation. We justify this mean specification based on the causality and correlation analysis and the optimality of the model based on Schwartz's information criterion.

Hence, the following univariate GARCH model is specified as the benchmark model for the TSE returns (see model 1 in Table 3.9).

$$R_t = a_{r,0} + D_{r,mon}MON_t + D_{r,hol}HOL_t + D_{r,asy}DR_{t-1} + D_{r,asyss}(DR_{t-1} * R_{t-1}) + \phi_{r,1}R_{t-1} + \sum_{p=1}^{3} \phi_{r,us,p}R_{us,t-1} + u_{rt} \quad (3.5.1)$$

where,

$$u_{r,t} \mid I_{t-1} \sim N(0, h_{rt}) \quad (3.5.2)$$

and,

$$h_{r,t} = \alpha_{r,0} + \alpha_{r,1}u_{r,t-1}^2 + \beta_{r,1}h_{r,t-1} + RD_{mon}MON_t + RD_{hol}HOL_t \quad (3.5.3)$$

where, $R_t$ is the TSE 300-TRI return and the dependent variable. $R_{us,t}$ is the return on the CRSP equally weighted index. $MON_t$ is the dummy variable equal to 1 for Monday and 0 otherwise. $HOL_t$ is similarly defined for days that follow a holiday in either market. $DR_{t-1}$ is a dummy variable equal to 1 if $R_{t-1}>0$ and 0 otherwise.

We consider various specifications for the conditional variance equation for the GARCH/GARCH-M functional form. We estimate Bollerslev's (1986) GARCH, Nelson's (1991) EGARCH, and the Glosten, Jagannathan, and Runkle (GJR), (1993) model with and without the M specification. Estimation is done by numerical maximization of the log-likelihood function using the algorithm suggested by Berndt, Hall, Hall and Hausman, hereafter BHHH (1974). Our results indicate that the GARCH (1,1) model is the optimal model based on the SIC criterion.

Model 1 in table 3.9 is the benchmark model. We find the expected negative weekend effect in the conditional mean equation. The asymmetry effect is positive and significant suggesting a possible momentum effect. The positive lag effect of U.S. returns is significant. The conditional variance results indicate a strong persistence similar to an Integrated GARCH (IGARCH) model. The diagnostics suggest that the standardized residuals are still significantly non-normal, being skewed and leptokurtic. The Ljung-Box statistic indicates that the model specification is an adequate fit for the TSE300 returns process as the higher order dependence in the squared residuals has been removed.

In models 2 to 7 in Table 3.9, we use a variant of Lamoureux and Lastrapes' (1990) methodology to examine whether volatility persistence in returns can be explained when U.S. stock returns or other trade variables are used as a proxy for information. This finding would imply that these variables proxy for information arrival time in the market. Lamoureux and Lastrapes' (1990) assume that returns are generated by a mixture of distributions with volume as the mixing variable, consistent with sequential information flow models. Their specification is as follows:

$$R_t = \mu_{t-1} + \varepsilon_t, \tag{3.5.4}$$

where,

$$\varepsilon_t | (V_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\ldots) \sim N(0, h_t) \tag{3.5.5}$$

and,

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \beta_1 h_{t-1} + \delta_1 V_t \tag{3.5.6}$$

Notice that in the case of GARCH models, the persistence of variance is measured by the sum of $(\alpha_1 + \beta_1)$. This sum tending to unity implies greater volatility persistence Assuming that the mixing variable is weakly exogenous, the persistence of variance should become insignificant if the mixing variable proxies for information arrival. The model predicts that if the trading variables or U.S. returns could be used as information proxies, then $\delta_1$ will be positive and significant and $\alpha_1$ and $\beta_1$ will be statistically insignificant.

Following the above approach, in models 2-7, we introduce the trade variables and lagged U.S. returns as mixing variables in the conditional variance equation (3.5.3). Model 2 in Table 3.9 indicates that U.S. returns have no impact on persistence suggesting that U.S. returns serve as a poor proxy for information arrival. Models 3-7 show that despite the introduction of the mixing variables, volume, value, transactions, size (share) and size (value), $\alpha_1$ and $\beta_1$ remain significant. In contrast to Lamoureux et al's. (1990) findings, the GARCH effects do not disappear. Hence these variables are not generating conditional variance in returns with sufficient persistence in them. We introduce each mixing variable separately in order to isolate the effectiveness of each in generating persistence in return volatility.

Second, using a slightly different methodology, we test Jones et al.'s (1994) findings that transactions per se and not their size generate volatility. Here, we introduce a combination of the trading variables in equation (3.5.3). From Table 3.10, Models 1 and 2 are consistent with Jones et al.'s (1994) findings that the volatility-size relation disappears when volume is included. Models 3 and 4 indicate that the value-volatility relationship dominates the size-volatility relation. Note however, that in contrast to Jones et al (1990), in models 5 and 6 we find that when volatility is conditioned on the number of transactions, the volatility-size relation is not rendered insignificant. Thus, our evidence suggests that the markets use information regarding transactions and size when pricing securities.

Overall, our results are complementary to Lamoureux et al. (1994) and Liesenfeld (1998) who find that a correlated mixing variable cannot account for all the persistence in variance of stock returns. From an economic viewpoint, given that the size of trades is positively correlated with the precision of information of informed traders, our finding that there is information content in both transactions and size of trades is consistent with both competitive and strategic models of trading.[29]

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[29] Both competitive and strategic models deal with asymmetric information. In competitive models, the size of trade is positively related to the precision of information possessed by informed traders [Grundy and McNichols (1989), and Kim and Verrecchia (1991)]. This is turn implies that informed traders prefer to trade large amounts at any given price. In strategic models, a monopolist informed trader may resort to several small-sized trades to camouflage his trading activity [Kyle (1985) and Admati and Pfleiderer (1988)]. However, Holden and Subrahmanyam (1992) point out that that with multiple informed traders, the size of trades or volume of the informed agent is positively related to the precision of information, thus implying a positive relation between volume and volatility.

### 3.5.2. Univariate GARCH Model of Trading Variables

Our motivation for using a GARCH specification for volume is the theoretical model of Foster and Viswanathan (1993) where it is shown that if public information is substantially different from what investors expect, then an increase in price volatility and trading volume occurs. Their model predicts conditional heteroskedasticity in trading volume and positive autocorrelation in volume.

Our conditional mean specification models the volume as a VAR process. We follow the approach of Gallant, Rossi and Tauchen (1992) in using the dummy variable specification for weekday, weekend, and holiday effects in the conditional mean. Since the correlation analysis in section 3.3 confirms a significant lead from U.S. returns to volume, we use lags of the U.S. return in our mean specification. We also include a lag term of the squared return in the mean equation to account for the lead effect of return volatility on the trading variable.

As in the case of returns, we use an indicator variable that takes a value of 1 if the preceding day's return was positive. We use this asymmetry variable in the volume conditional mean equation because our results suggest that this inherent asymmetry in returns also explains the asymmetry in volume. This asymmetry variable might reflect momentum trading. Finally in the variance equation, we introduce the square of the U.S. lagged return to proxy for news from the U.S. market. The optimal model is selected based on Schwartz's information criterion. Hence, the following univariate GARCH model is specified as the benchmark model for the trade variables (see models in Table 3.11).

$$V_t = a_0 + D_{v,mon}MON_t + D_{v,tue}TUE_t + D_{v,thu}THU_t + D_{v,fri}FRI_t + D_{v,hol}HOL_t + \phi_{v,r1}R_{t-1} + D_{v,asy}DR_{t-1} + D_{v,asyss}(DR_{t-1} * R_{t-1}) + \sum_{q=1}^{p} \theta_{v,q}V_{t-q} + \phi_{v,us}R_{us,t-1} + \phi_{v,rsq}R_{t-1}^2 + u_{vt} \tag{3.5.7}$$

where,

$$u_{vt} \mid I_{t-1} \sim N(0, h_{vt}) \tag{3.5.8}$$

and,

$$h_{vt} = \alpha_{v,0} + \alpha_{v,1}u_{v,t-1}^2 + \beta_{v,1}h_{v,t-1} + VD_{mon}MON_t + VD_{hol}HOL_t + V_{v,usq,t-1}R_{us,t-1}^2 \tag{3.5.9}$$

The estimates of the univariate GARCH (1,1) models for the trade variables are reported in Table 3.11. We use this specification to make it comparative with return volatility.

The results indicate the expected day of the week effect in the conditional mean equation as in Jain and Joh (1988). The asymmetry effect is positive and significant suggesting momentum. As in returns, the holiday effect is positive and significant in the conditional mean equation. The overall positive lag effect of US stock returns is significant. It may be noted that size (shares) unlike the other trade variables including size (value) does not display the asymmetry effect nor does it respond to lagged U.S. returns nor return volatility.

Notice that the persistence in volatility in the trade variables is less than that of returns. In fact, transactions seem to have the most persistence. Furthermore, the U.S.

volatility seems to accentuate the effect on the conditional variance equation for volume, value and transactions, supporting our earlier interpretation of the U.S. market as an information flow measure. The diagnostics suggest that the standardized residuals are still significantly non-normal, being leptokurtic. The Ljung-Box statistic indicate that the model specification is an adequate fit for all variables except transactions as the higher order dependence in the squared residuals has been removed.

Overall, the univariate analysis indicates that for both returns and the trade variables, U.S. returns serve as an information proxy. In both models the return asymmetry is positive and significant suggesting feedback trading. Furthermore, consistent with Liesenfeld (1998), we find that shocks in the variance process for returns are more persistent compared to the shocks in the volume process. In addition, we find that among the trade variables, the transactions variable is most persistent and size (share) the least informative. These results reinforce our earlier findings that various trading variables capture different or larger components of information.

### 3.5.3 Highlights of Differences in Stylized Facts between Canadian and U.S. Markets

Interday volatility patterns in volume demonstrate strong persistence and predictability, a finding not reported for U.S. data. We also find that the contemporaneous correlation between the U.S. and Canadian stock return series is positive and strongly significant. In fact, like TSE returns, U.S. returns are positively albeit, less strongly contemporaneously correlated with the three trading variables. Interestingly contemporaneous U.S. returns and the proxy Canadian return volatilities are negatively contemporaneously correlated. Overall, the correlation analysis suggests a difference with the U.S. market in that there is a unidirectional lead-lag effect from return volatility to volume in Canada. The analysis also suggests that in addition to volume, transactions, value, and size could also proxy for information. We also find that lagged U.S. returns could serve as an information measure for the Canadian markets.

We also report a threshold type nonlinearity in both returns and the trading variables in the Canadian markets. The Granger causality results indicate linear and nonlinear unidirectional causality from both returns and return volatility to volume, value, transactions, and size (value), consistent with noisy traders' feedback strategies. There is surprisingly no causality from returns to size (share) nor volatility to size (share).

In a departure from U.S. result findings, our results indicate that there is information content in both transactions and size of trades. Furthermore, the trading variables are unable to account for the persistence in variance of returns.

The differences between the U.S. and Canadian market aggregates could possibly arise due to increased institutional trading in the U.S., institutional constraints in Canada, thinner trading in Canada, and skewness of Canadian aggregates to resource stocks.

Overall, the preceding analysis implies that market participants draw inferences about the future value of an asset from trading and price history. There seems to be learning from the direction of trades and prices. It also lends support to the sequential information flow hypothesis and raises an interesting possibility that stock returns and volume react differently to information. Hence, the study of the joint dynamics of stock returns and trading variables should provide more information than univariate analysis. In the next section we present our results in a multivariate setting.

## 3.6. THE MULTIVARIATE MGARCH SPECIFICATION OF THE DYNAMICS OF STOCK RETURNS AND TRADING VARIABLES

The preceding analysis suggests that a model of the joint dynamics of stock returns and a trading variable (hereafter, volume in this section) should capture (i) the interdependence between returns, return volatility, and volume; (ii) the U.S. influence; (iii) the asymmetries; (iv) the time variation and persistence in the volatility of both stock returns and volume; (v) interdependencies of return and volume volatility as they adjust to new information and finally, (vi) the differential impact of the different trade variables.

A model that can accommodate inter-dependencies in both first and second moments would be the multivariate GARCH specification. First, it eliminates the two-step procedure, thereby avoiding the problem associated with estimated regressors. Second, it improves the efficiency and the power of the test of the estimates. Third, it is methodologically consistent with the notion that spillovers between volume and return are essentially manifestations of the impact of the same news. It also tests the possibility of asymmetries in the volatility transmission mechanism. The multivariate specification has been used effectively by among others by Baillie and Bollerslev (1987), Chan, Chan, and Karolyi (1991), Theodossiou and Lee (1993), and Karolyi (1995).

### 3.6.1. The Bivariate GARCH MODEL (MGARCH): Theory and estimation

Based on the preceding analysis, we propose the following general bivariate GARCH model for the joint processes governing stock returns and volume. The mean equations are specified in a similar fashion as in the univariate cases.

$$X_t = \mu + \sum_{i=1}^{p} \phi_p X_{t-p} + \sum_{q=1}^{q} \phi_{us,q} r_{us,t-q} + D_{mon} MON_t + D_{hol} HOL_t + D_{asy} DR_{t-1} + D_{ssasy}(DR_{t-1} * R_{t-1}) + \varepsilon_t \quad (3.6.1)$$

$$\varepsilon_t \mid I_{t-1} \sim N(0, H_t) \quad (3.6.2)$$

where $X_t = [R_t, \ Vt]'$ is the vector of returns and volume.

Conditional on this mean returns specification, the vector of innovations $\varepsilon_t$ is multivariate normally distributed with a time-varying conditional variance-covariance matrix $H_t$ given past information $I_{t-1}$. The economic literature details various formulations of $H_t$. One of the more general multivariate GARCH specifications is the positive definite GARCH model due to Engle and Kroner (1995) (MGARCH-EK) and its formulation is:

$$H_t = C'C + \sum_{i=1}^{q} F_i' \varepsilon_{t-1} \varepsilon_{t-1}' F_i + \sum_{j=1}^{p} G_j' H_{t-1} G_j \quad (3.6.3)$$

$C'C$ is a (2x 2) symmetric parameter matrix for the constants, where C is restricted to be upper triangular. F and G are (2 x 2) unrestricted parameter matrices with elements $f_{ij}$ and $g_{ij}$, respectively. This model allows $H_t$ to be a linear function of its own p past values, $H_{t-p}$ and Q past values of the squared innovations, $\varepsilon_{t-q} \varepsilon_{t-q}'$. This specification permits lagged own-market and cross-market influences in both returns' and volume's conditional variance and covariance through the squares and cross-products of both past period's conditional variances and innovations of both variables. Karolyi

(1995) used this specification to examine the international transmission of stock returns and volatility between the U.S. and Canadian markets. This model guarantees the positive definiteness of the conditional covariance matrices. According to Engle and Kroner (1995) the necessary and sufficient condition for this type of multivariate GARCH process to be covariance stationary is that all the eigenvalues of $(F\otimes F) + (G\otimes G)$ are less than one in modulus, where $\otimes$ denotes the Kroneker product.

A more parsimonious multivariate GARCH is Baillie et al.'s (1987) constant conditional correlation model (MGARCH-CC) previously used by Karolyi (1995) and Chan, Chan and Karolyi (1991). The conditional variances and covariances for a bivariate MGARCH-CC (1,1) would be specified as:

$$h_t = c + A\varepsilon_{t-1}^2 + Bh_{t-1} \tag{3.6.4}$$

and,

$$h_{ij,t} = \rho\left(h_{ii,t}h_{jj,t}\right)^{1/2}, -1 \le \rho_{ij} \le 1, \text{ for } i,j = 1,2 \text{ and } i \ne j \tag{3.6.5}$$

where,

$$H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix}, h_t = [h_{11t}, h_{22,t}]',$$

and,

$c = [c_1, c_2]'$ is a vector of constants.

A and B are (2 x 2) parameter matrices with elements $a_{ij}$ and $b_{ij}$, respectively.

In this representation, the conditional variance of each variable is modeled as a linear function of its past period's conditional variances and its own squared innovations as well as the past period's conditional variances and squared innovations of the second variable.

Unlike the general specification, this model assumes that the conditional correlation between the two variables is constant over time. Hence, this model assumes that all the variations in the conditional covariances over time are accounted for by changes in the corresponding conditional variances. This representation guarantees that the matrix $H_t$ is positive semidefinite.

A further simplification of the constant conditional correlation model is the one used by Theodossiou and Lee (1993), (MGARCH-CCTL). The conditional variance of each variable is modeled as a linear function its past period's conditional variances and squared innovations as well as the past period's squared innovations of the other variable. Hence, this model permits cross-market influences in the conditional covariances only through the past period's squared innovations which are taken to represent the past volatility shocks in the other markets. Notice that the parameter matrix B is restricted to be diagonal. A necessary and sufficient condition for the constant conditional correlation type of this multivariate GARCH process to be covariance stationary is that all the eigenvalues of (A + B) are less than one.

Under the assumption of conditional normality, the log likelihood function for the MGARCH models is given by

$$L(\Theta) = 3T\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}(\log|H_t| + \varepsilon_t' H_t^{-1}\varepsilon_t) \qquad (3.6.6)$$

where T is the sample size and $\Theta$ represents the unknown parameters in $\varepsilon_t$ and $H_t$. Joint estimation of the parameters of the mean and variance is done by numerical maximization of the conditional log-likelihood function in equation (3.6.6) using the BHHH algorithm.

### 3.6.2. Proposed Bivariate GARCH (MGARCH) MODEL of Stock Returns and Trading Variables

We report the results of the bivariate analysis in Table 3.12. We have performed the analysis for three bivariate combinations of returns with value, transactions and size (value). We choose these combinations because the preceding results suggest that volume and size (share) are more noisy proxies for information flow, compared to value, transactions and size (value). We would like to emphasize here that the bivariate system is a heavily parametrized system. In fact, even in the simplest specification of a bivariate MGARCH-TL (1,1), 51 coefficients are estimated including the coefficients in the mean equations. Thus, more complex specifications lead to more parameters and often do not converge. As such, we consider only the MGARCH (1,1) specification. The optimal model based on the SIC criterion for the return-value bivariate system is the MGARCH-CCTL (1,1). For comparison purposes, we also present the same bivariate specification for returns-transaction and returns-size (value) in Table 3.12.

Our general bivariate specification for returns and a trading variable is as follows:

Returns:

$$r_t = a_{r,0} + D_{r,mon}MON_t + D_{r,hol}HOL_t + \phi_{r,1}R_{t-1} + \sum_{p=1}^{3} \phi_{r,us,p} r_{us,t-1} + D_{r,asy}DR_{t-1} + D_{r,ssasy}DR_{t-1} * R_{t-1} + u_{rt} \quad (3.6.7)$$

where,

$$u_{rt} \mid I_{t-1} \sim N(0, h_{rt}) \quad (3.6.8)$$

and for the trading variable,[30]

$$V_t = a_{v0} + D_{v,mon}MON_t + D_{v,tue}TUE_t + D_{v,thu}THU_t + D_{v,fri}FRI_t + D_{v,hol}HOL_t + D_{v,asy}DR_{t-1} + D_{v,ssasy}DR_{t-1} * R_{t-1} + \sum_{q=1}^{5} \theta_q V_{t-q} + \sum_{m=1}^{2} \phi_{v,us,m} r_{us,t-m} + VRGARCH\sqrt{h_{r,t-1}} + u_{vt} \quad (3.6.9)$$

where,

$$u_{vt} \mid I_{t-1} \sim N(0, h_{vt}) \quad (3.6.10)$$

and the volatilities respectively for returns and the trading variables are

$$h_{rt} = \alpha_{r,0} + \alpha_{r,1}u_{r,t-1}^2 + \beta_{r,1}h_{rt-1} + \alpha_{rv}u_{v,t-1}^2 + \beta_{rv}h_{vt-1} + VRD_{mon}MON_t + VRD_{hol}HOL_t + V_{r,vol}V_{t-1} + V_{r,us,1}r_{us,t-1} + V_{asy,r}ASY_{r,t}u_{r,t-1}^2(ASY_{r,t} = 1, \text{if } u_{r,t-1} < 0) \; V_{asy,rv}ASY_{rv,t}u_{v,t-1}^2(ASY_{rv,t} = 1, \text{if } u_{v,t-1} < 0) \quad (3.6.11)$$

and,

$$h_{vt} = \alpha_{v,0} + \alpha_{v,1}u_{v,t-1}^2 + \beta_{v,1}h_{t-1} + \alpha_{vr}u_{r,t-1}^2 + \beta_{vr}h_{rt-1} + VVD_{mon}MON_t + VVD_{tue}TUE_t + VVD_{thu}THU_t + VVD_{fri}FRI_t + VVD_{hol}HOL_t + V_{v,us,1}r_{us,t-1}^2 + V_{asy,v}ASY_{v,t}u_{v,t-1}^2(ASY_{v,t} = 1, \text{if } u_{v,t-1} < 0) + V_{asy,vr}ASY_{vr,t}u_{r,t-1}^2(ASY_{vr,t} = 1, \text{if } u_{r,t-1} < 0) \quad (3.6.12)$$

[30] Notice that in equation (3.6.9), the last term with the coefficient VRGARCH allows the return volatility to impact the mean equation for the trading variable. We have incorporated this term to reflect the lead effect of return volatility on the trading variable.

$$h_{rvt} = \alpha_{r,v}\sqrt{h_{rt}h_{vt}} \qquad (3.6.13)$$

Notice that the return conditional variance equation has a lagged value term. This has been done based on our findings in section 3.5.1 regarding the explanatory power of the trading variable for return conditional volatility. In addition, we have included asymmetry terms in the conditional variance specification to confirm if the asymmetry evident in the mean equation is present in the variance too.

It may be observed from equations (3.6.12) and (3.6.13) that the bivariate model enables an analysis of the volatility relation between returns and the trading variable in two ways. First, the parameter given by $\beta_{vr}$ indicates the dependence of the conditional volatility in the trading variable on the conditional volatility of return. Similarly, $\beta_{rv}$ indicates the dependence of the conditional volatility in returns on the conditional volatility of trading variables. Second, $\alpha_{rv}$ measures the impact of squared lags of innovation in the trading variable on current return volatility. $\alpha_{vr}$ can be similarly interpreted. We now turn to the discussion of the results reported in Table 3.12.

### 3.6.3. Empirical Results

Based on the SIC criterion, MGARCH-CCTL(1,1) was the optimal model. This model incorporates return volatility spillovers to value volatility through past squared innovations of returns. For comparison purposes, we also present the same bivariate specification for returns-transaction and returns-size (value) in Table 3.12, where we report the estimated coefficients and the standard errors.

**Mean Equation Spillovers**

The important findings from the bivariate analysis for the conditional mean equations are the following. The seasonality effects in both the return and value mean equations are consistent with earlier empirical evidence. As before, the asymmetry effect is positive and significant in both returns and value mean equations. Relative to transactions and size (value), the asymmetry is strongest for value. The lag effect from returns to value is positive overall. The lag effect from U.S. returns to returns and volume is positive and significant. Finally the GARCH-M coefficient suggests that return volatility attenuates the trading variable's mean equation. Overall, we find significant spillovers from returns to volume in the mean equations.

The spillovers in the mean equation indicate that the role of volume as an information flow measure may be suspect. As returns and return volatility lead volume, a more appropriate information flow measure would be return volatility itself.

**Volatility Spillovers**

The major findings for the conditional variance equations are the following. For the return conditional variance equation, only own square innovations and conditional variances are significant, i.e. return volatility is not affected by volume volatility. On the other hand, volume volatility is affected by own and cross square innovations. The asymmetry in the return and value conditional variance equations is significant only for the return-value combination. The U.S. returns seem to attenuate the return conditional volatility. Notice, that while lagged value and transactions have signatory explanatory power for return conditional variance, size (value) does not. In addition, U.S. returns volatility seems to accentuate the conditional variance of volume.

Interestingly, the results for the return-size (value) and return-transactions bivariate analysis are similar, though one would intuitively expect them to be different since each represents complementary information regarding trading. Overall, the results indicate that the conditional volatility for the returns and volume are affected differently. The evidence suggests that a feedback system obtains from returns to the trading variables. The results for the volatility relations suggest that the pattern of new information flows may be more symmetric than that inferred from the causality returns.

If the GARCH parameterization adequately fits the data, the standardized residuals, squared standardized residuals and the cross-products of the standardized residuals of the estimated model should be uncorrelated. The results of the Ljung-Box tests indicate that the Ljung-Box statistics up to the eighth order serial correlation in the standardized residuals are all not statistically significant. The diagnostics from the cross-residuals show that the constant-correlation model is the correct specification. However, the LB (8) statistics for the squared standardized residuals are statistically significant. Thus, an additional source of persistence in return and the trading variable variance exists, that is not captured by this specification. Hence, our findings support those of Liesenfeld (1998) and Lamoureux and Lastrapes (1998) that the joint dynamics of returns and volume are not due to time series behavior of the information arrival process. A possible interpretation of the result is that the bivariate system is misspecified due to missing variables. In as much as our earlier results indicate that each trading variable has information that is not subsumed by the others, this type of misspecification is possible.

A second interpretation, following Liesenfeld (1998), is that the information-arrival process could have a long-run component that affects stock return volatility more than volume and a short run component that impacts volume more than the price changes. Overall, the bivariate specification is thus not able to adequately capture the higher dependencies in the squared residuals. Hence, we test the preceding two conjectures and present the results of our trivariate analysis in the next section.

### 3.6.4. Proposed Trivariate GARCH (MGARCH) MODEL of Stock Returns, Transaction and Value - Empirical Results

The bivariate specification was unable to capture the persistence in stock return volatility. Based on the preceding analysis, we propose a trivariate GARCH model for the joint processes governing stock returns, transactions and value. The mean and variance specifications are similar to the bivariate model, in that they capture the spillover effects of the cross square innovations. Furthermore, in this framework returns, value and transactions are endogeneously determined with each having information content that spills over into the mean and volatility specifications of the other. We report the results of the trivariate analysis in Table 3.13.

In this heavily parameterized system, many specifications failed to converge. We report the results for the MGARCH - CCTL (1,1) which was the optimal model based on the SIC criterion, among those that converged.

**Mean Equation Spillovers**

The important findings from the trivariate analysis for the conditional mean equations are similar to the findings from the bivariate analysis. The seasonality effects in both the mean equations are consistent with earlier empirical evidence. The asymmetry effect is positive and significant in all three mean equations, however the asymmetry is strongest for returns. The lag effect from returns to volume is positive overall. The lag effect from U.S. returns to returns and volume is positive and significant. As before, the GARCH-M coefficient suggests that return volatility attenuates the trading variable's mean equation. Overall, we find significant spillovers between returns to transactions and value in the mean equations.

**Volatility Spillovers**

The major differences from the bivariate analysis are the following. For the return conditional variance equation, own square innovations, cross-square innovations from transactions and own conditional variances are significant. For transactions, cross-square innovations from returns are significant, while for value cross-square innovations from transactions are significant. This indicates that transactions and value are endogeneously determined but value has no information content beyond that observed in the number of transactions. Notice, that this finding is consistent with Jones et al. (1994). It is also at variance with our earlier findings in the univariate analysis.

The results of the Ljung-Box tests indicate that the Ljung-Box statistics up to the eighth order serial correlation in the standardized residuals are not statistically significant for returns and value but are significant for transactions. From the diagnostics for the cross-residuals it is evident that the constant-correlation model is not an adequate specification. Moreover, the LB (8) statistics for the squared standardized residuals are statistically significant. Our results suggest that an additional source of persistence in return and the trading variable variance exists, that is not captured by this trivariate specification.

We also test Liesenfeld's (1998) conjecture that the information-arrival process could have a long-run component that affects stock return volatility more than volume and a short run component that impacts volume more than the price changes. Specifically we assume that transactions proxy for information arrival and use high and low pass filters on the transaction series. Hence the high-pass filtered series represents the short run component, that retained all the information pertaining to a periodicity of 5 days. The second filtered series, the low-pass filtered series retained information pertaining to a periodicity from 6-40 days and was void of any short-run dynamics and was termed the long-run component. Thus, this specification by using filtered series (results not reported here) captures short and long-run components. As before, this specification too is unable to capture the persistence of variance of returns.

In Table 3.14, we present the results of the trivariate analysis using returns, transactions and size (value). The findings, here are not substantially different from the preceding trivariate model. This is not surprising, since the variables essentially reflect the same information.

Our analysis thus suggests that a trivariate specification to model the dynamics of the financial markets cannot account for the persistence in the variance of stock returns.

Our findings imply that the information process affects trading variables and returns differently. An approach similar to a variance decomposition approach to arrive at the permanent and temporary components of the information arrival process may prove helpful.[31]

## 3.7. CONCLUSIONS

In this empirical study, we re-examine the short-run dynamic relationships among stock-returns, return volatility, and trading activity in the Canadian market in a multivariate framework with the intention of understanding better the role of volume in financial markets.

Our preliminary analysis indicates that transactions, value and size (value) are better proxies for information flow compared to volume and size (share). In a departure from U.S. findings, we also document a significant linear and nonlinear causality from stock returns and return volatility to volume, consistent with sequential information flow, positive feedback trading and noise trading. In addition, we find an inherent asymmetry in returns that impacts both returns and volume. Specifically price increases lead to increased trading in higher priced stocks the next day, which again supports positive feedback trading. We find a positive conditional volume-volatility relationship as in Tauchen et al. (1992). However, in contrast with Tauchen et al. (1992) our results do not suggest a positive conditional risk-return relationship after conditioning on lagged

[31] Alternatively a multivariate version of the artificial regression model proposed in chapter 2 of this thesis could be adopted.

volume. We also show that the U.S. stock returns serve as an effective public information measure for the Canadian market.

Another surprising finding is the heteroskedasticity in volume. In contrast to Lamoureux et al. (1990), the results suggest that the trading variables are not generating conditional variance in returns with sufficient persistence in them. In contrast to U.S. findings, our preliminary analysis indicated that there is information content in both transactions and size of trades. However, in a multivariate analysis, when returns, transactions and size are endogeneously determined, we find that size has no information content beyond that observed in the number of transactions.

The multivariate analysis reveals that the economic significance of trading volume as an information flow measure may be questionable. In fact, it seems that information affects returns and volume differently. Furthermore, as returns and return volatility lead volume, a more appropriate information flow measure would be volatility itself. This inference is also consistent with Whitelaw's (1994) findings that conditional volatility leads returns. Overall, the evidence suggests that a feedback system obtains from returns and return volatility to the trading variables consistent with traders whose feedback strategies based on prices appear to be driving volume. The results strongly support the sequential information flow and positive feedback trading hypotheses.

Finally, our multivariate specifications are unable to capture all the persistence in return variances. Our findings complement those of Liesenfeld (1998) and Lamoureux and Lastrapes (1994) that the joint dynamics of returns and volume are not due to time series behavior of the information arrival process. In as much as the short term component of the information arrival process affects volume more than it does returns, while the long term component affects returns more, an approach similar to a variance decomposition approach to arrive at the permanent and temporary components of the information arrival process may prove helpful.

Overall, the Canadian markets seem to have some significant differences from the U.S. markets despite the increasing integration of the markets. It is possible that the differences stem from the relatively thinner trading, fewer institutional trades, institutional constraints, and the smaller size of the market itself. Notwithstanding the differences, the surprising finding in this paper is the overwhelming evidence supporting the sequential information flow and positive feedback trading hypotheses.

**Table 3.1: Summary Statistics**
**January 1980 - December 1995 (3921 observations)**

Volume is the difference from the forty-day moving average of the natural log of the aggregate trading volume (000s) of the stocks in the TSE 300 composite index. Value is the difference from the forty-day moving average of the natural log of the aggregate trading value (000s) of the stocks in the TSE 300 composite index. Transaction is the difference from the forty-day moving average of the natural log of the aggregate number of transactions of all the stocks in the Toronto stock exchange. Size (share) is the difference from the forty-day moving average of the natural log of the aggregate trading volume divided by the aggregate number of transactions of all the stocks in the Toronto stock exchange. Size (value) is the difference from the forty-day moving average of the natural log of the aggregate trading value divided by the aggregate number of transactions of all the stocks in the Toronto stock exchange.

| Statistics[a] | CRSP Value-weighted returns with distribution | TSE 300 Index returns with distribution | Volume | Value | Size (Share) | Size (Value) | Transaction |
|---|---|---|---|---|---|---|---|
| Mean | 0.0591 | 0.0325 | 0.0185 | 0.0178 | 0.0128 | 0.0122 | 0.0057 |
| Variance | 0.6749 | 0.5403 | 0.0894 | 0.0971 | 0.0337 | .00348 | 0.0436 |
| t stat of mean | 4.5057** | 2.7706** | 3.8745** | 3.5871** | 4.3803** | 4.0915** | 1.6984* |
| Skewness | -0.4299** | -0.6208** | -0.058 | -0.1339** | 0.9824** | 0.1964** | -0.3218** |
| Kurtosis | 5.9721** | 8.2681** | 2.8280** | 2.3111** | 5.8805** | 0.8423** | 4.2445** |
| Autocorrelations: | | | | | | | |
| Raw Series | | | | | | | |
| ρ (lag 1) | 0.1073** | 0.2404** | 0.5063** | 0.5491** | 0.2741** | 0.3324** | 0.7371** |
| LB (6)[b] | 53.0612** | 243.1382** | 2121.3094** | 2788.9354** | 541.7706** | 966.0478** | 5673.9295** |
| LB (12) | 62.5433** | 245.7008** | 2237.4712** | 3076.5726** | 590.3324** | 1051.0543** | 6556.1896** |
| Squared Series | | | | | | | |
| ρ (lag 1) | 0.0810** | 0.1718** | 0.2740** | 0.3669** | 0.0547** | 0.1595** | 0.5174** |
| LB (6) | 380.2706** | 620.7533** | 563.8676** | 1089.5522** | 13.4641* | 140.7363** | 2169.1270** |
| LB (12) | 527.7127** | 814.8641** | 577.4322** | 1155.5372** | 16.7483 | 155.1143** | 2382.6783** |

[a] Statistics are based on all series adjusted for the effect of the October 1987 stock market crash.

[b] LB (6) denotes the Ljung-Box test statistic to test the significance of the autocorrelation of the first 6 lags. ρ (lag 1) denotes the autocorrelation coefficient at lag 1.

* indicates significance at the 0.10 level

** indicates significance at the 0.05 level

**Table 3.2: Sample contemporaneous cross-correlation coefficients**
**January 1980 – December 1995 (3921 observations)**

Volume is the difference from the forty-day moving average of the natural log of the aggregate trading volume (000s) of the stocks in the TSE 300 composite index. Value is the difference from the forty-day moving average of the natural log of the aggregate trading value (000s) of the stocks in the TSE 300 composite index. Transaction is the difference from the forty-day moving average of the natural log of the aggregate number of transactions of all the stocks in the Toronto stock exchange. Size (share) is the difference from the forty-day moving average of the natural log of the aggregate trading volume divided by the aggregate number of transactions of all the stocks in the Toronto stock exchange. Size (value) is the difference from the forty-day moving average of the natural log of the aggregate trading value divided by the aggregate number of transactions of all the stocks in the Toronto stock exchange

| | TSE 300 Index return with distribution | CRSP Value-weighted return with distribution | Absolute TSE 300 Index return with distribution | Squared TSE 300 Index return with distribution | Volume | Value | Size (Share) | Size (Value) | Transaction |
|---|---|---|---|---|---|---|---|---|---|
| TSE 300 Index return with distribution | 1.0000 | | | | | | | | |
| CRSP Value-weighted return with distribution | 0.7000** | 1.0000 | | | | | | | |
| Absolute TSE 300 Index return with distribution | -.0.0741** | -0.0792** | 1.0000 | | | | | | |
| Squared TSE 300 Index return with distribution | -0.1670** | -0.1443** | 0.8216** | 1.0000 | | | | | |
| Volume | 0.1381** | 0.0489** | 0.1843** | 0.1279** | 1.0000 | | | | |
| Value | 0.1589** | 0.057** | 0.1795** | 0.1081** | 0.9303** | 1.0000 | | | |
| Size (share) | 0.0920** | 0.0358** | 0.0524** | 0.0882** | 0.7239** | 0.5894** | 1.0000 | | |
| Size (value) | 0.1346** | 0.0521** | 0.0911** | 0.0270** | 0.6634** | 0.7592** | 0.8081** | 1.0000 | |
| Transaction | 0.1169** | 0.0386** | 0.1864** | 0.1371** | 0.7955** | 0.8139** | 0.1579** | 0.2398** | 1.0000 |

* indicates significance at the 0.10 level ** indicates significance at the 0.05 level

**Table 3.3: Sample cross-correlation coefficients January 1980 - December 1995 (3921 observations)**
Volume is the difference from the forty-day moving average of the natural log of the aggregate trading volume (000s) of the stocks in the TSE 300 composite index. Value is the difference from the forty-day moving average of the natural log of the aggregate trading value (000s) of the stocks in the TSE 300 composite index. Transaction is the difference from the forty-day moving average of the natural log of the aggregate number of transactions of all the stocks in the Toronto stock exchange. Size (share) is the difference from the forty-day moving average of the natural log of the aggregate trading volume divided by the aggregate number of transactions of all the stocks in the Toronto stock exchange. Size (value) is the difference from the forty-day moving average of the natural log of the aggregate trading value divided by the aggregate number of transactions of all the stocks in the Toronto stock exchange.

**Panel A: Cross-correlation of TSE 300 TRI index return**

| k | CRSP Value-weighted return with distribution | Squared TSE 300 Index return with distribution | Volume | Value | Transaction | Size (share) | Size (value) |
|---|---|---|---|---|---|---|---|
| -6[a] | -0.0115 | -0.0776** | 0.0759** | 0.1047** | 0.1204** | -0.0134 | 0.0401** |
| -5 | -0.0190 | -0.0738** | 0.1043** | 0.1370** | 0.1400** | 0.0105 | 0.0721** |
| -4 | -0.0363** | -0.0625** | 0.0832** | 0.1106** | 0.1244** | -0.0061 | 0.0454** |
| -3 | -0.0316 | -0.0430** | 0.0784** | 0.1123** | 0.1323** | -0.0228 | 0.0394** |
| -2 | -0.0043 | -0.1030** | 0.1158** | 0.1479** | 0.1595** | 0.0071 | 0.0685** |
| -1 | 0.0643** | -0.0438** | 0.1625** | 0.1924** | 0.1872** | 0.0517** | 0.1118** |
| 0 | 0.7000** | -0.1670** | 0.1381** | 0.1589** | 0.1169** | 0.0920** | 0.1346** |
| 1[b] | 0.2172** | 0.0990** | 0.0504** | 0.0551** | 0.0371** | 0.0398** | 0.0504** |
| 2 | 0.0595** | -0.0353** | 0.0042 | 0.0011 | -0.0005 | .0007 | 0.0024 |
| 3 | 0.0476** | -0.0022 | -0.0072 | -0.0051 | -0.0070 | 0.0037 | -0.0006 |
| 4 | 0.0066 | 0.0595** | 0.0035 | 0.0017 | 0.0009 | 0.0068 | 0.0039 |
| 5 | 0.0228 | -0.0040 | 0.0151 | 0.0148 | 0.0051 | 0.0189 | 0.0190 |
| 6 | 0.0002 | -0.0275 | 0.0003 | 0.0094 | 0.0142 | -0.0158 | -0.0002 |

**Panel B: Cross-correlation of Squared TSE 300 TRI index return**

| k | CRSP Value-weighted return with distribution | Volume | Value | Transaction | Size (share) | Size (value) |
|---|---|---|---|---|---|---|
| -6 | -0.0153 | -0.0388** | -0.0474** | -0.0591** | 0.0040 | -0.0130 |
| -5 | -0.0075 | -0.0476** | -0.0645** | -0.0578** | -0.0117 | -0.0430** |
| -4 | 0.0700** | -0.0087 | -0.0319 | -0.0393** | 0.0305 | -0.0093 |
| -3 | 0.0607** | -0.0221 | -0.0425** | -0.0281 | -0.004 | -0.0395** |
| -2 | -0.0093 | 0.0076 | -0.0107 | 0.0129 | -0.0023 | -0.0357** |
| -1 | 0.0998** | 0.0913** | 0.0743** | 0.1066** | 0.0274 | 0.0047 |
| 0 | -0.1443** | 0.1279** | 0.1081** | 0.1371** | 0.0524** | 0.0270 |
| 1 | -0.0020 | 0.0200 | -0.0082 | 0.0093 | 0.0221 | -0.024 |
| 2 | -0.0815** | 0.0194 | 0.0017 | 0.0141 | 0.0157 | -0.0129 |
| 3 | -0.0394** | -0.0144 | -0.0296 | -0.0175 | -.0035 | -0.0299 |
| 4 | -0.0592** | -0.0147 | -0.0324** | -0.0186 | -.0028 | -0.0334** |
| 5 | -0.0267 | -0.0328 | -0.0544** | -0.0354** | -0.013 | -0.0513** |
| 6 | -0.0682** | -0.0108 | -0.0282 | -0.0109 | -0.0051 | -0.0349** |

**Panel C: Cross-correlation of Volume**

| k | CRSP Value-weighted return with distribution | Absolute TSE 300 TRI index return | Value | Transaction | Size (share) | Size (value) |
|---|---|---|---|---|---|---|
| -6 | -0.0006 | -0.0126 | 0.1099** | 0.1674** | -0.0097 | -0.0039 |
| -5 | 0.0103 | -0.0363** | 0.2504** | 0.2669** | 0.0991** | 0.1195** |
| -4 | 0.0176 | 0.0026 | 0.2483** | 0.3071** | 0.0446** | 0.0709** |
| -3 | -0.0074 | -0.0075 | 0.2545** | 0.3283** | 0.0164 | 0.0574** |
| -2 | -0.0161 | 0.0152 | 0.3127** | 0.3988** | 0.0476** | 0.0758** |
| -1 | 0.0123 | 0.0273 | 0.5022** | 0.5589** | 0.1887** | 0.2130** |
| 0 | 0.0489** | 0.1843** | 0.9303** | 0.7955** | 0.7239** | 0.6634** |
| 1 | 0.1242** | 0.1494** | 0.5089** | 0.5330** | 0.2182** | 0.2532** |
| 2 | 0.0707** | 0.0290 | 0.3207** | 0.3706** | 0.0798** | 0.1208** |
| 3 | 0.0531** | -0.0047 | 0.2595** | 0.3024** | 0.0459** | 0.0948** |
| 4 | 0.0581** | 0.0075 | 0.2557** | 0.2684** | 0.0887** | 0.1267** |
| 5 | 0.0726** | -0.0296 | 0.2676** | 0.2446** | 0.1245** | 0.173** |
| 6 | 0.0551** | -0.0114 | 0.1384** | 0.1523** | 0.0075 | 0.0605** |

* indicates significance at the 0.10 level ** indicates significance at the 0.05 level.

a. Negative lags (k<0) or leads indicate cross-correlations ρ (TSE $return_t$, $Volume_{t-k}$) between future volume and current returns.

b. Positive lags (k>0) indicate cross-correlations ρ (TSE $return_t$, $Volume_{t-k}$) between past volume and current returns.

**Table 3.4: Asymmetric Effects – Canadian and U.S. Returns**
**January 1980 - December 1995 (3921 observations)**

$DR_t$ is an indicator variable. $DR_t=1$, if $Return_t > 0$, else 0. Standard errors are reported in parentheses.

| | TSE Return | CRSP VWTD. Return |
|---|---|---|
| Constant | -0.069** | 0.0194 |
| | (0.0232) | (0.0276) |
| Monday | -0.1443** | -0.0894** |
| | (0.0294) | (0.0339) |
| Holiday | -0.0928* | -0.0907 |
| | (0.053) | (0.061) |
| $Return_{t-1}$ | 0.0693** | 0.0373 |
| | (0.029) | (0.0319) |
| $DR_{t-1}$ | 0.1122** | 0.0489 |
| | (0.0318) | (0.0371) |
| $DR_{t-1}$* $Return_{t-1}$ | 0.2539** | 0.0983** |
| | (0.0433) | (0.045) |
| R-sq | 0.0753 | 0.0143 |
| Residuals: LB(8) | 10.3671 | 9.4058 |
| p-value | (0.2402) | (0.3092) |
| Squared residuals :LB(8) | 724.0553 | 434.8913 |
| p-value | (0.0000) | (0.0000) |

* indicates significance at the 0.10 level.
** indicates significance at the 0.05 level.

**Table 3.5: Asymmetric Effects Volume and Stock Returns[a]**
**January 1980 - December 1995 (3921 observations)**

Volume is the difference from the forty-day moving average of the natural log of the aggregate trading volume (000s) of the stocks in the TSE 300 composite index. $DR_t$ is an indicator variable. $DR_t=1$, if TSE-$Ret_t>0$, else 0. Standard errors are reported in parentheses.

| | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| | Volume | Volume | Volume |
| Constant | 0.0269** | 0.0179* | 0.0103 |
| | (0.0092) | (0.0105) | (0.0118) |
| Monday | -0.259** | -0.2588** | -0.261** |
| | (0.0123) | (0.0123) | (0.0122) |
| Tuesday | 0.0371** | 0.0371** | 0.0387** |
| | (0.0123) | (0.0123) | (0.0123) |
| Thursday | -0.0324** | -0.0324** | -0.0328** |
| | (0.0123) | (0.0122) | (0.0122) |
| Friday | -0.1233** | -0.1234** | -0.122** |
| | (0.0118) | (0.0118) | (0.0118) |
| Holiday | -0.1361** | -0.1359** | -0.1482** |
| | (0.0174) | (0.0174) | (0.0174) |
| $Volume_{t-1}$ | 0.4451** | 0.4448** | 0.429** |
| | (0.0153) | (0.0153) | (0.0157) |
| $Volume_{t-2}$ | 0.1031** | 0.1037** | 0.105** |
| | (0.0168) | (0.0168) | (0.0167) |
| $Volume_{t-3}$ | 0.0751** | 0.0751** | 0.0785** |
| | (0.0153) | (0.0153) | (0.0152) |
| \|TSE Ret\| | 0.0642** | 0.072** | 0.0813** |
| | (0.0064) | (0.0093) | (0.0094) |
| $DR_t$ | | 0.018* | 0.0165 |
| | | (0.0102) | (0.0101) |
| $DR_t$*\|TSE Ret\| | 0.0867** | 0.0695** | 0.0515** |
| | (0.0099) | (0.0138) | (0.014) |
| $\|TSE\ Ret\|_{t-1}$ | | | -0.0309** |
| | | | (0.0095) |
| $DR_{t-1}$ | | | 0.0137 |
| | | | (0.0101) |
| $DR_{t-1}$*$\|TSE\ Ret_{t-1}\|$ | | | 0.0654** |
| | | | (0.0139) |
| R-sq | 0.4293 | 0.4296 | 0.4378 |
| Residual: LB-8 lags | 14.5734 | 14.6246 | 13.5385 |
| p-value | 0.0680 | 0.0069 | 0.0946 |
| Squared-residual:LB-8lags | 140.6243 | 143.6700 | 156.5007 |
| p-value | 0.0000 | 0.0000 | 0.0000 |

**indicates significance at the 0.05 level

* indicates significance at the 0.10 level.

[a] Additional results (not reported here) from regressions (2.3.1-2.3.2) support Karpoff's hypothesis.

**Table 3.6: Asymmetric Effects – Various Trade Variables and Returns**
**January 1980 - December 1995 (3921 observations)**

Volume is the difference from the forty-day moving average of the natural log of the aggregate trading volume (000s) of the stocks in the TSE 300 composite index. Value is the difference from the forty-day moving average of the natural log of the aggregate trading value (000s) of the stocks in the TSE 300 composite index. Transaction is the difference from the forty-day moving average of the natural log of the aggregate number of transactions of all the stocks in the Toronto stock exchange. Size (share) is the difference from the forty-day moving average of the natural log of the aggregate trading volume divided by the aggregate number of transactions of all the stocks in the Toronto stock exchange. Size (value) is the difference from the forty-day moving average of the natural log of the aggregate trading volume divided by the aggregate number of transactions of all the stocks in the Toronto stock exchange.
$DR_t$ is an indicator variable. $DR_t$=1, if TSE-$Ret_t$ >0, else 0. Standard-errors are reported in parentheses.

| | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 |
|---|---|---|---|---|---|
| | Volume | Value | Trans | Size(share) | Size (value) |
| Constant | 0.0530** | 0.0539** | -0.0019 | 0.0467** | 0.0442** |
| | (0.0104) | (0.0102) | (0.0061) | (0.0072) | (0.0069) |
| Mon | -0.2688** | -0.2768** | -0.0726** | -0.1727** | -0.1731** |
| | (0.0126) | (0.0129) | (0.0072) | (0.0086) | (0.0089) |
| Tue | 0.0306** | 0.0407** | 0.0476** | -0.0109 | 0.0005 |
| | (0.0129) | (0.0132) | (0.0070) | (0.0087) | (0.0086) |
| Thu | -0.0413** | -0.0476** | -0.0152** | -0.0110 | -0.0190** |
| | (0.0123) | (0.0118) | (0.0063) | (0.0089) | (0.0083) |
| Fri | -0.1296** | -0.1368** | -0.0448** | -0.0575** | -0.0655** |
| | (0.0118) | (0.0117) | (0.0066) | (0.0090) | (0.0087) |
| Dhol | -0.1463** | -0.1402** | -0.0519** | -0.0648** | -0.0629** |
| | (0.0208) | (0.0213) | (0.0138) | (0.0134) | (0.0142) |
| Trade $Var_{t-1}$ | 0.4298** | 0.4571** | 0.6786** | 0.2523** | 0.2837** |
| | (0.0237) | (0.0239) | (0.0342) | (0.0215) | (0.0188) |
| Trade $Var_{t-2}$ | 0.1336** | 0.1434** | 0.0418 | 0.0938** | 0.1103** |
| | (0.0197) | (0.0208) | (0.0306) | (0.0183) | (0.0189) |
| $Tradevar_{t-3}$ | 0.0659** | 0.0823** | 0.0414** | 0.0414** | 0.0653** |
| | (0.0160) | (0.0157) | (0.0181) | (0.0160) | (0.0161) |
| $Tradevar_{t-4}$ | | | 0.0306* | 0.0385** | 0.0208 |
| | | | (0.0162) | (0.0153) | (0.0160) |
| $Tradevar_{t-5}$ | | | | | 0.0771** |
| | | | | | (0.0154) |
| \|TSE Ret\| | 0.0859** | 0.0916** | 0.0648** | 0.0212** | 0.0264** |
| | (0.0084) | (0.0078) | (0.0061) | (0.0053) | (0.0057) |
| \|TSE $Ret_{t-1}$\| | -0.0279** | -0.0375** | -0.0291** | -0.0009 | -0.0159** |
| | (0.0089) | (0.0091) | (0.0063) | (0.0048) | (0.0054) |
| \|TSE $Ret_{t-2}$\| | -0.0665** | -0.0800** | -0.0513** | -0.0161** | -0.0280** |
| | (0.0077) | (0.0075) | (0.0049) | (0.0048) | (0.0050) |
| $DR_t$*\|TSE Ret\| | 0.0649** | 0.0814** | 0.0389** | 0.0324** | 0.0449** |
| | (0.0112) | (0.0109) | (0.0074) | (0.0067) | (0.0070) |
| $DR_{t-1}$*\|TSE $Ret_{t-1}$\| | 0.07871** | 0.1023** | 0.0661** | | 0.0336** |
| | (0.0126) | (0.0129) | (0.0085) | | (0.0072) |
| R-sq | 0.4491 | 0.5123 | 0.6500 | 0.2088 | 0.2853 |
| residual:LB-8 lags | 11.4408 | 12.2369 | 12.4700 | 8.9133 | 2.5010 |
| p-value | (0.1205) | (0.1409) | (0.1314) | (0.3497) | (0.9617) |
| Squared-residuals: LB- 8lags | 150.2991 | 199.3736 | 396.5769 | 36.6837 | 70.3895 |
| p-value | (0.0000) | (0.0000) | (0.0000) | (0.0000) | (0.0000) |

* indicates significance at the 0.05 level

** indicates significance at the 0.05 level.

**Table 3.7: Linear Granger Causality Test Results**
**January 1980 - December 1995 (3921 observations)**

This table reports the results of the linear Granger causality tests. Lag lengths are set with Schwartz's information criterion. P-value denotes the marginal significance level of the computed F-statistic used to test the zero restrictions implied by the null hypothesis of Granger non-causality

| | | | |
|---|---|---|---|
| **A. TSE Returns and TSE Return Volatility** | | | |
| $H_0$: TSE stock volatility does not cause TSE stock returns | | $H_0$: TSE Stock returns do not cause TSE stock volatility | |
| F-stat | p-value | F-stat | p-value |
| 25.6865 | 0.0000** | 6.9204 | 0.0086** |
| **B. Volume and TSE Returns[a]** | | | |
| $H_0$: Volume does not cause TSE stock returns | | $H_0$: TSE Stock returns do not cause volume | |
| F-stat | p-value | F-stat | p-value |
| 0.4737 | 0.8931 | 68.6858 | 0.0000** |
| **C. Volume and TSE Return Volatility[b]** | | | |
| $H_0$: Volume does not cause TSE stock return volatility | | $H_0$: TSE Stock returns volatility does not cause volume | |
| F-stat | p-value | F-stat | p-value |
| 1.6405 | 0.0980* | 7.5116 | 0.0000** |
| **D. Size (Share) and TSE Returns** | | | |
| $H_0$: Size does not cause TSE stock returns | | $H_0$: TSE Stock returns do not cause size | |
| F-stat | p-value | F-stat | p-value |
| 0.6135 | 0.7197 | 1.7791 | 0.0993* |
| **E. Size (Share) and TSE Return Volatility** | | | |
| $H_0$: Size does not cause TSE stock return volatility | | $H_0$: TSE Stock returns volatility does not cause size | |
| F-stat | p-value | F-stat | p-value |
| 0.4967 | 0.8113 | 1.1492 | 0.3309 |

* indicates significance at the 0.10 level; ** indicates significance at the 0.05 level

[a] Transactions, Value and Size (value) have similar causal relationships with returns.

[b] Transactions, Value and Size (value) have similar causal relationships with return volatility.

**Table 3.8: Nonlinear Granger Causality Test Results**
**January 1980 - December 1995 (3921 observations)**

This table reports the results of the nonlinear Granger causality tests. Lag lengths are set with Schwartz's information criterion.
P-value denotes the marginal significance level of the computed F-statistic used to test the zero restrictions implied by the null hypothesis of Granger non-causality and the asymmetry variable.

| $H_0$: TSE stock return does not nonlinearly cause TSE stock return volatility | | $H_0$: TSE Stock returns volatility does not nonlinearly cause TSE stock return | |
|---|---|---|---|
| Asymmetry Indicator variable ($DR_{t-1}=1$, if TSE $R_{t-1}>0$) | p-value | Asymmetry Indicator variable ($DR_{t-1}=1$, if TSE $R_{t-1}>0$) | p-value |
| 0.1107 | 0.1387 | 0.0754 | 0.0160** |
| F-stat | p-value | F-stat | p-value |
| 4.5580 | 0.0105** | 38.0340 | 0.0000** |
| $H_0$: TSE stock return does not nonlinearly cause TSE volume[32] | | $H_0$: TSE volume does not nonlinearly cause TSE stock return | |
| Asymmetry Indicator variable ($DR_{t-1}=1$, if TSE $R_{t-1}>0$) | p-value | Asymmetry Indicator variable ($DVol_{t-1}=1$, if TSE $Vol_{t-1}>0$) | p-value |
| 0.0216 | 0.0430** | 0.327 | 0.3540 |
| F-stat | p-value | F-stat | p-value |
| 35.8900 | 0.0000** | 0.5122 | 0.8827 |
| $H_0$: TSE stock return volatility does not nonlinearly cause TSE volume | | $H_0$: TSE volume[33] does not nonlinearly cause TSE stock return volatility | |
| Asymmetry Indicator variable ($DR_{t-1}=1$, if TSE $R_{t-1}>0$) | p-value | Asymmetry Indicator variable ($DVol_{t-1}=1$, if TSE $Vol_{t-1}>0$) | p-value |
| 0.0572 | 0.0000** | -0.0611 | 0.4499 |
| F-stat | p-value | F-stat | p-value |
| 14.3431 | 0.0000** | 1.5334 | 0.1208 |

* indicates significance at the 0.10 level ** indicates significance at the 0.05 level

[32] Value, transaction and size (value) have similar unidirectional nonlinear causal relationships with returns. Size (share) has no unidirectional nor bidirectional nonlinear causal relationship with returns nor return volatility.
[33] In contrast to volume, value and transactions nonlinearly cause TSE return volatility. Both size variables do not nonlinearly cause TSE return volatility.

**Table 3.9: Univariate GARCH(1,1) Model for TSE-TRI return -I**
**January 1980 - December 1995 (3921 observations)**

$R_t$ is the TSE 300 -TRI return and is the dependent variable. $R_{us,t}$ is the return on the CRSP equally weighted index. $MON_t$ is the dummy variable equal to 1 for Monday and 0 otherwise. $HOL_t$ is similarly defined for days that follow a holiday in either market. $DR_{t-1}$ is a dummy variable equal to 1 if $r_{t-1}>0$ and 0 otherwise. Standard errors are reported in parentheses.

| Variables | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 | Model 7 |
|---|---|---|---|---|---|---|---|
| **Mean Eqn.** | | | | | | | |
| Constant | -0.0276 | -0.0327 | -0.0216 | -0.0201 | -0.0229 | -0.0266 | -0.0237 |
| | (0.0205) | (-0.0212) | (0.0200) | (0.0199) | (0.0201) | (0.0203) | (0.0201) |
| MONDAY | -0.1184** | -0.1070** | -0.1170** | -0.1171** | -0.1168** | -0.1174** | -0.1185** |
| | (0.0241) | (0.0244) | (0.0243) | (0.0243) | (0.0243) | (0.0240) | (0.0241) |
| HOLIDAY | -0.0239 | -0.0225 | -0.0172 | -0.0177 | -0.0168 | -0.0253 | -0.0251 |
| | (-0.0390) | (0.0384) | (0.0401) | (0.0398) | (0.0400) | (0.0392) | (0.0387) |
| $DR_{t-1}$(=1,if $R_{t-1}>0$) | 0.0699** | 0.0688** | 0.0642** | 0.0632** | 0.0656** | 0.0679** | 0.0670** |
| | (0.0273) | (0.0270) | (0.0268) | (0.0268) | (0.0270) | (0.0272) | (0.0269) |
| $R_{t-1}$ | 0.0719** | 0.0807** | 0.0801** | 0.0812** | 0.0783** | 0.0743** | 0.0763** |
| | (0.0363) | (0.0373) | (0.0365) | (0.0366) | (0.0367) | (0.0361) | (0.0361) |
| $DR_{t-1}*R_{t-1}$ | 0.1873** | 0.1830** | 0.1781** | 0.1784** | 0.1822** | 0.1828** | 0.1811** |
| | (0.0483) | (0.0468) | (0.0492) | (0.0493) | (0.0492) | (0.0486) | (0.0486) |
| $R_{us,t-1}$ | 0.0739** | 0.0658** | 0.0754** | 0.0743** | 0.0748** | 0.0752** | 0.0738** |
| | (0.0168) | (0.0168) | (0.0168) | (0.0168) | (0.0169) | (0.0168) | (0.0167) |
| $R_{us,t-3}$ | -0.0152 | -0.0131 | -0.0200 | -0.0192 | -0.0187 | -0.0169 | -0.0165 |
| | (0.0133) | (0.0134) | (0.0136) | (0.0135) | (0.0136) | (0.0135) | (0.0134) |
| $R_{us,t-3}$ | 0.0370** | 0.0367** | 0.0364** | 0.0360** | 0.0372** | 0.0362** | 0.0357** |
| | (0.0128) | (0.0129) | (0.0128) | (0.0128) | (0.0129) | (0.0128) | (0.0128) |
| **Cond. Var. Eqn** | | | | | | | |
| $\alpha_{r,0}$- Constant | 0.0061 | 0.0025 | 0.0046 | 0.0043 | 0.0065 | 0.0053 | 0.0035 |
| | (0.0032) | (0.0032) | (0.0033) | (0.0034) | (0.0034) | (0.0031) | (0.0032) |
| $\alpha_{r,1}$-lag sq. residuals | 0.0923** | 0.0814** | 0.0996** | 0.1008** | 0.1016** | 0.0939** | 0.0924** |
| | (0.0060) | (0.0056) | (0.0072) | (0.0071) | (0.0071) | (0.0064) | (0.0061) |
| $\beta_{r,1}$-lag Cond Var. | 0.8703** | 0.8855** | 0.8519** | 0.8524** | 0.8499** | 0.8654** | 0.8701** |
| | (0.0831) | (0.0075) | (0.0096) | (0.0094) | (0.0097) | (0.0087) | (0.0082) |
| Mon | 0.0347** | 0.0504** | 0.0546** | 0.0548** | 0.0506** | 0.0404** | 0.0431** |
| | (0.0141) | (0.0147) | (0.0147) | (0.0151) | (0.0148) | (0.0142) | (0.0144) |
| Holiday | 0.0953** | 0.0817** | 0.1340** | 0.1272** | 0.1204** | 0.1069** | 0.1044** |
| | (0.0173) | (0.0169) | (0.0178) | (0.0174) | (0.0180) | (0.0170) | (0.0167) |
| $R_{us,t-1}$ | | -0.0427** | | | | | |
| | | (0.0097) | | | | | |
| $R_{us,t-2}$ | | -0.0072 | | | | | |
| | | (0.0137) | | | | | |
| $R_{us,t-3}$ | | 0.0366** | | | | | |
| | | (0.0087) | | | | | |
| $Volume_t$ | | | 0.0443** | | | | |
| | | | (0.0056) | | | | |
| $Value_t$ | | | | 0.0406** | | | |
| | | | | (0.0052) | | | |
| $Transaction_t$ | | | | | 0.0422** | | |
| | | | | | (0.0066) | | |
| $Size(Share)_t$ | | | | | | 0.0460** | |
| | | | | | | (0.0100) | |
| $Size(Value)_t$ | | | | | | | 0.0565** |
| | | | | | | | (0.0098) |
| Std. Residuals: | | | | | | | |
| Skewness | -0.3934** | -0.32806** | -0.3700** | -0.3714** | -0.3726** | -0.3925** | -0.3908** |
| Kurtosis | 2.4188** | 2.17535** | 2.1801** | 2.1865** | 2.2288** | 2.3858** | 2.3596** |
| LB(8) | 8.5000 | 8.5677 | 8.3970 | 8.6054 | 8.6026 | 8.1613 | 8.3955 |
| Sq. Std. Residuals: | | | | | | | |
| LB(8) | 12.7071 | 11.9780 | 7.7323 | 8.2441 | 8.7295 | 10.5129 | 11.6922 |
| Log-likelihood | -125.1375 | -108.6130 | -107.5830 | -107.7382 | -114.5719 | -119.0757 | -115.0530 |

* indicates significance at the 0.10 level ** indicates significance at the 0.05 level

**Table 3.10 Univariate GARCH(1,1) Model for TSE-TRI return - II**
**January 1980 - December 1995 (3921 observations)**

$R_t$ is the TSE 300-TRI return and is the dependent variable. $R_{us,t}$ is the return on the CRSP equally weighted index. $MON_t$ is the dummy variable equal to 1 for Monday and 0 otherwise. $HOL_t$ is similarly defined for days that follow a holiday in either market. $DR_{t-1}$ is a dummy variable equal to 1 if $r_{t-1}$>0 and 0 otherwise. Standard errors are reported in parentheses.

| Variables | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 |
|---|---|---|---|---|---|---|
| **Mean Eqn.** | | | | | | |
| Constant | -0.0217 | -0.0209 | -0.0205 | -0.0201 | -0.0217 | -0.0201 |
| | (0.0200) | (0.0199) | (0.0199) | (0.0199) | (0.0120) | (0.0199) |
| MONDAY | -0.1173** | -0.1174** | -0.1174** | -0.1174** | -0.1173** | -0.1174** |
| | (0.0243) | (0.0243) | (0.0243) | (0.0243) | (0.0243) | (0.0243) |
| HOLIDAY | -0.0183 | -0.0191 | -0.0191 | -0.0189 | -0.0183 | -0.0189 |
| | (0.0401) | (0.0398) | (0.0397) | (0.0395) | (0.0401) | (0.0395) |
| $DR_{t-1}$(=1,if $R_{t-1}$>0) | 0.0645** | 0.0641** | 0.0638** | 0.0634** | 0.0645** | 0.0634** |
| | (0.0268) | (0.0268) | (0.0268) | (0.0268) | (0.0268) | (0.0268) |
| $R_{t-1}$ | 0.0799** | 0.0805** | 0.0808** | 0.0810** | 0.0799** | 0.0810** |
| | (0.0364) | (0.0364) | (0.0364) | (0.0365) | (0.0364) | (0.0365) |
| $DR_{t-1}$*$R_{t-1}$ | 0.1776** | 0.1771** | 0.1773** | 0.1781** | 0.1776** | 0.1781** |
| | (0.0491) | (0.0491) | (0.0492) | (0.0493) | (0.0491) | (0.0493) |
| $R_{us,t-1}$ | 0.0755** | 0.0750** | 0.0747** | 0.0741** | 0.0755** | 0.0741** |
| | (0.0169) | (0.0168) | (0.0168) | (0.0168) | (0.0169) | (0.0168) |
| $R_{us,t-3}$ | -0.0199 | -0.0195 | -0.0193 | -0.0188 | -0.0199 | -0.0188 |
| | (0.0136) | (0.0135) | (0.0135) | (0.0134) | (0.0136) | (0.0134) |
| $R_{us,t-3}$ | 0.0362** | 0.0358** | 0.0357** | 0.0357** | 0.0362** | 0.0357** |
| | (0.0048) | (0.0128) | (0.0128) | (0.0128) | (0.0128) | (0.0128) |
| **Cond. Var Eqn** | | | | | | |
| $\alpha_{r,0}$- Constant | 0.0040** | 0.0035** | 0.0034** | 0.0035** | 0.0040** | 0.0035** |
| | (0.0033) | (0.0033) | (0.0033) | (0.0034) | (0.0033) | (0.0034) |
| $\alpha_{r,1}$-lag sq resid | 0.0984** | 0.0984** | 0.0986** | 0.0992** | 0.0984** | 0.0992** |
| | (0.0071) | (0.0070) | (0.0069) | (0.0069) | (0.0071) | (0.0069) |
| $\beta_{r,1}$-lag Cond Var. | 0.8540** | 0.8554** | 0.8558** | 0.8562** | 0.8540** | 0.8562** |
| | (0.0095) | (0.0093) | (0.0092) | (0.0091) | (0.0095) | (0.0091) |
| Mon | 0.0546** | 0.0549** | 0.0548** | 0.0541** | 0.0546** | 0.0541** |
| | (0.0147) | (0.0150) | (0.0151) | (0.0151) | (0.0147) | (0.0151) |
| Holiday | 0.1342** | 0.1319** | 0.1299** | 0.1252** | 0.1342** | 0.1252** |
| | (0.0180) | (0.0175) | (0.0176) | (0.0173) | (0.0180) | (0.0173) |
| $Volume_t$ | 0.0404** | 0.0363** | | | | |
| | (0.0064) | (0.0068) | | | | |
| $Trans_t$ | | | | | 0.0404** | 0.0342** |
| | | | | | (0.0064) | (0.0068) |
| $Value_t$ | | | 0.0350** | 0.0342** | | |
| | | | (0.0056) | (0.0068) | | |
| $Size\text{-}Share_t$ | 0.0134 | | 0.0236 | | 0.0537** | |
| | (0.0128) | | (0.0119) | | (0.0113) | |
| $Size\text{-}Value_t$ | | 0.0232 | | 0.0165 | | 0.0507** |
| | | (0.0125) | | (0.0134) | | (0.0104) |
| Std. Residuals: | | | | | | |
| Skewness | -0.3714** | -0.3727** | -0.3732** | -0.3740** | -0.3713** | -0.3740** |
| Kurtosis | 2.1901** | 2.1937** | 2.1957** | 2.2017** | 2.1899** | 2.2018** |
| LB(8) | 8.3372 | 8.4011 | 8.4570 | 8.5867 | 8.3365 | 8.5868 |
| Sq. Std. Residuals: | | | | | | |
| LB(8) | 7.8563 | 8.0708 | 8.2344 | 8.6424 | 7.8537 | 8.6434 |
| Log-likelihood | -107.2474 | -106.4686 | -106.4690 | -107.2740 | -107.2474 | -107.2740 |

* indicates significance at the 0.10 level ** indicates significance at the 0.05 level

**Table 3.11: Univariate GARCH(1,1) Model for the trade variables**
**January 1980 - December 1995 (3921 observations)**

$R_t$ is the TSE 300 -TRI return. $RSQ_t$ is the squared TSE 300 -TRI return. $R_{us,t}$ is the CRSP equally weighted index. $MON_t$ is the dummy variable equal to 1 for Monday and 0 otherwise. $HOL_t$ is similarly defined for days that follow a holiday in either market. $DR_{t-1}$ is a dummy variable equal to 1 if $R_{t-1}>0$ and 0 otherwise. Standard errors are reported in parentheses.

| Variables | VOLUME | VALUE | TRANSACTION | SIZE (Share) | SIZE (Value) |
|---|---|---|---|---|---|
| **Mean Eqn.** | | | | | |
| Constant | 0.0800** | 0.0475** | 0.0100* | 0.0603** | 0.0402** |
| | (.0099) | (0.0103) | (0.0051) | (0.0078) | (0.0083) |
| MONDAY | -0.2722** | -0.2582** | -0.0747** | -0.1762** | -0.1731** |
| | (0.0119) | (0.0119) | (0.0059) | (0.0088) | (0.0096) |
| TUESDAY | 0.0450** | 0.0599** | 0.0415** | 0.0098 | 0.0031 |
| | (0.0127) | (0.0126) | (0.0058) | (0.0092) | (0.0094) |
| THURSDAY | -0.0372** | -0.0371** | -0.0198** | -0.0130 | -0.0164* |
| | (0.0116) | (0.0113) | (0.0058) | (0.0087) | (0.0088) |
| FRIDAY | -0.1378** | -0.1220** | -0.0495** | -0.0639** | -0.0663** |
| | (0.0108) | (0.0113) | (0.0053) | (0.0087) | (0.0092) |
| HOLIDAY | -0.1225** | -0.1194** | -0.0507** | -0.0625** | -0.0584** |
| | (0.0210) | (0.0211) | (0.0112) | (0.0130) | (0.0143) |
| $DR_{t-1}$(=1,if $R_{t-1}>0$) | -0.0029 | 0.0232** | 0.0062 | -0.0006 | 0.0131* |
| | (0.0114) | (0.0098) | (0.0049) | (0.0081) | (0.0070) |
| $R_{t-1}$ | 0.0749** | -0.0062 | 0.0161* | 0.0073 | -0.0127 |
| | (0.0132) | (0.0158) | (0.0082) | (0.0095) | (0.0112) |
| $DR_{t-1}*R_{t-1}$ | 0.0336** | 0.0845** | 0.0229 | 0.0031 | 0.0395** |
| | (0.0080) | (0.0261) | (0.0142) | (0.0048) | (0.0180) |
| $RSQ_{t-1}$ | 0.0243** | -0.0066 | 0.0026 | 0.0014 | -0.0036 |
| | (0.0043) | (0.0041) | (0.0029) | (0.0012) | (0.0024) |
| $R_{us,t-1}$ | 0.0187** | 0.0198** | 0.0163** | 0.0064 | 0.0074* |
| | (0.0063) | (0.0064) | (0.0033) | (0.0046) | (0.0044) |
| Trade $Variable_{t-1}$ | 0.4447** | 0.4749** | 0.5998** | 0.2719** | 0.2940** |
| | (0.0189) | (0.0186) | (0.0171) | (0.0192) | (0.0175) |
| Trade $Variable_{t-2}$ | 0.1023** | 0.0782** | 0.0702** | 0.0978** | 0.0855** |
| | (0.0182) | (0.0164) | (0.0221) | (0.0178) | (0.0163) |
| Trade $Variable_{t-3}$ | 0.1172** | 0.0812** | 0.0507** | 0.0539** | 0.0708** |
| | (0.0147) | (0.0158) | (0.0190) | (0.0163) | (0.0170) |
| Trade $Variable_{t-4}$ | | 0.0344** | 0.0442** | | 0.0239 |
| | | (0.0150) | (0.0152) | | (0.0163) |
| Trade $Variable_{t-5}$ | | | 0.0172 | | 0.0727** |
| | | | (0.0150) | | (0.0161) |
| Trade $Variable_{t-6}$ | | | 0.0057 | | -0.0084 |
| | | | (0.0146) | | (0.0161) |
| Trade $Variable_{t-7}$ | | | -.0010 | | 0.01884 |
| | | | (0.0157) | | (0.0155) |
| Trade $Variable_{t-8}$ | | | 0.0014 | | 0.0084 |
| | | | (0.0147) | | (0.0155) |
| Trade $Variable_{t-9}$ | | | 0.0064 | | -0.0253 |
| | | | (0.0144) | | (0.0158) |
| Trade $Variable_{t-10}$ | | | 0.0147 | | 0.0105 |
| | | | (0.0116) | | (0.0156) |

* indicates significance at the 0.10 level ** indicates significance at the 0.05 level

**Table 3.11: CONTINUED**
**Univariate GARCH(1,1) Model for the trade variables**
**January 1980 - December 1995 (3921 observations)**

$R_t$ is the TSE 300 -TRI return. $RSQ_t$ is the squared TSE 300 -TRI return. $R_{us,t}$ is the CRSP equally weighted index. $MON_t$ is the dummy variable equal to 1 for Monday and 0 otherwise. $HOL_t$ is similarly defined for days that follow a holiday in either market. $DR_{t-1}$ is a dummy variable equal to 1 if $R_{t-1}$>0 and 0 otherwise. Standard errors are reported in parentheses.

| **Cond. Var Eqn** | VOLUME | VALUE | TRANSACTION | SIZE (Share) | SIZE (Value) |
|---|---|---|---|---|---|
| $\alpha_{r,0}$-Constant | 0.0341** | 0.0331** | 0.0054** | 0.0214** | 0.0244** |
| | (0.0025) | (0.0025) | (0.0003) | (0.0021) | (0.0020) |
| $\alpha_{r,1}$-lag sq resid. | 0.2049** | 0.2000** | 0.3446** | 0.1018** | 0.0654** |
| | (0.0144) | (0.0144) | (0.0156) | (0.0094) | (0.0120) |
| $\beta_{r,1}$- lag Cond Var. | 0.0546 | 0.0930 | 0.2086** | 0.1447 | 0.0223 |
| | (0.0499) | (0.0498) | (0.0252) | (1.7755) | (0.0784) |
| Mon | 0.0037 | 0.0029 | 0.0020** | -0.0047** | -0.0006 |
| | (0.0024) | (0.0024) | (0.0006) | (0.0011) | (0.0012) |
| Holiday | 0.0305** | 0.0296** | 0.0153** | 0.0016 | 0.0074** |
| | (0.0090) | (0.0088) | (0.0019) | (0.0027) | (0.0034) |
| $Rsq_{us,t-1}$ | 0.0038** | 0.0038** | 0.0016** | -0.0004 | -0.0004* |
| | (0.0010) | (0.0010) | (0.0003) | (0.0002) | (0.0002) |
| Std. Residuals: | | | | | |
| Skewness | -0.0067 | 1.1611** | -0.1409** | 1.1611** | 0.4213** |
| Kurtosis | 1.7972** | 6.2980** | 3.1562** | 6.2980** | 1.2476** |
| LB(8) | 12.9655 | 13.8046 | 14.1684 | 13.8046 | 0.2782 |
| Sq. Std. Residuals: | | | | | |
| LB(8) | 10.5414 | 2.6484 | 48.6300** | 2.6484 | 11.3412 |
| Log-likelihood | 3939.1922 | 3911.3294 | 6457.4261 | 5158.1671 | 5237.4941 |

* indicates significance at the 0.10 level ** indicates significance at the 0.05 level

**Table 3.12: Bivariate GARCH Model of TSE300-TRI returns and Trading Variables with Constant Conditional Correlation, MGARCH(1,1); January 1980 - December 1995 (3921 observations)**

$$R_t = a_{r,0} + D_{r,mon}MON_t + D_{r,hol}HOL_t + D_{r,asy}ASY_t + D_{r,asyss}ASYSS_t + \phi_{r,1}R_{t-1} + \sum_{p=1}^{3}\phi_{r,us,p}R_{us,t-1} + u_{rt}$$

$$u_{rt} \mid I_{t-1} \sim N(0, h_{rt})$$

$$V_t = a_0 + D_{v,mon}MON_t + D_{v,tue}TUE_t + D_{v,thu}THU_t + D_{v,fri}FRI_t + D_{v,hol}HOL_t + \phi_{v,1}R_{t-1} + D_{v,asy}ASY_t +$$

$$+ D_{v,asyss}ASYSS_t + \sum_{q=1}^{p}\theta_{v,q}V_{t-q} + \phi_{v,us}R_{us,t-1} + VRGARCHM * \sqrt{h_{rt-1}} + u_{vt}$$

$$u_{vt} \mid I_{t-1} \sim N(0, h_{vt})$$

$$h_{rt} = \alpha_{r,0} + \alpha_{r,1}u^2_{r,t-1} + \beta_{r,1}h_{r,t-1} + \alpha_{rv}u^2_{v,t-1} + \beta_{rv}h_{v,t-1} + VRD_{mon}MON_t + VRD_{hol}HOL_t + V_{r,us,1}R_{us,t-1} +$$

$$+ V_{r,val,1}V_{t-1} + RASY_r u^2_{r,t-1}(RASY_r = 1, \text{if } u_{r,t-1} < 0) + RVASY_r u^2_{v,t-1}(RVASY_r = 1, \text{if } u_{v,t-1} < 0)$$

$$h_{vt} = \alpha_{v,0} + \alpha_{v,1}u^2_{v,t-1} + \beta_{v,1}h_{v,t-1} + \alpha_{vr}u^2_{r,t-1} + \beta_{vr}h_{r,t-1} + VVD_{mon}MON_t + VVD_{tue}TUE_t + VVD_{thu}THU_t + VVD_{fri}FRI_t +$$

$$VVD_{hol}HOL_t + V_{v,usq,1}r^2_{us,t-1} + V_{r,val,1}V_{t-1} + VASY_r u^2_{r,t-1}(VASY_r = 1, \text{if } u_{v,t-1} < 0) +$$

$$VRASY_r u^2_{r,t-1}(VRASY_r = 1, \text{if } u_{v,t-1} < 0)$$

$$h_{rvt} = \alpha_{rv,0}\sqrt{h_{rt}h_{vt}}$$

$R_t$ is the TSE 300 -TRI return. $R_{us,t}$ is the return on the CRSP equally weighted index.
$MON_t$ is the dummy variable equal to 1 for Monday and 0 otherwise. $TUE_t$, $THU_t$, $FRI_t$ are similarly defined. $HOL_t$ is similarly defined for days that follow a holiday in either market. $ASY_t$ is a dummy variable equal to 1 if $r_{t-1}>0$ and 0 otherwise. $ASYSS_t$ is a dummy variable equal to $ASY_t * r_{t-1}$. $V_t$, the trading variable is the difference from the forty-day moving average of the natural log of the trading variable of the stocks in the TSE 300 composite index.

**Table 3.12: CONTINUED**
**Bivariate GARCH Model of TSE300 -TRI returns and Trading Variables with Constant Conditional Correlation, MGARCH(1,1); January 1980 - December 1995 (3921 observations)**

| Mean Equation | | Returns & value | | Returns & Transactions | | Returns & Size(Value) | |
|---|---|---|---|---|---|---|---|
| | Variable | parameter estimate | Standard error | parameter estimate | Standard error | parameter estimate | Standard error |
| Return Mean Eqn | Constant | -0.0259 | 0.0213 | -0.0292 | 0.0212 | -0.0289 | 0.0209 |
| | MON | -0.1102** | 0.0246 | -0.1069** | 0.0241 | -0.1134** | 0.0244 |
| | HOL | -0.0186 | 0.0385 | -0.0206 | 0.0386 | -0.0268 | 0.0396 |
| | $R_{t-1}$ | 0.0900** | 0.0394 | 0.0829** | 0.0400 | 0.0835** | 0.0380 |
| | $ASY_t$ | 0.0637** | 0.0273 | 0.0665** | 0.0272 | 0.0691** | 0.0273 |
| | $SSASY_t$ | 0.1734** | 0.0494 | 0.1704** | 0.0504 | 0.1725** | 0.0489 |
| | $R_{us,t-1}$ | 0.0660** | 0.0173 | 0.0710** | 0.0170 | 0.0696** | 0.0172 |
| | $R_{us,t-2}$ | -0.0146 | 0.0140 | -0.0111 | 0.0137 | -0.0163 | 0.0139 |
| | $R_{us,t-3}$ | 0.0330** | 0.0131 | 0.0345** | 0.0132 | 0.0355** | 0.0130 |
| Trading Variable Mean Equation | Constant | 0.1269** | 0.0140 | 0.0475** | 0.0072 | 0.0707** | 0.0100 |
| | MON | -0.2570** | 0.0125 | -0.0756** | 0.0059 | -0.1718** | 0.0094 |
| | TUE | 0.0394** | 0.0118 | 0.0378** | 0.0056 | 0.0011 | 0.0088 |
| | THU | -0.0342** | 0.0121 | -0.0220** | 0.0058 | -0.0163* | 0.0090 |
| | FRI | -0.1281** | 0.0118 | -0.0529** | 0.0054 | -0.0645** | 0.0093 |
| | HOL | -0.1113** | 0.0205 | -0.0466** | 0.0119 | -0.0551** | 0.0132 |
| | $R_{t-1}$ | -0.0357** | 0.0116 | -0.0074 | 0.0070 | -0.0078 | 0.0075 |
| | $ASY_t$ | 0.0196** | 0.0098 | 0.0044 | 0.0052 | 0.0074 | 0.0068 |
| | $SSASY_t$ | 0.1518** | 0.0176 | 0.0703** | 0.0099 | 0.0349** | 0.0112 |
| | $V_{t-1}$ | 0.4014** | 0.0195 | 0.5758** | 0.0176 | 0.2879** | 0.0174 |
| | $V_{t-2}$ | 0.1072** | 0.0183 | 0.0858** | 0.0222 | 0.0916** | 0.0174 |
| | $V_{t-3}$ | 0.1024** | 0.0168 | 0.0465** | 0.0185 | 0.0781** | 0.0174 |
| | $V_{t-4}$ | 0.0192 | 0.0162 | 0.0487** | 0.0150 | 0.0216 | 0.0170 |
| | $V_{t-5}$ | 0.0385** | 0.0139 | 0.0191 | 0.0149 | 0.0738** | 0.0163 |
| | $V_{t-6}$ | | | 0.0070 | 0.0152 | -0.0081 | 0.0159 |
| | $V_{t-7}$ | | | 0.0138 | 0.0121 | 0.0150 | 0.0148 |
| | $Rr_{us,t-1}$ | 0.0172** | 0.0062 | 0.0156** | 0.0032 | 0.0081* | 0.0044 |
| | GARCH-M Sqrt(Ret Cond Var) | -0.1465** | 0.0179 | -0.0714** | 0.0085 | -0.0442** | 0.0124 |

* indicates significance at the 0.10 level ** indicates significance at the 0.05 level

**Table 3.12: CONTINUED**
**Bivariate GARCH Model of TSE300 -TRI returns and Trading Variables with Constant Conditional Correlation, MGARCH(1,1) ; January 1980 - December 1995 (3921 observations)**

| | | Returns & value | | Returns & Transactions | | Returns & Size(Value) | |
|---|---|---|---|---|---|---|---|
| Conditional Variance Eqn. | Variable | parameter estimate | Standard error | parameter estimate | Standard error | parameter estimate | Standard error |
| Return -Cond. Var | $\alpha_{r,0}$ | 0.0093** | 0.0032 | 0.0107** | 0.0034 | 0.0111** | 0.0036 |
| | $\alpha_{rv,0}$ | 0.1094** | 0.0155 | 0.0979** | 0.0148 | 0.0908** | 0.0169 |
| | $\alpha_{r,1}$ | 0.0831** | 0.0116 | 0.0961** | 0.0123 | 0.0955** | 0.0115 |
| | $\beta_{r,1}$ | 0.8603** | 0.0097 | 0.8411** | 0.0105 | 0.8557** | 0.0093 |
| | $VRD_{mon}$ | 0.0423** | 0.0148 | 0.0506** | 0.0159 | 0.0439** | 0.0150 |
| | $VRD_{hol}$ | 0.0819** | 0.0185 | 0.0889** | 0.0187 | 0.0994** | 0.0181 |
| | $V_{r,us,1}$ | -0.0251** | 0.0140 | -0.0257** | 0.0058 | -0.0231** | 0.0052 |
| | $Val_{t-1}$ | 0.0156** | 0.0067 | | | | |
| | $Trans_{t-1}$ | | | 0.0179** | 0.0082 | | |
| | $Size(Value)_{t-1}$ | | | | | 0.0094 | 0.0145 |
| | $ASY_r$ | 0.0251 | 0.0737 | 0.0241 | 0.0151 | 0.0078 | 0.0134 |
| | $ASY_{rv}$ | 0.0030 | 0.0312 | 0.0548 | 0.0598 | -0.1835 | 0.1469 |
| Volume - Conditional variance | $\alpha_{v,0}$ | 0.0302** | 0.0026 | 0.0055** | 0.0005 | 0.0119** | 0.0027 |
| | $\alpha_{v,1}$ | 0.1413 | 0.1852 | 0.3494** | 0.0160 | 0.0589** | 0.0105 |
| | $\beta_{v,1}$ | 0.1579** | 0.0558 | 0.1644** | 0.0246 | 0.4538** | 0.1020 |
| | $\alpha_{vr}$ | 0.0054** | 0.0022 | 0.0042** | 0.0007 | -0.0000 | 0.0007 |
| | $VVD_{mon}$ | 0.0047 | 0.0030 | 0.0023** | 0.0007 | -0.0009 | 0.0016 |
| | $VVD_{tue}$ | -0.0102** | 0.0026 | -0.0007 | 0.0005 | -0.0031* | 0.0017 |
| | $VVD_{thu}$ | 0.0017 | 0.0028 | -0.0004 | 0.0005 | 0.0030* | 0.0018 |
| | $VVD_{fri}$ | 0.0021 | 0.0026 | 0.0005 | 0.0005 | 0.0027* | 0.0016 |
| | $VVD_{hol}$ | 0.0291** | 0.0083 | 0.0176** | 0.0021 | 0.0054** | 0.0028 |
| | $V_{v,usq,1}$ | 0.0019* | 0.001 | 0.0006* | 0.001 | 0.0000 | 0.0004 |
| | $ASY_v$ | 0.0925** | 0.0270 | | | | |
| | $ASY_{vr}$ | -0.0057 | 0.0023 | -0.0030 | 0.0008 | -0.0002 | 0.0008 |
| Standardized Residuals: | | TSE300 return | Value | TSE300 return | Transactions | TSE300 return | Size (Value) |
| Skewness | | -0.4681** | 0.4168** | -0.4573** | -1.5653** | -0.4650** | 0.4287** |
| Kurtosis | | 4.1802** | 4.8351** | 4.1538** | 16.7370** | 4.1705** | 1.5424** |
| LB(8) | | 8.3582 | 7.2444 | 8.1625 | 61.9179** | 8.3849 | 1.7353 |
| Squared Standardized Residuals: | | | | | | | |
| LB(8) | | 217.0166** | 173.2840** | 212.5821** | 261.4426** | 211.8891** | 53.1409** |
| Cross- residuals | | 9.2636 | | 5.7595 | | 13.8473 | |
| Log Likelihood | | 3901.5388 | | 6410.4106 | | 5158.8753 | |

* indicates significance at the 0.10 level ** indicates significance at the 0.05 level

**Table 3.13: Trivariate GARCH Model of TSE300 -TRI returns, Transactions and Value with Constant Conditional Correlation, MGARCH(1,1); January 1980 - December 1995 (3921 observations)**

Return Mean Equation:

$$r_t = a_{r,0} + D_{r,mon}MON_t + D_{r,hol}HOL_t + D_{r,asy}ASY_t + D_{r,asyss}SSASY_t + \phi_{r,1}r_{t-1} + \sum_{p=1}^{3}\phi_{r,us,p}r_{us,t-1} + u_{rt}$$

$$u_{rt} \mid I_{t-1} \sim N(0, h_{rt})$$

Value and Transactions Mean Equation:

$$V_t = a_0 + D_{v,mon}MON_t + D_{v,tue}TUE_t + D_{v,thu}THU_t + D_{v,fri}FRI_t + D_{v,hol}HOL_t + \phi_{v,1}r_{t-1} + D_{v,asy}ASY_t +$$

$$+ D_{v,asyss}ASYSS_t + \sum_{q=1}^{p}\theta_q V_{t-q} + \phi_{v,us,m}r_{us,t-1} + VRGARCHM * \sqrt{h_{rt-1}} + u_{vt}$$

$$u_{vt} \mid I_{t-1} \sim N(0, h_{vt})$$

$$h_{rt} = \alpha_{r,0} + \alpha_{r,1}u_{r,t-1}^2 + \beta_{r,1}h_{r,t-1} + \alpha_{rv}u_{v,t-1}^2 + \beta_{rv}h_{v,t-1} + VRD_{mon}MON_t + VRD_{hol}HOL_t + V_{r,us,1}r_{us,t-1} +$$

$$+ V_{r,val,1}V_{t-1} + RASY_r u_{r,t-1}^2(RASY_r = 1, if\, u_{r,t-1} < 0) + RVASY_r u_{v,t-1}^2(RVASY_r = 1, if\, u_{v,t-1} < 0)$$

$$h_{vt} = \alpha_{v,0} + \alpha_{v,1}u_{v,t-1}^2 + \beta_{v,1}h_{v,t-1} + \alpha_{vr}u_{r,t-1}^2 + \beta_{vr}h_{r,t-1} + VVD_{mon}MON_t + VVD_{tue}TUE_t + VVD_{thu}THU_t + VVD_{fri}FRI_t +$$

$$VVD_{hol}HOL_t + V_{v,usq,1}r^2_{us,t-1} + V_{val,1}V_{t-1} + VASY_r u_{r,t-1}^2(VASY_r = 1, if\, u_{v,t-1} < 0) +$$

$$VRASY_r u_{r,t-1}^2(VRASY_r = 1, if\, u_{v,t-1} < 0)$$

$$h_{trt} = \alpha_{tr,0} + \alpha_{tr,1}u_{tr,t-1}^2 + \beta_{tr,1}h_{tr,t-1} + \alpha_{trr}u_{r,t-1}^2 + \beta_{trr}h_{r,t-1} + VtD_{mon}MON_t + VtD_{tue}TUE_t + VtD_{thu}THU_t + VtD_{fri}FRI_t +$$

$$VtD_{hol}HOL_t + V_{tr,usq,1}r^2_{us,t-1} + V_{trl,1}Tr_{t-1} + TASY_r u_{r,t-1}^2(VASY_r = 1, if\, u_{v,t-1} < 0) +$$

$$TRASY_r u_{r,t-1}^2(VRASY_r = 1, if\, u_{r,t-1} < 0) h_{rvt} = \alpha_{rv,0}\sqrt{h_{rt}h_{vt}}$$

$r_t$ is the TSE 300 -TRI return. $r_{us,t}$ is the return on the CRSP equally weighted index. $MON_t$ is the dummy variable equal to 1 for Monday and 0 otherwise. $TUE_t$, $THU_t$, $FRI_t$ are similarly defined. $HOL_t$ is similarly defined for days that follow a holiday in either market. $ASY_t$ is a dummy variable equal to 1 if $r_{t-1}$>0 and 0 otherwise. $ASYSS_t$ is a dummy variable equal to $ASY_t{*}r_{t-1}$. $V_t$, the trading variable is the difference from the forty-day moving average of the natural log of the trading variable of the stocks in the TSE 300 composite index.

**Table 3.13: CONTINUED**
**Trivariate GARCH Model of TSE300 -TRI returns, Transactions and Value with Constant Conditional Correlation, MGARCH(1,1) ; January 1980 - December 1995 (3921 observations)**

| Mean Equation | Variable | Returns parameter estimate | Returns Standard error | Transactions Variable | Transactions parameter estimate | Transactions Standard error | Value Variable | Value parameter estimate | Value Standard error |
|---|---|---|---|---|---|---|---|---|---|
| Return Mean Eqn | Constant | -0.0212 | 0.0217 | Constant | 0.0553** | 0.0071 | Constant | 0.1343** | 0.0140 |
| | MON | -0.1034** | 0.0248 | MON | -0.0778** | 0.0058 | MON | -0.2567** | 0.0123 |
| | TUE | | | TUE | 0.0313** | 0.0055 | TUE | 0.0236** | 0.0116 |
| | THU | | | THU | -0.0230** | 0.0057 | THU | -0.0381** | 0.0118 |
| | FRI | | | FRI | -0.0543** | 0.0054 | FRI | -0.1240** | 0.0121 |
| | HOL | -0.0334 | 0.0383 | HOL | -0.0477** | 0.0104 | HOL | -0.1050** | 0.0175 |
| | $r_{t-1}$ | 0.1057** | 0.0413 | $r_{t-1}$ | -0.0140** | 0.0067 | $r_{t-1}$ | -0.0338** | 0.0113 |
| | $ASY_t$ | 0.0602** | 0.0275 | $ASY_t$ | 0.0053 | 0.0053 | $ASY_t$ | 0.0131 | 0.0096 |
| | $SSASY_t$ | 0.1510** | 0.0515 | $SSASY_t$ | 0.0852** | 0.0095 | $SSASY_t$ | 0.1520** | 0.0169 |
| | $r_{us,t-1}$ | 0.0676** | 0.0172 | $r_{us,t-1}$ | 0.0135** | 0.0032 | $r_{us,t-1}$ | 0.0177** | 0.0060 |
| | $r_{us,t-2}$ | -0.0136 | 0.0141 | $r_{us,t-2}$ | | | $r_{us,t-2}$ | | |
| | $r_{us,t-3}$ | 0.0326** | 0.0134 | $r_{us,t-3}$ | | | $r_{us,t-3}$ | | |
| | | | | $V_{t-1}$ | 0.5296** | 0.0152 | $V_{t-1}$ | 0.3570** | 0.0142 |
| | | | | $V_{t-2}$ | 0.0922** | 0.0171 | $V_{t-2}$ | 0.1135** | 0.0142 |
| | | | | $V_{t-3}$ | 0.0397** | 0.0154 | $V_{t-3}$ | 0.0951** | 0.0141 |
| | | | | $V_{t-4}$ | 0.0467** | 0.0132 | $V_{t-4}$ | 0.0282** | 0.0136 |
| | | | | $V_{t-5}$ | 0.0348** | 0.0129 | $V_{t-5}$ | 0.0570** | 0.0130 |
| | | | | $V_{t-6}$ | 0.0033 | 0.0131 | $V_{t-6}$ | -0.0037 | 0.0130 |
| | | | | $V_{t-7}$ | 0.0211** | 0.0104 | $V_{t-7}$ | 0.0072 | 0.0120 |
| | | | | GARCH-M Sqrt(Lagged Cond. Var) | -0.0851** | 0.0085 | GARCH-M Sqrt(Ret Cond Var) | -0.1479** | 0.0180 |

* indicates significance at the 0.10 level ** indicates significance at the 0.05 level

Table 3.13: CONTINUED
Trivariate GARCH Model of TSE300 -TRI returns, Transactions and Value with
Constant Conditional Correlation, MGARCH(1,1) ; January 1980 - December 1995 (3921 observations)

| Conditional Variance Eqn. | | Returns | | | Transactions | | | Value | |
|---|---|---|---|---|---|---|---|---|---|
| Return -Cond. Var | $\alpha_{r,0}$ | 0.0142** | 0.0037 | $\alpha_{t,0}$ | 0.0065** | 0.0005 | $\alpha_{v,0}$ | 0.0296** | 0.0026 |
| | $\alpha_{rt,0}$ | 0.0986** | 0.0153 | $\alpha_{tv,0}$ | 0.6862** | 0.0083 | | | |
| | $\alpha_{rv,0}$ | 0.1163** | 0.0159 | | | | | | |
| | $\alpha_{r,1}$ | 0.0842** | 0.0109 | $\alpha_{t,1}$ | 0.2828** | 0.0274 | $\alpha_{v,1}$ | 0.0708** | 0.0176 |
| | $\beta_{r,1}$ | 0.8416** | 0.0109 | $\beta_{t,1}$ | 0.1326** | 0.0309 | $\beta_{v,1}$ | 0.2014** | 0.0595 |
| | $\alpha_{rv}$ | -0.0350 | 0.0617 | $\alpha_{tr}$ | 0.0022** | 0.0006 | $\alpha_{vr}$ | 0.0018 | 0.0017 |
| | $\alpha_{rt}$ | 0.4952** | 0.1883 | $\alpha_{tv}$ | -0.0045 | 0.0037 | $\alpha_{vt}$ | 0.1308** | .0663 |
| | $VRD_{mon}$ | 0.0450** | 0.0166 | $VTD_{mon}$ | 0.0025** | 0.0006 | $VVD_{mon}$ | -0.0028 | 0.0026 |
| | $VRD_{tue}$ | | | $VTD_{tue}$ | -0.0003 | 0.0006 | $VVD_{tue}$ | -0.0066** | 0.0024 |
| | $VRD_{thu}$ | | | $VTD_{thu}$ | 0.0005 | 0.0005 | $VVD_{thu}$ | 0.0022 | 0.0025 |
| | $VRD_{fri}$ | | | $VTD_{fri}$ | 0.0014** | 0.0005 | $VVD_{fri}$ | 0.0022 | 0.0025 |
| | $VRD_{hol}$ | 0.0777** | 0.0193 | $VTD_{hol}$ | 0.0102** | 0.0017 | $VRD_{hol}$ | 0.0074** | 0.0024 |
| | $V_{r,us,1}$ | -0.0289** | 0.0057 | $V_{r,us,1}$ | | | $V_{r,us,1}$ | 0.0142** | 0.0058 |
| | $V_{v,usq,1}$ | | | $V_{v,usq,1}$ | 0.0007** | 0.0003 | $V_{v,usq,1}$ | 0.0008 | 0.0007 |
| | $Val_{t-1}$ | 0.0099 | 0.0158 | | | | | | |
| | $Trans_{t-1}$ | -0.0108 | 0.0176 | | | | | | |
| | $Size(Value)_{t-1}$ | | | | | | | | |
| | $ASY_r$ | 0.0360* | 0.0158 | $ASY_t$ | 0.0064 | 0.0321 | $ASY_v$ | -0.0973** | 0.0280 |
| | $ASY_{rv}$ | -0.1615 | 0.1124 | $ASY_{tr}$ | -0.0016** | 0.0007 | $ASY_{vr}$ | -0.0025 | 0.0017 |
| | $ASY_{rt}$ | -0.1403 | 0.2759 | $ASY_{tv}$ | 0.0130** | 0.0057 | $ASY_{vt}$ | 0.4999** | 0.1151 |
| Standardized Residuals: | | TSE300 return | Transactions | Value | | | | | |
| Skewness | | -0.4448** | -1.3792** | -0.3029** | | | | | |
| Kurtosis | | 4.0894** | 14.5983** | 4.7077** | | | | | |
| LB(8) | | 7.1194 | 80.6078** | 33.5655 | | | | | |
| Squared Standardized Residuals: LB(8) | | 215.8006** | 268.1509** | 136.8594** | | | | | |
| Cross- residuals | | 10.3562 | 16.5792** | 112.2807** | | | | | |
| Log Likelihood | | 11622.5546 | | | | | | | |

* indicates significance at the 0.10 level ** indicates significance at the 0.05 level

**Table 3.14: Trivariate GARCH Model of TSE300 -TRI returns, Transactions and Size (Value) with Constant Conditional Correlation, MGARCH(1,1) ; January 1980 - December 1995 (3921 observations)**

| Mean Equation | Returns | | | Transactions | | | Size (Value) | | |
|---|---|---|---|---|---|---|---|---|---|
| | Variable | Parameter estimate | Standard error | Variable | parameter estimate | Standard error | Variable | parameter estimate | Standard error |
| Return Mean Eqn | Constant | -0.0245 | 0.0213 | Constant | 0.0349** | 0.0134 | Constant | 0.0686** | 0.0096 |
| | MON | -0.1031** | 0.0245 | MON | -0.0762** | 0.0060 | MON | -0.1714** | 0.0092 |
| | TUE | | | TUE | 0.0384** | 0.0057 | TUE | -0.0016 | 0.0088 |
| | THU | | | THU | -0.0232** | 0.0058 | THU | -0.0158* | 0.0088 |
| | FRI | | | FRI | -0.0529** | 0.0056 | FRI | -0.0646** | 0.0092 |
| | HOL | -0.0275 | 0.0381 | HOL | -0.0439** | 0.0111 | HOL | -0.0552** | 0.0131 |
| | $r_{t-1}$ | 0.0993** | 0.0414 | $r_{t-1}$ | -0.0052 | 0.0071 | $r_{t-1}$ | -0.0011 | 0.0068 |
| | $ASY_t$ | 0.0618** | 0.0272 | $ASY_t$ | 0.0049 | 0.0052 | $ASY_t$ | 0.0057 | 0.0066 |
| | $SSASY_t$ | 0.1548** | 0.0520 | $SSASY_t$ | 0.0642** | 0.0100 | $SSASY_t$ | 0.0277** | 0.0108 |
| | $r_{us,t-1}$ | 0.0727** | 0.0172 | $r_{us,t-1}$ | 0.0154** | 0.0032 | $r_{us,t-1}$ | 0.0077* | 0.0043 |
| | $r_{us,t-2}$ | -0.0114 | 0.0140 | $r_{us,t-2}$ | | | $r_{us,t-2}$ | | |
| | $r_{us,t-3}$ | 0.0349** | 0.0134 | $r_{us,t-3}$ | | | $r_{us,t-3}$ | | |
| | | | | $V_{t-1}$ | 0.5981** | 0.0187 | $V_{t-1}$ | 0.2839** | 0.0170 |
| | | | | $V_{t-2}$ | 0.0799** | 0.0223 | $V_{t-2}$ | 0.0863** | 0.0168 |
| | | | | $V_{t-3}$ | 0.0310** | 0.0193 | $V_{t-3}$ | 0.0841** | 0.0168 |
| | | | | $V_{t-4}$ | 0.0528** | 0.0157 | $V_{t-4}$ | 0.0190 | 0.0164 |
| | | | | $V_{t-5}$ | 0.0213* | 0.0153 | $V_{t-5}$ | 0.0720** | 0.0157 |
| | | | | $V_{t-6}$ | 0.0047 | 0.0150 | $V_{t-6}$ | -0.0084 | 0.0156 |
| | | | | $V_{t-7}$ | 0.0179 | 0.0120 | $V_{t-7}$ | 0.0144 | 0.0146 |
| | | | | GARCH-M Sqrt(Lagged Cond Var) | -0.0674** | 0.0088 | GARCH-M Sqrt(Ret Cond Var) | -0.0370** | 0.0128 |

* indicates significance at the 0.10 level ** indicates significance at the 0.05 level

**Table 3.14: Continued**
**Trivariate GARCH Model of TSE300 -TRI returns, Transactions and Size (Value) with Constant Conditional Correlation, MGARCH(1,1) ; January 1980 - December 1995 (3921 observations)**

| Conditional Variance Eqn. | | Returns | | | Transactions | | | Size (Value) | |
|---|---|---|---|---|---|---|---|---|---|
| Return - Cond. Var | $\alpha_{r,0}$ | 0.0171** | 0.0042 | $\alpha_{t,0}$ | 0.0048** | 0.0005 | $\alpha_{v,0}$ | 0.0146** | 0.0026 |
| | $\alpha_{rt,0}$ | 0.0995** | 0.0153 | $\alpha_{tv,0}$ | 0.1952** | 0.0157 | | | |
| | $\alpha_{rv,0}$ | 0.0939** | 0.0149 | | | | | | |
| | $\alpha_{r,1}$ | 0.0852** | 0.0126 | $\alpha_{t,1}$ | 0.3245** | 0.0337 | $\alpha_{v,1}$ | 0.0451** | 0.0123 |
| | $\beta_{r,1}$ | 0.8335** | 0.0114 | $\beta_{t,1}$ | 0.2185** | 0.0265 | $\beta_{v,1}$ | 0.3149** | 0.1036 |
| | $\alpha_{rv}$ | 0.0177 | 0.1049 | $\alpha_{tr}$ | 0.0037** | 0.0008 | $\alpha_{vr}$ | -0.0001 | 0.0008 |
| | $\alpha_{rt}$ | 0.8546** | 0.1955 | $\alpha_{tv}$ | -0.0027 | 0.0048 | $\alpha_{vt}$ | 0.0022 | 0.0198 |
| | $VRD_{mon}$ | 0.0408** | 0.0164 | $VTD_{mon}$ | 0.0023** | 0.0007 | $VVD_{mon}$ | 0.0003 | 0.0016 |
| | $VRD_{tue}$ | | | $VTD_{tue}$ | -0.0012** | 0.0006 | $VVD_{tue}$ | -0.0022 | 0.0016 |
| | $VRD_{thu}$ | | | $VTD_{thu}$ | -0.0005 | 0.0005 | $VVD_{thu}$ | 0.0026 | 0.0017 |
| | $VRD_{fri}$ | | | $VTD_{fri}$ | 0.0003** | 0.0005 | $VVD_{fri}$ | 0.0042** | 0.0015 |
| | $VRD_{hol}$ | 0.0832** | 0.0195 | $VTD_{hol}$ | 0.0126** | 0.0020 | $VRD_{hol}$ | 0.0019 | 0.0025 |
| | $V_{r,us,1}$ | -0.0289** | 0.0058 | $V_{r,us,1}$ | | | $V_{r,us,1}$ | 0.0142** | 0.0058 |
| | $V_{v,usq,1}$ | | | $V_{v,usq,1}$ | 0.0007** | 0.0003 | $V_{v,usq,1}$ | -0.0001 | 0.0004 |
| | $Val_{t-1}$ | -0.0108 | 0.0230 | | | | | | |
| | $Trans_{t-1}$ | 0.0041 | 0.0093 | | | | | | |
| | $Size(Value)_{t-1}$ | | | | | | | | |
| | $ASY_r$ | 0.0359** | 0.0165 | $ASY_t$ | -0.0052 | 0.0375 | $ASY_v$ | -0.0189 | 0.0232 |
| | $ASY_{rv}$ | -0.1615 | 0.1124 | $ASY_{tr}$ | -0.0024** | 0.0009 | $ASY_{vr}$ | -0.0002 | 0.0008 |
| | $ASY_{tv}$ | -0.1403 | 0.2759 | $ASY_{tv}$ | 0.0529** | 0.0096 | $ASY_{vt}$ | 0.1102** | 0.0362 |
| Standardized Residuals: | | TSE300 return | Transactions | Size (Value) | | | | | |
| Skewness | | -0.4415** | -1.4287** | 0.4182** | | | | | |
| Kurtosis | | 4.0132** | 15.3137** | 1.5129** | | | | | |
| LB(8) | | 6.8363 | 51.9423** | 2.4412 | | | | | |
| Squared Standardized Residuals: | | | | | | | | | |
| LB(8) | | 209.1662** | 316.5651** | 49.8723** | | | | | |
| Cross- residuals | | 5.7977 | 13.8550* | 8.3836 | | | | | |
| Log Likelihood | | 11717.4584 | | | | | | | |

* indicates significance at the 0.10 level ** indicates significance at the 0.05 level

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# CHAPTER 4

## THESIS CONCLUSION

The unifying theme of the thesis is the nonlinearity of asset returns and its implications for theoretical and empirical asset pricing. In chapter 2, we propose an "artificial regression model" as an alternative nonlinear specification for asset returns. The artificial regression model simplifies the econometric specification from a two-moment (time-varying) specification to one where only the first moment needs to be specified. The economic intuition underlying the model is that increasing the precision of information should shrink volatility and lessen heteroskedasticity in the error structure that is caused by misspecifying the nonlinear conditional mean. An important practical importance of the model is that it generalizes to a multivariate approach in a natural way and there is a major reduction in computational complexity in estimation. Given that many issues in finance particularly asset pricing deal with multivariate systems, our proposed framework has important practical implications. The results from the Monte Carlo studies and the empirical analysis support the proposed model.

In chapter 3, we investigate the inter-day dynamics of the Canadian market. In a departure from the U.S. markets the results strongly support the sequential information flow and positive feedback trading hypotheses. The multivariate analysis reveals that the economic significance of trading volume as an information flow measure may be questionable. Our multivariate specifications are unable to capture all the persistence in return variances and complement earlier studies that the joint dynamics of returns and volume are not due to time series behavior of the information arrival process. The results thus suggest that a potential model is the artificial regression model proposed in the preceding chapter.

# IMAGE EVALUATION TEST TARGET (QA-3)

150mm

6"

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