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GEOSTATISTICS OF THE ESTEVAN COAL SEAM IN THE BOUNDARY DAM MINE, SOUTHERN SASKATCHEWAN

by

PETER MACKENZIE WATERS

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE

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DEDICATION

To my family:

Chris, Jaime and Pam.

ABSTRACT

Traditionally, reserve estimates of plains coal have not included an error analysis, either of the input variables (average net thickness, area and density) or of the resulting tonnage. Relative sizes of the errors can be used to choose between different sources of data or between different methods of calculation; the better data and method should have a lower error. Density and seam thickness data require the use of French style geostatistics to determine their errors because they are regionalized variables.

Data on the production for two years from the Estevan Zone at the Boundary Dam Mine in southern Saskatchewan was studied to determine the sizes and sources of errors in the tonnage calculations. The results may be applicable to other plains coal mines.

Net thickness is the main variable studied geostatistically. Unfortunately, there was not enough density data to estimate a variogram, so this variable, along with the area data, were analysed using traditional statistics.

Two sources of thickness data were available: drill holes (at 150 m centres) and detailed (30 m spacing) measurements of the highwall. Two methods of estimating the average thickness were considered: ordinary kriging, which should be close to the best (=minimum variance) method, and the simplest, an arithmetic average. When comparing methods using pre-production data, the results were compared to the 'true' tonnage determined from detailed in-pit measurements.

The variogram models of the thickness were isotropic with two nested spherical structures and no nugget effect. The drill hole variogram had a higher sill and a longer range than the pit measurement variogram.

Density and thickness contribute about equally to the error of the tonnage. If the error in the area is controlled by the amount of ground that can be opened in one year, then the error of the area is an order of magnitude larger than the other errors. In this case, it does not matter what data or what method is used.

The drill holes produce a biased estimate, low by 6%, of the thickness when compared to the pit measurements. This is because the geological seam was logged in the holes not the seam as it would be mined. Better data to use is highwall measurements from an opened pit near the projected pit; the resulting estimate is unbiased with a slightly lower estimation variance.

The seam is so regular that the estimation errors are very small (1% to 5%). This means that the krige variance is only slightly smaller than the variance of the arithmetic average, which suggests that the extra effort required for geostatistical estimating is not justified. In this mine geostatistics is only useful for determining the proper variance of the thickness.

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List of Symbols

Geostatistical papers are notorious for loose notation, using a system which is unique to geostatistics. It can change several times within one paper, and sometimes the same symbol has two referents; for instance, "a" is the weight in a linear combination as well as the range of a transitional variogram. Context is often essential to unravel the formulas. I have attempted to follow the notation used by David (1977) within the limitations of the Scientific character set on the printer at the University of Alberta.

This study follows several conventions.
1. Regular font is used for scalar variables, such as the Lagrange parameter μ.
2. Lowercase **bold** is used for vector variables, such as the sample separation vector **h**; for instance |**h**| = h.
3. Uppercase *ITALICS* are used for matrices, such as the krige matrix *K*.
4. Square brackets [ ] are used to indicate the elements of an array or the argument of a function or operation; for instance $f[\chi]$ or $E[\mathcal{R}]$.
5. Any general variable is represented by χ or $\mathcal{R}$ as above.
6. All units are metric, though the mine still uses Imperial units.

The page refers to where the variable is first used or described.

Variable	Description	Page
σ	A standard deviation when used with an error	66
T	The estimated tonnage of coal in a domain	83
A	The area of the domain for which the tonnage is calculated	83
d	The average density of coal throughout the mine	84
R	The coal recovery	98
CLO	The coal-loaded-out	98
n	The exponent of a power model	127
Z*	The estimated value of Z	135
Z	A regionalized variable, thickness in this study. Usually it refers to the true value	135

Variable	Description	Page
*	This indicates an estimate, to differentiate it from the true value. In this study it is used only for the thickness	135
Σ_i	Abbreviation for the summation of all subscripted variables for i = 1 to n	135
n	The number of samples or sample pairs	135
a_i	The i'th weight	135
z_i	The i'th sample	135
$f[\mathcal{R}]$	An arbitrary function of $\mathcal{R}$	135
σ^2	The variance, in geostatistics it is always called the population variance	136
$m(\mathbf{h})$	The drift of the increments of a regionalized variable. It is not a constant but a function of $\mathbf{h}$	136
$E[\mathcal{R}]$	The expected value of $\mathcal{R}$	136
$\mathcal{R}$	A general variable	136
$Z(\mathbf{x})$	The value of the regionalized variable at location $\mathbf{x}$	136
$\mathbf{x}$	A location expressed as a vector. That is: $\mathbf{x} = [u,v]$, u is the easting and v the northing	136
$Z(\mathbf{x}+\mathbf{h})$	The value of the regionalized variable at location $\mathbf{x}+\mathbf{h}$, which is separated from location $\mathbf{x}$ by the vector $\mathbf{h}$	136
$\mathbf{h}$	The vector separating two samples or locations	136
$\gamma(\mathbf{h})$, γ	The variogram, or more correctly the semi-variogram	136
$VAR[\mathcal{R}]$	The variance of $\mathcal{R}$	136
H	The maximum distance used in the kriging equations	137
$N(\mathbf{h})$	The number of sample pairs separated by $\mathbf{h}$	137
$Z(\mathbf{x}_i)$	A sample at location $\mathbf{x}_i$	137

Variable	Description	Page
$\mathbf{x}_i$	A location in vector form. That is: $\mathbf{x}_i = [u_i, v_i]$	137
$Z(\mathbf{x}_i+\mathbf{h})$	A sample at location $\mathbf{x}_i+\mathbf{h}$, which is removed from sample $Z(\mathbf{x}_i)$ by the vector $\mathbf{h}$	137
a	The range of a transition variogram, the distance beyond which the variogram stays level and there is no correlation between samples	139
C	The sill of a transition model variogram. The height of the variogram beyond the range or the maximum variation between samples	139
l/a	The ratio of the support to the range, it measures the effect of regularization due to finite support	143
l	The size of the support of the data. The support is the size (and shape) of the sample over which the variable is averaged	143
Lag	is the length of the distance class used to calculate a variogram from irregularly spaced data. See Figure 1.5, p.145	143
ψ	Psi is half the width of the angular class used to calculate a variogram from irregularly spaced data. See Figure 1.5, p.145	143
ϕ	Phi is the direction of the distance class used to calculate a variogram from irregularly spaced data. See Figure 1.5, p.145	143
$d\gamma(h)/dh$	A first derivative, in this case the slope of the variogram in the direction of $\mathbf{h}$	146
C_0	The nugget effect, the sill of structures with a range less than the shortest sample spacing	146
C_t	The relative nugget effect, the ratio of the nugget effect to the overall sill	148
L	The maximum sample separation for which the variogram has been calculated	149
α	Significance level of a statistical test	165

Variable	Description	Page
σ_z^2	The variance of any estimation	200
V	The domain over which Z is averaged, in general it is a volume, in this study it is an area	201
$\bar{\gamma}(V_1,V_2)$	The average variogram between regions V_1 and V_2	201
$\bar{\gamma}(V,V)$	The average variogram between the domain and itself	201
$\bar{\gamma}(x_i,V)$	The average variogram between the sample x_i and the domain V	201
$\bar{\gamma}(x_i,x_j)$	The average variogram between samples x_i and x_j	201
$\partial\sigma_z^2/\partial a_i$	The partial derivative of σ_z^2 with respect to a_i	202
μ	The Lagrange parameter, it is the variance of the overall mean of the deposit	202
$\bullet$	Matrix multiplication	202
K	The krige matrix, most elements are the average variogram between each pair of samples, $\bar{\gamma}(x_i,x_j)$	203
A	The solution to the krige system of equations, a column vector of the Lagrange parameter and the required weights for the krige estimate	203
A^t	The transpose of A, the solution vector	203
D	A column vector of the average variograms between each sample and the domain, $\bar{\gamma}(x_i,V)$	203
K^{-1}	The inverse of the matrix K	203
Z_k^*	The krige estimator	203
σ_k^2	The variance of the krige estimate, usually called the krige variance	203
Z_x^*	The extension estimate, which is the arithmetic average of the data	203
σ_x^2	The extension variance, which is the	

Variable	Description	Page
	variance of the extension estimate of the data	203
χ_i	A general independent variable	218
T	A general dependent variable	218
$COV[\chi_i, \chi_j]$	The covariance between χ_i and χ_j	218
$\prod_i$	The product of all subscripted variables, for i = 1, 2, 3, ··· ,n	219
$\partial^2 T/\partial \chi_i^2$	Second partial derivative of T with respect to χ_i	219
$VAR[\chi]/\chi^2$	The relative variance of χ	219
L	The curvilinear length along a bench	226
t	The time required to remove a bench	226
α, β, θ	Regression parameters	226

1. INTRODUCTION

Matheron (1971, p.5) defines geostatistics as "···the application of the theory of the regionalized variables to the estimation of mineral deposits···" Geostatistical estimates of coal reserves have been made for the Boundary Dam Mine at Estevan in southern Saskatchewan (Figure 1.1). In this mine lignite is stripped from the Estevan Zone of the Paleocene Ravenscrag Formation.

1.1 OBJECTIVES OF THIS STUDY

The objective of this study is to examine the methods of determining reserve estimates in plains coal mines. The basic formula is:

Tonnage = average thickness x area x density

The overall objective is divided into two major portions.

1. Study the optimum drill hole spacing. It may be that hole separation could be increased for a less expensive sampling program with little loss in accuracy.
2. Study the methods used to estimate reserves, in particular:
   A. Choose the better of the two sources of data available on the average seam thickness over an area to be mined. The choice is between highwall measurements from a nearby pit and drill holes into the proposed pit.
   B. Test geostatistically-determined estimates to see if

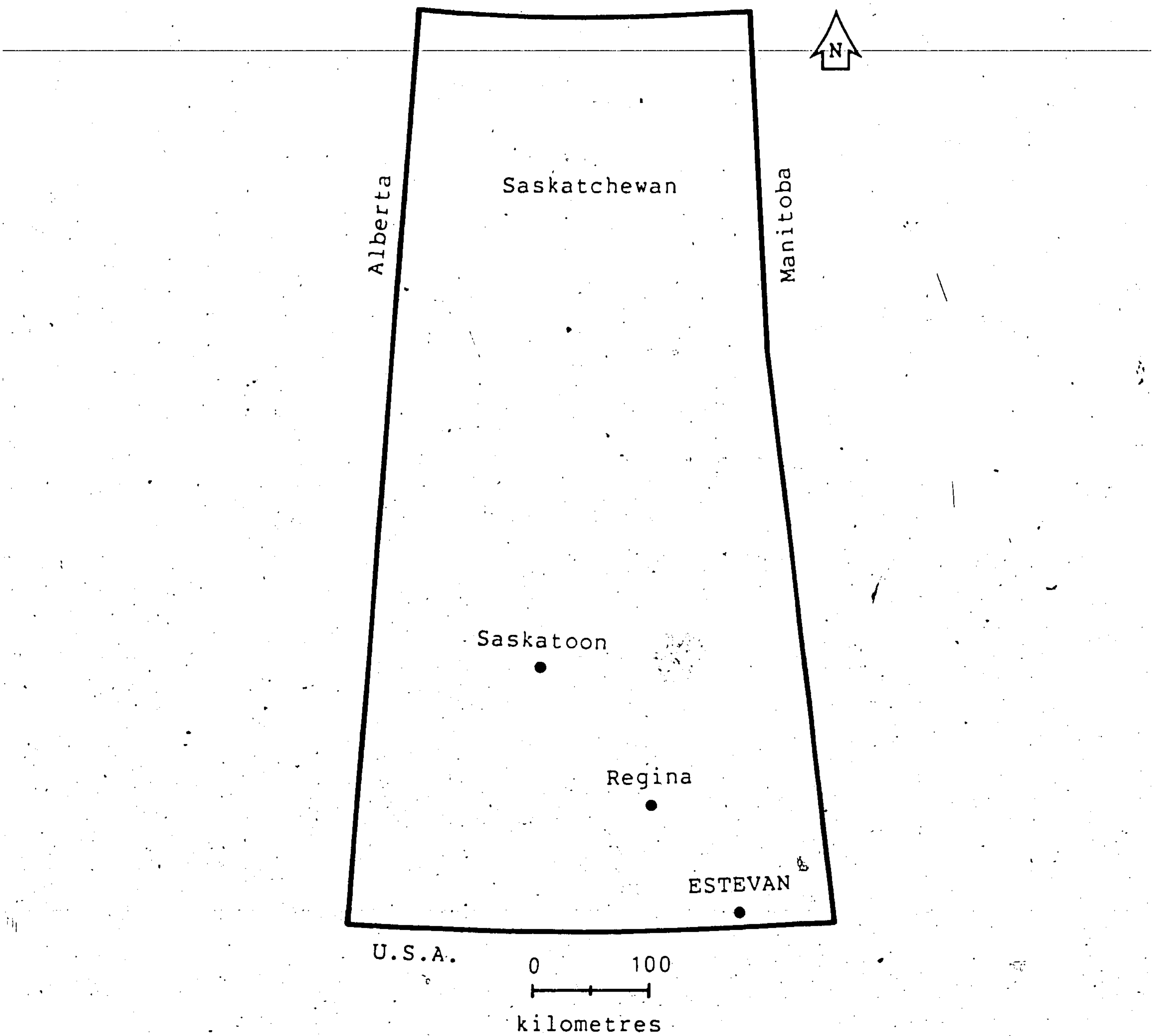


Figure 1.1 **Location map**

they are useful or better than the traditional methods. Geostatistical estimates are theoretically supposed to be the best, but usually at a considerable cost in effort. There may be little accuracy lost in using simpler methods, simpler even than the methods presently used at the mine.

C. Determine the size of the errors in the tonnage estimates and recovery estimates. This will indicate how much confidence to place in the numbers, how detailed to make any future calculations, and whether a given change in the recovery is real or just a statistical fluctuation.

D. Outline the major sources of the error in the tonnage figures. This should show where to concentrate effort if the accuracy of the estimates is to be refined. If the error of the reserves is dominated by the error of one input variable, say 'area', then improving the accuracy of that one will increase the accuracy of the estimate the most.

Determining the accuracy and errors above requires statistical analysis and because density and thickness are regionalized variables†, geostatistical analysis is required. All geostatistics must begin with variogram analysis, so a preliminary objective is to determine the experimental variogram and to model it.

† See Appendix 1.1.1 for a definition of a regionalized variable.

1.2 OUTLINE

A complete geostatistical study is done in several stages; beginning with a crude preliminary study, making many assumptions; finishing by refinements, correcting for each of the assumptions made in the first stage. This study is only the first stage, but the results were clear enough that the objectives were achieved without having to correct for all of the assumptions.

Tonnage (T) is calculated from the average† net thickness (Z)‡, density of the coal (d) and area of the pit (A) using:

$$T = Z \cdot d \cdot A \qquad \{1.1\}$$

Recovery (R) is calculated from the tonnage and the coal-loaded-out (CLO) using:

$$R = 100 \cdot CLO/T \qquad \{1.2\}$$

The main type of data is thickness measurements of the seam, which have two sources: daily measurements in the pit, and drill hole measurements. Most calculations were done with both to determine the best data to use. The other input variables (density, area and coal-loaded-out) are from

† 'Average' in this study, is rarely used the traditional sense of an arithmetic mean, but in the more general meaning of the 'best' single number with which to replace all the numbers in a subset of the data.
‡ Traditionally in geostatistics Z refers to a regionalized variable.

the mine records.

Only thickness was analysed geostatistically. Although density is regionalized, there is not enough data to estimate the variogram. Instead ordinary (or classical) statistics were used to analyse the density, and also the area and coal-loaded-out. Geostatistical analysis along with tonnage estimation form the bulk of this study.

Any statistical analysis begins with exploration of the data, that is, estimating the basic statistical properties of the data. For the thickness, these include drift and variogram analysis because geostatistics is used. Variogram analysis begins with the calculation of the raw variogram, correcting it to form the experimental variogram, which is modelled. The variogram model should closely estimate the true variogram that is required to determine the estimation variances.

Optimum drill ~~hole~~ spacing was considered in this study only as a trade-off of the expense of more closely spaced holes for a more accurate reserve estimate. In practice, other considerations also affect the choice of hole spacing. The trade-off was illustrated with a curve of the estimation variance against the spacing in the drilling grid. This curve shows at what spacing (70 to 220 m) there is a trade-off with the reserve accuracy. The particular balance between the two within this range must be left to the judgement of the mine engineer. The curve was calculated for a representative pit, approximately two year's production, and using a square grid of drill holes that was expanded for each successive point on the graph.

Coal tonnages, and in some cases recoveries, were estimated in several ways for several mined areas representing production for one month to two years. Tonnage and the resulting recoveries are most strongly affected by the average net thickness, on which most of the effort was spent. Tonnages for the area were calculated using the thickness determined in two ways: kriging, the best method from a geostatistical point of view, and an arithmetic average, the simplest method. Whenever an estimate was made, its variance was determined in order to compare the accuracy of the different methods. Finally, the error of the input variables was used to determine the sources of the tonnage and recovery errors.

Two types of tonnage were calculated: in-place coal and reserves. The in-place coal is the amount of coal that was in a bench prior to stripping and was estimated using thicknesses measured on the highwall after removal of the coal. It was used, along with the coal-loaded-out to determine the recovery. Reserves, on the other hand, were estimated for several locations after they were mined out, but using only pre-production data, either drill hole or nearby highwall measurements. These two numbers were compared to the 'true' reserve figure, which was estimated from post-production measurements of the highwall.

The error of the reserves for locations not yet mined depends heavily on the error of the area (VAR[A]) to be exposed. Two cases were considered:

1. VAR[A] = 0,
2. VAR[A] > 0.

The first case is when the area is given, such as a lease limit, and corresponds more closely to the strict meaning of geological reserves. In the second case, the area is determined by the amount of ground that can be exposed in a given unit of time, and so is highly variable.

1.3 PREVIOUS WORK

Little previous geostatistical work has been done that applies directly to this study. Most of the previous work is geological.

1.3.1 Previous Geostatistical Work

Techniques of geostatistics have only in the last five to ten years slowly made their way into North America after twenty to thirty years development in France and South Africa. Journel and Huijbregts (1978) and David (1977) have the most extensive bibliographies of case studies. These two, along with Clark (1979*a*), give the best practical descriptions of the methodology in English. Matheron (1971) describes the theory behind the applications.

Over 200 deposits have been analysed by geostatistics, (David, 1977, p.70), but there are few previous studies similar to this one. Journel and Huijbregts (1978, p.595) list 36 deposits that they used as examples of geostatistical analysis. Of these, half the studies are on tabular deposits of three types: bedded, residual soil and veins. Half of these tabular deposits are bedded, but only use thickness of the strata in conjunction with the accumulation†, never alone. None deals with the geostatistics of a coal mine.

What work has been done on coal mines has for the most part been done on coal quality. The only person to have done a geostatistical study of plains coal was Flint (1978). He is referred to often in this study, so when no year is suffixed, it should be taken to refer to his M.Sc. thesis, Flint (1978). He estimated the resources and their variances for three coal zones (including the Estevan Zone) in the Ravenscrag Formation, two of which are present over the entire Estevan Field.

† Accumulation is the average grade of an ore intersection times the length of the intersection.

Flint found the net coal thickness to be stationary and isotropic. The variogram model was spherical with ranges of 200 m, 800 m to 1000 m and 4000 m to 6000 m; and a fairly large nugget effect. The coefficient of variation of the tonnnage was about 20%, with over two thirds of the error derived from error in the determination of the area underlain by coal. The error in the density contributed a minor amount.

Sabourin (1975) examined the sulphur content in the Lingan Coal Mine of Cape Breton. Rendu and David (1979) studied the calorific value of the Hat Creek coal deposit of B.C. They introduced a new method of using geostatistics for studying thick sedimentary deposits by treating the deposit more like an irregular ore body than a regular strata-bound coal deposit. Irvine *et al.* (1978) have described a detailed coal resource inventory of the Ravenscrag Formation, using computerized traditional methods involving SURFACE II† and arbitrary distance-proportional weights, rather than geostatistics.

1.3.2 Previous Geological Work

Previous geological work in the area is extensive, beginning with preliminary studies done for the Palliser expedition in 1857. G.M. Dawson did the first serious geological investigations in 1875, but most of the geological

† SURFACE II is a grid manipulation and contouring package (Sampson, 1978).

work was done in the first third of this century on outcrops. More work has been done in the late sixties and early seventies using drill hole data. An account of previous work is detailed in Flint (1978, p.11). The most extensive bibliographies of the previous work are in Irvine *et al.* (1978) and Flint (1978).

1.4 REGIONAL, LOCAL AND MINE GEOLOGY

The Boundary Dam Mine is located in the plains physiographic region of central North America. The topography is mostly flat with low rolling hills with deep and sometimes broad valleys. Geologically, the study area is near the northern rim of the Williston Basin.

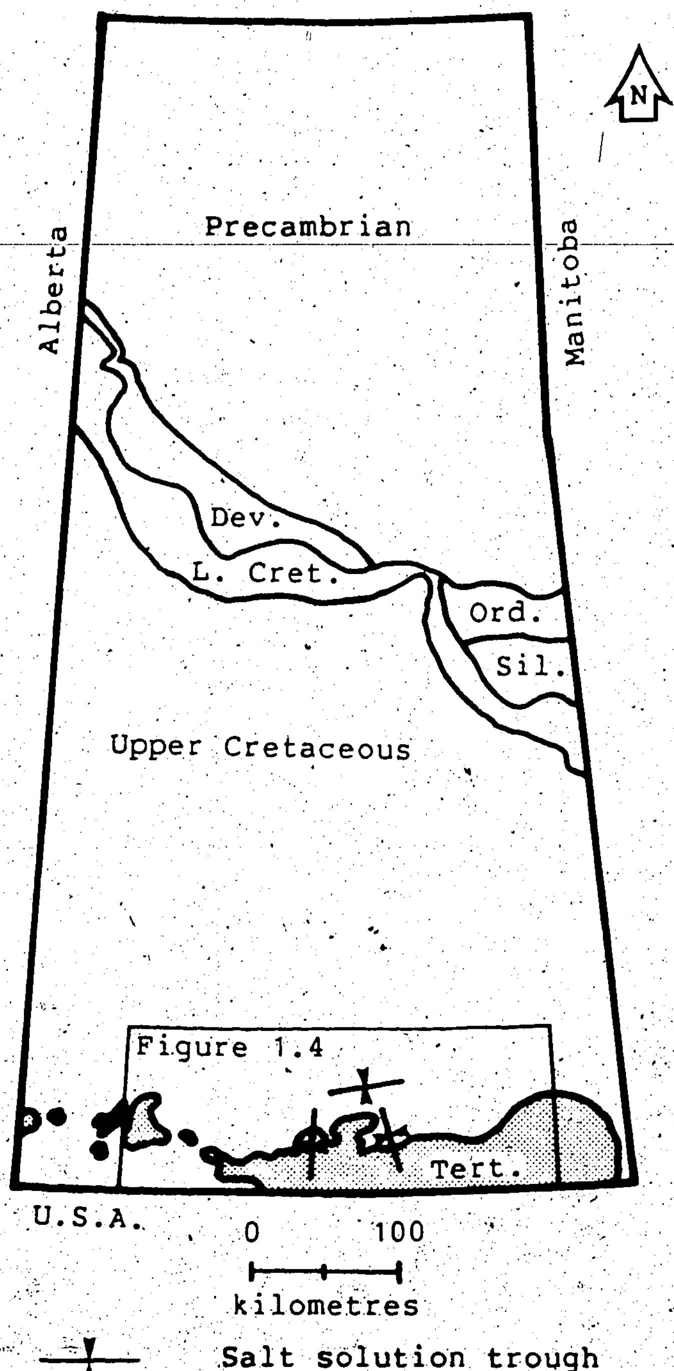
1.4.1 Regional Geology

The following discussion applies to the part of southern Saskatchewan underlain by the Ravenscrag Formation (Figure 1.2).

1.4.1.1 Regional stratigraphy

Paleozoic deposits, approximately 2000 m thick, are dominantly shelf carbonates, evaporites and basinal shales of Cambrian to Mississipian age (Figure 1.3), deposited over Churchillian basement.

A major unconformity separates Mississipian strata from the Jurassic bed. Several transgressions and regressions



Modified after Douglas (1970, Map 1250A)

Figure 1.2 Regional geology

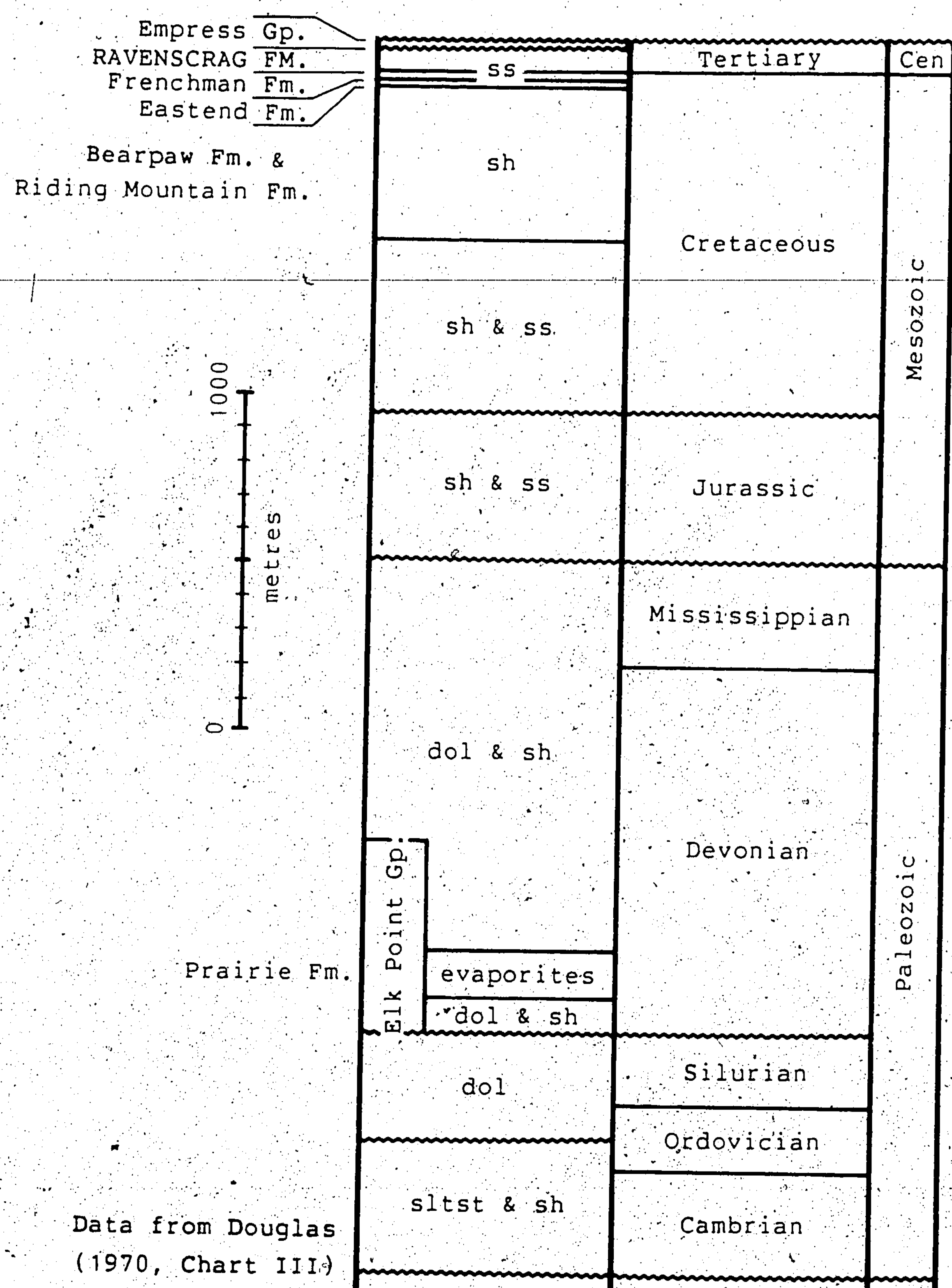


Figure 1.3 Regional stratigraphy

during the Mesozoic resulted the deposition of approximately 1500 m of clastics, both marine and non-marine, shed largely from the Cordilleran orogen to the west.

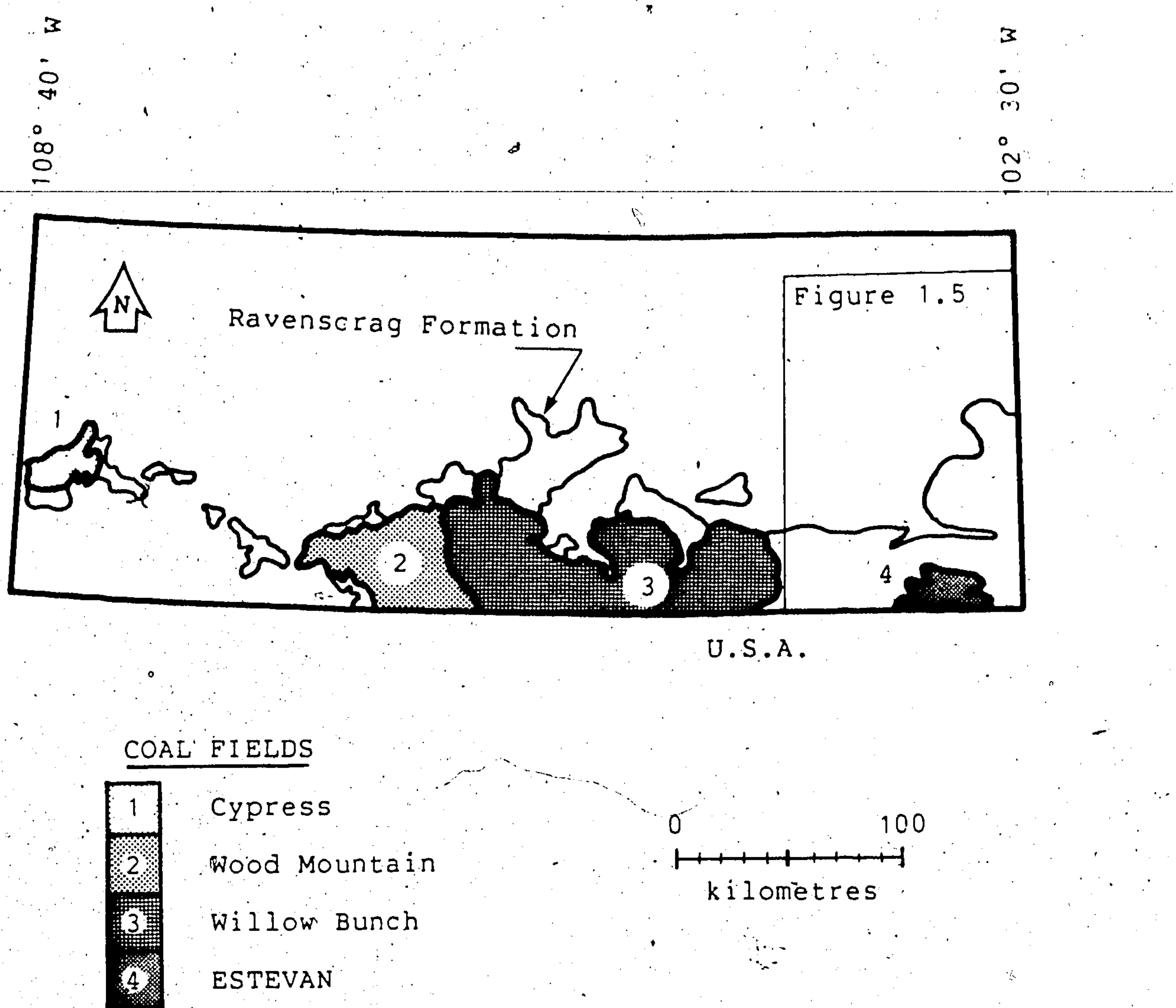
Cenozoic deposits are largely non-marine clastics containing several unconformities and one thin marine unit, the Cannonball Formation, which occurrs mainly in the adjacent United States. The Ravenscrag Formation of earliest Tertiary age (Danian and possibly a little later) was deposited over the central portion of the Williston Basin. The Ravenscrag coal measures are found in four coalfields (Figure 1.4):

1. Cypress,
2. Wood Mountain,
3. Willow Bunch,
4. Estevan.

Occasionally patchy Eocene to Pliocene deposits are found above the Ravenscrag Formation. A thin veneer of Pleistocene glacial deposits caps the sequence.

1.4.1.2 Regional structure

The strata form a broad syncline, which plunges gently at approximately 3 m/km to the south into the Williston Basin. Locally this pattern is modified by solution of evaporites in the Prairie Formation (Middle Devonian) and collapse of the overlying strata to form gentle synclines (Figure 1.2, p.11).



From Energy Mines and Resources (1982)

Figure 1.4 Coalfields in the Ravenscrag Formation

1.4.2 Local Geology

This description below applies to the Estevan Coal Field (Figure 1.5).

1.4.2.1 Local stratigraphy

Marine shales of the Riding Mountain and Bearpaw Formations (Figure 1.6) are overlain by the marginal marine Eastend Formation which in turn is overlain by the latest Cretaceous non-marine Frenchman Formation. The Cretaceous-Tertiary boundary is usually placed at the contact between the Frenchman and Ravenscrag Formations, though there is some disagreement. Total thickness of Cretaceous strata in the area is about 1000 m.

The Ravenscrag Formation may be as much as 550 m thick. The lower contact of the Ravenscrag Formation is conformable and usually defined as the base of the first coaly material above the Bearpaw or Riding Mountain shale. The upper contact is everywhere erosional.

Equivalents to the Ravenscrag Formation are parts of the Willow Creek and Porcupine Hills (Paskapoo) Formations in Alberta, and the Fort Union Group in the United States.

The lithology of the Ravenscrag Formation is "···grey and buff-weathering shales, fine sandstones, siltstones and sandy shales···" (Flint, 1978, p.16), usually poorly

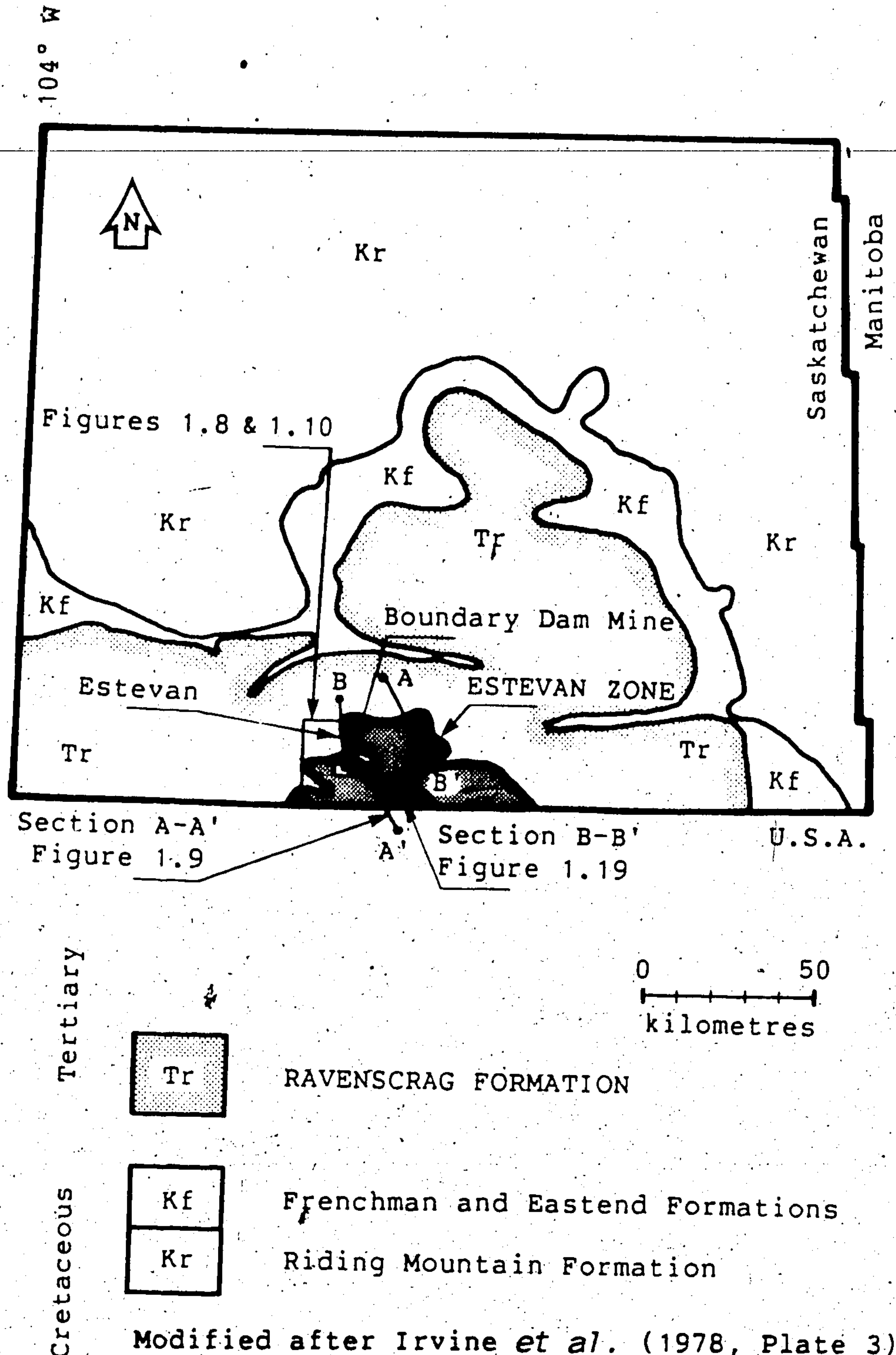
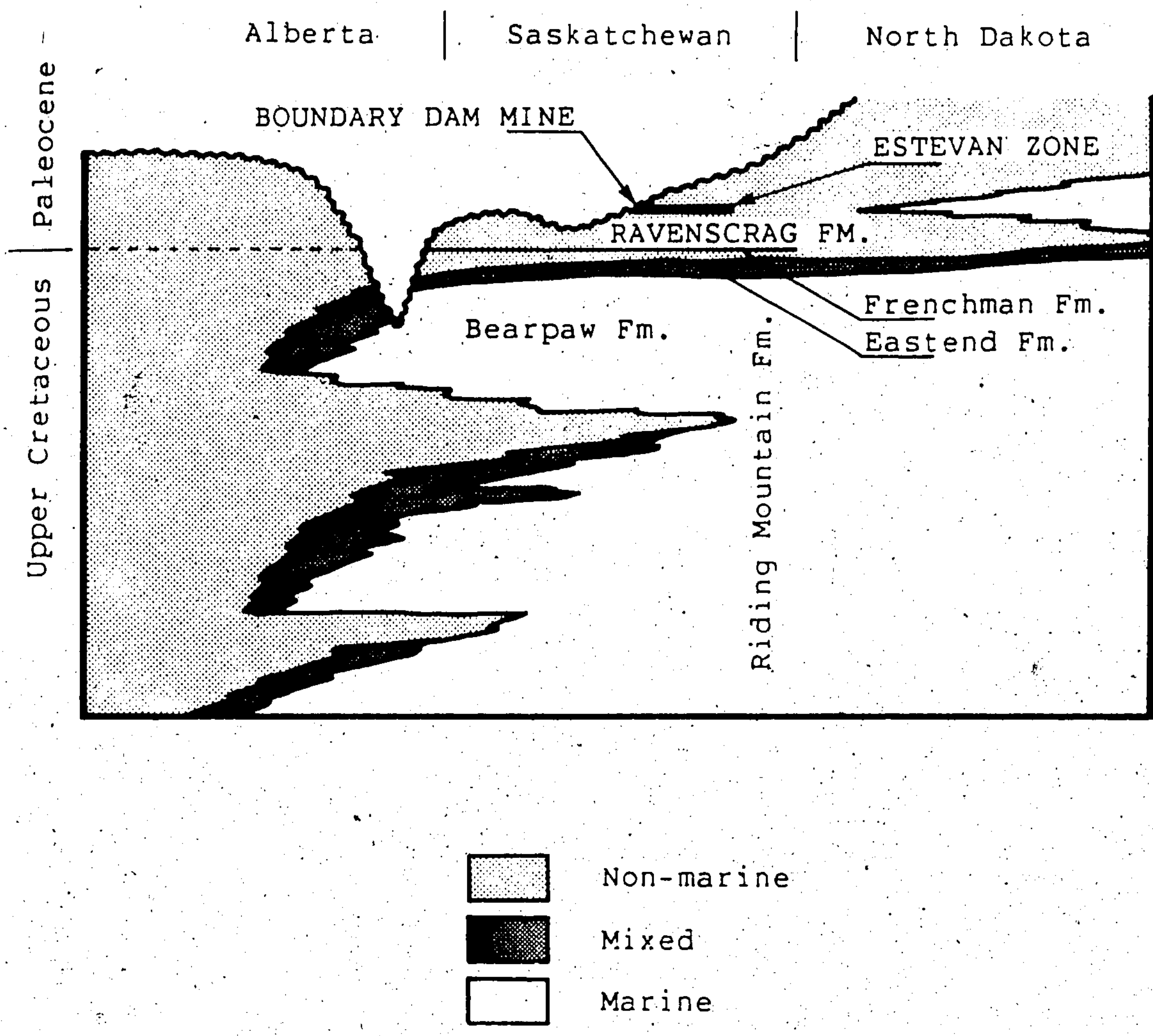


Figure 1.5 **Local geology**



Modified after Irvine *et al.* (1978, Figure 6, p.21)

Figure 1.6 **Local stratigraphy**

consolidated, commonly containing lignite seams in up to 19 zones.

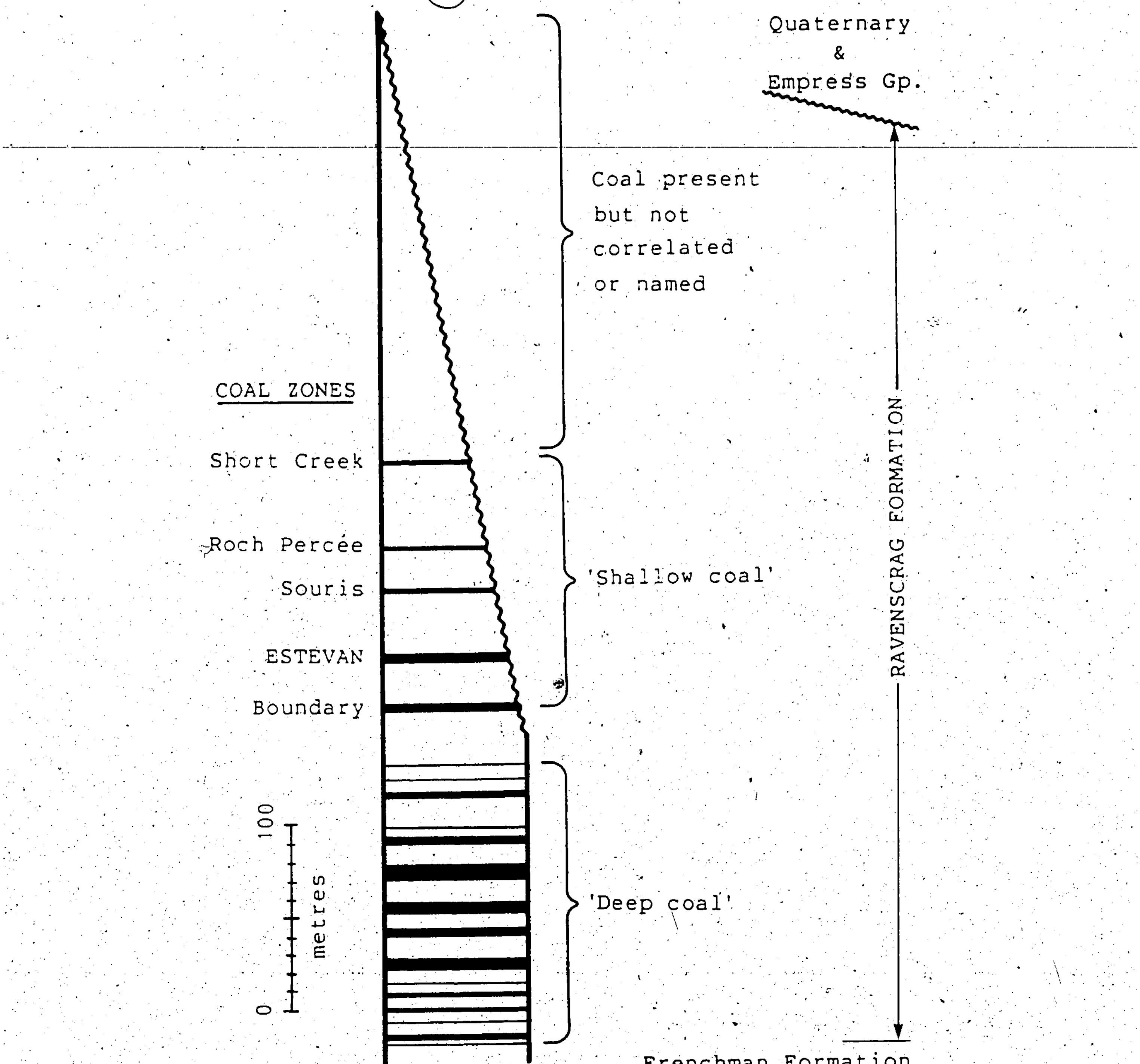
The depositional environment has been interpreted as an alluvial plain covered by forests, swamps and marshes in a humid temperate climate (Flint, 1978, p.17).

Irvine *et al.* (1978, p.49) have divided the coal zones in the Estevan Coal Field into two groups (Figure 1.7): fourteen zones of deep coal, too deep to mine at present; and five zones of shallow coal, which can be, or are being mined at present. The shallow coal zones (Figure 1.7, p.19 and Figure 1.8), in stratigraphic order from the top down are:

1. Short Creek,
2. Roch Percée,
3. Souris,
4. Estevan,
5. Boundary.

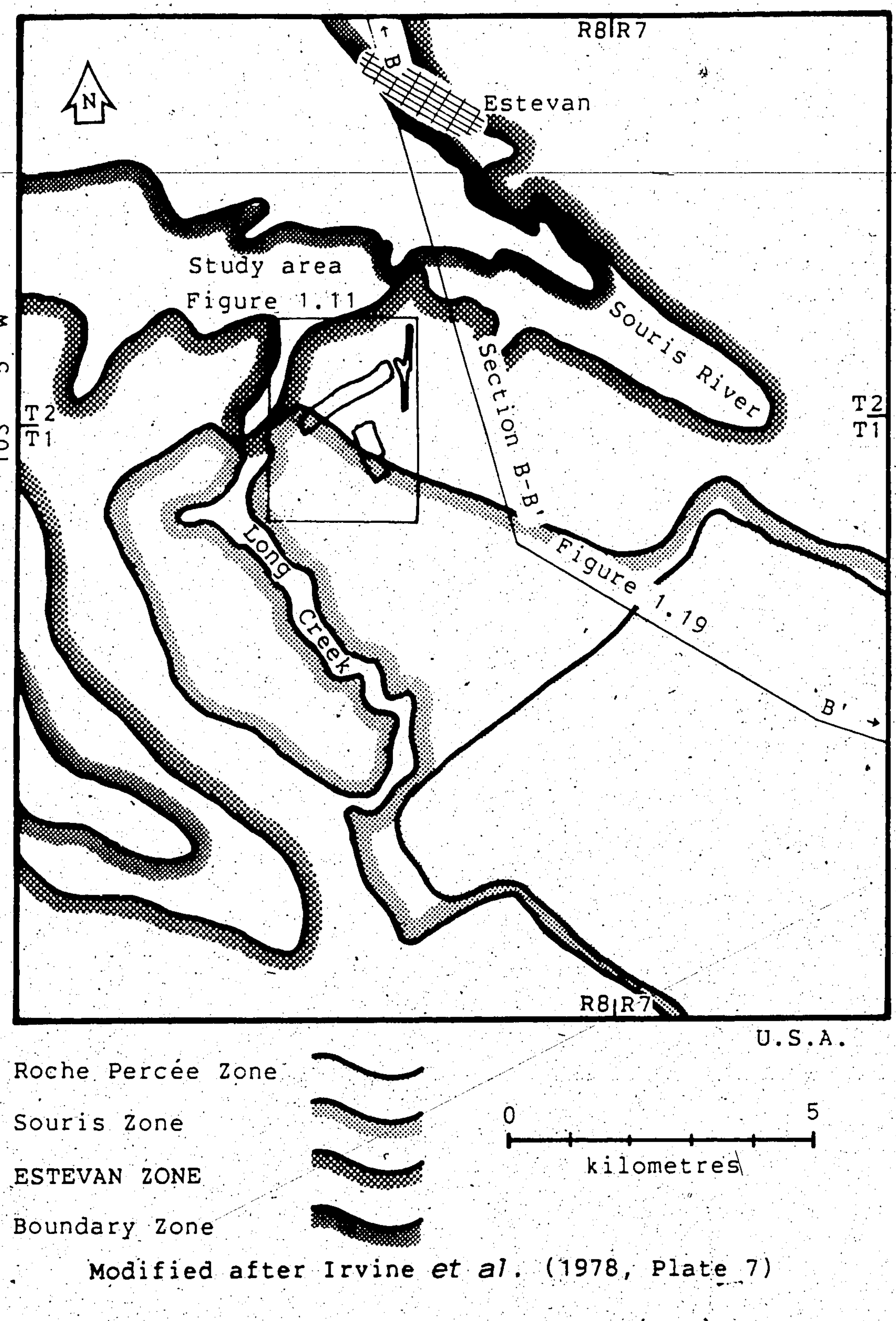
The Estevan and Boundary Zones, are the thickest, most widespread and most heavily mined.

The Ravenscrag Formation in the study area is erratically overlain by unconsolidated sands and gravels of the Plio-Pleistocene Empress Group. The whole area is covered by Pleistocene till except where it has been removed by fluvial erosion.



Data from Irvine *et al.* (1978, pp.49-58)

Figure 1.7 Coal zones of the Ravenscrag Formation in the Estevan Coal Field



Modified after Irvine *et al*. (1978, Plate 7)

Figure 1.8 Shallow coal subcrop in the Estevan area

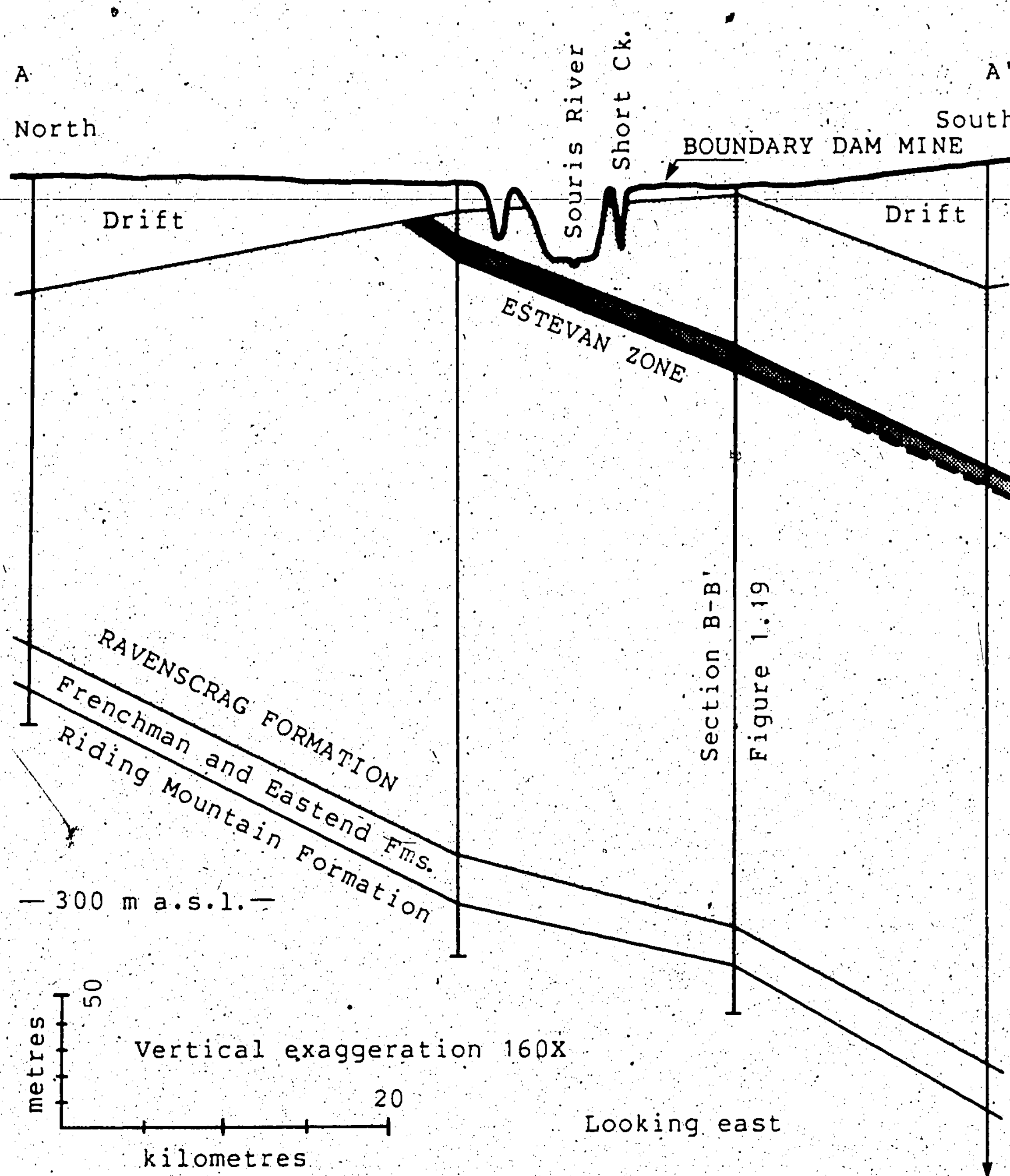
1.4.2.2 Local structure

The Ravenscrag Formation dips gently towards the southeast, but superimposed on it are a few very shallow southeast-plunging undulations (Figure 1.9). Solution of the Prairie Evaporite Formation has produced local collapse features.

In places the poorly-consolidated Ravenscrag Formation has been structurally disturbed by thrusting from a Pleistocene ice sheet. This disturbance takes the form of either disorganized strata or imbricate thrust sheets, usually within 25 m of the bedrock surface, though it can go deeper (Flint, 1978, p.153). Most ice-push features occur beyond the study area to the northeast.

1.4.3 Mine Geology

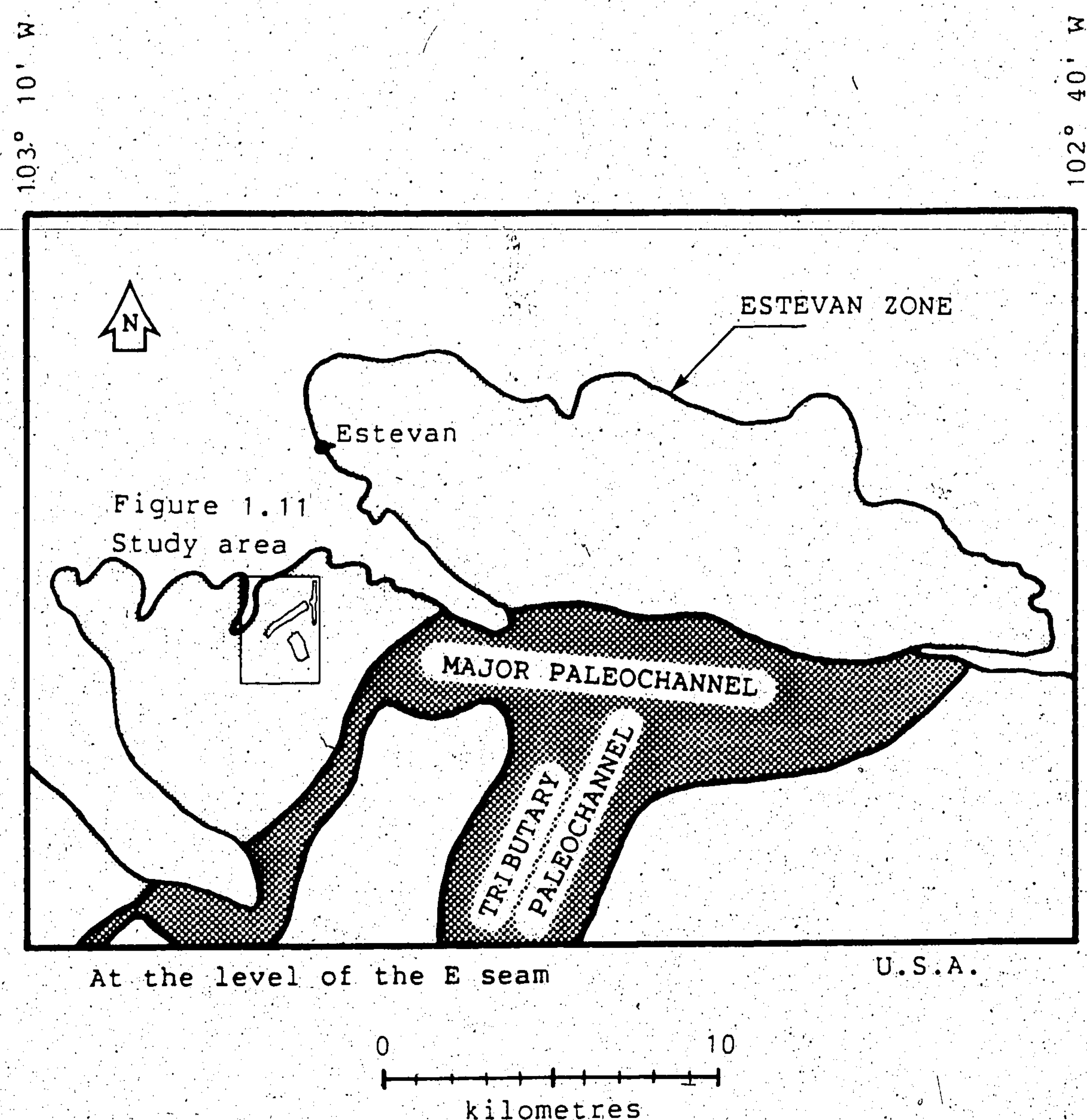
The Boundary Dam Mine is located at 103° 00' West Longitude, 49° 05' North Latitude or Range 8 on the boundary between Townships 1 and 2, West of the Second Meridian (Figure 1.8, p.20). The mine is approximately 3 to 7 km south of the town of Estevan and 8 to 12 km north of the U.S. border. It is bordered on the north by the Souris River Valley and on the west by the valley of Long Creek, both of which cut through the Estevan Coal Zone. The southeast boundary is formed by thinning of the coal towards a major northeast-trending, sand-filled paleochannel cut into the seam (Figure 1.10). The study area (Figure 1.11)



Section located on Figure 1.5

Data from Irvine *et al.* (1978, Plate 2)

Figure 1.9 Local cross section



From Irvine *et al.* (1978, Figure 37, p.62)

Figure 1.10 Location of the major paleochannels in the Estevan Zone

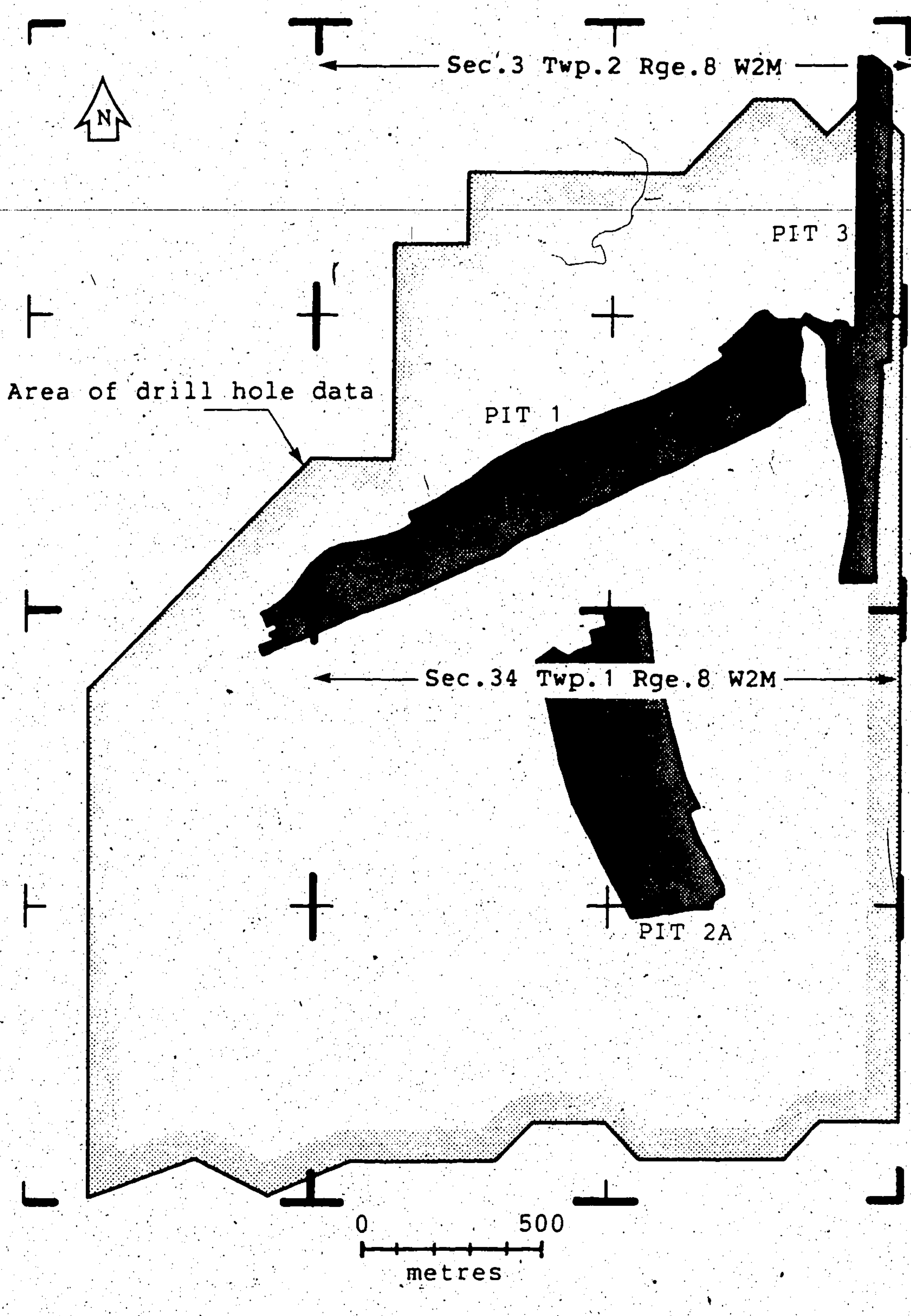


Figure 1.11 Pits used in the study

encompasses the three pits active in the mine from 1977 to 1978 along with the surrounding drill holes.

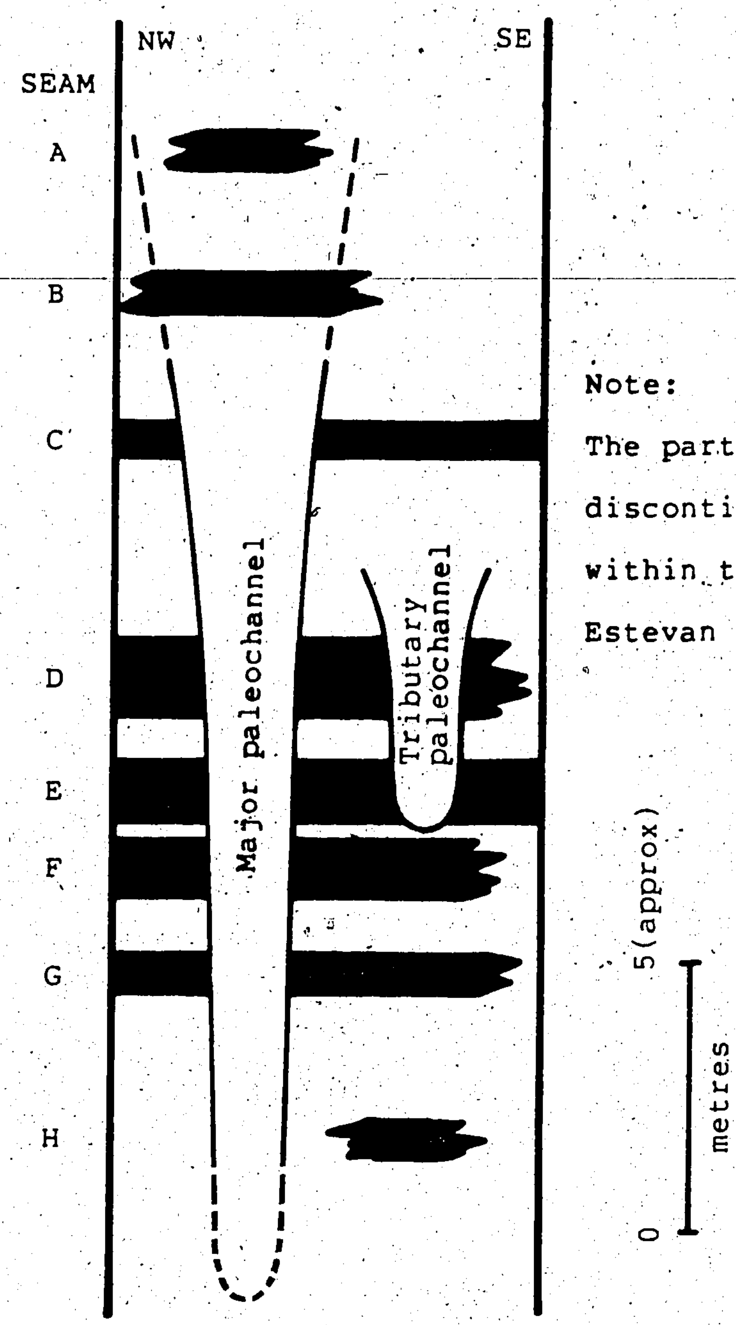
Topography over the mine is flat with about 5 m relief, but incised by the Souris River in a broad glacial meltwater channel, and Long Creek in a narrow Holocene ravine, both about 30 m deep.

1.4.3.1 Stratigraphy at the mine

Overburden, 15 to 20 m thick, is composed of glacial till, successively underlain by discontinuous deposits of sand and gravel of the Empress Group and poorly consolidated grey silty clay of the Ravenscrag Formation.

Seams in the Estevan Zone (Figure 1.12) are named A to H from the top down (nomenclature is after Irvine *et al.*, 1978, p.60). Seams A and B are thin, usually occurring as part of the fill in the major paleochannel. Seam C is a rider seam, too thin to mine in the Boundary Dam Mine. Seams D, E, F and G are mined, while seam H is a thin, discontinuous rider at the base of the zone.

Maceral content of the coal is 65% to 85% huminite, 10% to 25% inertinite and 5% to 15% exinite (Irvine *et al.*, 1978, Figure 13, p.46). The average proximate analysis of coal from the seam is listed in Table 1.1. Average gross thickness is 3.9 m and average net thickness is 3.3 m



Data from Irvine *et al.* (1978, pp.60-64)

Figure 1.12 Stratigraphic column of the Estevan Zone in the Boundary Dam Mine

Table 1.1 Typical proximate analysis of the coal from the Estevan Coal Field

Moisture	37.5%
Ash	9.1%
Volatile matter	24.6%
Fixed carbon	28.8%
Calorific value	15 100 kJ/kg
Rank	Lignite
Sulphur	0.3%

This data is from Adamcewicz (1963, p.101).

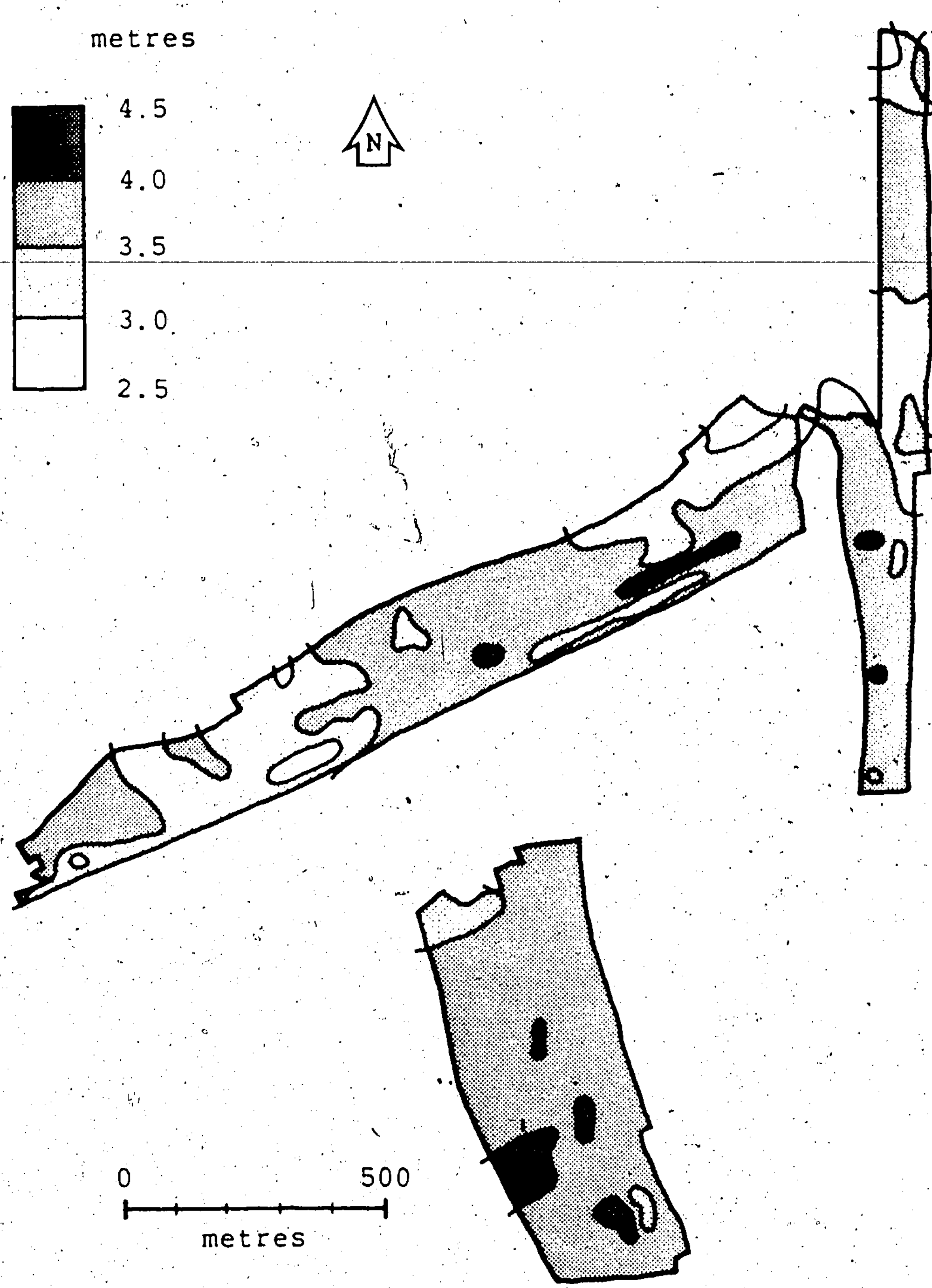


Figure 1.13 Isopach of the mined zone, net thickness

(Figure 1.13). The descriptions below refer primarily to the portion of the mine studied in this project. Two main partings† of grey and brownish clay are recognized by mine surveyors in the study area (Figure 1.14). The upper, probably D·E, parting is present in the southern part of the study area, wedges out towards the north (Figure 1.15 and Figure 1.16) and is 0.3 m thick and 0.9 m from the top of the mined coal. The lower parting, probably E·F, occurs in the north, wedging out towards the south and is 0.4 m thick and 2.1 m from the top of the mined coal. The F·G parting may have been measured in a few places in the south end of pit 3. If so, it is 0.1 m thick and 0.8 m from the base of the seam.

The pattern of splits‡ and partings seen by geologists in the drill holes from the study area is not quite the same as that seen by the mine surveyors. Different geologists seem to have their own criteria for identifying a parting and consequently, holes with one number of splits can be surrounded by infill holes with a different number of splits. The floor consists of poorly-consolidated grey clay.

A few linear areas (Figure 1.17) occur in the pit data where the parting and/or coal is thicker or thinner than it

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† A parting is a layer of rock within a coal seam.
‡ A split is a subdivision of a seam that is separated by a parting from another split.

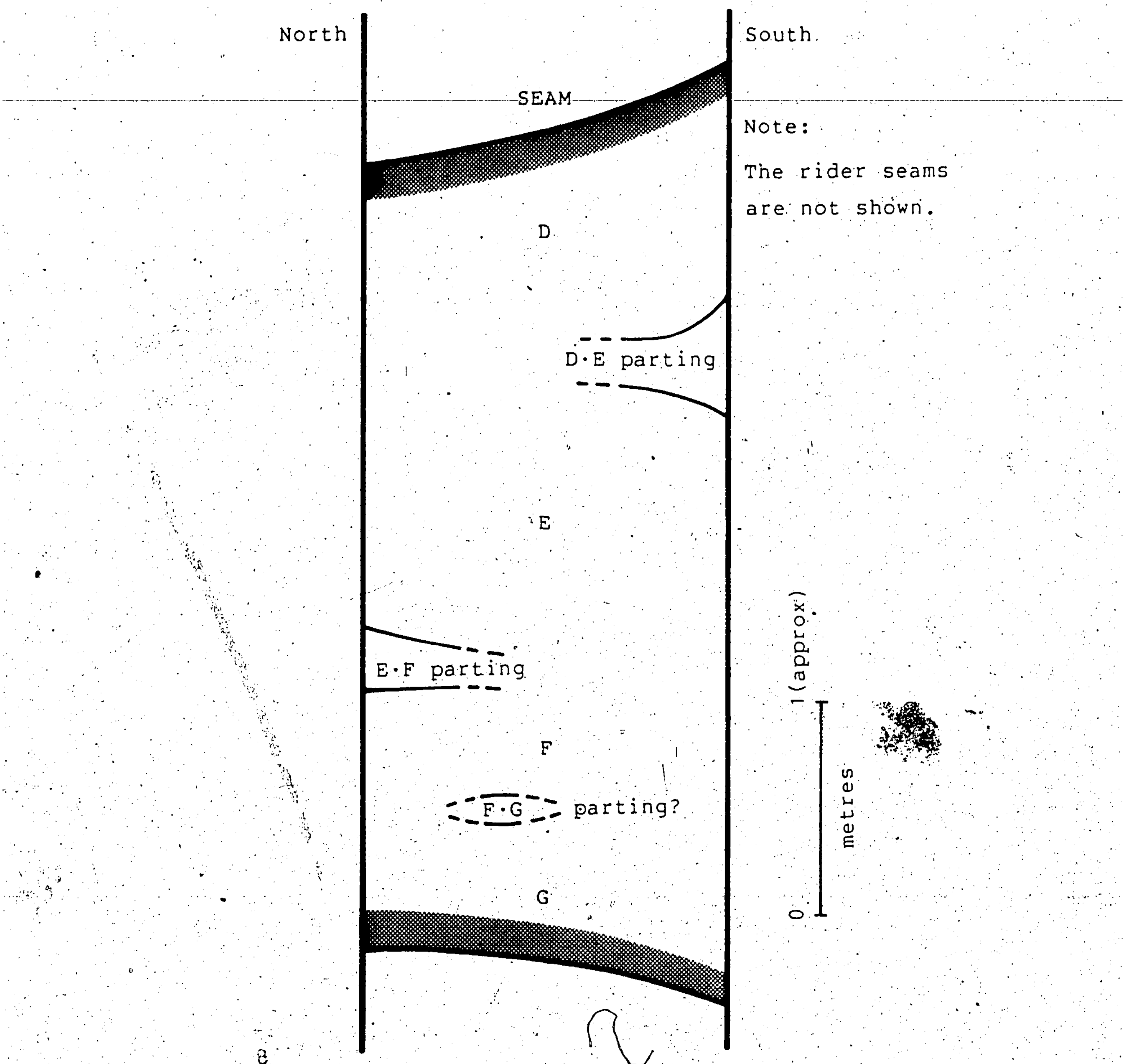
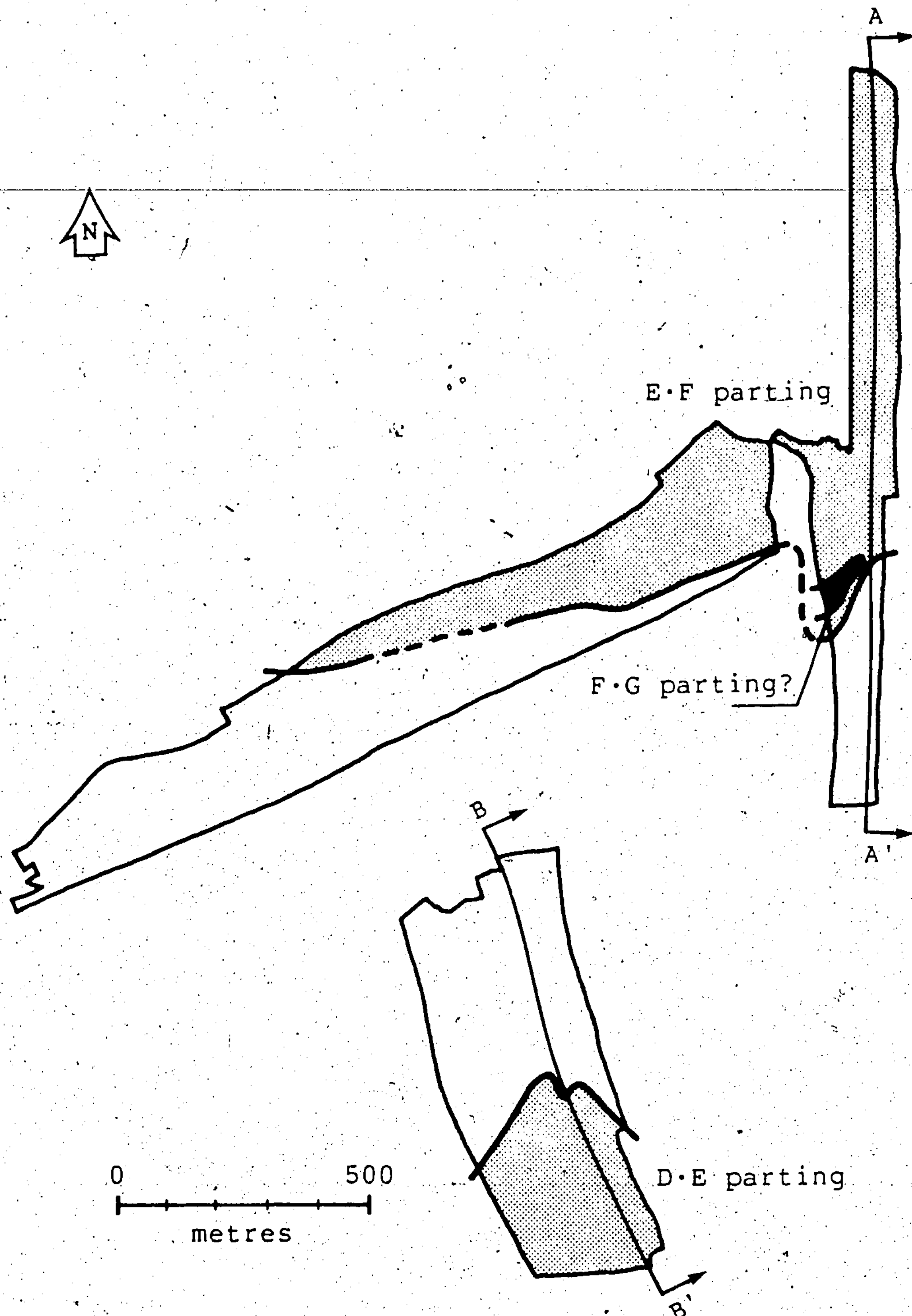


Figure 1.14 **Stratigraphic column for the study area**



Sections are shown on Figure 1.16

Figure 1.15 Location of the partings in the study area

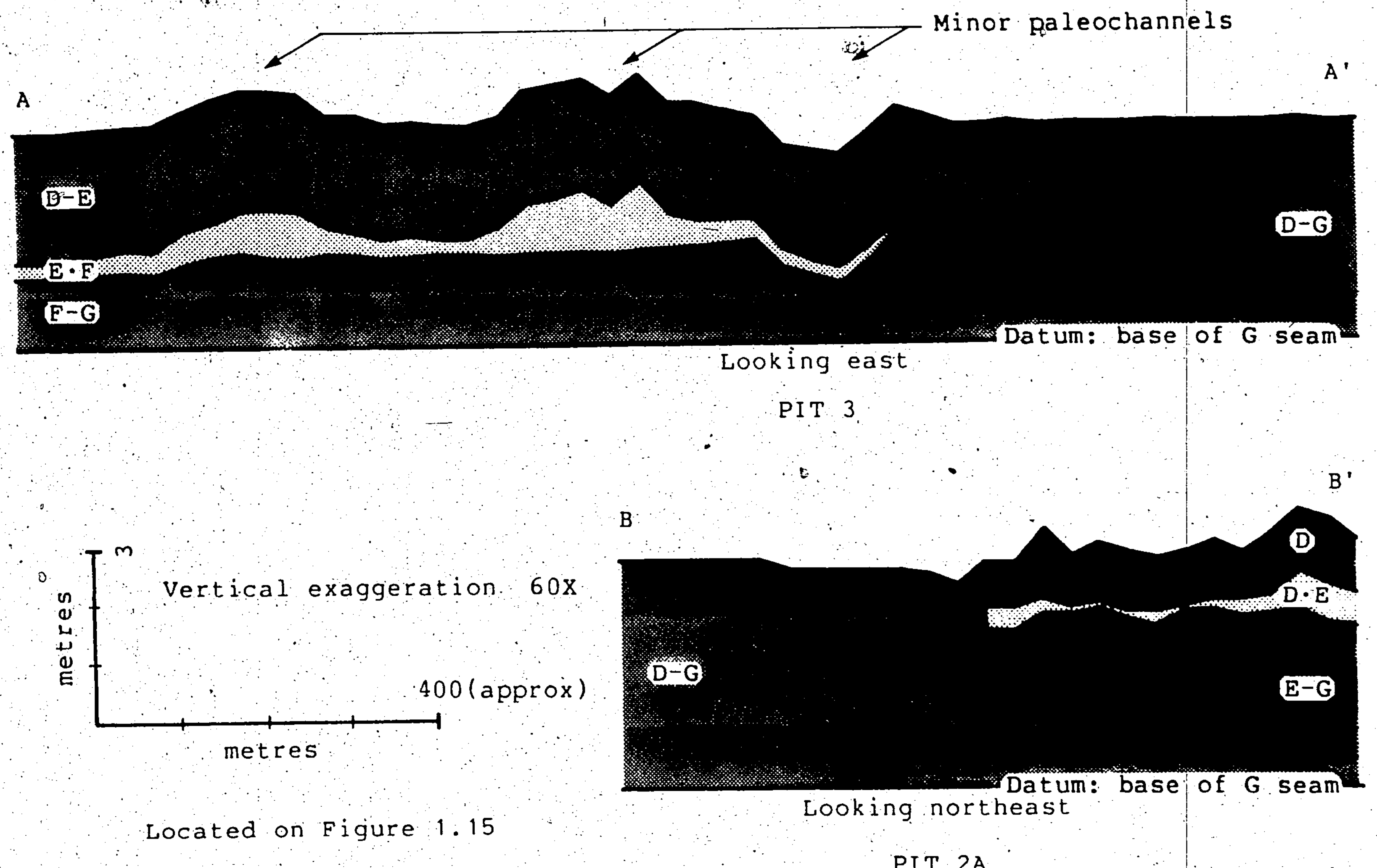


Figure 1.16 **Cross section through the study area**

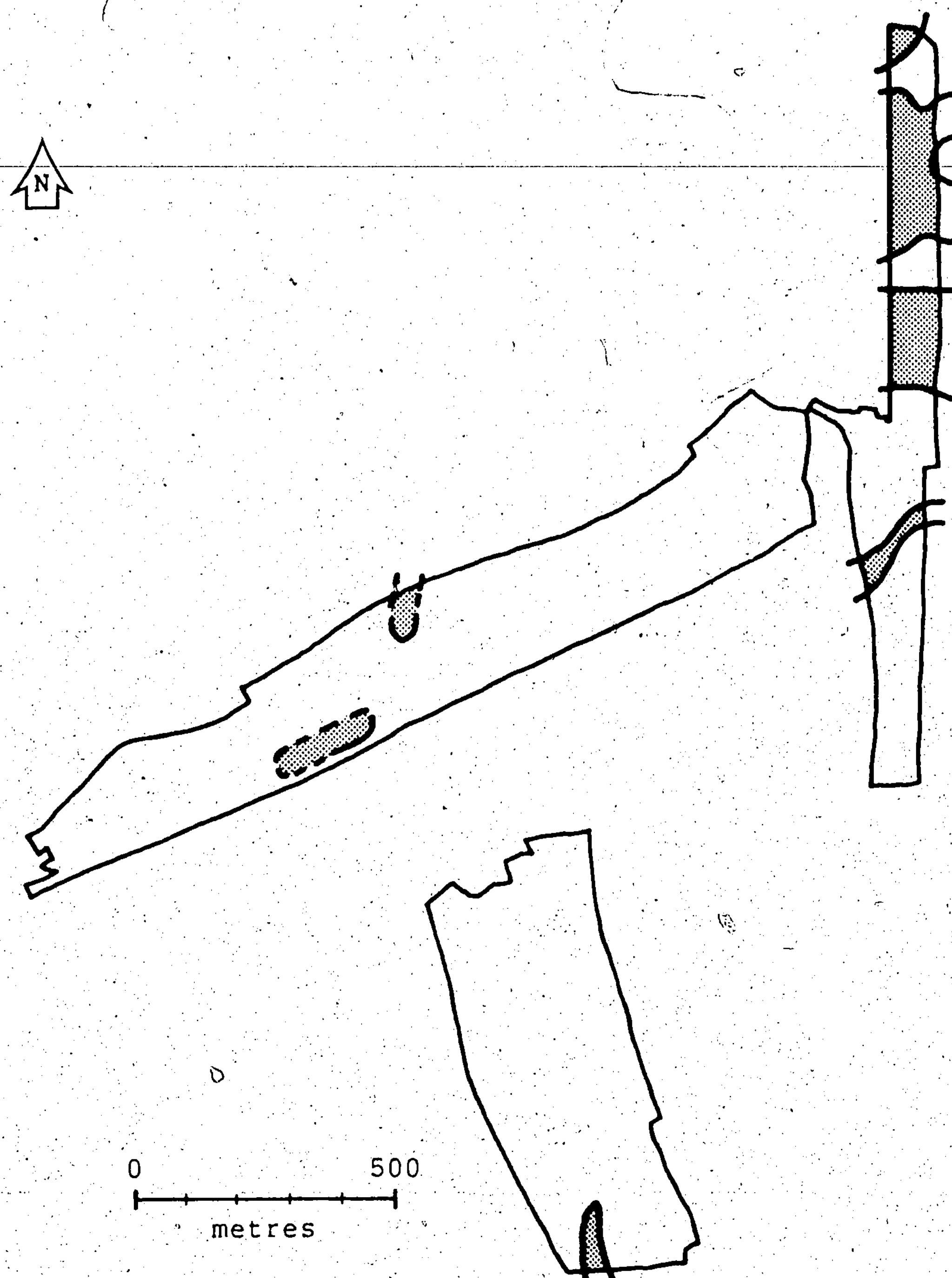


Figure 1.17 Location of the paleochannels in the pits of the study area

should be. These areas are probably minor paleochannels.

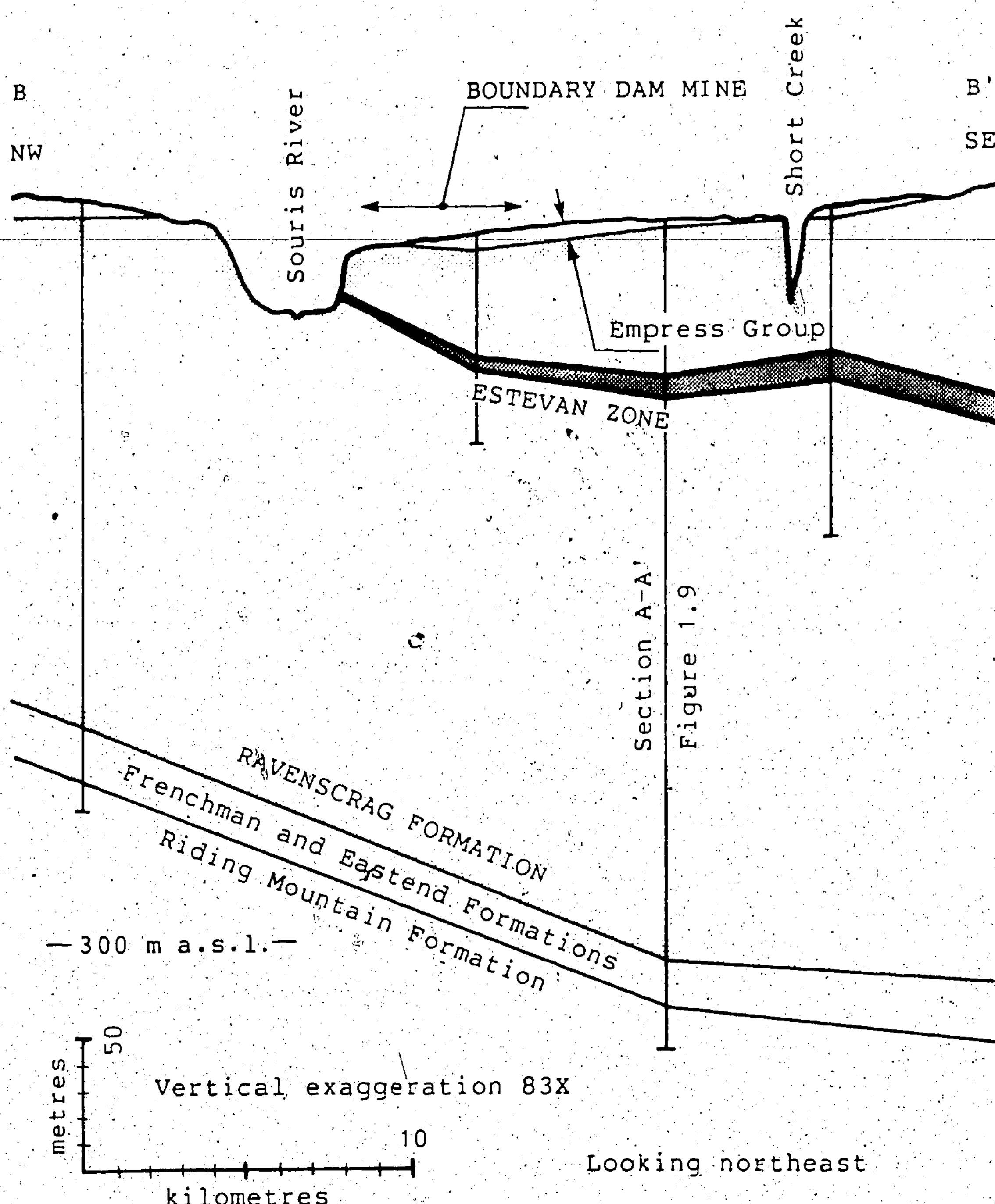
1.4.3.2 Structure at the mine

The coal zone dips gently to the southeast with minor undulations (Figure 1.18). No salt solution tectonics have been found, but there has been some collapse into a few of the old underground mines, particularly near the Souris Valley, outside and north of the study area.

1.5 COAL MINING IN SOUTHERN SASKATCHEWAN

Mining from small underground workings in the walls of the major river valleys began in the early 1870's. The first open pit mine started in 1927, and consolidation of the many small underground mines occurred about the time of the Second World War, with the conversion to strip mining complete by 1955. Six mines (Figure 1.19) operate in the area at present. Seventy percent of the coal mined in Saskatchewan is used for power generation and most of the rest is used in pulp and paper mills and for briquetting, (Adamcewicz, 1982, p.105). Coal from the Boundary Dam Mine†, with a nominal annual capacity of 1.8×10^6 tonnes, is used exclusively by the Saskatchewan Power Corp. in the mine-mouth Boundary Dam Power Station. A detailed history of coal mining in Saskatchewan may be found in Irvine *et al.* (1978, p.131).

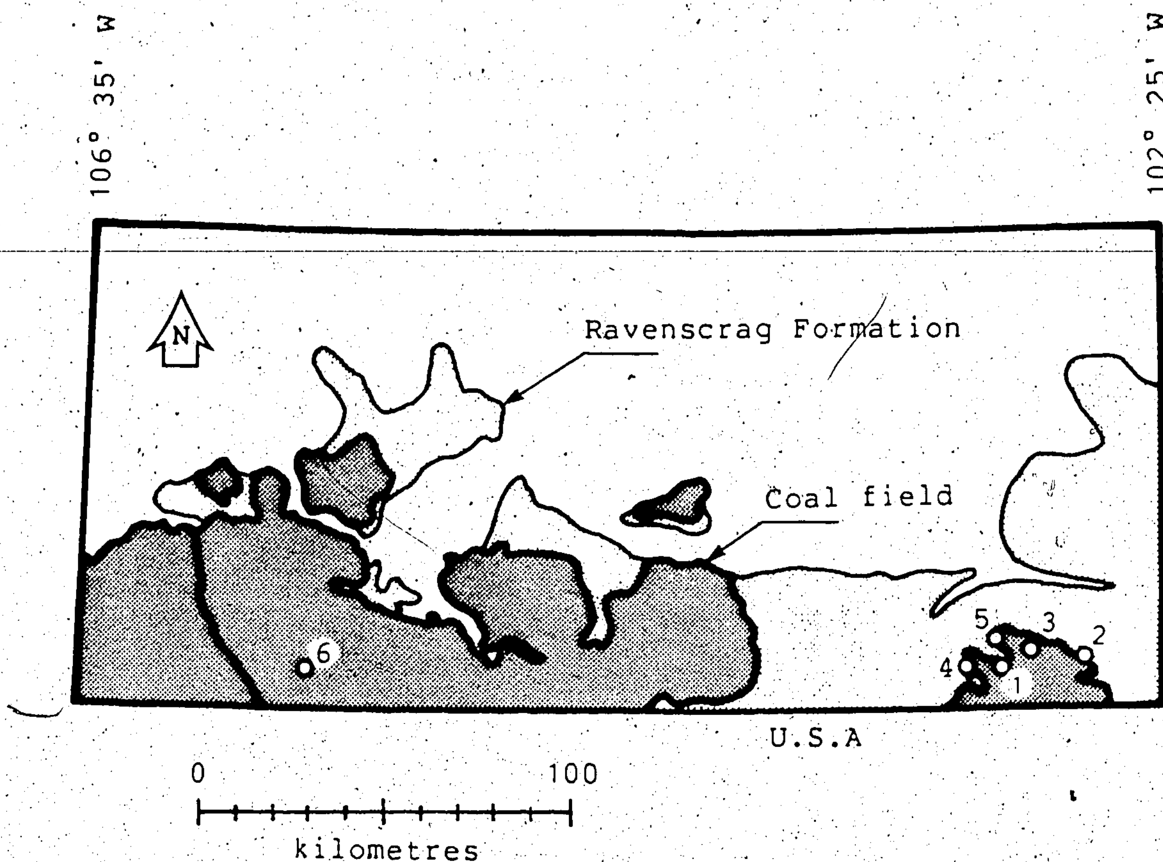
† The mine is owned by the Manitoba and Saskatchewan Coal Co., which is owned by Luscar Ltd.



Section located on Figures 1.5 and 1.8

Modified after Irvine *et al.* (1978, Plate 8)

Figure 1.18 Cross section through the mine



COAL MINES

1 BOUNDARY DAM MINE

2 Bienfait Mine

3 Klimax Mine

4 Utility Coals Ltd.

5 Souris Valley Mine

6 Poplar River Mine

Modified after Energy Mines and Resources (1982)

Figure 1.19 **Operating coal mines in Saskatchewan**

The Boundary Dam operation, like all those in Saskatchewan, uses strip mining techniques. Walking draglines, working from a prepared surface, remove the overburden in one pass and cast the spoil onto the adjacent bench from which the coal has been removed. Scrapers and graders clean off the surface of the seam and remove any partings thicker than about 1/3 m. Partings less than this are not selectively mined. Finally, the coal is loaded into trucks by electric shovels for transportation directly to the power station.

2. DATA ACQUISITION AND PRELIMINARY ANALYSIS

2.1 DATA SOURCE

Data for this study was acquired from the Boundary Dam Mine of the Manitoba and Saskatchewan Coal Company. Most of the data consists of seams and parting thicknesses, but also included are some mine records of coal density and coal actually loaded out of a pit during a particular time interval. The thicknesses have two sources:

1. Daily measurements of the mined seams in the pit highwalls by the mine surveyors (Figure 2.1). These is nominally 30 m apart along each bench, which are approximately 30 m wide. Data was collected from the three pits (1, 2A and 3) that were active in 1977 and 1978.
2. Production drilling on 100, 150 and 200 m grids (Figure 2.2), covering three sections that surround the pits.

In both cases the original blue-line drawings of the mine plans with data location and thicknesses were xeroxed because the original drawings were too large to fit on the digitizing tablet, a Summagraphics Bit Pad. The X and Y coordinates of each data location were digitized and the thicknesses entered simultaneously on the keyboard of the attached HP2648a graphics terminal. All thicknesses pertaining to the coal were entered, both partings and splits. The pit measurements either had no partings or one parting, rarely as many as two; whereas the drill holes had

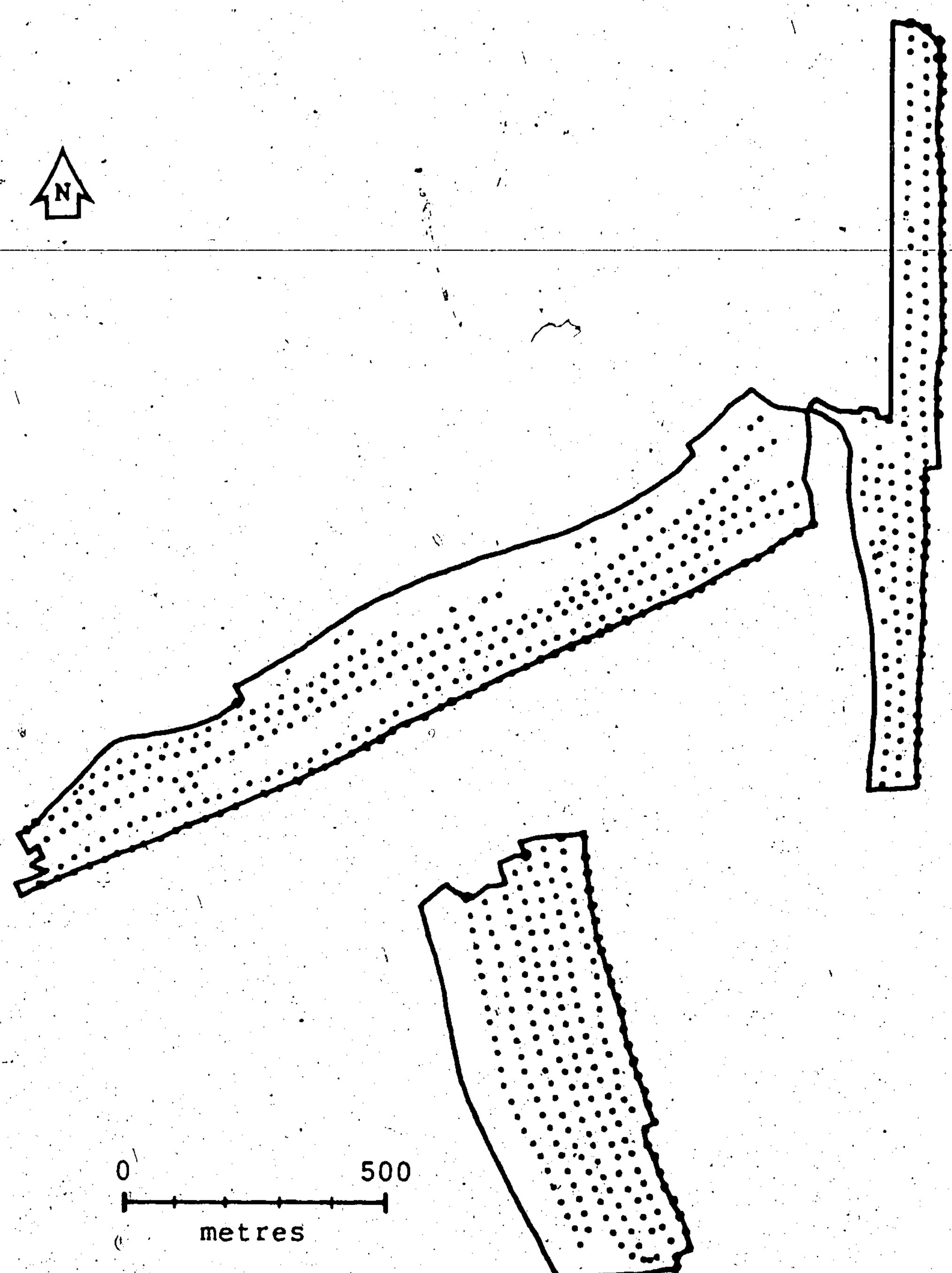


Figure 2.1 Location of the pit data

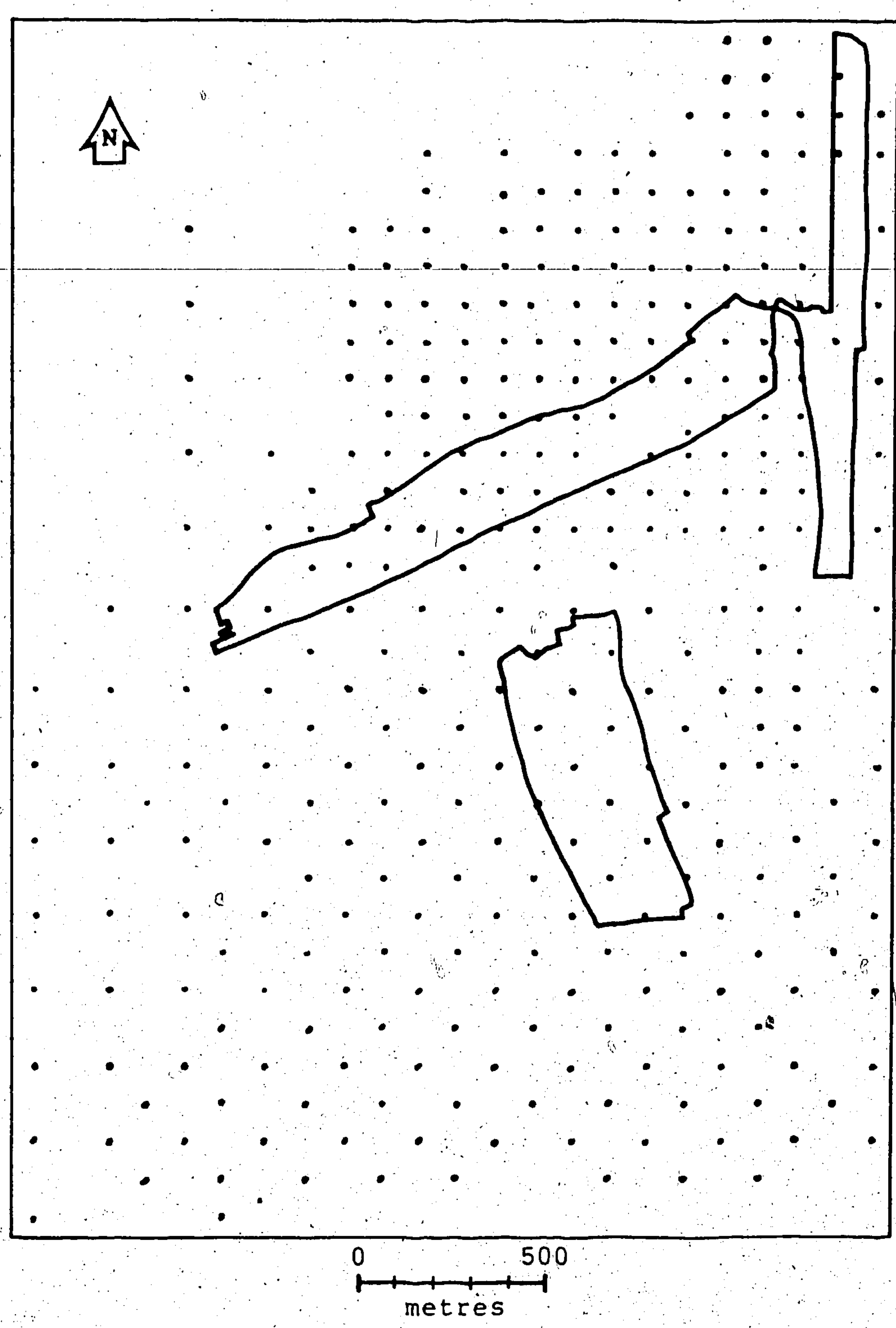


Figure 2.2 Location of the drill hole data

up to five partings. A program that calculated map coordinates from the pad coordinates also calculated net and gross coal thickness. See Appendix 1.1 for an example of the resulting data set. Outlines for the pits were also digitized at points no further than 30 m apart.

2.2 CLASSICAL STATISTICAL ANALYSIS

Classical statistics is non-geostatistical statistics or what is usually referred to as simply 'statistics' and which does not deal with any regionalized variables.† Any statistical analysis must use data that is as free as possible from all mistakes, so the data was cleaned, as outlined in Appendix 2.1.

Before any geostatistical analysis can be done, the data must be explored with classical statistics. Not only is it good practice, but the geostatistics used in this study make two assumptions that can be checked only with classical statistics. These assumptions are that the data is from one population that is normally distributed. In addition, the values of some classical statistical parameters affect some of those in geostatistics. The variogram sill is theoretically equal to the population variance, and the spatial distribution of the data determines how unbiased the arithmetic average is for a reserve estimate.

† Briefly, a regionalized variable is a variable whose statistics are affected by its location. See Appendix 1.1.1 for a more complete definition.

2.2.1 Statistics of the Net Thickness

Tonnages are calculated from the product of the density, area and the average net thickness of the mined seam. The last variable can be calculated by taking the net thickness before or after averaging. The latter is the better way of determining the average net thickness, as outlined in Appendix 2.2. To do this the net thickness is required at each sample† point. Three classical statistical properties of the net thickness were examined, its distribution, mean and variance.

2.2.1.1 Distributions of the thicknesses

If the thickness distributions are lognormal, then disjunctive kriging with de Wijsian variogram models should be used. If the population is normal, then more common variograms and kriging can be used. All the net thickness distributions are approximately normal except for some mixing of populations. Two methods were used to analyse the distributions: STPK and normal probability graphs.

STPK is an interactive statistics package that can compare the empirical quantiles of the data to the best-fit, theoretical quantiles for any of the following cumulative distribution functions: normal, gamma, chi-squared, F, Student's t, beta, Cauchy, Weibull, half normal, uniform,

† 'Sample' is used in this study to refer to any measurement of the seam thickness, either in a drill hole or on the highwall.

binomial, Poisson, geometric, and hyper-geometric.

The fit of the data was compared visually with each of the above distributions. The only good fits were the normal and gamma distributions (Figure 2.3). Only one pit is illustrated due to data limitations in STPK, but the other pits show a similar pattern. The gamma function fits a little better because it is a three parameter function and so can be skewed. However, the difference is negligible, so the distribution was taken to be normal. The next best fit was the Cauchy function (Figure 2.4); all the rest were much worse.

A program was written that plots the data on a cumulative normal probability graph both to visually estimate the normality of the data, and to find any mixing of populations. Data from one normal population will plot on a straight line with its location determined by the mean and its slope by the variance. Multiple normal populations will plot on straight line segments. All the pit data plots on reasonably straight lines except for tails toward the thin values (Figure 2.5). This indicates a slight to moderate skewness to the left (negative). The points in the tail are from discrete areas in the seam that are distinctly thinner than usual (Figure 1.16, p.32), areally associated and often linear (Figure 1.17, p.33), suggesting the effect of small paleochannels.

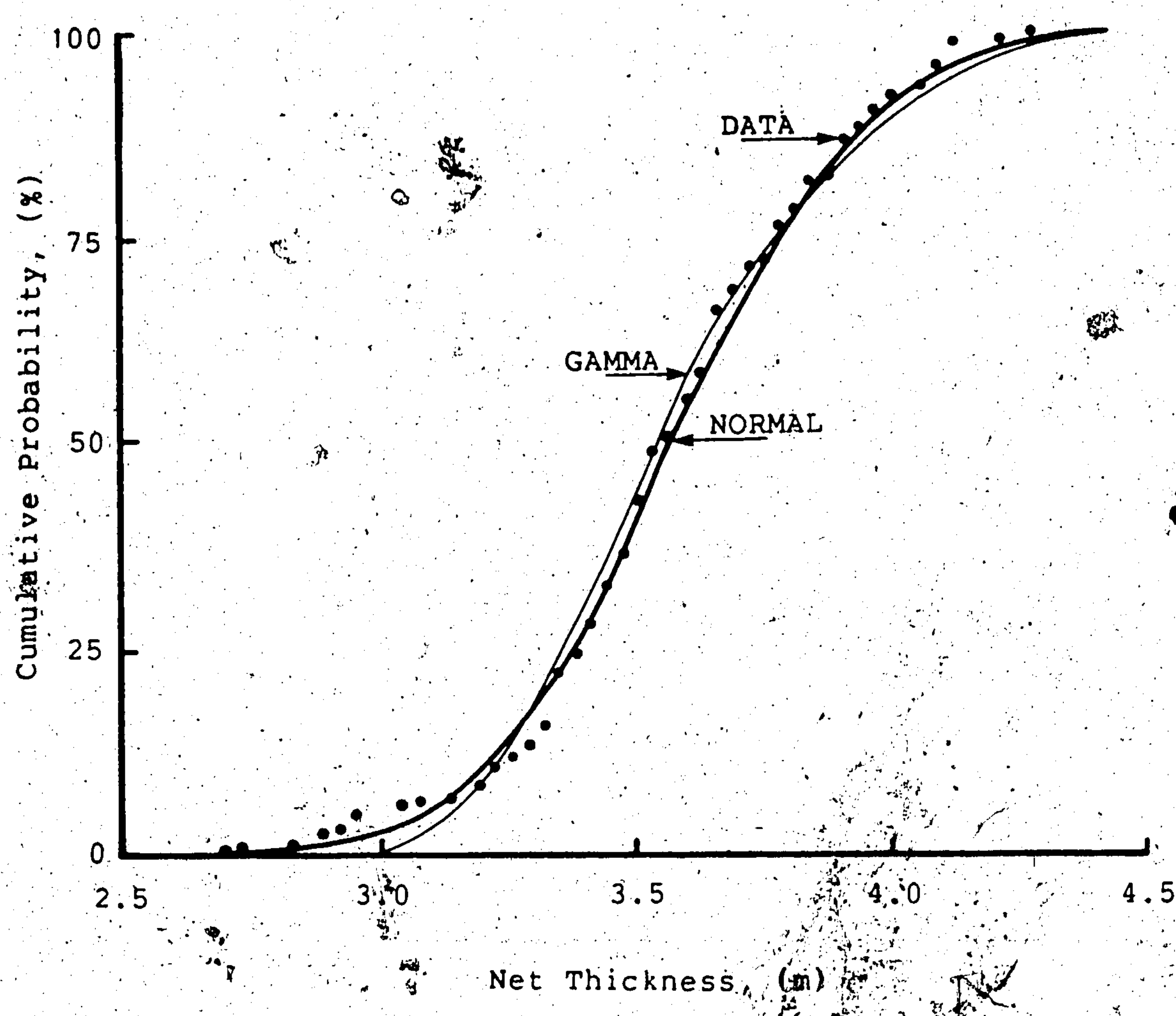


Figure 2.3 Best fit normal and gamma cumulative distribution functions for pit 1 data

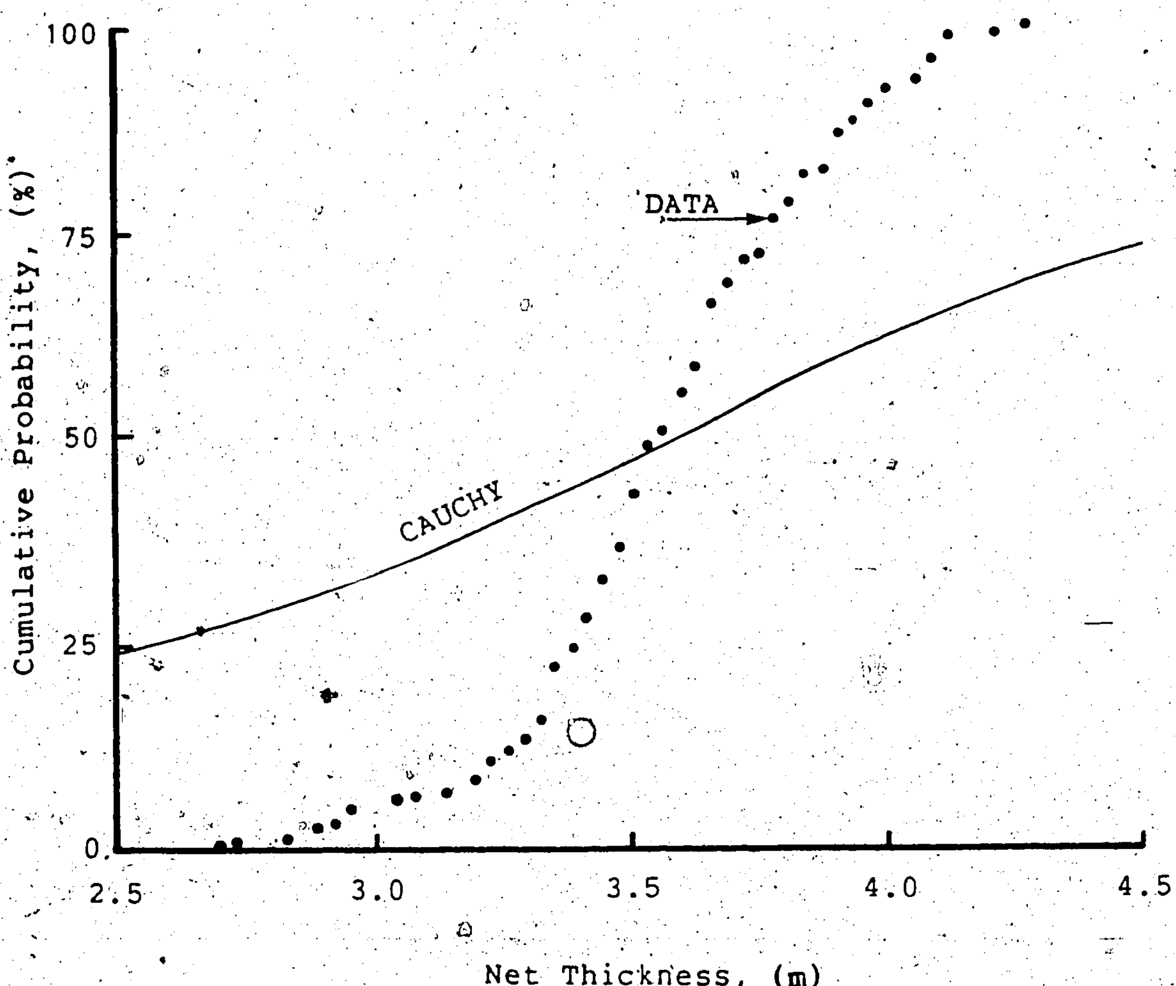


Figure 2.4 Best fit Cauchy cumulative distribution function for pit 1 data

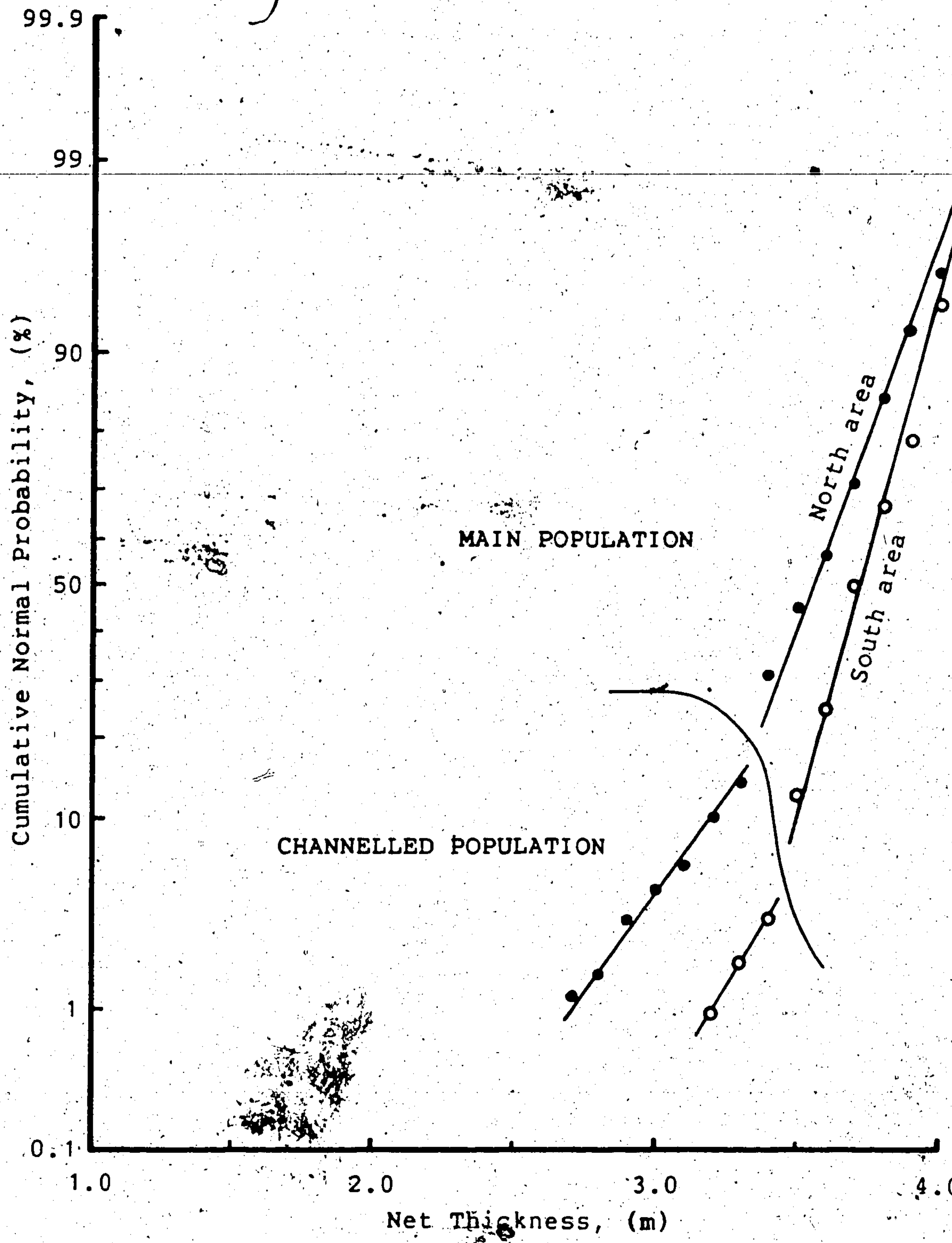


Figure 2.5 Cumulative normal probability curves for all of the pit data

The overall distribution of the data may be three parameter lognormal. However, an adequate interpretation is that there is a mixture of two populations:

1. most of the area, normally distributed,
2. channelled areas, probably normally distributed.

The pit data was considered to be one population since the departure from one population is small and the channelled areas could not be subdivided out and handled separately. These areas are too small to be treated on their own, as they contain less than the 30 to 50 sample minimum for calculating a variogram.

The drill data also plots as straight-line segments (Figure 2.6) but shows a much larger tail than the pit data (Figure 2.7). This longer tail is probably due to the larger sampled area having portions of the seam too severely channelled and thin to mine. The two populations in the drill hole data were too intimately mixed over the study area (Figure 2.8) to separate into simple areas.

2.2.1.2 Mean and variance of the thickness

The mean of the net thickness of all the pit data is 3.6 m and of the drill hole data is 3.3 m (Table 2.1). The drill hole average is less because some holes were drilled where the coal was too thin to mine, generally in the west of the study area. In addition the drill geologists on some holes may separate out partings that will not be selectively

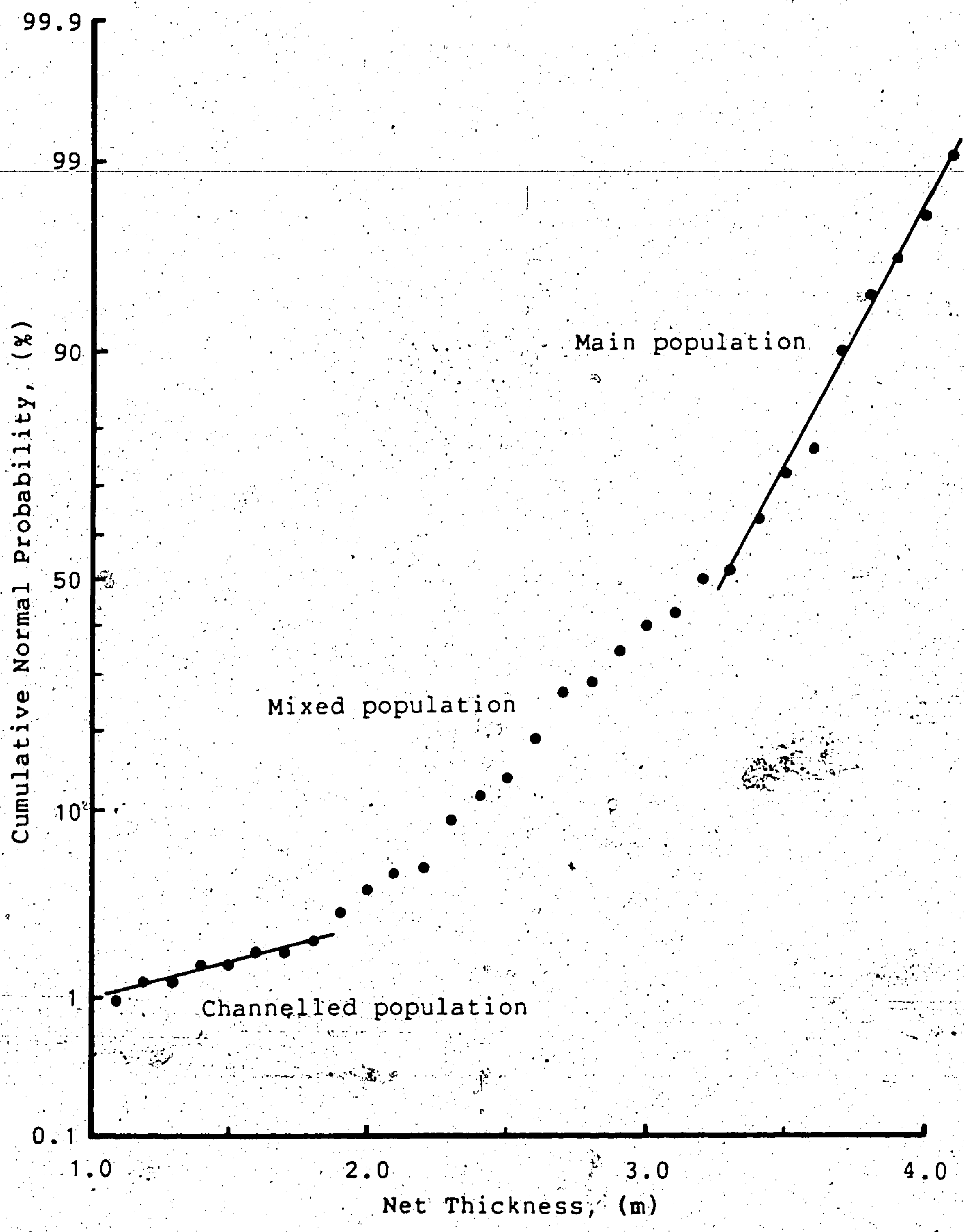


Figure 2.6 **Cumulative normal probability curve for the drill hole data**

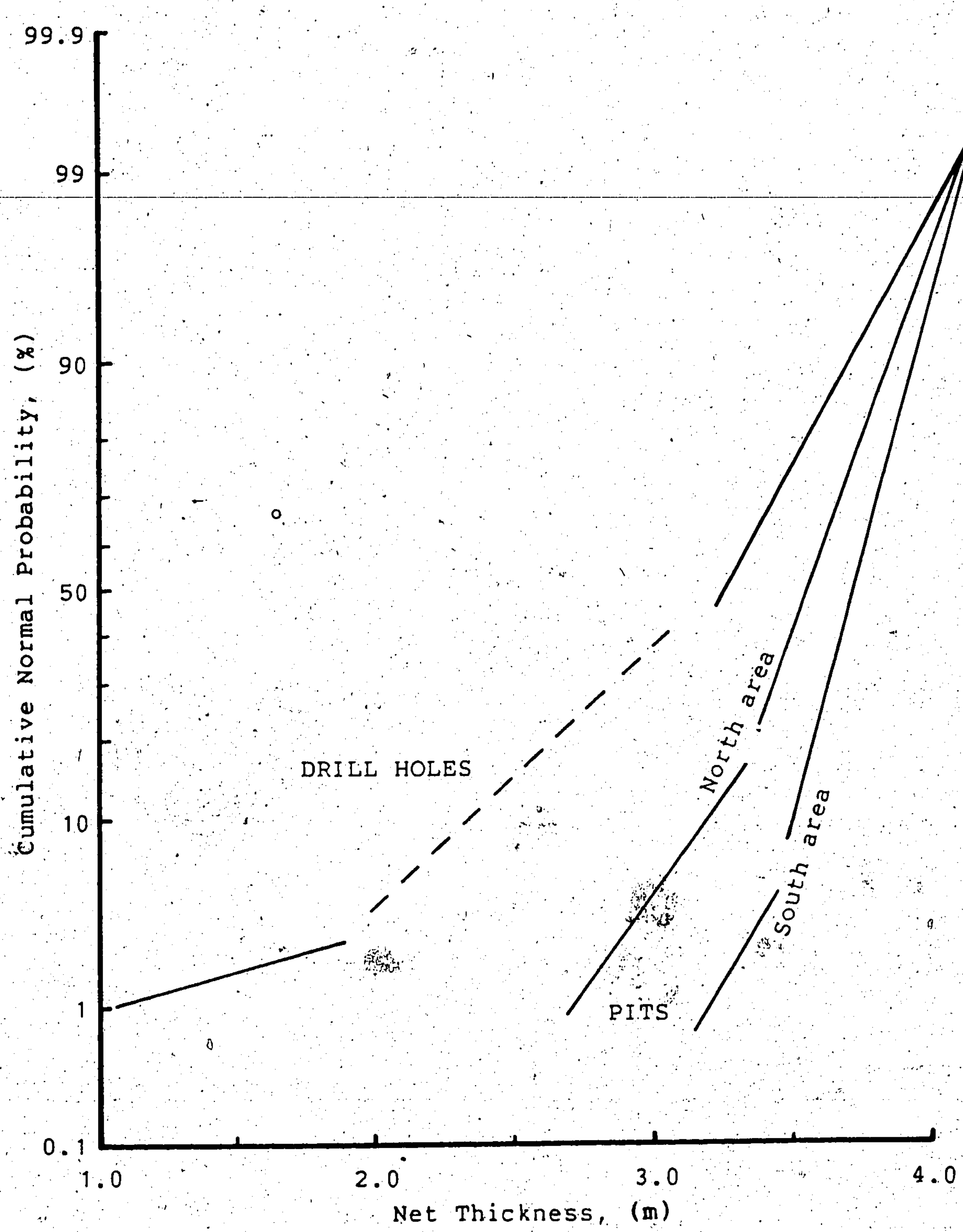


Figure 2.7 Comparison of the pit and drill hole cumulative normal probability curves

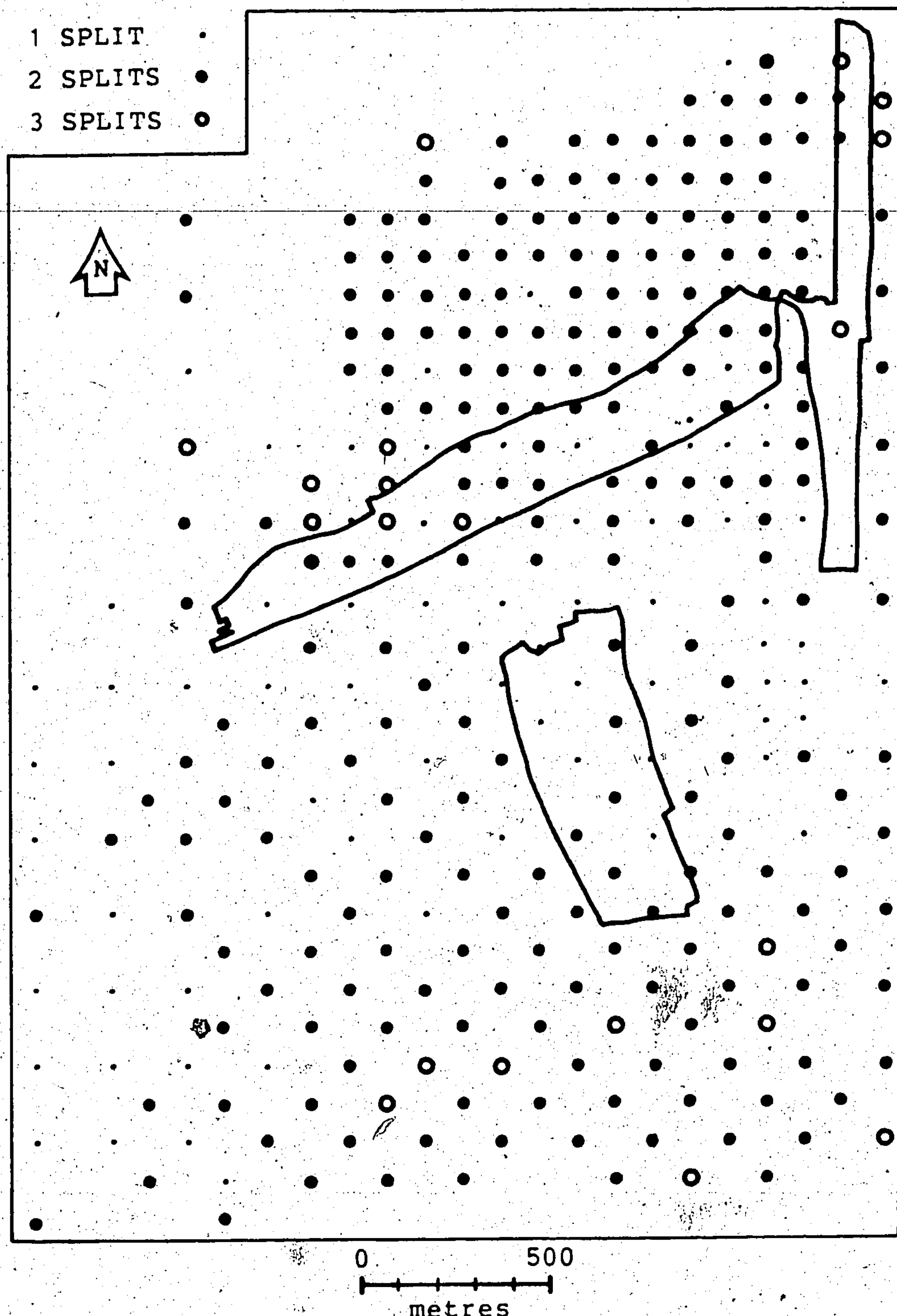


Figure 2.8 Location of the partings in the drill hole data

Table 2.1 Summary statistics of the thickness for each pit and the drill holes

Area	n†	Mean thickness (m)	Variance (m^2)	Standard deviation (m)
Pit 1	272	3.57	0.084	0.29
Pit 2A	201	3.76	0.036	0.19
Pit 3	158	3.61	0.063	0.25
All pits	631	3.64	0.073	0.27
Drill holes	336	3.29	0.278	0.53

† n = the number of samples.

mined.

In geostatistics, 'population variance' is the same as 'variance' in classical statistics. The overall variance of the pit thickness data is 0.073 m^2 and of the drill hole data is 0.278 m^2. The drill hole variance is larger for two reasons. Firstly, the geologists would be less likely to compare the measurements for consistency between holes. Whereas, the surveyors may make the pit thicknesses more consistent and less variable since they are usually done sequentially, in both time and space, on a continuous expo-sure. Secondly, there are more splits in the drill data (up to four partings) than in the pit data (rarely more than one parting). The extra partings will remove a more variable amount from the the thickness of the drill hole data.

2.2.2 Location of the Samples

The spatial distribution of the data was determined by visual inspection of the data posting†, which shows that the pit samples are on a semi-regular grid at approximately 30 m spacing with a few large areas of missing nodes (Figure 2.1, p.39). Postings of the drill data show that the holes are on three regular grids with several small areas of missing holes (Figure 2.2, p.40). The basic pattern is a 200 m square grid that has been infilled to a 100 m square grid in the northeast and a 150 m square grid in the southeast.

† Posting was done with SURFACE II (Sampson, 1978).

3. STRUCTURAL ANALYSIS OF THE DATA

The term 'structural' is used in this study in the geostatistical sense: it has nothing to do with faults or folds. The statistical structure of a regionalized variable is its spatial variability, namely its drift and particularly its variogram. The theory of structural analysis is outlined in Appendix 1.

3.1 VARIABLES USED IN THIS STUDY

The tonnages in this study are calculated by the product of three variables: thickness, density and area.

Thickness satisfies the conditions in Appendix 1.1.1 for a variable to be analysed by geostatistics and is the main geostatistical variable in this study. Thickness is a continuous variable distributed in two dimensional space and any linear combination of thicknesses is also a thickness.

Density also satisfies the same conditions: it is continuous, distributed in three dimensional space and any linear combination of densities is also a density. However, only three densities were determined and at least 30 to 50 measurements are required for geostatistical applications. Thus, even though density is regionalized and should be analysed by geostatistics, classical statistics had to be used.

Area is not a regionalized variable. It is not defined at a point and so does not meet the necessary conditions in Appendix 1.1.1. As a result it was analysed using classical statistics.

3.2 DRIFT ANALYSIS

Ordinary kriging, as used in this study, assumes that there is no drift. This assumption was tested by plotting on Figure 3.1 the one dimensional† drift values estimated by the variogram program.

The drift curves do not rise continually and have low maxima of 0.12 m (3% of the average thickness) for the pit data and 0.14 m (4%) for the drill hole data. Furthermore, the variograms (Figures 5.1 and 5.2 in Appendix 5.1) are bound and so show no drift effect. Both sets of data show little or no drift, so ordinary rather than universal kriging will suffice. Any small drift that is present should not affect the reserves estimates since David (1977, p.114) states that the estimation is very robust with respect to a drift.

3.3 VARIOGRAM ANALYSIS

Analysing the variogram is by far the larger and more important part of structural analysis, if there is no drift. Variograms should only be estimated over a homogeneous

† A description of how a one dimensional function can be estimated is in Appendix 1.3.3.2.

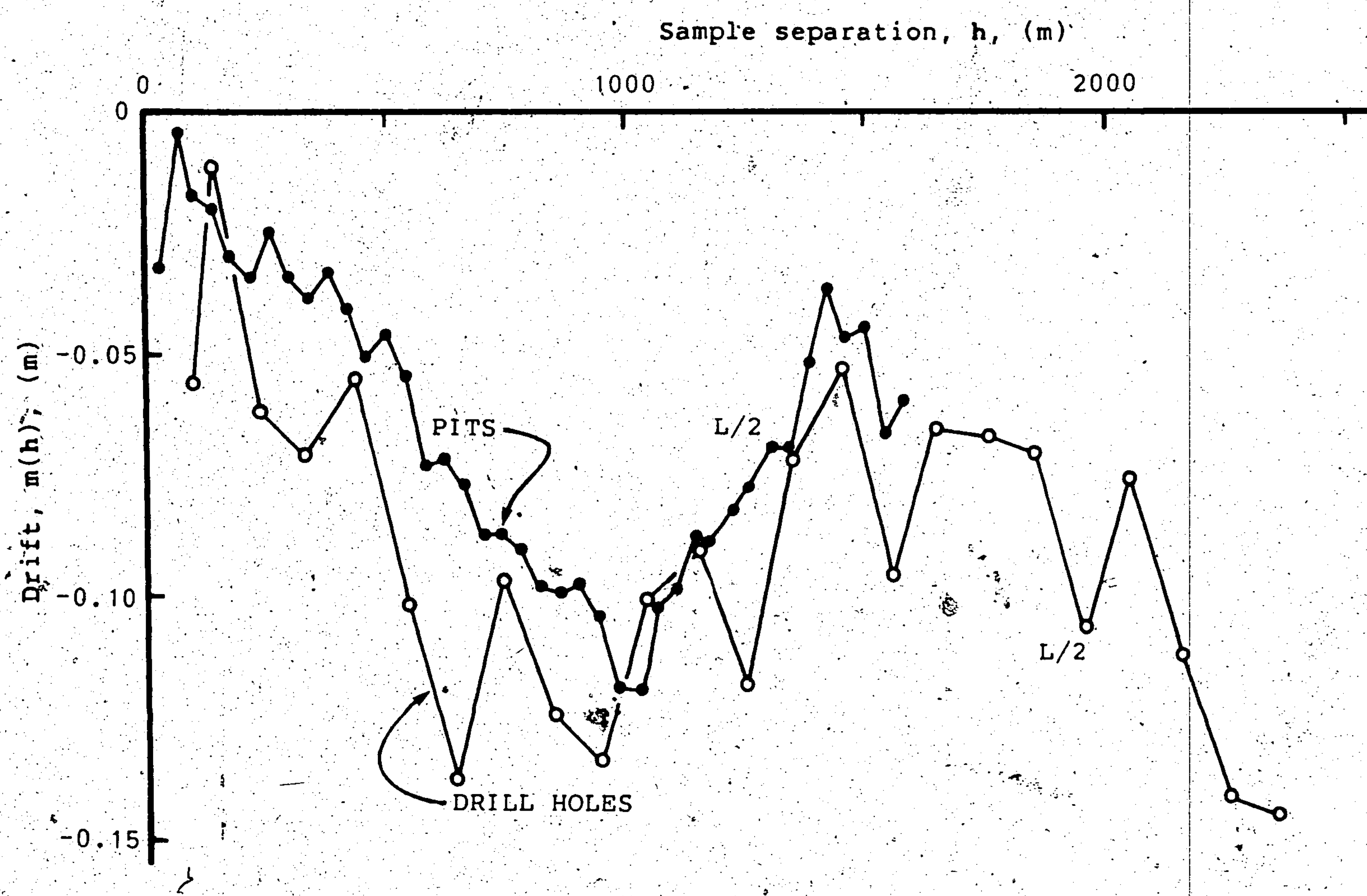


Figure 3.1 Drift of drill hole data and the combined pit data

region, which is an area of uniform geological, statistical and geostatistical properties. The pit data has two homogeneous regions; a north region consisting of pits 1 and 3, and a south region formed from pit 2A, while the drill holes had only one region. These regions are determined in Appendix 3. Usually variogram analysis is done in three stages:

1. calculation of the raw variogram,
2. regularization or correction of the raw variogram to form the experimental variogram,
3. modelling of the experimental variogram.

3.3.1 Calculating the Raw and Experimental Variogram

The raw variograms were calculated with the parameters discussed in Appendix 5.1 and drawn using the conventions listed in Appendix 1.3.1. Regularization is too small to be corrected for (Appendix 5.2), so the raw variograms are also the experimental variograms (Figure 3.2 and Figure 3.3), which were used for modelling. After modelling, the resulting variogram models were used as an input to the kriging program.

3.3.2 Modeling the Fundamental Variogram

Experimental point variograms are too ragged to use as input to the kriging program: a mathematical expression rather than the raw points is needed. Modelling is the process of fitting a mathematical curve to the calculated

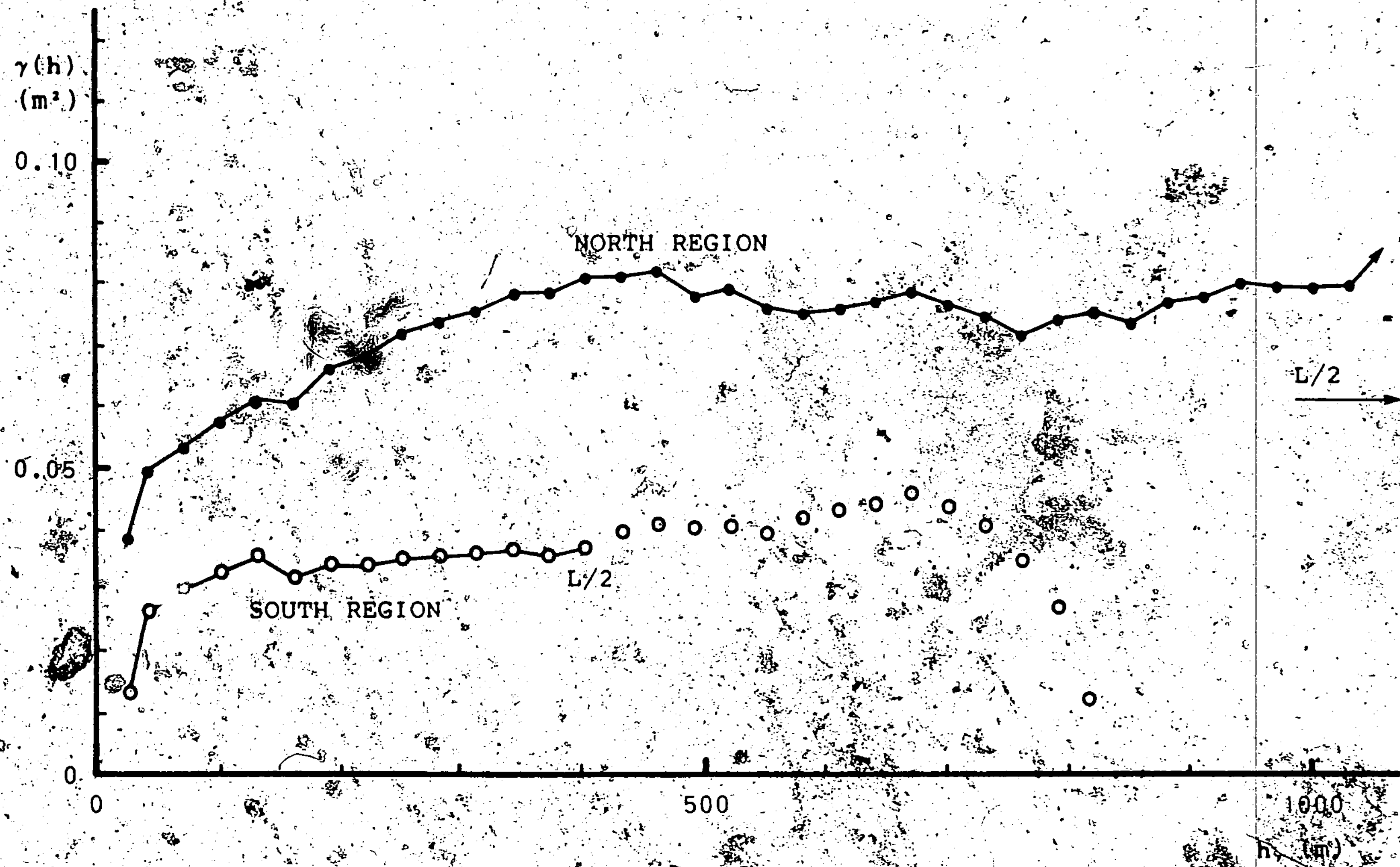


Figure 3.2 Experimental variograms of the pit data

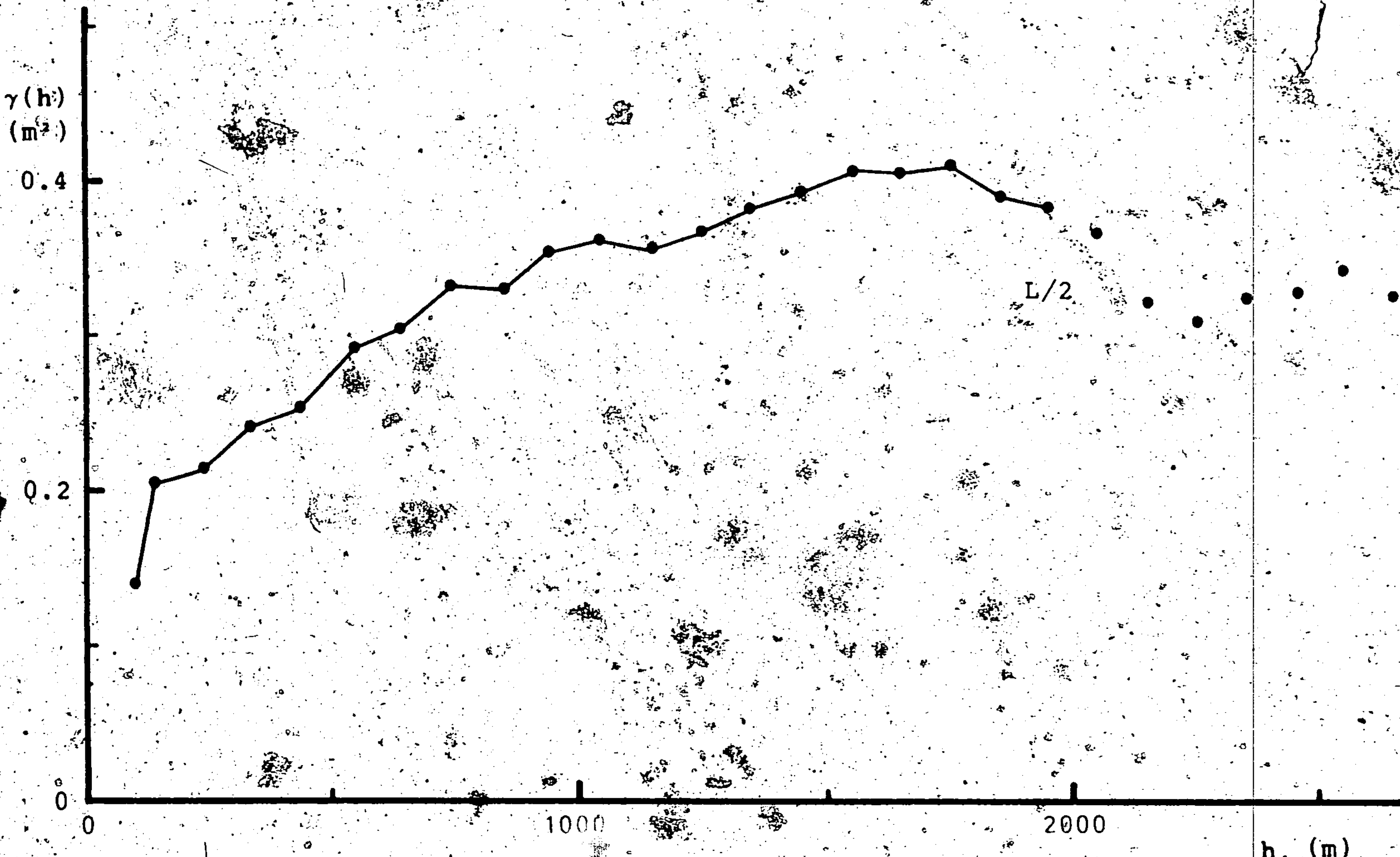


Figure 3.3 Experimental variogram of the drill hole data

points. It is largely an art because there are no algorithms to make a best fit of an ideal curve to the experimental variogram. Fortunately, "...a visual fit is usually sufficient..." according to David (1977, p.122). Parameters are chosen and a curve is calculated and compared to the experimental variogram. Then, the parameters are adjusted and the process repeated until a reasonable fit is achieved. Flint (1978) wrote a program to do this interactively; its operation is briefly outlined in Appendix 6. Some semi-automatic programs are on the market, but none is available on the University of Alberta computer.

3.3.2.1 Variogram models

Preliminary inspection of the experimental variograms, for both the pit (Figure 3.2, p.57) and drill hole data (Figure 3.3, p.58), shows that the models are from the transition class. For the reasons listed in Appendix 1.3.6 the spherical model was chosen for this study. The programs for calculating the variograms, modelling them and for kriging are based on the spherical model. Extra programming effort would have been needed to use any of the other two possible models mentioned below, with little prospect of increased accuracy.

Two nested, isotropic spherical models (Figure 3.4 and Figure 3.5), with no nugget effect are used for the modelled variogram. Their parameters are listed on Table 3.1. The

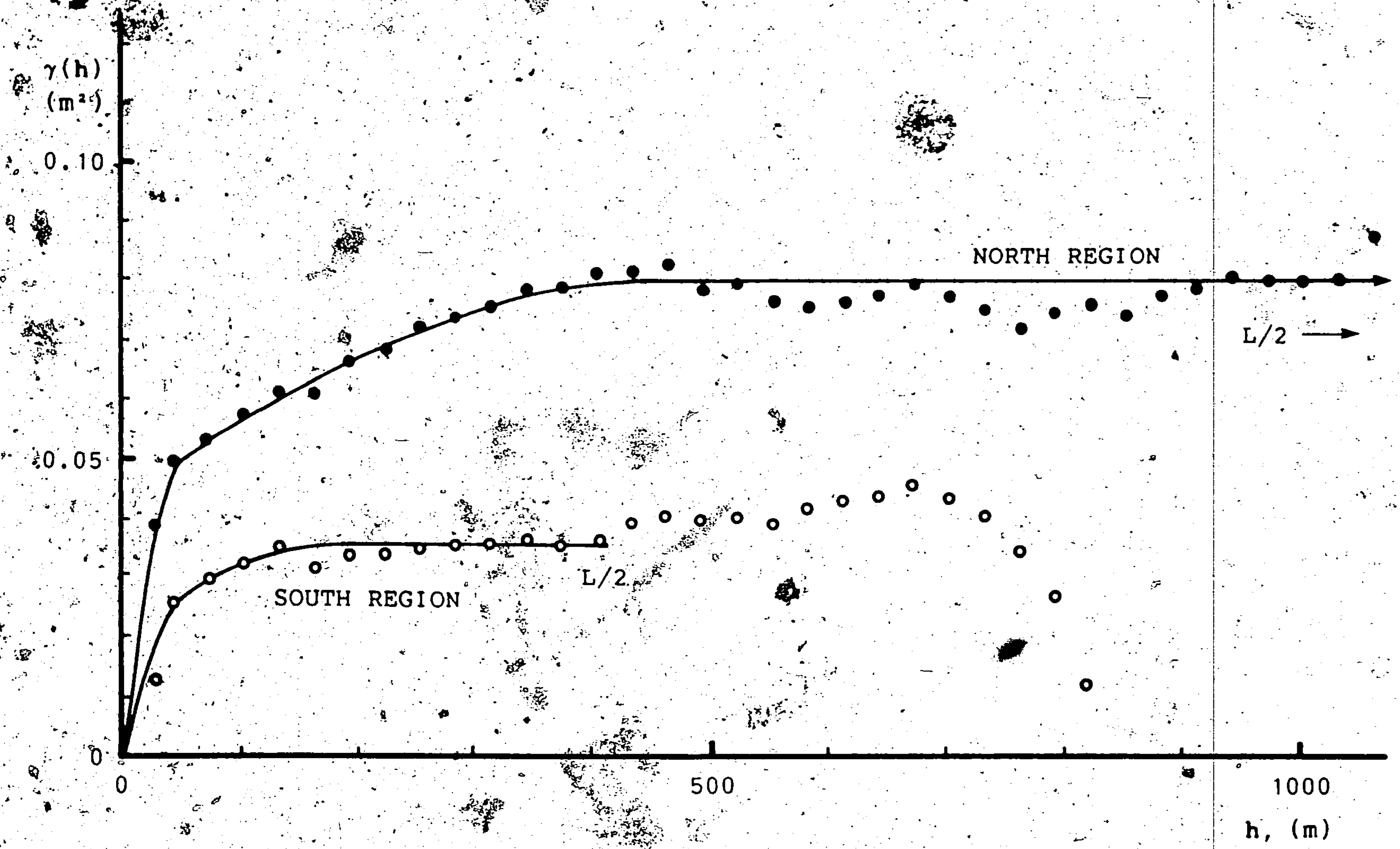


Figure 3.4 Variograms and models of the pit data

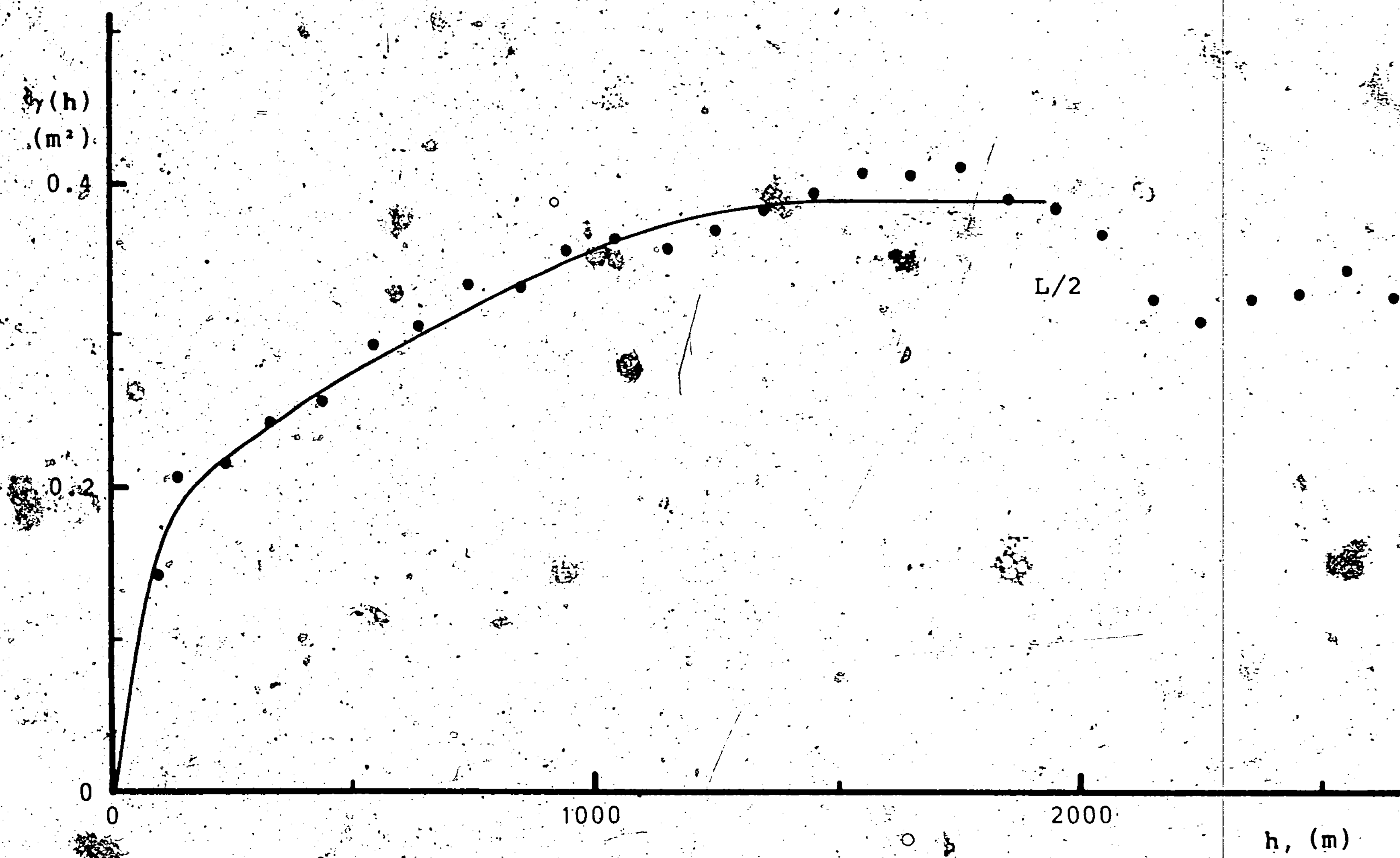


Figure 3.5 Variogram and model of the drill hole data

Table 3.1 Variogram model parameters

Parameter	South pit	North pits	All pits	Drill holes
Nugget effect, C_0	0.0	0.0	0.0	0.0
	Short range structure			
Sill, C_1	0.02 m^2	0.045 m^2	0.035 m^2	0.17 m^2
Range, a_1	50 m±	50 m±	50 m±	150 m
	Long range structure			
Sill, C_2	0.0158 m^2	0.035 m^2	0.035 m^2	0.22 m^2
Range, a_2	180 m	450 m	500 m	1500 m
Overall sill, C†	0.0358 m^2	0.080 m^2	0.070 m^2	0.39 m^2

† $C = C_0 + C_1 + C_2$

fit is good within the limitations of geostatistical modelling, but it is possible that a combination of a hole effect model plus a short range spherical model would also fit.

A hole effect model (Figure 1.9 in Appendix 1.3.6) was not used for the following reasons.

1. The spherical model fits well over the distances used in the kriging calculations: a few hundreds of metres for the pits and about 1000 m for the drill hole data.
2. The hole effect model would only improve the fit beyond these distances.
3. Journel and Huijbregts (1978, p.171) state that "For estimation purposes, an experimental hole effect that is open to doubtful interpretation, or not very marked, can simply be ignored..."
4. The modelling and kriging programs would have to be extensively modified to handle it.

3.3.2.2 Nugget effect

None of the models has a definite nugget effect. The intersection of the line through the first two points with the vertical axis was used to estimate C_0. The north pits (Figure 3.4, p.60) and all the pits together (Figure 3.6) may have a small nugget effect of about 0.01 m^2. The south pit and the drill hole variograms (Figure 3.2, p.57 and Figure 3.3, p.58) intersect the horizontal axis, which

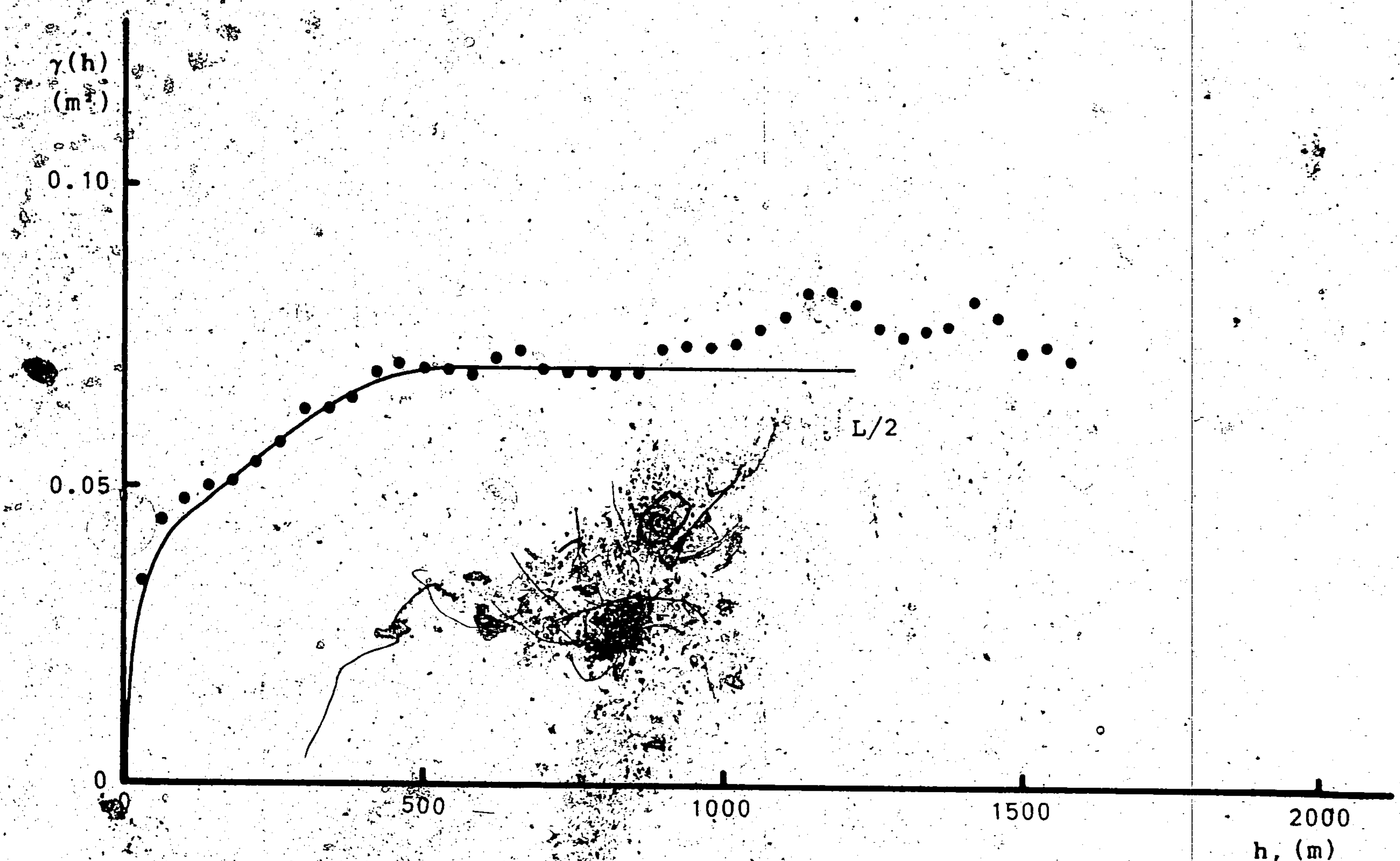


Figure 3.6 Variogram and model of all the pit data grouped together

suggests parabolic behaviour near the origin.

A nugget effect can have two causes: human nugget effect and a structural nugget effect.

David (1977, p.99) defines a nugget effect as "···the uncertainty on the value of the sample itself." A human nugget effect, though small, certainly exists in this mine. All readings, pit and drill hole, were nominally taken to the nearest 0.1 ft (0.03 m), which has an error of ±0.015 m. The mean square error is then:

$$(0.015\ \text{m})^2 = 0.000225\ \text{m}^2$$

This is an estimate of the human nugget effect (Gandin, 1963, p.29). Since ±0.1 ft is the best that the mine surveyors can hope to do it is a lower limit.

In practice, the surveyors and mine geologists were not always able to measure to the nearest 0.1 ft, but sometimes had to round to the nearest 0.2 ft, 0.5 ft or 1 ft when the precise position of the contacts was uncertain. If all readings were accurate to ±1 ft, then the human nugget effect would be 0.0225 m², which is an upper limit.

The true human nugget effect will be somewhere between 0.0002 m² and 0.02 m². The geometric mean of these two numbers, approximately 0.001 m², is a reasonable order of magnitude estimate of the human nugget effect. A more precise value of 0.0017 m^2 was calculated from estimates of the number of each type of approximation (±0.2 ft, ±0.5 ft and ±1 ft) that the surveyors made. This value is too small to model because the noise in the experimental variogram is larger by at least an order of magnitude.

A structural nugget effect probably does not exist. The drill hole data may be parabolic near the origin, which implies no nugget effect. The pit variograms also probably have no structural nugget effect for two reasons. Firstly, both pit variograms seem to have the same type of visible structure, both short and long range, even if the parameters vary. Therefore, it is likely that any invisible (nugget) structure would also be the same for both variograms. However, one pit variogram is parabolic while the other has an apparent nugget effect. This difference is probably random fluctuation with both true variograms having a simple linear behaviour near the origin. Secondly, any nugget effect would be due to a short range structure with a range less than 30 m^2 and a sill of approximately 0.02 m^2. This means the thickness would vary up to ±0.3 m (2σ†) within a distance of 30 m. This sort of variation cannot be seen in the continuous exposure along the highwall.

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† σ is used here and elsewhere in this study as an abreviation for the standard deviation.

3.3.3 Structural Interpretation of the Model

Structural interpretation is the interpretation of the variogram model parameters in geological terms.

3.3.3.1 Interpretation of the model parameters

Transition models usually apply to phenomena that are fairly regular and 'well behaved', as sedimentary thicknesses often are. The spatial structure includes a zone of influence, inside which the data is better correlated the closer they are to each other and beyond which the data is independent. Such data has a finite population variance, which is usually the case with thickness variograms.

The variograms show no anisotropies other than the slight effect due to measurement techniques in the pits discussed in Appendix 5.3. Most geological anisotropies in a tectonically undisturbed area will align with the preferred paleo-environmental directions; in this case, the major paleochannel (Figure 1.10, p.23). Neither the pit (Figure 3.7) nor the drill hole variograms (Figure 3.8) change depending on the direction with respect to the major paleochannel. This is probably because there is no strongly preferred direction of deposition in an essentially flat swamp. However, some anisotropy could be present nearer the paleochannel, or in other variables such as ash content.

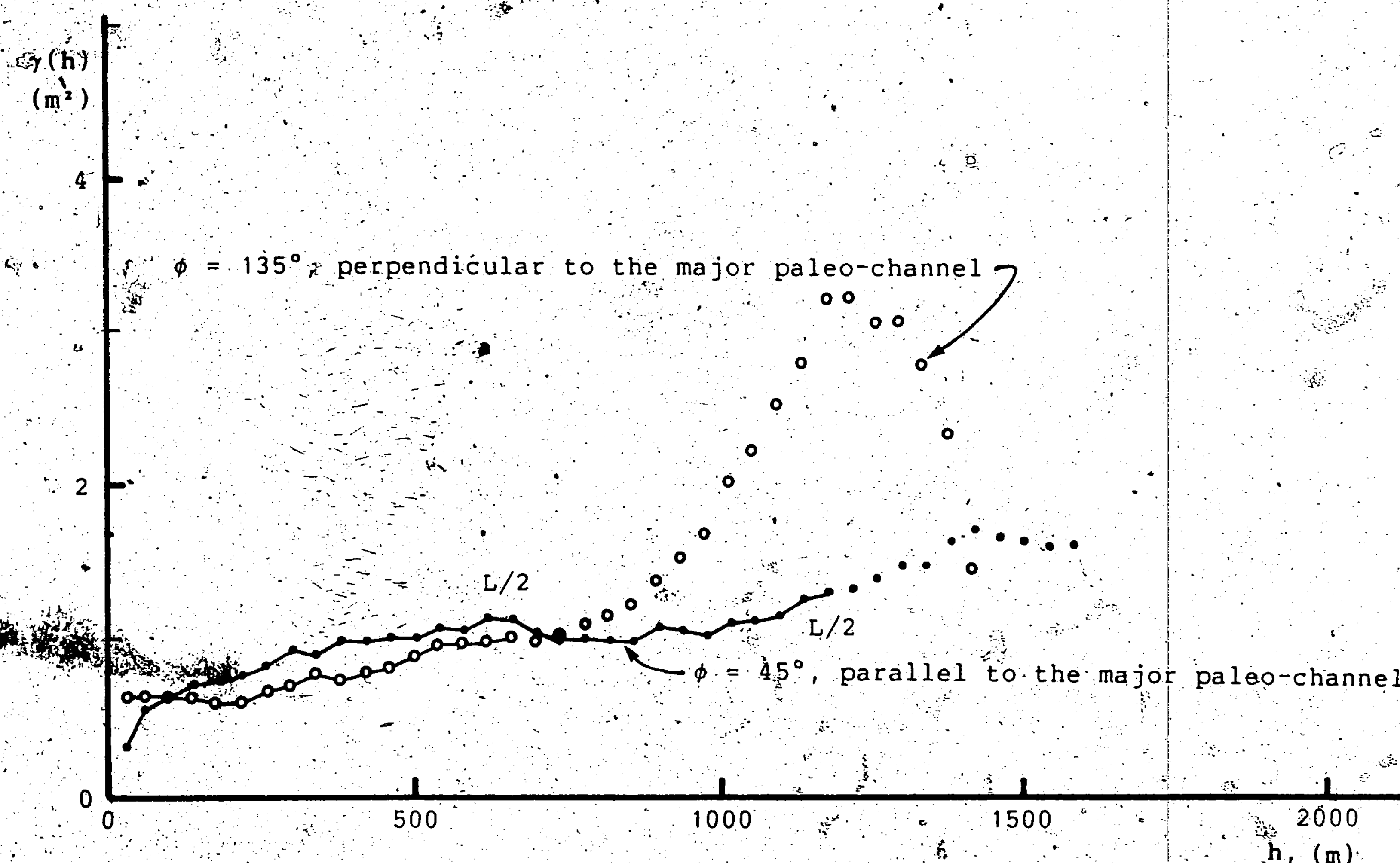


Figure 3.7 Pit variograms, parallel and perpendicular to the major paleochannel

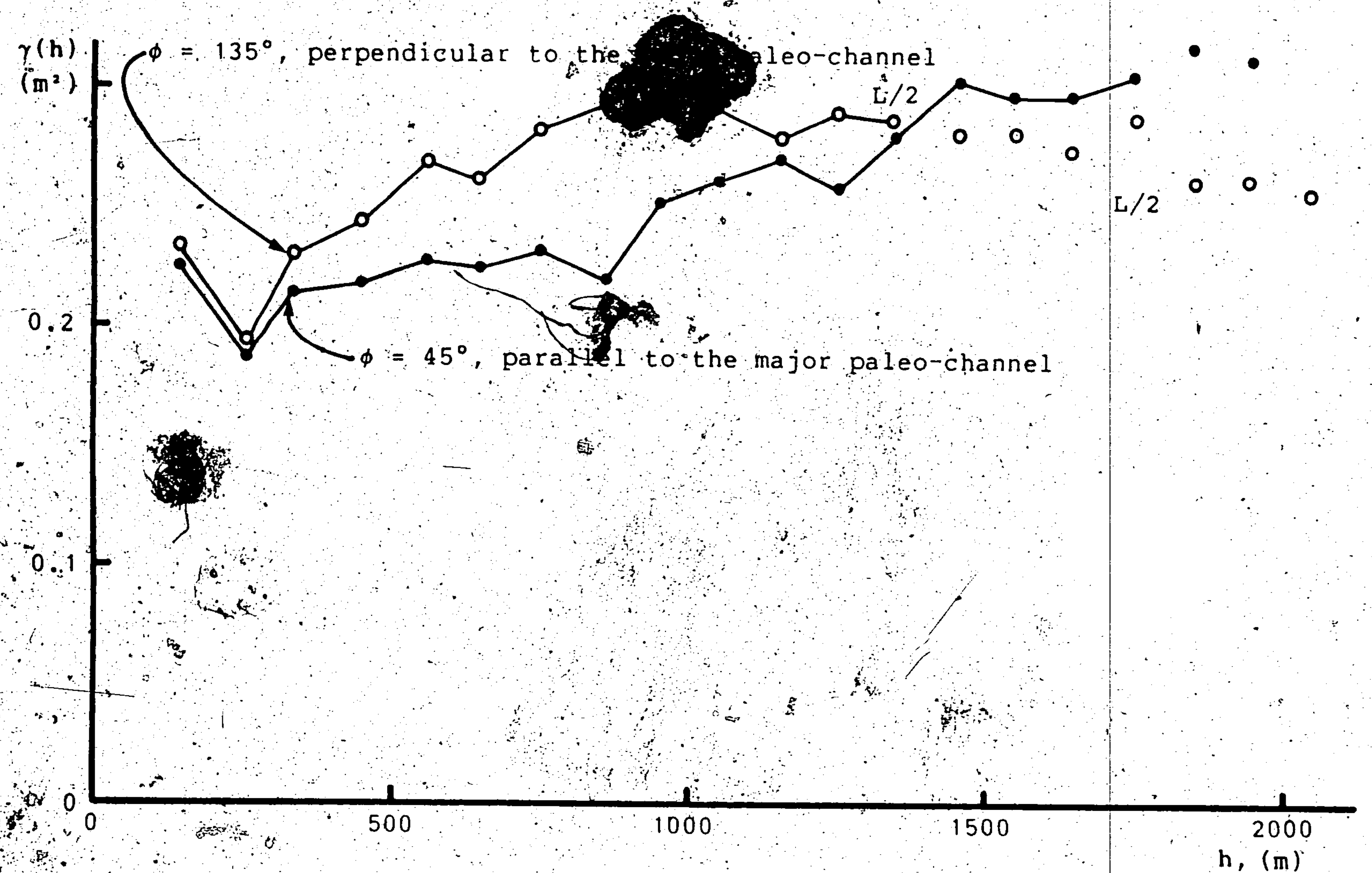


Figure 3.8 Drill hole variograms, parallel and perpendicular to the major paleochannel

As expected, the overall sill is well defined and generally equal to the population variance. The long range structure is probably due to the depositional history of the zone. When two drill holes are greater than 1500 m apart, the thicknesses are poorly correlated because they will tend to be in areas influenced by different partings.

The overall sill is larger for the drill hole model than for the pit models because of the differences in the population variance listed on page 52. The structures of the pit data are probably different because of different depositional micro-environments associated with the two partings. The sill is higher on the north variogram because of all the small paleochannels, which will increase the population variance.

3.3.3.2 Interpretation of the structures

The short range models may be due to the method of measurement or deposition. In all three cases, the short range is approximately equal to the sample spacing (Table 3.2). The differences between the range and the sample spacing are because the ranges are only a rough estimate using one to two points from the experimental variogram, while a good estimate requires at least three to four points. On the other hand, the short range sill (C_1) is always approximately equal to the long range sill (C_2) (Table 3.1, p.62), which suggests that the short range

Table 3.2 Comparison of the short ranges of the variogram models with the sample spacings

Area	Short range† (m)	Nominal sample spacing (m)
North pits	50±	30
South pit	50±	30
Drill holes	150	150±50

† From Table 3.1, p.62.

structures are also due to deposition rather than measurement effects.

The short range structure of the drill hole model is approximately the same as the long range structures of the pit models, taking into account the very differerent sources of the data. The discrepancy may be due to an error in the drill hole short range, which was estimated from only one point. The sills are also moderately close: 0.17 m^2 and 0.35 m^2.

Nugget effects (C_0) are not apparent on any of the variograms because the measurement errors are too small, about 0.001 m^2, and there is no visible structure with a range less than the sample spacing in the pits.

3.3.4 Comparison of the Models From This Study With Flint's models

Flint (1978, pp.107 and 108) modelled the variogram for the net thickness of coal over the full extent of the Estevan Zone. His model was isotropic, spherical and nested with the parameters listed in Table 3.3.

It should be possible to correlate the overlapping structures for two reasons. Firstly, the same variable was studied: the net thickness of the Estevan Coal Zone as measured in drill holes. Secondly, the magnitude of sample

Table 3.3 Comparison of the variogram model parameters from Flint (1978) with the drill hole model from this study

Parameter	Flint's model†	This study‡
	Short range structure	
Nugget effect, C_0	0.092 m^2	
Sill, C_1		0.17 m^2
Range, a_1	<250 m± ††	150 m
	Intermediate structure	
Sill, C_2	0.37 m^2	0.22 m^2
Range, a_2	1000 m	1500 m
	Long range structure	
Sill, C_3	0.51 m^2	
Range, a_3	4000 m	

† From Flint (1978, p.107).
‡ From Table 3.1, p.62.
†† This is the approximate minimum sample spacing and therefore, the maximum range for any short range structures.

separation overlaps: 250 m to 34 km in Flint's study and 30 m to 3600 m in this study. The only structures that overlap are Flint's short range structure and the long range structure from this study. Both the ranges (1000 m and 1500 m) and sills (0.37 m^2 and 0.22 m^2) are comparable. Flint's nugget effect corresponds to the short range structure of this study. Discrepancies in the correlation could be caused by the ragged nature of experimental variograms, particularly Flint's, which means that there will be some scope for estimating the parameters.

As a result of the comparison, an overall model for the Estevan Zone can be assembled from the models listed in Table 3.3, p.73 and Table 3.1, p.62. Model structures chosen as closest to the real structures were always from the more closely spaced data when there is a choice, since they are better estimates of the true structure. The best structures are the following:

1. short range model of the pit data,
2. long range model of all pit data grouped together,
3. long range model of the drill holes from this study,
4. Flint's long range model.

These structures have ranges of 50 m, 500 m, 1500 m and 4000 m. The last three are in an approximate geometric progression, each about one third the next larger. The exception is the first one (50 m) which is 10 times smaller than 500 m. This discrepancy may be due to the smallest structure being a measurement rather than depositional effect, as outlined on page 70 above. It could also be due to the pit data having a different progression than the drill hole data; in which case, the 500 m structure is also spurious and there is no geometric progression.

Serra (1968) studied the variability of iron concentration in the Lorraine basin of France from the scale of petrographic slides to the whole basin. He also found the models to be spherical with ranges in a geometric progression. This effect may be common in data that can be modelled over a wide variety of scales.

If all the ranges in the Estevan Zone are in a geometric progression, then the next larger one should be at about 12 km. Flint's experimental variogram (p.108) shows a possible structure at this range, but this is not certain because the data presented on the graph end at 14 km, leaving only a short distance to determine if it truly is another structure. In addition, the variogram is becoming unstable with large fluctuation errors, since L = 34 km, so that 12 km = L/2.8

4. SAMPLE SPACING STRATEGY

When estimating reserves, the usual objective is to obtain the most precise estimate for the least expense. In this study the precision of the estimate is measured in terms of its variance; the lower the estimation variance, the better the estimate.† The expense of an estimate is usually controlled by the number of samples, particularly when drilling, and the number of samples is determined by the sample spacing chosen. In general, the estimation variance will decrease in a nonlinear fashion‡ as the sample spacing decreases with the precise relationship depending on the variogram alone.

Since the variogram is known in this study, the effect of a sampling program on the estimation variance can be calculated prior to further sampling. For instance, it may be possible to reduce the amount of planned drilling with only a slight increase in the variance of the reserves estimate, depending on the form of the function between the estimation variance and the sample spacing. This function was determined by kriging the estimation variance for several sets of data with different sample spacings, then plotting the results. The variance versus sample-spacing curve could have been determined exactly using auxiliary

† This asssumes that the reserve estimate is unbiased.
‡ Non-linearity is due to the dimishing returns of more samples.

functions; instead, the numerical approximation by the computer program was used to demonstrate how this curve can be derived when the requirements of auxiliary functions are not met; namely irregular domains and sample locations.

The estimation variance depends solely on the variogram and the sample locations, or sample spacing if the data is on a regular grid. The sample values themselves do not affect the variance (see Equations {7.12} and {7.14} in Appendix 7.2.4), which is why the variance can be determined prior to drilling, once the variogram is known. For this study, as in most, the variogram was determined in another part of the deposit and the intrinsic hypothesis assumed to be true.

4.1 METHOD

The variance versus sample-spacing curve was calculated only for the drill holes, not the pit data. Measuring the highwall is cheap, so maximizing this sample spacing is not as critical as for the drilling. In addition, using the several hundred pit measurements required (see Appendix 8.4.1) to calculate the curve by computer would be very expensive, and by auxiliary functions, awkward and very tedious.

The data set used was artificially constructed with the sample locations on a perfectly regular square grid. This, rather than an actual drilling pattern, was used so that the results would not be affected by the idiosyncracies of a particular grid. The data values were arbitrarily set to one, since they had no effect. The theory of kriging and the operation of the kriging program are outlined in Appendix 7 and Appendix 8 respectively. Following are the input parameters for the kriging program.

1. The domain was rectangular, 300 by 500 m; approximately the size and shape of pit 2A or a little over two years of production.
2. The artificial drill hole grid was square, centred on, and aligned with the domain.
3. The variogram model used was the drill hole model in Table 3.1, p.62.

The krige and extension variances for a given sample spacing were computed using the modified kriging program with the best calculation parameters as determined in Appendix 8.4. The calculations were repeated for other sample spacings between 50 and 500 m by multiplying the sample coordinates in the original artificial data set by a scale factor. The artificial data and domain simplified this conversion.

This curve has been calculated for the thickness estimation variance, so that it would not be dependent on a particular density or area variance. The overall shape of the curve will not change from the one for the tonnage, since density (for this study) and area are not regionalized and their variances are not functions of the sample spacing.

4.2 RESULTS

The results of the calculations are plotted on Figure 4.1 and Figure 4.2, which show, as expected, the variance increasing with the spacing. The first figure is the general case for a seam with the same variogram; the second one is a particular case, which may be easier to use.

The krige variance is always less than the extension variance†, except where both are essentially zero at sample spacings less than 50 m. This means that a kriged estimate is always more accurate than the arithmetic mean, down to 50 m spacing.

If the arithmetic mean is used, which closely approximates the calculation method used at the mine‡, then there is a relatively rapid improvement in the reserves estimate as the hole spacing is decreased from 220 to 70 m. Within this distance the spacing can be chosen to trade off costs against estimate precision.

The optimum spacing drill hole spacing must be chosen by the mine engineer using this curve and how much he is willing to pay for a reserve estimate of a given precision.

† The extension variance is the variance of the extension estimate, which is the arithmetic mean of the data.
‡ See Section 5.1.2 for a description of the reserves calculation method used at the mine.

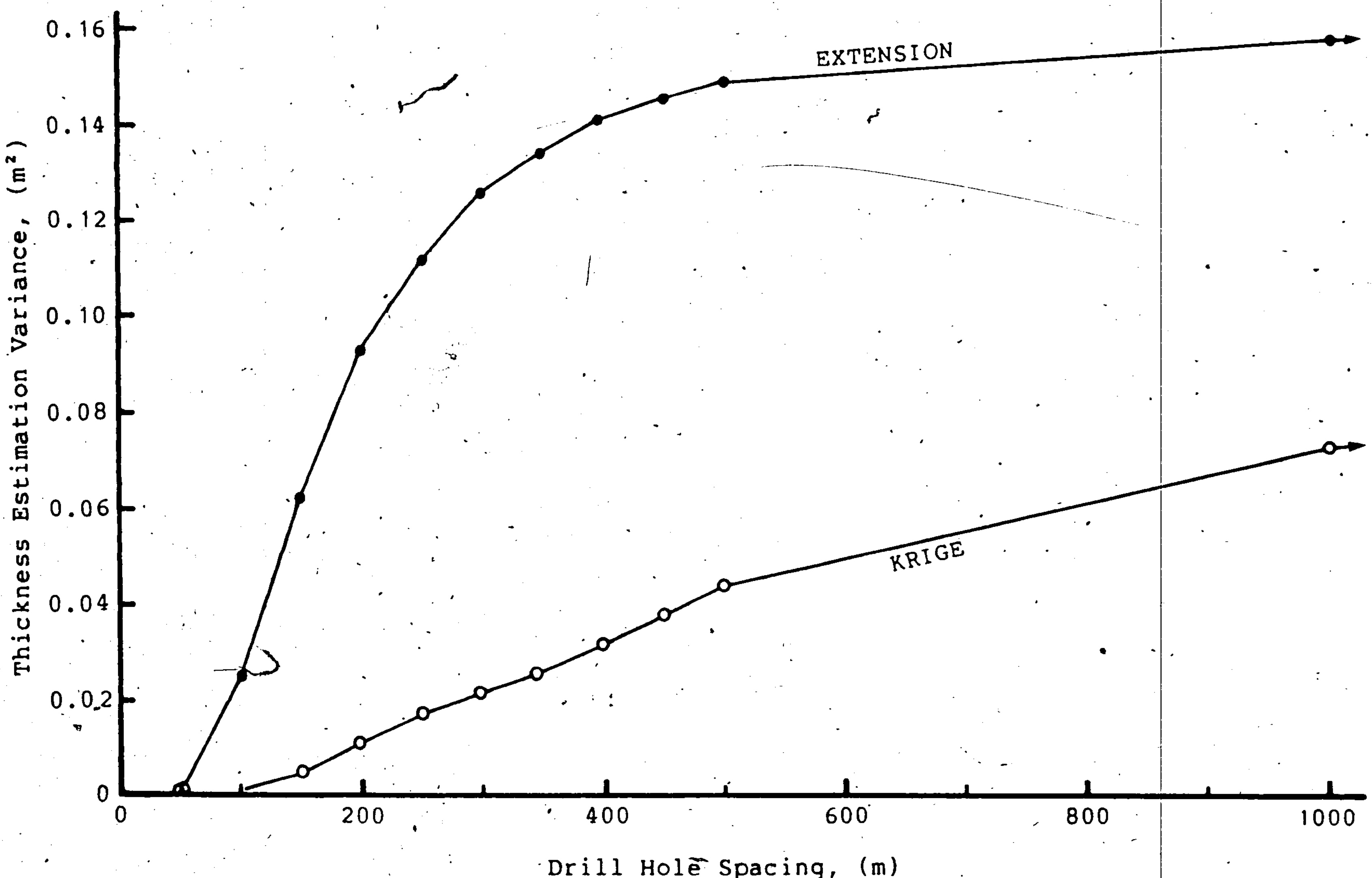


Figure 4.1 Thickness estimation variance versus drill hole spacing

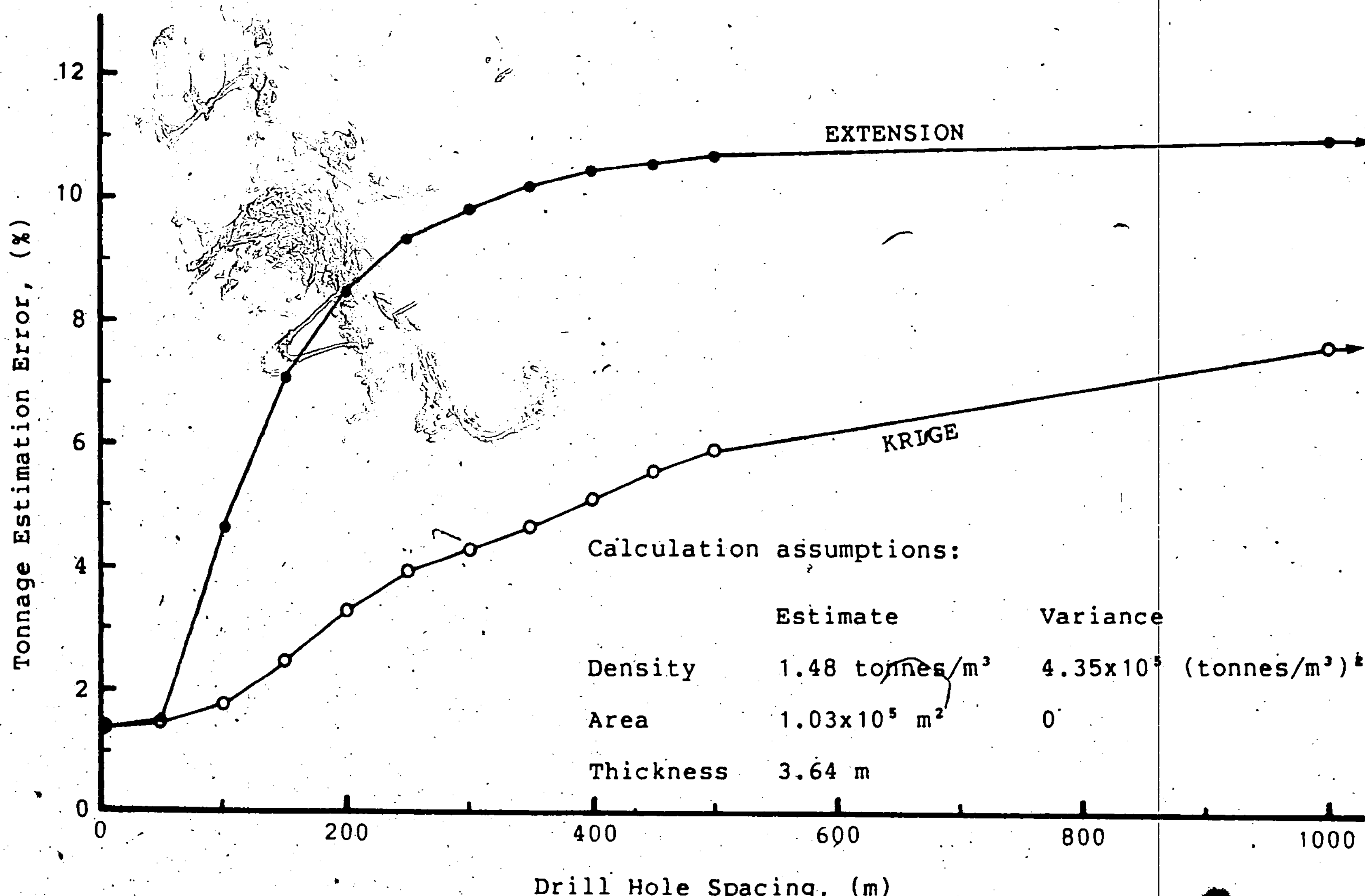


Figure 4.2 Tonnage error versus drill hole spacing

At the drill hole spacing of 100 to 150 m used in the mine (Figure 2.2, p.40), the extension estimate has about ten times the variance of the krige estimate with the same hole spacing. The same accuracy as the extension can be achieved by kriging samples approximately four times further apart, at 400 to 800 m spacing.

Minimizing the estimation variance is not the only criterion for choosing the hole separation. Closely spaced drilling is required at the Boundary Dam Mine to find and delineate coal lost to paleochannels and unrecorded underground mines along the Souris River Valley.

The variance/sample spacing curve depends only on the variogram. The drill hole variogram is only defined for $h < 2000$ m (Figure 3.5, p.61), so this variance/sample spacing curve says nothing about sample spacings beyond 2000 m. In fact it must rise, otherwise one drill hole would give as good an estimation variance as a 2000 m grid of holes.

5. CALCULATIONS OF PRODUCTION AND RESERVES

5.1 GENERAL METHOD

In this study calculations were done on two sets of ~~tonnage reserves:~~

1. this year's in-place coal, or the amount mined this year, which is used to estimate the coal recovery,
2. next year's in-place coal, or the amount to be mined next year, which is used to estimate reserves and future production.

In both cases the krige estimate (Z^*_k) and the extension estimate (Z^*_x) were calculated and the results compared to determine if the extra effort involved in geostatistical calculations is worth the increase in accuracy. Both tonnages were estimated for the same domain to compare the actual production with the forecast production. Finally, the sources and the relative sizes of the errors were determined.

5.1.1 Calculation of the Tonnage Estimate and Its Variance

The tonnage (T) is the product of the area, the density and the average thickness, or:

$$T = A \cdot d \cdot Z^* \qquad \{5.1\}$$

Where:

A = the area of the domain for which the tonnage is calculated.

d = the average density of the coal.

Z^* = the average net thickness of the seam, either Z^*_k or Z^*_x.

The variance of the tonnage is needed for several reasons:

1. comparing the accuracy of different methods for calculating the tonnage,
2. determining the sources and sizes of the errors in the tonnage calculations,
3. determining the range within which to expect next year's production to fall.

The tonnage variance is estimated by:

$$VAR[T] = d^2 Z^{*2} VAR[A] + A^2 Z^{*2} VAR[d] + A^2 d^2 VAR[Z^*] \quad \{5.2\}$$

Where:

$VAR[Z^*] = \sigma^2_k$ or σ^2_x depending on whether $Z^* = Z^*_k$ or Z^*_x.

This equation is developed in Appendix 9.1.

5.1.2 Calculation Methods Used at the Boundary Dam Mine

Reserves at the mine are calculated by the polygon method using drill hole data. In this method the seam is broken down into 'polygons of influence' around each hole, with all the polygons meeting halfway between the holes. The seam is assumed to be the same thickness throughout the polygon as at the hole. Total coal volume is the sum of the products of the area and thickness for each drill hole polygon.

In-place coal is calculated using the highwall thicknesses and a modification of the polygon method. The drill hole polygons are used, but the polygon thickness is an average of all the pit measurements inside the polygon, rather than the drill hole thickness. From there, the method is the same as for the drill holes. Recovery is calculated as the percent ratio of the coal-loaded-out to the in-place coal.

The calculation method used in this study is similar to the method used at the mine, except that the average thickness in this study is estimated twice (Z_k^* and Z_x^*) and the intermediate step involving the polygon of influence is bypassed. Since the pit data is fairly regularly distributed (Figure 2.1, p.39) and since the miner's average is weighted on the areas of the polygons, the extension estimate (Z_x^*) will give essentially the same answer as the method used by the mine engineers.

5.1.3 Calculation of the Density Estimate and Its Variance

The mine personnel have determined a 'tonnage correction factor' of 24 ft^3/ton, which is the reciprocal of the density (1.48 tonnes/m^3), but they have not calculated the error of this number.

The error in this density value was determined by recalculating the density using the original data collected by the mine engineers. They obtained these numbers by carefully measuring the volume and tonnage of the coal-loaded-out from three small (1×10^4 tonnes) sections of benches scattered over the mine. The variance of this data was estimated using classical statistics and weighting by the tonnage of each sample. The density and its error is in Table 5.1.

The coefficient of variation, sometimes called the relative standard deviation, is the square-root of the relative variance, or the estimate divided by the estimation standard deviation, and always expressed as a percent in this study. Since it is dimensionless, it is a straight forward way to compare the error in one quantity to the error in another quantity with a different dimension.

Density is a regionalized variable, so proper analysis would estimate its mean and variance using geostatistics. Unfortunately, the amount of data is far short of the required minimum of 30 samples† and barely enough to to do classical statistics. As result, the variance estimated above is probably too high, since the method makes an implicit assumption of a pure nugget effect model for the variogram.

† For a brief discussion of this limit see Appendix 5.1.

Table 5.1 **Data and statistics for the density estimate and error**

Density data

Location	Weight (tonnes)	Volume (m^3)	Density (tonnes/m^3)
Pit 1, cut 32, bottom coal	4723	3133	1.51
Pit 1, cut 32, top coal	14894	10192	1.46
Pit 3, cut 4†	9570	6375	1.50

Density statistics

Mean:	1.48 tonnes/m^3
Variance:	0.000435 (tonnes/m^3)^2
Coefficient of variation:	1.41%
Relative variance:	0.020%

† A thin (0.1 m) parting was removed from the volume and weight of pit 3, cut 4.

The density is assumed to be constant over the whole study area, so the same value is used for both this year's and next year's tonnages. In fact, it will not be constant, due to varying ash content, but there is not enough data to determine its spatial variability.

5.2 IN-PLACE COAL AND PRODUCTION FOR THIS YEAR

This year's in-place coal is needed to calculate the coal recovery, which is used by the mine staff to measure the efficiency of the operation.

5.2.1 Domains Chosen

The domains for calculating this year's in-place coal and recovery were selected on the availability of coal recovery data from the mine. Several sections of the mine had data on the coal-loaded-out as well as pit measurements, but the area of the domain could be determined accurately for only some of these. Dates on the mine plan, showing the location of some of the bench advances, often did not correspond to dates tabulated for the coal-loaded-out and therefore, for most benches, a direct comparison was not possible. Unfortunately, the rate of advance of the face is too erratic to make an accurate interpolation. For the remainder, a trial recovery was calculated, which should be in the range of about 70% to 100%. The recoveries fell into two groups: one within this range and the other grossly different. The second group was either less than 50% or, more commonly, far more than 100%, indicating that the wrong limits of the benches must have been used; consequently, these calculations were ignored. The following locations (Figure 5.1) had recoveries between 80% and 100% and therefore, were used for this study:

1. pit 1, 1977 benches,
2. pit 1, 1978 benches,
3. pit 2A, 1977 and 1978 benches less the box cut,
4. pit 3, bench 9.

5.2.2 Calculating the Average Thickness

The program modified from Flint (1978) calculated both a kriged average thickness and an arithmetic average thickness from the pit measurements; each is listed in Table 5.2. The average difference between the two estimates is about 1%, which is close to the size of the coefficient of variation of the estimates. Also the calculation error in the estimation variance (about 10%) is not much smaller than the difference between the coefficients of variation for the two estimates. As a result, the krige estimate does not make a large improvement over the extension estimate. The percent error of the krige estimate due to numerical approximations (that is the probable maximum difference between the calculated and the true value of the krige average) is about 0.05% (see Appendix 8.5). Such small errors of approximation, almost two orders of magnitude less than the estimation error, can be ignored.

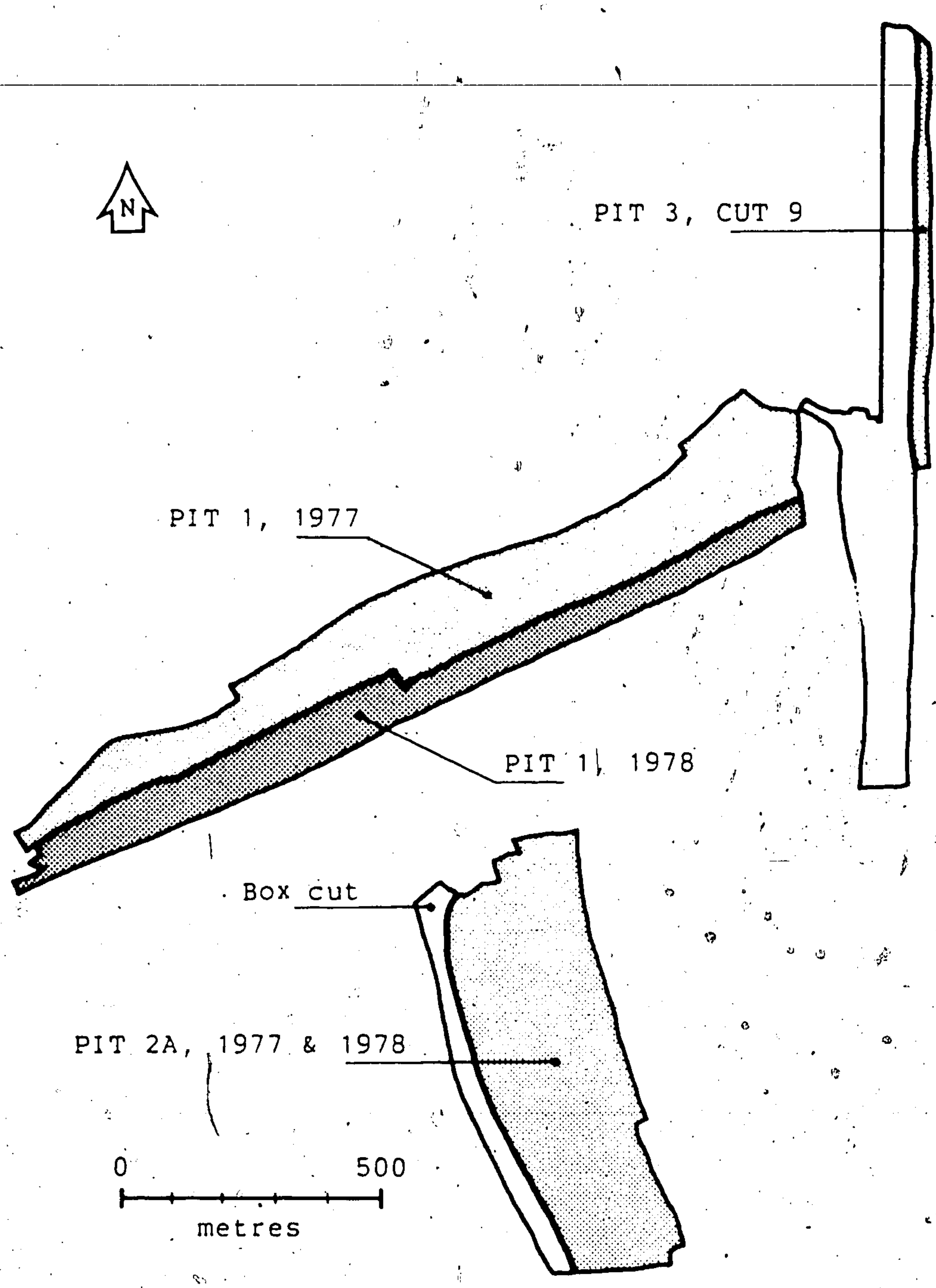


Figure 5.1 **Location of the domains for this year's in-place coal and production**

Table 5.2 Estimates and variances of the average seam thickness for this year's in-place coal

Domain	Estimate (m)		Coefficient of variation	
	Krige†	Ext.‡	Krige	Ext.
Pit 1, 1977	3.50	3.51	0.62%	0.62%
Pit 1, 1978	3.51	3.61	0.44%	1.0%
Pit 2A, 1977 & 1978	3.64	3.63	2.5%	3.0%
Pit 3, bench 9	3.50	3.61	0.4%	2.3%
Avg. coefficient of variation			1.0%	1.7%

† Krige estimate of thickness.
‡ Extension estimate of thickness.

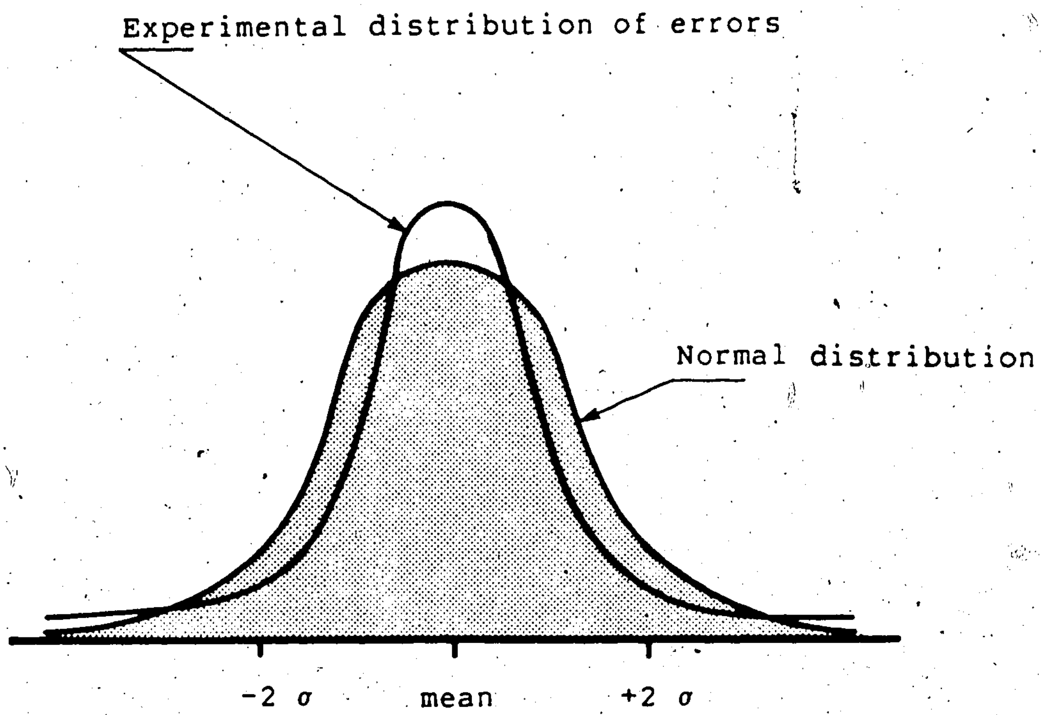
The coefficient of variation of a kriged estimate cannot be interpreted as it would be in classical statistics where, given a normal distribution, the true value will fall within 1σ of the estimate two thirds of the time. The experimental distribution of errors in geostatistics usually has a non-normal distribution, with larger tails than the Gaussian distribution (Figure 5.2). Nevertheless, the usual 2σ limits have generally been found to include the usual 95% of errors, according to Journel and Huijbregts (1978, p.50).

The error in the estimates for pit 2A are larger than the others because the model used had a nugget effect instead of a short range structure. The model was changed in order to experiment with the effect of a different structural interpretation. The estimate is largely unchanged but the estimation variance is increased by a factor of about two.

5.2.3 Area of the Domains

The area of each domain is also calculated by the kriging program. It takes as input the X and Y coordinates of the vertices of a polygon defining the domain of interest and calculates the area of this domain exactly. The areas of the domains are listed in Table 5.3.

The only source of error in the area is in the errors of digitizing the vertices, about 1 m $\pm 1\sigma$, which is discussed



From Journel and Huijbregts (1978, Figure II.13, p.50)

Figure 5.2 Idealized distribution of the errors for most geostatistical estimates

Table 5.3 Areas of the domains used in the calculations

Domain	Area (m^2) ($\times 10^5$)	Approximate production time (yr)
Pit 1, 1977 benches	2.583	1
Pit 1, 1978 benches	1.253	1
Pit 1, total	3.836	2
Pit 2A, 1977 & 1978 benches	2.023	2
Pit 3, bench 9	0.2539	1/8
Pit 3, total	1.555	1

in Appendix 2.1.1. The relationship expressing the error of the area as a function of the errors of the vertices was not used, but instead a subjective guess at the error was made. The precise error was not necessary since it is very much smaller than the errors in the density and thickness.

It is possible to put an upper limit on the error of the area. If one assumes that the errors at all vertices are perfectly correlated, and act together to make the polygon larger or smaller at the same time, then the coefficient of variation of the area of a typical domain in this study is about 5%. But the errors at the vertices should be independent, not correlated. Most polygons in this study have about 100 vertices and this number of independent errors will tend to cancel, making the error in the area much smaller than the upper limit given above. In actual mine practice, the error of the area would be less than the estimates in this study since there would be no digitizing errors, only surveyor's errors, which should be much smaller.

In order to determine the error of the area more closely, a subjective guess at the order of magnitude of the coefficient of variation was made A conservative estimate of 0.1% was chosen, which probably overestimates the true value. This produces a relative variance of $1\text{x}10^{-4}$%, effectively zero when compared to the other relative variances, which are usually of the order of 1%. Therefore, equation {5.2}, p.84 reduces to:

$$VAR[T]/T^2 = VAR[d]/d^2 + VAR[Z^*]/Z^{*2} \qquad \{5.3\}$$

5.2.4 Calculation of the Tonnage

The tonnage is calculated with equation {5.1}, p.83, using $Z^* = Z_k^*$ or Z_x^*, depending on whether the krige or extension estimate is used. The tonnage variance is estimated with equation {5.3}, p.96 slightly rearranged.

The results of the calculations are listed in Table 5.4. As with the thickness, there is no real difference between the tonnages estimated using the krige thickness and the extension estimate.

Using the kriged average of thickness the coefficient of variation of the tonnage is 1.9%. Using the arithmetic average, the coefficient of variation is 2.3%. The difference between these two numbers (0.4%) is close to the approximation error so kriging is barely more accurate than the arithmetic average.

The relative variance of the tonnage is the sum of the relative variances of the thickness and the density (equation {5.3}, p.96). The area contributes essentially nothing since its variance is so small. The density

Table 5.4 Tonnages of this year's in-place coal

Domain	Estimate (tonnes $\times 10^5$)		Coefficient of variation	
	Krige†	Ext.‡	Krige	Ext.
Pit 1, 1977	13.4	13.4	1.5%	1.5%
Pit 1, 1978	6.51	6.69	1.5%	1.7%
Pit 2A, 1977 & 1978	10.9	10.9	2.9%	3.3%
Pit 3, bench 9	1.32	1.36	1.5%	2.7%
Avg. coefficient of variation			1.9%	2.3%

† Calculated using the kriged estimate of thickness.
‡ Calculated using the extension estimate of thickness.

contributes 80% of the relative variance of the tonnage using the krige estimate and 30% to 60% of the relative variance of the extension tonnage. Thickness accounts for ~~the rest. Pit 2A has a large estimation error because of a~~ change in the variogram model as explained on page 92.

5.2.5 Calculation of the Recovery

The recovery (R) is the percentage ratio of the coal-loaded-out to the coal measured by the pit data.† It is calculated by:

$$R \cong 100 \cdot CLO/T \qquad \{5.4\}$$

Where:

CLO = the coal-loaded-out.

The coal-loaded-out is the tonnage of coal coming out of the mine as weighed at the scales. Values for the benches used in this section are listed in Table 5.5. The formula above is discussed further in Appendix 9.2.1.

The relative variance of the recovery estimate is estimated by:

$$VAR[R]/R^2 \cong VAR[T]/T^2 \qquad \{5.5\}$$

which is developed in Appendix 9.2.2. This equation means that the relative variance and the coefficient of variation

† The mine engineers also calculate recoveries using the drill holes. But these were not considered, other than to note that they were generally worse (more variable and often over 100%) than the recoveries using the pit measurements.

Table 5.5 Tonnages of coal-loaded-out from the mine records

Domain	Coal-loaded-out (tonnes)
Pit 1, 1977 benches	1 108 567
Pit 1, 1978 benches	587 774
Pit 2A, 1977 & 1978 benches	1 092 569
Pit 3, bench 9	128 760

are the same for both the recovery and the tonnage of reserves. Thus, all the remarks above about the error of the tonnage also apply to the recovery.

The recoveries from this study are listed in Table 5.6, which also has a comparison of the recoveries calculated by the mine personnel. The percent recovery averages about 90% according to all methods of calculation. The recovery calculated by the mine personnel using traditional methods is often within 1σ of the geostatistical estimates and always within 2σ, except for pit 2A because of the nugget effect used in the variogram model. Again, there is little improvement using a krige estimate except for a slight reduction of about 0.4% in the coefficient of variation. Even the relatively simple calculation mehod used at the mine is more complicated than necessary since there is little difference between their results and a simple average.

Pit 1 is 5σ (1978) to 10σ (1977) below 100%, so almost certainly the recovery is not perfect. The other two pits are within 1 or 2σ of 100% so all the coal may have been recovered. The recovery from pit 2A is slightly above 100% due to statistical fluctuation.

Table 5.6 Comparison of the percent recovery calculated by the mine personnel with the percent recovery estimated in this study

Domain	Mine† (%)	Estimate (%)		Coefficient of variation	
		Krige‡	Ext.††	Krige	Ext.
Pit 1, 1977	83.20	82.7	82.7	1.5%	1.5%
Pit 1, 1978	88.16	90.3	87.9	1.5%	1.7%
Pit 2A, 1977 & 1978	90.91	100.2	100.2	2.9%	3.3%
Pit 3, bench 9	96.62	97.5	94.7	1.5%	2.7%
Avg. coefficient of variation				1.9%	2.3%

† From the mine records as calcultaed by the mine engineers.
‡ Calculated using the kriged estimate of thickness.
†† Calculated using the extension estimate of thickness.

5.3 RESERVES AND PRODUCTION FOR NEXT YEAR

Next year's tonnages and variance are also determined by equation {5.1}, p.83 and equation {5.2}, p.84. The density is the same as that used for this year's in-place coal (estimated on page 85 above), but the thickness and area are arrived at differently.

The reserves are estimated for a domain already mined out, but as if production had not yet started. The estimation used only data that would have been available before excavation, either pit data (from nearby benches) or drill holes. These tonnage estimates can then be compared with the 'true' values, which are estimated by the best method possible (using kriged thicknesses of the highwall in the pit).

5.3.1 Domains Chosen

The domains used when estimating with the drill holes are all three pits (Figure 1.11, p.24):

1. pit 1, 1977 and 1978 benches,
2. pit 2A,
3. pit 3.

The domains used when estimating with the pit measurements are the two halves of pit 1 (Figure 5.1, p.90):

1. pit 1, 1977 benches only,
2. pit 1, 1978 benches only.

All these domains were in production for approximately one year (Table 5.3, p.94); as a result, they demonstrate the errors to be expected when estimating in-place coal or production for some future year.

5.3.2 Calculation of the Thickness

The average net thickness used to estimate next year's production can come from two sources: either drill holes into and around the projected pit, or pit measurements from a highwall beside or near the projected pit. Estimates using drill holes have the following *a priori* advantages.

1. Any drift will affect the results less since the estimates are interpolated rather than extrapolated as the pit measurements must be.
2. The estimates can be made wherever there are drill holes, not just near a previously opened pit.
3. There are fewer drill holes, so the computing will be cheaper.

Estimates using pit measurements have the following *a priori* advantages.

1. They are less biased since they have approximately the same mean as the 'true' value (Table 2.1, p.51), in contrast to the drill holes.
2. There are more measurements, so the estimate may have a lower variance.
3. The benches are generally advanced in a regular manner, so there is almost always an open bench near a proposed pit that is not too far in the future.

5.3.2.1 Next year's thickness using drill holes

The average net thickness for next year's in-place coal is determined from the drill hole measurements that are in or near the proposed pit.

Both the kriged and extension thickness were calculated and compared. As for this year's in-place coal, the 'true' thicknesses were determined by kriging all the pit measurements taken within the pit used as a projected domain. Kriging was used because it should give the best (in a geostatistical sense) determination of the actual thickness against which to compare the projected estimates. The coefficient of variation is also tabulated to show how much latitude there is in the 'true' value; usually about one third to one half of the variability in the estimates.

The results of the thickness calculations are listed in Table 5.7. Again, there is little to choose between krige and extension estimates except a slight increase in the estimation variance. However, both estimates consistently underestimate the 'true' thickness by 1.5σ to 2σ or about 6% on average. The 1σ range on the -6% figure is ±3%, which means that the bias is probably real. The bias is due to the drill hole thicknesses being thinner than the pit measurements on average (Table 2.1, p.51).

Table 5.7 Comparison of the 'true' thickness with the estimated thickness for next year, calculated using drill hole data

Domain	Estimate (m) Krige	Ext.	Coefficient of variation Krige	Ext.
Pit 1, total				
'True'†	3.52		0.99%	
Drill holes	3.35	3.37	1.5%	2.4%
Pit 2A, 1977 & 1978				
'True'†	3.77		0.37%	
Drill holes	3.60	3.57	2.9%	3.4%
Pit 3, total				
'True'†	3.61		0.48%	
Drill holes	3.26	3.22	3.6%	4.8%
Avg. coefficient of variation			2.7%	3.5%
Avg. error	-6.3±2.9%	-6.8±3.5%		

† 'True' thickness is kriged from pit measurements within the pit.

5.3.2.2 Next year's thickness using pit measurements

The same domains were estimated as above but this time using the thickness estimated from pit data of the benches near the domain.

The thicknesses of domains in pit 1 were estimated using two types of benches:

1. all the benches excavated in the previous year (the previous year's pit),
2. just one bench, which was considered for two cases:
   A. the bench immediately beside the proposed pit, bench 38. Since this bench straddles the pits for the two successive years, only the half that was outside the proposed pit was used.
   B. a bench more removed from the proposed pit, benches 36 and 39.

These benches and pits are shown on Figure 5.3. The average thickness along with the 'true' thickness determined from the measurements made in the pit are shown in Table 5.8.

In contrast to the drill holes, the estimates using nearby pit data are unbiased. The actual examples presented in Table 5.8, p.108 tend to overestimate the true thickness by about 4% but the 1σ scatter of the estimates (about ±5%) includes the 'true' value, thus the apparent bias is probably just a random fluctuation. Using data from a full year reduces the coefficient of variation by only 0.5% compared

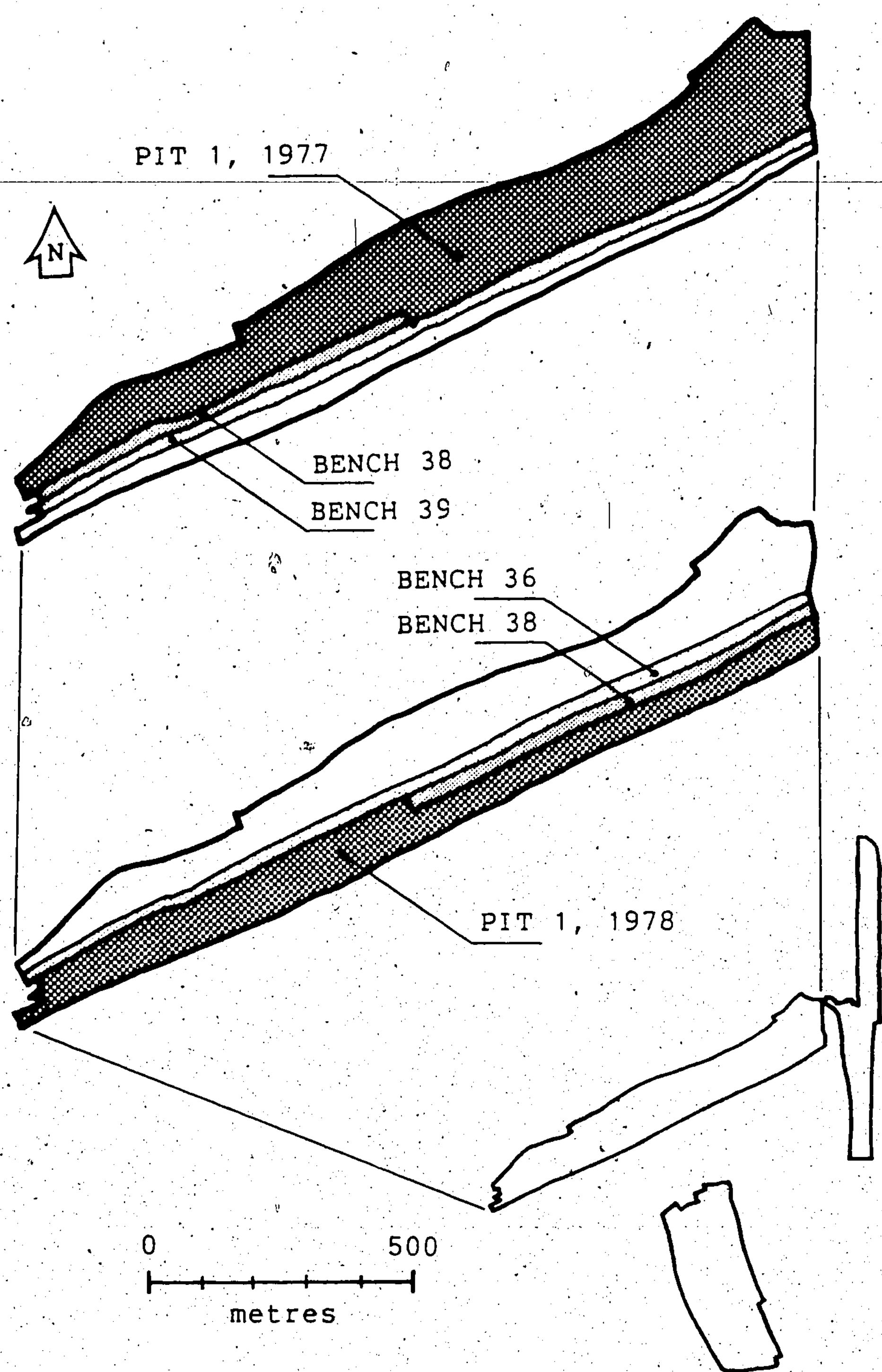


Figure 5.3 E ..ches used to estimate next year's in-place coal and production

Table 5.8 Comparison of the 'true' thickness with the estimated thickness for next year, calculated using nearby pit data

Domain	Estimate (m) Krige	Ext.	Coefficient of variation Krige	Ext.
Pit 1, 1977				
'True'†	3.50		0.62%	
Estimated by:				
1978 pit	3.66	3.61	1.6%	1.8%
Bench 38	3.81	3.94	2.0%	2.1%
Bench 39	3.37	3.35	2.2%	2.2%
Pit 1, 1978				
'True'†	3.51		0.44%	
Estimated by:				
1977 pit	3.67	3.59	1.0%	1.4%
Bench 38	3.75	3.94	2.0%	2.3%
Bench 36	3.57	3.58	1.7%	1.8%
Avg. coefficient of variation			1.7%	1.9%
Avg. error	3.8±4.4%	4.7±6.6%		

† From Table 5.2, p.91.

to the data from one bench. Also, it makes no difference to the coefficient of variation if the bench is beside (bench 38) or removed (benches 36 and 39) from the proposed pit. This insensitivity to the location of the data is because of the lack of a large drift and because the distance separating data from the pit to be estimated is generally between the long and short range structures of the variogram. This portion of the model has a low slope (Figure 3.4, p.60), so the $\bar{\gamma}$ terms and the resulting estimate will not change much with a change in sample separation. Finally, it makes little difference if the krige or extension estimate is used since the improvement using geostatistical methods is less than 1%.

The 1σ of the distribution of actual estimation errors is 5%, whereas the average coefficient of variation is 2%, for both krige and extension estimates. These two numbers should be the same; the difference suggests that the calculated estimation variances underestimate the 'true' estimation variance. However, the discrepancy may be a statistical fluctuation since it is close to the errors that are introduced by the calculation approximations. If the discrepancy is real, then there could be a local drift.

5.3.3 Area of the Domains

When considering the area and its variance for next year's tonnage, there are two possibilities. Firstly, the area can be assumed to be known perfectly. That is, given a domain, not necessarily for one year's production, what are the reserves underlying it? The domain could be an arbitrary block of land, a lease limit or some other predefined area. This domain would be used to estimate the in-place coal for next year. Alternatively, the area is what will actually be opened up next year, which is subject to considerable error. This domain would be used to estimate the projected production for next year.

For the first possibility, the area is perfectly known with no error, in particular VAR[A] = 0 and equation {5.3}, p.96 applies. For the second possibility, the variance is controlled by the efficiency of the excavation operation, hence very large. How to determine the variance in this second case is discussed in Appendix 10.

5.3.4 Calculation of the Tonnage

The tonnages are calculated in the same manner as the tonnages for this year's in-place coal. The tonnages calculated from the drill holes are listed in Table 5.9 and the tonnages from the pit measurements are in Table 5.10. In each table, the error of the tonnages is estimated for the two cases for the variance of the area:

1. VAR[A] = 0, when the area is predefined, for example, next year's in-place coal,
2. VAR[A] > 0, when the area is what will actually be

Table 5.9 Comparison of the 'true' tonnage with the estimated tonnage for next year, calculated using drill hole data

Domain	Estimate (tonnes x10⁵) Krige	Ext.	Coefficient of variation VAR[A] = 0 Krige	Ext.	VAR[A] > 0 Krige & Ext.
Pit 1, total					
'True'†	5.42		1.7%		31%
Drill hole	5.16	5.18	2.1%	2.8%	31%
Pit 2A, 1977 & 1978					
'True'†	5.80		1.5%		31%
Drill hole	5.54	5.49	3.2%	3.7%	31%
Pit 3, total					
'True'†	5.56		1.5%		31%
Drill hole	5.02	4.95	3.9%	5.0%	31%
Avg. coefficient of variation			3.0%	3.8%	31%
Avg. error	−6.3±2.9%	−6.8±3.5%			

† Calculated from Table 5.7, p.105.

Table 5.10 Comparison of the 'true' tonnage with the estimated tonnage for next year, calculated using nearby pit data

Domain	Estimate (tonnes x10^5)		Coefficient of variation		
			VAR[A] = 0		VAR[A] > 0
	Krige	Ext.	Krige	Ext.	Krige & Ext.
Pit 1, 1977					
'True'†	5.39		1.5%		31%
Estimated by:					
1978 pit	5.63	5.56	2.1%	2.3%	31%
Bench 38	5.86	6.06	2.4%	2.5%	31%
Bench 39	5.19	5.16	2.6%	2.6%	31%
Pit 1, 1978					
'True'†	5.40		1.5%		31%
Estimated by:					
1977 pit	5.65	5.52	1.7%	2.0%	31%
Bench 38	5.77	6.06	2.4%	2.7%	31%
Bench 36	5.49	5.51	2.2%	2.3%	31%
Avg. coefficient of variation			2.2%	2.4%	31%
Avg. error	3.8±4.4%	4.7±6.6%			

† Calculated from Table 5.8, p.108.

opened up next year, or next year's production. The 'true' tonnage is calculated from the 'true' thickness, which is kriged from the pit measurements inside the domain estimated. The area used is always the estimated area opened up in one year, 1.04×10^5 m^2 (see Appendix 10.3). Even though no pit has exactly this area, a constant area was used so that the tonnages could be compared easily, differing only in the thickness used.

The error of the projected production (VAR[A] > 0) is dominated by the error in the area that will be opened up during the next year. The proportion of the relative variance of the tonnage due to both the density and the thickness is so small (about 0.5%) that virtually any data and any calculation method is good enough for estimating.

The error of projected reserves (VAR[A] = 0) is similar to that for the in-place coal and recovery for this year. The density contributes about 20% to 40% of the relative variance, but slightly less if the extension estimate or the drill hole data is used, since both have a higher variance. Also, the krige coefficient of variation is less than 1% better than the extension coefficient of variation. Once again there is little improvement in the estimates using geostatistical methods.

6. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

6.1 SUMMARY OF RESULTS

6.1.1 Classical Statistics

In the Boundary Dam Mine area, seam net thickness measurements are fairly regularly distributed on square grids of 30 m for the pit data and and 100, 150 and 200 m for the drill holes.

Average net thickness of the seam in the pits is 3.6 m but thinner (3.3 m) when measured in the drill holes because more splits were logged than were selectively mined. The drill holes have about twice the coefficient of variation, probably due to more variable logging styles on the part of the drill geologists and more consistent reporting by the mine surveyors in the pits. Both kinds of data are approximately normally distributed with tails on the thin side, possibly due to paleochannels cutting into the coal. The tail on the drill hole distribution is large enough that a three-parameter lognormal distribution may fit better. The pit data could be separated into two populations (controlled by the partings), primarily on statistical evidence and secondarily on geology. On the other hand, the drill hole data was one homogeneous population.

6.1.2 Geostatistics

No drift was apparent in either set of data, except possibly at distances greater than the largest distance (about 2000 m) used in the kriging.

Transition variogram models with no drift were fit to the data. The best model of net thickness for both pit and drill hole data is the spherical model, usually as two nested structures. A hole effect model may be a better fit to data that includes the thickness of any rock, in particular, the partings and the gross thickness. There is also the possibility that a Gaussian model may fit the net thickness. Fortunately, kriging is insensitive to the model chosen so the spherical model gave good results.

No nugget effect was used, though it is possible to model the variograms with a small nugget effect. This change in the model has little effect on the estimate but a moderately large effect on the estimation variance.

The sill is approximately 0.07 m^2 for the pit measurements and about four times as high for the drill hole measurements. This higher variability is probably due to the random inclusion of thin rider seams and other differences in style of interpretation between geologists logging the holes.

Ranges from this study are of the order of several hundred metres. These, along with the ranges found by Flint (1978, p.107) form an approximate geometric progression: 50 m, 150 m, 1500 m, 4000 m (and 12 000 m?). The structures are probably due to depositional factors, though the 50 m range could be due to measurement methods.

Pit variograms show a small amount of zonal anisotropy perpendicular to the benches. The anisotropy ratio is about 1:2 for both the sills and the ranges, with the larger values lying in the direction perpendicular to the bench. This is probably due to the mine surveyors remembering the last measurements they took along the bench when confronted with a gradational contact. The drill hole variograms are isotropic, as is the variogram for the aggregate of all the pit data.

6.1.3 Drill Hole Spacing Strategy

Sampling strategy is determined only from the point of view of minimizing the estimation variance, without considering the necessity for outlining mining hazards. Sample spacing is a trade-off between cost and estimation error. Surveying the highwall is comparatively cheap, so the pit sample spacing was not analysed.

The error of any estimate will increase as the sample spacing increases. Figure 4.2, p.81 illustrates this relationship for the drill holes at the Boundary Dam Mine. If the extension estimate is used, essentially what is done at the mine, then the extension variance is only affected by drill hole spacings between 70 and 220 m. Beyond these limits the variance curve is flat with respect to spacing, which can therefore be maximized with no effect on the error of the estimate. On the other hand, the krige estimate has a lower variance and a steady trade off between spacing (or cost) and the error over the full range of practical sample spacing above 70 m.

At the drill hole spacing used at the mine (100, 150 and 200 m), the extension estimate has about three times the error of a kriged estimate using the same data spacing Conversely, for the same estimation variance, kriging requires samples four times further apart than the simple average of the data, the extension estimate. However, as will be seen in the next section, the percent errors are so small (1% to 5%) that there is not much practical difference between the two estimation methods.

6.1.4 Tonnage Calculations

Tonnages in this study were calculated two ways: by kriging and also by a simple arithmetic average of the thickness, similar to the method† used at the mine. As a result, the conclusions from this study should be applicable

† The mine engineers use a modification of the 'polygon of influence'.

to the regular method of calculation used by the mine engineers. Tonnages were calculated both ways for two cases: tonnage of coal that was in the pit, but is now recovered; and the tonnage of coal in a proposed pit.

6.1.4.1 In-place coal and recovery for this year

Conclusions for the reserves calculations are the same as those for the percent recovery, because the coal-loaded-out has negligible error. The recovery averages about 93% and ranges from 83% to 100%. Since the coefficient of variation for the recovery is about 2%, most of these are a statistically significant reduction from complete recovery.

Only a small improvement in the error can be made with the more sophisticated methods of geostatistics. The extension estimates (the arithmetic means) have only a slightly larger error (2.4%) than the krige estimates (1.9%). In all four areas, recoveries estimated both ways fall within 1 or 2σ of each other. Considering the extra effort of geostatistics, there is no advantage in kriging the estimate when a simple arithmetic average will suffice.

Density variation contributes 80% of the error in the tonnage or recovery when a kriged estimate is used and about half of the error when an extension estimate is used. The error of the average net thickness contributes the rest since the area is essentially error-free.

6.1.4.2 Reserves and production for next year

No large advantage is gained by using sophisticated calculations of average thickness and tonnage. The simple arithmetic mean of the thickness is adequate for calculating reserves. Kriging only reduces the estimation variance to 3% or less, from about 4% for the extension estimate. The reserves calculated using measurements from a nearby pit are an unbiased estimate of the 'true' tonnage to be expected in the projected pit. On the other hand, the drill holes consistently underestimate the the 'true' tonnnage by about 6% due to the drill hole net thickness being thinner on average than the pit measurements. The 'true' tonnage was determined by kriging highwall thicknesses from the pit. The largest error by far is introduced by the variable efficiency of the men and equipment, which can expose at one face somewhere between 7×10^4 and 13×10^4 m^2 of ground per year. All other sources of error in the projected tonnage are negligible by comparison. On the other hand, if the area to be mined is known without error, then the density contributes about a third of the relative variance with the rest coming from the thickness. Calculation errors, such as the numerical integration, are minute compared to these two.

6.2 CONCLUSIONS OF THIS STUDY

The following conclusions strictly apply only to the studied portions of the Boundary Dam Mine (pits active in 1977 and 1978). They can be applied to other portions of the mine if the properties of the data (particularly the net thickness variogram) do not change too much. Possibly the conclusions can be used at other plains coal mines with similar data properties, though this must be done with care and the results should be tested.

1. The error of the tonnage estimates increases when drill hole spacing increases from 70 to 220 m. Below 70 m spacing the error is negligible, while beyond 220 m and out to at least 2000 m the error is constant at about 10% and unaffected by the drill spacing. Between these limits hole spacing can be chosen from Figure 4.2, p.81 for the desired error. Other considerations, such as searching for potential mining problems, may determine the spacing actually used in practice.
2. Methods of determining reserve estimates could be altered slightly to reduce bias and also simplified without seriously increasing the error.
   A. Drill holes produce a reserve estimate that is slightly biased on the low side (by about 6%) and with a larger variance. The best data to use are highwall measurements from a pit or benches that are no more than a few hundred metres from the proposed pit.
   B. Geostatistical methods of estimating reserves of coal at the Boundary Dam Mine are only marginally better than the simplest method, which is arithmetic averaging of the data, essentially the method used

at the mine. The main advantage of the more sophisticated methods is a slightly smaller estimation variance, so it is doubtful if the large effort of geostatistical estimation is worth the minor improvement in the accuracy of the estimate. Geostatistics are only useful if the estimation variances are required, as they were in this study.

C. Errors in the tonnage and recovery estimates are equal and very low, about 4% to 6%. These are approximate 95% confidence limits, so that any change in the recovery of less than 4% has only a 1 in 20 chance of being real, rather than a random statistical fluctuation.

D. If the area of the proposed pit is given, and therefore error-free, density and thickness contribute about equally to the error of the tonnage, but the proportion of each can range from one quarter to three quarters. Calculation approximations are miniscule by comparison.

If the area of the proposed pit is unknown and must be estimated from the area that can be uncovered by the stripping operation in one year, then the error of this area is so large (about 30%) that it overwhelms all other sources of error. In this case, any data and any calculation method are adequate.

6.3 RECOMMENDATIONS

This section contains two sets of recommendations; beginning with further research that could be done in the area of this study, followed by suggested changes to the method of calculating reserves and recoveries at the Boundary Dam Mine.

6.3.1 Further Research

Research that could be done in the future falls into two areas. Firstly, a more thorough geostatistical analysis of this data or of the Boundary Dam Mine in general could be done, primarily to check some of the assumptions that had to be made. Secondly, more general geostatistical work could be done. This would explore some of the possibilities that were not pursued in this study.

6.3.1.1 Detailed research to extend this study

This study was detailed enough to determine the best method for calculating reserves and expected production; nevertheless, it was only a preliminary analysis of what would be a more complete geostatistical analysis. A much more involved study could be done to determine the validity of the assumptions and the sensitivity of the results, not for practical purposes but for the intellectual exercise. Suggestions for potential studies are listed below.

1. Regress the drill hole data against estimated thicknesses at the drill hole locations. The estimated

value should use the pit data in order to reduce the bias. This regression could depend on:

A. the location within the mine,

B. the number of splits,

C. the drilling program, either preliminary or infill. Controlling for the drilling program may control for the different drill geologists and their variable logging styles.

The drill holes, corrected by the regression, may produce a less biased estimate though the estimation variance may not be improved.

2. Study the thickness variogram in more detail.

   A. Take detailed measurements in the pit to eliminate the possibility of a nugget effect and to determine if the parabolic behaviour at the origin is real or not.

   B. Consider anisotropic models for each pit separately.

   C. Use other models, such as a Gaussian or a hole effect model.

   D. Check the intrinsic hypothesis by gathering and analysing data from more widely scattered portions of the mine. Does the variogram depend on location or the sedimentology of the floor, seam or roof?

3. Examine more closely the kriging and its effect on the results by considering the other methods to estimate the net thickness in Appendix 2.2.

   A. Use smaller domains with more local variogram models. For instance, krige the area underlain by a single split and combine several such areas. Next, compare this result to an overall estimate.

B. Use universal kriging, particularly split by split rather than on the total thickness as was used in this study. The individual splits tended to show more drift and a more individual variogram than the aggregate thickness.

4. Determine the density variogram and evaluate more precisely the effect of the density variance on the tonnage estimates.
5. Determine the variability of the coal-loaded-out and its effect on the variability of the recovery.
6. Determine explicitly the error in the area of a polygon and check that it really is small.

6.3.1.2 General geostatistical research

Below are listed some of the more general topics that could be done using this data, with or without geostatistics and with or without the data from Flint (1978).

Flint's kriging program could be expanded to handle other types of kriging:

1. universal kriging, in which case the drift could be compared to a trend surface analysis,
2. lognormal kriging,
3. punctual kriging,

4. random kriging,
5. blitz kriging,
6. changing data and/or model parameters interactively,
7. non-spherical models, in which case the modelling program would also have to be altered.

Universal and/or lognormal kriging could be used with the drill hole data, which exhibits a drift and may be lognormally distributed. If the kriging capability is expanded, then the variogram modelling program should be also.

The ranges of the variogram model from this study and Flint (1978, p.107) are in an approximate geometric progression, as are the ranges for the Lorraine iron deposits in France (Journel and Huijbregts, 1978, p.167). The estimates of the ranges could be improved with a merged data set. Then, if this progression is not just chance, it should be possible to predict the next larger and smaller ranges. There is already a suggestion of the next larger range in Flint (1978, p.108) at 12 000 m. These two predicted ranges could be checked, though gathering the data would be difficult in both cases. Estimating the shorter range would require detailed measurements in the pits, probably at several locations because the indeterminacy of the upper and/or lower contact may depend on the immediately adjacent facies. Estimating the larger range may call for unavailable or nonexistent data, necessitating the expansion of the variable under study to include other laterally equivalent coal zones.

Only one variable, the net thickness, was analysed geostatistically in this mine. Variograms could also be estimated for any of the variables in either the proximate, ultimate or maceral analysis. These could be compared to see if the model parameters, particularly the range, were similar and if any regularities had a geological control. More detailed data, such as proximate anlyses, could be cokriged† with more scattered data, such as ultimate analyses, to improve the estimate of the latter. In particular, the relationship between the density and the thickness could be determined.

Any of the above analyses could be done for several mines and any similarities compared across zones, formations or regions. In particular the variogram models could be compared, and if it turns out to be fairly constant, then it could be used for future exploration in mines where there was insufficient development to define a stable variogram.

The data set used for this study is large and quite detailed, ranging spatially from 30 to 2500 m. If it can be combined with Flint's data set, then the span is from 30 to 14 000 m, which appears to include several transition models

† Cokriging is the simultaneous kriging of two or more regionalized variables.

with ranges in a geometric progression. Such conditions can be modelled by one logarithmic scheme (Serra, 1968) and this could be explored. The data could possibly be fitted equally well with a generalized linear model (Figure 6.1) of the form:

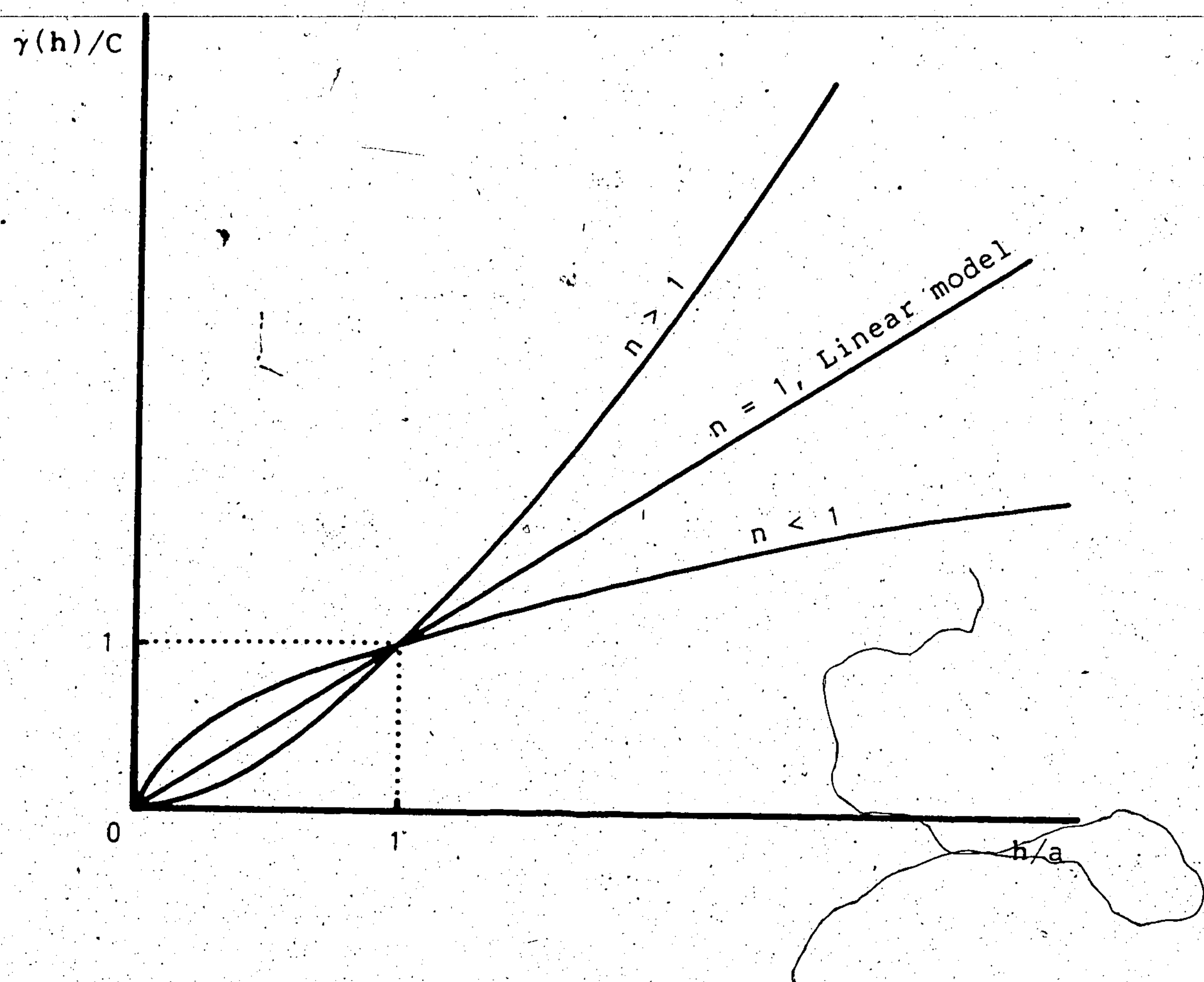
$$\gamma(h) = ah^n \; ; \; 0 < n < 2 \tag{6.1}$$

The generalized linear model with $n < 1$ is the theoretical variogram of an object with a fractal dimension, where the power (n) is the non-Euclidian dimension correction factor. A large span in order of magnitude is rare in most data sets, but is necessary for any fractional dimension analysis, which has not apparently been calculated for a coal seam or any stratum in general. The variogram and the non-euclidian dimension correction factor are both measuring approximately the same thing: the variability of the data. It would be interesting to compare the two approaches.

6.3.2 Suggestions For The Mine Engineers

This section contains suggestions to the operations personnel at the Boundary Dam Mine for possible changes in the way reserves are calculated. These suggestions may also be applicable to other plains coal mines, particularly if they have the same variogram.

1. Re-interpret the drill hole thicknesses to the thickness that will actually be mined, or determine a correction factor by regressing it with the pit data or coal-



$\gamma(h) = ah^n \; ; \; 0 < n < 2$

From Clark (1979*a*, Figure 1.6, p.9)

Figure 6.1 Generalized linear model

loaded-out.

2. When estimating reserves, use thicknesses measured from highwalls up to 500 m from the proposed pit. Those numbers are less biased than the drill holes
3. Include all the data inside the area to be estimated plus any data less than a fixed distance from the margins of the area. The distance is determined by the range of the variogram, about 500 m for the Boundary Dam Mine.
4. Average the net thickness over the whole seam. It is not necessary to break it into splits for separate calculations.
5. Use a simple arithmetic mean when calculating the average thickness of the seam over a given area. Even the best method, kriging, is only a marginal improvement over the simplest method, an arithmetic average, if the sample sites are fairly evenly distributed.
6. Do not bother doing calculations to a precision better than about 1% (other than guard digits), since this is the lowest error in the input data (net thickness, area and density) and some errors go up to 5%, particularly the area.
7. Refine the estimate of the density and its error, since it contributes about half of the error in most tonnage calculations and it should be fairly easy to lower the uncertainty.
8. For projected production, refine the estimate of average

man/machine productivity, or how much ground will be opened by the operation in a given amount of time. This contributes so much variability that if the error in this area cannot be greatly reduced, then any data and any calculation method is adequate.

9. Recovery is only accurate to about ±4% for 95% of the time; therefore, changes smaller than this are probably not real but random statistical fluctuations.

In general, the tonnage calculations can be simplified because the seam is so regular.

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2. David (1977) is the best description of the practice of geostatistics and a good introduction to the theory of regionalized variables.
3. Journel and Huijbregts (1978) has a good description of the practical aspects of geostatistics and the best readily accessible theory.
4. The fullest explanation of the theory is in the publications from Matheron's group at Fontainebleau, France. Unfortunately the English translations can lag the original French by up to 5 or 10 years.

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APPENDIX 1
OUTLINE OF THE THEORY OF STRUCTURAL ANALYSIS

The following is a brief introduction to elementary variogram theory, with the emphasis on those aspects required for this study. David (1977) gives a good introduction to the subject, while Journel and Huijbregts (1978) give a more complete description. For a definitive treatment one must read the papers of Matheron and his colleagues at Fontainebleau; for example Matheron (1972).

1.1 INTRODUCTION

The term 'geostatistics' is used in two senses: North American, or more general sense; and a French, or more specific sense. The North American usage of the word is analogous to the definitions of geophysics or geochemistry. It means the application of any method of statistics to a problem in geology. In contrast to this, the French usage of geostatistics refers to a particular method applied to a particular problem. The method is any application of Matheron's theory of regionalized variables. The problem is to determine the best estimate of the average of a regionalized variable over some area. 'Geostatistics' is used in this study in the French sense.

1.1.1 Regionalized Variables

A regionalized variable is one that is distributed over a region in space or time, which means it exists at every point in the region. In practical terms this means that all samples have a location and this location has an effect on the statistical properties of the variable. Usually the location is in 2 or 3 dimensional space, but if the location is in time, then signal theory is more commonly used. To be usable in geostatistics the variable must also be linearly additive, which means any linear combination of the variable must have the same meaning as the variable. Also most of the theory has only been worked out for continuous measures. Thickness and density are regionalized, area is not.

For the purposes of this study, classical statistics is defined as any non-geostatistical technique. Almost any statistical study must make assumptions; unfortunately, classical statistics makes one that is almost never true in geology. It assumes that the data used is independent; in other words, any one data value is not correlated† with any other data value. However, in geology the data at one location is usually correlated (sometimes very highly) with the

† 'Correlated' is generally used in its statistical, not geological, meaning for the remainder of this study.

data at nearby locations. Classical statistics can give an unbiased average, if the sample distribution in space is very regular, but it will not be the best, except by chance.

The best average is defined in terms of the BLUE, which is an acronym for *B*est *L*inear *U*nbiased *E*stimator.

Best means that the variance of the estimation is a minimum. This means that over the long term this estimate (Z^*) will come closer to the true value (Z) than any other estimate. The '*' indicates that the quantity is an estimate, usually of the thickness, while the symbol for the true value has no modifier.

Linear means that the estimator is a linear combination of the data. Mathematically the linear estimation is expressed as:

$$Z^* = \sum_i a_i z_i \qquad \{1.1\}$$

Where:
$\sum_i$ = the summation† for all i = 1 to n, n is the number of samples.
a_i = the i'th weight.
z_i = the i'th sample.
It is not necessary that the estimator be a linear combination; it can be any function of the data, such as:

$$Z^* = f[z_1, z_2, z_3, \cdots, z_n] \qquad \{1.2\}$$

Where:
$f[z_1, z_2, z_3, \cdots, z_n]$ = some arbitrary function of all sample values.
An arbitrary function will, in theory, give a better estimate with a lower variance. More often though, a simpler nonlinear formula is used:

$$Z^* = \sum_i f_i[z_i] \qquad \{1.3\}$$

Where:
$f_i[z_i]$ = some function of one sample value, the same function is used for each sample but with different parameters.
However, this method, nonlinear or disjunctive kriging, is difficult and the theory is not yet fully worked out. Linearity is a mathematical convenience for ease of computation and which also requires fewer assumptions about the statistical properties of the data.

An unbiased estimator is one that over the long term neither over- nor underestimates the true value. It is an

† Geostatistical convention is to use the abreviated form of the summation sign.

estimator because the objective is to statistically estimate an unknown average thickness.

1.1.2 Fundamental Measurements in Geostatistics

The two most important parameters of the distribution of a random variable are the first two moments, in classical statistics these are:

1. mean, μ,
2. variance, σ^2.

The corresponding parameters of a regionalized variable are:

1. drift, $m(\mathbf{h})$,
2. variogram, $\gamma(\mathbf{h})$.

In geostatistics these are not defined on the variable directly but on its increment, $Z(\mathbf{x}+\mathbf{h}) - Z(\mathbf{x})$. The drift is defined by:

$$m(\mathbf{h}) = E[Z(\mathbf{x}+\mathbf{h}) - Z(\mathbf{x})] \qquad \{1.4\}$$

Where:
$E[\mathcal{R}]$ = the expected value of $\mathcal{R}$.
$\mathcal{R}$ = a general variable.
$\mathbf{x}$ = the location expressed as a vector.† In particular: $\mathbf{x} = [u,v]$, u is the easting and v the northing.
$Z(\mathbf{x})$ = the value of the regionalized variable at location $\mathbf{x}$.
$Z(\mathbf{x}+\mathbf{h})$ = the value of the regionalized variable at location $\mathbf{x}+\mathbf{h}$, which is separated from location $\mathbf{x}$ by the vector $\mathbf{h}$.

The variogram, $\gamma(\mathbf{h})$, is defined by:

$$\gamma(\mathbf{h}) = \tfrac{1}{2}VAR[Z(\mathbf{x}+\mathbf{h}) - Z(\mathbf{x})] \qquad \{1.5\}$$

Where:
$VAR[\mathcal{R}]$ = the variance of $\mathcal{R}$.

The drift and variogram are functions of the vector between each sample pair. Usually they are presented as a graph of the statistic, plotted on the vertical axis, against the sample separation, along the horizontal axis, rather than the scalar representation used for the moments in classical statistics.

1.1.3 The Intrinsic Hypothesis

In order to model reality, particularly with mathematics, it is always necessary to make assumptions about the mathematical behaviour of the real world. The main assumption made in the geostatistics used in this study is the

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† The convention used in this study is to represent any variable that can be a vector in **bold face**.

intrinsic hypothesis. It assumes that the drift and the variogram remain constant over the study area.

If this hypothesis does not hold, then other more complicated methods in geostatistics must be used. In fact, the data in this study follow the slightly stricter assumption of weak, or second order stationarity, because the variogram has a sill and because $VAR[Z(\mathbf{x})]$ is defined. However, only the quasi-stationary assumption was tested since the data is only known to be stationary within a neighbourhood defined by $h < H$. H is the maximum distance used in the kriging equations, about 1000 m in this study. However, for simpicity the basic assumption used will be the intrinsic hypothesis.

1.2 DRIFT

The drift is shown as a graph of the average difference between all pairs of data separated by a vector plotted against the size of the vector. As suggested by David (1977, p.267) the drift is estimated using:

$$m(\mathbf{h}) \cong \{1/N(\mathbf{h})\}\sum_i[Z(\mathbf{x}_i+\mathbf{h}) - Z(\mathbf{x}_i)] \qquad \{1.6\}$$

Where:
$N(\mathbf{h})$ = the number of sample pairs separated by $\mathbf{h}$.
$Z(\mathbf{x}_i)$ = a sample at location $\mathbf{x}_i$.
$Z(\mathbf{x}_i+\mathbf{h})$ = a sample at location $\mathbf{x}_i+\mathbf{h}$, which is removed from sample $Z(\mathbf{x}_i)$ by the vector $\mathbf{h}$.

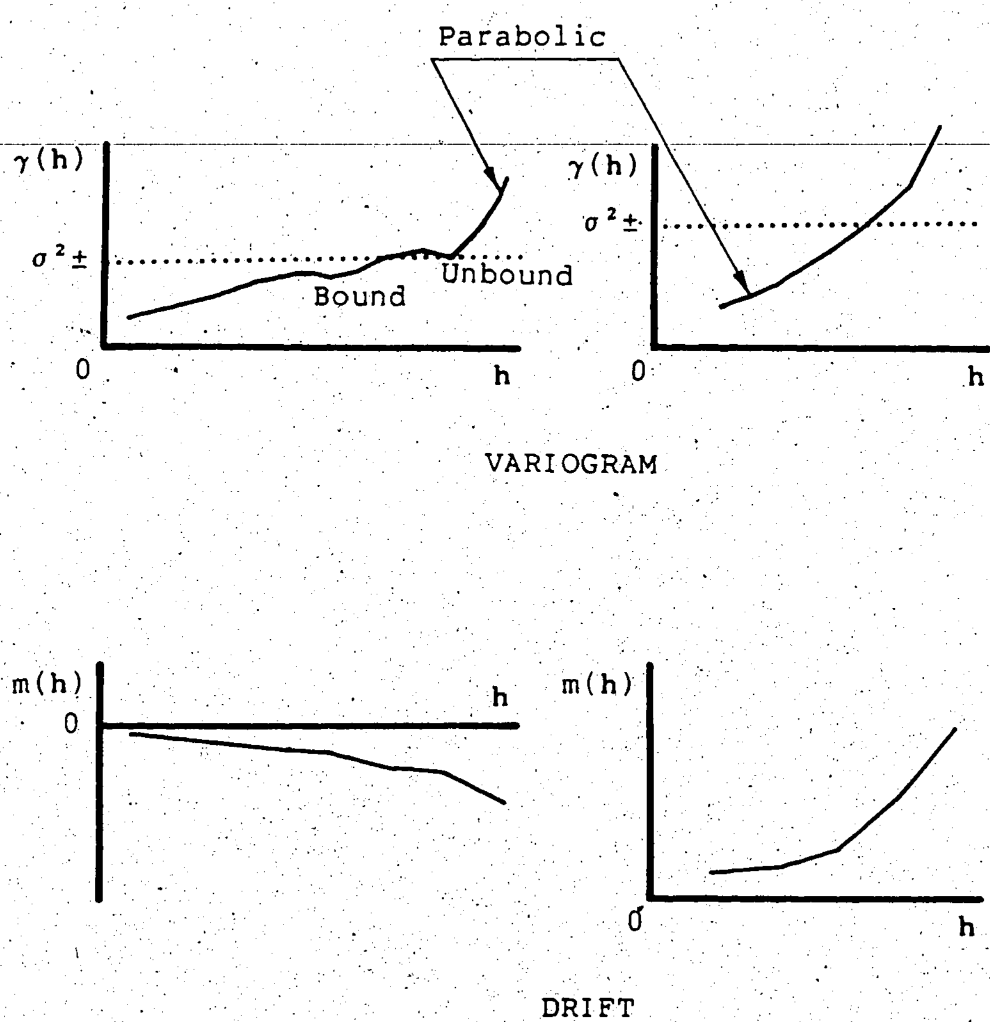
Note that, like all basic formulas in geostatistics, the formulas for the drift come in at least two varieties: a definition and a estimation formula. Equation {1.4}, p.136 defines the drift, but it must be translated into equation {1.6}, p.137 to actually calculate an estimate of the quantity. For the intrinsic hypothesis to hold the drift must be zero, or:

$$m(\mathbf{h}) = 0 \text{ ; for all } h \qquad \{1.7\}$$

According to Clark (1979*a*, p.23), the condition usually used in practice is:

$$m(\mathbf{h}) \simeq 0 \text{ ; for all } h < H \qquad \{1.8\}$$

If the intrinsic hypothesis does not hold, then ordinary kriging will give a biased estimate and universal kriging or the method of intrinsic random functions must be used to obtain an unbiased estimate. Examples that have a large drift are shown in Figure 1.1. The curve of the drift moves continously away from zero axis in either the positive or negative direction and with increasing slope. The variograms are unbound, which means they have no sill or steady horizontal portion for large h. The unbound portion is



Sulphur in a coal seam   Lead in a lead-zinc mine

Modified after David (1977, Figures 190-193, p.268)

Figure 1.1 **Examples of the drift and variogram from data with a large drift**

parabolic, with the slope increasing as h increases.

1.3 VARIOGRAM

'Variogram' is used in this study to refer to the semi-variogram, which is defined by equation {1.5}, p.136, whereas, the variogram is defined by the same formula, less the factor $\frac{1}{2}$. Most geostatistical authors use 'variogram', and a few even use both terms, when they mean 'semi-variogram'. Ambiguity is rarely a problem if care is taken. Assuming the variogram exists, a formula for estimating it is:

$$\gamma^{*}(\mathbf{h}) = \tfrac{1}{2}\{1/N(\mathbf{h})\}\Sigma[Z(\mathbf{x}_i+\mathbf{h}) - Z(\mathbf{x}_i)]^2 \qquad \{1.9\}$$

Where the terms are as defined for equation {1.6}, p.137 and γ^{*} estimates γ. By convention in geostatistics the '*' is usually dropped from γ^{*}, so γ refers both to the unavailable real variogram (γ) and to the estimate of it (γ^{*})

Variograms usually slope up to the right and a common behaviour is to level off after a certain distance (Figure 1.2) called the range (a), while the height at which it levels off is the sill (C).

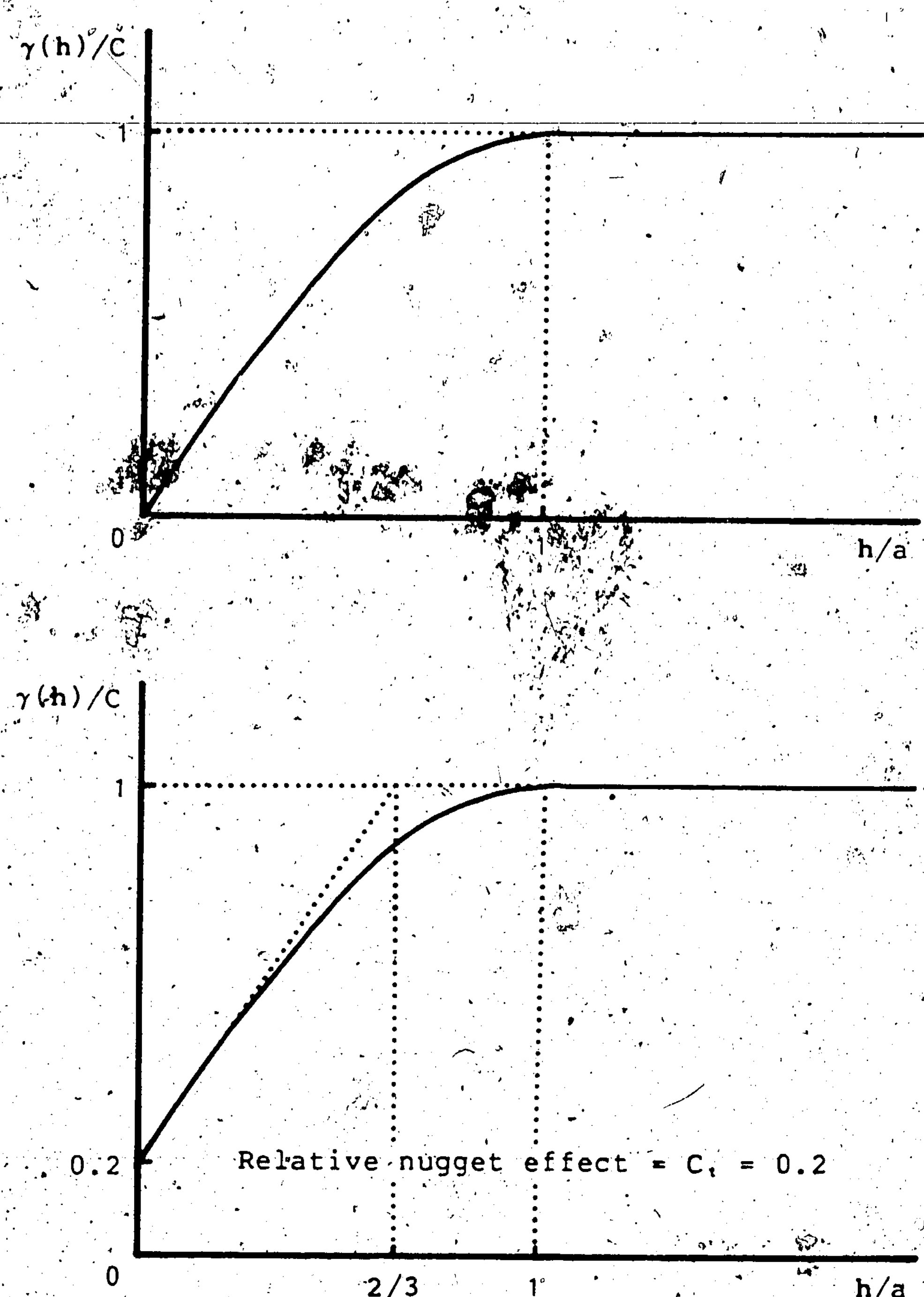
1.3.1 Conventions Used to Illustrate Variograms

Graphs of variograms in this study generally follow several conventions; one set for variograms of the coal thickness, another for theoretical variograms.

When the variogram is of data from this study, the units are always the same. The horizontal axis (the lag or **h**) is in metres, while the vertical axis (the variance of the nett† thickness) is in metres². Standardized scales for the axes would have facilitated easy visual comparison of variograms between figures, unfortunately, the pit variogram would have been too small, obscuring any detail. All calculated points are plotted, except those using fewer than 50 sample pairs. The plotted points are connected or the model is drawn out to L/2, beyond which there is no relationship between the ideal and the estimated variogram.‡ L is the maximum distance between sample pairs.

When the variogram is theoretical, such as an illustration of a model type, no data is used so the curve is drawn to the limits of the graph. Direct comparison of

† Two variograms also contain a gross thickness variogram.
‡ See Appendix 1.3.3 for a discussion of the variogram estimation errors.



Modified after Clark (1979*a*, Figure 1.3, p.7)

Figure 1.2 A transition variogram, spherical model different variograms is done by scaling the axes, the horizontal with the range (a) the vertical with the sill (C).

1.3.2 Anisotropic Variograms

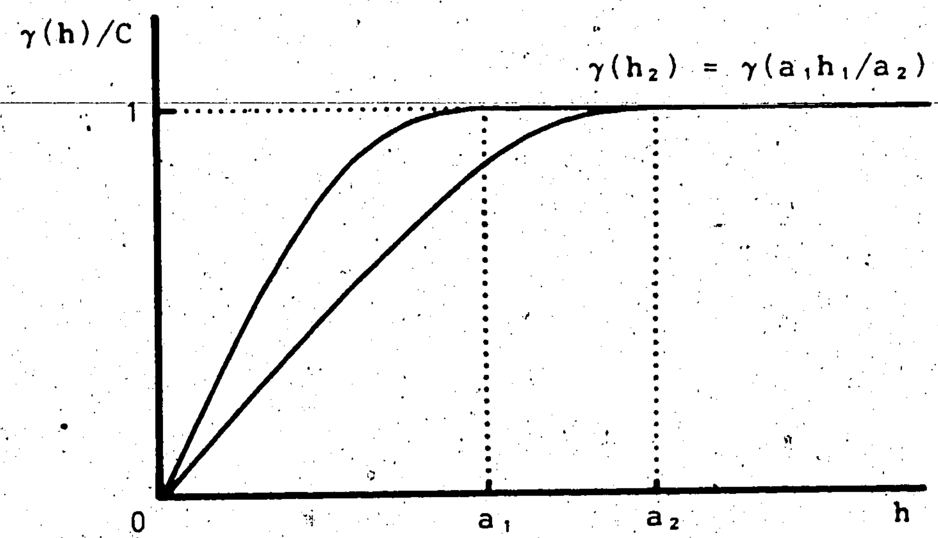
Anisotropy in a variogram is a substantial difference in the shape of the variogram depending on the direction of the vector **h**. In sedimentary variograms it is almost always present between the vertical and horizontal directions. This is because there is more variability vertically through the section than along it. It is also often present between directions parallel and perpendicular to the depositional strike, because of greater variability down the depositional dip. In variograms there can be two types of anisotropy (Figure 1.3): geometric anisotropy, sometimes called affine anisotropy; and zonal anisotropy, sometimes called stratified anisotropy. Zonal anisotropy is the general case, while geometric anisotropy is a particular case of it.

Geometric anisotropy is present when the variograms for different directions have the same sill but different ranges. A simple linear transformation or scale change of the coordinates of the data can correct for this, following which kriging can be done using an isotropic variogram. The linear transformation of the coordinates has the effect of stretching and/or shrinking the different ranges to the average range. Variograms parallel to the depositional strike often show a larger range than the perpendicular variograms.

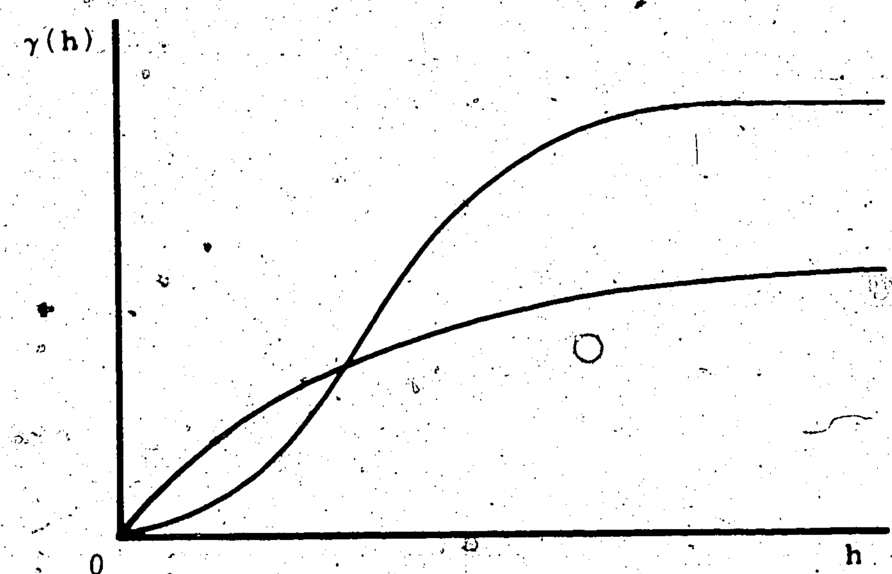
Zonal anisotropy is present when the variograms for the different directions have a different sill for bound models and usually also a different range or possibly a different model altogether. This cannot be corrected by a simple linear transformation of the data coordinates. Instead it is corrected by subdividing the data into groups that are each isotropic or geometrically anisotropic. Variograms parallel and perpendicular to the bedding often show zonal anisotropy.

1.3.3 Estimation Errors for the Variogram

The variogram as estimated by the data is rarely the variogram as required by the theory. Estimating a variogram produces a smoothed version, that must usually be corrected to remove the effects of the smoothing. This smoothing has sources: firstly, each sample almost always has a finite volume, resulting in a regularized variogram. Secondly, the data is grouped into distance and angle classes, analogous to, and with the same effect as the construction of a histogram.



GEOMETRIC (AFFINE) ANISOTROPY



ZONAL (STRATIFIED) ANISOTROPY

Modified after Olea (1975, Figures 2.2.2 and 2.2.3, p.22)

Figure 1.3 Types of variogram anisotropy

1.3.3.1 Regularization of the variogram

A regularized variogram results when the data has a finite support, rather than infinitesimal or point support. This means that each data value is the average of the variable over some small but finite volume or area, for instance the average grade of a piece of core. Support also includes the shape and orientation of the sample but this does not apply to thickness measurements, which are made at very close to an ideal point. This smoothing will tend to lower the sill and increase the range of the raw variogram with respect to the ideal point variogram (Figure 1.4). The effect of the regularization is measured by the ratio l/a, l is the size of the support, and a is the range. The larger l/a is, the more the smoothing, but unless $l/a > 0.1$ the smoothing will not usually be apparent.

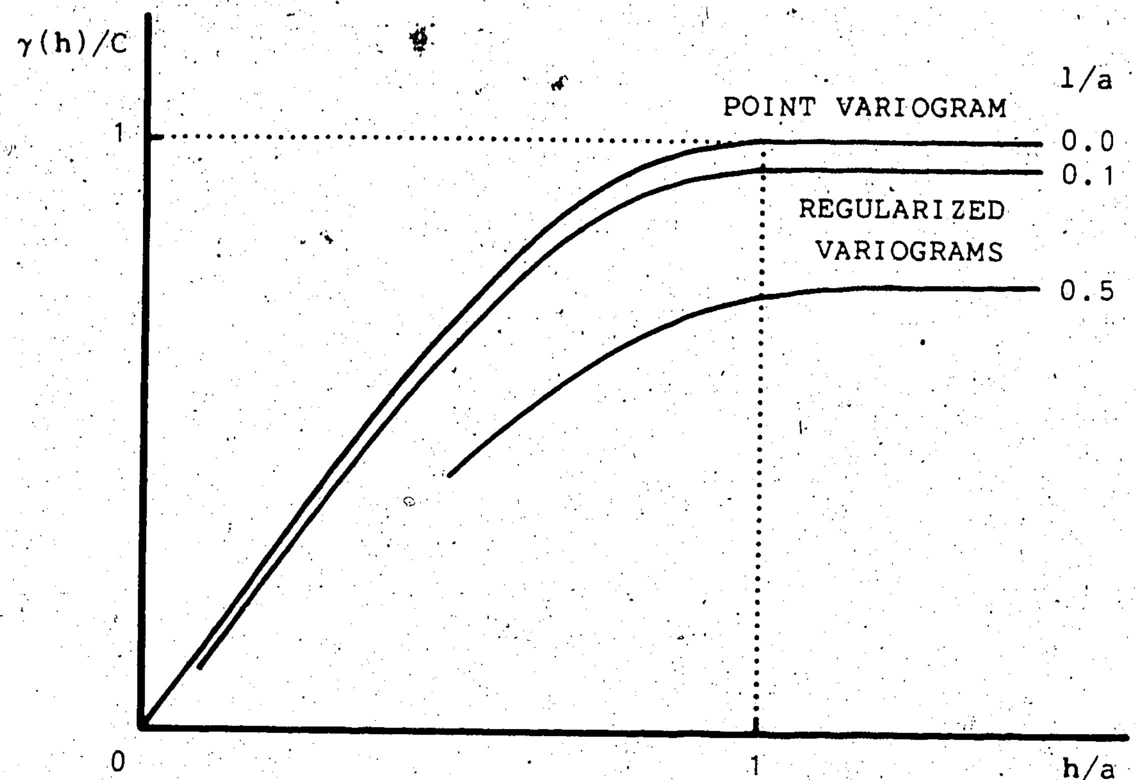
1.3.3.2 Grouping of the data

Data is often irregularly distributed on a plane, so it has to be grouped into classes in order to calculate a variogram. There are two ways that data must be grouped (Figure 1.5): lag (Δ**h**) determines the distance classes, and psi (ψ) determines the angular classes. Optionally, data can also be grouped into directional classes, determined by ϕ.

The lag (Δ**h**) is the linear regularization. It is the average separation between the points on the graph of a variogram. It is also the width of the distance over which **h** is averaged for one point on the experimental variogram. A larger value of the lag will smooth the variogram and detail will be lost, particularly near the origin. A smaller value will produce fewer data pairs for each point on the variogram, making it unstable and unreliable.

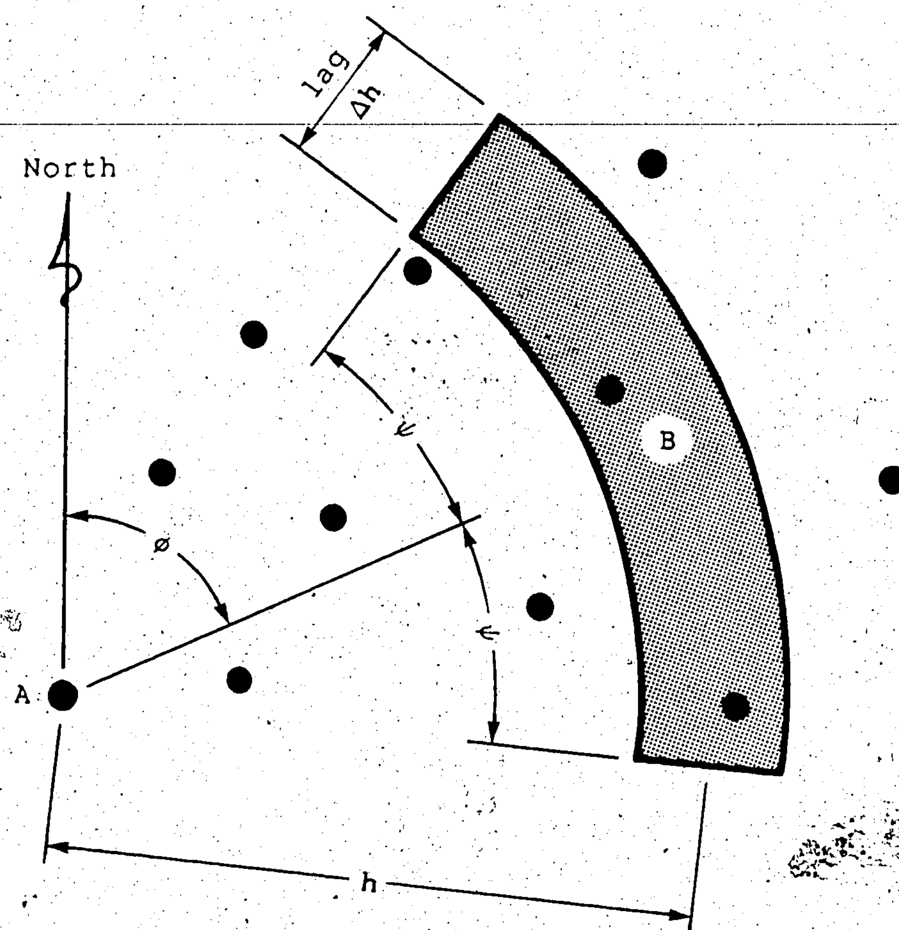
Psi (ψ) is the angular regularization. It is the half angular width of the directions of **h** within which data pairs are grouped to calculate a point on the variogram, and is the angular equivalent to the lag. If it is small, then few pairs can be found for the first points on the variogram, and these points on the variogram will not be stable. If it is large, then an excessive number of pairs will be found for the more distant points on the variogram, which will increase the computing costs. Large values will also tend to smooth any geometric anisotropies.

If $\psi = 90°$, then all pairs are used irrespective of the direction of **h**, and the average variogram will be estimated. This variogram is called the one dimensional variogram, even if the data is distributed in 2 or 3 dimensional space.



Modified after David (1977, Figure 100, p.131)

Figure 1.4 Effect of regularization on a spherical variogram



To calculate the variogram two points are needed.

The first point is at A.

The second point is any that falls inside box B.

Modified after Flint (1978, Figure 20, p.59)

Figure 1.5 Grouping of the data for calculating a variogram

The direction of the angular classes (ϕ) is used to search for any anisotropy. At least two directions should be chosen, perpendicular and parallel to the suspected anisotropy in the data. Good practice also suggests examining two more directions rotated 45° from the first two, in case the original ϕ's happen to bisect the anisotropic axes.

1.3.4 Behaviour of the Variogram Near the Origin

The variogram can have four different behaviours at the origin (Figure 1.6). In decreasing order of probability of occurance, these are:

1. linear behaviour,
2. nugget effect,
3. parabolic behaviour,
4. pure nugget effect.

Linear behaviour is the commonest property at the origin. The variogram is continuous, but not differentiable at the origin. This means the variogram intersects the origin with a non-zero slope:

$$\gamma(\mathbf{h}) = 0 \text{ ; as } h \rightarrow 0 \qquad \{1.10\}$$

$$d\gamma(\mathbf{h})/d\mathbf{h} > 0 \text{ ; as } h \rightarrow 0 \qquad \{1.11\}$$

Where:
$d\gamma(\mathbf{h})/d\mathbf{h}$ = the first derivative of $\gamma(\mathbf{h})$ with respect to $\mathbf{h}$. In this case it is the slope of the variogram in the direction of $\mathbf{h}$.

A nugget effect† is present when the variogram does not pass through the origin as on Figure 1.2, p.140. Theoretically, the variogram always passes through the origin, since the covariance between a sample and itself is zero. In practice, the experimental variogram often does not. This paradox is resolved by stating that the value of the variogram at zero is not the value of the variogram as it approaches zero:

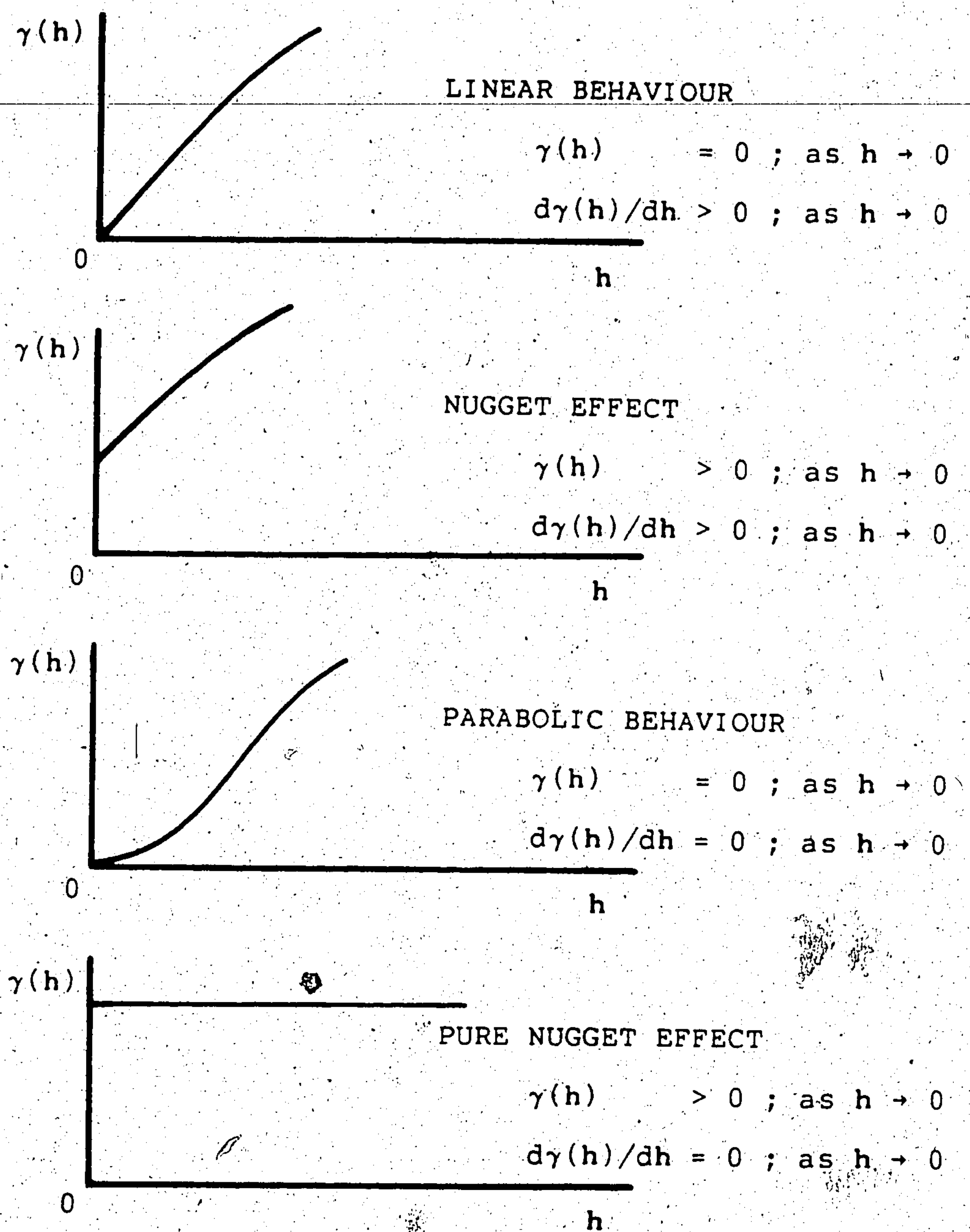
$$\gamma(\mathbf{h}) \neq \gamma(0) \text{ ; as } h \rightarrow 0 \qquad \{1.12\}$$

The value of the variogram as h → 0 is called the nugget effect, (C_0); it is defined by:

$$C_0 = \gamma(\mathbf{h}) \text{ ; as } h \rightarrow 0 \qquad \{1.13\}$$

The nugget effect is the random variance, uncorrelated from

† The term 'nugget effect' originated along with mining geostatistics in the goldfields of South Africa where nuggets of gold produce this effect on the variogram.



Modified after Matheron (1971, p.58)

Figure 1.6 **Types of variogram behaviour near the origin**

sample to sample, while the rest of the variance is correlated. When the nugget effect is present the slope of the variogram near the variance or Y axis is non-zero, as in the linear case in equation {1.11}, p.146.

The nugget effect adds a constant to the model:

$$\gamma(\mathbf{h}) = C_0 + \text{model} \qquad \{1.14\}$$

According to Rendu (1978, p.20), it can have two causes. The first is due to sampling or measurement errors, sometimes called the human nugget effect. The second is irregularities in the variable with a range shorter than the lag or the sample spacing. The relative nugget effect (C_r) is the ratio of the nugget effect to the sill:

$$C_r = C_0/C \qquad \{1.15\}$$

Where:
C = the overall sill, in particular: $C = C_0+C_1+C_2+ \cdots +C_n$
C_r can vary from 0, for no nugget effect to 1, for a pure nugget effect, which is discussed below.

Parabolic behaviour near the origin is rare in geostatistics since it is the property of a very regular regionalized variable. Sedimentary thicknesses and some meteorological data, according to Gandin (1965, pp.49 and 54), can often be this regular. The variogram is continuous at the origin (equation {1.10}, p.146), similar to the linear behaviour, but unlike the linear behaviour it has a zero slope at the origin:

$$d\gamma(\mathbf{h})/d\mathbf{h} = 0 \ ; \text{ as } \mathbf{h} \to 0 \qquad \{1.16\}$$

A pure nugget effect is the limiting model (with $C_r = 1$) for a transition variogram when the sample spacing is larger than the longest range. Of the several hundred deposits that have been analysed with geostatistics, only one showed a pure nugget effect. With this model classical statistics is as good as geostatistics since it gives an unbiased estimate with the same minimum estimation variance.

1.3.5 Confidence Intervals on the Fundamental Variogram

The experimental point variogram is not the true variogram that is required in the kriging calculations. The true variogram is defined using every point in an area of infinite extent. In contrast, the experimental variogram (γ^*) estimates the true variogram (γ) by using (1) a sample of all the potential data from (2) an area of limited extent. These two finite approximations produce the estimation error and fluctuation error respectively, which have been explicitly determined only for the case where the model is linear and the samples are normally distributed. In this study the samples are close to normally distributed, but the model is spherical rather than linear. The results, however, should give a rough indication for the present data.

The estimation error is due to a finite rather than an infinite number of samples being used to calculate the variogram. The error is usually small, particularly when the number of samples is large as in this case (n = 200 to 400); consequently, it can be safely neglected.

The fluctuation error is due to the variogram being calculated over a finite area rather than an infinite area. This error can be very large. In the linear/normal case outlined above, the error is small for small distances. However, when h becomes larger than about half the total distance over which the variogram has been calculated (L), then there is almost no relationship between the true and the experimental variogram (David, 1977, p.116)

Nothing can be done to improve the fluctuation error except to ignore all points with $h > L/2$ when modelling the variogram. More conservative geostatisticians use only the data with $h < L/4$, for instance Clark (1979*b*, p.92).

1.3.6 Variogram Models

The kriging system of equations needs the value of the variogram calculated between many pairs of points. A mathematical expression of the variogram is needed to do this, rather than the usually ragged set of points from the experimentally determined variogram. Modelling is the art of fitting a mathematically defined curve to the data, much as a histogram is fitted by a probability density function. Unfortunately, there are no automatic fitting methods. Davis and Borgman (1979) give a test for the goodness of the fit of a one dimensional variogram, but it is worked out in detail for a test of only one point at a time. More useful cases were not described. Another method is cross validation, which is choosing the model that produces the lowest estimation variance. Iterative visual fitting, the commonest method, was used in this study.

Not all functions may be used for a variogram model: they must be conditionally positive definite, which has two consequences. Firstly, any estimation variance is greater than or equal to zero, and secondly, the variogram must have less than a parabolic increase for large h, or:

$$\lim[\gamma(h)/h^2] = 0 \text{ ; as } h \to \infty \qquad \{1.17\}$$

All the models discussed in this study are conditionally positive definite.

Variogram models fall into two broad classes: transition models, and non-transition models. Transition models are bound variograms and therefore have a sill, while non-transition models are unbound and lack a sill. Preliminary inspection of variograms from the Boundary Dam Mine show that they are of the transition type so non-transition models will not be discussed further.

Transition models rise to a value, then stay steady at this value (Figure 1.2, p.140). The distance from the origin to the steady value is the range (a). The height at which it remains steady is the sill (C), which is theoretically equal to the population variance (σ^2) and usually taken to be equal to the sample variance (s^2). In other words:

$$\gamma(\infty) \cong s^2 \qquad \{1.18\}$$

If the sill and variance are not equal this can be a result of two causes: either the presence of drift in the data, or a hole effect in the variogram.

If there is a drift, then the variogram will continuously rise above the sill, approximately parabolically (Figure 1.1, p.138).

A hole effect can be produced when rich/thick zones alternate with poor/thin zones. If the alternation is approximately regular, then this periodicity will show as oscillations in the experimental variogram above and below the population variance (Figure 1.7).

There are three types of commonly-used transition model (Figure 1.8):
1. spherical model,
2. exponential model,
3. gaussian model.

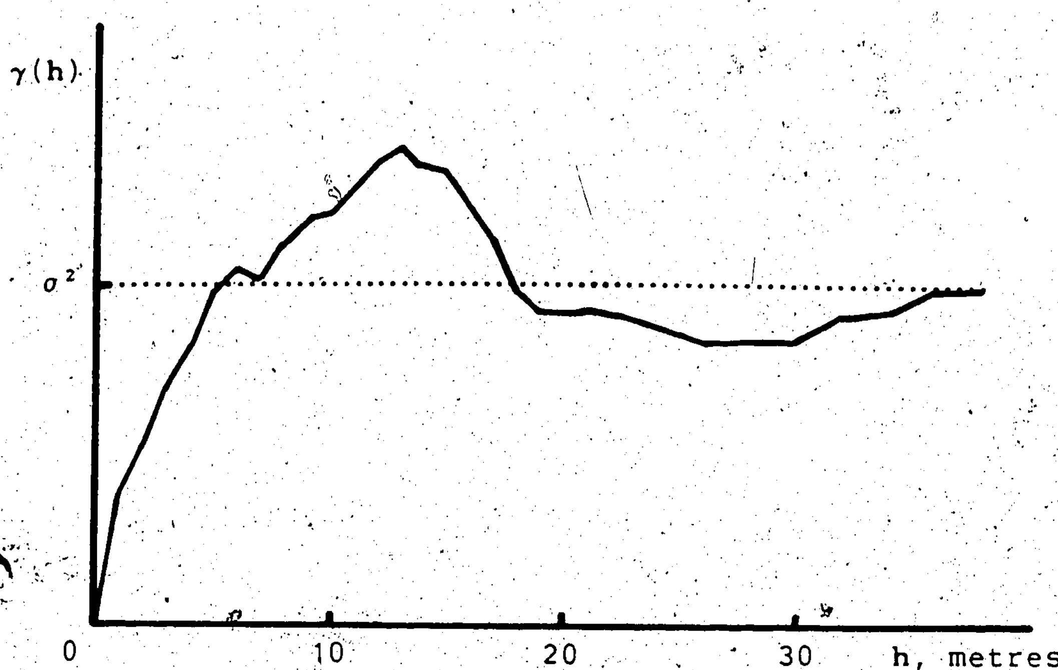
The spherical model† is by far the most commonly used model in geostatistics. Its formula is:

$$\gamma(h) = C\{3h/2a - h^3/2a^3\} \;;\ \text{for } h \le a$$

$$= C \;;\ \text{for } h \ge a \qquad \{1.19\}$$

Where:
C = the sill.
h = the sample separation.

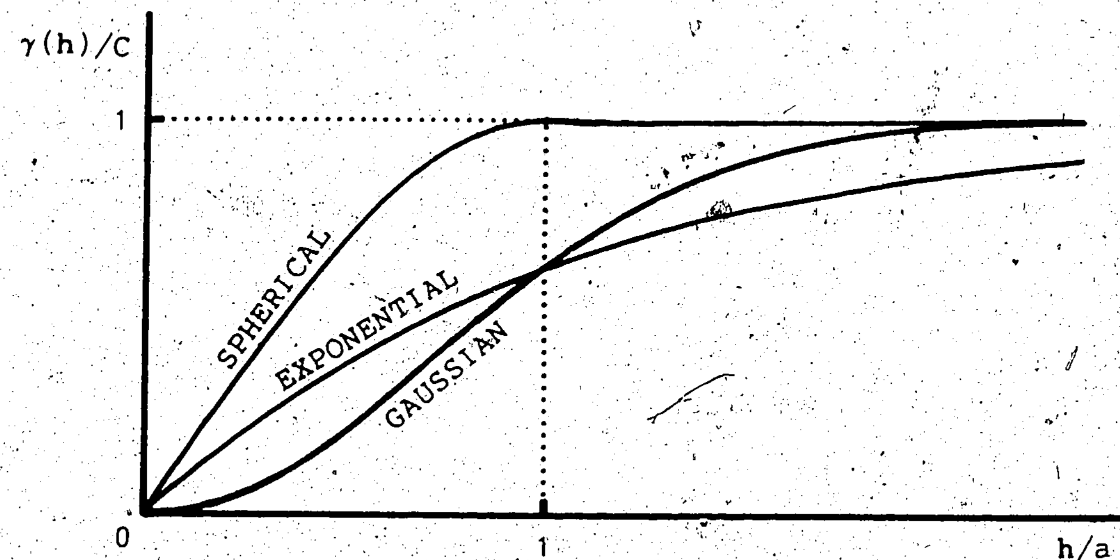
† The spherical model is occasionally called the Matheron model in honour of Georges Matheron, the father of theoretical geostatistics.



The hole effect is due to alternating rich and poor layers in a uranium mine.

Modified after David (1977, Figure 89, p.111)

Figure 1.7 An example of a variogram with a hole effect



SPHERICAL: $\gamma(h) = C\{3h/2a - h^3/2a^3\}$; for $h \le a$

$= C$; for $h \ge a$

EXPONENTIAL: $\gamma(h) = C\{1 - \exp[-h/a]\}$

GAUSSIAN: $\gamma(h) = C\{1 - \exp[-h^2/a^2]\}$

Modified after Rendu (1978, Figure 4.1, p. 18)

Figure 1.8 **Common types of transition model: spherical, exponential, gaussian**

a = the range.
Its ubiquity is a result of several reasons. Traditionally, everybody has used it, so people continue doing so. Considerable theoretical work has been done on it, and as a result many of the problems have been solved. It usually seems to fit the data better than the other two. The exponential model rises too slowly to the sill and the gaussian model is usually too flat or parabolic near the origin. David (1977, p.106) explains the origin of the term 'spherical'. "A process having a spherical variogram can be generated as follows: first take a random (Poisson) distribution of masses in space. Then assign to each point of space a grade given by the number of masses within a radius *a* of the point (in other words take the number of masses within a 'sphere')."

The exponential model is more commonly used in biometrics. In this science, data tend to be more regular, which causes the variogram to rise more slowly to the sill. The formula is:

$$\gamma(\mathbf{h}) = C\{1 - \exp[-h/a]\} \qquad \{1.20\}$$

The gaussian model is rarely used in geostatistics except where the parabolic behaviour at the origin is important. The formula is:

$$\gamma(\mathbf{h}) = C\{1 - \exp[-h^2/a^2]\} \qquad \{1.21\}$$

Occasionally it has been fit to the variogram of sedimentary thicknesses (Clark 1979*b*, p.94), since they are usually highly continuous and often parabolic near the origin. It has also been used in meteorology for sea level pressure (Gandin, 1965, p.54).

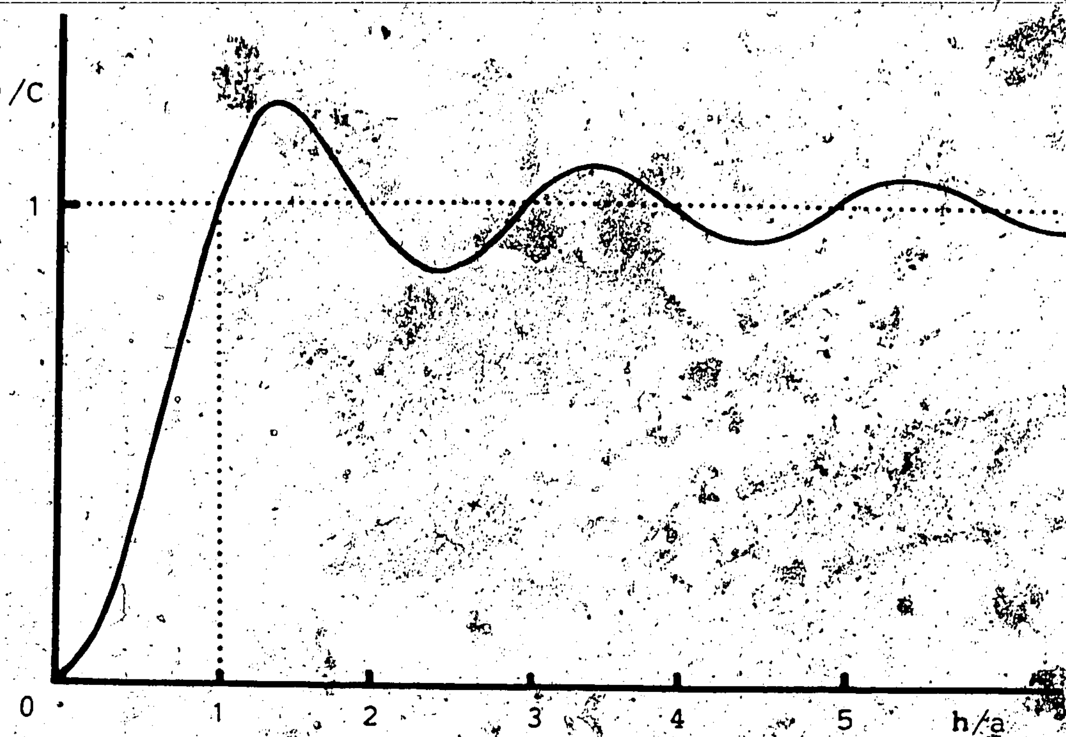
Hole effect models are another class of model, either transition or non-transition. They are usually trigonometric functions with a decaying amplitude. A common and simple transition model is the sine model (Figure 1.9) which is an inverted sinc function:

$$\gamma(\mathbf{h}) = C\{1 - \sin[\pi ah]/\pi ah\} \qquad \{1.22\}$$

Other hole effect functions are described by David (1977, p.111) and by Journel and Huijbregts (1978, pp.252 and 169).

A nested model is a linear combination of several simple models with different parameters (Figure 1.10): usually two models, very rarely three, almost never four. It has the formula:

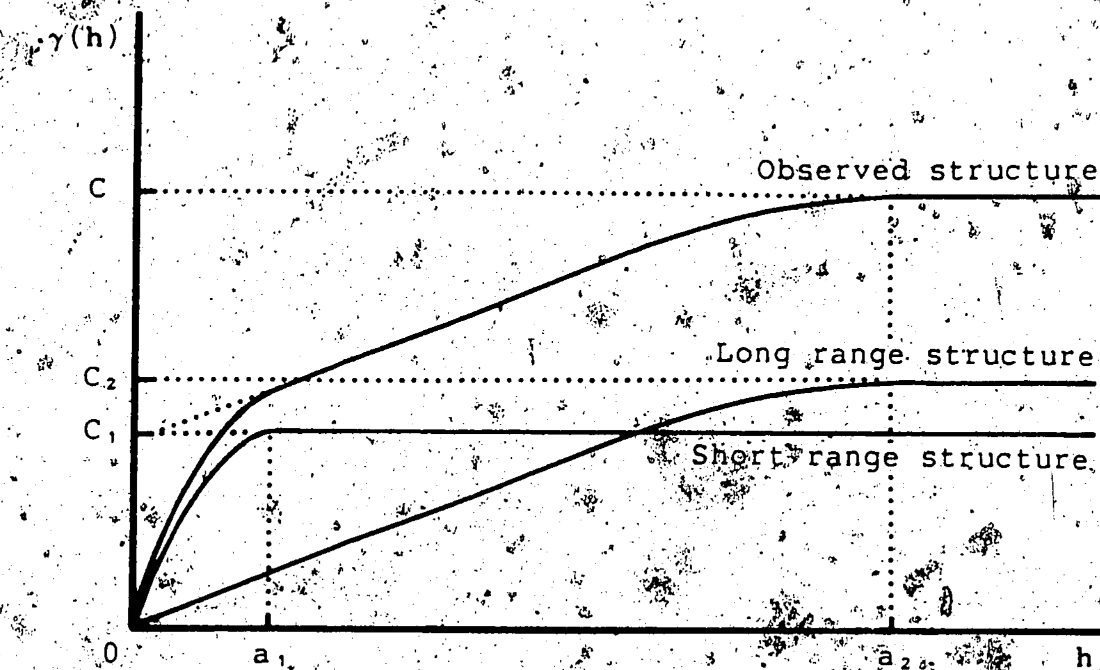
$$\gamma(\mathbf{h}) = \gamma_1(\mathbf{h}) + \gamma_2(\mathbf{h}) + \gamma_3(\mathbf{h}) + \cdots + \gamma_n(\mathbf{h}) \qquad \{1.23\}$$



$$\gamma(h) = C\{1 - \sin[\pi a h]/\pi a h\}$$

Modified after David (1977, Figure 88, p. 110)

Figure 1.9 The sine model, an example of a transition hole effect model



Observed structure =

long range structure + short range structure

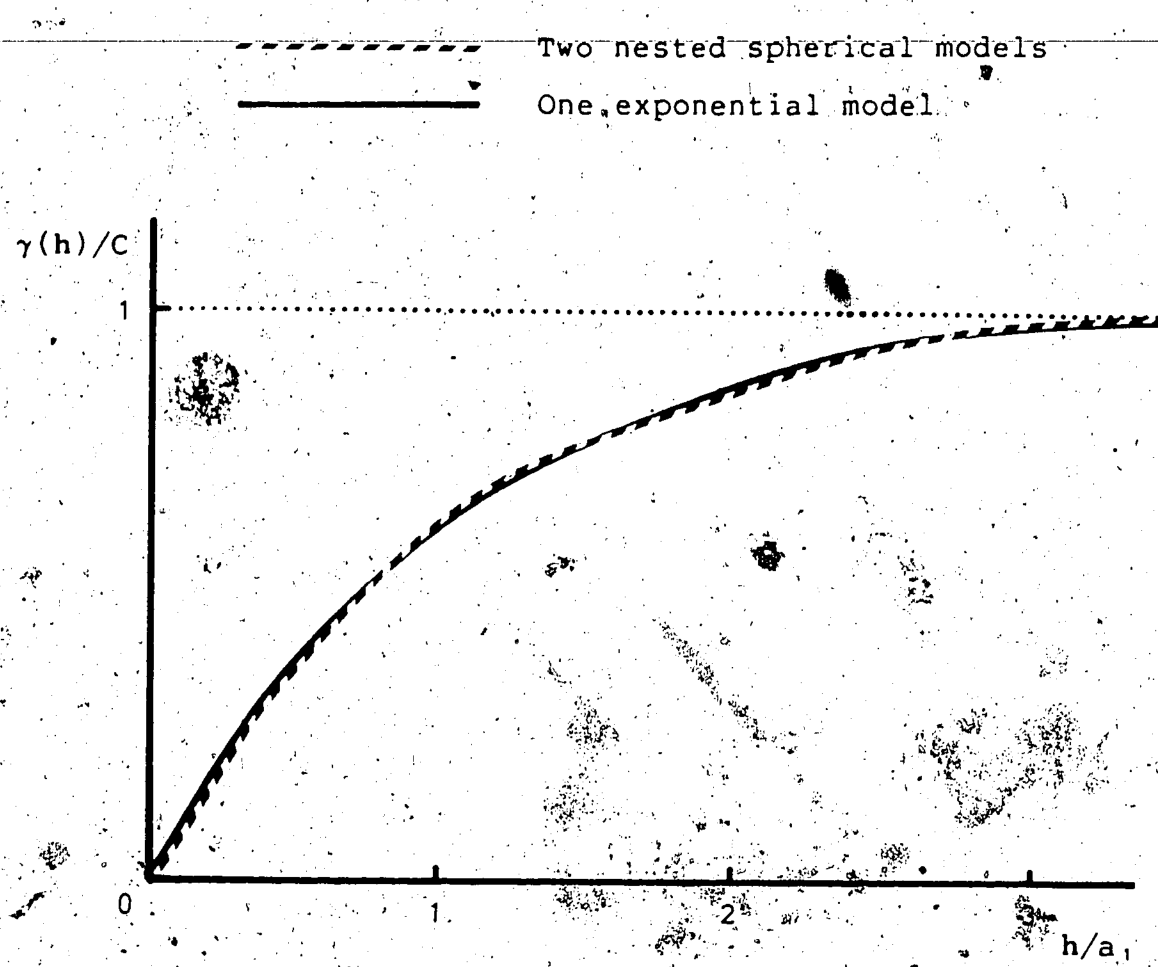
Modified after Rendu (1978, Figure 4.6, p.20)

Figure 1.10 Two nested models

Where:
$\gamma_i(h)$ = any valid variogram model.
Any linear combination of conditionally positive definite functions is also a conditionally positive definite function, so it also is a valid variogram model. The sills of each model are the combination weights.

1.3.7 Robustness of the Model

Geostatistical results are usually very robust with respect to the variogram model, both type and parameters (Journel and Huijbregts, 1978, p.167). Most models have similar graphs even though they may have different functions. For instance Journel and Huijbregts (1978, p.246) show some data that is fitted equally well with a logarithmic model, which has no sill, and two nested spherical models. They also show on page 234 how close an exponential and two nested spherical models can be (Figure 1.11), much closer than the scatter in most data.



From Journel and Huijbregts (1978, Figure III.30, p.234)

Figure 1.11 Comparison of an exponential and two nested spherical models

APPENDIX 2
PRELIMINARY DATA ANALYSIS

2.1 CLEANING THE DATA

Once the data was in machine readable form mistakes were checked for and corrected. Each data point, pit and drill hole, has three main variables: ID, location and thickness, all of which were checked. ID's not in sequential order after sorting indicated missing data which was redigitized.

2.1.1 Location of the Samples

Incorrect locations are due to either mistakes or errors. Mistakes, points digitized at a wrong location and missing or duplicate points, were corrected in two ways. Firstly, the posted data was overlaid on the original blue-line mine plan then missing or incorrectly located data redigitized. Secondly, duplicate points were found by visually scanning a sorted list of coordinates then eliminated.

Errors in location have several sources:

1. surveying errors,
2. slight scale changes introduced in the blue-line and xerox copying, or paper stretch,
3. digitizing errors, which are of three kinds:
   A. incorrect registration or location of the map on the table,
   B. not placing the mouse directly over the point,
   C. the tablet not recording the precise location of the mouse.

Nothing can be done to check the surveying errors; however, they are likely to be at least an order of magnitude less than the ±1 m error due to digitizing and so can be ignored. Scale change errors are automatically compensated for when registering the maps before digitizing.

The digitizing errors could not be totally ignored or eliminated; however, they could be measured. Several points, whose true locations were known, were digitized 100 times, to give an average $\pm 1\sigma$ error of approximately 1 m. This is small enough to be ignored for the variogram and kriging but not for the error of the area, which is examined in more detail on page 92.

2.1.2 Thicknesses of the Seams and Partings

Thicknesses, which may have been recorded incorrectly by the mine personnel, were checked by searching for extreme values. Only one fell beyond 3σ from the mean, but it was retained since it was in an area of channelling and rapid thickness variations.

Thicknesses that were incorrectly keyed into the computer were checked for by hand. Six readings in pit 2A were found to have a peculiar stratigraphy, where a thickness was recorded for the bottom coal and the parting but not for the top coal. These were discarded since it was not possible to determine the net coal thickness.

2.2 METHOD TO ESTIMATE NET THICKNESS

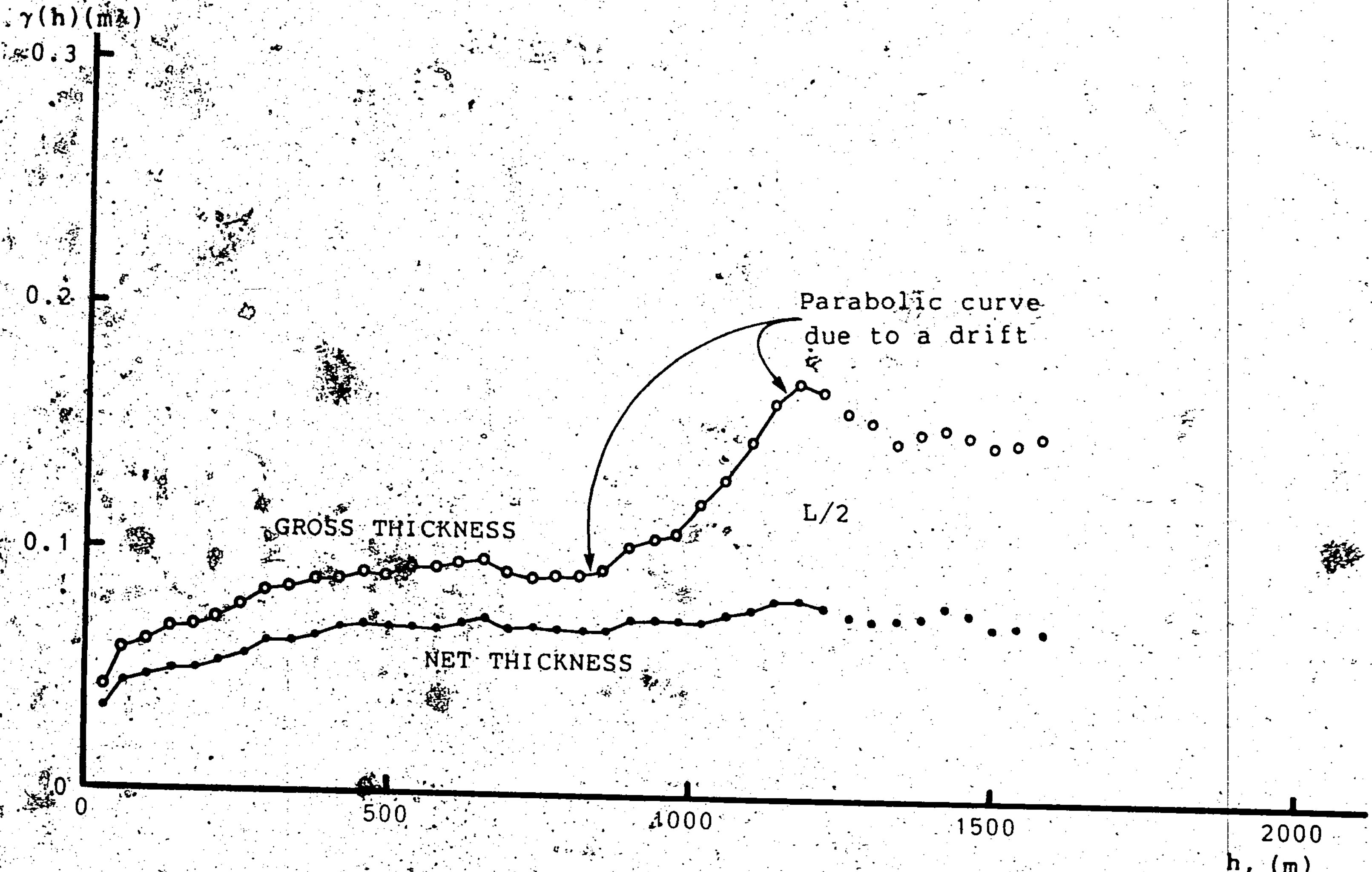
To calculate volume and tonnage it is necessary to estimate the average net thickness of the seam over a given area or domain.

For the pit data this can be done in three ways:
1. estimate average net thickness directly,
2. subtract the average parting thickness over the whole area from the average gross thickness,
3. add the average thicknesses of the bottom and top coal. Then calculate a weighted average of this and the average thickness where there is no parting.

Method 1 was chosen for the following reasons.
1. Calculating directly is simpler because less work is involved and it is less costly ~~as the~~ kriging program is expensive to run.
2. The net thickness variograms are simpler and better behaved than the gross thickness variograms, which have a strong drift (Figure 2.1 and Figure 2.2†). As a result the net variograms can be modelled more easily and more accurately, which will tend to produce a better estimate.
   A. Method 2 requires use of the parting variogram (Figure 2.3), which should be modelled by at least one and possibly two nested hole models.
   B. Method 3 requires the variograms for the bottom and top coal and for the seam with no parting (Figure 2.4), all of which are more ragged than the net thickness variogram (Figure 2.1, p.160 and Figure 2.2, p.161) and would probably require hole effect models.

For the drill hole data, method 1 or 2 above could be used, while method 3 would be impossible because the study area cannot be subdivided into simple contiguous regions of consistent stratigraphy (Figure 2.8, p.50). Method 2 would be very poor practice since the parting variogram would

† See page 139 for a description conventions used to illustrate variograms in this study.



re 2.1 Comparison of the gross and net thickness variograms for the pit data

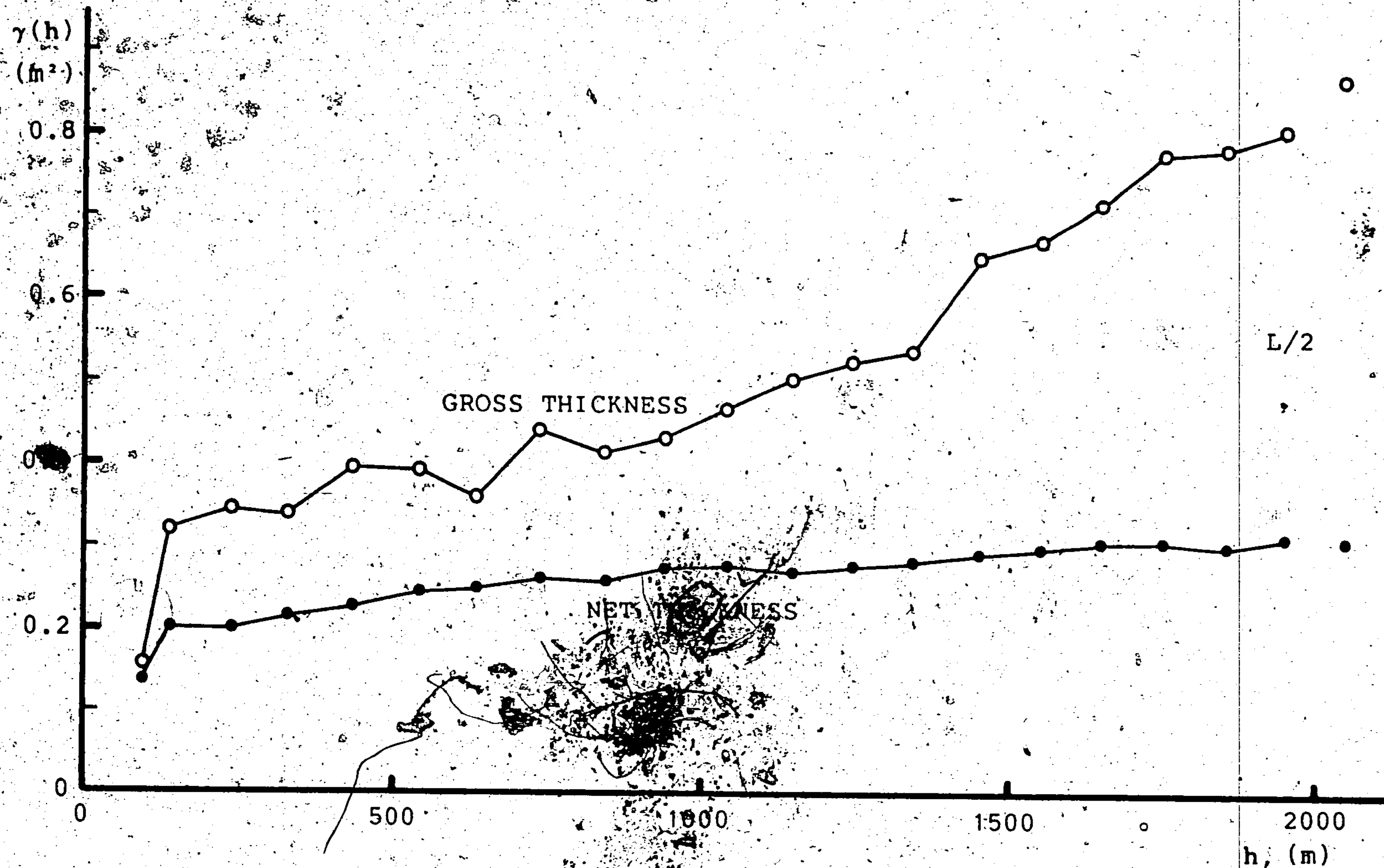


Figure 2.2 Comparison of the gross and net thickness variograms for the drill hole data

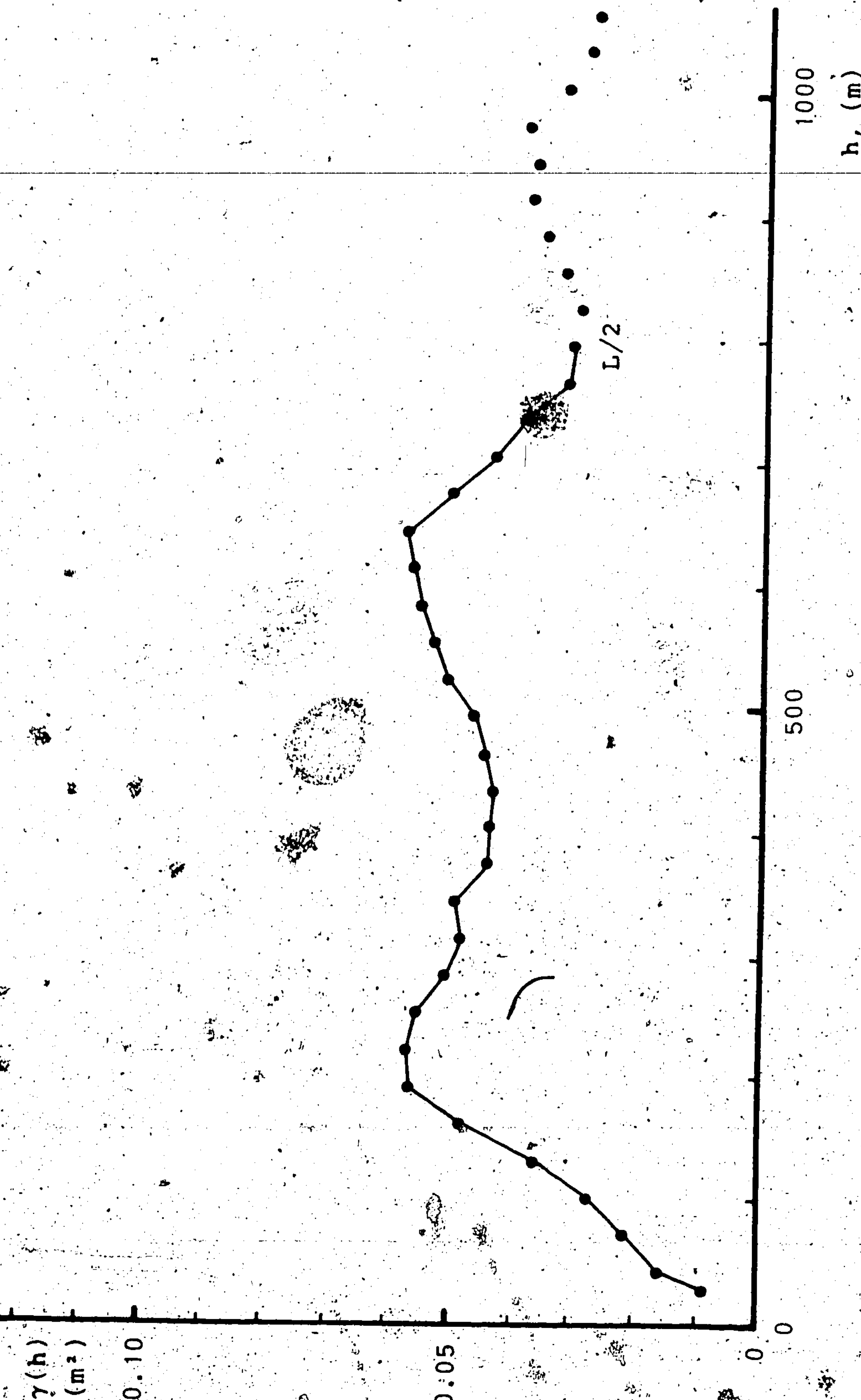


Figure 2.3 Variogram of the E•F parting

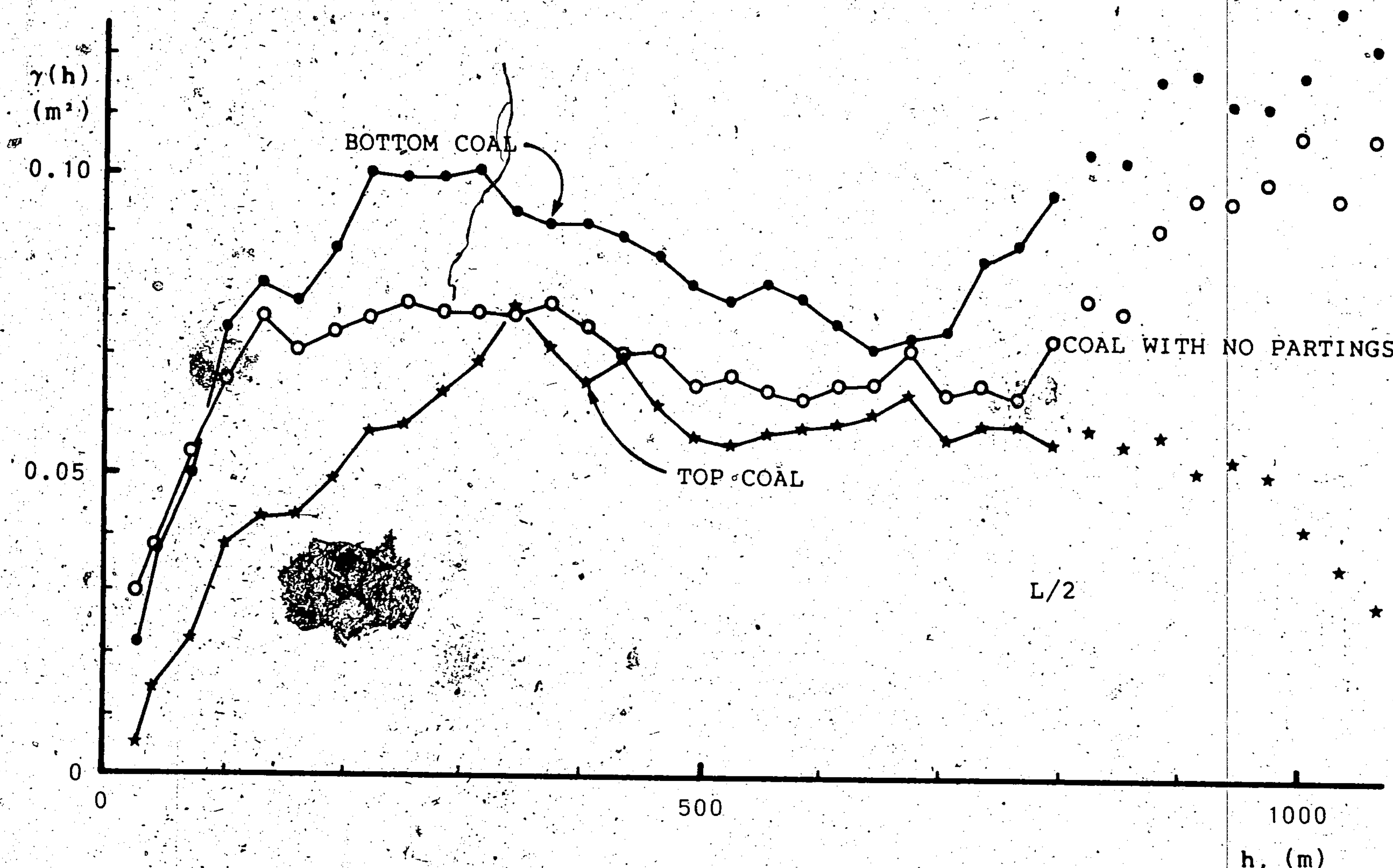


Figure 2.4 - Variograms for the north area, subdivided stratigraphically

reflect several partings, not a single phenomenon. The variogram would also have a larger sill, which increases the variance of the net thickness estimate. For these reasons plus the time and expense involved the drill hole net thickness was also estimated by method 1.

APPENDIX 3
SELECTING HOMOGENOUS REGIONS FOR ANALYSIS

Geostatistical analysis must be done on regions that are homogeneous both geologically and statistically. Although the geological evidence should have primacy over the statistical, in this study there are contradictions which must be resolved.

3.1 SELECTION CRITERIA

Homogeneous regions were selected using three criteria: the geology must be the same, the data must be from one population and the intrinsic hypothesis† must hold over the region.

The geology must be uniform for the results to be meaningful. For instance, it is not very useful to estimate the average thickness of a zone that includes both coal and a sand channel.

The region must also be from one statistical population if any statistical inferences are to be made. For instance it is difficult to make any inferences about a region that both includes and excludes a rider seam.

Finally, the geostatistical analysis assumes that over the region of interest the intrinsic hypothesis is true. This hypothesis assumes that there is no drift in the data and that the variogram is constant over the region under study.

The pit and drill hole data are not homogeneous, so were considered separately for the following reasons.

1. Both sets of data have different sources: the highwall and core or electric logs.
2. Both have different geology. Even though the measurements were made on the same zone, they were measuring two different things: the mined zone for the pit data, and the geological zone for the drill holes.
3. Both sets of data are from significantly‡ different populations (Table 3.1).
4. The variograms are different (Figure 3.1).

For each set of data, pit and drill hole, the regions of interest were selected using these steps:

1. choose a region of uniform geology;

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† See page 136 for a description of the intrinsic hypothesis.

‡ Generally whenever 'significant' is used in this study a significance test was done, in which case the confidence level will be reported in a footnote thus: $\alpha = 0.05$

Table 3.1 **Comparison of the summary statistics of the thickness for the pits and the drill holes**

Data	n	Mean thickness (m)	Variance (m^2)	t†	F
All pits	631	3.64	0.073		
				13.9	3.81
Drill holes	336	3.29	0.278		
Critical values ($\alpha = 0.05$)				1.96	1.3± ‡

† The t test for equal means assumes that the variances are equal, which is not the case here. However, the variances are not grossly different and the test statistic is so large (13.9) that the conclusion is probably correct. In either case the variances are significantly different, which is a sufficient condition for the populations to be different.

‡ The '±' on this and subsequent tables indicates that the value has been very crudely interpolated from the statistical tables.

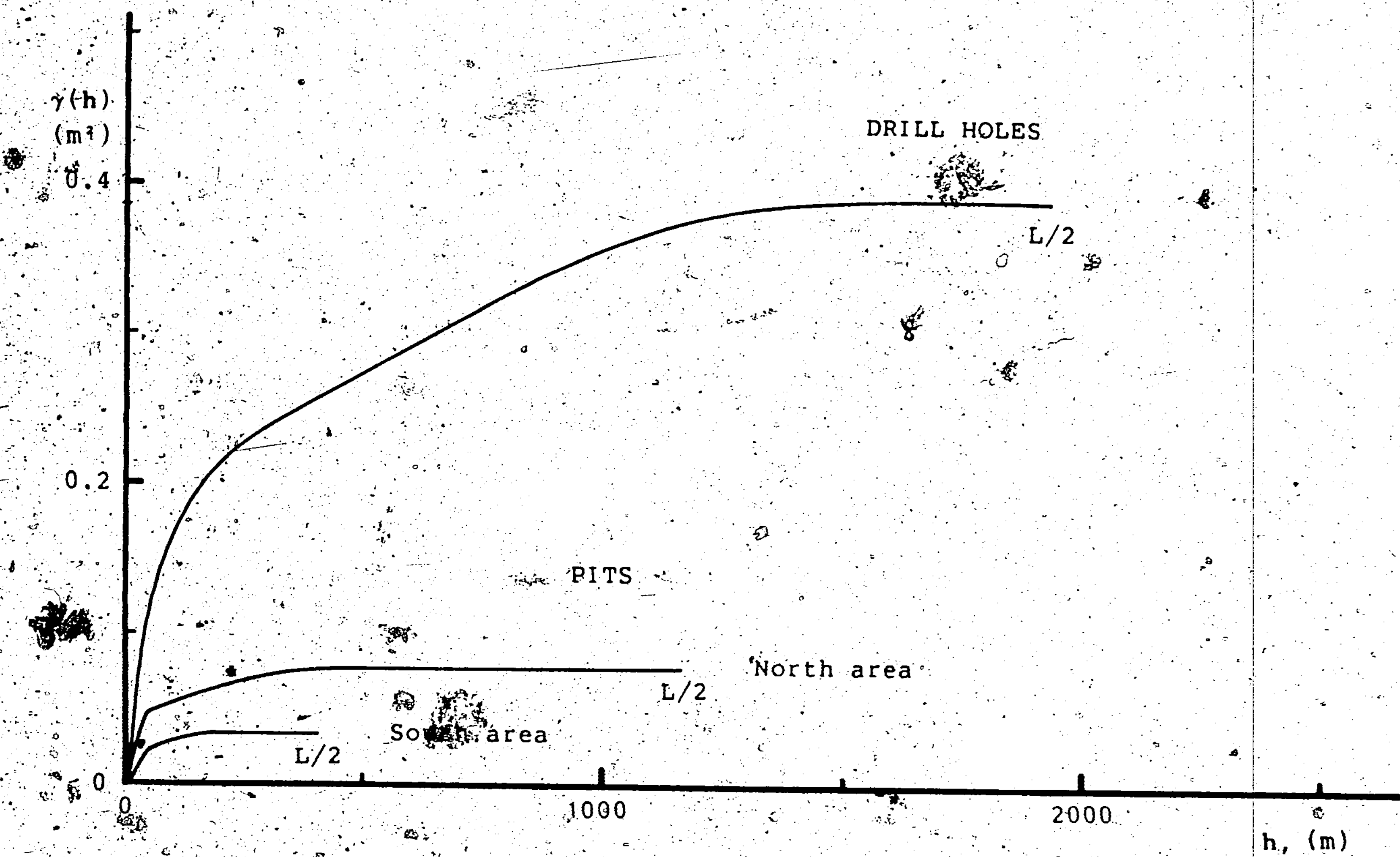

Figure 3.1   Comparison of the pit and drill hole variogram models

2. check that the data is from one population using classical statistics,
3. check that the intrinsic hypothesis is valid over the region.

An important assumption of both the t test and the F test is violated in this study for the thickness, almost always the variable to be tested. Each assumes that the data samples are random and independent. The data has not been selected at random, but on regular grids (Figure 2.1, p.39 and Figure 2.2, p.40). Neither is the thickness independent, it is regionalized with a variogram that cannot be modelled with a pure nugget effect. The test results are not strictly correct, but are calculated as a rough indication.

3.2 SELECTING HOMOGENEOUS PIT REGIONS

3.2.1 Geology of the Pit Regions

The only geological data available from the pits is the thickness of the coal and the partings. The thicknesses do not change drastically (Figure 1.13, p.28 and Figure 1.16, p.32). The study area was subdivided on the basis of presence/absence of the D·E and E·F partings (Figure 1.15, p.31). This resulted in three areas:
1. a north area split by the E·F parting,
2. a middle area of solid coal with no partings,
3. a south area split by the D·E parting.

Not enough points (3 to 7) are present on the F·G parting to separate it out from the north area.

The mine surveyors only measure a parting when it is selectively removed in mining. It is possible that the partings extend into the middle area but were too thin to separate out. Figure 2.8, p.50 shows many holes in the middle area that have a parting logged in them. The partings appear to terminate more by wedging out, as Figure 1.16, p.32 suggests, rather than by shaling out.

If this is the case, then the middle area will contain a mixture of two or three populations:
1. north area population,
2. a possible overlap or gap between the two wedge-edges of the partings from the north and south areas,
3. south area population.

The statistics of the middle area should be a mixture of the statistics of the north and south areas.

3.2.2 Classical Statistics of the Pit Regions

The cumulative normal probability curve of the middle area has several properties of mixed population. It plots midway between the curves for the north and south areas (Figure 3.2), with its ends almost parallel to the curve that dominates the extremity. In this case the thin end is sub-parallel to the north area curve. Finally, it is concave towards the higher-variance curve (north area)

Mixing is confirmed by calculating what the summary ~~statistics should be, assuming mixing with no overlap or~~ gap. The population mean and variance of the middle area should be functions of the population mean and variance of the north and south areas. This was checked by using the sample statistics.

The sample mean of the middle area ($\bar{Z}$) should be a simple weighted average of the means for the north and south areas, or:

$$\bar{Z} = k_n \bar{z}_n + k_s \bar{z}_s \qquad \{3.1\}$$

Where:
$\bar{z}_n$ = sample mean of the northern area.
$\bar{z}_s$ = sample mean of the southern area.
k_n = $n_n/(n_n + n_s)$, the proportion of all middle area samples that belong to the north area population.
k_s = $n_s/(n_n + n_s)$, the proportion of all middle area samples that belong to the south area population.
n_n = number of samples from the northern population in the middle area.
n_s = number of samples from the southern population in the middle area.

The variance of the middle area (S^2) should be a weighted average of the north and south variances corrected for the spread between the means, or:

$$S^2 = k_n s_n^2 + k_s s_s^2 + k_n k_s (\bar{z}_n - \bar{z}_s)^2 \qquad \{3.2\}$$

Where:
k_n, k_s, $\bar{z}_n$, $\bar{z}_s$ have the same meaning as in the formula for the mean.
s_n^2 = sample variance of the north area.
s_s^2 = sample variance of the south area.
The above two formulas are derived in Appendix 4.

The results of the calculations are shown in Table 3.2 on the line 'North+South mixed'. The calculations assume there are two populations in the middle area that meet midway between the recorded wedge-edges of the partings, which is between pit 2A and pits 1 + 3. Statistics measured for the middle area are the same† as the statistics estimated assuming a mixed population.

† $\alpha = 0.05$

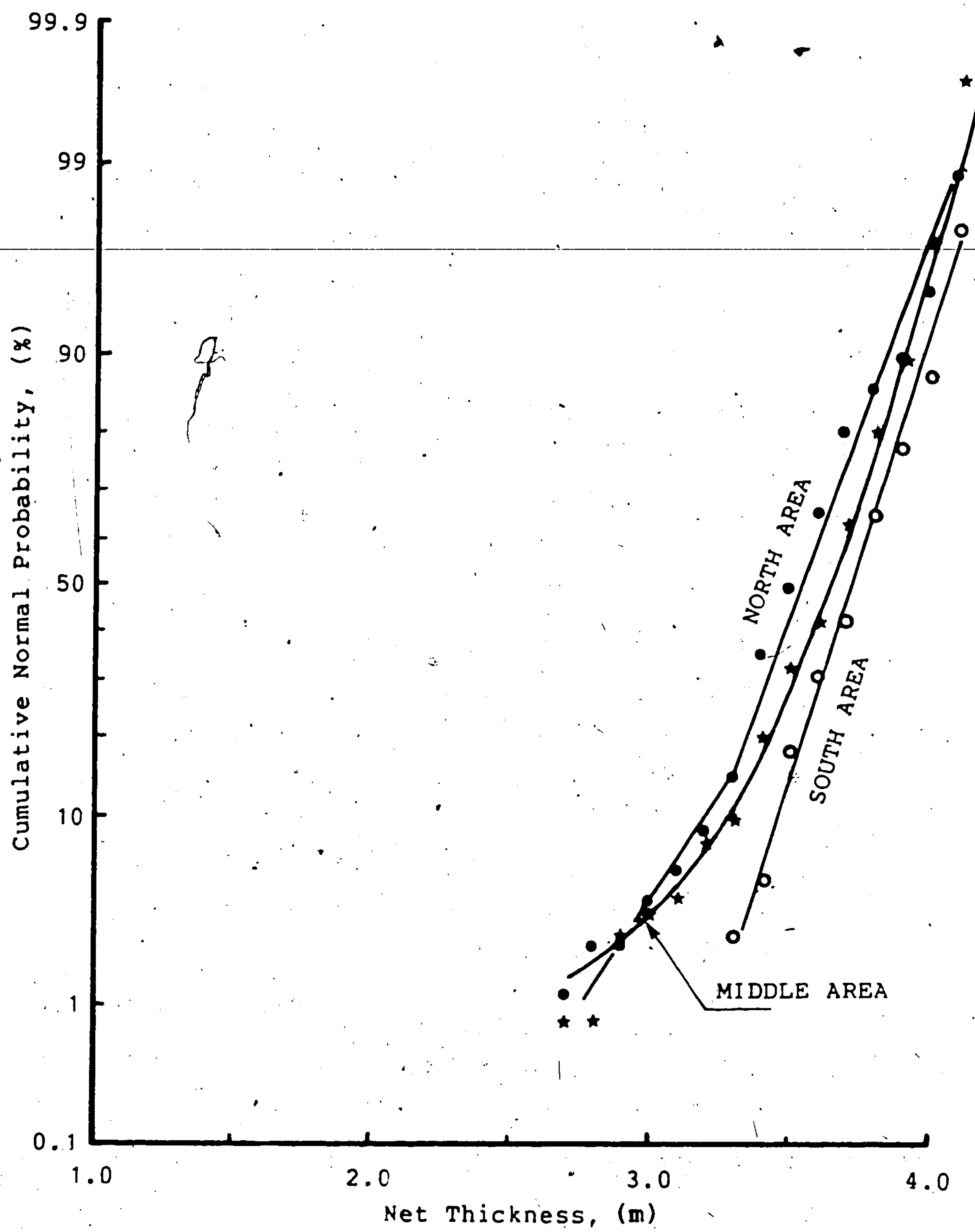

Figure 3.2 Cumulative normal probability curves from the north, middle and south areas of pit data

Table 3.2 **Comparison of the summary statistics of the thickness for the north, middle and south areas of pit data**

Area	n	Mean thickness (m)	Variance (m^2)	Standard deviation (m)
North	172	3.56	0.0736	0.271
South	79	3.77	0.0461	0.215
North+South mixed	251	3.63	0.0747	0.273
Middle	380	3.65	0.0660	0.257

Test statistics from comparing the middle area and the North+South mixed sample

t	0.52	
t crit(0.05)	1.96	
F		1.13
F crit(0.05)		1.3±

To further test the assumption of a mixed population the middle area was divided into two subareas: a north-mid subarea and a south-mid subarea (Figure 3.3). The sample statistics for these subareas were compared to each other and to those for the north and south areas (Table 3.3). The two subareas of the middle area are from significantly† different populations; but on the other hand, there is no significant‡ difference between the north area and the north-mid subarea or between the south area and the south-mid subarea.

3.2.3 Geostatistics of the Pit Regions

The variograms for the middle subareas were also compared to each other and to the variograms for the north and south areas. The variogram for the north-mid subarea is more similar to the variogram from the north area than to the one from the south area (Figure 3.4). The sills are almost the identical, though there is some difference in the range. The variogram from the north-mid subarea has a sill three times that from the variogram from the south area. The variogram from the south-mid subarea is most similar to the variogram from the south area (Figure 3.5). The ranges are similar and the sills are close. The variogram for the south-mid subarea has a different range and sill from the variogram for the north area.

3.2.4 Homogeneous Pit Regions

Since the middle area is composed of two mixed populations, it was subdivided between the pits and each portion combined with the north and south areas to give two homogeneous regions of pit data: a southern region, composed of pit 2A and a northern region, composed of pits 1 and 3. (Figure 3.6):

Within these regions the intrinsic hypothesis holds. There is no drift (Figure 3.7), except possibly in the northern region. This drift does not begin to become apparent until about 1000 m, which is at, or beyond the limit of any estimating done in this study. The two subregions (with and without a parting) within each homogeneous pit region have approximately the same variogram (Figure 3.4, p.175 and Figure 3.5, p.176); therefore, the variograms are probably stationary.

Even though the homogeneous regions have mixed geology (the partings wedge-out part way across), the statistics suggest that in fact the geology is uniform and that the

† $\alpha = 0.05$

‡ $\alpha = 0.05$ (0.02 for the F test on the southern variances)

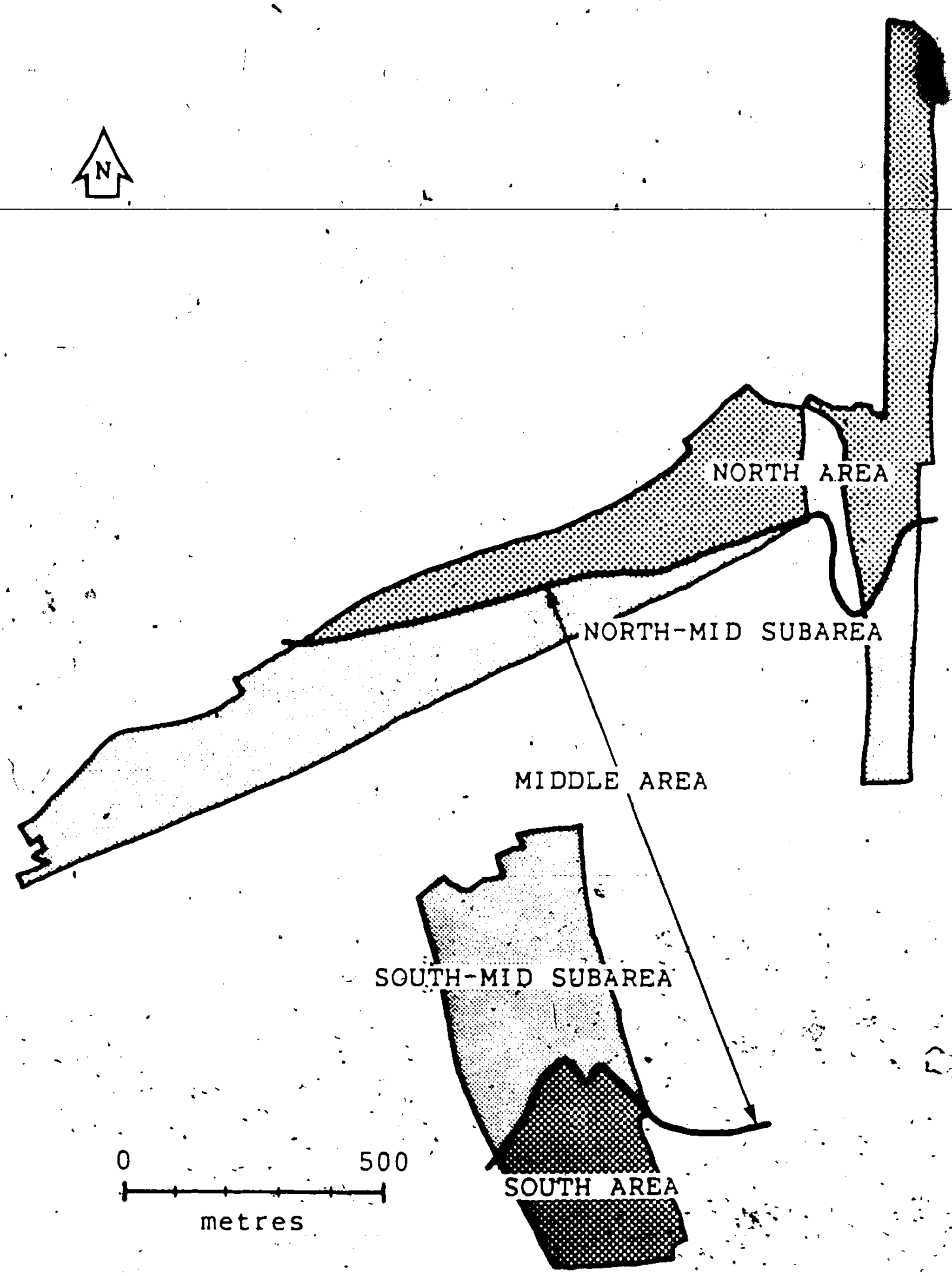

Figure 3.3 Location of the areas and subareas of the pit data

Table 3.3 **Comparison of the summary statistics of the thickness for the areas and subareas of pit data**

Area or Subarea	n	Mean thickness (m)	Variance (m^2)	t	F	F crit (0.05)
North	172	3.56	0.0736			
				1.85	1.03	1.3±
North-mid	258	3.61	0.0714			
				5.16†	2.57	1.3±
South-mid	122	3.75	0.0278			
				1.11	1.65	1.5± ‡
South	79	3.77	0.0461			
t crit (0.05)				1.96		

† See footnote † under Table 3.1, p.166.
‡ The variances are equal at $\alpha = 0.02$.

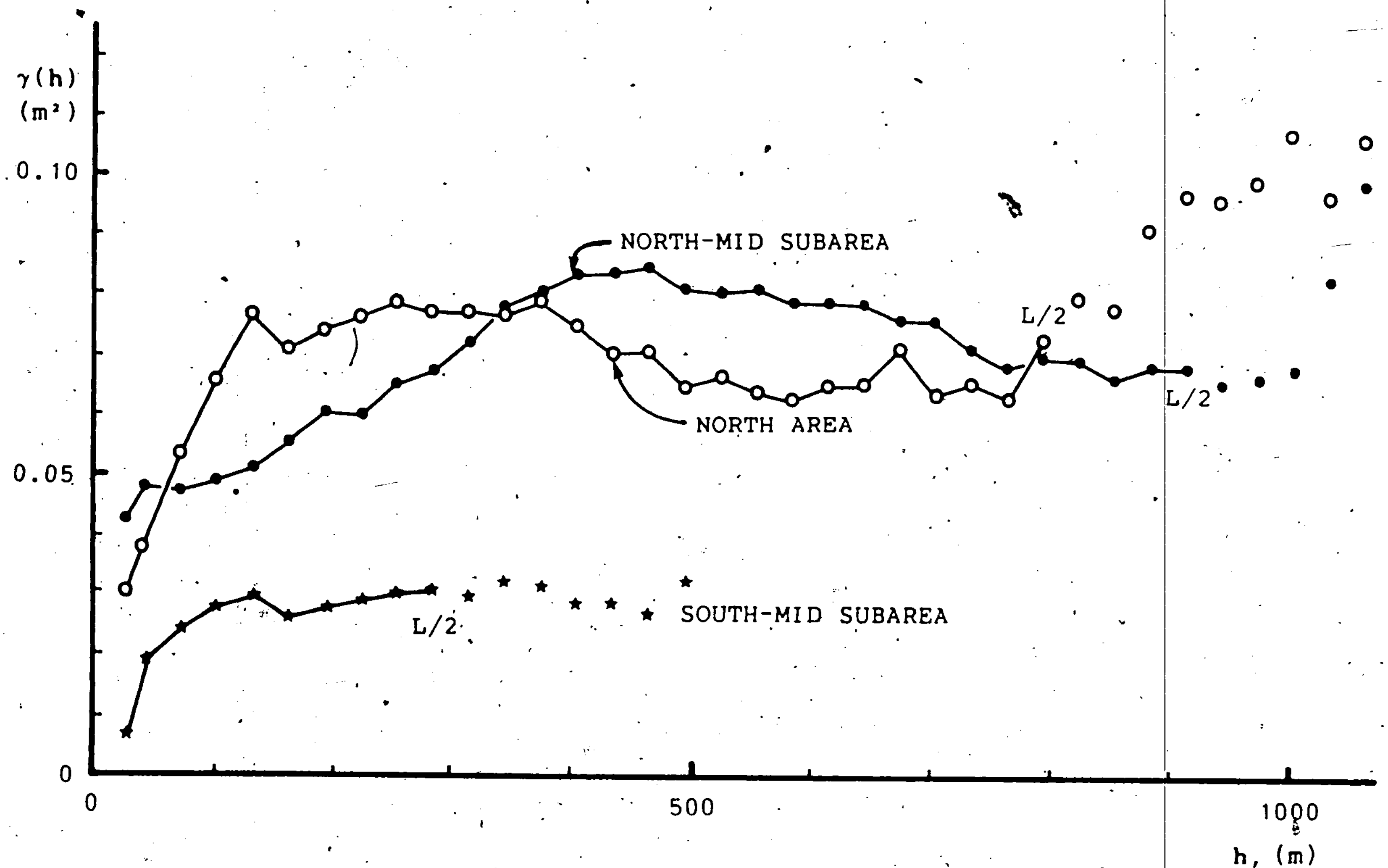

Figure 3.4 Comparison of the variograms for the north areas of the pit data

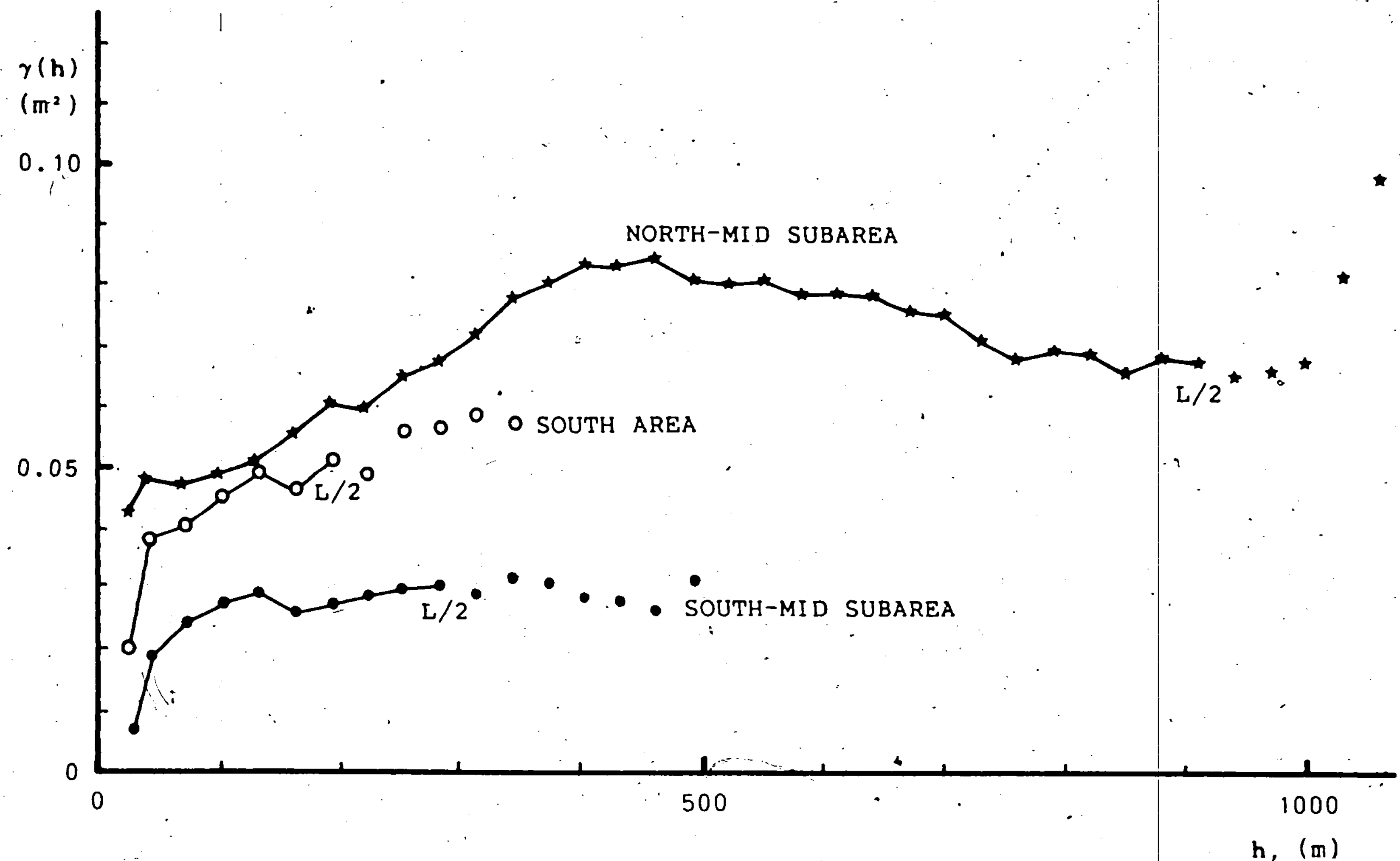

Figure 3.5 Comparison of the variograms for the south areas of the pit data

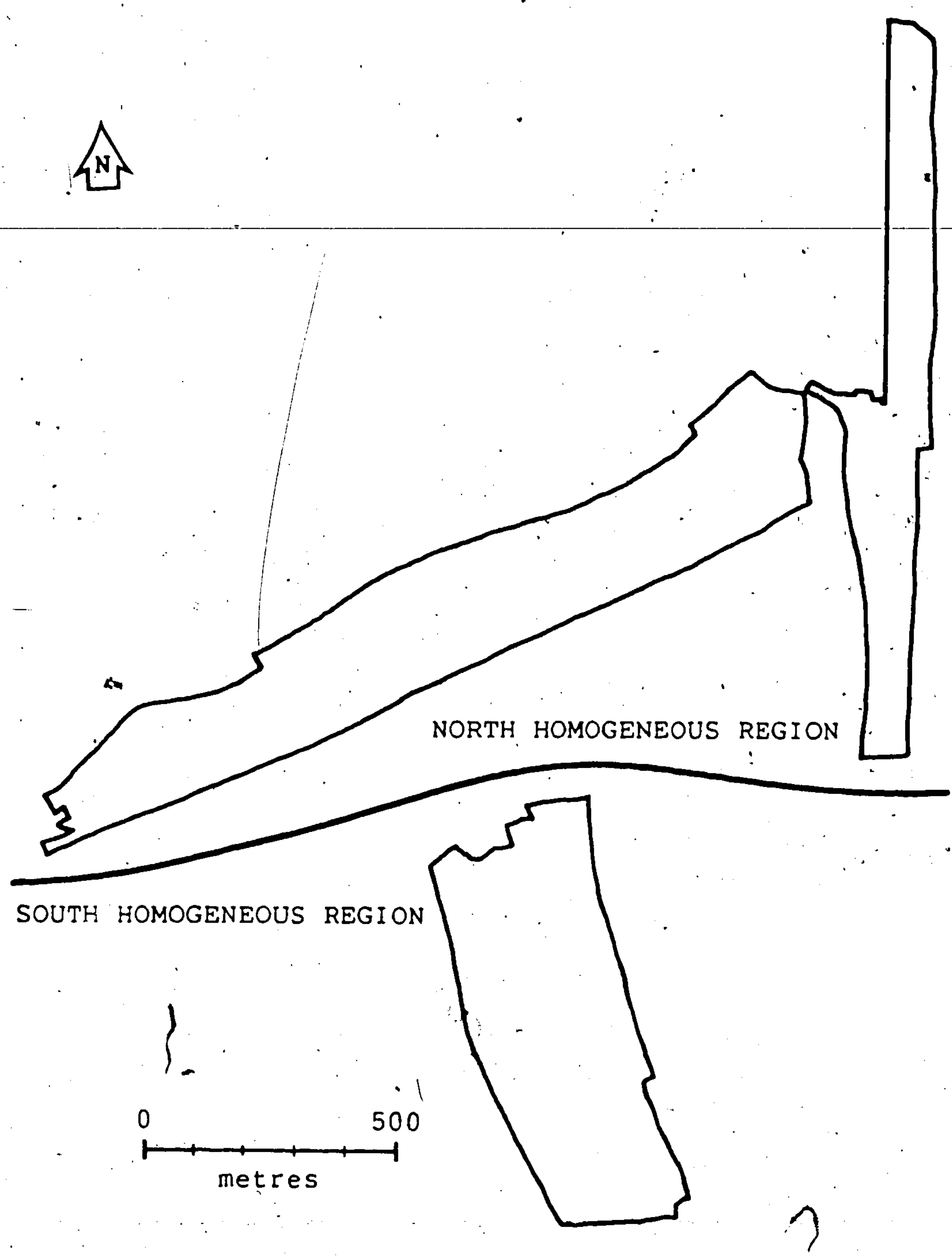

Figure 3.6 Location of the homogeneous regions of pit data

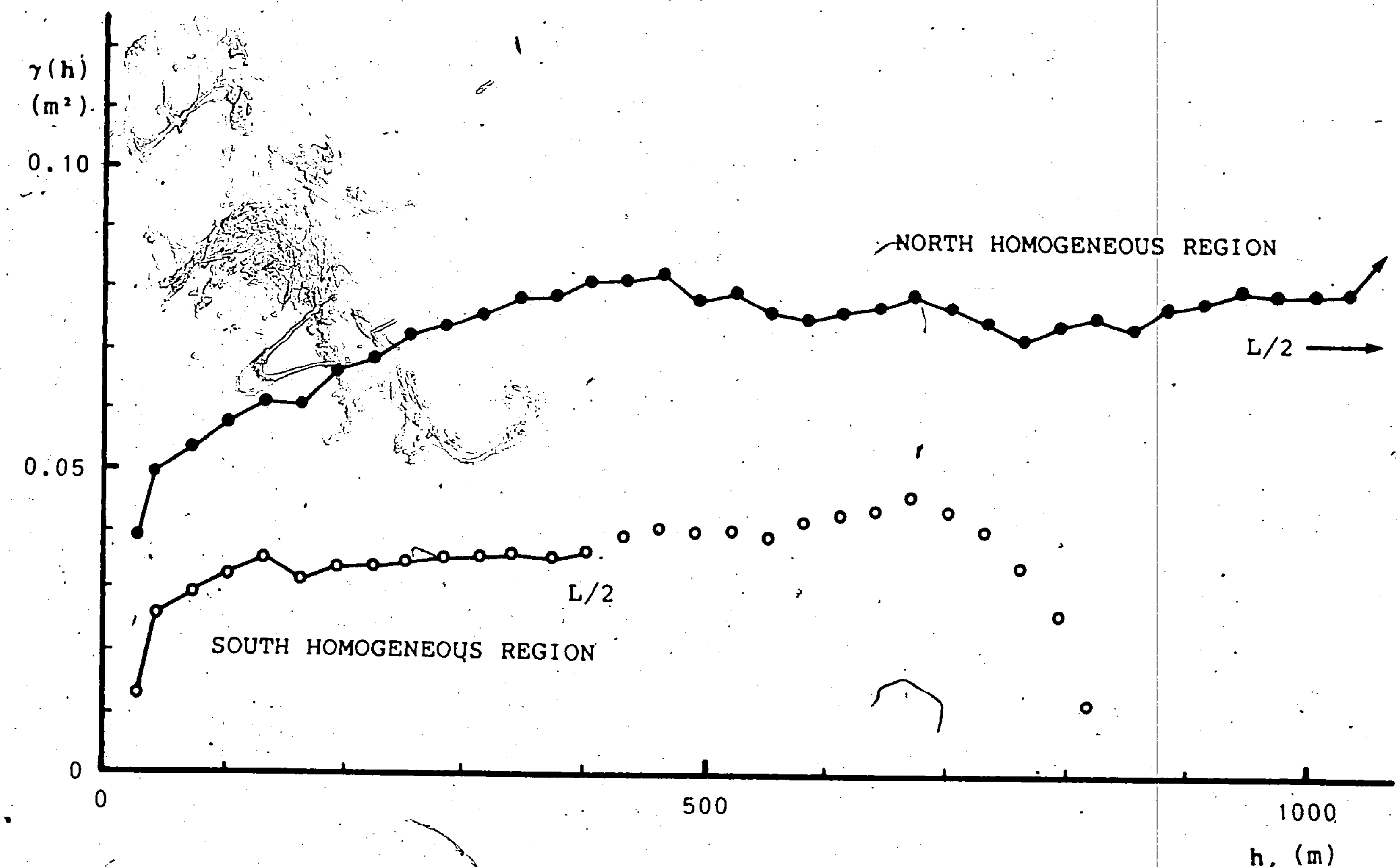

Figure 3.7 **Variograms of the homogeneous regions of pit data**

partings probably extends across the regions, but are too thin to selectively mine and measure in the centre of the study area.

These two homogeneous pit regions cannot be combined for these reasons.

1. The geology is different since there are different partings in each region (Figure 1.14, p.30; Figure 1.15, p.31 and Figure 1.16, p.32).
2. The populations are different since the means and variances are significantly† different (Table 3.4).
3. The intrinsic hypothesis does not hold since the variograms are different (Figure 3.7, p.178).

3.3 SELECTING HOMOGENEOUS DRILL HOLE REGIONS

The drill hole data cannot be subdivided into separate homogeneous regions. The geology varies slightly, but the classical statistics and the geostatistics are the same over the whole area.

There is a general tendency toward more splits in the north and south portions of the study area with fewer in the centre (Figure 2.8, p.50), but holes with different numbers of splits tend to alternate, so that it is not possible to separate areas underlain by a particular parting.

The drill holes were subdivided into subsets based on the number of splits, irrespective of their location and summary statistics were compared (Table 3.5). In general, there is very little difference; the means tend to decrease as more partings are separated out but there is no trend in the variances.

The statistics of each subset were compared against the population statistics estimated by the sample statistics of all the rest of the drill hole data without the subset in question (Table 3.6). This was repeated for each number of splits from one (with no partings) to five (with four partings). In general the comparison between the various subsets was either good (both mean and variance fit) or fair (either mean or variance did not fit); there are no gross discrepancies. The subsets of holes with one and three splits of coal did not fit very well, but the difference is not large. It is reasonable therefore from the standpoint of classical statistics to combine the drill hole data even though they include different numbers of splits.

If there are two populations of drill hole data similar to the pit data, then they should have about the same location as the pit populations. The drill holes were divided

† $\alpha = 0.05$

Table 3.4 **Summary statistics of the thickness for the homogeneous regions of pit data**

Region	n	Mean thickness (m)	Variance (m^2)	t	F
North	430	3.58	0.0759		
				385†	2.13
South	201	3.76	0.0356		
Critical values (α = 0.05)				1.96	1.3±

† See footnote † under Table 3.1, p.166.

Table 3.5 Summary statistics of the thickness for the drill holes, subdivided by the number of splits

Number of splits	n	Mean thickness (m)	Variance (m^2)
1	80	3.48	0.298
2	143	3.28	0.307
3	94	3.17	0.160
4	17	3.13	0.333
5	2	3.00	0.00577

Table 3.6 Comparison of the summary statistics of the thickness for the drill holes, subdivided by the number of splits

Number of splits	n	Mean (m)	Variance (m^2)	t	F	F crit (0.05)	Match
1	80	3.48	0.298	3.86	1.16	1.41	fair
All-1†	256	3.23	0.256				
2	143	3.28	0.307	0.21	1.19	1.3±	good
All-2	193	3.29	0.257				
3	94	3.17	0.160	2.61	1.98	1.39	poor
All-3	242	3.33	0.316				
4	17	3.13	0.333	1.29	1.22	1.83	good
All-4	319	3.30	0.274				
5	2	3.00	0.00577	0.76	48.4	3.71	fair
All-5	334	3.29	0.279				
t crit(0.05)				1.96			

† All-1 is all the drill hole data less those holes with one split, similarly for All-n.

into two areas (Figure 3.8), at approximately the same division used for the pit data (Figure 3.6, p.177). The two areas are statistically homogeneous (Table 3.7) since the means and variances are the same.† The variograms are essentially the same with no drift (Figure 3.9); ~~consequently, the intrinsic hypothesis is considered valid.~~

It may seem odd that the same phenomenon (the Estevan Coal Zone net thickness) measured one way (by drill holes) has one thickness population, but measured another way (in the pits) has two populations. This difference has two possible reasons. Firstly, the two phenomena measured are not really identical. The drill holes measure the whole zone as logged by a geologist, whereas the pit measurements describe only the mineable portion of the zone as seen by the excavation equipment operators. Secondly, the variance of the drill hole data is large enough to include two populations. At the 0.05 level, a difference in the means of 0.13 m between a separate north and south drill hole population would be barely significant. This is very close to the difference (0.18 m) between the north and south regions of the pit data (Table 3.4, p.180).

† $\alpha = 0.05$

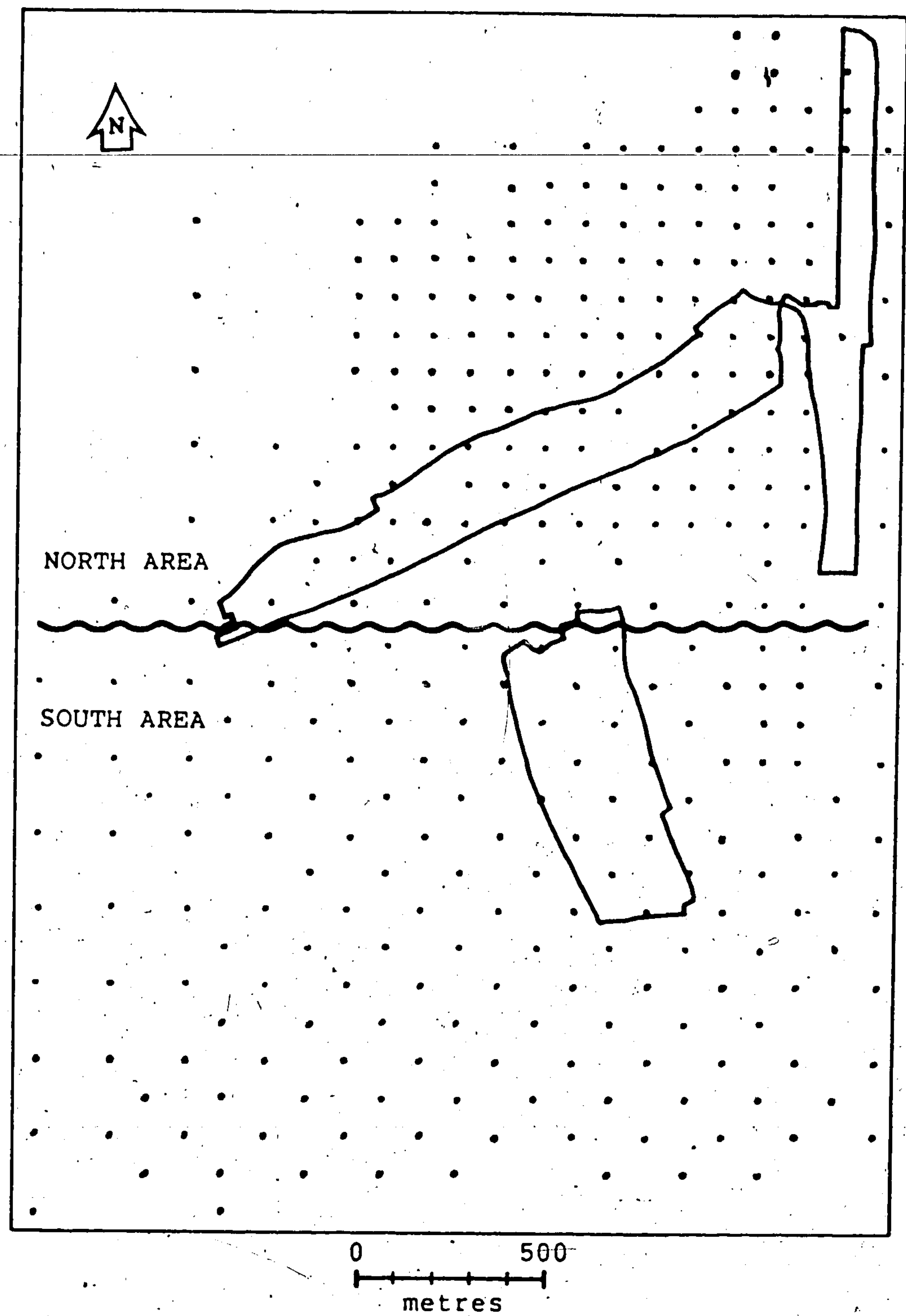

Figure 3.8 Location of the north and south areas of the drill hole data

Table 3.7 Comparison of the summary statistics of the thickness for the drill hole areas

Area	n	Mean thickness (m)	Variance (m^2)	t	F
North	169	3.12	0.313		
				0.92	1.18
South	167	3.18	0.369		
Critical values (α = 0.05)				1.96	1.3±

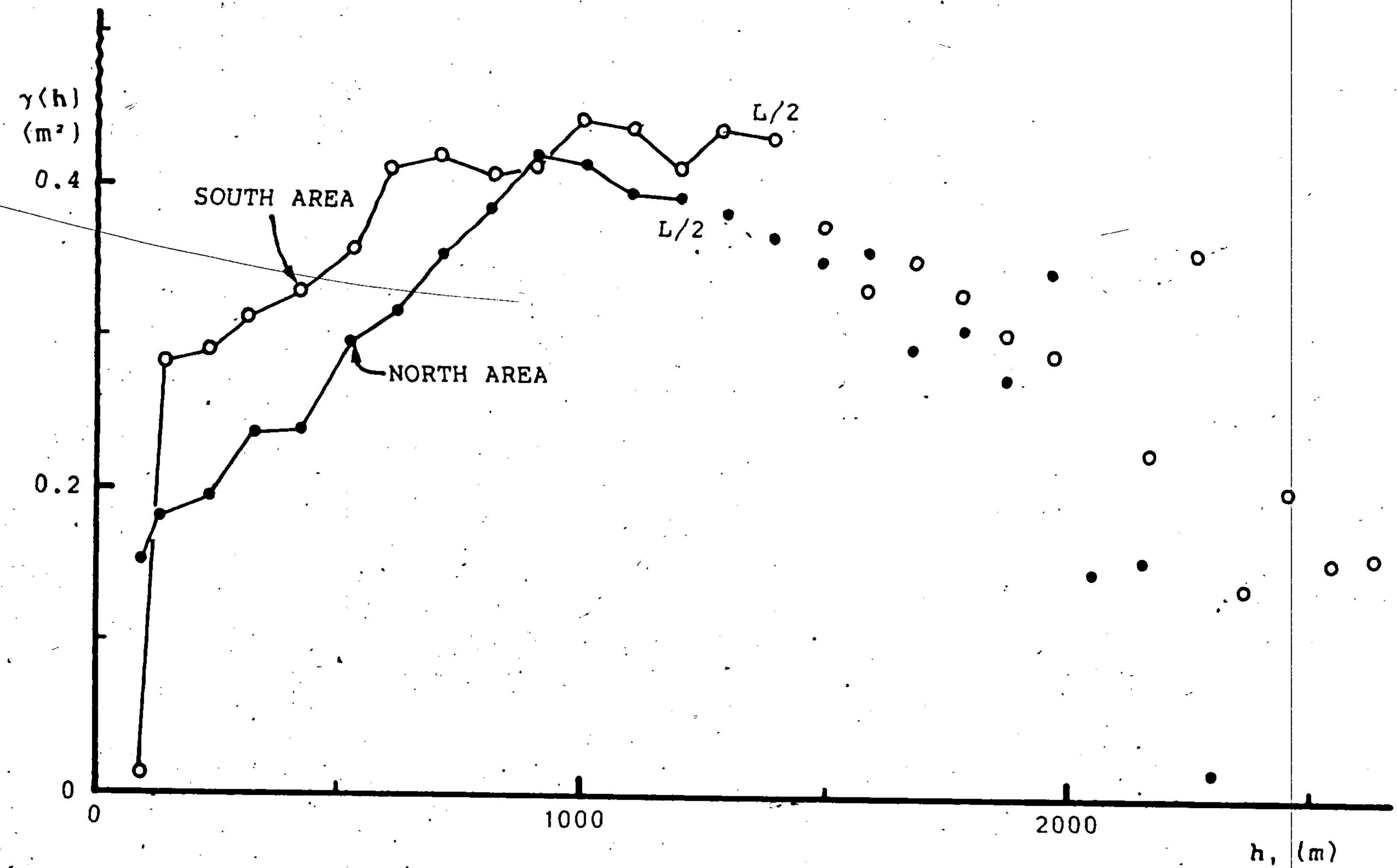

Figure 3.9 Comparison of the variograms from the north and south areas of drill hole data

APPENDIX 4
DERIVATION OF THE FORMULAS FOR A MIXED MEAN AND VARIANCE

It is assumed that the pit data from the middle area with no parting is made up of two populations: one from the northern and the other from the southern portion of the middle area.

4.1 MIXED MEAN

The overall sample mean of the data from the middle area is defined as:

$$\hat{z} = (1/n)\Sigma z_i$$

For simplicity Σ is unsubscripted, but it should be taken to mean the summation over all values of the subscripted variables. Separating the data from each population, this becomes:

$$\hat{z} = (1/n)\Sigma z_n + (1/n)\Sigma z_s \qquad \{4.1\}$$

The sample mean for the data from the north portion of the area only is defined as:

$$\hat{z}_n = (1/n_n)\Sigma z_n$$

This can be rearranged to:

$$\Sigma z_n = n_n \hat{z}_n \qquad \{4.2\}$$

Substituting this and the analogous equation for the south portion of the area into equation {4.1} above, it becomes:

$$\hat{z} = (1/n)n_n\hat{z}_n + (1/n)n_s\hat{z}_s$$

$$= (n_n/n)\hat{z}_n + (n_s/n)\hat{z}_s \qquad \{4.3\}$$

Define:

$$k_n = n_n/(n_n + n_s)$$

and since:

$$n = n_n + n_s$$

therefore:

$$k_n = n_n/n \qquad \{4.4\}$$

Substitute this and the southern analog into equation {4.3} above:

$$\bar{Z} = k_n \bar{z}_n + k_s \bar{z}_s$$

$\bar{z}_n$ is the mean of the sample from the northern portion of the middle area. But since it was assumed that this sample is drawn from the same population as the north area and since the sample sizes are large ($n > 100$), $\bar{z}_n$ is also the mean of the sample from the north area, similarly for $\bar{z}_s$.

4.2 MIXED VARIANCE

The overall sample variance of the data from the middle area is defined as:

$$S^2 = (1/n)\{\Sigma z_i^2 - (1/n)(\Sigma z_i)^2\}$$

Since the sample is large ($n > 100$) the biased form of the equation is as good as the unbiased form; therefore:

$$1/n \simeq 1/(n-1)$$

Separating out the data from each population as was done for the mean, this becomes:

$$S^2 = (1/n)\{\Sigma z_n^2 + \Sigma z_s^2 - (1/n)(\Sigma z_n + \Sigma z_s)^2\} \qquad \{4.5\}$$

The sample variance for the data just from the north portion of the middle area is defined as:

$$s_n^2 = (1/n_n)\{\Sigma z_n^2 - (1/n_n)(\Sigma z_n)^2\}$$

This can be rearranged to:

$$\Sigma z_n^2 = (1/n_n)(\Sigma z_n)^2 + n_n s_n^2$$

Substituting this and the south analog into equation {4.5} above along with some expansion, equation {4.5} becomes:

$$S^2 = (1/n)\{n_n s_n^2 + n_s s_s^2 + (1/n_n)(\Sigma z_n)^2 + (1/n_s)(\Sigma z_s)^2$$
$$- (1/n)[(\Sigma z_n)^2 + 2\Sigma z_n \Sigma z_s + (\Sigma z_s)^2]\}$$

Substituting equation {4.2} and equation {4.4} above into this with some further rearranging, it becomes:

$$S^2 = k_n s_n^2 + k_s s_s^2 + (n_n^2/nn_n)\bar{z}_n^2 + (n_s^2/nn_s)\bar{z}_s^2$$
$$- (n_n^2/n^2)\bar{z}_n^2 - (2n_n n_s/n^2)\bar{z}_n\bar{z}_s - (n_s^2/n^2)\bar{z}_s^2$$

Collecting similar terms this becomes:

$$S^2 = k_n s_n^2 + k_s s_s^2 + (n_n/n - n_n^2/n^2)\bar{z}_n^2$$
$$- (2n_n n_s/n^2)\bar{z}_n\bar{z}_s + (n_s/n - n_s^2/n^2)\bar{z}_s^2$$

Substituting equation {4.4} above into this, it becomes:

$$S^2 = k_n s_n^2 + k_s s_s^2 + (k_n - k_n^2) z_n^2 - 2k_n k_s z_n z_s + (k_s - k_s^2) z_s^2 \qquad \{4.6\}$$

Recall that:

$$k_n - k_n^2 = k_n(1-k_n) = k_n k_s \text{ ; since } 1-k_n = k_s$$

Substituting this and the south analog into equation {4.6} above, it becomes:

$$S^2 = k_n s_n^2 + k_s s_s^2 + k_n k_s z_n^2 - 2k_n k_s z_n z_s + k_n k_s z_s^2$$

This can be rearranged to:

$$S^2 = k_n s_n^2 + k_s s_s^2 + k_n k_s (z_n - z_s)^2$$

s_n^2 is the variance of the sample from the northern portion of the middle area. But since this sample is drawn from the same population as the north area and since the sample sizes are large ($n > 100$), s_n^2 is also the variance of the sample from the north area, similarly for s_s^2.

APPENDIX 5
CALCULATING THE EXPERIMENTAL VARIOGRAM

The raw variogram is calculated directly from the data, then the experimental variogram is calculated from the raw variogram by correcting for the smoothing.

5.1 CALCULATING THE RAW VARIOGRAM

The raw variogram is calculated using a program written by Flint (1978), which was modified from one written by David (1977, p.149). The program takes as input three parameters: lag (Δ**h**), phi (ϕ) and psi (ψ), all illustrated on page 141. The program breaks all the pairs of points into classes depending on the values of the lag, ϕ and ψ, then calculates an average value of the variogram for each class (using equation {1.9}, p.139), which becomes one point on the raw variogram. An example of some output from the variogram calculating program is listed in Appendix 12.

Not all points on the raw variogram are usable. If there are less than 30 to 50 pairs of samples used to calculate a point, then the value of the variogram is unstable (Journel and Huijbregts, 1978, p.194). Points with less than 50 pairs in the calculation were not plotted, fortunately the number of pairs was typically in the 1000's, except near the ends. Since the lag was chosen to be just greater than the nominal sample spacing, no points were lost at the beginning of the variogram. The only points discarded were at the far end when $h > L/2$ and therefore not useful anyway.

Various values of the parameters Δ**h**, ψ and ϕ were used to calculate the raw variograms in order to find the best numbers to use.

The lag (Δ**h**) of the pit data was varied from 320 to 20 m in geometric increments. As the lag became shorter, the variogram became more ragged and unstable (Figure 5.1 and Figure 5.2), particularly when it approached the minimum sample spacing of 30 m in the pits and 100 m in the drill holes. The reason is that fewer pairs were used to estimate each point on the variogram.

The most important part of a variogram is near the origin, where a nugget effect may be present. This part of the variogram can only be estimated by using a small lag. The lag must also be small enough to provide several points to estimate the shortest range; Journel and Huijbregts (1978, p.211) suggest Δ**h** < 1/3 or 1/4 of the short range; the ranges are 50 m for the pits and 150 m for the drill holes. This is a conflict between the requirements of a stable variogram (large Δ**h**) and an accurate determination of

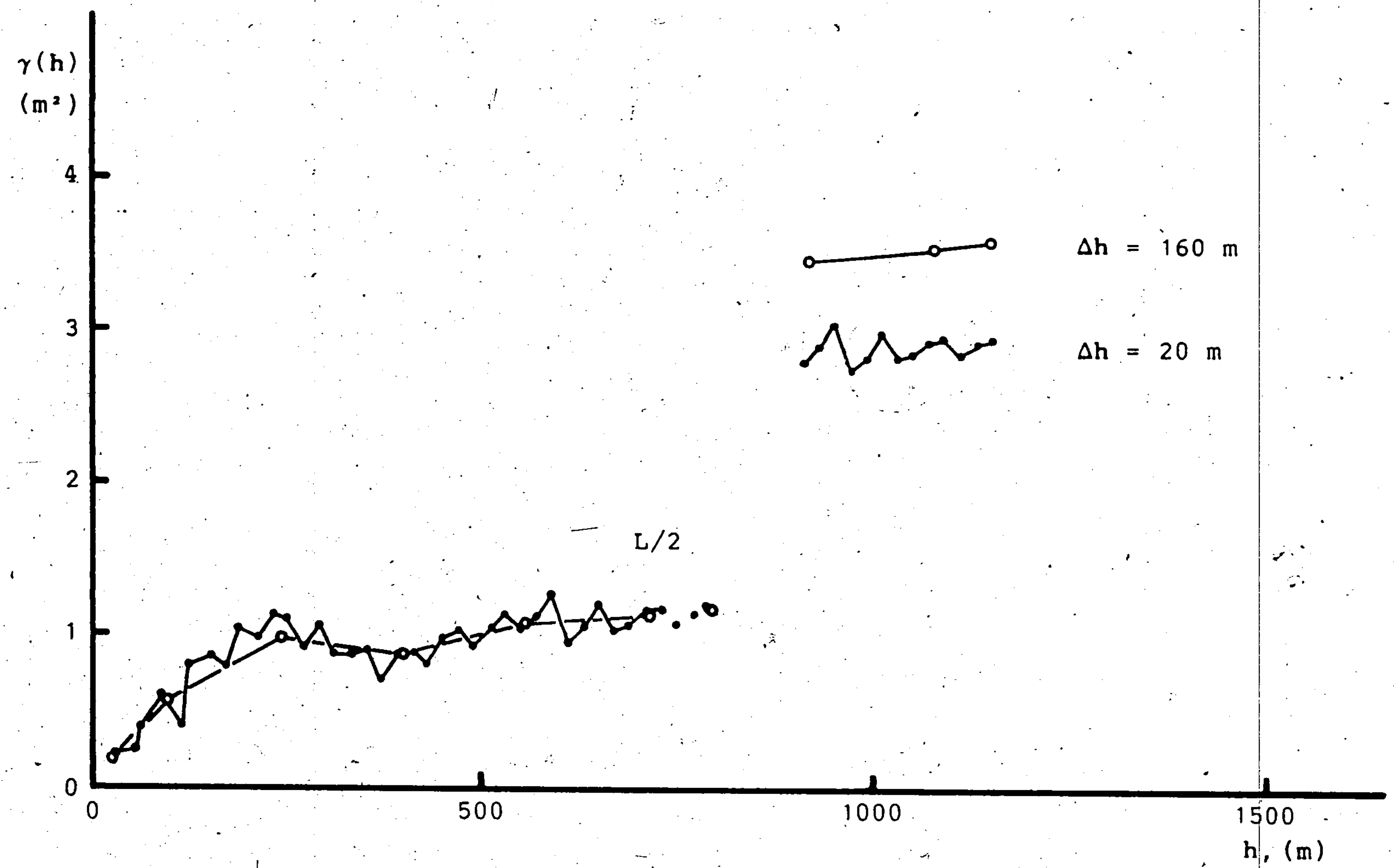

Figure 5.1 **Change in the variogram from pit 3 due to a change in lag**

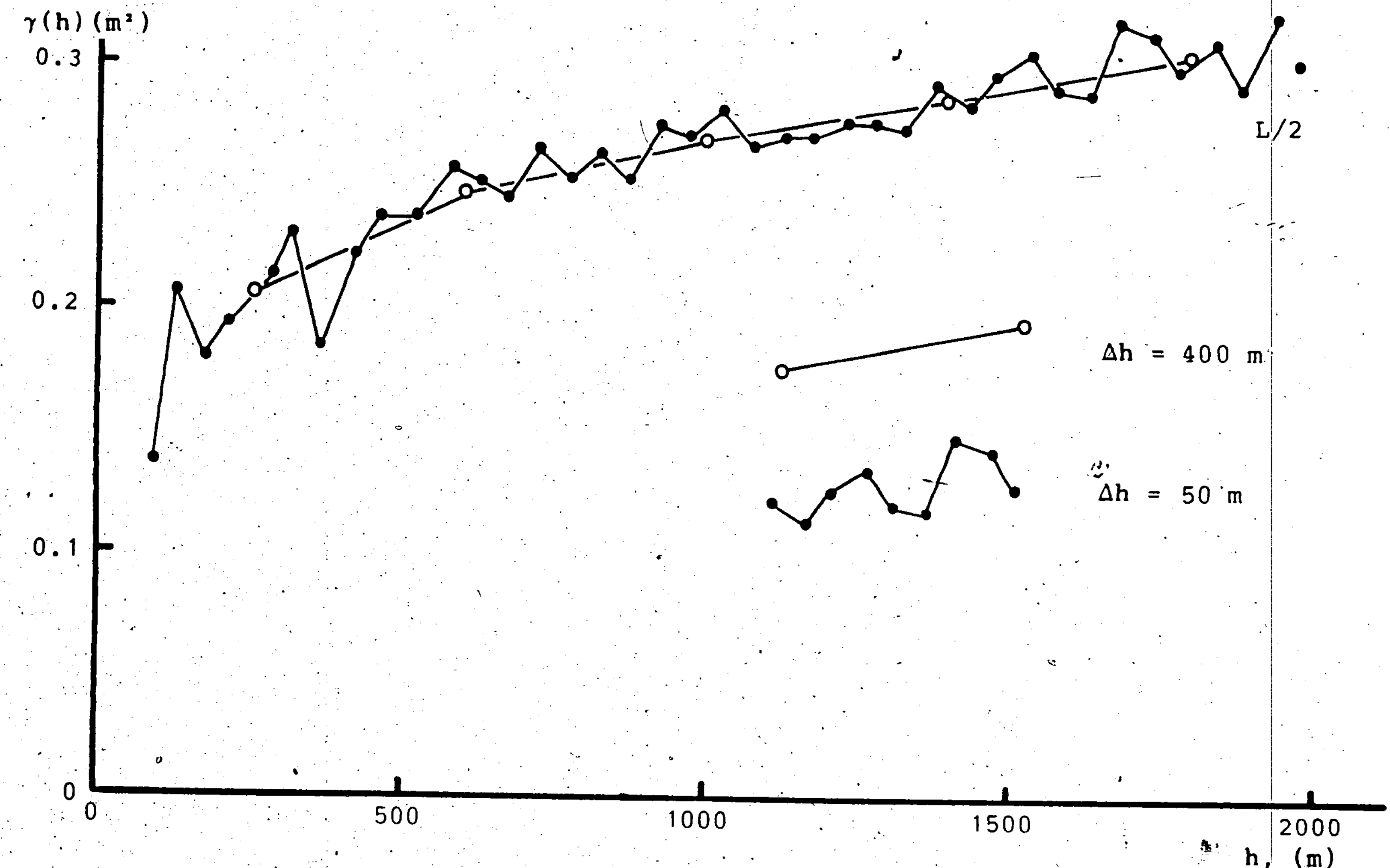

Figure 5.2 Change in the variogram for the drill holes due to a change in lag

C_0 (small Δh). The rest of the variograms in this study were estimated using a compromise value of 40 m for the pit data and 100 m for the drillholes. This did not produce a variogram which was too ragged and it showed the behaviour near the origin adequately.

The usual choice for the size of the angular classes (ψ) is 30°, but its value is not too critical. Again, data from pit 3 and the drill holes is used to illustrate the effect of ψ in Figure 5.3 and Figure 5.4. With ψ = 10° and 80°, there is little change and therefore, the standard ψ = 30° was used for all variograms in this study.

The last calculation parameter, ϕ, is the direction of the angular grouping and is used to any anisotropic structure to the data. Each anisotropic variogram was calculated along four directions, 0°, 45°, 90°, and 135° to the suspected anisotropic axes. The suspected axes were parallel to the benches (Figure 5.5) and parallel to the major paleochannel (Figure 3.7, p.68 and Figure 3.8, p.69).

5.2 CORRECTION OF THE RAW VARIOGRAM TO FORM THE EXPERIMENTAL VARIOGRAM

Smoothing of the variogram is the change in its shape due to the samples not being at mathematical points and to the grouping of the samples when calculating it. This change must be reversed to cacluate the experimental variogram from the raw variogram.

All thickness measurements in this study were taken almost at a single point. Each pit thickness was probably an average over the area (1 to 10 cm wide)† across which the mine surveyor visually estimates the limit of the seam; in this case l/a‡ is at most 0.1 m/100 m or 0.001. The drill hole thicknesses were averaged over the diameter of the core (0.1 m) or the depth of investigation into the rock by the geophysical tool (about 1 m), which gives l/a = 0.001 and 0.01 respectively. In each case l/a is so small that the raw variogram and the variogram corrected to a point support are indistinguishable; thus, the variograms were not corrected for finite support. The effect of different l/a ratios is illustrated on page 143.

Grouping smooths the variogram due to non-zero Δh and ψ. Experience has shown (David, 1977, p.146) that the corrections are minor if these two quantities are small. A small lag equals the average distance between samples and a small ψ is less than 45°. These constraints are satisfied

† All the calculations in this paragraph are order of magnitude only.
‡ A discussion of l/a can be found on page 143.

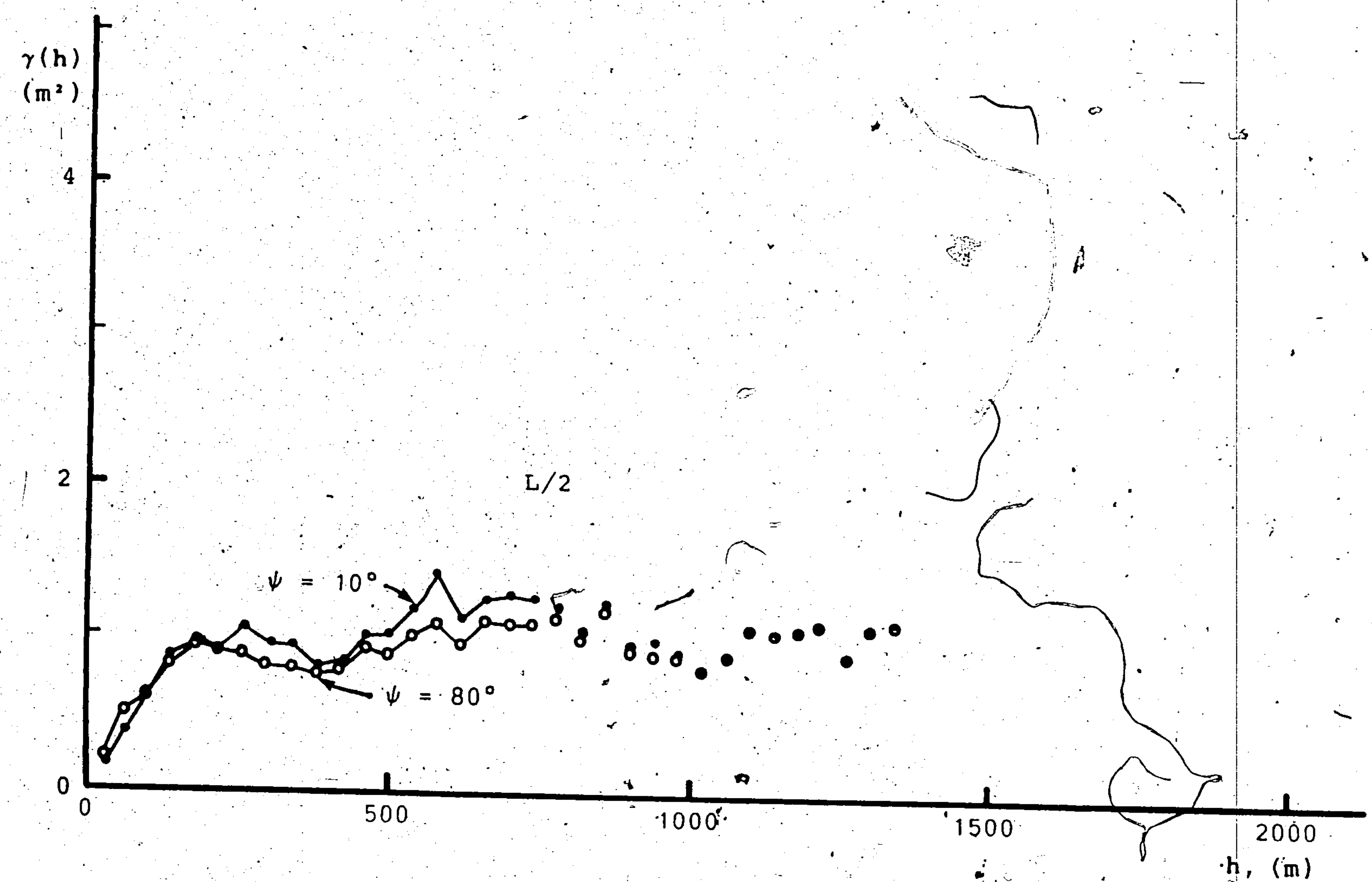

Figure 5.3 Change in the variogram from pit 3 due to a change in ψ

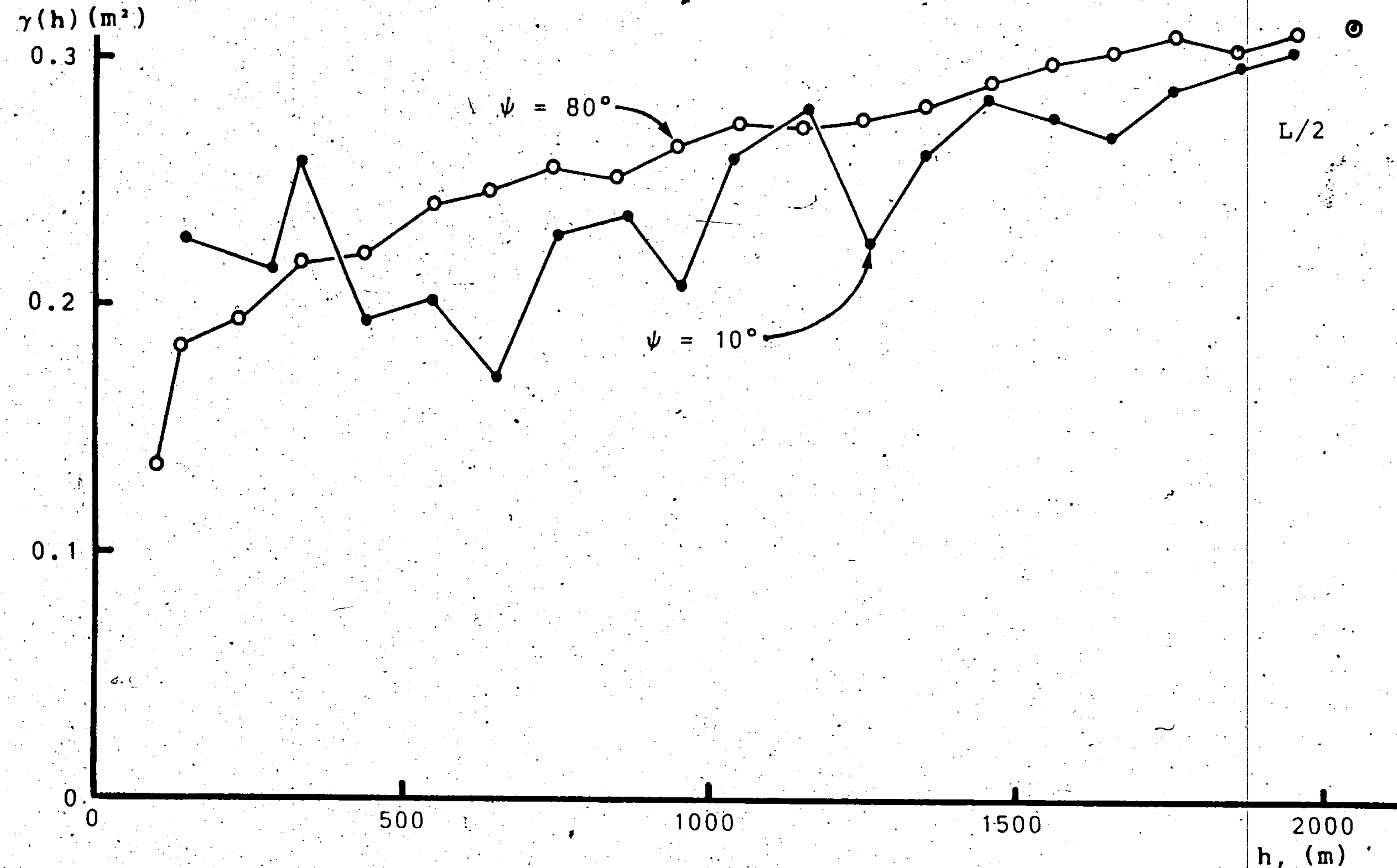

Figure 5.4 Change in the variogram for the drill holes due to a change in ψ

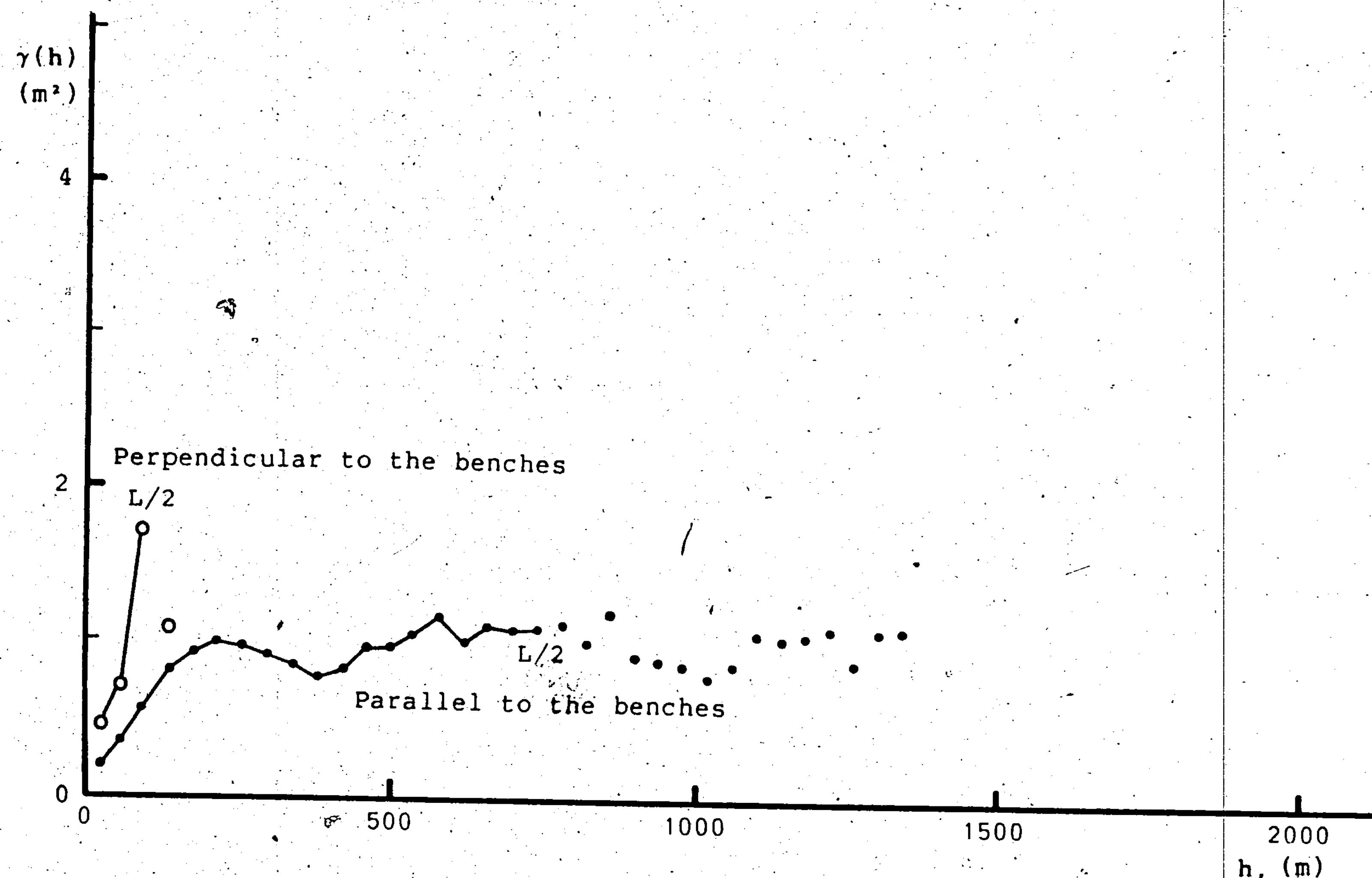

Figure 5.5 **Anisotropic variograms from pit 3, due to the measurement methods**

by the values chosen for the parameters in this analysis. The sample separation is 30 m in the pits and 100 to 200 m for the drill holes, while the lags used were 40 m and 100 m respectively. The angular regularization (ψ) was always 30°. The corrections can be computationally complex and the final results probably do not justify the extra effort. Since no corrections have to be made, the raw variograms are also the experimental point variograms (Figure 3.2, p.57 and Figure 3.3, p.58).

5.3 ANISOTROPY OF THE EXPERIMENTAL VARIOGRAM

The experimental drill hole variogram is isotropic, while the pit variograms show a slight zonal anisotropy with the axes perpendicular and parallel to the benches (Figure 5.5, p.196). When **h** is parallel to the benches, the sill may be lower and the range shorter than for the variogram perpendicular to the benches. However, the perpendicular variogram is so short and close to the L/2 value that the range may not be real. If it is real, then it is probably due to the surveyors' tendency to remember the last measurement along the bench and consciously or subconsciously adjust the current one, particularly when the contact is questionable. On the other hand, the closest measurement across the bench would have been taken several months earlier and would not have any mental effect on the present reading. This effect will increase the range and decrease the sill measured parallel to the benches, but it is not large, approximately half of the sill and maybe up to half of the range.

The variogram was considered to be isotropic for several reasons. Firstly, the anisotropy is not large and kriging is robust with respect to the variogram. Secondly, the range, with the largest anisotropy, does not affect the model as much as the sill with the lesser anisotropy. Finally, the pits are more or less perpendicular to each other (Figure 1.11, p.24), which means that the aggregate variogram of all pits together will tend to be isotropic.

The drill hole variogram is also isotropic for several reasons. Firstly, the drill holes were not logged sequentially in one direction, so there is no 'memory effect'. Secondly, any anisotropy due to location, for example perpendicular and parallel to the paleochannel or to the wedge-edges of the partings, is too small to be measured. This suggests that the underlying variogram in the pits may also be isotropic.

APPENDIX 6
VARIOGRAM MODELLING METHOD

Flint (1978) wrote a program to model variograms, which was modified to remove some of the inherent guessing in modelling. A range of points is input to the program, which then estimates the structures. Finally, the parameters are adjusted by trial and error to improve the fit.

For a simple variogram with one structure, the process is as follows.

1. The value of the population variance is used for the sill.
2. The first few points are input into the program, which fits a least squares line through them. This line intersects the sill at 2a/3 (Figure 1.2, p.140) and the vertical axis at the nugget effect (C_0).
3. The parameters are adjusted until a reasonable fit is achieved.

When there are several nested structures, as in this study, the best method has these steps.

1. The process above is begun at the largest structure with the sill equal to the population variance. This time though, the best fit line intersects the vertical axis at the sill of the next smaller structure (Figure 1.10, p.155).
2. The process above is repeated using the sill determined in the previous step. The last step determines C_0.
3. The initial guess at the parameters is adjusted to improve the fit.

The variogram parameters C_0, C and a, are not equally easy to estimate. The range is difficult because it is at a tangent point; furthermore, it is particularly difficult when the structure is defined by only one or two points, as are the short range stuctures in this study. A good estimate requires at least three or four points, but fortunately, the range does not greatly affect the model. On the other hand, the sill of the long range model is very easy to estimate, as are the other sills, though they all usually need some adjustment from the first estimates. The nugget effect has a large effect on the model and is the hardest to estimate in this study, as it is the result of an extrapolation of a few points to intersect the vertical axis at a small angle. It must usually be guessed by visual estimate rather than trusting the results of an algorithm.

APPENDIX 7
OUTLINE OF THE THEORY OF KRIGING

Kriging is a method for estimating the average value of a regionalized variable from data distributed over a region. Each data value can be either at a point or an average over a region. Similarly, the kriged average can be at a point or over a domain.

7.1 VARIETIES OF KRIGING

Many varieties of kriging are available, the choice depending on the assumptions made and the properties of the data. The three mentioned in this study are listed below.

1. Ordinary kriging: the expected value of the data is unknown but stationary. It is the same unknown value over the whole domain, which in mathematical terms is:

$$E[Z(x_i)] = m \qquad \{7.1\}$$

   This variety of kriging is used in the present study.
2. Universal kriging: the expected value of the data is both unknown and not constant but varies depending on the location, in other words there is a drift.
3. Lognormal kriging: this is ordinary kriging on lognormally distributed data.

7.2 OUTLINE OF THE THEORY OF ORDINARY KRIGING

The operation of kriging has two parts: estimation or the calculation of some weighted average value of a variable in an area, called the domain, followed by the estimation of the variance of this estimate.

The average value calculated by kriging is the BLUE. The calculation is done by finding those weights for averaging the samples that will minimize the variance of the estimate. This is accomplished by solving a system of linear equations called the kriging system. A detailed description of the system and its development is in Journel and Huijbregts (1978, p.303) and in David (1977, p.237).

The mining industry uses a variety of other estimators:
1. the polygon method, used at the Boundary Dam Mine,
2. simple average, called the extension estimator in geostatistics,
3. distance weighted averages.

These all use more or less arbitrary weights; therefore, they cannot guarantee a minimum variance estimator. In addition, they take into account neither the statistical structure of the data, in particular the variogram, nor the spatial arrangement of the data.

7.2.1 General Problem: to Find the BLUE

Samples $Z(\mathbf{x}_i)$, $i = 1, 2, 3, \cdots, n$ have been measured and Z is to be estimated: Z is the true average value over some domain. The estimate (Z^*) should be the best possible, which in this case is defined as the BLUE.† The estimator is determined by the krige system of equations which will be developed below by considering the BLUE in this order: estimate, linear, unbiased and best.

The objective is to estimate the true average value (Z) by Z^*, which will be some combination of the sample values $Z(\mathbf{x}_i)$ as in equation {1.2}, p.135.

Z^* could be any combination of $Z(\mathbf{x}_i)$; for example, equation {1.3}, p.135 or equation {1.2}, p.135. But in this study a linear combination is used:

$$Z^* = \sum_i a_i Z(\mathbf{x}_i) \qquad \{7.2\}$$

Where:
a_i = the weights for a linear combination.
A linear combination is used for the reasons listed on page 135 above and also because nonlinear estimates require extra assumptions beyond the intrinsic hypothesis which are difficult to verify. The problem now reduces to finding those weights (a_i) that will give the best estimate. If $a_i = 1/n$, then Z^* is the simple arithmetic mean of the sample data or the extension estimate.

The estimator should be unbiased, which means that in the long run Z^* will systematically neither over- nor underestimate Z. The unbiased condition is satisfied (David, 1977, p.238) if the intrinsic hypothesis is satisfied and if:

$$\sum_i a_i = 1 \qquad \{7.3\}$$

Finally, the estimate should be the best one, which is called the krige estimator (Z_k^*). The best estimator in this case is defined as the estimator with the minimum variance of estimation. This means that over the long run the estimate (Z_k^*) will come closer to the real value (Z) than any other estimate (Z^*).

The variance for any estimation (σ_z^2) is calculated by (David, 1977, p.28):

$$\sigma_z^2 = 2\sum_i a_i \bar{\gamma}(x_i, V) - \sum_i\sum_j a_i a_j \bar{\gamma}(x_i, x_j) - \bar{\gamma}(V,V) \qquad \{7.4\}$$

---

† The Best Linear Unbiased Estimator is defined on page 135.

Where:

V = the domain over which Z is averaged, in general it is a volume, but in this study it is an area.

$\bar{\gamma}(V_1,V_2)$ = the average variogram between regions V_1 and V_2.

This term is estimated by:

$$\bar{\gamma}(V_1,V_2) \cong (1/n_i n_j)\sum_i\sum_j \gamma(\mathbf{x}_i-\mathbf{x}_j) \qquad \{7.5\}$$

Where:

$\mathbf{x}_i$ is in V_1.

$\mathbf{x}_j$ is in V_2.

The summations are over all pairs of points, one from each region. The average variogram $\bar{\gamma}(V_1,V_2)$ is calculated by the following steps.

1. Consider all pairs of points, the first $\mathbf{x}_i$ in V_1 and the other $\mathbf{x}_j$ in V_2.
2. Calculate the vector separating them, $\mathbf{x}_i-\mathbf{x}_j$.
3. Calculate the variogram at this separation, $\gamma(\mathbf{x}_i-\mathbf{x}_j)$.
4. Average all these variogram values between each pair of points, $(1/n_i n_j)\sum_i\sum_j \gamma(\mathbf{x}_i-\mathbf{x}_j)$.

The two regions V_1 and V_2 can have these forms:

1. $V_1 = V_2$, in other words, the average variogram between all pairs of points in one area, the domain. This is called the average variogram of the domain and abbreviated to $\bar{\gamma}(V,V)$.
2. $V_1 = x_i$, in other words, one of the regions has been collapsed to a point x_i, usually a sample location. This is called the average variogram between the sample $Z(\mathbf{x}_i)$ and the domain and is written as $\bar{\gamma}(x_i,V)$. The average variogram is properly defined in terms of all points inside the sample being one of the pair of points. However, if the sample is small enough with respect to the domain, then one point in the centre of the sample is usually accurate enough. The smoothing introduced by this approximation is small. It is the same as the smoothing due to non-point support (Figure 1.4, p.144).
3. If both regions have been collapsed to points or samples at $\mathbf{x}_i$ and $\mathbf{x}_j$, then this is called the average variogram between the samples $Z(\mathbf{x}_i)$ and $Z(\mathbf{x}_j)$ and is written as: $\bar{\gamma}(x_i,x_j)$.

7.2.2 Auxiliary Functions

Calculating an average variogram that includes a non-point region is very difficult.† It has been explicitly solved for only a few cases, which must have all the following characteristics:

1. particular model, either the spherical, exponential,

† In the general case it involves a sextuple integral of the average variogram, one for each dimension in each region.

linear or log (de Wijsian),
2. rectangular domain,
3. simple spatial relationship between the sample and the domain. For instance the sample must be at the corner or along the side of the rectangular domain.

The resulting average variogram called auxiliary functions. Unfortunately, the samples and domains in this study do not fall into the simple patterns assumed by these functions, in which case it is necessary to evaluate the $\bar{\gamma}$ terms using numerical integration.

7.2.3 Kriging System of Equations

For the estimate to be the best, σ_z^2 must be a minimum with respect to the weights a_i, or:

$$\partial\sigma_z^2/\partial a_i = 0 \;; \text{ for } i = 1, 2, 3, \cdots, n \qquad \{7.6\}$$

Where:
$\partial\sigma_z^2/\partial a_i$ = the partial derivative of σ_z^2 with respect to a_i.
These equations, along with the unbiased condition of equation {7.3}, p.200, are n+1 equations in the n unknown a_i's. This over-determined system of equations is minimized using the standard Lagrange technique, which introduces an extra variable called the Lagrange parameter (μ) and minimizes a new system of n+1 equations in n+1 variables:

$$\partial[\sigma_z^2-2\mu(\sum_i a_i-1)]/\partial a_i = 0 \;; \text{ for } i = 1, 2, 3, \cdots, n$$

$$\partial[\sigma_z^2-2\mu(\sum_i a_i-1)]/\partial\mu = 0 \qquad \{7.7\}$$

These reduce to:

$$\sum_j a_i\bar{\gamma}(x_i,x_j) + \mu = \bar{\gamma}(x_i,V) \;; \text{ for } i = 1, 2, 3, \cdots, n$$

$$\sum_i a_i = 1 \qquad \{7.8\}$$

These can also be written in matrix form:

$$\begin{bmatrix} \bar{\gamma}(x_1,x_1) & \bar{\gamma}(x_1,x_2) & \cdots & \bar{\gamma}(x_1,x_n) & 1 \\ \bar{\gamma}(x_2,x_1) & \bar{\gamma}(x_2,x_2) & \cdots & \bar{\gamma}(x_2,x_n) & 1 \\ \vdots & & & & \\ \bar{\gamma}(x_n,x_1) & \bar{\gamma}(x_n,x_2) & \cdots & \bar{\gamma}(x_n,x_n) & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \bullet \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \\ \mu \end{bmatrix} = \begin{bmatrix} \bar{\gamma}(x_1,V) \\ \bar{\gamma}(x_2,V) \\ \vdots \\ \bar{\gamma}(x_n,V) \\ 1 \end{bmatrix}$$

or in simpler notation:

$$K \bullet A = D \qquad \{7.9\}$$

Where:
K = the krige matrix. See equation {7.5}, p.201 for a definition of the $\bar{\gamma}(x_i,x_j)$ terms.
A = $[a_1, a_2, \cdots a_n, \mu]'$
the solution, a column vector of the Lagrange parameter and the required weights for the krige estimate.
D = $[\bar{\gamma}(x_1,V), \bar{\gamma}(x_2,V), \cdots \bar{\gamma}(x_n,V), 1]'$
a column vector of the average variograms between each sample $\mathbf{x}_i$ and the domain V. See equation {7.5}, p.201 for a definition of the $\bar{\gamma}(x_i,V)$ terms.
This is called the krige system of equations. The required weights (A) are determined by solving this equation:

$$A = K^{-1} \bullet D \qquad \{7.10\}$$

Where:
K^{-1} = the inverse of K.
The components of A are μ and the weights (a_i) needed to calculate the reserve estimate that will minimize its variance.

7.2.4 The Estimators and Their Variances

The krige estimator (Z_k^*) is calculated by:

$$Z_k^* = \sum_i a_i Z(x_i) \qquad \{7.11\}$$

Where:
a_i = the elements of A, the solution vector, without μ.
The variance of the krige estimate (σ_k^2) is calculated by:

$$\sigma_k^2 = \sum_i a_i \bar{\gamma}(x_i,V) + \mu - \bar{\gamma}(V,V) \qquad \{7.12\}$$

Equation {7.4}, p.200 could also be used but that equation has more summations and is slower to compute.

The extension estimator (Z_x^*) or the arithmetic average of the data is calculated by:

$$Z_x^* = (1/n)\sum_i Z(\mathbf{x}_i) \qquad \{7.13\}$$

The extension variance (σ_x^2) is calculated by substituting $a_i = 1/n$ into equation {7.4}, p.200 to give:

$$\sigma_x^2 = (2/n)\sum_i \bar{\gamma}(x_i,V) - (1/n^2)\sum_i\sum_j \bar{\gamma}(x_i,x_j) - \bar{\gamma}(V,V) \qquad \{7.14\}$$

APPENDIX 8
DESCRIPTION OF THE KRIGING PROGRAM

This appendix contains an outline of the operation of the kriging program with the emphasis on the modifications made to original version from Flint (1978), brief lists of the input and output of the program, the best values to use for most of the input parameters, and the size of inaccuracies in the results due to the calculation method.

8.1 OPERATION OF THE KRIGING PROGRAM

The basic kriging program as written by Flint (1978) has the following steps:

1. input of the parameters and data as listed in Appendix 8.3.1 and Appendix 11 respectively.
2. calculation of the krige matrix *K*, and vector *D*, using these steps:
   A. construction of the numerical integration grid,
   B. computation of the $\bar{\gamma}$ terms by numerical integration,
   C. construction of *K* and *D* from the $\bar{\gamma}(x_i,x_j)$ and $\bar{\gamma}(x_i,V)$ terms respectively,
3. inversion of the matrix and calculation of the weights vector *A*, using IMSL subroutine LEQT1F,
4. calculation of the krige estimate Z_k^* and its variance σ_k^2,
5. output of the results as listed in Appendix 8.3.2 with an example in Appendix 13.

In addition, the program calculates the area of the domain and a default cell size.

8.2 MODIFICATIONS TO FLINT'S KRIGING PROGRAM

Flint's kriging program was modified to perform the numerical integrations twice instead of once, and also to calculate the extension estimate and its variance.

Auxiliary functions as described by Clark (1976) cannot be used because the domains to be estimated are irregular, not rectangular. Instead, to calculate $\bar{\gamma}(x_i,V)$ and $\bar{\gamma}(V,V)$, numerical integration must be done which introduces approximation errors.

This numerical integration is not the normal variety where a function is evaluated at each point in a region; instead it is the evaluation of a function (the variogram) for the distance between each pair of points in a domain. Integration is done by setting up a grid over the domain and evaluating the variogram either between the sample and each node for $\bar{\gamma}(x_i,V)$ or between each pair of nodes for $\bar{\gamma}(V,V)$. The smaller the grid cell size the closer the calculated value is to the true value. Normally the grid is completely inside the domain, and consequently, the calculated value will always underestimate the true value. This is because the grid will tend to use smaller **h**'s than if all of the points inside the domain were used. Shorter **h**'s mean that the variogram values will tend to be less and the average variogram lower.

The program was modified to calculate the value twice: once as above with all nodes just inside the domain and then again with the same grid extended by one node to just outside the domain. In the second calculation the average variogram should be overestimated, assuming no approximation errors. Thus, the two calculated values ought to bracket the true value if the cell size is small enough (about 20 m in this study). Finally, the program averages the two values for the best guess and reports the percent difference between the two estimates for judging the accuracy of the result.

Ordinarily, calculating the average variograms twice would not be necessary but in this study the seam thickness is so regular that the estimation variances are very small. The krige variance is largely† the result of the subtraction of two nearly equal large numbers: $\bar{\gamma}(x_i, V)$ and $\bar{\gamma}(V,V)$ in equation {7.12}, p.203. Slight approximation errors in these two numbers, particularly the latter, can produce large errors in the krige variance, to the point of a negative variance.

8.3 INPUT/OUTPUT OF THE KRIGING PROGRAM

Listed below is the input to, and the output from the modified kriging program.

8.3.1 Input Parameters and Data for the Kriging Program

Input may be entered interactively from terminal or from a file in batch. The input consists of:

1. sample measurements in a file which consist of:
   A. net thickness of the pit measurements or drill holes,
   B. location of the samples as X and Y coordinates,
2. the domain over which the average is estimated. This is in the form of a file of the X and Y coordinates of the digitized vertices of a polygon outlining the domain. The program also needs the FORTRAN format of the above two files.
3. a description of the variogram model, consisting of these parameters:
   A. the number of nested models, up to four, though only

† The Lagrange parameter (μ) is usually small in this study.

one or two were required in this study.
   B. C_0, the nugget effect,
   C. $C_1 \cdots C_4$, the sills of the nested models,
   D. $R_1 \cdots R_4$, the ranges of the models,
4. the cell size to be used in the numerical integration,
5. whether the elements of the inverted krige matrix are to be tested for accuracy and, if so, for how many digits.

8.3.2 Output From the Kriging Program

The program has two types of output: the kriged results and a list of the sample covariances, $\bar{\gamma}(x_i,V)$, with sample weights, a_i. These are listed below, with an example in Appendix 13.

The kriging results are presented (both on the screen and written to a file) in two portions:

1. an echo of the input parameters, including those listed above along with the following results of some intermediate calculations:
   A. the number of samples used,
   B. the area of the rectangle that encloses the domain. This is the size of the numerical integration grid; it helps in choosing an appropriate cell size that will balance computing speed and accuracy.
   C. a default cell size based on the size of the enclosing rectangle,
   D. the actual numbers of rows, columns and cells of the grid inside the domain,
   E. whether the accuracy test passed or failed for the specified number of digits.
2. the results of the kriging calculations:
   A. a calculation of the area of the domain over which the average was estimated,
   B. the kriged estimate,
   C. the extension estimate,
   Both of the estimates above include the following:
      a. the estimate itself; Z_k^* or Z_x^*,
      b. the estimation variance; σ_k^2 or σ_x^2,
      c. the covariance of the domain; $\bar{\gamma}(V,V)$,
      d. the average covariance of the samples to the domain; $\sum_i a_i\bar{\gamma}(x_i,V)$,
      e. the Lagrange parameter; μ.

The last three are used to calculate the estimation variance and their values are listed in order to judge the accuracy of the variance. If the values are large and close together, then the variance will be less accurate. Each of the above five values also has a percent difference reported. This is used to estimate the approximation error in the numerical integration.

The percent difference is also reported as a variance and standard deviation, calculated as if the over- and underestimation were a sample of two. This error in the estimate will add to the estimation variance for the total error in the estimate. However, the percent difference expressed as a variance was usually several orders of magnitude less than the estimation variance and for this reason, was neglected.

The sample covariances and weights are written to a file in two portions:

1. a brief echo of the important input parameters in order to identify the results,
2. the following information on each sample:
   A. ID number,
   B. location; x_i as X and Y coordinates,
   C. sample value; $Z(x_i)$,
   D. the covariance of the sample to the domain; $\bar{\gamma}(x_i, V)$,
   E. the krige estimate weight; a_i.

This output is used to verify the input and to determine the contribution of each sample to the estimate and estimation variance. Some samples may be screened or otherwise contributing little, and so could be dropped from future estimations.

8.4 SENSITIVITY OF THE KRIGING RESULTS TO THE INPUT PARAMETERS

Kriging and the program to carry it out are complex, as result, it is not possible to accurately predict the effects of input parameter variability on the output. Therefore, the program was tested using either artificial data with known results or data from this study for a more realistic test.

The following input parameters were empirically tested:

1. number of samples to be kriged,
2. accuracy of the variogram model parameters,
3. grid cell size.

The other inputs, the data and the domain, were thoroughly checked and assumed to be correct.

8.4.1 Number of Samples to be Kriged

The number of samples to include in the kriging is a compromise between accuracy and computing cost. More samples will produce a more accurate estimate, but the number of samples is one less than the number of simultaneous equations to be solved, so the costs rise rapidly with the number of samples included. All samples inside the domain should be used, plus those samples very near the domain. But data beyond the largest range is uncorrelated with the domain, and having little effect on the reserve estimate, can be ignored. Between these two limits, as samples get further from the domain, they are screened out, that is, their effect is reduced by closer samples (David, 1977, p.258). In this study the screen effect is strong because the relative nugget effect is low. The minimum number of samples that produces a stable estimate and variance was determined experimentally.

Computer hardware and software impose some upper limits on the number of samples. The program can handle a maximum of 200† samples on the regular CPU and 180‡ on the much cheaper array processor. Unfortunately, most domains studied had more than 180 samples. As the number of samples increases, the more ill-conditioned†† the krige matrix becomes, until eventually it is effectively singular and the subroutine cannot invert it. This was only a problem when kriging pit 1 because it is so large.

Each of the pits was kriged using all the drill holes inside the pit plus all the drill holes inside successively larger aureoles around the pit (Figure 8.1). The values of the krige and extension estimates and their standard deviations were plotted against both the distance of the aureole from the pit boundary and the number of samples kriged.

Pit 2A (Figure 8.2) is representative of all the pits. The krige estimate and variance settled down after a certain aureole size and number of samples was reached. Between the different pits, this point was more constant in terms of the aureole size than in terms of the number of samples (Table 8.1). The kriged estimate and variance settled down at about 40 samples. The extension estimate and variance do not settle down, possibly because the numbers become more biased with more samples, though 40 samples is also a reasonable number to calculate them as well.

Samples out to 200 m away from the domains were used as a conservative limit. When estimating with drill holes, this aureole around any pit or bench included the minimum of 40 samples required for stability. However, this criterion produced more than the software limit of 200 samples for estimates using pit data, so only the nearest 200 were used. If all available pit samples were used, the results should not change very much for the following reasons.

1. The seam is very regular and there is no drift.
2. The nugget effect is low, so distant samples will be

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† This limit is easily changed.
‡ This limit is impossible to change.
†† 'Ill-conditioned' means the matrix is nearly singular, making A very sensitive to small changes in the elements of K.

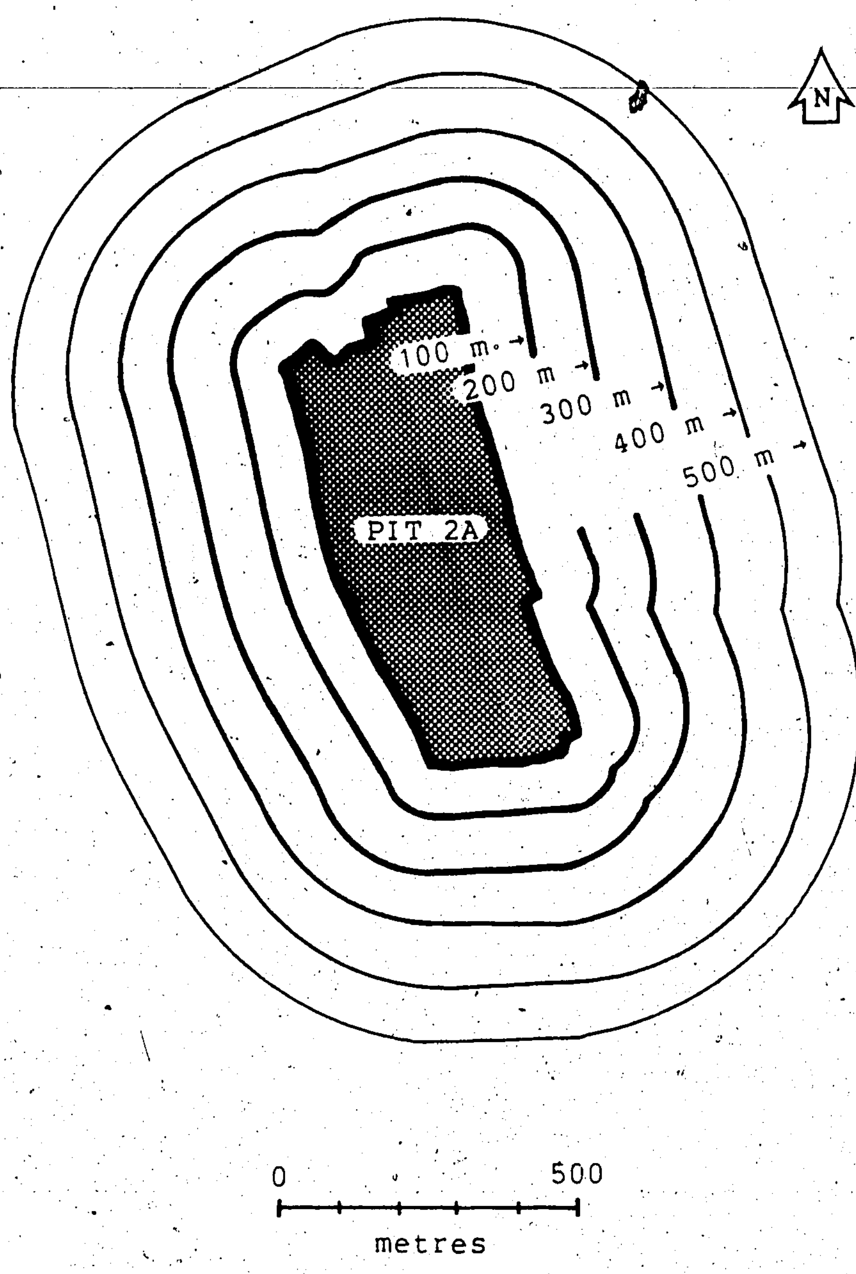

Figure 8.1 Aureoles of data used to determine the best number of samples for kriging

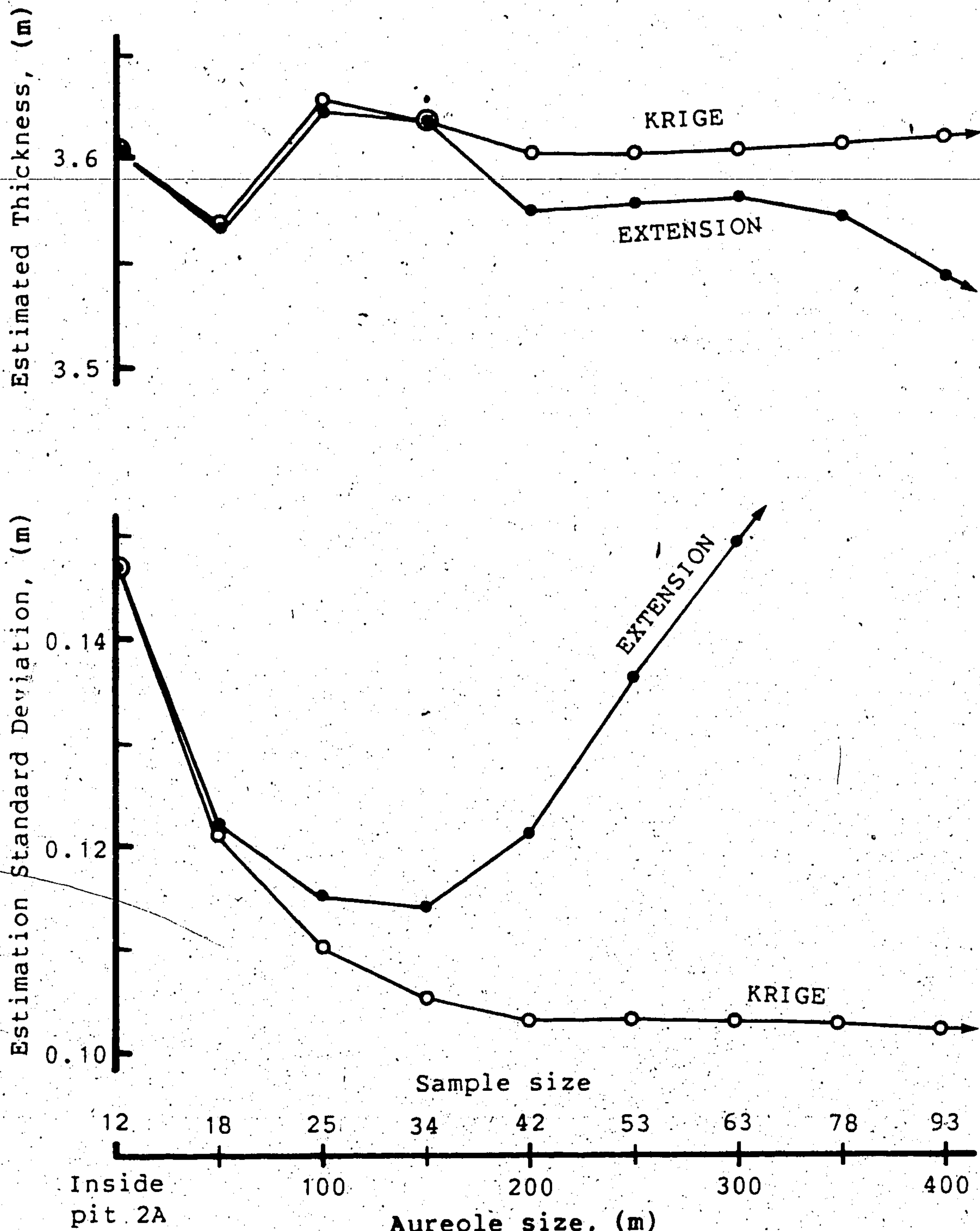

Figure 8.2 Thickness estimate and standard deviation versus the aureole size for pit 2A

Table 8.1 **Sample size and aureole distance required to produce a stable estimate**

Pit	Kriging		Extension	
	Sample size	Aureole distance (m)	Sample size†	Aureole distance† (m)
Pit 1	42	50	50	70
Pit 2A	38	200	26	150
Pit 3	42	200	38	150

† The sample size and aureole distance are averages of where the estimate and its variance stabilizes.

screened.

3. The estimate is stable for this number of samples (Figure 8.2, p.210).

8.4.2 Accuracy of the Variogram Model Parameters

Kriging is very robust with respect to the model chosen for the variogram (Journel and Huijbregts, 1978, p.233 and David, 1977, p.113) and is moderately robust with respect to the parameters chosen for the model. It was not possible to test the effect of different models on the results because both the modelling and the kriging program used can handle only a spherical model.

The effect of the parameters was tested experimentally. Different sets of parameters were input to the kriging program with all other inputs kept constant and the results compared for the pit data and again for the drill hole data (Table 8.2). Fairly large changes in the parameters could be made without large changes in the results. The estimates are within 0.1σ to 0.5σ of each other, or expressed in thicknesses they are within 1.5 cm over a total of 3.5 m; in other words identical. The estimation variances, expressed as a standard deviation, were much more variable, in some cases up to 50%. However, the values are so small that the absolute differences are also small.

8.4.3 Cell Size

Cell size is the distance between the nodes of the grid used for the numerical integration. The choice of the cell size is a compromise between increased accuracy and rapidly increasing computing costs as the cell size is reduced. The effect of the cell size was checked experimentally using data for which the results were determined by exact calculation using auxiliary functions. Brooker (1979), Clark (1979*a*, p.106) and some hand calculations provided the worked examples. The program actually uses the number of cells rather than the cell size. The number of cells was varied and the results compared to the exact values using auxiliary functions. Below about 50 cells the results were very erratic. Above about 200 cells the results were stable; for this reason, 200 to 300 cells were used in this study, which produces a cell size of about 20 m.

Flint (1978, p.67) also needed about 200 cells. Clark (1979*a*, p.121) states that by general agreement the number of cells should be in the range 64 to 100. David (1977, p.280) writes that 100 cells are necessary to calculate $\bar{\gamma}$ terms with a few percent precision but that fewer than 10 are needed for a stable estimate. This study required more cells because the thickness population variance is so low,

Table 8.2 Robustness of kriging with respect to the variogram model parameters

	Drill holes		Pits		
Model ⇒	This study	Flint†	North 1	North 2	South
C_0	0	0.092 m^2	0	0.03 m^2	0
C_1	0.26 m^2	0.37 m^2	0.35 m^2	0.04 m^2	0.07 m^2
a_1	150 m	1000 m	50 m	550 m	200 m
C_2	0.28 m^2	0.51 m^2	0.35 m^2		
a_2	1800 m	4000 m	550 m		
Kriged pit data					
Thickness	3.7625	3.7613	3.7615	3.7610	3.7619
Variance	0.00026	0.00095	0.00010	0.00030	0.000048
Standard deviation	0.016	0.031	0.010	0.017	0.007
Kriged drill hole data					
Thickness	3.6079	3.6133	3.6171	3.6173	3.5981
Variance	0.0097	0.0095	0.0027	0.0026	0.0017
Standard deviation	0.098	0.097	0.052	0.051	0.041

† From Flint (1978, p.107).

which makes the calculated results more sensitive to the inputs.

8.5 INACCURACY OF THE RESULTS DUE TO NUMERICAL APPROXIMATIONS

The accuracy of the computed results is affected by numerical approximations at two places in the program: numerical integration of the $\bar{\gamma}$ terms and inverting the krige matrix.

8.5.1 Inaccuracies Due to Numerical Integration

The accuracy of the numerical integration was tested using the same data as in Appendix 8.4.3. so that the computed value could be compared to the exact value. Krige estimates and variances were computed for several cell sizes (Figure 8.3 and Figure 8.4)†. The percent absolute difference between these results and the result calculated by Clark (1979*a*, p.109) using auxiliary functions was plotted as the 'error'. The 'difference' is the percent absolute difference between the two approximations computed by the machine; the over- and under-approximation as described on page 205. To judge the accuracy of the approximation for real data the 'error' must be known, but the machine can only compute the 'difference'. Fortunately, if a large enough number of cells is used the two machine answers should bracket the exact value. As a result the 'difference' should be greater than the 'error', so that the 'difference' can be used as a conservative estimate of the accuracy of the results.

The variance behaves as it should but the percent error of the krige estimate is somewhat erratic and not always less than the percent difference. This may be due to the particular location of the grid with respect to the domain. However, the 'error' and the 'difference' for the estimate are always within an order of magnitude of each other. The absolute values on the log-log plot are so small that the program can be considered to be accurate and the percent difference reported by the program a good indicator of its accuracy.

This study generally used about 200 cells for which the kriged average was always within 1% of the true value and usually closer. The krige variance was always within 10% of the true value and often within 2% of it.

The extension estimate was always exact of course. The extension variance is estimated in a similiar, though not

† Note that both of these figures are on a log-log scale.

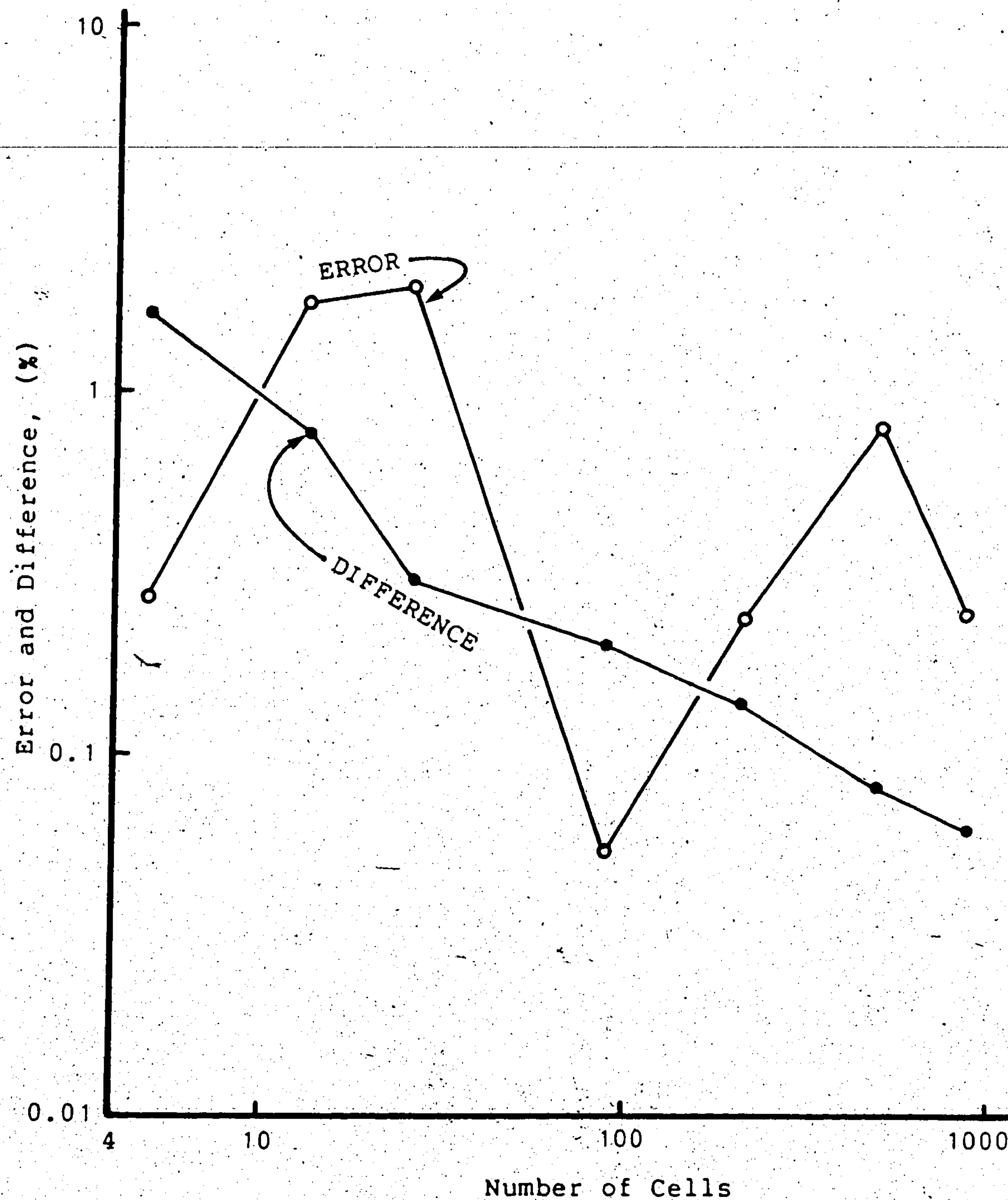

Figure 8.3 Calculation error of the krige estimate versus the number of grid cells

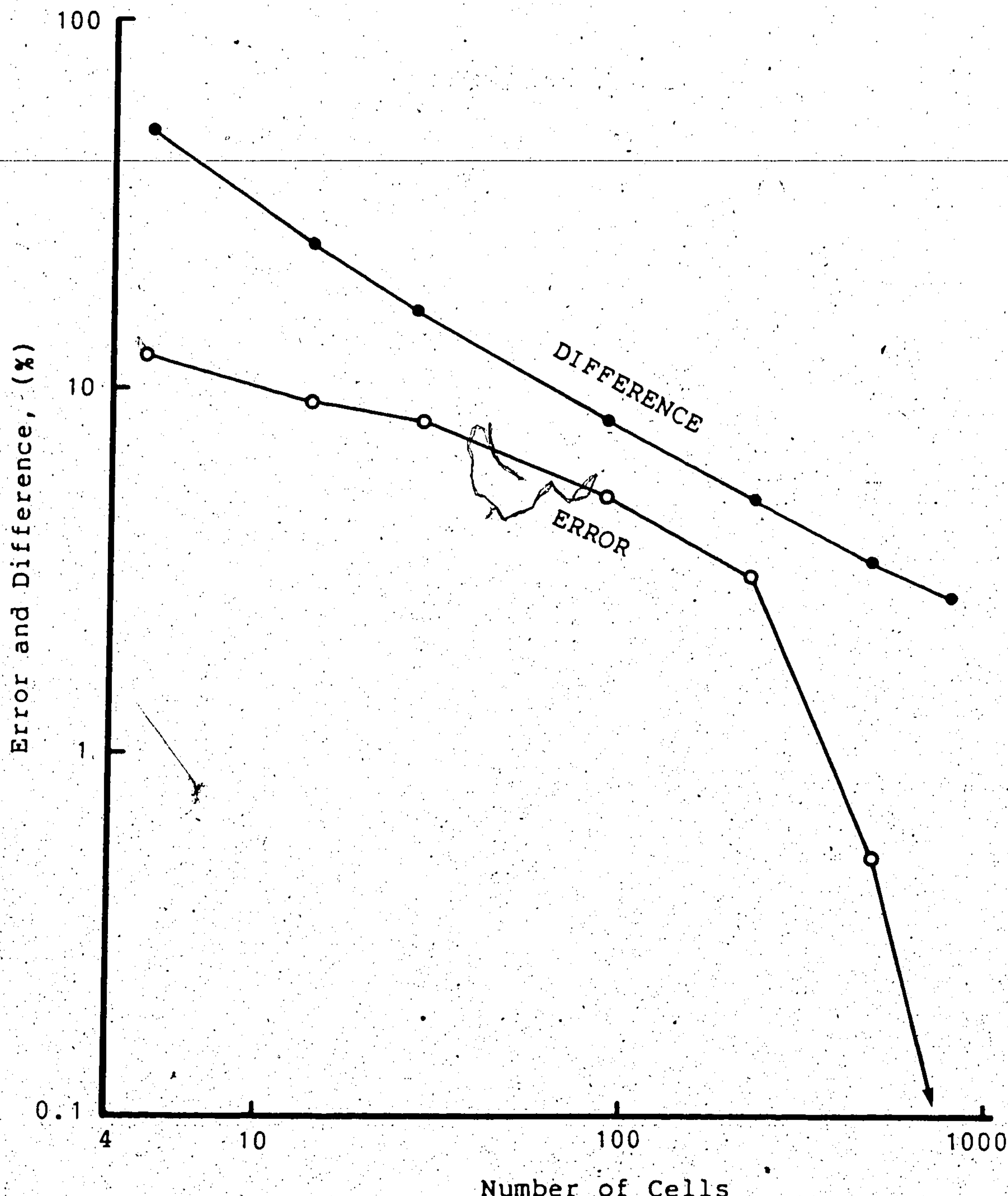

Figure 8.4 Calculation error of the krige variance versus the number of grid cells

identical, manner to the krige variance (equation {7.4}, p.200 and equation {7.12}, p.203 respectively). Therefore, it will have about the same size error as the krige variance.

8.5.2 Inaccuracies Due to Matrix Inversion

The accuracy of the krige matrix inversion is tested by the inversion subroutine and reported as the number of significant digits. As the number of samples increases the number of significant digits decreases; it is usually two digits (about 1%) and occasionally one or three digits (about 10% or 0.1%). This is the accuracy of the individual weights (a_i) calculated, of which there are 50 to 100 for a kriged average in this study.

The estimate is a linear combination of these a_i's and David (1977, p.281) states that "...linear estimates are deceptively robust." This is because the calculation errors in the weights should tend to cancel out, with the result that the calculation error of the kriged estimate and variance will be less than the significant digit error and much less than the inaccuracies due to numerical integration. For this reason and because the results are close to the exact values for the test data, any inaccuracies due to inversion of the krige matrix by the subroutine were ignored.

APPENDIX 9
FORMULAS FOR THE PROPAGATION OF ERRORS

Estimates and variances of the reserves (T) and the recovery (R) are calculated from parameter estimates of the input variables; A, d & Z and CLO & T respectively, using the formulas below. The general expression is considered with any dependent variable (T or $\mathcal{R}$) related to the input variables (χ_1, χ_2,$\cdots$,χ_n) by the relation:

$$T = f[\chi_1, \chi_2, \chi_3, \cdots, \chi_n] \tag{9.1}$$

or:

$$\mathcal{R} = f[\chi_1, \chi_2, \chi_3, \cdots, \chi_n] \tag{9.2}$$

Where:
T, $\mathcal{R}$, χ_i = general variables.
E[·] and VAR[·] are estimated for two cases, depending on whether the input variables (χ_i) are correlated (for $\mathcal{R}$) or not (for T). For the tonnage they are uncorrelated, but for the recovery they are highly correlated. These formulas, given below and developed by Hahn and Shapiro (1967, p.252), all assume that fourth ~~and~~ higher order terms effectively disappear. The high order terms in the formulas below are either zero or negligibly small, so higher terms can be ignored.

9.1 ESTIMATE AND VARIANCE WHEN THE INPUT VARIABLES ARE UNCORRELATED

The thickness (Z^*) has no drift; consequently, the expected value is everywhere the same, in which case it will not vary with the area of the domain. The density was treated as a non-regionalized variable, which implies that it too has no drift and therefore will not vary with the area of the domain. Finally, Z^* and d must be assumed to be uncorrelated since there is not enough density data to determine COV[Z^*,d]. In fact, there will be a slight correlation because portions of the seam with more clay bands will probably be both thicker and denser, since clay compacts less and is heavier than coal. However, the coal at the Boundary Dam Mine has a low ash content (Table 1.1, p.27) and therefore, the covariance term should be small. The input variables; A, d and Z^*, are uncorrelated and so any covariance terms vanish.

When the input variables are uncorrelated, that is:

$$\mathrm{COV}[\chi_i, \chi_j] = 0 \text{ ; for all } i \neq j$$

E[T] is estimated by:

$$E[T] = f[E[\chi_1],E[\chi_2], \cdots ,E[\chi_n]] + \tfrac{1}{2}\sum_i(\partial^2T/\partial\chi_i^2)VAR[\chi_i] + \cdots \qquad \{9.3\}$$

In this case, where T = T:

$$T = \prod_i \chi_i$$

therefore:

$$\partial^2T/\partial\chi_i^2 = 0$$

Which means that the estimate of E[T] reduces to:

$$E[T] = f[E[\chi_1],E[\chi_2], \cdots ,E[\chi_n]] \qquad \{9.4\}$$

The variance is estimated by:

$$VAR[T] = \sum_i(\partial T/\partial\chi_i)^2VAR[\chi_i] + \sum_i(\partial T/\partial\chi_i)(\partial^2T/\partial\chi_i^2)E[\chi_i-E[\chi_i]]^3 + \cdots \qquad \{9.5\}$$

Where:
$\partial^2T/\partial\chi_i^2$ = second partial derivative of T with respect to χ_i
As for equation {9.3}, p.219 second and higher order terms vanish to give:

$$VAR[T] = \sum_i(\partial T/\partial\chi_i)^2VAR[\chi_i] \ ; \text{ for } i = 1,\ 2,\ 3 \qquad \{9.6\}$$

Where:
T = T
χ_1 = A
χ_2 = d
χ_3 = Z^*, either Z_k^* or Z_x^*
This can be expanded to:

$$VAR[T] = d^2Z^{*2}VAR[A] + A^2Z^{*2}VAR[d] + A^2d^2VAR[Z^*] \qquad \{9.7\}$$

Where:
$VAR[Z^*] = \sigma_k^2$ or σ_x^2 depending on whether $Z^* = Z_k^*$ or Z_x^*.
An alternative version of the above is expressed in terms of the relative variances:

$$VAR[T]/T^2 = VAR[A]/A^2 + VAR[d]/d^2 + VAR[Z^*]/Z^{*2} \qquad \{9.8\}$$

Where:
$VAR[\chi]/\chi^2$ = the relative variance of χ.

Relative variance, always reported as a percent in this study, is a convenient way to express the error in a variable. This error can be split into simple additive contributions from several sources, unlike the raw variance.

9.2 ESTIMATE AND VARIANCE WHEN THE INPUT VARIABLES ARE CORRELATED

The mine engineers expend considerable effort to raise the correlation between the in-place coal and the coal-loaded-out as close to one as possible. Consequently, the recovery and its variance will be biased if calculated using formulas from the previous section. Unbiased values require the formulas from this section.

9.2.1 Mean of the Recovery

When some of the input variables are correlated, that is:

$$COV[\chi_i, \chi_j] > 0 \text{ ; for some } i \neq j$$

then $E[\mathcal{R}]$ is estimated by:

$$E[\mathcal{R}] = f[E[\chi_1], E[\chi_2], \cdots, E[\chi_n]] + \tfrac{1}{2}\sum_{i,j}\sum(\partial^2\mathcal{R}/\partial\chi_i\partial\chi_j)COV[\chi_i, \chi_j] + \cdots \qquad \{9.9\}$$

When χ_i = CLO, then $\partial^2 R/\partial CLO^2 = 0$, and all terms involving it vanish. When χ_i = T, the terms are negligibly small. This can be demonstrated with an order-of-magnitude calculation using the following data derived from Table 5.10, p.112.

1. $CLO \simeq T = 5\times10^5$ tonnes,
2. $VAR[T] = 1\times10^8$ tonnes^2,
3. $COV[T,CLO] \simeq 1\times10^{-1}$ tonnes^2.

COV[T,CLO] is estimated by remembering that the correlation coefficient between reserves (T) and the coal-loaded-out (CLO) will be approximately one, therefore:

$$1 \simeq COV[T,CLO]/\sqrt{(VAR[T]VAR[CLO])}$$

or:

$$COV[T,CLO] \simeq \sqrt{(VAR[T]VAR[CLO])}$$

The coal-loaded-out figures are reported to six significant figures (Table 5.5, p.99), which implies that:

$$VAR[CLO] \simeq 1\times10^{-10}$$

There are two terms involving T in equation {9.9}, p.220. When i = j then:

$$(\partial^2 R/\partial T^2)VAR[T] = (200/T^2)VAR[T] \simeq 1\times10^{-1}$$

and when $i \neq j$ then:

$$(\partial^2 R/\partial T\partial CLO)COV[T,CLO] \simeq -(100/T^2)COV[T,CLO] \cong -1x10^{-11}$$

Both terms are negligibly small when compared to a typical recovery of about 100% with an estimation variance of about 4%. As a result, equation {9.9}, p.220 reduces to:

$$E[\mathcal{R}] \cong f[E[\chi_1],E[\chi_2],\cdots,E[\chi_n]] \qquad \{9.10\}$$

9.2.2 Variance of the Recovery

The variance is estimated by:

$$VAR[\mathcal{R}] = \sum_i\sum_j(\partial\mathcal{R}/\partial\chi_i)(\partial T/\partial\chi_j)COV[\chi_i,\chi_j] + \Omega + \cdots \qquad \{9.11\}$$

Where:
$COV[\chi_i,\chi_j] = VAR[\chi_i]$; if $i = j$
Ω = higher order terms in $(\partial\mathcal{R}/\partial\chi_i)(\partial^2\mathcal{R}/\partial\chi_j\partial\chi_k)$
This formula makes the same assumptions as were made above, and are met for the same reasons. The first term can be decomposed into a sum of variances (when $i = j$) and a sum of true covariances (when $i \neq j$). Using the same data as was used for the mean, the true covariance terms are approximately $1x10^{-7}$. The partial derivatives in Ω evaluate to approximately $1x10^{-13}$, which the attendant expectations will not substantially increase. All these and higher order terms are negligible compared to a typical recovery estimation variance of 4%. Equation {9.11}, p.221 with the appropriate variable substitutions becomes:

$$VAR[R] \cong (CLO/T^2)^2 VAR[T] \qquad \{9.12\}$$

or in terms of the relative variance:

$$VAR[R]/R^2 \cong VAR[T]/T^2 \qquad \{9.13\}$$

APPENDIX 10
VARIANCE OF THE AREA OF PROJECTED PRODUCTION

The reserves estimates require the variance of all the input variables, in particular the area. If the area is not predefined, then it has an error associated with it and therefore, VAR[A] > 0 and equation {9.7}, p.219 or equation {9.8}, p.219 must be used. In this case it is necessary to estimate the area (and its variance) of the seam removed by the mining operation† in a given amount of time. One year is used as an example.

The mine plan (described on page 38 above) has some benches with dates on them showing the limits of excavation. These are rarely at one year intervals or any other constant unit of time. In order to be able to estimate the area uncovered in one year, and to be able to use all the available data, the amount excavated was regressed against the time to excavate for all the dated benches.

This method assumes that the dates on the map accurately show the extent of the excavation. There is no reason to suppose that the mine surveyors have been inaccurate. The date could mark the time of a survey rather than the time of the advance, but this seems unlikely. This method also assumes that the data shows the limits of coal removal and not just the limits of overburden removal. If the dates are for the latter, then the coal removal will usually lag by a fairly constant distance behind, and therefore, the length of coal removed along the bench will be about the same as the length of the overburden removed in the same time period. This possibility will increase the variability of the data.

The amount excavated can be measured using three methods, in order of increasing complexity these are:

1. Measure the length of the bench, then assume a constant width to convert to area.
2. Measure the area excavated directly.
3. Measure the volume of material excavated (mostly overburden), then assume a constant overburden depth to convert to area.

The method used should be simple, but more importantly it should produce as low a variance for the area as possible.

10.1 DIRECTLY MEASURING THE AREA OPENED UP

The area excavated on the dated benches could be measured directly. However, the extra effort of planimetry was not justified for the following reasons. Firstly,

† A description of the mining methods can be found on page 34 above.

planimetering is not very accurate, particularly on areas with a large perimeter/area ratio, such as the benches. Secondly, as shown below, regressing the length of the bench was accurate enough.

10.2 VOLUME AND LENGTH OF THE AREA OPENED UP

The most reasonable method is to regress the volume excavated against the time required, since the equipment moves a volume, not an area. The volume of the dated benches was determined from the length along the bench, the average width determined below and the overburden thickness, all of which were recorded on the mine plan.

The thickness of the seam has a low variance and is small (3 m) compared to the overburden thickness (40 to 70 m); consequently, it will not affect the variability of the total volume removed.

The overburden thickness is a regionalized variable; therefore, its average thickness under each dated bench could be determined by kriging. However, the thickness varies only slightly under each bench, so the extra effort did not seem worthwhile. The overburden measured by all holes in and immediately around the bench was arithmetically averaged for the overburden thickness in the bench. The volume of material removed daily was calculated, then the mean, variance and coefficient of variation of the volume of the benches from each pit were estimated. Similarly, the coefficient of variation of the bench length was calculated and compared in Table 10.1. The variability of the bench volume is as large as the variability of the bench length for each pit; thus, no increase in accuracy is gained by measuring the volume of the benches rather than the length of the benches.

10.3 REGRESSION OF THE LENGTH OF THE AREA OPENED UP

The area excavated each year was estimated by regressing the curvilinear length along each bench (L) against the time (t) required to dig it (Figure 10.1). The average length per day of each dated bench is unimodal; therefore, a single machine or several with the same capabilities must have been used. The dated benches that spanned more than one cut (4 out of a total of 13) were not significantly† below the regression line (two were above, two below); thus, the time to turn around the equipment and reverse the direction of excavation was not important and was not corrected for.

† $\alpha = 0.05$

Table 10.1 **Coefficients of variation of the bench length and bench volume**

Pit	Number of dated benches	Coefficient of variation Length	Volume
Pit 1	5	59%	60%
Pit 2A†	3	21%	21%
Pit 3	5	49%	58%
All pits	13	49%	58%

† The thickness was constant over the all of pit 2A so the variability will not be different between the length and the volume. It was included in order to compare the variabilty between pits to within pits.

8

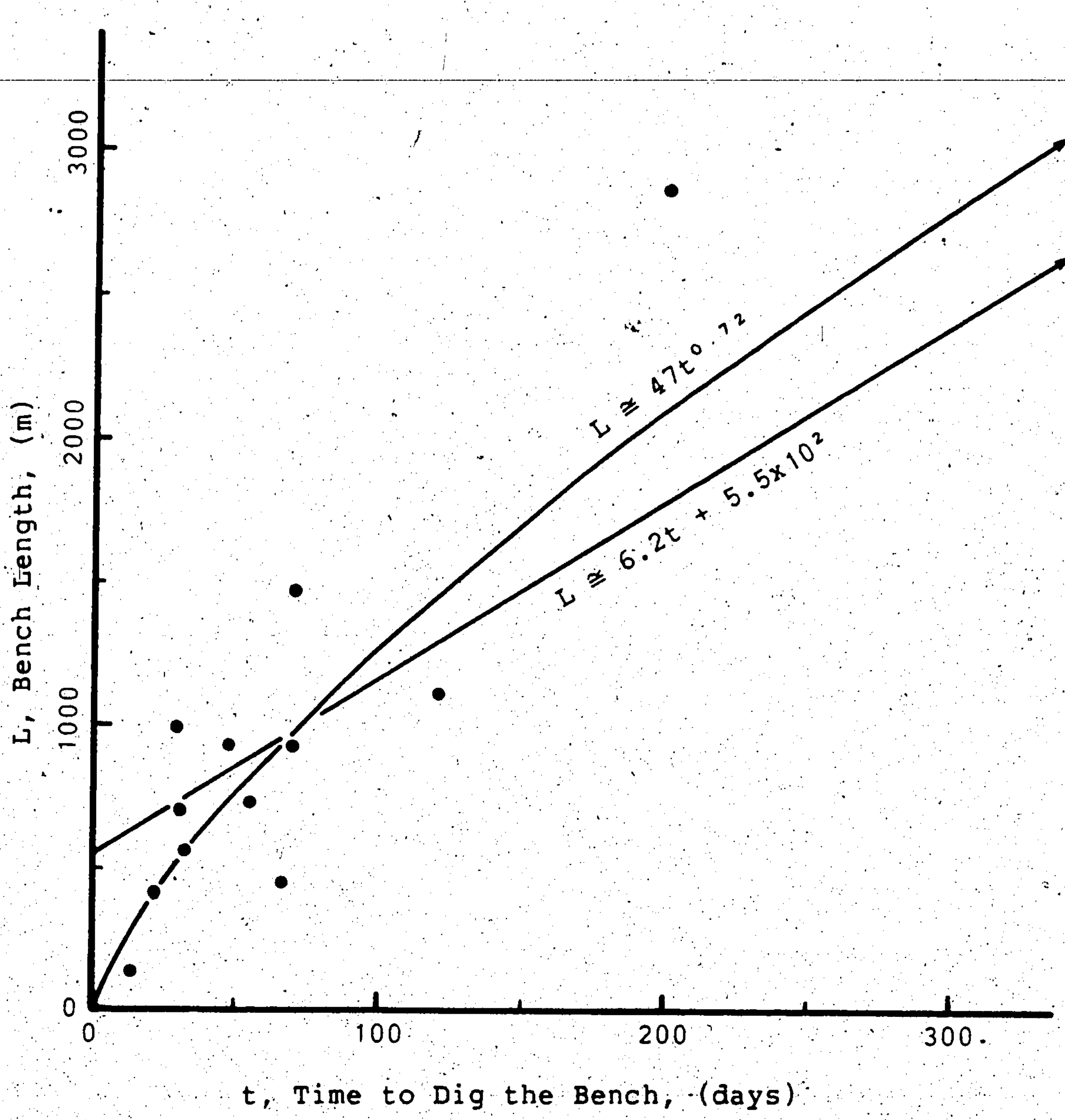

Figure 10.1 Regression of the bench length against the time to dig the bench

Two models were considered for the regression line, a linear model:

$$L = \beta t + a \qquad \{10.1\}$$

and a power model:

$$L = a t^{\theta} \qquad \{10.2\}$$

Where:
L = the curvilinear length along a bench.
t = the time required to remove a bench.
a, β, θ = regression parameters.

The best fit regression parameters are listed in Table 10.2. The confidence intervals on r^2 have a large overlap; therefore, the models fit equally well. The best fit equations are:

$$L \cong 6.2t + 5.5x10^{2} \qquad \{10.3\}$$

and:

$$L \cong 47t^{0.72} \qquad \{10.4\}$$

The linear model has a positive value of a ($5.5x10^2$), which is both significantly† non-zero and physically impossible. Even though the linear model has the advantage of simplicity, the power model was chosen for the following reasons.

1. The best fit straight line is incorrect because of its impossible intercept (a).
2. The data may be lognormally distributed since they are squeezed towards zero, which is an absolute bound on the data. Lognormally distributed data is linearly regressed by means of the power model. The power model is linearized by a log transformation to:

$$\ln[L] \cong \beta \ln[t] + \ln[a] \qquad \{10.5\}$$

3. The power curve has a more reasonable shape. Firstly, it passes through the origin (practically speaking). The true curve probably intersects the time axis at the time required to set up or turn around the operation at the end of a bench. Secondly, the curve is concave down, which implies that the longer the equipment is operating on a bench, the more likely a major breakdown becomes and the lower the average and instantaneous rate of advance becomes.

The estimated advance in one year is $3.28x10^3$ m, using equation {10.4}, p.226. The problem with using a power

† $\alpha = 0.05$

Table 10.2 **Regression parameters of the bench length with the time to dig the bench**

Parameter	Linear model	Power model
α	5.5×10^2 m	47 m/day
β	6.2 m/day	
θ		0.72
$+0.05$ CI	0.96	0.88
r^2	0.81	0.72
-0.05 CI	0.47	0.29

model is that the variance on an estimated advance is asymmetric. The standard deviation is $+1.04 \times 10^3$ m and -0.79×10^3 m. Unfortunately, equation {9.7}, p.219 requires a symmetric variance. Therefore, the absolute values of the plus and minus standard deviations were averaged and squared for an estimate (8.37×10^5 m^2) of the symmetric variance on the length. This is probably a conservative estimate since the variance using the linear model is an order of magnitude less: 7.44×10^4 m^2. The average bench length and its error are listed in Table 10.3.

A few hundred bench widths were measured at evenly distributed points on the mine plan. The statistics for the bench width were estimated using classical statistics. The average bench width and its error are listed in Table 10.3, p.229.

The average area excavated in one year was estimated using the preceding values for the length and width along with equation {9.7}, p.219, with the appropriate variable changes. The average and its error are listed in Table 10.3, p.229. The formula assumes that the length and the width are independent. In fact, they are not strictly independent, since in one year more of a narrower bench could be cut. This covariance should be small since the bench width is so regular. The width contributes 1.6% to the relative variance of the area with the rest coming from the length.

Table 10.3 **Statistics of the bench length, width and area**

	Mean	Variance	Coefficient of variation	Relative variance†
Bench length excavated in one year				
	3.28×10^3 m	8.37×10^5 m^2	28%	7.8%
Average bench width				
	31.8 m	16.2 m^2	13%	1.6%
Bench area excavated in one year				
	1.04×10^5 m^2	1.02×10^9 m^4	31%	9.4%

† The relative variances are smaller than the coefficients of variation because both are actually decimal fractions less than 1, but written as a percent for convenience.

APPENDIX 11
SAMPLE OF DATA

11.1 SAMPLE OF DATA FROM PIT 2A

ID = Sample number; the first digit is the bench number,
Coordinates = Metres north and east from the SW corner of NE-33-1-8-W2M.
F = A flag indicating the data source; 1=pit, 3=drill hole.
N = The number of splits.
All thicknesses are in metres; ***** indicates missing data.

ID	Coordinates X	Coordinates Y	F	N	Thickness gross	Thickness net	Coal bottom	Parting	Coal top
201	1750.4	844.7	1	1	*****	*****	*****		
202	1738.1	873.3	1	2	5.090	3.932	3.018	1.158	0.914
203	1725.3	900.8	1	2	4.572	3.962	3.048	0.610	0.914
204	1713.8	928.4	1	2	4.420	3.993	3.078	0.427	0.914
205	1700.1	956.1	1	2	4.298	3.932	3.018	0.366	0.914
206	1687.2	984.1	1	2	4.206	*****	*****	0.244	*****
207	1673.5	1012.6	1	2	4.359	4.115	3.261	0.244	0.853
208	1660.7	1039.0	1	1	3.962	3.962	3.962		
209	1647.2	1066.1	1	2	4.511	4.206	3.353	0.305	0.853
210	1631.2	1091.8	1	2	4.237	3.993	3.170	0.244	0.823
211	1616.2	1118.5	1	1	3.871	3.871	3.871		
212	1605.2	1146.4	1	1	3.840	3.840	3.840		
213	1591.1	1174.6	1	1	3.932	3.932	3.932		
214	1586.8	1206.8	1	1	3.871	3.871	3.871		
215	1579.2	1239.7	1	1	3.962	3.962	3.962		
216	1572.9	1265.8	1	1	3.962	3.962	3.962		
217	1562.3	1296.2	1	1	3.962	3.962	3.962		
218	1556.4	1324.0	1	1	3.780	3.780	3.780		
219	1551.3	1354.4	1	1	3.780	3.780	3.780		
220	1546.3	1383.3	1	1	3.566	3.566	3.566		
221	1539.9	1412.7	1	1	3.597	3.597	3.597		
222	1534.6	1442.6	1	1	3.627	3.627	3.627		
223	1528.3	1472.8	1	1	3.627	3.627	3.627		
224	1521.6	1501.9	1	1	3.231	3.231	3.231		
225	1520.5	1530.9	1	1	3.231	3.231	3.231		
301	1547.0	1553.6	1	1	*****	*****	*****		
302	1550.3	1526.8	1	1	3.353	3.353	3.353		
303	1558.7	1496.3	1	1	3.505	3.505	3.505		
304	1565.7	1466.7	1	1	3.658	3.658	3.658		
305	1574.8	1438.3	1	1	3.536	3.536	3.536		
306	1582.1	1408.1	1	1	3.566	3.566	3.566		
307	1590.3	1379.5	1	1	3.566	3.566	3.566		
308	1597.8	1349.4	1	1	3.597	3.597	3.597		
309	1604.2	1320.5	1	1	3.597	3.597	3.597		
310	1607.6	1289.8	1	1	3.566	3.566	3.566		

11.2 SAMPLE OF DRILL HOLE DATA

ID = The sample number, in sequential order, book-fashion, from the northwest corner of the study area
Coordinates = Metres north and east from the SW corner of NE-33-1-8-W2M.
F = A flag indicating the data source; 1=pit, 3=drill hole.
N = The number of splits.
All thicknesses are in metres; **** indicates missing data.

ID	Coords X	Y	F	N	Thickness gross	net	Coal bottom	Pt	Coal	Pt	Coal	Pt	Coal top
1	2021	3086	3	1	3.35	3.35	3.35						
2	2124	3087	3	2	3.20	3.04	0.30	0.15	2.74				
3	2226	3087	3	1	****	****	****						
4	2324	3088	3	4	4.08	2.62	0.54	0.12	0.54	0.82	0.79	0.51	0.7
5	1923	2989	3	3	3.41	2.95	0.30	0.15	0.45	0.30	2.19		
6	2021	2986	3	2	3.65	3.41	0.30	0.24	3.10				
7	2123	2986	3	3	3.84	3.17	0.48	0.15	0.64	0.51	2.04		
8	2219	2986	3	3	3.59	3.01	0.45	0.15	0.33	0.42	2.22		
9	2324	2987	3	4	3.53	2.25	0.36	0.42	0.45	0.54	0.79	0.30	0.6
10	2435	2987	3	4	3.87	3.17	1.28	0.18	0.30	0.21	0.76	0.30	0.8
11	1221	2883	3	2	2.89	2.43	0.45	0.45	1.98				
12	1424	2884	3	3	4.78	2.83	0.27	0.82	0.67	1.12	1.89		
13	1618	2884	3	2	4.11	3.10	0.67	1.00	2.43				
14	1720	2884	3	3	3.96	3.65	0.45	0.15	1.06	0.15	2.13		
15	1822	2884	3	3	3.96	3.53	0.39	0.21	0.85	0.21	2.28		
16	1922	2884	3	2	3.81	3.50	1.37	0.30	2.13				
17	2022	2883	3	2	3.96	3.23	0.73	0.73	2.49				
18	2124	2884	3	3	3.81	3.04	0.30	0.15	0.76	0.61	1.98		
19	2224	2882	3	3	3.84	3.26	0.36	0.12	0.67	0.45	2.22		
20	2325	2881	3	4	3.71	2.62	0.36	0.24	0.61	0.48	1.06	0.36	0.5
21	2433	2882	3	2	3.99	3.38	1.25	0.61	2.13				
22	1218	2783	3	2	2.92	2.31	0.48	0.61	1.82				
23	1421	2773	3	3	4.42	3.04	0.30	0.30	0.61	1.06	2.13		
24	1517	2783	3	2	3.81	2.28	0.45	1.52	1.82				
25	1616	2783	3	3	3.65	2.74	0.30	0.15	0.61	0.76	1.82		
26	1719	2784	3	3	3.81	3.20	0.30	0.30	0.61	0.30	2.28		
27	1821	2782	3	3	3.81	3.35	0.45	0.15	0.91	0.30	1.98		
28	1922	2782	3	2	3.65	3.50	1.67	0.15	1.82				
29	2021	2783	3	3	3.62	3.17	0.57	0.15	0.61	0.30	1.98		
30	2123	2783	3	3	3.35	2.80	0.15	0.15	0.67	0.39	1.98		
31	581	2679	3	2	3.04	2.74	0.76	0.30	1.98				
32	1018	2680	3	2	2.74	2.28	0.76	0.45	1.52				
33	1119	2680	3	2	2.98	2.37	0.54	0.61	1.82				
34	1218	2682	3	3	4.32	3.29	0.39	0.36	0.54	0.67	2.34		
35	1423	2680	3	2	3.04	2.74	0.76	0.30	1.98				

APPENDIX 12
SAMPLE OF COMPUTER PROGRAM OUTPUT

12.1 OUTPUT FROM THE VARIOGRAM PROGRAM

V A R I O G R A M

SAMPLE DATA FROM PIT 2A, AVERAGE VARIOGRAM

WITH A FIELD OF 90. DEGREES IN EACH DIRECTION

DATA FILE = SAMPLE.DATA

FORMAT = (7X,2(F16.5,1X),3X,1(8X),(1X,F7.3))

LAG IN METERS # 40.0000

NUMBER OF SAMPLES # 30

LOWER LIMIT FOR Z # 3.2310

UPPER LIMIT FOR Z # 4.2060

GENERAL MEAN OF Z # 3.7572

GENERAL VARIANCE OF Z # 0.0624

SAMPLE

360.000

DIST IN METRES	N.PR	DRIFT	VARIOGRAM	AVG DIST
0 ---- 40	33	-0.046	0.0090	31.6
40 ---- 80	47	-0.085	0.0177	58.1
80 ---- 120	48	-0.150	0.0261	96.7
120 ---- 160	48	-0.143	0.0293	138.7
160 ---- 200	29	-0.245	0.0478	180.0
200 ---- 240	31	-0.289	0.0643	216.6
240 ---- 280	34	-0.258	0.0546	259.5
280 ---- 320	22	-0.339	0.0787	301.0
320 ---- 360	24	-0.353	0.0842	336.5
360 ---- 400	27	-0.348	0.0889	379.4
400 ---- 440	20	-0.419	0.1097	423.2
440 ---- 480	15	-0.457	0.1366	459.0
480 ---- 520	19	-0.470	0.1361	497.8
520 ---- 560	14	-0.435	0.1086	542.2
560 ---- 600	8	-0.438	0.1045	581.7
600 ---- 640	10	-0.527	0.1589	617.6
640 ---- 680	4	-0.617	0.1974	656.6
680 ---- 720	2	-0.640	0.2067	686.4

12.2 OUTPUT FROM THE KRIGING PROGRAM

```
    D A T A   U S E D

DATA FILE USED =   SAMPLE.DATA
DATA FORMAT USED = (1X,I5,1X,2(F16.5,1X),3X,1(8X),(1X,F7.3))
THE NUMBER OF SAMPLES IS:   30

V A R I O G R A M   M O D E L S

THERE ARE  2 SPHERICAL MODELS
COEFFICIENT=        0.0200    RANGE=   50.0
COEFFICIENT=        0.0158    RANGE=  180.0
NUGGET EFFECT=      0.0        SILL=     0.0358
MAXIMUM RANGE=    180.0

D O M A I N

POLYGON FILE USED =    PIT.2A
POLYGON FORMAT USED = (7X,2(F16.5,1X))
AREA OF THE DOMAIN IS:                               202309.
AREA OF THE ENCLOSING RECTANGLE IS:                  374986.
EXTREMES OF THE ENCLOSING RECTANGLE ARE:
XMIN=      1483.38
XMAX=      1945.32
YMIN=       841.66
YMAX=      1653.42

E S T I M A T I O N   G R I D

ESTIMATED CELL SIZE IS:        29.99
CELL SIZE USED IS:             20.00
COLUMNS= 28
ROWS=    45
MAXIMUM NUMBER OF ELEMENTS=                  1260
PROBABLE NUMBER OF ELEMENTS IN DOMAIN=        679
EXTREMES OF THE ENCLOSING RECTANGLE ARE:
XMIN=      1460.00
XMAX=      1960.00
YMIN=       820.00
YMAX=      1660.00
ACTUAL NUMBER OF CELLS IN THE DOMAIN IS:    503 AND   604

MINOR WARNING: IER = 34
1 DIGITS TESTED FOR ACCURACY.
THE ACCURACY TEST FAILED.
```

R E S U L T S

AREA OF DOMAIN IS:
202308.5
PERIMETER OF DOMAIN IS:
2156.6
TOTAL NUMBER OF SIDES IS:
78
AVERAGE LENGTH OF A SIDE IS:
27.6

KRIGE RESULTS

	VAR.	S.D.	%DEV.
KRIGE Z VALUE IN THE DOMAIN: 3.789875	0.000001	0.000820	0.02
KRIGE VARIANCE: 0.002387	0.000000	0.000021	0.62
THE SUM OF WEIGHTS: 0.999998	0.000000	0.000001	0.00
COVARIANCE OF THE DOMAIN: 0.001277	0.000000	0.000136	7.55
AVG COV OF SAMPLE TO DOMAIN: 0.001234	0.000000	0.000061	3.52
LAGRANGE PARAMETER: 0.002344	0.000000	0.000054	1.63

EXTENSION RESULTS

ARITHMETIC MEAN OF Z IN THE DATA FILE: 3.757160			
EXTENSION VARIANCE: 0.002584	0.000000	0.000014	0.39
AVG COV SAMPLE TO SAMPLE: 0.003803			
COVARIANCE OF THE DOMAIN: 0.001277	0.000000	0.000136	7.55
AVG COV SAMPLE TO DOMAIN: 0.001248	0.000000	0.000061	3.46