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### UNIVERSITY OF ALBERTA

# **Two-Area Generation Reliability Assessment**

# **Including Load Diversity**

BY

### David R. Morrow

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfilment of the requirements for the degree of Master of Science

Department of Mechanical Engineering

EDMONTON, ALBERTA

Spring 1992



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#### UNIVERSITY OF ALBERTA

### FACULTY OF GRADUATE STUDIES AND RESEARCH

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### ABSTRACT

This thesis illustrates the generation reliability benefits that could result from increased cooperation between neighbouring utilities, in particular the Alberta Interconnected System (AIS) and British Columbia Hydro (BCH). SaskPower (SP) would benefit to a lesser degree from such cooperation. Specifically, the thesis develops a method of identifying a suitable level of mutual support between two interconnected systems. It examines the effects of load diversity which arise primarily from time zone differences and the benefits to system reliability derived from shared generation reserves.

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## List of Abbreviations

Abbreviation	Definition
AIS	Alberta Interconnected System
BCH	British Columbia Hydro
SP	SaskPower
СМН	City of Medicine Hat
EP	Edmonton Power
TAU	TransAlta Utilities
APL	Alberta Power Limited
CCES	City of Calgary Electric System
RD	City of Red Deer
LETH	City of Lethbridge
LRB	Load Ratio Block
TIEBLK	Name LOLE computer program calls LRB
LOLE	Loss of Load Expectation
LOLP	Loss of Load Probability
F&D	Frequency and Duration
LOEE	Loss of Energy Expectation
EUE	Expected Unsupplied Energy
COPT	Capacity Outage Probability Table
CAPT	Capacity Assistance Probability Table
EAU	Equivalent Assisting Unit
LDC	Load Duration Curve
FOR	Forced Outage Rate
ELCC	Effective Load Carrying Capability
FLCC	Firm Load Carrying Capability
SLCC	Surplus Load Carrying Capability
MW	Mega Watt
MWH	Mega Watt Hour

#### **CHAPTER 1** Introduction

### **1.0** General Considerations

Increased cooperation among electrical utilities with regard to providing emergency power to each other in time of need, and with regard to sharing of generation reserves have many potential benefits to the parties involved. These include: better utilization of available resources over a wider geographic area; the reduction of money spent to meet current and future expected demands placed on the generating utilities; less pollution, due in part to more efficient use of natural resources, particularly water powered systems; lower electrical rates which may result in a more competitive market location, inviting additional businesses to locate into the region; and the possibility of increased exports from the combined efforts of the cooperating utilities benefitting all parties at the expense of none.

Several disadvantages are immediately apparent as well. These include: regional economic stratification if one system is favoured over another; local job losses; possible loss of control over resources located within either utility's service area due to contractual obligations; and other issues well beyond the scope of this thesis. As a result, it is recognized that useful inter-utility cooperation need be fair and reciprocal. This study examines the type of interaction most likely to be employed first when considering inter-utility cooperation, namely, reserve sharing. The study focuses on an analysis of the reliability benefits that accrue from interconnecting with neighbouring utilities. Such reliability benefits include lower loss of load expectation, less required reserves and greater confidence in the ability of neighbouring utilities to provide support when called upon. Such benefits would most likely translate into economic benefits in the form of decreased capital spending for generation equipment. An analysis of these economic benefits are beyond the scope of this study, however, they are recognized as the driving force in conducting this research.

Each utility or utility group must carry suitable reserves (i.e., extra generation capacity) such that it may continue to operate in the face of expected forced outages [1], scheduled maintenance requirements, load uncertainty and generation resource uncertainty. Figure 1 details a typical allocation of the installed capacity into loaded capacity and reserves. In addition, each system experiences instantaneous random load variations which are caused primarily by customers starting and stopping, without warning, electrical equipment or lights. Each utility must carry a control margin slightly larger than this expected variation to ensure it has the ability to maintain stability within the electrical network.

The sharing of reserves involves determination of a suitable reserve level for each of the utilities. A system needs its maximum reserve level for only a very short period of time each year due to significant seasonal variations in the system load. Figure 2 shows a generic load curve for a utility, with maintenance and minimum reserve requirements detailed [2]. The distance between the capacity available, upper line including the blocks which represent units on maintenance, and the load level, bottom line, is the reserve available to the system at that specific time. The points of minimum reserve are shown on the figure. Since such minimum reserve is required only a few times a year, the additional reserve available at other times could be used



Figure 1 How Installed Capacity is Allocated

to support an interconnected utility.

The diversity in the load, forced outages and maintenance requirements means that neighbouring utilities have points of minimum reserve at different times of the year. Thus, one utility may be able to make use of some of the excess reserve of another interconnected utility at the time of its own minimum reserve. With suitable tie line interconnections to a neighbouring utility and a reserve sharing contract in place, a utility may have to provide only a portion of the required reserve from its own generating capacity and can make up the balance from its neighbour [3].

It is statistically unlikely that both parties would require high reserve usage simultaneously [3,4], hence, each of the interconnected neighbours benefits from the combined reserve provided by both, without a diminuation of expected power supply reliability [5]. In the event that large outages occur at the same time, each system





Figure 2 Annual Load and Capacity Model

does what it can to support the other without violating its own control margin. This support is on a "best efforts" basis and cannot be demanded of a neighbour.

A typical cost estimate of installing a natural gas turbine generating unit is in the order of \$450/KW of installed capacity. Thus a 100 MW plant may cost about \$45,000,000. Due to the large amount of capital spent on generation and transmission systems within any utility, the ability to determine a suitable level of reliance to place on ones' neighbours is paramount. Increased reliance on a neighbour through a tie line, within reasonable limits, will result in each utility having to provide less generation reserves, hence, less plant is required and less money is spent. The magnitude of the economics involved demonstrates the need for such determination of tic line reliance.

### 1.1 Description of the Alberta Electric System

The electrical systems used as a case study for quantitative examples in this thesis are the Alberta Interconnected System and British Columbia Hydro systems. The following is a description of the AIS and its interconnections with its neighbours.

The AIS consists of three major generating utilities, three major urban municipalities which buy bulk power and distribute within their areas and one smaller city, the City of Medicine Hat (CMH), which has both generation and distribution services. The three major generating utilities are: Edmonton Power (EP), TransAlta Utilities (TAU) and Alberta Power Limited (APL). The distribution municipalities are: The City of Calgary Electric Service (CCES), The City of Red Deer (RD) and The City of Lethbridge (LETH). EP, TAU and APL also provide distribution services to their respective customers. Figure 3 provides a graphic description of these relationships.

It has been recognized by the utilities and regulatory bodies within Alberta that the generation and transmission systems must be planned on a one system basis. Thus, each generating utility does not necessarily provide power for its customers only. In reality the load and generating resources for each generating utility have maintained a fair degree of balance at any time, i.e., each utility has generation facilities roughly able to meet its own load. However, due to the large incremental size of many of the units on the AIS, the individual utility load - resource balances have occasionally varied, with some over capacity or under capacity situations occurring for short times.





Figure 3 Representation of the Alberta Electric System including External Interconnections

### **1.2** Description of the Interconnections to Neighbours

The AIS is interconnected to BCH (through TAU) with two alternating current interconnections. As these two lines are physically close together, and one has a much higher rating than the other, AIS planners usually consider the two tie lines to be one interconnection for all intents and purposes. The BCH-AIS tie line is typically rated at about 800 MW based on thermal limitations of the line and adjoining equipment. The tie line has been in service since January 1986.

The AIS is also interconnected to SaskPower, through APL, with one direct current interconnection. This is rated at 150 MW and is limited to 125 MW by contract, which is scheduled to begin two way interchange on January 1, 1995.

#### **1.3** Scope of Thesis

This thesis develops a procedure for determining a suitable level of tie reliance based on the ability of each system to aid the other within the context of reserve sharing. It examines the effect of load diversity which arises as a result of the daily peak loads of the interconnected utilities not being coincident, that is, they do not occur at the same time. The level of reliance determined by the procedures developed in this thesis may be used for planning purposes to limit interaction between the parties involved to a sustainable level.

A quantitative determination of a suitable reliance on the BCH-AIS tie line will be completed. As part of this quantitative determination, a sensitivity analysis will be performed. Sensitivities examined include the effect of altering the reliability of the tie line, utilizing a simplified version of the load ratio block (LRB), including (or neglecting) transmission losses and altering the tie line thermal rating. The quantitative effect of removing the LRB will be examined, to assess the relative contribution of load diversity to this procedure. The LRB is a set of monthly factors which allow hourly load diversity to be incorporated into a daily Loss of Load Expectation (LOLE) analytical model.

#### **1.4** Expected Results

The procedure to be developed will detail a means of identifying a tie line reliance limit based on the ability of each utility to support the other, hence, it will provide insights into future cooperation agreements between the interconnected utilities.

This procedure does not involve strenuous computations, and is suitable for providing detailed insight into two area reliability benefits that accrue due to reserve sharing for interconnected systems which exhibit load diversity.

The effects of load diversity will be demonstrated and the approximate percent effect it has on the final tie reliance value will be computed. This will provide other researchers with an assessment of the effect of load diversity for similar systems, and form the basis for determining whether this effect is significant enough to warrant inclusion in future studies.

The AIS and BCH are presently involved in contract negotiations to agree on a reserve sharing limit for planning purposes. The results of this study will help to provide technical background and supportive details in the selection of the actual reserve sharing limit.

#### CHAPTER II Background Information

#### 2.0 Literature Review

#### 2.01 Single System Reliability Techniques

The computation of generation reliability indices for an electrical system is necessary to assess the adequacy of the system to meet the expected loads placed upon it. Prior to the late 1940s, generation planners used a system of deterministic criterion to indicate when a generation system was inadequate, i.e. needed expansion. Such deterministic techniques included using a fixed percentage of the load or installed capacity as a reserve against failure, or a reserve slightly larger than the largest unit on the system [6]. It was observed by generation planners that these criteria were insensitive to many common operational concerns and did not accurately reflect the reliability of the system for situations where the load factor of the system changed with time or the system was dominated by several large units.

It was recognized that a probabilistic approach would provide a more suitable measure of generation system adequacy. For a detailed discussion of the relative merits of probabilistic and deterministic criterion see reference [6]. A probabilistic index was first proposed by Calabrese [7] in 1947. His paper detailed a method for computing a probabilistic reliability index called the loss of load probability (LOLP) for a single system. This method, and the resulting index, were used to calculate a reserve level for the system based on its probabilistic characteristics.

Calabrese's method allowed one to prepare a capacity outage probability table (COPT) for the system. His technique often resulted in many irregularly spaced capacity states. This procedure for obtaining a COPT of a system without minimization of capacity states is termed "full convolution". The developed COPT was termed the *capacity* model. The expected daily peak loads that the system is expected to meet are collectively referred to as the *load* model. By noting the probability of occurrence for a capacity state equal to the expected load one could obtain an estimation of the LOLP for the system. Superimposing the load model onto the capacity model produces the LOLP *risk* model. In the Calabrese paper, unit maintenance requirements were not considered, nor was any method of minimizing capacity states or computation time.

In modern times the *probability* of loss of load is not usually computed but rather the *expectation* of loss of load for a given time period. This is known as the loss of load expectation. LOLE is the expectation of a number of daily loss of load probabilities, thus, LOLE = summation (LOLP).

By 1964, the principles of the probabilistic method were well established. However, the actual calculation was still somewhat cumbersome. Prior to 1964, the capacity model was constructed by convolving all the units into a table each time a new COPT was required. Maintenance was modelled by breaking the year into periods in which maintenance occurred, and periods in which there was no maintenance. A different COPT was required, and was created, for each time segment to reflect the capacity available during that time period. In 1964 Billinton [8] developed a method to speed up the required computations. He proposed progressive truncation of the COPT during construction. This permitted the elimination of capacity states which had probabilities of occurrence of less than some prespecif. 4 value, suggested at 10<sup>\*</sup>. In practice the LOLE results obtained using table truncation proved to be very accurate when compared with the full convolution model of Calabrese, as long as the truncation parameter was small (at least 10<sup>7</sup>). Billinton also introduced the concept of a capacity table step size, in which the COPT was constructed with capacity level increments of some predetermined equal size. This eliminated a large number of system capacity states, some of which were very close to each other. This segmentation of the COPT into equal discrete steps introduced a certain inaccuracy into the model. Billinton showed that there was a tradeoff between the accuracy achieved with a smaller step size and the saving in computational time gained with the larger step size. It was noted that the step size should not exceed the size of the smallest unit on the system.

In 1967, Billinton and Bhavaruju [3] included the effects of load uncertainty into the LOLE risk model. They showed that increased uncertainty in the forecasted loads resulted in higher LOLE, and hence, greater reserve requirements for a system.

Recursive algorithms for the creation and adjustment of the capacity model were introduced by Billinton [9] in 1970. In this paper Billinton described a technique for adjusting the COPT to reflect the addition *or removal* of a unit. He noted that for removal of a unit by his technique the table had to have equal steps between states in the COPT. This concept of COPT enhancement or reduction greatly simplified COPT construction for time periods in which maintenance was considered. This method has become known as the recursive technique for COPT preparation.

Booth introduced production simulation in 1971 [10]. In this paper Booth noted that the area under the probability-capacity curve represented the energy of the system. This in itself was not new, Baleriaux et. al. [11] had noted this as early as 1967. However, Booth formulated a technique of simulation which made use of the energy production of the system on a unit by unit basis. His recursive algorithm for

the adjustment of the probability distribution to reflect the removal of a unit is still in use today and forms the basis for many of the production simulation models currently available.

While it is possible to compute the LOLE of a system using this energy based method, it is recognized that the result is the same as the simpler LOLE technique [12], but that additional information is required about the energy capability of each unit. The result is that the field of reliability engineering for generation systems has split. To determine the simple adequacy of the system(s), the more usual LOLE technique is often used. To provide information on the energy production of the system(s), while maintaining the general probabilistic approach, one uses the production simulation techniques laid out by Booth.

Booth's work also made it possible to do a more detailed assessment of energy limited units, such as hydro or pumped storage. An energy limited unit is one for which a certain capacity may be achieved for only as long as the stored energy in the system can provide it. The most usual case is that of a hydro unit with water storage. The LOLE technique approximates the effects of energy limited units by adjusting the load model, not the capacity model as would be more appropriate. As the capacity model in the Booth simulation is energy based, it permits direct adjustment of the probability distribution for energy limited units. However, for many systems today, the modelling of energy limited units is not considered a serious impediment to the use of the LOLE technique and the added complexity and computational time required for energy simulation is often not constant is worth the trouble.

The energy simulation method introduced the concept of a second adequacy criterion for a system. The original criterion, LOLE, indicates the probability of not meeting a specified load as measured in MW. The energy based analysis indicates the probability of not producing a specified quantity of energy as measured in GWH. Thus, an energy based criterion was established. This second criterion is also recognized by the AIS planners and is checked periodically. The AIS system has been found to be primarily capacity constrained, hence, the energy criterion rarely affects any expansion plans.

In an attempt to reduce computing memory requirements and execution time, the concept of using a continuous distribution to model the COPT was proposed by Bhavaruju [13] in 1974. A normal distribution was used to model the probabilities in the capacity table. The results were not particularly accurate for smaller systems or those dominated by several large units, as in fact the probability distribution may contain discontinuities in such cases. However, for certain systems this technique has been shown to produce satisfactory results for daily engineering work. The cumulant technique, as it has come to be known, is much faster than the convolution procedure. Therefore a great deal of attention has been paid to this technique.

In 1979 Schenk and Rau [14] proposed the use of a Gram Charlier expansion of the normal distribution modelled with Fourier Transforms. This was found to yield better accuracy than a simple normal distribution as compared to the full convolution model by Calabrese, but still lacked precision for smaller systems. Hamoud and Billinton [15] in 1981 used the Fourier Transform methodology to incorporate load uncertainty into the method of cumulants. For cumulant models this Fourier Transform method eliminated the use of discrete steps in the load uncertainty that had been introduced by Billinton in 1967 [4].

In 1983 Hamoud and Neudorf [16] used a Fast Fourier Transform to model a folded normal expansion of the COPT. Accuracy was somewhat improved over previous cumulant techniques. For a reasonably large system it was shown that this cumulant technique yielded reliability indices within 1% of those produced with the convolution method, while the computational speed was shown to be up to twelve times faster.

Progress in the reliability field in the area of continuous functions to represent capacity probability tables and uncertainty is still progressing, but has the inherent problem that power systems actually operate at discrete levels.

One attempt to model the discrete nature of generation systems without constructing a COPT was made by Sutanto, Outhred and Lee [17] in 1989. In this approach they used the Z-transform to model the generating data directly as probability impulses. The Z-transform does for discrete distributions what the Fourier transform does for a continuous function, in that it permits a transformation of the data into a more convenient form for analysis.

Using the method of Sutanto et. al., the transformed probabilities can be convolved very accurately and efficiently and the result transformed back to the MW domain. It was found that this method produced identical LOLE values to the full convolution procedure as laid out by Calabrese, but yielded results about four times faster. This method also permits one to model multi-state units directly (without slowing computation time) as each level of capacity that the generation unit can operate at is considered as an integral part of the process of building the capacity model. With a typical multi-state LOLE model, the computation time increases somewhat as the extra states are convolved into the COPT.

A significant advantage of the Z-transform method is that generator data can be used directly; averaging of outage time and capacity levels are not required. Due to the inherent power of the technique, the Z-transform method may become one of the dominant techniques when it achieves a greater degree of sophistication. At the present time, maintenance and energy limited units have yet to be incorporated into the model. The development of this technique will be most interesting to observe.

The LOLE method is not the only probabilistic method used to determine system adequacy. In 1958, the basic concepts of the frequency and duration (F&D) method were first proposed by Halperin and Adler [18]. The approach was very cumbersome and further developments in the technique awaited the papers of Hall et. al. [19], Ringlee et. al. [20], Galloway et. al. [21] and Cook et. al. [22], in which recursive algorithms for building the capacity model and combining it with the load model were established.

The F&D indices provide information about the number of outages (frequency) and the average duration of the outages for the study period. This information is easy for the layman to understand as it relates directly to his or her perceived reliability. However, many planners distrust the use of F&D indices for generation planning because, for many systems, most of the electrical outages a customer experiences are a result of the distribution system, not the generation system. Hence, an index which apparently indicates the frequency of outages must be interpreted to mean generation outages, not electrical service outages. If one considers the transmission and/or distribution system in the analysis, F&D techniques do in fact indicate the customer outage rate and are therefore a direct indication of system reliability. F&D indices may be easier to understand than LOLE indices because LOLE is an expectation of load loss, based on probabilities, not a guarantee that load losses will occur at the computed LOLE level.

In 1972 Billinton and Singh [23] showed that F&D techniques were found to yield comparable reserve requirements and expansion sequences to those produced by loss of load methods. Partly as a result of these comparable results, and the extra computation required, F&D indices are not the method of choice for utility planners. However, they are computed by a number of programs in addition to LOLE. Occasionally the programs also produce energy indices such as Loss of Energy Expectation (LOEE) or Expected Unsupplied Energy (EUE). At the current time the F&D method is not being studied extensively as this particular branch of reliability has reached a rather mature level of development.

An alternate method of computing LOLE and F&D indices is to use Monte Carlo simulation. In this method a series of scenarios or snapshots of the system are obtained by hourly random drawings on the status of each generating unit [3]. Each snapshot produces a success or a failure to meet load. To obtain an expected value of the failure to meet load (i.e., LOLE), many runs are performed and a cumulative

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average is computed. When convergence to a prespecified confidence interval is obtained, the LOLE, F&D and energy indices are available to the analyst. This technique is the only one that provides a definitive distribution of the reliability indices of a system.

The most significant problem associated with Monte Carlo simulation is that the computational technique must converge, which can take considerable time. A more reliable system will converge more slowly than a less reliable system. A system dominated by several large units will converge less rapidly than a system with relatively similar sized units. The computation time involved in obtaining convergence for a single area system can be twice as long as for an analytical model. This is the most serious drawback to the use of Monte Carlo simulation. However, the computation time for Monte Carlo simulation is approximately linear with the number of units on the system(s). As a result of this linearity, Monte Carlo simulation will often produce results quicker for interconnected systems. This is especially true when three or more systems are interconnected within one model.

In North America, Monte Carlo simulation is used by very few utilities [24]. It is used much more extensively in Europe and South America. The majority of North American utilities prefer to use the analytical approach. For simpler planning exercises some utilities and power pools still use a deterministic approach which has been calibrated by a probabilistic model.

### 2.02 Two Area Reliability Techniques

In 1963 Cook et. al., [5] extended the preliminary work of Calabrese to a system of two interconnected sub-systems. The basic probability tables of two interconnected systems were incorporated in such a way that a two dimensional array of outage probabilities was created. By examining the risk model of one of the systems, one could determine whether that system was deficient or had an excess of power. If it was deficient, one would examine the other system to see if it had sufficient reserves to cover the deficiency. One also had to ensure that the tie line capacity between the two systems was able to handle the deficiency. One of the limiting features of this two dimensional array technique was its lengthy procedure.

This same paper introduced some other important ideas. It noted that load diversity would be a contributing factor in determining the net overall ability of one system to aid the other. This well be reflected in a lower required reserve level for the combined system. It was also noted in the paper that monthly load duration curves (LDCs) would preserve seasonal load diversity. The load diversity noted on an hourly basis was incorporated into the model by means of a simple percentage of peak loads that reflected the coincident load levels. i.e., System A might be said to be at 80% of its peak load when System B was at peak.

The paper detailed the computation of LOLE for each sub-system and for the combined system, a new innovation for that time, and laid the groundwork for a number of questions about the priority of assistance for multi-area reliability studies. However, the technique was limited by the fact that they used a load modification technique to incorporate maintenance requirements rather than the more rigorous capacity model adjustment.

A second paper on generating capacity reliability for two interconnected systems was prepared by Billinton and Bhavaruju in 1967 [4]. It made use of the same two dimensional probability table as Cook et. al. [5]. Billinton's approach was to obtain a series of curves to represent the load carrying capability of the systems being studied as a function of the tie capacity. It was found that an "infinite" [4] tie capacity could be determined beyond which any increase in tie capacity resulted in little or no increase in the ability of one system to support the other, or in other words, the LOLE of each system remained essentially unchanged.

The paper mentioned load diversity in the preamble but made no attempt to quantify it. Instead the paper concentrated on the effects of load uncertainty and laid out a detailed method for analyzing this effect. Further, the paper used the assumption that the annual LOLE be calculated using twelve Decembers. For a Canadian utility, December typically contributes the greatest amount to system unreliability due to high loads experienced. Hence, the use of twelve Decembers results in a pessimistic computation of the actual system LOLE. In most studies performed today the actual distribution of loads for a complete calendar year is included. Finally, the paper stated that a tie line Forced Outage Rate (FOR) of 0.01 resulted in values of support essentially unchanged from that of a perfect tie line. This statement is not supported by this author's experience in LOLE computations. In fact, a decrease of the tie FOR to 0.0001, (near perfect tie line), results in a noticeable difference in the LOLE for the AIS. Billinton's finding may have been due to the small size of the tie line and/or the relative inaccuracy of the computational methods of that time.

The effects of correlated loads (load diversity) were addressed in the work of Billinton and Singh [23] in 1972. However, the load diversity was represented by either zero correlation or 100% correlation. In reality neither is found to be the case, but rather some unknown correlation level which may fluctuate with time.

An assessment of the effects of operating policy on the use of a tie line was presented by Pang and Wood [12] in 1975. They concerned themselves primarily with multi-area considerations and attempted to identify the overall effect of load loss sharing policies on the flow of power over an interconnecting tie line. One such load loss sharing policy is "no load loss sharing" in which support from the assisting system is restricted so as to ensure that the assisting system can experience no loss of load as a result of supporting its neighbour. In this paper it was stated that load diversity was determined to be a more significant factor to the calculated reliability level than the effects of various operating policies. Again, the load correlation was considered to be either zero or fully correlated. Pang and Wood also showed that the expected unsupplied energy of the system, computed as a derivative of LOLE, was the same as that computed by the simulation technique of Booth [10]. This supported the geoncept of the energy criterion as a measure of system adequacy.

In 1981 Billinton et. al. [25] formally described the equivalent assisting unit (EAU) approach. The concept was used by Billinton and Singh [26] as early as 1971, although the term "equivalent assisting unit" was not employed at that time. The EAU method is an extension of the two dimensional probability table method. It permits a multi-state representation of the assisting system to be included in the risk model of the assisted system. This differed from the two dimensional probability table method in that both systems are not considered explicitly at the same time.

The EAU method is not restricted to two area analysis but is particularly suited for this. Three and four area studies may be performed providing the systems are configured radially. In a two area study it is assumed that all support will be provided to the other utility. The EAU method allows a great many assistance states to be included into the supported system and as such, provides a very detailed model of the support available from the assisting system.

Billinton's paper made no mention of load diversity, and used assistance tables prepared for each month. It should be possible to account for load diversity directly in the EAU approach by permitting hourly EAU tables to be entered into an hourly LOLE model. The computation time for such a complete enumeration technique would be vast and the technique would not be workable for actual studies as it would take far too long to achieve an answer. As the EAU method is central to the method of including load diversity as presented in this thesis, a more complete description of the EAU method is given in Sections 2.2 and 2.3.

Within a daily LOLE model, the load level used for the creation of the CAPT is the daily peak. Even in an hourly model, the CAPT is generated by using the daily peak load [44]. The level of assistance at daily peak load represents a lower limit to the amount of help available throughout the day. It is recognized that this produces a pessimistic assessment of the amount of available support [44] based on hourly loads, however, the simplifying assumption is justified on practical grounds, otherwise, the

convolution of the many hourly CAPTs would be too lengthy and the procedure unusable.

In practice the COPT of a system is not constant throughout the year, but must be adjusted for units that are removed for maintenance. For each time interval of consistent capacity a COPT is created. There are at least as many CAPTs as COPTs, however, even if the COPT is constant, each different daily peak that is used to compute a daily LOLE produces a new CAPT. To minimize the creation of CAPTs, the following technique has been used by Ruiu et. al. [44] and others. For each daily peak in the specified study period, usually one week, one CAPT is created. The probabilities of each capacity state in the CAPT are divided by the number of days in the period. Then, the seven CAPTs for the week are added up. The resulting CAPT stands as an average CAPT for the week. This technique greatly minimizes the number of CAPTs which must be handled.

In 1982 Rau et. al. [27] proposed a method of incorporating correlated demands, i.e. load diversity. They used the method of cumulants which was subsequently shown by Sutanto et. al. [17] to have reduced accuracy. Rau et. al. used the Gram Charlier approximation of the normal distribution to approximate a continuous function of the capacity model. The authors correlated the loads of two systems on an hourly basis for one year, averaged the result, and arrived at an equivalent joint discrete load distribution. They chose to use this one average daily load distribution to model the load diversity for the entire year. The concept of using only one day of hourly load distribution seems to ignore the reality that load diversity is not constant for an entire year and introduces an assumption that is not required in the methodology proposed in this thesis. The usefulness of Rau et. al.'s method was in utilizing an expected daily load curve approach instead of assigning a level of correlation. It would not be hard to implement many daily load curves into a model such that each day is represented individually, however the authors chose not to do this. The model used by Rau et. al. was an hourly model, from which they computed the load diversity available for each hour of the day, something that no daily model could properly reflect.

An attempt to define the effective load carrying capability of tie lines was proposed by Deb [28] in 1984. This paper also considered the loads of the two interconnected systems to be correlated. Deb was primarily interested in the reduction in new required generating plants as a result of increased tie reliance. Deb compared the effective load carrying capability (ELCC) of the system for the case of assistance from the neighbouring utility and for the case of addition of a new generating plant. He defined the ELCC as the load at which the LOLE would be equal to a prespecified criterion, for that case 0.1 days per year was chosen. The AIS planners call this same quantity the Firm Load Carrying Capability (FLCC).

Deb's calculation used the M-slope method detailed in 1970 by Garver [29], to ascertain the expected contribution of a new unit. Garver found the M-slope of the entire COPT changes only slightly as single units are brought on or off line. The Mslope can therefore be used to estimate the contribution of a single new unit before it is added by extrapolating the system size while maintaining the original M-slope. Deb used a bivariate normal distribution to model the joint load distribution and the method of cumulants based on hourly loads to assess the system LOLE. The M-slope method seems to be computationally inefficient, but it does allow one to formulate a closed form solution for a two party interconnected system.

Load diversity was modelled within the bivariate normal distribution, similar to the technique described above in the paper by Rau et. al. [27]. Here again, a single daily distribution of load correlation was used to model the load diversity for the two systems for the year. Deb recognized that his method was quite approximate, but argued that for practical engineering work it was accurate enough.

Interaction among utilities involves the transfer of energy from one party to another, at some time, for a specific duration and magnitude. It is not necessary that the magnitude of the support remain constant for the entire duration of support time. At another time the flow may be reversed, the magnitude of the help changed and the duration modified to suit the needs at that time. This transfer of energy requires use of the interconnecting tie line(s) and is restricted to some reasonable limit based on tie line capabilities or the abilities of the utilities to aid one another. This represents a maximum reliance that should be placed on the neighbouring utility through the tie line.

The ability of one utility to aid another was discussed by Puntel [30] in his unpublished paper of 1990. Puntel mentioned that the ability of a supporting system to provide aid could form one limitation to the overall interaction throughout a multiarea system. Puntel did not quantify this limit directly but rather showed that operating policy had a significant effect on the overall reliability indices for the interconnected multi-area system.
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Stoll [31] discussed the concept of system ability to support others by considering three cases. In the first case the supported system was relatively heavily loaded and required as much help as possible. Assuming that the tie line was large enough to conduct such a level of support the limit was therefore found to lie with the supporting system. The effective support through the tie line was limited by the upper limit of the CAPT of the supporting system. In the second case the supported system required assistance of about the magnitude of the tie line thermal capacity. In this case, the ability of the supporting system to provide support would be less of a factor, as the tie thermal limit also acted to limit interaction some of the time. In the third case, the supported system was relatively lightly loaded and required little assistance. In this case the ability of the supporting system was of little consequence as the support was not required to overcome expected outages in the highly reliable system, and as such the effect of support through the tie was negligible.

### 2.03 Multi Area Reliability Techniques

Use of the LOLE technique seemed limited to two area studies, or multi-area studies in which the system could be arranged into a radial configuration, i.e. one in which there is a centrally located system interconnected with other systems, but these "planetary" systems are not directly interconnected with each other. A number of researchers had experimented with various ways to transform a non-radial interconnected system into equivalent radial systems for analysis [32,33]. These methods proved to be less than satisfactory, as the assumptions necessary to transform the system into radial subsystems resulted in different answers depending on exactly what subsystems were selected.

True multi-area assessment was first introduced by Pang and Wood [34], in 1974, with the development of the linear flow model. This acted as an electrical transportation model and allowed the risk models of all the interconnected systems to interact with each other simultaneously. This iterative technique of determining which system was in a loss of load situation and which system could provide support, was dependent on the concept of grouping failure states. It avoided the extremely lengthy task of complete enumeration of all possible states in the entire interconnected system. For a system of N units, there are up to  $2^{N}$  states in the system probability table. The linear flow model permitted the reduction of state enumeration by observing that a number of states would occur if certain conditions were met. Pang et. al. were able to identify a minimum number of significant states, called cut sets, that were contributors to system failure. Each of these minimal cut sets would produce an LOLE for the system for all the states that fell within that set. The summation of all the LOLE's of the minimal cut sets would produce the combined system LOLE. Individual system LOLE values were then obtained from the probability table for that system.

The field of multi-area reliability has received a great deal of attention. Pang, Clancy, Wood, Dhar, Billinton, Singh, Patton, Shahidehpour, Oliveira, Garver and others [35,36,37,38,39,40,3,41,42,43,34,12] were instrumental in preparing a series of papers which dealt with many of the problems and considerations specific to multiarea reliability analysis. For a more complete understanding of multi-area reliability the reader is referred to the above references.

This concludes the discussion of the relevant literature. From the above literature review it has been observed that the subject of load diversity has not been covered in detail in the literature, and where it has been treated, the analysis has included some simplifying assumptions about the load correlation or means of incorporation into the risk model of the systems in question.

In particular the concept of incorporating load diversity by adjusting the Capacity Assistance Probability Table (CAPT) has not been treated in the literature. The ability of the supporting system to provide assistance is contained in the CAPT produced by the EAU method. However, the current use of the EAU method does not recognize the additional assistance that may be available as a result of load diversity. Thus, it is recognized that a quantitative assessment of the ability of the supporting system to provide assistance, while recognizing load diversity, could be employed in the risk model for increased confidence in the results. This thesis formulates and details one procedure for including the effects of load diversity into the risk model of the supported system and thereby determining an upper limit of reliance that should be placed on the supporting system through the interconnecting tie.

### 2.1 General Considerations

It has long been recognized by other researchers that load diversity warrants further study. From the literature review it has been noted that load diversity has not been intensively studied.

A brief assessment of the magnitude of the possible benefits can be quickly undertaken. Consider a situation where the supporting system was at 95% of its daily peak load when the supported system required assistance. For the AIS, a typical maximum daily load is in the order of 5000 MW. Hence, a 5% load reduction (from 5000 to 4750) could mean about 250 MW of extra support available to the supported system. At \$450/KW installed capacity, this could translate into about \$112,500,000. It is recognized that in a reliability assessment, one cannot segregate the 250 MW so simply, however, this provides a first, very brief, view of the upper limit of including the effect of load diversity.

### 2.2 Loss of Load Expectation (LOLE) Index Method

In a single area LOLE representation, all transmission limitations within the area are ignored. The generating and transmission systems are considered to be equally reliable in all geographic parts [30]. In effect it could be considered to be one bus connecting the load centers and generation resources of the entire system. This is depicted in Figure 4. Eliminating the transmission system as a factor in the reliability analysis of a system results in a risk assessment of the system based only on the generating capacity and the loads placed on the system. Under such a condition, the LOLE of a system is defined as the probability of not being able to serve firm load,

i.e. the probability of meeting a specified capacity state less than the load for the system.



Figure 4 Conventional LOLE System Model

The mathematics of computing LOLE are based upon the probability of generating units being unavailable to meet the expected demand. The method of calculating the LOLE of a system is fully described in the literature [2,31]. The following example briefly describes a simplified LOLE calculation for a system with three generating units. In this example the COPT is built up in successive stages. For simplicity the FOR's of the three units are all assumed to be 2%.



Figure 5 Sample 3 Generating Unit LOLE Problem

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 Table 1
 Required Data for Sample Three Unit LOLE Calculation

Unit data:	Unit	1	2	3
	Capacity (MW)	3.0	3.0	5.0
	FOR	0.02	0.02	0.02

# Table 2Individual Capacity Probability Tables

Unit ore	State	MW out	Prob.	Cum. Prob.
	1	0	0.9800	1.0000
	2	3	0.0200	0.0200
Unit three	State	MW out	Prob.	Cum. Prob.
	1	0	0.9800	1.0000
	2	5	0.0200	0.0200

Note: Units 1 & 2 are identical.

Table 3Capacity Outage Probability Tables

Add unit 2 to unit 1:

	State	MW out	Prob.	Cum. Prob.
	1	0	0.9604	1.0000
2 unit COPT	2	3	0.0392	0.0396
	3	6	0.0004	0.0004

Add unit 3 to the other two:

	State	MW out	Prob.	Cum. Prob.
	1	0	0.9412	1.0000
3 unit COPT	2	3	0.0384	0.0588
	3	5	0.0192	0.0204
	4	6	0.0004	0.0012
	5	8	0.0008	0.0008
	6	11	0.0000	0.0000

Now, assuming a load of 8 MW on this system, the following example demonstrates the means of determining the system LOLE index and of separating the table to obtain the CAPT, which can be used for assistance to an interconnected system. The example system LOLE for a load of 8 MW is the cumulative probability of having more than 3 MW of capacity on outage. This translates directly into the probability of having less than 8 MW on line, hence, the LOLE is 0.0204.

Table 4	Samp	le results of	LOLE Calcula	tion, COPT with Load Identified
State	MW ou	t MW on	Prob.	Cum. Prob.
1	0	11	0.9412	1.0000
2	3	8	0.0384	0.0588
3	5	6	0.0192	0.0204 < LOLE
4	6	5	0.0004	0.0012
5	8	3	0.0008	0.0008
6	11	0	0.0000	0.0000

Table 5Capacity Assistance Probability Table for Sample Three Unit Problem

	State	MW out	Assistance (MW)	Prob.
CAPT for load	1	0	5	0.9412
of 6 MW.	2	3	2	0.0384
	3	5	0	0.0204

The CAPT for a load of 6 MW is obtained from the COPT at the point of 6 MW of available capacity. After the tie line limitations are imposed on this CAPT, the resulting EAU can be used to represent this system in the risk model of an interconnected utility. Note that a different CAPT would be obtained for loads of 6 MW or 5 MW. The tie line limitation is imposed by noting that any support level in the CAPT above the thermal rating of the tie line would be limited to the thermal rating. Hence, all probabilities above the thermal rating are added together to form the probability of achieving the thermal rating. See Table 6 for an example EAU assuming a tie thermal rating of 2 MW.

Table 6Equivalent Assisting Unit Representation for Sample Three Unit<br/>Problem

	State	Assistance (MW)	Prob.
EAU assuming	1	2	0.9796
2 MW tie.	2	0	0.0204

This principle of generating alternate CAPTs will be exploited as the means of incorporating load diversity into the risk model of the supported system in this thesis.

### 2.3 Equivalent Assisting Unit Representation of a System

It is possible to simulate support through a tie line as a simple two or three state configuration within the risk model of the supported system. This approach is often used for day to day work within the generation planning profession, however, the model is not particularly robust, and ignores the many discrete capacity states that the supporting system may actually assume while providing support.

To avoid this limitation Billinton [25] described a method of including a multistate representation of the supporting system into the risk model of the supported system. This model may assume hundreds or thousands of states as the need demands. This representation is known as the "equivalent assisting unit" approach and is well described in Billinton's book [2], chapter 4.3.

As shown in the example of Section 2.2, the EAU technique uses the COPT of the supporting system to develop an assistance table, the CAPT. This region of the COPT lists the probabilities of meeting load levels above that required to service the expected load in the supporting system. Since these load levels are above the supporting system load requirement, they can be used to support another system if suitable interconnections exist. Thus, the CAPT obtained for a given load level represents the ability of the supporting system to accommodate possible capacity deficiencies in the supported system. CAPT minimization [44] is employed to produce a single CAPT. This average CAPT is the assistance table used to produce the EAU, which in turn models the support to the interconnected utility.

Load diversity is incorporated by adjusting the loads used for the creation of the daily CAPTs, before averaging, from which the period CAPT is obtained.

### 2.4 LOLE Model Used in this Thesis

Computer models that employ hourly load information have only recently become commercially available. An example of such a model is the program GRIP, developed by Associated Power Analysts, which was first made available in 1984 [44]. The computer model in use by Alberta utilities, for the AIS, is entitled LOLE [45], and was developed prior to hourly models being commonly available to utility planners. The program, LOLE, uses daily peak load information which allows the program to operate much faster than an hourly model, as only 1/24th of the load information is required. However, it does not allow easy representation of hourly variations, such as load diversity. In order to make the results of this study directly comparable to published AIS results, the LOLE method based on daily peak loads was used.

The LOLE model used for this analysis was developed for the AIS by Moneco Engineering in 1972 [45]. It has been revised a number of times since original release [46] to incorporate additional features or to make the input easier to manage.

### **CHAPTER III** Techniques of Analysis

### **3.0** Objectives of Analysis

Day to day adequacy reviews of the AIS system require detailed knowledge of the AIS generation system and some knowledge of the utilities that could assist the AIS in times of trouble. For rapid analyses to be performed, a simple assistance model must be used to represent BCH support available through the tie line. The current AIS tie line model allows only two states: operating at a predetermined level, or forced off line. The rationale behind such a simplified tie model is that with an entire provincial system supplying power, the tie should be able to be energized to some prespecified level most of the time. This assumption is generally well received by planners and regulators, provided that the prespecified level is chosen with care. This raises the question of how the tie reliance level is established. It is known that the tie reliance level cannot exceed the reserve sharing limits of either province [2], or the thermal rating of the tie line. Thus it is essential to establish the reserve sharing limits of the provinces.

To determine a suitable reserve sharing limit between two interconnected provinces, a two-area LOLE analysis of the systems must be performed. One method of accomplishing this is to prepare a representation of the assisting system which can be used directly in the supported system's risk model. The level of reliability calculated for the assisted system will then include the support from the assisting utility. This method is known as the equivalent assisting unit method [2]. The EAU method makes use of the natural diversity of generator outages to provide support. However, in addition to such capacity model diversity, diversity is also found between the loads experienced by the provinces.

Load diversity arises when two interconnected systems do not experience peak loads at the same time. There are two broad classes of load diversity, hourly load diversity and seasonal load diversity. Seasonal diversity may occur between interconnected utilities which have peak loads in different seasons, possibly summer and winter. An example of this would be a southern utility that peaks in the summer due to air conditioning loads and a northern utility with an annual peak in December caused by low light levels and heating load requirements.

Hourly load diversity is any combination of interconnected system loads that result in peak loads at different hours of the day. It can be made up of weather variations, diverse load use patterns due to industry or other demands, or could occur because two systems have similar load patterns but are in different time zones. Such a case occurs between many utilities connected on an east-west basis. This often happens in Canada because of the large areas the individual utilities serve, and because Canada is arranged in an east-west manner and spans several time zones. In particular, the AIS and BCH are in different time zones, with a one hour time difference between them.

To affect a daily LOLE calculation, load diversity must alter the daily peak or span more than one day. Otherwise the effects due to load diversity cannot be readily distinguished as a daily LOLE model is incapable of recognizing hourly variations within any day unless they affect the daily peak. A daily LOLE model will be able to capture seasonal load diversity, as the daily loads have the ability to register seasonal variations.

To include the effects of hourly load diversity in a daily LOLE model, it is necessary to make special provisions to recognize load diversity. This thesis develops a method of incorporating hourly load diversity into the reserve sharing analysis using a daily LCLE model. This permits both capacity and load diversity to be included in the analysis.

The method to be developed is expected to be of considerable benefit to many generation planners who do not have chronological, hourly models, to be able to incorporate hourly load diversity into a daily model. This methodology can be incorporated into other daily LOLE models relatively easily, and is not specific to the computer model used for this analysis.

The initial idea for the proposed procedure to determine tie reliance was taken from consultation with other generation planners in Alberta, from the idea of an "equivalent assisting unit" as described by Billinton [2], and from the concept of altering the load model to include load diversity somewhat similar to that proposed by Rau et. al. [27].

## 3.1 Multi-State Representation of Assisting Province

The current two state AIS tie model is incapable of accurately reflecting the many possible states a system may be in and still provide some degree of support. Thus, a multi-state representation of the supporting province is required. The EAU approach readily provides such a model with the thermal rating of the tie line providing a maximum level of tie assistance. It is known that the thermal rating of a tie line varies with weather conditions and loading, however, for use in a study of this nature it suffices to use an average expected thermal rating. If is were deemed significant, then several thermal ratings could be incorporated to reflect the thermal rating under different conditions.

In the literature [2], the load level chosen as the point of separation of the CAPT from the supporting system COPT is that of daily peak demand of the supporting system. However, recognizing load diversity, the actual load level experienced in the supporting system at the time of daily peak in the supported system will be the coincident load. This is generally lower than the daily peak load. The CAPT prepared using the daily peak would, therefore, be smaller and less able to meet deficiencies than the CAPT prepared using a lower, more realistic load level, the coincident load. The additional support available, as a result of the differences between these two CAPTs, represents the benefits of considering load diversity.

Figure 6 shows the loads experienced by the AIS and BCH for one specific day, January 2, 1987. The time axis for Figure 6 is local time for both systems. From this graph it is clear that a high correlation exists between hou<sup>-1</sup>y load patterns experienced in the two provinces. Further, the daily peak appears to occur at about 18:00 hours local time in both provinces.

Figure 7 shows the same AIS-BCH loads using the time frame of the AIS. To accomplish this, the BCH loads are shifted by one hour to reflect the time zone





Figure 6 AIS - BCH Hourly Peak Loads

difference. It can be seen that plotting coincident BCH loads has a significant effect on the total load experienced by the two area system at any hour. Upon examining the loads at the time of the AIS daily peak, the BCH coincident load is seen to be about 200 MW lower than the BCH daily peak. This difference demonstrates load diversity, which can be a substantial factor in determining the ability of one system to aid another.

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Figure 7 BCH - AIS Coincident Hourly Loads

# 3.2 Overview of Technique used to Incorporate Load Diversity into the Reserve Sharing Limit Determination

To provide information on the ability of one system to provide assistance to another, one must know what generation units will be available at the time of support. If the support is to be based on currently existing systems, then a projection of future generation plants (generation sequence) is not required. However, a study of the current systems would not permit one to include new generation deferrals as a result of reserve sharing. As a result of this consideration, the time period chosen for this study was 1993-1997, which entails estimating the generation units available in that time period. Thus, a suitable generation sequence must be ensured for each province prior to the CAPT being prepared.

The following is a brief description of the steps required to determine the reserve sharing limit of the interconnected system while incorporating load diversity. Steps 1 and 2 of the procedure must be performed for each of the twelve months of the year. Six years of historical load data are available for analysis, thus, there are about 180 (6 x 30) daily peak and coincident loads for each month for load ratio calculations. Each of the following steps will be discussed in detail in the sections that follow:

- Step 1) From the hourly historical load data base, determine the daily peak load and hour of occurrence for the supported system for each day of a month.
  - Determine the coincident load in the supporting system at the hour of the supported system daily peak and compute the ratio of coincident load to daily peak load in the supporting system for each day.
- Step 2) Using the coincident to daily peak load ratios developed in Step 1, develop a set of five load ratio levels with associated probabilities of occurrence that are representative of the distribution of the approximately 180 ratios available for each month. These five levels and probabilities make up the LRB required to reflect load diversity in the supporting system.

- Step 3) Prepare LDCs, maintenance requirements, unit generating statistics and FORs for the supported and supporting systems for each month of the year into a working data file.
- Step 4) Using the LRB from above, and assuming a level of tie dependence on the supported system, prepare a generation sequence, which in turn is used to prepare the CAPT of the supporting system.
  - Using the tie thermal limit and the tie FOR, modify the CAPT to produce the EAU representation of the supporting system. Note the SLCC of the supporting system for later comparisons.
- Step 5) Perform an LOLE analysis of the supported system using the EAU from the supporting system. Note the SLCC of the supported system for later comparison. Find the difference between this SLCC and that produced without external assistance. This gives one value of reserve sharing for this set of generating units and configuration of supported and supporting systems.
- Step 6) To allow the supported and supporting systems to achieve equilibrium, increase the assumed tie dependence (in step 4) and produce new CAPTs, (300 MW steps used in the sample case study). This simulates the condition that 300 MW of tie support was assumed in the preparation of the CAPT.

- Produce a new EAU, compute the SLCC of the supported system again. Compute the incremental gain (or loss) in SLCC of the supported system from the zero tie dependence case.
- Rerun Step 6 with increasing tie dependence assumed until the tie assumption inherent in the supporting system equals the incremental SLCC gain (determined reserve sharing) of the supported system. The interconnection will then be at equilibrium.
- Step 7) From a plot of the assumed tie dependence in the supporting system versus the gain in SLCCs of the supported system, the equilibrium reserve sharing limit is determined. This curve forms the upper limit of reserve sharing possible for the interconnected system for that configuration of supporting and supported systems.
  - To determine if the balanced reserve sharing limit is the appropriate limit to use for the two way tie reliance limit, one must perform the entire procedure with the roles of supporting and supported systems reversed. The lower of the two balanced reserve sharing limits becomes the tie reliance limit.

Equilibrium of the supported and supporting system must be achieved as the assumed field support available at the time of creating the generation sequence affects the number of new plants added to the AIS system. This in turn affects the ability of the AIS system to aid BCH, assuming this tie support is not available, which it would

not be if BCH was in need of assistance. Since the level of reserve sharing is computed with an assumed sequence in place, the reserve sharing is actually dependent on the assumed tie dependence. Hence, the solution must be determined iteratively. Equilibrium is achieved when the commendate dependence (support available at time of sequence generation) equals the computed reserve sharing limit. The procedure for determining the reserve sharing limit is described in detail in Section 3.5.

The following discussion provides details for each of the seven steps above.

### 3.3 Assessment of Coincident / Daily Peak Load Ratios

The objective of this step was to provide information to the risk model of the supporting system that would allow the daily peak load (generated from the LDC internally) to be converted to the coincident load, which is used to prepare the CAPT. To adjust the internally prepared daily peak load to approximate the coincident load requires an adjustment ratio. Thus, a ratio must be calculated that is representative of the expected difference between the daily peak load and the coincident load. Since the precise relationship between these two levels cannot be known in advance, a distribution of coincident load and daily peak load ratios was obtained from historical data. These ratios, along with their probability distribution, are made into a set of adjustment factors, which has been named a "load ratio block" (LRB). This specifically is a set of up to five adjustment factors for each month of the year that transforms the supporting system's daily peak load to an estimation of the coincident load that would occur at the time the CAPT is required.

The LRB factors permit the computation of up to five coincident loads, each probability of occurring taken from the probability distribution of the LRB

with a probability of occurring taken from the probability distribution of the LRB. One CAPT is prepared for each coincident load and the CAPTs are combined into a single daily CAPT by using an expected average of each assistance level multiplied by its respective probability for that CAPT. The daily CAPTs are used to produce the monthly CAPT which is changed into the EAU of the supporting system by altering (truncating) the CAPT to reflect the transmission limitation of the tie line, and by modifying the CAPT to reflect the probability of the tie line being available. The supporting EAU is then ready for use in the supported system's risk model.

It is well known that the reliability response of a generation system is highly non-linear, hence, it was determined that more than one estimation of the coincident load should be made for each daily peak load. Based on the work of Billinton and others on multiple state representation [46], and considering the additional calculational legwork required to capture many load levels, it was felt that a distribution of five coincident levels would represent a good compromise between accuracy and computational difficulty. It was further determined that the results obtained with different numbers of coincident load levels should be examined so as to determine whether what number of load levels represents an optimum.

A study of the frequency of simultaneous daily peaks between the two provinces was performed to determine if, in fact, the coincident load is less than the daily peak load a significant proportion of the time. From the results, seen in Table 7, it can be seen that most of the time the coincident load is less than the daily peak load in the supporting system. In either case, with the AIS supporting BCH and for BCH supporting the AIS, the coincident load was also the daily peak 490 days out of six years of daily data, i.e. 22.4% of the time.

	AIS at time	of	BCH at time of	
Month	BCH Daily l	Peak %	AIS Daily Peak	%
Jan	25	1.1	24	1.1
Feb	69	3.2	69	3.2
Mar	71	3.2	71	3.2
Apr	55	2.5	54	2.5
May	60	2.7	62	2.8
Jun	39	1.8	39	1.8
Jul	43	2.0	44	2.0
Aug	31	1.4	31	1.4
Sep	46	2.1	45	2.1
Oct	47	2.1	47	2.1
Nov	1	0.05	1	0.05
Dec	3	0.14	3	).14
Totals	490		490	

Table 7Frequency of Coincident Loads that are also Daily Peaks

An examination of many individual days of hourly load data yielded the conclusion that the daily load curves for the AIS and BCH often appear to have a similar shape, see Figure 6, with a midday peak around noon and a larger hump at the daily peak, about 18:00 hours. The similarity in load profiles helps explain the relatively low incidence of coincident load being the daily peak load, as was shown in Table 7. This is particularly true in the winter months, the months of most need. The relatively small frequency of coincident loads that are daily peak loads and the similarity in load shape patterns indicates that most of the hourly load diversity comes

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from the time zone difference. If other significant factors were acting on the load data of two systems, the resultant profiles should exhibit more variation from hour to hour and day to day. For the AIS - BCH interconnected system it is, therefore, reasonable to conclude that most load diversity arises from the one hour time shift.

To prepare the LRBs, the ratio of daily peak to coincident peak was examined for each day of the historical load data from the years 1982 to 1987. As the LOLE program operates on a monthly basis, the LRBs were arranged to allow different ratios for each month. This eliminates the assumption that one distribution of ratios is valid for the entire year. For each month, approximately 180 ratios of coincident to daily peak were obtained, about 30 days per month for 6 years. To obtain these ratios, the supported system's daily peak and the supporting system's coincident load were arranged in tables such as Table 8, which is specifically for the BCH 1000 at time of AIS daily peak. Table 8

BCH loads at time of AIS Daily Peaks

Dav	214	BCH			AIS			-	AIS E	CH	AIS	всн
Day					1984						1987	1987
1	3348	4059	3139		3364					4293	3934	4731
2	3634		3328	4172	3798			5432		5737	4344	5604
3	3676	4738	3779	4771	3979	5106	4118	5434	4459	5630	4213	5192
4	4032	5372	3843	4885	3955	4889	4067	5491	4288	5287	4246	5126
5	3991	5388	3832	4809	3949	4988	3933	4831	4262	5359	4575	5607
6	3884	5689	3824	4977	3884	4892	3856	5079	4617	5690	4633	5669
7	3996	5474	3668	4779	3775	4485	4323	5338	4502	5557	4621	5687
8	3877	5189	3401	4286	3741	4496	4343	5323	4498	5484	4591	5767
9	3681	4756	3414	4463	4192	5168	4416	5449	4533	5576	4490	5709
10	3722	4917	4001	4833	4208	5009	4415	5320	4309	5403	4189	5319
11	3916	5181	3854		4138							5094
12	3904	5205	3767	4616	4102	4940	3929	4735	4224	4870	4611	
13	3754	5044	3719		4091		3:12	4809		5482		
14	3968	4849	3645	4557	3950	4637	4266	5459	4490	5455	4612	5676
15	3832	5176	3403					5180				
16	3645	4901	3389								4561	5777
17	3526	4829	3800	4737	4242	5228	4157	5249	4357	5410	4275	5523
18	3981	5177	3732	4589	4260	5449	4190	5116	4125	5132	4324	5268
19	3961	5115	3738	4600	4228	5360	4150	5032	4125	476.3	6 4672	5827
20	4004	5161	3770	4578	3949	5533	4130	4807	4504	5387	4584	5664
21	3993	5352	3658	4533	3735	4971	4295	5358	4523	5548	3 4602	5578
22	3887	5394	3529	4219	3599	4491	4199	4940	4619	5554	4595	5694
23	3733	5248	3503	4364	3964	5169	4110	5506	4521	5255	5 4449	5750
24	3554	4864	4020	4957	4055	4981	4079	5281	4355	5522	2 4252	5593
25	3767	5432	3863	4858	3973	4703	3946	5463	4080	4715	5 4167	5467
26	3739	5317	3864	4738	3972	5083	3782	2 4878	3 4101	5073	3 4606	5 5977
27	3834	5094	3721	4453	3743	4966	5 3806	5 5169	4485	5453	3 4533	5883
28	3807	5240	3529	4761	3545	4171	4249	9 5453	4500	) 5274	4 4511	5576
29	3744	5178	3359	4449	3479	4528	3 4361	5083	4543	5403	3 4467	7 5653
30	3497	4870	3276	6 4371	3757	4811	4329	5802	2 4498	3 539	1 4360	) 5612
31	3389	4748	3651	4449	3729	5287	7 4345	5 5673	3 4256	5 536	9 4089	5210

From the load tables, ratios of coincident load divided by daily peak load were obtained. A short excerpted sample of a ratio table is found in Table 9.

## Table 9BCH-AIS Coincident Unitized Loads

BCH Coincident to Daily Peak Ratios at time of AIS peak

0.9618       1.0000       1.0000       1.0000       1.0000       1.0000       0.9474       0.9816       0.9067       0.958         0.9434       1.0000       1.0000       0.959       0.9746       0.9589       0.9854       0.9684       1.0000       0.9361       0.9189       0.9709         0.9491       0.9888       1.0000       0.9951       1.0000       0.9839       0.9840       0.9667       0.9958       1.0000       0.9524       0.9583         0.9401       1.0000       0.9981       1.0000       0.9955       0.9814       0.9756       0.9828       0.9928       0.9831       0.9119       0.9549         0.9375       0.9833       0.9804       1.0000       0.9432       1.0000       0.9820       1.0000       0.9644       0.9293       0.9566	
0.9491 0.9888 1.0000 0.9951 1.0000 0.9839 0.9840 0.9667 0.9958 1.0000 0.9524 0.958 0.9401 1.0000 0.9981 1.0000 0.9955 0.9814 0.9756 0.9828 0.9928 0.9831 0.9119 0.9549	1
0.9401 1.0000 0.9981 1.0000 0.9955 0.9814 0.9756 0.9828 0.9928 0.9831 0.9119 0.9549	5
	1
0.9375 0.9833 0.9804 1.0000 1.0000 0.9432 1.0000 0.9820 1.0000 0.9644 0.9293 0.9566	)
	)
0.9659 0.9868 1.0000 0.9960 1.0000 0.9898 0.9983 0.9887 1.0000 0.9987 0.9089 0.947	7
0.9658 1.0000 1.0000 0.9824 1.0000 1.0000 0.9743 1.0000 0.9571 0.9948 0.9352 0.956	1
0.9516 1.0000 0.9943 0.9835 1.0000 0.9684 0.9918 0.9566 0.9802 0.9873 0.9154 0.993	3
0.9510 1.0000 1.0000 1.0000 0.9695 0.9952 0.9959 0.9776 0.9812 0.9123 0.9430	0
0.9681 1.0000 1.0000 1.0000 1.0000 0.9948 0.9970 1.0000 1.0000 1.0000 0.9357 0.956	1
0.9527 1.0000 0.9942 1.0000 0.9770 1.0000 0.9971 0.9870 0.9840 1.0000 0.9178 0.962	9
0.9680 0.9935 0.9992 0.9880 0.9571 0.9980 0.9916 0.9868 0.9373 0.9246 0.9467 0.972	4
0.9557 1.0000 1.0000 0.9546 0.9725 1.0000 0.9692 1.0000 0.9839 0.9423 0.9221 0.970	1
0.9406 0.9957 1.0000 0.9687 0.9755 0.9917 0.9935 0.9874 1.0000 0.9768 0.9300 0.964	2

To convert the approximately 180 ratios per month to the maximum of five ratios permitted by the LRB, a distribution of the ratios was obtained. The distributions were often skewed, with more points near 1.0. Table 10 gives a sample distribution for January and shows the mean and standard deviation of the entire table, the frequency of points between range limits, the probability of being in a given range and the expected value of the ratios that fell within a given range.

# Table 10Distribution of Ratios, BCH at time of AIS Peak

Mean 0.952 St. Dev 0.0233

Poin	ts Jan	Ranges Free	quency	Probability	Expected Ratio
1	0.9618	0.00-0.92	6	0.032	0.915
2	0.9434	0.92-0.94	52	0.280	0.931
3	0.9491	0.94-0.96	76	0.409	0.950
4	0.9401	0.96-0.98	27	0.145	0.967
5	0.9375	0.98-0.9999	1	- 0.134	1.000
6	0.9659	0.9999-1.0	24		
7	0.9658				
8	0.9516				
9	0.9510	186.	.0000	1.0000	
10	0.9681				
11	0.9527				
12	0.9680				
13	0.9557				
14	0.9406				
15	0.9601				
16	0.9565				

The five expected ratios generated from Table 10, along with the probability of being in the respective range are used to create the LRB. Table 11 gives the five level (multi-level) LRBs for each province in the required format for the LOLE program. The program accepts the keyword TIEBLK to signal the input of an LRB. The second column of the table is the month number, with January given number 1. The third column is the highest load ratio followed by the probability of it being encountered. The next eight columns are for the other four load ratios and their

associated probabilities.

Table 11Load Ratio Blocks, Multi-level

AIS at time of BCH Daily Peak.

P4 LR5 LR1 **P1** LR2 P2 LR3 P3 LR4 P5 TIEBLK 1 0.991 0.581 0.973 0.376 0.955 0.043 0.000 0.000 0.000 0.000 TIEBLK 2 0.995 0.822 0.973 0.166 0.948 0.012 0.000 0.000 0.000 0.000 TIEBLK 3 0.995 0.828 0.974 0.151 0.951 0.021 0.000 0.000 0.000 0.000 TIEBLK 4 0.995 0.672 0.972 0.217 0.951 0.067 0.934 0.039 0.914 0.005 TIEBLK 5 0.995 0.796 0.974 0.097 953 0.038 0.931 0.065 0.914 0.004 TIEBLK 6 0.994 0.744 0.976 0.089 0.949 0.078 0.926 0.078 0.901 0.011 TIEBLK 7 0.994 0.780 0.973 0.097 0.954 0.054 0.931 0.048 0.914 0.021 TIEBLK 8 0.993 0.720 0.972 0.161 0.949 0.065 0.933 0.048 0.919 0.006 TIEBLK 9 0.995 0.661 0.974 0.133 0.947 0.072 0.930 0.111 0.904 0.023 TIEBLK 10 0.993 0.780 0.973 0.167 0.952 0.043 0.935 0.010 0.000 0.000 TIEBLK 11 0.987 0.317 0.971 0.600 0.954 0.083 0.000 0.000 0.000 0.000 TIEBLK 12 0.986 0.376 0.972 0.586 0.953 0.038 0.000 0.000 0.000 0.000

## BCH at time of AIS Daily Peak

TIEBLK 1 1.000 0.134 0.967 0.145 0.950 0.409 0.931 0.280 0.915 0.032
TIEBLK 2 0.995 0.805 0.973 0.142 0.952 0.024 0.928 0.029 0.000 0.000
TIEBLK 3 0.995 0.774 0.973 0.167 0.953 0.032 0.928 0.022 0.916 0.005
TIEBLK 4 0.994 0.739 0.972 0.156 0.951 0.039 0.932 0.056 0.914 0.010
TIEBLK 5 0.996 0.694 0.971 0.172 0.952 0.086 0.931 0.032 0.916 0.016
TIEBLK 6 0.993 0.661 0.971 0.278 0.951 0.050 0.935 0.011 0.000 0.000
TIEBLK 7 0.994 0.720 0.971 0.199 0.951 0.075 0.910 0.006 0.000 0.000
TIEBLK 8 0.993 0.683 0.971 0.250 0.956 0.086 0.938 0.011 0.000 0.000
TIEBLK 9 0.995 0.522 0.971 0.250 0.954 0.094 0.933 0.034 0.000 0.000
TIEBLK 10 0.995 0.586 0.972 0.204 0.949 0.097 0.929 0.097 0.914 0.016
TIEBLK 11 0.988 0.017 0.967 0.211 0.951 0.456 0.932 0.267 0.913 0.049
EBLK 12 0.993 0.048 0.967 0.312 0.951 0.527 0.936 0.086 0.906 0.027

Table 12 gives a simplified LRB based only on the mean value of all dat: Table 10, i.e. a single expected load ratio for each month, as opposed to the distribution in Table 11. The single level LRBs (Table 12) could be used in place of the more complex multi-level LRB given in Table 11 if it was found that are reserve sharing results do not vary significantly for the systems being studied.

AIS Single Level LRB.	BCH Single Level LRB.
LR1 P1	LR1
TIEBLK 1 0.983 1.000	0.952
TIEBLK 2 0.990 1.000	0.989
TIEBLK 3 0.991 1.000	0.983
TIEBLK 4 0.984 1.000	0.985
TIEBLK 5 0.987 1.000	0.984
TIEBLK 6 0.983 1.000	0.984
TIEBLK 7 0.985 1.000	0.986
TIEBLK 8 0.983 1.000	0.984
TIEBLK 9 0.979 1.000	0.983
TIEBLK 10 0.988 1.000	0.978
TIEBLK 11 0.975 1.000	0.948
TIEBLK 12 0.976 1.000	0.955

Table 12Load Ratio Blocks, Single Level

The multi-level LRBs will be used for tie reliance determination in this thesis. The single level LRB will be studied as one of the sensitivities performed. This concludes the discussion of the creation of the adjustment factors necessary to permit the LOLE analysis to recognise load diversity at the hour of daily peak, thus, steps 1 and 2 in the procedure have been completed.

### 3.4 Using the LRB in the Preparation of the CAPT

The first step in preparing the data specific to the study systems for analysis was the production of the LDCs used within the LOLE model. These LDCs are used to determine the expected daily load from the monthly peak. The AIS standard preparation procedure uses five years of historical load data to prepare an average LDC for each month. An example of the load profile for one specific month, January 1987, for BCH is shown in Figure 8. This figure shows the characteristic shape of an LDC in that it shows the usual rapid dropoff at the high end, followed by a relatively linearly decreasing portion in the middle, followed by a rather sharp dropoff at the low end.

A similar curve of five years of averaged data was created for each month. Then a set of points (usually between 6 and 9, to a maximum of 24), were used to provide a curve fit approximation to the average LDC for each month. The LDC generation procedure is external to the LOLE program used in this study and was performed by a companion data preparation program to LOLE, called PERCARDS [47]. As most LDCs have a rather typical profile, it is known from experience that about 3 points are needed to model each of the two curved ends, with the large relatively linear region in the middle being mapped by about 2 or 3 values. An





**Figure 8** January 1987 Load Duration Curve for BCH example of the final LDC for BCH that was used in the LOLE program is shown in Figure 9.

Having produced the LDCs for the program LOLE, the next step was to make an assumption about the tie dependence. This assumption is required because the tie reliance determined from the reserve sharing limit depends on the assumed generation available, which is dependent on the amount of tie line support one can expect to receive.

Using the assumed tie reliance, a generation sequence was prepared such that the expected LOLE of the system in the test years was below the criterion set by the respective provincial system planners, 0.2 days/year for the AIS and 0.1 days/year for





Figure 9 5 Year Average, January Load Duration Curve for BCH

BCH. Next, the loads were increased to use up any surplus and bring the system to criterion level. This step is necessary because surplus generation is a saleable commodity and must not be included in the reserve sharing limit determination.

With the expected generation available determined from the assumed tic reliance, it was possible to prepare the required EAU and thereby compute the reserve sharing limit and hence the tie reliance limit.

For the first iteration a tie reliance of zero was assumed. This provided an extreme value (endpoint) of computed tie reliance, which is an optimistic maximum. When the tie reliance was set to zero, the internal generation sequence created for

either province provided all load and required reserves without external assistance. Thus the generation system would be overbuilt for that case, as in reality the generation system would be built assuming some level of reserve sharing.

To facilitate the convergence of the two s stem interaction, the assumed tie dependence was increased in steps of 300 MW, new sequences generated, from which new tie reliances were determined as detailed in Section 3.5 below. Convergence was obtained from a plot of assumed dependence versus computed tie reliance, when the value of the assumed tie dependence equalled the computed tie reliance.

When preparing the CAPT of the supporting system, the LRB for that system was used to provide the coincident loads needed to incorporate load diversity. The LOLE program performed this step internally. The master CAPT was then truncated to reflect the maximum thermal limit of the tie line and the tie forced outage rate was incorporated to reflect the probability of the line being functional. The resulting EAU prepared for use in the supported system's risk model therefore included effects of the ability of the supporting system to provide assistance, the size of the tie line and the probability of the tie line being available to conduct power. The EAU also retained the same equal sized, MW increments that were present in the CAPT. For the AIS and BCH systems, the step size was chosen to be 5 MW. The EAU for either provincial system was a series of capacity states at 5 MW intervals with the accompanying probability of encountering that capacity state.

During the construction of the CAPT with the five level LRB, care must be exercised when combining the five individual CAPTs into the master CAPT. One must ensure that the master CAPT sums to 1.0. As mentioned previously, each of the five ratios in the LRB was used to compute one daily CAPT for the supporting system. These CAPTs were multiplied by their respective ratio's probability of occurrence. The five CAPTs were combined together beginning at the common point for each table, zerc MW of support. Each 5 MW increment was averaged, line by line, starting at zero MW of assistance and proceeding to the top of the table. This last step allowed the distribution of ratios to be merged into one daily CAPT, from which the monthly CAPT was prepared. This in turn generated the single EAU for

use in the supported system's risk model.

To eliminate the possibility of the automatic maintenance scheduler causing variations within the study, a suitable maintenance schedule was first developed with the automatic maintenance scheduler. All subsequent computer runs used this maintenance schedule so as to eliminate maintenance variability as a factor in the LOLE calculations for any of the sensitivities studied.

The required data on unit generating statistics, forced outage rates and seasonal ratings was obtained from published AIS and BCH data.

## 3.5 Reserve Sharing and Tie Line Reliance Determination

Having completed the analysis of assistance available given several presumed tie line dependence levels, the balanced reserve sharing limit was determined from this data. The increase in the SLCC of the supported system, from the case with no external assistance, provided the required information, i.e. the calculated increase in SLCC that the supported system had, as a result of sharing reserves with the supporting system, as compared with the SLCC of approximately zero, which is what was experienced without any external support. The calculated SLCC increases were plotted against the assumed tie dependence. When the two values reach equality, the balanced reserve sharing limit had been reached; the effect of presumed tie support equalled the computed assistance. In the case of the AIS and BCH it would be expected that the balanced reserve sharing limit would be less than the thermal limit of the interconnection as the latter is a relatively large percentage of either systems' total size.

One balanced reserve tharing limit was determined for the case of the AIS supporting BCH and a second limit was determined for BCH supporting the AIS. The principle of reciprocity demanded that each system assist the other equally, as measured in MW. The final agreed tie reliance limit between two utilities could not exceed either reserve sharing limit. Thus, the weaker system must limit tie reliance because its lesser ability to assist the other, or utilize support from the other, will be reflected in the lower reserve sharing limit achieved.

An example demonstrating the determination of a reserve sharing limit and tie reliance limit determination methodology is found in Figure 10. The data for this example is generic only and is found in Table 13. Figure 16 shows the balanced reserve sharing for the AIS supporting BCH case, the lower line. This forms the computed tie reliance limit for this example, with a value of 375 MW.

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Table 13	Example Data	for Reserve	Sharing	Computation

Assumed tie	AIS supporting	BCH supporting
dependence	BCH	AIS
0	500	600
300	400	500
600	300	400



Figure 10 Example Tie Reliance Determination

To compute the balanced reserve sharing limit one must examine the point where the increase in SLCC experienced by BCH equals the presumed tie interaction between the provinces. This is the equilibrium condition. Any point where the tie line assumption equals the increase in load carrying capability of the supported system (increase in SLCC), constitutes such a balanced form of intertie support. This can be modelled by placing a line on Figure 10 starting at the point 0,0 and concluding at 800,800. The oints along this line represent equal support levels being assumed and actually increased.

The balanced reserve sharing limit from Figure 10 is found by locating the position where the increased SLCC curve intersects the equal support criterion line. The value read from the x axis (or the y axis) is the amount of support BCH receives (reserve sharing), given that the AIS and BCH systems were designed assuming that same tie line support.

# 3.6 Inclusion of Transmission Losses on Tie Support

It is recognized that conducting electricity results in transmission line losses. which result in real power losses for the system. For the case of mutual support between the AIS and BCH, these losses have been determined by the AIS and BCH utilities to be in the order of 14% for electrical energy sent from the AIS to BCH, and about 2% for energy sent from BCH to the AIS. The difference results from the geographic locations of the resource and load centers.

The 14% loss to BCH would mean that only 86% of the power put on the line by the AIS would assist BCH, the balance would be lost enroute. Since the thermal tie capacity is set at 800 MW at the Cranbrook interchange, the losses could be considerable, up to 112 MW. A potential loss of such a size could significantly affect the results of this study, hence, it was determined that the effects of transmission losses must be accounted for in the risk model.
The losses were incorporated by adjustment of the CAPT prior to the EAU being created. Each 5 MW increment of the CAPT would be diminished by the transportation factor equally. Thus, for a factor of 0.86 (the AIS supporting BCH), the increments received by BCH would be 4.3 MW (5 x 0.86). The probability for each of the capacity states would remain unchanged, just the state step size itself would be reduced by the line loss. However, the program LOLE, would not allow the step size to be altered from that chosen for the COPT step size. This necessitated the conversion of the reduced steps to "equivalent probability" full sized (5 MW) steps. This was performed using the same technique the LOLE method uses to reduce capacity states in the convolution process. Specifically, the loss reduced MW step increments were broken down into the two closest full sized states, with the probability apportioned between these states using a linear interpolation [2]. See Table 14 for a sample calculation.

Table 14	Example	Computation	of Equivalent	CAPT Leve	İs
----------	---------	-------------	---------------	-----------	----

Origia	ral	Step size of	New full	Modified
State	Prob.	Power received	size steps	Probability
0	0.0011	0	0	0.0018
5	0.0053	4.3	5	0.0055
10	0.0034	8.6	10	0.0024

The probabilities computed for the modified CAPT, which produces the EAU, result in the transmission losses being included in a very realistic way. The assisting province sees a full sized load due to providing support, while the assisted province

receives a reduced support level. An assessment of ignoring transmission losses is included in Chapter 4 as well, to demonstrate its significance on the study.

This concludes the discussion of the techniques developed and used within this thesis.

#### CHAPTER IV Results and Discussion

#### 4.0 Tie Reliance Limit Determination

The results of reserve sharing were computed for the time period of the test study, 1993-1997, and an effective tie reliance limit was determined. These results were obtained with the methodology described in Section 3.5.

Figure 11 shows the increase in SLCC of the BCH system with AIS support. The three computed delta SLCC values result from the three cases wherein the AIS system was designed with the inherent assumption of 0, 300 and 600 MW of the line support from BCH (or another utility). For the case with BCH supported by the AIS the balanced reserve sharing limit was computed as 481 MW.

The first point (left point) plotted on Figure 11 shows that for an AIS system built with no dependence on a tie interconnection, the BCH system received 666.5 MW of firm power support from the AIS. The second point on the curve was for the AIS system constructed assuming a tie dependence of 300 MW. This resulted in a support level diminished from the first case to 554.0 MW. The last point was for a tie dependence of 600 MW and indicated a reserve sharing benefit of 432.2 MW for BCH. It can be seen that the graph has downward curvature. As dependence increased, the ability of the supporting province to provide assistance decreased.

Using the method of Section 3.5 and examining Figure 11, one can see the point of intersection is about (481,481). This represents the maximum reserve sharing possible with the AIS supporting BCH as determined by the ability of the AIS to provide assistance over and above its own load needs. Note that the limit of 481 MW





was well below the thermal rating of the tie line, which was set at 800 MW at the receiving end.

Figure 12 shows the tie reliance determination in the case of BCH supporting the AIS. As expected, the larger size of the BCH system gives it more ability to aid the AIS under the zero tie dependence case (left point) and so the assistance curve starts somewhat higher than it did on Figure 11. However, the weaker AIS system, especially noticed with the tie dependence of 600 MW, caused a rapid diminishment of the reserve sharing benefits available to the AIS. The weakness of the AIS, when a tie dependence of 600 MW was assumed, was such that the support of BCH through reserve sharing could not overcome the inability of the AIS to supply firm power.







The reciprocal support was determined from the lower of the two balanced reserve sharing limits above. For the AIS - BCH interconnection the limiting quantity was 368 MW, so the tie reliance limit should be set at 368 MW.

Operationally, mutual support must occur at different times as the tie cannot be used in both directions at the same instant. This assumption is based on the natural diversity of the outage probability table and is recognized within the technical literature as a valid assumption [2,30,31]. The following section discusses some of the sensitivities performed to examine this reliance method.

### 4.1 Factors affecting Tie Reliance Limit Determination

### 4.11 Effect of Single Equivalent LRB

As the process of grouping the unitized load ratios was dependent on the analyst's selection of suitable ranges, it was deemed useful to compare the results achieved with the multi-level LRB with those obtained using only the expected value of all unitized load ratios as a single equivalent CAPT adjustment factor, i.e. the single level LRB. The results of utilizing the single LRB are shown in Figures 13 and 14. In either case, the AIS supporting BCH or vice versa, the single equivalent LRB resulted in assistance curves slightly above the five level LRB.

From Figure 13, a balanced reserve sharing limit of about 484 MW was obtained for the AIS supporting BCH. The reserve sharing obtained from Figure 14, BCH supporting the AIS, was about 378 MW. This is quite close to that obtained using the five level LRB, about 368 MW. Both results support the notion that for practical purposes, for the AIS - BCH tie line, a single level LRB would produce good results with less effort. It is therefore recommended that for future studies a single LRB be produced to incorporate load diversity.

It can further be noted that the difference between the two lines on Figure 13 is approximately constant. This indicates that the single level LRB is equally useful as an estimation of load diversity support for any presumed tie dependence, i.e. it is not a function of presumed tie dependence.





### Figure 13 Reserve Sharing using Single Level LRB, AIS supporting BCH

#### 4.12 Effect when Load Diversity Excluded

It was deemed of interest to ascertain the overall effect of using the LRB to simulate load diversity, in the determination of the tie reliance limit. To find the relative contribution of the LRB, a sensitivity was performed with the LRBs removed. All other factors remained unchanged. The results of this are show in Figures 15 and 16.

From Figure 15, the case with the AIS supporting BCH, the effect of removing the load diversity factors was to lower the assistance curve by about 50



Figure 14 Reserve Sharing using Single Level LRB, BCH supporting AIS

MW. This translated into a balanced reserve sharing limit of about 431 MW. This represents a considerable decrease in the assistance that could be expected through the tie when load diversity was included, and indicates the relative contribution of load diversity to reserve sharing.

From Figure 16, with BCH supporting the AIS, the same trend of decreased reserve sharing can be observed. In this case, the reserve sharing decreased from 368 to 293 MW, a difference of 75 MW. It is interesting to note that this difference is not the same as for the case of the AIS supporting BCH. Thus, it can be seen that generalizations about the effects of the load diversity are not straight forward.





Figure 15 Effect of Load Diversity on Reserve Sharing, AIS supporting BCH

Inclusion of the LRBs, to model load diversity, made a significant impact on the final tie reliance values determined using this reciprocal technique. In this example the tie reliance was reduced from 368 to 293 MW when load diversity was excluded. As a result of the 75 MW contribution to tie reliance directly attributable to load diversity, it is recommended that load diversity be included in any interconnection study for which load diversity is apparent in the load data.



Figure 16 Effect of Load Diversity on Reserve Sharing, BCH supporting AIS

#### 4.13 Effect when Tie FOR Reduced

One may assume that it would be of great benefit to have many tie lines attached to a given area. This would provide additional means of support to that area. However, if the utility is also obliged to support its neighbours to the extent that it receives support, then its ability to support external ties likely becomes the limiting factor. It may, therefore, be incorrect to assume that more tie lines automatically results in increased support. Additional tie lines between two interconnected utilities would have the effect of providing redundancy to the interconnecting transmission system and of changing the thermal rating of the tie line. Both of these factors were used in the determination of the EAU.

The result of tie redundancy would be an effective tie forced outage rate much lower than any one line could attain, that is, an interconnection system much more reliable than could normally be expected of a single line.

To determine the effect of the tie reliability assumption on the reliance determination, the tic FOR parameter was altered from 1.0% (normal level) to 0.01%, a near perfect reliability level.

From Figure 17, the reserve sharing limit for the case of 0.01% tic FOR, AIS supporting BCH, was determined to be about 500 MW. The result represents an increase of 19 MW, or 4.0% of expected tie support. This is not a particularly significant change and indicates that reserve sharing, for this particular case, is not very sensitive to changes in the forced outage rate of the tie line.

For the case of BCH supporting the AIS, Figure 18, the reserve sharing limit was established at 414 MW, which would also become the tie reliance. This represents a tie reliance increased by 46 MW, or 12.5%, from the normal tie reliability levels. The tie reliability parameter chosen thus has a noticeable effect on the tie reliance and should not be neglected. Future studies should be done to ascertain the tie line forced outage rate with a greater degree of confidence to justify the use of 1.0% as the FOR for normal tie operation. For the AIS - BCH tie line, the





Figure 17 Effect of Tie FOR on Reserve Sharing, AIS supporting BCH

degree of confidence in the accuracy of the current tie FOR value is not very high.





Figure 18 Effect of Tie FOR on Reserve Sharing, BCH supporting AIS

#### 4.14 Effect when Tie Size Increased

The second effect of adding extra tie line(s) would be to increase the thermal rating of the interconnection system. To ascertain the degree that tie reliance depends on the tie size chosen, the tie line thermal rating was changed from 800 MW to 1000 MW. The results of this are shown in Figures 19 and 20.

From Figure 19, with the AIS supporting BCH, the reserve sharing was determined to be 486 MW. This compares with the control case of 481 MW, a rather small increase of 5 MW in firm support for a change in the size of 200 MW.



# Figure 19 Effect of Tie Size on Reserve Sharing, AIS supporting BCH

In the case of BCH supporting the AIS, Figure 20, the tie reliance was established at 404 MW. This compares with the 368 MW limit computed earlier for the control case. This 36 MW increase in firm support is only 18% of the 200 MW tie size increase. However, the 36 MW represents about 9.8% of the expected tie reliance. Thus, the size of the tie line used in this study does significantly affect the final tie reliance result. As such, further work may be required to accurately determine the tie size. The current fie AIS - BCE tie restriction of 800 MW is based on thermal considerations, some of which could possibly be altered with an equipment upgrade. It is recommended that the possibility of upgrading equipment be





# Figure 20 Effect of Tie Size on Reserve Sharing, BCH supporting AIS

investigated. The potential 36 MW of increased firm reliance would represent the upper limit of benefits available for a tie size increase of 200 MW, and the costs of re-rating the tie line thermal rating could be justified against the costs of providing 36 MW of firm support by other means.

#### 4.15 Effect when Transmission Losses Excluded

As mentioned earlier, a means of accounting for transmission losses was included in this study, so as to more accurately model the support available between the two provinces. All the reserve sharing values obtained above had the transmission losses accounted for.



Figure 21 Effect cf Transmission Losses on Reserve Sharing, AIS supporting BCH

Figure 21 shows the effects of excluding transmission losses for the case with the AIS supporting BCH. Here one can see that the 14% loss of transmitted power experienced when sending electrical energy to the major load centers of BCH is noticeable, but less significant than expected. The reserve sharing for this no-loss case is about 500 MW, an increase of 19 MW from the base case of 481 MW.

The result of the transmission loss on reserve sharing is less marked for the situation wherein BCH supports the AIS, as the line losses are only about 2%. Figure

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Fagure 22 Effect of Transmission Losses on Reserve Sharing, BCH supporting AIS

22 indicates that a tie reliance of 369 MW would be obtained for this case, as compared to 368 MW in the control case. Thus, in this study, neglecting the effects of long distance transmission losses would be quite in significant.

The reason for this insignificance, which was not expected, can be attributed to the rather large reserves available throughout most of the year within each province. The available reserves are compiled in the CAPTs. The procedure of converting the CAPTs to EAUs used herein fixed the tie thermal rating at the *outlet* of the interconnection. This allowed the sending system to put more power on the line to compensate for losses. In the AIS - BCH case, the CAPTs of the supporting systems allowed the higher demands to be met through available reserves and hence, the effect of losses was minimized.

If the system were modelled v is the tie thermal rating set at the beginning of the interconnection, all support above 800 MW would not have been available and the results of the transmission losses would have been more noticeable. The procedure for adjusting the diminished capacity states would still be valid, however, the 800 MW limitation would have been performed first, with the final EAU computed with the loss reduced MW support level. This would be only 688 MW instead of 800 for the AIS support  $\leq 0.0^{\circ}H_{\odot}$  is 0.86 = 688; or 784 MW for BCH supporting the AIS. Future enserts to determine the exact location of the thermal capacity bottleneck need to be performed. Chee this is ascertained, the modelling procedure can be modified to reflect the known location, and the results obtained will then correspond more closely to the actual system under investigation. Table 15 contains a summary of the tie reliance determinations discussed above, and also has the specific values of reserve sharing computed for the tie dependence levels of 0, 300 and 600 MW for each case studied.

Fig. Run Particulars No.	Balanced Res. Sharing Limit	Values ai G	assumed 300	tie levels 600
			<b></b>	
AIS supporting BCH				
11 Five level LRB	481	666.5	554.0	432.2
13 Single level LRB	484	668.6	558.2	436.8
15 Lord Diversity exc	luded 431	629.9	487.5	357.8
17 'Tie FOR reduced	500	738,2	595.3	452.4
19 Tie Size increased	486	723.7	569.6	434.9
21 Trans. Losses excit	uded 500	679.8	581.3	460.4
BCH supporting AIS				
12 Five level LRE	368	722.1	417.9	195.2
14 Single level LRB	378	739.5	436.4	214.6
16 Load Diversity exc	luded 293	597.1	286.9	60.3
18 Tie FOR reduced	414	803.6	498.5	275.0
20 Tie Size increased	404	790.3	481.4	256.8
22 Trans. Losses excl	udot 369	723.7	410.5	106.7

Table 15Summary of Tie Reliance Calculations

It has been shown that the effect of load diversity, as expressed by the coincident two province load level actually experienced, signifier untry affects the level at which the CAPT table is cut from the COPT, and the resulting tie reliance determined. In previous studies performed with western provinces of Canada data [48,49], the effect of load diversity on reserve sharing was assumed to be minimal

and was neglected. These studies demonstrated little benefit to be gained from increased cooperation between neighbouring utilities apart from emergency support. The results of this thesis indicate that disregarding load diversity would likely result in a pescimistic assessment of reserve sharing. As such, the results of the above studies [48,49] may be deficient in having reached conclusions which neglected the reality of load diversity.

#### CHAPTER V Conclusions

This thesis has developed and demonstrated the use of a technique for determining tie reliance based on identifying the limit of reserve sharing available between two neighbouring systems. It was found that the ability of the supporting system to help and the relative strength of the supported system were significant factors in determining the tie reliance. This procedure included the effects of locd diversity between the supported and supporting utilities, and assessed the relative impact of load diversity on the results. The thesis further demonstrated the benefits to the utilities that could be expected upon entering into a reserve sharing agreement, and provides a way to determine an appropriate reciprocal level of such reserve sharing to be used for planning purposes for generation deferrals.

It was determined that a significant portion of the effects of load diversity between the provinces can be attributed to the one hour time difference that exists between these two provinces. Knowledge of this factor may prompt utility planners to emphasize reserve sharing between systems in differing time zones, and thus affect the way utilities plan their generation systems in the future.

The level of tie reliance, as determined using this proposed technique with a five level load ratio block, (capacity assistance probability table adjustment factors), was determined to be 368 MW, and was limited by the ability of the AIS to meet its own load while receiving support from BCH, after having deferred 368 MW as a result of presumed tie dependence. If the tie line were not relied upon for the 368 MW, each utility would have to provide about this same amount of firm power to

maintain their level of reliability. To accomplish this, they would need to build additional generating plants. Thus, reserve sharing represents a very significant tool for utility planners to make use of when contemplating how best to meet future generation requirements. This point is well recognized within the AIS and BCH, and the two parties have entered into a formal reserve sharing agreement in 1991.

The contribution to reserve sharing by load diversity was assessed. Neglecting load diversity, by removing the CAPT adjustment factors (LRBs), resulted in a tie reliance of 293 MW, which represents a 20.4% decrease in the reserve sharing capability. Thus, recognizing load diversity has the effect of adding 75 MW of firm power to both systems. This is only slightly less that the amount of firm power received from a standard 100 MW generation plant, currently valued at about \$45,000,000. Therefore, if load diversity is apparent in the load data, it cannot be neglected without seriously compromising the results of the interconnection study.

A simplified analysis, using a single equivalent CAPT adjustment factor (single level LRB) produced a tie reliance of 378 MW. This compares with 368 for the five level LRB, and indicates that using the single level LRB to reflect coincident load levels is sufficient. As it is relatively simple to compute the single level LRB and to modify most any LOLE program to use the LRB, the use of the single LRB is recommended.

Several of the key parameters that affect the construction of the EAU were examined. These parameters include transmission losses, tie reliability and tie size. Neglecting transmission losses resulted in a tie reliance of 369 MW instead of 368 MW; for the case with BCH supporting the AIS, using 2% line losses. This insignificant change is due in part to applying the thermal limitation at the end of the tie line. It is anticipated that applying the limitation at the beginning of the tie line or elsewhere would show losses to be more significant.

Altering the tie size by 200 MW resulting in an increase in tie reliance of 36 MW, about 9.8%. The upper limit of the effect of increased tie reliability was determined to be 46 MW, about 12.5%. Changing either of these parameters causes enough variation in the final tie reliance limit to warrant further investigation into possible ways of improving them. The gain in firm power could be used as a basis to assess the morits of various plans of altering these parameters.

It is hoped that further studies, utilizing this technique for inclusion of load liversity effects into a risk model, could be performed to extend this technique to three or more areas. It is hoped that generation planners will recognize, and make use of, these technique is for incorporating the effects of load diversity (an hourly effect) into a daily LOLE model. These effects can be combined into a multi-state approximation of a tie line to replace the very simplistic two state tie line models currently in use in many LOLE analysis.

#### REFERENCES

- 1. Schoonyan, G. "Presentation of fulti-Area Generation Planning Reliability made to the EUPC Generation Planning Committee" August 24, 1990.
- 2. Billinton, R. and R. N. Allan, "Reliability Evaluation of Power Systems", Plenum Press, 1984, Chap. 2 and 4.
- 3. Haringa, G. E., G. A. Jordan and L. L. Garver, "Application of Monte-Carlo Simulation to Multi-Area Reliability Evaluations", IEEE Transactions, January 1991, 0895-0156/91/0100-0021.
- 4. Billinton, R. and M. P. Bhavaruju, "Loss of load approach to the evaluation of generating capacity reliability of two interconnected systems", CEA Transactions, Spring Meeting, March 1967.
- Cook, V. M., C. D. Galloway, M. J. Steinberg and A. J. Wood, "Determination of reserve requirements of two interconnected systems", AIEE Transactions, PAS-82, Part III 1963, pp 110 116.
- 6. Garver, L. L., "Reserve Planning for Interconnected Systems", Power Engineering, 1976.
- 7. Calabrese, G., "Generating Reserve Capacity Determined by the Probability Method", AIEE Transactions 66, 1947, pp 1439-50.
- 8. Billinton, R., "Application of Probability Techniques in the Evaluation of Generating Capacity Reliability", CEA Western Zone, System Planning and Operating Section, Spring Meeting 1964.
- 9. Billinton, R., "Power System Reliability Evaluation", Gorden and Breach, New York, 1970
- Booth, R. R., "Power System Simulation Model Eased on Probability Analysis", Paper 71C 26 PWR-II-C, IEEE PICA Conference 1971, Boston, Mass.
- Baleriaux, H., H. Jamoulle and Fr. L. de Guertechin, "Simulation de l'exploitation d'un parc de machines du production d'electricite couplees a des stations de pompage", Revue E (albé de S.R.B.E.), 2013, no. 7, 09:3-24, 1967.

- 13. Bhavaruju, M. P., "An Analysis of Generating Capacity Reserve Requirements", Paper # C-74-1554, SEE Transactions, 1974.
- Schenk, K. F. and N. S. Rau, the ations of Fourier Techniques for the Assessment of Reliability of Generation Systems", Paper # A79 103-3, IEEE Winter Meeting 1979, New Yv.
- Hamoud, G. and R. Billinton. "Uncertainty Aspects in LOLE Calculations for Two Interconnected Syster. Paper # 81 WM 071-0, IEEE Winter Meeting 1981, Atlanta, Georgia.
- Hamoud, G. and E. Neudorf, "Use of the Cumulant Method in Computing the Frequency and Duration Indices of Single and Interconnected Systems", Paper # 83 WM 052-8, IEEE Winter Meeting 1983, New York.
- Sutanto, D., H. R. Outhred and Y. B. Lee, "Probabilistic Power System Production Cost and Reliability Calculation By the Z-Transform Method", IEEE Winter Meeting 1989, New York.
- 18. Halperin, H., and H. A. Adler, "Determination of Reserve Generating Capacity", AIEE Transactions, PAS-77, 1958 pp 530-44.
- Hall, J. D., R. J. Ringlee and A. J. Wood, "Frequency and Duration Methods for Power System Reliability Calculations: Part I--Generation System Model", IEEE Transactions, PAS-87, 1968 pp 1787-96.
- 20. Ringlee, R. J. and A. J. Wood, "Frequency and Duration Methods for Power System Reliability Calculations: Part II--Demand Model and Capacity Reserve Model", IEEE Transactions, PAS-88, 1969 pp 375-88.
- Galloway, C. D., L. L. Garver, R. J. Ringlee and A. J. Wood, "Frequency and Duration Methods for Power System Reliability Calculations: Part III-Generation System Planning", IEEE Transactions, PAS-88, 1969 pp 1216-23.

- Cook, V. M., R. J. Ringlee and A. J. Wood, "Frequency and Duration Methods for Power System Reliability Calculations: Part IV--Models for Multiple Boiler Turbines and for Partial Outage States", IEEE Transactions, PAS-88, 1969 pp 1224-32.
- Billinton, R. and C. Singh, "Generating Capacity Reliability Evaluation in Interconnected Systems using a frequency and duration approach, Part III-Correlated Load Models", IEEE Transactions, PAS-91 1972, pp 2143-53.
- 24. Sanghvi, A. P., N. J. Balu and M. G. Lauby, "Power System Reliability Planning Practices in North America", IEEE Winter Meeting 1991, New York.
- Billinton, R., C. L. Lee, P. R. S. Kuruganty and P. R. Thompson, "Interconnected System Reliability Evaluation Concepts and Philosophy", CEA Transactions, 20, Part III, 1981.
- 26. Billinton, R. and C. Singh, "Generating Capacity Reliability Evaluation in Interconnected Systems using a frequency and duration approach, Part I-Mathematical Analysis", IEEE Transactions, PAS-90 1971, pp 1646-54.
- Rau, N. S., C. Necsulescu, K. F. Schenk and R. B. Misra, "Reliability of Interconnected Power Systems with Correlated Demands", Paper # 82 WM 155-0, IEEE Winter Meeting 1982, New York.
- 28. Deb, R. K., "Effective Load Carrying Capability of Interties", IEEE Winter Meeting 1984, Dallas, Texas.
- Garver, L. L., "Reserve Planning Using Outage Probabilities and Load Uncertainties", IEEE Transactions, vol. PAS-89, April 1970, no. 4, pp 514-521.
- 30. Puntel, R. W. and B. W. Erickson, "Factors Affecting Multi-Area Generation Reliability Evaluations", Power Technologies Inc. 1990, (Unpublished)
- 31. Stoll, H. A., "Least Cost Utility Planning", John Wiley and Sons, 1989, Chap. 9 and 11.
- 32. Vassell, G. S. and N. Tibbetts, "Analysis of Generating Capacity Reserve Requirements for Interconnected Power Systems", IEEE Transactions, Vol PAS-91, 1972, pp 638-649.

- 33. Spears, H. T., K. L. Hicks and S. T. Lee, "Probability of Loss of Load for Three Areas", IEEE Transactions, Vol PAS-89, 1970, pp 521-526.
- C. K. Pang and A. J. Wood, "Multi-Area Generation System Reliability Calculations", Paper # T74 342-2, IEEE Summer Meeting 1974, Anaheim, California.
- 35. Shahidehpour, S. M., J. R. Tudor and C. O. Harbout, "ML'ti-Area Power Reliability Evaluation", Proceedings of the Annual Reliability and Maintainability Symposium, 1982, pp 132-136.
- 36. Clancy, D. P., G. Gross and F. F. Wu, "Probabilistic Flows for Reliability regaluation of Multi-Area Power System Interconnections", Electric Power and Energy Systems, Vol. 5, No. 2, April 1983, Butterworth & Co.
- Oliveira, G. C., S. H. Cunha and M. V. Pereira, "A Direct Method for Multi-Area Reliability Evaluation", IEEE Transactions on Power Systems, Vol. 2, No. 4, November 1987, pp. 934-942.
- 38. Lee, F. N., "Multi-Area Reliability Evaluation A New Approach", IEEE Transactions on Power Systems, Vol. 2, No. 4, November 1987, pp. 848-855.
- Singh, C. and A. Lago-Gonzalez, "Improved Algorithms for Multi-Area Reliability Evaluation Using the Decomposition-Simulation Approach", Paper No. 88 SM691-8, 1988, IEEE/PES Summer Power Meeting, Portland, Oregon, July 24-29, 1988.
- 40. Shahidehpour, S. M., "Reliability Evaluation of a Three-Area Power System", Electric Power System Research, 10 1986, pp 227-233.
- 41. Dhar, S. B. and C. K. Pang, "Interconnected Power System Reliability Evaluation", 1981 Reliability and Maintainability Symposium, Philadelphia, Pa.
- 42. Dhar, S. B. and C. K. Pang, "Interconnected Power System Planning Criteria Evaluation", Paper # A80 081-0, IEEE Winter Meeting 1980, New York.
- 43. Pang, C. K. and S. B. Dhar, "Reliability and Sensitivities Studies of Interconnected Power Systems", Paper # A79 104-1, IEEE Winter Meeting 1979, New York.

- 44. Ruiu, D., A.D. Patton and C. Singh, "Generation Expansion Planning using a Simultaneous Assessment of Several Adequacy Indices", CEA March 1985 Meeting, Montreal.
- 45. LOLE Program Operating Manual, 1972 Moneco Engineering.
- 46. LOLE Program Update Operating Manual, 1981 Moneco Engineering.
- 47. PERCARDS Data Preparation Program Operating Manual, 1972 Moneco Engineering.
- 48. Kuruganty, P. R. S., P. R. Thompson and R. Billinton, "A Reliability Assessment of the Western Canada Grid", IEEE Transactions on Power Apparatus and Systems, Vol. PAS-102, No. 9, September 1983.
- 49. SaskPower Manitoba Hydro Generation Coordination Study, 1989.