Confirmation of quasi-perpendicular shock reformation in two-dimensional hybrid simulations

Xingqiu Yuan,[1,2] Iver H. Cairns,[1] Larisa Trichtchenko,[2] Robert Rankin,[3] and Donald W. Danskin[2]

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[1] Shock reformation involves regions of a shock undergoing periodic collapse and redevelopment on a time scale close to the ion cyclotron period. Reformation is often observed in one-dimensional (1-D) hybrid and particle in cell (PIC) simulations of quasi-perpendicular collisionless shocks provided the Alfvén Mach number M_A and ion plasma beta β_i are sufficiently high and low, respectively. Initial 2-D PIC simulations showed some evidence for shock reformation, with ion reflection providing the main energy dissipation mechanism, while recent spacecraft observations showed a reforming shock with large amplitude whistler waves in the foot region. While recent spacecraft observations showed an case with reforming shock crossing with whistler waves dominated in the foot region. However, recent 2-D hybrid and PIC simulations suggest that reformation does not occur in exactly perpendicular 2-D shocks. This paper re-examines shock reformation in quasi-perpendicular shocks using 1-D and 2-D hybrid simulations. We find that 2-D quasi-perpendicular shocks (θ_{bn} = 85°) indeed undergo cyclic reformation providing M_A and β_i are high and low enough, respectively. For low $M_A \leq 4$, 2-D quasi-perpendicular shocks are found to be quasi-stationary, despite 1-D simulations predicting reformation, confirming and extending recent work for perpendicular 2-D shocks. The dynamics of reformation are quite different in 2-D than in 1-D: in 2-D large amplitude whistler waves grow in the shock foot, have amplitudes of order the downstream magnetic field, and affect the reformation. The whistlers have almost zero phase speeds in the shock frame and oblique wave vectors with respect to the upstream magnetic field. The predicted reformation period increases in 2-D compared with 1-D and increases nonlinearly as M_A decreases towards the reformation threshold.
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1. Introduction

[2] Collisionless shocks are fundamental plasma phenomena that are widely important in laboratory, space, and astrophysical plasmas. They form via nonlinear steepening of waves and are usually generated by supersonic relative plasma motions. Previous 1-D particle in cell (PIC) and hybrid (particle ions but a massless electron fluid) simulations have revealed that high Mach number, collisionless, shocks are not always steady [*Leroy*, 1983; *Quest*, 1985; *Lembege and Dawson*, 1987; *Hellinger et al.*, 2002; *Hada et al.*, 2003; *Scholer et al.*, 2003; *Scholer and Matsukiyo*, 2004; *Yuan et al.*, 2007]. Instead the shock front can periodically collapse and redevelop on a time scale close to the ion cyclotron period $T_{ci} = 2\pi/\Omega_{ci}$, where Ω_{ci} is the ion gyro-frequency. This process is called shock reformation. Accumulation of the reflected ions in time is believed responsible for the cyclical development of shock front on spatio-temporal scales of order T_{ci} and the "convected ion gyroradius" v_{sw}/Ω_{ci} for flow speed v_{sw} [*Krasnoselskikh et al.*, 2002; *Hada et al.*, 2003].

[3] Evidence for shock nonstationarity exists for laboratory experiments [*Morse et al.*, 1972], but is difficult to identify unambiguously in space. *Krasnoselskikh et al.* [1991] interpreted low frequency waves observed by Prognoz-10 and AMPTE-UKS in terms of reformation. Cluster observations reported by *Horbury et al.* [2001] show evidence for significant wave activity in the foot and downstream region, with a relatively stable ramp, but do not rule out reformation. Their shock parameters were Alfven Mach number $M_A \sim 5$, ion plasma beta $\beta_i = 0.1$, and angle $\theta_{bn} = 89°$ between the shock normal and upstream magnetic field vector $\mathbf{B}_0$. In contrast, *Lobzin et al.* [2007] found evidence of reformation for a shock with M_A = 10, β_i = 2.0, and θ_{bn} = 81°: as well as different magnetic profiles for the 4 Cluster spacecraft, they found bursty ion reflection and quasiperiodic variations of Langmuir-like waves excited by bursty electrons reflected by the shock, both with periods of order Ω_{ci}^{-1}. They also reported large amplitude whistler waves. However, the mechanisms responsible for the shock reformation are still topics of current debate.

[4] Some early 1-D [*Biskamp and Welter*, 1972; *Lembege and Dawson*, 1987] and 2-D PIC [*Lembege and Savoini*, 1992] simulations reported reformation of quasiperpendicular shocks due to accumulation of reflected ions. However, *Hellinger et al.* [2007] presented high-resolution 2-D hybrid and PIC simulations with quasi-stationary shock solutions despite reformation occurring in 1-D hybrid simulations for the same shock parameters (M_A = 3.6, β_i = 0.2, β_e = 0.5, and θ_{bn} = 90°). They claimed that 2-D exactly perpendicular shocks do not reform because of large amplitude whistler waves generated in the foot region. These whistlers are generated for a wide range of shock parameters in their 2-D simulations, but not in 1-D simulations, providing the mesh

[1]School of Physics, University of Sydney, Sydney, New South Wales, Australia.
[2]Geomagnetic Laboratory, Natural Resources Canada, Ottawa, Ontario, Canada.
[3]Department of Physics, University of Alberta, Edmonton, Alberta, Canada.

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size is fine enough. Generated in the foot, the whistlers develop amplitudes large enough to compete with the shock front itself. In their picture, the whistlers dominate the dynamics of the entire shock and stop cyclic shock reformation. Clearly, *Hellinger et al.'s* [2007] work suggests that reforming shock solutions are an artifact of 1-D simulations or else due to 2-D simulations with low resolution and/or other computational issues. However, this appears to contradict the recent experimental evidence and other simulations referenced above.

[5] This paper re-examines shock reformation using 1-D and 2-D hybrid simulations with a very fine mesh. We find that 2-D quasi-perpendicular (θ_{bn} = 85°) shocks indeed undergo cyclic reformation providing M_A and β_i are high and low enough, respectively. However, at low enough M_A, 2-D quasi-perpendicular shocks do not reform, despite 1-D simulations predicting reformation. It is consistent with recent quasi-stationary 2-D PIC and standard hybrid simulation results for exactly perpendicular shocks, but extended to the quasi-perpendicular regime, and found to be below the $M_A - \beta_i$ reformation threshold in 2D. The dynamics of reformation are quite different in 2-D than in 1-D: in 2-D large amplitude whistler waves grow in the shock foot, have amplitudes of order the downstream magnetic field, and affect the reformation. The whistlers have almost zero phase speeds in the shock frame and oblique wave vectors with respect to the upstream magnetic field. The predicted reformation period increases in 2-D compared with 1-D and increases nonlinearly as M_A decreases towards the reformation threshold. This paper is organized as follows. In Section 2, we briefly describe the numerical method. The main results are presented in Section 3. The last section contains a discussion and summary.

2. Numerical Method

[6] In this paper, we use standard 1-D and 2-D hybrid codes to study shock reformation. The 2-D code is the natural extension of the 1-D hybrid code of *Yuan et al.* [2007]. These codes calculate the plasma velocity and electromagnetic fields in 3-D. The equations of ion motion are integrated using the standard leapfrog method, and fields are updated by a predictor-corrector method with time step dt. The ion density and currents are smoothed using three- and five-point averages in space at every time step for the 1-D and 2-D runs, respectively. The simulations are performed in a normal incidence frame. For the 1-D runs, $\mathbf{B}_0$ lies in the $x - y$ plane at an angle θ_{bn} with respect to the x-axis. In 2-D, $\mathbf{B}_0$ lies in the $x - y$ plane at an angle θ_{bn} with respect to the negative x-axis. The simulation domain has lengths of $80c/\omega_{pi}$ (ω_{pi} is the ion plasma frequency) and $30c/\omega_{pi}$ along the x and y axes, respectively. The mesh sizes in x and y are both $0.1c/\omega_{pi}$. The initial setup has 150 ions per cell.

[7] The shock is generated by the piston method [*Quest*, 1985] with a high-speed plasma injected from the left-hand boundary and specularly reflected at the right-hand boundary. The simulations use a time step $dt = 1.0 \times 10^{-4}\,\Omega_{ci}^{-1}$, artificial resistivity $\eta = 1.0 \times 10^{-2}\mu_0 v_A^2 \Omega_{ci}^{-1}$, and an adiabatic electron pressure model. Here v_A is the upstream Alfvén speed. This paper's simulations have different M_A, but the same values of β_i = 0.15, β_e = 0.2, and θ_{bn} = 85° except where specified otherwise.

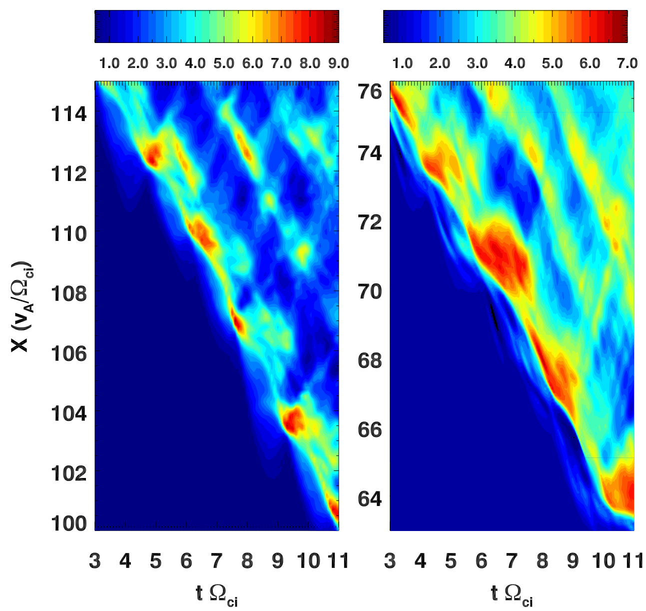

Figure 1. Relative magnetic field (left) B_z/B_0 in 1-D and (right) B_y/B_0 in 2-D simulations at $y = n_y/2$ as a function of t and x. The shock parameters are $M_A = 5.6$, $\beta_i = 0.15$, $\beta_e = 0.2$, and $\theta_{bn} = 85°$.

3. Simulation Results

[8] Figure 1 clearly demonstrates shock reformation in both 1-D and 2-D hybrid simulations for M_A = 5.6. The shock propagates upstream from the right to the left of the simulation box as time t increases. Both the 1-D and 2-D simulations show that the shock front periodically collapses and develops on a time scale close to T_{ci}. Reformation starts with the foot region extending further upstream by reflecting upstream incoming ions and increasing the magnetic field strength. Then the foot develops into a new steep ramp by accumulating the reflected ions, after which the previous shock ramp collapses. The shock reformation period is about $1.6\Omega_{ci}^{-1}$ in 1-D but about $2.6\Omega_{ci}^{-1}$ in 2-D, a considerable difference.

[9] Figure 1's 2-D simulations show that waves are generated quasi-periodically in the foot region, attempt to propagate upstream, but are absorbed by the advancing shock structure. Upstream disturbances are also visible in the 1-D simulations, obeying similar temporal variations but without the clearly defined spatial variations in x seen in the 2-D runs. The 1-D and 2-D runs have the same, very fine, mesh size and shock parameters, so the different wave characteristics are due to the dimensionality.

[10] Consider now simulations with low M_A = 3.6. While the 1-D simulation predicts shock reformation (not shown here), Figure 2's 2-D results show that the 2-D shock is quasi-stationary, with the ramp always present and only small variations in the magnitude of the magnetic field in the ramp and overshoot region. Waves are still visible in the foot, now attempting to propagate upstream and not appearing to be caught by the shock. Reflected ions still gyrate

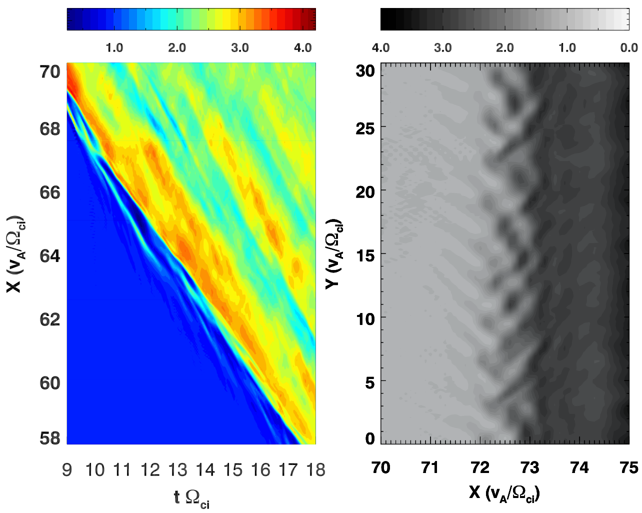

Figure 2. Relative magnetic field B_y/B_0 at (left) $y = n_y/2$ as a function of t and x and (right) as a function of x and y at $t = 3.1/\Omega_{ci}$. Shock parameters are $M_A = 3.6$, $\beta_i = 0.15$, $\beta_e = 0.2$, and $\theta_{bn} = 85°$.

upstream for half a gyroradius and then move downstream. Performing the simulation for the parameters of *Hellinger et al.*'s [2007] Figure 1 ($\theta_{bn} = 90°$, $\beta_i = 0.2$, and $\beta_e = 0.5$) gives similar results to Figure 2. These calculations thus confirm *Hellinger et al.*'s [2007] quasi-stationary simulation run and extends the result to the quasi-perpendicular regime $\theta_{bn} = 85°$. The small variations in magnetic field along the ramp in Figure 2 (left) appear somewhat periodic, suggesting that the parameters are near a threshold for reformation. Indeed, simulations with higher M_A, as in Figure 1, yield reforming shocks.

[11] Figure 3 compares the shock reformation periods T_{ref} 1-D and 2-D simulations for different M_A but $\beta_i = 0.15$, $\beta_e = 0.2$ and $\theta_{bn} = 85°$. At large $M_A > 7$, T_{ref} is very similar in 1-D and 2-D. However, at intermediate M_A (4.6 and 5.7), T_{ref} is much larger in 2-D than in 1-D, increasing nonlinearly as M_A decreases below $M_A = 5$. This behavior is consistent with a threshold existing for reformation near $M_A \sim 4$, as inferred by comparing Figures 1 and 2. Moreover, it demonstrates that reflection alone (a 1-D process) does not cause reformation.

[12] Snapshots of the magnetic fields for 2-D simulations are shown in Figure 4 for Figure 1's reforming shock. The

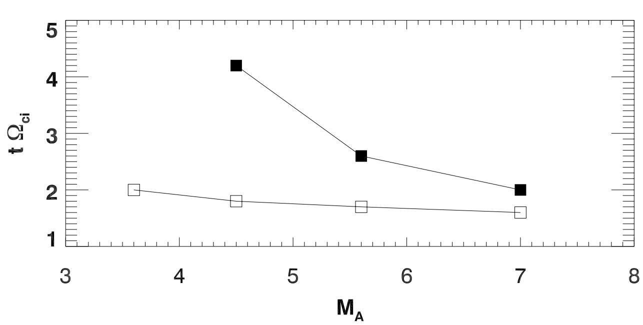

Figure 3. Comparison of the reformation period τ in 1-D (open squares) and 2-D (solid squares) simulations for different M_A but constant $\beta_i = 0.15$, $\beta_e = 0.2$ and $\theta_{bn} = 85°$.

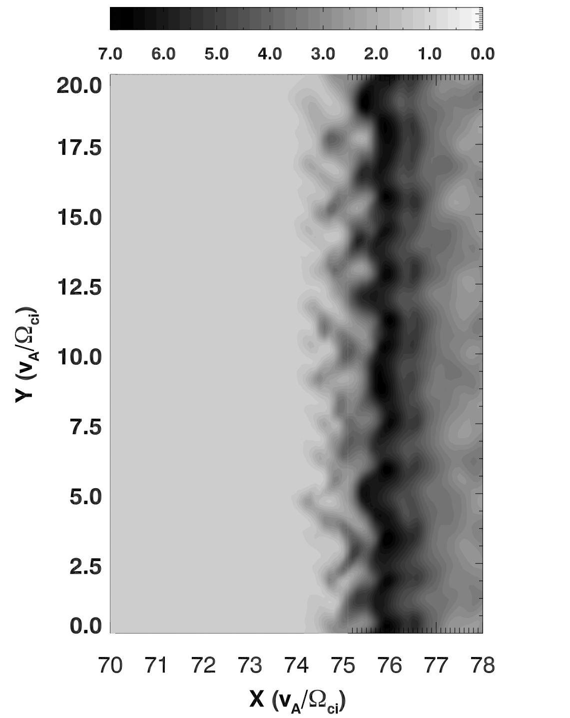

Figure 4. $B_y(x, y)/B_0$ from 2-D simulation at $t = 3.0/\Omega_{ci}$ for Figure 1's shock parameters.

1-D run predicts a well-defined foot, ramp, overshoot and under-overshoot structure without strong waves in the foot (not show here). However, in the 2-D run plasma waves dominate the foot. The waves start near the ramp and grow as they propagate into the foot, as the shock also moves upstream. The wave vectors are highly oblique with respect to both $\mathbf{B}_0$ and the shock normal, and the waves are almost phase standing in the simulation frame, as found by *Hellinger et al.* [2007]. Their angular frequency ω and wavelength λ are approximately $0.75\Omega_{ci}$ and $1.3V_A/\Omega_{ci}$, respectively, corresponding to whistler waves. Their maximum amplitude is about $\delta B \approx 0.6B_{\max} \approx 3.5B_0$.

[13] Strong whistler waves are also found in Figure 2 (right) for a 2-D shock near the reformation threshold, dominating the foot with maximum amplitudes $\delta B \approx 0.7B_{\max} \approx 2.5B_0$. They have $\omega \approx 0.8\Omega_{ci}$ and $\lambda \approx 1.3V_A/\Omega_{ci}$. Similarly, 2-D runs with lower $M_A = 2.6$ but otherwise identical parameters still find both gyrating ions and whistlers, the latter with $\delta B \approx 0.8B_{\max} \approx 3B_0$. However, simulations with high $M_A = 6.0$ show two wave populations coexisting in the foot: small scale length whistler waves and large scale macroscopic shock ripples. The large scale waves propagate along the shock front with typical $\lambda \approx 4.0V_A/\Omega_{ci}$ for $M_A = 5.6$ (not shown here). While it appears that the whistlers do not cause or inhibit reformation, the role of plasma waves on shock reformation needs more detailed investigation.

4. Discussion and Summary

[14] In this paper standard 1-D and 2-D hybrid simulations are used to study the reformation of high M_A, low β, closely perpendicular collisionless shocks. With a very fine mesh size, strong short wavelength whistler waves are

generated in the 2-D runs for a wide range of shock parameters, consistent with *Hellinger et al.*'s [2007] simulations, but extending them into the quasi-perpendicular regime. However, in 1-D simulations with the same mesh size, only weak perturbed magnetic fields due to reflected upstream ions are found in the foot region.

[15] The simulations demonstrate that 2-D quasi-perpendicular high Mach number shocks do undergo cyclic shock reformation provided that M_A is high enough, just as for 1-D shocks. *Hellinger et al.*'s [2007] low $M_A = 3.6$ quasi-stationary perpendicular shock solution is confirmed and extended to the quasi-perpendicular regime $\theta_{bn} = 85°$, but is found to be very close to the threshold for reformation. In the 2-D simulations, large amplitude whistlers develop and strongly influence the foot and ramp. They appear to play an important role, changing the shock reformation period from the 1-D value. In our simulations, when M_A is low (≤ 4) the amplitude of the whistlers is comparable with the maximum magnetic field strength in the shock ramp and so the whistlers can strongly affect the shock dynamics. However, even though reformation stops for sufficiently low M_A, the whistlers are present and have similar magnitudes for both quasi-stationary and reforming shocks. This appears to be inconsistent with *Hellinger et al.'s* suggestion that the whistler waves stop the shock reformation processes.

[16] At higher M_A, the whistler amplitude increases in absolute terms but decreases relative to the shock's maximum magnetic field. Here it appears (not shown) that the shock dynamics is likely dominated by ion reflection and effects not related to the whistlers. Indeed, at high enough M_A a second wave population (the large-scale shock ripples mentioned above) develops and coexists with the whistlers. The effects of waves on shock reformation require further detailed investigation.

[17] Clearly the whistlers and shock structures found in the 2-D runs should be compared with space observations and both linear and nonlinear instability theory. However, before doing so we should recognise that reformation may be as different in 3-D from 2-D as it is different between 2-D and 1-D. Similarly, considering a broader range of θ_{bn} and including kinetic electron physics (and β_e variations) may lead to significant differences [cf. *Scholer and Matsukiyo*, 2004]. Further research is required to investigate these aspects.

[18] In conclusion, the results in this paper show that quasi-perpendicular shocks in 2-D hybrid simulations do reform if M_A is high enough and β is low enough, bringing simulations, observations, and theory into qualitative agreement. Importantly the reformation period increases significantly in 2-D compared with 1-D, unless M_A is relatively large. It is confirmed that strong whistler waves are present in the 2-D simulations, but not in 1-D, and that they likely affect the shock dynamics but do not stop reformation. Finally, more extensive simulations should be performed in 2-D and 3-D for a wide range of shock parameters and the role of whistler waves on shock reformation studied further.

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I. H. Cairns, School of Physics, University of Sydney, Sydney, NSW 2006, Australia. (cairns@physics.usyd.edu.au)

D. W. Danskin, L. Trichtchenko, and X. Yuan, Geomagnetic Laboratory, Natural Resources Canada, Ottawa, ON K1A 0Y3, Canada. (ddanskin@NRCan.gc.ca; ltrichtc@NRCan.gc.ca; xyuan@NRCan.gc.ca)

R. Rankin, Department of Physics, University of Alberta, Edmonton, AB T6G 2G7, Canada. (rankin@phys.ualberta.ca)