

---

BY PERMISSION OF THE COUNCIL  
EXCERPT FROM *GÉOTECHNIQUE*, JUNE 1963

# STABILITY CHARTS FOR EARTH SLOPES DURING RAPID DRAWDOWN

BY  
NORBERT MORGENSTERN, B.A.Sc.(Eng.)

PUBLISHED BY  
THE INSTITUTION OF CIVIL ENGINEERS  
GREAT GEORGE STREET · LONDON S.W.1.  
1963

*The rights of publication and of translation are reserved*

---

*The Institution of Civil Engineers as a body is not responsible either for the statements made  
or for the opinions expressed in the following pages.*

PRINTED IN GREAT BRITAIN BY WILLIAM CLOWES AND SONS, LIMITED  
LONDON AND BECCLES

# STABILITY CHARTS FOR EARTH SLOPES DURING RAPID DRAWDOWN

by

NORBERT MORGENSTERN, B.A.Sc.(Eng.)

## SYNOPSIS

Stability charts are presented to facilitate the computation of the factor of safety of earth slopes during rapid drawdown. As the reservoir level is lowered, the factor of safety decreases if it be assumed that no dissipation of pore pressure occurs during drawdown. Pore pressures during drawdown have been estimated by assuming that  $\bar{B}$  is unity and stability calculation for the range of sections and soil parameters commonly encountered in earth dam practice have been carried out using an electronic computer in order to obtain the data given in the charts.

Des cartes de stabilité sont présentées pour faciliter le calcul du facteur de sécurité des pentes en terre pendant un affaissement rapide. A mesure que le niveau du réservoir baisse, le facteur de sécurité diminue s'il est supposé qu'aucune disparition de la pression interstitielle n'a lieu pendant l'affaissement. L'évaluation de la pression interstitielle pendant l'affaissement a été effectuée en supposant que  $\bar{B}$  est l'unité et les calculs de la stabilité pour la gamme des sections et les paramètres relatifs au sol dont l'usage est habituel dans les travaux ayant trait aux barrages en terre ont été exécutés en utilisant une machine à calculer électronique afin d'obtenir les données apparaissant sur les cartes.

## INTRODUCTION

A sufficient number of failures of earth dams under drawdown conditions have been recorded to demonstrate that it is important, if not critical, to investigate the stability of the structure under these conditions. The details of four earth dam failures which occurred due to the drawdown of the reservoir have been given by Mayer (1936).<sup>1</sup> Drawdown failures have also been described by Schatz and Boesten (1936), Reinius (1948), Sherard (1953), and others.

A list of failures of earth dams attributed to the conditions set up during drawdown is given in Table 1. It is of interest to note that failure of the upstream slope may occur as a secondary failure following a breach of the dam due to overtopping or piping and the sudden release of the reservoir. The Utica Dam and Aiai-ike Dam are examples of this type of failure. In the case of the Utica Dam, the damage to the upstream slope was much more extensive than to the downstream slope. Sudden drawdown may also induce slides in the natural slopes of the reservoir area. Jones, Embody and Peterson (1961) have recorded that landslides on the banks of Franklin D. Roosevelt Lake are more numerous after a lowering of the level of the water impounded behind the Grand Coulee Dam. In addition, Koppejan, van Wamelen, and Weinberg (1948) have suggested that the establishment of a drawdown mechanism during tidal recession is one of the causes of coastal flow slides.

## PORE PRESSURE DURING DRAWDOWN

Prior to the lowering of the reservoir, the pore-pressure distribution in an earth slope is governed by the equilibrium conditions for the flow of water through porous media. Laplace's equation holds and the pore pressure may be determined readily by conventional methods. The effect of a rapid drawdown is twofold. First, it establishes new boundary conditions for the flow of water through the dam and an unsteady state is established while the phreatic

<sup>1</sup> The references are given on p. 131.

line adjusts to a new equilibrium position. The stress changes due to drawdown also cause changes in the pore pressures.

In the case of free-draining fills of low compressibility, such as clean sands and gravels, the pore pressures can be estimated by constructing a flow net satisfying the new boundary conditions. The most critical distribution exists immediately after drawdown. With time, the pore pressures will decrease and the factor of safety of the upstream slope will increase.

**Table 1**  
**Some drawdown failures of earth dams**

Name	Height: (ft)	Upstream slope	Soil properties	Notes and references
Cercey .. ..	37.7	2.4:1	$c' = 2.6$ per sq. in. $\phi' = 26^\circ$	Drained shear box tests carried out many years after failure, Mayer (1936)
Wassy .. ..	54.0	1.5:1	$c' = 2.8$ per sq. in. $\phi' = 23^\circ$	do.
Grosbois .. ..	57.0	1.9:1	$c' = 3.6$ per sq. in. $\phi' = 25.7^\circ$	do.
Charmes .. ..	55.7	1.9:1	$c' = 4.1$ per sq. in. $\phi' = 26.6^\circ$	do.
Bear Gulch .. ..	63.0	3:1	—	Sherard (1953)
Belle Fourche ..	122.0	2:1	$c = 7.9$ per sq. in. $\phi = 9.7^\circ$	Undrained direct shear tests after failure, Sherard (1953)
Brush Hollow ..	73.0	3:1	$c_u = 13.5-28.4$ per sq. in.	Unconfined compression tests after failure, Sherard (1953)
Mount Pisgah ..	76.0	1.5:1	—	Sherard (1953)
Utica .. ..	70.0	2:1	—	Reinius (1948)
Eildon .. ..	90.0	1.35:1	—	Schatz and Boesten (1936)
Aiai-ike .. ..	42.5	1:1-2:1	$c' = 1.5$ per sq. in. $\phi' = 18^\circ$	Consolidated undrained triaxial tests, Akai (1958)
Fruitgrower's ..	36.0	3:1	—	Sherard (1953)
Forsyth .. ..	65.0	2:1	—	Sherard (1953)
Standley Lake ..	113.0	2:1	—	Sherard (1953)
Willingdon .. ..	55.0	2:1	—	Rao (1961)
Palakmati .. ..	46.0	2:1-3:1	—	Rao (1961)

The determination of drawdown pore pressures in free-draining fills has been discussed by Reinius (1948) and will not be considered further.

In fill material with low permeability, considerable time must elapse for the pore-pressure distribution to readjust to the new conditions obtaining after drawdown. Furthermore, impervious fills are characteristically compressible, and the change in loading during drawdown induces changes in the shear stress that affects the magnitude of the residual pore

pressures extant after drawdown. The problem of the determination of pore pressures in impervious, compressible fills has been treated by Bishop (1952, 1954).

The pore pressures after drawdown may be predicted from triaxial test data relating the change in pore pressure to the reduction in the principal stresses. A simple slope under reservoir loading is shown in Fig. 1. The pore pressure at any point in the upstream slope is given by  $u_0$ :

$$u_0 = \gamma_w(h_f + h_w - h') \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where  $\gamma_w$  denotes the bulk density of water.

After drawdown, there will be a change in pore pressure  $\Delta u$ , and the pore pressure becomes:

$$u = u_0 + \Delta u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now the change in pore pressure may be related to the change in major principal stress,  $\Delta\sigma_1$ , by the following expression (Bishop, 1954):

$$\Delta u = \bar{B} \cdot \Delta\sigma_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Assuming that the major principal stress is equal to the weight of material above the point under consideration, the change in major principal stress after drawdown is:

$$\Delta\sigma_1 = -\gamma_w h_w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and the residual pore pressure after drawdown is:

$$u = \gamma_w[h_f + h_w(1 - \bar{B}) - h'] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

It is evident that the magnitude of the pore pressure after drawdown; hence the factor of safety of the upstream slope depend upon the value of  $\bar{B}$ . The lower the value of  $\bar{B}$  the lower will be the factor of safety. Neglecting the influence of  $h'$  leads to a conservative estimate of the pore pressures.

The pore-pressure ratio,  $\bar{B}$ , can be determined experimentally from triaxial tests in which the reduction of the major principal stresses during drawdown is simulated. The testing procedure has been described by Bishop and Henkel (1957) and test data on two earth fills have been reported by Fraser (1957). In the case of a glacial moraine and a boulder clay, both compacted at 1% above optimum moisture content, the values of  $\bar{B}$  were 1.05 and 1.14, respectively.

The relationship between  $\bar{B}$  and the pore-pressure coefficients  $A$  and  $B$  has been given by Skempton (1954):

$$\frac{\Delta u}{\Delta\sigma_1} = \bar{B} = B \left[ 1 - (1 - A) \left( 1 - \frac{\Delta\sigma_3}{\Delta\sigma_1} \right) \right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

For soils close to saturation, the upper limit of the magnitude of both  $A$  and  $B$  is unity. During drawdown, the minor principal stress decreases more than the major principal stress and the incremental change in principal stress ratio is greater than one. With  $B$  approximately equal to one, and  $A$  less than one,  $\bar{B}$  must be greater than one, which is usually the case if the shear stresses are sufficiently large to endanger the stability of the dam during drawdown. Therefore it is reasonable to assume, as proposed by Bishop (1952, 1954) that for a conservative estimate of pore pressure after a rapid drawdown,  $\bar{B}$  may be taken as unity, and the experimental data quoted above confirm this assumption.

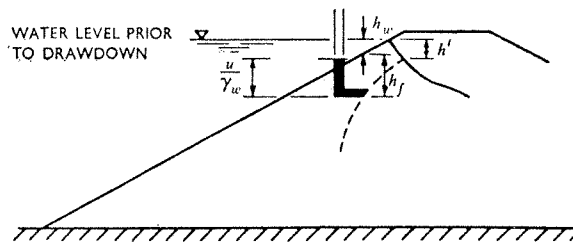


Fig. 1. Pore pressures in upstream slope prior to drawdown

It is important to investigate the extent to which field observations corroborate this assumption. In the case of the Alcova Dam (Glover, Gibbs, and Daehn, 1948), drawdown pore pressures, predicted using the assumptions that  $\bar{B}$  is one together with values of  $h'$  obtained from measurements during the steady seepage condition, agreed on an average within 6% of those measured.

Another set of measurements obtained during drawdown of the Glen Shira Dam (Paton and Semple, 1960) indicate that  $\bar{B}$  was less than one, although the fill was placed wet of optimum.\*

Since the dam had a concrete core wall, it seems possible that the upstream slope never became fully saturated and the residual air content caused the low values of  $\bar{B}$  that were observed. Nevertheless, an analysis using a value of  $\bar{B}$  equal to one gave a value of the factor of safety approximately 10% less than that based upon the measured pore-pressure values, indicating that the effects of dissipation and low values of  $\bar{B}$  in part cancelled each other.

Further data has been provided by Lewis (1962) who reports that measurements in two dams during drawdown at points close to the upstream surface indicated values of  $\bar{B}$  close to unity.

Although the evidence available is by no means conclusive, it appears reasonable to take

$\bar{B}$  as unity, at least for preliminary design calculations. It is important to accumulate further field evidence to clarify the behaviour of earth dams during drawdown, particularly for soils with a significant residual air content. For slopes composed of such soils the following solution, based upon the assumption that  $\bar{B}$  is unity, may not be applicable.

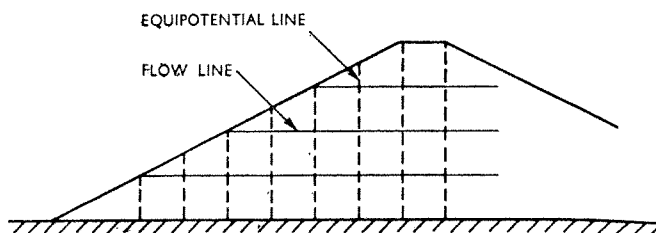


Fig. 2. Assumed pore pressure distribution after complete drawdown

### STABILITY CHARTS

In the computation of the data for the construction of the stability charts it has been assumed that the slope is homogeneous and constructed of a single material with effective stress strength parameters  $c'$  and  $\phi'$ . The earth slope is seated on a rigid, impermeable base and before drawdown the reservoir is at crest level. This condition is called full submergence. During drawdown  $\bar{B}$  has been taken to be unity and the effect of  $h'$  has been neglected. Furthermore, no dissipation during drawdown is assumed to occur. The bulk density of the fill is taken to be twice that of water and the residual pore pressure is then given by:

$$u = \gamma_w h_f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where  $h_f$  denotes the height of fill above the point under consideration, and  $\gamma_w$  is the bulk density of water.

The flow lines and equipotentials consonant with equation (7) are shown in Fig. 2. It is of interest to note that Terzaghi and Peck (1948) have suggested that such a flow net may be

\* Bazett (1960) has also given results that indicate low values of  $\bar{B}$  upon drawdown. However, an estimate of the factor of safety of the section with the observed pore pressures gives such a low value that it is suggested that it would be unwise to infer any conclusions from this data at the present time.

used for an approximate estimate of the drawdown pore pressures in rigid fills as well as compressible fills after complete and instantaneous drawdown.

The minimum factor of safety for circles tangent to a specified level for a given slope with particular values of the strength parameters varies with the depth to drawdown. As the distance from the crest level to the drawdown level is increased, assuming no dissipation, the factor of safety is reduced. Computing the factor of safety for a range of drawdown levels allows this relationship to be determined.

Four slope inclinations have been selected for analysis and expressed in terms of the cotangent of their inclination to the horizontal  $\beta$ ; they are 2:1, 3:1, 4:1, and 5:1. The factor of safety of these slopes at varying drawdown levels has been determined for a range of shear-strength parameters. The investigation has been concerned with values of  $\phi'$  of 20°, 30°, and 40° and three values of cohesion, as expressed by the dimensionless ratio  $c'/\gamma H$ , equal to 0.0125, 0.025, and 0.05. These values represent most of the range encountered in drawdown problems. The use of the ratio  $c'/\gamma H$ \* to simplify the presentation of the data has been discussed in detail by Bishop and Morgenstern (1960).

Referring to Fig. 3, we introduce the drawdown ratio,  $L/H$ , where  $L$  is the distance from the top of the dam to the drawdown level and  $H$  is the height of the dam. For all combinations in the range of shear strength parameters chosen, the minimum factor of safety for each slope has been determined for values of the drawdown ratio of 0,  $\frac{1}{4}$ ,  $\frac{1}{2}$ , and 1. Calculations have been carried out using an electronic computer with the programme devised by Little and Price (1958). The procedure was similar to that adopted by Bishop and Morgenstern (1960) in the computation of stability coefficients. To maintain consistency with the assumed value of  $c'/\gamma H$  all slip circles investigated were tangent to the base of the section. The factor of safety for intermediate levels of tangency can then be obtained by adjusting the magnitude of  $c'/\gamma H$  and  $L/H$  to their appropriate values as demonstrated in an example given in a subsequent section.

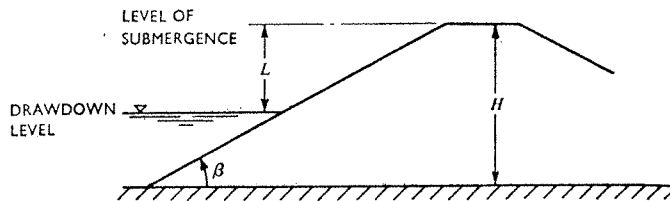


Fig. 3. Drawdown ratio  $\frac{L}{H}$

The stability charts showing the variation of the factor of safety with drawdown ratio are given in Figs 4, 5, and 6. Fig. 4 shows these relationships for a value of  $c'/\gamma H$  equal to 0.0125 over the range of slope inclinations and  $\phi'$  covered by the solution. Similarly, Figs 5 and 6 pertain to values of  $c'/\gamma H$  of 0.025 and 0.05, respectively. Intermediate values can be obtained by interpolation.

Before illustrating the use of these charts with some examples, it is of interest to compare the factor of safety given by them for the complete drawdown case with that given by the method described in Taylor (1948). Taylor suggested that his general solution in terms of total stresses could be used to determine the factor of safety for complete drawdown ( $L/H = 1$ ) if the angle of shearing resistance mobilized were obtained on the basis of the following expression:

$$\phi_w = \frac{\gamma'}{\gamma} \phi_m \quad \dots \dots \dots (8)$$

where  $\phi_w$  denotes the mobilized angle of shearing resistance after drawdown,  $\phi_m$  denotes the mobilized angle of shearing resistance and is equal to  $\tan^{-1} (\tan \phi' / F)$ ,  $\gamma'$  denotes the submerged density of the soil, and  $\gamma$  denotes the bulk density.

\*  $c'$  denotes the apparent cohesion in terms of effective stresses,  $\gamma$  is the bulk density of the soil, and  $H$  denotes the height of the section from foundation level to crest.

An example has been calculated for which  $\beta$  is 4:1,  $\phi' = 30^\circ$ , and  $c'/\gamma H = 0.025$ . Both Taylor's method and the stability charts gave a factor of safety of 1.56 for complete and instantaneous drawdown. Taylor's solution cannot, however, be used for intermediate drawdown levels.

Fig. 7 shows that for an intermediate value of  $L/H$  equal to  $\frac{1}{2}$  and for a typical case, the variation of the factor of safety with  $c'/\gamma H$  is substantially linear. Therefore reasonable

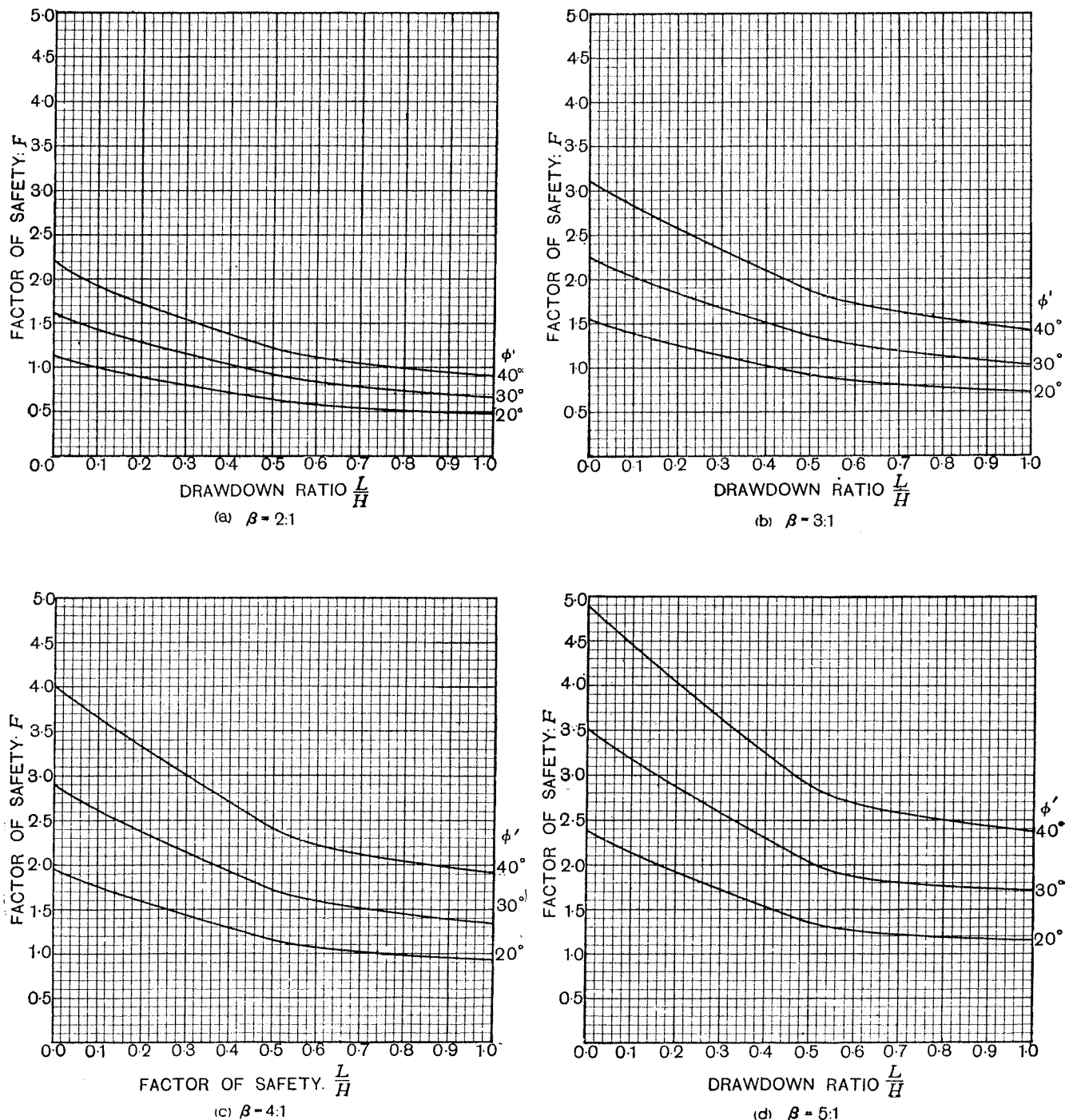


Fig. 4. Drawdown stability chart for  $\frac{c'}{\gamma H} = 0.0125$



results may be expected by extrapolating beyond the range of the stability charts to values of  $c'/\gamma H$  not covered by the figures. Figs 4, 5, and 6 do not provide directly the final solution to the problem of finding the minimum factor of safety for a partial drawdown. If the drawdown level is above the base of the section, the critical circle may be tangential to a level above the base as illustrated by the difference between the following two examples.

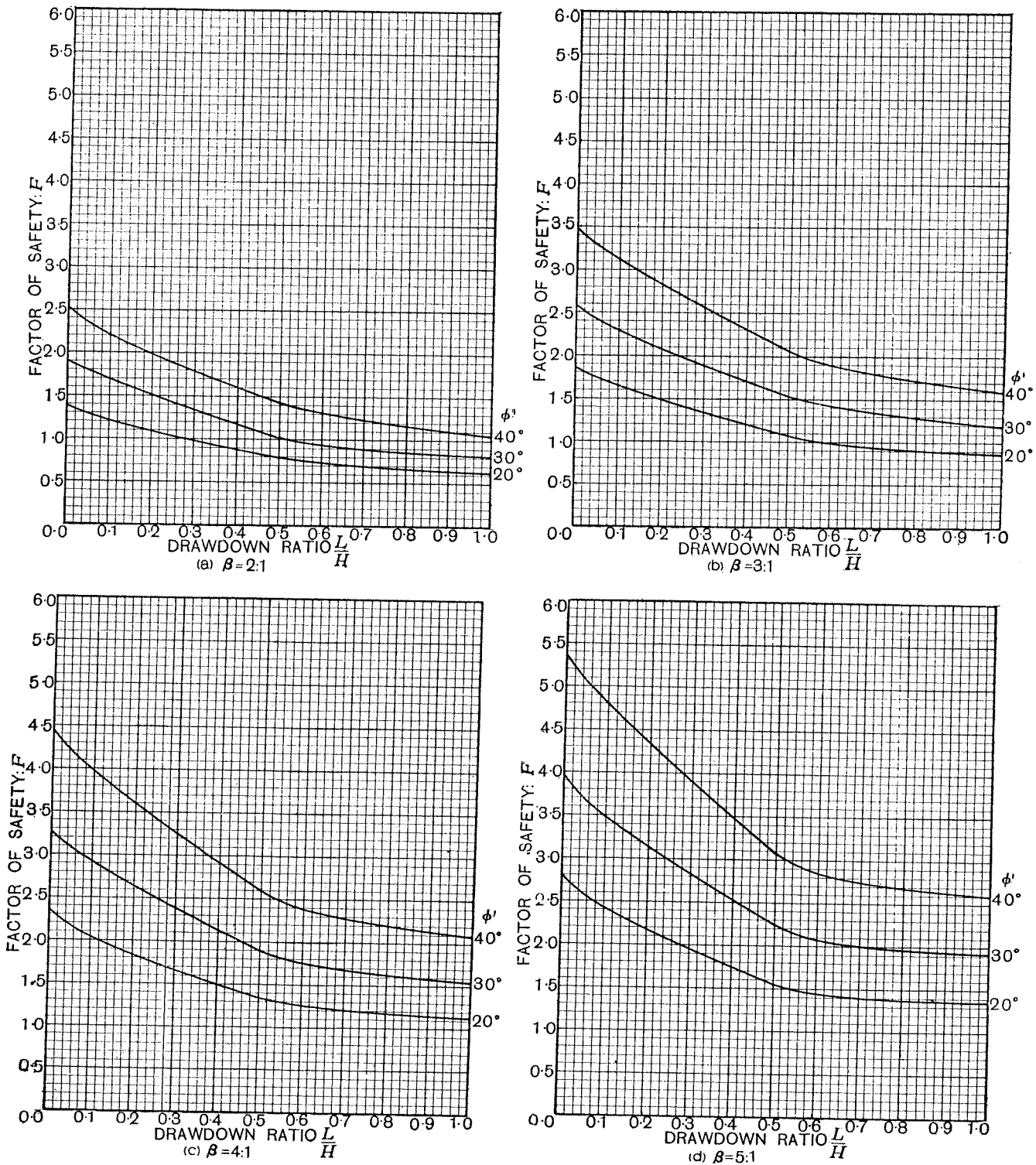


Fig. 5. Drawdown stability chart for  $\frac{c'}{\gamma H} = 0.025$

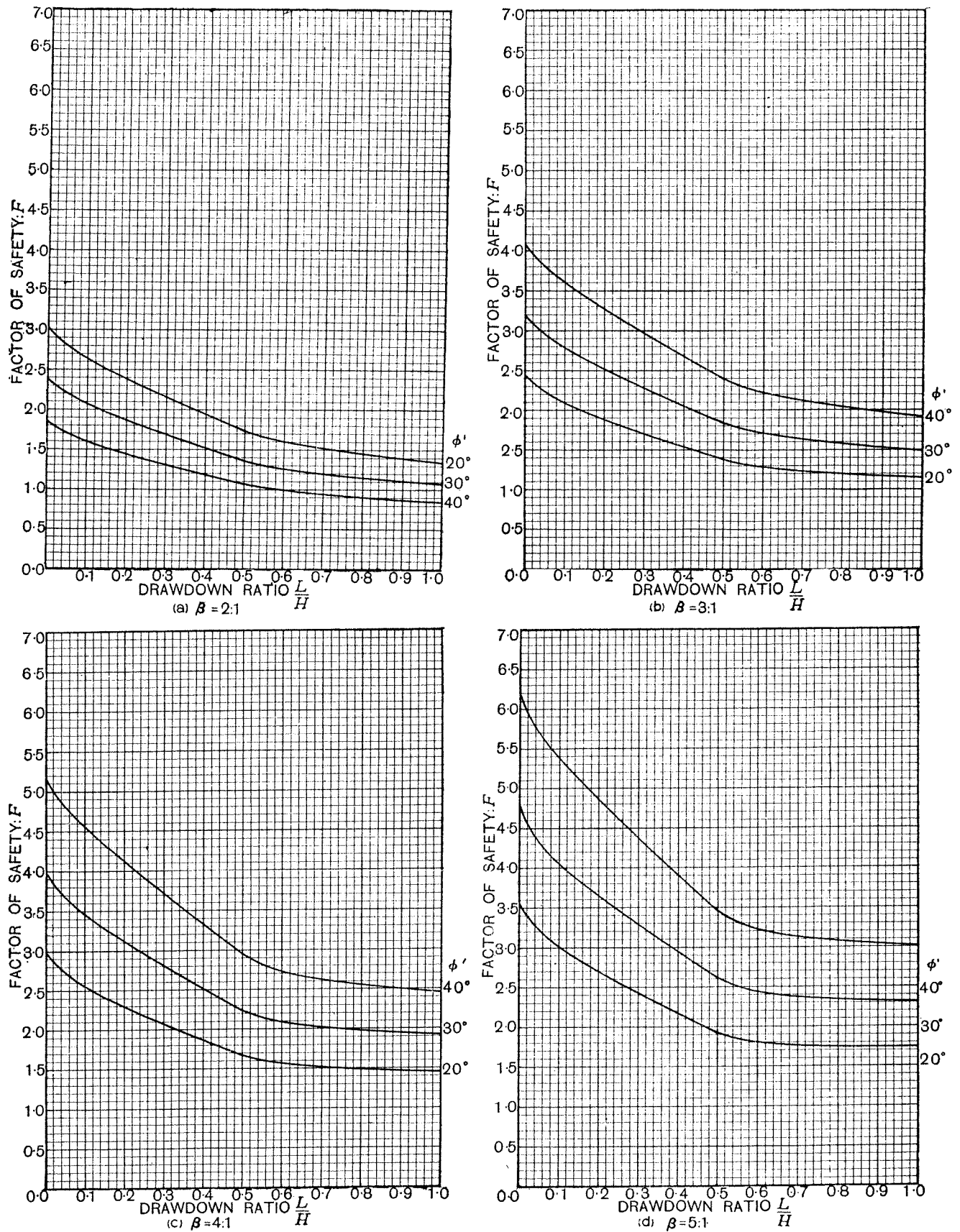


Fig. 6. Drawdown stability chart for  $\frac{c'}{\gamma H} = 0.05$

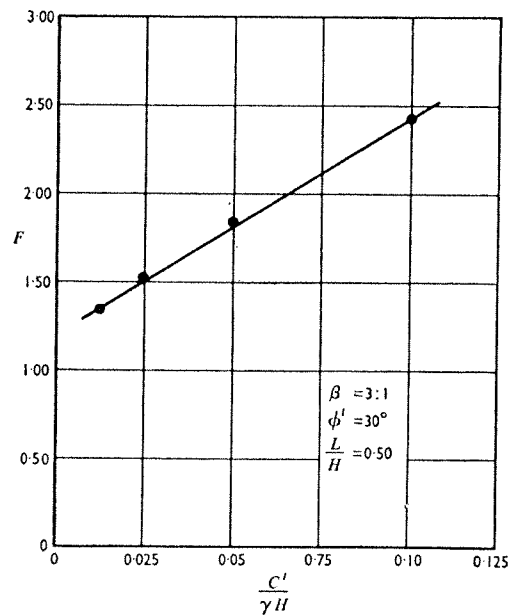


Fig. 7. Linear variation of Factor of Safety with  $\frac{c'}{\gamma H}$

### Example 1

It is required to find the minimum factor of safety for a complete drawdown of the section shown in Fig. 8. From the data given in the figure:

$$\beta = 3:1$$

$$\frac{c'}{\gamma H} = 0.025$$

$$\phi' = 30^\circ$$

The factor of safety is directly obtainable from Fig. 5, with  $L/H = 1$ ,

$$F = 1.20.$$

It is evident that the critical circle is tangent to the base of the dam and no other level need be investigated since this would only raise the effective value of  $c'/\gamma H$ , resulting in a higher factor of safety.

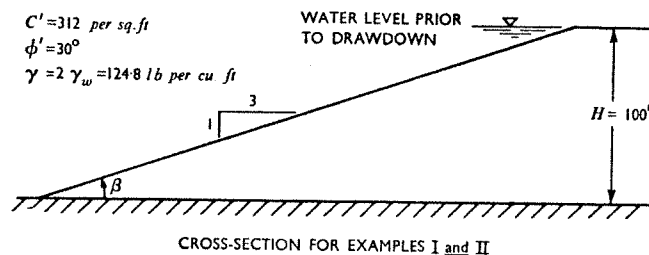


Fig. 8. Cross-section for examples I and II

*Example II*

It is required to find the minimum factor of safety for a drawdown to a level at the mid-height of the section shown in Fig. 8.

(i) Considering slip circles tangential to the base of the dam, the effective height of the section  $H_e$  is equal to its actual height and therefore:

$$\frac{c'}{\gamma H_e} = 0.025,$$

and

$$\frac{L}{H_e} = 0.50.$$

With  $\beta = 3:1$  and  $\phi' = 30^\circ$  the minimum factor of safety for this family may be obtained directly from Fig. 5 and:

$$F = 1.52$$

(ii) Considering slip circles tangential to the mid-height of the dam, the effective height of the section is equal to one half the actual height:

$$H_e = \frac{H}{2}$$

Therefore:

$$\frac{c'}{\gamma H_e} = 0.05$$

and:

$$\frac{L}{H_e} = 1.00.$$

The minimum factor of safety for this family may be obtained from Fig. 6 giving:

$$F = 1.48.$$

(iii) For slip circles tangential to a level  $H/4$  above the base of the dam, the effective height of the section is equal to three-quarters of the actual height:

$$H_e = \frac{3}{4}H.$$

Therefore:

$$\frac{c'}{\gamma H_e} = 0.033,$$

and

$$\frac{L}{H_e} = 0.67.$$

The minimum factor of safety for this family must be obtained by interpolation. From Fig. 5, with  $c'/\gamma H_e = 0.025$ :

$$F = 1.31,$$

and from Fig. 6, with  $c'/\gamma H_e = 0.05$ ,

$$F = 1.61.$$

Interpolating linearly for  $c'/\gamma H_e = 0.033$ , it is found that the minimum factor of safety for this family is:

$$F = 1.31 + \frac{0.30}{3} = 1.41.$$

Although a slightly lower value could perhaps be found, further refinements are unwarranted. This example demonstrates that for partial drawdown, the critical circle may often lie above the base of the dam and it is important to investigate several levels of tangency for the maximum drawdown level. In the case of complete drawdown for slopes

considered in the preceding solution the minimum factor of safety is always associated with circles tangent to the base of the slope and the factor of safety at intermediate levels of drawdown need not be investigated. This will not be the case when the pore-pressure distribution during drawdown differs significantly from that assumed in this solution.

### CONCLUSIONS

Stability charts that show the variation in factor of safety with drawdown level have been presented for homogeneous slopes. The analyses have been carried out in terms of effective stress and it has been assumed that  $\bar{B}$  is unity during drawdown. Existing experimental and field data though not conclusive suggest that this is a safe assumption for saturated soils and fill compacted wet of optimum moisture content, at least for preliminary design purposes. It has also been assumed that  $h'$  may be neglected and that no dissipation occurs during drawdown. The latter two assumptions are conservative. The range of the charts may be extended by extrapolation. For the case of complete drawdown, the minimum factor of safety is given directly by the charts but in the case of partial drawdown computations must be carried out as illustrated in the second example because it is not known to what depth the critical slip circle is tangent.

### ACKNOWLEDGEMENT

The Author is grateful to Dr V. E. Price and the English Electric Company for providing computing time on DEUCE.

### REFERENCES

- AKAI, K., 1958. "On the character of seepage waters and their effect on the stability of earth embankments." *Bull. No. 24, Disaster Prevention Research Inst., Kyoto, Japan.*
- BAZETT, D. J., 1960. *Discussion in Proc. Conf. Pore-Pressure*, p. 134, Butterworths, London.
- BISHOP, A. W., 1952. "The stability of earth dams." *Ph.D. Thesis, Univ. of London.*
- BISHOP, A. W., 1954. "The use of pore-pressure coefficients in practice." *Géotechnique*, 4:4:148-152.
- BISHOP, A. W., and D. J. HENKEL, 1957. "The measurement of soil properties in the triaxial test." *Edward Arnold, London.*
- BISHOP, A. W., and N. MORGENSTERN, 1960. "Stability coefficients for earth slopes." *Géotechnique*, 10, 4:129-150.
- FRASER, A. M., 1957. "The influence of stress ratio on compressibility and pore-pressure coefficients in compacted soils." *Ph.D. Thesis, Univ. of London.*
- GLOVER, R. E., H. J. GIBBS, and W. W. DAEHN, 1948. "Deformability of earth materials and its effect on the stability of earth dams following a rapid drawdown." *Proc. Second Int. Conf. Soil Mech.*, 5:77-80.
- JONES, F. O., D. R. EMBODY, and W. L. PETERSON, 1961. "Landslides along the Columbia River Valley, Northeastern, Washington." Professional Paper No. 367, U.S. Geological Survey, Washington.
- KOPPEJAN, A. W., B. M. VAN WAMELEN, and L. J. H. WEINBERG, 1948. "Coastal flow slides in the Dutch province of Zeeland." *Proc. Second Int. Conf. Soil Mech.*, 5:89-92.
- LEWIS, J. G., 1962. Personal communication.
- LITTLE, A. L., and V. E. PRICE, 1958. "The use of an electronic computer for slope stability analysis." *Géotechnique*, 8:3:113-120.
- MAYER, A., 1936. "Characteristics of materials used in earth dam construction—stability of earth dams in cases of reservoir discharge." *Proc. Second Congr. Large Dams*, 4:295-327.
- PATON, J., and N. G. SEMPLE, 1960. "Investigation of the stability of an earth dam subject to rapid drawdown including details of pore-pressures recorded during a controlled drawdown test." *Proc. Conf. Pore-Pressure, Butterworths, London.*
- RAO, K. L., 1961. "Stability of slopes in earth dams and foundation excavations." *Proc. Fifth Int. Conf. Soil Mech.*, 2:691-695.
- REINUIS, E., 1948. "The stability of the upstream slope of earth dams." *Bull. No. 12, Swedish State Ctte. for Bldg Research.*
- SCHATZ, O., and H. BOESTEN, 1936. "Gebrochene Staudammen." ("Broken earth dams.") *Der Bauingenieur.*
- SHERARD, J. L., 1953. "Influence of soil properties and construction methods on the performance of homogeneous earth dams." Tech. Memo. 645, U.S. Bureau of Reclamation.
- SKEMPTON, A. W., 1954. "The pore-pressure coefficients  $A$  and  $B$ ." *Géotechnique*, 4:4:143-147.
- TAYLOR, D. W., 1948. "Fundamentals of soil mechanics." Wiley, New York.
- TERZAGHI, K., and R. B. PECK, 1948. "Soil mechanics in engineering practice." Wiley, New York.

