# Effects of Social Comparison on Incentive Contracts and Performance Evaluation

by

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# Abstract

Humans are known to gain or lose utility by comparing their payoffs with those of their peers. I analyze how such social preferences affect incentive contracts and performance evaluation in principal-agent settings. In my models, agents dislike getting worse off than their peers (I say they have a sense of ‘envy’). In some cases, I also assume agents either like or dislike getting better off than their peers (I say they have a sense of either ‘greed’ or ‘guilt’). This dissertation analyzes three different settings. In each setting, a principal hires two agents who work on productive tasks. First, I examine how joint-production settings are affected by social preferences. I show that the principal is indifferent about how envious the agents are, but she prefers guilty agents to greedy ones. Second, I show that envious agents can be efficiently incentivized by aggregated performance evaluation. This result supports the use of fixed-wage contracts and inflated ratings. Third, I analyze a situation in which an older agent is envious of his younger peer, but not vice versa. I show that (i) the optimal contract is characterized by pay inversion, granting a higher expected wage to the young agent than to the old agent, and (ii) the principal is worse off with unilaterally envious agents than bilaterally envious ones. The former result shows a benefit of causing pay inversion. The latter shows a cost of age diversity.

# Preface

This thesis is an original work by Eiji Ohashi. Chapter 2 is forthcoming for publication as "Team Incentives and Peer-Regarding Agents" in the *Journal of Theoretical Accounting Research*.

# Acknowledgments

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# Chapter 1

# Introduction

Humans are known to be affected by *social comparison* (Galinsky and Schweitzer 2015, Chapter 1). According to this notion, people compare themselves with other people to evaluate how well they are doing things. By comparing themselves with more fortunate people, they tend to feel unhappy and motivated to work harder. Conversely, by doing so with less fortunate people, they tend to feel happy and complacent. For example, Clark and Oswald (1996) find that workers tend to feel more satisfied with their jobs if they get a higher income than their peers. They also show that workers tend to be more strongly affected by relative income than by absolute income. Furthermore, humans' strong reaction to relative payoffs is believed to have an evolutionary origin. Brosnan and de Waal (2003) report that monkeys — nonhuman primates — also react to their relative payoffs.

In this dissertation, I analyze how social comparison affects incentive contracts and performance evaluation in principal-agent settings. I consider agents who gain or lose utility based on not only their own payoffs but also their peers'. In my analysis, agents lose utility when their peers are paid more than themselves (I say they have a sense of 'envy'). In some cases, I also assume agents either gain or lose utility when they are paid more than their peers (I say they have a sense of 'greed' or 'guilt').

I present three main findings from Chapters 2 to 4. In each chapter, I analyze a model of a principal and two agents. Chapters 2 and 3 examine how social preferences affect incentive contracts when the principal hires homogeneous agents and uses team performance evaluation and individual performance evaluation, respectively. Chapter 4 examines a contract under relative performance evaluation when agents are heterogeneous.

In Chapter 2, I analyze the optimal contract when the principal evaluates the agents' performance based on group-level output. I show that (i) envy is irrelevant for the principal, and (ii) greed increases the risk of the contract, while guilt decreases it. These results imply that the principal should hire agents who have a strong sense of guilt, but she can ignore how envious the agents are.

In Chapter 3, I show that envious agents can be efficiently motivated to work hard even if the performance measure is coarse. Information economics predicts that a principal will prefer a finer performance evaluation system to a coarser one. Nevertheless, coarse performance evaluation is often used in practice. To explain this seeming contradiction, I construct a model of a principal and envious agents. I show that a coarse evaluation system can do as well as a finer one if agents are sufficiently envious, i.e., if they incur large utility loss when they are paid less than their peers. This result supports the use of coarse performance evaluation that aggregates signals of good performance, in the form of fixed wages or inflated ratings.

In Chapter 4, I analyze a situation where envy arises one-sidedly between two agents. Research has shown that older people tend to feel envious of their younger peers, but not vice versa. To understand how such unilateral envy affects optimal incentive contracts, I model a contract for young and old agents. In my model, the old agent has a sense of envy, i.e., he incurs utility loss when his younger peer gets paid more than himself. I show that the optimal contract can be characterized by pay inversion: The principal offers a higher expected wage to the young agent than to the old agent. This result is supportive of causing pay inversion versus increasing wages based on seniority. Further, I show that the principal is worse off with unilaterally envious agents than bilaterally envious ones. This result implies that an opportunity cost exists when a firm assigns comparable positions to age-diverse workers rather than workers similar in age.

# Chapter 2

# Team Incentives and Peer-Regarding Agents

## 2.1 Introduction

Research has shown that people gain or lose utility by comparing themselves with their peers (this process is called social comparison; see Galinsky and Schweitzer 2015, Chapter 1). For example, Clark and Oswald (1996) show evidence that workers tend to feel more satisfied with their jobs if their peers' income is smaller. Moreover, they find that workers' welfare is more strongly affected by relative income than by absolute income. These findings imply that the relative welfare of members of a group should be taken into consideration when one considers group incentives.

In this paper, I analyze how group incentives are affected by social preferences. I model a group of two agents who have peer-regarding utility functions as in Fehr and Schmidt (1999). Each agent derives utility from his own payoff (i.e., the utility from compensation, minus the cost of effort) and from social preferences. Each agent loses utility when he is worse off than his peer (I say he has a sense of 'envy'). On the other hand, each agent either gains or loses utility when he is better off than his peer (I say he has a sense of either 'greed' or 'guilt').[1]

A number of studies have examined how contracts are affected by agents' peer-regarding preferences (e.g., Itoh 2004; Rey-Biel 2008; Neilson and Stowe 2010; Bartling and von Siemens

[1]Some researchers (e.g., Bartling and von Siemens 2010) use the term 'compassion' rather than 'guilt' to mean the disutility caused by advantageous inequality, i.e., the situation in which the agent is better off than his peer. Baumeister et al. (1994) reason that people feel guilt when they believe they are overrewarded compared to other people. Fehr and Schmidt (1999) assume this type of agent (who also has a sense of envy) to explain cooperative behavior among people, as in the ultimatum game.

2010; Bartling 2011). Among these studies, my analysis is closely related to Itoh (2004), Bartling and von Siemens (2010), and Bartling (2011). They analyze optimal contracts for peer-regarding agents when the principal can design the contracts based on relative performance evaluation. In their model, the principal adjusts the optimal contract depending on how envious, greedy, or guilty the agents are. In particular, the principal may use team performance evaluation in some cases, i.e., the principal may pay wages based on the team's aggregate output, ignoring what each agent produces at a disaggregate level. In their model, team performance evaluation arises endogenously, as the principal's optimal choice. On the other hand, I analyze optimal contracts when the principal can use only a team performance measure. In other words, unlike Itoh (2004), Bartling and von Siemens (2010), and Bartling (2011), I impose team performance evaluation exogenously.

Although I analyze a restricted setting compared with the previous studies, analyzing such a situation is important in accounting contexts. In cost accounting settings, there are situations in which costs are not traceable to individual cost objects (see Horngren et al. 2015). When costs are not traceable to individual agents, the principal may find it too costly to estimate individual agents' contributions to the group's production. In this case, the principal will offer compensation to each agent based on an output from the group of agents. In other words, the principal will need to use team performance evaluation. This paper examines how peer-regarding preferences affect optimal contracts in this setting.

My results can be summarized in two points. First, I show that agents' envy is irrelevant. The literature has found envy affects optimal contracts in important ways (e.g., Itoh 2004; Rey-Biel 2008; Neilson and Stowe 2010; Bartling and von Siemens 2010; Bartling 2011), but envy has no impact in my setting. This result also contradicts the intuition that agents are affected by envy. For example, Galinsky and Schweitzer (2015, 15) note on envy that "looking up makes us feel worse, but can motivate us to strive harder." In other words, when envy exists, people lose utility, but they are motivated to work harder. One might believe that the principal needs to consider these effects. I show, however, the principal doesn't need to do so when she is constrained to use team performance evaluation.

Second, I show that guilt and greed affect how risky the optimal contract is for the agent. If agents have a stronger sense of guilt (or a weaker sense of greed), agents will get more stable wages. If they have a stronger sense of greed (or a weaker sense of guilt), they will get more volatile wages. Further, if the contract is monotonic in the performance signal, the principal will become better off as agents become guiltier (or less greedy).

My results have practical implications on what type of agent the principal should hire. When the principal can use only team performance evaluation, she should choose agents who have a strong sense of guilt. She can ignore how envious the agents are.

## 2.2 Model Description

A risk-neutral principal ("she") hires two risk-averse, peer-regarding agents ($i = 1, 2$; "he" in singular). I assume that each agent has a utility function $U^i(\cdot)$ that depends not only on his payoff (his wage minus effort cost) but also on his peer's. Specifically, I consider a peer-regarding utility function as in Fehr and Schmidt (1999):[2]

$$\begin{aligned} U^i(w^i, e^i, w^j, e^j) =& u(w^i) - C(e^i) \\ & - \alpha \max\big\{[u(w^j) - C(e^j)] - [u(w^i) - C(e^i)], 0\big\} \\ & - \beta \max\big\{[u(w^i) - C(e^i)] - [u(w^j) - C(e^j)], 0\big\}, \end{aligned} \tag{2.1}$$

where $w^i$ is the wage payment to the agent $i$, $e^i$ is the level of agent $i$'s effort, $u(\cdot)$ is utility from the wage and is an increasing and strictly concave function, and $C(e_i)$ is the cost of effort.

Parameters $\alpha$ and $\beta$ in (2.1) represent how sensitive the agent $i$ is to inequality. The parameter $\alpha \geq 0$ represents the strength of envy. On the right-hand side of (2.1), the third term means that the agent $i$ loses utility when his peer $j$ is better off than himself. On the other hand, the fourth term means that the agent $i$ either gains or loses utility when he is better off than his peer $j$. The parameter $\beta$ is positive if the agent $i$ loses utility when he is better off than his peer $j$ (I say agent $i$ has a sense of 'guilt'). The parameter $\beta$ is negative if the agent $i$ gains utility when he is better off than his peer $j$ (I say agent $i$ has a sense of 'greed').[3] I also assume $-1 \leq \beta \leq 1$. In words, the effect of greed or guilt on utility is at most as strong as that of the actual payoff net of effort cost,

[2]In Fehr and Schmidt (1999), players are assumed to be risk-neutral, so their utility equals their monetary payoffs. I modify the Fehr and Schmidt (1999) utility function so that agents in my model are risk-averse with respect to their wages.

[3]Fehr and Schmidt (1999) assume $\beta \geq 0$. I also consider negative $\beta$ to study agents who like getting ahead of their peers.

$u(w^i) - C(e^i)$. Finally, I assume that the two agents are working closely enough — so that each agent can observe, or form a belief about, his peer's wage and effort.

The game proceeds as in Figure 2.1. The principal offers each agent $i$ a contract $w_i$, and each agent can either reject or accept it. If both agents accept it, they join the firm. Otherwise, the game ends. After joining the firm, agents ($i = 1, 2$) simultaneously choose to work hard ($e^i = h$ for high effort) or not ($e^i = l$ for low effort). Then the firm produces output $y$, based on which the principal pays wages to the agents.

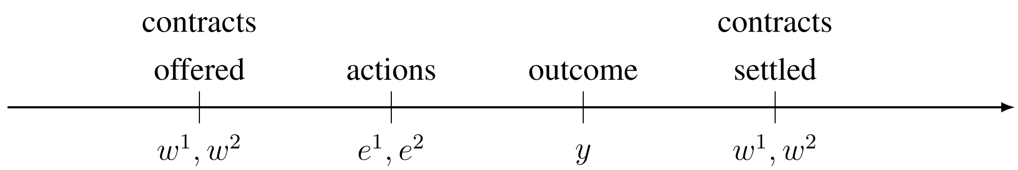


Figure 2.1: Timeline of events

Each agent produces his output independently, and the individual output is either high or low. Each agent produces high output with probability $p > 0$ (low output with probability $1 - p > 0$) when he works hard. When he doesn't work hard, he produces high output with probability $q > 0$ (low output with probability $1 - q > 0$), where $q < p$. I assume that the principal cannot observe how much each agent contributes to the production process. Hence, the principal uses team performance evaluation, i.e., the contract is based only on the joint output. This situation is common in accounting and management contexts. For example, when researchers in the same institution collaborate on a research project, the institution may not be able to tell how much each researcher is contributing to the project. The joint output $y$ of the firm is $G$ (good) if the two agents produce high output, $M$ (moderate) if only one of the agents produces high output, $B$ (bad) if both of the agents produce low output. The principal pays each agent a wage $w_y$, where $y = G, M, B$. For simplicity, the cost of effort is assumed to be $c > 0$ when the agent makes a high effort, 0 when he makes a low effort.

I assume that the principal wants high expected output. Equivalently, since $p > q$, she wants both agents to work hard $(e^1, e^2) = (h, h)$. Given this assumption, the firm is faced with constant expected output. Hence, profit-maximization in this model is equivalent to cost-minimization. In other words, the principal minimizes the expected wage payment to each agent:

$$\min_{w_G, w_M, w_B} p^2 w_G + 2p(1-p)w_M + (1-p)^2 w_B, \tag{Obj}$$

where (Obj) means the objective function of the principal's problem. Given $(e^1, e^2) = (h, h)$, the output is $G$ with probability $p^2$, $M$ with probability $2p(1-p)$, and $B$ with probability $(1-p)^2$.

The principal considers participation and incentive constraints. The participation constraint for each agent $i, j = 1, 2$ where $i \neq j$ is:

$$E(U^i(w^i, e^i = h, w^j, e^j = h)) \geq U, \tag{2.2}$$

where $U$ is the agent's reservation utility. I can rewrite (2.2) as:

$$p^2 u(w_G) + 2p(1-p)u(w_M) + (1-p)^2 u(w_B) - c \geq U, \tag{PC}$$

where (PC) means the participation constraint. Neither $\alpha$ nor $\beta$ appears in (PC). Intuitively, given $(e^1, e^2) = (h, h)$, both agents incur the same cost of effort $c$ and get equal wages, so no inequality arises.

The incentive constraint for each agent $i, j = 1, 2$ where $i \neq j$ is:

$$E(U^i(w^i, e^i = h, w^j, e^j = h)) \geq E(U^i(w^i, e^i = l, w^j, e^j = h)). \tag{2.3}$$

The left-hand side of (2.3) is the same as that of (2.2) and hence that of (PC). The right-hand side of (2.3) is:

$$\begin{aligned} & pq[u(w_G) - \beta \max\{[u(w_G) - 0] - [u(w_G) - c], 0\}] \\ & \quad + p(1-q)[u(w_M) - \beta \max\{[u(w_M) - 0] - [u(w_M) - c], 0\}] \\ & \quad + (1-p)q[u(w_M) - \beta \max\{[u(w_M) - 0] - [u(w_M) - c], 0\}] \\ & \quad + (1-p)(1-q)[u(w_B) - \beta \max\{[u(w_B) - 0] - [u(w_B) - c], 0\}] \\ = & pqu(w_G) + (p + q - 2pq)u(w_M) + (1-p)(1-q)u(w_B) - \beta c. \end{aligned} \tag{2.4}$$

Given $(e^i, e^j) = (l, h)$, the output is $G$ with probability $pq$, $M$ with probability $p(1-q)+(1-p)q$, and $B$ with probability $(1-p)(1-q)$. In (2.4), $\beta$ is present but $\alpha$ is absent. By deviating from high to low effort, each agent can become better off than his peer by the reduced cost of effort $c$. Hence, (IC) is affected by the agents' sense of greed or guilt $\beta$. Nevertheless, given joint production as a team, each agent cannot become worse off than his peer by deviating to low effort, i.e., by free-riding on his peer's effort. Since envy is relevant only when the agent is worse off than his peer, (IC) is unaffected by the agents' sense of envy $\alpha$.

Consequently, (2.3) can be rewritten as:

$$\begin{aligned} & p^2 u(w_G) + 2p(1-p)u(w_M) + (1-p)^2 u(w_B) - c \\ & \geq pqu(w_G) + (p+q-2pq)u(w_M) + (1-p)(1-q)u(w_B) - \beta c, \end{aligned}$$

or equivalently:

$$\begin{aligned} & p(p-q)u(w_G) - (2p-1)(p-q)u(w_M) \\ & \quad - (1-p)(p-q)u(w_B) - (1-\beta)c \geq 0, \end{aligned} \tag{IC}$$

where (IC) means the incentive constraint.

## 2.3 Analysis

I note that $\alpha$ doesn't appear in either the objective function (Obj) or the two constraints (PC) and (IC). Hence, it is irrelevant to the optimal contract.

**Proposition 2.1.** *The strength of envy $\alpha$ doesn't affect the optimal contract.*

The intuition behind Proposition 2.1 is as follows. Since the contract is based on the joint output, the two agents always get identical wages. Under the strategy $(e^1, e^2) = (h, h)$, they incur the same cost $c$ and get paid an identical wage regardless of the realized outcome. Hence, under $(e^1, e^2) = (h, h)$, the third and fourth terms on the right-hand side of (2.1) are irrelevant. Agents can get unequal payoffs only when one of them deviates from $(e^1, e^2) = (h, h)$, i.e., chooses to free-ride on the other agent's effort. By free-riding, however, each agent will be strictly *better off* than the other agent by $c$. The agent cannot become worse off than the other agent. Since the strength of envy $\alpha$ matters only when the agent is worse off than the other agent, it doesn't affect the optimal contract.

Envy is known to impact optimal contracts (e.g., Itoh 2004; Rey-Biel 2008; Neilson and Stowe 2010; Bartling and von Siemens 2010; Bartling 2011), so one might expect that envy is also important in my model. In Itoh (2004), for example, envy affects the incentive and participation constraints of the principal's cost-minimization problem. In his model, envy motivates the agents to work hard, but it demotivates the agents to participate in the firm. In my model, however, these effects do not arise. When the principal is constrained to use team performance evaluation, each agent is at least as well off as his peer whether he follows the equilibrium strategy $(e^1, e^2) = (h, h)$ or not. Consequently, envy has no impact on the optimal contract.

I now analyze how guilt (or greed) $\beta$ affects the optimal contract. The optimal contract requires that (PC) and (IC) be binding constraints.[4] Hence, it satisfies:

$$p^2 u(w_G) + 2p(1-p)u(w_M) + (1-p)^2 u(w_B) - c = U, \tag{2.5}$$

$$\begin{aligned} & p(p-q)u(w_G) - (2p-1)(p-q)u(w_M) \\ & - (1-p)(p-q)u(w_B) - (1-\beta)c = 0, \end{aligned} \tag{2.6}$$

From (2.5) and (2.6), I can derive the following property of the optimal contract:

**Lemma 2.1.** *The optimal contract satisfies the following equations:*

$$pu(w_G) + (1-p)u(w_M) = U + \frac{p\beta - q + 1 - \beta}{p-q} c \tag{2.7}$$

$$pu(w_M) + (1-p)u(w_B) = U + \frac{p\beta - q}{p-q} c. \tag{2.8}$$

[4]The argument for the binding constraints is as follows. Suppose (PC) is non-binding. Then, the principal can set new wages $(w_H^{new}, w_M^{new}, w_L^{new})$ so that $u(w_y) - \epsilon = u(w_y^{new})$ for $y = H, M, L$, where $\epsilon$ is a positive number. By doing so, she can reduce the expected wage without affecting (IC), because coefficients of $u(w_H)$, $u(w_M)$, and $u(w_L)$ in (IC) sum up to zero. This argument contradicts that the original contract is optimal. On the other hand, suppose (IC) is non-binding. In this case, the principal will minimize the risk of the contract by setting fixed wages: $w_H = w_M = w_L$. When $\beta = 1$, this solution makes (IC) binding. When $\beta < 1$, the agents will deviate from $e_H$ to $e_L$, the less-costly strategy. This argument contradicts that the principal wants high effort and high production output.

*Proof.* Multiplying both sides of (2.6) by $(1-p)/(p-q)$ and adding the product to (2.5), I get:

$$\begin{aligned}[p(1-p)+p^2]u(w_G)+[-(2p-1)(1-p)+2p(1-p)]u(w_M)\\ -\frac{1-p}{p-q}(1-\beta)c-c=U.\end{aligned} \tag{2.9}$$

Rearranging the terms in (2.9), I obtain:

$$pu(w_G)+(1-p)u(w_M)=U+\frac{(1-\beta)(1-p)+(p-q)}{p-q}c,$$

or equivalently:

$$pu(w_G)+(1-p)u(w_M)=U+\frac{p\beta-q+1-\beta}{p-q}c.$$

Similarly, multiplying both sides of (2.6) by $-p/(p-q)$ and adding the product to (2.5), I get:

$$\begin{aligned}[p(2p-1)+2p(1-p)]u(w_M)+[p(1-p)+(1-p)^2]u(w_B)\\ +\frac{p}{p-q}(1-\beta)c-c=U.\end{aligned} \tag{2.10}$$

Rearranging the terms in (2.10), I obtain:

$$pu(w_M)+(1-p)u(w_B)=U+\frac{p-q-p(1-\beta)}{p-q}c,$$

or equivalently:

$$pu(w_M)+(1-p)u(w_B)=U+\frac{p\beta-q}{p-q}c.$$

□

Given $\beta \leq 1$, the right-hand side of (2.7) is at least as large as that of (2.8). The left-hand side of (2.7) is each agent's expected utility from the wage conditional on him producing high output. When the agent produces high output, he gets $u(w_G)$ when his peer produces high output (i.e., with probability $p$), $u(w_M)$ when his peer produces low output (i.e., with probability $1-p$). Similarly, the left-hand side of (2.8) is each agent's expected utility from the wage conditional on him producing low output. When the agent produces low output, he gets $u(w_M)$ when his peer produces high output (i.e., with probability $p$), $u(w_L)$ when his peer produces low output (i.e., with probability $1-p$).

The right-hand sides of (2.7) and (2.8) are affected by $\beta$. The right-hand side of (2.7) is deacreasing in $\beta$ because the first derivative of it with respect to $\beta$ is $(p-1)c/(p-q) < 0$. The right-hand side of (2.8) is increasing in $\beta$ because the first derivative of it with respect to $\beta$ is $pc/(p-q) > 0$.

**Proposition 2.2.** *If $\beta$ increases, each agent gets (i) lower expected utility conditional on him producing high output and (ii) higher expected utility conditional on him producing low output.*

The intuition behind Proposition 2.2 is as follows. If the principal can use only a team performance measure, the two agents get paid identical wages. When agents become guiltier, i.e., when $\beta$ increases, each agent dislikes free-riding on his peer's effort. Hence, the principal can impose lower risk on the agents to motivate them to work hard. When $\beta = 1$, the right-hand side of (2.7) is the same as that of (2.8). In this case, the agents are motivated to work hard solely by the sense of guilt, so the principal needs to impose no risk on the contract. On the other hand, when agents become greedier, i.e., when $\beta$ decreases, each agent likes free-riding on his peer's effort. As a result, the principal needs to impose higher risk on the contract to prevent the agents from shirking.

The result in Proposition 2.2 seems to suggest that the principal is better off when $\beta$ increases, i.e., when agents have a stronger sense of guilt. The proof is mathematically intractable, but a wide range of numerical analysis is consistent with this conjecture. Further, this conjecture is always valid as long as $w_G > w_B$.

**Proposition 2.3.** *Suppose the optimal contract satisfies $w_G > w_B$. Then, as $\beta$ increases, the cost of the contract decreases.*

*Proof.* By using (2.7) and (2.8), respectively, I can express $w_G$ and $w_B$ in terms of $w_M$:

$$w_G = u^{-1}\left(-\frac{1-p}{p}u(w_M) - \frac{c[\beta(1-p)-1+q]}{p(p-q)} + \frac{U}{p}\right) \tag{2.11}$$

$$w_B = u^{-1}\left(-\frac{p}{1-p}u(w_M) + \frac{c(p\beta-q)}{(1-p)(p-q)} + \frac{U}{1-p}\right) \tag{2.12}$$

From (2.11) and (2.12), I can rewrite the cost of the contract as follows:

$$\begin{aligned}
&p^2 w_G + 2p(1-p)w_M + (1-p)^2 w_B \\
=&p^2 u^{-1}\left(-\frac{1-p}{p}u(w_M) - \frac{c[\beta(1-p)-1+q]}{p(p-q)} + \frac{U}{p}\right) \\
&+ 2p(1-p)w_M \qquad (2.13)\\
&+ (1-p)^2 u^{-1}\left(-\frac{p}{1-p}u(w_M) + \frac{c(p\beta - q)}{(1-p)(p-q)} + \frac{U}{1-p}\right).
\end{aligned}$$

I calculate the first derivative of (2.13) with respect to $\beta$:

$$\begin{aligned}
&p^2\left(-\frac{(1-p)c}{(p-q)p}\right)\frac{1}{u'(w_G)} + (1-p)^2 \frac{pc}{(p-q)(1-p)}\frac{1}{u'(W_B)} \\
=&\frac{cp(1-p)}{p-q}\left[\frac{1}{u'(w_B)} - \frac{1}{u'(w_G)}\right] < 0,
\end{aligned}$$

where the first expression follows from differentiation of inverse functions and (2.11) and (2.12); the final inequality from concavity of $u$ and $w_G > w_B$. Hence, the expected payment is decreasing (i.e., the principal's welfare is increasing) in $\beta$. □

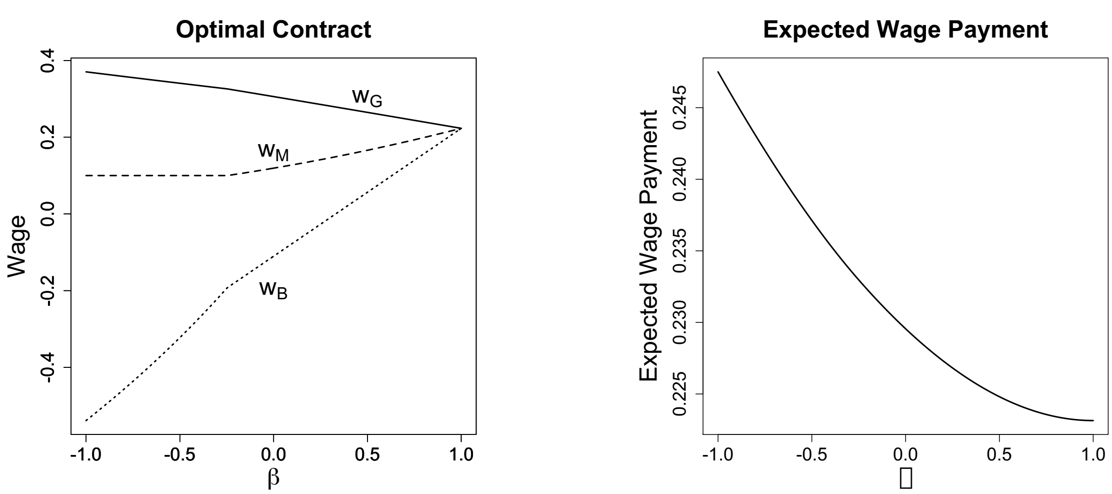


Figure 2.2: Numerical Analysis

Figure 2.2 shows a numerical example. In this example, I assume $p = 0.8$, $q = 0.2$, $c = 1$, $U = 1$, and $u(w) = 10 - 10\exp(-w)$. The graph on the left in Figure 2.2 shows how the optimal contract changes in $\beta$. The solid line shows $w_G$, the dashed line $w_M$, and the dotted line $w_B$. The optimal contract is monotonic in the performance signal: $w_G > w_M > w_B$. As $\beta$ increases, $w_G$, $w_M$, and $w_B$ approach each other, so the contract becomes less risky. This result is consistent with Proposition 2.2. When $\beta = 1$, the three wages $w_G$, $w_M$, and $w_B$ take the same value, so the principal imposes no risk on the contract. In this case, the agents are motivated to work hard solely be the sense of guilt, i.e., because they want to avoid free-riding on the other agent's high effort.

The graph on the right in Figure 2.2 shows how the cost of the contract changes in $\beta$. As in Proposition 2.3, the expected wage payment decreases in $\beta$. As $\beta$ increases, i.e., as agents become guiltier, they dislike free-riding on their peer's effort. Hence, the principal incurs a smaller risk premium.

The results from Proposition 2.1 to Proposition 2.3 are useful when one considers the optimal choice of employees. My results indicate that the principal should hire agents who have a strong sense of guilt, or a weak sense of greed. The principal can ignore how envious the agents are. As in Merchant and Van der Stede (2017, Chapter 3), my findings emphasize the importance of selecting the right personnel when designing incentive systems.

My model is also descriptive of a situation in which production of the team is non-separable. In some production processes, 'individual output' of each agent may not be observable to anyone. For example, when multiple researchers brainstorm ideas on a research project, there may not be identifiable 'individual output' of each researcher. When no one can observe an agent's individual output, the interpretation of (2.7) and (2.8) (hence Proposition 2.2) does not carry over. Nevertheless, Proposition 2.1 and Proposition 2.3 continue to hold.

## 2.4 Conclusions

In this paper, I show how team incentives are affected by social preferences. In cost accounting settings, costs may not be traceable to each agent. In this case, the principal may find it too costly to estimate how much each agent contributes to the production process. Then she will have to use team performance evaluation. When the principal evaluates only an aggregate signal from the team, the crux of the problem is free-riding. By free-riding, each agent cannot get worse off than his peer. Hence, envy is irrelevant to the contract. On the other hand, if the agents have a stronger sense of greed (guilt), the principal offers more volatile (stable) wages. When the contract is monotonic in the performance signal, the principal is better off as agents become guiltier.

These findings are helpful when one considers the optimal choice of employees. When the principal must use team performance evaluation, she should hire agents who have a strong sense of guilt, but she can ignore how envious the agents are.

I analyze how an accounting situation is affected by social preferences. Nevertheless, more research is needed on how social preferences such as envy and greed affect accounting activities. Future research could examine how assuming peer-regarding preferences will change what is known about, for example, disclosure, earnings management, and performance evaluation.

# Chapter 3

# Coarse Performance Evaluation for Envious Agents

## 3.1 Introduction

A basic insight of decision theory and information economics is that finer information systems are preferred to coarser ones (see Marschak and Miyasawa [1968] for discussion). In reality, however, coarse performance evaluation is often used to motivate workers in agency situations (Holmström and Milgrom 1991; Martin and Bartol 1998). In this study, I show that a principal can achieve an optimal contract with a coarse performance evaluation system as well as a finer one if agents are sufficiently envious, i.e., if they incur large utility loss when they are paid less than their peers.

One example of coarse performance evaluation is an "almost" fixed wage schedule. Under this type of schedule, each worker is paid a fixed wage as long as he doesn't get the worst performance rating. If he gets the worst rating, he is punished by getting a pay cut or being dismissed from the job. This type of contract is based on coarse evaluation, namely, whether the agent gets the worst rating or not. Holmström and Milgrom (1991) show that a fixed wage contract can be explained by a multi-task agency model with measurable and unmeasurable attributes of performance. I provide an alternative explanation of a fixed wage contract by using a single-task agency model. I show that the optimal contract is characterized by an "almost" fixed wage when agents are envious enough.

Another example of coarse performance evaluation is *rating inflation* in the workplace. Under inflated performance ratings, the principal labels both outstanding and average performance "outstanding," i.e., two (or more) distinct positive signals are collapsed into one coarse signal. Rating inflation is often believed problematic, and researchers have proposed remedies for it (e.g., Martin and Bartol 1998; Roch 2005). Contrary to this argument, I show that a principal uses inflated ratings to motivate envious agents efficiently. This result is consistent with the argument of Longenecker, Sims, and Gioia (1987), who reason that evaluators cause rating inflation, in part, to increase the subordinate's motivation and performance. My analysis provides an economic model that supports their argument. On the other hand, Grund and Przemeck (2012) and Golman and Bhatia (2012) also provide economic models to explain why rating inflation arises. These two studies consider an altruistic evaluator. Grund and Przemeck (2012) also consider inequality-averse workers as in my model. In Grund and Przemeck (2012) and Golman and Bhatia (2012), rating inflation is modeled as leniency bias, i.e., workers receive higher performance ratings than they deserve. Unlike these studies, however, I treat rating inflation as an example of coarse performance evaluation, i.e., aggregated signals. Consequently, my result differs from Grund and Przemeck (2012) and Golman and Bhatia (2012) in that rating inflation arises even if the evaluator is not altruistic.

I employ a formal analysis to study the balance of countervailing effects of envy. I consider a model of a principal and two agents who have a sense of envy. Each agent observes the other agent's wage, and loses utility when his peer is paid a larger wage than himself.[1] Envy affects the optimal contract in a non-trivial way. On the one hand, as agents become more envious, they are motivated to work hard by envy. The principal will need to pay a smaller expected compensation, and this effect will become stronger when the performance measure becomes finer. On the other hand, as agents become more envious, they will require higher expected payments that compensate for utility loss from envy. The principal will need to pay a larger expected compensation, and this effect will also become stronger when the performance measure becomes finer. I analyze how envy affects the optimal contract under two different performance evaluation systems: a fine performance measure and a coarse performance measure. I show that a fine performance measure has no incremental value over a coarser one when agents are sufficiently envious.

This study is closely related to the literature on incentives for peer-regarding agents. Itoh

[1]In reality, agents may not be able to observe their peers' wages, but they can usually form beliefs about them. Hence, my model can be interpreted as having agents who form beliefs about their peers' wages, and lose utility when they believe they have been paid less than their peers.

(2004), Bartling and von Siemens (2010), and Bartling (2011) show that a principal may aggregate multiple agents' performance signals into a signal of the team when agents have peer-regarding preferences. My analysis also shows that signals are collapsed for peer-regarding agents, but within each agent's performance measure. Unlike Itoh (2004), Bartling and von Siemens (2010), and Bartling (2011), I examine whether a principal is willing to collapse a given evaluation system for each agent into a coarser one. Neilson and Stowe (2010) show that a principal may set lower piece rates for peer-regarding agents. My analysis also shows that a principal may use weak incentives for peer-regarding agents. Nevertheless, unlike Neilson and Stowe (2010), I characterize the optimal contract for peer-regarding agents by aggregated signals, which are descriptive of "almost" fixed wages and inflated ratings.

This paper is tied to the literature on performance measure aggregation. Accounting researchers have studied various forms of aggregation. Examples are linear aggregation of underlying signals (Banker and Datar 1989), intertemporal aggregation (Arya, Glover, and Liang 2004), the use of a group-level measure rather than individual measures (Arya and Mittendorf 2011), and aggregation of performance measures in multi-task settings (Şabac and Yoo 2018). I consider another form of aggregation. In my model, signals of good performance (say "good" and "moderate" signals) are aggregated into one coarse signal for envious agents. This form of aggregation is consistent with what happens under "almost" fixed wage contracts and inflated ratings.

This paper offers a new insight into benefits of aggregation. Aggregation is known to be beneficial because processing a lot of information is costly (i.e., because humans are constrained by their bounded rationality). Accounting researchers have also identified less obvious benefits. For example, aggregation is known to limit cherry picking, serve as a substitute for commitment, and curb managerial slack (see Arya and Glover [2014] and the references therein). In my model, aggregation is beneficial because it mitigates envy: Agents have less opportunity to feel envious when their performance measures are coarser. Hence, coarse performance evaluation is optimal for envious agents.

My analysis is concerned with a central issue of managerial accounting: performance evaluation. My findings indicate that firms may not necessarily have to "fix" incentive systems that treat outstanding and average performance equally, e.g., fixed wages and inflated ratings. I show this type of coarse performance evaluation is optimal when workers are sufficiently envious.

## 3.2 Model

A risk-neutral principal employs two risk-neutral agents. I assume the agents have a sense of envy, so they dislike becoming worse off than their peers. I modify the peer-regarding utility function of Fehr and Schmidt (1999) so that agents in my model (i) have only a sense of envy (but not greed or guilt: they are indifferent about becoming better off than their peers) and (ii) compare gross wages, i.e., ignore effort costs in their social comparison:[2]

$$U^i(w^i, e^i, w^j, e^j) = w^i - C(e^i) - \alpha \max\{0, w^j - w^i\}, \tag{3.1}$$

where $w^i$ is agent $i$'s wage, $e^i$ is agent $i$'s effort, and $C$ is the cost of effort. The exogenous parameter $\alpha \geq 0$ represents how envious the agents are. I assume each agent can either observe his peer's wage, or form a belief about it. Hence, (3.1) means each agent loses utility from envy if he observes (or believes) his peer is paid a larger wage than himself.

The game proceeds as in Figure 3.1. The principal chooses a performance evaluation measure, either a fine one or a coarse one. Then, the principal offers each agent $i$ a contract $w^i$. Each agent $i$ decides whether to accept it. If both of them accept the contract, they are in the firm. Each agent $i$ privately takes an action $e^i$, based on which he independently produces a signal $y^i$. Finally, the principal pays $w^i$ based on $y^i$. I assume that each contract is based on the agent's own signal, i.e., I assume individual performance evaluation (IPE).

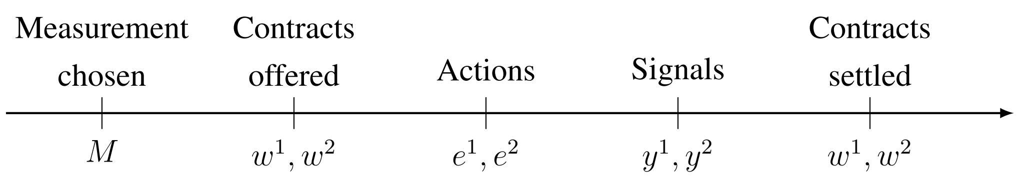


Figure 3.1: Timeline of events

There are two ways to interpret my model and its IPE assumption. Firstly, my model can be

[2] In other words, agents in my model compare gross wages rather than net wages. This assumption is natural because humans are subject to self-serving bias. For example, Messick and Sentis (1979) suggest workers who make a lower effort than their peers tend to compare gross wages rather than net wages in their fairness consideration. I analyze a moral-hazard situation in which the principal designs a contract to prevent the agents from making a low effort. Hence, it is natural to assume they compare gross wages rather than net wages when they choose to work hard or not. On the other hand, if both agents make a high effort, they incur the same cost of effort. Hence, their participation decision to the firm is unaffected whether they compare gross wages or net wages.

interpreted as having a principal and only one agent, and the latter compares his wage with the wage that was expected. In other words, the agent forms a belief about what an agent can typically get (in reality, this belief could be based on, e.g., the latest data on average salary of the industry or the workplace). The agent compares the wage he has got with the wage a typical agent would get, and incurs negative utility when the former is lower than the latter. This idea is consistent with what social psychologists call relative deprivation (see, e.g., Smith, Pettigrew, Pippin, and Bialosiewicz [2012]). Secondly, the IPE assumption can be interpreted as the principal using IPE for multiple agents. It is known that relative performance evaluation is not widely used in practice (Merchant and Van der Stede 2017, 528), and many business people seem to be fixated on IPE. For example, one business consultant complains that "[t]his [ranking employees based on their performance] is yet another example of a lazy policy that avoids *the hard and necessary work of evaluating each individual objectively, based on his or her merits* [emphasis added]" (Bradberry 2015). In other words, it is often believed that employees should be evaluated based on their individual performance. My IPE assumption is descriptive of a principal fixated on IPE.

I assume that the principal can use two types of performance evaluation, a fine measure and a coarse measure. Under the fine measure, each agent produces a good ($g$), moderate ($m$), and bad ($b$) performance signal with probability $p_h$, $q_h$, and $1 - p_h - q_h$, respectively, when he makes a high effort $e_h^i$. When the agent makes a low effort $e_l^i$, he produces each performance signal with probability $p_l$, $q_l$, and $1 - p_l - q_l$, respectively. The principal pays $w_y$ to the agent $i$ when the agent produces a signal $y = g, m, b$.

On the other hand, the principal can also use a coarse performance measure. In this case, each agent produces a good ($G$) and bad ($B$) signal with probability $\overline{p}_h = p_h + q_h$ and $1 - \overline{p}_h = 1 - p_h - q_h$, respectively, when he makes a high effort $e_h^i$. When the agent makes a low effort $e_l^i$, he produces each performance signal with probability $\overline{p}_l = p_l + q_l$ and $1 - \overline{p}_l = 1 - p_l - q_l$, respectively. This definition of coarse performance evaluation matches what happens under fixed wage schemes or inflated ratings, in which the best signals are collapsed. I later show that this way of coarsening the fine measure is optimal when agents are envious enough. Further, the principal pays $w_y$ to the agent $i$ when the agent produces a signal $y = G, B$.

For simplicity, the reservation utility of each agent is assumed to be $0$. The cost of effort is assumed to be $0$ when the agent makes a low effort, and $c > 0$ when he makes a high effort. I focus on the efficient contract that motivates both agents to choose high effort.

## Parameter Assumptions

To ensure the optimal contract is monotonic in the performance signal, I assume:

$$p_h - p_l > 0 \tag{3.2}$$

$$q_h - q_l > 0 \tag{3.3}$$

$$2p_h + q_h - 1 > 0 \tag{3.4}$$

$$p_h + 2q_h - 1 > 0. \tag{3.5}$$

Intuitively, (3.2) and (3.3) mean that each agent produces a good and a moderate signal, respectively, with higher probability when he works hard than when he shirks.[3] Further, (3.2) and (3.3) ensure $\overline{p}_h > \overline{p}_l$. The third assumption (3.4), equivalent to $p_h > 1 - p_h - q_h$, means that a high-effort agent is more likely to produce a good signal than a bad signal. Similarly, the fourth assumption (3.5), equivalent to $q_h > 1 - p_h - q_h$, means that a high-effort agent is more likely to produce a moderate signal than a bad signal.

Finally, I assume $p_l/p_h$ is small enough relative to $q_l/q_h$. Under this assumption, the principal can motivate the agents to work hard more efficiently by using a good signal than a moderate signal when the presumed order of the payment is $w_b < w_m < w_g$ and $\alpha$ is not too large. I also need this property even when the presumed order of the payment is different from $w_b < w_m < w_g$. Specifically, I assume the parameters $(p_h, p_l, q_h, q_l)$ satisfy the following inequalities:

$$\frac{p_l}{p_h} < \frac{q_l}{q_h} - \alpha^*(1 - p_h - q_h + p_l + q_l - \frac{q_l}{q_h}) \tag{3.6}$$

$$\frac{p_l}{p_h} < \frac{q_l}{q_h} - \alpha^{**}(\frac{p_h}{q_h} + 1)(q_h - q_l), \tag{3.7}$$

where $\alpha^* = p_l/(p_h(1 - p_l))$ and $\alpha^{**} = (p_l + q_l)/((p_h + q_h)(1 - p_l - q_l))$.

[3] Although (3.3) might look restrictive, I impose it to simplify the analysis. In undocumented numerical analysis, I can replicate the qualitative insight of the model even with parameters that violate (3.3).

## 3.3 Analysis

### Benchmark: The Coarse Measure

Under the coarse measure, the principal minimizes the expected wage payment:

$$(1-\overline{p}_h)w_B + \overline{p}_h w_G, \tag{OBJc}$$

subject to limited liability, participation, and incentive constraints. The limited liability constraints are:

$$w_B \geq 0 : w_G \geq 0. \tag{LLc}$$

The participation constraint for agent $i$ is:

$$E[U^i(w^i, e^i_h, w^j, e^j_h)] \geq 0. \tag{3.8}$$

The incentive constraint for agent $i$ is:

$$E[U^i(w^i, e^i_h, w^j, e^j_h)] \geq E[U^i(w^i, e^i_l, w^j, e^j_h)]. \tag{3.9}$$

These constraints (3.8) and (3.9) are sensitive to how the payments $(w_B, w_G)$ are ordered. To rewrite these constraints into specific expressions, I use the monotonicity property of the model:

$$w_B \leq w_G. \tag{3.10}$$

In my model, the principal prefers the monotonic ordering (3.10) to the ordering $w_G \leq w_B$. See the Appendix for the proof.

By using (3.10), I rewrite (3.8) as:

$$(1+\alpha\overline{p}_h)(1-\overline{p}_h)w_B + \overline{p}_h(1-\alpha+\alpha\overline{p}_h)w_G - c \geq 0,$$

or equivalently:

$$\Big(1 - \frac{\alpha}{1+\alpha\overline{p}_h}\Big)\overline{p}_h(1+\alpha\overline{p}_h)(w_G - w_B) + w_B \geq c. \tag{PCc}$$

I also rewrite (3.9) as:

$$(1+\alpha\overline{p}_h)(\overline{p}_h-\overline{p}_l)(w_G-w_B)-c\geq 0. \tag{ICc}$$

Using (ICc), I can evaluate the left-hand side of (PCc) as follows:

$$\begin{aligned}\left(1-\frac{\alpha}{1+\alpha\overline{p}_h}\right)&\overline{p}_h(1+\alpha\overline{p}_h)(w_G-w_B)+w_B\\ &\geq\left(1-\frac{\alpha}{1+\alpha\overline{p}_h}\right)\frac{\overline{p}_h}{\overline{p}_h-\overline{p}_l}c+w_B.\end{aligned} \tag{3.11}$$

On the right-hand side of (3.11), if $(1-\alpha/(1+\alpha\overline{p}_h))$ is greater than $(\overline{p}_h-\overline{p}_l)/\overline{p}_h$, the first term is greater than $c$, i.e., the participation constraint (PCc) is satisfied. In this case, the principal sets $w_B=0$. Otherwise, $w_B$ must be positive.

Hence, when $\alpha$ satisfies:

$$\alpha\leq\frac{\overline{p}_l}{\overline{p}_h(1-\overline{p}_l)}, \tag{3.12}$$

only the incentive constraint (ICc) and the limited liability constraint for $w_B$ are binding. The optimal contract is:

$$\begin{aligned}w_B&=0\\ w_G&=\frac{c}{(1+\alpha\overline{p}_h)(\overline{p}_h-\overline{p}_l)}.\end{aligned} \tag{3.13}$$

As $\alpha$ increases, $w_G$ decreases. Hence, the principal's expected cost decreases in $\alpha$. Intuitively, as agents become more envious, they are more strongly motivated to work hard by envy. The principal can offer smaller $w_G$ because she can exploit this inventive effect of envy without considering the cost of envy to each agent. The participation constraint (PCc) is slack, so the cost of envy to each agent is absorbed by the agent's rents.

When $\alpha$ is larger than $\overline{p}_l/((1-\overline{p}_l)\overline{p}_h)$, both the incentive constraint (ICc) and the participation constraint (PCc) are binding. The optimal contract is:

$$\begin{aligned}w_B&=\frac{-\overline{p}_l+\alpha(1-\overline{p}_l)\overline{p}_h}{(1+\alpha\overline{p}_h)(\overline{p}_h-\overline{p}_l)}c\\ w_G&=\frac{1-\overline{p}_l}{\overline{p}_h-\overline{p}_l}c,\end{aligned} \tag{3.14}$$

where $w_B < w_G$ holds because $\overline{p}_h > \overline{p}_l$.

It is straightforward to see $dw_B/d\alpha > 0$ and $dw_G/d\alpha = 0$. Hence, the principal's expected cost increases in $\alpha$. Intuitively, as agents become more envious, they become more reluctant to join the firm because they suffer more from envy. As a result, the principal increases $w_B$ to satisfy the participation constraint. As the principal increases $w_B$, she must (i) increase $w_G$ to motivate the agents to work hard. Nevertheless, as agents become more envious, they are more motivated to work hard because of envy. Then the principal can (ii) decrease $w_G$. These two effects (i) and (ii) balance in my model, and the principal sets $w_G$ constant.

These results are different from Itoh (2004), who also analyzes the optimal contract for peer-regarding agents with two signals and two levels of effort. Itoh (2004) considers the optimal contract under relative performance evaluation, but I assume individual performance evaluation. Hence, the optimal contract in my model reflects the agent's own signal, but not his peer's, as in (3.13) and (3.14).

## The Fine Measure

If the principal uses the fine measure, she minimizes the expected wage payment:

$$(1 - p_h - q_h)w_b + q_h w_m + p_h w_g, \tag{OBJf}$$

subject to limited liability, participation, and incentive constraints. The limited liability constraints are:

$$w_b \geq 0 : w_m \geq 0 : w_g \geq 0. \tag{LLf}$$

The participation constraint for agent $i$ is:

$$E[U^i(w^i, e_h^i, w^j, e_h^j)] \geq 0. \tag{3.15}$$

The incentive constraint for agent $i$ is:

$$E[U^i(w^i, e_h^i, w^j, e_h^j)] \geq E[U^i(w^i, e_l^i, w^j, e_h^j)]. \tag{3.16}$$

These constraints (3.15) and (3.16) are sensitive to how the payments $(w_b, w_m, w_g)$ are ordered. To rewrite these constraints into specific expressions, I use the monotonicity property of the model:

$$w_b \leq w_m \leq w_g. \tag{3.17}$$

In my model, (3.17) works as if it were a constraint. The monotonic ordering (3.17) is preferred to any other possible ordering of $w_b$, $w_m$, and $w_g$, regardless of $\alpha$. In other words, the principal cannot improve her welfare by deviating from $w_b \leq w_m \leq w_g$. See the Appendix for the proof.

By using (3.17), the participation constraint (3.15) can be rewritten as:

$$PC_b w_b + PC_m w_m + PC_g w_g - c \geq 0, \tag{PCf}$$

where:

$$\begin{aligned} PC_b &= (1 - p_h - q_h)[1 + \alpha(p_h + q_h)] \\ PC_m &= q_h + \alpha q_h(-1 + 2p_h + q_h) \\ PC_g &= p_h + \alpha p_h(-1 + p_h). \end{aligned}$$

The incentive constraint (3.16) can be rewritten as:

$$IC_b w_b + IC_m w_m + IC_g w_g - c \geq 0. \tag{ICf}$$

where:

$$\begin{aligned} IC_b &= -[(p_h - p_l) + (q_h - q_l)][1 + \alpha(p_h + q_h)] \\ IC_m &= (q_h - q_l)(1 + \alpha p_h) + \alpha q_h[(p_h - p_l) + (q_h - q_l)] \\ IC_g &= (p_h - p_l)(1 + \alpha p_h). \end{aligned}$$

**Proposition 3.1.** *Under the fine measure, there exist two thresholds $\alpha^*$ and $\alpha^{**}$, where $\alpha^* < \alpha^{**}$, such that the optimal contract is characterized as follows:*

*(i) for $0 \leq \alpha \leq \alpha^*$, $w_b = w_m = 0 < w_g$,*

*(ii) for $\alpha^* \leq \alpha \leq \alpha^{**}$, $w_b = 0 < w_m < w_g$, and*

*(iii) for* $\alpha^{**} \leq \alpha$*,* $0 < w_b < w_m = w_g$*.*

**Proof:** See the Appendix. □

The specific expressions of the two thresholds in Proposition 3.1 are:[4]

$$\alpha^* = \frac{p_l}{p_h(1-p_l)}$$
$$\alpha^{**} = \frac{p_l + q_l}{(p_h + q_h)(1 - p_l - q_l)}.$$

It is easy to verify that $\alpha^* < \alpha^{**}$. I note that $\alpha^{**}$ is the same as the threshold in the coarse measure contract, stated in (3.12), because $\overline{p}_h = p_h + q_h$ and $\overline{p}_l = p_l + q_l$.

When $\alpha$ satisfies:

$$0 \leq \alpha \leq \alpha^*, \tag{3.18}$$

only the incentive constraint (ICf) and the limited liability constraints for $w_m$ and $w_b$ are binding. In this case, the optimal contract is:

$$\begin{aligned} w_b &= w_m = 0 \\ w_g &= \frac{c}{(p_h - p_l)(1 + \alpha p_h)}. \end{aligned} \tag{3.19}$$

When $\alpha$ is smaller that $\alpha^*$, $w_g$ decreases as $\alpha$ increases. Intuitively, as agents become more envious, the principal can offer smaller $w_g$ because each agent is more strongly motivated to work hard by envy. The participation constraint (PCf) is slack, so the cost of envy is absorbed by the agent's rents.

The contract (3.19) could be regarded as coarse performance evaluation, because $w_b = w_m$. Nevertheless, this "coarsening" result is trivial. In basic models of risk-neutral agents, optimal contracts usually require that the wage payment for the worst signals be zero (or the limited liability amount: see e.g., Laffont and Martimort [2002] for discussion). In my model, $w_b = w_m = 0$ when $\alpha$ is zero. This property also holds when $\alpha$ is small, because the contract for $\alpha = 0$ is qualitatively robust for a small change in $\alpha$.

When $\alpha$ satisfies:

$$\alpha^* \leq \alpha \leq \alpha^{**}, \tag{3.20}$$

[4]These are the same $\alpha^*$ and $\alpha^{**}$ in the parameter assumptions (3.6) and (3.7).

the incentive constraint (ICf), the participation constraint (PCf), and the limited liability constraint for $w_b$ are binding. In this case, the optimal contract is:

$$\begin{aligned} w_b &= 0 \\ w_m &= \frac{IC_g - PC_g}{PC_m IC_g - PC_g IC_m} c \\ w_g &= \frac{PC_m - IC_m}{PC_m IC_g - PC_g IC_m} c. \end{aligned} \tag{3.21}$$

When agents are sufficiently envious, the participation constraint is binding because envy eliminates the agents' rents. The principal mitigates the agents' utility loss from envy by setting $w_m$ positive. In this arrangement, each agent loses less utility from envy when the agent gets $w_m$ while his peer gets $w_g$.

Finally, when $\alpha$ satisfies:

$$\alpha > \alpha^{**}, \tag{3.22}$$

the incentive constraint (ICf) and the participation constraint (PCf) are binding, and the monotonicity property (3.17) for $w_m$ and $w_g$ holds with equality. The optimal contract is:

$$\begin{aligned} w_b &= \frac{-q_l - p_l + \alpha(1 - p_l - q_l)(q_h + p_h)}{[1 + \alpha(p_h + q_h)](p_h + q_h - p_l - q_l)} c, \\ w_m = w_g &= \frac{1 - p_l - q_l}{p_h + q_h - p_l - q_l} c. \end{aligned} \tag{3.23}$$

This contract coincides with the contract under the coarse measure, stated in (3.14), because $\overline{p}_h = p_h + q_h$ and $\overline{p}_l = p_l + q_l$.

When agents are envious enough, they are reluctant to join the firm because they lose utility from envy. The principal sets $w_b > 0$ to motivate the agents to participate in the firm. Further, the principal eliminates a pay gap between $w_m$ and $w_g$ to reduce the utility loss from envy. In this arrangement, an agent who gets $w_m$ does not suffer from envy when his peer gets $w_g$.

Hence, if agents have a strong sense of envy, the principal cannot improve her welfare by refining a coarse "good" signal into two signals (say, "moderate" and "good").

**Corollary 3.1.** *When $\alpha \geq \alpha^{**}$, the optimal fine-measure contract can also be achieved under the coarse measure.*

In practice, coarse performance evaluation as in Corollary 3.1 takes the form of either fixed wage schemes or inflated ratings. Under a fixed wage scheme, each agent is paid a fixed wage unless he performs poorly (i.e., unless he produces the worst signal). The principal ignores whether each agent produces a good or moderate signal. On the other hand, under inflated ratings, both good and moderate signals are labeled "good." The principal stops distinguishing these two signals.

## Numerical Example

I construct a numerical example as follows. I assume that the performance signal follows a binomial distribution with size two (so that there are three possible outcomes) and probability 0.6 if the agent works hard; 0.2 if he doesn't. In other words, I assume the following probability parameters:[5]

$$\{(1-p_h-q_h), q_h, p_h\} = \{0.16, 0.48, 0.36\}$$
$$\{(1-p_l-q_l), q_l, p_l\} = \{0.64, 0.32, 0.04\}.$$

Hence, $1-\overline{p}_h = 0.16$, $\overline{p}_h = 0.84$, $1-\overline{p}_l = 0.64$, and $\overline{p}_l = 0.36$. Further, I assume $c = 0.5$.

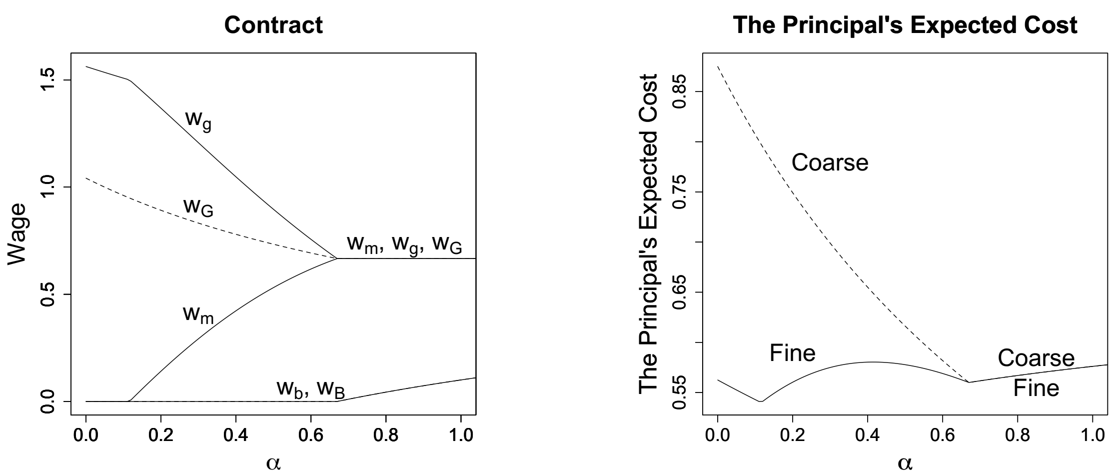


Figure 3.2: Comparing the two performance measures

The graph on the left in Figure 3.2 shows how the wage contract changes as $\alpha$ increases under the coarse measure and the fine measure. The solid lines represent $w_g$, $w_m$, and $w_b$. The dashed

[5]I can replicate the qualitative insight of the analysis for a wide range of parameter choices. I present a result that uses a binomial distribution here to reduce arbitrariness of the parameter choice. A binomial distribution resembles a normal distribution, so I have normally distributed production in mind.

lines represent $w_G$ and $w_B$, though the latter is invisible on the graph because it is identical to $w_b$. In both contracts, the payments for the worst outcome, $w_b$ and $w_B$, start to rise at $\overline{p}_l/((1-\overline{p}_l)\overline{p}_h) = \alpha^{**} = 0.6696429$. When $\alpha$ is larger than this value, the contract under the fine measure is identical to that under the coarse measure. Specifically, $w_m = w_g = w_G = 0.6666667$, and $w_b = w_B > 0$.

The graph on the right in Figure 3.2 shows how the principal's expected cost changes as $\alpha$ increases. The solid line shows the expected cost under the fine measure, and the dashed line under the coarse measure. In either case, envy affects the expected cost in two ways: (i) it reduces the expected cost because it makes the incentive constraint easier to satisfy, i.e., because agents are motivated to work hard by envy (I call this cost-reducing effect of envy the *incentive effect*); and (ii) it increases the expected cost because it makes the participation constraint harder to satisfy, i.e., because agents need to be compensated for their utility loss from envy (I call this costly effect of envy the *participation effect*).

The cost of the contract under the coarse measure has only one turning point. When $\alpha \leq \alpha^{**}$, envy reduces the cost of the contract. In this case, envy has the incentive effect, but the participation effect is absent because the participation constraint (PCc) is slack. On the other hand, when $\alpha \geq \alpha^{**}$ under the coarse measure, envy increases the cost of the contract. In this case, the participation effect dominates the incentive effect.

Under the fine measure, the principal's expected cost changes in a non-trivial way. When $\alpha \leq \alpha^* = 0.1157407$, the expected cost is decreasing in $\alpha$ because envy has the incentive effect. In this area, the participation constraint is slack, so the participation effect of $\alpha$ is absent. As $\alpha$ increases from $\alpha^*$, the principal's expected cost starts to increase in $\alpha$. In this area, the participation constraint starts to bind, and the participation effect of $\alpha$ dominates its incentive effect. As $\alpha$ reaches around 0.4, the principal's expected cost starts to decrease in $\alpha$. In this area, the participation effect of $\alpha$ is weaker than when $\alpha$ was smaller, because $w_g$ and $w_m$ are sufficiently close now — agents incur less utility loss from envy by comparing $w_g$ and $w_m$. Consequently, the incentive effect of $\alpha$ dominates its participation effect until $\alpha$ reaches $\alpha^{**}$. Finally, when $\alpha \geq \alpha^{**}$, the expected cost increases in $\alpha$ because the participation effect of $\alpha$ dominates its incentive effect.

Under the fine measure, the principal's expected cost attains its global minimum at either $\alpha = \alpha^*$ or $\alpha = \alpha^{**}$, depending on the parameter values. By a straightforward calculation, if $p_h q_l(1 - p_h - q_h) > p_l q_h(1 - p_l - q_l)$ as in my example, the principal's expected cost is minimized at $\alpha = \alpha^*$;

otherwise, at $\alpha = \alpha^{**}$.

## 3.4 Robustness Analysis

### When More Than Three Signals Are Available

I conjecture that my result will hold even when the information system can produce more than three signals. For example, when there are four signals, numerical analysis shows a similar pattern to that in the three-signal case. See Figure 3.3. I assume each agent produces a signal ranging from 4 (worst) to 1 (best). The probability parameters are again created by binomial distributions: from the worst to best signals, they are assumed to be (0.008, 0.096, 0.384, 0.512) when the agent makes a high effort, and (0.512, 0.384, 0.096, 0.008) when he makes a low effort. I assume everything else is analogous to the three-signal case.

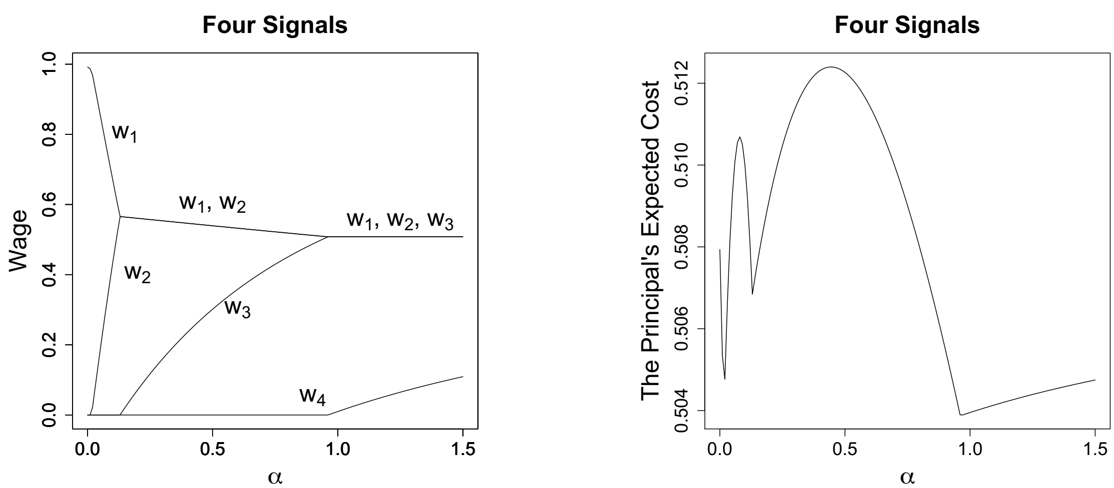


Figure 3.3: When Four Signals Are Available

The graph on the left in Figure 3.3 shows how the contract changes as $\alpha$ increases. As $\alpha$ increases, signals are collapsed one after another. Each time a collapsing happens, the two "best" signals — the signals for the two highest levels of performance — are collapsed into one. When $\alpha$ becomes high enough, only two signals remain.

The graph on the right in Figure 3.3 shows how the principal's expected cost changes as $\alpha$ increases. In this example, the expected cost attains its minimum around $\alpha = 0.97$. The optimal $\alpha$

is one of local optimal points, depending on the parameter values.

## The Effects of Greed and Guilt

In this section, I assume agents have a sense of not only envy but also either greed or guilt. Instead of (3.1), I use a utility function as in Fehr and Schmidt (1999):[6]

$$U^i(w^i, e^i, w^j, e^j) = w^i - C(e^i) - \alpha \max\{0, w^j - w^i\} - \beta \max\{0, w^i - w^j\},$$

where the exogenous parameter $\beta$ represents how much utility the agent $i$ gains or loses by getting a higher wage than his peer $j$. When $\beta < 0$, the agent has a sense of greed, so the agent likes getting a higher wage than his peer. When $\beta > 0$, the agent has a sense of guilt, so the agent dislikes getting a higher wage than his peer. I also assume $|\beta| < \alpha$, i.e., each agent is affected more by a sense of envy than a sense of either greed or guilt. I analyze the model numerically, using the same numerical parameters as in Section 3.3.

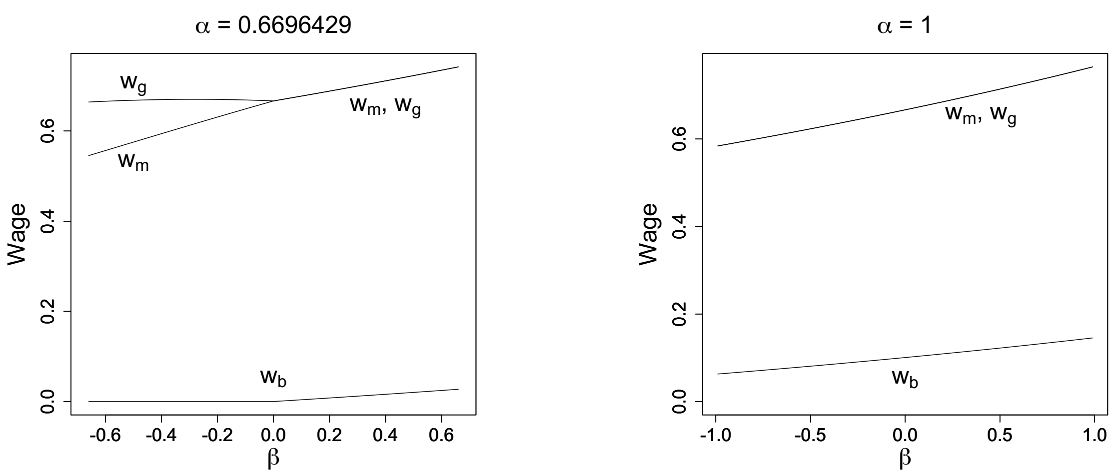


Figure 3.4: The Effects of Greed and Guilt

The graphs on the left and right in Figure 3.4 show how the contracts change in $\beta$ when $\alpha = \alpha^{**} = 0.6696429$ and $\alpha = 1$, respectively. The graph on the left in Figure 3.4 shows that when $\alpha = \alpha^{**}$, coarse performance evaluation is preferred for guilty agents ($\beta > 0$), but fine performance

[6]Fehr and Schmidt (1999) assume $\beta$ is negative. Unlike Fehr and Schmidt (1999), I assume $\beta$ can also be positive so that I can study the effects of greed.

evaluation is preferred for greedy agents ($\beta < 0$). Intuitively, guilty agents prefer contracts that are likely to produce equal outcomes. They dislike the inequality that may arise under the fine measure, when they get $w_g$ and their peer $w_m$. On the other hand, greedy agents prefer contracts that give a higher wage for a good signal than for a moderate signal. They like the inequality that arises when they get $w_g$ and their peer $w_m$.

Nevertheless, this coarsening or refining effect of $\beta$ disappears when $\alpha$ is large enough. The graph on the right in Figure 3.4 indicates that when $\alpha = 1$, coarse performance evaluation is preferred for all $\beta \in (-1, 1)$. When $\alpha$ is large enough, the coarsening effect of envy dominates the refining effect of greed.

## 3.5 Conclusions

I show that a principal optimally aggregates signals of good performance when agents are highly envious. My model explains why this type of coarse performance evaluation is often used in practice, contrary to the conventional wisdom that predicts the use of fine performance evaluation. My analysis illustrates a possible common mechanism for two seemingly different phenomena: a fixed wage contract and rating inflation. They are examples of coarse performance evaluation, which is optimal when agents are sufficiently envious.

My analysis has a number of limitations. Most importantly, I assume that the principal uses individual performance evaluation. The result may not hold if I allow relative performance evaluation. I also make a number of simplifying assumptions such as binary effort, independent production functions, and risk-neutral agents.

Despite these limitations, my analysis has a number of practical implications. I predict that fine performance evaluation is used in unique, off-the-beaten-track jobs. If agents do not have their peers of whom they feel envious, the principal uses fine performance evaluation. On the other hand, I predict that a principal uses coarse performance evaluation in common jobs in which each agent can observe, or form a belief about, how much a typical agent earns. In such jobs, agents are subject to envy, so the principal uses coarse performance evaluation. In the latter case, organizations don't necessarily have to "fix" evaluation systems that aggregate positive signals, in the form of fixed wages or inflated ratings. This type of coarse performance evaluation helps motivate envious workers efficiently.

## 3.6 Appendix

### Proof of the Monotonicity of the Coarse-Measure Contract

The optimal contract under the coarse measure is monotonic in the performance signal, i.e., $w_B < w_G$. To see this point, consider a contract that satisfies $w_G < w_B$ under the coarse performance evaluation. Then, the incentive constraint can be rewritten as:

$$[1 + \alpha(1 - \overline{p}_h)](\overline{p}_h - \overline{p}_l)(w_G - w_B) \geq c. \tag{3.24}$$

There is no valid contract that satisfies (3.24), because $w_G < w_B$ and $\overline{p}_h > \overline{p}_l$ render the left-hand side of (3.24) negative.

### Proof of the Optimal Fine-Measure Contract

I first derive the optimal contract under the fine measure, assuming the contract is monotonic in the performance signal. I then show that the optimal contract is monotonic.

I derive the optimal fine-measure contract by the Kuhn-Tucker approach. I can write the Lagrangian as:

$$\begin{aligned}\mathcal{L} =&(1 - p_h - q_h)w_b + q_h w_m + p_h w_g \\ &+ \lambda_{IC}[IC_b w_b + IC_m w_m + IC_g w_g - c] \\ &+ \lambda_{PC}[PC_b w_b + PC_m w_m + PC_g w_g - c] \\ &+ \lambda_{GM}(w_g - w_m) + \lambda_{MB}(w_m - w_b).\end{aligned} \tag{3.25}$$

The Kuhn-Tucker conditions are:

$$\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial w_b} \geq 0;\ w_b \geq 0;\ \frac{\partial \mathcal{L}}{\partial w_b} w_b = 0\\
&\frac{\partial \mathcal{L}}{\partial w_m} \geq 0;\ w_m \geq 0;\ \frac{\partial \mathcal{L}}{\partial w_m} w_m = 0\\
&\frac{\partial \mathcal{L}}{\partial w_g} \geq 0;\ w_g \geq 0;\ \frac{\partial \mathcal{L}}{\partial w_{gg}^y} w_{gg}^y = 0\\
&\frac{\partial \mathcal{L}}{\partial \lambda_{IC}} \geq 0;\ \lambda_{IC} \leq 0;\ \frac{\partial \mathcal{L}}{\partial \lambda_{IC}} \lambda_{IC} = 0\\
&\frac{\partial \mathcal{L}}{\partial \lambda_{PC}} \geq 0;\ \lambda_{PC} \leq 0;\ \frac{\partial \mathcal{L}}{\partial \lambda_{PC}} \lambda_{PC} = 0\\
&\frac{\partial \mathcal{L}}{\partial \lambda_{GM}} \geq 0;\ \lambda_{GM} \leq 0;\ \frac{\partial \mathcal{L}}{\partial \lambda_{GM}} \lambda_{GM} = 0\\
&\frac{\partial \mathcal{L}}{\partial \lambda_{MB}} \geq 0;\ \lambda_{MB} \leq 0;\ \frac{\partial \mathcal{L}}{\partial \lambda_{MB}} \lambda_{MB} = 0,
\end{aligned} \tag{KTC}$$

where $\lambda_{IC}$, $\lambda_{PC}$, $\lambda_{GM}$, and $\lambda_{MB}$ are the Lagrange multipliers of the incentive constraint, the participation constraint, the monotonicity $w_g \geq w_m$, and the monotonicity $w_m \geq w_b$.

**Claim 3.1.** *When $\alpha \leq p_l/(p_h(1-p_l))$, the optimal contract is as in (3.19).*

**Proof:** Suppose the incentive constraint (ICf) and the limited liability constraints for $w_m = 0$ and $w_b = 0$ are binding. Then, $\partial\mathcal{L}/\partial\lambda_{IC} = 0$ implies $w_g = c/IC_g$, as in (3.19). Further, $\partial\mathcal{L}/\partial\lambda_{PC} > 0$ implies $\alpha \leq p_l/(p_h(1-p_l))$.

I now check the remaining (KTC) conditions. First, $\partial\mathcal{L}/\partial w_g = 0$ leads to $\lambda_{IC} = -p_h/((p_h - p_l)(1+\alpha p_h))$, which is negative because of (3.2). Second, $\partial\mathcal{L}/\partial w_b > 0$ as long as $IC_B < 0$, which in turn holds because of (3.2) and (3.3). Finally, $\partial\mathcal{L}/\partial w_m > 0$ is equivalent to $IC_g/p_h > IC_m/q_h$. This inequality holds because of (3.7). □

**Claim 3.2.** *When $p_l/(p_h(1-p_l)) \leq \alpha \leq (q_l+p_l)/((1-p_l-q_l)(q_h+p_h))$, the optimal contract is as in (3.21).*

**Proof:** Suppose the incentive constraint (ICf), the participation constraint (PCf), and the limited liability constraint for $w_b = 0$ are binding. Then, $\partial\mathcal{L}/\partial\lambda_{IC} = 0$ and $\partial\mathcal{L}/\partial\lambda_{PC} = 0$ lead to $w_m$ and $w_g$ stated in (3.21). The numerator of $w_m$ is non-negative if, and only if, $p_l/(p_h(1-p_l)) \leq \alpha$. Further, the numerator of $w_g$ is greater than that of $w_m$ if, and only if, $\alpha \leq (q_l+p_l)/((1-p_l-$

$q_l)(q_h + p_h))$. In this case, the numerator of $w_g$ is positive. I now have $IC_g > 0$, $IC_g > PC_g$, and $PC_m > IC_m > 0$, so the common denominator of $w_m$ and $w_g$ is also positive.

I now check the remaining (KTC) conditions. First, $\partial\mathcal{L}/\partial w_g = 0$ and $\partial\mathcal{L}/\partial w_m = 0$ lead to $\lambda_{PC} = (p_h IC_m - q_h IC_g)/(PC_m IC_g - PC_g IC_m)$, which is negative if, and only if, $IC_g/p_h > IC_m/q_h$. This inequality holds because of (3.7). Furthermore, I can derive $\lambda_{IC} = -(p_h PC_m - q_h PC_g)/(PC_m IC_g - PC_g IC_m) < 0$. Finally, I can verify $\partial\mathcal{L}/\partial w_b = 1 - p_h - q_h + \lambda_{IC} IC_b + \lambda_{PC} PC_b > 0$. □

**Claim 3.3.** *When $\alpha \geq (q_l + p_l)/((1 - p_l - q_l)(q_h + p_h))$, the optimal contract is as in (3.23).*

**Proof:** Suppose the incentive constraint (ICf), the participation constraint (PCf), and the monotonicity property for $w_m$ and $w_g$, i.e., $w_m \geq w_g$ are binding. Then, $\partial\mathcal{L}/\partial\lambda_{IC} = 0$, $\partial\mathcal{L}/\partial\lambda_{PC} = 0$, and $w_m = w_g$ lead to the contract stated in (3.23). The denominators of the contract in (3.23) are positive given $p_h + q_h > p_l + q_l$. The numerator of $w_b$ is non-negative if, and only if, $\alpha \geq (q_l + p_l)/((1 - p_l - q_l)(q_h + p_h))$.

I now check the remaining (KTC) conditions. First, $\partial\mathcal{L}/\partial w_g = 0$, $\partial\mathcal{L}/\partial w_m = 0$, and $\partial\mathcal{L}/\partial w_b = 0$ lead to $\lambda_{PC} = -1 < 0$ and $\lambda_{IC} = \alpha(p_h + q_h)(p_h + q_h - 1)/(IC_m + IC_g) < 0$. Second, $\lambda_{GM} < 0$ is equivalent to:

$$\frac{PC_g - p_h}{IC_g} < \frac{PC_m + PC_g - p_h - q_h}{IC_m + IC_g},$$

which in turn holds because of (3.4). □

## Proof of the Monotonicity of the Fine-Measure Contract

I show that the optimal contract under the fine measure is monotonic in the performance signal, i.e., $w_b < w_m < w_g$. Nevertheless, before I show this point rigorously, I sketch the principal's expected cost for each possible ordering of the contract in Figure 3.5, using the same numerical parameters as in Section 3.3. The dashed lines in Figure 3.5 are the principal's expected costs when I don't assume $w_b \leq w_m \leq w_g$; The solid line when I assume $w_b \leq w_m \leq w_g$. For simplicity, I let $ijk$ denote $w_i \leq w_j \leq w_k$.

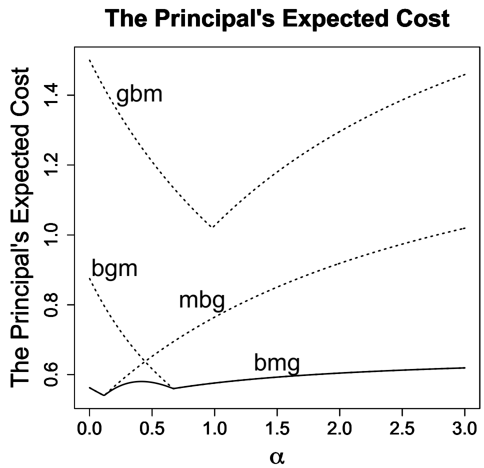


Figure 3.5: Possible Orderings

I prove these points below in turn.

- There are no feasible contracts under $gmb$ and $mgb$.
- When $\alpha$ is small, the principal is indifferent between $mbg$ and $bmg$. The principal prefers $bmg$ to $mbg$ when $\alpha$ is larger than $\alpha^*$.
- When $\alpha$ is large, the principal is indifferent between $bgm$ and $bmg$. The principal prefers $bmg$ to $bgm$ when $\alpha$ is smaller than $\alpha^{**}$.
- The principal incurs a higher expected cost under $gbm$ than under $bmg$.

***The Fine Measure with*** $w_g \leq w_m \leq w_b$

Suppose $w_g \leq w_m \leq w_b$. The incentive constraint is:

$$IC_b^{gmb} w_b + IC_m^{gmb} w_m + IC_g^{gmb} w_g \geq c, \tag{3.26}$$

where:

$$\begin{aligned} IC_b^{gmb} &= (-p_h - q_h + p_l + q_l)[1 + \alpha(1 - p_h - q_h)] \\ IC_m^{gmb} &= (q_h - q_l)[1 + \alpha(1 - p_h - q_h)] - \alpha q_h(p_h - p_l) \\ IC_g^{gmb} &= (p_h - p_l)[1 + \alpha(1 - p_h)]. \end{aligned}$$

In (3.26), $IC_b^{gmb}$ is negative, and $IC_g^{gmb}$ is positive. The coefficients of $w_b$, $w_m$, and $w_g$ in (3.26) sum up to zero, so the left-hand side of (3.26) can be rewritten and evaluated as:

$$\begin{aligned}&IC_b^{gmb}w_b + (-IC_g^{gmb} - IC_b^{gmb})w_m + IC_g^{gmb}w_g\\=&IC_b^{gmb}(w_b - w_m) + IC_g^{gmb}(w_g - w_m) < 0.\end{aligned}$$

Hence, there is no $gmb$ contract that satisfies (3.26).

***The Fine Measure with*** $w_m \leq w_g \leq w_b$

Suppose $w_m \leq w_g \leq w_b$. The incentive constraint is:

$$IC_b^{mgb}w_b + IC_m^{mgb}w_m + IC_g^{mgb}w_g \geq c, \tag{3.27}$$

where:

$$\begin{aligned}IC_b^{mgb} &= (-p_h - q_h + p_l + q_l)[1 + \alpha(1 - p_h - q_h)]\\IC_m^{mgb} &= (q_h - q_l)[1 + \alpha(1 - q_h)]\\IC_g^{mgb} &= (p_h - p_l)[1 + \alpha(1 - p_h - q_h)] - \alpha p_h(q_h - q_l).\end{aligned}$$

In (3.27), $IC_b^{mgb}$ is negative, and $IC_m^{mgb}$ is positive. The coefficients of $w_b$, $w_m$, and $w_g$ in (3.27) sum up to zero, so the left-hand side of (3.27) can be rewritten and evaluated as:

$$\begin{aligned}&IC_b^{mgb}w_b + IC_m^{mgb}w_m + (-IC_m^{mgb} - IC_b^{mgb})w_g\\=&IC_b^{mgb}(w_b - w_g) + IC_m^{mgb}(w_m - w_g) < 0.\end{aligned}$$

Hence, there is no $mgb$ contract that satisfies (3.27).

***The Fine Measure with*** $w_m \leq w_b \leq w_g$

Suppose $w_m \leq w_b \leq w_g$. The participation constraint is:

$$PC_b^{mbg}w_b + PC_m^{mbg}w_m + PC_g^{mbg}w_g \geq c,$$

where:

$$\begin{aligned}
PC_b^{mbg} &= (1 - p_h - q_h)(1 + \alpha p_h - \alpha q_h) \\
PC_m^{mbg} &= q_h + \alpha q_h - \alpha q_h^2 \\
PC_g^{mbg} &= p_h - \alpha p_h + \alpha p_h^2.
\end{aligned}$$

The incentive constraint is:

$$IC_b^{mbg} w_b + IC_m^{mbg} w_m + IC_g^{mbg} w_g \geq c,$$

where:

$$\begin{aligned}
IC_b^{mbg} &= -(p_h + q_h - p_l - q_l) - \alpha p_h (p_h - p_l) - \alpha (1 - q_h)(q_h - q_l) \\
IC_m^{mbg} &= (q_h - q_l)[1 + \alpha(1 - q_h)] \\
IC_g^{mbg} &= (p_h - p_l)(1 + \alpha p_h).
\end{aligned} \tag{3.28}$$

I note that $PC_g^{mbg}$ and $IC_g^{mbg}$ are the same as $PC_g$ and $IC_g$, respectively, in the $bmg$ case. The limited liability constraints are the same as in Section 3.3: $w_b, w_m, w_g \geq 0$.

Suppose $\alpha$ satisfies:

$$0 \leq \alpha \leq \alpha^*. \tag{3.29}$$

This threshold in (3.29) is the same as that in (3.18) in the $bmg$ case. Then the contract is:

$$\begin{aligned}
w_b &= w_m = 0 \\
w_g &= \frac{c}{IC_g^{mbg}}.
\end{aligned} \tag{3.30}$$

The contract (3.30) coincides with that in (3.19).

When $\alpha$ satisfies:

$$\alpha > \alpha^*,$$

the contract is:

$$\begin{aligned} w_b = w_m &= \frac{IC_g^{mbg} - PC_g^{mbg}}{IC_g^{mbg}} c \\ w_g &= \frac{1 + IC_g^{mbg} - PC_g^{mbg}}{IC_g^{mbg}} c. \end{aligned} \tag{3.31}$$

The contract (3.31) is weakly costlier than (3.21) or (3.23) in the $bmg$ case. This point can be argued by revealed preference. The contract (3.31) is feasible under $bmg$, but it doesn't coincide with the optimal $bmg$ contract in (3.21) or (3.23).

***The Fine Measure with*** $w_b \leq w_g \leq w_m$

Suppose $w_b \leq w_g \leq w_m$. The participation constraint is:

$$PC_b^{bgm} w_b + PC_m^{bgm} w_m + PC_g^{bgm} w_g \geq c,$$

where:

$$\begin{aligned} PC_b^{bgm} &= (1 - p_h - q_h)(1 + \alpha p_h + \alpha q_h), \\ PC_m^{bgm} &= q_h - \alpha q_h + \alpha q_h^2, \\ PC_g^{bgm} &= p_h + 2\alpha p_h q_h + \alpha p_h^2 - \alpha p_h. \end{aligned} \tag{3.32}$$

The incentive constraint is:

$$IC_b^{bgm} w_b + IC_m^{bgm} w_m + IC_g^{bgm} w_g \geq c, \tag{3.33}$$

where:

$$\begin{aligned} IC_b^{bgm} &= (-p_h - q_h + p_l + q_l)(1 + \alpha p_h + \alpha q_h), \\ IC_m^{bgm} &= q_h + \alpha q_h^2 - q_l - \alpha q_h q_l, \\ IC_g^{bgm} &= p_h + 2\alpha p_h q_h + \alpha p_h^2 - p_l - \alpha p_l q_h - \alpha p_h p_l - \alpha p_h q_l \end{aligned}$$

I note that $PC_b^{bgm}$ and $IC_b^{bgm}$ are the same as $PC_b$ and $IC_b$, respectively, in the $bmg$ case. The limited liability constraints are the same as in Section 3.3: $w_b, w_m, w_g \geq 0$.

When $\alpha$ satisfies:

$$0 \leq \alpha \leq \alpha^{**}, \tag{3.34}$$

the contract is:

$$\begin{aligned} w_b &= 0, \\ w_m = w_g &= \frac{c}{(1 + \alpha p_h + \alpha q_h)(p_h + q_h - p_l - q_l)}, \end{aligned} \tag{3.35}$$

which is the same as the contract under the coarse measure when $\alpha$ is small — see (3.13). The threshold in (3.34) is also the same as in the coarse measure case and the $bmg$ case, as in (3.12) and (3.22).

The contract (3.35) is weakly costlier than (3.19) or (3.21) in the $bmg$ case. This point can be argued by revealed preference. The contract (3.35) is feasible under $bmg$, but it doesn't coincide with the optimal $bmg$ contract in (3.19) or (3.21).

On the other hand, when:

$$\alpha \geq \alpha^{**},$$

the contract is the same as in (3.23):

$$\begin{aligned} w_b &= \frac{-q_l - p_l + \alpha(1 - p_l - q_l)(q_h + p_h)}{[1 + \alpha(p_h + q_h)](p_h + q_h - p_l - q_l)} c, \\ w_m = w_g &= \frac{1 - p_l - q_l}{p_h + q_h - p_l - q_l} c. \end{aligned}$$

Hence, when $\alpha$ is large, the optimal contract is the same for $bmg$ and for $bgm$.

***The Fine Measure with*** $w_g \leq w_b \leq w_m$

Suppose $w_g \leq w_b \leq w_m$. The participation constraint is:

$$PC_b^{gbm} w_b + PC_m^{gbm} w_m + PC_g^{gbm} w_g \geq c,$$

where:

$$
\begin{aligned}
PC_b^{gbm} &= (1 - p_h - q_h)(1 - \alpha p_h + \alpha q_h), \\
PC_m^{gbm} &= q_h - \alpha q_h + \alpha q_h^2, \\
PC_g^{gbm} &= p_h + \alpha p_h - \alpha p_h^2.
\end{aligned} \tag{3.36}
$$

The incentive constraint is:

$$
IC_b^{gbm} w_b + IC_m^{gbm} w_m + IC_g^{gbm} w_g \geq c,
$$

where:

$$
\begin{aligned}
IC_b^{gbm} &= -(p_h + q_h - p_l - q_l) - \alpha(1 - p_h)(p_h - p_l) - \alpha q_h (q_h - q_l), \\
IC_m^{gbm} &= (q_h - q_l)(1 + \alpha q_h), \\
IC_g^{gbm} &= (p_h - p_l)(1 + \alpha - \alpha p_h).
\end{aligned} \tag{3.37}
$$

I note that $IC_g^{gbm}$ and $IC_m^{gbm}$ are positive, $IC_b^{gbm}$ negative.

When $\alpha$ satisfies:

$$
0 \leq \alpha \leq \frac{q_l}{q_h(1 - q_l)},
$$

I can derive the contract as:

$$
\begin{aligned}
w_b &= w_g = 0 \\
w_m &= \frac{c}{IC_m^{gbm}}.
\end{aligned} \tag{3.38}
$$

If $\alpha$ satisfies:

$$
\alpha \geq \frac{q_l}{q_h(1 - q_l)},
$$

I can derive the contract as:

$$
\begin{aligned}
w_m &= \frac{IC_m^{gbm} - PC_m^{gbm}}{IC_m^{gbm}} c \\
w_b = w_g &= \frac{1 + IC_m^{gbm} - PC_m^{gbm}}{IC_m^{gbm}} c.
\end{aligned} \tag{3.39}
$$

The contract in (3.38) and (3.39) is weakly costlier than the $bmg$ contract in (3.19), (3.21), and (3.23). This point can be argued by revealed preference. The contract in (3.38) and (3.39) is feasible under $bgm$, but it doesn't coincide with the optimal $bgm$ contract. The principal weakly prefers the $bgm$ contract to $gbm$. Further, the optimal $bgm$ contract is feasible under $bmg$, but it doesn't coincide with the optimal $bmg$ contract. The principal weakly prefers the $bmg$ contract to $bgm$. By transitivity, the principal weakly prefers the $bmg$ contract to $gbm$.

# Chapter 4

# Unilateral Envy, Pay Inversion, and the Cost of Age Diversity

## 4.1 Introduction

Pay and age are related in various ways in organizations. Under a *seniority system*, older employees are paid more than younger ones. On the other hand, *pay inversion* arises when newly hired employees are paid more than their longer-serving peers. In this paper, I address the following questions: 1) What is the optimal contract for young and old workers? 2) Is it efficient to promote age diversity for incentive contracts? These questions need careful analysis because age-based reward structures create envy among workers.

Research has shown older people are more likely to be envious of their younger peers than the other way around (Henniger and Harris 2015). When younger people compare themselves with their older peers, they expect they can become like their older peers in the future. Even if younger people are less successful than their older peers at present, they may not suffer from envy. On the other hand, when older people compare themselves with their younger peers, they will feel unpleasant if they are less successful than their younger peers.[1] To the best of my knowledge, the literature has yet to examine how such unilateral envy affects incentives. This consideration

[1] For example, Galinsky and Schweitzer 2015 note "[w]e expect to see an elder brother achieve success before the younger, and when the opposite occurs, discontent can follow" (p. 35). They reason that older people tend to feel envious of their younger peers than the other way around.

is important because the population is aging worldwide (United Nations 2019), so young and old workers are likely to work together more frequently.

Research has investigated how envy can affect incentives for homogeneous peer-regarding agents. When all employees can be envious of their peers, envy is known to either increase or decrease the cost of providing incentives, depending on the assumptions about, for example, the agents' risk preferences, limited liability, and the shape of the contract (Itoh 2004; Bartling and von Siemens 2010; Bartling 2011). In these models, (i) agents are motivated to work hard by a sense of envy, but (ii) they must be compensated for utility loss from envy. One might expect this second, costly effect of envy would be the primary effect when there are heterogeneous, age-diverse agents with unilateral envy. In practice, a worker's wage tends to rise with seniority (e.g., Hutchens 1989; Topel 1991). Intuitively, one might reason that the principal would want to reduce the costly effect of envy by using a seniority system, i.e., granting higher wages to envious old agents than to younger ones. In this paper, however, I show this intuition is mistaken.

I analyze the optimal contract for a group of young and old agents. In my model, only the old agent has a sense of envy, so he incurs utility loss when his younger peer gets paid more than himself. I show that the optimal contract is characterized by pay inversion, i.e., granting a higher expected wage to the young agent than to the old agent. Further, I show that the principal is better off when agents are bilaterally envious than when they are unilaterally envious.

My analysis makes two main contributions. First, it reveals a possible reason for causing pay inversion versus using seniority-based pay in the workplace. Pay inversion is considered problematic in organizations like universities (Richardson and Thomas 2013; Glassman and McAfee 2005; McAfee and Glassman 2005; McNatt, Glassman, and McAfee 2007). Nevertheless, I show that the principal can achieve the optimal contract by causing pay inversion when the older agent can be envious of his younger peer but not vice versa. This result will help organizations to design incentives for a group of age-diverse workers.

Second, my analysis offers a new insight into costs of age diversity. Research has identified costs of age diversity. For example, a group of diverse workers are likely to suffer from communication difficulties and differences in preferences (see Backes-Gellner and Veen [2013] and the references therein). I show another cost of age diversity. In my analysis, the principal is better off when agents are bilaterally envious than when they are unilaterally envious. Bilaterally envious agents are descriptive of workers similar in age, and unilaterally envious agents are descriptive of age-diverse workers (Henniger and Harris 2015). Hence, my analysis shows an opportunity cost of assigning comparable positions to age-diverse workers rather than workers similar in age.

## 4.2 Model

A risk-neutral principal employs two risk-neutral, effort-averse agents. One of the agents is *young*, and the other is *old*. The agents are symmetric in all respects, except that the old one has a sense of envy. The young agent has the following utility function:

$$U^y(w^y, e^y) = w^y - C(e^y), \tag{4.1}$$

where $w$, $e$, $C$ are wage, effort, and the cost of effort, respectively. The superscript $y$ is for the young agent.

On the other hand, the old agent has a sense of envy. He has a utility function similar to the one introduced by Fehr and Schmidt (1999):[2]

$$U^o(w^o, e^o, w^y) = w^o - C(e^o) - \alpha \max\{0, w^y - w^o\}, \tag{4.2}$$

where the superscript $o$ is for the old agent. When the old agent gets paid a smaller wage than does the young agent, he loses utility from envy. The strength of envy is denoted by $\alpha > 0$.

The game proceeds as in Figure 4.1. The principal offers each agent $i \in \{y, o\}$ a contract $w^i$. Each agent $i$ decides whether to accept it. If both of them accept the contract, they are in the firm. Each agent $i$ privately takes an action $e^i$ (either high or low effort; $e^i \in \{h, l\}$), based on which he independently produces a signal $s^i$ (either good or bad signal; $s^i \in \{g, b\}$). Finally, the principal pays $w^i(s^i, s^j)$, where $i \neq j$.

[2]Unlike Fehr and Schmidt (1999), I assume the old agent reacts to disadvantageous inequality $w^y > w^o$, but not to advantageous inequality $w^o > w^y$. The psychology literature has yet to document how young and old people react to advantageous inequality.

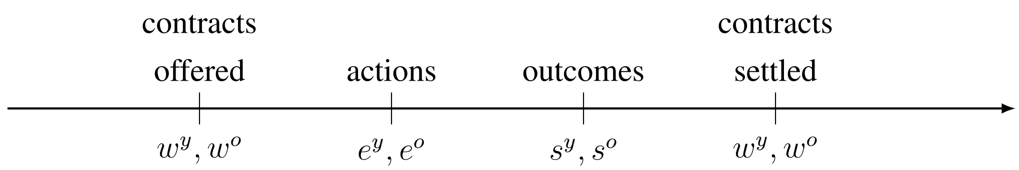


Figure 4.1: Timeline of events

The two agents are assumed to have identical and independent production technology. I assume identical production technology because empirical evidence is mixed on the relationship between age and productivity (see Ng and Feldman [2008] for a meta-analysis). Each agent $i$ produces a good signal $g$ with probability $p$ when he makes a high effort. Similarly, I assume each agent produces a good signal $g$ with probability $q$, where $q < p$, when he makes a low effort. The principal offers each agent $i$ a contract $(w^i_{gg}, w^i_{gb}, w^i_{bg}, w^i_{bb})$, where the first and second subscripts of each element are the agent $i$ and $j$'s signals. For example, when the old agent produces $g$ and the young agent produces $b$, the former will get $w^o_{gb}$ and the latter will get $w^y_{bg}$ (not $w^y_{gb}$).

For simplicity, the reservation utility of each agent is assumed to be $0$. The cost of effort is assumed to be $0$ when the agent makes a low effort, and $c > 0$ when he makes a high effort. I focus on the efficient contract that motivates both agents to choose high effort. All proofs are relegated to the Appendix.

The principal minimizes the expected compensation:

$$\begin{aligned} &p^2(w^o_{gg} + w^y_{gg}) + p(1-p)(w^o_{gb} + w^y_{bg}) \\ &\quad + (1-p)p(w^o_{bg} + w^y_{gb}) + (1-p)^2(w^o_{bb} + w^y_{bb}), \end{aligned} \tag{Obj}$$

subject to limited liability, participation, and incentive constraints. The limited liability constraints are, for $i = y, o$:

$$w^i_{gg} \geq 0 : w^i_{gb} \geq 0 : w^i_{bg} \geq 0 : w^i_{bb} \geq 0. \tag{LL}$$

The participation constraints are:

$$E[U^y(w^y, e^y = h)] \geq 0, \tag{PCy}$$

$$E[U^o(w^o, e^o = h, w^y)] \geq 0. \tag{PCo}$$

The incentive constraints are:

$$E[U^y(w^y, e^y = h)] \geq E[U^y(w^y, e^y = l)], \tag{ICy}$$

$$E[U^o(w^o, e^o = h, w^y)] \geq E[U^o(w^o, e^o = l, w^y)]. \tag{ICo}$$

I can rewrite (PCy) as:

$$p^2 w^y_{gg} + p(1-p)w^y_{gb} + p(1-p)w^y_{bg} + (1-p)^2 w^y_{bb} - c \geq 0. \tag{PCy*}$$

I can rewrite (PCo) as:

$$\begin{aligned} &p^2[w^o_{gg} - \alpha \max(0, w^y_{gg} - w^o_{gg})] \\ &\quad + p(1-p)[w^o_{gb} - \alpha \max(0, w^y_{bg} - w^o_{gb})] \\ &\quad + p(1-p)[w^o_{bg} - \alpha \max(0, w^y_{gb} - w^o_{bg})] \\ &\quad + (1-p)^2[w^o_{bb} - \alpha \max(0, w^y_{bb} - w^o_{bb})] \ - c \geq 0. \end{aligned} \tag{PCo*}$$

I can rewrite (ICy) as:

$$\text{LHS of (PCy*)} \geq qpw^y_{gg} + q(1-p)w^y_{gb} + (1-q)pw^y_{bg} + (1-q)(1-p)w^y_{bb}. \tag{ICy*}$$

Finally, I can rewrite (ICo) as:

$$\begin{aligned} \text{LHS of (PCo*)} \geq\ &qp[w^o_{gg} - \alpha \max(0, w^y_{gg} - w^o_{gg})] \\ &+ q(1-p)[w^o_{gb} - \alpha \max(0, w^y_{bg} - w^o_{gb})] \\ &+ (1-q)p[w^o_{bg} - \alpha \max(0, w^y_{gb} - w^o_{bg})] \\ &+ (1-q)(1-p)[w^o_{bb} - \alpha \max(0, w^y_{bb} - w^o_{bb})]. \end{aligned} \tag{ICo*}$$

I note that (ICy*) and the limited liability constraints (LL) imply (PCy*). The right-hand side of (ICy*) is greater than or equal to zero because of the limited liability constraints (LL). Therefore, when (ICy*) holds, (PCy*) holds. I hereafter ignore (PCy*).

I now rewrite (PCo*) and (ICo*). To simplify the analysis, I use the following property of the model.

**Lemma 4.1.** *For any optimal contract ($w^o$, $w^y$) that fails to satisfy:*

$$w^i_{bg} = w^i_{bb} = 0 \text{ for both agents } i = y, o, \tag{4.3}$$

*there exists another optimal contract ($\hat{w}^o$, $\hat{w}^y$) that satisfies* (4.3).

Lemma 4.1 means that I can assume (4.3) without loss of generality. I hereafter assume (4.3).

To rewrite (PCo*) and (ICo*), I need to consider two cases; when $w^o_{gg} \geq w^y_{gg}$ and when $w^o_{gg} \leq w^y_{gg}$. When $w^o_{gg} \geq w^y_{gg}$, I can rewrite (PCo*) and (ICo*), respectively, as:

$$p^2 w^o_{gg} + p(1-p)w^o_{gb} - p(1-p)\alpha w^y_{gb} - c \geq 0, \tag{4.4}$$

$$p(p-q)w^o_{gg} + (p-q)(1-p)w^o_{gb} + p(p-q)\alpha w^y_{gb} - c \geq 0. \tag{4.5}$$

On the other hand, when $w^o_{gg} \leq w^y_{gg}$, I can rewrite (PCo*) and (ICo*), respectively, as:

$$p^2(1+\alpha)w^o_{gg} + p(1-p)w^o_{gb} - p^2\alpha w^y_{gg} - p(1-p)\alpha w^y_{gb} - c \geq 0, \tag{4.6}$$

$$\begin{aligned} &p(p-q)(1+\alpha)w^o_{gg} + (p-q)(1-p)w^o_{gb} \\ &\quad - p(p-q)\alpha w^y_{gg} + p(p-q)\alpha w^y_{gb} - c \geq 0. \end{aligned} \tag{4.7}$$

I note that (4.4) and (4.5) are easier to satisfy than are (4.6) and (4.7), respectively. One can verify this point by evaluating the difference between (4.4) and (4.6); and between (4.5) and (4.7). Suppose the principal minimizes (Obj) given (4.4) and (4.5). If the contract obtained this way satisfies $w^o_{gg} \geq w^y_{gg}$, it is preferred to any contract under $w^o_{gg} \leq w^y_{gg}$, because in the latter contract, the principal is faced with stricter constraints. A contract under $w^o_{gg} \leq w^y_{gg}$ is costlier to the principal, because envy that stems from the pay gap $w^y_{gg} - w^o_{gg} \geq 0$ makes the old agent's incentive and participation constraints harder to satisfy.

## 4.3 Optimal Contract

### 4.3.1 Benchmark: No Envy

When $\alpha = 0$, the agents are symmetric in all respects. The principal minimizes:

$$p^2(w^o_{gg} + w^y_{gg}) + p(1-p)(w^o_{gb} + w^y_{gb}), \tag{Obj$^0$}$$

subject to participation, incentive, and limited liability constraints. For each agent $i = y, o$, these constraints are:

$$p^2 w^i_{gg} + p(1-p)w^i_{gb} - c \geq 0, \tag{PC$^0$}$$

$$\text{LHS of (PC}^0) \geq pq w^i_{gg} + (1-p)q w^i_{gb}, \tag{IC$^0$}$$

$$w^i_{gg} \geq 0; w^i_{gb} \geq 0. \tag{LL$^0$}$$

I note (IC$^0$) and (LL$^0$) imply (PC$^0$). The optimal contract for each agent $i = y, o$ is non-unique. It is characterized by $w^i_{gg} \geq 0$, $w^i_{gb} \geq 0$, $w^i_{bg} = w^i_{bb} = 0$, and:[3]

$$p(p-q)w^i_{gg} + (p-q)(1-p)w^i_{gb} = c. \tag{4.8}$$

In particular, one possible contract is, for each agent $i = y, o$:

$$\begin{aligned} w^i_{gg} &= 0, \\ w^i_{gb} &= \frac{c}{(1-p)(p-q)}, \end{aligned} \tag{4.9}$$

and $w^i_{bg} = w^i_{bb} = 0$. I show this specific contract is qualitatively similar to the optimal contract when $\alpha$ is positive but small.

[3]This result also appears in Itoh (2004, p. 34).

### 4.3.2 Unilateral Envy

When $\alpha > 0$, the optimal contract takes one of three forms, depending on $\alpha$. I characterize them in turn, from Proposition 4.1 to Proposition 4.3.

**Proposition 4.1.** *When $0 < \alpha \leq (1-p)q/p$, the optimal contract is characterized by $w^i_{bg} = w^i_{bb} = 0$ for $i = y, o$, and:*

$$\begin{aligned} w^y_{gg} &= 0 \\ w^y_{gb} &= \frac{c}{(1-p)(p-q)}. \end{aligned} \tag{4.10}$$

*The optimal combination of $w^o_{gg} \geq 0$ and $w^o_{gb} \geq 0$ is non-unique and is characterized by:*

$$p(p-q)w^o_{gg} + (p-q)(1-p)w^o_{gb} = c(1 - \frac{\alpha p}{1-p}). \tag{4.11}$$

When $\alpha$ is small, (ICo*) and (ICy*) are binding, but (PCo*) is slack. The form of the contract for the young agent (4.10) is the same as (4.9) in the benchmark setting with $\alpha = 0$. When $0 < \alpha \leq (1-p)q/p$, however, (4.10) is uniquely determined. The intuition is as follows. The young agent is indifferent between $w^y_{gb}$ and $w^y_{gg}$. Nevertheless, the principal sets $w^y_{gb}$ positive and $w^y_{gg}$ zero, because unlike $w^y_{gg}$, large $w^y_{gb}$ motivates the envious old agent to work hard to avoid the pay gap for the case $(s^y, s^o) = (g, b)$.

Because of this motivating effect of envy, the principal can reduce the expected payment to the old agent. It is easy to calculate that the expected payment to the old agent is $pc(1 - \alpha p/(1 - p))/(p - q)$, which is decreasing in $\alpha$. Envy helps motivating the old agent, so it reduces the cost of the old agent's contract. I can also calculate that the old agent's expected utility (rather than payment) is $pc(1 - \alpha/(1 - p))/(p - q) - c$, which is decreasing in $\alpha$ and is zero when $\alpha = (1-p)q/p$, i.e., on the upper threshold in Proposition 4.1. As long as the old agent's expected utility is positive, the participation constraint of the old agent is slack. The principal can exploit the motivating effect of envy without considering the old agent's utility loss from envy.

On the other hand, the expected payment to the young agent is $pc/(p - q)$, which is larger than that to the old agent. Unlike the old agent, the young agent doesn't have a sense of envy, and his expected compensation is unaffected by $\alpha$. The young agent's expected utility (the expected compensation minus the cost of effort) is $qc/(p-q)$ and is constant.

The principal is indifferent about the choice between $w^o_{gg}$ and $w^o_{gb}$, as long as the contract satisfies (4.11). The young agent is indifferent about this choice, because he doesn't care how much the old agent is paid. The old agent is also indifferent because of the following reason. When $\alpha$ is positive but small enough, the contract satisfies $w^o_{gg} \geq w^y_{gg} = 0$ and $w^o_{gb} \geq w^y_{bg} = 0$. As long as the old agent produces a good signal $s^o = g$, he is paid at least as much as the young agent. In this case, he is free from a sense of envy, so he is indifferent about the young agent's signal. In particular, the principal can set $w^o_{gg} = w^o_{gb}$, so that the old agent's pay is independent from the young agent's signal given $s^o = g$. This indifference result in (4.11) is qualitatively the same as that in (4.8) in the benchmark setting.

**Proposition 4.2.** *When $(1-p)q/p \leq \alpha < (1-p)/p$, the optimal contract is characterized by $w^i_{bg} = w^i_{bb} = 0$ for $i = y, o$, and:*

$$\begin{aligned} w^y_{gg} &= \frac{\alpha p - (1-p)q}{\alpha p^2 (p-q)} c \\ w^y_{gb} &= \frac{qc}{\alpha p (p-q)}. \end{aligned} \tag{4.12}$$

*The optimal combination of $w^o_{gg} \geq 0$ and $w^o_{gb} \geq 0$ is non-unique and is characterized by $w^o_{gg} \geq w^y_{gg}$ and:*

$$p(p-q) w^o_{gg} + (p-q)(1-p) w^o_{gb} = c(1-q). \tag{4.13}$$

When $\alpha$ is of moderate size, all three constraints (ICy*), (ICo*), and (PCo*) are binding. As in the previous case with $0 < \alpha \leq (1-p)q/p$, the principal exploits the motivating effect of envy by setting $w^y_{gb}$ positive. Unlike the previous case, however, the old agent has no excess utility left beyond the level of reservation utility. The principal needs to consider the cost of envy for the old agent. As $\alpha$ increases the old agent's participation constraint becomes increasingly difficult to satisfy because of envy. This negative effect arises from $w^y_{gb}$, but not from $w^y_{gg}$ as long as $w^y_{gg} \leq w^o_{gg}$. By increasing $w^y_{gg}$ and decreasing $w^y_{gb}$, the principal can reduce the cost of envy for the old agent. Hence, $w^y_{gb}$ is decreasing in $\alpha$, and $w^y_{gg}$ is increasing in $\alpha$. On the other hand, by the same reasoning as in the previous case with $0 < \alpha \leq (1-p)q/p$, the principal and the two agents are indifferent about the choice of $w^o_{gg}$ and $w^o_{gb}$, as long as the contract satisfies (4.13).

When $\alpha$ is of moderate size as in Proposition 4.2, the expected payment to the old agent is $pc(1-q)/(p-q)$, which is independent of $\alpha$. As $\alpha$ increases, (i) the old agent is motivated to work hard by envy, but (ii) $w^y_{gb}$ decreases, so this motivating effect is dampened. At the same time, (iii) the old agent suffers more from envy and he needs to be compensated more to satisfy the participation constraint, but (iv) $w^y_{gb}$ is decreases and $w^y_{gg}$ increases, so this negative effect is dampened. The effects (i) and (iii) are cancelled out by (ii) and (iv) respectively, and increasing $\alpha$ doesn't have an overall effect on the cost of the old agent's contract. I also note that the expected utility of the old agent is zero for any $\alpha \in [(1-p)q/p, (1-p)/p)$.

On the other hand, the expected payment to the young agent is $pc/(p-q)$, which is independent of $\alpha$ and is the same as in the previous case with $0 < \alpha \leq (1-p)q/p$. The principal changes the contract for the young agent (4.12) depending on $\alpha$. Nevertheless, the principal does so only to incentivize the envious old agent efficiently. The young agent's expected compensation is independent of $\alpha$. Further, the expected utility of the young agent is $qc/(p-q)$, the same as in the previous case with $0 < \alpha \leq (1-p)q/p$. The young agent is unaffected by a change in the old agent's sense of envy $\alpha$.

The optimal contract in Proposition 4.2 requires $w^o_{gg} \geq w^y_{gg}$. This inequality ensures that increasing $w^y_{gg}$ produces no envy for the old agent. Recall that the principal needs to increase $w^y_{gg}$ as $\alpha$ increases. Because $w^o_{gg} \geq w^y_{gg}$ and $w^y_{gg}$ is increasing, the principal loses freedom to choose $w^o_{gg}$ and $w^o_{gb}$ in (4.13) as $\alpha$ increases. When $\alpha \geq (1-p)/p$, the principal can only find an optimal contract with $w^o_{gg} \leq w^y_{gg}$. The structure of the contract changes as follows.

**Proposition 4.3.** *When $\alpha \geq (1-p)/p$, the optimal contract is characterized by $w^i_{bg} = w^i_{bb} = 0$ for $i = y, o$, and:*

$$w^o_{gg} = \frac{\alpha p + p - q}{p^2(1+\alpha)(p-q)}c,$$
$$w^o_{gb} = 0.$$

*The optimal combination of* $w_{gb}^{y} \geq 0$ *and* $w_{gg}^{y} \geq w_{gg}^{o} \geq 0$ *is non-unique and is characterized by:*

$$w_{gb}^{y} \geq w_{gg}^{y} - \frac{\left(\alpha - \frac{q}{p}\right)c}{p(p-q)\alpha}, \tag{4.14}$$

$$pw_{gg}^{y} + (1-p)w_{gb}^{y} = \frac{c}{p-q}. \tag{4.15}$$

When $\alpha$ is large, (ICy*) and (PCo*) are binding, and (ICo*) can be rewritten as (4.14). The contract satisfies $w_{gg}^{y} \geq w_{gg}^{o}$ and $w_{gb}^{y} \geq w_{bg}^{o} = 0$. If the young agent produces a good signal $s^y = g$, the old agent incurs utility loss from envy regardless of his own signal. To minimize the old agent's utility loss from the pay gap $w_{gg}^{y} - w_{gg}^{o} \geq 0$, $w_{gg}^{o}$ is set positive and $w_{gb}^{o}$ zero. I also note $dw_{gg}^{o}/d\alpha > 0$. Intuitively, when $\alpha$ increases, the old agent loses more utility from envy. To compensate for this utility loss, the principal must increase $w_{gg}^{o}$. The expected utility of the old agent is zero for any $\alpha \geq (1-p)/p$.

The principal chooses $w_{gb}^{y}$ to incentivize the old agent by envy. More specifically, $w_{gb}^{y}$ has to be sufficiently large as in (4.14).[4] An increase in $w_{gb}^{y}$ motivates the envious old agent to work hard to avoid the pay gap for the case $(s^y, s^o) = (g, b)$. On the other hand, (4.14) also implies $w_{gg}^{y}$ has to be sufficiently small. The contract specifies $w_{gg}^{y} \geq w_{gg}^{o}$. Hence, an increase in $w_{gg}^{y}$ can intensify utility loss from envy when the old agent is successful in his production. In short, it demotivates the old agent.

The principal is indifferent about the choice of $w_{gg}^{y}$ and $w_{gb}^{y}$, as long as the contract satisfies (4.14) and (4.15). Given (4.14), the expected impact of envy on the old agent is the same for a unit increase in either $pw_{gg}^{y}$ or $(1-p)w_{gb}^{y}$. To see this point, consider a situation where the young agent produces a good signal $s^y = g$. If the old agent also produces a good signal $s^o = g$, with probability $p$, the impact of envy depends on $w_{gg}^{y}$. If the old agent produces a bad signal $s^o = b$, with probability $1-p$, the impact of envy depends on $w_{gb}^{y}$.

When $\alpha$ is large as in Proposition 4.3, the expected payment to the old agent is $c[(\alpha p + p - q)/(1+\alpha)]/(p-q)$, which is increasing in $\alpha$ and converging to the expected payment to the young agent $pc/(p-q)$. Unlike the previous case in Proposition 4.2, the contract in Proposition 4.3 has an additional source of envy: $w_{gg}^{y} \geq w_{gg}^{o}$. As $\alpha$ increases, the old agent needs to be compensated

[4] I note that $w_{gb}^{y}$ doesn't necessarily have to be greater than $w_{gg}^{y}$. The second term of the right-hand side in (4.14) can be either positive or negative, depending on the parameter assumptions.

heavily for his utility loss from envy. The cost of the old agent's contract increases in $\alpha$.

On the other hand, the young agent, who doesn't have a sense of envy, faces a constant expected compensation $pc/(p-q)$ and expected utility $qc/(p-q)$. These values are the same as in the previous two cases with $0 < \alpha \leq (1-p)q/p$ and $(1-p)q/p \leq \alpha < (1-p)/p$.

For any $\alpha > 0$, the expected payments to the two agents have the following *pay inversion* property:

**Corollary 4.1.** *When $\alpha > 0$, the expected payment to the young agent is greater than that to the old agent.*

Intuitively, the principal can use the old agent's envy to motivate him at a lower cost. When envy is absent, both agents earn rents. When the old agent is envious, the principal can create a contract that causes envy and reduces the old agent's rents. The expected payment to the old agent is smaller because the old agent is motivated to work hard by envy.

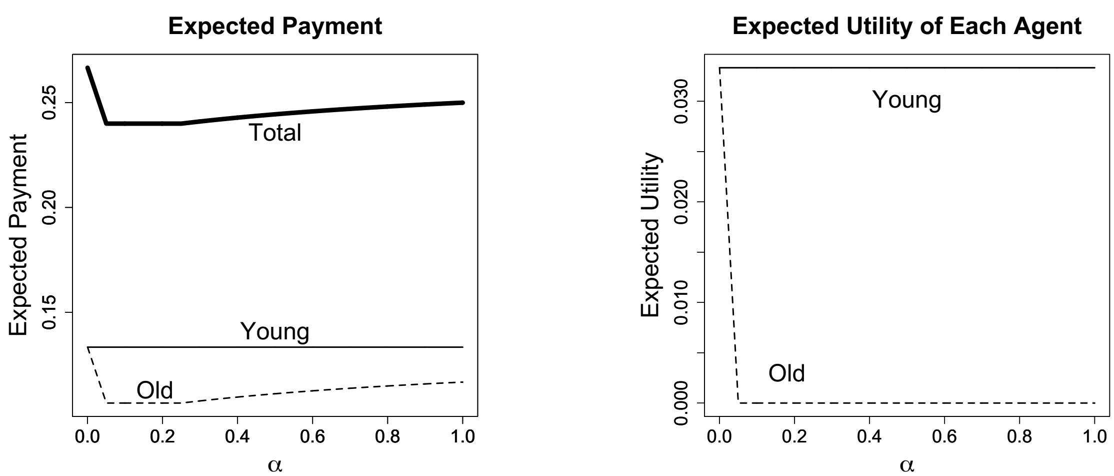


Figure 4.2: Expected Payment and Utility

Figure 4.2 shows a numerical example of how each agent's expected payment (the graph on the left) and expected utility (the graph on the right) change in $\alpha$. The thin lines show those of individual agents: the solid lines for the young agent, the dashed lines for the old agent. On the graph on the left, the thick solid line shows the total expected payment for the principal. In this example, I set $p = 0.8$, $q = 0.2$, and $c = 0.1$.

### 4.3.3 Unilateral versus Bilateral Envy

I compare the principal's welfare in my model with that in Itoh (2004, Section 4), who considers homogeneous, bilaterally envious agents. Itoh (2004, Section 4)'s agents are descriptive of workers similar in age, because people tend to feel envious of their peers who are similar in age to themselves (Henniger and Harris 2015). My model is identical to that of Itoh (2004, Section 4), except agents are unilaterally envious in my model. When agents are bilaterally envious, each of them has a utility function as in (4.2) with an identical sense of envy $\alpha$.

Straightforward calculations show that the principal is better off when envy exists bilaterally as in Itoh (2004, Section 4) than when it exists unilaterally as in my model. Moreover, this relation holds even if the strength of envy $\alpha$ is different across the two settings,[5] as long as $\alpha$ in each setting is not too small — specifically, as long as $\alpha \geq (1-p)q/(p(1-q))$ in the bilateral case and $\alpha \geq (1-p)q/p$ in the unilateral case.

**Proposition 4.4.** *Suppose $\alpha \geq (1-p)q/(p(1-q))$ when agents are bilaterally envious, and $\alpha \geq (1-p)q/p$ when agents are unilaterally envious. The principal is better off when she employs bilaterally envious agents than unilaterally envious ones.*

Proposition 4.4 implies that there exists a cost of age diversity. The principal should fill two comparable positions with two agents who are around the same age rather than age-diverse agents. When envy exists bilaterally, the principal can exploit the two agents' sense of envy. On the other hand, when envy exists unilaterally, the principal can do so only for the old agent.

[5]It is natural to assume $\alpha$ is different across the two settings. In psychology, the strength of envy is known to be affected by whom people compare themselves with (Smith and Kim 2007).

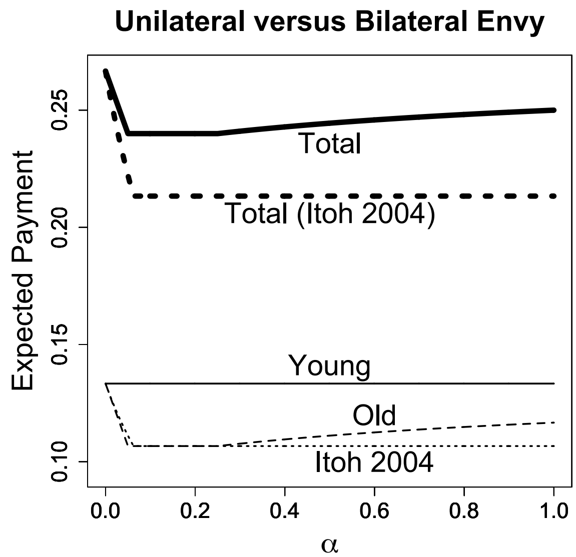


Figure 4.3: Comparing Unilateral and Bilateral Envy

Figure 4.3 shows a numerical example. In this example, I use the same set of numerical parameters as in Figure 4.2. Figure 4.3 is constructed based on the graph on the left in Figure 4.2, by adding lines showing expected payments of the contract when envy exists bilaterally as in Itoh (2004, Section 4). The dotted lines show the expected cost of the contract when envy exists bilaterally. The thicker line shows the total expected payment to the two agents, and the thinner one shows the expected payment to each agent.

## 4.4 Conclusions

This paper studies the optimal contract for young and old agents. I assume that the old agent can be envious of the young agent, but not vice versa. I show that 1) the principal optimally causes pay inversion when agents are unilaterally envious, and 2) the principal prefers bilaterally envious agents to unilaterally envious ones. The first result implies that there are circumstances in which a firm should cause pay inversion rather than increase wages based on seniority. The second result implies that an opportunity cost exists when a firm assigns comparable positions to age-diverse workers rather than workers similar in age.

My analysis imposes a number of simplifying assumptions. Most importantly, it studies a single-period contract. In reality, however, the principal may want to reward an agent's long-term contribution to the firm. Future research can extend my model to analyze a long-term contract for unilaterally envious agents.

## 4.5 Appendix

### Proof of Lemma 4.1

In this section, I prove that I can assume $w^y_{bg} = w^y_{bb} = w^o_{bg} = w^y_{bb} = 0$ without loss of generality. I prove this fact with the following steps. First, I remove as many max operators as possible from (PCo*) and (ICo*). Specifically, I show that I can assume $w^y_{gb} \geq w^o_{bg}$; $w^o_{bb} \geq w^y_{bb}$; and $w^o_{gb} \geq w^y_{bg}$. Second, I show in turn that $w^o_{bb}$, $w^y_{bb}$. $w^o_{bg}$, and $w^y_{bg}$ can be assumed to be zero.

In Lemma 4.2 to Lemma 4.4, I show that I can assume $w^y_{gb} \geq w^o_{bg}$; $w^o_{bb} \geq w^y_{bb}$; and $w^o_{gb} \geq w^y_{bg}$ without loss of generality.

**Lemma 4.2.** *If there is an optimal contract ($w^o$, $w^y$) that satisfies $w^y_{gb} < w^o_{bg}$, there exists a contract ($\hat{w}^o$, $\hat{w}^y$) that satisfies $\hat{w}^y_{gb} \geq \hat{w}^o_{bg}$ and gives the principal the same expected payoff as the contract ($w^o$, $w^y$).*

*Proof.* Suppose the contract ($w^o$, $w^y$) satisfies $w^y_{gb} < w^o_{bg}$. I consider two cases: $w^y_{gg} \geq w^o_{gg}$ and $w^y_{gg} \leq w^o_{gg}$.

When $w^y_{gg} > w^o_{gg}$, I consider a new contract ($\hat{w}^o$, $\hat{w}^y$) that is identical to ($w^o$, $w^y$) except:

$$\hat{w}^y_{gb} = w^y_{gb} + \frac{d}{p(1-p)}$$
$$\hat{w}^y_{gg} = w^y_{gg} - \frac{d}{p^2},$$

where $d > 0$ can be any positive number as long as the new contract ($\hat{w}^o$, $\hat{w}^y$) satisfies $\hat{w}^y_{gb} \leq \hat{w}^o_{bg} = w^o_{bg}$ and $\hat{w}^y_{gg} \geq \hat{w}^o_{gg} = w^o_{gg}$. In words, I increase $w^y_{gb}$ and decrease $w^y_{gg}$ without changing the order of the elements of ($w^o$, $w^y$). When the principal deviates from ($w^o$, $w^y$) to ($\hat{w}^o$, $\hat{w}^y$), her expected cost (Obj) is the same. The left-hand side of (PCo*) increases by $\alpha d$. The left-hand side of (ICo*) increases by $\alpha d$, the right-hand side by $(q/p)\alpha d$, so the new contract ($\hat{w}^o$, $\hat{w}^y$) satisfies (ICo*). Finally, (ICy*) is unaffected. I can set the value of $d$ so that the contract ($\hat{w}^o$, $\hat{w}^y$) satisfies either $\hat{w}^y_{gb} = \hat{w}^o_{bg}$ or $\hat{w}^y_{gg} = \hat{w}^o_{gg}$. If I achieve $\hat{w}^y_{gb} = \hat{w}^o_{bg}$, I get $\hat{w}^y_{gb} \geq \hat{w}^o_{bg}$. If I achieve $\hat{w}^y_{gg} = \hat{w}^o_{gg}$ but not $\hat{w}^y_{gb} = \hat{w}^o_{bg}$, I move on to the procedure for the case when $w^y_{gg} \leq w^o_{gg}$.

When $w^y_{gg} \leq w^o_{gg}$, I consider a new contract ($\hat{w}^o$, $\hat{w}^y$) that is identical to ($w^o$, $w^y$) except:

$$\hat{w}^o_{bg} = w^o_{bg} - \frac{d}{p(1-p)}$$
$$\hat{w}^o_{gg} = w^o_{gg} + \frac{d}{p^2},$$

where $d > 0$ can be any positive number as long as the new contract ($\hat{w}^o$, $\hat{w}^y$) satisfies $\hat{w}^y_{gb} = w^y_{gb} \leq \hat{w}^o_{bg}$ (notice that $\hat{w}^y_{gg} = w^y_{gg} \leq \hat{w}^o_{gg}$ always holds). In words, I decrease $w^o_{bg}$ and increase $w^o_{gg}$ without changing the order of the elements of ($w^o$, $w^y$). When the principal deviates from ($w^o$, $w^y$) to ($\hat{w}^o$, $\hat{w}^y$), her expected cost (Obj) is the same. The left-hand side of (PCo*) is unaffected. The right-hand side of (ICo*) decreases by $d(p-q)/(p(1-p))$, so the new contract ($\hat{w}^o$, $\hat{w}^y$) satisfies (ICo*). Finally, (ICy*) is unaffected. The principal can set the value of $d$ so that the contract ($\hat{w}^o$, $\hat{w}^y$) satisfies $\hat{w}^y_{gb} = \hat{w}^o_{bg}$. □

**Lemma 4.3.** *If there is an optimal contract ($w^o$, $w^y$) that satisfies $w^o_{bb} < w^y_{bb}$, there exists a contract ($\hat{w}^o$, $\hat{w}^y$) that satisfies $\hat{w}^o_{bb} \geq \hat{w}^y_{bb}$ and gives the principal the same expected payoff as the contract ($w^o$, $w^y$).*

*Proof.* Suppose the contract ($w^o$, $w^y$) satisfies $w^o_{bb} < w^y_{bb}$. I consider a new contract ($\hat{w}^o$, $\hat{w}^y$) that is identical to ($w^o$, $w^y$) except:

$$\hat{w}^y_{gb} = w^y_{gb} + \frac{d}{p(1-p)}$$
$$\hat{w}^y_{bb} = w^y_{bb} - \frac{d}{(1-p)^2},$$

where $d > 0$ can be any positive number as long as the new contract ($\hat{w}^o$, $\hat{w}^y$) satisfies $\hat{w}^o_{bb} = w^o_{bb} \leq \hat{w}^y_{bb}$. When the principal deviates from ($w^o$, $w^y$) to ($\hat{w}^o$, $\hat{w}^y$), her expected cost (Obj) is the same. The left-hand side of (PCo*) is unaffected. The right-hand side of (ICo*) is unaffected, so the new contract ($\hat{w}^o$, $\hat{w}^y$) satisfies (ICo*). Finally, (ICy*) is satisfied. The principal can set the value of $d$ so that the contract ($\hat{w}^o$, $\hat{w}^y$) satisfies $\hat{w}^o_{bb} = \hat{w}^y_{bb}$. □

**Lemma 4.4.** *If there is an optimal contract ($w^o$, $w^y$) that satisfies $w^o_{gb} < w^y_{bg}$, there exists a contract ($\hat{w}^o$, $\hat{w}^y$) that satisfies $\hat{w}^o_{gb} \geq \hat{w}^y_{bg}$ and gives the principal the same expected payoff as the contract ($w^o$, $w^y$).*

*Proof.* Suppose the contract ($w^o$, $w^y$) satisfies $w^o_{gb} < w^y_{bg}$. I consider a new contract ($\hat{w}^o$, $\hat{w}^y$) that is identical to ($w^o$, $w^y$) except:

$$\hat{w}^y_{gb} = w^y_{gb} + \frac{d}{p(1-p)}$$
$$\hat{w}^y_{bg} = w^y_{bg} - \frac{d}{p(1-p)},$$

where $d > 0$ can be any positive number as long as the new contract ($\hat{w}^o$, $\hat{w}^y$) satisfies $\hat{w}^o_{gb} = w^o_{gb} \leq \hat{w}^y_{bg}$. When the principal deviates from ($w^o$, $w^y$) to ($\hat{w}^o$, $\hat{w}^y$), her expected cost (Obj) is the same. The left-hand side of (PCo*) is unaffected. The right-hand side of (ICo*) decreases by $\alpha d(p-q)/(p(1-p))$, so the new contract ($\hat{w}^o$, $\hat{w}^y$) satisfies (ICo*). Finally, (ICy*) is satisfied.

The principal can set the value of $d$ so that the contract ($\hat{w}^o$, $\hat{w}^y$) satisfies $\hat{w}^o_{gb} = \hat{w}^y_{bg}$. □

From Lemma 4.2 to Lemma 4.4, I can rewrite (PCo*) and (ICo*) as:

$$\begin{aligned} &p^2[w^o_{gg} - \alpha \max(0, w^y_{gg} - w^o_{gg})] + p(1-p)w^o_{gb} \\ &+ p(1-p)[(1+\alpha)w^o_{bg} - \alpha w^y_{gb})] + (1-p)^2 w^o_{bb} - c \geq 0, \end{aligned} \tag{PCo**}$$

$$\begin{aligned} \text{LHS of (PCo*)} \geq\ & qp[w^o_{gg} - \alpha \max(0, w^y_{gg} - w^o_{gg})] + q(1-p)w^o_{gb} \\ &+ (1-q)p[(1+\alpha)w^o_{bg} - \alpha w^y_{gb}] + (1-q)(1-p)w^o_{bb}. \end{aligned} \tag{ICo**}$$

I show in turn that $w^o_{bb}$, $w^y_{bb}$. $w^o_{bg}$, and $w^y_{bg}$ can be assumed to be zero in Lemma 4.5 through Lemma 4.7.

**Lemma 4.5.** *If there is an optimal contract ($w^o$, $w^y$) that satisfies $w^o_{bb} > 0$ or $w^y_{bb} > 0$, there exists a contract ($\hat{w}^o$, $\hat{w}^y$) that satisfies $\hat{w}^o_{bb} = \hat{w}^y_{bb} = 0$ and gives the principal the same expected payoff as the contract ($w^o$, $w^y$).*

*Proof.* Suppose at least one of $\hat{w}^o_{bb}$ and $\hat{w}^y_{bb}$ is positive in the contract ($w^o$, $w^y$). I consider a new contract ($\hat{w}^o$, $\hat{w}^y$) that is identical to ($w^o$, $w^y$) except:

$$\hat{w}^o_{bb} = 0$$
$$\hat{w}^o_{gb} = w^o_{gb} + \frac{1-p}{p} w^o_{bb}$$
$$\hat{w}^y_{bb} = 0$$
$$\hat{w}^y_{bg} = w^y_{bg} + \frac{1-p}{p} w^y_{bb}.$$

When the principal deviates from ($w^o$, $w^y$) to ($\hat{w}^o$, $\hat{w}^y$), her expected cost (Obj) is the same. All the constraints continue to hold. □

**Lemma 4.6.** *If there is an optimal contract ($w^o$, $w^y$) that satisfies $w^o_{bg} > 0$, there exists a contract ($\hat{w}^o$, $\hat{w}^y$) that satisfies $\hat{w}^o_{bg} = 0$ and gives the principal the same expected payoff as the contract ($w^o$, $w^y$).*

*Proof.* Suppose the contract ($w^o$, $w^y$) satisfies $w^o_{bg} > 0$. I consider two cases: $w^y_{gg} > w^o_{gg}$ and $w^y_{gg} \leq w^o_{gg}$.

When $w^y_{gg} > w^o_{gg}$, I consider a new contract ($\hat{w}^o$, $\hat{w}^y$) that is identical to ($w^o$, $w^y$) except:

$$\hat{w}^o_{bg} = w^o_{bg} - \frac{d}{p(1-p)}$$
$$\hat{w}^o_{gg} = w^o_{gg} + \frac{d}{p^2},$$

where $d > 0$ can be any positive number as long as the new contract ($\hat{w}^o$, $\hat{w}^y$) satisfies $\hat{w}^y_{gg} \geq \hat{w}^o_{gg}$. When the principal deviates from ($w^o$, $w^y$) to ($\hat{w}^o$, $\hat{w}^y$), her expected cost (Obj) is the same. The left-hand side of (PCo*) is unaffected. The right-hand side of (ICo*) decreases by $(1+\alpha)d(p-q)/(p(1-p))$, so the new contract ($\hat{w}^o$, $\hat{w}^y$) satisfies (ICo*). Finally, (ICy*) is unaffected. The principal can set the value of $d$ so that the contract ($\hat{w}^o$, $\hat{w}^y$) satisfies either $\hat{w}^o_{bg} = 0$ or $\hat{w}^y_{gg} = \hat{w}^o_{gg}$. If I achieve $\hat{w}^y_{gg} = \hat{w}^o_{gg}$ but not $\hat{w}^o_{bg} = 0$, I move on to the procedure for the case when $w^y_{gg} \leq w^o_{gg}$.

When $w^y_{gg} \leq w^o_{gg}$, I consider a new contract ($\hat{w}^o$, $\hat{w}^y$) that is identical to ($w^o$, $w^y$) except:

$$\begin{aligned}
\hat{w}^o_{bg} &= w^o_{bg} - \frac{d}{p(1-p)} \\
\hat{w}^y_{gb} &= w^y_{gb} - \frac{d}{p(1-p)}, \\
\hat{w}^o_{gg} &= w^o_{gg} + \frac{d}{p^2} \\
\hat{w}^y_{gg} &= w^y_{gg} + \frac{d}{p^2},
\end{aligned}$$

where $d > 0$ can be any positive number as long as the new contract ($\hat{w}^o$, $\hat{w}^y$) satisfies $\hat{w}^o_{bg} \geq 0$ (in this case, it always satisfies $\hat{w}^y_{gb} \geq 0$ by Lemma 4.2). When the principal deviates from ($w^o$, $w^y$) to ($\hat{w}^o$, $\hat{w}^y$), her expected cost (Obj) is the same. The left-hand side of (PCo*) is unaffected. The right-hand side of (ICo*) decreases by $d(p-q)/(p(1-p))$, so the new contract ($\hat{w}^o$, $\hat{w}^y$) satisfies (ICo*). Finally, (ICy*) is unaffected. The principal can set the value of $d$ so that the contract ($\hat{w}^o$, $\hat{w}^y$) satisfies $\hat{w}^o_{bg} = 0$. □

**Lemma 4.7.** *If there is an optimal contract ($w^o$, $w^y$) that satisfies $w^y_{bg} > 0$, there exists a contract ($\hat{w}^o$, $\hat{w}^y$) that satisfies $\hat{w}^y_{bg} = 0$ and gives the principal the same expected payoff as the contract ($w^o$, $w^y$).*

*Proof.* Suppose the contract ($w^o$, $w^y$) satisfies $w^y_{bg} > 0$. I consider two cases: $w^y_{gg} > w^o_{gg}$ and $w^y_{gg} \leq w^o_{gg}$.

When $w^y_{gg} > w^o_{gg}$, I consider a new contract ($\hat{w}^o$, $\hat{w}^y$) that is identical to ($w^o$, $w^y$) except:

$$\begin{aligned}
\hat{w}^y_{bg} &= w^y_{bg} - \frac{d}{p(1-p)} \\
\hat{w}^y_{gb} &= w^y_{gb} + \frac{d}{p(1-p)}, \\
\hat{w}^o_{gg} &= w^o_{gg} + \frac{d}{p^2} \\
\hat{w}^o_{gb} &= w^o_{gb} - \frac{d}{p(1-p)},
\end{aligned}$$

where $d > 0$ can be any positive number as long as the new contract ($\hat{w}^o$, $\hat{w}^y$) satisfies $\hat{w}^y_{gg} \geq \hat{w}^o_{gg}$ and $\hat{w}^y_{bg} \geq 0$ (in this case, it satisfies $\hat{w}^o_{gb} \geq 0$ by Lemma 4.4). When the principal deviates from ($w^o$, $w^y$) to ($\hat{w}^o$, $\hat{w}^y$), her expected cost (Obj) is the same. The left-hand side of (PCo*) is unaffected. The right-hand side of (ICo*) decreases by $\alpha d(p-q)/(p(1-p))$, so the new contract ($\hat{w}^o$, $\hat{w}^y$) satisfies (ICo*). Finally, (ICy*) is satisfied. The principal can set the value of $d$ so that the contract ($\hat{w}^o$, $\hat{w}^y$) satisfies either $\hat{w}^y_{gg} = \hat{w}^o_{gg}$ or $\hat{w}^y_{bg} = 0$. If I achieve $\hat{w}^y_{gg} = \hat{w}^o_{gg}$ but not $\hat{w}^y_{bg} = 0$, I move on to the procedure for the case when $w^y_{gg} \leq w^o_{gg}$.

When $w^y_{gg} \leq w^o_{gg}$, I consider a new contract ($\hat{w}^o$, $\hat{w}^y$) that is identical to ($w^o$, $w^y$) except:

$$\begin{aligned}
\hat{w}^y_{bg} &= w^y_{bg} - \frac{d}{p(1-p)} \\
\hat{w}^o_{gb} &= w^o_{gb} - \frac{d}{p(1-p)}, \\
\hat{w}^y_{gg} &= w^y_{gg} + \frac{d}{p^2} \\
\hat{w}^o_{gg} &= w^o_{gg} + \frac{d}{p^2},
\end{aligned}$$

where $d > 0$ can be any positive number as long as $\hat{w}^y_{bg} \geq 0$. When the principal deviates from ($w^o$, $w^y$) to ($\hat{w}^o$, $\hat{w}^y$), her expected cost (Obj) is the same. The left-hand side of (PCo*) is unaffected. The right-hand side of (ICo*) is also unaffected, so the new contract ($\hat{w}^o$, $\hat{w}^y$) satisfies (ICo*). Finally, (ICy*) is satisfied. The principal can set the value of $d$ so that the contract ($\hat{w}^o$, $\hat{w}^y$) satisfies $\hat{w}^y_{bg} = 0$. □

## Proof of Proposition 4.1 through Proposition 4.3

I derive the optimal contract by the Kuhn-Tucker approach. I consider two cases, $w^o_{gg} \geq w^y_{gg}$ and $w^o_{gg} \leq w^y_{gg}$.

Recall that (4.4) and (4.5) are easier to satisfy than are (4.6) and (4.7), respectively. I show that the principal can minimize (Obj) given (4.4) and (4.5) with slack $w^o_{gg} \geq w^y_{gg}$ when $\alpha < (1-p)/p$.

**The case when $w^o_{gg} \geq w^y_{gg}$**

When $w^o_{gg} \geq w^y_{gg}$, the constraints are (ICy*), (4.4), and (4.5). I can write the Lagrangian as:

$$\begin{aligned}\mathcal{L} = &- p^2(w^o_{gg} + w^y_{gg}) - p(1-p)(w^o_{gb} + w^y_{gb}) \\ &+ \lambda^y_{IC}[p(p-q)w^y_{gg} + (p-q)(1-p)w^y_{gb} - c] \\ &+ \lambda^o_{IC}[p(p-q)w^o_{gg} + (p-q)(1-p)w^o_{gb} + \alpha p(p-q)w^y_{gb} - c] \\ &+ \lambda^o_{PC}[p^2 w^o_{gg} + p(1-p)w^o_{gb} - \alpha p(1-p)w^y_{gb} - c],\end{aligned} \tag{4.16}$$

where $\lambda^i_{IC}$ and $\lambda^i_{PC}$ are the Lagrange multipliers of the incentive constraint and the participation constraint, respectively, of an agent $i = y, o$. The Kuhn-Tucker conditions are:

$$\begin{aligned}&\frac{\partial \mathcal{L}}{\partial w^o_{gg}} \leq 0;\ w^o_{gg} \geq 0;\ \frac{\partial \mathcal{L}}{\partial w^o_{gg}} w^o_{gg} = 0 \\ &\frac{\partial \mathcal{L}}{\partial w^o_{gb}} \leq 0;\ w^o_{gb} \geq 0;\ \frac{\partial \mathcal{L}}{\partial w^o_{gb}} w^o_{gb} = 0 \\ &\frac{\partial \mathcal{L}}{\partial w^y_{gg}} \leq 0;\ w^y_{gg} \geq 0;\ \frac{\partial \mathcal{L}}{\partial w^y_{gg}} w^y_{gg} = 0 \\ &\frac{\partial \mathcal{L}}{\partial w^y_{gb}} \leq 0;\ w^y_{gb} \geq 0;\ \frac{\partial \mathcal{L}}{\partial w^y_{gb}} w^y_{gb} = 0 \\ &\frac{\partial \mathcal{L}}{\partial \lambda^y_{IC}} \geq 0;\ \lambda^y_{IC} \geq 0;\ \frac{\partial \mathcal{L}}{\partial \lambda^y_{IC}} \lambda^y_{IC} = 0 \\ &\frac{\partial \mathcal{L}}{\partial \lambda^o_{IC}} \geq 0;\ \lambda^o_{IC} \geq 0;\ \frac{\partial \mathcal{L}}{\partial \lambda^o_{IC}} \lambda^o_{IC} = 0 \\ &\frac{\partial \mathcal{L}}{\partial \lambda^o_{PC}} \geq 0;\ \lambda^o_{PC} \geq 0;\ \frac{\partial \mathcal{L}}{\partial \lambda^o_{PC}} \lambda^o_{PC} = 0.\end{aligned} \tag{KTC}$$

I first consider the case when (PCo*) is slack. Among three constraints, (PCo*) is the only constraint that can be redundant. From (KTC), I can derive:

$$\begin{aligned}\lambda^y_{IC} &= \frac{p - p^2 - \alpha p^2}{(1-p)(p-q)} \\ \lambda^o_{IC} &= \frac{p}{p-q} \\ w^y_{gg} &= 0 \\ w^y_{gb} &= \frac{c}{(1-p)(p-q)}\end{aligned} \tag{4.17}$$

Further, $w_{gg}^o$ and $w_{gb}^o$ satisfy:

$$p(p-q)w_{gg}^o + (p-q)(1-p)w_{gb}^o = c(1 - \frac{\alpha p}{1-p}).$$

I note that (4.4) is slack and $\lambda_{IC}^y > 0$. The slack inequality (4.4) is equivalent to $\alpha \leq q(1-p)/p$, and $\lambda_{IC}^y > 0$ is equivalent to $\alpha \leq (1-p)/p$. Hence, this contract arises if, and only if, $\alpha \leq q(1-p)/p$. I also note that the principal can construct an optimal contract so that $w_{gg}^o \geq w_{gg}^y$ is slack.

I now turn to the case when all the constraints (ICy*), (PCo*), and (ICo*) are binding. From (KTC), I can derive:

$$\begin{aligned} \lambda_{IC}^y &= \frac{p}{p-q} \\ \lambda_{IC}^o &= \frac{p(1-p)}{p-q} \\ \lambda_{PC}^o &= p \\ w_{gg}^y &= \frac{\alpha p - (1-p)q}{\alpha p^2 (p-q)} c \\ w_{gb}^y &= \frac{qc}{\alpha p(p-q)}. \end{aligned} \tag{4.18}$$

Further, $w_{gg}^o$ and $w_{gb}^o$ satisfy $w_{gg}^o \geq w_{gg}^y$ and:

$$p(p-q)w_{gg}^o + (p-q)(1-p)w_{gb}^o = c(1-q). \tag{4.19}$$

This contract is optimal if, and only if, $q(1-p)/p \leq \alpha < (1-p)/p$. The lower bound $q(1-p)/p$ can be derived by noting $w_{gg}^y \geq 0$. The upper bound is $(1-p)/p$ because $w_{gg}^o \geq w_{gg}^y$ and (4.19) imply $w_{gg}^y \leq (1-q)c/(p(p-q))$, but $w_{gg}^y$ is increasing in $\alpha$ and is $(1-q)c/(p(p-q))$ when $\alpha = (1-p)/p$. The principal can find an optimal contract with slack $w_{gg}^o \geq w_{gg}^y$ when $\alpha < (1-p)/p$.

### The case when $w_{gg}^o \leq w_{gg}^y$

When the principal cannot find an optimal contract by assuming $w_{gg}^o \geq w_{gg}^y$, she chooses one by assuming $w_{gg}^o \leq w_{gg}^y$. When $w_{gg}^o \leq w_{gg}^y$, the constraints are (ICy*), (4.6), and (4.7). I can write the

Lagrangian as:

$$\begin{aligned}\mathcal{L} = &- p^2(w^o_{gg} + w^y_{gg}) - p(1-p)(w^o_{gb} + w^y_{gb}) \\ &+ \lambda^y_{IC}[p(p-q)w^y_{gg} + (p-q)(1-p)w^y_{gb} - c] \\ &+ \lambda^o_{IC}[(1+\alpha)p(p-q)w^o_{gg} + (p-q)(1-p)w^o_{gb} \\ &\qquad - \alpha p(p-q)w^y_{gg} + \alpha p(p-q)w^y_{gb} - c] \\ &+ \lambda^o_{PC}[(1+\alpha)p^2 w^o_{gg} + p(1-p)w^o_{gb} \\ &\qquad - \alpha p^2 w^y_{gg} - \alpha p(1-p)w^y_{gb} - c].\end{aligned} \tag{4.20}$$

The Kuhn-Tucker conditions are the same as (KTC). I can derive:

$$\begin{aligned}&\lambda^y_{IC} = \frac{1}{p-q}\Big[\frac{\alpha p}{1+\alpha} + p\Big] \\ &\lambda^o_{IC} = 0 \\ &\lambda^o_{PC} = \frac{1}{1+\alpha} \\ &w^o_{gg} = \frac{c(\alpha p + p - q)}{(1+\alpha)p^2(p-q)} \\ &w^o_{gb} = 0.\end{aligned} \tag{4.21}$$

Further, $w^y_{gg}$ and $w^y_{gb}$ satisfy $w^o_{gg} \le w^y_{gg}$ and:

$$\begin{aligned}&w^y_{gb} \ge w^y_{gg} - \frac{\left(\alpha - \frac{q}{p}\right)c}{p(p-q)\alpha}, \\ &p(p-q)w^y_{gg} + (p-q)(1-p)w^y_{gb} = c.\end{aligned}$$

I note that the contract in Proposition 4.2 also satisfies these conditions when $\alpha = (1-p)/p$. In other words, on the threshold $\alpha = (1-p)/p$, the contract in Proposition 4.2 is an example of the non-unique contract in Proposition 4.3.

## Proof of Proposition 4.4

I restate the optimal contract in Itoh (2004, Section 4) using the notation in my model. When $\alpha \geq (1-p)q/(p(1-q))$, bilaterally envious agents are offered the following contract:

$$\begin{aligned} w_{gg} &= \frac{c(1-p)q}{(p-q)p^2}\Big[\frac{p(1-q)}{(1-p)q} - \frac{1}{\alpha}\Big], \\ w_{gb} &= \frac{qc}{\alpha p(p-q)}, \\ w_{bg} &= w_{bb} = 0. \end{aligned} \tag{4.22}$$

The expected cost of the contract (4.22) for the two agents is:

$$2p^2 w_{gg} + 2p(1-p)w_{gb} = \frac{2p(1-q)c}{p-q}. \tag{4.23}$$

I now calculate the expected payment for unilaterally envious agents. When $(1-p)q/p \leq \alpha \leq (1-p)/p$ as in Proposition 4.2, it is:

$$p^2 w^y_{gg} + p(1-p)w^y_{gb} + p^2 w^o_{gg} + p(1-p)w^o_{gb} = \frac{(2-q)pc}{p-q}. \tag{4.24}$$

When $\alpha \geq (1-p)/p$ as in Proposition 4.3, it is:

$$p^2 w^y_{gg} + p(1-p)w^y_{gb} + p^2 w^o_{gg} = \frac{c}{p-q}(2p - \frac{q}{1+\alpha}). \tag{4.25}$$

Straightforward calculations show (4.23) is smaller than (4.24) and (4.25) even if $\alpha$ is different across the two models.

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