

A Model for the Frequency of Long Periods of Drought at Forested Stations in Canada

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ABSTRACT

Long dry spells (sequences of dry days) are rare events, but they are important because they correlate significantly with the area burned during bad wildfire years. Previous attempts to model the frequency of dry spells have been successful for spells of short duration, but have failed for prolonged dry spells.

In the current study, an empirical method has been developed that yields a realistic estimate of the probability of a spell of any duration. The theoretical framework proposes that the data can be explained partly by the dichotomy of weather into blocked and nonblocked westerly flows. A bimodal distribution of dry consecutive days is a consequence of this dichotomy.

The transitional probability of a dry day following k dry days generally peaks at $k = 1$, declines to a shoulder for small k values, and then rises slowly to an asymptotic value that must be estimated from sparse and highly irregular data.

1. Introduction

During a study of the relationship between various meteorological variables and the area burned by wildfire on a monthly basis by province (Flannigan and Harrington 1988), it was discovered that the area burned correlated significantly with the duration of dry spells, but did not correlate with precipitation. Consequently, a statistical analysis of the duration of dry spells at forested stations in Canada was carried out in an attempt to estimate the return period of long dry spells indicative of "bad" fire years.

The literature on the probability of sequences of rain days or dry days is voluminous. A brief review begins here with the pioneering work of Newnham (1916), who showed that the probabilities of rain or no-rain days were not constants, but depended on the condition on the previous day. For example, the transitional probability of rain after a dry day at Aberdeen, Scotland, was 0.50, but increased to 0.67 if the previous day had been wet. After two wet days at Aberdeen, the probability of rain rose to 0.70 and after three wet days it was 0.76, remaining constant thereafter. Probabilities that take the preceding weather into account are called transitional probabilities. For Aberdeen, the transitional probabilities can be written as $p(r|d) = 0.50$, $p(r|rd) = 0.67$, $p(r|rrd) = 0.70$, and $p(r|rr \dots rd) = 0.76$.

A number of models have been devised in an attempt to predict the occurrence of rare events. Eggenberger

and Polya (1923) modified the Poisson series in a manner that allowed the mean to increase slowly with dry spell duration. Their model succeeded in fitting rare events, but failed to provide a good fit to the data when the sequences were not rare events.

Williams (1947) found that a logarithmic series conformed closely to the distribution of dry-day sequences, but not to wet-day sequences. However, his curves show that even for dry-day sequences, the fit becomes poor when the sequence becomes long.

Markov chain¹ models were first used to estimate the distribution of wet- and dry-day sequences by Gabriel and Neumann (1957, 1962). They were able to show that a first-order Markov chain adequately described the distributions of wet- and dry-day sequences during the rainy season in Tel Aviv, Israel, but failed for the dry-day sequences during the summer dry season.

Berger and Goossens (1983) compared the adequacy of a number of models in their description of the distribution of dry-day sequences at Uccle, Belgium. The models tested were the modified Poisson series of Eggenberger and Polya (1923), the logarithmic series of Williams (1947), a geometric series used by Neumann (1955), a Markov chain model described by Eriksson (1965), and a modified Markov chain model derived by Dingens et al. (1970). Berger and Goossens (1983) found that the model of Dingens et al. (1970) provided

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¹ A first-order Markov chain is one in which the probability of the occurrence of a dry day, for example, is dependent only upon whether the previous day was wet or dry. A second-order Markov chain is one in which the two previous days must be considered, and so forth.

the best fit, but did not significantly improve upon the simple Markov chain model of Eriksson (1965). Goossens and Berger (1984) determined that the order of the Markov chain providing the best fit to data from 15 stations in Belgium lay between three and five, depending on the location of the station.

A similar method was applied by Chin (1977) to data from climatological stations across the United States. He divided the year into winter and summer seasons and found that the best fit to the data was obtained with a first-order Markov chain in winter and a second-order Markov chain in summer.

The models of both Eriksson (1965) and Dingsen et al. (1970) were applied by Harrington and Flannigan (1987) to 28 years of data (1953–80), collected during the forest fire season, April to September inclusive, from 41 stations located in forested regions of Canada. The order of the Markov chain providing the best fit to the data is shown in Fig. 1. Although the Markov chain models fit most of the data at a confidence level of greater than 95%, it became apparent that the models were consistently underestimating the transitional probabilities of long periods of drought. This failure to predict the frequency of long droughts, also noted by Gabriel and Neumann (1957), made the simple Markov chain models inappropriate for estimating the likelihood of bad forest fire years.

This paper demonstrates an empirical method for

estimating transitional probabilities for long dry spells and offers a theoretical framework for the method.

2. Preparation of data

Data from the 41 stations shown in Fig. 1 were classified as rain days or dry days depending on whether rainfall amounted to more or less than 1.5 mm, respectively, during the regular 24-h observation period. The 1.5-mm threshold for precipitation had been found to be more significant for fire prediction than the 0.25-mm threshold used by most authors. In forested areas, much of the first 1.5 mm is lost through interception by the trees (Flannigan and Harrington 1988). Use of the 1.5-mm threshold rather than 0.25 mm led to an increased frequency of longer dry-day sequences.

Transitional probabilities of a dry day following k dry days were computed using the equations

$$p(k) = \frac{N(k+1)}{N(k)}, \quad k = 1, 2, \dots \quad (1)$$

and

$$p(0) = P(d|r) = \frac{\text{total dry spells}}{\text{total rain days}}, \quad (2)$$

where $N(k)$ is the cumulative occurrence of all dry spells of k or more days duration. A five-point binomial



FIG. 1. Forested stations in Canada. Numbers indicate the order of the Markov chain providing the best fit to the distribution of transitional probabilities for sequences of dry days.

filter was applied to smooth all of the transitional probabilities with the exception of the first three points.

Data for North Battleford, Saskatchewan, are presented in Table 1 and a typical plot of the transitional

TABLE 1. Observed sequences of exactly k dry days, cumulative frequencies of k or more dry days, and the transitional probability of at least one more dry day during the period from April to September inclusive at North Battleford, Saskatchewan, 1953–80.

Spell duration (k)	Frequency		Transitional probability $p(k)$
	Exact n	Cumulative N	
1	133	674	0.803
2	94	541	0.826
3	72	447	0.839
4	57	375	0.848
5	64	318	0.799
6	42	254	0.835
7	31	212	0.854
8	34	181	0.812
9	17	147	0.884
10	21	130	0.838
11	13	109	0.881
12	15	96	0.844
13	9	81	0.889
14	8	72	0.889
15	7	64	0.891
16	7	57	0.877
17	11	50	0.780
18	5	39	0.872
19	4	34	0.882
20	1	30	0.967
21	2	29	0.931
22	2	27	0.926
23	6	25	0.760
24	3	19	0.842
25	3	16	0.813
26	0	13	1.000
27	0	13	1.000
28	2	13	0.846
29	0	11	1.000
30	2	11	0.818
31	0	9	1.000
32	1	9	0.889
33	0	8	1.000
34	1	8	0.875
35	1	7	0.857
36	1	6	0.833
37	0	5	1.000
38	0	5	1.000
39	0	5	1.000
40	0	5	1.000
41	0	5	1.000
42	0	5	1.000
43	3	5	0.400
44	0	2	1.000
45	0	2	1.000
46	0	2	1.000
47	1	2	0.500
48	1	1	0.000

Total dry spells: 674
 Total rain days: 945
 $p(d|r) = 0.7132$
 Total days: 5124
 Total dry days: 4179

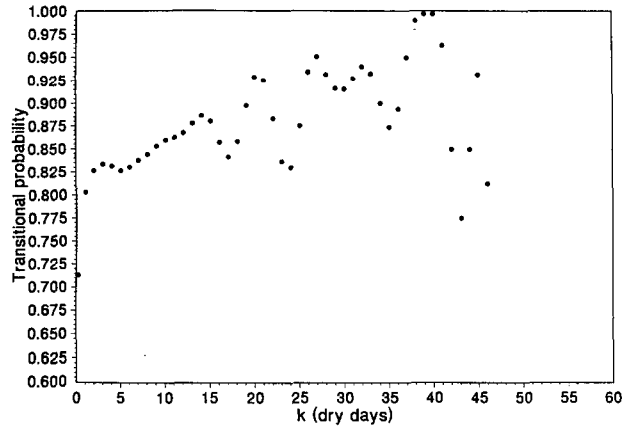


FIG. 2. Transitional probability of a dry day following k dry days at North Battleford, Saskatchewan.

probability of a dry day following k dry days is shown in Fig. 2. The transitional probability of a dry day following a rain day is quite low at 0.7132. After one dry day, it climbs to 0.803 and after two dry days to 0.826, where it reaches a shoulder. Subsequently, it climbs to higher values. As the data become sparse toward longer dry spells, the transitional probabilities become more and more erratic, eventually losing all significance as predictors.

Suppose that a third-order Markov chain was found to be valid for North Battleford dry-day sequences. In that case, the probability of 40 or more dry days following a rain day would be 0.000715, that is, $[p(d|r)p(d|dr)p(d|ddr)p(d|ddd)]^{37}$ from Table 1. During the 28 years of observations, there were 945 rain days and, therefore, a likelihood of 0.67 occurrences if the third-order Markov chain model is accepted. In fact, five occurrences were observed. The general rising trend in transitional probability with sequence length illustrated in Fig. 2 and consequent underestimation of the probability of the occurrence of long sequences was observed in all but 6 of the 41 datasets. Thus, the single Markov chain model was inadequate for the purpose of this study, namely to determine the probability and return period of long sequences of days without rain.

In the following sections, a simple theoretical explanation is offered for the observed rising trend in transitional probability with increasing duration and an empirical curve that provides an adequate fit to the data is developed based on the theory.

3. Theory

To explain the rising trend in the transitional probability of dry-day sequences, it is proposed that the curves are composed of two Markov chains. One chain corresponds roughly to low-amplitude westerly flows of relative frequency f . The other chain corresponds

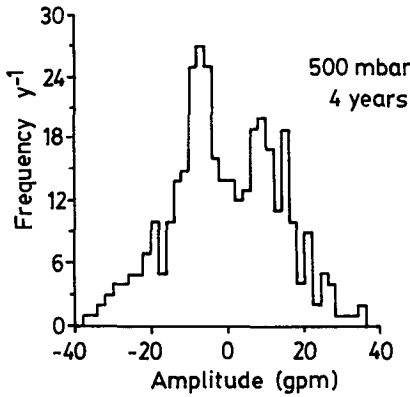


FIG. 3. Frequency distribution of the amplitude of the wave packet (in geopotential meters) formed from very long waves in westerly flows. Redrawn from Hansen (1986).

roughly to high-amplitude ridges in the westerlies or blocking conditions of relative frequency $1 - f$. The fraction f , however, is not to be considered fixed by nature but will be allowed to vary from close to 1.0 for short periods and down toward zero for the longer persistence of dry days. The existence of bimodal flow patterns in the westerlies has been amply demonstrated by Sutera (1986) as shown in Fig. 3.²

Under ideal conditions, that is, when there is a pure bimodal distribution of flow patterns, the probability of a dry day following a rain day can be written as

$$p'(d|r)f + p''(d|r)(1 - f) = \hat{p}(1), \quad (3)$$

where the single prime, double prime, and circumflex denote normal flows, blocking flows, and observations, respectively. Suppose that both Markov chains are third order. In that case, $p'(d|ddd)$ and $p''(d|ddd)$ will be constants throughout the recorded time series. The ratio of the two, $p''(d|ddd)/p'(d|ddd) = \alpha$, should also be a constant. Hence,

$$p''(d|ddd) \left[\frac{f}{\alpha} + 1 - f \right] = \hat{p}(k), \quad k \geq 4. \quad (4)$$

If $\hat{p}(3)$ is measured and the asymptotic value $p''(d|ddd)$ estimated, then the term in square brackets is known and

$$\alpha = \frac{f}{f - \beta} \quad \text{and} \quad f = \frac{\beta\alpha}{\alpha - 1}, \quad (5)$$

where $\beta = 1 - (\delta/\gamma)$, $\delta = \hat{p}(3)$, and $\gamma = p''(d|ddd)$.

Suppose, for example, that δ for North Battleford is estimated to be 0.830 and γ to be 0.925, then β will be 0.1027 and the value of α for various values of f will be as shown in Table 2.

The frequency of normal flows f cannot be constant in this model because if it were, Eq. (4), now used for prediction, would reduce to $p(k) = p'(d|ddd)$, a constant. At this point, we resort to empiricism. Our assumption of a double Markov process forces α to be constant. Therefore, if $\hat{p}(k)$ is to increase with increasing k , f , the fraction of nonblocking flows, must gradually decline, approaching zero as $\hat{p}(k)$ approaches its asymptotic value. It is reasonable to assume that the decline in f is inversely proportional to α and that α must be adjusted by a factor to account for the fact that meteorological circulations are not completely bimodal. Let

$$f = \frac{f_\epsilon}{(\alpha - t)^{k-\epsilon}}, \quad (6)$$

where ϵ is the value of k at the shoulder where measured transitional probabilities are known with a fair degree of confidence, and $\alpha - t$ is arbitrarily adjusted to fit the observations.

The predicted transitional probability that a day will be dry given a preceding sequence of k dry days will then be

$$p(k) = \gamma \left[1 - \frac{\beta}{(\alpha - t)^{k-\epsilon}} \right], \quad (7)$$

where $\beta = 1 - (\delta/\gamma)$ as for Eq. (5) and δ is the transitional probability at the shoulder, where $k = \epsilon$. For values of $k \leq \epsilon$, the observed transitional probabilities were accepted as the best estimates of the true probabilities.

Observed transitional probabilities for dry spells longer than about 10 days are extremely erratic, as illustrated by the North Battleford data in Fig. 2. To smooth the data without introducing artificiality, the transitional probabilities from several similar curves were averaged. Stations were grouped on the basis of having nearly equal transitional probabilities at the

TABLE 2. Values of f and α for North Battleford.

f	α
1.0	1.114
0.8	1.147
0.6	1.207
0.4	1.345
0.2	2.056
0.1027	—

² The very long waves referred to in the caption for Fig. 3 have wavenumbers 2-4, that is, there are two to four waves encircling the globe. When the wave amplitude is high, the winds tend to have strong components from the north or south and storm systems move slowly eastward. When the amplitude is low, the westerly component is stronger and storm systems proceed eastward more rapidly. Over North America, the high pressure ridge aloft is generally centered between the West Coast and the eastern side of the Rocky Mountains. An upper-level trough usually lies over eastern North America. The high-amplitude phase associated with strong ridging or blocking has a characteristic time scale of 10 days.

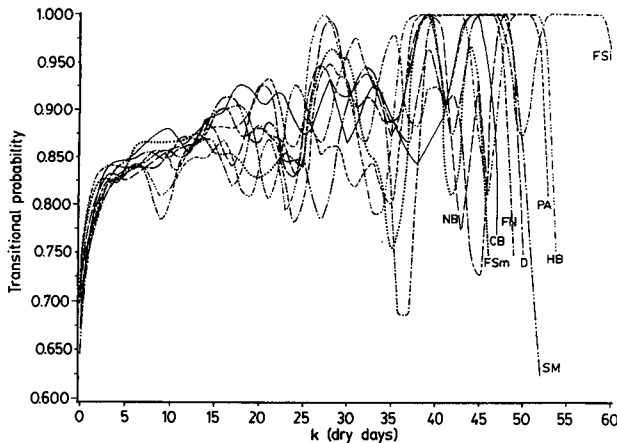


FIG. 4. Transitional probabilities (group 3 curves) for nine stations in western Canada: Fort Nelson (FN), Smithers (SM), Cranbrook (CB), Dawson (D), Fort Simpson (FSi), Prince Albert (PA), Hudson Bay (HB), North Battleford (NB), and Fort Smith (FSm).

shoulder of the curve. These stations also tended to be located in the same general geographic areas and were subject to the same large-scale flow patterns, similar frequencies of blocking, and therefore, similar asymptotic transitional probabilities. Group 3 curves for nine stations in western Canada are shown in Fig. 4. Their values were averaged and smoothed using a five-point binomial filter on all but the first three points (Fig. 5). The upward trend in transitional probability is clearly evident. Values along the curve can reasonably be estimated to about 40 days. The asymptotic value, although obscure, was estimated as 0.925 based on this curve and those of other western Canadian groups.

4. Results and discussion

The data for 41 Canadian stations were divided into seven groups based on the transitional probability at

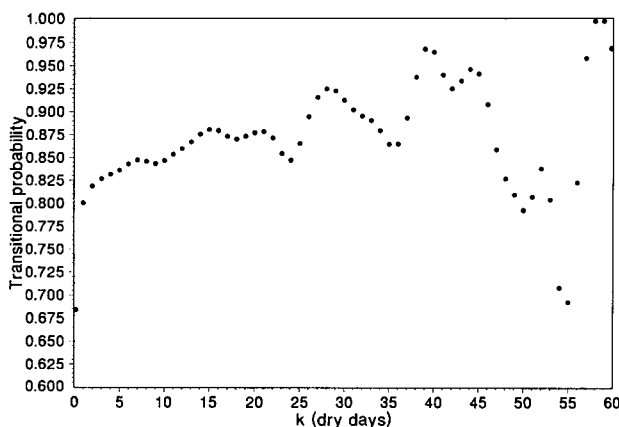


FIG. 5. Average transitional probability for the group 3 curves shown in Fig. 4.

TABLE 3. Canadian stations grouped by the height of the characteristic shoulder.

Group	Location of station
1	Yellowknife, N.W.T.
2	Victoria, B.C.; Williams Lake, B.C.; Whitehorse, Y.T.
3	Fort Nelson, B.C.; Smithers, B.C.; Cranbrook, B.C.; Dawson, Y.T.; Fort Simpson, N.W.T.; Prince Albert, Sask.; Hudson Bay, Sask.; North Battleford, Sask.; Fort Smith, N.W.T.
4	Thompson Man.; Dauphin, Man.; Winnipeg, Man.; Cold Lake, Alta.; Slave Lake, Alta.; Fort McMurray, Alta.; Whitecourt, Alta.; The Pas, Man.; Rocky Mountain House, Alta.
5	Kenora, Ont.; Sioux Lookout, Ont.; Armstrong, Ont.; Earlton, Ont.; Thunder Bay, Ont.; Petawawa, Ont.; Muskoka, Ont.; Timmins, Ont.; Bisset, Man.
6	Goose Bay, Nfld.; Gander, Nfld.; Val d'Or, Que.; Roberval, Que.
7	Lansdowne House, Ont.; Kapuskasing, Ont.; Maniwaki, Que.; Truro, N.S.; Fredericton, N.B.; Charlo, N.B.

the shoulder of each curve. The locations of the groups (Table 3) conform reasonably well with the pattern of rainfall received in July. Yellowknife, by itself in group 1, with less than 50 mm of rainfall, is the driest station, but not much drier than Victoria, Williams Lake, and Whitehorse in group 2. Within the large area covered by group 3, which receives July precipitation ranging from 50 to 75 mm, is group 4, a strip of moister boreal forest receiving more than 75 mm of rainfall. East of Lake Winnipeg, group 5 stations receive July precipitation ranging from 75 to 100 mm, except in the vicinity of Toronto where it is drier. Group 6 and 7 stations are located in northern Ontario, Quebec, and the Atlantic provinces, where precipitation is more reliable and generally exceeds 100 mm. The distribution of transitional probabilities in six of the eastern stations appears to be represented best by a single Markov chain. These are placed in group 7, although their geographical location suggests that they should belong in the same group as the other eastern Canadian stations.

Analyses were performed on the average transitional probability curve for each group. The values of δ , chosen arbitrarily as the value of the transitional probability on the fifth day; γ , the asymptotic value chosen sub-

TABLE 4. Shoulder transitional probabilities δ , asymptotic values γ , and slope factors $\alpha - t$ for the Canadian station groups.

Group	δ	γ	$\alpha - t$
1	0.890	0.925	1.030
2	0.866	0.925	1.070
3	0.836	0.925	1.050
4	0.801	0.925	1.040
5	0.750	0.829	1.090
6	0.715	0.775	1.120
7	0.737	0.737	1.000

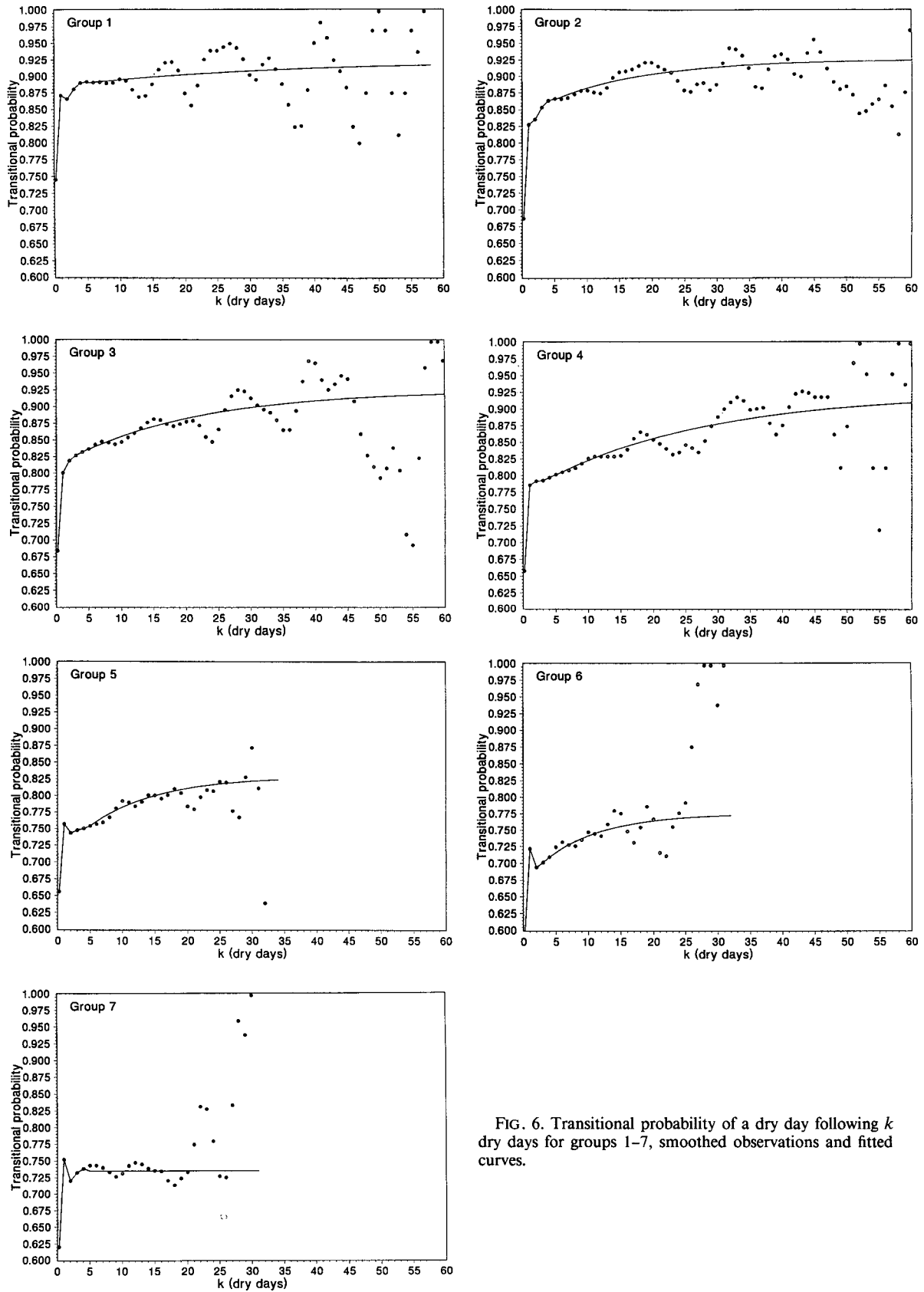


FIG. 6. Transitional probability of a dry day following k dry days for groups 1–7, smoothed observations and fitted curves.

jectively; and $\alpha - t$, adjusted to provide the best eyeball fit to the mean curves are shown in Table 4. Fitted curves for each group are shown in Fig. 6.

The curves in Fig. 6 have several features in common. They start with a relatively low probability at $k = 0$; rise rapidly after one dry day and slowly thereafter; reach a leveling off point, termed a "shoulder"; and then gradually rise to an assumed asymptotic value. Several groups exhibit a spike at $k = 1$. This spike is attributed to the unlikely occurrence of cyclonic storms or low pressure troughs following each other during the summer season with an intervening gap of a single dry day. Only group 7 appears to fit a single Markov chain model.

The probability of a dry day after rain, the transitional probability at the shoulder, and the asymptotic value of long sequences of dry days are all high in the west and declining toward the east. Although all of the curves become highly irregular when data become scarce, there is no consistent evidence of a maximum transitional probability followed by a decline as sequences become long. The transitional probabilities appear to continue increasing toward some unknown, but here estimated, asymptotic value.

Data covering longer periods than the 28 years covered in this study might be expected to provide more dependable estimates of transitional probabilities. Four transitional probability graphs for long-term stations are shown in Fig. 7. Victoria, British Columbia, with 90 years of records, exhibits a transitional probability curve closer to that of Yellowknife (group 1) than that of group 2 to which it originally belonged. The observations do not seriously conflict with an estimated asymptotic value of 0.925. Scott, Saskatchewan, with 70 years of records, is fit closely by the group 3 curve to about 15 years, differs considerably to about 25 years, and then conforms reasonably well toward the end of the curve. Winnipeg, Manitoba, with 110 years of records, is fit reasonably well by the group 4 curve to about 35 years. Finally, Toronto, Ontario, with 140 years of records, is fit reasonably well by the group 5 curve, except near the shoulder where it demonstrates a slightly higher probability of drought. Toronto is, in fact, drier than the other Ontario stations of group 5 (Environment Canada 1986), which would explain the higher frequency of short periods of drought. Toronto is, however, under the same large-scale flow pattern as the other Ontario stations and, therefore, should have

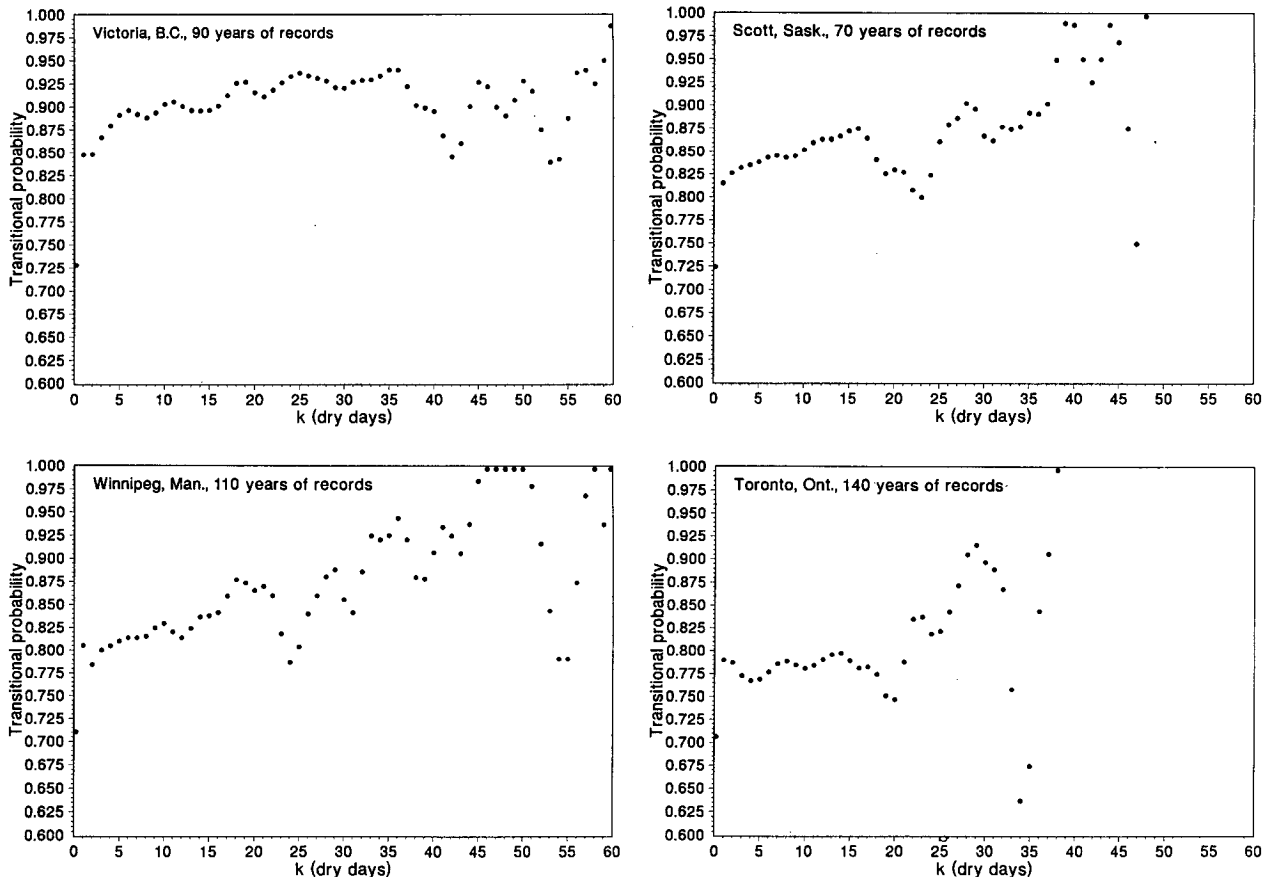


FIG. 7. Transitional probabilities for four long-term stations: Victoria, British Columbia; Scott, Saskatchewan; Winnipeg, Manitoba; and Toronto, Ontario.

TABLE 5. Probability P of n or more dry days following a rain day and return period R (years) for sequences of n or more dry days.

n		Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7
1	P	0.745	0.687	0.684	0.658	0.656	0.594	0.735
	R	0.056	0.049	0.041	0.036	0.031	0.028	0.025
2	P	0.649	0.568	0.547	0.517	0.497	0.428	0.540
	R	0.064	0.059	0.052	0.046	0.041	0.039	0.034
3	P	0.562	0.475	0.448	0.409	0.369	0.297	0.397
	R	0.074	0.071	0.063	0.058	0.055	0.056	0.046
4	P	0.494	0.405	0.370	0.324	0.276	0.208	0.292
	R	0.084	0.083	0.076	0.073	0.073	0.080	0.063
5	P	0.440	0.350	0.308	0.258	0.207	0.148	0.215
	R	0.094	0.096	0.092	0.091	0.098	0.113	0.086
6	P	0.391	0.302	0.258	0.207	0.156	0.106	0.158
	R	0.106	0.111	0.110	0.114	0.129	0.158	0.117
7	P	0.349	0.263	0.216	0.167	0.119	0.076	0.116
	R	0.119	0.128	0.131	0.142	0.170	0.219	0.159
8	P	0.311	0.229	0.183	0.135	0.091	0.056	0.085
	R	0.133	0.147	0.155	0.175	0.222	0.302	0.216
9	P	0.278	0.201	0.155	0.110	0.070	0.041	0.063
	R	0.149	0.168	0.183	0.215	0.287	0.411	0.294
10	P	0.248	0.177	0.132	0.090	0.055	0.030	0.046
	R	0.167	0.191	0.214	0.262	0.368	0.557	0.399
20	P	0.085	0.057	0.032	0.016	0.006	0.002	0.002
	R	0.487	0.593	0.879	1.517	3.542	9.413	8.680
30	P	0.031	0.022	0.010	0.004	0.001	0.000	0.000
	R	1.318	1.549	2.812	6.322	26.734	131.700	188.646
40	P	0.012	0.009	0.004	0.001	0.000	0.000	0.000
	R	3.372	3.692	7.758	21.284	184.596	1731.223	4099.900
50	P	0.005	0.005	0.001	0.000	0.000	0.000	0.000
	R	8.279	8.426	19.489	62.072	1230.601	>10 000	>10 000
60	P	0.002	0.002	0.001	0.000	0.000	0.000	0.000
	R	19.714	18.817	46.355	164.572	8105.232	>10 000	>10 000

the same asymptotic value. The longest dry spell at Toronto was 40 days. This corresponds well with the return period of 184 years calculated for stations in group 5 (Table 5) and would correspond even more closely if the higher shoulder at Toronto was taken into consideration.

Table 5 shows the probability that a sequence of dry days will continue for at least n ($n = k + 1$) days, where $P(n) = P(k)p(k)$ and the return period R in years for sequences of n or more dry days. The abrupt decrease in the probability of long spells of dry weather when crossing from the Prairie provinces into Ontario should be noted. Long dry spells not only provide dry fuel but also ample time for fire ignition, fire spread, and one or more periods of strong wind. It is reasonable to expect that wildfire will be more difficult to control and will burn much larger areas in western Canada than in the east. This is clearly indicated by fire statistics (Harrington 1982).

5. Conclusions

The probability of long periods of dry weather is exceedingly difficult to predict because of the lack of data, even for stations that have been in operation for more than a century. A method is proposed in which data from several stations with similar probability dis-

tributions are combined. From these data, the transitional probability at the shoulder and at the asymptote of the distribution are estimated. The observed curve is approximated by an empirical function. An approximate distribution of the frequency of long sequences of dry days is computed from the fitted curve.

The method described throughout this paper is empirical in nature, hanging loosely on a theoretical framework. The theory calls for a bimodal distribution of weather types leading to a double Markov process. It is acknowledged that reality cannot be described so simply and that strict adherence to such a model must fail. By modifying the model empirically, a fit to the data has been obtained that yields reasonable estimates of the probabilities and return periods of long sequences of dry days.

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