Culturally Responsive Teaching Through the Adinkra Symbols of Ghana and its Impact on Students' Mathematics Proficiency

BY

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ABSTRACT

Ghana's chief examiners, for the core and elective mathematics for the 2016 examinations, suggested that teachers should take the necessary steps to make the teaching of mathematics more practical and related to real-life problems. However, the Ghanaian mathematics curriculum hardly uses real-life or local examples to illustrate mathematics concepts. Educators and researchers of ethnomathematics and culturally responsive pedagogy encourage the inclusion of students' cultural knowledge in their classroom learning. They stress that students' culture has a significant influence on their mathematics learning and using students' cultural background and experiences for mathematics instruction has a positive effect on their learning and their life in general (Bonotto, 2010; Delpit, 1995; Gay, 2010; Sharma & Orey, 2017; Weldeana, 2016). Therefore, this study investigated how the use of culturally responsive teaching through ethnomathematics could impact students' mathematics learning in a classroom setting.

Ethnomathematics (D'Ambrosio, 1990), culturally responsive pedagogy (Gay, 2010), and a framework of mathematics proficiency (Kilpatrick et al., 2001) informed this design-based research study, which involved the participation of five mathematics teachers. In the first phase of the study, the mathematics teachers and I investigated the mathematics concepts in the images and the creation process of the Adinkra symbols (ethnomathematics).

The result of the study of the Adinkra symbols revealed that there are mathematics concepts that could be gleaned from the images and the craftsmen's creation process of the Adinkra symbols. The findings also revealed that the investigation of the Adinkra symbols provided an opportunity for teachers to realize the connections between various mathematics concepts. Again, it enabled mathematics teachers to connect mathematics of the Adinkra

symbols to the school curriculum and the teaching and learning of mathematics. The study of the Adinkra symbols also enabled mathematics teachers to develop classroom teaching strategies that were consistent with culturally responsive pedagogy. That is, this phase of the study increased the mathematics teachers' knowledge base for teaching.

The acquired mathematics knowledge, observed in the Adinkra symbols, was used to design culturally responsive teaching activities for students at the junior and senior high schools in Ghana. Four lessons were designed for the senior high school (SHS) class on the topics: reflection, translation, rotation, and multiple transformations, and four lessons were designed for the junior high (JHS) class on the topics: ratio and proportion, angles, enlargement, and rotation.

To investigate how the use of culturally responsive teaching through ethnomathematics could impact students' learning, the designed activities were implemented in one JHS class (form two or Grade 8) and one SHS class (form two or Grade 11). The implementation of the activities was guided by culturally responsive pedagogy theory. Some of the culturally responsive pedagogy features employed in the study included: the use of the local language for instruction, students working in small cooperative groups, discussions of the meanings, and social values of the Adinkra symbols, using the images of the Adinkra symbols as mediating tools, and using Adinkra symbols as context for problems/tasks.

The strands of mathematics proficiency of Kilpatrick et al. (2001), which define mathematics learning, namely: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition, were used to assess students' learning that occurred during the implementation of the activities designed.

The results revealed that it is viable to implement culturally responsive activities in the classroom using students' cultural artifacts like the Adinkra symbols as mediating tools to promote students' learning. All the five strands of mathematics proficiency were observed to have occurred in the lessons, indicating that the use of ethnomathematics and culturally responsive pedagogy indeed promoted students' mathematics learning. It was observed that the use of the local language and small group work promoted students' engagement and interactions, which made it possible for all the strands of proficiency to emerge. The use of the Adinkra symbols, as context and as mediating tools, provided models for students to relate the concepts to, and that also motivated them to stay on task, as they experienced the application of mathematics in their own culture. Lastly, not only did the students learn mathematics through the Adinkra symbols, but they also learned social and moral values, and some of the social values they learned could be related to the productive disposition strand of Kilpatrick et al. (2001).

PREFACE

This thesis is original research by Mavis Okyere. This research project under the name "The use of sociocultural artifacts for mathematics instruction: An example of the Adinkra symbols of the Akans of Ghana" and its data collection, received ethics approval from the University of Alberta Research Ethics Board, study number Pro00096990 on February 10, 2020, and from the Municipal Director of Ghana Education Office of the Municipality where the study took place on October 22, 2020. The permission letter is in Appendix B.

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CHAPTER ONE

Introduction

Bishop (1994) hypothesized that any formal mathematics education is a process of cultural interaction, and every child experiences some degree of cultural conflict in his/her learning process. He outlined the mathematical conflicts identified by UNESCO (1989) as language, geometric concepts, calculation procedures, symbolic representations, logical reasoning, attitudes, objectives, cognitive preferences, values, and beliefs. If Bishop's hypotheses are credible, then it could be said that differences in the representation and use of mathematics concepts in Ghanaian socio-culture activities and school/curriculum representation of mathematics concepts may influence students' learning. However, the effect of the differences can be reduced if an attempt is made to reconcile the everyday mathematics encountered at home and the formal mathematics learned at school.

Davis et al. (2011) in their study on "cultural influences on students' conception of fractions" in Ghana revealed how primary school students in the community found it difficult to understand the different fractions because in their culture every fraction is recognized as "*fa*" meaning part. There is no distinction between one-half, one-third, and so on. This is one example of a cultural conflict between school mathematics and culture. Another example in the Akan communities is that there are no distinctions between two-dimensional figures and three-dimensional figures. A circle or round object is known as *kanko* or *krukruwa* (in the Twi language) and these two words are also used to describe the sphere and the oval (*kanko* is usually used to describe people sitting in a circle or objects arranged in a circle, and *krukruwa* is used to describe a round object). Nonetheless, both words are used to describe circles or round objects. For example, if an Akan person says *na adea krukruwa bi da kwan no mu,* it will mean there was a circular or spherical object laying on the road. Without reconciling this prior knowledge with the formal knowledge on shapes, confusion may arise in students' thinking, thereby hindering their ability to distinguish such shapes.

My aim in conducting this study was to investigate how the use of the Adinkra symbols of Akans of Ghana for teaching mathematics concepts related to them could promote students' development of mathematics proficiency. By mathematics concepts, I refer to the concepts delineated in the school curriculum of Ghana. There is no consensus on the definition of mathematic; however, according to Bishop (1988), traditional mathematics (academic) was developed based on six activities found across cultures which are counting, locating, measuring, designing, playing, and explaining. Hence, any of these activities involve mathematics.

This chapter includes a brief description of the demographics of Ghana and the formal education system. It also presents research studies conducted in Ghana about students' mathematics learning, the autobiographical origins of the study, the relevance of the study, the statement of the problem, the purpose of the study, and research questions.

Location and Demographics of Ghana

Ghana is located along the Gulf of Guinea in the West Africa sub-region (see Figure 1.1). It has a land area of 227,533 Km^2 and water area of 11,000 km^2. Ghana shares borders with three countries. On the north, it shares a border with Burkina Faso, on the west with Ivory Coast, and on the east with Togo. The Gulf of Guinea and the Atlantic Ocean are to the south of Ghana (see UNESCO, 2015-2016 report).

Figure 1. 1

Location of Ghana in Relation to West Africa

From *Political Map of West Africa,* by Nations online project 1998-2019 (https://www.nationsonline.org/oneworld/map/west-africa-map.htm). In the public domain.

Ghana was a former British colony. It is also the first West African country to obtain independence on 6th March 1957. Ghana has been politically stable since 1992 when it returned to constitutional rule, though the country experienced political instability between the 1960s and the 1990s.

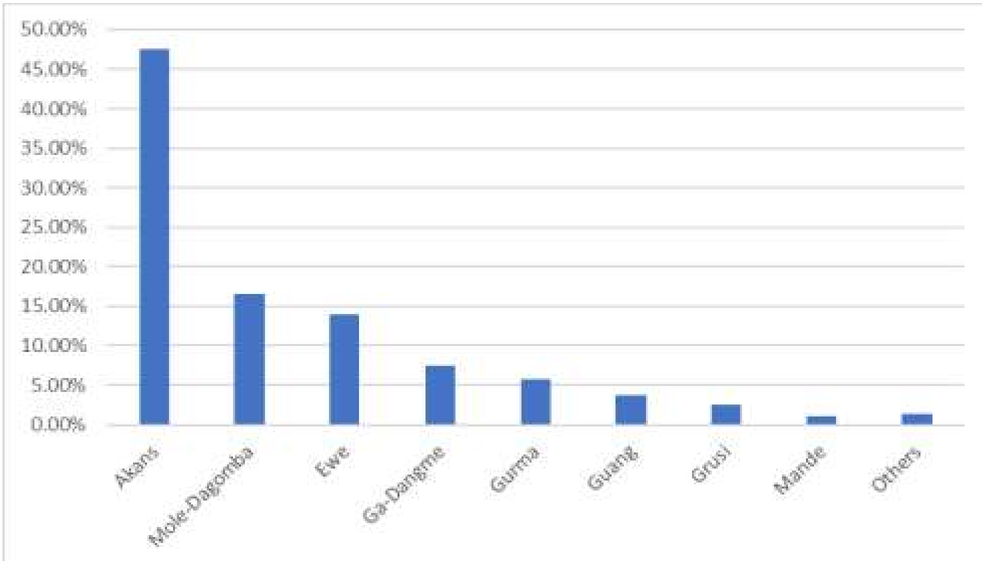
The country is separated into 16 governmental regions. For local government, there are 254 districts which include 150 ordinary districts, 98 municipal districts, and six metropolitan districts. The ordinary districts have a population of less than 75,000, municipal districts have a population between 75,000 and 95,000, and metropolitan districts have a population higher than 250,000. The capital of Ghana is Accra. There are two major religions in Ghana: Christianity and Islam.

There are more than 60 ethnic groups in Ghana, but the major tribes are Akans, Mole-Dagomba, Ewe, Ga-Dangme, Gurma, Guang, Grusi, and Mande (Nsiah, 2018). Each of these ethnic groups has subdivisions; however, the subdivisions share a common language, history, and culture. Figure 1.2 shows the percentage distribution of the various ethnic groups in Ghana as given by Nsiah (2018).

There are about 49 languages and dialects spoken in the country. However, English is the official language. Of the 49 languages and dialects, only nine of them are government-sponsored and include Akan, Dagaare/Wale, Dagbani, Dangme, Ewe, Ga, Gonja, Kasem, and Nzema. One of these nine languages is studied in every school, depending on the location of the school. The Akan language (also called *Twi,* which is the language of the Asantes) is the predominant one and is spoken in all parts of Ghana. Most television programs are presented in the Akan language. The region and ethnic group (the research site) chosen by the researcher is informed by the knowledge of the predominant ethnic groups and languages (details of the research site are presented in Chapter Four). The Akan ethnic groups of Ghana consist of the Agona, Akwapim, Akyem, Asante, Denkyire, Wassa, Kwahu, and Fanti. The Akans presently occupy nine of the 16 administrative regions of Ghana. The Akans share a common culture (food, clothing, chieftaincy, religion, etc.). Although they speak different dialects of the Akan language, each ethnic group understands the dialect of the other.

Figure 1. 2

Percentage Distribution of the Major Ethnic Groups in Ghana



The Formal Education System in Ghana

Ghana presently follows an 11-year free basic education program comprising two years of kindergarten, six years of compulsory primary school (Grade 1-6), and three years of compulsory junior high school (Grade 7-9). Basic education is followed by a three-year senior high school or vocational/technical education program, and then a 3- or 4-year tertiary education (universities, polytechnics, and colleges). To progress from primary school to junior high school (JHS) does not require external examinations, so it is expected that every child gains access to junior high school without any hindrance. However, to progress from junior high to senior high, and from senior high to tertiary level requires a student to pass a national examination (Adu-Gyamfi et al., 2016; Kuyini, 2013). At the end of junior high school (Grade 9) students write the Basic Education Certificate Examination (BECE). If they pass the BECE, students can continue with three years of senior high school (grade 10-12) or technical/vocational education (Adu-Gyamfi et al., 2016; Kuyini, 2013).

Senior high school education (Grades 10-12) is not compulsory, yet it has been subsidized by the government since its inception in 1990 and was declared free by the government in September 2017. Students take the West African Senior School Certificate Examination (WASSCE) at the end of the 12th grade. Success in this examination allows students to attend universities and other tertiary education institutions.

The local language spoken within the community where a school is located is used as the medium of instruction at the kindergarten and lower primary level (Grades 1-3). This has been the policy on language for instruction since 1970. Local languages are supposed to be used as the medium of instruction at the lower primary levels, but most private schools including the international schools use the English language for instruction at the lower primary level, and books including textbooks, pupils' workbooks, and teachers' handbooks except Ghanaian language books are written in English.

Over the last decade, the government of Ghana has strengthened its commitment to "education for all" through measures such as the abolition of school fees, the capitation grant (an amount of money per student given to a school), improvements in infrastructure, supply of textbooks, and other teaching/learning materials including computers. According to a UNESCO 2015-2016 report, Ghana developed a policy framework called the Education Sector Plan (ESP) 2010-2020. This plan helped to strengthen some existing educational programs such as the school feeding program, access to a functional literacy program, education and training for employability programs, and education for development and sustainability programs. One of the four thematic areas outlined in the Education Sector Plan (ESP) to achieve Ghana's Millennium Development Goals (MDGs) was quality of education. One of the policy goals on quality of education was to improve the quality of teaching and learning to enhance pupil/student achievement (MoESS, 2006). Through this ESP policy framework and the government's investment in education, Ghana has made some strides in the expansion of and access to basic and secondary education.

The Education Sector Performance report for 2018 recorded an increase in enrolment across all levels: kindergarten through senior high school from 2016/17 to 2017/18, and an increase in the percentage of trained teachers (teachers with formal qualifications in education) across all levels of basic education in 2017/18 compared to 2016/2017 (Ministry of Education, MOE, 2018).

Although Ghana has made considerable strides in the expansion and access to basic and secondary education, the problem of quality of education persists, especially for mathematics and science. For example, the Ministry of Education (2015) report indicated that textbook to student ratios at the secondary school level declined for English, Mathematics, and Science subjects in 2014/15 compared to 2013/2014. The 2018 report of the ministry also indicated that the country has not yet met the textbook to students' ratio of 1:1. The decline in school resources could be a contributing factor to the decline in students' performance on the core mathematics examination as reported by the chief examiner of West African Examination Council (WAEC) for the May/June 2018 examination. However, other variables could contribute to educational achievement which the various educational reforms in Ghana since 1970 have not yet considered. The Republic of Ghana (2010) report noted that "it is becoming clear that further progress cannot be achieved by a simple algorithm between inputs, resources, standards, and projected educational outcomes" (p. 2). This statement shows educational outcomes are not only dependent on resources and inputs but also influenced by other variables which have not yet been determined by the various policies. It is evident from the above discussions that even though access to education for the masses is increasing steadily, the expected outcomes are not forthcoming. The Republic of Ghana (2010) report again pointed out that educational outcomes are also determined by deep-seated social and other external factors that cannot be easily changed by some short-term, well-intended actions. The implication is that social and cultural contexts could be the variables that could influence students' learning outcomes, especially in mathematics which have not yet been studied in Ghanaian educational research.

Research Studies on Students' Mathematics Learning and Achievement in Ghana

Many studies have been conducted in Ghana into variables that account for students' mathematics achievement. Among the variables that have been identified and studied concerning students' mathematics performance are; the quality of instruction (Ampiah, 2008; Butakor et al., 2017; Enu et al., 2015; Fletcher, 2005), mathematics curriculum and its assessment (Anamuah-Mensah & Mereku, 2005; Butakor et al., 2017), school resources (Ampiah, 2008; Enu et al., 2015), teacher quality and motivation (Butakor et al., 2017; Enu, et al, 2015), and students' mathematics self-concept (Bruce, 2016; Butakor et al., 2017; Enu et al., 2015; Mensah et al., 2013).

Some educators have developed theories of pedagogy that are adapted to students' cultures. They consider ways to improve the teaching and learning process through the link between the student's culture and the taught subject (for example, Bishop, 1988; D'Ambrosio, 1990, 2001; Gay, 2010; Ladson-Billings, 1995a; Sterenberg, 2013). In these theories, it is necessary to integrate student culture into the existing mathematics curriculum. However, a review of Ghana's mathematics curriculum from primary to upper secondary education shows that except for measurement in primary and lower secondary programs that suggest the use of the arbitrary unit (non-standard) for measurement before introduction to the Imperial System (SI units), no other topic is linked to the cultural context of Ghana. The perspectives of the culturally relevant pedagogy and ethnomathematics suggest the need for mathematics curricula to reflect not only school mathematics but the mathematics within society as well. A sociocultural approach to mathematics learning is to locate mathematical ideas within culturally organized activities and bring them into the formal school curriculum.

D'Ambrosio (1990, 1994, 2001) and Bishop (1988) are of the view that mathematics, as a body of knowledge, developed out of the human search for solutions for problems they encountered, hence mathematics is embedded in human activities. In these contexts, D'Ambrosio was referring to formal mathematics as in school mathematics. Harouni (2015), for instance, described how mathematics education which aimed to develop mathematically literate people who will use their knowledge in solving everyday life problems has evolved over the years from its source in human activities to a basic form that has no social power because it has lost its original emphasis and purpose. Harouni, however, noted that there is nothing arbitrary in mathematics and that to get the meaning of mathematics is to establish a relationship between the arbitrary and to what the arbitrary refers. In my view, since the formal school mathematics curriculum developed out of cultural contexts, there are still cultural contexts that can be used to support the teaching and learning of formal mathematics. This view is not emphasized in the existing Ghanaian curriculum, and only a few studies have been conducted on the cultural influence on students' mathematics understanding in Ghana.

Some Views About Mathematics Learning

Davis (2016) is of the opinion that although the Ghanaian mathematics curriculum does not have a place for out-of-school mathematical conception and representations, this does not stop students from drawing on their out-of-school mathematical conceptions in their school mathematics activities. Hence, the author argued that educators should find ways of adequately integrating students' home mathematical experiences/conceptions into the school mathematics curriculum.

Ginsburg (2006) for instance, is of the opinion that competent educators start planning for instruction by carefully observing children playing or engaging in other activities to identify their daily mathematics and then interpret the mathematics underlying behaviors, and how they integrate into key mathematical concepts and programs. When these are identified, educators create activities to integrate new concepts with children's prior knowledge. Thus, educators observe students' problem-solving, document what children say, do, and represent to plan and decide how to respond, challenge, and expand students' thinking. Therefore, knowledgeable educators will help students turn their everyday mathematics into a more formalized understanding that can be applied in other situations. Some mathematics educators refer to this as "mathematization" which requires teachers and students to abstract, represent, elaborate on informal experiences, and create models of their everyday activities (Freudenthal, 1991; Gravemeijer & Terwel, 2000; Menon, 2013; Wheeler, 1982).

Freudenthal (1991) identified two types of mathematization: horizontal and vertical. He characterized horizontal mathematization as movement from the world of life into the world of symbols and vertical mathematization as a movement within the world of symbols (see also Gravemeijer & Terwel, 2000; Menon, 2013). Horizontal mathematization includes converting a problem in a real-life situation to a known mathematical problem, or vice versa. The use of a formula to solve a mathematical problem could be regarded as an example of vertical mathematization. It could be said that horizontal mathematization is missing in Ghanaian mathematics classrooms because local examples of Ghanaian culture are not seen in the curriculum. Freudenthal (1991) suggests that a good mathematics instruction/lesson should have elements of both horizontal and vertical mathematization.

Lesh et al. (1987) described five representations for learning and solving mathematical problems. Their model shows that in teaching/learning, students should be made to move among five representations of the concept (real-life, manipulative, pictures, verbal symbols, and written symbols). That is, they should translate whatever concept they are learning from one representation to another. For example, translating from real-world to oral language and/or to written language. They believed that the more ways a student is given to reflect on and test an idea, the better chance there is for the idea to be formed correctly and integrated into a rich web of ideas. Their research revealed that children who have difficulty translating a concept from one representation to another will also have difficulty solving problems and understanding computations. They concluded that strengthening the ability to move between and among these representations improves the growth of children's concepts (Lesh et al., 1987). However, it is rare to see all the representations in Ghanaian mathematics curriculum materials and so in classrooms. The representations commonly used in Ghanaian classrooms are pictures/diagrams, written symbols, and oral language. In my experience as a mathematics teacher, I will say it is rare to see Ghanaian students representing mathematical concepts in the real world because the concepts they are learning, as presented in their textbooks, are far removed from the local context, so this influences how they will use different representations for mathematics concepts due to their limited access to these concepts through their daily activities.

Literature shows that elements of mathematics can be found in every culture and many concepts in the formal mathematics curriculum could be influenced by culture. Examples include number and numeration, measurement, shape, space, and symmetry (Fouze & Amit, 2018;

Gerdes, 1988a, 2012; Muhtadi et al., 2017; Weldeana, 2016; Zaslavsky, 1994). Muhtadi, et al. (2017), for instance, demonstrated how the Sudanese use the concept of area and calculate areas of plots of lands by converting every shape of a plot of land to a square or rectangle and determining its area using a unit of measurement called *bata.*

Before formal education was introduced in Africa, Africans had been engaged in buying and selling (which involves the use of numbers and counting) and calculating land measurement for farming and building. The ideas of shape and space could be seen in African architecture and other art forms. Thus, students experience these concepts by taking part in various activities in the home before they come to the mathematics classroom to be introduced to the formal curriculum. The knowledge gained from these everyday activities may thus impact their concept formation in the mathematics classroom. The students' prior knowledge may either enhance or hinder their learning of the formal curriculum content. It is, therefore, useful to conduct a research study to identify mathematical concepts that are applied in Ghanaian sociocultural activities and artifacts and teach those concepts using cultural activities/artifacts.

D'Ambrosio (1990, 2001), for example, suggested that all societies have developed mathematical practices appropriate to their lives and their cultures, yet little information about these cultural practices has entered the school mathematics curriculum. However, I believe that if mathematics educators could investigate mathematical practices within their localities and employ them in their teaching, those local cultural practices could gradually make their way into the curriculum. In Ghana, for example, there are a number of mathematical-oriented games in the Akan culture. Games such as *ampe* (a cultural game for young girls where they jump and play with both the left and the right feet, and in so doing, they simultaneously count) and *oware* (another cultural game for both adults and children among the same Akans where they count numbers using pebbles and other forms of counters) have some mathematics ideas inherent in them. One other area in which mathematics ideas are applied is in Ghanaian art. Examples can be found in the handwoven and hand-printed clothes of Ewes and the Akans. In these examples, there is an application of forms of symmetries. All these have underlying cultural mathematical imprints; nevertheless, none of these ideas is related to in the formal mathematics curriculum in Ghanaian schools. It is against this background that this study explored mathematical concepts inherent in the Adinkra symbols of the Akans and investigated how the use of the symbols to teach those concepts could enhance students' mathematics learning. The Adinkra symbols and their semantic usage will be presented in Chapter 3.

Autobiographical Origins of the Research Study

As a researcher, situating myself in the study enables the reader to understand how my own experiences might have influenced the choice of subject and methodology (Merriam, 2009; Patnaik, 2013). Narrating my lived experiences that led to this study has the potential of adding to the credibility and depth of the research itself (Creswell, 2007; West, 2009; Maxwell, 2008). By reflecting on my lived experiences as a researcher, I also become aware of my own worldview, which is key to interpreting data in qualitative research (Maxwell, 2008). That is, narrating my lived experiences that led to the study facilitates the reader's understanding of the perspectives that led to the analysis and interpretation of the data (Maxwell, 2008; Patnaik, 2013). In this section, I will give an account of my lived experiences that led to the conception of this study.

This study aimed to explore strands of mathematical proficiency that students would develop by using the Adinkra symbols in teaching/learning mathematics concepts. Two issues led to this study. The first was my experience as a mathematics teacher and the second was an experience I had with my first two children.

I was among the students considered to be brilliant in mathematics. I remember how my junior high school mathematics teacher held me in high esteem because of my excellent performance in mathematics. I completed junior high school with distinction in the Basic Education Certificate Examination (BECE) and was ranked “1 out of 9” in mathematics with “1” representing a score of between 75-100 and “9” for a score of between 44-0. I did well in mathematics not because I understood everything taught but because of the interest I had in the subject and my consistent practice of the procedures taught.

With this excellent achievement, I entered senior high school and was placed in the science program with an elective mathematics option because I obtained grade 1 in mathematics at the BECE (in Ghanaian senior high schools two mathematics courses are studied: Core mathematics, which is compulsory for every student, and elective mathematics, which is optional for students in the science, economics, and geography programs). Therefore, at senior high school, placement is based not only on students’ interests but on their performance in the BECE.

Unfortunately for me, I was the only female in the elective mathematics class, and because of the African belief that mathematics is for boys, I was shunned by the male students in the class, no one was willing to study with me or share any material with me. If I had any difficulty understanding a topic taught in class, I had to figure it out by myself. It was in senior high school that I realized that just following the procedures and practicing them does not work all the time. I needed to understand the reasons behind the procedures. I had to develop my own way of conceptualizing what was taught. The most disturbing part for me at that time was that if I didn’t understand what was taught, I wouldn’t copy the notes given in class. I realized that my interest in the subject, particularly in elective mathematics, was going down. I remember going to the elective mathematics class, a couple of times, with a storybook to read while the teacher was teaching.

I was fortunate to meet a female teacher in my final year. She taught elective mathematics in a different way than the two teachers I had met in my previous classes. My interest in the subject developed again as I could understand most of the things this teacher taught me. Maybe, it was because I was able to relate to her as a female. She could connect the topics in the core mathematics class to those in the elective mathematics class, she sometimes told us about the application of the topics to real-life situations, though most of them were outside my local context, and far beyond my level of comprehension at that time, but it made me believe that what I was learning was at least useful and might be useful to me in later life.

With this encouragement in my heart, I persisted and continued to work hard. I wrote my Senior Secondary Certificate Examination (SSCE), now called West African Senior Secondary Certificate Examination (WASSCE- because it is a common examination for West African countries) and was able to pass. However, my grade in core mathematics was better than that of elective mathematics. Nevertheless, it was a pass mark that enabled me to pursue mathematics at the tertiary level.

I persisted in mathematics because I liked the subject and since I was doing well in it, naturally, everyone encouraged me to study mathematics. Becoming a mathematics teacher and pursuing mathematics to this level is both a result of my experiences as a mathematics student and my interest in the subject. My female elective mathematics teacher was also a particular source of inspiration to me to become a mathematics teacher. I entered university to be trained as a mathematics teacher to prepare me with the best knowledge and strategies for teaching mathematics. I will not say this aim was not realized because I could see that my students, most of the time were able to understand what they did not understand before taking my class in mathematics. What was challenging to me was finding examples in our local context that are related to the topics we are covering in class.

On completion of my university degree, I was posted to teach mathematics at one of the senior high schools in the then Brong Ahafo region of Ghana. As a qualified mathematics teacher, I thought I had adequate content and pedagogical knowledge to teach mathematics.

I had some obstacles to face because, at that time, it was unusual to find a female mathematics teacher in a senior high school, but I was confident because I had enough knowledge to teach the high school mathematics content. I was doing well by the standards of educational authorities, and I was complimented by both teachers and students.

Being new and the only female among four mathematics teachers, I tried to apply all the principles in my teaching that I had learned at the university. One of the things that I usually struggled with was how to find relevant examples from the local environment to support my explanations of mathematics concepts. I was able to find examples for some topics but not for all. To my surprise, even the mathematics teachers who were there for years before I joined them could not help me with this matter.

In my second year of teaching in this school, I was assigned to teach logarithm in one of the second-year classes. I looked everywhere to find relevant examples but could not get any related to the local context. So, I went by the usual method of reviewing previous knowledge related to the topic and connecting that to the new topic. I had already treated exponents (indices) with them so that was what connects to the logarithm.

So, the lesson was started by reviewing indices and making a connection to the logarithm. Most students were responding to questions, so in my mind, I thought everything was going on as planned until a student raised his hand to ask me a question. Following is the exchange between the student and me in the class:

> Student: So, after doing all this what next?
>
> Teacher: What do you mean by what next?
>
> Student: I mean what is the use of logarithm in my life?
>
> Student: Don't you think we are learning so many useless things in mathematics? My brother said all the mathematics he learned at the university; he is not applying any of them in his current job.
>
> Teacher: I can't think of any use of logarithm now, but I know it is applicable in real life.

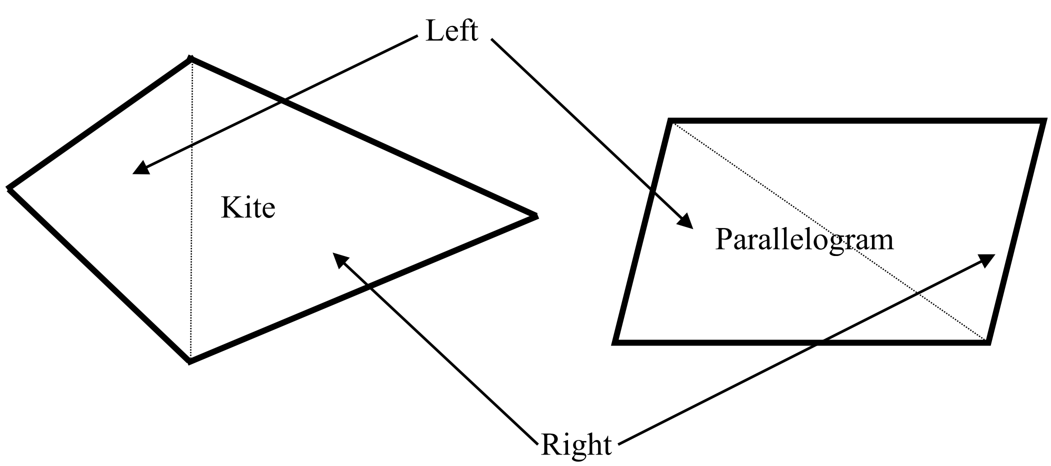
I reflected on this exchange for a while, but I could not find an answer for him until I left the school the following term for further studies. I had been wondering about this problem and asking myself: Do Ghanaians use enough mathematics in everyday activities as a society? Or is school mathematics different from the mathematics used in the everyday life of Ghanaians?

The second experience that led to this study stemmed from an interaction with my own children. My children (a 9-year-old girl and a 7-year-old boy) were playing in our living room one day, then they said they were going to make kites (children in Ghana usually make kites, attach them to a thread and throw them into the air).

Initially, I was not paying attention to what they were doing but an argument ensued between them that caught my attention. They argued about the kite being the same or different from the parallelogram. The boy said it's the same as the parallelogram and the girl said no, it differs from the parallelogram in that the sides of the parallelogram are not equal but with the kite some sides are equal. The boy responded that it is not true because a parallelogram has some of the sides to be equal too. The girl responded again saying "I know that the parallelogram has some sides to be equal, but I mean if you take one side of the parallelogram, say the left, the two sides on the left are not the same but with the kite, the sides on the left are the same and those on the right are the same. So, if you bring the two short sides of the parallelogram to the right side and the two long sides to the left side you get the kite. She drew the two shapes on a sheet and the boy agreed that those are the shapes. So, at this point, I got interested and wanted to understand what she meant by the left side or the right side. I called them to the dining table where I was working to explain to me the left and the right sides of the shapes. They showed me something in the form of Figure 1.3.

Figure 1. 3

Parallelogram and Kite Differentiated by the Children



I used that opportunity to explain to them the adjacent and opposite sides of a polygon and they were able to explain that the kite has two adjacent sides equal, and the parallelogram has two opposite sides equal. I began to wonder how similar artifacts could be used in the classroom. I particularly regretted not asking my junior high school students to bring their constructed kites to class when discussing the properties of the 2D and 3D shapes. Since that day I have been thinking about how to incorporate students' daily experiences and common materials (artifacts) around them in mathematics instruction.

The desire to search for sociocultural elements in my students' local environment that relate to mathematics continued to grow and I decided to undertake this research study as my PhD thesis. However, I had difficulty deciding which aspect of the Ghanaian culture I should consider; should it be games, drumming, trade, clothing, or what? As I continued to contemplate on the aspect of the culture to consider, my supervisor assigned a task to me to look for literature on Islamic tiling and geometry. My search in the literature on Islamic tiling opened a new insight into our culture that I had not considered before, that of the Adinkra symbols. I told myself that if mathematics concepts can be learned from Islamic tiling, then surely there should be mathematics in the designs of the Adinkra symbols and how the symbols are stamped on clothes.

The Relevance of the Topic to Education

The current mathematics curriculum for the basic and senior high schools of Ghana does not use the sociocultural context of the people of Ghana. This may be because Ghana inherited the British education system, and the mathematics curriculum may be reflecting that of the British. I do not agree with authors who think formal school mathematics concepts are Eurocentric (example Gerdes, 1996), I do, however, believe that mathematics concepts found in the primary and secondary mathematics curricula exist in every culture, maybe in different forms (and in Ghanaian culture as well); therefore, those cultural examples can be used to demonstrate the concepts in curriculum materials. I am of the view that the findings of this study may provide an example of the Ghanaian contexts that can be used in the teaching and learning of mathematics in our basic and secondary schools. This will challenge mathematics educators in the country and inform policymakers about the possibilities of including our cultural knowledge in the school mathematics curriculum.

Given that literature on Ghanaian cultural practices and their connections to mathematics is virtually non-existent, the findings of this study will provide information about Ghanaian culture and mathematics for the research community in Ghana and Africa at large. This study will pave a way for dialogue on culture and mathematics and may lead to further research to guide the development of a culturally appropriate mathematics curriculum.

As an education researcher, I hold the view that ideas never stand-alone but emerge from a context, hence the context used to present mathematical ideas is relevant. Many educators are of the view that immersing students in situations that can be related to their own direct experiences (context) is more consistent with a willingness to make sense and allows them to deepen and broaden their understanding of the scope and usefulness of mathematics as well as learning ways of thinking mathematically about situations (for example, Mukhopadhyay et al., 2009; Orey & Rosa, 2007; Zaslavsky,1988). It is the aim of this study for the findings to contribute to the contextual relevance of mathematics teaching and learning, especially among Ghanaians where the textbook examples reflect the colonial perspective and experiences.

In my opinion, culture identifies differences in our view of the world and hence of mathematics. If mathematics is meaningful to students' cultures, they will acquire mathematics knowledge from their own cultural perspectives. Giroux (1996) pointed to the need to consider ethnic and cultural differences in the curriculum. Giroux was of the opinion that educators need to appreciate how different discourses offer students varied ethical references for structuring their own relationship to the broader society. This study, therefore, investigated the strands of mathematical proficiency that occurred in mathematics lessons that used the Adinkra symbols as context.

Some studies suggest that students' culture has a great influence on their school and classroom discourses and that the use of students' cultural contexts in presenting mathematics concepts provide a viable way for students to function in the classroom. Mathematics educators who share this perspective say that beliefs are an essential aspect of the determination of meaning in general and the definition of mathematical meaning in particular (for example, Abraham & Bibby, 1988; Cobb, 1986). Teachers in Ghana, particularly, stand to benefit from this study because it will help them to identify Adinkra symbols they can use to illustrate mathematics concepts, this will enable them to guide their students' discussions on those topics, relating them to their culture (Adinkra symbols), in their respective levels or classes. Based on research literature that suggests the context and application of knowledge must be relevant to students, and that students must be encouraged to recognize that mathematical practices and ideas flow from the real needs and interests of human beings (see Davis, 2018: Gutierrez, 2012; Osher et al., 2018; Zaslavsky, 1994), the findings of this study will reveal the significance of mathematics to the culture of Ghanaians and will help make mathematics meaningful to students and help to promote their interest in studying the subject since they will have a context to relate the school mathematics to.

In light of the current advocacy for finding mathematical knowledge from Indigenous and marginalized cultures, to include these concepts into school curricula, and to open up schools to diversities in our societies today (Pais, 2011), I believe this study will make an immense contribution to both curriculum development and pedagogy in mathematics education. Many mathematics educators are calling for a more culturally sensitive view of mathematics to be incorporated into the school curriculum (for example, Mukhopadhyay et al., 2009; Orey and Rosa, 2007). Orey and Rosa (2007) were of the view that culture influences how we acquire and use our own mathematical knowledge. It is necessary to create and integrate mathematical material related to different cultures and take advantage of students' personal experiences in a mathematics curriculum, as it is possible to apply multicultural methods or ethnomathematical strategies in teaching and learning mathematics. The findings of this study could be useful to support mathematics teaching, not only in Ghana or Africa but in other developed countries such as Canada, the United Kingdom, and the United State of America, that host a large number of immigrants.

The Statement of the Problem

The perspectives of ethnomathematics and culturally relevant pedagogy suggest the need for mathematical programs to reflect, not only school mathematics, but also mathematics within society. However, the Ghanaian mathematics curriculum has failed to reflect its cultural elements. Babbit et al. (2015) noted that the Ghanaian national curriculum has made considerable effort to include local cultural resources, like the Adinkra symbols, in its humanities curriculum, however, it has failed in doing the same in the case of mathematics and science. The problem of the Ghanaian mathematics curriculum materials not using Ghanaian examples, therefore, makes a research project investigating mathematics ideas/concepts inherent in the cultural practices, and the way that those ideas could be used to design a curriculum/lesson important. This will at least draw the attention of stakeholders to the fact that mathematics exists in our culture and could be used in school pedagogy to enhance the understanding of mathematics concepts.

Many studies have been conducted in Ghana on factors that impact students' mathematics learning. However, to the best of my knowledge, only a few of them mentioned culture as a variable that affects students' mathematics learning and how the cultural elements could be incorporated into school pedagogy (for example Davis, 2016, 2017; Davis et al., 2011). This may be due to the mathematics inherited from our British colonial administrators, and for that reason, mathematics educators in Ghana may still have the notion that school mathematics is free of their own cultural elements. However, several researchers have pointed to the fact that cultural values, cultural artifacts, or sociocultural life in general, play a significant role in teaching and learning of mathematics (see Muhtadi et al., 2017; Nurhasanah et al., 2017; Presmeg, 2007; Sharma & Orey, 2017).

Ghana's chief examiners for both the core and elective mathematics for the 2016 examinations suggested that "teachers should take the necessary steps to make the teaching of mathematics more practical and also relate to real-life problems. This will help the students to appreciate the topics being taught" (WAEC, 2019, p. 2). In my view and from my experience as outlined in the autobiography, one way of making mathematics practical and relating it to real-life is to make use of the students' cultural experiences. It is against this background that this study investigated mathematics concepts of the Adinkra symbols of Ghana, developed mathematics activities around the symbols, and demonstrated the concepts to students through culturally responsive teaching to find out how the use of the Adinkra symbols could help students in developing their mathematical proficiency. Babbitt et al. (2015) noted that in terms of monetary wealth, Ghanaians may be considered poor, but they are culturally rich. It is therefore important to use what the country has, which is ‘our cultural assets’, to improve students’ learning.

The WAEC chief examiners’ report on the 2016 core mathematics paper, for instance, showed that students had weaknesses in understanding the concept of plane geometry, recalling, and applying circle theorem to solve related problems, and finding the equation of a circle (WAEC, 2019). The report on elective mathematics also revealed that students showed weaknesses in solving problems involving geometry and trigonometry; constructing the locus of points equidistant from two intersecting lines, using the concept of vectors to find the coordinates of points and the midpoint of a given vector. Similar weaknesses are reported in the WAEC chief examiners’ report for 2020 core and elective mathematics. The weaknesses listed above, which were identified by the chief examiners of the 2016 and 2020 mathematics examinations, are related to geometry and many geometry ideas/concepts can be identified with many of the Adinkra symbols of the Akans of Ghana. I believe that if teachers draw on the Adinkra symbols in teaching geometric concepts identified with them some of these weaknesses may be eliminated.

Purpose of the Study

Many educators believe that mathematics teaching can be more effective and will yield equal opportunities if teaching is based on the cultural knowledge and experiential background of the students (see Gay, 2010; Ladson-Billing, 1995a, 1995b; Muhtadi et al., 2017; Nurhasanah et al., 2017; Presmeg, 2007; Sharma & Orey, 2017). Hence, a designed-based research study that used an intervention to the curriculum to facilitate culturally responsive teaching was implemented. **This research study investigated strands of mathematical proficiency that could be developed by students participating in culturally responsive lessons involving the**

use of the Adinkra symbols. This is to inform curriculum developers, and teachers, that there are mathematical elements in the Ghanaian cultures that could be incorporated in the teaching and learning of mathematics to improve students' learning.

Research Questions

The study investigated the following questions:

1. What mathematics concepts could be related to the:
    a. images of Adinkra symbols?
    b. creation process of Adinkra symbols?
2. What strands of mathematical proficiency could secondary school students develop through culturally responsive teaching involving the use of Adinkra symbols in teaching mathematics?
3. How does the use of culturally responsive pedagogy, involving Adinkra symbols, promote secondary school students' development of mathematical proficiency?

The process of the inquiry raised a subsequent question that has been incorporated into the study:

1. What features of culturally relevant pedagogy emerged from teachers' deliberate efforts to incorporate the Adinkra symbols into mathematics lessons?

Organization of the Study

This paper is organised into eight chapters. In Chapter 1, I described the demographics of Ghana, highlighting the major ethnic groups and languages in the country and the formal education system in the country. A brief autobiography that led to the research study is also presented. The statement of the problem was presented discussing the problem of students' poor performance in mathematics. The significance and the purpose of the study, as well as research questions, are also presented in Chapter 1.

In Chapters 2 and 3, I reviewed theories that illuminated the problem and previous research related to culture and mathematics. The theoretical lenses that were drawn to support the study were discussed. These were theories and research supporting sociocultural views of learning. In Chapter 2, I discussed the sociocultural theory of learning, culturally responsive pedagogy, and students' mathematics proficiency. In Chapter 3, I presented a perspective on mathematics and culture (ethnomathematics), and mathematics concepts identified in sociocultural objects. Semantic usage of the Adinkra symbols and mathematics concepts related to the symbols were also presented in this chapter. In Chapter 4, I described how the study was conducted. The research design, research site and participants, data sources, data analysis procedure, ethical considerations were described in Chapter 4.

Chapter 5 is where I presented mathematics concepts that were identified by investigating images and creation processes of Adinkra symbols and the themes of discussions that mathematics teachers had about the Adinkra symbols. The culturally responsive features implemented in the lessons are presented in Chapter 6. In Chapter 7, I have presented the mathematical proficiency students demonstrated during the intervention and how the features of culturally responsive teaching implemented contributed to the strands of mathematics proficiency students demonstrated. The last chapter presents the summary conclusions and recommendations from the findings.

CHAPTER TWO

Social Nature of Learning

Since this study investigated the use of a sociocultural object for mathematics instruction, it was appropriate to discuss theories about the sociocultural views of learning. In this chapter, I discussed the sociocultural theory of learning of Vygotsky and the culturally responsive pedagogy of Gay, and also explain how mathematics learning was defined in this study.

Sociocultural Theory of Learning

The sociocultural theorists believe that the society within which an individual is born and raised largely influences the development of the individual. The sociocultural theory emphasizes the importance of interaction between the individual and the culture in which the individual dwells. These philosophers view knowledge construction beyond the internal/innate abilities of the individual. Specifically, these theorists hold the view that the innate ability of an individual is socially constructed through the participation of the individual in his/her cultural activities. Thus, the formation of the mind is socially mediated.

The sociocultural theory emerged from the work of psychologist Lev Vygotsky (1934/1987) who believed that the society at large and the individuals the child interacts with, including teachers, peers, and other adults greatly influence the development of what he referred to as “higher-order functioning” of the child. Vygotsky (1978) distinguished two qualitative lines of development from the general process of development; the elementary process which has a biological origin and the higher psychological functions which have a sociocultural origin. Hassan (2002) referred to these two lines as the natural line and the social line. The author stated that the natural line is responsible for the elementary mental functions while the social line enables higher mental functions. Vygotsky believed that, in its own structure, the elementary process (natural line) does not manifest the qualities that are distinctive of human mental functions. Human qualities are introduced into the mental functions through the intervention of the social line (higher psychological functions), which transforms the natural functions into a higher level of mental activity (Hassan, 2002). Vygotsky (1978) revealed that in the history of behavior these transient systems are situated between the given biological and the culturally acquired. He referred to this process as the “natural history of the sign”.

It can be said from the above discussions that the sociocultural theory places much emphasis on the role played by the culture and social interaction between the child and significant others around the child in the cognitive and behavioral development of the child. This theory suggests that learning occurs in social interactions and that schools, and for that matter the curriculum, should emphasize the social in teaching and learning in the school setting.

Similar to the two lines of development, Vygotsky (1986) also identified two types of concepts: scientific concepts and every day or spontaneous concepts. He argued that every day and scientific concepts emerge under entirely different inner and outer conditions, and the relation of the child’s experience of scientific concepts differs greatly from its relation to everyday concepts. He explained that everyday concepts develop spontaneously without any form of systematic instruction. The scientific concepts, on the other hand, come about and develop under the conditions of systematic cooperation between the child and the teacher. That is a scientific concept results from systematic instruction. He suggested that the development and maturation of the child's higher mental functions are products of such cooperation. I am inclined to say that Watson’s notion of school mathematics is similar to Vygotsky’s definition of scientific concepts. School mathematics is seen as the establishment of mathematical knowledge within the school, stipulating the forms of engagement in the formal context of teaching mathematics (Watson, 2008). By this definition, it can be said that the mathematics knowledge acquired in schools through rigorous activities and instructions can be said to be scientific knowledge (concept).

Vygotsky (1986) stated that although the development of everyday/spontaneous concepts occurs in the child earlier than scientific concepts, scientific concepts advance earlier than spontaneous concepts because they benefit from systematic instruction and cooperation. He argued that as long as the curriculum provides the necessary material that supports the development of scientific concepts, the development of spontaneous concepts will always trail behind the development of scientific concepts. However, he explained that the development of spontaneous concepts must reach a certain level for the child to be able to absorb a related scientific concept. Scientific thoughts systematically and gradually reduce to concrete phenomena, while the development of spontaneous concepts goes from phenomena to generalizations. Vygotsky pointed out that it is essential first to bring spontaneous concepts to a certain level of development that ensures that scientific concepts are slightly above the spontaneous ones before the scientific concept can be learned. This suggests that scientific concepts cannot be developed in the absence of spontaneous concepts.

According to Vygotsky (1986), the acquisition of scientific concepts is accomplished through the mediation provided by spontaneous concepts already acquired. Vygotsky argued that these two forms of conceptual development are both important in the formation of what he referred to as mature concepts. To Vygotsky, a scientific concept only comes to life and finds a broad range of applications when it is blended with spontaneous concepts.

It can be argued from the above discussions that everyday/spontaneous mathematics can be blended with school mathematics (which can be seen in the view of Vygotsky as scientific concepts) to provide concreteness to students' concept formation. It can be implied from Vygotsky's discussions on everyday/spontaneous and scientific concepts that everyday concepts form the foundation for the development of scientific concepts, hence, using everyday activities within a student's culture to introduce the student to mathematics concepts is a way of providing grounds for the formation of the concepts.

Another concept of Vygotsky that is important for discussion on sociocultural theory is his concept of Zone of Proximal Development (ZPD). He defined the ZPD as "the distance between the actual level of development as determined by independent problem-solving and the level of potential development determined by problem-solving under adult guidance or in collaboration with more able peers" (Vygotsky, 1978, p. 86). Vygotsky believed that the actual level of development characterizes the retrospective of mental development, while the ZPD characterizes mental development prospectively. The ZPD shows what level of development the child is developing into, not the development the child has achieved. That is, the ZPD defines those functions that are not yet matured but that are in the process of maturing. Vygotsky argued that the essence of learning is to create a zone of proximal development. He explained that learning arouses a variety of internal development processes which only function when children interact with people around them and cooperate with their peers, and these processes become part of the children's independent development once they are internalized.

The cultural interpretation of Vygotsky's zone of proximal development as given by Davydov and Markova (1983) is based on his distinction between scientific and everyday concepts. Vygotsky argued that these two conceptual systems; scientific concepts, developed from above and everyday concepts, develop from below, reveal their real nature in the interrelationships between real development and the zone of proximal development (Vygotsky, 1986). The cultural interpretation of ZPD is given as “the distance between the cultural knowledge provided by the socio-historical context—usually made accessible through instruction and the everyday experience of the individual” (Davydov & Markova, 1983, as cited in Daniels, 2017, pp. 5-6). The cultural interpretation of the ZPD suggests that the socio-historical context of the knowledge teachers wish to transmit to their students be related to the students’ everyday experiences.

Vygotsky (1986) pointed out that children might have difficulties dealing with problems related to life situations because they are not aware of their everyday concepts and, therefore, cannot operate with them at will, as the tasks demand. He explained that the strength of scientific concepts lies in their conscious and deliberate nature, and everyday concepts, on the other hand, are strong with respect to situational, empirical, and practical. This shows how crucial it is to bring together students’ everyday concepts and scientific concepts through the use of students’ everyday activities and materials within their sociocultural environment in classroom instruction. One of the aims of the Ghanaian senior high school core mathematics syllabus is that the student will be able to “use mathematics in daily life by recognizing and applying appropriate mathematical problem-solving strategies” (MOE, 2010, p. ii). However, from Vygotsky’s point of view, as shown above, this cannot be possible if an effort is not made by mathematics educators to blend the students’ everyday concepts (found in their everyday out-of-school activities) and scientific concepts (school mathematics).

As mentioned earlier, it is claimed that a ‘mature concept’ is achieved when the scientific and everyday forms have merged. Thus, the smaller the distance between an individual’s everyday concepts and his/her scientific concepts, the more mature the individual’s concept formation is. In my view, the merging of the students’ everyday concepts (used in their sociocultural environments or out-of-school situations) with the scientific concepts (school mathematics) will not only help students to form mature concepts but also help them to develop a positive perception about school mathematics.

Ghanaian students weak formation of some mathematics concepts, as mentioned by the WAEC chief examiners for 2016 and 2020 core and elective mathematics examinations, and their negative perception and anxiety about mathematics and low self-concept regarding mathematics (Bruce, 2016; Butakor et al., 2017; Enu et al., 2015; Mensah et al., 2013) may be attributed to the fact that the examples of concepts presented in mathematics curriculum and classroom instructions are far removed from their everyday concepts or activities. The cultural interpretation of the ZPD could be related to Freudenthal’s definition of horizontal mathematization, both concepts stress the importance of using students’ everyday activities or concepts in mathematics instructions. The concept of ZPD and that of horizontal mathematization show the need to bridge the gap between students’ out-of-school mathematics and in- school mathematics in order for students to develop an understanding of mathematics.

Relating to Vygotsky’s ideas of everyday or spontaneous and scientific concepts, and his ZPD is the notion of mediation. The definition of the ZPD and scientific concept suggests human mediation. Mason (2000) noted that in Vygotsky's view, the teacher is to lead the learner to higher levels of thinking through the interpretation of events or things, and the teacher stands in the middle of the learner and the knowledge to be learned. Hasan (2002) pointed out that:

> the noun ‘mediation’ is derived from the verb mediate, which refers to a process with a complex semantic structure involving the following participants and circumstance that are potentially relevant to this process: [1] someone who mediates, i.e. a mediator; [2] something that is mediated; i.e. a content/force/energy released by mediation; [3]

someone/something subjected to mediation; i.e. the "mediatee" to whom/which mediation makes some difference; [4] the circumstances for mediation; viz, (a) the means of mediation i.e. modality; (b) the location i.e. site in which mediation might occur (Hasan, 2002, para. 20).

Following Hassan's mediation process, in the context of this study, during the implementation of the intervention, the mediation process involved the following participants and settings; (a) mathematics teachers—mediators, (b) mathematics concepts—something to be mediated (those that were identified from the Adinkra symbols), (c) students—'mediatees', (d) mathematics classrooms—location and (e) Adinkra symbols—modality. Hassan's inclusion of the means of mediation as part of the mediation process is similar to what Vygotsky (1978) called signs and tools (psychological tools) that play mediating roles in learning. He gave their distinctions as follows:

> The most difference between sign and tool, and the basis for the real divergence of the two lines is the different ways that they orient human behavior. The tool's function is to serve as the conductor of human influence on the object of activity; it is externally oriented; it must lead to changes in objects. It is a means by which human external activity is aimed at mastering, and triumphing over nature. The sign, on the other hand, changes nothing in the object of a psychological operation. It is a means of internal activity aimed at mastering oneself; the sign is internally oriented (Vygotsky, 1978, p. 55).

Vygotsky was of the impression that the essence of human memory is the fact that people remember actively with the help of signs (marks). He suggested that the use of signs extends the operation of memory beyond the biological dimensions of a person's nervous system and allows the use of artificial or self-produced stimuli. He indicated that the use of signs leads humans to a specific structure of behavior that breaks with biological development and creates new forms of a psychological process based on culture. Hassan (2002) noted that the notion of tools is crucial to the process of mediation. He stated that tools are not given by nature, but they are artificial stimuli created by human beings in the course of their social life and have significant implications in their social life. He explained that to say that a tool is a mediator means to say someone uses it to convey some force or energy to the business at hand.

It can be inferred from the above discussions that, to be able to merge the everyday and scientific concepts in order to develop 'mature concepts' or to determine the child's ZPD requires human mediation or tool mediation, or both. Teachers usually mediate students' mathematics concept learning through the use of tools such as calculators, abacus, base blocks, etc. Vygotsky (1978) also noted that the use of tools broadens the range of activities within which the new psychological functions may operate. In my opinion, the Adinkra symbols could be used as tools to extend curriculum activities on the mathematics concepts that were identified with them. They could also be used as signs to aid students' memory of the mathematics concepts. Teachers could lead students to use such cultural artifacts as signs or as tools in line with Vygotsky's definition of tools and signs. What about using them as both signs and tools for the same concept? I have the impression that the use of such sociocultural artifacts can serve both purposes. It can be used to extend teaching/learning activities and at the same time, it will have an impact on the internal organization of students' mental structures that will aid in memory of whatever will be learned. Vygotsky referred to the combination of tools and signs as a 'higher psychological function' or 'higher behavior'.

Vygotsky noted that the semiotic means that can be used for mediation include language, various counting systems, mnemonic techniques, algebraic symbol systems, works of art, writing, diagrams, maps, and mechanical drawings, and all kinds of conventional signals. From a semiotic perspective, Radford stated “the world of nature has been transformed into a species-made niche, and it is in this artificial niche that we come to know, think, act, and feel” (Radford, 2013, p. 405). Radford continued to explain that the human mind and reality are structured in such a way that there is a link between the human capacity of language and symbolism and the human capacity to comprehend reality. In the same way, to develop students’ capacity in mathematical language and its symbolism requires the use of artifacts that students can feel and think about in relation to mathematics. Educators recognize that group work and physical involvement in learning materials can greatly increase the understanding and retention of difficult concepts, hence mathematics manipulatives—such as pattern blocks and number lines—are increasing their way to the classrooms and children's museums (Radford, 2013). I wonder, how can materials that have social or cultural significance in a community be used to mediate mathematics learning by students? Can they also find their way into the mathematics classrooms? Can such artifacts enhance group work and students’ involvement in mathematics lessons? In this study, the Adinkra symbols which could be regarded as an artwork and symbols for communication among Akans were used to mediate students’ learning of mathematics concepts that were found to be related to the symbols.

In a nutshell, the socio-culturalists’ position, from the discussions provided so far, indicates that culture provides a means for students to develop conceptual understanding in mathematics and that there are everyday concepts in students’ contextual activities that could be merged with the school mathematics (scientific concepts) to help develop mature concepts in students. Teachers and tools (artifacts) serve as mediators in the quest to develop mature concepts. Since students' contextual activities can be used in teaching, it presupposes that, contextual tools (cultural tools) could be used for mediation since most contextual activities usually involve the use of tools. This study investigated the effect that the Adinkra symbols used as a mediating tool had on students' mathematics learning.

Lerman (2001) noted that it was not until the end of the 1980s that mathematics educators began to develop a theoretical framework that focuses on the social origins of knowledge and consciousness which have been emphasized by Vygotsky. Two of such theories are culturally responsive pedagogy and ethnomathematics. These two theories are used as the theoretical framework for the study. In the next section, the culturally responsive pedagogy is discussed, ethnomathematics will be discussed in the upcoming chapter.

Culturally Responsive Pedagogy

The theory of culturally responsive pedagogy, also known as culturally sensitive pedagogy, postulates that the discontinuity between the school culture and the home and community cultures of students is an important factor that affects their academic achievement. Bridging the gap between students' home culture and school culture, by reflecting and drawing on the students' culture, will consequently increase the academic achievement of these students (Gay, 2010). Ladson-Billings (1994) referred to this as culturally relevant pedagogy. She defined it as a pedagogy that empowers students intellectually, socially, emotionally, and politically by using their cultural references to convey knowledge, skills, and attitudes to them. There are other variations of culturally responsive pedagogy such as cultural regenerative teaching and cultural restorative teaching. Cultural regenerative pedagogy seeks to reclaim educational practices that indigenous people have lost (Lee, 2005) by reinstituting such practices.

This version of culturally responsive pedagogy does not fit my study because the Adinkra symbols were not something employed in teaching mathematics before. Another variation of the culturally responsive pedagogy is the cultural restorative (Wood, 2016) aimed to construct educational practices and methodologies and use them to refine existing practices to build a safe community of healthy relationships. It does this by studying the cultural practices and behavior of the students and by examining the practices of other schools. Though my study investigated the mathematics that can be learned from and with the cultural artifacts of the students and how the artifact can be used to support students' learning of mathematics, I am not in the position to refine the existing practices of the schools to include the artifact in their curriculum. The findings of the study could, however, inform educators of how the Adinkra symbols could be used to enhance students' learning.

The culturally responsive pedagogy or culturally relevant pedagogy defined as the use of cultural knowledge, past experiences, frameworks, and performance styles of various ethnic students that constitute the class to make learning encounters more relevant and effective for them (Gay, 2010) is the approach that best fits this study. Ladson-Billing (1995a) also acknowledged that culturally relevant teachers use student culture as a vehicle for learning. The Adinkra symbols were not artifacts that were originally used to teach mathematics but rather; they exist as part of the Ghanaian culture—used to communicate ideas and values; Ghanaian students already have knowledge about them, and they can describe most of them. Hence, in this study, students' knowledge about the Adinkra symbols was drawn to support teaching and learning of mathematics concepts that were found to be related to the Adinkra symbols.

Although this theory was developed in the United States of America, which is a more multicultural state than Ghana and the two countries also differ greatly in their historical and economic backgrounds, the theory equally applies to the Ghanaian situation because of the country's continuous adoption of foreign educational policies at the expense of its own social and cultural needs. The mathematics curriculum of Ghana is developed to meet international standards; however, the presentation of contents and assessment prescribed in the curriculum is far removed from the Ghanaian context. This makes this theory relevant to mathematics education in Ghana.

Gay (2010) stated that culturally responsive pedagogy aims to empower and transform learners. She explained that it aims at helping students connect academic mathematics to other forms of mathematics. It connects school mathematics to the 'sociocultural-ethical' aspects of their home culture, enables teachers to practice equitable pedagogical practices that cater for all learners, and allows both students and teachers to acknowledge and celebrate their own and each other's cultural background (see also Ladson-Billings, 2014, 2021). Culturally responsive mathematics teaching involves incorporating relevant aspects of the students' culture into classroom mathematics activities to stimulate students' thinking.

Culturally responsive pedagogy is based on a number of premises. One of the premises of culturally responsive teaching that resonates with this study is that culture is at the heart of all we do in education, whether that is curriculum, instruction, administration, or performance assessment. It notes that even without being aware of it, culture determines how we think, believe, and behave, and these, in turn, affect how we teach and learn (Gay, 2010).

Pai et al. (2006) explained that education is a sociocultural process. Therefore, a critical examination of the role of culture in human life is essential for understanding and controlling educational processes. This premise of culturally responsive teaching acknowledges a cultural context of teaching and learning, and views the inclusion of the students' culture as essential in improving student academic success (see also Eglash, 1997; Orey & Rosa, 2007; Stoicovy, 2002)

Flippo et al. (1997) stated that the relationship between literacy and culture is bidirectional. Not only will cultural diversity mediate the acquisition and expression of literacy, but literacy education will also influence and mold an individual's cultural identity. Thus, there is a two-way interaction between culture and literacy, hence one cannot advance in the absence of the other. Learning cannot be successful if it is devoid of the student's culture.

It has been noted that a mismatch between school culture and the culture of the students creates the potential for misunderstanding of actions and misinterpretation of communication between teacher and student. This misunderstanding and miscommunication or lack of cultural harmonization increase the possibility of failure for students who lack the cultural knowledge to navigate the unstated culture and norms of the school (Delpit, 1995; Irvine, 1990). Those students who become successful in school bring to school values that the school considers appropriate. Irvine stated that those students who fail to assimilate or switch to the dominant culture of the school are at a greater risk of failing (Irvine, 1990; 2003). Gay (2010) also shares a similar sentiment by noting that cultures of schools and different ethnic groups do not always synchronize, and this disconnection can interfere with students' academic achievement in part because how some ethnically diverse individuals usually engage in intellectual processing, self-presentation, and task performance is different from the processes used in schools. Teachers, therefore, need to understand different cultural intersections and incompatibilities, minimize the tensions, and bridge the gaps among different cultural systems.

Mukhopadhyay et al. (2009) argued that ignoring students' culture demonstrates a lack of respect for students and means that students' learning should be treated independently of their roles as citizens, in which they must contribute to society using the knowledge gained in education (see also D'Ambrosio, 1990). This supposes that the isolation of students' learning from their culture deprives them of contributing immensely to the development of their society. Culturally responsive pedagogy, therefore, emphasizes the importance of placing culture at the center of the analysis of strategies for improving the performance of underachieving students, or of explicitly acknowledging that it is already there and broadening the center of educational practices to make it culturally pluralistic rather than homogenous (Gay, 2010). Culturally responsive pedagogues believe that congruency between how the educational process is ordered and delivered, and the cultural frames of reference of diverse students will improve school achievement for students (Mukhopadhyay et al., 2009).

Another premise of culturally responsive pedagogy that informed this study is that effects of innovations that appear to be yielding desired results may not stand the test of time if the innovative programs attempt to deal with an academic performance by dissociating it from other factors that affect achievements such as culture, ethnicity, and personal experience (Gay, 2010). Gay explained that such innovative programs may not stand the test of time because they unconsciously cause students to compromise their ethnic and cultural identity to attain academic achievement.

As a country, Ghana has gone through a number of reforms, all in an attempt to improve students' performance and consequently increase productivity. However, the performance of students particularly in mathematics continues to decline. For instance, as a follow up to the New Educational Reform Program of 1987, the Basic Education Sector Improvement program (BESIP) which had free compulsory and Universal Basic Education (fCUBE) as a major component was launched in 1996 (MOE, 1996). The fCUBE program had three broad objectives, namely quality teaching and learning, management for efficiency and effectiveness, and access and participation. Decades after the implementation of this program, the objectives have not been achieved since much learning has not occurred as expected. In fact, in terms of access to education, especially to the less privileged (as already shown in Chapter 1), Ghana has made significant progress since 2000. Enrolment continues to go up with gender parity almost equal in most basic schools. However, this has not translated into the quality of learning, especially in mathematics in our basic and high schools. This is evident in the fact that the majority of our students continue to fail in our national examinations (West Africa Senior School Certificate Examination, WASSCE, and Basic Education Certificate Examination, BECE) particularly in mathematics. All this presupposes the need in Ghana to take a second look at the mathematics curriculum that appears to be divorced from the Ghanaian cultural context.

Vygotsky's zone of proximal development, which has been explained in the previous section, suggests that it is important for learners to engage in social interactions with experts, and teachers should use culturally developed sign systems and culturally appropriate artifacts as psychological tools for instruction (Vygotsky, 1978). Some authors are of the view that in culturally relevant mathematics pedagogy, teachers construct bridges between the home culture and school learning of the students, where it promotes the background experience and knowledge of learners (Orey & Rosa, 2010; Sharma & Orey, 2017; Villega & Lucas, 2002).

Nuri et al. (2006) suggested that to be a culturally proficient teacher, one needs to teach in a manner that builds an understanding of the teacher's and learner's world that produces a value of diversity. Is the world of mathematics teachers and students made up of only dominoes, calculators, computers, and so on usually found in schools? Of course, not, there are other things that mathematics teachers and students interact with within or outside school that can be used in classrooms to learn mathematics. For this reason, this study investigated the use of the Adinkra symbols for their potential to promote students' development of mathematics proficiency.

Towards Culturally Responsive Teaching. Gay, a culturally responsive pedagogue, has proposed five components that need to be considered when preparing for culturally responsive teaching, and these elements were used to guide the planning and the implementation of the intervention in this study.

Develop Culturally Diversity Knowledge Base. Gay (2002) states that effective and culturally sensitive teaching requires sufficient pedagogical content knowledge, as well as explicit knowledge of ethnic diversity (see also Gay, 2010). Part of this knowledge includes understanding the cultural characteristics and contributions of different ethnic groups, including the "cultural values, traditions, communication methods, learning styles, contributions and relational patterns of ethnic groups" (Gay, 2002, p.107).

Design Culturally Relevant Curriculum. In addition to acquiring a knowledge base on ethnic and cultural diversity, teachers must learn to translate it into culturally sensitive curriculum projects and teaching strategies (Gay, 2002, 2010). The author suggests that culturally receptive teachers should know how to determine the multicultural strengths and weaknesses of curriculum designs and teaching materials and make the necessary changes to improve their overall quality, and teachers need to know how to use cultural scaffolding when teaching students. That is, teachers are to use their students' cultures and experiences to expand their intellectual horizons and academic achievements (Gay, 2002, 2010).

Cultural Communication. Teachers must be familiar with communication styles, including the linguistic structures of ethnic communication. The author is of the view that students who are told not to use their communication styles may be intellectually silenced because they are denied the use of their natural ways of speaking; their thinking, intellectual effort, and academic effort also decline (Gay, 2002, 2010; Ladson-Billings, 1995b).

Cultural Congruity in Classroom Instruction. In order to teach to fit students' culture, teaching techniques must match students' learning styles (Gay, 2002, 2010; Ladson-Billings, 1995a, 1995b). Gay (2002) explains that teachers need repertoires of multicultural examples to use in teaching ethnically diverse students. Gay gave examples like the use of ethnic architecture, fabric designs, and recipes in teaching geometry (Gay, 2002, 2010). The author also notes that interaction in the classroom greatly affects students' learning; hence, the teacher needs to understand the kind of interactions preferred in the culture of the students to make use of such interactions in the classroom.

In summary, students' culture has been identified to have a significant influence on their academic success. Cultural incongruence between school and home negatively affects students' learning. Students' academic learning would be enhanced if elements of their culture could be drawn upon and used in the classroom. In the next section, I explain how students' mathematics learning was defined in this study.

Understanding Students' Mathematics Proficiency

Culture is seen to impact students' learning and neglecting students' culture in school instruction can reduce their motivation to learn and thus serve as a barrier to their achievement (Averill et al., 2009; Delpit, 1995; Gay, 2010; Irvine, 1990). Ethnomathematics and culturally responsive pedagogy theorists strongly argue that incorporating students' culture into the school subjects has a greater chance of improving their achievement (Gay, 2010; Weldeana, 2016). Others also argue that the use of students' cultural experiences and artifacts promote understanding of mathematics concepts and help students realize the relevance of mathematics to their culture (Scarlatos, 2006; Zaslavky, 1988). It also promotes high thinking skills, logical reasoning, and creativity, and increases students' ability to solve problems (Weldeana, 2016). All these claims suggest that the use of students' cultural knowledge and sociocultural artifacts have a great impact on students' learning of mathematics, and hence, achievement. These claims of culture contributing to mathematics understanding, high thinking skills, logical reasoning, creativity, and problem-solving abilities is similar to saying that the students' culture influences students' mathematics learning.

Prediger et al. (2015) noted that the learning theory of Vygotsky is focused on what it means to know and understand as it relates to learning. Some mathematics educators have developed theories on what it means to do mathematics (Niss, 2015) and what it means to learn mathematics (Kilpatrick et al., 2001). Since this study was focused on how the use of a cultural artifact in mathematics instruction could promote students' learning, the strands of mathematical proficiency of the National Research Council (Kilpatrick et al., 2001), which stipulates what it means to learn mathematics served as a framework for data gathering during the implementation of the intervention, to investigate the strands of proficiency that occurred in the intervention.

The Mathematics Learning Study Committee, under the auspices of the National Research Council, Kilpatrick et al., (2001), in the attempt to describe what successful learning of mathematics is, theorized five strands of mathematical proficiency that could be used to define learning outcomes for students of all grade levels. The committee indicated that the strands are interdependent; hence mathematical proficiency cannot be achieved by focusing on one or two of the strands. The five strands of mathematical proficiency as identified by the committee are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

Conceptual Understanding

According to Kilpatrick et al. (2001), conceptual understanding is the comprehension of mathematical concepts, operations, and relations. The authors show that conceptual understanding is an integrated and functional grasp of mathematical ideas, it enables students to know more than isolated facts and methods, and students with conceptual understanding know why a mathematical idea is important and can identify contexts in which the idea is useful. Kilpatrick et al, explained that students with significant conceptual knowledge attributes are likely to retain mathematical ideas and knowledge more easily. The understanding and remembering of the methods as well as the reconstruction of the method, in case of forgetting, are easily and effectively accessible.

Fisher et al. (2012) noted that for children to understand mathematics and use it meaningfully, they must engage in meaningful activities on a personal level that facilitates the learning process. Following Fisher et al's view, if the mathematics activities with the Adinkra symbols were meaningful to the participating students, then their conceptual understanding would be enhanced through the activities. Fraivillig (2001) also noted that frameworks for effective teaching to support children's conceptual understanding also emphasize the need for tasks that are mathematically challenging and meaningful.

Ally (2011) found that despite continued persuasion and education of teachers about the need for students to develop conceptual understanding, in the normal mathematics classroom, teachers do not frequently provide opportunities for this to happen. It is against this background that this study designed interventions based on an artifact in the students' sociocultural environment to provide a meaningful mathematics engagement with students and found out whether the intervention helped to promote the students' conceptual understanding.

Procedural Fluency

Procedural fluency refers to knowledge of procedures, knowing when and how to use them appropriately, and the ability to perform them flexibly, accurately, and efficiently (Kilpatrick et al., 2001). Ally (2011) indicated that procedural fluency has two parts: (a) knowing the formal language and identifying the representations and (b) knowing the rules and the step-by-step procedure needed to complete mathematical tasks. Ally (2011) found from a study of how the strands of mathematical proficiency are promoted in a district in South Africa that the trend in the district is to teach procedural fluency skills with limited moments of conceptual understanding (see also Engelbrecht, 2005).

> Students who experience skills-focused instruction tend to master the relevant skills, but do not do well on tests of problem solving and conceptual understanding. Students who study more broad-based curricula tend to do reasonably well on tests of skills (that is, their performance on skills-oriented tests is not statistically different from the performance of students in skills-oriented courses), and they do much better than those students on assessments of conceptual understanding and problem solving (Schoenfeld, 2007, pp. 63-64).

These researchers have observed the tendency of promoting the procedural fluency strand in mathematics classrooms at the expense of the remaining four strands which in turn is negatively affecting students' success with mathematics. "Procedural fluency and conceptual understanding are often seen as competing for attention in school mathematics. But pitting skill against understanding creates a false dichotomy" (Kilpatrick et al., 2001, p. 122). That is, Kilpatrick et al. (2001) also observed that there is an interconnection between procedural knowledge and conceptual understanding, the two should move together.

Researchers have observed an interrelationship between conceptual understanding and procedural fluency such that conceptual understanding enables someone to access the relevant ideas and use them to solve the problem in a specific way (procedure fluency) (Dias Corrêa, 2017; Groves, 2012; Kilpatrick et al., 2001). That is, procedural fluency enhances conceptual understanding through reflecting and reasoning about how the procedure works. Ally (2011) also noted that insufficient procedural fluency is likely to result in learners having difficulty understanding mathematics concepts or experiencing obstacles in the solution of problems. Thus, Ally also affirms that procedural fluency and conceptual understanding are interrelated.

Strategic Competence

Strategic competence refers to the ability to formulate mathematical problems, represent them in different means, and devise ways to solve them (Kilpatrick et al, 2001). Strategic competence is one’s ability to formulate, represent, and solve mathematical problems. From the definition, it can be said that strategic competence requires conceptual understanding and procedural fluency. A students’ understanding of mathematical ideas and procedures enhances his/her capacity to formulate and use mathematical strategies to attack a problem.

Kilpatrick et al. (2001) stated that strategic competence is “similar to what is generally called problem-solving and problem formulation. Students need to encounter situations in which they need to formulate the problem so that they can use mathematics to solve it” (p.124). In such problem-solving situations, the student is required to devise strategies to solve the problem, evaluate the effectiveness of the strategies in every stage of the solution process until a solution is reached. Strategic competence requires procedural fluency as well as a certain level of conceptual understanding for it to be manifested (Ally, 2011; Özdemir & Pape, 2012).

Özdemir and Pape (2012) found the following strategies to enhance strategic competence (a) allowing autonomy and shared responsibility during the early stages of learning, (b) focusing on student understanding, (c) creating contexts for students to learn about strategies and to exercise strategic behavior, and (d) helping students to personalize strategies by recognizing their ideas and strategic behaviors.

Adaptive Reasoning

Kilpatrick et al. (2001) stated that “Adaptive reasoning refers to the capacity to think logically about the relationships between concepts and situations. Such reasoning is correct and valid, stems from careful consideration of alternatives, and includes knowledge of how to justify the conclusions” (p. 129). It is one’s capacity for logical reasoning through reflection, explanation, and justification. Ally (2011) noted that support for learners’ mathematical thinking appears in the form of reasoning, explanations, justification, and arguments amongst many other forms of mathematical practice, and that explaining a procedure, justifying a mathematical idea, or reasoning during computation underpins mathematical understanding and learning. From Ally’s statement, it can be said that the strand of conceptual understanding is required for adaptive reasoning to happen.

Dias Corrêa (2017) said that adaptive reasoning is likely to occur once the first three strands discussed have been reached. Other authors have also noted that the adaptive reasoning strand is the strand that holds the other strands together (Ally, 2011; Kilpatrick et al., 2001). “Conceptual understanding provides metaphors and representations that can serve as a source of adaptive reasoning, which, taking into account the limitations of the representations, learners use to determine whether a solution is justifiable and then to justify it” (Kilpatrick et al., 2001, p.

131). This view of Kilpatrick et al. is emphasizing the connection between conceptual understanding, strategic competence, and adaptive reasoning.

Kilpatrick et al, (2001) stated that instruction that emphasizes thinking strategies are the ones that enable students to develop the strands of proficiency in a unified manner. Özdemir & Pape (2012) were also of the view that classrooms that allow whole-class discussions, enabling students to explain and justify their methods enhance the adaptive reasoning strand.

Productive Disposition

> If students are to develop conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning abilities, they must believe that mathematics is understandable, not arbitrary; that, with diligent effort, it can be learned and used; and that they are capable of figuring it out. Developing a productive disposition requires frequent opportunities to make sense of mathematics, to recognize the benefits of perseverance, and to experience the rewards of sense making in mathematics. (Kilpatrick et al., 2001, p.131).

The productive disposition strand is about personal issues relating to the students' personal values and attitude toward mathematics, their perceived ability and confidence in mathematics, and their recognition of mathematics as worthwhile. The disposition toward mathematics influences students' willingness to engage in mathematics activities (Ally, 2011; Dias Corrêa, 2017). The recognition of performance and encouragement of working autonomously are contributing factors of motivation to persist in mathematics (Ally, 2011).

Ally (2011) found evidence that shows that links between the mathematics taught, and the learner's out-of-school experiences promote the productive disposition strand and that this link needs to be made explicit in mathematics lessons. The author was of the view that such connections ultimately influence the learners' conceptions of mathematics understanding and the learners need to see that mathematics in the class and life in the real world are connected. This study aimed at investigating the strands of mathematical proficiency that could occur in students' participation in an intervention that was designed around their everyday sociocultural artifact (the Adinkra symbols).

Connection to the Ghanaian Context

Although the strands of mathematics proficiency framework were developed in the North American context, it provided a way of communicating out from the Ghanaian mathematics curricular context. The Ghanaian mathematics curricular documents do not outline these strands of mathematical proficiency; however, all the five strands could be inferred from the general aims of the mathematics curriculum of the JHS and the SHS. A perusal of the general aims of the Ghanaian mathematics syllabus for the JHS (MOE, 2012) and that of SHS core mathematics (MOE, 2010) show that being mathematically literate is more than being able to perform computational algorithms. It goes beyond the ability to deal with the four operations on numbers to a more complex knowledge of recognizing and applying mathematics in different situations. In Table 2.1, I have outlined the general aims of the JHS and the SHS mathematics syllabus and mapped them to the strands of mathematical proficiency discussed. The connection between the general aims of the Ghanaian mathematics curricula for the junior and senior high levels, and the strands of mathematical proficiency is the justification for the use of the strands of mathematical proficiency as a framework for interpreting students' mathematics learning in this study.

Table 2. 1

The General Aims of the Ghanaian JHS Mathematics and the SHS Core Mathematics Curricula and Strands of Mathematics Proficiency They Imply

Aims	Strand of proficiency
• Develop the skills of selecting and applying criteria for classification and generalization (SHS). • Use mathematics in daily life by recognizing and applying appropriate mathematical problem-solving strategies (SHS). • Help children develop a variety of problem-solving strategies involving mathematics (JHS).	Strategic competence
• Communicate effectively using mathematical terms, symbols, and explanations through logical reasoning (SHS). • Develop the ability to think and reason logically (JHS).	Adaptive reasoning Conceptual understanding
• Use the calculator and the computer for problem solving and investigations of real-life situations (SHS). • Help children become mathematically literate in a world which is technologically and information oriented (JHS). • Understand the process of measurement and use appropriate measuring instruments (SHS).	Procedural fluency Conceptual understanding
• Develop the ability and willingness to perform investigations using various mathematical ideas and operations (SHS). • Develop in children the skills, concepts, understandings, and attitudes which will enable them to cope confidently with the mathematics of everyday life (JHS).	Productive disposition Conceptual understanding
• Work cooperatively with other students in carrying out activities and projects in mathematics (SHS). • Develop the values and personal qualities of diligence, perseverance, confidence, patriotism, and tolerance through the study of mathematics (SHS). • Develop interest in studying mathematics to a higher level in preparation for professions and careers in science, technology, commerce, industry, and a variety of work areas (SHS). • Appreciate the connection among ideas within the subject itself and in other disciplines, especially Science, Technology, Economics and Commerce (SHS). • Help children appreciate the value of mathematics and its usefulness to them (JHS). • Foster a sense of personal achievement and to encourage a continuing and creative interest in mathematics (JHS).	Productive disposition

Based on the general aims of the Ghanaian mathematics curriculum for the JHS and the SHS, in this study, the following strands of mathematical proficiency with their given indicators were used to describe students' learning during the intervention.

1. Conceptual understanding: Identifying the extent of concepts implied by a mathematical problem or statement, using appropriate ideas and strategies in solving problems effectively using mathematical terms, and generating new knowledge. Understanding and knowing why a procedure works.
2. Strategic competence: Devising and executing a strategy to solve problems, classifying using criteria, generalizing based on the used criteria, and devising a strategy to represent a mathematical idea.
3. Adaptive reasoning: Thinking logically about relationships among concepts and situations, giving logical explanations and justifications to express thoughts and understanding of mathematical ideas through oral and written language, mathematical symbols, diagrams, tables, and graphs. Transferring concepts from one situation to the other. Interpreting such representations as graphs, diagrams, and tables. Relating or comparing situations and extending a mathematics concept to another context.
4. Procedural Fluency: Selecting the appropriate mathematical procedures and correctly applying appropriate procedures, theorems, and tools in solving problems.
5. Productive disposition: Recognizing the usefulness of mathematics to different fields of life and positive self-concept in mathematics. Willingness to persist in solving mathematical problems. Recognizing the application of mathematics in other fields of study and everyday life, including recognizing mathematics in objects. Personal qualities

of diligence, perseverance, confidence, patriotism, and tolerance. Ability to work cooperatively with other students.

In the upcoming chapter, I have presented a perspective of mathematics learning that stipulates that there are mathematics ideas in students' cultures that could be used to enhance their mathematics learning in schools.

CHAPTER THREE

Mathematics and Culture

As discussed in the previous chapter, for effective culturally responsive teaching, educators need a repertoire of knowledge about the culture. One of the aspects of this knowledge for mathematics instruction is knowledge about mathematics ideas inherent in students' culture. In this chapter, I discuss ethnomathematics which is a perspective on mathematics in cultures and give examples of mathematics concepts that have been identified in various cultural artifacts and practices. I will also present the Adinkra symbols and mathematics concepts different authors have identified with some of them.

Ethnomathematics

Before the emergence of the field of ethnomathematics, the prevailing view about mathematics as in the school curriculum was that it is a cultural-free subject (Presmeg, 2007) and this idea was reinforced by the way mathematics was taught in schools. Many researchers have pointed out that mathematics is inherent in many activities of life of people everywhere. Many concepts found in conventional school mathematics can be recognized in many human activities carried on outside of the school (Presmeg, 2007; Nurhasanah et al., 2017). Ethnomathematics "in part" seeks to blend school mathematics with everyday mathematics practices from the culture of the students. Muhtadi et al. (2017) recognized that mathematics is a form of culture integrated into many aspects of society, wherever we are, there is mathematics. A study of mathematics in sociocultural artifacts or cultural activities is referred to as ethnomathematics (D'Ambrosio, 1990, 2001; Rosa & Orey, 2011).

Ubiratan D’Ambrosio, the intellectual father of ethnomathematics was the first to conceptualize the term ethnomathematics as “‘ethno’ [socio-culture and natural], ‘mathema’ [explaining, understanding, coping] and ‘tics’ [arts, techniques]. Thus, ethnomathematics is the art or technique of explaining, knowing, and understanding the diverse cultural contexts of mathematics” (D’ Ambrosio, 1990, p. 22). Ethnomathematics has gone through a number of definition changes and different authors have different views about what ethnomathematics is. Some early writers on the subject expressed the view that ethnomathematics came to challenge the Eurocentric nature of academic mathematics (Ascher & Ascher, 1997; Gerdes, 1996). Another view of the term is that ethnomathematics is the study of different forms of mathematics from different modes of thought (D'Ambrosio, 1994). These views of the term ethnomathematics express the notions that academic mathematics as we have now is purely developed from the Western cultures and that there are other forms of mathematics in other cultures that need to be discovered and studied. I do not, however, agree with the fact that academic mathematics is Eurocentric in nature, though I believe that academic mathematics does not present all forms of mathematics knowledge in the world and there are more to be discovered, however, that was not the focus of this study.

In the context of this study, ethnomathematics is defined as an approach to teaching and learning mathematics that builds on the student’s previous knowledge, background, the role that the environment plays in terms of content and method, and the past and present experiences from the immediate environment (D’Ambrosio, 2001). This definition suggests that ethnomathematics is not contesting mathematics as a body of knowledge as in academic mathematics (it being Eurocentric or not) but rather, it is contesting the fact that mathematics is regarded as a cultural-free subject. Further, it advocates that academic mathematics developed out of human activities (context), and hence, teaching and learning of mathematics should be situated in context to aid understanding.

The definition of ethnomathematics by D'Ambrosio (2001) corresponds to the educational dimension of ethnomathematics defined by Rosa and Orey (2016) as the dimension that does not reject academically acquired knowledge and behavior but integrates human values such as respect, tolerance, acceptance, kindness, dignity, integrity, and peace in the teaching and learning of mathematics to humanize it and give it life. The authors explain that this dimension of ethnomathematics promotes the strengthening of academic knowledge by helping students understand ideas and procedures in the context of their daily life activities.

Watson (2008) explanation of the worst form of school mathematics teaching is that content is imposed on students, there is a phenomenon of cognitive intimidation that does not help to develop students' natural ways of thinking, nor does it lead to competence, it has no purpose, context, and meaning. This problem of school mathematics is what ethnomathematics and culturally responsive pedagogy seek to address by drawing upon students' cultural expertise in teaching mathematics content.

Hofstede (1986) argued that culture influences mathematics through such cultural traits as geometric shapes, values, artifacts, and symbols. Ethnomathematics enables the mathematical concepts inherent in cultural practices to be discovered and shows that all people develop a special way of doing mathematics. This suggests that having students discover mathematics in their own cultures is a real way to bring life to mathematics and give students the chance to see the significance of mathematics. Teaching mathematics through culture is integrating students' cultural knowledge in classroom discourses to enhance students' mathematical understanding.

The motivation for this investigation arose from the fact that there is a great deal of literature on the extent to which mathematics education now reflects culturally-based inequities (D'Ambrosio, 2007a, 2007b; Bishop, 1988; Boaler, 2002; Weissglass, 2002) and a growing acceptance that teachers must draw learning contexts from students' culturally situated expertise, interest, knowledge, and experiences (Bonotto, 2001, 2003, 2005, 2010; Gay, 2002, 2010; Orey & Rosa, 2007; Rosa & Orey, 2011; Sharma & Orey, 2017). The student's culture has been identified as one of the factors influencing the study of mathematics, and individuals from different cultural groups have different worldviews which are fundamental and therefore, do not disappear rapidly (Sharma & Orey, 2017). This suggests differences in students' culture and the culture of the mathematics classroom, including teaching and learning methods, could create classroom tension, thereby inhibiting students' ability to comprehend what is taught in class.

Different authors have noted that one of the strengths that children bring to the classroom is their cultural knowledge and teachers can use children's cultural knowledge to stimulate learning or ignore it and actively deplete motivation to learn, thus, adding another barrier to achievement (Averill et al., 2009; Bonotto, 2010; Delpit, 1995; Gay, 2010; Irvine, 1990; Sharma & Orey, 2017; Weldeana, 2016). Gay (2010) for instance, has argued that ignorance of cultural diversity of students has contributed to the underachievement of Black and other minority groups in the United States. It is for this reason that the ethnomathematics program advocates for the studies into mathematics inherent in cultural practices of diverse ethnic groups and incorporating them in the school mathematics curriculum. Weldeana (2016) stated that mathematics needs to expand its parameters to become more inclusive of the mathematics found in the world that the students inhabit. I believe that one way of doing this is to include students' experiences in mathematics discourses in order to help students develop a greater interest in mathematics and recognize that mathematics extends beyond the classroom and has real-world importance.

Brandt and Chernoff (2014) also argued that mathematics is found nearly everywhere one looks, and hence, we need to find ways of including these other ideas into our classrooms. I believe ethnomathematics is one way of including these alternative viewpoints into classroom mathematics. Ethnomathematics presents the mathematical concepts of the curriculum so that they relate to the cultural origins of the students, increasing their ability to make meaningful connections and deepen their understanding of mathematics (Bonotto, 2010; Rosa & Orey, 2011; Sharma & Orey, 2017). This study investigated mathematics concepts that could be recognized in the creation process and appearance of the Adinkra symbols of Ghana and how the symbols could be used to teach those concepts to enhance students' learning. For these reasons, the ethnomathematics perspective was employed to investigate the mathematics in the Adinkra symbols for the possibility of using the symbols for mathematics instruction.

Critiques of the Ethnomathematics Program

One of the critiques of the ethnomathematics program states that if an ethnomathematics curriculum is implemented then mathematics as an academic discipline is likely to become accessible only to the most privileged of society, and other students will learn multicultural arithmetic by solving problems as a skill of life or just venturing into geometric aesthetics (Rowlands & Carson, 2002). It follows that ethnomathematical ideas in school can serve as a factor of exclusion; because students of "mainstream culture" (students who follow the formal school mathematics curriculum) continue to learn the academic mathematics that allows them to compete in an increasingly mathematical world, while the students of the 'ethnic group' (those who follow an ethnomathematics curriculum) learn mathematics that is only related to their culture (Rowlands & Carson, 2002; Horsthemke & Schäfer, 2007). Another argument against ethnomathematics is that formal mathematics, like science, is an artificial cultural system that has evolved through deliberately directed goals, teaching it with any degree of efficiency requires an artificial setting, hence the school (Rowlands & Carson, 2002). This position of Rowlands and Carson implies that employing students' cultural activities and artifacts in mathematics lessons will undermine the artificial nature of both the classroom and mathematics thereby inhibiting students' learning of mathematics. However, other authors on the subject have contested these views. For instance, Pais (2011) argued that the pedagogical implications of ethnomathematics focus on how to bring cultural knowledge to the classroom to enable meaningful education; how to bridge the gap between students' culture and institutional knowledge (see also Adam et al., 2003). Thus, ethnomathematics is not necessarily advocating for discovering and teaching 'new' mathematics to students, but rather, to uncover what the various cultural groups have that could be related to the formal academic knowledge to use the cultural knowledge to help students understand the academic knowledge.

The use of sociocultural artifacts in the classroom faces opposition from some educators of academic mathematics. Since many examples of ethnomathematics are hands-on and can be taught in group project settings, many teachers and educators feel that ethnomathematics is a hindrance, or is time-consuming, it prevents them from covering the requirements found in the curriculum (Brandt & Chernoff, 2014). Mukhopadhyay et al. (2009) also suggested that ethnomathematics may be confronted with opposition from people such as historically oppressed group leaders and educational activists in developing countries because these people believe and fear that too much emphasis on ethnomathematics and pedagogy can hinder the educational, intellectual, and technological progress of their community. However, the advantages outweigh the disadvantages, and teachers are encouraged to make use of sociocultural artifacts in designing and implementing their lessons. Bonotto (2005), for example, stated that teaching students to critically interpret the reality in which they live and to understand their codes and messages in order not to be excluded or misled must be an important goal of compulsory schooling. Bonotto, therefore, argued that the widespread use of appropriate artifacts could be a useful tool for creating a new link between mathematics in school and everyday life, incorporating everyday life experiences and students' reasoning. In this way, we present mathematics as a means of interpreting and understanding reality and increasing the possibilities of observing mathematics outside the school context.

Advocates' Views About Ethnomathematics

The use of students' sociocultural experiences in teaching and learning mathematics which is part of the research, the philosophical, and a pedagogical program known as ethnomathematics has been identified by many advocates to have numerous advantages. Presmeg (1998) believed that implementing ethnomathematics in mathematics classrooms could promote social justice through the building of tolerance and sensitivity. Similarly, D'Ambrosio (2007a) contended that ethnomathematics has the potential to create equity and social justice. He argued that mathematics is the universal mode of thought, and the universal struggle of humans is to survive with dignity, so it stands to reason that ethnomathematics holds the key to linking these two ideas to promote peace and social justice. Thus, ethnomathematics helps students to feel accepted and accept others, thereby fighting against racism (Bonotto, 2010; D'Ambrosio, 2007b, 2017; Lal, 2014).

Weldeana (2016) argued that current mathematics curricula should incorporate students' cultural background to offer opportunities for all students to learn and achieve as well as to promote students' spiritual, moral, social, and cultural development, and prepare all students for the opportunities, responsibilities, and experiences of adult life. Ethnomathematics has the potential to help engage, inspire, and empower students; it provides students with hands-on connections to the mathematics curriculum through their own cultural and historical backgrounds; through the use of ethnomathematics in the classroom students develop a desire to learn, and their self-confidence grows; ethnomathematics has positive effects on students' problem-solving abilities; it assists students in seeing the relevance of mathematics to their culture, and subsequently, to use this link to assist in learning mathematics (Bonotto, 2010; Brandt & Chernoff, 2014; Fouze & Amit, 2018; Weldeana, 2016). Thus, the affective dimensions of the teaching and learning process, including motivation, engagement, and persistence on task are more likely to be enhanced through the use of ethnomathematics and for that matter sociocultural artifacts.

Weldeana (2016) outlined the following as the benefits of using cultural artifacts for mathematics instruction.

> [It] improves the learning of academic content and promotes higher level thinking skills; provides students with the opportunity to think logically and creatively; fosters students' achievement and improves their ability to solve mathematical problems; improves students' strategies for acquiring information; develops personal and social skills; boosts students' self-esteem; improves students' ability to work with others during learning; helps students experience self-reliance; increases gender relations; allows students' own decision-making; increases students ability in making connections between everyday practice and school mathematics; improves students ability in finding relevant meaning to

many abstract mathematical ideas taught in schools; and ensures high pledge of cultural consideration (pp. 148-149).

Taking up Weldeana's claims, my study investigated the strands of mathematical proficiency that could arise in culturally responsive teaching/learning activities involving the use of the Adinkra symbols. If the use of sociocultural artifacts enhances students' learning of academic subjects like mathematics, then it suggests that the use of the sociocultural artifacts for mathematics instruction will enhance students' development of mathematics proficiency (Kilpatrick et al., 2001).

Ethnomathematics and Culturally Responsive Pedagogy

The ethnomathematics perspective is that mathematics is inherent in every culture, and wherever people are there are mathematical ideas ingrained in their cultural practices. The ethnomathematics program, therefore, advocates that mathematics inherent in the different cultures should be investigated and included in the school mathematics curriculum. In a similar vein, culturally responsive pedagogy is also promoting the inclusion of students' cultural experiences into the school and classroom instruction, that is, ethnomathematics can be seen as a dimension of engaging in culturally relevant pedagogy.

Gay (2002, 2010) is of the view that for culturally responsive teaching to be possible, teachers need to acquire knowledge about the culture of the ethnic groups that make up the class. In my opinion, for culturally responsive mathematics teaching, the first and foremost knowledge the teacher needs is the ethnomathematics knowledge of the cultures of the ethnic groups.

Culturally relevant curriculum in schools aims to integrate students' cultural mathematics knowledge through ethnomathematics (Balamurugan, 2015; François & Van Kerkhove, 2010; Rosa & Orey, 2016). That is, culturally responsive pedagogy is consistent with D'Ambrosio

(2001)'s definition of ethnomathematics as an approach to teaching and learning mathematics which builds on a student's previous knowledge and background, the role that the environment plays in terms of content and method, and the past and present experiences from the immediate environment. Therefore, it is valuable for mathematics teachers who wish to practice culturally responsive pedagogy to explore and identify from the students' sociocultural background and culture mathematics knowledge that could be related to the curricular content they are to share with the students (Abdularahim & Orosco, 2019; Gay, 2002, 2010; Ladson-Billings, 1995a, 2006; Villegas & Lucas, 2002, 2007). Through ethnomathematics, mathematics concepts in the students' cultures are unearthed and integrated into the school mathematics curriculum for culturally responsive pedagogy. Thus, ethnomathematics provides a context for culturally responsive teaching. Rosa and Gavarrete (2016) noted that ethnomathematics presents possibilities for educational initiatives and new curriculum objectives based on culturally relevant pedagogies. Therefore, culturally responsive teachers support students' learning by helping them to build links between what they already know about a topic (through their cultural experiences) and what they need to learn about it.

Mathematics Identified in Some Sociocultural Artifacts and Practices

According to Bishop (1988), traditional mathematics or mathematics as cultural knowledge is developed based on six universal activities: counting, involving tallying, or using objects and strings to record, number words and names; locate and learning how to encode and navigate in the environment; measuring, including measurement units and methods; design and how to design objects, artifacts, and technologies; play and how to play, including games and activities; and searching and explaining a theory or model of connection. It can therefore be said that mathematics is inherent in almost every human activity.

Many researchers in mathematics education have conducted studies on mathematics concepts that can be found in sociocultural activities and artifacts. Some examples of artifacts introduced into classroom activities with brief specific content learning goals are outlined below, most of them have similar implications or mirror the activities identified by Bishop to be related to cultural mathematics. Bonotto (2010) identified the following sociocultural artifacts and the mathematical concepts that they could be used to teach/learn:

> some supermarket bills to introduce some aspects of multiplicative structure of decimal numbers; some menus of restaurants and pizzerias to enhance the understanding of decimal numbers; a cover of a ring binder to introduce the concept of surface area; a weekly TV guide to develop the concept of equivalence between time intervals expressed in different ways; advertising leaflets containing discount coupons for supermarkets and stores to develop the concept of percentage; an informational booklet issued by "Poste Italiane" to estimate and discover area and length dimensions of some envelopes; the maps of cities to introduce and practice with the system of Cartesian coordinates (Bonotto, 2010, p. 23).

These artifacts listed by Bonotto are not associated with a particular cultural group or age group; however, it is important to note that the artifacts were not originally made for the purpose of teaching mathematics. This suggests that many other artifacts that are common in the students' environment that were not created for pedagogical purposes could be related to and used for mathematics teaching.

Gerdes has done extensive work on sociocultural artifacts and mathematics concepts, particularly in African cultures. Gerdes (1988a) demonstrated how the Pythagoras theorem can be derived from a widespread decorative motif of the basketry designs of the Salish Indians of

British Columbia (they call it '*star* pattern'), the Porno Indians of California (they call it '*deer-back* or *potato forehead')* and on a plaited mat of Angolans (they call the motif *Tchokwe* meaning Tortoise).

Gerdes (1988b) also revealed how peasant Mozambicans apply Euclid's fifth postulate (simplified as the rectangle axiom by Alexandrov) to construct the rectangular bases of their houses. He used the two techniques employed by the peasant Mozambicans to teach rectangle axioms to teacher trainees (pre-service teachers). He also demonstrated how the hexagonal patterns in Mozambican baskets and fish traps could be used to teach other geometric theorems such as the sum of interior angles of a regular polygon, similar triangles, and circular functions. He noted that by this process of rediscovering the mathematical thinking hidden in these baskets and fish traps, and other traditional production techniques, the future teachers feel stimulated to reconsider the value of cultural heritage. He emphasized that, geometrical thinking was not and is not alien to African culture and that "'unfreezing of culturally frozen mathematics' can serve, in many ways, as a starting point and source of inspiration for doing and elaborating other interesting mathematics" (Gerdes, 1988b, p.153).

Gerdes (1988c, 2012) illustrated the possibility of using the Angolan sand drawings to teach the concepts of arithmetic progression, Pythagorean triples, symmetry and similarities, greatest common divisor (Euclidean algorithm), and Euler graphs.

Sharma and Orey (2017) also identified the following mathematical ideas to be associated with a drum (*dhol*— a traditional drum of the Nepal people in southern Asia) for teaching geometric concepts such as cylinder, circle, lines, and angles as well as mathematical concepts such as measurements. Weldeana (2016), using various artifacts from the Ethiopian context, suggested that parallel lines and congruent angles, surface area and volume calculations can be drawn from cane weaving and pottery. Fouze and Amit (2018) recognized the following mathematical processes to be linked with the Bedouin folklore games *Ta'ab* and *Mozkat:* Counting and adding of numbers, permutations, development of winning strategies by playing multiple times & reflecting, communicating of thought, and having informal experience with the concept of probability by attending to outcomes of tossing the sticks.

From Cultural Object to Pedagogical Object

The construction and use of artifacts are typical of human activity and their use also contributes to our cognitive development (Bussi et al., 2012; Radford, 2013). Artifacts such as the abacus, compass, graphic papers, Base 10 blocks, etc. have been in mathematics classes for centuries. Bussi et al. (2012) referred to any artifact used for teaching and learning as a semiotic mediation tool. Thus, the artifacts mentioned here are called semiotic mediation tools. These artifacts in the mathematics class were purposefully built to teach mathematical concepts such as counting, addition, subtraction, and so on. The question is whether Adinkra symbols created for a purpose other than mathematics education can be used to teach mathematical concepts.

Presmeg et al. (2016) noted that mathematical concepts are the result of continuous refinement of physical objects. They gave the example of the concept of the cylinder which developed out of the Greek culture of craftsmen who roll things, called *kulindros* (rollers). If mathematical concepts have been developed from objects or physical activities, then this implies that these objects can be used to teach mathematical concepts. However, since the objects are not originally intended for mathematics teaching activities, their use requires some considerations. Although the Adinkra symbols were created to represent Akan values and beliefs, and their creation and their use are generally outside of school mathematics, this study assumed that the symbols could be used to communicate mathematical ideas which are related to them.

Authors, such as Wake (2014) found a fundamental difference between the nature of the role of mathematics in school and social contexts as a workplace with great impact on mathematics as practiced. The mathematics found in the sociocultural context differ in many respects from school mathematics and therefore it is important to consider ways of merging the two contexts. According to Presmeg (2006, 2007), to bridge the gap between students' everyday out-of-school practices and in-school mathematics, a teacher must follow a semiotic framework that uses meaning strings (chains of signifiers). According to her, this could be done by a process of chaining signifiers in which each sign 'slides under' the subsequent signifiers. Presmeg (2007) further stated that in this process, objectives, discursive patterns and the use of terms and symbols all evolve into classroom mathematical practices in order to preserve the essential structure and certain meanings of the initial activity.

Presmeg indicated that by using a semiotic chain, a sequence of abstractions is created, preserving the important relationships of students' daily practices, and that this chain has as its last link a mathematical concept that is desirable for students to learn. Using this process, a teacher can use the chains of signifiers as a pedagogical model to develop a mathematical concept starting with an everyday situation and linking it into a series of steps with formal school mathematics (Presmeg, 2006, 2007).

Presmeg (2006, 2007) illustrated two possible ways to merge school mathematics and students' daily activities: (1) Start with a daily practice that makes sense for participants, then see what mathematical notions emerge from the thread as it develops and (2) focus on a mathematical concept that needs to be taught and then look for a starting point in students' everyday practices, which can lead to this concept at different levels of the chaining process.

Bussi et al. (2012) also gave a teaching-learning process for semiotic mediation that starts with the emergence of students' personal meanings in relation to the use of the artifact. However, the authors state that the appearance of evidenced texts and their evolution into mathematical/scientific texts must be promoted by the teacher through specific social activities. Their teaching and learning process through the use of semiotic is as follows:

1. Activities with the artifact. This type of activity is the beginning of a cycle and is based on tasks to be performed using an artifact. Situations are designed to promote the emergence of signals regarding the use of the artifact.
2. Semiotic activities mainly through written productions with drawings. For example, students may be asked to write individual reports on an earlier activity with the artifact or to produce activity and artifact drawings related to their own experience and include any questions. The writings and drawings produced by students can become objects for discussion in the following collective work.
3. Production of texts by the whole class (discussion). Discussions form the core of the semiotic process upon which the teaching-learning activity is based. Written texts of students or other texts can be analyzed and commented on. The main objective of the teacher in such a discussion is to promote the transition to mathematical meanings, taking into account individual contributions and exploring the semiotic potential arising from the use of the artifacts.

Some authors show that sometimes we may have to modify some artifacts by, for example, taking some data out of it before it can be used as a tool for teaching and learning, creating new mathematical goals and providing students with basic experience in mathematical modelling (Bonotto, 2005; Rosa & Orey, 2010).

Bonotto also suggested other ways of incorporating sociocultural artifacts in the classroom. It can be done by asking children:

> i) to select other artifacts from their everyday life, ii) to identify the embedded mathematical facts (see also Gellert & Jablonka, 2007), iii) to look for analogies and differences (e.g., different number representations), iv) to generate problems (e.g., discover relationships between quantities) the children should be encouraged to recognize a great variety of situations as mathematical situations, or more precisely mathematizable situations (Bonotto, 2010, p. 23).

The dual nature of the artifacts, that is belonging to the world of everyday life and to the world of symbols, allows movement from situations of normal use to the underlying mathematical structure and vice versa, in agreement with 'horizontal mathematization' (Freudenthal, 1991).

From all the discussions on ethnomathematics, it can be deduced that mathematics, as a subject, originated from human experiences and actions. Therefore, the subject becomes meaningful when it is connected to its source. Mathematics curriculum devoid of human experiences, which is the life source of mathematics, becomes meaningless to students as well as to teachers who are supposed to facilitate the students' learning. Many educators, therefore, advocate for the connection between school mathematics and everyday mathematics in our sociocultural environment.

Although the context of school differs from the home or out-of-school context, teachers are encouraged to bridge the two contexts so that students will have meaningful mathematics learning and be able to transfer their school mathematics knowledge to their real-life experiences. It is suggested that learning processes arising from the sociocultural context can be introduced in the mathematics classroom by modifying the sociocultural artifacts to suit classroom learning goals, creating new mathematical goals, and allowing students to identify on their own, embedded mathematical facts in a sociocultural activity or artifacts may help to bridge the two contexts (Bonotto, 2003, 2005; Orey & Rosa, 2010).

The use of sociocultural artifacts in teaching and learning mathematics is found to have numerous benefits in mathematics education and education in general. It is believed that learning school mathematics through sociocultural artifacts helps to fight against racism and increases social justice as students learn to accept themselves and each other (Bonotto, 2010; D'Ambrosio, 2007b, 2017; Lal, 2014). Again, it has an affective advantage of motivating students to learn on their own and persist with challenging mathematics problems. Students become appreciative of the importance of mathematics in their lives. It could also help to raise students' achievement levels in mathematics (Fouze & Amit, 2018; Weldeana, 2016).

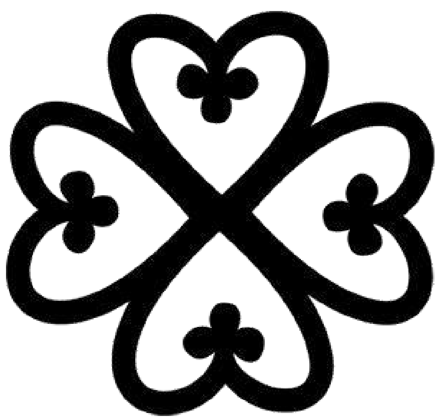
This study is particularly important to Ghanaian mathematics teachers and students since the current mathematics curriculum overemphasizes non-Ghanaian experiences, which could be a possible contribution to the poor performance of students in mathematics. The Adinkra symbols have been recognized as part of the Ghanaian national culture and hence is included in the social studies curriculum, it is also recognized internationally as symbols of African identity (Babbit et al., 2015). Hence, in my opinion, the use of the symbols in mathematics instruction could be beneficial, not only to Ghanaian students; but to students in other parts of the world where the Adinkra symbols are common by helping them develop their mathematics proficiency.

The Adinkra Symbols: Semantic Usage and Related Mathematics Concepts

In the Akan language (Twi), Adinkra means 'bidding farewell or goodbye'. *Nkra* means message. Bidding farewell in the Akan cultural context involves stressing/telling the person an important message you want that person to know/remember or deliver to someone else. Adinkra symbols are visual symbols created by the Akans to represent certain concepts, beliefs, and values. The symbols are used to communicate messages that are part of the daily lives of the Akans. Thus, each symbol has meaning that it communicates to its viewers. For example, the Adinkra symbol in Figure 3.1 is called Nyame dua (God's tree) and means "God's presence and protection". Danzy (2009) and Arthur (2017) recognized the symbols as a writing system, an ideographic script, a script with symbols representing ideas. The symbols are part of the artwork of the Akans and are a well-known craft among the Asantes in particular, which is transferred from one generation to another through apprenticeship.

Figure 3. 1

Image of the Nyame Dua (God's Tree) Symbol



From *West African wisdom: Adinkra symbols & meanings*, by Well-Tempered Web Design 2001-2007
(http://www.adinkra.org/htmls/adinkra_index.htm copyright 2001-2007). In the public domain.

Semantic Usage of the Adinkra Symbols

The Adinkra symbols may not be known to many people, however, in Ghana, they are everywhere, on chairs, doors, buildings, wrappers, and clothes. Adinkra symbols are also used on pottery, wood carvings, and metalwork; and are now incorporated into modern commercial designs, logos, and tattoos because of their meanings. Most institutions in Ghana have an Adinkra symbol in their logo to express their ideology.

The Adinkra symbols are based on human body parts, geometric and abstract figures, and fauna showing how animals play important roles as models of all that are instinctive (Arthur, 2017; Danzy, 2009). Arthur (2017) explained that the head is used in Adinkra to signify that one person cannot rule a state. The eye signifies love, sleepiness, the fragility of the physical body, agreement, vigilance, agitation, etc. The heart signifies love, patience, and devotion. Thus, Adinkra symbols are graphic designs of vines, birds, animals, human body parts, and geometric shapes, all of which represent more than their image and are understood within the context of Asante/Akan culture.

Some of the geometric shapes used in Adinkra symbols include squares, rectangles, triangles, circles, and semi-circles. Each shape connotes an idea. For example, the circle symbolizes the presence and power of God and the sanctity of the male aspect of society (Arthur, 2017; Larbi, 2009). The square and the rectangle represent the sanctity of the male aspect of God and man, and it also symbolizes the territorial power and dominance of the male ruler (Antubam, 1963 as cited in Arthur, 2017; Larbi, 2009). Squares and rectangles are used to depict qualities such as perfection, wisdom, honesty, justice, courage, fairness, mercy, etc. Semi-circles or the crescent moon represent the female aspect of fertility, tender kindness, grace, sereneness (Arthur, 2017). Similarly, the author revealed that the crescent moon symbolizes the feminine aspect of society and its influence on life as a whole; it is seen as a symbol of maternal protection and feminine charm, and its meanings include beauty, female tenderness, and gracefulness. When combined with a star, the crescent represents female faithfulness in love (Antubam 1963 as cited in Arthur, 2017).

Mathematical Ideas Identified With Some Adinkra Symbols

Some authors have already identified elements of transformational geometry with the Adinkra symbols (see Abiola & Biodun, 2010; Arthur, 2017; Mireku, 2014). However, none of these authors tried to develop and teach lessons on the identified topics using the symbols. Abiola and Biodum (2010), for example, recognized that the *kuntankantan* – (Egocentricism) symbol has the concepts of bilateral symmetry, quadrilateral, and similarities associated with it. The authors also noted that the *bi nka bi* (no one bites the other) symbol also demonstrates topological transformations (points that are arbitrarily close in the figure are also arbitrarily close in the image after transformation). Table 3.1 shows images of Adinkra symbols and mathematics concepts associated with them.

Arthur (2017) has also identified bilateral symmetry with the *Woforo dua pa na yepia wo* (it is only when you climb a good tree that someone will assist you). He also identifies radial symmetry with the *Adinkrahene* (chief of Adinkra) symbol and *Mako nyinaa mpatu mmere* (all fruit on the same tree does not ripen simultaneously). The author also recognized the following transformational geometries with the named Adinkra symbols:

Rotation: *Bese saka* and *Nkontimsefuo pua*

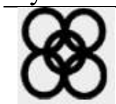
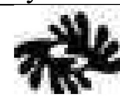
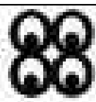
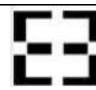
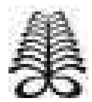
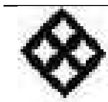
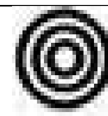
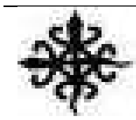
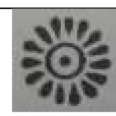
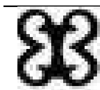
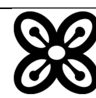
Reflection: Funtunfunefu denkyemfunefu and *Pempamsie*

Translation: *Aban* and *Ntesie-mate masie*

Scaling: Aya and *Sumpie*

Table 3. 1

Some Adinkra Symbols and Types of Transformations Identified With Them

Adinkra symbol	Name	Mathematics concepts	Adinkra symbol	Name	Mathematics concepts
	*Kuntankantan* (Egocentricism)	bilateral symmetry, quadrilateral and similarity ideas associated with it		*Bi nka bi* (no one bites the other) *Bi nka bi* (no one bites the other)	Topological transformation
	*Ntesie- mate masie* (I heard and kept it)	Translation		*Woforo dua pa na yepia wo* (when you climb a good tree)	Bilateral symmetry
	*Nkontimsefuo pua* (hairstyle of court attendance)	Rotation		*Aya* (fern)	Scaling
	*Aban* (Fence)	Translation		Adinkrahene (Chief of Adinkra)	Radial symmetry
	*Funtunfunefu denkyemfunefu* (Siamese crocodile)	Reflection		*Mako nyinaa mpatu mmere* (All fruit on the same tree does not ripen simultaneously)	Radial symmetry
	*Pempamsie* (sew in readiness)	Reflection		*Bese saka* (bunch of cola nut)	Rotation
	*Sumpie* (platform)	Scaling			

The sumpie and mako nyinaa mpatu mmere symbols were taken from *Cloth as metaphor: (Re)reading the Adinkra cloth symbols of the Akans of Ghana* (2nd ed. pp. 162-163), by G. F. K. Arthur, 2017. Bloomington: iUniverse. The other images are from *West African wisdom: Adinkra symbols & meanings*, by Well-Tempered Web Design 2001-2007 (http://www.adinkra.org/htmls/adinkra_index.htm copyright 2001-2007). In the public domain.

Babbit et al. (2015) in their paper "Adinkra Mathematics: A Study of Ethnocomputing in Ghana" noted that despite the clear presence of mathematical concepts, such as the geometric transformations, the Cartesian plane, and the basic calculations used by the craftsmen in the making of the symbols of Adinkra, none of these resources is used in the teaching of mathematics and science in Ghanaian classrooms. The Culturally Situated Design Tools (CSDT) website (https://csdt.rpi.edu/culture/adinkra/geometry.html), also demonstrates reflection, translational, rotational, and dilation geometry associated with the Adinkra symbols *Funtunfunefu denkyemfunefu*, *Ntesie-mate masie*, *Nkontim* and *Aya* respectively. The same website shows the Adinkra computing software designed to draw various Adinkra symbols. This software will draw the Adinkra symbol you select, the dimensions of the symbols can be changed as you wish. The software interface has options for the rotation, reflection, and translation of the symbols, you can perform any of these transformations on your drawn symbol. As demonstrated, over the years many mathematical concepts have been identified with the Adinkra symbols, however, the various reforms on school curriculum have not considered relating the Adinkra symbols to the mathematics curriculum.

Babbit et al. (2015) used a quasi-experimental design to teach logarithmic functions associated with the spirals in some Adinkra symbols to junior high school students using the 'Adinkra computing' software. While this is a good example for teaching a concept associated with the symbols, I am wondering how most Ghanaian students and teachers will benefit from it since the majority of junior high schools, even in the urban centers of Ghana do not have computers. Again, the evaluation of the study was done using a t-test to compare the overall performance of the control and the experimental group on the scores on pre-test and post-test. The finding reveals that the experimental group performed higher than the control group. However, the study did not investigate specific strands of proficiency that the experimental group exhibited more than the control group.

All these transformational ideas are associated with the named Adinkra symbols; however, the Ghanaian mathematics curriculum developers and mathematics educators have not made any effort to include them in the school mathematics curriculum, nor have they investigated how their use in teaching will affect students' learning. For this reason, this study investigated how the use of Adinkra symbols through culturally responsive teaching could impact students' development of mathematics proficiency. Since the study was to find out how the use of the Adinkra symbols for mathematics instruction would impact students' mathematics proficiency, there was the need to situate the study in a classroom and implement interventions that were designed based on the Adinkra symbols. In the following chapter, I describe how the research study was conducted.

CHAPTER FOUR

Research Methodology

This study explored strands of mathematical proficiency that students could develop by participating in culturally responsive teaching through ethnomathematics of the Adinkra symbols, and it also investigated how culturally responsive strategies used in the intervention promoted the strands of mathematical proficiency. The Adinkra symbols are used in the culture of the Akans to communicate ideas and values. In this research, the symbols were deemed to have mathematical concepts in them, hence, the symbols were investigated for their mathematical concepts, and then used to communicate the mathematical concepts that were found to be related to them. The study involved the researcher and mathematics teachers: studying images of Adinkra symbols for mathematics concepts relating to them, observing an Adinkra craftsman doing his work for mathematics concepts he employed, designing activities to incorporate the mathematics of Adinkra symbols into lessons for junior and senior high school students, implementation of the lessons, and the revision of the lessons based on reflections on the lessons and students' learning. In this chapter, I have described the research approach that was employed for the study, the research site, the procedures used for gathering data, and the procedures for data analysis. The ethical consideration of the study is also presented in this chapter.

Research Approach

Emerging from the theory of socio-culturalist: ethnomathematics and culturally responsive pedagogy, this study intended to identify strands of mathematical proficiency that students could develop by participating in lessons that use the Adinkra symbols to demonstrate mathematics concepts to them. Since the study involved designing and teaching mathematics lessons with the Adinkra symbols as the main teaching materials, a design-based research approach was employed for the study.

Design-based research, also known as developmental research or formative research (for other labels see Prediger et al., 2015; Van Den Akker et al., 2006), is a methodology that seeks to advance theory and practice. Design-based research emphasizes the need to develop a theory and to design principles that guide, inform, and improve practice and research in educational contexts (Anderson & Shattuck, 2012; Brown, 1992; Cobb et al., 2016; Shavelson et al., 2003). With respect to this definition, this study intended to investigate if the use of interventions with Adinkra symbols for culturally responsive teaching would promote students' mathematics proficiency and how the use of culturally responsive strategies enhanced the students' mathematical proficiency. The proponents of the ethnomathematics program and the culturally responsive pedagogy argue that including students' culture will improve students' learning (for example Bonotto, 2010; Delpit, 1995; Gay, 2010; Irvine, 1990; Rosa & Orey, 2011; Sharma & Orey, 2017). It is based on this that this study intended to design and implement an intervention that makes use of well-known cultural artifacts (Adinkra symbols) of the Akans of Ghana to illustrate mathematics concepts. The aim is to inform educators of the possibility of using the symbols to enhance students' learning by informing educators about the kind of mathematics learning, defined in terms of mathematical proficiency the intervention could promote and by developing a local instructional theory that would guide instructional practices on the use of the Adinkra symbols for mathematics instruction.

As Bell (2004) noted, "education is an interventionist, designed enterprise by its very nature" (p. 246). It is, therefore, appropriate for a study that was intended to identify strands of mathematical proficiency that students could develop by the use of cultural artifacts in mathematics instruction to adopt an intervention approach which, in this case, is the design-based research method. Cobb et al. (2016) also emphasized that the intent, when conducting design-based research, is to investigate the possibility of improving learning.

Design-based researchers incorporate specific theoretical statements about teaching and learning and help us understand the relationships between instructional theory, designed artifacts, and practice; the design is at the heart of efforts to foster learning, create usable knowledge, and advance theories of learning and teaching in complex environments (The Design-Based Research Collective, 2003). This research aims to contribute to the various theories about leveraging students' cultural backgrounds in mathematics teaching to promote mathematics proficiency. That is the study intended to find out how the use of Adinkra symbols for culturally responsive teaching of mathematics could help promote students' mathematics proficiency. As already mentioned, this research is informed by the culturally responsive pedagogy theory, and the ethnomathematics program using the mathematics proficiency theory as a conceptual framework for classroom data analysis. The findings of this study will, therefore, contribute to the knowledge on the practicability of using students' cultural knowledge in mathematics instruction and how students could benefit from that. Bell (2004) also explained that design-based research is based on the notion that we can learn important things about nature and conditions of learning by trying to design and support educational innovations in everyday environments.

The Design-Based Research Collective (2003) stated that design-based research tests the principles of anchored teaching, which include the belief that learning should be contextualized and the idea that mathematics learning should be more related to the experiences of the students. Thus, the methodology was applied to this study to investigate the possibility of using students' background experiences from their culture to support their understanding through mathematics teaching as the culturally responsive pedagogy and ethnomathematics program advocate.

In my opinion, the study did not find an objective reality, but it found out truths—about the Adinkra symbols and their relationships to mathematics teaching and learning. Thus, the study was based on the relativist stand of knowledge which according to Mayan (2009), is that multiple realities exist, and that "truth" is dependent on experiences. This is to say that truth changes and develops depending on one's experiences. There is no intent to generalize from this study out of context, but the findings will contribute to the ongoing discussion and emergent understanding of culturally relevant pedagogy and ethnomathematics.

The relative position is consistent with the design-based research method that was employed for the study. The methodology recognizes that its results cannot be generalized out of context (Cobb, McClain, et al., 2003; Kelly, 2006). This is to say that the methodology recognizes the existence of multiple realities and acknowledges that truth is context dependent.

According to Gallagher and Fazio (2017), design-based research is based on the premise of collaboration between teachers and researchers, hence, as the researcher, I needed to collaborate with mathematics teachers to generate mathematics concepts from the Adinkra symbols and design the classroom activities. Throughout the study, from the identification of the mathematics concepts in the Adinkra symbols to the designing and the implementation of the interventions, I was involved as a participant as well as a researcher. Thus, the emic approach to research was employed as there were interactions between the participants (mathematics teachers and students) and me. A similar approach was used by Dias Corrêa (2017) in research on "Students' Mathematical Understanding and Mathematical Proficiency Through Mathematical Modelling". Since the design-based research methodology was employed for the study, the researcher collaborated with the mathematics teacher and students to select the tasks that were used in the classroom and how they would be integrated into the teacher's lessons and students' learning. In the case of Cobb, McClain, et al. (2003), the three researchers collaborated to conduct the design experiment on students learning of statistical covariation, the class teacher did not take part in the intervention, hence, one of the researchers assumed teaching responsibility, with another as a teaching assistant during the intervention. In this study, I collaborated with the mathematics teachers in investigating the Adinkra symbols for their related mathematics concepts, selecting concepts and the Adinkra symbols that were used for the design, and I also worked with the teachers to design the teaching interventions. During the implementation of the intervention. I acted as a participant-observer by observing class activities and interactions, acting as a teaching assistant to the teachers, and as an active listener as they reflected on the lessons.

Characteristics of Design-Based Research

Authors who have conducted design-based research or design experiments have identified features that set design-based research apart from other research methodologies. Since the aim of conducting design-based research is to investigate the possibilities of improving students' learning or teachers' practices, the methodology involves the designing and implementation of an intervention. Every design research aims at designing and implementing activities in the real-world context to study how the activities will impact learning (examples include: Anderson & Shattuck, 2012; Cobb, Confrey, et al., 2003; Cobb et al., 2016; McKenney & Reeves, 2012; Van Den Akker et al., 2006). In this study, interventions, consisting of activities, were designed around the Adinkra symbols to illustrate mathematics concepts that were identified with the

Adinkra symbols. The researcher, together with the mathematics teachers, designed the interventions.

In design-based research, designs or interventions are improved through iterative cycles of analysis, revision, and re-implementation. That is to say that the design interventions evolve (Anderson & Shattuck, 2012; Cobb et al., 2016; Gallagher & Fazio, 2017; McKenney & Reeves, 2012; The Design-Based Research Collective, 2003; Shavelson et al., 2003; Van Den Akker et al., 2006). Unlike in scientific experiments where hypotheses are fixed and tested once, in design-based research, hypotheses/conjectures are not fixed and are tested multiple times to refine them (Bakker & Van Eerde, 2015). In the case of this study, the researcher and teachers modified instructional activities based on what was encountered in the classroom. The researcher and mathematics teachers had briefing sessions before every lesson and debriefing sessions after every lesson to discuss what went well and what needed modifications, subsequent activities were modified accordingly before implementation.

Another feature of design-based research is that it is theory-oriented. Many authors on this methodology indicate that design-based research is informed by theoretical propositions, and that, it also contributes to theory building (Cobb, McClain, et al., 2003; Ford & Forman, 2006; McKenney & Reeves, 2012; Prediger et al., 2015; The Design-Based Research Collective, 2003; Molina et al., 2007; Van Den Akker et al., 2006). This is one difference between design-based research and action research, design-based research is aimed at meeting a local need as well as advancing a theory (Barab & Squire, 2004). This study is informed by two theories that are believed to emerge from the sociocultural theory: ethnomathematics and culturally responsive pedagogy and the strands of mathematics proficiency by Kilpatrick et al. (2001). The findings of this study would provide a local instruction theory on how culturally responsive pedagogy through ethnomathematics of the Adinkra symbols could promote strands of mathematics proficiency. Cobb et al. (2016) stated that the aim for conducting design-based research is to produce a theory, hence, it is important when planning design-based research, to place the design within a larger theoretical context. Theory and theorizing in design-based research will be discussed in an upcoming section.

Design-based research is also process-oriented. It involves the active role of students, with support from teachers, with a focus on understanding and improving interventions (Cobb, McClain, et al., 2003; Prediger et al., 2015; Van Den Akker et al., 2006). To understand how the intervention affects students' learning, there should be a setting for students to actively participate in the implementation of the intervention. The implementation of the intervention in this study consisted of students actively involved in activities designed around the Adinkra symbols. Also, teachers were actively involved in the lessons by facilitating students' discourses and engagement in the activities during the classroom implementations. The participating mathematics teachers collaborated with the researcher to design instructional activities and conjectures that guided the classroom implementation of the activities. Teachers' insights from the classroom interactions with students were used to revise the design activities and conjectures.

Design experiments need a setting that allows the researcher to investigate how learning occurs, hence, the design is expected to have practicability for users in real contexts (Anderson & Shattuck, 2012; McKenney & Reeves, 2012; The Design-Based Research Collective, 2003; Van Den Akker et al., 2006). That is, learning takes place in a real context, therefore, design-based research aimed at investigating students' learning must take place in a real context (Molina et al., 2007). This study took place in the context of mathematics classrooms (junior high and a senior high school class) where students were involved in mathematics activities based on the

Adinkra symbols. The teaching activities that were developed around the Adinkra symbols will have a practical use, not only for this study but also for other mathematics teachers within the region to adapt and use in their classrooms. A local instructional theory that will emerge from the study, could guide future studies on cultural artifacts and mathematics learning.

Design-based researchers, such as Anderson and Shattuck (2012), Gallagher and Fazio (2017), McKenney and Reeves (2012), and Molina et al. (2007), all acknowledged that design-based research involves collaboration between different people; students, teachers, researchers, administrators, etc. Classroom design-based research, unlike action research, where the teacher is also the researcher (Anderson & Shattuck, 2012), usually involves collaboration between teachers and researchers. This research study involved collaboration between the researcher, the Adinkra craftsman, students, and mathematics teachers at the JHS and the SHS level. The researcher, together with mathematics teachers and a craftsman, worked together to identify mathematics concepts that could be inferred in the designing process of the Adinkra symbols. Mathematics teachers and I investigated images of the Adinkra symbols for their related mathematics concepts. The researcher and the mathematics teachers then developed teaching/learning activities on the identified concepts using the Adinkra symbols related to the concepts as context and teaching materials. Students were engaged in these activities through the guidance of their mathematics teachers.

Cobb et al. (2016) explained that a research team conducting design-based research collects data that will allow it to document both the process of students' learning in the classroom sessions as well as the enacted support for the students' learning. Design-based researchers, therefore, make use of more than one method for data collection (Anderson & Shattuck, 2012; Brown, 1992; Gallagher & Fazio, 2017). More than one method was used in this study to gather data on the process of the design/intervention and its implementation, including observational protocols, reflective journals, working papers, video and audio recordings, and students' written work to gather data.

Authors such as McKenney and Reeves (2012), Kelly (2004), Shavelson et al. (2003) recognized that another feature of the design-based research is that it is multileveled. Design-based studies usually do not consider only instructional practices happening at the classroom level, but they also consider other higher-level variables that could influence instructional practices. For example, Shavelson et al. (2003) explained that design studies are multileveled because they link classroom practices to events or structures in the school, district, and community. Since this study found mathematics in a cultural artifact and brought the artifact to the classroom for mathematics learning, the study linked a community practice of designing Adinkra symbols to the classroom practice of teaching and learning mathematics. An Adinkra craftsman from the community collaborated with mathematics educators to find mathematics employed in the creation process of Adinkra symbols so that those mathematics concepts could be taught with the Adinkra symbols as mediating tools.

Critiques of Design-Based Research

A number of critiques have been raised against design-based research. One such critique of the design-based research method is its credibility. The credibility and trustworthiness of results of design-based research are often contested because of the possibility of researcher bias. It is argued that researchers may influence the results since they are involved in the conceptualization, designing, and implementation of interventions; hence, the findings of design-based research may not be valid and reliable (Anderson & Shattuck, 2012). However, The Design-Based Research Collective (2003) pointed out that collaboration (in this study between teachers and the researcher) and the typical iterations of design-based research ensure or strengthen the credibility of design-based research results. The reliability of design-based research findings is promoted through triangulation from multiple sources of data and the repetition of analyzes across phases (The Design-based Research Collective, 2003; Swan, 2014).

Prediger et al. (2015) observed that critics of this research method argue that the method can't yield the dual goals—design goal and theoretical goal. However, Cobb et al. (2009), and Molina et al. (2007) have shown that, in this methodology, instructional design and research are interdependent; the design of learning situations or the learning environment, serves as a context for the research, and daily analysis of the classroom situations are conducted to improve the design. Van Den Akker et al. (2006) also made it clear that since design research is context-bound, design-based research does not emphasize isolated variables but rather is context-bound and hence does not provide context-free-generalizations (see also Cobb, McClain, et al., 2003). This presumes that the theories that are developed out of designed-based research are usually context-bound, which may be the reason why they are referred to as 'local instructional theory'. They say theories that are based on accounts of specific systems, they explain how the design goals were achieved in the context of the intervention. That is, the theories are only accountable to the activities of the particular design, they explain the activities of the design (Cobb, McClain, et al., 2003; Gravemeijer & Cobb, 2006; Molina et al., 2007).

Another critique observed by Prediger et al. (2015) is that the method does not involve the use of perfectly prescribed instructional materials in prescribed ways. However, Anderson and Shattuck (2012) noted that design-based interventions are rarely designed and implemented in a perfect way; thus, there is always room for improvement in design and subsequent evaluation. This evolution over multiple iterations is only one of the challenges of the methodology, as it is difficult to know when (or if ever) the research is completed. As already mentioned, one of the features of this method is that it evolves. Hence, teachers involved in design interventions are to adapt to the needs of their students and the requirements of their classroom environment.

In this study, the interventions were implemented in four weeks in four evolving episodes. Precise and completely reliable instructional theory about the use of the Adinkra symbols might not have emerged by the end of the four weeks, however, I believe that the conjectures that were generated were good enough to inform a study involving the use of the Adinkra symbols for culturally responsive mathematics instruction and contribute to theories about the use of sociocultural artifacts for mathematics instruction. Zack and Reid (2003) had a similar view by recognizing that in mathematics learning, 'good enough' understanding enables the students to strive on, and that research is conducted by acknowledging that final theories-of cannot be attained but 'good enough' theories-for are possible to attain.

Theory and Theorizing in Design-Based Research

A theory is an account of what happened, how it happened and may also account for why it happened, and the process of accounting for what, how, and why something happened is referred to as theorizing (Charmaz, 2014). Charmaz is therefore of the opinion that theorizing is a social action that researchers build together with others in a given place and time.

Prediger et al. (2015) noted that all high-quality research is based on theory, and it also aims to contribute to theory development. The theorizing process in design-based research usually occurs at the level of local instructional theories, which usually begins with a conjectured local instruction theory, which then evolves with the experiments. This is similar to constructivist grounded theorists' idea that their theories evolve through reflexivity on the research process and product (Charmaz, 2014). The purpose of design research theories is not to accept or reject theoretical elements, as is the case in experimental research, but to revise, refine, or improve them (Cobb, 2000; Prediger et al, 2015). Confrey and Lachance (2000) had a similar view. The authors indicated that in instructional design research, theorizing occurs in an evolutionary sense of refining and revising categories and conjectures. In other words, local theories are produced by iterations of a basic intervention in some classrooms (Mckenney & Reeves, 2012).

Cobb et al. (2016) indicated that theorizing in design research is a process-based explanation that considers causality as referring to causal mechanisms and processes involved in specific events and situations. Kelly (2004) believed that design research requires 'argumentative grammar' to justify its claims, the author explains that argumentative grammar is the logic behind the use of the method and supports reasoning about data. In other words, theorizing in design research requires the researcher to provide reasons for specific activities and also to show how students' thinking about the activities evolved. Again, this idea is similar to Charmaz's (2014) views that theory has an internal logic, and grounded theorists develop theories because they define and establish relationships between experiences and events.

According to Cobb et al. (2016), the argumentative grammar that links data to analysis and final statements and assertions (theories) requires design researchers to (a) demonstrate that students have developed specific forms of mathematical reasoning that should be attributed to their participation in the designed activities (see also Ford & Forman, 2006), (b) document how these forms of reasoning emerged through the reorganization of prior reasoning, and. (c) identify the specific aspects of the classroom learning environment needed to foster the emergence of these forms of reasoning.

Cobb et al. (2016) contended that this process is a basis for the argumentative structure underlying design research. They note, however, that in a single study it is not always possible to develop a solid instructional theory, especially if the research base that the researchers can rely on to formulate their initial conjectures for the design is thin. It may be for this reason that Swan (2014) said that theories that result from design studies should not be judged by their claims to be 'truth' but rather by their claims to be 'useful'. I believe that the theories that resulted from this study will be useful to other researchers who may wish to conduct similar studies with cultural artifacts and also to me in my future design studies on Ghanaian cultural artifacts.

Theories derived from designed interventions are focused on explaining how and why a particular design works in a particular way (McKenney & Reeves, 2012; Swan, 2014); hence, theories emanating from design studies are context-bound, and if a researcher wishes to adopt an instructional theory to another study, the researcher must differentiate the necessary and the contingent aspects of the design before using the theory (Cobb et al., 2016).

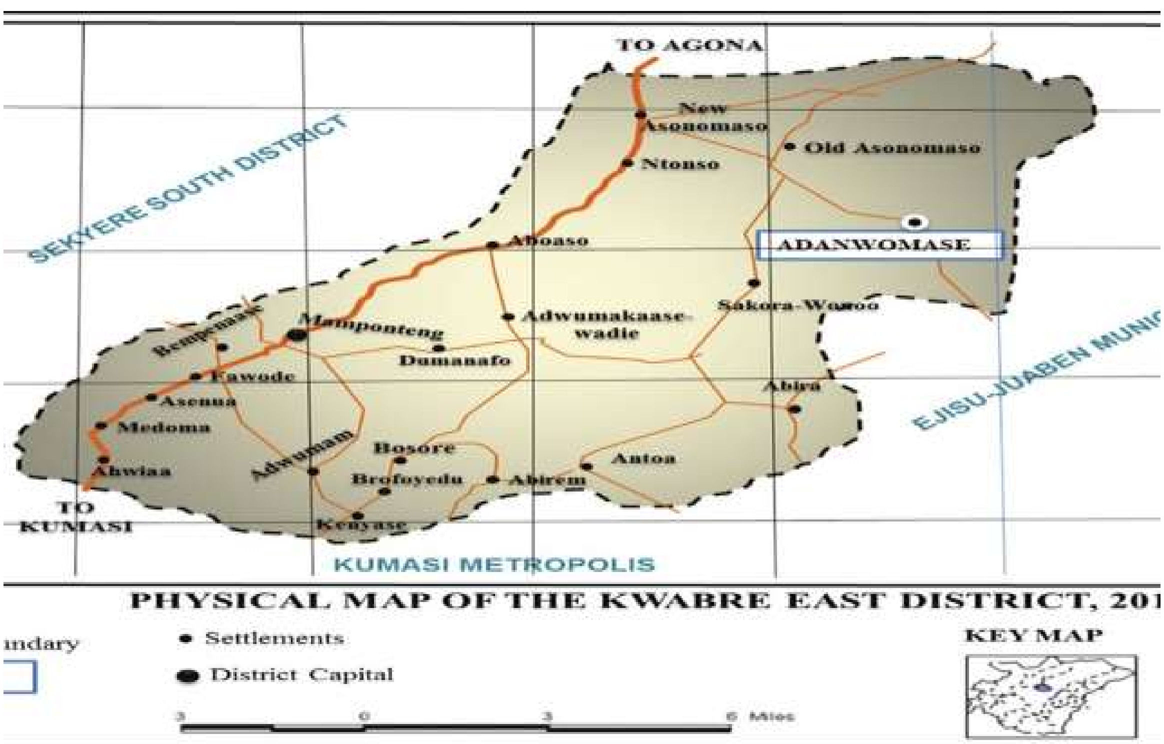
Research Site

The study took place within a municipality in the Kumasi sub-metro area where Adinkra making is a common craft. The study was situated within this municipality because it included an Adinkra craftsman and schools (students and mathematics teachers) to work with. Kumasi is considered the cultural center of Ghana, making it a privileged place to learn about the rich culture of the Asante Kingdom.

Pre-colonial Kumasi had a system of 'wards' that were inhabited by business people with a common trade. For instance, goldsmiths, coffin makers, and umbrella makers all practiced and lived within a particular section of town (Schmidt, 2005). According to Schmidt (2005), the villages within the immediate vicinity of Kumasi were also responsible to the palace to fulfill special tasks—a village may owe certain personal services to the Asantehene, the palace, or the Asante Empire in general. For example, the village of Bantama was responsible for supplying and housing soldiers. As these villages became incorporated into the city and became neighborhoods, these tasks were often continued; today Bantama is the location of the army barracks (Schmidt, 2005). This geographical distribution of specialized skills and commercial enterprises in designated neighborhoods still exists in modern-day Kumasi. The villages (now towns) which are suburbs of Kumasi are known for their artisans and handicrafts. Each town is unique and tends to specialize in its own craft. This study was located within the Kwabre East municipality of the Kumasi metropolis. See Figure 4.1 for the map of the municipality.

Figure 4. 1

The Map of Kwabre East Municipality of the Asante Region, Ghana



From "The impact of cultural values on the development of the cultural industry: Case of the Kente textile industry in Adanwomase of the Kwabre East District, Ghana", by M. O. Asibey, K. O. Agyeman, & V. Yeboah, 2017. *Journal of Human Values, 23*(3), p. 206 (https://doi.org/10.1177/0971685817713282)

This municipality includes a town called Ntonso. Ntonso is a historical town where *Kente* cloth weaving and the carving and printing of the Adinkra symbols on cloths are known to be a common craft. The crafts of carving and printing of Adinkra cloths are male-dominant crafts in this town. Ntonso, nicknamed Adinkra village, is about 22 km north of central Kumasi. Ntonso craftsmen carve the Adinkra symbols into calabash gourds and attach them to bamboo sticks and then, using a black ink made from a tree (*badie* tree - bridelia ferruginea), they stamp the symbols on clothes. Figure 4.2 shows photos of Adinkra symbols carved into a calabash gourd. Although some Adinkra artisans can be found at the National Cultural Center in Kumasi and other parts of Kumasi, and even outside Kumasi, it was important to locate the study at the original home of the Adinkra symbols where the symbols are used in the daily lives of the people, and also for the purpose of getting schools (teachers and students) to work with.

Figure 4. 2

Images of Adinkra Symbols Carved Into a Calabash Gourd

Source: (Mavis's research data)

Gaining Access to Research Site

This study required access to two sites; 1) schools— to get mathematics teachers and students to work with, and 2) craftsmen's workshops—to get the craftsman who demonstrated the designing of the Adinkra symbols to participating mathematics teachers and me. My first point of call at the research site was the Municipal Education office to explain the study to the

Municipal Director of Education and seek permission (see the letter in Appendix A) to visit schools to speak with mathematics teachers and students. The Municipal Director of Education received me warmly and advised that I first meet some mathematics teachers and get their consent to participate before I come for the permission letter that will enable me access to students (He advised that I do not need permission to meet teachers, but I will need permission to meet students and use school facilities). He gave me the contact number of one mathematics teacher in a secondary school in the municipality. This opened a way for me to meet the Adinkra craftsman and other mathematics teachers within the municipality.

I met this teacher and explained the purpose of the study to him, and he introduced me to other mathematics teachers in his school. The teacher also introduced me to an Adinkra craftsman at the Adinkra village who agreed to demonstrate the creation of the symbols to us. I also made contact with teachers at the junior high level in the same town. Through these initial contacts, I had the opportunity to meet mathematics teachers in other towns in the municipality because their friends informed them about the study and gave me their contacts. Because I met teachers in their respective schools to describe the different activities with the Adinkra symbols, all teachers and administrators welcomed me anytime I went to the schools. However, I could not just contact students to take part in the study. The established protocol included the following steps: 1) Approval for this research proposal from the ethics board of the University of Alberta, 2) Written approval from the Municipal Director of Education (Appendix B) of the research site instructing sampled schools to grant me access to classrooms and students for the study, 3) Contacts to the parents by phone to explain the study to them (parent's consent form can be seen in Appendix E), and 4) Assent forms (Appendix F) were given to the students whose parents consent to the study.

Selecting Participants

The study could not have been possible without the collaboration with mathematics teachers, students, and Adinkra craftsmen. A craftsman was needed to demonstrate to the researcher and mathematics teachers how the Adinkra symbols are designed. Luckily for me the first craftsman I was introduced to could draw the symbols and he consented to participate in the study.

Six certified mathematics teachers were selected for the study. Three teachers from the junior high level who were teaching in the same school and three teachers from the senior high level (two from the same school and one from another school) consented to participate (Appendix G shows teachers' consent form). They were given pseudonyms: Mr. Obeng, Mr. Yeboah, Mr. Antwi; and Mr. Abu, Mr. Antwi, Mr. Oti, respectively. These teachers were selected because they had more than two years of teaching experience (their teaching experiences ranged between 4 -12 years) and they all held a degree in education. However, Mr. Abu opted out of the study after our second meeting, but his ideas were still used as part of the data because the teachers' ideas were considered collective, as they were building on each other's ideas, especially, during the conversations and debriefing meetings. I selected teachers from the two levels because junior high and senior high are two different levels in the Ghanaian education system. It is expected that mathematics teachers at the senior high schools have higher qualifications than the mathematics teachers at the junior high schools (teachers at the senior high school must hold at least a first degree in the subject of their specialty, while the teachers at the junior high schools have a diploma or certificate "A"). However, it was found that all the teachers who participated in my study from both the junior and senior high levels had a bachelor's degree in Education. One teacher from each level was selected by the participating teachers, based on their commitment and contributions at the pre-intervention stage of the study, to implement the intervention. Classes and students selected for the intervention were those whose mathematics teachers were chosen to teach the lessons in their classes. Selected students required their parents' consent to participate. The remaining students took part in the intervention, but their views were not included in the data. In all, 22 students from the SHS class of 31 and all 24 students from the JHS class participated in the study.

Selecting the Adinkra Symbols for the Study

According to Arthur (2017), there are about 450 different Adinkra symbols, the study could not include all the 450 symbols. The Adinkra craftsman gave me a sheet that contained 48 images of Adinkra symbols, this was presented to teachers for teachers to select the common ones among their community. In all the teachers selected 27 of the symbols. Images of these 27 symbols were used by mathematics teachers to study the mathematics concepts that could be related to them (see Appendix H for images of the 27 symbols). Each teacher was asked to identify 10 symbols that had a mathematics concept in them and to name the concept. In total, 22 Adinkra symbols were identified by the mathematics teachers to have mathematics concepts in them. From these 22 Adinkra symbols, 12 were selected by the teachers for the craftsman to demonstrate their drawings.

Designing and Implementing the Intervention

Gravemeijer and Cobb (2006), outlined three phases of the design experiment which were followed in this study:

1. Phase one-preparation for the experiment: Provisional learning activities are designed in conjunction with conjectured learning processes that anticipate how students' reasoning might evolve through their involvement in the instructional activities being

developed (Bakker & Van Eerde, 2015, Cobb, McClain, et al., 2003, Gravemeijer & Cobb, 2006).

2. Phase two-conduct the experiment: The activities and conjectures developed during the first phase are implemented. The goal is to test and improve the activities and conjectured theory of instruction. That is, data is generated at this stage on the issues identified as the theoretical intent of the experiment at the beginning of the experiment (Bakker & Van Eerde, 2015, Cobb, McClain, et al., 2003; Gravemeijer & Cobb, 2006).
3. Phase three-retrospective analysis: All data sets generated during phase two are analyzed. The data sets are analyzed chronologically to address the research concept and develop a local instruction theory (Bakker & Van Eerde, 2015; Cobb, McClain, et al., 2003; Gravemeijer & Cobb, 2006).

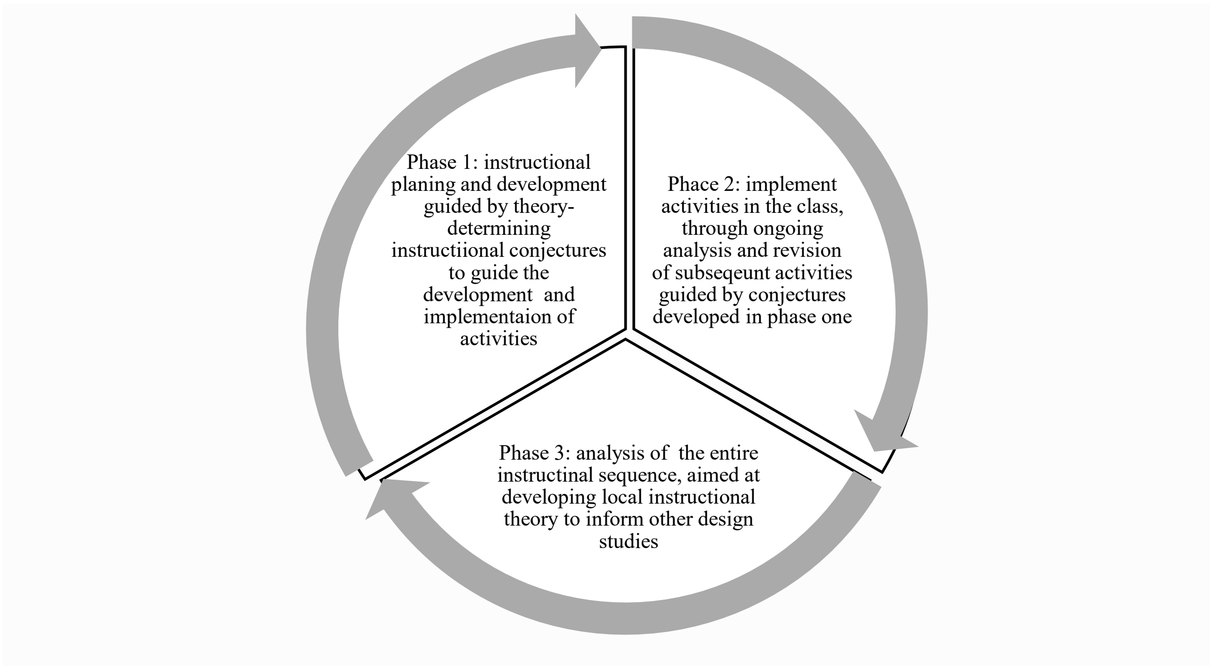
Bakker and Van Eerde (2015) considered these phases to be cyclic. It can be regarded as a cyclic process because theories developed in the last phase (retrospective analysis) may inform other studies and the process begins again. This view of the phases as a cyclic process can be illustrated as in Figure 4.3.

Phase One: Preparing for the Intervention

A number of activities were carried out before the intervention to inform the design of the intervention. The first activity was the observation of the images of the Adinkra symbols by the teachers and the researcher to identify and describe the mathematics that can be seen in them. The second activity was the meeting with the Adinkra craftsman to gain first-hand knowledge about the designing of the symbols to identify the mathematical concepts he employs in designing the Adinkra symbols. The next activity was mapping the identified mathematics concepts in the Adinkra symbols to the junior and senior high school mathematics curriculum for the teachers to select four concepts they would use with their students. Data generated in these prior activities provided knowledge upon which the intervention was developed.

Figure 4. 3

Cyclic Process of Design-Based Research



Observation of Images of the Selected Adinkra Symbols. The mathematics teachers were given the images of the 27 selected Adinkra symbols (Appendix H) to examine for two weeks and to identify mathematics concepts inherent in the symbols. The images of the symbols were used since the 2D form of the symbols is more common to students as they are seen in clothes, on doors, chairs, walls, etc. The mathematics teachers and I kept a journal to record and describe what we saw in each Adinkra symbol as mathematically related.

The researcher and the teachers met at the end of two weeks to have a conversation on what was noticed about each symbol which was related to school mathematics concepts. The conversations were about "what mathematics do you see in the symbols? How can you describe that concept(s) in the symbol(s)? or how can one identify that concept(s) from the symbol(s)?"

The conversations were videotaped since teachers mostly described the concepts seen in the Adinkra symbols with drawings. Teachers' journal records of mathematics concepts they saw in the symbols were also copied.

Meeting With the Adinkra Craftsman. The researcher and participating mathematics teachers met the Adinkra craftsman who consented to describe the creation of the Adinkra symbols. Since the interest was not on the carving of the symbols into the calabash, but rather the drawing of the symbols before they are carved, the craftsman was asked to demonstrate only the drawings. The craftsman demonstrated the drawing of the Adinkra symbols to us, and we inferred mathematical ideas that were demonstrated in the designing process of the selected Adinkra symbols. These meetings were videotaped and photos at the different stages of the creation of the Adinkra symbols were taken as the craftsman illustrated how they are created. We used observational protocols (Appendix I) to record the observation data. The advantage of using this technique to gather data is that it provided the opportunity for us to record information as it occurred (Creswell, 2012). During the observation, we asked the craftsman questions for clarifications on issues concerning the creation of the symbols, the meanings of some of the symbols, who can use a symbol, and when to use a particular Adinkra symbol. The researcher and the mathematics teachers also discussed the mathematics concepts that were observed in the creation process of the Adinkra symbols to synchronize them with what the teachers found from studying the images of the symbols. Data gathered at this stage informed the designing of the instructional activities for the study.

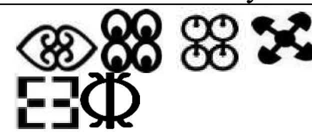
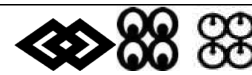
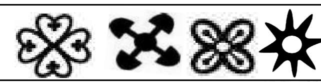
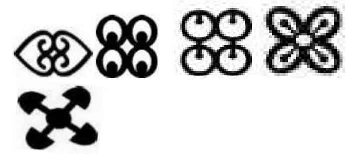
Curriculum Document Analysis. Another activity carried on before the intervention was a discussion with the mathematics teachers on the prescribed curriculum content and matching them with the concepts identified from the symbols. This enabled us to determine at which level the concepts are taught and the depth to which students are to be exposed to the concepts. This was to guide the design of the curriculum (activities) around the Adinkra symbols for the concepts that were identified. We examined the junior high school mathematics syllabus (MOE, 2012) and the senior high school core mathematics syllabus (MOE, 2010) to match the concepts obtained from the Adinkra symbols to the curriculum to identify the levels at which students are to learn the concepts. Four of these concepts/topics for each level were selected for the intervention.

Teachers Selected the Concepts and the Adinkra Symbols for the Lessons. As was mentioned, the mathematics teachers and I first identified the ethnomathematics in the Adinkra symbols and the next stage was to use the knowledge obtained from investigating the Adinkra symbols for culturally responsive teaching. Teachers then selected the concepts they recommended that their students learn. At the time of the study, only form two students were available for the senior high level (because form three students had already completed the term, and new students had not yet been admitted into form one). For the junior high level, the first- and second-year groups were available (the third-year group had completed waiting to enter SHS), hence, the teachers selected concepts that fell within the curriculum for these levels.

The senior high school (SHS) teachers selected transformation: reflection, translation, and rotation. Table 4.1 shows the concepts selected by the senior high school teachers with their curriculum objectives that were considered in the lessons and the related Adinkra symbols selected by the teachers for the lessons. From the core mathematics syllabus for the SHS (MOE, 2010), I observed that reflection and translation fall under *Rigid motion I* and are to be treated in the first year, which suggested that they might have taught that already.

Table 4. 1

The Concepts Selected for the SHS Lessons With Their Curriculum Objectives and Related Adinkra Symbols

Concept	syllabus objectives (MOE, 2010, p.21-22, 46)	Related Adinkra symbols
Reflection	1. identify and explain the reflection of an object in a mirror line. 2. describe the image points of shapes in a reflection.	
Translation	1. identify and translate an object or point by a translating vector and describe the image.	
Rotation	1. identify the image of an object (or point) after a rotation about the origin (or point).	
Multiple transformations	1. Describe the transformations that maps an object T onto different images of it, X, Y and T in a plane.	

The images of the symbols are from *West African wisdom: Adinkra symbols & meanings*, by Well-Tempered Web Design 2001-2007 (http://www.adinkra.org/htmls/adinkra_index.htm copyright 2001-2007). In the public domain.

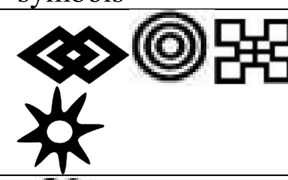
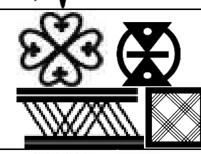
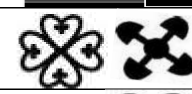
When I asked the teachers, Mr. Antwi responded:

> You see, we said earlier that our teaching is exam-driven, so, here we combine all the concepts under rigid motion and treat them in SHS two or SHS three, depending on the teacher who takes them from SHS one. Mr. Antwi

Mr. Antwi, head of the department for mathematics, explained that though the syllabus split some concepts into two or three years for the different levels, it is the practice of the department that all sub-topics of a concept are treated together at any level of the students. So, for example, percentages are split into two: Percentage 1 for SHS one and percentage II for SHS two, however, both concepts will be treated together, either in SHS one or SHS two. He continued to explain that the syllabus split transformation into sub-topics for the different levels, under the headings *rigid motion I and II*, however, they bring all concepts on rigid motion together and treat them together either in SHS two or SHS three. The other teacher, Mr. Oti, who was teaching the class at the time of the study, and who taught them from SHS one affirmed this and said: "I was planning to treat rigid motion in the third year, but I want to treat the three concepts with them now using the Adinkra symbols".

Table 4. 2

The Concepts Selected for the JHS Lessons With Their Curriculum Objective and Related Adinkra Symbols

Concept	Syllabus objective (MOE, 2012, p. 36- 37, 45-46, 56-58)	Related Adinkra symbols
Ratio and proportion	express two similar quantities as a ratio express two equal ratios as a proportion. use proportion to find lengths, distances, and heights involving scale drawing.	
Angles	discover that the sum of angles on a straight line is 180°and angles at a point is 360°calculate the sizes of angles between parallel lines.	
Rotation	identify a rotation of an object (shape) about a center and through a given angle of rotation.	
Enlargement	carry out an enlargement on a geometrical shape given a scale factor. determine the scale factor given an object and its image. state the properties of enlargements, with respect to their similarity, congruence, and orientation.	

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Mr. Oti also said that he wants the fourth lesson to be the combination of the three transformations. I asked the junior high school (JHS) teachers to consider the concepts they would recommend for their lessons with the students. Table 4.2 depicts the concepts together with the syllabus objectives and the related Adinkra symbols that the teachers selected for the concepts.

Mr. Obeng was the most willing to teach the lessons with his class, so the other two teachers asked him to select the concepts he wanted his students to learn so that they could assist to plan the lessons together. Mr. Obeng selected *Angles*, *Ratio and Proportion,* and *Rotation* using the Adinkra symbols. Mr. Oteng reminded him that *Rotation* is to be treated in JHS three. To this, Mr. Yeboah remarked: “This is their final term in JHS two, when we reopen for next term, they are going to JHS three, so I think treating a JHS three topic with them is okay”. The teachers agreed that it is okay to treat *Rotation* which was a JHS three concept according to the curriculum with the JHS two students. Having agreed on that, Mr. Oteng then asked that we include *Enlargemen*t to the list of the concepts for students. The teachers selected the four concepts—*Ratio and Proportion, Angles, Rotation, and Enlargement*. They also selected the Adinkra symbols to be used for each concept.

Preparing the Lessons. The mathematics teachers from each level selected four of the concepts related to the selected Adinkra symbols that they recommended to work with their students. We selected four concepts for each level because of the time frame of the study (it was barely three months left for the school term to end). Four lessons, each with one concept, were designed for each level. The researcher and the mathematics teachers determined the instructional starting points (ZPD) for the selected concepts (see Cobb, Confrey, et al., 2003; Gravemeijer & Cobb, 2006; Scott & Palinecsar, n.d), and then determined the learning objectives. This was done by studying the instructional objectives in the curriculum (syllabus) outline for those concepts and how the concepts were elaborated in the syllabus and textbooks, then establishing the relevant goal for the intervention, which is mathematics proficiency, and then establishing what students already know on which the new knowledge can be built (Bakker & Van Eerde, 2015). At this point, teachers were introduced to the five strands of mathematical proficiency (Kilpatrick et al., 2001) to design activities around the Adinkra symbols with the aim of helping students develop mathematical capabilities within those five strands. The researcher and teachers established possible means of supporting the learning process to enhance the development of mathematical proficiency.

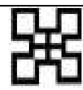
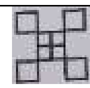
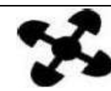
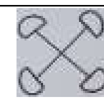
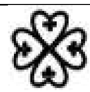
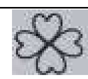
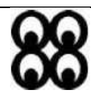
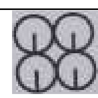
Decisions about how to design successful lessons, and the format for the lesson plans were made, the concepts were then shared among ourselves, each person prepared one lesson using the related Adinkra symbols and brought it forward to the group for discussion. I prepared the *Translation* lesson activity for SHS and the *Enlargement* lesson and worksheet for JHS. Mr. Antwi prepared the *Reflection* lesson and Mr. Oti prepared the *Rotation* and the *Multiple transformations* lessons for the SHS level. Mr. Obeng prepared the lesson plan and worksheet for *Ratio and proportion*, Mr. Yeboah prepared the lesson plan on *Angles* and Mr. Oteng prepared the *Rotation* lesson plan for the JHS level. Anderson and Shattuck (2012), McKenney and Reeves (2012), and Van Den Akker et al. (2006) all indicated that design experiments are expected to have practicability for users in real contexts. Hence, at the debriefing meetings on the planned lessons, we also tried to do the activities to test if they were possible for implementation. See Appendix J for the lesson plans and Appendix K for the worksheets. The lesson plan followed the usual lesson plan structure in Ghanaian schools: Introduction, class activities (development), and evaluation exercise.

Teachers Modified Some of the Adinkra Symbols. Researchers in ethnomathematics and mathematical modeling have observed that sometimes the sociocultural artifact we wish to use for mediation may have to be modified before using it for instruction (e.g., Bonotto, 2005; Rosa & Orey, 2010). The case for the use of the Adinkra symbols as mediation tools (Vygotsky, 1978) for mathematics instruction was not different. To be able to conveniently use some of the

Adinkra symbols, teachers thought they had to be slightly modified. Minor modifications were made to some of the Adinkra symbols before they were used in the lesson activities. One example was with the *Nsaa* (handwoven blanket) symbol. The *Nsaa* symbol has gaps at the common vertices of the large and small squares as shown on the left side of Table 4.3.

Table 4. 3

Original Forms of the Adinkra Symbols and Their Modified Forms

Name of symbol	Original form	Modified form	Name of symbol	Original form	Modified form
Nsaa (a hand woven branket)			*Akoma ntoasoɔ* (Extension of hearts)		
Nyame dua (God's tree)			*Asaase yƐ duru* (The earth is heavy)		
Mate masie (I keep what I hear)					

Original images of the symbols are from *West African wisdom: Adinkra symbols & meanings*, by Well-Tempered Web Design 2001-2007 (http://www.adinkra.org/htmls/adinkra_index.htm copyright 2001-2007). In the public domain.

We modified it to remove the gaps for activity on ratio and proportion, and enlargement with the junior high students. In the rotation lessons, with both the junior and the senior high students, we designed the activities that made use of the *Nyame dua* (God's tree) symbol such that the final image that students will form will not include the cross sign in the hearts. Also, we created the reflection assignment that was given to the senior high students around the *Asase yƐ duru* (the earth is heavy, or the earth has a weight) symbol such that the resultant image the students will form will exclude the spirals.

Phase Two: Conducting the Experiment

Every design-based research project aims to design and implement activities in the real-world context to study how the activities will influence learning (Anderson & Shattuck, 2012; Cobb, Confrey, et al., 2003; Cobb et al, 2016; McKenney & Reeves, 2012; Van Den Akker et al., 2006). One junior high teacher and one senior high teacher were selected to implement the lessons in their respective classrooms. Each lesson was based on one of the four selected concepts for each level.

The teaching and learning process with the Adinkra symbols followed Bussi et al. (2012) model of semiotic mediation where; the teacher designs activities with the artifact with signals regarding its use, followed by semiotic activities through writing reports where students write reports on the activity with the artifact, and the final stage which is the production of text by the whole class. In both the JHS and the SHS classes, students went through the activities designed for the intervention in groups and wrote down their reports. Each group (as in the case of the SHS) or individuals from the groups (in the case of the JHS) shared their reports which were used by the teacher for whole-class discussions.

The researcher observed the lessons, taking note of what was happening in the classroom (fieldnotes). The lessons were videotaped for further analysis. When students were engaged in small group activities, each group discussion was videotaped for analysis. Mathematics teachers and I also had a 20-minute briefing session before a lesson to discuss observations that someone had made about the lesson plan and what needed to be emphasized in the lesson. At the end of every lesson, the students' classwork was collected by the researcher to make copies and return to them at the next meeting. The participating teachers also kept a journal to write their reflections at the end of every lesson. These reflection notes were focused on what went well and what needed further attention and modification.

After each class, the researcher and the teachers held a debriefing meeting to discuss their reflections on the lessons and make recommendations for revision to the next lessons including discussions on classroom interactions that favored or did not favor the realization of the objectives of the lesson. The discussions at these sessions were used to refine the subsequent lesson activities before the activities were implemented (Cobb, 2000; Cobb et al., 2016). That is, the working papers were revised during the debriefing sessions if necessary.

Methods for Gathering Data During Phases One and Two. Data that were generated in phases one and two are indicated in Table 4.4 and Table 4.5 respectively, explaining when and why they were generated. Multiple forms of data were generated to ensure that retrospective analysis of the data from the experiment will result in rigorous, empirically grounded assertions (Cobb, Confrey, et al., 2003). Journals, field notes, video and audio recordings, and students' written work and drawings were used as data sources.

Table 4. 4

Data Obtained From the Preparation Activities (Phase One)

Data sources	When and why
Journals	During the investigation of the images of the Adinkra symbols, teachers kept a journal describing mathematics concepts seen in the images/appearance of Adinkra symbols (Cobb, McClain, et al., 2003; Prediger et al., 2015). This was used as data for answering research question one (a).
Observational notes	When observing the craftsman creating the selected Adinkra symbols. The observation notes describe mathematics concepts seen through the process of creating the Adinkra symbols. Also, at the debriefing sessions with the teachers to discuss mathematics in the Adinkra symbols, the researcher took notes (Gravemeijer & Cobb, 2006). This was data for answering research questions one (a and b).
Debriefing session/conversations (audio/videotapes)	Debriefing sessions were held for teachers to describe the mathematics concepts in the appearance of symbols and how the concepts can be demonstrated through the symbols. We also had a debriefing session on the mathematics seen in the creation process of the Adinkra symbols and synchronized concepts identified in the images and those from the creation process. Also, we had conversations on how the concepts are connected to the school curriculum, and strategies to make the intervention a success. These conversations were videotaped or audiotaped (Cobb, McClain, et al., 2003; Gravemeijer & Cobb, 2006). This was used as a data source for research questions one (a and b) and the research question that emerged.
Working papers	These were generated in phase one and revised through the different episodes in phase two. This included the learning goals, instructional starting points, and conjectures about how the learning is to occur and means of supporting students' learning (Cobb, Confrey, et al., 2003; Bakker et al., 2015). Apart from using the working papers to guide the implementation of the experiment, it was also used with other data sources to answer research question three, and as a data source for the research question that emerged in the course of the study.

Table 4. 5

Data Generated During the Implementation (Phase Two)

Data source	When and why
Fieldnotes	During the implementation phase, the researcher observed and took notes of the sequence of happenings and interactions in the classroom (Gravemeijer & Cobb, 2006). This was the data source for research questions two and three.
Reflective journals	Reflections were written by the researcher and teachers after every lesson on both productive moments and moments of failure. This was used to refine the lessons for reteaching. The reflections show what works, why it works, and how it works (Cobb, McClain, et al., 2003; Prediger et al., 2015), the reflective journals provided data for research question three.
Videotapes	Students' small group activities were videotaped. Video recorders were placed at the center of the table of each group to record what the group was doing, so that we could record the kind of interactions and explanations that were taking place among group members (Cobb, McClain, et al., 2003; Gravemeijer & Cobb, 2006). The video recordings were transcribed and used as a data source for research questions two and three. Lessons were also videotaped. These video recordings were transcribed to support the field notes describing teacher – students' interactions and used as a data source for research questions two and three.
Audiotapes	Before lessons were taught, the researcher and teachers held 20 minutes briefing sessions, and 40 minutes debriefing sessions also took place at the end of every lesson to discuss the reflections and to effect the necessary changes to subsequent lessons. These meetings were audiotaped. Teachers were interviewed at the end of the lesson implementations about the affordances and constraints of the intervention and these interviews were audiotaped and transcribed for analysis (See Appendix L for interview questions) and used as the data source for research question three (Cobb, McClain, et al., 2003; Gravemeijer & Cobb, 2006).
Students' written work	Since most of the strands of proficiency were demonstrated through students' solutions to problems, it was wise for me to collect and make copies of students' classwork. The students' exercise books were collected and returned to them after copies had been made. The students' class works were analyzed for strands of mathematical proficiency that were demonstrated in their solutions (Cobb, McClain, et al., 2003; Gravemeijer & Cobb, 2006; Prediger et al., 2015). This provided another data source for research question two.

Phase Three: Retrospective Data Analysis

Since design-based research consists of contingencies of events where events are informed and constrained by earlier events, design-based researchers employ a procedure known as retrospective analysis to analyze all data gathered during the experiment (Bakker et al., 2015; Cobb, Confrey, et al., 2003; Gravemeijer & Cobb, 2006). The retrospective analysis involves the systematic analysis of extensive, longitudinal data sets generated in the course of the design experiment so that the resulting claims are trustworthy (Cobb, Confrey, et al., 2003). Thus, the data sets obtained from the research study were analyzed chronologically. The analysis followed Roper and Shapira (2000) data analysis steps of (a) coding for descriptions, (b) sorting for patterns, (c) identification of outliers or negative cases, (d) generalizing (in the context of the study) with constructs and theories and, (e) memoing-including reflective remarks. The interpretations of the findings are supported with data from the different phases.

Research Question One. To answer the research question: What mathematics concepts could be related to the Adinkra symbols? data sources that were meant for research question one journals, my field notes, videos, and observational protocols used to observe the creation of the Adinkra symbols were analyzed. Video recordings of conversations on the mathematics in the images, and in the creation of the selected Adinkra symbols, as well as audio transcripts of teachers' conversations after meeting the craftsman were transcribed and the transcripts were coded. The codes were based on mathematics concepts that were identified in sentences.

The coded transcripts, together with the teachers' journals describing the mathematics concepts in named Adinkra symbols and observational protocols and field notes I made from the conversations, were organized together according to mathematics concepts identified, not according to the Adinkra symbols. That is, the data were sorted and categorized according to the mathematics concepts that were identified in them. For example, when the craftsman was creating the Mate masie symbol, after drawing the square he said, “I will divide it into two at the middle from top to down, … divide it into two again at the middle from left to right”. This sentence was coded “symmetry”. In the teachers’ conversations after meeting the craftsman, one teacher made this observation: “We found transformational symmetries in the images, and in the creation, the craftsman created symmetries”. This statement was coded “transformation”. However, in the report, I merged the two ideas under the heading “transformational symmetries” because they are interrelated mathematics concepts. It is the type of symmetry that defines the kind of transformation, and again, the two concepts were found to be associated with the same Adinkra symbols. Narratives on each mathematics concept identified to be related to the selected Adinkra symbols were given, based on the video/audio transcripts, teachers’ journals, and my field notes. Again, the transcripts and my field notes were used to describe the themes of conversations teachers had regarding the selected Adinkra symbols. This result is presented in Chapter 5.

Research Questions Two and Three. To answer the questions: What strands of mathematical proficiency could students develop through culturally responsive teaching involving the use of Adinkra symbols in teaching mathematics concepts related to them? And how does the use of culturally responsive pedagogy involving Adinkra symbols promote students’ development of mathematical proficiency? Data that were generated during the implementation stage of the intervention were analyzed chronologically, episode by episode. The result is presented in Chapter 7.

To determine the strands of mathematics proficiency demonstrated by students during the intervention, videos of students’ group work were transcribed and these transcripts, together with students' written work/drawings, were organized into tables and interpreted according to Kilpatrick et al. (2001) model of indicators of strands of mathematics proficiency aligned with the aims of the Ghanaian mathematics curriculum. The indicators used to analyze strands of mathematical proficiency in the students' work are defined in Table 4.6 (adapted from Dias Corrêa, 2017). For briefness, for each lesson, I selected the group of students who presented much information in their conversations (as in the video transcripts) and their written work. So, data from one group of students is used as a representative of learning that occurred in each lesson. In the JHS class, students' solutions to the evaluation exercises were used for them. In the case of the SHS students, I used in-class activities for them instead of the evaluation exercises due to time constraints. Therefore, the evaluation exercises were their homework.

To interpret strands of mathematical proficiency demonstrated by students in the lessons, tables were used as presented in Table 4.7 (the other tables can be found in Appendix M). The tables are divided into four columns, the first column has numbers labeling stages in the students' solutions. Using a method developed by Dias Corrêa (2017), I categorized the students' solutions into three stages: stages 1, 2, and 3, representing the beginning, the middle, and the concluding phases of the solutions. The second column shows students' conversations as they worked through the solution to the problem (note that the students' conversations happened in both the English and the Twi languages, but to make the reading easier, I have translated everything to English. Sentences that were not completely given in English are italicized. The third column is images of students' written solutions, and the last column is the strand of mathematical proficiency that I have inferred from the students' conversations and written solutions.

Table 4. 6

Strands of Mathematics Proficiency and Their Indicators for Analysis of Students' Work

Conceptual understanding	Identifying the extent of concepts in a mathematical problem or statement. Using appropriate ideas and strategies in solving problems. Effectively using mathematical terms, ideas, and operations. Generating new knowledge. Making sense of a mathematical problem. Interpreting such representations as graphs, diagrams, and tables. Understanding why the procedure/theorem works.
Strategic competence	Devising a strategy to solve problems. Executing the strategy to solve the problem. Classifying using criteria. Generalizing, based on the used criteria. Devising a strategy to represent a mathematical idea.
Adaptive reasoning	Thinking logically about relationships among concepts. Thinking logically about relationships among situations (relate or compare situations). Giving logical explanations and justifications to express thoughts and their understanding of mathematical ideas. Transferring ideas from one situation/context to the other.
Procedural fluency	Selecting appropriate mathematical procedures and theorems in solving problems. Correctly applying appropriate procedures and theorems and tools in solving problems. Selecting appropriate mathematical tools in solving problems. Knowing how the tool works.
Productive disposition	Recognizing the usefulness of mathematics. Having a positive self-concept in mathematics. Willingness to persist in solving mathematical problems. Recognizing and respecting others in mathematics (ability to work in cooperation).

The color codes used for the different strands, as in Table 4.6, were used to code different parts of the students' conversations that indicate a particular strand. If a strand is identified in column four but its color is not found in column two, then it implies that I inferred it from the students' written solutions.

Table 4. 7

JHS Group Five's Solution to an Enlargement Problem With the Akoma (Heart) Symbol

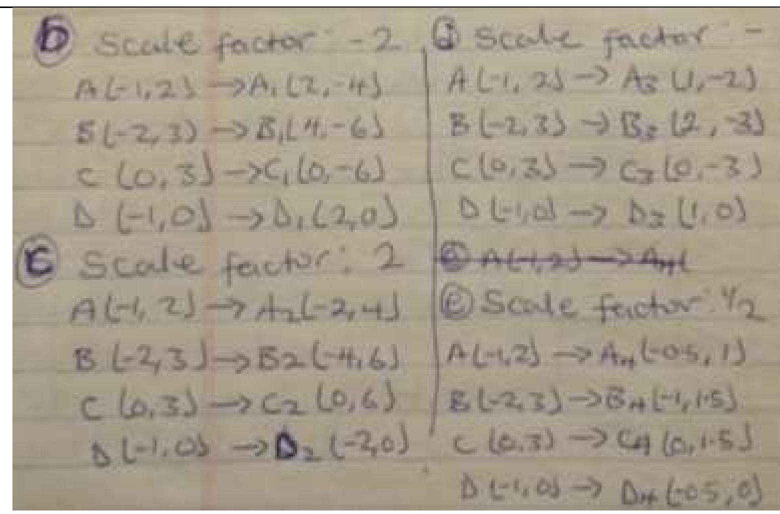
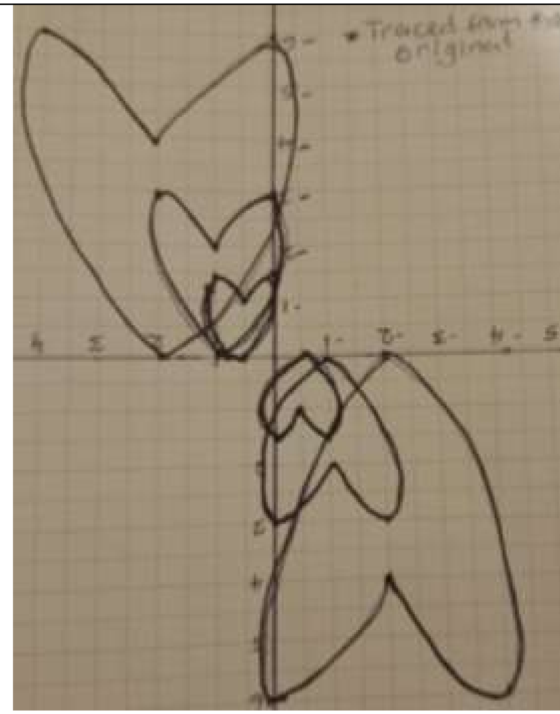
Stage	Students' conversations	Students' written solutions	Strands of mathematical proficiency
1	Amoako: We need to draw the x and the y-axes with the scale given. Boadu: *Okay, I will draw the axes then we draw the object first.* Amoako: Locate the coordinates of the points so that we draw the first *Akoma.* Boafo: *Sir, is explaining something let's listen to him and do what he says.* Osei: Join the points to form the heart as Sir did.		Strategic competence Conceptual understanding Procedural fluency
2	Osei: Let's find the image coordinates first before we go to the graph again. Amoako: Are we not supposed to work everything on the graph? Boafo: *He, is right oh, we need the image points else we can't draw the images on the graph.* Boadu: *We find the image coordinates by multiplying the object coordinates by the scale factor. Each person should do one from b to f and then I will write your answers here.* Boadu: *I have done b and c, give me the results for the d, e and f.*		Strategic competence Adaptive reasoning Procedural fluency Productive disposition Conceptual understanding

Table 4.7 Continued

2	Boafo: *Now, locate the points on the graph. F or d, point A, neg one and the two goes to point A one, one and negative two.* Boadu: Let's do it in order, from b to f.		
3	Amoako: Let everyone locate the points they calculated and draw the *Akoma*. Boadu: *Okay, let me do my b and c and I will pass it to you to do yours. But make sure you draw the heart well. If you can't draw it, after locating the points give it to me to draw the hearts through them.* Boafo: *This is interesting, with all the negative scale factors the heart turned upside down.* Osei: *This means, scale factors can be used to enlarge different Adinkra symbols to form another design.* Boadu: Now, what name do we give to the design? Amoako: *Since it is like the heart is growing from small to big, let's call it 'love grows'.* Osei: *We can call it love differs from person to person*. Boafo: *But the Akoma means patience, not love, let's call it you need big patience in difficult situations.* Boadu: *If you agree with me let's combine love and patient and call it "grow in love and patience".* Osei: *Yes, that name sounds good enough, and it contains what we all said.*		Productive disposition Conceptual understanding Adaptive reasoning Procedural fluency

The inferences about students' learning that were generated from the videotapes and the students' written work, as well as my field notes and teachers' reflective journals in one episode, were examined against those generated in the other episodes (Bakker & Van Eerde, 2015; Cobb, McClain, et al., 2003) to make final claims about strands of mathematical proficiency that occurred during the experiment. The final claims were justified by backtracking through the data sources from the four episodes of the experiment of each level (JHS and SHS) **(**Gravemeijer & Cobb, 2006). Results on the strands of mathematical proficiency that emerged in the intervention are presented in Chapter 7.

Research question three about how the use of the Adinkra symbols through culturally responsive instruction led to the emergence of the strands of mathematics proficiency was answered through memoing. The interpretations made about the strands of mathematics proficiency that emerged (research question two) were examined against the working papers, noting how each implemented feature of the intervention contributed to the emergence of the strands of mathematical proficiency. First, audio recordings of the briefing and debriefing meetings during the implementation stage, and the teachers' interviews and video recordings of lessons, were transcribed verbatim. The transcribed data, together with my field notes and the teachers' reflective journals, were analyzed for relationships and explanations on the affordances of the intervention. The transcribed data, teachers' reflective journals, and my field notes were used to support the explanations about how the implemented features contributed to students' mathematics proficiency as reported in Chapter 7.

The Question that Emerged in the Inquiry. A question drawn in the process of the inquiry is "What features of culturally relevant pedagogy emerged from teachers' deliberate efforts to incorporate the Adinkra symbols into mathematics lessons?" To answer this question, I categorized the teachers' ideas that were expressed during the briefing sessions on the preparation of the lessons into topics as language, group work, independent inquiry, and Adinkra symbols. The transcripts were coded, subsequently modified, and then used as the topics in the report. However, the topic "Adinkra symbols' was split into five as the context of tasks, significance of the symbols, Adinkra search, posted images, and renaming. For example, the question "Can we allow the students to talk about the significance of the meanings of the symbols?" was coded "significance", and the statement "I think we will need to post the images of the symbols with transformational symmetries on the classroom walls with their names and meanings" was coded "posted images" I then compared the teachers' suggested ideas on how the lessons could be implemented to ensure the emergence of students' mathematics proficiency against various views of sociocultural theory and culturally responsive pedagogy. This was done to ensure the suggested ideas were consistent with the culturally responsive views. Literature that describes various strategies that can be employed for culturally responsive teaching and those that support sociocultural views on learning were sought for and used to justify the suggested ideas of the teachers to answer this question. These features of culturally responsive teaching employed in the study are presented in Chapter 6.

Ensuring Credibility of the Results

O'Leary (2014) stated that one way to ensure the credibility of a qualitative study is through the use of triangulation. That is using more than one source of data to confirm the authenticity of each source (see also Creswell, 2014; Swan, 2014; The Design-Based Research Collective, 2003). During the retrospective analysis, more than one source of data was used in both phase one and phase two of the intervention. The results obtained from each phase are credible because they were generated from multiple sources. Again, inferences generated and tested for specific episodes were tested in the other episodes. For example, inferences that were derived on mathematical proficiency in episode two were also tested for in the other episodes (Bakker & Van Eerde, 2015; Cobb & Whitenack, 1996).

Another method that is used to test the credibility of research findings is member checking. A participant is asked to check that interpretation of events, situations, and phenomenon gels with the interpretations of insiders (Creswell, 2014; O'Leary, 2014). Cobb (2000) shares a similar view by saying that the trustworthiness of design research results is enhanced by the extent to which it is critiqued by other research team members. Results obtained in phase one were confirmed by mathematics teachers before conjectures about the lessons and the activities were developed. These conjectures were also tested in the different episodes and refined continually until the last episode (Bakker & Van Eerde, 2015; Cobb, & Whitenack, 1996).

Again, all topics/concepts discussed with the students were based on the teaching syllabus of the JHS and SHS levels. The eight lesson activities were not designed by the researcher alone. Each participant designed at least one of the teaching activities, and the activities were tested by all participants before the implementation. All these enhanced the credibility and validity of the results of the study.

Ethical Concerns

Flinders (1992) stated that we need ethical guidance in seeking to protect the research participants. Flinders offered a framework that was used to guide this study. The four frameworks that Flinders proposes are utilitarian, deontological, relational, and ecological.

In utilitarian ethics, we are looking to produce the greatest good for the greatest number of people; it involves seeking consent, avoiding harm, and assuring participants of confidentiality of the study (Flinders, 1992). Before embarking on this research study, I obtained ethics approval from the University of Alberta's research ethics board. Upon arriving in Ghana, I sent the University of Alberta's approval letter attached to a "request for permission letter" to the Ghana Education Service office of the municipality within which the research was situated to seek authorization to carry on the study with mathematics teachers and students. Consent letters explaining the purpose of the study and requests for voluntary participation from mathematics teachers were given to teachers who showed interest to participate. Also, oral consent was sought from parents of students whose teachers implemented the intervention in their classrooms, in addition, the students had to assent to participate in the study before they could participate. Parents' oral consents were sought through telephone conversations explaining the study and its purpose. A teacher introduced me to the Adinkra craftsman, and I introduced the study to him and sought his consent to participate in the study (see Wilson, 2008). The consent forms explained the purpose of the study, and assured participants of the confidentiality of the study, and assured them that the study was for research purposes only and that they would be free to opt-out at any point of the study if they wished.

Deontological refers to the fact that moral conduct cannot be fully justified in terms of consequences and must be consistent with "standards such as justice and honesty" (Flinders, 1992, p. 104). Deontological ethics is concerned with reciprocity, avoidance of wrong, and fairness. As the researcher in this study, I was engaged in dialogue with the participants and paid attention to my interactions with them, I believe that I was influenced and they were also influenced by the interactions, and by that, together with participants, we co-created mathematical knowledge from the Adinkra symbols, and designed interventions on this mathematical knowledge. Craftsmen were informed about the potential value of children using the Adinkra symbols for mathematics learning. As Gallagher and Fazio (2017) said, researchers of design-based studies need to invest in building an interpersonal relationship with participants in order to gain their trust and commitment (see also Cobb, Confrey, et al., 2003).

Relational ethics places our attachments and regard for others at the center of our considerations. It involves collaboration with participants, avoidance of imposition, and confirmation of findings by participants (Flinders, 1992). Thus, being the researcher and also acting as a participant, I was enabled to build a relationship with the other participants which means a community of researchers/participants was built for this study (Wilson, 2008). As mentioned earlier, since the researcher was in collaboration with the mathematics teachers, it was important for me not to impose my views/interpretations on them, but rather, through dialogue, we all came to a shared conclusion on issues. That is, the relationship between the researcher and participants was sustained through negotiations (Cobb, Confrey, et al., 2003; Gallagher & Fazio, 2017). For this reason, all the participating teachers came to a unanimous agreement on the two teachers who will implement the lessons in their classrooms. The other teachers willingly volunteered as teaching assistants when the lessons were being implemented in their respective levels. Also, through dialogue, the mathematics teachers, students, and I established their classroom norms.

Ecological ethics takes account of the environment in which individuals are working and is concerned with cultural sensitivity and responsive communication (Flinders, 1992). One important thing I had to keep in mind throughout the study was to show the people that I respected them and their views. In the culture of the Akans, men are meant to be respected (all my teacher participants were men) so to get their commitment throughout the intervention, I had to show them respect by respecting their views. Students' views were also respected, and opportunities were given to them in the course of the intervention to express their views and they were encouraged to respect each other's views. Again, the Adinkra symbols are highly adored in the communities, hence, I, as the researcher, had to show the same respect to the Adinkra symbols.

In this chapter, I have described the methods employed for data collection and analysis, In the upcoming chapters, I will present the findings of the study in the different phases. I have supported my interpretations with data from the different phases of the study for the reader to ascertain the trustworthiness of the claims.

CHAPTER FIVE

Mathematics Observed in the Images and the Creation of Adinkra Symbols

The educational dimension of the ethnomathematics program is to find mathematics inherent in cultural objects and practices and include them in the school curriculum (Rosa & Orey, 2016). This study involved teachers (research participants) and the researcher investigating mathematics inherent in the Adinkra symbols, ways to incorporate their mathematics in lessons and to investigate the students' mathematics proficiency that could emerge from the lessons. The symbols are found on wall paintings and in carved wood and metals. They are also stamped on cloths used for many purposes. The Adinkra symbols are prominent visual representations of the values and beliefs of Akans of Ghana. The symbols date back to pre-colonial times and continue to be replicated by craftsmen using their traditional practices. Recognizing that there are likely to be important differences between the mathematics identified from images of the Adinkra symbols and the actual creation of the symbols, there were two stages dedicated to investigating the mathematics of the symbols. The first stage involved the teachers and the researcher studying images of Adinkra symbols for the mathematics that could be related to them. The second stage involved the observation of an Adinkra craftsman (who creates the symbols out of calabash which is used for stamping the symbols onto fabrics) creating Adinkra symbols. The purpose of these exercises was to identify Adinkra symbols that could be used as mediating tools for the teaching and learning of mathematics (Bussi et al., 2012; Radford, 2013). This chapter integrates and presents the findings from these two exercises of the study.

Mathematics teachers investigated printed images of 27 Adinkra symbols that they selected to be common within the community. Each teacher then identified 10 of the symbols that they deemed to have inherent mathematics concepts. In total, the teachers identified 22 of the 27 Adinkra symbols to have mathematics concepts related to them. The concepts they identified included: transformational symmetries, ratio, and proportion, similar shapes, scaling (enlargement and reduction), linear and geometric sequence, angles, area of sectors of circles, decimal numbers, and percentages. These 22 Adinkra symbols and the concepts that were found to be associated with them can be found in Appendix N. Not all 22 Adinkra symbols that were discussed with their mathematics concepts were used in the lessons the teachers created. For brevity, I will discuss only the concepts that were used in the intervention.

The concepts that were taught in the interventions using Adinkra symbols are presented in this chapter. For each of the concepts, the teachers' mathematical inquiry into the Adinkra images is presented followed by how the concept was also observed in the craftsman's creation process. Because the intention of the study is to better understand the possibility of using ethnomathematics as one dimension of engaging in culturally relevant pedagogy, I will also present, in this chapter, the topics of discussions that the teachers had about the Adinkra symbols. The results of the investigations revealed that, in the course of the teachers' interactions with the Adinkra symbols and discussions with each other and the researcher they made connections to the curriculum, to students' learning and teaching. The teachers were observed to do mathematics with the Adinkra symbols as they explored them, and they also learned mathematics and social values of the Adinkra symbols from the craftsman.

I start the discussion of the mathematics concepts that the teachers found about the Adinkra symbols with transformational symmetries.

Distinguishing Transformational Symmetries From Adinkra Symbols

Like other researchers who have identified symmetry as an element of several Adinkra symbols (Abiola & Biodun, 2010; Arthur, 2017, Mireku, 2014), the teachers in this study also discussed the symmetry of Adinkra symbols. Table 5.1 presents images of Adinkra symbols, and the type of symmetry found in them.

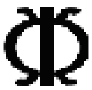
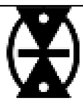
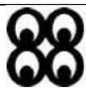
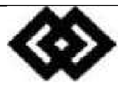
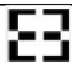
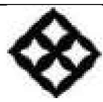
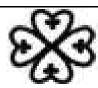
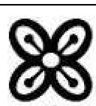
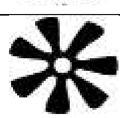
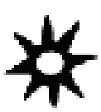
One of the teacher's descriptions of the *Wawa aba* symbol makes use of the definition of symmetrical shape as a shape that has one half of it the same as the other half. He used the metaphor of folding to describe that when the *Wawa aba* symbol is folded along the vertical line the halves will fall exactly on each other. He also identified the line of symmetry as the line of reflection, and finally, indicated that the symbol could be formed by reflecting half of it using the y-axis as the reflection line. Mr. Obeng said the following about the *Wawa aba* symbol (See Table 5.1 for the Wawa aba symbol).

> We can say (pointing to the vertical line in the middle of the *Wawa aba* symbol) that the symbol has a reflection in the line y-axis. That is reflection under rigid motions. When you divide or fold the symbol along this line, one half of it will fit exactly on the other half. The symbol can be formed using the y-axis as the line of symmetry and drawing each half on the opposite sides of the y-axis. Mr. Obeng

Unlike the junior high school teachers, who were more interested in the lines of symmetries in the symbols and only defined symmetry with reflection, the senior high teachers started their discussion of the symbols using transformational symmetry. They talked about the symbols according to the kind of transformation they found in each symbol. They used the language of reflectional, translational, and rotational symmetries to discuss the Adinkra symbols. For example, Mr. Abu who opened the discussion had this to say about the *Aban* symbol: "I observed that the *Aban* symbol is made up of four congruent rhombuses placed in different positions. Hence, it can be thought of to have the concept of translation of the basic shape, rhombus".

Table 5. 1

Adinkra Symbols and the Kinds of Transformations Identified With Them

Symbol	Name	Mathematics concepts
	*Wawa aba (*Seed of Wawa tree)	Reflection
	*Mmere dane* (time changes)	Reflection
	*Mate masie* (I keep what I hear)	Reflection and translation
	*M'aware wo* (I will marry you)	Reflection and translation
	*Apa* (handcuff)	Reflection and translation
	*Woforo dua paa* (When you climb a good tree)	Reflection
	*Aban* (Fence)	Translation and reflection
	*Nyame dua* (God's tree)	Rotation and reflection
	*Akoma ntoasoɔ* (extension of hearts)	Rotation and reflection
	*Bese saka* (Bunch of cola nut)	Reflection and Rotation
	*Ananse ntontan* (Spider web)	Rotation and reflection
	*Nsoroma* (Star)	Rotation and reflection

Images are from *West African wisdom: Adinkra symbols & meanings*, by Well-Tempered Web Design 2001-2007 (http://www.adinkra.org/htmls/adinkra_index.htm copyright 2001-2007). In the public domain.

The junior high school teachers described this same symbol simply by means of "four lines of symmetry." The senior high school teachers extended the idea of symmetry by pointing out that the symbol has four congruent shapes (shapes of equal sizes), positioned in different locations; hence, this symbol illustrates the translation of a basic shape. They identified the following symbols, *Mate masie, M'aware wo, Apa* and *Aban* also to have translational symmetries.

The next transformational symmetry they discussed was reflectional symmetry, observed in the following symbols: *Wawa aba, Woforo dua pa a, Mmere dane, M'aware wo, Mate masie, and Apa*.

Asserting that the *Mate masie* and *Apa* symbols (Figure 5.1) also have reflectional symmetry prompted me to ask teachers to reflect on how it was possible that the symbols could have both reflection and translation. These responses emerged:

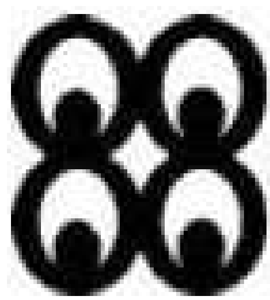
> I am looking at the *Apa* symbol, it has a vertical line of reflection (tracing the line with a pen). If you observe the symbol well, whichever is the first rhombus, or the object was moved or dragged horizontally to form the second rhombus of the image. This will imply that all y-coordinates remain unchanged, but the x-coordinates changed. So, in my view, any time one of the coordinates of the translation vector is zero, we get an image that will look like the reflection of the object. Mr. Antwi

> Yeah, the same thing applies to the *Mate masie* symbol, you see that it has a vertical line of symmetry or mirror line, the left side can be considered as the image of the right side under a reflection in the y-axis. If we consider the left as the image of the right under translation too, you can say that the object was dragged horizontally to the left (dragging

the hand on the table to the left), hence, the translation vector was $\begin{pmatrix} x \\ 0 \end{pmatrix}$, meaning there won't be a change in the y-coordinates. That means if the translation vector has the x-coordinate to be zero too, it can be regarded as a reflection in the x-axis. Mr. Oti

Figure 5. 1

Images of the Apa (left) and Mate Masie (right) Symbols



From *West African wisdom: Adinkra symbols & meanings*, by Well-Tempered Web Design 2001-2007
(http://www.adinkra.org/htmls/adinkra_index.htm copyright 2001-2007). In the public domain.

The teachers found the *Mate masie*, *M'aware wo*, *Aban,* and *Apa* symbols to have both reflectional and translational symmetries. What the two teachers said as presented above, indicates that some translations can be viewed as reflections, and this occurs whenever one of the components of the translation vector is zero. If the translation vector is $\begin{pmatrix} x \\ 0 \end{pmatrix}$ *or* $\begin{pmatrix} 0 \\ y \end{pmatrix}$, the result, after translation, will appear as a reflection in the y-axis or x-axis respectively. However, we observed during the lesson preparation stage that, though the symbols can be used to present both the concept of reflection and translation, the idea that when a translation vector is $\begin{pmatrix} x \\ 0 \end{pmatrix}$ *or* $\begin{pmatrix} 0 \\ y \end{pmatrix}$, the image can be said to be the reflection of the object, does not hold. You have to choose your coordinates carefully for each of the two transformations.

Lastly, the teachers observed that the *Akoma ntoasoɔ, Bese saka, Nyame dua*, *Nsoroma,* and *Ananse ntontan* symbols also have both rotational and reflectional symmetries. Based on these observations, teachers concluded that since these symbols have transformational symmetries in them, it is possible to form the symbols by the transformation of a basic shape/initial shape through rotation, reflection, or translation.

In what follows, I have presented the creation of three Adinkra symbols that were identified by teachers to have reflectional, translational, and rotational symmetries respectively, and then discussed how teachers found these symmetries in the creation process. As explained above, the teachers observed that the symbols could be created by transformation, but did the craftsman actually employ transformations to create the symbols? Let's look at the creation steps of the symbols by the craftsman.

Creation of the Mate Masie (I Keep What I Hear) Symbol

According to the craftsman (CM), the *Mate masie* symbol means "to learn to keep information secret". He said that in the Akan community, a wise person is regarded as someone who does not spread rumors but ponders over the information before reacting to it. He explained that the *Mate masie* symbol can be used by anyone to signify wisdom and knowledge.

The following steps were involved in creating the *Mate masie* symbol, as demonstrated in Figure 5.2 below.

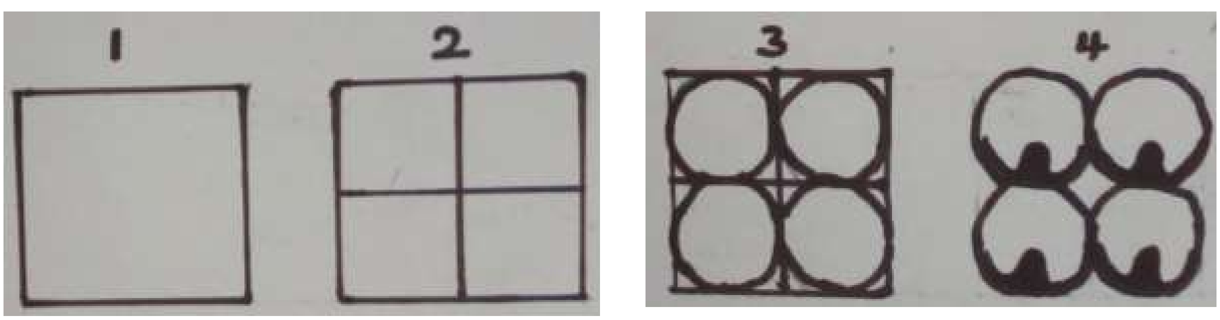
1. The craftsman drew a square with the help of a straight edge which was about 25cm-30cm long. The length of the square was equal to the length of the straight edge.

When asked if they use specific measurements, he responded "*Nea me twa no wo kora so de yɛ ntoma no susua sene wie.* (Those I draw on the calabash for stamping on cloths are smaller than this one)". The response shows that the size depends on how large you want the final symbol to be.

2. After drawing the square, he estimated half of each side and drew lines to divide the square vertically and horizontally. Four smaller squares that share a common vertex were formed out of the large square.
3. At the four corners of each smaller square, he created arcs with the freehand such that a circle is formed in each square.
4. Using a duster, he rubbed the unwanted parts of the squares and added bars from the circumference to about 2/3 of the radius of the circles to form the *Mate masie* symbol.

Figure 5. 2

The Stages in the Creation of the Mate Masie Symbol



Creation of the Apa (Handcuff) Symbol

The craftsman said that the *Apa* symbol was initially created by Adinkra craftsmen to signify slavery because of the colonial rule in Ghana, but its meaning has been changed to signify law and order, and justice. He also said the *Apa* symbol is used to admonish people to refrain from deviant behaviors, and there is a proverb attached to this Adinkra symbol which says: *“ɔne ko a ne pa da wo nsa no, na n’akoa ne wo* (you are a slave to the one whose handcuffs you wear)”.

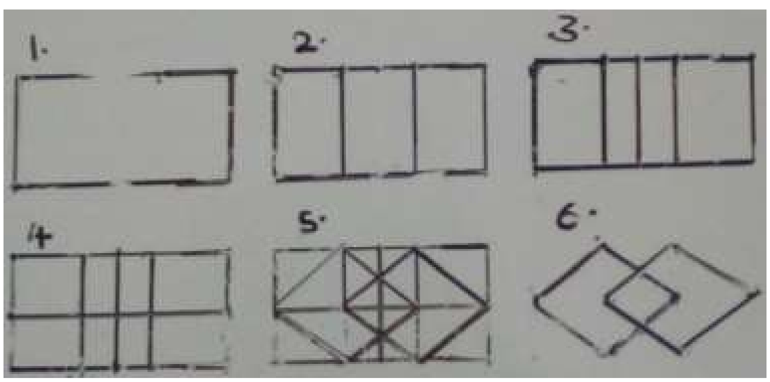
To create this symbol, he said:

> *Me drɔɔ sokwaɛ nso ntwimu baako ware kyɛn ntwimu baako* (I am drawing a square, but the length of one side is longer than the other) *ɛfa no a ɛware no mɛkyɛ mu mmiennsa pɛpɛɛpɛ* (I will divide it into 3 equal parts through the longer side). CM

1. The craftsman drew a rectangle with the help of a straight edge.
2. He estimated the length of the rectangle into thirds and divided it into three equal parts breadthwise, to form three congruent rectangles as shown in step 2 of Figure 5.3.
3. He then estimated the length of the middle rectangle into halves and divided the middle rectangle into two equal parts, using the straight edge.

Figure 5. 3

The Stages in the Creation of the Apa Symbol



4. The craftsman now estimated and divided the original rectangle into two halves, lengthwise, using the straight edge. Eleven intersections were formed at this stage.
5. The craftsman used the straight edge to draw lines to join pairs of intersections as shown in step 5 of Figure 5.3. The craftsman erased the parts he did not need to make the *Apa* symbol.

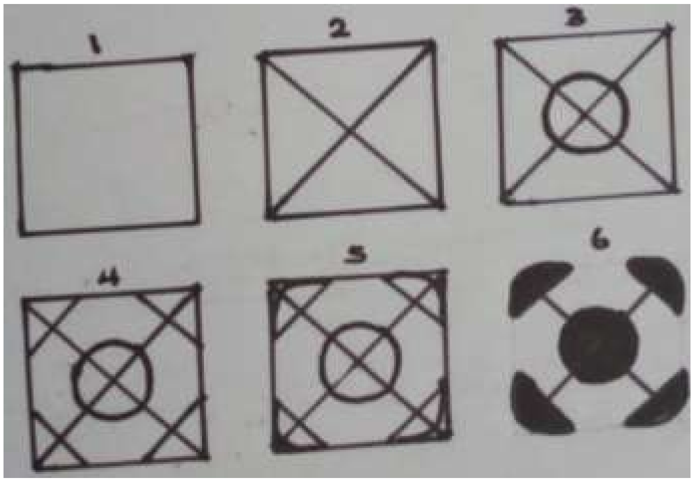
Creation of the Akoma Ntoasoɔ (Extension of Hearts) Symbol

The craftsman explained that the *Akoma ntoasoɔ* symbol signifies seeking ones' accord or making a treaty with someone. It is used to advise people to be committed to agreements they have entered into with others. For example, it tells married couples that each partner should play their part in the marriage agreement. This Adinkra symbol is created from a square. Figure 5.4 shows the stages in the creation of the *Akoma ntoasoɔ* symbol.

1. The craftsman first drew a square with the help of the straight edge, such that the length was as long as the length of the straight edge.
2. Using the straight edge, he drew the diagonals of the square to intersect each other.
3. And then, using the intersection point of the diagonals as the center and a radius of about 1/3 the length of the semi-diagonal of the square, he drew a circle with the help of the compass.
4. From the corners of the square, the craftsman estimated 1/3 the length of the semi-diagonals and placed marks at those points, then he used the straight edge again to draw lines at each corner of the square such that the lines pass through the points he marked and make triangles at each corner.
5. With freehand, he created arcs from one end of the new lines to the other end of the lines to touch the corners of the square to form a semi-circle at each corner. The unwanted parts of the square were then rubbed off. The craftsman now shaded the circle and the semi-circles to form the *Akoma ntoasoɔ* symbol which signifies understanding.

Figure 5. 4

The Stages in the Creation of Akoma Ntoasoɔ Symbol



From the creation of the symbols presented, teachers were of the view that the craftsman had the understanding of symmetry, and that he employed this mathematics concept in creating the symbols.

> He used the symmetries in the figures to create symmetry in the symbols. For example, in the *Mate masie* symbol, he created the vertical symmetry of the square and that resulted in the symbol having a vertical line of symmetry. Mr. Obeng
>
> It is the same concept we found in the images. The diagonal symmetries of the squares that the craftsman created resulted in the image that has rotational symmetry, and the vertical and horizontal lines of symmetry he created resulted in reflection and translational symmetry. Mr. Antwi

The teachers explained, as in the statements above, that the diagonal symmetries of the squares the craftsman created, as in the creation of the *Akoma ntoasoɔ* symbol, resulted in the formation of symbols that were found to have rotational symmetries, and the vertical and horizontal lines of symmetries of the square and rectangles created by the craftsman, resulted in symbols with reflectional and translational symmetries. That is, in the creation process, the craftsman demonstrated an understanding of the mathematical concept of symmetry. He did not necessarily use the different transformations, as observed by the teachers, to create the symbols. He knew that the square has both reflectional symmetry and rotational symmetry, and he employed this knowledge to create different Adinkra symbols. He used the square to craft Adinkra symbols that exhibit reflectional and translational symmetries such as the *Mate masie* symbol by creating the vertical line of symmetry*,* as well as those that exhibit both reflectional and rotational symmetries such as the *Akoma ntoasoɔ* symbol by using the diagonal symmetry lines. With the *Apa* symbol, he used the vertical and the horizontal symmetries of the rectangle to create the symbol that has both vertical and horizontal symmetries. Again, after partitioning the rectangle into three equal parts, he partitioned the middle rectangle into two, and that line of symmetry was used to ensure the overlapping of the two rhombuses that makes the *Apa* symbol appear as formed by translation.

The Concepts of Ratio and Proportion, and Area of a Sector of a Circle

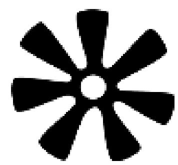
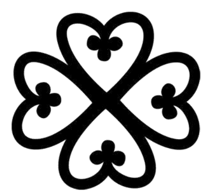
Ratio and proportion are mathematics concepts that were identified by the teachers to be related to the *Adinkrahene, Nsoroma*, and *Ananse ntontan* symbols. The images of the Adinkra symbols that teachers identified for possibilities of using them for ratio and proportion, and the area of a sector of a circle are presented in Table 5.2. The idea of ratio came up first with the *Nsoroma* and *Ananse ntontan* symbols. Teachers realized that these symbols form a circle and that each blade or hands (a basic shape in each of the symbols) represent a sector, hence, there is a relationship between the area of the entire circle and that of each sector, and also between the total angle of the circle and the angle of each sector.

> I realized that they have ratios and proportions in them. In each of them, the total area or the total angle of a circle which is $360°$ was divided by eight to get the ratio the circle is

to be divided. That is to say that the ratio obtained by dividing the area of the circle into eight will be proportional to the ratio obtained by dividing the total angle into eight. Mr. Yeboah

Table 5. 2

Adinkra Symbols for Ratio and Proportion, and Area of a Sector

Adinkra symbol	Mathematics concept	Adinkra symbol	Mathematics concept
 *Adinkrahene* (Chief of Adinkra)	Ratio and proportion	 *Ananse Ntontan* (Spider's web)	Ratio and proportion, Area of a sector of a circle
 *Nsoroma* (Star)	Ratio and proportion, Area of a sector of a circle	 *Nyame dua* (God's tree)	Ratio and proportion, Area of a sector of a circle

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This teacher considered the eight basic shapes (hands or blades) in the *Nsoroma* symbol but neglected the spaces between them; however, another teacher prompted him by saying that,

> I am going back to the ratios, in the *Ananse ntontan* symbol, we see that there are seven equal spaces between the seven blades. And the size of the spaces is equal to the size of the blades. Therefore, the total angle, or area, of the circle was divided into 14 but not seven. That means the ratio between the angles of the sector and that of the circle is $\frac{360}{14}:360$. Mr. Obeng

When Mr. Obeng said this, we all agreed, I then provided the following analysis to the teachers:

For the *Ananse ntontan* symbol which has 14 divisions, the ratio between the angle for each sector and the total angle of the circle is 25.71:360. That is, $\frac{360°}{14} = 25.71°$ as given by Mr. Obeng. Also, if the area of each sector, which was not known, is represented by A_1 and the total area of the circle is A, then according to the teacher's explanation, we can compare these two areas as A_1: A. According to the statement given by Mr. Yeboah above, these two ratios are proportional, that is, we can equate the two ratios to have:

$$A_1 : A = 25.71 : 360$$
$$\frac{A_1}{A} = \frac{25.71}{360}$$
$$\Rightarrow A_1 = \frac{25.71}{360} \times A$$

This is the proportional relationship the teachers and I found to exist between the areas of the sectors in the *Ananse ntontan* symbol and the area of the circle that was used to create it.

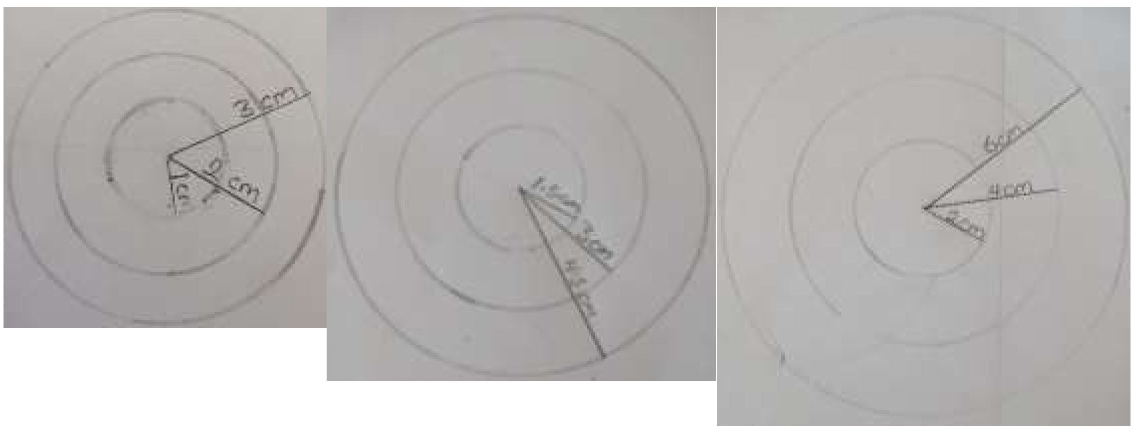
From the relation established above about the *Ananse ntontan* symbol, the teachers and I concluded that this Adinkra symbol and other symbols like the *Nsoroma* and the *Nyame dua* symbols, which are also formed from sectors of a circle, can be used to help students understand the relationship between the area of a sector and that of its circle.

Teachers observed that the *Adinkrahene* symbol is made up of three concentric circles whose radii are in a certain ratio. It was agreed that since the circles are of equal distances from each other, it implies their radii are set in a ratio. This was investigated by drawing different sizes of *Adinkrahene* symbol with the condition that the distance between successive circles is equal to the radius of the innermost circle. I have reproduced three of the teachers' drawings in

Figure 5.5. The investigation confirmed that the radius of the circles, when compared to each other, are in the ratio 1:2:3, starting from the innermost circle to the outermost circle. This ratio was used to draw more *Adinkrahene* symbols, and it was concluded that this ratio must exist in every *Adinkrahene* symbol, no matter the size.

Figure 5. 5

Three Different Sizes of Adinkrahene Symbol Illustrating the Ratio of 1:2:3 in the Radii of the circles



For example, if I start with the radius of the innermost circle as 4cm because the *Adinkrahene* symbol has the distance between the circles to be equal to the radius of the innermost circle, the radius of the second circle will be 8cm and that of the third circle will be 12cm. Comparing these radii gives 4:8:12 which reduces to 1:2:3. The creation of the *Nsoroma* symbol is presented here to show how the concept of ratio and proportion, which led to the concept of the area of a sector, was observed in the creation process.

Creation of the Nsoroma (Star) Symbol

The craftsman created the *Nsoroma* (star) symbol from a circle. See Figure 5.6 for the stages in the creation of the *Nsoroma* symbol. According to the craftsman, this symbol has a proverb attached to it that says: "*ɔba nyankonsoroma te Nyame na ɔnnte ne ho so* (The child of God's illumination reflects God, but not his own)"

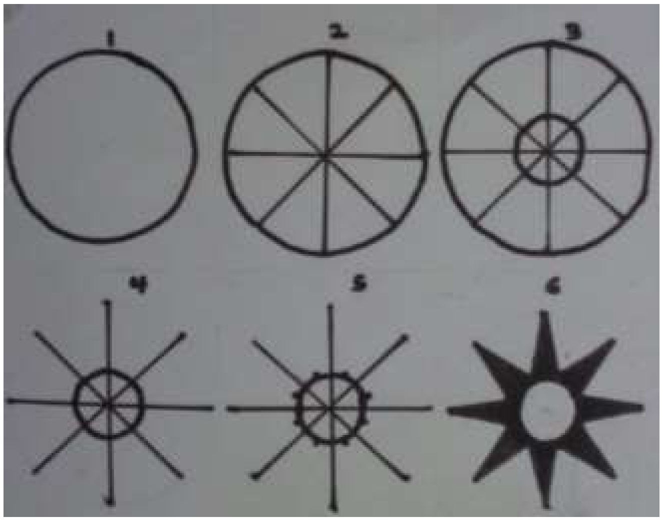
He explained that the *Nsoroma* symbol is used to admonish people to reflect on the nature and character of God, and it is also used to signify God's blessings. He further explained that this symbol, together with the *Adinkrahene* symbol, is printed on cloths for chiefs only. That is, a common person cannot put on attire or cloth that has a combination of the two symbols to a durbar of chiefs. To form this Adinkra symbol:

1. The craftsman first drew a circle with a compass using an estimated radius.
2. Using the straight edge, he divided the circle into eight parts. He did this by first dividing the circle into four parts by drawing two diameters of the circle such that they intersect at right angles. These four parts were then divided, each into two parts to form eight equal parts.

   Sɛ deɛ me yɛɛ Nyame dua no, saa ara na wei nso m'ɛkyɛ sɛɛkre no mu mmienu ne tenten so afa mfininfin pɛpɛɛpɛ, na m'asan akyɛ mu mmienu bio afiri nkyɛn mu afa mfininfin. Afei ɛnkyɛmu nnan a m'anya no m'ɛkyɛ ɛbiara mu mmienu (I am doing the same thing I did when creating the *Nyame dua* symbol, I will divide the circle into two vertically through the center and divide it again from the side through the center (horizontally), then, I will divide each of the four parts into two). CM
3. The craftsman then estimated about 1/3 the radius of the circle and used that radius to draw another circle within the first circle such that they share a common center.
4. The craftsman then rubbed the first circle and the drawing appeared as spokes of a bicycle wheel.
5. On the inner circle, he estimated and marked points such that each point divides the arc of the sector into two.

Figure 5. 6

The Stages in the Creation of the Nsoroma Symbol



6. To complete the drawing, the craftsman drew lines using the straight edge, from the points he marked to the top end of the spokes, such that they appear as triangles. He erased the diameters used to create the sectors and shaded the triangles to form the *Nsoroma* -star symbol.

The junior high school teachers made the following observations about the creation of the *Nsoroma* symbol:

> As we said when using the images, the craftsman has confirmed our observation by the strategy he used to divide the circle. Dividing the circle into halves, fourths, and then eighths. In each case we can say, the area of the circle, as well as, the angle of the circle, is divided into halves, then fourths, and then eighths. Mr. Obeng
>
> Yes, that is it. So, each sector is part of the area of the circle as well as the angle formed at the center of the circle. That is why they can be compared. Mr. Oteng

The teachers noted that, just as they described in the image of the *Ananse ntontan* symbol*,* the craftsman employed ratio to divide the circle into sectors, and since each sector is a portion of the circle, the area of the sectors can be compared to those of its circle, and the angle of a sector can be compared to the angle of the circle. Another observation made by the senior high school teachers about the creation of the symbol was that symmetries are created in shapes using the ratio 1:1.

> The craftsman divided the circle vertically into two, that ratio is one to one. The same one to one ratio was used to divide it horizontally, and then the same ratio is used to divide each quarter into halves again. Mr. Oti
>
> I have been thinking about this; you can see that he used the ratio of one to one in creating all the shapes with symmetry, I have not thought about that before we observed the craftsman's creations. Mr. Antwi

Mr. Oti's statement shows that the craftsman employed ratios throughout the creation process to divide the circle into eight sectors, and the ratio that was used to accomplish that task was 1:1. Mr. Antwi also noted that ratio was used to create symmetries. That is, the teachers observed the connection between the mathematical concepts of ratio and symmetry through the work of the craftsman. It can be said that ratios and proportions were critical in the creation of the Adinkra symbols that the teachers identified to have transformational symmetries.

Similar Shapes, Enlargement, and Reduction (Scaling)

The *Nsaa* symbol is made with two sets of squares—four large squares surrounding four smaller squares. Each set is of equal size. A teacher pointed out that if we take a pair of squares (a large square and its adjacent smaller square), we can compare their sizes, and since each set is made up of squares of the same size, it implies the ratio between each pair of adjacent squares is the same; hence, the pairs are proportional to each other.

> Talking about ratios, there is also a ratio implied in the *Nsaa* symbol or let's say the concept of enlargement in the *Nsaa* symbol. Note that, the four large squares are similar to the four smaller squares. The large squares are equal to each other, and the smaller squares are also equal to each other. This implies that a constant ratio exists between each pair of large and small squares. That is, to create the *Nsaa* symbol, you need the measure of either the set of large or small squares, and then with a constant ratio, you enlarge or reduce the sides to get the measure of the other set of squares. Mr. Obeng

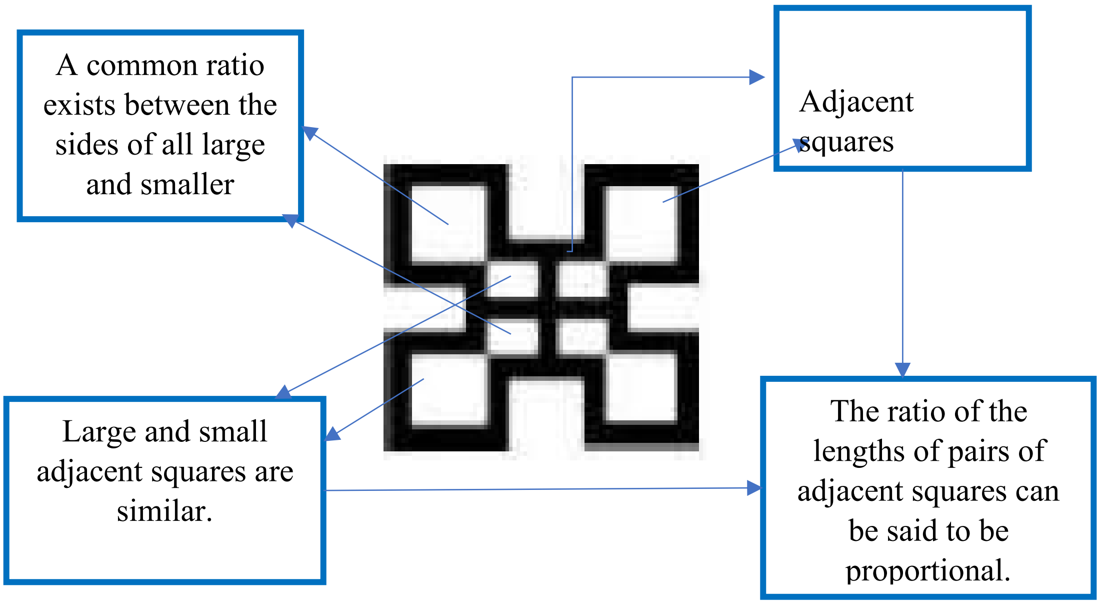
Mr. Obeng, in this statement, is referring to the mathematical concept of similarity between squares, which is that all squares are similar to one another and since the *Nsaa* symbol is made up of squares of two sizes, the symbol is made up of similar shapes. Again, similar shapes have a common ratio between corresponding sides. In the *Nsaa* symbol, there is a common ratio between the length of the sides of every pair of a large and small square.

Together with the teachers, we drew the following conclusions. The common ratio existing between the squares could be referred to as the scale factor that is used to enlarge the smaller squares to get the larger squares or reduce the larger squares to get the smaller squares. Therefore, knowing the lengths of the smaller squares, with a given ratio (scale factor), the length of the larger squares could be determined and vice versa, and the *Nsaa* symbol can be drawn. Again, since enlargement and reduction (scaling) always result in the formation of similar shapes, similar squares will be formed after enlarging the smaller square or reducing the larger square with a scale factor. Figure 5.7 shows the *Nsaa* symbol with the concepts related to it.

Since the ratio of corresponding sides in similar shapes is proportional, and since we have established that the ratio of the lengths of every pair of larger squares to the lengths of their adjacent smaller squares is proportional to each other, it presupposes that the squares in the *Nsaa* symbol are similar; hence, the teachers and I concluded that the *Nsaa* symbol can be used to introduce the concept of similar figures/shapes to students.

Figure 5. 7

The Nsaa Adinkra Symbol and its Related Mathematics Concepts



The *Nsaa* symbol can be created by determining the length of the sides of the larger squares or smaller squares and using a scale factor to reduce or enlarge the length of the sides.

Image of *Nsaa* is from *West African wisdom: Adinkra symbols & meanings*, by Well-Tempered Web Design 2001-2007 (http://www.adinkra.org/htmls/adinkra_index.htm copyright 2001-2007). In the public domain.

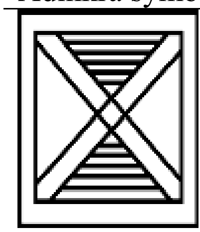
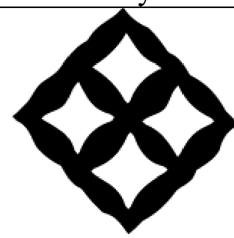
Similar shapes were also found in the *Mframadan* and the *Aban* symbols (Table 5.3). The *Mframadan* symbol has four similar triangles and the *Aban* symbol has four congruent squares which are similar to the symbol formed by the four together. Mr. Oti notes:

> The *Aban* symbol also has the concept of similar shapes. The reason is: The symbol itself is a rhombus (tracing the outline of the symbol with a pen) and there are four smaller

rhombuses in it (pointing to each small rhombus). We can say that the smaller rhombuses are similar to the large rhombus, which is the symbol itself. Mr. Oti

Table 5. 3

The Mframadan and Aban symbols

Adinkra symbol	Mathematics concept	Adinkra symbol	Mathematics concept
 *Mframadan* (Wind resistance house)	Ratio and proportion Similar shapes	 *Aban* (Fence/fortress)	Ratio and proportion, Similar shapes

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Teachers also observed the concept of scaling in the *Aya* symbol. To investigate the *Aya* symbol, lines were drawn to join the ends of each pair of leaves on the stalk, as shown in Figure 5.8, and the lengths of these lines were determined with the ruler. The ratio of the lengths of successive leaves was determined, and we obtained similar values which were all approximately 1.2. It was agreed that this *Aya* symbol was possibly not drawn to scale (with a correct measurement); if it was, then probably a constant ratio would have been determined. It was concluded that the *Aya* symbol can be drawn by determining the length of the top pair of leaves or the bottom pair of leaves, and then with a common ratio (set by the one drawing) the lengths of the remaining seven pairs of leaves could be determined. That is, the symbol could be formed by scaling (reduction or enlargement) of succeeding leaves with a common ratio. Arthur (2017) also identified enlargement to be related to this symbol.

Figure 5. 8

Aya Symbol With Straight Lines Drawn to Join the Tips of Each Pair of Leaves

Source: (Okyere's research data)

The creation of the *Nsaa* symbol, by the craftsman, is presented next with the concepts observed in the creation. The stages in the creation can also be seen in Figure 5.9.

Creation of the Nsaa (Hand-Woven Blanket) Symbol

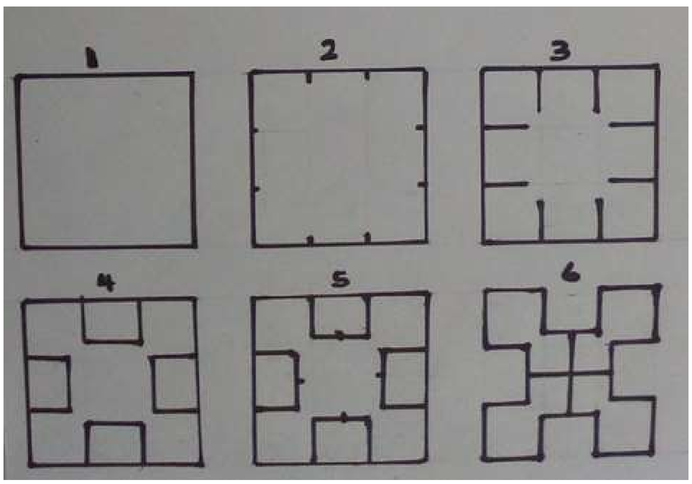
The craftsman said *Nsaa* is a hand-woven blanket that was common in the olden days. And the *Nsaa* blanket was known for its quality; hence, it is associated with the aphorism "*Nea onim Nsaa na oto nea ago* – (he who knows *Nsaa* will even buy the old/used one)". He used this aphorism to explain that the older the *Nsaa* blanket becomes, the more comfortable it becomes. Hence, the symbol is used to signify quality, durability, and excellence. This is how he created the symbol:

1. Using a straight edge, the craftsman drew a square.
2. With the same straight edge, he estimated and divided the sides into three by marking points on the sides such that three equal portions were obtained.

3. Within the square, he extended lines from the points he marked to about 1/3 the length of the square as shown in stage 3 of Figure 5.9.
4. He now joined the ends of these lines to form four rectangles.
5. He marked the midpoints of the widths of the rectangles, which were inside the square and drew a line to join opposite points.

Figure 5. 9

The Stages in the Creation of the Nsaa Symbol



6. He erased the width of the rectangles, which were on the sides of the square, to form the *Nsaa* symbol.

Teachers identified similar shapes to be associated with the *Nsaa* symbol, and it was said that the corresponding sides of these shapes will have a common ratio between them. That is, the idea of ratio and proportion came up because of the similar shapes found in the image of the symbol, but after observing the craftsman, it was noted that he employed the concept of ratio to create similar shapes in the symbols. This is how geometric similarity is related to the concept of ratio. Each result in the other. Two junior high school teachers made these observations about the creation of the *Nsaa* symbol:

Let's look at the creation of the *Nsaa* symbol, he used one to one to one to divide the sides of the square into three equal parts. And then the middle portion was used to create the four smaller squares by dividing each portion also by the ratio one to one. Mr. Yeboah

That is why the resulting shape has similar squares in it and the smaller squares appear as the reduction of the larger square. Mr. Oteng

The teachers noted that, after dividing the sides of the squares in the creation of the *Nsaa* symbol by a given ratio, similar squares were formed such that one set of squares appeared as resulting from the scaling of the other set of squares. That explains why the concept of the common ratio between corresponding sides, and enlargement and reduction was associated with the image of the *Nsaa* symbol. Here, we see another application of ratio, by the craftsman, to create another mathematics concept. Just as he used ratios to create symmetries, he also used ratios to create similar shapes.

Angle Properties of Parallel Lines

Angles could be associated with most of the Adinkra symbols; however, angle properties of parallel lines were evident in two Adinkra symbols, namely, the *Nhwimu* and the *ɔwɔ foro adobe* symbols shown in Figure 5.10 below. These two Adinkra symbols are designed with sets of parallel lines and transversals. It was noted that the two Adinkra symbols could be used to present the angle properties of parallel lines because of their appearance (they are made up of parallel lines). The following observation was made by one of the senior high school teachers:

The persistence, I mean the *ɔwɔ foro adobe* symbol can be thought of to have the concepts of angles. We can identify the following angles in it: angles at a point, corresponding angles, vertically opposite angles, co-interior angles, etc. Mr. Oti

Figure 5. 10

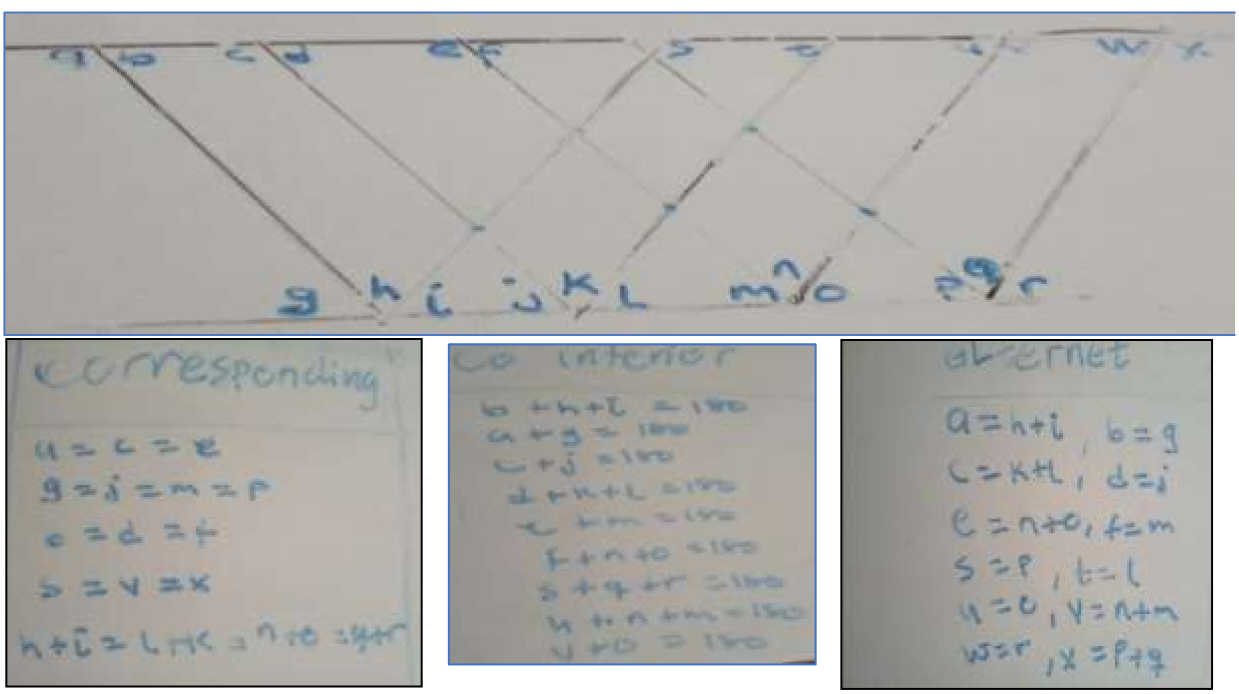
The Nhwimu (on the left) and the Ɔwɔ Foro Adobe (on the right) Symbols

The images are from *West African wisdom: Adinkra symbols & meanings*, by Well-Tempered Web Design 2001-2007 (http://www.adinkra.org/htmls/adinkra_index.htm copyright 2001-2007). In the public domain.

Mr. Antwi drew the *ɔwɔ foro adobe* symbol on the board with two sets of transversals, each set has four parallel lines (the original symbol has three sets of transversals each with four parallel lines). Figure 5.11 below illustrates how Mr. Antwi illustrated the various angles that could be identified in the *ɔwɔ foro adobe* symbol.

Figure 5. 11

The Ɔwɔ Foro Adobe With Some Angles Mark on it



What could make the use of this symbol unique for teaching angle properties of parallel lines could be the fact that some of the angles are split into two by some of the transversals, making some angles to be equal to the sum of two angles. From Figure 5.11, Mr. Antwi shows that angle ‘c’ is an alternate angle to $k+l$; hence, $c=k+l$. Again, angle ‘d’ is a co-interior angle to angle $k+l$, and since co-interior angles are supplementary, it implies $d+(k+l)=180°$.

Teachers noted that this illustration is not found in the textbook example that always shows the parallel lines with two transversals; hence, using the *ɔwɔ foro adobe* symbol would be helpful to students. Initially, Mr. Antwi drew only two transversal lines to the parallel lines, and Mr. Oti told him to draw more transversals by saying:

> This is what is in the textbook and the syllabus, but I think we can use this *ɔwɔ foro adobe symbol* to extend students’ thinking. What about drawing the four transversals in each set as in the symbol. Do that and let’s look at what happens. Mr. Oti

Next, I present the creation of the *ɔwɔ foro adobe* symbol which shows how the angle properties of parallel lines were observed in the process.

Creation of the Ɔwɔ Foro Adobe (The Snake Climbs the Raffia Palm) Symbol

This symbol was not created from a basic shape. The symbol is made up of two parallel lines and three sets of transverse lines. Before creating this symbol, the craftsman explained that the raffia palm, is a tree with thorns, so ordinarily, it is not expected that the snake (a crawling animal) should be able to climb it, but with tactics and perseverance, the snake is found on top of this tree. Therefore, this symbol is used to encourage people to be persistent and diligent, even when they face challenges in life endeavors. That is, the symbol is used to signify perseverance. The different stages in its creation are demonstrated in Figure 5.12. Steps to create this symbol:

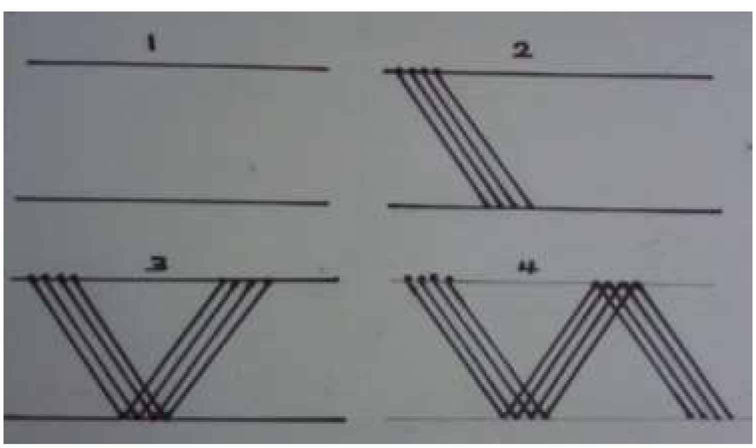
1. The craftsman created this symbol by first drawing a horizontal line with a length twice the length of the straight edge he was using. Then with a distance equal to the length of the straight edge, he marked a point below the line, dragged the straight edge forward and made another mark, then dragged it again to another point and made another mark there. He then drew a line joining these three marks parallel to the first line.
   He then marked four points which were about 0.5cm apart on the top line (first line). From these points, he drew oblique lines to meet the other parallel line by just slanting and dragging the straight edge.

When drawing the slanted lines, he emphasized that you need to be certain that the set of four lines are inclined at the same angles to the horizontal lines. I am inferring this condition from his statement below:

> *Ɛwɔ sƐ wohwƐ sƐ lines no nyinaa nkyiayƐ mu no yƐ pƐ, Ɛno nti na me mpagya dua no na metwe no ase no* (you must make sure the lines are inclined at the same angle, that is why I am not lifting the straight edge but dragging it). CM

Figure 5. 12

The Stages in the Creation of the Ɔwɔ Foro Adobe Symbol



1. From the ends of the set of the four lines, he extends another set of four lines to meet the top parallel line, again, being certain that their angles of inclination are the same.
2. From the ends of the new set of four lines (on the top parallel line), he extends another set of parallel lines to meet the second parallel line at the same angles of inclination.

The teachers and I made the following conclusions about the creation of this symbol. In the creation of the *ɔwɔ foro adobe* symbol, the craftsman applied the concept of corresponding angles by slanting and dragging the straight edge to different locations in drawing the sets of four transversal lines, such that the lines are inclined at the same angle. Angles at a point, alternate angles, and co-interior angles were found in the image of the symbol; however, from the creation process, the craftsman only applied the concept of corresponding angles to create the symbol, and that resulted in the formation of the other types of angles.

> There are other math applications in the *ɔwɔ foro adobe* symbol, first parallel lines, and then slanting the straight edge to draw the transversals shows the application of corresponding angles. Mr. Yeboah

Nature of Discussions Teachers had About the Adinkra Symbols

The meetings I had with the teachers about the mathematics they identified in the Adinkra symbols resulted in discussions that encompassed many aspects of mathematics teaching. I interpreted and classified the teachers' discussions under the following topics: doing mathematics, making connections to the curriculum, and making connections to teaching and students' learning. I also observed that the teachers learned mathematics as well as values of the Adinkra symbols from the craftsman's creation of the symbols.

Teachers Doing Mathematics With Adinkra Symbols

Taton (2015) suggested that a professional development program for teachers should allow teachers to do mathematics. Although this was a research study, it could also function as a professional learning opportunity for the teachers who participated. Teachers had an opportunity to do mathematics with the Adinkra symbols. There are instances in which they recognize mathematics concepts in an Adinkra symbol, and there are other instances in which they conjectured, analyzed, and justified their conjectures. For example, when teachers were identifying Adinkra symbols with rotational symmetries, they mentioned the *Nyame dua, Nsoroma,* and *Ananse ntontan* symbols to have rotation in them and Mr. Oti said:

> The same applies to the *Akoma ntoasoɔ* and *Bese saka* symbols, they are also formed by 90° rotations. Since the syllabus demands that we teach rotation of 90°, 180°, and 270° clockwise, and anti-clockwise, these symbols can be used to present it. Mr. Oti

To justify his claims, he demonstrated rotation in the *Bese saka symbol* by drawing the x-axis and the y-axis and formed one oval in the fourth quadrant with one coordinate as the origin and the other coordinates as (-3,3), (-2,1), and (-1,2) and rotated it 90°, 180°, and 270° clockwise to form the complete *Bese saka* symbol using the rotational transformation rules. Figure 5.13 shows the image of the *Bese saka* symbol, and what the teacher formed on the x-axis and the y-axis is shown in Figure 5.14.

In Mr. Oti's illustration in Figure 5.14, though the resultant image formed is not exactly as the *Bese saka*, the concept he wanted to prove was shown clearly by him using the mathematical formula for clockwise rotation to find the coordinates of the image points of the oval and drawing the images of the oval under 90°, 180°, and 270° rotations to form the complete symbol. Davis and Simmt (2006) also supported developing pre-service mathematics teachers' courses around mathematics doing to foreground their qualities for mathematics teaching. That is, the teachers' qualities for teaching were enhanced through the opportunity they had to do mathematics with the Adinkra symbol.

Figure 5. 13

Image of the Bese Saka Symbol

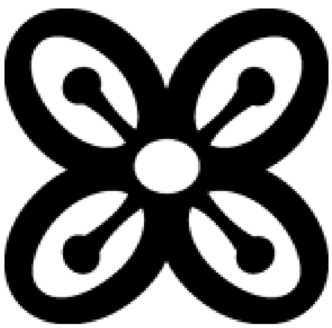


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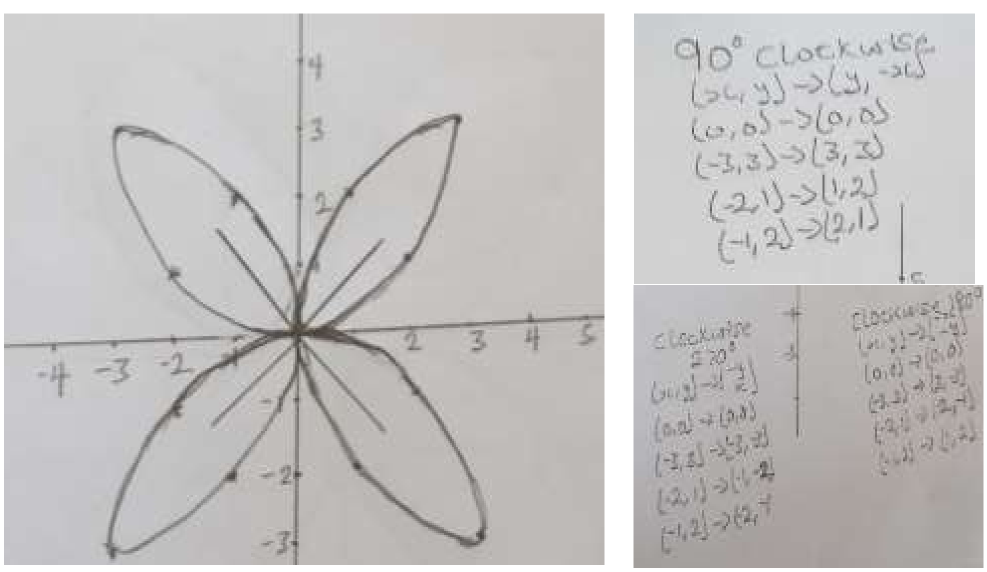
Figure 5. 14

Mr. Oti's Illustration of Rotation in the Bese Saka Symbol



Mathematics education researchers suggest that students, when doing mathematics, should reason and justify their reasons (for example, Kilpatrict, et al., 2001, Niss, 2015; Silver &

Stein, 1996). The teachers in each group worked together, raised questions and concerns, inquired, and collaboratively built on each other's ideas. There are instances where they disagreed and debated and finally agreed on an idea. There are other instances where an argument was refuted because it could not be justified. Teachers explained and justified their ideas and used mathematical reasoning to refute some ideas.

The sociocultural theory, which is the basis of this study, shows that human learning occurs through social interactions. Learning first occurs through interaction with people. Teachers collaborated to learn mathematics from and with the Adinkra symbols. The teachers collaborated as they interacted and negotiated meanings. The mathematical ideas presented in the preceding section did not come from only one person, rather the mathematics was created by the teachers working together.

Many mathematical ideas that came up revealed other mathematical ideas. For example, the finding that similar shapes are related to the *Nsaa* symbol led to the emergence of the concept of ratio, which also led to the idea of scaling (enlargement or reduction). Boesen et al., (2014) explained that students should be able to connect mathematical ideas to other concepts in mathematics. The idea of ratio and proportion in the *Nsoroma* symbol led to the concept of the area of a sector. The teachers in the study had a repertoire of mathematical concepts, and as they investigated the symbols, they came to realize how the concepts were interrelated. Thus, they were building connections between different mathematical concepts. Making connections between ideas is one of the aims of the SHS mathematics curriculum (MOE, 2010). Teachers were making connections between different mathematical concepts, inferring that they were in a position to communicate this objective to their students. Through this experience with the

Adinkra symbols, teachers learned mathematics by developing new relationships between concepts.

Just as students are expected to communicate in mathematical language when doing mathematics (Niss, 2015; Turner, 2011), the teachers also had the opportunity to communicate mathematically about the Adinkra symbols. Working with the Adinkra symbols offered the teachers an opportunity to communicate mathematics outside the classroom, and also to communicate about a non-mathematical object. The teachers used mathematical terms such as mirror line, y-axis, co-ordinates, and transversal, just to mention a few, to describe some Adinkra symbols. That is, terms that would only be understood by mathematicians were used to describe symbols created by a non-mathematician.

Teachers Making Connections to the Mathematics Curriculum

The mathematics concepts that the teachers could identify from the Adinkra symbols are those concepts that they are already familiar with from the curriculum. Teachers' mathematical knowledge is based on the content of the curriculum that they are expected to share with their students. It is also based on the curriculum they have learned as students. The teachers did not identify a single example of a concept that was beyond the junior and the senior high school levels. This shows how familiar they are with the mathematics curriculum, and how capable they are to link the curriculum content to other situations and objects which are found outside the mathematics classroom. Thus, they could draw a link between their everyday experience with Adinkra symbols and the school mathematics curriculum. In Mr. Oti's statement in the preceding section, he made a connection to the curriculum by identifying that the mathematics syllabus requires students to learn the rules for clockwise and anti-clockwise rotations through 90°, 180°, and 360° about the origin, so, he also formed his version of the *Bese saka* symbol around the origin.

As already mentioned, ethnomathematics seeks to find mathematics in the culture of people and blend it with school mathematics. The role of the teacher is to interpret and bring the curriculum into the classroom. Alsubaie (2016), for example, noted that teachers are the most important persons when it comes to curriculum implementation. As implementors of the curriculum, they interpret and identify models and language to be used to communicate the curriculum to students. The teachers did not only identify mathematics concepts in the Adinkra symbols, but they also thought of what the symbols could be used for regarding the school curriculum. For example, when we were discussing the *Adinkrahene* symbol, one of the junior high school teachers said:

> By using '*a*' (as radius of the innermost circle) as Mr. Obeng has it, we can employ the *Adinkrahene* to teach *Relations and Functions*. For example, we can set the sequence *a*, 2*a*, 3*a,* 4*a* and 5*a,* which are the radii of the circles, as the domain and ask students to find the co-domain, we can say the circumference. Mr. Yeboah

This teacher is linking his observations on the *Adinkrahene* symbol to curriculum content he is expected to address with the students. In this sense, the teacher shows that the *Adinkrahene* symbol could be used as a model to communicate the concept of *Relations and Functions* to students. He shared the view that the pattern obtained from the *Adinkrahene* symbol can be related to the topic *Relations and Functions*. Again, he is aware that the pattern was obtained from circles (it is the radii of the different circles in the *Adinkrahene* symbol); hence, he says they can set the pattern as a *domain* and the *co-domain* should be something related to the circle, such as the circumference. He is aware that the circumference is a function of the radius, hence, he relates the pattern obtained from the radii of *Adinkrahene* circles to the circumference of the circle. That is the level of knowledge the teacher has regarding mathematics concepts that are part of the school curriculum. A culturally relevant curriculum aims to integrate students' cultural mathematics knowledge through ethnomathematics (Balamurugan, 2015; François & Van Kerkhove, 2010; Rosa & Orey, 2016). Teachers identified mathematics they already knew in the cultural symbols of the students and linked them back to the school mathematics curriculum that they are studying with their students.

Teachers Making Connections to Teaching and to Students' Learning

The teachers understood how content is presented in the school curriculum (syllabus and textbooks) and could identify possibilities for extending students' thinking, by extending what is in the curriculum with the Adinkra symbols. When the *ɔwɔ foro adobe* symbol was being investigated for angle properties of parallel lines, one of the teachers went to the board to draw it but with only two transverse lines and the other teacher prompted him saying:

> This is what is in the textbook and the syllabus, but I think we can use this *ɔwɔ foro adobe symbol* to extend students' thinking. What about drawing the four transversals in each set as in the symbol. Do that and let's look at what happens. Mr. Oti

Students' learning is the primary concern of teachers. In teachers' instructional decision-making, the students are paramount. Teachers are always concerned about the methods and strategies they can employ to enhance students' learning and achievement. In this statement, Mr. Oti demonstrates the understanding of his students' thinking and imagines possible extensions. He has ideas about what he can do with the Adinkra symbol to help students learn and get a better understanding than they might have by using what is demonstrated in the textbook. Probably, he will accept this symbol for instruction, not only for the fact that it has a mathematics concept, but rather, for the fact that it could be used to extend his students' thinking about the particular mathematics concept. Rosa and Orey (2016) argued that a cultural connection is a key aspect in developing new strategies for the mathematics teaching and learning process, as it allows students to perceive mathematics as an important part of their cultural identity (see also Lunney Borden & Wagner, 2011; Gay, 2010: Ladson-Billing, 1995a; Orey & Rosa, 2007, Rosa & Orey, 2011).

Another example was when teachers were investigating the *Nyame dua* symbol for the mathematical idea of rotation, and one of them was wondering if students would be able to form the *Nyame dua* symbol on the grid, this was the response from another teacher:

> It will not be as big as you say, since the heart is considered as a basic shape, and they talk about it right from the KG. What we need to give to students is the coordinates of points in the plane such that when we tell students to join them to form the heart, they can easily follow the points to know which one should be joined to which one to form it. Maybe the teacher will draw a heart on the grid and identify the coordinates of the points that when given to children will make it easy for them to draw it on the grid. Mr. Antwi

The purpose of developing a curriculum, including the teacher's designed activities for a lesson, is to meet the needs of students, which is 'learning'. "Curriculum development should be viewed as a process by which meeting student needs leads to improvement of student learning" (Alsubaie, 2016, p. 107). When teachers are planning to teach, they consider what and how they could help their students achieve the stated objectives, by finding what students already know, and by building connections to it. In the statement of Mr. Antwi above, he notes that students already have a knowledge of the 'heart' shape as a basic shape, from kindergarten. Now he considers how they (teachers) could lead students to draw the heart on the grid with given points, and subsequently, form the *Nyame dua* symbol on the grid through rotation of the heart.

What Teachers Learned From the Adinkra Craftsman

From our interactions with the craftsman and the conversations I had with the teachers about the processes of creating the Adinkra symbols, and about the lesson preparations, I observed that teachers learned two things from the craftsman: the first is the values the Adinkra symbols signify and the second is mathematics.

Teachers Learned Values of the Adinkra Symbols From the Craftsman

In the teachers' approach to the images of the Adinkra symbols, teachers referred to the meanings of the Adinkra symbols only once. It could be interpreted that the task that was given to them to investigate the images for mathematics concepts did not ask for the values of the symbols. Teachers, in their journals and their conversations, regarded the images as academic objects and forgot about the fact that the original intended use of the Adinkra symbols was to communicate the social values and spirituality of the Akans. That is, the teachers were more mathematical about the images of the Adinkra symbols than the meanings and values they signify.

The craftsman, on the other hand, started his discussion about the Adinkra symbols by telling us the names of the symbols, their meaning, when to use them, and who uses which of them. In some cases, he mentioned how the symbol emerged and told us proverbs that are associated with some of the symbols before demonstrating the creation of the symbols. It can be said from the teachers' investigations with the symbols and the craftsman's discussions that, while the meaning of the symbols and the procedure to create them were significant to the craftsman, the teachers were more interested in the mathematics they are familiar with and how it can be seen in the images of the symbols.

Gay (2002, 2010) noted that the knowledge teachers need for culturally responsive teaching includes knowing the cultural values of ethnic groups. The craftsman made it possible for us to reconnect to the social values of the Adinkra symbols as we connected the symbols to the mathematics we know. Teachers appreciated the social values of the symbols and suggested, as will be explained in the next chapter, that we should also discuss the social values of the symbols with the students in the lessons. One of the teachers even connected the social values of the symbols to one of the aims of the mathematics curriculum. Thus, the Adinkra craftsman made it possible for us to rethink the curriculum and how it relates to our cultural values.

Mathematics Teachers Learned Mathematics From the Adinkra Craftsman

One will wonder how teachers could learn mathematics from a craftsman who does not even know that he is using mathematics in his creations. From the teachers' conversations after meeting the craftsman, and a comment one teacher made in the interview after the intervention, I could infer that they made new realizations about the mathematics knowledge they have acquired over the years and taught their students.

Teachers developed an understanding of mathematics concepts and their interconnections by observing the creation process of the Adinkra symbols. From the statements of Mr. Antwi and Mr. Oti given under the creation of the *Nsoroma* symbol, it can be said that the teachers understood how the two concepts; ratio and symmetry, are related. The teachers learned from the craftsman that symmetries in shapes are created using the ratio 1:1. Though mathematics teachers have knowledge about symmetry and could identify shapes with symmetries and probably have formed symmetrical shapes before, and they also know ratios and can use ratios to compare quantities, they have never thought of how the ratio concept is connected to symmetries and only came to this realization from observing how symmetries are created in the Adinkra symbols by the Adinkra craftsman.

In the interview session after the intervention, when I was asking teachers about the affordances of the intervention, I was only interested in what happened in the classroom, however, a teacher referred to the craftsman's creation and said:

> I don't know if it is a result of the curriculum or our fault as teachers. For instance, we studied lines of symmetries and how to identify them in primary school, and then we studied the types of transformation at the JHS and SHS, but we never talked about how these two are related. I only learned how the different lines of symmetries lead to the different transformations after observing the creation of the *Mate masie* and the *Akoma ntoasoɔ* symbols. This study has really helped us, we have examples to make connections to when teaching. Mr. Yeboah

This teacher says he has learned how the different lines of symmetries of a shape like that of the square as used in the creation of the two symbols, *Mate masie* and the *Akoma ntoasoɔ* lead to reflection and rotation respectively and he can refer to them when teaching. He also admits he has acquired knowledge about examples he can use to teach mathematics concepts.

The teachers learned, from the creations by the craftsman, that the concept of similar shapes results from ratios. One of the teachers made the following comment when we were discussing the ratio in the creation of the *Nsaa* symbol:

> Let me confess, I have never thought of a scale factor as a ratio before. All my life as a student and now a mathematics teacher, what I know is to find the image of the object under enlargement with a scale factor of this or that. … Oh, we have a lot to learn. It

appears we can learn more about how the concepts are similar through their applications in real- life, like how the craftsman is creating these symbols with ratios and proportions. Mr. Oti

This teacher also admits that though he knows scale factors and when to apply them, he did not have an understanding that scale factors are ratios until he observed the creation of the Adinkra symbols.

Kilpatrick (2001) and Kilpatrick et al. (2001) noted that the quality of mathematics instruction is dependent on the teacher's mathematical knowledge (see also Hill et al., 2005; Petrou & Goulding, 2011; Shulman, 1987). Important components of the teacher's mathematics knowledge, to facilitate their students' learning of mathematics, include having models to represent mathematics concepts (Davis & Simmt, 2006; Fennel, 2006; Hill et al., 2005; Lesh et al., 1987), and knowing how the different concepts in mathematics are interconnected (Eli et al., 2013; Mhlolo et al., 2012; Monroe & Mikovch, 1994).

Fennel (2006) stated that representation should be an important element of lesson planning and that one of the questions teachers must ask themselves, when planning lessons, is what models or materials (representations) will help me to convey the mathematical focus of the lesson? (See also Davis & Simmt, 2006; Mhlolo et al., 2012). The teachers acquired knowledge on examples of Adinkra symbols that could be used as models for the different mathematics concepts.

Mhlolo et al. (2012) in their classroom observation of four Grade 11 teachers found that, in most cases, teachers' mathematical connections were either imperfect or superficial, and this compromised the students' opportunities for making meaningful mathematical connections, and for developing an understanding of how different concepts are alike. That is, a teacher's lack of knowledge on connections between the different mathematics concepts can hinder students' mathematics understanding and their ability to make connections between concepts. However, from the teachers' comments presented and discussed above, teachers after observing the creation of the Adinkra symbols came to appreciate how some concepts are related, and this knowledge will help them in their teaching. It can be said from the above discussions that the mathematical understanding of the teachers has been expanded or enhanced by careful observation of the ethnomathematics in the objects (Adinkra symbols), as well as in the process of the creation of the objects.

In this chapter, I have presented the mathematics concepts that the mathematics teachers and I identified with the images, and in the creation process of the Adinkra symbols, and the topics of discussions teachers had about the Adinkra symbols. The chapter that follows presents how the knowledge acquired in this chapter was used to design culturally responsive interventions for students.

CHAPTER SIX

Implemented Features of Culturally Responsive Pedagogy

Advocates for culturally responsive pedagogy are of the view that the sociocultural knowledge of students that teachers incorporate into classroom teaching helps students make meaningful connections to the content of learning (Orosco & O'Connor, 2011). That is, culturally responsive pedagogy is consistent with D'Ambrosio (2001)'s view of ethnomathematics as an approach to teaching and learning mathematics that builds on a student's previous knowledge and background, the role that the environment plays in terms of content and method, and the past and present experiences from the immediate environment. Therefore, it is valuable for teachers to explore and identify students' sociocultural knowledge that could be related to the curricular content they are to share with students.

One of the contentions for culturally responsive pedagogy stressed by different authors is to include in the curriculum content issues and topics that relate to students' background experiences and culture (Abdularahim & Orosco, 2019; Gay, 2002, 2010; Ladson-Billings, 1995a, 2006; Villegas & Lucas, 2002, 2007). This is how culturally responsive pedagogy is well aligned with the use of ethnomathematics in the classroom. In Chapter Five, I reported on teachers' exploration of the Adinkra symbols of Ghana for mathematics concepts in the images of the Adinkra symbols and from the practices of an Adinkra craftsman creating the Adinkra symbols (ethnomathematics, D' Ambrosio, 1990; 2001; Rosa & Orey, 2011, 2016), for the purpose of using them as mediating tools in mathematics instruction. Through ethnomathematics, the participating teachers and I unearthed mathematics in the students' culture

(Adinkra symbols). This ethnomathematics (discussed in Chapter Five) was used as the starting point for us to plan for culturally responsive teaching.

Gay (2010) stated, "culturally responsive teaching encompasses curriculum content, learning context, classroom climate, student-teacher relationships, instructional techniques, classroom management, and performance assessments" (p. 39). That is, we planned activities for the intervention in the senior and junior high mathematics classrooms using the Adinkra symbols. After the teachers from each level selected the four concepts from the ethnomathematics found in the Adinkra symbols, the teachers and I planned lessons so that both the high school and junior high school teachers could teach students mathematics concepts with the symbols. Another aspect of culturally relevant pedagogy was reflected in the decisions about the learning context and classroom climate, including student-teacher interactions. The following section presents the culturally responsive teaching decisions that were made, and implemented, to enhance students' meaning making in mathematics, which was investigated in terms of the strands of mathematics proficiency.

Planning the Intervention

Designing the intervention was the last activity to complete the first phase of this study. After the teachers selected the mathematics concept for students to learn through the Adinkra symbols, I met with them to plan the lessons. I also introduced them to the conceptual framework I would be using in the analysis, the strands of mathematics proficiency as defined by Kilpatrick et al. (2001), and by relating each strand to the aims of the Ghanaian mathematics curriculum/syllabus. As described in Chapter Two, I found the aims of the Ghanaian mathematics curriculum, for the junior and senior high school levels, well-aligned with the strands of mathematics proficiency, although, they are not labeled as such. Because the purpose of the study was to investigate how the use of a sociocultural object for culturally responsive teaching could occasion the development of mathematics proficiency, I thought it would be wise for teachers to be aware of Kilpatrick et al. (2001)'s work, and to see how the strands of mathematics proficiency are similar to the aims of their mathematics curriculum. For instance, aims of the SHS and the JHS mathematics curriculum that imply strategic competence include:

- Develop the skills of selecting and applying criteria for classification and generalization (SHS) (MOE, 2010).
- Help children develop a variety of problem-solving strategies involving mathematics (JHS) (MOE, 2012).

When the aims of the curriculum were discussed with the teachers, by relating them to the strands of mathematics proficiency, the JHS teachers did not indicate any form of disagreement. However, the SHS teachers felt some incongruity between the aims and how content is presented in the curriculum:

> We teach according to the syllabus, and as we have seen, the general aims of the syllabus are similar to the proficiency you have mentioned. However, the same syllabus does not present the content in a way that will allow the achievement of these aims in the classroom. Mr. Antwi
>
> The type of exams we take here is also a contributing factor. No exam question will ask students to apply their knowledge to solving an everyday problem. Even, if you use an everyday example to teach, such examples will not be used in the WASSCE examination. In the end, you will be assessed by how well your students passed the WASSCE, not what they learned. … but I think this study is part of the process of change. Mr. Oti

Their comments revealed the fact that, although they are aware of the aims of the mathematics syllabus, their classroom work was to help students pass examinations. The statements also indicate that the teachers stress procedural fluency in their teaching. There was some agreement, by the end of our discussions that the aims of the syllabus should be their focus when teaching. We also discussed how to facilitate a successful lesson to enable students to comprehend the mathematics concepts through the use of the Adinkra symbols. The ideas that were raised in our discussions about the various supports that were needed to enhance students' learning, and how learners will interact among themselves, and with the teachers, during the lessons, led to decisions for the lessons. Our working papers, created during the discussions, document our initial conjectures for the implementation of the intervention (Bakker & Van Eerde, 2015, Cobb, McClain, et al., 2003, Gravemeijer & Cobb, 2006).

In this section, I present the decisions made and show how they are consistent with culturally responsive pedagogy and reflect a deliberate attempt to teach differently, using the cultural artifacts. The first decision made was to convert the curriculum or content objectives to a culturally responsive objective, so that culturally responsive means could be used to achieve the stated objectives for the topics, as well as for the aims of the curriculum. All the measures taken in the planning stage and implemented are illustrated in Figure 6.1.

Figure 6.1 illustrates how the aims of the curriculum relate to the content, and the objectives for the content (in other words, each informs the other). These content objectives were restated to reflect the cultural aspect we intended for the lessons (hence the restated objectives might correlate the aims of the curriculum). Hence, two-directional arrows were used to connect the aims, the content objectives, and the culturally responsive content objectives. Again, the stated culturally objects informed the culturally responsive teaching that, in return, reflected the achievement of the culturally responsive objectives as shown by a two-directional arrow. Lastly, the success of the culturally responsive teaching was dependent on five main elements: Adinkra symbols as context of activities, use of local language, Adinkra-related activities, teachers' facilitation, and cooperative groups. Each of these elements is discussed in detail in this chapter.

Teachers Adapt the Mathematics Curriculum Objectives for the Lessons

The teachers being aware of the relevance of instructional objectives in lesson planning and knowing that the lesson activities cannot be designed without stated instructional objectives, suggested the need to state objectives for each of the topics selected. In culturally responsive teaching,

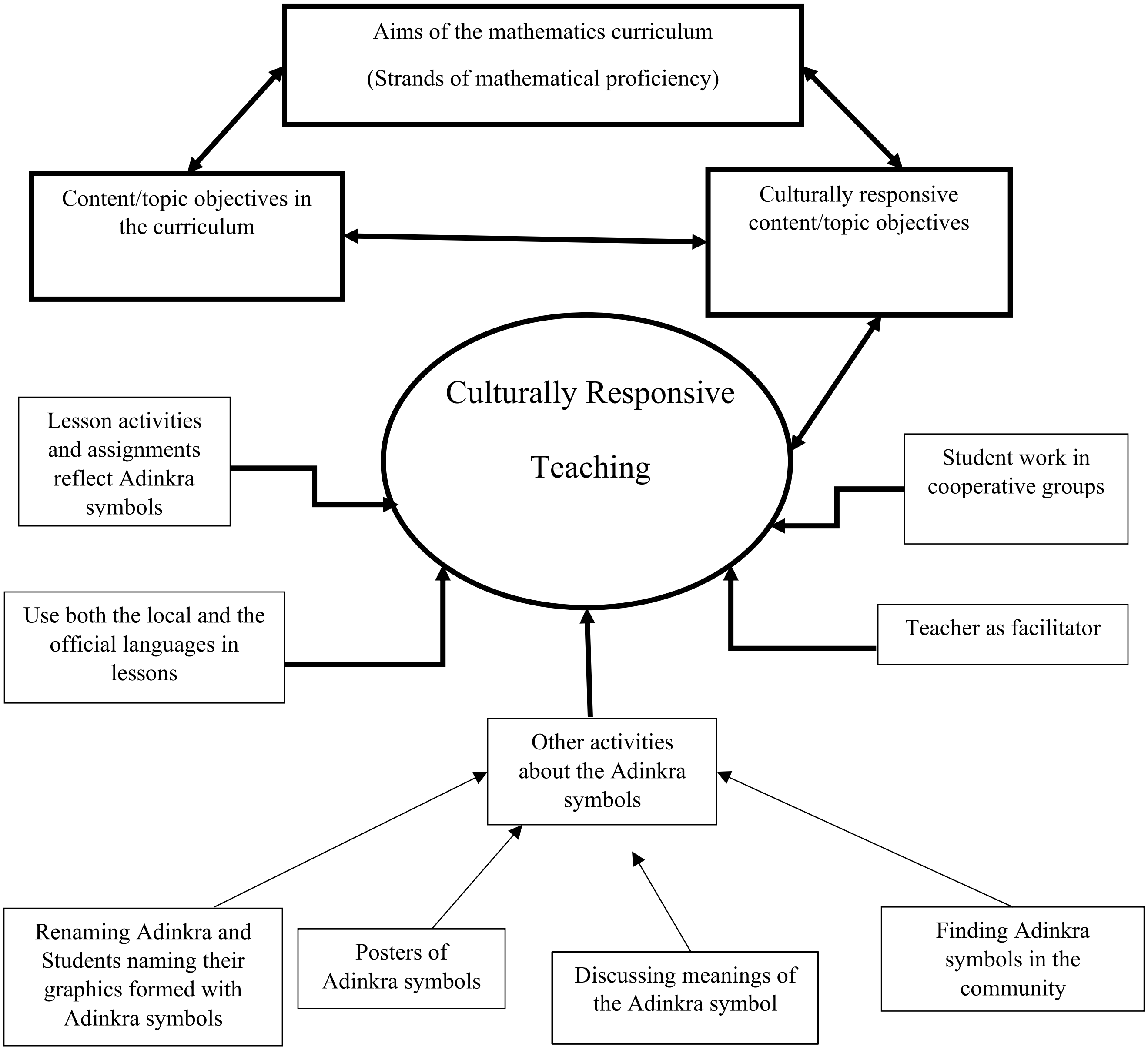
> The "knowledge of interest" is information about ethnically diverse groups; the "strategic thinking" is how this cultural knowledge is used to redesign teaching and learning; and the "bounds" are the reciprocity involved in students working with each other, and with teachers, as partners to improve their achievement (Gay, 2002, pp.109-110).

It can be inferred from Gay's statement that for effective culturally responsive teaching to take place, after identifying the elements of the students' culture you wish to use in the class, you must redesign the curriculum using the cultural knowledge you have gathered about the students.

The teachers and I had already gathered the mathematical knowledge that relates to the Adinkra symbols, and we wished to present this mathematical knowledge to students through culturally responsive teaching; hence the first thing that we did was modify and restate the instructional objectives from the curriculum to match with the intended teaching and learning materials.

Figure 6. 1

Components of the Model for Culturally Responsive Teaching in This Study



For example, for the lesson on translation, the syllabus had only one instructional objective; "identify and translate an object, or point, by a translating vector and describe the image" (MOE, 2010, p. 22). The textbook stated three objectives for the topic: 1) to identify and translate an object under a given translation vector, 2) to locate the image points of an object under a translation vector, and 3) to perform translations of shapes using a given vector (Asiedu et al., 2017, p.362). Since the objectives from the textbook imply the stated objective in the curriculum/syllabus, we modified the textbook objectives for the lesson on translation as follows:

1. Draw the *Apa* symbol on a graph and locate the coordinates of the vertices, for the original rhombus and its image.
2. Deduce the translation vector for the translation of the original rhombus to the image to form the *Apa* symbol.
3. Perform translations of shapes using a given translation vector.

With the objectives modified to suit the instructional material (which incorporated Adinkra symbols), the classroom activities for the lessons, as well as the evaluation questions (questions used to assess students' learning at the end of lessons), were described in the context of the Adinkra symbols. The Adinkra symbols that are related to the concepts were used in designing the activities for the lessons and as artifacts during the intervention for students to interact with (Scott & Palincsar, n.d.; Steele, 2001; Vygotsky, 1978). Fisher et al. (2012) indicate that for students to understand and meaningfully use mathematics, they must engage in meaningful activities on a personal level that facilitates the learning process. Because the Adinkra symbols are artifacts the students are already familiar with, it was expected that the use of the Adinkra symbols, as a context for mathematics problems, will make the activities not just challenging, but also meaningful to the students. Gerofsky (2004) mentioned that the genre of a mathematics problem must address the culture of the audience the problem is meant for, and this is exactly what we used the knowledge we gained by investigating the Adinkra symbols. We created problems and activities around the Adinkra symbols for the mathematics concepts to which they were related. Kilpatrick et al. (2001) also noted that using problem-solving strategies in the classroom provides a context in which all the strands of mathematics proficiency have the potential to be developed. Hence, creating activities and problems around the Adinkra symbols is expected to help with the development of the students' mathematics proficiency.

Students to Identify and Name Adinkra Symbols in the Community

The teachers suggested that the students' minds be prepared for the lessons by asking them to find Adinkra symbols in the community, record their names, and draw them. The teachers were of the view that, since the symbols have not been viewed mathematically before if students go out and find Adinkra symbols in the community, it will prompt them to think about them mathematically before the lessons.

> I think we should ask students, as early as possible, probably, a week before the lesson, to draw the Adinkra symbols they know and what they see around them. There are some symbols on the walls at the durbar grounds. We can ask them to pass there on their way home, to observe them and draw those they can draw for class discussion. In my opinion, that will make them think about the symbols before the lessons begin. Mr. Obeng
>
> Better still, I think we can ask students to do a tour of the environment and find the Adinkra symbols around with their names and meanings before we post ours in the classroom. Students can go around and note the names of symbols they see around. Mr. Antwi

These two suggestions show how the JHS and SHS teachers intended to help students become more aware of their environment, and how the Adinkra symbols are part of it. They expressed a desire for students to familiarize themselves with the symbols before the symbols would be brought to the classroom for instruction. From the comment of the junior high school teacher, it can be inferred that teachers are associated with the subject they teach, hence, a mathematics teacher asking students to find Adinkra symbols around them will cause the students to think about the symbols mathematically. I thought to myself, the students would be curious about why the mathematics teacher is asking them to look for the Adinkra symbols, hence, the curiosity to find out more about the symbols in the classroom with the teacher will be heightened. It would also enable those students who were not familiar with the Adinkra symbols to get acquainted with them before the lesson began.

This decision to encourage students to observe the symbols in their environment before using them for mathematics instruction is consistent with using ethnomathematics in the classroom, and the culturally responsive pedagogy views that learning should be contextualized and related to the experiences of the students (Gay, 2002, 2010; Rosa & Orey, 2011, 2016). In other words, asking the students to find Adinkra symbols in their environment created another opportunity for students to familiarize themselves and connect with the cultural symbols that are prevalent in their society.

Students' engagement with classroom tasks is recognized to have a positive impact on their mathematics learning (Fredricks et al., 2018; Kilpatrick et al., 2001; Park, 2005). To motivate students' engagement, teachers are required to create a bridge between what students are familiar with and what they are to learn. The Adinkra symbol search activity was used as part of the process of creating the bridge. As Mr. Obeng remarked, "it will prepare their minds for the lesson", and if students are mentally ready for a mathematics lesson, we know their engagement with the activities will be high. The Adinkra symbols, gathered by the students, were used by the teachers to introduce the purpose of the intervention lessons to them. In the JHS first lesson, the teacher asked the students to draw the Adinkra symbols they found and then questioned them about their process. Students showed their symbols as we went around to view them. The following conversations ensued between the teacher and the students.

Mr. Obeng:	Ernest, did you use any math ideas in drawing the symbols?
Ernest:	Eei, I did not use any math. Oh, I don't know.
Gladys:	Sir, I drew the *Gye Nyame* (Except God), I used the circle and used division to divide it into two and use that line for the fist.
Pokua:	Sir, I drew the *Mate masie* (I keep what I hear) with four circles.
Adjei:	I used the square to form the *Woforo dua pa a (when you climb a good tree).*

Using these exchanges, the teacher introduced the purpose of the lessons by saying, "The craftsman who creates these symbols also used mathematical ideas. In these few days, we are going to learn mathematics through the Adinkra symbols."

Students to Discuss the Significance of the Adinkra Symbols

The aim of educating our children includes the transfer of our cultural values and beliefs to them (Bishop, 1988), and teachers are agents in this process. Another idea that came up in the teachers' discussions was the need to stress the significance of the symbols to the students during the lessons.

> In every lesson, can we allow the students to talk about the significance of the meanings of the symbols? If you are using the *Woforo dua pa a* symbol, for example, let students talk about how its meaning is significant to them as students, us as teachers and parents, the leaders of the school or the society, etc. Mr. Antwi
>
> One of the aims of the mathematics syllabus is to help our students to develop tolerance, diligence, … I am not sure if I am saying it as it is, but there is something like that and

these are the values that the Adinkra symbols signify. So, in my opinion, we should also stress the meanings and what the symbols signify in the teaching. Mr. Oteng

The productive disposition will also be achieved by the 1st idea you came up with that we should ask students for the significance of the symbols to them. Mr. Oti

Here, we see how the teachers were willing to emphasize the Adinkra symbols, the values the symbols signify, and to use them to achieve a stated aim of the curriculum. In Mr. Antwi's suggestion, he emphasized that we let students relate the meanings to themselves as students, to them as teachers and parents, and to the leaders of the society. So, *Woforo dua pa a* - (when you climb a good tree) which is used to signify *encouraging someone who is taking a good pursuit in life*, could mean to students that education is a good pursuit in life and therefore they should encourage each other not to give up. Learning about the proverbs and meanings of the Adinkra symbols, and relating them to their own lives, is a means of encouraging cultural integrity among the students (Ladson-Billings 1995b). Mr. Oti, in response to the suggestion of Mr. Antwi, also linked the significance of discussing the meanings of the Adinkra symbols to the curriculum by noting that it will help students to enhance their productive disposition. Ladson-Billings (2006) explained the cultural competence component of culturally relevant pedagogy as "helping students to recognize and honor their own cultural beliefs and practices while acquiring access to the wider culture" (p. 36).

Discussing the significance of the Adinkra symbols in mathematics lessons might lead students to accept the Adinkra symbols to be worthwhile, and to adopt the values the symbol is expected to enact on the lives of society. Culturally relevant pedagogy is a "theoretical model that not only addresses students' achievement, but also helps students to accept and affirm their cultural identity while developing critical perspectives that challenge inequities that schools (and other institutions) perpetuate" (Ladson-Billings, 1995a, p.469). Discussing the significance of the Adinkra symbols with the students will not necessarily help students to attain the mathematical knowledge expected of them, but rather, it could help them become more human by learning human values like honesty, dignity, patience, and so on, from the Adinkra symbols that will be used in the lessons.

In the lesson on *Ratio and Proportion*, the following exchanges between teacher and students marked the introduction. Most of the exchanges were in the Twi language, I have translated them for the reader.

Mr. Obeng: What is the main occupation of men in this town?

Akotie: Kente weaving and stamping cloth making.

Mr. Obeng: What are some of the things you see in these clothes?

Bruce: Colors and symbols

Mr. Obeng: What is the name of the symbols in the clothes?

Diana: Adinkra symbols

Mr. Obeng: Name some of the Adinkra symbols you see in the clothes and give their meanings.

Akotie: *Gye Nyame* (Except God), it means only God has power.

Bruce: *Nsoroma* (Star), stands for a blessed person.

Elvis: *Funtunfunefu denkyemfunefu* (Siamese crocodile), is used to say we are all one.

Mr. Obeng: So, what does that tell you as a student?

Yaw: We are the same people so we should not fight each other over land or properties.

Serbeh: We should help someone who needs help because he/she is our relative.

Bruce: We depend on one another so helping another is helping yourself.

It took some time to name and give the meanings of the Adinkra symbols, with the teacher occasionally asking students to tell how the meanings were significant to their lives. Most of the students' activities also required them to give the meanings of the symbols they used or name the symbol they created using mathematics ideas (see lesson plan in Appendix J or students' work in Appendix M).

Students to Rename the Adinkra Symbols and Name their Graphics

Another idea that emerged in the planning stage of the lessons was the idea of renaming the Adinkra symbols. This idea came from the senior high school teachers after they had worked with the craftsman. "The craftsman said new symbols are created, and the symbols become accepted based on their beauty, names, and meanings. How do we make use of this idea in the lessons?" (Mr. Antwi). Similarly, Mr. Oti suggested that we ask students to rename Adinkra symbols and give the meaning of the names. This was a response to Mr. Antwi's earlier suggestion that students should be allowed to generate non-mathematical ideas about the Adinkra symbols in the lessons. He said: "I think the students should be allowed to demonstrate their reasoning not only in terms of mathematics but also about the names and values of the symbols. What ideas could students come out with regarding the Adinkra symbols?"

This demonstrates a re-orientation of the teachers' views about their teaching. Teachers who were initially talking about how their teaching is examination-driven were now suggesting that students should be allowed to rename Adinkra symbols and reason about their names and values of their names in mathematics lessons. I mentioned this to the junior high school teachers, they agreed that they could ask students to name the graphics that the students created in the course of the lessons. Traditional views of mathematics teaching appear to marginalize students as capable of knowledge creation (Watson, 2008). One of the arguments for ethnomathematics and culturally responsive pedagogy is the origin, nature, and source of knowledge (D'Ambrosio & Rosa, 2017; Gay 2010). In the classroom, for example, students regard teachers as the source of knowledge (Baki 1997; Watson, 2008). Asking students to rename and give the reason they are giving those names to symbols, and the meanings of the names they are giving is a form of power-sharing. Students begin to realize that they are also capable of reasoning, just as the Adinkra craftsman could reason to give names and meanings to the symbols. D'Ambrosio (2007a) argues that the purpose of ethnomathematics is to create equity and social justice, and Gay (2010) also stated that culturally responsive pedagogy is intended to challenge the issue of power. Creating a culturally relevant classroom environment could help students appreciate the fact that they have the power to be a source of knowledge.

By allowing students to name the graphics they create and renaming the Adinkra symbols, is a way to ensure equity in knowledge sharing. That is, knowledge is coming from the students as well as from the teachers. In this sense, students become not only consumers of knowledge, but producers as well (Gay, 2010). This idea was infused in most of the students' activities with the Adinkra symbols as can be seen in the lesson plan (Appendix J) and students' classwork (Appendix M). The SHS students renamed the Adinkra symbols used in their activities and named the graphics they created in their evaluation exercises. The JHS students were asked to name the graphic they created in the enlargement (scaling) lesson.

Printed Images of Adinkra Symbols to be Used in the Lessons

The use of tools for mediation implies that students actively interact with a semiotic artifact to gain knowledge. The Adinkra symbols that are related to the concepts were used in designing the activities for the lessons, and as artifacts during the intervention for students to interact with (Scott & Palincsar, n.d.; Steele, 2001; Vygotsky, 1978). The Adinkra symbols were the main teaching/learning materials in this intervention, so it was important to provide samples of the Adinkra symbols in the classroom. As explained in Chapter Four, the 2D form of the symbols is the most common form within the community, hence, images of the symbols that were used to develop the lessons were posted in the classroom, and copies were given to students for reference when necessary (see Appendix O for the images). As can be seen in the students' data reported in Appendix M, Students did refer to them in the course of the implementation. The SHS students' entire lessons required them to refer to and use these images. For instance, in the lesson on reflection, this is how the teacher instructed the students to use the images:

Mr. Oti: I want you to try this out, draw lines through the middle of the printed images of the Adinkra symbols we have given you, and tell me which ones could satisfy the description of the image we observed in the mirror.

After a few minutes, the students made the following responses:

Group 5 member: *Nsaa, Woforo dua paa and Apa*

Group 3 member: *Asase ye duru and Nyame dua*

Group 2 member: *Akoma ntoasoɔ, Nsaa they have said the rest.*

Group 4 member: *Sir, those with circles can we do it for them? We can't see the side which is turned upside or reverse.*

Mr. Oti: *Yes, someone should explain why it is possible.*

Group 4 member: *It is okay sir; it is because of how circles appear, that is why we couldn't see the reverse of it. The Mate masie and M'aware wo too have it. We drew vertical lines through them*

Ally (2011) defined conceptual knowledge to include the ideas of the nature of topics, exemplified in the use, illustration, or representation of concrete and semi-concrete models, referring to the underlying structure of mathematics. The images of the Adinkra symbols served as models of the concepts that were related to them. It was expected that, since the students had models to connect to the concepts they were learning, it would enhance their conceptual understanding. Kilpatrick et al. (2001) has stressed how important it is to use different models of representations for mathematics concepts. For instance, the authors note: "In becoming proficient problem solvers, students learn how to form mental representations of problems, detect mathematical relationships, and devise novel solution methods when needed." (Kilpatrick et al., 2001, p. 126).

The teachers felt the use of the images of the Adinkra symbols as mediating tools in the intervention would help students form mental images of the mathematics concepts they were related to, and these mental images would enable them to relate the mathematics concepts they learned from the images to similar objects they encounter in the future. Thus, not only will the use of the images of the Adinkra symbols enhance students' conceptual understanding, but it will also enhance their adaptive reasoning and productive disposition.

The Local Language to be Used for Interaction in Math Lessons

Ghana has about 49 languages with English as the official language. English is used for classroom instructions from primary level four to the tertiary level. Therefore, the language for mathematics instruction at the junior and the senior high levels is English. This notwithstanding, the teachers in the study argued for the use of the local language of the community in addition to the English language within these lessons that used the Adinkra symbols.

> Since we want to encourage them to discuss, and to enable them to participate well in the lessons, I think we should allow them to speak Twi in the classroom, since most of them are not fluent in the English language. Mr. Oti
>
> In our teaching, we have to accept students' responses given in our language, you know most of the students may have an idea but won't be able to express it in the English language, so we have to use both languages in the teaching of these lessons. Mr. Oteng

The teachers' main argument is that the students have difficulty expressing themselves well in the English language. Their comments show the level of the teachers' sensitivity to their students' language needs. Gay (2010) emphasized that teaching and learning are most effective for students when classroom communication is culturally responsive, and also, that communication is the essential way in which humans make meaningful connections with one another, whether in teaching or learning. The teachers were concerned about students participating in the lessons. They noted that the use of the English language alone in the lessons could be a hindrance to students' participation; hence, teachers proposed that we use both languages in the lesson delivery. I wondered if teachers realized that, in our interactions with the images of the Adinkra symbols, we often shared our ideas in both the English and Twi language, therefore the students may also need to use both languages so that they can express their thoughts.

Although Kilpatrick et al. (2001) did not explicitly discuss communication as a strand of mathematics proficiency, they do note that the quality of mathematics instruction is dependent on interaction and engagement with mathematics tasks. The teachers recognized this and noted in our discussions that the students would be able to interact well among themselves, and with the subject matter if they were allowed to use their local language. Teachers and students switching into their local language could enable active interactions and enhance students' conceptual discourses (Setati & Adler, 2000).

Gay (2010), using the Sapir Whorf hypothesis on the relationship between language and thought, stated that language is a way of defining experience, thinking, and knowing. Students' experiences in the lessons, and their thoughts about the Adinkra symbols used, could only be expressed through language, and the teachers noted that the students could express themselves better in their local language. Mr. Antwi noted that if it is desired that students participate well in the lessons, then the local language should be used in the lessons. Students participating, or becoming involved in the lessons, means students express their views to each other and the teacher in class discussions, and respond to the teacher's questions and those of other students in the class, and they also ask questions. The teachers had the perception that if this level of involvement or engagement with the mathematics task could be achieved in the lessons, they would need to make use of the language with which the students are most fluent.

The teacher's activities in lessons were to include giving directions, responding to students' questions, asking students questions, and giving feedback to students. These all require the use of language as a medium. That is to say, the medium of communication is very important in teaching and learning. The teachers noted that the mother tongue of the community is the most effective means of communication in lessons; hence, they proposed it should not be ignored in the lessons. For teachers to be able to determine what students know and can do, as well as what they are capable of knowing and doing, is a function of how well teachers can communicate with the students (Gay, 2002). It was agreed that the local Ghanaian language and the official language (English) must be used together in the lessons. Ladson-Billings (1995b) has illustrated that students who are allowed to use both their first language and their official language become better users of both languages. Other authors have noted that the use of students' native language in the mathematics classroom supports students' ability to achieve academic success (Barwell et al., 2016; Irizarry, 2007; Lee, 2010; Nieto, 2002). That is, the local language of the community, which is the Twi language, was used together with the English language at the implementation stage.

Students to be Organized in Small Groups to Learn With the Adinkra Symbols

Researchers have observed that building a sense of community and connectedness among students is essential for culturally responsive teaching (Abdulrahim & Orosco, 2019; Ladson-Billings, 1995a, 1995b, 2006). These researchers hold the view that, in culturally responsive teaching, students should be allowed to work in cooperative groups, and should be held accountable for each other's success. I had planned to suggest to teachers that students work in groups, and the teachers also suggested that we allow students to work in groups. Their reasons were that, when students work in groups, they can assist each other, one person may be competent in something more than the others in the group, and that competency could be an asset for the entire group.

> By the nature of the study, I think if the students also work together in pairs or in groups as we did, they will be able to learn a lot from each other by studying the symbols together, so Madam, if it is okay for you, let's allow them to work in groups or pairs. Mr. Obeng
>
> *Afei nso,* (also) we have to let them work in groups. If they work in groups, within each group there may be one or two people who will be able to write what they say in English.

> I am saying this because Madam Mavis said she will be making photocopies of their classwork. Mr. Oti

Mr. Obeng's reason was that students will learn from each other because each person may come up with ideas that will help the members to learn together. The teachers' suggestion of asking students to work in groups is surprising since mathematics classes rarely include group work, and at the same time, not surprising, since group work is consistent with the communal living among Ghanaians, especially, in semi-urban and rural communities, like where the study was situated. Ladson-Billings (1995b) recognized that letting students work in cooperative groups in the classroom is a good teaching strategy (Steele, 2001; Vygotsky, 1978). For example, Davis and Renert (2013) observed that mathematics ideas emerge as collective practices of the classroom community (see also Davis & Simmt, 2006). Asking students to work in cooperative groups could therefore enhance the emergence of their mathematics ideas. Kilpatrick et al. (2001) are of the view that mathematics learning involves the interaction among students and teachers around the subject matter. Fredricks et al. (2018) also found out that students' engagement in mathematics and science is high in classrooms where there is peer engagement. That is, working in cooperative groups is expected to increase students' engagement and consequently, increase their mathematics proficiency. Reflecting on group work in the context of this study reveals that asking students to work in cooperative groups is another means of bridging the gap between the students' cultural experience and classroom mathematics activities. Teachers decided that the students would be put in groups to interact with the subject matter, and collectively to develop their mathematics understanding.

Teachers Should be Facilitators in the Math Lessons

Sociocultural theory suggests that the teacher must mediate between students' personal meanings, students' collective meanings, and the scientific knowledge the students are supposed to learn (Hassan, 2002; Scott & Palincsar, n.d.; Steele, 2001; Vygotsky, 1978). Another important decision made by the teachers was how they were going to lead students in the lessons to achieve the stated objectives. With teachers growing awareness toward student-centered learning and conceptual understanding, it was agreed that students be allowed to make deductions by themselves, with minimal teacher guidance. Mr. Antwi suggested: "Since the syllabus demands that students know the rules for the different transformations, I suggest that they will be allowed to find the rules by themselves … with you giving clues". The teachers indicated that it is very important for students to learn the rules for the different transformations. They referred, many times, to the fact that their teaching is examination driven, hence knowing the rules for the different transformations was necessary for the students to succeed in their examinations. However, they believed that students could be led to deduce the rules by themselves. Hence, the teachers proposed that the students be led to deduce the transformation rules through the Adinkra symbols. In the senior high school lessons, the teacher was to facilitate the students' learning by giving them cues to follow.

Teachers supported students' learning by scaffolding through the use of clues. The teachers gave prompts to the students as they investigated the symbols for the mathematics concepts being taught. That is, the prompts were the scaffolds that the teachers used to lead students to the desired goals. In the lesson on translation with the senior high school students, the teacher gave the following clues to students.

I want every group to draw the x and the y-axes on the graph sheet given to you using any scale you want. Look for the *Apa* symbol on the sheet that has images of Adinkra symbols. Draw the A*pa* symbol on your graph sheet, anywhere on the graph provided it falls on the grid. What are the shapes that form the *Apa* symbol? Label the first rhombus you drew as the object and the second one as the image. I want you to do this, label the vertices of the object *ABCD* and the corresponding image points *A'B'C' and D'*. Write down the coordinates of the vertices of the object and that of the image. Compare the coordinate of vertices of the object to their corresponding image points and write down your observation.

The junior high school teachers had a similar view, that the teacher should lead students, through questions and response drills, until the students get to the stage in the lesson that they could continue on their own before the students continue to work in groups. The junior high teacher used scaffolding to guide students in instances where he found the task to be challenging to the students. For example, in the enlargement task involving the *Akoma* - Heart symbol, to assist the students to draw the heart on the graph, the teacher provided the following assistance for students to draw the first heart:

He drew a heart on the board and asked the students to observe the '*kink*' in the shape and gave the following prompts:

a. You are going to draw the *Akoma* - heart symbol on your graph to pass through the points $A(-1,2)$, $B(-2,3)$, $C(-0,3)$ *and* $D(-1,0)$, locate the points on your graph.
b. Having these points on your graph sheet, where do you think the *kink* will be located?

c. Now, use the freehand to join the points to form the *Akoma*-heart symbol.

That is, in this task, the teacher thought drawing the heart to pass through the given points could be challenging to students, so he had to provide scaffolding to get them to draw the first heart (though two groups had already done it before the teacher gave the prompts).

The teachers' view of this proposal led to the preparation of worksheets for the junior high school students. For the first three lessons, the teacher was to guide the students through the sheets, in both languages, on every item on the worksheet, except the question for evaluation, before the students work on the items. One of the junior high teachers followed up on a comment from another teacher, "she also mentioned a point on logical reasoning and justification which is also in our syllabus, so when teaching, don't forget to ask "why" questions. Every response they give to a question should be followed by why". That is, the teachers noted the importance of asking for justification from students to enhance students' reasoning abilities; the teachers who facilitated the lessons were therefore encouraged to require justification from the students.

"Teachers must scaffold*,* or build bridges, to facilitate learning" (Ladson-Billing, 1995a, p. 481). It can be said that both the junior high and the senior high teachers were prepared to scaffold the students' mathematics learning but by different means. Student-centered mathematics classrooms have been found to allow for students' engagement, thereby increasing their learning (Fredricks et al., 2018). Özdemir and Pape (2012), for example, have observed that classroom instruction that gives students cognitive autonomy helps students develop their strategic competence. Consequently, teachers, assuming the role of facilitators in the intervention, are expected to increase students' engagement with the tasks, enhance their strategic competence and adaptive reasoning.

To summarize, in order to achieve the content objectives and the broader aims of the curriculum (strands of mathematics proficiency) through culturally responsive teaching, the following decisions were made and implemented:

- Teachers designing and using classroom activities and assignments that reflect the Adinkra symbols,
- students finding Adinkra symbols within the community,
- posting images of Adinkra symbols in the classroom, and copies for students,
- teacher and students discussing the meanings of the Adinkra symbols,
- students renaming the Adinkra symbols and naming their created graphics,
- teachers and students using both the local language and the official language,
- students engaging in cooperative group work, and
- teachers acting as facilitators in the lessons.

Gay (2001) explained that personal, moral, social, political, cultural, and academic knowledge and skills are taught simultaneously in culturally responsive teaching. The strategies I have discussed above regarding the measures planned and implemented in the lessons; were made to achieve all the goals of culturally responsive teaching in the lessons. In the next chapter, I present the students' learning that occurred in the intervention and how these implemented ideas contributed to students' learning.

CHAPTER SEVEN

Mathematics Proficiency that Emerged in the Implementation Phase

In this study, teachers used the Adinkra symbols as mediating tools for culturally responsive teaching, enabling the researcher to investigate the kinds of learning that occurred in the lessons. In Chapter Five, I presented the results of the ethnomathematical investigations of the Adinkra symbols, and in Chapter Six, I described the culturally responsive intervention and its implementation in the two levels (JHS and SHS).

In this chapter, I present the students' learning that occurred during the implementation of the lessons, using the strands of mathematics proficiency described by Kilpatrick et al. (2001). I identified indicators of Kilpatrick et al. (2001)'s strands of mathematics proficiency as presented in Table 7.1 with the indicated color codes. These color codes were used to code extracts from students' oral discussions when solving problems in small groups. These, together with my field notes, teacher's reflective journals, and teachers' interview responses were used to describe the strands that emerged in the lessons and to illustrate how the culturally responsive intervention contributed to their emergence. Using the color-coded extracts, and the students' written work, a table was constructed for each lesson by selecting the work of one group of students per class, for a total of eight tables. Table 7.2 is an example. The other tables referenced in this chapter can be found in Appendix M. The groups selected were the ones that provided enough information in their oral discourses and written documents that enabled me to make sense of their work. Students made use of the local language and the English language in the lessons. In the tables, I have translated their conversations into English, with sentences which were expressed using the two languages together, or only the local language italicized.

Table 7. 1

Strands of Mathematical Proficiency and Their Indicators Used to Analyze Students' Work

Conceptual understanding	Identifying the extent of concepts in a mathematical problem or statement. Using appropriate ideas and strategies in solving problems. Effectively using mathematical terms, ideas, and operations. Generating new knowledge. Making sense of a mathematical problem. Interpreting such representations as graphs, diagrams, and tables. Understanding why the procedure/theorem works.
Strategic competence	Devising a strategy to solve problems. Executing the strategy to solve the problem. Classifying using criteria. Generalizing, based on the used criteria. Devising a strategy to represent a mathematical idea.
Adaptive reasoning	Thinking logically about relationships among concepts. Thinking logically about relationships among situations (relate or compare situations). Giving logical explanations and justifications to express thoughts and their understanding of mathematical ideas. Transferring ideas from one situation/context to the other.
Procedural fluency	Selecting appropriate mathematical procedures and theorems in solving problems. Correctly applying appropriate procedures and theorems and tools in solving problems. Selecting appropriate mathematical tools in solving problems. Knowing how the tool works.
Productive disposition	Recognizing the usefulness of mathematics. Having a positive self-concept in mathematics. Willingness to persist in solving mathematical problems. Recognizing and respecting others in mathematics (ability to work in cooperation).

Table 7. 2

SHS Group Five's Deductions *on* Reflection Using *the Woforo Dua Pa a* (*When You Climb a Good Tree*) *Symbol*

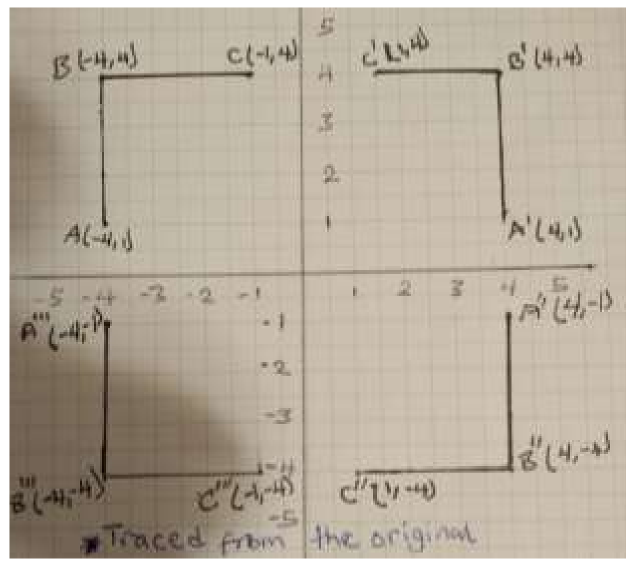
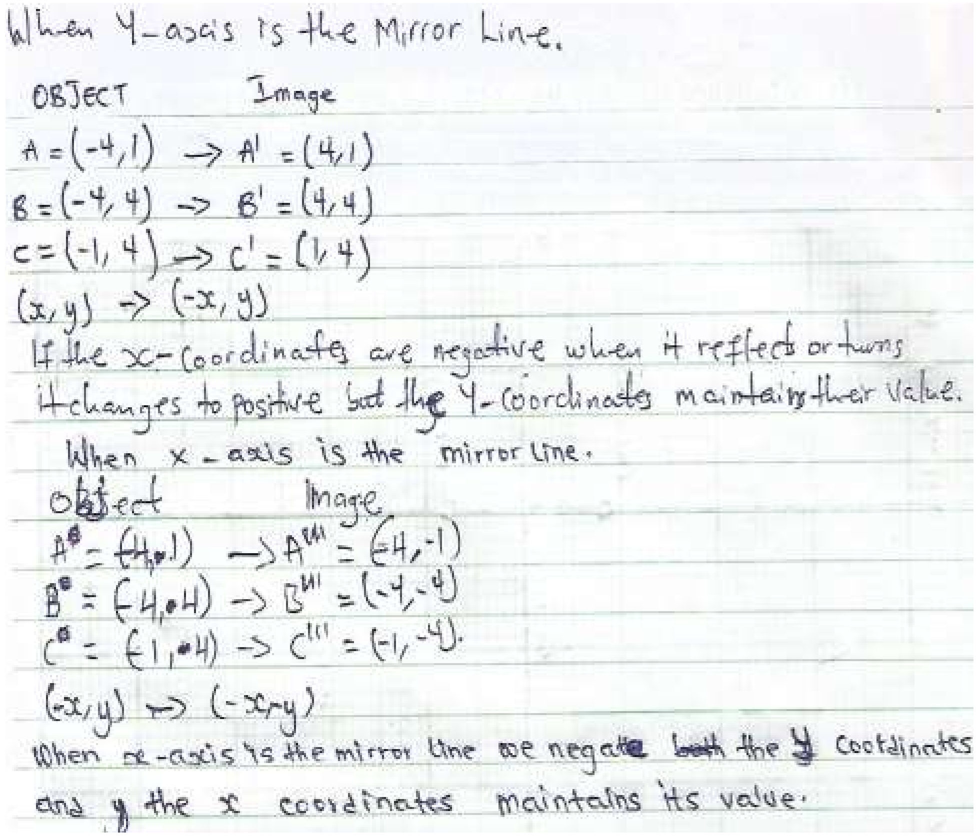
stage	Students' conversations	Students written solution	Strands of mathematical proficiency
1	Esther: *We need* to draw it such that we can easily read points on it. Justina: *We need to draw the axes to pass through the middle exactly so that this part will be equal to this part* (pointing to the left part and right part of the Adinkra symbol) *and this part will be equal to this part* (pointing to the top and bottom parts of the Adinkra symbol)		Strategic competence Adaptive reasoning Conceptual understanding Procedural fluency
2	Prince: Now, we have to label some points and find their coordinates. Esther: *From what he said, let's take one of the four parts as the object and label the corners ABC.* Esther: *For the images, let's use one prime for one side, two primes for another side and three primes for the last side.* Enoch: When we are labelling, we need to remember the image we saw in the mirror. Justina: *Yes, we have to label it such that the labels at the other part of the axes will look like the image of the quadrilateral in the mirror.* Enoch: Write the coordinate of each point against it on the graph. Justina*: I don't even understand why they call this symbol Woforo dua paa. It doesn't look like a tree.*	 	Strategic competence Adaptive reasoning Procedural fluency

Table 7.2 Continued

3	Esther: We can now map the points to their images and compare them. Prince: When we look at the y-axis as the mirror line, the image of point A are the same values of A, but the x-coordinate is negated. Enoch: That is true, the same thing is happening with the B and C coordinates, the x-coordinates become negative of that of the object x-coordinate. Esther: So, when the x-coordinate is negative, it becomes positive when it is reflected on the y-axis. Justina: Write it down and let's go to the x-axis as a mirror line. Justina: It is interesting, now the image points have the y-coordinates also becoming negative. Esther: *Yes, they were positive in the object coordinates and become negative in the image coordinates.* Enoch: *That is, with the image points, the x-coordinates were the same as in the object, but the y-coordinates became negative.* Prince: *Now we have found the reflection rules, what does the meaning of the symbol apply to us?* Justina*: It can mean if you want people to support you, do good things.* Enoch: Support those who are taken good courses. Esther: *That also means we should rebuke those who have bad behaviors.*	 When Y-axis is the Mirror Line. OBJECT Image A = (-4,1) -> A' = (4,1) B = (-4,4) -> B' = (4,4) C = (-1,4) -> C' = (1,4) (x,y) -> (-x,y) If the x-coordinates are negative when it reflect or turns it changes to positive but the Y-coordinates maintain their value. When x-axis is the mirror line. Object Image A = (4,1) -> A''' = (4,-1) B = (4,4) -> B''' = (-4,-4) C = (1,4) -> C''' = (-1,-4). (x,y) -> (-x,-y) When x-axis is the mirror line we negate ~~both~~ the y coordinates and the x coordinates maintains its value.	Conceptual understanding Adaptive reasoning Procedural fluency Productive disposition

Learners' Mathematics Proficiency

Kilpatrick et al. (2001) identified five strands of mathematical proficiency that describe successful learning of mathematics. The five strands were used to describe the students' learning that occurred in the intervention. The eight tables, created to describe students' strands of mathematical proficiency that emerged in the intervention, as reported in Appendix M, confirm that students worked on all the strands of mathematical proficiency: conceptual understanding, strategic competence, procedural fluency, adaptive reasoning, and productive disposition, in all the episodes during the implementation of the intervention. How each strand emerged in the students' learning through the intervention is presented here:

Conceptual Understanding

Kilpatirck et al. (2001) explained that conceptual understanding is ones' ability to grasp mathematical concepts, operations, and relations. In this study, as explained in Chapter Two, indications of conceptual understanding were:

- identifying the extent of concepts in a mathematical problem or statement,
- using appropriate ideas and strategies in solving problems,
- effectively using mathematical terms, ideas, and operations,
- generating new knowledge,
- making sense of a mathematical problem,
- interpreting such representations as graphs, diagrams, and tables, and
- understanding why a procedure/theorem works.

The intervention presented opportunities for students' conceptual understanding to emerge. Students' conceptual understandings were mainly identified from the transcripts of their group conversations. Hence, I presume that those understandings were afforded by those conversations as students discussed and explained the mathematics of the Adinkra symbols in their small groups. In other instances, their conceptual understandings were seen in their attempts to explain and justify solution methods in whole-class discussions. That is, access to their conceptual understanding was mainly through their explanations in oral discourses (Dias Corrêa, 2017; Özdemir & Pape, 2012).

Throughout the intervention lessons, students' conceptual understanding emerged in different ways. In most instances, students started the investigation of a problem by identifying the content in the problem that would enable them to make sense of the problem. By doing so, they identified the extent of concepts implied in the given problem. For example, in the JHS students' solution to a problem on ratio relating to the *Adinkrahene* symbol, (as presented in Appendix M, Table M1, stage 1), they commenced their solution by identifying the meaning from the problem:

Yaw: The sum of the radius is sixteen centimeters, and the ratio is one to two to three.

Bismack: *We need to find the sum of the ratios too to be equated to the sixteen centimeters.*

Yaw: *No, we will not equate it to six, we have to write each ratio as the fraction of the sum.*

Yaw: That means the radius of the inner circle will be one out of six of the sum of the radii, which is sixteen and the middle circle will be two over six times the sixteen.

Diana: Yes, we will have one over six multiplied by eight.

There were other instances in which students asked questions of each other to better understand the mathematical meaning behind a task. An example can be seen in the conversation among Ernest, Bruce, and Afriyie (Appendix M, Table M2, stage 1):

Ernest: What is the meaning of inclined?

Bruce: *If we look at the ɔwɔ foro adobe symbol, I think the sixty degrees inclined is the angle the transversals make with the horizontal lines.*

Afriyie: *There are two angles at each point where the transversal meets the horizontal, so do we need to divide the sixty degrees into two?*

Bruce: *No, that can't be possible, because at the points of the intersection of the transversal and the horizontal lines form angles on a straight line which is one hundred and eighty degrees, I think we should measure one of the angles to be sixty degrees.*

In this example, we see how students used questions to seek the meaning of the term "inclined". Their colleague referred to the Adinkra symbol they were supposed to create, and he explained the possible meaning of the word. This was followed by another question from another group member. At that point, Bruce used the image of the Adinkra symbol as context, and his understanding of angles on a straight line to explain the possible meaning of inclining the transversals 60° to the horizontal.

There were opportunities given to students to select and use appropriate ideas. That is, their conceptual understanding was also demonstrated in their use of appropriate ideas and strategies for solving the problems they encountered. For example, in Appendix M, Table M4, stage 1 and 2, Edna gave this explanation for anti-clockwise rotation:

Edna: *We now move to the other part of the question, if we are moving ninety degrees anticlockwise, the A points will move like, the negative three on x go to negative three on y, and the positive one on y to negative one on x (she demonstrates it on paper as she speaks to explain to her colleagues)*

Edna: *So, we are doing the same for one hundred and eighty degrees anticlockwise, every point will turn one hundred and eighty degrees from left for y points and from down for x points.*

It can be seen, from the students' comments and from classroom discussions that the students effectively used mathematical terms and ideas. From the conversations presented above, we see students using the correct terms, such as "one hundred and eighty degrees", "anti-clockwise", "inclined", and "transversal". In each case, they were able to express their understanding based on the context of the problem. For example, Edna explained that moving one hundred and eighty degrees anti-clockwise, y-points will move from left and the x-point will move from down. The student said this because the object they were rotating was found in the second quadrant, where y-coordinates are positive; hence moving anticlockwise, the y-coordinates will move to the left. Though the student could have used the same idea of *left* to explain that of the x-coordinates, the student sees the x-coordinates to be moving down, based on her orientation. The students' representations, as in Appendix M, Table M4, column 3, stages 1 and 2 shows that the student understands anti-clockwise rotation $180°$. Let's look at another example, from the SHS class, explaining the concept of translation vector in Appendix M, Table M5, stage 2:

Oppong: *So, the translation vector is the number of points to move left or right, up, or down*.

Atta: Yes, left or right is the horizontal movement and up or down is the vertical movement.

Christy: When the x-component is positive we will move right, when it is negative, we move left.

Oppong: That means a positive y component will go up and a negative y will go down.

Here, students made use of the idea of movement, and the direction of the movement, to explain what a translation vector is. In Oppong's statement, he explains the idea of vector as magnitude and direction, by saying "the number of points" (magnitude) "to move left or right, up or down" (direction). Students used this understanding of the translation vector to perform translations of shapes on the graph, as can be seen in Appendix M, Table M5, stage 3 and column 3.

Another form of conceptual understanding, that emerged in the intervention, is generating new ideas. The intervention enabled students to come out with new mathematical knowledge. For example, in the SHS translation activity with the *Apa* symbol, during the group presentations, Group Three members made the following observations:

> We have also seen that for the object and its image formed, $\overrightarrow{AB}$ is parallel to $\overrightarrow{A'B'}$, $\overrightarrow{CD}$ is parallel to $\overrightarrow{C'D'}$, $\overrightarrow{AC}$ is parallel to $\overrightarrow{A'C'}$ and $\overrightarrow{BD}$ is also parallel to $\overrightarrow{B'D'}$. When we expressed them as component vectors and compared them, we found $k = 1$ in each case, which means they are also equal to each other.

When the teacher asked why they decided to do that, a member of the group replied:
"When we realized the sides were parallel, we wanted to find out if *k* will be the same as one of the components of the translation vector we obtained between the object and the image".
The *k,* used by the students here, refers to the scalar multiplier between two parallel vectors. This made the teacher consolidate the students' learning by discussing properties of translation with

the students, which were not part of the curriculum objectives. The teacher shaped the ideas that the students generated to lead them to accurate mathematical thinking (Stein et al., 2008).
The teacher explained that, if two lines are parallel (as is the case of the sides of the rhombus in the *Apa* symbol), and they are translated by the same translation vector, their images are also parallel. Another example, in the JHS whole-class discussion on ratio and proportion activity with the *Nsaa* symbol, Abena had this insight; "if we find the ratio between the sides of the large squares, we get one, and the ratio between the sides of the smaller squares is also one". The teacher asked the class why this is so, and Ernest responded that it is so because the sides are equal, and a number divided by itself is one. From the student's response, the teacher explained that when two shapes are congruent, the ratio between their corresponding sides is one.

Again, in the intervention, students had the opportunity to interpret graphs and drawings. The entire SHS lessons were based on transformations and students located Adinkra symbols on the graph and used their drawings to find the rules of transformations. This presupposes that the students could figure out graphical representations of transformations, and they could interpret such representations based on the type of transformation observed in the graph. For example, in the activity on multiple transformations with the *Mate masie* symbol (presented in Appendix M, Table M7, column 1, stage 2), Group four members had the following conversations about the images and the object:

Antwi: *For object and image I, the coordinates of the A and the A prime shows reflection in the x-axis, because the x-coordinate which is negative four remains negative four in A prime, but the y-coordinate negative two becomes positive two*.

Konlan: *No, it can't be a reflection, it is not a reflection because the line in the circle would have appeared at the top in the image and not down.*

Siaw: I think the object was dragged vertically to the top, so it is a translation.

Serwaa: I agree with you, let's find the translation vector.

Antwi: The same movement is happening from J to K or K to J, that is also translation.

That is, the students used their understanding of the various transformations and their reasoning about how the shapes appear, to interpret the drawing on the graph sheet. Another example could be the identification of the different types of angles formed in the *ɔwɔ foro adobe* symbol by the JHS students, as illustrated in Appendix M, Table M2.

The intervention also afforded students an opportunity to explain why a procedure or strategy works. In constructing the *ɔwɔ foro adobe* symbol, Alex convinced his colleagues to use the set square by saying: "The sixty-degree set square has a sixty degrees angle at this corner and thirty degrees at this corner (showing the sixty-degree set square, and pointing to the corners), and we have used it to draw parallel lines before". Alex is saying that the sixty-degree set square will help them draw the parallel lines that are inclined to the horizontal at 60°. That is, he demonstrated his understanding of the concepts of parallel lines inclined at 60° and could relate it to the function of the sixty-degree set square. This implies he also knows why the tool will work in the case of the problem they were presented with. The way the students communicated their solutions, as shown in Appendix M and the extracts given in this narrative, shows that the culturally responsive intervention provided opportunities for students conceptual understanding to emerge, and their conceptual understanding was demonstrated through their explanations of mathematical procedures and strategies they employed. That is, students' conceptual understanding was enhanced in the activities.

Procedural Fluency

Procedural fluency refers to students' knowledge of procedures, knowing when and how to use them appropriately, and the ability to perform them flexibly, accurately, and efficiently (Kilpatrick et al., 2001). The following indicators were used to identify the emergence of procedural fluency:

- selecting appropriate mathematical procedures and theorems to solve problems,
- correctly applying appropriate procedures and theorems,
- selecting appropriate mathematical tools to solve problems, and
- using the tool correctly or knowing how the tool works.

Most of the students' procedures and strategies were demonstrated in the written work and drawings as shown in the Tables in Appendix M. Here, I will refer to some scenarios in their conversations.

The intervention afforded students the opportunity to select the mathematical methods and theorems they deemed appropriate and apply them in their solution processes. For example, when resolving fractions in a problem related to the *Adinkrahene* symbol, Comfort said "multiply by the LCM, let's just cross multiply". That is, Comfort knew the method of LCM and when it is applicable, she also knew the alternative and easier way is to do what is called "cross multiplication". The students applied this instrumental procedure (Skemp, 1978) correctly to solve the problem. When the JHS students were finding the images of the *Akoma* symbols through scaling, Boadu told the group members the method they could use to get the coordinates of the images. "We find the image coordinates by multiplying the object coordinates by the scale factor". Here, again, they applied a procedure correctly to obtain the image coordinates.

Throughout the intervention, students had the opportunity to choose their procedures and moments to apply them. They also described and performed the procedures correctly, as verified in the narratives on students' conceptual understanding, and in the written work as shown in Appendix M. In all the graph work on transformation, students correctly used the procedure of writing coordinates of a point labeled on the objects and their corresponding image points.

Another dimension of procedural knowledge that can be observed is the students' use of mathematical instruments. Students chose the right mathematical instruments to use in their drawings. For example, when the JHS students were drawing the *ɔwɔ foro adobe* symbol, they appropriately chose the ruler to draw the two horizontal lines, using the correct mathematical procedure of drawing a horizontal line to another line, and they used the ruler to measure and mark four points, 1cm apart, from the left end of the top horizontal line. They correctly measured an angle of 60° at the first point they marked, using the protractor, and again, they used the ruler to draw a straight line through the points. When Alex suggested to his group members to use the set square, as narrated under conceptual understanding, he demonstrated that he knew how the set square works. When the group members drew the second set of transversals with the sixty-degree set square, Bruce said "Let's measure with the protractor and see if the angles are one twenty and sixty degrees". This shows that they knew that the protractor is used to measure angles and they selected it to correctly measure the angles formed at the intersections of the transversals and the horizontal lines. As demonstrated in this narrative, students' procedural fluency was enhanced in the intervention through the opportunities they had to select and use appropriate mathematical procedures and methods in solving problems, and also in selecting and correctly using appropriate mathematical tools.

Strategic Competence

Strategic competence refers to the ability to formulate mathematical problems, represent them in different means, and devise ways to solve them (Kilpatrick et al., 2001). The indicators used for this strand included:

- devising and executing a strategy to solve problems,
- classifying using criteria,
- generalizing based on a used criterion, and
- devising a strategy to represent a mathematical idea.

The culturally responsive teaching intervention provided opportunities for students to develop strategies to understand, represent, and solve problems, and to test their solutions to mathematical problems relating to the Adinkra symbols.

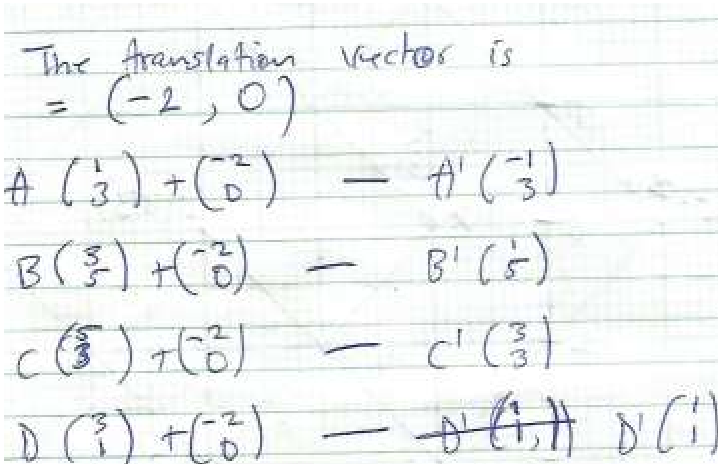
In the JHS students' activity on the *ɔwɔ foro adobe* symbol, they needed to comprehend the word "inclined" in order to make meaning of the problem. The image of the *ɔwɔ foro adobe* symbol provided the students with the context for working on their understanding of "inclined", as they incorporated what they knew about transversals. That is, the images of the Adinkra symbols, as mediating objects, were used by students to make sense of the problems. Bruce said: "*If we look at the ɔwɔ foro adobe symbol, I think the sixty degrees inclined is the angle the slanted lines (transversals) make with the horizontal lines".* Thus, in this case, the strategy the students used was to use the image of the symbol to interpret the problem presented to them. In many examples of the students' activities, we see the students using the images of the symbol, not only as a strategy to comprehend the problem but also to reason about the strategy to employ to solve the problem.

The intervention provided students with the opportunity to select and adopt different strategies in solving problems. In the JHS students' solution to a ratio problem associated with the *Adinkrahene* symbol (Appendix M, Table M1, stage 2, and column 3), the students represented the radius of the middle and the outer circles with variables and modeled the proportional relations $(1:2:3=2:x:y \Rightarrow 1:2=2:x; 1:3=2:y; 2:3=x:y)$. This is a demonstration of a blend of strategic competence, conceptual understanding, and adaptive reasoning. In the reflection activity of the SHS students, they adopted a strategy to distinguish the different images of the object. Esther suggested, "*For the images, let's use one prime for one side, two primes for another side, and three primes for the last side"* (see Appendix M, Table M5, stage 1. Column 2). Adopting Esther's suggestion to distinguish the different images helped the students to organize their results according to which type of reflection a particular image falls under.

An example of students using a strategy to test their solution can be seen in the SHS activity on translation with the *Apa* symbol. After they found the translation vector, they verified the translation vector by adopting a strategy of adding translation vectors to the object coordinates to verify that they will get image coordinates that corresponded to that on their graph sheet as shown in the extract Figure 7.1, taken from Group Two's written work in Table M5 Appendix M.

Figure 7. 1

A Method Used by Group Two Members to Verify a Translation Vector



The intervention also offered opportunities for students to devise strategies to organize and represent their mathematical ideas. For example, in the SHS activities on reflection with the *Woforo dua pa a* symbol, the students employed the strategy of using arrows to map the object coordinates to the image coordinates. That is, the arrows were used to indicate the mathematical relationship between the object coordinates and the image coordinates. They then used symbols to generalize their deduction. For example, under the y-axis as mirror line, students wrote $(x, y) \rightarrow (-x, y)$. Another example can be seen in the JHS solution to the enlargement problem with the *Akoma* symbol (Appendix M, Table M3). The students organized their results under each scale factor using the correct mathematical representation of using arrows to show the direction of the transformation. For example, the students identify the image coordinate of the point $A(-1, 2)$ under enlargement with a scale factor -2, and correctly, they represented the relationship mathematically as $A(-1, 2) \rightarrow A_1(2, -4)$.

Although not explicitly demonstrated through students' conversations, their reasoning and explanations indicate that criteria were used in selecting the methods and procedures, as well as the mathematical tools they employed, at the different stages of the solution process. For example, in the JHS students use of the sixty-degree set square to draw the transversals in the *ɔwɔ foro adobe* symbol, Afriyie remarked "The angles are correct, it works, and it is the easiest, you don't need to measure the angles, you just slide or turn the set square". So, in this case, their criteria for selecting the set square and the ruler over the protractor and the ruler is the fact that (1) the set square works and (2) it is easier to use than using the protractor and the ruler together, so they see the use of the set square to be more efficient than the protractor. Another example about the use of criteria is demonstrated in the SHS students' activity on multiple transformations. We see, from the extract of students' conversation below (taken from Appendix M, Table M7) that the students used both the rules of transformation and the appearance of the shapes to determine the kind of transformation used to form the images. This demonstrates how strategic competence is highly interrelated with adaptive reasoning.

Antwi: *For object and image I, the coordinates of the A and the A prime shows reflection in the x-axis, because the x-coordinate which is negative four remains negative four in A prime, but the y-coordinate negative two becomes positive two.*

Konlan: *No, it can't be a reflection, it is not a reflection because the line in the circle would have appeared at the top in the image and not down.*

While Antwi, used the transformation rule to show that the image resulted from the reflection of the object, Konlan proved otherwise using the appearance of the image of the object. That is, arguing about the object and its image using the transformation rules alone would have justified Antwi's statement, however, the appearance of the object and its image disproved it. Subsequently, students were using both the rule of transformation as well as how the object and its images appeared to establish the kind of transformation forming the images from the object.

From the narrative on strategic competence, it can be concluded that the intervention offered students opportunities to develop and execute their own strategies to solve problems. I found examples of students using these opportunities to apply criteria and generalize based on the criteria through reasoning, and they devised strategies to represent and present their mathematical ideas.

Adaptive Reasoning

Adaptive reasoning refers to the capacity to think logically about the relationships among concepts and situations. Such reasoning is correct and valid, stems from careful consideration of alternatives, and includes knowledge of how to justify the conclusions (Kilpatrick et al., 2001, p. 129). Wood (1998) notes that there is a prevalent tendency in mathematics classrooms for teachers to focus on having students generate correct responses rather than eliciting their underlying reasoning. This was not the case in this study, as one of the decisions made was to require students to justify their responses and explain why they think their response was also correct, and teachers ensured this at the implementation stage. The indicators for adaptive reasoning were:

- thinking logically about relationships among concepts,
- thinking logically about relationships among situations (relate or compare situations),
- giving logical explanations and justifications to express thoughts and their understanding of mathematical ideas, and
- transferring ideas from one situation/context to the other.

The intervention provided opportunities for students to demonstrate their capability to think logically about the relationship among concepts. For example, in the SHS activity on rotation with the *Nyame dua* symbol, students made use of their understanding of clockwise and anticlockwise rotations to establish that 270° anticlockwise and 90°clockwise are the same. In addition, they logically established a relationship between 270° and 90° clockwise. Esi notes "two seventy degrees clockwise is the opposite of ninety degrees clockwise, x will take the negative of y, and y will take x".

Again, students also had an opportunity to logically establish relations among situations. In the JHS solution to the angle problem on the *ɔwɔ foro adobe* symbol, students reasoned about how they would label a few angles with letters to respond to the question. Afriyie suggested that "we have labeled the first two one twenty and sixty degrees, so let's label one angle that is corresponding, alternate or co-interior to these two". They related the unknown angles to the already known angles and using their understanding of the different angle properties of parallel lines, they were able to deduce the angle measure of the unknown angles.

One of the SHS teachers noted in his reflective journal that "because we did not focus the exercise only on correct answers, but also on why they had the answers, students were also thinking and asking their colleagues why they had certain answers". This observation by the teacher can be confirmed by the following extract from group four members of the SHS class conversation on multiple transformations using the *Mate masie* symbol (Appendix M, Table M7, stage 1, column 2).

Konlan: Object to I is reflection, and object to K is also reflection.

Serwaa: *We can't just say it is reflection or translation, we need to show why it is.*

Siaw: *I think we have to use the coordinates of the point given or label some coordinates and use them to prove.*

Antwi: It is okay for us to use only the one given, let's use that.

From this extract, we see that Konlan identified images of an object as resulting from reflection. However, his colleague (Serwaa) thinks that cannot be accepted unless they provide evidence to support that the images resulted from a reflection of the object. Siaw and Antwi then suggested means by which they can provide evidence to support their facts. Students' emergent reasoning grew in the course of the lessons. There are many examples, as shown in students' conversations, presented in the Tables in Appendix M, that indicate students' reasoning about the strategies, concepts, and procedures they employed.

Another example of adaptive reasoning can be seen in the JHS tasks on the *Adinkrahene* symbol. In the JHS students' task on calculating radii of circles with given ratios and using that information to draw *Adinkrahene* symbol (Appendix M, Table M1), after calculating the radii, and the students knowing that the calibration on their rulers will not allow them to measure lengths that are in two decimal places, they decided to round off the values they obtained from their calculations to one decimal place, when Comfort expressed "*let's round the answers to one decimal so that we can measure from the ruler*". This illustrates adaptive reasoning because they had to adapt their values to meet the capacity of their measuring instrument (context).

Students' reasoning also happened in the form of questioning. For example, Antwi asked this question about their investigations with the *Mate masie* symbols, "*Does it mean some translations look like reflections? Because if the middle line was not there, we would have considered image I as the reflection of the object by looking at the coordinates*". And this prompted the students to also investigate the image they considered as a reflection of the object for the possibility of it also resulting from translation. The students brought this up during their presentation and the teacher led them to use the coordinates of an object and its image, in a reflection exercise they had done previously, to prove if reflections are the same as translations.

The students then concluded that, since they could not obtain the same translation vector for all the corresponding points of the object and its image, reflection is not the same as translation.

Again, the intervention afforded students chances to give logical explanations and justifications to express their thoughts and their understanding of mathematical ideas and used their understanding to transfer ideas from one situation/context to the other. For example, using the translation vector $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ to translate image O formed on a graph, Nketiah said this; "this one, we will move the points three steps left horizontally and two steps up vertically", this they did to obtain the image on the same sheet. This is a combination of adaptive reasoning and conceptual understanding. To conclude, it can be said that the intervention promoted students' adaptive reasoning by providing opportunities for them to think logically about relations and situations, give logical explanations to justify their mathematical ideas, methods, and strategies, and also transfer ideas from one context to another.

Productive Disposition

The productive disposition strand relates to students' personal values and attitudes toward mathematics, their perceived ability and confidence in mathematics, and their recognition of mathematics as worthwhile. The disposition toward mathematics has been found to influence students' willingness to engage in mathematics activities (Ally, 2011; Dias Corrêa, 2017). Indicators used for this strand were:

- recognizing the usefulness of mathematics,
- having a positive self-concept in mathematics,
- willingness to persist in solving mathematical problems, and
- recognizing and respecting others in mathematics (ability to work in cooperation).

The culturally responsive teaching interventions promoted students' self-confidence in mathematics. Students demonstrated that they have the capability to learn mathematics. This can be observed in phrases such as "we can …", "let us …", "we will …" In addition, students demonstrated productive disposition by showing their confidence and willingness to persist, even if their initial attempts fail. This is demonstrated by statements such as the following:

Ernest: We can always try something, if it doesn't work, we erase it and try something else.

Afriyie: Yes, we are drawing the *ɔwɔ foro adobe*, which means we should continue to try no matter the difficulty.

Students' willingness to try new ideas also indicates their confidence in mathematics. Based on the JHS students' readiness to try new ideas, they went ahead to try the set square conjecture as proposed by Alex, and after trying, they tested it by measuring the angles with the protractor to confirm their results. The same can be said about the SHS students, as they also demonstrated a willingness to try new ideas:

Sarfo: We can always try something different.

Adu: Yes, there are so many ways of killing the cat. We can use different means to get the same results.

The intervention provided opportunities for students to demonstrate their confidence in mathematics and their belief that their mathematical capabilities could grow. After creating the *Adinkrahene* symbol, Diana appreciated their drawing, saying with laughter "Our *Adinkrahene* is more beautiful than the image on the sheet"; they also related to the meaning of the symbol, which is creativity, and Bismack said they are creative because they have created *Adinkrahene* in the mathematics classroom. This statement also indicates that the intervention offered students the opportunity to develop their sense of agency. They were able to assert ownership of their knowledge and creation. This was more evident in the whole class discussions and group presentations. For example, when Group Three members of the SHS class were about to present their new observation about the *Apa* symbol, as presented under conceptual understanding, the presenter made this comment: "*This one is a special discovery by our group, it is our own finding different from what we were all investigating.*" That is, the group members asserted ownership of their new findings and they were proud to have discovered something about the symbol which no other group discovered.

The intervention also made students recognize mathematics as worthwhile as they began to see the usefulness of mathematics in their everyday cultural objects. They recognized the fact that mathematical ideas are used in creating the Adinkra symbols.

Serwaa: In the translation activity we did, I thought, only translation was used to create the *Mate masie* not knowing the other transformations can also be applied to it.

Osei: This means scale factors can be used to enlarge different Adinkra symbols to form another design.

In the statements of both Serwaa and Osei, they recognized how transformation is useful in their community. They have seen and recognized how mathematics can be applied in the creation of Adinkra symbols, which give livelihood to members of their community. In the whole-class discussion on the *Nsaa* symbol, in the lesson on ratio and proportion, a funny response was given by Obed to the teacher's question on why the craftsman maintained equal ratios in the creation of the *Nsaa* symbol, made the entire class burst into laughter. Obed responded, “to avoid improper fraction”. Probing further, it was realized that the student meant the equal ratios helped the craftsman to maintain balance in the symbol.

Mr. Obeng: What do you mean to avoid improper fraction?

Obed: *He used equal ratios so that no part of the symbol will be too heavy for the other part to carry.*

Obed: *If the ratios are not equal one part of the symbol will be bigger than the other part. If we use the horizontal line, the top could be bigger than the down part.*

Mr. Obeng: Okay. What Obed means is that the craftsman used equal ratios so that there will be balance in the different parts of the symbol.

That is, through the activity on the *Nsaa* symbol, students learned the usefulness of ratios and proportions in creating designs like the Adinkra symbols.

Not only did students see mathematics to be worthwhile, but they also saw mathematics to be interesting, as they observed interesting patterns in their drawings. For instance, Boafo made this observation: “This is interesting, with all the negative scale factors the heart turned upside down”. The teacher strengthened this interesting observation by Boafo in the whole class discussions by asking students what they observed about the image and the object when the scale factor was positive, negative, magnitude less than one, greater than one, and when it is one.

Recognizing and respecting others in mathematics, or the ability to work in a group is originally not identified by Kilpatrick et al. (2001) as part of productive disposition. I included it as part of the productive disposition strand based on an aim of the Ghanaian mathematics curriculum that stipulates that a student will be able to work in cooperation with others to carry out mathematical investigations (MOE, 2010). Analyzing this aim, I concluded that ones’ ability to work in cooperation with others is dependent on one’s beliefs about his/her and others’ capabilities; hence, it is linked to productive disposition. This was identified in students' conversations by words such as "us", "we", "our", that is, first-person pronouns that signify collectiveness. The small group work used in this intervention afforded that. It can be seen in the students' conversations, as shown in the Tables in Appendix M, that students frequently used such pronouns to indicate their togetherness. This also shows their respect for each other's views. There are other instances, where students demonstrated their belief in each other's abilities by sharing tasks among themselves. For example, in the JHS students' activity on enlargement, Boadu said "*we find the image coordinates by multiplying the object coordinates by the scale factor. Each person should do one from b to f, and then I will write your answers here*". Having said this, each member selected a point and found the image coordinates. When they were locating the points on the graph, Amoako suggested everyone locate the points they calculated and draw the *Akoma*. This shows the level of commitment they demonstrated throughout the intervention and how they believe each person is mathematically capable. Students' interest in the lessons was evident in their passionate engagement with the activities through their group discussions.

The Interconnectedness of the Strands of Mathematics Proficiency

Kilpatrick et al. (2001) acknowledged that the five strands of mathematical proficiency are interconnected, and it appears their interconnections have been manifested in the students' data presented here. Taking the strands apart from the students' data was a challenging one because one statement of a student could imply more than one strand. As I analyzed and coded students' conversations for mathematical proficiency, I realized there is a connection between conceptual understanding and procedural fluency. I was able to code many more statements for conceptual understanding than for procedural fluency and regarded the written work as procedural fluency. Then, I realized that learners cannot accomplish a task or procedure correctly without an understanding of the procedure, hence their understanding expressed in their conversations was what led to the procedure in their written work. Graven and Stott (2012) also noted, when analyzing oral responses of students to a numeracy instrument, an overlap of learners' method and response as both procedural and conceptual understanding. They developed a procedural fluency scale to assess students' procedural fluency and observed that, as flexibility and efficiency were high, conceptual understanding and procedural fluency became increasingly intertwined. Their observation implies that allowing students some form of flexibility in approaching a task will help both with their conceptual understanding and their procedural fluency, and this was confirmed in this study.

Another connection, in the strands of mathematical proficiency I observed from the students' data, is between strategic competence and adaptive reasoning. It was difficult for me to categorize the two strands because behind every strategy the students employed is a reason. This is to say that adaptive reasoning goes hand in hand with strategic competence, and strategic competence is mostly merged in students' reasoning. In most cases, in the students' classroom data, students give the reason for a particular strategy in their conversations before they execute the strategy, that is how the two strands are related.

Again, I see a connection between conceptual understanding and adaptive reasoning, and between procedural knowledge and strategic competence. Let me use one example to explain. Making conclusions about drawings is an indicator of conceptual understanding; however, it would only be possible when the students can justify that conclusion from the drawing, which is adaptive reasoning. That is, students' reasoning is made possible through their conceptual understanding, and their conceptual understanding is demonstrated through reasoning. Also, every strategy the students devise, after explaining why that strategy works and executing it reveals the students' procedural fluency.

The productive disposition strand was the most difficult strand for me to identify, and it can be seen from the tables in Appendix M, that I was able to identify a few from the students' conversations. Dias Corrêa (2017) had a similar observation about this strand. She noted that students do not usually, or openly, express productive disposition. Groth (2017), in a report on how two prospective teachers used the strands of mathematical proficiency to analyze and reflect on qualitative classroom data, also found that productive disposition posed a challenge to the preservice teachers to identify. The question I asked myself was, should productive disposition always be expressed verbally, or there are other means that students exhibit productive disposition. My conclusion was that productive disposition need not be expressed only verbally; it can also be expressed in one's actions. From the analysis of the students' data, I found the following to be indicators of productive disposition: (1) students showing confidence in their mathematics abilities, (2) their willingness to stay on a task no matter the challenge they face, (3) students' willingness to try something different when they fail, (4) students appreciating their effort and their product, (5) students expressing ownership of their knowledge, (6) students respecting each other, (7) students recognizing and appreciating each other's view, (8) students recognizing mathematics as an interesting subject, and (9) students identifying the application of mathematical concepts in different situations. Dias Corrêa (2017) had similar conclusions about this strand in her study on students' mathematical proficiency in the context of mathematical modelling. It can be said from this that the productive disposition strand is the force that moves the students, and also sustains them to have the desire to persist in mathematics activities. The productive disposition strand makes it possible for the other strands to grow.

Lastly, I realized from the students' data that, in the mathematics classroom, procedural fluency is usually identified through actions, like measuring, drawing, writing, and solving problems on a sheet of paper, while the oral discussions are very important in identifying the other strands of mathematical proficiency, namely: conceptual understanding, adaptive reasoning, strategic competence and sometimes, productive disposition. Dias Corrêa (2017) had a similar observation from analysis of students' solutions to modeling tasks and notes that:

> Adaptive reasoning was expressed when students explained their thoughts or options to their peers, or when they were thinking out loud as a way of understanding their ideas. It was not usual for students to write down their reasoning. In general, they wrote down the procedures derived from their reasoning processes (p. 120).

This shows how important it is to emphasize discussions in the mathematics classroom. Although communication is not explicitly identified as a strand of proficiency by Kilpatrick et al. (2001), I believe it is implied in the sense that you cannot witness all the strands without it. The implication is that, instead of always looking for students' written answers and procedures on a sheet of paper, it is helpful to allow them to talk through the procedure that led to the answer they had to get to their understanding behind the procedure. Even though there was not a follow-up interview on students' written work, the classroom discussions of their results illustrated many of the things they said in their conversations, this means if you are not able to take a video or audio record of students' conversations, allowing them to share their results with their colleagues in the class will be an opportunity to help them to develop their mathematical proficiency. Many mathematics educators, particularly in design-based research and sociocultural theorists, have emphasized classroom discourses and socio-mathematical norms as very important (Cobb, Confrey, et al., 2003; Cobb, McClain, et al., 2003; Lerman, 2001; Van

Den Akker et al., 2006; Yakel & Cobb, 1996). In this intervention, the level of autonomy given to students over their approach to the task offered them the opportunity to bring their understanding to bear on the solutions, and they were able to demonstrate their conceptual understanding and reasoning behind the procedures they opted to use. The whole-class discussion that took place after every activity was to allow students to explain their solution methods, with emphasis on justifying why they think their methods work or why their solution is also correct with teachers consolidating students' learning at this point. Özdemir and Pape (2012) observed that using whole-class discussions, that allow students to explain and justify their solutions, helps students to demonstrate their adaptive reasoning.

Impact of the Culturally Responsive Teaching

Many forms of teaching can contribute to the emergence of a learner's mathematical proficiency, so what was particular to this intervention, in terms of cultural relevance and how those particularities contributed to the learner's mathematical proficiency identified in the research? Some strategies for teaching were identified by teachers in the planning stages of the study. These can be understood in terms of culturally relevant pedagogy. The use of the students' local language, working in groups, teachers facilitating lessons, and the focus on the cultural artifact (Adinkra symbols) were deliberately planned for and implemented by the teachers in this intervention. Each of the elements can be seen to have had an impact on the students' learning during the intervention.

The Use of Both the English and the Twi Languages

As already mentioned, many educational researchers have pointed to the importance of the use of students' native/local language in classroom interactions (Barwell et al., 2016; Irizarry, 2007; Lee, 2010; Nieto, 2002). At the planning stage of the lessons, teachers agreed that students would be allowed to use their local language (Twi), in addition to the English language, to allow for greater participation in the lessons.

Unlike most lessons in Ghanaian secondary school classrooms, the students mostly spoke Twi during the lessons. Giving students explicit permission to use the Twi language was a move the teachers made toward culturally responsive pedagogy. Students' engagement with the classroom activities was high. I observed students to be more comfortable when speaking in the Twi language than when speaking in English. The use of the local language in the class seemed to create a relaxed and friendly atmosphere for students. The level of participation, as could be seen in the students' conversations in column two of the tables in Appendix M, was high, indicating how students could freely air their views. This was possible because of the use of the local language. The Twi language provided students with a chance to share their ideas and thoughts with their colleagues. A student may begin a sentence in English but the sentence will end in the Twi language, and they code-switch from Twi to English, especially when using mathematical terms. Thus, students spoke in Twi and inserted English mathematical terms mid sentences. A teacher in his reflective journal said: "Students made use of mathematical terms in English even when they were speaking Twi, because they are mostly taught mathematics in English and that the only name that they know to represent the concept is the English name". This view of the teacher is similar to the finding of Moschkovich (2007), that bilingual students will talk about a topic in their second language if they have not received instruction on the topic in their first language. That is, if the students know the topic in their first language, they will also talk about the topic in their first language. I also indicated in my fieldnotes that, in my experience as a native speaker of the Twi language, this could be attributed to the fact that mathematical terms are hardly expressed in single words in the Twi language.

When the teacher asks a question in the SHS class, a student who wishes to answer but cannot express him/herself in English will first ask: “Sir, may I speak in Twi?”, and when the teacher says yes, the student will continue to give the response in Twi. This is probably because of the demand on the students at this level to speak only English. Even though they were told at the beginning of the lessons that they could speak Twi in the class, they still sought permission from the teacher to respond to his questions in Twi. However, in their exploratory group activities, they mostly used Twi. This finding resonates with that of Setati and Adler (2000) and Setati et al. (2002). This suggests that the emergence of the students’ proficiency was enhanced by the use of their local language in the intervention.

Planas and Civil (2013) found language as a resource for thinking and doing, particularly for the learning and teaching of mathematics. I could probably have not been able to access the students’ strands of mathematical proficiency if they had been made to speak English only in class, because students mostly explained concepts, described mathematical situations, and justified their answers in Twi. If I could identify any strand at all, it might have been procedural fluency, which could be seen in their paper and pencil work, if students were made to use only the English language in class. Setati and Adler (2000) had also observed that students in bi/multilingual classrooms where English is not their first language make use of the English language in short phrases, single words, or recall of procedures in the public domain, but their exploratory talks are in their main language(s). That is students’ conceptual understanding, adaptive reasoning, productive disposition, and strategic competence emerged and could be accessed together with their procedural fluency because they were allowed to use their local language.

A JHS teacher, in the interview, remarked in an answer to a question on the affordances of the intervention, “did you see how students in the class were talking when it came to the whole class discussions? If they were made to speak in English, only a few people would have been heard in the class”. This is an indication that some students do not participate in classroom activities, not necessarily because they cannot reason or do not understand what is being discussed, but because they lack the language to express their thoughts. Opoku-Amankwa (2009) described a classroom situation in a Ghanaian English-only school where students pretend to be learning by responding “Yes madam” and teachers pretend to be teaching by ignoring students who do not respond to questions and interact with only a few students who could respond in English. The author also gave an example of a student who came to class late, and when the teacher asked him why he was late, the student did not respond. After the class, the author met the student and asked the student, in the Twi language, why he came to class late, and the student gave details on why he happened to be late. Then the author asked him again, why he did not say the same thing to the teacher when the teacher asked him why he was late, the response of the student was “I cannot say it in English and Madam will insist I say it in English”. It was likely that this situation described by Opoku-Amankwa (2009) could have repeated itself in our lessons if we had asked students to speak in English only. Voices of a few of the students would have been heard in the class, and teachers might not have any option other than to concentrate on the few who would be able to interact in the English language. A parallel observation was made by Planas and Civil (2013), in their analysis of two sets of data from mathematics lessons in two languages (Catalan or Spanish). They observed that there are differences in the mathematical participation of students based on which language they use; students’ participation was high when the language of instruction was their home language. Setati and Adler (2000) and Setati et al. (2002) had similar observations about the use of the students' local language. The use of the local language did not only enable students to interact with each other to engage in the mathematical investigations with the Adinkra symbols, but it also allowed all the five strands of proficiency to be manifested by students.

The Small Group Work

Small cooperative group work is a pedagogy that is being encouraged in the classroom, especially in mathematics classrooms, by many educational researchers (Abdulrahim & Orosco, 2019; Ladson-Billings, 1995a, 1995b, 2006; Mcleod, 2018; Steele, 2001). However, the Akan community believes in collaboration, and encourages the community to work together through such sayings as "*Prayɛ, sɛ woyi baako a na ebu, wokabomu a emmu* (a stick of broom will break but a bunch of them tied together can never be broken)", "*Ti koro nko agyina,* (one head does not constitute a council)" or "*Baanu so a emmia* (When two people carry, it does not hurt)". I think this could be a reason to have an aim for the mathematics curriculum for students to work in collaboration. Therefore, consistent with culturally responsive teaching and the communal living in the Akan society of Ghana, and in the semi-urban area where the study was situated, students were made to work in small groups during the intervention. Steele (2001) stated that "to give opportunities for children to build taken-as-shared meanings, teachers need to plan contexts that encourage children's active involvement and mental activity and provide social learning situations in which communication takes place" (p. 405). The small group cooperative learning that was used in this intervention provided social learning situations for students where they interacted and shared ideas.

In column two of the tables in Appendix M, we see the level of interaction that occurred among group members. It can be seen, from the tables used to present students' data that, strands of mathematical proficiency were developed at the group level before the individual level. That is, the members of the group came together to solve the problems with each member exhibiting his/her mathematical abilities. Together, they used their mathematical proficiency to solve the problems at the group level, and this provided opportunities for individuals to achieve the strands of mathematical proficiency as they learn from each other's ideas. The strength of a member became the strength of the group, an idea of a member became the idea of the group. Students recognized and respected each other's views and together, they solved the problems. Vygotsky's sociocultural theory emphasizes that cognitive development results from social interactions, implying that when students work in groups, group members learn from each other through their interactions, and less capable learners learn from more capable learners (McLeod, 2018).

In the group work, we see students explaining their thoughts and justifying their ideas to each other, this shows adaptive reasoning. Steele (2001) believed that explaining thought to others becomes reasoning for oneself. While explaining themselves, they make use of their conceptual understanding and adaptive reasoning. The SHS teacher, who implemented the lessons, said in the interview:

> The group work solved a lot of problems for me. I did not have to respond to many questions from the students, because they had colleagues in the groups to respond to their questions, if not I would have been talking throughout the lessons. Mr. Oti

This response indicates that the small group work was a resource for students to learn from each other and respond to each other's questions. It lessened the burden of the teacher, and it made the students independent of the teacher. In my observation, I could say that the collaborative work also helped the students to persist on tasks, as they pulled each other along throughout their solution processes.

Teachers as Facilitators

In this study, students were to develop their own strategies to tackle a problem. During the main activities of the lessons, teachers were only to give cues to students when necessary, and students had to make most of the decisions for themselves. In the rotation lesson at the JHS class, for example, the teacher led the class to discover the rotation rules, however, the students were free to use any method they thought possible to tackle the evaluation exercise. We see in Appendix M, Table M4, how the Group Two members developed their own strategy to rotate the different points to get their images. In the debriefing session after the JHS lesson on rotation, Mr. Oteng said:

> The students could use their own methods to find the coordinates of image points through rotation with the different angles because they were not restricted to using the rules. Some groups even completed the entire task straight on the graph without finding the image coordinates somewhere as we usually do. In my view, students understood what they were doing, and it also shows they have knowledge and abilities beyond what we teach them, but they do not demonstrate them because we don't give them the chance.

The teacher's statement emphasizes the fact that the intervention offered students the opportunity to exhibit their mathematical capabilities. The autonomy given to students about their choice of methods or procedures helped them to come up with new strategies and ideas that were beyond what the teachers were expecting from the students. Kilpatrick et al. (2001) noted that for students to be able to solve non-routine problems, teachers must give them flexibility in approaching the problem.

In the SHS lessons, though the different groups were investigating the same Adinkra symbols for transformation rules, the groups were given the option to choose how to orient their symbols, the size, and where they position them on the graph, and that resulted in them having the autonomy in making their own deductions. Özdemir and Pape (2012) for example, revealed that one way of supporting students to develop strategic competence is to allow autonomy and shared responsibilities. Teachers allowed students independence to make their decisions on how to approach a problem, and within the groups, students had a form of shared responsibility in the sense that each member was to contribute to the solution of the problem at hand. Thus, students create their own knowledge and develop mathematical meanings as they learn to explain and justify their thinking to others in the group (Steele, 2001). Allowing students to work in groups facilitated this decision of the teachers to give students the autonomy to make decisions, and this offered opportunity to students to claim ownership of the knowledge they generated. Graven and Stott (2012) had a similar observation and concluded that giving students a choice of procedures enhances both their procedural fluency and conceptual understanding. The level of independence given to the students enabled them to develop their own strategies to solve problems, explain to each other why and how a strategy works or does not work, using their conceptual knowledge and reasoning about situations.

The Use of the Adinkra Symbols

Teachers provided contexts for the mathematics concepts taught in the lessons. The use of the Adinkra symbols provided a context for mathematics learning. At the same time, the classroom activities were designed around the Adinkra symbols so that students would realize how the concepts are useful in real life. This notwithstanding, teachers took it upon themselves to introduce every lesson, if not with an Adinkra symbol, with materials that are common in the students' everyday experience, before allowing students to go into the class activities that were designed around the Adinkra symbols. For example, the JHS teacher used a milo tin and a plastic cup to demonstrate to students the meaning of ratio. The SHS teacher was aware that students use mirrors in their homes. The teacher came to class with a mirror to reflect a quadrilateral drawn on a sheet of paper to introduce reflection to students. These demonstrations enhanced students' understanding of the topics. All classroom activities, as can be seen in the lesson plan (Appendix J) and worksheets (Appendix K), and the evaluation exercises were developed around the Adinkra symbols.

The Adinkra symbols, as mediating objects, mediated the students' mathematics concept learning, creativity, and moral values. The images of the Adinkra symbols, posted in the classrooms and those handed to the students, provided students with models to refer to when solving the problems. In the SHS activities, they had to refer to the symbols to transfer them to the grid sheet. You can also see, in the JHS solution to the problem on the ɔwɔ *foro adobe* symbol (Appendix M, Table M2), how they referred to the symbol to explain the statement "inclined at 60° to the horizontal". Students' mathematical conversations were all around the symbols involved in the tasks. It can be said that students' solution strategies as well as their explanations were always based on a particular symbol involved in the task. The use of the symbols as contexts for the mathematics tasks posed to students helped them to develop across all strands of mathematical proficiency. Throughout the lessons, students paid attention to the beauty and the values of the Adinkra symbols, as well as the mathematics concepts in them.

Another important observation that could be made from the students' activity with the Adinkra symbols is the fact that students made use of their understanding of mathematics concepts to rename Adinkra symbols, and to name their created graphics. Before the students renamed an Adinkra symbol, they related the appearance of the symbol to mathematics concepts. For example, in the SHS students' activity with the *Mate masie* symbol, which signifies knowledge, the students renamed the symbol by first noting that the symbol is made up of the concept of circles, and relating the circles to the human head said "*the four circles represent human head because knowledge is kept in the head*" and again by their understanding of the mathematics concept of four as a quantity being three more than one, said, "*I think because of the four circles we can say that knowledge is acquired best in groups just as we are doing now*". Likewise, in the JHS students naming of the graphic they obtained by enlarging the *Akoma* symbol with different scale factors, from the students' abstraction of small and big, they said "*since it is like the heart is growing from small to big*, *let's call it grow in love and patience*". This is a different use of mathematics concepts in real life. Here, apart from students using mathematics ideas to create the designs, they also referred to mathematics concepts to name the designs. That is, they made use of their conceptual understanding and adaptive reasoning about the appearance of the symbols to give them names.

The discussion of the meanings of the Adinkra symbols is a form of a moral lesson (and learning of cultural values) in the mathematics classroom. In the students' conversations, they referred to the meanings of some of the Adinkra symbols. In JHS Group One's solution to the problem on the *ɔwɔ foro adobe* symbol, one of them said: "Yes, we are drawing the *ɔwɔ foro adobe*, which means we should continue to try no matter the difficulty". This student is relating to the meaning of the *ɔwɔ foro adobe* symbol, which is persistence. The values of the symbols the students learned also supported their productive disposition. In the JHS class, the teacher discussed the moral values of the Adinkra symbols with the students as part of the introduction, while in the SHS lessons, as part of the students' activities, they were to relate the meanings of the Adinkra symbols to their values as students. In the SHS Group Two discussions on the meaning of the *Apa* symbol, a student said the symbol means "*you have to abide by the rules of* *any society you belong to*". Thus, not only did the students learn mathematics through the Adinkra symbols, but they also learned social values through them. Some of the values they learned could be related to the productive disposition strand of mathematics proficiency. As mentioned above, the meaning of the *ɔwɔ foro adobe* symbol, which is persistence, is encouraging students to persist in challenging mathematical tasks. Another example of students being mathematically productive was them relating to the meanings of the symbols. One group of students wrote "*The Adinkrahene means creativity, we are, therefore, creative because we have created Adinkrahene with mathematics".* In the SHS activity on the *Mate masie* symbol, Group Four members renamed the *Mate masie* symbol as *Adwene ntoatoa* - put minds together. This name encourages collaboration among the students in mathematics. Gay (2010) notes that culturally responsive teaching simultaneously develops along with academic knowledge, social consciousness, and cultural affirmation (see also Gay, 2002; Ladson-Billing 1995b, 2021). Pinxten and François (2011), in their argument for what they refer to as "multimathemacy" stated that "In an era where 'intelligence' is now advocated in the plural, involving also such capabilities as emotional and spiritual intelligences, it would be absurd to keep restricting that human capability to formal procedures alone" (p. 269). The use of the Adinkra symbols in the mathematics classroom promoted the development of academic mathematical knowledge, as well as, the Akan social values, which includes their spirituality.

In a study of four Grade 6 classes in South Africa, for evidence of the promotion of the strands of mathematical proficiency, Ally (2011) revealed that occasions that showed evidence of developing productive disposition were mostly due to the inclusion of the real world, or out of school situations that teachers incorporated in their lessons. That is, using examples of students out of school experiences for mathematics instruction is beneficial for students' learning. Most likely, what the use of the Adinkra symbols added to the lessons was the fact that the students' interest to know what mathematics is in the symbols, motivated them to stay and continue the investigations until they found what needed to be found. Students' excitement, especially when they created something out of the Adinkra symbols, cannot be overemphasized. When the JHS students finally created the *ɔwɔ foro adobe* symbol, one of them shouted, saying "we have finally climbed the tree oh!!, and it is beautiful". In the whole-class discussion, one student commented: "Sir, you have to ask the Adinkra people to employ us to draw the symbols for them with mathematics because ours is more beautiful". In this intervention, all the strands did occur, I believe it is the productive disposition strand that made it possible for the other strands to be demonstrated.

Kilpatrick (2011) revealed that in the mathematics learning study group that resulted in Kilpatrick et al. (2001) strands of mathematics proficiency, while teachers were of the view that productive disposition towards mathematics is critical for students' mathematics learning, some mathematicians resisted having it as one of the strands of mathematical proficiency. Teachers, who are in the classroom, have witnessed the importance of the affective component in learning mathematics. In this intervention, the Adinkra symbols brought some form of an affective domain that motivated the students to stay on the task and learn mathematics through the Adinkra symbols. Mr. Antwi said this in the interview:

> I think students enjoyed the lessons because the concepts were developed from something they could relate to. Even the students who were not part of the study enjoyed the lessons because of how they could relate the transformation rules to the Adinkra symbols, something they know already. Let me say this, you see, in most of our teachings, we only mention examples of things that a concept is related to, but we have not used the

> examples in lessons as we did in this your research, that is why I think the students showed much interest in the lessons and focused on the activities. Mr. Antwi

From this teacher's response, the use of the Adinkra symbols, as context for the topics, provided opportunities for students to realize the application of mathematics in their daily life, and that motivated them to stay focused on the activities.

It can be seen, in the results presented in this chapter, that students' mathematics proficiency was indeed promoted in the intervention, and that each of the implemented features of culturally responsive pedagogy had a positive influence on students' mathematics proficiency. In the next chapter, I present the summary, conclusion, and recommendations that are based on the findings.

CHAPTER EIGHT

Summary, Conclusion, and Recommendations

This chapter will give a summary and conclusions of the main findings of the study and their recommendations for mathematics education and curriculum development, and future research.

Summary and Conclusion

In my study, I investigated the kinds of mathematics learning that could occur in mathematics lesson activities involving the use of Adinkra symbols (sociocultural artifacts) through culturally responsive pedagogy. Before a sociocultural artifact can be used for mathematics instruction, the mathematics concepts that are related to it or that can be learned from it must be established. Identifying the mathematics concepts in cultural artifacts, or cultural practices is termed ethnomathematics (D'Ambrosio, 1990; Rosa & Orey, 2011).

I began the study with five mathematics teachers (3 JHS and 2 SHS), to find related mathematics concepts of Adinkra symbols to be used as mediating tools in the design of lesson activities. The teachers and I came together and studied images of Adinkra symbols for their related mathematics concepts, and also observed a craftsman creating the selected symbols to infer mathematics concepts employed in the creation processes. After we identified the mathematics concepts in the Adinkra symbols, JHS and SHS teachers each selected four concepts for lessons to be designed using the Adinkra symbols for teaching. Together, the teachers and I designed the activities to include the Adinkra symbols related to the concepts for implementation in the classrooms. I believe the results of this study serve two purposes: (1) to illustrate how teachers can incorporate cultural artifacts in a purposeful and meaningful way as an element of culturally relevant teaching and learning, and (2) to serve as a model for further studies on culture and mathematics teaching.

Identifying Mathematics From Adinkra Symbols and Ethnomathematics

The teachers identified a number of concepts associated with the Adinkra symbols from their observations of printed symbols and from observing the creation of the symbols by a craftsman. These concepts included: transformational symmetries, ratio and proportion, area of a sector, similar shapes, scaling, and angle properties of parallel lines. Other authors have also identified transformational symmetries with some of the Adinkra symbols noted in this study (Abiola & Biodun, 2010, Aurthur, 2017, Muireku, 2014). Although no researcher before this study has associated ratio and proportion with an Adinkra symbol, Dabbour (2012) identified proportions as an underlying concept in Islamic art. It can be said that the Adinkra craftsman also used ratios to create proportional balance in the different parts of the shapes. Arthur (2017) identified scaling in the *Aya* symbol, which implies similar shapes and ratios can be inferred from that, however, only in this study has similarity and ratio been identified explicitly. So far, I have not found any reference in the literature for my study that explicitly discussed angles with the Adinkra symbols without referring to rotational transformation. However, in the creation processes of the symbols, we observed that different kinds of angles were formed at some stages in the creation. We found angle properties of parallel lines with two Adinkra symbols: *ɔwɔ foro adobe* and *Nhwimu*. Identifying this range of concepts in the symbols themselves, and the work of the craftsman creating the symbols, was a critical part of the teachers' work to integrate these cultural artifacts into their lessons.

Researchers of the ethnomathematics program have stressed the need for teachers to investigate and incorporate mathematics ideas from cultures of different ethnic groups into their mathematics teaching. What is not revealed, by these researchers, is the fact that as teachers investigate the cultures of the ethnic groups for their mathematics concepts, the teachers can learn more than just identifying the mathematics in the cultural practices. The study revealed that the investigation of the Adinkra symbols afforded teachers the opportunity to: (1) do mathematics with the symbols, (2) make links between the mathematics they observed in the symbols and the mathematics in the curriculum, (3) establish links with what they have observed and how that could be employed in teaching to enhance students' learning, (4) make connections between different mathematics concepts, (5) relate the meanings of the Adinkra symbols to the aims of the mathematics curriculum, and (6) develop culturally responsive teaching ideas. I believe that the teachers' "mathematics-for-teaching" as Davis and Simmt (2006) described, was expanded through the investigation of the ethnomathematics in the Adinkra symbols.

The research into ethnomathematics does not define specific ways for investigating cultural practices and artifacts for their related mathematics concepts. How the teachers in this study came to the "knowledge of" the mathematics concepts in the Adinkra symbols illustrates how studying the symbols themselves for mathematics and observing the creation of those symbols provided different observations of mathematics. This strategy of both observing the final products (Adinkra symbols themselves), and how they are created, could be one of the ways the symbols could be investigated for their mathematics.

Affordances of the Culturally Responsive Intervention on the Emergence of Students' Mathematics Proficiency

This study confirms that mathematics classes, based on culturally responsive pedagogy, can play a prominent role in increasing the emergence of students' mathematics proficiency. Decisions for mathematics lessons led to culturally responsive strategies that could promote students' mathematics proficiency and were implemented in the classrooms. The implemented culturally responsive strategies were:

- using the local language together with the English language as a medium of instruction,
- using the Adinkra symbols in the lessons,
- having students work in small cooperative groups, and
- teachers facilitating students' learning.

Of all the decisions taken and implemented in the classroom to enhance students' learning, the use of the Ghanaian language (Twi) facilitated access to all students' learning that occurred in the lessons through group conversations, whole-class discussions, and group presentations. Affordances of each of the implemented ideas are presented in Figure 8.1. The aim is not to highlight them as the best culturally responsive teaching practices; however, the study suggests they are promising practices for a teacher who wishes to use an artifact, like Adinkra symbols, for mathematics teaching. Again, as demonstrated in the previous chapter, it should be noted that, in the analysis of students' mathematics proficiency that emerged in the intervention, for each episode, only one group was selected based on the amount of data they provided in writing and conversations. That is to say that other tables could be created that may present different elements of mathematical proficiency. However, this conclusion is based on the data presented.

Figure 8. 1

Implemented Strategies and Their Affordances and Strands of Mathematics Proficiency They Promoted

The Use of the Ghanaian Language. Other studies, on the promotion of the strands of mathematics proficiency in classrooms, have revealed that mathematics teachers tend to emphasize procedural fluency at the expense of the other four strands (Ally, 2011; Engelbrecht, 2005; Schoenfeld, 2007). However, in the case of this study, it was observed that all five strands emerged. I believe that I was able to access all the strands due to the use of the local language in the lessons. Students could express their mathematical ideas and reasoning in the lessons through their local language, Twi. This is an indication that the language barrier could be a contributing factor to why mathematics teachers focus on procedural fluency. English, which is the official language of Ghana, is the second language of Ghanaian teachers and students, therefore, the teachers and students may have deficits in the English language, hence, using it for instruction becomes difficult, and may cause teachers to emphasize procedural knowledge in the mathematics classroom at the expense of the other strands of proficiency. Opoku-Amankwa (2009) for example, illustrated how English-only communications in a Ghanaian classroom created anxiety, and most students failed to speak because they could not express themselves in English. Similar observations were made by Erling et al. (2012) that: 1) the language of instruction can constitute a barrier to good pedagogy, 2) teachers' competency in the language of instruction is an important factor in teaching, and 3) students' deficit in the language of instruction can limit opportunities for communication.

The level of participation and students' engagement in the lesson activities in this intervention could be attributed to the use of the local language. It can therefore be said that students may become passive in the mathematics classroom and show a lack of interest in the subject partly because the local language is not used. Students involved in this intervention were more relaxed in the lessons and contributed to building on each other's ideas. An argument in favor of the use of students' first language in education has been on the learning benefits that all the students may get from having the opportunity to use the language they know and understand in the development and communication of their mathematical thinking (Planas & Civil, 2013). I am, therefore, appealing to those in authority to reconsider the language policy, especially concerning mathematics teaching at all levels.

The Use of the Adinkra Symbols. In this intervention, the Adinkra symbols were used in four main ways: 1) using images of the symbols as teaching/learning material, 2) as a context of students' mathematical tasks/problems, 3) discussion of the values of the Adinkra symbols, and 4) renaming of the symbols. The images of the Adinkra symbols, posted in the classroom and those handed to students during class activities, served as models for students to refer to when necessary. Using the images as examples of the mathematics concepts for students enhanced their conceptual understanding.

The notion of everyday and scientific concepts (Vygotsky, 1986) and Zone of Proximal Development (Vygotsky, 1978) suggest mediating tools (Hassan, 2002; Vygotsky, 1978), hence we needed to have mediating tools in the intervention. The images of the Adinkra symbols, which were found to be associated with the concepts, were used as mediating tools that mediated students' learning of mathematics concepts. This enabled students to make sense of the mathematics problems/tasks, and they served as representations of the mathematics concepts students learned. The five representations for learning and solving mathematical problems by Lesh et al. (1987) indicated that in teaching/learning, students should move among five representations of the concept (real-life situations, manipulatives, pictures, verbal symbols, and written symbols) and that students should translate whatever concept they are learning from one representation to another. It was observed that students referred to the images mostly in their initial attempts to make sense of the problems and after creating Adinkra symbols, through the use of mathematics ideas to compare their creation with the images. Through this act of comparing and talking about the graphics they created compared to the images, students strengthen their mental representation of the mathematics concepts they used to create the graphics (Adinkra symbols). Again, it can be seen in the transformation activities, that students' mathematical ideas resulted from studying the images and recreating them on the cartesian plane. The use of the images, as learning materials for students, therefore, promoted their conceptual understanding, adaptive reasoning, strategic competence, and productive disposition.

As illustrated in the literature, the cultural interpretation of the ZPD (Davydov & Markova, 1983 cited in Daniels, 2017) and the concept of horizontal mathematization (Freudenthal,1991), all indicate that the context of the knowledge we want students to learn must be related to their experiences; hence, the Adinkra symbols related to the mathematics concepts used in the lessons were used as context to design students' classroom activities as well as the evaluation exercises. That is, just as teachers found themselves in the curriculum they developed, students also found themselves in the curriculum. Students found an element of their culture (and an element of their identity), which is the Adinkra symbols that give livelihood to their community, in their classroom mathematics activities. Finding their identity in the mathematics classroom motivated them to fully engage with the mathematics tasks they were assigned. Students, being fully engaged in mathematics tasks, promoted the emergence of all the strands of mathematics proficiency. The students reasoned, based on the context, and developed strategies that were consistent with the problems at hand; and using their understanding of relevant mathematics ideas and procedures, they solved the problems. Researchers have observed that one advantage of using students' cultural artifacts for mathematics learning is that students find connections to the curriculum through their culture (Bonotto, 2010; Rosa & Orey, 2011; Sharma & Orey, 2017).

The use of the Adinkra symbols, as context for mathematics tasks, revealed to students the application of mathematics in their environment. Students realized how different mathematics ideas are related to different Adinkra symbols, and they used mathematics ideas to create some of the Adinkra symbols. The use of the Adinkra symbols to design students' intervention activities made students begin to realize mathematics as sensible and worthwhile. Other researchers had similar observations about the impact that the use of students' cultural experiences in the mathematics classroom has on their learning (see Bonotto, 2005, 2010: Rosa & Orey, 2011; Sharma & Orey, 2017; Weldeana, 2016; Zaslavsky, 1988).

The teachers' decision to discuss the meanings and the values of the Adinkra symbols as part of the culturally responsive strategy was meant to help students develop an interest in their cultural values and to learn social values from the symbols. The students' classroom work and conversations revealed that the students connected to the meanings of some of the symbols in their interaction with the activities on the Adinkra symbols. In particular, the students connected to the meanings of two Adinkra symbols: *ɔwɔ foro adobe* and *Adinkrahen*e, in mathematically productive ways. Students related to the meaning of the *ɔwɔ foro adobe* symbol to indicate they will persist in solving challenging mathematical tasks, and to the *Adinkrahen*e symbol to say they are creative in mathematics. Thus, students engaging in discussing the meanings of the Adinkra symbols also contributed to the emergence of their productive disposition.

It is seen, in the students' attempt to rename the symbols that not only did they apply reasoning, but it was found that they also used their knowledge in mathematics to rename Adinkra symbols and to name their created graphics. That is, this activity also enhanced the students' adaptive reasoning, conceptual understanding, and productive disposition. The teachers' decision to ask students to rename the Adinkra symbols used in their activities, and to name their created graphics and give their meaning, was one of the attempts to encourage students to know that they too can develop knowledge that could also be meaningful and acceptable. It was a form of helping students to have a sense of self-confidence, knowing that they can also come out with names and meanings of symbols that are reasonable to the members of the classroom community. This aim was achieved as demonstrated in the confidence with which students gave out their given names and their meanings. In that attempt, students also demonstrated ownership of their created knowledge (names and meanings of graphics). Fouse and Amit (2018) for instance, were of the view that culturally responsive teaching and ethnomathematics promote students' self-confidence in mathematics (see also Lunney Borden & Wagner, 2011; Wagner & Lunney Borden, 2012; Weldeana, 2016).

Another important observation that could be made about the names students gave to the Adinkra symbols and their created graphics, and their meanings are that the names and meanings signified social values. That is, students' class activities were also linked to the learning of social values through discussing the already known meanings of the Adinkra symbols, and also through their attempts to name and give meanings to their graphics. Students gave names like: "*Adwene ntoatoa* - put minds together", "grow in love and patience", these names signify social values that are encouraged in different ethnic groups. We need such values to sustain the world. Learning these values is equipping them for social life in the present and in adulthood. Weldeana (2016) noted that incorporating students' cultural background into the mathematics curriculum offers opportunities for all students to learn and achieve, as well as promoting their spiritual, moral, social, and cultural development and preparing them for opportunities, responsibilities, and experiences of adult life. These benefits of culturally responsive teaching are what Ladson-Billings (1994) used to explain culturally relevant pedagogy. The author explains that culturally relevant pedagogy is a pedagogy that empowers students intellectually, socially, emotionally, and politically by using their cultural references to convey knowledge, skills, and attitudes to them. Renaming Adinkra symbols and naming their created graphics was an intellectual activity for students that enhanced their reasoning and use of their mathematics knowledge; it also helped them to have a sense of agency.

The Use of Small Cooperative Groups. Communal living is a way of life in Ghana, especially in semi-urban towns and villages; hence, we decided to make use of cooperative groups in the lessons. The small cooperative groups also enhanced students' engagement and interaction with peers; the students became independent of the teacher and relied on each other for support. This enhanced their mathematics proficiency at the group level, before the individual level, as they learned from each other's ideas and built on each other's ideas. Since the students became independent of the teacher, they changed their identity as doers and learners. That is, this culturally responsive strategy fostered students' self-reliance. They were able to successfully claim ownership of the mathematics knowledge and graphics they created during the intervention. The small group work increased students' confidence in their mathematical ability and creativity. The collective generation of mathematical ideas by the students in the intervention prompted the emergence of all the five strands of mathematics proficiency, as seen in their conversations and written work.

Teachers Facilitating Students' Learning. Unlike the usual Ghanaian mathematics classroom where the teacher stands in front of the class and gives information to the students, and the students look up to the teacher for knowledge (Watson, 2008), in this study, teachers gave students the autonomy to make decisions about how they tackled mathematics problems/tasks. Teachers allowed students to choose their procedures and methods. This independence made students dependent on their peers in the group for answers to their questions. Through this experience, interaction with peers and engagement with tasks were enhanced. Butakor et al. (2017) identified the lack of engagement of students in mathematics lessons in

Ghanaian classrooms as one of the causes of low performance of students in TIMSS 2011 (see also Anamuah-Mensah & Mireku, 2005; Fletcher, 2005). The high level of engagement and interaction among peers, which was enhanced by the fact that the teachers were only facilitators of the lessons, contributed to the students' ability to develop ideas that were beyond what was planned for them to achieve. It can also be said that the decision of teachers to give autonomy to students was enhanced by the small collaborative groups that were employed in the intervention and made it possible for students to collectively contribute their mathematics proficiency in solving the tasks.

Based on these findings, relating to how each of the implemented strategies contributed to the emerged students' proficiency, I speculate that without implementing the elements of the culture for culturally responsive teaching, the observed mathematics proficiency would not have occurred. Having knowledge of mathematics in the cultural artifact and using that knowledge is not enough, it is also important to incorporate knowledge about the values, learning approaches, interaction patterns, and language of the ethnic group of the students in the lessons. Using the traditional approach of mathematics teaching described by Watson (2008), which is common in Ghanaian secondary schools, to implement mathematics ideas in the culture of the students might not have much impact on students' learning.

My speculation, as stated above, is aligned with Bishop (1994) who hypothesized that any formal mathematical education is a process of cultural interaction and that every child experiences some degree of cultural conflict in his/her learning process. The conflicts Bishop identified included: language, geometric concepts, calculation procedures, symbolic representations, logical reasoning, attitudes, objectives, cognitive preferences, values, and beliefs. It can be said that in this design-based research, most of the conflicts, including language, geometric concepts, cognitive preferences, and values, were resolved through the incorporation of knowledge about the cultural practices of the community in the classrooms. Mukhopadhyay et al. (2009) stated that ignoring student culture demonstrates a lack of respect for students and means that students' learning should be treated independently of their role as citizens, in which they must contribute to society using the knowledge gained in education. Respect for students was high in this culturally responsive intervention, as students learned mathematics through their cultural way of being from their cultural elements, something that provides livelihood to most members of their community. This means students can contribute to the community through the knowledge they acquired, as one student asked the teacher to tell the Adinkra people to employ them (students) to create the symbols for the craftsmen.

Challenges in Using the Culturally Responsive Intervention

Some challenges were encountered during the study; however, these challenges could be overcome. The first challenge was getting teachers' commitment to work to the end of the study. The study caused considerable strain on teachers as it required significant involvement. Studying the Adinkra symbols for their mathematics concepts and planning the lessons was challenging and time-consuming for them. During the implementations too, there were briefing and debriefing sessions to discuss what happens in lessons. All these activities were demanding for mathematics teachers who had other responsibilities. It was the responsibility of the researcher, therefore, to encourage teachers throughout the study and to allow flexibility for them to decide when the next activity should take place. Allowing them to make most of the decisions secured their commitment to the end.

There was another constraint that related to the designed activities and the students' class time. The number of activities involved in the JHS worksheets, including the evaluation exercises, always exceeded the two mathematics periods that were offered by the headteacher for each topic. Though we tried out the activities before the implementation, we did not anticipate that this was going to happen. Luckily, the headteacher allowed us to extend the periods further to complete each activity. A researcher, who wants to employ such an intervention, should consider the length of the activities and the class period permitted for the research. I will encourage anyone, who wishes to adopt the lesson plan and the activity of the junior high class, to split it into two lessons or perform the activities on the worksheet in one lesson and the evaluation in another lesson.

The SHS teachers also observed that the group presentations extended the class time beyond the two periods. This concern can also be resolved by making use of whole-class discussions instead of group presentations.

Another challenge I faced was the novel COVID-19 pandemic, which caused schools to be closed in the first stage of the study. This challenge was not study-related, though it had a strain on the study. Ghanaian secondary schools (JHS and SHS) were closed for about five months. When the secondary schools resumed, I had to work extra hard with teachers to complete the data collection by the close of the term in December 2020. Our usual weekly meetings had to be changed, especially during the lesson preparations, to meeting twice or thrice a week.

Culturally Responsive Teaching Through Ethnomathematics is a Viable Approach in Mathematics Classrooms

The result of the study demonstrates that it is possible to employ culturally responsive and ethnomathematics ideas in the classroom to increase students' mathematics proficiency. As shown in Chapters Six and Seven, the culturally responsive lesson activities were implemented in two Ghanaian classrooms, JHS Form 2 (Grade 8) and SHS Form 2 (Grade 11) with 31 and 24 students respectively. Apart from investigating the Adinkra symbols for their mathematics concepts and designing classroom activities that could ensure the emergence of students' mathematics proficiency, no special equipment or arrangements were made in the classroom. It is not expensive to incorporate culturally responsive ideas in the mathematics classroom. The classroom setting used for this intervention was much the same as the usual Ghanaian classroom, except that, images of the Adinkra symbols were posted on walls of the classrooms (which are not usually found in classrooms). Normal Ghanaian classrooms, with desks and chairs, do not hinder the use of culturally responsive teaching. The mathematics content that was supposed to be covered by the teachers and the students was covered, using culturally responsive teaching.

The study has also revealed to curriculum developers, in-service mathematics teachers, and pre-service mathematics teachers that mathematics in the cultural practices of the Ghanaian community (ethnomathematics) could be investigated for and included in the mathematics curriculum. It may be that the only investment that needs to be made to develop a culturally responsive mathematics curriculum and use culturally responsive teaching strategies in the classroom, would be the investigation of the mathematics in the cultural artifacts and practices, by engaging a knowledgeable community member (in the case of this study Adinkra craftsman) and designing activities with the ideas that would be obtained from the investigations. Gerdes (1998) emphasized that uncovering the mathematical knowledge of a cultural group requires a respectful and attentive focus on encoded ideas of its material culture. That is, the mathematical ideas in cultural practices and materials are not all that obvious, it requires time and energy to investigate the materials and the practices to uncover them. This could be done through teachers' professional development programs and as projects for pre-service mathematics teachers.

As has been revealed in the study, teachers investigating the ethnomathematics in cultural materials and practices helped them get a better understanding of their mathematics concepts, recognize how the different mathematics concepts are interrelated, and identify models from the culture to represent the school mathematics content. Hence, including such investigations into teachers' professional development programs, and pre-service mathematics teachers' curriculum, would promote teachers' mathematics understanding, and increase their knowledge base for mathematics instruction, at the same time as illustrating a practice that they could employ throughout their careers to gain new knowledge and ideas for teaching.

Finally, this study has revealed to mathematics educators a viable way of promoting the emergence of all the strands of proficiency, and how they can be accessed and assessed. The study has demonstrated how different culturally responsive strategies helped promote students' mathematics proficiency, and how students' proficiency was accessed and assessed through observing students' solution processes and through class discussions (group conversations, whole-class discussions, and group presentations). Teachers can use the indicators of the strands of proficiency outlined in this study, and tables created to illustrate strands of mathematics proficiency that emerged in the group work, to guide them to identify students' mathematics proficiency. Pre-service teachers can practice assessing students' mathematics proficiency by using these indicators during their teaching practice, as some of the strands are difficult to identify (Dias Corrêa, 2017; Groth, 2017). Lastly, the study has pointed to the importance of the language of communication in all these processes, that the use of the students' home language is beneficial for culturally responsive teaching.

Recommendations to Ghanaian Curriculum Developers and Teacher Educators

The results confirm that there could be value for Ghanaian curriculum developers to investigate and include ideas of our culture into the mathematics curriculum for the primary and secondary levels. Instead of casually mentioning in the curriculum that teachers should refer to real-life examples, curriculum developers should illustrate the mathematics concepts in the curriculum with examples that are related to Ghanaian culture and are meaningful to both teachers and students. The Curriculum Research and Development Division (CRDD), under the

Ministry of Education, together with mathematics educators from the different levels of education should consider investigating the culture of Ghana for examples of the Ghanaian culture that can be incorporated into the curriculum.

Based on the findings of the study, I am recommending that Ghanaian mathematics teacher educators consider the use of elements of Ghanaian culture for teacher education, both for pre-service and in-service teachers. The curriculum developers should not only include elements of the culture in the curriculum of pre-service teachers but also include projects in the curriculum that will enable pre-service teachers to investigate their cultures for mathematics knowledge and aspects of the culture that can be incorporated in their teaching. Through such projects, the pre-service teachers would become aware of the mathematics in Ghanaian culture. Pre-service teacher lecturers, at the various education colleges and universities, could include aspects of the culture into their teaching to create awareness that there are mathematical elements in the Ghanaian culture that can be employed for teaching different mathematics concepts. This could be a way of preparing the pre-service teachers for culturally responsive teaching by having them learn mathematics from the elements of their culture. Similarly, Ghana Education Service could use ethnomathematics and culturally responsive approaches to conduct professional development workshops for mathematics teachers both at the primary and secondary school levels, so that teachers will have the opportunity to also learn mathematics from their culture. Professional development workshops can focus on artifacts, such as Adinkra symbols, for teachers to investigate their related mathematics concepts.

Recommendations for Further Studies on the Adinkra Symbols

My interest in this study was to investigate how culturally responsive teaching through ethnomathematics of the Adinkra symbols could promote students' mathematics proficiency. My aim was to inform curriculum designers, teacher educators, and school teachers of the possibility of including such cultural artifacts in the teaching of mathematics for the benefit of students. As I reflected on the study, I identified the following to be limitations of the study that I will have to consider in my further studies on the Adinkra symbols:

1. Absence of an agent from the Ghana Education Office in the study: Since the study was to inform curriculum developers about the possibility of incorporating knowledge of the Adinkra symbols into the school mathematics curriculum, I think it would have been advantageous to have included someone from the municipal education office in the study. I could have sought consent from the mathematics coordinator of the municipal education office to be part of the study. The Education Office of the Municipal District is the agent that reports issues, in the district, to the national level. If I had included someone from that office in the study, that person could have helped to communicate the findings directly to the national level, so that curriculum developers would get to know of it from their agent, rather than from me as the researcher. I, therefore, recommend that further studies on the Adinkra symbols for curriculum innovation should include an agent of the Ghana Education office.
2. Students were not interviewed to find their reactions about the intervention: Students were the main recipient of the intervention, therefore, I think it would have been beneficial if I had interviewed them to get their views about the intervention, how the intervention affected their mathematics learning, why they said what they said in the conversations, or wrote what they wrote. That could have enhanced the report on the students' mathematics proficiency. I am, therefore, suggesting that further studies on the use of the Adinkra symbols for teaching intervention should include an interview of the students to determine how the intervention contributed to their learning and their general perception of the intervention.

3. Not all Adinkra symbols were studied for their mathematics concepts. Arthur (2017) indicates that there are about 450 Adinkra symbols. In this study, only 27 symbols were studied. Therefore, I recommend that a study should be conducted to expand the number of the Adinkra symbols to include more than the number used in this study. That could probably lead to identifying more mathematics concepts with the Adinkra symbols than were identified in this study.
4. The intervention was implemented in only two classrooms. I think a large-scale replication of the intervention in Ghana would give a better picture of how culturally responsive teaching through the Adinkra symbols contributes to the development of students' mathematics proficiency.

In conclusion, I want to emphasize that the findings of my research are relevant for the mathematics education research community, curriculum developers, and pedagogy in mathematics education. The findings of this study are relevant for the research communities of ethnomathematics and culturally responsive pedagogy. The study has revealed how these two perspectives work together to enhance students' learning, hence, for the ethnomathematics and culturally responsive research communities, in their search for cultural knowledge that could impact students' learning positively, this study could be a helpful guide. The findings are also relevant to curriculum developers as they seek to find mathematical knowledge of various ethnic groups to include in the mathematics curriculum for various levels for a culturally responsive mathematics curriculum. Again, the findings are an important contribution to pedagogy. The study revealed viable ways to include ethnomathematics and culturally responsive ideas into the mathematics classrooms. The study established a link between ethnomathematics, culturally responsive teaching, and strands of mathematics proficiency, showing how ethnomathematics leads to culturally responsive teaching ideas and how implemented culturally responsive ideas led to the emergence of students' mathematics proficiency, and also how emergent strands of mathematics proficiency of students were accessed and assessed. These ideas are helpful for mathematics pedagogy both in teaching and in assessment.

What I Have Learned From the Study

My experience as a mathematics student and teacher has made me believe that guiding students through the usefulness of mathematics could be one of the ways for meaningful mathematics teaching and learning. Hence, I began to look for means by which students' learning could be meaningfully facilitated. Fascinated by the ethnomathematics perspective, I decided to investigate mathematics in the cultural symbols of the Akans of Ghana called Adinkra symbols, for the possibility of incorporating the symbols in mathematics lessons to enhance students' learning. There are a few things I have learned through this research journey that I would like to highlight.

I was initially struggling between using culturally responsive teaching as the basis of the research or ethnomathematics because the two theories seemed to be about the same idea. I remember the challenge I faced in my candidacy exam is distinguishing the two theories. Through this study, I have realized that there cannot be culturally responsive pedagogy in mathematics without ethnomathematics study of the cultures of the students. I observed that the investigation of the ethnomathematics of the Adinkra symbols provided the opportunity for teachers to make connections between the ethnomathematics of the Adinkra symbols and the school mathematics curriculum and teaching and learning. Teachers came to know the aspects of the culture of the community and knowledge of the Adinkra symbols that can be incorporated into the classroom for culturally responsive mathematics teaching. From my observation of the students' learning that emerged in the intervention, I confirmed that culturally responsive teaching practices also present opportunities for the emergence of the strands of mathematics proficiency.

Upon reflection, I have come to believe that the mathematics teachers were able to develop these ideas because of their interaction with the Adinkra symbols and the Adinkra craftsman. For example, during the investigations of the Adinkra symbols, the teachers collectively developed the mathematics ideas related to the Adinkra symbols, especially, during the conversation sessions on the symbols. In the conversations as we were code-switching between the English and the Twi language, we made use of the images of the Adinkra symbols and different stages of the drawings by the craftsman, and the craftsman discussed the meanings of the Adinkra symbols with us. The necessary ideas needed for culturally responsive teaching of a particular group of students can be identified from the ethnomathematics of their cultures. I have learned that it is not enough to inform teachers about culturally responsive practices to employ in their teaching, but they must also experience what to do through ethnomathematics investigations. Teachers should be engaged in identifying the mathematics in their students' cultures (ethnomathematics), and through that experience, they will come to know the elements of the students' cultures to be included in the classroom for culturally responsive teaching. Asking teachers to investigate mathematics in their cultures and use it to redesign the mathematics curriculum for teaching is a way of acknowledging their professional knowledge and capabilities for creating ways to implement the school mathematics curriculum. After discovering the ethnomathematics of the Adinkra symbols and deciding on the aspects of the knowledge to include in the lessons, teachers prepared the classroom activities. They saw themselves in the curriculum and found a connection with their identities in the curriculum they developed because the ideas in the lessons were from them. I have come to understand that teachers are learners and curriculum designers as well.

Many educational research studies involve collaboration among teachers because collaborative research has been found to be an effective means for generating insights into teaching and research (Cowie et al., 2015). Through this study, I have realized that collaborative research studies become successful when the participants (teachers) share an interest in the topic of investigation. Although I had contacted several teachers, explained the purpose of the study to them, and sought their consent to participate, I had only five teachers to work with to the completion of the study. These five teachers showed genuine concerns about their students' learning and were willing to investigate their culture for mathematics ideas that they could use to improve their students' learning. The study was demanding in terms of time and required activities, but these five teachers traveled with me to the end. This has made me assert that research collaborations are only possible when the participants hold similar beliefs, values, and interests as the researcher. In other words, this study's success (at least in part) is because the participating teachers, like me, had the same interest in investigating and including the mathematics of the Adinkra symbols into their teaching.

In addition to the participants sharing the same interest as the researcher, I observed that mutual respect is also paramount for the study to be fruitful. One person cannot impose his/her views on others. Teachers and the researcher in the study negotiated knowledge through numerous conversations/debates, allowing us to learn from each other.

Although it is obvious teachers learned a lot from the study—they enhanced their practice through the study, I observed that teachers were building on what they already know. The teachers' professional knowledge, and knowledge they have acquired through integration in the culture, helped them build new knowledge through investigating the Adinkra symbols, and then developing the culturally responsive curriculum and implementing it.

The investigation into the Adinkra symbols with the teachers and the craftsman has also revealed to me that in ethnomathematics studies the contribution of a knowledgeable community member is paramount. Many things about the Adinkra symbols would not have been known to us (the researcher and participating teachers) if the Adinkra craftsman had not been involved in the study. In addition to gaining mathematical knowledge from his creations, we also learned how some symbols emerged and the values the symbols are used to depict.

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APPENDICES

Appendix A: Request for Permission Letter

[redacted]

FACULTY OF EDUCATION
DEPARTMENT OF SECONDARY EDUCATION
347 Education South, 11210 - 87 Ave
Edmonton, Alberta, Canada T6G 2G5
Tel: 780.492.3674 | Fax: 780.492.9402
educ.sec@ualberta.ca | secondaryed.ualberta.ca

The Municipal Director
Ghana Education Service
Kwabire East

Dear Sir/Madam

REQUEST FOR PERMISSION TO CONDUCT RESEARCH WITHIN SCHOOLS IN THE MUNICIPALITY

My name is Mavis Okyere, A Ghanaian PhD candidate at the Department of Secondary Education in the Faculty of Education at University of Alberta, Canada and a lecturer at the Catholic University College of Ghana, Sunyani-Fiapre.

As the heading refers, I am conducting a research study titled "The use of socio-cultural artefacts for mathematics instruction: An example of the Adinkra symbols of the Akans of Ghana". This study is a partial fulfilment of the requirement for the degree of Doctor of Philosophy in Secondary Education at the Department of Secondary Education in the Faculty of Education, University of Alberta, Canada.

This study is to investigate mathematics concepts that are related to the Adinkra symbols and the possibility of using the Adinkra symbols for mathematics lessons on the concepts that could be identified. The participants will be mathematics teachers and students at the junior high and senior high schools. The participating teachers will be part of focus group discussions on the Adinkra symbols and lesson preparations and presentations. The students will take part in the lessons that will be developed around the Adinkra symbols.

Please, find the attached the approval letter from the Research Ethics Board of the University of Alberta, Canada.

I would be pleased if my request will meet your warm response. Thank you.

Yours Sincerely,

[redacted]

Mavis Okyere

The plan for this study has been reviewed for it adherence to ethical guidelines by the Research Ethics board at the University of Alberta. For questions regarding participant' rights and ethical conduct of the research contact the Research Ethics office at +1 780 492 2615. For other questions regarding the research contact Mavis Okyere (okyere@ualberta.ca; +1 780 803 2858 or +233 244 285 810) or Prof. Elaine Simmt (esimmt@ualberta.ca; +1 780 492 0998)

Appendix B: Permission Letter From the Municipal Director of Education

GHANA EDUCATION SERVICE

KWABRE EAST MUNICIPAL EDUCATION OFFICE

P.O BOX 30
MAMPONTENG-GHANA
TELEFAX 03220-74577
DIRECTOR'S OFFICE 03220-70647
E-mail – kwabreeast@yahoo.com

OUR REF NO. GES/ASH/KEM/INTRO/91/V.3/4

Your Ref :..

REPUBLIC OF GHANA

DATE: 22ND OCTOBER, 2020

ALL HEADS OF INSTITUTION
KWABRE EAST

INTRODUCTORY LETTER

This is to introduce Madam Mavis Okyere, a PhD student at the University of Alberta, Canada. She is conducting a research study on "The use of Socio-Cultural Artifacts for Mathematics instruction: An example of the Adinkra symbols of the Akans of Ghana.

I am by this letter requesting heads of sampled Schools that, she has been granted permission to conduct the said research. Kindly grant her all the needed assistance.

Counting on your usual co-operation, while you continue to stay safe during this COVID -19 period.

Thank you.

[redacted]

DORA ASARE (MRS.)
AG. MUNICIPAL DIRECTOR OF EDUCATION

Appendix C: Letter of Information

[redacted]

FACULTY OF EDUCATION
DEPARTMENT OF SECONDARY EDUCATION
347 Education South, 11210 - 87 Ave
Edmonton, Alberta, Canada T6G 2G5
Tel: 780.492.3674 | Fax: 780.492.9402
educ.sec@ualberta.ca | secondaryed.ualberta.ca

Letter of information

Study ID Pro00096990

Introduction

My name is Mavis Okyere. I am a PhD student in the Faculty of Education, Department of Secondary Education at the University of Alberta. As part of my PhD work, I am conducting a study on the topic "The use of socio-cultural artefacts for mathematics instruction: An example of the Adinkra symbols of the Akans of Ghana". I am by this letter inviting in-service mathematics teachers, Adinkra craftsmen and students to participate in the study.

Purpose and procedure

The purpose of this study is to investigate how the use of the Adinkra symbols for mathematics instruction promotes students' development of mathematics proficiency. Your participation in this study is voluntary and you are not obliged to participate. Your participation will include any or all of the following:

If you are a mathematics teacher you will be asked to:

1. Identify mathematics concepts in the Adinkra symbols.
2. Observe Adinkra craftsmen for any mathematics demonstrated in creating the Adinkra symbols.
3. Design lesson activities around the Adinkra symbols to teach the identified mathematics concepts.
4. Implement the mathematics lessons in your class.
5. Write reflective notes on the effectiveness and constraints encountered during the lessons (if you implement it).

6. Participate in an interview after your lessons to further elucidate information on the affordances and constraints of using the Adinkra symbols for mathematics instruction (if you implement the lesson).

If you are Adinkra craftsman, you will be asked:

7. To demonstrate to myself and up to six teachers how the Adinkra symbols are created. if I can take photographs of you as you work and the crafts you make that include the Adinkra symbols.

If you are a student, you will be asked to:

8. Do a card sorting task, once before the less and again after the lesson.
9. Be video and audio recorded as you participate in lessons designed around the Adinkra symbols in your mathematics class
10. Provide your classwork and exercises done during the lessons so they can be photographed and then returned to you.

I will collaborate with mathematics teachers in conducting all the aspects of the study. All interactions including classroom activities, interviews and debriefing sessions will be video or audio recorded. I will collect reflective journals that will be kept by mathematics teachers in the course of the study. Student class work during the period of the intervention will be collected to make copies and return to them in a week's interval.

Voluntary participation

You are not under any obligation to participate in the study. By participating in the study, you consent that your contributions be used for my PhD research. Be aware that the results of the study may also be presented at conferences and/or may appear in academic journals. If after participating in the study (observations, lesson planning, implementing the lessons and reflecting on the lessons) you do not want your contributions to be used for my research or for publication, you must inform me within 2 weeks after the study.

Confidentiality and anonymity

Your anonymity will be protected in the research report, publications and any public presentation by using pseudonyms. The names of schools, teachers and students, and craftsmen will be replaced with pseudonyms in any publication. Data gathered from participants will be treated collectively rather than individually. All materials gathered during the study; video and audiotapes, journals, protocol and students' classwork will be kept in a secure place and protected by passwords on my computer. I request that participants respect the privacy of each other.

Risk and Benefits

By the nature of the study, I do not foresee any potential risk for participants. Adinkra craftsmen will benefit by the fact that, they will get to know that they are unknowingly using mathematics in their craft making and that their crafts can be used in schools for teaching. Getting to know this about their work could be a positive thing for them. Teachers participating in this study stand the chance to benefit by enhancing their knowledge in mathematics and its application in everyday activities and art. They will also acquire knowledge on using socio-cultural artefact like the Adinkra symbols to aid students learning. Students who will be part of the intervention stand to enhance their mathematical understanding by learning mathematics content through the use of a common artefact in their environment.

Remuneration

Participating teachers will receive GH₵50 (equivalent to about Canadian $13) for transportation and snack for attending meetings such as debriefing sessions and observations of craftsmen.

Craftsmen who will demonstrate the designing of the symbols will receive the same amount for snack.

Contact information

For any question about the study, you may contact the researcher Mavis Okyere (okyere@ualberta.ca; mavisokyere020@gmail.com; +233 244285810/+1 7808032858) or Professor Elaine Simmt (esimmt@ualberta.ca; +1 780 248-1812). The plan for this study has been reviewed by a Research Ethics Board at the University of Alberta. If you have questions about your rights or how research should be conducted, you can call (780) 492-2615. This office is independent of the researchers.

Sincerely,

[redacted]

Mavis Okyere

Appendix D: Consent Form for Adinkra Craftsman

Consent statement

This letter of information regarding the research study on "The use of sociocultural artifacts for mathematics instruction: An example of the Adinkra symbols of the Akans of Ghana" has been read to me. I agree to participate in the research study and provide consent to demonstrate how the Adinkra symbols are created to mathematics teachers. And agree that photos of my crafts and of the creating process can be taken.

_____________________________ ____________________ _______________

Name of craftsman Signature Date

Appendix E: Consent Form for Parents

Consent statement: I am the parent/ legal guardian of the student named below, and I have read and retained a copy of this information letter regarding the research study on "The use of sociocultural artifacts for mathematics instruction: An example of the Adinkra symbols of the Akans of Ghana. I give my consent for my child to participate in the following:

	Yes	No
Pre-sorting and post-sorting activities		
Be video and audio recorded during lesson activities, and that the recorded videos/audios could be used by other participating teachers in the study during debriefing sessions		
Use his/her class work (products)		

_______________________________ ____________________ __________________

Name of parent/guardian Signature Date

Name of child ____

Appendix F: Assent Form for Students

Assent form for students

I have been informed and my parents are aware of this research study on "The use of sociocultural artifacts for mathematics instruction: An example of the Adinkra symbols of the Akans of Ghana. I know that even though my parents gave permission, this is completely voluntary, and I can decide not to participate simply by telling my teacher or the researcher I do not want to participate.

I agree to participate in the following:

	Yes	No
Pre-sorting and post-sorting activities		
Be video and audio recorded during the study, and participating teachers can watch/listen to the recordings at debriefing meetings		
The researcher can use my classwork (products) developed in the study		

______________________________ ____________________ ________________

Name of student Signature Date

Appendix G: Consent Form for Mathematics Teachers

Consent statement

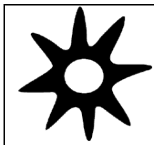
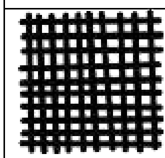
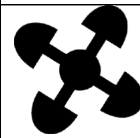
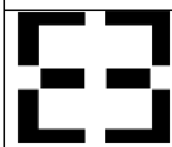
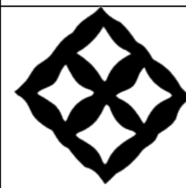
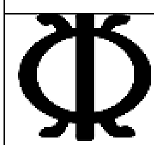
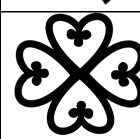
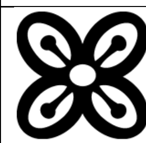
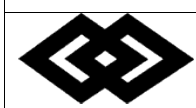
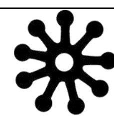
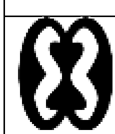
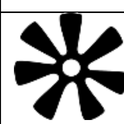
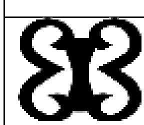
I have read and retained a copy of this letter of information regarding the research study on "The use of sociocultural artifacts for mathematics instruction: An example of the Adinkra symbols of the Akans of Ghana". I agree to participate in the research study and provide consent for the following:

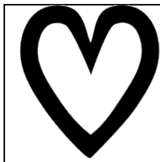
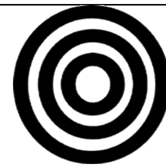
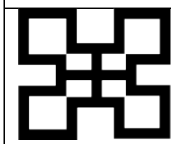
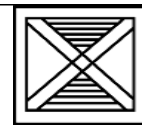
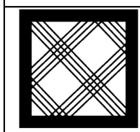
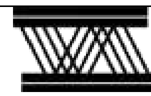
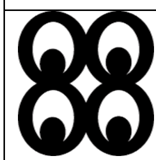
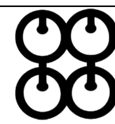
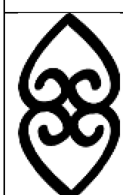
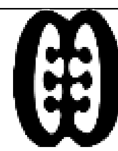
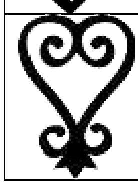
	Yes	**No**
Identifying mathematics concepts in the Adinkra symbols.		
Observing Adinkra craftsmen for any mathematics demonstrated in creating the Adinkra symbols.		
Designing Lesson activities around the Adinkra symbols to teach the identified mathematics concepts		
Implementing the mathematics lessons in my class.		
That the lessons will be video and audio recorded, and other participating teachers can use the recordings at debriefing sessions.		
Participating in debriefing sessions		

____________________________ ____________________ ________________

Name of teacher Signature Date

Appendix H: The 27 Adinkra Symbols Selected by Teachers to be Common in the Community

	*Nsoromma* (Star) Meaning: Child of the heavens, favour, goodwill, or blessings		*Mmere Dane* (Time changes) Meaning: the temporariness of good times
	*Kete pa* (Good bed) Meaning: a good marriage		*Akoma Ntoasoɔ* (Extension of the hearts) Meaning: Charter, agreement, understanding
	*Woforo Dua Pa a* (When you climb a good tree) Meaning: support for good cause		*Aban* **(**Fortress**)** Meaning: Safety or protection
	Wawa aba **(**seed of the wawa tree**)** Meaning: hardiness, toughness, and perseverance		Nyame dua (God's tree (sacred stump) Meaning: God's presence and protection
	Gye Nyame **(**Except God**)** Meaning: God is supreme		*Bese Saka* (Bunch of kola nut) Meaning: Abundance, affluence
	*Apa* **(**Handcuffs**)** Meaning: slavery, law, and order		Fofo (a yellow-flowed plant) Meaning: jealousy, envy
	*Nkonsonkonson* (chain links) Meaning: hope, unity, and human relations		*Ananse ntontan* (Spider's web) Meaning: Wisdom and creativity
	*Pempamsie* (Sew in readiness) Meaning: Readiness, steadfastness		*Aya (*Fern) Meaning: Endurance, resourcefulness

	*Akoma* **(**Heart**)** Meaning: Patience and tolerance		*Adinkrahene* (Chief of Adinkra symbols) Meaning: Leadership creativity and charisma
	*Nsaa* (A type of woven blanket) Meaning: Excellence, genuineness, authenticity		*Mframadan* **(**Wind resistance house**)** Meaning: Resilience and readiness to face the ups and downs of life
	*Nhwimu* (crossed divisions made on Adinkra cloth before printing) Meaning: Skilfulness and precision		*ɔwɔ foro adobe* (Snake climbs the raffia tree) Meaning: Persistence
	*Mate masie* (What I hear, I keep) Meaning: Wisdom, knowledge, and prudence		*Ma ware wo* (I will marry you) Meaning: Commitment and perseverance
	*Asase ye duru* (the earth has weight) Meaning: Providence and the divinity of mother earth		*Ase ne tekrema* (the Teeth and the tongue) Meaning: Friendship and human interdependence
	*Sankofa* (Go back and get it) Meaning: learn from the past		

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Appendix I: Observational Protocol for Observation of Adinkra Craftsman

Setting:

Role of observer

Time:

Duration of observation

Description of activities	Reflective notes: possible mathematics concepts implied

Appendix J: JHS and SHS Lesson Plans

JHS Lesson One

Topic: Ratio and Proportions

Objective: By the end of the lesson the student will be able to:

1. Express sides of different parts of Adinkra symbols as ratios of each other.
2. Deduce from the *Nsaa* symbol the meaning of proportion and tell why it is important in art.
3. Solve problems involving proportions in selected Adinkra symbols.

R. P. K: students have been measuring and comparing lengths, weight, volume etc.

LTM: different types of tins, and rulers of different lengths, Adinkra symbols

Introduction:

Present different sizes of containers to students and ask them to compare them in terms of similarities and differences.

1. Let students talk about the amount of liquid each container can hold, how many of the smallest tin will fill the largest etc.
2. Let them also compare the lengths of the rulers.
3. Ask students why we can not compare the length of a ruler to the amount of liquid in any of the tins.

Development

1. Let students measure the lengths and widths of their exercise books, classroom widows and express them as ratios, Length: width.
2. Present students with the worksheet that has the *Nsaa*- excellence symbol and ask them to compare the sides of the squares that have been labelled with the same letter.
3. Assist students to express the lengths of the larger and the smaller squares as ratios in the form $a : b \ or \frac{a}{b}$, and use that to explain what proportion is.
4. Discuss how proportion is important in creation of the Adinkra symbols.
5. Let students express the number of girls and the number of boys in their classroom as a ratio.
6. Using students' response from '5', explain to students that ratio can be used to share/divide quantities such as lengths and angles. Using how the Adinkra craftsman divided lengths of squares and rectangles in creating different Adinkra symbols.
7. In the *Mframadan* symbol on their worksheet ask them to guess the ratio used to divide the length of the rectangle, and the middle line.
8. Let students continue to solve the problem on their worksheet involving the *Apa* symbol.

Core points:

1. Ratio is the comparison of two or more quantities of the same unit. For example, we can compare the lengths of a triangle to its height, but we cannot compare the length to any of its angles because they are of different units.
2. Proportion is when we set two ratios to be equal.

3. Also let students have time to talk about the social values of the symbols used in this lesson and relate them to the learning of mathematics, school life etc.
4. Proportion can be used in art to make the work beautiful.

Evaluation Exercise

1. In constructing the *Adinkrahene* symbol, craftsmen use the ratio 1:2:3 to divide the total radius set to construct the three circles starting from the innermost one. If a craftsman wishes to construct *Adinkrahene* such that the total radius of the three circles will be 16cm. find the radius of each circle.
2. If the innermost circle of Adinkrahene is 3cm, find the radius of the outermost circle.
3. Draw an *Adinkrahene* such that the total radius for the three circles is 8cm.
4. What does the *Adinkrahene* symbol mean to you?

JHS Lesson Two

Topic: Angles

Objectives: Using various stages of the creation of *the Mmeredane, Nyame dua,* and *Nhwimu* symbols, students will deduce:

1. Angles on a straight-line sum up to 180◦.
2. Angles surrounding a point sum up to 360◦.
3. The angle properties of parallel lines and apply these properties to find angles in quadrilaterals.

LTM: Selected Adinkra symbols, ruler, and protractor

R.P.K.: Students can identify angles and use the protractor to measure angles.

Introduction: Let students explain how angles are formed and name the types of angles they know.

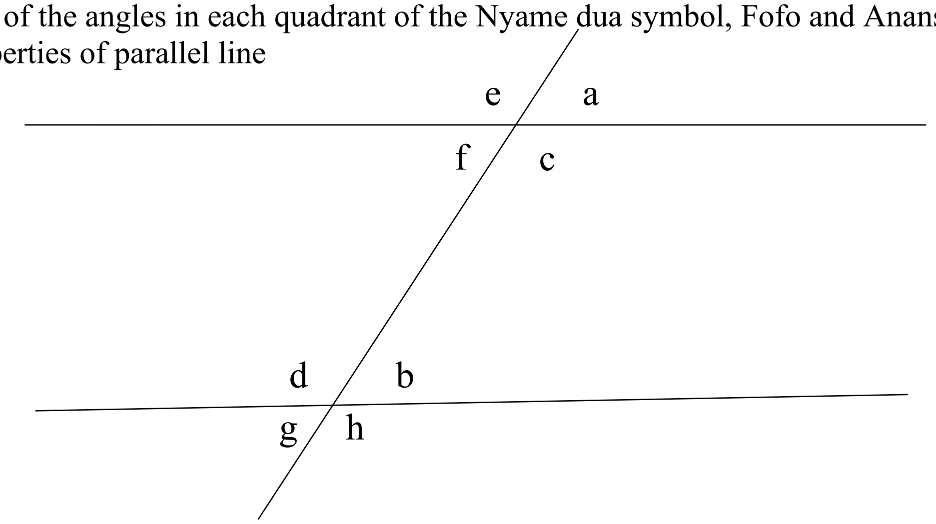
Development:

1. Using the *Mmere dane* Adinkra symbol, let students:
   a. identify angles that are found on the same straight line.
   b. measure all the angles shown on the top of the horizontal line and sum them up.
   c. let students also measure angles shown on one side of one of the diagonal lines and sum them up.
2. Using the Mmere dane symbol, again assist students to measure all the angles formed at the center and sum them up.
3. Extend their thinking by asking them to measure the angle in each quadrant of the God's protection Adinkra symbol. Let them guess what the sum will be
4. Let them guess the sum of all angles in the *Ananse ntontan, Fofo* and *Nsoroma* Adinkra symbols and explain why they think so.
5. Assist students to draw two parallel lines and two transversals across them (part of the *ɔwɔ foro adobe* symbol).
6. Lead them to measure the angles surrounding each point of intersection.
7. Let students compare the angles and group them according to those that are equal.
8. Lead them to establish those that are:

   a. corresponding angles
   b. Alternate angles
   c. Co-interior angles etc.
9. Using the *Nhwimu* Adinkra symbol, let students find the angles that are corresponding, alternate, and co-interior.

Core points

1. Discuss the meanings of the *Mmere dane, Ananse ntontan, Fofo, ɔwɔ foro adobe, Nhwimu* and *Nsoroma* Adinkra symbols with the students.
2. The sum of angles on any side of the horizontal line is 180°,
3. The sum of all the angles on any side of the diagonal line is also 180°
4. The sum of the angles around the central point in Mmere dane symbol is 360°. So is the sum of the angles in each quadrant of the Nyame dua symbol, Fofo and Ananse ntontan.
5. Properties of parallel line



   a. Corresponding angles are equal. Ie. a = b, c = h, e = d, and f = g
   b. Alternate angles are equal. i.e. c = d and f = b
   c. Vertically opposite angles are equal, e = c, a = f, d = h, b= g
   d. Co-interior angles sum up to 180°, $c + b = 180^\circ$, $f + d = 180^\circ$
6. Stress the fact that artisans make use of the mathematical idea of angles in creating many designs.
7. Let students complete the activity on the *Nhwimu* symbol.

Evaluation exercise

1. Using the idea of properties of parallel lines and angles on a straight line, with the help of your mathematical instruments, draw the *ɔwɔ foro adobe* symbol such that the transversals are inclined 60° to the horizontal lines. Set the intervals between the transversals at 1cm.
   a. By labelling angles formed in your drawing with letters, identify and give the measure, a pair of:
      i. Corresponding angles
      ii. Alternate angles
      iii. Co-interior angles
      iv. Vertically opposite angles

JHS Lesson Three

Topic: Enlargement

Objectives:
1. Determine the scale factor in a given *Nsaa* symbol.
2. Use a scale factor to draw *Nsaa* given the length of one of the squares.
3. State the properties of enlargements, concerning similarity, congruence, and orientation.
4. Transfer this idea to enlarge Adinkra symbols on the Cartesian plane.

R. P. K.: Students have seen enlarged pictures before.

LTM: The *Nsaa, Akoma* and *Aya* symbols. And two images of the same object with one being the enlargement of the other.

Introduction:

Present the two pictures of the same object (but different sizes) to students and ask them to talk about their similarities and differences.

Development

1. On the students' worksheet is the *Nsaa* Adinkra symbol, assist them to measure and compare the sides of the smaller squares to the sides of the larger square adjacent to them.
    a. Let them find the ratio of the larger square to the smaller square. Is this ratio common to all the larger squares and their respective adjacent smaller squares?
    b. Let them draw *Nsaa* which has the sides of the smaller squares given using the common ratio they obtained in (a)
2. On the students' worksheet, the top two pairs of leaves of the *Aya* Adinkra symbol are drawn, assist students to measure and find the ratio of the lengths of the two pairs of leaves.
3. Let students use this ratio as a scale factor to determine the lengths of the remaining leaves that will be located at the indicated points. Let them draw the leaves to complete the A*ya* symbol.
4. With the idea of a scale factor and its function established, let students draw a line or shape eg square or rectangle on a graph sheet and label the vertices.
5. Lead students to enlarge/reduce their shape using a scale factor of 1, -1, 3, -3, ½ and -1/2.
6. Let students discuss what happens to the image when the scale factor of enlargement is positive, negative, and a fraction.

Core points

1. Discuss with students the significance of the *Aya, Akoma* and the *Nsaa* Adinkra symbols.
2. The *Nsaa* Adinkra symbol is obtained by enlarging the smaller squares to get the larger ones, or by reducing the larger squares to get the smaller ones using a common ratio called the scale factor.
3. Emphasis the use of the common ratio (scale factor) as important for enlargement and reduction.
4. Enlargement is used in making many artistic designs including the creation of some Adinkra symbols and in textile designs.

5. Allow students to discuss how they were able to obtain the lengths of the remaining leaves on the *Aya* symbol on their worksheet.
6. Discuss enlargement under various conditions:
    a. When the scale factor is negative, the image is located at the opposite side of the object with the image turned upside down.
    b. When the scale factor is positive, the image is located in the same quadrant as the object, the image is in the same orientation as the object.
    c. When the scale factor is greater than 1 or less than negative one, the image size is larger than the object. The object and the image are similar.
    d. When the scale factor is between 1 and -1 (fraction) the image size is smaller than the object.
    e. When the scale factor is 1 or -1 the object and the image are congruent.

Evaluation exercise

1. Using a scale of 2cm: 2 units draw the x-axis and the y-axis on a graph sheet.
    a. On the same graph sheet, draw the "Akoma-heart" Adinkra symbol to pass through the points $A(-1,2)$, $B(-2,3)$, $C(0,3)$ *and* $D(-1,0)$
    b. Draw the image of the same Akoma symbol in (a) with a scale factor of -2
    c. Draw the image of the same Akoma symbol in (a) with a scale factor of 2.
    d. Draw the image of the same Akoma symbol in (a) with a scale factor of -1.
    e. Draw the image of the Akoma symbol in (a) with a scale factor of ½.
    f. Draw the image of the Akoma symbol in (a) with a scale factor of -½.

If the graphic you have formed on your graph is supposed to be used for a GTP print, how will you name it and why that name?

JHS Lesson Four

Topic: Rotation

Objectives: using the *Nyame dua* symbol, the students will be able to:

1. Locate and state the coordinates of images of points under anti-clockwise and clockwise rotations through 90◦, 180◦ and 270◦ about the origin.
2. State and draw the images of shapes under clockwise and anticlockwise rotations through 90◦, 180◦ and 270◦ about the origin.

LTM: *Bese saka*, *Nyame dua*, *Nsoroma* and *Ananse ntontan* Adinkra symbols.

R.P.K: Students can give examples of objects that turn in everyday life.

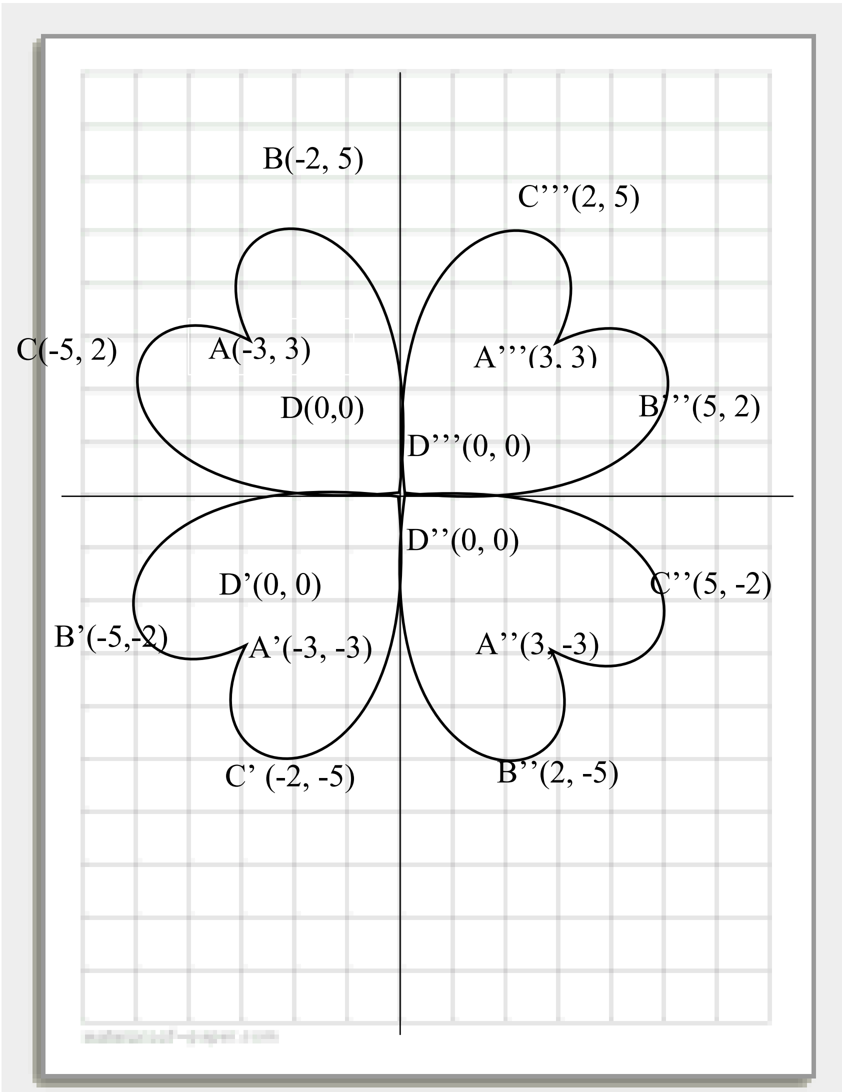
Introduction

Let students name some objects in their homes that turn.

Development

1. Using students' responses from 'introduction', explain to them that rotation is the turning of an object about a fixed point/center/origin through an angle.
2. Let students identify some Adinkra symbols that have rotated shapes in them.

3. For each example mention by students in (2) let them identify the center of rotation and guess the angle of rotation.
4. Using the *Nyame dua* symbol on a graph sheet, assist students to identify images of points marked through a rotation of 90◦, 180◦ and 270◦ clockwise and anti-clockwise about the origin.



Clockwise rotations	**Anti-clockwise rotation**
90◦ clockwise	**90◦ anti-clockwise**
$A(-3,3) \rightarrow A'''(3,3)$	$A(-3,3) \rightarrow A'(-3,-3)$
$B(-2,5) \rightarrow B'''(5,2)$	$B(-2,5) \rightarrow B'(-5,-2)$
$C(-5,2) \rightarrow C'''(2,5)$	$C(-5,2) \rightarrow C'(-2,-5)$
$D(0,0) \rightarrow D'''(0,0)$	$D(0,0) \rightarrow D'''(0,0)$
That is the rule for the rotation is $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y \\ -x \end{pmatrix}$	That is the rule for the rotation is $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ x \end{pmatrix}$

180◦ clockwise $A(-3,3) \to A''(3,-3)$ $B(-2,5) \to B''(2,-5)$ $C(-5,2) \to C''(5,-2)$ $D(0,0) \to D''(0,0)$ That is $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -x \\ -y \end{pmatrix}$	**180◦ anti-clockwise** $A(-3,3) \to A''(3,-3)$ $B(-2,5) \to B''(2,-5)$ $C(-5,2) \to C''(5,-2)$ $D(0,0) \to D''(0,0)$ That is $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -x \\ -y \end{pmatrix}$
270◦ clockwise $A(-3,3) \to A'(-3,-3)$ $B(-2,5) \to B'(-5,-2)$ $C(-5,2) \to C'(-2,-5)$ $D(0,0) \to D'(0,0)$ That is $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -y \\ x \end{pmatrix}$	**270 ◦anti-clockwise** $A(-3,3) \to A'''(3,3)$ $B(-2,5) \to B'''(5,2)$ $C(-5,2) \to C'''(2,5)$ $D(0,0) \to D'''(0,0)$ That is $\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} y \\ -x \end{pmatrix}$

5. Using isosceles triangle on a graph sheet, assist students to rotate it 90◦, 180◦ and 270◦ clockwise or anti-clockwise about the origin to form a 4-point star.
6. Let students compare the coordinates of the points in (5) in the clockwise and anti-clockwise rotations through 90◦, 180◦ and 270◦ about the origin.

Core points

1. Let students talk about their personal values and how they relate to the meanings of the Adinkra symbols they have identified to have rotational symmetries.
2. Discuss the two conditions needed for rotation - the angle the plane is to be rotated through and the point about which it is to be rotated - point of rotation. with rotation some points will remain in the same position after rotation].

Evaluation exercise

1. Using a scale of 2cm: 1unit on both axes, draw an arc to pass through points $A(-3,1)$, $B(-3,3)$ *and* $C(-1,3)$. Join A to C with a straight line. locate the mid-point of $\overline{AC}$, what are the coordinates of the mid-point of $\overline{AC}$. Join the mid-point of $\overline{AC}$ to the origin.
    a. Rotate the entire shape formed through a clockwise or anti-clockwise of 90◦, 180◦ and 270◦ about the origin.
    b. What Adinkra symbol have you formed? What is the meaning of this symbol in your community?

SHS Lesson one
Topic: Reflection

Objectives:

1. Identify at least three (3) Adinkra symbols with reflection symmetry in them and identify their mirror lines.
2. Deduce the rules for reflections with $x-axis\ and\ y-axis$ as mirror lines using the *Woforo dua pa a* symbol.
3. Reflect an object given $x-axis\ and\ y-axis$ as mirror lines.

Related previous knowledge, RPK

Students do observe images in mirrors.

Introduction:

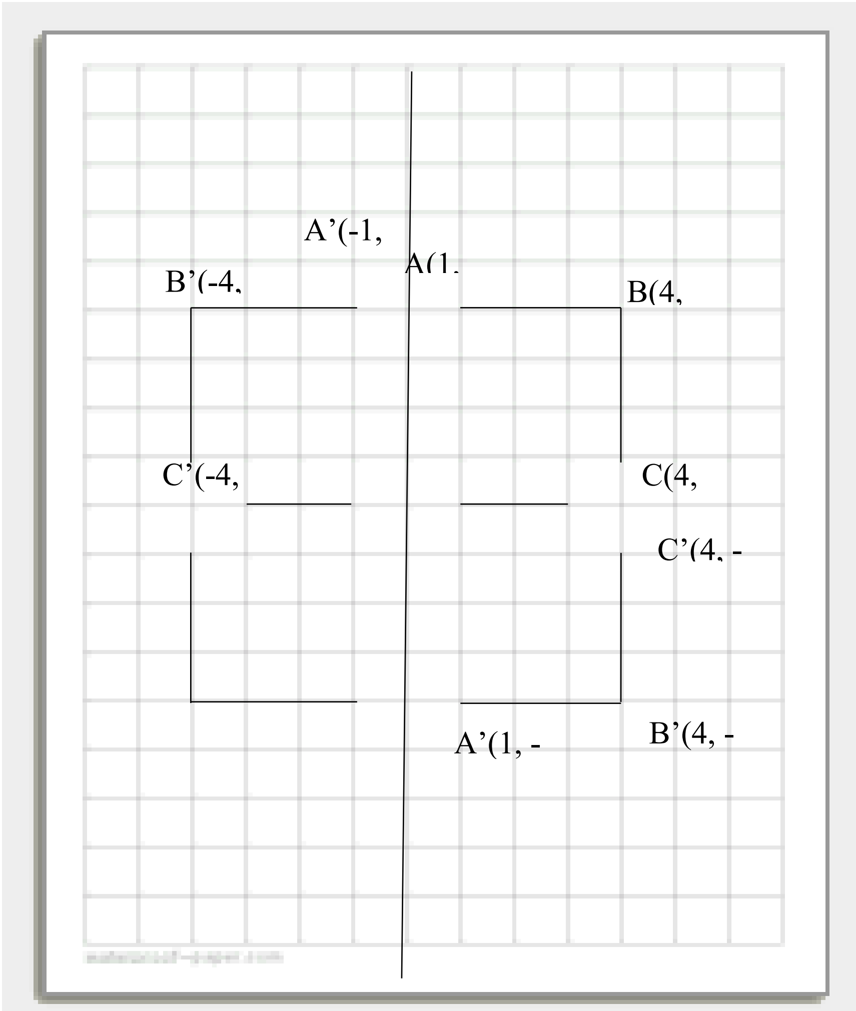
Students describe the images of objects seen in a mirror.

Activities:

1. Present a set of Adinkra symbols to students and ask them to identify at least 3 of them with reflection properties and identify their mirror lines.
2. Assist students to draw the *Woforo dua pa* symbol and draw any one of the mirror lines through it.
3. Ask students to identify the coordinates of the three points on one side of the mirror line and that of their corresponding points on the other side of the mirror line.
4. Ask students to compare the coordinates of the corresponding points. Lead them to establish the rules for reflection on a vertical and horizontal mirror line.

Hints:

1. A reflection mirrors a whole plane and its points. To do a reflection of a plane, you need the mirror line. After reflection, it is possible for some points to remain in the same location.
2. Explain what a mirror line is using the mirror to illustrate.
3. Identify Adinkra symbols with reflection properties and talk about their meanings and values attached to them. Especially the meaning of the *Woforo dua paa* symbol which is used in this activity.
4. Let students deduce the rules of reflection and employ them to draw their own Adinkra symbols encourage them to name their symbols and give their meanings.



On the grid is the *Woforo dua pa a* symbol.
If we consider the top part of the symbol and the vertical axis as the mirror line, if the right-hand side is the object and the left-hand side as the image, it can be observed that:

$A(1,4) \rightarrow A'(-1,4)$

$B(4,4) \rightarrow B'(-4,4)$

$C(4,1) \rightarrow C'(-4,1)$

It can be seen that in this transformation, the x-coordinates of the image is the negative of the original x-coordinate, the y-coordinate remains unchanged. That is, for reflection in the vertical mirror line (y-axis)

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ y \end{pmatrix}$$

Taking the right-hand side again and the horizontal line through the center of the Adinkra symbol as the mirror line, we have:

$A(1,4) \rightarrow A'(1,-4)$

$B(4,4) \rightarrow B'(4,-4)$

$C(4,1) \rightarrow C'(4,-1)$

That is, when the horizontal line (x-axis) is the mirror line the image formed has its y-coordinates to be the negative of the original y-coordinates. $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ -y \end{pmatrix}$

Evaluation:

(a) Using a scale of 2cm to 1unit on both axes, locate the points $A(-2.5,0), B(-1.5,-1), C(-0.5,0)$ *and* $D(-1.5,-3)$. Use your freehand to join A to B, B to C, C to D, D to A to form the Akoma – heart symbol.
(b) Draw the image of the shape formed by reflecting the shape using the x-axis as mirror line such that $A \rightarrow A'$, $B \rightarrow B'$, $C \rightarrow C'$ *and* $D \rightarrow D'$
(c) What is the name of the Adinkra symbol formed? What is it social value?
(d) Reflect the entire Adinkra symbol formed in (b) using the y-axis as mirror line. What are the coordinates of the vertices of the image of the Adinkra symbol?
(e) How will you name the entire figure formed and why?

SHS Lesson Two
Topic: Translation

Objectives:

4. draw the *Apa* symbol on a graph and locate the coordinates or the points of the vertices for the original rhombus and its image.
5. deduce the translation vector for the translation of the original rhombus to the image to form the *Apa* symbol.
6. perform translations of shapes using a given translation vector.

Related previous knowledge:
Students can identify congruent shapes and have been performing sliding of objects in their everyday lives.

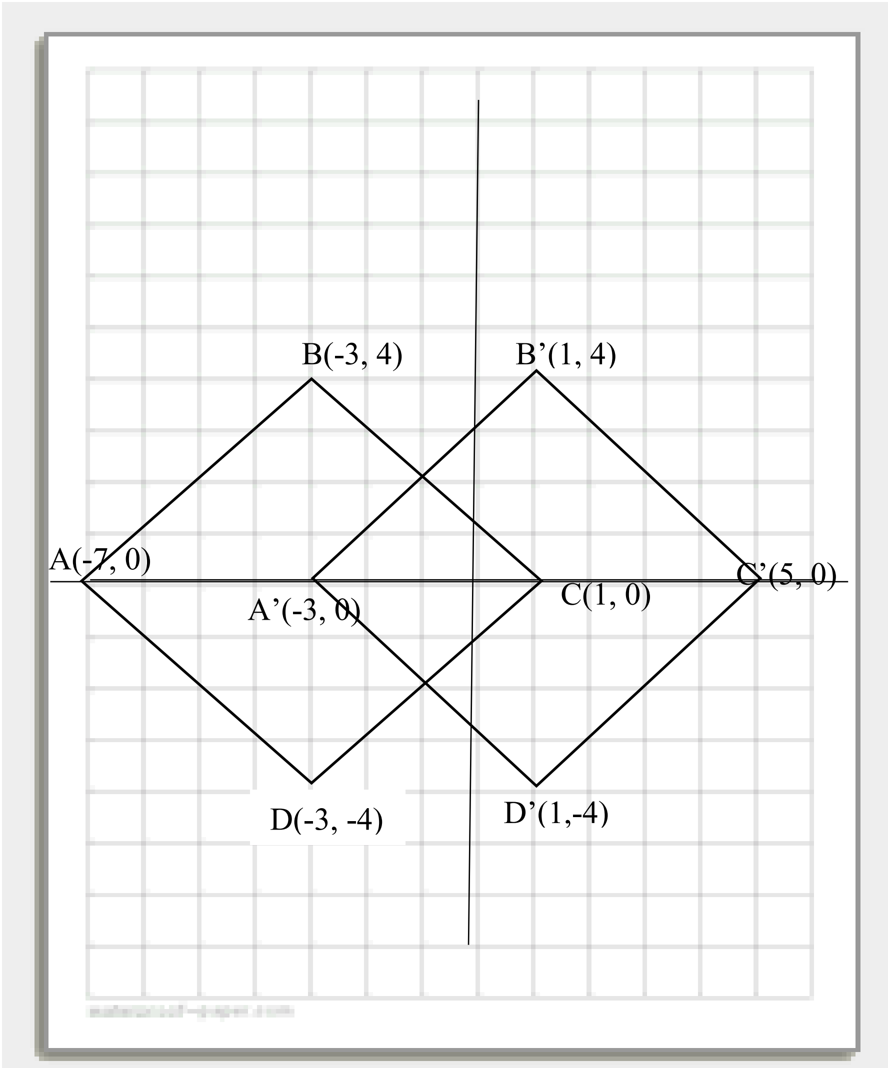
Introduction:
Students explain congruent shapes and identify among various Adinkra symbols those that are made up of congruent shapes.

Activities:

1. Lead students to draw a rhombus on a grid sheet and label its vertices (original object).
2. Ask students to look at the *Apa* Adinkra symbol and draw the congruent of the rhombus drawn in activity 1 (image of the first rhombus) such that the *Apa* Adinkra symbol is formed on the graph.
3. Ask students to identify the coordinates of the vertices of the rhombuses.
4. Guide students to compare/contrast the coordinates of the vertices of the original rhombus and its image to deduce the displacement vector for the translation of the original rhombus to its image to form the *Apa* symbol. Let students talk about the values of the *Apa* symbol.
5. Discuss translation with students and the needed requirements for a translation to be possible.
6. Let students translate a given shape using a given vector.
7. Given a shape and a displacement vector, let students translate the shape to form its image. With the same vector let them translate the image to another position. Let students continue the translation of the images till they get to the end of the graph sheet.
8. Discuss how the Adinkra symbols are printed on cloths.

Hints:

1. A translation moves a plane and its points the same distance in the same direction. To translate one needs the distance and the direction the points are to be moved, represented by a vector sometimes referred to as the displacement vector or the translation vector. With translation, no point remains in the same location after translation.
2. Let them explain why some students obtained negative translation vectors and others obtained positive translation vectors.
3. Let students explain the mathematical terms: vertices, coordinate of the vertices, displacement vector etc.
4. Extend their thinking by continuous translation of the images of a shape with the same or different displacement vector and relate to the stamping of the Adinkra cloth.
5. Let students talk about how the concept of translation is employed in drawing different Adinkra symbols and in other artwork.
6. Let students talk about the meanings of the Adinkra symbols that students will identify to have translational symmetry. Especially those used in the activities.



The coordinates of the original rhombus (object) are:

A(-7, 0)
B(-3, 4)
C(1, 0)
D(-3, -4)

The coordinates of the image of the rhombus are:
A'(-3, 0)
B'(1. 4)
C'(5, 0)
D' (1, -4)

Comparing the coordinates of the object to that of the image, it can be observed that the values for the horizontal movement (x-coordinates increased by 4, while that for the vertical movement (y-coordinates) remained the same. That is, the image is formed by moving each point in the positive direction in the x-axis 4 steps forward with no movement in the y-axis. That is the displacement/translation vector is $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

Evaluation

(a) Using a scale of 2cm to 1 unit on both axes, on a grid paper draw a circle that passes through the points $A(-4,2), B(-2,0), C(0,2)\ and\ D(-2,4)$. Join point B to the center of the circle.

(b) Using a displacement vector of $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ draw the image of the circle such that the points $A \rightarrow A'$, $B \rightarrow B'$, $C \rightarrow C'$ and $D \rightarrow D'$. Join B' to the center of the circle.

(c) Draw the image of the circle that passes through points $A'B'C'D'$ to pass through the points $A''B''C''D''$ using the translation vector $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$.

(d) With a displacement vector $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ draw the image of the circle such that $A'' \to A'''$, $B'' \to B'''$, $C'' \to C'''$ and $D'' \to D'''$

(e) What is the name of the Adinkra symbol you have formed? What is its social value?

SHS Lesson Three
Topic: Rotation

Objectives:

1. Identify at least three Adinkra symbols with rotational symmetry.
2. Estimate with some degree of accuracy the angles through which the shapes in the Adinkra symbols identified in (1) were rotated to form the symbol.
3. Deduce the rules for clockwise and anticlockwise rotations through $90°, 180°, 270°$ using the *Nyame dua* symbol.

Related Previous Knowledge

Students have been performing rotations in real-life. Example, turning of a door to open or close it.

Introduction

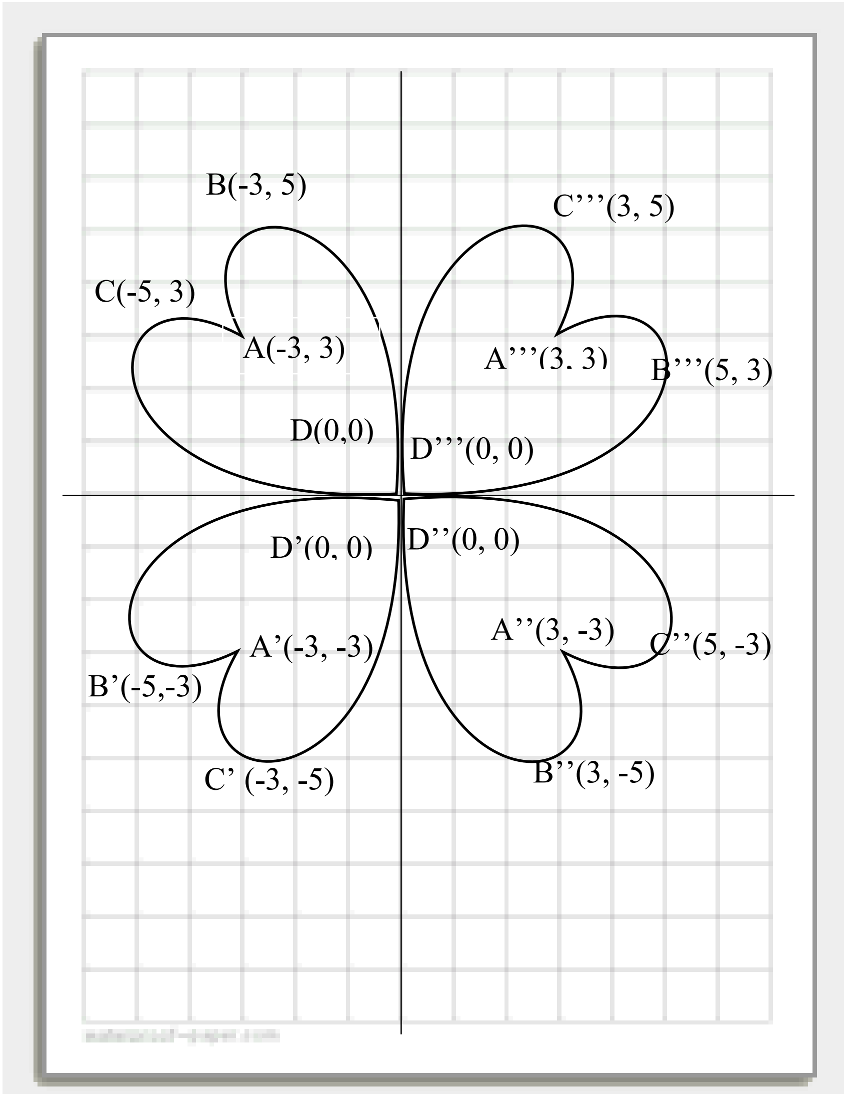
Let a student open a door and the other students guess the angle through which the door has been turned.

Activities

1. Let students select Adinkra symbols with rotational symmetries and describe the order of rotational symmetry each of them has.
2. Assist students to draw one of the hearts in *Nyame du - God's tree* symbol and label four points on it with their coordinates.
3. Guide students to rotate this shape to form the complete *Nyame dua* symbol.
4. Assist students to describe the various angles through which the heart shape was rotated to form the Adinkra symbol.
5. Lead students to compare the coordinates of the four points in the original heart and that of its images after the various rotations (90◦, 180◦, 270◦ both clockwise and anti-clockwise).

Hint

1. Let students talk about their personal values and how they relate to the Adinkra symbols they have identified to have rotational symmetries.
2. Let students name/rename the Adinkra symbol used in the activities and tell the meaning of the name.
3. Let them distinguish rotation from the other transformations [rotation rotates the plane and all points in it. The two conditions needed for rotation are the angle the plane is to be rotated through and the point about which it is to be rotated --point of rotation. Like reflection, with rotation some points will remain in the same position after rotation]
4. Let students justify their responses to questions.

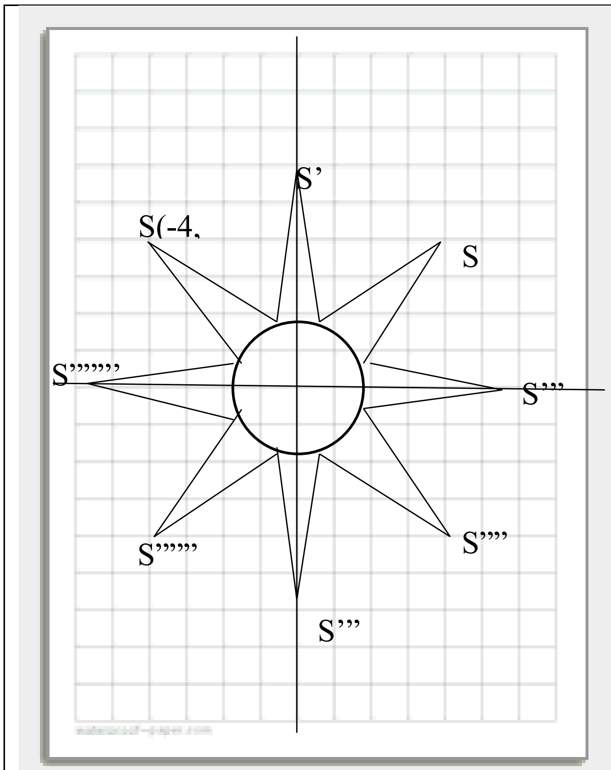


Clockwise rotations	**Anti-clockwise rotation**
90◦ clockwise $A(-3,3)\rightarrow A'''(3,3)$ $B(-3,5)\rightarrow B'''(5,3)$ $C(-5,3)\rightarrow C'''(3,5)$ $D(0,0)\rightarrow D'''(0,0)$ That is the rule for the rotation is $\begin{pmatrix} x \\ y \end{pmatrix}\rightarrow\begin{pmatrix} y \\ -x \end{pmatrix}$	**90◦ anti-clockwise** $A(-3,3)\rightarrow A'(-3,-3)$ $B(-3,5)\rightarrow B'(-5,-3)$ $C(-5,3)\rightarrow C'(-3,-5)$ $D(0,0)\rightarrow D'''(0,0)$ That is the rule for the rotation is $\begin{pmatrix} x \\ y \end{pmatrix}\rightarrow\begin{pmatrix} -y \\ x \end{pmatrix}$
180◦ clockwise $A(-3,3)\rightarrow A''(3,-3)$ $B(-3,5)\rightarrow B''(3,-5)$ $C(-5,3)\rightarrow C''(5,-3)$ $D(0,0)\rightarrow D''(0,0)$	**180◦ anti-clockwise** $A(-3,3)\rightarrow A''(3,-3)$ $B(-3,5)\rightarrow B''(3,-5)$ $C(-5,3)\rightarrow C''(5,-3)$ $D(0,0)\rightarrow D''(0,0)$

That is $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \end{pmatrix}$	That is $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ -y \end{pmatrix}$
270◦ clockwise $A(-3,3) \rightarrow A'(-3,-3)$ $B(-3,5) \rightarrow B'(-5,-3)$ $C(-5,3) \rightarrow C'(-3,-5)$ $D(0,0) \rightarrow D'(0,0)$ That is $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ x \end{pmatrix}$	**270 ◦anti-clockwise** $A(-3,3) \rightarrow A'''(3,3)$ $B(-3,5) \rightarrow B'''(5,3)$ $C(-5,3) \rightarrow C'''(3,5)$ $D(0,0) \rightarrow D'''(0,0)$ That is $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y \\ -x \end{pmatrix}$

Evaluation

1.



a. On the grid sheet, an Adinkra symbol is drawn. What is the name of the Adinkra symbol and its social value?
b. To form this symbol, the point S(-4, 4) indicated on the grid is rotated many times. What is the angle of rotation for each image of S? What is the order of rotation?
c. Find the coordinates of all the images of S that corresponds to $90°, 180°, 270°$ anti-clockwse and clockwise.

2. Draw your own Adinkra symbol by drawing a basic shape on a grid and rotate it through an angle about the origin. Indicate the coordinates of three vertices on your shape and the angle of rotation. After rotation indicate the corresponding vertices on the image. How will you name your Adinkra symbol? What will be its social value?

SHS Lesson Four
Topic: Multiple Transformations

Objective

1. Identify Adinkra symbols that could be formed by different transformations.
2. State a single transformation rule for two or more transformations.
3. Design their own Adinkra symbols through reflecting, rotating and translation (not necessarily in that order) i.e. students perform multiple transformations of a shape to form their own Adinkra symbols.

Related Previous Knowledge

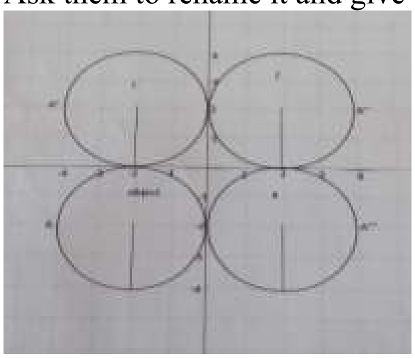
Students can perform single transformations.

Introduction

Let students describe various transformations that can be used to obtain the *M'aware wo*, *Mate masie, Bese saka* and *Apa* Adinkra symbols

Activities

1. Give students copies of the grid paper with the *Mate masie* symbol drawn on it.
2. Ask students to give the meaning of this symbol and relate it to their values as students.
3. Ask them to rename it and give the meaning of the name.



4. By ordering the images, let students describe the possible transformations that can be used to form each image.

Example: discuss the possible transformation that can be used to obtain I, J, and K from the object. Or J from I, I from K etc.

5. By skipping some images let students describe a single transformation that can be used to obtain some image.

Example: how can the image J be obtained from the object without going through I or K

6. Let students draw their own Adinkra symbols using reflection, translation, and rotation (in any order they prefer).
7. Discuss with students how transformations are used in art (particularly, in Ghanaian prints).

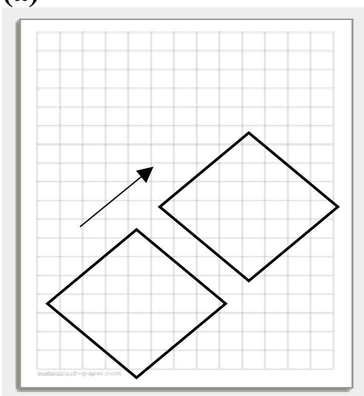
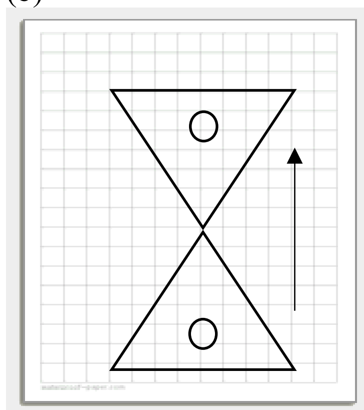
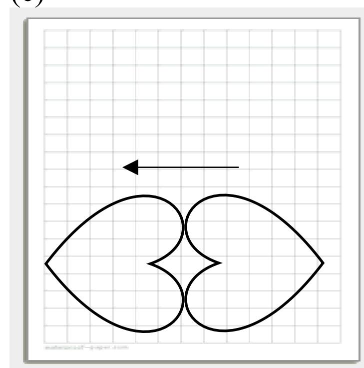
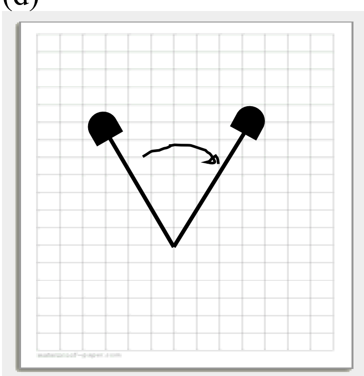
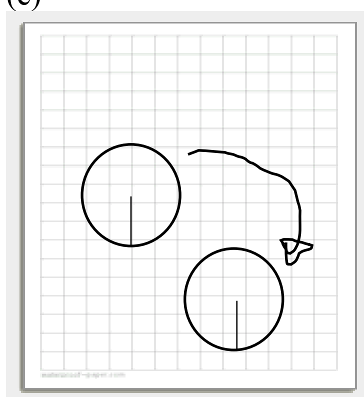
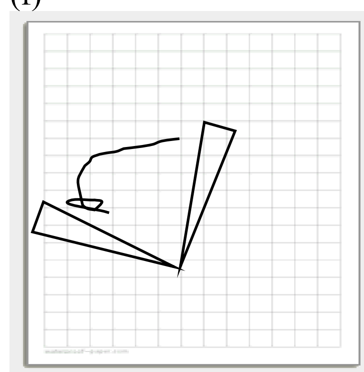
Hint

1. Let students distinguish and talk about differences and similarities if any between rotation, reflection, and translation.
2. Let students justify their responses to questions.
3. Let them continue to talk about the social values of the Adinkra symbols.

Evaluation

1. On the grid sheets provided, an original shape (object/pre-image) has been relocated (transformed). An arrow is used to indicate where the object was moved from to form the

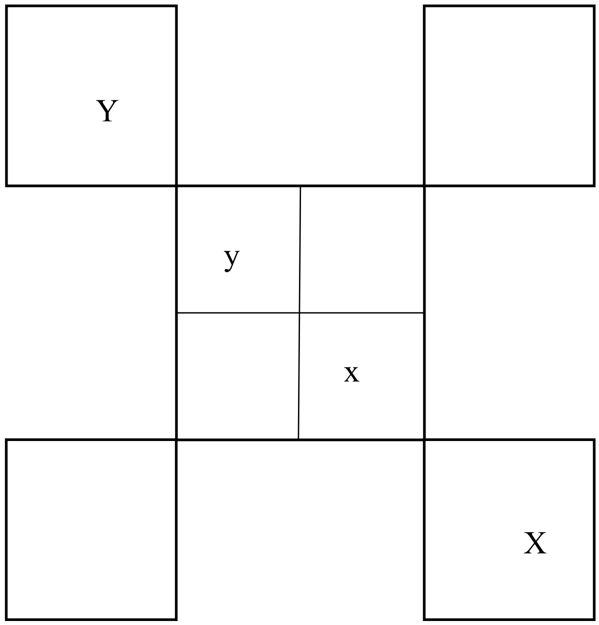
image. You are to indicate the kind of transformation that took place on each grid explaining why you think that is the transformation on the grid.

(a) 	(b) 	(c) 
(d) 	(e) 	(f) 

Appendix K: JHS Worksheets

Worksheet One
Topic: Ratio and proportion

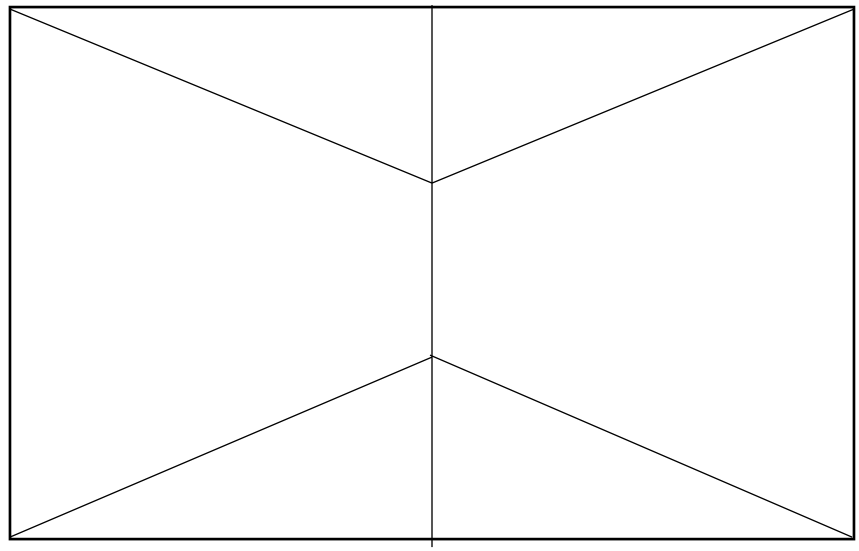
1. In the *Nsaa* Adinkra symbol below, measure the lengths of the larger squares marked with letters and their adjacent smaller squares and express them as ratios.



Ratio	Length of larger X	Length smaller x	Length of larger Y	Length of smaller y
a. $\text{length of larger square} / \text{length of smaller square}$				
b. $\text{length of smaller square} / \text{length of larger square}$				
What can you say about the ratios				

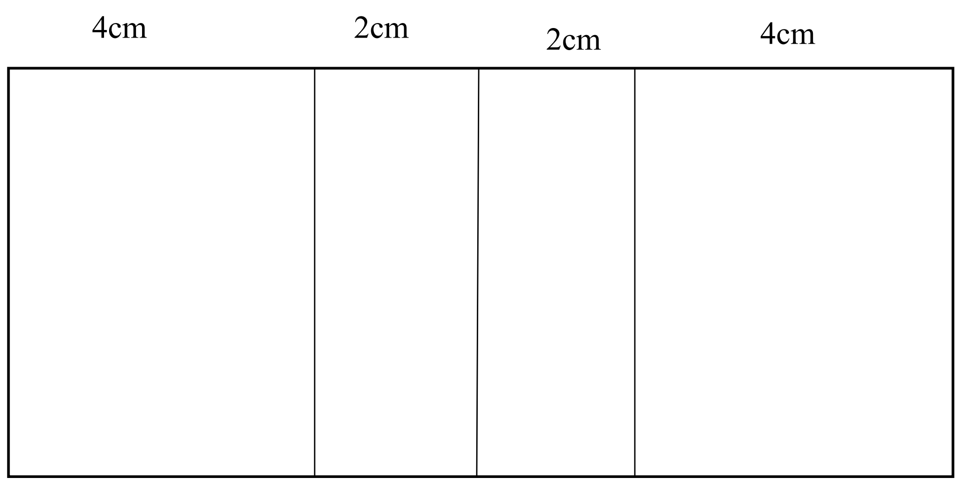
c. Why did the craftsman create the *Nsaa* symbol such that these ratios are equal?

2. The diagram below shows one stage of the creation process of the *Mframadan* Adinkra



a. What was the ratio used to divide the length of the rectangle into two?
    i. How did you get your answer?
b. What ratio is used to divide the middle line where the diagonals meet?

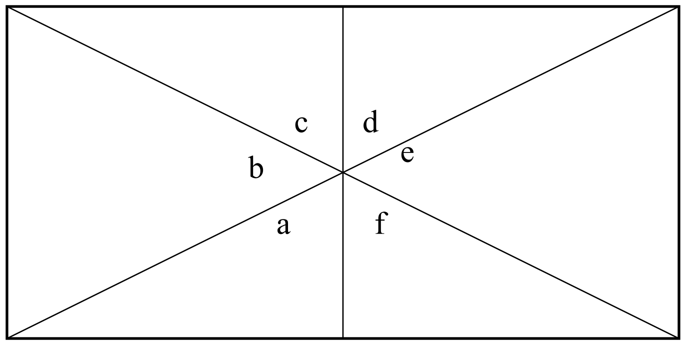
2. In constructing the *Apa* Adinkra symbol the length of the rectangle is divided as shown in the figure below:



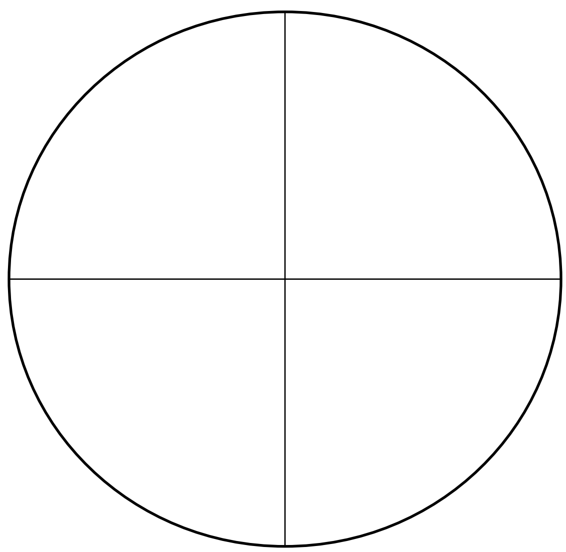
a. express the lengths of the four portions as a ratio.
b. Using the ratio obtained if a craftsman wishes to obtain an *Apa* from a rectangle of length 15cm what will be the length of each portion.
c. How does the meaning of this symbol apply to your personal values?

Worksheet Two
Topic: Angles

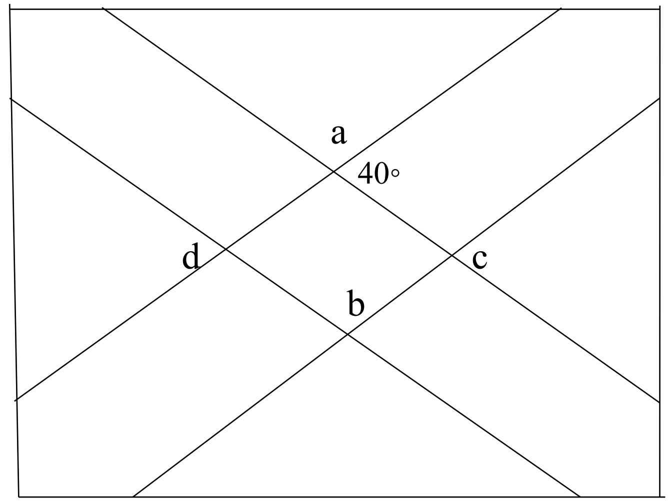
1. The figure below shows one stage in the creation of the *Mmeredane*



   a. What are the angles found on the left of the vertical line? Measure and sum them up
   b. What is the sum of the measure of the angles at the right of the vertical line?
   c. What do you think will be the sum of the angles on each side of any of the diagonals? Why?
   d. Now measure and sum all the angles that surround the central point of the figure.
2. Look at the diagram below it is used to create the *Nyame dua*-God's tree Adinkra symbol. What do you think the angles around the central point will sum up to? Measure them and sum them up. Was your guess, right?

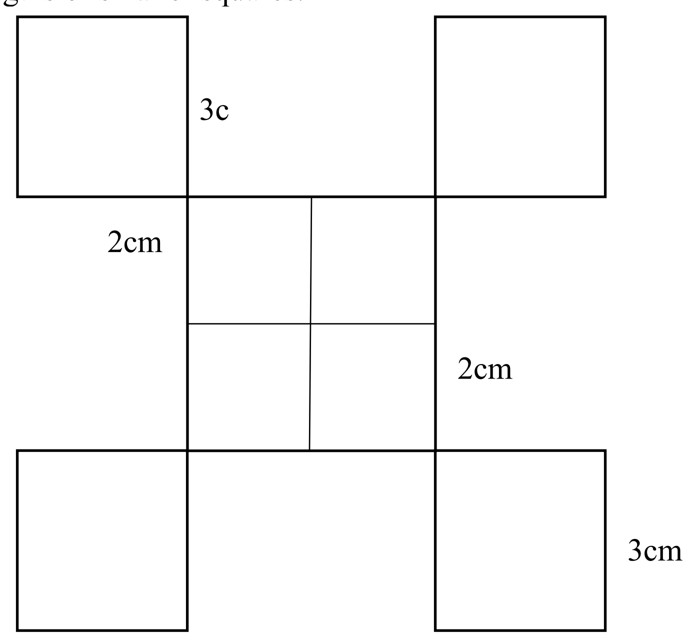


3. The diagram below shows part of the *nhwimu* Adinkra symbol with some angles shown. Find the measure of the angles that have been indicated by letters.



Worksheet Three
Topic: Enlargement

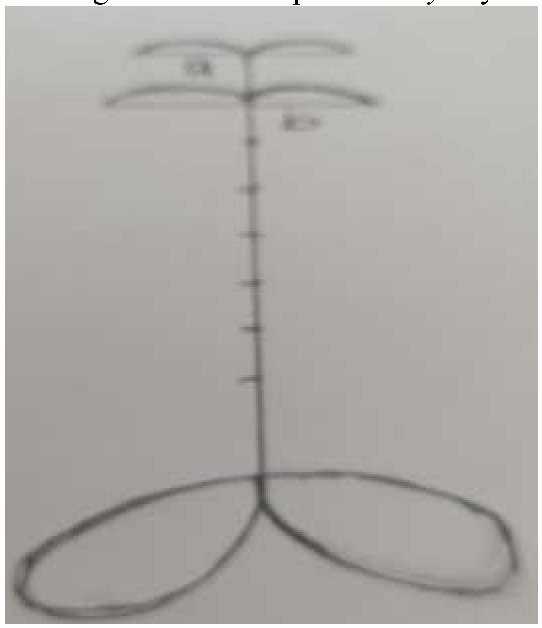
1. In the *Nsaa* Adinkra symbols below, find the ratio of the lengths of the larger squares and the corresponding lengths of smaller squares.



   a. What can you say about the two ratios? What is the importance of this ratio?
   b. Using the ratio obtained between the large and smaller squares, draw your own *Nsaa* symbol such that the lengths of the smaller squares is 1.5 cm.
   c. If you are to rename this symbol, what name will you give to it? What is the meaning of that name?
2. Look at the figure below is an uncompleted drawing of the *Aya* symbol, measure the length of leave "a" and leave "b" by measuring the length of the line that joins the tips of the leaves together. Find the ratio b:a or b/a and use this ratio as the common ratio (scale

factor) to determine the lengths of the remaining leaves that will be drawn on the points indicated.

a. Draw the remaining leaves to complete the *Aya* symbol.



b. How did you obtain the lengths of the remaining leaves from the lengths of the existing leaves?

c. How will you enlarge a line joining the points are $A(1,1)$ *and* $B(2,2)$ using the same scale factor you used in (a)? you can locate the points on the attached graph sheets and draw the line.

d. Enlarge this line with a scale factor of $3, -3, \frac{1}{3}$ *and* $-\frac{1}{3}$.

Appendix L: Teachers' Interview Questions

Semi-structured interview questions for teachers

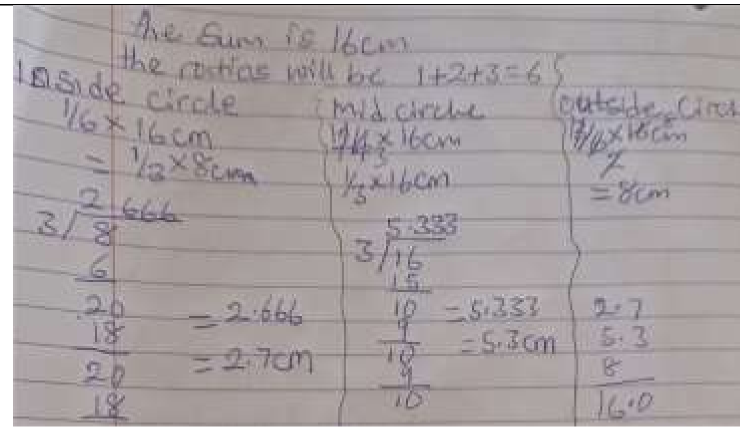
The purpose of this interview is to help evaluate the effectiveness of using the Adinkra symbols for mathematics instruction to promote student learning in mathematics classrooms. The responses you will give to the questions will remain anonymous and confidential.

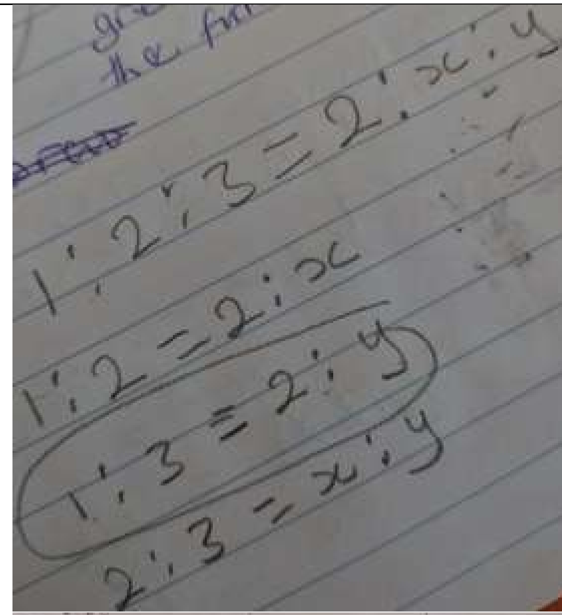
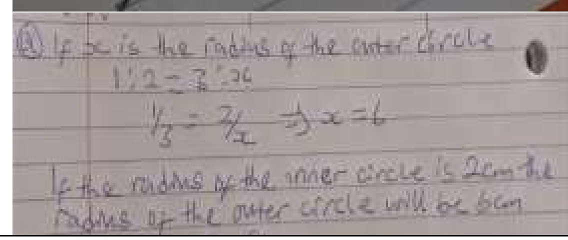
1. Did you benefit in any way by participating in this study? Why/why not
2. Was the study beneficial to your students? Why/why not
3. What aspects of the lessons had a greater effect on students' learning?
4. What are the constraints of the use of the Adinkra symbols for mathematics instruction?
5. What measures did you use to reduce the effects of the constraints?
6. Will you recommend the use of the Adinkra symbols for teaching to any mathematics teacher? Why/why not?
7. Is there anything that mathematics teachers should pay particular attention to when using the Adinkra symbols for mathematics instruction?

Appendix M: Strands of Mathematics Proficiency that Emerged in Students' Work

Table M1

JHS Group Four's Solution to a Ratio and Proportion Problem on Adinkrahene Symbol

Stage	Students' conversations	Students' written solution	Strands of mathematics proficiency
1	Yaw: The sum of the radius is sixteen centimeters, and the ratio is one to two to three. Bismark: *We need to find the* sum of the ratios *too to be equated to the* sixteen centimeters. Yaw: *No, we will not equate it to six*, we have to write each ratio as the fraction of the sum. Diana: Yes, so the inside circle will be one over six. Yaw: That means the radius of the inner circle will be one out of six of the sum of the radius, which is sixteen and the middle circle will be two over six times the sixteen. Diana: Let's do the calculations.		Conceptual understanding Adaptive reasoning Procedural fluency

Stage	Students' conversations	Students' written solution	Strands of mathematics proficiency
2	Ataa: *If we let the radius of the middle and the outside circles be x and y, we can write them as ratio two is to x is to y and equate it to the ratio in the question*. Bismack: *We will express the ratio in the question and the ratio we obtained using x and y as proportion* Ataa: I have done it here, from that, I had three proportions look at it Diana: *Let's follow what she said, she is right, if we let the radius of the outer circle be x we will have one is to two equals three is to x.* Bismack: Write it and let's solve for x Comfort: We can multiply by the LCM, let's just cross multiply.	 	Adaptive reasoning Strategic competence Conceptual understanding Productive disposition Procedural fluency

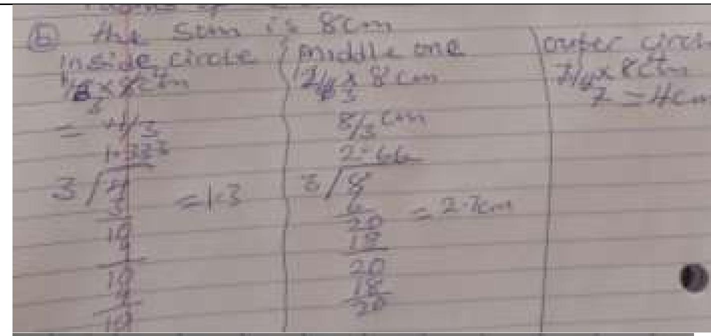
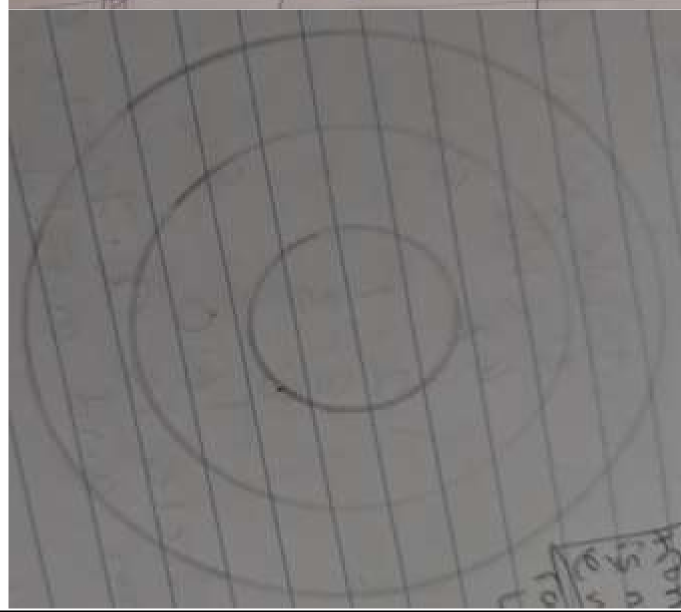
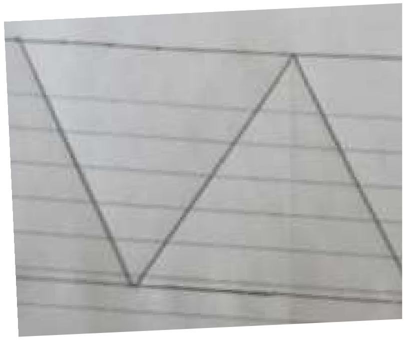
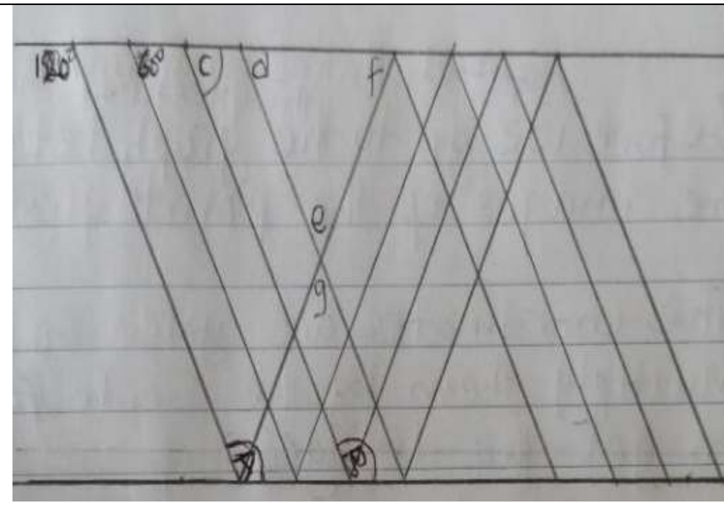
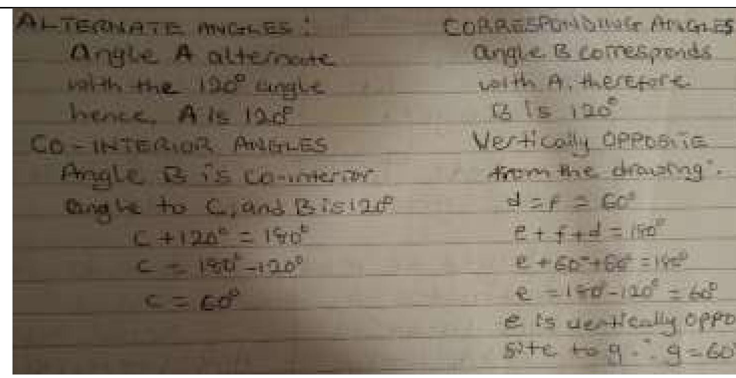
Stage	Students' conversations	Students' written solution	Strands of mathematics proficiency
3	Ataa: *This one, we have to use the same method we used in the first one.* Diana: Yes, we will have one over six multiplied by eight. Comfort*: Let's round the answers to one decimal* so that *we can measure from the ruler.* Yaw: *Let's start with the small one, measure with the compass and draw.* Diana: *Our Adinkrahene is even beautiful than what is on the sheet (laughter)* Bismack: *The Adinkrahene means creativity, we are, therefore, creative because we have created Adinkrahene with mathematics.* Ataa: *It also means greatness, if we live good lives, we will be great people in the future.*	 	Adaptive reasoning Conceptual understanding Strategic competence Productive disposition Procedural fluency

Table M2

JHS Group One's Solution to Angle Problem Developed Around the ɔwɔ Foro Adobe Symbol

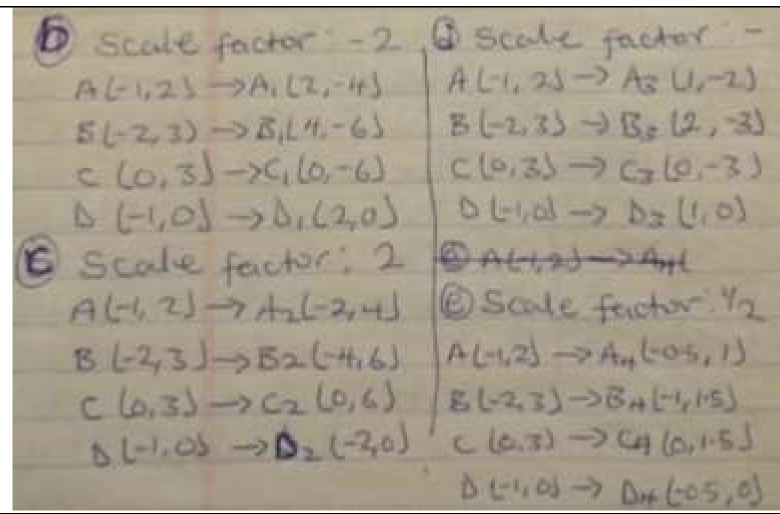
Stage	Students' conversations	Students' written solutions	Strands of mathematics proficiency
1	Ernest: What is the meaning of inclined? Bruce: *If we look at the ɔwɔ foro adobe symbol, I think the sixty degrees inclined is the angle the slanted lines (transversals) make with the horizontal lines.* Afriyie: *There, are two angles at each point where the transversal meets the horizontal, so do we need to divide the sixty degrees into two?* Bruce: *No, that can't be possible, because at the points of the intersection of the transversal and the horizontal lines form angles on a straight line, which is one hundred and eighty degrees, I think we should measure one of the angles to be sixty degrees.* Elvis: *Yes, let's try it, if we get the first one wrong, we erase it and try again.*		Conceptual understanding Adaptive reasoning Strategic competence Productive disposition
2	Afriyie: Let's draw the two horizontal lines. Ernest: *The question says the intervals between the transversals is one centimetre, let's measure and mark them on the top or bottom lines.* Elvis: *Do we have to mark the points on both lines?* Ernest: *I don't think so, because the second and the third transversals will start from where the first ones end.* Elvis: *We can start marking the points one centimetre from the left end of the horizontal line.* Bruce: *Use the protractor to measure sixty degrees on the first point and draw a line through it to the two horizontal lines.* Afriyie: *The transversals are from top to down, down to top and top to down again, so let's measure sixty degrees at the end of this transversal and draw a line through the point to the top horizontal.* Ernest: Yes, then we do the same thing from the top to down. Alex*: I think we can use the sixty degrees set square here to make it easy for us.* Bruce: Em, can the set square be used for these transversals? Alex: *The sixty degrees set square has sixty degrees angle at this corner and thirty degrees at this corner* (showing the sixty degrees set square, and pointing to corners)*, and we have used it to draw parallel lines before.*		Procedural fluency Adaptive reasoning Strategic competence Conceptual understanding

Stage	Students' conversations	Students' written solutions	Strands of mathematics proficiency
2	Ernest: We can always try something, if it doesn't work, we erase it and try something else. Afriyie: *Yes, we are drawing the ɔwɔ foro adobe, which means we should continue to try no matter the difficulty.* Bruce: Let's measure with the protractor and see if the angles are one twenty and sixty degrees. Afriyie: *The angles are correct, it works, and it is the easiest, you don't need to measure the angles, you just slide or turn the set square.* Bruce: *Okay then let's use it, but can we use it to start the first transversal on the second point or we need to measure that one.* Alex: I think it can be used to draw all of them. Bruce: *Then Elvis, continue the drawing with the set square.* Afriyie: *Give it to me, let me try it.*		Strategic competence Procedural fluency Productive disposition
3	Ernest: Let's label the angles, we know they will be sixty and one twenty degrees. Alex: Must we label all the angles? Afriyie: *We have labeled the first two, one twenty and sixty degrees, so let's label one angle that is corresponding, alternate or is co-interior to these two.* Ernest: *Why are you labeling with capital letters, Sir Eric said we label angles with small letters.* Afriyie: I wrote with a black pen I can't clean it.		Conceptual understanding Strategic competence Adaptive reasoning Procedural fluency

Stage	Students' Conversations	Students' written solutions	Strands of Mathematics proficiency
	Bruce: It will not spoil anything it is okay. Afriyie: The angle I have labeled A is an alternate angle to the one twenty degrees, so A is one twenty. Bruce: label this angle B (pointing to the angle) the whole angle is corresponding to A, it is one twenty degrees. Ernest: Make this one C (pointing to an angle in the diagram) it is a co-interior angel to B, they will sum to one hundred and eighty degrees. Alex: *The last one, vertically opposite angles, if you look at the drawing they are formed at the intersections of the transversal lines, label those on the first intersection and we will find their value.* Afriyie: I have labeled them, e and g, and have labeled these, d and f, (pointing to the angles) so that they form angles in a triangle. Bruce: So, d plus e plus f should be one hundred and eighty, they are in a triangle. Ernest: And d is corresponding to c, which means d is sixty degrees. Alex: What about f? Bruce: *I am not sure but looking at how the line is slanted the f part will be sixty degrees and the other angle will be the one twenty.* Ernest: *Let's use that, if we sum and we don't get one eight degrees then we are wrong.*		Conceptual understanding Strategic competence Adaptive reasoning

Table M3

JHS Group Five's Solution to an Enlargement Problem With the Akoma Symbol

Stage	Students' conversations	Students' written solution	Strands of mathematics proficiency
1	Amoako: We need to draw the x and the y-axes with the scale given. Boadu: *Okay, I will draw the axes then we draw the object first.* Amoako: Locate the coordinates of the points so that we draw the first *Akoma*. Boafo: *Sir, is explaining something let's listen to him and do what he says.* Osei: Join the points to form the heart as Sir did.		Strategic competence Conceptual understanding Procedural fluency
2	Osei: Let's find the image coordinates first before we go to the graph again. Amoako: Are we not supposed to work everything on the graph? Boafo: *He, is right oh, we need the image points else we can't draw the images on the graph.* Boadu: *We find the image coordinates by multiplying the object coordinates by the scale factor. Each person should do one from b to f and then I will write your answers here.* Boadu: *I have done b and c, give me the results for the d, e and f.*	 (b) Scale factor: −2 A(−1, 2) → A_1(2, −4) B(−2, 3) → B_1(4, −6) C(0, 3) → C_1(0, −6) D(−1, 0) → D_1(2, 0) (c) Scale factor: 2 A(−1, 2) → A_2(−2, 4) B(−2, 3) → B_2(−4, 6) C(0, 3) → C_2(0, 6) D(−1, 0) → D_2(−2, 0) (d) Scale factor: − A(−1, 2) → A_3(1, −2) B(−2, 3) → B_3(2, −3) C(0, 3) → C_3(0, −3) D(−1, 0) → D_3(1, 0) (e) Scale factor: 1/2 A(−1, 2) → A_4(−0.5, 1) B(−2, 3) → B_4(−1, 1.5) C(0, 3) → C_4(0, 1.5) D(−1, 0) → D_4(−0.5, 0)	Strategic competence Adaptive reasoning Procedural fluency Productive disposition Conceptual understanding

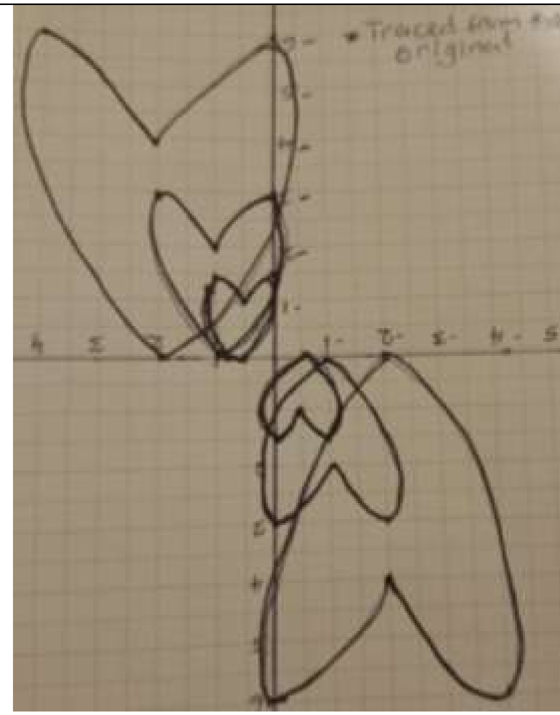
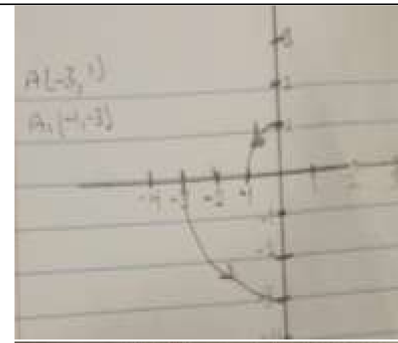
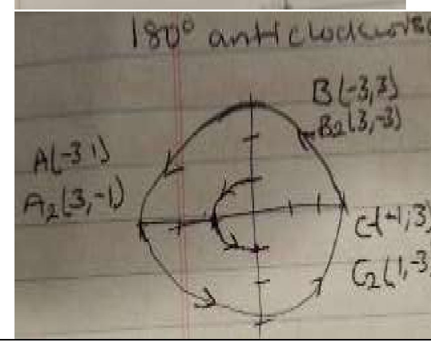
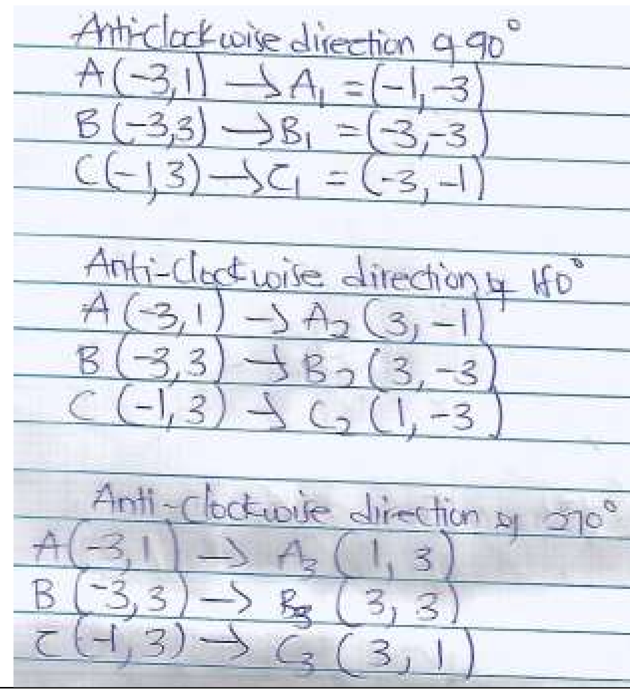
Stage	Students' conversations	Students' written solution	Strands of mathematics proficiency
2	Boafo: *Now, locate the points on the graph. F or d, point A, neg one and the two goes to point A one, one and negative two.* Boadu: Let's do it in order, from b to f.		
3	Amoako: Let everyone locate the points they calculated and draw the *Akoma*. Boadu: *Okay, let me do my b and c and I will pass it to you to do yours. But make sure you draw the heart well. If you can't draw it, after locating the points give it to me to draw the hearts through them.* Boafo: *This is interesting, with all the negative scale factors the heart turned upside down.* Osei: *This means, scale factors can be used to enlarge different Adinkra symbols to form another design.* Boadu: Now, what name do we give to the design? Amoako: *Since it is like the heart is growing from small to big, let's call it 'love grows'.* Osei: *We can call it love differs from person to person*. Boafo: *But the Akoma means patience, not love, let's call it you need big patience in difficult situations.* Boadu: *If you agree with me let's combine love and patient and call it "grow in love and patience".* Osei: *Yes, that name sounds good enough, and it contains what we all said.*		Productive disposition Conceptual understanding Adaptive reasoning Procedural fluency

Table M4

JHS Group Two's Solution to a Rotation Problem on the Akoma Ntoasoɔ Symbol

Stage	Students' conversations	Students' written solution	Strands of mathematics proficiency
1	Adansi: Isaac draw the axes Edna: What are you doing, you have to understand the question, it says two centimeters to one unit. Adansi: Draw the axes and let's locate the first points given to us. Isaac: Yes, we need to finish the main question before we go to the a and b. Serbeh: *The question says we should join A, B and C with an arc, are we using the hand or the compass?* Isaac: Let's try with the compass, if it does not work then we use our hands. Edna: *We now move to the other part of the question, if we are moving ninety degrees anticlockwise, the A points will move like, the negative three on x go to negative three on y, and the positive one on y to negative one on x* (she demonstrates it on paper as she speaks to explain to her colleagues)	 	Procedural fluency Conceptual understanding Strategic competence Adaptive reasoning

Stage	Students' conversations	Students' written solution	Strands of mathematics proficiency
2	Adansi: I think I understand this one, do another point and let's see. Serbeh: *Yes, I also understand this but are we not supposed to use the rules Sir wrote on the board?* Isaac: *He said, you can use it if you want, and you can also use your ownunderstanding to do it.* Edna: If everyone understands it, then someone should do for point C. Isaac: I got C one, to be neg three and neg one. Edna: *So, we are doing the same for one hundred and eighty degrees anticlockwise, every point will turn one hundred and eighty degrees from left for y points and from down for x points.* Edna: We have the image points, now we can transfer them to the graph sheet.		Conceptual understanding Productive disposition Adaptive reasoning Procedural fluency

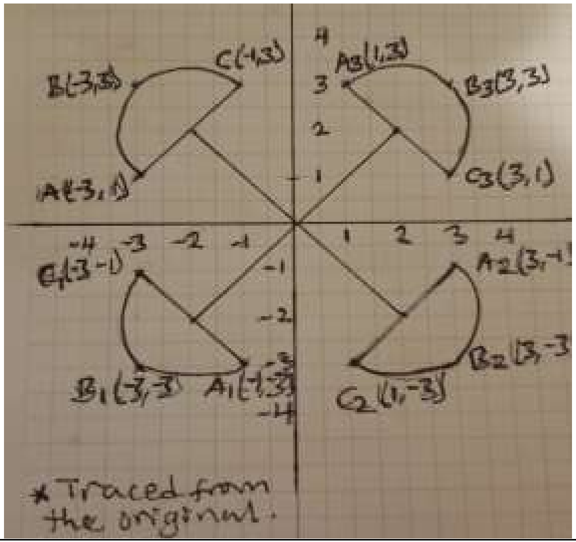
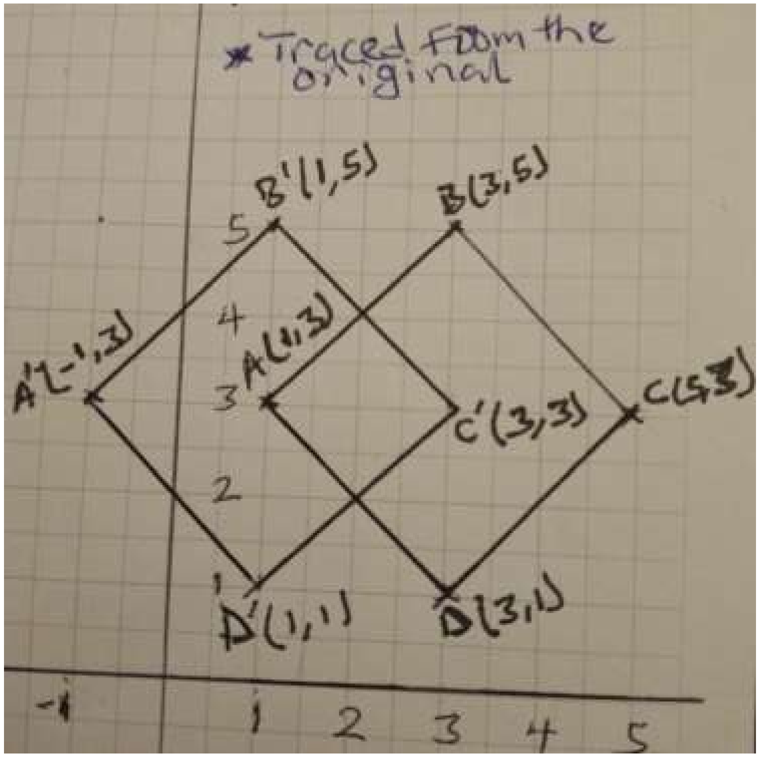
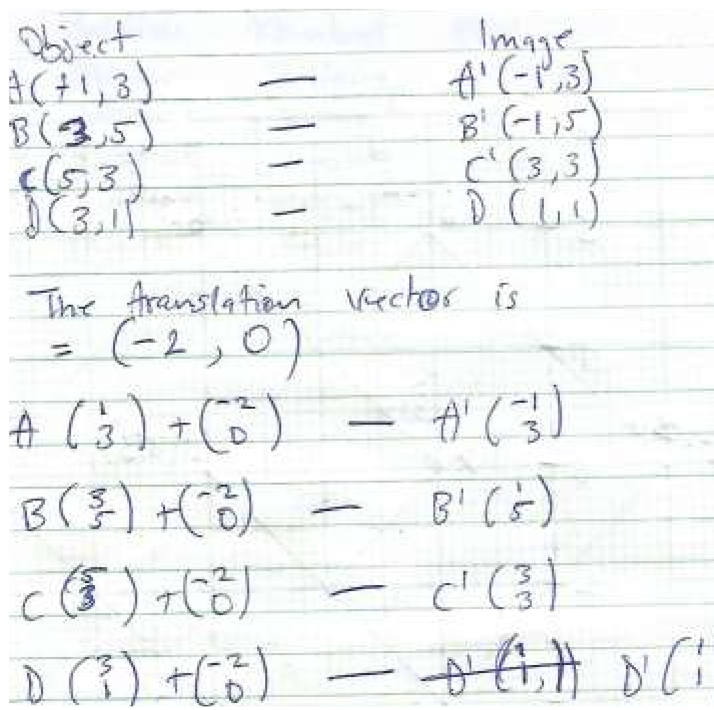
Stage	Students' conversations	Students' written solution	Strands of mathematics proficiency
3	Adansi: *Sir should have asked us to include the circle to make the complete Akoma ntoasoɔ symbol.* Serbeh: *This one, they want us to see how mathematics is used to create the Adinkra symbols, you can include the circle on your graph when you are copying the activity.*		Adaptive reasoning Productive disposition

Table M5

SHS Group Two's Work on Translation Using the Apa Symbol

Stage	Students' conversations	Students' written solution	Strands of mathematics proficiency
1	Oppong: We will draw the axes and then the *Apa* on it. Atta: What scale should I use? Boakye: Use two centimeters to one unit so we can have a large diagram. Christy: It is a rhombus, like the square so make sure the sides are equal. Atta: *Let's follow Mr Oti's instructions to do the labelling.* Nketiah: Yes. So, our first rhombus will be labelled ABCD, and the second one A prime, B prime, C prime and D prime. Christy*: Check it well, we are assuming the first rhombus was drag to the second one, so the image points should be similar to the object points. They should be at the same points as the object.* (what he meant was that they should see the second rhombus as resulting from dragging the first rhombus to that position, so they should name the image points to correspond to that of the objects) Atta: *Now we have labelled the points, we need to relate the object points and the image points and analyze them and see the difference as we did in reflection.*		Strategic competence Adaptive reasoning Conceptual understanding Procedural fluency

Stage	Students' conversations	Students' written solution	Strands of mathematics proficiency
2	Oppong: *In the image, the x-values are changing but the y-values are the same as that of the object.* Christy: *We have seen that, but we need to explain how they are changing.* Boakye: The x-coordinates are changing by minus two. Oppong: *So, what do we say the translation vector is because he is asking us to look for the translation vector.* Christy: *I think we should look at the graph again and check the movement because that is what Sir is asking us to do now.* Boakye: *Okay, x is the horizontal movement and y is the vertical movement, let's check from the drawing and see.* Atta: From A to A prime, we move two steps to the left and stopped there, we did not move up or down. Boakye: *I think it is the same as the B and C, from C to C prime we move horizontally to left two points, no movement vertically.* Christy: *So, the two steps to the left show what we had as minus 2 or maybe because it is the direction, let's say negative two,* and no *movement vertically is zero movements, therefore, the translation vector is negative two and zero.* Oppong: *So, the translation vector is the number of points to move left or right, up, or down* Atta: Yes, left or right is the horizontal movement and up or down is the vertical movement. Christy: When the x-component is positive we will move right, when it is negative, we move left. Oppong: That means a positive y component will go up and a negative y will go down. Boakye: The others verified their results by adding the translation vector to the object coordinates to get the image coordinates, let's check ours if it will give us the image coordinate.		Adaptive reasoning Conceptual understanding Procedural fluency Strategic competence

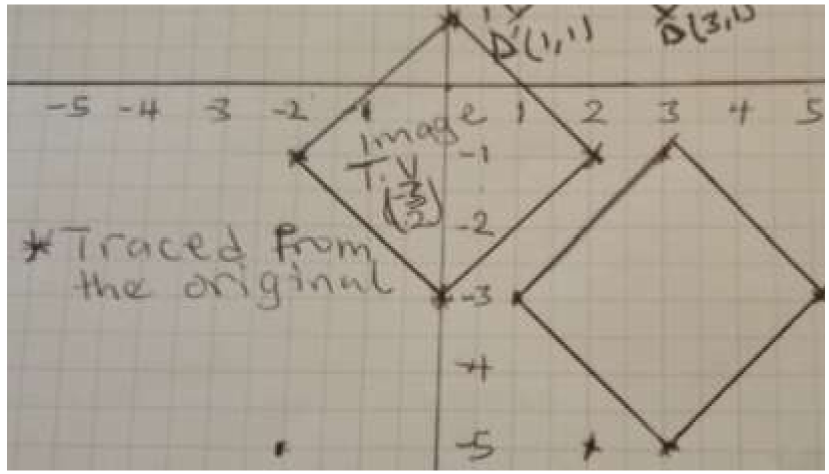
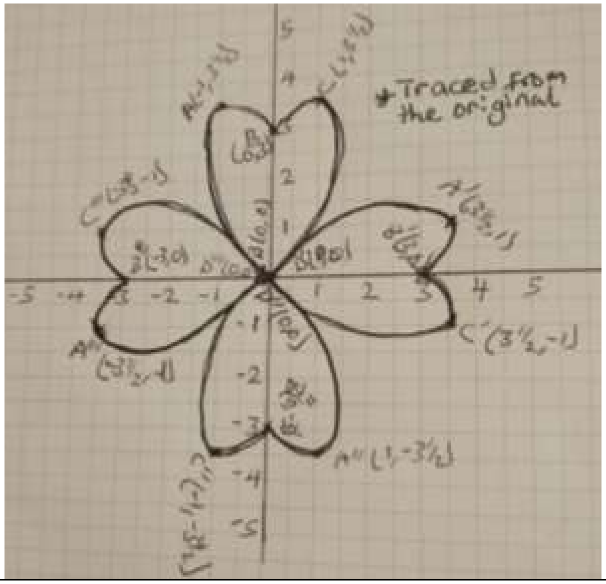
stage	Student's conversations	Students' written solution	Strands of mathematical proficiency
3	Boakye: *Let's translate the first rhombus with the vector zero and negative six and see where the image will come to.* Oppong: With this vector, the x points will remain the same and the y points will move downwards six points. Boakye: *On the graph, since there is no horizontal movement, we will move all points six points down.* Atta: I see, now let's move this with a vector that does not have a zero component. Christy: Let's use negative three and positive two to translate the new image. Nketiah: This one, we will move the points three steps left horizontally and two steps up vertically. Oppong: *Now the meaning of the Apa symbol to us. It means we have to abide by the rule, we have set in this classroom.* Boakye: *Not only that, every other rule, in the school, at home everywhere.* Christy: *It simply means you have to abide by the rules of any society you belong to.*		Conceptual understanding Adaptive reasoning

Table M6

SHS Group One's Deductions of Rotation Rules From the Nyame Dua Symbol

Stage	Students' conversations	Students' written solution	Strands of mathematical proficiency
1	Adu: Let's draw the axes and form the *Nyame dua* around the axes. Gifty: *I think it* will *be easier if we draw the symbol first on the graph and draw the axes through it to divide each part into two.* Esi: The topic is not reflection, it is rotation. Gifty: *Yes, I know, but I think we will be getting the same results once the axes are drawn.* Sarfo: *We can always try something different.* Adu: *Yes, there are so many ways of killing the cat. We can use different means to get the same results.* Kyere: *We can try the different method, but we need to make sure the hearts are drawn equally such that the axes can divide the Nyame dua into equal parts*.		Strategic competence Conceptual understanding Productive disposition
2	Sarfo: The top one is the one I drew first, so let's label that as the object O. Esi: Okay, now we have to locate some points on it as Sir Oti has done and find their coordinates. Kyere: *We should bear in mind this time we are doing turning so before we label the images, we have to examine them very well to see where the points will be if the object is turned to that position*. Sarfo: *Yes, our drawing is different from what Mr Oti has drawn so if we follow his labels, we may get it wrong.*		Strategic competence Conceptual understanding Adaptive reasoning Productive disposition Procedural fluency

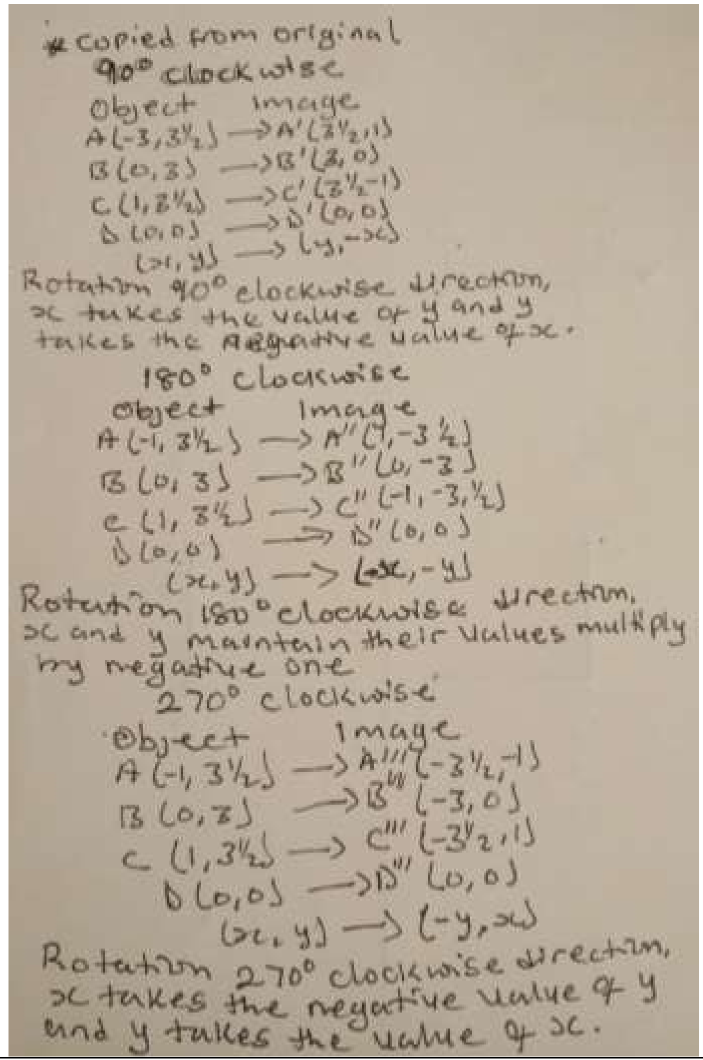
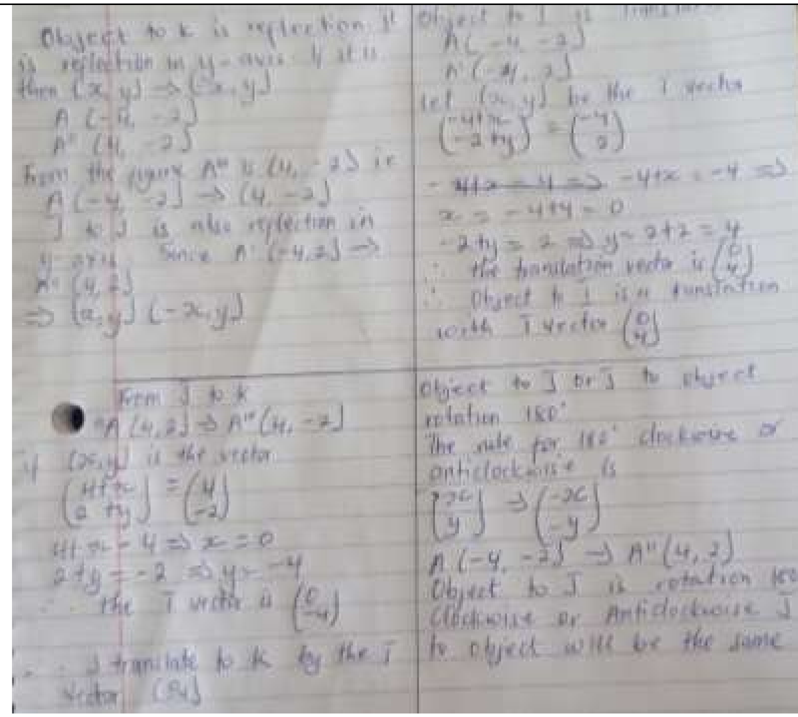
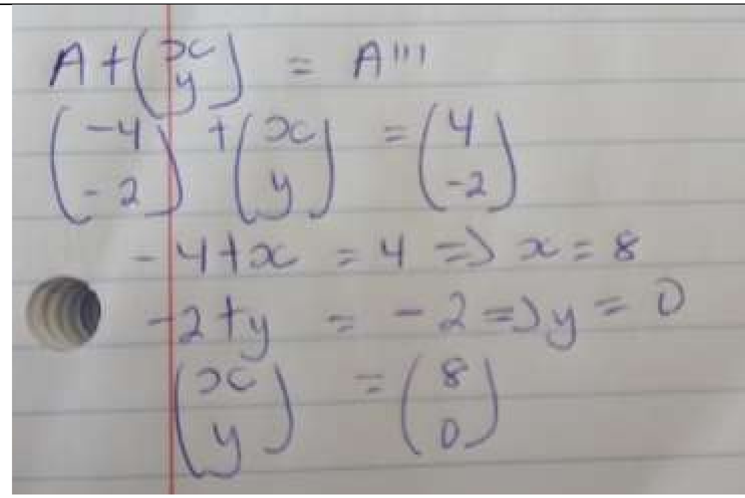
Stage	Students' conversations	Students' written solutions	Strands of mathematics proficiency
3	Kyere: Write the mapping for the objects and their images. Adu: We have to write for the different angles and deduce the rule. Esi: Each person has the mapping, so some people should do ninety degrees clockwise and anticlockwise, and others we do one eighty degrees and two seventy degrees. Sarfo: *There is no need to do both because if we look at the drawing ninety degrees clockwise will take us to the same image as two seventy degrees anticlockwise, so let's do for only clockwise. Turning one eighty degrees clockwise or anticlockwise will take you to the same position.* Kyere: *Rotation ninety degrees clockwise, the image coordinates are like this: x take the y coordinate of the object point and the y takes the negative value of x. that is for the image, you interchange the coordinates of the object and then you negative the y-component.* Gifty: In the one-eighty rotation, the image takes the negative values of the object coordinates. Esi: Two seventy degrees clockwise is the opposite of ninety degrees clockwise. x will take the negative of y and y will take x. Sarfo: *For the meaning of this symbol, I don't think there is any other meaning apart from God is with us.* Gifty*: But we are not looking for a different meaning, we are to say how the meaning applies to us.* Adu: *Yes, so it means God is with us every day we shouldn't fear*. Gifty: It also means, God is watching so we should be careful of what we do.		Strategic competence Productive disposition Adaptive reasoning Conceptual understanding Procedural fluency

Table M7

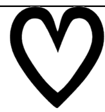
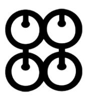
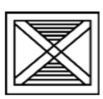
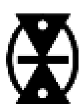
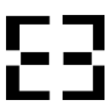
SHS Group Four's Solution to Multiple Transformations Problem on the Mate Masie Symbol

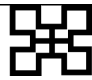
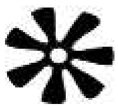
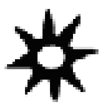
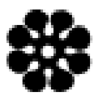
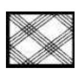
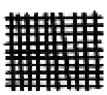
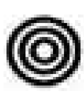
Stage	Students' conversation	Students' written solution	Strands of mathematics proficiency
1	Konlan: First of all, *Mate masie* means what? Serwaa: It means we should look for knowledge. Antwi: *What it means to you, not just the meaning*. Serwaa: *You can say it if you have an idea.* Siaw: *She is right. We can say as a student you have to seek knowledge*. Serwaa: *Aha, are we not in school to seek knowledge? Maybe we can add that we should continue to seek knowledge after completing SHS.* Keneth: Can we move on to the transformations? Antwi: We have to do the renaming first. Konlan: *I think they used the four circles to represent human heads because knowledge is kept in the head.* Keneth: Why did they use four circles; they could have used one. Serwaa: *Forget about why they drew the symbol with four circles and let's look for our own name for the symbol.* Keneth: *I think because of the four circles we can say that knowledge is acquired best in groups just as we are doing now.* Antwi: You have given the meaning, now the name. Siaw: *Base on the meaning he gave, let's call it Adwene ntoatoa* – (put minds together). Serwaa: We can now look at the transformations. Konlan: Object to I is reflection, and object to K is also reflection. Serwaa: *We can't just say it is reflection or translation, we need to show why it is*. Siaw: I *think we have to use the coordinates of the point given or label some coordinates and used them to prove.* Antwi: It is okay for us to use only the one given, let's use that.		Adaptive reasoning Strategic competence

Stage	Students' conversations	Students' written solutions	Strands of mathematics proficiency
2	Antwi: *For object and image I, the coordinates of the A and the A prime shows reflection in the x-axis because the x-coordinate which is negative four remains negative four in A prime, but the y-coordinate negative two becomes positive two.* Konlan: *No, it can't be a reflection, it is not a reflection because the line in the circle would have appeared at the top in the image and not down.* Siaw: I think the object was drag vertically to the top, so it is translation. Serwaa: I agree with you, let's find the translation vector. Antwi: The same movement is happening from J to K or K to J, that is also translation. Konlan: *From object to K is reflection, the point A on the left of the object appeared on the right in the image*. Serwaa: Okay, let's use the coordinates of A and the image coordinates on K to check. Siaw: That means from the image to the image J is also a reflection. Serwaa: *From object to J or J to object, if we move to K and then to J it would have been translation, this one I don't know.* Konlan: *It is rotation, you said it moved to K and then to J, from object to K and to J is one hundred and eighty rotation.* Antwi: Anti-clockwise or clockwise? Keneth: Both are the same, let's check with the rule and see.		Conceptual understanding Adaptive reasoning Strategic competence Procedural fluency

State	Student's Conversation	Students' Written solution	Strands of mathematics proficiency
3	Antwi: *Does it mean some translations look like reflections? Because if the middle line was not there, we would have considered image I as the reflection of the object by looking at the coordinates.* Serwaa: *It is true, okay, we have agreed that from object to K is a reflection. Let's check and see if we can still find a translation vector from the coordinates.* Antwi: We have gotten a translation vector. Does it mean every reflection is also a translation? Keneth: *We may need to try more examples before we can conclude that. We can also raise it when we are presenting or work.* Antwi: Let's conclude what we were doing on the rotation. Serwaa*:* *It worked, it is one hundred- and eighty-degrees rotation. In the translation activity we did, I thought, only translation was used to create the Mate masie not knowing the other transformations can also be applied to it.*		Adaptive reasoning Conceptual understanding Productive disposition Procedural fluency

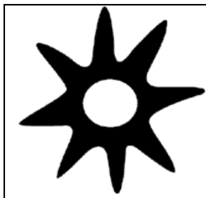
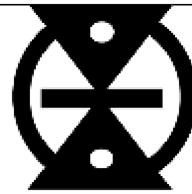
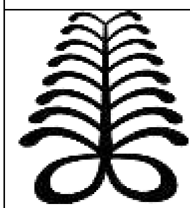
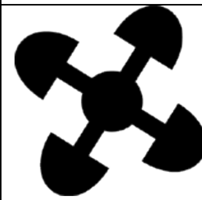
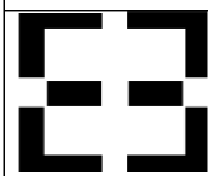
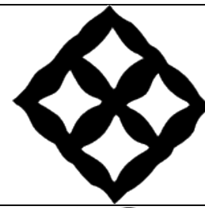
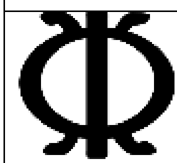
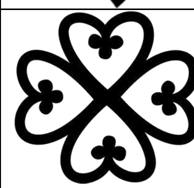
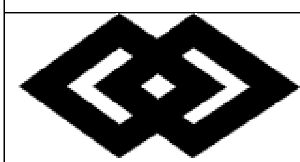
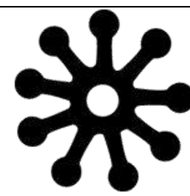
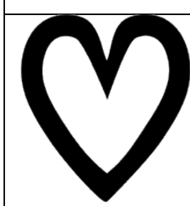
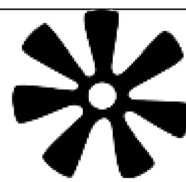
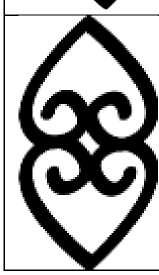
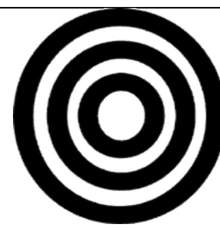
Appendix N: Images of Adinkra Symbols and the Mathematics Concepts Associated With Them

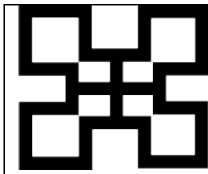
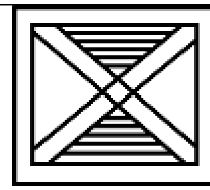
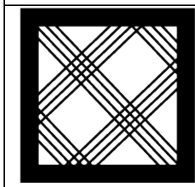
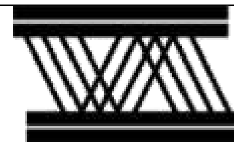
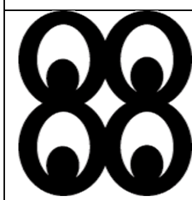
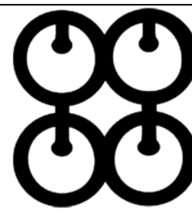
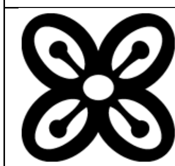
	Symbol	Name	Mathematics concepts
1		*Akoma* (heart)	Reflection
2		*Aya* (fern)	Reflection, ratio and proportion, scaling
3		*Mate masie* (I keep what I hear)	Reflection and translation
4		*M'aware wo* (I will marry you)	Reflection and translation
5		*Wawa aba* (Seed of Wawa tree)	Reflection
6		*Mframadan* (Wind resistance house)	Reflection, congruent and similar shapes
7		*Asase yɛ duru* (the earth is heavy/ has weight)	Reflection
8		*Mmere dane* (time changes)	Reflection, congruent shapes
9		*Apa* (handcuff)	Reflection, translation, ratio and proportion
10		*Woforo dua paa* (When you climb a good tree)	Reflection
11		*Nyame dua* (God's tree)	Rotation and reflection
12		*Aban* (fence)	Translation and reflection, similar shapes

13		*Nsaa* (hand woven blanket)	Reflection, ratio and proportion, similar shapes, and scaling
14		*Akoma ntoasoɔ* (linked hearts)	Rotation and reflection
15		*Bese saka* (Bunch of cola nut)	Reflection and Rotation
16		*Ananse ntontan* (Spider web)	Rotation, ratios, and area of sectors
17		*Nsoroma* (Star)	Rotation, ratio, and area of sectors
18		*Fofo* (Yellow flower plant)	Rotation, ratio, and area of sectors
19		*Nhwimu* (Division)	Angle properties of parallel lines
20		*ɔwɔ for adobe* (the snake climbs the rafia tree)	Angle properties of parallel lines
21		*KƐtƐ pa* (Good mat)	Decimals, multiplication, percentages, and proportions
22		*Adinkrahene* (Chief of Adinkra)	Ratio and proportion, and linear sequence

Images are from *West African wisdom: Adinkra symbols & meanings*, by Well-Tempered Web Design 2001-2007 (http://www.adinkra.org/htmls/adinkra_index.htm copyright 2001-2007). In the public domain.

Appendix O: Images of Symbols Posted in Classrooms, and Handed to Students

	*Nsoromma* (Star) Meaning: Child of the heavens, favour, goodwill, or blessings		*Mmere Dane* (Time changes) Meaning: the temporariness of good times
	*Aya* (Fern) Meaning: Endurance, resourcefulness		*Akoma Ntoasoɔ* **(Extension of the hearts)** Meaning: Charter, agreement, understanding
	*Woforo Dua Pa a* (When you climb a good tree) Meaning: support for good cause		*Aban* **(**Fence/Fortress**)** Meaning: Safety or protection
	*Wawa aba* **(**seed of the wawa tree**)** Meaning: hardiness, toughness, and perseverance		*Nyame dua* (God's tree/ sacred stump) Meaning: God's presence and protection
	<u>*Apa*</u> **(**Handcuffs**)** Meaning: slavery, law, and order		*Fofo* (a yellow-flowed plant) Meaning: jealousy, envy
	*Akoma* **(**Heart**)** Meaning: Patience and tolerance		*Ananse ntontan* (Spider's web) Meaning: Wisdom and creativity
	*Asase ye duru* (the earth has weight). Meaning: Providence and the divinity of mother earth		<u>*Adinkrahene*</u> (Chief of Adinkra symbols) Meaning: Leadership creativity and charisma

	*Nsaa* (A type of woven cloth) Meaning: Excellence, genuineness, authenticity		*Mframadan* (Wind resistance house) Meaning: Resilience and readiness to face the ups and downs of life
	*Nhwimu* (crossed divisions made on Adinkra cloth before printing). Meaning: Skilfulness and precision		*ɔwɔ foro adobe* (Snake climbs the raffia tree) Meaning: Persistence
	*Mate masie* (What I hear, I keep) Meaning: Wisdom, knowledge, and prudence		*Ma ware wo* (I will marry you) Meaning: Commitment and perseverance
	*Bese Saka (*Bunch of kola nut) Meaning: Abundance, affluence		

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