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Shear Alfvén waves on stretched magnetic field lines near midnight in Earth's magnetosphere

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Abstract. The spectrum of toroidal standing shear Alfvén waves (SAWs) is derived for a stretched magnetic field line geometry that is approximated by the Tsyganenko 96 magnetic field model. The model is applied to field lines near the midnight meridional plane. It is demonstrated that the fundamental mode frequency for typical ambient parameters of Earth's magnetosphere is in the 1-4 mHz range. This frequency is in agreement with the measured frequency of magnetospheric field line resonances, and is up to an order of magnitude smaller than for a dipolar magnetic field. We conclude that stretching of Earth's magnetic field offers a possible explanation for anomalously low frequency SAWs observed within diffuse auroral emission (H_β) produced by energetic proton precipitation in the inner plasma sheet.

Introduction

The hypothesis that some classes of discrete auroral arcs are associated with isolated, narrow band field line resonances [*Samson et al.*, 1996; *Trondsen et al.*, 1997], is gaining more support in spacecraft and ground based observations [*Lotko et al.*, 1999], and in model development [*Streltsov and Lotko*, 1999; *Rankin et al.*, 1999]. Standing toroidal shear Alfvén wave (SAW) field line resonances (FLRs) can provide a self-consistent and comprehensive explanation of energy accumulation near magnetic surfaces where the frequency of global scale compressional waves matches the local field line eigen-frequency. FLR formation also explains the subsequent structuring and narrowing of SAWs, and the formation of density cavities. With dispersive effects, SAWs excited in FLRs lead to electron acceleration in parallel electric fields that can be attributed to discrete auroral arcs.

Spectral analysis of ground based optical and magnetometer data have determined that in the vicinity of the midnight sector, the FLR frequency range is $1-4$ mHz at latitudes corresponding to dipole L-shells varying between $L=6$ to 10. However the standing SAW frequencies calculated for the dipolar magnetosphere on these L-shells are larger by an order of magnitude. This problem has been addressed by a number of authors by considering more realistic magnetic field configurations [*Warner and Orr*, 1979; *Singer et al.*, 1981; *Walker et al.*, 1992], the presence of heavy (oxygen) ions in the plasma sheet [*Waters et al.*, 1996; *Steltsov and Lotko*, 1999; *Rankin et al.*, 1999], or by invoking wave coupling with the slow ion acoustic mode [*Bhattacharjee et al.*, 1999].

In this letter we demonstrate that field line stetching in the midnight region during weakly to moderately active magnetospheric conditions can explain the low FLR frequencies observed in this region. We present calculations of the spectrum of linear toroidal Alfven eigenmodes at midnight using magnetic field configurations defined by the Tsyganenko 96 model, [*Tsyganenko*, 1996]. The input magnetospheric conditions to the model (plasma density and magnetic field) will be shown to be consistent with observations. The longer length of the field line and the much smaller magnetic field in the plasma sheet contribute to the resulting 10-fold decrease in FLR frequency from the dipolar case.

Other authors have noted problems with anomalosuly low FLR frequencies. [*Warner and Orr*, 1979] and [*Singer et al.*, 1981] demonstrated that the FLR frequency decreases in a geometry where magnetic field line stretching can occur. However, the WKB approach used in [*Warner and Orr*, 1979] is not appropriate for the fundamental mode, while [*Singer et al.*, 1981] used an old magnetic field model by Olson and Pfitzer, and did not extend their calculations to the high latitude nightside magnetosphere. Assuming dipole-like field lines, [*Walker et al.*, 1992] estimated that the stretching of field lines by 1.4 to 2 times their normal lengths is required for FLR frequencies to match observations in the post midnight-sector. By combining a presumed plasma density profile together with the T87 magnetic field model [*Waters et al.*, 1996] matched observed dayside plasma densities in some specific cases. However for other cases the required plasma sheet density was an order of magnitude above the satellite measurements.

Although the above mentioned studies explain FLR frequencies in specific regions of the magnetosphere, none can explain observations of 1 to 3 mHz FLR frequencies at L-shell values between 5 and 7 in the near midnight region of the magnetosphere during relatively inactive periods. For example, [*Samson et al.*, 1996] present ground based observations of FLR frequencies in the $1-3$ mHz range at L-shells between 6 and 7 during a $19.5-23.5$ magnetic local time (MLT) interval, while [*Lotko et al.*, 1999] observed a 1.3 mHz FLR at 22.4 MLT at $L=5.6$. Recent attempts to explain such observations by varying the plasma sheet density [*Streltsov and Lotko*, 1999; *Rankin et al.*, 1999] and by invoking wave coupling with the slow ion acoustic mode [*Bhattacharjee et al.*, 1999] have been unsuccessful. Below, we present results from a model of FLRs on stretched magnetic field lines and show that even during quiet times, there

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is sufficient stretching to explain anomalously low FLR frequencies near midnight on L-shells down to $L = 6$.

Equilibrium state of the stretched magnetosphere

The formal theory of standing SAWs has not been considered for a magnetic geometry other than dipolar because of the absence of an appropriate equilibrium model for Earth's magnetosphere. However, thermal effects are not particularly important for SAWs: the eigenmode frequency depends on the ambient magnetic field and plasma density, and thermal pressure and electron inertia effects define relatively small dispersion corrections, [*Hasegawa*, 1976]. Magnetic field observational data are summarized in the recent T96 [*Tsyganenko*, 1996] model of the magnetosphere. Our first objective is therefore to adapt the T96 model with stretched magnetic field lines to calculations of the FLR frequency. Since the main problem is to explain observed FLR frequencies in the nightside magnetosphere, we assume that the magnetosphere is axially symmetric, and that magnetic field lines in the meridional z, x-plane are described by the T96 model for the midnight sector. The azimuthal component of the geomagnetic field is relatively small, and is neglected in our analysis. We parametrize the stretched magnetic field configuration in a spherical coordinate system: $R(L, \theta)$ is the distance from the center of the Earth to a point of the magnetic field line, $B_0(L, \theta)$ is the corresponding parallel magnetic field, L is the dipolar magnetic shell number, and θ is the polar angle. In a dipolar magnetosphere, $R(L, \theta) = LR_e \sin^2\theta$, where R_e is Earth's radius.

For the considered stretched magnetic field configuration to be in equilibrium with the enclosed plasma, it is necessary to satisfy the force balance equation, $\mathbf{J} \times \mathbf{B} = \nabla P$ where $J = \mu_0^{-1} \nabla \times \mathbf{B}$, and therefore to find the plasma pressure P. This is a complicated equation, as discussed by many authors [*Zakharov and Shafranov*, 1982]. However, the plasma density is constant along most of the magnetic field line, and to first order we do not need the plasma pressure . We therefore assume that an equilibrium configuration with the given magnetic field line geometry exists, and we will use the force balance equation only to estimate the magnitude of the ion temperature in the plasma sheet.

Field line resonances on stretched magnetic lines

The eigenmode equation for standing toroidal shear Alfvén waves has been derived for the dipolar case in [*Cummings et al.*, 1969] and by many other authors. For an azimuthal perturbation of the magnetic field, $B_\phi = (S/h_\phi)\exp(-i\omega t)$ at a given magnetic shell L this equation reads:

$$-\omega^2 S = \frac{h_\phi}{h_\psi} \partial_l V_A^2 \frac{h_\psi}{h_\phi} \partial_l S \tag{1}$$

where $V_A(l) = B_0/\sqrt{\mu_0 \rho}$ is the local Alfvén velocity and l is the coordinate along the magnetic field line, where

$$dl/d\theta = \sqrt{R^2 + (dR/d\theta)^2} \tag{2}$$

and $l = 0$ in the equatorial plane. According to [*Singer et al.*, 1981], Eq. (1) takes exactly the same form in any curvilinear coordinate system with an appropriate definition of the metric coefficients: here, $h_\phi(l) = R(L, \theta(l)) \sin\theta(l)$ is the compression coefficient between magnetic field lines in the azimuthal direction and $h_\psi(l) = (B_0(l)\, h_\phi)^{-1}$ is the compression coefficient between flux surfaces. All these functions can be calculated for a given equilibrium plasma configuration. One has only to find the function $\theta(l)$ by solving Eq. (2) with the appropriate function $R(\theta)$ taken from the T96 model, and then to calculate the metric coefficients.

For modeling, we consider a two species plasma with a constant density of hydrogen, $n_H = 1$ cm^{-3}, and a cold oxygen component of atmospheric origin with a density $n_O = 10^4$ cm^{-3} at the ionospheric ends which decreases exponentially with altitude over a scale length $h_O = 600$ km: $n(l) = n_H + n_O \exp[-(l_{max} - |l|)/h_O]$. This density profile reproduces a characteristic peak of the Alfvén velocity at an altitude of 10 – 15 thousand km. The presence of oxygen does not affect significantly the calculated FLR frequency. The ionosphere is assumed to be highly conducting, and for consistency with observations we account for finite Pederson conductivity, $\Sigma_P = 10$ S. Then, the boundary condition for Eq. (1) at $l = \pm l_{max}$ reads: $\partial_l S = \pm i S\omega/\mu_0 \Sigma_P V_A$. Following the method described in [*Rankin et al.*, 1999], we calculate the SAW damping perturbatively from :

$$\gamma = \frac{1}{2\mu_0 \Sigma_P} \frac{[S^2 h_\psi/h_\phi]_{l_{max}}}{\int_0^{l_{max}} dl\, S^2 h_\psi/h_\phi}. \tag{3}$$

The frequency of the first antisymmetric Shear Alfvén eigenmode is shown in Fig. 1 as a function of what would be a dipolar L-value at low altitude. Each circle marked on the figure corresponds to a particular field line in Fig. 2a (pressure of 2 nPa) and Fig. 2b (pressure of 3 nPa). For a dynamic solar wind pressure of 2 nPa, the FLR frequency is 1.4 mHz at $L = 6.1$ and the damping time is 122 min. In a dipolar magnetosphere with the same parameters, the corresponding FLR frequency is around 27 mHz. With a higher dynamic pressure of 3 nPa, the $L = 6.1$ field line eigenfrequency drops to 0.84 mHz, as shown on the lower curve in Fig. 1. Alternatively, one can say the FLR shifts to lower L-shell, which is $L = 5.9$ in the example with a 2 nPa solar wind pressure. This corresponds to a southward motion of the FLR of about 0.5°.

The $L = 6.1$ field line eigenfrequency of 0.8 – 1.4 mHz is consistent with observations [*Samson et al.*, 1996], which

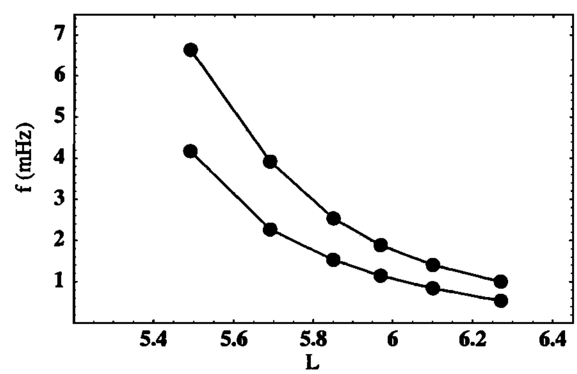

Figure 1. Dependence of the FLR eigenmode frequency as a function of the dipolar L-shell value for a dynamic solar wind pressure of 2 nPa (upper curve) and 3 nPa (lower curve). The field line profiles for each of the two cases are sketched in Fig. 2a and Fig. 2b, respectively.

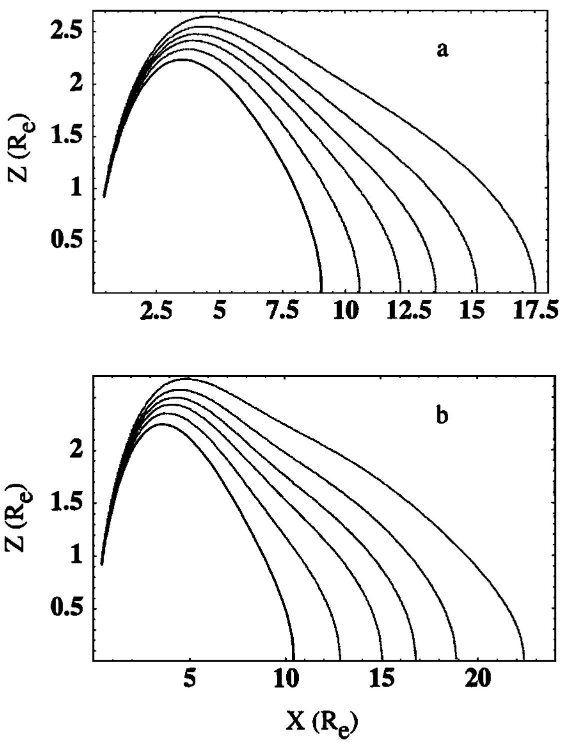

Figure 2. a) Meridional cross section of magnetic field line profiles for magnetospheric activity defined by the T96 model. The dynamic solar wind pressure is (a) 2 nPa, and (b) 3 nPa with $D_{st} = -30$ nT, $B_y = 0$ and $B_z = -3$ nT. The L-shell values near the ionosphere correspond to $L = 5.49$, 5.69, 5.85, 5.97, 6.1, and 6.27, respectively.

often show a preponderance of wave activity in that range. For a solar wind pressure of 2 nPa, Fig. 1 shows that the frequency changes with L-shell from approximately 7 mHz for $L = 5.5$ to 1.4 mHz for $L = 6.3$. At a solar wind pressure of 3 nPa, the frequency varies from 4 mHz to 0.5 mHz over the same range. Thereafter, the frequency changes more gently with L-shell, which may explain why wave power near 1 mHz is often observed over a range of latitudes. As mentioned above, the frequency decrease, as compared to the dipolar case, is due to the smaller z-component of the magnetic field in the equatorial plane, and the longer field line. According to Fig. 2a, the field line that corresponds to $L = 6.1$ at low altitude, threads to a distance of approximately 16 R_e in the equatorial plane where the magnetic field is 4.8 nT. It has a length of 31.4 R_e. The length of the corresponding dipolar line is about 15 R_e and the magnetic field in the equatorial plane at a distance of 6 R_e is approximately 90 nT.

Now we consider the wave properties expected for dipolar and stretched magnetic field lines at a solar wind pressure of 2 nPa. For this situation, the eigenfunction $S(l)$ for the stretched and dipolar geometries is shown in Fig. 3a. It has the interpretation of parallel current per unit of magnetic flux tube. In Fig. 3b, the azimuthal magnetic field is normalized to an amplitude of 200 nT at the foot of the magnetic field line. All other quantities are scaled correspondingly. Although the profiles of the eigenfunction $S(l)$ and the wave magnetic field B_ϕ for the stretched and dipolar cases are similar, the current density, Fig. 3c, is much larger in the stretched case because of the larger contraction factor, $B_0(l)/B_0(0)$, of the flux tube cross-section: it is 1200 for the dipolar case and more than 10^4 for the stretched geometry.

Equation (1) provides important relations between the measurable quantities, B_ϕ, E_r, and $J_{\|}$, which can be compared with observations to infer the consistency of the

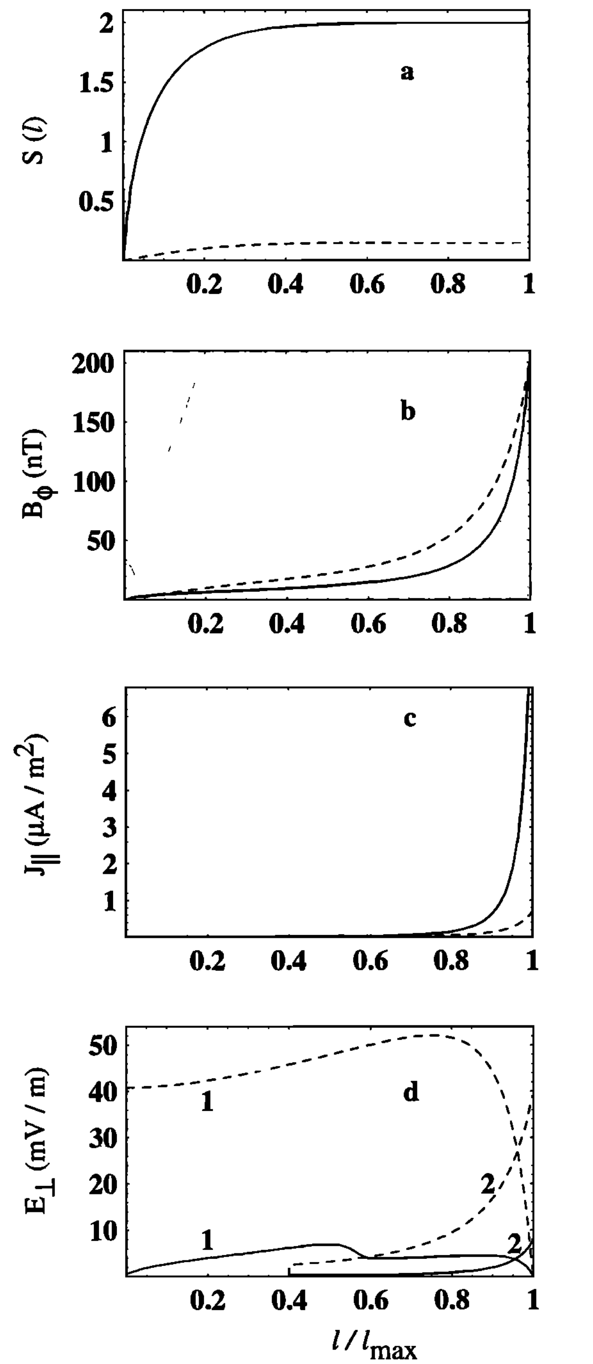

Figure 3. Dependence of the parallel current per unit magnetic tube (a), azimuthal magnetic field (b), parallel current density (c), and the perpendicular electric field (d), 1, and its quadrature component (d), 2 on the distance along the magnetic line $L = 6$ for T96 parameters defined in the text. The magnetic field is normalized to 200 nT at the foot of the magnetic field line. All other quantities are scaled correspondingly assuming the WKB approximation in the radial direction with $k_\perp R_e = 1$. Dashed lines correspond to the dipolar case.

model. In particular, recent analysis [*Tikhonchuk et al.*, 1999] shows that model predictions of B_ϕ, E_r, and $J_\|$, based on Eq. (1), are in agreement with observations [*Lotko et al.*, 1999] within a factor of 2. It can be seen from panels c and d of Fig. (3) that the stretched geometry results in a larger parallel electric current and a smaller radial electric field. Near the ionosphere, the quadrature component of the radial electric field dominates because of finite ionospheric conductivity.

Discrete arcs in all-sky images are often immersed in diffuse H_β (red emission at 630 nm) associated with energetic proton precipitation. Precipitation is expected when the magnetic curvature near the equatorial plane becomes comparable to the proton gyroradius and is consistent with magnetic field stretching. For the conditions of Fig. 2a at $L = 6.1$, the field line radius of curvature in the equatorial plane, 0.13 R_e, is comparable to a 1 keV proton gyroradius at the equator. Therefore, protons can easily fill the loss cone and excite diffuse auroral emission. Of significance is that the 1.3 mHz FLR reported in [*Lotko et al.*, 1999] is located poleward of the equatorward edge of the diffuse proton emissions, indicating that the resonant field line maps well into the plasma sheet where strong curvature is predicted [*Samson*, 2000]. For higher latitudes, protons are probably too cold ($T_i < 500$ eV) to excite optical emission.

Conclusions

In this article, we have used the Tsyganenko 1996 (T96) magnetic field model to calculate properties of standing SAWs in the midnight sector of Earth's magnetosphere. Our inputs to the T96 model correspond to a quiet to moderately active magnetosphere: solar wind pressures of 2 nPa and 3 nPa, respectively, with $D_{st} = -30$ nT, $B_y = 0$ and $B_z = -3$ nT. For these conditions, field line stretching offers a plausible explanation of anomalously low shear wave eigenfrequencies reported in ground based observations. For a field line that projects to a magnetic latitude of 66.1° ($L = 6$), the eigenfrequency is in the range of $0.8 - 1.4$ mHz. This frequency range is observed near the equatorward border of the auroral oval, and is more than an order of magnitude smaller than for a dipolar field. Although we have chosen fairly small L-shells, our results should remain valid at higher latitudes since the amount of stretching needed to lower the eigenfrequencies will be less. With the solar wind conditions defined in the text, our calculations suggest there may be enough stretching to reduce frequencies for MLT's up to 3 hours from midnight. The range of local times for significant stretching will increase with solar wind pressure.

For our chosen magnetospheric conditions, and magnetic field lines that map to dipolar $L-$shells between $L = 5.5$ and $L = 6.3$, the equatorial footprint is found to be well within the plasma sheet, where ions are hot, around 1 keV. This is consistent with ground based observations of auroral emissions that are attributed to standing wave field line resonances. The discrete auroral emissions are embedded within a band of diffuse proton (H_β) precipitation that is caused by non-adiabatic orbits (pitch angle scattering) in strong magnetic field curvature in the plasma sheet. For 1 keV proton temperatures in the plasma sheet, we find that the equatorial radius of curvature of a stretched $L = 6.1$ field line becomes comparable to the proton gyroradius. Thus, the frequency of the observed SAWs is consistent with their being excited on field lines that are significantly distorted well within the plasma sheet.

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References

Bhattacharjee, A., C. A. Kletzing, Z. W. Ma, N. F. Otani, and X. Wang, Four-field model for dispersive field line resonances: Effects of coupling between shear-Alfvén and slow modes, *Geophys. Res. Lett. 26*, 3281, 1999.
Cummings, W. D., R. J. O'Sullivan, and P. J. Coleman, Jr., Standing Alfvén waves in the magnetosphere, *J. Geophys. Res., 74*, 778, 1969.
Hasegawa, A., Particle acceleration by MHD surface wave and formation of aurora, *J. Geophys. Res., 81*, 5083, 1976.
Lotko, W., A. V. Streltsov, and C. W. Carlson, Discrete auroral arc, electrostatic shock and suprathermal electrons powered by dispersive, anomalously resistive field line resonance, *Geophys. Res., Lett., 25*, 4449, 1998.
Lysak, R. L., Electrodynamic coupling of the magnetosphere and ionosphere, *Space Sci. Rev., 52*, 33, 1990.
Rankin, R., J. C. Samson, and V. T. Tikhonchuk, Discrete auroral arcs and nonlinear dispersive field line resonances, *Geophys. Res., Lett., 26*, 663, 1999.
Samson, J. C., Private Communication, 2000.
Samson, J. C., L. L. Cogger, and Q. Pao, Observations of field line resonances, auroral arcs, and auroral vortex structures, *J. Geophys. Res., 101*, 17,373, 1996.
Singer, H. J., D. J. Southwood, R. J. Walker, and M. G. Kivelson, Alfvén wave resonances in a realistic magnetospheric magnetic field geometry, *J. Geophys. Res., 86*, 4589, 1981.
Streltsov, A. and W. Lotko, Small scale "electrostatic" auroral structures and Alfvén waves, *J. Geophys. Res., 104*, 4411, 1999.
Tikhonchuk, V. T., R. Rankin, and J. C. Samson, Ponderomotive effects and kinetic dispersion in field line resonances, *AGU Fall meeting*, San Francisco 1999, SM31A-02.
Trondsen, T. S., L. L. Cogger, and J. C. Samson, Asymmetric multiple auroral arcs and inertail Alfvén waves, *Geophys. Res., Lett., 24*, 2945, 1997.
Tsyganenko, N. A., Effects of the solar wind conditions on the global magnetospheric configuration as deduced from data-based models, in: *Proc. of Third Int. Conf. on Substorms (ICS-3) Eur. Space Agency Spec. Publ., ESA SP-389*, 181, 1996.
Walker, A. D. M., J. M. Ruohoniemi, K. B. Baker, R. A. Greenwald, and J. C. Samson, Spatial and temporal behavior of ULF pulsations observed by the Goose Bay HF radar, *J. Geophys. Res., 97*, 12,187, 1992.
Warner M. R. and D. Orr, Time of flight calculations for high-latitude geomagnetic pulsations, *Planet. Space Sci., 27*, 679, 1979.
Waters C. L., J. C. Samson, and E. F. Donovan, Variation of plasmatrough density derived from magnetospheric field line resonances, *J. Geophys. Res., 101*, 24,737, 1996.
Zakharov, L. E., V. D. Shafranov, Equilibrium of the plasma with the current in toroidal systems, in: *Reviews of Plasma Physics*, vol. 11, Consultants Bureau, New York, 1986.

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