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SETS, OPERATORS AND VOICE LEADING IN ANTON WEBERN'S OPUS 12/i

BY

HENRY KLUMPENHOUWER

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF MUSIC

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TO

ALFRED FISHER

Abstract

Early twentieth-century music in the Austro-Germanic tradition has long posed problems for the theoretical musicologist, and despite recent successes scored by the application of set-theory and prolongational analysis (derived from Schenker's important statements about tonal music), our understanding of these pieces is still so dim, that they have only recently been described as "mysterious, doorless houses."[1]

This study examines an example of this repertoire (Webern's opus 12/i) with a view, not to establish a hybridization or potpourri of analytical techniques, but rather to investigate the particular kinds of information yielded by these two analytical perspectives, and their relation to each other.

In particular, it is demonstrated that in Webern's opus 12/i structural importance is placed upon pitch-class sets belonging to a limited collection of set-types, which are related by a small number of operators in an almost motivic fashion. From the set-theoretical data, a fundamental structure is constructed which, while certainly not of the order and universal power of the Schenkerian Ursatz, permits an analysis of directed motion in Webern's opus 12/i from a position of some speculative confidence.

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[1]William Benjamin, "Ideas of Order in Motivic Music," Music Theory Spectrum 1 (1979), 23.

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To Christopher Lewis; friend, expert craftsman of "cradles."

To Alfred Fisher, to whom I humbly dedicate this scholarship; a rabbi of the order of Schenker; a prophet of the order of Schoenberg; a brother of the order of no other.

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Introduction

For the past twenty-five years, early twentieth-century music in the Austro-Germanic tradition has been the focus of a great deal of scholarly theoretical attention. Notwithstanding the amount and quality of analytical as well as purely scholarly activity, there seems to be little consensus on the supremacy of any one analytical approach, at least to the degree enjoyed, for example, by Heinrich Schenker's statements on tonal music. Indeed, the division of opinion is even mirrored in the annoying lack of universal agreement on a suitable appellative for this body of works.[1]

Aside from the lack of unanimity, it must be stressed that certain analytical positions have enjoyed some early success in achieving at the very least a limited understanding of atonal music. Most notable of these are the set-theoretical approach, stemming principally from the work of Milton Babbitt, David Lewin and Allen Forte; and a progressive branch of Schenkerian analysis represented chiefly by Felix Salzer, Roy Travis and Joel Lester.[2] The compass of this study shall be to examine Anton Webern's Lied, Der Tag ist vergangen opus 12/i (1915), with a view to discovering the nature and extent of the information borne by these two analytical traditions.

Since the primary thrust and the particular talent of the set-theoretical approach is one of precise classification, the principal issue is the definition of the attributes required for membership in a particular set or class. Because the collecting of pitches into sets is carried through to three levels - to the determination of sets of sets of sets of pitches (i.e. pitch-class set-types) - there are necessarily three correspondingly different species of criteria for set membership.

The first of these arranges all pitches into one of twelve classes according to octave relatedness. In other words, each pitch is arranged into one of twelve distinct and non-intersecting sets defined as (assuming enharmonic equivalency) the set of all C's, the set of all C#'s, the set of all D's and so on. These pitch-classes are notated and valued by the integers 0, 1, 2, ... 11, respectively. Consequently, the interval between pitches of different classes must be expressed as a numerical difference, paralleling Ellis's logarithmic measure of the intervals of the equally tempered duodenary scale.[3]

Ignoring the question of order of constituent elements, the complete inventory of sets of these pitch-classes comprises 4,096 (2^{12}) unique members. Disregarding the two extreme sets - one empty and one containing the entire domain of discourse - the remaining 4,094 are grouped into classes or types according to relatedness under some definable operator or similarity of interval content. These operators or functions - transposition, inversion and multiplication - seek to construct consistent and exhaustive relationships between the pitch-class elements of one set and those of another. Put another way, the application of one of these operators to a given set generates a second, in a way that a comprehensive one-to-one correspondence between the contents of the two collections may be drawn. Consequently, these operators meet Suppes's definition of "functions as binary relations which relate to each element of their domains a <u>unique</u> element of their counterdomains";[4] and may accordingly be expressed as equations,[5] where x represents each element of the original set; and n=0 to 11, with all products reckoned modulo 12.

1) Transposition $Tn(x) = n + x$

2) Inversion $TnI(x) = n - x$

3) Multiplication $TnM(x) = n + 5x$

and, by combining 2) and 3),

4) Multiplicative Inversion $TnMI(x) = n - 5x$ or $= n + 7x$

The result of each operator in the complete inventory of 48 (4 basic operators X 12 values for n) is catalogued in sets of mapping cycles, with each set unique to a particular operator (see ex. I-1).[6] In addition to providing a complete picture of the effect of any operator on members of any pitch-class, mapping cycles eloquently demonstrate Suppes's definition. Specifically, under all of the functions listed in ex I-1, 1) no element maps onto more than one other element, and 2) the entire domain of discourse is accounted for by each function.

Mapping cycles also manifest relationships between certain functions. Pairs of operators whose cycles are <u>in a simple</u> retrogradient relationship negate the effects of each other, and are said to be each other's inverse. An operator whose cycles are non-retrogradable - i.e. one that possesses no mapping cycle of more than two elements - is said to be reflexive.

The questions of precisely which of the raw operators (transposition, inversion and multiplication) may legitmately define

Example I-1: Operators and Mapping Cycles

n	Transposition Tn(x)=n+x		Inversion TnI(x)=n-x		Multiplication TnM(x)=n+5x		Multiplicative Inversion TnMI(x)=n-5x or n+7x	
0	(0)(1)(2)(3)(4)(5)(6)(7)(8)(9)(B)(H)	*	(0)(1-H)(2-B)(3-9)(4-8)(5-7)(6)	*	(0)(1-5)(2-B)(3)(4-8)(6)(7-H)(9)	*	(0)(1-7)(2)(3-9)(4)(5-H)(6)(8)(B)	*
1	(0-1-2-3-4-5-6-7-8-9-B-H)	H	(0-1)(2-H)(3-B)(4-9)(5-8)(6-7)	*	(0-1-6-7)(2-H-8-5)(3-4-9-B)	7	(0-1-8-9-4-5)(2-3-B-H-6-7)	5
2	(0-2-4-6-8-B)(1-3-5-7-9-H)	B	(0-2)(1)(3-H)(4-B)(5-9)(6-8)(7)	*	(0-2)(1-7)(3-5)(4-B)(6-8)(9-H)	*	(0-2-4-6-8-B)(1-9-5)(3-H-7)	B
3	(0-3-6-9)(1-4-7-B)(2-5-8-H)	9	(0-3)(1-2)(4-H)(5-B)(6-9)(7-8)	*	(0-3-6-9)(1-8-7-2)(4-H-B-5)	9	(0-3)(1-B)(2-5)(4-7)(6-9)(8-H)	*
4	(0-4-8)(1-5-9)(2-6-B)(3-7-H)	8	(0-4)(1-3)(2)(5-H)(6-B)(7-9)(8)	*	(0-4)(1-9)(2)(3-7)(5)(6-B)(8)(H)	*	(0-4-8)(1-H-9-7-5-3)(2-6-B)	8
5	(0-5-B-3-8-1-6-H-4-9-2-7)	7	(0-5)(1-4)(2-3)(6-H)(7-B)(8-9)	*	(0-5-6-H)(1-B-7-4)(2-3-8-9)	H	(0-5-4-9-8-1)(2-7-6-H-B-3)	1
6	(0-6)(1-7)(2-8)(3-8)(4-B)(5-H)	*	(0-6)(1-5)(2-4)(3)(7-H)(8-B)(9)	*	(0-6)(1-H)(2-4)(3-9)(5-7)(8-B)	*	(0-6)(1)(2-8)(3)(4-B)(5)(7)(9)(H)	*
7	(0-7-2-9-4-H-6-1-8-3-B-5)	5	(0-7)(1-6)(2-5)(3-4)(8-H)(9-B)	*	(0-7-6-1)(2-5-8-H)(3-B-9-4)	1	(0-7-8-3-4-H)(1-2-9-B-5-6)	H
8	(0-8-4)(1-9-5)(2-B-6)(3-H-7)	4	(0-8)(1-7)(2-6)(3-5)(4)(9-H)(B)	*	(0-8)(1)(2-6)(3-H)(4)(5-9)(B)(7)	*	(0-8-4)(1-3-5-7-9-H)(2-B-6)	4
9	(0-9-6-3)(1-B-7-4)(2-H-8-5)	3	(0-9)(1-8)(2-7)(3-6)(4-5)(H-B)	*	(0-9-6-3)(1-2-7-8)(4-5-B-H)	3	(0-9)(1-4)(2-H)(3-6)(5-8)(7-B)	*
B	(0-B-8-6-4-2)(1-H-9-7-5-3)	2	(0-B)(1-9)(2-8)(3-7)(4-6)(5)(H)	*	(0-B)(1-3)(2-8)(4-6)(5-H)(7-9)	*	(0-B-8-6-4-2)(1-5-9)(3-7-H)	2
H	(0-H-B-9-8-7-6-5-4-3-2-1)	1	(0-H)(1-B)(2-9)(3-8)(4-7)(5-6)	*	(0-H-6-5)(1-4-7-B)(2-9-8-3)	5	(0-H-4-3-8-7)(1-6-5-B-9-2)	7

The value for n of an operator's inverse is indicated in the column at the right of that operator's cycles. Reflexive operators are indicated by asterisk. Note that, in order to maintain the use of single characters, B replaces 10, and H replaces 11.

Although parentheses show the boundaries of a particular cycle, the cycle is to be considered as an infinite series, with the first element following the last. In each series, the result of the corresponding operator appears to the right of the operand. Thus, the result of applying, for example, T7M to a member of the pitch-class 8 yields a pitch of the class H (i.e. 11). This same operator applied to pitch-class H (remembering that the cycle is an infinite series) yields a pitch of class 2. Put another way, the product of T7M applied to pitch-class 8 (thus n=7, x=8 and T7M(8)=7+5(8)) will be found to the right of the integer 8 in the set of mapping cycles located at the co-ordinate n=7 in the multiplication column. If a particular cycle has only one element (like all the cycles of T0), then the associated operator applied to a member of that pitch-class will yield a member of the same pitch-class.

set-membership, and whether similarity of interval content alone may do so, are issues that still incite vigorous sparring.[7] Nevertheless, there seems to be general agreement, if only for the sake of nomenclatural convention, that set-type membership be determined by relatedness under transposition or inversion. In any case, the principal problem here for set-theorists is that the collection of pitch-class sets related under transposition or inversion does not exhaust the collection of pitch-class sets with similar interval content.[8] In other words, while a number of pairs of pitch-class set-types, described initially by David Lewin and later designated by Allen Forte as Z pairs,[9] are related by the similarity of their interval vectors, they are unrelated under the functions of transposition or inversion. The aggregate of Z pairs is not however a homogeneous collection, and the polarity of operational unrelatedness and similarity of interval content is not resolvable in any consistent manner.

Presumably, this gap may be mediated either by accepting functions in addition to transposition and inversion as set-type builders, or by accepting that similarity of interval content may, at least in the case of hexachordal Z pairs, be due to their complementary relationship coupled with the equality of their cardinalities. The mechanics of this are clarified by an engaging demonstration commonly known as the hexachord theorem.[10] The proof begins by taking the level of intersection between a given pitch-class set and a transpositionally related form as a metonym for the interval content associated with members of that set-type, thus seizing upon the fact that the degree of intersection between two transpositionally related sets equals the frequency within the set-type of intervals of the class notated by that integer which identifies the transpositional operator that relates the two sets.

The level of intersection between a particular set (indicated here by the letter A) and the complement or residue of a transpositionally related pitch-class set (indicated as $\overline{TnA}$) equals the number of elements in the original set (i.e. its cardinality), minus the level of intersection between the original set and the transpositionally related set (rendered as $\#(A \cap TnA)$).[11] This is translated as (1).

(1) $\#(A \cap \overline{TnA}) = \#A - \#(A \cap TnA)$

Further, the level of intersection between the complement of the original set ($\overline{A}$), and the complement of the transpositionally related set ($\overline{TnA}$) equals the cardinality of $\overline{A}$ minus the level of intersection between $\overline{TnA}$ and A.

(2) $\#(\overline{A} \cap \overline{TnA}) = \#A - \#(A \cap \overline{TnA})$

Substituting from (1), we derive (3).

(3) $\#(\overline{A} \cap \overline{TnA}) = \#A - (\#A - \#(A \cap TnA))$

Reducing (3) renders (4),

(4) $\#(\overline{A} \cap \overline{TnA}) = \#A - \#A + \#(A \cap TnA)$.

And hence,

(5) $\#(\overline{A} \cap \overline{TnA}) - \#(A \cap TnA) = \#A - \#A$.

Recalling that #(A TnA) and #(A TnA) stand as metonyms for the interval vectors of pitch-class set A, and of its complement, respectively, it is clear that the difference between corresponding values in their interval vectors may be determined by the difference of their cardinalities.

Consequently, if the value of cardinality for a given pitch-class set is 6, the cardinal value of its complement will also be 6 and the difference between corresponding values in their vectors will be zero. The main point here, of course, is that this theorem offers, arguably, a satisfactory explanation for the disparity between similarity of interval content and operational relatedness, since it demonstrates that, at least for hexachords, these need not be seen as necessarily linked characteristics.

Another tack for spanning this gap is the acceptance of multiplicative functions as valid definers of membership at the set-class level, and indeed for the sole tetrachordal and two of the pentachordal Z pairs this class of operators does provide a specific value for relatedness. Nevertheless, multiplicative operators are not capable of uniquely and completely providing mappings between all Z pairs, even if hexachordal pairs are disregarded. Not only is the relationship between the 5-Z12/5-Z36 pair left undefined, but multiplicative operators pair set-types with dissimilar interval content. Moreover, within the field of hexachordal Z pairs, multiplication class functions, while mapping five of the fifteen pairs with similar interval content, map members of eight of the pairs onto set-types other than their Z partners, and leave the members of the final two pairs, like the (5-Z12/5-Z36) couple, unrelated by any operator to their respective Z, or any other, set-type (see ex. I-2).

While a single comprehensive and unassailable keystone relationship dovetailing the two possible criteria for set-type membership seems elusive, we are nevertheless empowered by an entire

Example I-2: <u>Z Related Pairs of Hexachordal Sets - Types</u>
(Brackets indicate complementary set-types)

	Z-pair			Relationship
1)	4-Z15	/	4-Z29	multiplicative
2)	5-Z12	/	5-Z36	undefined
3)	5-Z17	/	5-Z37	multiplicative
4)	5-Z18	/	5-Z38	multiplicative
5)	6-Z3 ⌉	/	6-Z23 ⌉	multiplicative; non-complemenatary
6)	6-Z36 ⌋	/	6-Z47 ⌋	" "
7)	6-Z4 ⌉	/	6-Z26 ⌉	" "
8)	6-Z37 ⌋	/	6-Z48 ⌋	" "
9)	6-Z10 ⌉	/	6-Z46 ⌉	" "
10)	6-Z39 ⌋	/	6-Z24 ⌋	" "
11)	6-Z13 ⌉	/	6-Z50 ⌉	" "
12)	6-Z42 ⌋	/	6-Z29 ⌋	" "
13)	6-Z12	/	6-Z41	non multiplicative; complementary
14)	6-Z17	/	6-Z43	" "
15)	6-Z11	/	6-Z40	multiplicative; complementary
16)	6-Z6	/	6-Z38	" "
17)	6-Z44	/	6-Z19	" "
18)	6-Z45	/	6-Z23	" "
19)	6-Z49	/	6-Z28	" "

array of heuristic relational possibilities.

Returning then to the initial discussion, set-theory provides standards for membership at the level of sets of pitches (octave relation), and at the level of sets of sets of sets of pitches, or, more commonly, pitch-class set-types (complementation, functional relatedness, similarity of interval content). Since set membership at the outer level is a pre-analytical given, the principal analytical concern is the construction of a view of a given piece of atonal music that expresses how it coheres in the categories of the third level. In other words, the analyst travels the territory of the middle level to span the "foreground level" (the piece, expressed in pitch-classes) and the "background" system of possible relationships (relatedness of collections under operators, etc.). As Daniel Starr has expressed it,

> I find it both fruitful and intuitive to conceive of an operation-object duality, in which "operation" is a concept subject to general discussion, while "object" arises from the discussion of specific works. Thus, to approach various general aspects of twelve-tone or related types of musics, I stress what I consider the operations that we apply to sets, rows, partitions, rather than those objects themselves, or, for that matter, their classification, which is the topic most often considered. (12)

Ex. I-3: Set Membership Criteria

First Level	sets of pitches (pitch-classes)	object
Second Level	sets of sets of pitches (pitch-class sets)	object
Third Level	sets of sets of sets of pitches (pitch-class set-types)	operation

Despite the remarkable technical edifice constructed by twenty-five years of set-theoretical speculation, its approach is censurable on the grounds that the data yielded by its application - however cogent - provide neither an agreeable perceptional metaphor nor a perceptional aid. That is, by relying on cognitive rather than aural-emotive faculties, it transgresses the accepted, if not sophistical, demand that analytical statements concerning a given work be ultimately accountable to observation, however this is construed in musical categories. Indeed, since set theory seems innately dependant on relationships among sets of pitch-classes, it tends to view a work atemporally, and hence presents itself as a rigorously static approach.

In contrast, prolongation analysis successfully incorporates a consideration of temporal process while it accepts the authority of aural "intuition." Interestingly, the fontanel of the approach is the flaccidity of its technology. To be sure, Joel Lester has presented a discussion of prolongational "operations" (passing and neighbor notes) in uniquely atonal terms by, for example, speculating that these take advantage of the uniform grade of the chromatic scale and generally divide tonal spaces into equal parts.[13]

Notwithstanding these useful speculative statements, since the prolongation analysis of atonal music lacks a universal theoretical framework, such as the classical Ursatz, analytical statements made from its perspective can hardly be more than personal assertions. Clearly this is because, as Carl Schachter has pointed out, "Without some sense of the background, one can't begin to understand the foreground."[14] In other words, the criterion for determining what is prolongational and what is structural at a certain level arises, not from within that level itself, but from its relationship to a background fundamental structure.

Without an <u>Ursatz</u> of some kind, all that remains is a sketching technique - however precise and compelling.

The incisiveness of Schenker's formulation of the tonal fundamental structure (<u>Ursatz</u>) lies in his understanding of this particular aspect of its analytical function. It is never presented simply as a hypothesis developed from observed data, although clearly at the outset Schenker devised it inductively.[15] But by Schenker's transformation of the <u>Ursatz</u> from a hypothesis into an <u>a priori</u>, the thrust of analysis shifts from endeavoring to discredit the <u>Ursatz</u> as a legitimate inference to a determination of how it is expressed by a given tonal work. Its elevation from a probable inference to a necessary one, from a supposition of the characteristics of tonality to a definition of its features, permits the process of analysis not only to account for a perception of a piece but also to heighten it.

Naturally, the absence of a universalized fundamental structure for atonal music requires us to develop other methods for discovering the background for each atonal example anew. In other words, prolongational analysis of atonal music has yet to reach the watershed that will provide a given universal for all atonal works. Nevertheless, if we believe these pieces to be meaningful from the perspective of structural levels, and that they possess depth with respect to voice leading and prolongations, then they are clearly governed by a fundamental structure of some kind. The critical issue here is that it cannot be a pre-analytical given.

This paper will seek to demonstrate that these two analytical perspectives, their respective technical flaws notwithstanding, provide a well-appointed apparatus of undeniable power and elegance, and permit the analyst to make speculative as well as purely analytical statements with conviction and authority. What is presented here however is not a "hybrid" technology - and indeed this is the crux of the matter - for neither approach is completely definable by its technical gear. Rather, each represents what may more properly be termed a *Musikanschauung*. Their respective technical aspects merely serve to view a piece in a particular way. It will be demonstrated that the discovery of the background fundamental structure for Webern's opus 12/i is facilitated by, and perhaps only possible from the data reaped by an analysis of its sets.

Chapter II

Sets and Mappings in Webern's Opus 12/i

We can begin by noting that the gross formal division of the Lied "Der Tag ist vergangen"[16] into two sections, indicated by a slackening of the rate of the basic pulse to a full stop at the end of m. 12, and a metrical broadening of m. 13, corresponds to the formal disposition of the folksong text into strophes. Notably, the binary formal design of the song and these two methods of articulating this division are introduced in the three prefatory measures. There is a possible further formal segmentation of each section into two sentences, each of these in turn comprised of a further subdivision into two phrases. Since these formal segments are clearly and unambiguously articulated by surface characteristics, these statements appear to be merely observational. Nevertheless their real importance lies in providing boundaries, or at least interruptions, of musical processes. More precisely, the most meaningful task is to determine how these formal divisions are articulated in set-theoretical and voice-leading terms.

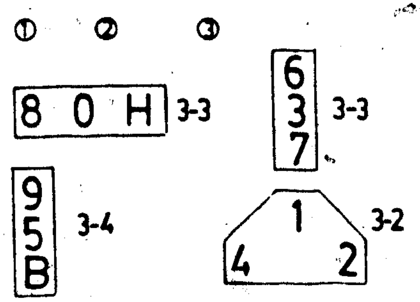
Example II-1: Formal Disposition of Opus 12/i

m.				
1 - 3	[introduction]			
4 - 5	Der Tag ist vergangen,	antecedent phrase	antecedent sentence	antecedent strophe
6 - 8	die Nacht ist schon hier,	consequent phrase		
7 - 9	gute Nacht, O Maria,	antecedent phrase	consequent sentence	
10-12	bleib ewig bei mir.	consequent phrase		
13-14	Der Tag ist vergangen,	antecedent phrase	antecedent sentence	consequent strophe
14-16	die Nacht kommt hierzu,	consequent phrase		
16-18	gib auch den Verstorbnen,	antecedent phrase	consequent sentence	
19-23	die ewige Ruh.	consequent phrase		

16 The designations "antecedent" and "consequent" are meant only in an ordinal sense here.[17]

The introductory three measures contain a representative of each of the twelve pitch-classes, arranged in four non-intersecting trichords which divide the musical space of the first three measures geometrically into four quadrants. (The integers 10 and 11 are represented in all examples by the letters B and H, respectively.)

Example II-2: <u>Trichords, mm. 1-3</u>



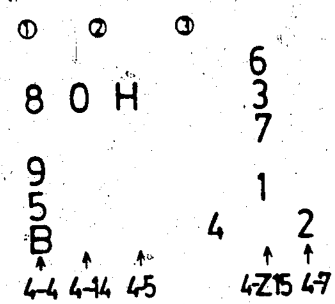
These trichords may be coupled vertically, horizontally, or according to gestural similarity to form three different partitionings of the introduction into hexachords of complementary set-types.

Example II-3: <u>Hexachords, mm. 1-3</u>

1) (0,8,H) ∪ (9,5,B) // (6,3,7) ∪ (4,1,2) = 6-Z36 // 6-Z3
2) (0,8,H) ∪ (6,3,7) // (9,5,B) ∪ (4,1,2) = 6-Z44 // 6-Z19
3) (0,8,H) ∪ (4,1,2) // (9,5,B) ∪ (6,3,7) = 6-Z10 // 6-Z39

The unions of melodically articulated pitch-classes (8, 0, H, 1, 2) with the two chordally stated trichords (9, 5, B) and (6, 3, 7) form the following tetrachords:

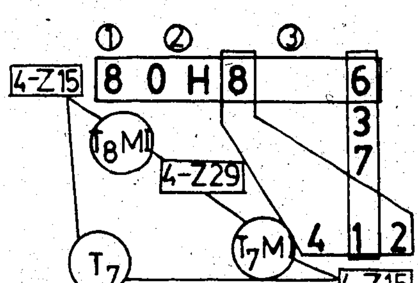
Example II-4: Tetrachords, mm. 1-3



While these tetrachords are neither transpositionally nor inversionally related, the first two do belong to set-types which are equivalent under multiplication. More specifically, these two tetrachords map onto each other under the reflexive operator T_8 M.

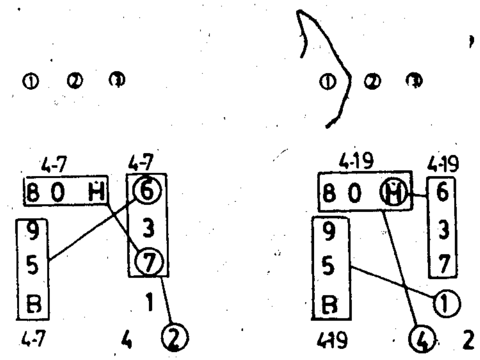
Continuing to build tetrachords through the union of the primary trichords with single pitch-classes yields an additional member of the set-type 4-Z15, along with a member of its multiplicatively related set-type 4-Z29. If the collection is viewed as a series of sets and operators, each set intersects only one member with either of the others; the result is a composite operator of T_7.

Example II-5: 4-Z15 and 4-Z29 Tetrachords, mm. 1-3



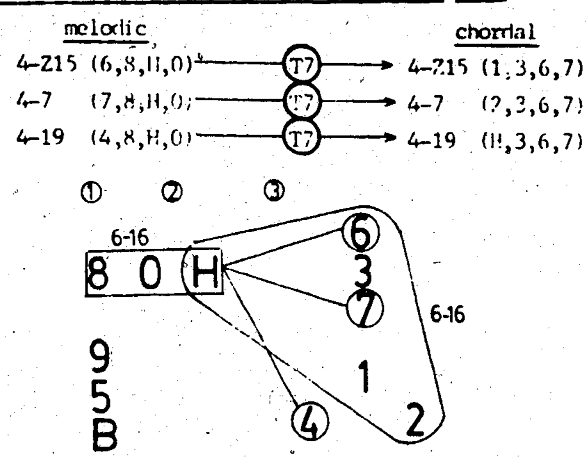
Two additional members of the set-type 4-7, and three members of the set-class 4-19 may be similarly derived from the union of primary trichords with single pitch-classes.

Example II-6: <u>4-7 and 4-19 Tetrachords, mm. 1-3</u>



Remarkably, the initial members of the set-types 4-Z15, 4-7 and 4-19, each one articulated melodically, all map onto equivalent members at the end of the introduction, each presented vertically, under the operation T_7. Not surprising, the hexachord formed by the union of these melodically articulated tetrachords, a member of the set-class 6-16, maps onto the union of the chordally presented ones under the same operator.

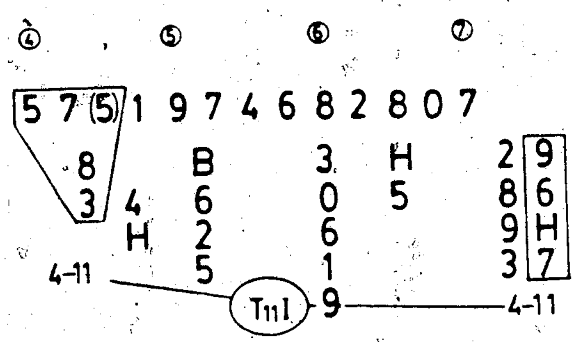
Example II-7: <u>T_7 - Related Tetrachords, mm. 1-3</u>



The First Strophe (mm. 4 - 12)

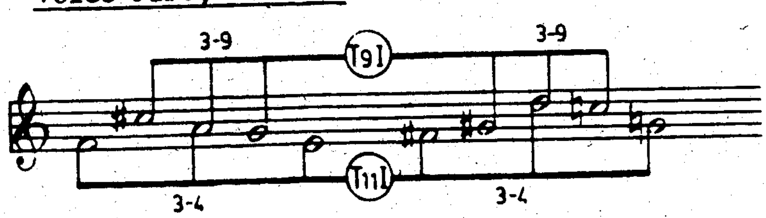
The first line of the <u>Lied</u>, comprising two vocal phrases counterpointed by three piano gestures, begins and concludes in mm. 4 and 7 with inversionally related members of the tetrachordal 4-11.

Example II-8: <u>T_2I - Related 4-11 Tetrachords, mm. 4, 7</u>



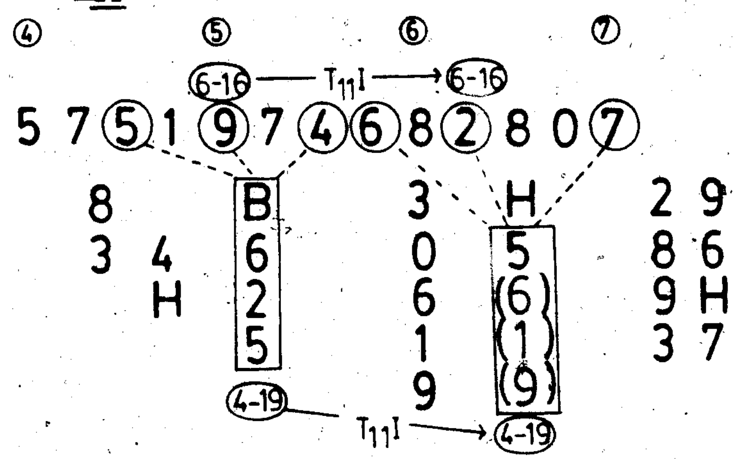
The inversional relatedness of these tetrachords is much more than a matter of local concern, for this class of operator pervades the first line to the degree that it characterizes the relationship between mm. 4 and 5, and mm. 6 and 7. Specifically, the two vocal phrases may be seen as interlocking trichords of the set-types 3-9 and 3-4 that map onto their fellows under the inversional operators T_9I and $T_{11}I$ respectively. Note that in each phrase, the elements of the pair of trichords are presented in alternation, with the central pitch of each configuration shared between them.

Example II-9: <u>Voice Part, mm. 4-7</u>



The operator $T_{11}I$ also relates the two tetrachords of the set-type 4-19 that begin and conclude the interior piano gesture in mm. 5 and 6. Remarkably, the unions of each of these tetrachords with corresponding $T_{11}I$-related trichords of the set-type 3-4 build two $T_{11}I$-related hexachords of the 6-16 set-class, which was similarly formed in the introductory measures (cf. exx. II-7, and II-10).

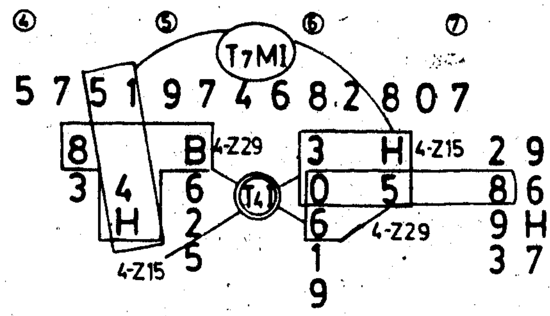
Example II-10: <u>$T_{11}I$ - Related Tetrachords and Hexachords, mm. 5, 6.</u>



In addition, both halves of this sentence contain, in mm. 4 and 6, a member of the tetrachordal set-type 4-Z15 and a member of its multiplicatively equivalent set-type 4-Z29. Like the other pitch-class sets discussed in the first sentence, the sets of the same class are inversionally related, both under T_4I. Notably, like the sets of these set-types in the introduction, the first member of the 4-Z29 set-type (m. 5) maps onto the final 4-Z15 tetrachord (m. 6) under the operator T_7M1 (cf. exx. II-5, II-11).

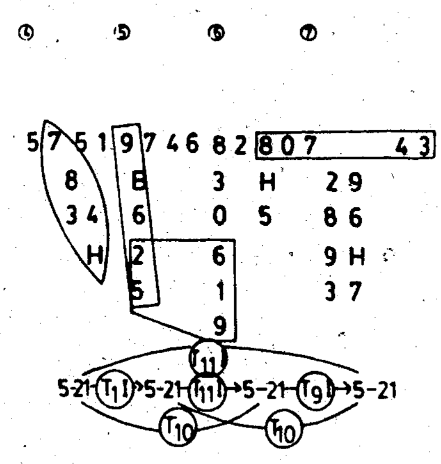
Both degree and species of relatedness between pairs of tetrachords of the same set-type are paralleled between sets of larger cardinality. In the first sentence there are four members of the pentachordal set-type 5-21 which, when viewed as a series, form a chain

Example II-11: Tetrachords of the Classes 4-Z15 and 4-Z29, mm. 5, 6



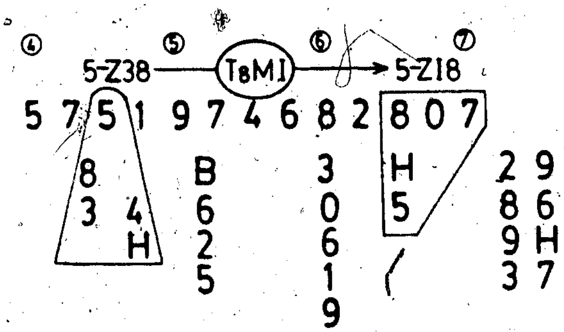
of inversionally related sets. More important, the first and fourth sets, along with the second and third, like the two members of the set-type 4-19 and the two members of the trichordal set-type 3-4 constitute pairs of $T_{11}I$-related sets. Note that any two members of the set-type 5-21 that are related by $T_{11}I$, intersect maximally; or, put another way, exist in an Rp relation. The first and second pentachords map onto the third and fourth respectively under T_{10}. In contrast to $T_{11}I$-related members of this set-class, 5-21 pentachords related by T_{10} intersect no elements.

Example II-12: Pentachords and Mappings, mm. 4-7



Finally, the tetrachords of the set-types 4-11 and 4-19 in m. 5 frame a member of the pentachordal set-type 5-Z38, while their inversionally related set-class fellows in mm. 6 and 7 frame a pentachord that is related to the 5-Z38 set under the operator T_8 MI.

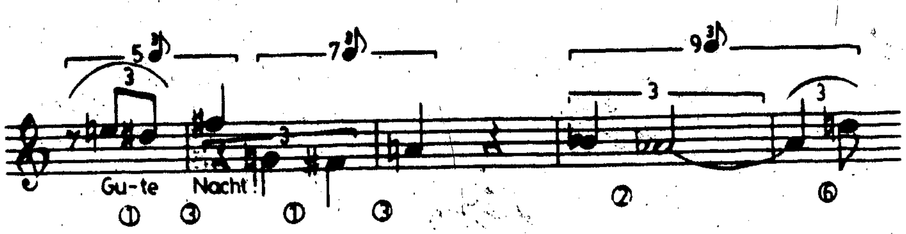
Example II-13: Pentachords of the Set-Types 5-Z38 and 5-Z18, mm. 4-7



From the perspective of sets of pitch-classes then, the first sentence may be viewed as a series of inversionally, or, in the case of the last two pentachords, multiplicatively-inversionally related pairs of tetrachords and pentachords, nested palindromically around a newel-point located between the third and fourth eighth notes in m. 5.

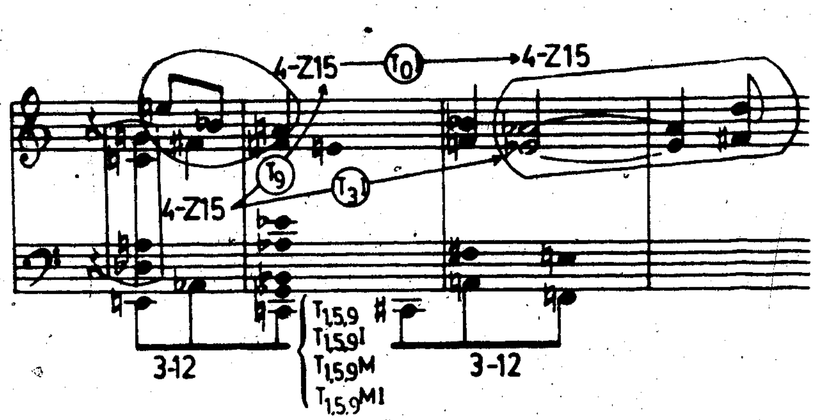
The piano and voice correspond more closely in the next five measures than in the initial sentence, where the two vocal phrases were in metrical contrast to the three piano gestures. Here, in mm. 7-12, both instruments articulate the subdivision of the second sentence in the middle of the seventh measure. The uppermost voice of the piano part in mm. 8-11 derives its melodic contour from the impetus of the vocally presented "Gute Nacht" motive in mm. 7 and 8, with each successive restatement beginning a minor third higher than the last. In addition, each is rhythmically augmented, while the trailing echo of the motive in mm. 10 and 11 transforms it by a doubling of the melodic intervals.

Example II-14: Motivic Transformations, mm. 7-11



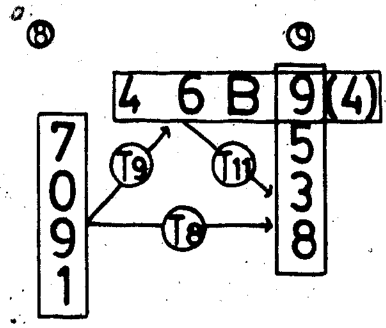
The two piano restatements of the motive that articulate the phrase are framed, like the two phrases of the initial sentence, by inversionally related members of the same set-type. More specifically, m. 8 contains two transpositionally related tetrachords of the set-type 4-Z15 which map onto the final two dyads in the right hand of the piano in mm. 10 and 11 under the operators T_3I and T_0I respectively. Moreover, each piano restatement of the motive is supported in the bass by a trichord of the type 3-12, which, because of the internal symmetry of the set-class to which they belong, are related under one quarter of the 48 possible operators.

Example II-15: Sets and Mappings, mm. 8-11



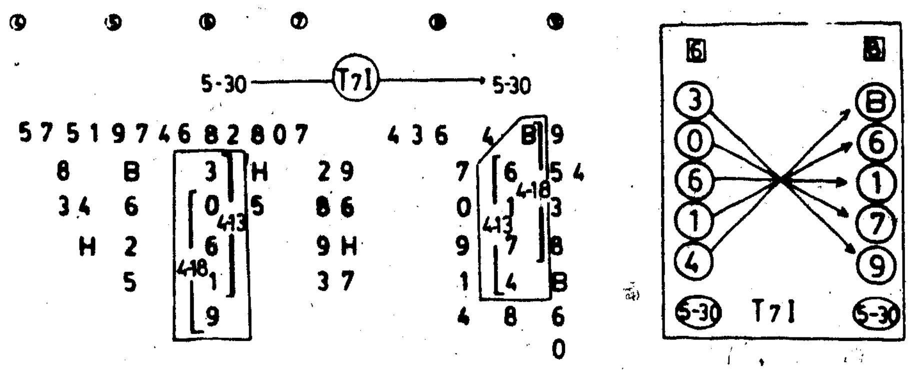
While the two phrases of the first sentence draw meaning almost exclusively from their relationship to each other, suggesting that the harmonic logic flows continuously through the sentence, here in mm. 8-11, each of the two phrases possesses an internal coherence, in spite of the implications of the framing 4-Z15 tetrachords and the continuity of the process of motivic transformation. Specifically, the first phrase of the second sentence (mm. 8, 9) begins, ends, and is spanned by members of the set-type 4-Z15. While the members of this set class in the first sentence, and the two that frame the second sentence, are inversionally related, the three 4-Z15 tetrachords in mm. 8 and 9 are related under transposition.

Example II-16: Tetrachords of the Set-Type 4-Z15, mm. 8, 9



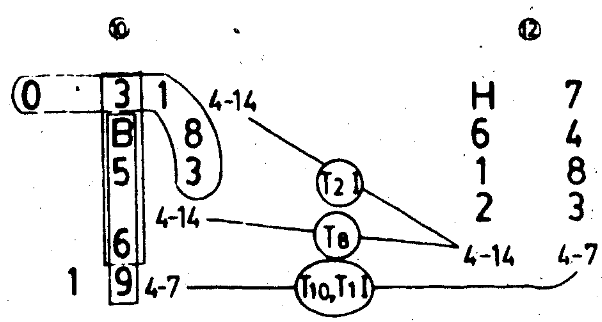
The two vertical tetrachords of this set-type are separated by tetrachords of the set-types 4-18 and 4-13, whose union forms a pentachord of the type 5-31. Chords of these set types play a similar role in m. 5 as the harmonic newel of the first sentence around which the inversionally related sets are canonically arranged. In addition to being operationally derivative under T_7I these sets in m. 8 represent a retrograded mapping of their equivalents in m. 5.

Example II-17: Sets of the Classes 4-18, 4-13 and 5-31, mm. 6, 8



The second phrase of the second sentence is likewise framed (on the first beat of m. 10 and the last eighth of m. 12) by tetrachords of the set-type 4-7. In addition, the tetrachord immediately preceding the final tetrachord in m. 12 is a member of the set-type 4-14, and is operationally related to tetrachords of m. 10.

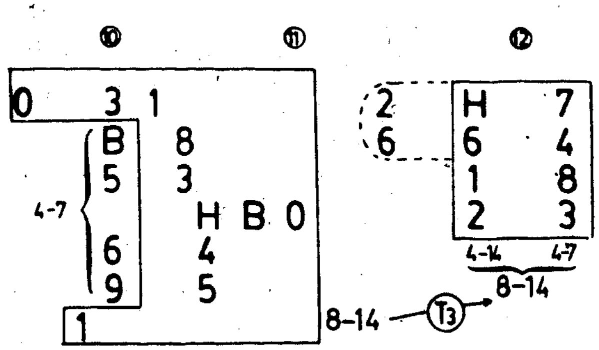
Example II-18: Tetrachords of Classes 4-7 and 4-14, mm. 10, 12



The precise operational relationships between members of the same set-type here are of more than passing importance. First, the operators T_2I and T_8, which map the two members of the set-type 4-14 onto the tetrachord of the same class in m. 12, respectively describe the relationship between the framing tetrachords of the set-type 4-11 in the first sentence (see ex. 8), and between the similarly functioning tetrachords of the 4-Z15 set-class in the first phrase of the second sentence. Second, because the set-type 4-7 is internally symmetrical, the first tetrachord of this class on the first beat of m. 10 maps onto the last tetrachord of the entire strophe in m. 12 under both T_{10} and T_1. These two operators are foreshadowed in the relationships among the quartet of pentachords of the set-type 5-21 in the first sentence (see ex. II-12).

An operational connection between mm. 10 and 12 is not restricted to sets of smaller cardinality. The union of the final 4-7 tetrachord with the 4-14 tetrachord immediately preceding it produces an octachord belonging to the latter's complementary set-class 8-14. A transpositionally related member of this octachordal set-class is formed by the collection of all pitches in m. 10, excluding the initial tetrachord of the set-type 4-7 in the piano part on the first beat of that measure.

Example II-19: Octachords of the Class 8-14, mm. 10-12.



Remarkably, the inclusional/exclusional relationships between tetrachords of the classes 4-7 and 4-14, and between tetrachords of the class 4-7 and octachords of class 8-14, 4-14's complementary set-type, are counterparalleled in the two halves of the phrase. More precisely, the tetrachords of the class 4-7 and 4-14 on the initial beat of m. 10 intersect maximally, while the 8-14 octachord in this segment of the phrase is formed by maximally excluding the 4-7 tetrachord. In m. 12, the second segment of the phrase, the tetrachord of this set-type is maximally included in the octachord of the class 8-14, while the tetrachords of the classes 4-7 and 4-14 are completely non-intersecting. Regarding each half of the phrase, m. 10 and m. 12, as universes, the complement of the 4-7 tetrachord in the first forms an octachord of the class 8-14, while in m. 12 the complement of the 4-7 tetrachord is a tetrachord of the class 4-14, 8-14's complementary set-type.

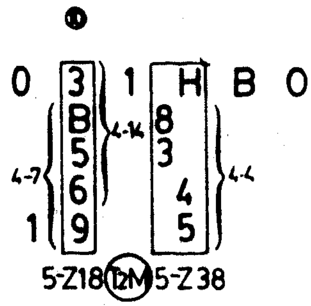
Example II-20: <u>Complementation and Inclusion, mm. 10, 12</u>

4-7 maximally includes 4-14	4-7 maximally excludes 4-14
4-7 maximally excludes 8-14	8-14 maximally includes 4-7
($\overline{4\text{-}7}$ = 8-14)	($\overline{4\text{-}7}$ = 4-14)

Finally, the union of the tetrachords of the classes 4-7 and 4-14 in m. 10 forms a pentachord of the set-type 5-Z18, which maps under the operator T_2M onto the union of the piano tetrachord with the vocal line on the second beat of the same measure.

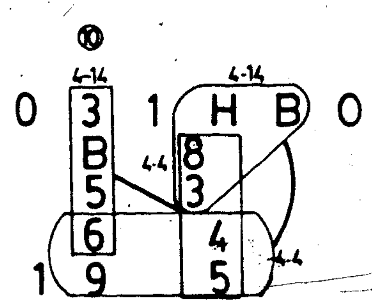
Notably, this same operator describes the relationships not only

Example II-21: 5-Z18 and 5-Z38 Pentachords, m. 10



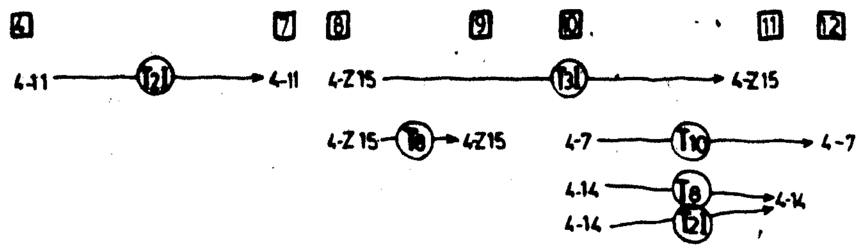
between the 4-14 and 4-4 subsets of these two pentachords, but also between two other sets of these same types. That is, the 4-4 tetrachord formed by the left hand of the piano in m. 10 maps onto the 4-14 tetrachord formed by the union of the dyad in the right hand of the piano with the final two pitches of the vocal line on the second beat of m. 10.

Example II-22: T_2M-Related Tetrachords, m. 10



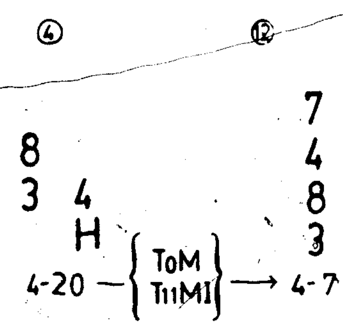
Taking stock of the data presented thus far, it is clear that each of the sections in the first strophe exhibits a degree of internal sequential sense and relational harmonic coherence. Formal musical space is marked out through a framing of each sentence, and of each of the internal phrases in the second sentence by operationally related pitch-class sets.

Example II-23: Framing Tetrachords, First Strophe, mm. 4-12



Logical relationships among sets and a logical succession of set-types are not confined to smaller formal divisions, and indeed the coherence among these formal units is similar to that within them. In particular, the entire first strophe is framed by piano tetrachords in mm. 4 and 12 belonging respectively to the multiplicatively related classes 4-12 and 4-7.

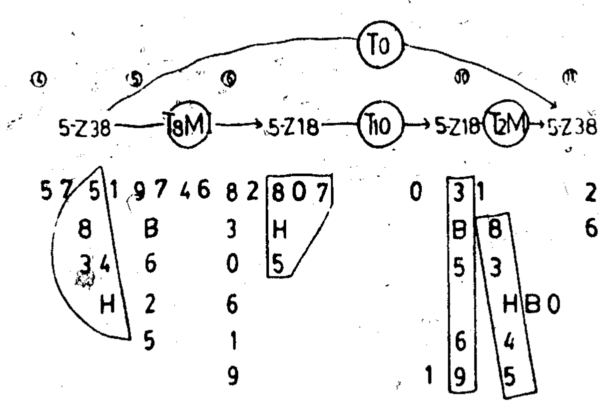
Example II-24: Multiplicatively Related Tetrachords, mm. 4, 12



Dispersed throughout the first strophe are multiplicatively related members of the set-classes 5-Z18 and 5-Z38 (see exx. II-13 and II-21).

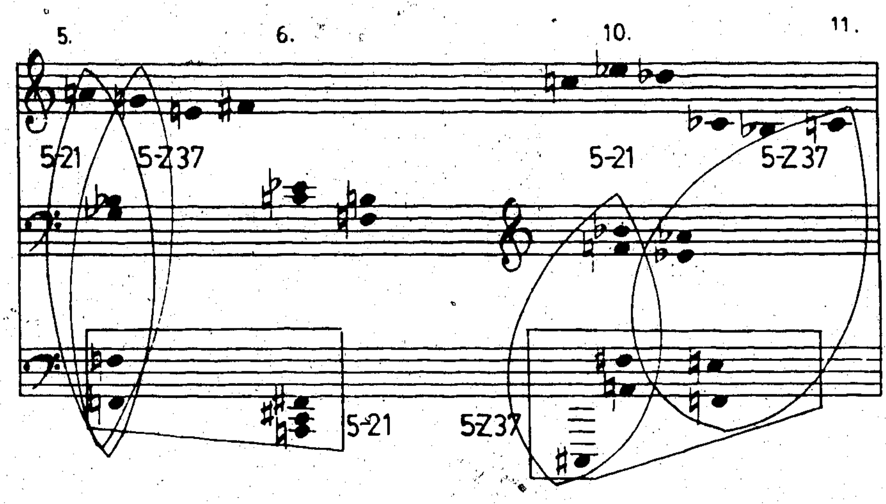
In addition to the meaning these pitch-class sets have for the smaller formal units in which they are found, the two pairs of pentachords exist in a special relationship with each other. Specifically, the member of the 5-Z18 set-class in m. 6 maps onto its set-class fellow on the first beat of m. 10 under the operation T_{10}. The pentachord of the set-type 5-Z38 and the beginning of the first strophe in m. 4 maps onto the pentachord of the same set-type on the second beat of m. 10 under T_0. In effect, then, these operators along with those that relate the multiplicatively related pairs in eahc sentence, may be viewed as a succession of functions performed upon the initial pentachord, resulting in a return of its original pitch-class collection.

Example II-25: 5-Z18 and 5-Z38 Pentachords, mm. 4-6, 10



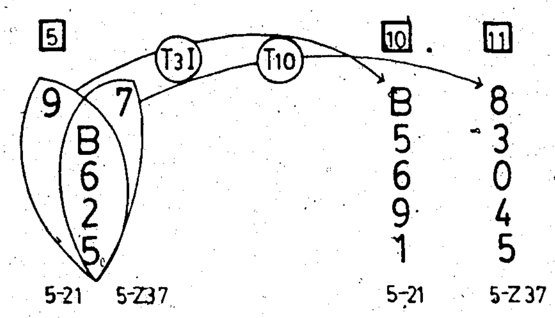
A similar operational logic is evident among two trios of pentachords of the classes 5-21 and 5-Z37 in mm. 5-6 and 10-11, and warrants some discussion. First, each place contains a vertically articulated member of the set-classes 5-21 and 5-Z37. While these pentachords are supported in the left hand of the piano in mm. 5-6 by a pentachord of the type 5-21, the underlying pentachord in mm. 10-11, segmented in precisely the same manner in the left hand of the piano, belongs to the set-class 5-Z37.

Example II-26: 5-21 and 5-Z37 Pentachords, mm. 5-6, 10-11



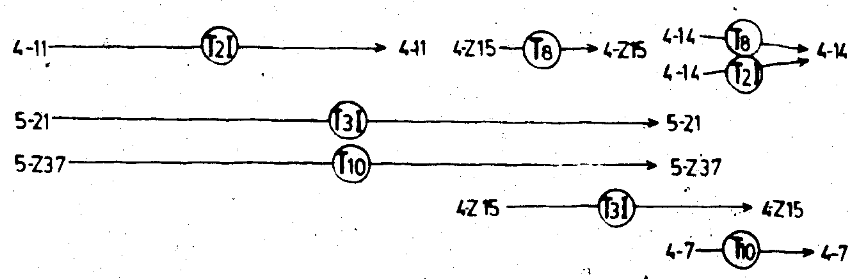
Second, the first vertically articulated member of the set-type 5-21 in m. 5 maps onto its set-class relative in m. 10 under the inversional operator T_3I, while the similarly articulated member of the set-type 5-Z37, like the 5-Z18 pentachord in ex. II-25, maps onto its fellow class-member under the transpositional operator T_{10}.

Example II-27: Pentachords and Mappings, mm. 5, 10 - 11



Both of these operators are recollective of the relationships between sectionally delineating tetrachords in the first strophe. The operators defining the relationship between the two framing tetrachords of the class 4-11 in the first sentence, and between the similarly functioning tetrachords of the set-class 4-Z15 in the first phrase of the second sentence, are the same operators that map the two members of the set-type 4-14 at the beginning of the second phrase onto the penultimate tetrachord of the sentence in m. 12. Similarly, the two operators that map the pentachords of the classes 5-21 and 5-Z37 in m. 5 onto their set-type relatives at the strophe's conclusion (see ex. II-27) also define the relationships between 1) the two 4-Z15 tetrachords that frame the entire second sentence; and 2) the two 4-7 tetrachords that, along with the tetrachords of the class 4-14, frame the second phrase of the second sentence.

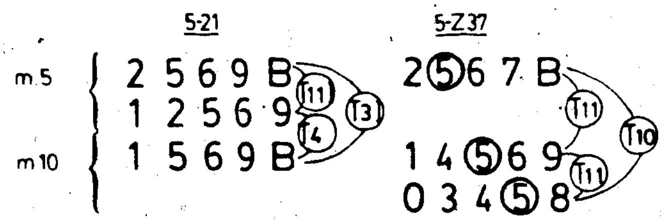
Example II-28: <u>Parallel Operators, First Strophe</u>



Finally, there is a degree of order in the level of pitch-class invariance among members of both pentachordal set-types 5-21 and 5-Z37 as a result of their operational relationships. The pentachord of the set-type 5-21 in m. 10 shares in the relationship of maximal intersection that exists between the two members of this set-type in m. 5. All three of these 5-21 pentachords contain the trichordal subset

(5, 6, 9) plus two elements of the trichord (1, 2, B). In contrast, the trio of 5-Z37 pentachords, with each successive member of the set-type derived from its predecessor by the function T_{11} , holds only pitch-class 5 invariant.

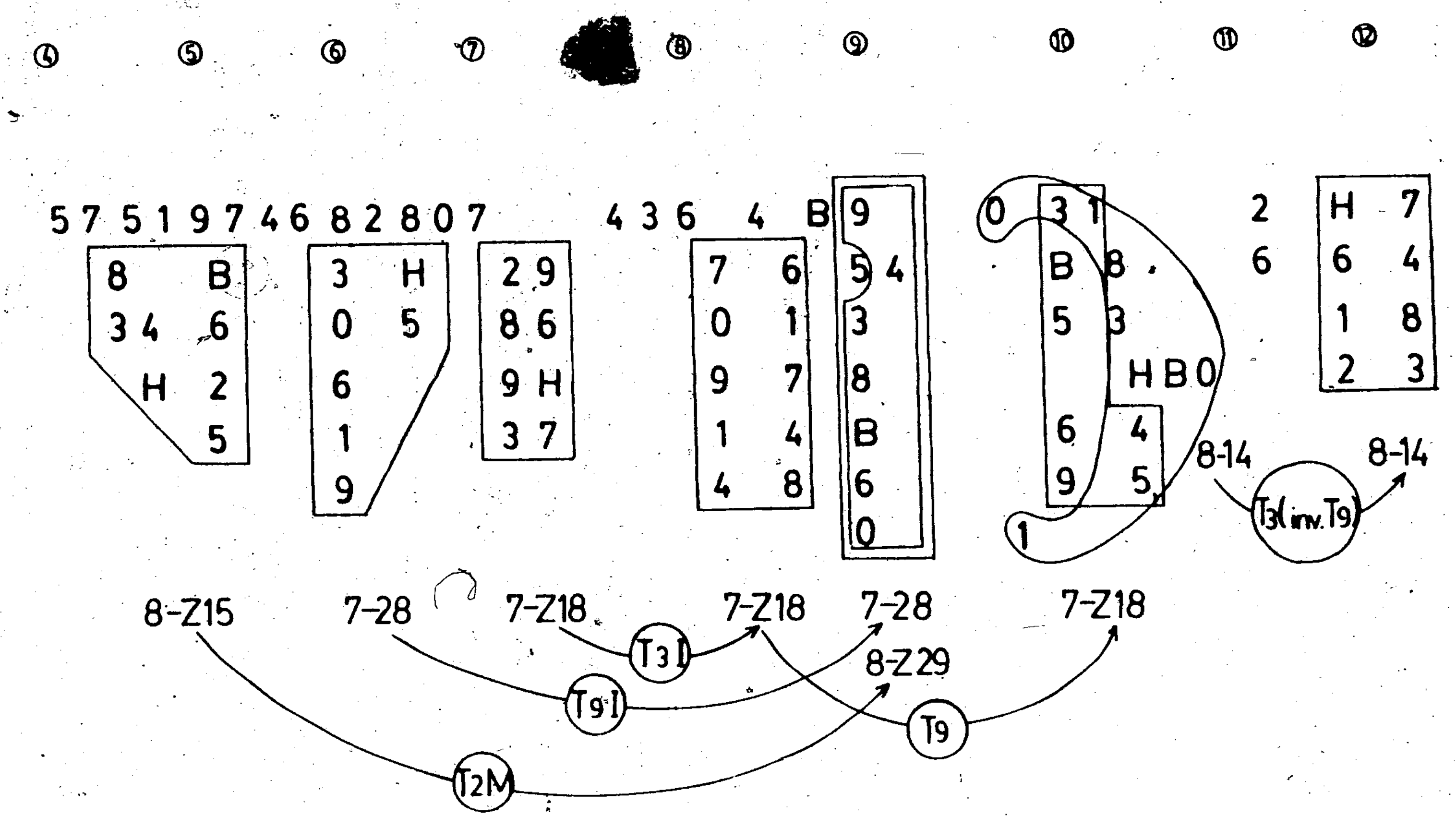
Example II-29: Invariance Among 5-21 and 5-Z37 Set-type Members



Operational connections may also be drawn in mm. 4-12 among segmentations larger than pentachords and tetrachords. Except for the two members of the septachordal set-class 7-28, the complementary set-types of these septachords and octachords have proven to be important within the smaller formal divisions throughout the first strophe (see ex. II-30). The succession of set-types and operators shown in ex. II-30 deserves some additional comment. First, the arrangement of the first six segmentations (8-Z15, 7-28, 7-Z18, 7-Z18, 7-28, 8-Z29) corresponds quite clearly to the model of palindromically nested and inversionally related sets of pitch-classes presented by the first sentence. Here, the conclusion of the first sentence and the beginning of the second serve as the axis around which the sets are anticlinally arranged.

Second, each phrase of the second sentence begins with a member of the septachordal set-class 7-Z18. More important however, T_9, the

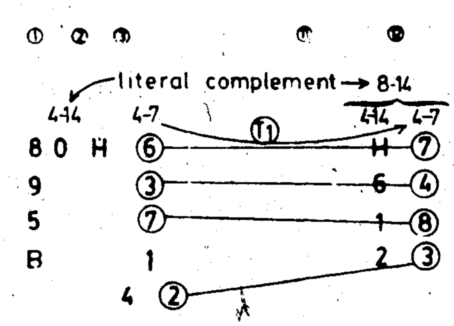
Example II-30: Large Sets, mm.4-12



operator that maps the septachord of this class at the beginning of the first phrase of the second sentence onto its set-class fellow at the beginning of the second phrase in m. 10, is inversionally related to T_3, the operator that maps the octachord of the class 8-14 in mm. 10-11 onto its relative in m. 12. In other words, while the succession of large set-types and the pitch-class membership of these sets in the first sentence are inverted in the first phrases of the second sentence, reflecting a similar processing of elements between two phrases of the first sentence, it is the mapping operation itself which is inverted between the two phrases of the second sentence.

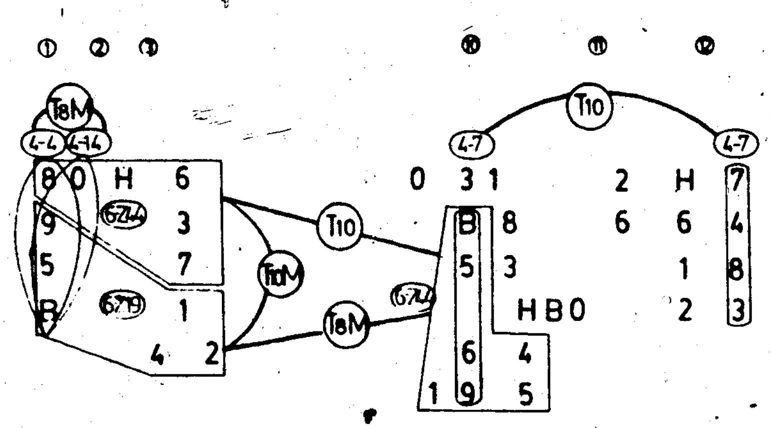
Though the final 8-14 octachord in mm. 11 and 12 refers operationally back to the preceding measure, certain characteristics of its pitch-class content exhibit a clear connection with the three introductory measures. The complement of the octachord in the final measures of the first strophe (0, 5, 9, B), forms a tetrachord of the class 4-14 which is identical in pitch-class membership to the second vertically articulated tetrachord in the first measure. Moreover, the final tetrachord of the first strophe (m. 12) is an order-preserving T_1 transposition of the introduction's final vertical sonority (m. 3).

Example II-31: <u>4-14 and 4-7 Tetrachords; Relationships, mm. 1-3, 11-12</u>



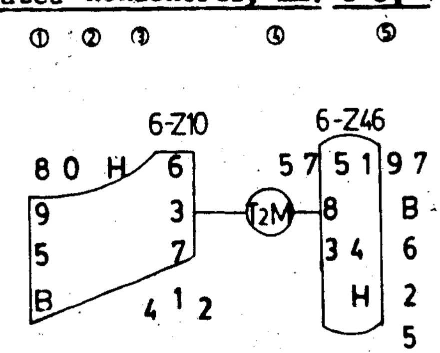
Of course, the second phrase also refers to the introductory measures by the many sets belonging to the multiplicatively related set-classes 4-14 and 4-4 (see exx. II-4 and II-23). This allusion is strengthened by the presence in m. 10 of a hexachord of the set-type 6-Z44 (formed by the union of the 5-21 and 5-Z37 pentachords), which stands in an operational relationship with the hexachords of classes 6-Z44 and 6-Z19 resulting from a segmentation of the opening measures according to register (see ex. II-3). The hexachord of the class 6-Z19 in mm. 1-3 maps onto the 6-Z44 hexachord in m. 10 under the multiplicative operator T_8M, the same function which relates the initial two vertical sonorities - tetrachords of the classes 4-4 and 4-14 - in m. 1. The literal complement of this 6-Z19 hexachord maps onto the 6-Z44 hexachord maps onto the 6-Z44 hexachord in m. 10 under T_{10}, the transpositional operator relating, among others, tetrachords of the class 4-7 that frame the second phrase of the second sentence. In effect, the very relationships which define internal operational cohesiveness in the introduction and in the second phrase of the second sentence also establish a connection between these formal units.

Example II-32: <u>Hexachords, Tetrachords and Operators, mm. 1-3, 10-12</u>



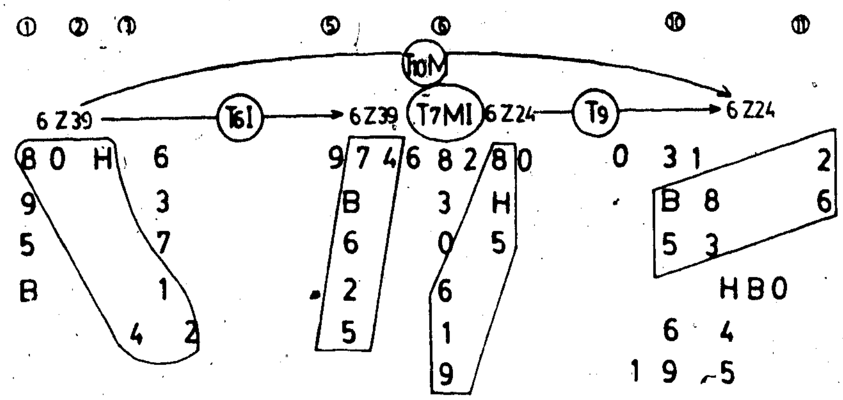
The hexachords in the introduction formed by segmentation according to gestural similarity are also operationally related to pitch-class sets in mm. 4-12. These hexachords, members of the classes 6-Z10 and 6-Z39, stand only in a complementary relationship, in contrast with hexachords of the classes 6-Z19 and 6-Z44 which are, in addition, operationally related under multiplication. Hexachords of the classes 6-Z10 and 6-Z39 are, however, multiplicatively related to hexachords of the classes 6-Z46 and 6-Z24 respectively. These latter set-types are, in turn, complementary and a complex of four hexachordal set-types is therefore formed, whose membership is defined by either a complementary relationship (and thus, as demonstrated by the hexachord theorem, similarity of interval content), or an operational relationship. From the point of view of operators, the hexachord of the class 6-Z10 in mm. 1-3 is related by T_2M to a hexachord of the set-type 6-Z46 in m. 4. This operator is the same as that which maps the octachord of the set-type 8-Z15 in mm. 4-5 onto the 8-Z29 octachord in m. 9 (see ex. II-30), and, in addition, maps the tetrachords of the class 4-14 and the pentachord of the class 5-Z18 in the first half of m. 10 onto, respectively, the tetrachords of the class 4-4 and the pentachord of the class 5-Z38 in the second half of that measure (see exx. II-21, II-22).

Example II-33: <u>T_2M-Related Hexachords, mm. 1-3, 4</u>



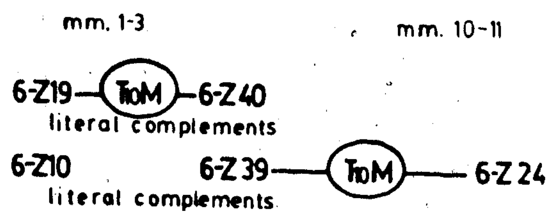
The residual hexachord in the introduction, a member of 6-Z10's complementary set-class 6-Z39, enjoys both inversional and multiplicative relations with hexachords in mm. 5, 6, and 10-11.

Example II-34: Operationally Related Hexachords, mm. 1-3, 5, 6, 10-11



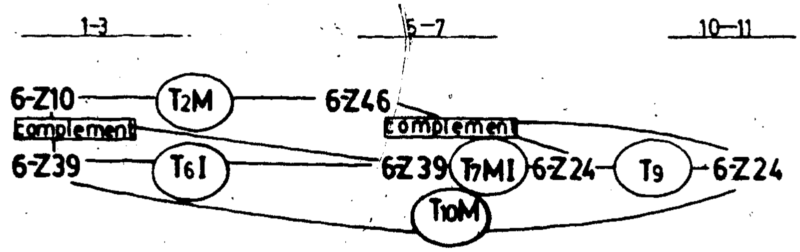
The multiplicative operations here merit some additional comment. First, the hexachord of the class 6-Z39 in m. 5 maps onto the adjacent hexachord in m. 6, a member of the class 6-Z24, under the operator T_7MI, the same operator that maps the two tetrachords of the class 4-Z29 (mm. 1-3, and m. 5) onto the tetrachords of the class 4-Z15 in mm. 3 and 6 respectively (see exx. II-5 and II-11). More important here, however, is that $T_{10}M$, the operator that maps the hexachord of the class 6-Z39 in the introduction onto the 6-Z24 hexachord formed by the right hand of the piano in mm. 10 and 11 - or, put another way, the composite operator of the four hexachords in ex. II-34 - is the same function that maps the hexachord of the class 6-Z19 in the introduction onto its literal complement (see ex. II-32). That is, $T_{10}M$ is the operational agent whereby the hexachords of the set-types 6-Z19 and 6-Z40 in the introduction, due to their vector <u>and</u> operational relatedness, realize their potential for literal complementation; and although the literally complementary hexachords of the classes 6–Z10 and 6–Z39 in the introduction do not possess the same ability to map onto each other, the operator $T_{10}M$ nevertheless describes the accumulation or composite of operators relating sets that are operationally related to 6–Z39 throughout the first strophe.

Example II-35: Literal Complements and Operators, mm. 1-3, 10, 11



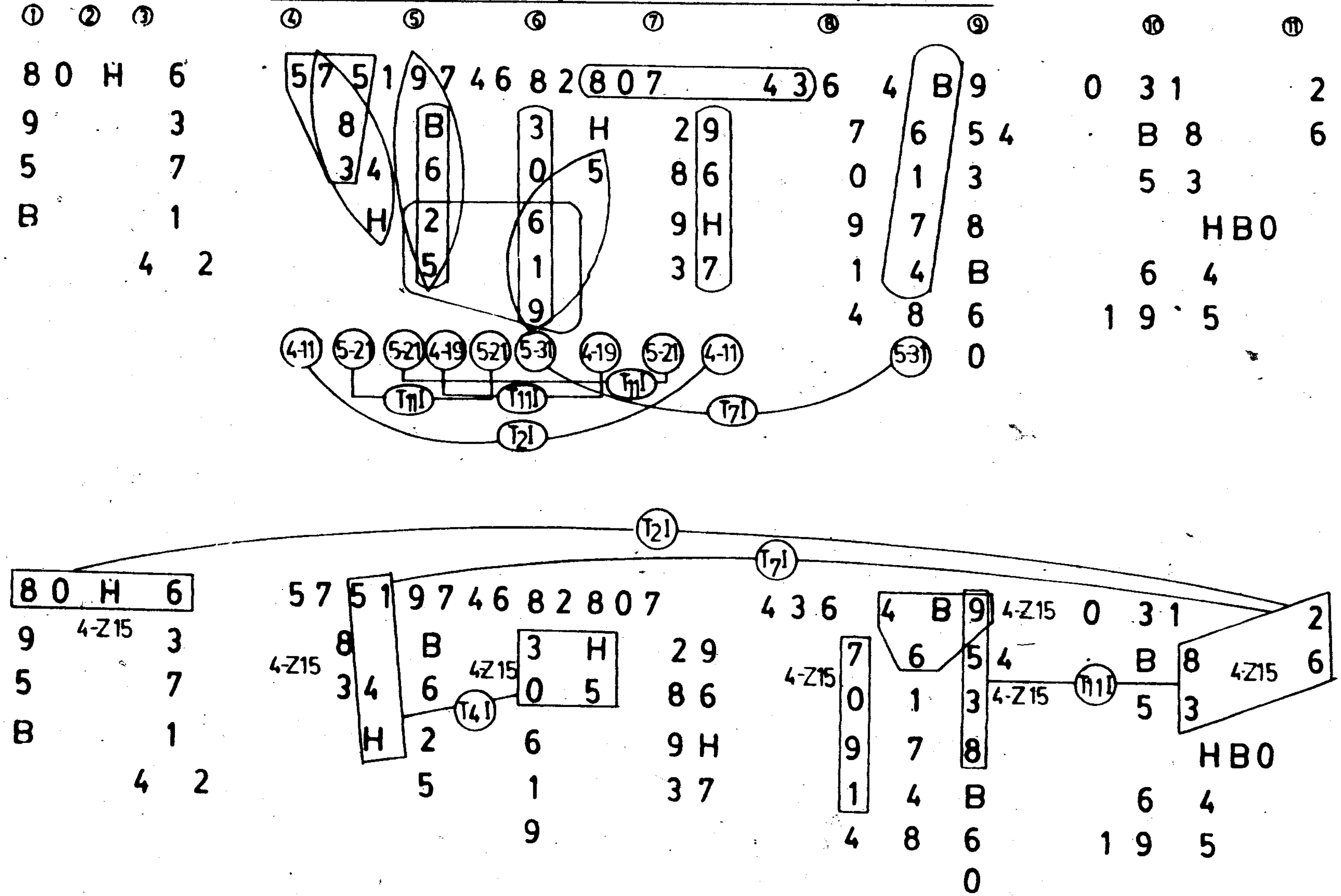
In any case, the operational and complementary relationships among members of the small complex of set-types 6–Z10, 6–Z39, 6–Z24 and 6–Z39 is sketched in ex. II-36.

Example II-36: Complementary and Operationally Related Hexachords, mm. 1-3, 5-7, 10-11



A few final observations about the harmonic logic in the introduction and the first strophe concern the relationships among members of the set-class 4-Z15 throughout the initial twelve measures (see exx. II-5, II-11, II-15 and II-16). First, the initial member of

Example II-37: Relationships Among 4-Z15 Tetrachords, mm.1-12



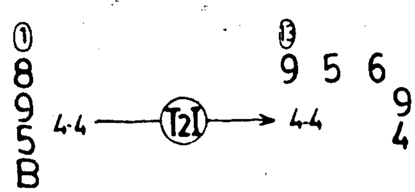
this set-type in the introduction maps onto the final tetrachord in mm. 10–11 under the inversional operator T_2I, the same operator that relates the similarly framing tetrachords of the class 4–11 in the first sentence of the strophe. In addition, $T_{11}I$, the operator that relates not only the interior pair of canonically nested pitch-class sets of the type 4–19, but also both pairs of like-arranged pentachords of the class 5–21 in mm. 4–7, relates the concluding 4–Z15 tetrachord of the second sentence's first phrase (m. 9) onto the concluding 4–Z15 tetrachord of the entire sentence in mm. 10–11. Finally, the inversional operator T_7I, which maps the pentachord of the class 5–31, the focus of the palindromic arrangement of inversionally related tetrachords of the first sentence, onto the 5–31 pentachord in m. 8 (which, in turn serves as a mediant between the framing tetrachords of the first phrase of the second sentence), also describes the relationship between the first strophe's framing tetrachords in mm. 5 and 10–11.

The Second Strophe (mm. 13 - 23)

The form of the folksong lyric suggests that the music of the second strophe should in some way parallel that of the first. What is of primary interest is precisely how this might be expressed in the disposition of pitch-class sets and the operational relationships among them.

The beginning measures of the second strophe (mm. 13–16) comprise, in a reversal of the corresponding sentence of the first strophe, three vocal gestures counterpointed in the piano by two gestures, or, more precisely, two gestural complexes. Unlike the texturally dense music presented by the piano thus far, the weave of the piano's music is loosened as the sentence progresses, and indeed it completely unravels towards the end of the sentence in m. 16. In addition to the textural differentiation between the corresponding sentences in the two strophes, the tetrachordal set-type 4-11 as an initiating harmony is replaced in m. 13 by the set-class 4-4. Notably, this tetrachord in m. 13 is related to the initial vertical sonority under T_2I, the same operator which describes the relationship between the two framing tetrachords of the set-type 4-11 in the first sentence of the first strophe, as well as the entire strophe's framing tetrachords of the set-class 4-Z15 in mm. 1 and 11 (see ex. II-37).

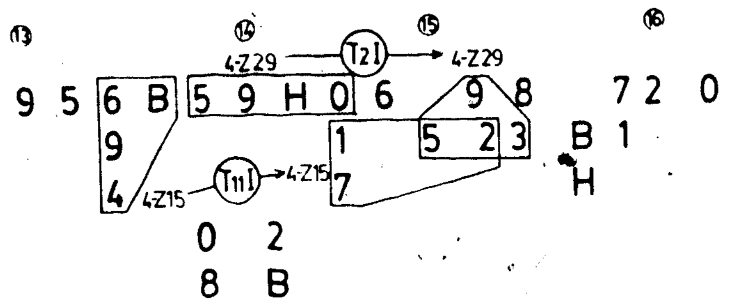
Example II-38: <u>4-4 Tetrachords, mm. 1 and 13</u>



Notwithstanding the clear textural differences and dissimilar initial harmonies, the first sentences of the two strophes may be shown to possess the same set-theoretical components and processes. As in the first strophe, the first sentence again divides into two halves, with pitch-class sets in the first half standing in an inversional operational relationship with pitch-class sets in the second. A member of each of the multiplicatively related set-classes 4-Z15 and 4-Z29 is present in the first half of the sentence and maps onto an operational relative in the second half under the inversional operators $T_{11}I$ and T_2I

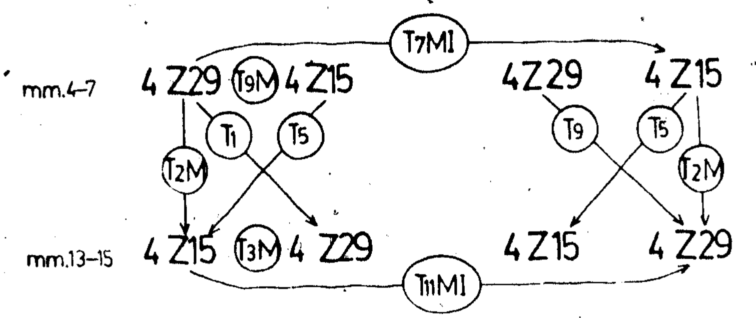
respectively. Remarkably, these are the functions which relate palindromically nested pairs of tetrachords of the classes 4-19 and 4-11 in the first sentence of the first strophe (see exx. II-8 and II-10).

Example II-39: <u>4-Z15 and 4-Z29 Tetrachords, mm. 13-15</u>



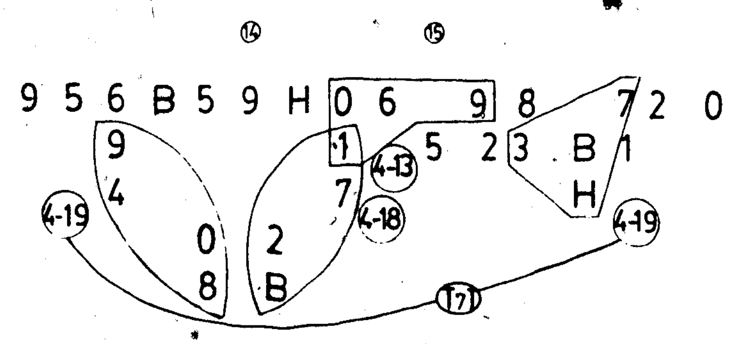
While the initial tetrachord of this group of pitch-class sets in mm. 4-7, a member of the set-class 4-Z29, maps onto the tetrachord of its multiplicatively related set-type 4-Z15 in the first phrase of the sentence under T_9M, and onto the tetrachord of this set-class in the second phrase under T_7M_I, both the succession of set-types and mapping operations are literally reversed in the corresponding measures of the second strophe. That is, the initial pitch-class set of the quartet in mm. 13-16 is of the set-type 4-Z15 and maps onto its multiplicative relative in the first half of the sentence under T_3M (the inverse operator of T_9M), and onto the 4-Z29 tetrachord in the second half under $T_{11}MI$ (the inverse operator of T_7MI). Finally, the tetrachords of the classes 4-Z15 and 4-Z29 in the first sentence of the first strophe are transpositionally related to their set-type counterparts in mm. 13-15, while both the initial 4-Z29 tetrachord in m. 4 and the concluding tetrachord of the set-type 4-Z15 in m. 6, map onto their correspondent multiplicative relatives in mm. 13 and 15 under T_2M. It is this operator which relates the two large segmentations in mm. 4-5 and 9 belonging to the octachordal set-classes which are complementary to the set-types 4-Z15 and 4-Z29 (see ex. II-30).

Example II-40: <u>4-Z15 and 4-Z29 Tetrachords, mm. 4-7, 13-15</u>



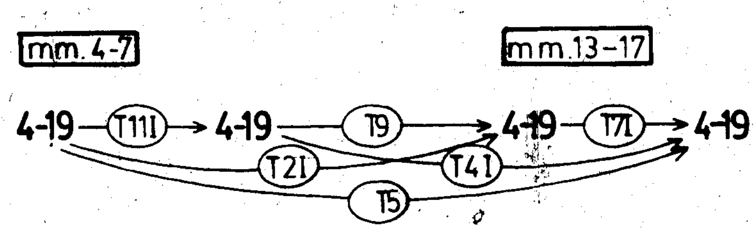
In addition to tetrachords of the classes 4-Z29 and 4-Z15, the first sentences of each strophe hold in common inversionally related tetrachords of the class 4-19 separated by tetrachords of the classes 4-13 and 4-18 (see exx. II-10 and II-17).

Example II-41: <u>4-19, 4-13 and 4-18 Tetrachords, mm. 14-15</u>



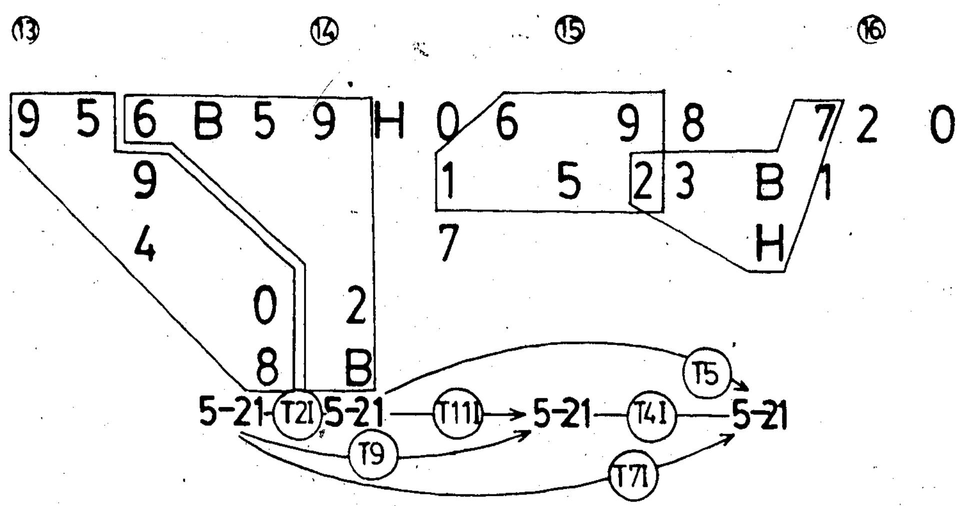
Considered as a group, the four members of the 4-19 set-class in the first sentence of each strophe not only exhibit the same degree of relational order that exists among the tetrachords of the set-types 4-Z29 and 4-Z15 in ex. II-40, but are in fact related under precisely the same operators. The two 4-19 tetrachords in the first strophe map onto their correspondent equivalents in the second strophe under the operators T_2I and T_4I respectively (see ex. II-39), and onto the reciprocal members in the second sentence under the transpositional operators T_5 and T_9, which do map the first 4-Z15 tetrachord and the second 4-Z29 tetrachord in the first sentence of the first strophe onto similarly placed tetrachords in the second strophe (see ex. II-40). Further, the four inversional operators of this scheme ($T_{11}I$, T_2I, T_4I and T_7I), are those which describe relationships among tetrachords of the set-type 4-Z15 throughout the first twelve measures (cf. exx. II-37 and II-42).

Example II-42: Relationships Among 4-19 Tetrachords, mm. 4-7, 13-16



The first sentence of the second strophe, like mm. 4-7, contains four members of the set-type 5-21. More important, the relationships among these four pitch-class sets are precisely the same as those among the four members of the set-class 4-19 in the first sentence of each strophe (cf. exx. II-42 and II-43).

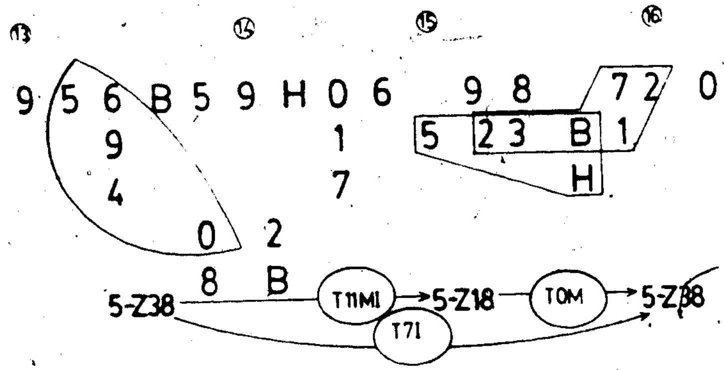
Example II-43: Pentachords and Operators, mm. 13-16



The inversional operator T_7I, in addition to relating the outer pair of the four pentachords of the set-type 5-21 and the two members of the tetrachordal set-class 4-19, maps a pentachord of the set-type 5-Z38 in m. 13 onto its relative in m. 15. Between these inversional relatives is a pentachord belonging to the multiplicatively related set-type 5-Z18. This pitch-class set is derived from the first 5-Z38 under the operator $T_{11}MI$, which also relates the framing tetrachords of the set-types 4-Z15 and 4-Z29 in these measures, and maps onto the last 5-Z38 pentachord in m. 15 under T_0M. Remarkably, these two multiplicative operators map the first strophe's initial tetrachord in the piano in m. 4 onto the last in m. 12 (see ex. II-24).

At the end of the first stophe's first sentence (m. 7), two tetrachords of the classes 4-9 and 4-11 close the formal unit. Here in the second strophe, two members of these same set-classes repeat this function in m. 15. In addition, while the union of the two tetrachords

Example II-44: 5-Z18 and 5-Z38 Pentachords, mm. 13-16

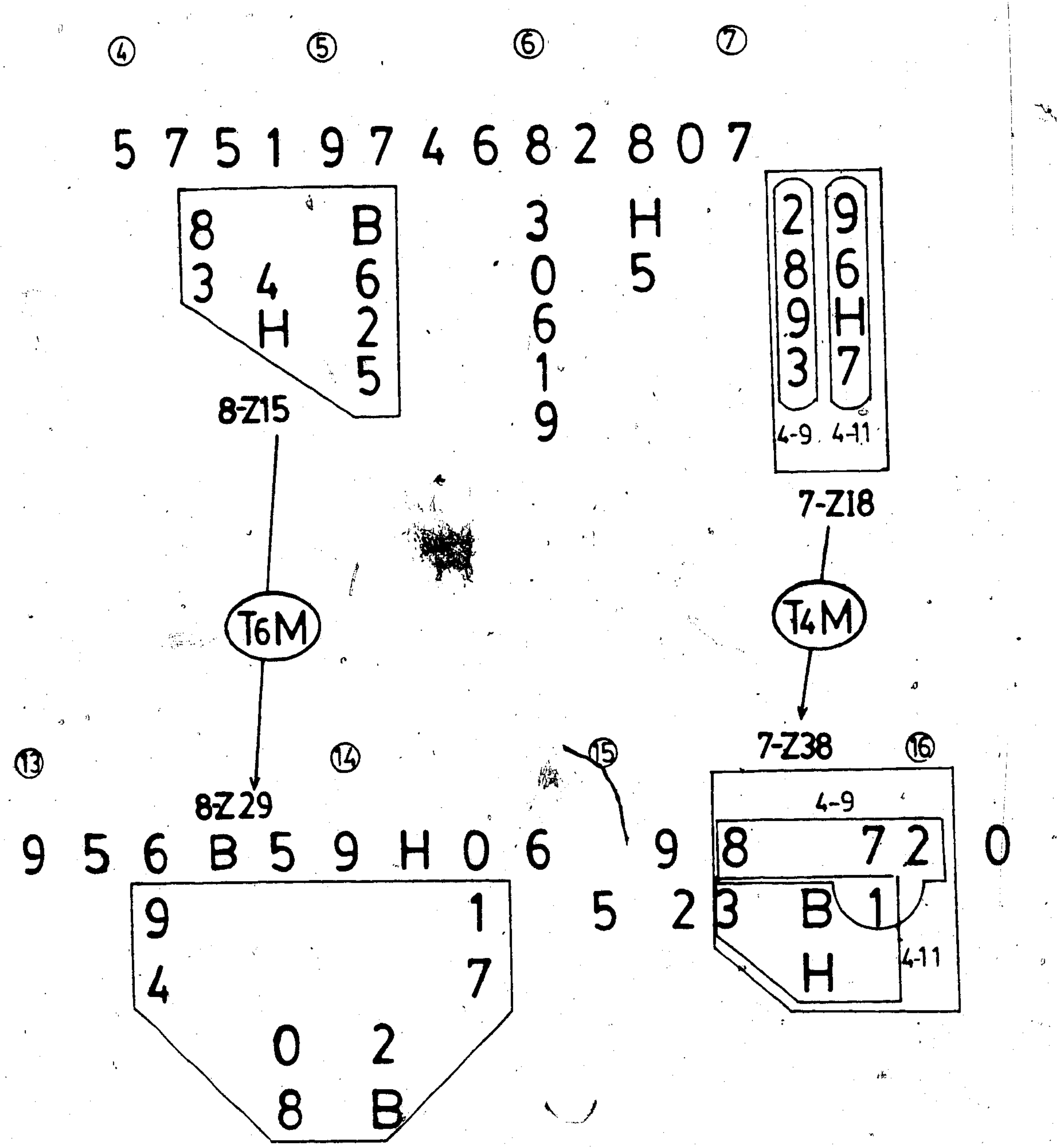


in m. 7 forms a septachord of the class 7-Z18, in m. 15 the union of the corresponding tetrachords forms a septachord of the multiplicatively related set-type 7-Z38. A multiplicative function also maps the first eight pitches played by the piano in mm. 4 and 5 onto the similarly presented pitches at the corresponding point in the first sentence of the second strophe (see ex. II-45).

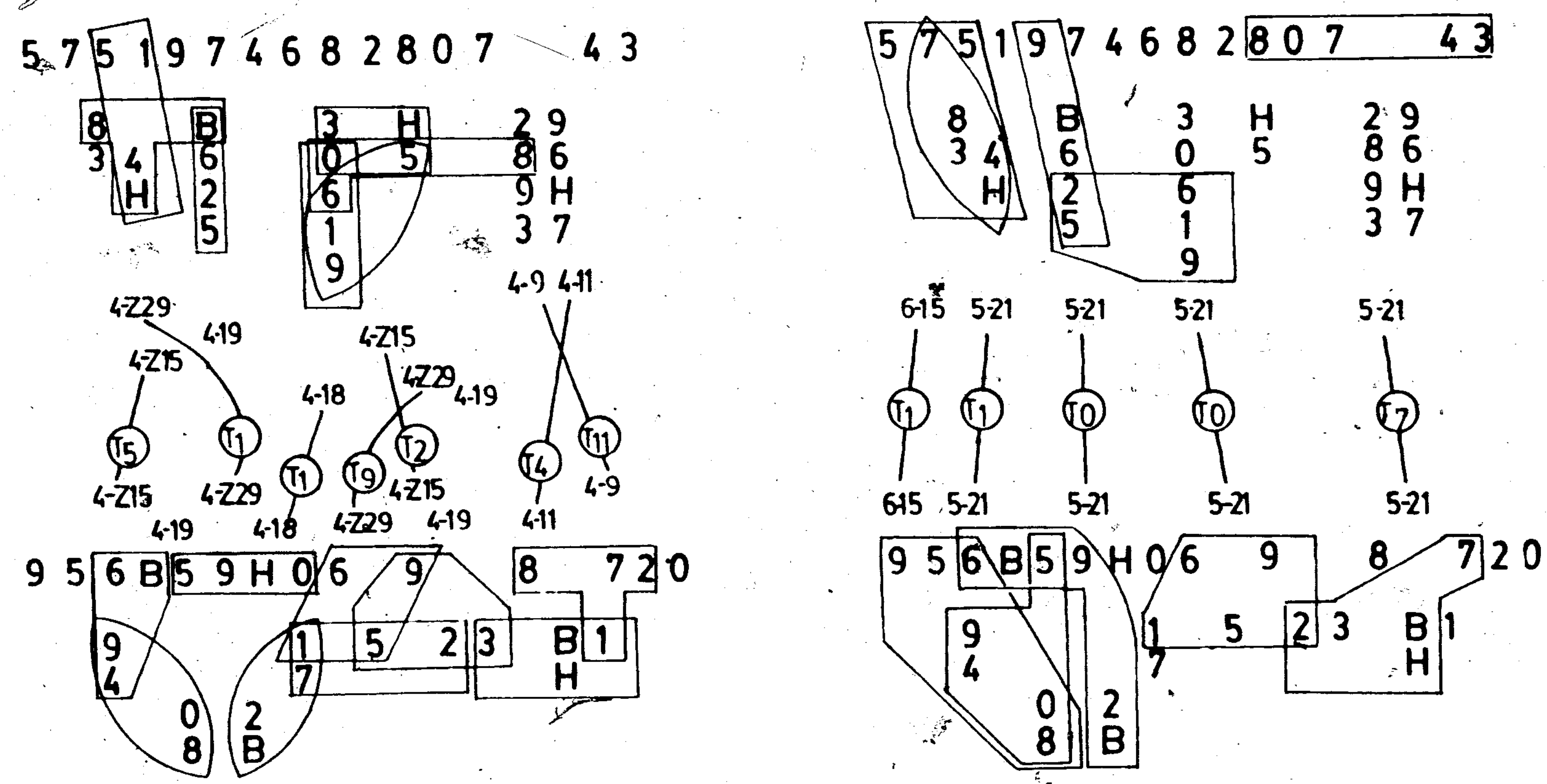
Clearly then, the first sentences of both strophes comprise a great deal of the same harmonic components. More precise analytical assertions may, however, be made. The first sentence of the second strophe maintains the succession of set-types of the first strophe's initial sentence and, equally important, these pitch-class sets are related to each other under the same class of operators that relates their corresponding relatives in mm. 4-7. Finally, the members of these set-types in the first strophe map onto their second strophe equivalents almost exclusively under transpositional operators (see ex. II-46).

The second sentence of the second strophe (mm. 16-23) comprises

Example II-45: Multiplicatively Related Sets, mm. 4-7,13-16



Example II-46: Pitch-Class Sets and Mappings, mm. 4-7,13-16



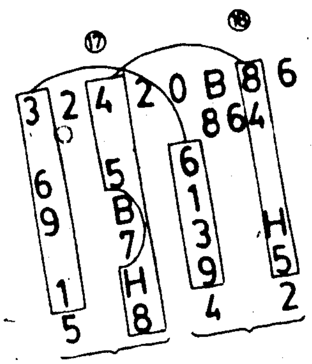
two distinct vocal phrases (mm. 16-18, 18-23) each in turn counterpointed by a pair of gestures in the piano. The first pair (mm. 16-18) is melodically derived from the same motive that generates the piano gestures in the second sentence of the first strophe (mm. 7-11). Moreover, this motive undergoes precisely the same manipulation through an augmentation of interval class and durational value that transformed it in mm. 7-11.

Example II-47: Motivic Transformation, mm. 7 - 11, 16 - 18

Each of the piano gestures in mm. 16-18 is supported by two chords. From a perspective of pitch-class content, there is a similarity between each group's first chord, which in both cases contains a member of the pitch-classes 1, 3, 6 and 9 (forming a tetrachord of the set-type 4-27), and between the second chord of each group, whose intersection set includes members of the pitch classes 4, 5, 8 and 11 (forming a tetrachord of the 4-18 set-class).

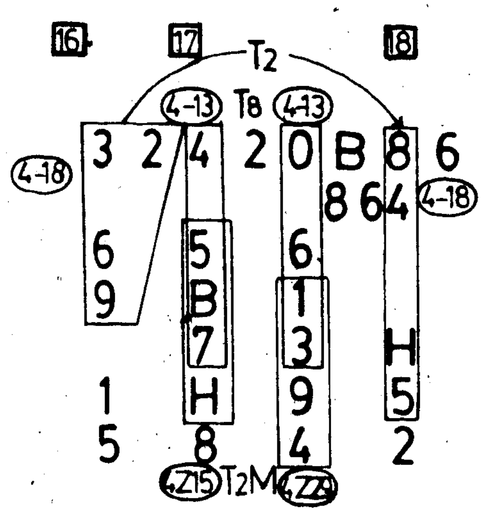
From the point of view of set-types and mappings, however, there is a connection between the exterior and interior pairs of chords, in

Example II-48: <u>Pitch-Class Intersection, mm. 16 - 18</u>



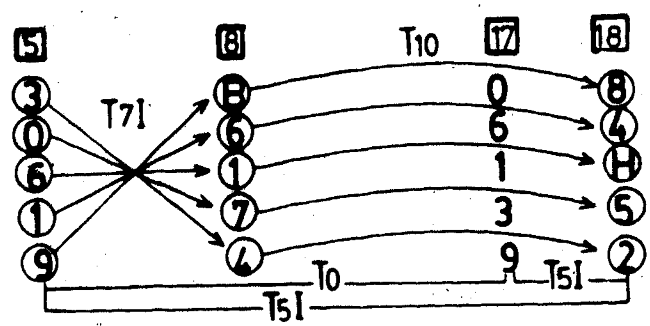
contrast to the similarity between odd and even pairs suggested by pitch-class intersection. Like the preceding formal units, and in particular the corresponding phrase of the first strophe in mm. 7-9 (see ex. II-16), the boundaries of the first phrase of the second strophe's second sentence are marked out by operationally equivalent tetrachords. Specifically, the uppermost four pitches on the last beat of m. 16 form a tetrachord of the set-type 4-18 which maps under the transpositional operator T_2 onto the uppermost four pitches of the last vertical sonority of the phrase on the first beat of m. 18. Notably, this final 4-18 tetrachord is comprised of members of pitch-classes held in common between the even chords in this phrase (see ex. 48). The two chords in m. 17 nested within the outer verticalities in mm. 16-18 are also operationally equivalent. The first of this pair possesses a tetrachord of the set-type 4-Z15 which maps onto the bottom four pitches of the following verticality under the multiplicative operator T_2M, which also defined the relationship between sets of the types 4-4 and 4-14, and of the types 5-Z18 and 5-Z38 in the second phrase of the first strophe's second sentence (see exx. II-21, II-22). In addition, both of the chords in m. 17 contain, as a subset, a member of the set-type 4-13, and these tetrachords are related under the transpositional operator T_8. This same operator relates the framing 4-Z15 tetrachords in the first phrase of the first strophe's second sentence (see ex. II-16), and the framing tetrachords of the set-type 4-14 in that sentence's second phrase (see ex. II-18).

Example II-49: Nested Pairs of Operationally Related Tetrachords, mm. 16-18



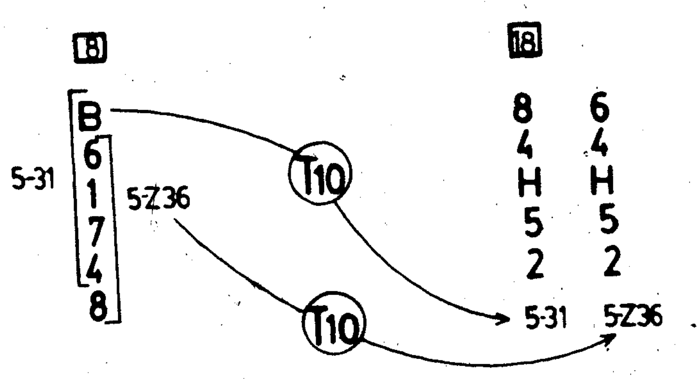
A third member of the tetrachordal set-type 4-13 occurs as the chord articulated by the piano on the first beat of m. 18 and forms in union with the voice part a pentachord of the set-type 5-31, related under T_5I to the uppermost five pitches of the chord which precedes it. More important, the pentachord in m. 18 is operationally equivalent to pentachords of the same type in mm. 5 and 8 (see ex. II-17). Remarkably, the order-sensitive mapping that takes place between the members of the 5-31 settype is extended to the pentachord in m. 18. Also, the composite operation of the three 5-31 pentachords in mm. 5, 8 and 18 is T_5I, which relates the 5-31 pentachord in m. 18 and its equivalent in m. 17.

Example II-50: 5-31 Pentachords, mm. 5, 8, 17 and 18



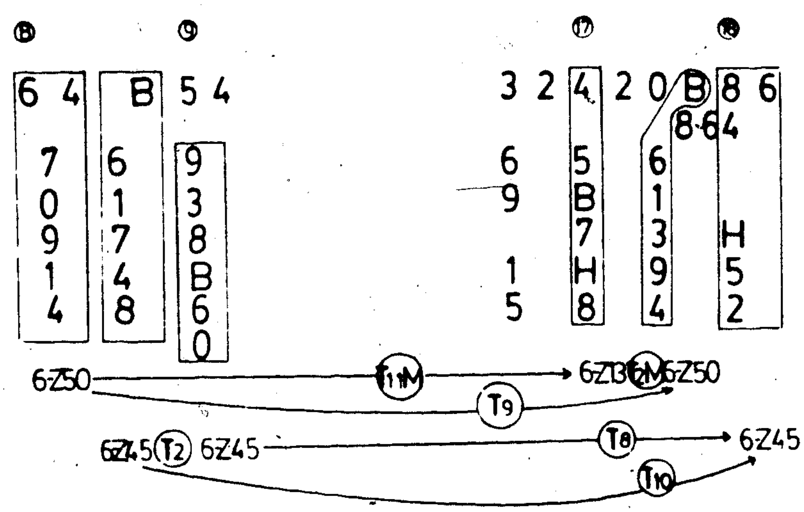
In addition, the final vertical sonority in the first phrase of the second strophe's second sentence (m. 18), a member of the set-type 5-Z36, can be derived from the final piano chord in m. 8 under the operator T_{10}, the same operator that relates the two 5-31 pentachords which maximally intersect with the pentachords of the 5-Z36 set-type (cf. ex. II-50, II-51).

Example II-51: 5-Z36 and 5-31 Pentachords, mm. 8, 18



Needless to say, the operator T_{10} also relates the hexachords of the class 6-Z45 formed by the unions of the 5-Z36 and 5-31 pentachords in mm. 8 and 18. Another hexachord of this set-type immediately follows the 6-Z45 hexachords in m. 8, and can be derived from it by the function T_2, the inverse operator of T_{10}. In addition, the hexachord immediately preceeding the 6-Z45 hexachord in m. 8 is operationally related to the two hexachords immediately preceding the last hexachord of the class 6-Z45 on the first beat of m. 18.

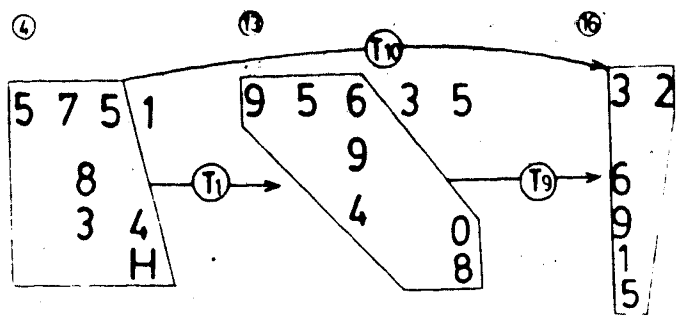
Example II-52: Operationally Related Hexachords, mm. 8-9, 17-18



Notably, like the pitch-class sets in the first sentence in strophe one, the pitch-class sets in the first phrase of the second sentence of strophe one map onto corresponding sets in the parallel section of the second strophe under either transpositional or multiplicative operators.

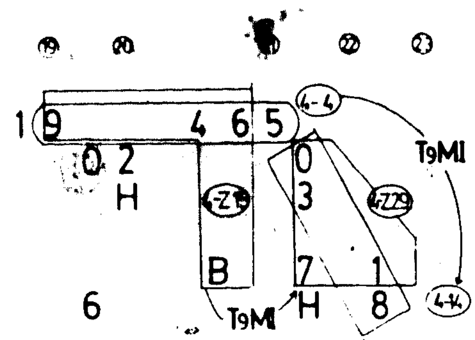
The initial hexachord of this section (in m. 16), a member of the set-class 6-15, is also operationally related to sets in the previous measures. Two members of this set-type, related to each other under T_1, occur as the initial six pitches of the first sentence of each strophe in mm. 5 and 13 (see ex. II-46). Remarkably, these two hexachords map respectively onto the first six pitches of the second strophe's second sentence under the transpositional operations T_9 and T_{10}, the composite operations of the two trios of hexachords in ex. II-52.

Example II-53: <u>6-15 Hexachords, mm. 4, 13 and 16</u>



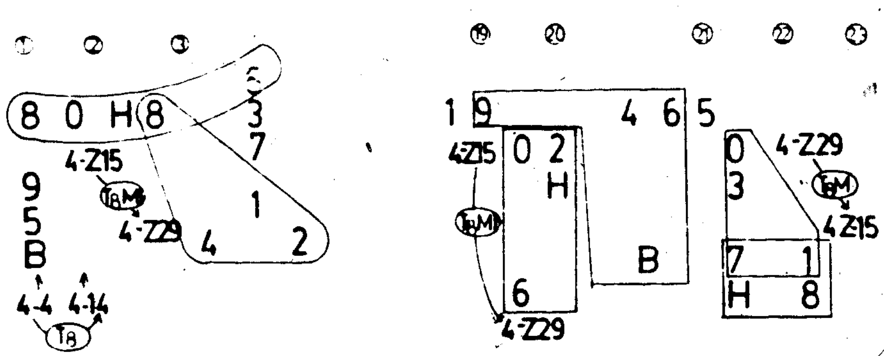
Like the first phrase of the second strophe's second sentence, the second (mm. 19-23) begins with a tetrachord belonging to the set-type 4-18. T_3I, the operation which maps the initial 4-Z15 tetrachord in the second sentence of the first strophe onto the sentence's last member of this set-type in mm. 10-11 (see ex. II-15), relates these two 4-18 set-class tetrachords as well. While the 4-18 tetrachord is operationally unrelated to tetrachords later on in this second phrase, the harmonic logic of mm. 19-20 is, nevertheless, like the preceding formal units, still demonstrably focused around a central axis, here located at the utterance of "<u>Ruh</u>" in the voice in m. 21. Both halves of the phrase contain a member of each of the multiplicatively related set-types 4-Z15 and 4-Z29. Also multiplicatively related are the tetrachords belonging to the set-classes 4-4 and 4-14 presented as the last four pitches in the voice (mm. 20-21) and the final two dyads articulated by the piano's right hand in 21-23 respectively. The latter two tetrachords are related under the operations T_9MI, the same operator which maps the 4-Z15 tetrachord in the first half of the phrase onto the tetrachord of the set-type 4-Z29 in the second.

Example II-54: T_9MI-Related Tetrachords, mm. 19-23



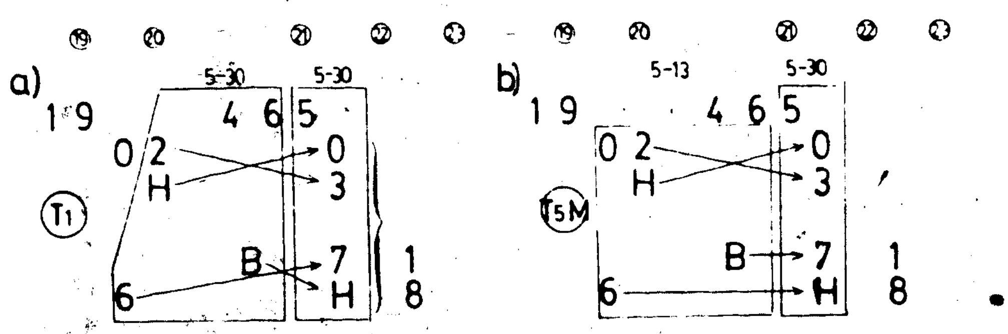
The similarity of relationships between sets of the types 4-4 and 4-14 and of the types 4-Z15 and 4-Z29 extends beyond this phrase however. The two operations which relate the initial two tetrachords of the set-types 4-4 and 4-14 and the two sets belonging to the classes 4-Z15 and 4-Z29 in the introduction (see Ex. II-4) are recollected here in mm. 19-23 in the relationships between sets of the latter two set-classes within the two halves of the phrase.

Example II-55: Tetrachords of the Classes 4-4, 4-14, 4-Z15 and 4-Z29, mm. 1-3, 19-23



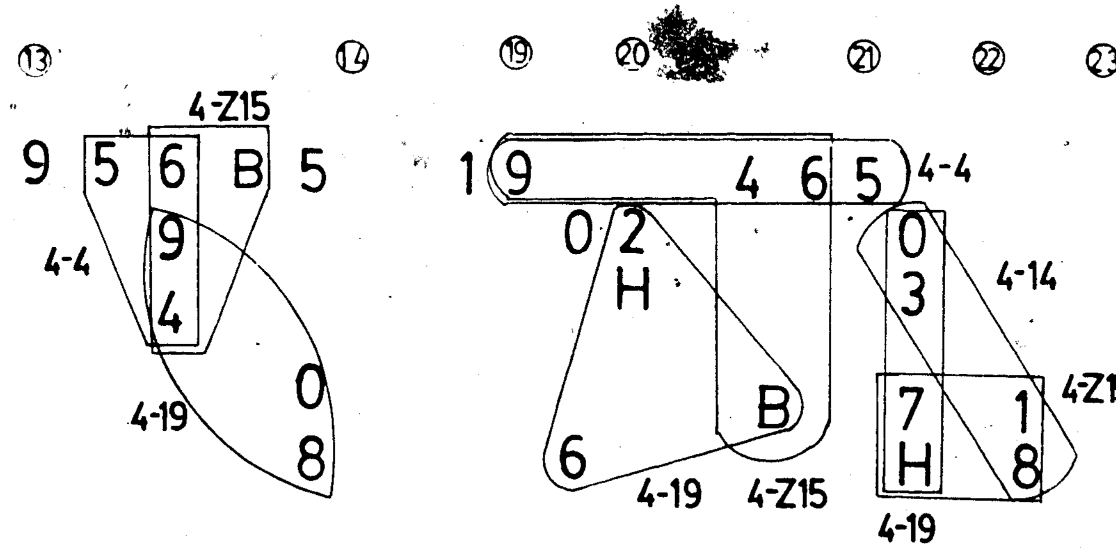
The division of the second phrase is most dramatically articulated by the mappings, under the transpositional operator T_1, of the final vertical sonority in m. 20 - a member of the set-class 5-30 - and, under the multiplicative operator T_5M, of the entire piano part in mm. 19-20, forming a pentachord of the class 5-13, onto the first vertical sonority in m. 21. Both of these operators are order-sensitive with respect to the dyadic content of the two 4-19 tetrachordal subsets of the related pentachords. Consequently, these two tetrachords (mm. 20, 21) are related under both T_1 and T_5M.

Example II-56: <u>T_1- and T_5M-Related Sets, mm. 19-23</u>



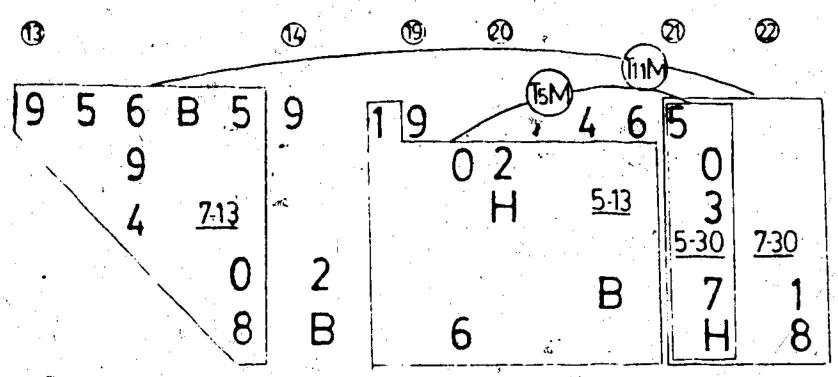
A tetrachord belonging to the 4-19 set class occurs also at the beginning of the first phrase on the third beat of m. 16 as the supporting harmony to the motivically derived melody and maps onto the two 4-19 tetrachords in mm. 19 and 20 under T_5 and T_6 respectively. Consequently, while tetrachords belonging to the set-type 4-18 enfold the first phrase of the second sentence and initiate its second phrase, 4-19 class tetrachords demarcate the entire second sentence. Sets of pitch-classes in the second phrase of the second sentence are also related to sets found at the very beginning of the second strophe. First, the initial four pitches presented by the piano in m. 14 are, as a set, related transpositionally to the T_1- related 4-19 tetrachords in mm. 19-20 and m. 21, both also articulated solely by the piano. In addition, the tetrachord formed by the union of the initial piano dyad and the first three pitches in the voice in mm. 13-14 is identical with respect to pitch-class content to the 4-4 tetrachord formed by the last four pitches presented in the voice in mm. 19-20. Naturally, then, the 4-4 tetrachord in mm. 13-14 maps onto the multiplicatively related tetrachord of the set-type 4-14 in mm. 21-23 under the same operator that relates this 4-14 tetrachord to the 4-4 tetrachord in mm. 19-20. A similar situation exists among the tetrachords of the class 4-Z15 in m. 14, m. 21 and mm. 22-23 (see exx. II-54 and II-55).

Example II-57: Related Sets, mm. 13-14, 19-23



Operational relationships between the beginning and the end of the second strophe extend as well to larger segmentations. The union of the first vocal phrase in mm. 13-14 with the initial piano tetrachord in m. 14 forms a septachord of the class 7-13, and maps under the multiplicative operator $T_{11}M$ onto the septachord of the class 7-3 formed by the final seven pitches of the piece in mm. 21-23. Remarkably, this function is the inverse of T_5M, the operator that relates the pentachords in mm. 19-21 belonging to set-types related complementarily to the original two septachordal set-types (see ex. II-56 b).

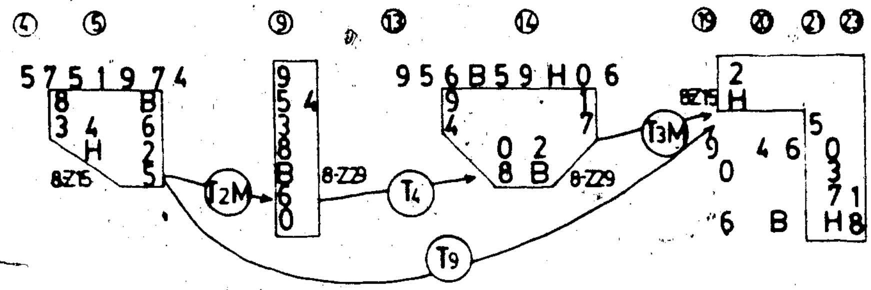
Example II-58: <u>T_5M- and $T_{11}M$-Related Sets, mm. 13-14, 21-23</u>



The octachordal superset of the 7-30 septachord in mm. 20-23, a member of the set-class 8-Z15, is related, like its seven-element subset, under a multiplicative function to a set of pitch-classes at the beginning of the second strophe. More important, this octachord in mm. 20-23 completes a series of four members of the multiplicatively related set-types 8-Z15 and 8-Z29 in mm. 4-5, 9, 13-14 and 20-23, whose compound operator is T_9 (see exx. II-30, II-45).

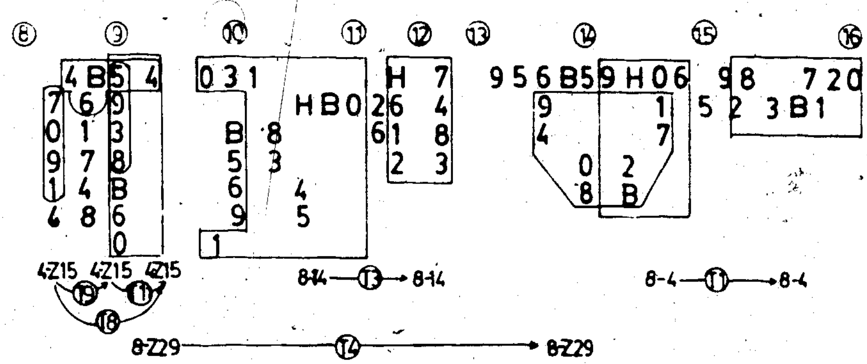
Notably, while the 8-Z29 tetrachord in m. 9 is followed in the second phrase of the second sentence (first strophe) by two T_3-related

Example II-59: 8-Z15 and 8-Z29 Octachords, mm. 4-23



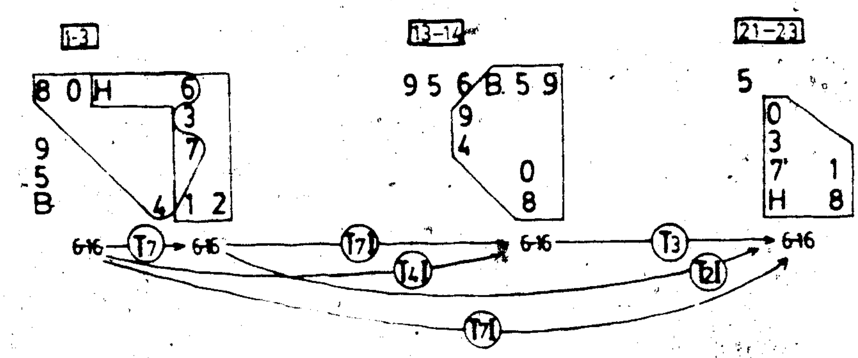
octachords of the class 8-14, the octachord of this set-type in mm. 13-14 is followed by two octachords, related under T_1, belonging to 8-14's multiplicatively related set-class 8-4. The operators relating these three pairs of octachords - T_4, T_3, and T_1 - are the inverse operators of those that describe the relationships among the three members of the set-type 4-Z15 that frame and span the first phrase of the first strophe's second sentence in mm. 8-9.

Example II-60: Sets and Mappings, mm. 8-16



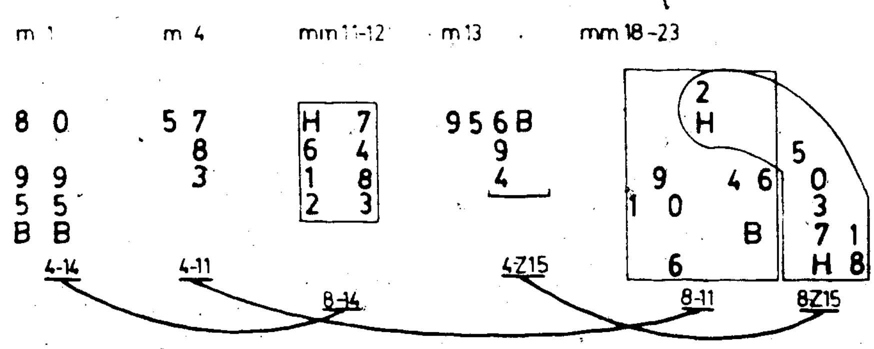
The final six pitches presented by the piano in mm. 20-23 form a hexachord of the class 6-16 and also completes a series of pitch-class sets related in a significant way. The first two are the T_7-related products of the respective unions in mm. 1-3 of melodically articulated tetrachords of the set classes 4-Z15, 4-7 and 4-19, and the chordally articulated tetrachords of these same set-types (see ex. II-7). The third is formed by the union of the first piano tetrachord in the second strophe and the vocal line in m. 14. The eight operations among these four hexachords are inversionally related to those which related the quartet of 4-19 tetrachords in the first sentence of strophes one and two, and the four pentachords in the second strophe's first sentence belonging to the 5-21 set-type (see exx. II-42 and II-43).

Example II-61: 6-16 Hexachords, mm. 1-3, 13-14, 21-23



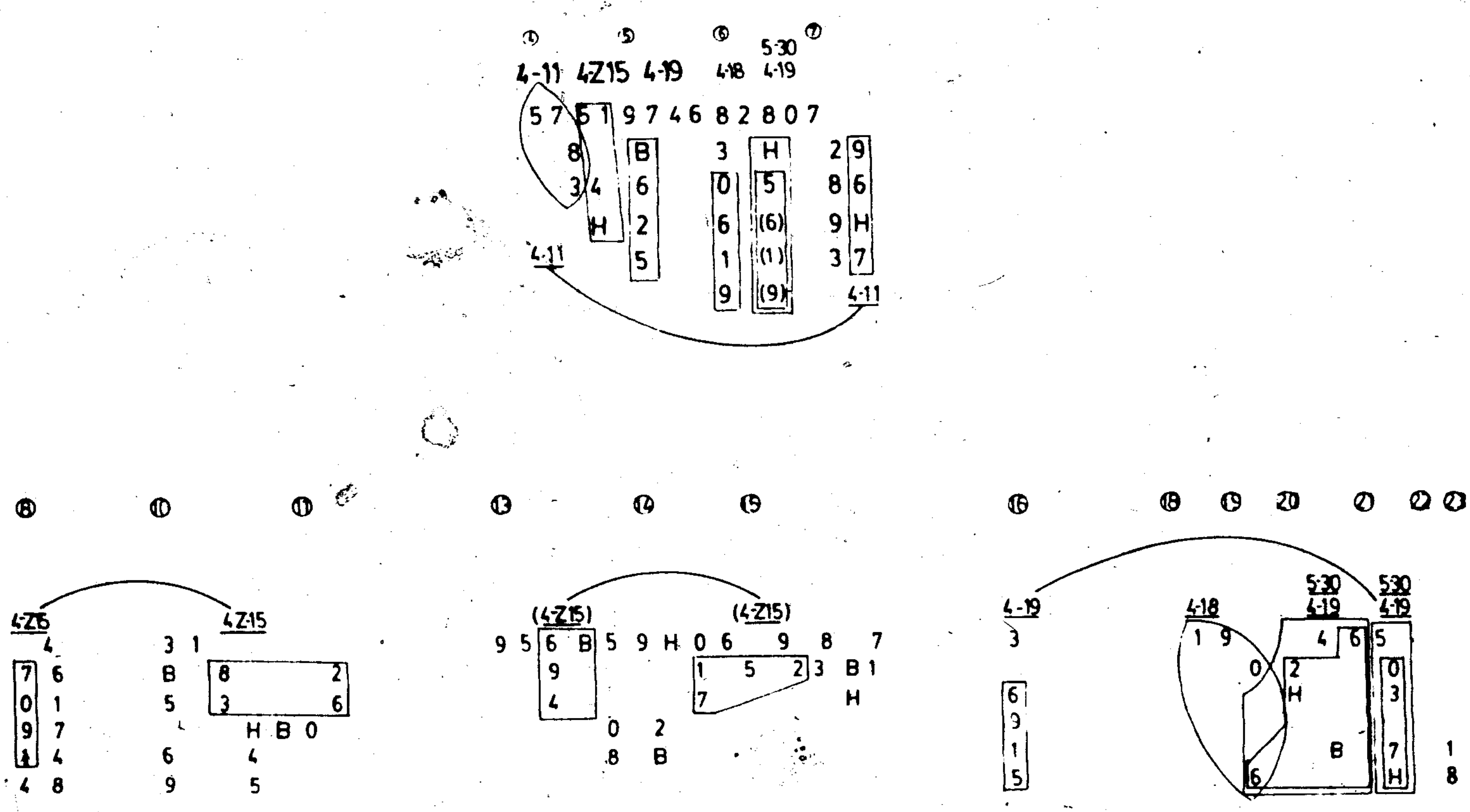
The large segmentations that comprise the final phrase of this song are also complementarily related to important sets of pitch-classes throughout the piece. It was noted above that the final 8-14 octachord of the first strophe is the literal complement of the second vertical sonority in the introduction. Remarkably, the octachord formed by the nine pitches in mm. 18-19 which make up the first half of the final phrase form an octachord of the 8-11 set-class which is the literal complement of the initial tetrachord of the first strophe. Similarly, the 8-Z15 octachord that comprises the last half of the final phrase is related in precisely the same way to the first tetrachord in the second strophe.

Example II-62: Literally Complementary Sets



The idea of operationally related framing tetrachords is clearly of fundamental significance. The succession of set-types whose members frame the four sentences of the two strophes recalls the succession of set types in the first phrase of the first sentence. That is, the ordering of the set-types 4-11, 4-Z15 and 4-19 in mm. 4-5 is repeated by the framing 4-11 tetrachords of the first sentence; the framing 4-Z15 tetrachords of the second sentence of the first strophe and the first sentence of the second strophe; and the tetrachords of the set-type 4-19 that frame the second strophe's second sentence. Moreover, the two tetrachords in m. 6 that follow the initial three are identical in pitch-class membership to the 4-19 and 4-18 tetrachords that initiate the two phrases of the last sentence. Finally, the last pentachord in the first sentence (m. 5) is a member of the same set-type to which the T_1-related closing pentachords in mm. 20 and 21 belong (see ex. II-63).

Example II-63: Repetition of Set-Types



I began this analysis by noting the formal demarcations of the lyric and speculating about a correspondence between these units and the boundaries of the harmonic processes. Another more remarkable and more subtle association between text and music, one that belies the inane *Volkstuemlichkeit* of the folktext, may be asserted. In the first sentence of both strophes there is an inversion of semantic field across the two phrases. *Der Tag* becomes *die Nacht*, and *ist vergangen* becomes *ist schon hier* or, in the case of the first sentence of the second strophe, *kommt hierzu*. Likewise, in the first sentence of each strophe, pitch-class sets in the first phrase are related inversionally to sets in the second phrase. In addition, from the first strophe to the second, there is a transposition, even a metaphoric multiplication of the image of sleep (*Gute Nacht, O Maria, bleib ewig bei mir*) to one of death in the second (*Gib auch den Verstorbnen, die ewige Ruh*). Appropriately, functions of transposition and multiplication describe too the relationship between pitch-class sets in the first strophe and corresponding sets in the second.

Chapter III

Directed Motion in Webern's Opus 12/i

The set-theoretical analysis provides a great deal of data cogent to a discussion of structural levels and directed motion in opus 12/i. Most powerful is that which proffers important clues to the content of the fundamental structure.

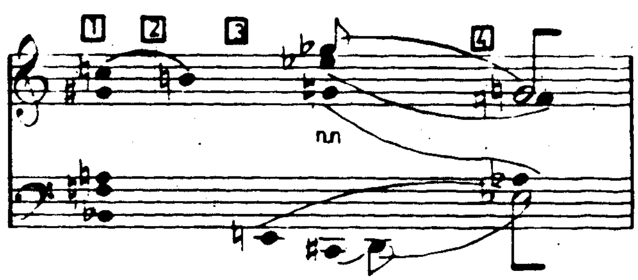
First, a comparison of the two T_1-related tetrachords of the class 4-7 that conclude respectively the introduction (m. 3) and the first strophe (m. 12); the tetrachord of the class 4-11 that initiates the first strophe (m. 4); and the 4-Z15 tetrachord in m. 13 and its rearticulation in m. 20, ultimately provides the shape of the structural matrix, or, as Schenker liked to put it, the Origin of the piece. A reduction of these chords to one octave and specifically a consideration of their outer voices point clearly to a fundamental line G^4-Gb^4 ultimately achieving F^5 in m. 21, counterpointed by a fundamental bass motion Eb^3-Eb^3 finally resolving, like the soprano, to F^5 in m. 21. The makeup of the inner voices seems less clear. In spite of the unassailably important stature of the 4-Z15 tetrachord in the second strophe, the role played by F^4 in m. 4, and the manner of articulation, suggest that F^4 which immediately precedes Gb^4 in m. 13 is similarly an inner voice of the structural harmony at that point. In any case, the function of Bb^4 in m. 13, in spite of its membership in the 4-Z15 tetrachord, seems more likely to be that of a covering-tone. Finally, the background of $\left(\begin{smallmatrix} G - G\flat \\ E\flat - E\natural \end{smallmatrix} > F\right)$ cannot of course be generalized for all atonal pieces in the manner of a true <u>Ursatz</u>. Nevertheless, like its tonal counterpart, this fundamental structure does provide, for this specific piece, the theoretical underpinning necessary to make compelling analytical statements (see ex. III-1).

Example III-1: Background, opus 12/i

Introduction and First Strophe, mm. 1-2

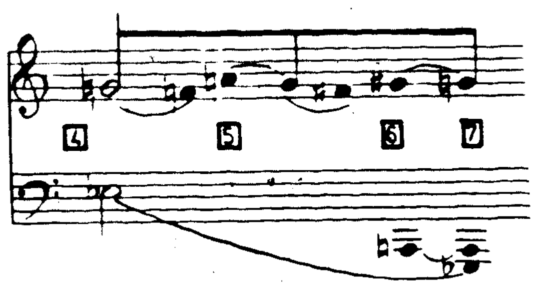
The first three measures exhibit little extended melodic movement and musical space is very tenuously marked out, with the final tetrachord (of the class 4-7) the focal point of the introduction. The neighbor function of this chord to the 4-11 tetrachord in m. 4 is heightened by the bass melody which accompanies it. The structural Eb in m. 4 is prepared through the provision of both an upper chromatic neighbor (E^2), and lower one (D^2) in m. 3. The latter is imbued, by its own lower chromatic neighbor C#^{3} (displaced an octave), with additional momentum upwards. Note too, that this neighbor contains, as its alto and tenor voices, the soprano and bass tone of the structural chord which it embellishes.

Example III-2: Sketch, mm. 1-4



The bass line of the first sentence (mm. 4-7) prolongs the structural Eb^3 through a symmetrical division of the octave space by A^1 in m. 6, while the structural control of the melody note G^4 is extended across the two phrases through its embellishment by what Joel Lester terms "double-symmetrical tones."[18] In the initial phrase (mm. 4-5), there are G^4's upper and lower whole tone neighbors A^4 and F^4. In the second phrase (mm. 5-7), the base interval of symmetry is reduced to a semitone, and G^4 is embellished by its chromatic upper and lower neighbors $F\#^4$ and $G\#^4$. Not only do these diminutions increase the strength of melodic will or intention towards G^4 in m. 7, but reflect, at a more-to-the-foreground level, the essence of the fundamental structure.

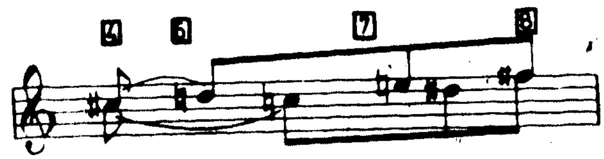
Example III-3: <u>Prolongation of Structural Tones, mm. 4-7</u>



The constellation of covering tones in mm. 6-7 illustrates a slightly different application of symmetrical motion and the convergence of two lines upon a single goal. In this case, tones are arranged in two dyads, with the upper note of each (D^5 and E^5) marking out a motion in whole-tones toward $F\#^5$ in m. 8. This line is counterpointed by motion in minor thirds, from C^5 through $D\#^5$ to $F\#^5$. Consequently, $F\#^5$ in m. 8 presents itself as the conflux of these two short spans. $C\#^5$ in m. 4 is clearly prefatory to the D/C dyad in m. 7 and may be interpreted as both a lower chromatic neighbor to D^5 and an upper chromatic neighbor to C^5.

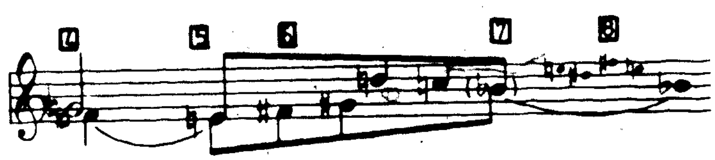
The melody of the second phrase of the first sentence may also be understood as whole-tone motion emanating in the alto voice's E^4 in

Example III-4: Covering Tones, mm. 4-8



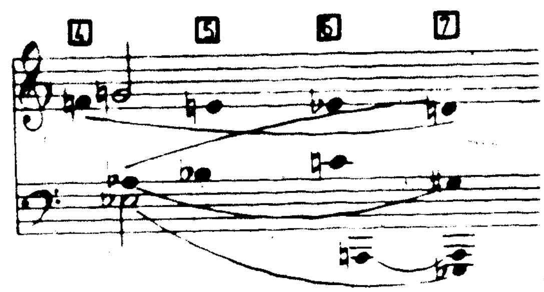
m. 5 and converging from two directions upon an implied Bb^4 in m. 7 - an implication strengthened by this note's actual presence in the following measure. The origin of this motion, E^4 in m. 5, seems to have its own source in the alto voice F^4 in m. 4, mirroring in these first measures of the first strophe the large-scale motion of the alto voice in mm. 4-12 (cf. ex. III-1).

Example III-5: Covering Tones (Alternate version), mm. 4-9



Voices woven more deeply into the fabric of the music demonstrate the same melodic logic displayed by the outer voices. F^4 and Ab^3, the alto and tenor voices respectively of the initial structural harmony in m.4, approach D^4 in m. 7, not only from both directions, but with intervals of motions of different size, though equally symmetrical. The third-span, F ^{4} to D^4, traversed by semitones, is counterpointed by a tritone-span from Ab^3, marked out in whole-tones. The unfolding of the interval Ab^3 to D^4 culminates in a sounding of both pitches together, coincident with the bass melody Eb^1 and its embellishing tone A^1.

Example III-6: Inner Voice Spans, mm. 4-7

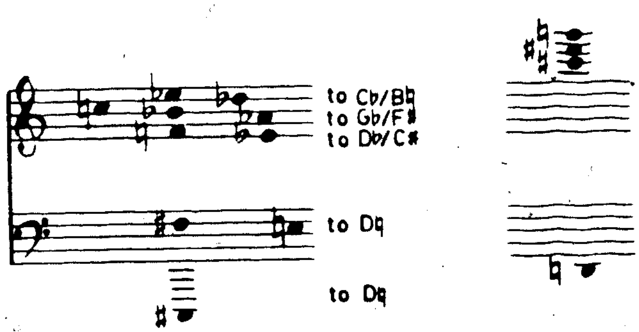


The second sentence (mm. 8-12), linked to the first by an arpeggiation of the introduction's 4-7 tetrachord, prolongs, by means of a descending octave progression in whole-tones through an implied D^5 in m. 8, the pitch-class F#. This is counterpointed in the bass by contrary melodic motion from C^2 to $C\#^1$ - each of these in turn prolonged by symmetrically related pitches - finally achieving D^1 in m. 11. The importance of the F#-D dyad as a local goal of the second sentence is underscored by an anticipatory articulation in the right hand of the piano in m. 11. Moreover, the manner in which the bass note D^1 is achieved in m. 11 is an elegant recollection of the preparation of the structural Eb^3 in m. 4. That is, the thrust toward D^1 is heightened by the provision, in addition to the chromatic lower neighbor $C\#^1$ (itself prefaced by its similarly related tone), of the chromatic upper neighbor Eb^1 at the end of the first sentence (m. 7).

B^6 and $C\#^6$, the other tones in the neighboring harmony in mm. 11-12 are, like $F\#^6$ and D^1, also objects of symmetrical voice-leading.

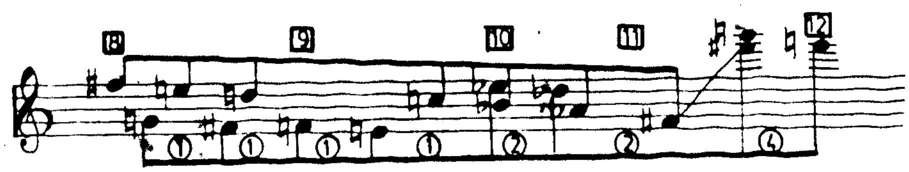
Example III-7: Sketch, mm. 3-13

Example III-8: Voice-Leading, mm. 10-11



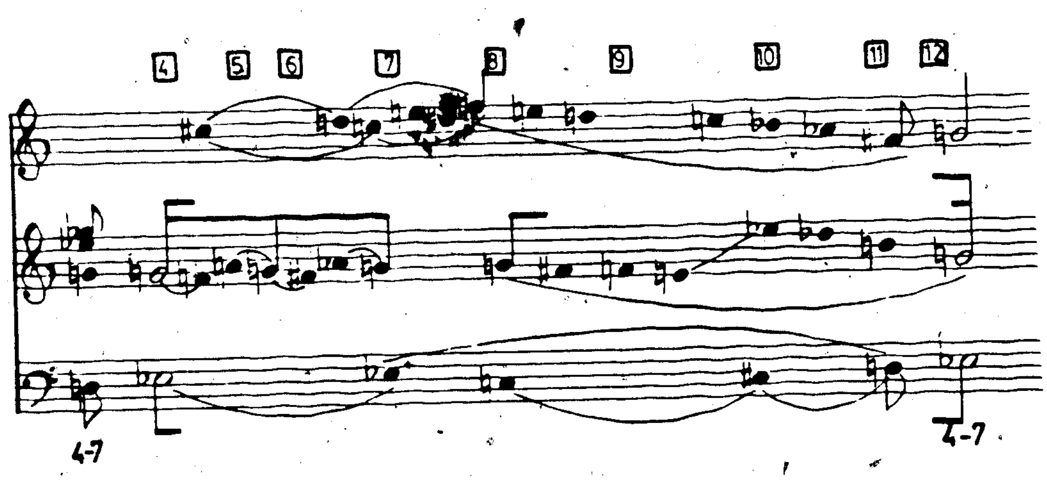
The tones in m. 10 covering the principal octave descent from F#[5] in m. 8, and which lead to B[6] in m. 11, may be seen to have their origin in G[4] in m. 8, the top voice of the piano. The line begins by chromatically traversing a minor third to E[4] in m. 9. After a simple ascending register transfer, the progress continues toward B[6] in m. 11 in whole steps, where it descends the tonal space of a major third to Db in m. 12. Consequently, the pace of the passing motion broadens with regularity - in a way curiously reflective of the motivic transformation of this section (see ex. II-14) - from semitones (G-F#-F-E-Eb) to whole-tones (Eb-Db-B) to a single major third (B-G), with each doubling of interval size underscored by an ascending register transfer.

Example III-9: <u>Sketch, mm. 8-12</u>



Thus the fundamental line's G, embellished in the first sentenced by two double-symmetrical tones and prolonged in the second by an octave span, also presents itself as the resolution of the covering line, beginning in the first sentence with a span from $C\#^5$ to $F\#^5$, then prolonging F# with a whole-tone octave descent in the second sentence, resolving ultimately as the chromatic lower neighbor to G^6 in m. 12.

Example III-10: <u>Fundamental Line and Covering Tones</u>, mm. 4-12

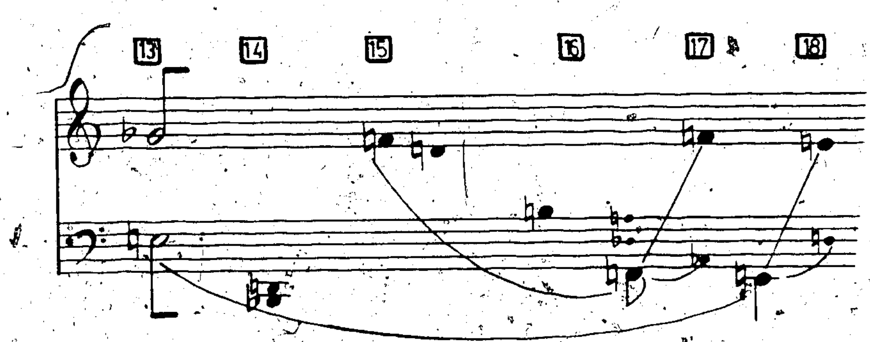


The Second Strophe , mm. 13 - 23

The unravelling of the musical fabric and the increasing wispiness of the piano gestures in the first sentence of the second strophe would seem to inhibit an understanding of the manner in which the structural Gb and E are prolonged. Happily, the dissipation of musical material notwithstanding, the melodic threads are still comprehensible.

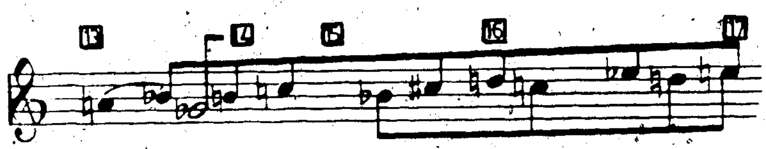
The prolongation of E is especially clear when the first phrase of the second sentence (second strophe) is also considered. As in the first sentence, the process is initiated by a symmetrical division of the octave space by a tritone-related passing tone. A prolongation of the upper chromatic neighbor F through a series of minor thirds is nested within (mm.15-16), before the octave is finally spanned in m.17.

Example III-11: Prolongation of Bass E, mm. 13-18



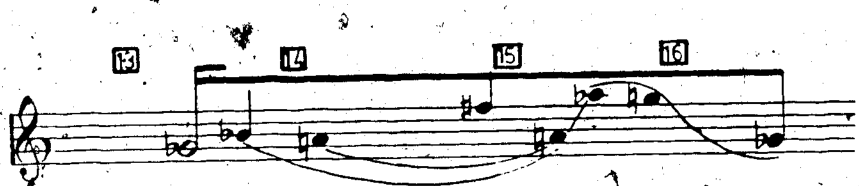
This supports two progressions through the tonal space of an augmented fourth, both covering the principal prolongation of the structural Gb. The first begins with the covering tone Bb^4 in m. 13 and traverses a path to E^5 in m. 16 in semitones. This progression is joined midway to its goal by another Bb^4 (m. 15), which moves toward E^5 in whole-tones. This is, of course, a recollection of the activities of the covering tones in the parallel passage of the first strophe (cf. exx. III-3 and III-11).

Example III-12: Covering Tones, mm. 13-17



The first Bb^4 (m.13) also seems to initiate a semitonal descent to Gb^5, carried out across two registral shifts, finally achieving Gb^5 in the alto voice of the piano on the first beat of m.16.

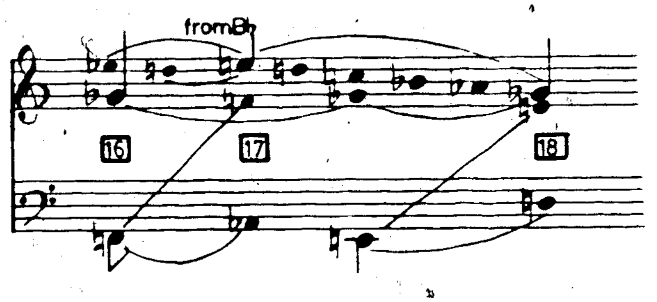
Example III-13: Unfolding of Bb-Gb, mm.13-17



The business of the first phrase of the second sentence (mm. 17-18) is quite clearly 1) to resolve in the bass, the chromatic upper neighbor F to E in m. 17; and 2) to unfold the whole-tone E-Gb through a desending seventh progression. Consequently, Bb^4 in m. 13 progresses toward Gb, not only in the semitonal descent, but also by the more circuitous route ending in the unfolded seventh from E^5 to Gb^4 (mm. 17-

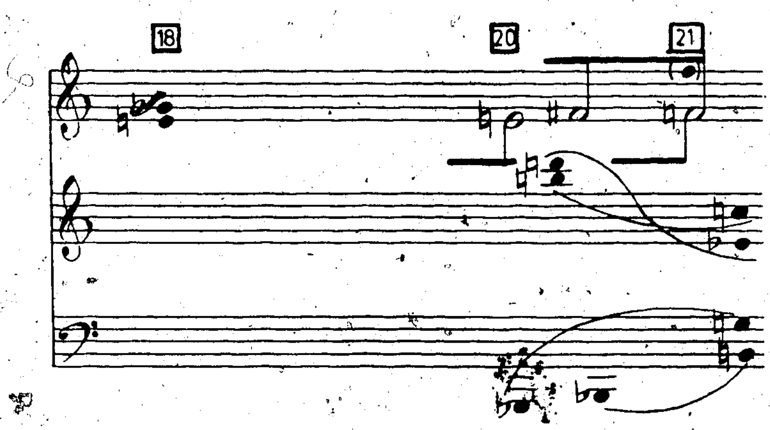
18). Moreover, the structural dyad has, by this point, unequivocally placed itself at the centre of attention.

Example III-14: Sketch, mm. 16-18



The last phrase (mm. 18-23) of course concerns the resolution of the structural Gb-E dyad in F. This is clearly articulated and presents no real problem. The notes in the piano supporting this final descent of the fundamental line (mm. 20-21) may be seen, as demonstrated in the set-theoretical discussion of this section (see ex. II-56), as quadruple chromatic lower neighbors to the 4-19 tetrachord supporting the resolved F in m. 21.

Example III-15: Sketch, mm. 18-21



While F^5 is the resolution of logical voice-leading, it also completes the span of a twelfth beginning in $D\flat^4$ in m. 18, and divided symmetrically by A^4 in mm. 19-20 into two equal leaps of a major sixth. Moreover, F^5, again in connection with the A^4 that precedes it, presents itself as the conclusion to a kind of syllogism of motivic processing (see discussion accompanying ex. II-47). First, it was noted that piano gestures in m. 17-18 are shaped by the motive presented initially in the voice in mm. 7-8, and, as they are in the second sentence of the first strophe, the basic intervals of the motive are transformed through a doubling of the values of their interval classes. In addition, while each of the three statements of the motive in the first strophe begins on the pitch a minor third higher than the preceding statement, the initial pitch of the second of the two statements in the second strophe is a minor third lower. Carrying the two processes (doubling of basic interval class, initiating a minor third lower than the preceding statement) to one more articulation of the motive, we derive the pitches A^5-F^5-F^6. Strikingly, by the inflection of this motivic conclusion through the substitution for F^5 of its upper and lower chromatic neighbors E^5 and $F\#^6$, motivic logic is absorbed and transformed by the demands of counterpoint.

Example III-15: <u>Motivic Transformation</u>

* * *

The results of this analysis of Webern's opus 12/i beg comparison to analyses of tonal music from the same perspective. An enumeration of some differences between the two provides a matrix for further speculation.

First, in Webern's opus 12/i, there seems to be an emphasis at the foreground level on linear passing motion, almost to the exclusion of arpeggiation and, to a lesser degree, neighbor relationships. This is reflected in the content of the fundamental structure. Unlike the tonal Ursatz, which comprises a fundamental line counterpointed by a bass arpeggiation, the governing background progression here consists of two fundamental lines. Notably, however, contrary motion (ascending bass, descending line) and the confluence of the two voices in one "tonic" pitch, are characteristics of the tonal Ursatz that are maintained here. Similarly, as in tonal music, the diminutions of the voice, at least at the foreground level, involve mostly large intervals, while the motion of upper lines is generally carried out in semitones and whole-tones.

Second, series of apparent foreground neighbors seem more meaningfully interpreted as two co-existing principal lines than as one embellished melody.

Finally, although the idea of tonic and dominant functioning sonorities, expressed, for example, as opposed set-types, appears to be unsubstantiated, certain set-types do seem to possess some syntactical significance. One need only think of the tetrachords of the class 4-7 at the end of the introduction and the first strophe, and of the class 4-Z15, to which belong the penultimate chord in the introduction, and the first and last chords in the second strophe.

By now it is clear that without the benefit of the set-theoretical data to give direction, the analysis of directed motion in Webern's opus 12/i would be radically different in its conclusions, its methodology and its intent. First, structural weight would more likely be placed upon F^4 at the beginning of both strophes (mm. 4 and 13) with G^4 and Gb^4 in these places regarded as prolongational pitches. More important, however, is that the principal activity of the analysis would become one of evaluating and ranking pitches in order to discover a fundamental progression without, of course, the proper speculative tools. The background becomes merely a result instead of an a priori enabling the analyst to construct a convincing and compelling interpretation.

REFERENCES

Chapter I

1. Atonal, free atonal, pantonal, motivic radical European, dodecaphonic, and twelve-tone have been suggested as adjectives. Cognizant of the possible charges of catachresis, and against the better judgement of the editor of the second edition of Fowler's Modern English Usage, I shall use the more vague "atonal" here.

2. This paper presumes a knowledge of definitions germane to these analytical approaches. Useful, short introductions include: David Beach, "Pitch Structure and the Analytical Process in Atonal Music," Music Theory Spectrum 1 (1979), 7-22 and James Baker, "Schenkerian Analysis and Post-Tonal Music," in Aspects of Schenkerian Theory, ed. by David Beach. (New Haven: Yale University Press, 1983).

3. Harvard Dictionary of Music, 2nd ed., s.v. "Intervals, Calculation of."

4. Patrick Suppes, Introduction to Logic, (New York: Van Nostrand Rheinhold Company, 1957), 230.

5. John Rahn, Basic Atonal Theory, (New York: Longman, 1980), 40 ff.

6. Derived from Daniel Starr, "Sets, Invariance and Partitions," Journal of Music Theory 3 (1978), 1-42.

7. Robert Morris, "Set Groupings: Complementation and Mappings Among Pitch-Class Sets," Journal of Music Theory 26 (1982), 101ff.

8. Morris, "Set Groupings," 107. "Not being able to specify the rules by which Z related sets map onto each other implies that we do not fully understand the relation between our perceptions (because interval content should strongly correlate to aural similarity) and the structure of the pc set universe."

9. David Lewin, "The Intervallic Content of a Collection of Notes," Journal of Music Theory 4 (1960), 98-101; and Allen Forte, The Structure of Atonal Music, (New Haven: Yale University Press, 1973), 21-22.

10. Cf. Eric Regener, "On Allen Forte's Theory of Chords," Perspectives on New Music 13 (1974), 207 ff., and Morris, "Set Groupings," Journal of Music Theory 26 (1982), 107. The notations used here are derived from Morris's formulation of the theorem.

11. This a restatement of the simple observation that since TnA and its complement ($\overline{TnA}$) exhaust the entire domain of discourse, the members of A that are not in TnA must be in $\overline{TnA}$.

12. Starr, "Sets," 1.

13. Joel Lester, "A Theory of Atonal Prolongations as Used in the Analysis of the Serenade, Opus 24 by Arnold Schoenberg," (Ph.D. diss., Princeton University, 1970), 2 ff.

14. Carl Schachter, " A Commentary on Schenker's Free Composition," Journal of Music Theory 25 (1981), 132. Also see, Heinrich Schenker, Free Composition (der freie Satz), trans., ed. by Ernst Oster (New York: Longman, 1979), 18.

15. Heinrich Schenker, Das Meisterwerk in der Musik, Band II, (Munich: Drei Masken Verlag, 1926). repr. (Hildesheim: Georg Olms Verlag, 1974), 41.

Chapter II

16. According to Hans Moldenhauer, the first draft of the Lied is dated January 13, 1915. It remained unpublished until May, 1922 when it "had an advance printing . . . as an insert to Musikblätter des Anbruch. Webern's copy of this publication, found in his estate, contains numerous corrections and additions. These were later incorporated in the Universal Edition release." Hans Moldenhauer, Anton von Webern: A Chronicle of his Life and Work, (New York: Knopf, 1979), 265.

17. Heinz-Klaus Metzger describes a similar formal disposition for Webern's opus 15/iv. See Heinz-Klaus Metzger, "Analyse des Geistlichen Liedes, Op. 15, Nr.4," Die Reihe 2 (1955), 80.

Chapter III

18. Lester, "Atonal Prolongations," 4.

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A P P E N D I X

"Der Tag ist vergangen," opus 12/i

by Anton Webern

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