University of Alberta

Routing Strategies for Multihop Wireless Relaying Networks

by

Ramin Babaee

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Examining Committee

Norman C. Beaulieu, Electrical and Computer Engineering

Masoud Ardakani, Electrical and Computer Engineering

Ehab S. Elmallah, Computing Science

To my beloved wife

and

to my dear family

Abstract

Multihop routing is an effective method for establishing connectivity between the nodes of a network. End-to-end outage probability and total power consumption are applied as the optimization criteria for routing protocol design in multihop networks based on the local channel state information measurement at the nodes of a network. The analysis shows that employing instantaneous channel state information in routing design results in significant performance improvement of multihop communication, e.g., achieving full diversity order when the optimization criterion is outage performance. The routing metrics derived from the optimization problems cannot be optimized in a distributed manner. Establishing an alternate framework, the metrics obtained are converted into new composite metrics, which satisfy the optimality and convergence requirements for implementation in distributed environments. The analysis shows that the running time of the proposed distributed algorithm is bounded by a polynomial.

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Table of Contents

1 Introduction **1**

1.1 Multihop Networks 3

1.2 Network Architecture Layering 3

1.3 Cross-Layer Design 6

1.4 Related Works 6

1.5 Thesis Motivation and Contributions 7

1.6 Thesis Outline 10

2 Joint Routing and Power Allocation Optimization **11**

2.1 System Model and Problem Formulation 12

2.2 Optimal Routing Using Mean CSI 15

2.2.1 Equal Power Allocation 17

2.2.2 Optimal Power Allocation 19

2.3 Optimal Routing Using Instantaneous CSI 19

2.3.1 Equal Power Allocation 21

2.3.2 Optimal Power Allocation 22

2.3.3 Extension to Multihop Diversity Networks 23

2.4 Low Complexity Routing Algorithms 23

2.4.1 Local Search Based Routing 23

2.4.2 Distributed Routing 24

2.5 Simulation Results 25

2.5.1 Outage in Linear Networks 25

2.5.2 Comparison of the Proposed Routing Algorithms 29

2.5.3 Outage in Two-Dimensional Networks 30

2.6 Summary 31

3 Distributed Optimal Outage Efficient Routing **35**

3.1 System Model 36

3.2 Problem Formulation . . . 37
3.2.1 Amplify-and-Forward . . . 37
3.2.2 Decode-and-Forward . . . 39
3.3 Distributed Routing Algorithm . . . 40
3.3.1 Definitions . . . 40
3.3.1.1 Amplify-and-Forward . . . 42
3.3.1.2 Decode-and-Forward . . . 43
3.3.2 Routing Table Construction . . . 44
3.3.3 Packet Forwarding . . . 47
3.3.4 Optimality and Consistency . . . 48
3.4 Complexity Analysis . . . 50
3.5 Simulation Results . . . 51
3.5.1 Scenario 1 . . . 51
3.5.2 Scenario 2 . . . 55
3.6 Summary . . . 56

4 Power-Optimized Routing With Bandwidth Guarantee 59
4.1 Network Model . . . 60
4.2 Problem Formulation . . . 60
4.2.1 Decode-and-Forward . . . 60
4.2.2 Amplify-and-Forward . . . 61
4.3 Distributed Routing Algorithm . . . 63
4.3.1 Definitions . . . 63
4.3.2 Routing Table Construction . . . 65
4.3.3 Packet Forwarding . . . 67
4.3.4 Power Allocation . . . 68
4.4 Performance Evaluation . . . 68
4.5 Summary . . . 70

5 Conclusions and Future Work 72
5.1 Concluding Remarks . . . 72
5.2 Future Research Directions . . . 73

References 75

A Proof of Convexity 79

List of Tables

3.1 Routing table of Node s, constructed based on the proposed distributed optimal outage efficient routing algorithm for the network illustrated in Fig. 3.2 46

List of Figures

1.1 A multihop network where the source node communicates with the destination via v_3 and v_{N-2}. 4

1.2 The OSI reference model. 5

2.1 A linear multihop network. 13

2.2 The optimal number of hops for OR-MCSI with EPA in an equally spaced linear network. 18

2.3 The outage probability as a function of average source-destination SNR for the proposed routing algorithms in linear multihop networks ($N = 6$). 29

2.4 The average number of hops as a function of average source-destination SNR for the proposed routing algorithms in linear multihop networks ($N = 6$). 30

2.5 The average number of hops for different values of α in linear networks ($N = 6$ and $\Gamma = 15$ dB). 31

2.6 The outage probabilities of the proposed routing algorithms versus path loss exponent in linear networks ($N = 6$ and $\Gamma = 15$ dB). 32

2.7 Comparison of the outage probability of OR-ICSI for several numbers of nodes in linear multihop systems without diversity to the outage probability of linear multihop diversity systems. 33

2.8 Outage performances of optimal routing, local search based routing, and distributed routing in linear networks. 33

2.9 Outage performance comparisons of the proposed routing strategies with EPA for different numbers of nodes in linear networks. 34

2.10 The outage probability of the proposed approximate distributed routing algorithm for several numbers of nodes in two-dimensional networks. 34

3.1 A 4-node multihop network which shows that the routing metrics obtained for outage efficient routing in AF and DF networks are not isotone. 41
3.2 Network example for the proposed outage efficient routing. 46
3.3 The outage probability of the proposed optimal distributed routing algorithm for several numbers of nodes in the network. 52
3.4 The average number of hops of the proposed optimal distributed routing algorithm for several numbers of nodes in the network. 53
3.5 Comparison of the outage probability of the proposed algorithm in multihop systems without diversity to the outage probability of multihop diversity systems ($N = 10$). 54
3.6 Comparison of the average-case running time of the proposed routing algorithm in AF and DF multihop networks for a wide range of N. The routing algorithm is run for two cases, finding the optimal paths to one destination and all the nodes in the network. 55
3.7 Comparison of the average number of efficient paths between source and destination in AF and DF multihop networks for a wide range of N. 56
3.8 A random network topology with 60 nodes. Each node can only communicate with the nodes within a disk with the radius of 25. . . . 57
3.9 The optimal outage efficient paths from all the nodes in the network to arbitrary destination node d. The thickness of the first link of a node-destination route is proportional to the spectral efficiency of that path. 57

4.1 A multihop network which shows that the routing metrics obtained for power-optimized routing in AF and DF networks are not isotone. 63

4.2 The average total power of different routing schemes for different values of R in random two-dimensional networks with 10 nodes. 70
4.3 The average total power of different routing schemes for several numbers of nodes in the network assuming $R = 1$. 71

List of Algorithms

2.1 Summary of the approximate distributed routing strategy for OR-ICSI with EPA in DF networks from node t_i's view 26

2.2 Summary of the approximate distributed routing strategy for OR-ICSI with OPA in DF networks from node t_i's view 27

3.1 Summary of the distributed optimal outage efficient routing in AF networks 47

3.2 Summary of the distributed optimal outage efficient routing in DF networks 48

4.1 Summary of the power-optimized routing algorithm in DF networks 67

List of Symbols

A	Total number of available paths between source and destination	13
α	Path loss exponent	13
β_{t_i}	Portion of total transmitting power assigned to node t_i	14
D	Distance between source and destination	12
d_{ff}	Far-field distance	13
d_{v_i,v_j}	Distance between nodes v_i and v_j	13
$\mathcal{E}$	Set of all the links in the network	12
G_{t_i}	Amplification gain at i-th AF relay in a multihop network	37
Γ	Average link SNR between source and destination	26
γ	Network signal-to-noise ratio	14
γ_d	Destination SNR	38
γ_{t_i}	Instantaneous received SNR at node t_i	22
$\bar{\gamma}_{v_i,v_j}$	Average SNR of the link between nodes v_i and v_j	14
h_{v_i,v_j}	Channel gain between nodes v_i and v_j	13
k	Number of hops	13
L	Lagrangian function	60
λ	Lagrange multiplier	60
M	Number of intermediate relays in local search routing	24
N	Number of nodes in the network	12
N_0	Noise power	14
$\Omega(p)$	Composite routing metric of path p	42
$\omega(p)$	Routing metric of path p	38

p	Path	12
$\mathcal{P}(v_j, v_j)$	Set of all possible paths from v_i to v_j	12
$P_{\text{outage},p}$	Outage probability of transmission over path p	14
P_{total}	Total available transmitting power	14
P_{t_i}	Transmitting power of node t_i	14
R_p	Capacity of transmission over path p	15
$\sigma^2_{v_i,v_j}$	Mean channel power gain between nodes v_i and v_j	14
$\mathcal{V}$	Set of all the nodes in the network	12
v_0	Source node	12
v_i	i-th node in the network	12
v_{N-1}	Destination node	12

List of Abbreviations

AF	Amplify-and-forward	3
CSI	Channel state information	6
DF	Decode-and-forward	3
EPA	Equal power allocation	15
LSR	Local search routing	24
MAC	Medium access control	4
MIMO	Multiple-input multiple-output	1
MRC	Maximal ratio combining	23
OPA	Optimal power allocation	15
OR-ICSI	Optimal routing using instantaneous CSI	21
OR-MCSI	Optimal routing using mean CSI	16
OSI	Open systems interconnection	3
SNR	Signal-to-noise ratio	14
TDMA	Time division multiple access	12

Chapter 1

Introduction

Wireless communication networks are expected to provide high data rates for both real-time applications such as conferencing, and non real-time applications such as web browsing. In contrast to wired networks, which have a fixed communication infrastructure, wireless networks suffer from unpredictable topology changes which are mainly due to the nodes' mobility. The uncertainty caused by mobile users combined with the fading effect of rich scattering environments severely degrades the performance of wireless systems and makes the quality of service requirement of intended applications difficult to satisfy. Several solutions have been proposed to alleviate these shortcomings. A physical layer approach, known as multiple-input multiple-output (MIMO), is to employ multiple antennas at the transmitter and receiver side. A MIMO system effectively combats the severe effect of multipath fading by diversity combining of independently fading signal paths. In other words, the basic idea behind diversity is to receive several replicas of a signal through multiple independent fading channels. In diversity combining, the diversity order is defined as the number of independent fading paths between the transmitter and receiver, e.g., the diversity order of a MIMO system with L uncorrelated antennas at the transmitter and M uncorrelated antennas at the receiver is $L \times M$.

Although MIMO systems significantly enhance the performance of wireless communications, using more than one antenna is sometimes not practical due to the size limitation faced by the mobile users. An effective solution to this issue is cooperative diversity which can be achieved by the collaboration of the users in the network [1],

[2]. In cooperative networks, the available radio resources are shared among network users (nodes) to combat multipath fading and signal shadowing to achieve a superior quality of service. To this end, cooperative systems utilize an effective technique called user diversity in which several replicas of a signal are transmitted to the destination through some intermediate nodes. At the destination, the received signals are combined in an appropriate manner to achieve a resultant signal that is less likely to be faded. In cooperative networks, diversity is obtained by the presence of at least one other node called a relay which receives the transmitted signal from the source and forwards it to the destination. Therefore, the destination receives the desired signal through both the source and the relay(s). The requirement of multiple flows of information from source to destination may not be always satisfied.

Multihop communication is a simple topology of cooperative networks, which is an effective method for establishing connectivity between the nodes of a network where direct transmission from source and relays to destination is not feasible or power efficient. Multihop wireless routing is widely used in many applications owing to its simple and economical implementation. In a multihop wireless system, the data is carried forward from source to the corresponding destination through some intermediate nodes. Multihop wireless networks such as ad hoc networks, wireless mesh networks, and sensor networks are emerging technologies that require a careful design due to their self organization characteristic. These types of networks are dynamically configured and the nodes in the network are responsible for establishing connectivity among themselves. This requirement makes the design of these networks challenging.

In addition to a multihop routing, the performance of a network can be further enhanced by utilizing a cross-layer design. Compared to the classical layering of networks, which designs each layer separately, a cross-layer framework uses the beneficial interactions of the different communication layers and provides a wider variety of design objectives and improved network performance. For example, the broadcast nature of wireless communications and channel state information, which are physical layer feature and parameter, respectively, can be employed by the network layer to achieve more robust and effective routing algorithms.

In this thesis, we propose different routing strategies for multihop wireless relaying networks. Our routing algorithms use the channel state information measured at the physical layer to find outage efficient or power efficient paths.

1.1 Multihop Networks

As mentioned earlier, multihop networks are a simple form of cooperative networks where the data transmission is accomplished by the help of some intermediate relaying nodes. A multihop communication is illustrated in Fig. 1.1 where nodes v_3 and v_{N-2} are relaying information from v_0 to v_{N-1}. Each relay terminal processes the received signal from the immediate preceding node and forwards it to the next relay node in the next time slot. Generally, to provide orthogonal subchannels, the m-th relay sends its information to the $(m+1)$-th relay terminal in the $(m+1)$-th time slot. Since the channels are orthogonal and at each moment only one node is transmitting, there is no interference in the network. The processing method at the relay depends on the functionality of the relay node and can be either amplify-and-forward (AF) or decode-and-forward (DF). In AF relaying, each relay amplifies the received signal and forwards it to the next terminal. In DF relaying, the relay node decodes its received signal and then re-encodes and re-transmits. AF relaying schemes have the advantage of lower processing complexity compared to DF relaying because decoding at the relays is not implemented. Generally, AF schemes are more difficult to analyze although DF may perform better in more applications [3]–[5].

1.2 Network Architecture Layering

To reduce the complexity of design, a communication network is organized as a stack of layers. Each layer is a collection of similar functions, which offers certain services to the layers above it. The protocols defined in each layer control the partial functionality of the communication process which should be designed appropriately according to the network design objective(s). The open systems interconnection (OSI) model is a reference model developed by the ISO (International Organization for Standardization), as a conceptual framework of standards for a communication network [6]. This model consists of seven layers and is depicted in Fig. 1.2. We review the main tasks of each layer briefly.

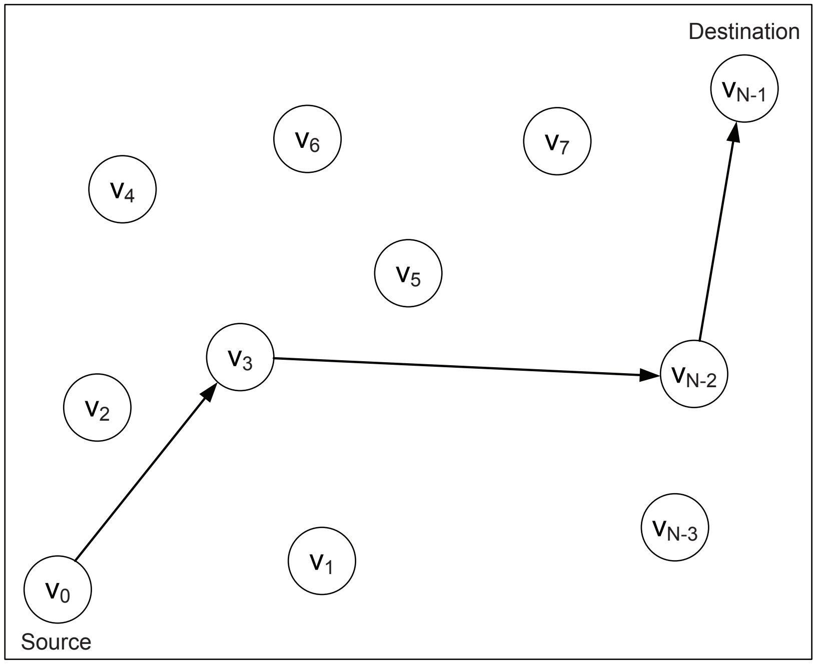

Fig. 1.1. A multihop network where the source node communicates with the destination via v_3 and v_{N-2}.

The *physical layer* is responsible for transmitting raw bits over a communication media. Modulation and demodulation mechanisms, channel coding, and power assignment are the main tasks performed in this layer. Also, the capability of full duplex or half duplex operation is defined in the physical layer.

The *data link layer* provides a reliable data transmission between the network entities by arranging physical layer bits into a sequence of frames and adding some redundant bits for error checking. If a frame is received in error at the destination, a re-transmission is requested by the link layer. This procedure is performed until the frame is received error-free. If the communication media is shared among the transmitters, a sublayer of the link layer, called medium access control (MAC), is responsible for controlling the access of the nodes in the network to the channel.

The main function of the *network layer* is to route the data packets in the network. To route the packets, a distributed routing protocol is implemented in this layer to find the best paths. The outcome of route computations is a routing table at each terminal in the network, which specifies the next node that a data packet must be forwarded to. Moreover, the quality of service provided by the network,

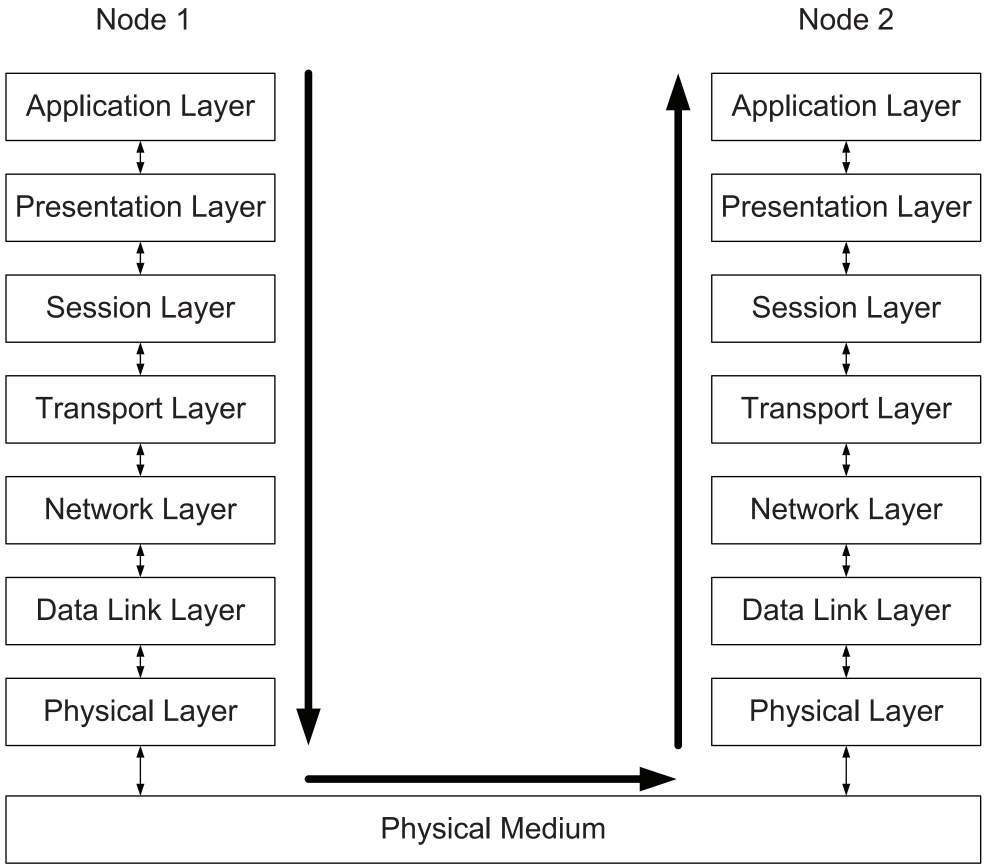

Fig. 1.2. The OSI reference model.

e.g., delay, is the issue of the network layer.

The *transport layer* provides reliable data transfer services to the higher layers by ensuring complete, error-free, and sequential transfer of data between the end-nodes. The reliability of a link is controlled through flow control, error control, and segmentation/desegmentation.

The *session layer* allows applications on separate nodes to share a connection called a session. This layer establishes, manages and terminates connections between applications at the end-nodes. It also controls the dialog between two processes by determining which terminal can transmit or receive at each moment during the communication.

The *presentation layer* deals with the syntax and semantics of the information transmitted. This layer makes the communication between nodes with different data structures possible by transforming the data received from the network into a format acceptable in the application layer or vice versa. File format conversion, data encryption, and data compression are some of the possible functions used in this layer.

The *application layer* contains a variety of applications that an end user interacts with. Some examples of widely used application layer protocols are hypertext transfer protocol (HTTP), file transfer protocol (FTP), and simple mail transfer protocol (SMTP).

1.3 Cross-Layer Design

The traditional layered approach design of communication networks simplifies the process of design and implementation of networks. However, strict separation of network functions in the layered structure does not fully exploit the capabilities of a communication network. A cross-layer design violates the strict layering of the traditional approach by employing possible interactions of different layers to achieve the design objective(s).

For wireless networks, although the separate design of layers is convenient, it is not the most efficient. This is because of the influence of the layers on each other. As previously mentioned, wireless networks with mobile users suffer from time-varying channels and limited power resources. Also, unlike the wired networks, these networks share the wireless links. Therefore, it is important to make the network utilization highly adaptive to the channel conditions. A cross-layer optimization framework which jointly considers physical layer with higher layers is an effective solution to the issues arising from the traditional layered design.

1.4 Related Works

The physical layer of multihop systems has been widely investigated in the literature [3], [4], [7]–[14]. However, there are limited studies on cross-layer design of multihop networks in which channel state information (CSI) is used to improve the routing protocol. The joint optimization of power allocation and routing for a multihop network is studied in [15] in which the authors consider a fixed number of hops and utilization of mean CSI knowledge for routing. Efficient relay selection algorithms for multihop networks in which spatial diversity is exploited are proposed in [16]–[20].

While [15], [21], and [22] used the end-to-end outage probability as the performance metric for optimization, some other optimization criteria such as spectral efficiency, bit-error rate (BER), and total power were employed in [23]–[26],

It has been shown that routing with knowledge of channel state information can significantly improve the quality of communications resulting in lower power consumption and higher reliability [16], [21], [23], [27]. In [19], the authors solve the problem of joint routing and power allocation for cooperative diversity networks using dual decomposition. Also, simultaneous routing and resource allocation is obtained for a general wireless network in [28] via dual decomposition. In both of these works, the optimization problem is solved based on using an iterative distributed algorithm, which may take up to hundreds of iterations to approach relatively close to the optimal solution. Reference [23] studies spectrally efficient routing in decode-and-forward (DF) multihop networks, but the authors obtained a routing metric which cannot be implemented in an optimal distributed approach. Also, the metric cannot be applied to centralized environments because of the exponential computational complexity of the exhaustive search method. Hence, [23] proposed an efficient suboptimal algorithm for implementation in distributed environments. Also, the authors in [21] and [22] investigate the optimal routing for cluster based DF networks by considering the fading characteristics of the potential relay channels in the routing protocol design. However, the proposed distributed algorithms are not optimal and the optimal solution can only be achieved with a centralized exhaustive search. The work in [29] provides a polynomial time algorithm for solving the problem introduced in [23]. The proposed algorithm can only be implemented in centralized environments in which knowledge of the channel gains of all the network links is available at a centralized controller. Also, the solution to the problem is only applicable to DF systems and cannot be extended to AF networks.

1.5 Thesis Motivation and Contributions

Recently, network path selection has been proposed to achieve full diversity order while simplifying the physical layer design of transceivers through eliminating the need to implement complicated space-time coding [16]. In other words, a full diversity ordered can be achieved by employing an appropriate routing strategy, without the need to receive multiple replicas of the transmitted signal at the destination. The goal of this thesis is to study the routing problem in wireless networks with an emphasis on distributed implementation offering another form of diversity, *route selection diversity*, to be employed in multihop transmission systems. Motivations for this thesis followed by our contributions are outlined in the following.

- **Joint routing and power allocation for multihop networks**

  Power efficient communications can significantly improve the performance of wireless networks and increase the lifetime of mobile users, because they have limited power resources. Several power allocation schemes have been proposed in the literature for multihop networks [12], [13], [30]. However, the optimal power assignment is obtained for a fixed path, without considering the routing problem. The work in [15], which jointly considers power allocation and routing, relies on the mean CSI and a fixed number of hops. Therefore, in Chapter 2 of this thesis, we study the problem of finding the optimal route for a multihop wireless network with selective decode-and-forward relaying strategy under the total power constraint. The objective function of interest is the end-to-end outage probability from source to destination, which is to be minimized, assuming that equal or optimal power allocation is employed in the physical layer. A realistic channel model which includes the path loss and fading is considered. More specifically, we determine the optimal routing algorithm using either mean CSI or instantaneous CSI without fixing the number of relays for transmission. It is shown that routing with knowledge of CSI can improve the quality of communication resulting in lower power consumption or higher reliability. Although incorporation of a strategy for determining the best relay nodes can significantly enhance the performance, the overhead of centralized implementation is not negligible. For amenable implementation in large networks, we propose a near optimal distributed routing protocol based on the Bellman-Ford routing algorithm [31].

- **Distributed optimal outage-efficient routing algorithm for multihop networks**

Routing is the key functionality of establishing connectivity between the nodes of a network. In a routing protocol, optimal paths are found based on optimizing a routing metric in the network. References [21] and [23] have studied the routing problem in multihop transmission systems, but their proposed distributed algorithms are not optimal. Also, the routing metrics obtained in Chapter 2 for deployment in decode-and-forward relaying systems cannot be optimized in distributed manner, i.e., they do not converge onto optimal routes. The disadvantage cannot be remedied by exhaustive search because an exhaustive search is not feasible owing to the factorial growth of the search space with the number of nodes in the network. The approximate distributed protocol proposed for outage efficient routing is not optimal and the gap between its performance and that of the optimal approach grows as the number of nodes increases. Thus in Chapter 3, we establish an alternate framework by converting the obtained metric into a new composite metric which is suitable for distributed implementation and can be executed in polynomial time. The new composite metrics and packet forwarding algorithm satisfy the optimality and consistency requirements for distributed implementation. It is proved that the proposed distributed algorithm achieves full diversity order and converges onto optimal paths in any network topology.

- **Power-optimized routing with bandwidth guarantee in multihop networks**

  In Chapter 4 of this thesis, we study the problem of joint routing and power allocation optimization with bandwidth guarantee for DF and AF multihop wireless networks. Using the idea presented in Chapter 3, a distributed solution is proposed for power-optimized routing with desired end-to-end capacity constraint based on the local instantaneous channel state information measurement at the nodes of the network. The proposed distributed routing algorithm is optimal in DF networks and performs close to optimal in AF networks.

1.6 Thesis Outline

The remainder of this thesis is organized as follows. In Chapter 2, we describe the system model and problem formulation of optimal outage efficient routing in multi-hop networks. The problem is solved for two general cases; when either mean CSI or instantaneous CSI is available. For each case, equal-power and optimal-power allocation is considered. Then, a local search based algorithm and an approximate distributed protocol is proposed, which both have a polynomial complexity but substantial loss in performance. In Chapter 3, the routing metrics derived for optimal outage routing in AF and DF networks are converted into new composite metrics. Then, a routing algorithm is proposed to optimize the composite metrics obtained in an optimal distributed approach. The problem of minimum power routing with a desired end-to-end capacity is solved in Chapter 4. Finally, Chapter 5 concludes the thesis and discusses possible future research in this field.

Chapter 2

Joint Routing and Power Allocation Optimization [1]

Route selection diversity is a new approach used at the network layer of a communication system to mitigate the effect of fading in wireless networks [16]. In this chapter, employing channel state information at the network layer, efficient routing protocols are proposed for equal-power and optimal-power allocation in a decode-and-forward multihop network in fading. The end-to-end outage probability from source to destination is used as the optimization criterion. The problem of finding the optimal route is investigated under either known mean CSI or known instantaneous CSI. The analysis shows that the proposed routing strategy achieves full diversity order, equal to the total number of nodes in the network excluding the destination, only when instantaneous CSI is known and used. The optimal routing algorithm requires a centralized exhaustive search which leads to an exponential complexity, which is infeasible for large networks. An algorithm of polynomial complexity for a centralized environment is developed by reducing the search space. An approximate distributed approach based on the Bellman-Ford routing algorithm is proposed which achieves a good implementation complexity-performance trade-off.

The remainder of this chapter is organized as follows. The system model and problem formulation are described in Section 2.1. Optimal route selection algorithms

[1] A version of this chapter has been published in part in *IEEE Transactions on Wireless Communications* [32] and *Proceedings of IEEE Wireless Communications and Networking Conference (WCNC)* [33].

using mean and instantaneous CSI are proposed in Sections 2.2 and 2.3, respectively. Section 2.4 presents a local search based method for the centralized strategy and a distributed routing algorithm. Section 2.5 includes simulation results along with some discussion. Finally, Section 2.6 concludes the chapter.

2.1 System Model and Problem Formulation

Consider a linear multihop network in which all the nodes in the network lie in a line from source to destination. To avoid interference, the MAC sublayer is designed in time division multiple access (TDMA) mode which allows each node to transmit its information in orthogonal time-domain subchannels. Also, it provides a symmetric channel gain between the nodes, i.e., the channel response from node v_i to node v_j is equal to that of the link in the opposite direction. We simplify the system by assuming equal duration time slots.[2] The nodes are working in half-duplex mode which precludes them from transmitting and receiving simultaneously. As the main contribution of this chapter, we focus on a system which does not exploit multihop diversity. However, we extend some of the results to multihop diversity systems in Section 2.3.3. Pure multihop systems have the advantages of lower implementation complexity as well as better power conservation. Spatial reuse is not considered in this work and therefore, during each particular time slot, only one node is transmitting. Also, we assume each node in the network is equipped with a single omnidirectional antenna although the results of this work can be extended to more general cases where multiple antennas are used.

We present our wireless network model as an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V}$ and $\mathcal{E}$ denote the set of nodes and wireless links in the network, respectively. Our model consists of N nodes ($N = |\mathcal{V}|$)[3] which are numbered from v_0 to v_{N-1}. So, the set of nodes is denoted as $\mathcal{V} = \{v_0, v_1, ..., v_{N-1}\}$ where v_0 is the source node and v_{N-1} is the corresponding destination node. The distance between source and destination is D. Let $\mathcal{P}(v_0, v_{N-1}) = \{p : p = \langle t_0, ..., t_{k_p-1}, t_{k_p} \rangle\}$ denote the set of all possible paths from source to destination where $t_0 = v_0$, $t_{k_p} = v_{N-1}$, and

[2] While the durations of the time slots can also be optimized, such an approach makes the study much more complex and is beyond the scope of this work.

[3] $|\mathcal{V}|$ denotes the cardinality of set $\mathcal{V}$.

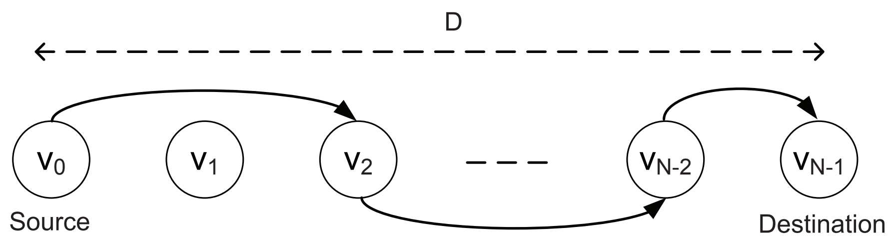

Fig. 2.1. A linear multihop network.

$t_i \in \{v_1, v_2, \cdots, v_{N-2}\}$, $1 \leq i \leq k_p - 1$. Each path is specified by a set of nodes that includes source and destination. Let the cardinality of route p be $k_p + 1$ where k_p is equal to the number of hops of route p, which can be any integer between 1 and $N - 1$ corresponding to direct transmission and multihop transmission using all the nodes of the network, respectively. For notional convenience, we drop the index p, and denote the number of hops as k. Note that k is not a constant and its value is a function of the corresponding path. Fig. 2.1 illustrates a linear multihop network and a path connecting source to destination which can be represented as $\langle v_0, v_2, v_{N-2}, v_{N-1} \rangle$. In general, the total number of available paths between source and destination with no backward in a linear network with N nodes is $A = 2^{N-2}$.

In contrast to disk models in which each node can only communicate with the nodes inside a disk [11], a more realistic channel model is exploited here. We consider both path loss and multipath fading in our propagation model by assuming independent and non-identical Rayleigh models for the fading on the links of the network. Let h_{v_i,v_j} denote the channel gain between nodes v_i and v_j given as

$$h_{v_i,v_j} = \delta \lambda_{v_i,v_j} \left[\max\left(d_{v_i,v_j}, d_{ff}\right) \right]^{-\alpha/2} \tag{2.1}$$

where λ_{v_i,v_j} is a zero-mean, circularly symmetric, complex Gaussian random variable with unit variance, d_{v_i,v_j} is the distance between v_i and v_j, d_{ff} is the far-field distance which is a function of the largest dimension of an antenna and the carrier wavelength [34], and the path loss exponent is denoted by α. A typical value of α is in the range of 2 to 4 where 2 is for propagation in free space with a maximum distance of 100 m and larger values represent relatively lossy environments such as urban areas [35]. Also, δ is a constant which is assumed to be equal 1 throughout this thesis. Different values of δ will only change the scale of the total transmission power and do not affect the relative performance comparisons among the proposed schemes. Typically, d_{v_i,v_j} is much larger than d_{ff}; therefore, (2.1) is simplified to

$$h_{v_i,v_j} = \delta\lambda_{v_i,v_j} d_{v_i,v_j}^{-\alpha/2}\ , \quad d_{v_i,v_j} > d_{ff}. \tag{2.2}$$

At each node, zero-mean complex additive white Gaussian noise with variance $N_0/2$ per dimension is considered to corrupt the signal. It is assumed that the total available power for allocation among the nodes is P_{total}. The intuition behind the assumption of a total power constraint is to provide fair comparison among routes with different numbers of hops. If we only consider individual node power constraints, then routes with different numbers of hops consume different amounts of power. Let P_{t_i} denote the power allocation for the transmitting node t_i. We consider β_{t_i} to be the portion of the total power devoted to node t_i, hence $P_{t_i} = \beta_{t_i} P_{\text{total}}$ where $0 \le \beta_{t_i} \le 1$. So, the power allocation of route p can be represented by the vector $\beta_p = [\beta_{t_0}, ..., \beta_{t_{k-2}}, \beta_{t_{k-1}}]$. The total power constraint which ensures that the overall power is fixed in the system is given by $\sum_{i=0}^{k-1} \beta_{t_i} = 1$. For simplicity of notation, we define $\gamma = \frac{P_{\text{total}}}{N_0}$. Including path loss and fading, the average signal-to-noise ratio (SNR) of the link between v_i and v_j is given by

$$\bar{\gamma}_{v_i,v_j} = \beta_{v_i} \sigma_{v_i,v_j}^2 \gamma \tag{2.3}$$

where σ_{v_i,v_j}^2 is the mean channel power gain between nodes v_i and v_j.

Our goal is to find the path with the minimum end-to-end outage probability from source to destination. The solution requires an optimization over all the paths connecting these two nodes subject to the total power constraint. Therefore, the optimization problem is given by

$$\min_{\beta_p, p} P_{\text{outage},p} \tag{2.4a}$$

$$\text{subject to} \begin{cases} \sum_{i=0}^{k-1} \beta_{t_i} = 1 \\ 0 \le \beta_{t_i} \le 1 \text{ for } 0 \le i \le k-1. \end{cases} \tag{2.4b}$$

The power constraints in (2.4b) impose a maximum power constraint P_{total} for each individual node. In this chapter, we assume that all the transmitting terminals in the network are able to provide the maximum power P_{total}.

The solution to this problem depends on the availability of channel state information at each time slot. In order to solve (2.4), we consider two general cases. First, we use only the variances of the channel gains, which we refer to as the mean CSI case, to obtain the optimal solution. Then, supposing that the absolute values of the channel gains are available at each moment, which is referred to as the instantaneous CSI case, a more efficient routing protocol is proposed. For each case, two power allocation schemes are considered, equal power allocation (EPA) in which the transmitting nodes employ the same amount of power and optimal power allocation (OPA) which efficiently distributes the power among the nodes to improve the outage performance.

2.2 Optimal Routing Using Mean CSI

This section investigates joint power allocation and optimal path selection for minimizing the end-to-end outage probability when only mean CSI is available. In other words, only the large scale effect of channel gains (path loss) is taken into account in this section.

As shown in [2], the achievable end-to-end data rate of a decode-and-forward multihop communication over route p, R_p, is the minimum of the rate at each of k hops,

$$R_p = \min_{i=1,\ldots,k} R_i. \tag{2.5}$$

So, R_p represents the maximum rate of transmission over that route for which all the relays and destination can reliably decode the received signals. Outage occurs when the capacity of the source-destination pair falls below a specified threshold, namely path spectral efficiency R. The outage probability is given by [4]

$$\begin{aligned} P_{\text{outage},p} &= \Pr[\min_{i=1,\ldots,k} R_i < R] \\ &= 1 - \prod_{i=1}^{k} \Pr[R_i > R] \end{aligned}$$

$$
\begin{aligned}
&= 1 - \prod_{i=1}^{k}(1 - P_{\text{outage},i}) \\
&= 1 - \prod_{i=0}^{k-1}\Big(1 - \Pr[\frac{1}{k}\log_2(1+\gamma\beta_{t_i}|h_{t_i,t_{i+1}}|^2) < R]\Big) \\
&= 1 - \prod_{i=0}^{k-1}\Big(1 - \Pr[|h_{t_i,t_{i+1}}| < \sqrt{\frac{2^{kR}-1}{\gamma\beta_{t_i}}}]\Big) \\
&= 1 - \exp\big(-\sum_{i=0}^{k-1}\frac{2^{kR}-1}{\gamma\beta_{t_i}\sigma^2_{t_i,t_{i+1}}}\big)
\end{aligned}
\tag{2.6}
$$

where $P_{\text{outage},i}$ is the outage probability at the i-th hop. Since the system is working in TDMA mode and spatial reuse is not exploited, the communication process lasts k time slots. Therefore, there is a factor of $\frac{1}{k}$ for the rate of each hop. If a node cannot decode the message, there is no detour, rather, outage occurs. Note that eq. (2.6) indicates that in a decoded multihop relaying system, an outage event occurs when an outage occurs at any intermediate terminal along the multihop path. Substituting (2.6) in (2.4), the optimization problem is equivalent to

$$
\min_{\beta_p,p}\sum_{i=0}^{k-1}\frac{2^{kR}-1}{\beta_{t_i}\sigma^2_{t_i,t_{i+1}}} \tag{2.7a}
$$

$$
\text{subject to}\begin{cases}\sum_{i=0}^{k-1}\beta_{t_i}=1 \\ 0\le\beta_{t_i}\le 1 \text{ for } 0\le i\le k-1.\end{cases} \tag{2.7b}
$$

Therefore, the optimal outage probability can be computed as

$$
P^{\text{opt}}_{\text{outage}} = 1 - \exp\big(-\frac{1}{\gamma}\min_{\beta_p,p}\sum_{i=0}^{k-1}\frac{2^{kR}-1}{\beta_{t_i}\sigma^2_{t_i,t_{i+1}}}\big). \tag{2.8}
$$

It is inferred from (2.8) that the optimal outage probability achieved by optimal routing using mean CSI (OR-MCSI) decays as γ^{-1} for large SNR. Consequently, the diversity order offered by this scheme is always 1 and optimal routing and power allocation can only improve the diversity processing gain.

Note that direct transmission is a special case of multihop communication when $k = 1$. So, its outage performance can be obtained by assigning all the power to the source node, namely,

$$P_{\text{outage, DT}} = 1 - \exp\big(-\frac{2^R - 1}{\gamma} D^{\alpha}\big). \tag{2.9}$$

2.2.1 Equal Power Allocation

Assuming that an EPA strategy is used to distribute power among the nodes, one has $\beta_{t_i} = \frac{1}{k}$ for $0 \leq i \leq k-1$. Therefore, the routing algorithm is the solution to the optimization problem

$$\begin{aligned} p^{\text{opt}} &= \arg \min_{p \in \mathcal{P}(v_0, v_{N-1})} \sum_{i=0}^{k-1} \frac{2^{kR} - 1}{\beta_{t_i} \sigma^2_{t_i, t_{i+1}}} \\ &= \arg \min_{p} \; k(2^{kR} - 1) \sum_{i=0}^{k-1} d^{\alpha}_{t_i, t_{i+1}}. \end{aligned} \tag{2.10}$$

Special case: When the nodes are equally spaced, $d_{t_i, t_{i+1}} = \frac{D}{k}$ for $0 \leq i \leq k-1$. Therefore,

$$p^{\text{opt}} = \arg \min_{p} \; (2^{kR} - 1) k^{2-\alpha}. \tag{2.11}$$

Using (2.11), the most outage efficient linear network in which the nodes are spaced equally can be found. In other words, the optimal number of hops can be obtained when the design parameter is k, namely,

$$k^{\text{opt}} = \arg \min_{k} \; (2^{kR} - 1) k^{2-\alpha}. \tag{2.12}$$

In [26] and [36], the solution to a slightly different version of this expression was found for the case that the optimization objective is spectral efficiency. Following the approach taken in [26], the optimal number of hops for a given R which yields the smallest outage probability can be approximately computed as

$$k^{\text{opt}} \approx \Big[\frac{\alpha - 2 + \omega\big((2-\alpha) e^{-\alpha+2}\big)}{R \ln 2} \Big]_{+} \tag{2.13}$$

where $\omega(x)$ is the principal branch of the Lambert W function [37] and $[x]_{+}$ is the closest positive integer to x.[4] Fig. 2.2 shows the numerically computed k^{opt} as a

[4]Our simulation results indicate that the rounding function provides closer answers to the exact solution of eq. (2.12) compared to ceiling or floor functions.

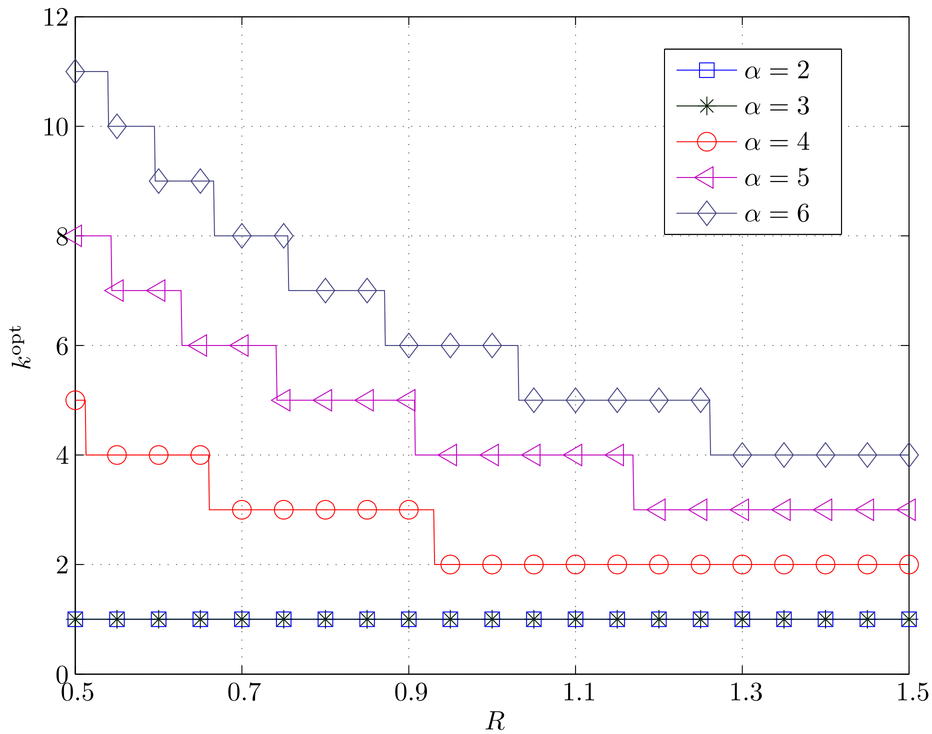

Fig. 2.2. The optimal number of hops for OR-MCSI with EPA in an equally spaced linear network.

function of R for different values of α. It is observed that as α increases, the required number of hops for optimal routing increases. Intuitively, the faster the attenuation of the channel gains with distance, the greater the number of nodes that will be needed to compensate for the path loss. Note that for the cases of $\alpha = 2$ and $\alpha = 3$, direct transmission is the best routing strategy. Generally, the power assigned to each node in EPA is proportional to $\frac{1}{k}$ and the average channel power gain changes as k^{α} with k. So, the average per-hop SNR increases as $k^{\alpha-1}$ with an increase in k. Although the average per-hop SNR is a linear function of k and k^2 in the cases of $\alpha = 2$ and $\alpha = 3$, respectively, this increase is not enough to compensate the effect of the $\frac{1}{k}$ factor in the transmission rate in eq. (2.6). Consequently, the best route for communication between source and destination is the direct link. We point out that an explicit optimal solution for k^{opt} is only available for the case when the nodes are equally spaced. For a different network topology, i.e., when the nodes are distributed randomly, simulation must be used to find the optimal number of hops.

2.2.2 Optimal Power Allocation

For each path p, we allocate the optimal power to the nodes and then, the optimal route is found by a global search over all the paths. OPA requires knowledge of all the channel gains and can be implemented in a centralized way. It was shown in [30] that the OPA for the routing path p is computed by

$$\beta_{t_i}^{\text{opt}} = \frac{1}{\sigma_{t_i,t_{i+1}} \sum_{i=0}^{k-1} 1/\sigma_{t_i,t_{i+1}}} \qquad 1 \leq i \leq k. \tag{2.14}$$

Using (2.14), the optimization problem is simplified to

$$p^{\text{opt}} = \arg\min_{p} \; k(2^{kR} - 1)\Big(\sum_{i=0}^{k-1} d_{t_i,t_{i+1}}^{\alpha/2}\Big)^2. \tag{2.15}$$

In the case of equally spaced nodes, $\sigma_{t_i,t_{i+1}}^2$ is the same for $0 \leq i \leq k-1$. So, in this case optimal power allocation is equivalent to uniform power allocation.

2.3 Optimal Routing Using Instantaneous CSI

In the previous section, we employed only path loss to design the optimal routing algorithm. But multipath fading may be even more important in determining the channel attenuations and should be considered in protocol design. Here, we suppose that instantaneous CSI is available and can be employed for designing a more efficient routing strategy. One can rewrite the outage performance as

$$\begin{aligned} P_{\text{outage},p} &= \Pr[\, R_p < R] \\ &= \Pr[\min_{i=0,\ldots,k-1} \; R_i < R] \\ &= \Pr[\min_{i=0,\ldots,k-1} \; \frac{1}{k}\log_2(1+\gamma\beta_{t_i}|h_{t_i,t_{i+1}}|^2) < R]. \end{aligned} \tag{2.16}$$

Eq. (2.16) indicates that outage happens when the achievable rate of the weakest channel of p becomes less than R. Returning to the main optimization, we have

$$\begin{aligned} \left\{p^{\text{opt}}, \beta_p^{\text{opt}}\right\} &= \arg\min_{\beta_p,p} P_{outage,p} \\ &= \arg\min_{\beta_p,p} \Pr[\min_{i=0,\ldots,k-1} \; \frac{1}{k}\log_2(1+\gamma\beta_{t_i}|h_{t_i,t_{i+1}}|^2) < R] \end{aligned}$$

$$= \arg\max_{\beta_p, p} \Big[\min_{i=0,\ldots,k-1} \frac{1}{k} \log_2(1 + \gamma\beta_{t_i} |h_{t_i,t_{i+1}}|^2) \Big]. \tag{2.17}$$

Therefore, the main requirement for solution is to find the channel with the lowest received SNR among the hops of a route for all paths. Then, the route with the maximum achievable rate is chosen. So, the optimal outage probability is given by

$$P_{\text{outage}}^{\text{opt}} = \Pr\Big[\max_{\beta_p, p} \frac{1}{k} \log_2(1 + \min_{i=0,\ldots,k-1} [\gamma\beta_{t_i} |h_{t_i,t_{i+1}}|^2]) < R \Big]. \tag{2.18}$$

Similar to the observations made in [21], it is not trivial to derive a closed-form expression for $P_{\text{outage}}^{\text{opt}}$ in general because the routes share some common links and are not independent.

Special case: We consider a network with three nodes. Since there are only two independent paths between source and destination terminals, a closed-form expression for the optimal outage can be derived. Transmission can follow the direct link from source to destination and a relayed path through the middle node, corresponding to the routes $p_1 = \langle v_0, v_2 \rangle$ and $p_2 = \langle v_0, v_1, v_2 \rangle$, respectively. So, the outage probability is given as

$$\begin{aligned} P_{\text{outage}}^{\text{opt}} &= \Pr\big[\max\, [R_{p_1}, R_{p_2}] < R \big] \\ &= P_{\text{outage},p_1} \times P_{\text{outage},p_2} \\ &= \Big(1 - \exp\big(-\frac{2^R - 1}{\gamma}\big)\Big) \\ &\times \Big(1 - \exp\big(-\frac{2^{2R} - 1}{\gamma\beta_{v_0} d^{-\alpha}} - \frac{2^{2R} - 1}{\gamma\beta_{v_1} (D-d)^{-\alpha}}\big)\Big) \end{aligned} \tag{2.19}$$

where d is the distance between source and relay and consequently $D - d$ is the distance between relay and destination. To derive the outage probability of transmission over p_2, the total power is divided between v_0 and v_1. But for the case of direct communication, all the power is assigned to the source node. For large values of SNR, using a Taylor series expansion, it can be shown that

$$P_{\text{outage}}^{\text{opt}} \approx \frac{(2^R - 1)(2^{2R} - 1)}{\gamma^2} \Big(\frac{d^\alpha}{1 - \beta_{v_1}} + \frac{(D-d)^\alpha}{\beta_{v_1}} \Big). \tag{2.20}$$

Clearly, the solution in (2.20) achieves full diversity order for a three-node network since γ^2 appears in the denominator.

Generally, in a network with N nodes there are at least $N-1$ independent paths between source and destination. Direct source-destination and $N-2$ source-relay-destination routes do not include any shared links. Note that outage occurs when all the routes are in outage. Therefore, the probability of end-to-end outage is less than the probability of outage for the aforementioned $N-1$ routes. So,

$$\begin{aligned} P_{\text{outage}}^{\text{opt}} &\leq P_{\text{outage},\langle v_0, v_{N-1}\rangle} \times \prod_{i=1}^{N-2} P_{\text{outage},\langle v_0, v_i, v_{N-1}\rangle} \\ &= \Big(1 - \exp\big(-\frac{2^R - 1}{\gamma}\big)\Big) \\ &\times \prod_{i=1}^{N-2} \Big(1 - \exp\big(-\frac{2^{2R} - 1}{\gamma \beta_{v_0} d_{v_0,v_i}^{-\alpha}} - \frac{2^{2R} - 1}{\gamma \beta_{v_i} d_{v_i,v_{N-1}}^{-\alpha}}\big)\Big). \end{aligned} \tag{2.21}$$

For large SNR,

$$P_{\text{outage}}^{\text{opt}} \leq \frac{(2^R - 1)}{\gamma^{N-1}} \prod_{i=1}^{N-2} (2^{2R} - 1)\Big(\frac{d_{v_0,v_i}^{\alpha}}{1 - \beta_{v_i}} + \frac{d_{v_i,v_{N-1}}^{\alpha}}{\beta_{v_i}}\Big). \tag{2.22}$$

Since the maximum diversity order offered by a network with $N-1$ transmitting nodes is $N-1$ and the upper bound obtained for the optimal outage probability decays as $\frac{1}{\gamma^{N-1}}$, optimal routing using instantaneous CSI (OR-ICSI) always achieves full diversity order with any reasonable power allocation scheme.

2.3.1 Equal Power Allocation

Employing equal powers at each hop, the optimization problem is

$$p^{\text{opt}} = \arg\max_{p} \Big[\frac{1}{k} \log_2\big(1 + \frac{\gamma}{k} \min_{i=0,\dots,k-1} |h_{t_i,t_{i+1}}|^2\big)\Big]. \tag{2.23}$$

Generally, the outage probability of transmission over route p depends on the link with the weakest channel gain. So, the bottleneck link is determined for each path and, then a comparison among all the paths is made to find the optimal relays.

2.3.2 Optimal Power Allocation

The optimal power allocation can be formulated as

$$\max_{\beta_p} \Big[\min_{i=0,\ldots,k-1} \frac{1}{k} \log_2(1+\gamma\beta_{t_i}|h_{t_i,t_{i+1}}|^2) \Big] \tag{2.24}$$

which is equivalent to

$$\max_{\beta_p} \Big[\min_{i=0,\ldots,k-1} \beta_{t_i}|h_{t_i,t_{i+1}}|^2 \Big]. \tag{2.25}$$

Using the method of Lagrange multipliers [38], the optimal power allocation coefficients can be found as

$$\beta_{t_i}^{\text{opt}} = \frac{1}{|h_{t_i,t_{i+1}}|^2 \sum_{i=0}^{k-1} |h_{t_i,t_{i+1}}|^{-2}}. \tag{2.26}$$

Then, the received SNR at each hop of p is

$$\gamma_{t_i} = \frac{\gamma}{\sum_{i=0}^{k-1} |h_{t_i,t_{i+1}}|^{-2}} \tag{2.27}$$

which is constant for all the hops. The optimal power allocation scheme alleviates the shortcoming of the EPA scheme by eliminating the dependency of the performance on the bottleneck link. The power is efficiently allocated among the nodes of a route so that all the links have equal SNR. Substituting the optimal power in (2.17), the min operator will have the same values of argument for all the hops of each route. Consequently, the optimization can be simplified to

$$p^{\text{opt}} = \arg\max_{p} \Big[\frac{1}{k} \log_2(1+\frac{\gamma}{\sum_{i=0}^{k-1} |h_{t_i,t_{i+1}}|^{-2}}) \Big]. \tag{2.28}$$

The proposed routing algorithms, OR-MCSI and OR-ICSI, both require a centralized environment and global channel information to be executable. In principle, the exhaustive search method over 2^{N-2} different paths is not feasible in networks with large numbers of nodes, even if a centralized implementation is possible.

2.3.3 Extension to Multihop Diversity Networks

In this subsection, we extend the results of Section 2.3.1 to multihop diversity networks. The nodes cooperating in a multihop diversity system combine and process the signals received from all their preceding terminals. However, the relays in a multihop system without diversity only consider the signal from the immediate preceding node. In general, the signals received at node t_m through its preceding nodes t_0, t_1, ..., t_{m-1} are combined using maximal ratio combining (MRC). Then, the output signal is decoded, re-encoded, and broadcast in the $(m+1)$-th time slot. Assuming equal power allocation, the instantaneous received SNR at t_i, $i = 1, ..., k$ is given by [39]

$$\gamma_{t_i} = \frac{\gamma}{k} \sum_{j=0}^{i-1} |h_{t_j,t_i}|^2. \tag{2.29}$$

So, the optimal route is obtained as

$$p^{\text{opt}} = \arg\max_{p} \Big[\min_{i=0,\dots,k-1} \ \frac{1}{k} \log_2 (1 + \frac{\gamma}{k} \sum_{j=0}^{i} |h_{t_j,t_{i+1}}|^2) \Big]. \tag{2.30}$$

Comparing eqs. (2.30) and (2.23), the superior performance of the multihop diversity system over that of the corresponding system without diversity is easily appreciated.

2.4 Low Complexity Routing Algorithms

In this section, we propose two near optimal algorithms with polynomial time complexity. First, we discuss how to simplify the proposed algorithm via a local search instead of an exhaustive search, and then a distributed algorithm based on the Bellman-Ford routing protocol is presented to apply the proposed outage efficient routing algorithm to large scale networks.

2.4.1 Local Search Based Routing

As already remarked, the complexity of the proposed optimal routing algorithm is $\mathcal{O}(2^N)$ which increases exponentially with the number of nodes. Therefore, a solution with a reasonable complexity is needed to take advantage of the high efficiency of the proposed algorithm. A brute-force search provides a solution based on searching among all the possible routes with any number of hops. The number of intermediate relays can be limited to an upper bound, namely M, to reduce the number of paths and hence the search space. If we consider in particular the OR-ICSI, at least a search over direct transmission and source-relay-destination routes is necessary to provide full diversity order. Consequently, M must be greater than 0 and less than $N-1$; therefore $1 \leq M \leq N-2$. Generally, employing this method, the total number of possible paths reduces to $\sum_{k=0}^{M} \binom{N-2}{k}$ which is equal to 1 and 2^{N-2} for $M=0$ and $M=N-2$, respectively. In the cases of $M=1$ and $M=2$, the algorithm has complexity of $\mathcal{O}(N)$ and $\mathcal{O}(N^2)$, respectively. In general, the complexity of local search routing (LSR) for a fixed value of M is $\mathcal{O}(N^M)$. This is a significant reduction in the search space compared to exhaustive search. The performance of LSR for different values of M will be investigated by simulation. A closed-form expression for the outage performance of LSR is only available for the case of $M=1$. The upper bound of $P_{\text{outage}}^{\text{opt}}$ in eq. (2.21) for OR-ICSI is equal to the exact outage performance of LSR with $M=1$.

2.4.2 Distributed Routing

In this section, we examine the trade-off between implementation complexity and outage performance. In a large network, each node must be able to compute the best paths to the other nodes in the network by exchanging certain information with its neighbors. A distributed routing protocol can efficiently facilitate the information exchange and hence the signal transmissions in the network.

There are two important properties, monotonicity and isotonicity, that ensure a dynamic routing protocol converges onto optimal paths in a network [40]. Defining the total order as the greater-than relation over nonnegative numbers, the monotonicity implies that the path metric does not increase when a new link is concatenated. In other words, along any path starting from the origin, the metric value is non-increasing. Isotonicity implies that the metric relationship between two different paths with the same origin and end holds if both of them are extended by a common link. Monotonicity leads to protocol convergence in any network and isotonicity assures that the routing algorithm converges onto optimal routes. Unfortunately, the routing metrics in (2.23) and (2.28) are not isotone but it can be simply verified that they are monotone. Therefore, these routing metrics can be used in any network and they converge onto suboptimal paths.

The proposed distributed routing protocols for OR-ICSI using EPA and OPA are based on the Bellman-Ford routing algorithm. First, all the nodes in the network must initialize a module to determine their channel gains to their neighbors and hence update their initial routing table. Then, each node should exchange its routing information (represented by the metric vector here) with its neighbors. The received metric vectors are employed at each node to find the best outage efficient path to possible destination nodes. After the routing tables are completed and the new routes are explored, each node sends its updated metric vector to its neighbors. This procedure is repeated periodically.

In our algorithm, the link metric between nodes v_i and v_j is defined as the power gain of the corresponding channel, $|h_{v_i,v_j}|^2$. The path metrics employed are given by (2.23) and (2.28). Each routing table entry corresponds to one destination node and contains three parameters, i.e., path signature [40], next hop, and number of hops to destination. We define the path signature as $\min_i |h_{t_i,t_{i+1}}|^2$ for EPA and $\sum_i \frac{1}{|h_{t_i,t_{i+1}}|^2}$ for OPA. A brief description of the proposed protocols is presented in Algorithms 2.1 and 2.2.

2.5 Simulation Results

In this section, simulation results are presented to evaluate the outage performances of the proposed routing strategies and to validate our theoretical analysis.

2.5.1 Outage in Linear Networks

In this subsection, we consider a linear multihop network in which the nodes are randomly distributed with uniform distribution. The outage probability is averaged over 10^4 network realizations. We assume a path loss with multipath fading model described in eq. (2.1), taking the path loss exponent $\alpha = 4$ and the far-field distance $d_{ff} = 0.1$. For our simulations, the number of nodes in the network, N, is equal to 6, the path spectral efficiency, R, is set to 1 bit/s/Hz, and the distance between source and destination $D = 100$, unless specified otherwise. Also, note that the simulation

Algorithm 2.1 Summary of the approximate distributed routing strategy for OR-ICSI with EPA in DF networks from node t_i's view

Initialization:

For all neighbors t_k, the routing table entries are set as
  $M(t_i, t_k) = |h_{t_i,t_k}|^2$;   // path signature
  $H(t_i, t_k) = t_k$;   // next hop
  $N(t_i, t_k) = 1$;   // number of hops to t_k
End For
Node t_i's metric vector to itself is set to $M(t_i, t_i) = \infty$; $H(t_i, t_i) = t_i$; $N(t_i, t_i) = 0$;

Exchanging metric vectors and updating routing table:

If the received metric vector from t_k contains a new destination node t_w, the new node is added to the routing table as
  $M(t_i, t_w) = 0$;
  $H(t_i, t_w) = \varnothing$;
  $N(t_i, t_w) = 0$;
End If
For all destination nodes t_j
  If $(H(t_i, t_j) = t_k)$
    $M(t_i, t_j) = \min\left[|h_{t_i,t_k}|^2, M(t_k, t_j)\right]$;
    $N(t_i, t_j) = 1 + N(t_k, t_j)$;
  End If
End For

Finding the best path:

For all destination nodes t_j
  $C_{t_i,t_j} = \frac{1}{N(t_i,t_j)} \log_2(1 + \frac{\gamma}{N(t_i,t_j)} M(t_i, t_j))$;
  For all neighbors t_k
    $temp = \frac{1}{1+N(t_k,t_j)} \log_2(1 + \frac{\gamma}{1+N(t_k,t_j)} \min[|h_{t_i,t_k}|^2, M(t_k, t_j)])$;
    If $(temp > C_{t_i,t_j})$
      $M(t_i, t_j) = \min\left[|h_{t_i,t_k}|^2, M(t_k, t_j)\right]$;
      $H(t_i, t_j) = t_k$;
      $N(t_i, t_j) = 1 + N(t_k, t_j)$;
    End If
  End For
End For

results of OR-ICSI and LSR are obtained based on an exhaustive search.

Fig. 2.3 shows the end-to-end outage probability achieved by the proposed routing protocols. In addition, the outage performance of direct transmission is plotted for comparison. The outage is plotted as a function of Γ which is the average link SNR between source and destination in direct transmission and is given as $\Gamma = \gamma\sigma^2_{v_0,v_{N-1}}$. As shown in the theoretical analysis, the optimal routing algorithm which uses knowledge of the instantaneous channel conditions achieves full diversity order equal to

Algorithm 2.2 Summary of the approximate distributed routing strategy for OR-ICSI with OPA in DF networks from node t_i's view

Initialization:

For all neighbors t_k, the routing table entries are set as
  $M(t_i, t_k) = \frac{1}{|h_{t_i,t_k}|^2}$;
  $H(t_i, t_k) = t_k$;
  $N(t_i, t_k) = 1$;
End For
Node t_i's metric vector to itself is set to $M(t_i, t_i) = 0$; $H(t_i, t_i) = t_i$; $N(t_i, t_i) = 0$;

Exchanging metric vectors and updating routing table:

If the received metric vector from t_k contains a new destination node t_w, the new node is added in the routing table as
  $M(t_i, t_w) = \infty$;
  $H(t_i, t_w) = \varnothing$;
  $N(t_i, t_w) = 0$;
End If
For all destination nodes t_j
  If ($H(t_i, t_j) = t_k$)
    $M(t_i, t_j) = \frac{1}{|h_{t_i,t_k}|^2} + M(t_k, t_j)$;
    $N(t_i, t_j) = 1 + N(t_k, t_j)$;
  End If
End For

Finding the best path:

For all destination nodes t_j
  $C_{t_i,t_j} = \frac{1}{N(t_i,t_j)} \log_2(1 + \frac{\gamma}{M(t_i,t_j)})$;
  For all neighbors t_k
    $temp = \frac{1}{1+N(t_k,t_j)} \log_2(1 + \frac{\gamma}{\frac{1}{|h_{t_i,t_k}|^2}+M(t_k,t_j)})$;
    If ($temp > C_{t_i,t_j}$)
      $M(t_i, t_j) = \frac{1}{|h_{t_i,t_k}|^2} + M(t_k, t_j)$;
      $H(t_i, t_j) = t_k$;
      $N(t_i, t_j) = 1 + N(t_k, t_j)$;
    End If
  End For
End For

$N - 1$ for both equal and optimal power allocation schemes. This is also observed in Fig. 2.3. Furthermore, optimal power allocation offers significant reduction in outage probability relative to EPA. The mean CSI only depends on the location of the nodes. So, while the nodes in the network are fixed, OR-MCSI finds the same path irrespective of channel gain variations. Thus, as observed in Fig. 2.3, it is expected that OR-MCSI achieves the same diversity order as direct transmission. EPA for this case shows a slight performance loss compared to OPA. EPA and OPA implementations of OR-MCSI enhance the network performance little compared to direct communication.

The average number of hops versus Γ is illustrated in Fig. 2.4. Interestingly, OR-MCSI has the same average number of hops for all values of SNR. OR-ICSI, which outperforms OR-MCSI significantly, employs fewer nodes to send the data from source to destination. As the SNR increases, the average number of hops for OR-ICSI goes to one. In other words, at large SNR the direct source-destination link becomes advantageous to a multihop approach. In the case of OR-ICSI, OPA uses more relays on average compared to EPA. Also, the results in the figure show that the average number of relays used for OR-ICSI is fewer than 2. The number of helping nodes, k, significantly affects the spectral efficiency, as it appears in the denominators of the routing metrics (2.23) and (2.28). On one hand, when the number of hops gets larger, the distance between the transmitting nodes decreases, and so the quality of the links is improved. On the other hand, transmission over more hops reduces the spectral efficiency. Therefore, if the improvement of channel quality is dominant in the spectral efficiency, i.e., the path loss exponent is sufficiently large, the optimal routing chooses the routes with more relaying nodes and the number of relaying hops increases. This conclusion is consistent with the results of Section 2.2.1. Also, the results in Fig. 2.5, which show the average number of hops versus the path loss exponent, validate this statement.

The effect of path loss exponent on the efficiency of the proposed routing protocols is examined in Fig. 2.6 when $\Gamma = 15$ dB. Obviously, the larger the value of α, the greater the attenuation of the transmitted signals. Therefore, the outage probability of the proposed optimal routings decreases as α increases.

Fig. 2.7 compares the performance of OR-ICSI for several numbers of nodes in multihop systems without diversity to multihop diversity systems. A slight performance improvement is achieved in multihop diversity systems. The comparison between the outage performances of EPA and OPA in multihop systems shows a 3 dB diversity processing gain gap between them. Also, the full diversity order achieved by both power allocation schemes is evident in the figure.

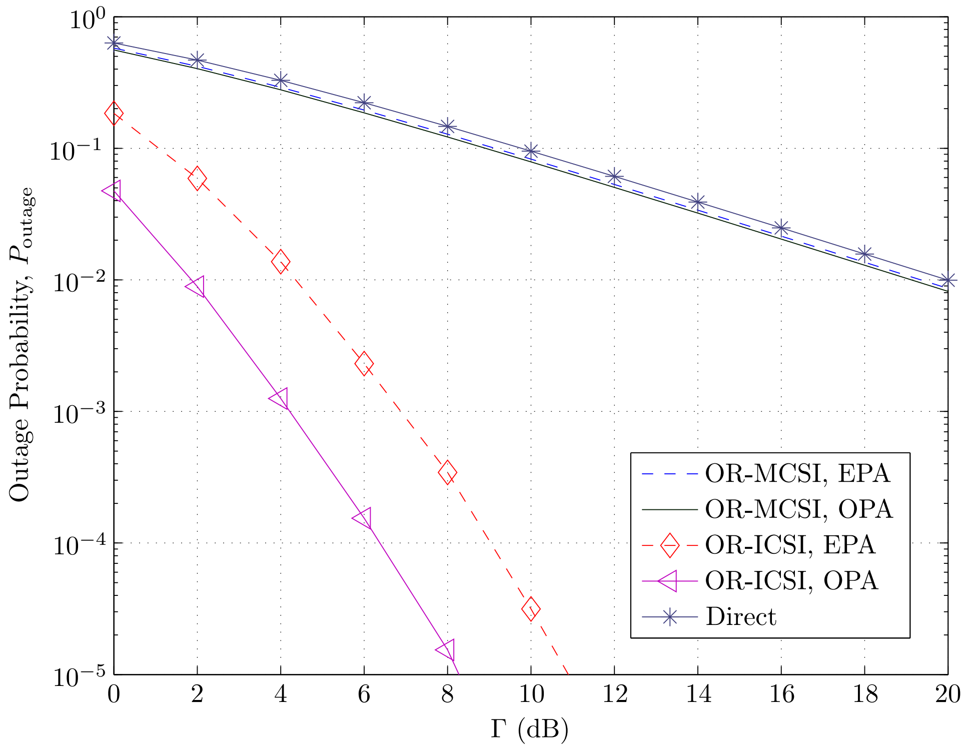

Fig. 2.3. The outage probability as a function of average source-destination SNR for the proposed routing algorithms in linear networks ($N = 6$).

2.5.2 Comparison of the Proposed Routing Algorithms

In this subsection, we investigate the outage performances attained by the proposed protocols in linear networks. The system parameters are the same as in Section 2.5.1. Fig. 2.8 presents the outage probability of OR-ICSI, local search based routing, and distributed routing as a function of Γ for both EPA and OPA. The LSR algorithm performs as well as the optimal routing for $M = 3$ where the routes with a maximum number of 3 relays are considered only. As shown in Fig. 2.8, when M increases, the performance of LSR approaches that of optimal routing. Obviously, in the case of $M = N - 2$, it performs exactly the same as optimal routing. The distributed routing shows a slight performance loss compared to optimal routing; this is mainly due to the algorithm finding suboptimal solutions.

The end-to-end outage probability of the proposed algorithms with EPA is illustrated in Fig. 2.9 for several values of N. The distance between source and destination is not fixed in Fig. 2.9 and is chosen randomly from $[50, 1000]$ for each network realization. Those of the proposed algorithms which use instantaneous CSI

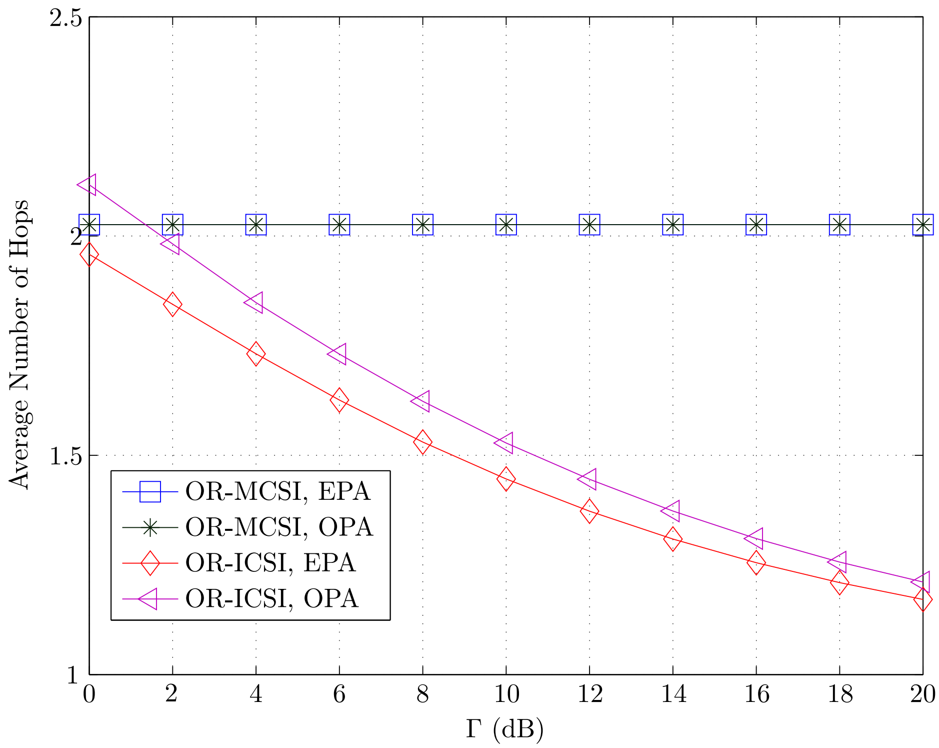

Fig. 2.4. The average number of hops as a function of average source-destination SNR for the proposed routing algorithms in linear networks ($N = 6$).

achieve full diversity order. For small values of N, LSR and distributed routing perform as well as OR-ICSI. However, when N increases, a small performance gap appears. A 1 dB gap between optimal and distributed routing is observed in the case of $N = 17$ at an outage of 10^{-4}. LSR outperforms distributed routing and performs close to optimal routing even in large networks.

2.5.3 Outage in Two-Dimensional Networks

In this subsection, a two-dimensional random network is considered where the source and destination nodes are located at $(0, 0)$ and $(100, 100)$, respectively. The other nodes in the network are uniformly distributed along the x and y axes. The outage performance is averaged over 10^4 network realizations. The number of nodes, path loss exponent, and spectral efficiency are the same as in Section 2.5.1. Fig. 2.10 presents the outage probability of distributed routing with EPA and OPA versus Γ for different values of N. According to the results in Fig. 2.10, the same conclusions drawn for linear networks can be also drawn in this case. Full diversity order is

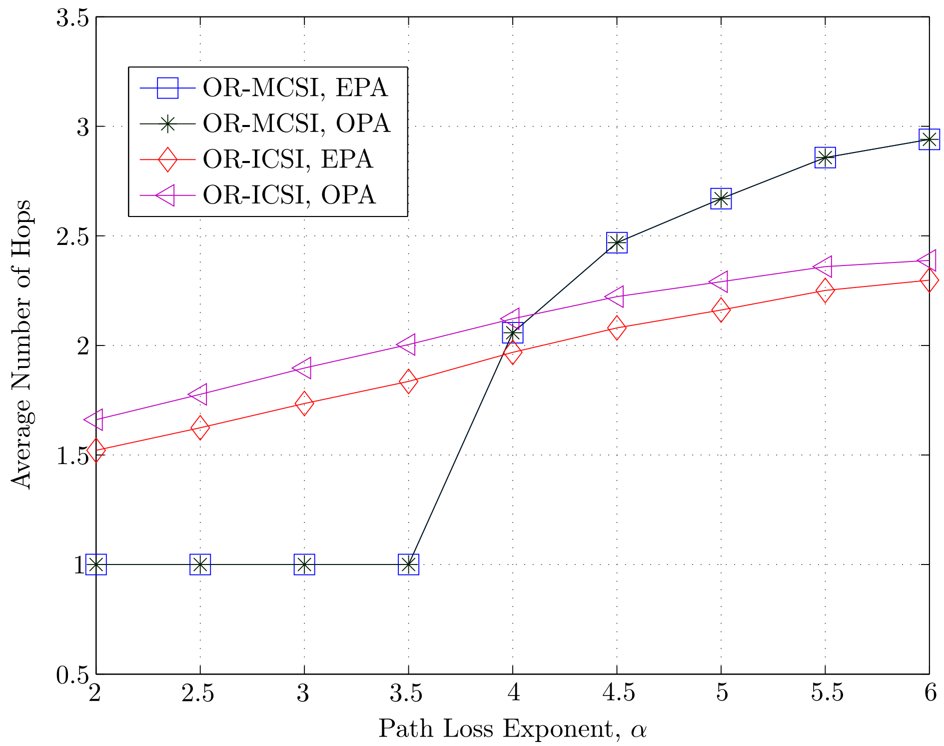

Fig. 2.5. The average number of hops for different values of α in linear networks ($N = 6$ and $\Gamma = 15$ dB).

attained based on using instantaneous CSI for finding an outage efficient route. Fig. 2.10 shows that the proposed distributed algorithms perform in two-dimensional networks as well as in linear networks.

2.6 Summary

An efficient routing algorithm with the goal of minimization of the end-to-end outage probability from source to destination was proposed for a decode-and-forward multihop relaying network operating in fading. We developed our routing algorithm for both conditions when mean or instantaneous CSI are available. The proposed routing strategy was simplified for both cases of optimal and equal power allocation. It was proved that only the scheme which uses instantaneous channel responses can achieve full diversity order regardless of power allocation. Through optimal power allocation, significant improvement in outage probability is obtained at the expense of a more complex power allocation scheme. Also, we showed that routing with mean CSI does not improve the diversity order compared to a direct link. Since the opti-

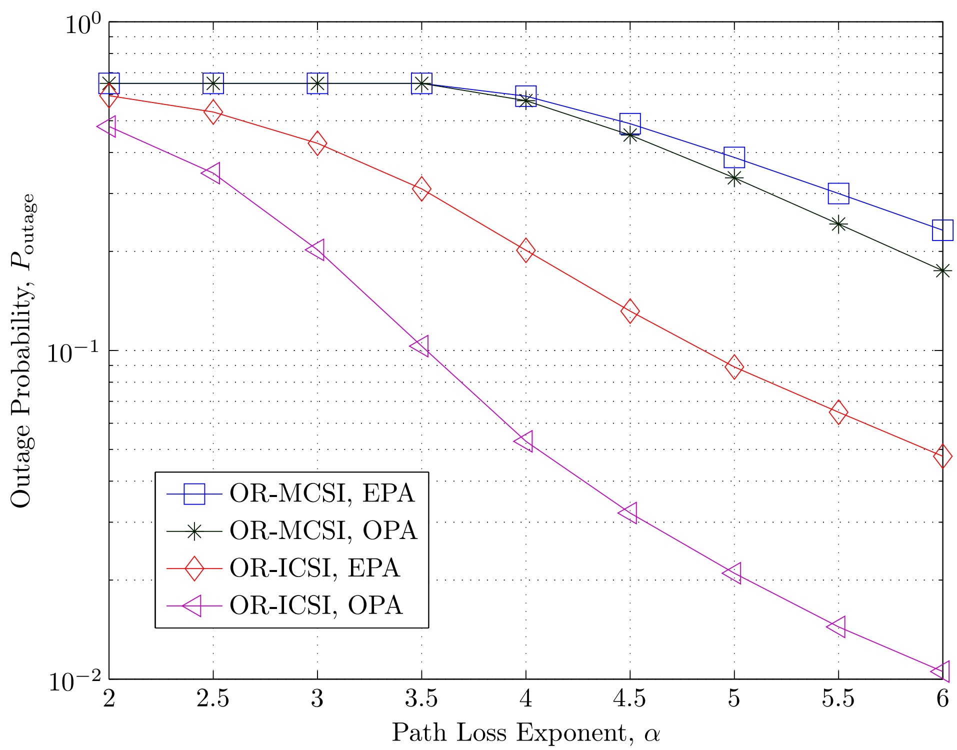

Fig. 2.6. The outage probabilities of the proposed routing algorithms versus path loss exponent in linear networks ($N = 6$ and $\Gamma = 15$ dB).

mal routing requires a brute-force search, which is not practical in large networks, a local search based routing which restricts the search space to routes with up to M relays was also considered. The simulation results showed that $M = 3$ leads to a good trade-off between complexity and performance. Moreover, a distributed routing scheme based on the Bellman-Ford routing algorithm was proposed and its performance was investigated.

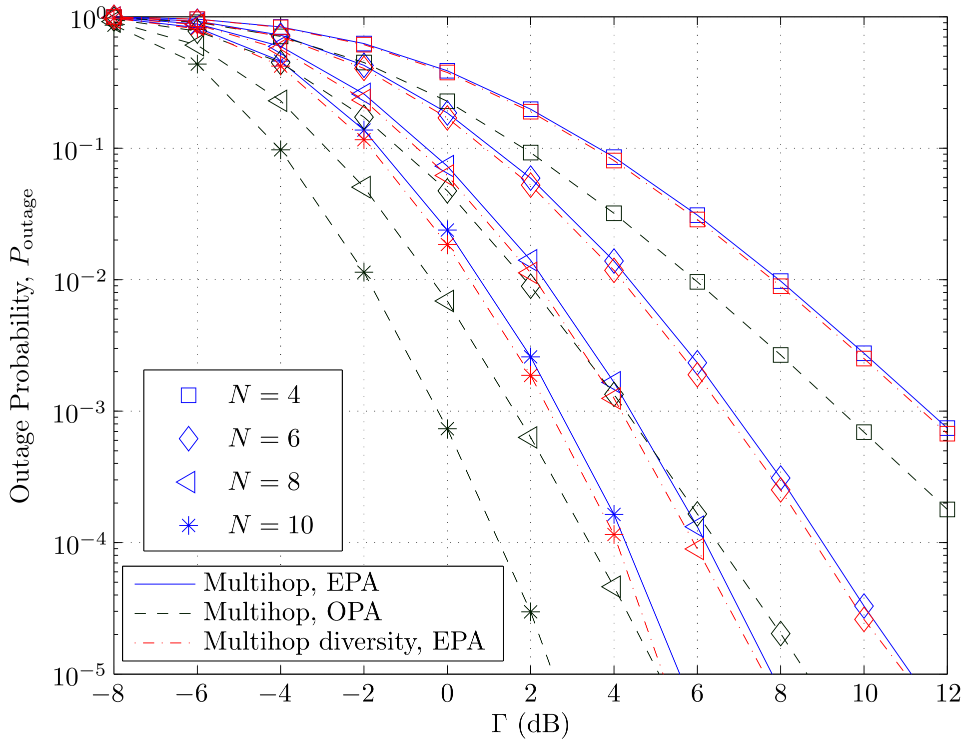

Fig. 2.7. Comparison of the outage probability of OR-ICSI for several numbers of nodes in linear multihop systems without diversity to the outage probability of linear multihop diversity systems.

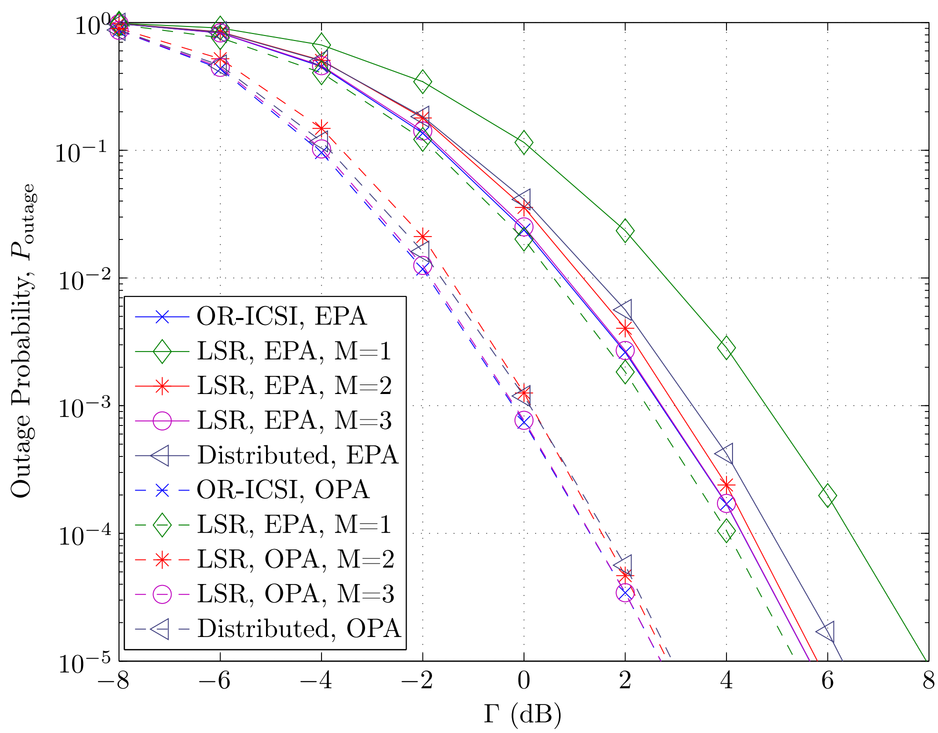

Fig. 2.8. Outage performances of optimal routing, local search based routing, and distributed routing in linear networks.

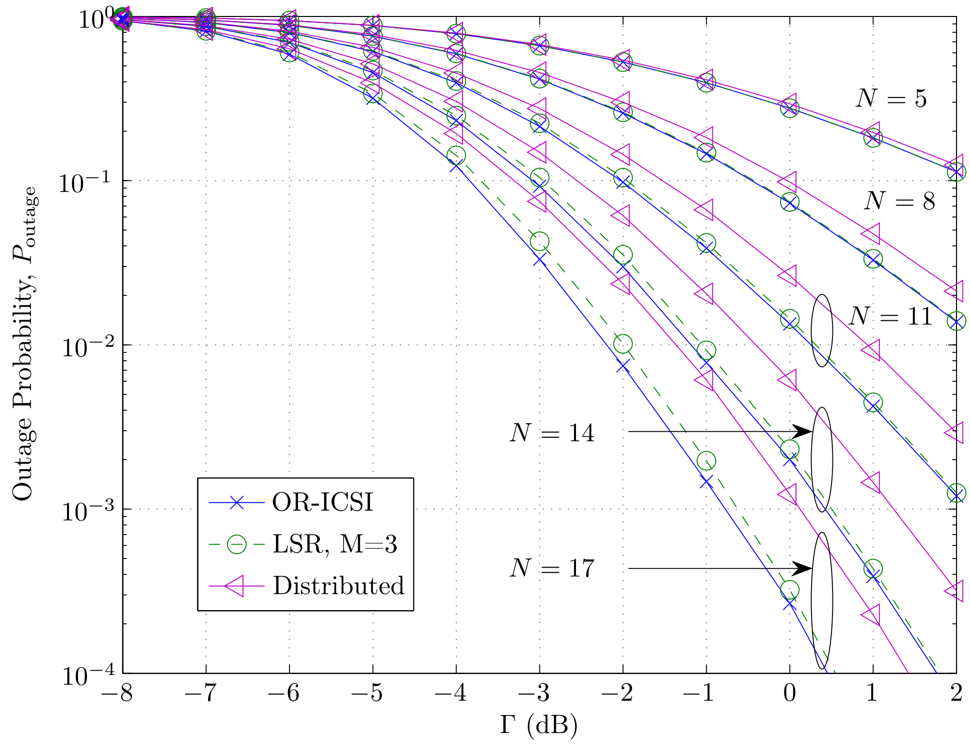

Fig. 2.9. Outage performance comparisons of the proposed routing strategies with EPA for different numbers of nodes in linear networks.

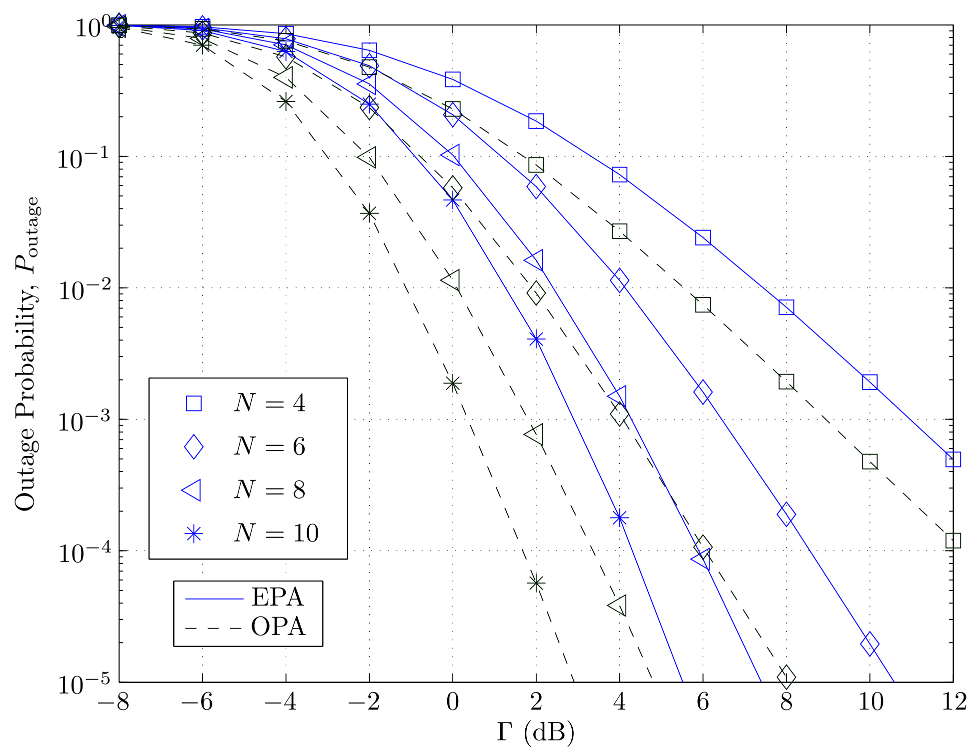

Fig. 2.10. The outage probability of the proposed approximate distributed routing algorithm for several numbers of nodes in two-dimensional networks.

Chapter 3

Distributed Optimal Outage Efficient Routing [1]

The self configuration characteristic of wireless networks necessitates the utilization of a distributed routing algorithm, since a centralized approach is not feasible. Using the broadcast advantage of wireless communications together with instantaneous channel measurements, an optimal distributed routing algorithm is proposed for finding the most outage efficient routes in AF and DF multihop networks. As discussed in Chapter 2, design of optimal outage efficient routing leads to a routing metric which is not implementable in an optimal distributed approach and using the Bellman-Ford algorithm for the routing metric obtained results in a significant performance loss compared to the optimal approach. In this chapter, we establish an alternate framework by converting the obtained metrics into new composite metrics which are suitable for distributed implementation, and can be executed in polynomial time. We focus on the essential properties of routing algebra and prove that our proposed composite metrics satisfy the optimality and consistency requirements. It is proved that the new optimal outage efficient routing algorithm achieves full diversity order.

The remainder of this chapter is organized as follows. The system model and assumptions are described in Section 3.1. In Section 3.2, we formulate the problem

[1] A version of this chapter has been published in part in *Proceedings of IEEE Global Telecommunications Conference (GLOBECOM)* [41] and has been submitted to *IEEE Transactions on Communications* [42].

of finding the optimal path with minimum end-to-end outage probability for deployment in AF and DF networks. An optimal distributed route selection algorithm, routing table construction, and packet forwarding algorithm are proposed in Section 3.3. The computational complexity of the proposed routing algorithm is assessed in Section 3.4. Section 3.5 presents simulation results along with some discussion. Finally, Section 3.6 concludes the chapter.

3.1 System Model

We consider a two-dimensional multihop relaying network operating in TDMA. Consequently, there is only one data transmission during each particular time slot. Also, TDMA provides symmetric channel gains among the nodes. The nodes are working in half-duplex mode and each node is equipped with a single omnidirectional antenna. In our model, the nodes are distributed uniformly random and are numbered from v_0 to v_{N-1}. So, the set of nodes is denoted as $\mathcal{V} = \{v_0, v_1, ..., v_{N-1}\}$ where v_0 is the source node and v_{N-1} is the corresponding destination node. Let $\mathcal{P}(v_0, v_{N-1}) = \{p : p = \langle t_0, ..., t_{k-1}, t_k\rangle\}$ be the set of all possible paths connecting source to destination where $t_0 = v_0$, $t_k = v_{N-1}$, and $t_i \in \{v_1, v_2, \cdots, v_{N-2}\}$, $1 \leq i \leq k-1$. Note that the number of hops of path p is equal to k. In general, the maximum number of available paths between source and destination in a two-dimensional network, L, is equal to the sum of all path permutations with any length. So, in a network with N nodes,

$$A = \sum_{i=0}^{N-2} \frac{(N-2)!}{i!} \tag{3.1}$$

which can be upper-bounded by $(N-2)! \times e$ where $e = \exp(1)$.

In our propagation model, we assume independent and non-identical block Rayleigh fading on the links of the network and our analysis considers both path loss and multipath fading. Let h_{v_i,v_j} denote the channel gain between v_i and v_j modeled as a zero-mean, circularly symmetric, complex Gaussian random variable. The effect of path loss is captured in the variance of the channel complex gains, namely,

$$\sigma^2_{v_i,v_j} = \left[\max(d_{v_i,v_j}, d_{ff})\right]^{-\alpha} \tag{3.2}$$

where d_{v_i,v_j} denotes the distance between v_i and v_j, d_{ff} is the far-field distance, and α is the path loss exponent. Typically, d_{v_i,v_j} is much larger than d_{ff}; therefore, (3.2) is simplified to

$$\sigma^2_{v_i,v_j} = d^{-\alpha}_{v_i,v_j} \ , \quad d_{v_i,v_j} > d_{ff}. \tag{3.3}$$

At each node, zero-mean complex additive white Gaussian noise with variance $N_0/2$ per dimension is considered to corrupt the signal. In this chapter, we consider the case in which all the nodes in the network use the same amount of power P for transmission [23], [29]. For simplicity of notation, we define $\gamma = \frac{P}{N_0}$.

3.2 Problem Formulation

Our goal is to find the path with the minimum end-to-end outage probability from source to destination. The solution requires an optimization over all the paths connecting these two nodes. Therefore, the optimization problem is given by

$$p^{\text{opt}} = \arg \min_{p \in \mathcal{P}(v_0, v_{N-1})} P_{\text{outage},p} \tag{3.4}$$

where $P_{\text{outage},p}$ is the outage probability of transmission over path p and p^{opt} denotes the optimal route.

3.2.1 Amplify-and-Forward

In an amplify-and-forward network, the AF relay amplifies the received signal. The power gain of the relay is adapted based on the channel gain to provide constant output power P which is given by [2]

$$G_{t_i} = \frac{P}{P|h_{t_{i-1},t_i}|^2 + N_0}. \tag{3.5}$$

The outage probability of an AF multihop transmission over path p with k hops is computed as

$$P_{\text{outage},p} = \Pr\left[\frac{1}{k}\log_2(1+\gamma_d) < R\right] \tag{3.6a}$$

where γ_d is the SNR at the destination node which can be derived as [4]

$$\gamma_d = \Big(\prod_{i=0}^{k-1}(1+\frac{1}{\gamma|h_{t_i,t_{i+1}}|^2})-1\Big)^{-1}. \tag{3.6b}$$

The optimization problem can be re-written as

$$\begin{aligned} p^{\text{opt}} &= \arg\min_p P_{\text{outage},p} \\ &= \arg\min_p \Pr\Big[\frac{1}{k}\log_2(1+\frac{1}{\prod_{i=0}^{k-1}(1+\frac{1}{\gamma|h_{t_i,t_{i+1}}|^2})-1}) < R\Big] \\ &= \arg\max_p \Big[\frac{1}{k}\log_2(1+\frac{1}{\prod_{i=0}^{k-1}(1+\frac{1}{\gamma|h_{t_i,t_{i+1}}|^2})-1})\Big]. \end{aligned} \tag{3.7}$$

We define the AF routing metric of path $p = \langle t_0, t_1, ..., t_k\rangle$ as

$$\omega_{\text{AF}}(p) = \frac{1}{k}\log_2(1+\frac{1}{\prod_{i=0}^{k-1}(1+\frac{1}{\gamma|h_{t_i,t_{i+1}}|^2})-1}) \tag{3.8}$$

which is to be maximized in the network. Then, the optimal outage probability is given by

$$P_{\text{outage}}^{\text{opt}} = \Pr\Big[\max_p[\frac{1}{k}\log_2(1+\frac{1}{\prod_{i=0}^{k-1}(1+\frac{1}{\gamma|h_{t_i,t_{i+1}}|^2})-1})] < R\Big]. \tag{3.9}$$

Since the paths are not independent and share common links, an exact expression for $P_{\text{outage}}^{\text{opt}}$ is not derivable. But, an upper bound can be found by considering independent routes from v_0 to v_{N-1}. Assuming all the nodes in the network can communicate directly with a sufficiently low rate [23], the direct source-destination path and $N-2$ dual-hop paths through an intermediate relay form $N-1$ independent paths between v_0 and v_{N-1}. Note that outage occurs when all the routes are in outage. Therefore, the optimal outage probability is less than the probability of outage for the aforementioned $N-1$ routes. So,

$$P_{\text{outage}}^{\text{opt}} \le P_{\text{outage},\langle v_0,v_{N-1}\rangle} \times \prod_{i=1}^{N-2} P_{\text{outage},\langle v_0,v_i,v_{N-1}\rangle} \tag{3.10a}$$

where

$$P_{\text{outage},\langle v_0,v_{N-1}\rangle} = 1 - \exp\left(-\frac{2^R-1}{\gamma\sigma^2_{v_0,v_{N-1}}}\right) \tag{3.10b}$$

and

$$P_{\text{outage},\langle v_0,v_i,v_{N-1}\rangle} = \Pr\left[\frac{1}{|h_{v_0,v_i}|^2} + \frac{1}{|h_{v_i,v_{N-1}}|^2} + \frac{1}{\gamma|h_{v_0,v_i}|^2|h_{v_i,v_{N-1}}|^2} > \frac{\gamma}{2^{2R}-1}\right]. \tag{3.10c}$$

For asymptotic behavior where $\gamma \to \infty$, we can use [2, Lemma 1] and obtain

$$P_{\text{outage},\langle v_0,v_i,v_{N-1}\rangle} \approx \frac{2^{2R}-1}{\gamma}(\sigma^{-2}_{v_0,v_i} + \sigma^{-2}_{v_i,v_{N-1}}). \tag{3.11}$$

So,

$$P^{\text{opt}}_{\text{outage}} \leq \frac{1}{\gamma^{N-1}}(2^{2R}-1)^{N-2}(2^R-1)\sigma^{-2}_{v_0,v_{N-1}}\prod_{i=1}^{N-2}(\sigma^{-2}_{v_0,v_i} + \sigma^{-2}_{v_i,v_{N-1}}). \tag{3.12}$$

Since outage probability decays proportional to γ^{N-1}, full diversity order is achieved based on using instantaneous CSI for finding the most outage efficient route.

3.2.2 Decode-and-Forward

In a decode-and-forward transmission, the DF relay decodes the received signal, re-encodes and forwards it to the next hop. As shown in Section 2.3, one can write the outage performance as

$$P_{\text{outage},p} = \Pr\left[\min_{i=0,\ldots,k-1}\left[\frac{1}{k}\log_2(1+\gamma|h_{t_i,t_{i+1}}|^2)\right] < R\right]. \tag{3.13}$$

Eq. (3.13) indicates that outage happens when the achievable rate of the weakest channel of p becomes less than R. Returning to the main optimization, we have

$$\begin{aligned} p^{\text{opt}} &= \arg\min_p P_{\text{outage},p} \\ &= \arg\min_p \Pr\left[\min_{i=0,\ldots,k-1}\left[\frac{1}{k}\log_2(1+\gamma|h_{t_i,t_{i+1}}|^2)\right] < R\right] \\ &= \arg\max_p \left[\min_{i=0,\ldots,k-1}\frac{1}{k}\log_2(1+\gamma|h_{t_i,t_{i+1}}|^2)\right]. \end{aligned} \tag{3.14}$$

We have proved in Chapter 2 that DF multihop transmission with instantaneous CSI-aided routing also achieves full diversity order. The routing metric for DF multihop networks is given as

$$\omega_{\mathrm{DF}}(p) = \frac{1}{k} \log_2(1 + \gamma \min_{i=0,..,k-1} |h_{t_i,t_{i+1}}|^2). \tag{3.15}$$

The most straightforward solution to eqs. (3.7) and (3.14) is to try all the possible paths between source and destination (all paths permutations of any length) and find the path with the maximum metric (brute-force search). The running time of this algorithm lies within a polynomial factor of $\mathcal{O}((N-2)!)$ which is unfeasible for even a small number of nodes, e.g., $N = 10$. So, the need arises to design an optimal distributed routing algorithm, which not only makes distributed implementation feasible, but which also decreases the computational complexity.

3.3 Distributed Routing Algorithm

The concept of a routing metric from an algebraic point of view can be represented by the quadruplet $(\mathcal{P}, \oplus, \Omega, \preceq)$ where $\mathcal{P}$ denotes all the paths in the network, $\oplus$ is the concatenation operation of two paths, Ω denotes the metric function which maps a path to a metric value, and $\preceq$ is the order relation defined over the set of metric values.

3.3.1 Definitions

In this subsection, we summarize the essential properties of a routing algebra, monotonicity and isotonicity, as introduced in [40], [43], [44]. Then, we present the new composite metrics and prove that they hold the required properties.

Definition 1. *A routing metric is monotone if and only if* $\Omega(p) \preceq \Omega(p \oplus l)$ *for all* $p \in \mathcal{P}$ *and* $l \in \mathcal{E}$*. In other words, monotonicity states that the routing metric does not decrease when concatenated with a new link.*

Definition 2. *A routing metric is isotone if and only if* $\Omega(p_1) \preceq \Omega(p_2)$ *implies that* $\Omega(p_1 \oplus l) \preceq \Omega(p_2 \oplus l)$ *for all* $p_1, p_2 \in \mathcal{P}$ *and* $l \in \mathcal{E}$*. Isotonicity means that the order relationship between two paths with the same origin and end is preserved when*

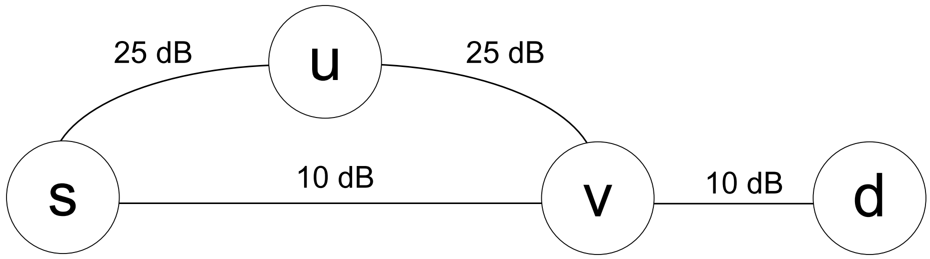

Fig. 3.1. A 4-node multihop network which shows that the routing metrics obtained for outage efficient routing in AF and DF networks are not isotone.

both of them are concatenated with a new common link. In other words, any subpath between two nodes on an optimal path is optimal among all the routes connecting those nodes.

If we define $\preceq$ as the greater-than-or-equal-to relation over real numbers, it can be simply verified that the routing metrics proposed in eqs. (3.8) and (3.15) are monotone. As the transmitted signal travels over a greater number of relays, the destination SNR becomes smaller and the denominators of eqs. (3.8) and (3.15) get larger. So, the routing metric of a path becomes smaller as it is extended by a new link. Consequently, the routing metrics proposed in eqs. (3.8) and (3.15) are monotone but not necessarily isotone. A simple 4-node network is illustrated in Fig. 3.1 where each link is associated with its corresponding SNR. We define $p_1 = \langle s, u, v \rangle$ and $p_2 = \langle s, v \rangle$ as the possible routes connecting s to v. Assuming the nodes of the network employ amplify-and-forward relaying, one can compute $\omega_{\mathrm{AF}}(p_1) = 3.65$ and $\omega_{\mathrm{AF}}(p_2) = 3.45$ using eq. (3.8), which shows that p_1 is a better path than p_2. Now, define q_1 and q_2 as the routes of p_1 and p_2 extended by link l, respectively, where l is the direct link between v and d. The calculations show that q_2 is more desirable than q_1 from s to d because $\omega_{\mathrm{AF}}(q_1) = 1.12$ and $\omega_{\mathrm{AF}}(q_2) = 1.26$. This example shows that the routing metric ω_{AF} is not isotone. Also, one can verify that in case of DF relaying for this example, the same conclusion can be made for ω_{DF}. So, we propose a new composite metric which satisfies the required properties and which has the property that its overall value can be easily computed from its component partial metrics.

In Fig. 3.1, the best route connecting s to v is the dual-hop transmission through node u. As a result of the lack of isotonicity, if v advertises only $\langle v, u, s \rangle$ to its neighbors, then d cannot find its optimal path to s since $\langle v, u, s \rangle$ is not on its optimal path. So, a new algorithm is required in which more than one path is advertised. In the following, we propose a new routing scheme that determines which paths must be advertised to satisfy the optimality requirement.

Definition 3. *We say that the path p is more effective than path q if and only if $\Omega(p) \prec \Omega(q)$ where the relation $\prec$ is defined such that $\Omega(p) \prec \Omega(q)$ if $\Omega(p) \preceq \Omega(q)$ and $\Omega(p) \neq \Omega(q)$. Also, we say that p is an efficient path if and only if there is no path q with the same origin and end for which $\Omega(q) \prec \Omega(p)$.*

In the following, we propose two distinct composite metrics for AF and DF relaying networks that determine which paths must be advertised to satisfy the optimality requirement.

3.3.1.1 Amplify-and-Forward

Definition 4. *For a given path $p = \langle t_0, t_1, ..., t_k \rangle$, we define the new composite metric for AF networks as $\Omega_{AF}(p) = (\omega^{(1)}_{AF}(p), \omega^{(2)}_{AF}(p))$ where $\omega^{(1)}_{AF}(p) = \prod_{i=0}^{k-1}(1 + \frac{1}{\gamma_{i+1}})$, $\omega^{(2)}_{AF}(p) = k$, and $\gamma_i = \gamma|h_{t_{i-1},t_i}|^2$. Also, we define the order relationship as $\Omega_{AF}(p) \preceq \Omega_{AF}(q)$ if and only if $\omega^{(1)}_{AF}(p) \leq \omega^{(1)}_{AF}(q)$ and $\omega^{(2)}_{AF}(p) \leq \omega^{(2)}_{AF}(q)$.*

Note that not every pair of routing metrics are comparable under the order relationship defined in Definition 4; so it is a partial ordering.

Theorem 1. *The proposed composite metric is monotone.*

Proof. Let $p = \langle t_0, t_1, ..., t_k \rangle$, so $\omega^{(1)}_{\mathrm{AF}}(p) = \prod_{i=0}^{k-1}(1 + \frac{1}{\gamma|h_{t_i,t_{i+1}}|^2})$ and $\omega^{(2)}_{\mathrm{AF}}(p) = k$. Let l be the link between t_k and the new node t_{k+1}. Then,

$$\begin{aligned} \omega^{(1)}_{\mathrm{AF}}(p \oplus l) &= \prod_{i=0}^{k}(1 + \frac{1}{\gamma|h_{t_i,t_{i+1}}|^2}) \\ &= \omega^{(1)}_{\mathrm{AF}}(p) \times (1 + \frac{1}{\gamma|h_{t_k,t_{k+1}}|^2}) \\ &> \omega^{(1)}_{\mathrm{AF}}(p) \end{aligned} \tag{3.16}$$

and

$$\omega^{(2)}_{\mathrm{AF}}(p \oplus l) = \omega^{(2)}_{\mathrm{AF}}(p) + 1 > \omega^{(2)}_{\mathrm{AF}}(p). \tag{3.17}$$

So, $\Omega_{\mathrm{AF}}(p) \prec \Omega_{\mathrm{AF}}(p \oplus l)$. □

Theorem 2. *The proposed composite metric is isotone.*

Proof. Let $p_1 = \langle s, t_1, ..., t_{k-1}, d\rangle$ and $p_2 = \langle s, u_1, ..., u_{n-1}, d\rangle$ such that $\Omega_{\mathrm{AF}}(p_1) \preceq \Omega_{\mathrm{AF}}(p_2)$. Let l denote the link between d and the new node w. Then,

$$\omega_{\mathrm{AF}}^{(1)}(p_1 \oplus l) = \omega_{\mathrm{AF}}^{(1)}(p_1) \times (1 + \frac{1}{\gamma|h_{d,w}|^2}) \tag{3.18}$$

$$\omega_{\mathrm{AF}}^{(1)}(p_2 \oplus l) = \omega_{\mathrm{AF}}^{(1)}(p_2) \times (1 + \frac{1}{\gamma|h_{d,w}|^2}). \tag{3.19}$$

Since we know that $\omega_{\mathrm{AF}}^{(1)}(p_1) \leq \omega_{\mathrm{AF}}^{(1)}(p_2)$, we can conclude $\omega_{\mathrm{AF}}^{(1)}(p_1 \oplus l) \leq \omega_{\mathrm{AF}}^{(1)}(p_2 \oplus l)$ from eqs. (3.18) and (3.19). Also, $\omega_{\mathrm{AF}}^{(2)}(p_1) \leq \omega_{\mathrm{AF}}^{(2)}(p_2)$ results in $\omega_{\mathrm{AF}}^{(2)}(p_1 \oplus l) \leq \omega_{\mathrm{AF}}^{(2)}(p_2 \oplus l)$. So, $\Omega_{\mathrm{AF}}(p_1 \oplus l) \preceq \Omega_{\mathrm{AF}}(p_2 \oplus l)$. □

3.3.1.2 Decode-and-Forward

Definition 5. *The DF composite metric is defined as* $\Omega_{DF}(p) = (\omega_{DF}^{(1)}(p), \omega_{DF}^{(2)}(p))$ *where* $\omega_{DF}^{(1)}(p) = \min_{i=0,...,k-1} \gamma_{i+1}$ *and* $\omega_{DF}^{(2)}(p) = k$*. Also, the order relationship is defined as* $\Omega_{DF}(p) \preceq \Omega_{DF}(q)$ *if and only if* $\omega_{DF}^{(1)}(p) \geq \omega_{DF}^{(1)}(q)$ *and* $\omega_{DF}^{(2)}(p) \leq \omega_{DF}^{(2)}(q)$*.*

Theorem 3. *The proposed composite metric is monotone.*

Proof. Similar to the proof of *Theorem* 1, one can write

$$\begin{aligned}\omega_{\mathrm{DF}}^{(1)}(p \oplus l) &= \min_{i=0,...,k-1,k} \gamma|h_{t_i,t_{i+1}}|^2 \\ &= \min\left(\omega_{\mathrm{DF}}^{(1)}(p), \gamma|h_{t_k,t_{k+1}}|^2\right) \\ &\leq \omega_{\mathrm{DF}}^{(1)}(p)\end{aligned} \tag{3.20}$$

and

$$\omega_{\mathrm{DF}}^{(2)}(p \oplus l) = \omega_{\mathrm{DF}}^{(2)}(p) + 1 > \omega_{\mathrm{DF}}^{(2)}(p). \tag{3.21}$$

So, $\Omega_{\mathrm{DF}}(p) \prec \Omega_{\mathrm{DF}}(p \oplus l)$. □

Theorem 4. *The proposed composite metric is isotone.*

Proof. Similar to the proof of *Theorem* 2, we have

$$\omega_{\mathrm{DF}}^{(1)}(p_1 \oplus l) = \min\left(\omega_{\mathrm{DF}}^{(1)}(p_1), \gamma|h_{d,w}|^2\right) \tag{3.22}$$

$$\omega_{\mathrm{DF}}^{(1)}(p_2 \oplus l) = \min\left(\omega_{\mathrm{DF}}^{(1)}(p_2), \gamma |h_{d,w}|^2\right). \tag{3.23}$$

Assuming that $\Omega_{\mathrm{DF}}(p_1) \preceq \omega_{\mathrm{DF}}(p_2)$, we have $\omega_{\mathrm{DF}}^{(1)}(p_1) \geq \omega_{\mathrm{DF}}^{(1)}(p_2)$. Using eqs. (3.22) and (3.23), we write $\omega_{\mathrm{DF}}^{(1)}(p_1 \oplus l) \leq \omega_{\mathrm{DF}}^{(1)}(p_2 \oplus l)$. Also, $\omega_{\mathrm{DF}}^{(2)}(p_1) \leq \omega_{\mathrm{DF}}^{(2)}(p_2)$ leads to $\omega_{\mathrm{DF}}^{(2)}(p_1 \oplus l) \leq \omega_{\mathrm{DF}}^{(2)}(p_2 \oplus l)$. Therefore, $\Omega_{\mathrm{DF}}(p_1 \oplus l) \preceq \Omega_{\mathrm{DF}}(p_2 \oplus l)$. □

3.3.2 Routing Table Construction

Having proved the monotonicity and isotonicity properties of the proposed composite metrics, we are now able to develop a distributed routing algorithm to identify the most outage efficient (maximum spectrally efficient) paths from each node to possible destination nodes in the network. In our algorithm, each node stores a routing table which contains the required information for routing information exchange and packet forwarding. If a node finds a new efficient path to another node or the composite metric of an existing path is changed, it must advertise this path information to its neighbors according to the corresponding entry in its routing table. Each path is identified by the quadruple $(s, d, \xi(p), \Omega(p))$ where $\xi(p)$ is the immediate next hop of route p after the source node, s denotes the source node and d is the destination node. Every node stores all the efficient paths either received from its neighbors or found by itself in its routing table. So, more than one entry might be stored in the routing table for each destination node.

First, each node is required to launch a module to explore its channel gains to its neighbor nodes. Since the network is working in TDMA mode and the channel gains are symmetric, the assumption that every two neighbors find the same value of $\omega^{(1)}(p)$[2] for their direct link is valid. So, every node initializes its routing table by entering the neighbors' information as $(s, u, u, (\omega^{(1)}(p), 1))$ where $\omega_{\mathrm{AF}}^{(1)}(p) = 1 + \frac{1}{\gamma |h_{s,u}|^2}$ and $\omega_{\mathrm{DF}}^{(1)}(p) = \gamma |h_{s,u}|^2$.

When s receives an advertisement $(u, d, \xi(q), \Omega(q))$ from node u, it first updates all its routing table entries which are destined for d and whose next hop is u. Then, s constructs the new path p from s to d whose next hop is u. It compares $\Omega(p)$ with all the stored paths from s to d to find and remove the routes less effective than p. Comparing the new route with the previously explored routes, node s can determine

[2] For simplicity of notation, we write $\omega^{(1)}$ and $\omega^{(2)}$ for referring to the first and second component of both Ω_{AF} and Ω_{DF}, respectively.

whether the new path is an efficient path or not. If it is, it will store the information in its routing table and will advertise it to its neighbors. If p is not an efficient path, that path information is discarded. The values of the composite metric $\Omega(q)$ are computed as

$$\omega_{\mathrm{AF}}^{(1)}(p) = \omega_{\mathrm{AF}}^{(1)}(q)(1 + \frac{1}{\gamma|h_{s,u}|^2}) \tag{3.24}$$

$$\omega_{\mathrm{AF}}^{(2)}(p) = \omega_{\mathrm{AF}}^{(2)}(q) + 1 \tag{3.25}$$

and

$$\omega_{\mathrm{DF}}^{(1)}(p) = \min\left(\omega_{\mathrm{DF}}^{(1)}(q), \gamma|h_{s,u}|^2\right) \tag{3.26}$$

$$\omega_{\mathrm{DF}}^{(2)}(p) = \omega_{\mathrm{DF}}^{(2)}(q) + 1 \tag{3.27}$$

for AF and DF networks, respectively. We use a simple 4-node network illustrated in Fig. 3.2 to explain more precisely the process of constructing the routing table. We are, in particular, interested in finding the routing table of node s to d. Suppose that the nodes in the network employ amplify-and-forward relaying. First, all the nodes create initial entries in their routing table by entering the information of direct links to their neighbors. Then, the information is advertised. For instance, v advertises $(v, d, d, (1.03, 1))$ to u, s, and d. Once u knows that there is such a path from v to d, it creates the entry $(u, d, v, (1.13, 1))$ and generates an advertisement. Meanwhile, s compares $(s, d, v, (1.13, 2))$ with $(s, d, d, (1.32, 1))$ and finds out the new route is also an efficient path. Now, the routing table of s to d has two efficient paths. Having received u's new advertisement, s computes the composite metric of the new path $q = \langle s, u, v, d\rangle$ as $\Omega_{\mathrm{AF}}(q) = (1.49, 3)$. Then, it compares this metric to the previously known paths and realizes that $p = \langle s, d\rangle$ with metric $\Omega_{\mathrm{AF}}(p) = (1.32, 1)$ is more effective than the new path. Therefore, the new route is discarded. The final version of node s's routing table is shown in Table 3.1.

A summary of the proposed algorithms for routing table construction in AF and DF networks is given in Algorithms 3.1 and 3.2, respectively. Note that when s receives a path advertisement from u to d, and if there is not any path from s to d in the routing table of s, it only adds the newly discovered path and generates an advertisement. Also, when a path in the routing table of a node is not an efficient path anymore, for example as a result of node failure or channel gain change, and is

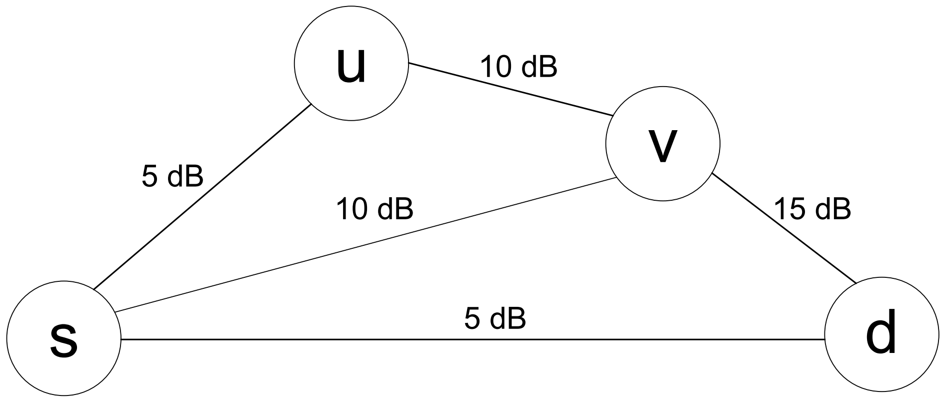

Fig. 3.2. Network example for the proposed outage efficient routing.

TABLE 3.1
ROUTING TABLE OF NODE s, CONSTRUCTED BASED ON THE PROPOSED DISTRIBUTED OPTIMAL OUTAGE EFFICIENT ROUTING ALGORITHM FOR THE NETWORK ILLUSTRATED IN FIG. 3.2

destination	next hop	metric $\Omega_{\text{AF}}(p)$	
		$\omega_{\text{AF}}^{(1)}(p)$	$\omega_{\text{AF}}^{(2)}(p)$
u	u	1.32	1
	v	1.21	2
v	v	1.1	1
d	d	1.32	1
	v	1.13	2

subject to removal, that node must make an advertisement with the previous path information but with $\omega_{\text{AF}}^{(1)}(p) = \infty$ or $\omega_{\text{DF}}^{(1)}(p) = 0$. This is necessary for the other nodes in the network in order to consider this change and find new paths.

In wireless networks, the channel gain among the mobile nodes changes with time. Therefore, the metrics stored at each node must be updated and sent to the neighbors for routing table update. The proposed routing algorithm achieves the optimal performance as long as the channel gains do not change significantly in the duration of protocol convergence. But in case of fast channel changes, e.g. fast fading environments, the protocol may not be able to converge fast enough, and thus, there is a performance loss compared to the optimal routing. Note that the size of a network plays an important role in the routing protocol convergence time, and therefore a larger network will converge slower than a small one.

3.3.3 Packet Forwarding

In traditional hop-by-hop routing algorithms, the originating node puts only the destination address in the data packet. Then, intermediate nodes select the next node based on their routing table, which stores the next hop for reaching each destination terminal. Since the isotonicity property is not valid for the main metrics, the optimal paths from intermediate nodes to destination do not necessarily lie on the source-destination optimal route. A consistent packet forwarding scheme is essential to ensure that the data packet travels on the optimal route.

As described in Section 3.3.2, the process of routing table construction may result in several potential paths to each destination node. Suppose node s wants to send some information to node d. In order to determine the optimal path among the available routes stored in s's routing table, it needs to compute the main metric using the composite metric $\Omega(p)$. Then, the path with maximum resulting capacity is selected,

$$p_{\mathrm{AF}}^{\mathrm{opt}} = \arg\max_{p} \frac{1}{\omega_{\mathrm{AF}}^{(2)}(p)} \log_2 (1 + \frac{1}{\omega_{\mathrm{AF}}^{(1)}(p) - 1}) \tag{3.28}$$

Algorithm 3.1 Summary of the distributed optimal outage efficient routing in AF networks

/* Node s receives a path advertisement $q = \big(u, d, \xi(q), (\omega_{\mathrm{AF}}^{(1)}(q), \omega_{\mathrm{AF}}^{(2)}(q))\big)$ from its neighbor node u. */

if there is a path from s to d with $\omega_{\mathrm{AF}}^{(2)}(p^*) = \omega_{\mathrm{AF}}^{(2)}(q) - 1$ **then**
    $\omega_{\mathrm{AF}}^{(1)}(p^*) = \omega_{\mathrm{AF}}^{(1)}(q)(1 + \frac{1}{\gamma|h_{s,u}|^2})$
    $found = 1$
$q^* = \langle s, u\rangle \oplus q$
$\Omega_{\mathrm{AF}}(q^*) = \big(\omega_{\mathrm{AF}}^{(1)}(q)(1 + \frac{1}{\gamma|h_{s,u}|^2}), \omega_{\mathrm{AF}}^{(2)}(q) + 1\big)$
for each path p from s to d in the routing table of s **do**
    if $(\Omega_{\mathrm{AF}}(q^*) \prec \Omega_{\mathrm{AF}}(p))$ **then**
        Remove p from the routing table
        Advertise $\big(s, d, \xi(p), (\infty, \omega_{\mathrm{AF}}^{(2)}(p))\big)$
    else if $(\Omega_{\mathrm{AF}}(p) \prec \Omega_{\mathrm{AF}}(q^*))$ **then**
        if $(found = 1)$ **then**
            Remove p^* from the routing table
            Advertise $\big(s, d, \xi(p^*), (\infty, \omega_{\mathrm{AF}}^{(2)}(p^*))\big)$
        return
Add and advertise $\big(s, d, u, \Omega_{\mathrm{AF}}(q^*)\big)$

$$p_{\mathrm{DF}}^{\mathrm{opt}} = \arg\max_{p} \frac{1}{\omega_{\mathrm{DF}}^{(2)}(p)} \log_2\left(1 + \omega_{\mathrm{DF}}^{(1)}(p)\right). \tag{3.29}$$

Once the optimal path is determined, s puts the destination address and $\omega^{(2)}(p^{\mathrm{opt}})$ in the header of the data packet and sends it to the next hop, namely u, based on the corresponding entry in its routing table. Upon receiving the packet, u must determine the next hop to forward the data. Using the information in the packet header, u can extract the destination and number of hops and find the path with $\omega^{(2)}(q) = \omega^{(2)}(p^{\mathrm{opt}}) - 1$ in its routing table. Having found the route entry, u updates the number of hops field of the packet header with $\omega^{(2)}(p^{\mathrm{opt}}) - 1$ and forwards it to the next node.

3.3.4 Optimality and Consistency

In this subsection, we prove that the proposed routing and packet forwarding protocols satisfy the optimality and consistency requirements for distributed implementation.

Theorem 5. *The proposed distributed routing algorithm converges onto optimal paths.*

Algorithm 3.2 Summary of the distributed optimal outage efficient routing in DF networks

---

/* Node s receives a path advertisement $q = \big(u, d, \xi(q), (\omega_{\mathrm{DF}}^{(1)}(q), \omega_{\mathrm{DF}}^{(2)}(q))\big)$ from its neighbor node u. */

if there is a path from s to d with $\omega_{\mathrm{DF}}^{(2)}(p^*) = w_2(q) - 1$ **then**
  $\omega_{\mathrm{DF}}^{(1)}(p^*) = \min(\omega_{\mathrm{DF}}^{(1)}(q), \gamma|h_{s,u}|^2)$
  $found = 1$
$q^* = \langle s, u\rangle \oplus q$
$\Omega_{\mathrm{DF}}(q^*) = \big(\min(\omega_{\mathrm{DF}}^{(1)}(q), \gamma|h_{s,u}|^2), \omega_{\mathrm{DF}}^{(2)}(q) + 1\big)$
for each path p from s to d in the routing table of s **do**
  if $(\Omega_{\mathrm{DF}}(q^*) \prec \Omega_{\mathrm{DF}}(p))$ **then**
    Remove p from the routing table
    Advertise $\big(s, d, \xi(p), (0, \omega_{\mathrm{DF}}^{(2)}(p))\big)$
  else if $(\Omega_{\mathrm{DF}}(p) \prec \Omega_{\mathrm{DF}}(q^*))$ **then**
      if $(found = 1)$ **then**
        Remove p^* from the routing table
        Advertise $\big(s, d, \xi(p^*), (0, \omega_{\mathrm{DF}}^{(2)}(p^*))\big)$
      return
Add and advertise $\big(s, d, u, \Omega_{\mathrm{DF}}(q^*)\big)$

Proof. Suppose that the optimal path between t_0 and t_k is unique and can be represented by $\langle t_0, t_1, ..., t_k\rangle$. We use induction to prove that each on-path node t_i must advertise the subpath $\langle t_i, t_{i+1}, ..., t_k\rangle$ information to its neighbors, i.e., t_{i-1}.

As the initial step of induction, it is obvious that t_{k-1} advertises the direct link $\langle t_{k-1}, t_k\rangle$ to t_{k-2}. For the induction step, assume that t_{i+1} has made an advertisement for path $\langle t_{i+1}, t_{i+2}, ..., t_k\rangle$ to t_i. Now, if t_i does not advertise path $p_1 = \langle t_i, t_{i+1}, ..., t_{k-1}, t_k\rangle$, an efficient path $p_2 = \langle t_i, s_{i+1}, ..., s_{k-1}, t_k\rangle$ must exist that is more effective than p_1. Using the isotonicity property of Ω, which was proved previously, $p_2 \oplus q$ must also be more effective than $p_1 \oplus q$ where $q = \langle t_0, t_1, ..., t_i\rangle$. This leads to the result that $\langle t_0, t_1, ..., t_k\rangle$ is not the optimal path, which contradicts our basic assumption. □

Lemma 1. *If two distinct paths p_1 and p_2 with the same origin and end have the same value of $\omega^{(2)}$, they cannot be efficient paths simultaneously.*

Proof. It is obvious that we have either of these relationships: $\omega^{(1)}(p_1) < \omega^{(1)}(p_2)$ or $\omega^{(1)}_{\mathrm{AF}}(p_1) \geq \omega^{(1)}(p_2)$. Since $\omega^{(2)}(p_1) = \omega^{(2)}(p_2)$, we will have either $\Omega(p_1) \prec \Omega(p_2)$ or $\Omega(p_1) \succeq \Omega(p_2)$ which proves that one of the paths is more effective than the other. □

Theorem 6. *The proposed distributed routing algorithm satisfies the consistency requirement.*

Proof. Suppose that t_0 wants to send some information to t_k through path $p = \langle t_0, t_1, ..., t_k\rangle$. Our routing algorithm is consistent if node t_1 forwards the received data to t_k through $\langle t_1, t_2, ..., t_k\rangle$. According to Lemma 1, there is only one path in the routing table of t_1 which corresponds to destination t_k with $\omega^{(2)}(q) = \omega^{(2)}(p) - 1$. Since t_0 has discovered path p which goes through the on-path node t_1, t_1 must have previously advertised $\langle t_1, t_2, ..., t_k\rangle$. Considering these facts, one reaches the conclusion that t_1 forwards the received data according to its unique entry, namely q. □

3.4 Complexity Analysis

In this section, we investigate the computational complexity of the proposed distributed routing algorithm.

Suppose that node s wants to find its optimal path to d in a network consisting of N nodes. Let Q denote the average number of neighbors of each node. In the case that all the nodes can communicate directly, Q is equal to $N-1$. The proposed distributed routing algorithm requires the flow of routing information from d to s. In the first iteration of the algorithm, all the direct links are known. At the next step, the nodes find dual-hop paths and efficient routes among them are stored in the routing tables. Consequently at step m, all the m-hop efficient paths are assessed. The worst-case scenario occurs when s can reach d via an efficient path with the length of $N-1$ hops. Thus, the algorithm is required to be run $N-1$ times so that the source node obtains the information of the aforementioned path. In each iteration of the algorithm, the nodes exchange the paths information with their neighbors and each node updates the entries of its routing table which correspond to node d. It was proved in Lemma 1 that from each node to a specific destination, it is not possible that there are two efficient paths with the same value of $\omega^{(2)}$. So, considering the fact that each node can reach any node in a connected network with at most $N-1$ hops, the maximum number of efficient paths from each node to d is the maximum value of $\omega^{(2)}$ which is $N-1$. Consequently, the required memory for storing the information of efficient paths from each node to d is $\mathcal{O}(N)$. As a node receives an advertisement from its neighbor, it has to update the metric of previously known paths if that route had been previously discovered. Then, it compares the new path with the other efficient routes in the routing table to check if the new route is efficient. Since there are $\mathcal{O}(N)$ paths, the computational complexity of determining whether the new path is efficient is $\mathcal{O}(N)$. The number of efficient paths advertised by the neighbors of each node is unknown. So, we can roughly consider $N-1$ as an upper bound for the number. Consequently, the upper bound for the total number of received advertisements at each node is $\mathcal{O}(NQ)$. Finally, the algorithm running time is $\mathcal{O}(N^2Q)$ at each node and $\mathcal{O}(N^3Q)$ for the whole network. Note that the complexity derived is based on the worst-case scenario, rough upper bounds, and is only valid for one destination node.

3.5 Simulation Results

In this section, simulation results are presented to evaluate the outage performance of the proposed routing strategy and to validate our theoretical analysis. Two different network topologies are considered. In the first scenario, all the nodes can communicate directly. But in the second scenario, only the nodes within a disk can be reached directly [11]. For both simulation scenarios, the path loss exponent, α, is assumed to be 4, the path spectral efficiency, R, is set to 1 bit/s/Hz, and the far-field distance is considered to be 0.1.

3.5.1 Scenario 1

In this subsection, we consider the case where all the nodes in the network can communicate directly. This assumption allows us to ascertain the full diversity order achieved by the proposed routing protocol and to verify the analysis presented in Section 3.2. The nodes are located at random positions in a two-dimensional network. The locations of source and destination nodes are fixed at $(0, 0)$ and $(100, 100)$, respectively. The outage performance is averaged over 10^5 random and independent network realizations. In each realization, the vertical and horizontal coordinates of remaining nodes are chosen randomly according to a uniform distribution. The channel among the nodes is generated based on the model given in Section 3.1.

Fig. 3.3 depicts the end-to-end outage probability for several numbers of nodes in the network for both AF and DF transmission systems. In addition, the outage performance of direct transmission is plotted for comparison. The outage is plotted as a function of Γ which is the average link SNR between source and destination in direct transmission. The figure clearly demonstrates the full diversity order achieved by the proposed routing algorithm. The greater the number of nodes in the network, the more efficient routes are discovered, which in turn significantly enhances the end-to-end performance. The figure shows that the DF system outperforms AF in all SNR regimes. But in high SNR, amplify-and-forward and decode-and-forward have roughly the same performance, which can be clearly observed for the cases of $N = 4$

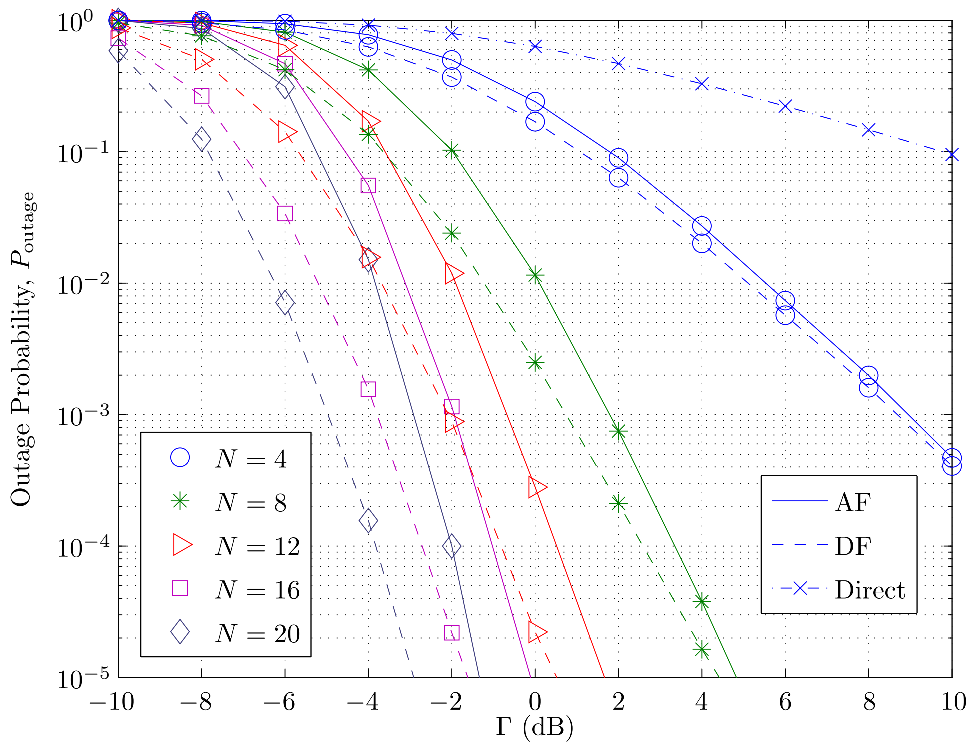

Fig. 3.3. The outage probability of the proposed optimal distributed routing algorithm for several numbers of nodes in the network.

and $N = 8$.

The average number of hops versus Γ is illustrated in Fig. 3.4 for $N = 4$, 12, and 20. The simulation results show that the optimal paths have greater numbers of hops at lower SNRs. But in high SNR regimes, the optimal paths employ fewer relays for data transmission. As the SNR increases, the average number of hops goes to one. In other words, the direct source-destination link becomes preferable to a multihop approach. This leads to the conclusion that the high SNR performance of AF relaying is approximately equal to that of DF relaying, which is consistent with the results found in Fig. 3.3. Interestingly, optimally routed decode-and-forward employs more relays in comparison with amplify-and-forward. Moreover, the effect of N on the average number of hops can be observed in Fig. 3.4. As N increases, the average number of hops also increases but not linearly. There is a smaller increase as N changes from $N = 12$ to $N = 20$ in comparison with the case as N changes from $N = 4$ to $N = 12$. Fig. 3.4 further indicates that even for large numbers of nodes in the network, the average number of hops quickly falls below 2, and

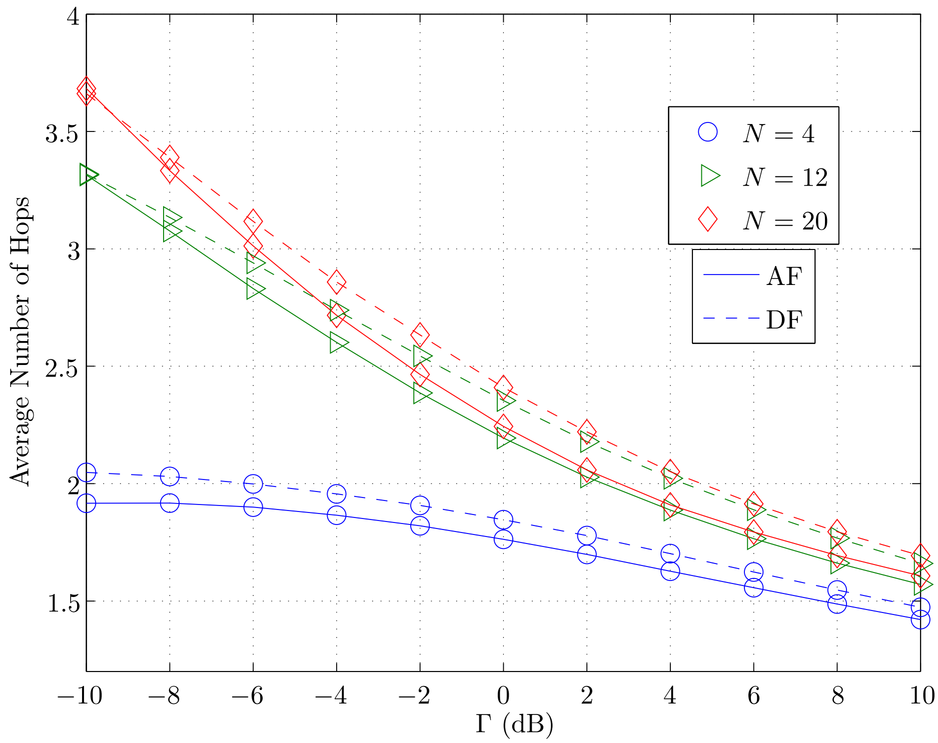

Fig. 3.4. The average number of hops of the proposed optimal distributed routing algorithm for several numbers of nodes in the network.

approaches 1 even for small values of SNR, about 2 dB or greater, on the direct transmission link. Comparing the results in Fig. 3.4 with the results in Fig. 3.3, one concludes that multihop wireless relaying networks offer substantial reduction in outage performance at a relatively low cost of additional network hops.

Fig. 3.5 compares the performance of the proposed routing algorithm in multihop systems without diversity to multihop diversity systems. The simulation results for multihop diversity networks are obtained based on running the same routing protocol but employing maximal ratio combining of the signals received at each relay from the preceding terminals. The figure shows that if the nodes use MRC strategy, not only the routing algorithm performs very well but also there is a slight performance improvement. Note that the performance achieved for the case of multihop diversity systems is not optimal. The optimal strategy can be found by obtaining the appropriate metric for routing in multihop diversity networks and applying the same method used in this chapter for converting the obtained metric into a composite metric.

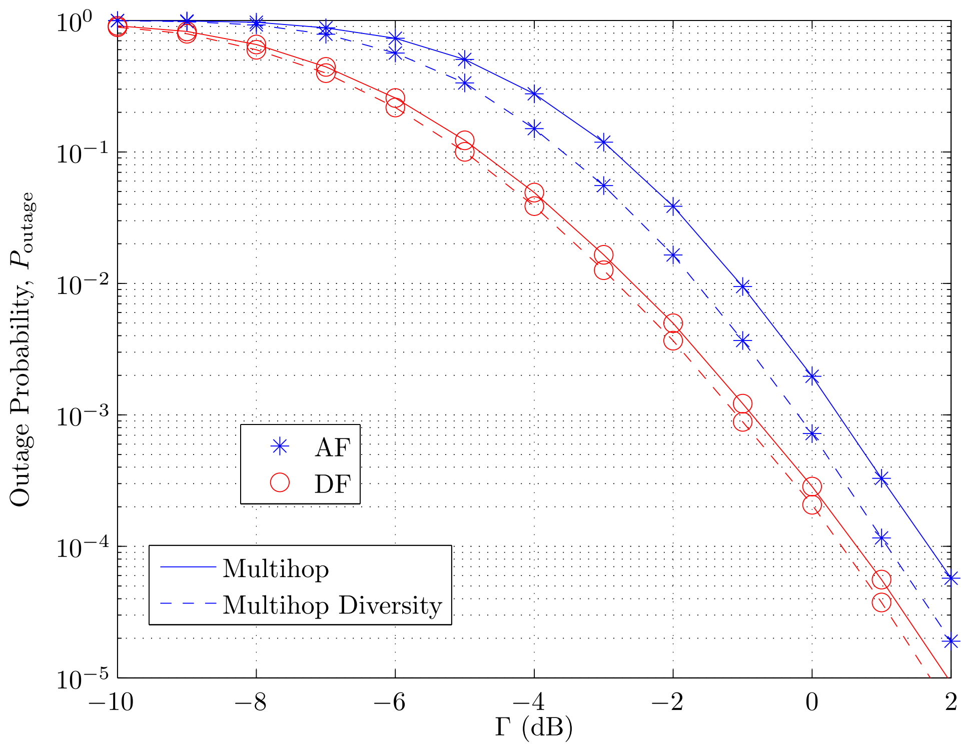

Fig. 3.5. Comparison of the outage probability of the proposed algorithm in multihop systems without diversity to the outage probability of multihop diversity systems ($N = 10$).

Fig. 3.6 presents the running time of the proposed routing algorithm for AF and DF systems as a function of N when $\Gamma = 5$ dB. The algorithm running time is averaged over 10^4 network realizations. Note that the routing protocol is run for two cases, finding the routes to one destination node and all the nodes in the network. A curve has been fitted to the simulation result of each transmission case. For decode-and-forward, a 3rd order polynomial was tightly fitted for the case of one destination. But for amplify-and-forward, we were not able to find any well fitted polynomial with maximum order of 4, which is the order of worst-case running time. Empirically, we found that the function $N^3 \log(N)$ best fits the average complexity in AF networks. Although we proved the worst-case complexity of optimal routing is $\mathcal{O}(N^4)$, interestingly the average-case complexity for DF and AF multihop is $\mathcal{O}(N^3)$ and $\mathcal{O}(N^3 \log(N))$, respectively. Following the same approach, we found the average complexity of the algorithm for finding the optimal paths to all the nodes as $\mathcal{O}(N^4)$ and $\mathcal{O}(N^4 \log(N))$ in DF and AF networks, respectively. As evident in the figure, the running time of a DF network is consistently lower than that of an AF network.

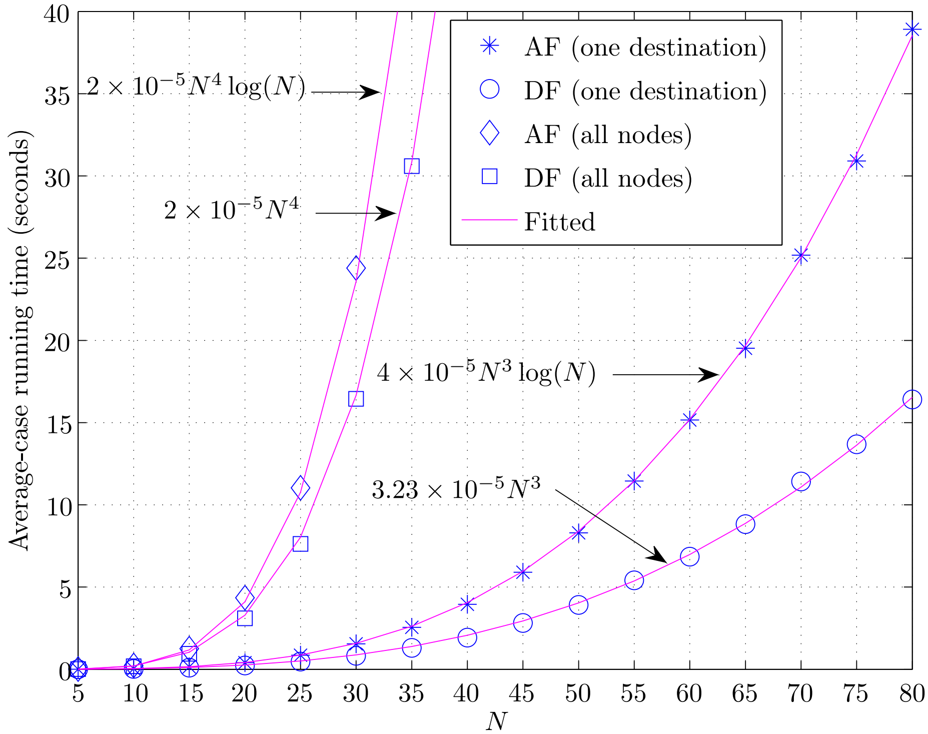

Fig. 3.6. Comparison of the average-case running time of the proposed routing algorithm in AF and DF multihop networks for a wide range of N. The routing algorithm is run for two cases, finding the optimal paths to one destination and all the nodes in the network.

Our simulation results show that the main reason for this difference is the number of efficient paths found at each node to reach the destination. The average number of efficient paths stored in the routing table of the source node is depicted in Fig. 3.7, which shows that this number is larger in AF networks, compared to DF networks. Unfortunately, analytical derivation of an expression for the average-case running time for the proposed algorithm is not possible since no knowledge is available about the statistics of the number of efficient paths at each node.

3.5.2 Scenario 2

In this scenario, we consider the same network settings used in Section 3.5.1, except that two nodes can only communicate directly if the distance between them is less than 25. The channel model among the nodes with direct access is Rayleigh fading. We do not fix the location of source and destination. A random network topology generated with 60 nodes is plotted in Fig. 3.8. Note that the depicted graph is connected so that there is a path from any node to any other node in the network.

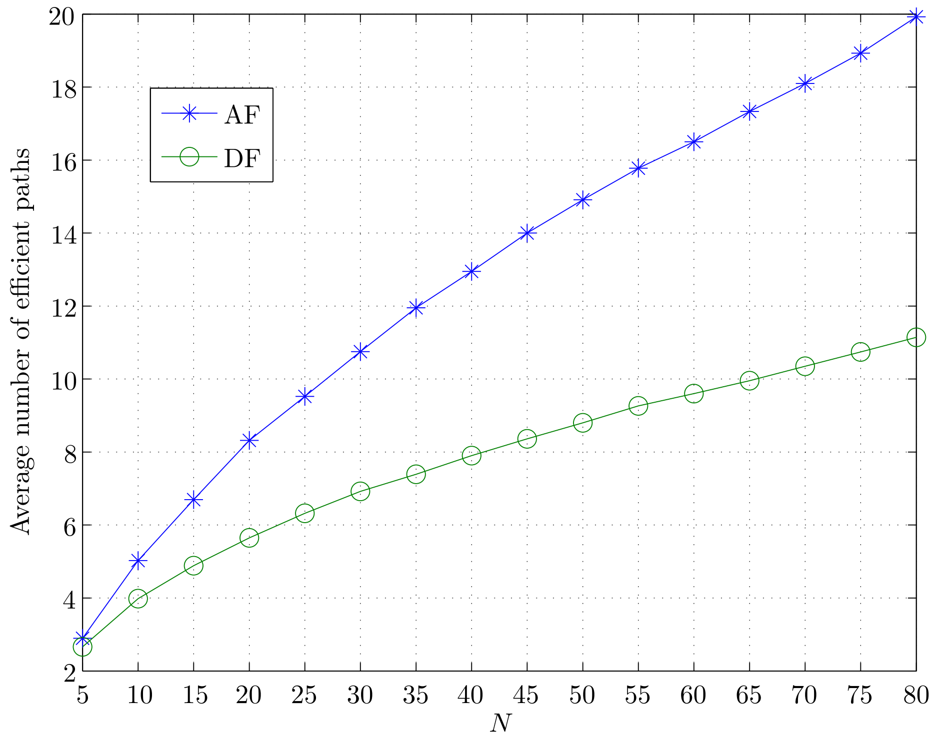

Fig. 3.7. Comparison of the average number of efficient paths between source and destination in AF and DF multihop networks for a wide range of N.

The proposed algorithm is run for the network shown in Fig. 3.8, assuming the nodes are amplify-and-forward and $\Gamma = 0$ dB. Fig. 3.9 illustrates the optimal routes from all the nodes in the network to the arbitrary destination node d. The thickness of the first link of each path represents proportionally the spectral efficiency of that route. Clearly, the nodes with further distance from d reach the destination with less spectral efficiency. In a connected network, if the routing metric is isotone and monotone, the graph of optimal routes to a destination is a spanning tree which does not include any cycle. But in this work, since the main routing metric (3.8) is not isotone, the drawn graph is not a tree. For example, it can be observed that there are two paths from u to d where one of them is node s's optimal route and the other one is u's path to d.

3.6 Summary

In this work, we applied the end-to-end outage probability as the optimization criterion for routing protocol design in AF and DF multihop transmission systems based

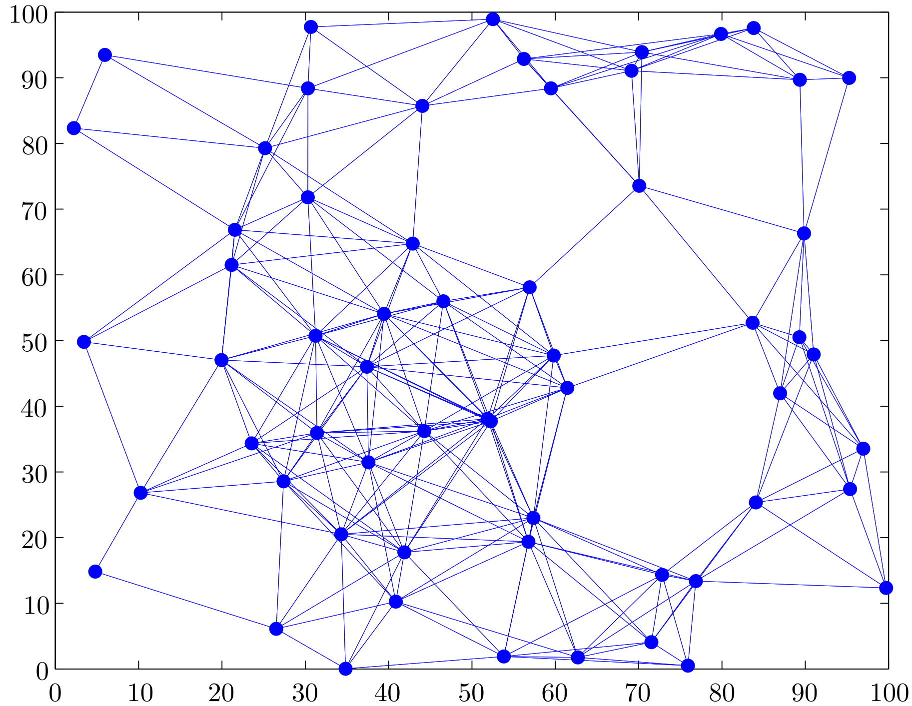

Fig. 3.8. A random network topology with 60 nodes. Each node can only communicate with the nodes within a disk with the radius of 25.

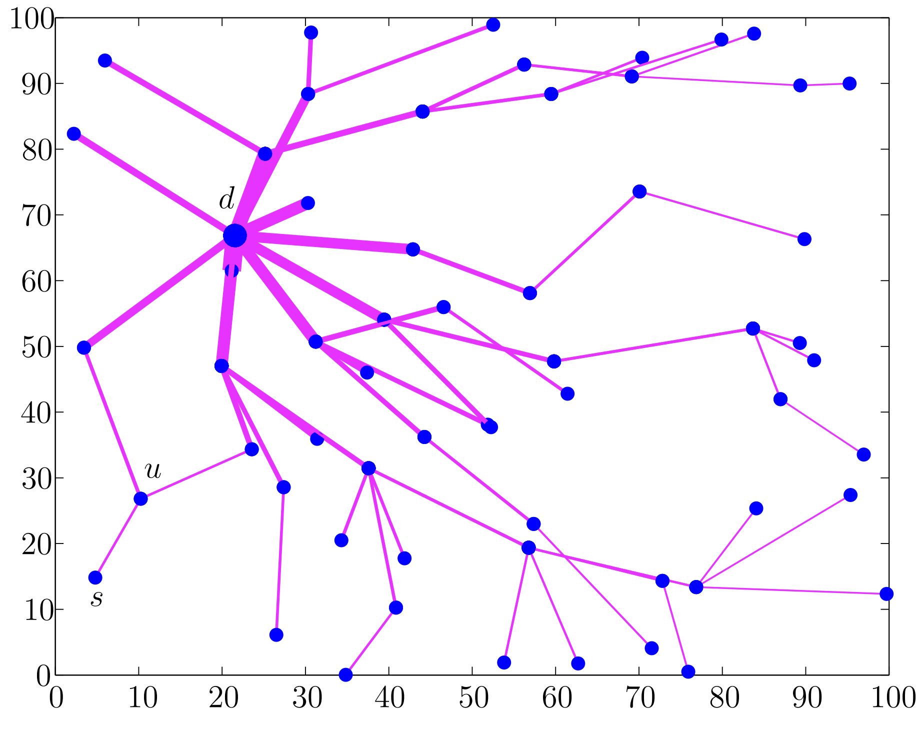

Fig. 3.9. The optimal outage efficient paths from all the nodes in the network to arbitrary destination node d. The thickness of the first link of a node-destination route is proportional to the spectral efficiency of that path.

on the availability of CSI at the network layer. The assessed routing metrics for outage optimization were not isotone and did not qualify for optimal distributed implementation. Novel composite metrics were proposed which satisfy consistency and optimality requirements for distributed implementation. Moreover, it was proved that the optimal routing achieves full diversity order. Although the worst-case complexity of the proposed algorithm is $\mathcal{O}(N^4)$, the simulation results showed that the average-case running time of the algorithm for one destination is $\mathcal{O}(N^3 \log(N))$ and $\mathcal{O}(N^3)$ in AF and DF networks, respectively.

Chapter 4

Power-Optimized Routing With Bandwidth Guarantee [1]

In this chapter, we investigate a cross-layer optimization framework for total power minimization subject to a desired end-to-end capacity constraint for AF and DF multihop networks. The derived metrics cannot be optimized in a distributed approach; so, we apply the method used in Chapter 3 to convert the metrics obtained into new composite metrics which satisfy the essential properties for routing protocol convergence and optimality. Then, we develop a distributed method for finding the best path between the nodes. The proposed scheme does not require topology information and is only based on the local instantaneous channel conditions as measured at each node.

The remainder of this chapter is organized as follows. The network model is described in Section 4.1. In Section 4.2, we formulate the problem of minimum total power routing for optimization. A distributed route selection scheme, routing table construction, packet forwarding algorithm, and power allocation are proposed in Section 4.3. Section 4.4 presents simulation results for network performance evaluation along with some discussion. Finally, Section 4.5 concludes the chapter.

[1] A version of this chapter has been submitted to *IEEE International Conference on Communications (ICC)* [45].

4.1 Network Model

Our network model consists of N nodes, numbered from v_0 to v_{N-1}, where v_0 and v_{N-1} are the source and destination nodes, respectively. For simplicity, we consider routing for one source-destination pair. We consider a network with TDMA protocol without spatial reuse. A path is identified by a sequence of connected nodes, $p = \langle t_0, ..., t_{k-1}, t_k \rangle$ where t_0 and t_k are the origin and end nodes, respectively, and k is the length of path p. Let $\mathcal{P}(v_0, v_{N-1})$ be the set of all possible routes from source to destination. The channel gain between v_i and v_j is represented as h_{v_i,v_j}. The power assigned to node t_i is denoted as P_{t_i}. At each node, additive white Gaussian noise with unit variance is considered to corrupt the signal.

4.2 Problem Formulation

In this section, we formulate the problem of minimum power routing in DF and AF relaying networks to find appropriate path metrics for distributed implementation.

4.2.1 Decode-and-Forward

The optimal power routing in DF systems under the end-to-end capacity constraint between source and destination can be formulated as

$$\begin{aligned} &\min_{p \in \mathcal{P}(v_0, v_{N-1})} \sum_{i=0}^{k-1} P_{t_i} \\ &\text{subject to} \quad \min_{i=0,...,k-1} \left[\frac{1}{k} \log_2(1 + P_{t_i} |h_{t_i,t_{i+1}}|^2) \right] \geq R. \end{aligned} \tag{4.1}$$

Since the objective function is linear and the constraint can be expanded to k convex constraints, the optimization problem is convex and, thus, has a unique solution. Using the Lagrange multiplier method [38], one derives the Lagrangian L associated with the problem (4.1) as

$$L = \sum_{i=0}^{k-1} P_{t_i} + \sum_{i=0}^{k-1} \lambda_i \Big(R - \frac{1}{k} \log_2(1 + P_{t_i} |h_{t_i,t_{i+1}}|^2) \Big) \tag{4.2}$$

where λ_i is the Lagrange multiplier. Taking the derivative of L with respect to P_{t_i}, $i = 0, ..., k-1$, and solving the set of equations obtained with respect to the KKT conditions, one derives

$$P_{t_i} = \frac{2^{kR} - 1}{|h_{t_i,t_{i+1}}|^2}. \tag{4.3}$$

Substituting the obtained power in the minimization function of (4.1), we obtain the optimization problem as

$$\min_{p \in \mathcal{P}(v_0, v_{N-1})} \sum_{i=0}^{k-1} \frac{2^{kR} - 1}{|h_{t_i,t_{i+1}}|^2}. \tag{4.4}$$

Therefore, the routing metric for DF networks, which is to be minimized, is given as

$$\omega_{\text{DF}}(p) = \sum_{i=0}^{k-1} \frac{2^{kR} - 1}{|h_{t_i,t_{i+1}}|^2}. \tag{4.5}$$

4.2.2 Amplify-and-Forward

The optimal power routing in AF networks subject to the capacity constraint can be expressed as

$$\begin{aligned} &\min_{p \in \mathcal{P}(v_0, v_{N-1})} \sum_{i=0}^{k-1} P_{t_i} \\ &\text{subject to} \quad \frac{1}{k} \log_2(1 + \gamma_d) \geq R \end{aligned} \tag{4.6a}$$

where γ_d is the destination SNR and is given as [4]

$$\gamma_d = \Big(\prod_{i=0}^{k-1} (1 + \frac{1}{P_{t_i} |h_{t_i,t_{i+1}}|^2}) - 1\Big)^{-1}. \tag{4.6b}$$

The problem in (4.6a) can be reformulated as

$$\begin{aligned} &\min_{p \in \mathcal{P}(v_0, v_{N-1})} \sum_{i=0}^{k-1} P_{t_i} \\ &\text{subject to} \prod_{i=0}^{k-1} (1 + \frac{1}{P_{t_i} |h_{t_i,t_{i+1}}|^2}) \leq 1 + \frac{1}{2^{kR} - 1}. \end{aligned} \tag{4.7}$$

Since the objective function is linear and, as proved in Appendix A, the constraint is convex, the optimization problem (4.7) is convex. To solve (4.7), one writes the Lagrangian as

$$L = \sum_{i=0}^{k-1} P_{t_i} + \lambda\Big(\prod_{i=0}^{k-1} (1 + \frac{1}{P_{t_i} |h_{t_i,t_{i+1}}|^2}) - \varphi\Big) \tag{4.8a}$$

where

$$\varphi = 1 + \frac{1}{2^{kR} - 1}. \tag{4.8b}$$

Note that φ is not fixed and is a function of k; therefore, its value may change as the path changes. Following the approaches taken in Section 4.2.1 for solving a convex optimization problem, one can obtain the power allocation of node t_i as

$$P_{t_i} = \frac{\sqrt{1 + 4\mu|h_{t_i,t_{i+1}}|^2} - 1}{2|h_{t_i,t_{i+1}}|^2} \tag{4.9a}$$

where μ is the solution to

$$\prod_{i=0}^{k-1} (1 + \frac{2}{\sqrt{1 + 4\mu|h_{t_i,t_{i+1}}|^2} - 1}) = \varphi. \tag{4.9b}$$

So, optimization problem (4.7) can be reformulated as

$$\min_{p \in \mathcal{P}(v_0, v_{N-1})} \sum_{i=0}^{k-1} \frac{\sqrt{1 + 4\mu|h_{t_i,t_{i+1}}|^2} - 1}{2|h_{t_i,t_{i+1}}|^2} \tag{4.10}$$

where μ can be computed for each path using eq. (4.9b). The only possible method for solving (4.10) is an exhaustive search; for each possible path between source and destination, μ must be calculated, and then the optimal path which minimizes the total power can be found. Solving the problem with this approach results in a centralized algorithm where all the network information, such as the channel gains and noise powers, should be available at a particular terminal and the solution should be transmitted back to the corresponding nodes. The high complexity implementation of this centralized algorithm along with the huge amount of overhead makes it inappropriate for wireless networks with mobile nodes.

We use an approximation for the instantaneous SNR at the destination to derive a routing metric suitable for distributed implementation. As shown in [4], γ_d in (4.6b) can be well approximated as

$$\gamma_d \approx \Big(\sum_{i=0}^{k-1} \frac{1}{P_{t_i}|h_{t_i,t_{i+1}}|^2}\Big)^{-1} \tag{4.11}$$

particularly for large values of SNR. Substituting the approximation (4.11) in (4.6a)

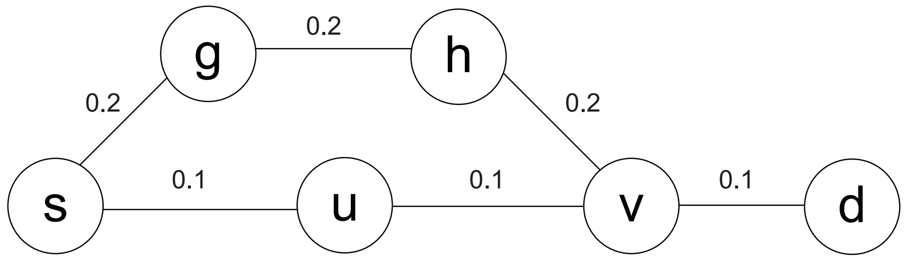

Fig. 4.1. A multihop network which shows that the routing metrics obtained for power-optimized routing in AF and DF networks are not isotone.

and solving the resulting optimization problem, we have

$$P_{t_i} = \frac{2^{kR}-1}{|h_{t_i,t_{i+1}}|}\sum_{i=0}^{k-1}\frac{1}{|h_{t_i,t_{i+1}}|}. \tag{4.12}$$

So, the routing metric which is to be minimized in the network is given as

$$\omega_{\mathrm{AF}}(p) = (2^{kR}-1)\Big(\sum_{i=0}^{k-1}\frac{1}{|h_{t_i,t_{i+1}}|}\Big)^2. \tag{4.13}$$

Note that as a result of using an approximation for the destination SNR, the routing metric obtained does not lead to optimal paths in the network. Note further that the approximation in eq. (4.11) is an upper bound for γ_d. Therefore, the actual capacity achieved by employing the metric (4.13) is less than or equal to the optimal value R. We will investigate the accuracy of this approximation and the achievable capacity in Section 4.4.

4.3 Distributed Routing Algorithm

In this section, we present a distributed solution to the joint routing and power allocation optimization problem for DF and AF multihop networks.

4.3.1 Definitions

The routing metrics obtained as eqs. (4.5) and (4.13) for DF and AF networks, respectively, satisfy the monotonicity property but they are not isotone. A multihop network is illustrated in Fig. 4.1 where each link is associated with its channel gain. We assume the nodes employ the DF relaying strategy. We define $p_1 = \langle s, g, h, v\rangle$

and $p_2 = \langle s, u, v \rangle$ as the available routes connecting s to v. One can calculate that $\omega_{\text{DF}}(p_1) = 525$ and $\omega_{\text{DF}}(p_2) = 600$ assuming $R = 1$, which indicates that p_1 is a better path than p_2. Consider q_1 and q_2 as the routes of p_1 and p_2 extended with link $\langle v, d \rangle$, respectively. The calculations show that q_2 is a better path than q_1 since $\omega_{\text{DF}}(q_1) = 2625$ and $\omega_{\text{DF}}(q_2) = 2100$. In other words, the best route between s and v is through the nodes g and h, while multihop transmission through u and v is optimal for connecting s to d. This example shows that the routing metric (4.5) is not isotone. One can verify that the corresponding metric for routing in AF networks is not isotone for the network in Fig. 3.1 when taking $R = 0.4$.

Definition 6. *For a given path $p = \langle t_0, t_1, ..., t_k \rangle$, we define the composite metric for routing in DF networks as $\Omega_{DF}(p) = (\omega_{DF}^{(1)}(p), \omega_{DF}^{(2)}(p))$ where $\omega_{DF}^{(1)}(p) = \sum_{i=0}^{k-1} \frac{1}{|h_{t_i,t_{i+1}}|^2}$ and $\omega_{DF}^{(2)}(p) = k$. We define the order relationship as $\Omega_{DF}(p) \preceq \Omega_{DF}(q)$ if and only if $\omega_{DF}^{(1)}(p) \leq \omega_{DF}^{(1)}(q)$ and $\omega_{DF}^{(2)}(p) \leq \omega_{DF}^{(2)}(q)$.*

Definition 7. *For a given path $p = \langle t_0, t_1, ..., t_k \rangle$, we define the composite metric for routing in AF networks as $\Omega_{AF}(p) = (\omega_{AF}^{(1)}(p), \omega_{AF}^{(2)}(p))$ where $\omega_{AF}^{(1)}(p) = \sum_{i=0}^{k-1} \frac{1}{|h_{t_i,t_{i+1}}|}$ and $\omega_{AF}^{(2)}(p) = k$. The order relationship is the same as that of DF relaying.*

Theorem 7. *The proposed routing metrics are monotone.*

Proof. We consider DF relaying. Let $p = \langle t_0, t_1, ..., t_k \rangle$ denote an arbitrary route in the network, so $\omega_{\text{DF}}^{(1)}(p) = \sum_{i=0}^{k-1} \frac{1}{|h_{t_i,t_{i+1}}|^2}$ and $\omega_{\text{DF}}^{(2)}(p) = k$. Let l be a link from node t_k to node t_{k+1}. Then,

$$\begin{aligned} \omega_{\text{DF}}^{(1)}(p \oplus l) &= \sum_{i=0}^{k} \frac{1}{|h_{t_i,t_{i+1}}|^2} \\ &= \omega_{\text{DF}}^{(1)}(p) + \frac{1}{|h_{t_k,t_{k+1}}|^2} \\ &> \omega_{\text{DF}}^{(1)}(p) \end{aligned} \tag{4.14}$$

and

$$\omega_{\text{DF}}^{(2)}(p \oplus l) = \omega_{\text{DF}}^{(2)}(p) + 1 > \omega_{\text{DF}}^{(2)}(p). \tag{4.15}$$

Thus, $\Omega_{\text{DF}}(p) \prec \Omega_{\text{DF}}(p \oplus l)$ and the Theorem is proved for the case of DF relaying.

The proof of the monotonicity property of the AF relaying routing metric is similar.

□

Theorem 8. *The proposed routing metrics are isotone.*

Proof. We consider DF relaying. Let $p_1 = \langle s, t_1, ..., t_{k_{p_1}-1}, d\rangle$ and $p_2 = \langle s, u_1, ..., u_{k_{p_2}-1}, d\rangle$ such that $\Omega_{\mathrm{DF}}(p_1) \preceq \Omega_{\mathrm{DF}}(p_2)$. Let l denote the link between d and the new node w. Then,

$$\omega_{\mathrm{DF}}^{(1)}(p_1 \oplus l) = \omega_{\mathrm{DF}}^{(1)}(p_1) + \frac{1}{|h_{d,w}|^2} \tag{4.16}$$

$$\omega_{\mathrm{DF}}^{(1)}(p_2 \oplus l) = \omega_{\mathrm{DF}}^{(1)}(p_2) + \frac{1}{|h_{d,w}|^2}. \tag{4.17}$$

Since we know that $\omega_{\mathrm{DF}}^{(1)}(p_1) \leq \omega_{\mathrm{DF}}^{(1)}(p_2)$, we can conclude $\omega_{\mathrm{DF}}^{(1)}(p_1 \oplus l) \leq \omega_{\mathrm{DF}}^{(1)}(p_2 \oplus l)$ from eqs. (4.16) and (4.17). Also, $\omega_{\mathrm{DF}}^{(2)}(p_1) \leq \omega_{\mathrm{DF}}^{(2)}(p_2)$ results in $\omega_{\mathrm{DF}}^{(2)}(p_1 \oplus l) \leq \omega_{\mathrm{DF}}^{(2)}(p_2 \oplus l)$. Thus, $\Omega_{\mathrm{DF}}(p_1 \oplus l) \preceq \Omega_{\mathrm{DF}}(p_2 \oplus l)$ and Theorem 2 is proved for the DF case. One can prove similarly that the isotonicity property also holds for the proposed routing metric in AF networks. □

4.3.2 Routing Table Construction

In a distributed routing algorithm, each node has a routing table which stores the information required for routing message exchanges with the neighbors and packet forwarding. Each entry in a routing table corresponds to one path from that node to a particular destination terminal in the network. In our algorithm, some other information such as source node, destination node, and the next hop in addition to the metric of a path are stored in each route entry to identify a route uniquely. As a result, each path is identified with the 4-tuple $(s, d, \xi(p), \Omega(p))$ where $\xi(p)$ is the immediate next hop of route p after the source node, s denotes the source node, d is the destination node, and $\Omega(p)$ is the routing metric associated with path p. In our protocol, each node stores all the efficient paths from itself to possible destination nodes in its routing table.

As the initialization step of our algorithm, each node launches a module to obtain its channel responses to its neighbors. Once the gains are obtained, each terminal enters the information of its direct path to its neighbors in the routing table as

$(s, u, u, (\omega^{(1)}(p), 1))$ where $\omega_{\text{DF}}^{(1)}(p) = \frac{1}{|h_{s,u}|^2}$ and $\omega_{\text{AF}}^{(1)}(p) = \frac{1}{|h_{s,u}|}$. Then, each node advertises this information to its neighbors.

Similar to the procedures described in Section 3.3.2, once node s receives an advertisement $(u, d, \xi(q), \Omega(q))$ from its neighbor node u, it updates all its routing table entries which are destined for d and whose next hop is u. Then, a new path p from s to d is constructed as the concatenation of q with the direct link $\langle s, u \rangle$. Node s compares $\Omega(p)$ with all the stored paths from s to d to find and remove the paths less effective than p. Then, it compares the composite metric of the new route with that of the previously explored routes and determines whether the new path is an efficient path or not. If it is, it will store the information in its routing table and will advertise it to its neighbors. The values of the composite metric $\Omega(p)$ are computed as

$$\omega_{\text{DF}}^{(1)}(p) = \omega_{\text{DF}}^{(1)}(q) + \frac{1}{|h_{s,u}|^2} \tag{4.18}$$

$$\omega_{\text{DF}}^{(2)}(p) = \omega_{\text{DF}}^{(2)}(q) + 1 \tag{4.19}$$

and

$$\omega_{\text{AF}}^{(1)}(p) = \omega_{\text{AF}}^{(1)}(q) + \frac{1}{|h_{s,u}|} \tag{4.20}$$

$$\omega_{\text{AF}}^{(2)}(p) = \omega_{\text{AF}}^{(2)}(q) + 1. \tag{4.21}$$

for DF and AF relaying schemes, respectively.

A summary of the proposed algorithm for routing table construction in DF networks is given in Algorithm 4.1. Note that when s receives a path advertisement from u to d, and if there is not any path from s to d in the routing table of s, it only adds the newly discovered path and generates an advertisement. Also, when a path in the routing table of a node is not an efficient path anymore, for example as a result of node failure or channel gain change, and is subject to removal, that node must make an advertisement with the previous path information but with $\omega^{(1)}(p) = \infty$. This is necessary in order for the other nodes in the network to consider this change and find new paths.

4.3.3 Packet Forwarding

As previously discussed, more than one efficient path may be stored in the routing table of a node for each destination node. Once source node s needs to transmit some data to destination node d, it must first find the optimal route from the stored efficient paths. So, it needs to compute the overall value of the main metrics (4.5) or (4.13) from the component partial metrics of the proposed composite metrics. Then, the path with the minimal total power is selected according to

$$p_{\text{DF}}^{\text{opt}} = \arg\min_{p} \, (2^{\,\omega_{\text{DF}}^{(2)}(p)R} - 1)\omega_{\text{DF}}^{(1)}(p) \tag{4.22}$$

$$p_{\text{AF}}^{\text{opt}} = \arg\min_{p} \, (2^{\,\omega_{\text{AF}}^{(2)}(p)R} - 1)\big(\omega_{\text{AF}}^{(1)}(p)\big)^2. \tag{4.23}$$

Once the optimal path is determined, s puts the destination address and $\omega^{(2)}(p^{\text{opt}})$ in the header of the data packet and sends it to the next hop, namely u, based on its corresponding entry in its routing table. Having received the packet, u must determine the next hop to forward the data. Using the information in the packet header, u can extract the destination and number of hops and find the path with $\omega^{(2)}(q) = \omega^{(2)}(p^{\text{opt}}) - 1$ in its routing table. Having found the route entry, u updates the number of hops field of the packet header with $\omega^{(2)}(p^{\text{opt}}) - 1$ and forwards it to

Algorithm 4.1 Summary of the power-optimized routing algorithm in DF networks

/* Node s receives a path advertisement $(u, d, \xi(q), (\omega_1(q), \omega_2(q)))$ from its neighbor node u. */
if there is a path from s to d with $\omega_2(p^*) = w_2(q) + 1$ **then**
    $\omega_1(p^*) = \omega_1(q) + \frac{1}{|h_{s,u}|^2}$
    $found = 1$
$q^* = \langle s, u \rangle \oplus q$
$\Omega(q^*) = (\omega_1(q) + \frac{1}{|h_{s,u}|^2}, \omega_2(q) + 1)$
for each path p from s to d in the routing table of s **do**
    if $(\Omega(q^*) \prec \Omega(p))$ **then**
        Remove p from the routing table
        Advertise $(s, d, \xi(p), (\infty, \omega_2(p)))$
    else if $(\Omega(p) \prec \Omega(q^*))$ **then**
            if $(found = 1)$ **then**
                Remove p^* from the routing table
                Advertise $(s, d, \xi(p^*), (\infty, \omega_2(p^*)))$
            return
Add and advertise $(s, d, u, \Omega(q^*))$

the next node. Note that there is only one path to d in the routing table of u with $\omega^{(2)}(q) = \omega^{(2)}(p^{\text{opt}}) - 1$. This is because of the fact that two distinct routes with the same value of $\omega^{(2)}$ and destination cannot be simultaneously efficient as one of them is necessarily more effective than the other.

An advantage of the proposed algorithm is that the routing messages exchanged among the nodes and the routing tables constructed at the terminals are independent of the desired end-to-end capacity R. As a result, all the nodes in the network can find their efficient paths to the destination nodes without knowledge of R. Only once a node is required to initiate a data transmission to another node, does it need to determine the desired transmission rate and then the optimal path. The optimal path is always among the efficient paths stored in the routing table. Therefore, each of the efficient paths might be optimal for a particular range of R.

4.3.4 Power Allocation

In our distributed algorithm, each on-path relaying node must be able to calculate its transmitting power on its own. Considering eqs. (4.3) and (4.12), each relay requires some information about the path from s to d (not only its path to the destination), e.g., the total number of hops, the summation of the inverse of the channel gains, and the desired transmission rate R. This information is only known at the source node. Therefore, it is required that the source puts this information in the header of the packet so that each node along the path can compute its optimal power assignment. For DF relaying networks, the quantity $\left(2^{\,\omega_{\text{DF}}^{(2)}(p^{\text{opt}})R} - 1\right)$ in (4.3) is sufficient for all the relays to find their power level. In the case of AF systems, the source must put the quantity $\left(2^{\,\omega_{\text{AF}}^{(2)}(p^{\text{opt}})R} - 1\right)\omega_{\text{AF}}^{(1)}(p^{\text{opt}})$ in (4.12) in the packet header. Given this information at the relay nodes along the optimal path, each node can easily compute and adjust its transmitting power.

4.4 Performance Evaluation

We consider a fully connected wireless network with random topology in which all the nodes can communicate directly with each other. The nodes are located at random positions in a two-dimensional network and the locations of source and destination nodes are fixed at $(0, 0)$ and $(100, 100)$, respectively. The channel gain for the link between v_i and v_j is modeled as

$$h_{v_i,v_j} = K\big[\max(d_{v_i,v_j}, d_{\min})\big]^{-\alpha/2} \tag{4.24}$$

where $K = 10^4$, d_{v_i,v_j} is the distance between the nodes, $d_{\min}$ is a minimum distance considered to be equal to 0.1, and α is the path loss exponent which is assumed to be 4 in the simulations. The proposed distributed routing algorithm is run for each network realization and the optimal path between source and destination is found. The total power is averaged over 10^4 random and independent network realizations.

Fig. 4.2 depicts the average total power of the proposed distributed routing algorithm and the optimal solution for AF systems as a function of R when $N = 10$. Moreover, the required power for direct transmission between source and destination is plotted for further comparison. According to the figure, as the desired capacity R increases, the total power needed for establishing transmission also increases. The figure shows that for high values of R, the power grows exponentially with R, which is consistent with eqs. (4.5) and (4.13). Multihop communication in DF relaying outperforms multihop communication in AF relaying as Fig. 4.2 shows a 5 dB gap at low values of R. As discussed in Section 4.2.2, using an approximation for the destination SNR in AF networks results in lower than achievable capacity. As R increases, the required total power also increases, and thus the approximation in (4.11) becomes increasingly more accurate as the SNR increases. So, the achievable capacity of the proposed distributed algorithm for AF networks approaches R for large values of SNR. Our simulation results show that the distributed AF scheme achieves on average an end-to-end capacity of 0.97 bit/sec/Hz when $R = 1$. This is why the total power of the distributed algorithm is less than that of the optimal solution for AF relaying. The performance of the proposed algorithms for both DF and AF networks approaches that of direct transmission for large values of R. In other words, for large values of R, direct transmission is preferable to a multihop communication in both DF and AF networks.

Fig. 4.3 presents the average total power versus the number of nodes in the network assuming $R = 1$. Note that the calculation of the optimal solution for AF

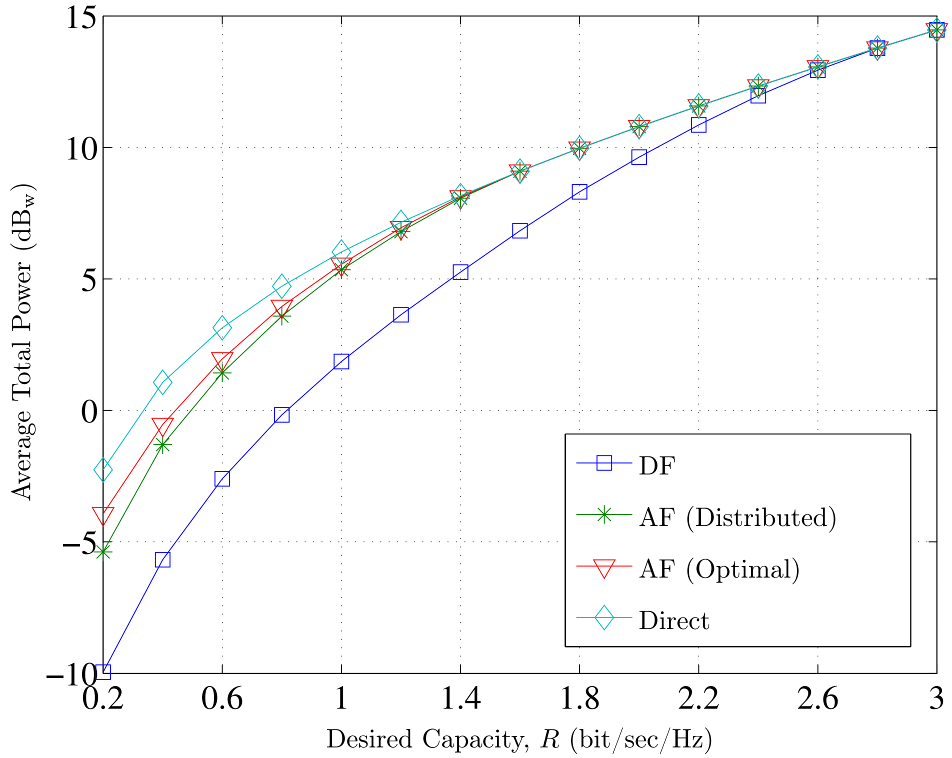

Fig. 4.2. The average total power of different routing schemes for different values of R in random two-dimensional networks with 10 nodes.

networks is not feasible because of the factorial growth of computational complexity with N. The superior performance of relaying in DF networks compared to that of AF networks is evident in this figure. As the number of nodes grows, the total power decreases and the gap between the performances of DF and AF systems becomes larger.

4.5 Summary

In this chapter, we studied the minimum total power routing in DF and AF multihop wireless networks subject to a desired end-to-end capacity constraint. A distributed solution for the joint routing and power allocation optimization problem was proposed based on the local instantaneous channel state information measurement at the nodes of the network. The proposed solution is optimal in DF networks but for the case of AF networks, it performs close to the optimal approach. The optimal solution for AF networks is not feasible due to its high computational complexity.

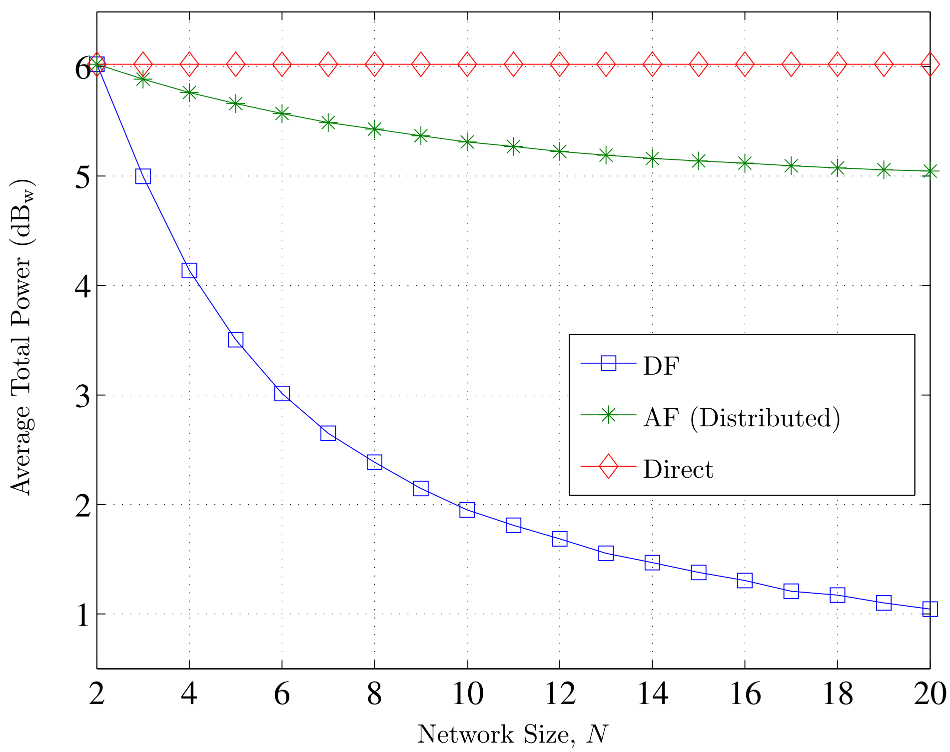

Fig. 4.3. The average total power of different routing schemes for several numbers of nodes in the network assuming $R = 1$.

Chapter 5

Conclusions and Future Work

In this chapter, we conclude this thesis and remark on possible future directions of research for this field based on the results of this work.

5.1 Concluding Remarks

This thesis presented optimization frameworks for routing in multihop networks. In Chapter 2, we studied the problem of finding the optimal outage efficient route between two nodes considering a total power constraint. We solved the problem for two cases, known mean CSI and known instantaneous CSI. The scheme using mean CSI achieves a diversity order of 1 whereas the analysis shows that using instantaneous CSI for route selection offers full diversity order. For each case, equal-power and optimal-power allocation were investigated. Simulation results demonstrated the superior performance of OPA over that of EPA. We extended the results obtained for multihop networks to the case of multihop diversity networks. Analysis proves that multihop diversity networks outperform multihop networks without diversity. In order to find the optimal path among all the paths connecting two nodes, a brute-force search is required which has an exponential running time. A near optimal local search based algorithm with polynomial complexity was proposed by limiting the search space to routes with up to a certain number of relays. Although simulation results presented a reasonable complexity-performance trade-off, the local search based algorithm is not appropriate for routing in distributed environments, such as ad-hoc networks. For implementation in distributed networks, an approximate routing protocol was proposed to optimize the obtained routing metrics based on the Bellman-Ford routing protocol. Although the proposed routing scheme is practical, it is not optimal.

In Chapter 3, we implemented the proposed outage efficient routing in an optimal distributed way. Based on the routing algebra theory, a routing metric must satisfy two essential properties to be maximizable. The routing metrics acquired in Chapter 2 hold only one of the properties. Therefore, we introduced composite metrics for AF and DF networks whose overall value can be easily computed from their component partial metrics and which satisfy the necessary properties and thus, are maximizable. In addition, a consistent packet forwarding scheme was proposed to ensure that the data packets travel on the optimal route. The complexity of the proposed algorithm was assessed by simulation based on the average-case running time. It is shown that the average-case running time of the algorithm is $\mathcal{O}(N^3 \log(N))$ and $\mathcal{O}(N^3)$ for one destination in AF and DF networks, respectively.

Another optimization framework, intended to minimize the total power in multihop networks subject to a desired end-to-end capacity, was presented in Chapter 4. We were able to find an explicit routing metric only for the case of DF networks. However, using an approximation for the destination SNR, an approximate metric was obtained for routing in AF systems. Similar to the approaches taken in Chapter 3, the obtained metrics were converted to composite metrics and their performance was investigated.

5.2 Future Research Directions

In this work, we only considered one transmission in the network. However, a more practical setting is that there are multiple simultaneous flows which should be routed through the network and the power at each node should be shared to transmit data of these different flows.

While this work, as well as most of the published works in the area of multihop and cooperative networks, assume the availability of perfect channel state information, it might be more realistic to consider imperfect channel knowledge. It is important to investigate the performance of the proposed algorithms under this assumption. Also, finding robust algorithms for routing in multihop networks with imperfect channel knowledge is an interesting topic of research.

Although in this thesis, we mainly focused on multihop systems without diversity, one can apply the results obtained to multi-relay cooperative networks and multihop diversity systems. Also, we only assessed outage-efficient and power-optimized routing in this work, yet, some other criteria may be considered for optimization, e.g., BER as future research.

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Appendix A

Proof of Convexity

In order to determine the convexity of the constraint function of (4.7), we need to prove that its Hessian matrix is positive semi-definite [38]. Defining the constraint function f as

$$f = \prod_{i=0}^{k-1} (1 + \frac{1}{P_{t_i}|h_{t_i,t_{i+1}}|^2}), \tag{A.1}$$

the second-order partial derivatives of f can be derived as

$$\frac{\partial^2 f}{\partial P_{t_i}^2} = \frac{2}{P_{t_i}^3|h_{t_i,t_{i+1}}|^2} \prod_{l \neq i} (1 + \frac{1}{P_{t_l}|h_{t_l,t_{l+1}}|^2}) \tag{A.2}$$

$$\frac{\partial^2 f}{\partial P_{t_i} \partial P_{t_j}} = \frac{1}{P_{t_i}^2 P_{t_j}^2 |h_{t_i,t_{i+1}}|^2 |h_{t_j,t_{j+1}}|^2} \prod_{l \neq i,j} (1 + \frac{1}{P_{t_l}|h_{t_l,t_{l+1}}|^2}). \tag{A.3}$$

The Hessian matrix can be decomposed as

$$\nabla^2 f = \mathbf{x}\mathbf{x}^T + \mathbf{Q} \tag{A.4}$$

where

$$\mathbf{x} = \Big[\frac{1}{P_{t_1}^2|h_{t_1,t_2}| + P_{t_1}}, \ldots, \frac{1}{P_{t_k-1}^2|h_{t_{k-1},t_k}| + P_{t_k}}\Big]^T \tag{A.5}$$

and

$$Q_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ \frac{1+2P_{t_i}|h_{t_i,t_{i+1}}|^2}{P_{t_i}+P_{t_i}^2|h_{t_i,t_{i+1}}|^2} & \text{if } i = j. \end{cases} \tag{A.6}$$

Since $\mathbf{Q}$ is a diagonal matrix with positive elements on its diagonal, $\nabla^2 f$ is positive semi-definite, and thus f is convex.