University of Alberta

A Case Study of Teaching and Learning Mathematics Online

by

Julie Genevieve Peschke

A thesis submitted to the Faculty of Graduate Studies and Research

in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

Department of Secondary Education

Spring 2014

Edmonton, Alberta

Dedication

This thesis is dedicated to all of my students, past and present, from whom I have learned so much about the teaching and learning of mathematics as a subject of study, as a language and as an art form.

Abstract

The culture of the young may increasingly be seen as a harbinger enticing us to follow a pathway from which will emerge the re-conceptualized educational practices of a new century. This research set out to discover where the highways of the Internet would lead me, the researcher-practitioner, in terms of the teaching and learning of mathematics in an online world of my own design. The venue was a virtual environment which had shifted from inter-active to inter-personal, from digital text to digital dialogue, and which had the potential to morph the traditional student role into a more engaging self-organizing instructor role. My primary research questions became: What would be the *lived experiences*, both cognitive and affective, of adult learners taking an online program in developmental mathematics, without any face-to-face contact or means of support? Moreover, would our learning together be hindered or be enhanced by this disassociation of persons in both space and time? The research took the form of an empirical case study by way of narrative inquiry but was encapsulated within a hermeneutical interpretative framework. The findings led me to a re-conceptualized notion of a face-to-face classroom ensconced within a virtual world – a place of learning, even deep learning, a place of community with its resonances and its discordances, and an enduring legacy.

Preface

Open universities around the world generally seek to provide almost unrestricted access to educational opportunities for those who are not able or willing, for one reason or another, to attend traditional university classroom courses. Some institutions have a policy of no entrance requirements for introductory courses at the first-year level. This has created a dilemma for Departments of Mathematics, Faculties of Science, and certain other academic disciplines which rely heavily on their students already having acquired a relatively solid base of certain fundamental mathematical skills. The need for developmental mathematics programs, which are targeted at those making the transition from school-level to university-level mathematics, has almost become an imperative in such institutions. With the burgeoning use of the Internet worldwide, there is also a growing demand for such programs to be online.

So what is generally accepted to be encompassed within the *online learning paradigm*? It is a web-based teaching and learning environment that is electronically supported: virtual classrooms, Internet-based learning management systems with multi-media digital materials and meeting spaces. Learning in these spaces is often self-paced and self-directed but not always. In the case of formal, accredited education programs, learning content and activities are often professionally designed and mediated by an instructor.

As a faculty member of an open and distance education institution, I was concerned by a lack of preparedness for mathematics-related courses by a certain segment of the student population. The University administration had already begun to examine the statistics surrounding the numbers of students who were not completing their courses successfully, particularly at the introductory level. Mathematics courses were singled out as being those with some of the highest student attrition rates. It became the onus on the course coordinators and the instructors of such programs to rectify the situation. At that juncture, I began to search for an online solution to our collective dilemma.

This research is a direct result of my journey to find a possible solution.

TABLE OF CONTENTS

A CASE STUDY OF TEACHING AND LEARNING MATHEMATICS ONLINE

The Nutshell

What follows is the narrative – a travelogue if you will – of one particular journey into the virtual world of e-teaching and e-learning. It is the story of my online teaching experience with a rather diverse group of adults, each of them wishing to return to the formal world of mathematics education after a hiatus of many years. The motivation for the project was to provide me, the **practitioner-researcher**, with an opportunity not only to experience the lived reality of teaching and learning in a fully online environment but also to document my findings and observations. I had taught in face-to-face classrooms for a number of years in a traditional university setting and had always tried to be sensitive to my students' reactions both to the subject matter and to the presentation of the mathematics itself. In the distance-learning institution in which I now worked, I was less sure of what would be 'successful' in terms of pedagogical approach and presentation for both me and my students. This project was to be the testing ground.

The path of my journey began, ironically enough, with a detour. Before I could proceed with any research at all, I had to build my own classroom – something I had never had to do before. That secondary excursion led me to:

1. the many aspects of learning online (Chapter 1),
2. the stories of those who had gone before (Chapter 2),
3. a deep examination of my own perspectives of the teaching and learning of an abstract subject in an abstract world (Chapter 3), and
4. my ultimate choice regarding online pedagogy (Chapter 4).

Once that virtual and philosophical stage had been set, what remained was to choose the research method of inquiry and the means of documenting my findings (Chapter 5). My overriding purpose for doing this research, as a **teacher-practitioner**, was to foster in those who volunteered to be a part of this experiment a greater understanding of foundational mathematical ideas so that they could progress to university-level, math-related courses with greater ease. This was not to be a formal course of studies for credit. It was to be a broad review of topics and concepts whose understanding was generally considered essential to the grasp of higher-level mathematical ideas. My primary research question was: what would be the lived experiences, both cognitive and affective, of such a group of adult participants taking a fully online program in developmental mathematics, without any face-to-face contact or means of support? (Chapter 6) As a point of clarification, the use of either of the terms, *remedial* or *developmental mathematics*, is intended to be a “catchall phrase to denote a sequence of coursework, which gradually increases in difficulty, used to bring underprepared students up to the level of academic competency necessary to be successful at the college level” (Feldman & Zimbler, 2012, p. 4).

I had come to the conclusion that this kind of research was both necessary and informative to current and future educational practice. Both Feldman and Zimbler (2012) had regarded developmental education as being a key element of support for many students either entering or returning to college. Practically, however, they saw such programs as problematic with many challenges. “One underlying problem has been the paucity of rigorous research into the effectiveness of various developmental education approaches” (Feldman & Zimbler, 2012, p. 3). The consequences of that, they felt, had resulted in many developmental programs which were, in effect, navigated without a guiding compass. They had understood that students' needs did vary and that there probably could be no single correct approach to the design and implementation of any remedial learning initiatives. In fact, they had discovered that a variety of both traditional and more innovative approaches to developmental education had been successful.

When I began my investigation into the creation of a fully online remedial mathematics program some six years ago, the primary pedagogical issues of such programs, in my opinion, had had to do with their form (the HOW of presentation) and their content (WHAT to include). In the ensuing years, the path to that end turned out to be rather long and circuitous, crossing a number of disciplinary divides. However, it ultimately culminated in a multi-layered, blended model of both asynchronous and synchronous e-learning paradigms and a study of student responses and learning experiences within it.

Justification for this kind of qualitative research into learning experiences of developmental mathematics students has been corroborated, not only by Feldman and Zimbler (2012), but also by Landry (2012). Landry had mentioned that the primary focus, thus far, of *developmental* education research literature had been on success rates and the comparison of student course outcomes in online versus face-to-face classrooms. Many had been large-scale quantitative analyses. However, Landry (2012) went on to add:

> Lacking in the literature, however, are studies that look at the specific instructional methods used in developmental mathematics classrooms. In particular, the body of literature concerning developmental mathematics lacks studies conducted by researcher-practitioners who study adult learners within the context of the classroom. (p. 5)

Hopefully my research, with its broad philosophical base and the documenting of individual stories in an online setting, will help to fill a part of that void and provide some insights into the personal experiences of both the teacher and the learners in a particular kind of remedial mathematics environment. In this research, the pedagogy of teaching mathematics online was oriented more towards mastery and curiosity of the subject matter rather than summative assessments of learning outcomes. Therefore, there were no formally administered tests or exams to determine relative achievement of understanding in the group.

At the outset of the project, I told the twenty or so volunteers who came that I was there, not just to review basic mathematical ideas and methods, but to teach them how to 'think mathematically' (a term which will be defined in more detail in Chapter 2). While a certain degree of the committing definitions to memory and the understanding of certain algorithmic methods were important in the learning of this sometimes enigmatic subject, I was convinced that the development of a way of thinking was the key to their success in any future mathematical endeavours. From the outset, it appeared that everyone had come to learn something – especially I, the **researcher-teacher-student**. We were, collectively, on a path into the ethereal world of mathematical thinking within the abstracted spaces of online interaction.

This adventure was to be what I would call an exposition of a particular CASE of the *virtual reality* of educational experience – in particular, that of the learning of mathematics online. And that is where I will begin – with the prediction of a technological virtual existence beyond the imaginations of most by the mid-twentieth century.

CHAPTER 1 MY VIRTUAL WORLD

The year was 1962. The message was proclaimed:

> The next medium, whatever it is—it may be the extension of consciousness—will include television as its content, not as its environment, and will transform television into an art form. A computer as a research and communication instrument could enhance retrieval, obsolesce mass library organization, retrieve the individual's encyclopedic function and flip into a private line to speedily tailored data of a saleable kind. (McLuhan, 1962, pp. 52-53)

One cannot explore the phenomenon of online learning without some reference to the Edmonton-born Canadian, Marshall McLuhan. He was an educator, a philosopher and a scholar whose writings on communication media theory were prophetic, probing, particularly provocative and sometimes perplexing. His enigmatic aphorism 'the medium is the message' (McLuhan, 1964, p. 203) and his prediction of the World Wide Web in his book *The Gutenberg Galaxy: The Making of Typographic Man* (see the above quotation), almost twenty years before it was even invented, compel us to examine his work concerning the influence of communication media, as it relates to the effects of online pedagogy on both instructor and student in this new age of digitalization.

McLuhan even used the notion of '*surfing*' to refer to rapid, irregular and multi-directional movement through a heterogeneous body of documents or knowledge. One of McLuhan's (2011) section headings declared: "Peter Ramus and John Dewey were the two educational SURFERS or wave-riders of antithetic periods, the Gutenberg and the Marconi or electronic" (p. 164). In his view, technology was not necessarily hard-wired to mechanical means or to digital processes. Technology, based on McLuhan's sense of the word, was, according to Gordon (2011), any extension of the human self created by self for self. Furthermore, McLuhan (2011) had recognized the power inherent in any widely accepted technology when he claimed that: "Every technology contrived and OUTERED by man has the power to numb human awareness during the period of its first interiorization" (p. 174).

Today, half a century later, it is common to talk about *virtual worlds*, *virtual presence* and *virtual reality* (VR), embodied, if you will, in a 'second life' experiential bit-byte form. As McLuhan (2011) was well aware, we are living in an ever dissolving and transfiguring world: "When technology extends ONE of our senses, a new translation of culture occurs as swiftly as the new technology is interiorized" (p. 47). Accordingly, the media of pedagogy have significantly altered from the pencil, paper, book and blackboard classrooms of yesteryear to the world of the hyphenated-e counterparts of that same nomenclature. One of McLuhan's (2011) overriding questions was: "Does the interiorization of media such as LETTERS alter the ratio among our senses and change mental processes" (p. 28)?

I prefer to use another mathematical construct to ask the same question. Was the ongoing use of a particular medium a single parallax of moving object and stationary viewer or was it a double parallax of moving object and metamorphosing participant? McLuhan (1964) believed that it was the latter. He justified his claim that the medium is the message "because it is the medium that shapes and controls the scale and form of human association and action" (p. 203). Part of the motivation for my thesis was to probe for some insight into this and other questions in the context of Internet pedagogical exchange and possibly to find some insight into the effects of learning mathematics using digital technologies.

And so, before moving into the task of teaching in an e-setting, I took heed of a cautionary note from Dominique Scheffel-Dunand (2011):

> And to the twenty-first-century reader gazing at the ongoing displacement of the monochromatic type on a page by interactive, multidimensional graphics and sound bites in digital scholarly publishing, we say that designing a post-Gutenbergian model of knowledge and scholarly communication does not entail obliterating the past. (p. xLvii)

I was aware that I, in this new adventure, could not be hard-wired to the past. Nonetheless I decided to keep in mind my face-to-face experiences in the classroom as I moved cautiously into this rather alien virtual world of e-teaching and e-learning. To begin, I needed to know what people, in general, understood by the term 'virtual.' Establishing definitions and terminology had always been a priority in my classroom teaching.

Meanings and Nuances of the Virtual

The definition of the adjective 'virtual' had, at the beginning of the twenty-first century, become almost synonymous with objects and lived experiences in the digital world of computers. However, the word 'virtual' has and has had quite a wide diversity of meanings, some quite general and others specific to certain disciplines of study. For example, the general meaning of ‘virtual’ is the appearance, essence, or effect, of something though not formally recognized as being an actual fact, form, or name,[1] such as a virtual dictator or virtual revolution. It can also take on the meaning of that which exists in the mind, especially as a product of the imagination, such as a virtual experience.

In the physical sciences, the meaning of ‘virtual’ extends to either relating to, or being a hypothetical particle whose existence is inferred from indirect evidence (for example, a virtual photon[2]). Of particular interest to me was what physicists called a ‘virtual image.’

> The kind of magnified, right-side-up image that you see when you look at an object though a double-convex lens is a virtual image. It’s “imaginary.” Your eyes see it through the lens, but the image can’t be cast on any other surface. (Townsend & Hurd, 1973, p. 82)

The idea that an image can be seen but not realized on any surface means that the configuration of a double-convex lens creates an effect which registers in the visual cortex of the brain but is not physically tangible in nature. Later in Chapter 2, this metaphor of a ‘virtual image’ will be applied to the cognitive processes of teaching and learning.

In the subjects of Electronics or Computer Science, the word ‘virtual’ can apply to an array of many things, all having tethers to the digital world. They include: [3]

- of or relating to a computer technique by which a person, wearing a headset or manipulating a mouse/joy stick, has the experience of being in an environment created by the computer, and of interacting with and causing changes in it (hardware and software);
- of, relating to, or using virtual memory (hardware);
- being on, simulated, or carried on by means of a computer or computer network (medium of activity, such as virtual shopping or virtual conversations);
- occurring or existing primarily online (object space, such as an e-book);
- of, relating to, or existing within a virtual reality (a subjective experience within a virtual world or while on a virtual tour).

Keeping in mind the many and varied nuances of the adjective 'virtual,' one is led to an examination of ways one can experience aspects of the virtual worlds in which we may find ourselves.

Reality versus Virtual Experience: a Personal Perspective

Without venturing into philosophical, ontological or phenomenological theories of what constitutes one's reality, my view of a personal reality is a *live*, in-person, experience registered by one or more of the five senses, transformed into memory, and able to be recalled from memory. These experiences, to me, are the ***as-it-happens*** events of our personal existence and are highly individual. Many are transformative. For example, the act of winning a large sum of money in a lottery may be said to become part of that person's reality. It will, no doubt, be remembered for a very long time and may alter the way that person perceives the world in which he or she lives from that time onward. In short, a person's reality has an element of continuity to it through memory, regardless of the duration of the experience itself.

Virtual experiences, *before the advent of the digital world*, were personal but vicarious experiences registered by some subset of all five senses *and the imagination*, transformed into memory, and then able to be recalled from memory. These are the ***as-if-we-were-the-other*** experiences of our personal lives. In fact, the word 'vicarious' has the meaning: felt or undergone at second hand through sympathetic participation in another's experiences. In particular, *living* the experience of another (whether a contemporary or an imagined character) often happens through some kind of communication medium. For example, engaging in a conversation, hearing a story told on the radio, watching a movie on the big screen or reading a book can all lead to vicarious or virtual personal experiences.

It is this latter meaning of lived, but second hand, personal sensorial experiences *through an external medium* which has prompted me to propose that a second layer meaning of the virtual reality (VR) of the twenty-first century is a *personal experience* registered by some subset of all five senses, transformed into memory, and able to be recalled from memory, but *lived* through a virtual object or within a virtual world through interaction by way of keyboard, mouse, joy stick, monitor, microphone, or web cam. We might call these the ***as-if-we-were-there*** experiences of our lives.

The New World Encyclopedia describes the virtual reality (VR) of the digital age as the VIRTUAL CONTAINER of interaction:

> **Virtual Reality** (**VR**) is technology which allows a user to interact with a computer-simulated environment through one's senses. . . . Most current virtual reality environments are primarily visual in nature, displayed either on a computer screen or through stereoscopic displays, but some simulations include additional sensory information, such as sound. . . . Users can interact with a virtual environment or a virtual artifact (VA) either through the use of standard input devices such as a keyboard and mouse, or through multimodal devices such as a wired glove, the Polhemus boom arm, and omnidirectional treadmill. The simulated environment can be similar to the real world, as in simulations for pilot or combat training, or it can differ significantly from reality, as in VR games. games. (Retrieved December 19, 2013, from http://www.newworldencyclopedia.org/entry/Virtual_reality)

Others, however, focus more on the VIRTUAL EXPERIENCES within the VIRTUAL CONTAINER as being the 'virtual reality' or 'VR' of the virtual environment. While the virtual environment is merely a simulation of a real or imagined world, the reactions and responses of persons in that environment are actual, as-it-happens, experiences which can impact the emotions and even transform the thinking of those inhabiting those virtual worlds. For example, the National Centre for Supercomputing Applications (NCSA) of the University of Illinois has described VR as follows:

> The experience of venturing inside a computer-generated "virtual" world, of immersion in data, is what happens in "virtual reality" or "VR." Through various display devices, data are transformed into 3D images you not only see but also hear or even touch. (Retrieved November 17, 2013 from http://archive.ncsa.illinois.edu/Cyberia/VETopLevels/VR.Overview.html#toc)

Burbules (2006), in his article "Rethinking the Virtual," stated that he preferred to use the term 'the virtual' rather than 'VR'. He de-emphasized the role of the technology and placed the focus on the Immersive qualities of a virtual experience.

> Yet the key feature of the virtual is not the particular technology that produces the sense of immersion, but the sense of immersion itself (whatever might bring it about), which gives the virtual its phenomenological quality of an "as if"

> experience. When we think of the virtual in this way, we see that all sorts of things can create this sense of 'as if': watching a film, reading a book, listening to music, or just being caught up in a reverie or conversation. (Burbules, 2006, p. 37)
>
> The virtual should not be understood as a simulated reality exposed to us, which we passively observe, but a context where our own active response and involvement are part of what gives the experience its veracity and meaningfulness. Hence the virtual is better seen as a medial concept, neither real nor imaginary, or better, both real *and* imaginary. (Burbules, 2006, p. 38)
>
> . . . the virtual is a *medial* concept between the patently made and the apparently real. (Burbules, 2006, p. 39)

It is evident that there are a variety of perspectives as to what the term 'virtual reality' (VR) may encompass, particularly as to whether it is narrowly or broadly defined or simply a misnomer, as Burbules (2006, p. 38) has suggested.

I tend to embrace both the VR CONTAINER and the VR EXPERIENTIAL views of 'virtual reality' as being integral to any study of e-teaching and e-learning. My personal perspective is that online educational experience is a nested or layered virtual world encompassing not only the virtual environment of interaction (the VR CONTAINER) but also the virtual experiences garnered through the external medium while inhabiting that world (the VR EXPERIENCE). For purposes of this research, it will be in this context and through these lenses that my journey into the online world of teaching and learning will be viewed.

My notion of immersion in the learning of mathematics takes into account the transformative quality of the deep understanding of its concepts, its memorability and the basis for the continuity of that now personal experience and knowledge into the future – or the act of reproduction. This is reminiscent of the notion of a virtual image – the idea that an image patently made by light passing through a double-convex lens is only apparently real because it can be seen by the eye and registered in the imagination but cannot be substantiated. The communication processes of teaching and learning act like a double-convex lens reflecting back one on another which then create the virtual internal 'image' of knowledge, a transformative but ethereal state of cognition which can be realized in the behaviours of the individual in the future but has no tangible form.

It was this aspect of participant virtual experiences which I wanted to explore in my research. Mesa et al. (2012) claimed that:

> . . . we believe that unless we understand the conditions in which teaching and learning of mathematics happens in community colleges, it will be difficult, if not futile, to attempt sustainable changes of current practices, . . . (p. 4)

This claim of Mesa and colleagues for face-to-race classrooms, I believed, would necessarily be applicable to online places of learning as well. Therefore, it was important for me to experiment with teaching in an online environment and to discover how students were reacting to that environment. In particular, one of my research questions was: In such faceless classrooms, could students become immersed in the mathematics to the extent that they could carry forward their learning into other realms of endeavour successfully?

The issue was that online classrooms generally didn't come ready-made with published or personal texts. They had to, at least to some extent, be created. Therefore, towards the ultimate goal of designing an online place of learning, it became important for me to examine the medium more closely, keeping in mind McLuhan's message, so as to acquaint myself with its characteristics.

Virtual Spaces as Layered Media

Any virtual space, meaning that which exists in the mind or as a product of the imagination, may be thought of as both hardware and software. Though I see this as a valid statement in the virtual cognitive space of the mind (the synapses and structure of the brain as the 'hardware' and the sensory perceptions gleaned from external stimuli as the 'software'), in what follows, virtual spaces will be those intangible spaces which are intermediated through the hard-wiring of a computer or computer network.

Virtual classrooms accessed through the Internet, while visually observed on the computer screens of the participants, are media without any tangible content. They have no desks, no chairs, no textbooks, no whiteboards, no Smartboards, no pencils, no pens, no crayons, no felt markers, no erasers, and no paper. However, they do have intangible content – presented as layers and layers of technological communication media. In some cases, participants can 'write' on digital whiteboards across distances of space and time zones by manipulating their mice and tapping on their keyboards. There can also be button tools which simulate the functionality of crayons having various colours, pens of varying nib widths, a typewriter, and an eraser. The use of headphones or microphones can permit audible conversations to take place in real time across continents. Such virtual spaces of layered media are no less VR-environments of interaction than if all of the participants had been wired to play a virtual game in a virtual fantasy wonderland of dungeons and dragons or personalized avatars. In fact, in some of those classrooms, there may have been those who indeed have felt that they had enrolled in such a fantasmagorical world of war games with technical sand traps, climbing precipices and enemy agents. Certainly there are those who think that the study of mathematics is a terrifying experience.

The point is that, by the mere visual and virtual presence of teacher/mentor and one or many students on a series of computer screens, a 'classroom' of formal learning (in the sense of group or instructor-moderated) could be created. This was simply another kind of virtual reality container – not a game, not a shopping tour but a classroom.

My task for this research was to create such an environment for the teaching and the learning of developmental mathematics.

My Virtual Endeavour

Initially, my efforts to build such a pedagogical container evolved into the creation of a web site dedicated to the learning of foundational mathematical concepts, called the Remedial Math Centre or RMC, for short. This web site was to be the virtual space of my research study. It was to embody my pedagogical approach to mathematical thinking and learning at the time (see Chapters 3 and 4). Moreover, it was to be the testing ground for my research (see Chapters 5 and 6) on how to present mathematics in a virtual world so that it could be effective, efficient and engaging. Its original design was born of many hours of reading and reviewing the literature surrounding:

- adults learning mathematics (Morgan, 2003; O'Donoghue, 2000; Cobin et al., 2000; Miller-Reilly, 2000; Helme, 1994; Rogers, 1986);
- differences between online as distinguished from physically face-to-face learning (Hrastinski, 2008; Tang & Byrne, 2007; Chan & Waugh, 2007; Zhao et al., 2005;

Magagula & Ngwenya, 2004; Anderson, 2004, 2002; Moore, 2002; Haythornthwaite & Kazmer, 2002);

- the importance of self-directed learning (Bell, 2007; Candy, 1991; Bruner, 1966); and
- the multitudinous particulars of visuospatial issues and navigational aspects inherent in both physical and virtual environments learning (Barbour, 2007; Mayer, 2005; Shah & Miyake, 2005; Halpern & Collaer, 2005; Shah & Freedman, 2003).

The insights afforded by these writings are outlined in greater detail in Chapter 2. More information on the structure, the design and the content of the RMC web site, part of which was inspired by and adopted from these many incursions into the relevant literature, may be found in my article entitled *"Moving Ahead to the Future by Going Back to the Past: Mathematics Education Online"* (Peschke, 2008).

A brief history on how this web site actually came to be a reality is described in what follows. It begins with the reasons why I, as a university faculty member, was interested in developmental mathematics.

Paths to Personal Pedagogy

We locate ourselves, at any specific juncture, through a confluence of our past history, present circumstances and future visions. In essential ways, research projects are often extensions of our egocentric internal identities defined through external media. Our topics echo the rhythm of our heartbeats of interest and concern. Routes to discovery begin within familiar surroundings but venture into uncharted terrains. Preferred methods often imitate the beaten and familiar paths we have trod numerous times before. Our goals reflect a desire to clarify once murky and impenetrable waters. This is no less true of me as it is of others. I had always found the study of mathematics comforting, challenging and fascinating. Its comfort lay in its steady state of forms and algorithms for which 'truth' had meaning quite impervious to diversity of opinion. Its challenge surfaced every time ideas could not be fully penetrated within a single round of concentrated thought. Its fascination for me was bound up in the wonder that it could provide abstract symbolic solutions to seemingly chaotic regularities of a truly palpable world, whose timeless processes were always teasingly beyond our full knowing and our complete grasp. And so it was that I grew up loving this abstracted way of thinking and eventually came to teach it.

Throughout the days when I taught the subject in traditional face-to-face university classrooms, my students who had had difficulties with comprehension and mastery of some mathematical concepts seemed to recognize a certain empathy within me for their struggles. Whether it was because of this tacit expression of caring or some other indicator of trustworthiness or simply willingness, I could not say, but I was given numerous opportunities to participate in remedial work, both one-on-one and in group-based projects. It is the latter which became the focal point of this virtual endeavour.

The opportunity to undertake the creation of a remedial mathematics web site (not a trivial task from the perspectives of either the presentation or the navigational structure) came as an initiative of the open and distance education institution for which I worked.

The Acorn

In June of 2007, I was provided with seed money to design and create a web site – simply called the Math Site – illuminating various approaches to mathematical thought which would generate interest in the subject itself, by putting a 'human face' on it. The intention was to untangle some of its intricacies and open some doors to its mysteries. At the time, I was of the firm belief that the teaching of remedial mathematics should be an integral part of such a site and so the Remedial Math Centre (RMC) came to be included in the original design of the Math Site itself.

At first, my idea for such a virtual learning environment was relatively simple – tutorials in some form or other and a means of providing practice and self-assessment. It was really just to be a database of what are typically called 'learning objects' behind a nicely designed navigational menu. The research component of it would involve a quantitative study to measure the efficacy of these objects in terms of communicating the underlying ideas and generating understanding of the mathematical tools. During my first year of exposure to educational research at the University of Alberta, however, I was pushed firmly outside of this familiar zone.

There has been a recurring theme within the research educational literature highlighting the importance of the individual and his or her *experience* in the classroom setting. Certainly John Dewey (1938) had provoked a lot of thought about it. As a lecturer, I had modified the presentation of the mathematical ideas towards the group. Now, as the designer of an online pedagogical site, my first vision of its initial form was also to be directed to the group as my lectures had been – the anonymous, amorphous group. However, I soon began to realize that, if the teaching of developmental mathematics online were to ‘succeed’ at all, the environment had to be customized to individual needs in all of their diversity and complexity. I could not rely on immediate individual participant feedback, as I had during my one-on-one tutoring sessions or even in one of my university classrooms. Even as face-to-face classroom instructors teach to the group, but modify for the individual, so must I in my design of this pedagogical space. I began to conceive of an online setting which was more personalized than a set of objects, even with a textualized human face. Moreover, I felt strongly that it had to be founded upon an underlying philosophy of what I believed mathematical reasoning was all about and how this way of thinking could be achieved in such a setting. The question remained: How could this be achieved? This led to my exploratory journey through the literature of past findings and experiences.

CHAPTER 2 SETTING the VIRTUAL STAGE

The task of creating a virtual learning site would not be an easy one. A number of issues had to be investigated before even beginning to create an online environment conducive to effective learning for an adult population of students.

Initial Perspectives from the Literature

Adults Learning Mathematics

It was O'Donoghue (2000) who described the task of teaching adults mathematics without a modicum of embellishment:

> The inescapable reality for those working with adults is that mathematics as seen through the eyes of adult learners is the antithesis of what tutors would like. Mathematics for these learners is difficult to learn; evokes negative emotions; is associated with failure; presents an obstacle to job promotion; constitutes a bar to further education; and perpetuates inequality in society. Already the educational die is loaded against success for a significant proportion of adult learners of mathematics. (p. 230)

O'Donoghue was referring to adult education as involving those adult learners returning to formal education after some years away from classroom learning. He also suggested that there was a dearth of research on this kind of adult education because such students were not distinguished from their traditional-age counterparts in the same classrooms of instruction.

Another issue surrounding adults learning mathematics at the time was noted by Cobin et al. (2000):

> Difficulties in tracing activity by adults learning mathematics also occur because adult mathematics education may be subsumed within literacy or other community development programmes where mathematics learning is seen as a secondary concern. (p. 20)

As a consequence, little has been documented on adults learning developmental mathematics. Nonetheless, O'Donoghue (2000) acknowledged that the 'Challenge' (with a capital 'C') for mathematics educators was "to find effective ways to teach mathematics to a diverse population of adult learners with mixed attitudes towards mathematics and different aspirations, who are underprepared for post-secondary education in mathematics" (p. 231).

Rogers (1986) listed some practical guidelines for teaching adult learners, which included moving from concrete examples to generalizations (rather than the reverse), relating new material to learners' existing experiences and knowledge base, encouraging them to be active learners, and designing learning scenarios which would capture their interest and motivate them to engage with the task. He based these guidelines on characteristics of observed adults' self-directed learning activities.

Miller-Reilly (2000) experienced some mixed responses from the students enrolled in the Mathematics I programme introduced at the University of Auckland in 1994. It was specifically designed for adults who were returning to university and for those who had been previously unsuccessful at mathematics. Based on the realistic mathematics education approach in the Netherlands (de Lange, 1987), the underlying philosophy of the Mathematics I programme lay in the recognition that learners are able to construct their own understanding of mathematical concepts by working together in groups on real-world problems. The students in the course were encouraged to work together on open-ended investigations involving scenarios about environmental issues, maps, packaging and medicine. The intent was to promote their "learning to communicate mathematically in order to increase their mathematical confidence and their ability to mathematise" (Miller-Reilly, 2000, p. 257). At the end of the course, a survey of the students showed that 82% of the older students (over age 30) indicated their confidence in learning mathematics had improved over the duration of this programme, compared with 25% of the younger students (between 20 and 24 years of age). When asked if they had felt more capable of doing mathematics or if the course had improved their attitude towards mathematics, 55% of the older group, versus 25% of the younger group, responded positively. It was suggested that students' beliefs about how to learn mathematics effectively may have caused the negative reactions in the younger group of adults. Miller-Reilly herself attributed the success of the programme (78% pass rate for the older group; 65% for the younger group) to the alignment of the approach with adult learning styles.

Not all researchers, however, were enamoured with scenario-based mathematical teaching and learning (having students learn mathematics primarily through the working out of examples or problems in various real-world contexts). Helme (1994), in her study of female adult mathematics students, found that unfamiliar contexts to some students created a kind of cultural alienation for them, that not everyone was motivated by solving applications of mathematics in different disciplines, and that students differed in terms of their preference for familiar or unfamiliar scenarios.

The research literature on adults learning mathematics was sparse. Coben et al. (2000) described it as an “emerging research domain, interdisciplinary within the social sciences (as is its ‘parent’ field, ‘education’) and spanning the sub-fields of mathematics education and adult education” (pp. 50-51). In 2009, the U.S. Department of Education, Office of Planning, Evaluation, and Policy Development (2009) had published a meta-analysis and review of online learning studies. They conducted a systematic search of the research literature from 1996 through to July 2008 and identified more than one thousand empirical studies of online learning. They were subsequently screened according to the following criteria. Each study selected for consideration had to:

1. involve learning that took place either entirely or partially over the Internet;
2. involve controlled conditions contrasting either online and face-to-face (category 1) or blended and face-to-face learning environments (category 2);
3. report a learning outcome that was measured for both treatment and control groups;
4. use a controlled design (experimental or quasi-experimental); and
5. report sufficient data for proper statistical analysis.

After this screening, only two of the remaining studies dealt with situations involving adults learning mathematics online – one algebra program and one course in statistical hypothesis testing. Studies which measured only non-learning outcomes (for example, those that dealt with attitude, level of learner/instructor satisfaction, retention) were excluded from the meta-analysis. This rather comprehensive 2009 U.S. Department of Education study helped convince me that my research project concerning adult experiences of learning mathematics in an online setting was timely and, hopefully, would serve to fill a void in this still emerging research domain.

Despite the dearth of studies about adults learning mathematics at a distance (in particular, those who were returning to their studies after some years of absence from a formal education classroom), Evans (2000, p. 44) had claimed that a good measure of agreement existed about the affective factors which might be expected to influence thinking and performance in mathematics in older students and adults. His list included: mathematics anxiety, confidence (self-image, self-efficacy, and locus of control), prior conceptions of the usefulness and the difficulty of mathematics, and whether the mathematics was considered interesting and/or enjoyable. Therefore, in order to understand my potential volunteer participants better, I decided to include opportunities for my project participants to discuss their personal attitudes about the subject and their past experiences learning it, both one-on-one in conversation and in writing.

Facets of e-Learning: the importance of 'place'

Wood (2013) corroborated what Graham (2012) had claimed: that place matters in any learning situation.

> . . . learners need a place to learn (especially those without comfortable home or school environs) and with that place, they need mentors and guides, not teaching assistants, facilitators, or monitors. (Wood, 2013)

This also applies to e-learners. Virtual spaces are initially empty containers. This is why online educators are, in many cases, forced to choose the space and built the 'place' of learning before they can begin teaching. There is generally no prescribed room in which to conduct their classes. In addition, learning materials, at least in this relatively early stage of the adoption of e-classrooms, need to be created, not just reproduced from existing texts. For this reason, the journey I took to create my own version of a learning place is documented in this research. This was an integral part of my personal experience of teaching online.

Choice of virtual container. The first task in my project was to choose an appropriate container to house the learning objects. Essentially there were two options open to me at the time: use the University's course management system (CMS) as the teaching platform for the project or build an independent venue outside of that encompassing structure.

Morgan (2003) had published the findings of a research study conducted by ECAR (EDUCAUSE Center for Applied Research) in 2002 on the faculty use of course management systems. EDUCAUSE is a nonprofit association whose mission is to advance higher education by promoting the intelligent use of information technology. The location of this mixed methods study was the University of Wisconsin System (UWS) with 6500 faculty and more than 150,000 students, encompassing two doctoral /research institutions, eleven universities that award bachelor's and master's degrees, thirteen freshman-sophomore campuses and UW-Extension outreach programs. This research encompassed six different CMSs: WebCT (22%); Blackboard (75%); Promethues; LearningSpace; eCollege; FirstClass. Both quantitative and qualitative data were collected by polling faculty and monitoring students' usage of CMS systems in 5160 courses during the fall of 2002.

In terms of using a course management system (CMS) as a pedagogical container, faculty had expressed several concerns. The primary concern was loss of control over the instructional environment because of the technology layer between them and their instructional materials. Rigidity of structure with limited customization capabilities within the CMS was perceived by many as an imposed, whether conscious or unconscious, move towards standardization of "one of the institution's core and most highly personalized processes" (Morgan, 2003, p. 12) – that of teaching and learning. This perception was held especially so by faculty involved in fully online courses. One faculty member commented: "The inflexibility of the structure gets in the way of good pedagogy" (Morgan, 2003, p. 10). On the other hand, some others saw a kind of 'accidental' pedagogy inherent in CMSs, although they conceded that it would be difficult to measure or prove. They felt that such structures encouraged better course organization, greater transparency and accountability.

Of significance to my research was the finding that among many faculty members, there was a pervading sense that the linearity of presentation and the rigid structure of the CMS software hampered true creativity and spontaneity of an essentially dynamic exchange between instructor and student. This was a viewpoint also reiterated in Lane (2008). Lane's research, in particular, pointed out that CMSs are not readily amenable to the inclusion of multi-modal learning experiences involving audio, visual, or mixed-media formats. Because of these and other limitations, she concluded that any attempt to offer such a comprehensive pedagogical experience to students at a distance would not be served well by course management systems. Lane ultimately advocated a *building it your own way* alternative. In fact, this is what I ended up doing.

Some years after the ECAR 2002 study, Chan and Waugh (2007) undertook a qualitative study with a group of adult distance learning students at the Open University of Hong Kong. These participants had all been enrolled in a foundational course in mathematics and had come from a wide variety of educational backgrounds. Chan and Waugh's primary purpose in this research was to identify factors which affected students' participation, or lack of it, in the online learning environment of WebCT, the course management system used by the University. It should be noted that neither mathematical symbols nor Chinese characters were supported in this software at the time of the study. This lack of mathematical symbolic support in the course materials may have been a covariant factor in hindering or even discouraging students from engaging with the content but could not be measured statistically.

Chan and Waugh found that, of the large majority (88-90%) of students who had accessed the site, only a very small percentage (6-8%) of them had taken full advantage of its offerings. The results of the study indicated that students' reasons for accessing the site had mainly to do with the ease of getting updated information and the ease of contacting tutors and peers, some finding that kind of immediate communication particularly helpful to them in doing the assignments and preparing for exams. However, eighty-two percent (82%) of those who did use the site did not engage with it further. Reasons provided by those students focused primarily on the issue of content. In a number of cases, the content provided was helpful neither to the understanding of their studies nor to the completion of their assignments. Some felt that the online information was simply a duplication of their other course materials. Still others found it inconvenient to use the site more fully because of lack of time. Some found the software to be user-unfriendly and there were also those who preferred to read paper copies over their digital counterparts. Whatever the reasons, these adult distance learning students at the Open University of Hong Kong did not find the online presentation of the developmental mathematics course compelling enough pedagogically to use it extensively for learning purposes.

What I learned from Chan and Waugh's 2007 study was the utmost importance to create an attention-grabbing but engaging online 'text' which covered the basics of a comprehensive remedial mathematics program but was geared primarily to the learning of the subject matter. Such a web site could not simply be a digital replica of other published texts or course materials. It had, as much as was possible, to use the full capabilities of the Internet medium to enhance and extend learning.

Because I had really enjoyed the flexibility, the spontaneity and the multi-tasking of teaching in a face-to-face environment, I decided against embedding the learning materials for the project into the University's course management system. I opted, instead, for a web site of my own making with an accompanying third-party self-assessment tool. I have always held to the philosophy that both the teaching and the learning of mathematical ideas and constructs are more effective if fluid and dynamic, rather than rigid and too highly structured – even though the subject itself is often portrayed as being just the diametrical opposite. Therefore, the underlying navigational map of the web site, rather than being an indexed or chronicled list of options and tasks (as would have been the case in a CMS), was to be constructed as an open lattice which would allow for multiple avenues of entry and exit, thereby interlaced with threads of continuity stretching across its entire map.

Now that the decision had been made to create a web site for the learning of developmental mathematics, I had to come up with some ideas as to how this could be constructed – other than the fact that it would structurally be in the form of an open lattice. Because I had wanted to include animations and video as a way to engage viewers and, hopefully, to enhance the understanding of the students, my examination of the literature turned to visuospatial[4] aspects of learning mathematical ideas online.

Visuospatial considerations. One of my earliest literature reviews on e-learning concerned its visuospatial dimensions. The primary purpose of my delving into the literature on visuospatial matters was to glean the wisdom from a wide variety of research findings across diverse disciplines on visuospatial thinking in order to understand better its role in e-learning, in general, and in the online learning of mathematics, in particular. Base-level visuospatial thinking encompasses object recognition, visual attention, and scene perception. At the core, however, of higher-order visuospatial thinking is the ability to form a mental image from visual input and be able to manipulate that image in accordance with specified, non-arbitrary tasks (Mayer, 2005, p. 477).

Shah and Miyake (2005, p. xii) noted that the age of information technology has placed new demands on people's visuospatial thinking skills, heretofore not seen as intrinsic to basic educational curricula. For example, the public-at-large is now required to navigate through virtual environments on a regular basis and many are faced with having to comprehend and use interactive displays, some animated and others in three dimensions. The good news for educators has come from Richard Mayer (2005) who had discovered that "appropriate visuospatial thinking during learning can enhance the learner's understanding, and multimedia presentations can be designed to prime appropriate visuospatial thinking during learning" (p. 479).

Halpern and Collaer (2005), who also conducted research into visuospatial issues of learning, described visuospatial thinking this way:

> Visuospatial information processing involves an interplay of multiple cognitive processes, including visual and spatial sensation and perception, a limited capacity visuospatial working memory[5], and longer-term memories where visual and spatial information may be encoded in many ways. (p. 171)

Their 2005 research analysis suggested that differences in learning in some areas of mathematical thinking may have had to do with:

1. how learners perceive visually and spatially,
2. how they encode that information into a mental image[6] using both immediate stimuli and long-term memory constructs,
3. their capacity to hold that 'image' in memory and do appropriate tasks with it, and
4. Gender.

In fact, Halpern and Collaer's research had provided evidence that there were substantive differences in visuospatial abilities based on gender differences. They found that, when processing visuospatial tasks:

> Females tend to use analytic or detail-oriented and verbal strategies, and males tend to use holistic and schematic strategies, which is probably the reason why females have better memory for landmarks along a route and males travel through complex environments more quickly and with fewer errors. (Halpern & Collaer, 2005, pp. 203-204)

Moreover, this team of researchers also found evidence that these differences between females and males could be responsible for some of the same proportion of gender differences found in the solving of mathematical problems containing spatial information and requiring visuospatial thinking. Therefore, I decided that both verbal detail and top-down schematics would be a part of the presentation of both the materials and the navigational structure of the web site intended to be my online 'text' of the developmental mathematics project.

It was, however, a review article by Shah and Freedman (2003), entitled "Visuospatial Cognition in Electronic Learning," which was to have significant ramifications on the design and use of graphics and animation materials on the RMC web site. The insights I garnered from this article became the basis for most of the design elements incorporated into its structure and building blocks, all in an effort to enhance students' cognition of the mathematics presented there. I decided that the RMC web site was to be much more than plain text linearly presented on a screen with a few static diagrams strategically placed as if it were simply an e-text equivalent of its Gutenbergian face-to-face classroom counterpart.

In their 2003 article, Shah and Freedman gave an overview of the research literature surrounding possible cognitive benefits of visualizations (whether static, animated, or interactive) in e-learning environments, possible cognitive constraints of their use and how to overcome them, and how individual differences in learning style (visual versus verbal) and cognitive abilities (in particular, visuospatial) may interact with the learning process from such visualizations. The authors found evidence in the literature that the offering of instructional materials in both visual and verbal (or textual) forms does, in fact, promote learning (Clark & Paivio, 1991; Mayer, 2001). Presentation in dual sensory modalities is generally considered a superior way of presenting information because the information disseminated is then encoded in two modes of sensory perception as opposed to one. This would give students more than one way of accessing the information in memory. In addition, it was believed that, because visual presentations (as compared to textual or verbal presentations) are externalized in some sense, the cognitive load is lessened due to the reduced need to hold the mental representation or the processing of it in memory (Hegarty & Steinhoff, 1997; Zhang, 1997).

The collective voices from Shaw and Freedman's literature review also had indicated that interactive or animated displays may require deeper processing because they stimulated comparison and integration with other materials as well as their own prior knowledge (Mayer, 2002; Wiley & Voss, 1999; Rieber, 1990). It was found that the attractiveness of some visually presented materials can be motivational for students

(VanderStoep, Fagerlin, & Feenstra, 2000; Rieber, 1990). However, in some studies, animations and other visualizations were **not** the preferred choice of learning (Pane, Corbett, & John, 1996). The conclusions were that visualizations can make complex information easier to comprehend and are of particular benefit in demonstrating cause-and-effect relationships and presenting changes over time (Larkin & Simon, 1987; Park & Gittelman, 1992; Zhang & Norman, 1994; Robinson & Skinner, 1996; Oestermeier & Hesse, 2000; Mayer, 2001). They may also be used to advantage in facilitating problem-solving (Oestermeier & Hesse, 2000).

Shaw and Freedman (2003) did point out one important constraint in the use of more complex visualizations, especially those containing auditory and/or textual information. That had had to do with the amount of cognitive demand placed on a learner's working memory. This part of the cognitive function was considered to be highly limited in terms of the collective amount of detail which can be actively held and processed simultaneously. Visualizations which are complex or 'busy' in their design may place an inordinate memory load on the learner. The literature had outlined three basic principles for keeping the students' cognitive load to within reasonable limits. They were: 1. eliminate less relevant information, 2. keep information to be integrated relatively close to the primary focus, and 3. present information in a manner requiring minimal cognitive computation for comprehension.

The research on visuospatial cognition mentioned in Shah and Freedman's 2003 article had also concluded that learning style and inherent visuospatial ability influenced the degree to which visualizations were effective for learners (Hegarty & Kozhevnikov, 1999; Gyselinck, Comoldi, Dubois, De Beni, & Ehrlich, 2002; Kozhevnikov, Hegarty & Mayer, 2002). Visual, as opposed to verbal, learners and those with a high degree of visuospatial ability, not surprisingly, benefited from them. In summing up, Shah and Freedman recommended that e-learning content designers should concentrate on the creation of visualizations aimed at those with a low level of visuospatial ability, rather than increasing their technological sophistication.

From this survey of the research on visuospatial issues involved in online learning, it therefore became of paramount importance to construct the learning objects with minimal cognitive load for most viewers. I also decided to use colour-coding in some tutorial explanations so as to facilitate understanding by providing visual continuity, thereby reducing working memory load.

Structure and design. Barbour (2007) laid out several guidelines for effective web-based content for secondary school-level students. This research was based upon the initiative of the Centre for Distance Learning and Innovation (CDLI) formed by the Government of Newfoundland and Labrador in 2001 whose mandate was to provide opportunities for web-based *asynchronous* learning to students in rural communities throughout the province. Typically, a student's CDLI involvement constituted at most two courses undertaken in an unsupervised distance education room, using a combination of *synchronous* instruction via the web conferencing system, Elluminate Live (audio-visual instruction with opportunity for inter-communication), in addition to some *asynchronous* instruction using the course management system WebCT.

The content developers for CDLI had already taken into account research on the design and content of distance education projects which had highlighted the importance of providing interactivity and enough flexibility for learner autonomy/control. From this base, Barbour (2007) was able to glean a list of seven guidelines for content creators in regards to the structure and design of educational content. The guidelines included:

1. Plan out the entire project with all details before embarking on any development;
2. Use a simple consistent navigation design, but diverse content presentation;
3. Supply content summaries and adapt content to students' real life situations as much as possible;
4. Provide clear instructions and clear, but realistic, expectations;
5. Refrain from using too much text. Consider visuals or alternate presentations for textual materials;
6. Make smart use of interactive multimedia; and
7. Develop content for the average learner and those who require reinforcement of basic skills.

However, to make 'smart use' of interactive multimedia was not as simple as the statement itself.

A cognitive theory of multimedia learning. Multimedia presentations utilize dual-channel, visual/pictorial and auditory/verbal, processing of information which may include printed or spoken text and static or dynamic graphics. While printed text may initially be instantiated into a mental construct in the visual /pictorial channel, it may subsequently be processed in the auditory/verbal channel. The processing of auditory, verbal and visual, and pictorial material is done using channels which are separate from one another (Baddeley, 1986, 1999; Paivio, 1986) and each has certain limitations in its cognitive load capacities at any given time (Baddeley, 1986, 1999; Sweller, 1999).

Mayer (2005) proposed a cognitive theory of multimedia learning, concluding that “meaningful learning occurs when learners engage in active processing within the channels including selecting relevant words and pictures, organizing them into coherent pictorial and verbal models, and integrating them with each other and appropriate prior knowledge” (p. 481). Succinctly expressed, five cognitive processes were involved in learning activities. They included the selection of both images and words, the organization of each of those selections and ultimately the integration of those images and words. Each cognitive process would require visuospatial information processing and verbal thinking.

The underlying premise of Mayer’s research was that the activation of two modalities of sensory perception, thereby allowing students to integrate both pictorial and verbal representations mentally, could result in more in-depth understanding of the subject matter. Therefore, in online learning environments, he concluded that it was important to utilize proven design principles in order to foster and enhance visuospatial thinking. Consequently, Mayer (2005, p. 486) laid out various design principles for multimedia instructional materials:

1. contiguity (present words and pictures concurrently, rather than successively);
2. coherency (avoid irrelevant - even if seductive - words, sounds, images or graphics);
3. modality (design materials so that cognitive load is shared between two sensory channels, instead of one);
4. redundancy (avoid dual-channel encoding for same content);
5. the personalization principle (use conversational style rather than a formal style for narration or text);
6. interactivity (allow viewers to control the presentation rate); and

7. the signaling principle (signal narration by an outline, headings, pointers, voice intonation, voice emphasis, or highlighting).

The end result of all of this investigation was the decision to adhere to Barbour's (2007) seven (7) guidelines when first designing the layout of the RMC web site and Mayer's (2005) seven (7) principles for the design of any instructional multimedia content. A detailed account of the navigational structure and content of the site may be found in my article entitled "Moving ahead to the future by going back to the past: Mathematics education online" (Peschke, 2008).

Shifting Focus from Learning Object to Learning Subject

Now that the container of a web site had been chosen, themes of further inquiry from the documented research would involve the perspectives and the experiences of the learners in other online environments and the indicators of student satisfaction or students' successful completion of a course of studies in such virtual settings. I had hoped that lessons learned in other e-venues may contribute to a greater understanding of core issues which may affect students' learning experiences in my own. It was these enlarged, more telescopic, visions which I wanted to bring into my particular research inquiry: *How could one create a viable, economic and effective online classroom of remedial mathematics for adult learners wishing to pursue post-secondary programs of study?*

While, in a broad sense, the project involved a multi-layered study of the experiences of online teaching and learning of remedial mathematics, the research would also focus on ways in which a depersonalized environment, comprising various digital learning objects and venues, could be made more personable – yet still be an effective learning and communication space for adult students of mathematics – without becoming a media circus. This move to examine the more personal aspects of the web-based learning mathematics prompted me to seek out research studies which examined some of the affective factors for predicting learning outcomes and achievement. As noted previously, Evans (2000, p. 44) had spotlighted a number of such factors which might be expected to influence thinking and performance in mathematics in older students and adults. Among those factors, Evans included mathematics anxiety, confidence level (self-image, self-efficacy, and locus of control), prior conceptions of the usefulness and the difficulty of mathematics, and whether the mathematics was considered interesting and/or enjoyable. This led to the inclusion of motivational materials and options for finding study buddies or speaking with a professional counselor (Peschke, 2008).

Wadsworth, Husman, Duggan and Pennington (2007) had conducted a study with a similar group of students in a comparable online setting to those who were in my project, except that, in my case, no formal assessments of learning would be required of the participants. I had decided very early on that my research focus was on learning and the experiences of learning from the participants' points of view, not only those gleaned from my assessments. The participants in the Wadsworth et al. study were a group of college students in an online developmental mathematics course. Approximately one-half of the students were in the course for the first time; the remaining half had taken the course before but had not received credit for it. The overall distribution of female to male students in the program was in a ratio of 60:40. The content of the course was delivered entirely online through the use of a hypermedia-based software package which offered lectures on video, practice exams, machine-graded homework with instant feedback and major chapter exams. This was a self-paced course and required the students to regulate their time and the environment in a way not typical in classroom settings. Three online surveys given to the students in the final week of the semester were the instruments of data collection for this study. It should be noted that responses to the complete set of surveys were obtained from less than eighteen percent (18%) of the possible participants.

In the Wadsworth et al. 2007 study, students' learning strategies were assessed using the Learning and Study Strategies Inventory (LASSI) diagnostic measure of strategies and skills (see Appendix A) which provides standardized scores and national norms for ten (10) different scales: attitude, motivation, time management, anxiety, concentration, information processing, selecting the main idea, use of support /materials, self-testing and test strategies. In these tests, self-efficacy was measured by the students assessing their own confidence level in being able to solve specific types of mathematics problems. Achievement was measured by points granted for quizzes, tests, time spent in the mathematics lab and some extra credit for filling out the surveys.

The results of the Wadsworth et al. study showed very little difference between males and females in terms of the final grade received and their subscale scores on the

LASSI. A multiple regression analysis indicated that four types of learning strategies – motivation, concentration, information processing, and self-testing – along with self-efficacy were the only significant predictors of the final grade. The findings of this study also indicated that the more participants used self-testing strategies, the lower their achievement scores. Poor achievers also scored below the 50 percentile in seven (7) out of ten (10) of the LASSI subscales.

The pre-2007 research literature cited by Wadsworth and colleagues in this study had been conducted in traditional face-to-face classroom settings. At the time, Wadsworth et al. had had suspicions that successful strategies in classroom settings may not always transfer over to e-learning environments. However, when analyzing their own research findings, they concluded that students' study strategies and self-efficacy applied to Web-based learning were not substantially different from student learning in traditional remedial courses. The incorporation of metacognitive tools integrated into online software was recommended as being helpful to keep students on track and remind them to use learning techniques such as note taking and reflection. They also added a cautionary note that, with only eighty-nine (89) out of five hundred and eleven (511) students responding to the surveys, this study did not provide sufficient evidence for generalization and that the sample may be skewed towards those students who were more highly motivated to succeed. However, the Wadsworth group also cited Chang's (2005) research study. This latter study had indicated that students enrolled in a web-based course which included self-regulated learning strategy instruction were more positively motivated and more self-directed than those without such instruction.

The relatively high correlation between the LASSI subscales and achievement prompted me to examine the LASSI tests. I then decided to give the participants in the project an option to write these metacognitive tests and discover their own learning strategies by way of a link in the RMC web site (at the cost of the University). In addition I decided to include information about the Felder-Silverman Model of learning styles for mathematics (see Appendix B) for those who were interested. The web site was to be both a learning and a meta-learning place where those who came had the option to discover more than just mathematical content.

After the decisions on the asynchronous portion of the research project had, more or less, been made, my last step in setting the virtual stage was the inclusion of synchronous meetings with all the participants. There was and still is evidence to show that a blended e-learning model has its merits. Wood (2013) claimed that “Memorable experiences come from synchronized encounters -- dorms, registrations, classes, exams, projects, guest lectures, homecomings, study groups, commencements“. Wood also quoted a University Ventures Fund newsletter (2012) suggesting that “a course is not a book but a journey, led by an expert.”

Blending Synchronous into Asynchronous

In my journey to find the best possible stage for the e-learning of mathematics, I decided to shift the focus from interactive to inter-personal. This move had been primarily motivated by my wish to diminish the feelings of isolation which students may experience in virtual places and to foster a sense of social connection for those who came – notwithstanding the asynchronous communication devices of discussion forums or email. This was essentially a re-conceptualization of the 1970’s open classroom[7] in a new but equally fluid setting because participants would be allowed to move freely from venue to venue, choosing asynchronous texts or synchronous discussions at will.

Speaking mathematically in such forums is difficult, at best, and sometimes simply impossible because of the absence, in such venues, of a math palette of symbols inherent in mathematical discourse. Besides all that, fostering a sense of a social connection, community and comfort for groups of mathematics learners still remains one of the challenges of teaching the subject itself – no matter where the interaction takes place. Voorhees (2007) had claimed: “The online environment magnifies the challenges of getting to know one’s students and of getting students to feel connected to the class”. Haythornthwaite and Kazmer (2002) observed that, for groups of online learners:

> Isolation can be overcome by more continued contact, particularly synchronously, and by becoming aware of themselves as members of a community rather than as isolated individuals communicating with the computer. (p. 459)

Hrastinski (2008) had noted that there were basically two types of e-learning: the asynchronous model and the synchronous model. Prior to 2008, teaching and learning in virtual environments relied mainly on asynchronous communication means. However, since that time, improvement in technology and increasing band width capabilities had opened up opportunities for synchronous e-learning initiatives and these were growing in popularity.

In his Ph.D. thesis examining student involvement in synchronous online education, Hrastinski (2007) talked about “two dimensions of online student participation, which were labelled personal participation and cognitive participation” (p. 121), both an integral part of the learning process. He defined ‘participation’ following Wenger’s (1998) definition as “a process of taking part and also to the relations with others that reflect this process” (p. 55). Hrastinski (2007) went on to explain that participation “is a complex process that combines doing, talking, thinking, feeling and belonging” (p. 30). The results of his thesis suggested that: “Synchronous communication seems particularly useful for supporting task and social support relations, and to exchange information with a lower degree of complexity” (Hrastinski, 2007, pp. 121-122).

In his follow-up paper in 2008, Hrastinski asserted that asynchronous and synchronous models supported different purposes in e-learning environments. Asynchronous materials and communication methods, he suggested, provided the groundwork for students’ cognitive participation by enabling increased reflection time and allowing for deeper processing of information. Synchronous sessions and communication methods, on the other hand, encouraged students’ personal participation by fostering somewhat increased psychological arousal, motivation and convergence on meaning. He concluded that synchronous communication had the potential to foster e-learning communities.

Whether a learning community could be forged within a group of mathematics students in an online setting was of great interest to me as a researcher. If so, what human interaction ingredients would contribute towards the creation of this sense of community? How enduring could it be? How was it the same or different from other learning communities? These were some of my research questions surrounding the lived realities of online teaching and learning. By including synchronous live sessions using web conferencing software as another layer of media in my project, I could investigate this and, perhaps, find some answers into these probing questions.

Already Anderson (2004) had highlighted a fundamental difference in the nature of community within face-to-face educational settings in comparison to a geographically disparate virtual phalanx of participants. The latter configuration of learners was characterized by its lack of mutual presence in space and time, thus attenuating any social links or relationships which may normally be formed between them. Anderson (2004) suggested that, "it may be more challenging than we think to create and sustain such communities" (p. 5). It should be noted, however, that he had also previously suggested that:

> Sufficient levels of deep and meaningful learning can be developed, as long as one of the three forms of interaction (student-teacher; student-student; student-content) is at very high levels. The other two may be offered at minimal levels or even eliminated without degrading the educational experience. (Anderson, 2002, p. 3)

Therefore, those who would interact with the Remedial Math Centre web site at a high level could potentially have a fulfilling educational experience without a lot of student-teacher or student-student interaction. Others, of necessity would need to involve themselves with either teacher or fellow student within the synchronous venues in order to achieve a satisfying learning experience. I believed that some of these differences in engagement with the various online offerings of the project would have to do with the students' perception of their own innate abilities and their preferred or, at least, habituated student-instructor relationship.

Candy (1991) had suggested that one of the many challenges for a project which was primarily learner-controlled was the delicate renegotiation and "reshaping of the foundational underpinning of the teacher-learner relationship" (p. 223). This, of course, was going to extend to the teaching-learning dynamics of the online venues in my virtual learning space, in particular.

Lessons Learned

Based upon the findings of the many and diverse research findings in the literature review, the final choice of the virtual stage of the project learning space was set up as an alternating triad of media layers: a web site of independent study, a test bank of auto-marked self-assessment exercises, and a weekly series of synchronous tutorials. The construction of the RMC web site (Peschke, 2008) was ultimately detached from any institutional course structural container. A venue of reading, listening, reiterative practice, guidance and self-assessment formed a loosely spread out lattice with multiple avenues of entry and exit to be chosen or ignored by personal preference. The mathematical content was presented primarily through interactive Flash tutorials (some audio-backed, others with animated captioning) and a third-party self-assessment tool correlated with the topics covered. Historical notes of interest, vignettes and discussions about scientific applications to the mathematics were interspersed throughout the site. Navigational menus were designed to provide a continuous thread of coherency from topic to topic throughout the entire venue. Tutorials and other learning objects were created so as to keep the cognitive load for learning the material at a minimum. Metacognitive activities were included in the menu options so that students interested in their personal learning styles and learning strategies could test themselves in a more formal setting using various assessment instruments. This was to be totally optional in nature. At no time would I strongly encourage participants to avail themselves of these tests. My methodology was non-interventionist. This was to be an open forum of learner-mediated, self-paced learning.

Coupled with the RMC web site and the self-assessment tool, a regular weekly series of live synchronous sessions, with me as instructor, were offered as an e-version of the face-to-face classroom. The mathematical content of both the asynchronous and synchronous virtual environments were founded upon my personal pedagogy of many years, but laid out and presented using modern technologies. Within this broadened and blended pedagogical e-setting, I hoped to be able to emulate the student experiences of face-to-face classroom learning without having to simulate.

Up to this juncture, my search through the literature had always been directed to specific online learning cases and general pedagogical principles for digital objects of instruction. The medium of the Internet itself and its intrinsic effect on our psyches, primarily, and our learning habits, incidentally, had remained in the background as the proverbial *elephant in the room*, ubiquitously present but never acknowledged. Because McLuhan (2011, pp. 52-53) had predicted the existence of a medium (we now call the Internet) which would be transformative to the societies in which its adoption became a norm, I went the extra mile of the journey, so to speak, to discover what he had had to say about media in general and their place in the socialization of our world. His prognostication did come true some twenty years after his declaration of it and the medium of the Internet has, since that time, definitely shrunk our planet into a global village. Therefore, before considering the potential impact of adopting the Internet for educational purposes on a grand scale, I wanted to look at another technology which had transformed education in Europe many centuries ago. I will begin with McLuhan's message to us in 1964.

McLuhan's Virtual Message

> The medium is the message. This is merely to say that the personal and social consequences of any medium -- that is, of any extension of ourselves -- result from the new scale that is introduced into our affairs by each extension of ourselves, or by any new technology. (McLuhan, 1964, p. 203)

McLuhan, in his book, *Understanding Media: The Extensions of Man*, published in 1964, claimed that any mass communication media carried with it a message independent of the authors' texts. His memorable aphorism "the medium is the message" (see the quote above) abruptly stole the focus of attention away from the content as the message of the medium to the populace and sharply re-oriented it to the message of the medium as being its own intrinsic effect on the populace. He asserted that the impact on both the individual and the society as a whole was orchestrated by the characteristics of the medium itself. Each such medium then was a kind of virtual message in and of itself.

McLuhan was not the first to propound such theories but he was the first to call this subliminal impact of technology on wo(men) a 'message.' Many other scholars, including Chaytor (1945), Hall (1959), Innis (1950), Usher (1959) and Carpenter (1960), had formed a collective mid-twentieth-century viewpoint of how various technologies of the past had played a role in shaping our experiences and even in how we ultimately thought and learned as a culture. McLuhan (2011) argued that:

> If a technology is introduced either from within or from without a culture, and if it gives new stress or ascendancy to one or another of our senses, the ratio among all of our senses is altered. We no longer feel the same, nor do our eyes and ears and other senses remain the same. (pp. 28-29)
>
> When technology extends ONE of our senses, a new translation of culture occurs as swiftly as the new technology is interiorized. (p. 47)

The story of the introduction of the movable-type printing press in Europe was a case in point. This technology forced the ascendancy of the eye over the ear in terms access to information and subsequently in how that information was then disseminated. The adoption of educational materials generated by the printing press had the effect of not only granting greater access to the once esoteric knowledge of academicians and scientists but also, thereby, linearizing learning to some degree. The printed word also had an incidental message – that of its being the indisputable truth, hence silencing some vocal critics. In fact, it was precisely this technological device which fundamentally transformed educational practices in the Western European universities of the time, transferring their power from the hand of the scribe and the voice of the orator to the mechanized, metallic movements of the printing press. So what has been the cultural legacy of the Gutenberg press?

The Message of the Gutenberg Printing Press: a sensorial twist

Prior to the invention of the metal movable-type printing press in Europe by Johannes Gutenberg around 1439, books of learning were created using manuscript methods – meticulously hand written and drawn as personalized works of art shaped by the tactility of the author or scribe. This single invention spread rapidly throughout Europe and is credited with revolutionizing thinking and learning from that time forward as being a major contributor to the development of the Renaissance[8], the Reformation[9], the Age of Enlightenment[10] and the Scientific Revolution.[11] Today, the legacy of the printed word is the groundwork for the modern knowledge-based economy[12] and the spread of learning to the masses across continents and oceanic divides. In particular, it is said that the Scientific Revolution "was sparked by the publication in 1543 of two works that changed the course of science: Nicolaus Copernicus's *De revolutionibus orbium coelestium* (*On the Revolutions of the Heavenly Spheres*) and Andreas Vesalius's *De humani corporis fabrica* (*On the Fabric of the Human body*)." (Retrieved January 2, 2012 from http://en.wikipedia.org/wiki/Scientific_Revolution)

As mentioned earlier, the incidental effect of the Gutenberg technology, on the societies which embraced it, was to promote the pre-eminence of the visual over the aural. Within one hundred years of the invention, scholarly learning in Europe had metamorphosed into tomes of linearly read texts and was much less moderated by the messier forms of debate or dialogue. Nonetheless, the transition to the new technology of the time did not immediately obliterate the oral traditions of the past. The principles of rhetoric from the works of Cicero[13] were not altogether forgotten. Thomas Wilson

(1524 – 1581), an English diplomat, judge and author, is now remembered for his influential texts on logic and rhetoric in the English language: *Logique* (1551) and *The Arte of Rhetorique* (1553). What McLuhan called 'The Gutenberg Galaxy' had simply textualized oratory on a massive scale. Rhetoric, along with grammar and logic (dialectics), still remained the classical three arts of discourse in Western education until the late nineteenth century. In its written form, the classical rhetorical education focused on the five canons of composition: invention, arrangement, style, memory and delivery. Those of us, who were educated in the West during the twentieth century, are aware that Rhetoric, as a formal subject of study, is almost unheard of today. The sensorial twist had completed its turn by the beginning of the twenty-first century. The memorization of long passages of poetry or prose or even mathematical tables, the formal study of the grammatical structure of the English language, and the systematized elements of good debate had been almost expunged from most education curricula – returning to a more medieval-like creative free-form expressiveness, both written and oral without the structure of rhetoric, logic or grammatical precision.

What has indeed happened is as McLuhan had concluded. New technologies, once internalized, alter the ratios of our collective senses by either anesthetizing or truncating other ones in favour of the one that becomes the predominant one. The result is a collective truncated educational experience in some dimensions. After having lived in the Gutenberg Galaxy for so many generations, the ratios of our senses have been transformed under its influence and this, in turn, has altered our cultural perceptions of what is important or essential in the education of our young. In fact, it can alter how we see the world around us and make a profound difference in the cultural legacy of a society.

The Gutenberg press was not the first of technologies which had both enlarged and delimited the knowledge base of human kind. Consider, for example, the contribution of the Ancient Greeks to science and mathematics. The technology (where technology, in its broadest meaning, is simply an extension of the human self created by self for self) they used in the teaching and learning of geometry was the *abakon*, a sand-strewn planar tablet. The Ancient Greeks tended to be more tactile than visual in their learning of the spatial characteristics of the universe around them. The compilation of geometric ideas and results by Euclid in the treatise entitled *Elements* (circa 300 B.C) is testament to the impact of Ancient Greek thinking on future generations for millenia. Nevertheless, Ivins (1946) had suggested that:

> . . . again and again during a period of six or seven centuries they went right up to the door of modern geometry, but that, inhibited by their tactile-muscular, metrical ideas, they were never able to open that door and pass out into the great open spaces of modern thought. (p. 58)

Rather, it was René Decartes, a seventeenth century French philosopher and mathematician, who was instrumental in visualizing a plane which could be alphabetized, thereby freeing geometry from its strictly Euclidean tactile dimensions. For the Ancient Greeks, the ascendancy of the tactile over the visual had had an effect on its legacy of knowledge and learning which it had passed onto future generations. It took the message of the Gutenberg era against the backdrop of the development of algebraic thinking inherited from earlier Arab cultures which ultimately resulted in the conceptual foundations of modern geometry. This had paved the way to non-Euclidean geometries and a greater understanding of our physical and cosmic surroundings.

On the other hand, some educators have recognized the effects of visual predominance over tactile-audile sensory perceptions and have taken steps to counterbalance them. Carl Orff, a 20th-century German composer, developed an influential approach to music education for children. He refused to allow children who had already learned to read and write into his music and dance school. He felt that their audile-tactile sensibilities had already been somewhat compromised by the visual bias of the textual learning.

In fact, not just our senses are affected by a technology. Even, in some cases, our functionality can be modified by the continual stress of one sense over the other. Chaytor (1945) was one of the first to write about the effect of print, over centuries of inculcation, on long-term memory:

> Our memories have been impaired by print; we know that we need not "burden our memories" with matter which we can find merely by taking a book from a shelf. When a large proportion of a population is illiterate and books are scarce, memories are often tenacious to a degree outside modern European experience. (p. 116)

Chaytor also went on to add:

> Nothing is more alien to medievalism than the modern reader, skimming the headlines of a newspaper and glancing down its columns to glean any point of interest, racing through the pages of some dissertation to discover whether it is worth his more careful consideration, and pausing to gather the argument of a page in a few swift glances. Nor is anything more alien to modernity that the capacious medieval memory which, untrammeled by the associations of print, could learn a strange language with ease and by the methods of a child, and could retain in memory and reproduce lengthy epic and elaborate lyric poems. (p. 10)

Then, only fifty years later, in 1995, just as we were about to face the turn of yet another century, another hugely significant, potentially global, technology, the Internet, was introduced into the public arena. McLuhan had warned of the tacit implications and messages embodied by technologies into societies as exemplified by the Ancient Greeks and the Medieval Europeans. Since his death in 1980, McLuhan's message still rings true. However, I feel that it is imperative upon us, to analyse the inherent messages streaming from these new media, to make predictions as to how they might mold our thought and education processes and to suggest ways of balancing the sensorial experience. The remedy, even according to McLuhan (2011), was not to oppose all forms of technology which threatened to engulf us and cause such sensorial imbalances. "Our need today is, culturally, the same as the scientist who seeks to become aware of the bias of the instruments of research in order to correct that bias. (McLuhan, 2011, p. 36)

Why was I then examining the Gutenberg legacy at this juncture before this research project began to take shape? The primary motive was my growing perception that the Internet medium was a digital cosmos embracing the Gutenberg galaxy as one of its texts. The study of McLuhan's work had had a residual effect on my own perspective of teaching and learning using yet another technology, one which will soon encompass the entire planet and which will have global ramifications for the cultures of the future. His virtual message sensitized me to the potential impact of any one technology on the process of learning itself and how we use it for educational purposes. In my opinion, the greatest danger is *not* the medium itself; the greatest danger is *not* to be aware of its power and its effect on both person and culture.

The Danger of the Machine

One of my colleagues told me this story a few years ago. What it illustrates is that the repetitive use of some form of medium certainly can alter the ways we interact with the world around us and can even change, quite dramatically, our perceptions of our interrelation with its objects and its peoples.

> After many months of faultless working, my wife's iPad suddenly froze one night. "It's a computer," I told her, "computers sometimes crash. Just reboot it, everything will be fine." (it was fine, of course, in seconds.)
> But....it's...not...a...computer," she said very slowly, seriously and deliberately, a slightly concerned look on her face, as though I had just told her that the dog was a robot. She was, of course, correct. Anonymous (2011)

Werner Heisenberg, a German physicist, Nobel laureate and one of the founders of Quantum Mechanics, is well known for his articulation of the 'Uncertainty Principle' in particle behavior. In layman's terms, if we try and precisely predict the position of a moving particle, its momentum (the direction and magnitude of its movement) cannot be determined with the same precision. There is always an element of uncertainty in particle behavior. In the same way, the effect of innovation in technologies, despite the intentions of its creators, can never be fully predicted with exactitude. Heisenberg (1958), in discussing the interaction between humankind and the natural world in which we breathe and move, wrote:

> In this connection it has often been said that the far-reaching changes in our environment and in our way of life wrought by this technical age have also changed dangerously our ways of thinking, and that here lie the roots of the crises which have shaken our times and which, for instance, are also expressed in modern art. (p. 20)

Heisenberg's comment, though uttered more than half-a-century ago, indicated that he had already observed the deeply embedded impact of the technical age on the thinking of those societies which had subscribed to it, both for good and for ill. Only four years later, McLuhan had reiterated that very sentiment when he declared that the medium was the message.

Part of the danger of the machine may be attributed to a metamorphosis from an affective association of human kind with an object/activity of personal interest or need to an arm's length transaction – leading to a disassociation of the human from the natural world and the basic rhythms of the life cycle. This disconnection can then burgeon into a lack of appreciation of the object or activity, sometimes leading to complete disregard. Consider the examples of the combine versus the harvest; the television versus the sport; email or telephone versus live conversation; ritual versus religious belief; book learning versus real knowledge; and weaponry versus bare-hand combat. This concern about disassociation within our world is not only a modern phenomenon. Heisenberg (1958, p. 20), in that same treatise, quoted the Chinese sage, Chuang-Tzu, who, in the sixth century B.C., was alleged to have said: "I have heard my teacher say that whoever uses machines does all his work like a machine. He who does his work like a machine grows a heart like a machine, ... "

There certainly has been, and perhaps still is, a general concern about the long-term effects of a life in cyberspace (the world of electronic communication) on its inhabitants, particularly children. If Benedikt (1991) was correct in his claim that cyberspace "has a geography, a physics, a nature, and a rule of human law" (as cited by Anderson, 2004, p. 48), then, for me, it was important to be aware of the possible pitfalls, especially when designing learning materials. If technology truly affects the person in deeply resonant ways including instinctual reactions to stimuli, then it will affect his/her learning and thought processes. Zajonc (1980) suggested that affective reactions can occur without extensive perceptual and cognitive encoding - and can be made sooner and with greater confidence than cognitive judgments. Many have experienced long term intransigent reactions born only of first impressions.

So how dangerous would this Internet technology be if we set our stage in an entirely virtual world? Would our teaching and learning flourish or be hindered by this disassociation of persons in both space and time? That was the next leg of the journey of discovery.

RISKS and CHOICES: in the online learning of mathematics

The Message of Digital Internet Technology

Unlike the ordered presentation in a Gutenbergian textbook, educational materials on the Internet generally take the form of a loosely connected assemblage of patchwork-quilt-like units, often asymmetric in design. Each individual piece may be well crafted and provide insight into its central theme. However, to stitch them together into a cohesive, meaningful whole can be quite difficult to achieve. Moreover the twodimensionality of the interlocking shapes defies any measure of deep understanding of that overall picture. One of the qualities of the Internet medium is that it almost forces an educator into presenting a topic or a course of studies using such a mosaic style of exposition, rather than an elongated linear manuscript, paginated and indexed.

Elena Lamberti, in her essay entitled 'Not Just a Book on Media: Extending *The Gutenberg Galaxy*' (2011), talked about the mosaic style of exposition with its patchwork quilt of discontinuities combining knowingness and unknowingness. She cautioned that, if the rhetorician's intention was not respected and adhered to, "we risk turning the complex architecture into a trivial version of the reader's digest" (p. xLi). She then extended the analogy to the mosaic learning platform of the Internet:

> A similar risk is always present when we explore the World Wide Web: we jump from a *fragment* to the next one, but if we do not make the interval *resonate*, we flatten our understanding and turn knowledge into *information*; we also tend to oversimplify complex issues, and neglect history or the nature of our stratified memories. (p. xLii)

The mosaic style of presentation, which may be applied in any disciplinary discourse, consists of discrete units of information whose assemblage is meant to create a picture in the mind's eye which then acquires meaning through the interplay with its own ground. One of the risks of presenting on the World Wide Web has to do with fragmentation of subject matter – ideas in isolation without connective tissue. It is easy for a creator of a web site to fall prey to this kind of thin exposition of complex phenomena – and for very good reasons – because of space and time issues in creating pedagogical materials. File sizes must be within certain confines and multimedia presentations generally take a lot of time, not to mention expertise, to develop. More than that, the nature of surfing over the network reinforces this style of touching down to glean bits and bytes of information (the mosaic tiles) without acquiring deeper understanding (the big picture). This is, in effect, the message of Internet technology: it inculcates the illusion that one can learn at the speed of light and sound because that is exactly how fast information reaches us over the wire.

In fact, some of the already observed effects of online learning, which I have noticed in myself and in some students include the following phenomena in the affective domain of the human psyche: **Expectation of entertainment** – the surfing crowd, having fun, riding the waves with no need for serious navigation**; Reduction of attention span** due to the kinetic and frenetic effect of skimming pages; **Knowledge as bit-byte morsels of information** leading one to expect ease of digestion of that knowledge; **Collapsing structure of the knowledge pyramid** weakened by a fast-forward pace of learning necessitating fast food or pre-digested chunks, thereby glossing over theoretical underpinnings of more expansive understanding; **Truncated memory** due to accumulation of unrelated facts and to the readily available (24-7) access to the information; **Belief of verifiably false proclamations of truth** as a result of fly-by traversing the globe without touching down to absorb the culture of the grounded place; **Disengagement from the subject matter** – too busy with the pyrotechnics of the medium including being bedazzled by their flash or by the ways to get them to 'go off,' so to speak; **A mindset that discovery is merely accidental** – at best fortuitous – but not achieved through thought, concentration, experimentation and/or simply determination; **Expectation of immediate gratification leading to impatience** with the learning process exhibited by inflammatory frustration the second something doesn't work properly – the five minutes something isn't immediately understood; **Rudderless learning** caused by isolation from the expert, the mentor and other learners with no timely personal feedback.

Another message of ubiquitous Internet technology may turn out to be a contraction of an individual's rich language skills, both in grammar and in composition, or even a significant change in the language and social structure of the society itself. What verbal conversation affords with relative ease cannot be accomplished as readily by a hand, even two hands, on the keyboard. Four students (unnamed) at James Madison University researched the effects of extensive Internet communications on language, social norms and etiquette. In their article entitled, 'The Effect of the Internet on Language and Communication,' they suggested that the Internet has "greatly affected the languages and styles we use to communicate" (Retrieved on October 29, 2013 from http://wikibin.org/articles/the-effect-of-the-internet-on-language-and-communication.html). Not only email correspondences but also texts sent through instant messaging devices are more terse and informal than letters sent by post. These latter communications, they claimed, have already had a profound influence on language constructs:

> Being able to type to friends at real time has vastly changed the language of every country in which AIM [America Online Instant Messaging] exists. In order to quickly produce a message to be read, typing had to improve, shortcuts had to be found, and rules had to be lifted. It is the abbreviations, internet slang, and loosened grammar rules that have affected languages so greatly. . . . The loosened rules of spelling and grammar have also greatly influenced language. Being more focused on the content and subject and having to respond as quickly as possible to friends online has allowed for very common misspellings of words to become acceptable if the message still pulls through. Also, an entire new alphabet, known as leet, or 1337, has been created. Short for “elite,” this alphabet rewrites words using numbers and symbols that resemble the shapes of the letters in the English alphabet. Although this alphabet is more prevalent in online video games, it is making its way into AIM and other online instant message devices as well. (Retrieved on October 29, 2013 from http://wikibin.org/articles/the-effect-of-the-internet-on-language-and-communication.html)

Though these forms of expression are still text-driven, the speed of communication (a kind of Mach 1) has warped the content to a greater or lesser degree. In instant messaging correspondences, one is expected to write at the pace at which one speaks. Herein is the twist of sensorial ratios evident in this kind of Internet exchange. In email correspondence and in online discussion forums, language is often tersely expressed in phrases, rather than sentences, and even those phrases are often just a sequence of linguistic ellipses. Such virtual interchanges, if they were to become the norm for a society’s twenty-first century conversations, would inevitably strip the richness of a language structure down to a more code-like form. After all, computer languages from their very inception were all in code, with perhaps one or two notable exceptions. As the expression of language is foundational to teaching and learning by way of exposition and explanation, this effect of Internet technology may break up the educational process into bite-size bits, thus inhibiting deep learning.

The legacy of this technology has not yet been fully realized. Nonetheless it was important for me, in this early twenty-first century experiment of teaching mathematics through this medium, that I try to soften at least some of these mind sets with my students before they became interwoven into the fabric of their e-learning paradigm.

Mollifying the Message of Digital Internet Technology

Why was it so important to slow down the hyper-media messages of the Internet? – because, in teaching and in learning, one must take the time to pause, to ponder and to think. Einstein expressed his opinion of what it meant to *think*:

> What precisely is thinking? When at the reception of sense impressions, a memory picture emerges, this is not yet thinking, and when such pictures form series, each member of which calls for another, this too is not yet thinking. When however, a certain picture turns up in many of such series then – precisely through such a return – it becomes an ordering element for such series, in that it connects series, which in themselves are unconnected, such an element becomes an instrument, a concept. (A. Einstein, Autobiographical Notes, p.7) (as cited by Barbatis et al., 2012, p. 20)

In his personal notes, Einstein had alluded to the thinking process as beginning with the ability to form a mental image from sensory input. A series of such mental constructs in the memory still would not, in his opinion, constitute thinking. In his mind, thinking would take place when the mind was able to provide an order or relation to a number of such a series of mental images so as to create a connection between them. At this moment of consolidation, a concept of the mind – otherwise expressed as a composition of mental imagery – would have been substantiated. That, to him, was thinking. By taking Einstein's lead, we may define mathematical thinking as a duplication of this thinking process with higher-order visuospatial thinking at the outset forming the basis of the mental images thus formed. In the subsection of this thesis entitled "Visuospatial considerations" (see Chapter 2), higher-order, visuospatial thinking was described as the ability to form a mental image from visual input and to be able to manipulate that image in accordance with specified, non-arbitrary tasks. The series of mental visuospatial images would become a collage of mosaic pieces of mathematics in its elementary forms (a historical factoid, a definition, a diagram of explanation, an algorithm or procedure, a description of an application). Mathematical thinking would then become the ability to create meaning from the many mosaic tiles by juxtaposing them together into a single entity of understanding or a coherent, recognizable, conceptual image.

Therefore, if we teach students how to *think mathematically*, not merely expose them to mathematical symbol and method, then they will have the means to carry that *way of thinking* forward into other venues of learning and achievement. The Internet was not the problem; its natural form was a mosaic. To teach online with the purpose of encouraging the learners to think, the educator had to provide ways for students to link the mosaic pieces together – then give them time to form their own mental construct of those ideas as the students, themselves, glued them together into a coherent whole. The final step in setting my virtual stage was to take the time to connect the mosaic pieces of mathematical methods, definitions and topics together. I could do that in any synchronous session extemporaneously. However, this had to be built into the online text of the project, the RMC web site.

With every mosaic piece on that site, there would be an introduction to the topic at hand which presented the historical context or instances of real world applications of the mathematics discussed. The menu structure would afford the student options either to surf back to review the mathematical foundations underlying the topic or to surf ahead to extensions or derivatives of the ideas presented. The ever-present horizontal navigational menu bar would hold the underlying juxtaposed patchwork-quilt shapes of mathematical thought together as a sliding scale of historical discovery or as a cohesive whole body of knowledge. Animations with pyrotechnics, considered to be distractions from the learning process, would be deliberately understated. Any of them which were included would be placed strategically for pedagogical purposes only. There would be introductions, summaries, exercises and conclusions, all of them intended to pull the materials together in an effort to provide a kind of mental gravitational force to help coalesce the processes of mathematical insight.

In the synchronous tutorial sessions, I would be the virtual instructor. I would slow the tutorials down to a pace which was suitable for most, if not all, of the participants. The presentation was not to be a fragmented array of web-related applications available to me. Rather, I would retain a kind of technological simplicity for the participants as much as possible. I would not use the ‘flash card’ style of pre-

prepared pages to present the mathematics. Rather, I decided to restrict myself to the traditional way of presenting mathematics on a board (in this case a virtual whiteboard) at my personal speed of writing so as to avoid the surfing phenomenon. I also chose to incorporate a review of previously covered topics into each session, thereby providing time to answer questions extemporaneously from the group. The web site would always be there as a back-up e-text for them as a means to encourage their cognitive participation outside of our virtual gatherings. In an effort to maintain continuous threads of both cognitive and personal participation between the live sessions, I would be available to answer any questions they might have via email. Those were my choices in order to correct the bias which inherently was characteristic of much participant/learner interaction over the Internet – disjointed, sometimes splintered and even truncated. An ironic outcome of my efforts to slow the pace, perhaps not surprisingly, was a call, by at least one of the participants, to speed up. He would intermittently interject into our group discussions: “Mach 1” (the speed one must travel to break the sound barrier).

As to the issue of use of language in the virtual world, my personal efforts to reduce an insidious metamorphosis to truncated communication took the form of retaining the full natural language structure, which I had learned during my school days, in all of my correspondence, both personal and professional. Even in the teaching of mathematics, a coded language in its own right, I have always tried to overlay it with complete natural language sentences with its attendant punctuation, spelling and grammatical structure. This has been an act of will over convenience, not to be supercilious or overly pedantic, but in a concentrated effort to retain my natural language skills in a world of coded messages.

However, as an educator, one of the most far reaching messages of the adoption by a society of unstructured, *learning by surfing*, online pedagogy is what I would call a ‘virtual education’ – in the sense of having the appearance of understanding and knowing but not achieving knowledge in actual fact. This is the great paradox of the Internet. It is a technology with so much potential to expand our individual and collective knowledge but, through its inherent characteristics, it has the effect of contracting and fragmenting our understanding. It can teach us much but, if unguided, we will learn little. It is a proliferation of many topics (sometimes only a web page in length) but not in enough detail. That was one of the lessons I learned about my web site. It was structured as a connected series of top-down, somewhat terse, overviews of various mathematical concepts. One could learn a lot if one took the time to think about its content – *with patience*. However, it was not enough to teach the mathematics in real depth, particularly with those having inculcated surfing habits.

Learning over the Internet can also lead to a mindset that knowledge is readily attainable 24-7 at the *touch* of one's fingertips, simply to be gleaned and not earned. This may lead to a nomadic virtual world existence without the construction of applicable knowledge assets, defined by many to be education for the accumulation of expertise for the purpose of employment or engagement in the knowledge economy. The point is that a professional person is not merely an Internet surfer who can find sites, as needed, and then apply the skills found therein with expertise. This misconception can lead to an increasing belief that one is self-taught without acknowledging or even recognizing one's interdependence with the community, both past and present. This ultimately will create an island of self-perpetuated knowledge experts or egocentric sages, wearing thin, if not thread-bare, cloaks of wisdom. By reading and then quoting or paraphrasing, one can simulate wisdom or in-depth knowledge without the reality. This is particularly true of Internet writings. It is in the oral accounts that real distortion can take place; it is also in the oral that real clarification can take place. It is in the written that thought must take place to be considered credible beyond the moment.

For all of these reasons, part of the research was to discover if deep learning in mathematics, what I refer to as mathematical thinking, could be achieved by an engaged learner in such an online setting. Could the attention and interest of the participants be captured long enough in this abstracted environment for this to take place? Could a community of inquiry[15] as outlined in Garrison, Anderson and Archer's article, entitled "Critical inquiry in text-based environment: Computer conferencing in higher education" (2000), be achieved? The *Community of Inquiry* theoretical framework of Garrison, Anderson and Archer (2000) depicted a process of creating a deep and meaningful (collaborative-constructivist) learning experience through the development of three interdependent self-intersecting elements – social presence, cognitive presence and teaching presence. Those were pressing issues for which I had, at the time, no answers. The Internet medium did present its difficulties in online learning environments but so may the participants in this virtual world of faceless interaction.

Mediating the Affective Mine Field

Barbatis et al. (2012) had also recognized that "the problem we as mathematics teacher-researchers have set up for ourselves to solve is the absence of learner interest in mathematics. An absence perhaps initiated and definitely exacerbated by prior exposure and repeated sense of failure" (p. 19). One of the issues in that regard, which Barbatis and colleagues (2012) had highlighted, was the need to foster the creative process in such classrooms by embedding creative thinking into, not only, the design of the materials, but also the teaching and learning activities of the classroom. In addition to having to teach remedial mathematics learners how to think mathematically, it was advisable to provide opportunities for them to think creatively.

The notion of creative thinking put forward by Barbatis and his research colleagues was based upon Jacques Hadamard's analysis of the creative process in mathematics and science. Hadamard had been a French mathematician, a contemporary of Albert Einstein's, who had surveyed about one hundred of the leading physicists of his time as to how they did their work. In his book *Psychology of Invention in the Mathematical Field* (1954), Hadamard had described mathematical thought processes as being introspective and wordless, but accompanied by mental images representing the entire solution to a problem. Of course, this reminds us of Einstein's notion of the thought process, itself – as being the compilation of a seemingly unrelated series of mental constructs which suddenly metamorphose into a single cohesive whole supplying meaning and import to the problem at hand. Hadamard (1954) had concluded that the creative process of problem solving involved four stages of mental activity:

1. Insight into the problem through past associations and mental constructs;
2. A period of Incubation which would take the form of a temporary break or mental rest from the active process of trying to solve the problem. This may also allow an exploratory phase where the imagination cycles through the many series of mental constructs already present in the memory and tentatively places these images in varying sequences;

3. Illumination is that moment when all the mental constructs coalesce into that order which provides the solution;
4. Verification is that necessary process of cognitive and practical testing of the solution which is demonstrable to others.

In my mind, the ‘insight’ stage could be accomplished by an instructor providing a diverse environment for learning mathematics -- from using different approaches to the same problem to recycling past ideas which had been easier to understand. The ‘incubation’ stage could be fostered by promoting a non-judgmental exchange of ideas amongst learners and by suggesting that the learners take time to ponder, to rest and then to return to the problems to be solved. The ‘illumination’ stage was highly personal and would differ for each of the participants. The verification’ stage was what all mathematics students ask for and that is that the instructor provides them with definitive and detailed solutions.

The research findings of Barbatis et al., in their paper entitled ‘Engagement in Creative Thinking,’ were published in 2012. This was a couple of years after I had conducted my own research regarding the choice of online environment in which to document the learning experiences of a group of remedial students of mathematics. Nonetheless, in retrospect, I have concluded that my own choices and teaching methods at the time did form a multimedia learning environment which allowed for each of the creative process stages to occur.

The first choice to promote deeper understanding and the creative processes had been to use and explore multiple virtual models of communication and intercourse for the experiment, McLuhan’s so-called ‘technique of the suspended judgment’ (2011, p. 81). Coupled with the, essentially text-based, web site, I would provide opportunities for the participants to attend synchronous tutorials, engage in online discussions, or visit chat rooms. Thomas Aquinas, a thirteen century Italian Dominican priest, had allegedly noted that neither Socrates nor Jesus had committed their teachings to writings because the kind of interplay of minds that takes place in teaching cannot be captured solely, or adequately, by the written word. This would be one means of providing continuity in a discontinuous medium. The online presentations would be triangulated and tessellated providing homogeneous segments of knowledge with dialogue and query. To subjugate the machine effects of frenetic surfing, I would encourage meditation of the ideas, personal practice, and group revisits to certain topics. This would allow individual and collective background processing, a rumination, so to speak, of the subject matter leading more thorough digestion of the material. In the live tutorials, the layering of voice over text and animation would provide the aural, the visual, the kinetic and even tactile (navigation) modalities of learning synchronously. The imperfect recall of just print or just voice in isolation would be ameliorated by this multi-sensory experience, thus providing an opportunity for deeper learning. To avoid the effect of long-term memory loss in such a pedagogical environment, I would encourage the participants to commit some of the important content to memory for future use in their mathematical studies. Finally, to counter balance any negative or timid mind sets about the mathematical ideas which the participants may be carrying into the venue, I would continue my pedagogical policy of expecting the best from all of my students, providing assistance when requested, providing praise when merited, providing caution when required and never patronizing them by assuming that they were, even the least little bit, incapable.

The virtual stage was now set.

CHAPTER 3 MATHEMATICS: THE HEART OF THE MATTER

In any educational endeavour, the subject matter and its messages to and through both instructor and student permeate any learning environment with an ether of both overtones and undertones. For example, how a mathematics teacher perceives the mathematical content will be communicated implicitly to her or his students in both overt and covert ways. My view of mathematics as a whole body of knowledge has shaped and modified my approach to the teaching of this subject. This chapter is an attempt to articulate my perspectives of this academic discipline which will explicate, to some degree, my pedagogical approaches to the learning of its rigours and enigmas.

If every medium is, by osmosis, its own message, then what about the information sent through it? Our common sense tells us that the content, whether read, heard, watched or actively played, sends overt and covert communications to the receivers. The impact of the content on an individual will vary with the ways in which it is designed and broadcast. McLuhan was certainly more preoccupied with the characteristics of each technological medium than the content that was being communicated through it. However, he also proposed that a characteristic of every medium is that its content is another medium nested within itself affecting the human psyche in its own particular way (McLuhan, 1964, pp. 8-9). This leads us to a discussion of mathematics – the subject matter of my virtual endeavour.

Anyone who has ever studied or simply read a mathematical document, whether in a classroom, in a library or on the Internet, eventually comes to discover that its symbolic language generally has had an impact on his/her psyche – sometimes leaving an indelible impression which can never quite be expunged. The messages of mathematics have long evoked a pendulum swing of emotive responses in those who have studied it. In my experience, it has variously been seen as: a science; a philosophy; a succinct code for describing the physical laws of our universe; a bunch of unintelligible hieroglyphics; a subject to be hated; a subject to be feared; a subject to be avoided at all costs; a complex discipline of thinking to be conquered; and even a subject beloved. Reactions to this content medium can be extreme, both positively and negatively, but they are usually never neutral.

Dimensions of Mathematics: the icon of the polytope

If you were given the task of choosing one image as being a visual representation of the word '*mathematics,'* what would you pick? Personally, I would envision a *topological polytope embedded in n-space*. For the uninitiated, this sequence of words may mean absolutely nothing. Therefore, I will now attempt to describe – without diagram or imagery – the notion of a topological polytope .

To envision this geometric but abstract object, it is necessary to go through a nested layering of definitions to get to its real meaning. First let's assume that everyone knows what a ***polygon*** is. Examples of polygons include triangles, rectangles, rhombuses, five-pointed stars, octagons or any "closed plane figure bounded by straight line segments as its sides" (Bronshtein et al., 2003, p. 137). From those two-dimensional planar objects, let's extrapolate to the third dimension. A ***polyhedron*** is a three-dimensional solid object comprised of a collection of contiguous polygons (called its faces) which are usually joined along their edges of the same length and which enclose a 3-dimensional interior. A cube, a rectangular box or a pyramid are examples of polyhedra. Without having the luxury of producing pictures of polyhedra in this print-only exposition, I will now attempt to take the proverbial quantum leap of abstraction from which some students of mathematics do not fully recover. I will push the number of dimensions up to four (4) or five (5) or even more by jumping out of visual geometry into the abstract world of algebraic formulations. A ***polytope***[16] is a more general version of a polyhedron which can be defined in arbitrarily many (or *n*-) dimensions having the same properties as its lower dimensional counterparts. These objects are no longer visible to the naked eye – only to the eye of the imagination. In one sense of the word 'virtual,' a polytope is a virtual entity. Further allow some curvature to the previously planar faces of this multi-dimensional object and you have a ***topological polytope***. By doing so, we have moved into the realm of topology,[17] a mathematical discipline which examines the nature of spaces which can be transformed in a smooth continuous manner without ripping or tearing them.

So why do I think that a topological polytope is a metaphor for the word '*mathematics?'* Embedded in this icon are the mathematical disciplines of geometry, number theory, algebra, topology and ultimately analysis. This montage from the simple to the complex, from the concrete to the abstract, characterizes almost all of mathematical discovery and thought.

The Human Faces of Mathematics: how to meld them into a virtual world

When I was given some funding to build a web site about mathematics in 2007, the only real directive I was given was to put a human face on the abstraction of the subject matter. (See Chapter 1, the subsection entitled "The Acorn.") What follows are my musings about the human side of mathematical thought.

MATHEMATICS: a language with alphabet, grammar and composition

I see the discipline of mathematics comprising a strata of languages - a layer of natural language over an esoteric notational language having its own alphabet, grammar and rules of composition for its paragraphs and essays. Its peculiar grammar of nouns, adjectives, verbs, tenses, clauses and sentences are expressed through its symbols drawn from the Roman, the Greek and even the Hebrew alphabets. For example, I interpret the mathematical expression: $\mathbf{f}(x) = (x^2 - 2x\,\pi)/(x\text{-}5)$, $x\ \varepsilon$ R as a mathematical sentence in which the subject is the noun ' **f** ' modified by the adjective ' (x) '; the verb, ' = ' , is in the present tense and the passive voice; the predicate clause is the right-hand side of the equation; and a dependent clause ' x ε R ' is appended at the end of the sentence. Moreover, quite ordinary English words, such as 'field,' 'ring,' 'group,' 'manifold,' and numerous others, may and do have very distinctive mathematical meanings which bear absolutely no relation to their ordinary English language counterparts. Suffice it to say that mathematics has a vocabulary and grammar of its own and those who wish to speak its language must understand the individual words and phrases of its structure. I find that a primary task in mathematical discussions is often dedicated to definition, clarification and exemplification of its esoteric vocabulary.

Language, however, is more than just a list of nouns and the infinitives of action. Further in my analysis of the language of mathematical discourse, I began to perceive that its verbs were, so to speak, 'declined' through its peculiar subjunctive and conjunctive relations. Its *subjunctives* bear a resemblance to the grammatical subjunctive mood of a verb. They express the possible, the conditional or the dependent relations, rather than the absolute. The essays of mathematics (that is, the major theorems, their corollaries and ensuing propositions) are often initially formulated as conjectures or hypotheses in terms of subjunctive relationships. If something may be assumed under certain circumstances (its initial conditions), then such and such is true.

In the same way, the *conjunctives* of mathematical discourse have a commonality of purpose with the conjunctions of ordinary language as grammatical constructs which connect words, phrases or clauses in both meaning and construction. Rather than being just qualifying adjectives or quantifying adverbs, the conjunctives of mathematics take the form of its algorithms, its methods and its minor derivations. These are the aspects of the language which are rehearsed throughout the majority of mathematics classrooms worldwide. If vocabulary forms the base of the language and the subjunctives articulate its statements of truth, then the conjunctives are the ways and means whereby these statements may be justified, clarified and exemplified. They are the glue between the conjecture or the hypothesis and the definitive solution.

What has emerged in my consciousness over my teaching career is the central role of the appropriate use of language in fostering communication of ideas and in avoiding confusions of understanding by either misuse of or inattention to the same – particularly in media which do not afford immediate feedback. This has alerted me to the necessity of adopting whole mathematical language analysis as a mandatory part of every web-based learning initiative. Such wholeness entails an emphasis on the proper definitions of terms and the contexts in which they have meaning. Of equal importance is that students of mathematics reflect on the differences between particular and general truths and thereby understand the import of a counterexample. Finally, I feel that it is necessary for learners to be able to express, in context, the conjunctive forms of the language coupled with the facility to use them in the appropriate subjunctive relationships. This is all part and parcel of 'speaking mathematically,' as Pimm (1987) so nicely coined the term. Such facility of expression is, in my opinion, a quintessential component of all learning environments in the subject.

Why language is so central in the teaching of mathematics and why special attention has been made to its importance in the creation of the online materials is because meaningful communication of these ideas to a diverse group of adult learners will be a measure of their success as virtual *places* of learning. Without lucid teaching materials, both written and oral, pedagogical web sites are only momentary landing platforms of an Internet surfer; they are not *places* of learning. See the section in Chapter 2, subsection "Facets of e-Learning: the importance of 'place'."

MATHEMATICS: a tool of science to code the universe

Mathematics can be thought of as the foundational language of the scientific disciplines though each of the natural sciences will undoubtedly have its own stratum of esoteric vocabulary and symbolism. Some consider it a science in its own right. Strictly speaking, pure mathematics (that is, mathematics not born of application) is not a science because it is not empirical in nature. In general, it does not collect physical data to formulate its hypotheses, nor to justify its conclusions. Nonetheless, its ability to describe the quantum earthliness of our planet, its position in the Sun's solar system, and the unboundedness of the surrounding and encompassing Cosmos of which it is an iota part is testament to its being the succinct language of much of scientific endeavour from our earliest historical times. It is the mathematical sentences and compositions which have provided both form and meaning to the invisible laws of human physical experience. Cases in point are Einstein's equation $E = m c^2$ which explains the relationship and equivalence between mass and energy in the Universe around us and Maxwell's system of equations which consist of four equations governing the behavior of electric and magnetic fields in our world. J. C. Maxwell, a nineteenth century physicist, formulated these mathematical equations in 1860 basing them on experimental evidence of electric and magnetic phenomena which had been gathered at the time.

This foundational role which mathematics has played in many of the sciences, not to mention other disciplines of study, prompted me to dot the landscape of the Remedial Math Centre (RMC) web site with tidbits of discussions of the applications of mathematics in fields as diverse as the physical sciences, finance, commerce, sports and meteorology.

MATHEMATICS as poetry – with its powers of expression

John Hollander (1961), in his book *The Untuning of the Sky,* talked about poetry in its written form:

> If a poem is to be treated as a highly complex utterance in a spoken language, its written form becomes a simple coding of it, word by word, onto a page. (p. 7)

How better to describe the mathematical equations of both Einstein and Maxwell mentioned earlier and of many other scientists both before and after them! Such mathematical statements are truly simple codings of highly complex theoretical utterances in a spoken language. In other words, they are scientific poems which collectively describe our universal physical laws governing our existence on this planet.

Personally, I feel that poetry is meant to be read aloud, with rests and long dwells on the rhyme with tempo and pause. For that reason, considering the succinctness of mathematical expression and its inherent complexity, in particular as lines of poetry, I decided that the voice components of my virtual stage of learning would be articulated at a leisurely pace. This unhurried rhythm was not only to apply to the live session discussions but also to the web site presentations. To that end, animated tutorials with audio backing were slowly added to the site, either complementing or replacing their silent predecessors. Ultimately, all such tutorials on the site would have a voice overlay.

MATHEMATICS as art form – an abstract work of art

Surprisingly, the metamorphosis of mathematical thought from its earliest formulations to its ultimate post-eighteenth century set-theoretic abstractions may be viewed, *from my personal perspective*, as a philosophical precursor of the development of some of the more celebrated modern European art forms throughout the nineteenth and twentieth centuries. There is more art in the learning and teaching of mathematics than meets the eye and there is more mathematics in some art forms than we can imagine.

Dating back to prehistoric times, **graphic art** has always been a part of the human landscape. Mathematics viewed as a form of mental graphic art is generally thought of as a kind of science which is used to construct the architecture of space in a somewhat stylized but rigid, highly structured and technically precise manner. More than that, some artists have used mathematical ideas to inspire their art. For example, M. C. Escher, a twentieth-century Dutch graphic artist, explored the idea of infinity not only using hyperbolic tessellations (or regular tilings of the hyperbolic plane) in his wood cuttings “Circle I – IV” but also using conformal maps of elliptic curves in his lithograph “Print Gallery.”

Realism, in the realm of the arts, was a movement which had its beginnings in France in the 1850s. Its artists sought to depict the locality of their multi-dimensional personal world with truth and accuracy – using brush strokes on a planar surface. The realism of mathematics may be found in the meticulous ways in which it is capable of describing our physical universe, explaining how it works and accurately portraying its dimensions – using textual language rather than brush strokes. Those who subscribe to mathematics as merely Realism generally believe that its primary function in education pertains to its applications in the real world. Other forms or expressions of mathematical thought may be viewed as abstract nonsense and serve no useful purpose.

Impressionism and its derivative, **Pointillism**, were both art movements in the latter part of the nineteenth century initiated by a group of French artists. Both presented the landscape, not as a viewer's visual reality, but as a tesselation of impressions of the eye through light, shadow and the imagination. Impressionist techniques involved using small, thin, but visible brush strokes on the canvas. Pointillism, on the other hand, achieved a similar, but not the same, effect with a pixilation of an image into small distinct dots of pure, not mixed, colours. These art forms relied on the ability of the human eye and the human imagination to blend the color spots into a complete and coherent image when viewed at a distance.

Earlier I had discussed the notion of mathematical thinking. In fact, Einstein described the thinking process as the metamorphosis of the reception of a series of sense impressions into a connected image forming a concept. (See Chapter 2, subsection "Mollifying the Message of Digital Internet Technology.") When teachers of mathematics teach the subject matter as Impressionists or Pointillists, they assiduously focus on the 'microscopic' details. After several repetitions and fine tunings of the same information, we teachers expect that the students will have done the deeper thinking so that their virtual impressions of the individual tesserae have melded those dots of information into a complete image of understanding, context and meaning.

It is said that one of the most influential European art movements of the twentieth century was **Cubism**. If Impressionism was the first layer of abstraction of the art of Realism, then Cubism was the second layer. The Cubist art form was the result of the artist's dissection of 3-dimensional objects into non-uniform and irregular chunks and then reassembling them without regard to their original order or any attempt to reconstruct a recognizable image of the object in question. It was an artist's rendition of a world blown apart and then, by gravity, resettled into a chaotic pile of remnants.

However, the artist picked up those fragments, rearranged them a second time and then glued together into an object of art for others to ponder its purpose.

For some, this is what modern mathematics embodies – an unintelligible jumble of abstract forms from which its historical realism cannot be extracted. However, mathematics in its sentences and its paragraphs and its essays demands structure. Most often, the uninitiated eye cannot detect the underlying geometry or numerical truth by looking at its second abstracted form, however nicely composed. Nonetheless, with time and effort, the mathematician can take the jumble of the symbolism and reconstruct the original historical premise. The First Fundamental Theorem of Calculus is an example of mathematical Cubism. It was first formulated and proved by a seventeenth century mathematician called James Gregory, centuries before the advent of Cubism as an artistic form.

The mathematics exemplifying the philosophies of the twentieth century art movements of **Surrealism** (Einstein's four-dimensional model of the space-time continuum), **Futurism** (the Frenet-Serret moving frame) and **Vorticism** (mathematical String Theory) have moved into the realms of differential geometry and quantum physics. The mathematics, as well as the art forms, have become ever more complex, mystifying and seemly impenetrable as we probe deeper and deeper into our expanding universe.

MATHEMATICS as philosophy – a way of viewing the world around us

From the above discussion of mathematics as an art form, we are really talking about the underlying philosophical perspectives of the subject's artisans and how they have chosen to analyse and depict the human experience. Similar philosophies have penetrated not only the artistic domain but also the worlds of music and literature. A philosophy is simply an articulated way of viewing the world by interacting with and within it. There is one significant difference between mathematical artistry and its humanistic forms. Mathematical statements, unless designated as hypotheses or conjectures, generally encompass a verifiable, universal truth and are subject to an unforgiving scientific scrutiny which is generally not true of other philosophies.

How we subscribe to the artistry and philosophy of mathematics will have a virtual effect on our students. These subliminal perspectives embedded in the mind will embody themselves into our methods of teaching and thereby form yet another virtual medium infused into the content which will have its own message to our students. In addition, if we view the study of mathematics in these humanistic ways, then we must be cognizant of the fact that it will take time to probe the many facets of its polyhedral surface to achieve deeper understanding of even some of the seemingly simplest concepts in its domain. My awareness of both the fundamental simplicity of mathematics and its ultimate complexity had shaped my pedagogy in face-to-face classrooms. Now I had to present them to a virtual audience with virtual tools. Would my presentations differ significantly or would I draw from the past to forge the path to the future?

The Parallax of Perception: a blend of mathematics, philosophy and language

Parallax, in my rear-view mirror, is a distillate extracted from a complex confluence of philosophy, mathematics and language: three major disciplines of learning which touch on the human soul, its perceptions of its spatial home, and its means to communicate its feelings, imaginations, and observations. While in geometry *parallax* is objective, in language, it is subjective. Mathematicians define it as the change of angular position of two lines of sight of a single object relative to each other as seen by an observer, caused by a change in the *observer's* position. The observer often perceives it as a change in the position of the object, rather than a change in her/his own perspective of that object. In contemporary writing, the meaning of *parallax* is singularly subjective. It refers to a piece of writing told from a different perspective than that of another or to similar stories from approximately the same time period. Finally, in philosophy, the word *parallax* merges the two themes and creates an enigmatic objective/subjective perspective of knowing and understanding which is highly individual. The nineteenth century German philosopher, Georg Hegel (1967), proposed a conception of knowledge and mind ultimately linked to a person's reality which encompassed the notion of 'identity in difference.' He maintained that the mind internalizes and then externalizes itself in various forms and mental objects which are created from its own sensory perceptions outside or opposed to itself. At the point of initial externalization, it may be likened to an aesthetic object outside of self. However, when it recognizes itself in these external manifestations, an identity of mind and 'other-than-mind' takes place. At this juncture, it is metamorphosed into, what Madelaine Grumet (1999) named, an aesthetic object reflected into internal autobiography. This parallax of perception of object and the mind's subjective externalization of it alters the observer's position. This new position founded on the identity of difference thereby transforms the individual's body of knowledge and becomes an integral part of it. Some believe that this parallax is the true knowing and understanding of the perceived world around us. This again reminds us of Einstein's description of what constituted 'thinking' – that final coalescence of recurring mental images into a cohesive, substantive concept called knowing. (See Chapter 2, subsection "Mollifying the Message of Digital Internet Technology.")

I would like to carry this metaphor of parallax into the teaching, learning and understanding of mathematical constructs and ideas, whether abstract or geometric. Here the objective parallax is the viewing of the object by mathematician, instructor or student of the construct, as a collective of the definitions, theorems, conceptual ideas, and theories integral to its formulation. These perspectives alter slightly over time, the longer one grapples with its complexities. The language parallax happens through the communication process in how each of these components of structural essence are re-viewed subjectively through the diverse perspectives of others and self so as to enable and enhance their comprehension. The philosophical parallax occurs when colleagues or students, having observed the object, form mental abstracts of these perceptions, externalize them and ultimately identify with their meaning so as to achieve an individualized knowledge and understanding of them. This third parallax most often is a blend of the objective (the percept formed from successive snapshots of the eye) and the subjective, both the student's subjective of prior knowledge or belief and the presenter's subjective of personal perspective. A 'percept', as defined by Reisberg and Heuer (2005, p. 43), is an internal representation of a stimulus perceived by the eye. It is a depiction rather than a mere transcription or copy of the stimulus because the representation is both organized and interpreted by the viewer. It is essentially the subjective mental construct of the object viewed.

Visuospatial Imagery and Mathematical Reasoning: a case for hypermedia[18]

From the time in the early nineteenth century when Euclid's fifth postulate,[19] the parallel axiom, was proven not to be universally true, there was a prevailing suspicion amongst eminent mathematicians that visual constructs for justification, understanding and proof of mathematical ideas were inadequate. This ushered in an era in which all theorems had to be analytically or geometrically proved with rigour and, consequently, pushed the analytic, abstract presentations of mathematical ideas to the forefront of explanations in higher-level mathematics classrooms. From that time onward, it was felt that the purest form of mathematical reasoning and the mathematical concepts themselves were singularly abstract and visualizations for justification or proof of them were rejected. More recently, however, psychologists and scientists are taking a new look at the role of visualization in the formulation and comprehension of physical space and spatial forms, in particular cognitive mappings[20] (Larkin & Simon, 1987; Gollege, 1998; Shah & Miyake, 2005), including the ability to think about space metaphorically (Gattis, 2001) as well as supporting a natural way of thinking about other domains in spatial terms (Bloom et al., 1999). Behrmann and Tipper (1999) did a study that suggested that objects and locations are coded in the brain simultaneously using several reference systems, one that involves the object of observation (location and shape), a second that involves the viewer (position and parallax) and a third that focuses on the environment containing both. Tversky (2005) in her article “Functional Significance of Visuospatial Representations,” concluded:

> The space of external representations, of pictures, maps, charts, and diagrams, serves as cognitive aids to memory and information processing. To serve those ends, graphics schematize and may distort information. (p. 1)
>
> All in all, the neuroscientific evidence shows that the brain codes many aspects of space, notably, the things in space, their spatial relations in multiple reference frames, and interactions with space and with things in space. (p. 26)

Thus visuospatial imagery may be a conduit for mathematical reasoning from perception to processing into cognition. And we are back to Einstein’s way of thinking. This is a philosophical parallax – a change of perspective of what constitutes knowledge.

Marcus Giaquinto (2005), Professor of Philosophy and Logic at University College London, in his article “From Symmetry Perception to Basic Geometry”, put forward a theory in regard to the process which leads to a mathematical understanding what the geometric object called a ‘square’ is. He suggested that:

> . . . the starting point is visual perception of basic shapes: . . . I hold that in having geometrical concepts for shapes, we have certain belief-forming dispositions; these dispositions can be triggered by experiences of seeing or

> visual imaging, and when they are, we acquire geometrical beliefs. I further claim that the beliefs thus acquired constitute knowledge, in fact synthetic *a priori* knowledge, provided that the belief-forming dispositions are reliable. (p. 31)

George Lakoff (1987) and Mark Johnson (1987) too had distanced themselves somewhat from the view that reasoning is 'abstract and disembodied' towards a view of reasoning as 'imaginative and embodied' born of real-life experience garnered through the physical senses, but abstracted through mental processes via metaphor.

A poignant example of this was cited by Davis and Maher (1997, p. 106). It was the story of a grade 4 student, named Brandon, who was asked by his teacher to solve the following problem: How many different pizzas can be made if every pizza has cheese, but to this you can add whichever of the toppings green peppers, sausage, mushrooms, and pepperoni you wish, and in any combination you wish? His solution consisted of a matrix, or rectangular array, of 0's and 1's where each row represented a pizza and each column represented one of the toppings. A matrix entry of 0 indicated the topping was missing; an entry of 1 meant it was present. One can certainly view this abstraction as being 'embodied' or 'born of real-life experience.' In addition, no one will deny that it was 'imaginative.'

Quintessential to Brandon's abstract construction was what English (1997) and others have called 'reasoning by analogy,' defined as the transfer of structural information from one system, the base, to another system, the target, through a mapping process which embodies the relational correspondences between the two systems, even though the systems may be quite different in design and origin. The matrix abstraction was the result of the matching process that Brandon made from 'looking at' his various pizza options. The toppings were either there (1) or not there (0). In that sense, his mental model inherited the relational structure of his physical and perceptual experience. This mathematical creation evolved through a parallax of interpretation born of necessity to fulfill a task: that is, describing his viewing of an ordered pizza from a different perspective.

It is also generally accepted that mathematicians themselves sometimes rely on analogy and visual imagery for insight and understanding. These processed images become the preliminary percepts which ultimately point the way to the distillation of ideas into hypotheses and the formulations of their theoretical underpinnings. Whether one calls them ‘image schemata’ as Johnson (1987) and Lakoff (1987) or mental models, these mental constructs which are metamorphosed from visuospatial snapshots of the eye, as structures of order and relation, emerge from one’s own experience and are modified by it. This bears out the importance of visuospatial imagery in mathematical reasoning and the formation of its theoretical foundations – no less for the student of mathematics than for the matured mathematician.

This dominance of the ‘visual’ in reasoning and problem solving suggests that the hypermedia of virtual worlds, enriched with three-dimensional imagery, animations and videos, may not be as deficient as was once believed in the realm of teaching and learning, particularly of mathematics because of its inherent abstractness.

The Parallax of Language in the Teaching of Mathematics

The role of the instructor, as interpreter, brings another parallax into play in the learning process. It is that of perspective, often through analogy, metaphor and metonymy. Analogies, as we saw in Brandon’s solution of the pizza problem, capture structure, pattern and relationship.

Mathematical metaphors, some educators have suggested, stretch well beyond being merely implied comparisons. They believe that the power of a metaphor lies in the way the human mind conceptualizes one mental domain in terms of another. Lakoff (1994) conceived of metaphors as cross-domain mappings between source and target systems whose connection was usually implicit, rather than explicit. This characterization is certainly exemplified in certain of the grounding metaphors utilized in the pedagogical explanations of the notion of a function as slot machine, number systems as sets, the real numbers as a line, the irrationals as point-cuts and numerous others. Lakoff and Núñez (1997) mentioned the particularly powerful metaphors of Cantor: namely, infinity as a point (which evolved out of his work on transfinite numbers and the notion of number as a point) and his metaphor of ’equinumerosity is equipollence’ utilized in his discussion on the cardinalities of infinite sets. This kind of reasoning has been employed in classrooms worldwide to assist students in the understanding of the more formal aspects of mathematical language and its meanings.

Metonymy, on the other hand, forms the basis of all mathematical symbolic notation and many of its diagrams, the naming of one thing for that of another. These metonymies of symbolism give mathematics its power to embody a sophisticated philosophical idea into a single letter and to ‘crunch’ complex relationships into a diagram of arrows on a page.

In this way, the parallax of language is used by instructors and mentors to contract the compilation of philosophical, mathematical, and scientific thought over literally millennia into a finitely constructed package so as to make it cognitively accessible, sometimes even to the youngest and not yet fully initiated thinkers in our public school classrooms. Moreover, in many cases, the vehicle is through visuospatial imagery.

After some reflection, I became convinced that the virtual worlds of multimedia could certainly become the ideal spaces through which to impart the multi-layered mathematical visions of our realities – whether concrete or abstract. It is my personal belief that mathematics is best taught within a pyramid-like structural framework (a polytope if you will) which is striated by its thickened layers of the philosophical over the artistic form over the strata of languages over the visuospatial imagery over the dogma which forms much of its foundational base. It was these perspectives which have melded themselves into the theory and practice of the blended model of e-learning which ultimately became the choice of online environment for my project.

CHAPTER 4 THEORY, PHILOSOPHY and PRACTICE

Pedagogy and Applied Design

My initial conception of online teaching was that it would be essentially the same, at least in principle, as face-to-face teaching. To me, teaching was basically a conversation with, or an address to, an audience delivered with some artistic flare and dramatic overtones so as to hold the audience captive. From my earliest days in the classroom, as educator, I had always thought of teaching as a bit of theatre. I had perceived the differences between the two educational environments, online and face-to-face, as resulting from the differing ratios of the senses in each of them, not particularly enforcing a major change in theoretical pedagogy. The facial expressions and gestures of my in-person classroom experiences would simply have to be articulated in another way. Because of the nature of the subject matter, the nature of the medium, and human nature, I felt that the absence of the teacher's physical persona in the online forum would, no doubt, demand the inclusion of some kind of kinesthetic drama in the presentation so as to establish my virtual presence in the group. I was well aware that no-one would be coerced to stay; no-one would be compelled to listen or to read except out of interest; and many could be driven away at the first hiccup of difficulty because of innate fear and awe of the subject itself. How could I negotiate the teaching dynamic in the virtual setting so as to be most effective pedagogically and yet retain their curiosity – especially with a group of volunteers?

At the time, I felt that I had only two options for the layout and discussion of the mathematical materials. They were:

1. a top-down presentation – as a distillation of the essence of a topic with its primary defining characteristics, thereby affording students an opportunity to fly over the forest without trying to find one's way through its underbrush; or
2. a mosaic – which would promote the discovery of relations and principles but without the attendant structure in which they reside.

I chose to use both types of teaching methods so as to give a more complete picture of the options available to learn mathematics in a virtual setting.

The Remedial Math Site was essentially an attempt to provide a **top-down look** at the basic mathematical skills needed to transition into university-level math-based courses. It was intended to be review – not a bottom-up teaching forum. The paucity of space and time forbade it from becoming a full-blown course offering. As none of its visitors were obligated to be there, the site relied upon invitation and encouragement to entice them to stay. Its menus tried to offer not only a variety of options to enhance the students' own meta-learning of their learning habits but also to provide a mechanism for anxiety-ridden students to be counseled one-on-one. Flashes of theatre would be placed at various junctures throughout the site, taking the form of anecdotes, surprising applications of the mathematics or videos about celebrated mathematicians and their inventions.

The **mosaic approach**, on the other hand, would be laid out in the live weekly sessions with its microscopic details, its connections and embeddings to other mathematical or scientific ideas, and its telescopic summaries. In this way, three levels of learning could be addressed:

1. the probe or gloss to detail – like the action of diving being surrounded by the medium but not fully understanding its nature because the immediate focus is on the techniques of the manoeuver;
2. the meaning – like the action of surfacing and then swimming or floating with a growing sense of the medium itself and its depths; and
3. the big picture – like the action of flying over the medium with a view of its overall structure and its spatial place in the environment of mathematical thought.

My pedagogical path of enquiry, therefore, would take the form of a climbing helix which would, from time to time, spiral down to self-intersect and then rise again to even higher levels of sophistication, only to be repeated again and yet again at later junctures. Because much of the more detailed explication would be conveyed primarily in definitions and symbols, I decided to probe McLuhan's notion of the mosaic of language for possible insights into how to layer the presentations with richer forms of language constructs.

McLuhan's Mosaic: Its Implications for Learning

McLuhan's work (1962, 1964) on the virtual properties of communication media had already ignited a spark (actually a fire) of interest within me in regards to the future of learning in an online world. His notion of a mosaic of language to impart the deeper meaning(s) of a text had deeply resonated within me. I became convinced that these ideas, if played out in the realm of mathematical thought, had the potential to lead to a multi-dimensional understanding of the concepts. Lamberti (2011) had also interpreted McLuhan's mosaic as a tool to enhance our knowledge and our apprehension of writings and texts through the "interplay of ancient wisdom and cognitive stimulation" (Lamberti, 2011, p. xxxv). Further, she suggested that McLuhan's mosaic was "born out of the meeting of orality and literacy" (p. xxxv) and played out by the juxtaposition and the connection of differences. In my understanding of it, this kind of mosaic was a path for a scholar to take towards a Gadamerian "fusion of horizons"[21] whereby a scholar, when interpreting a text, finds a way to articulate the text's history and meaning within his/her own background of understanding. In fact, McLuhan himself said: "NEW MEDIA DO NOT REPLACE EACH OTHER, THEY COMPLICATE EACH OTHER" (as cited by Lamberti, 2011, p. xLi).

Therefore McLuhan's mosaic was not just a new cloak of communication which would employ different media models in an effort to offer more lively and immediate experiences of the text(s). His mosaic was to be a kind of *hypertext*[18] which allowed readers/learners the means to link to and retrieve information in a non-sequential manner. This kind of hypertext was not simply a wired network leading to writings of Others continents away – though that too would eventually become a reality in our networked world. Rather, it was more an internal network tapping into an individual's inner storehouse of embedded knowing. Words had the capacity to portray analogies between apparently unrelated things. Language could also conjure up past experiences and bring to the surface prior knowledge. McLuhan's mosaic of presentation, Lamberti (2011) suggested, was the task of the media grammarian, not the media designer, "to unpack all the compressed learning embedded in language" (p. xxxv).

So, what would it mean to 'unpack' the language of mathematics in such an environment? After much reflection, I decided that a partial answer to the dilemma of complexity was to provide a multimedia of perspectives from the many facets of this multi-dimensional topological polytope, described in Chapter 3. The grammatical, the scientific, the poetical, the artistic and the philosophical aspects of the subject content should somehow be woven through its exposition as striations through the pedagogical pyramid of learning.

The implications of McLuhan's mosaic for the teaching of mathematical concepts online were two-fold. The web site of learning would be a mosaic of media containing a hypermedia environment of videos, applications in science, auto-marked exercises of practice and other web sites of learning. The University Math Site, which was the public face of the Remedial Math Centre, would become a showcase of mathematics as a human endeavour. Its essays, videos and power point presentations were intended to give a visitor glimpses of these many facets. However, there had to be more than that. Part of the exposition had to be presented as a mosaic of languages of import to the mathematical texts. Historically, much of the study of mathematics has had its roots in antiquity and the ultimate continuum of mathematical thought has been based on and gleaned from those early writings through observation and innovation, propelled by need or imagination. For McLuhan, to keep the mosaic from decaying into ruinous heaps of broken tiles, these tethers to the past were essential threads of connection in order to maintain the cohesiveness of the exposition. This, indeed, was a question of grammar – not of stratified media flash. As an instructor online, I felt I had to become more of a grammarian. In the classroom there had always been reminders that the form of the language was as important as the content.

Some of the probes in the formal learning of mathematics were, precisely, the tasks of solving problems involving procedural thinking – necessarily repetitive, but changing, and often not embedded in the universal context of place. For example, the solving of linear and quadratic equations with the many approaches and techniques to do so, the factoring of polynomials, the rote exercise of arithmetic operations on relatively small numbers are all probes of mathematical thinking and learning. Each is a mosaic piece meant to hone a particular skill in mathematical methodology. This is at the apprenticeship level of a craft. We all want our students to be masters of the craft but it is so easy to forget that mastery can only develop after years of garnering a collection of smaller apprenticeship skills and slowly melding them together. The lesson learned by the mosaic was that each individual piece is important but, in order to achieve a whole understanding, there had to be hyperlinks (digitally and metaphorically) to some of the triangulated faces of the mathematics polytope and to the human faces of mathematical endeavour – the linguistic, the scientific, the poetical, the artistic and the philosophical. If even only one is missing, the art of the mosaic is unfinished.

From Compilation to Composition

How then should one paint the mathematical mural of ideas to the twenty-first century student? Should it simply be a multimedia presentation of a multi-dimensional object with a multi-faceted surface?

Although unpacking language can be a constructive process bringing richness out of meagerness, supplying texture through collage and allowing breadth by way of compilation, the artistic pedagogue must, at some juncture, create a composition. There are many ways to achieve this. Relating each mosaic piece to its neighbours, having a map showing the connections and the relationships, placing the ideas into a historical context were all options open to online interpretations. In short, as suggested in Chapter 3, build the polytope from the ground up ensuring that the dimensional edges adjoined and its vertices aligned into a coherent image. Be the camera which pans a whole object with its various faces in both space and time. Topologize it (that is, make it less rigid and round its edges) by performing unusual transformations on it which would meld smoothness and curvature into its straight-line rigidity. Don't obliterate the past. The first layer of the mathematical pyramid was laid in antiquity. It is the solidity of this base which has kept the knowledge pyramid at right angles to its earthly orientation and its structural girders from collapse. Introduce dialectics, rhetoric and even debate if warranted. Explicate by grammar and logic. Use the building blocks of both tessera and strata. Permit, in some cases, triangulation of results. By all means, definitely take snapshots but, in the end, ***paint a tableau****.*

The Centrality of Language: as the Heart Beat of Communication and Community

John Dewey said it elegantly in the first chapter of his book entitled *Democracy and Education* (1916):

> What nutrition and reproduction are to physiological life, education is to social life. This education consists primarily in transmission through communication. Communication is a process of sharing experience till it becomes a common possession. (p. 12)

Burbules (2006), some ninety years later, addressed the notion of communication in a faceless world. He thought of the online environment as being a potential *space* of collaboration for those who engaged with it: "The fact that they inhabit a shared space is essential for this collaboration to work" (2006, p. 44). Burbules's concept of immersion

(2006, p. 40), through interest, involvement, imagination and interaction in and within such an environment, exhibits common threads with Dewey's process of sharing experience until it becomes a common possession.

The issue is: How can one share experiences if there is no togetherness of place? McLuhan highlighted the issue another way: "WHEN YOU ARE ON THE PHONE OR ON THE AIR YOU HAVE NO BODY" (as cited by Lamberti, 2011, p. xxix). Both of these instances of technological communication take *place* in a kind of ethereal "zone of between," to call on a phrase of Ted Aoki's (2005a, p. 161). Even though persons involved exclusively in these kinds of media interchanges have no visual impact on each other, yet it is possible to develop, over a period of time with repeated interactions, a recognizable, personal identity over both telephone and radio through one's voice, through certain behaviours (such as if one laughs a lot or meanders in thought pattern) and through the medium of language expressed through that voice. Consider the experience of attending teleconference meetings. Such technologies have simply altered the ratios of the senses from a mix of the major senses, as in face-to-face situations, to a single sense – that of hearing. When you are communicating over the Internet (without web cams), you also have no body; in particular, you are faceless. Therefore I began to ask: What kind of personal identity and impact, if any, can one have in such environments? How can 'no body' become 'somebody' in a physical vacuum?

Moreover, even a sense of community of spirit and purpose can be fostered and felt during teleconference calls and in on-the-air environments. For example, certain radio broadcasters have regular listeners who tune in either to listen and or to phone in to chat and exchange ideas on the topic at hand. There can be camaraderie and conflict of opinion but they are all engaged by the talk of the moment. Could this 'sense' of community ever transpire in a digital learning space and, if so, how would it take place?

Language: the FACE of the Virtual World

The 'FACE' in face-to-face situations, whether it be meetings, classrooms or other encounters, is generally experienced through a strata of sensorial perceptions:

1. the tangible or physical presence of persons there;
2. the intangible presence of persons there (persona);
3. physical appearance;
4. physical body language as seen through the eye or as felt by bodily contact;

5. spoken language (culture and tone); and
6. the kinetics of personality through the expressions of body language and the spoken word.

Such face-to-face spaces are diffused with a mélange of sensorial experiences through the visual, auditory, tactile, kinesthetic, and haptic responses we humans register internally and project externally upon the environments which we inhabit, particularly the communal ones.

The virtual or intangible qualities of 'FACE' in these same communal gatherings are the non-physical characteristics of presence which impact the affective domain of the human psyche. The most prominent of those qualities is the written/spoken language (culture and tone), thereby giving rise to the intangible or emotive personas of those present. These personas may often be defined by the role a character plays in the environment and their actions therein. Behaviors are selected according to the desired impression an individual wishes to create when interacting with other people. However, perhaps the most pervasive and persistent of the virtual qualities is the emotive impact of personality through the expressions of both written and spoken language and physical behaviours within the group setting.

I perceive the 'FACE' in *virtual* spaces to consist entirely of the *virtual qualities* of FACE which are present in physical environments: language (email, chat, or voice if audio used), then persona, and ultimately personality. What is noteworthy is that the intersection of the tangible face and the intangible face consists only of language, either vocal or textual. We are FACE-TO-FACE through our language in virtual environments (without web cams).

The Experience of Communication

What Dewey (1916) said about education bears repeating: "This education consists primarily in transmission through communication. Communication is a process of sharing experience till it becomes a common possession" (p. 12). The essence of communication, according to Dewey, is the attainment of a common possession by the participants in the educational process. This may be interpreted as a mutual understanding or meeting of minds as to the meaning of the expositor's message and how it has been interpreted by the listener, observer or reader towards a Gadamerian-like 'fusion of horizons.' It should be noted that:

> The basic model of understanding that Gadamer (1989) finally arrives at in *Truth and Method* is that of conversation. A conversation involves an exchange between conversational partners that seeks agreement about some matter at issue; consequently, such an exchange is never completely under the control of either conversational partner, but is rather determined by the matter at issue. (Retrieved on October 10, 2013 from the Stanford Encyclopedia of Philosophy http://plato.stanford.edu/entries/gadamer/#HapTra)

This progression from conversation → communication → common understanding → knowledge is not a two-becoming-one alloy of experience but is rather indicative of two fusing at their horizons of experience. In fact, Gadamer claimed that language (understood as dialogue or conversation) was the universal horizon of hermeneutic experience. (Retrieved on October 10, 2013 from http://plato.stanford.edu/entries/gadamer/#HapTra)

My claim was that, in the virtual world of online mathematical education, language may be unpacked using McLuhan's (2011) mosaic and, thereby, embodied in virtual settings through the virtual FACE. Language, understood as dialogue or conversation, whether oral or written, was the heart beat of the communication of ideas (Dewey, 1916; Gadamer, 1989) leading to a community of common spirit and understanding. As such, language stood at the core of almost all educational endeavour. My virtual endeavour was to become a testing ground for this hypothesis.

Melding Theory and Philosophy with Practice: the choice of online pedagogy

The ultimate design of my virtual project which included the RMC web site and the pedagogy of the live tutorial sessions was, as previously discussed, based upon my view of the learning of mathematical ideas. This view may be encapsulated within a three-fold definition of the word 'parallax' (see *The Parallax of Perception* in Chapter 3). This *parallax* of philosophy, mathematics and language formed the philosophical foundation of the web site used as a backdrop text for the participants in the project. Its design and construction began with the learning objects viewed as visuospatial imagery having a certain kind of cognitive load. It was hoped that this imagery, gleaned from the senses, though primarily visual, would be embedded into mental constructs by all who interacted and engaged with them. These virtual entities were then intended to nurture the development of mathematical reasoning and thought processes. In order to embue and nuance these mathematical objects with meaning and resonance, they would be overlaid, using my version of a McLuhan-like mosaic, with historical accounts, commentary, application and animation. Those participants who experienced their own philosophical parallax of personalized interpretation and formulation of mathematical ideas converging into mathematical meaning would likely also have experienced a shared common understanding in Dewey's (1916) sense, a Gadameran (1989) fusion of horizons, and a sense of immersion to a Burbulean (2006) place.

In order to fine tune the mosaic, I took a serious look at the ratio of senses built into the various media of the project. I ultimately decided that, in both the embodied live sessions and the more static RMC web site, special attention would be paid to the visual aspects of presentation in regards to placement, size, colour, and impact. I included additional layers of audio and kinesthesia afforded by both dialogue and animation, which these online environments afforded with relative ease. It was my belief that, in that way, the development of mathematical reasoning could be facilitated. Moreover, it was hoped that the discussions would stimulate the participants' curiosity and lead to their probing further into the ideas even outside the confines of the project venues. Keeping in mind McLuhan's "the medium is the message," I believed that the anticipated change in the ratios of the senses had the potential to place those persons learning mathematics online in a somewhat advantageous position over some others whose only experiences of mathematics were the explicated Gutenberg press equivalents of text and diagram. To investigate the validity of my assumption, I returned to the research literature to find the experiences and stories of other virtual-Face teaching endeavours. As it turned out, the indicators of that hypothesis not being false were already beginning to show themselves.

Surfing Back to the Past and Ahead to the Future on the Digital Wave

As already outlined in Chapter 2, many of the decisions involving the choice and structural design of the online learning environment for the project were a result of a fairly expansive literature review. This cross-disciplinary purview had drawn upon scholarly works of recognized importance in research areas as diverse as adults learning mathematics, cognitive load aspects of educational hypermedia learning, issues and properties of online web-based learning, as well as self-directed learning. It also took into account students involved in distance learning initiatives around the world.

What was evident from the research was that students' needs and expectations within this kind of pedagogical setting, from Newfoundland, Canada (Barbour, 2007) to Hong Kong, China (Chan & Waugh, 2007) to Swaziland in Africa (Magagula & Ngwenya, 2004), were consistent across the continents of the globe. For example, Bell (2007) had suggested that students learning online required more learner control and self-direction than those involved in traditional classroom-based instruction. His research examined the effects of both self-regulated learning (SRL)[22] and epistemological beliefs (EB)[23] on individual learner levels of academic achievement in an online asynchronous setting. Each of these categories had already been identified individually by, at the time, recent research in educational psychology as being related to academic success in learner-controlled educational settings. It was discovered that, for students to be successful self-regulators of their own learning, they had to possess certain attributes or mindsets which included (a) intrinsic motivation, (b) expectation that one's efforts to learn will bring positive outcomes, (c) confidence in one's abilities to perform and complete an academic task, (d) the ability to monitor one's progress toward goal completion, (e) the ability to control one's effort and attention, and (f) the ability to manage time and resources for learning and studying. The effect of computer self-efficacy, reasons for taking an online course, prior college academic achievement, and parental level of education were held as covariant factors (CF) in the analysis. The findings of Bell's research indicated that, of all the self-regulated learning and epistemological belief factors internal to the student's personality and situation, only expectancy for success in learning, prior academic achievement (measured by the student's grade point average), and the interaction term (the cross product of these two variables) were significant predictors of learning achievement in *asynchronous* online courses.

What I garnered from Bell's (2007) study was that when students come to learn, if those expectations are met, then one could reasonably predict greater student satisfaction and success with their learning, no matter which environment had been chosen for that purpose. This had been a study in which success had been measured quantitatively by the researcher. This study did not address the students' perceptions of their own learning achievement in such asynchronous spaces. That being the case, the task for me was to enhance the learning environment itself in ways so as to build the confidence of individuals as they interacted within it. My wish was that the participants' personal expectations for learning would be more than just satisfied, regardless of their prior academic achievement record. I would also include a means of participant self-assessment as to their achievements in learning during the project.

It was Moore (2002) who had suggested that online learning promoted a higher level of reasoning and thinking skills. In addition, other research (Tang & Byrne, 2007; Magagula & Ngwenya, 2004) had indicated that the academic achievement of distance learners in direct comparison to their classroom counterparts had been, even at that time, at least as good, if not better in some instances. This convinced me that pursuing this mode of delivery in the learning of mathematics had considerable merit. The challenge then was:

1. to garner a list of effective learning strategies (Wadsworth, Husman, Duggan, & Pennington, 2007),
2. to provide opportunities for students to apply their learning (Lim & Kim, 2003),
3. to incorporate other technologies into the structure of the system to provide a greater degree of personalization, interoperability and flexibility (Sun, Joy, & Griffiths, 2007),
4. to create a pedagogical structure which was transparent to the user (Barbour, 2007),
5. to build-in a more definitive social and cognitive 'presence' of the instructor in the environment (Moore, 2002; Hrastinski, 2008), and
6. to provide alternate types of learning objects suitable for various learning styles (Shah & Freedman, 2003).

Simpson, Stahl, & Francis (2004) had laid out the all-encompassing task – the umbrella task, so to speak – by arguing that high-risk undergraduate students would best be served through a comprehensive cognitive-strategies curriculum. This would become my version of McLuhan's mosaic for the learning of developmental mathematics online.

The outcome of the amalgamation of personal and other viewpoints from the literature was the decision to create the remedial mathematics space of learning as a web site detached from any institutional course structural container. Within this broadened pedagogical environment, I hoped to provide a similitude (that innate sensory experience of similarity) of open face-to-face classroom of learning without having to simulate (by means of external and artificial reproduction). In particular, it was a setting which was as flexible, as interactive, and as free-form as any other space of learning, limited only by the imagination of its creator. The RMC web site, in fact, had been constructed in the likeness of what Anderson (2004) called the second model of e-learning associated with independent study learning whose tools of instruction sometimes included computer-assisted tutorials, drills, simulations and even virtual labs. The addition of the synchronous sessions with the potential for a high level of interactivity was in keeping with John Dewey's approach to education. Anderson (2004) reminded us of that:

> As long ago as 1916, John Dewey referred to interaction as the defining component of the educational process that occurs when the student transforms the inert information passed to them from another, and constructs it into knowledge with personal application and value. (p. 8)

Implications for Online Pedagogy

If one is to better the future of teaching and learning online, then I believe that one is obligated to learn from the past and from others who have gone before. This, of course, has implications for one's own future endeavours. What follows are those considerations of pedagogical nature which I took to heart.

Self-directed learning: how much, how little. Candy (1991) wrote a seminal book on self-direction for lifelong learning. He delineated four distinct meanings of 'self-direction': personal autonomy, self-management, learner-control and autodidaxy. The middle two are of relevance to my research, because the visitors to the Remedial Math Centre would require some of both aspects of those kinds of self-direction. My online educational stage was intended to lean towards a Platonic model of learning with the 'virtual teacher' as mentor, guide, developer of learning skills and problem-solver. Though it did offer information and organized instruction, it fell more within the purview of Candy's (1991) independent study model which was:

> . . . characterized by a high degree of learner-control over many instructional elements, including the setting of objectives and student choices about individual pacing, content and methodology, and assessment of learning outcomes. (p. 13)

Variance in each of these four dimensions of learner-control: pace, choice, method, and content were on a sliding scale continuum with each pedagogical situation positioned somewhere between the theoretical absolutes of complete teacher domination to autodidaxy.

Over two decades after Candy's 1991 book was published, George (2012) had reiterated the theme: "student autonomy should be taken into consideration when choosing strategies through which students are motivated for achievement" (p. 1). The case of an independent, non-course related, online learning environment demanded that the Remedial Math Centre lie heavily weighted on this continuum towards a significant amount of learner-control but pulled back from an extreme position because, as Bruner (1966) pointed out in regard to the teaching of mathematics:

> A body of knowledge, enshrined in a university faculty and embodied in a series of authoritative volumes, is the result of much prior intellectual activity. To instruct someone in these disciplines is not a matter of getting the learner to commit results to mind. Rather, it is to teach him or her to participate in the process that makes possible the establishment of knowledge. We teach a subject . . . to get a student to think mathematically for himself or herself . . . Knowing is a process, not a product. (p. 72)

In that sense of knowing, there would be limits as to how much self-directed learning could take place in virtual isolation. For that reason, it was my strong conviction that the teaching of foundational mathematical ideas should never be completely severed from the mathematical expert, though it is of considerable benefit to students if those ties are kept loose and flexible so as to encourage as much learner participation and autonomy as is possible under the circumstances. I have always tried to build both emotional and intellectual strength in my students' learning of mathematics. I have always felt that I had only taught them well if, at the end of our limited journey together, they were able to carry on their studies with confidence and solid understanding. Therefore, in order to provide enough guidance so that students did not encounter too many pathways leading to some kind of virtual obscurity and confusion, I would include enough navigational and instructional guidance without providing too much explicit direction, which tends to discourage learner curiosity and choice. The challenge lay in striking a balance.

To that end, my research would attempt to probe student reaction and involvement within the combination of the two chosen virtual venues so as to determine whether, in fact, these environments did facilitate self-directed learning. The important issue of the impact of the instructor's virtual presence – not merely cognitive presence - in online settings would also have to be addressed. Was it imperative for students' engagement and ultimate success? Was it preferable? Was it negotiable, depending on the participant? Or was it is merely redundant? The responses to these questions would be a measure of the teacher-student dichotomy on the self-directed learning scale.

The students and their reactions. Candy (1991) had probed the research literature on self-directed learning and had come to the following conclusion: "Self-direction is not a quality that exists in the person or the situation independently but rather is a result of the interaction between a person and a situation' (p. 247). If this were indeed the case, then learner interaction in an online environment would inevitably take on different complexities from that of regular classroom learning because of the much broader expectations and demands of self-direction in a virtual world.

Hirumi (2002) proposed three levels of learner interaction in online learning: learner-self, learner-other, and learner-instruction. Ally (2004) expanded on this and suggested four levels: learner-interface, learner-content, learner-support and learner-context. Learner-interface interaction is most often physical; learner-content interaction in the pedagogy of mathematics is mainly cognitive; learner-support interaction is social; but learner-context is sensorial and belongs to the aesthetic.

Although I would have no means to advise students who, potentially at least, were not well-suited to learning mathematics online, I was nonetheless interested in types of learners who were prime candidates for self-directed learning and those who were not. Fry (1972, as cited by Kanuka & Nocente (2003)) researched the relationship between students' levels of aptitude and inquiry (or self-directedness) and learning outcomes in both learner-controlled and expert-controlled environments. The results of this study showed that high-aptitude-high-inquiry students learned significantly better in a learner-controlled situations; high-aptitude-low-inquiry students fared better under a high degree of expert control; and the results varied with low aptitude students.

Those adult learners who volunteered for my developmental mathematics learning project would likely be quite diverse in their academic backgrounds, their prior knowledge of mathematics and their levels of aptitude and inquiry for the subject matter. How could I, as a virtual-face instructor, accommodate such diversity in a setting with no eye contact? How could I document their lived experiences, both cognitive and affective, in this kind of environment? Therefore, to probe for these multi-faceted and highly individual participant experiences, I conducted three (3) surveys during the course of the project: the Pre-Project Survey (PPS), the First Impressions Survey (FIS) and the Learning Modes Survey (LMS) (see Appendices C, D and E, respectively). The first survey was an effort to garner relevant information about the participants' past experiences and attitudes towards the study of mathematics. The remaining two surveys elicited personal, ingenuous responses to four kinds of learner interaction in online settings (Ally, 2004): learner-interface, learner-content, learner-support and learner-context. My hopes were that an analysis of these personal reflections would reveal some insights into what their virtual realities had been in this medium. What had the message of the medium been for each of these learners?

The role of Cognitive Psychology[24] in hypermedia environments. Cognitive psychology, being the study of all mental activities related to acquiring, storing, and using knowledge, spans the entire spectrum of conscious and unconscious mental activities: sensation and perception, learning and memory, thinking and reasoning, decision making, and problem solving. Its scope is broad and encompasses the learning experiences of all of human kind. Cognitive load theory asserts that the capacities and limitations of the human memory system must be taken into account during the process of instructional design in order to produce optimal learning materials and environments (Boechler, 2008b, p. 3283). Cognitive load theory for educational hypermedia provides guiding principles to instructional designers and educators alike in the creation of such materials (Mayer, 2005). Boechler (2008a, p. 1199) pointed out that individual personal differences such as learning style, prior topic knowledge, level of interest, and gender, as well as the structural characteristics of the objects themselves all influenced student performance and behavior. In hypermedia settings, learners may, in fact, require different kinds of interface support. When designing such environments, it was important to understand the relationships between individual spatial and reading skills and the types of such support networks. By incorporating such mechanisms into the architecture of this medium, students of differing and diverse abilities may find it an optimum interface for the learning of mathematics. Boechler (2008a) also suggested that "Cognitive and learning theories can provide guidance for exploring the interactions that occur between the interface characteristics and individual differences for both navigation and learning outcomes" (p. 1202).

Though visuospatial issues and cognitive theories of multimedia learning have been discussed at length previously, I felt it was important to highlight once again the major role that Cognitive Psychology has in the development and design of online educational environments. The attention paid to such theories may make the difference for some between the experience of a simple space of educational materials and a learning place as a fusion of horizons.

Learning Spaces to Learning Places

Hoyles (1985) had recognized that children have a need to articulate their thought structures - even the more abstract ones. She spotlighted the significance of discussion in a socio-cultural framework as a contributing factor to the development of a child's mathematical knowledge. This is certainly no less true for some adults, particularly those learning at a distance. Therefore, it was important that adult learners, as well, be provided with opportunities to verbalize, or express in natural language, their thought processes of learning. This exercise would help learners to analyze the coherency and resonance of meaning of their own thinking. If a student's thoughts could stand up in the arena of public scrutiny, this could be the first step to deeper understanding.

Consequently, my attention turned towards an emerging vista of technological advances which were capable of opening avenues of genuine exchange by allowing a digital kind of mathematical discourse. Several means of 'over-the-wire' communication could now become a part of this 'faceless' space in the form of what I term 'digital dialogues.' Faceless it might remain, but it would no longer be voiceless. Text-based student discussion forums would allow the participants to meet and converse asynchronously. In addition, the possibility of synchronous meetings via web conferencing software between an instructor and an interested group of students had opened the way for an audio-based, digital hands-up, exchange of ideas in addition to the use of a virtual whiteboard to present detailed solutions to exercises and pictorial explanations. This full compilation of technologies could then be regarded as a virtualFace-to-virtualFace e-version of the face-to-face classroom with its various e-counterparts of presentation, demonstration, repetition, practice, assessment, personal guidance capabilities and **conversation**. It was Gadamer (1989) who regarded conversation and dialogue as the universal horizon of hermeneutic experience.

The inclusion into my virtual stage model of the synchronous space meetings was intended to be the remedy for those who would require such virtual ‘closeness’ of human presence in the learning process. Ironically, this addendum to my original ideas of online learning spaces as being primarily web sites or highly structured course management systems turned out to be, in the final analysis, a core component of my study.

CHAPTER 5 RESEARCH STRUCTURE and METHOD

The creation of my virtual stage of teaching and learning, encompassing both the RMC web site itself and the ensuing mosaic of layered asynchronous and synchronous technologies, had drawn upon theories and research findings of two distinct but overlapping theoretical frameworks, that of self-directed learning paradigms and that of the domain of Cognitive Psychology, particularly cognitive load theory, the latter undergirded with sensitivities towards visuospatial issues. However, because the focus of the research was to explore participant *experiences* within a particular virtual learning environment, the primary research was to take the form of an empirical research case study by way of narrative inquiry but ensconced within a hermeneutical interpretative framework. Stake (1995) described it this way:

> Case study is the study of the particularity and complexity of a single case, coming to understand its activity within important circumstances. (p. xi)
> The qualitative researcher emphasizes episodes of nuance, the sequentiality of happenings in context, the wholeness of the individual. (p. xii)

I, the researcher, would fill the role of an interpreter, not an explicator. The methods of inquiry would be born of investigative strategies and observations but realized as a telling of thematic narratives.

Ontological and Epistemological Considerations

Gadamer (1989) believed that the role of hermeneutics in the human sciences was not to be equated with the role of research methods in the natural sciences. Hermeneutics was not a highly structured, scientifically articulated method of discovery with rules of engagement. Rather,

> According to Gadamer, language is a medium of hermeneutic experience and is a vehicle of our understanding of the world. The universal aspect of hermeneutics as a realm of philosophical inquiry is that language is the being of everything which can be understood. Hermeneutics is not merely a method of interpretation, but is an ontological relationship between an interpreter and a language which is to be interpreted. (Scott, 2003)

This is reminiscent of the notion of language as being not only central to the art (and science) of communication but also the very heart beat of community (see Chapter 4). Nonetheless, we cannot extricate ourselves from ourselves. Our personal experiential realities are highly individual and layered by biology, environment, culture and personal history. Though, as researchers, we may cast a benevolent outward eye towards the unfamiliar, our interpretations of the observed, especially in terms of behaviour, beliefs, and inherent meanings, are inextricably embedded into our own historicity. Gadamer (1960) stated that: “Hermeneutic work is based on a polarity of familiarity and strangeness” (p. 295). To engage in it is, by an act of will, to open up a tiny bit of the researcher-self, to project our internal focus onto that alien influence which will bequeath to us its etchings, and then to rewrap that altered vision around the inward self as we distance ourselves again for reflection and for analysis. The part of the observed Other which is retained in the middle passage between our projected focus outward and the subsequent rewrapping inward will determine the extent of our ‘indentification’ (rather than ‘identification’) with the unfamiliar and the degree to which the understanding of it has been achieved. This reciprocal process of our sensory projection and the subsequent reflection of its multi-faceted image onto one’s self again is an internal ‘fusion of horizons.’ In this case, the word ‘horizon’ is defined to be the outer extents of one’s personal understanding of the world around us – in other words, that circular band of distinction between the “I” and the “Other” (Connelly & Clandinin, 1990, p. 9). I interpret the hermeneutical endeavour as a kind of transformative learning process, a metaphorical double convex lens, so to speak, which modifies our perspectives in much the same way as described by Dewey (1916).

The researcher’s horizon of pre-conceived prejudices, where ‘prejudice’ is defined by Gadamer as “a judgment that is rendered before all the elements that determine a situation have been finally examined” (1960, p. 270), meets, mingles and mixes with the horizon of the articulated experiences of the Other, also suffused with fore-meanings and distortions of self-perception. The permeability of our projected self-imaginings to the imprint or colouring of the narrative text of the Other will determine the extent of this osmotic infusion of mutual understandings. This entire hermeneutic process can have many iterations in which “interpretation begins with fore-conceptions that are replaced by more suitable ones” (Gadamer, 1960, p. 267). The hermeneutic dialogue is both temporal and transforming. The constant in this dynamic process is the researcher’s task of recognition of his or her own fore-conceptions in order to diminish their impact on the researcher self. How much a researcher can truly know or understand about the Other is framed within this context of the hermeneutical circle of iterative exchanges.

It is within this hermeneutical framework that the collected narratives and survey responses of my case study research were to be pondered, analyzed, thematically categorized and interpreted. Its power to transform (generate understanding) and its limitations to enlighten (open the imagination) would be determined by the personalities involved in the study. That ability or inability, that willingness or reticence, those ingenuous tendencies or self-preservation instincts could either open or constrict the aperture of viewing by both the researcher and the researched. Those behaviours would ultimately determine the outcomes of this research in terms of whether there would be any kind of mutually shared reality towards a common understanding of mathematics and each other in this educational project.

The Relevance of Axiology

In keeping with the meaning of the Greek word *αίσθητικός* (translated into English as 'aesthetics') – that is, having to do with the senses or sensitivity – the intent of my research was to capture not only the perceptions but also the inherent sensitivities of the visitors to this new world of learning. Their visual, auditory, and kinesthetic responses may, in fact, trigger certain cognitive and emotional reactions. Significant human values are often intermeshed in both the ethical and the aesthetic domains of any classroom. Notions of ease of use and navigation, familiarity with past mathematical fore-meanings, finiteness of task (in the sense of doable), adequacy of direction, reliability of presentation, variety of form, and sense of community would be addressed in my research. The theory of environmental aesthetics suggests that a "normative dimension suffuses the perceptual range, and this underlies positive or negative value judgments of an environment" (Berleant & Carlson, 2007, p. 16). This theoretical framework further embraces the tenet that "our sensory experiences are mediated by our ideas and our understanding" (Berleant & Carlson, 2007, p. 20). Therefore, questions must be formulated, not only to elicit responses but also to probe for prior beliefs and the formation of judgments about their personal experiences so as to imbue the superficial sensorial surfaces with thicker layers of meaning. The three (3) surveys mentioned in Chapter 4 were the instruments I chose to probe for some of these emotive impulses in the deeper layers of the human psyche.

Laying the Framework for Analysis

As mentioned earlier, the structure of the research was to take the form of an empirical case study bounded by the size of the group and the time frame in which the study would be conducted. I decided that the time frame of the study would be the four-month period from January through April 2010. Its setting was to be the virtual stage I had developed during the three previous years. The sample would be drawn from those University students and staff wishing to upgrade their math skills in preparation for further university-level study programs. It was a sample of convenience and I had hopes that it would snowball if enough volunteers could not be obtained. I thought an initial sample size of ten or more participants would be desirable so as to obtain a good cross section of individuals at varying levels of cognitive understanding. Taking into account those who may withdraw during the course of the study, I considered a residual number of about eight participants to be ideal for analysis. This was the case in question.

The data would be drawn from a variety of field texts to be collected before, during and after the duration of the project. Such texts would include email letters, the transcripts of synchronous online meetings, text-based online discussion forums and three (3) surveys with open and leading questions. The interview questions would include queries surrounding 'Who are you?' and 'What have your experiences been?' See Appendices C, D and E for detailed lists of survey questions which were presented to each of the participants.

The live tutorials would be fashioned after my past face-to-face classroom pedagogy but overlaid with several forms of online media to provide more texture to the learning of an abstract discipline. These virtualFace-to-virtualFace meetings with the instructor/researcher would give the participants an opportunity to voice their concerns, to put forward their queries and to express their ideas in a synchronous setting.

The form and the sporadic nature of the interview-like conversations recorded during the live meetings were to be fashioned to elicit participants' sensory experiences of both the online venues and the innate meanings which they may have embodied from them. I use the word 'sensory' in its broadest form, including the physical senses of vision and hearing, the kinesthetic aspects of navigating and interacting, as well as the ensuing perceptions of personal meaning and the acquisition of knowledge. The latter would include the sense of belonging, the sense of isolation, the sense of support, and the sense of safety. Questions would also call for participant descriptions of present and past mind-sets during their rather brief sojourns in this virtual world. What were the emotional, psychological, and cognitive experiences of students in this rather singular kind of pedagogical place? In addition to that, a student discussion forum would also be available online for those wishing to touch base from time to time, in an asynchronous manner, with their fellow participants. Those Internet connections between individuals were intended to foster feelings of 'connectedness' one to another, so that all could sense an equality of position, a mutual caring, and a common purpose. They also provided both structured and unstructured possibilities for dialogue and debate.

The continuous and iterative process of **collaboration** and **corroboration** would take place not only through email and conversational exchanges between researcher and participants clarifying meanings and interpretations but also through a broad sweeping view across all of the field texts. The cross-referencing of responses from the initial description of their ideal classroom setting through the synchronous encounters of the tutorials to the textual conversations of the entire forum would take into account the multiple perspectives of those who had given voice and substance to their experiences. These exchanges were also a kind of **validation**.

Methods of Inquiry

Negotiating the Zone of Between

In the world of mathematics education, we often formulate both a collective and a personal ideal of what it means to be a good educator and many of us strive to emulate its quintessence in the brief encounters we have with our students. Such meetings are often attempts to reach out to them and more fully understand their somewhat alien realities. In accordance with Aoki (2005), teachers actually *live* in the "zone of between" (p. 161) of curriculum of study as an architectural plan and curriculum of study as an experiential form. In fact, many of us constantly strive to traverse the bridge between the mythical ideals associated with each. My orchestrated online multimedia setting, too, was designed in that same spirit, first as a curriculum of study and then as a dynamic virtual space of mathematical discourse with instructor presence, learner presence, texts, learning materials, discussion, lessons, homework, testing and opportunity for counseling.

Garrison, D. R., Anderson, T., & Archer, W. (2000), in their article entitled "Critical inquiry in a text-based environment: Computer conferencing in higher education," proposed a theoretical framework for what they termed a Community of Inquiry. An educational community of inquiry was defined to be a group of individuals who collaboratively engaged in purposeful critical discourse and reflection in order to construct personal meaning and confirm mutual understanding in ways emulating Gadamer's (1989) hermeneutical process and Dewey's (1916) notion of communication. In that sense, my project participants and I would form a mathematical community of inquiry with our discourse being both written and oral. The model Garrison et al. (2000) put forward as representing that framework was visualized as a Venn diagram of three mutually intersecting and interacting 'presences' of experience in educational settings: that of the cognitive, the social, and the pedagogical or teaching presence. At the core of that Venn diagram was the common educational experience of that learning environment.

The three 'presences' have been defined variously and subsequently as follows:

- ***Cognitive Presence*** was characterized by the "extent to which learners are able to construct and confirm meaning through sustained reflection and discourse in a critical community of inquiry . . . . cognitive presence reflects higher-order knowledge acquisition and application and is most associated with the literature and research related to critical thinking" (Garrison, Anderson, & Archer, 2001, p. 1).
- ***Social presence*** was defined by Garrison (2009) as "the ability of participants to identify with the community (e.g., course of study), communicate purposefully in a trusting environment, and develop inter-personal relationships by way of projecting their individual personalities" (p. 352).

Akyol, Garrison, & Ozden (2009) went on to distinguish three categories of social presence: affective expression, open communication, and group cohesion.

- ***Teaching presence*** was considered, by Anderson, Rourke, Garrison, & Archer (2001), to encompass "the design, facilitation, and direction of cognitive and

> social processes for the purpose of realizing personally meaningful and educationally worthwhile learning outcomes" (p. 5).

The first type of teaching presence was essentially the instructional design, organization and presentation of the materials or content to be discussed; the second was the role of instructor as facilitator to encourage dialogue involving all members of the group; the third had to do with there being some direct instruction taking place at appropriate junctures to provide guidance and expertise. Facilitating discourse was thought to be critical to maintaining students' interest, motivation, and engagement.

At the hub of all of these presences lay the essence of educational experience formed by the interlocking and overlapping of the three domains. The core of my research endeavour was, in fact, situated precisely at that hub. It melded the selection and structure of the content (the web site) with the task of setting the climate for dissemination and exchange (the form of the live sessions), and finally was overlaid with the ways the ensuing discourse could be facilitated and negotiated (the teaching /learning interchange amongst all the participants). For that reason, the Community of Inquiry framework ultimately formed the loosely followed thematic base for the analysis of the experiential narratives in the project. I would explore the narratives for evidence of the three elements in the framework to discover if there had been sufficient qualities of salient interaction in each of the three domains to warrant the virtual presences forming a veritable Community of Mathematical Inquiry. This was the '*zone of between'* for both teachers and learners – an extension of Aoki's original metaphor.

The investigative techniques were qualitative in nature and took the form of a **Narrative Inquiry**[26] within the educational **Community of Inquiry** framework but under the umbrella of the hermeneutical interpretative tradition. The purpose was to glean a thematic narrative of the participants' lived experiences - a rich telling of events, of perceptions, of emotions, of meanings, of actions, of reactions, and of interactions embodying their highly personal realities while inhabiting this re-conceptualized learning environment. In addition, the granting of voice and story to their fears and frustrations while learning this, for many, difficult subject may, in fact, serve to release telescopic[27] insights into the often unspoken tethers which trammel their venturing into such an ethereal meeting of cognitive horizons. Hopefully, we together would negotiate a mutual crossing of the great divide between us as a shared conversation – a so-called 're-storying' – thus empowering each of us to effect a fusion of our individual horizons so as to understand the Other. It was intended to be, for *both* participant and researcher, that Gadamerian process (1960) to "dissociate with one's past to associate with the experience of the other" (pp. 266-267).

Telling the Story: Towards the Virtual Experience of the Third "I"

Connelly and Clandinin (1990, p. 9) talked about the multiple "I's" present in the inquiry into narrative. They focused on the "I" variously as the researcher, the teacher, the person, the narrative critic, the theory builder, and other roles inherent in the persona of Inquirer. However, the "Other," signifying the group of participants in the study, also constituted multiple "I's," both individually and collectively. These were the second category of "I's" in the research endeavour. Further to Connelly and Clandinin's classifications, I would like to introduce what I call the Third "I". They are the readers of the final written narrative, the interested audience of the research work. They too form a collection of multiple "I's." It is to the Third "I" that the final narrative will be told as a re-storying of participants' lived experiences hopefully resonating into a vicariously lived experience for those of the Third "I". If the narrative is skillfully told, it will constitute yet another fusion of horizons.

I see narrative inquiry as a transitive process from participant to researcher to audience – another virtual medium. At each juncture, for the narrative to have validity, there must be a kind of osmotic experiential connection for those involved. Going back once again to McLuhan's message, the medium of narrative has its own peculiar impact and message. The narrative itself will be written mainly in the inductive mode whereby the stories and the data speak for themselves to the audience of interested readers. This will be the *accounting of* the participants' momentary realities. However, the *accounting for* such perceptions and emotive responses will draw upon a deductive analysis of significant events and encounters.

Validation

It was also Connelly and Clandinin (1990) who observed that the hermeneutic nature of narrative inquiry across the individual and group divides of the many "I's" of participant, researcher and audience somehow demanded a redefinition of what is considered to be 'valid' or 'reliable' data in this research paradigm. They and others had put forward criteria of verisimilitude or transferability as being alternative ways of determining ‘validity’ or ‘reliability’ of personal story, keeping in mind the ever-present principle of temporal defeasibility (Chatman, 1981) because of sometimes not-so-subtle differences between events-as-lived and events-as-told. I paid attention to these observations and suggestions. Over time, I began to perceive validation of case-study narrative research as best measured by the extent of the ‘transfusion’ of horizons from the Participant “I’s” to the Research “I” to the Third “I”. As with all such hermeneutical endeavours, this kind of validation would also be infused with “I”-like interpretation and conclusion. Therefore, it ultimately came down to a question of critical mass as to whether such a piece of research could be considered fertile soil for a grounded theory or simply an interesting special case with its *small* universalities and its *large* peculiarities. Perhaps that is all we can expect in a world of many realities and multiple ways of knowing.

Ethics: from the Sampling to the Narrative

Because the research study sample of participants consisted of volunteers to be drawn from those wishing to upgrade their math skills, the pre-conceptions I had had about such participants became the first stage to examine in regards to the researcher’s personal ethical process. I could not assume that everyone had had difficulty with mathematics as a subject of study. Some would only come to review. It would be presumptuous of me to expect everyone to reveal and openly discuss deeply personal past classroom experiences. While it was true that there may be those for whom this openness would be cathartic, others may only be further traumatized by the revival of such negative memories. It became of utmost importance to treat such volunteers in a nonjudgmental manner and with respect, for some would probably, as in the hermeneutical circle of interaction, open themselves up to reveal their innate fears and vulnerabilities in this particular realm – an endeavour which was intricately interlaced with aspects of personal self-esteem and identity.

In my opinion, volunteers for education-based research studies are, in some sense, placing themselves ‘at the mercy’ of their observers. Therefore, in an attempt to avoid any unseemly accusations of beguilement or even open fraud, I decided it was paramount to provide a full disclosure of the nature of the research. The participants should come into such a project with their eyes opened and open. To that end, a letter of invitation with full disclosure (see Appendix F) was sent to a targeted audience of already enrolled remedial mathematics students and, in addition, a posting of that same invitation was posted on the RMC's web site itself. That letter contained the following information: the purpose and length of the study; the expectations of the researcher in terms of participant involvement; the option of participants to withdraw at any time including the 'final date of veto' (the time beyond which it would not be possible to remove personal data from the final written Narrative); the kinds of data to be collected and how long they would be kept; how the data would be used; to whom the results of the study would be disseminated; AND the complete and utter assurance of anonymity and confidentiality. The latter was to be achieved by a process of double pseudonyms. Each participant was asked to choose a fictitious name at the outset of the course of the study. As all interactions were online, no-one had to know the real identity of the other. As a further precautionary measure, the final narrative would use a different, but correlated, set of pseudonyms for most of the participants so that the chorus of actors themselves would have some difficulty recognizing each other in the ensuing drama of their interaction.

The continuous and iterative process of collaborative exchange was to take place between researcher and participants in order to clarify meanings and interpretations and to ensure authenticity of their individual stories. However, during this collaboration, each participant could opt to veto any or all of her or his story at any time, except beyond the letter of invitation's final date of veto. I was of the opinion that any narrative inquiry probing for participant sensitivities should be done in such a manner so as to afford a sense of graciousness and gratitude towards those who had opened for viewing their personal and often very private moments. The final narrative was, therefore, to be written as the researcher "I" vicariously living out the experiences of the Other "I" with as much verisimilitude and authenticity as was possible. Its re-storying was envisioned to be a woven fabric of silken threads imbued with empathy and connectedness to each of the participant realities as voiced by the individuals themselves.

Here is the telling of that story.

CHAPTER 6 WHERE PHILOSOPHY and REALITY MEET

THE CASE STUDY - PART I

My particular case was a study of the learning of remedial mathematics within a virtual learning space using two different layers of online environments. The two venues included:

- an asynchronous web site of interactive tutorials which showcased topics from within a typical developmental mathematics curriculum, coupled with a third-party self-assessment tool of associated exercises and tests, and
- a series of paced, weekly, synchronous teaching sessions by way of web conferencing.

The study took place from January through April, 2010, and was subsequently extended, by request of some of the participants, through to the end of July 2010.

Stake (1995) had suggested a categorization of individual case studies into being either intrinsic or instrumental depending on the level of the researcher's personal interest or stake which she/he had in the given situation or activity. He had delineated an intrinsic case from the others as follows:

> The case is given. We are interested in it, not because by studying it we learn about other cases or about some general problem, but because we need to learn about that particular case (Stake, 1995, p. 3).

In that sense, my case study was intrinsic for the following reasons:

1. the web site was a creation of my own making;
2. I personally taught the live tutorials over the research project time frame; and
3. I wanted to know more about the lived experiences, both cognitive and affective, of a group of adult participants learning remedial mathematics in this environment.

In my experience, there have been too many students who have been less than enamoured of the subject matter of mathematics and who are even terrified of it. O'Donoghue (2000) had also observed that:

> The inescapable reality for those working with adults is that mathematics as seen through the eyes of adult learners is the antithesis of what tutors would like. (p. 230)

This lack of engagement, sometimes even total disengagement, can occur in diverse classroom settings and continues to be a topic of much concern because of the changing face of education using technology. Questions have been put forward as to the effect of technology on the learning process and whether any deep learning of the subject matter can be achieved at all in a fully online environment. These sorts of issues had generated the primary questions for this research project.

In my case study, three kinds of experiences of the group members were examined: the cognitive, the social and the pedagogical for their virtual presence within the collective of the online venues. In addition, three levels of cognitive, social and pedagogical engagement were studied: those who ultimately disengaged completely; those who engaged by purpose; and those who were fully engaged by passion. See Chapter 5 for a full explanation for the research methods of inquiry and the numerous field texts used to collect the qualitative data for the study. While I had initially intended to interview each willing participant, one-on-one, about his/her experiences, I chose not to pursue that avenue of data collection about two months into the project. I decided that I would capture a truer picture of the participants' virtual realities if I just watched, listened, and read *their* personal narratives. When one is face-to-face, even in the virtual world, one has a tendency to say what we think the other wants to hear. Therefore, the gathering of data was to be non-interventive and empathic in the way Stake (1995) had suggested regarding case study researchers:

> We try hard to understand how the actors, the people being studied, see things. Ultimately, the interpretations of the researcher are likely to be emphasized more than the interpretations of those people studied, but the qualitative case researcher tries to preserve the *multiple realities*, the different and even contradictory views of what is happening. (p. 12)

I wanted to be that kind of researcher.

The main thrust of the research centred around the metamorphosis from a learning *space* to a learning *place* as put forward by Burbules (2006, p. 49). Could it happen at all and, if so, how would it take shape? A learning *space* was, according to Burbules, a context which was distinct from the learner but in which the learner was a recipient of information. A learning *place* was a context in which the learner became embedded or immersed so that the acquisition of the information there was absorbed, by a kind of osmotic process, into the learner's psyche as transformed, personalized knowledge to be carried forward with them into the future.

In my particular case study, the two main issues were etic issues (that is, brought forward by the researcher); the sub-issues were emic (that is, the issues of the actors in the case which emerged during the project). Each is listed and described below with the associated research questions.

ISSUE 1: There was to be an investigation of human self- interaction towards the learning of mathematics. Would there be enough salience of both **cognitive presence** and **teaching presence** so that the learning of mathematics online was considered to be as effective as in face-to-face instructional situations? How would the participants compare the two learning environments? Did it simulate the traditional classroom experience? Could higher learning or, what I call, mathematical thinking (see Chapter 2, subsection "Mollifying the Message of Digital Internet Technology"), occur in such virtual settings?

Cause-effect questions: Did the fact that students and instructor have no face-to-face communication affect the perception of the quality of teaching and learning mathematics online? Did the online environment change the way students learn mathematics? Did the online environment change the pedagogy of the instructor?

ISSUE 2: There was to be an exploration into the notion of 'community' formed by human interaction with others in a fully online setting. Could there be enough **social presence** generated by the members of a group in such a virtual learning space so that they felt part of a meaningful community? If so, what human interaction ingredients contributed towards the creation of this sense of community? How enduring could it be? How was it the same or different from other learning communities?

Cause-effect questions: Does the fact that students and instructor have no face-to-face communication hinder the creation of a sense of community in virtual worlds of learning? What are some of the essential qualities of online learning environment communities?

Sub-ISSUE 1: Self-identity (personal historicity) towards individual engagement with the subject matter

Sub-ISSUE 2: Coping (personal) towards engagement with the context of the environment in the context of self-identity

Sub-ISSUE 3: Belonging (social acceptance by others in the context and the group politic) towards the formation of community

IN MY CASE, the tension between the case itself (the two e-layers of the environment) and the issues (essentially dynamic and highly personal) led to my focusing on the synchronous mode of interaction, during the live session tutorials. I confess that I really had no inkling whatsoever as to how the online experience would play out – both for myself and those who agreed to experiment with me. My first major concern was that I would have too few volunteers. There were grave suspicions at that time that online environments for the teaching and learning of mathematics were substantially different from the traditional face-to-face classroom settings. If, in fact, that were true, my study would be reduced to the articulation of those differences. I also suspected that there would be little or no sense of community generated amongst the volunteers. After all they would simply be a roster of names (pseudonyms, no less) on a computer screen. In my experience, adult-only mathematics classes were often non-chatty affairs anyway. We were all too busy trying to penetrate the mystery of the enigmatic, but dry, subject matter to indulge in idle chat. For some, the notion of having been subjected to this ordeal was enough to promote a kind of resentful non-participation. Therefore, the personal interaction between the attendees of the live tutorials could be expected to be more formal than friendly; ergo, the dialogue would be limited to asking questions and providing answers. However, I did believe that learning could happen but I wondered if any really deep learning towards genuine mathematical thought would, or even could, take place.

Blending Two Virtual Worlds

As discussed in Chapter 1, there are many meanings of the adjective '*virtual*,' some depending on the context. Most often, a *virtual world* is a computer-dependent space of human interaction using keyboards, mice, headsets and other hand-held devices. The term, 'virtual reality' (or VR, for short), can be just another term for a *virtual world* but can also refer to the personal human interaction and experience in a *virtual world*.

This case study is a blend of two *virtual worlds* and the attendant *virtual experiences* of those inhabiting those worlds. One is the virtual world of the Internet in which the word *virtual* will mean occurring or existing primarily online (for example, our web conferencing meeting space) in the sense of a computer-dependent space of human interaction. The second is the virtual world of the learning process in those spaces. The primary focus of this case study will encompass the experiential *virtual realities* of both of these *virtual world*s. This will comprise NOT ONLY the personal actions, interactions and reactions of the participants in the rooms and hallways of that meeting space and its extensions to the *virtual worlds* of email and discussion forums BUT ALSO the impact of those experiences on them in the personal world of the inner self.

All this philosophizing, but where is its merit? The theoretical, philosophical base of this case study was my own *virtual world* of the interdependent processes of learning and teaching in which I had been involved over years of teaching experience. The pedagogical edifice was a construction of personal theories and methods laid stratum by stratum and interlaced with mosaic pieces of individual encounters in face-to-face classrooms and private one-on-one tutorials. This study was intended to be an analysis as to whether those same theories and methods could be implemented in an educational *virtual world* of teaching mathematics online – and to what success. It must be said that this entire philosophy had been molded, jiggered and jolleyed (to use a potter's terms) by my personal view of mathematics and the various ways I had tried to foster mathematical thinking in a classroom setting.

A Rocky Start

The project had a rocky start. Having to obtain ethics approval from two institutions took time – time I didn't really have. I couldn't post an invitation to participate in the project until I had received final approval from both. Therefore, it wasn't until December 23, 2009 when my invitation (see Appendix F) was posted on the University MATH SITE and its attendant Remedial Math Centre site. The project was to begin less than two weeks later, on January 5, 2010! To be a participant in this research project, each interested party was expected to:

1. sign and return two consent forms, one for each of the universities I was representing;
2. choose a non-identifying pseudonym by which they would be identified throughout the entire project;

3. obtain a non-identifying email address through which all correspondence relating to the project would be communicated;
4. fill out a Pre-Project Survey which garnered some, what I thought was, relevant information about their past experiences and attitudes towards the study of mathematics;
5. be willing to provide feedback on their experiences learning mathematics online in this setting, either in survey form or through conversation; and
6. dedicate a minimum of several hours a week for one semester (January 5 to mid-April 2010) to engage with the RMC web site, to try out the self-assessment facility and to attend at least some of the weekly synchronous meetings using the web-conferencing tool.

Because of the very short time frame (over the Christmas Holiday period no less), the limited exposure of its posting, and the significant commitment involved, I had visions of not having enough participants. I had ideally wanted about fifteen – though that, according to my thesis supervisor at the time, was the absolute upper limit for good qualitative research of this nature. He had thought that a group of eight or ten participants was more realistic in terms of sample size. However, by January 8, there were twenty-one interested persons. I sent out a ‘Welcome to the Project’ letter to each. At the time, I was so busy trying to make the transition to the online project as seamless as possible for everyone concerned, including myself, that I didn’t pause to wonder how many of those first eager twenty-one would finish it. On January 6, 2010, I had the project invitation posting removed from the MATH SITE.

When the first of the live tutorials began on January 14, 2010, sixteen people attended the orientation. They were (in pseudonym):

> Rocky (I did mention we had a Rocky start); Chloe; Saldare (Sal); Daisy; Anton; Harvey; Bill; Janette; Lillie; Ursula; Anna; Phoebe; Abigail; Carl; Georgiana; and Ruth.

Marie, the seventeenth participant, had said she would not be attending the first tutorial. Clarissa, the eighteenth member of our group, had indicated that she would not be able to attend most of the live tutorials but she would follow along with the web site and the recordings of the live sessions as they were made available. Meg A and

Merrieanne did not show at all. Kevin decided to watch the archived recordings of each as they were made available.

I lost two of the attendees immediately afterward. Anton wrote to say that he felt overwhelmed and that he didn't have enough time to devote to it. I encouraged him to stay but to no avail. Daisy simply disappeared, despite an 'I am missing your presence' email to her in early February. It is so easy to fade away in virtual worlds (to be nobody in a no-body world) and equally difficult to make one's presence felt in such ethereal circumstances (to be somebody in a no-body world). To obtain two sets of consent forms was a bit of a hassle but not prohibitively so. Three of the eighteen had to be excluded from the group of participants because I did not receive both consent forms. Essentially, therefore, the project began with fifteen active participants.

My Promise

During the first live session, I promised to take them as far as they wanted to go within the curriculum topics of the project.

The Orientation: for both Participant and Presenter

Invitations to the live tutorials were sent out by email to the participants in advance. These RSVP's, so to speak, contained the title of the topic, the URL address of the virtual 'room' in which it would take place and the time it was to begin. When I arrived for the first tutorial, there were some fourteen persons already there; two others arrived a little late. I learned later that one person in the group had had connection problems and had been unable to 'find' the room or had been barred from entering. It turned out that the issue had to do with her computer. She then had taken the computer to have the problem analysed and it was sorted out. She was in attendance at the tutorial the following week.

It was as much an orientation for me as for them. The web conferencing software was new to me as well. I had only had a couple of test sessions with a colleague so as to familiarize myself with the interface. Consequently, I decided to keep the interaction within this virtual world as simple as possible. It was rather like moving into a new home. It would take time to adjust and feel comfortable in this new space. Though there were whiteboard text and freehand drawing facilities available, including a small palette of mathematical symbol inserts, I chose to present the mathematics by sharing my desktop with all participants and using an application called an *Equation*

Editor through Microsoft Word documents. That way the mathematics would be properly annotated and the language clearly and precisely 'written.' The use of an *Equation Editor* was also new to me so I kept even the presentation of the mathematics as simplistic as possible. Teaching mathematics online was a huge experiment on my part. In some ways I was as naïve as some of the participants in terms of the use of this web conferencing software. I too would be testing the ease with which it could be done and if it could be done in an effective manner. By presenting all the mathematics dynamically as I walked and talked them through it, I had hoped it would seem more like a face-to-face classroom, albeit without faces, in which the instructor writes on the whiteboard or Smartboard and is simultaneously in conversation with the students as required. The pace would be moderated by the speed with which I was able to 'write' using the software. I was probably a little slow at first but, over time, I actually became quite proficient with it. No one complained about the presentation being too slow – at first.

Several pedagogical issues surfaced almost immediately.

Issues of Diversity

After that first live online tutorial, it was evident that this group had a lot of diversity in terms of mathematical backgrounds and ultimate goals. Two participants, in particular, stood out – Saldare and Phoebe. Both had strong mathematical backgrounds. Phoebe was a high school teacher and simply wanted some extra help on the more advanced topics in the senior high school curriculum. Saldare, we called him Sal, was an independent business man who was a welder by trade. He simply wanted a math review of almost the same topics as Phoebe. He had already completed Math 30 just six years before and had done well in it. On the other hand, there were those in the group who wanted to start with a review of basic arithmetic.

So you see my dilemma. If I kept them in one group, I would probably waste the time of some and go too fast for others. I decided to split them into three groups: one (simply called Group 1) for those who wanted to start with the basics (up to and including the Grade 10 curriculum); a second (Group 2) for those whose goal was to take an introductory statistics course (up to and including Grade 11, Pure Math 20); the third (Group 3) for the very few who wanted to solidify some of the topics at the senior high school level (up to and including Grade 12, Pure Math 30). Instantly, my project had expanded into three web conferencing tutorials per week, more than I had anticipated. However, I had promised at the outset that I would provide them with a relatively solid base for entrance into university-level mathematics courses. I did ultimately fulfill that promise but it took six months, instead of the anticipated three. Sal stayed to the end of the six months. Phoebe, however, did not attend any of the live tutorials after the first one because the administration of her school had transferred her into the middle school program and she was having difficulty keeping up with the project (REF: February 3, 2010 email). I did not hear from her again.

Issues of Time

Scheduling live tutorials for a group located across three or four time zones (from the Maritimes to Alberta) can be almost impossible. Some of the participants were shift workers and simply couldn't attend all of the live sessions because of changing work commitments. One worked in a camp in Fort McMurray and didn't get back home until almost 7:00 pm my local time; he preferred the tutorial to be between 7:00 pm and 8:00 pm MST. Seven o'clock in Edmonton was ten o'clock in Halifax. To begin a tutorial at ten o'clock in the evening seemed rather late, for my liking and the attendees who lived in Nova Scotia.

The solution turned out to be one mid-day tutorial and two early evening tutorials spread over three days of the week. All participants were very cooperative, not complaining if they started a tad too early for them or, in most cases, lasted rather late in the evening.

Issues of Technology

Surprisingly, there turned out to be not many technological glitches at all. I certainly expected more than actually materialized. The fact that the Firefox browser did not support the live web conferencing software and that everyone had to use Internet Explorer did not seem to present a problem either. As to the conferencing software itself, remember I had tried to keep its use very basic and that proved to be an astute decision. The Chat Box was used by all without any difficulty. At first, the only microphone in use was my own. Gradually I allowed up to four microphones for those wishing to use them. Most of the participants preferred to use the Chat Box – something I found surprising, especially in a math environment where its symbolism was quite challenging to put into textual keyboard-dependent 'writing.'

I had anticipated sound problems – background noise, not being able to be heard, garbled conversation, the sound kicking out altogether at various times, echo and the like. This had been born of my previous experiences while attending web conferencing meetings or lectures with various brands of software. If any technological functionality were going to create a problem for the session, it had always been sound. For the project, therefore, I had purchased a very good set of headphones and that proved to be a wise move. All participants could hear me clearly with no background noise unless other microphones had been left on simultaneously, thus creating a reverberation echo in the system. However, that was easily solved by requesting that each participant turn the microphone off after speaking.

Throughout the entire project, the only real problem of a technological nature was that, from time to time, a participant would suddenly disappear. Remember that this was a virtual world. The disappearance took the form of a person's name being erased, without warning, from the roster of attendees. Suddenly, he or she would be gone. Most often, the person would reappear a few minutes later by refreshing the browser and logging in again. However, there were occasions when even that did not happen. The attendee simply vanished without a trace! Those persons who were able to re-enter the room again generally could not explain why they had been 'kicked out.' Of course, neither could I. Those who did not reappear again had said that the system refused to let them in a second time. This phenomenon did not happen often but it did reoccur from time to time. I did not find any rational explanation for this strange, selective behavior of the virtual world until much later when I was told that the University had purchased only a fixed number of the web-conferencing session places, across all of the virtual rooms, simultaneously. When the collective roster of all the synchronous sessions taking place at any given time was over the specified limit, the next person trying to join one of those meetings or the last person having joined was, without further ado, unceremoniously, quickly, and silently removed from the scene. In all of the one hundred or more hours we spent meeting online, there was just one instance when the software crashed and we all had to clamber back in a few minutes later. That occurred months into the project and, by that time, we collectively had attained a comfort level with this virtual world so as not to be unduly phased by such events. We really had metamorphosed this alien virtual space into a Burbulean place of relative comfort.

Issues of Learning Style and Comfort

Despite the fact that the project had been designated as a research project about the online learning of mathematics, two participants asked for the name of a paper-based text to accompany the online components. That had been their preferred venue for learning mathematics. In one case, the concern had had to do with eyesight. She was a government worker and spent all day in front of a computer screen. Her eyes were fatigued when she went home in the evening and she wanted a book from which to study the mathematics presented in the tutorials. For both of the participants in question, the lattice-style RMC web site, even though the topic list was in a linear order, did not seem to be a sufficient guide for their learning.

I was not able to accommodate their requests for paper-based materials because the project was about online learning and I had not prepared for a hard-copy text to be used. However, I did note their mutual concern about completely online course materials for learning.

Issues of a Haunting Past

Two of the group participants had told me, in confidence, that a teacher in school had discouraged them from continuing any further studies in mathematics because of lack of ability. Both of these participants were now adult women, one still single with some rather serious health concerns; the other married with two children, the younger of which having just been diagnosed as autistic. These school-age experiences still haunted them. Two weeks into the project, both admitted to me that they were overwhelmed by the material and profoundly discouraged, despite the fact that I had started with a review of basic arithmetic. Interestingly, despite these early debilitating experiences, both set aside their mounting concerns and delved into the mathematical venues again with a great passion, almost a desperation, to succeed and to conquer these deep-seated fears of all things mathematical. Both were good students. Each worked very hard and always came to every tutorial if it were at all possible, and sometimes at personal sacrifice.

A Chronicling of Events

In addition to Phoebe, three more participants had essentially left the project by early February, 2010. Lillie and Anna, after brief engagements with the first assignments, left without any kind of explanation, despite my efforts to touch base with them on an individual basis. Kevin wrote to say that his wife had just given birth to their second child and his house was a bit of a ‘Zoo’ at the time.

The remaining eleven participants included nurses, librarians, government workers, business men, artists, at-home mothers, accountants, and personnel in educational institutions. One of the participants had a physical disability which affected the mobility of the hands but, interestingly, never used a microphone during the live tutorials – only the keyboard Chat facility. Another participant had had a problem with stuttering but had conquered it through self-help methods. Also of note is that this latter individual made use of the microphone primarily, but not exclusively, during the web conferencing sessions. A third member of the group had ongoing health problems which could be debilitating at times, thereby preventing them from attending some of the live sessions; a fourth was looking after an elderly parent (who died before the project ended) and two teenage children, all the while working full time; a fifth was caring for six children and was also engaged in full time employment taking on several work contracts at a time. Not one of them made mention of these life situations publically. I only found out about these issues privately by telephone or by email throughout the course of the project. (Note that, in order to protect the identity of these various individuals, no gender pronouns were indicated.)

The Personalities and Their Stories

The remaining eleven participants were subdivided into streams. I list them by group participation.

GROUP 3 consisted of those who wanted to achieve a relatively sophisticated understanding of most topics within the senior high school level pure mathematics curriculum. It was the ‘Group of Three.’ There were three participants in this group until the project formally ended in late-April. Two remained for the ensuing ten live sessions in the post-project period from May through to the end of July.

- **Ursula**: “I have struggled with math all my life. We have a strong **dislike** for each other” (REF: PPS).

Ursula was there to fulfill a dream – to prepare herself to take and be successful in passing an introductory statistics course at the University. She came into the project at a severe disadvantage academically with haunting ghosts from the past. After a case of chicken pox in Grade 3, she had missed learning about division and felt that she had never truly caught up with the rest of her classmates by the end of the school year. In high school, she had switched from trade school to the academic stream but hadn't learned any kind of algebra or geometry. At the very beginning of the project, she telephoned me to say that she already felt completely overwhelmed and then confided to me that she had always been told all her life by her teachers that she did not have the ability to go to university. Though she had, at the time, a somewhat negative attitude toward the subject and an array of physical, emotional and psychological hurdles to overcome, it turned out that she had an achiever's personality and a positive winning attitude.

On a regular basis throughout the entire project, she engaged with all three forms of online learning which had been provided: the RMC website, the online self-assessment testing facility, and a remarkable attendance record for the web conferencing sessions – thirty-eight out of the possible forty-five sessions over all groups during the main part of the project from January 14 through April 26, 2010.

Ursula was a happy, diffident, and quiet observer during the live sessions. However, she would respond to group-directed questions, always exhibiting a certain kind of engagement with the material at hand. She was the participant whose attendance seemed to be interrupted by events around her. Sometimes her computer crashed and she had had to find a way to scramble back into the session; sometimes she came late having just arrived home from her other night class; sometimes she had had to deal with her little ones getting out of bed and needing her attention; one night her neighbourhood had suffered a brown out; another night there had been a fire next door. Despite everything, she always came, no matter what. I call that *engagement*. I don't believe it was ever easy for her to attend but she was one who let nothing stop her from coming.

Ursula also reached out to other members of the group, always in a playful way. She obviously liked the interaction with those struggling to understand the mathematics much as she was herself. She would openly admit to being lost or to having not understood something at all. This kind of ingenuousness, no doubt, helped put others at ease. In the Group 3 tutorials, in particular, her friendly teasing and banter with some of the other participants was enjoyable and fun to watch – a light relief to the seriousness and unembellished backdrop of mathematical pedagogy.

Over time, her doggedness of attendance showed a remarkable determination and a personality which was quietly strong – having the tensile strength of malleable steel. On February 10, 2010, she had written me saying: “PS I am really struggling with this stuff. I am hoping that one day soon it will all sink in before I have to take the stats class”! She had understood that it took time to digest some mathematical ideas and she was determined to give herself as much exposure to the concepts as possible in order to allow it to, as she put it, ‘sink in.’ This, I believe, proved to be the single most pertinent personal trait which was to determine whether her expectations going into the project would be fulfilled. I always called her a trouper – a designation I give to those who have literally awed me with their sheer determination to surmount and cross a myriad of hurdles between them and the finish line of the ‘race’ ahead.

A year or so after the project had ended, Ursula wrote to me saying that she had since achieved her goal – that of successfully completing an introductory Statistics course at a distance. (REF: March 5, 2012 email)

- **Rocky Math** or simply**, Rocky**, introduced himself to me using a metaphor:

  Math has always been on the dark side of the moon for me. I know it exists; I know people who go there without difficulty; it is even a place I would like to visit myself. But I don't know how to get there. At least not yet. (REF: PPS)

Rocky had joined the project in the hopes that he could gain enough skill and confidence in mathematics so that he would be able to enjoy learning it. He anticipated that this would be somewhat like learning to dance. Once he knew the steps, it would open up a whole new world. He attributed his difficulty with mathematics in school, now many years ago, to ‘poor instruction,’ at least poor instruction for him. Over time, he began to feel that he was not ‘unequal to the task of learning math’ but that the instruction had to be tailored to him.

Rocky was, in my opinion, a steady slogger, actually another trouper in the group, who set his own pace and kept going. I privately called him the Rocky of Gibraltar. “You can always count on me” (REF: June 21, 2010 tutorial), he would say. He proved to be focused on his goal, diligent in keeping to the task, and exacting in detail. In the early days of the project, he was the quintessential observer. He ALWAYS did the homework and was ALWAYS engaged with the subject matter. Though neither ostensibly introverted nor extroverted and not particularly community-oriented for learning ('Having other students in the class was not helpful to me' (REF: LMS)), towards the end of project, he did admit in one tutorial, when he and I were the only ones there, that he did miss the entertainment which the others in the group had provided. Sometimes, he even took part in the repartee which sometimes erupted in the sessions when fatigue had set in to the group.

Rocky attended all twenty-five (fifteen project and ten post-project) of the Group 3 tutorials from its inception to its conclusion. It was he who decided when the post-project period would end in late July 2010. By that time, he had moved the indicator of teacher/student control continuum to the diametrically opposite side of student/teacher direction and pace. By then, he was also quite capable of second guessing me and being correct in his conclusions even when I was teaching a new topic. He had become a true mathematical thinker of some sophistication – one of my goals for the project. (See Chapter 6, subsections "Analysis of the FORGING of a Virtual Place" and "The Teaching/Learning Dynamic: a System of Differential Equations.")

- **Sal**: "Without sound my brain wants to wander off" (REF: PPS).

Sal wanted to come away from the project with a good mathematical foundation so that he could feel confident in pursuing university-level mathematics courses. In my view, Sal was like a mathematical air force pilot, both in the sense that he was the highest flyer in the group of participants and that he was, metaphorically speaking, a gust of fresh air which swept into each session he attended – a wind current always pushing against the slower moving craft in a fluid pool of communal knowledge. He had already attained an applied knowledge of high school mathematics which was nothing short of impressive. When asked in a survey if he would classify himself as a math person, he responded in the affirmative and added:

> I use geometry to figure out the height of buildings and sine and cosine to figure out angles to help me get distances. I work on vessels and tanks a lot and a knowledge of arcs and circles often comes in handy. (REF: PPS)

An ebullient, open and friendly person, Sal loved to engage in banter with both me and other participants. He told jokes, teased others in the live sessions, and offered answers and solutions all at the same time. (See Chapter 6, subsections "Analysis of the FORGING of a Virtual Place" and "The Teaching/Learning Dynamic: a System of Differential Equations.")

As early as the end of January, he had created a web site for the rest of the participants out of the sheer love of learning mathematics and of wanting others to engage in his enthusiasm for the subject. He offered to help those who required extra instruction by forming a study buddy group for the project participants. From time to time, he made suggestions of ways to organize mathematical material which could aid others in a process of deeper learning of the concepts. His ingenuous and ingenious contributions were variously viewed by other members of the group: some with admiration for his expertise, some with suspicion about his motives (altruistic or self-aggrandizing), and some with rejection of his offers. (See Chapter 6, subsections "The Virtual Schoolyard Skirmish – No Witnesses" and "The BREAKING of Trust.")

Up until the end of March, Sal attended both Group 1 and Group 3 tutorials. After that, he attended only the Group 3 and that sporadically due to very heavy work commitments. At the beginning of July, he had enrolled in an introductory Calculus program. Had I just moved the group a little faster along, perhaps he would have remained more engaged in those final days. I often wonder if I had had done that, would I then have left the other participant floundering? As an instructor, I have always endeavoured to keep all participants with me as much as possible even at the expense of slowing another's progress – a kind of 'No one left behind' policy, which is probably not realistic in practice. Every teacher faces those decisions. Every dynamical system is unique. Every outcome is moot. (See Chapter 6, subsection "*The Teaching/Learning Dynamic: a System of Differential Equations*.")

GROUP 2 consisted of those participants who wanted to obtain a solid mathematical base for the purpose of taking an introductory statistics course. I called them my 'Stats Ladies.'

The formation of this group of participants was an unexpected turn of events in this project. I had never intended to be teaching statistical methods to any of the participants – only the foundational pure mathematics which is covered in the high school curriculum. Therefore, these live tutorials were deliberately oriented more towards providing an overview of some elementary descriptive statistics and an introduction to the theory of probability than shoring up the algebra and analysis skills which make this applied mathematics subject coherent and possible. This was an on-the-fly solution which I had felt was necessary in order to prepare this group for the mathematics they would need. As such, the web site was not a suitable vehicle of learning for the most part because its content centred primarily around topics in pure mathematics. In addition, while I did many fully worked-out examples during the sessions and gave a few questions for homework, the online self-assessment exercises did not test any statistical concepts. These 'deficiencies' were noted by some in the surveys. Nonetheless, as an instructor trying to meet the needs of a very diverse group, I felt that it was the best solution possible under the circumstances. The 'Stats Ladies' included the following participants:

- **Ursula**, remember, came to tutorials in all three (3) groups.
- **Marie:** "I am a 'math person' waiting to be discovered" (REF: PPS).

Marie was also one of the quieter observers in the group. She was a regular name on the roster but said little during the live sessions (nineteen of them). She only emailed me once or twice asking mostly about dates and times. However, from the written opinion surveys, one can glean a little of the person behind the name. I sensed that she was a practical, logical, disciplined, and self-motivated person. Her reason for being a participant in the project was to review mathematical concepts so that she would feel comfortable in a Masters of Business Administration (MBA) program.

Although she side-stepped all questions on the surveys about personal feelings in mathematical venues, both past and present, Marie did mention her age as being a factor in her learning. She attributed her age, not the subject matter, nor the virtual environment, as being a reason for her not being able to assimilate information as quickly as before (REF: February 2, 2010 email). Her personal philosophy that "anyone can learn math provided they are willing to practice" (REF: PPS) seemed to be indicative of her determination to succeed despite any past experiences or obstacles.

Moreover, Marie displayed a no-nonsense, kind of 'face-the-music' attitude towards her learning online.

> The key is having a goal, and a schedule – you simply adapt to the environment. I am also taking a traditional university course. (REF: LMS)

Her statements above, from my reading of them anyways, seemed to exhibit a subtle underlying stratum of steely grittiness not to let emotions get in the way of the end game. I interpreted "–you simply adapt to the environment" as an act of discipline on her part.

Over time, she did interact more within the group dynamic but she self-admittedly relied on others in the group to ask the appropriate questions: "I was less likely to ask a question – however there were more brave participants – and my questions were asked, and answered"(REF: LMS). Marie stayed until mid-April (the formal end of the project), at which time she took a short holiday break before embarking on her MBA program. Just recently I was informed that she is just about to finish her MBA program.

- **Janette:** "I still love math but feel it is beyond me" (REF: PPS).

Even though she loved mathematics, Janette did not classify herself as a math person because of her girlhood experiences in school. When she was in grades 9 and 10, she said she was a whiz at math. After entering grade 11, her bubble burst. As soon as the mathematics became more complex, she nearly failed. She attributed part of the traumatic experience to her inability to understand her teacher's English due to his being a new immigrant to Canada at the time and having a thick accent. She, however, did continue on in the applied mathematics program in grade 12 and tried, from that time onward, to stay away from all things mathematical. (REF: PPS)

Her goals for the project were two-fold: firstly, to conquer her fears about math; secondly, to be able to read statistically-based research projects with intelligence (REF: PPS). She attended the live tutorials regularly, some twenty-one of them. She was a good participant, asked questions and did homework. In the March 31, 2010 live session, she said she had experienced a 'light bulb' moment in her understanding of relative frequency empirical probability and that it was a lesson worth waiting for. However, it was also during that tutorial that she mentioned that she had sort of ground to a halt in her project studies because she could not keep up with everything going on in her life at the time. She explained that she had had a family to look after and that she had also formally enrolled in a statistics course at the beginning of February. Unfortunately, that was the last live session she attended.

After some enquiry, I discovered that she had had some urgent family matters which had prevented her from participating after that. She was never able to return even though, in late April, she wrote to say that she would try and catch up. I heard from her again in mid-November of that year. She had dropped out of her statistics course because she had become a full-time caregiver to a family member but she wanted to continue with a review using the RMC web site and the archived recordings of the live tutorials.

- **Abigail:** "It's been 29 years since I did Math. Loved Math and still do" (REF: PPS).

Abigail's motivation to become a participant in the project was to become better prepared for her statistics course. She explained:

> I am thinking this will be quite a challenge and hope I will be able to grasp it. I feel I need more guidance as well as independence as I feel kind of lost starting this. (REF: PPS)

Abigail attended about ten (10) of the live tutorials. While she didn't ask many questions of her own, she did respond to general queries put forward by me and others to the entire group. Just a few weeks into the project, she mentioned that, on her first visit to the RMC web site, she had really liked the site and found it easy to navigate (REF: FIS). However, she hadn't looked at the topics list. Now she wished she had and had done the tutorials as a preparation for the live sessions because she found herself behind from the very beginning. It was interesting to note that, from such an intrepid beginning, this lady had a spectacular finish. At the end of project, she enrolled in an introductory statistics program and achieved a grade of A.

- **Ruth:**

> I never had the opportunity to develop the basic skills needed for more advanced math. To compensate for my lack of knowledge in mathematics I have learned to try and use logic where possible instead of rules that are foreign to me. (REF: PPS)

Ruth was an extremely busy woman with the simultaneous responsibilities of caring for a large family (six children), juggling numerous work contracts and participating in online educational studies. She was the one, who during the first tutorial when a group of participants wanted to stay past 8:00 pm Edmonton time, excused herself by saying: "I'm at 10 I have to get up at 5 tomorrow". She would also email me at 5:30 am (her local time) to ask questions and discuss mathematical issues. She was a meticulous, assiduous and responsive learner. Her goal in being part of this "experiment," as she called it, was to get a very good base in mathematics so that she could continue her studies in higher level mathematics (REF: PPS).

When coming into the project, she had indicated that she had never been exposed to the subject of algebra throughout her entire school experience. In fact, she admitted that she had not done much in mathematics at all in her school days because: "The (work) material we covered was not well taught with ancient books" (REF: LMS). Nonetheless, just before dawn one February morning, she wrote to me saying:

> Finally, I wanted to share a little story I had last week concerning the use of algebra at work. I was trying to figure out which payroll deductions were remitted by the employer and which by the employee since the person before me did not leave proper records. All I knew was that a total of $42.40 was remitted to Revenue Quebec for QPIP (Quebec Parental Insurance Plan). I also knew that the employer's portion was 1.4 more than the employees. I was able to come up with the following formula 42.40 = X + X*1.4. Since I did not know how this can be solved, I put it into an algebraic calculator that I found on-line and it worked. X =17.67. (REF: Feb. 8, 2010 email)

Even at that early stage in the project, JP 6151 had already assimilated some of the skills in algebra which she had just been taught. This, I thought, was quite remarkable. She was already beginning to think mathematically.

Ruth continued to attend the live tutorials regularly until they finished in late April 2010. She was always an active participant, asking questions and answering others' questions. Her final words to me were in a note addressed to me at the end of a survey:

> By being part of this project I discovered how much I really enjoy learning math. I will use my newly acquired knowledge as a stepping stone to reach even higher. (REF: LMS)

- **Georgiana:** “I feel my son in grade 7 knows more about math than I do” (REF: PPS).

Perhaps that had indeed been true when she embarked on this project. However, by its end, she had gleaned enough understanding and knowledge of the mathematical background necessary to do very well, actually – exceptionally well – in the Introductory Statistics course she took just after the project ended.

Her motivation for signing up for the project had had primarily to do with the fact that she had done poorly on a mathematics diagnostic readiness test for an introductory course in statistics. Her goal was to become more comfortable with mathematics after some twenty-three (23) years of not having to use many math-related concepts. She described herself as an active, visual learner – a “picture girl” she called herself – who found learning easier if she could picture the concepts (REF: PPS). For example, when she read instructions on how to put things together, it was the picture directions to which she migrated.

She also classified herself as one who could independently process the information as directed and subsequently complete the assignments as required with little personal contact with a tutor (REF: LMS). That was perhaps why Georgiana was not one to contact me by email very much and was simply a quiet listener during most of the live tutorials which she attended regularly for the duration of the project.

- **Clarissa** was my mysterious participant. In the surveys, she revealed nothing about her personal life.

On the very day the invitation was posted to the Internet, Clarissa emailed me saying that she was interested in being a volunteer for the research project. She wanted to refresh her skills to prepare for the math course that she needed to take for her program.

She was a quintessential distance learner. In January, she had written to say that she had had a scheduling conflict with at least one of the tutorials and would have to come late to a second one. She did attend six (6) of the Group 2 tutorials but worked mostly on her own with the archived recordings. By early May, her last request was for me to send her the URL’s for the recorded sessions she had missed during the previous couple of weeks.

GROUP 1 contained all of the participants who wanted to begin at the beginning and start from the rules of arithmetic. Many of the participants attended some of the Group 1 tutorials in order to orient themselves. Over time, a growing number migrated to the Group 2 or the Group 3 series.

Ursula attended everything and so was a regular Group 1 participant. Ruth, Janette, Marie, Sal, and Rocky were among those who eventually found their places elsewhere. Those who remained only Group 1 participants were Carl, Bill, and Chloe. There were others who attended only once or twice but, for various reasons, dropped out of the project almost as soon as they had entered it.

- **Carl:**

  I was an arts major in university and am currently a visual artist. Apparently, spatial awareness (eg composition) and math skills resides in a similar part of the brain; I'm not afraid of math, so much as totally ignorant and curious. (REF: PPS)

Carl came to the project with a high school background in which students had been divided into streams. The teachers, apparently, had decided who were the math kids and who were the liberal arts kids. He described this as "the most discouraging experience – I was a stronger reader than math person at the time, and ended up not taking any more than the bare minimum" (REF: PPS). He said he literally did not have a single memory of a high school math class. His goal in participating in the project was to assess his own math skills with the ultimate aim of being confident enough to take a university-level math or science course.

He had attended six (6) of the live tutorials during which he had been an enthusiastic participant who had asked a lot of questions. In early February, he began to miss tutorials, emailing me before or after each, citing heavy work commitments as the reason for his continued absence. In fact, he was getting ready for an art exhibition of his work in March. He wondered whether his sporadic attendance would work for him. I wrote him and said it could if he had the time and the commitment to watch the recordings, do some homework, and email me with his problems. In fact, that was too optimistic. He faded away from the project but on the 4th of March, he invited me to the art show in which his work was to be featured. I wrote to him thanking him for the invitation with some comments about links between art and mathematics.

Unfortunately, I was not able to travel to New York to attend. That was the last I heard from Carl. His parting words to me were:

> And I agree - art and math/science are cousins; angry cousins, but related no less. I in fact use a lot of left-brain stuff in my work. If you check out my website, I have a body of work that looked at astronomical and geophysical themes, with some use of science. Too bad I didn't like math then! (REF: March 4, 2010 email)

- **Bill:** "While in post secondary education, I did not find math difficult but I had really little interest in the subject" (REF: PPS).

Bill was, from my perspective, the only surfer in the group. His primary goal had been to revisit math concepts that built upon and supported daily business activities, such as accounting, finance, economics and marketing. Perhaps his earlier disinterest in mathematics from high school days still influenced his approach to the learning of the subject.

> I recall those high school math topics that I realized I would never use in the real world and thus did not give much effort to these topics. . . . Just a lack of interest in math. (REF: PPS)

It hadn't helped that, from the very beginning of the project, the allotted times of the tutorials were not suitable for his schedule (too late in the day). Consequently, he sporadically attended only five (5) live tutorials and worked mostly on his own with the topic tutorial notes I sent to everyone along with the URLs of the archived recorded sessions. After a change of work schedule in the early part of March, I did not hear much from him again. Because Group 2, in essence the Applied Math group, had been geared towards statistics and not business mathematics, I deduced that his ultimate disengagement came from not being given what he had come to find.

- **Chloe:** "I am no longer terrified of math but I am very weak. I just hope that I can keep up and contribute to the project" (REF: PPS).

Chloe had come to gain a foundation in basic mathematical skills up to and including a pure mathematics grade 11 background level. She began the project as an enthusiastic, engaged and dedicated learner. She had a keen analytic mind and appeared to be a reasonably objective and intuitive observer of her surroundings and her relationship to them. She made use of all the three forms of e-learning provided by the project and she attended the live tutorials until mid-February. Then, from my perspective at the time, something catastrophic happened in her life. Her engagement and overt enthusiasm deteriorated into a kind of panic or, at best, profound discouragement. After that, she threatened to quit many times (as early as one month into the project) and eventually did disengage completely from the learning of the subject matter. However, she did not formally withdraw from the project, no doubt because of my sincere request for her feedback on her experience learning mathematics online and her equally sincere desire to contribute to the project. Her story is a tragedy exemplifying why many have so much trauma and terror associated with this beautiful, enigmatic and profoundly challenging subject. (See the Chapter 6, the subsection entitled “Anatomy of the BREAKING of a Virtual Engagement.”)

THE CASE STUDY - PART II

This section of the case study contains an accounting of and a hermeneutic accounting for the events which happened over the five and one-half months we were together. The lenses of the analysis will focus on two issues:

1. The PERSONS: the engagement or disengagement of individual participants with the mathematics toward deeper learning in the context of Burbules’s (2006, p. 40) notion of ‘immersion;’
2. The PLACE: evidences, in this virtual space of learning, of the three essential presences as put forward by Garrison et al. (2000) in their Community of Inquiry model. These presences included: the cognitive, the teaching and the social interactions which became either a positive force for learning or a deterrent.

All of this would, of course, be viewed with an eye towards the research issues of the lived experiences, both cognitive and affective, of such a group of adult participants within such an online setting, its sub-issues and its questions. The narratives of all eleven participants were taken into account in the thematic analyses which follow. However, the experiences of three of those individuals have been related in more detail because of their more intense involvement within the project itself, with the participants and with the provided layers of media learning.

Engagement or Disengagement: What made the difference?

It should be remembered that this project was entirely voluntary. The only thing keeping any of the participants there was either the perception that they, in fact, were progressing towards their goals mathematically or sheer interest. There were no obligations to attend – no fees to lose – no exams to pass – no formal credit or recognition of achievement even if they did very well. So why did they stay when they did and what made them leave when they didn't?

Early Indicators and Omens of the Future

In each of the following episodes of interaction either during the live sessions or by email correspondences, though I didn't really think of it at the time, there were portents of how certain participants would involve themselves in the project and how others would eventually fade away.

January 21, 2010: After 2 hours and 20 minutes into the group session, some participants were still interested in staying longer. Sal wanted 'to stay and play.' In fact, Sal and Janette did stay and play. Rocky just asked: 'Do we have homework?' That became his hallmark from beginning to end. At the end of absolutely every session, without fail, he asked about the homework.

January 22, 2010: A SURPRISE – an offer to help comes from a fellow participant! Sal had created a Google-docs web site, both personalized and math-specific, in which all the project participants could engage, *if so desired*. He had told me about it and I offered to back the initiative. To that end, I sent out an invitation to all participants about the site highlighting its offerings of an equation editor, extra help and discussion on the project topics. This was followed up by an email from Sal himself.

I was shocked and delighted. I had not expected this. Wow! ….. student engagement at its finest. Moreover, Sal's mathematical background was strong enough to fill the role of a study buddy/guide to some of the weaker students. In addition, he invited them to send pictures and personal biographies for inclusion on the site. This online invitation to form a virtual community of inquiry, outside of the project venues, for those in the project was something I simply had not anticipated. However, again to my surprise, reaction to this ingenuous offer was checkered. Some were enthusiastically positive; others rebuffed the initiative as either intrusive on their time or unsatisfactory to their needs.

January 26, 2010: Before the tutorial officially began, I had asked the early arrivers as to whether they had gone through the RMC web site tutorials. Chloe said she had but then added: "read does not equal understand." I tried to encourage her by saying that it takes time to digest certain mathematical ideas. (Mathematical thinking required ‘incubation’ time, as Jacques Hadamard (1954) had pointed out.) When I further asked if anyone had done the homework, some had and some hadn’t but Chloe, who hadn’t, said: “I would get zero out of it regardless.”

Nonetheless, all participants engaged in a good discussion of the topic covered. Three of them – Carl, Janette and Ruth - asked a lot of questions seeking answers and solutions.

And then more aid for the struggling was offered……

January 28, 2010: During the very first Group 1 tutorial, there had been a call from some of the attendees for more live tutorials. I had absolutely no more time to devote to such sessions. Sal, who had, throughout the previous tutorials, suggested ‘tricks of the trade’ for learning mathematics and added helpful commentary, offered to provide some online live practice sessions for the group. This was in addition to his web site.

January 29, 2010: Sal wrote that there had been some interest from the group and then asked for advice on which topics to include.

January 29, 2010: The very next morning, two letters arrived on my desktop by email. One asked: “Doesn't having a second " teacher" i.e. Sal undercut your process evaluation/ thesis”? The second remonstrated:

> I don't quite know how to put this, but has Sal high jacked this project?
>
> I have no idea who this person is; I have no idea of his qualifications; I don't know why he is offering to help us in math; and I don't know why he is even a participant because he certainly doesn't seem to need any remediation in math.

This vehement a reaction on the part of two of the participants was definitely unexpected but I pondered carefully over both emotive reactions. Of course, this would have ramifications on the integrity of my research which I, in my enthusiasm, had not considered. I responded to the situation by writing a letter to Sal explaining why such live tutorials could not be held by non-AU personnel. Essentially it was because I was not able to be in attendance as Moderator. I thanked him for all of his work and then encouraged him to continue his collaborations with those who wished to participate through his web site. Sal took it graciously.

February 24, 2010: Another email arrived, this time from Bill.

> I have not extensively engaged in this course to the fullest extent as I have a number of other courses on the go, and hence, prioritization of time and effort leaves a less than full commitment to this course. To offset this limited commitment, I have sought to make it a point to tune into your live sessions. I had to miss two, one because of a final exam that I was preparing for, and the other due to family commitments.

This letter was indicative of a typical scenario as to why certain participants did not or could not fully engage with the project in terms of attendance and mathematical involvement. More often than not, it was a case of a different set of priorities or a plethora of more pressing commitments. However, there was one participant for whom these were **not** cited as the reasons for eventually withdrawing – with regret. This is her story.

Anatomy of the BREAKING of a Virtual Engagement

Chloe was the one real tragedy in this tale of the virtual. In what follows TA, short for teaching assistant, is the terminology she used for the self-assessment tool.

Her past history – in her own words:

> I will begin by saying that I have never been part of what I would consider a successful math classroom. I am defining the word "classroom" to mean a group of students with similar backgrounds and goals. (REF: LMS)
>
> Prior to 2005, I was terrified of mathematics. My background was limited to the use of a calculator and even basic arithmetic was beyond me . . . I was so fortunate to have found a very experienced professor who agreed to doing some private tutoring with me. By tailoring the math to what I needed, I was able to complete the course . . . (REF: PPS)

She felt that exam anxiety was not an issue with her. She had assessed herself, by the Felder-Silverman Learning Styles Model (see Appendix B) as an active**,** sensing**,** verbal and "extremely so" sequential learner (REF: LMS).

> My learning style is such that I must be able to reproduce, and preferably understand, a step-by-step method for solving a problem. If it is not understood, it has not been fully learned and I find it difficult to move on. (REF: PPS)

She admitted to being a print-based learner but, on her first visit to the RMC web site, she said she went straight to the topics page to see what was available, found what she was looking for, settled down to work, and "surfaced again a couple of hours later" (REF: FIS). She said that her learning style had an inherent quality of 'impatience' to it.

That alone should have alerted me to a potentially negative response to this mode of learning – but, at the time, it didn't. First of all, her mathematical background was very weak by comparison to some others in the group. Secondly, she was a book person; online environments were not her preferred venue for learning. Thirdly, she was a self-described impatient learner. The learning of mathematics to a level beyond factum and method, in my experience, required a necessary rumination process of digestion before one could grasp the larger context of meaning. Impatience with that process may lead to frustration and discouragement. Moreover, if one could not jump over that temporary hurdle to move beyond it, disengagement with the task could easily result.

Her Initial expectations of the environment:

> I would like to see focused classes. It would be helpful to know the expected content of the project. . . . It needs to be very clear, at the outset, which parts of the project are too advanced for a Math 100 level student. (REF: PPS)

Her initial expectations of the instructor:

> A critical component of teaching is the ability to build trust. This helps to reduce or eliminate fear. The professor needs to be able to determine where the students are at in the learning process. He or she needs to teach towards the middle but also accommodate those that are either faster or slower than the average. (REF: PPS)

Her Initial engagement with the RMC web site – in her own words:

> virtually every week, often several times . . . prior to when I unofficially withdrew from the project due to incapability. I was initially gearing for 20+ hours per week. (REF: LMS)

Her commentary on the RMC web site itself as a learning tool was generally positive – very comprehensive, topics well sequenced, design easy to follow, and the color coding of the tutorials pleasing visually (REF: LMS). In addition, she appreciated the inclusion of the 'in a nutshell' component which had helped her to emphasize the importance of certain kinds of information. She had liked having the ability to turn off the animations and the option of obtaining a PDF version which allowed the material to be printed. The only negatives were those of a technical nature, some of which, in the interim, have been made possible, corrected, or improved.

Her initial engagement with the self-assessment tool:

Chloe was one of the very few participants who engaged extensively with the assessment tool. This software tool did have some problematic bugs in it which had not been fully eliminated by the time the project had begun. However, other, more patient, participants were able to side-step or ignore these relatively minor issues and not let it affect their learning of the mathematics presented. Some others had simply ignored the self-assessment tool altogether and, in order to check their understanding, relied exclusively on the feedback I gave on the assignments sent to the participating members at the end of every tutorial. Nonetheless I had sensed that the assessment tool issues did play a role in Chloe's ultimately very negative reaction to the project itself. That is why I have included them here.

When working in the assessment tool, she talked about the short time-out period of the program. She had vision issues and often needed to copy a question so that she could enlarge it in Word to increase the font size. She would complain that, by the time she had worked out the question, the tool would have timed out. I am assuming that her printer, at the time, did not have the facility to print an enlarged copy of the exercise. This had been a built-in option open to anyone using that tool.

Finally the grading scheme which I had put in place for the self-assessment exercises had been very problematic for this participant – though none of the others had expressed any issues with it. I had explained to the group during the very first tutorial that the standards were set very high (particularly in the foundational number theory sections) so that they would recognize the importance of practicing these skills until almost perfect. Moreover, I clarified that no-one was bound to achieve the desired score before moving on. It was to be a judgment call for the student. Chloe, very unfortunately, took the commentary, which had only been intended to be a generic set of advisories, to a personal level. Despite my reminders to her about the reasons behind the high standards, she did not take them to heart and, in her analysis of the project venues, wrote:

> Grading in . . . TA, if different from the norm, should somehow be communicated to the student BEFORE registration. . . . The standard grade for

> University math courses appears to be 50%. To receive a grade of 86% in basic arithmetic, in material that was new to me, and be told that something to the effect of "practice until you receive a better score" may be unrealistic. There also may not be the interest to attempt to receive a better score.
>
> The comments in the grading portion of TA border on the inane. For example: your score is 1 out of 10. You might want to do this one again. Really? Is this not self-evident in an adult environment? Maybe another choice would be to withdraw from the project/course. Or, exponents are important. Please, redo this exercise until you are comfortable. (the inference is that the student can become comfortable). (REF: LMS)

I have pondered over this reaction many times and, since then, have eliminated almost all such commentary from the exercises and tests in the self-assessment setting. I realized that this was a tool for self-assessment – not my assessment.

I did, however, make an even more serious mistake in one of the assessment commentaries. I suggested that one of the topics in the foundational number theory section was 'easy' to master. Chloe nailed that as a judgment call on the learning of others and she was right. It was a judgment call. What may be easy to one is not necessarily easy to all. Nonetheless, after having vetted her frustrations with the self-assessment tool experience, she did qualify her reaction to the system itself:

> The concept . . . is great, but practically it is not there yet. These exercises caused considerable frustration. Again, if I had been able to keep up, I might not feel this to the same extent. (REF: LMS)

Her initial engagement with the live session tutorials:

Chloe:

> At the beginning when I was able to keep up and could ask focused questions, I found that the live tutorial helped to get the materials to gel. Feedback was instantaneous". (REF: LMS)

One of her criticisms of the live sessions had had to do with the interaction among participants and even exchanges between me and other attendees.

> I actually find the ability to "chat" with other students to be problematic. I am not in class to "chat" but rather to learn on a topic that has been announced for that evening's class. When I was attending the live tutorials, I did find that the

> classes easily get off topic and in some cases the topic is not addressed at all (i.e. the session was to be on algebra but most of the lesson was spent reviewing previous homework). (REF: LMS)

I admit that, sometimes there had been a little bit of extemporaneous, back-and-forth kibitzing amongst those present – something I enjoyed because I have found math classes to be generally rather serious affairs. So many careers have been dependent on a student's having been successful in some mathematics course or other. Covering the curriculum material to achieve even a basic understanding had always been trammeled by time limitations and, therefore, there never had been much time to chat 'idly.' For the most part, in order to keep all participants with me, at least in some measure, the sessions were begun with reviewing previous homework and ended with a continuation of the next topic *provided that* we got that far along in the discussion. Within the Group 1 setting, some students had needed a lot of review and had asked for extra examples or fully worked out solutions to the homework. I believe this is what she meant by 'the classes easily get off topic.' That was not the first time that my 'no person left behind' policy had not been appreciated by participants in that group.

The pace of the tutorials had also been a huge issue for Chloe:

> The online tutorials were a detriment to the learning process . . . Perhaps if I had the ability to keep up and understand I would feel differently. (REF: LMS)

At first, she would do the homework, submit it, and want it marked privately, rather than seeing the solutions gone through in the tutorials – even though I had been willing to do so, if asked. Because of her reticence to ask for solutions during the live sessions, she would try to format the questions and her solutions in a text or Word document. The homework did not actually require any special formatting program though I did use an Equation Editor to present the mathematics online and in the assignments, for more realistic viewing of the symbolism involved. Chloe, in her desire for perfection and exactness, wanted to do the same. Somehow, whatever she tried in presenting her own solutions to the questions apparently had taken too much time and this, naturally, had increased her frustration.

All of this preamble regarding Chloe's mathematical background, her preferred learning style, and her expectations of learning – coupled with an exacting personality who strived for perfection and excellence in all that she did – may provide some insights into the following series of events which transformed an enthusiastic, engaged learner into a profoundly discouraged drop-out. She did not drop out of the project but she did, consequently, sever her connections to any mathematical study since that time.

Chronology of events leading to the breaking of an engagement:

January 14, 2010 She was a fully engaged learner at the beginning of the project. As was her wont, she always went directly to the matter at hand. She wanted to test herself using the self-assessment tool: “oops. Forgot to mention that I have already been played with TA (and love it with a couple of concerns) . . . “.

January 15, 2010 Already, the day after the first live session, she was handing in the assigned homework and proud of herself for having done so:

> Here is homework number 1 - it took way longer than it should have - probably 2.5 hours - but I also find the formating challenging. What I don't believe is that I sat down quite happily. . . . Definitely back in the saddle and happy.

January 17, 2010, 10:43 am She wrote to express happiness and appreciation for being part of the project.

The emotional roller coaster ride begins:

This is the stage of learning when heightened emotions begin to sabotage one’s clarity of thought.

January 17, 2010, 11:01 am Chloe wrote me after having had problems accessing the assessment tool site because of a password issue:

> This is, unfortunately, where I don't think that technology is a good means for learning. . . . Like most of our group, I only have weekends to do this and if I can't get into it, I also can't keep up.

Subsequently, I looked up her login ID; it was only one digit different from the one she had tried to use. The problem was that, for someone whose patience had already been stretched to its maximum, the simple and common mistake of changing one digit in a password can push one over the edge and set the mind into a turmoil.

January 17, 2010, 12:36 pm I then found out that she was concurrently dealing with serious personal issues.

January 17, 2010, 2:11 pm Less than two hours later, she wrote to say:

> OK, I think that this is unrealistic. I have 85.24% and am told that is not a pass and I need to do more. . . . I think I am out of my ability range. Oh well. Will come back later.

January 17, 2010, 7:47 pm I responded to her afternoon email, reiterating that there were no passes or fails on the self-tests and exercises. There were only recommendations and explanations. I encouraged her to keep going and not be deterred by the artificially high assessment standards. It was best to ignore them.

January 19, 2010 She was back on an even keel, once again focusing on her goal.

> I want to say just how much I appreciate the . . . TA questions on definitions. Very few (like maybe 2 of the 9) were familiar to me. The more that it is apparent that my background is seriously lacking, the more I value getting the help when I needed it. There was nothing then . . . and now I owe it to myself to complete the basics. Chloe

January 20, 2010 I announced that an RMC project discussion forum had been set up as a place for the participants to chat together, exchange ideas, and maybe find a Study Buddy. This had been requested by a number of members of the group. Chloe's initial interest in having this kind of peer contact quickly faded when the registration process with the University's Student Union required more personal information than she was willing to divulge.

The 'more hurry, less speed' phenomenon:

January 22, 2010 In response to Chloe's expressing a lot of frustration with the self-assessment exercises, I asked whether she had watched the Demo about how to use it which was available on the web site and which I recommended all participants to read. She had not; she had gone directly to the exercises.

That very day I sent out an email to all participants that the live tutorials were going to be split into three streams. She immediately responded very positively and expressed an interest in maintaining contact with other participants after the research project was over. From this, though she had classified herself as an independent learner, it was clear to me that she was ready to seek out some social contact with her fellow participants.

> This is excellent. After the pilot is over, . . . we can choose to (or choose not to) know each other and maybe even become study buddies in an informal way.

> Excellent, just excellent. From my point of view (and it appears one shared by others), this resolves most (maybe all) of my concerns with the live tutorials. Bounce ... bounce ... bounce CHLOE

Then came the FATEFUL DAY …….

January 25, 2010 An offer to help came from a fellow participant!

Sal had announced the creation of his web site in which all of the project participants could engage and interact. Now Chloe may be able to find that study buddy or mentoring guide she had initially wanted. However, I left the decision to join this outside venture to each participant without any recommendation.

The beginning of self-doubt:

January 26, 2010 After struggling for days over a concept involving exponents, Chloe wrote:

> Even without the pressure not sure that I can achieve grade 10. . . . I will go back again but I really am frustrated with myself.

That day there had been six back-and-forth exchanges by email with her. I tried to encourage her:

> Chloe, Just take each step one at a time. You will get there but it may take longer than April 2010. You know all about dogged determination. AND you know that it can produce amazing successes. Julie

January 27, 2010 Her spirits rallied again. We were both struggling: she with an ever diminishing level of confidence in her own ability to meet her goals; I with growing concern that discouragement would completely choke out any bit of self-confidence in herself that she still had. She wrote:

> OK, this was a more productive 15 minutes. I can understand the 4 but I have one question. . . . I will be okay and yes dogged persistence pays huge dividends. Guess I need to live up to my reputation.

Discouragement - Public and Private Enemy #1

Chloe, herself, explained the difference between fear and discouragement when it came to the learning of mathematics.

February 14, 2010:

> I have discovered that, too (sic) me, it was far less stressful to be fearful than it is to be discouraged. They are very different emotions. If a person is afraid, they

> are likely not going to expect to understand. What little understanding there is is a plus (regardless how minor). Discouragement draws from the expectation that something is possible/probably when it likely is not. CHLOE

However, once the tiniest seed of self-recrimination is planted in the consciousness, it can grow exponentially. Every frustration in personal performance becomes a reason to disengage from the pain of the experience.

Not being able to keep pace

February 1, 2010 Chloe indicated that she had taken advantage of Sal's web site offer. She found it no longer useful because it did not have all the answers to the examples and some of the examples were not precisely what had been covered in the tutorials. She also talked about not understanding enough to be able to answer my questions which I typically asked during the live sessions. She had always wanted to be ready to participate. The inability to keep pace with the others and with me became a source of continuous and ever increasing doubt as to her abilities to finish the project. She wrote:

> I will NOT be able to answer your questions. . . . This is taking a fairly large toll on me at the moment. I was trying, and no I am not scared ... I am just very discouraged with myself.

Expectations dashed (She called herself a realist)

February 1, 2010 4:12 pm She wrote me saying:

> At the moment, I have zero on my homework and don't know how to even seriously tackle it (I actually thought that most of it was right). My hours were at least double - and one week triple ... to what I expected. For zero percent. And admittedly, if something isn't included in a tutorial, I don't want to try to answer test questions on it. I did improve a bit on . . . TA but not enough to know what I was doing . . .

February 1, 2010 4:37 pm I **dashed to the rescue** by creating a summary document on exponentials (this had been the problem topic for a number of the participants) and writing an open letter to all the participants:

> Hi Everyone, but particularly to the GROUP 1 participants:
>
> Exponentials in one full swoop, as it is given here (not over years of schooling), is not that easy to grasp for the uninitiated. Don't become discouraged if you don't pick it up immediately. I have concluded that that emotion called

> 'Discouragement' is your Public and Private Enemy #1. It keeps you silent in public and it stresses you out in private. Both are detrimental to any learning process. This is my best advice at the moment. Just keep going. Julie

Throwing out a life line ……. Was learning at a distance part of the problem?

February 3, 2010 In an effort to discover the effect of distance in the learning of mathematics, I took the opportunity to ask Chloe, who had been in both face-to-face and distance learning venues for a long time, to explain the difference from her perspective.

> Hi Chloe: Online teaching/learning is not quite as personal – but only because instructor/student are not in the same place at the same time. That lends another dimension to it. Are you able to describe what constitutes that extra dimension in face-to-face classroom situations?

This was her response.

> . . . in one word, emotional, including body language. When I am on very familiar territory, I love distance education because I don't have to deal with the "people" issues. But in a new territory, like math, I still need more support.

Simply put, she was missing that extra individual support – a kind of personal gyroscope which could bring her sense of self-confidence back to a state of stability and equilibrium in order to continue with the task ahead. This was precisely the kind of support I could not give to any one individual in this project setting. Apparently other efforts to provide at least some help from her fellow participants in this virtual setting were not considered adequate. Nonetheless, she rallied again and continued on with the learning of fractional exponents.

Latent feelings resurface: a sense of betrayal suffuses the conscientiousness

February 1, 2010:

> And I guess I will never get over my bitterness toward the Alberta Public School system that thought that keeping a child with their peers was more important than education. Chloe

February 6, 2010:

> As for the "streams" in public school, based on my grade 9 mark, I was put into Math 10. We both know that I could not even do basic arithmetic. From there I

> moved to math 23 "passed" at 45% and that was what was needed to "graduate". Children don't realize what they haven't been taught. Chloe

Beginning to compare one's own progress with that of others: realism or folly?

February 4, 2010 Chloe attended the live tutorial that evening and fully participated by asking a number of questions using the Chat Box. I tried my best to answer her questions clearly and with explanations.

Directly afterward, she wrote to me. On a number of previous occasions, she had made mention of how others in the group seemed not to be understanding or following the discussion at hand. The issue NOW was that these others seemed to be understanding and she was not.

> Tonight is a perfect example of the limits of technology (not the limits of you). I could not explain my question to get an answer that made sense to me. And this time I tried. But I don't think that you understood what I was trying to ask. .And further, it wasted 25 minutes of time.
>
> If I can just sit and listen, not ask or answer questions, I might be able to see the end of it . . . . I'm happy for Carl, who is now understanding it and Janette who feels comfortable. But personally I understood little but will go back. I am extremely discouraged at the moment and in tears as is the usual.

Despite deepening discouragement, she plodded on and continued to hand in assignments. The time-outs in the self-assessment tool, the back and forward arrows on diagnostic tests, and other technical issues produced heightened irritation in her responses. Because none of these had been ever mentioned by any of the others in the project, I could only conclude either that the Internet connections she had had to deal with were inadequate to the task or that the use of the software was problematic for her. I did mark her assignments, made comments on where her thinking had gone wrong and provided accolades when she had solved them correctly. I even offered her a private online tutorial geared to deal with her personal issues of learning in regards to exponentials.

The first threat to withdraw:

February 6, 2010:

> And undoubably I will continue. Rerun the tutorials several times a week, in tears. . . . I don't believe that math can be effective taught to groups online, …. At least not with students that have little, if any, previous skills in the subject. Not sure where (or if) I am heading but I am feeling way out of my depth. . . . I have to seriously consider whether to "unofficially" withdraw. Chloe

February 7, 2010:

> Before I cancel this account, I will give myself a week to determine whether I have the ability to continue or if I am just wasting my and others time. I will not be responding to emails prior to Feb 11. Chloe

The breaking point!

Ghosts of the past had come back to haunt Chloe. Her self-recrimination deepened.

February 8, 2010:

> This [tutorial] finished it for me. I cannot even understand when your notes are in front of me. As family have said to me, I only have a degree because I had extra help. Otherwise I would not be capable of it. And the sad thing is that they are right.
>
> I feel very inadequate. If you wish to send me the tutorial links for group 1 after it is recorded, I might listen but after having listened several times to last week's tutorial, I would not be able to go to the online class and be silent. I will be asked if I understand and I can either say no or lie and say yes. Neither is productive.
>
> And again it makes me "different" from the rest of the class. And I suspect that I would not get enough out of it to be worth your time.

Disengagement! ……. dropping the project pseudonym:

February 9, 2010 11:37 am Chloe wrote to me using her real name: "I find it very difficult to do anything when I am constantly in tears".

February 9, 2010 10:18 pm

> I am afraid that I will be stressed to the extreme for a long time. The less math can do, I do the more inadequate I feel.

The BACKLASH – against herself first and then others:

February 10, 2010 6:35 am:

> I feel that I have let both you and I down. I really felt that my math background was sufficient that I could keep up with others that have similar backgrounds. . . . What is even more frustrating to me is that I can understand some of what is in front of me until I actually go and do it.

February 12, 2010: Chloe wrote me to comment on the camaraderie amongst the others in a tutorial she had not attended but had only watched in its archived version.

> I am extremely glad that I didn't attend last night. This is the first time I have seen this type of behavior and am less than impressed. Oh well. I am referring to the joking around among the participants . . . in a nutshell it probably won't matter to me because I do not foresee going back into the live tutorials but I did find it rather unnecessary.

February 17, 2010: Chloe wrote again:

> People issues, to me, refer to those that don't take education or class seriously. I am not interested in playing games and joking around in class. But that is my personality. And last week, if I had been there, I would have said something and I would likely have got into trouble in the process. It is a disadvantage to me to have various levels of backgrounds - to the extreme - in group 1 online sessions. There is no doubt that we have a person who is way above the level of group 1. And that is a perception . . . for whatever a perception may be worth.

The diversity of mathematical backgrounds in the group sessions was, for her, a real disadvantage to her learning. She was always keenly aware of others in the group and their relative progress. Her own insecurity about her capability to succeed in mathematics caused her to reach out for what she considered to be more reliable comparisons of relative achievement. However, even during a tutorial she hadn't attended but had only watched, her perception that one 'keener' who had also started with a limited background had monopolized my time also irritated her.

March 21, 2010:

> It is also obvious that we have a "keener" with limited background (like myself) who is doing well and is making a lot of requests, both during the live tutorials and to the researcher. Somewhere there needs to be restrictions that prevent the perception of a monopoly. One of the difference between face-to-face and distance education is that there are not the group means to object to this type

> of behavior. Further, in a small adult class, which is being recorded, there are students that will not openly object. (REF: LMS)
>
> While it was possible to ask questions (which are answered without judgment), it became humiliating to ask a question (and still not understand). (REF: LMS)

The aftermath:

Chloe, from February 11, 2010 onward continued to request the links to the archived recorded sessions though she did not attend any of them. She still asked math questions via email. To her great credit, she persisted with an enormous show of courage in her private studies to the end of the project.

February 18, 2010

> Success! . . . I am just finishing afternoon break and between it, my morning break, and noon hour, I have 90% on the Maple TA and here is my homework (which I hope is right). CHLOE

March 10, 2010 Technical issues continued to dog her efforts to send her homework to me.

> OK, if this file attaches, this is as much as I can do before tomorrow (yes, I realize that it is probably not right). I still don't understand the division questions . . . I am afraid that I am losing interest in the project.

Unfortunately, I couldn't open the file of math questions and solutions which had been attached to her email. I wrote her back, explaining the situation. She responded with:

> The only option for me to communicate, short of Canada Post which I am not willing to do, is to use the equation editor. I DO NOT HAVE A SCANNER AND I AM NOT SURE THAT I AM WILLING TO ASK OUR DIRECTOR IF I CAN SCAN FROM THE COPIER HERE. . . . My interest level is declining on a rapid basis.

There was a continuous stream of emails from that time onward and a number of threats to withdraw from the project formally – though she never did actually withdraw. She maintained a running email correspondence with me until the main part of the project ended in mid-April.

WHY? WHAT HAPPENED?

Clues to the reasons for such a disengagement with the content and with the social structure of the group have been taken from her own commentary and are summarized below. She actually never did disengage with me. Even after the end of the project, she wrote me periodically to keep in touch and to explain what had happened in retrospect.

The inability to keep pace with the group:

> Whether the amount of information to be covered in a week is realistic, I cannot say but it was certainly heavier than I could do in a 20+ hour per week commitment. (REF: LMS)
>
> Generally I was happy with the information in the RMC tutorials. My problems were in my lack of ability to learn and move forward in a paced environment. (REF: LMS)
>
> It will always bother me that I couldn't keep up with the project but I am also not surprised. It was more than time for me to decide to be alone rather than either compete with others or see what others can do. (REF: April 22, 2010 email)

Inadequate mathematical foundation for the task:

> I guess I should have been more aware that I needed private math help for one University course and should not have expected to be able to keep up in the project (prior to 2005, I could not even add, multiple, subtract or divide). I simply over estimated my own ability. (REF: LMS)

Feelings of humiliation when not understanding immediately – in front of the group:

> These online tutorials (when I was still attending) actually de-motivated me and created a feeling of humiliation when it was obvious that I was not keeping up. I don't handle public humiliation well. (REF: LMS)

The desire for support from her peers but reluctant to accept it:

> However, in a subject where I am weak or failing, I need more support, preferably from students that have similar backgrounds to myself. (REF: LMS)

Perceptions of 'inadequacy' – to the task:

> I guess, though, that I thought that I was further along than I actually am. (February 16, 2010 email)
>
> And I do still believe that I have a learning disability that makes it more difficult for me. (REF: February 17, 2010 email)

Profound discouragement born of unfulfilled expectations:

> Discouragement draws from the expectation that something is possible/probably (sic) when it likely is not. (REF: February 14, 2010 email)

I responded to her letter by saying that both fear and discouragement, in uncontrolled amounts, could debilitate the best of minds and make it impossible for the person to learn anything at all. I was not at all convinced that she simply couldn't learn this – that is, when her mind was at ease and she took it one step at a time *with patience*. She had mentioned about not being able to see some of the smaller print. That may have been a factor in her increasing frustration as well. (REF: February 15, 2010 email)

Feelings of 'differentness' – from the group:

> I am referring to being asked and not able to correctly answer questions in a live tutorial when there is evidence that the others are not having difficulty. You, of all people, know how much I yearn to be like everyone else. (REF: February 17, 2010 email)

Mental Paralysis – the inability to move past the hurdle:

From late January to the end of April, Chloe requested help on exponentials, particularly with fractional powers. She never seemed to venture beyond this topic, giving it time to gel, and, in the meantime, tackling another. In retrospect, this could have been as a result of her implacable linear sequential learning style. She did say:

> My learning style is such that I must be able to reproduce, and preferably understand, a step-by-step method for solving a problem. If it is not understood, it has not been fully learned and I find it difficult to move on. (REF: PPS)

However, I do feel that another phenomenon had also occurred which almost everyone experiences when faced with profound feelings of helplessness or uncontrolled fear. There is neuroscientific, medically supported, evidence that the human brain shuts down some of its synapses under these circumstances – leaving it in a heightened survival mode but reducing its capacity for cognitive learning (Caine & Caine, n.d.). In short, in this frame of mind, the person unknowingly disables her/himself with a kind of mental paralysis until those intense emotions of fear or helplessness subside.

Effect of distance in learning:

> OK, Julie, this further underlines for me the inadequacy of distance education for mathematics. This is now the third time that I have asked

> and I do not understand. That is fine. I am just too stupid to have followed what was done in class (and I was in the live tutorials at the time). This material on exponents was in the first few weeks (less than a month) of the project start. Yes, a student is expected to put the concepts together. This is great if a student understands. I don't. (REF: April 13, 2010 email)

Chloe, as far as I am aware, did not pursue her dreams of continuing on in a university-level mathematics course. She chose to pursue other interests.

Analysis of the FORGING of a Virtual Place

Fortunately, the collective tale of this online learning adventure has not yet been fully told. The majority of those who engaged with the mathematics and stayed to the end of the project did fulfill their individual goals. What follows are the stories of two participants whose experiences of learning mathematics online, in some sense, were nothing short of phenomenal.

Rocky and Sal were the only participants to continue through the post-project period from late April through to the end of July 2010. The commitment each had exhibited was remarkable. Both had contributed to the learning of the others by offering suggestions of good web sites, techniques for learning mathematics, and ideas during the live sessions. Both had achieved a level of mathematical thinking from which each could pursue the math-related courses of their dreams.

By July 12, 2010, some six months from the start of the project, Sal and Rocky were engaged to the highest level of comfort in the virtual environment and in control of the subject matter. Each had demonstrated a relatively sophisticated understanding of the topics covered. Rocky was ready to learn mathematics on his own using his own methods of doing so (REF: July 12, 2010 tutorial). Sal had been prepared to do that just a short while into the project, perhaps even at its beginning - but work commitments had prevented his doing a lot of the homework to excavate deeper into the theoretical base of the underlying mathematical bedrock. However, Sal was quick to comprehend and had a reasonably solid foundation from the outset. All he needed was guidance and some filling in of the blanks in his understanding. In that respect, both Rocky and Sal were now 'equals' in terms of the comprehension of the topics covered.

Rocky's leap into the abstract thinking of the higher mathematical world was, in my mind at least, quite amazing. On March 23, 2010, Rocky had complained:

> I found the general mathematical statements to be of no use. For example, when you use **n** for any number and **n + 1** to mean any number plus one, I just become more confused. For me it would make more sense to master a particular mathematical process first and then deduce a general mathematical statement. (REF: LMS)

Yet only four months later, during the July 26, 2010 tutorial, that is precisely how he was introduced to logarithms. By then, Rocky was able to accept a generalized statement of a mathematical definition and convert the generality into specifics for purposes of evaluation and discussion of particular examples without any prompting from me, except the occasional *virtual* nod.

The achievements of these two alone convinced me that deep learning of mathematical ideas was indeed possible in an online setting. Those processes of engagement of which Burbules (2006, p. 40) spoke – interest, involvement, imagination, and interaction which he had put forward as the ways to achieve immersion in a pedagogical environment – were evident throughout both of Rocky's and Sal's highly individual, yet interconnected, journeys throughout this experiment. Each, in his own way, had cleared the path from virtual educational *space* to a *place* of profound understanding. Each had demonstrated a keen interest in the mathematics itself. Each was involved rather intensely in the learning process, interacting with the subject matter, with each other, and with others who attended from time to time. Each had shown a level of curiosity and imagination which took them beyond the confines of the live sessions, where we all convened once a week, into the expanding cosmos of the World Wide Web. The interaction between the two of them slowly developed into a virtual bond characterized by a wee bit of friendly competition, a good dose of camaraderie, and ultimately respect for one another as persons of learning. They had learned together, taught one another, exchanged ideas, and collaborated outside of the meeting place.

The next section articulates their individual experiences as each progressed towards that Burbulean *place* of immersion in a totally virtual world.

The TEACHINE/LEARNING DYNAMIC: a System of Differential Equations

Both learning and teaching are dynamic processes. In fact, this interpenetrating dynamism is the hub of educational experience for each and all concerned in any learning environment. This hub lies at the centre or the vortex, if you like, of the three interactive presences: the cognitive, the teaching and the social as described in the Garrison et al. (2000) Community of Inquiry model mentioned in Chapter 5, section entitled "Negotiating the *Zone of Between*." I like to think of this three-way interaction of reciprocity as a system of pedagogical differential equations, which models the learning dynamic of a group of individual persons (the particles), each having a set of pre-determined conditions (initial position, directional force and velocity of intent). The dynamic path of progress of each person will vary depending on a number of factors. They include:

1. the starting conditions (the teaching presence of the instructor and the collection of cognitive presences each member brings to the group dynamic);
2. the direction and speed of the underlying current of the fluid pedagogical environment (the interacting teaching/cognitive presences); and
3. the jostling of the particles one with another and with the streams of consciousness[28] of that group as each seeks to expose their thoughts to text for others to examine (the social presence intermingling with the cognitive responses).

Every educational group endeavour may be thought of as such a dynamical system. There are generally predictable flows with minor aberrations in such modeled systems but an unexpected event can interrupt that continuity and throw the system into a chaotic state. If the momentum of the flow is strong enough, it will recover from the event keeping all of its constituent particles moving together as an intact body, even if shook up a little bit. If, on the other hand, the flow of the current is not cohesive enough, the intrusion will break the continuum at the expense of the moving group. The outcome is generally unpredictable and becomes an individual story for each such occurrence.

The teaching/learning dynamic is both personal (its particle-wise initial conditions) and communal (its set of differential forms). The goal of the primary educator is to maintain a continuity which carries all towards a common final destination. In essence, this kind of dynamical system is a collection of virtual paths from learning spaces to learning places. Each of the participants starts at her/his own unique position but is motivated in a specific direction depending on the goal. Each has an inner energy or style which carried them, independent of the group around them. Moreover, each is also born by the momentum of the group dynamic but, nonetheless, traverses a personalized path. This defines a dynamical system of human action, interaction and reaction. What follows are the stories of two individual journeys of learning in an online space which led to the forging of a virtual place of learning.

Individual journeys on the wave of the future. ROCKY's DYNAMICAL SYSTEM: **Rocky** was the 'homework' person of the entire group. He would ask, in almost every live tutorial, either "What is the homework"? or "Do we have any homework"? If the topic of conversation ever strayed away from mathematics for too long, it was Rocky who brought us all back on track. He was certainly one of the participants about whom there was so much to say in terms of his interaction with the subject matter and coincidentally with me. He wrote to me almost every week about things mathematical.

Initial position (level of mathematical knowledge):

He did not consider himself a 'math person.'

> I never wanted to do math or science because all they did was frustrate me. (REF: PPS)

Initial velocity (learning style and personality):

He classified himself as being an active**,** sensing**,** visual**,** sequential learner (REF: LMS) (see Appendix B).

> I am a very independent person and I have done self study work in the past. (REF: PPS)
>
> As for learning strategies, I know my strategy is to keep doing something new until it becomes second nature. (REF: FIS)

Initial direction (towards his goal):

He joined the project in the hopes that he could gain enough skill and confidence in the study of mathematics that he would be able to enjoy it. In particular, he wanted to glean enough mathematical expertise to be confident enough to take university-level math courses. (REF: PPS)

Rocky's definition of having really understood a concept in mathematics:

> One way that I know whether I understand something or not is whether I am able to present a clear explanation of it to someone else. Only if I can share a concept with someone else (and that someone else then understands it) do I know I understand it. (REF: PPS)

He always went straight to the homework. 'That will be the test that I understand it or not. It's your job to be picky'. (REF: PPS)

The suitability of the pedagogical environment:

Rocky used a lovely metaphor to sum up his ideal mathematics learning environment:

> Because math is such a strange land for me, I would like to have someone to act as interpreter. I would depend on the guide to plan the overall trip. But at each stop along the way, I would like a chance to meet the locals and turn to my interpreter only as needed. (I hope I am not overworking the travel metaphor.) (REF: PPS)

His initial reaction to the interactive online web site was rather negative but he did sporadically visit the site nonetheless. In essence, the top-down pedagogical approach in these web site tutorials was not suited to his style of learning as a sequential, rather than a global learner of the subject matter. (REF: FIS, LMS)

The dynamism of the live tutorials (another virtual space), on the other hand, seemed more to his liking than the relatively static RMC web site environment. He did express some criticism of the web conferencing software itself but the highly interactive nature of the sessions did more to enhance his learning than deter it. As the "homework" man, he had had lots of chances to practise the new skills. Because he was a regular attendee at the live tutorials, he used me as his interpretive guide for confirmation, correction, and re-direction. That he had appreciated:

> I could interact with the instructor as much as I needed and I could listen to the recordings as often as necessary. It is a great combination. (REF: LMS)

His Journey:

Rocky attended all twenty-five (25) of the Group 3 tutorials from its inception to its conclusion. His engagement with the subject matter took hold very early on in the project. In the February 16, 2010 tutorial, he and I were chatting a bit before the others arrived. During that conversation, he mentioned that he had already advanced his math skills beyond the level he had had before and that being part of the project had peaked his curiosity about other aspects of mathematics, so much so that he had searched out other online math web sites to find out more. It should be noted that that particular evening session took place during the time of the Vancouver Winter Olympics. I happened to comment that some of our participants probably would rather watch the Olympics than come to a math tutorial. To my surprise, Rocky's response was: "I'll take the math tutorial any day." As a math instructor, of course, I was a little taken aback.

However, the truth of the matter was that this was an early indication of Rocky's growing personal engagement with the learning of mathematics. It was exhibited both in terms of his increasing comfort within the virtual meeting space and his involvement with subject matter itself. The metamorphosis of formal observer into Rocky's PLACE had already begun. Moreover, the light bulb moments of insight (such as his saying that two things can be equal but not the same) were already manifesting themselves in a subject he had, just a month before, called "the dark side of the moon for me." (REF: PPS)

March 8, 2010 tutorial

Rocky and Sal were the only attendees that evening. While I was trying to open an application that was taking some time to accomplish, there were some quiet moments. The two participants then started to talk about when their interest in mathematics had begun. Rocky told Sal that he had struggled with math all his life and then added: "It's strange that at this point I'm starting to get some enjoyment out of it".

March 22, 2010 tutorial

In this tutorial, I had worked through a very challenging quadratic polynomial, from its irrational roots, to the intervals where it was positive and where it was negative, and finally to its graph. After we got the final answer, Rocky said: "I know that none of you is going to believe this but that is actually the answer I got. …. Just not sure how I did it".

At this point, Rocky was to the stage where he could follow some rather complex procedural instructions to get to an answer without really understanding the process itself or why it worked. That is the first stage of mathematical learning – procedural accuracy. The deeper understanding into why the process works would come more gradually. Nonetheless, all this theory which had been driving him crazy had taken a small hold in his learning.

March 29, 2010 tutorial

This was when Rocky was showing signs of his beginning to think ahead mathematically on his own. He sent some questions to me before the session so that I could respond to them during the session. However, his questions were leading into the topic of functions which we had not covered yet. This personal exploration into the unknown exhibited an even deeper engagement with the mathematics than he had exhibited prior to that time.

ENCOUNTERING a STEEPER INCLINE on the LEARNING CURVE

April 6, 2010 tutorial

Rocky admitted to having had a difficult week. He hadn't really grasped what had been going on in the last session on rational equations and inequalities. Formally he was able to work through the exercises and get correct answers. However, he had still not made the connections to a deeper context of meaning and understanding. This too was indicative of the move to a more advanced stage of learning. He was no longer satisfied with just having been able to provide correct solutions to the homework exercises.

In fact, during that same session, he had worked with the Tartaglia-Cardano formula for finding roots of a certain kind of cubic equation. This, by the way, was not part of the curriculum but was introduced as a point of interest because a number of the Group 3 participants were anxious to dig deeper into the solutions of polynomials. Rocky had actually worked through the formula found on a web site and had found a discrepancy with it. He was correct in his conclusion. There had been some problem with the statement regarding this rather complex formula as cited. To discover this, he had had to make connections back to quadratic equations. Rocky's thinking was, at that stage, beginning to show more of a intuitive/global relational style of learning, rather than a sensing/sequential procedural style he had claimed to have had at the outset of the project. This change of mathematical thinking in less than three months of engagement was a real leap in the learning process. Rocky's quiet, studious ways and his persistence had already pushed him across the borders of his known territory into an alien land – that land he had wanted to visit but hadn't known how to get there.

> I know people who go there without difficulty; it is even a place I would like to visit myself. But I don't know how to get there. At least not yet. (REF: PPS)

He was quite relieved that his conclusion about the formula on the web had been correct. I reassured him that he was really quite good at algebra and that he could now trust himself a bit more in terms assessing whether he was right or wrong when his answers didn't jive with others.

ARRIVAL in the PROMISED LAND

Also during that April 6th tutorial, I had introduced functions as a set of ordered pairs and had talked, in great generality, about their domains, ranges, values, algebraic graphs and geometric graphs. As was typical of the participants in the group, Rocky listened politely. In some sense, he had already arrived at this 'strange land,' as he had called it, of deeper understanding in mathematics. However, even he, after this barrage of abstraction, confessed to being "blown away by all the theory and the abstract thinking required here." He added that he considered himself an abstract thinker in a lot of ways BUT this approach to mathematics had really blown him away. I explained that I used the top-down approach[29] to the teaching of mathematics in which the instructor introduces the general definitions of the terms or the major theorems surrounding the topic before going through examples as to how the theory worked. The primary reason why I had chosen this approach with the Group 3 participants in the project was because of severe time restrictions to cover the necessary materials. I simply didn't have the luxury of time to build up the theory, example by example to an understanding of why definitions and theorems were formulated to capture the essence of what the theory implied. Rocky, at this point, I think, began to see the logic behind the pedagogy and promised to press on though he did qualify the terms of engagement: "OK, I shall press on. I make no guarantees but I shall press on." And he did just that.

REAFFIRMING THE VOWS OF COMMITMENT

April 12, 2010 Group 3 tutorial

It should be noted that mid-April was the formal end of the project. At this juncture, I approached the group about continuing on. I reiterated my promise to go as far in the curriculum as they wanted to go. In the Group 3 tutorial, we still had not covered function theory adequately, nor discussed important classes of functions, such as the exponentials or the logarithms. Rocky was still on board: "I am here to the bitter end if you are." During the rest of the project and the post-project period, he would make this *space* his *place* of residence.

April 19, 2010 Group 3 tutorial

Rocky was the only participant present. As soon as he entered the room and we had exchanged greetings, he asked whether I had heard from the others. Because neither Sal nor Ursula could come that evening, Rocky and I pressed on together in a sliding student-teacher continuum in which the student now determined the direction and the teacher simply supplied the necessary content to complete the student's understanding of the issues. Without others present to change that balance, this is how we proceeded from that time onward when we (teacher-student) were alone online together. It was a back-and-forth kind of tennis match without the competitive spirit, rather more like a practice of skills. He would come with questions; the ball was now in my court; I would respond and send it back to him; and thus it continued until we had both had had enough practice for that evening's session.

It was during that session that Rocky reiterated his dream to take Calculus someday – but added that he "wanted to get through this first." I reassured him that he was doing extremely well and that he shouldn't worry about getting through this.

AT THE TIP OF THE ICEBERG

Rocky

> It seems like the more math I learn that more I realize there is to learn. It feel like I am at the tip of the tip of the tip of the iceberg.

I think that once we know how much we don't know then that is the beginning of being an educated person.

THE SLIPPERY SLOPES OF THE ICEBERG

May 3, 2010 Group 3 tutorial

When I asked Rocky at the start of the session how his week was, he said that, since the project had started, it was first time that he had really felt frustrated. He had had a really hard time with the homework I had given him the last time. I told him that there was a remedy to frustration. I would work with him to clarify the issues with which he had been struggling.

I had given a question in the homework which required the learner to find the answer more by experimentation and discovery than by an application of a previously discussed method. I assured him that his frustration was natural, especially because he had not come to the answer he had expected which had been obtained by his working previous examples of the same type. Under such circumstances, the typical reaction is for learners to give up. Rocky did admit, after he saw the solution, that he had been well on his way to getting the correct solution but had given up too easily. Now he was really in the land of mathematical thinking and climbing his way on the slippery and craggy slopes of the surface of the ICEBERG. He had now been exposed to mathematics which had not been wrapped up in a neat little package. In addition, during that same session, Rocky had demonstrated his ever expanding fusion of horizons by solving an exercise I had given the group using a different interpretation than the one I had initially presented.

However, despite the frustration he had felt at the beginning of the evening, towards the end of the session, that feeling of discouragement had probably abated somewhat for he reassured me and himself by saying:

> I tend to be rather obsessive in that respect. I have to understand A before I go on to B. . . . Well, perhaps I can still learn math. I still have hope. This project has been a real boost to my self-confidence in terms of learning math because I have always had a struggle with it. To experience success is really wonderful for me.

The tutorial ended with his requesting extra reading to do and the promise that he would read it religiously.

PASSING THE TORCH

May 17, 2010 Group 3 tutorial

It never ceased to amaze me how dedicated some and, really, if I think about it, most, if not all, of the participants were to the project. They often came under personal situations of duress and busyness to spend a few hours together to talk mathematics. I think that is what fascinated me the most. I was not, by any stretch of anybody's imagination, entertaining them. The subject matter has had and still has a reputation for being, at best, dry and, at worst, terrifying. I was trying to make it interesting within the confines of what one could do in the online world but, even then, those options were severely limited in such a setting. Yet they came and they came regularly. Rocky was the prime example of this kind of dedication from project beginning to project end.

That evening, Rocky had actually hurried back from the airport to attend the group session. He had warned me by email that he might be a little late – but he was definitely arriving. Then, to top it all off, the web conferencing software wouldn't let him into the tutorial right away without a password – something that had never happened before. However, he persisted and finally he entered the room – albeit a bit 'frazzled' – as he later confided to me.

Rocky had often asked for me to translate mathematical symbol-ladened sentences into English. This time, I asked him whether he would be willing to talk me through an example explaining what a vertical translation of a function was. When he had completed his explanation, I asked him what his understanding of the horizontal translations of functions was. He continued. Finally, I suggested that he learn about non-rigid transformations on his own (something I had not discussed prior to that time but had supplied a few notes of explanation) and come back next week with his personal take on how to work with these transformations mathematically. He would have to teach me about them – not I him. That was the day I passed the teaching torch to Rocky. The teacher-student continuum had moved completely to the other end of the spectrum. Rather than I always teaching him, he was now going to teach me.

Though he said he had been tired that evening (naturally!) and not his best, before we parted, he expressed his appreciation that I had maintained the commitment to the project for him and he promised to press on. This was a declaration that he was keeping his commitment as well.

May 25, 2010 Group 3 tutorial

What I had considered a rather bold act of transfer done in the previous tutorial had had an unexpected outcome. His first words were: "Well, I feel a lot better this time than I did last time." He had gone through everything that he had been struggling with during the previous week – and then added, optimistically, "and well, I think I actually understand it". Not only had he done everything that I had asked of him, but also he had taken the initiative to proceed on this own through the operations on functions! His solutions to the homework proved that his understanding of both rigid and non-rigid transformations was really quite solid. He did admit to having experienced some self-doubt as we went through his homework. He was always wondering if he had really understood it as well as he had, at first, claimed. This kind of self-doubt is perfectly natural after having just 'taken the lead role,' so to speak, in this moving continuum of learning/teaching. However, what I deduced from his experience was that he had taken that torch and run with it.

May 31, 2010 Group 3 tutorial

It seemed that, after this point, I became more a facilitator than a teacher. Rocky slowly began to set the agenda for the sessions - not just suggest it. As an explorer in this now not so alien land, he said he had discovered all kinds of way to learn mathematics – in particular, trigonometry. Wow! He had been learning trigonometry on his own or, maybe, still with Sal. Sal and he had begun to collaborate about trigonometric concepts early on in the project – as early as mid-February. I deduced that Sal was now perhaps too busy to engage with this extra-curricular activity and Rocky had had to proceed on this own.

ONE STAGE HIGHER on the PYRAMID of UNDERSTANDING

June 7, 2010 Group 3 tutorial

Rocky came to the session very excited about having understood functions. At one point during the tutorial, after I had introduced some very concise mathematical symbolism for the real number system, Rocky admired it by saying that the symbolism made the statement more elegant. He thought it was enormously elegant. This marked another change of thinking. From the one who always felt more comfortable with the natural language explanations of any mathematical sentences or phrases, he was now beginning to see the merit of the symbolic language as being more concise and yet expressing a whole idea. This was another step up in the understanding of the language of mathematics and his burgeoning appreciation of it.

June 21, 2010 Group 3 tutorial

What was interesting to note was that, initially, Rocky saw potential in the online assessment facility for the learning of mathematics in the virtual world but was not particularly enamoured of the RMC web site. I had sent out the Learning Modes Survey (LMS) (see Appendix E) in late March. In it, I had asked the question: “What role do you feel these online interactive web site tutorials played in your learning process”? Rocky’s response to that question was “None at all” and asserted that calling them ‘interactive’ was a misnomer because he couldn’t question them and get answers. They were more like a book. He even suggested some other web sites which were better for learning – at least for his style of learning.

However, less than three months later, in an analysis of his own learning during the project, he said that he had used the project as his curriculum and this had been the framework on which his learning had been founded and fleshed out. “I have learned a great deal” (June 21, 2010 Group 3 tutorial). He had learned from the instructor; he had learned from others; he had learned from textbooks; he had learned from videos; he needed a context. BUT the glue that held it together was the collection of the project materials which had reflected and had articulated the curriculum. These project materials look many forms: the web site itself, the online self-assessment exercises, the archived live tutorial sessions, the notes from the live sessions which included full solutions to the many worked-out examples and homework questions during those sessions. These collectively formed the layered multimedia text of the context. It was Rocky who had taken advantage of all three presences in that environment (cognitive, teaching and social) and had even taken excursions outside of it to obtain a rich educational experience at the hub of Garrison et al.’s model of the Curriculum of Inquiry.

Another revelation was that it was it had been his son who had encouraged him to volunteer for this project. Moreover, he had just retired from the workplace only two weeks before. This open sharing moment had simply confirmed what I have believed for some time now. Age is not negatively correlated to the ability to learn mathematics. It is more a question of attitude and dedication to the task. We humans have a tendency to give up too easily if the going gets tough.

By July 12, 2010 some six months from the start of the project, Rocky was ready to learn mathematics on his own using his own methods of doing so.

July 26, 2010 Group 3 tutorial

It was the first lesson on logarithms. Rocky was the sole participant and had never been introduced to this concept before. During the live session Rocky and I talked about COMMUNITY and its effect on learning, about the moving scale on the TEACHER/ STUDENT CONTINUUM of control and direction in the learning process of the STUDENT, and about Rocky’s personal learning style.

However, we began with the definition and evaluation of the log of the number 16 to the base ½. I first explained the concept in general terms. Rocky said that even though he hadn’t known what to do at first glance:

> As soon as you express it in English, I am able to figure it out. Then I can see the relationship between the English words and the mathematical expression I am seeing. Having it written out in English helps me to do it.

I then wrote out the mathematical sentence in English nomenclature and had him work through the example himself. After doing so, he commented: “I am able to understand it because I worked it out myself.” This teaching technique of layering the natural language over the symbolic language was how Rocky preferred to learn mathematics. It certainly was one of the ways in which we, together, had utilized McLuhan’s mosaic of language to impart the deeper meanings of a text. (See Chapter 4, the subsection entitled “McLuhan’s Mosaic: Its Implications for Learning.”)

Then, without any prompting, Rocky went on to say: “On the next question, let’s see if I can express it in English myself.” By doing this, he had again moved the register on the teacher/student continuum to the other side and had taken over the control and direction of the pedagogy. The next example was more complicated. He was asked to evaluate the logarithm of the cube root of 25 to the base 5. This was the cogent moment. Could he do this? Remember, he had not seen this concept before and the definition of logarithm with its intricate connection to exponentials was not necessarily easy to grasp in that short a time. Did he have enough foundation and mathematical analytic thinking to explain it to me as if I were the student and he the instructor? And, yes, he did. He had already gained enough comfort in the environment and enough mathematical aplomb to be able to work through a verbal explanation of this example on his own, in front of me, without interjection on my behalf. Of course, I was pleased. This was what I had spent a great deal of effort trying to achieve with all of the participants at whatever level each had been situated at the time.

However, what did take me by surprise was his ensuing remark: “All the time I was talking, I was writing things out”. Of course, in this virtual world, I was unable to see him doing that. In effect, he was imitating the teaching process he had watched throughout the project. I had always typed out the mathematics as I walked through an example and the participants had watched me do this many, many times. There were no cuts and pastes. There were no pre-prepared lists of solutions or examples. The examples could be those I thought were relevant to the process or one a participant had, on the fly, wanted to see explained. The dynamism of the process and the continual moving scale of teacher/student control and direction had been assimilated by Rocky. He had, in fact, throughout a short six-month period, moved from a reticent observer with a 'rocky' mathematical background (his full pseudonym had been Rocky Math) to one who could teach others a new mathematical concept in a coherent, mathematically precise manner with all the proper nomenclature and symbolism. He was able to do all of this having just a mathematical definition of explanation as an introduction.

MY PROMISE FULFILLED

It was Rocky who decided when the post-project period would end in late July 2010. By that time, he had become a true mathematical thinker of some sophistication at that level – one of my goals of the project for all of the participants. The date was July 26, 2010.

ROCKY's FINAL POSITION

Rocky achieved his goal, not only in terms of a deep understanding of the subject matter but of preparing himself for university-level mathematics courses. As far as I know, he took some time off at the end of the project to pursue another of his absorbing interests.

SAL's DYNAMICAL SYSTEM

Sal was a social and cognitive presence in his own right. As mentioned earlier, Sal loved to engage in conversation, either on or off topic. He told jokes, teased others in the live sessions, and offered answers and solutions all at the same time. He was as socially gregarious as Rocky was the quietly contemplative.

Initial position (level of mathematical knowledge):

Sal told me that, when he was in high school, he did not have much interest in mathematics. However, as he got older, he started enjoying math more. Its appeal to him was that he found it so ordered and logical (REF: March 8, 2010 tutorial). He came into the project having taken a Math 31 class at another institution but, as he put it: "I did not do as well at as I thought I should have and could have".

Initial velocity (learning style and personality):

Sal had classified himself as being a reflective**,** intuitive**,** verbal**,** global learner (REF: LMS) – in fact, the diametric opposite of Rocky's learning style in the Felder-Silverman model of learning mathematics (see Appendix B). When Sal entered the project, he exhibited a good amount of confidence in his mathematical skills but was intelligent enough to know that he had just tapped the surface of this discipline which had had five millenia of recorded history. He was definitely a mover and a shaker in this project, maybe in part because of his personal philosophy.

> I believe anybody can learn math and no one is limited to a certain level of math education, students are only limited by what they believe they can accomplish. (REF: PPS)

Initial direction (where he was headed):

His goal was to come away from the project with a good foundation from which to further his math education into Calculus, Statistics, and Linear Algebra and feel confident that he could succeed in higher level mathematics courses.

Sal's definition of having really understood a concept in mathematics:

When I can state succinctly and in my own words the correct concept, it usually means I have understood the material.

> I think math is a step-by step method for solving a question. Often I find I have a question but I'm limited in solving it by my existing math knowledge. I usually get the answer but is there a better or shorter way to get the answer.
> If I can visualize the concept (use it in solving a problem) it becomes much more clear to me. I am always looking for ways to apply and use what I learn. I have trouble with some word problems. (REF: LMS)

He also felt that understanding a mathematical concept meant being able to piece together different mathematical ideas and see their connections and interrelatedness. Sal was definitely a global learner in mathematics.

The suitability of the pedagogical environment:

Sal's notion of an 'ideal' math class was expressed as follows:

> Concepts are well explained and easy access to other math resources or topics that help further explain the material. A good amount of problems to work through and instructor input on homework and problems. . . . Good instructors are an absolute must as well. There really is no substitute for an instructor that not only understands the material he/she also knows how to teach the material has a good knowledge of their students. (REF: PPS)

He valued the independence of being able to learn at his own pace but also having access to a good instructor when the need arose. He did not like classes where students were left mostly to learn on their own. He also preferred to have alternative resources to draw on.

I tried to be responsive to the needs of each of the students. However, if there had been any one participant for whom I had 'failed' to provide an adequate learning environment during that project, it was Sal. The RMC web site was still at too elementary a level to be of use to him, other than refreshing some of the finer details of the mathematical theory. He commented:

> I found the tutorials gave more of a brief cursory explanation of the material. I would have liked a more in depth treatment of the material. I found myself getting frustrated because the tutorials were geared to a introductory level of understanding. I found myself using other resources to find a more in depth treatment of the material. (REF: LMS)

He had wanted to cover conics, sequences and series and trigonometric identities. I had not anticipated covering those topics except at the end of the program. I did promise to discuss them in detail during the live tutorials but it took longer than the three months allotted for the project to get to them. When I finally did, Sal had already become too busy to participate meaningfully, though he did come from time to time.

Sal's Journey:

At the end of January, Sal had already created a web site of learning for the rest of the participants out of the sheer love of learning mathematics and wanting others to engage in his enthusiasm for the subject. He offered to help those who required extra instruction by forming a study buddy group for the project participants. From time to time during the live sessions, he had also made suggestions of ways of organizing mathematical material which could aid others in a process of deeper learning of the concepts.

His contributions to the group sessions were inevitably infused with enthusiasm, always mixing a little bit of fun with the seriousness of the subject matter. He was famous for his non sequiturs. Sometimes, it was to communicate to me that I should speed up the presentation – the warp 5 Chat Box imperatives. There were, of course, those inaudible chats with fellow participants in which he indulged so as to keep himself engaged when he was experiencing overwhelming fatigue. Other times, his mind had leaped ahead and he was thinking of the mathematical implications of the topic we were discussing. An example of this was when he asked about the latus rectum when we hadn't really got past the idea of the graph of a quadratic equation. The latter fast forwards were the norm for Sal – always pushing the current to cover more territory at a faster pace.

March 8, 2010 tutorial

Sal had sent me homework to mark. I didn't get it marked before the tutorial so, at the beginning of the session, I explained why in the form of a mildly teasing complaint.

Julie You did problems that I don't think I assigned and then you did far more than expected ….. and now I have to read … and work through all your calculations. It looks good though.

Sal lol ….. Just send it back and say that it looks good. ….

Julie No, no, I will mark it.

I did not want to discourage any student from submitting homework even though it was extra work for me. I then gave him my assessment of his understanding of the mathematics. By then, he had had an excellent grasp of the algebra and was moving towards functions and transformation of functions on his own – having created his own colour-coded diagram of the transformations of functions. Closer to the end of that evening's tutorial, Sal had made a PDF file of this diagram and had sent it to each of us in the session. I then opened it on my desktop so that all could see it and we, as a group, discussed it together. During the project, I always made every effort to incorporate students' mathematical initiatives into the project tutorials or venues so that they could share, if they so desired, the instructor's podium.

By March 15, 2010, Sal was talking about his work taking him all over Canada and the United States. He said that, to date, he had clocked 82,000 air miles. That very day, he had travelled from Calgary to Fort McMurray and then had joined us for the evening tutorial. It became clear that he probably could not carry this kind of schedule for the duration of the entire project.

March 22, 2010 Group 3 tutorial

Sal had a very active and curious mind about all things mathematical. He obviously had surfed the Web and other places for new concepts. Sometimes he would come to class and ask about them. For example, during this tutorial, we were discussing the solving of quadratic equations and what the solutions to those equations meant graphically. Suddenly, Sal interjected that he had used Descartes' Rule of Signs on both the quadratics but concluded that the information gleaned was too general to be of specific use in our examples. He was correct, of course. However, he had inadvertently pushed the group into the concept of complex numbers – a topic which was not going to be covered in the project. Later in the tutorial, he suddenly asked if I was going to discuss the 'latus rectum.' This was a leap into the subject of the conics sections. Towards the end of the evening, he offered to read up on Tartaglia and bring his findings back to the group.

This was an integral part of Sal's style of learning. I don't think that others really realized that, for him, investigation of other topics was the way he put concepts together and made linkages between the understanding of one and the knowledge of another. This was very much in keeping with his personal style of learning – the reflective-intuitive-verbal-global approach. His wanting to share and discuss these issues was his way of giving purpose to his research other than just for personal knowledge. He also believed that: "Sometimes the best way to learn mathematics is to explain it to someone else" (REF: January 14, 2010 tutorial). This was just his way of being able to explain it to a receptive audience for feedback.

By late March, unbeknown to the group at the time, Sal's ubiquitous presence during these tutorials was about to come to an end. The indicators had already been evident. On March 22, 2010, he had mentioned that he had worked fourteen hours per day during the preceding two weeks, without a day off. Despite that, he had always come to the tutorials and had always contributed to them – at the same time making light of the fact that he hadn't perhaps done the homework, even if others had.

Sal did not attend the tutorials again until June 7, 2010. In the interim, he had written to say that he was still working thirteen hours per day, was arriving home after 8:00 o'clock in the evening and simply could not come.

THE RETURN OF THE WANDERER

June 7, 2010 Group 3 tutorial

Surprise….. Surprise! Sal showed up at the tutorial that evening. Rocky and I were there to greet him. We had really missed his presence for the last few months or so. Sal explained that he had been very, very busy. He was between jobs that evening and had to leave again the next day for another job in Saskatchewan. He had just finished a contract as a foreman. Then I told him that it was fortuitous that he showed up tonight because this was the very night I was going to start conics – the topic he had wanted to cover way back in March. Even though he would be working twelve hour shifts, he said he would make an effort to show up for the conics tutorials as much as he could. Sal worked hard but he confirmed that evening that he was still committed to the project. Finally, I was able to cover some of the material he had specifically come to review. I felt really good about that. When I started to discuss inverses of functions that evening, it was the first time I heard Sal say that he had not really understood something and had struggled with the concept.

Sal wow you just cleared up more confusion and frustration than you know
i never knew this ….. I've seen them but always struggled until now

He had also asked for homework to practice the skills required to solidify this concept in his understanding. The facts that this material had been relatively new to Sal as well as his request for homework to do meant that he was now fully engaged again. My guess was that he would remain with us as long as he had the time to attend.

TRIUMPH OVER TROUBLED WATERS

June 14, 2010 Group 3 tutorial

When I arrived at the live session that evening, Sal was already there. He had written a list of quadratic equations in two variables on the whiteboard and was trying to classify them as conics sections. I handed the podium over to him and asked him to explain his thinking in terms of these questions. It was now Sal's turn to be the instructor.

When Rocky arrived a few minutes later, Sal was essentially at a point mathematically where the topic material was relatively new, as it was for Rocky, and they were now on an even plane of learning. Both were interested in going over the homework. Both wanted to start from the basics for the conic sections. It was probably one of the first sessions where Sal was not wanting to 'warp 5' the progress of the lesson.

June 21, 2010 Group 3 tutorial

During this tutorial, Sal's imagination began to fire up again. He then suddenly announced: "I'm going to do a YouTube video on conics at some point soon".

The positive side of this project for him was his announcement in the June 21, 2010 tutorial that he had been promoted to a foreman's position for that work contract because of the knowledge of mathematics he had learned in the project (REF: June 21, 2010 tutorial on Conics 2). I was delighted to hear that. I said that this project was really a success story for him then and he agreed.

I had always, prior to that point, really felt quite badly that he had had to wait so long for much of the material he had initially wanted to review and learn. He had been patient and had attended virtually all of the Group 3 tutorials until late March. While this material was mostly within his grasp, he always contributed ideas and answers – not to mention camaraderie and humour. He was always upbeat but, underneath it all, I sometimes sensed a certain kind of impatience that his needs were not entirely satisfied. His active and vocal participation was a group dynamic which I personally enjoyed and encouraged from all. However, all did not share my enthusiasm. There were those in Group 1 who exhibited irritation and opposition to his enthusiastic involvement, in forms both tacit and overt, both toxic and benign. I am ashamed to have to admit that, in retrospect, I am not at all certain that I handled this series of interruptive events in the best possible way. They didn't disintegrate the dynamic into chaos but no-one was quite the same subsequent to the happenings – not Sal, not I, not those who formed the 'Opposition.' How cognitive diversity can create division in a group and precipitate varying levels of disengagement from the pedagogical task is a tale yet to be told. See the subsections on "The BREAKING of Trust" and "The Virtual Schoolyard Skirmish – No Witnesses."

Up until the end of March, Sal had attended both Group 1 and Group 3 tutorials. After that, he attended only the Group 3. Had I just moved the group a little faster along, perhaps he would have remained more engaged in those final days of summer. I often wonder, if I had done that, would I then have left the other participants floundering? As mentioned before, I have always tried to keep all participants with me as much as possible even at the expense of slowing another's progress. Though this is perhaps not always realistic in practice, every teacher faces those decisions. Every dynamical system is unique. Every outcome is moot.

SAL's FINAL POSITION

For Sal, from the first day of the project, this online environment had been his *place* of learning. He simply moved around in it as if it had always been his home territory. Sal did achieve his goal. Before the end of the project, he had enrolled in the introductory Calculus course at the University. He was already on his way.

The Virtual Classroom

Domains of Learning

Benjamin Bloom (1956), in his seminal work involving a group of college educators, identified three domains of learning. The cognitive domain constituted the development of mental skills towards the accumulation of knowledge and a deep understanding of the concepts. The affective domain touched on aspects of the social psychology of beliefs, emotions and experiences surrounding the learning process (in short, attitude) and their effect on the learning process. The third domain, called the psychomotor domain, dealt with the development of manual or physical skills. The learning of mathematics involves a highly complex interweaving of both the cognitive and affective domains of learning but in which the psychomotor domain has little, if any, import other than the facility to write or type and, in some cases, to draw. Therefore, in what follows, the learning spaces to which I refer are spaces of cognitive and affective learning.

The Changing SPACE of 'Face-to-Face' Classrooms

The Wikipedia Encyclopedia defines a classroom simply as a room in which teaching or learning activities can take place, and further clarifies that a room is any distinguishable space within a structure. My interpretation of the word 'classroom' is that it is a 'room' in which the underlying intent of those gathering there is either to teach or to be taught or both. While it is generally understood that classrooms can be situated either indoors or outdoors and can be designed in a variety of ways, most often, however, until recently at least, a 'room' was generally assumed to be a delineated and relatively confined physical space in one geographical location.

In this era of the twenty-first century, however, while the definition of 'room' hasn't really changed much, the generally acceptable kinds of structures which may contain classrooms have altered quite dramatically. For example, the human-made structure in which the teaching and learning took place in this research was located in an intangible 'room' in cyberspace, defined as an electronic medium of computer networks over which human communication is possible. The ‘room’ in that structure was visualized only in the mind’s eye through a contiguous series of images on the computer screens of individuals located at as many different locations across the North American continent as there were people-presences in the room. Our virtual meeting space was incorporeal but was equipped with:

- a writing board (a virtual whiteboard instead of a Smartboard, a blackboard or a green board or a flip chart);
- tools for writing and drawing (a virtual typewriter instead of pens and pencils) and buttons which could be clicked to enable drawing or placement of symbols;
- a text box for sending notes over the wire to another person in the ‘room,’ the virtual counterpart of either passing a slip of paper from hand-to-hand or texting in a physical meeting room; and
- a microphone facility so that everyone in the ‘room’ could hear what you were saying even if you were four thousand miles apart physically.

This was, more appropriately, not our classroom but our classroom interFACE. The term *interface* has a poignant, but subtle, symbolic connotation in the context of virtual environments. In the computing world, it is a set of media which enables a user-machine connection. It is both physical (the hardware with its loaded software) and virtual in as much as it is a medial concept. Each of the participants in the group had such a personalized interFACE. However, the interaction of teaching and learning took place in what Aoki (2005a) called the “zone of between” (p. 161), a metaphor alluded to in the previous chapter. One of my research questions was concerned with ways in which such interfaces could transform our notions of classroom in this next century.

The Changing FACE of ‘Face-to-Face’ Classrooms

The positive impact of ‘face-to-face’ on the learning process has been a fundamental tenet of the teaching profession for a long time. I myself have been a proponent of such dogma. Such beliefs have, in many cases, centred around both the cognitive and the affective aspects of learning, particularly in the sciences and their applications. One argument put forward supporting face-to-face learning environments over virtual encounters was the claim that computer-generated laboratory scenarios, simulations, animations or even explanations could not emulate the ‘face-to-face’ of learning by way of physical activities with mentors and peers. Another concern was that the virtual environment would lack the necessary social component to keep students motivated and attentive. Some feared that deep learning of the more difficult concepts could not occur at all without the instructor being there in person as a focal point of expertise. Because online presentation of the materials was necessarily thin in approach, these were perceived as genuine concerns. Anyone who has attended or taught in a face-to-face class has experienced the dynamism which can occur in such settings, not necessarily all positive for learning but, nonetheless, transforming. Fox (1995) even suggested that any person-machine interaction in a virtual environment would necessarily preempt the person-to-person connection intended to be the purpose of the virtual gathering.

> While all this razzle-dazzle connects us electronically, it disconnects us from each other, having us "interfacing" more with computers and TV screens than looking in the face of our fellow human beings. (Fox (1995) as cited in Smith & Kollock, 1999, p. 168)

How could this dichotomy of personal FACE and interFACE lead to the kind of internal connections which infuse physical classrooms?

I claim that, even in the realm of face-to-face classrooms, there is a dichotomy of FACE into the tangible FACE and the intangible FACE of human presence (see Chapter 4, the subsection "Language: the FACE of the Virtual World"). The tangible FACE consists of the qualities of 'face' which are attributable to the physical being, otherwise expressed as the body of human presence. As discussed earlier, the impact, and there is one, of this face on a community of learners is generally experienced through:

- the tangible presence of persons in a physical location;
- the physical appearances of those present (static); and
- the physical body language (active) as seen through the eye or as felt by bodily contact, such as a frown, a smile, a roll of the eyes, a shrug of the shoulders, the making of a fist, an embrace and such like.

On the other hand, the intangible FACE or the virtual FACE, as I like to call it, consists of the virtual (or medial) qualities of 'face' which are those attributable to human expression or projection - in other words, the import of human presence projected towards others in the group. The impact of such qualities on the affective domain of the human psychology may be felt through:

- the spoken/written dialogue (modified by culture, tone and intent), which then gives rise to the intangible or emotive personas of both instructor and student, often defined by the role a character plays or wishes to play in that setting; and
- the kinetics of personality mediated through the expressions of both spoken/written, which tend to determine and define the emotive impact of personality on the individual and on the group in a learning situation.

Behaviors are selected according to the desired impression an individual wishes to create when interacting with other people.

The virtual FACE in every classroom. If the processes of thinking/learning and communication to a common understanding are virtual, medial, hermeneutic things as Dewey (1916), Einstein (cited by Barbatis, 2012), Gadamer (1989) and Aoki (2005a) have all suggested in a variety of educational contexts, then **every classroom of learning is essentially a *virtual* Face-to-*virtual* Face encounter**. The only difference between the physical and the virtual meeting spaces is that, in the latter without web cams, we are face-to-face only through our *virtual faces*. What is noteworthy is that the virtualFace consists primarily of the constructs of language, either vocal or textual. Gadamer (1989) had understood the transformative power of language through dialogue and conversation to knowledge and greater understanding. Our personas which project to others through expressions of language and behavior reflect the new notion of the face-to-face classroom, whether through social gatherings or social media involvement, hand-written or email exchanges, corridor or chat room discussions, lecture halls or synchronous web-conferencing meeting rooms.

While critics of online communication have cited the truncation of the richness of human interaction through the absence of the physical face, others have lauded that obliteration of image in community-like involvement. The absence of image can, in fact, be a catalyst for broader interaction between more diverse groups of individuals. I certainly found that to be the case in the live sessions of the project. There were no strata of social norms to divide us. What we revealed of ourselves was a personal choice. Our one and only common denominator was our collective desire to learn the mathematics at hand. In that, we were united as a single group. Smith & Kollock (1999) have also pointed out that physical appearance, rather than being a boon, can also be a barrier to communication. Smith & Kollock (1999, p. 184) noted a number of concurring opinions:

> The Net's relative lack of social richness can foster contact with more diverse others. The lack of social and physical cues online makes it difficult to ascertain whether another Net member has similar social characteristics or attractive physical characteristics. (Sproull and Kiesler, 1986)
>
> . . . As one pooch in a *NewYorker* cartoon says to another, "On the Internet, nobody knows if you're a dog." (Steiner, 1993)
>
> . . . This allows relationships to develop on the basis of communicated shared interests rather than be stunted at the onset by perceived differences in social status. (Hiltz and Turoff, 1993; Coate, 1994)

All of this suggests a more expansive notion of what a face-to-face classroom connotes to a world living out their lives in virtual spaces.

The Virtual Classroom – a Personal Perspective

In what follows, what I will refer to as a *virtual* FACE-to-FACE classroom is a meeting room in cyberspace in which interaction occurs through the interface of that virtual world and learning and teaching is mediated through the *virtual* FACEs of its participants.

The RMC interactive web tutorials, even coupled with the self- assessment tool, could not be considered a candidate for a virtual classroom in our virtual face-to-face sense of the word with the imperative of a group interaction among learners and mentor(s). This web site of online learning was not designed to be a communal meeting space. Only the live sessions (held at least once every week for each group) could be construed as a gathering point suggesting the notion of a 'classroom,' meaning a communal space where two or more persons meet to teach and learn together. Therefore, to discover how the various participants in the project described their experiences of learning in the virtual web conferencing room, data was collected from surveys and chat during the live sessions about personal thoughts or experiences while learning mathematics during those sessions.

Beyond the virtual circumstances in which we met, one of the primary objectives of this research was to determine whether this incorporeal space with no fixed address (other than its electronic URL locator) could act as a reliable facsimile for the ubiquitous face-to-face classrooms in which most of us have taken our early training in mathematics. The notion of 'classroom' has a resonance beyond the spatial room. The word itself is also embued with a powerful symbolic meaning through the embodiment of the collective experiences and processes of learning having taken place in such spaces. What is integral to a 'room' metamorphosing into a 'classroom' are the experiential aspects of learning and teaching in that meeting space. The 'class' part of classroom is merely a conjunction of terms used to symbolize the transformation from a delineated space set up for learning activities into a place of engagement having a *classroom climate.*[30] The 'class' conjunct may also be considered a metaphor of the educational process itself from its spatial meeting location, wherever that might be, to the place of transformative learning by way of immersion (embodiment through osmosis, not submersion) achieved through participant interest, involvement, imagination and interaction (Burbules, 2006). This is a person-content-person relationship which may also be construed as the student's engagement with the learning materials individually and collectively. How an instructor can foster this kind of learning is another issue. The question of whether our virtual 'room' was actually a 'classroom' for the project participants was one of my primary interests in this study.

Because any such encounters are diffused with a mélange of sensorial experiences from all of the group participants, such a diverse and multi-faceted collection of exchanges are experienced, nonetheless, in highly personal and individual ways. Therefore, the assessments of those participants in terms of whether they considered the 'room' in which they met together as a 'classroom' in that metaphorical sense would also be viewed from a highly personal and individual perspective. Here were their conclusions.

Participant Assessments of Educational Place in a Virtual World

The participants were surveyed about their classroom experiences both in the past and within the online setting of the research project. This is a compendium of the relevant questions and their responses from the Learning Modes Surveys (REF: LMS).

> *QUESTION: At any time during this project, did you ever feel you were a part of a mathematics classroom with fellow students and teacher – even though you could not see each other face-to-face?*

Marie and **Clarissa** simply said 'yes.' Others had more to say.

Abigail Yes, I did. I felt it was even better than a class because it was in my kitchen.

Georgiana Definitely, the sessions that I attended were also attended by most of the same students. Therefore, we as students, in a mathematical way got to know each other, weather (sic) in a problem solving way or mathematical ability setting. The instructor was pleasant, polite, and willing to answer any questions without making anyone feel inadequate.

Ruth Yes very much so. By [the instructor] greeting each participant upon joining and inquiring how they were doing, it felt like we were a group doing a course together despite the geographical differences. By seeing other people's questions & answers, I felt them being "present."

Ursula ….. always. The instructor always welcomed each student when we came to the tutorial classes, when we were not there she was aware and when we said good bye there was always the personal acknowledgement that you were leaving.

Rocky I felt that way all the time.

Sal I felt like part of a community right from the very first class. We had some friendly banter going on between the students from the first class that continued in every class. We had in almost every class a bit of humor. It was the most enjoyable and productive math class I have ever taken.

Chloe No, and I see this as a draw-back. Lack of cohesiveness negatively impacts learning that is to occur in a group situation. I actually find the ability to "chat" with other students to be problematic.

Julie, as instructor Whenever I teach mathematics, whether to a group of one-hundred and seventy-five students in a theatre setting, or forty students in a physical classroom, or one student in a private tutorial or any number of students in an online setting, I feel as if I am in a classroom.

*QUESTION: How was the experience of learning mathematics during the live sessions **similar** to a face-to-face classroom?*

Issues addressed in this regard were the physical presence of instructor and fellow students, the paced time frame of the sessions, and the ways of presenting the materials.

Abigail They are similar to the classroom in that you can ask questions and things are explained and I think because of the format, we may ask more questions or may ask questions we would not have asked if we were in a face-to-face setting because of fear of “looking bad.”

I can’t say enough about these tutorials. The pluses are many+++ The coverage of the topics was excellent and we would go over them as often or many times as necessary as well as being able to contact Julie as often as needed. We could ask questions as we went through the tutorial and the answers were presented immediately.

Clarissa Being able to ask questions when they arose so I wouldn't forget what I wanted to ask. The ability to repeat the steps to a problem while it was still fresh in my head and the "classroom atmosphere" that the tutorials provided. I find the settings very similar and very helpful with the ability to ask questions and learn in the comfort of my own home.

Georgiana The ability to ask questions and get an immediate response was great, as in a class room setting. I think that being taught through a virtual classroom was less intimidating when it came time to ask a question or something that I did not understand. It was great! I do not think I would have learned as much or would have been as committed to the study if I did not attend the tutorials.

Ruth Thank goodness the live tutorials were not similar to the classroom I have been to. By scheduling the live tutorials I was ‘forced’ to attend unlike the other mediums where I simply did not find or make the time to practice. The fact that there was a teacher that could be heard made all the difference to me. Problems & questions were addressed in “real time” and being able to “see” what was being taught was very important to me.

Marie This experience has been more positive than a traditional class with 50 students. The small number of students, and Julie was very approachable and flexible in her dealings with students. I think that this environment is helpful to those who are not confident in math. . . . I do think it is more difficult to ask for clarification of minor points.

Ursula I find that this experience is a more positive one than actual class. The teacher teaches you don’t have to worry about distractions, or someone making noise and not listening while you are trying to listen. Felt more involved in the learning and was able to see what was being taught right in front of me and was at a pace I could follow.

Rocky The subject matter is the same (or similar) to what I have been exposed to. Having someone to ask questions of was the same. I could interact with the instructor as much as I needed. Having other students in the class was not helpful to me.

Sal I found the tutorials almost exactly like a classroom setting. Except the commute to class was a lot easier. I was sipping coffee, and snacking and I wasn't a distraction to anyone doing it. The structured format of the . . . live tutorials, was wonderful. That we could ask questions at any point, the input of the other students, and the detailed treatment of the subjects all helped me immensely in this program.

Chloe I will begin by saying that I have never been part of what I would consider a successful math classroom. I am defining the word "classroom" to mean a group of students with similar backgrounds and goals.

Julie, as instructor I have taught mathematics at a variety of levels in many face-to-face classes at the post-secondary level. This was my first online classroom experience. I found the pedagogical process much the same. The technological aspects of the online setting were not as troublesome as I had expected and, therefore, the technological hurdle, so to speak, compared favourably to other hurdles which are present without computer involvement in a classroom setting. For example, having to draw diagrams of 3-dimensional surfaces on a physical vertical classroom board, if done in real time, also presents some difficulties. Each forum has its challenges and its benefits.

What was of great surprise to me was that, in this online teaching experience, I formed more lasting and bonding connections to my students than I have ever done in a face-to-face classroom. The absence of the physical component allowed me to work at a higher comfort level with the participants because we were communicating in a more informal way. My teaching persona was less distant and more accessible than in a regular classroom.

*QUESTION: How was the experience of learning mathematics during the live sessions **different** from a face-to-face classroom?*

It should be noted that the live presentations of the mathematical ideas differed only from an in-person class presentation in the fact that the writing/drawing was done through a virtual interface, meaning on a virtual whiteboard instead of a physical black/Smart board with a virtual keyboard rather than chalk or felt markers. The giving out of notes and assigned homework were a part of each of the sessions. The backup text, if one can call it that, was the RMC website in which the topics were placed in some kind of linear order of mathematical understanding but were not presented as a curriculum of studies. At least a couple of participants mentioned the need of a textbook with the structure of a full course program. However, the website was not designed to be a text for a course per se; it was created as a review of topics relevant to the students' understanding of mathematical ideas before proceeding to math-related courses at the university-level.

Group perspective –

Many differences were put forward by the participants, but only one had had to do with learning. That came from Abigail: "The recording of the sessions is a big plus because you usually don't get a chance to relook at a class!!" (REF: LMS) The differences were rather numerous. They included: the ability to attend the class from almost anywhere; there being no travel time to get to class; the fact the chatting in the class was *inaudible*; the ability to eat and drink in the class; having to deal with technical issues; the ability to enter and leave without disturbing the others; the phenomenon of being 'ejected' from the class without having done anything disruptive; the access to archived recordings of the entire session for review; not having to worry about distractions or someone making noise while you are trying to listen.

Ironically, *only one* of the participants did specify one difference between the two teaching/learning paradigms as being the lack of physical face-to-face presence. Perhaps it was the proverbial 'elephant in the room' which was universally tacitly acknowledged but not vocalized. It was Chloe who saw this aspect as being a primary distinction between the two venues of learning:

> The first difference is that in a face-to-face class there is more chance for an emotional component. Body language in the class, in addition to being physically present with others, provides alternative avenues for educational support. There is student contact either before or after the class. . . . Body language and thought processes are far more apparent. Because students are physically in the same place at the same time, it is easier to reach out to others or to offer assistance. In contrast, an online tutorial limits personal interaction, with body language limited to more subtle behaviors like silence.

Julie, as instructor I saw all of the articulated differences of the participants as being based on the interface of the environment – that is, aspects of the technology used to bring about such a meeting across time zones and even countries. However, other than that, there were really no or few ostensible differences in the affective domain of learning. People chatted, learned, bantered, teased, coalesced into interest groups, fought, vied for attention, felt intimidated, remained silent, totally withdrew, and exhibited frustration at not understanding and exhilaration at finally understanding. Our personalities, our personas, our thought processes (which are all 'virtual' in the sense of having no tangible reality) held fast and were present in this in this virtual disembodied but not dispirited human environment.

Most of the participants engaged enough and met together with adequate frequency so that the thin, formal, almost empty space of interaction evolved into an inhabited, even vibrant learning place of learning and comfort. How did that happen? The answer is woven into the cohesive fabric of the coil of community.

Virtual Community in a Virtual Space

Bransford, Brown and Cocking (2000) identified four interlocking lenses through which one may view learning environments. They considered a learning environment to be effective if it was community-centred (social component), knowledge-centred (critical thinking subject-content component), learner-centred (awareness of individual learner's needs component) and assessment-centred (evaluation, motivation, and giving direction to learners component). Note the order in which these four lenses were placed. There are those who envision the learning/teaching process, wherever it may take place, as a community endeavour. Lipman (1991) talked about a 'community of inquiry.' Garrison, D. R., Anderson, T., & Archer, W. (2000) proposed a theoretical framework for such a Community of Inquiry with its interlocking spheres of the cognitive, the social and the teaching virtual presences. Wenger et al. (2002) put forward the idea of a community of practice. So let's probe into the definitions and the dimensions of 'community' over the Internet – this Einstein-like model of the e-universe across its four-dimensional space-time continuum.

The word '*community*' has its etymological roots in the Latin word '*communitas'* (*cum* meaning 'with or together' and *munus* meaning 'gift'). It is an encompassing term for an organized society helping each other. In biology, a community is a group of interacting living organisms sharing a populated environment. The biological definition can, of course, be extended to human populations thereby leading to our collective common notions of what constitutes a community as a social unit sharing a common interest, purpose or set of values. That which is common to all is the hub of the tenuous set of individual threads interwoven together into an identifiable entity – a kind of fabric, if you will. However, communities, if they are to survive, require a means to communicate and share amongst its members and the willingness on the part of each to do so. It is the latter which gives the fabric of the community its cohesiveness and its collective meaning. Psychologists, McMillan and Chavis, in a seminal 1986 study, identified four elements inherent in one's having a 'sense of community': membership (belonging), influence, integration and fulfillment of needs, and shared emotional connection.

Since the advent of the Internet, the concept of 'community' has been released from its inherent geographical limitations. Now people can gather *virtually* in an online setting and share common interests regardless of physical location. One of the primary issues I had wanted to explore was whether those who met together in a virtual space to learn mathematics would ever achieve a meaningful sense of community and, if so, what human interaction ingredients would contribute towards its creation in such a setting?

The Conduit of Community and Community as Conduit

In that expanded context of community, what I will term a *virtual community* is a gathering of persons each using a computer interface to interact one with another through their virtualFaces. The phrase 'virtual community,' to have any significant meaning as a coalescing of persons towards a common purpose, must have three components: the notion of the container in which it maintains its existence, the symbolism inherent in its meaning as a human community, and the conduit from one to the other. These virtual components (container – conduit – symbolic meaning) form a coiled twist of the linguistic conduit metaphor (symbol/word – conduit – meaning) to which Pimm (1987) referred in his book *Speaking Mathematically*:

> This way of speaking about the relationship between symbols and their meaning reflects the *conduit metaphor* (Reddy, 1979), which identifies

> linguistic expressions as containers for ideas, and communication as their transmission. (p. 15)

It should be noted that even the linguistic conduit metaphor is also a virtual thing because it is a medial concept from the patently made (word/symbol) to the apparently real (import/meaning attributed to it).

The CONTAINER of a virtual community is its interFace, as discussed in the foregoing section “The Changing SPACE of ‘Face-to-Face’ Classrooms,” In other words the online setting of interaction. In my mind, the interFace of a virtual community is inextricably linked to its meaning because it defines and delineates the kinds of human interaction which can take place there. The SYMBOLIC MEANING of a virtual community is the virtual concept distilled from its corresponding linguistic metaphor. This is derived from either the word ‘community’ infused with all of its tacit implications or an iconic image of community which communicates the same. It is noteworthy that many iconic symbols of community show a circle of stylized human figures, each having arms spread with finger tips touching. Another common icon representing a community is a radial disc, each of its radii emanating from the centre and reaching out to an individual human point on its circular boundary. The implicit deeper meanings of human interaction and connection and togetherness are evident in the symbolism. The CONDUIT from the container to the symbolic realization is two-fold embracing both the human interaction through the interface of language and the internalization of that interaction by each member of the group. The latter was the *embodiment*, so to speak, of the human exchange affecting the cognitive and the affective domains of the human psyche. This was the context within which the *virtual community* of this project would be examined.

One of my overriding questions was: Could the human interaction in the virtual space of my project ever metamorphose into a virtual community? Moreover, could it become a place of community in the sense put forward by McMillan and Chavis (1986)? Would each of the participants in this virtual classroom experience a sense of belonging (membership), feel as if they were important to the group (influence), have the opportunity to participate and achieve their goals (integration and fulfillment of needs), and develop some kind of shared emotional connection?

The first step in answering these questions was to think about how a community with all of its nuances of meanings could be formed. I ultimately came to think of '*community*' as a quality of human interaction, consisting of fourteen cohesive human behaviours forming a CONDUIT from the CONTAINER to the SYMBOLIC MEANING, which coalesces its members into a unity through an invisible gravitational force. I believed that the secret gravitational force which creates a whole object/ community from a tenuous assemblage of individually connected links is TRUST.

In mathematical terms, a community could be thought of as a torus, a 2-dimensional surface embedded in 3-dimensional space resembling the outer surface of a donut. Each of the cohesives of community is an interactive human behaviour, interestingly enough, whose nomenclature begins with the letter C. These individual C's may be linked, top to tip, with its neighbours in a circular coiling pattern. When fully joined together, this coil of touching C's could be wound around a torus-like object. With a sufficient degree of trust, this coil could coalesce into a solid whole.

Cohesive Components of Community – the Conduit from Container to Embodiment

Human interaction can be both formal in nature and also spontaneous. Formal or symbolic action and interaction comprise those behaviours which preserve the group structure. Spontaneous reaction and interaction are instinctual and intuitive.

What follows is an examination of the components of community which were exhibited in the virtual space of live session tutorials and its attendant forums. Both the four (4) symbolic and the ten (10) spontaneous interactions amongst the participants who met there will be considered integral parts of the conduit of human behaviours leading towards or away from a sense of community.

Symbolic Interaction in a Virtual Space

The notion of teacher/student interaction in a classroom has the expectation that everyone has gathered together either to teach or to learn or both, that everyone will make some effort to do that, that everyone will attend regularly and that each is committed, at least in some measure, to the task. These expectations I call formal or symbolic behaviours within a classroom structure. Even if each member relates only in a formal way with the subject matter and with the others in the group, these interactions do lead to a community of learners – albeit not necessarily in the deeper meanings of the word 'community' as set out by McMillan and Chavis (1986).

So what are the defining characteristics of a community in a virtual environment? Sal had expressed this way:

> I felt like part of a community right from the very first class. We had some friendly banter going on between the students from the first class that continued in every class. We had in almost every class a bit of humor. (REF: LMS)

Exactly what had created such a sense of connection within a group of complete strangers who had never met or mingled and who had never seen each other before, during or even since? This preliminary sense of comfort was probably initiated by the symbolic capital-C behaviours of classroom interaction. They included:

- **The C of Coherency** of purpose

Each participant became involved with the project to further her/his mathematical knowledge. (See Chapter 6, subsection "The Personalities and Their Stories.")

- **The C of Common Language**

Each participant moved together with the instructor to build a common understanding of mathematical terminology and meanings so as to be able to converse in a common language and, thereby progress to their goals. Even Clarissa, the only participant who could not attend any of the tutorials, seemed to keep pace with the group through the archived versions.

- **The C of Continuity** of presence

Those participants who attended regularly over the course of the main project were those who formed the community. The intermix of personalities and the extent of the interaction determined the nature of the community thus formed.

Those who stayed for the post-project period formed a very close-knit bonded community with deeper links to the subject matter and to each other. This small group seemed to be those who *engaged by passion* with the mathematics itself.

The members in Groups 1 and 2 interacted more formally with the mathematics and with each other. They tended to engage in personal conversation less than the other group. Communication links generally went radially through me, as centre, and then back to each of them, rather than from one to the other along the circumference of the community circle. These groups were a bit larger in number and those who attended seemed to be those who *engaged by purpose* with the mathematical subject matter.

- **The C of Commitment** to common goals

Those participants who attended, who endeavoured to learn the materials presented (either online or offline), who asked questions, who offered suggestions for answers and who did homework were the ones who were committed to achieving their goals. With few exceptions, these participants did achieve them.

Spontaneous Interaction in a Virtual Space

It is generally the spontaneous behaviours in teaching/learning environments which lead to the emotional actions and reactions in such settings, both in response to the subject matter and towards others in the group. Some suffer from severe and persistent math anxiety for years; others dedicate their lives to the pursuit of mathematical knowledge as a career choice. These extreme reactions, in many cases, have their roots in the spontaneous interaction of an individual's childhood classroom experiences.

According to McMillan and Chavis (1986), the four essentials to one's having a 'sense of community' in any group setting are derived more from spontaneous interaction than formal interaction amongst its members. Each member must feel that they are accepted as a member and are important in some way to the group dynamic. More than that, each must engage with the group's activities to the extent that the achievement of individual needs or goals are met. Finally, a sense of community implies some kind of positive emotional connection one with the other whether out of respect, admiration, appreciation, empathy or liking. When most of these behaviours are exhibited in a group dynamic, then the group of individuals coalesces into a cohesive community. It should be noted that if a violation of one or more of these expectations occurs, the community may fissure into interest groups, undergo a reduction in its membership, or splinter completely.

Because the quintessential qualities of community are human behaviours, virtual environments through the interface of language also have the potential to be subject to a similar array of human experiences as do physically-located communities (with the exception of bodily contact). One of the issues of my research into online environments had to do with whether the absence of body language in a virtual world would make the interaction so sterile that such gatherings could not form a genuine community of inquiry.

In the context of my virtual classroom of mathematics, the spontaneous behaviours which formed the capital-C cohesive elements binding all the participants into a capital-C Community of Inquiry as described by Garrison et al. (2000) included: Comfort, Conversation, Collaboration, Coping, Connection, Corroboration, Cooperation , Contribution, Camaraderie, and Caring.

The C of Comfort – in the space and within the group. Even in the first live tutorial, on January 14, 2010, some felt quite comfortable in the space. There were fourteen who attended. For some unknown reason, even though exactly one-half of them had been correctly registered in the participant roster under their personal pseudonyms, the other seven had been put in as Rocky 1 to Rocky 7. However, not wishing to suggest that this kind of 'Rocky' start was an omen for the project, I, instead, made some spurious comment about the participant roster looking a bit like a list of movies. Well this broke the ice a little bit and then one of the Rocky's said:

Rocky 7 Hi this is Janette …. Yes, I am on my son's computer

Rocky 3 yeah we gotta do something about all these rockies

Rocky 1 Just don't get rid of the real Rocky... which is me!

About twenty-five minutes into the session, Rocky 3 began to write on the Whiteboard while I was answering questions from the rest of the group. Eventually, using screen captures, this particular Rocky put a coffee cup on the Whiteboard for all to see. Suddenly, others started to place things on the Whiteboard, asking questions on how to erase, how to get symbols, how to . . . etc. etc. At the end of the first live session, some of the parting comments were:

Ruth I loved it. Tonight is the first time I enjoyed math. thanks Julie

Rocky 3 it was fun

Rocky 7 Thanks Julie, looking forward to the homework.

As mentioned earlier, even though the tutorials were rather late in the evening for the participants living on the East Coast, almost half of the participants in total wanted to stay an extra hour after the tutorial had officially ended at 7:30 pm Edmonton time. This was the case in many of the tutorials throughout the project. I would always give the option of staying a little longer in case anyone wanted to clarify some issues. Of course, those who could not stay or did not want to stay did not miss anything because the recordings could be watched at a later date. Nonetheless, a surprising number stayed – sometimes up to an hour longer. I attributed that to participant engagement and a general feeling of comfort with both the interFace and the group interaction of our virtualFaces. People were learning something and were having their questions answered.

Discomfort – in the space and within the group. One of the very few discordant notes within that community had had to do with the diversity of mathematical backgrounds in it. For this reason, as mentioned earlier, I decided to split the group into three streams based on goals and expectations of the participants. I wanted each student to maximize their experiences in terms of comfort, belonging and learning. In the larger group, I could not both go slow enough for some and fast enough for others.

That proved to be a good decision. The very next day after announcing this decision, I received the following email from one of the participants:

> I am really interested in this because I could really feel the tension of those who were more advanced in the course and then I would hesitate to ask or further my questions. I have an understanding of some things but sometimes I am missing one basic concept that makes moving forward confusing. Sorry if I was the cause of slowing down the group with my questions this week. So thank you for this chance to work at my level and focus.

Had I not divided the group into its interest streams, this level of discomfort felt by some in the live sessions, could have jeopardized the entire learning process for certain participants. In fact, for Chloe, this was definitely a factor in her ultimate disengagement from the learning of mathematics and helped to destroy any sense of community she may have formed with the group during that time. Unfortunately, even after the division into streams, she could not recoup her enthusiasm for the project objectives which she had previously exhibited. The elements of belonging (in this case, mathematically) and having influence (interpreted as importance or genuine place in the group), as McMillan and Chavis (1986) had suggested, were integral to members developing a sense of togetherness. Chloe had lamented being ‘different’ from the others:

> I am referring to being asked and not able to correctly answer questions in a live tutorial when there is evidence that the others are not having difficulty. You, of

all people, know how much I yearn to be like everyone else. CHLOE (REF: February 17, 2010 email)

The C of Conversation/Communication. I began each live tutorial with a welcome to all those who were there. This was extended to any of the late comers as they arrived at various times throughout the tutorial time period. This simple, but sincere, gesture was noted by quite a number of the participants in the surveys. Eventually other participants began welcoming the late comers to the tutorials. Most, after a bit, always said goodbye to everyone who attended that evening.

As early as the third tutorial, on January 25, 2010, participants had had enough sense of community with one another to break out into conversation between themselves, of which I had no part. For example, Carl and Anna had this spontaneous discussion:

Carl In other words, negative exponents always express fractions?

Anna reverse fractions

Carl It'll work? …. Right ….. "reverse fractions" - you mean reverse of what they are in the first place? Something like that?!

Anna thats what I meant

Carl thnks

Anna np

The barrier of merely formal interaction had already been broken.

The sounds of silence. Silence is a very powerful communicator. There is a saying that actions speak louder than words but so too can silence. Everyone, no doubt, has experienced awkward silences in a group setting when no one knows exactly what to say to break the so-called deafening silence which looms as an impenetrable wall between the individuals in it. Silence by one may generate conversation from others. Silence by two or more participants may be born of some common connection one to another. Silence in both instances may also be indicative of disconnection to the group itself. There are various kinds of silences which can occur in educational settings. During the project tutorials, they displayed themselves in various ways. The most obvious was the absence of an attendee’s name in the Chat Box.

Silent observers can communicate many things. Sometimes it is simply the result of quiet reflection or abject fatigue. In other instances, silence may be indicative of a person's feelings of inadequacy to the task at hand, nervousness to speak in group settings, or profound resentment of an offence taken. Introvert personalities often remain silent in group settings. Context is paramount in the interpretation of silence. The kinds of silence which expressed themselves in our virtual classroom included:

- **The Silence of Reticence**

Marie admitted: **"**I was less likely to ask a question – however there were more brave participants – and my questions were asked, and answered" (REF: LMS).

- **The Silence of Frustration**

Ursula was generally not reluctant to ask questions. However, over time I began to notice that her silence in the group sometimes meant that she did not feel confident enough to speak to the topic – particularly if others had seemed to grasp it. This was evident in one exchange between her and Sal during the February 22, 2010 Group 1 session.

Ursula ok that is what I thought …….. just wanted to make sure

Sal oh ok sorry

Ursula don't be so keen

Julie He's a keener alright.

I went on to explain that I would be pulling everything together soon using graphs.

Ursula I will be pulling something

Sal what?

Ursula hair out

What was noteworthy was that, after that seemingly capricious bit of banter, Ursula never spoke again until the end of the session, almost one hour later, when she simply said 'Goodnight.'

At the time, I didn't think anything of it. Silence, at least on the part of students learning mathematics, is not uncommon in group settings. In retrospect, however, maybe her last comment was an attempt to tell the rest of us about her increasing frustration at not being able to cope with the topic when others seemed to be far in advance of her level of expertise at the time. Her silence thereafter was, perhaps, her way of communicating her own feelings of inadequacy to the task.

Nonetheless, pushing frustrations aside, Ursula continued to attend the tutorials in solidarity with her goals and her classmates. She did, however, make a friend that night. Sal and Ursula had many conversations over the ensuing months during the live sessions. They had a personal rapport which was evident. Her commitment to the learning of this difficult subject was palpable. I sometimes thought of Ursula's quiet teasing interludes of interaction as a complement to Sal's ebullience.

- **The Silence of Comfort and Common Purpose**

At the beginning of the April 6, 2010 live tutorial before the others had come, Rocky and I began to play around on the whiteboard drawing symbols and figures independently – in complete silence. This kind of silence was born of comfort with one another's presence; this was not an awkward silence. The episode was spontaneous, each participant introspective but linked by a common mind set.

The C of Collaboration. Collaboration is a group process of sharing information and working together to a common end. Two such collaborations formed between participants in the project. One was relatively brief. After the January 21, 2010 tutorial, Sal and Janette stayed late to work together on the Whiteboard. She may also have worked with him on his Google Docs web site, but I was not privy to that information. The second was a mathematical collaboration between Sal and Rocky.

In the February 8, 2010 Group 3 tutorial, Sal and Rocky had come early and had started to chat together. Sal was a goldmine of information and Rocky had a lot of motivation to glean as much knowledge as he could in the time allotted.

Rocky I am thinking of problems where you have one dimension of a triangle and one angle and you have to figure of the other angles and sides.

Sal oh those are fun problems I am pretty good with those ……. i can put some trig stuff up on the web site roicky …….. rocky i mean

Rocky thanks, Sal.

Sal trig identities are even more fun

As I found out much later, Rocky and Sal did work together on trigonometry through Sal's Google Docs web site.

This was not the only instance of these two participants collaborating with one another. Sal and Rocky had come early to the March 8, 2010 tutorial. When I arrived they were chatting about the merits of using graphing calculators and the merits of learning how to graph functions by hand. On March 22, 2010, when I entered the room for our session that evening, I found these same two participants working through a factoring problem together on the whiteboard. As far as I am aware, collaborations in the group often had Sal as a part of them, no doubt because of his relative expertise in that setting.

The C of Coping – as a form of Commitment. In my mind, one of the tests of a community is what happens when members encounter difficulties. In the case of this particular project, one of the major threats to the formation of a community was the lack of motivation to continue if and when hurdles had to be overcome – hurdles such as not having enough time to attend or not having enough energy and concentration to keep current with the mathematics being taught. Generally speaking, most do not find mathematics entertaining or relaxing. It is hard work and people have to be highly motivated to pursue mathematical knowledge. I would venture to suggest that everyone in that final group of participants had had to cope with some life circumstance or other just to finish their journey. Many showed an exceptional amount of courage in doing so.

- **Coping with work and family obligations**

Most of the participants in the project talked about work and family commitments. Some worked shifts; others traveled extensively for their jobs; some did both. One had six children to care for; another had an autistic child; a third had an aged dying father; a fourth had chronic health issues. Virtually all of them were returning to study mathematics after many, many years away from a school setting. Yet they came to these voluntary evening online tutorials week after week. They coped with the family matters and the work obligations in ways which enabled them to attend. I found this kind of commitment, despite all the demands for their time and attention, quite remarkable.

Ursula had arrived about half an hour late to the March 29, 2010 tutorial. Sal had showed up even later. While I was explaining the algebraic side of finding asymptotes of rational functions, Sal suddenly interjected (maybe he was bored; maybe his mind was wandering; maybe he was trying to ward off fatigue):

Sal i get to go home tommorrow night and see my wife and kids **–** 5 dys at ghome

Julie And then you go back?

Sal yes **–** back to work for 24 days

Ursula with all the traveling does any place feel like home?

Sal its difficult to maintain roots ….. very hard on a family

Ursula I just worked long hours and many days in a row and did not see my husband for weeks

Sal so you know Ursula

Ursula it makes math more fun

Sal I come for the math but also the facetiuos banter

Rocky what does the cup up mean?

Rocky always brought the conversation back to mathematics. It seemed that, despite the obvious fatigue both Sal and Ursula were experiencing that evening, each came, even if late, to connect to the community of which each felt themselves to be an integral part. They were getting to know each other a little bit more and liked to be there – not just for the math but for each other. Rocky sometimes took part in the banter but mostly watched until he decided to call us all to order again.

- **Coping with distractions**

During the February 16, 2010 Group 3 tutorial, Ursula excused herself momentarily. She had had to go and tend to a little one who was out of bed. She returned some minutes later. Such distractions do not occur in regular classrooms. She wasn't obligated to return to the session. We all would have understood why she had been detained. But, no, she came back to join us and stayed to the end.

Another unusual incident occurred during the March 15, 2010 Group 3 tutorial. About one and a half hours into the session, Ursula simply dropped off the roster. It is true that in an online environment a participant can disappear suddenly without a trace. In some cases, no one will even notice it – at least, not right away. We would never have known what had happened if Ursula hadn't suddenly 'beamed up' again into our midst a few minutes later. The town where she lived had just had a blackout. She materialized on the Chat Box with an apology: "sorry the whole house crashed ….. the whole house, tv, lights, ….. it's all good now."

- **Coping with the frustrations of learning mathematics**

Probably everyone, at some time or other, has had to cope with some anxiety or frustration when learning mathematics. The only difference in this particular setting was that none of the participants were under any obligation to stay. They had volunteered.

During the February 25, 2010 Group 1 tutorial, some of these cares had bubbled to the surface.

Ruth It's trial and error but I'm learning.

Janette I am in a similar boat. It is crazy and I never seem to ever get the head above water - maybe my nose

Ursula I think I am ahead and then the bomb drops

I did have concerns that some, at the first hint of trouble, would silently disappear without a trace. That actually never happened. Those who did withdraw most always wrote to me to explain why.

- **Coping with fatigue**

Sal, from April through to the end of July, fought fatigue during the evening classes. In mid-to-late March, he had mentioned that he had worked fourteen (14) hours per day for the previous two weeks, without a day off. Another time he had explained that he had been working thirteen (13) hours a day. After finishing work, he then had had to get back to his digs from camp and grab something to eat before he could settle in to the session. The fact that this project was not a for-credit program attests to the commitment Sal felt toward his own learning and to his fellow colleagues. He coped with his physical and mental fatigue by playful teasing with those in attendance, by witty remarks, and by using plays on words in the mathematical content. This tactic kept everyone entertained and probably even contributed to their engagement in the learning process, including his own.

He, of course, was not the only one experiencing some kind of fatigue during these sessions. For some, the tutorial sessions were rather late in the evening. For example, the earliest an evening tutorial began was at 6:00 pm Edmonton time. For the participants in the Maritimes, this was 9:00 pm. I would venture to say that pretty well everyone in the project had had very busy schedules, work commitments, family obligations and even other course work to complete. Their tenacity to stay with the project was a testament to their dedication and commitment to their learning. This alone was a common characteristic of this group and may have contributed to their patience with one another and, ultimately, their sense of community.

The C of Connection – to the others. A person's feeling a sense of belonging to a group is sometimes both a spontaneous and instantaneous response to another person or a situation. You just know that you relate on a deeper level to another person or you just feel good about being in a certain place. These are the covert links to the space, to the group and to the individual. However the connections which are built over time between and amongst individuals and places are the ones which weave the fabric of community. These take the form of reaching out to others in an effort both to be understood by the other ones and to understand them – and sometimes even offering to extend the personal relationship outside of the meeting space.

The strongest connections formed between participants in this project took place in the Group 3 tutorials. In Groups 1 and 2, the connections were more formal, on the one hand, and more tacit on the other. Therefore, the evidence of the communal ties formed during this project will be illustrated mainly from the Group 3 interactions.

Both Ursula and Sal had, from the beginning, always sought connection with the group and participated in the collective dynamic but in entirely different ways. Ursula wanted to be there every time even if the mathematics was beyond her level. This was particularly true in the Group 3 tutorials. Her philosophy was that eventually some of it would sink in. In that setting, she didn't ask many questions but she reached out to others in the form of playful banter with those in attendance. Sal, on the other hand, actively engaged in the subject matter and tried very hard to draw others into this engagement. His web site, created specifically for the other participants, was a case in point. He also sought a deeper connection to the other participants in the group.

February 1, 2010 and **February 2, 2010 tutorials**

When Rocky said goodbye to the group, Sal called out: "Email me Dude."

March 8, 2010 tutorial

Sal suggested to the entire group to meet outside of the online space. He announced that he and his family were going to be moving to the Edmonton area around the end of April. Then he said to Rocky:

Sal maybe we can get together for coffee and learn some math

Rocky That's a good idea.

Sal maybe we could do one of these Monday night sessions together …. That would be kind of fun …. we could really bug Julie then.

Rocky (giggle) …. and gang up on her

Sal yeah …. Gang up on her **–** I think I make things difficult enough for Julie.

Rocky I think she appreciates it.

Sal you think so ….. I hope so

This short exchange showed the deepening feelings of comfort, the increased desire to collaborate and the light-hearted fun each member of the group had had with the others.

This kind of connection among the participants was a hallmark of this group's interaction one with another. At the very end of that evening's tutorial, Sal, knowing that Rocky *always* asked for homework, mischievously suggested:

Sal give us lots of homework …… but give Rocky more .

What had begun as a virtual meeting of minds had transformed into an offer of a face-to-face meeting of personalities. Because Sal's plans to move to the Edmonton area did not transpire, the physical meeting did not, unfortunately, take place. It should be noted that, in any event, only Ursula could not have joined the face-to-face meeting because she lived some 2500 miles away.

Though Rocky was very much an independent learner ('Having other students in the class was not helpful to me' REF: LMS), the efforts of the others to connect on a more personal level did have an effect on him. This came to the fore during the April 6, 2010 Group 3 tutorial. Rocky and I were the only ones present. The whole tutorial had been centred around mathematics. About an hour into the session, he suddenly said:

> It just occurred to me that we haven't had Sal and Ursula to keep us entertained tonight, have we?

By April, when one member of the group was not present, there was not only a glaring blank on the attendance roster (not surprising in such a small group) but also the sense of an emptiness left by her/his lack of cognitive and social presence at that session. It wasn't quite the same when all were not there. Both Rocky and I, who attended everything all of the time, had a sense of Sal and Ursula as persons – even in this virtual venue. Of course, I missed their not being there. In addition, I acknowledged to Rocky that when they were there, they did entertain us royally – not to mention help us learn.

In early June 2010, when Sal started to attend the tutorials again after an extended absence, the connection and comfort between Sal and Rocky in the live sessions was open and evident. The comfort level was high; connections had been established; and camaraderie was now an ever present part of the interaction within this close-knit group. I think that, in every dimension of learning, we had formed a Garrison et al. (2000) Community of Inquiry in which the sliding scale of student /instructor role was moving from one to the other depending on the topic and the direction of the learning. Rocky or Sal increasingly set the direction and the pace in the tutorials. I simply moved back and forth with them, filling in when either needed assistance. This had actually been taking place all along but the longer we met together, the smoother the transitions became because of the deepening comfort-level with one another and the subject matter. Collectively, we meshed at the intersection of our individual teaching, cognitive and social presences.

During the June 21, 2010 Group 3 tutorial, another event of significance occurred. Sal put forth yet another invitation to Rocky to meet outside of the virtual world in which each of us had come to know the other.

Sal Rocky and I have yet to do coffee

Rocky I think we should. I think we should somehow. I don't know if we would have to wait until the project is over or not but I would really like that. Here's my phone number, Sal.

The circle of connection was now complete. It had extended itself beyond the borders of the virtualFace classroom and into a face-to-face meeting place.

The C of Corroboration – from the others. Most everyone, no matter the age or gender, needs some kind of positive reinforcement in validating their existence and/or their accomplishments in groups which are of importance to them. Children seek corroboration from their parents; students look to their teachers for feedback on their learning; members of social groups desire acceptance from other members; research scientists require validation from their colleagues; and the list goes on and on. Desiring the feedback of others to test the connection to others or provide some evidence of acceptance by others is a part of human existence.

It is also generally accepted that negative reinforcement in learning situations does affect the process of learning in many individuals. It can either motivate the student to perform better or it can discourage him/her from continuing to engage. In some extreme cases, it can have a deleterious effect on a person's notion of self-worth.

Knowing this, I have always tried to encourage my students by telling them when they did well and when their work was exceptional. However, in this virtual classroom, I made a special effort to corroborate their achievements because they could not see the subtle nuances of teacher-pride in their successes displayed through body language – for example, a smile, a thumbs-up, a pat on the back, and so on. Therefore, the fact that I constantly gave positive feedback to the learners, both privately through email and publically during the live sessions, was not exceptional.

However, even some of the participants corroborated the achievements and the importance of others being in the group. Marie and others had mentioned how fabulous Sal's web site had been. Rocky had acknowledged the importance of Ursula and Sal being in the group sessions. During the June 21, 2010 Group 3 session, Sal had commented: "i like rocky he has questions it helps to build my understanding too." Sal's contribution to Rocky's learning had been going on almost since the beginning of the project. At this stage of the project, Rocky's questions were supplementing and complementing Sal's own viewpoint and understanding of some topics. The often dual perspectives precipitated by the 'opposite' learning styles, on top of my own presentation, created a rich, diversified learning experience for all of us. It also served as an inspiration for further investigation. At the end of the tutorial, Sal said he was going to make a You Tube video on conics. When Sal announced that, Rocky said that he was really looking forward to it.

In that same June 21, 2010 session, Rocky and Sal mentioned that each had wanted to do a Calculus course with the mathematical base they had hoped to acquire during the project. Sal had come into the project with that specific goal but Rocky had not had the courage to even suggest it. Rocky had come into the project calling mathematics that 'strange land.' On that evening, it was more a *home*land to him. It had become Rocky's *place*.

The C of Cooperation – with the others.

- **Cooperation by accommodating the needs of others**

In mid-February, Sal wrote to say that he would be working seven days a week, ten hours a day for the next six to eight weeks, and would not be home until after 6:00 pm. Because of this, attending any of the tutorials would be difficult. Because Sal could no longer make the 7:00 pm start time for the tutorials, Rocky was agreeable to starting them later so that Sal could attend the entire session. More than that, in June when Sal started attending the sessions again, Rocky went the second mile to accommodate Sal's busy schedule. Rocky agreed to another change in the start time: from 7:00 to 7:30 pm because Sal was finding it difficult to get home before 7:00 pm after working thirteen hours (13) per day.

- **Cooperation by being prepared for the next session**

My assessment was that all of the participants cooperated with me through their attendance and their active participation in the teaching/learning process. However, there was a point of contention by at least two members of the group who felt that some of the group held others back in their learning by not having their homework done by the next class. This would necessitate my having to go over some of the questions in more detail than otherwise necessary – thereby resulting in fewer new topics being covered in that tutorial. Over time, this can lead to a significant, for some, reduction in the amount of mathematics presented.

The C of Contribution. Even if it is not possible to contribute to a group at times, it is also the willingness that counts. Anyone who asked questions, offered answers, and engaged in dialogue with me, the instructor, was a major contributor to the sense of community during any of the live tutorials. These actions solidified the common purpose of the group and enhanced the communication within the group. By engaging themselves that way, tacit connections were formed between them. The opening up of self to others, thereby risking being wrong in front of the group and exposing vulnerability, was one of the most potent evidences that a community of participants was slowly forming even if we would never meet face-to-face.

However, there were a few instances where the contribution went beyond general participation. In one instance, Rocky mentioned that Sal had asked him if he knew of any good Pre-Calculus books. Rocky provided the name of one which he had found useful (REF: March 1, 2010 Group 3 tutorial). In another case, I had asked the group for ideas to make the RMC web site better for learning. Rocky was also particularly helpful in that regard. During the March 29, 2010 session, he mentioned some and then later emailed me the URL's of a number of math web sites which had been beneficial in his learning of the subject matter.

However, by far the most ambitious contribution to the group was Sal's Google Docs web site. It was designed specifically for group participation and for the purpose of engaging and helping others understand the mathematics covered during the tutorials. Moreover, it was created in a spirit of cooperation with the goals of the project.

The C of Camaraderie. Where there is comfort and connection in a group, there is often camaraderie. Of the many instances of playful teasing and repartee exchanged throughout the live sessions, two of them are detailed below.

- **Meanings of a KISS**

During the January 21, 2010 live tutorial, while working through the power rule of exponents, a few of the participants were not following along with my explanation – try as I might. I do admit that exponentials are a mathematical hurdle for many students. So, in order to indicate the usefulness of the exponential notation, I told them that the goal was to keep the presentation smaller, neater, nicer, less complicated with fewer brackets and as uncluttered as possible. The whole aim of the game in mathematics was to keep everything as simple as possible. It had enough complexity in its own right. Sal then interjected with:

Sal yes the KISS rule

Julie Yes, keep it simple ….. What's the double S mean?

Sal stupid

Julie (giggle) stupid?…. No it doesn't. …. Stupid?…. No I don't believe it. …… Keep it smart and simple. That's the KISS Rule.

Sal I like to say smarty

Rocky sweetheart

Ruth I learned it as short and simple

Sal well im a welder ….. so

This banter on the play of words on the acronym KISS was not the only example of the camaraderie that would be pervasive throughout the entire series of the live sessions. It is an interesting rather unique quality of web conferencing environments that participants can 'talk in class' without having to be told to be quiet because the conversation is completely inaudible.

- **The PROSELYTE – how to build conformity in diversity**

During the February 22, 2010 Group 3 session, I had just shown the group a visual, diagrammatic device for simple factoring a quadratic polynomial into two linear factors. This prompted another extemporaneous dialogue between Rocky and Sal which highlighted their two distinct personalities and which demonstrated a kind of friendly tug-o-war of ideas – with each holding fast to his choice based on his personal learning style. Ursula, the third member of the group that evening, remained silent throughout the conversation. Sal, as was quite typical, began the dialogue.

Sal cool …….. thats a neat way of doing it ……. but im still gonna do it my way …..hehe

Rocky That is awsome!

Sal I di like it your way its very cool I will remember it and use it sometimes

Rocky i am converted.

Sal lol Rocky ……. Rocky the proselite ……. lol ……… lol

Rocky i am easily moved.

This incident illustrated that community does not necessarily require conformity to hold the group together.

During the Group 3 tutorials, there was always a bit of chit-chat, banter, and outright teasing. Sal and Rocky were usually on microphones; Ursula used only the Chat Box. However, there were times when all used the Chat Box from time to time, even I – if only to test it for utility. I soon found out how easy it is to make typographical errors. Therefore, in the Chat Box conversations quoted herein, all the errors have been left in place.

I am going to stop there as far as recounting anecdotes illustrating the camaraderie amongst the participants involved in this project. So much more could be said about the depth of community experienced by all in that virtual classroom. As I write, I too feel some nostalgia for those hours of friendly interaction and fun which we all experienced while we were learning mathematics.

The C of Caring. The showing of care or concern for another without body language often finds its outlet through verbal and oral expressions of the connection and community one feels for the other. However, it can also play out in the language of disappointment and anger. As to the occurrences of the latter, none were evident during the live sessions. As far as I am aware, there were no words of disappointment and/or anger exchanged between participants. Those which were expressed were done in emails between the individuals involved. See the sections on 'The BREAKING of Trust' and 'The *VIRTUAL* SCHOOLYARD SKIRMISH' – no witnesses.

During the March 22, 2010 Group 3 tutorial, the members of the group agreed to attend an extra session on rational expressions so that we could move on at a quicker pace. We were collectively trying to coordinate a suitable date and time for all concerned. Each deferred to the other in terms of date and time, making sure that everyone could attend and that the time was not too late for Ursula (because her time zone was two hours ahead of the rest of us). I felt that these were nice gestures of caring for the other – wanting all to be there and at a suitable time for all – a genuine community spirit.

All participants (except the early drop outs) remained committed for at least three and a half months; two stayed almost six months. Why? With one exception, my only conclusion is encapsulated in the essence of 'community,' articulated through the capital C's of both formal and spontaneous interaction AND reinforced by:

1. a group of determined, goal-oriented personalities,
2. the perception that the project was worthwhile in helping them to attain their goals,
3. positive connection with the others in the group, and
4. an engagement with the subject matter.

In short, the enduring sense of community came from the presence of the essentials of McMillan and Chavis's 1986 model: integration with the group, engagement with its activities, fulfillment of personal needs and shared emotional connection.

A Final Observation

The strong commitment to the group of some can generate a commitment in others. Towards the middle of April, when the formal commitment for the project was officially at an end, I offered to continue with anyone who wanted to continue. I would fulfill my promise to walk with them to their journey's end in terms of the set curriculum which had been outlined at the beginning of the project. During the May 17, 2010 tutorial, before we parted that evening, Rocky expressed his appreciation that I had maintained the commitment to the project for him and he promised to press on. This was a declaration that he was extending his commitment as well.

The Adhesives of Community – the Glue that Holds It All Together

All of the fourteen (14) individual components which form the conduit from the symbolism of community to its embodiment will not hold together unless there is a binding connection between instructor/mentor and each participant (the radian disc icon of community) and amongst the participants themselves (the circle of linked human figures icon). It is *trust* that keeps each person in contact with his/her neighbours. It is each individual's connection to the instructor tethered by trust which maintains the cohesiveness of the community as a body of learners. Chloe said it explicitly: "A critical component of teaching is the ability to build trust. This helps to reduce or eliminate fear" (REF: PPS).

The BREAKING of Trust

I have always tried to be trustworthy in my teaching. If I had said something was true, I generally had previously convinced myself mathematically that it indeed was a valid statement. If, during a class, I was asked a question whose answer I did not immediately know, I would say that I didn't know but would look it up or work it out and then tell them the answer on or before the next class. If I made a promise, I would try my very best to keep it. This was no less true in this virtual world than it had been in face-to-face teaching. The binding force of an educational community is the trust built between instructor and students as long as there no abiding acrimony amongst its students. Even if fissures of dissent or disappointment appear in the group dynamic, the community will hold together by the force of the trustworthiness of its leader(s). To some extent, a breach or perception of a breach of trust can be repaired but it always leaves a scar.

The call for more live tutorials on January 28, 2010, as chronicled earlier, led to an offer by Sal to supervise some of them as a kind of math lab instructor. I agreed, in part, because I had really wanted to promote a sense of *community* with these geographically dispersed persons who only knew each other in pseudonym. Getting students to participate in math classes has generally always been a task and most efforts to do so, at least in my experience, have been less than desired. Needless to say, I was delighted at the offer.

The reaction from at least two of the participants was swift and vocal. The very next morning, January 29, 2010, I received two emails: one protesting that there were now two teachers involved, the other querying the qualifications of Sal as a math instructor. It was made clear to me that some of the participants did not *trust* that Sal was qualified to tutor in this setting. I was forced to see the error of my unabated enthusiasm for this venture and quickly backed away from anyone in the project acting as a 'lab assistant.' That, however, was not the only rebuff that Sal had to suffer for his enthusiasm and engagement. The second came from me!

Sal's web site initiative had carried on independently. Responses to it were varied. There were those who had been very enthusiastic about it. Marie, Harvey and Anna all wrote me saying how wonderful this was. There was also a bit of a kickback to it which happened behind the scenes of the project stage of which I had had no knowledge at the time (see 'The Virtual Schoolyard Skirmish – No Witnesses'). However, one incident between instructor and participant created a breach which I tried immediately to repair but I had the nagging feeling that a scar (however so tiny) had been left in its place. The trust was not as implicit, nor as freely given, as before the event.

February 5, 2010

Sal had started to assign exercises to the group of participants who were involved with his web site. Because of the negative reaction to his being a kind of lab assistant in the project when he had come in as a project participant himself, I emailed him saying:

> You may post any of your self-designed exercises and solutions to the web site which you have created, but please don't use this project as a teaching tool for yourself. There have been some complaints that you are trying to be an assistant instructor. . . .
>
> I do not wish to deter your efforts as a student to collaborate with other students but do not assume the role of a certified teacher of mathematics. . . .
>
> Thanks for your cooperation. Julie

February 6, 2010 Sal wrote me saying:

> Sal I won't be able to make it for any tutorial next week. I will try to make it for a portion of them. Good thing you are recording them. Julie

February 7, 2010 I felt that I had been too harsh. I wanted to repair any breach that may have been created by my coming down too hard on him. I wrote back immediately saying that I hoped he could attend at least some of them because I didn't want to lose his input in the live tutorials:

> Sal Your contribution has always been good and very valuable. The only issue has been boundaries. No offence was intended by my last email – only cautionary because of the two complaints. If I seemed too sharp and I may have been, I apologize for it. I think your web site is remarkable and I hope you continue it. Julie

February 8, 2010

> Julie Your right I need to be more aware of boundaries. I apologize that I did step over my boundaries, I recognize it and will keep appropriate boundaries. And no I don't think you were harsh. Sal

It was then, however, when I heard *his* side of the virtual schoolyard skirmish.

The Virtual Schoolyard Skirmish – No Witnesses

February 8, 2010: Sal went on to explain that he had written some of the participants in order to confirm a correct email address to which he could send updates about his Google Docs web site. This is what he related: (for privacy purposes I have replaced each of the participant's names with yet another pseudonym)

> Julie From PARTICIPANT A I got back a nasty reply. I was only trying to let everyone know about google docs and the web site. No one else responded like either PARTICIPANT A or PARTICIPANT B. I get that they may be having a difficult time with learning, perhaps busy schedules and or other stresses of life or whatever reason but I absolutely know I've done nothing to deserve their abuse. I've kept all of PARTICIPANT B's emails and all my emails to her. Your welcome to see for yourself. That she has been mostly abusive towards me in most of her correspondence to me. My wife read those emails and was quite upset with how abusive PARTICIPANT B was. You'll also see in those emails I have been more than civil and encouraging, and I have most certainly done nothing to deserve her abuse. . . . I know enough that you gotta let some stuff roll off like water off a ducks back. I apologize if I seem a little upset, its because I am a little perturbed. Also somewhat puzzled at that kind of reaction. Sal

Sensing his hurt, I responded immediately:

> Sal I have been very supportive of your endeavour as a means of creating come kind of community for the project participants and as an extra source of help and encouragement. I also know that others have appreciated it. This kind of reaction I did not anticipate at all.
>
> Thanks for bringing this forward. I do hope you do continue with your web site. For those having the time and inclination to engage with you and your site, I am certain that it will be a mutual benefit. Don't let these incidents sour you in any way. It would be a great loss for the rest of us. Julie

Sal, in fact, did carry on his web site with the participants as a parallel stream to the project tutorials. I did not interfere nor did I participate. This was a student initiative for other students of basic mathematics. However, when these incidents came to light, I tried to investigate – to no avail. None of the correspondences were made public to me and I myself had not been party to it. Neither PARTICIPANT A nor PARTICIPANT B had contacted me about this exchange of words. I can only gather that neither antagonist had pulled any punches. In addition, neither the reaction nor the responses of the protagonist could be determined.

Nonetheless, I was still uneasy. I instinctively felt that some damage had been done. There had been too many punches in too short a time not to have left a mark. I personally believed that Sal's motives were more altruistic than self-serving. Others, apparently, did not. My intuition on this incident had been right. After that, Sal's enthusiasm and engagement had cooled. He did not attend the tutorials as before. I had made a number of attempts to touch base with him, when he broke his silence:

February 20, 2010

> Julie I have been thinking about it and have come to the realization that the abusive comments of a couple of students did have some small effect on me, but I am and was able to let it run off quite quickly. However your comment that I was using this as a forum to teach was just so far out in left field, that I was surprised by that statement. I did in my excitement probably over step my bounds and I apologized for that but I was not trying to teach nor usurp you. I enjoy math and am an enthusiastic student of math, that enthusiasm spills over into anything I do in life that involves math.
>
> Since that statement I have been more purposefully tepid and somewhat less

> enthusiastic in regards to both the live tutorials and the web site. My enthusiasm was dampened by that statement. My intent was merely to reinforce my own understanding and retention and if someone else is helped in their learning by my process of learning, well, I think that's great.
>
> I thought that if a few people are offended with me participating, I just won't come. And that thought was somewhat reinforced by your statement that I was using this as a forum to try and teach. I enjoy learning and the format of this math class adds to that enjoyment for me.
>
> If I wanted to teach I would be far more patient with beginning topics, but I am not. I want to get to more advanced stuff because I am concerned with my own learning. I am glad we have a few students who are at a more advanced level as well and they catch on quite quickly to new concepts. It is my hope that they will help to bring things along at a more rapid pace so that we can get to some more advanced stuff. I say this so that you know I do not want to teach I just want to learn.
>
> I should have said right away that I was bothered by what your email said, I was trying to forget it and let it go. It was right after that email that my participation in the RMC class changed. After thinking about it today I realize that as well. You can't deal with what you are not aware of and in that regard until I tell you, you remain unaware and unable to address the situation. Sal

Yes, I had been too harsh. The connection between us had been bruised and was still painful. I responded by saying:

> Sal So I was too harsh in my email. I do apologize for that - again. The two email letters about your trying to be another instructor in the class, I guess, made me more sensitive to everything connected with this project. I did notice that you did change at that point - but you still kept coming.
>
> I must say that, prior to that, I had interpreted your enthusiasm and involvement in the true spirit in which it was given - as an enthusiastic student of mathematics. I admire that so much and I welcome it. Because it was misinterpreted by some of the participants as your being a kind of 'show off', I, unfortunately, let that sway my opinion a little - at least, enough, to talk to you about it. However, I was too harsh. I know that now.

> I also know that once bitten, twice shy. So I guess I cannot expect that you will recover your unbridled enthusiasm for the project. That will, no doubt, be the consequence of all of this. However, if you can afford me that one chance you have afforded others, then I would be truly grateful. I value, not only your presence in the tutorials because you have a lot to offer, but also your feedback. There are many lessons to learn about interaction and online interaction in teaching and learning situations. My research is all about that - and my reactions are also part of that research.
>
> Take care, Sal, and keep that enthusiasm for life always. It is a truly rare gift. Julie

NONETHELESS THE TRUST HAD BEEN DAMAGED – but not irreparably so. Bruises do heal over time. Sal did continue through the project until late July, though his involvement was not quite what it once was – mostly due to his extraordinarily busy schedule. I tried on a number of occasions to 'make it up' to him by offering him the opportunity to present some topics during the live sessions. He never refused – but either his work commitments precluded his participation or the other participants did not want to deviate from the curriculum of the project.

I never really understood the negative response of some of the other participants to Sal's initiatives. I can only assume that he was resented for his knowledge which was more comprehensive than the others' at the time. Perhaps, there had also been some covert competitiveness that did not surface in the live sessions themselves. Moreover, enthusiasm can sometimes be mistaken as bombastic behavior.

Reminiscences of the Journey

At the end of the project, I spent many hours ruminating on all the happenings which had occurred over those seven months in that, now far away, world of my own creation. The collective experience was, I thought, a bit like a journey on a space ship – in the cosmos far from our earthy home – with a bunch of strangers having only weightless forms – all having volunteered for the voyage – now pushed together into an insular cubicle of habitation – with the sole purpose of conducting an experiment – involving a specified task – for a prescribed period of time. We could only hear each other and write to each other as we went about our daily work. We had been warned that the experiment may not be successful – but we were all willing to try.

Now, having come back to earth again, all that I had were my memories of it and some logs of our conversations, which, if one wanted to deny the journey's existence, for purposes of disinformation, could have been said to have been made in any recording studio. I had no visual mementoes of the voyage, no photographs for remembrance, no ostensible evidence that I, in fact, had even gone at all. And yet, it was not a dream. It, actually, had its own reality – a virtual reality if you like – which had been transforming for all who had been there. Its effect had been internalized. Its legacy would impact our lives for many years to come.

CHAPTER 7 LOOKING BACK to FACE the FUTURE

As previously mentioned in Chapter 6, at the beginning of the project, I was so busy trying to make the transition to the online venues as seamless as possible for all concerned, that I didn't pause to wonder how many of those first eager twenty-one volunteer participants would finish it. Well, in fact, eleven of them did persevere: Rocky, Sal, Chloe, Bill, Marie, Ursula, Janette, Ruth, Abigail, Georgiana, and Clarissa. Of those eleven, all but one or two of them had achieved their immediate goals, either by the end of the project itself or in the ensuing few years following it.

Shedding Light on the Research Questions

My primary research question had always been: what would be the lived experiences, both cognitive and affective, of such a group of adult participants taking a fully online program in developmental mathematics, without any face-to-face contact or means of support?

Some Thematic Analysis

The narrative research inquiry and analysis of the field texts of the case study followed two themes:

1. THEME 1: There was to be an investigation of human self-interaction towards the learning of mathematics. The framework would be within the context of the teaching and cognitive presences in the Community of Inquiry model proposed by Garrison et al. (2000).
2. THEME 2: There was to be an investigation into the evidences of social presence felt by the participants in that learning environment leading to the formation of a meaningful community with its four essential elements of membership (belonging), influence, integration and fulfillment of needs, and shared emotional connection (McMillan & Chavis, 1986). Sub-themes which emerged from the various texts were issues of self-identity, coping with the environment and the subject matter, and a personal sense of belonging to the group.

REFLECTIONS on THEME 1

Would there be enough salience of both **cognitive presence** and **teaching presence** so that the learning of mathematics online was considered to be as effective as in face-to-face instructional situations? How would the participants compare the two learning environments? Did it simulate the traditional classroom experience? How was it the same or different from other learning communities?

The case study, chronicled and hermeneutically analysed in Chapter 6, was an attempt to articulate the kaleidoscopic core of both cognitive and affective interactions and responses of those in the group who had engaged with the environment, with the subject matter and with each other. The conclusion I drew was that our group of twelve (12) members was really not very different from other learning communities. We did have our drop outs – ten (10) of them. We did have our issues: diversity of mathematics background, lack of time, idiosyncrasies of technology, the correlation of learning style to learning, and inculcated emotional baggage. We did have our successes (Chapter 6, subsections “The Personalities and Their Stories“ and “Analysis of the FORGING of a Virtual Place“). We had our fun times (Chapter 6, subsection “The C of Camaraderie”). We had our tragedies (Chapter 6, subsections “Anatomy of the BREAKING of a Virtual Engagement“ and “The BREAKING of Trust”). We did have disagreements (Chapter 6, subsection “The Virtual Schoolyard Skirmish – No Witnesses”) but we also had times when we truly appreciated one another (Chapter 6, subsections “The C of Corroboration” and “The C of Caring”). Our individual personas and personalities did impact themselves through all the media in our synchronous environment and through our correspondences. We had all become somebody’s in a no-body world. Our language had resonated our inner stories. All of the essentials of a classroom (cognitive, teaching and social presences) had been salient in our midst. All participants, with few exceptions, achieved academically what they had set out to do. In the final analysis, yes, the project experience had been a veritable classroom. Almost all had compared it favourably with a good face-to-face classroom experience (Chapter 6, subsection “Participant Assessments of Educational Place in a Virtual World“).

I had also asked: Could mathematical thinking be fostered and developed towards a deeper understanding of mathematical ideas in such a setting? The conclusion was, to my great surprise, “Yes!” Chapter 6, subsection “The Teaching/Learning Dynamic: a System of Differential Equations” details both Rocky and Sal’s cognitive journeys into the world of higher-level mathematical thinking. Both were empowered by an individual and collaborative, passionate engagement with the subject matter. Both had reached and had extended their horizons. Both had transformed our online space into their personalized Burbulean (2006) *place* of learning (Chapter 6, subsection "Analysis of the FORGING of a Virtual Place").

REFLECTIONS on THEME 2

Could there be enough **social presence** generated by the members of a group in such a virtual learning space so that they felt part of a meaningful community? If so, what human interaction ingredients contributed towards the creation of this sense of community? How enduring could it be?

All of the essentials of community formation had been witnessed or felt during our times together – both the positive and the not-so-positive human responses to interaction. See Chapter 6, subsections "Cohesive Components of Community – the Conduit from Container to Embodiment" for detailed accounts of both symbolic and spontaneous interactions within the group which had contributed to the formation of the four essential characteristics of a meaningful sense of community (McMillan & Chavis, 1986). The fourteen (14) Capital-C's outlined in that subsection were the evidences of this. Each participant either tacitly or openly expressed a sense of membership or belonging to the whole (Chapter 6, subsections "The C of Comfort" and "The C of Cooperation"). I made every effort to involve each participant in conversation, dialogue and decision making (Chapter 6, subsections "The C of Conversation" and "The C of Corroboration"). Each participant had been encouraged to ask questions, make suggestions and even participate in some presentations so that each would feel that she/he was an important component of the group's learning dynamic (Chapter 6, subsections "The C of Contribution" and "The C of Collaboration"). Those who had engaged in the session discussions, had done the homework and had asked for solutions would have, no doubt, experienced some integration into the group's activities and a fulfillment of personal needs through their interaction with them. The Chapter 6, subsections, "The C of Caring" and "The C of Camaraderie," highlight some of the internal, but shared, emotional connections which came to the surface between certain of the participants.

Not to be ignored or disregarded were the incidents of disconnection which also formed part of the fabric of meaningful communities. See Chapter 6, the subsection entitled "The Adhesives of Community – the Glue that Holds It All Together," for incidents of participants who withdrew from the group and why these connections had been strained through the interaction with either the participants, the environment or the subject matter. It must be noted that this ungluing of communal connections is a very real part of having formed a sense of community. The trauma of the experience exhibited by those participants was evidence of that. Had they not felt a bond to the group, the withdrawal would not have been as traumatic or painful.

So, the answer to the research question was: Yes, enough **social presence** was and, in the future, could be generated by the members of a group in such an online environment so that they did and could feel part of a vibrant community. I attributed that to participants' engagement with the subject matter and a general feeling of comfort with both the interFace and the group interaction of our virtualFaces. Most of the participants interacted enough and attended with sufficient frequency so that an almost empty space metamorphosed into an inhabited, energetic place of learning and comfort. People were stretching the horizons of their mathematical knowledge and they were even having a bit of fun doing it. What was important was that most were having their expectations fulfilled and, in the end, did achieve their goals.

The only difference was that we never could or would see each other or meet after class at a local café or pub to continue the conversations. This was the factor which would preclude our being able to determine how enduring such a virtual community of learners could be. I cannot speak for the others, but I do know that even today, some three years later, I still have some kind of email contact with four of the participants in that 2010 odyssey.

The Importance of Place in a Virtual World

The importance of a mutual meeting place for a community of inquiry had been highlighted by various researchers (Wood, 2013; Graham, 2012) as being paramount in the learning process. In addition, both Dewey's (1916) definition of communication as the result of mutually shared experiences and Gadamer's (1989) notion that we acquire knowledge by a mutual fusion of horizons through conversation had suggested that a meeting place for dialogue and debate constituted an essential aspect of a teaching/learning environment. That is why it was so imperative for me to investigate and determine the choice of the virtual stage – or the kind of CONTAINER where we would meet and mingle – very, very carefully. It had to be conducive to both conversation and visual presentation for higher-order learning to occur. See Chapters 2 and 4.

Educational Web Sites: Love 'em or Hate 'em Why?

Because a pedagogical web site without live tutorial support is a rather insular learner-negotiated study space, before the project even began (see Chapter 2), I had already anticipated that there would be those who would engage with it and those who would not – in part, due to the absence of a virtual instructor-presence. That pre-cognition turned out to be true but it came with a surprising twist.

From the raw statistics alone, six of the eleven liked the site itself without reservations. Three had positive things to say about it but made recommendations to improve it. One did not engage with it at all. One actively disliked it, at least at first. By contrast, the self-assessment tool was ignored for various reasons by six of the group participants. Two actively liked the tool. Two could see its potential but had some comments about its not-so-desirable characteristics. One had a lot of negative things to say but still could anticipate how it could be used as a positive force for learning the subject if the wrinkles were ironed out. In the final analysis, no-one disliked the tool so much that they felt it had no place in a learning space of this kind. On a Likert scale, the experiences with both venues of the web site appeared to be much the same.

All of my investigations surrounding on-screen visuospatial and navigational issues, the importance of aesthetics, and the absolute necessity of considering the cognitive load of computerized learning objects (as well as entire environments) paid off in spades. It had been very important to me that the site was easy to navigate and intuitive in the sequences of learning topics. I had carefully constructed the tutorials in 'bite size' chunks of learning so as to keep the cognitive load as light as possible (not an easy task). We are, after all, whether we admit to it or not, a 'bits and bytes' generation of learners.

Moreover, I had incorporated both colour coding, anecdotes, historical brush strokes and splashes of animation into the presentations. I had even added some documents on meta-learning for those who felt very intimidated or apprehensive about the learning of mathematics in general. In addition to all of that, I had thought it important to supply links to a self-assessment tool which correlated with the topics on the web site. Only one participant made mention of investigating any of the meta-learning documents. Otherwise, everyone else primarily noticed the structural components of the site. Comments about the RMC web site itself included: well organized, easy to navigate, user friendly, clean presentation, easy on the eyes, comprehensive, logical, informative, well sequenced, provided alternative ways of presentation, interactive, lots of resources and supporting materials, always there for reference. (REF: the three (3) project surveys: PPS, FIS and LMS)

Those who had used the self-assessment software said that:

- It had helped them to guage their progress;
- it had enabled them to test themselves at their own pace, without the pressure of a timed test, and with immediate feedback; and
- the hints and explanations provided had helped in their learning.

To my delight, one of the participants commented:

> I have visited a lot of internet math sites and this one the [..] Remedial Math Centre site is, by far, the very best. I use the tutorials on RMC in conjunction with a few good search engines and I find that method very good. (REF: LMS)

Of course, I was delighted that someone had found this site very good for learning mathematics but I also understood that the learning style of this participant had matched my top-down approach in the structure of the interactive tutorials and the navigational layout of the site. What had become apparent to me was that self-regulated online learners prefer web sites which somewhat match, at least in part, their preferred learning styles (see Chapter 6, subsection “The Teaching/Learning Dynamic: a System of Differential Equations ROCKY’S DYNAMICAL SYSTEM”).

There were, of course, recommendations for the improvement of the web site and the self-assessment software. I had solicited them because I needed the input of these participants to give me clues as to how this could be done. Until the beginning of the project, I had only looked at it through my own biased perspective. As mentioned in Chapter 5, I agreed with Gadamer (1960, p. 295) that “Hermeneutic work is based on a polarity of familiarity and strangeness.” Therefore, I needed the individual “I’s” of Connelly and Clandinin (1990) for validation. These other “I’s” told me the following: (All are direct quotes, with a few corrected typographical errors, from the correspondences or completed surveys of the participants without reference to specific persons.)

- I found the site very basic; however, it provided lectures by topic and the assignments, and that is most important.
- For a quick review it would be OK but when I am actually learning this for the first time I think it would be easier if it would be more repetitious.
- more audio …. to hear things explained
- I think a study guide or text would also help.
- Some sort of glossary, for an entry level course, would be helpful.
- list recommending text books, instructors' notes, external links
- ways of getting to something specific (a search engine)
- I would like to be able to print off all the module to have in a book.
- Preferred hard copy over e-text.
- I would have liked a more traditional means of testing and exercises such as graded individual and group assignments as well as graded written tests.

I felt that all of the suggestions, particularly the suggestion to have all of the tutorials voice enhanced, were valid and even highly desirable. The preference of a hard-copy text could be accommodated in a course setting blended with the live sessions but not on the web site itself as an open accessible place of review for University students and staff. The inclusion of formal tests could definitely be a part of any online learning experiment but this project was not intended to have the formal structure of a course. I purposely omitted a formal testing component. It was to be an adventure in various forms of online learning without the pressure of quantitative assessment. I was, however, flattered that some of the participants did consider the presentations to have the cohesiveness of a course in mathematics.

Issues of web site learning. Another issue which arose about web site learning had to do with those who preferred paper texts over e-texts for reading. Janette, Clarissa, Chloe and Marie all preferred to study from hard-copy materials, rather than online expositions. Janette had explained that she worked all day on a computer and, therefore, preferred paper-texts when studying in her 'spare time.' Clarissa explicitly asserted:

> I wanted to print out the "in a nutshell" section for myself for each topic because I don't like reading on the computer . . . but I would prefer to be able to sit somewhere more comfortable and read through it. (REF: PPS)

So did Chloe: “I am well aware that I am a print based learner” (REF: FIS) and Marie: “I miss having a text for reference. I find having pages printed rather disorganized and messy” (REF: February 2, 2010 email).

Synchronicity: Must It be a Part of the Paradigm?

The synchronicity of presentation of the subject matter with an actively present instructor – even if only virtually so – was considered a desirable part of the online venue of learning. In fact, I have concluded that, whenever possible, synchronous communication should be a media layer of every online learning environment in a subject like mathematics. Whether the ‘live’ is sequenced at regular intervals over a set period of time or is provided as an option to drop-in extemporaneously would be a matter of choice or convenience.

Ten of the eleven participants liked the live sessions. Some of them were unreservedly enthusiastic about the huge role these live encounters had had in their learning of a subject which had previously given them a lot of anxiety and concerns. One minor criticism had had to do with using the Chat Box but that was a matter of choice. All participants had had the option of using a microphone. However, one of the participants reacted so negatively to the synchronous sessions that she almost quit the entire project over the trauma it had caused her. Her vitriol at the time was sharp and pointed. (See Chapter 6, the subsection entitled “Anatomy of the BREAKING of a Virtual Engagement”). Despite all of that, she did qualify her dislike of the medium.

> The online tutorials were a detriment to the learning process for the reasons stated . . . . Perhaps if I had the ability to keep up and understand I would feel differently. These online tutorials (when I was still attending) actually de-motivated me and created a feeling of humiliation when it was obvious that I was not keeping up. (REF: LMS)

The archived recordings of the live tutorials were also very much appreciated and used for catch up and review purposes. This feature of online learning is what makes it advantageous, in some cases, over learning in a face-to-face situation. In regards to the role the live synchronous sessions played in the participants’ learning of mathematics, this is what they said:

Bill In terms of the live instruction, I find that it is very useful and I do believe that this is a venture that [the University] should continue with. (REF: by email, February 24, 2010)

Janette I just wanted to say that I loved the tutorials and the interaction with you online. (REF: LMS)

Georgiana It was great! I do not think I would have learned as much or would have been as committed to the study if I did not attend the tutorials. (REF: LMS)

Chloe There is also no consistency as to who is in the class. Add the anonymity and there is no cohesiveness. (REF: LMS)

Abigail

> I can't say enough about these tutorials. The pluses are many+++ The coverage of the topics was excellent and we would go over them as often or many times as necessary as well as being able to contact Julie as often as needed. We could ask questions as we went through the tutorial and the answers were presented immediately. When I didn't get to the tutorial, I would listen to the recording when it was convenient for me and this was also nice because you could pause it and restart it. I also would listen to the tutorials when I may have gotten a little lost during the presentation. (REF: LMS)

Rocky I could interact with the instructor as much as I needed and I could listen to the recordings as often as necessary. It is a great combination. (REF: LMS)

Ursula The teacher teaches you don't have to worry about distractions, or someone making noise and not listening while you are trying to listen. . . . having the notes to refer back to, the live recordings to go back to. (REF: LMS)

Clarissa

> Being able to ask questions when they arose so I wouldn't forget what I wanted to ask. The ability to repeat the steps to a problem while it was still fresh in my head and the "classroom atmosphere" that the tutorials provided. Also having access to the recording was very important to me if I was unable to attend OR simply wanted to re-watch the tutorial. (REF: LMS)

Sal

> The structured format of the [Univeristy] live tutorials, was wonderful. I found them absolutely indispensible. That we could ask questions at any point, the

> input of the other students, and the detailed treatment of the subjects all helped me immensely in this program. I would recommend a . . . live aspect in every course that [the University] offers. (REF: LMS)

Ruth

> The fact that there was a teacher that could be heard made all the difference to me. Problems & questions were addressed in 'real time' and being able to 'see' what was being taught was very important to me. By scheduling the live tutorials I was 'forced' to attend unlike the other mediums . . . (REF: LMS)

Issues of the virtual interface. Two of the live session attendees had concerns about the functionality of the web conferencing software. One made a good observation about the time it took for students to interact using the Chat Box:

> During the course of a 90 minute session, a lot of time is spent in the interactive texting from the student. If the sessions had a large number of students, and some energy students (keeners), the lesson plan might significantly slow down." (by email, February 2010)

In small classes, one remedy for this is to encourage participants to use a microphone but even that has time issues. No matter which means of live online communication one opts for, band widths will always play a role. For very large classes, in either case, the amount of dialogue would have to be limited somewhat. This, however, is also true in large face-to-face lecture theatres and even small face-to-face classrooms.

Another participant made mention of the 'general clumsiness' of the web conferencing program in terms of presenting the mathematics:

> All of the cutting, copying, pasting, inserting, deleting, and scrolling up and down got in the way of the material being presented. I don't think that the program actually lends itself to the presentation of mathematics." (REF: LMS)

The individual was referring to the step-by-step, in real time, presentation of the solutions to the mathematics problems using a mathematics equation editor. This kind of software presents the mathematics in a format which emulates the type-setting in a hard-copy text. It was not the online equivalent of writing on the black/white/smart board in a face-to-face classroom which is a free-hand style of writing/drawing. In any event, trying to write with a mouse within the dimensions of a screen-sized board was simply not an option. Much more mathematics could be presented in an optimal way using an equation editor within a Word document. However, the downside of that was lack of board space to leave part of it visible to the students when progressing though a long solution. While I understand that the movement on the screen could have been obtrusive for some, I simply could not offer a remedy for the issue at the time. Part of online teaching and learning includes having to make allowances for the idiosyncrasies of the particular media used to present the materials.

Speaking or texting: more than just a personal choice. In the live sessions, all participants had a choice of communicating via texting through the Chat Box or via a microphone. With the exceptions of Sal and Rocky, everyone else chose the chat facility. Ruth gave the following reasons for her choosing not to use a microphone:

> I wouldn't because I would be concerned about the background noise. I would have to ask my kids to be very quiet. . . . I also find that written questions are more thought out vs. talking can get drawn out and off topic. (REF: March 18, 2010 Group 1 tutorial)

Rocky preferred speaking over writing on the whiteboard (REF: April 6, 2010 Group 3 tutorial). He found writing on the virtual whiteboard frustrating. However he always used both the microphone and the Chat Box – the microphone when he was talking about mathematics, the Chat Box when he was conversing with this fellow participants. Sal, on the other hand, worked with all three media on a regular basis.

Garrison, Anderson, & Archer (2001) had pointed out that the choice of either speaking or texting in an online environment was not just a matter of participant preference.

> The issue of text versus speech was also raised as being of particular importance with regard to higher-order learning in Fabro & Garrison (1998). The present authors argue that differences in the nature of spoken and written communication is (sic), in fact, a key to understanding computer-mediated communication and specifically the use of computer conferencing in a critical community of inquiry (Archer, Garrison, & Anderson, 2000). (p. 3)

Garrison et al. (2001) went on to say that:

> There is sufficient evidence to suggest that writing has some inherent and demonstrable advantages over speech when one person or a group is engaged in rigorously thinking through a problem. As noted by Bean (1996), writing has

long been used as both a process and product of rigorous critical thinking. The written word serves best to mediate recall and reflection, while spoken word functions most effectively to mediate action – usually in a face-to-face context (Wells, 1999). (p. 3)

The experiences of the participants in the project seemed to corroborate these research findings. Sal, in particular, often used the whiteboard to work out some mathematical problems in front of the group. Rocky preferred to speak when discussing mathematical ideas but took personal notes throughout the live tutorials. Everyone else wrote their questions out in the Chat Box rather than use a microphone. I, as instructor, used the Chat Box only for social chatting. When I was 'speaking mathematically,' then speaking as I wrote was the preferred form of communication because of the analytic thinking involved and, in our case, a lot of detailing of definitions and meanings. Whether I used the equation editor or the whiteboard, this virtual writing gave all of the participants time to reflect on its meaning and follow its logic as I wrote it out. Moreover, we, collectively, could then view it and examine it for meaning and accuracy.

Web Site or Web Talk?

The RMC web site had been intended only as a review of basic mathematical skills for those making the transition to university-level math-related courses. Nonetheless, I was very curious about the reaction of the participants to this site. To that end, I had posed the following question in the Learning Modes Survey (LMS) in order to elicit their experiences while using it.

Question: *If there had been no live tutorials to pace your program, what is your assessment of the RMC web site and the self-assessment tool as a self-regulated remedial mathematics learning site intended to provide a more solid mathematics background to those who are preparing for future math courses?*

Seven participants ventured to make a judgment call on the fully learner-controlled option offered in this project.

Abigail At the start I used this site quite a bit. As we moved forward, I used it less because I found the . . . live [sessions] excellent and we could go over questions as often or as many times as needed.

Ruth I would do it as a last resort but I know that it would never take the place of a teacher imparting her knowledge and skills in particular regarding mathematics.

Janette I liked the tutorials but I needed the live tutorial to really synthesize the learning. I think a study guide or text would also help. (REF: FIS)

Clarissa I would say that it would be far more challenging. The live tutorials kept me motivated to keep working ESPECIALLY because it was not for credit and without the live tutorials I think I likely would have stopped. The online [web site] tutorials were very helpful but if I didn't understand the process from those it isn't possible to be shown an alternative method of doing it. I suppose if I e-mailed my tutor but still it wouldn't be in the form of a live tutorial where I could as (sic) the how's and why's of each step.

Chloe Generally I was happy with the information in the RMC tutorials. If a student is looking for an informal means of learning, the RMC website and the [self-assessment tool] might be fine. It has math instruction that can be done semi-independently, assuming that the student has some background.

Sal Without the live tutorials I do not think I would have continued in the course. The RMC web site is a good site to gain a basic foundational understanding of pre university math topics. It has more to do with where I was at and what I needed.

Rocky In their present state they wouldn`t help me at all. I would have given up on them. Everything I have learned in this project has been due to two things: the online tutorials and the inspiration and curiosity that the tutorials have inspired in me to learn math from other sources.

The jury had reached a decision. For a course of instruction, the web site, coupled with the self-assessment tool, was clearly not adequate. As a top-down review, it was incomplete. As a remedial source, it did not provide enough mosaic detail to be of a great help to those really needing it. This was yet another confirmation that synchronicity seemed to be an essential component of mathematics instruction in virtual classrooms – at least for the development of a true community of inquiry.

The Million Dollar Question

> *QUESTION:* *In the future, if you had a choice of either a classroom setting or an online live tutorial setting to learn mathematics, which would you choose and why?* (*Assume that both options were available to you at the time.*)

The responses from the group, as usual, formed an interesting array of perspectives.

Georgiana Online because it fits into my busy life.

Rocky I would choose the online option provided the lecture notes and the recordings were made available right after each session.

Chloe . . . the value of distance education may depend on the abilities of the student and on the particular discipline.

Marie The key is having a goal, and a schedule – you simply adapt to the environment. I am also taking a traditional university course. This remedial math course has provided a very flexible and effective way to help students return to the learning of math so they can pursue educational goals. I am very grateful for this opportunity.

Ursula Live tutorial by far. I felt that the atmosphere was more supportive for learning than in a regular class. Less distractions that you normally have to deal with in a live classroom.

Abigail i would choose the online setting.

Sal I would like a combination of both online and the possibility of a physical classroom. I think, for those who could, meeting in a physical group and still doing online tutorials for those who not able to attend physically would be great.

Ruth I would definitely choose an online tutorial. Leaving home in the evening is very difficult if not impossible to me. Also after a long day at work I can make myself comfortable while at the same time being part of the 'class'.

And, from my perspective, the most surprising of all:

Clarissa If it was for credit I would choose online if it was not for credit I would choose a classroom.

From this kaleidoscope of viewings, I began to see that this virtual classroom of learning had evolved over time into a dynamic interplay between and amongst all of the members in the group – whether online or offline. It had become a place of learning for the majority of the participants (see Chapter 6, the subsection entitled "Participant Assessments of Educational Place in a Virtual World"). Interactions had ventured beyond the confines of our meeting place into the virtual schoolyard. There had, at least from the perspectives of the majority of the participants, been enough learning and interaction in the live sessions to create an ethos which is traditionally associated with face-to-face classroom experiences.

However, their learning had not been restricted to the project venues. Some had surfed out to the Internet to explore the vast array of spaces available to them.

Others had consulted hard-copy books because they needed such a structured text. Still others had used a different assessment tool which was more familiar to them. Some had even virtually ‘attended’ most of the live sessions through its archived recordings and still had considered themselves to be a member of the group in that classroom. However, there had been a constant. It was brought to my attention from quite a number of attendees, that even in this setting with only virtual presences, I, the instructor had been there for them – not only by way of active live instruction but through email correspondences and telephone calls in real time. This had been the glue holding all of us together into a cohesive community of inquiry. In these loosely tethered settings, apart from the presentation of the subject matter, a central role of the instructor had been to set the tenor and the tone of the primordial ethos thus taking shape.

Drawing Conclusions: Past and Present

David Jaffee (2003), a Sociology Professor from the University of North Florida, suggested that:

> A central sociological proposition is that structural environments influence the social perceptions, roles, and relations of human actors. As increasing numbers of students and faculty find themselves operating in virtual learning environments, we might also expect to find some changing instructional dynamics. (p. 2)

Gadanidis and Geiger (2010) had also anticipated a potential change in pedagogy with the shift to Internet technologies: “Our thinking about mathematics education has been disrupted and reorganized as we use and think with the technology of the Web” (p. 95). In fact, they had envisioned the reorganization of mathematics education as being more than simply a structural move from the printing press to the world of hypermedia.

> The Web’s shift from text-based, read-only communication to multimodal, read/write communication is not simply a quantitative change: it is not just a case of having more communication modes. It is a qualitative change, analogous to the change that occurred when we moved from an oral to a print culture. However, our understanding of what this change implies for mathematics education (and education in general) is emergent and not fully conceptualized or articulated. In this context of increased social affordances of communication

technologies, which are becoming commonplace in out-of-school settings, and are beginning to enter educational settings, it is important to consider what educational theories of social interaction might be developing alongside the technological changes. (Gadanidis & Geiger, 2010, p. 95)

QUESTION: Does the online environment change the pedagogy of the instructor?

The answer as to whether my pedagogy changed in response to the virtual environment is two-fold: not really but definitely yes. Working in a web conferencing, synchronous environment really didn't change my underlying philosophy or approach to the pedagogy of mathematics which I had previously used in face-to-face classrooms. The virtualFace of language remained much the same as it had been before but customized, as was always the case, to the students with whom I was to engage. The difference had had to do with the additional media I could draw upon to illustrate and explicate certain of the ideas, thereby affording a much richer presentation with more texture and colour. Whether the change from a physical to a virtual setting had changed the teaching /learning dynamics significantly was to be determined by the participants themselves. From the set of responses to all of the research questions, I concluded that this virtual experience had not been significantly different from those I had had in face-to-face university classrooms – with TWO NOTABLE EXCEPTIONS – the teacher-student continuum of pedagogical control during the project and the teacher-student continuum of social connection after the project had ended.

Changing Rules of Engagement in a Virtual World

I was now convinced that my virtual layered-media environment had indeed emulated the traditional face-to-face classroom experience. The evidence was there from the field texts and the analysis. Yet, technology aside, it wasn't a traditional classroom. It had a different flavor. This was a new era of pedagogy. Those who worked alongside me in this virtual experiment had not echoed the previously articulated concerns of some who had feared that the online social/cognitive experience would be necessarily truncated in comparison to that of physical face-to-face class environments. True, in our classroom, there had been no eyes to read into the soul; there had been no body language to enrich the dialogue. Yet that didn't seem to make much difference to our social presences, nor to our learning.

Anagnostopoulos, Basmadjian, and McCrory (2005) wrote an article about the de-centering of the teacher in virtual classrooms and its ramifications on the re-construction of social norms in such environments. Anagnostopoulos et al. (2005) pointed out that, in contrast to face-to-face settings where the ethos of social space and interaction was well-defined within the overriding cultural norms and expectations, online classrooms were still open to re-interpretation.

> Instead, teachers and students must deliberately consider how and when they will enter into the virtual classroom and where and how they will locate themselves and each other within it. (pp. 1699-1700)

Well, we twelve had done just that.

In structured face-to-face settings, there had always been broadly defined rules of engagement – even if more fluid now than in the past. In virtual spaces, however, there have been very few confines of interaction. For example, in this virtual environment, there was no Gutenberg text common to all learners. There was no coercion to engage with any particular form of the available layered 'media texts.' Each participant had quite simply 'done their own thing' with the spaces. Even I had experimented with media and pedagogy. We were children of a new age. Perhaps other types of questions should have been posed to each individual. For example, maybe I should have asked: What did social presence mean to you in this context? or How did you immerse yourself in your learning? But I didn't because it had never occurred to me to ask that of any student before. Short, Williams, and Christie (1976, p. 65) had defined social presence as ''the degree of salience of another person in an interaction and the consequent salience of an interpersonal relationship'' (as cited by Anagnostopoulos et al., 2005, p. 1700). Certainly we collectively had experienced that salience one-on-one to a greater or a lesser extent (see Chapter 6). We each had defined our own level of engagement and interaction with the content and with the others. There were really no rules and certainly no expectations as to how and how much each participant should engage. I must admit that, before the project began, I had absolutely no idea what to expect in this developmental mathematics 'classroom.' I simply moved with the moment and hoped that I could achieve my goal and my promise to those who came. 'Success' was not a word I could even define in this context because I had had no intention of examining them formally.

What, for me, was a rather surprising outcome in the research project was the extent to which so many of the participants engaged in their learning and went on to pursue their dreams. Feldman & Zimbler (2012) had outlined some of the essential ingredients for remedial course programs to be considered 'successful' to the task:

> It is critical for developmental programs to create an environment in which students can receive personalized advisement and be placed in an educational setting designed to elicit student success. The more students feel that their education is relevant and personally beneficial, the greater the chances that they will continue on their developmental tracks.
>
> Essential to this process is continuous communication and the encouragement of student feedback. It is crucial for developmental programs to provide enough individualized attention and support that a struggling student feels comfortable turning to the college's support network for assistance, preempting the decision to leave school. (p. 14)

Perhaps that contains one of the secrets of good pedagogy in remedial mathematics settings – both online or offline. Shower them with attention and provide an array of options but allow each student to be free to choose his or her individual path of learning. In other words, loosen the rules of engagement.

From the experiences of my eleven (11) eye witnesses, I have concluded that:

1. The learning of mathematics online can be as effective as in face-to-face instructional situations.
2. Deep learning can occur in such virtual settings.
3. Students can experience the learning of mathematics in both the same and in expanded ways online, as distinguished from other learning environments.

The free-form, information-rich, setting of the Internet affords an infinity of avenues for students to pursue their learning goals. With some guidance from a mentor, such explorations can greatly enhance the students' educational experiences and free them from the emotional tight-jackets of the past – in particular the affective trammels of a negative self-image, lack of confidence in one's ability and the sense of having no control.

Changing Connections in a Virtual World

> *QUESTION: So, did the fact that my project participants had no face-to-face communication with me affect the quality of teaching and learning mathematics online?*

From my perspective, I felt that the tenor of my teaching was pretty well on par with the ways I had taught mathematics in face-to-face classrooms in the past with many of the same results. One thing was surprising, however. I felt a deep connection with each of these virtual attendees – even long after the project had finished. This may have been due, in part, to the fact that this classroom was a place of research and constituted, for me, a deeper probe into what makes good pedagogy in virtual environments. Moreover, because it had involved my own creation (the RMC web site), it thereby became a personal enterprise, unlike the physical classrooms in which I had taught for many years. Despite that, my personal experience of 'knowing' the Other had been through the medium of language and all that it conveyed cognitively, psychologically and emotionally – a real parallax of human encounter – a *virtual viewing* of the Other, so to speak. There had been no physical or social status barriers between us – no road blocks of preconception from societal norms. The absence of the physical component, in fact, allowed me to work at a higher comfort level with the participants because we were communicating in a more informal way. My teaching persona was less aloof and more accessible than in a regular classroom. The result was that, in this online teaching experience, I formed more lasting and bonding connections to my students than I have ever done in a face-to-face classroom.

Finding the Constants in a Virtual World

In 1999, four years after the opening of the Internet to the public arena, Benbunan-Fich and Hiltz observed the following:

> Students report being both more satisfied and learning more from online classes in which both the teacher and the students make themselves socially present, though social presence has not been found to predict substantive engagement with ideas or the development of meaningful discourse. (as cited by Anagnostopoulos et al., 2005, p. 1700)

At that relatively early time in the partially digitalized universe, the virtual presences of both teacher and learner were considered to be a factor in student satisfaction and learning. At that time, however, Benbunan-Fich and Hiltz had not been able to determine if those combined social presences had had a significant impact on the students' engagement with the course material.

Zhao et al. (2005) undertook a meta-analytical study of research on distance education as compared to its face-to-face counterpart. In their study, distance education was defined to include both traditional forms of it (such as correspondence courses and television broadcasts) and the more recent forms, now commonly referred to as online learning, Web-based learning or e-learning. The purpose of their study was to identify factors which had contributed to the effectiveness of learning at a distance. They reported that past research in the discipline had supported the conclusion that there were no significant differences between the outcomes of distance education compared to face-to-face instruction. Out of an initial sample of 8840 potentially relevant articles, 51 journal articles were actually examined, all of which contained empirical evidence as to the reasons for the effectiveness of distance education. However, only one of those was involved the teaching of mathematics. In addition, they found that the year 1998 had been a watershed year in terms of distance modes of education beginning to outperform face-to-face classrooms in term of effectiveness.

> Studies published before 1998 did not seem to find a significant difference between distance education and face-to-face education, whereas studies published in and after 1998 found distance education to be significantly more effective than face-to-face education. (Zhao et al., 2005, p. 1855)

Their own study, however, concluded that there still not seem to be a significant difference in learning outcomes between the two pedagogical paradigms, thereby confirming the claims of previous researchers. However, they did find a wide variety of differences as to the factors used to determine 'effectiveness' across the breadth of the research studies themselves. Two-thirds of the studies showed that distance education produced better student outcomes; the remaining one-third found just the opposite. To understand this disparity, they examined what effects were measured to obtain those results. They looked at: grades (including quizzes), student attitude and beliefs, student satisfaction, and student participation, student evaluation of learning or metacognition, instructor involvement, student prior education level and whether the outcome had been based on researchers' observations alone.

Of all of the measured factors, Zhao and colleagues found that **the most significant moderator of effectiveness was instructor involvement**. When instructor involvement in distance education was low (taken as being present in less than or equal to 40% of the time devoted to course content presentation or consultation), the outcomes in distance learning were not as good as in face-to-face instruction. As instructor involvement increased, distance education effectiveness also increased but achieved its maximum positive effect in the middle range of 50% to 70% of contact time. I interpreted this as meaning that every mode of learning at a distance, including online learning, must have a good degree of instructor presence to be effective. The teacher involvement should not totally dominate the interaction and learning process, such as in the case of a series of videos or presentations with minimal student interaction. Time must be allotted and granted both outside and inside the online classroom for students to explore and learn on their own.

This finding seemed to be corroborated by the participants in my project. While more than half appreciated the RMC web site without reservation and used it, more than 90% attributed more of their learning and their motivation to stay with the project to their having attended the live tutorial sessions – again confirming the importance of a solid and readily available teaching presence in online mathematics education. They overwhelmingly voted for the live tutorials over the RMC web site as a better online environment for learning mathematics. In addition, from the collective opinions gathered, I feel that the synchronous sessions not only motivated, but enhanced, active engagement with the mathematical content within and even without the project curriculum of studies.

Another interesting finding of Zhao et al. (2005, p. 1857) was that a student's prior education level was significant in predicting the effectiveness of distance over face-to-face learning. For students having a high school diploma, distance education showed significantly better outcomes than their face-to-face counterparts. If students had already achieved a college degree, the differences in outcomes between the two were not significant. The participants in my research were, for the most part, at the high school level in terms of the learning of mathematics. This perhaps accounted for some of their overall enthusiasm for the online learning model.

In 2006, Abrami and Associates, was commissioned by the Canadian Council on Learning to conduct a *Review of e-Learning in Canada: A Rough Sketch of the Evidence, Gaps and Promising Directions*. They concluded:

> Overall, we know that research in e-learning has not been a Canadian priority; the culture of educational technology research, as distinct from development, has not taken on great import. . . . We noted that there are gaps in areas of research related to early childhood education and adult education. (p. ii)

In order to fill this gap, they also suggested some implications for both K-12 practitioners and post-secondary institutions. Although, during their study, some educators had suggested that e-learning had the potential to transform the learning process in general, Abrami et al. (2006) claimed that there was limited *empirical* research to assess the benefits. To that end, they called for more quantitative research as to the attainment of learning outcomes in online environments. Abrami et al. (2006) also had recognized the need "to determine the feasibility and effectiveness of such things as learning objects and multimedia design" (p. iii), adding that:

> Properly implemented computer mediated communication can enrich the learning environment; help reduce low motivation and feelings of isolation in distance learners. (p. iii)

The findings of my own case study research with adult students learning mathematics online have actually supported Abrami et al.'s (2006) conclusions that the design of multimedia and learning objects must be carefully examined for efficacy and potential cognitive overload. Most of the participants in my project themselves corroborated the latter conclusion of the Abrami research group. Computer-mediated environments can and do enrich learning (Chapter 6, subsection "Participant Assessments of Educational Place in a Virtual World").

Interestingly enough, in 2009, some three (3) years after the Abrami et al. study, the U.S. Department of Education, Office of Planning, Evaluation, and Policy Development had also conducted a meta-analysis and review of online learning studies. Their analysis of 51 study effects, 44 of which were drawn from research with older learners, found that: "Students who took all or part of their class online performed better, on average, than those taking the same course through traditional face-to-face instruction" (U.S. Department of Education, 2009, p. xiv). Baran, E., Correia, A., &

Thompson, A. (2013) lent further support to the Zhao et al. (2005) finding that instructor involvement was one of the primary factors in the effectiveness of online learning:

> Research has found that teaching presence was a significant predictor of students' perceived learning, satisfaction, and sense of community (Gorsky & Blau, 2009; LaPointe & Gunawardena, 2004; Russo & Benson, 2005). (p. 3)

From the body of literature with its paucity of mathematics-oriented research cases, coupled with my own research involving only a small sample of convenience drawn from a population of volunteers having had experiences of both face-to-face and virtualFace-to-virtualFace classroom settings, I dared not draw any kind of all-encompassing conclusions. However, what I could deduce was that e-environments for the learning of mathematics have the potential to be viable alternatives to face-to-face settings for a growing number of students worldwide.

The remaining questions to be explored were: What are the essential qualities of online environments for engaged and deep learning to take place? How does one structure them? Moreover, if instructor presence is the one constant in effective learning online, then how does the instructor mediate his or her place in those venues in order to maximize the students' learning?

Redefining and Redesigning 'Face-to-Face' in a Virtual World

Changing the shape of a cultural paradigm of centuries of learning must be approached with some caution. I certainly did not come into this research project thinking that I knew all the answers. I wasn't even sure that I knew any of them. All of my previous pedagogical experiences in the online world of formal education had been as an interested observer. Candy (1991) had already known that over two decades ago when he said:

> It is relatively easy to advocate a change from one instructional approach to another, but a different matter to see it through in practice. This is particularly so when the change involves, as the transition from teacher-control to learner-control does, a reshaping of the foundations underpinning the teacher-learner relationship. (pp. 223-224).

Then, the first step in changing the instruction approach is for the instructor to define the CONTAINER as a mosaic of layered-media suitable to the task. Components may include any or all of the following:

- a curriculum of study posted online; coupled with
- a series of virtualFace-to-virtualFace synchronous meetings of both students and instructor (mediated by some computer interFace, through the constructs of language and embellished by persona and personality); and
- a layer of communication or social media options such as email, discussion forums, collaborative projects (e.g. Wikipedia), blogs (e.g. Twitter), social networking sites (e.g. Facebook), virtual game-worlds (fill in your favourite one), virtual social worlds (e.g. Second Life) and content communities (e.g. Second Life); with the backing of
- a variety of learning materials which may be a combination of e-learning objects, multimedia presentations, e-books, videos, animations, web sites, a list of e-readings and digitally archived live sessions or lectures of interest; and flushed out with a
- a means of online assessment, either summative or formative, correlated to the curriculum.

From that base, future e-learning instructors can begin their work in configuring and reshaping the CONTAINER to allow for their personal pedagogical dictates and preferences.

Boechler (2008a, p. 1199) found that individual personal differences such as learning style, prior topic knowledge, level of interest, and gender, as well as structural characteristics of the objects themselves all influenced student performance and behaviour in hypermedia settings. She suggested that learners may require different kinds of interface support. Therefore, when considering the design of a multi-layered virtual classroom, it was prudent to understand the relationships between individual spatial and reading capabilities and the types of support networks available to develop and enhance those very skills. Boechler (2008a) also suggested that "Cognitive and learning theories can provide guidance for exploring the interactions that occur between the interface characteristics and individual differences for both navigation and learning outcomes" (p. 1202). Gadanidis and Geiger (2010) had highlighted yet another important consideration in the learning and teaching of mathematics in a virtual world:

> In addition, socio-culturalism acknowledges the role of both cultural tools, such as language representations and sign systems, and physical artefacts, such as

> calculators and computers, in mediating learning. The appropriation of tools into accepted modes of reasoning and discourse is also an important part of the enculturation process. Thus, a learner must acquire more than facility with such tools: they must also appropriate new modes of reasoning, argumentation and knowledge validation in which digital tools are seamlessly integrated. (p. 96)

However, despite all efforts to accommodate this new mode of interaction, the virtualFace-to-virtualFace classroom may not serve the best interests of all. Fry (1972) researched the relationship between students' levels of aptitude and inquiry (or self-directedness) and learning outcomes in both learner-controlled and expert-controlled environments. The results of Fry's study were that high-aptitude-high-inquiry students learned significantly better in learner-controlled situations; high-aptitude-low-inquiry students fared better under a high degree of expert control; and the results varied with low aptitude students. I believe that this is definitely applicable to web-based learning in a subject as complex and as esoteric as mathematics. Fry's insight helps to account for, in part, the various preferences of the project participants as related to their starting points of mathematical knowledge at the outset of the project (Chapter 6, subsection "The Personalities and Their Stories").

Anderson (2002) maintained that web-based configurations of learning could produce sufficient levels of deep and meaningful learning provided that at least one of the three forms of interaction in such environments (student-teacher or '*teaching presence'* in the Community of Inquiry model; student-student or '*social presence*;' student-content or '*cognitive presence'*) were at a significantly high enough level. The other two may be offered at minimal levels or even eliminated without degrading the educational experience. In fact, I tend not to agree with this conclusion when it comes to the learning of mathematics. While student-student interaction can promote the acquisition of mathematical knowledge and understanding, if this kind of social presence were to be the predominant one and the teacher and cognitive presences were minimal, then my guess would be that not a lot of mathematics would have actually been learned in such a setting. More than that, Kreber and Kanuka (2006) had suggested that research conducted over the previous two decades had "revealed that higher-order learning is not easily achieved in the online classroom" (p.121), citing

(Garrison, Anderson, & Archer, 2001; Gunawardena, Lowe, & Anderson, 1997; Kanuka & Anderson, 1998).

Therefore, one of the goals of the virtual classroom practitioner should be to provide a balance of the three presences in the layered space of interaction so as to encourage all three forms of interaction from as many of the participants as possible. Of note, is that past research has shown that, the one constant in effective online teaching/learning environments is a salient teaching presence (Benbunan-Fich & Hiltz, 1999; Zhao et al., 2006; Baran et al., 2013). The conclusions which I too have drawn from this case study support the conclusion that a synchronous teaching presence should be a part of the online paradigm of distance education in order to maximize its effectiveness, particularly in the learning of mathematical ideas. This **new form of blended e-learning**, constructed as a mosaic of asynchronous and synchronous encounters, may be renegotiated by teacher and students for each such environment. The components of interaction which took place during the time frame of my research project actually differed from student to student, as is generally the case in any learning environment. All three forms of interaction were offered to each: cognitive presence by way of the RMC web site, the self-assessment tool and the plethora of Web-based educational math sites; teaching presence by way of the live tutorials and their archived versions; and social presence through online discussion forums, emails and the live sessions. How each of us engaged was a personal choice and it differed for each participant. However, there was one *omni-presence* interwoven through the fabric of the cognitive, the teaching and the social spaces of engagement; it was the instructor who had created the materials, intermediated the live sessions and set the tone of the social component of interaction.

The Nature of Virtual Community

Once one has a structural framework for creating a virtualFace-to-virtualFace learning environment, the next stage of development has to do with designing ways to generate a classroom atmosphere. Conrad (2005) defined a community as “a general sense of connection, belonging and comfort that develops over time among members of a group who share purpose and commitment to a common goal” (p.1). How does one generate a community of learners in a virtual space? Of course, the first step is to build the CONTAINER or the layered space which provides opportunities for all three interactions to occur. The next step is to create the CONDUIT. Garrison, Anderson, and Archer (2000) defined the affective goals of the educational process as the importance placed on the participants enjoying the group interaction and finding it personally satisfying so that they would remain in the cohort of learners for the duration of the course program. They claimed that, to attain both the affective goals and the desired cognitive outcomes in any venue of learning, "social presence is a direct contributor to the success of the educational experience" (Garrison et al., 2000, p. 89). They considered the evidence of emotional expression, open communication and group cohesion as earmarks of the existence of social presence in such settings. Anderson (2004), on the other hand, suggested that this may be easier said than done in virtual settings:

> In short, it may be more challenging than we think to create and sustain these communities, and the differences—linked to a lack of placedness and synchronicity, that is, mutual presence in time and place—may be more fundamental than the mere absence of body language and social presence. (pp. 5-6)

Already back in 2004, Anderson had had an inkling that mutual presence in time (synchronicity) and mutual presence in place (the CONTAINER) were the missing ingredients of asynchronous online communities of inquiry. I do believe that, even without the mutual placedness of synchronicity, e-learning can provide a suitable environment for the learning of some mathematical topics, even without the formation of a community of learners. However, for most, in order to achieve a deep understanding of the underlying mathematical ideas and connections, there is growing evidence from a variety of researchers, as cited above, that the live as-it-happens social and cognitive interaction of both instructor and participants is generally considered a predictor of effectiveness in virtual classrooms. Fixed place and time cannot always be discarded as superfluous in learning. Anderson (2004) concluded that:

> The flexibility of virtual communities allows more universal participation, but a single environment that responds to all students does not exist; thus, the need for variations that accommodate the diverse needs of learners and teachers at different stages of their life cycles. (pp. 5-6)

In this new blend of distanced educational encounters defined as the virtualFace-to-virtualFace classroom, virtual mutual placidness (synchronicity) should be included as part of the mosaic. This certainly will go a long way to provide a more stable environment of learning where a community can evolve towards MEANING.

I maintain that even small sample qualitative experiments of learning, such as this case study, can provide insights into what makes certain kinds of e-learning effective. Although Case Study as a research method may seem a poor basis for generalization, the activities or problems or responses which come up time and again in such cases, can be places where certain generalizations can be drawn, the so-called 'petite generalizations' of Stake (1995, p.7). Certain phenomena elaborate or exemplify grander issues and therefore, sometimes, may be used to modify the 'grand generalizations.'

For most of the project participants, it was the in-the-'same'-place and at-the-'same'-time live sessions which had been the central social space of our virtual environment located in cyberspace. For a detailed account of the form and nature of our little community, see the section entitled '"Virtual Community in a Virtual Space" in Chapter 6. How had it evolved? On my part, I, as instructor, had made an effort to incorporate as many as possible of the capital C's of 'Community' into my interaction with the participants. I welcomed each individual as they entered the virtual room where our live meetings took place. As much as possible, I had endeavoured to engage each in some kind of dialogue about both personal and mathematical considerations. I tried to set the tone of openness of expression by being open and honest myself. I reassured them that *all* of their concerns and questions were important to our collective understanding of the mathematics. It was of ultimate importance to me that no answer or query be judged to be too trivial or too obvious to warrant a response. Somehow that virtualFace of the instructor began to be emulated within the group. Others started to be more open in their responses and exhibit more comfort to ask for clarification of the mathematics under discussion. We grew *together* in our understanding of the mathematics, from the most basic to the more complex. In our humble way, I was convinced that we had formed a virtual Community of Inquiry in the sense of Garrison, Anderson and Archer (2000). Our community intermingled within all of the essential presences – cognitive, teaching and social – exhibiting both the positive and negative effects of such interactions. The fact that we had formed a real community was corroborated by the majority of the participants in this project. The synchronicity had been the magic ingredient in making our space into a place of learning.

For that reason, it is my strong conviction that the teaching of foundational mathematical ideas should never be completely severed from the mathematical expert, though it is of considerable benefit to students to make those ties loose and flexible so as to encourage as much learner participation and autonomy as is possible under the circumstances. I have always tried to build both emotional and intellectual strength in my students in their learning of mathematics. I feel that I have only taught them well if, at the end of our limited journey together, they are able to carry on their studies with confidence and solid understanding. Therefore, in order to provide enough guidance so that students do not encounter too many pathways leading to virtual obscurity, the adult educator should include enough navigational and instructional guidance without providing too much explicit direction, thereby discouraging learner curiosity and choice. The challenge lies in striking a balance.

Striking the Balance: the New Blend of Learning

Akyol, Garrison and Ozden (2009) examined the notion of a community of inquiry in both fully online and blended environments. They had considered 'blended' to mean a mixture of both computer-mediated learning and physical face-to-face instruction. The results of their research suggested that the blended course had distinct advantages over the online course. They concluded that blended learning with the face-to-face component reduced the time required to form a cohesive group. The researchers also found that the blended version promoted higher levels of inquiry, thereby allowing more time for the integration and resolution of the formation of knowledge. Finally, it was determined that the addition of the face-to-face meeting with the computer-mediated components satisfied the needs of more students because that combination provided multiple avenues of communication. Akyol, Garrison and Ozden (2009) further added that their findings had provided support for the assertion of Garrison and Kanuka (2004) that the blended learning environment is particularly effective in supporting a community of inquiry.

This is encouraging for the future of 'strictly' online learning because now e-environments can accommodate a virtual face-to-face synchronous meeting with a salient teacher and social presence, accompanied by that immediate feedback mechanism which students of mathematics so often ask for in their e-learning classrooms. My 'solution' for the construction of effective online environments for the learning of mathematics is the virtualFace-to-virtualFace classroom, as outlined in Chapter 6, section "The Virtual Classroom." In my opinion, it is the new paradigm of blended learning and it is fully online.

Facing the Future

As more and more educational institutions embrace e-learning as a viable option of pedagogy in order to reach out to more diverse groups of students, the 2009 *Chronicle of Higher Education* research report, entitled *The College of 2020*, predicted that "students will demand more online courses in the near future (Van Der Werf & Sabatier, 2009)" (as cited by Baran et al., 2013, p. i). As new technologies emerge every year and more educators are called upon to venture into the territory of the virtual classroom, it is prudent that more research, both qualitative and quantitative, be carried out in this new blend of learning. For example, what, if any, roles do hand-held devices and social networks, not to mention improved and innovative softwares, play in this extended 'face-to-face' environment?

Major (2010) contended that an articulation and examination of instructor experiences in experimental models of distance learning should be part of the research of the future:

> Investigating how faculty experience online teaching is critical to understanding new practices and patterns of behavior that occur in the technology-mediated environment.
>
> Therefore, systematically examining the experiences of faculty who have taught online can tap a potentially rich vein of information about online distance education and provide important insights into its viability as an instructional medium. (p. 2161)

Anderson, Rourke, Garrison and Archer (2001) had, many years before, posited that the role of the instructor was central and pivotal for the effectiveness and saliency of the endeavours of any online community. It was the instructor who was to establish and maintain a teaching presence through the design and organization of the space of learning, who was to facilitate the interaction within that space and who was to provide the direction of both cognitive and social processes in order to create such "personally meaningful and educationally worthwhile student outcomes" (p. 5) in that space. How individual teachers can accomplish this in an online setting is another avenue of present and future research. Baran, Correia, and Thompson (2013), when they interviewed six online teachers (who had been nominated as exemplary instructors) as to the reasons for their successes in a virtual world, discovered the following:

> The findings of this study indicated that when teachers described their successful practices, they often linked them to their changing roles and new representation of their "selves" within an online environment. Their portrayal of the teacher self, both built on a plethora of previous experiences and reformed with the affordances and limitations of the online environment, went through a process whereby teachers were constantly challenged to make themselves heard, known, and felt by their students. (p. 1)

Yes, it is true. One of the challenges for any instructor in this forever mutating world of online education is how to become a 'somebody' in a 'no-body' world and encourage your students to strive to do the same.

Other areas of exploration in e-learning have been suggested by Anagnostopoulos, Basmadjian, and McCrory (2005, p. 1700):

> Little attention is given to how the actions of the participants form the classroom environment and what characteristics that environment takes on as a result of teacher and student actions. Further, little attention is given to specifying the structure of the online classroom environment itself and how it shapes teacher and student interactions. (p. 1700)

Questions about the future FACE of learning still remain. How can a meaningful community of inquiry be achieved? What are its shapes and substances? What, if any, are its limitations?

Are there any definitive answers? Perhaps not prescriptive answers per se – but we have indicators which point in certain directions to the answers themselves, if they exist at all. An answer seems so permanent, thereby necessitating no further explanation or exploration. In fact, it may take a global village, a term attributed to Marshall McLuhan himself, to find a satisfactory mosaic of appropriate and valid responses to these probing questions. In suggesting this, I am forecasting into the twenty-first century what McLuhan (2011) said of the twentieth:

> The method of the twentieth century is to use not single but multiple models for experimental exploration – the technique of the suspended judgment. (p. 81)

ENDNOTES

Chapter 1

1. Retrieved on February 19, 2013 from http://www.thefreedictionary.com/virtual

2. A **photon** is an elementary particle, the quantum of light and all other forms of electromagnetic radiation, and the force carrier for the electromagnetic force, even when static via virtual photons. Retrieved on September 14, 2013 from http://en.wikipedia.org/wiki/Photon

3. Retrieved on February 19, 2013 from http://www.thefreedictionary.com/virtual and http://www.merriam-webster.com/dictionary/virtual

Chapter 2

4. The word '**visuospatial**' is an adjective meaning: of or relating to visual perception of spatial relationships among objects. Retrieved on September 15, 2013 from http://www.thefreedictionary.com/visuospatial

5. **Visuospatial image**: as defined by Presmeg (1997, p. 303), is a "mental construct depicting visual or spatial information".

   Visuospatial perception: is "one component of cognitive functioning which refers to the ability to process and interpret visual information about where objects are in space." Retrieved on September 25, 2013 from the University of Alberta Dictionary of Cognitive Science, http://www.bcp.psych.ualberta.ca/~mike/Pearl_Street/Dictionary/contents/V/vspperc.html

6. **Working memory**, the more contemporary term for short-term or primary memory, "is conceptualized as an active system for temporarily storing and manipulating information needed in the execution of complex cognitive tasks (e.g., learning, reasoning, and comprehension)". Retrieved on September 25, 2013 from the University of Alberta Dictionary of Cognitive Science, http://www.bcp.psych.ualberta.ca/~mike/Pearl_Street/Dictionary/contents/V/vspperc.html

7. The **open classroom** was a student-centered classroom design format popular in the United States in the 1970s. The idea of the open classroom was that a large group of students of varying skill levels would be in a single, large classroom with

several teachers overseeing them. Retrieved on March 1, 2013 from http://en.wikipedia.org/wiki/Open_classroom

8. The **Middle Ages** was an era in European history which spanned the fifth to the fifteenth centuries. It was divided into three epochs: the Early (from the fall of the Roman Empire to the end of the tenth century), the High (beginning in the year 1000 A.D. and lasting through the thirteenth century) and the Late (from the early years of the fourteenth century to the late fifteenth century). Retrieved on October 2, 2013 from http://en.wikipedia.org/wiki/Middle_Ages

9. The **Renaissance** was a cultural movement which began in Italy in the fourteenth century, spreading throughout Europe until the late sixteenth century.

10. The **Reformation** was a schism in the Roman Catholic Church, sparked by Martin Luther in 1517 and resulting in the formation and spread of various Protestant churches within Europe in the sixteenth century.

11. The **Age of Enlightenment** was an intellectual social movement in the seventeenth and eighteenth centuries which began in Europe and spread to the American colonies.

12. The **Scientific Revolution** was the era of the dawning of modern science in Europe with its breakthroughs in mathematics, physics, astronomy, biology, medicine and chemistry. It began more or less in the late sixteenth century and lasted until the late eighteenth century.

13. The **Knowledge-Based Economy** is the use of knowledge technologies to produce economic benefits, as well as job creation. (Retrieved on February 22, 2013 from http://en.wikipedia.org/wiki/Knowledge-based_economy)

14. **Cicero** was a Roman philosopher, linguist, translator and orator of the first century B.C.

15. An **educational community of inquiry** is a group of individuals who collaboratively engage in purposeful critical discourse and reflection to construct personal meaning and confirm mutual understanding.

Chapter 3

16. A **polytope** is defined by Coxeter (1973, p. 118) as the general term of the sequence "point, line segment, polygon, polyhedron, ...," or more specifically as a finite region of n-dimensional space enclosed by a finite number of hyperplanes.

17. **Topology** is the mathematical study of the properties that are preserved through deformations, twistings, and stretchings of objects. Tearing, however, is not allowed. (Retrieved on October 6, 2013 from http://mathworld.wolfram.com/Topology.html)

18. **Hypertext** is a method of storing data through a computer program - thus allowing a user to create and link fields of information at will and to retrieve the data nonsequentially.

Hypermedia is a logical extension of the term hypertext, in which graphics, audio, video, plain text and hyperlinks intertwine to create a generally non-linear medium of information. This contrasts with the broader term *multimedia*, which may be used to describe both non-interactive linear presentations as well as hypermedia. (Retrieved on March 3, 2013 from http://en.wikipedia.org/wiki/Hypermedia)

19. **Eudlid's fifth postulate**, the parallel axiom, states that If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. (Retrieved on October 7, 2013 from http://mathworld.wolfram.com/EuclidsPostulates.html)

Chapter 4

20. **Cognitive mapping** is a construct that encompasses those processes that enable people to acquire, code, store, recall, and manipulate information about the nature of their spatial environment.

21. **Fusion of horizons: "**The notion of 'horizon' employed here derives from phenomenology according to which the 'horizon' is the larger context of meaning in which any particular meaningful presentation is situated. Inasmuch as understanding is taken to involve a 'fusion of horizons', then so it always involves the formation of a new context of meaning that enables integration of what is otherwise unfamiliar, strange or anomalous. In this respect, all understanding involves a process of mediation and dialogue between what is familiar and what is alien in which neither remains unaffected". Retrieved on October 8, 2013 from the Stanford Encyclopedia of Philosophy, Hans-Georg Gadamer, http://plato.stanford.edu/entries/gadamer/#HapTra .

22. **Self-regulated learning (SRL)** is an element of social cognitive learning theory that states that learner achievement is affected by the learners' behaviours and motivations and by aspects of the learning environment (Bell, 2007).

23. **Epistemological beliefs (EB**) are beliefs held by individuals about knowledge and learning (Bell, 2007).

24. **Cognitive Psychology** is "concerned with the internal processes involved in making sense of the environment, and deciding what action might be appropriate. These processes include attention, perception, learning, memory, language, problem solving, reasoning and thinking." (as cited by All-About-Psychology, Retrieved January 20, 2013, from http://www.all-about-psychology.com/cognitive-psychology.html)

Chapter 5

25. **Case Study:** 1. A detailed analysis of a person or group, especially as a model of medical, psychiatric, psychological, or social phenomena.(Retrieved October 12, 2013 from http://www.thefreedictionary.com/case+study)

26. **Narrative inquiry:** is a discipline within the broader field of qualitative research. Narrative inquiry uses field texts, such as stories, autobiography, journals, field notes, letters, conversations, interviews, family stories, photos (and other artifacts), and life experience, as the units of analysis to research and understand the way people create meaning in their lives. These mosaic pieces are then woven into narratives as a means of telling individual personalized stories of the participants.

27. Viewed from a distance to encompass the whole because "Narrative explanation derives from the whole" (Connelly & Clandinin, 1990, p. 2).

Chapter 6

28. **Stream of consciousness** is a literary technique, used primarily in poetry and fiction, which seeks to portray an individual's point of view by giving the written equivalent of the character's thought processes, either in a loose internal interior monologue, or in connection to his or her sensory reactions to external occurrences. (Retrieved on March 24, 2013 from http://www.newworldencyclopedia.org/entry/Stream_of_consciousness)

29. T**op-down approach** is defined as an approach to a problem that begins at the highest conceptual level and works down to the details.

30. C**lassroom climate** is defined loosely as a particular combination of variables that work together to promote learning in a comfortable environment. (Retrieved on March 24, 2013 from http://www.cte.cornell.edu/teaching-ideas/building-inclusive-classrooms/classroom-climate.html)

REFERENCES

Abrami, P. C., Bernard, R. M., Wade, A., Schmid, R. F., Borokhovski, E., Tamim, R., . . . Peretiatkowicz, A. (2006). A Review of e-Learning in Canada : A Rough Sketch of the Evidence, Gaps and Promising Directions. *Canadian Journal of Learning and Technology, 32*(3).

Ally, M. (2004). Foundations of educational theory for online learning. In T. Anderson & F. Elloumi (Eds.), *Theory and practice of online learning* (pp.1-32). Athabasca: Athabasca University Press.

Akyol, Z., Garrison, D.R., & Ozden, M.Y. (2009). Online and blended communities of inquiry: Exploring the developmental and perceptional differences. *The International Review of Research in Open and Distance Learning (IRRODL), 10* (6). Retrieved March 14, 2013, from http://www.irrodl.org/index.php/irrodl/article/view/765/1436

Anagnostopoulos , D., Basmadjian, K.G., & McCrory, R.S (2005). The decentered teacher and the construction of social space in the virtual classroom. *Teachers College Record, 107* (8), pp. 1699–1729.

Anderson, T. (2002). Getting the mix right: An updated and theoretical rationale for interaction. *ITFORUM, Paper #63.* Retrieved March 11, 2013, from http://it.coe.uga.edu/itforum/paper63/paper63.htm

Anderson, T. (2004). Towards a theory of online learning. In T. Anderson & F. Elloumi (Eds.), *Theory and practice of online learning*. Retrieved May 15, 2009, from http://cde.athabascau.ca/online_book/ch2.html

Anderson, T., Rourke, L., Garrison, D. R., & Archer, W. (2001). Assessing teaching presence in computer conferencing context. *Journal of Asynchronous Learning Networks, 5*(2), 1-17. Retrieved April 1, 2013, from http://auspace.athabascau.ca/bitstream/2149/725/1/assessing_teaching_presence.pdf

Angeli, C., Valanides, N., & Bonk, C.J. (2003). Communication in a web-based conferencing system: The quality of computer-mediated interactions. *British Journal of Educational Technology*, *34*(1), 31-43.

Aoki, T.T. (2005a). Teaching as indwelling between two curriculum worlds. In W.F. Pinar & R.L. Irwin (Eds.), *Curriculum in a New Key: The collected works of Ted T. Aoki* (pp.159-165). New Jersey: Lawrence Erlbaum Associates.

Aoki, T.T. (2005b). The sound of pedagogy in the silence of the morning calm. In W.F. Pinar & R.L. Irwin (Eds.), *Curriculum in a New Key: The collected works of Ted T. Aoki* (pp.389-401). New Jersey: Lawrence Erlbaum Associates.

Baddeley, A.D. (1986). *Working memory*. Oxford, England: Oxford University Press.

Baddeley, A.D. (1999). *Human memory*. Boston, MA: Allyn & Bacon.

Baran, E., Correia, A., & Thompson, A. (2013). Tracing successful online teaching in higher education: Voices of exemplary online teachers. *Teachers College Record, 115* (3).

Barbatis, P., Leyva , A., Prabhu, V., Sidibe, A., & Watson, J. (2012). Engagement in creative thinking. *Mathematics Teaching-Research Journal Online, 5* (2).

Barbour, M. (2007). Principles of effective web-based content for secondary school students: Teacher and developer perceptions [Electronic version]. *Journal of Distance Education, 21*(3), 93-115.

Behrmann, M., & Tipper, S.P. (1999). Attention accesses multiple reference frames: Evidence from neglect. *Journal of Experimental Psychology: Human Perception and Performance, 25*, 83-101.

Bell, P.D. (2007). Predictors of college student achievement in undergraduate asynchronous web-based courses [Electronic version]. *Education*, *127*(4), 523-534.

Benbunan-Fich, R., & Hiltz, S. R. (1999). Education applications of CMCS: Solving case studies through asynchronous learning networks. *Journal of Computer Mediated Communication, 4*(3). Retrieved May 12, 2005, from http://jcmc.indiana.edu/vol4/issue3/benbunan-fich.html

Benedikt, M. (1991). Cyberspace: Some proposals. In M. Benedikt (Ed.), *Cyberspace: First steps* (pp. 119–224). Cambridge, MA: MIT Press.

Berleant, A., & Carlson, A. (2007). Introduction. In A. Berleant & A. Carlson (Eds.), *The aesthetics of human environments*. Peterborough, Canada: Broadview Press.

Bernstein, R. (2002). The constellation of hermeneutics, critical theory and deconstruction. In R. Dostal (Ed.), *The Cambridge Companion to Gadamer* (pp. 267-282). Cambridge, UK: Cambridge University Press.

Bloom, B. S. (1956). *Taxonomy of Educational Objectives, Handbook I: The Cognitive Domain.* New York, NY: David McKay Co Inc.

Bloom, P., Peterson, M. P., Nadel, L., & Garrett, M. (Eds). (1999). *Language and space*. Cambridge, MA: MIT Press.

Boechler, P. M. (2008a). Supporting navigation and learning in educational hypermedia. In L. Tomei (Ed.). *Online and distance learning: Concepts, methodologies, tools, and applications*. Hershey, PA: Idea Group Publishing.

Boechler, P. M. (2008b). Understanding cognitive processes in educational hypermedia. In L. Tomei (Ed.). *Online and distance learning: Concepts, methodologies, tools, and applications*. Hershey, PA: Idea Group Publishing.

Bransford, J., Brown, A., & Cocking, R. (2000). *How people learn: Brain, mind experience and school*. Washington, DC: National Research Council. Retrieved October 31, 2013, from http://www.nap.edu/openbook.php?isbn=0309070368

Bronshtein, I. N., Semendyayev, K. A., Musiol, G., & Muehlig, H. (2003). *Handbook of Mathematics (4th ed.).* Berlin, DE: Springer.

Bruner, J.S. (1966). *Toward a Theory of Instruction.* New York, NY: Norton.

Burbules, N.C. (2006). Rethinking the virtual. In J. Weiss et al. (Eds.), *The international handbook of virtual learning environments*. New York: Springer, pp. 37-58.

Caine, R.N., & Caine, G. (1991). *Making connections: Teaching and the human brain.* Alexandria, VA: Association for Supervision and Curriculum Development.

Caine, R.N. & Caine, G. (n.d.). Mind/brain learning principles. *21st century learning initiative*. Retrieved April 6, 2008, from http://www.21learn.org/arch/articles/caine_principles.html

Candy, P.C. (1991). *Self-Direction for Lifelong Learning.* San Francisco: Jossey-Bass Inc.

Cantor, G. (1955). *Contributions to the founding of the theory of transfinite numbers*. (P. Jourdain, Trans.). New York, NY: Dover. (Original work published 1895 & 1897)

Carpenter, E. S. (1960). Eskimo. *Explorations*, *9,* 75-6. Toronto, CA: University of Toronto Press.

Chan, M.S., & Waugh, R.F. (2007). Factors affecting student participation in the online learning environment at the Open University of Hong Kong. *Journal of Distance Education*, *21*(3), 23-38.

Chang, M. (2005). Applying self-regulated learning strategies in a Web-based instruction: An investigation of motivation perception. *Computer Assisted Language Learning, 18*(3), 217-230.

Chatman, S. (1981). What novels can do that films can't (and vice versa). In W.J.T. Mitchell (Ed.), *On narrative* (pp.117-136). Chicago: University of Chicago Press.

Chaytor, H.J. (1945). *From script to print: An introduction to medieval vernacular literature*. Cambridge, UK. Heffer and Sons, pp. 99-102, 105-106.

Cheng, H., Lehman, J., & Armstrong, P. (1991). Comparison of performance and attitude in traditional and computer conferencing classes. *The American Journal of Distance Education, 5*(3), 51-64.

Clandinin, D.J., & Rosick J. (2007). Mapping a landscape of narrative inquiry: Borderland spaces and tensions. In D.J. Clandinin (Ed.), *Handbook on narrative inquiry: Mapping a methodology* (pp.117-136). Thousand Oaks, California: Sage Publications.

Clark, J. M., & Paivio, A. (1991). Dual coding theory and education. *Educational Psychology Review, 3,* 149-210.

Coben, D., O'Donoghue, J., & FitzSimons, G.E. (Eds.). (2000). *Perspectives on adults learning mathematics: Research and practice*. Dordrecht: Kluwer Academic.

Connelly, F.M. & Clandinin, D.J. (1990). Stories of experience and narrative inquiry. *Educational researcher*, 2-14.

Conrad, D. (2005). Building and maintaining community in cohort-based online learning. *Journal of Distance Education, 20*(1), 1-21.

Coxeter, H. S. M. (1973). *Regular Polytopes (3rd ed.).*New York, NY: Dover.

Davies, B., & Davies, C. (2007). Having, and being had by, "experience": Or, "experience" in the social sciences after the discursive/ poststructuralist turn. *Qualitative Inquiry*. Retrieved August 25, 2008, from http://qix.sagepub.com/cgi/content/abstract/13/8/1139

Davis, R.B., & Maher, C.A. (1997). How students think: The role of representations. In L. English (Ed.), *Mathematical reasoning: Analogies, metaphors, and images* (pp. 3-18). New Jersey: Lawrence Erlbaum Associates.

de Lange. J. (1987). *Mathematics, insight and meaning: Teaching, learning and testing mathematics for the Life and Social Sciences.* Utrecht: Vakgroep Onderzoek Wiskundeonderwijs en Onderwijscomputercentrum, Rijksuniversiteit.

Dennis, D. (2000). The role of historical studies in mathematics and science educational research. In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 799-813). *New Jersey: Lawrence Erlbaum.*

Dewey, J. (1916). *Democracy and Education: an introduction to the philosophy of education*. New York, NY: Macmillan Publishing.

Dewey, J. (1938). *Experience and Education*. New York, NY: Macmillan.

Einstein, Albert (1954). *Short History of Music*. New York, NY: Vintage Books.

English, L.D. (1997). Analogies, metaphors, and images: Vehicles for mathematical reasoning. In L. English (Ed.), *Mathematical reasoning: Analogies, metaphors, and images* (pp. 3-18). New Jersey: Lawrence Erlbaum Associates.

Evans, J. (2000). *Adults' mathematical thinking and emotions: A study of numerate practices.* London: Routledge/Falmer.

Fabro, K.R., & Garrison, D.R. (1998). Computer conferencing and higher-order learning. *Indian Journal of Open Learning*, *7*(1), 41-54.

Fauvel, J., & van Maanen, J. (Eds.). (2000). *History in mathematics education: The ICMI study.* Dordrecht: Kluwer Academic.

Felder, R.M., & Spurlin, J.E. (2005). Applications, reliability, and validity of the index of learning styles. *International Journal of Engineering Education*, *21*(1), 103-112.

Feldman, R.S., & Zimbler, M. (2012). Improving college student success: The challenges and promise of developmental education. McGraw-Hill Research Foundation. Retrieved May 17, 2013, from http://mcgraw-hillresearchfoundation.org/wp-content/uploads/2012/09/Development-Education-FeldmanFinal1242981312-2.pdf

Fowler, D. (1988). Perils and Pitfalls of History. *For the Learning of Mathematics*, *11*(2), 15-16.

Fry, J.P. (1972). Interactive relationship between inquisitiveness and student-control of instruction. *Journal of Educational Psychology*, *63*(5), 459-465.

Gadamer, H.G. (1989). *Truth and Method*. (J. Weinsheimer & D. Marshall, Trans.). New York, NY: Crossroad Publishing. (Original work published 1960)

Gadanidis, G., & Geiger, V. (2010). A social perspective on technology-enhanced mathematical learning: from collaboration to performance.(Report). *Zdm*, (1), 91.

Garrison, D. R. (2009). Communities of inquiry in online learning: Social, teaching and cognitive presence. In C. Howard et al. (Eds.), *Encyclopedia of distance and online learning* (2nd ed., pp. 352-355). Hershey, PA: IGI Global.

Garrison, D. R., Anderson, T., & Archer, W. (2000). Critical inquiry in text-based environment: Computer conferencing in higher education. *The Internet and Higher Education, 2*(2–3), 87–105. Retrieved March 14, 2013, from http://www.communitiesofinquiry.com/sites/communityofinquiry.com/files/Critical_Inquiry_model.pdf

Garrison, D. R., Anderson, T., & Archer, W. (2001). Critical thinking and computer conferencing: A model and tool to assess cognitive presence. *American Journal of Distance Education*, *15*, 7–23.

Gattegno, C. (1963). *Modern mathematics with numbers in colour*. Reading, England: Educational Explorers Ltd.

Gattis, M. (Ed.). (2001). *Spatial schemas in abstract thought*. Cambridge. MA: MIT Press.

George, M. (2012). Autonomy and Motivation in Remedial Mathematics. *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies, 22* (4), 255-264. Retrieved May 17, 2013, from http://www.tandfonline.com/doi/abs/10.1080/10511970.2010.497958

Giaquinto, M. (2005). From symmetry perception to basic geometry. In P. Mancosu, K. Jorgensen, & S. Pedersen (Eds.), *Visualization, explanation and reasoning styles in mathematics* (pp 13-30). Dordrecht, The Netherlands: Springer.

Golledge, R.G. (Ed.). (1998). *Wayfinding behavior: Cognitive mapping and spatial processes.* Baltimore, MD: Hopkins Fulfillment Service.

Gordon, W.T. (2011). Compass for the Voyage to a World of Electric Words. In M. McLuhan, *The Gutenberg Galaxy* (2011 edition) (pp. vii – xxiii). Toronto, Canada: University of Toronto Press.

Graham, G. (2012). How the embrace of MOOCs could hurt Middle America. *Chronicle of Higher Education*, October 5.

Grumet, M.R. (1999). Autobiography and Reconceptualization. In W.F. Pinar (Ed.), *Contemporary curriculum discourses: Twenty years of JCT* (pp. 25-29). New York, NY: Peter Lang.

Gyselinck, V., Comoldi, C, Dubois, V., De Beni, R., & Ehrlich, M.F. (2002). Visuospatial memory and phonological loop in learning from multimedia. *Applied Cognitive Psychology, 16,* 665-685.

Hadamard, J. (1954). *The psychology of invention in the mathematical field*. New York, NY: Dover.

Hall, E.T. (1959). *The Silent Language.* New York, NY: Doubleday, pp. 5, 236.

Halpern, D. F., & Collaer, M. L. (2005). Sex differences in visuospatial abilities: More than meets the eye. In P. Shah & A. Miyake (Eds.), *The Cambridge handbook of visuospatial thinking* (pp. 170 – 212). New York: Cambridge University Press.

Haythornthwaite, C., & Kazmer, M. M. (2002). Bringing the Internet home: Adult distance learners and their Internet, home, and work worlds. In B. Wellman & C. Haythornthwaite (Eds.), *The Internet in everyday life* (pp. 431-463). Malden, Massachusetts: Blackwell Publishing.

Hegarty M., & Kozhevnikov, M. (1999). Spatial abilities, working memory and mechanical reasoning. In J. Gero & B. Tversky (Eds.) *Visual and spatial reasoning in design.* Sydney, Australia: Key Centre of Design and Cognition.

Hegarty, M., & Steinhoff, K. (1997). Individual differences in use of diagrams as extemal memory in mechanical reasoning, *Learning and Individual Differences, 9,* 19-42.

Hegel, G. (1967). *Phenomenology of Mind*. (J. B. Baillie, Trans.). London: Harper & Row. (Original work published 1807)

Heisenberg, W. K. (1958). *The Physicist's Conception of Nature*. London: Hutchinson, p. 34.

Helme, S. (1994). *Mathematics Embedded in Context: The Role of Task Context: in Performance, Task Perceptions and the Solution Methods of Adult Women Students*. Unpublished Masters of Education Thesis Australian Catholic University.

Hirumi, A. (2002). A framework for analyzing designing, and sequencing planned e-learning interactions. *The Quarterly Review of Distance Education, 3*(2), 141-160.

Hollander, J. (1961). *The Untuning of the Sky*. Princeton, NJ: Princeton University Press, pp. 68-9, 230.

Hoyles, C. (1985). What is the point of group discussion in mathematics?. *Educational Studies in Mathematics, 16*(2), 205-214.

Hrastinski, S. (2007). Participating in synchronous online education. Retrieved April 21, 2013, from http://www.lu.se/lup/publication/599311

Hrastinski, S. (2008). Asynchronous & synchronous e-learning. *EDUCAUSE Quarterly, 4*, 51-55.

Innis, H. (1950). *Empire and communications.* Oxford: Oxford University Press, pp. 30, 56, 132.

Ivins, W. Jr. (1946). *Art and geometry: A study in space intuitions*, Cambridge, Mass: Harvard University Press.

Jaffee, D. (2003). Virtual transformation: web-based technology and pedagogical change. *Teaching Sociology*. Retrieved March 8, 2013, from http://www.unf.edu/~djaffee/virtualtran.htm

Johnson, M. (1987). *The body in the mind: The bodily basis of meaning, imagination, and reason*. Chicago, IL: The University of Chicago Press.

Joseph, Miriam (2002). *The Trivium: The Liberal Arts of Logic, Grammar, and Rhetoric: Understanding the nature and function of language*. Philadelphia, PA: Paul Dry Books.

Kanuka, H. (2006). A Review of e-Learning in Canada: A Rough Sketch of the Evidence, Gaps and Promising Directions: A Commentary. *Canadian Journal of Learning and*

Technology, 32(3). Retrieved November 1, 2009, from http://www.cjlt.ca/index.php/cjlt/article/view/31/29

Kanuka, H., & Nocente, N. (2003). Exploring the effects of personality type on perceived satisfaction with Web-based learning in Continuing professional development. *Distance Education, 24*(2), 227-245.

Keller, J.M. (1987). The systematic process of motivational design. *Performance & Instruction, 269*(10), 1-8.

Kim, C. & Hodges, C. B. (2012). Effects of an emotion control treatment on academic emotions, motivation and achievement in an online mathematics course. *Instructional Science Springer*, *40*, 173-192. Retrieved May 16, 2013, from http://link.springer.com/article/10.1007/s11251-011-9165-6#

Kozhevnikov, M., Hegarty, M., & Mayer (2002). Revising the visualizer/verbalizer dimension: Evidence for two types of visualizers. *Cognition and Instruction, 20,* 47-78.

Krathwohl, D.R., Bloom, B.S., and Masia, B.B. (1964). *Taxonomy of educational objectives: Handbook II: Affective domain*. New York, NY: David McKay Co.

Kreber, C., & Kanuka, H. (2006). The scholarship of teaching and learning and the online classroom. *Canadian Journal of University Continuing Education, 32*(2), pp. 109–131.

Lakoff, G. (1987). *Women, fire, and dangerous things*. Chicago, IL: The University of Chicago Press.

Lakoff, G. (1994). What is a metaphor?. In J.A. Barnden & J. Holyoak (Eds.), *Advances in connectionist and neural computation theory* (Vol. 3, pp. 203-258). Norwood, NJ: Ablex.

Lakoff, G., & Núñez, R.E. (1997). The metaphorical structure of mathematics: Sketching out cognitive foundations for a mind-based mathematics. In L. English (Ed.), *Mathematical reasoning: Analogies, metaphors, and images* (pp. 21-89). New Jersey: Lawrence Erlbaum Associates.

Lamberti, E. (2011). Not just a book on media: Extending the Gutenberg Galaxy. In M. McLuhan, *The Gutenberg Galaxy* (2011 edition) (pp. xxv – xLv). Toronto, Canada: University of Toronto Press.

Landry, S. T. (2012). A study of the impact of instructional approach on community college students' problem solving and metacognitive abilities in the developmental mathematics course, Introductory Algebra. *University of New Orleans Theses and Dissertations.* Paper 1547. Retrieved May 17, 2013, from http://scholarworks.uno.edu/td/1547

Lane, L.M. (2008). Toolbox or trap? Course management systems and pedagogy. *Educause Quarterly*, 2. Retrieved February 22, 2009, from http://connect.educause.edu/term_view/E-Learning

Larkin, J. H., & Simon, H. A. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science, 11,* 65-99.

Leclercq, D.J. (1961). *The love for learning and the desire for God*. (C. Misrahi, Trans.). New York, NY: Fordham University Press. (Original work published 1957)

Lim, D. H., & Kim, H. (2003). Motivation and learner characteristics affecting online learning and learning application. *Journal of Educational Technology Systems*, *31*(4), 423-439.

Lipman, M. (1991). *Thinking in education*. Cambridge: Cambridge University Press.

Magagula, C M., & Ngwenya, A P. (2004). A comparative analysis of the academic performance of distance and on-campus learners. *Turkish Online Journal of Distance Education, 5*(4). Retrieved November 2, 2007, from ERIC database.

Major, C.H. (2010). Do virtual professors dream of electric students? University faculty experiences with online distance education. *Teachers College Record, 112*(8), 2154–2208.

Mayer, R. E. (2001). *Multimedia learning.* New York, NY: Cambridge University Press.

Mayer, R. E. (2002). Cognitive theory and the design of multimedia instruction: An example of the two-way street between cognition and instruction. *New Directions for Teaching and Learning, 89,* 55-71.

Mayer, R.E. (2005). Multimedia learning: Guiding visuospatial thinking with instructional animation. In P. Shah & A. Miyake (Eds.), *The Cambridge handbook of visuospatial thinking* (pp. 477-508). New York, NY: Cambridge University Press.

McLuhan, Marshall (2011). *The Gutenberg Galaxy : The making of typographic man*. Toronto, Canada: University of Toronto Press. (Original work published 1962)

McLuhan, Marshall (1964). *Understanding media: The extensions of man*. New York, NY: McGraw Hill.

McMillan, D.W., & Chavis, D.M. (1986). Sense of community: A definition and theory. *American Journal of Community Psychology, 14*(1), 6-23.

Mesa, V., Sitomer, A., Strom, A., & Yannotta, M. (2012). Research Commentary: Moving from anecdote to evidence: The need for a research agenda in community college mathematics education. Retrieved on May 20, 2013, from http://deepblue.lib.umich.edu/handle/2027.42/94208

Miller-Reilly, B.J. (2000). Exploration and Modelling in a University Mathematics Course: Perceptions of Adult Students. In D. Coben, J. O'Donoghue, & G.E. FitzSimons (Eds.), *Perspectives on adults learning mathematics: Research and practice* (pp. 251-269). *Dordrecht: Kluwer Academic.*

Moore, M.B. (2002). What does research say about the learners using computer-mediated communication in distance learning? [Electronic version]. *The American Journal of Distance Education*, *16*(2), 61-64.

Moore, M.G. (1975). Cognitive style and telematic (distance) learning and teaching. *ICCE Newletter*, *5*(4), 3-10.

Moore, M.G., & Kearsley, G. (1996). *Distance education: A systems view*. Belmont, CA: Wadsworth.

Morgan, G. (2003). Faculty use of course management systems. *Educause Quarterly*, 2. Retrieved February 24, 2009, from http://www.educause.edu/ResearchStudies/1010

Mousavi, S., Low, R., & Sweller, J. (1995). Reducing cognitive load by mixing auditory and visual presentation modes. *Journal of Educational Psychology, 87*, 319-334.

Murgatroyd, Stephen (2012). Getting to know your students: Preparing to develop and teach online and blended courses. *Contact North*. Retrieved December 11, 2012, from http://www.contactnorth.ca/tips-tools/getting-know-your-students

O'Donoghue, J. (2000). Perspectives in teaching adults mathematics. In D. Coben, J. O'Donoghue, & G.E. FitzSimons (Eds.), *Perspectives on adults learning mathematics: Research and practice* (pp. 229-234). *Dordrecht: Kluwer Academic.*

Oestermeier, U., & Hesse, F. W. (2000). Verbal and visual causal arguments. *Cognition, 75,* 65-104.

Paivio, A. (1986). *Mental representations: A dual-coding approach*. Oxford: Oxford University Press.

Pane, J. F., Corbett, A. T., & John, B. E. (1996). Assessing dynamics in computer-based instruction. In M. J. Tauber (Ed.), *Proceedings of the ACM conference on human factors in computing systems* (pp. 797-804), Vancouver, Canada: ACM.

Park, O., & Gittelman, S. S. (1992). Selective use of animation and feedback in computer-based instruction. *Educational Technology, Research, and Devetopment, 40,* 27-38.

Peschke, J. (2008). Moving ahead to the future by going back to the past: Mathematics education online. *Journal of Computers in Mathematics and Science Teaching, 28*(2), 123-133.

Picciano, A. (2002). Beyond student perceptions: Issues of interaction, presence, and performance in an online course. *Journal of Asynchronous Learning Networks, 6*(1), 21-40.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. London: Routledge.

Pimm, D. (1995). *Symbols and meanings in school mathematics*. London: Routledge.

Presmeg, N.C. (1985). *The role of visually mediated processes in high school mathematics: A classroom investigation.* Unpublished doctoral dissertation, University of Cambridge.

Presmeg, N.C. (1997). Generalization using imagery in mathematics. In L. English (Ed.), *Mathematical reasoning: Analogies, metaphors, and images* (pp. 299-312). New Jersey: Lawrence Erlbaum Associates.

Reisberg, D., & Heuer, F., B. (2005). Visuospatial images. In P. Shah & A. Miyake (Eds.), *The Cambridge handbook of visuospatial thinking* (pp 35-80). New York: Cambridge University Press.

Rieber, L. P. (1990). Using computer animated graphics with science instruction with children. *Joumal of Educational Psychology; 82,* 135-140.

Robinson, D. H., & Skinner, C. H. (1996). Why graphic organizers facilitate search processes: Fewer words or computationally efficient indexing? *Contemporary Educational Psychology, 21,* 166-180.

Rogers, A. (1986). *Teaching adults*. Milton Keynes, England: Open University Press.

Sepänmaa, Y. (2007). Multi-sensoriness and the city. In A. Berleant & A. Carlson (Eds.). *The aesthetics of human environments*. Peterborough, Canada: Broadview Press.

Scheffel-Dunand, D. (2011). The Invisible and the visible: Intertwining figure and ground in The Gutenberg Galaxy. In M. McLuhan, *The Gutenberg Galaxy* (2011 edition) (pp. xLvii – Lix). Toronto, Canada: University of Toronto Press.

Scott, A. (2003). *H.-G. Gadamer's Truth and Method*. Retrieved April 25, 2013, from http://www.angelfire.com/md2/timewarp/gadamer.html

Shah, P., & Freedman, E. G. (2003). Visuospatial cognition in electronic learning. *Journal of Educational Computing Research*, *29*(3), 315-324.

Shah, P., & Miyake, A. (2005). Preface. In P. Shah & A. Miyake (Eds.), *The Cambridge handbook of visuospatial thinking* (pp xi-xv). New York, NY: Cambridge University Press.

Simpson, M.I., Stahl, N.A., & Francis, M.A. (2004). Reading and learning strategies: Recommendations for the 21st century. *Journal of Developmental Education, 28*(2), 2-15.

Smith, M., & Kollock, P. (1999). *Communities in cyberspace*. London: Routledge.

Spurlin, J. E. (2007). The premise of this book is that as computing becomes widely available to teachers and students, education will be transformed [Review of the book *Ubiquitous computing in education: Invisible technology, visible impact*]. *Educause Quarterly*, 2. Retrieved February 15, 2009, from http://connect.educause.edu/term_view/E-Learning

Stake, R. E. (1995). *The art of case study research*. Thousand Oaks, CA: Sage Publications.

Sweller, J. (1999). *Instructional design in technical areas*. Camberwell, Australia: ACER Press.

Sun S., Joy M., & Griffiths N. (2007). The use of learning objects and learning styles in a multi-agent education system [Electronic version]. *Journal of Interactive Learning Research, 18*(3), 381-399.

Tang, M., & Byrne, R. (2007). Regular versus online versus blended: A qualitative description of the advantages of the electronic modes and a quantitative evaluation [Electronic version]. *International Journal on ELearning*, *6*(2), 257-267.

Thomson, G. (1984). The cognitive style of field dependence as an explanatory construct in distance education dropout. *Distance Education, 5*(2), 286-293.

Townsend, R.D., & Hurd, P.D. (1973). *Energy, matter, and change: Excursions into physical science*. Glenview, Illinois: Scott, Foresman and Company.

Tversky, B. (2005). Functional significance of visuospatial representations. In P. Shah & A. Miyake (Eds.), *The Cambridge handbook of visuospatial thinking* (pp. 1-34). New York, NY: Cambridge University Press.

Tzanakis, C., & Arcavi, A. (2000). Integrating history of mathematics in the classroom: an analytic survey. In J. Fauvel & J. van Maanen (Eds.), *History in mathematics education: The ICMI study* (pp. 201-240). *D*ordrecht: Kluwer Academic.

U.S. Department of Education, Office of Planning, Evaluation, and Policy Development. (2009). *Evaluation of evidence-based practices in online learning: A meta-analysis and review of online learning studies.* Washington, D.C.: U.S. Department of Education. Retrieved November 25, 2009, from http://www.ed.gov/rschstat/eval/tech/evidence-based-practices/finalreport.pdf

University Ventures Fund (2012). Synchronicity. *UV Newsletter*, 21.

Usher, A. P. (1959). *History of mechanical inventions.* Boston, MA: Boston Press, pp. 7-8, 143, 173.

VanderStoep, S. W., Fagerlin, A., & Feenstra, J. S. (2000). What do students remember from introductory psychology? *Teaching of Psychology. 27,* 89-92.

Voorhees, D. (2007). The online environment magnifies the challenges of getting to know one's students and of getting students to feel connected to the class [Review of the book *Teach beyond your reach*]. *Educause Quarterly*, 2. Retrieved February 15, 2009, from http://connect.educause.edu/term_view/E-Learning

Von Bonsdorff, P. (2007). Urban richness and the art of building. In A. Berleant & A. Carlson (Eds.). *The aesthetics of human environments*. Peterborough, Canada: Broadview Press.

Wadsworth, L.M., Husman, J., Duggan, M.A., & Pennington, M.N. (2007). Online mathematics achievement: Effects of learning strategies and self-efficacy [Electronic version]. *Journal of Developmental Education*, *30*(3), 6-13.

Weinstein, C.E., Husman, J., & Dierking, D.R. (2000). Interventions with a focus on learning strategies. In M. Boekaerts, P.R. Pintrich, & M. Zeidner (Eds.), *Handbook of self-regulation* (pp. 727-747). San Diego: Academic Press.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge: Cambridge University Press.

Wenger, E., McDermott, R., & Snyder, W. (2002). *Cultivating communities of practice: A guide to managing knowledge*. Cambridge, MA: Harvard Business School Press.

Wiley, J., & Voss, J. F. (1999). Constructing originals from multiple sources: Tasks that promote understanding and not just memory for texts. *Journal of Educational*

*Psychology, 91,*301-311.

Wood, M. T. (2013). Opportunities in online education – staying ahead of the curve: The case of the MOOC (PowerPoint slides). Retrieved May 22, 2013, from http://www.cic.org/News-and-Publications/Multimedia-Library/CICConferencePresentations/2013%20Presidents%20Institute/Concurrent%20Sessions/Recent%20Developments%20in%20Online%20Education_%20Wood%20Handout.pdf

Zajonc, R. B. (1980). Feelings and thinking: Preferences need no inferences. *American Psychologist*, 35(2), pp. 151–175.

Zhang, J. (1997). The nature of external representations in problem solving. *Cognitive Science, 21,* 179-217.

Zhang, J., & Norman, D. A. (1994). Representations in distributed cognitive tasks. *Cognitive Science. 18,* 87-122.

Zhao, Y., Lei, J., Yan, B., Lai, C., Tan, H.S. (2005). What makes the difference? A practical analysis of research on the effectiveness of distance education. *Teachers College Record , 107* (8), pp. 1836–1884

Zywno, M.S. (2003). A contribution to validation of score meaning for Felder-Soloman's index of learning styles. *Proceedings of the 2003 American Society for Engineering Education Annual Conference and Exposition* (session 2351). Nashville, TN. Retrieved November 14, 2007, from http://www.asee.org/acPapers/2003-532_Final.pdf

Credit to the following websites, as pieces of a twenty-first century mosaic of educational import, which have been created, built and maintained by a community of volunteers whose primary purpose was to share a truly remarkable pool of human knowledge:

http://en.wikipedia.org/wiki/

http://www.thefreedictionary.com/

http://dictionary.reference.com/

www.merriam-webster.com/dictionary/

APPENDIX A

Learning And Study Strategies Inventory test: the LASSI Scales

The Learning And Study Strategies Inventory (LASSI) diagnostic test was developed at the University of Texas by Claire E. Weinstein, Ann C. Schulte, and David R. Palmer in relation to the learning and study habits of college students. It attempts to uncover a person's overt and covert thoughts, beliefs, attitudes and behaviours which affect the way they learn and study and which may be altered if desired.

This 80-question assessment test focuses on three aspects strategic to successful learning: skill, will, and self-regulation. The latter, self-regulation, is what I call the 'way'. Eight questions of this test are directed to each of the ten, what they call, LASSI scales:

1. Skill component: information processing scale, selecting main ideas scale, test strategies scale;
2. Will component: anxiety, attitude, motivation scales;
3. Way component: the scales of concentration, self testing, study aids, and time management.

Although there is some overlap and interaction among and between the components and the individual scales, each of them makes a difference in the outcome of one's learning endeavours.

As with any statistical measurement, especially those based on self-evaluation, it is important to address the issue of reliability of the scale data. One of the most common indices of reliability of a sum scale is Cronbach's *coefficient alpha.* In short, if all items are perfectly reliable and measure the same thing (true score), then the coefficient alpha is equal to 1. It should be noted that the coefficient alphas for the LASSI scales range from a low of .73 to a high of. 89. Therefore, they are reasonably to quite reliable.

APPENDIX B

LEARNING STYLES for MATHEMATICS: Felder-Silverman Model

Richard Felder, a Professor of Chemical Engineering at North Carolina State University, and Linda Silverman, an educational psychologist, together formulated this model of learning styles, now referred to as the **Felder-Silverman model**. They identified these four dimensions of learning, suggesting that everyone is a special mix of both styles in each of these dimensions.

- **Active** (learn by doing)
  or Reflective (learn by first thinking it over)
- **Sensing** (better at learning facts)
  or Intuitive (learn by discovering possibilities and relationships)
- **Visual** (learn by pictures, diagrams or demonstrations)
  or Verbal (learn by reading or listening to text)
- **Sequential** (learn in logical sequences of linear steps)
  or Global (learn material almost randomly and then suddenly see the big picture)

APPENDIX C

RESEARCH PROJECT: PRE-PROJECT SURVEY

You are not obligated to answer any questions which may make you feel uncomfortable. These are questions surrounding 'How would you describe yourself?' Please answer in the spaces provided as best as you can. Feel free use more or less space than is indicated.

1. Would you describe yourself as a 'math person'? Please explain why or why not.
2. In the past, when you felt you had really understood a mathematical idea, was it because:
    a. you could answer short-answer questions about it? **Yes/No**
    b. you could reproduce a step-by-step method for solving a question? **Yes/No**
    c. you could visualize the concept in such a way so that the idea made sense to you? **Yes/No**
    d. you could reproduce a method for solving a word problem involving a 'real-life' situation? **Yes/No**
    e. you could apply the concept to something else that interested you? **Yes/No**
    f. you were able to piece together different mathematical ideas and see their connections and interrelatedness? **Yes/No**

You may comment on any or all of the above, but please explain your choice(s).

3. Describe any feelings of inadequacy in the learning of mathematical concepts you may (or do not) feel coming into this project.
4. Describe any past experiences which may have discouraged you from or encouraged you to the taking more courses in mathematics, science, engineering, business, or other math-related disciplines.
5. Describe what particular aspects constitute your 'ideal' of a math class.
6. Describe what particular aspects constitute your 'not-so-ideal' math learning experience.
7. How would you define a self-directed learner? In which ways, would you describe yourself as a self-directed learner?
8. In learning mathematics concepts, how much independence do you generally prefer to have regarding how you learn it or in what way or order you prefer to learn it?

9. In learning mathematics concepts, to what degree to you prefer to learn ‘on your own’ versus learning with the help of a readily available instructor?
10. What are your expectations coming into this project? Discuss particular objectives and goals you wish to achieve while being a part of this experiment.

APPENDIX D

RESEARCH PROJECT: FIRST IMPRESSIONS SURVEY

You are not obligated to answer any questions which may make you feel uncomfortable. These are questions surrounding 'What were your first impressions of the RMC web site?' Please answer in the spaces provided as best as you can. Feel free to use more or less space than is indicated.

Please be honest. There were no expectations on my part as to what anyone 'should' have done, as this site was and is intended to be a self-directed learning site which attempts to accommodate the various needs of a diverse group of students.

1. When **you first logged onto the site**, what was (were) the first thing(s) you noticed? What stood out above anything else ? What pleased you, if anything? What didn't you particularly like, if anything? Think about colours, layout, size, presentation, etc.
2. **During your first visit**, did you
   a. read the RMC opening paragraph? **Yes/No**
   b. read the RMC opening page? **Yes/No**
   c. read the '**Mapping Your Learning Strategy**' paragraph on the opening page? **Yes/No**
   d. go straight to the menu to look around and see what was available? **Yes/No**
   e. go straight to the Diagnostic Test and work through it? **Yes/No**
   f. follow the '**Words of Encouragement**' link? **Yes/No**
   g. read the '**Words of Encouragement**' page? **Yes/No**
   h. follow the '**Remedial Math Centre Discussion Forum**' link? **Yes/No**
   i. follow the '*Exam Anxiety*' menu link? **Yes/No**
   j. follow the '*Learning Styles'* menu link? **Yes/No**
   k. write the Learning Styles Test? **Yes/No**
   l. follow the '*Learning Strategies'* menu link? **Yes/No**
   m. check out the '*Important Links'* menu page? **Yes/No**
   n. check out the '*Topic List'* page? **Yes/No**

   You may comment on any or all of the above by indicating why you followed the links you did, if the page did or did not catch your interest and why, if the page did not present what you expected, explaining why.

3. **During your first visit**, did you go to any of the tutorial topic pages and read them? If so, which ones and why? If you wish, you may comment on those tutorials you read or listened to.
4. This web site is a human-made environment where students will come to stay a while, no matter how briefly. Of all the human-designed and human-constructed environments that you can think of, what does this web site remind you most of?
5. Describe any impressions of the site and/or its contents which I have not anticipated in the questions above. For example, did you find the easy to navigate or did anything strike you as odd or did you feel overwhelmed at any point during your first visit?

APPENDIX E

RESEARCH PROJECT: LEARNING MODES SURVEY

You are not obligated to answer any questions which may make you feel uncomfortable. These are questions surrounding 'the three modes of learning mathematics online offered in the RMC project'. Please answer in the spaces provided as best as you can. Feel free to use more or less space than is indicated.

Please be honest. There were no expectations on my part as to what anyone 'should' have done or thought, as this site was and is intended to be a self-directed learning site which attempts to accommodate the various needs of a diverse group of students.

PART 1: QUESTIONS ABOUT YOUR PREFERRED WAY OF LEARNING

1. Indicate, from the list below, which style of learning is your preferred one. Choose one category out of each of the four groups which you think best describes how you learn mathematics.

Active (learn by doing) _________________________

or Reflective (learn by first thinking it over) ________________________________

Sensing (better at learning facts) _________________________

or Intuitive (learn by discovering possibilities, connections, and relationships)

____

Visual (learn by pictures, diagrams or demonstrations) _________________________

or Verbal (learn by reading or listening to text)

____

Sequential (learn in logical sequences of linear steps) __________________________

or Global (learn material almost randomly and then suddenly see the big picture)

____

2. Approximately when were you first exposed to the subject of algebra in your mathematical education?
   a. before the age of 14 ________________________________
   b. between 14 and 18 years of age ________________________________
   c. while in university or college ________________________________

d. after your post-secondary education ______________________

e. when I came to this project ____________________________

3. Approximately when did you first learn the formalized grammatical structure of your native language?

a. before the age of 14 ____________________________

b. between 14 and 18 years of age ____________________________

c. while in university or college ____________________________

d. after your post-secondary education ______________________

e. not up to the present time ____________________________

4. When you were taught your native language in the school system, how were you taught it?

a. phonetically (parsing words syllable by syllable) ________________________

b. as whole language (learning word by word) ___________________________

c. other (please explain) ____

PART 2: QUESTIONS ABOUT THE THREE MODES OF LEARNING MATHEMATICS ONLINE (offered in the RMC project)

Interactive Tutorials (on the RMC web site):

1 Approximately **how often** did you use the online interactive tutorials?

a. never ____________________________

b. sporadically ____________________________

c. from time to time but in a regular sequence ___________________________

d. regularly but not every single week ______________________

e. virtually every week, often several times ____________________________

2. Which aspects of these tutorials **helped you the most** in your learning of the mathematical ideas presented? Discuss things such as coverage of a topic, the animation of the instruction, the colour coding of the materials, the ability to move at your own pace, the ability to repeat the tutorial as often as desired, and other issues (perhaps inclusion of motivational items) relevant to your particular learning process. Please provide examples where appropriate.

3. Which aspects of these tutorials **helped you the least** in your learning of the mathematical ideas presented? Discuss things which were not particularly important and state why.
4. Which aspects of these tutorials may have **hindered you** in your learning of the mathematical ideas presented? Discuss things which frustrated or irritated you and explain why. Could you suggest a better or an alternative format which would alleviate these irritants? Please provide examples where appropriate.
5. What role do you feel these online interactive tutorials played in your learning process?

Live Tutorials (using Elluminate Live):

1. Approximately **how often** did you **either attend or listen to** the recordings of the Elluminate Live tutorials?
   a. never ________________________________
   b. sporadically ________________________________
   c. from time to time but in a regular sequence ________________________________
   d. regularly but not every week __________________________
   e. virtually every week, often several times ________________________________
2. Which aspects of these tutorials **helped you the most** in your learning of the mathematical ideas presented? Discuss things such as coverage of a topic, how the material was presented, the ability to ask questions for immediate feedback, the ability to repeat the tutorial as often as desired by having access to the recording, the ability to chat with the other participants, and other issues relevant to your learning process. Please provide examples where appropriate.
3. Which aspects of these tutorials **helped you the least** in your learning of the mathematical ideas presented? Discuss things which were not particularly important and state why.
4. Which aspects of these tutorials may have **hindered you** in your learning of the mathematical ideas presented? Discuss things which frustrated or irritated you and explain why. Could you suggest a better or an alternative format which would alleviate these irritants? Please provide examples where appropriate.

5. All of you have been in face-to-face classroom settings in mathematics. Now you have experienced some online synchronous live tutorials. Describe ways you find them similar to the classrooms you have been in and articulate any of the differences you noticed between them. Provide examples, where appropriate.
6. What role do you feel these live online tutorials played in your learning process?
7. At any time during this project, did you ever feel you were a part of a mathematics classroom with fellow students and teacher – even though you could not see each other face-to-face? Please explain and provide examples where appropriate.
8. In the future, if you had a choice of either a classroom setting or an online live tutorial setting to learn mathematics, which would you choose and why? Assume that both options were available to you at the time.

Assessment Tool (Maple TA online):

1. Approximately how often did you use the online assessment tool, Maple TA, to practice and self-test?
   a. never ________________________________
   b. sporadically ________________________________
   c. from time to time but in a regular sequence ______________________________
   d. regularly but not every week _________________________
   e. virtually every week, often several times ________________________________
2. Which aspects of this assessment tool **helped you the most** in your learning of the mathematical ideas presented? Please provide examples where appropriate. Assume that all the technological glitches can be ironed out over time, such as marking questions incorrect when they provide a correct answer identical to yours.
3. Which aspects of this assessment tool **helped you the least** in your learning of the mathematical ideas presented? Discuss things which were not particularly important and state why.
4. Which aspects of this assessment tool which may have **hindered you** in your learning of the mathematical ideas presented? Discuss things which frustrated or irritated you and explain why. Could you suggest a better or an alternative format which would alleviate these irritants? Please provide examples where appropriate.

5. What role do you feel these practice exercises and tests played in your learning process?

Comparing All Three: Interactive Web Site, Live, Assessment Tool

1. In a formal course for credit, if **only two** of these modes of learning were to be offered online and the remaining one was either email or paper-based, which two would you prefer? And why?
2. In a formal course for credit, if **only one** of these modes of learning were to be offered online and the remaining two were either email or paper-based, which one would you prefer? And why?
3. If there had been no live tutorials to pace your program, **what is your assessment** of the RMC web site and the Maple TA Assessment Tool as a self-regulated remedial mathematics learning site intended to provide a more solid mathematics background to those who are preparing for future math-courses? Please explain your conclusions.

APPENDIX F

Recruitment and Consent Letter

Dear Prospective Participant,

We are conducting a research project to solicit your reaction to the enhanced Remedial Math Centre web site as an effective means of delivering online remedial mathematics lessons which include topics leading to a Pure Math 30 level of understanding of mathematics. The project timeframe is one regular university semester from January 5 to mid-April 2010.

The web site offers self-help menu items about style and ingrained habits of learning, a mathematics diagnostic test, a fairly comprehensive list of topics covering basic mathematics skills from grade 10 through grade 12. with options for animated tutorials, practice exercises, self-assessment, a weekly live online tutorial session with the instructor/researcher, and a student discussion board to exchange greetings and ideas with other students in the project.

For the duration of the project, the volunteer participants will be asked to describe their experience of this online learning environment by personal narrative. Email responses, online anonymous interviews and completion of pre-and post-project online survey forms will be used to glean opinions, reactions, and emotional responses to this particular format of a mathematics 'classroom.'

The first task given to each participant of the study will be to describe his or her notion of an 'ideal' or, alternatively, 'not so ideal' mathematics classroom. Each will then be given access to the web site and will be encouraged to experience it as a new medium of classroom experience.

To preserve anonymity, each volunteer will be asked to chose her/his own 'handle' (pseudonym for the project) and will be encouraged to obtain an email address which reflects this handle. The handle name will be used in all correspondence and online interactions.

The filling out of the online pre-project and post-project survey forms at the beginning and towards the end of the project will be voluntary, identifiable only by the participants' handles.

If needed, technical assistance will be provided so that the use of the various menus and learning objects of the web site is made as easy as possible. No particularly sophisticated computer expertise will be required, but you should have basic computer skills so that you can negotiate around a web site with some ease. Access to an internet connection is necessary; however, a high-speed connection is recommended.

All data will be treated confidentially and anonymity will be maintained as only your 'handle' will appear in any document or any collected data. All data collected will be kept in a secure location under secured electronic storage for a period of five years. After five years, the data will be destroyed by shredding any hard copies generated and deleting the data files from the computer storage.

Your participation is voluntary. You also have the right to withdraw from the study at any time without penalty. All information will be kept confidential except when legislation or a professional code of conduct requires that it be reported.

To inform the academic community of the results of the study, the results may be published in a professional journal and presented at a conference. If you have any questions now or during the research period please feel free to contact me by email at __________. The University Research Ethics Board has reviewed this research study and may be reached by e-mailing ____________ or calling ___________ if you have questions or comments about your treatment as a participant.

If you decide you would like to be part of this great experiment, please sign the letter below and return it to me. Thank you for considering it.

____

Julie Peschke

I have read and understood the information contained in this letter, and I agree to participate in the study, on the understanding that I may refuse to answer certain questions, and that I may withdraw at any time. __________________________ __________________ Name Student ID. # __________________________ Date